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OBLIGATIONS, SOPHISMS
AND INSOLUBLES

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The focus of the paper is a sophism based on the proposition 'This is Socrates' found in a short treatise on obligational *casus* attributed to William Heytesbury. First, the background to the puzzle in Walter Burley's traditional account of obligations (the *responsio antiqua*), and the objections and revisions made by Richard Kilvington and Roger Swyneshed, are presented. All six types of obligations described by Burley are outlined, including *sit verum*, the type used in the sophism. Kilvington and Swyneshed disliked the dynamic nature of the *responsio antiqua*, and Kilvington proposed a revision to the rules for irrelevant propositions. This allowed him to use a form of reasoning, the "disputational meta-argument", which is incompatible with Burley's rules. Heytesbury explicitly rejected Kilvington's revision and the associated meta-argument. Swyneshed also revised Burley's account of obligations, formulating the so-called *responsio nova*, characterised by the apparently surprising thesis that a conjunction can be denied both of whose conjuncts are granted. On closer inspection, however, his account is found to be less radical than first appears.

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1. Medieval Logic[1]

Medieval logic inherited the legacy of Aristotle: first, the *logica vetus*, Aristotle's *Categories* and *De Interpretatione*, which together with some of Boethius' works were all the logic the Latin West had to go on around 1100. Subsequently, over the following century, came the recovery of the *logica nova*: the rest of Aristotle's *Organon*, all of which was available in Latin by 1200. The medievals' own original contribution began to be formulated from around 1150 and came to be known as the *logica modernorum*. It consisted of a theory of properties of terms (signification, supposition, appellation, ampliation, restriction etc.); a theory of consequences; a theory of insolubles; and a theory of obligations. This development was arguably stimulated by the theory of fallacy, following recovery of *De Sophisticis Elenchis* around 1140.[2]

My focus will be on some logicians at the University of Oxford, mostly at Merton College, in the early fourteenth century, in particular, Walter Burley (or Burleigh), Richard Kilvington, Roger Swyneshed and William Heytesbury. I will contrast three different approaches to the theory of obligations found in these authors, and some of the reasons for these contrasts. The standard theory of obligations, the *responsio antiqua*, was codified by Burley in a treatise composed in Oxford in 1302. Kilvington and Swyneshed objected strongly to certain aspects of Burley's theory, and proposed their own revisions. Heytesbury reverted to the standard theory. These objections were often motivated by certain sophistical arguments, and in return, the discussion of sophisms in treatises on them such as Kilvington's are ridden through with obligational terminology, as are other treatises such as those on insolubles. One such sophism is found in a short treatise (an abridgement, or perhaps an early version, of ch. 2 of Heytesbury's *Regulae*) found in Oxford MS Can. lat. 278 f. 70r. The sophism occurs as part of an obligational *casus*. Burley's solution to a related sophism raises wider issues of signification and truth, connected with

[2] See [De Rijk, 1962].

the diagnoses of the insolubles found in Bradwardine and Buridan, which I have discussed elsewhere and to which I will briefly allude.[3]

2. Obligations

Obligations were a species of disputation. Their purpose has been variously described as pedagogical exercises;[4] tools for solving sophisms and insolubles;[5] experiments with counterfactual reasoning;[6] a theory of belief revision;[7] a theory of thought-experiments;[8] a sophisticated theory of argumentation and disputation;[9] and games of consistency maintenance.[10]

I believe we should understand obligations as logical exercises, exercises in logical disputation. No record of any actual disputation, rather than discussion of the theory of obligations, has survived. Nonetheless, the medieval philosophical literature is rife with reference to obligational terminology. In my view, we should accept what is said in perhaps the longest passage describing the purpose of obligational disputations that we have, found in an anonymous treatise of the 1330s, the *De Arte Obligatoria* in a Merton College MS:[11]

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[3] See, e.g., [Read, 2002].

[4] Contemporary treatises so describing obligations include those of Nicholas of Paris (see [Braakhuis, 1998]), Ralph Strode (see [Spade, 1977]), the anonymous *Obligationes Parisienses* (see De [Rijk, 1975]), and the anonymous *De arte obligatoria* (see [Kretzmann and Stump, 1985]). Modern scholars holding this view include Romuald Green [Green, 1963], Mary Anthony Brown [Brown, 1966], Charles Hamblin [Hamblin, 1970] and Jennifer Ashworth [Ashworth, 1985].

[5] Contemporary treatises expressing this view include William of Sherwood, *Tractatus Sorbonnensis de Petitionibus Contrariorum* (in [De Rijk, 1976]). A modern exponent is Eleonore Stump [Stump, 1981].

[6] The only text where this view is expressed is, I believe, Richard Kilvington's *Sophismata* (see § 6 below). This interpretation has been advocated by Paul Spade [Spade, 1982] following a suggestion by Norman Kretzmann in an unpublished lecture (see [Spade, 1982: 3 fn. 6]).

[7] So argue [Lagerlund and Olsson, 2001].

[8] According to Mikko Yrjönsuuri [Yrjönsuuri, 1993].

[9] See [Hajo Keffer, 2001].

[10] Suggested by Chris Martin [Martin, 1993], and defended by Catarina Dutilh Novaes [Dutilh Novaes, 2005].

[11] [Kretzmann and Stump, 1985: 251]: "Haec ars informat respondentem ut advertat quid conceditur et negatur, ne duo repugnantia concedat infra idem tempus. Aristoteles enim in *Elenchis* docet arguentem multa proponere, ut de propositorum responsione ob multitudinem respondens non recolens redargueretur. A quo in parte haec ars ordinem traxit, ut advertentes

"This art trains the Respondent so that he pays attention to what is granted and denied, in order not to grant two incompatible things at the same time. For in *De Sophisticis Elenchis*, Aristotle teaches the arguer to put forward many things so that the Respondent who does not remember because of the large number may be refuted as regards his response to the things put forward. It is partly from this that the art has derived its structure, so that as long as we pay attention we may keep ourselves from being tricked. Just as it is important for a liar to have a good memory in order to make claims without asserting contraries, so for someone who is good at responding it is appropriate that he respond formally regarding the things admitted, granted and appropriately denied and remembered."

Disputations played a large role in medieval teaching, and the terminology of disputations, and of obligations, permeates the scholarly literature. Indeed, we know from contemporary reports that disputations were frequently practiced as a way of training students. The prevalence of the terminology of obligations is good evidence that obligational disputations also played a large role in medieval pedagogical practice. It is also a practice from which we have much to gain in our philosophical understanding of logic.

