RECONSTRUCTION IN PHILOSOPHY OF MATHEMATICS

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Throughout his work, John Dewey seeks to emancipate philosophical reflection from the influence of the classical tradition he traces back to Plato and Aristotle. For Dewey, this tradition rests upon a conception of knowledge based on the separation between theory and practice, which is incompatible with the structure of scientific inquiry. Philosophical work can make progress only if it is freed from its traditional heritage, i.e. only if it undergoes reconstruction. In this study I show that implicit appeals to the classical tradition shape prominent debates in philosophy of mathematics, and I initiate a project of reconstruction within this field.

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1. Introduction

In recent years, a renewed attention has been paid to John Dewey's logical works, notably LW12, as a significant resource for current philosophy of science[1]. It has been perceived that widely debated issues concerning realism or the truth of scientific theories can be fruitfully re-examined along the lines suggested by Dewey. It has not been so far suggested, however, that a systematic reconstruction of current philosophical debates on the basis of Dewey's logic is possible and desirable and that it will have to encompass philosophy of mathematics as well as philosophy of science.

My goal in this study is to initiate a project of reconstruction in philosophy of mathematics by outlining its initial steps with respect to a class of contemporary debates. I offer an explication of the reason why the task of reconstruction is needed and worthwhile, as well as an indication of the manner in which it should proceed. In doing so, I hope to offer concrete proof of the effectiveness of Dewey's ideas when adopted critically to investigate specific issues in current philosophy. Although my discussion is largely self-contained, it is assumed to take place within the framework of LW12[2].

2. The task of reconstruction

John Dewey's motivation for reconstruction in philosophy stems from what he regards as a proliferation of puzzling questions within this discipline, whose distinctive feature is that they prove insoluble by the manner in which they have been set up. Reconstruction is called for because philosophical work has to be reorganised in such a way that it can escape artificial problems and, thus, irrelevance. In order for reconstruction to be possible, the source of artificial problems has to be identified.

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[1] See in particular Brown (2012) and Godfrey-Smith (2002, 2010).

[2] In particular, I work with the account of propositions offered in Chapter 15 of LW12 and with the account of mathematical discourse offered in Chapter 20 of the same text.

Dewey traces the source back to a deeply ingrained contradiction pervading modern philosophical thought. The poles of this contradiction are an attachment to a traditional, pre-modern theory of knowledge as apprehension of ultimate, immutable Being on the one hand, and the acknowledgment of the significance of scientific inquiry on the other hand. A contradiction arises because, briefly put, modern scientific inquiry owes its effectiveness to a manner of acquiring knowledge that is at variance with the pre-modern conception. The latter, whose original, systematic expression Dewey finds in Plato and Aristotle[3], requires drawing a sharp ontological divide between what is precarious and subject to change on the one hand, and what is absolutely invariable and exempt from modification on the other. Only the latter is recognised as the proper object of knowledge. To know is then to apprehend or assimilate an antecedently given reality that is ultimate and self-sufficient[4]. Knowledge so conceived issues only in the internal modification of the knowing agent[5], leaving ultimate reality unchanged. It follows that the aim of knowledge is to get hold of the unblemished picture of immutable Being or to identify oneself with its synthetic unity. Ultimate reality can, in other words, be an object of aspiration and contemplative attention but not a partner in any transactions.

By contrast, Dewey characterises modern scientific inquiry as a problem-solving activity that involves an enquirer and her surroundings in controlled processes of change. Its import is revealed by the consequences it can settle through the intelligent management of given existential conditions[6]. Scientific theories or propositions take part in this enterprise as instruments of intelligent management, as opposed to representations of fundamental realities or highest Being. The pre-modern conception of knowledge has little to do with this picture. Despite this, it has exercised a persisting influence on the manner in which philosophical reflection problematised the materials

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[3] See e.g. MW12: 140-143 and LW4: 13.

[4] As pointed out in e.g. LW4: 12.

[5] See LW4: 17 and LW12: 161.

[6] For Dewey's discussion of the structure of inquiry, see, in particular, Chapter 6 of LW12.

of scientific inquiry. Such influence has led to the construction of several artificial problems[7].

