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NONINDEXICAL CONTEXT-DEPENDENCE AND THE INTERPRETATION AS ABDUCTION APPROACH

Abstract
Inclusive nonindexical context-dependence occurs when the preferred interpretation of an utterance implies its lexically-derived meaning. It is argued that the corresponding processes of free or lexically mandated enrichment can be modeled as abductive inference. A form of abduction is implemented in Simple Type Theory on the basis of a notion of plausibility, which is in turn regarded a preference relation over possible worlds. Since a preordering of doxastic alternatives taken for itself only amounts to a relatively vacuous ad hoc model, it needs to be combined with a rational way of learning from new evidence. Lexicographic upgrade is implemented as an example of how an agent might revise his plausibility ordering in light of new evidence. Various examples are given how this apparatus may be used to model the contextual resolution of context-dependent or semantically incomplete utterances. The described form of abduction is limited and merely serves as a proof of concept, but the idea in general has good potential as one among many ways to build a bridge between semantics and pragmatics since inclusive context-dependence is ubiquitous.

Keywords
contextualism, contextuals, interpretation, higher-order logic, preference upgrade

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1. Introduction

According to the interpretation as abduction view the "best" or most plausible interpretation of a linguistic expression is inferred by means of an abductive reasoning scheme. This account of interpretation has first been laid out by Hobbs et al. (1993) but since then has only rarely been applied in formal pragmatics.[1] Based on previous work (Rast 2009, 2010) in this article a form of abductive inference is implemented in type theory and then used for resolving nonindexical context-dependence. Since type theory is used the account can be easily transferred, sometimes without further modifications, to common semantics frameworks like Montague Grammar (Montague 1974, 1979), Combinatory Categorial Grammar (Steedman 1996, 2000), and Type-logical Semantics (Morrill 1994; van Benthem 1995; Carpenter 1997).

The article comprises three parts. In section 2, higher-order logic (simple type theory) is briefly laid out, which then serves as a basis for the subsequent formal treatment of abduction in section 3. Both what we call *classical abduction* and variants like causal abduction are discussed. It is also argued in this section that abduction ought not be conflated with inference to the best explanation and that some dynamics of plausibility update is needed in order to make abduction (in the form presented here) a useful tool. In section 4, the approach is applied to the modelling of forms of nonindexical context-dependence. As it will turn out, interpretation as abduction is particularly well-suited for modelling how an agent arrives at an interpretation when the interpreted content of an utterance implies the literal meaning of the corresponding sentence (*inclusive context-dependence*). When this relation does not hold, like for example in case of ambiguities and semantic transfer, not much is gained by abduction.

2. Formal prerequisites

The account of abduction that will be introduced in section 3 is based on plausibility and its revision implemented in higher-order logic (type theory), which is also commonly used for linguistic theorizing in general semantics. In principle the use of type theory makes it possible to compute semantic representations using a Categorial Grammar. For brevity, we will not give full grammars for fragments in what follows and instead work with semantic representations at sentence level. However, the ability to link up tools from formal epistemology – in this case,

[1] See the mostly critical exposition in Norvig and Wilensky (1990) and the honorable mention in Aliseda-Llera (1997: 23).

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plausibility models and their revision – with traditional sentence-level semantics is one of the primary motivations for choosing type theory as the base language.

2.1. Type theory

A more or less standard type theory is used, which is similar to what can be found in the seminal literature like Benzmüller et al. (2008). The main purpose of this section is to provide as many details as needed for later laying out the suggested abduction mechanism and giving examples.

Types are written in the style of Montague. For example, *et* is a function from individuals of type *e* to truth values of type *t*. (In the style of Church, Henkin, and Andrews this function would have type *o*ι, where ι is the type of individuals.) So *e* is the type for individuals and *t* for truth values – *T* and *F* in the classical setting, which will be assumed throughout this article. In addition to this, *c* is used as type for intensional states (possible worlds, contexts, situations). If α and β are types, then (α β) is a compound type. Parentheses in types may be left out; in that case, right-associativity is assumed: *ttt* means *t*(*tt*).

Basic terms are variables or constants that are sequences of alphanumeric letters. We use *x*, *y*, *z* as variables of type *e*, *s*, *t*, *u*, *v* as variables of type *c*, and *P*, *Q* as variables of type *ct* unless noted otherwise. If *A* is a term of type (βα) and *B* is of type β, then (*AB*) is a compound term of type α. If *x* is a variable of type α and *B* is a term of type β, then (λ*xB*) is a compound term of type (αβ). For any terms *A*, *B* of type α, there is a term $Q^{\alpha(\alpha t)}$ which will be interpreted as equality. If *A* is of type (α*t*) then (ι*A*) is of type α.