3. The *Responsio Antiqua*

The classic account of obligations, the *responsio antiqua*, is found in the treatise on obligations by Walter Burley. He was born in Yorkshire, England, around 1275. We find him as Master of Arts at Merton College in Oxford University, by 1301. He wrote treatises on *Suppositions* and *Obligations* in 1302. He went to the University of Paris before 1310 and stayed there until 1326 or 1327, when Edward III came to the throne of England. His most famous logic text is *De Puritate Artis Logicae*, a title best translated as 'On the Essentials of the Art of Logic', written in the mid-1320s.[12] Burley was a member of the intellectual circle surrounding Richard de Bury, the Bishop of Durham. Reports say that Bury had the largest private library in England at that time. Burley was envoy to the papal court for Edward III from 1327. His many works include commentaries on Aristotle. He died around 1344/5.

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non indeceptos servemus. Sicut decet mendacem esse bene memorem ut non contraria licet affirmat asserat, ita bene respondentem iuxta admissa et concessa et nagata convenienter et memorata formaliter convenit respondere."(P. 243).

    [12] On the translation of Burley's title, see [Spade and Menn, 2003].

An obligational disputation is a disputation between an Opponent and a Respondent. Burley distinguishes six types of obligation:[13] *Institutio* (or *Impositio*), where the Respondent is obligated to use a term with a new meaning; *Petitio*, where the Respondent is obligated to act in a certain way; *Positio*, where the Respondent is obligated to grant a particular proposition, the *positum*; *Depositio*, where the Respondent is obligated to deny a certain proposition, the *depositum*; *Dubitatio*, where the Respondent is obligated to doubt a given proposition, the *dubitatum*; and finally, *Sit verum*, where the Respondent is obligated to respond as if he variously knew, doubted or was ignorant of the *positum*. The primary type, which takes up half of Burley's treatise, is *positio*. Burley and earlier writers distinguish two types of *positio*, *positio possibilis* and *positio impossibilis*.

In *positio* the Opponent presents a *casus*, that is, a hypothetical background situation; a *positum*, that is, a proposition which may be accepted or rejected by the Respondent; and a sequence of propositions which may be granted, denied or doubted (or in later texts, distinguished as ambiguous) by the Respondent, according to the rules of *positio*. The obligation ends either when the Respondent grants and denies the same proposition (or grants a contradiction), or when the Opponent says '*cedat tempus*', i.e., time's up. There may follow an analysis of how well the Respondent responded.

The basic rules of *positio*, according to the *responsio antiqua*, are as follows. In possible *positio*, the *positum* should be accepted only if it could be true. If the proposition follows from or is inconsistent with the *positum* and/or something already granted/denied, it is said to be "relevant" (*pertinens*), otherwise "irrelevant" (*impertinens*). If it is relevant, it is "obligated" and should be granted if it follows (*pertinens sequens*), denied if it is inconsistent (*pertinens repugnans*), while if it's irrelevant, it is not obligated and (in accord with the *casus*) should be granted if (known to be) true, denied if (known to be) false, and doubted if it is not known whether it is true or false.

Here is an example of possible *positio*:

0. *Positum*: 'Every man is running'	Accepted
1. 'Every man is running'	Granted (the *positum*)
2. 'You are running'	Denied (irrelevant and false)
3. 'You are a man'	Denied (true, but inconsistent with the *positum* and the opposite of what has been denied)

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[13] [Burley, 1988] contains an English translation of a little over half of Burley's treatise, edited in full in [Green, 1963].

The early treatises, up until the time of Burley and Ockham, also accepted impossible *positio*, where the *positum* is impossible. However, the *positum* must not be explicitly contradictory; it must be credible. E.g., 'God is not God', or 'A man is an ass' can be accepted.[14] Not every consequence should be granted; e.g., in impossible *positio* one must not use the "rule of the Adamites",[15] *viz* that from the impossible anything follows, nor the rule that what is necessary follows from anything. But one can use syllogistic inferences and rules of transposition. Ockham says that "by such *positio* one opens the way to recognising which inferences are good and self-evident and which are not."[16]

Catarina Dutilh Novaes [2007, p. 161] has argued that the Respondent always has a winning strategy, at least in possible *positio*. The reason is Lindenbaum's Lemma, that any consistent set of propositions has a maximal consistent extension.[17] The construction in Lindenbaum's Lemma is very similar to the way an obligational disputation develops. We start with the *casus* together with the *positum*. These will often be inconsistent, but if the *positum* is impossible it should be rejected. Let Π be the set of common knowledge, amended in light of the *casus*; let p_0 be the *positum*; and let $\Sigma_0 = \{p_0\}$. Then consider each proposition p_{n+1} in turn:

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[14] See, e.g., [De Ockham, 1974: III-3 c. 42]: "Impossible positio is when some impossible proposition is posited. It must be realised that it is often useful to posit an impossible proposition and to accept an inference in which an impossible proposition is inferred from another impossible one. For example, it is useful to posit this proposition, 'A man is capable of braying' and to make this inference: 'If a man is an ass, a man is capable of braying'. Similarly, it is useful to posit this proposition, 'God is not God' and to make this inference: 'If there is no God, God is not God'... [or] ...'God is not three persons, so God is not God'." ("Positio impossibilis est quando ponitur aliqua propositio impossibilis. Et sciendum est quod multum refert ponere unam propositionem impossibilem et accipere unam consequentiam in qua unum impossibile infertur ex alio impossibili. Sicut multum refert ponere istam propositionem 'homo est rudibilis' et inferre istam consequentiam 'si homo est asinus, homo est rudibilis'. Similiter multum refert ponere istam propositionem 'Deus non est Deus' et inferre istam consequentiam 'si Deus non est, Deus non est Deus' ... 'Deus non est tres personae, igitur Deus non est Deus'.") As a negative proposition, 'God is not God' is true if its subject term is empty.

[15] See [Anonymous, 2001: 218]. Adam of Balsham taught at the school of the Petit Pont in Paris in the 12th century, and became famous for the argument that anything follows from a contradiction. See, e.g., [Martin, 1986: 571].

[16] [De Ockham, 1974: III-3 c. 42 P. 741]: "Per talem enim positionem aperitur via ad sciendum quae consequentiae sunt bonae et evidentes et quae non sunt evidentes."

[17] See, e.g., [Mendelson, 1979: 66-7] Lemma 2.11.

- If p_{n+1} is *pertinens sequens* (i.e., $\Sigma_n \vdash p_{n+1}$), p_{n+1} is consistent with Σ_n, so by the rules $p_{n+1} \in \Sigma_{n+1}$ and Σ_{n+1} is consistent if Σ_n is
- if p_{n+1} is *pertinens repugnans* (i.e., $\Sigma_n \vdash \neg p_{n+1}$), p_{n+1} is not consistent with Σ_n, so by the rules $\neg p_{n+1} \in \Sigma_{n+1}$ and Σ_{n+1} is consistent if Σ_n is
- if p_{n+1} is *impertinens*, then by the rules $p_{n+1} \in \Sigma_{n+1}$ if and only if $p_{n+1} \in \Pi$

Note that Σ_n may well be inconsistent with Π (over consequences of accepting p_0). But since p_{n+1} is *impertinens* only if p_{n+1} is consistent with Σ_n, the obligational rules guarantee that Σ_n is consistent for all n. Note in particular that the construction, as in Lindenbaum's Lemma, is entirely syntactic, building a set of propositions Σ_n. There is no reference to any semantic interpretation or model.

