Zeno Beach

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> Οἴονταί τινες, βασιλεῦ Γέλων, τοῦ ψάμμου τὸν ἀριθμὸν ἄπειρον εἶμεν τῷ πλήθει.
>
> There are some, King Gelon, who think that the number of the sand is infinite in multitude.
>
> — Archimedes, *The Sand-Reckoner*[1]

When Aristotle discusses infinity in *Physics* 3. 4–8, he is normally taken to assert that no infinite quantity of any kind, be it magnitude or plurality, can exist in actuality. On the other hand, it is generally acknowledged that practically all of Aristotle's *arguments* against infinite quantity concern magnitude, not plurality.[2] This has left commentators facing a puzzle. Did Aristotle know of good reasons for denying the actual existence of infinite pluralities? And if so, what were these reasons? In this paper I will examine this question, and try to deepen the puzzle that it poses for existing interpretations of Aristotle.

More specifically, I am interested in whether Aristotle knew of good reason to deny the possibility of an actually infinite plurality *of extended things*. Pluralities of extended things are important because they pertain to the metaphysics of continua and the parts of continua. Aristotle

1. C. Mugler (ed.), *Archimède* (Paris, 1971), 134.2–3, tr. Heath.

2. D. Bostock, 'Aristotle, Zeno, and the Potential Infinite', *Proceedings of the Aristotelian Society*, 73 (1972), 37–51 at 38; E. Hussey, *Aristotle's* Physics*, Books III and IV* [*Physics III–IV*] (Oxford, 1983), 82; U. Coope, 'Aristotle on the Infinite', in C. Shields (ed.), *The Oxford Handbook of Aristotle* (Oxford, 2012), 267–86 at 268.

emphasizes in *Physics* 6, and elsewhere, that continuous things such as lines, motions, and times are infinitely divisible into parts.[3] Furthermore, as it seems to me, his manner of writing and arguing in *Physics* 6 suggests that, whenever a thing is divisible into a part with a certain position and size, there already exists a part of the thing with that position and size. These parts may have a distinctive mode of being (perhaps they exist in capacity, not in actuality), but at any rate Aristotle proceeds as if parts are there to be identified and labeled with letters, have determinate positions and sizes, and are determinately non-identical with one another.[4] It would seem to follow that, for every continuous thing, the answer to the question 'how many parts?' is that there are actually infinitely many parts. If Aristotle had reason to reject this outcome, it would be useful to know exactly what his reasons were.

My own view is that Aristotle did not know of good reasons to deny the existence of an actual infinite plurality of extended things. I also think that Aristotle did not, in fact, deny the existence of such a plurality.[5] I will try to support these views in the present paper. Since negative claims

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3. *Post. An.* 2. 12, 95^{b}9–10, 29–30; *GC* 1. 2, 315^{b}26 ff.; *De caelo* 1. 1, 268^{a}6–7; 3. 7, 306^{a}26–8; *Metaph.* Δ 6, 1016^{b}24–31; I 1, 1053^{a}24; *Phys.* 1. 2, 185^{b}10–11, 16–19; 3. 6, 206^{a}9–11; 3. 7, 207^{b}25–7; 6. 1, 231^{b}11–12, 15–16; 6. 2, 232^{a}23–5, 232^{b}23–5; 6. 3, 234^{a}10–11; 8. 5, 257a33–4.

4. I make this claim about *Physics* 6 only, at least for now (but see *Phys.* 8. 8, 262^{a}22–4, 28–9 for quantification over potential midpoints and for the assignment of a name, 'B', to a potential midpoint). Coope argues for the opposite interpretation of *Phys.* 3, 4, and 8, in U. Coope, *Time for Aristotle:* Physics *IV.10-14* [*Time*] (Oxford, 2005), 9–13. If the present paper succeeds, then the force of Coope's argument is significantly weakened.

5. I argue for the latter claim in 'Aristotle's Actual Infinities'.

like these are hard to prove, however, my ambitions will be limited. I will be looking at putative *reasons* for finitism, not arguing about whether or not Aristotle *endorsed* finitism. Nor, of course, can I claim to deal with every possible reason Aristotle may have had. I will focus on three prominent arguments, each of which can claim ancient provenance and each of which has been attributed to Aristotle by some commentators. For each argument, I will aim to show that it relies on at least one premise that is not supported in Aristotle's extant writings. In two cases, I will try to show that Aristotle would positively have rejected the argument.

For readers who are interested in viewing Aristotle as an accepter of actual infinite pluralities, the arguments of this paper should make this view of him more plausible. For committed finitist interpreters, I hope that my discussion of the evidence will still be useful. It will, I hope, advance our understanding of the history of arguments on the topic of infinity, and of the considerations that may and may not have motivated Aristotle to take whichever view he took.

To pursue this project, it will be useful to work with an example. So let us take Zeno Beach. On Zeno Beach, there is a grain of sand the size of a poppy seed. And a grain of sand half the size of a poppy seed. And a quarter, and an eighth, and a sixteenth, and so on. For every natural number n, there is a grain of sand on Zeno Beach that is 2^nth the size of a poppy seed. (Archimedes, by the way, assumes to the contrary in his *Sand-Reckoner* that a maximum of 10,000 grains of sand can be lined up within the diameter of a poppy seed (145.9–11 Mugler).) Our question is whether Aristotle knew of any good reason to deny the possibility of Zeno Beach.

I have selected an example involving infinitely many distinct whole bodies—grains of sand—rather than an example of something with infinitely many parts. This is because I want to avoid, as far as possible, issues of interpretation raised by Aristotle's statements about the metaphysical status of parts, their actuality or potentiality, their priority or posteriority to wholes, and so on. I hope that if we first get more clarity on his views about infinity, then we will subsequently be in a better position to interpret his views about parts and wholes.

The three arguments to be considered in this paper may be summarized as follows:

1. If Zeno Beach existed, the beach would be infinitely large.
2. If Zeno Beach existed, it would be impossible to walk across the beach.
3. If Zeno Beach existed, there would be a plurality that is both lesser than and equal to another plurality.

Each of these arguments rests on some highly general assumption or assumptions about quantities, namely: (1) that infinitely many things, each of finite size, will collectively be infinite in size; (2) that it is impossible to have performed every step in an infinite sequence of steps; (3) that (*i*) proper parts are lesser than wholes and (*ii*) something infinite is not lesser than anything.

Before I address these and other such general arguments, there are some physical considerations about the possibility of my example that need to be addressed.

1 PHYSICAL OBJECTIONS

1.1 How did the grains get there?

One might wonder how infinitely many grains of sand could have come to be on Zeno Beach. Did someone split a grain of sand in half, and then split the half in half, and so on? That would require the completion of an infinite sequence of separate actions, and this, Aristotle thinks, is impossible (*Post. An.* 1.3, 72^{b}10–11; 1.22, 82^{b}38–9, 83^{b}6–7, 84^{a}2–3; *Phys.* 8. 8, 263^{a}18–22; *Metaph.* Γ 4, 1007^{a}14–15).

I can suggest two options for the history of Zeno Beach. The first is that it has always been the way it is. Infinitely many grains of sand have always been there. In Aristotle's view, the world has existed from eternity, hence not everything that exists has come into being. There is no need to suppose that the sand on the beach resulted from any process, infinite or otherwise.

The second option is that, at some point, something struck a pebble and the pebble shattered into infinitely many pieces all at once. Things do not have to be divided into one piece at a time. Why not into infinitely many pieces simultaneously? This option, again, avoids the need for an infinite sequence of steps to have been completed.

1.2 Natural minima

A different doubt is whether Aristotle thought it is physically possible for whole bodies of arbitrarily small size to exist, and to avoid immediate destruction. A basis for this doubt may perhaps be located in a passage in *Physics* 1. 4, where Aristotle develops objections to Anaxagoras' doctrine of total mixture. Anaxagoras held, according to Aristotle's report, that every body is a mixture of infinitely many elements, in such a way that every body has every kind of homoiomerous body mixed in with it. In two of Aristotle's arguments against this doctrine (187ᵇ22–34, 187ᵇ35–188ᵃ2), Aristotle uses the premise that there is a greatest and smallest possible size that a body of flesh can have. He gives an argument for this premise[6] as follows:

T1 ἔτι δ' εἰ ἀνάγκη, οὗ τὸ μόριον ἐνδέχεται ὁπηλικονοῦν εἶναι κατὰ μέγεθος καὶ μικρότητα, καὶ αὐτὸ ἐνδέχεσθαι (λέγω δὲ τῶν τοιούτων τι μορίων, εἰς ὃ ἐνυπάρχον διαιρεῖται τὸ ὅλον), εἰ δὴ ἀδύνατον ζῷον ἢ φυτὸν ὁπηλικονοῦν εἶναι κατὰ μέγεθος καὶ μικρότητα, φανερὸν ὅτι οὐδὲ τῶν μορίων ὁτιοῦν· ἔσται γὰρ καὶ τὸ ὅλον ὁμοίως. σὰρξ δὲ καὶ ὀστοῦν καὶ τὰ τοιαῦτα μόρια ζῴου, καὶ οἱ καρποὶ τῶν φυτῶν. δῆλον τοίνυν ὅτι ἀδύνατον σάρκα ἢ ὀστοῦν ἢ ἄλλο τι ὁπηλικονοῦν εἶναι τὸ μέγεθος ἢ ἐπὶ τὸ μεῖζον ἢ ἐπὶ τὸ ἔλαττον.

Furthermore, if it is necessary that, when the *part* of something admits of being any size whatsoever, great or small, then the thing itself also admits this (I mean the sort of part that is present in the whole and into which the whole is divided), then, if indeed it is im-

6. Perhaps, as some commentators hold, this passage also serves as a self-standing argument against the doctrine of Anaxagoras, in addition to establishing a premise that will be used in subsequent arguments. So J. Murdoch, 'The medieval and renaissance tradition of *minima naturalia*', in C. Lüthy, J.E. Murdoch, and W.R. Newman (eds.), *Late Medieval and Early Modern Corpuscular Matter Theories* (Leiden; Boston, 2001), 91–131 at 97; J. Mcginnis, 'A Small Discovery: Avicenna's Theory of Minima Naturalia', *Journal of the History of Philosophy*, 53 (2015), 1–24 at 3-4.