For instance, the centrality of the concepts of particle and force in XVII century natural philosophy could be interpreted, along pre-modern lines, as the discovery that reality is fundamentally a system of mechanical interactions between material bodies. Once materialistic metaphysics had pinned down the essential characters of reality, the presence of affectional and volitional objects in ordinary human dealings could be taken as a perplexing problem, capable of animating indefinitely protracted disputes[8]. The ensuing dialectic, disengaged as it was from the practice of specific, limited enquiries, could not satisfy any expectation for a definite outcome.

Whenever philosophical reflection integrates a pre-modern conception of knowledge into the analysis of materials belonging to scientific inquiry, similar predicaments arise. Aspects of inquiry are exploited as cues to metaphysical conundrums that cannot be resolved, while they implicitly lead, among other things, to a dismissal of independent analytical efforts directed towards a better understanding of scientific practice and its liberation from metaphysical dogmatism. Dewey calls for reconstruction under these circumstances. His goal is to take leave of metaphysical disputes irrelevant to inquiry and replace this activity with the practice of inquiry itself. For this to be possible, a preliminary critical work is needed, which identifies the prepossessions animating existing philosophical debates and shows that their plausibility depends on neglect or misrepresentation of the context of inquiry itself.

Prominent debates in philosophy of mathematics call for reconstruction in Dewey's sense, animated as they are by the pre-modern conception of knowledge. This study is mainly devoted to showing that this is the case and to providing a definite orientation for reconstructive work. The critical analysis I articulate in the following sections can easily be applied to other topics in contemporary philosophy of mathematics and in philosophy of science.

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[7] See LW1: 107-114.

[8] See LW4: 33 and LW1: 110.

3. The indispensability argument

A matter of continued concern in contemporary philosophy of mathematics is the ontological status of mathematical entities. The possibility of developing this concern rests on the presumption that the references to entities such as numbers, lattices, graphs et cetera, as they are encountered in mathematical statements, have existential import. If this is the case, mathematical entities are to be conceived as entities that exist apart from ordinary experience: they are not items with which daily commerce is to be had under changing conditions, but eternal realities that cannot be located in the spatio-temporal continuum within which empirical change takes place. On this view, mathematical knowledge is the apprehension of immutable mathematical realities and, as such, it provides an ineffable connection between experience and transcendence. It is clear how profoundly the pre-modern conception of knowledge discussed in the previous section is in operation here.

Philosophers who view mathematical propositions in the manner just described are mathematical realists or, as they are sometimes called, Platonists. A widely discussed attempt on the part of Platonists to establish the correctness of their position, upon which I shall focus, invokes the pervasiveness of mathematical propositions within scientific discourse as evidence for its central ontological claim.

This kind of strategy is of special interest because it relies on the ancient conception of the object of knowledge as ultimate and immutable and seeks to reconcile this view with the results of scientific practice, whose significance it acknowledges as a matter of course. I argue that, if the latter acknowledgment is serious, the ancient conception must be abandoned, because it is untenable in the light of scientific practice. With it must also go the speculative effort proposed by the Platonist as worthy of being pursued.

Platonists make the application of mathematics in empirical science serve their cause by locating its significance in the context of a particular argument, usually traced back to the writings of Quine and

Putnam[9], and known as the indispensability argument. Its canonical formulation[10] (a variant will be examined in the next section) runs as follows:

> (P1) We ought to have ontological commitment to all and only the entities that are indispensable to our current best scientific theories.
> (P2) Mathematical entities are indispensable to our best scientific theories.
> (C) We ought to have ontological commitment to mathematical entities.

The most important statement in this argument is (P2), since it mentions mathematical entities, whose existence the Platonist intends to prove, as well as their ineliminable role within scientific theories. A trivial, but necessary, remark is that mathematical propositions, rather than entities, figure in scientific theories. Thus, at best, references to mathematical entities or, more precisely, mathematical terms, may be indispensable. Although this looks like a statement of fact, it raises a crucial issue, which goes unnoticed if no attention is paid to the scientific enterprise as a form of inquiry, i.e. as an activity aiming at the resolution of problematic situations. If mathematical subject-matter is to play any useful role within inquiry, then it must serve the purpose of attacking problematic situations and supporting their resolution or reorganisation.