4. Other Types of Obligation

Burley describes five other types of obligation. The first he mentions is *institutio*, sometimes called *impositio*.[18] For example, let A signify 'man' in a false proposition, 'ass' in a true proposition and the disjunctive term 'a man or not a man' in a doubtful proposition:

1. 'You are A'	???
2. *Cedat tempus*	

This places you in a dilemma. For either you are A or not. If you are A, 'You are A' is true and irrelevant, so you should grant it when under the obligation, and then A signifies 'ass', so you would grant that you are an ass. If you are not A, 'You are A' is false and irrelevant, so you should deny it when under the obligation, and then A signifies 'man', so you would deny that you are a man. If you doubt it, you doubt it when under the obligation, and then A signifies 'man or not man', so you would doubt whether you are a man or not.

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[18] See, e.g., [Spade, 1977: §III, P. 258].

Burley's response is:

"An *institutio* should never be accepted when what the proposition signifies depends on the truth or falsity of the proposition in which it is used."[19]

The second type of obligation that Burley describes is *petitio*. For example, suppose that I require (*peto*) you to grant that a man is an ass (§2.05):

1. 'You grant that a man is an ass'	???
2. *Cedat tempus*	

If you grant this, you grant what is false when not obligated to do so, so you responded badly. If you deny it, you were obliged to grant that a man is an ass and you've denied it, so you responded badly. Burley's solution (§2.13) is that you should deny 'You grant that a man is an ass', for you were obligated to grant that a man is an ass, not to grant that you grant that a man is an ass. *Petitio* can be subsumed under *positio*. For example, instead of requiring that you grant *p*, simply posit 'You grant *p*'.

Depositio in the mirror image of *positio*. Once accepted, the *depositum* should always be denied. Since it should always be denied, whatever implies the *depositum* must also be denied.[20] E.g.,[21]

0. *Depositum*: 'You respond badly or you should deny that you respond badly'	Accepted (call it *A*)
1. '*A* is deposited to you'	Granted (irrelevant and true)
2. 'You should deny *A*'	Granted (follows from 1)
3. 'You should deny that you respond badly'	???
4. *Cedat tempus*	

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[19] [Burley 1988: §1.02]; cf. [Green, 1963: II P. 35]: "Numquam est institutio admittenda ubi significatum vocis dependet ex veritate vel falsitate propositionis in qua ponitur." Swyneshed's response, according to the *responsio nova* that we will consider in §8 below, is to accept the obligation and deny 'You are *A*'. See [Spade, 1977: §40]. For although *A* signifies 'man' in a false proposition and 'You are *A*' is false, and it would follow that you deny you are a man (that is, grant that 'You are *A*' is false), you can deny the conjunctive antecedent while granting the conjuncts.

[20] Obviously, one should accept the *depositum* only if its falsehood is consistent with the *casus*.

[21] [Green, 1963: II P. 89] (§4.24).

If you grant (3), you grant something that implies the *depositum*, contrary to the rule for *depositio*. If you deny it, you deny something that follows from what you have granted, *viz* 'You should deny *A*', for you should deny the parts of any disjunction you should deny.

Burley's solution (§4.25) is that '*A* is deposited to you' should have been denied at line 1, for it already implies the *depositum*, and is not irrelevant. That is, '*A* is deposited to you' implies that you should deny *A*. So you should deny that you respond badly. So either you respond badly or you should deny that you respond badly, which is the *depositum*.

Dubitatio is, as the name implies, a species of obligation in which the *dubitatum*, the *obligatum*, should be doubted. Hence:[22]

"One must respond to the *dubitatum*, what is equivalent to it, what is contradictory to it, what is false and follows from it, and what is true and implies it, by saying one is in doubt." E.g., suppose Socrates is white and that you know this.[23]

0. *Dubitatum*: 'Socrates is white'	Accepted
1. 'You are in doubt whether Socrates is white'	Denied (irrelevant and known to be false)
2. 'You know that Socrates is white'	Denied (implies the *dubitatum*)[24]
3. 'Socrates is not white'	???
4. *Cedat tempus*	

If you grant (3), you grant the opposite of the *dubitatum*. If you deny it, you deny something that follows from the opposites of what has been denied, for the contradictory of (3) and the contradictory of (2) imply (1), so the contradictory of (1) and the contradictory of (2) imply (3).

Burley's solution is that 'You are in doubt whether Socrates is white' should have been doubted at line 1, for it cannot be granted, since it is false and known

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[22] [Burley, 1988: §5.05]; cf. [Green, 1963: II P. 90]: "Ad dubitatum et ad suum convertibile et ad suum contradictorium et ad suum consequens, si sit falsum, et ad suum antecedens, si sit verum, respondendum est dubie."

[23] [Burley, 1988: §5.07].

[24] Burley says that (2) should be denied because it implies the dubitatum. But that is only why it cannot be granted. It cannot be doubted since that was ruled out at line 1.

to be false, and it cannot be denied, since its denial implies the contradictory of (2) which together imply (3), the opposite of the *dubitatum*.

The final species of obligation is *Sit Verum* ('Let it be true'), which Burley describes as follows:[25]

"It is usually said that *sit verum* creates an obligation on a mental state, and since mental states are of three kinds, namely, the states of knowledge, of doubt and of ignorance, this obligation is of three kinds, either through a verb of knowing, or through a verb of doubting, or through a verb of ignorance. For example, 'Let it be true that you know you are running', or 'Let it be true that you doubt you are running'."

Again, *sit verum*, like *petitio*, can be subsumed under *positio*. For example, instead of letting it be true that you don't know whether *p*, simply posit 'You don't know whether *p*'.

5. Problems with the *Responsio Antiqua*

Burley observes that in possible *positio*, the Respondent can be forced to grant any other false proposition compatible with the *positum*. E.g., to prove you are a bishop:[26]

0. *Positum*: 'You are in Rome'	Accepted (possible)
1. 'You are not in Rome or you are a bishop'	Granted (irrelevant and the first disjunct is true)
2. 'You are a bishop'	Granted (follows from the *positum* and what was granted)

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[25] [Green, 1963: II P. 94-5] (§6.01): "Ideo dicitur communiter qoud 'sit verum' obligat ad statum mentis, et cum triplex sit status animae, scilicet, status scientis, dubitantis et ignorantis, tripliciter fit haec obligatio, aut per verbum sciendi, aut per verbum dubitandi, aut per verbum ignorandi. Verbi gratia: sit verum te scire te currere; vel: sit verum te dubitare te currere."

