Coherence, Probability and Explanation

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ABSTRACT: Recently there have been several attempts in formal epistemology to develop an adequate probabilistic measure of coherence. There is much to recommend probabilistic measures of coherence. They are quantitative and render formally precise a notion – coherence – notorious for its elusiveness. Further, some of them do very well, intuitively, on a variety of test cases. Siebel, however, argues that there can be no adequate probabilistic measure of coherence. Take some set of propositions ***A***, some probabilistic measure of coherence, and a probability distribution such that all the probabilities on which ***A***'s degree of coherence depends (according to the measure in question) are defined. Then, the argument goes, the degree to which ***A*** is coherent depends solely on the details of the distribution in question and not at all on the explanatory relations, if any, standing between the propositions in ***A***. This is problematic, the argument continues, because, first, explanation matters for coherence, and, second, explanation cannot be adequately captured solely in terms of probability. We argue that Siebel's argument falls short.

Recently there have been several attempts in formal epistemology to develop an adequate *probabilistic* measure of *coherence*.[1] The basic idea behind this approach is that the degree to which a set of propositions ***A*** is coherent – the degree to which the propositions in ***A*** "mutually support each other" or "hang together" – is a function of various probabilities involving the propositions in ***A***. A brief illustration is in order. Suppose ***A*** consists of propositions A_1 and A_2, and take some probabilistic confirmation measure c.[2] Then, on

[1] See e.g. Douven and Meijs (2007), Fitelson (2003), Meijs (2006), Olsson (2002), Roche (2013), Schupbach (2011), and Shogenji (1999).
[2] For discussion of, and references regarding, the main probabilistic confirmation measures in the literature, see Crupi et al. (2007), Eells and Fitelson (2002), and Festa (1999).

one approach to developing a probabilistic measure of coherence, the degree to which $\boldsymbol{A}$ is coherent, $COH_c(\boldsymbol{A})$, is given by:

$$COH_c(\boldsymbol{A}) = 1/2 \cdot [c(A_1, A_2) + c(A_2, A_1)].$$

$COH_c(\boldsymbol{A})$ is the (straight) average of (i) the degree to which A_1 is supported by A_2 and (ii) the degree to which A_2 is supported by A_1, where support – as measured by c – is a function of various probabilities involving A_1 and A_2.[3] There is much to recommend probabilistic measures of coherence. They are *quantitative* and render formally precise a notion – coherence – notorious for its elusiveness.[4] Further, some of them do very well, intuitively, on a variety of test cases.[5] Siebel (2005, 2011), however, argues that there can be no adequate probabilistic measure of coherence.[6] Take some set of propositions $\boldsymbol{A}$, some probabilistic coherence measure, and a probability distribution such that all the probabilities on which $\boldsymbol{A}$'s degree of coherence depends (according to the measure in question) are defined. Then, the argument goes, the degree to which $\boldsymbol{A}$ is coherent depends *solely* on the details of the distribution in question and *not at all* on the explanatory relations, if any, standing between the propositions in $\boldsymbol{A}$. This is problematic, the argument continues, because, first, explanation matters for coherence, and, second, explanation cannot be adequately captured solely in terms of probability. We aim to show that Siebel's argument falls short.

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[3] See Douven and Meijs (2007) for details on how to generalize from the case where $\boldsymbol{A}$ consists of *two* propositions to the case where $\boldsymbol{A}$ consists of n propositions.

[4] Swain's complaint (about BonJour 1985) in the following passage is typical:

> One of the most disappointing features of BonJour's book is the lack of detail provided in connection with the central notion of coherence. No effort is made at defining this concept. Instead, we are given several rather vaguely formulated conditions which loosely characterize coherence. (1989, p. 116)

BonJour himself later notes that "the precise nature of coherence remains a largely unsolved problem" (1999, p. 124).

[5] See the references given in note 1. See also Koscholke (unpublished).

[6] The version of the argument given in Siebel (2011) is identical in all essential respects to the version given in Siebel (2005: 356−358). All subsequent references to Siebel are to Siebel (2011).