> possible for an animal or plant to be any size whatsoever, great or small, it is apparent that none of their parts admits of this either. For the whole, too, will be likewise. But flesh and bone and such things are parts of an animal, and fruits are parts of plants. It is clear, then, that it is impossible for flesh or bone or something else to be any size whatsoever, either in the direction of the greater or of the smaller. (*Phys.* 1. 4, 187^{b}13–21)

Since there is a greatest and least possible size of an animal or plant, Aristotle argues, there is a greatest and least possible size of every kind of body into which an animal or plant can be divided. Aristotle applies this result to flesh, bone, fruit, blood, and brain (cf. 188^{a}3 for the last two). It is sometimes suggested that he thought his result applied to every kind of physical body whatsoever.[7] Certainly this passage served as a source for medieval doctrines of so-called *minima naturalia*.[8] But it is doubtful, at best, whether Aristotle himself would have extended the doctrine more widely than to the parts of living things.[9] And, to the best of my knowledge, sand is not a

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7. R. Glasner, 'Ibn Rushd's Theory of Minima Naturalia', *Arabic Sciences and Philosophy*, 11 (2001), 9–26 at 13-14

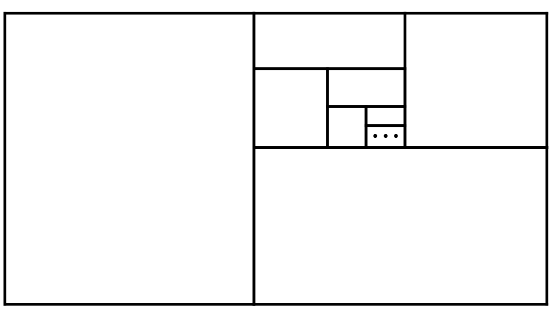
8. For the history of these later doctrines, see for example A. Maier, 'Das Problem des Kontinuums in der Philosophie des 13. und 14. Jahrhunderts', *Antonianum*, 20 (1945), 331–68 at 350-61; P. Duhem, *Medieval Cosmology: Theories of Infinity, Place, Time, Void, and the Plurality of Worlds* (Chicago, 1985), 35-45; Glasner, 'Ibn Rushd's Theory of Minima Naturalia'; Murdoch, 'The medieval and renaissance tradition of *minima naturalia*'; Mcginnis, 'A Small Discovery: Avicenna's Theory of Minima Naturalia'.

9. G. Muskens, 'Is bij Aristoteles van minima naturalia sprake?', *Studia catholica*, 21 (1946), 173–75 at 174; E.J. Dijksterhuis, *De mechanisering van het wereldbeeld* (Amsterdam, 1950), 25

part of any living thing: that is, no animal or plant can be divided in such a way as to yield a body of sand. (This is true even if sand is an ingredient in some mixture that is present in a plant or animal.)

A different sort of objection may be raised. Aristotle states that when a body is broken into very tiny pieces, and these pieces are in contact with another kind of body (such as air or water), then they will quickly be transformed by and assimilated to the other kind of body (see, for example, *GC* 1.2, 317^a27–9). How, then, can the exceedingly tiny grains of sand on Zeno Beach survive for any length of time?

In answer to this objection, we may suppose that the tiny grains of sand on Zeno Beach are buried somewhat below the surface, in such a way that each of the grains smaller than some safe size is surrounded by, and in contact with, nothing but other grains of sand. For example, we could suppose that the sand is packed together like this:



We might even allow some tiny air gaps between the tiny grains of sand. Provided that the bits of air are roughly as small as the grains of sand that they separate, I think such a configuration will make for a stable equilibrium.

In sum, I do not see any insurmountable physical objections to the existence of Zeno Beach. If there is a serious objection, it is more likely to derive from a more general argument against the existence of infinite pluralities of magnitudes. In any case, it is the more general arguments that I am interested in, because these are the ones that could be adapted into arguments against an infinite plurality of parts of a magnitude. So let us now turn to arguments with this level of generality.

2 Impossible numbers

I begin with a pair of arguments in Aristotle that are not, I believe, intended as arguments against the existence of actually infinite pluralities, but which have often been taken as such. I here give a brief discussion of the issue, since I have discussed it in more detail elsewhere.[10] Aristotle twice argues, namely, that there is no such thing as an infinite *number*. The arguments are as follows:

T2 ἀλλὰ μὴν οὐδ' ἀριθμὸς οὕτως ὡς κεχωρισμένος καὶ ἄπειρος· ἀριθμητὸν γὰρ ἀριθμὸς ἢ τὸ ἔχον ἀριθμόν· εἰ οὖν τὸ ἀριθμητὸν ἐνδέχεται ἀριθμῆσαι, καὶ διεξελθεῖν ἂν εἴη δυνατὸν τὸ ἄπειρον.

But indeed, neither is there an infinite number, in such a way as to be separate and infinite. For a number, or that which has number, is countable. If, then, it is possible to count

10. 'Aristotle's Actual Infinities', Section 2.

that which is countable, then it would also be possible to go through the infinite. (*Phys.* 3. 5, 204^{b}7–10)

T3 ὅτι μὲν τοίνυν ἄπειρον οὐκ ἐνδέχεται, δῆλον (οὔτε γὰρ περιττὸς ὁ ἄπειρός ἐστιν οὔτ' ἄρτιος, ἡ δὲ γένεσις τῶν ἀριθμῶν ἢ περιττοῦ ἀριθμοῦ ἢ ἀρτίου ἀεί ἐστιν.

It is clear that the number cannot be infinite; for an infinite number is neither odd nor even, but the generation of numbers is always the generation either of an odd or of an even number. (*Metaph.* M 8, 1084^{a}2–4)

Now, if it could be assumed that every plurality is a number (or that, whenever there are some things, there is a number of all the things), then these arguments would show that there are no infinite pluralities. However, Aristotle does not identify plurality with number. He regards the former as the genus of the latter (*Metaph.* Δ 13, 1020^{a}13; I 6, 1057^{a}2–3). Moreover, he holds that every genus extends more widely than each of its species (*Topics* 4. 1, 121^{b}11–14; 4. 2, 123^{a}6–7; 4. 3, 123^{a}30; 4. 6, 128^{a}22–3; *Metaph.* Δ 3, 1014^{b}12–14). Hence, he must deny that every plurality is a number.

Next, consider the central premises of Aristotle's two arguments. They are, in **T2**, that every number is countable, and, in **T3**, that every number is either odd or even. Would Aristotle accept the premise that every *plurality* is countable, or that every *plurality* is either odd or even? He could not accept either of these premises.[11] He could not accept the premise that every plurality

11. Aristotle has been thought to imply that every plurality is countable in *Metaph.* Δ 13, 1020^{a}8–9, but I do not

is countable, because if something is countable then it is measurable by a unit, and Aristotle gives 'measurable by a one' as the differentia of *number* over against the genus of plurality (*Metaph.* I 6, 1057ª3–4). Since 'the genus must be said more widely than the differentia, and must not partake of the differentia' (*Top.* 4. 2, 123ª6–8), and since plurality is the genus of number, it is not the case that every plurality is measurable by a one. Any pluralities that are not measurable by a one (perhaps because they are infinite) are not countable.[12] Turning to **T3**, Aristotle could not accept the premise that every plurality is either odd or even, because he writes in the *Posterior Analytics* that odd and even should be defined as attributes of number (*Post. An.* 1. 4, 73ª37–40; 1. 22, 84ª14–17). From this it follows that nothing other than a number is even or odd.[13] So, given that plurality is the genus of number and extends more widely than it, he cannot hold that every plurality is either even or odd.

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think this is correct. The passage is discussed in 'Aristotle's Actual Infinities', Section 2.3.

12. For the claim that countability implies measurability, see H. Bonitz (ed. and comm.), *Aristotelis Metaphysica* (Bonn, 1848), vol. 2, 257. What does it mean to be measurable? Menn explains the notion as follows: 'X is measured by Y iff X is the sum of finitely many equal constituents each equal to Y' (S. Menn, *The Aim and Argument of Aristotle's Metaphysics* [*Metaphysics*] (Internet resource), retrieved 2018, Iγ2a, 7). Mueller gives a similar explanation: 'one positive integer measures a second when it divides the second evenly, i.e., when the second positive integer can be segregated into some number of parts each equal to the first. The notion of measurement can, of course, be applied geometrically as well, and Euclid does do so' (I. Mueller, *Philosophy of Mathematics and Deductive Structure in Euclid's Elements* (Cambridge, Mass., 1981), 61).

13. 'Nothing outside number is odd', *Post. An.* 2. 13, 96ª31–2.

In Aristotle's usage, 'number' signifies a special kind of plurality, and it is its special number-like properties that guarantee finitude. A further sign of this is that Aristotle uses the phrase 'infinite in plurality' a fair number of times, but never—well, hardly ever—uses the phrase 'infinite in number'.[14] It is worth observing that Plato's *Parmenides* is even more consistent on this point of usage.[15] (Later writers, including Archimedes, use the phrase 'infinite number', but for Aristotle this is practically a contradiction in terms.) Given this fact, Aristotle's arguments about number should not be read as arguments against the existence of actual infinite pluralities.

3 The beach would be infinitely large

I now turn to an argument that dates back at least to the ancient commentators Themistius and Simplicius. These commentators adopt the premise that if there are infinitely many things, and each one has finite size, then all of them together are infinitely large. Given this premise, the assumption that there is an infinite plurality of things with finite size leads immediately to the con-

14. ἄπειρα τὸ πλῆθος: *GC* 1. 1, 314a22; 1. 8, 325a30; *Phys.* 1. 4, 187b34; 2. 5, 197a16; Fragment 5.32.208.17. πλήθει ἄπειρα: *De caelo* 3. 4, 303a5–6; *Metaph.* α 2, 994b28; I 6, 1056b29; *Phys.* 3. 4, 203b34–204a1; *Pol.* 1. 8, 1256b35. ἄπειρον κατὰ πλῆθος: *Phys.* 1. 4, 187b8, 10. **Total: 12**. ἄπειρα τὸν ἀριθμόν: *DA* 1. 5, 409b29; *SE* 1, 165a12. **Total: 2**. For context, note that on the whole, 'number' appears more often than 'plurality' in Aristotle.

