edge of the other. What good is a political teaching that has no ground in ontology, epistemology, or cosmology? What explanatory power do such theories have if they cannot make sense of the totality of political phenomena, the most primary fact of ordinary experience? This simple insight is, I believe, a profound one because the courage of the philosophical Eros is equally as requisite as the ratiocinative power of *dianoia* for giving a logos of the totality of Being, if not more so in late Modernity, although Plato places the two on exactly even footing at (344a2-b1). Otherwise stated, it would seem that Plato teaches that the philosophical Eros aims at unifying the unstable dualism of theory and practice, of theoria and phronesis. This has the consequence that the topos of the unity of theory and practice is the human soul and not the Hegelian Absolute Spirit. This ineffable and unperfectable unity can be regarded as an image of Wisdom understood as the unreachable goal of philosophy; as an image of a complete and fully discursive Logos of the totality of Being; or as an image of absolute self-consciousness beyond the unending quest for the dialectical extension of self-knowledge. That is to say, philosophy aims not simply at a coherent whole of theory and practice but rather at a good and coherent whole of these two moments of thought and action. In one last re-formulation due to my colleague Barry Gilbert, the conditions of the possibility of ordinary experience are not the same as the conditions of the good ordering of ordinary experience. But they are the same in the teleological sense that the philosophical Eros aims at a good unity of theory and practice, as is clear from the fact that we distinguish between better and worse attempts at such unification. This contradiction, this torn-harmony, is the Truth at which Philosophy aims.

The Synthetic Relation in Hume

Stephanie Rocknak

Introduction

Here we will see that contrary to the party line, Hume's notion of a "relation" should be understood, in *all* cases, as a peculiar non-necessary *synthetic* relation; unique but similar in a certain constructive sense to what I characterize as a mathematical notion of synthesis.[1] And most controversially, I argue that this non-necessary synthetic notion of a relation includes Hume's *arithmetical* relations, which have typically been interpreted as either "*analytic*", necessary, or both.[2]

But before I launch into the specifics of this argument, let me at least sketch the context of the broader project it is imbedded in - my dissertation. Doing so might help the reader better understand the deeper historical and philosophical significance of understanding *all* relations in Hume as not only synthetic, but as non-necessary as well.

In its most general sense, my dissertation concerns the identification of certain fundamental aspects rooted in Hume's epistemology with equally fundamental aspects rooted in Quine's epistemology. In particular, I show that both men, for various reasons, are forced to depend on a very *weak* notion of belief to account for a specific, but entirely pervasive epistemological problem: the problem of induc-

1 Let it be noted that this paper is just a *sketch* of what will appear in my dissertation. This is the case due to length restraints.

2 See for starters, W. V. Quine's paper "Two Dogmas of Empiricism", in: *From a Logical Point of View*, 2nd Ed., Cambridge 1980. Note in particular the line: "Kant's cleavage between analytic and synthetic truths was foreshadowed in Hume's distinction between relations of ideas [arithmetic] and matters of fact [non-arithmetic]. See also Alexander Rosenberg's paper "Hume and the Philosophy of Science", pp. 80-81, in the *Cambridge Companion to Hume*, edited by D.F. Norton, Cambridge University Press, 1994. See also Galen Strawson's and Antony Flew's respective books: *The Secret Connexion; Causation, Realism and David Hume*, Oxford 1989 and *David Hume; Philosopher of Moral Science*, Basil Blackwell, 1986. For somewhat similar to what I argue here, both claim that Hume thought necessity was psychological in all cases. As we will see, I attempt to take this a step further and show that necessity drops out altogether - even in a psychological sense.

tion. For according to both Hume and Quine, we may only justify the general truths/knowledge that both admit we *do* possess,[3] in terms of purely psychological,[4] non-rational belief.

As part of showing that both Hume and Quine rely on this weak notion of belief, I must show how and why they rule out other possible answers to the problem of induction. These answers include, among others,[5] an appeal to the *a priori* or similarly, to the necessary.[6] For if Hume and Quine *did* appeal to the *a priori* or the necessary, they could have subsequently turned to mathematical probability[7] to answer the problem of induction.[8]

3 Of course, Hume and Quine are both *skeptical* philosophers in regard to just what extent we *do* actually possess this knowledge. However this is another story and one that does not directly concern us here.

4 Roughly, this is opposed to relying on "*rational* belief", which Keynes, one of the originators of probability theory, thought probability turned on. Note: "When we argue that Darwin gives valid grounds for our accepting his theory of natural selection we do *not* simply mean that we are *psychologically inclined* to agree with him; it is certain that we also intend to convey our belief that we are acting *rationally* in regarding his theory as probable." (*A Treatise on Probability* by John Maynard Keynes, London, 1957, p. 5). See at least Chapter 1 of my dissertation for more detail.

5 As I see it, there are three fundamental approaches to this problem: 1. We may appeal to *a priori* premises to formulate some kind of Principle of Induction. These premises may be thought of as "*external*", e.g. The Law of the Universality of Nature, or "*internal*", e.g. "innate ideas or rules" such as Kant's "principle of universal causation". Logical or mathematical laws may fall into either of these categories; where in either case, one or both may or may not be understood as rationality *par excellence*. 2. Rather than appealing to *a priori* premises, we may instead appeal to a Popperian notion of "falsification" which actually denies that there *is* induction; representing as well, a rejection of probability theory that might rely on *either a priori* principles *or* "inductive principles". This is Popper's notion of "deductivism", as first spelled out in *The Logic of Scientific Discovery*. 3. Or, rather than appealing to any of the above, we may appeal to a psychological ability to simply *believe* that some general truths must hold as such; this belief being a function of repetition, or as Hume would have it, custom. Or likewise, as Quine would have it, community affirmation. See at least the Introduction and Chapter 1 of my dissertation for more detail.

6 Although useful distinctions can and have been made between the necessary and the *a priori*, we need not spell them out in this paper.

7 Here we should also note that claiming that general knowledge is *probable* is distinct from claiming that general knowledge may be *strictly deduced*, or formally put, is distinct from claiming that $Sx \therefore \forall x\, Sx$ is valid. We should note that $Sx \therefore \forall x\, Sx$ (which reads informally: Because x is an S in at least one case, then in *every* case x must be an S) is *not* truth preserving. This is the case because inductive inferences introduce new information. As such, inductive inferences may generate a True-False conditional, which as shown in elementary logic, is false. For instance, if I claim that [A] I have been sneezing all morning, therefore [B] I have caught a cold, it may be the case that [A] is indeed true, but that [B] is *not* true because my sneezing has actually been caused by allergies, rather than the common cold. Thus we would have a True-False conditional.

With this very general overview in mind, realize that by showing that Hume, like Quine, rejects the broad epistemological notion that there are "two distinct kinds of knowledge" - i.e., quite roughly, that he rejects a substantive distinction between "*analytic*" and "*synthetic*" knowledge, where the former is *a priori* and/or necessary and the other is *not* - we see precisely why Hume, like Quine, may *not* appeal to the notion of necessity nor to the *a priori* to account for the problem of induction.[9]

Thus, in short, by showing that *all* relations in Hume are a particular kind of *non*-necessary synthetic, we see two things: [A] Precisely why, like Quine, Hume may not appeal to either *a priori* premises (e.g. "The Law of the Universality of Nature") or mathematical probability to account for induction. [B] Like Quine, but long *before* Quine, Hume attempts to dismantle the analytic/synthetic distinction, making Quine's famous paper "The Two Dogmas of Empiricism" seem more like a fashionable re-write than the bold and original piece it is heralded to be.

8 One must turn to necessary, or *a priori* premises in probability theory to avoid infinite regress. This is the case simply because if one appeals to *non*-necessary premises, i.e. *inductive* premises to justify a theory of induction, one must then somehow justify *them* and so on. However, if one does appeal to necessary premises, such as the Principle of Non-Sufficient Reason, and The Principle of Indifference, as Bernoulli and Keynes respectively did, then it seems one's only recourse is to dogmatically accept these premises. For how else could one justify them? In fact, it is worth noting how Popper phrased this very problem in *The Logic of Scientific Discovery*: "At this stage I can disregard the fact that the believers in inductive logic entertain an idea of probability that I shall later reject as highly unsuitable for their own purposes ... I can do so because the difficulties mentioned are not even touched by an appeal to probability. For if a certain degree of probability is to be assigned to statements based on inductive inference, then this will have to be justified by invoking a new principle of induction, appropriately modified. And this new principle in its turn will have to be justified, and so on. Nothing is gained, moreover, if the principle of induction, in its turn, is taken not as 'true' but only as 'probable.' In short, like every other form of inductive logic, the logic of probable inference, or 'probability logic,' leads either to infinite regress, or to the doctrine of *a priorism*. [*The Logic of Scientific Discovery*, Routledge, 1995, p. 30]

9 However, Hume *does* have a notion of "pragmatic" mathematical necessity. But this is not be confused with a "psychological" sense of necessity that, as such, must necessarily hold in all our thought processes. In fact, this notion has entirely misled scholars to believe that Hume thinks there is a substantive "analytic/synthetic" distinction, including Flew and G. Strawson. But this is explained elsewhere in my dissertation.

Part I. A Brief History of the Notion of a "Relation"

To properly understand the pervasively non-necessary *synthetic* character of relations in Hume, we should first broadly canvas the history of the philosophical relation. Doing so will help inform my final definition of the synthetic relation in Hume, and further in my dissertation, an almost identical relation in Quine.

However, let it be openly admitted that the history of relations is obviously much richer than I will make it out to be here. But for our somewhat restricted purposes, we need only involve ourselves with the some of the very elementary shifts, some of the very elementary concerns.

These concerns are only three in number: [1] We will see that the "Early" (i.e. ancient and medieval) and "Modern" (i.e. Descartes through Locke) conception of the relation was based on understanding it primarily in terms of *subjects* and *predicates*. However, the "Contemporary" (i.e. the late 19th and 20th century "analytic") notion of a relation is, for the most part, *not* formulated in terms of *subjects* and *predicates*. [2] We will see that as such, the "Early" and "Modern" notions of a relation were *relative* while the "contemporary" notion seems to have fundamentally *lost* this sense of relativity. [3] With the exception of Kant, no philosopher explicitly speaks of a relation that may somehow be understood as *constructing* new objects, although relative relations seem to at least *guarantee* the existence of other objects.

Generally speaking, these concerns historically unfold as follows: From Plato to Russell, the debate has waxed and waned regarding whether a relation is an independent object or is in some way inherent in, or predicated of, one or more of the terms[10] it is said to hold between. In other words: Is the relation R between n terms, where n is = 2[11]· some *independent* object holding between a and b? Or does it somehow exist *just* in a as some kind of predicated property and/or quality[12] such as, say, being "the mother of?" Further, if so, does it exist in *both* a and b in terms of concomitant properties and/or qualities inherent in both? And as such, may a relation be construed as a *predicate* of both subjects a and b? But how would we do

10 I have nothing technical in mind by "term" here. It is merely a place holder for whatever things relations may hold between.

11 We will not take on the problem of *self*-relation here, where n would of course, = 1.

12 There have been numerous debates over whether a given thing has "properties" as opposed to "qualities", or neither. Here however, I will not enter into this debate since it does not affect my main argument. So rather than committing myself to either a "property" or a "quality", I will say throughout 'property and/or quality,' and intend both to mean *predicates* that hold of the given term or terms.

that? For instance, does "the mother of" exist in a as a certain property and/or quality while the "daughter of" concomitantly exists in b as a certain property and/or quality? Further, if the claim *is* made that R somehow exists in a *and* in b, what exactly, if any, is the connection between the qualities and/or properties existing in a and b? For instance, does such a situation engender anything akin to a *necessary* connection between at least those two inherent and related "aspects" of a and b? For example, if some a is said to have the quality and/or property of being "a mother of", *must* there also be some object b that has the property and/or quality of being the "daughter or son of a?" Further, can some x in a go a step beyond guaranteeing the *existence* of b such that it is y and somehow *construct* a b such that b is a y? Or, as we will see to be the case with Russell, shall we simply disregard all the questions noted above as curious artifacts of a distinctly subject-predicate architecture? Instead, shall we simply posit a relation as an entirely independent entity that, as such, entertains no substantive dependence on the terms it holds between?