First, consider the thesis that explanation cannot be adequately captured solely in terms of probability. Siebel contends that explanation is hyperintensional whereas probability is merely intensional. He writes:

> The short version of this proof [i.e., proof to the effect that explanation cannot be adequately captured solely in terms of probability] is that 'explanation' is a hyperintensional notion whereas 'probability' is merely intensional. This is to say, whereas logically equivalent propositions do not always have the same explanatory force, they have to be treated as equal with respect to their probability. (pp. 264-265)

Siebel gives a barometer case to illustrate. Let "D", "H_1", and "H_2" be understood as follows:

- D My barometer falls;
- H_1 My barometer is exposed to a drop in atmospheric pressure; and if a barometer is exposed to a drop in atmospheric pressure, it falls;
- H_2 My barometer falls; and my barometer is exposed to a drop in atmospheric pressure, or my barometer does not fall; and if a barometer is exposed to a drop in atmospheric pressure, it falls.

H_1 and H_2 are logically equivalent to each other, hence, as probability is merely intensional, H_1 and H_2 are identical in their "probability profiles", i.e. a probability involving one of them equals the corresponding probability involving the other. But, whereas H_1 explains D, H_2 does not, since D is an essential conjunct of H_2; the second and third conjuncts of H_2 do not entail, together or separately, that the barometer in question is exposed to a drop in atmospheric pressure, and thus do not entail, together or separately, D.

We grant for the sake of argument that explanation is hyperintensional whereas probability is merely intensional, and that this is illustrated by Siebel's barometer case. Our view is that, even granting all this, it does not follow that there can be no adequate probabilistic measure of coherence.

Siebel disagrees. He writes:

> If probabilistic accounts cannot cope with explanation, they will hardly be able to deal with coherence because, as BonJour (1985) and many others have pointed out, coherence is a function of explanation. Among other things, explanatory relations between the elements of a system increase its coherence. […] That is, in order to gain control over *coherence* with purely probabilistic means, it is required that *explanation*

be captured solely in terms of probability. Since the latter is impossible, there is also no hope for the former. (p. 266, emphasis Siebel's)

The charge is that probabilistic measures of coherence run afoul of theses such as:

(1) If H_1 explains D, whereas H_2 does not, then, *ceteris paribus,* $\{H_1, D\}$ is more coherent than $\{H_2, D\}$.[7]

Take a case, such as Siebel's barometer case, where H_1 and H_2 are logically equivalent to each other, but H_1 explains D whereas H_2 does not. Since H_1 and H_2 are logically equivalent to each other, $\{H_1, D\}$ and $\{H_2, D\}$ are identical in probability profile (i.e. any probability involving the propositions in the one set equals the corresponding probability involving the propositions in the other set). Any probabilistic measure of coherence, the argument goes, thus implies – contra (1) – that $\{H_1, D\}$ and $\{H_2, D\}$ are equal in coherence.

That, then, is Siebel's argument against probabilistic measures of coherence. The question now is whether Siebel's argument succeeds.

(1) perhaps has some initial plausibility (though this depends on how the ceteris paribus clause is understood). But when it is realized (or assumed) that explanation is hyperintensional whereas probability is merely intensional, things are quite different: (1) is highly suspect. Consider a case where H_1 and H_2 are logically equivalent to each other, and yet H_1 explains D whereas H_2 does not. Suppose, as in Siebel's barometer case, H_1 entails D and so confers on D a probability of 1. Then, as H_1 and H_2 are logically equivalent to each other and thus have the same probability profile, H_2 too confers on D a probability of 1. H_2 fails to explain D, but what matters for *coherence*, it seems, is simply the fact that each of H_1 and H_2 confers on D a probability of 1. In other words, given that each of H_1 and H_2 confers on D a probability of 1, it matters not at all for coherence – for mutual support or hanging together – that D is explained by H_1 but not by H_2.[8]

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[7] Strictly speaking, Siebel appeals to a slightly different thesis: If H_1 explains D_1 and D_2, whereas H_2 explains only D_1, then, *ceteris paribus,* $\{H_1, D_1, D_2\}$ is more coherent than $\{H_2, D_1, D_2\}$. But Siebel (personal communication) does accept (1) and does hold that probabilistic measures of coherence run afoul of (1). What we say about (1) can be said *mutatis mutandis* about the slightly different thesis just mentioned. We focus on (1), and not on the slightly different thesis, in part because (1) is the simpler of the two theses.