[26] [Burley, 1988: §3.61].

Or the trick (*cautela* – [Green, 1963: §3.145]) can be pulled like this (§3.62):

0. *Positum*: 'You are in Rome'	Accepted
1. ' "You are in Rome" and "You are a bishop" are alike in truth-value'	Granted (irrelevant and true – they are both false)
2. 'You are a bishop'	Granted (follows from the *positum* and what was granted)

Moreover, Burley's theory is dynamic – the response can depend on the order in which propositions are proposed: e.g.,

0. *Positum*: 'You are in Rome'	Accepted
1. 'You are a bishop'	Denied (irrelevant and false)
2. ' "You are in Rome" and "You are a bishop" are alike in truth-value'	Denied (inconsistent with the *positum* and the opposite of what has been denied)

Recall that in the previous example, when proposed in the opposite order, (1) and (2) were granted. Indeed, responses can change (§3.87):

0. *Positum*: 'The king is sitting or you are running'	Accepted
1. 'The king is sitting'	Doubted (irrelevant and unknown)
2. 'You are running'	Denied (irrelevant and false)
3. 'The king is sitting'	Granted (follows from the *positum* and the opposite of what has been denied)

However, although what has been doubted can later be granted or denied, grant can never turn into denial or vice versa.

Burley emphasizes that "all responses must be for the same instant." (§3.84) For suppose at the start of the obligation, you are sitting, but having granted the irrelevant proposition 'You are sitting', you then stand up. Should you now deny 'You are sitting' ? If so, you have denied something you earlier granted, and so you have responded badly. But if you grant it, you may have granted something irrelevant and false, and again you have responded badly. Burley's answer is that you should grant it, even though it is now false, for it was true when you granted it, and "all responses must be for the same instant."

It was usual to take the instant to be the start of the *obligation*. Suppose we call the instant A (§3.82):

0. *Positum*: 'The Antichrist exists'	Accepted
1. 'The Antichrist exists at A'	Denied (the Antichrist exists only in the future)
2. 'It is A'	Denied (inconsistent with the *positum* and the opposite of what has been denied)

However, although we must deny that it is A, that does not mean that 'It is A' is false. We are often obliged to grant falsehoods and deny truths.

Pragmatic inconsistency results from the following obligational sophism (§3.17):

0. *Positum*: 'Nothing is posited to you'	Accepted
1. 'Everything that follows from the *positum* must be granted'	Granted (it's a rule)
2. 'Something follows from the *positum*'	Granted (follows from what has been granted)
3. 'Something was posited to you'	???
4. *Cedat tempus*	

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If you grant it, you grant the opposite of the *positum*, so you respond badly. If you deny it, you deny something that follows, so again you respond badly. Burley says (§3.19) that step 1 should be denied: the rule is that *if* something follows from the *positum* it should be granted.

6. The Oxford Calculators

Many writers disliked the dynamic nature of Burley's theory and the fact that responses could change, including Richard Kilvington, one of the Oxford Calculators. Kilvington was the son of a priest from the diocese of York (probably in Kilvington, near Thirsk). He was Master of Arts in 1324-5, probably at Oriel College, and wrote his *Sophismata* in the mid-1320s. He was Doctor of Theology by 1335, and also a member of Richard de Bury's household and active in circle of Edward III. Dean of St Paul's Cathedral, 1354, he died in a second phase of the Black Death in 1361.

The Oxford (or Mertonian) Calculators were a group of mathematical physicists, many working at Merton College, from the early 1320s until the height of the Black Death in 1348-9. Their main interest was natural philosophy, but treated in the context of logical disputations: beginning, ceasing, motion, velocity, calculation, infinity, continuity.

Kilvington's *Sophismata* is replete with the terminology of disputations, in particular, obligational terms (grant, deny, doubt). Take his 47th sophism. First, he shows that you know that the king is seated:[27]

0. *Positum*: If the king is seated, you know that the king is seated, and if the king is not seated, you know that the king is not seated	Accepted (possible)
1. Either you know he is seated or you know he is not	Granted (follows from the *positum* given Excluded Middle)
2. You know that the king is not seated	Denied (irrelevant and false)
3. You know that the king is seated	Granted (follows from 1 and the opposite of 2)

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[27] [Kretzmann and Kretzmann, 1990b: S 47 c].

Then, from the same *positum*, he shows that you do not know that the king is seated (S47 d):

1´. Either you know he is seated or you know he is not	Granted (as before)
2´. You know that the king is seated	Denied (irrelevant and false)
3´. You know that the king is not seated	Granted (follows from 1´ and the opposite of 2´)
4´. You do not know that the king is seated	Granted (follows from 3´)

We appear to have contradicted ourselves.

Kilvington considers three different responses to the sophism. The third response seems to consist in refusing to accept the *positio*. Kilvington rejects this response. The second response is Kilvington's preferred response – more below. Before he comes to that, Kilvington notes that in the second stage of the proof, at line 2´, we denied what we had already granted at line 3 in the first proof. So we should then have granted 'You know the king is seated' as *pertinens sequens*, and the second proof would have failed. Hence we should grant the sophism (i.e., 'You know the king is seated': S47 e):

0. *Positum*: 'If the king is seated, you know that the king is seated, and if the king is not seated, you know that the king is not seated'	Accepted
1. 'Either you know he is seated or you know he is not'	Granted
2. 'You know that the king is not seated'	Denied (irrelevant and false)
3. 'You know that the king is seated'	Granted (follows from 1 and the opposite of 2)
4. 'You do not know that the king is seated'	Denied (incompatible with 3)

Kilvington rejects this response: if we had given the second proof first, he says, this response would then instruct us to deny the sophism (S47 f):

0. *Positum*: 'If the king is seated, you know that the king is seated, and if the king is not seated, you know that the king is not seated'	Accepted
1. 'Either you know he is seated or you know he is not'	Granted
2. 'You know that the king is seated'	Denied (irrelevant and false)
3. 'You know that the king is not seated'	Granted (follows from 1 and the opposite of 2)
4. 'You know that the king is seated'	Denied (incompatible with 3)

In Kilvington's solution to this sophism, he revises what he calls the "common usage" of the notion of irrelevant proposition (*loquendo de impertinenti ut communiter sumitur*). He focuses on Burley's "trick" for making the respondent grant any other false compatible proposition (S47 q):[28]

"Thus I say as regards a familiar example, ... that when 'You are in Rome' has been posited, it is not the case that anything false that is compatible with it can be proved, such as 'You are a bishop' and the like. And the reason is that once it is posited that you are in Rome, you would not grant this: '"You are in Rome" and "You are a bishop" are alike [in truth-value]' unless you were a bishop."