With this general picture in mind, let's now take on just a bit more detail; restricting ourselves to just some of the more influential western philosophers:

Plato: It's not completely clear just how we may read Plato regarding these issues; his intentions are not entirely consistent when understood in terms of the entire Platonic/Socratic corpus. For instance, in the *Phaedo*, Plato appears to understand "greatness" and "smallness" as properties and/or qualities that *inhere* in individuals, rather than as independent entities.[13] As such, we should understand that Plato formulates the relation here in terms of a broad subject-predicate framework; for roughly but accurately speaking, it is only a *predicate* that may be said to *inhere in*, or be in some *subject*. We must also note that the attributes that hold of a given term a in the *Phaedo* seem to bear a particular kind of relationship of "towardness" to b, if only in the sense that greatness is great only *relative* to some thing b that is

13 Note: Socrates: "when you say that Simmias is greater than Socrates and smaller than Phaedo, [do you not] say that there is *in* Simmias greatness and smallness? [*Phaedo*, D102, B-C. Translated by H. N. Fowler, Loeb Classical Library, Harvard University Press, Cambridge, MA, 1990]. For further discussion on the inherent nature of relations in the terms in the *Phaedo*, see also J. R. Weinburg's book ***Abstraction, Relation and Induction; Three Essays in the History of Thought***, University of Wisconsin Press, Madison & Milwaukee, 1965, p. 66. See also F.M. Cornford, ***Plato and Parmenedies***, London 1939, p. 78, and R. Hackforth's *Plato's Phaedo*, Cambridge 1955, p. 155.

smaller than a.[14] In this very loose sense, it may seem that Plato thinks here (*qua* Socrates) that some thing a may only be "great" or "small" if some other thing b exists that is respectively, smaller or greater than a. Thus, such a property and/or quality in a seems to *guarantee* the existence of a b with an appropriate property and/or quality; for some thing a is only x (e.g. "smaller") *relative* to some thing b. And we see that if a is in fact x (e.g. "smaller"), then it follows that there *must* be b such that it is y (e.g. "bigger"). However, there is no indication in the *Phaedo* that the existence of such an a introduces or in any way *generates* the existence of an appropriate b.

Further, in the *Parmenedies*, Plato once again rejects the idea that relations are independent entities, i.e., "forms".[15]

However, in the *Sophist*, Plato (*qua* the Eleactic Stranger) claims that the relation "difference" *is* in fact some kind of "form".[16] We could interpret this to mean that the relation difference - D - *is* some independent entity holding between a and b while both a and b somehow "participate" in D, but D is *not* predicated of either a and b. Thus here, Plato seems to gesture towards what we will later see to be the 19th-20th century logical notion of an independent *non*-predicated notion of a relation. However, as noted in the passage cited in footnote 16 of this paper, Plato takes this "form" of the relation D to exist only *relative* to other forms, what he

14 Note: "But" said Socrates, "you agree that the statement that Simmias is greater than Socrates is not true as stated in those words. For Simmias is not greater than Socrates by reason of being Simmius, but by reason of the greatness he happens to have; nor is he greater than Socrates because Socrates is Socrates, but because Socrates has smallness *relatively* to his greatness"[*Phaedo*, D 102 C]

15 Note the *Parmenedies*, 149 eff.

16 Note: "Stranger: Then we shall call 'the other' a fifth class? Or must we conceive of this and 'being' as two names for one class? Theat.: Maybe. Str. But I fancy you admit that among the entities some are always conceived as absolute, and some as relative. Theat.: Of course. Str.: And other is always relative to other, is it not? Theat.: Yes. Str. It would not be so, if being and the other were not utterly different. If the other, like being, partook of both absolute and relative existence, there would be also among the others that exist another in relation to any other; but as it is, we find that whatever is other is just what it is through compulsion of some other. Theat.: The facts are as you say. Str.: Then we must place the nature of a 'true other' as a fifth among the classes in which we selected our examples. Theat.: Yes. Str.: And we shall say that it permeates them all; for each of them is other than the rest, *not by reason of its own nature, but because it partakes of the idea of the other.*" [Emphasis my own, *Sophist* 255 c and d. Loeb Classical Library, translated by H.N. Fowler, Harvard University Press, Cambridge MA, 1987]. For further discussion regarding the idea that Plato takes a relation to be an independent entity in the *Sophist*, see also Cornford's *Plato's Theory of Knowledge*, p. 282.

refers to in the *Sophist* as "absolute" forms [lines 255 c]. Thus, it seems that if we accept the existence of form D, i.e. the relative relation D, then certain absolute forms *must* also exist. For once again the reasoning is: Without absolute forms, or perhaps even other relative forms, the relative form D would have no forms to be relative *to*. However, there is once again no indication that the existence of form D somehow introduces, or generates the existence of these various other "absolute" forms.

With just these three dialogues in mind,[17] we may say that with the exception of the *Sophist* Plato thought:

[1] Relations are qualities and/or properties that hold of, or are *predicated* of the given terms. In other words, he seems to broadly understand a relation in terms of *subjects* and *predicates*.

[2] As such, relations are *relative*. That is, some thing a may only said to be x in relation to some b that is a y. Thus it follows that if some thing a is indeed x then there must also be some thing b that is y.

Aristotle: We now turn to a quick overview of Aristotle. And again, it is openly admitted that Aristotle's thoughts on relations are more complex than I will make them out to be here. But as with Plato, there are certain fundamental points that, with a good historical conscience, we can simply highlight and move on. For instance, as most philosophers know, Aristotle classified relations as a category in the *Categories*. As such, relations were something that could be said of substance. And this means that relations must somehow *inhere* in, or are predicated of substances, a point Aristotle never turns his back on. Note: "all the other things [besides primary substance] are either *said of* the primary substances or *in them* as subjects"[18]

We must also note that Aristotle thought relations are accidental aspects of substance; how some substance may be related to something else is not to be understood as part of the *essential* nature of a substance. For instance, in order for the reader to be *you*, it is not essential that this piece of paper is currently in your line of vision. That is, it is not essential that the paper be *related* to you in this fashion, much less at this time and in this place. However, it is important to note that

17 Granted, my discussion of Plato is far from comprehensive. However, for our very general purposes, it is enough.

18 [emphasis my own] *Categories*, Ch. 5 2a1 35-36.

because substances are *real* things - that is, not in any way to be understood as ideal and/or as mental constructions - the relations that inhere in them are to be understood as *real* as well; the real cat is *really* wild, and is *really* standing in the kitchen. In other words, these aspects of the cat are *not* constructs of the perceiver's mind.

With these basic points in mind, let's now briefly focus our attention on a well-known passage in the *Metaphysics*, Book V, Chapter 15 concerning the tri-partite notion of a "relative". Doing so will simply assure us of the *relative* notion of a relation in Aristotle:

> *Things are relative (1) as double to half and treble to a third, and in general that which contains something else many times to that which is contained many times in something else, and that which exceeds to that which is exceeded; (2) as that which can heat to that which can be heated, and that which can cut that which can be cut, and in general the active to the passive; (3) as the measurable to the measure and the knowable to the knowledge and the perceptible to perception.*[19]

In other words, Aristotle tells us here that there are three fundamental ways in which we may understand how a thing is said to be "relative" to some other thing. Or in other words, how a thing is said to *relate* to some other thing. Mark Henniger aptly characterizes these classifications in his book *Relations: Medieval Theories 1250-1325*, as respectively, "numerical, causal and psychological".[20] For our purposes, we should realize that in general, all three of these types of characterizations may be understood as follows: Each claims that a thing **a** may be said to be **x** *only if* there is some thing **b** that is **y** that **a** may be said to be **x** in relation to. That is, the fact that **a** is an **x** in terms of the three characterizations noted above, *guarantees the existence* of **b** being a **y**.

For instance, some **a**, where **a** is a number, may only be a half (i.e. **x**) *in relation to*, or *relative to*, **b** where **b** is a whole (i.e. a **y**) and is also a number: "number is always commensurable, and number is not said of the non-commensurable".[21] Thus, if we have some thing that is said to be a half, we *must* also have some thing that is said to be a whole.

Similarly, a thing **b** (the passive patient) may only be said to have the potential of a certain property and/or quality of being *actualized*, e.g., cut, if some thing **a** (the

19 *Metaphysics*, Book V, Ch 15 1020b 25-31.

20 *Relations: Medieval Theories 1250-1325*, Mark G. Henniger. Clarendon Press, Oxford, 1989.

21 *Metaphysics*, V15 1021a 5.

active agent) has the property/quality of being able to cut it: "The active and the passive imply an active and a passive capacity and the actualization of the capacities".[22]

And finally, a thing **b** (again, the passive patient) may only be *known* if there is some **a** (the active agent) that does the know*ing*.[23] Note:

> *that which is measurable or knowable or thinkable is called relative because something else is related to it. For the thinkable implies that there is a thought of it.*[24]

Thus, in short, for our purposes, we see that according to Aristotle:

[1] Relations inhere in substances. That is, as was the case with most of Plato's work, Aristotle thought we should understand relations in terms of *subjects* and *predicates*.

[2] Relations are real.

[3] Relations are accidental.

[4] Relations are *relative*. That is, a certain thing **a** may only be an **x** (e.g. respectively, a half, actually cut, or known) if there is some thing **b** that is **y** (e.g. respectively a whole, something capable of cutting, or something that can know).

However, as was the case with Plato, nowhere do we see evidence that the existence of an **a** that is **x** in any way *generates* or *introduces* the existence of a **b** that is **y**. For example, although we might observe some thing that has the capacity to be cut, say a tomato, although this means there must also exist some thing **b** such that it has the property or quality of being able to *cut* the tomato, this object **b** did not

22 *Metaphysics* V 15 1021a 15-16, 20-25.

23 Merely for the sake of accuracy, we should note that Aristotle *does* make a distinction in this chapter between the first two characteristics of relatives (the numerical and the causal) and the last (the psychological) by pointing out that although a number that is a half must be so in relation to a whole, and something that has the capacity to be cut may only *realize* that capacity if some thing exists such that it can cut it, a half and a potentially-cut thing are not as such because other things stand in relation *to them*. It is instead, he claims, the other way around. That is, a half is a half because it is related *to* a whole, not because a whole is related to *it*. But as for knowledge, he claims that some thing **a** may only be known if some thing is actually related *to it* by knowing it. That is, knowledge in this sense becomes passive while the knower is active. Note: "Relative terms which imply number or capacity, therefore, are all relative because their very essence includes in its nature a reference to something else, not because something else is related to *it*; but [the opposite is the case regarding "psychological" relatives]" [*Metaphysics* V 15 1021a 26-31].

24 *Metaphysics* V 15 1021a 26-31.

somehow "come into existence" merely because the tomato has the capacity to be cut.