A number of notational conventions shall ease readability. Instead of *((QA)B)* we write *A*=*B*. Moreover, the usual logical functors $\neg^{tt}$, $\wedge^{t(tt)}$, $\vee^{t(tt)}$, $\rightarrow^{t(tt)}$, and $\equiv^{t(tt)}$ are written in infix notation, i.e. *(A∧B)* is written instead of *((∧A)B)*, and so forth. Finally, traditional operator-argument syntax will be used in some examples, but only when it increases readability. For instance, *pred*(*s*, *a*, *b*) will be written instead of ((*pred s) a)*.

To deal with the iota operator we assume that ꟻ is an alien element ꟻ ∈ D_α for any type α except *t* (cf. Rast (2010)).[2] A standard model consists of a collection of non-empty domains D_α for simple types α, where D_t={T, F}, $D_{(\alpha\beta)} = D_\beta^{D_\alpha}$ for compound types (αβ), and an interpretation function $I^g(.)$ from terms to their

[2] In the literature it also common to assume that the result is unspecified when the conditions of the iota operator are not fulfilled, see e.g. Church (1940), Henkin (1950) and Carpenter (1997). A more elegant way to deal with undefined terms can be found in Farmer (2008).

denotation in dependence of a variable assignment *g* satisfying the following requirements:

(1) $I^g(x) = g(x)$ if *x* is a variable

(2) $I^g(c^\alpha) \in D_\alpha$ if *c* is a constant

(3) $I^g(A^{(\alpha\beta)}) \in D_{(\alpha\beta)}$

As an additional model constraint it is stipulated that $I^g(A^{(\alpha t)})$ (ⅎ)=*F* for any α, so any term interpreted as a function from α to a truth value yields falsity when its argument is ⅎ. Let $\|.\|^{M,g}$ be the evaluation function for terms and *g*[*x*/*c*] be the same assignment as *g* except that *g*(*x*)=*a*. Types are left out where they can be inferred from the context. Complex terms are evaluated as follows (cf. the seminal Church (1940) and Henkin (1950)):

(4) $\|c\|^{M,g} = I^g(c)$ for any variable or (nonlogical) constant c

(5) $\|(AB)\|^{M,g} = \|A\|^{M,g}(\|B\|^{M,g})$ (functional application)

(6) $\|(\lambda x^\alpha A^\beta)\|^{M,g} =$ the $f \in D_{(\alpha\beta)}$ s.t. for any $a \in D_\alpha$, $f(a) = \|A\|^{M,g[x/a]}$

(7) $\|(\iota x^\alpha A^t)\|^{M,g} = a$ if there is a unique $a \in D_\alpha$ s.t. $\|A\|^{M,g[x/a]} = T$, ⅎ otherwise

(8) $\|(A = B)\|^{M,g} = T$ if $\|A\|^{M,g} = \|B\|^{M,g}$, *F* otherwise

(9) $\|\neg A\|^{M,g} = T$ if $\|A\|^{M,g} = F$, *F* otherwise

(10) $\|(A \wedge B)\|^{M,g} = T$ if $\|A\|^{M,g} = T$ and $\|B\|^{M,g} = T$, *F* otherwise

The Verum *T* may be defined as $(=^{tt} =^{(tt)(tt)t} =^{tt})$, an identity statement that is always true, and the universal quantifier can be defined as $(\forall(\lambda x A)) = (\lambda x T)$. The existential quantifier and the remaining truth-functions are defined as usual.

2.2. Plausibility

The particular implementation of abduction used in this article will be based on a preorder relation. Preorders are commonly used for representing preferences – see van Benthem and Liu (2005), Liu (2008) – and have also been used for

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representing graded belief by Lang and van der Torre (2007), Baltag and Smets (2006, 2011), and Rast (2010). In what follows, a preorder represents the plausibility that an agent associates with different states (possible worlds, scenarios, situations, ...), which are represented by entities of type *c*. Let $\leq$ be a function of type *ecct*, written in infix notation as $s_1 \leq_{x,u} s_2$, with the reading for agent *x* in *u*, state s_1 is at least as plausible as s_2. Since $\leq$ is a total preorder with respect to its second and third argument it generates a strict order and an equivalence relation. Let $\sim_{x,\,u}$ be the corresponding equivalence relation and $<_{x,\,u}$ the strict part of the relation. As is laid out in more detail in Rast (2010), in order to implement the preorder and make it usable for the current purpose of modeling subjective plausibility the following model constraints need to be satisfied:[3]

(11) $\forall s[s \leq s]$
(12) $\forall suv[(s \leq u \wedge u \leq v) \rightarrow s \leq v]$
(13) $\forall P[\exists v(Pv) \rightarrow \exists s(Ps \wedge \neg\exists t[Pt \wedge t < s])]$