Once this is acknowledged, it is legitimate to ask how mathematical subject-matter can guide intervention on specific empirical problems. It appears at least doubtful that it should do so by a sudden shift of attention from the terms of the problem at hand, which are empirical, to an altogether unrelated ontological realm, in which the eternal relations of non-empirical entities are crystallised. To invoke the structural resemblance between these non-empirical entities and empirical ones in order to legitimise an appeal to the latter would be, on the one hand, to identify the stability of experimental control or methodical action with a feature of fleeting events and, on

[9] Among others, Quine (1976, 1980) and Putnam (1971).
[10] It is taken from Colyvan (2011), 49.

the other hand, to render the *ontological* appeal to extra-natural entities superfluous, if the patterns they display do have empirical realisations directly amenable to study[11].

Thus, to accept (P2), given a cursory look at the structure of inquiry is, at the very least, to adopt a conception of the successful application of mathematics that turns it into a miraculous occurrence[12], as opposed to the fruit of deliberate and focussed reflection. It is nothing short of miraculous that mathematics should be effective insofar as it conveys no information upon the terms of the problems it is invoked to resolve. It is more plausible to think that its effectiveness depends on what it can do as an instrument capable of managing information for the sake of definite purpose: this type of function does not call for a supernatural reality supporting its performance.

The last conclusion is strengthened by any explicit analysis of the functions performed by mathematical resources within scientific inquiry. Without going into detailed illustrations, it is possible to show why by means of a few remarks and a couple of brief examples. Intelligent conduct within inquiry demands deferring overt action in favour of strategic planning: for this to be possible, symbols have to be introduced, since it becomes necessary to talk about envisaged occurrences and future ways of acting, as opposed to handling given existences at once. Thus, within inquiry, the terms of a problem have to be symbolised and, once symbolised, they may be subjected to a formal treatment oriented towards the resolution of the problem itself.

It is then possible to regard references to mathematical entities as modes of treatment of the terms of a problem, i.e. ways of putting available evidential materials into a form amenable to particular trains of thought governed by mathematical propositions. For example, to assign a street network a directed graph, in the context of an application of mathematics, is to declare how streets (seen as directed

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[11] It is noteworthy that Field (1980) makes use of the correspondence between mathematical and empirical structures to mount an argument against Platonism.

[12] Platonists have not hesitated to take it in this way: see in particular Colyvan (2001).

edges or, if two-way, pairs of directed edges) and their crossings (seen as vertices) are going to be reasoned about[13]. Similarly, to assign a 3-simplex to an election involving three candidates[14] is to declare how voter preferences can be studied and classified. Examples could be multiplied at will. Items like graphs and simplices are not, in applied capacity, nouns, but adverbs: they describe selected modes of operation, not entities foreign to the problematic situation under study[15].

When this conception of terms occurring in mathematical propositions involved in scientific applications is available, (P2) loses its force. What this premiss can now convey is that, at most, mathematical terms prove strategically crucial in problem-solving because they select modes of operation that are used to develop in reasoning the terms of the problem at hand. Ontological considerations are not relevant to this process. To defend their relevance is to defend the supernatural where only natural processes are at play.

In view of this discussion, (P1) appears to be a hasty statement. There is no obligation to attach an ontological commitment to any term whatsoever that happens to enter the formulation of a scientific theory before carrying out a study of the particular functions performed by kinds of terms in inquiry. The latter study should be the primary goal of philosophical reflection, since the indispensability argument is of highly uncertain force before that study is carried out: it remains undecided what force its premisses exactly carry and whether or not they are pointing to an interesting problem. In view of the foregoing discussion, which is an immediate articulation of Dewey's ideas, it is clear that the premisses in question may seem compelling because no sufficiently thorough study of the application of mathematics as a complex of functions supporting enquiries is

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[13] This is done in models of municipal street-sweeping. See e.g. Tucker and Bodin (1976).

