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Chapter 1
Conditionals, Support and Connexivity

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Abstract
In natural language, conditionals are frequently used for giving explanations. Thus the antecedent of a conditional is typically understood as being connected to, being relevant for, or providing evidential support for the conditional's consequent. This aspect has not been adequately mirrored by the logics that are usually offered for the reasoning with conditionals: neither in the logic of the material conditional or the strict conditional, nor in the plethora of logics for suppositional conditionals that have been produced over the past 50 years. In this paper I survey some recent attempts to come to terms with the problem of encoding evidential support or relevance in the logic of conditionals. I present models in a qualitative-modal and in a quantitative-probabilistic setting. Focussing on some particular examples, I show that no perfect match between the two kinds of settings has been achieved yet.

1.1 Introduction

Specialised logics for indicative and subjunctive conditionals have been developed for more than 50 years. Their analyses all focussed on *suppositional conditionals*. Suppose the antecedent and check whether the consequent is true or believed to be true. If yes, the conditional is true or accepted (from now on, I will use 'true^{A}' as an abbreviation for 'true or accepted' or 'true or believed to be true'). If no, it isn't. Suppositional conditionals thus allow for truth or acceptance by "inertia". If the consequent of a conditional is true^{A}and its antecedent does not interfere with the consequent's truth^{A}, this is enough for the conditional to be true^{A}. In recent years, however, an increasing number of authors have argued that it should not be enough. They argued that the mere truth^{A} of A and C does not justify the acceptance of 'If A then C'. Many conditionals as used in natural language are meant to express the fact that the antecedent is positively relevant to the truth^{A} of the consequent. In the area of conditionals, however, the notion of relevance is claimed by distinct traditions, and may mean very different things.[1]

One tradition concerns so-called *relevant logics*, designed to formulate a more demanding notion of implication than Lewis's strict implication (cf. Meyer and Routley 1973). The idea of relevant implication is based in large part on the idea that the antecedent and the consequent of an acceptable conditional $A \rightarrow B$ must be connected in terms of contents or topics. For example, a conditional of the form $(p \wedge \neg p) > q$ need not be valid, despite having an impossible antecedent, since the consequent shares no content with the antecedent. Various constraints can be imposed on the idea of content-sharing, leading to various non-classical logics of relevant implication (see Mares 2020 for an overview, and Weiss 2019 on the use of analytic entailment to deal with Sextus' so-called fourth conditional). We will not follow this interpretation of 'relevance' in this paper.[2]

Another approach to the notion of relevance is in terms of the idea that the antecedent of a conditional should *make a difference* to the truth, assertability or probability of the consequent. Consider a conditional like "if Leicester is in England, then Russell was an English philosopher". The conditional sounds odd because the truth of the antecedent does not matter at all to the truth of the consequent (Leicester is not mentioned in Russell's autobiography or in Monk's biography of Russell). The idea of difference-making is related to the idea of positive statistical relevance that was prominent in debates on probabilistic causation and explanation in the 20th century (Suppes, Salmon), and it can be expressed in various ways:

$$A \left\{ \begin{array}{c} \text{supports} \\ \text{explains} \\ \text{is evidence for} \\ \text{is a reason for} \\ \text{is positively relevant to} \\ \text{speaks in favour of} \\ \text{is connected to} \\ \text{has a link to} \\ \text{makes a difference to} \end{array} \right\} C, \quad \text{or} \quad C \text{ is dependent on } A.$$

I will use 'A supports C' or the noun 'support' as the generic expressions covering these readings.

Of course, the idea of support itself can be formulated in several ways. But loosely speaking, support can only go in one direction, relevance cannot be both positive and negative. More precisely, we can say:

[1] I set aside a discussion of "biscuit conditionals", so-called after J. L. Austin's (1956) example "there are biscuits on the sideboard if you want them". Following Iatridou (1991), such conditionals have occasionally been called "relevance conditionals" in the linguistics literature.

[2] In Section 1.3.3, though, we will briefly touch upon a related idea.

1. Nothing can speak in favour of its own negation.
2. If something speaks in favour of something else, it cannot speak in favour of the latter's negation.
3. If something speaks in favour of something else, the former's negation cannot speak in favour of the same thing.

These statements should resonate in the connexive logician's ears. If we interpret them as applying to conditionals, they correspond closely to the celebrated theses associated with Aristotle, Boethius and Abelard, except that they spoke of something *following from* something else. So perhaps we should trace the idea that the antecedent of a conditional *supports*, or *speaks in favour of*, its consequent. This idea has indeed been recently advocated in a research community that is quite separate from the community of connexive logicians. This paper reviews some of the approaches that have been taken and that can be taken, as well as the logics they deliver. The field surveyed here is just beginning to take shape and is still rather heterogenous. So not all aspects can be treated uniformly and in the same detail. But I try to tell my story as coherently as I can.

1.2 Qualitative accounts

1.2.1 The suppositional interpretation of conditionals

In the following I will presuppose that antecedents A and consequents C of conditionals are *factual* sentences (no occurrence of any non-Boolean conditional connective). I do not exclude, and I do not commit myself to, the view that conditionals have truth values. So I will not say anything about nested or otherwise embedded conditionals.

Covering both indicative and subjunctive conditionals, this is the idea of the suppositional interpretation:

(SC) A *suppositional conditional "If A, then C"* $(A > C)$ is true$^{\mathrm{A}}$ in a state σ just in case

$$C \text{ is true}^{\mathrm{A}} \text{ in } \sigma \text{ revised by the supposition that } A.$$

This definition is broadly in the tradition of Frank Ramsey, Ernest Adams, Robert Stalnaker and David Lewis. Strengthening the Antecedent, Contraposition and Transitivity are all valid for material and strict conditionals, but to the founders of conditional logic—Adams, Stalnaker and Lewis—, they are paradigmatically *invalid* for both indicative conditionals and counterfactuals as used in natural language. Still there are many principles that remain valid and make up attractive logical systems.

In the following, I will formulate logical principles in a way that avoids embeddings of conditionals. My presentation is thus closer to the nonmonotonic logics in the style of Kraus, Lehmann and Magidor (1990) than to the usual presentations of conditional logics, with the exception that the conditional connectives we'll use ('$>$', '$\rightarrow$', '$\gg$', '$\ggg$', '$\rhd$' etc.) are elements of the object language, not of the meta-language. Each of those connectives can lay some claim on being a formal representation of (some) natural language conditionals 'If ... then'.

A principle of the form 'If Φ, then Ψ' is to be read as follows: *for every state* σ, if the conditionals mentioned in Φ are all true^{A} in σ, then the conditionals mentioned in Ψ are true^{A} in σ. (Sometimes Φ or Ψ refer to some conditionals that are *not* true^{A}.) The variables A, B and C range over propositional sentences without any occurrences of conditional connectives.

System P: the conservative core logic of suppositional conditionals

The following principles have often been considered to form the conservative core of reasoning with conditionals (Adams 1975, Burgess 1981, Veltman 1985, Pearl 1989, Kraus, Lehmann and Magidor 1990):

(Ref) $A > A$. (Reflexivity)

(LLE) If $A \dashv\vdash B$, then $A > C$ iff $B > C$. (Left logical equivalence)

(RW) If $A > B$ and $B \vdash C$, then $A > C$. (Right weakening)

(And) If $A > B$ and $A > C$, then $A > B \wedge C$.

(Or) If $A > C$ and $B > C$, then $A \vee B > C$.

(Cut) If $A > B$ and $A \wedge B > C$, then $A > C$.

(CMon) If $A > B$ and $A > C$, then $A \wedge B > C$. (Cautious monotonicity)

Given a suppositional conditional, one can define a strong, "outer" necessity $\Box A$ as $\neg A > \bot$ and a weak, "inner" necessity $\boxdot A$ as $\top > A$, essentially following Lewis (1973b, pp. 22, 30). '$\Box$' can be read either as a metaphysical or as a doxastic necessity operator, '$\boxdot$' either as a truth or as a belief operator. Notice that $\Box A$ implies $\boxdot A$.[3]

System R: a popular logic of suppositional conditionals

System **R** adds to system **P** the principle of Rational Monotony, which entails, with system P in the background, the weaker Disjunctive Rationality and the still weaker Negation Rationality.

(NRat) If $A > C$ and $A \wedge B \not> C$, then $A \wedge \neg B > C$. (Negation rationality)

(DRat) If $A \vee B > C$ and $A \not> C$, then $B > C$. (Disjunctive rationality)

(RMon) If $A > C$ and $A \not> \neg B$, then $A \wedge B > C$. (Rational monotonicity)

System S: the strongest popular logic of suppositional conditionals

System **S**, named after Stalnaker (1968) who pioneered it, is a further strengthening of System **R**. It is usually presented by adding the axiom of Conditional Excluded Middle (CEM): $A \rightarrow C$ or $A \rightarrow \neg C$. We will get back to this principle when we consider the principle of 'Negation Commutation' in section 1.3.6. But for reasons that will become clear later (in section 1.2.3), I want to replace (CEM) by a slightly more

[3] Suppose that $\neg A > \bot$. Then by (RW), $\neg A > A$. By (Ref), $A > A$. So by (Or) and (LLE), $\top > A$.

complex, but more versatile variant which is equivalent to (CEM) in the context of System **P**:

(ROr) If $A \rightarrow B \vee C$, then $A \rightarrow B$ or $A \rightarrow C$. (Right Or)

(ROr) was discussed and recommended as an alternative to (CEM) by Hawthorne and Makinson (2007, pp. 269–271, 280–282). (ROr) implies (RMon) in the context of System **P**. Suppose $A > C$ and $A \not> \neg B$. By (Ref) and (RW), we have $A > B \vee \neg B$. So by (ROr), $A > B$. But then, by (CMon), $A \wedge B > C$.

The semantics for suppositional conditionals satisfying the conditions of systems **P**, **R** and **S** can be given in terms of selection functions σ selecting the "best" accessible elements of a set W of possible worlds according to a partial order, a weak order and a linear order, respectively. In the infinite case, the orders have to be well-orders. I want to allow here for both a doxastic interpretation and a metaphysical interpretation of σ. In the *doxastic interpretation*, σ represents an agent's doxastic state. The guiding idea here is to think of σ as a plausibility function representing our dispositions to restrict attention to certain possible worlds, given certain suppositions. Intuitively, for a subset V of W, σ selects the set $\sigma(V)$ of *most plausible* elements of V. For our purposes, it is sufficient that σ is defined on the algebra of propositions in W that are expressible by factual sentences of our language. Let $|A|$ denote the set of worlds at which A is true. An agent whose dispositions are represented by σ and who makes the supposition that A restricts attention to the worlds in $\sigma(|A|)$ for her further reasoning. $\sigma(|A|)$ can be understood as the strongest believed proposition after (hypothetically) revising her beliefs by A. $\sigma(|\top|) = \sigma(W)$ can be identified with the strongest proposition currently believed to be true by the agent. In the *metaphysical interpretation*, σ represents the similarities between possible worlds, and $\sigma(|A|)$ can be understood as the set of A-worlds that are *most similar* or *"closest"* to the actual world. $\sigma(|\top|)$ can be thought to represent the worlds that are closest to the actual world w_0, and one often wants to assume that $\sigma(|\top|) = \{w_0\}$. If there are no doxastically or metaphysically accessible worlds in $|A|$, then no worlds get selected and $\sigma(|A|) = \emptyset$.[4]

Non-standard inference schemes

The following inference schemes are typically *not* included in the most popular logics of conditionals. We use the generic arrow '$\rightarrow$' here, rather than the more specific corner '$>$' that is reserved as a symbol for the suppositional conditional.

(Arist1) Not $A \rightarrow \neg A$. (Aristotle's first thesis)

(Arist2) Not both $A \rightarrow C$ and $\neg A \rightarrow C$. (Aristotle's second thesis)

(Ablrd) Not both $A \rightarrow C$ and $A \rightarrow \neg C$. (Abelard's first principle)

(MP) If $A \rightarrow C$ and $\boxdot A$, then $\boxdot C$. (Modus Ponens)

(MT) If $A \rightarrow C$ and $\boxdot \neg C$, then $\boxdot \neg A$. (Modus Tollens)

[4] All of this is too brief to be fully accurate. For more extensive background information, see Nute and Cross (2001).

(DA) If $A \to C$ and $\boxdot\neg A$, then $\boxdot\neg C$. (Denying the Antecedent)

(AC) If $A \to C$ and $\boxdot C$, then $\boxdot A$. (Affirming the Consequent)

(ConjSuf) If $\boxdot A$ and $\boxdot C$, then $A \to C$. (Conjunctive Sufficiency)

(LRLE) If $A \dashv\vdash B$ and $C \dashv\vdash D$, then $A > C$ iff $B > D$. (Left Right Log. Equ.)

(ConjRat) If $A \to B \wedge C$, then $A \to B$ or $A \to C$. (Conjunctive Rationality)

(ACRW) If $A \to A \wedge B \wedge C$, then $A \to A \wedge C$. (Anteced. Conj. Right Weakening)

(Ant) If $A \to C$, then $A \to A \wedge C$. (Antecedence)

(Cnsv) If $A \to A \wedge C$, then $A \to C$. (Conservativity)

(AaOE) $A \to C$ iff ($A \to A \wedge C$ and $A \to A \vee C$). (And-and-Or Equivalence)

(Cntrpos) If $A \to C$, then $\neg C \to \neg A$. (Contraposition)

(CondPerf) If $A \to C$, then $\neg A \to \neg C$. (Conditional Perfection)

(AOT) $A \to A \wedge C$ iff ($\neg A \nrightarrow \neg A \vee C$ and not $\Box A$). (And-Or Toggling)

(AOT′) ($A \to A \wedge C$ and not $\Box\neg A$) iff $\neg A \nrightarrow \neg A \vee C$. (And-Or Toggling′)

The first three principles—(Arist1), (Arist2) and (Ablrd)—are well-known theses of connexive logic; the terminology used here follows McCall (2012, p. 416).[5]

The principles of the second group—(MP), (MT), (AC) and (DA)—have been widely studied in the psychology of reasoning. In the versions presented here, they make use of the inner modality $\boxdot A$. The same holds for Conjunctive Sufficiency which I have added to this group.

The principles of the last group—(LRLE), (Conjunctive Rationality), (ACRW), (Ant), (Cnsv), (AaOE), (Cntrpos), (CondPerf), (AOT) and (AOT′)—have turned out to be relevant in the study of difference-making conditionals, dependence conditionals and evidential conditionals (see sections 1.2.2–1.2.4 below). (ConjRat), (ACRW) and (Cnsv) are all weakenings of (RW). The term 'Antecedence' as used here goes back to Bochman (2001, p. 140). In labelling the converse inference by the term 'Conservativity', I am using the latter in a more specific sense than other authors.[6] Antecedence follows from (AaOE). It will turn out that (Ant) is valid, but (Cnsv) is invalid for support conditionals. Contraposition is of course valid for material and strict conditionals, but usually taken to be invalid for suppositional conditionals. The term 'Conditional Perfection' is due to Geis and Zwicky (1971). (CondPerf) does *not* imply that conditionals are *bi*conditionals (for which $A \to C$ would have to be equivalent to $C \to A$), unless in addition Contraposition is available. (AOT) and (AOT′) are very similar to each other, since both say that the truth$^{\mathrm{A}}$ (truth or acceptance) of $A \to A \wedge C$ is almost equivalent to the falsity$^{\mathrm{A}}$ (falsity or rejection) of

[5] These names have become fairly standard by now. Martin (1987, pp. 379–381) originally had the numbering of the Aristotelian theses the other way round, and he has kept it in later papers.

[6] Keenan and Stavi (1986, p. 275) and van Benthem (1986, pp. 8, 77) used 'Conservativity' for the *conjunction* of (Ant) and (Cnsv). A little earlier, van Benthem (1984, p. 311) had used 'Antecedence' for the same conjunction.

$\neg A \rightarrow \neg A \vee C$—but only almost, since some limiting cases concerning the (partial) contingency of A have to be attended to. In (AOT), $A \rightarrow A \wedge C$ is a little stronger, and in (AOT′) it is a little weaker than $\neg A \nrightarrow \neg A \vee C$. This somewhat quirky distinction will turn out to be significant later (in Sections 1.2.3 and 1.3.5.2).

Why are the non-standard schemes not part of the standard systems for suppositional conditionals? There are two very different reasons for this. There is a group of schemes that are derivable in the standard systems: (MP), (MT), (ConjSuf), (LRLE), (ConjRat), (ACRW), (Ant), (Cnsv) and (AaOE) are all derivable in **P**.[7] The complementary group is simply invalid with respect to the semantics of **P** (and of **R** and **S** for that matter): these are (Arist1), (Arist2), (Ablrd), (DA), (AC), (Cntrpos), (CondPerf), (AOT) and (AOT′).[8] While suppositional conditionals characteristically validate (RW), (ConjSuf) and (Cnsv) (which is a weakening of (RW)), we will see that support conditionals characteristically violate these principles.

