A Formal Model of Primitive Aspects of Cognition and Learning in Cell Biology as a Generalizable Case Study of Peircean Logic

Timothy M. Rogers
Trinity College, University of Toronto, Canada timothy.rogers@mail.utoronto.ca

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Abstract

A formal model of the physical processes of digestion in a hypothetical cell is developed and discussed as a case study of how the threefold logic of Peircean semiotics works within Rosen's paradigm of relational ontology. The formal model is used to demonstrate several fundamental differences between a relational description of biological processes and a mechanistic description. The formal model produces a logic of embodied generalization that is mediated and determined by the cell through its interactions with the environment. Specifically, the synchronization of the functions of pattern recognition and semantic attribution results in an open and adaptive learning system that is stabilized by a hermeneutic circle. The relational principles of biosemiotics demonstrated through this case study are applicable to other biological systems, as well as to the relational ontology of systems theory and relativistic quantum theory.

Keywords

Peircean logic, cognition, relational ontology, semiotic freedom, systems theory, logic of distinctions

Introduction

Theses on Biosemiotics: Prolegomena to a theoretical biology (Kull et al., 2009) sets out a theoretical framework or paradigm for characterizing, researching and understanding biological organisms and systems as biosemiotic processes involving representation, signaling, and goal-directed function. Foundational to the theoretical paradigm is the threefold logic of Peircean semiotics, which is a singular expansion of the structural binary logic of objects and classes, and which discloses a dynamical and more encompassing relational logic of signs and categories.

In Life Itself: A Comprehensive Inquiry into the Nature, Origin, and Fabrication of Life, Rosen (1991) establishes core mathematical underpinnings of the biosemiotic paradigm, using mathematical category theory, which involves components, directed relations and mappings. In Rosen's formulation of relational biology, a component is the basic unit of biological organization; it is a dynamical function that formally maps inputs to outputs analogous to the way signs map signifiers (as inputs) to what they signify (as outputs). That is to say, components and signs are both formulated through the inferential logic: if (input) then (output). Components, as relational entities, exist by way of a system of entailments or directed inferences that collectively form a relational model characterizing the organization of processes in the biological organism or system. The relational model is both dynamically open to develop and respond to changes in the environment and inferentially closed by way of a self-defining circular topology of entailments.

Rosen's rigorous approach places biosemiotics on a solid mathematical foundation and unpacks hidden or obscured metaphysical commitments characterizing the theoretical paradigm of exclusively external selection that describes

biological organization through mechanistic determinism combined with random variations. Rosen's analysis of the organization of inferential entailments in relational models also demonstrates limitations inherent to deterministic models of information processing used in biology that may make them unsuitable for modelling key characteristics of biological organization, such as the ability to anticipate changes in the environment and the ability to create new types of functional ordering. As Gare (2019) has argued, Rosen's relational biology provides a strong defense of the scientific merit of the theoretical paradigm of biosemiotics. Further, Vega (2021) has developed an integrated account of Rosen's relational ontology of components and mappings combined with Peirce's agentic logic of signs.

Building on this work, in this paper Rosen's method of relational modelling (Rosen, 1991) is used to describe a concrete example of the embodiment of Peircean semiotic logic in a primitive biological system. The resulting formal model, which only involves physical processes and their relations, is explored as a representative case of generalizable principles underwriting the relational logic of agency in biosemiotics. Thus, the formal model iconically instantiates biosemiotically mediated action or *intention* in a natural system that is taken to be governed only by the laws of physics. The formal model demonstrates the principle of distinguishing distinctions that Mayer-Foulkes (2023) argues is intrinsic to the nature of living beings.

The representative formal model is used as a learning heuristic to probe three interrelated questions relevant to the paradigm of biosemiotics:

- How are higher order functional components formed in biological cells from lower order biomolecular components?
- How does the inferential logic of Peircean semiotics operate as a logic of embodied physical processes?
- How is a relational model stable yet also responsive and adaptive?

The formal model is also used to demonstrate several fundamental differences between a relational approach to modelling biological processes and a mechanistic approach. These fundamental differences have implications for developing a coherent understanding of relational ontology that is relevant to other fields of research, including systems theory (Zwick, 2023) and relativistic quantum theory (Rogers, 2022).

Overview

As the study of processes mediated by signs, semiosis is most often associated with human language and mind-dependent communication. However, it can also be used to characterize the flow of information through biological processes that do not involve human agents. This more general case involves biological organisms, such as plants or animals, that possess an interiority that exists by way of a disjointed, yet resonant, relationship with an exteriority. The resonant relationship is asymmetric or oriented, such that the interior of the organism is categorically differentiated from its exterior. A categorical difference¹ involves a *relationship* between two categories that cannot be subsumed into either category and therefore always exceeds both categories. The asymmetric relationship, often called *intentionality*, is mediated by sign-processes that involve agency and choice for the biological organism. While the application of semiosis in biological systems is well developed (Favareau and Kull, 2024), the extent to which semiosis is relevant for understanding inorganic natural systems, such as those described by quantum theory, remains an open question (Deely, 2001; Deacon 2011; Rogers 2022). By focusing on the physics of a simple biological cell, the formal model developed in this paper can be placed on this boundary between the organic and the inorganic, understood as two different categories.

Often the subject of interest in biosemiotics is the content of the interpreted world or *Umwelt*² of the biological organism which involves the interior representation of repeating exterior patterns that have significance for the

¹ A category unites constituent objects or entities or concepts by common type or sameness or homogeneity or equality. A categorical difference involves an unresolved relationship of the same to an irreducible Other—often a *qualitative* difference. While a categorical difference cannot be resolved by way of the two constituent categories, the relationship may be resolved through a higher-level category that unites the two lower-level categories *and their relations*. Duality is an iconic example of a categorical difference in physics.

² Umwelt refers to the fraction of the external world that an organism can recognize and act upon. An Umwelt is a "coarse grained" interpretation of aspects of the external world (exteriority) that is formed by the particular biology

organism. In this paper, by contrast, the subject of interest is the formal condition of possibility for the formation of an Umwelt. Central to this subject is the threefold logic of the sign. A sign is a *vehicle* of representation that can mediate the categorical difference between exteriority and interiority by mediating a relationship between an external *object* (that which is represented by the sign) and an internal response or *interpretant* (that which is signified by the object to the biological organism). Through sign processes, the object is conditioned by the interpretant and the interpretant is conditioned by the object. In the context of a biological cell, the sign can form an identity operator that resonantly relates a process in the exterior world of the cell to a categorically different process in the cell's interior world. In this way, external objects, as forms, manifest exteriorly for the organism insofar as interior responses manifest interiorly as formal interpretations of those external objects. As will be explored in this paper, this resonant possibility—*the process of distinguishing*—becomes the formative principle for an Umwelt (Mayer-Foulkes, 2023).

While signs may be physical entities as in the case of a biological cell, signs can also be mathematical entities. Usually in mathematics, a sign represents a classical mathematical object that is not a sign. Such a classical mathematical object can be called an *element*. Elements form sets. Most, if not all, of theoretical physics can be reduced to mathematical formulations based on set theory (Augenstein, 1996), which does not involve categorical differences. Set theory involves collections of discrete and distinguishable elements (Simons, 2005), each of which might be said to exist in-itself as a classical object. In set theory all elements are a priori distinguished and there is no possibility of a formative process of distinguishing. To the extent that theoretical physics reduces to set theory, it only allows for such classical objects or elements as formal types in nature and it does not allow for formative or emergent processes whereby something new in nature comes into distinction that wasn't always already formulated by the theory as a totalizing construct. However, a sign is categorically different from a classical object or element because, in addition to its being-in-itself, a sign relates irreducibly to something other-than-itself. In fact, the beingin-itself of the sign is forged from its essential relationship to something other than itself and so the sign can never be said to rest-in-itself as might be said of a classical object (Levinas, 2002). This relationship to Other—intrinsic to the nature of the sign and constitutional for mathematical category theory—is not constitutional in mathematical set theory. The formalism of signs therefore offers a different metaphysical perspective on the ontological foundations of physics and the natural sciences, not unlike the metaphysical perspective from systems theory described by Zwick (2023). It is this new perspective that is explored heuristically in this paper through the development of a formal model of sign processing in a simple biological system that is taken to be governed by the laws of physics.

A challenge to entering into this new metaphysical perspective on natural systems comes from recognizing that relation cannot merely be imported into the formalism as another constitutional element. Such a move would reduce back to set theory. Instead, relation must be formulated as involving the categorical difference that lacks distinction in set theory. The strategy used in this paper, which is motivated by the problem of time in the theory of special relativity (Rogers, 2022; pp 196-211), is to formulate relation through three mutually interdependent categorical aspects, namely light-like relations, time-like relations, and space-like relations. These three categorical aspects are treated in the manner of Peirce's threefold typology of categories³: Firstness, Secondness, and Thirdness, respectively. Following the categorical distinctions set forth by Peirce (1887-8), light-like relation is the aspect, called identity, which is "simply in itself, not referring to anything nor lying behind anything". Time-like relation is the aspect, called difference, through which something "is what it is by force of something to which it is second". Space-like relation is the aspect, called sameness or equality, through which something "is what it is owing to things between which it mediates and which it brings into relation to each other". Treated in this way, relation opens the description of the natural system to three categories of order—analogous to Bohm's three levels of generative, implicate and explicative order (Bohm and Peat, 2000)—which also follow Peirce's categorical typology. The asymmetrical aspect of relation, which characterizes the difference between mathematical category theory and set theory, further opens to a distinction between effective causation as successive progression (eg. time-like procession) and formal causation as structural scaffolding (eg. space-like symmetries), a distinction that is not possible within classical mathematical formulations of physics that involve only space-like relations (Unger and Smolin, 2015). While not unified in themselves, these two categories of causation can be unified through a higherlevel category that subsumes the two categories and their relations, namely the category of final causation (rest). In

of the organism (interiority) and, in turn, formulates what the organism can do within it. See Favareau and Kull (2004) for an introduction to the concept of Umwelt, as well as other major concepts and terms used in biosemiotics. ³ Category types are distinguished *relationally* with respect to one another, unlike classical types that can be defined axiomatically.

the development of the formal model in this paper, this paradigmatic and unifying movement of final causation (Deacon, 2011)—which involves increase towards completion (fulfilment or unfolding)—is brought about through a relation of "synchronicity" which is light-like. Likewise, the logical unfolding in this paper is intended to develop organically towards a final form, rather than through systematical construction, as key formal components of the logic are distinguished and brought into relation with other formal components towards the fulfilment of an integrated whole⁴.

