ESSAYS IN FORMAL METAPHYSICS

BY

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ABSTRACT OF THE DISSERTATION

Essays in Formal Metaphysics

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The goal of my dissertation is to bring insights from branches of logic that are not well-discussed in the literature, notably modal model theory, to bear on questions in the metaphysics of time and modality. This occurs on both the meta-level and on the level of first order philosophical questions.

On the meta-level, I mount a defense of the ongoing usefulness of modal logic, considered as a branch of mathematics, in the face of recent views in metametaphysics that consider modal tools too crude to usefully state metaphysical theses or adjudicate metaphysical disputes. In doing so, I draw on the study of expressive power in languages and Bayesian epistemology to formulate a new criterion for ideological parsimony: if two ideologies are expressively equivalent, then they are equally parsimonious. After explicating this principle, I show how it blocks arguments against the use of modal logic (among other consequences for parsimony arguments in the literature). I go beyond purely negative arguments by then showing how to use modal logic to study things other than necessity and possibility, and use it to unearth a hitherto unappreciated parallel between grounding and provability.

On the first order level, I focus on A-theories of time. Tense logic and modal logic are mathematically similar; their model theory is typically studied together. I address several problems with A-theories. First, I argue that the standard way of setting up tense logic is hostile to open future views, and propose an alternative that is not. I show that my alternative can provide a logical setting for evaluating arguments about whether the future is open, and prove that the standard setup is a special case of my framework. Second, I argue that (a) presentists can consistently adopt a counterpart theory of identity across time, and (b) that they can solve several problems if they do so.

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Dedication

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Introduction

Metaphysics and logic are old companions in arms. Logical tools provide the means to express metaphysical views with the clarity and precision of mathematics, and metaphysics provides the conceptual backdrop necessary to interpret the logician's formalism. It has been common in the history of philosophy for developments in logic to drive developments in metaphysics, such as the developments in modal metaphysics in the late 20th century driven by Kripke's "possible worlds" model theory for modal logic; it has also been common for the desire for the tools to explore a metaphysical view to spur developments in logic, such as A.N. Prior's invention of tense logic in his quest to develop his temporal metaphysics.

My dissertation stands in this tradition. My goal is to leverage work in logic, especially modal model theory, that has been underappreciated to date amongst metaphysicians in order to gain insight into metaphysical issues. My focus in this dissertation has been on modality and on the philosophy of time. In particular, the first two chapters address methodological issues in how we think about the tools of metaphysics, developing a novel theory about the use of parsimony in metaphysical debates and addressing the place of modal logic in the metaphysician's toolkit. The second two chapters focus on refining views in the philosophy of time, in particular versions of the "dynamic" or "A-theory." In them, I argue that a prominent type of A-theory - presentism - can and should be tied to a stage-theoretic account of identity over time which is usually not associated with it, and I work to expand the system of logic Prior pioneered, tense logic, in a way that allows it to express the full variety of views about the structure of time that contemporary metaphysicians have developed.

Chapter 1. Ideological Innocence

Chapter 1 draws on the study of the expressive power of languages to provide a novel partial analysis of the theoretical virtue of parsimony. Theoretical virtues are features of theories in virtue of which that theory is, *ceteris paribus*, more likely to be true. Throughout the history of science, parsimony has been an important ingredient in theory change, typically for the better. But a precise statement of the parsimony principle has eluded us.

Following W.V.O Quine, philosophers recognize two kinds of theoretical commitment: ontology and ideology. Ontology is the stuff that must exist if a theory is true. Cars, tables, persons, electrons, and numbers are all examples of ontology. Ideology is the conceptual apparatus required in order to state the theory. Ideology is expressed in linguistic devices such as quantifiers, predicates, operators, and truth-functional connectives. Since commitments comes in two varieties, parsimony comes in two varieties: ontological parsimony and ideological parsimony. My focus in this chapter is on ideological parsimony.

I propose a criterion for when adding ideology to a theory's commitments does not offend against parsimony - a criterion of ideological innocence. I call it the expressive power innocence criterion, and it says: if the ideologies of two theories are expressively equivalent, then the theories are equally ideologically parsimonious. I defend this criterion with the argument from accuracy. The argument begins by noting that if one theory has more of a theoretical virtue than another, then it is *ceteris paribus* more likely to be true. But then, if some rule of parsimony counts theory 1 as more parsimonious than theory 2 despite the theories having expressively equivalent ideologies, we will be able to find a theory in the ideology of theory 2 that is logically equivalent to theory 1. Since logically equivalent theories are equally likely to be true, this rule of parsimony won't make theories with more of it more likely to be true, and so won't be the analysis of a theoretical virtue.

After defending the expressive power innocence criterion, I explore its upshots for the metaphysics literature, arguing that while some prominent arguments from ideological parsimony aren't blocked by it, others are.

Chapter 2. Modal Logic for Post-Modal Metaphysics

In chapter 2, I apply the result from chapter 1 to a particular debate: the debate over the use of modality and modal logic in formulating and defending metaphysical views. Recently, the use of these modal tools has come under some criticism. In particular, Ted Sider has cited ideological parsimony as a reason to regard modality as non-fundamental, and along with others has argued that it is too crude for use in formulating metaphysical theses. My aim on this chapter is to show the limits of this critique, and to argue that it does not fully apply to modal logic. Modal languages can be interpreted in many ways; even if concepts like 'possibility,' 'necessity,' and 'actuality' need supplementation or replacement, modal logic thought of as a mathematical tool still has much to offer. Defending my claims takes us on a tour through a branch of modal logic not often used in contemporary metaphysics, modal correspondence theory. After reviewing some of its most basic results, we will see that modal logic is a much more versatile tool than post-modalists have given it credit for. As an example, I will show how modal logic can be used to study the recently much-discussed 'grounding relation.' After setting up the basic framework, I will use the modal logic of grounding to study property inheritance principles and to show a surprisingly tight parallel between grounding and provability. I end the study with some negative results.

Chapter 3. Presentist Counterpart Theory

Presentism is a thesis about ontology. The only concrete things are present things. Counterpart theory is a theory about persistence through time. Things persist by having counterparts at future times. Counterparts are not identical; but they are similar enough in the right respects to count as the things they are counterparts of when reckoning persistence through time. These views are not commonly found together; they are generally taken to get along about as well as the Hatfields and the McCoys. But in this chapter, I argue that a presentist can and should be a counterpart theorist.

I begin by presenting two problems for presentism. and then showing how to solve them with a presentist counterpart theory. The first problem: a revived version of the problem of temporary intrinsics. David Lewis, Berit Brogaard and others have argued that presentists should adopt a kind of tense operator known a span operator. Span operators allow presentists to talk about things that were true at different times under the scope of one operator. For example, 'in 1809' is a span operator, and would allow the presentist to say 'In 1809, both Lincoln and Darwin were born' without quantifying over past individuals like Lincoln and Darwin or a past year in which they were both born. Presentists need span operators because they are needed to say some true things that eternalists with quantification over times can say but that presentists without span operators cannot. But span operators revive the problem of temporary intrinsics. The second problem: many presentists still want to maintain the truth of singular propositions about past and future individuals. But I argue that the presentist has no explanation for why multiple singular propositions can be about the same individual, while the non-presentist has an easy one. After introducing the presentist-friendly counterpart theory, I show how it solves both problems and thereby recommends itself to the presentist.

Chapter 4. Logic and the Open Future

In the fourth chapter, I develop a logical framework for studying temporal relations that generalizes and improves upon the existing standard framework. There are two main motivations for developing the framework. First: following the polish logician Lukasiewicz, I want to have a tense logic that allows for a third truth value that can be sensibly interpreted as 'indeterminate.' There are perfectly good views of the structure of temporal relations that imply that contingent propositions about the future have indeterminate truth-value, and the existing standard framework for tense logic has no room for these views, ruling them out in its minimal setup. Second: I want a framework general enough that we can find within it classes of models that correspond to all coherent combinations of the following three debates about the structure of temporal relations.

First: the open/closed debate. Defenders of an open timeline say that there are not contingent facts about the future. Propositions like 'there will be a sea battle tomorrow' are not true (false) until tomorrow arrives and sea battle does (not) happen. Defenders of a closed timeline say no. They think that there is a fully determinate suite of contingent facts about the future.

Second: the determinist/indeterminst debate. Determinists say facts about the past, plus facts about the laws of nature (or causal laws, or...) entail all facts about the future. Indeterminists disagree; they think that the past and the laws of nature (or causal laws, or...) do not entail all the facts about the future. If there is a fully determinate suite of facts about the future, they do not simply follow from the history and the laws.

Third: the branching vs. linear debate. Everett's interpretation of quantum mechanics says that we live in a branching spacetime, where every decision, every event that could happen one way or another, creates a new branch, so that everything happens on some branch or other. Traditionalists have thought of time as a line, with alternate possibilities failing to occur, rather than occurring on different branches of our spacetime structure.

After arguing that there are six coherent ways of combining these views about the structure of temporal relations, I introduce my framework and show how it can provide different classes of models for each one. Then I prove that the standard framework is a special case of my framework, when you make particular substantive metaphysical assumptions.

Chapter 1

Ideological Innocence

1.1 Introduction

Quine taught us that theoretical commitments come in two varieties: ontology and ideology.[1] A theory's ontology is the entities that must exist if the theory is true: things like chairs, gods, electrons, incars, and Eiffel-tower-noses. A theory's ideology is the primitive notions or concepts employed in its most perspicuous statement: things like quantifiers, operators, predicates, and connectives. Good theories minimize their commitments. Better theories minimize their commitments more. This minimization of commitments is typically codified in the theoretical virtue of parsimony, which may be split into two components: ontological parsimony and ideological parsimony. My interest here is in ideological parsimony, There is, unfortunately, no widely accepted theory of what ideological parsimony amounts to. I have no such theory on offer. But I do wish to defend a condition for when additional ideology does not offend against parsimony; a criterion of ideological innocence, so to speak. I will argue that when adding ideology does not increase expressive power, it does not count against a theory's parsimony.

In defense of my proposal I offer what I call the argument from accuracy. The argument takes a specific conception of how epistemic theoretical virtues do their job and combines it with a theory of epistemic reasons to show that any adequate analysis of ideological parsimony must include my expressive power innocence criterion.

After giving the argument from accuracy, I explore the consequences of my proposal for the literature, passing judgment on several prominent arguments, before we consider

[1] Quine [1951], [1983].

several objections.

1.2 The Expressive Power Innocence Criterion

Expressive power is a property of languages. For our purposes, we can think of a language as a set of symbols plus some formation rules. Roughly, a language's expressive power is the range of things it can be used to communicate. Two languages are expressively equivalent when they can communicate the same things. A bit more precisely: two languages are expressively equivalent when there exists a meaning-preserving map from sentences of the first to sentences of the second such that the sentences paired up are true in all and only the same models, which for our purposes we can think of as mathematical structures that together with a semantics give the meaning and truth conditions of the sentences of a language.[2] We can think of a theory as a set of sentences in some language.

We can compare the expressive power of ideologies by comparing the expressive power of languages whose syntax includes only symbols representing their primitives and the things that can be defined out of them. Given some ideology I_n, we call the language containing symbols only for its primitives and what can be defined using them L_{I_n} its perspicuous language. We can then say that one ideology is expressively equivalent to another just in case their perspicuous languages are expressively equivalent. We can extend this notion of a perspicuous language directly to a specific theory's ideology as follows: A language L_{T_i} is perspicuous for a theory T_i just in case it contains symbols only for the ideology that appears in T_i (give or take convenience items like scope indicators). I am not going to go into detail on the technical aspects of expressive equivalence; the interested reader is referred to Kocurek [2017] and Pelletier & Urquhart [2003] for a formal discussion. We can now state the expressive power innocence criterion that I will be defending:

[2] In this sense, the notion of expressiveness in play here is propositional and intensional. I do not mean to claim that all good notions of expressiveness are like this, only that this is the one I will be using.

EXPRESSIVE POWER INNOCENCE CRITERION: some ideology I_k and some other ideology I_j are equally parsimonious if their perspicuous languages L_{I_j} and L_{I_k} are expressively equivalent.[3]

The expressive power innocence criterion gives us a sufficient condition for when arguments from ideological parsimony fail. They fail when the perspicuous language of T_i is expressively equivalent to the perspicuous language of T_j.[4]

1.3 The Argument from Accuracy

Now we can turn to the central argument of the paper: the argument from accuracy. The argument proceeds from a specific conception of the way in which epistemic virtues do their work and a specific theory of epistemic value to the conclusion that any adequate analysis of ideological parsimony must concur with the EXPRESSIVE POWER INNOCENCE CRITERION in all of its verdicts. I will not be able to offer full-blooded defenses of the theories of theoretical virtue and epistemic value that I will rely on, but these defenses can be found elsewhere. Instead, I will give a brief explanation of each before showing how they can be combined to produce an argument for the EXPRESSIVE POWER INNOCENCE CRITERION.

[3]Nota Bene: This definition does make ideological parsimony relative to a class of models and choice of semantics, but I think this is a harmless relativism. It is relative in the sense that anything that is relative to meaning is relative. In most cases we care about, there will be a clear 'right' class of models to use in making the comparison: namely, those we used when determining validities and analyticities for the languages. For example, for first order theories, the 'right' class of models will be models of predicate logic where the predicates of the theories stand in any common-ground analytic relations.

[4]As noted in fn. 3, expressive equivalence is always relative to the class of models used to give meaning to the language(s). So in order to use the criterion effectively, we will need to compare expressive power relative to the appropriate class of models and semantics. Spelling out which class of models and semantics is 'appropriate' will depend so much on the theories in question and the context of the debate that I doubt much can be said about it at this level of abstraction. But minimally, it should be a class of models and semantics where any common-ground validities and analyticities are respected.For example: if both theorists agree that 'bachelor' is equivalent to 'unmarried man,' then models including married bachelors are not appropriate.