The first two conditions for reflexivity and transitivity simply ensure that $\leq$ is a preorder. The last condition is a well-foundedness condition that ensures that (for any agent and base situation) any non-empty intension P contains $\leq$ -minimal elements. If there is more than one such element, say *s* and *t*, then $s \sim t$ for the respective agent at the respective base state, which means that the agent in the base state considers *s* and *t* equally plausible. Given all this, a function that yields the maximum elements with respect to an agent and given base state may be defined:

(14) $MIN := \lambda CxuP.\iota Q\forall s[(Ps \wedge \neg\exists t[Pt \wedge Cxuts \wedge \neg Cxust]) \equiv Qs]$,

where *C* is a variable for preorders of type *ecct* such as $\leq$. The result of this operation is the proposition (intension) *Q* that is true of any *P*-state that *x* considers most plausible in *u*. (We write $MIN_{\leq}$ for $(MIN\ \leq)$, where $\leq$ is of type *eccct*, and leave out the argument when $\leq$ has not been modified.) Notice that $MINxu(\lambda s.T)$ represents *x*'s strong unconditional belief at *u*, i.e. whatever he considers most plausible in state *u*.

In Rast (2010) plausibility was used in a two-dimensional setting to investigate various notions of interpretation. Here, for simplicity only the one-dimensional case will be considered, i.e. the primary semantic objects at sentence level are terms of type *ct* rather than *cct*, and the focus of this article is on the abductive flavor of such an approach.

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[3] The auxiliary arguments *x*, *u* are left out for simplicity. The conditions must hold for any agent and any base state.

3. Abduction

Abduction has sometimes been labelled “inference to the best explanation”. This cannot be taken literally, though. It would be neat if the logician could provide the physicist with a tool that gives him the best explanation of a given phenomenon; so when the question was, say, “Which explanation is better, string theory with 9 or with 11 dimensions?” the logician’s answer would be “the one with 11 dimensions”. It is obvious that the logician has no authority to give such an answer and even *if* such an inference tool was possible (for which there is no indication), then it would have to be powered by deep physical knowledge in addition to general logical rules.

If the phrase “best explanation” is used at all, then the explanation in question must be understood as the explanation that currently, for a given agent or group of agents at a given time and on the basis of given beliefs, appears to be the most plausible one – but even then the modelling of inference to the best explanation will remain vacuous as long as no positive account of what decides good from bad explanations is given. The same problem occurs with abduction in general. The crucial question concerning abduction is how an agent ideally and rationally arrives at judgments about which states are more plausible than others and how he updates these judgments in light of new evidence. From the abundant literature on qualitative belief upgrade and quantitative update (Bayesianism, ranking theory, Dempster-Shafer theory, etc.) it is apparent that no satisfying general answer to this question can be expected anytime soon – and perhaps the question cannot be answered in general.

Nevertheless, the lack of a reliable “logic of discovery” does not in principle speak against abduction as an inference scheme. For one thing, an abductive inference mechanism can be a valuable tool even when it is just used in a descriptive, analytic way. Secondly, agents and humans in particular *do* consider certain states subjectively more plausible than others and the way in which they heuristically combine evidence cannot solely be based on inductive reasoning because of the sheer computational complexity of probabilistic inference.[4] Thirdly, once a way of upgrading subjective plausibility is available and sufficiently motivated, abductive inference on the basis becomes less problematic. Any account of graded belief gives birth to a form of abduction. Whether the abduction mechanism is reasonable primarily depends on the reasonableness of the underlying notion of graded belief and its upgrade. Although addressing the question in general as to *how* agents update subjective plausibility estimates in

[4] Pollock is well-known for this critique on inductive reasoning and Bayesianism in particular, see e.g. Pollock (2008).

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light of new evidence would go far beyond the scope of this article, some minimal dynamics will be laid out in section 3.3 as a proof of concept.

3.1. Classical abduction

The term *classical abduction* is here understood as any method intended to give a solution to the following problem. Let $\vdash$ be a consequence relation, Γ be a set of presumed (hypothetical) premises $P_1, \ldots, P_n$, C be a conclusion, and $P \vdash C$ for each $P \in \Gamma$. Problem: Find the most plausible $P \in \Gamma$!

One may call C the explanandum and the abduced content the explanans. Before going into the details, some things should be clarified. First, the premises are hypothetical in the sense that they might be false. The reasoner considers these premises as potential explanations but the abductive inference itself is not generally truth-conducive. Of course, the reasoner must still choose the premise he considers most plausibly to be true; the trivial case when the consequence relation is vacuously true because the premise is false does not interest him. Second, since we are not looking for a proof-theoretical account of abduction in this article the deductive consequence relation can be replaced by a use of the conditional in the object language.