1.2.2 The difference-making interpretation of conditionals

Rott (1986) proposed another definition of relevance in a belief revision framework, a criterion picked up by Spohn (2013), and proposed independently in various accounts of causality. Arguing in particular against Conjunctive Sufficiency (see Rott 1986, pp. 350–352), Rott suggests that a support conditional $A > C$ is accepted if and only if $C \in Bel * A$ and $C \notin Bel * \neg A$ (where '$*$' denotes a belief revision operator). This is a relevantised variant of the Ramsey Test, ruling that the conditional is accepted provided the revision by A results in the belief that C, but the revision by $\neg A$ does not result in the belief that C. Using the terminology of *difference-making conditionals*, Rott (2022b) calls the violation of RW the hallmark of conditionals that encode the idea that the antecedent is positively relevant to the consequent.

As already mentioned, the most widely known conditional logics are all concerned with suppositional conditionals. Support conditionals are different. One characteristic is that they don't—and shouldn't—validate Conjunctive Sufficiency. The *violation of Strengthening the Antecedent* can be taken to be the hallmark of such conditionals. From 'If you bring your letter to the post office, it will be delivered' one must not conclude 'If you bring your letter to the post office and burn it there, it will be delivered.' As a kind of dual thesis, Rott (2022b) holds that the *violation of Weakening the Consequent* is the hallmark of difference-making conditionals, and the same holds for dependence conditionals. From 'If you pay an extra fee, the letter will be delivered by express' it would be odd to conclude 'If you pay an extra fee, the letter will be delivered.'

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[7] Here is a proof for (MP) and (MT). From $A > C$, we get $A > (A \supset C)$. From this and $\neg A > (A \supset C)$, we get $\top > (A \supset C)$. From the latter and $\top > A$ [or $\top > \neg C$], we get $\top > C$ [or $\top > \neg A$], as desired.

[8] (Arist1) and (Ablrd) (but not (Arist2)) are derivable in **P** if they are restricted to possible antecedents, or more exactly, to antecedents A such that $\neg\Box\neg A$ (outer modality).

(DMC) A *difference-making conditional "If A, then C"* ($A \gg C$) is true$^{\mathrm{A}}$ in a state σ just in case

$$C \text{ is true}^{\mathrm{A}} \text{ in } \sigma \text{ revised by the supposition that } A, \quad \text{and}$$
$$C \text{ is not true}^{\mathrm{A}} \text{ in } \sigma \text{ revised by the supposition that } \neg A.$$

This is essentially the *Relevant Ramsey Test* (Rott 1986; 2022b). Semantically, in terms of orders, it means that for every $A \wedge \neg C$-world, there is a 'better' $A \wedge C$-world, but not for every $\neg A \wedge \neg C$-world, there is a 'better' $\neg A \wedge C$-world. It has the following important property: A state that happens to have A and C true$^{\mathrm{A}}$ does not automatically have $A \gg C$ true$^{\mathrm{A}}$—no Conjunctive Sufficiency! More generally, the truth$^{\mathrm{A}}$ of $A \gg C$ is completely independent of the truth$^{\mathrm{A}}$ of A and of C taken individually. In this sense, difference-making conditionals are "purely relational". Even when A entails C, there is no guarantee that $A \gg C$ is true$^{\mathrm{A}}$ (no supra-classicality). For it may be that C is true$^{\mathrm{A}}$ "anyway", i.e., on the supposition that $\neg A$, too. This is the case when C is a tautology, but there are many other such cases.

A difference-making conditional respects, in a qualitative way, both of the two notions of confirmation of Carnap (1962, p. xvi), firmness and increase in firmness: C is not only true$^{\mathrm{A}}$ given A, but the supposition of A also increases the truth$^{\mathrm{A}}$ value of C vis-à-vis the supposition of $\neg A$.

The strong, outer necessity '$\Box$' and the weak, inner necessity '$\boxdot$' (again in the sense of Lewis 1973b) can be expressed in the pure language of difference-making conditionals: $\Box A$ is expressible by $\neg A \gg \bot$, $\boxdot A$ by 'not $\bot \gg A$'. The suppositional conditional $A > C$ can be expressed by '$A \gg A \wedge C$ or ($\Box A$ and $\boxdot C$)'.[9]

The Relevant Ramsey Test yields a logic of difference-making conditionals that invalidates all of the principles of system **P**, except for (LLE) and (And). This is not a bad thing at all, but just reflects the different meaning of the support conditional. If we are losing so much, however, is there anything left? It turns out that the answer is 'yes'. We start with the basic part of the logic of difference-making conditionals.[10]

Theorem 1 *Difference-making conditionals satisfy the following basic conditions: (LRLE), (ConjRat), (AaOE), (AOT), and in addition:*

(Cnst) *not* $\boxdot\bot$; *(Consistency)*

(IN) $\boxdot(A \vee C)$ *iff* ($\boxdot A$ *or not* $A \gg A \vee C$); *(Inner Necessity)*

(OND) $\Box(A \wedge B)$ *iff* ($\Box A$ *and* $\Box B$). *(Outer Necessity Distribution)*

The (And) principle follows from this set of axioms. Except for (LLE), it is the only condition that carries over from suppositional conditionals to difference-making conditionals. However, it is not needed if we endorse the above collection of axioms.

The conditions mentioned in Theorem 1 can be taken to be the axiomatisation of difference-making conditionals. Difference-making conditionals also satisfy (MP), (MT), (AC) as well as a weaker form of (DA) (Rott 2022b, p. 145):

[9] Compare Rott (2022b, pp. 149–150).

[10] For proofs of results similar to Theorems 1–4, the reader is invited to compare Raidl (2021a; 2021b) and Rott (2022b).

($\gg$DA^w) If $A \gg C$ and $\boxdot\neg A$, then not $\boxdot C$.

One can also prove the converse of Theorem 1.

Theorem 2 (Representation theorem for difference-making conditionals)

Let $\gg$ be a conditional that satisfies the conditions mentioned in Theorem 1. Then there is a selection function σ that generates $\gg$ by means of (DMC).

For the representation theorem, one can define σ by $\sigma(|A|) = \bigcap\{|C| : A \gg A \wedge C$ is true$^{\mathrm{A}}$, or $\Box A$ and $\boxdot C$ are true$^{\mathrm{A}}\}$.

The supplementary part of the logic of difference-making conditionals concerns conditionals with compound antecedents. Many of the principles of systems **P** and **R** are valid for difference-making conditionals if they are restricted to non-beliefs (*restricted validities*):

($\gg$Or^r) If $A \gg C$ and $B \gg C$ and not $\boxdot(A \vee B)$, then $A \vee B \gg C$.

($\gg$Cut^r) If $A \wedge B \gg C$ and $A \gg B$ and not $\boxdot A$, then $A \gg C$.

($\gg$CMon^r) If $A \gg C$ and $A \gg B$ and not $\boxdot A$, then $A \wedge B \gg C$.

($\gg$RMon^r) If $A \gg C$ and not $A \gg \neg B$ and not $\boxdot A$, then $A \wedge B \gg C$.

Moreover, many of the principles of systems **P** and **R** are valid if they are modified by repeating the antecedent as a conjunct of the consequent (*modified validities*). This in effect amounts to a de-relevantisation of difference-making conditionals:

($\gg$Or*) If $A \gg A \wedge C$ and $B \gg B \wedge C$ and not $\Box(A \vee B)$, then $A \vee B \gg (A \vee B) \wedge C$.

($\gg$Cut*) If $A \gg A \wedge B$ and $A \wedge B \gg A \wedge B \wedge C$, then $A \gg A \wedge C$.

($\gg$CMon*) If $A \gg A \wedge B$ and $A \gg A \wedge C$, then $A \wedge B \gg A \wedge B \wedge C$.

($\gg$RMon*) If not $A \gg A \wedge \neg B$ and $A \gg A \wedge C$, then $A \wedge B \gg A \wedge B \wedge C$.

Connexivity. The difference-making conditional $\gg$ satisfies Aristotle's theses (Arist1) and (Arist2), as well as a restricted form of Abelard's thesis:

($\gg$Ablrd^r) Not both $A \gg C$ and $A \gg \neg C$, unless $\Box\neg A$ and neither $\boxdot C$ nor $\boxdot\neg C$.

'Full connexivity' with unrestricted ($\gg$Ablrd) is obtained for difference-making conditionals in Rott (2022a) where a slightly different background theory for belief revision is used ("conceivability-limited belief revision"). In this paper it is argued that the three connexive principles *should not be restricted* in any way if the interpretation of conditionals as support conditionals is to be taken seriously.

1.2.3 The dependence interpretation of conditionals

Here is a closely related, yet different idea of capturing the support relation between antecedent and consequent.

(DpC) A *dependence conditional "If A, then C"* ($A \ggcurly C$) is true$^{\mathrm{A}}$ in a state σ just in case

$$C \text{ is true}^{\mathrm{A}} \text{ in } \sigma \text{ revised by the supposition that } A, \quad \text{and}$$
$$\neg C \text{ is true}^{\mathrm{A}} \text{ in } \sigma \text{ revised by the supposition that } \neg A.$$

The second clause for the dependence conditional is stronger than that for the difference-making conditional unless $\sigma(\neg A)$ is empty (i.e., unless A is a necessity). Semantically, in terms of orders, it means that for every $A \wedge \neg C$-world, there is a 'better' $A \wedge C$-world, and for every $\neg A \wedge C$-world, there is a 'better' $\neg A \wedge \neg C$-world. Notice that $A \ggcurly C$ is equivalent to $\neg A \ggcurly \neg C$ (CondPerf). The idea of dependence conditionals is similar to David Lewis's (1973a, pp. 562–563) famous reduction of causal dependence to counterfactual dependence: If A and C describe the occurrences of two events a and c, then c causally depends on a if and only if the counterfactuals $A \mathrel{\Box\!\!\rightarrow} C$ and $\neg A \mathrel{\Box\!\!\rightarrow} \neg C$ are both true. Lewis's counterfactuals are non-epistemic suppositional conditionals in our sense.

We can express strong, outer necessity '$\Box$' and weak, inner necessity '$\boxdot$' in the pure language of dependence conditionals: $\Box A$ can again be expressed by $\neg A \ggcurly \bot$, but now $\boxdot A$ is expressed by $\top \ggcurly A$. The suppositional conditional $A > C$ can be expressed by $A \ggcurly A \wedge C$.[11]

This is the basic part of the logic of dependence conditionals:

Theorem 3 *Dependence conditionals satisfy the following basic conditions: (LRLE), (Ref), (And), (ACRW), (AaOE), (CondPerf), and in addition:*

(Cnst′) *not* $\boxdot\bot$*;*

(IN′) $\boxdot(A \vee C)$ *iff* $(\boxdot A$ *or* $A \ggcurly A \vee \neg C)$*;*

(OND′) $\Box(A \wedge B)$ *iff* $(\Box A$ *and* $\Box B)$.

The conditions mentioned in Theorem 3 can be taken to be the axiomatisation of dependence conditionals. The most conspicuous property of the dependence conditional $\ggcurly$ is (CondPerf) which follows immediately from definition (DpC). But it does not imply that $\ggcurly$ is a biconditional. The meaning of $A \ggcurly C$ is still very different from that of $C \ggcurly A$. Except for (Ref) and (LLE), (And) is the only condition that carries over from systems **P** and **R** to dependence conditionals. It is not redundant in the above collection of basic axioms.

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[11] Compare Rott (2022b, p. 151).

Dependence conditionals satisfy (MP), (MT), (DA) and (AC). The condition (ACRW) is also valid for difference-making conditionals. (ConjRat) and (AOT) do not generally hold for dependence conditionals.[12]

The supplementary part of the logic of dependence conditionals which concerns conditionals with compound antecedents is similar to, and slightly more straightforward than, that of difference-making conditionals.

Connexivity. Dependence conditionals $\ggcurly$ satisfy the fully unrestricted versions of Aristotle's theses (Arist1) and (Arist2) and Abelard's thesis (Ablrd) (for all this, see Rott 2022b, p. 151).

One can also prove the converse of Theorem 3.

Theorem 4 (Representation theorem for dependence conditionals)

Let $\ggcurly$ be a conditional that satisfies the conditions mentioned in Theorem 3. Then there is a selection function σ that generates $\ggcurly$ by means of (DpC).

For the representation theorem, one can define σ by $\sigma(|A|) = \bigcap\{|C| : A \ggcurly A \wedge C$ is true$^{\mathrm{A}}\}$.

Now that we know that difference-making conditionals and dependence conditionals are different in that they satisfy different sets of axioms, we can ask whether there are conditionals that satisfy both sets of axioms. The first answer to this question is 'no'. On the one hand, the difference-making conditional $\top \gg \top$ can never be true$^{\mathrm{A}}$, while the dependence conditional $\top \ggcurly \top$ is valid, i.e., invariably true$^{\mathrm{A}}$(Rott 2022b, pp. 144, 151).

We can conclude from (CondPerf) and (AOT′) that $A \not\ggcurly A \wedge \neg C$ implies $A \ggcurly A \wedge C$. Since $A \ggcurly A \wedge C$ is equivalent to the suppositional $A > C$, this can be interpreted as a kind of Conditional Excluded Middle for de-relevantised conditionals. World-selection functions σ selecting only singleton sets (for any input, also for the input W) generate conditionals satisfying very nearly both sets of axioms, for the simple reason that for such functions, the truth or acceptance of $\neg C$ in σ revised by a supposition is normally equivalent to the falsity or non-acceptance of C in σ revised by the same supposition. This only fails for the limiting case of metaphysically or doxastically impossible suppositions, on which both C and $\neg C$ are true$^{\mathrm{A}}$(in which case σ delivers the empty set).

If we are ready to replace (AOT) by (AOT′), then we can define a special nontrivial conditional that satisfies both sets of axioms. Call a (suppositional, difference-making or dependence) conditional *Stalnakerian* if it is based on a singleton-valued selection σ that is determined by a *total* well-ordering of the accessible worlds in W (cf. section 1.2.1).

Theorem 5 *(i) A Stalnakerian dependency conditional satisfies all the axioms for dependence conditionals plus (Right Or).*
(ii) A Stalnakerian dependency conditional also satisfies all the axioms for difference-making conditionals, with the exception that (AOT′) has to be substituted for (AOT).

[12] Concerning (ConjRat): From $\neg A > \neg(B \wedge C)$, it does not follow that either $\neg A > \neg B$ or $\neg A > \neg C$. Concerning (AOT), the left-to-right direction: $A > C$ does not preclude $A > \neg C$. Concerning its right-to-left direction: $A \not> \neg C$ does not preclude $A \not> C$.

Proof. (i) Given Theorem 3, we only have to deal with (ROr). Suppose that $A \succcurlyeq B \vee C$. From $A > B \vee C$, we can infer, by (ROr) for $>$, that $A > B$ or $A > C$. From $\neg A > \neg(B \vee C)$, we can infer, by (RW) for $>$, that both $\neg A > \neg B$ and $\neg A > \neg C$. So $A \succcurlyeq B$ or $A \succcurlyeq C$.

(ii) We prove that the axioms for difference-making conditionals with (AOT) replaced by (AOT′) are all derivable from the axioms for Stalnakerian dependence conditionals.

(LRLE) and (AaOE) are already axioms for dependence conditionals.

(ConjRat) follows from (CondPerf) and (ROr): If $A \succcurlyeq B \wedge C$, then $\neg A \succcurlyeq \neg(B \wedge C)$ by (CondPerf), so $\neg A \succcurlyeq \neg B \vee \neg C$, so, by (ROr), $\neg A \succcurlyeq \neg B$ or $\neg A \succcurlyeq \neg C$. Thus, by (CondPerf) again, $A \succcurlyeq B$ or $A \succcurlyeq C$.

(AOT′), from left to right: Suppose that $A \succcurlyeq A \wedge C$ and $A \not\succcurlyeq \bot$. Using (And) and (LRLE), we can conclude that $A \not\succcurlyeq A \wedge \neg C$. By (CondPerf and (LRLE), this gives us $\neg A \not\succcurlyeq \neg A \vee C$, as desired.