1.3.1 Virtue-Probabilism

Arguments from theoretical virtue attempt to show that one theory is better than its rival(s), even though all theories are consistent with the data. We can divide theoretical virtues between *epistemic* and *pragmatic* virtues. Epistemic virtues provide epistemic reasons in favor of the theories that possess them. Following Ernest Sosa, I define an epistemic reason as a reason to affirm in the effort to be right, reliably enough.[5] Pragmatic virtues provide reasons to use a theory, truth be damned. I have nothing against pragmatic virtues, but I am only interested in ideological parsimony as an epistemic virtue. Thus, I will argue that any analysis of ideological parsimony that violates the EXPRESSIVE POWER INNOCENCE CRITERION is inconsistent with regarding ideological parsimony as an epistemic virtue. For those skeptical or agnostic about whether ideological parsimony makes theories more likely to be true, think of my argument as partially showing what ideological parsimony would have to be in order for it to do the work of an epistemic virtue.

Epistemic likelihood is given by probability functions. There are a number of arguments for this conclusion, which I won't rehearse in detail here. They include Cox's Theorem,[6] which lays down some intuitive axioms that govern the term 'plausibility' and uses them to derive the laws of probability; Dutch Book arguments, which are traditionally used to show that anyone whose degrees of belief don't conform to the axioms of probability can be baited into a series of bets that guarantee a loss, but have been adapted for purely epistemic purposes[7]; and Accuracy-Dominance arguments, which show that anyone whose degrees of belief are not a probability function is dominated with respect to distance from truth by a probability function.[8] But an important feature of probability functions is that they assign equal values to logically equivalent theories. Thus, if two theories are logically equivalent, one cannot be more likely than

[5] Sosa [2015].

[6] Cox [1946]

[7] Maher [1997], Christenen [1996]

[8] Joyce [1998], [2009], Easwaran and Fitelson [2015], Pettigrew [2016], Leitgeb and Pettigrew [2010a], [2010b].

the other.[9] This gives us our first premise in the argument from accuracy.

FIRST PREMISE: Logically equivalent theories are equally likely to be true.

1.3.2 The Value of Truth

Epistemic virtues are the ones that make a theory more likely to be true. They are characteristic of theories we affirm in the effort to be right, reliably enough. This makes truth the only value that epistemic virtues are sensitive to and gives us our next premise.

SECOND PREMISE: Theories that are equally likely to be true are equally epistemically virtuous.

Starting with Goldman, there is a tradition in epistemic value theory called *veritism* that takes truth to be the sole epistemic virtue. I don't quite need full blown veritism here; it is sufficient for my purposes that theories that are equally likely to be true are equally virtuous. But arguments for veritism are arguments for my premise.[10] It is important to note that for me, and for these arguments, it is the accuracy of the mental state that is its measure of value, not its role in facilitating the future learning or getting closer to the truth of other propositions, at least for synchronic assessment. Veritism is not equivalent to the claim that a theory/mental state has epistemic value only if it's true. There are sensible ways of measuring the relative accuracy of a theory/mental state even when it is not perfectly accurate. The veritist claim is that epistemic virtue covaries with relative accuracy.

Although I will not be giving a full defense of this premise, it is necessary to say a bit more about epistemic value. In his *Writing the Book of the World*, Ted Sider

[9] This places my view of the inferential role of theoretical virtues in the same company as Bayesian approaches. I only need probabilism, not full blown Bayesianism, to make the argument work, but arguments for Bayesianism will be arguments for the premise I need. See McGrew [2003] and Climenhaga [2017] for a Bayesian case; Douven [2016] for opposition.

[10] See Goldman [1999]; see also Konek and Levinstein [Forthcoming] for the development of a veritist epistemic decision theory.

has proposed that the use of joint-carving ideology is itself epistemically valuable.[11] He makes this point by comparing 'regular' color predicates with 'grueified' ones. The color of the world's supply of emeralds and sapphires may be truly described using the familiar 'green' and 'blue,' or Nelson Goodman's 'bleen' and 'grue.'[12] But, he contends, the blue/green description is clearly better. This betterness is explained by blue and green being more fundamental concepts (or more structural, as he puts it) than grue and bleen. He later argues that it is epistemically better to know more fundamental (or structural) truths, a kind of epistemic goodness that cannot be explained in veritistic terms.[13]

With Sider, I agree that some concepts carve more natural joints than others. And, as one interested in truth, I agree that it is good to know which concepts those are. In that sense, if we think there are facts about which concepts carve at nature's joints, knowing those facts is desirable from a veritist perspective. But it is not a special value. I do not think that the superiority of the blue/green description is explicable in terms of epistemic value over and above the value of truth. Rather, I think the added value is a pragmatic matter. We think in terms of blue and green. It is therefore less of a cognitive strain for us to use the blue/green description. If a community naturally thought in terms of grue/bleen, they would be right to say that the grue/bleen description is better. It may be that we are right and they are wrong (or that they are right and we are wrong) about which concepts are more natural, but I maintain that even if we were to learn that grue and bleen carved closer to the joints in nature than blue and green do, we would be right to maintain our verdict that the blue/green description is superior for us.

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[11] Sider [2011].

[12] Reminder: something is grue iff it is observed before 1/1/2028 and it is green, or it is not observed before 1/1/2028 and it is blue; something is bleen iff it is observed before 1/1/2028 and it is blue, or it is not observed before 1/1/2028 and it is green.

[13] Sider [2011] section 4.2.

1.3.3 Bringing It All Together

The argument from accuracy will show that any adequate analysis of ideological parsimony as an epistemic virtue must respect the EXPRESSIVE POWER INNOCENCE CRITERION. The argument itself is fairly straightforward.

We start with a theorem of the probability calculus. If $\phi \leftrightarrow \psi$ is valid (that is, if ϕ and ψ are logically equivalent), then $\Pr(\phi) = \Pr(\psi)$. Thus, if ϕ is more likely to be true than ψ is, then $\phi \leftrightarrow \psi$ is false in some model(s). Consequently, if T_1 is more epistemically virtuous than T_2, then T_1 and T_2 are not logically equivalent. Epistemic virtues do not divide logical equivalents.[14]

Any attempt to explicate ideological parsimony that does not respect the EXPRESSIVE POWER INNOCENCE CRITERION will divide logical equivalents. Suppose the perspicuous languages of two collections of ideology, I_1 and I_2, are expressively equivalent, but some proposed criterion of ideological parsimony deems I_1 more parsimonious than I_2. Now take a theory T_1 whose ideology just is I_1. By assumption, there exists a function $Tr()$ from the perspicuous languages for I_1 to that of I_2. But this means that there exists a theory, $Tr(T_1)$, whose ideology just is I_2 and is logically equivalent to T_1. So the proposed criterion divides logical equivalents, and consequently cannot be the analysis of an epistemic virtue. Although the EXPRESSIVE POWER INNOCENCE CRITERION does not aspire to analyze ideological parsimony, it does create a necessary condition for any analysis of ideological parsimony that could be the analysis of an epistemic virtue.[15]

The argument from accuracy is the primary reason to accept the EXPRESSIVE POWER INNOCENCE CRITERION. We shall now turn to consequences of accepting it and objections that might be launched against it.

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[14] A wrinkle: we may be uncertain whether two ideologies are expressively equivalent. If so, then we may still be rational in assigning different probabilities to logically equivalent theories; Bayesian superbabies may be logically omniscient, but we aren't. This is an instance of the problem of logical learning. The best framework for modeling logical learning is Garrabrant et al.'s [Ms] "Logical Induction," in which the probabilities of logical equivalents converge in the limit. This is enough for the argument. As we learn more logic, we approximate the Bayesian ideal.

[15] My thanks to Veronica Gomez for pointing out this feature of EXPRESSIVE POWER INNOCENCE CRITERION to me.

1.4 Rivals

There are several proposed criteria for ideological parsimony in the literature. For now, I am going to evaluate them as attempts to analyze an epistemic virtue (see §5.5 for an alternative interpretation). So analyzed, all of them fall prey to the argument from accuracy. As we have seen, if a proposed explication of a theoretical virtue divides logically equivalent theories, then it is not the explication of an epistemic virtue. The best way to test this is to compare minimal pairs: theories that are as similar as possible while differing with respect to the proposed virtue. I will use this minimal pair test to show that some intuitive explications of ideological parsimony that conflict with the expressive power innocence criterion fail; whatever goodness they capture, it is not epistemic.

We can use this test to eliminate rival explications of ideological parsimony that are somewhat intuitive and conflict with the expressive power innocence criterion. In particular, we can eliminate the COUNTING CRITERION, the KINDS COUNTING CRITERION, and the MERE DELETION criterion.[16] We will do this by providing logically equivalent theories that satisfy each, showing that ideological parsimony so-explicated is not an epistemic virtue. But first, we state the criteria. In doing so, it is important to recall that I am considering (and arguing against) these as guides to truth, criteria that makes theories that have them more probable than otherwise. There might be other interpretations (as non-epistemic virtues) of these same parsimony principles that are beyond the scope of my arguments:

THE COUNTING CRITERION: Some ideology I_j is more parsimonious than some other ideology I_k if I_j has fewer bits of ideology than I_k.

THE MERE DELETION CRITERION: If I_j is obtained from I_k by deleting some bit of

[16] Although it tends to be the initial heuristic used, I've yet to encounter anyone actually accepts the counting criterion; arguments against it may be found in Cowling [2013], Sider [2013], and Goodman [1951]; Sider [2013] employs the mere deletion criterion for fundamental theories, while Cowling [2013] proposes the kind counting criterion].

ideology, then I_j is more parsimonious than I_k.

THE KINDS COUNTING CRITERION: Some ideology I_j is more parsimonious than some other ideology I_k if I_j includes fewer kinds of ideology than I_k.

These are the criteria. Now for the theories. We begin with a pair that takes down both COUNTING CRITERION and the MERE DELETION CRITERION. The first is a mereological theory with parthood taken as primitive. The second is a mereological theory with both parthood and overlap taken as primitive. Since they both have a finite ideology, the first theory satisifies both the COUNTING CRITERION and the MERE DELETION CRITERION relative to the second. But since both are axiomatizations of classical mereology, they are logically equivalent when the standard interdefinition of parthood and overlap is added (or: in the class of models where the interdefinition of parthood and overlap is a theorem). This shows that both criteria cut too finely to be epistemic virtues. Note that both unrestricted fusion axioms are actually axiom schemata, which we are using to avoid adding plural quantification to the ideology.

MEREOLOGICAL THEORY ONE:[17]

i. All predicate logic tautologies — TAUT

ii. AXIOM: $\forall x Pxx$ — PART REFLEXIVITY

iii. AXIOM: $\forall x \forall y ((Pxy \wedge Pyx) \rightarrow x = y)$ — ANTISYMMETRY

iv. AXIOM: $\forall x \forall y \forall z ((Pxy \wedge Pyz) \rightarrow Pxz)$ — TRANSITIVITY

v. AXIOM: $\forall x \forall y (\neg Pxy \rightarrow \exists z \forall w (Pwz \leftrightarrow (Pwx \wedge \neg \exists t (Ptw \wedge Pty))))$ — REMAINDER

vi. AXIOM: $\exists x \varphi \rightarrow \exists z (\forall y (\varphi \rightarrow Pyz) \wedge \forall w \forall y ((\varphi \rightarrow Pyw) \rightarrow Pzw))$ — UNRESTRICTED FUSION

MEREOLOGICAL THEORY TWO:

[17] Courtesy of Varzi and Cotnoir [ms.]

i All predicate logic tautologies TAUT

ii AXIOM: $\forall x Oxx$ OVERLAP REFLEXIVITY

iii AXIOM: $\forall x Pxx$ PART REFLEXIVITY

iv AXIOM: $\forall x \forall y((Pxy \wedge Pyx) \rightarrow x = y)$ ANTISYMMETRY

v AXIOM: $\forall x \forall y \forall z((Pxy \wedge Pyz) \rightarrow Pxz)$ TRANSITIVITY

vi AXIOM: $\forall x \forall y(\neg Pxy \rightarrow \exists z \forall w(Pwz \leftrightarrow (Pwx \wedge \neg \exists t(Ptw \wedge Pty))))$ REMAINDER

vii AXIOM: $\exists x \varphi \rightarrow \exists z(\forall y(\varphi \rightarrow Pyz) \wedge \forall w \forall y((\varphi \rightarrow Pyw) \rightarrow Pzw))$ UNRESTRICTED FUSION

As we can see, THEORY ONE is an axiomatization of classical mereology using the ideology of quantificational logic with identity and a primitive parthood predicate, while THEORY TWO is an axiomatization of classical mereology using the ideology of quantificational logic with identity plus both primitive parthood and primitive overlap predicates. But since they are both axiomatizations of classical mereology, they are logically equivalent in models where P and O take their intended interpretations (that is: models where $\forall x \forall y(Pxy \leftrightarrow \forall z(Ozx \rightarrow Ozy))$ is true; adding this equivalence to PART REFLEXIVITY allows us to derive OVERLAP REFLEXIVITY). But the ideology of THEORY ONE is more parsimonious than the ideology of THEORY TWO by both the COUNTING CRITERION and the MERE DELETION criterion. Both have a finite ideology, and that of THEORY ONE may be obtained from THEORY TWO by deleting the 'Overlap' relation. But since classical mereology doesn't get any more likely when reaxiomatized with different primitives, neither COUNTING CRITERION nor MERE DELETION CRITERION are the right way to explicate ideological parsimony.

The fall of the mere deletion criteria is striking. When we think of ontological parsimony, it seems like some analogue of the mere deletion criterion (perhaps applied to ontological kinds or fundamental ontology) is extremely plausible, so if we thought that ideological parsimony operated similarly to ontological parsimony, we would have expected something along these lines to be right. Even if it wasn't the whole story, it seemed like a good place to start. But the argument from accuracy says otherwise. This tells us that we should not expect too much from analogy between ontological and ideological parsimony.

It will require a different example to show how the KIND COUNTING CRITERION fails. Here we must be a bit less precise, because we do not have a rigorous definition of an ideological kind on hand. In what follows, however, I will rely on only two claims: (i) adding a modal operator to a truth-functional propositional language adds a new kind of ideology, and (ii) 'truth-function' is a kind of ideology. Since the primary defender of the KIND COUNTING CRITERION endorses (i)[18], that leaves (ii) as the only risky commitment I must make. But (ii) strikes me as fairly low risk. Insofar as I have any intuitive grasp on the notion of an ideological kind (and thus am willing to countenance them sans definition), truth-functions form one. With (i) and (ii) in hand, we can use Godel's interpretation of intuitionistic propositional logic in the modal propositional system S4 and its converse proved by Tarski and McKinsey to generate counterexamples to the KIND COUNTING CRITERION.[19]

The language of intuitionistic propositional logic contains proposition letters and truth-functional connectives.[20] The language of (basic) modal logic adds a modal operator to its base of propositional variables and truth-functional connectives. Thus, it contains more kinds of ideology than intuitionistic propositional logic. But we can find ever so many S4-theories that are logically equivalent to one in the language of intuitionistic propositional logic. The KIND COUNTING CRITERION divides many logical equivalents.