15. Plato, *Parmenides*, 132b2, 143a2, 144a6, 144e4–5, 145a3, 158b6, 158c6–7, 164d1, 165c2. **Total: 9**. No occurrences of 'infinite in number'. For context, note that, in total, 'plurality' appears 22 times, while 'number' appears 25 times in Plato's *Parmenides*.

clusion that there is an infinite magnitude. Since Aristotle argues that there cannot be an actual infinite magnitude (*Phys.* 3. 5), he may conclude that there is no actual infinite plurality of things with finite size. In the case of Zeno Beach, the argument works out to this: infinitely many grains of sand, each of which has finite size, would collectively be infinitely large; but it is impossible for there to be an infinitely large beach; hence, Zeno Beach is impossible.

Here is how Themistius and Simplicius present the argument:

T4 τὸ δὲ ἐξ ἀπείρων συγκείμενον ἄπειρόν ἐστι κατὰ μέγεθος, τοῦτο δὲ ἀδύνατον ἐπιδεικνύντες ἄρτι πεπαύμεθα.

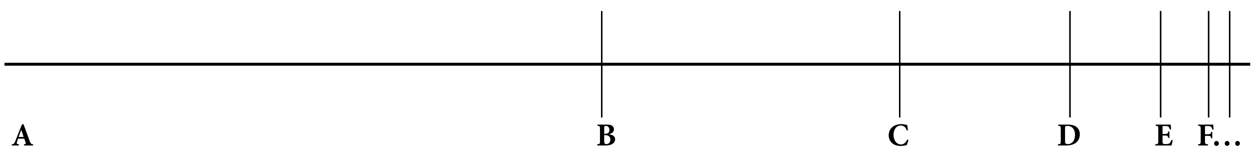
Something composed of infinitely many things is infinite in magnitude, and we have just left off showing that this is impossible. (Themistius, *In Phys.* 91.29–30)

T5 ἐὰν δὲ γένηται ἐνεργείᾳ ἄπειρα διαιρέματα μεγέθη ἔχοντα, καὶ μέγεθος ἄπειρον ἂν γένοιτο (τὸ γὰρ ἐξ ἀπείρων τῷ πλήθει μεγεθῶν ἄπειρον ἔσται μέγεθος, ὡς εἴρηται πολλάκις), ἀποδέδεικται δὲ διὰ πολλῶν μὴ ὂν ἄπειρον μέγεθος

If infinitely many parts having magnitude actually come to be, then an infinite magnitude would come to be as well (for a magnitude composed of infinitely many magnitudes will be an infinite magnitude, as has been said many times). But it has been shown through many arguments that there is no infinite magnitude. (Simplicius, *In Phys.* 492.16–19)

The argument appears to be a direct descendant of an antinomy propounded by Zeno. Zeno had argued that 'if there are many' then 'it is necessary that they are both small and large: so small as to have no magnitude, so large as to be infinite.'[16] Zeno's argument on the 'large' side of the antinomy contains an inference from the existence of infinitely many things with size to the existence of something with infinite size (or of some things that, collectively, have infinite size). Themistius and Simplicius think that Aristotle accepted this Zenonian inference as valid.

Let us pause to clarify the nature of the inference. Consider a series of lines AB, BC, CD, … laid end to end, with lengths 1 unit, 1/2 unit, 1/4 unit, and so on.



If we take any finite number of lines in this series, no matter how many, the sum of their lengths will be less than 2 units. Aristotle was aware of this fact (*Phys.* 3. 6, 206^{b}7–9). Converging series of the kind had been used by mathematicians in the preceding generation or two (see Euclid's *Elements* XII).[17] But what happens if we take the sum of *all* the lines in the series, all infinitely many of them? Is there such a sum, and if so, how big is it? According to the Zenonian inference, the sum of infinitely many lines is an infinite length, even in the case where the sum of *any finite*

16. Diels-Kranz Fragment B1, preserved in Simplicius *In Phys.* 141.6–8

17. Euclid, of course, wrote after the time of Aristotle, but he is thought to have organized and recorded the work of earlier mathematicians including Eudoxus and Theaetetus. See for example B.L. van der Waerden, *Science Awakening* (Groningen, 1954); W.R. Knorr, *The Evolution of the Euclidean Elements: A Study of the Theory of Incommensurable Magnitudes and Its Significance for Early Greek Geometry* (Dordrecht, 1975); M.F. Burnyeat, 'The Philosophical Sense of Theaetetus' Mathematics', *Isis*, 69 (1978), 489–513.

number of these lines is below a fixed finite upper bound.[18] Themistius and Simplicius accept this, and they think that Aristotle accepted it too.

Two arguments in the *Physics*, however, show to my mind that Aristotle would not have gone along with this. In these two arguments, Aristotle wants to draw an inference from the existence of an infinite plurality of magnitudes to the existence of an infinite magnitude. In both cases, before he draws the inference, Aristotle first goes out of his way to secure an additional assumption or lemma: namely, that there is a finite lower bound on the magnitudes being summed. This is evidence that he did not think that just any old infinite plurality of magnitudes will sum to an infinite magnitude. We may note, of course, that there is no finite lower bound on the sizes of grains of sand on Zeno Beach.

The first of the two arguments is in *Physics* 1. 4. It is directed against Anaxagoras' view that all things are mixed together in such a way that every kind of stuff is mixed into, and can be extracted from, every other kind of stuff. The final part of Aristotle's argument runs as follows:

18. See D. Furley, 'Aristotle and the Atomists on Infinity', in I. Düring (ed.), *Naturphilosophie bei Aristoteles und Theophrast* (Heidelberg, 1969), 85–96 at 87. G. Vlastos, 'Zeno', in W. Kaufmann (ed.), *Philosophic Classics* (Englewood Cliffs, 1961), 27–45 at 32-33 regards Zeno's inference as a straightforward mathematical error, as if Zeno did not notice that in some cases—including his own Dichotomy of the line—there is an upper bound on finite sums. It is possible that Zeno made this error, or wanted his hearers to make this error. But it is also possible that he and/or his hearers believed there is a discontinuity between sums of finitely many terms and sums of infinitely many terms.

T6 ἀφαιρεθείσης γὰρ ἐκ τοῦ ὕδατος σαρκός, καὶ πάλιν ἄλλης γενομένης ἐκ τοῦ λοιποῦ ἀποκρίσει, εἰ καὶ ἀεὶ ἐλάττων ἔσται ἡ ἐκκρινομένη, ἀλλ᾽ ὅμως οὐχ ὑπερβαλεῖ μέγεθός τι τῇ μικρότητι. ὥστ᾽ εἰ μὲν στήσεται ἡ ἔκκρισις, οὐχ ἅπαν ἐν παντὶ ἐνέσται (ἐν γὰρ τῷ λοιπῷ ὕδατι οὐκ ἐνυπάρξει σάρξ), εἰ δὲ μὴ στήσεται ἀλλ᾽ ἀεὶ ἕξει ἀφαίρεσιν, ἐν πεπερασμένῳ μεγέθει ἴσα πεπερασμένα ἐνέσται ἄπειρα τὸ πλῆθος· τοῦτο δ᾽ ἀδύνατον.

If flesh is extracted from water, and again other flesh comes to be from the remainder by separation, then, even if the flesh separated out is continually smaller and smaller, still *it will not fall below a certain magnitude*. So on the one hand, if the process comes to a stop then everything will not be in everything else (for there will be no flesh in the remaining water); but if it does not come to a stop, and further extraction is always possible, then there will be an infinite multitude of *finite equal parts* in a finite quantity, and this is impossible. (*Phys.* 1. 4, $187^{b}27$–34, emphasis added)

Prior to this passage, Aristotle has argued for the lemma that all animal and plant parts, including flesh, have a finite maximum and finite minimum possible size (see T1 above). This entitles him to his claim that, if flesh is repeatedly separated out from water, the portions of flesh that are separated out 'will not fall below a certain magnitude'. As far as I know, Aristotle does not claim anywhere else in his writings that there are minimum possible quantities of flesh or other stuffs. The argument for this lemma appears to have been specially devised for the present context. This indicates that Aristotle recognized the importance of the lemma to his argument here. It is hard to see why he would have taken the trouble to include it, if he thought that infinitely many magnitudes always sum to an infinite magnitude, even in the absence of a finite lower bound.

The second argument is in *Physics* 7. It aims to establish that there are no infinite chains of moved movers. Having assumed for *reductio* that there is such an infinite chain, Aristotle proceeds in a key part of the argument as follows:

T7 ἐπεὶ δὴ ἄπειρα τὰ κινοῦντα καὶ τὰ κινούμενα, καὶ ἡ κίνησις ἡ ΕΖΗΘ ἡ ἐξ ἁπασῶν ἄπειρος ἔσται· ἐνδέχεται μὲν γὰρ ἴσην εἶναι τὴν τοῦ Α καὶ τοῦ Β καὶ τὴν τῶν ἄλλων, ἐνδέχεται δὲ μείζους τὰς τῶν ἄλλων, ὥστε εἴ τε ἴσαι εἴ τε μείζους, ἀμφοτέρως ἄπειρος ἡ ὅλη· λαμβάνομεν γὰρ τὸ ἐνδεχόμενον.

Since the movers and moved items are infinitely many, the motion EFGH composed out of all their motions will be infinite: for it is possible that the motions of A, B, and the others are equal, and it is possible that the motions of the others are greater than the motion of A. Consequently, *whether they are equal or greater*, in both cases the whole is infinite. For we suppose what is possible. (Phys. 7. 1, 242^b45–50, emphasis added)

In this argument, Aristotle first asserts that it is *possible* for a certain motion (namely, 'the motion of A') to be a lower bound on the greatness of all the motions. He then explains that we may 'suppose what is possible', thereby introducing the assumption that all the motions are actually equal to or greater than the motion of A. In the analysis given by Rosen and Malink,[19] this is labeled an

19. J. Rosen and M. Malink, 'A Method of Modal Proof in Aristotle', *OSAP*, 42 (2012), 179–261 at 205-13

'assumption for the possibility rule', and it introduces a distinctive sort of modal inference. The logical structure of Aristotle's argument is made significantly more complex by the use of this assumption. As in the refutation of Anaxagoras, it is difficult to see why Aristotle would have included it, unless he thought it was required in order to derive the result that the sum of the motions is infinitely great.