In the present setting all states are already ordered by their plausibility for an agent in a given state, hence classical abduction is basically just an application of the *MIN* operator. For reasons to be laid out below, let us, however, introduce a binary relation R between states under which the abduction takes place; this relation plays a role similar to an accessibility relation in normal modal logic. The abduction operation is then given the following function:

(15) $\downarrow := \lambda CxuP.MIN_C xu\lambda s\forall t[Rst \rightarrow Pt]$

This function takes any relation C of the same type as $\leq$. As a notational convention, let us write $\downarrow_{\leq}$ for $(\downarrow \leq)$ and leave out the preorder relation when it is the original $\leq$. Given some agent x, base state u, and explanandum P of type ct the above function yields a proposition of type ct that contains those states s of which the agent considers it to be most plausible (in u) that a state t that is R-reachable from s is a p-state. Notice that the well-foundedness condition (13) ensures that the result is not empty as long as P is not empty and there are P-states that can be reached by R. Generally, R should fulfill the condition that for every non-empty P there are some s, t such that Rst and Pt. If R is taken as the identity relation (15) represents classical abduction.

An example shall illustrate the account. Suppose Hercule Poirot arrives at a crime scene and comes to believe that Lady Buttersworth (a) has been murdered by someone:[5]

(16) $P^* := \lambda s \exists x[murder(s, x, a)]$

Suppose that only three persons can have murdered the lady, Sir Price (b), Kenneth Thompson (c), or Lydia Patterson (d). To each of them corresponds one of the following propositions:

(17) $P_1 := \lambda s.murder(s, b, a)$
(18) $P_2 := \lambda s.murder(s, c, a)$
(19) $P_3 := \lambda s.murder(s, d, a)$

As a notation for the states, let us write sequences of digits 1 and 0 where the first digit is 1 when P_1 yields true in the state (0 otherwise), and likewise the second digit stands for P_2, and the third for P_3. Let us assume as an example that Poirot (e) in state u0 has the following preferences: $110 \sim 100 \sim 010 < 111 \sim 101 \sim 011 < 001 < 000$. This means that he considers it most plausible that P_1, P_2 or both of them are true while P_3 is false, he considers it more likely that any of the P_1, P_2 are true in combination with P_3 than the state in which P_3 alone is true and he considers it least likely that none of the premises holds.
When R is just the identity relation it follows from definition (15) that $\downarrow_{\leq} eu_0 P^*$ determines the $\leq_{e,u_0}$-minimum with respect to P^*. Given the above preferences the result is a proposition Q of type ct such that $\|Q\| = \{110, 100, 010\}$. In other words, Hercule Poirot believes that P_1 or P_2 (or both of them) and not P_3 are the most plausible explanation for the conclusion P^*. Consequently, he will start his investigation by taking a closer look at what Sir Price and Kenneth Thompson did during the night when Lady Buttersworth was murdered.

Speaking of an “explanation” in this context is justified only to a certain extent. Problems resulting from the use of the classical truth-functional conditional have to be ignored. More importantly, the account is static and therefore appears to be counter-intuitive or at least somewhat pointless. The abduction has only made explicit what Hercule Poirot already believed anyway. However, this is less problematic as it might seem at first glance. Like in the case of deduction, the static

[5] Tense is ignored, as this would require a two-dimensional semantic representation, and the example is simplified in many other respects.

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nature of abduction only appears counter-intuitive when real agents are mixed up with ideal epistemic agents. Human agents – even Hercule Poirot! – are resource-bounded and both deduction and abduction take time. So it has taken Poirot some efforts to arrive at the abduced content, even though these efforts are not part of the logical representation of the scenario. In contrast to this, ideal agents are not resource-bounded. How to make an epistemic logic with abduction (or deduction, for what it is worth) resource-bounded is an issue of its own and the fact that Hercule Poirot is not resource-bounded in the current implementation should not play a role in evaluating the account's faithfulness. The same mechanism as laid out above could serve as a basis for abduction in a logic with resource-bounded agents.

A second point worth noting is that abduction in the above sense is *ignorance-preserving*, a requirement put forward by Gabbay and Woods (2005: 42-44). While it will take any resource-bounded agent some efforts to obtain the abduced content, an ideally-rational non-resource-bounded agent does not learn anything new from an abduction. The same can, of course, be said about deduction under ideal rationality assumptions. Hence, the principle difference between abduction and deduction is that the latter is truth-preserving while the former is not, and the principle similarity between abduction and deduction is that in contrast to induction neither of them gives an ideal agent any new insights into the world.