(AOT′), from right to left: Suppose that $\neg A \not\succcurlyeq \neg A \vee C$. Then, by (CondPerf), $A \not\succcurlyeq A \wedge \neg C$. Since $A \succcurlyeq A$, by (Ref), we also have $A \succcurlyeq (A \wedge C) \vee (A \wedge \neg C)$. Thus, by (ROr), $A \succcurlyeq A \wedge C$. For the second part of this direction, note that $A \not\succcurlyeq A \wedge \neg C$ implies $A \not\succcurlyeq A \wedge C \wedge \neg C$ by (ACRW), or equivalently, $A \not\succcurlyeq \bot$, by (LRLE).

(Cnst), which in the language of difference-making conditionals means $\bot \succcurlyeq \bot$, follows from (Ref).

(IN), which in the language of difference-making conditionals is equivalent to $\bot \succcurlyeq A \vee C$ iff $\bot \succcurlyeq A$ and $A \succcurlyeq A \vee C$.

From left to right. Suppose that $\bot \succcurlyeq A \vee C$. Then, by (CondPerf), $\top \succcurlyeq \neg A \wedge \neg C$. First we can use (LRLE) and (ACRW) to get $\top \succcurlyeq \neg A$. Second, suppose for reductio that $A \not\succcurlyeq A \vee C$. Then we can use (AOT′) and get $\neg A \succcurlyeq \neg A \wedge C$. So, by (CondPerf), $A \succcurlyeq A \vee \neg C$. By (IN′), we get $\top \succcurlyeq A \vee C$. On the other hand, we have $\top \succcurlyeq \neg A \wedge \neg C$. So by (And) and (LRLE), we have $\top \succcurlyeq \bot$, contradicting (Cnst′).

From right to left. Suppose that $\bot \succcurlyeq A$ and $A \gg A \vee C$. From the former we get, by (CondPerf), $\top \succcurlyeq \neg A$, and from the latter we get, by (IN′), $\top \succcurlyeq A \vee \neg C$. Thus by (And) and (LRLE) $\top \succcurlyeq \neg A \wedge \neg C$. So, by (CondPerf), $\bot \succcurlyeq A \vee C$.

(OND) is identical to (OND′), since the meaning of '$\Box$' is the same. $\Box$

Concerning part (i) of Theorem 5, we note that the condition (ROr) is valid for all suppositional, difference-making and dependence conditionals that are Stalnakerian.[13] And based on part (ii) of this theorem, we can say, loosely speaking, that the union of the axioms for difference-making conditionals and those for dependence conditionals are 'almost consistent'. Our second answer to the question raised above thus is 'almost yes'.

Theorem 6 (Representation theorem for Stalnakerian dependence conditionals)

Let $\succcurlyeq$ be a conditional that satisfies the conditions mentioned in Theorem 3, (ROr) and

($\succcurlyeq$Or*) *If $A \succcurlyeq A \wedge C$ and $B \succcurlyeq B \wedge C$, then $A \vee B \succcurlyeq (A \vee B) \wedge C$.*

[13] By contrast, (CEM) is invalid for Stalnakerian difference-making and Stalnakerian dependence conditionals, as is witnessed by any state in which both $A > C$ and $\neg A > C$ are true/accepted.

($\gg$CMon*) *If* $A \gg A \wedge B$ *and* $A \gg A \wedge C$ *, then* $A \wedge B \gg A \wedge B \wedge C$.

Then there is a Stalnakerian selection function σ *that generates* $\gg$ *by means of (DpC).*

Sketch of proof: We build on Theorem 7.2 and its proof in Rott (2022b), from which we know that the conditional $>$ defined by $A > C$ iff $A \gg A \wedge C$ is a suppositional conditional satisfying System **P**. The selection function σ is defined by putting $\sigma(|A|) = \bigcap\{|C| : A \gg A \wedge C \text{ is true}^{\mathrm{A}}\} = \bigcap\{|C| : A > C \text{ is true}^{\mathrm{A}}\}$. What remains to show here is that $>$ satisfies (ROr). Let $A > B \vee C$, i.e., $A \gg A \wedge (B \vee C)$. By (LRLE), this means that $A \gg (A \wedge B) \vee (A \wedge C)$. Thus, by (ROr) for $\gg$, $A \gg A \wedge B$ or $A \gg A \wedge C$. Using the definition of $>$, again, this is equivalent to $A > B$ or $A > C$. Now we have shown that $>$ thus defined is a Stalnakerian suppositional conditional, and it is known that such conditionals can be generated by the Stalnakerian selection function σ.

1.2.4 The contraposition interpretation of conditionals

Vincenzo Crupi and Andrea Iacona (2022a) have worked out another idea of capturing the notion of epistemic support. They start out their explication of ‘evidential conditionals’ referring to an incompatibility between the antecedent and the negated consequent of such conditionals. They take the term ‘incompatibility’ from Chrysippus, but offer their own interpretation:

> The core idea of the evidential account is that a conditional $A \rhd C$ is true if and only if A and $\neg C$ are incompatible in the following sense: if A is true, then C cannot easily be false, and if C is false, then A cannot easily be true.[14]

Crupi and Iacona then go on and analyse the first part of the quoted passage by the Ramsey Test for the suppositional conditional $A > C$, and the second part by what they call the Reverse Ramsey Test for this conditional, which is identical with the ordinary Ramsey test for $\neg C > \neg A$. This conjunction they call *the Chrysippus Test* (Crupi and Iacona 2022a, p. 2901). The following representation of the evidential conditional is a good rendering of the essence of Crupi and Iacona’s idea. Even though I have serious doubts whether their approach captures the notion of evidential support (cf. Rott 2022c), I will stick to their term ‘evidential conditional’, hoping to be excused if I apply quotation marks from time to time.

(CpC) An *evidential conditional “If A, then C”* ($A \rhd C$) is true$^{\mathrm{A}}$ in a state σ just in case

$$C \text{ is true}^{\mathrm{A}} \text{ in } \sigma \text{ revised by the supposition that } A, \text{ and}$$
$$\neg A \text{ is true}^{\mathrm{A}} \text{ in } \sigma \text{ revised by the supposition that } \neg C.$$

[14] Crupi and Iacona (2022a, p. 2900), notation adapted.

Semantically, in terms of orders, (CpC) means that for every $A \wedge \neg C$-world, there is a 'better' $A \wedge C$-world *and* a 'better' $\neg A \wedge \neg C$-world. The second clause used in (CpC) is logically independent of the second clauses used in (DMC) and in (DpC).

The strong, 'outer' necessity '$\Box A$' can be expressed again by $\neg A \rhd \bot$, or equivalently, by $\top \rhd A$. However, the weak, 'inner' necessity '$\boxdot A$' cannot be expressed in the pure language of evidential conditionals.[15]

When introducing their modal account, Crupi and Iacona considered only few of the principles of Systems **P** and **R**, namely the valid (LLE) and the invalid (RW). They also mentioned (MP) as valid. Characteristically, Crupi and Iacona's account also validates Contraposition. The validity of the principles (And), (Or) and (CMon) was verified by Raidl (2019), the validity of (NRat) and (DRat) and the invalidity of (Cut), and (RMon) by Rott (2020). This holds against the background of a very well-behaved selection function that validates the principles of System **R** for the suppositional conditional $>$.

Crupi and Iacona (2022a, pp. 2913–2914) also show that their conditional satisfies restricted variants of Aristotle's theses and Abelard's thesis. The restricting conditions are that $\neg\Box\neg A$ for (Arist1) and (Ablrd) and that $\neg\Box C$ for (Arist2). Crupi and Iacona emphasise that the restricted principles, 'unlike their unrestricted counterparts, imply no revision of classical logic.' It is easy to check the validity of (MT) and the invalidity of (AC) and (DA). Let us collect what we know in another theorem:

Theorem 7 *Evidential conditionals satisfy the following conditions: (LRLE), (Ref), (And), (Or), (CMon), (NRat), (DRat), (Cntrpos), (Ant) and moreover (MP), (MT) and restricted versions of (Arist1), (Arist2) and (Ablrd). On the other hand, they do not satisfy (RW), (Cut), (RMon), (AC), (DA), (ConjSuf) and (Cnsv).*

We can conclude that Crupi and Iacona's 'evidential conditionals' obey a very well-behaved logic. A completeness theorem for truth-valued, fully embeddable and iterable evidential conditionals is proved by Raidl, Iacona and Crupi (2022).

1.2.5 Definable conditionals

Eric Raidl (2021a; 2021b) has developed a sophisticated general technique of transferring completeness results for the well-known and "well-behaved" suppositional conditionals $>$ to relevance-expressing conditionals that are definable in terms of $>$. He is thus able to cover the proposal of Crupi and Iacona as well as those of Rott

[15] This can be seen as follows. Consider the propositional language with two atoms p and q, suppose that there are only three accessible worlds w_1, w_2 and w_3 satisfying $p \wedge q$, $p \wedge \neg q$ and $\neg p \wedge \neg q$, respectively. Consider two different states characterised by the total orders $w_1 \prec_1 w_3 \prec_1 w_2$ and $w_3 \prec_2 w_1 \prec_2 w_2$. The semantics for $\rhd$ makes it clear that the two orders satisfy exactly the same evidential conditionals (essentially $p \rhd q$ and its consequences according to Theorem 9). However, we have $\boxdot_1(p \wedge q)$ and not $\boxdot_1(\neg p \wedge \neg q)$, but the reverse is true for $\boxdot_2$. Given what we said above, it follows that suppositional conditionals are not definable from evidential ones, nor are difference-making conditionals or dependence conditionals. The converse definitions are all possible.

in a unified manner and presents logics of *definable conditionals* (or *strengthened conditionals*) as modal logics with full embeddings and iterations of conditionals.

Difference-making conditionals:
$A \gg C \quad \text{iff}_{def} \quad A > C \text{ and } \neg A \not> C\,.$

Dependence conditionals:
$A \ggg C \quad \text{iff}_{def} \quad A > C \text{ and } \neg A > \neg C\,.$

Crupi and Iacona's 'evidential conditionals':
$A \rhd C \quad \text{iff}_{def} \quad A > C \text{ and } \neg C > \neg A\,.$

Raidl's completeness theorems concern conditionals with a truth-value semantics.

1.2.6 Iterated change interpretations of conditionals

A different way of modifying the truth or acceptance conditions of conditionals in such a way as to guarantee the relevance of the antecedent to the consequent and to avoid Conjunctive Sufficiency consists in first retracting belief (or disbelief) in the antecedent or the consequent, and only then making the suppositional test as formulated in (SC). This idea in particular avoids (ConjSuf). It can be found with some variations in Rott (1986), Vidal (2016; 2018) and Andreas and Günther (2019).

Rott (1986, pp. 352–355) suggests "contracting" the agent's belief set by the *consequent* of a "universal conditional" before the conditional gets evaluated by a suppositional test with respect to its antecedent. Vidal (2016, pp. 293–299; 2018, pp. 131–133) requires that the *antecedent* be 'inhibited' or 'neutralized' in a first step for the conditional 'If A, C' (without 'then' preceding the consequent), and that *both the antecedent and the consequent* be inhibited in a first step for the conditional 'If A, then C'. We take a closer look at Andreas and Günther who also propose to retract both the antecedent and the consequent before the antecedent is supposed to be true again:

> First, suspend judgement about the antecedent and the consequent. Second, add the antecedent (hypothetically) to your stock of explicit beliefs. Finally, consider whether or not the consequent is entailed by your explicit beliefs. (Andreas and Günther 2019, p. 1248)

The positive relevance of A for C is intended to be guaranteed by an 'agnostic move' with respect to both A and C and finding that subsequently A is sufficient to reinstate C. This interesting idea is similar to one of Vidal's proposals. Two problems make it difficult to work out a logic for such conditionals: the simultaneous retraction of two sentences (the antecedent and the consequent), and the iteration of belief revision operators (a hypothetical addition after the retraction). Both problems have been dealt with in the belief revision literature ('multiple contraction', 'iterated revision'), but there are no generally agreed solutions to either of them. Andreas and Günther identify the simultaneous retraction of two sentences with a retraction of their disjunction[16] and so get around the first problem. The problem of iterated belief change operations is not pressing from them, since the antecedent of the conditional can be added in the second step without any inconsistencies. Like Rott and Vidal, Andreas and Günther don't investigate systematically the logical principles validated by their conditionals.

1.3 Quantitative-probabilistic accounts

1.3.1 Concepts of validity in quantitative-probabilistic accounts

Values and unvalues, support values and unsupport values. The quantitative accounts we will consider are not interested in the truth or falsity of conditionals, and they don't treat them as (un-)acceptable or (un-)assertable *simpliciter*. Sentences are assigned *degrees* of acceptability or assertability. For this, we employ *valuation functions* V over the set of sentences of the language. For a factual sentence, such an acceptability or assertability value is simply its probability (the idea of support isn't applicable here).[17] For a suppositional conditional, it has traditionally been identified with the conditional probability of the consequent given the antecedent, and we'll stick to the tradition. For a support conditional, we will use measures of evidential support as the values of its acceptability or assertability.

We adopt the convention that conditionals don't get negative support values.[18] When the antecedent is negatively relevant to the consequent and a measure is used that then takes a negative value, we set the support value $V(A \rightarrow C)$ of the conditional to 0. Support values will always be between or equal to 0 and 1. We assume with the literature (Adams, Berto and Özgün, Crupi and Iacona) that the *unvalue* (a cover term for the *un*certainty value, the *un*acceptability value, the *un*assertability value, ...) of a conditional is equal to 1 minus its support value: $U(A \rightarrow C) = 1 - V(A \rightarrow C)$.

The support value of a negated conditional $A \nrightarrow C$ is defined to be 1 minus the support value of the unnegated conditional $A \rightarrow C$, that is, $V(A \nrightarrow C) = U(A \rightarrow C)$.

We will look at four particular suggestions for measures of evidential support. But before we do that, we need to be clear how we can possibly make use of values representing degrees of evidential support in the definition of the validity of inferences. Once we have the numbers, what are we going to do with them?

Validity. The key idea is simple. Validity in deductive logic is preservation of truth. As we won't commit us the the view that conditionals have truth values, we will take preservation of acceptability or assertability for the definition of validity.

[16] In general, this does not seem to give the same results. For example, if one is convinced that a crime was committed by a single perpetrator, suspending judgment on the innocence of *each* of two suspects individually does not entail suspending judgment on the claim that *at least one* of them is innocent.

[17] This is also what Crupi and Iacona (2022c, p. 49) do.

[18] This choice is made mainly for reasons of simplicity.

We discuss two basic implementations of this idea. Both implementations make use of *thresholds*. In the first one, the threshold for acceptability is the same for the premises and the conclusion, the second one allows varying thresholds (typically higher thresholds for the premises than for the conclusion).[19]

Validity 1. An inference is valid_1 iff the value of the conclusion is not lower than the value of every single premise (the unvalue of the conclusion is not higher than the unvalue of every single premise). Equivalently, iff the following condition holds: the value of the conclusion is at least the minimal value of the premises (the unvalue of the conclusion is at most the maximal unvalue of the premises). And again equivalently, iff the following condition holds: for every threshold value t, whenever the premises are all acceptable in the sense that their values are above t, then the conclusion is acceptable in the same sense, too (whenever the unvalues of all premises are below t, then the unvalue of the conclusion is below t, too). So this concept of validity is clearly an *acceptance-preservation concept*.

Validity 2. An inference is valid_2 iff the the following holds: for every target threshold $t = 1 - \varepsilon$ for the conclusion there is threshold $t' = 1 - \delta$ for the premises such that whenever all premises have a value of at least t', the conclusion has a value of at least t (or equivalently: for every target threshold $t = \varepsilon$ for the conclusion there is threshold $t' = \delta$ for the premises such that whenever all premises have an unvalue of at most t', the conclusion has an unvalue of at most t). This notion of validity may also be regarded as a kind of acceptance-preservation concept, one which resorts to acceptance thresholds t' for the premises that are in general higher than the acceptance threshold t for the conclusion. The idea is due to Ernest Adams (1965; 1966). For him, the values of factual sentences A and conditionals $A > C$ are plain and conditional probabilities ($\Pr(A)$ and $\Pr(C \mid A)$), and their unvalues are "uncertainties" (i.e., improbabilities $1 - \Pr(A)$ and $1 - \Pr(C \mid A)$, respectively). He takes the probability of a conditional to be a measure of its acceptability[20] and the 'whenever' in the definition above is actually a quantification over all probability functions. He calls the resulting logic a *logic of high probability*: "as the premisses *approach* certainty the conclusion's probability must approach certainty too" (Adams

[19] Threshold ideas transferring ideas of Section 1.2 directly to the probabilistic setting might look like this. Given a contextually fixed threshold $t \in (0, 1)$, a conditional $A \rightarrow C$ is acceptable (w.r.t. t) iff

(i) $\Pr(C \mid A) \geq t$, but $\Pr(C) < t$.
(ii) $\Pr(C \mid A) \geq t$, but $\Pr(C \mid \neg A) < t$.
(iii) $\Pr(C \mid A) \geq t$ and $\Pr(\neg C \mid \neg A) \geq t$.
(iv) $\Pr(C \mid A) \geq t$ and $\Pr(\neg A \mid \neg C) \geq t$.