Method of relational modelling

Rosen's approach to theoretical biology involves relational modelling. With relational modelling, a distinction is made between a real biological organism in the world (a Natural system) and a logical map of the relations among functional components of the Natural system (a Formal model). The Formal model is developed by a theorist to understand something about the Natural system. The Formal model attempts to capture the "organizational logic" of the Natural system by focusing on how functional processes (called components) might work together in an integrated or wholistic way. The Formal model must stand on its own as an integrated system of inferential entailments. Rosen's proposed modelling relation between a Natural system and a Formal model is shown in Figure 1

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⁴ Abductive movements of logical reasoning (belonging to the category type of *Firstness*) are explicitly brought into relation with deductive (belonging to the category type of *Thirdness*) and inductive (belonging to the category type of *Secondness*) logical processes.

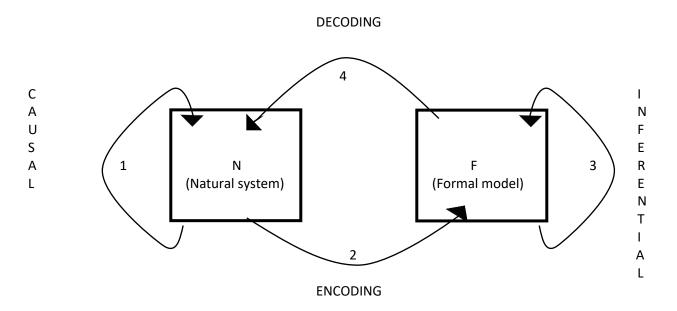


Fig. 1: The Modelling Relation according to Rosen (1991; Figure 3H.2, p60). The modelling relation connects a Natural system N with a Formal model F. The procession of events or phenomena in N (biological organism and its ambience) is governed by causal entailments in the world that impinge upon N (arrow 1). The Formal model F is governed by logical inferences or entailments hypothesized by the theoretical modeller that impinge on the components and relations in F (arrow 3). Comparison of the two entailment structures (Natural causes and Formal inferences) requires a mapping that encodes the phenomena in N into the propositions in F (arrow 2) and a mapping that decodes propositions in F back to phenomena in N (arrow 4)

In Figure 1, the Formal system F is a model of the Natural system N if the causal entailment of the relations among phenomena in N (arrow 1) is the same as what is obtained by encoding of the phenomena in N to F (arrow 2), then determining the inferential entailments within F for the represented phenomena from N (arrow 3), and finally decoding the result back to N (arrow 4). That is to say: for any phenomenal relationship in N, the relational entailment I equals the composite relational entailment I expective I entailments are what a theorist would predict about the Natural system based on the Formal model. The establishment of a modelling relation between N and F brings their respective entailment structures into coincidence. According to Rosen (1991; p61), "the modelling process compares causal entailment in N with inferential entailment in F; if we are successful in establishing such a relation, then F is the model; N is a *realization* of that model".

Importantly, Rosen's approach to relational modelling is not reductional. For example, the functional components in the Formal model do not reduce to particular material elements in the Natural system. They are taken to represent responsive, adaptive, dynamical processes within the Natural system that have no necessary material existence if abstracted from their interrelated functionality within the Natural system. Additionally, the whole Natural system maps to the whole Formal model. This allows the theorist to ask what makes the Natural system an integrated whole in terms of the formal organization of processes within any subsystem(s) and its ambience. Moreover, the inferential relations in the Formal model involve a topology of closed circles or loops which stabilizes dynamical processes as integrated wholes. Such inferential entailments are taken to model causal loops in the Natural system. Finally, the Natural system maintains open relations involving the ambience such that sub-systems cannot be isolated or abstracted from their environments and still remain intact.

The method of this paper is to use Rosen's approach of the modelling relation to develop a Formal model of primitive aspects of cognition in a hypothetical cell and to use this Formal model to address the three questions about biosemiotics posed in the introduction. From the Natural systems side of the relational modelling, research is showing that single cells possess functional capacities that are characteristic of cognition, including registering information from the environment, memorizing significant experiences from the past, and acting purposefully to endure (Dodig-Crnkovic, 2022). Moreover, the embodied realization of cognitive functioning appears to be fundamental to living organisms and is not contingent on the existence of a particular material element, such as a brain (Dodig-Crnkovic, 2022). From the Formal system side, cognition has long been recognized as characteristic of the fundamental relationship between semiosis and function in biological organisms (Kull et al., 2009). It involves an inferential structure of "if-then" that couples perception signs (as premises) to action signs (as conclusion). "Perception signs grant (with some probability) that something is the case, and consequently, the organism 'makes a decision' to act on the basis of this information" (Kull et al., 2009; p170).

The primary focus of this paper is on the development of the Formal model, which is a relational model involving a rudimentary system of inferential entailments that demonstrates primitive aspects of cognitive functioning. The Formal model is developed using the logic of Peircean semiotics with the intention that this Formal model might also demonstrate of some of the foundational principles of semiotics in biological systems (Kull et al., 2009; Gare, 2019). Whether or not this Formal model is actually realized in a Natural system is left as an open experimental question.

The Formal model stands as an integrated whole. The development of the Formal model in this paper is intended as a logical narrative that looks at some functional aspects of the Formal model in a simpler context in order to understand how the functions work. This movement from simpler to more complex contexts is a heuristic device used to help understand how the Formal model works as a whole. The logical narrative, which describes the transformation of a material biochemical pathway into a formal cellular communication channel, is not intended as either a necessary or sufficient natural system narrative of cellular evolution.

Setting up the Formal model

In the development of the Formal model, it is assumed that there is an existing level of order involving organic and inorganic molecules and their interactions that follows the natural laws of physics and biochemistry. The Formal model describes how this biomolecular level of order might be organized within the cell into functional processes at the cellular level of order. In the Formal model, the cellular level of order is not determined by the biomolecular level of order *alone*. A further process of semiotic mediation is involved that connects the biomolecular level of order—involving biochemical components and processes—with the cellular level of order—involving functional components and processes. As conjectured in the development of the Formal model, the semiotic mediation between the two levels of order is enacted by *cyclical* biomolecular processes within the cell. Synchronization of these cyclical actions imparts constrained indeterminism or semiotic freedom to the cellular level of functional order such that the cell can adapt, develop and learn through its interactions with the environment.

Let's begin with the core assumption of the existence of a biological cell with a semi-permeable membrane. The semi-permeable membrane establishes the context for the Formal model. The goal of the Formal model is to demonstrate, through a system of inferential entailments, how this hypothetical cell might acquire a rudimentary function that has properties associated with cognition. Specifically, how might this cell acquire the function of discriminating food⁵ sources as food sources in its environment?

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⁵ In developing the Formal model, the terms "food" and "digestion" are used metaphorically to foreground a process that is not yet more fully distinguished or specified biologically. "Food" refers to an unspecified component in the ambiance that the cell needs to sustain itself. "Digestion" refers to an unspecified process of integrating an exterior component interiorly. In this way, a general formal process—relating the unspecified external component to the unspecified internal function, is articulated that can then be used differentially to distinguish diverse examples of "food" that involve respective "digestion" processes, where "food" refers to the necessary and more specific external component and "digestion" refers to the corresponding internal and more specific biological function that

Let's also introduce the following terminology, adapted from Rosen (1991) and Rogers (2022), which will be used throughout the paper to differentiate a relational approach to modelling biological (or physical) systems from the much more common mechanistic approach:

- Classical description—refers to the dominant interpretative framework of determinate mechanism used in biology and physics that is based on the binary logic of objects and classes. The classical description applies when a determinate mechanistic approach is used to model biological organisms that involves determinate lawful regularity without arbitrariness or metaphysical indetermination.
- Semiotic description—refers to the interpretative framework of relational biology described by Rosen (1991). This framework for modelling natural systems is based on the *threefold* logic of signs and categories. The semiotic description applies when a *relational approach* is used to model biological organisms.

Please note that the terms *classical description* and *semiotic description* refer to idealizations (i.e. a formal ontology of objects and classes and a formal ontology of signs and categories, respectively). The classical description, which applies to Newtonian mechanics and traditional binary logic, involves an ontology of abstractable "things" in the Natural system that become formal "objects" in the Formal system. Most importantly, and in contrast to Rosen's relational ontology, the classical description treats *time as space*. In this paper, the classical description is used as a counterpoint to show (relationally) how the semiotic description is ontologically different and can manifest new logical processes that are not describable classically, such as emergent levels of order. Most models in biology are an undifferentiated combination of both the classical description and the semiotic description, although they tend to rely on the classical description *alone* to model *dynamical processes*. This paper does not intend to provide a critique of such combined approaches, apart from pointing out that more robust dynamical phenomena can be modelled if the semiotic description⁶ is fully deployed, such as pattern recognition and adaptive learning.