The Medievals: With this broad overview of Aristotle in mind, let us now turn to a brief look at the medievals, who were significantly influenced by Aristotle's work on relations. However, regardless of this singular influence, they did not share a singular vision. For instance, although like Aristotle none thought relations were entirely *mind*-dependent, they did furiously disagree regarding *where* we should locate relations and what exactly took place between the two or more terms. Henniger characterizes this dispute in terms of two demands:

> *Many medieval thinkers believed that there were two demands made on an adequate theory of relation. The first demand is historically conditioned, a product of the pervasive Aristotelianism in the late medieval period. The second is what I might call transhistorical, being present in an adequate theory or relation in whatever era.*
>
> *The first demand, resulting from a substance-accident ontology, was to treat a real relation as an accident existing in one subject. The second demand was to do justice to a relation's character as somehow involving more than one thing. If one's theory must respond to both of these demands, one can conceive of a real relation as existing in one thing, yet depending on and somehow 'referring to' another thing. In scholastic terminology, (esse-in) and a being-toward (esse-ad).*[25]

Henniger explains that we must understand the medieval *modus operandi* in terms of how a philosopher handled these two demands, which, as noted, was not always the same. Further, if a philosopher did not remain somewhere within the confines of the two demands cited above, we should *not* classify him as a "traditional" medieval thinker - despite the fact that he worked in the medieval era.

For instance, to touch on the more "notables", Thomas Aquinas and Henry of Ghent each had a sympathetic foot solidly placed in both demands cited above, despite certain subtler differences that we won't go into here.[26] As such, following Henniger, we may characterize them as "traditional" medieval thinkers. Meanwhile, Richard of Mediavilla did *not* explicitly conform to these demands, but neverthe-

[25] Henniger, p. 175

[26] See Henniger's book for more detail.

less, he did not deviate entirely from them. Thus, we may characterize him too as a "traditional" medieval thinker. And finally, William of Ockham should be understood as straying far enough from these demands to classify him as "*non*-traditional" medieval thinker.

To inform our brief discussion of these four thinkers, we should first review the medieval concept of a "foundation": If some accidental property, say a, inheres in **a**, and some accidental property, say b, inheres in **b**, then it was typically said that a and b serve as the *foundations* for a relation R holding in **a** and a co-relation R' holding in **b**.[27] For instance, consider a situation where **a** is a mother and **b** is a daughter of **a**. In this case, **a** and **b** are related in terms of, respectively, R "being the mother of", and R', "being the daughter of". The fact that **a** and **b** have the accidental properties of being respectively, "the mother of **b**" and "the daughter of **a**" are the respective *foundations* a and b for the relation R and its co-relation R.'

This is somewhat similar to how I loosely characterized the situation earlier in terms of Plato and Aristotle. For there I noted that in order for some **a** to be an **x**, a **b** must be a **y**. For we can roughly identify what we referred to earlier as **x** and **y**, with, respectively, the medieval notions of a and b, keeping in mind that both models are based on understanding relations *qua* subjects and predicates. However, earlier we did *not* see direct evidence for such "co-relations". But what exactly these foundations were and if there are indeed such things as "co-relative" relations, were some of the larger bones of medieval contention; the details of which we will not concern ourselves with here.

However, broadly speaking, realize that Aquinas and Henry of Ghent both thought that a given thing **a** had a "foundation" a and thus both thought that a relation "inhered" in **a** in terms of a. This "inherence" constituted what they referred to as the *esse-in* of a relation; as such we may understand the relation in terms of a *predicate*. Meanwhile, both also believed that such a relation somehow "pointed towards", or as I have explained above, somehow guaranteed that **b** have b as a foundation, and thus guaranteed that **b** have a co-relation R.' This constituted the *esse-ad* aspect of the relation R. But again, rather than going into any more of the detail regarding this, simply because for our purposes we needn't, we may conclude that both Aquinas and Ghent thought that:

[1] There are substances; they are real.

[27] See Henniger, p. 5

[2] Relations inhere in them accidentally, but they are also real; they have extra-mental being. This real inherence constitutes the *esse-in* of a relation. Thus, once again, a relation should be understood in terms of a subject-predicate framework.

[3] Such real (accidental) relations also point "towards" something else. This "towardness" constitutes the *esse-ad* of a relation and guarantees that **b** will have a foundation b for a co-relation **R**.'

[4] Thus, as such, relations are ***relative.*** That is, in order for **a** to have the foundation a, **b** must concomitantly have the foundation b.

Noting this, realize that at least Mediavilla argued that relations ***do*** have foundations but relations do not "inhere" in them. In short, this means that Aquinas's and Henry of Ghent's notion of *esse-in* was not only under attack but the notion of a relation ***qua*** predicate was as well. For instead, he argued, one must somehow understand the relation as "existing" somewhere "between" both terms. However, he also argued that it is ***neither*** a completely independent entity, ***nor*** is it completely "inherent" in both terms.[28] But rather than digging into precisely what ***that*** could amount to here, we may simply conclude that Richard of Mediavilla thought that:

[1] There are substances; they are real.

[2] A relation is ***real.***

[3] A relation is a relative ***thing.*** That is, it is ***not*** to be identified with its foundation. Further, it must be understood as obtaining between ***two*** terms. As such, Richard does not view a relation in terms of two accidental foundations that must inhere in the two terms. However, mysteriously enough, it is not ***entirely*** independent of either, and thus the notion of a relation as a predicate is not left ***entirely*** behind.

As for the ***non***-traditional medievals, i.e. those who did not adhere to either of the two demands cited above (the *esse-in* and *esse-ad*), we have, most notably, William of Ockham. For he did not think that a relation "inhered" in **a**. That is, in medieval terms, **a** did not have a foundation. In fact, Ockham writes: "I say that a relation does not have a foundation nor is that word 'foundation of a relation' found in the philosophy or Aristotle, nor is it a philosophical word."[29] Instead, Ockham argued that although a certain term **a** that has a certain property and/or quality **x** may, as such, infer the existence of **b** such that **b** is a **y**, it does ***not*** infer that there is some relation **R** that exists that is ***distinct*** from **a** and **b**, while somehow

28 See Henniger pp. 59-68, 178.

29 *Quodlibet*, VI q.10, as translated by Henniger, p. 181.

simultaneously being "founded" in both **a** and **b**. In short then, we may conclude that Ockham thought:

[1] There are substances; they are real.

[2] A relation is real.

[3] But a relation does not ***inhere*** in either of the two terms, ***or*** somewhere "between" them as a third thing.

[4] Instead, a relation may either be understood as a ***concept*** that relates two terms **a** and **b**, ***or*** it may be understood as the class of all things that fall under such a relation.[30]

[5] A relation is ***relative.***

In short, we may conclude that as initially noted in the beginning of our discussion of the medievals, none thought that relations were entirely mind-***dependent*** - despite their various differences.[31] That is, relations may have held ***of*** things (Aquinas/Henry of Ghent) or somewhere, somehow, "in-between" things (Mediavilla), or denoted ***classes*** of things (Ockham) but none are to be understood as entirely ***rational*** or mental, constructions. Further, with what appears to be the exception of Ockham, all formulations of a relation were based on a ***subject-predicate*** framework. Also, despite Ockham's apparent deviation from the subject-predicate framework, all defined the notion of a relation as ***relative.*** And finally, there is no evidence in any of those noted above that a relation could in any way ***construct*** new individuals, or terms.

The Moderns: Now we turn to a general overview of the modern understanding of relations, where "modernity" is understood as beginning with Descartes. For our purposes, we need only highlight the dramatic change that the philosophical construal of a relation underwent in the modern era. Simply put: they were no longer "real" in the simple sense that they were ***not*** mind-***independent.*** For instance, Des-

30 See Henniger, p. 119-149.

31 Note Henniger's characterization of the intellectual situation: "Despite the variety of theories, no one held that real relations are completely mind-dependent ... This is not surprising. Given the pervasive Aristotelianism, it would be extremely difficult to deny all extra-mental reality to relations, for the scholastics interpreted Aristotle as explicitly teaching that relation is one of the ten categories of extra-mental being. More fundamentally, both Aristotle's and the medievals' thought is pervaded by the notion of an extra-mental order, whether this be the Greek cosmos or the medieval universe. In the thirteenth and early fourteenth centuries the principal problem was not whether relations have extra-mental reality, but rather what specific type of extra-mental reality is to be accorded to them" (Henniger, p. 174).

cartes,[32] Leibniz,[33] Spinoza,[34] Hobbes,[35] and for the most part, Locke[36] and Berkeley,[37] all understood relations as mind-*dependent* objects. However, *crucial* to note for our purposes, none of these philosophers thought that relations were actually *constructed* by the mind. Instead, they represented certain pre-established *aspects* of the mind.[38] As such, for these philosophers, relations were neither inherent in one or more of the terms as *real* qualities and/or properties of those terms (e.g.. Plato, Aristotle, Aquinas), *nor* did they exist as mind-independent objects holding between the given terms (e.g. Mediavilla), or mind-independent classes (e.g. Ockham).

However, like those philosophers preceding them, they too placed relations within the general subject-predicate framework, although, ironically enough, the metaphysical notion of *substance*, i.e. the paradigmatic notion of a *subject*, had been systematically eschewed by at least Locke and Berkeley.[39]

However, Leibniz is a partial exception to the rule regarding an adherence to the subject-predicate framework. For instance, not coincidentally, Russell cites the fol-

32 See for instance, *The Rules For the Direction of the Mind*, and the *First Meditation* where Descartes claims that God places reason in us, and thus, the ability to conceive of and make relations.

33 Leibniz's work is not available to me now, so at this point we will just have to trust Russell's interpretation of Leibniz in his book *The Philosophy of Leibniz*: "But relations, though founded in things [for Leibniz], derive their reality from the supreme reason (N.E. p. 235: G.V. 210); God sees not only individual monads and their various states, but their relations also, and in this consists the reality of relations (G.H 438) … relations and aggregates have only a mental truth; the true proposition is one ascribing a predicate to God and to all other who perceive the relation" (Russell, The *Philosophy of Leibniz*, p. 14)

34 See *Metaphysical Thoughts*, Part I, Ch. 5 and the *Short Treatise*, Part I, Ch. 10.

35 See the *Leviathan*, Chapters 1-5, in particular, Chapter 5 on "Reason".

36 See *An Essay Concerning Human Understanding*, ed. P. H. Niddich, Oxford, 1975, bk. II Chapter 25, paragraphs 1, 5, and 7.

37 See the Principles of Human Knowledge .

38 Even Hobbes, Locke and Berkeley did not think that we *construct* relations. See at least Bk IV, chapter 1, "Of Knowledge in General" in Locke's *An Essay Concerning Human Understanding*. Here, Locke talks of *innate* ways that we may compare ideas to acquire knowledge. Also realize that Berkeley ultimately attributed all rationality (and thus the ability to relate) to God. See the *Principles* for more detail.

39 See respectively, Chapter XXIII of "Of Our Complex Ideas of Substances", in An Essay Concerning Human Understanding, and Commonplace Book, p. 20, A Treatise Concerning the Principles of Human Knowledge, Part I, §37, and Three Dialogues Between Hylas & Philonous, Third Dialogue, p. 455 (all from Berkeley's Complete Works, Vol. I, ed. A. C. Fraser, Oxford at the Clarendon Press, 1901).

lowing passage of Leibniz in his book *The Philosophy of Leibniz*. For as we will see further on, it is Russell who, in light of modern logic, would later insist that a relation is an *independent* entity, and as such, is not to be understood in terms of a subject-predicate framework. Note this passage:

> *The ratio or proportion between two lines L and M may be conceived three several ways; as a ration of the greater L to the lesser M; as a ration of the lesser M to the greater L; and lastly, as something abstracted from both, that is, as the ratio between L and M, without considering which is the antecedent, or which is the consequent; which is the subject and which the object … In the first way of considering them, L the greater is the subject, in the second M the lesser is the subject of that accident which philosophers call relation or ratio. But which of them will be the subject, in the third way of considering them? It cannot be said that both of them, L and M together, are the subject of such and accident: for if so, we should have an accident in two subjects, with one leg in one, and the other in the other; which is contrary to the notion of accidents. Therefore we must say that this relation, in this third way of considering it, is indeed out of the subjects; but being neither a substance, nor an accident, it must be a mere ideal thing, the consideration of which is nevertheless useful.*[40]

In other words, Leibniz claims here:

[1] There are three ways to conceive of the "ratio or proportion" or in other words, the *relation* between the lines L and M of different lengths. They are:

> [A] We may conceive of it in terms of the relation "greater than" where L is greater than M. In this case, L is the subject and has, in the traditional Aristotelian sense explained earlier, the accidental property of "greater than".
>
> [B] Or, we may conceive of it in terms of "smaller than". In this case, M is the subject and has the accidental property of "smaller than".
>
> [C] Or, finally, we may conceive of it as somehow inhering in both C and M where neither is understood as the subject or as the object. Instead both C and M have equal status.