That is, according to Mikko Yrjönsuuri's interpretation,[29] Kilvington proposes that one should respond to irrelevant propositions not by reference to their actual truth-value (as far as we know it), but to what their truth-value would be if the *positum* were true. The upshot is that how one responds does

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[28] Cf. [Kretzmann and Kretzmann, 1990a]: "Unde dico – in communi exemplo, ut facilius intelligatur – quod posita ista 'Tu es Romae', non contingit probare aliquod falsum sibi compossibile, ut istam: 'Tu es episcopus', et similia. Et causa est quia posito quod tu esses Romae, non concederes istam '"Tu es Romae" et "Tu es episcopus" sunt similia' nisi fores episcopus."

[29] [Yrjönsuuri, 1994: 121].

not depend on intervening responses, and so not on their order, but only on the *positum*.

Kretzmann and Spade's theory of obligations as experiments in counterfactual reasoning has some plausibility in describing Kilvington's theory. Nonetheless, I think it is certainly wrong as a general account of obligations in other authors, such as Burley. Moreover, I think it is also a misunderstanding of Kilvington's approach. What Kilvington is doing is to use the practice of obligations as a method of proof and disproof in sophisms, as noted by Stump. The theory of obligations, as we find it in Burley (the *responsio antiqua*) is unsuited for this task. For example, Kilvington is fond of a type of reasoning called by Yrjönsuuri [Yrjönsuuri, 1994: 130] (following Kretzmann) "the disputational meta-argument":

This inference is (known to be) valid

The premises are in doubt

So the conclusion cannot be denied.

For if one denies the conclusion of a valid argument (which one knows to be valid), one must deny at least one of the premises, so the premises (as a whole) cannot be in doubt. But this is inconsistent with Burley's rules, as we saw where the Respondent can be led to deny something he had earlier doubted. We have no record of Kilvington's theory of obligations other than what we can infer from the rather oblique remarks he makes in the final pages of the *Sophismata*. But Kilvington uses this disputational meta-argument again and again throughout Sophisms 45-48, for example (S47 i):

0. *Positum*: 'If the king is seated, you know that the king is seated'	Accepted
1. 'The king is seated'	Doubted
2. 'You know that the king is seated'	???

According to Burley's rules, 2 should be denied as irrelevant and false (for although it follows from 0 and 1, 1 was not granted). However, the disputational meta-argument says 2 should not be denied, since it follows validly from 0 and 1, and 1 has been doubted. Kilvington writes (S47 i):[30]

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[30] My own translation. Cf. [Kretzmann and Kretzmann, 1990a]: "Si immediate post positionem casus proponeretur ista 'Rex sedet', ista foret dubitanda a te. Igitur cum haec propositio 'Tu scis regem sedere' sit consequens ad istam ut nunc 'Rex sedet' per casum, igitur si pro eodem instanti proponeretur tibi haec propositio 'Tu scis regem sedere', ista non foret a te

"If immediately after the positing of the hypothesis, 'The king is seated' were proposed, it should be doubted by you. Then, since 'You know that the king is seated' is a consequence, by the hypothesis, of 'The king is seated', therefore, if 'You know that the king is seated' is proposed to you for the same instant, it should not be denied. For otherwise it would follow that for some instant of the response there would be a good consequence and the antecedent should be doubted and the consequent denied, which is clearly not consistent."

Kilvington's solution is that we should express doubt about 'You know the king is seated' as well as about 'The king is seated'. He explains this by distinguishing two senses of 'doubt'. Note that we can clearly contrast 'granted' (*concedenda*) with (known to be) 'true' (*vera/scita*), and 'denied' (*neganda*) with (known to be) 'false' (*falsa/nescita*). Kilvington is pointing to similar contrast between *dubitanda* and *dubia*: just because I am obliged to express doubt about something doesn't mean I do actually doubt it. Hence, one cannot infer from an obligation to express doubt about something that one doesn't know it (S47 dd):[31]

"It does not follow that because the proposition 'The king is seated' should be doubted by me (*a me dubitanda*) that the proposition is in doubt for me (*mihi dubia*) … Nor does it follow that because this proposition should be doubted by me that it is not known by me."

In the present case, 'You know the king is seated' should be doubted (*dubitanda*), that is, one should express doubt about it, even though you know the king is seated (if he is). Just as one often has to grant a proposition which one knows to be false (or about which one is in doubt), or deny one that one knows to be true (or again, about which one is in doubt), so too one may have to express doubt about a proposition that one knows to be true (or to be false). He writes (S47 dd-ee):[32]

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neganda. Consequentiam probo; quia aliter sequeretur quod pro aliquo instanti responsionis aliqua consequentia foret bona, et antecedens foret dubitandum et consequens negandum – quod non apparet conveniens."

[31] Cf. [Kretzmann and Kretzmann, 1990a]: "Nec sequitur 'Haec proposito "Rex sedet" est a me dubitanda; igitur haec proposito "Rex sedet" est mihi dubia' ... non sequitur 'Haec propositio est dubitanda a me; igitur haec propositio non est scita a me'." Stump appears to misunderstand Kilvington's point when she writes [Stump, 1989: 223]: "The point of S47 is to justify the apparently paradoxical claim ... that one may doubt and know the same proposition."

[32] Cf. [Kretzmann and Kretzmann, 1990a]: "Quia propositio est dubitanda in casu quando scitur, et ideo est dubitanda aliquando quando nescitur a me utrum sciatur ... Ad argumentum concedendum est quod tu scis regem sedere vel tu scis regem non sedere. Sed minor coas-

"For [sometimes] a proposition must be doubted when it is known, and sometimes it must be doubted when it is not known whether it is known ...To the argument, it must be granted that you know that the king is sitting or you know that the king is not sitting. But the minor premise that was joined to it, namely, 'You do not know that the king is not seated', must be doubted. For if the king is seated, you do not know that the king is not seated (by the hypothesis), and the antecedent ['The king is seated'] must be doubted, so the consequent must be doubted too."

William Heytesbury rejected Kilvington's revision of the obligational rules. Heytesbury was another of the Oxford Calculators. Fellow of Merton College by 1330, he composed his *Rules for Solving Sophisms* 1335. During the 1330s he also wrote a treatise on *On Compounded and Divided Senses*, a collection of *Sophismata*, another called *Sophismata Asinina* where each sophism is tailored to establish the paradoxical conclusion, 'You are an ass', and the pedagogical treatise on consequences, *Iuxta Hunc Textum*. He was Doctor of Theology by 1348 and Chancellor of the University of Oxford from 1352 till perhaps 1354 and again from 1370-2. He died in 1372-3.

At the end of his treatise 'On Compounded and Divided Senses', Heytesbury considers the now-familiar example:[33]

"It often happens ... that a proposition is altogether irrelevant when proposed in the first place but relevant enough in the second and third place ... For example, let this disjunctive proposition be posited: 'The king is seated or you are in Rome.' If the proposition 'The king is seated' is then proposed, it must be doubted since it is doubtful and irrelevant. Let the proposition 'You are in Rome' be proposed then. Since it is false and irrelevant, it must be denied. And if 'The king is seated' is then proposed again, it must be granted; for it follows from the *positum* together with the opposite of [a proposition] that was correctly denied."