I conclude that, in Aristotle's view, it is not the case that every infinite plurality of magnitudes sums to an infinite magnitude. In his view, an infinite magnitude is yielded by the sum of infinitely many *equal* magnitudes, or, more generally, by infinitely many magnitudes that share a finite lower bound.

This interpretation is supported by a remark in *On Sense and Sensibles:* 'the continuous is cut into infinitely many *unequals*, but into finitely many *equals*' (*De Sensu*, 445^{b}27–8). Zeno's antinomy about size, then, would not be a reason for Aristotle to deny the possibility of Zenos' Beach.

4 It would be impossible to walk across the beach

A different sort of argument against the possibility of Zeno Beach might be proposed on the basis of one of Zeno's paradoxes of motion.[20] In order to traverse any line, says Zeno, one has to traverse half the line, and half of what remains, and half of what remains after that, and so on. If

20. Bostock, 'Aristotle, Zeno, and the Potential Infinite', 40; Coope, 'Aristotle on the Infinite', 269 n. 4

anything is to move, then, it will have to traverse infinitely many lines in a finite time. But this is impossible.

Aristotle's first reply, in *Physics* 6, is to say: yes it is possible. It is possible because a finite time contains an infinity of parts to match the infinitely many parts of a line (*Phys.* 6. 2, 233ᵃ21–30). In *Physics* 8. 8, however, Aristotle changes his tune. He says that the real issue is not whether it is possible to do something infinite *in a finite time*, but whether it is possible to 'have gone through' (διεξελθεῖν) infinitely many things, period. The outline of his solution to this revised statement of the problem appears as follows:

T8 ὥστε λεκτέον πρὸς τὸν ἐρωτῶντα εἰ ἐνδέχεται ἄπειρα διεξελθεῖν ἢ ἐν χρόνῳ ἢ ἐν μήκει, ὅτι ἔστιν ὥς, ἔστιν δ' ὡς οὔ. ἐντελεχείᾳ μὲν γὰρ ὄντα οὐκ ἐνδέχεται, δυνάμει δὲ ἐνδέχεται.

> Consequently, to the questioner who asks whether it is possible to go through infinitely many things, either in a time or in a length, we should say that in a way it is, and in a way not. For if they are in actuality it is not possible, but if they are in potentiality it is. (*Phys.* 8. 8, 263ᵇ3–6)

In this passage, Aristotle says that it is not possible to go through infinitely many things—presumably including temporal intervals and instants, parts of a line, and points[21]—if all those things are in actuality, but only if they are in potentiality. This suggests an argument for the proposition that it would be impossible to move across Zeno Beach. Given the assumption that

21. These are, I take it, the things in a time and in a length that one might suppose to be infinitely many.

motion is in fact possible everywhere, it would follow that Zeno Beach does not exist. Here is how the argument might go:

1. If it is possible to move across Zeno Beach, then it is possible to traverse a line on Zeno Beach that passes over infinitely many grains of sand.
2. Necessarily, each grain of sand on Zeno Beach exists in actuality (since it is divided from its neighbors).
3. Necessarily, wherever a line passes over an actually existing grain of sand, the line contains a part that exists in actuality, bounded by the two points where the line crosses the boundary of the grain of sand.
4. Necessarily, if a line passes over infinitely many actually existing grains of sand, then the line contains infinitely many parts that exist in actuality. (From 3)
5. Necessarily, when something traverses a line, it goes through all the parts of the line.
6. If it is possible to move across Zeno Beach, then it is possible to go through infinitely many parts of a line, all of which exist in actuality. (From 1, 2, 4, 5)
7. It is not possible to go through infinitely many parts of a line that all exist in actuality.
8. It is not possible to move across Zeno Beach. (From 6, 7)

Variations of the argument may be produced in which the talk of parts is replaced by talk of midpoints, or temporal intervals or instants. In any of its variants, the nub of the argument is the pair of premises in lines 3 and 7: line 3 generates infinitely many items, and line 7 says that they cannot be gone through.[22] The problem with attributing the argument to Aristotle is that Aristotle would not accept both premises together. The Aristotle of *Physics* 6 may have accepted line 3, but, as we observed above, he denies the premise in line 7 (*Phys.* 6. 2, 233ᵃ21–3). The Aristotle of *Physics* 8. 8 accepts line 7, but, as I will now argue, he does so on the basis of views that undermine the premise in line 3. For according to *Physics* 8. 8, a line traversed by motion has no actually existing parts unless the moving thing halts on the line; and Aristotle surely does not believe that moving things halt at every body they move across.

To see that the Aristotle of *Physics* 8. 8 would not accept line 3, let us begin with what he says there about points on a line. Early in *Physics* 8. 8, Aristotle argues for the following claim:

> HALT AT MIDPOINT: if an object O moves along a line AC, and B is actually a midpoint on AC, then O halts at B.

We can see him endorsing HALT AT MIDPOINT in the following passages:[23]

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22. I here state the variations on line 3 for midpoints, temporal intervals, and instants. For *midpoints*, line 3 would read: 'Necessarily, wherever a line passes over an actually existing grain of sand, the line contains midpoints in actuality where the line crosses the boundary of the grain of sand.' For *temporal intervals*: 'Necessarily, whenever something moves over an actually existing grain of sand, the time of the motion contains a part that exists in actuality, during which the thing (or its foremost point) is directly above the grain of sand.' For *temporal instants*: 'Necessarily, whenever something moves over an actually existing grain of sand, the time of the motion contains instants that exist in actuality, at which the thing (or its foremost point) crosses the boundary of the grain of sand.'

23. In addition to passages **T9** and **T10**, see also *Phys.* 8. 8, 262ᵇ24–6 and 262ᵇ31–263ᵃ2.

T9 ὥστε τῆς εὐθείας τῶν ἐντὸς τῶν ἄκρων ὁτιοῦν σημεῖον δυνάμει μέν ἐστι μέσον, ἐνεργείᾳ δ᾽ οὐκ ἔστιν, ἐὰν μὴ διέλῃ ταύτῃ καὶ ἐπιστὰν πάλιν ἄρξηται κινεῖσθαι.

Consequently, of the points within the extremes of the straight (line[24]), any point at all is a midpoint in potentiality, but *it is not a midpoint in actuality unless* (the moving thing) divides (the line) here, that is, *halts* and begins again to move. (*Phys.* 8. 8, 262ᵃ22–5)

T10 ὅταν δὴ χρήσηται τὸ φερόμενον Α τῷ Β μέσῳ καὶ τελευτῇ καὶ ἀρχῇ, ἀνάγκη στῆναι διὰ τὸ δύο ποιεῖν.

When a moving body, A, *uses point B as a midpoint*, i.e., as an end and as a beginning, it is necessary for A to *halt* because it makes the point two. (*Phys.* 8. 8, 262ᵇ5–6)

When it comes to Aristotle's *justification* for HALT AT MIDPOINT, it is a matter of controversy how that should be understood.[25] Fortunately, we do not need to resolve the controversy here.

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24. One might consider supplying 'motion' rather than 'line': both are feminine nouns that would agree with εὐθεί-ας. However, I have not found any passage where Aristotle speaks of a 'straight motion'. He speaks of motions as being 'in', 'on', or 'along' something straight (ἐν, ἐπί, κατά), indicating that the adjective 'straight' itself describes lines, not motions. (Contrast the usage attributed to Chrysippus in Stobaeus, *Ecl.* I 19 3, 10–11.)

25. Aristotle's main argument admits of a fairly straightforward interpretation, but one on which he makes a mistake: Bostock, 'Aristotle, Zeno, and the Potential Infinite', 42; R. Sorabji, 'Aristotle on the Instant of Change – I', *Proceedings of the Aristotelian Society, Supplementary Volumes*, 50 (1976), 69–89 at 85; M.J. White, *The Continuous and the Discrete: Ancient Physical Theories From a Contemporary Perspective* [*Continuous and Discrete*] (Oxford : New York, 1992), 54–55; J. Rosen, 'Physics V–VI versus VIII: Unity of Change and Disunity in the Physics' ['Physics V–VI

What matters for our purposes is simply the fact that he endorses the claim; this is, I think, uncontroversial.[26] Aristotle's initial purpose in introducing the claim is to argue that continuous motion along a straight line cannot contain reversals of direction: if an object turns around, he argues, it must do so at an actual, and not merely potential, midpoint; from here he uses HALT AT MIDPOINT to infer that it will halt at the midpoint, and hence that it will not move continuously (*Phys.* 8. 8, 262^{b}30–263^{a}3). This argument serves the overall aim of the chapter, which is to show that circular locomotion is the only kind of motion that can be continuous and eternal.

Immediately after the argument just described, Aristotle says that the application he has made of HALT AT MIDPOINT can be adapted to answer Zeno's paradox. Zeno's paradox-mongers,

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versus VIII'], in M. Leunissen (ed.), *Aristotle's Physics: A Critical Guide* (Cambridge, 2015), 206–24 at 220–22. Some commentators have developed more or less elaborate interpretations in order to cure or obscure Aristotle's apparent error. See for example White, *Continuous and Discrete*, 55–58; J. Bowin, 'Aristotle on the Unity of Change: Five *Reductio* Arguments in *Physics* VIII 8' ['Unity of Change'], *Ancient Philosophy*, 30 (2010), 319–45 ; D. Blyth, *Aristotle's Ever-Turning World in Physics 8: Analysis and Commentary* [*Ever-Turning World*] (Leiden, 2016), 247–48; C. Cohoe, 'Why Continuous Motions Cannot be Composed of Sub-Motions: Aristotle on Change, Rest, and Actual and Potential Middles', *Apeiron*, 51 (2018), 37–71.