The ideas of (ii)–(iv) are similar to difference-making, dependence and evidential conditionals, respectively. As far as I know, none of these rules was suggested in the literature, so I will not further discuss them in the following.

[20] In empirical tests, Skovgaard-Olsen, Singmann and Klauer (2016) found that depending on whether the antecedent is positively relevant, negatively relevant or irrelevant to the consequent, the probability of a conditional may or may not be considered equal the conditional probability by the subjects. In other words, relevance considerations interact with assignments of probabilities to conditionals. Their main finding was that Adams' Thesis appears to hold good only when there is a relation of positive relevance.

1996, p. 9). Adams had an excellent justification for this definition. For purely factual, i.e., non-conditional sentences, this second concept of validity coincides with the usual concept of logical entailment in classical logic.

Adams also made a surprising discovery. Given his interpretations of values and unvalues, validity in this second sense can equivalently be defined as follows: An inference is valid$_2$ iff the unvalue of the conclusion is not higher than the sum of the unvalues of the premises.[21] Given the above relation between values and unvalues, this is equivalent to saying: the value of the conclusion is not lower than the sum of the values of the premises minus $k-1$, where k is the number of premises. Adams used the ϵ-δ-definition in his early work and later switched to the uncertainty-sum condition as his official definition of validity.

Let us thus call validity in the second sense the *unvalue-sum concept* or simply the *sum concept of validity* and contrast it with validity in the first sense by calling the latter the *unvalue-max concept* or simply the *max concept of validity*. These names refer to unvalues rather than values. Adams's project, which results in system **P**, is a classic by now. Hawthorne (1996) and Hawthorne and Makinson (2007) have have pursued a similar project using the max concept rather than the sum concept and studied the weaker system **O**. Being more permissive than the max concept, the sum concept in general generates stronger and more 'well-behaved' logics than the former.

Validity for Horn schemes. We consider only inference schemes with one or two premises (see Section 1.2.1). For one-premise Horn schemes the two validity concepts coincide.[22] For two-premise Horn schemes of the form

(H) If $A_1 \to C_1$ and $A_2 \to C_2$, then $A_3 \to C_3$,

however, they come apart.

Validity 1: $U(A_3 \to C_3) \leq \max\{U(A_1 \to C_1),\ U(A_2 \to C_2)\}$.
$V(A_3 \to C_3) \geq \min\{V(A_1 \to C_1),\ V(A_2 \to C_2)\}$.

Validity 2: $U(A_3 \to C_3) \leq U(A_1 \to C_1) + U(A_2 \to C_2)$.
$V(A_3 \to C_3) \geq V(A_1 \to C_1) + V(A_2 \to C_2) - 1$.

Validity for non-Horn schemes. Non-Horn inference schemes can be presented in different ways.[23] The versions of the simple case that is relevant to us are

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[21] I have been unable to find a crisp and direct proof of this equivalence in the work of Adams. Adams's proof takes a detour via so-called p-orderings which he invented in 1966. In Adams (1996, pp. 14–15), he refers back to Adams (1986, pp. 273–275), which in turn refers to Adams (1981, pp. 171–175), which in turn appeals to a non-trivial result of Adams (1966, probably Theorem 7). In his textbook, Adams (1998, pp. 137, 152) just states the equivalence and gives no proof. In his earlier book, Adams (1975, p. 57) had said that the fact that the ε-δ rule implies the uncertainty-sum criterion is an "easy consequence of basic definitions plus previous theorems". In retrospect, this appears to be overly optimistic.—For the rich variety of equivalents to Adams's rule, see Leitgeb (2016).

[22] Horn schemes are closure conditions. See Hawthorne and Makinson (2007, p. 250).

[23] The non-Horn schemes we have listed in Section 1.2.1 are (NRat), (DRat), (RMon), (ROr), (ConjRat), and the right-to-left directions of (AOT) and (AOT′).

(NH1) If $A_1 \rightarrow C_1$ and $A_2 \nrightarrow C_2$, then $A_3 \rightarrow C_3$.

on the one hand and

(NH2) If $A_1 \rightarrow C_1$, then $A_2 \rightarrow C_2$ or $A_3 \rightarrow C_3$.

on the other. These forms are usually considered to be equivalent. However, it turns out the definition of validity of a non-Horn scheme is sensitive to the form of presentation. Let us try out both of the validity concepts above.

Validity 1 for (NH1): $U(A_3 \rightarrow C_3) \leq \max\{U(A_1 \rightarrow C_1),\ 1 - U(A_2 \rightarrow C_2)\}$.
$V(A_3 \rightarrow C_3) \geq \min\{V(A_1 \rightarrow C_1),\ 1 - V(A_2 \rightarrow C_2)\}$.

Validity 2 for (NH1): $U(A_3 \rightarrow C_3) \leq U(A_1 \rightarrow C_1) + 1 - U(A_2 \rightarrow C_2)$.
$V(A_1 \rightarrow C_1) \leq V(A_2 \rightarrow C_2) + V(A_3 \rightarrow C_3)$.

Validity 1 for (NH2): $U(A_1 \rightarrow C_1) \geq \min\{U(A_2 \rightarrow C_2), U(A_3 \rightarrow C_3)\}$.
$V(A_1 \rightarrow C_1) \leq \max\{V(A_2 \rightarrow C_2),\ V(A_3 \rightarrow C_3)\}$.

Validity 2 for (NH2): Not applicable.

Validity 1 for (NH1) is not symmetric in $A_2 \rightarrow C_2$ and $A_3 \rightarrow C_3$, and it does not seem to make much sense. But validity 1 for (NH2) is kind of dual to validity 1 for Horn schemes and gives a neat and reasonable condition. Validity 2 for (NH1) is symmetric in $A_2 \rightarrow C_2$ and $A_3 \rightarrow C_3$ and is in a way dual to validity 2 for Horn schemes. But validity 2 is not even applicable to (NH2), since there no relevant sum that could be taken. So we conclude that the choice of the validity concept and the choice of the way of presenting non-Horn inference schemes are dependent on each other. It is recommended to take the formulations using V rather than U and apply Validity 1 to the form (NH2) and Validity 2 to the form (NH1).[24] In later sections, we will opt for the sum concept of validity and thus apply it to scheme (NH1).

Adams as well as Hawthorne and Makinson had suppositional conditionals in mind. In the remainder of this paper, we want to have a look at similar ideas for support conditionals. I do not know of any attempt to work out the max concept of validity for support conditionals and set that concept aside for later research. However, the unvalue-sum concept of validity has been employed recently, namely in Crupi and Iacona's (2022c) probabilistic investigation into evidential conditionals. They applied it to a measure that is different from the conditional probability measure. Although Adams's justification is not available for their measure,[25] we will follow Crupi and Iacona and see what results the unvalue-sum rule yields when applied to other measures of evidential support.

There is yet another interesting probabilistic property of inference schemes, viz., *probabilistic informativeness* (Pfeifer and Kleiter 2006; 2009). For Pfeifer and Kleiter, the probability of a conditional $A \rightarrow C$ is the conditional probability $\Pr(C \mid A)$,

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[24] Notice that for the validities of Non-Horn rules determined in this way, the sum concept makes more inferences valid than the max concept. This is just as it is for Horn rules.

[25] And their name 'uncertainty' for their unvalue measure $U = 1 - V$ may be a misnomer. Cf. Section 1.3.5.3 below.

and an inference scheme is probabilistically *informative* if and only if definite probability values of the premises constrain the coherent assignment of a probability value to the conclusion (the conclusion cannot take *any* probability value). Pfeifer and Kleiter compare probabilistic informativeness to classical deductive validity as well as to Adams's P-validity. All principles of system **P** are informative, while the paradigmatic non-validities of suppositional conditionals (Strengthening the Antecedent, Transitivity and Contraposition) are not informative. On top of this, the classically invalid schemes of Denying the Antecedent and Affirming the Consequent (and Aristotle's first thesis as well as Boethius's thesis) are all informative.[26] This seems to place the idea of Pfeifer and Kleiter into the vicinity of the conditionals surveyed in this paper. However, since the relationship between evidential support and probabilistic informativeness is unclear and since the latter has not been claimed to be a notion of validity,[27] we will not further discuss the conditional logic of this kind of informativeness.

1.3.2 Suppositional conditionals

Adams (1965; 1966; 1975; 1981; 1986; 1996; 1998) produced a large and trailblazing oeuvre spanning over more than three decades. He pioneered the study of suppositional (indicative) conditionals in a probabilistic framework, but at the same time he showed us—as early as 1966!—how to cross the quantitative-qualitative watershed. He considered conditional probabilities as degrees of acceptability or assertability (and refrained from specifying rules for plain acceptance), and he employed the sum concept of validity (see Section 1.3.1). The logical system that he obtained is now known as 'system **P**'. It consists of Horn rules only. Horn rules are closure conditions saying which conditionals can and should be accepted if other conditionals are already accepted.

Hawthorne (1996) and Hawthorne and Makinson (2007) continued the tradition initiated by Adams, but with important changes. They adopt a simple acceptance condition for a conditional $A \to C$, given a probability function Pr and a threshold value $t \in (0, 1]$:[28] either $\Pr(A) = 0$ or the conditional probability $\Pr(C \mid A)$ must be equal or greater than t. Acceptance preservation then in effect means that they employ the max concept of validity (calling it 'probabilistic soundness'). Now the question is whether the set of accepted conditionals given Pr and t satisfies certain closure properties most of which are among the inference principles we have listed.

Hawthorne and Makinson find that (And), (CMon), (Cut) and (Or) are all violated, but that the logic they obtain includes (Ref), (LLE), (RW), as well as weakened

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[26] See Pfeifer and Sanfilippo (2021).

[27] Informativeness is not preservation of truth or acceptability. It rather preserves information about the probability of propositions in the sense that if there is perfect such information about the premises, then there is at least *some* such information about the conclusion.

[28] Here I gloss over the fact that they actually talk about nonmonotonic consequence relations rather than about conditionals.

versions of (And), (OR) and (CMon),[29] as well as (NRat). They study the two sound, but incomplete systems called **O** and **Q**, with **Q** being **O** extended by (NRat).[30] The most striking feature of the systems is perhaps that they do *not* include (And). But this is not surprising, once one realises that the (conditional) probabilities of both B and C may well lie above a threshold t that the (conditional) probability of $B \wedge C$ falls short of. Like Adams's account, Hawthorne and Makinson's is not intended to encode the idea of support or relevance in conditionals.

1.3.3 Topical conditionals

Berto and Özgün (2021) require topical or thematic relevance of the antecedent for the consequent. Their valuations of conditionals are supposed to represent degrees of acceptability and unacceptability (Berto and Özgün 2021, p. 3711):

$$V(A \rightarrow C) = \begin{cases} \Pr(C \mid A) & \text{if } \Pr(A) > 0 \text{ and the topic of } C \text{ is included in the topic} \\ & \text{determined by } A, \\ 0 & \text{otherwise.} \end{cases}$$

$$U(A \rightarrow C) = 1 - V(A \rightarrow C).$$

Then Berto and Özgün use the sum-concept of validity (p. 3712), which is here identical with Adams's uncertainty-sum rule: an inference from $A_1, \ldots, A_k$ to A_{k+1} is valid iff $U(A_{k+1}) \leq U(A_1) + \ldots + U(A_k)$. They show that their account makes Strengthening the Antecedent, Transitivity and Contraposition are all invalid.

Theorem 8 (Berto and Özgün)

This account validates (Ref), (RW-And),[31] (And), (Or), (Cut) and (CMon). But its topic-relativity makes it hyperintensional, and so it does not satisfy (LLE) or (RLE), and thus not (RW).

Berto and Özgün do not require positive evidential relevance (evidential support) of the antecedent for the consequent. Their idea may be taken as a generalisation of the variable-sharing idea of relevance logic. Here the connection is provided by the semantic contents of A and C themselves.[32] In contrast, the connection required by the approaches we focus on in this paper is assumed to be provided by the (metaphysical or epistemic) context—the state that gets revised by the supposition that A and also other suppositions. We will not further elaborate on topical conditionals here.

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[29] The weakened versions are (WAnd) If $A \rightarrow C$ and $(A \wedge \neg B) \rightarrow B$, then $A \rightarrow (B \wedge C)$; (XOr) If $A \rightarrow C$ and $B \rightarrow C$ and $\vdash \neg(A \wedge B)$, then $(A \vee B) \rightarrow C$; (VCM) If $A \rightarrow (B \wedge C)$, then $(A \wedge B) \rightarrow C$.

[30] Paris and Simmonds (2009) showed that **O** is incomplete and presented a (very complicated) complete extension. Hawthorne's (2014) strengthened system $\mathbf{O}^{+}$ is more elegant, but it is still unknown whether it is enough.

[31] (RW-And) is a particular form of (RW): If $A \rightarrow (B \wedge C)$, then $A \rightarrow B$ and $A \rightarrow C$.

[32] This is simplifying matters. In addition to their central topic function, Berto and Özgün make use of a function that takes into account the context as well and determines, together with the antecedent, a set of background assumptions.

1.3.4 Evidential support conditionals based on high probability combined with positive relevance

As far as I know, the first author to work out a logic of support conditionals was Igor Douven (2008, 2016). His *Evidential Support Thesis* is a quantitative combination of the two notions of confirmation of Carnap (1962, p. xvi), firmness and increase in firmness. This is how it reads:

> A conditional $A \to C$ is acceptable iff $\Pr(C\,|\,A) > t$ for some threshold t and $\Pr(C\,|\,A) > \Pr(C)$. (Douven 2008, p. 28; 2016, p. 108)

This combines two criteria, high conditional probability and positive relevance.[33] Douven's main argument in support of the positive relevance condition is that high probability is not enough for assertion. He gives the following example: "If Chelsea wins the Champion's league, then there is at least 1 head in the first 100,000 tosses." Though the probability of the consequent given the antecedent is high, the antecedent does not increase the probability of the consequent. It is utterly irrelevant, and the conditional may be called a "non-interference conditional". Notice, by the way, that a difference-making conditionals in the sense of Section 1.2.2 encodes the same twofold idea as Douven's conditional in a qualitative setting: C is true$^{\mathrm{A}}$ after a revision by A, and A is indeed positively relevant to the truth$^{\mathrm{A}}$ status of C.

Douven defines validity in terms of acceptability-preservation according to his epistemic support criterion for all thresholds $t \in [0.5, 1)$. He establishes the validity and invalidity of many principles (see Douven 2016, Theorem 5.2.1). We focus on the main principles of our lists above.[34] The resulting logic is rather weak.

Theorem 9 (Douven)

Paired with validity as acceptability preservation, this account validates (Ref), (LRLE), (Ant) and a restricted form of (Ablrd). It invalidates (RW), (And), (Or), (Cut), (CMon) and (Cntrpos).

The most surprising fact is perhaps the violation of (And) which is essentially due to having a high-threshold criterion in the valuation of conditionals and not employing the uncertainty-sum rule. Finding a sound and complete axiomatisation for Douven's logic is still an open problem.

[33] In the terminology introduced in the next section, the second part of this definition is expressible by the difference measure: $d(A, C) > 0$. But the requirement of positive relevance could equivalently be expressed using the Delta measure: $\Delta(A, C) > 0$. Douven's earlier evidential support theory of conditionals is actually more complicated (Douven 2008, p. 30), but he appears to withdraw this complication later (Douven 2016, pp. 120–122).

[34] But we should add that Douven's account validates the principles (WAnd) and (XOr) mentioned in footnote 29. (VCM) is invalid.

1.3.5 Measures of evidential support as evaluations of conditionals

We will follow a generic approach in the remainder of this paper. First, we take a probability measure Pr as given and define a measure of evidential support *ES* based on Pr. Intuitively, $ES(A, C)$ is the degree to which A supports C. The support value $ES(A, C)$ will be less or equal to 1, and it will be positive if and only if A is positively relevant to C. Here A is called *positively relevant* (or *negatively relevant* or *irrelevant*) to C if and only if $\Pr(C \mid A) > \Pr(C)$ (or $\Pr(C \mid A) < \Pr(C)$ or $\Pr(C \mid A) = \Pr(C)$, respectively). We will look at four different, simple and intuitive measures of evidential support that have been proposed in the literature. My presentation, however, will be biased. I will put a special focus on at the second of these measures, because it allows an instructive comparison with the difference-making and dependence conditionals of the qualitative setting.