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[18]Cowling [2013].

[19]See Godel [1933] and McKinsey and Tarski [1948] for the proofs. For those surprised by this translatability, it will be helpful to recall a few facts. First: the basic idea behind intuitionist logic is provability. The intuitionist only accepts theorems that she has a constructive proof of. Thus, in her mouth, $\neg P$ means 'there is no constructive proof of P.' This is why she rejects excluded middle: there is a third option between P being true and there being no constructive proof that P: namely, that P is both true and lacking a constructive proof. Second: there is a provability interpretation of the modal logic formalism, where $\Box P$ means 'it is provable that P' and $\diamond$ P means 'it is consistent that P.' Concatenating the provability-interpreted box with classical negation yields a meaning of 'it is not provable that P' for the expression $\neg \Box P$, which is not that far from the meaning of intuitionistic negation. This is the central insight of both the Godel and the Tarski-McKinsey constructions.

[20]It requires an infinity of truth-values to characterize intuitionistic logic truth-functionally. See Kleene [1937].

1.5 Extant Parsimony Arguments

Next we will look at some consequences of adopting EXPRESSIVE POWER INNOCENCE CRITERION for several metaphysical debates. We will examine the following cases: Ted Sider's mereological nihilism vs. its traditional rivals, modal theories with primitive actuality vs. modal theories without it, David Lewis's attempt to reduce modality to quantification over worlds, and an argument against the "moving spotlight" theory of time.

Before we start, it will be worth highlighting a companion of the EXPRESSIVE POWER INNOCENCE CRITERION that we will rely on throughout this section. As stated, the EXPRESSIVE POWER INNOCENCE CRITERION only has something to say when we are dealing with ideologies whose perspicuous languages are expressively equivalent. But when we are in a situation where the perspicuous language of one theory is expressively equivalent to a fragment of the perspicuous language of another, we can state a companion of the EXPRESSIVE POWER INNOCENCE CRITERION that does apply:

FRAGMENTARY EXPRESSIVE POWER INNOCENCE CRITERION: some ideology I_k is not more parsimonious than some ideology I_j if its perspicuous language L_{I_k} is expressively equivalent to a fragment of that ideology's persicuous language L_{I_j}.

The Argument from Accuracy also supports FRAGMENTARY EXPRESSIVE POWER INNOCENCE CRITERION. Given a theory T in L_{I_k}, we can find a logically equivalent theory in the fragment of L_{I_j} that L_{I_k} is expressively equivalent to (and so also in L_{I_j} itself) that is logically equivalent to T. So, by the same reasoning as that in §3.3, any explication of ideological parsimony that violates FRAGMENTARY EXPRESSIVE POWER INNOCENCE CRITERION will divide logical equivalents.

1.5.1 Nihilism Old and New

Very generally, mereological nihilists deny that anything is a part of anything other than itself. Traditionally, this means replacing UNRESTRICTED FUSION with an axiom that denies the existence of proper parts. But traditional nihilists do not dispute axioms like TRANSITIVITY and REFLEXIVITY. Parts may well have these properties, if there were any. Thus, a traditional nihilist mereology has a very similar axiomatization to a traditional universalist mereology, with changes only to the axioms that say which composites exist. We give one below:

i All predicate logic tautologies TAUT

ii AXIOM: $\forall x Pxx$ REFLEXIVITY

iii AXIOM: $\forall x \forall y \forall z((Pxy \wedge Pyz) \to Pxz)$ TRANSITIVITY

iv AXIOM: $\forall x \forall y(Pxy \to x = y)$ NIHILISM

Ted Sider's mereological nihilism departs from this tradition.[21] Sider argues for the wholesale elimination of mereological ideology. Thus, he does not simply claim that nothing has a proper part; he claims that 'proper part' has no place in a fundamental theory of the world. While this has a similar effect on his ontology - it lacks composites - as traditional nihilism would, it changes his mereological theory and therefore his ideology considerably. The traditional nihilist theory above has the same ideology as the universalist's; where it differs is in ontology, in the things it claims exist. Sider's mereological theory, by contrast, omits all of the distinctively mereological ideology and consequently the axioms formulated in it. We give it below:

i All predicate logic tautologies TAUT

This is intuitively an advance in ideological parsimony. Sider's ideology, lacking the ability to say anything about parts, seems simpler than that of a theory which has lots to say about what parts would be like, were there to be any.

Since Sider's ideology is only the bare logical vocabulary of predicate logic, it is expressively weaker than that of the traditional nihilist, lacking any equivalent way to express the 'parthood' predicate. So by the lights of FRAGMENTARY EXPRESSIVE

[21] Sider [2013].

POWER INNOCENCE CRITERION, the traditional nihilist's theory is not more parsimonious than Sider's. A parsimony argument for Sider's nihilism survives the test. Our criteria have nothing to say about when one ideology is more parsimonious than another. They cannot make a parsimony argument, they can only refute one. But the fact that they do not refute a parsimony argument is a point in its favor.

1.5.2 Modal Theories and Actuality

We can find a second test case in modal metaphysics. One of the longest-running disputes among modal metaphysicians has been over the status of actuality. Reductionists like David Lewis[22] and Robert Adams[23] explain actuality in terms of something else (indexicality, truth) while primitivists such as Phillip Bricker[24] take actuality as a simple, unanalyzed property of worlds. There is, *prima facie*, an argument from ideological parsimony in favor of the reductionists. The EXPRESSIVE POWER INNOCENCE CRITERION and its fragmentary cousin do not stand in its way.

For simplicity, let us imagine two modal metaphysicians who disagree only about the status of actuality. They agree on which modal logic is correct, and they agree about what's possible/necessary. Thus, we can may describe their theories as below:

MODAL THEORY ONE

i All S5 tautologies — TAUT

ii Possibility postulates — POSS

MODAL THEORY TWO

i All S5 tautologies — TAUT

ii Possibility postulates — POSS

[22] Lewis [1986]

[23] Adams [1974]

[24] Bricker [2006].

iii Hybrid Axioms HYBRID

iv Actuality Postulates ACT

The first theory includes axioms for modal logic and axioms for which things are possible (necessities may be derived from these two). I will assume these can all be described in the langauge of propositional modal logic (nothing much turns on whether the language is propositional or first order, so we can count this as a simplifying assumption). The second theory includes the same modal logic and theory of possibility, but it adds in axioms for the logical behavior of an actuality operator and a substantive theory of what's actual. The language of this theory is thus a simple hybrid language: that of propositional modal logic enriched by an actuality operator.

It seems fairly clear that the ideology of MODAL THEORY TWO is more complicated than that of MODAL THEORY ONE. Aside from additional postulates, the actuality operator requires new axioms to describe its logical behavior. And the FRAGMENTARY EXPRESSIVE POWER INNOCENCE CRITERION agrees. It is a well-known result that simple modal logic with actuality operators (known as hybrid logic) is expressively superior to simple modal logic, which is a fragment of it, in both the propositional and first-order cases.[25] Thus, the addition of an actuality operator to the ideology of MODAL THEORY ONE is not judged innocent by our criteria.

1.5.3 Modal Reduction and Quantification

In both the metaphysics of modality and the metaphysics of time, we find debates between those who prefer primitive operators such as 'possibly' and 'was' and those who prefer to reduce those operators to quantification over worlds/times. A typical telling of the situation goes something like this: reductionists purchase an improvement in ideological parsimony in the coin of ontology; primitivists have a leaner ontology but at the price of a bloated ideology.

The FRAGMENTARY EXPRESSIVE POWER INNOCENCE CRITERION calls this telling into question. Basic modal and tense logics are provably expressively weaker than

[25]see Kocurek [2016] for the first order case, Areces and ten Cate [2007] for propositional.

two-sorted predicate logics with quantification over worlds/times.[26] In fact, they are expressively equivalent to fragments of them. Furthermore, proposals to increase the expressive power of modal/tense logics to capture some of the things that can be done with quantification over worlds/times do not generally achieve full expressive equivalence, and I know of no proposal that would achieve expressive superiority. Consequently, the FRAGMENTARY EXPRESSIVE POWER INNOCENCE CRITERION says that there is no successful argument from ideological parsimony for reductionism about time/modality over primitivism. Since anything the primitivist can say in her language has a translation in the language of the reductionist, anything expressible in her ideology is also expressible in the ideology of the reductionist. While our criterion falls silent on whether the primitivist's ideology is more parsimonious, the equivalence between the primitivist's language and a fragment of the reductionist's is enough to show that it is not less parsimonious.

This places the primitivist in good position as regards parsimony over the reductionist. By most accounts of ontological parsimony, her view is more parsimonious. By any adequate account of ideological parsimony, her view is at least not less ideologically parsimonious. If theoretical economy is the main motivation for reductionism, this leaves the reductionist in an awkward position. It does not, of course, constitute anything like a conclusive argument for primitivism. But it leaves the reductionist with more work to do than is generally thought. It is also a surprising and interesting result of accepting the FRAGMENTARY EXPRESSIVE POWER INNOCENCE CRITERION and a good case of it helping us make progress in first order metaphysical disputes.

1.5.4 The Moving Spotlight

Another argument from ideological parsimony may be found in the philosophy of time, targeting "moving spotlight" theories of time.[27] This argument does not fail, but its non-failure is interesting. It will turn out that some of the moving spotlighter's

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[26] See Blackburn and von Bentham [2006], Hodkinson and Reynolds [2006].

[27] Various versions of this theory may be found in Broad [1923], Cameron [2015], and Skow [2015].

alleged excess machinery is innocent, but some of it is not.

Moving spotlight theories address two debates in the philosophy of time (among other things). The first is over ontology. Presentists think that only the present time and present things (give or take a few abstracta) exist. Their opponents, eternalists, think that all times - past, present, future - and all of the things - past, present, future - exist and are on an ontological par. As a result, presentists and eternalists account for truths about the past and future in very different ways. In order to see how, we'll start with a fairly ordinary, present-tense truth: the sun is shining. This is true just in case there is a sun, and that sun shines at the time of utterance (which for some eternalists may be the value of a covert variable in the expression's logical form, while for presentists it will be picked out by the tense indexical NOW which is implicitly part of the English verb 'is'). On that, everyone agrees. For eternalists, truths about the past and future are very much like this truth. For an eternalist, 'the sun was shining' will be true just in case, at a time earlier than ours, there exists a sun and that sun is shining. She can do this because she believes that all times - past, present, future - and all things - past, present, future - exist, just as much as we and our shining sun do now. A presentist has to say something different, since she does not think that past times and past things, or future times and future things, exist. So she introduces tense operators. A tense operator attaches itself to a tenseless sentence and commands us to evaluate its truth not at the present but in the past or the future. Thus, a sentence like 'the sun was shining' becomes PAST (sun shine), true just in case the sun was shining in the past. This allows the presentist to state facts about the past (or future) without an existing past or future. In order to rid herself of that ontology, she has adopted the ideology of tense operators.

The second debate is over change. A-theorists believe that change of some sort is a deep feature of the world, and this is ultimately to be accounted for by change from past to present to future, making the distinction between past, present, and future an important feature of the world. Their opponents, B-theorists, think that change is shallow. Change for a B-theorist is merely difference along a time-like dimension. For example, according to a B-theorist, all it is for me to change from sitting to standing is for a past version (I use 'version' here to remain neutral between stage theories and worm theories) of me to be sitting while a present one is standing. According to an A-theorist, however, there is only one version of me, which once sat and now stands.

The moving spotlighter combines an eternalist ontology with an A-theorist's approach to change, but only for a few very special properties. Like the eternalist, the moving spotlighter places all times on a par: past, present, and future. And like the B-theorist, she accounts for most change by difference along a time-like dimension. I change from sitting to standing by having a sitting version succeeded by a standing version. But she adds something: a few special properties that change in the A-theorists' sense. The tense properties. This is what gives the view its name. The moving spotlighter believes in an objective present, a primitive property of 'being present' that is true of one time and then another. This property "lights up" each time in succession, making it the case that what is past, present and future changes not simply by being arranged in an 'earlier than' relation, but by being before, at, or after a time that is objectively present. Thus, the moving spotlighter supplements the B-theorists' quantification over times with tense operators that track the movement of the spotlight (change in the facts about the objective present).

Sider has questioned whether the small amount of "genuine" or A-theory-ish change that the moving spotlighter manages to secure is worth the cost of the extra ideology - the addition of tense operators and the presentness property.[28] But the tense operators, it turns out, are free. Quantified tense logic is expressively equivalent to a fragment of two-sorted quantificational logic, and there is no known way to extend it to the entire language.[29] Consequently, the FRAGMENTARY EXPRESSIVE POWER INNOCENCE CRITERION gets them off the hook. However, the same cannot be said for the 'is present' predicate. Just as the Siderian nihilist can remove the mereological predicates and achieve an expressive decrease against the regular mereologist, the eternalist can

[28] Sider 2011

[29] Hodkinson and Reynolds [2006]

dispense with the 'is present' predicate and achieve an expressive decrease against the moving spotlighter.

1.5.5 A Fundamentality Interpretation

Before we depart our discussion of the impact of the EXPRESSIVE POWER INNOCENCE CRITERION on the literature, it is good to consider another interpretation of parsimony arguments to make clear the scope of the critique. So far, I have explored parsimony arguments as arguments for the truth of a theory. But it might be used in other ways; it may be used only as a guide to the fundamentality (or fundamental truth) of a theory. According to this version of the parsimony principle, more parsimonious theories need not be more likely to be true, but are more likely to be fundamental. Two theories may both be true, but the fact that one is more parsimonious gives us reason to think that its ideology is more fundamental. This allows rivals to the EXPRESSIVE POWER INNOCENCE CRITERION to avoid the argument from accuracy, because it is possible to have two logically equivalent theories, one of which is more fundamental than the other.[30]

I think there is something to the fundamentality interpretation, so I will not attempt to argue that it is not a route to viable parsimony arguments. But I want to be clear on its possible justification. Attempts to justify the principle of parsimony have varied from brute intuition to theological arguments about the kind of world a creator-god would make to appeals to scientific practice or the history of science.[31] I don't think it intuitive *a priori* that the world should be parsimonious, and I find the theological arguments dubious, so I think the best of these justifications is from the history of science. There seems to be a property of theories - call it parsimony or simplicity - that better theories tend to have. The catch for the fundamentality interpretation: in key motivating cases, the theories being compared are a true theory and a false one (or a theory that is closer to truth and one that is further from it), not a more fundamental

[30] In personal correspondence, Sider has indicated that this is how his use of the MERE DELETION CRITERION in his [2013] is intended.