[2] Seemingly convinced that [C] is the correct construal, Leibniz then concludes: But if both L and M are *subjects*, then both cannot have the "accidental

40 D. pp. 266-7; G. vii. 401, as cited by Russell in *The Philosophy of Leibniz*.

qualities" of, respective, "greater than", or "smaller than". This is the case he says, because "that would be contrary to the notion of accidents". In other words, Leibniz seems to think here that we may *not* understand relative relations as accidentally inhering between two terms. For as soon as we claim that some thing **b** *must* have a property that is concomitant to the given property in **a**, we remove the "accidental" aspect from the particular quality and/or property in **b** and give it a *necessary* aspect. Thus he concludes, *this* relation must be understood as some *third* thing, if only as a "mere ideal thing".

And thus, it seems that at least in *this* case, Leibniz thinks that a relation is *not* to be understood in terms of a *predicate* inhering in some given subject. However, Russell thinks Leibniz simply writes this peculiarity off as some kind of a quirk, since he is simply unable to extricate himself from the subject-predicate framework. Note Russell's comment: "It appears that [Leibniz] is unable to admit, as ultimately valid, any form of judgment other than the subject-predicate form."[41]

Keeping this "quirk" of Leibniz's in mind, we may summarize the modern conception of a relation as follows:

[1] In some cases, there are real substances, in others, there are not.

[2] In all cases, relations are mind-dependent, but in no cases are relations *constructed* by the mind.

[3] In most cases (with the slight exception of Leibniz) relations are based on a subject-predicate model.

[4] In no cases does a relation in any way *construct* new individuals or terms.

Kant: Skipping Hume, whose thoughts on relations we will consider in detail in Part III, let's now take a brief look at Kant's notion of a relation in The *Critique of Pure Reason*. As most philosophers know, like his more immediate predecessors and colleagues noted above, Kant quite famously understood relations as mind-*dependent*. Further, with his notion of a judgment, Kant too adhered to the subject-predicate framework.[42]

For instance, taken in their broadest sense, Kant defines relations as certain *a priori* "logical functions"[43] that lie behind all possible judgments. As such, relation

41 Russell, *The Philosophy of Leibniz*, p. 13.

42 See Kant's numerous discussions of judgments throughout the *Prolegomena* and the First *Critique*.

43 See A79/B105, Critique of Pure Reason.

establishes itself as one of the sub-categories of Categories, which is in turn split into three sections: [1] The relation "Of Inherence and Substance" and [2] The relation "Of Causality and Dependence" and [3] The relation "Of Community (Reciprocity between agent and patient)".[44] Further, this meant that certain relations for Kant must be understood as pure[45] *a priori* concepts.

However, here we will not concern ourselves with the subtler differences between these kinds of relatives and what exactly it meant for them to be mind-dependent. However, it *is* essential for us to highlight Kant's understanding of the mathematical synthetic *a priori* relation.[46] For although I will return to this relation in more detail in the next section, let us note now that for Kant, a *mathematical* synthetic *a priori* relation between two terms **a** and **b** is a relationship that, through our formal, intuitive comprehension of **a**, actually *introduces*, or *generates* **b**. This is what Kant means throughout the First *Critique* by synthetic *a priori construction*.[47] Recall for instance where he speaks of such mathematical constructions: "A concept of space and time, as quanta, can be exhibited *a priori* in intuition, that is, *constructed*, either in respect of the quality (figure) of the quanta, or through number in their quantity"[48] and "I *construct* a triangle by representing the object which corresponds to [the given concept] either by imagination alone, in pure intuition"[49].

Thus, for our purposes, we may simplify the situation as follows: Kant thought relations were:

[1] Mind-*dependent*, similar to the moderns noted above.

44 Critique of Pure Reason, A80/B106

45 Recall that for Kant in the First *Critique*, only the Categories are *pure a priori* concepts. All other *a priori* concepts may be *a priori*, but are nonetheless *derivative* from the Categories. Thus, there may be some *a priori* concepts that are relations, but are nonetheless not included as one of the three kinds of Categories. See A81/B107, where he speaks directly of this distinction.

46 We must keep in mind that Kant had *two* senses of the synthetic *a priori* in mind in the First *Critique*: The *philosophical* synthetic *a priori* and the *mathematical* synthetic *a priori*. See my paper "The Distinction Between Mathematics and Philosophy in the First *Critique*: an Account of Kant's Two Fundamental Senses of the Synthetic *A Priori*" (unpublished).

47 For further discussion on this matter, see my paper "The Distinction Between Mathematics and Philosophy in the First *Critique*:..." and Jaakko Hintikka's *Logic, Language Games and Information; Kantian Themes in the Philosophy of Logic*, Oxford University Press, 1979 (reprint), Chapters VI, VIII, and IX.

48 [Emphasis my own] *First Critique*, A720/B748

49 [Emphasis my own]*First Critique*, A713/B741

[2] As is the case with most of his predecessors, Kant's notion of a relation must be understood in terms of subjects and predicates.

[3] However, he was the first to *explicitly*[50] claim that a mathematical synthetic *a priori* relation between two terms a and b resulted in the *introduction* of b into our thought via our intuitive comprehension of **a** - a process we will discuss in somewhat more detail in the next section.

The Contemporaries: At the turn of the century, a *logical*, or *formal* notion of relations was developed by, most notably, DeMorgan,[51] Peirce,[52] Frege[53] and Russell.[54] With these men, we see perhaps the most dramatic shift in the construal of a relation. For generally speaking, the 19-20th century's logical formalization of a relation gave it a certain independence that it hadn't seen since Plato's *Sophist* and to a certain degree, William of Ockham. For according to these logicians, to claim that **aRb** is at the very least to *infer* that there are not only *three* independent entities at work, **a**, **b** and **R**,[55] but also, that these are mind-*independent* entities. And bringing the new-found independence of the relation to its mature form was Frege's logical formulation of predicate calculus, first seen in the *Begriffshrifft* of 1879. Here, a formal and decisive distinction was made between a predicate and a relation.[56]

50 As we will see in detail in the next section, many before Kant thought as much as well. However, they did not *explicitly* say as much, nor did they try to carefully spell out the distinction between *this* kind of relation *qua* the introduction of objects in opposition to the purely *philosophical* relation where objects are *not* introduced.

51 See the *Transactions* of the Cambridge Philosophical Society (1864) vol. 10, pp. 173-230, 331-358, 428-487.

52 See at least *Collected Papers*, Vol. II, *Elements of Logic*, ed. by C. Hartshorne and Paul Weiss, Cambridge 1932.

53 See at least the *Begriffshriffi*, 1879.

54 See *Principia Mathematica*, by A.N. Whitehead and B. Russell, Cambridge University Press, 1903. 2nd ed., New York, 1938.

55 Some claim that the *Tractatus* Wittgenstein was the exception; allegedly being unwilling to grant relations an independent existence. See at least: G.E.M Anscombe, *An Introduction to Wittgenstein's Tractatus*, 2nd, revised ed. Hutchinson, London, 1963, Irving Copi's "Objects, Properties and Relations in the *Tractatus*", *Mind*, vol. 67, 1958, pp. 145-65, George Pitcher's *The Philosophy of Wittgenstein*, Prentice-Hall, Englewood Cliffs, 1964, pp. 113-18. However, Jaakko Hintikka and Merrill B. Hintikka disagreed. See Chapter 2 of *Investigating Wittgenstein*, Basil Blackwell Ltd. reprinted 1989.

56 See also J. S. Mill, *A System of Logic Ratiocinative and Inductive*, ed. J.M. Robson, in *Collected Works of J. S. Mill*, vol. 7, Buffalo, 1973, bk. 1, chap. 7.

Briefly, let's consider Russell in more detail, who, compared to the other three men just mentioned, was the most concerned with and subsequently, wrote the most about the *philosophical* nature of relation. For instance, in Russell's whirlwind tour on the *Theory of Knowledge*, written in just over 30 days but later abandoned thanks to Wittgenstein's scorn, we see Russell talk of "bare relations" and our "acquaintance" with them. Note, for instance, the following passages:

> *An* entity *which* can *occur in a complex as "precedes" occurs in "A precedes B" will be called a* relation [57]

> *we are forced to the conclusion that the knowledge which we indubitably possess concerning relations involves acquaintance, either with the bare relations themselves, or at least with something equally abstract; and by "something equally abstract I mean something which is determinate when the relation is given, and does not, like a complex, demand some further datum.* [58]

That is, here Russell characterizes relations as *independent* entities, which, as such, we must be "acquainted"[59] with along with the terms they hold between in order to *know*, e.g., aRb. Further, elsewhere in the *Theory of Knowledge*, Russell characterizes relations as mind-independent, or "external" entities.[60] Russell also claims in the *Theory of Knowledge* that we must be acquainted with the "logical form" of **aRb** to know that **aRb** *is* in fact **aRb** and not **bRa**.[61] For instance, we must be able to distinguish between "the man rides the horse" from "the horse rides the man".

But we need not discuss any more of the details of Russell's theory of acquaintance here. What we *do* need to highlight are the following points: In *The Theory of Knowledge*, Russell took a relation to be a mind-independent entity. Paring this fact with the fact that he *also* claimed that to know that **aRb**, we must: [1] Be "acquainted" with all *three* entities, **a**, **b** and **R**, and [2] Be "acquainted" with the "logical form", it follows that knowing **a** alone and any properties and/or qualities it may have did *not* guarantee knowing **R** *or* that there must be some **b** such that **b** is related to **a** in terms of **R**. That is, knowing **a** and all its various qualities/properties

57 Emphasis my own, *Theory of Knowledge*, Routledge, 1984, p. 84.

58 Emphasis my own, *Theory of Knowledge*, p. 84.

59 See at least p. 35 of the *Theory of Knowledge* for Russell's definition of "acquaintance".

60 See pp. 42-43, 54 of the *Theory of Knowledge*.

61 See p. 99 of the *Theory of Knowledge*.

does not in any way *guarantee* that we know that there must also be some b such that b is related to a in terms or R. For it seems that according to Russell, if I know that Mary is a mother, this does not mean that I also know that out there somewhere, there must be Mary's son or daughter. Instead, I must actually be "acquainted" with the son or daughter to know this. Likewise, I must also be somehow acquainted with the relation of "motherhood".

In short then, Russell's theory of acquaintance:

[1] Is based on the logical construal of a relation as a *non*-predicate.

[2] Thus, it seems that it *wipes out* what we have seen to be the concomitant relationship between terms that hold of a "relative" relationship.

[3] Regardless, relations were indeed *real* for Russell.

[4] Finally, there is no indication that any kind of relation may serve to *construct* new objects or terms.

We may loosely refer to this conception of relations as the "contemporary" notion. And surely, it is decidedly *unlike* what we have seen to be the case thus far, as well as unlike what we will see to be the case with Hume - and further on in my dissertation, Quine as well.