In the second step, we use the measure of epistemic support to define a valuation of conditionals that lies between 0 and 1.[35]

$$V(A \rightarrow C) = \begin{cases} ES(A, C) & \text{if } A \text{ is Pr-contingent and not negatively relevant to } C, \\ 0 & \text{otherwise.} \end{cases}$$

Here a sentence A is called *contingent* with respect to a probability function Pr, or more briefly Pr-*contingent*, if $0 < \Pr(A) < 1$. The lower line is in part motivated by the strong intuition that non-contingent sentences cannot offer support to anything. If an agent is certain that A is true or false, then the supposition that A cannot raise or lower the credence of C.[36]

1.3.5.1 The difference measure

For every sentence A with $0 < \Pr(A)$, the difference measure is defined by

$$d(A, C) = \Pr(C \mid A) - \Pr(C).$$

MacColl (1906, p. 129, variables exchanged) seems to be the first author advocating $d(A, C)$. He called it a measure of 'the dependence of the statement C upon the statement A'. Other proponents include Carnap (1962), Eells (1982), Jeffrey (1992) and Earman (1992). Fig. 1.10 gives a graphical indication of what $d(A, C)$ means. The difference measure has been discussed many times, and we need not say more about it here.

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[35] One might also consider the use of negative valuations of conditionals, but I will not follow such a path here.

[36] Crupi and Iacona (2022c, p. 50) have another rule. For them an A with zero probability is maximally supportive to everything, and a C with unit probability receives maximal support by everything.

The valuation of conditionals should conform to the scheme above: $V(A \to C) = d(A, C)$ if A is Pr-contingent and positively relevant to C, and $V(A \to C) = 0$ otherwise.

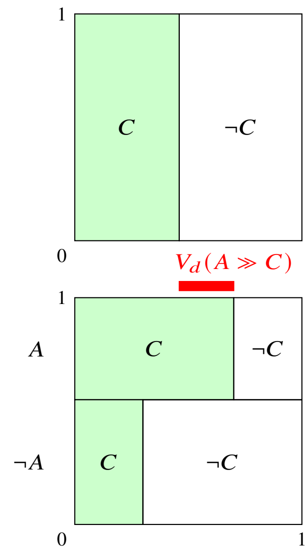

Fig. 1.1: $V_d(A \gg C) = d(A, C)$.

1.3.5.2 The ΔP measure

The ΔP measure can be defined, for every Pr-contingent sentence A and every sentence C, as follows:

$$\Delta(A, C) = \Pr(C \mid A) - \Pr(C \mid \neg A).$$

The notation used here drops the letter 'P'. Notice that this measure is 'purely relational' in that it is completely independent of the probability of A (except that A has to be contingent). This contrasting of the suppositions of A and $\neg A$ is close to the contrasting of the same suppositions in—qualitative—difference-making and dependence conditionals.

The ΔP measure has been popular in the psychology of causality or 'contingency' (Jenkins and Ward 1965, p. 2; Rescorla 1968, p. 1). It was called the *measure for a one-way dependency* (Shanks 1995b, p. 22), the *cue-outcome contingency* (Shanks 1995a, p. 259) or the *contingency* (or *contrast*) between candidate cause A and effect C (Cheng 1997, p. 367). Within philosophy of science, the same measure

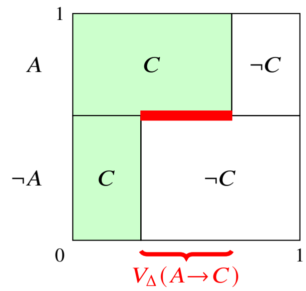

Fig. 1.2: $V_\Delta(A \to C) = \Delta(A, C)$.

was suggested by Christensen (1999) and Joyce (1999, pp. 203–213; 2003) under the names of *potential further support* and *effective (increment of) evidence*, respectively. Skovgaard-Olsen, Singmann and Klauer (2017, p. 1211) found empirical "evidence for high agreement between ΔP and ratings of perceived relevance and reason relations and suggest [. . .] that ΔP is a better predictor than the difference measure."[37]

Application as a valuation for the acceptability or assertability of conditionals:

$$V_\Delta(A \to C) = \begin{cases} \Delta(A, C) & \text{if } \Delta(A, C) \geq 0 \text{ and } A \text{ is Pr-contingent,} \\ 0 & \text{otherwise.} \end{cases}$$

This will be my favoured measure for the acceptability or assertability of support conditionals because it embodies the same contrast idea that we found, among the qualitative approaches, in difference-making conditionals and dependence conditionals.

Here is a list of useful facts about the V_Δ values of some special conditionals.

Lemma 1 *(i)* $B \vdash C$ *does not imply* $V_\Delta(A \to B) \leq V_\Delta(A \to C)$.

(ii) $V_\Delta(A \to A) = 1$ *if* A *is* Pr-*contingent, and* $V_\Delta(A \to A) = 0$ *otherwise.*

(iii) $V_\Delta(A \to C) = 1$ *iff* ($\mathrm{Pr}(C \mid A) = 1$ *and* $\mathrm{Pr}(C \mid \neg A) = 0$).

(iv) $V_\Delta(A \to \top) = 0$.

(v) $V_\Delta(A \to \bot) = 0$.

(vi) $V_\Delta(\top \to C) = 0$.

[37] Notice that $\Delta(A, C)$ is positive (negative, equal to zero) iff $d(A, C)$ is.—Skovgaard-Olsen et al. (2017, p. 1203) maintain that Spohn (2012) predicted 'a high correlation between ΔP [. . .] and perceived relevance and between ΔP and ratings of reason relations'. Van Rooij and Schulz (2019, p. 58) claim that Spohn (2013) proposed that 'the acceptability of the conditional correlates with ΔP' (notation adapted). I could not verify either of these claims. But cf. the discussion in Spohn (2012, pp. 106–107).

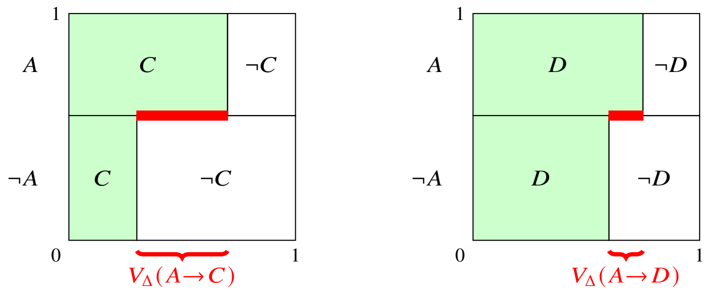

Fig. 1.3: Example of how Right Weakening may get violated: Supposing $C \vdash D$, it is still easily possible that $V_\Delta(A \to C) > V_\Delta(A \to D)$ and hence $U_\Delta(A \to C) < U_\Delta(A \to D)$. The figures indicate that the proposition $[C]$ is contained in the proposition $[D]$; the sizes of the areas correspond to the respective probabilities.

(vii) $V_\Delta(\bot \to C) = 0$.

(viii) $V_\Delta(A \to A \wedge C) = \mathrm{Pr}(C\,|\,A)$ *if* A *is* Pr-*contingent, and* $V_\Delta(A \to A \wedge C) = 0$ *otherwise.*

(ix) $V_\Delta(A \wedge C \to C) = \mathrm{Pr}(\neg C\,|\,\neg A \vee \neg C)$ *if* $A \wedge C$ *is* Pr-*contingent, and* $V_\Delta(A \wedge C \to C) = 0$ *otherwise.*

(x) $V_\Delta(A \to A \vee C) = \mathrm{Pr}(\neg C\,|\,\neg A)$ *if* A *is* Pr-*contingent, and* $V_\Delta(A \to A \vee C) = 0$ *otherwise.*

(xi) $V_\Delta(A \vee C \to C) = \mathrm{Pr}(C\,|\,A \vee C)$ *if* $A \vee C$ *is* Pr-*contingent, and* $V_\Delta(A \vee C \to C) = 0$ *otherwise.*

(xii) $V_\Delta(\neg A \to \neg C) = V_\Delta(A \to C)$.

(xiii) $V_\Delta(A \to A \wedge C) \geq V_\Delta(A \to A \wedge B \wedge C)$.

(xiv) $V_\Delta(A \to A \wedge C) + V_\Delta(A \to A \vee C) = V_\Delta(A \to C) + 1$ *if* A *is* Pr-*contingent, and* $V_\Delta(A \to A \wedge C) + V_\Delta(A \to A \vee C) = V_\Delta(A \to C)$ *otherwise.*

(xv) $V_\Delta(A \to B) + V_\Delta(A \to C) \leq V_\Delta(A \to B \wedge C) + 1$.

(xvi) *Not always* $V_\Delta(A \to B \wedge C) \leq V_\Delta(A \to B) + V_\Delta(A \to C)$.

(xvii) $V_\Delta(\neg A \twoheadrightarrow \neg A \vee C) = V_\Delta(A \to A \wedge C)$ *if* A *is* Pr-*contingent.*

(xviii) $V_\Delta(A \to \neg C) \leq V_\Delta(A \twoheadrightarrow C)$. *And* $V_\Delta(A \to \neg C) \geq V_\Delta(A \twoheadrightarrow C)$ *if and only if* A *is* Pr-*contingent and* $|\mathrm{Pr}(C\,|\,A) - \mathrm{Pr}(C\,|\,\neg A)| = 1$.

Proof. (i) For a counterexample, see Fig. 1.3.

(ii) If $0 < \mathrm{Pr}(A) < 1$, we have $\mathrm{Pr}(A\,|\,A) = 1 \geq 0 = \mathrm{Pr}(A\,|\,\neg A)$. So $V_\Delta(A \to A) = \mathrm{Pr}(A\,|\,A) - \mathrm{Pr}(A\,|\,\neg A) = 1 - 0 = 1$. However, if $\mathrm{Pr}(A) = 0$ or $\mathrm{Pr}(A) = 1$, then $V_\Delta(A \to A) = 0$ by definition.

(iii) From left to right: Let $V_\Delta(A \to C) = 1$. Then $\Pr(C \mid A) - \Pr(C \mid \neg A) \geq 0$ and indeed $\Pr(C \mid A) - \Pr(C \mid \neg A) = 1$. The last equation gives us $\Pr(C \mid A) = 1$ and $\Pr(C \mid \neg A) = 0$.

From right to left: Let $\Pr(C \mid A) = 1$ and $\Pr(C \mid \neg A) = 0$. So $\Pr(C \mid A) - \Pr(C \mid \neg A) \geq 0$. Thus $V_\Delta(A \to C) = \Pr(C \mid A) - \Pr(C \mid \neg A) = 1 - 0 = 1$.

(iv) First suppose that $0 < \Pr(A) < 1$. Since $\Pr(\top \mid A) = 1 \geq \Pr(\top \mid \neg A)$, we have $V_\Delta(A \to \top) = \Pr(\top \mid A) - \Pr(\top \mid \neg A) = 1 - 1 = 0$. We get the same result for $\Pr(A) = 0$ or $\Pr(A) = 1$.

(v) First suppose that $0 < \Pr(A) < 1$. Since $\Pr(\bot \mid A) \geq 0 = \Pr(\bot \mid \neg A)$, we have $V_\Delta(A \to \bot) = \Pr(\bot \mid A) - \Pr(\bot \mid \neg A) = 0 - 0 = 0$. We get the same result for $\Pr(A) = 0$ or $\Pr(A) = 1$.

(vi) Since $\Pr(\top) = 1$, we have $V_\Delta(\top \to C) = 0$ by definition.

(vii) Since $\Pr(\bot) = 0$, we have $V_\Delta(\bot \to C) = 0$ by definition.

(viii) First suppose that $0 < \Pr(A) < 1$. Since $\Pr(A \wedge C \mid A) \geq 0 = \Pr(A \wedge C \mid \neg A)$, we have $V_\Delta(A \to A \wedge C) = \Pr(A \wedge C \mid A) - \Pr(A \wedge C \mid \neg A) = \Pr(C \mid A)$. Second, suppose that $\Pr(A) = 0$ or $\Pr(A) = 1$. Then $V_\Delta(A \to A \wedge C) = 0$.

(ix) First suppose that $0 < \Pr(A \wedge C) < 1$. Since $\Pr(C \mid A \wedge C) = 1 \geq \Pr(C \mid \neg(A \wedge C))$, we have $V_\Delta(A \wedge C \to C) = \Pr(C \mid A \wedge C) - \Pr(C \mid \neg(A \wedge C)) = \Pr(\neg C \mid \neg A \vee \neg C)$. Second, suppose that $\Pr(A \wedge C) = 0$ or $\Pr(A \wedge C) = 1$. Then $V_\Delta(A \wedge C \to C) = 0$.

(x) First suppose that $0 < \Pr(A) < 1$. Since $\Pr(A \vee C \mid A) = 1 \geq \Pr(A \vee C \mid \neg A)$, we have $V_\Delta(A \to A \vee C) = \Pr(A \vee C \mid A) - \Pr(A \vee C \mid \neg A) = \Pr(\neg C \mid \neg A)$. Second, suppose that $\Pr(A) = 0$ or $\Pr(A) = 1$. Then $V_\Delta(A \to A \vee C) = 0$.

(xi) First suppose that $0 < \Pr(A \vee C) < 1$. Since $\Pr(C \mid A \vee C) \geq 0 = \Pr(C \mid \neg(A \vee C))$, we have $V_\Delta(A \vee C \to C) = \Pr(C \mid A \vee C) - \Pr(C \mid \neg(A \vee C)) = \Pr(C \mid A \vee C)$. Second, suppose that $\Pr(A \vee C) = 0$ or $\Pr(A \vee C) = 1$. Then $V_\Delta(A \vee C \to C) = 0$.

(xii) First, suppose that $0 < \Pr(A) < 1$ and $\Pr(C \mid A) \geq \Pr(C \mid \neg A)$. Notice that the latter is equivalent to $\Pr(\neg C \mid \neg A) \geq \Pr(\neg C \mid A)$. Thus $V_\Delta(\neg A \to \neg C) = \Pr(\neg C \mid \neg A) - \Pr(\neg C \mid A) = 1 - \Pr(C \mid \neg A) - (1 - \Pr(C \mid A)) = V_\Delta(A \to C)$.

Second, suppose that $\Pr(A) = 0$ or $\Pr(A) = 1$ or $\Pr(C \mid A) < \Pr(C \mid \neg A)$. Notice that the latter is equivalent to $\Pr(\neg C \mid \neg A) < \Pr(\neg C \mid A)$. Thus $V_\Delta(\neg A \to \neg C) = 0 = V_\Delta(A \to C)$.

(xiii) If $0 < \Pr(A) < 1$, we get, using (viii), that $V_\Delta(A \to A \wedge C) = \Pr(C \mid A) \geq \Pr(B \wedge C \mid A) = V_\Delta(A \to A \wedge B \wedge C)$. Otherwise all probabilities are 0.

(xiv) First suppose that $0 < \Pr(A) < 1$. Then we get, using (viii) and (x), that $V_\Delta(A \to A \wedge C) + V_\Delta(A \to A \vee C) = \Pr(C \mid A) + \Pr(\neg C \mid \neg A) = \Pr(C \mid A) + (1 - \Pr(C \mid \neg A)) = V_\Delta(A \to C) + 1$. Second, suppose that $\Pr(A) = 0$ or $\Pr(A) = 1$. Then all probabilities are 0.

(xv). Suppose that $0 < \Pr(A) < 1$. First let $\Pr(B \mid A) \geq \Pr(B \mid \neg A)$, $\Pr(C \mid A) \geq \Pr(C \mid \neg A)$ and $\Pr(B \wedge C \mid A) \geq \Pr(B \wedge C \mid \neg A)$. Then $V_\Delta(A \to B) + V_\Delta(A \to C) = (\Pr(B \mid A) - \Pr(B \mid \neg A)) + (\Pr(C \mid A) - \Pr(C \mid \neg A)) = (\Pr(B \mid A) + \Pr(C \mid A)) - (\Pr(B \mid \neg A) + \Pr(C \mid \neg A)) \leq (\Pr(B \wedge C \mid A) + 1) - \Pr(B \wedge C \mid \neg A) = V_\Delta(A \to B \wedge C) + 1$.