Placing the subject

The modelled cell is categorically separated from its environment or ambience by way of its membrane such that we can consider the cell to have a differentiated *interior* that is in relationship with an *exterior* as shown in Figure 2.

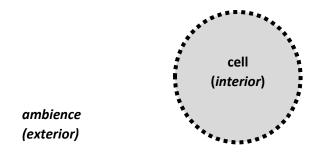


Fig. 2 Established Context. A semi-permeable membrane differentiates interior and exterior through their relation

subsumes the external component. It is the form of the relation that remains constant and will allow us transform a material pathway of biochemical components into a formal pathway of cellular communication signals.

⁶ At the level of fundamental physics, the semiotic description is a singular expansion of the classical description to include formal processes of synchronization; while the classical description is a reduction of the semiotic description in the limit of infinitely fast mediation of synchronization (eg. the speed of light $c \rightarrow \infty$) and vanishing of the

Through biochemical pathways that penetrate the membrane, molecular complexes in the ambience can cause chemical changes in the interior of the cell. This is an effective causal mechanism, whereby one molecule or molecular complex impacts another (by movement and/or chemical reactions) which then impacts another and so on. This effective causal mechanism, characterized by temporal succession, is assumed to follow the natural laws of physics and biochemistry.

Consider the case of a molecular complex X in the ambience which is a particular nutrient needed by the cell. Suppose there is a biochemical pathway from the ambience to the interior of the cell that brings the nutrient inside the cell and activates a process D whereby the cell digests⁷ the nutrient. As shown in Figure 3, we can represent this series of effective causes that start with the nutrient in the ambience and end with the activated state of digestion in the cell as a single (composite) effective causal pathway from X to D. X can be taken as the source for which D is the terminus. Note that all of these processes happen successively through the determinate laws of physics and biochemistry—this is what the term effective cause means.

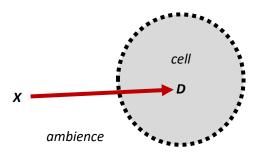


Fig. 3: Directed inference as primary relation Directed relationship (red arrow) between the external source (\mathbf{X}) of an effective causal pathway and the internally received effect or terminus (\mathbf{D})

For this case, there is logical relationship of inference between X and D. We might say that this case *realizes* a relationship of inference: *if* X *then* D. The logical relation of inference is a relation of implication or *entailment*. It tells us that the first term (X) implies or entails the second term (D) as a lawful consequence. This is a directed relationship that takes us *from* the first term *to* the second term. It can be represented by the directed arrow \rightarrow



In the classical description, we would say that the object X causes the event D. That is to say, there exists a deterministic law of physics that determines a fixed relationship between the state of the object X at one time and the event of D at a later time. This deterministic law relates the particular material object X (the particular molecular complex) to the resulting material event D (an event that is experimentally determinable through physical or material measurement processes). In the classical description, the law is taken as fully deterministic. The state of the object X determines the event D because there is a deterministic physical law that entails this relationship in time. Furthermore, when a fully deterministic law governs the whole Natural system, then the relationship between X and

⁷ See footnote 5 for a discussion of the specification of the process of digestion.

D is reversible⁸. This is equivalent to saying that the (fundamental) mechanistic laws obey time reversal invariance⁹. The entailment "if X then D", further entails "if D then X". Reversible relationships define sameness or equality ¹⁰ of material states and events. Thus, the classical description necessarily results in models of natural phenomena that are fully determined; such models are called *mechanistic* by Rosen (1991). Because the classical description assumes that natural laws are fully deterministic, we might also say that the classical description *seeks* fully deterministic laws of entailment of material objects and events in the world.

In the semiotic description, we might say that the component X is the signifying cause that triggers the process D. The component X is the source of the signifying cause and the component which has the function of digestion D is the receiver of the signifying cause. In this way of speaking there is a forward pointing (in time) biochemical law of determination 11 between X and D that takes us from X to D. Because of this effective-causal entailment, there is also the possibility of a reverse relation that is a *semantic* relation between D and X. The reverse relation would be interpreted as: X can be taken as a sign of D for the cell, where X is the sign-object, D is the sign-interpretant and the vehicle of their relatedness is the called the $sign^{12}$. The semantic content of the relation is the form of the

⁸ In this case X and D are particulars that depend on the totality of all component relations at a single instant in time (i.e. points in the phase space of the whole Natural system). The classical description, if applied to the whole Natural system, usually involves a further artifice of partitioning out a subsystem-in-itself that is modelled by states from an environment-in-itself that is modelled by dynamical laws governing the subsystem (Rosen, 1991; pp98-103) without the possibility of a return cycle between the subsystem and the environment (i.e. without agency of the subsystem). Moreover, taken as a (closed) totality, a classical description cannot entail embedded semantic relations nor the flow of information unless determinate mechanism is supplemented by the artifice of arbitrary random noise (Rogers, 2022; pp 438-59). The semiotic description, on the other hand, allows for a "semiotic cut" that relationally differentiates interior from exterior (eg. subsystem from ambience). This "semiotic cut" can introduce agency, irreversibility of time, branching of temporal trajectories, abstracting of spatialized patterns and other non-classical aspects to the model and can allow for directed logical inferences that are not reversible. Indeed, a semiotic cut between interpreter and interpreted necessarily introduces an artifice of arbitrariness or metaphysical indetermination into any classical description along with irreversibility of time. Arbitrariness (indetermination) and directed flow (directed or irreversible relation) come from an uninterpreted grounding of the lower level of order (eg. the explicated biomolecular level of order) and implicative reference to wholism at a higher level of order (eg. the implicated cellular level of order). Thus the semiotic description involves three interwoven categories of order similar to Bohm's categories of order (Bohm and Peat, 2000). The explicative category of order (biochemical level) that is given and external. The implicate category of order (the cellular level) that is internal, unfolding and involves subjective agency. The generative category of order (synchronicity) that brings wholeness and final cause. These correspond to the Peircean typology of categories, namely, Thirdness, Secondness, and Firstness, respectively. For more on the relation between semiosis, Peircean categories and Bohmian levels of order in physics, see Rogers, 2022; pp 325-37.

The important point here is that spatial relations are reversible, but temporal relations are directed (i.e. irreversible). In the classical description, only reversible relations (i.e. spatial relations) can be modelled and, as a result, time proper is replaced by a spatialized dimension. The elimination of directed relations (i.e. temporal relations) is the crucial ontological critique Rosen (1991) makes regarding a mechanistic approach. It is also the crux of the ontological critique of modern physics made by Unger and Smolin (2015).

 $^{^{10}}$ A relationship of sameness or equality forces both X and D to remain in the same causal domain, namely the domain of material objects.

¹¹ Unlike with the mechanistic approach, with the relational approach natural laws are not *necessarily* deterministic (although they may be). For this reason, the relational description is well disposed to handle laws of probability like those found in quantum theory.

¹² As Peirce emphasized, this must be understood as an irreducibly threefold relation. Gare (2019) provides a good overview of Peirce's notion of a sign and its relationship to Rosen's relational biology. He quotes Peirce's most general definition of a sign as that which "mediates between an object and an interpretant; since it is both determined by the object *relatively to the interpretant*, and determines the interpretant *in reference to the object*, in such wise as to cause the interpretant to be determined by the object through the mediation of the "sign"" (Gare, 2019; p61). What the sign communicates is a Form—"a sign may be defined as a Medium for the communication of Form" (Peirce as quoted by Gare, 2019; p75). For further discussion of Peirce's notion of the sign in the context of natural systems, see Rogers, 2022; pp18-44.

physical law relating X and D – the form that triggers a digestive response. In our case, X is a nutrient that the cell needs to feed itself. D is the function of feeding. Therefore, X can be taken as a sign of food for the cell. X signifies something (food) for the cell by way of the process D.

Lets assume, consistent with the necessary condition of the classical description (applied to the whole Natural system), that the biochemical laws governing the Natural system, including the relation between X and D, are fully deterministic.

In the classical description, we would speak of a law of determination that tells us the way in which X is equal to D. If we were to speak of signs, we would likely say D is a sign of X. That is because the event D is the *effect* for which the event X is the *cause*. By calling D a sign, we mean that because of the event D, we can refer to another event or state X, prior in time, which caused D. In the classical description effects are signs of causes, and the mechanistic approach involves seeking to identify and quantify such effective causes and their formal laws of entailment by way of their signs. But it must be kept in mind that when we use signs in this way, we mean that they are signs for us. That is to say, in the classical description, the signs are signs for an observer who is assumed to stand outside of the whole of Figure 2 (cell and its ambience) and observe the system from a timeless vantage. The interpreter of signs is called the subject. In the classical description, the subject stands outside of the embodied world (and therefore any ambience of that world that may contain a cell) and, indeed, outside of (represented) time. Often, in the classical description, the placement of the subject is not made explicit and, by default, there is an assumption of universality for the sign, as if the subject played no role in the interpretation. As a consequence, it seems to make no sense to speak of signs for a cell. Furthermore, attributing significance to signs for cells appears to impart something to the cell that it doesn't have, namely the ability to stand outside of itself and observe itself and its ambience from a timeless vantage.

To relate the classical description to the semiotic description it is important to place the subject (the interpreter) in relation to the Formal model by identifying its proper vantage point. In the classical description, the subject (the biologist) stands outside the Formal mechanistic model. In the semiotic description, the subject (the cell) is a part of the model.