[31] Sober [2015] Ch. 1 has a good summary.

and a less fundamental theory.

The prime example of a simpler theory triumphing over a more complex one comes from the debate between Ptolemaic geocentric theories of the solar system and Copernican heliocentric ones. When Copernicus first advanced his theories, he did not have any observation that his system accounted for but his Ptolemaic rivals could not. The two rivals were observationally equivalent. Instead, Copernicus was able to account for the observations with a simpler model, using fewer free parameters and explaining things that Ptolemy could not. This ultimately marked his theory as more correct than his rivals'. If the scientific virtue we are latching onto with our parsimony talk is one that tracks fundamentality, then the heliocentism vs. geocentrism case cannot be brought out in its justification. Ptolemaic theories were not even derivatively true (or somehow derivable from the also-false Copernican theory). The stakes in this debate were not fundamentality, but truth.

But there are some cases from the history of science that might be more favorable to the fundamentality interpretation. Sider discusses the case of Newtonian vs. Galilean spacetime. According to Newtonian theories of spacetime, space and time are absolute, and therefore there is absolute position and motion. By contrast, Galilean spacetime does not accept absolute position or motion. Everything is relative to an inertial frame. Unlike geocentric and heliocentric models of the solar system, Newtonian and Galilean models of spacetime do not contradict each other. Newtonian models add an absolute coordinate system to Galilean models. But it could be that what is fundamental is only what we find in the Galilean theory, while some convention fixes a preferred frame that gives rise to the in-some-sense-absolute spacetime of the Newtonian theory. In this case, parsimony would be a guide to fundamentality (although in fact there is no such convention).

It would require a more exhaustive examination of the history of parsimony arguments in the sciences than I have space to undertake to settle whether they tend to be guides to fundamentality rather than guides to truth. For now, I am content to acknowledge the limits of my arguments and register some skepticism that the fundamentality interpretation ultimately will be the one the history of science ends up vindicating.

1.6 Objections

Finally, we will consider two objections. The first comes from a special class of languages: those with an 'is primitive' operator.[32] The ability to include axioms in a theory that say what the theory takes as primitive poses a threat to the claims of equivalence that drive the argument from accuracy. I respond by arguing that a theory is not equivalent to a cousin theory with the same axioms but an additional one that says what the theory's primitives are. The second comes from Nelson Goodman, who considered an expressive power analysis of parsimony but rejected it as undermining the goal of seeking a parsimonious set of primitives out of which to build a theory's ideology. I respond by showing that Goodman and I have incompatible conceptions of the role of parsimony as a theoretical virtue. Goodman's is pragmatic while mine is not.

1.6.1 Higher-Order Languages With a 'Primitive' Operator

So far, we have been working with cases where the perspicuous languages are fairly well understood, with a wide body of model-theoretic results to draw on. Now we will consider theories formulated in languages that are less well-studied, but seem fairly natural for our purposes: languages where we are allowed to list our primitives in the object-language. In these languages, we introduce an explicit 'is primitive' operator $\mathfrak{p}$, which allows us to give a list of symbols which stand for the ideological primitives in a theory. A theory in a language like this can then have a 'primitives axiom,' that is: an axiom which lists the theory's primitives. Any language can be extended with a 'primitive' operator, and so any theory can be supplemented with a primitives axiom. If we are required to list our primitives in a new axiom, this poses a threat to the argument I've given for the EXPRESSIVE POWER INNOCENCE CRITERION. Languages that merely

[32]Something similar to the operator could be accomplished with an 'is primitive' predicate. What I say about the languages with the operator applies *mutatis mutandis* to those with the predicate.

allow us to list our primitives aren't a problem. But if forced to include a primitives axiom, mereological theories 1 and 2 from §4 are no longer logically equivalent,[33] and thus the argument from accuracy will no longer apply. Indeed, any two theories that differ in their primitives will no longer be logically equivalent. This puts rivals to the expressive power approach back in play.

My response takes the form of a challenge: "Sez Who?" Why must our theories specify their primitives in the object-language? It will unquestionably be true about our theories that they employ such-and-so primitives. But there are lots of things that will be true about our theories that we do not need to say in the object-language, or add special axioms that break natural equivalences to account for. To take a somewhat absurd example: every theory is stated in a certain number of characters. So there will be some truth about each theory which says how many characters are in its statement. But it would be a bit absurd to insist that our theorizing take place in a language with apparatus designed to talk about the character-count of theories, so that each theory is supplemented with a 'character-count axiom,' thereby breaking logical equivalences. What makes a 'primitives axiom' different from a 'character-count' axiom, such that we ought to include one in our theories?

I can think of two potentially promising responses to the "Sez Who" challenge. First: the Closure Response. The closure response answers the challenge with: logic. It's fairly standard to think of theories as closed under logical consequence. If, therefore, it turned out that every theory in a language that includes a $\mathfrak{p}$-operator, as a matter of logical closure, includes a primitives axiom, then it looks like the only thing stopping us from specifying our primitives in our theories is our refusal to include a $\mathfrak{p}$-operator in our language. This refusal on the part of the EXPRESSIVE POWER INNOCENCE CRITERION defender then comes to look awfully convenient, and somewhat arbitrary.

Second: the Equivalence Response. The equivalence response answers the challenge

[33] Perhaps. As Arc Kocurek has noted in personal correspondence, a genuine primitives operator only produces sentences that are true in all models or in none, and so can be rendered expressively inert by mapping sentences involving it to either the True or the False as appropriate. I'm going to spot the advocate of a primitives axiom a way out of this objection, although I don't know what that way would be, and offer a less formal response. But if my response fails, let the record show that I endorse Kocurek's response as a backstop.

with: because a theory is always equivalent to its expansion with a primitives axiom. Inquiry into the conditions under which two theories are equivalent is ongoing. But there is a fair case to be made that it is something less stringent than logical equivalence. If so, then this leaves open the possibility that, under a plausible and well-defined notion of theoretical equivalence, a theory and its expansion with a primitives axiom are equivalent. 'Because they are equivalent' seems like a good reason to replace a theory with its primitives-axiom-supplemented twin.

I respond to these with a dilemma. We can partition the space of possible logical relationships between a theory and its primitives-axiom-enhanced cousin as follow: (i) the two theories could be logically equivalent, (ii) the enhanced theory could be logically stronger, (iii) the unenhanced theory could be logically stronger, or (iv) they could be logically incomparable. We can rule out (iii) and (iv) pretty easily. In order to compare the two at all, we must use the language of the enhanced theory and its attendant consequence relation. If we were to use the language of the unenhanced theory, the enhanced theory would be gibberish. Since the enhanced theory is a superset of the unenhanced theory, it will entail the unenhanced theory according to any sane consequence relation. That leaves two options: the enhanced theory is logically stronger, or the two theories are logically equivalent.

If both theories are logically equivalent, then the objection fails. Logical equivalence is transitive. So if T_1 and T_2 are equivalent, and the enhanced versions of T_1 and T_2 are logically equivalent to T_1 and T_2 respectively, then the enhanced versions of T_1 and T_2 will be logically equivalent to each other. Thus, if the objection is to work at all, the enhanced theory must be logically stronger than the unenhanced version. This undermines the closure response. A theory must be logically equivalent to its deductive closure, because by definition it mutually entails its own deductive closure.

That leaves only the equivalence response. There are approaches to theoretical equivalence that allow for theories that are not logically equivalent to be theoretically equivalent nonetheless. We find several proposed formal criteria of theoretical equivalence in the literature.[34] This opens the door for a theory and its primitives-axiom-supplemented twin to be deemed theoretically equivalent despite not being logically equivalent.

Ideally, we would be able to test for this mathematically. But two things prevent us from doing that. First: there is no standard formal criterion for theoretical equivalence. That question remains open. Second: we do not have a good semantics for the $\mathfrak{p}$-operator. Fortunately, we can introduce some non-formal considerations that push us toward rejecting the claim of equivalence.

Which primitive concepts are the correct ones is a fact about the world. A theory can be true while taking things as primitive that should not be so taken.[35] If this is true, then it looks like a primitives axiom is a genuine addition to a theory. It says more about the world than the theory itself does. And if that is true, it tells against regarding the theories as equivalent. One says more than the other. Absent a powerful reason - such as the verdict of a well-regarded formal criterion of theoretical equivalence - to think otherwise, this should be enough to reject the equivalence response. Circumstances in which equivalent theories are not logically equivalent are very specific. We should not just suppose ourselves to be in them.

Of course, nothing I have said here *disallows* a language from having an 'is primitive' operator and theories from having a primitives axiom. But if those theories neither follow from nor are equivalent to their primitives-axiom-lacking cousins, the argument from accuracy still stands. Suppose we are interested in what's fundamental and want theories that list their primitives. Presumably equivalences among those will be much more rarer. But when they do occur, the EXPRESSIVE POWER INNOCENCE CRITERION will still apply.

[34] See Barrett and Halverson [2016] for a good summary.

[35] Sider [2011] presents a compelling case for these theses.

1.6.2 A Goodmanian Objection

Nelson Goodman considered a similar approach to simplicity in *The Structure of Appearance*.[36] He proposed and rejected something more ambitious than the EXPRESSIVE POWER INNOCENCE CRITERION: he considered an analysis of simplicity in terms of expressive power, where one theory is simpler than another if the language of its ideology is expressively weaker. However, he rejected this analysis by observing that it provided no motivation for replacing defined terms with primitives. Since, he argued, the main goal of finding a simpler theory was to find primitives that could be used to define all desired predicates, the analysis of simplicity in terms of expressive power undercut the original motivation to seek simpler theories.

Although the pragmatist Goodman wouldn't be interested in our project of explicating a distinctively epistemic virtue, it's worth seeing he provides the basis for an objection to our criteria. We can see how the objection works most clearly with an example. Consider the following two sets of primitives for a modal language.

MODAL PRIMITIVES ONE: $\Box, \Diamond, \rightarrow, \wedge, \vee, \neg$, and a countable infinity of propositional variables.

MODAL PRIMITIVES TWO: $\Box, \rightarrow, \neg$ and a countable infinity of propositional variables.

All of the primitives in MODAL PRIMITIVES ONE can be defined out of primitives in MODAL PRIMITIVES TWO. Thus, if an expressive power analysis of ideological parsimony is right, the two sets of primitives are equally parsimonious. Likewise, if the EXPRESSIVE POWER INNOCENCE CRITERION is right, the extra primitives are ideologically innocent. And what goes for these goes for anything else definable from the primitives in MODAL PRIMITIVES TWO. We could add a primitive for all 16 bivalent truth functions and still not increase expressive power. And so, Goodman says, we might as well never define anything. Replacing a defined notion with a primitive one

[36] Goodman [1951].

will never increase our expressive power.

If Goodman is right that the task of finding the simplest ideology for our theory is tantamount to finding the best base of primitives from which to define everything else, then this objection is fatal. But for reasons articulated in §3, he cannot be correct. Merely replacing primitives with defined notions that have the same truth conditions cannot make a theory more likely to be true, and so cannot be a way to increase ideological parsimony in the sense we are interested in–as an epistemic improvement. Of course, there are many contexts when reducing out number of primitives is useful. Two examples: first, when doing metatheory, it is often helpful to state our theory using the fewest kinds of lexical item (especially when things need to be proven by induction); second, sometimes we search for ways to define some terms in terms of others in order to find out which theories/languages are equivalent in the first place. But that usefulness is merely pragmatic. It's not the virtue we're looking for.

1.7 Conclusion

I have argued for a partial analysis of ideological parsimony, the EXPRESSIVE POWER INNOCENCE CRITERION. The criterion says that if the primitive ideology of one theory is expressively equivalent to that of another, then neither ideology is more parsimonious to the other. This has the consequence that additional ideology that does not increase expressive power is innocent. It can be added without taking a hit to virtuousness. I have argued for this criterion in two ways. First: I have shown that popular criteria for ideological parsimony that conflict with it divide logically equivalent theories, and therefore doing better by their lights need not make a theory more likely to be true. Second: I have given some intuitive cases where one theory does have a simpler ideology than another, and in all of those cases the simpler theory was expressively weaker. Next, I considered two objections, both of which failed. I conclude that EXPRESSIVE POWER INNOCENCE CRITERION is true, and we are one step closer to an analysis of ideological parsimony.

Chapter 2

Modal Logic for Post-Modal Metaphysics

2.1 Introduction

Modality and modal logic were key tools of the late 20th century renaissance in metaphyics. But recently, a group of metaphysicians has argued that they are not well suited to the formulation of metaphysical theses and adjudication of metaphysical arguments. Dubbed 'postmodal' by Ted Sider, they tend to level a similar general criticism: modal tools are too crude, too coarse-grained, to formulate explanatory metaphyiscal theses or to decide metaphysical disputes. I disagree.

While I do think that the postmodalist critique is apt when applied to some of the modal tools, I do not think it is correctly applied to modal logic. Throughout the paper, I will rely on an important distinction between (metaphysical) modal concepts on the one hand and modal logic on the other. Modal concepts are those like possibility, necessity, and actuality. They have a familiar fixed meaning and are (occasionally) creatures of ordinary thought and natural language. Modal logic, by contrast, is a branch of mathematical logic. It studies modal logics. A modal logic is a logic (typically defined as the deductive closure of a set of sentences) in a modal language - one that uses (at least) box and diamond operators. Examples include modal propositional logic, quantified modal logic, and tense logic. So understood, modal logic is the study of a kind of formalism, with no fixed interpretation. It is a creature of mathematics.

With this distinction in mind, I will examine the arguments against a modal toolkit and show that even if they apply to modal concepts, they do not apply to modal logic. I do not claim that their authors meant them to; the goal is simply to show their limits. In the course of examining these arguments, I will make two major claims. First: that modal logics are expressively equivalent to fragments of quantificational logic. Second: that modal logic can be used to give a logic for the grounding relation.