[14] The geometric treatment of voting alluded to is due to Donald Saari and developed e.g. in Saari (1995).

[15] Note in this connection Dewey's remark that the referents of abstract terms are modes of operating, in LW12: 350.

available. In reconstructed philosophy of mathematics, this task takes centre stage, if only as a preliminary to making well-founded assertions about the employment of mathematical resources within scientific practice.

4. The enhanced indispensability argument

The main purpose of the foregoing discussion was twofold. On the one hand, it aimed at detecting, with respect to a philosophical topic of current interest, fragments of the conception of knowledge and of the object of knowledge that prompted Dewey's call to reconstruction in philosophy. On the other hand, it aimed at showing that, since this conception of knowledge can be enforced only if the context of inquiry and its purpose are held in abeyance, a reinstatement of the latter context suffices to motivate and to direct reconstruction. Thus, the philosophical content at variance with the structure of inquiry is set aside in favour of a philosophical task directly connected with the structure of inquiry.

In the illustration of this process offered in section 3, I attempted to show that the canonical indispensability argument in philosophy of mathematics presumes for mathematical statements employed in scientific inquiry a position that must be in sharp conflict with the role they actually play in it. When this role is clarified, the initial presumption can no longer be upheld. A discussion of the indispensability argument is to be replaced by a study of the functions performed by mathematical resources within inquiry.

This outcome seems to have been partially perceived by the proponents of indispensability arguments. In particular, Alan Baker framed what has come to be known as an enhanced indispensability argument[16], motivated by a recognition that not every occurrence of mathematical terms in discourse relevant to scientific enquiries may carry an ontological commitment to transcendent mathematical realities[17]. The modification of the indispensability argument

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[16] See e.g. Saatsi (2011).

[17] Baker (2005), 224.

demanded by this recognition goes in the direction of a search for substantive employment of mathematical resources in scientific practice. Substantive, however, simply means 'unambiguously carrying ontological commitment'.

It is conjectured that, when mathematical resources are used in an explanatory capacity, substantive commitment should be guaranteed. Since, however, explanatory capacity does not, on its own, provide an automatic or dependable lead to ontological commitment, the search for `genuinely' mathematical explanations[18], as opposed to spurious ones, is in question. In this context, 'genuine' means, again, an 'unambiguously carrying ontological commitment'. Thus, if one replaces 'scientific theory' with 'genuine explanation' in the indispensability argument from section 3, one obtains an enhanced indispensability argument.

The discussion from section 2 suffices to show that enhanced indispensability arguments trigger an indefinite search for something that cannot be found, as long as one remains within the compass of ordinary scientific research, as opposed to the reaches of mystical contemplation. Insofar as the goal of indispensability arguments is to identify ontological commitment, it fundamentally differs from the goal of inquiry, which is to adopt certain symbolic instruments in order to resolve problematic situations. The idea that such instruments should promote an effective way of handling the terms of a problem precisely because they refer to something alien to it is not directly entertained by Platonists. What Platonists defend is the thought that mathematical resources prove effective and that there is no better way of interpreting mathematical statements than one taking them as pointers to supernatural realities. From the point of view of reconstruction, the latter statement does not expound a view but highlight a conflict. It is the conflict between the pre-modern view of the object of mathematical knowledge as an unchanging, self-contained reality, and the modern recognition that mathematical resources are extensively used to advance empirical investigations and thus function cooperatively within specialised activities wholly included in the natural world.

[18] Baker (2005), 233-236.

The proponents of enhanced indispensability arguments have not acknowledged the presence of this conflict because it has seemed to them clear that certain traits of mathematical treatment, notably abstractness and generality, cannot be ascribed to empirical particulars. The seemingly natural conclusion is that they must be features of abstract, mathematical objects. It is for instance argued[19] that mathematical objects ensure scope generality, in the sense that they identify patterns to which a variety of empirical instances conform, as well as topic generality, in the sense that the same mathematical entity (say, a graph-theoretical structure) can be applied to disparate situations.