Furthermore, in the classical description, the semantic content of a sign is constrained to one causal domain, namely the domain of material causes. Semantic content or meaning comes *from* the material objects and events *to* the subject. Signs are fixed and universal signs *of* material object-states and events in the world and semantic content comes from these material states and events like an output from a source. This one-way movement from material sources defines the process of *literal* interpretation. When semantic content comes exclusively from material sources, the sign refers literally within the lawful structure of relations that universally determine the world in which the cell is found. The sign becomes an indicator of the universal form of a material entity in the world. The literal use of signs allows the cell to function in the material world. The semantic content of literally interpreted signs is *determinate* by way of the structural syntax of signs, defined by physical laws of determinate mechanism.

In the semiotic description, semantic content can also move from the subject to the object. This may seem to you like a very strange statement at first. What I mean is that a material object (eg. a functional component of the semiotic system) can be taken by the subject as a sign with potential semantic content available for the subject to work out in developing its interpretation of the world¹³. A given determinate sign (with a literal material referent) is used by the cell as an iconic sign of a significant pattern in the ambience that has yet to be determined. In this case, semantic content might be said to "flow" into the sign like input to a receiver. Semantic content comes simultaneously from the domain of material causes and the domain of formal causes. The semantic content has the indeterminate meta-form of a likeness yet to be determined. The flow of semantic content is towards a final cause,

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¹³ For a discussion of this use of signs in the context of quantum theory, see (Rogers, 2022; pp 374-393).

which is the abstracted form of the pattern¹⁴. The metaphorical use of signs¹⁵ (Ricoeur, 1979) by the cell allows it to discern *potential* formal patterns in the ambience that may have significance for maintaining its integrity as a whole. The semantic content of metaphorically¹⁶ interpreted signs is *indeterminate*. An expanded syntax is needed to define the referent form literally. Therefore, signs bear the potential for creative discovery in the relational approach¹⁷.

In the semiotic description, we would say that the biochemically entailed inference X D, between the ambience and the cell, which is a consequence of physical laws, implies that X can be taken as a sign of the function D for the cell. However, at this point X only bears the potential or possibility of being a sign for the function D. This potential is not realized through the biochemical pathway $alone^{18}$. The reason why is because the (materially constrained) pathway is a directed relation $in \ time$ of successive entailments. The material object-state X results in the later material-event D, but the sematic import of X is not realized until D happens, which is the event of triggering the digestive function. For the event D, X is an event in the past whose semantic import is only realized after the fact of D, so to speak. The cell remains a part of the materially constrained causal pathway and has no vantage to relate to the effectively-entailed (entailed through physical laws) inference X D directly or immediately, $as \ a \ whole$. It is because the semantic relationship is only potential and not actual that the classical description rejects such an interpretation in principle.

Yet the approach of relational biology does allow the semantic relation to become actual *for the cell*. In order for this to happen, however, there needs to be a semiotic scaffolding (Favareau, 2015) or syntax (Ricoeur, 1979) that can contain the biochemically entailed semantic relation as a *proximate* relation *for the cell* and bring it into a system of formally or syntactically-entailed relations. By proximate, we mean an immediate relation between X and D that is not separated by a temporal series of successive intermediaries—a *semantic event*¹⁹.

In the development of the Formal model below, the *material* pathway that brings the material biochemical component X into proximity with the material biochemical event D (Figure 3) will be transformed, by the semantic event, into a *formal* pathway that communicates a semantically-constrained *signal* about X to the functional component of the cell that corresponds to D (Figure 10).

¹⁴ In the classical description the final cause of the sign is taken to be the same as the material cause (which also makes the effective cause and the formal cause the same). In the semiotic description, the final cause comes from beyond the domains of material, effective and formal causes, such that all four domains only have unity through their inter-relatedness.

¹⁵ The metaphorical use of signs allows the cell to discern *meaningful patterns*, which is the basis of generalization. Relational biology uses the mathematical theory of categories which is a theory of organization *per se* based on relations of likeness. For more on how metaphorical use is a *relational concept* (and not just a literary trope), see (Ricoeur, 1979).

¹⁶ The word metaphor references relation (not entities) and can only be spoken about metaphorically. Metaphor refers to a property of relation, called resemblance, that comes from the way in which one entity, as a sign, can be brought into proximity and substitute for another (Ricoeur, 1979). Through the metaphorical use of signs, *a pattern of sameness* can be distinguished within difference. In the context of biological organisms, the metaphorical use of signs can only be understood from a dynamical, processual perspective that brings the parts into an essential and irreducible relation with the whole.

¹⁷ This distinction of the literal and metaphorical is like Levinas' (2002) distinction of the *Said* and the *Saying* and Kristeva's (1986) distinction of the *Symbolic* and the *Semiotique*.

¹⁸ That is to say, it is not realized in the domains of material and effective causes alone.

¹⁹ Following Ricoeur (1979), a *semantic event* is "something that can be identified and re-identified [...] one must adopt the point of view of the reader or hearer and treat the novelty of an emerging meaning as his work within the very act of hearing or reading [...] metaphorical attribution is essentially the construction of the network of interactions that causes a certain context to be one that is real and unique". (Ricoeur, 1979; p 98).

Realization of semantic relations

Recall that in the mechanistic approach, the arrow indicating the relation from X to D is reversible because the physical law determining the relation between X and D is fully deterministic.

$$X \rightarrow D$$
 implies $X \leftarrow D$

This establishes a relationship of sameness or symmetry between X and D. All material (and temporally bound) object-states and events are taken to be symmetrically related in the classical description and there is only one category of relatedness, namely relationships of *sameness* or equality. We will call this category *space-like*, for reasons which will hopefully become apparent later. The mechanistic approach results in spatialized models or fixed and timeless *structures*²⁰.

In the relational approach, the asymmetric directedness of a (potential or actual) semantic relation—the relation of *reference*—is not reversible. It belongs to a *new* category of relatedness which is *time-like*²¹. The relationship characterized by time is not reversible. Therefore, in order to speak of laws of (syntactical) determination in the semiotic description, we need to find a way to incorporate *symmetrical* relations into our model. The symmetrical relations will form the syntax (or semiotic-scaffolding) for the interpretation of signs by the cell. A simple example of a symmetrical relation that is actualized in the world is a chemical bond. The symmetrical chemical bond establishes a resonant channel of communicative (i.e. semantic) exchange through mutual interaction between two molecular complexes **A** and **B** that *endures in time* as shown in Figure 4.

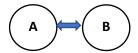


Fig. 4: Space-like relations (in blue) yield enduring co-presence (mutual interaction). A chemical bond forms an enduring relationship of sameness between two or more components

Therefore, in the relational approach we maintain two distinct categories of relations²². *Space-like* relations are characterized by sameness or equality. They involve two or more components in temporally enduring relations of

²⁰ The category of space (Peirce's category of Thirdness) reduces all causes to formal causes, that is to say pure syntax with no sematic import. In the relational approach, *time does not reduce to space*.

²¹ The category of time (Peirce's category of Secondness) maintains an *irreducible relation* of the same with the Other. This is a purely semantic *excess* that cannot be contained by syntax (Levinas, 2002).

The two categories of time-like and space-like relations correspond to the theories of *substitution* and *interaction* in the theory of metaphor as discussed by Ricoeur (1979). They also correspond to the relations characterizing time and space in modern theories of physics as discussed in Rogers (2022; pp. 460-79). Space-like relations of interaction form a structure of syntax but cannot incorporate new semantic content because they cannot reference outside of the syntactical structure. Time-like relations (namely reference) can point outside of a given system of entailments to reference something new and currently not present in the syntactical structure. It is the ability to entrain *absence or negation* (Deacon, 2011; Rogers, 2022; pp 374-93) and bring it into a syntax of determination that sets apart time-like relations from space-like relations. This introduces final causes (Deacon, 2011) which cannot be referenced through pure syntax, which is to say they cannot be referenced in the classical description. It also introduces the possibility of emergence as a movement from a lower level of organization (eg. biomolecular components) to a higher level of organization (eg. functional components of an organism). For more on the role of final causes in the organization of physical and biological processes, see (Deacon, 2011).

mutual exchange²³. These relations are characterized by *laws of determination*. They are *formal* or *syntactical*. *Time-like* relations are characterized by directedness or difference and involve succession²⁴. In these relations, there is an unreciprocated movement *from* one component *to* another component. Time-like relations are characterized by their potential for *semantic significance*. With time-like relations the movement *from* can be thought of as an output of one component for which the movement *to* becomes an input to another component. Time-like relations are successive, so that any *input* to a particular component will result in an *output* that is available to another component as shown in Figure 5.

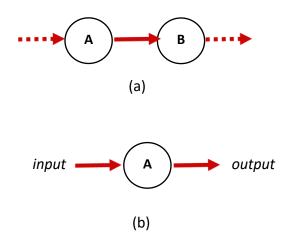


Fig. 5: Time-like relations (in red) yield successive progression or movement

- (a) Time-like relation (solid red arrow) in which the output of A becomes the input for B. This type of relation is characterized as *proximity and substitution* by Levinas (2002)
- (b) With time-like relations each component is the receiver of an input and the source of an output

To explore how the categories of space-like and time-like relations are interrelated in relational biology, lets return to our original model of a cell within an ambience (Figure 2). Suppose that the nutrient X in our Formal model can be biochemically bound to another molecular complex M inside the cell to form a hybrid biomolecular complex M_x as shown in Figure 6.

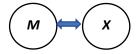


Fig. 6: Hybrid biomolecular complex M_x binding the nutrient X to the biomolecular complex M

²³ They are characterized by the logical operator AND.

²⁴ They are characterized by the logical operator OR.