There is still something missing in the picture as a whole, though. As mentioned earlier, an account based on a plausibility preorder remains unsatisfying unless it also gives at least a partial answer to the question how that ordering arises in the first place. This issue will be addressed in section 3.3. Before turning to dynamic aspects of the account let us briefly take another look at the static approach, though.

3.2. Variations of a theme

The above scheme is fairly general and allows for the implementation of various forms of abduction. For example, R could express causality. When $R(s_1, s_2)$ holds between two states s_1 and s_2 this relation could be read as "s_1 causes s_2". Minimally, s_1 must then temporally precede s_2.[6] Definition (15) does not need to be changed in order to account for this interpretation, apart from

[6] A temporal ordering of states is needed for the tenses anyway and must allow for expressing all relations between time intervals such as s_1 temporally overlaps s_2 or s_1 temporally predeces s_2. See Allen (1983) and van Benthem (1991) for details.

imposing additional constraints on R when it is interpreted as a causal relation between states.

Since a given sentence, say φ, is usually true in a variety of states (unless it expresses a contradiction), the intension P representing φ in the presumed one-dimensional semantic setting will usually also be true at a variety of states. Correspondingly, the abduced proposition in the above version of abduction might encode a variety of *different* singular causes; each state in the abduced proposition corresponds to a (potentially infinite) description of a constellation of reality that as an ensemble causes a state that makes the given proposition true. What these descriptions have in common depends on the properties and the intended reading of the causal relation.

Someone might have concerns about the adequacy of expressing causality as a relation between two states, and these concerns might very well be justified. In the long run, it seems unlikely that questions concerning the adequacy of the underlying representation for modeling causality can be ignored. However, these questions are orthogonal to the question of how to implement abductive inference in the first place. If semantic entities of type c turned out not to be suitable for a certain view of causality, some more appropriate representation would need to be chosen. As long as this representation is based on semantic entities that can be ranked according to their plausibility, and causality can be expressed as a relation between two of them, the above approach can be used to represent causal abduction.

It is a virtue of the current approach that it is neutral both with respect to the exact nature of the relation used as a basis for abduction and with respect to implementation details of the semantic entities (of type c) used. One might consider and compare other relations between states and explore their conceptual usefulness for playing the role of R in (15). Any non-empty binary relation between two states can be used as a basis for the above type of abductive inference, and one may introduce as many of these relations and corresponding types of abduction as needed into the object language on the basis of suitable meaning rules, model constraints, or axiom schemes. Likewise, entities of type c can have nearly any kind of fine-structure; as long as it makes sense to say that an agent can compare any two of them, they can serve as a basis for (15).

3.3. Plausibility upgrade

As mentioned before, just introducing abduction of the static kind laid out above leads to a form of “vacuous modelling”. *If* an agent can already decide between any two states which one is more plausible, *then* it takes him no effort to come up with the most plausible cause or reason for a proposition. But *how* does

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the agent get to these preferences in the first place? In the following paragraphs a way of upgrading a given plausibility ordering on the basis of new evidence is laid out in order to illustrate that it is possible to give an answer to this pressing question. The upgrade mechanism that will be used is based on work by von Benthem and Liu (2005), Liu (2008), and Baltag and Smets (2006, 2011). Both its implementation and the underlying idea are straightforward. New information, which could be based on perceptual evidence or on testimony, is represented by a proposition *P* of type *ct*. An epistemic agent then revises his plausibility relation in such a way that *P*-states are preferred over ¬*P*-states:

(20) $REV := \lambda x u_0 P C \iota D \forall y u s t[\text{ if } u_0 = u \wedge x = y \wedge Ps \wedge \neg Pt \text{ then } Dxust \wedge \neg Dxuts \text{ else } (Dyust \equiv Cyust) \wedge (Dyuts \equiv Cyuts)],$

where "if A then B_1 else B_2" abbreviates $(A \rightarrow B_1) \wedge (\neg A \rightarrow B_2)$ and *C*, *D* are of type *eccct*. The operation characterized by (20) is known as *lexicographic upgrade* in the semantic belief revision literature. As Baltag and Smets (2011) lay out, both more conservative and more radical upgrades could also be implemented. That there are many ways of upgrading belief ought not come as a surprise, as the research on traditional, syntactic AGM-style belief revision has shown that there are usually, in non-trivial cases, many different ways to contract or revise beliefs.[7] For our purposes, lexicographic upgrade seems to be the most suitable one. Rast (2010) uses it in a two-dimensional semantic setting and proves that the properties of the original ordering relation (reflexivity and transitivity) are preserved under upgrades. The above operation really yields a revised preorder, which may in turn be revised, allowing any number of subsequent revisions when new evidence comes in.