26. It is agreed to by all the commentators cited in n. 25. Likewise Themistius *In Phys.*, 228.32–229.3 Schenkl; Simplicius *In Phys.*, 1282.3–5, 15–16 Diels. Sarah Waterlow (Broadie) makes no mention of halting in her discussion of the arguments of *Physics* 8. 8 (S. Waterlow, *Nature, Change, and Agency in Aristotle's Physics* (Oxford, 1982), 149–53). But she also writes that 'we need not follow Aristotle step for step' (149), so her omission need not mean she would deny that the actual steps of the argument, as written by Aristotle, include the claim about halting.

he explains, include two sorts of questioner. One questioner asks whether, whenever an object traverses something, it must also traverse its half (but the halves are infinitely many, and it is not possible to have gone through what are infinitely many); the other adds the supposition that, while an object moves, each half that it traverses is counted (so that if the motion is completed, an infinite number will have been counted, and this is impossible). Aristotle seems to say that both of these questioners, to whom he refers as 'the counter and the divider into halves' (263ᵃ25–6), assume that there are actually existing midpoints, points that they 'treat as two' in their discussion. In effect, then, they illicitly assume that the motion and the line under discussion are not single or continuous (263ᵃ21–7). Aristotle continues:

T11 ἡ γὰρ συνεχὴς κίνησις συνεχοῦς ἐστιν, ἐν δὲ τῷ συνεχεῖ ἔνεστι μὲν ἄπειρα ἡμίση, ἀλλ' οὐκ ἐντελεχείᾳ ἀλλὰ δυνάμει. ἂν δὲ ποιῇ ἐντελεχείᾳ, οὐ ποιήσει συνεχῆ, ἀλλὰ στήσει.

For a continuous motion is motion over a continuous (line), and in what is continuous there are infinitely many halves, but not in actuality but in potentiality. *If he makes (them be) in actuality, he will not make (the motion) continuous, but rather he will halt it.* (*Phys.* 8. 8, 263ᵃ27–30)

If someone makes the halves of a line exist in actuality, then the person halts the motion. In this context, 'make' seems to mean stipulate or posit, and 'halt' seems to mean posit as containing a halt.[27] Aristotle is describing an interlocutor, not a physical actor. His claim is that, where a

27. This is suggested by Blyth: 'In *thinking of* the body as at the point, someone must *think of* the point as actual

motion traverses a line, the assumption that the line has actually existing parts entails the conclusion that the moving thing halts along the way. In this passage, Aristotle extends his earlier claim about midpoints (HALT AT MIDPOINT) to cover halves, and, presumably, parts more generally, of a line. We may formulate the claim of **T11** as follows:

> HALT AT HALF: if an object O moves along a line AD, and BC is an actually existing part or half (of a half of a half of a…) of AD, then O halts at the limits of BC.

This, then, yields Aristotle's revised solution to Zeno's paradox. When a questioner poses the paradox, by describing an infinite sequence of points or line-parts and asking whether it is possible to have gone through them all, he is—perhaps unwittingly—positing a situation in which something moves, halts, and moves again. He is asking (again, perhaps unwittingly) whether it is possible to *move and halt* infinitely many times. If an object moves continuously, without halting, then the points and line-parts described by the questioner do not exist in actuality, and the moving object does not go through them. (More precisely, the moving object does not go through them *per se*; it does go through infinitely many line-parts accidentally, but Aristotle evidently regards this as unproblematic.[28]) Contrary to appearances, then, the questioner is not rais-

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and so of the body as stopped there', Blyth, *Ever-Turning World*, 236, emphasis added. For the use of 'make' (ποιεῖν) in the sense of 'posit', see H. Bonitz, *Index Aristotelicus* (Berlin, 1870), 609^{a}15–48.

28. *Phys.* 8. 8, 263^{b}6–9. To understand what this means, we may compare Aristotle's statement that when something turns white, and someone is thinking of the color white, the thing changes into *what is being thought of* accidentally, insofar as being thought of is an attribute of the color: *Phys.* 5. 1, 224^{b}18–22. (This passage in *Phys.* 5. 1 is dis-

ing difficulties about anything that genuinely occurs when something undergoes a single continuous motion.

This solution, based on HALT AT MIDPOINT and HALT AT HALF, precedes and explains what Aristotle says in **T8**, including his admission of the premise that it is impossible to go through infinitely many parts of a line that all exist in actuality. Now let us see what this means for the possibility of motion across Zeno Beach.

The crucial question is this. Would Aristotle think that whenever something moves across a surface composed of distinct bodies, it halts along the way? Picture Socrates strolling across a mosaic floor. Does Socrates stop every time he passes over the edge of a pebble? Surely not, and I cannot believe that Aristotle would have thought so. Perhaps someone will say that Socrates' motion is discontinuous on account of the pebbles, even if the motion is not broken up by any periods of rest.[29] In principle, this might be so: Socrates could perform something analogous to a relay race, a series of motions that succeed each other without temporal gaps, but which are not one and continuous with each other (cf. *Phys.* 5. 4, 228^{a}26–^{b}1). However, it seems far-fetched to insist that this is the only way of getting across a tiled floor. It is especially implausible if, instead of thinking about Socrates, we consider a heavenly body such as the moon. It seems possible that the outer reaches of the sublunary realm contain some number of bodies that are in contact with

cussed in J. Rosen, 'Motion and Change in Aristotle's *Physics* 5. 1', *Phronesis*, 57 (2012), 63–99 at 72–78.)

29. Such a view has been suggested in conversation. Additionally, Blyth suggests at one point that we can think of something as stopped 'just for an instant' (Blyth, *Ever-Turning World*, 241).

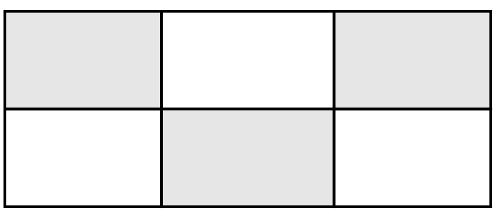
the sphere of the moon. It also seems possible that there is only a single body there. So the moon in its circling might be going round the surface of a single body, or it might be going round a number of bodies and crossing the boundaries between them. I cannot believe Aristotle thought that the continuity of the moon's motion is contingent on which of these two situations obtains. The moon moves continuously, regardless of whether it moves across multiple bodies below it.[30] Hence, the moon does not go through any actually existing midpoints or line-parts, even if it moves over multiple bodies.

If this is correct, then Zeno's paradox of motion does not present any obstacle to a stroll across Zeno Beach. On an ordinary beach, our motion is not halted or made discontinuous at every crossing from one grain of sand to another. Neither will it be so on Zeno Beach. So long as we walk continuously, we do not go through any actually existing midpoints or parts of a line, no matter how many grains of sand pass beneath our feet. This means that we can walk across infinitely many grains of sand without going through more than one actually existing line.

30. Each heavenly sphere undergoes a single eternal motion (*Metaph.* Λ 8, 1073^{a}26–31), and if a motion is single then it is continuous (*Phys.* 5. 4, 228^{a}20–22).

4.1 A consequence for the metaphysics of lines

Our discussion of *Physics* 8. 8 has an implication for the metaphysics of lines. It is sometimes said that, according to Aristotle, every line is an edge of, or division in, some body.[31] *Physics* 8. 8 is evidence against the attribution of such a view to Aristotle. To see why, consider the following example. Suppose that twelve distinct cubes of Jell-O (firm enough to keep their shape, soft enough to be moved through) are arranged in contact with each other, as in the following illustration. (This is a view from above; there are six more cubes below the ones shown.)



It is possible for an ant to burrow through our arrangement from left to right, along the central axis. It seems entirely possible for an ant to perform such a motion continuously and without stopping. If it does so, then, according to *Physics* 8. 8, the ant traverses a single, continuous line with no actually existing parts or midpoints. However, there is no single edge of, or division in, any body that extends the full length of our arrangement through its center from left to right. There are twelve edges located on the line that the ant traverses (three groups of four co-located

31. For example, Hussey writes that points and lines 'actually exist only if they are physically realized limits of actually existing physical bodies' (Hussey, *Physics III–IV*, xxi; see also pp. 177, 179). Similarly C. Pfeiffer, *Aristotle's Theory of Bodies* (Oxford, 2018), 110.

edges), and each of them extends only a third of the way along the path of the ant's motion. Thus the line that the ant traverses will not be identical with, nor composed out of, any of these (if it were composed of them, it would have actually existing parts). Therefore, according to the doctrine of *Physics* 8. 8, it is possible for there to be a line that is not an edge of, or a division in, a body.[32]

5 Infinite parts

The last argument that I want to consider draws again on an ancient idea, but, as far as I have seen, no ancient commentator applied this idea directly to the interpretation of *Physics* 3. The idea is that an infinite plurality would necessarily have another infinite plurality as a proper part, but that this is impossible. Aristotle would thus have been moved by an argument having the following two premises:

32. I am not sure what the balance of evidence is in Aristotle's other writings. The view that all lines are limits or divisions of bodies is voiced at *Metaph.* B 5, 1002^{b}8–11, and *Metaph.* K 2, 1060^{b}12–17; but the former belongs to a book of *aporiai*, which often draws on Academic positions that Aristotle rejects, and the latter is (*a*) a quasi-doublet of the former, and (*b*) perhaps not written by Aristotle (P. Aubenque, 'Sur l'inauthenticité du livre K de la Métaphysique', in *Zweifelhaftes im Corpus Aristotelicum: Studien zu einigen Dubia* (Berlin, 1983), 318–44). In the *Topics*, Aristotle speaks of people who define a line as the limit of a plane ('everyone' does it, he says), but he says that this is *not* a proper definition of the essence, because lines are prior to planes (*Top.* 6. 4, 141^{b}19–24).

A. Infinite part: If there is an infinite plurality, then there is an infinite plurality that is a proper part of an infinite plurality.

B. No infinite part: If something is infinite, then it is not a proper part of anything (at least, not of anything of the same kind as itself).[33]

These premises entail that there are no infinite pluralities. Let us call this the Argument from Infinite Parts. This argument is more general than the other two, since it applies to all pluralities whatsoever, not only to pluralities of things with size or position such as the grains of sand on Zeno Beach.

Commentators typically justify premise A by construction: having posited an infinite plurality, they describe how to construct either (*i*) an infinite proper part of the posited plurality, or (*ii*) an infinite plurality of which the posited plurality is a proper part. Premise B, on the other hand, is typically inferred from a pair of more basic premises:

B1. Part less than whole: If X is a proper part of Y, then X is lesser than Y (with respect to the kind of quantity proper to Y).