Suppose further, that the cell possesses an enzyme I that can release the nutrient X from this chemically bound complex. For example, this enzyme may be part of the *last step* in the biochemical process that connects X to D (see Figure 3) as shown in Figure 7.

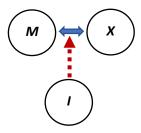


Fig. 7: Enzyme I that unbinds the nutrient from M_x

The enzyme I interacts with M_x and the effect of I is to release X. Once X is released, it can effectively cause D as show in Figure 8.

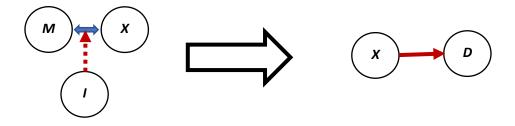


Fig. 8: Enzyme I can realize a semantic relationship bound in M_x like a memory

But the relationship between X and D, as a relationship of components, is now *immediate* in the sense that there is a *direct* connection between X and D that happens *within* the cell. Note how this is different from the *mediated* connection of successive temporal steps in a biochemical pathway from the ambience to within the cell that we started with in Figure 2. The potential semantic relation in Figure 2 between X and D has been *realized* (made real) by the cell in Figure 7 as a *semantic event*. It has the *form* of triggering the digestive function.

We can also say that I interprets the semantic relationship between X and D that is potentially present²⁵ in M_x . That is to say, I releases X which then effectively causes D. We can express this by the following logical inference:

If I, then
$$M_x \rightarrow [X \rightarrow D]$$
 Relation 1

What is important here for the semiotic description is that the biomolecular complex M_x is an *iconic sign*²⁶ of the unmediated *relationship* between X and D, a relationship that resides in the domain of effective causes. In the classical description, there is no way to reference such a *relationship* because the classical description only references *material objects*. It is not possible to speak of the *significance* of the relationship for the organization of processes in the mechanistic approach.

Recognition of general types

There is another important semantic function that the semiotic description can capture but the classical description cannot, namely the process for categorization. To see how categorization works in the relational approach, suppose the enzyme I in the cell can also be chemically bound to a new component R which sits in the membrane of the cell. We will call this component a receptor. Let the receptor have the property that it can bind to multiple *particular* molecular complexes f_i , after which it releases the enzyme I as shown in Figure 9.

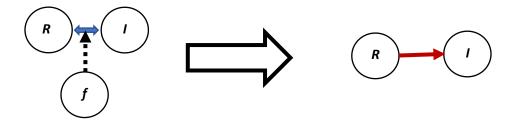


Fig. 9: Receptor R releases Enzyme I in presence of molecule of type f

In Figure 9, the receptor \mathbf{R} relates multiple molecular complexes to a single process, namely the release of enzyme \mathbf{I} . Therefore, it can map a set of particular biomolecular complexes to a single general type \mathbf{f} that is significant for the cell. Of course, the type of we have in mind is food for the cell. This type will be realized if those molecular complexes that can bind to \mathbf{R} are also nutrients for the cell.

A simple organization of processes that can provide a rudimentary cognitive function for a biological cell

Combining the two processes of categorization (Figure 9) and semantic realization or attribution (Figure 8) provides us with a model system of *organized processes* that function to recognize food for the cell.

²⁵ The potential of the original semantic relation has been stored in the cell like a memory. This is what allows the enzyme I to reference an immediate relationship of semantic significance.

²⁶ At the biochemical level of order, the sign is literal and indexical, but at the cellular level of order the sign becomes an iconic sign of a *relationship* as discussed in the next section.

The formal logic of this model is that the receptor R recognizes biomolecular complexes in the ambience that belong to a type that we have called food. The receptor releases an enzyme I that can *interpret* this recognition as a process that is significant for the cell. The enzyme I activates an archived memory M_x that releases X which then causes the activation or triggering of the digestive function D. Thus whenever R encounters food-complexes, it will trigger a digestive response.

No longer is the pathway between X and D materially constrained, rather a semantically-informed signal about X is communicated to D.

The digestive trigger can then become an *internal* sign for a process D^* that will enable the cell to *anticipate* the presence of food. Let's call this anticipatory process "food seeking". Anticipation is possible because the internal sign D provides a functional model of the *relationship* between the function of food recognition in the ambience and the biochemical event of triggering internal digestion²⁷:

f is to D*X is to D(in the semiotic systemas(through the biochemical pathway of physical laws)
Relation 2

The development of the internal sign involves a bifurcation²⁸. This bifurcation is made possible if there is a process that returns X to M to create a *cycle* involving the circular movement of X between M_x and D. The original digestive process, as a succession of events, can then be parsed into a *triggering event* D followed by a subsequent digestive process D^* . The triggering event becomes an *indexical sign* that can be interpreted by D^* . In the beginning, D^* is constituted by the successive biochemical processes of digesting the particular molecule X. However, the relation of the internal sign D to the process D^* introduces a degree of arbitrariness that can free up the subsequent process D^* such that it can take on new functionality by deferring digestion of X. Specifically, D^* may involve a new function of seeking out an external food source whose presence in the ambience has been "detected" by the indexical triggering event D.

The food seeking process D^* links the triggering of a digestive process D back to the presence of food f in the ambience by way of the functional model (which relates relations). Therefore, the presence of food in the ambience f directly entails the food-seeking process D^* . Figure 10 shows the organizational structure of processes for this anticipatory function.

²⁷ The functional model, like an analogy, involves a relation of similarity *between two relations*.

²⁸ It is beyond the scope of this paper to describe in detail how such a bifurcation process can happen within the semiotic description. However, Kull (2016) has identified the core principles of semiotic bifurcation in a paper on the formation of biological species.

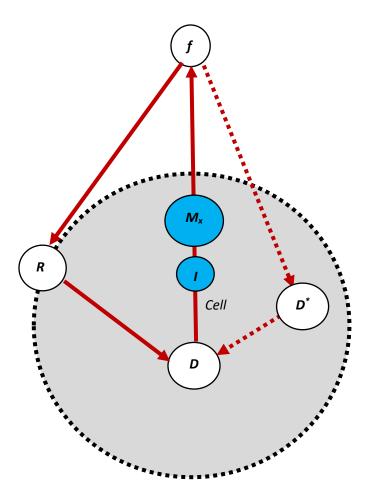


Fig. 10: The organization of an anticipatory digestive process (Rosen, 1991; Figure 10C.6, p251) A food source in the environment f activates the process of recognition by categorization R. The recognition process activates the triggering of a digestive response D through the mediation of an enzyme I. The digestive response is semantically related to the food source by way of a biomolecular memory M_x . The triggering of a digestive response D (which becomes a deferred process) becomes a signal for activating a process of food seeking by the cell D^* .

Notice that the directed red arrows in Figure 10 now relate semiotic *functions* within the organization of processes which is a higher level of organization than the original level of biomolecular components (Figure 2). The blue circles indicate biomolecular components (from the original level of organization) that have been co-opted to perform semantic functions at the higher level of organization. Food from the ambience is recognized and interpreted, the interpretation activates a memory that results in the triggering of a (deferred) digestive response from the cell. This trigger activates the food seeking function.

One remarkable aspect of this semiotic description is that the system can respond to new sources of food. The original nutrient that triggered the digestive process has been subsumed into an organization of functional

components (processes) that can control the digestive function. The original relation mediating the material biomolecular complex X to the internal functioning of the cell D (the effective cause of uncontrolled digestion) has become an archived memory for the cell that functions as an $iconic \, sign$ that triggers a response that in turn becomes an internal $indexical \, sign$ for the development of an anticipatory function of food seeking. Perhaps the cell will continue to evolve and adapt to new environments such that it no longer needs (or perhaps even recognizes) the original nutrient X. Nonetheless, the biomolecular complex X plays an important function for the digestive process because it archives the original material process (Figure 2) that was formerly uncontrolled digestion. A mechanistic approach cannot adequately explain how a material cause becomes a functional cause.

Having explicated the digestive response in terms of relationships between different functional components as illustrated in Figure 10, it now becomes possible to impose upon the cell a new mechanistic model at this higher level of order among functional components. The *explicative order* (Bohm and Peat, 2000) of biomolecular components that we began with thereby becomes the receptive ground for an *implicate order* (Bohm and Peat, 2000) of functional components within a hierarchy of orders. While a classical description can be re-inscribed on the higher order as a *structure*, it cannot relate the two levels of order directly. The semiotic description, however, can represent mediation between the two hierarchical levels of order *in time* and thereby offers a different way of understanding biological ordering. Specifically, if we focus on the role of the enzyme *I* in the developmental process that leads to the (classically) explicated order²⁹ of cellular functions, it becomes apparent that this biomolecular component mediates between the development of the function of recognition and the development of the function of food seeking, like an identity operator. Figure 11 shows the hierarchical ordering of the synchronization of developmental processes that is mediated by the enzyme *I*.

²⁹ In the semiotic description, the cellular level of order remains an implicate order in the language of Bohm and Peat (2000). In the classical description, there is only one category of order that applies (Thirdness) and it corresponds to an explicative order in the language of Bohm and Peat (2000). Therefore, in the classical description we can describe an explicative order at the biochemical level or we can describe an explicative order at the cellular level, but we cannot coherently include both levels of order together in the same deterministic model.

