On rules of inference and the meanings of logical constants

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1. Introduction

In the theory of meaning, it is common to contrast truth-conditional theories of meaning with theories which identify the meaning of an expression with its use. One rather exact version of the somewhat vague use-theoretic picture is the view that the standard rules of inference determine the meanings of logical constants. Often this idea also functions as a paradigm for more general use-theoretic approaches to meaning. In particular, the idea plays a key role in the anti-realist program of Dummett and his followers.

In the theory of truth, a key distinction now is made between substantial theories and minimalist or deflationist views. According to the former, truth is a genuine substantial property of the truth-bearers, whereas according to the latter, truth does not have any deeper essence, but all that can be said about truth is contained in T-sentences (sentences having the form: 'P' is true if and only if P).

There is no necessary analytic connection between the above theories of meaning and truth, but they have nevertheless some connections. Realists often favour some kind of truth-conditional theory of meaning and a substantial theory of truth (in particular, the correspondence theory). Minimalists and deflationists on truth characteristically advocate the use theory of meaning (e.g. Horwich). Semantical anti-realism (e.g. Dummett, Prawitz) forms an interesting middle case: its starting point is the use theory of meaning, but it usually accepts a substantial view on truth, namely that truth is to be equated with verifiability or warranted assertability. When truth is so understood, it is also possible to accept the idea that meaning is closely related to truth-conditions, and hence the conflict between use theories and truth-conditional theories in a sense disappears in this view.

2. Carnap's challenge

The view that the standard rules of inference (or, sometimes, just the introduction rules) determine the meanings of logical constants seems to be rather widely held:

(RIDMLC) The Rules of Inference Determine the Meanings of Logical Constants.

The idea is certainly natural and may indeed appear in a sense quite uncontroversial, given the completeness of all standard systems of rules of inference for both propositional and (first-order) predicate logic.

But the popularity of such views notwithstanding, certain facts undermine them. Namely, although this is apparently very little known, Carnap (1943) put forward considerations to the conclusion that in a definite sense, it is not true that the standard rules of inference manage solely to determine the meanings of logical constants. It can be shown that no ordinary formalization of logic, and not the standard rules of inference (of the natural deduction) in particular, is sufficient to 'fully formalize' all the essential logical properties of logical constants. That is, they do not exclude the possibility of interpreting logical constants in any other than the ordinary way. As Carnap's arguments seem to have been almost entirely ignored,[1] it is perhaps proper to remind the philosophical community of them.[2]

Let us consider the following principles that seem to be essential for negation and disjunction. They follow, in classical logic, from the semantical definitions of connectives (e.g. in terms of truth tables), but they also hold in intuitionistic logic, where truth is identified with provability (Carnap only considered explicitly classical logic.) A and B are arbitrary sentences:

(N1) A is true $\Rightarrow \neg A$ is false.
(N2) A is false $\Rightarrow \neg A$ is true.

(D1) A is true and B is true $\Rightarrow (A \vee B)$ is true.
(D2) A is true and B is false $\Rightarrow (A \vee B)$ is true.
(D3) A is false and B is true $\Rightarrow (A \vee B)$ is true.
(D4) A is false and B is false $\Rightarrow (A \vee B)$ is false.

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[1]   There are, however, some important exceptions: Carnap's results are taken into account at least in Smiley and Shoesmith 1978, Belnap and Massey 1990, Smiley 1996, Rumfitt 1997, Rumfitt 2000, and Humberstone 2000. These works certainly deserve to be read in this connection, though the aims and emphasis of them are quite different from mine here. Unfortunately, they have not changed the overall situation to a large extent; it still seems that Carnap's challenge is not sufficiently known – in particular among those who should be most concerned with it, i.e. the advocates of RIDMLC. More generally, the large literature on the meanings of logical constants is basically quiet about it.

[2]   I'll consider only the case of propositional logic. Carnap also gives examples of non-normal interpretations for quantification theory. But propositional logic suffices for making the point.

Yet the standard formalizations of logic (rules of inference) do not rule out non-normal interpretations which violate these principles. There are (in the context of propositional logic) two different kinds of non-normal interpretations of connectives allowed by the ordinary rules of inference: there may either be a violation of (N1) or a simultaneous violation of (N2) and (D4). (As it happens, the rules of inference guarantee that (D1)–(D3) cannot be violated.) In the first kind of non-normal interpretation, where (N1) is violated, for any sentence A, both A and $\neg A$ are true, and moreover, every sentence is true. In the second kind of non-normal interpretation, where both (N2) and (D4) are violated, both A and $\neg A$ are false, but $(A \vee \neg A)$ is true; in that case, there are infinitely many true and infinitely many false sentences (see Carnap 1943: Chapter C).

In sum, although the standard rules of inference completely formalize logical truth and logical consequence, they do not fully represent all logical properties of the connectives. Thus, for instance, it arguably belongs to the logical properties of disjunction that a sentence of disjunction with two false components is false. Nevertheless, this property is in no way represented in ordinary formalizations of logic. Similarly, it is certainly a part of the intended meaning of negation that a sentence and its negation cannot both be true, and further that they cannot both be false either.[3] But, again, these properties are not in any way represented in the usual rules of inference.