[8] The same is true in cases where H_1 and H_2 confer on D a probability less than 1 (assuming there can be cases of this sort).

What seems correct, more generally, is this: the probabilistic facts in a given case "screen off" the explanatory facts from the coherence facts, in that given the probabilistic facts, the explanatory facts have no impact on the coherence facts. This thesis is "the Screening-Off Thesis". Our view is that (1) is implausible insofar as it runs counter to the Screening-Off Thesis.

It should be noted that the Screening-Off Thesis allows that the explanatory facts play a role in determining the coherence facts *by playing a role in determining the probabilistic facts*. If, though, the Screening-Off Thesis is correct, it follows that when, if at all, the explanatory facts have an impact on the coherence facts, they do so *only indirectly through their role in determining the probabilistic facts*.[9]

It might be argued, contra the Screening-Off Thesis, that (in the case where H_1 and H_2 are logically equivalent to each other, and yet H_1 explains and entails D whereas H_2 merely entails D) H_1 stands in *two* coherence-increasing relations to D (an entailment relation and an explanatory relation) whereas H_2 stands in just *one* (an entailment relation), and that $\{H_1, D\}$ is therefore greater in coherence than $\{H_2, D\}$. Perhaps it is correct that H_1 stands in two coherence-increasing relations to D whereas H_2 stands in just one. It does not follow, however, that $\{H_1, D\}$ is greater in coherence than $\{H_2, D\}$. The one relation (the entailment relation), it seems, renders irrelevant for coherence the other relation (the explanatory relation) so that the latter relation confers no advantage on $\{H_1, D\}$ over $\{H_2, D\}$.

Siebel, at any rate, provides no argument against the Screening-Off Thesis. Nor does BonJour (1985) or anyone else (as far as we are aware).[10] Consider, for instance, the following passage from BonJour (1985):

> As I have already suggested by mentioning the ideal of unified science, the coherence of a system of beliefs is enhanced by the presence of explanatory relations among its members.
>
> Indeed, if we accept something like the familiar Hempelian account of explanation, this claim is to some extent a corollary of what has already been said. According to that account, particular facts are explained by appeal to other facts and general laws from which a statement of the explanandum fact may be deductively or probabilistically inferred; and lower-level laws and theories are explained in an analogous fashion by showing them to be deducible from more general laws and theories. Thus

[9] It should also be noted that the Screening-Off Thesis is distinct from the considerably stronger thesis that explanatoriness is evidentially irrelevant; see Roche and Sober (2013) for discussion.

[10] See e.g. Harman (1986), Lycan (1988), and Thagard (1992).

> the presence of relations of explanation within a system of beliefs enhances the inferential interconnectedness of the system simply because explanatory relations *are* one species of inference relations. (pp. 98-99, emphasis BonJour's)

BonJour's point is that explanatory relations are inferential relations, i.e. relations by virtue of which one or more propositions can be inferred, deductively or probabilistically, from one or more propositions, and so, as with inferential relations of any sort, explanatory relations are coherence-increasing (where coherence is a matter of "inferential connectedness").[11] This point is distinct from, and does not require, the point that while all entailment relations – whether explanatory or not – are inferential relations and, thus, are coherence-increasing, explanatory entailment relations do more for coherence than do non-explanatory entailment relations. Likewise with respect to explanatory inductive relations and non-explanatory inductive relations. BonJour's point is consistent with the Screening-Off Thesis.

BonJour makes a second point concerning coherence and explanation. He writes:

> For my purposes, an anomaly is a fact or event, especially one involving some sort of recurring pattern, which is claimed to obtain by one or more of the beliefs in the system of beliefs, but which is incapable of being explained (or would have been incapable of being predicted) by appeal to the other beliefs in the system. (Obviously such a status is a matter of degree.) The presence of such anomalies detracts from the coherence of the system to an extent which cannot be accounted for merely by appeal to the fact that the belief in an anomalous fact or event has fewer inferential connections to the rest of the system than would be the case if an explanation were available. The distinctive significance of anomalies lies rather in the fact that they undermine the claim of the allegedly basic explanatory principles to be genuinely basic, and thus threaten the overall coherence of the system in a much more serious way. (1985, p. 99)