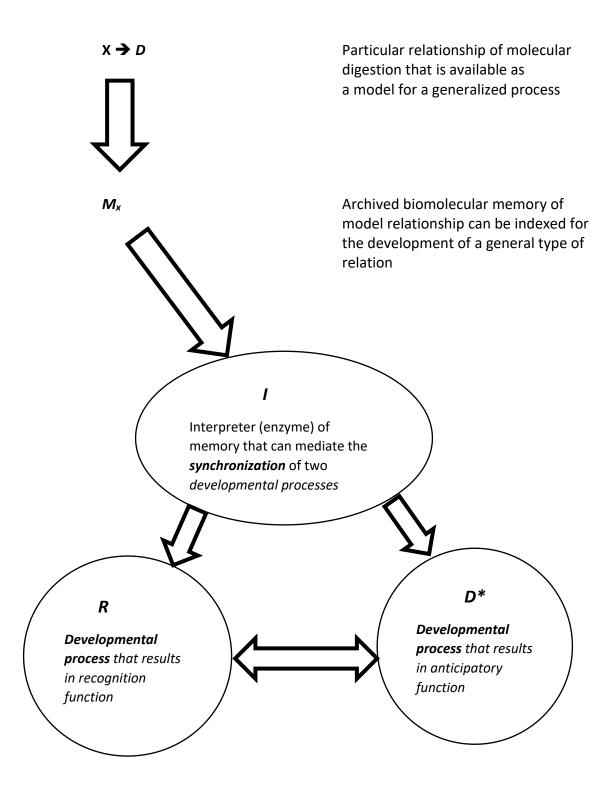


Fig. 11: The synchronization of developmental processes mediated by an interpreter

Thus, the relational approach allows us to investigate the development and adaptation of the cell in relation to its ambience in a way that is not possible with the mechanistic approach because development and adaptation are based on *temporal* relations that are not representable within the structural framework of determinism.

Synchronization of developmental and adaptive processes is made possible by closed circles of inferential entailment in the Formal model. Referring back to Figure 10, this closure manifests in our model as a lemniscate joining the two circular movements $f \rightarrow R \rightarrow D \rightarrow f$ and $f \rightarrow D^* \rightarrow D \rightarrow f$. The first circular movement entails a receptive aspect to the whole model that is informed by effective causes (biochemical processes) in the Natural system. It constitutes the model through syntax or structure or form. The second circular movement entails a creative aspect to the whole model that is informed by formal causes. It governs the model by constraining functional processes to reference semantic events, thereby generalizing them for the cell.

The hermeneutic circle and adaptive learning

In order to grasp how the Formal model "works", it is necessary to let go of the assumption that the Natural system (see Figure 1) is fully determined by the effective causal processes of biochemistry. Let's introduce a certain degree of indetermination—of flex and $slop^{30}$ —in the underlying processes of biochemistry that operate at the lower level of organization (i.e. the biochemical level).

Within the classical description, this constrained indeterminism might loosely be said to come from thermal vibrations, theoretically excluded quantum fluctuations, chemical bond flexibility, or other (relatively) random processes that are not part of the Formal model because they have no interpretable meaning for the level of organization of interest³¹. Yet, because a classical description is fully deterministic with respect to the underlying physical laws of mechanics, there is no proper way to understand such constrained indeterminism apart from an assumed randomness that is imposed externally upon the mechanistic model³². Within the semiotic description, however, the concept of constrained indeterminism is subsumed under a much more expansive concept of *semiotic freedom* (Favareau, 2015; Kull, 2023a and 2023b). Semiotic freedom does not reduce to any concept of "noise" or "randomness" within a framework of determinate mechanism. It comes from an arbitrariness in semiotic *relations* that is not representable within determinate mechanism (Kull, 2023a and 2023b). As a relational concept, semiotic freedom can imply choice in the form of the presentation to an agent of simultaneously available possibilities (Kull 2023a). It can lead to creative freedom of semiotic agents (Favareau, 2015; Kull, 2023a and 2023b).

Because of semiotic freedom, the creative or governing aspect of the whole system of entailments in the Formal model is able to "guide" the receptive aspect towards an organization of processes that accommodates generalization. The lemniscate of entailments acts like a *seed* which draws the *whole* Natural system, as its develops, towards an "attractor" which is represented by the Formal model.

Drawing from the field of Artificial Intelligence (AI), the Formal model has a functional organization that is similar to a formal learning system based on reservoir computing involving generalized synchronization (Verzelli et al., 2021). The learning system involves an initial training phase of *listening* and *fitting*. During this phase, the cell creates a meaningful representation of the ambient molecular components (the ambient input) impinging on the recognition function by releasing the enzyme *I* whenever the input matches a specific food source *fi*. This response

³⁰ The terms "flex" and "slop" come from Smith (1998).

³¹ In fact, some form of metaphysical indetermination (such as an external randomness beyond the laws of determinate mechanism) *necessarily* comes from the semiotic cut between interpreter and interpreted in the classical description, whether or not this disjuncture is part of the model. See footnote 2.

³² However, if the underlying physical laws are based on a theoretical paradigm consistent with a relational ontology, such as relativistic quantum theory, then constrained indeterminism is found at all levels of order, including the underlying Heisenberg indeterminism of quantum mechanics (Rogers, 2022; pp 226-98). Logically, the difference is that determinate mechanistic models only involve Peirce's category of Thirdness (which is the category of spatial relations in theories of physics) while relational models involve all three categories of Firstness, Secondness, and Thirdness (which are the categories of light-like, time-like, and space-like relations in theories of physics) (Rogers 2022; pp 460-79).

is followed by the food seeking response, which is a function that "computes" the desired output of the cell. The cell's output is the attribution of food source to the molecule(s) impinging from the ambience. That is to say, the cell responds yes or no to the input from ambience, based on whether or not that input is the food source f_I that generates the full digestive response \bar{D}^* (i.e. food seeking plus subsequent digestion³³). The successful completion of digestion provides the confirmation that the recognition function has correctly identified f_I. This initial training phase of listening and fitting matches the recognition function to the food seeking function as already discussed above. Together they form a synchronization function, whereby the food seeking function forms an input echo to the recognition function. The synchronization function maps the input from the ambience (unknown source), through the recognition function (potential recognition of food source), to the completion of the digestive response (confirmation of food source). The learning goal of the training phase is to produce an accurate prediction of food from the unknown sources impinging from the ambience. Once the training phase is completed, the recognition function can accurately "predict" when an unknown source from the ambience is food of the specific type-token f_L . In this way, the synchronization function asymptotically approaches a "synchronization manifold" where the final output of the cell (digestion) depends only on the environmental inputs (unknown sources). In the subsequent predicting phase, the cell can expand its predictions beyond the specific original training type-token f_I to identify new type-tokens of food $\{f_2, f_3, f_4, \ldots\}$, belonging to the same functional category defined by the full digestive response, based on the whole history of food sources encountered by the cell.

In the Formal model of the Natural system, synchronization can create a hermeneutic circle of return that further enables adaptive learning by the cell in response to changes in the ambience. The hermeneutic circle is fulfilled if we postulate a process whereby the confirmation of digestion releases the enzyme I back to the recognition function thus preparing the synchronization function for a new iterative cycle of response. In the first instance, this process of return may operate on average within a cell that possesses an ensemble of recognition and digestion componentsan uncontrolled process of return. The hermeneutic circle of adaptive learning has the following general form. A pattern recognition function (the recognition function) releases an output label I in response to an input from the ambience that is a potential food source of the (open) category f. The label becomes an input to the food seeking function that triggers a food seeking response. If the food seeking function is successful in finding and digesting food, the label *I* is returned to the pattern recognition function with the semantic attribution of food, thus positively reinforcing the pattern recognition function with respect to the unknown source. In this way, the recognized pattern enters into a syntax of meaningful representation which is defined by the synchronization function. If the digestive function is unsuccessful, the pattern recognition function does not receive positive feedback and the unknown source does not enter into the syntax of semantic representations for the cell. Co-ordinated by the representing label I, the hermeneutic circle of return allows the cell to tentatively predict new food sources and bring them into the syntax of semantic attribution³⁵ if they successfully complete the digestive response. Both the recognition function and the semantic attribution function (namely the full digestive response) can be adaptive and change with time as long as the circle of return remains intact as shown in Figure 12. As argued by Smith (2019), AI systems based on machine learning do not have this capacity for adaptive learning through participatory engagement with the ambience.

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³³ Note that the full digestive response (D^*) is now differentiated from the digestive trigger (D) by the bifurcation described earlier. The food seeking function and the full digestive response are the same function after the bifurcation. But the digestive function has acquired a new anticipatory function of food seeking. To simplify the discussion, this new composite function is referred to as the food seeking function.

³⁴ The synchronization manifold plays the role of an "attractor" for the dynamics that has the property that the response of the cell is determined only by the environmental input. This is the property that allows the cell to expand its basis of predictions $\{f\}$ beyond f_1 to other food sources f_2, f_3, \ldots . The nature of the synchronization manifold is further explored in the discussion section of the paper.

³⁵ Semantic attribution comes from the fact that the hermeneutic circle signifies something external for the cell, namely an input of type $\{f\}$, where $\{f\}$ is *defined by the cell* through the food seeking function D^* .

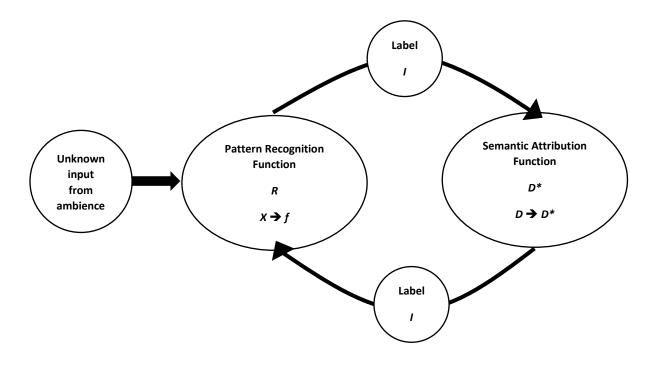


Fig. 12: The hermeneutic circle

Through the hermeneutic circle, the pattern recognition function R is synchronized with the semantic attribution function D^* by way of the label \underline{I} . The circle of entailments allows the system to generalize a particular iconic relation $X \twoheadrightarrow D$ (an internal model of a significant relationship of the ambience to the cell) into a general relation of functional types $f \twoheadrightarrow D^*$ that is expandable and adaptive.