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sumpta est dubitanda – scilicet, haec: 'Tu non scis regem non sedere'. Quia si rex sedet, tu non scis regem non sedere – per casum – et antecedens est dubitandum; igitur consequens est dubitandum."

[33] [Kretzmann and Stump,1988: 432-4]. Cf. [Heytesbury, 1494: f. 4rb]: "Accidit tamen frequenter quod propositio primo loco posita est impertinens omnino: sed in secundo et tertio loco satis pertinens est ... Verbi gratia: ponatur ista disiunctiva. rex sedet vel tu es rome: tunc si proponatur illa. rex sedet: cum ipsa sit dubitanda et impertinens debet dubitari. Deinde proponatur illa. tu es rome. cum ipsa sit falsa et impertinens debet negari. et tunc si proponatur iterum ista. rex sedet. debet concedi cum ipsa sit sequens ex posito cum opposito bene negati."

Again, in ch. 2 of his *Regulae*, 'De Scire et Dubitare' ('On "Know" and "Doubt"'), he addresses Kilvington's revision directly. He presents the following argument in the mouth of an opponent:[34]

> "For you could not then correctly deny that you know the king to be in London, because, according to you, this follows: 'The king is in London, and you believe unhesitatingly that the king is in London; therefore, you know that the king is in London.' The major premise is in doubt for you, and the minor is the hypothesis; therefore, the consequent is not to be denied by you."

This is an instance of Kilvington's meta-argument:

0. *Positum*: 'You believe unhesitatingly that king is in London	Accepted
1. 'The king is in London'	Doubted
2. 'You know that the king is in London'	???

Heytesbury's response is that it must be denied since it is false and irrelevant. Thus Heytesbury explicitly rejects Kilvington's revision.

7. The Closure Principle

In a MS held in the Bodleian,[35] we find a short treatise entitled '*Casus Obligationis*' attributed to Heytesbury. It presents five *casus* corresponding to five of the seven in 'De Scire et Dubitare'. They correspond to arguments 2, 4, 5, 6 and 7 in 'De Scire'. The first and longest argument in 'De Scire' does not appear, where, as we have seen, Heytesbury directly tackles Kilvington's proposed revision to the response to irrelevant propositions, and rejects his disputational meta-argument. Arguments 2, 5 and 7 are versions of the Hooded Man,[36] and are diagnosed as confounding the compounded sense with the

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[34] [Kretzmann and Stump, 1988: 447]. Cf. [Heytesbury, 1494: f. 13vb]: "Non enim poteris tunc bene negare quod tu scis regem esse londonis, quia sequitur per te: rex est londonis et credis absque hesitatione quod rex est londonis, igitur scis quod rex est londonis. Maior est tibi dubia et minor est casus, igitur consequens non est a te negandum."

[35] Oxford Bodleian MS Canon. lat. 278, f. 70ʳ.

[36] See Aristotle, *De Sophisticis Elenchis* ch. 24 (179b1-4): "In the case of the man approaching, or the hooded man, 'to be approaching' is not the same as 'to be Coriscus', so that

divided sense. For example, in 5, the last case, even if there is nothing you doubt to be human (the divided sense), one can still doubt 'This is human' (the compounded sense). In argument 6, the *casus* is rejected as contradictory. Finally, we come to the third argument, argument 4, which seems to turn on the closure of signification under consequence. The reasoning is much clearer in the shorter version, where Heytesbury is seen to reject the suggestion that signification is closed under consequence.[37]

He writes:[38]

"I admit all of [the objection] up to the claim that I know the proposition 'This is Socrates' signifies precisely that this is Socrates or that this is Plato. I deny that. Nor does it signify in this way primarily and principally. Instead, it primarily and principally signifies that this is Socrates."

That this is Socrates or Plato does indeed follow from this being Socrates. But Heytesbury refuses to accept that 'This is Socrates' signifies, at least primarily and principally, that this is Socrates or Plato.

That signification is closed under consequence, and that Upward T- Inference must accordingly be qualified, was the basis of Thomas Bradwardine's iconoclastic solution to the insolubles in the early 1320s.[39] Heytesbury distinguishes a proposition's signifying "as its words usually suggest" ("*sicut verba illius communiter pretendunt*")[40] and its signifying such and such precisely, and followed Bradwardine in claiming that no proposition can precisely signify its own falsehood.[41] Suppose Socrates says 'Socrates says what is false' and nothing else. 'Socrates says what is false' certainly signifies that Socrates says what is false as the words usually suggest, primarily and principally. Yet it can't signify only that, but must signify more. Casting his account of insol-

suppose I know Coriscus, but do not know the man who is approaching, it still isn't the case that I both know and do not know the same man."

[37] This *casus* (or sophism) is discussed by Spencer Johnston in his paper '"This is Socrates": a Mertonian sophism about signification' in this volume.

[38] [Kretzmann and Stump, 1988: 460]. Cf. [Heytesbury, 1494: f. 15rb] regulae: "Admitto totum usque ad hoc quod dicitur quod scio quod haec propositio hoc est sortes significat precise quod hoc est sortes vel quod hoc est plato. Illam nego, nec illa primo et principaliter sic precise significat. Sed primo et principaliter significat quod hoc est sortes."

[39] See [Bradwardine, 2010] and [Read, 2002]. Upward T-Inference is the principle that a proposition is true if things are as it says they are, so called by [Maudlin, 2004: 112].

[40] [Heytesbury, 1494: f. 6ra]. Spade's translation reads: "as its words commonly pretend": [Heytesbury, 1979: §§44-45, P. 46].

[41] See [Bradwardine, 2010: ad A.4.3].

ubles in the language of obligations, Heytesbury claims that one is under no obligation to say what it signifies precisely:[42]

> "[Assume] that Socrates says only 'Socrates says what is false' ... If someone asks under this *casus* what the proposition uttered in this way by Socrates signified other than that Socrates is saying what is false, I say to him that the Respondent does not have to solve or to give his determination for that question. For from the *casus* it follows that the proposition signifies otherwise than that Socrates is saying what is false, but the *casus* does not specify what that is; hence, the Respondent does not have to give any further determination for that question."

Bradwardine was clear what else insolubles signified, namely, their own truth. Heytesbury, writing some ten years later, is more cautious, and refuses to be drawn on what else insolubles signify. Nonetheless, he agrees, they cannot signify precisely what the words usually suggest. Robert Fland, writing some years after Heytesbury, contrasts these two approaches:[43]

> "There are two theories [of insolubles] which claim that an insoluble signifies other than things are. The first claims that an insoluble signifies other than things are and specifies what its secondary signification is. The second theory claims that an insoluble signifies other than things are (but) the respondent does not have to specify what it signifies that is other than things are."