So in short, we may summarize the fundamental points regarding the history of relations as follows: The "Early" philosophers, with few exceptions, e.g. Ockham, all based their understanding of the relation in terms of some kind of subject-predicate paradigm. As a result, relations were also construed as *relative*, where the existence of one predicate in a subject a guaranteed the existence of another subject b with a certain predicate inhering in it. The "Modern" philosophers thought as much as well, but the relation was understood as mind-*dependent*, although not *constructed* by the mind. Further, with the exception of Kant, none of the "Early" or "Modern" philosophers *explicitly* explained that certain relations could guarantee not only the existence of other individuals with certain predicates (properties and/or qualities), but could actually *introduce* them. And finally, with the advent of modern logic, and thus with what I loosely refer to here as the "contemporary" philosophers, the notion of a relation was no longer understood in terms of the subject-predicate framework. Instead, it was construed as a logical, mind-independent entity. As a result, the relation lost its stronger sense of relativity, and thus, its ability to guarantee the existence of other objects, much less *introduce* them.

With this general background in mind, let's now turn to a synopsis of a few philosophers/mathematicians who *did* think a relation could actually construct new objects or terms.

Part II. A Certain Kind of Synthesis

The notions of "analysis" and "synthesis" have changed throughout history as the notion of a relation changed. This is the case partly because for some proposition or sentence to be characterized as "analytic", or "synthetic" it must be done by determining whether it displays an analytic or synthetic *relation*.

However, it is not feasible nor fruitful for me to summarize all the significant variations of the "analytic" and the "synthetic" here. I refer the reader instead to some of the better books and papers on this subject.[62] Here, I will focus just on canvassing a particular kind of relation that involves the introduction of new objects; which after Kant, I will call "synthesis", merely because he was the first to explicitly examine it as a method *per se*. However, we will see that as early as Pappus, the Greek geometer, the notion of "*analysis*" concerned the introduction of new "individuals". Further, in all cases, we will see that this type of synthesis is *mathematical*.[63]

To guide our way through these examples, let me first present a general paradigm of what I mean by the introduction, or construction of new objects in terms of a relation: If we have a certain hypothesis, say, 2+2=4, we will typically want to prove that it is true. Now, some might argue that it simply follows from the *rules* of mathematics that 2+2=4. Thus, in this very general sense, one might say that given these rules, which have apparently been deemed *necessary* in some sense, it necessarily follows that 2+2=4. In turn, one might call 2+2=4 an "analytic" truth, i.e. a truth that necessarily "follows from the rules". As such, this "analytic" truth has nothing to do with "experience", nor anything else (e.g. "intuition"). However, some might argue that 2+2=4 *doesn't* simply follow mechanically from the rules. Instead, one might argue that one must appeal to "something else" to prove that both 2+2=4 is true. One might, the argument continues, have to actually *construct* a picture in one's head of 4 things and two sets of 2 things. This *construction* is what Kant (very) roughly referred to as *synthesis*, and as such, removes the entirely mechanical "rule-following" aspect from the process of proving that 2+2=4.

62 See for starters, J. Hintikka's book ***Logic, Language Games and Information; Kantian Themes in the Philosophy of Logic***, Chapter 6.

63 It is worth briefly noting that given what we saw in Part I of this paper, it is clear that there is some historical tension between the ***philosophical*** construal of a relation and the ***mathematical*** construal - which may indeed by echoed in Kant's distinction between the philosophical and mathematical synthetic ***a priori***. However, examining this tension in detail is not our concern here, so let it simply stand noted and we will move on.

Instead, the process demands that the given thinker formally intuits the information needed to prove the hypothesis true. In a bit more detail this means: Given one's knowledge of 2 and the operation +, one, so to speak, "dips into" a "third thing" i.e. formal intuition, to *construct* an object such that it is = to 2+2, i.e. the object 4. However, according to Kant, it is still *necessary* that 2+2 = 4 because, very roughly speaking, one appeals to *formal* intuition, which, crudely put, keeps relations *necessary*. With this in mind, realize that the *relation* here is the mathematical relation of =. Further, because we must *construct* an object, e.g. 4, to prove that this relation is true, we must understand this relation to be *synthetic*. Thus, in *this* sense, synthetic relations necessarily involve the *construction* of new objects.

So, beginning with Pappus, I will give another brief historical background, but now just in terms of a mathematical process of constructing new objects used to prove that certain relations are true. As such, none of the following methods are mechanical, and further, all appeal to some "third thing" (e.g. in Kant's case, formal intuition) to create the needed objects.

Pappus First realize that Pappus was a geometer - he worked with geometrical *objects*, not with propositions *about* objects. As such, his proofs operated in terms of the inter-relations holding between properties of geometrical objects. Hintikka and Remes effectively show us that this is the case with a re-creation of a Pappian proof in chapter III of *The Method of Analysis*.[64] With these interdependencies in mind, we should note that the "secret" of Pappian analysis is to put the information we are given in our theorem or problem to use. Thus, if we wanted to give a proof of a particular theorem using the Pappian method, say $A \supset B$ - that is, if we wanted to show the relation É between A and B was true - we must somehow use the information given about the interdependencies between the *objects* that A and B speak about. However, the notation '$A \supset B$' does not typically represent a relation between objects, but instead, represents a relation between propositions. So instead of attempting to formalize Pappus's method in terms of statement logic, as many have done,[65] we should instead appeal to first-order predicate logic. Doing as much

64 See pp. 22-29. Note that the discussion of Pappus above is almost entirely dependent on the work done in *The Method of Analysis*.

65 See H. Hankel from *Zur Geschichte der Mathematik in Alterum and Mittelaltus*, George Olms, Hildesheim, 1965 (reprint of the original ed. of 1874), pp. 139-140.

will simply help us to better understand what the "introduction of new individuals" amounts to, at least formally.

Accordingly, Hintikka and Remes suggest that we formally express the theorems needed to be analyzed as follows:

[1] $(x_1) \supset (x_k)\ (A(x_1 \supset x_2) \supset B(x_1 \supset x_n))$

However, we need to realize that this characterization won't quite do because Pappus didn't work with *generalizations* about geometrical objects. Instead, he worked with particular but nevertheless indeterminate objects. To capture this, we need only strip the quantifiers from our conditionals and replace all our bound variables with variables that did not occur free anywhere in our quantified statements. In other words, we instantiate:

[1'] $A(a_1 \supset a_k) \supset B(a_1 \supset a_k)$

However, the formalization '$A(a_1 \supset a_k)$ and $B(a_1 \supset a_k)$' still does not capture what's going on here. For also typically put to task in these proofs was an independent set of axioms and theorems, generally Euclid's. Let's refer to this set with 'E.'

So we know that by using $A(a_1 \supset a_k)$, $B(a_1 \supset a_k)$ and E, the Greek analyst had to somehow show how $B(a_1 \supset a_k)$ is *related to* $A(a_1 \supset a_k)$ in terms of other objects. That is, the Greek analyst had to "bridge the gaps" between the two with objects $A_1(a_1 \supset a_k) \supset B_n(a_1 \supset a_k)$.

In short, this means that when Pappus filled in the gaps above with $A_1(a_1 \supset a_k) \supset B_n(a_1 \supset a_k)$, he did not *derive* them in the ordinary sense from $A(a_1 \supset a_k)$, E and $B(a_1 \supset a_k)$. Instead, using natural deduction, he derived them in the sense that they necessarily "went together "with $A(a_1 \supset a_k)$, $B(a_1 \supset a_k)$ and E.

And thus, in this sense, Pappus used a method that "introduced new objects". However, as noted above, Pappus referred to this as "analysis", not "synthesis". For according to Pappus, the *synthetic* procedure was a process of simply arranging all the information introduced by *analysis* into a deductive proof. Regardless of this trivial terminological artifact, we see here that in order to prove that a given **a** is indeed related to a given **b**, i.e. that $A(a_1 \supset a_k) \supset B(a_1 \supset a_k)$ is true, we:

[1] Set up the problem as if it has already been solved, i.e. that $A(a_1 \supset a_k) \supset B(a_1 \supset a_k)$ *is* true.

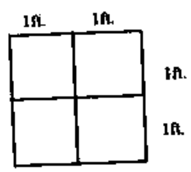
[2] Appealing to what Pappus never explicitly defines, but what is seemingly "mathematical intuition", *and* the knowledge that we have of $A(a_1 \supset a_k)$, E and $B(a_1 \supset a_k)$, we construct new individuals.

[3] Thus, the given relation is proved to hold in this particular instance between the particular objects $A(a_1 \supset a_k)$ and $B(a_1 \supset a_k)$.

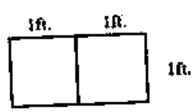
Plato We see a similar process of the mathematical introduction of individuals in the *Meno*, lines 82B - 85A. Here, Socrates, intent on showing that knowledge is ultimately derived from recollecting knowledge from the soul, leads a slave-boy through the following problem. *Socrates'* point is to show that with the proper questions, the slave-boy may reach a certain state where he can begin to "recollect" the knowledge his soul is already equipped with. *My* point is to show that this method of "recollection" is a mathematical method quite similar to that sketched above in terms of Pappus; a process of "filling in the gaps" that does not work according to explicit rules. For in each case where Socrates draws a new line or lines, or has the slave-boy imagine a new line or lines, Socrates and the slave boy ***introduce*** or ***construct*** new elements into the proof, drawing on a "third thing", i.e. knowledge inherent in the soul.

The Problem: How long are the sides of a square that has the area of 8 square feet?

[1] First, Socrates draws a square in the sand and gets the slave boy to agree that all sides are equal. Then he draws two lines through the center points of each side to get the following picture



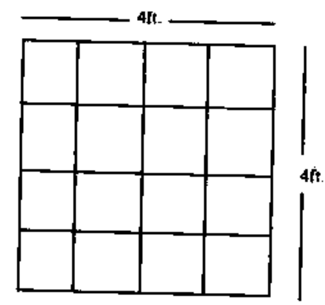
[2] Following, Socrates asks, if each side is 2 feet long, what is the area of the square? After asking the slave boy to construct a figure in his head where one side is one unit and the other 2 units and then asking him to determine the area of *that*, the slave boy answers that the area of the above square is four.



The imagined

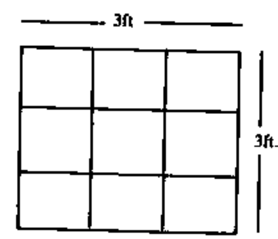
[3] Then Socrates asks: So how long would each side be if a given square had the area of 8 square feet?

[4] The slave boy replies that the sides would have to be ***double*** that of the square in [1] above. Socrates determines that the boy cannot realize his mistake unless he constructs such a square for him, which he does:



[5] Now the slave boy actually *sees* that the area of the square in [4] is 16 and not 8. So they return to the problem, reviewing what they have seen so far. They conclude that a square with the area of 8 square feet must have sides that are ***longer*** than a square with the area of 4 square feet and ***shorter*** than a square with the area of 16 square feet.

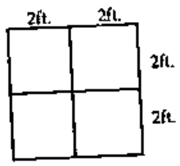
[6] So the slave-boy guesses that an 8ft. square square must have sides that are 3 feet long. Once again, Socrates must construct such a square to show the slave-boy that he is mistaken:



[7] Socrates notes at approximately this point that all of these "discoveries" made by the slave-boy are contingent on a certain state of mind: "Now do you [Meno] imagine he would have attempted to inquire or learn what he thought he knew,

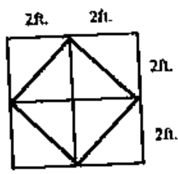
when he did not know it, until he had been reduced to the perplexity of realizing that he did not know, and had felt a craving to know?"[66]

[8] Then they return to the original 2 x 2 square. Following, Socrates adjoins three others to it to form a square of 4 x 4:



[9] Socrates now asks the slave-boy how much larger this new square is compared to the original 2 x 2 square. The slave-boy answers: 4 times as large. But Socrates says, we need one that is only 2 times as large to get a square with area 8.