These features are not distinctive of the application of mathematics and they are not to be ascribed to entities. For instance, an evacuation procedure is scope general in the sense that it identifies a pattern of interactions transferrable to distinct venues of a similar kind. Physical exercise is topic general in the sense that it applies to disparate goals, medical, agonistic or spiritual. If generality is to be of any use, it cannot pertain to entities but to activities and procedures. Reasoning itself may be one such procedure and mathematical reasoning one special form thereof. The generality of mathematical reasoning becomes the trait of an entity only when the fact that certain interactions can be liberated from particular occurrences and formulated as procedures involving generic conditions is hypostatised into the quality of an ultimate object that cannot pertain to any particular object encountered in experience.

When the adoption of mathematical means is not understood as an activity within inquiry but as a self-contained appeal to eternal truths, generality may at first look as if it could be conceived as a quality of mathematical entities foreign to empirical problems. If, however, it can be so conceived, it immediately becomes a source of perplexity, since it is disconnected from the more precarious pursuit it was intended to support. It must be brought to bear on it and there is no a priori reason safely to rest in the conviction that this can be done by clinging to an ontology that does not offer any possibility of interaction with empirical traits. Reconstruction begins with noting

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[19] Baker (2017), 200-201.

that, within the dynamics of enquiry, features peculiar to extra-natural mathematical entities cannot prove helpful in practical situations thanks to their thorough irrelevance to them. Conjectures about such inexplicably effective entities are put aside in favour of a more straightforward examination of the place occupied by mathematical propositions within problem-solving activities.

Even though enhanced indispensability arguments encourage the hypostatisation of strategies within inquiry as traits of objects foreign to all empirical inquiry, which reconstruction must undo, they have the merit of pointing to more clearly defined goals for reconstruction than canonical indispensability could do. These goals are the analysis of generality, abstraction and explanatory function in mathematised empirical inquiry.

The manner in which the latter goals are to be pursued can, to some extent, be determined contrastively, i.e., by looking at the way in which they are pursued under the controlling influence of a pre-modern conception of knowledge. Whenever philosophical work evinces attachment to such conception, it does not merely provide a misleading suggestion. As soon as it is compared against the context of inquiry, it also offers useful indications as to what information concerning the conduct of scientific practice was omitted or misrepresented and needs to be reinstated or faithfully portrayed. The act of reinstatement or rectification does not coincide with a simple dismissal of the earlier philosophical effort but with a more effective reorganisation of this effort that can shed greater light on the structure of its object, namely scientific practice.

By contrast, to neglect the task of reconstruction where it should be engaged in, is to cloud what would have been a sharper picture of scientific practice with ideas ill-suited to it. Such undesirable outcome is not merely achieved by forgetting about the context of inquiry and deploying an old-fashioned ideal of knowledge in its place, but also by selecting certain features of inquiry, which are later hypostatised and treated as metaphysical entities or metaphysical truths.

This kind of proceeding is instructively exemplified by some recent work concerning mathematical explanation, intended to characterise it independently of any preoccupations with indispensability. The characterisation of interest has been proposed by Marc Lange[20]. Its critical discussion is the subject of the next section.

5. Distinctively mathematical explanation

Marc Lange's recent account of mathematical explanation presupposes a hierarchy of laws exhibiting various levels of necessitating strength. Within this hierarchy, mathematical necessity exercises a stronger constraint on a phenomenon to which it applies than, in particular, physical necessity does[21].

The view defended by Marc Lange is that explanation has a distinctively mathematical character when it describes a configuration of empirical traits as the result of sufficiently strong, real necessitation. In the next subsection, I shall show that this view is arrived at by committing what may be called the fallacy of selective emphasis. This is the hypostatisation of a distinct element or moment of inquiry, which is first isolated as significant and then identified with ultimate reality[22]. In subsection 5.2 I shall provide further elaboration on the particular manner in which Lange commits the fallacy and offer a few remarks on the ensuing misrepresentation of scientific practice.