The above forgotten facts present a serious challenge for RIDMLC, the view that rules of inference determine the meaning of logical constants – a problem one to which should at least give serious consideration.

3. Rules and truth

What then is the moral of all the above? This depends on various other general philosophical background assumptions. There are several different alternatives here:

(a) A radical formalist may just deny the very meaningfulness, or at least relevance, of the notions of truth and falsehood (and consequently, of (N1)–(D4)) and insist that his use-theoretical approach is a genuine alternative to the truth-conditional approach and that it would beg the question to appeal to (N1)–(D4) against it. With respect to this sort of radical view, the above considerations have no force. But such a view is in itself

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[3] Carnap (1943: 100) calls the former, quite naturally, the principle of (excluded) contradiction and the latter, less happily, the principle of excluded middle.

very controversial and problematic. I do not believe that any contemporary adherent of RIDMLC accepts such a radical formalism, certainly not intuitionists such as Dummett, Prawitz and their followers. Consequently, I shall not consider this extreme view any further.

(b) One might introduce the minimalist or deflationist notion of truth and falsehood, and try to argue that they suffice for avoiding Carnap's problem and to save RIDMLC – more on this below.

(c) One may alternatively assume that there is some sort of match between the proof-theoretical meaning-giving rules of inference and semantical notions of truth and falsity (possibly understood, as by intuitionists, in terms of provability), and that the former simultaneously determines a suitable notion of truth (possibly a non-realist, epistemic or verificationist one, which equates truth with provability). In particular, something like this seems to be the view of Dummett, Prawitz and their followers. For this kind of view, Carnap's problem seems to pose a real challenge.

4. Relation to bivalence and LEM

A central aim of the anti-realist program of Dummett, Prawitz and others is to rebut the Law of the Excluded Middle (LEM) by undermining the Principle of Bivalence. However, their fundamental equivalence,

Bivalence holds ⇔ LEM is true,

only holds under two default assumptions:

(i) A is true or B is true $\Rightarrow (A \vee B)$ is true (which is stronger than our D1 above, and entails it).
(ii) A is false $\Rightarrow \neg A$ is true. (i.e. our (N2) above)

But it is unclear at least how an adherent of RIDMLC (such as Dummett, Prawitz and co.) is allowed to appeal to these principles. The rules of inference do not suffice to entail them, as Carnap's results show, so from where does one get them, without begging the question?

5. Minimalism and deflationism

If the use theory of meaning (and RIDMLC in particular) is understood as a competitor of the truth-conditional theory of meaning, it would be somewhat problematic if an adherent of the former would appeal to a substantial theory of truth in order to solve Carnap's problem. However, one may think that at least an appeal to the minimalist or deflationist view on truth is unproblematic. According to deflationism, the meaning

of truth and falsity is exhausted by the following self-evident equivalences:

(T1) A is true $\Leftrightarrow A$,
(T2) A is false $\Leftrightarrow \neg A$.

Alternatively, one might take falsity as a defined notion, in which case (T1) would suffice, e.g. as follows:

A is false $\Leftrightarrow_{\text{def}} (\neg A)$ is true.

With the aid of not much logic, these schemes seem to give us what we want (i.e. principles (N1)–(D4)):

(N1) A is true $\Rightarrow A \Rightarrow \neg\neg A \Rightarrow \neg A$ is false;
(N2) A is false $\Rightarrow \neg A \Rightarrow \neg A$ is true;
(D1)–(D3) A is true or B is true $\Rightarrow A \vee B \Rightarrow (A \vee B)$ is true;
(D4) A is false and B is false $\Rightarrow \neg A \wedge \neg B \Rightarrow \neg (A \vee B)$
                $\Rightarrow (A \vee B)$ is false.

However, a closer look shows that this does not really help, but that the problem has just moved to the next level. Namely, nothing here rules out e.g. the possibility that a sentence A, or every sentence, is both true and false. In other words, the problem now occurs on the level of truth as the problem is that true and false are certainly assumed to be mutually exclusive properties, but nothing here guarantees that. Everything assumed so far allows that true and false overlap or are even coextensive.

At this point, someone might suggest that we just add to our theory the exclusion assumptions:

(E1) A is true $\Leftrightarrow$ A is not false
(E2) A is false $\Leftrightarrow$ A is not true.

At first sight, this would appear to solve the problem. But in fact, this does not help either. More formally, these read simply as:

 A is true $\Leftrightarrow \neg(A$ is false$)$
A is false $\Leftrightarrow \neg(A$ is true$)$.

And because we just have not been able to fix the meaning of the negation in the intended way, any such principles, which essentially use negation, cannot help either. This becomes especially evident when, using T-equivalences, one replaces 'A is true' by A, and 'A is false' by $\neg A$; we then get:

 $A \Leftrightarrow \neg\neg A$
$\neg A \Leftrightarrow \neg A$.

These are, however, already provable tautologies of (classical) propositional logic (i.e. derivable with the help of the standard rules of inference) and cannot possibly help in solving Carnap's problem.

6. Conclusion

Carnap's forgotten result poses at least prima facie a deep problem for RIDMLC. Now I do not wish to go as far as to claim that RIDMLC stands conclusively refuted. But certainly the above-considered facts deserve attention, and those who build their philosophical theories on RIDMLC owe us at least an explanation of how such problems could be circumvented.[4]

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