[10] So Socrates draws more lines to get:



[11] Then he asks, how big is the new tilted square? When the slave-boy responds that he doesn't know, Socrates asks him: Isn't each little 2 x 2 square cut in *half* by the diagonal lines? The slave-boy agrees. Then Socrates asks: So how many half-spaces are there? The slave-boy answers 4. Following, when Socrates asks what the total area of the interior square is, the slave boy can answer 8. Thus the answer to the problem is: The length of the side of a square with the area of 8 square feet is the length of the diagonal of a 2ft. x 2ft. square.

[12] Socrates concludes: "'So that he who does not know about such matters, whatever they be, may have true opinions on such matters, about which he knows nothing?' Meno: "Apparently." Socrates: "And at this moment those opinions have just stirred up in him like a dream; but if were repeatedly asked these same questions in a variety of forms, you know he will have in the end as exact an understanding of the them as anyone.'"[67]

---

66 Meno, 84c.

67 *Meno* 85c

Thus, in short, the slave-boy, with the help of Socrates, must "recollect" geometrical knowledge in terms of how newly-constructed lines and shapes are related to each other to get the "final recollection", i.e. the solution to the problem. Thus:

[1] Given certain geometrical objects drawn before him, the slave-boy must, using his knowledge of them, appeal to some deeper knowledge in his soul and in turn:

[2] *Imagine*, or *see* certain additional objects in order to understand what the relation of the area of a 4x4 square is to one of its sides.

[3] Finally, the slave-boy must (with the help of Socrates) actually construct the solution (see [11] above).

For without such constructions, he would have been unable to determine what this relation is. So once again, we see that the proper comprehension of a relation is dependent on a process of constructing addition objects given some "third thing", i.e. knowledge visually recollected from the soul. As such, this is not a process of "rule-following".

Descartes Now note the following passage from Descartes *La Geometrie*:

> *If then, we wish to solve any problem, we first suppose the solution to be already effected, and give names to all the lines that seem needful for its construction - to those that are unknown as well at to those that are known. Then, making no distinction between known and unknown lines, we must unravel the difficulty in any way that shows most naturally the relations between these lines, until we find it possible to express a single quantity in two ways. This will constitute an equation... We must find as many such equations as there are supposed to be unknown lines.*[68]

Here, we see evidence that Descartes is relying on a method strikingly similar to the method we saw at work in not only Pappus,[69] but also, Plato. For as indicated in this passage, Descartes appears to think that one must:

[1] Assume that the given problem has already been solved.

[2] In turn, different from his predecessors, he suggests that we translate a given geometrical problem into algebraic terms.

---

68 (6:372).

69 See Hintikka's paper *A Discourse on Descartes' Method* for further discussion on this point; the interpretation of Descartes' method discussed above relies entirely on the work done in this paper.

[a] To do this, one must set up equivalence relations between the different angles and lines of geometrical objects in terms of polynomial dependencies.

[b] However, crucial to note, to think in terms of such dependencies is to remain thinking about the interdependencies of geometrical objects, but now characterized in algebraic terms.

[3] We then must symbolize (or "name", as Descartes says above) not only every known line that constitutes a given geometrical object, but every ***unknown*** line as well.

[4] Following, we algebraically define our unknown lines in terms of our known lines. In other words, we set up equivalence relations between our knowns and unknowns.

[5] If we can solve these equations, we have found our unknowns (lines constituting geometrical objects) and thus, we have "discovered" all that we need to construct our problem.

With this in mind, let's recall the main points of *Pappian* analysis:

[1]' On must first assume that our given problem or theorem has already been solved.

[2]' Analysis is concerned with geometrical *objects.*

[3]' When we analyze, we work to fill in the "gaps" between a consequent B of a given problem or theorem and a set E of axioms and previously proved theories, with the antecedent A by using not only the information given to us by A &E but by *B* also.

[4]' More often than not, the information given by E, A and B is not enough so we introduce *new* geometrical elements, i.e., auxiliary constructions.

With this in mind, realize that the similarities between steps [1] and [2] of the Cartesian versus the Pappian steps of [1]' and [2]' are self-evident. However, the connection between Descartes' steps [3]-[5] and Pappus's steps [3]-[4] warrant some explanation: Descartes' algebraic unknowns, Hintikka explains in the *Discourse on Descartes' Method,* function in much the same way Pappus's auxiliary constructions did: When the unknowns are solved, they, like auxiliary constructions, introduce the new information needed to solve the problem at hand. And, also like auxiliary constructions, these unknowns are introduced in terms of a dependent (concomitant) relationships. In algebraic terms, we can refer to these relationship as functional dependencies. Also, again parallel to the role auxiliary constructions play in Pappian analysis, we are finished with our proof when we have solved for enough

unknowns. Thus, in both cases, we "bridge the gaps" using information that is both given to us initially and through the *introduction of new objects.*

New let's recall the main points in the method of recollection given in the *Meno:*

[1] We assume that somewhere in our soul, the given problem has already been solved.

[2] Recollection (as in the case of geometry) is concerned with being able to correctly construct new objects (lines).

[3] When we make the "final recollection", i.e. solve the problem, we have done so by constructing new objects. In this way, we "fill in the gaps".

Thus, so far, the fundamental similarities in method should be self-evident; all three procedures sketched thus far for proving a given problem are not mechanical, and thus, are not carried out by following self-evident rules. As such, I characterize all three in this sense as *synthetic,* and likewise, dependent on the construction of new objects to determine that a given relation does in fact hold between the given terms.

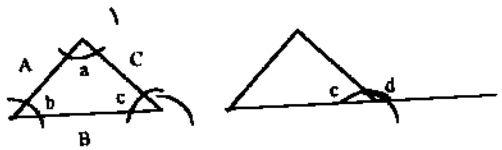
Kant: Now recall Kant's illustration of the methodological distinction between the philosopher and the mathematician beginning at line [716/B744] of the *Critique of Pure Reason.* Here we see Kant's notion of the synthetic ***a priori qua*** the introduction of new objects clearly at work. For Kant, the main point of this thought-experiment was to show that the ***mathematician's*** synthetic ***a priori*** method allowed for the introduction of new objects through an appeal to formal intuition while the ***philosopher's*** synthetic ***a priori*** method did ***not.***

The Problem: What exactly is the relation of the sum of a triangle's angles to two right angles?

Step 1: Apparently from previous constructions, the geometer knows that 90° + 90° "is exactly equal to the sum of all the adjacent angles which can be constructed from a single point on a straight line" [A 716/b744] With this quality of previous-given "quanta"[70] in mind, he extends the bottom of a given triangle to get two adjacent angles, c and b, which equal 180.°

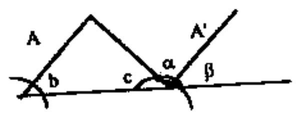
[70] 'Quanta' is the word Kant uses to refer to mathematical objects of intuition in the First *Critique.* See all of the *Transcendental Doctrine of Method,* Chapter 1, *The Discipline of Pure Reason,* Section 1, *The Discipline of Pure Reason in its Dogmatic Employment* for more detail.

That is, he imagines, or constructs, a new "quantum" in his formal intuition - an extension. Concomitantly, this gives him two more "quanta", the angles c and d.



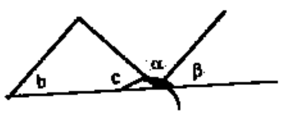
Step 2: Then he draws a line parallel to A through d, i.e. A.'

That is, he constructs, or imagines, still another "quanta".



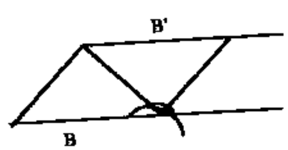
Step 3: So we have the angles c, a and b, where b must be equal to angle b.

That is, the constructed "quanta" in Step 2 concomitantly constructed 2 more "quanta": angles a and b. And mathematician also somehow observes a necessary ***quality***[71]of the "quanta" the "fact" that b is equal to b.



Step 4: Following, he draws a line parallel to B at the top of the triangle, i.e. B.'

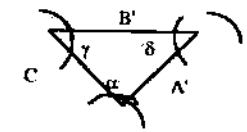
That is, he constructs still another "quanta".



Step 5: We now have a second triangle with angles a, d, g.

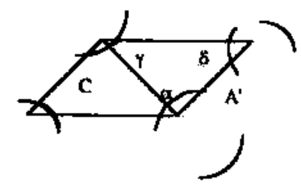
That is, the geometer has been concomitantly "given" more "quanta" thanks to Step 4.

71 Again, see my paper on Kant for more detail.



Step 6: We can show, simply by cutting and pasting (i.e. ***somehow*** "observing") that angle d = b, and thus = b, a = a and g = c.

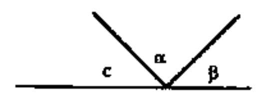
That is, somehow, the geometer conceptualizes the qualities of the quanta at hand.



αδγ =abc

Step 7: Thus, angles a, d, g = a, b, c which = c, a, b the latter set being = 180° as noted in Step 1.

That is, again, there is a "recognition" of certain "qualities" of the given quanta.



cαβ=180

Step 8: Thus, because triangle a, d, g is congruent with triangle a, b, c, given Step 6, we have shown, via "construction", that our original triangle must be made up 180,° i.e. the sum of two right angles.

That is, there is a final "recognition" of the "qualities" of the given quanta. Or in Kant's own words: "In this fashion, through a chain of inferences guided throughout by intuition, he arrives at a fully evident and universally valid solution of the problem" [A 717/B745]

In short then, similar to what we saw to be the case with Pappus, Plato and Descartes, Kant construes the mathematician as:

[1] Initially considering ***both*** the hypothesis and the conclusion.

[2] In turn, appealing to formal intuition, he "fills in the gaps". He does so by introducing, or ***constructing***, new objects.

As such, this thought-example represents a paradigmatic example of Kant's notion of the synthetic *a priori*.

In short then, what Pappus initially referred to as *analysis* later became what Plato called *recollection* in the *Meno*, what Descartes called enumeration/deduction, and what Kant called the mathematical synthetic *a priori*. But here, our concern is not to overload ourselves with any more of the specifics regarding this issue than we already have. Rather, it is enough to historically identify what I will simply characterize as the mathematical notion of construction *qua* the introduction of new objects - which in Pappus's case represented an appeal to the ancient version of "postulates", in Plato's case, an appeal to knowledge in the soul, in Descartes' case, "intuition", and in Kant's case, an appeal to "formal intuition".

In the next section, I will argue that Hume too, whether overtly, entirely intentionally or not, employed a particular version of *this* sense of synthesis *qua* the introduction of new "individuals" throughout his epistemology when it came to explaining the origins of *both* the philosophical relation *and* the mathematical relation. However, crucial to note, *unlike* the notions sketched above in terms of Pappus, Descartes and Kant, Hume's synthesis is not *necessary*, nor is it strictly *mathematical*.

Part III. Relations in Hume

Given what we've seen in Parts I and II, to determine exactly what a relation was for Hume, we need to ask and answer the following questions:

A. Does Hume have an "Early" or "Modern" subject-predicate conception of a relation in mind?

B. Or, like the "contemporaries", does Hume think a relation is some kind of independent entity that we may perceive as such, or be "acquainted with" as such?

C. Or does Hume have some kind of *constructive* notion of a relation in mind, similar to the mathematical sense of synthesis canvassed in Part II?

Here, we will see that the answers are, respectively:

A.' *No*. Hume does not think that a relation should be understood in the "Early" or "Modern" subject/predicate sense that we saw outlined in Part I; as such, his notion of a relation should in no way be understood as "relative" in the sense given in Part I.

B.' *No*. Hume does not think that relations may be perceived as independent entities in the sense that Russell and his logical contemporaries conceived of them.

C.' *Yes*. Hume does have a conception of a relation in mind somewhat similar to how mathematical objects were shown to be introduced in Part II. However, as noted, for Hume, such *constructed* relations apply to *both* philosophical *and* mathematical thought. Further they are *not* necessary. And as such, Hume does *not* make a substantive distinction between "analytic" (necessary) truths and "synthetic" (non-necessary) truths. For according to Hume, *all* relations are *themselves* synthetic, and thus, all truths are *synthetic* as well.