Defending these claims will take us on a tour through a branch of modal logic not often used in contemporary metaphysics, *modal correspondence theory*. After reviewing some of its most basic results, we will see that modal logic is a much more versatile tool than is often appreciated among metaphysicians.

As a demonstration, and to buttress my response to the post-modalists, I will show how modal logic can be used to study the grounding relation. After setting up the basic framework, I will use the modal logic of grounding to study property inheritance principles and to show a surprisingly tight parallel between grounding and provability. I end the study with some negative results.

2.2 Post-modal Metaphysics

The term 'post-modal' comes from Ted Sider. He uses it to describe the new additions to the metaphysician's toolkit that have come in the wake of the hyperintensional turn, as well as the critique of the previously-preferred modal tools that they seek to supplement or surpass.[1] The primary use of these new tools has been to study metaphysical explanation and metaphysical dependence. They include Sider's structure operator, the grounding relation, Karen Bennett's building relations, and Kit Fine's non-modal notion of essence. Very roughly, they seek to explain how less fundamental facts, concepts, or things are related to and explained by more fundamental facts, concepts, or things. I will not try to give much of an analysis of the post-modal toolkit here.[2] Instead, I will turn to the critique of the modal toolkit offered by post-modalists as justification for the new tools they introduce and use.

But before we get too deep into the critique of modal tools, we'd best get a sense of what they are. Modal tools are roughly those primarily associated with notions like possibility and necessity. These include (but are not limited to): conceptual items like

[1]Sider [Ms].

[2]The interested reader is referred to the literature, especially: Audi [2012], Bennett [2016], Fine [1994], [2001], [2012], Nolan [2014], Rosen [2010], Schaffer [2009], [2016], and Sider [2011].

consistency, entailment, intension, and supervenience; ontology such as possible worlds and possibilia; ideology such as that distinctive to strict implication and box/diamond languages; and systems of formal reasoning such as modal and counterfactual logics. After reviewing some of the arguments that these resources are not enough for the work of metaphysics, I will begin my campaign on behalf of modal logic as a valuable part of an up to date metaphysical toolkit.

Sider summarizes the general thrust of the post-modalist critique as follows: "A vague theme has been that modal concepts are too crude for many purposes, in that even after modal questions are settled, there remain important questions that can be raised only by using the post-modal tools.[3]" Here I will look into some of the specific arguments and claims that have been made by post-modalists. My objective will be to show that they don't apply to modal logic, considered as a mathematical tool, not necessarily under the its common interpretation as the logic of possibility and necessity. I make no claim that the authors intended them to; I mean only to show that they don't.

2.2.1 Modal Notions are Not Fundamental

In *Writing the Book of the World*, Ted Sider argues for a notion called 'structure.' Structural concepts are those that carve nature at its joints: they describe reality not only truly, but perspicuously. The business of metaphysics, according to Sider, is to determine which of various candidate-concepts are structural (equivalently: fundamental). The goal of metaphysics is to give an account of the structure of reality, analyzing the non-structural in terms of the structural. Combined with Sider's argument that modal notions are non-fundamental, we have the first post-modalist attack on modal tools to examine. For the purposes of this section, I will accept Sider's account of the goals of metaphysics and address his argument that modal notions are not fundamental.

Why are modal tools non-fundamental? Because they are not required in formulating our most fundamental theories. Sider writes:

[3] Sider [Ms].

> The good reason for opposing modal primitivism is simply: ideological economy. Modal talk is certainly common in ordinary and special-science discourse. But we do not generally take notions from these high-level domains as good candidates for being metaphysically basic...they are unneeded for the most fundamental inquiries of mathematics and physics...[s]ince modality is unneeded for the most fundamental inquiries, it too is metaphysically nonfundamental, however conceptually fundamental it may be.[4]

This argument is in essence an argument from ideological parsimony. A theory's ideology, as Quine taught us, is the fundamental concepts or notions needed to state the theory. There's no concensus on how to measure ideological parsimony,[5] but it is safe to assume that it involves eliminating ideology. Here Sider marks modal ideology for elimination.

I think this is a fine type of argument. But I don't think it applies to modal logic.[6] As Sider makes clear elsewhere, he does consider quantificational logic fundamental.[7] As we shall see in the §3, modal logics are in general expressively equivalent to (fragments of) quantificational logics. Increasingly sophisticated modal logics are equivalent to increasingly large fragments of quantificational logic. This, I claim, means that helping ourselves to modal languages is no more ideologically pricey than helping ourselves to quantificational languages. Both, as we shall see, describe the same mathematical structures. In fact, since we probably don't need the full expressive power of quantificational logic - I've yet to meet a physicist with a use for 34,890-adic relations - it may be that some modal logic is equivalent to the fundamental-inquiry-useful fragment of quantificational logic. If so, it may well be the quantifiers on the outside looking in.

I've made two key claims in response to Sider. First: that expressively redundant ideology is cost-free. Second: that modal operators are expressively redundant in the

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[4] Sider [2011].

[5] See Goodman [1951], Sider [2013], Cowling [2013], and removed for blind reivew for discussion.

[6] Or if it does, it is only because some non-fundamental notions are okay as metaphysical tools, and modal logic is among them.

[7] Sider [2011].

presence of quantifiers and predicates. I've defended the first claim in detail elsewhere,[8] but will recapitulate the main argument here. The defense of the second may be found in §3.

My first claim can be encapsulated in the EXPRESSIVE POWER INNOCENCE CRITERION. Before we state the criterion, a little terminology. An ideology is a collection of notions or concepts. These must be expressed in language. We will call a langauge that contains symbols that express all and only the concepts/notions in an ideology that ideology's *perspicuous language*. The *expressive power* of a language is the ideas that language can talk about. Two languages that can express all and only the same things are expressively equivalent. With these definitions, we can state the criterion:

EXPRESSIVE POWER INNOCENCE CRITERION: some ideology I_k and some other ideology I_j are equally parsimonious if their perspicuous languages L_{I_j} and L_{I_k} are expressively equivalent.

This tells us: if we have two ideologies whose perspicuous languages can state all and only the same things, then neither ideology is more parsimonious.

The primary reason to accept the EXPRESSIVE POWER INNOCENCE CRITERION is the argument from accuracy. The argument from accuracy has two key premises. The first: ideological parsimony is an epistemic virtue. Epistemic virtues are those that make theories that have them CETERIS PARIBUS more likely to be true. Equally epistemically virtuous theories are equally likely to be true. The second: likelihood to be true obeys the probability calculus. The probabiliy calculus says that if ϕ and ψ are logically equivalent, then they have the same probability. And if likelihood to be true obeys the probability calculus, it follows that logically equivalent theories are equally likely to be true. Epistemic virtues do not divide logical equivalents.

With that in mind, we can argue for the EXPRESSIVE POWER INNOCENCE CRITERION by *reductio*. Suppose we have two ideologies that are expressively equivalent yet I_1 is

[8] Rubio [Ms].

more parsimonious than I_2. Then there will be a theory whose ideology is I_1 that is logically equivalent to a theory whose ideology is I_2, but more epistemically virtuous than it (I am assuming that the two are even on all other virtues). So an epistemic virtue divides logically equivalents. Contradiction.

Thus, if modal logics are expressively equivalent to fragments of quantificational logics (we will see in §3 that they are), then they are ideologically innocent for those like Sider who have already accepted the ideology of quantificational logic.

2.2.2 Modal Notions Cannot Capture Dependence Relations

Perhaps the most common post-modalist complaint is that the modal toolkit cannot capture dependence relations. Thus, in his "On What Grounds What," Jonathan Schaffer writes:

> [S]upervenience analyses of grounding all fail. There are two evident and systematic problems with using supervenience to simulate grounding. The first is that supervenience has the wrong formal features: supervenience is reflexive, and non-asymmetric, while grounding is irreflexive and asymmetric. The second problem is that supervenience is an intensional relational while grounding is hyperintensional. For instance, there are substantive grounding questions for necessary entities (like numbers), but supervenience claims go vacuous for necessary entities...There have been other attempts to analyze grounding, including those centered around existential dependence counterfactuals...But such counterfactuals are problematically contextually variable, and the analysis goes vacuous on necessary entities.[9]

Kit Fine lodges a similar complaint:

> [T]here would appear to be something more than a modal connection in each case. For the modal connection can hold without the connection signified by 'in virtue of' or 'because'. It is necessary, for example, that if it is snowing then $2 + 2 = 4$ (simply because it is necessary that $2 + 2 = 4$), but the fact that $2 + 2 = 4$ does not obtain in virtue of the fact that it is snowing; and it is necessary that if the ball is red and round then it is red but the fact that the ball is red does not obtain in virtue of its being red and round. In addition to the modal connection, there would also appear

[9]Schaffer [2009].

> to be an explanatory or determinative connection...[10]

A number of others make similar comments.[11]

As before, I accept the argument for most of the modal toolkit. But I think modal logic is an exception. Grounding, explanation, and dependence are relations, and so like any other relation they can be studied with modal logic. Indeed, §4 is devoted to a preliminary investigation of grounding between objects (ontological dependence, perhaps) using modal logic, and will present a number of novel results, positive and negative. Even though it cannot define dependence, it can still illuminate it.

2.2.3 Modal Notions Cannot Distinguish Between Necessities and Between Impossibilities

Modal tools do not distinguish between necessities and between impossibilities. In an intensional context, any two necessary truths and any two necessary falsehoods are substitutable *salve veritate*. Daniel Nolan makes this point when it comes to counterpossible conditionals:

> The first example I wish to discuss is the example of counterpossible conditionals, particularly counterfactual counterpossible conditionals. Some seem correct, and some incorrect, and many of them are about the non-representational world. "If there was a piece of steel in the shape of a 36 sided platonic solid, it would have more sides than any piece of steel in the shape of a dodecahedron" seems true, but it is false that if there were such a 36 sided steel platonic polyhedron, it would have fewer sides than a dodecahedron.[12]

A similar point holds for impossibilities. The properties 'being a married bachelor' and 'being the largest natural number' have the same intension - the null intension - and

[10]Fine [2012]

[11]Nolan [2014], Rosen [2009], Audi [2012], Bennett [2016], more?

[12]Nolan [2014].

yet it is perfectly intelligible to think of them as different properties. A radical change to how we do mathematics may make a largest natural number thinkable, but it would not alter the marital status of bachelors. Similarly, a radical change to our concepts of human relationships that allowed married men to remain bachelors would not suddenly impose a largest natural number.

Again I concede the objection for most of the modal toolkit, but I plead an exception for modal logic. There is a purely formal sense in which necessities and impossibilities are equivalent in modal logic. Two propositions that are true at the same points in a model may be substituted *salve veritate*. But it is entirely the decision of the theorist to decide what the informal content of those propositions is. If we wish 'x is a married bachelor' and 'x is the largest natural number' to come apart in truth value, we need only assign them as the informal content of propositional variables that are not all true at the same points. This may be useful to do if we are reasoning about possibilities for our mathematical system but have no desire to change the concepts we use to structure human relationships.

In fact, we can find examples of this in the literature. Most notably, Kurt Godel's interpretation of intuitionistic propositional logic in the modal system S4, and Dan Garber's solution to the problem of old evidence. We'll review each, starting with Garber.

Garber's Solution to the Problem of Old Evidence

Bayesianism is a powerful tool for modeling scientific inference. It uses real numbers between 0 and 1 to track degrees of belief or credences in various propositions (notably including hypotheses and evidence), and imposes some evaluative norms of rationality on them. Exactly which norms are imposed varies by theorist, but the following are generally accepted:

PROBABILISM: the credence function $cr(-)$ should be a probability function.

CONDITIONALIZATION: $cr_1(-) = cr_0(-|e)$, where e is a proposition whose content is everything learned between t_0 and t_1.

Furthermore, there is a standard Bayesian account of when some proposition is evidence for another:

EVIDENCE: e is evidence for h when $cr(h|e) > cr(h)$

Before we get into the problem, a bit of technical background about probability functions.[13]

Probability functions are functions from a set of sentences (including the tautology) closed under disjunction and negation (so the standard language of propositional logic) to the unit interval [0,1] that follow the *kolmogorov axioms*:

NORMALITY: $pr(\top) = 1$

NON-NEGATIVITY: $\forall\phi\ pr(\phi) \geq 0$

ADDITIVITY: If ϕ and ψ are mutually exclusive, then $pr(\phi \vee \psi) = pr(\phi) + pr(\psi)$.[14]

In addition, we define conditional probability with the *ratio formula*.[15]

RATIO FORMULA: If $pr(\psi) > 0$ then $pr(\phi|\psi) = \dfrac{pr(\phi \,\&\, \psi)}{pr(\psi)}$.

Most notably for our purposes, probability functions treat all tautologies equivalently,

[13] The presentation here is brief and compressed. For a thorough, easygoing treatment, the reader for whom probability theory is unfamiliar is advised to consult Hacking [2001].

[14] The additivity principle I've given here is finite additivity. In contexts where we are countenancing an infinite algebra, most Bayesians wish to impose the stronger countable additivity axiom. But countable additivity is somewhat controversial and irrelevant to our discussion, so I will ignore it.

[15] The ratio formula also has its detractors, most notably Hajek [2003]. But it is fairly standard, and its use is inessential to the problem.

and require that all of them receive probability 1. Thus, the probability of any proposition, conditional on a tautology, is simply its prior probability.

Although this framework has numerous successes, it faces problems when dealing with tautologies. One of these, first raised by Clark Glymour, is known as the problem of old evidence. It is best introduced with an example. When Einstein proposed his theory of relativity, one of its first great triumphs came when the new approach to gravitation was able to predicted observations of Mercury's orbit that had bedeviled Newtonian mechanics. Any good model of the confirmation of relativity should report that observations of Mercury's orbit were evidence (and good evidence at that) for Einstein's new theory. Unfortunately for Bayesianism, it is unable to do this. The relevant observations were old news when Einstein formulated his theory. Thus, a good Bayesian agent would have already learned them, assigning them a credence of 1. As one can easily verify, when $cr(-)$ is a probability function and $cr_0(e) = 1$, then $cr_0(-|e) = cr_0(-)$. Conditionalizing on old evidence has no effect on credence.