Fland thinks these two theories are equally good and both better than any other response to the insolubles (§15). The first view (clearly Bradwardine's) says that insolubles are implicitly conjunctive, signifying in addition to their primary and principal signification their own truth. The second view (clearly Heytesbury's) says that, faced by, e.g., the proposition 'A is not true', which is itself A, one need not specify what A signifies other than that A is not true, although one must grant that it does not signify precisely that A is not true. It must signify more than that, and everything it signifies must obtain in order

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[42] [Heytesbury, 1979: §§50, 51]. Cf. [Heytesbury, 1494: f. 6va]: "Supposito quod Sortes solummodo dicat illam propositionem: sortes dicit falsum ... Si autem queratur in isto casu quid significavit ista propositio sic dicta a sorte aliter quam quod sortes dicit falsum: huic dicitur quod respondens non habet istam questionem solvere seu determinare: quia ex casu sequitur quod illa propositio aliter significat quam quod sortes dicit falsum: sed casus non certificat quid illud sit ideo non habet respondens quesitum illud ulterius determinare."

[43] [Spade, 1978: §8, P. 63]: "Unde duae sunt positiones quae ponunt quod insolubile significat aliter quam est. Prima ponit positio quod insolubile significat aliter quam est et certificat quae est sua significatio secundaria. Secunda positio ponit quod insolubile significat aliter quam est; respondens non habet certificare quid significat aliter quam est."

for it to be true. Nonetheless, Heytesbury refuses to follow Bradwardine in using the closure principle to discover what more it signifies, that is, what it signifies precisely or secondarily.

8. The *Responsio Nova*

In his treatise on 'Obligations', Fland tells us that there was a different way of responding to obligations, a "new response", the "*responsio nova*."[44] He gives an example:

0. *Positum*: 'Every man is running'	Accepted
1. 'Every man is running'	Granted (the *positum*)
2. 'You are a man'	Granted (irrelevant and true)
3. 'You are running'	Denied (false and irrelevant)

Why is 'You are running' irrelevant? Not because it does not follow from the *positum* and what has been granted. He says it does. But he denies that the conjunction of (1) and (2) should be granted, that is, one can deny a conjunction both of whose conjuncts have been granted:[45]

"This [new] response puts forward these two rules. The first is: A conjunction may be denied each of whose parts should be granted. The second is that a disjunction may be granted each of whose parts should be denied."

Jennifer Ashworth [1986] shows that the author of the *responsio nova* was Roger Swyneshed. Indeed, Paul of Venice plays on the name, speaking of *oppinionem illorum quos porcinos vocat* ("the opinion of those whom he [the master he is criticizing] calls 'swinish')."[46]

Roger Swyneshed (or Suisset) is not to be confused with the better-known Merton Calculator, Richard Swyneshed (or Swineshead). Roger Swyneshed studied at Oxford under Thomas Bradwardine and Richard Kilvington. He wrote treatises on *Insolubles* and *Obligations* between 1330 and 1335 (and

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[44] [Spade, 1980: §14, P. 45]: "*Est tamen una alia responsio quasi nova ...* "

[45] [Spade, 1980: §17]: "Une illa responsio ponit tales duas regulas. Prima est: Utraque pars copulativae est concedenda, quae copulativa est neganda. Secunda est quod disjunctiva est concedenda cujus utraque pars est neganda."

[46] [Paul of Venice, 1988: 323].

also a treatise on *Consequences* now apparently lost). He was the author of *Descriptiones motuum* (or *De motibus naturalibus*), a treatise on natural changes, including locomotion. Subsequently, he became Master of Theology (though his *Sentences*-lectures seem not to have survived). He too was a member of Richard de Bury's circle, and a Benedictine monk of Glastonbury. He died about 1365. The following touching epigram has been preserved:

Subtle Swyneshed, denizen of Glastonbury,

Indeed a monk of fond memory,

Whose fame of industry has not perished,

Suffered the poor to live in peace.[47]

In his treatise on *Obligationes*, Swyneshed presents the two striking theses mentioned by Fland:[48]

"Having granted the parts of a conjunction, the conjunction need not be granted, nor having granted a disjunction, need either of its parts be granted."

Yet in an obligational disputation, one must normally grant whatever follows from what has already been granted. What is Swyneshed's new theory of obligations? Why does Swyneshed offer a new theory? Is Swyneshed's theory a logical heresy?

Fland's central example continues like this:[49]

0. *Positum*: 'Every man is running'	Accepted
1. 'Every man is running'	Granted (the *positum*)
2. 'You are a man'	Granted (irrelevant and true)
3. 'You are running'	Denied (irrelevant and false)
4. 'Every man is running and you are a man, so you are running'	Granted (since it is valid)
5. 'Every man is running and you are a man'	Denied (irrelevant and false)

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[47] From Richard Trevytlam OFM, *De laude universitatis Oxoniae*, in [Weisheipl, 1964]: "Subtilis Swynyshed, proles Glastoniae, / Revera monachus bonae memoriae, / Cuius non periit fama industriae,/ Sinebat pauperes in pace vivere."

[48] [Spade, 1977: 257]: "Propter concessionem partium copulativae non est copulativa concedenda nec propter concessionem disjunctivae est aliqua pars ejus concedenda."

[49] [Spade, 1980: §14 P. 45].

6. 'Not every man is running or you are not a man'	Granted (equivalent to the opposite of (5))
7. 'Not every man is running or you are not a man, but you are a man, so not every man is running'	Granted (since it is valid)
8. 'Not every man is running or you are not a man, and you are a man'	Denied (inconsistent with the *positum*)

(1), (2) and (5) prove Swyneshed's first thesis, and (6) his second thesis.

Swyneshed's *responsio nova* differs from Burley's *responsio antiqua* in several respects. First, Swyneshed, and the *nova responsio* in general, recognises only *positio*, *impositio* and *depositio*. (As we saw, *petitio* and *sit verum* can be subsumed under *positio*.) Next, Swyneshed makes a sharp distinction between the *positio* and the *positum* (and in general, between the *obligatio* and the *obligatum*). Swyneshed also characterizes possible *positio* differently from Burley. Most importantly, he characterizes "relevance" differently. Moreover, responses to irrelevant propositions need not be for the same instant, but only for the present. Finally, he characterizes success and failure (winning and losing) differently.