To give an example of how the upgrade operation works, consider Hercule Poirot's case of section 3.1 again. In his initial epistemic state he is able to abduce that Sir Price or Kenneth Thompson are the most plausible candidates for being the murderer of Lady Buttersworth, corresponding to the abduced content {110, 100, 010}. Now suppose Sir Price says that he went to a meeting of the Royal Geographic Society the evening during which the crime took place, expressed formally as an intension of type *ct* denoted by *Q*, but Poirot does not believe Sir Price initially. Suppose, for instance, that Poirot (*e*) has held the conditional belief that $\forall s[Qs \rightarrow \neg murder(s,b,a)]$, but also believed *Q* is not the case (the *Q*-states are not among the most plausible ones). He believed that Sir Price did not go to the meeting. Later testimony confirms Price's alibi, though, and so Poirot learns that *Q*

[7] Overviews of belief revision can be found in Alchourrón et al. (1985), Gärdenfors (1988), Gärdenfors and Rott (1995), and Hansson (1999b).

holds. Consequently, he also learns that the actual state is in $\lambda s.\neg murder(s,b,a)$. So a revision by Q, given the conditional belief $\forall s[Qs \rightarrow \neg P_1 s]$, will narrow down the abduced content to {010}. The P-states 110 and 100 are no longer in the minimum – in this particular example, they are no longer in $\downarrow_{(REVeu_0 Q)} eu_0 P^*$. In other words, Poirot abduces that the murderer is Kenneth Thompson.

Of course, in reality the deductive structure of Poirot's beliefs would be much more complicated and he would likely have to take into account a vast number of alternatives. There are many differences between abduction in the above sense and actual abductive reasoning by humans. Human reasoners like Poirot can imagine new alternative scenarios that would explain a certain phenomenon and assess their plausibility on the basis of past experiences and rich background knowledge about how the world (usually) works. Regarding background knowledge and ontology the difference is merely one of scale: A complex background ontology could be expressed in the current setting without any problems. However, the heuristics – or intuitions, as some might say – that humans use to assess the plausibility of a scenario is hard to formalize; and even if a reasonable approximation was found it could turn out to be difficult to express it in the current setting for technical reasons. For example, in a qualitative account distance measures between the plausibilities of different scenarios are not readily available, and so a heuristics making use of such measures could not be expressed without substantial changes. Generally speaking, it seems fair to say that modeling how humans come up with new explanations goes far beyond the scope of logic.

Despite these obvious limitations the above account illustrates that abduction is not (entirely) pointless. As long as there *is* a rational way of assessing the plausibility of new evidence, abduction can be regarded as an inference scheme that makes use of this information. Different ways of assessing plausibility give rise to different versions of abduction, and so to some extent the usefulness of abductive inference schemes of the sort laid out above hinges on the power and rationality constraints of the underlying belief upgrade mechanisms. Here we have only taken a look at lexicographic upgrade, which suffices to show that there *are* rational ways of revising graded belief in light of new evidence.

4. Interpretation as abduction: the case of nonindexical context-dependence

Let us now turn to an application of abduction on which the remainder of this article will focus: interpretation as abduction. The main idea of this approach is as follows. Whenever an utterance can be interpreted in various ways the hearer will choose the interpretation he considers most plausible at a given moment. This way

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of using abduction goes back to Hobbs et al. (1993) who analyze reports like "disengaged compressor after lube oil alarm". (This and other examples resulted from their practical work on the implementation of an automated report analyzing system.) In their approach, values are assigned directly to formulas that represent partial interpretations of a report and it is then determined how subformulas combine in a way that maximizes the value of a formula representing one interpretation of the whole report. We will illustrate the interpretation as abduction approach in the present semantic setting on the basis of examples of what one may call *nonindexical linguistic context-dependence*.

4.1. Semantic incompleteness: missing arguments

Here is a typical case of nonindexical context-dependence:

(21) John is ready.

This and similar examples have been discussed extensively in the philosophical literature on linguistic context-dependence, see for example Recanati (2004), the contributions in Preyer and Peter (2005, 2007) and Baptista and Rast (2010). Example (21) is in the present tense and therefore indexical, because the present tense semantically depends on the time of utterance. But according to a view defended by Bach (2005) it is also semantically incomplete. Just from the meanings of the expressions involved a listener cannot derive for what John is ready. Moreover, according to Bach it also makes no sense to claim that someone can be ready *simpliciter* as opposed to being ready for this or that.[8]

Following Bach in this initial assessment of the linguistic data (but diverging from his view in other respects, particularly his views about the nature of semantic incompleteness), Rast (2009) has laid out how examples like the one above may be modelled from a truth-conditional perspective. According to this view, expressions like *ready* are syntactically complete and semantically incomplete in sentences like (21). They lack an argument and can be formally represented by open formulas. In a first interpretative step, the open variables in these formulas then need to be bound by the existential quantifier (*existential completion*). In a second step, the agent abduces the most plausible interpretation. While in a more realistic model the hearer's assumptions about what the speaker believes would need to be taken into

[8] Cappelen and Lepore have suggested that there is such a thing as being ready simpliciter or being tall simpliciter. See Bach (2007a, 2007c) for some (in our point of view) convincing critique of this position.

account, these details shall be ignored in what follows and as we focus on abductive inference instead.