Second, let $\Pr(B \mid A) \geq \Pr(B \mid \neg A)$, $\Pr(C \mid A) \geq \Pr(C \mid \neg A)$ and $\Pr(B \wedge C \mid A) < \Pr(B \wedge C \mid \neg A)$. Then $V_\Delta(A \to B \wedge C) = 0$ and $V_\Delta(A \to B) + V_\Delta(A \to C) \leq$ *(as before)* $(\Pr(B \wedge C \mid A) + 1) - \Pr(B \wedge C \mid \neg A) < 1 = V_\Delta(A \to B \wedge C) + 1$.

Third, let $\Pr(B\,|\,A) < \Pr(B\,|\,\neg A)$ or $\Pr(C\,|\,A) < \Pr(C\,|\,\neg A)$. Then $V_\Delta(A \rightarrow B) = 0$ or $V_\Delta(A \rightarrow C) = 0$, and the claim is trivial.

Now suppose that $\Pr(A) = 0$ or $\Pr(A) = 1$. Then all probabilities are 0, and the claim is trivial.

(xvi) For a counterexample, see Fig. 1.8 below, where $V_\Delta(\neg A \rightarrow \neg B) = V_\Delta(\neg A \rightarrow \neg C) = 0.3$ and $V_\Delta(\neg A \rightarrow (\neg B \wedge \neg C)) = 0.7$. Note that for the probability function Pr specified, $\neg A$ is positively probabilistically relevant to $\neg B$, to $\neg C$ and to $\neg B \wedge \neg C$.

(xvii) First suppose that $0 < \Pr(A) < 1$. Using (viii) and (x), we get $V_\Delta(\neg A \nrightarrow \neg A \vee C) = 1 - V_\Delta(\neg A \rightarrow \neg A \vee C) = 1 - \Pr(\neg C\,|\,A) = \Pr(C\,|\,A) = V_\Delta(A \rightarrow A \wedge C)$.

(xviii) We first verify that $V_\Delta(A \rightarrow \neg C) \leq V_\Delta(A \nrightarrow C)$. Suppose that $0 < \Pr(A) < 1$. If $\Pr(\neg C\,|\,A) - \Pr(\neg C\,|\,\neg A) \geq 0$, then $V_\Delta(A \rightarrow \neg C) \geq 0$. But in this case $V_\Delta(A \nrightarrow C) = 1 - V_\Delta(A \rightarrow C) = 1 - 0 = 1$. If, on the other hand, $\Pr(\neg C\,|\,A) - \Pr(\neg C\,|\,\neg A) < 0$, then $V_\Delta(A \rightarrow \neg C) = 0$. Suppose, on the other hand, that $\Pr(A) = 0$ or $\Pr(A) = 1$. Then $V_\Delta(A \rightarrow \neg C) = 0 < 1 = V_\Delta(A \nrightarrow C)$.

Now we check the second claim. From left to right. Suppose that $V_\Delta(A \rightarrow \neg C) \geq V_\Delta(A \nrightarrow C)$. We have already seen in the first step that this implies that A is $\Pr(A)$-contingent. If, on the one hand, $\Pr(\neg C\,|\,A) - \Pr(\neg C\,|\,\neg A) \geq 0$, then $V_\Delta(A \nrightarrow C) = 1 - V_\Delta(A \rightarrow C) = 1$, so we must have $V_\Delta(A \rightarrow \neg C) = 1$, too. By (iii), this means that $\Pr(\neg C\,|\,A) = 1$ and $\Pr(\neg C\,|\,\neg A) = 0$, or equivalently $\Pr(C\,|\,A) = 0$ and $\Pr(C\,|\,\neg A) = 1$. If, on the other hand, $\Pr(\neg C\,|\,A) - \Pr(\neg C\,|\,\neg A) < 0$, then $V_\Delta(A \rightarrow \neg C) = 0$, so we must have $V_\Delta(A \nrightarrow C) = 0$, too. The latter is equivalent to $V_\Delta(A \rightarrow C) = 1$, and by (iii), this means that $\Pr(C\,|\,A) = 1$ and $\Pr(C\,|\,\neg A) = 0$. Either way, $|\Pr(C\,|\,A) - \Pr(C\,|\,\neg A)| = 1$.

From right to left. Suppose that $0 < \Pr(A) < 1$ and $|\Pr(C\,|\,A) - \Pr(C\,|\,\neg A)| = 1$. If, on the one hand, $\Pr(C\,|\,A) = 1$ and $\Pr(C\,|\,\neg A) = 0$, then $V(A \rightarrow C) = 1$, $V(A \nrightarrow C) = 0$ and $V(A \rightarrow \neg C) = 0$. If, on the other hand, $\Pr(C\,|\,A) = 0$ and $\Pr(C\,|\,\neg A) = 1$, then $V(A \rightarrow C) = 0$, $V(A \nrightarrow C) = 1$ and $V(A \rightarrow \neg C) = 1$. Either way, $V_\Delta(A \rightarrow \neg C) \geq V_\Delta(A \nrightarrow C)$. □

We again set the unvalue of a conditional to 1 minus its support value. This gives us:

$$U_\Delta(A \rightarrow C) = 1 - V_\Delta(A \rightarrow C) =$$

$$\begin{cases} \Pr(\neg C\,|\,A) + \Pr(C\,|\,\neg A) & \text{if } \Pr(\neg C\,|\,A) + \Pr(C\,|\,\neg A) \leq 1 \text{ and } A \text{ is Pr-contingent,} \\ 1 & \text{otherwise.} \end{cases}$$

Here is a corresponding list of useful facts about the U_Δ unvalues of some special conditionals.

Lemma 2 *(i) $B \vdash C$ does not imply $U_\Delta(A \rightarrow C) \leq U_\Delta(A \rightarrow B)$.*

(ii) $U_\Delta(A \rightarrow A) = 0$ if A is Pr*-contingent, and $U_\Delta(A \rightarrow A) = 1$ otherwise.*

(iii) $U_\Delta(A \rightarrow C) = 0$ iff ($\Pr(C\,|\,A) = 1$ and $\Pr(C\,|\,\neg A) = 0$).

(iv) $U_\Delta(A \rightarrow \top) = 1$.

(v) $U_\Delta(A \rightarrow \bot) = 1$.

(vi) $U_\Delta(\top \to C) = 1$.

(vii) $U_\Delta(\bot \to C) = 1$.

(viii) $U_\Delta(A \to A \wedge C) = \mathrm{Pr}(\neg C \mid A)$ *if* A *is* Pr*-contingent, and* $U_\Delta(A \to A \wedge C) = 1$ *otherwise.*

(ix) $U_\Delta(A \wedge C \to C) = \mathrm{Pr}(C \mid \neg A \vee \neg C)$ *if* $A \wedge C$ *is* Pr*-contingent, and* $U_\Delta(A \wedge C \to C) = 1$ *otherwise.*

(x) $U_\Delta(A \to A \vee C) = \mathrm{Pr}(C \mid \neg A)$ *if* A *is* Pr*-contingent, and* $U_\Delta(A \to A \vee C) = 1$ *otherwise.*

(xi) $U_\Delta(A \vee C \to C) = \mathrm{Pr}(\neg C \mid A \vee C)$ *if* $A \vee C$ *is* Pr*-contingent, and* $U_\Delta(A \vee C \to C) = 1$ *otherwise.*

(xii) $U_\Delta(\neg A \to \neg C) = U_\Delta(A \to C)$.

(xiii) $U_\Delta(A \to A \wedge B \wedge C) \geq U_\Delta(A \to A \wedge C)$.

(xiv) $U_\Delta(A \to A \wedge C) + U_\Delta(A \to A \vee C) = U_\Delta(A \to C)$ *if* A *is* Pr*-contingent, and* $U_\Delta(A \to A \wedge C) + U_\Delta(A \to A \vee C) = U_\Delta(A \to C) + 1$ *otherwise.*

(xv) $U_\Delta(A \to B) + U_\Delta(A \to C) \geq U_\Delta(A \to B \wedge C)$.

(xvi) *Not always* $U_\Delta(A \to B \wedge C) \leq U_\Delta(A \to B) + U_\Delta(A \to C) - 1$.

(xvii) $U_\Delta(\neg A \nrightarrow \neg A \vee C) = U_\Delta(A \to A \wedge C)$ *if* A *is* Pr*-contingent.*

(xviii) $U_\Delta(A \nrightarrow C) \leq U_\Delta(A \to \neg C)$. *And* $U_\Delta(A \nrightarrow C) \geq U_\Delta(A \to \neg C)$ *if and only if* A *is* Pr*-contingent and* $|\,\mathrm{Pr}(C \mid A) - \mathrm{Pr}(C \mid \neg A)| = 1$.

Proof. The claims follow immediately from the corresponding claims of Lemma 1 and the definition $U_\Delta(A \to C) = 1 - V_\Delta(A \to C)$. □

The most striking parts of Lemmas 1 and 2 are perhaps parts (xii), which show that $U_\Delta(A \to C) = U_\Delta(\neg A \to \neg C)$. So it is immediately clear that Conditional Perfection is unrestrictedly valid. This property makes the ΔP measure particularly interesting for a comparison with the qualitative dependency conditional which is notable for its satisfying (CondPerf) (see Section 1.2.3).

But we can say much more than that. Combined with the unvalue-sum concept of validity, the measure V_Δ validates the basic properties of dependence conditionals (except for the left-to-right direction of (IN′)) and some, but not all those of difference-making conditionals. Because we have no definition of an outer modality in the probabilistic context, we replace conditions of the form '$\Box A$' in (AOT), (AOT′) and (OND) by the maximally uncertain '$\neg A \to \bot$'. For the inner necessity operator $\boxdot$ in (MP), (MT), (DA), (AC) and (ConjSuf), we use the same value as for factual sentences: $V(\boxdot A) = V(A) = \mathrm{Pr}(A)$.

Theorem 10 *If the valuation* V_Δ *is combined with the unvalue-sum rule for validity, then the following Principles are all valid: (Ref) restricted to* Pr*-contingent sentences,* *(And), (ACRW), (CondPerf), (AaOE), (AOT), (AOT′), (Ant), (Cnst′), the right-to-left direction of (IN) and (IN′) and (OND). Moreover, (MP), (MT), (DA) and (AC) are all valid. On the other hand, (RW), (Or), (Cut), (CMon), (NRat), (DRat), (RMon), (ROr), (ConjSuf), (ConjRat) and (Cnsv) and the left-to-right direction of (IN) and (IN′) are invalid.*

Proof. (Ref) restricted to Pr-contingent sentences follows from part (ii) of Lemma 2 (we take $U_\Delta(A \rightarrow A) = 0$ as sufficient for treating $A \rightarrow A$ as the conclusion of a valid inference from zero premises). (And) follows from (xv), (ACRW) from (xiii), (CondPerf) from (xii),

The two parts of the left-to-right direction of (AaOE) follow from (viii) and (x) of Lemma 2, respectively (hint: pay special attention to the case where A is negatively relevant to C), and the right-to-left direction of (AaOE) follows from (xiv).

(AOT) and (AOT′) follow from (xvii) and (v) of Lemma 2.

(OND) is satisfied trivially since all necessity statements get maximum 'uncertainty', by (v).

For the failure of (RW), see Fig. 1.3.

For the failures of (Or), (Cut), (CMon), (NRat), (DRat), (RMon), (ROr) and (ConjSuf), see Figures 1.4–1.9. By (CondPerf), the counterexample to (ROr) is at the same time a counterexample to (ConjRat) (with negations flipped, cf. the proof of Lemma 1 (xvi)).

For (MP), (MT), (DA) and (AC), first note that $U_\Delta(A \rightarrow C) \geq \min\{\Pr(\neg C \mid A) + \Pr(C \mid \neg A), 1\}$, and thus (a) $U_\Delta(A \rightarrow C) \geq \Pr(A \wedge \neg C)$ and (b) $U_\Delta(A \rightarrow C) \geq \Pr(\neg A \wedge C)$. For (MP), we need to show that $\Pr(\neg A) + U_\Delta(A \rightarrow C) \geq \Pr(\neg C)$, and this follows from (a). For (MT), we need to show that $\Pr(C) + U_\Delta(A \rightarrow C) \geq \Pr(A)$, which follows from (a), too. For (DA), we need to show that $\Pr(A) + U_\Delta(A \rightarrow C) \geq \Pr(C)$, and this follows from (b). For (AC), we need to show that $\Pr(\neg C) + U_\Delta(A \rightarrow C) \geq \Pr(\neg A)$, which follows from (b), too.

For (Ant) and (Cnsv), it is straightforward to check that $V_\Delta(A \rightarrow A \wedge C) -$

$$V_\Delta(A \rightarrow C) = \begin{cases} \Pr(C \mid \neg A) & \text{if } \Pr(C \mid A) \geq \Pr(C \mid \neg A) \text{ and } A \text{ is Pr-contingent,} \\ \Pr(C \mid A) & \text{if } \Pr(C \mid A) < \Pr(C \mid \neg A) \text{ and } A \text{ is Pr-contingent,} \\ 0 & \text{otherwise.} \end{cases}$$

So in any case $V_\Delta(A \rightarrow A \wedge C) \geq V_\Delta(A \rightarrow C)$, and sometimes $V_\Delta(A \rightarrow A \wedge C) > V_\Delta(A \rightarrow C)$. The claims regarding (Ant) and (Cnsv) then follow immediately.

For (Cnst′), note that $U(\top \nrightarrow \bot) = 0$.

For the right-to-left direction of (IN) and (IN′), note that $U(\boxdot(A \vee C)) = \Pr(\neg A \wedge \neg C)$ is not higher than $U(\boxdot A) = \Pr(\neg A)$ and $U(A \rightarrow (A \vee \neg C)) = U(A \nrightarrow (A \vee C)) = \Pr(\neg C \mid \neg A)$. A counterexample to all variants of the left-to-right direction of (IN) and (IN′) (some of them are non-Horn!) is given by the following probability function: $\Pr(A \wedge C) = 0.5$, $\Pr(A \wedge \neg C) = 0$, $\Pr(\neg A \wedge C) = 0.4$ and $\Pr(\neg A \wedge \neg C) = 0.1$. □

We note that the non-Horn principles of (NRat), (DRat), (RMon) and (ROr) are invalid even on the more liberal concept of Validity 2 applied to the schemes of the form (NH1), as explained in Section 1.3.1.

$C/\neg C$	B	$\neg B$
A	0.49 / 0	0 / 0.21
$\neg A$	0.02 / 0.21	0.04 / 0.03

	V_Δ	U_Δ
$A \to C$	0.5	0.5
$B \to C$	0.57	0.43
$A \vee B \to C$	0	1

Fig. 1.4: Counterexample to (Or) for conditionals based on V_Δ and the unvalue-sum rule.

$C/\neg C$	B	$\neg B$
A	0.384 / 0.096	0 / 0.32
$\neg A$	0.01 / 0.01	0.09 / 0.09

	V_Δ	U_Δ
$A \to B$	0.5	0.5
$A \wedge B \to C$	0.61	0.39
$A \to C$	0	1

Fig. 1.5: Counterexample to (Cut)

$C/\neg C$	B	$\neg B$
A	0.24 / 0.24	0.32 / 0
$\neg A$	0.002 / 0.018	0.018 / 0.162

	V_Δ	U_Δ
$A \to B$	0.5	0.5
$A \to C$	0.6	0.4
$A \wedge B \to C$	0	1

Fig. 1.6: Counterexample to (CMon)

1.3.5.3 The Rips measure

An alternative approach was recently proposed by Crupi and Iacona (2022c). From the perspective of the present paper, Crupi and Iacona's approach is the most ambitious and advanced one so far, because they explicitly aim at offering both a qualitative and a probabilistic approach to support conditionals (their 'evidential conditionals'). They use the Rips measure of confirmation. For all sentences A and C with $0 < \Pr(A)$ and $\Pr(C) < 1$, it defines

$$ri(A, C) = \frac{\Pr(C \mid A) - \Pr(C)}{1 - \Pr(C)}.$$

This is a normalisation of $d(A, C)$. The measure ri was proposed in the statistics literature as a "measure of conditional agreement" by Coleman (1966) and Light (1969; 1971) (see Bishop, Fienberg and Holland 1975). Rips (2001) used it as a measure of "argument strength". Crupi, Tentori and Gonzalez (2007) and Crupi and Tentori (2013) advocated it as a measure of empirical support that they propose to read as a "measure of partial entailment". Notice that the Rips measure may be equivalently written as $ri(A, C) = 1 - \frac{\Pr(A \wedge \neg C)}{\Pr(A) \cdot \Pr(\neg C)}$, which makes it transparent that $ri(A, C) = ri(\neg C, \neg A)$.