33. I add the parenthetical qualification because it might be acceptable, to take an example, for an infinite *line* to be part of a *plurality* of lines. In a case like this, the respect in which the whole is greater than the part (e.g., plurality) is different from the respect in which the part is infinite (e.g., length).

B2. No lesser infinites: If something is infinite, then it is not lesser than anything (with respect to the kind of quantity proper to itself).[34]

If something is infinite, then, by B2, it is not lesser than anything, and hence, by B1, it is not a proper part of anything (at least, not of anything of the same kind as itself).[35]

An argument along these lines is presented by Alexander of Aphrodisias in his commentary on Aristotle's *Metaphysics* Γ. Alexander is offering extra support to Aristotle's assertion that no name signifies infinitely many things. He gives the following argument:

T12 εἰ τὸ ἄνθρωπος ἄπειρα σημαίνει, ἤτοι καὶ τὸν οὐκ ἄνθρωπον ὁ ἄνθρωπος σημαίνει … ἢ εἰ τοῦτο ἄτοπον, ἔσται τῶν ἀπείρων τι πλέον. τοῖς γὰρ ἀπείροις, ἃ ἐσήμανεν ὁ ἄνθρωπος, προστεθὲν τὸ οὐκ ἄνθρωπος πλείω αὐτὰ ποιήσει· οὕτω τε ἔσται καὶ τὰ ἄπειρα τινῶν ἐλάττω.

If 'human' signifies infinitely many things, then either 'human' signifies also what is not human, …, or, if this is absurd, there will be something more than the infinitely many. For if the not-human is added to the infinitely many things that 'human' was supposed to

34. The parenthetical specification is required for reasons mentioned in n. 33. If, for example, a line is part of a plurality of lines, then the line is lesser than the plurality *with respect to plurality*. It is not thereby lesser than anything with respect to length, the kind of quantity proper to lines.

35. To state the inference more carefully, if something is infinite then, by B2, it is not lesser than anything with respect to its own proper kind of quantity. Hence, by B1, it is not a proper part of anything that has the same proper kind of quantity as itself.

signify, it will make them more. Thus it will also turn out that the infinitely many are fewer than some things. (Alexander *In Metaph.*, 279.4–9 Hayduck)

In this argument, Alexander justifies a version of premise A (INFINITE PART) by construction: supposing there to be an infinite plurality of things signified by 'human', he constructs a plurality of which it is a proper part by adding the not-human to it. He applies B1 (PART LESS THAN WHOLE) when he asserts that the latter plurality is 'more' than the former and that the former is 'fewer' than the latter. The final sentence in the passage, 'the infinitely many are fewer than some things', is treated as an impossible consequence which concludes Alexander's *reductio*. In treating this consequence as impossible, Alexander relies on B2 (NO LESSER INFINITES). Thus, Alexander here gives a version of the Argument from Infinite Parts.

A version of the Argument from Infinite Parts is also given by Philoponus in a passage from his commentary on *Physics* 3. In this passage, Philoponus argues against Aristotle, and in favor of his own Christian belief, that the cosmos had a beginning in time.[36] He argues that impossible consequences follow from the assumption that past time is infinite:

T13 καὶ συμβήσεται δὲ οὐ μόνον πολλαπλασιάζεσθαι τὸ ἄπειρον, ἀλλὰ καὶ ἀεὶ αὔξεσθαι καὶ ἄπειρον ἀπείρου μεῖζον εἶναι. τοῦ γὰρ ἀριθμοῦ τῶν ἐνιαυτῶν ἀπείρου ὄντος, ὁ τῶν μηνῶν ἀριθμὸς πολλαπλασιάζει τὸ ἄπειρον· δωδεκάκις γὰρ ἕξει τὸ ἄπειρον. ἀλλὰ μὴν ὁ τῶν ἡμε-

36. Philoponus gives similar arguments in other texts as well; they are discussed in R. Sorabji, *Time, Creation, and the Continuum: Theories in Antiquity and the Early Middle Ages* [*Creation*] (Ithaca, N.Y., 1983), 214-18.

ρῶν πολλῷ πλέον, ὁ δὲ τῶν ὡρῶν ἔτι πολὺ μᾶλλον. … εἰ τοίνυν ἀδύνατον ἢ αὔξεσθαι ἢ πολλαπλασιάζεσθαι τὸ ἄπειρον, ἢ ἐνεργείᾳ ἄπειρον ἀριθμὸν γενέσθαι, ἀδύνατον ἄναρχον εἶναι τὸν χρόνον.

> It will result not only that the infinite is multiplied, but also that it is continually increased, and that an infinite is greater than an infinite. For if the number of years is infinite, then the number of months multiplies the infinite, since it has twelve times the infinite. And indeed the number of days is many more, and that of hours many more still. … If, then, it is impossible for the infinite to be increased or multiplied, or for there to be an actually infinite number, then it is impossible for time to be without beginning. (Philoponus, *In Phys.* 467.25–9, 468.2–4)

Here Philoponus asserts the impossibility of two operations on infinite pluralities: multiplication, and 'increase' or addition. In both cases, the impossibility consists at least partly in the fact that the operation would yield a plurality greater than an infinite plurality ('that an infinite is greater than an infinite'). Thus, Philoponus seems to rely on B2 (NO LESSER INFINITES). Furthermore, Philoponus says that, if the past is infinite, then an infinite number is increased with the passage of time (486.1–2). Here it is plausible to think of him as establishing A (INFINITE PART) and then drawing an inference based on B1 (PART LESS THAN WHOLE): the plurality of days that were past as of yesterday is a proper part of the plurality of days that are past as of today, and therefore the former plurality is less than the latter. Thus Philoponus, like Alexander, presents a version of the Argument from Infinite Parts.

Neither Alexander nor Philoponus claims to be expounding any particular passage in Aristotle. Still, they may well think that they are arguing from premises that Aristotle accepted. More recently, some commentators have attributed the Argument from Infinite Parts to Aristotle himself. According to Scholz, Aristotle employed a variation on the argument in order to disprove the existence of infinite magnitude. In Scholz's reconstruction, Aristotle's overall argument takes the form of a dilemma whose first horn is:

> The actual infinite consists of parts that are themselves actually infinite. But then the whole is no longer greater than the part; and this is absurd.[37]

Here, premise A belongs to the description of the first horn of the dilemma. Premise B2 justifies the claim that 'the whole is no longer greater than the part', and B1 justifies the claim that 'this is absurd'.

According to Sorabji, Coope, and others, a version of this argument moved Aristotle to deny the existence of infinite collections or pluralities. In the words of Coope, an infinite collection 'would have parts that were also infinite collections… But Aristotle has already argued that it is

37. H. Scholz, 'Warum haben die Griechen die Irrationalzahlen nicht aufgebaut?' ['Irrationalzahlen'], in *Die Grundlagenkrisis der griechischen Mathematik* (Charlottenburg, 1928), 35–72 at 55, my translation. Scholz adds in footnote 1: 'So interpretiere ich den m. E. bisher nicht verstandenen entscheidenden Satz Phys. Γ 5, p. 204a 25: πολλὰ δ' ἄπειρα εἶναι τὸ αὐτὸ ἀδύνατον.'

impossible for something to be composed of infinite parts.'[38] (Whereas Scholz attributes B1 and B2 to Aristotle, Coope attributes B (NO INFINITE PART) to him without providing an account of his reasons for accepting it.)

The Argument from Infinite Parts is a fairly persuasive argument, and one with a long afterlife.[39] Unlike the previous two arguments, there is no clear evidence that Aristotle would have rejected any of its premises. Nevertheless, there is also no good evidence that Aristotle accepted all of its premises. In particular, as far as I have found, Aristotle never endorses or relies on B1 (PART LESS THAN WHOLE) as applying to parts of an infinite whole.

Some commentators think that Aristotle makes an Argument from Infinite Parts in the following passage:

T14 ὥστ' ἢ ἀδιαίρετον ἢ εἰς ἄπειρα διαιρετόν· πολλὰ δ' ἄπειρα εἶναι τὸ αὐτὸ ἀδύνατον.

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38. Coope, *Time*, 10-11.

39. Variations on the argument appear, for example, in Galileo and in Bolzano: G. Galilei, 'Discorsi e dimostrazioni matematiche intorno à due nuove scienze', in A. Favaro, I. Del Lungo, V. Cerruti, G.V. Schiaparelli, and U. Marchesini (eds.), *Le opere di Galileo Galilei* (Firenze, 1898), 38–362 at 78-79; B. Bolzano, *Paradoxien des unendlichen* (Berlin, 1889), 27-28. Galileo accepts B2 and rejects B1, saying that no infinite quantity is greater than, less than, or equal to any other (p. 79). Bolzano accepts B1 and rejects B2, saying that a one-to-one correspondence between infinite pluralities does not guarantee their equality (pp. 30-1). Since Cantor, we have good reason to deny both B1 and B2.

> Consequently, it is either indivisible or it is divisible into infinites. But it is impossible for the same thing to be many infinites (*Phys.* 3. 5, 204ᵃ24–5).

These commentators read the claim 'it is impossible for the same thing to be many infinites' as an assertion of premise B (NO INFINITE PART).[40] It is possible to read the passage this way, but other readings are also possible and, I believe, more plausible.[41] Let us briefly review the context of the passage, and then survey some possible interpretations of it. Aristotle is arguing against a view that he has previously attributed to the Pythagoreans and to Plato (203ᵃ4, 204ᵃ33), according to which the infinite is a substance and a principle in its own right (as opposed to being a quantitative attribute of things). His argument has two steps. First he argues that, according to the view he is attacking, the infinite must be indivisible. Second, he asserts that nothing indivisible can be infinite. Text **T14** completes the first of these two steps.

Aristotle has asserted that if the infinite is a substance and principle, then, if the infinite is divisible into parts, every part of it will be infinite (204ᵃ22). In **T14** he says that this is impossible because 'it is impossible for the same thing to be many infinites'. Our question is, what does it mean to be many infinites, and why does Aristotle feel entitled to assert, without argument, that this is impossible? Here are some options.

40. Menn, *Metaphysics*, Iβ3, 24 n. 51, Iβ4, 14; Scholz, 'Irrationalzahlen', 55 n. 1; Sorabji, *Creation*, 211-12; Coope, *Time*, 11 n. 23; Bowin, 'Unity of Change', 328.