Suppose for the sake of the argument that the complement of *ready* can be represented by an entity of type *ct*. Given that, the existential completion of example (21) is:

(22) $\lambda s \exists X^{ct}[ready(s, John, X)]$

To this existential completion abduction is applicable. Suppose a hearer accepts, perhaps tentatively, (22) as the literal semantic content of (21). Let $S = \{s \mid \|(22)\|^{M,g}(s) = T\}$. Then any $s_1, s_2 \in S$ are comparable to each other by $\leq$ for that agent in the given base situation and there is a $\leq$ -minimal subset of *S* that represents the abduced content. Suppose $C_1, \ldots, C_4$ are constants of type *ct*. The step of abducing more particular content then results from the hearer's ability to compare the set of states characterized by each of $\lambda s.ready(s, John, C_1) \ldots \lambda s.ready(s, John, C_4)$ and find the most plausible one on the basis of the existential completion (22).

There are two things worth noting about existential completion. First, the fact that an agent computes and accepts an existential completion must also be regarded an interpretational process. For under certain (rare) circumstances it is possible to interpret (21) as $\lambda s \exists Q[Q(X, ready(s, John, X))]$, where Q is a variable for a quantifier of the appropriate type. This allows for readings such as *John is ready for everything* or *John is ready for most actions*. Using higher-order quantification in this way rebuts a counter-argument raised by Bach (2007b) in a slightly different context, that of arguing against Borg's version of semantic minimalism, namely that sometimes an interpretation may involve quantifiers other than the existential one. The present approach is in our point of view compatible with Bach's main tenets as long as starting with the existential completion of a semantically incomplete utterance is regarded a simplification that most of the time yields the right result but might sometimes fail. When it fails, the literal content can still be represented as a higher-order existential completion of the form λs. ∃ Q(Qx[...]).

Secondly, the above suggestion is not compatible with another view laid out in Bach (2004, 2005) that semantically incomplete utterances are usually not truth-bearers. In a higher-order setting an appropriate existential completion is always available (by quantifying over quantifiers, etc.) and so it is always possible to find a proposition that faithfully represents the literal meaning of an utterance. Semantic incompleteness ought not be conflated with sub-propositionality. To give a more drastic example, consider the following one-word utterance:

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(23) John: Fire!

The existential completion of this utterance expresses a proposition that is true or false, and there are many ways to represent it. Consider, for example, the following terms:

(24) $\lambda s \exists x[burn(s, x)]$
(25) $\lambda s \exists s'[fire(s') \wedge partOf(s', s)]$

Many other representations are conceivable and which of them is the most adequate one shall not interest us here.[9] Now if a Bach-style minimalist insisted that (24) or (26) are not adequate, a higher-order existential completion could be used instead. In any case the interpretation as abduction view is applicable. For example, if (24) is taken as the meaning of (23), the abduced content could be equivalent to $\lambda s.burn(s, c)$, where c represents a particular ashtray.

There is a deeper issue with the approach that ought to be mentioned, though. The hearer must also *accept* the existential completion. Accepting involves a check against previous beliefs and qualitative models of this checking are underdeveloped. Seminal approaches to non-prioritized AGM belief revision like Makinson (1997) and Hansson (1999a) are not directly transferable to the current semantic approach and more research is needed in this area.

4.2. Quantifier domain restriction

The abduction method can be applied to many other cases of nonindexical linguistic context-dependence. Consider, for example, quantifier domain restriction (QDR) as in the following sentence:

(26) Every beer is in the fridge.

This example may be represented in higher-order logic (among other, mostly equivalent ways) by introducing a fresh context variable for each nominal during semantic construction. Let us for the sake of simplicity assume a Russellian

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[9] An answer to this question depends on the structural constraints imposed on D_c, as for example (25) requires that states are ordered mereologically. Ideally, the underlying logic ought to be partial like in Muskens (1995) because this turns states into non-maximal truth-makers akin to situations.

account for *the fridge*, where the determiner is also treated like a quantifier. Given all that, the result of semantic construction can (roughly) be represented as:[10]

(27) $\lambda s \forall x[(beer(s,x) \wedge X(s,x)) \rightarrow locatedIn(s,x,\iota x[fridge(s,x) \wedge Y(s,x)])]$

