Tree-ring semantics

B. Rabern | *University of Edinburgh*

01/03/2017 | 02:31

According to dendrochronology or tree-ring analysis, a science anticipated by da Vinci[1], the growth rings of a tree carry *information*. For example, as is well known, the number of growth rings in a tree cross-section represent the age of the tree. Here is a standard cross-section of a tree showing its center pith, a number of growth rings, and its outer bark:

Our aim here is to lay the groundwork for formal tree-ring analysis combining data from dendrochronology with formal techniques from semantics. We will present the basic syntax of, and basic compositional semantics of tree-ring structures.

First we define the formal tree-ring syntax. There are three basic symbols:

pith: $\bullet$

rings: ()

bark: {}

The well-formed ring-structures of the language are divided into *ring-sentences* and *ring-terms*. They are provided by the following grammar, where each ϕ is a ring-sentence and each α is a ring-term:

$\alpha ::= (\bullet) \mid (\alpha)$
$\phi ::= \{\alpha\}$

Thus each ring-sentence is composed of bark encompassing a well-formed ring-term, where ring-terms are composed of any number of growth rings around a center pith. For example, a well-formed ring-sentence is the following: $\{((\bullet))\}$.

For the semantics let a model $\mathfrak{A} = \langle \mathbb{N}, W, T, A \rangle$, where $\mathbb{N}$ is the natural numbers, W is a set of worlds and T is a set of times, and A is a set of individuals (or *trees*). Given this we provide the following lexical entries.

$[\![\bullet]\!] = 0$

$[\![()]\!] = \lambda n.n + 1$

$[\![\{\}]\!] = \lambda n.\{\langle w, t, a\rangle : a \text{ is } n \text{ years old in } w \text{ at } t\}$

[1] "Li circuli delli rami degli alberi segati mostrano il numero delli suoi anni, e quali furono più umidi o più secchi la maggiore o minore loro grossezza" (Leonardo da Vinci, *Trattato della Pittura*, 1817).

Composition proceeds via functional application.

> `Composition rule:` If χ is ring structure composed of immediate parts $\{\beta, \gamma\}$, and $[\![\gamma]\!]$ is in the domain of $[\![\beta]\!]$, then $[\![\chi]\!] = [\![\beta]\!]([\![\gamma]\!])$.

For example, consider the following sentence of the tree-ring language: $\{(((((((\bullet)))))))\}$. Given the semantics above we compute its truth conditions as follows:

$$
\begin{aligned}
[\![\{(((((((\bullet)))))))\}]\!] &= [\![\{\}]\!]\big([\![(((((((\bullet)))))))]\!]\big) \\
&= [\![\{\}]\!]\big([\![()]\!]\big([\![((((((\bullet))))))]\!]\big)\big) \\
&= [\![\{\}]\!]\big(1 + \big([\![((((((\bullet))))))]\!]\big)\big) \\
&= [\![\{\}]\!]\big(1 + \big([\![()]\!]\big([\![(((((\bullet)))))]\!]\big)\big)\big) \\
&= [\![\{\}]\!]\big(1 + \big(1 + \big([\![(((((\bullet)))))]\!]\big)\big)\big) \\
&= [\![\{\}]\!]\big(1 + \big(1 + \big([\![()]\!]\big([\![((((\bullet))))]\!]\big)\big)\big)\big) \\
&= [\![\{\}]\!]\big(1 + \big(1 + \big(1 + \big([\![((((\bullet))))]\!]\big)\big)\big)\big) \\
&= [\![\{\}]\!]\big(1 + \big(1 + \big(1 + \big([\![()]\!]\big([\![(((\bullet)))]\!]\big)\big)\big)\big)\big) \\
&= [\![\{\}]\!]\big(1 + \big(1 + \big(1 + \big(1 + \big([\![(((\bullet)))]\!]\big)\big)\big)\big)\big) \\
&= [\![\{\}]\!]\big(1 + \big(1 + \big(1 + \big(1 + \big([\![()]\!]\big([\![((\bullet))]\!]\big)\big)\big)\big)\big)\big) \\
&= [\![\{\}]\!]\big(1 + \big(1 + \big(1 + \big(1 + \big(1 + \big([\![((\bullet))]\!]\big)\big)\big)\big)\big)\big) \\
&= [\![\{\}]\!]\big(1 + \big(1 + \big(1 + \big(1 + \big(1 + \big([\![()]\!]\big([\![(\bullet)]\!]\big)\big)\big)\big)\big)\big)\big) \\
&= [\![\{\}]\!]\big(1 + \big(1 + \big(1 + \big(1 + \big(1 + \big(1 + \big([\![(\bullet)]\!]\big)\big)\big)\big)\big)\big)\big) \\
&= [\![\{\}]\!]\big(1 + \big(1 + \big(1 + \big(1 + \big(1 + \big([\![()]\!]\big([\![\bullet]\!]\big)\big)\big)\big)\big)\big)\big) \\
&= [\![\{\}]\!]\big(1 + \big(1 + \big(1 + \big(1 + \big(1 + \big(1 + \big([\![\bullet]\!]\big)\big)\big)\big)\big)\big)\big) \\
&= [\![\{\}]\!]\big(1 + \big(1 + \big(1 + \big(1 + \big(1 + \big(1 + \big(0\big)\big)\big)\big)\big)\big)\big) \\
&= [\![\{\}]\!]\big(7\big) \\
&= \{\langle w, t, a\rangle : a \text{ is 7 years old in } w \text{ at } t\}
\end{aligned}
$$

Thus we get the desired result that the sentence is true at a world centered on a tree and time just in case the tree is 7 years old in the world at that time.

Note that this initial investigation is quite limited in its ambitions. We have only addressed one aspect of tree-ring analysis. Our treatment only captures the core chronological information imparted by rings, but trees say more.

> When a tree is cut down and reveals its naked death-wound to the sun, one can read its whole history in the luminous, inscribed disk of its trunk: in the rings of its years, its scars, all the struggle, all the suffering, all the sickness, all the happiness and prosperity stand truly written, the narrow years and the luxurious years, the attacks withstood, the storms endured.
>
> (Hermann Hesse, 1920 *Wandering*)

The more complex information concerning environmental conditions or cross-dating via ring width, etc., if it is even properly construed as semantic (instead of pre-semantic or post-semantic), must await future work.[2]

[2]Contact: `brian.rabern@gmail.com`