To show why Hume must be read in this way, in §1 I will give a detailed account of how Hume defines relations throughout his epistemological work.[72] Second, in §2, given what we will see in §1, I will show why all my answers to the questions asked above must be the case.

§1 The exegetical work

To understand just what a relation is for Hume, we must first realize he claims that with very few exceptions, we may *never* initially perceive *any* relationships as *such*, particular *or* general.[73] Rather, to speak somewhat crudely, we *create* them. In other words, the very *idea* "relation" for Hume is a product of the interrelation between the imagination and belief.

To understand this process in more detail, we need to first realize why Hume thinks only very few "relations" may be *directly* perceived as such.

The only relations we may initially perceive are, *sometimes*: just contiguity and resemblance[74] *other times*: resemblance, contrariety and degrees in quality,[75] and still

---

72 Hume's work may be roughly divided into four broad categories: epistemology/metaphysics, ethics, political/economics and history. Unlike some, e.g. Anthony Flew in *Hume's Philosophy of Belief: A Study of his First Inquiry* (London, 1961), I am convinced that Hume's ideas regarding epistemology did *not* significantly change from the *Treatise* to the *Enquiry*, his two major epistemological works. Rather, his fundamental arguments remained the same. For a brief but convincing account why, see John B. Stewart's introduction to *An Enquiry Concerning the Principles of Morals* (Open Court, 1994). As a result, I will speak of his epistemological work in the *Treatise* and the *Enquiry* in one breath and with a good conscience.

73 This is opposed to, of course, the notion that the mind may somehow grasp a relation, whether it in inheres in a subject as some kind of property or whether it is some kind of independent entity (cf. Part I).

74 See *The Treatise*, 2nd ed., edited by L.A. Selby-Bigge, Oxford at the Clarendon Press, 1992, pp. 99-101, pp. 282-284

75 See *The Treatise*, pp. 70, 79, 464.

other times, contiguity and succession.[76] I say "sometimes" because Hume vacillates regarding what relations are to be included on this list. Regardless of this detail, the significance of which we won't go into here, we should realize that these "relations" are quite vague, somewhat "weak" and Hume can only very briefly explain them in terms of "intuition". Note:

> *[resemblance, contrariety, and degrees in quality] are discoverable at first sight and fall more properly under the province of intuition rather than demonstration ...[and when we make decisions regarding these relationships] we always pronounce [them] at first, sight without any enquiry or reasoning*[77]

That is, according to this passage, and elsewhere throughout his work,[78] Hume claims we can directly perceive the relation of say, a resembles b, with*out* having to have repeated impressions[79] of a "resembles" b. In this sense, resemblance, and the other relations noted above, are *fundamental* relationships for Hume - we may *intuit* the truth that a relationship of resemblance holds between at least some impressions.

With this in mind, now note the following passage:

> *tho' I cannot altogether exclude the relations of resemblance and contiguity from operating on [the imagination] ... 'tis observable that, when single, their influence is very feeble and uncertain. As the relation of cause and effect is requisite to persuade us of any real existence, so is this persuasion requisite to give force to these other relations*[80]

76 See *The Treatise*, p. 168.

77 Emphasis my own, *Treatise*, p. 70.

78 See *The Treatise*, p. 168, p. 464, and *The Enquiry*, ed. by Antony Flew, Open Court, La Salle, Illinois, 1994, pp. 66-67.

79 Recall that an "impression" for Hume is a singular "occurrence" *either* the senses or what Hume calls "reflexion". Note: "Impressions may be divided into two kinds, those of *sensation and* those of *reflexion*. The first kind arises in the soul originally, from unknown causes. The second is derived in a great measure from our ideas and that in the following order. An impression first strikes upon the senses, and makes us perceive heat and cold, thirst or hunger, pleasure or pain of kind or other. Of this impression there is a copy taken by the mind, which remains after the impression ceases; and this we call an idea. This idea of pleasure or pain, when it returns upon the soul, produces the new impressions of desire and aversion, hope and fear, which may be properly called impressions of reflexion, because derived from it" (*Treatise*, pp. 7-8). See Chapter 1 of my dissertation for more detail.

80 Emphasis my own, *Treatise*, p. 109.

Here, Hume notes that the relation of *cause and effect* not only has *more* influence on the imagination than resemblance and contiguity have, but also that the relation of cause and effect *strengthens* our "intuitive" sense of resemblance and contiguity.

To properly understand this, we should ask: Why would Hume say that: **A.** The imagination is influenced by the non-intuitive relation of cause and effect as well as the intuitive relations of resemblance and contiguity? and **B.** How is it that the influence of the *non-intuitive* relation of cause and effect on the *intuitive* relations resemblance and contiguity *strengthens* the latter?

We may answer these questions as follows: Hume claims **A.** because it is the *imagination* that allows us to develop *other* relationships, *including* the relationship of cause and effect; imagination is the fundamental *creator* or *constructor* of *all* relations, apart from those relations of resemblance and contiguity - and whatever else Hume may occasionally include in this "fundamental relation" category (cf. p. 28 of this paper). This is the case, simply speaking, because if I have two impressions, say a and b, and I constantly perceive them as "related" in a particular way, it is only the *imagination* that allows us to "determine" *what* this relationship could be. For we do *not* perceive relationships as such, other than those already noted above. However, Hume is then of course faced with the question: "Well how would the imagination even begin to "know" how to imagine relationships? Might it not come up with something entirely idiosyncratic, if it's not *regulated* in some fashion?" The answer is: as noted in the last passage cited above, it *is* regulated, by our direct perception of those relationships of resemblance, contiguity *and* cause and effect; thanks to *them*, we imagine in a "resembling", "contiguous", or "cause and effect" way. This is why the imagination *must* be influenced by these fundamental relations; *they guarantee regularity*. We may see how Hume could have conceived of this to be the case with the following diagram/thought-experiment:[81]

81 After creating this simple diagram in an earlier paper of mine, "Hume's Method in Book I of the *Treatise*, a Glancing Prelude to Quine", I discovered that Popper created something very similar in the *Logic of Scientific Discovery*, p. 421. There however, Popper uses the diagram to argue against what he calls the "fundamental doctrine which underlies all theories of induction ... the *doctrine of the primacy of repetitions*" (*Logic of Scientific Discovery*, p. 420). Following, he distinguishes two variations of this doctrine. In the first variation, repeated instances may serve to *justify* an already formulated *universal law*. In the second variation, although repeated instances do *not* justify an already established universal law, they generate certain *beliefs* that there are indeed universal laws that hold in regard to certain repeated instances. Popper attributes this latter version to Hume. Following, he argues that with consideration of his diagram, which is similar enough to mine, both variations of the

Fig. 1 Fig. 2 Fig. 3

Given figures 1-4, ask yourself, if you were to draw the next image, Fig. 5, what would you draw? Most likely, all readers would agree that the next image should be any shape with a smaller circle in the upper higher right hand corner. That is, one would draw a shape that is - what? - *related* in a certain way to Figures 1-4. For convenience's sake, let's call this relation 'X.' Now according to Hume, one cannot claim that [s]he was born knowing X, and nor can [s]he claim that [s]he simply *saw* X, i.e. simply had an impression of it. For if the former was the case, [s]he would be claiming that [s]he can have an idea which was not ultimately derived from impressions, which, according to Hume is impossible.[82] And if the latter was the case, I could have just as easily drawn only *one* shape and expected you to explain the relation X to me, e.g. recall Fig 1:

Fig. 1

"doctrine of the primacy of repetitions" are undermined. This is the case he argues, because the comprehension of certain similarities (and thus repetitions of similar instances of forms) depends on what *point of view* the viewer either has or chooses to take. This means, says Popper, that "it is logically necessary that points of view, or interests, or expectations, are logically prior, as well as temporally (or causally or psychologically) prior, to repetition. But this result destroys both the doctrines of the logical and of the temporal primacy of repetitions" (*LSD*, p. 422). However, we must realize that Hume did *not* abide by Popper's second variation of the "doctrine of the primacy of repetitions", according to Hume, as noted in the main body of this paper, we do not experience repeated instances and then somehow come to believe in a universal law *without* the aid of some pre-supposed regulatory power. For there is the *imagination* which does indeed regulate and is indeed presupposed; allowing Hume to agree that certain "points of view", however vague they might be, *are* indeed presupposed.

82 Recall that according to Hume, *all* ideas must have their origins in impressions. See at least Book 1, Part I, Section 1 of the *Treatise*.

Looking at just this image, and asked to draw Figure 2, how could one draw a set of images related in terms of X? Rather than doing that, one could just as easily drawn a set of figures that had the same smaller-to-large spatial relation. For instance, one could have drawn:

Fig. 1 Fig. 2

Or had the same size. For instance:

Fig. 1 Fig. 2

Or had the same darkness. For instance:

Fig. 1 Fig. 2

And so on. Thus, to imagine X, it seems one needs repeated impressions of the association that appears to constitute it, i.e. you needed to have impressions of figures 2-4. However, how *many* associations one needs to imagine a relation is not an issue for Hume. Also, crucial to note, one must be able to recognize *regularity*, i.e. our ability to imagine the relation X is *regulated* by our impression of the relation regularity.

Indeed, this regulated nature of the imagination is why, according to Hume, we can understand each other, and why we develop the same relations although we may have very different experiences. This is, I am sure, what Hume meant when he wrote:

> *We are only to regard [imagination] as a gentle force, which commonly prevails, and is the cause why, among other things, languages so nearly correspond to each other*[83]

With this regulatory power of the imagination in mind, now let me emphasize that, as noted in the passage cited above, the relationship of cause and effect *also* affects the imagination. Note:

> *The qualities, from which [these] associations [of the imagination] arise, and by which the mind is after this manner convey'd from one idea to another are three, vis. Resemblance, Contiguity in time or place and Cause and Effect.*[84]

Now realize that this is the case simply because the relation of cause and effect is the *strongest* regulation placed on our imagination, and *thus* of our ability to imagine or construct *all other* relations. For recall that as noted in the passage cited from p. 109, pages 28-29 of this paper, that Hume specifically says that the relation of cause and effect must regulate the imagination when it comes time for the imagination to imagine other relations. This is the case, again, because as noted, contiguity and resemblance are really too "feeble". It is crucial for us to realize then, *that the relation of cause and effect does all the fundamental regulatory work on the imagination; it allows it to construct relations as it does.*

In addition, resemblance and contiguity are also "influenced/regulated" by cause and effect, as noted in question B. above. Thus, in this sense, the imagination and thus, the faculty that *creates all* relations is "doubly" affected by the relation of cause and effect. Somewhat visually, see that this works as follows:

1. [cause and effect] (influences/regulates →) [resemblance and contiguity].
2. But [resemblance, contiguity *and* cause and effect] (also influences/regulates →) [the imagination]
3. In turn [the imagination] (imagines/regulates →) [all *other* relations]

However, let me be careful to note that *before* the relation of cause and effect may have such an influence on either those intuited relations or the imagination, we must first *develop* the relationship of cause and effect, because we surely don't

[83] Emphasis my own, *Treatise*, pp. 10-11.

[84] First emphasis my own, *Treatise*, p. 11.

have an impression of it.[85] To do this, the psychological/epistemological procedure is basically as follows:

1. We are indeed born with the ability to "intuit" resemblance and contiguity (and sometimes, Hume thinks, those other "fundamental" relations noted above).

2. As noted, these relations inform, or regulate, our ability to imagine, and at first, with*out* the help of the relation of cause and effect.