BonJour has in mind cases where (speaking in terms of propositions and not in terms of beliefs) the anomalous proposition is *not* inferentially connected to the hypothesis, and where, moreover, the anomalous proposition *casts doubt on* the *truth* of the hypothesis (and thus on the hypothesis's status as genuinely explanatory and basic). Cases of this sort differ in crucial respects from cases where H_1 and H_2 are logically equivalent to each other, and each of H_1 and H_2 entails D, and yet H_1 explains D whereas H_2 does not. Cas-

[11] BonJour (1985, pp. 99-100) goes on to consider and reject the view that *only* explanatory relations are coherence-increasing.

es of the latter sort are cases where the anomalous proposition – D relative to H_2 – *is* inferentially connected to the hypothesis and thus does *not* cast doubt on the truth of the hypothesis. BonJour's point about anomalies and coherence poses no threat to the Screening-Off Thesis.

Clearly, information about explanatory relations can *convey information* about coherence. It is plausible that, even if not all explanatory relations are entailment relations (and even if not all entailment relations are explanatory), at least some explanatory relations are entailment relations (see Hempel 1965), and that information about explanatory entailment relations provides information about coherence. This rather transparent fact is not at odds with the Screening-Off Thesis.

Siebel appeals to a second *ceteris paribus* thesis connecting explanation and coherence:

(2) If H explains D_1 better than D_2, then, *ceteris paribus*, $\{H, D_1\}$ is more coherent than $\{H, D_2\}$.

The worry, presumably, is that there are cases where D_1 and D_2 have the same probability profile (perhaps because D_1 and D_2 are logically equivalent to each other), and yet, though H explains both D_1 and D_2, H explains D_1 better than D_2 – cases therefore where probabilistic coherence measures run afoul of (2). Siebel, however, never argues that there can be such cases. Furthermore, (2) runs counter to the Screening-Off Thesis and so, like (1), is implausible.

It should be noted that even if the Screening-Off Thesis is false, it might be that Siebel's argument fails nonetheless. The key here is that even if, as Siebel argues, explanation cannot be adequately captured solely in terms of probability, it might be that "explanatory power" can be (see Schupbach and Sprenger 2011, and Crupi and Tentori 2012) – that, where H explains D, H's explanatory power with respect to D is solely a matter of probability. Suppose now H_1 and H_2 are logically equivalent to each other but H_1 explains D whereas H_2 does not. Suppose, further, contra the Screening-Off Thesis, $\{H_1, D\}$ is greater in coherence than $\{H_2, D\}$. Then, many probabilistic coherence measures—the ones on which the Screening-Off Thesis holds—are inadequate. But perhaps not all possible probabilistic coherence measures are inadequate. Take some probabilistic measure of explanatory power ε and some confirmation measure c, where ε and c share the same *range*. Further, let

$$f_{\varepsilon,c}(A_1, A_2) = \begin{cases} \varepsilon(A_1, A_2) & \text{, if } A_1 \text{ explains } A_2 \\ c(A_1, A_2) & \text{, otherwise} \end{cases}.$$

The degree to which $\{A_1, A_2\}$ is coherent can then be measured as follows:

$$COH_f(A_1, A_2) = 1/2 \cdot \left[f_{\varepsilon,c}(A_1, A_2) + f_{\varepsilon,c}(A_2, A_1)\right].$$

$\{H_1, D\}$ and $\{H_2, D\}$ have the same probability profile. But, since H_1 explains D whereas H_2 does not, and since, thus, $f_{\varepsilon,c}(H_1, D) = \varepsilon(H_1, D)$ whereas $f_{\varepsilon,c}(H_2, D) = c(H_1, D)$, it might be that $COH_f(H_1, D) > COH_f(H_2, D)$. This will depend on the chosen measures of explanatory power and confirmation.

Any such probabilistic measure of coherence runs counter to the Screening-Off Thesis and thus, we hold, is implausible. The point remains, however, that even if the Screening-Off Thesis is false because of what it implies with respect to cases where H_1 and H_2 are logically equivalent to each other but H_1 explains D whereas H_2 does not, it does not follow immediately that there can be no adequate probabilistic measure of coherence.[12]

We conclude that Siebel's argument fails. Even if explanation cannot be adequately captured solely in terms of probability, it might well be that there can be an adequate probabilistic coherence measure.