$C/\neg C$	B	$\neg B$
A	0.28 / 0.13	0.35 / 0.21
$\neg A$	0 / 0.01	0 / 0.02

	V_Δ	U_Δ
$A \rightarrow C$	0.65	0.35
$A \wedge B \not\rightarrow C$	0.91	0.09
$A \wedge \neg B \rightarrow C$	0	1

	V_Δ	U_Δ
$A \rightarrow C$	0.65	0.35
$A \not\rightarrow \neg B$	1	0
$A \wedge B \rightarrow C$	0.09	0.91

Fig. 1.7: Counterexample to (NRat) (and thus also (DRat)) as well as to (RMon)

$C/\neg C$	B	$\neg B$
A	0.05 / 0.2	0.2 / 0.05
$\neg A$	0.1 / 0	0 / 0.4

	V_Δ	U_Δ
$A \rightarrow B \vee C$	0.7	0.3
$A \not\rightarrow B$	0.7	0.3
$A \rightarrow C$	0.3	0.7

Fig. 1.8: Counterexample to (ROr)

	C	$\neg C$
A	0.81	0.09
$\neg A$	0.08	0.02

	V_Δ	U_Δ
A	0.9	0.1
C	0.89	0.11
$A \rightarrow C$	0.1	0.9

Fig. 1.9: Counterexample to (ConjSuf)

Crupi and Iacona (2022c) recommend to apply *ri* as a valuation for the degree of assertability of conditionals. They suggest a slight deviation from our generic method in that they stipulate that $V(A \rightarrow C) = 1$ if $\Pr(A) = 0$ or $\Pr(C) = 1$, but this is a minor difference that we are going to neglect.[38]

$$V_{ri}(A \rightarrow C) = \begin{cases} ri(A, C) & \text{if } \Pr(C \,|\, A) \geq \Pr(C) \text{ and } A \text{ and } C \text{ are Pr-contingent,} \\ 0 & \text{otherwise.} \end{cases}$$

This yields an unvalue score for a conditional which Crupi and Iacona call "uncertainty" (even though it is not the same as improbability, as in Adams's work).

$$U_{ri}(A \rightarrow C) = 1 - V_{ri}(A \rightarrow C) =$$

$$\begin{cases} \frac{\Pr(A \wedge \neg C)}{\Pr(A) \cdot \Pr(\neg C)} & \text{if } \Pr(C \,|\, A) \geq \Pr(C) \text{ and } A \text{ and } C \text{ are Pr-contingent,} \\ 1 & \text{otherwise.} \end{cases}$$

[38] As mentioned above, I think that 0 is the more adequate value here, since intuitively A can lend no support to C when either $\Pr(A) = 0$ or $\Pr(C) = 1$. This also allows us to use the property of Pr-contingency in the case distinction, without any change involved.

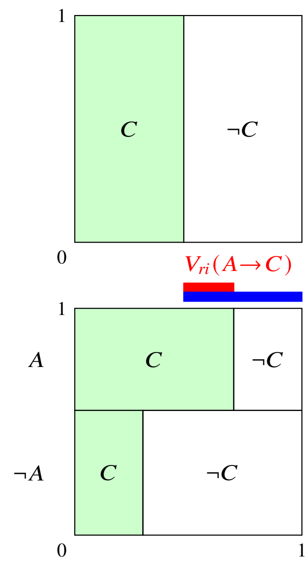

Fig. 1.10: $V_{ri}(A \to C) = ri(A, C)$.

Crupi and Iacona then employ the unvalue-sum concept of validity: an inference from $A_1, \ldots, A_k$ to A_{k+1} is valid iff $U_{ri}(A_{k+1}) \leq U_{ri}(A_1) + \ldots + U_{ri}(A_k)$.

We now turn to the properties yielded by their interpretation of conditionals. Most of the following theorem is due to Crupi and Iacona (2022c).[39]

Theorem 11 *If the valuation V_{ri} is combined with the unvalue-sum rule for validity, then the following principles are all valid: (Ref) restricted to* Pr*-contingent sentences, (LLE), (And), (Or), (CMon), (NRat) and (Cntrpos). Restricted variants of Aristotle's first and second theses and Abelard's thesis are valid, with the extra premises being $\neg\Box\neg A$ for both (Arist1) and (Ablrd) and $\neg\Box B$ for (Arist2). Moreover, (MP), (MT) and (Ant) are valid.*

On the other hand, (RW), (Cut), (DRat), (RMon), (ROr), (DA), (AC), (ConjSuf), (ConjRat) and (Cnsv) are invalid.

Proof. We only look at a selection of parts of the theorem. For (Ref), note that $V_{ri}(A \to A) = \frac{\Pr(A\,|\,A) - \Pr(A)}{1 - \Pr(A)} = 1$ if A is Pr-contingent. So in this case, $U_{ri}(A \to A) = 0$ which we take as sufficient for treating $A \to A$ as the conclusion of a valid inference from zero premises. However, if A is not Pr-contingent, then $U_{ri}(A \to A) = 1$, and the conditional $A \to A$ is unacceptable.

For a counterexample to (DRat) and (ConjRat), see Fig. 1.11.

[39] Crupi and Iacona (2022c) did not consider (Ref), (DRat), (ROr), (MT), (DA), (AC), (ConjRat), (Ant) and (Cnsv). For the necessity operator $\Box$, they use the values $V(\Box A) = 1$ if $\Pr(A) = 1$, and $V(\Box A) = 0$ otherwise.

$C/\neg C$	B	$\neg B$
A	0.01 / 0.24	0.24 / 0.01
$\neg A$	0.24 / 0.01	0.01 / 0.24

	V_{ri}	U_{ri}
$A \vee B \rightarrow C$	0.31	0.69
$A \nrightarrow C$	1	0
$B \rightarrow C$	0	1

	V_{ri}	U_{ri}
$\neg C \rightarrow \neg A \wedge \neg B$	0.31	0.69
$\neg C \nrightarrow \neg A$	1	0
$\neg C \rightarrow \neg B$	0	1

Fig. 1.11: Counterexample to (DRat) and (ConjRat) for conditionals based on V_{ri} and the unvalue-sum rule.

For a counterexample to (ROr), we can use the one given in Fig. 1.9 and calculate that $V_{ri}(A \rightarrow B \vee C) = \frac{0.9-0.55}{1-0.55} = 0.78 > 0.23 + 0.23 = \frac{0.5-0.35}{1-0.35} + \frac{0.5-0.35}{1-0.35} = V_{ri}(A \rightarrow B) + V_{ri}(A \rightarrow C)$ (decimal numbers rounded).

For the restricted versions of (Arist1), (Arist2) and (Ablrd), see Crupi and Iacona (2022c, pp. 49, 54, 62–63).

For (MP) and (MT), first note that (a) $U_{ri}(A \rightarrow C) \geq \Pr(A \wedge \neg C)$. For (MP), we need to show that $\Pr(\neg A) + U_{\Delta}(A \rightarrow C) \geq \Pr(\neg C)$, and this follows from (a). For (MT), we need to show that $\Pr(C) + U_{\Delta}(A \rightarrow C) \geq \Pr(A)$, and this follows from (a), too.

For a counterexample to (DA) and (AC), consider the case where $\Pr(A \wedge C) = 0.09$, $\Pr(A \wedge \neg C) = 0.01$, $\Pr(\neg A \wedge C) = 0.51$ and $\Pr(\neg A \wedge \neg C) = 0.39$. First, note that $\Pr(A) + U_{ri}(A \rightarrow C) = 0.1 + \frac{0.01}{0.1 \cdot 0.4} = 0.35 < 0.6 = \Pr(C)$, which shows that (DA) is invalid. Second, note that $\Pr(\neg C) + U_{ri}(A \rightarrow C) = 0.4 + \frac{0.01}{0.1 \cdot 0.4} = 0.65 < 0.9 = \Pr(\neg A)$, which shows that (A) is invalid.

For (Ant) and (Cnsv), note first, $V_{ri}(A \rightarrow A \wedge C) = \frac{\Pr(A \wedge C \mid A) - \Pr(A \wedge C)}{1 - \Pr(A \wedge C)} = \frac{\Pr(\neg A)\Pr(A \wedge C)}{\Pr(A)\Pr(\neg(A \wedge C))}$ if $\Pr(A) > 0$ and $\Pr(A \wedge C) < 1$. A few elementary transformations show that $V_{ri}(A \rightarrow A \wedge C) - V_{ri}(A \rightarrow C) =$

$$\begin{cases} \frac{\Pr(A \wedge \neg C) \cdot \Pr(\neg A \wedge C)}{\Pr(A) \cdot \Pr(\neg C) \cdot \Pr(\neg A \vee \neg C)} & \text{if } \Pr(C \mid A) \geq \Pr(C), \Pr(A) > 0 \text{ and } \Pr(C) < 1, \\ \Pr(C \mid A) \cdot \Pr(\neg A \mid \neg A \vee \neg C) & \text{if } (\Pr(C \mid A) < \Pr(C) \text{ or } \Pr(C) = 1) \\ & \quad \text{and } \Pr(A) > 0 \text{ and } \Pr(A \wedge C) < 1, \\ 0 & \text{otherwise.} \end{cases}$$

So in any case $V_{ri}(A \rightarrow A \wedge C) \geq V_{ri}(A \rightarrow C)$, and sometimes $V_{ri}(A \rightarrow A \wedge C) > V_{ri}(A \rightarrow C)$. The claims regarding (Ant) and (Cnsv) then follow immediately. □

Some comments are in order. The logic of probabilistically interpreted conditionals identified by Crupi and Iacona is comparatively strong, it satisfies quite a number of principles of system **P** and Contraposition on top of that. The failures of Conjunctive Sufficiency and (RW) are welcome features, and the failures of (Cut) and (DRat) are not that bad either. Crupi and Iacona (2022a, p. 2914) point out that the restricted connexive principles mentioned in Theorem 11, “unlike their unrestricted counterparts, imply no revision of classical logic." The invalidities mentioned in the theorem of course remain invalid if the unvalue-maximum concept of validity is applied rather than the unvalue-sum concept.

Crupi and Iacona's surprising finding was that their modal account and their probabilistic account validate exactly the same subset of the set of principles they looked at. All together, this amounts to a list of about 15 principles the validity of which comes out the same in their modal and probabilistic accounts. This observation is indeed striking, and Crupi and Iacona hence claim that the "probabilistic semantics [...] converges with the modal semantics [...] in all relevant respects" (2022b, p. 646) and that they imply "exactly the same pattern of results" (2022a, p. 2919).

It must be stressed again that Crupi and Iacona's accounts are logically very well-behaved, much more well-behaved than, for instance, the logics of Rott's difference-making and dependence conditionals and of Douven's evidential conditionals. This makes them worthy objects of logical investigations. However, I do not think that Crupi and Iacona's undertaking is completely successful. First, intuitively Contraposition is neither sufficient nor necessary for evidential support. Second, a closer look has revealed that the two logics are different after all: the modal account validates Disjunctive Rationality and Conjunctive Rationality, but the probabilistic account doesn't validate them (see again Fig. 1.11). Third, the theoretical role of Contraposition is different in the modal account (here it is supposed to capture the idea of evidential support) and in the probabilistic account (here it is employed just to choose the authors's favourite measure of evidential support). These criticisms are discussed in Rott (2022c).

1.3.5.4 The Sheps measure

In the same way in which Crupi and Iacona normalised Douven's difference measure, van Rooij and Schulz (2019; 2021; 2022) argue that a better predictor of the acceptability of conditionals is given by a normalised measure of contingency, that is, by a measure that I will call the *Sheps measure*. For sentences A and C with $0 < \Pr(A) < 1$ and $\Pr(C \mid \neg A) < 1$, it defines

$$sh(A, C) = \frac{\Pr(C \mid A) - \Pr(C \mid \neg A)}{1 - \Pr(C \mid \neg A)}.$$

This measure is a normalisation of $\Delta(A, C)$. In philosophical discussions, the measure *sh* is not usually included among measures of evidential support. It is discussed and advocated in analyses of causation by Mindel Sheps (1958; 1959, 'relative difference'[40]), I. J. Good (1961, p. 47, 'quasiprobability'), Patricia Cheng (1997, p. 374, 'causal power'), Judea Pearl (1999, p. 113; 2000, p. 292, 'probability of sufficiency').

[40] Neither the formula nor the term 'relative difference' has a prominent status in the work of the Canadian physician, biostatistician and demographer Sheps. Both are highlighted by Fleiss (1973, pp. 60–61) who decidedly attributes them to Sheps.—Incidentally, Sheps's name has been frequently misspelt in the literature as 'Shep' since 1999.

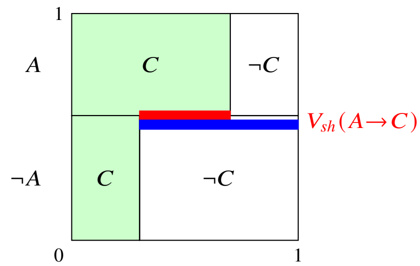

Fig. 1.12: $V_{sh}(A \to C) = sh(A, C)$.

An argument for using it as a measure of confirmation was recently given by Igor Douven (2021).

Van Rooij and Schulz (2019, p. 58; 2021, p. 440; 2022) recommend to apply *sh* as a valuation for the degree of acceptability or assertability of conditionals:

$$V_{sh}(A \to C) = \begin{cases} sh(A, C) \text{ if } \Pr(C \mid A) \geq \Pr(C \mid \neg A) \text{ and } A \text{ is } \Pr\text{-contingent}, \\ 0 \qquad\qquad \text{otherwise.} \end{cases}$$

According to Van Rooij and Schulz, a conditional is acceptable or assertable if and only if this measure is high. They argue that this rule is a matter of conversational implicature rather than semantic content, and that it can account for various intuitions about the assertability of conditionals and explain empirically found differences in the acceptance of causal and diagnostic conditionals. They do not, however, present a logic for conditionals conforming to this idea.

1.3.5.5 High evidential support entails high conditional probability

Speaking with Carnap again, does sufficient increase in firmness give enough firmness? Won't the support criterion let room for the possibility that given the antecedent, the consequent is still not probable enough to be accepted? Fortunately, the answer is no. We can specify the numerical relations between our four measures of evidential support.[41]

Theorem 12 *(i) The relationships between the four measures of evidential support are as indicated in Fig. 1.13. In particular, all support values $ES(A, C)$ are below the conditional probability* $\Pr(C \mid A)$.
(ii) The Delta and Rips measures are incomparable, with $\Delta(A, C) \gtreqless ri(A, C)$ if and only if $\frac{\Pr(C)}{\Pr(\neg C)} \gtreqless \frac{\Pr(C \mid \neg A)}{\Pr(\neg C \mid A)}$.

[41] After having completed this section, I discovered that similar results were obtained by Douven (2021).

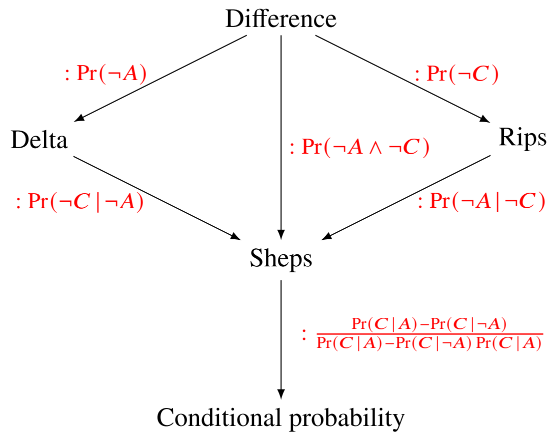

Fig. 1.13: Relations between the four measures $d(A,C)$, $\Delta(A,C)$, $ri(A,C)$ and $sh(A,C)$ and the conditional probability $\Pr(C\,|\,A)$. Arrows point to measures that yield higher values.

So if a conditional gets a high support value in one of the four senses, the conditional probability of the consequent given the antecedent is at least as high.