Semiosis and Cognition

The diagram of the Formal model in Figure 12 depicts a *symbolic* sign in Peircean semiotics in terms of a relational ontology of functional components³⁶. The label I is the *name* of the sign. The *external* pattern recognized by I, from the unknown ambient inputs, is the sign-object. The semantic attribution by I is the interpretant. Unlike the case with mechanistic determinism, in the relational approach of the semiotic description, the symbolic sign does not represent a fixed material object in the ambience. Rather it represents a *formal*, *abstracted pattern that has significance for the cell* as defined by the semantic attribution function. What the cell comes to represent from its ambience is therefore determined by the cell's adaptive functions and is not something universally represented simply by its material "givenness" in the ambience. Furthermore, both the pattern and the semantic attribution can develop over time to meet the changing needs of the cell. In this sense, we might say that the Formal model enacts primitive processes of *uncontrolled* cognition. Some of the key aspects of cognition that are exhibited by the Formal model include the ability to register and represent abstract information (i.e. formal patterns), the ability to archive memory, the ability to anticipate and act purposefully.

The symbolic sign enacts the logic of a conditional³⁷: If R then D^* , in which both R and D^* are variable functions. The label I fixes the conditional structure of the formal logic. The conditional structure not only synchronizes the two functions, it also constrains valid adaptations in such a way that R and D^* can change arbitrarily as long as the hermeneutic circle of input-output is maintained by the label I. The two functions form a duality whose property is that they mutual constrain one another while allowing for some degree of internal freedom. This is a fundamentally threefold relation that is not representable through the binary logic of mechanistic determinism. Constraints create a condition of possibility (Zwick, 2023; Deacon 2011) whereby semiotic freedom can creatively guide the organization of functional processes for the cell.

The label I indexes the semantic attribution of the sign as its significance unfolds in time. In the Formal model, I first picks out the original embodied material relation $X \rightarrow D$ that represents *uncontrolled* digestion by way of the archived memory M_x . The *duality* of the recognition function and semantic attribution function, acting in synchronicity, then allows the *particular material relation* (the iconic relation) to become generalized to an abstract conditional relation of *formal types*. In this way the *implicit* pattern in the iconic relation that is of significance to the cell is unfolded or *explicated*; the pattern *develops* over time. This process is fundamentally temporal and has no correlate in determinate mechanism.

Additionally, the symbolic sign mediates the emergence of *controlled* digestion from *uncontrolled* digestion. Control is made possible because the organization of functional processes in the cell by the sign creates a "downward" pressure on the biomolecular level to co-opt the arbitrariness in biomolecular bonds (thermal vibrations, quantum fluctuations, chemical bond flexibility, etc.) into determinate patterns of significance for the interpretation of the cell. Such control is a *formal cause* that goes from the higher level of cellular order to the lower level of biomolecular order. This type of cause is invisible within the classical description of determinate mechanism because only effective causes are allowed in that description.

It should be pointed out that the threefold logic of the sign is much more fundamental and applicable in nature than the particular Formal model of that logic presented here. Referring to Figure 12, for example, we might consider this to depict a pre-cursor to a bicameral brain where the pattern recognition function sits in the right hemisphere, the semantic attribution function sits in the left hemisphere, and the labels are signals crossing the *corpus callosum*. This could create a condition of possibility for the internalization and control of the primitive aspects of cognition demonstrated in the Formal model, resulting in what is commonly referred to as cognition in vernacular language.

³⁶ In terms of the diagrammatical representation of the formal structure of the hermeneutic circle the label I is arbitrary as is the case with a symbolic sign. However, there remains a functional relationship between the label I and the memory M_x at the biochemical level of order that is historically established and therefore not arbitrary. Moreover, the whole hermeneutic circle is a general type that relates pattern recognition to semantic attribution, where the functions of pattern recognition and semantic attribution can be distinguished specifically according to the specific of the cell. That is to say, the hermeneutic circle connects a specific category of pattern recognition to a specific type of functional response irrespective of the specificity. A given cell may instantiate numerous hermeneutic circles, each corresponding to a specific cellular component or function. See also footnote 5.

³⁷ Quantum coupling in physical processes similarly enacts the logic of a conditional (Rogers 2022, pp 374-93).

Discussion

In a recent paper, Vega (2021) has provided a detailed discussion of the connections between component mapping, functionality and semiotic relations that underwrite the integration of Rosen's relational biology with Peircean semiotics. His analysis carries forward to this model with one key exception. In his treatment, the biological organism is taken to be a system that is closed with respect to effective causation *alone*. This assumption is similar to reducing the organization of the functional components in Figure 10 to an *explicative order* that is fully determined by effective causes. The assumption further cuts off or dissociates the biochemical level of molecular interaction from the cellular level of functional interaction, resulting in two separate and physically unrelated realms of effective causation. As a result, formal and effective causes become synonymous and one can no longer use the Formal model to investigate the synchronization of levels of order.

The approach used in this paper maintains a dynamical interaction between the biomolecular and the cellular levels of semantic organization. This is possible because the closure of causation involves an inter-change between formal and effective causation as discussed above. To use the language of Vega's paper (2021), there are two types of closure involved in the Formal model: 1) a local closure that involves effective causes and defines the semiotic relation and 2) a global closure that involves formal causes and defines the cell. But the Formal model is not closed with respect to either 1) or 2) alone; rather it is closed with respect to a dynamical interchange between 1) and 2). Topologically, the Formal model involves a lemniscate that relates formal and effective causes to create a closed *double loop*, rather than a closed *circle* of effective causation alone. This further implies that formal causation and effective causation must be handled separately and do not reduce to one another in the Formal model as they do in Vega's discussion.

The pivot sign for this model that distinguishes it from one in which there is closure with respect to effective causation is played by the enzyme *I*. As mediator, the enzyme synchronously links two otherwise disconnected semantic orders—the lower-level biomolecular order and the higher-level cellular order. It establishes the *metaphorical relations* discussed above. As argued by Kull (2023b), relations *between* logically incompatible rules or codes operate in a region where "non-contradiction is not yet set"; such "betweenness" is the necessary condition for semiotic agency and freedom. In this light, we might say that the enzyme, as an individuated mediator, is a semiotic agent of the cell that operates between the biomolecular order and the cellular order.

Within the relational ontology of the Formal model, the enzyme I plays the role of naming. Naming is a purely semiotic concept that has no correlative in the classical description in which naming has always already happened. As a name or sign-vehicle, the enzyme I maintains a constant relation between the external (and generalized) object of food f in the environment and the internal interpretant of the food seeking function D^* . This threefold relation between sign-vehicle, object, and interpretant endures through multiples iterations of particular encounters that the cell experiences with its ambience. Naming is the identity operator of the symbolic sign.

Similarly, in establishing control over the digestive process, the biomolecular component X plays the semiotic role of iconic likeness generation. Insofar as it becomes internalized by the cell to create a dynamic cycle between the biomolecular memory M_x and the triggering event D, X becomes a semiotic agent that represents type-tokens for generalization.

Naming and likeness belong to a category of relations, provisionally called Return relations, that are *light-like*. Return relations (which belong to Peirce's category of Firstness) are semiotic identity operators. They enable, unite and complete *time-like* relations (which belong to the category of Secondness) and *space-like* relations (which belong to the category of Thirdness). With Return relations the *semiotic* identity of the cellular semiotic agent (which is different from its biomolecular function) remains constant while the semiotic agent moves iteratively around a cycle of discrete and separated functional contexts. Thus identity in Return relations is formed through *participatory action*. As freely created cyclical actions, Return relations also have a property of logical ambiguity or indeterminacy—sometimes the semiotic agent completes the cycle and sometimes it does not. Therefore, although semiotic identity operators are constrained in the Formal model, they are not *determined* by the Formal model. Rather semiotic identity operators act as transcendental signifiers pointing to something beyond the Formal model. They create the condition of possibility for the emergence of new ordering, for example.

Through Return relations involving indeterminate cyclical action, semiotic agents break the successive chain of effective causation at the biomolecular level that is formed by time-like relations (see Figure 5). They become semiotic identity operators that form a network of cyclical nodes at the higher level of cellular functional organization. These nodes can be proximately connected³⁸ by space-like formal relations (See Figure 4) to index a fixed *structure* of functional components of the cell. The proximate connectivity of this structured network of cyclical nodes has the logical form

$$[X]_c \rightarrow [I]_c = [X]_c \leftarrow [I]_c$$
 Relation 3

Here $[X]_c$ and $[I]_c$ are the Return cycles of the semiotic agents X and I, respectively. Semiotic agents are able to creatively enact this higher-level order because they *simultaneously* participate in the effective causal chains of the lower-level order of biomolecular components and the formal causal structure of the higher-level order of functional components of the cell. This is the way in which the biomolecular level of order can creatively form a higher-level structure of functional cellular components while at the same time the functional cellular organization is able to influence the lower-level biomolecular order. The proximate connection of the semiotic nodes in the functional structure of the cell thereby becomes the generative origin of biomolecular *communication channels* mediating the flow of semantic information between functional components of the cell.