41. I am in agreement with D. Bostock, 'Time and the Continuum', *OSAP*, 6 (1988), 255–70 at 264.

Interpretation 1: Aristotle's language in the passage is not typical of claims about wholes and parts in his writings. He does not normally say that a thing *is* its parts, but that it is (composed) *out of* its parts.[42] Thus, the simple accusative plural ('many infinites') in the predicate expression is unusual. The emphatic τὸ αὐτό ('the same thing') in the subject expression is also unusual. More typical for Aristotle, if he were making a claim about the parts it is possible to have, would be to say, 'it is impossible for something to be (composed) out of many infinites': ἐκ δὲ πολλῶν ἀπείρων εἶναί τι ἀδύνατον.[43]

Given the language of the passage, perhaps the impossibility Aristotle has in mind is not about parts at all. He might instead be thinking about the multiplication of a Platonic Form. The language in **T14** is reminiscent of the following bit of anti-Platonic argument from the *Metaphysics*:[44]

T15 ἔτι πολλὰ ἔσται αὐτὸ τὸ ζῷον·

Furthermore, Animal Itself will be many. (*Metaph.* Z 14, 1039^{b}9)

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42. For some examples of ἐκ + genitive, paired with talk of divisibility, see *GC* 1. 2, 316^{a}16 ff. In *Topics* 6. 13, Aristotle says that the parts are not, in general, the same as the whole (150^{a}15–16), and he distinguishes between the claim that something is 'these' and the claim that it is '(composed) out of these' (ταῦτα vs. τὸ ἐκ τούτων, 150^{a}22). I am not aware of any passages that would provide a parallel for talk of something as 'being' the parts into which it is divisible, though there may be such passages.

43. Compare, for example, 'ἀδύνατον ἐξ ἀδιαιρέτων εἶναί τι συνεχές' at *Phys.* 6. 1, 231^{a}24.

44. Compare also *Phys.* 185^{b}25–7; Plato's *Soph.* 251b2–3; *Parm.* 129b6–7, 143a8–9, 144e5–7.

In **T15**, as in **T14**, we have αὐτό ('same', 'itself') in the subject and a simple plural πολλά ('many') in the predicate. The situation described in **T15** is one in which there are multiple Forms of the very same thing: more than one Animal Itself. This is an unwelcome or impossible consequence for Plato, since Plato standardly opposes a Thing Itself to its many participants, holding that, for any F, if there is an F Itself then the F Itself is one.[45] We might speculate that Aristotle is deriving the same sort of impossibility in **T14**. We could explain Aristotle's use of the phrase τὸ αὐτό as an abbreviation for τὸ ἄπειρον αὐτό, 'the Infinite Itself'. Aristotle would be arguing that, if the Infinite Itself is divisible, then every part of it is—not just infinite, but—an Infinite Itself.[46] Hence, there would be more than one Infinite Itself, and this is impossible.

On this interpretation, infinity as such plays no special role in the argument for indivisibility: the same reasoning would apply to any Form that is a principle. Infinity will, however, play a role in the second step of the overarching argument, when Aristotle argues that something indivisible cannot be infinite.

Interpretation 2: Immediately before **T14**, Aristotle has argued that the kind of infinite under discussion, if it were divisible, would be such that every part of it is infinite. It would not merely have some infinite parts; it would have exclusively infinite parts, and no finite parts. We might read **T14** as a claim about parts indeed, but as an assertion of the impossibility of this more spe-

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45. *Rep.* 479d10, 479e2, 493e2–494a1

46. If we end up favoring this line of interpretation, we would need to spell out a justification for this claim. For now, I am content to sketch the interpretation as one of a handful of possibilities.

cific kind of situation. Admittedly, **T14** does not speak explicitly of being *nothing but* infinites. But Aristotle has emphasized this aspect of the situation immediately beforehand and will do so again immediately afterward: ‘anything whatsoever of it that is taken will be infinite’ (204^a22); ‘just as air is part of air, so infinite of infinite, if it is a substance and principle’ (204^a26–7). Now, when Aristotle himself reasons about infinite things, he apparently assumes that it is always possible to take a finite part of something infinite.[47] So he presumably held that it is impossible for something to have exclusively infinite parts, and no finite parts. He can very reasonably have held this even if he allowed the possibility of something’s having one or more infinite parts in addition to having finite parts. Under this interpretation, then, **T14** does not suggest an inclination towards the Argument from Infinite Parts.

Interpretation 3: Aristotle may be saying that it is impossible for something to have infinite parts, plural, while not denying the possibility of having an infinite part, singular.[48] Specifically, he may be saying this because he thinks it is impossible to *construct* a plurality of infinite parts, thinking that one infinite part will never leave room for the construction of another infinite part. This would explain why he gives his argument only for that one special case where he can *infer*

47. For example: ‘of the (infinite) time C, let something finite be taken, CD’ (εἰλήφθω δέ τι τοῦ χρόνου πεπερασμένον, ἐφ’ ᾧ ΓΔ), *Phys.* 6. 2, 233^a35–$^b1$; ‘let there be taken away from the infinite (magnitude) a finite magnitude, BD’, (ἀφῃρήσθω οὖν ἀπὸ τοῦ ἀπείρου πεπερασμένον μέγεθος ἐφ' ᾧ τὸ ΒΔ), *De caelo* 1. 6, 273^a29–30.

48. D. Bostock, ‘Aristotle on continuity in Physics VI’, in L. Judson (ed.), *Aristotle’s Physics: A Collection of Essays* (Oxford: New York, 1991), 179–212 at 180 n. 3.

the existence of multiple infinite parts, as opposed to constructing them. It explains why he does not give similar arguments against ordinary infinite magnitudes, times, or pluralities, but only against something whose very essence is to be infinite.

According to this interpretation, Aristotle's argument in **T14** is closely related to an argument he gives later in the chapter, during his discussion of infinite body. In the later passage, Aristotle states that it is impossible for more than one element (earth, air, fire, or water) to be infinitely large. He explains his position as follows:

T16 ἕκαστον δ' ἄπειρον εἶναι ἀδύνατον· σῶμα μὲν γάρ ἐστιν τὸ πάντῃ ἔχον διάστασιν, ἄπειρον δὲ τὸ ἀπεράντως διεστηκός, ὥστε τὸ ἄπειρον σῶμα πανταχῇ ἔσται διεστηκὸς εἰς ἄπειρον.

It is impossible that each (element) is infinite. For a body is that which has extension in every direction, and infinite is that which is infinitely extended, so that an infinite body will be extended to infinity in every direction. (*Phys.* 3. 5, 204^{b}19–22)

According to this passage, a body qualifies as infinite only if it extends to infinity in every direction. Based on the surrounding text, Aristotle seems to think that such an infinite body might leave room for other, finite, bodies (presumably they would form pockets within the infinite body).[49] But he evidently thinks it could not leave room for a second infinite body. Nowadays, we

49. Aristotle argues that an infinite body would have infinite power (for heating or cooling, and for making wet or dry), and would therefore destroy any finite bodies of different kinds, *Phys.* 3. 5, 204^{b}14–19. The fact that he gives this

know how to divide infinite bodies into multiple infinite parts (for example, nested spirals might each of them extend infinitely in every direction), but Aristotle appears to have thought that this cannot be done. He did know, of course, how to divide time into infinite past and infinite future, and could surely contemplate similar divisions in magnitudes. But he seems to regard the resulting sorts of partially bounded items as infinite only 'in a way' (πῇ), not infinite properly speaking (*De caelo* 1. 12, 283ᵃ10).

Aristotle's argument in **T16** and, according to the present interpretation, in **T14** as well, is the inverse of the Argument from Infinite Parts as explained above (pp. 30 ff.). The Argument from Infinite Parts, as adapted to the present case involving plural parts, would consist in the construction of two or more infinite parts, so as to justify what we may call premise A' (INFINITE PARTS [PLURAL]), followed by a principled argument for what we may call premise B' (NO [PLURAL] INFINITE PARTS). Aristotle, to the contrary, gives an argument of principle for premise A', and relies on considerations about (in)constructibility when he endorses premise B'. This means that his grounds for endorsing premise B' block the typical route to a justification of premise A'. As long as he believes premise B', he will not endorse premise A' (at least, not for ordinary infinite magnitudes and pluralities); and, if someone convinces him of premise A' in the typical way, namely by construction (by dividing an infinite body into nested spirals, for example, or dividing

argument suggests that, from a purely spatial point of view, he thinks an infinite body can coexist with distinct finite bodies. The impossibility of the situation is physical, not purely quantitative.

the numbers into even and odd), then he will stop believing premise B'. We do not, then, have the makings of an Argument from Infinite Parts, according to the present interpretation.

Interpretation 4: Finally, we come to the interpretation recommended by the commentators mentioned above (n. 40). According to this interpretation, Aristotle in **T14** expresses a principled belief in the impossibility of having infinite parts, designed to figure in a form of the Argument from Infinite Parts. According to some of these commentators, Aristotle is motivated by the conviction that a whole must be greater than any of its parts (B1, PART LESS THAN WHOLE).[50] For these commentators, the underlying view of **T14** must be that it is impossible to have even one infinite part.[51] Other commentators do not give an account of Aristotle's underlying reasons, and it is unclear whether they think that one or many infinite parts are needed to generate the impossibility claimed by Aristotle.[52]

This interpretation faces some disadvantages relative to the other three. One disadvantage is that it does not explain Aristotle's use of the plural 'infinites' and the word 'many', given that some of its adherents positively think that a single infinite part is impossible, and the others at any rate do not explain what is specially impossible about plural infinite parts.

50. Menn, *Metaphysics*, Iβ4, 14; Scholz, 'Irrationalzahlen', 55

51. Bowin also appears to interpret the doctrine of **T14** as denying the possibility of even one infinite part (Bowin, 'Unity of Change', 238 n. 13).