Proof. The arrows from Difference to Rips and from Delta to Sheps are true by definition. The arrow from Difference to Delta is easy (!). In the following, I assume that that A is Pr-contingent and positively relevant to C and that the denominators are all non-zero. We give a direct proof of the arrow from Rips to Sheps.

$$
\begin{aligned}
\frac{ri(A,C)}{sh(A,C)} &= \frac{\Pr(C|A) - \Pr(C)}{\Pr(\neg C)} \cdot \frac{\Pr(\neg C|\neg A)}{\Pr(C|A) - \Pr(C|\neg A)} \\
&= \frac{(\Pr(A \wedge C) - \Pr(A)\Pr(C)) \cdot \Pr(\neg C)|\neg A)\Pr(A)\Pr(\neg A)}{\Pr(A)\Pr(\neg C) \cdot (\Pr(\neg A)\Pr(C \wedge A) - \Pr(A)\Pr(C \wedge \neg A))} \\
&= \frac{(\Pr(A \wedge C) - \Pr(A)\Pr(C)) \cdot \Pr(\neg C \wedge \neg A)}{\Pr(\neg C) \cdot ((1 - \Pr(A))\Pr(C \wedge A) - \Pr(A)\Pr(C \wedge \neg A))} \\
&= \frac{(\Pr(A \wedge C) - \Pr(A)\Pr(C)) \cdot \Pr(\neg C \wedge \neg A)}{\Pr(\neg C) \cdot (\Pr(A \wedge C) - \Pr(A)\Pr(C))} \\
&= \frac{\Pr(\neg C \wedge \neg A)}{\Pr(\neg C)} = \Pr(\neg A|\neg C)\ .
\end{aligned}
\tag{1.1}
$$

Second, we verify that the conditional probability is even higher than the Sheps measure, that is, the support measure with the highest values we have considered.

$$\frac{sh(A,C)}{\Pr(C\,|\,A)} = \frac{\Pr(C\,|\,A) - \Pr(C\,|\,\neg A)}{(1-\Pr(C\,|\,\neg A))\cdot\Pr(C\,|\,A)}$$

$$= \frac{\Pr(C\,|\,A) - \Pr(C\,|\,\neg A)}{\Pr(C\,|\,A) - \Pr(C\,|\,\neg A)\Pr(C\,|\,A)} \leq 1. \tag{1.2}$$

Finally, we compare the Delta and Rips measures and verify that they are incomparable. From their relation to the difference measure, is is clear that $\frac{\Delta(A,C)}{ri(A,C)} = \frac{\Pr(\neg C)}{\Pr(\neg A)}$. So $\Delta(A,C) \geq ri(A,C)$ iff $\Pr(A) \geq \Pr(C)$, or equivalently, iff $\Pr(A\wedge\neg C) \geq \Pr(\neg A\wedge C)$. An essential difference between Δ and ri is this. Suppose we have the contrasting conditional probabilities $\Pr(C\,|\,A)$ and $\Pr(C\,|\,\neg A)$, and we keep them fixed while varying $\Pr(A)$ (have another look at Fig. 1.12 to visualise what we are doing). Then $\Delta(A,C)$ does not change at all, while $ri(A,C)$ decreases if $\Pr(A)$ is increased, and vice versa.[42]

1.3.6 Connexive principles on the probabilistic account

The main historical sources of connexive logic are Aristotle and Boethius and, to a somewhat lesser extent, Abelard. Connexive logic is most frequently taken to be defined by the idea that certain 'theses' or 'principles' should be theorems of propositional logic—even though they are non-theorems in the propositional logic that we today call classical. I repeat my non-committal formulations of the three central principles of connexivity for the reader's convenience:

(Arist1) Not $A\to\neg A$. (Aristotle's first thesis)

(Arist2) Not both $A\to C$ and $\neg A\to C$. (Aristotle's second thesis)

(Ablrd) Not both $A\to C$ and $A\to\neg C$. (Abelard's first principle)

Aristotle's second thesis and Abelard's principle are remarkably similar to each other and exhibit an appealing symmetry: the former concerns a negated antecedent, the latter a negated consequent. (Ablrd) is widely regarded as defining the very concept of connexive logic, while there is only little discussion about (Arist2) in the connexive logic community. This seems unjustified to me, both from a historical and from a systematic point of view. The historical origin of connexive logic is often located in Aristotle's *Analytica Priora* 57b3–14 where he argues for (Arist2) (or something very similar) on the grounds of (Arist1) (or something very similar), and neither Boethius nor Abelard had any intention of denying what Aristotle had said. Systematically, the justification of (Ablrd) by means of (a version of) (Arist1) runs exactly parallel to the justification of (Arist2) by means of (the twin version of) (Arist1).[43] So perhaps the preference that contemporary connexive logicians give to

[42] To be precise, if $r = \Pr(C\,|\,A)$ and $s = \Pr(C\,|\,\neg A)$, then $ri(A,C) = 1 - \frac{1-r}{1-s-\Pr(A)(r-s)}$ which is a decreasing function of $\Pr(A)$.

[43] Compare in particular Abelard (1956, p. 288–292) and the discussions by McCall (1963, p. 21) and Martin (1986, p. 568; 2004, p. 190).

(Ablrd) over (Arist2) is only a matter of historical contingency. In any case, (Arist2) looks as plausible as (Ablrd) if conditionals are interpreted as support conditionals.

As pointed out by Crupi and Iacona (2022c, p. 57), Adams's suppositional conditional (see Section 1.3.2) already satisfies restricted variants of Aristotle's first thesis and Abelard's thesis, with the restricting conditions being that $\neg\Box\neg A$. However, the restrictions are significant, and Aristotle's second thesis (Arist2) does not even hold in a restricted form. The situation changes quite dramatically if we employ measures of evidential support rather than conditional probabilities.

Theorem 13 *All conditionals $\rightarrow$ based on quantitative measures of evidential support satisfy the connexive principles (Arist1), (Arist2) and (Ablrd).*

Proof. It is very easy to see this, considering the fact that all values and unvalues lie in the interval $[0, 1]$. For (Arist1), we note that always $U(A \rightarrow \neg A) = 1$, because a Pr-contingent A is always negatively relevant to $\neg A$. For (Arist2), we need to verify that $U(\neg A \nrightarrow C) \leq U(A \rightarrow C)$. If A is contingent and positively relevant to C, then $U(\neg A \nrightarrow C) = V(\neg A \rightarrow C) = 0$, because $\neg A$ is negatively relevant to C. If, on the other hand, A is not contingent or not positively relevant to C, then $U(A \rightarrow C) = 1$. For (Ablrd), we need to verify that $U(A \nrightarrow \neg C) \leq U(A \rightarrow C)$. If A is contingent and positively relevant to C, then $U(A \nrightarrow \neg C) = V(A \rightarrow \neg C) = 0$, because A is negatively relevant to $\neg C$. If, on the other hand, A is not contingent or not positively relevant to C, then $U(A \rightarrow C) = 1$. □

The case of the ΔP measure is special because we have $U_\Delta(A \rightarrow \neg C) = U_\Delta(\neg A \rightarrow C)$, and so the claims of (Arist2) and (Ablrd) are virtually equivalent.

The thesis of *Negation Commutation* says that the negation of a conditional is equivalent to the conditional with a negated consequent. It was attributed, somewhat controversially, to Boethius by Kneale and Kneale (1962, p. 191). It was called a "hyper-connexive" thesis by Sylvan (2000, pp. 82, 89) and advocated, for instance, by Adams (1965), Cooper (1968), Belnap (1973), Wansing (2005) and Cantwell (2008). It is instructive to break Negation Commutation into two parts:

(NC-Ex) If $A \rightarrow \neg C$, then $A \nrightarrow C$.

(NC-In) If $A \nrightarrow C$, then $A \rightarrow \neg C$.

We have just shown that (NC-Ex), which is equivalent to (Ablrd), holds for support conditionals. But this is not the case for its converse (NC-In), which is very close to Stalnaker's Conditional Excluded Middle (CEM).[44] For example, it is easy to show that the conditional characterised by the ΔP measure and the unvalue-sum rule does not satisfy (NC-In). This follows immediately from the second part of Lemma 2 (xviii). That (NC-In) fails to hold for support conditionals is hardly surprising.

[44] Even Lewis, who did *not* endorse (CEM), wrote: "The law of Conditional Excluded Middle is plausible because it explains why we do not distinguish, in ordinary language, between the external negation of a whole conditional $\neg(A \,\Box\!\!\rightarrow C)$ and the internal negation of the consequent $A \,\Box\!\!\rightarrow \neg C$." (Lewis 1973b, p. 79, notation adapted)

From the fact that A does not support C, one cannot conclude that A supports $\neg C$. A may well be neutral vis-à-vis C.[45]

1.4 Conclusion

Whether in logic or in the psychology of reasoning, the investigation of relevance measures and of the resulting inferences is currently a very active field in the study of conditionals. In this paper I have surveyed a variety of conditionals capturing the idea that the antecedent lends evidential support to the consequent. In terms of their logical behaviour, they are separated from the commonly studied suppositional conditionals by a resolute rejection of Conjunctive Sufficiency and a consequential rejection of Right Weakening (and even of the weaker principle of Conservativity). An overview of the logical properties of the 'support conditionals' that are most important from our perspective is given in Table 1.1 (on a float page at the end of the paper).

All support conditionals have strong connexive properties, but some of them are restricted. In this way, the concept of evidential support offers a new interpretation of 'connexivity', or a new justification for the central connexive principles, namely Aristotle's first and second theses and Abelard's thesis. This interpretation strongly suggests that the connexive principles should not be restricted and that Aristotle's second thesis is on an equal footing with Abelard's thesis.

It has been my aim to offer a presentation that goes across the 'qualitative/quantitative watershed' (Hawthorne and Makinson 2007).[46] The first role model for this project is of course Ernest Adams who, from 1966 on (clearly before Stalnaker and Lewis, long before Kraus, Lehmann and Magidor), built a beautiful bridge between his probabilistic and a qualitative (modal) account of suppositional conditionals.[47]

My second role models for conditionals as representing evidential support are Vincenzo Crupi and Andrea Iacona. They have challenged the idea that support conditionals behave in rather unusual and unruly ways, and they came forward with the bold and exciting project of not only attacking systematically the idea of evidential support in conditionals, but also doing it twice over: in a modal and in a probabilistic setting. Their logics, in which contraposition plays a central role, are surprisingly well-behaved. But as explained in Rott (2022c), the modal and probabilistic logics

[45] The proof of Lemma 1 (xviii) shows that the effect of (NC-In) would actually force us to conclude from the fact that A is irrelevant (or negatively relevant) to C that it is *maximally* relevant to $\neg C$. This is a peculiarity of the present account that limits itself to the interval $[0, 1]$.

[46] Hawthorne and Makinson (2007, p. 248) identify (And) as the "watershed condition" separating qualitative and (some) quantitative approaches to suppositional conditionals. Interestingly, (And) is valid for all qualitative *and probabilistic* support conditionals considered in the present paper.

[47] As Adams (1981, p. 171; 1986, pp. 271–273; 1998, p. 133, 136) pointed out repeatedly, his 'P-orderings' (Adams 1966, pp. 282–315), later also called or 'order-of-magnitude probability orderings', are the converses of Lewis's '(normal) possibility orderings' of possible worlds. This seminal part of Adams's work tends to be forgotten nowadays.

do not perfectly match, and it is doubtful whether their 'Chrysippus test' captures the intuitive notion of evidential support. In the present paper I have tried to do something similar to what they did for difference-making and dependence conditionals by invoking, on the quantitative side, the ΔP measure of evidential support and the unvalue-sum rule for validity. I think that the notion of evidential support is more adequately captured by this group of conditionals than by Crupi and Iacona's conditionals. However, as we have seen, the logics emerging at the qualitative and the probabilistic sides do not perfectly match here either.

I have discussed four well-known, simple measures of evidential support. The two most complex ones of them, the Rips and the Sheps measures, were suggested to be employed in the recent literature on conditionals. I have advocated the use of a simpler one, ΔP, which encodes the idea of contrasting the suppositions A and $\neg A$ in a way that is similar to the way difference-making conditionals and dependence conditionals encode it on the qualitative side. Applied together with the unvalue-sum rule defining validity, the ΔP measure results in a logic of conditionals that satisfies at the same time And-Or-Toggling (AOT), a central condition of the logic of difference-making conditionals, and Conditional Perfection (CondPerf), which is crucial for dependence conditionals. The axiom sets of the latter two kinds of conditionals are strictly speaking incompatible, but we found that they are, in a sense, 'almost' compatible. This suggested looking at Stalnakerian dependence conditionals as the closest approximation to the probabilistic side. The logic of conditionals based on ΔP, however, is still different, because the principle of Conjunctive Rationality (ConjRat) is not validated.

Plenty of work is left for the future. There are many more measures of empirical support in the probabilistic literature that can be investigated concerning their suitability for the interpretation of support conditionals and the logic they give rise to (see, e.g., the lists in Crupi, Tentori and Gonzalez 2007, p. 230; Sprenger and Hartmann 2019, p. 56, Merin 2021, pp. 269–270). At the same time, a lot more work seems needed in order to justify carefully—or perhaps modify—several elements of our analysis. Key points are: (i) the meaning of 'unvalue', of which 'uncertainty' is only a very special interpretation; others include 'unacceptability' and 'unassertability'; (ii) the use of the equation $U(A) = 1 - V(A)$ for the unvalue U when V is not a (conditional) probability; (iii) the application of the unvalue-sum rule in the definition of validity when V is not a (conditional) probability; (iv) the restriction of support values and unvalues to the interval [0,1]; and finally, (v) the identification of the value of a rejected (not accepted, 'negated') conditional with the unvalue of the accepted conditional.

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	$>$	$\gg$	$\succ\!\!\succ$	$\succ\!\!\succ^+$	V_Δ	$\rhd$	V_{ri}
(Ref)	$\sqrt{}$	–	$\sqrt{}$	$\sqrt{}$	r	$\checkmark$	r
(LLE)	$\sqrt{}$	$\checkmark$	$\checkmark$	$\checkmark$	$\checkmark$	$\checkmark$	$\checkmark$
(LRLE)	$\checkmark$	$\sqrt{}$	$\sqrt{}$	$\sqrt{}$	$\checkmark$	$\checkmark$	$\checkmark$
(RW)	$\sqrt{}$	–	–	–	–	–	–
(And)	$\sqrt{}$	$\checkmark$	$\sqrt{}$	$\sqrt{}$	$\checkmark$	$\checkmark$	$\checkmark$
(Or)	$\sqrt{}$	r / *	r / *	r / *	–	$\checkmark$	$\checkmark$
(Cut)	$\sqrt{}$	r / *	r / *	r / *	–	–	–
(CMon)	$\sqrt{}$	r / *	r / *	r / *	–	$\checkmark$	$\checkmark$
(NRat)	[$\checkmark$]				–	$\checkmark$	$\checkmark$
(DRat)	[$\checkmark$]				–	$\checkmark$	–
(RMon)	[$\sqrt{}$]	[r / *]	[*]	[*]	–	–	–
(CEM)	[$\checkmark$]	–	–	–	–	–	–
(ROr)	[$\checkmark$]	–	–	$\sqrt{}$	–	–	–
(Arist1)	–	$\checkmark$	$\checkmark$	$\checkmark$	$\checkmark$	r	r
(Arist2)	–	$\checkmark$	$\checkmark$	$\checkmark$	$\checkmark$	r	r
(Ablrd)	–	r	$\checkmark$	$\checkmark$	$\checkmark$	r	r
(MP)	$\checkmark$	$\checkmark$	$\checkmark$	$\checkmark$	$\checkmark$	$\checkmark$	$\checkmark$
(MT)	$\checkmark$	$\checkmark$	$\checkmark$	$\checkmark$	$\checkmark$	$\checkmark$	$\checkmark$
(DA)	–	r	$\checkmark$	$\checkmark$	$\checkmark$	–	–
(AC)	–	$\checkmark$	$\checkmark$	$\checkmark$	$\checkmark$	–	–
(ConjSuf)	$\checkmark$	–	–	–	–	–	–
(Cntrpos)	–	–	–	–	–	$\checkmark$	$\checkmark$
(CondPerf)	–	–	$\sqrt{}$	$\sqrt{}$	$\checkmark$	–	
(ConjRat)	$\checkmark$	$\sqrt{}$	–	$\checkmark$	–	$\checkmark$	–
(ACRW)	$\checkmark$	$\checkmark$	$\sqrt{}$	$\sqrt{}$	$\checkmark$	$\checkmark$	
(AaOE)	$\checkmark$	$\sqrt{}$	$\sqrt{}$	$\sqrt{}$	$\checkmark$	–	
(AOT)	–	$\sqrt{}$	–	–	$\checkmark$	–	
(AOT′)	–	–		$\checkmark$	$\checkmark$	–	
(Supraclass)	$\checkmark$	–	–	–	–	$\checkmark$	$\checkmark$
(Ant)	$\checkmark$	$\checkmark$	$\checkmark$	$\checkmark$	$\checkmark$	$\checkmark$	$\checkmark$
(Cnsv)	$\checkmark$	–	–	–	–	–	–

Table 1.1: Properties of the conditionals considered in this paper. $\succ\!\!\succ^+$: Stalnakerian dependence conditional. $\checkmark$: valid; $\sqrt{}$: valid and used as an axiom; $-$: not valid; r: valid in restricted form; *: valid in modified form; [$\checkmark$]: valid under special constraints on σ.

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