Acknowledgments

We thank Jakob Koscholke, Michael Roche, Mark Siebel, and two anonymous reviewers for helpful comments or discussion. This work was partly supported by Grant SI 173/1-1 to Mark Siebel from the Deutsche Forschungsgemeinschaft (DFG) as part of the priority program "New Frameworks of Rationality" (SPP 1516).

[12] It might be countered that *by definition* a probabilistic coherence measure implies that any two sets having the same probability profile also have the same coherence value. Fine. The important point is that even if extant probabilistic coherence measures are false because they run counter to theses such as (1) and (2), there are coherence measures very much in the spirit of extant probabilistic coherence measures and on which two sets can have the same probability profile and yet differ in coherence because of differences concerning explanation. Such measures, as with extant probabilistic coherence measures, are quantitative and render formally precise the notion of coherence.

References

BonJour, L. (1985). *The structure of empirical knowledge*. Cambridge, Mass.: Harvard University Press.

BonJour, L. (1999). The dialectic of foundationalism and coherentism. In *The Blackwell guide to epistemology*, ed. J. Greco and E. Sosa, 117−42. Malden: Blackwell.

Crupi, V., and Tentori, K. (2012). A second look at the logic of explanatory power (with two novel representation theorems). *Philosophy of Science* 79: 365−85.

Crupi, V., Tentori, K., and Gonzalez, M. (2007). On Bayesian measures of evidential support: Theoretical and empirical issues. *Philosophy of Science* 74: 229−52.

Douven, I., and Meijs, W. (2007). Measuring coherence. *Synthese* 156: 405−25.

Eells, E., and Fitelson, B. (2002). Symmetries and asymmetries in evidential support. *Philosophical Studies* 107: 129−42.

Festa, R. (1999). Bayesian confirmation. In *Experience, reality, and scientific explanation*, eds. M. Galavotti and A. Pagnini, 55−87. Dordrecht: Kluwer.

Fitelson, B. (2003). A probabilistic theory of coherence. *Analysis* 63: 194−99.

Harman, G. (1986). *Change in view: Principles of reasoning*. Cambridge, Mass.: MIT Press.

Hempel, C. G. (1965). Aspects of scientific explanation. In his *Aspects of scientific explanation and other essays in the philosophy of science*, 331−496. New York: Free Press.

Koscholke, J. (unpublished). Last measure standing: Evaluating test cases for probabilistic coherence measures.

Lycan, W. (1988). *Judgement and justification*. Cambridge: Cambridge University Press.

Meijs, W. (2006). Coherence as generalized logical equivalence. *Erkenntnis* 64: 231−52.

Olsson, E. J. (2002). What is the problem of coherence and truth? *Journal of Philosophy* 99: 246−72.

Roche, W. (2013). Coherence and probability: A probabilistic account of coherence. In *Coherence: Insights from philosophy, jurisprudence and artificial intelligence*, eds. M. Araszkiewicz and J. Savelka, 59−91. Dordrecht: Springer.

Roche, W., and Sober, E. (2013). Explanatoriness is evidentially irrelevant, or inference to the best explanation meets Bayesian confirmation theory. *Analysis* 73: 659−68.

Schupbach, J. N. (2011). New hope for Shogenji's coherence measure. *British Journal for the Philosophy of Science* 62: 125−42.

Schupbach, J. N., and Sprenger, J. (2011). The logic of explanatory power. *Philosophy of Science* 78: 105−27.

Shogenji, T. (1999). Is coherence truth conducive? *Analysis* 59: 338−45.

Siebel, M. (2005). Against probabilistic measures of coherence. *Erkenntnis* 63: 335−60.

Siebel, M. (2011). Why explanation and thus coherence cannot be reduced to probability. *Analysis* 71: 264−66.

Swain, M. (1989). BonJour's coherence theory of justification. In *The current state of the coherence theory: Critical essays on the epistemic theories of Keith Lehrer and Laurence BonJour, with replies*, ed. J. Bender, 115−24. Dordrecht: Kluwer.

Thagard, P. (1992). *Conceptual revolutions*. Princeton: Princeton University Press.