This is because probability functions assign all tautologies probability 1. Facts of the form '$h \vdash e$' automatically receive probability 1. There is no logical learning. As a result, learning that Einstein's theory predicts Mercury's orbit, which should be a crucial piece of evidence in favor of the theory, is not even counted as evidence for it by the Bayesian model.

In response, Garber proposed to separate the logical relations recognized by the model from those we wished to study with the model.[16] As he notes, the content of the atomic sentences is assigned extra-sytematically. And so, if we wish to study the impact of learning logical relations within a Bayesian theory, we should extra-systematically assign the logical relations of interest to some atomic sentences. We can then assign them probabilities less than 1, and model the impact of learning them within the framework. My suggestion for studying necessary truths (and impossibilities) within the mathematical framework of modal logic trades on the very same distinction between logical relations internal to a model and the content assigned extra-systematically to

[16] Garber [1983]

atomic sentences. In the same way that Garber showed us to study logical learning with probability theory, we can use modal logic to study relationships between necessities and between impossibilities.

The S4-Intuitionism Equivalence

A second example, this one of someone using modal logic to study non-classical logic: Godel's S4 interpretation of intuitionistic propositional logic. Kurt Godel proved that we can interpret intuitionistic propositional logic in S4 with the box read as 'it is provable that'; McKinsey and Tarski proved the converse.[17] In the Godel intuitionistic interpretation, S4-non-equivalent formulae of the modal language are mapped to (some) classically equivalent propositions of the language of propositional logic. First, we give the Godel translation. Next, we give a simple example.

Intuitionist formula ϕ	Godel Interpretation $GI(\phi)$
Atomic P	P
$\neg\phi$	$\neg\Box GI(\phi)$
$\phi \vee \psi$	$\Box GI(\phi) \vee \Box GI(\psi)$
$\phi \wedge \psi$	$\Box GI(\phi) \wedge \Box GI(\psi)$
$\phi \rightarrow \psi$	$\Box GI(\phi) \rightarrow \Box GI(\psi)$

Putting together Godel's and McKinsey and Tarski's theorems, we get: Intuitionistic Proposional Logic $\vdash \phi$ iff S4 $\vdash GI(\phi)$. Now consider the classically equivalent formulae P and $\neg\neg P$. Their Godel Interpretations are P and $\neg\Box\neg\Box P$, which are not equivalent in S4. Modal logic is good for reasoning about more situations than those in which co-intensional (even classically equivalent) propositions are substitutable *salve veritate*.

This concludes our brief sweep through some of the more prominent arguments for a post-modal metaphysics toolkit. My project here has been defensive: granting many of the criticisms post-modalists have made, I have argued that they do not discount modal logic. In the process, I made several promises. The remainder of this essay is delivery

[17] Godel [1933], McKinsey and Tarski [1948].

on those promises, starting with a very brief introduction to modal correspondence theory and the relationship between modal and quantificational logic.

2.3 Modal Correspondence Theory: A Very Brief Introduction

Modal Correspondence Theory studies the relationship between propositional modal languages and quantificational languages. It begins with a simple observation: the same sorts of mathematical structures can be used to give the semantics for both kinds of language. These structures go by a number of names, but are most commonly called *relational structures* or *Kripke models*. Before we look at their details, it is worth laying out the modal and quantificational languages we are interested in in detail.[18]

First, we give the modal language. For our purposes, we will restrict our attention to the basic modal language, which contains only one modal operator. However, we can add as many modal operators as needed.

Item	Symbol
Propositional Variables	$A,\ B,\ C,\ D,\ldots$
Boolean Connectives	$\neg, \vee$
Scope Indicators	), (
Modal Operator	$\Diamond$

We can introduce the other Boolean connectives (conjunction, implication, etc) by their usual abbreviations. We also introduce the modal operator $\Box$ as $\neg \Diamond \neg$. The rules for well-formedness are as follows:

(i) Propositional Variables are well-formed

(ii) If ϕ is well-formed, so is $\neg\phi$

(iii) If ϕ is well-formed and ψ is well formed, so is $\phi \vee \psi$

(iv) If ϕ is well-formed, then $\Diamond\phi$ is

[18] The presentation in this section follows that of Blackburn and van Bentham fairly closely. See Blackburn et al [2006]. See also Blackburn et al [2002].

Next, the quantificational language, known also as the first-order correspondence language:[19]

Item	Symbol
One-Place Predicate for each Propositional Variable in $\mathfrak{L}^{\Diamond}$	A, B, C, D,...
Boolean Connectives	$\neg, \vee$
Scope Indicators	), (
Quantifier	$\exists$
Variables	x, y, z,...
Two-Place Predicate	R

Again, we introduce the other Boolean connectives by their usual abbreviations. The rules for well-formedness follow:

(i) φx is well-formed, where φ is an 1-place predicate and x is a variable.

(ii) $\varphi x_1 x_2$ is wellformed when φ is a 2-place predicate and x_1, x_2 are variables.

(iii) If ϕ is well-formed, so is $\neg\phi$

(iv) If ϕ is well-formed and ψ is well formed, so is $\phi \vee \psi$

(v) If ϕ is well-formed and x is a variable, $\exists x \phi$ is well-formed

The formulae from (i) and (ii) are called atomic.[20] When a variable x occurs in a formula ψ but not in any subformula of ψ of the form $\exists x \phi$, it is called free in ϕ. Otherwise, it is bound. A formula with no free variables is called closed; otherwise, it is called open.

With the two languages on the table, we can now introduce Kripke models and show how they can be used to give the semantics for each. This will lay the foundation for what is known as the standard translation, an algorithm for moving from sentences in

[19]For modal languages with more than one basic modal operator, we introduce a different two-place relation for each basic operator; note that I will only be discussing first-order correspondence. There is also a correspondence with second-order logic, which is beyond the scope of this paper.

[20]Usually atomic formulae are given with n-place predicates. We will not anything beyond a 2-place predicate in what follows, and giving them separate clauses eases the statement of our semantic clauses.

the propositional modal language to sentences in its first order correspondence language in a way that preserves truth in the model (we will see a theorem that says that a sentence from the modal language is true at a world in a model just in case its standard translation is true in the model with the 'right' assignment).

Models are triples $\langle \mathfrak{W}, \mathfrak{R}, \mathfrak{V} \rangle$. $\mathfrak{W}$ is a set of points (or worlds, or times, or objects, or....). The most common interpretation of the elements of $\mathfrak{W}$ is as possible worlds, leading Kripke's semantics for modal logic to commonly be called 'possible worlds semantics.' But because I will sometimes be interpreting them as objects, I will use the more neutral 'points.' Next, $\mathfrak{R}$ is a binary relation over $\mathfrak{W}$. It is generally called the accessibility relation. In the interpretation of modal logic where its operators stand for necessity and possibility, the accessibility relation codes facts about relative possibility. We don't want to assume that what is possible is necessarily possible (or possibly possible, or possibly necessary, or...), so we interpret $\mathfrak{R}$ as telling us which worlds are possible from which. In other interpretations of modal logic, we think of it as encoding other relations of interest, as we shall see. For example, in epistemic logic, $w\mathfrak{R}v$ holds when v is an epistemic alternative to w. Finally, $\mathfrak{V}$ is a valuation function that assigns each propositional variable a subset of $\mathfrak{W}$, which we can think of as the points at which the proposition is true.

When we use the Kripke models to give the semantics for modal logic, our basic notion of truth is world-relative. Formulae are evaluated at a world w in a model $\mathfrak{M}$. When ϕ is true at w in $\mathfrak{M}$, we will write '$\mathfrak{M}, w \vDash \phi$.

Sentence	Truth-Condition
Atomic ϕ	$\mathfrak{M}, w \vDash \phi$ iff $w \in V(\phi)$
$\neg\phi$	$\mathfrak{M}, w \vDash \neg\phi$ iff $\mathfrak{M}, w \nvDash \phi)$
$\phi \vee \psi$	$\mathfrak{M}, w \vDash \phi \vee \psi$ iff $\mathfrak{M}, w \vDash \phi$ or $\mathfrak{M}, w \vDash \psi$
$\Diamond\phi$	$\mathfrak{M}, w \vDash \Diamond\phi$ iff $\exists v\, w\mathfrak{R}v$ and $\mathfrak{M}, v \vDash \phi$

Now we give the semantics for the first-order correspondence language. Remember our goal is to use the very same Kripke models $\langle \mathfrak{W}, \mathfrak{R}, \mathfrak{V} \rangle$ that we used to give the semantics for modal logic in order to give the semantics for a quantificational logic.

The key to doing this is take the pieces of a typical model of quantificational logic and find a piece of the Kripke model that can do the same job. Ignoring constants, an interpretation for quantificational logic has a domain of discourse which contains the objects that the language is talking about and a specification that says which elements of the domain (or n-tuples of elements of the domain) fall into the extensions of the predicates. In our Kripke model, the set of worlds $\mathfrak{W}$ naturally suggests itself as a domain of quantification. It is a set of objects. In fact, there's no technical reason that the members of $\mathfrak{W}$ must be worlds. It's natural to think of them that way when we are interpreting the box and diamond of the modal language as representing possibility and necessity. But they can be other things if we wish to interpret our modal language differently. This will be important later, when we discuss the modal logic of grounding. For now, the key point is that $\mathfrak{W}$ will behave like a domain of discourse.

Next, we need something that specifies the extensions of predicates. Our first-order correspondence language has two kinds of predicate: monadic predicates, and one two-place predicate. The accessibility relation $\mathfrak{R}$ is well-suited to give the extension of a two place predicate. It is, after all, just a dyadic relation on $\mathfrak{W}$. And the valuation function $\mathfrak{V}$ is well-suited to give the extension of a bunch of monadic predicates. A valuation function just assigns worlds to sentence letters, dividing $\mathfrak{W}$ into a bunch of sets. That is exactly what we need to give the extensions of our monadic predicates.

Like in modal logic, in a quantificational logic truth is relative. But instead of being relative to a world, it is relative to an assignment. Assignments are functions g from variables to objects in the domain of quantification. For our first-order correspondence language, they work as follows. A monadic open sentence φx is true relative to a model $\mathfrak{M}$ and assignment g iff $g(x)$ is in $\mathfrak{V}(\varphi)$ according to $\mathfrak{M}$. A dyadic open sentence (and recall that since we only have one 2-place predicate in our language, all of our dyadic open sentences will involve it) $\varphi x_1 x_2$ is true relative to a model $\mathfrak{M}$ and assignment g iff $g\langle(x_1), g(x_2)\rangle$ is in $\mathfrak{R}$ according to $\mathfrak{M}$.[21] Two assignment functions g and g' are called

[21] To those familiar with the standard semantics for first order logic, this definition will look inelegant. That's because usually the valuation function assigns extensions to all predicates. But in Kripke models, the valuation function only deals with the extensions of propositional variables; accessibility relations are separate. Also, since we are ignoring constants, assignment functions only assign values to variables.

x-variants of each other just in case they assign each variable the same object except possibly x.

Sentence	Truth-Condition
Monadic Atomic φx	$\mathfrak{M}, g \vDash \varphi x$ iff $g(x) \in V(\varphi)$
Polyadic Atomic $\varphi x_1 x_w$	$\mathfrak{M}, g \vDash \varphi x_1 x_2$ iff $\langle g(x_1), g(x_2)\rangle \in \mathfrak{R}$.
$\neg\phi$	$\mathfrak{M}, g \vDash \neg\phi$ iff $\mathfrak{M}, g \nvDash \phi$
$\phi \vee \psi$	$\mathfrak{M}, g \vDash \phi \vee \psi$ iff $\mathfrak{M}, g \vDash \phi$ or $\mathfrak{M}, g \vDash \psi$
$\exists x\phi$	$\mathfrak{M}, g \vDash \exists x\phi$ iff for some $w \in W$, $\mathfrak{M}, g' \vDash \phi$ where g' is an x-variant of g and $g'(x) = w$

So far our semantics has given a notion of truth that is relative. In the case of modal logic, relative to a world. In the case of quantificational logic, relative to an assignment. We can use these definitions to say when a formula is simply true in a model. For modal logic, we say that a formula ϕ is true in a model $\mathfrak{M}$ just in case $\mathfrak{M}, w \vDash \phi$ for every world in $\mathfrak{M}$. For quantificational logic, we can say that a formula ϕ is true in a model $\mathfrak{M}$ just in case $\mathfrak{M}, g \vDash \phi$ for every assignment g on $\mathfrak{M}$.

Having shown how the same kinds of model can serve in the semantics for both modal logic and quantificational logic, we are now in a position to state the standard translation. The standard translation is a function mapping formulae of the modal language to formulae of the first-order correspondence language in such a way that one and the same triple $\langle \mathfrak{W}, \mathfrak{R}, \mathfrak{V}\rangle$ models the one iff it models the other. The formulae of the first-order correspondence language to which modal formulae are mapped all include one free variable. We will write $ST_v(\phi)$ to indicate the standard translation of ϕ with free occurrence of v.

Item	Symbol
Modal formulae ϕ	First-Order Formula $ST_x(\phi)$
P	Px
$\neg\phi$	$\neg ST_x(\phi)$
$\phi \to \psi$	$ST_x(\phi) \to ST_x(\psi)$
$\Diamond\phi$	$\exists y(Rxy \wedge ST_x(\phi)[y/x])$

The atomic and boolean clauses should be fairly straightforward, but the clause for the diamond requires some elaboration. The notation $\phi[y/x]$ means 'the formula ϕ with all free occurences of x replaced by y.' It's important that y be a *new* variable. It can't appear free in ϕ before the substitution. It's also important that the variable we replace x with in $ST_x(\phi)$ be the same variable that the quantifier at the front of the formula binds, so that when we make the substitution the quantifier binds every free variable in $ST_x(\phi)$, leaving the first x as the only free variable in the formula.