5.1. Explanation and inquiry

In order to provide instances of mathematical explanation, Lange must isolate certain resolved situations, with their terms identified and their import known, i.e. their consequences settled. Under these conditions, an explanatory demand is the request of a *rationale* for the consequences so settled. Lange provides more or less sophisticated examples: since the exact same ideas apply to all of them, it will suffice to discuss only the simplest one[23]. A mother seeks evenly to distribute

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[20] In Lange (2013, 2016).

[21] See Lange (2013), 505 and Lange (2016), 31.

[22] For a discussion of selective emphasis, see LW1:31-32.

[23] Lange (2013), 495 and Lange (2016), 19.

twenty-three strawberries among her three children. She then realises that twenty-three is not a multiple of three. This is regarded as a distinctively mathematical explanation of failure to allocate the fruit in the desired manner. On Lange's view, divisibility absolutely constrains the allocation of discrete units. It is in force as a constraint even if one could envisage a scenario where physical laws had been altered.

The significance of constraint, as well as its mathematical connotation, are not in question. Lange is certainly correct to emphasise them. He runs into troubles by interpreting them along metaphysical lines. To clarify this point and to identify the specific problem that affects Lange's account, some close analysis of his proposed example is required.

The mother of three, whose plight Lange discusses, faces the problem of distributing some strawberries among her children. She needs to tackle this problem intelligently. The fact that twenty-three strawberries cannot be evenly divided, when regarded as units, both restricts her allocation strategies and directs her towards a viable one. The appeal to divisibility is for her an immediate development of evidential materials in a form more suitable to the resolution of a problem that presently matters to her. This is because the mother's initial observation, spelled out in terms of divisibility, identifies a hinderance only subject to a particular way of singling out the terms of the problem: if strawberries are the units of allocation, then even allocation is not possible.

A proposition about divisibility here is a way clarifying what the successful lines of action are, by pointing out what action will be unsuccessful and by suggesting that success may be achieved by choosing the terms of the problem in such a way that divisibility no longer matters. The import of an appeal to divisibility is the partial result that, for allocation to be even, either strawberries are not to be regarded as units (slices might) or more of them should be bought, or fewer allocated or, finally, the arithmetical notion of even divisibility discarded. Since the controlling practical concern is with fair allocation, the same amount of strawberries measured in grams might be the objective of allocation. In this case, the three children may possibly receive the same amount of strawberries, but different numbers of them.

Such pedantic analysis has been gone through simply to emphasise, as forcefully as possible, that the significant content of the basic mathematical considerations in which the mother of Lange's example engages, i.e. the content that is consequential to her pursuit, is a discrimination of alternative courses of action. Discrimination includes the possibility of modifying the terms of the problem. Their initial, tentative position, under which strawberries, as opposed to e.g. slices thereof, were units of allocation, allows progress in problem resolution by pointing to an obstruction and calling for further reflection. The fact that, when strawberries are conceived as units and even allocation as allocation of these units in equal number, something cannot be done with them, is just a way of spelling out the relevance of the conceptions initially entertained to the problematic situation at hand.

Strawberries are tentatively treated as units and it emerges that something cannot be done with them if they are so treated. This impossibility is an obstacle within an envisaged or attempted transaction. It is not surprising, but crucial to bear in mind, that transactions – because they are not delusional episodes in which desire attains complete fulfilment without resistance – involve effort, frustration and suffering. These features of transactions, as they occur within inquiry, can be meaningfully isolated. Mathematical instruments may facilitate their isolation, as in the example just discussed.

When, however, this straightforward fact of inquiry is singled out and hypostatised into a metaphysical reality, i.e. law-like necessitation, the fallacy of selective emphasis is committed. Absolute reality takes the place of a salient trait of experience.

The concrete basis of Lange's account is the fact that the constraints encountered as inquiry progresses are adversities or advantages emerging in the course of purposeful interaction. They are recognised and dealt with as obstructions and opportunities that present themselves in a given pursuit. Mathematical instruments that figure in applications are designed or adapted to support any such pursuit by highlighting adversities and developing advantages into strategies of action. If they were powerless to do so, they would be of no use in scientific inquiry and, consequently, never taken up or overhauled.