The Formal model is a logical depiction of the embodiment of the process of distinguishing distinctions as discussed and developed by Mayer-Foulkes (2023). The mutual relation between the memory cycle and the hermeneutic circle (Relation 3) is the condition of possibility for establishing what Mayer-Foulkes calls "Bet structure", namely a recurrence relation between two sequences. Bet-structure establishes validity conditions for the Formal model as outlined in the Appendix.

Generalization

In analogy with the category of signifiers used by Godel to prove the incompleteness of Number theory (Hofstadter, 1980; Goldstein, 2005), let's define a transcendental signifier as a signifier that represents a truth condition that is not decidable within the binary logic of *any* formal model of the Natural system³⁹. In the Formal model, Relation 3 becomes the relevant truth condition of the transcendental signifiers $[I]_c$ and $[X]_c$ – the condition of synchronicity.

Furthermore, we can express the relational logic of the functional structure of the Formal model in terms of a memory *label* \mathbf{M} in the following way. Relation 1 (on page 11), characterizing the memory \mathbf{M} , can be re-expressed as:

$$[I]_c \rightarrow M = [X]_c$$
 Relation 4

Relation 2 (on page 13), characterizing the functional model, can be re-expressed in terms of the memory M as:

$$[I]_c \rightarrow f :: D^* = M$$
 Relation 5

Combining Relation 4 and Relation 5 results in an embedded relation of cycles:

³⁸ For a discussion of the nature of this proximate relation, see (Rogers 2022; pp 129-147).

³⁹ This is the condition of physical embodiment. It applies from the cellular level of functional organization through the biomolecular level of order down to the level of quantum physics. The renormalization of self-energy in quantum electrodynamic theory (QED) would be another example of a transcendental signifier.

$$\{ f :: D^* = [X]_c \}_{[I]_c}$$
 Relation 6

Relation 6 states that the cycle enacted by the semiotic agent X is the same as the generalized relationship of food in the environment f to the food seeking function D^* in the context of the cycle enacted by the semiotic agent I. The context of the cycle enacted by the semiotic agent I is defined by Relation 3. It is a state of synchronicity between the creative governing cycle $[I]_c$ and the responsive cycle $[X]_c$.

As a result of this synchronic coupling mediated by the transcendental signifiers, the formal logic of the recognition function and the semiotic attribution function form a duality of constrained *signifiers* that reference a general form of pattern in the ambience whose significance is defined by the action of the cell. Thus, the coupled transcendental signifiers create a resonant channel for the flow of semantic information (i.e. significant patterns) from the ambience to the cell. This resonant channel is constituted from biomolecular processes that have become cellular level *signifiers* of the Formal model. That is to say, the biomolecular semiotic agents, as processes with a degree of semiotic freedom, form communication channels for the flow of information at the higher level of cellular order. The Formal model acts as a final cause (Deacon, 2011) that guides the formation of communication channels by reinforcing the coupling between the memory cycle and the hermeneutic circle.

For the Natural system side of the modelling relation (Figure 1), the Formal model leads to the following hypothesis that is open to experimental verification⁴⁰. In some cells a hermeneutic circle is established between a biomolecular recognition process on the cell membrane and an internal anticipatory digestive process, which is shepherded by an enzyme. Within this cycle, a memory cycle is established that forms a trigger for the anticipatory digestive response whenever the memory cycle is activated. The synchronization of these two cycles allows the cell to anticipate and adapt to new food sources in the environment because the trigger indexes change in the recognition process to change in the anticipatory digestive process.

The underlying logic of the Formal model is necessarily incomplete and it is this incompleteness that allows for creative emergence of hierarchical levels of order. But this also means that the Formal model can only provide a partial or limited description of the Natural system. More fundamentally, the emergence of transcendental signifiers (Return relations) in the threefold logic of Peirce makes explicit the logical incompleteness that is always present in any formal model. By contrast, as Rosen (1991) shows, in mechanistic models logical incompleteness can easily be obscured or overlooked resulting in a false supposition that the classical description can describe a totality. Incompleteness has important metaphysical implications that are beyond the scope of this paper to discuss (Levinas, 2002; Kristeva, 1986; Zwick, 2023; Deacon, 2011; Rogers, 2022).

Appendix: Synchronicity generates validity conditions for the integrity of the modelled system as a whole

The Formal model is characterized by a network of cyclical nodes that are proximately connected by space-like relations. The synchronicity condition for the cyclical nodes is defined by Relation 3 in which the logical inference is reversible. As outlined below, synchronicity generates validity conditions for inputs from the ambience that are determined by the temporal development of the functional processes of the cell *as a whole*. The generation of validity conditions is analogous to the logical basis of distinguishing distinctions as developed by Mayer-Foulkes (2023).

Consider $[X]_c$, the Return cycle of the semiotic agent X. This cycle is completed in response to unknown inputs from the ambience which form a time-like sequence as show in Figure A1.

⁴⁰ That is to say, the structure of the conditional relation between the two functional processes can be interrogated experimentally (see Figure 12).

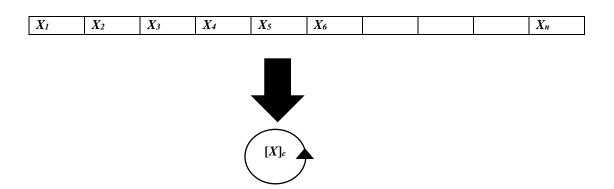


Fig A1. Inputs to the Return cycle of semiotic agent X Unattributed inputs from the ambience X_i form a time-like sequence indexed to the Return cycle $[X]_c$.

Similarly, the Return cycle $[I]_c$ is completed in response to inputs from $[X]_c$. The relevant output from the semiotic agent X that forms a *potential* input to $[I]_c$ is the successful completion of the cycle $[X]_c$. However, only those outputs from $[X]_c$ that further generate a completed cycle of the semiotic agent I register as valid inputs to $[I]_c$. This valid input can be labelled as a memory M as shown in Figure A2.

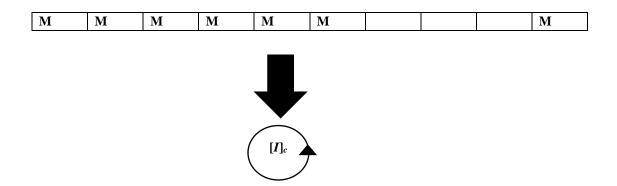


Fig A2. Inputs to the Return cycle of semiotic agent I The input—from the Return cycle $[X]_c$ to the Return cycle $[I]_c$ —forms a time-like sequence of identical inputs indexed to the Return cycle $[I]_c$. The indexical label M represents those cases in which Return cycle $[X]_c$ is in *synchronicity* with $[I]_c$ such that the output from $[X]_c$ is *validated* by the completion of $[I]_c$.

Therefore, the two Return cycles generate two *different* time-like sequences, each indexed to its respective Return cycle. In the classical description, there is only one time sequence possible which is the universal or absolute time sequence that applies to all components of the system *as well as to the observer and the ambience*. This universal time is the only meaningful notion of synchronicity for the classical description. That is to say, in the classical description the world is taken to be instantaneously and infinitely synchronized at all levels of order to a universal temporal sequence. In the semiotic description, the two different time-*like* sequences of the Return cycles $[I]_c$ and $[X]_c$ can be synchronized by way of an internal indexical label M, where M represents a *completed process*, as shown in Figure A3.

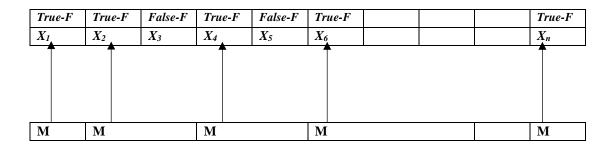


Fig A3. The synchronization of the two different time-*like* sequences from Figure A1 and Figure A2 The internal label **M**—representing completed Return cycles of $[I]_c$ —selects true tokens of the general type F from among the unattributed inputs to the Return cycle $[X]_c$.

When the two time-like sequences are synchronized internally as in Figure A3, the condition of synchronicity between $[X]_c$ and $[I]_c$ (Relation 3) picks out true tokens of the general type F (i.e. "food") from the unknown inputs and rejects false tokens of the general type F. But the Return cycle $[I]_c$ is also taken to be in synchronicity with the cell as a whole. This state of synchronicity involves the coordination of the network of cyclical nodes that regulate functional processes and define the functional level of order for the whole cell. The coordinated network of nodes creates formal constraints which entrain and entail semiotic freedom. The coordination governs the creative aspect of the formal model by imparting formal constraints that pertain to functioning of the whole cell. Through synchronicity, the network holds in tension the structured semiotic scaffolding of actualized semantic relations and the dynamical potential of novel metaphorical relations. The internal label M represents a cyclical biochemical process⁴¹, formally selected by the whole cell like a memory, through which external inputs (as processes) to the cell are brought into resonance with internal processes within the cell. The label formally indexes external patterns in the environment to internal patterns within the whole cell. That is to say, M is an indexical sign that represents patterns in the environment (sign-objects) by patterns in the cell (sign-interpretants). This representation governs the receptive aspect of the formal model by indexing internal processes to external processes.

Therefore, the condition of validity for a token X_n to belong to the category F is that the token enter into the synchrony of cellular functional processes $as\ a\ whole$. False tokens do not enter into the syntax of the whole cellular semiotic system because they remain "out of phase" with the overall synchrony and therefore uninterpretable. Moreover, because the external token X_n is represented internally by the selected process M, the condition of validity is *relational*, connecting *objective* processes in the external environment to *subjective* processes within the cell. In this way the whole cell as a coordinated synchrony of cyclical actions can adaptively respond to its changing environment.

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⁴¹ M represents a Peircean *interval*.

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