52. Sorabji, *Creation*, 211–12; Coope, *Time*, 10–11; Coope, 'Aristotle on the Infinite', 267.

A second, related, disadvantage is that the interpretation does not explain why Aristotle confines his argument to the case of the essentially infinite, and does not explicitly employ it against ordinary infinite magnitudes or pluralities. Aristotle knows how to construct at least *one* infinite part of an infinite quantity, since he knows how to take a finite part (see n. 47) and he knows that doing so leaves an infinite remainder.[53] So, if he thought that an impossibility follows from the existence of a single infinite part, it would have been useful for him to say so. (Not, of course, that Aristotle wrote down everything he believed; an absence is not probative. Still, on the topic of infinity Aristotle is generous with his arguments, both in *Phys.* 3. 5 and in *De caelo* 1. 5–7, and does not give the impression of holding anything back.) And if he did not think so, then we want an account of what is wrong with two or more infinite parts, that was not already wrong with one.

In all, I see little reason to prefer this interpretation over the other three options, and some reason against preferring it. Therefore, I do not think **T14** can be regarded as evidence that Aristotle would endorse the Argument from Infinite Parts.

53. If the remainder were finite, then the remainder together with the part taken would be finite (*De caelo* 271^{b}21–3); but that is the original quantity, which was supposed to be infinite. Also, a finite quantity is exhausted by repeated removal of equal finite quantities (*Phys.* 1. 4, 187^{b}25–6), while an infinite quantity is not (*Phys.* 8. 10, 266^{a}20–1). But if the removal of finite quantity X from infinite quantity Y left as remainder a finite quantity Z, then Y and Z would both alike be exhausted by repeated removal of X.

Before closing, let us look more closely at premise B1 (PART LESS THAN WHOLE). As mentioned above, Scholz and Menn attribute this premise to Aristotle. However, I have not been able to find evidence for such an attribution in Aristotle's texts. And there is reason to doubt that he would have accepted a principle like this which gives purely quantitative grounds for a rejection of the infinite. For Aristotle holds that time is infinite.[54] And, although there are physical and metaphysical differences between time and other quantities such as magnitudes and pluralities, Aristotle applies the same sorts of *quantitative* reasoning to them all.[55]

Aristotle appears to assume that finite wholes are greater than any of their parts. But never, as far as I have found, does he do so for infinite items. Instead, he occasionally employs a rather more roundabout technique to show, of certain parts of an infinite item, that they are less than infinite. The technique resembles an argument from proportion. It begins from the assumption that there is a correspondence between two types of quantity: for example, between times and the distances traversed in them, or between bodies and their weights. Aristotle then introduces

54. See for example *De caelo* 2. 1, 283^{b}29, 284^{a}9-10; 2. 6, 288^{b}10-11. Aristotle speaks of infinite time as a completed totality, as the lifetime of the cosmos, at *De caelo* 1. 9, 279^{a}25–8. I do not understand or accept the claim of Cooper that, in Aristotle's view, future and past time are '*indeterminately* finite', finite even while '[t]here is always more time past before any "now" than any finite period we may specify' (J.M. Cooper, 'Aristotelian Infinites', *OSAP*, 51 (2016), 161–206 at 185, emphasis original).

55. See for example *Phys.* 4. 12, 220^{b}24–31; *Phys.* 6 passim.

the premise that a greater (or lesser) quantity of the one type always corresponds to a greater (or lesser) quantity of the other type. Here are examples of such 'greater-greater' premises:[56]

T17 πᾶν τὸ κινούμενον ἐν χρόνῳ κινεῖται, καὶ ἐν τῷ πλείονι μεῖζον μέγεθος.

Everything that moves moves in time, and moves a greater distance in more time. (*Phys.* 6. 7, 237^{b}23–4)

T18 τὸ γὰρ τοῦ ἐλάττονος βάρος ἔλαττον.

The weight of the lesser (body) is lesser. (*De caelo* 1. 6, 273^{a}31–2)

From here, Aristotle assumes (always for *reductio*, as far as I have found) that an infinite quantity of one type corresponds to a finite quantity of the other type: for example, that an infinite time is taken to traverse a finite distance, or that an infinite body has finite weight. He takes a part of the finite quantity, and infers that this part is lesser than the whole of which it is part. (Thus, he assumes that a finite whole is greater than its proper part.) He then infers that the corresponding part of the infinite quantity is less than infinite, based on the 'greater-greater' premise. That is, Aristotle does not infer that a part of the infinite is lesser from the bare fact that it is a part.[57] He seems to go a little bit out of his way to avoid relying on such an inference. In-

56. See, in addition, *Phys.* 8. 10, 266^{a}18 (a greater body imparts motion for more time); 266^{a}26–7 (a greater power produces an equal effect in less time); 266^{b}7–8 (there is more power in a greater magnitude).

57. Thus, I disagree with Bowin's claim that the argument of *Phys.* 6. 7, 238^{a}9–11, relies on 'the assumption that infinite times cannot have infinite times as subsets' (Bowin, 'Unity of Change', 328 n. 13).

stead, he infers that it is lesser from the fact that it *corresponds to something lesser* than what is corresponded to by the infinite of which it is part.

As an example of the pattern of argument just described, we may take a passage in *Physics* 6. 7. Aristotle has assumed for *reductio* that something traverses a finite distance, AB, in an infinite time, CD. He has asserted that a greater distance is traversed in a greater time (**T17** above). He now argues that every proper part of AB will be traversed in a finite time:

T19 εἰλήφθω δή τι τοῦ AB διαστήματος, τὸ AE, ὃ καταμετρήσει τὴν AB. τοῦτο δὴ τοῦ ἀπείρου ἔν τινι ἐγένετο χρόνῳ· ἐν ἀπείρῳ γὰρ οὐχ οἷόν τε· τὸ γὰρ ἅπαν ἐν ἀπείρῳ. καὶ πάλιν ἕτερον δὴ ἐὰν λάβω ὅσον τὸ AE, ἀνάγκη ἐν πεπερασμένῳ χρόνῳ· τὸ γὰρ ἅπαν ἐν ἀπείρῳ.

Let something of the interval AB be taken, AE, which will measure out AB. Then this AE came about in an individual time out of the infinite: for it cannot have come about in an infinite time, since in an infinite time the entirety came about. And again if I take another equal to AE, it is necessary that it came about in a finite time, since in an infinite time the entirety came about. (*Physics* 6. 7, 238^a6–11)

Aristotle's argument in this passage can be reconstructed as follows:

1. X traverses finite distance AB in infinite time CD (assumption)
2. AE is a proper part of AB (assumption)
3. AE is lesser than AB (from 2, FINITE PART LESS THAN FINITE WHOLE)
4. The time in which X traverses AE—call it KR—is lesser than CD (from 1, 2, **T17**)

5. KR is finite (from 4, NO LESSER INFINITES)

According to this reconstruction, Aristotle relies on premise B2 (NO LESSER INFINITES), but he does not rely on B1 (PART LESS THAN WHOLE) in connection with infinite wholes. Similar reconstructions may be given of arguments in *Physics* 8. 10 and in *De caelo* 1. 6 (see n. 56 and **T18**).

In sum, Aristotle's writings do not warrant attributing premise B (NO INFINITE PARTS) or B1 (PART LESS THAN WHOLE) to him. Therefore, the texts do not warrant the view that he accepted the Argument from Infinite Parts.

6 CONCLUSION

6.1 Silence

Prominent philosophers before Aristotle, including Leucippus, Democritus, and Anaxagoras, believed in an infinite plurality of principles: infinitely many atoms or infinitely many homoiomerous stuffs. Aristotle engages fairly extensively with their views. If he had known of a general argument against the possibility of actually infinite pluralities, such an argument would have been highly relevant to his discussions of these thinkers. Aristotle does not give any such general argument. Instead, he makes points such as the following:

1. If the same results can be derived from finitely or infinitely many principles, then it is better to posit finitely many principles, and the fewer the better. (*De caelo* 3. 4, 302^{b}26–30, 303^{a}17–18; *Phys.* 1. 6, 189^{a}14–16; 8. 6, 259^{a}8–12)

2. There are finitely many differences between kinds of body. (*De caelo* 3. 4, 302^b30–303^a3, 303^a19–20)

3. There are finitely many kinds of simple locomotion, corresponding to finitely many places that can serve as the goals of such locomotion. (*De caelo* 3. 4, 303^b5–8; *Phys.* 3. 5, 205^a29–31)

4. The infinite *qua* infinite is unknowable. (*Phys.* 1. 4, 187^b7; 1. 6, 189^a12–13)

None of these points is a reason to deny the existence of an actually infinite plurality of bodies or other particulars. Particulars are not, in general, principles (1). Infinitely many particulars could belong to a finite number of kinds, and have finitely many kinds of simple locomotion (2, 3). Particulars are not objects of scientific knowledge according to Aristotle, so it is not a problem if they are unknowable (4).[58]

Now, just because Aristotle did not give a certain argument in one of his surviving texts, we cannot securely infer that he rejected or was ignorant of the argument. I do not want to put too much stock in the argument from silence that I have just suggested. On the other hand, one sometimes hears arguments from silence on the other side: 'if Aristotle thought that infinite plu-

58. Universals are finite and knowable, while particulars are infinite and unknowable: *Post. An.* 1. 24, 86^a3–10. Hence I do not think Lear was right to say that for Aristotle 'the possibility of philosophy—of man's ability to comprehend the world—depends on the fact that the world is a finite place containing objects that are themselves finite' (J. Lear, 'Aristotelian Infinity', *Proceedings of the Aristotelian Society*, 80 (1979), 187–210 at 202, quoted approvingly by J. Bowin, 'Aristotelian Infinity', *OSAP*, 32 (2007), 233–50 at 233).

ralities are acceptable, why didn't he say so explicitly?' Therefore, I offer my argument from silence as a counterweight to those arguments from silence.

6.2 Closing

We have examined three ancient arguments that might be given for the impossibility of an actually infinite plurality of extended things. I have argued that we should not attribute any of these arguments to Aristotle. These were not, of course, the only possible finitist arguments that might be given, but I took them to be the most prominent and likely possibilities.

At some point in antiquity, it seems to have become common to deny the possibility of actually infinite pluralities. Certain arguments for this denial seem to have become widely influential. And both the denial and the arguments seem to have been associated with the name of Aristotle from a quite early date. And yet, as far as I can see, Aristotle's own extant writings do not support the view that he himself made the denial or endorsed the arguments. If this is correct, then I think there are interesting historical questions to pursue here.

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