To transform the above set of ordinary features of inquiry into evidence for the existence of metaphysical necessities, is effectively to dismiss inquiry as a source of knowledge and reinstate in its place an anachronistic conception of knowledge as the apprehension of a fundamental, unchanging reality constituted by eternal laws holding the cosmos together. What is a feature of inquiry is thus transformed into an absolute feature of reality that must escape inquiry, since eternal and universal laws, unlike manageable interactions between particulars, are never to be encountered in experience.

Lange's account of distinctively mathematical explanation requires that the latter transformation be effected. Various undesirable consequences follow: one of them consists in the deletion of the role of laws as instrumentalities allowing the resolution of gross qualitative events for the sake of tighter control[24]. Focus on laws as the ultimate bounds locking Nature into an immutable order excludes a more productive focus on the function of laws in inquiry. The latter is contrastively singled out as the objective of philosophical reconstruction. The particular way in which it is forgotten against the background of Lange's account is the subject of the next subsection.

5.2. Laws and necessity

Lange's conception is not only erected on the fallacy of selective emphasis, but on an iteration thereof. In its first stage, the application of selective emphasis in Lange's study of explanation isolates obstructions or advantages within inquiry and identifies them with signs of necessitating constraints or laws. In the iterated stage, the distinctive methods (mathematical or non-mathematical) whereby obstructions and advantages may be detected are isolated and hypostatised as distinct orders of laws.

[24] Cf. LW12: 449.

This is why Lange can work with a hierarchy of stronger and stronger necessitation, where mathematical necessity is in particular stronger than physical necessity. Behind the distinction one may easily discover features of enquiries concerning mathematical or physical subject matter that undergo a process of hypostatisation.

To clarify the point, consider a concrete example of inquiry from mathematical logic, revolving around the question about which subsets of the real numbers endowed with addition and multiplication are first-order definable. The question confronts an investigator with an indeterminate situation, whose full resolution will issue in a specific characterisation of the relevant subsets[25]. It is clear that the conceptions leading to the construction of the indeterminate situation given at the start of inquiry, e.g. the notion of a real number or the logical notions of a first-order language and of definability, are the results of previous enquiries, which have arisen and developed independently of physical subject-matter. In Dewey's terminology, such enquiries proceed independently of existential content[26]. They take as initial materials the objects of earlier reflection into relationships between formal languages and models. The latter are given only in the sense that they result from trains of thought that can be developed out of an axiomatic system (e.g. the theory of sets ZFC, conceived of as the axiomatised semantic meta-theory in use), not in the sense in which the components of an experimental setup are given. In a situation of this type, no treatment of a model-theoretical problem needs to attract the contents of physical subject-matter in order to be carried to a close.

What the last remarks highlight is that the independence of mathematical results from physical considerations is a consequence of the disjoint trajectories followed by the way actual investigations have been set up[27]. To think of independence as the fact that certain eternal mathematical truths about definable subsets of the reals would

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[25] A set is first-order definable in the given structure if, and only if, it is a union of intervals with algebraic endpoints.

[26] See e.g. LW12: 392.

[27] Obviously, this is not to say that they cannot be integrated at a later stage, in the face of a distinctive, new problematic situation.

continue to hold even where physical truths differed from those familiar at present is to misrepresent the matter. Misrepresentation is achieved through the metaphysical hypostatisation of one selected feature of actual, distinct enquiries, namely, the fact that, along their career, they do not need to rely upon one another. This simple fact is metaphysically sanctified when it is transformed into the assertion that mathematical necessity is stronger than physical necessity.

Because the latter assertion is the cornerstone of Lange's analysis of distinctively mathematical explanations, it follows that its endorsement makes any attempt at understanding the role played by laws within scientific inquiry more arduous, by involving it into undesirable metaphysical detours, each of which replaces the career of investigation with absolute features of Nature.