According to Swyneshed's rules for *positio*, the *positum* should be accepted only if it is contingent, that is, if responses to it outside the obligation would change as the facts change. If a proposition follows from or is inconsistent with the *positum* (regardless of what has subsequently been granted or denied), it is said to be "relevant" (*pertinens*), otherwise "irrelevant" (*impertinens*). If it is relevant, it is "obligated" and should be granted if it follows, denied if it is inconsistent, and if it's irrelevant, is not obligated and (in accord with the *casus* and how things are at that instant) should be granted if (known to be) true, provided that is not inconsistent with the *positio* (the *obligatio*), denied if (known to be) false, provided that is not inconsistent with the *positio* (the *obligatio*), and doubted if it is not known whether it is true or false. The obligation ends when either the Respondent grants and denies the same proposition (unless it is irrelevant), or when the Opponent says '*cedat tempus*'.

The *responsio nova* deals with many of the apparent problems that we noted with Burley's theory. Not every false proposition (compatible with the *positum*) need be granted: the "tricks" (*cautelae*) introduced by Burley no longer work, since they only require the false proposition to be granted be-

cause it follows from the *positum* in conjunction with a true irrelevant proposition which has been granted. Responses change only when the facts change: relevance is determined only by the *positum* and not by any irrelevant propositions subsequently proposed. Order does not affect responses: again, since relevance only looks back to the *positum*, it cannot depend on any subsequent responses or their order. Finally, the pragmatic inconsistency introduced by *posita* such as 'Nothing is posited to you' is excluded by treating them as irrelevant and evaluating them as if the *positio* never was.

Jennifer Ashworth [Ashworth, 1986] showed that each *responsio* had strong support in subsequent decades. The *responsio antiqua* was maintained by Ralph Strode, Albert of Saxony, John Wyclif, Richard Brinkley, William Buser, Marsilius of Inghen, John of Holland, Peter of Mantua, Peter of Candia, and Paul of Venice; while the *responsio nova* was advocated by Robert Fland, Martinus Anglicus, the anonymous authors of the *Tredecim questiones*, a *Commentary on Marsilius*, the *Tres sunt modi*, another *Obligationes* treatise, the *secundum usum Oxonie*, and Richard Lavenham. The majority of surviving treatises reject Swyneshed's innovation. But his ideas still influenced those who rejected it.

9. Heresy or Orthodoxy?

How heretical is Swyneshed's logic? Can a conjunction be false even though both its conjuncts are true? No: that is to confuse granting with being true, denial with being false: the Respondent may be obliged to grant a proposition which is false (e.g., the *positum*). He may be obliged to deny a proposition which is true (e.g., if it is incompatible with the *positum*). He may even be obliged to doubt a proposition (i.e., to say 'I doubt it') which he knows to be true or false, as we saw in § 6 above.

Mikko Yrjönsuuri [1993, P. 317] suggested a book-keeping metaphor to explain the logic involved in Swyneshed's *nova responsio*. Catarina Dutilh Novaes [2006, P. 137] formalized Yrjönsuuri's account. Let P^+ represent what is relevant and follows from the *positum* (*pertinens sequens*); let P^- represent what is relevant and inconsistent with the *positum* (*pertinens repugnans*); and let I represent what is irrelevant (*impertinens*). She sets out the tables for conjunction and disjunction as follows:

φ_n	P^+	P^+	P^+	P^-	P^-	I
φ_m	P^+	P^-	I	P^-	I	I
$\varphi_n \wedge \varphi_m$	P^+	P^-	I	P^-	P^-	I
$\varphi_n \vee \varphi_m$	P^+	P^+	P^+	P^-	I	I

Representing these as 3x3-matrices with some re-ordering, we obtain Kleene's strong matrices:[50]

$\vee$	P^+	I	P^-
P^+	P^+	P^+	P^+
I	P^+	I	I
P^-	P^+	I	P^-

$\wedge$	P^+	I	P^-
P^+	P^+	I	P^-
I	I	I	P^-
P^-	P^-	P^-	P^-

Thus a conjunction can be irrelevant, and so denied (when known to be false), although its conjuncts are, respectively, *pertinens sequens* (hence granted, though known to be false) and *impertinens* (irrelevant, hence granted, since known to be true). So Swyneshed's logic is thoroughly orthodox, as are Kleene's matrices. What Kleene was doing was showing how to combine partial information, the output of partial functions. If we know a conjunct is false, we already know that the conjunction is false, but if one conjunct is unknown, the other true or unknown, the value of the conjunction is open. Kleene's matrices are regular in that, in the light of more information, no fixed value (true or false) will change, though indeterminacy may be determined one way or the other. Swyneshed's analysis shares these characteristics. Irrelevant conjuncts may be granted or denied in the light of external information, but that does not affect the response to the conjunction, which will also be irrelevant and so determined in the same way. But when the conjuncts are relevant, the conjunction will be denied if any conjunct should be denied, granted only if both are granted.

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[50] See [Kleene, 1952: § 64].

10. Conclusion

The function of obligational disputations was to test students' ability to handle logical inferences – to use logic in practice. This interpretation is supported by the very few texts which describe their function. It is not surprising that we have no record of any actual disputation: one doesn't need to engage in these often short exchanges; just thinking about them trains one to think logically. Swyneshed's *responsio nova* seems radical and iconoclastic, in, e.g., denying conjunctions both of whose conjuncts have been granted. But it is important to distinguish 'true' from 'granted', 'false' from 'denied', and 'doubtful' from 'doubted', and when we do so, Swyneshed's theory is thoroughly mainstream. Examination of the subtleties of obligational disputation shows that it does inculcate close attention to logical relationships. We see this practical training preserved in the use of obligational terminology in other logical treatises, e.g., on insolubles. Obligational disputations show logic at its most direct and realistic – as a practical discipline.

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Предметом исследования в статье является софизм, основанный на пропозиции «Это – Сократ». Он содержится в небольшом трактате о предписаниях, принадлежащем, как считается, Уильяму Гейтсбери. Прежде всего исследуется предыстория и контекст традиционной версии диспутов с предписаниями (*responsio antiqua*) у Вальтера Бурлея, затем анализируются возражения и поправки, предложенные Ричардом Килвингтоном и Роджером Суайнсхедом. Характеризуются все шесть видов предписаний, описанных Бурлеем, включая *sit verum*, т.е. тот вид, который послужил основой для софизма. Килвингтону и Суайнсхеду претила динамичность *responsio antiqua*, поэтому Килвингтон решил пересмотреть правила, в соответствии с которыми высказывание могло быть признано нерелевантным. Это позволило ему использовать такой способ построения рассуждения который был несовместим с правилами Бурлея, – «диспутативный метааргумент». Гейтсбери не принял реформу Килвингтона и предполагаемый ею метааргумент. Суайнсхед также подверг пересмотру предложенную Бурлеем версию диспутов с предписаниями, сформулировав так называемую *responsio nova*, исходным пунктом которой является кажущееся парадоксальным положение: не следует принимать конъюнкцию, две части которой мы уже приняли. При ближайшем рассмотрении, однако, версия Суайнсхеда оказывается не столь радикальной, как казалось сначала.

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Диспуты с предписаниями, софизмы и инсолюблии

(*на английском языке*)