In order to illustrate the standard translation at work (especially the somewhat complex clause for the diamond), we will show the translation of the 4 axiom: $\Diamond \Diamond P \rightarrow \Diamond P$:

$$\begin{aligned} ST_x(\Diamond \Diamond P \rightarrow \Diamond P) &= ST_x(\Diamond \Diamond P) \rightarrow ST_x(\Diamond P) \\ &= \exists y(Rxy \wedge ST_y(\Diamond P)) \rightarrow \exists z(Rxz \wedge ST_z(P)) \\ &= \exists y(Rxy \wedge \exists w Ryw \wedge ST_w(P)) \rightarrow \exists z(Rxz \wedge Pz) \\ &= \exists z(Rxz \wedge \exists y(Rzy \wedge Py)) \rightarrow \exists y(Rxy \wedge Py) \end{aligned}$$

Note how, in the final formula, the only free variable is x, and that in the translation of the antecedent (where we had nested diamonds), there is a new variable for each diamond, and that we have the same number of existential quantifiers as we had diamonds in the modal formula.

We have now introduced the standard translation and given an example of how it works. But how do we know that it is a good translation? Good translations preserve information; in the ideal case, where S is a sentence, the translation of S into a new language conveys exactly the same information that S does. In our case, we can make this precise: it is provable that, given a Kripke model $\mathfrak{M}$, a formula ϕ of the modal language is true at some point w in it if and only if $ST_x(\phi)$ is true when w is assigned to x. Now we can give the theorem:

THEOREM: For any formula ϕ of the modal language, Kripke model $\mathfrak{M}$, point w in

$\mathfrak{M}$, and assignment function g such that $g(x) = w$: $\mathfrak{M}, w \vDash \phi$ iff $\mathfrak{M}, g \vDash ST_x(\phi)$.

Proof: By induction on the structure of ϕ.[22]

Because Kripke models (also known as relational structures) can serve as models for modal and for quantificational logics, we can think of these different logics as giving different perspectives on the same thing. Quantificational logics provide an external perspective. Their formulae (once an assignment has been fixed) are true or false *simpliciter*. Using quantifiers, variables, and predicates, they transparently describe relational structures as a distant observer, telling us what objects there are and what relations they stand in. The quantifiers range over the entire domain, and so every object is relevant to the truth of quantified formulae.

By contrast, modal logics provide an internal perspective. Their formulae are true only relative to specific points inside the model. With a simple syntax of only operators, propositional variables, and boolean connectives, it is not at all transparent that the information they convey is information about relational structures. And for formulae containing modal operators, only part of the structure is relevant to truth; namely, the accessible points. As Blackburn et al put it:

> The function of the modal operators is to permit the information stored at other states to be scanned —but, crucially, only the states accessible from the current point via an appropriate transition may be accessed in this way....the reader who pictures a modal formula as a little automaton standing at some state in a relational structure, and only permitted to explore the structure by making journeys to neighboring states, will have grasped one of the key intuitions of modal model theory.[23]

And, as they note, this piecemeal way of scanning the model is why modal logics have one of their best features, one that quantified logics lack: decidability.

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[22] See Blackburn *et al* [2002], exercise 2.4.1.

[23] Blackburn et al [2002].

As the translation shows, modal logics correspond to fragments of quantificational logics. They do not get us extra expressive power. But that does not impugn their usefulness. What they lack in expressive power, they make up for in other desirable properties. Most notably: they are often decidable, whereas standard quantificational logics are not. This makes them more user-friendly, and allows us to use them to obtain results that would be more difficult to obtain otherwise. But not to be forgotten: their operators behave like bounded quantifiers, giving a simple way to quantify over only objects that stand in interesting relations to each other. While this too can be done in the standard quantificational setting, the resulting formulae tend to be longer and less tractable. There is much to be said for a simple, compact syntax.

2.4 Logic of Grounding

The correspondence between modal and quantificational logic suggests that modal logic will be useful anywhere that quantificational logic is. Its simplicity and certain model-theoretic advantages such as decidability suggest that it might allow us to discover things that we would not otherwise think of. This has been done for in a number of cases already: epistemic logic, description logic, mereotopological modal logic, tense logic, and others.[24]

The key to using modal logic to study a relation that we are interested in is the accessibility relation. Changes in the formal properties of the accessibility relation change which formulae of modal logic we take as axioms. Because of this, modal logicians have introduced the idea of a *frame*. A frame $\mathfrak{F}$ is a model without its valuation function; thus, it is a set of points $\mathfrak{W}$ and an accessibility relation $\mathfrak{R}$. It will sometimes be helpful to think of a frame as a class of models which share a set of points and an accessibility relation but differ in their valuation function. We can then use the idea of truth in a model to define truth in a frame. A formula is true in a frame if it is true in every model in the frame. We will be interested in classes of frames where every frame in the class has some formal property of interest. A formula is true in a class of frames

[24]See Blackburn et al [2002], Blackburn et al [2006], Burgess [2009], Nenov and Vakarelov [2008].

if it is true in every frame in the class. Table 7 tells us how manipulating the formal properties of the accessibility relation determines which formulae are axioms. When a given formula is an axiom when we give the accessibility relation a certain property, we say that that formula is the characteristic axiom for the frames with that property.

Name	Properties of $\mathfrak{R}$	Characteristic Axiom
K	None	K: $\Box(\phi \rightarrow \psi) \rightarrow (\Box\phi \rightarrow \Box\psi)$
T	Reflexive	T: $\phi \rightarrow \Diamond\phi$
B	Symmetric	B: $\phi \rightarrow \Box \Diamond \phi$
4	Transitive	4:$\Diamond \Diamond \phi \rightarrow \Diamond\phi$
5	Euclidean	5: $\Diamond\phi \rightarrow \Box \Diamond \phi$

When a given collection of formulae are axioms when the accessibility relation has a certain property or properties, we say that collection of formulae axiomatizes the logic of frames with those properties. For example, below we see the axioms for the class of all frames (whose characteristic axiom is K):

1. AXIOM: all propositional tautologies
2. AXIOM K: $\Box(\phi \rightarrow \psi) \rightarrow (\Box\phi \rightarrow \Box\psi)$
3. INFERENCE NEC: $\phi \vdash \Box\phi$
4. INFERENCE MP: $\phi, \phi \rightarrow \psi \vdash \psi$

This collection is known as the minimal normal modal logic.

One last distinction. We have talked about the formulae that are axioms when the accessibility relation has certain formal properties (equivalently: are valid in frames whose accessibility relations have certain formal properties). We will refer to the deductive closure of these formulae as the ‘logic of’ that class of frames. They axiomatize the set of sentences that are true in every frame in the class. Sometimes, an additional relationship holds between a class of frames and a set of formulae: the formulae in the set are not only true in every frame in the class, they are true only in frames in the class. When this happens, they are said to define that class of frames, or to axiomatize its cannonical logic. Defining a class of frames is more than just being a theorem of its logic. A set of formulae could be theorems of the logic of a class of frames (by being true in every frame in the class and sufficient to derive every other formula that is) but not define it, because they are also true in some frames outside the class. This will be important later, because not all classes of frames can be defined by a set of formulae.

How does this help us with grounding? When we wish to use modal logic to study a single given relation, we can study the logic generated by the class of frames whose accessibility relation has the same formal properties as the relation we are trying to study. In our particular case, we are interested in grounding, and so we will be studying the logic generated by the class of frames whose accessibility relation has the typical formal properties of the grounding relation.

Grounding was introduced as a relation of metaphysical explanation or of ontological dependence.[25] Some authors posit it as a relation between facts (or sentences), others as one between objects, others as what might best be described as between aspects of objects.[26] Efforts to give a logic of ground due to Fine, Correia, and others typically focus on grounding as a relation between facts (or sentences).[27] To my knowledge, other than Schaffer's general treatment of explanatory relations using structural equation models[28], no attempt has been made to give the logic of ground as a relation between objects.[29] I will use a modal logic to do so. When I do, we will see surprising similarities between objectual ground and provability that have not previously been appreciated. We will also be able to use modal logic to explore structural property inheritance principles, a longstanding open question in the grounding literature. This shows that modal logic can offer insights into even the most quintessentially post-modal notions.

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[25]Fine [2001], [2012], Schaffer [2009], Rosen [2010].

[26]Schaffer [2016] gives a fuller inventory.

[27]Correia [2010], Fine 2012b

[28]Schaffer [2016]

[29]Partisans who wish to reserve the term 'ground' for a relation between facts may substitute 'ontological dependence' for 'ground' in the following without.

Grounding between objects is a relation of ontological dependence. More fundamental things ground less fundamental things, explaining their existence and property-possession. The paradigm case is the relationship between a singleton set and its element; {Socrates} exists because Socrates does, and not the other way around. This illustrates the distinctively post-modal property of ground: it is *hyperintensional*. The existence of Socrates entails the existence of {Socrates} (assuming standard set theory with urelements), and the existence of {Socrates} entails the existence of Socrates. The grounding relation is more fine-grained than entailment. To ground something is more than to entail its existence; it is to explain it.

Since grounding is a kind of explanatory relation, and we tend not to like explanatory circles, it is generally taken to be asymmetric. If o grounds o^*, then o^* does not ground o. It follows from its asymmetry that grounding is also irreflexive, since irreflexivity is the special case of asymmetry where $o = o*$. A final example illustrates the last standard property of ground: transitivity. Consider the set {{Socrates}}, the singleton set of singleton Socrates. It seems fairly natural to include Socrates in the explanation of why {{Socrates}} exists, even though the direct ground is {Socrates}. Examples like this have led metaphysicians to think that grounding is transitive; if o grounds $o*$, and $o*$ grounds $o**$, then o grounds $o**$. Of course, each of these is controversial (to varying degrees), but they're standard enough to start the discussion.

A final pair of properties of interest: well-foundedness and converse well-foundedness. A number of authors think that grounding is well-founded.[30] Defining well-foundedness formally requires second-order resources and gets a bit complicated, but the idea is fairly straightforward: well-founded relations don't have maximal chains that descend without limit.[31] In the case of grounding, then, this corresponds to the existence of a fundamental 'level' of objects: a class of things that exist, in whom all grounding chains terminate, and whose existence is not grounded in anything else. These are typically thought of as the basic particles or entities of physics, but gods, turtles, and

[30] Cameron [2008], Audi [2012], Fine [2012a] Schaffer [2016]

[31] See Dixon [2016] and Rabin and Rabern [2016] for extended discussion of well-foundedness principles as they relate to the grounding relation.

the cosmos as a whole are also candidates. Converse well-foundedness is the opposite of well-foundedness: converse well-founded relations lack infinitely ascending chains. In the case of grounding, then, this implies (but does not follow from) the existence of a top 'level' of objects: a class of things that are not themselves the grounds of anything else, and to which all grounding chains lead. The cosmos (if it is not fundamental) or a universal fusion are prime candidates. Although converse well-foundedness has seen little discussion in the literature, it's an interesting property and one that turns out to induce an unexpected logic.

Having been introduced to the grounding relation, we can now show how to use modal logic to study it. Things begin, as always, with Kripke models. When we are using them to study metaphysical modality, we interpret the points in $\mathfrak{W}$ as possible worlds at which sentences have truth values, so that $\mathfrak{V}$ says which atomic sentences are true at which worlds. This is not forced by the mathematics; we do it to match the mathematical tool to the conceptual machinery we are working with. We'll be using the same formal tools to study grounding, but we need a new way linking it up to the conceptual machinery. Grounding is a relation between objects. So we shall interpret the points in $\mathfrak{W}$ as objects, some of which ground others. Sentences are not typically true or false at objects. Instead, objects have properties. So we shall interpret the valuation function $\mathfrak{V}$ as assigning extensions to properties. And the accessibility relation, $\mathfrak{R}$, will serve as a mirror to the grounding relation.

We can think of an atomic sentence being true at a point in the model as the object the point represents having the property the sentence represents. When one point is accessible from another point, we can think of it is being grounded in that point. This allows us to interpret the box as 'for all objects grounded in this object,' so that $\Box P$ is true at a point iff just in case P is true at all points grounded in it, and to interpret $\Diamond P$ as 'for some object grounded in this one,' true at a point when P is true at some point grounded in it. In essence, we are using the same formalism we always do for modal logic, but we are interpreting it with the ideology of grounding, rather than the ideology of modality.[32]

We are assuming that the basic grounding relation is transitive and asymmetric. Thus, its weakest logic will be stronger than the usual weakest normal modal logic. Typically, as we add properties to the accessibility relation, we add axioms to the logic that define those properties. In the case of transitivity, the relevant axiom is 4: $\Box\phi \leftarrow \Box\Box\phi$. But sometimes, there is no axiom of modal logic that defines a property of interest. This is well known in the case of asymmetry and irreflexivity.

THEOREM: No formula of modal logic defines asymmetrical or irreflexive frames.

Proof This follows from Segerberg's Bulldozer Theorem, see Segerberg [1971] p. 80.

We have proven that no modal formula defines asymmetry. This means that there is no canonical set of formula for the grounding frames. But it does not mean that there is no modal logic of grounding. We can still give a set of formulae that axiomatize all modal formulae that true in transitive, asymmetric frames. But they will also be true in some frames that fall outside the class. They give its logic, but they don't define it.

Since no formula defines asymmetry, the basic modal logic of ground (MLG) is K4 (not to be mistaken for the better-known S4, which is the result of adding the T axiom to K4). K4 is the logic resulting from adding the 4 axiom to the basic system K.[33] We give its axiomatization here:

1. AXIOM: all propositional tautologies

2. AXIOM K: $\Box(\phi \to \psi) \to (\Box\phi \to \Box\psi)$

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[32] Although properties may seem like a bit of an odd candidate for the semantic value of a sentence, it's not unheard of. In his [1979], David Lewis used property self-ascriptions as the value of sentences like 'I am making a mess,' and noted that one could see coarse-grained propositions as a special kind of property. Assignments of non-standard semantic values of this kind are familiar in the literature on mereotopological modal logic. See, e.g., Nenov and Varakelov [2008], where the valuation function assigns propositional variables to sets of sets.

[33] Nota Bene: we used D alongside the grounding properties in our proof that asymmetry is undefinable. But since grounding isn't serial, D has no place in our logic. We just needed it for the reductio.

3. AXIOM 4: $\Box\phi \rightarrow \Box\Box\phi$

4. INFERENCE NEC: $\phi \vdash \Box\phi$

5. INFERENCE MP: $\phi, \phi \rightarrow \psi \vdash \psi$

By finding modal formulae which define other properties of ground and then adding them to K4, we can find more specific logics. We can also produce negative results by showing that first-order formulae defining interesting properties of ground have no modal definition. We can also find or test *inheritance principles* by seeing if their formalizations are theorems of K4, or whatever other modal logic we are using. To this we now turn.