This criticism cannot only be voiced from the standpoint of Dewey's logical work[28] but it is also implicit in much later philosophical work on natural laws. A notable example is provided by the writings of Nancy Cartwright, who extensively emphasises the intimate connection between the notion of physical law and the tight delimitation of an experimental setup shielded from external interferences[29]. When Cartwright's analysis is read from the standpoint of LW12, its most important result is that the very conception of a law arises within inquiry and cannot be ascribed to a universal regularity that is observable or significant apart from deliberate efforts aimed at experimental control and from technical restrictions of empirical possibilities. To revive a notion of law as a universal constraint that is actualised under a variety of contingent conditions, as Lange seeks to do, is to dismiss the structure of inquiry as an object of philosophical reflection in order to replace it with a conception that, being in essence pre-modern, is also pre-scientific.

6. Prospects

Work in philosophy of mathematics is often profound and insightful.

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[28] Especially Chapter 22 of LW12.

[29] In this connection, see especially Chapter 3 of Cartwright (1983).

The critical remarks proposed here are intended to suggest that its level of depth and insight can easily increase where metaphysical presuppositions incompatible with the structure of inquiry are in operation. This is the sign that a reconstructive task is needed, as a result of which greater insights may be obtained and hinderances to understanding may be removed.

Reconstructive activity, as pointed out in this paper, is especially needed in connection with philosophical work dealing with the application of mathematics. Its first order of business is to replace debates concerning the ontological import of mathematical propositions with an analysis of their functions within the context of scientific enquiries.

Although the required analysis cannot be fully carried out here, it seems appropriate to describe its general orientation. Because the goal of any inquiry is the resolution of an indeterminate situation, culminating in overt action aimed at modifying initially given existential conditions, the functions of mathematical resources are to be understood in relation to this goal.

Apart from their specific characterisation, these functions play an intermediate role, in the sense that they are performed once a situation has been problematised and its terms can be put into a specific symbolic form amenable to mathematical treatment, which is in turn guided by mathematical propositions. The results of mathematical treatment are also intermediate, since they lead to the formulation of plans of action that either trigger further development of symbolic form or prelude to intervention.

This picture is very rough but it sets the task of discerning the functions of mathematical treatment in the course of inquiry. Once this is done, mathematical resources can be looked at as instrumentalities aiding problem-solving, as opposed to descriptions of ultimate traits of self-sufficient realities. When viewed as such descriptions, or attempted descriptions, they institute a separation between formal models and their targets, with the attending problem of deciding what kind of bridge may be invoked to make models relevant. Moreover, descriptions that do not match the respective targets, e.g. on account of idealisations, appear as imperfect, false or distorted pictures thereof. The artificial puzzle arises of accounting for the usefulness or effectiveness of models that are cut off from their targets and in addition misrepresent them.

If, on the contrary, following a reconstructive approach, mathematical ideas come to be studied as instruments of symbolic intervention that help develop the terms of a problem into a resolution thereof, the generic notion of a formal model is to be replaced by the distinct notion of a complex of functions or a site of symbolic interventions that advance problem-solving. The problem of the relation between a mathematical model and what it seeks to describe is replaced by the analysis of the manner in which mathematical techniques promote interaction with an indeterminate situation.

The puzzle of useful yet hopelessly inaccurate descriptions of phenomena is replaced by the analysis of idealisations or other assumptions as strategies employed to open a line of attack on particular problems. The effectiveness and insufficiencies of these plans are evidently a matter of philosophical interest.

It is to be expected that paying a closer attention to scientific inquiry, as implied in the execution of a reconstructive task, should eliminate a number of puzzles in favour of a more lucid and more nuanced account of scientific practice, which can serve the purpose of providing the working scientists themselves with a sharper and more serviceable understanding of their activities and goals. This is a task of some importance, because it helps prevent the dogmatic habit of thinking promoted by the uncontrolled, because unsuspected, influence of philosophical prepossessions from the past on present common sense.

It was perceptively remarked by Dewey that many philosophical ideas of the past survive as "the presupposed background, the unexpressed premises, the working (and therefore controlling) tools of thought and action"[30]: it is a worthwhile task of philosophical critique to recognise their persistence and encourage progress beyond them.

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[30] EW 4: 62.

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