Again, the existential completion is computed:

(28) $\lambda s \exists X \exists Y \forall x[(beer(s,x) \wedge X(s,x)) \rightarrow locatedIn(s,x,\iota x[fridge(s,x) \wedge Y(s,x)])]$,

and the result of abduction might, for example, turn out to be more specific than (28) and equivalent to

(29) $\lambda s \forall x[(beer(s,x) \wedge C_1(s,x)) \rightarrow locatedIn(s,x,\iota x[fridge(s,x) \wedge C_2(s,x)])]$,

where C_1, C_2 are constants of the appropriate type. Note that no linguistic constraints are put on the abduced content. This means that the substitution instances for the variables in the above formula could stand for arbitrarily complex content of the appropriate type. For example, (29) with respect to an agent *a* with complex background beliefs and a base state s_0 could have the reading "every (bottle of) beer that *a*'s friend John has bought yesterday at a gas station just outside the town where *a* lives is located in the (one and only) fridge in the kitchen of John's apartment situated to the left of *a*'s body axis at the time of s_0 and the place of s_0". This insensitivity to syntax is one of the main advantages of a semantic approach over broadly-conceived syntactic accounts of abduction.

4.3. Resolving ambiguities

So far interpretation as abduction has turned out to be rather powerful. Let us now turn to cases when abduction can be used, but the net gain is relatively low. Take a look at the following textbook examples of ambiguity, the first illustrating quantifier scope ambiguity and the second lexical ambiguity:

(30) Every sailor loves a woman.

[10] Stanley and Szabó have laid out a difference between quantifier domain restriction and nominal restriction and argued for the latter – see Stanley and Szabó (2000), Stanley (2002). Since in the above example it is assumed that a new variable is introduced for every nominal in the restriction of a determiner it is closer to Stanley's than to ones that restrict the domain tout court.

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(31) John went to the bank.

Before turning attention to the second example, consider the familiar readings of (30):

(32) $\lambda s \forall x[sailor(s, x) \rightarrow \exists y(woman(s, y) \wedge love(s, x, y))]$
(33) $\lambda s \exists y[woman(s, y) \wedge \forall x(sailor(s, x) \rightarrow love(s, x, y))]$

Since (33)u_0 implies the weaker (32)u_0 for given base state u_0, the result of abducing $\downarrow_{\leq} au_0$(32) for some agent a in u_0 may turn out to be equivalent to (33). So the weak reading could to be taken as the default content of (30), which in this case is also the strongly preferred reading. The strong reading represents the rare case when the respective agent considers only those states most plausible that satisfy (33).

However, the fact that the above example “works” is more or less an accident of the quantification involved. The mechanism does not help in dealing with the interpretation of ambiguities in general. To see this, let us call an ambiguity of a sentence with literal, existentially completed meaning φ and readings ψ_1, ..., ψ_n *inclusive* if and only if $\psi_i \rightarrow \varphi$ holds in intended models ($1 \leq i \leq n$). It is fairly obvious that inclusive ambiguities can generally be understood in the way laid out for (30). However, other examples such as (31) are not inclusive. General world knowledge suggests that the sets of states satisfying the two readings of (31) – one involving a river bank and the other a financial institution – are independent from each other in the sense that no one implies the other. In that case a compound disjunctive meaning would have to be taken as the basis for abductive inference. Applying abduction to such a disjunction would only yield a pyrrhic victory. In natural language processing the combinatory complexity of ambiguities is frowned upon and avoided by deriving semantically underspecified meanings (see e.g. Egg et al. (1998) and Copestake (2001)), and there is no obvious way how to implement semantic abduction of the above sort on the basis of such underspecified meanings. The example illustrates that the interpretation as abduction view is not very well-suited for non-inclusive context dependence.

5. Conclusion

Abductive inference has been implemented in higher-order logic on the basis of a total preorder relation that reflects an agent’s ability to compare doxastic alternatives according to their subjective plausibility. It has been argued that abduction of this sort must be kept apart from inference to the best explanation. As long as only a static model is taken into account, abduction on the basis of plausibility is problematic because all the work is done by the underlying plausibility relation, which does not explain how agents arrive at their judgments about the plausibility of different scenarios. This shortcoming can be fixed by introducing a rational way of updating a plausibility relation in the light of new evidence. Lexicographic update has been discussed as an example of such an operation.

In the second part of the article it has been illustrated how phenomena of linguistic context-dependence can be modelled on top of traditional semantic representations by assuming that agents choose the one they consider most plausible among various alternative interpretations. This method yields the best results when the underlying context-dependence is inclusive. Since this form of context-dependence is very common, the interpretation as abduction view is broadly applicable to the problem of modelling interpretation under ideal rationality assumptions.*

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