3. We perceive certain impressions constantly conjoined, thanks to our abilities to identify, re-identify, note contiguity, and of course, remember.[86]

4. In turn *belief* is *developed;* we see a and b constantly conjoined so we begin to *believe* that they are associated in a contiguous fashion. For essential to note, we are born with *no* beliefs, but rather with an ability *to* believe; belief may only arise from repetition of the perception of certain impressions "arranged" in certain ways. Note:

> *[it is] clearly prove[d] that a present impression with a relation of causation may enliven any idea, and consequently produce belief or assent, according to the precedent definition of it* [87]
>
> *Now as we call everything CUSTOM, which proceeds from a past repetition, without any new reasoning or conclusion, we may establish it as a certain truth, that all the belief which follows upon any present impression, is deriv'd solely from that origin* [88]
>
> *belief joins no new idea to the conception. It only varies the manner of conceiving, and makes a difference to the feeling or sentiment. Belief, therefore, in all matters of fact arises only from custom, and is an idea conceived in a peculiar manner* [89]
>
> *belief, which attends experience, is explained to be nothing but a peculiar sentiment, or lively conception produced by habit* [90]

[85] Recall of course, the fact that we do *not* have an impression of cause and effect is perhaps *the* fundamental claim in Hume's epistemology; representing of course, the root of the problem of induction.

[86] Keep in mind the role of memory in Hume, which due to length restraints, I can't discuss in detail here. See at least Book I, Part I, Section III of *The Treatise.*

[87] *Treatise*, p. 101.

[88] *Treatise*, p. 102.

[89] Abstract of a Treatise of Human Nature, p. 36.

[90] Abstract of a Treatise of Human Nature, p. 39.

as it is impossible that this faculty of imagination can ever, of itself, reach belief, it is evident that belief consists not in the peculiar nature or order of ideas, but in the manner of their conception, and in their feeling to the mind. I confess, that is impossible perfectly to explain this feeling or manner of conception [91]

the sentiment of belief is nothing but a conception more intense and steady than what attends the mere fictions of the imagination, and that this manner of conception arises from a customary conjunction of the object with something present to the memory or senses[92]

5. In turn, we imagine and then come to believe in the relationship of *cause and effect.*

6. Following, when we believe that a and b must *always* be associated in such a contiguous, i.e. "cause and effect" fashion, we *imagine* the relationship of *necessity.* For like all other relations, necessity is an *imagined* relationship for Hume. Like-wise, *how* necessary a relation is simply equitable to how much we *believe* in it. Likewise, how *probable* some relationship is, is equitable to how much we believe in it; the "upper limit" of belief, so to speak, being the belief in a *necessary* relationship.

Note:

For after a frequent repetition, I find, that upon the appearance of one of the objects, the mind is determin'd by custom to consider its usual atten- dant, and to consider it in a stronger light upon account of its relation to the first object. "'Tis this impression, then, or determination, which affords me the idea of necessity.[93]

The idea of necessity arises from some impression. There is no impression convey'd by our senses, which can give rise to that idea. It must, therefore, be deriv'd from some internal impression, or impression of reflexion. There is no internal impression, which has any relation to the present business, but that propensity, which custom produces, to pass form an object to the idea of its usual attendant. This therefore is the essence of necessity. Upon the whole, necessity is something, that exists in the mind, not in objects;

91 Enquiry, p. 92.

92 Enquiry, p. 93.

93 *Treatise*, p. 156.

nor is it possible for us to ever form the most distant idea of it, consider'd as a quality in bodies. Either we have no idea of necessity, or necessity is nothing but that determination of the thought to pass from causes to effects and from effects to causes, according to their experien'd union .[94]

With this in mind, it should be clear why Hume does not appeal to or employ a strict sense of necessity in the sense that it is *a priori.* Thus we must realize that Hume's necessity admits of "degrees" of belief, with its "upper limit" representing something like "necessity", which is merely strong *belief,* not *a priori* necessity. Thus, it simply follows that there are *no a priori* necessary truths for Hume, and thus we have sufficient evidence that:

A. Hume thought *all* knowledge is fallible in terms of *a priori* or "strict" certainty because

B. There is *no a priori* sense of necessity because:

C. Necessity is merely a function of *belief.*

Concomitantly we see that:

D. The relation necessity is derived from cause and effect because:

E. We initially came up the relation necessity when we came to *believe* that a certain cause *necessarily* caused a certain effect. And thus, Hume's comment noted above: "*Either we have no idea of necessity, or necessity is nothing but that determination of the thought to pass from causes to effects and from effects to causes, according to their experien'd union* " However, this is *not* to say that Hume didn't think that necessity could not then be applied to relations *other* than that of cause and effect.

Visually, with Approach/Argument #1 in mind, see that this entire process proceeds as follows:

1. We are born with the capability to remember, imagine, have impressions and ideas, and recognize the relations of resemblance and contiguity.

2. [resemblance and contiguity] (influence/regulate →) [the imagination]

3. [the memory] (remembers →) [certain impressions]

4. [the ability to believe] (comes to believe that →) [these relations are related in a certain way]

5. [the imagination (given 2)] (imagines the →) [the relation of cause and effect]

6. [the imagination] (imagines that →) [the relation of cause and effect is necessary]

94 *Treatise*, p. 166.

7. [the ability to believe] (believes that →) [the relation of cause and effect is necessary]

8. [cause and effect] (influences/regulates →) [resemblance and contiguity].

9. In turn [resemblance, contiguity *and* cause and effect] (influences/regulates →) [the imagination]

10. In turn [the imagination] (imagines/regulates →) [all *other* relations]

11. [the imagined relation necessity] (may be applied to →) [all those other relations].

Thus, with the exception of those relations noted in 2, all relations (comparisons) are dependent on (regulated by) the relation of cause and effect.

With this in mind, realize that it simply follows that mathematical necessity must also be *probable* for Hume. In fact, note that *directly after* the long passage cited on page x, where Hume explicitly links necessity to cause and effect, he writes:

> *Thus as the necessity, which makes two times two equal to four, or three angles of a triangle equal to two right ones, lies only in the understanding, by which we consider and compare these ideas.*[95]

And then later:

> *There is no Algebraist nor Mathematician so expert in his science, as to place entire confidence in any truth immediately upon his discovery of it, or regard it as any thing, but a mere probability. Every time he runs over his proofs, his confidence encreases; but still more by the approbation of this friends; and is rais'd to its utmost perfection by the universal assent and applauses of the learned world. Now 'tis evident, that this gradual encrease of assurance is nothing but the addition of new probabilities, and is deriv'd from the constant union of cause and effects, according to past experience and observation.*[96]

That is, according to these passages, and others throughout Hume's work, mathematical necessity is indeed a function of "consider[ing] the union of two or more objects in all past instances."[97] Thus even *mathematical* comparisons, or in other words, mathematical reasoning, indeed, *all* reasoning for Hume, relies on the

95 *Treatise*, p. 166.

96 *Treatise*, pp. 180-181.

97 *Treatise*, p. 166.

notion of cause and effect, leading him to claim: "we must consider custom ... to which I attribute *all* belief and reasoning."[98]

Thus, we see that Hume thinks *there is no sharp division* between mathematical (and logical) knowledge and all other knowledge,[99] although Hume does *claim* that to most of us, there certainly *appears* to be a difference. This alleged difference is representative of what I loosely refer to as Hume's *pragmatic* sense of necessity which I discuss elsewhere in my dissertation.

§2 Evaluations

With §1 in mind, we may now make the following conclusions in light of Parts I and II of this paper:

A. According to Hume, a given relation may only be *imagined* to hold between some **a** and some **b** (with the exception of those noted above). Further, it may only be imagined as such with the help of constant conjunction and belief, as explained in detail in §1. As a result, for Hume, a relation could never be construed as being in any sense connected to a real property, or predicate of a subject; it is *imagined*, a construction of the mind. Further, a relation for Hume could never be construed as being *relative* in the sense that some thing **a** with a property **x** *infers*, much less *guarantees* that there also be some **b** with a concomitant property *y*; there is no *inference* from any **a** to **b** in Hume, we are thoroughly dependent on constant conjunction instead - i.e. we associate **a** with **b** only because we have seen them together a certain number of times. Thus, in short, Hume does *not* adhere to the traditional paradigm of relation *qua* subject-predicate.

B. According to Hume, we don't directly perceive relations, with the exception of those very few noted in §1. From this it follows that relations are *not* mind-independent entities that we may be "acquainted with", or logically conceive of as such. Thus, Hume did not belong to what I loosely characterized as the "contemporary" school of thought regarding a relation in Part I either.

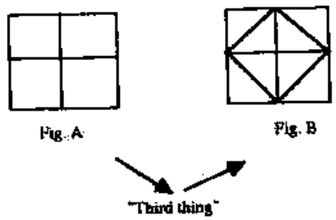
C. As noted, with very few exceptions, almost all relations are neither perceived nor are we born knowing them. They are instead, as has been shown above, *constructed*. However, as suggested above, the parallel between Hume's conception of construction is not exactly identical to the mathematical construction of objects noted earlier. Rather, the parallel must be understood in a slightly different sense.

98 Emphasis my own, *Treatise*, p. 115.

99 That is, there is no substantive distinction between "matters of fact" and "relations of ideas".

To see how, let's review what's going on here in terms of a couple of simple diagrams. First, recall the general process of mathematical synthesis as I characterized it in Part II:

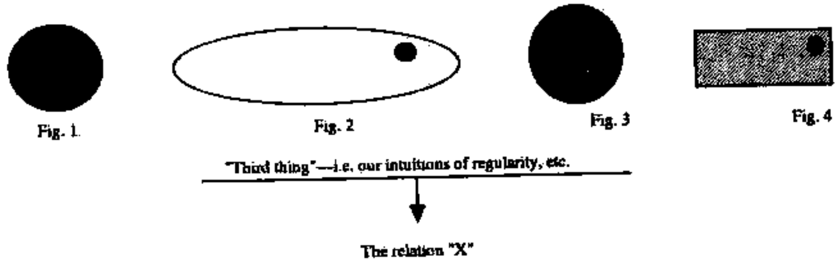
[1] a. Recall that in all cases, from Pappus to the contemporaries, relations are already "given" (to be filled in in Part II) With this in mind, note the following diagram that generalizes the constructive process.



b. Here, with our knowledge of figure A in mind, we appeal to some "third" source, e.g. formal intuition or knowledge inherent in the soul, to *create* or *construct* figure B, and thus, ultimately solve the given problem.

Now recall the general process behind the Humean construction of a relation:

[2] a. Keep in mind that for Hume, relations are *not* given. But the process of *creating*, or *constructing* them is in fact parallel to the process described above. Recall our earlier thought-experiment:



b. That is, we experience a given series of impressions constantly conjoined. Appealing to some "third thing", e.g.. our intuitions of resemblance, contiguity,

etc., we imagine, or in other words, *create, or construct* the relation, which we may understand in this case, as an imagined "*object*".[100]

Thus in short: All relations for Hume, again with those very few exceptions noted above, are bred from a synthesis that is not entirely unique, but has shown itself in terms of mathematical construction throughout the history of philosophy. However, we also see that although Hume too appeals to "intuition" as a "third thing," it is "feeble" and does *not* guarantee the *necessity* of any relation. Instead, *all* relations for Hume are contingent.

Concluding Remarks

As promised at the very beginning of this paper, we may now conclude the following:

[A] Hume did not and could not appeal to the either the *a priori* and/or the "necessary" to solve the problem of induction. For as he saw it, all relations are synthetic non-necessary constructions. As a result, any proposition, including *mathematical* propositions, must be synthetic and non-necessary. Concomitantly, Hume could not appeal to probability theory to account for the problem of induction.

[B] Long before Quine, and for reasons very *similar* to Quine, Hume questioned the "analytic/synthetic" distinction.

100 The further question is: Just how much of an "object" is a relation in Hume? Is it in any way analogous to a set-theoretical object - i.e. a set that is in fact to be identified with a given relation R?