2.4.1 Inheritance Principles

One of the more promising use of a modal logic of ground is to explore structural principles of property inheritance. The conditions under which some object passes properties on to the objects it grounds remains a longstanding open question in metaphysics. But certain formulae of MLG are naturally construed as inheritance principles: namely, those with a conditional as the main connective. Take, as a simple example, the 4 axiom, with o as the point of evaluation:

AXIOM 4: $\Box\phi \rightarrow \Box\Box\phi$

4 says: If all objects grounded in o bear ϕ, then all objects grounded in all objects grounded in o bear ϕ. When grounding is transitive, it's fairly clear why this is true.

We can use MLG both to test inheritance principles that we formalize, and to discover new inheritance principles by finding theorems of the logics. For example, the following is a theorem of K:

PRINCIPLE: $\Diamond(\phi \rightarrow \psi) \leftrightarrow (\Box\phi \rightarrow \Diamond\psi)$.

Proof: Hughes and Cresswell [1996].

We may read this as: if some grounded object is such that if it is ϕ then it is ψ, then if every grounded object is ϕ then some grounded object is ψ. It's difficult to find too many more inheritance principles in relatively weak logics like K4. The more interesting properties we give to grounding, the stronger our logic becomes and the more inheritance principles we can discover.

2.4.2 Grounding and Provability

Like asymmetry, well-foundedness and converse well-foundedness lack a defining modal formula. However, when converse well-foundedness is combined with transitivity, there is a defining formula, which (combined with the observation that transitivity and converse well-foundedness imply asymmetry) leads us to the first surprising insight from the modal logic of ground: one of its axioms is characteristic of the logic of provability in arithmetic.

It's well-known that Lob's formula defines transitive, converse well-founded frames.[34]

LOB'S FORMULA: $\Box(\Box\phi \to \phi) \to \Box\phi$

This is surprising, because Lob's formula is characteristic of provability logic. When the box is interpreted as 'it is provable that' and the diamond as 'it is consistent that,' the logic of provability is obtained by adding Lob's formula to K. Thus, on the supposition of converse well-foundedness of the grounding relation - that is, on the supposition that reality has a 'top' level, a class of objects that do not themselves ground further objects - grounding and provability have the same logic.

The exact formal parallel allows us to import results from provability to grounding. A first interesting result we can import from provability: no formula where a diamond

[34]See, e.g., Boolos [1993].

has widest scope is a theorem. Even $\Diamond\top$.[35] This makes sense when you think about the properties we have assigned the grounding relation. Since we are assuming a top level to reality, objects that do not themselves ground further objects, and are interpreting the diamond as 'there is a grounded object such that,' every model will have objects which do not ground other objects and therefore at which every formula where a diamond has widest scope is false.

With new axioms come new theorems, some of which may be interesting inheritance principles. Here, for instance, is the dual of a theorem of provability logic from Boolos[36]:

PRINCIPLE: $P \to ((P \wedge \neg \Diamond P) \vee \Diamond(P \wedge \neg \Diamond P))$

This says: if an object bears P, then either it bears P and grounds no object that bears P, or it grounds an object that both bears P and grounds no object that bears P.

The formal parallel between grounding and provability is useful for more than just importing results and finding inheritance principle. Taking a step back, we can find philosophical upshots from the existence of the parallel itself. In recent work, Jonathan Schaffer has defended the thesis that explanation has a tripartite structure.[37] Explanations proceeds from sources to results via linking principles. This is meant to hold for causation, for grounding, and for logical/mathematical explanation, which he understands as an explanatory proof. One of the arguments he advances for the unity of explanation is that all types of explanation have uniform formal features: they are transitive, irreflexive, and asymmetric.

However, the logical/mathematical case introduces a bit of a wrinkle. As Schaffer notes, it's not always clear which propositions to regard as axioms and which as derived results.[38] In part because of this, it's not at all clear that the relation 'there exists

[35] Boolos [1993].

[36] Boolos [1993].

[37] Schaffer [Ms].

[38] Schaffer [Ms] fn 6.

an explanatory proof from' between sets of formulae is asymmetric. For an example, consider two classic mathematical principles: Zorn's Lemma, and the Axiom of Choice, both of which are crucial to achieving important results, especially when working with infinite numbers:[39]

ZORN'S LEMMA: Let P be a set partially ordered by R. If every chain in P has an upper bound in P, then P has a maximal element.

AXIOM OF CHOICE: Suppose that F is a set of non-empty sets. Then there exists a function $h : F \mapsto \Cup F$ such that for every $A \in F$, $h(a) \in A$. We call h a choice function for F.

These two principles are equivalent. And arguably both the proofs that AXIOM OF CHOICE implies ZORN'S LEMMA and vice versa are explanatory. Each has a good case for being regarded as an axiom.[40] And there any number of other examples in logic and mathematics of this sort.

This threatens to undermine Schaffer's argument for a unified structure to explanation. Schaffer uses the formal parallels between various types of explanatory relation to argue for a single, unified notion of explanation that underwrites more specific versions such as grounding and causation. If we can have symmetrical explanations in the mathematical case, it becomes harder to argue that it is of a kind with causation and grounding.

The grounding-provability parallel can help deflect this worry. Even if the relation 'there exists an explanatory proof from' doesn't have the right formal features, there is still a deep formal parallel between provability in formal systems and grounding/causation. One revealed by the modal logics of the 'it provable that' and 'for all grounded objects' operators.

[39]Statement of these principles based on Goldrei [1996].

[40]For choice, it's in the name. Zorn also orignally called his principle an axiom. See Goldrei [1996]

2.4.3 Negative Results

I have argued that modal logic is a useful tool for metaphysics beyond the metaphysics of modality (or metaphysical arguments that make crucial use of possibility or necessity claims). But like any tool, its uses are circumscribed. There are some things that it can't do. I will conclude my brief study of the modal logic of ground with some negative results, showing where this tool won't be useful.

There are limits to the frame properties that can be defined in the basic modal language. We have seen a case where this is provable: the asymmetry case. In fact, because frame definability in modal logic has been extensively studied, we have necessary and sufficient conditions for when a property of accessibility relations can be defined by a formula of modal logic. This is given by the Goldblatt-Thomasson theorem, which we will state but will only elaborate on as needed.

GOLDBLATT-THOMASSON THEOREM: A first-order frame property is modally definable iff it is preserved under taking disjoint unions, generated subframes, bounded morphic images, and reflects ultrafilter extensions.

Proof: Blackburn *et al* [2002].

Disjoint unions, generated subframes, bounded morphic images, and ultrafilter extensions are all set-theoretic operations on frames. They can get a bit complicated, so we will define them only as needed to get our results.[41] However, we can think of them as tests for modal definability. If a property of accessibility relations passes all four tests, then there is a modal formula that defines it. Unfortunately, there is no fully general way to compute the formula. But if it fails even one test, we know that it is not definable. Of course, sometimes properties that are not themselves definable become definable in combination with other properties (as in the case of transitivity and converse well-foundedness). So in order to generate our negative results, we show that

[41] The interested reader may consult Blackburn and van Bentham [2006] for definition and elaboration.

properties of interest fail at least one of the four tests given in the Goldblatt-Thomasson Theorem.

The first property we will consider is Priority Monism.[42] Priority Monism is the thesis that exactly one thing - typically the cosmos as a whole - is fundamental. Everything else is grounded in that one thing. We can give this a fairly simple first-order characterization:

PRIORITY MONISM: $\exists x \forall y((y \neq x) \rightarrow Rxy)$

The class of frames satisfying this property are the priority monist frames. But PRIORITY MONISM is not closed under disjoint unions. A frame $\mathfrak{f}$ is the disjoint union of two frames $\mathfrak{F}^*$ and $\mathfrak{F}^{**}$ just in case:

1. the sets of points $\mathfrak{W}^*$ and $\mathfrak{E}^{**}$ in $\mathfrak{F}^*$ and $\mathfrak{F}^{**}$ have null intersection

2. The set of of points $\mathfrak{W}$ in $\mathfrak{F}$ is the union of $\mathfrak{W}^*$ and $\mathfrak{W}^{**}$

3. The accessibility relation $\mathfrak{R}$ in $\mathfrak{F}$ is the union of the accessibility relation $\mathfrak{R}^*$ in $\mathfrak{F}^*$ and the accessibility relation $\mathfrak{R}^{**}$ in $\mathfrak{F}^{**}$

Informally, the disjoint union of two frames is the frame that results from combining their sets of points - which we assume have no members in common - and then taking an accessibility relation where one point accesses another only if it does so in one of the two frames being joined together. We can now give the result.

THEOREM: The property PRIORITY MONISM is not closed under disjoint union.

Proof: Consider two frames $\mathfrak{F}$ and $\mathfrak{F}^*$, defined as follows. The set of points in $\mathfrak{F}$ is the even numbers, and the accessibility relation works as follows: $\mathfrak{R}xy$ iff x is less than y. Thus, 2 will access every other point, and no point will access 2. In similar fashion, the set of points in $\mathfrak{F}{\ast}$ is the odd numbers, and the accessibility relation works as

[42] Schaffer [2010], [2013] gives a full explanation and defense.

follows: $\mathfrak{R}xy$ iff x is less than y. Thus, 1 will access every other point, and no point will access 1. It is clear that both models satisfy PRIORITY MONISM. But their disjoint union $\mathfrak{F}^{**}$ will not. The points in their disjoint union will be the natural numbers, but no even numbers will access any odd numbers and no odd numbers will access any even numbers. Thus, 1 and 2 will provide counterexamples to PRIORITY MONISM.

□

It follows immediately from the Goldblatt-Thomasson Theorem that PRIORITY MONISM is not modally definable.

The opposite of priority monism is priority pluralism. As the name suggests, priority pluralism posits multiple fundamental entities. It, too, can be given a tidy first-order definition:

PRIORITY PLURALISM: $\exists x \exists y \forall z(\neg Rzx \wedge \neg Rzy \wedge x \neq y)$.

Also like priority monism, it does not have a modal definition. The proof will give us a chance to use a different operation from the Goldblatt-Thomasson theorem. We will show that PRIORITY PLURALISM is not closed under generated subframes.

Before we can define a generated subframe, we need to introduce a bit of terminology. Given a relation $\mathfrak{R}$ over a set $\mathfrak{W}$ (say, the set of points in a Kripke frame and its accessibility relation), we say that $\mathfrak{W}'$ is an $\mathfrak{R}$-closed subset of $\mathfrak{W}$ if, when v is in $\mathfrak{W}'$, so is any u such that $\mathfrak{R}vu$. Basically, an $\mathfrak{R}$-closed subset of $\mathfrak{W}$ is a subset of $\mathfrak{W}$ that includes all points accessed by any of its members. With that in place, we can now say: a frame $\mathfrak{F}$ is a generated subframe of a frame $\mathfrak{F}^*$ under the following conditions:

1. The set of points $\mathfrak{W}$ in $\mathfrak{F}$ is a proper subset of the set of points $\mathfrak{W}^*$ in $\mathfrak{F}^*$
2. The accessibility relation $\mathfrak{R}$ in $\mathfrak{F}$ is an $\mathfrak{R}$-closed subset of $\mathfrak{W}^*$

We will now show that PRIORITY PLURALISM is not closed under generated subframes.

THEOREM: the property PRIORITY PLURALISM is not closed under generated subframes.

Proof: We can use the same frames we used in the previous proof. It's clear that $\mathfrak{F}^{**}$ satisfies PRIORITY PLURALISM, with 1 and 2 both fundamental. And $\mathfrak{F}$ is a generated subframe of $\mathfrak{F}^{**}$. It satisfies condition 1, since the even numbers are a proper subset of the natural numbers. And since we constructed $\mathfrak{F}^{**}$ so that no even number accesses any odd, they make up an $\mathfrak{R}$-closed subset, satisfying condition 2.

□

It then follows from the Goldblatt-Thomasson theorem that PRIORITY PLURALISM is not modally definable.

2.5 Conclusion

I have argued that modal logic deserves a place in the toolkit of post-modal metaphysics. First, I reviewed the main arguments in favor of a post-modal toolkit, arguing that they did not exclude an ongoing place for modal logic alongside the newer tools. In the course of that argument, I defended two key claims: first, that modal logic is expressively equivalent to (fragments of) quantificational logic, a result familiar from modal correspondence theory; second: that modal logic can be used to illuminate the relation of ontological dependence. In so doing, I hope to have shown how a familiar tool can be put to fruitful new uses in the study of metaphysics.

Chapter 3

Presentist Counterpart Theory

3.1 Introduction

Presentists think that only those things which exist now exist simpliciter. Those past are no more, those future yet to be. This naturally leads to questions about what makes truths about the past or future true. Presentism stands in contrast to eternalism, which embraces the unqualified existence of all things past, present, or future. If eternalism is true, truths about the past and future are no more problematic than truths about the present. Just as it is presently true that Ginsberg is a justice of the United States Supreme Court because there exists such a person as Ginsberg and she occupies the office of Supreme Court Justice, it was true in the past that Marshall was a justice of the United States Supreme Court because at some earlier time there exists such a person as Marshall and he occupies the office of Supreme Court Justice. Likewise, in the future Ginsberg will still be a justice of the United States Supreme Court because at some time later than now there exists such a person as Ginsberg and she occupies the office of Supreme Court Justice. Unfortunately, this pleasingly symmetrical treatment of past, present, and future truths is unavailable to presentists. Presentism admits no concrete past or future times, and therefore no past or future people (Ginsberg, Marshall, or anyone else) to occupy offices at them.

This general problem for presentism is known as the grounding or truthmaker problem. Presentists have made various replies, but the one I am interested in here invokes primitive tensed facts. According to tense-primitive presentism, facts about the past and future are "grounded" in facts about what the world was and will be like. It is true that Marshall once was and Ginsberg will still be supreme court justices not because of some facts about them existing at other times and the properties they have, but because it is true now that they will/did exist and occupy their office(s).

I am going to raise two problems for the tense-primitive presentist. The first is an old one: the problem of temporary intrinsics, which emerges when one and the same thing has incompatible properties at different times, even though that thing at one time is identical to itself at the later time when it has the incompatible property. It is generally thought that presentism is a solution to the problem. But I will argue that, when presentism has solved a different problem having to do with what can be expressed in its signature tense logic, the problem re-emerges.

The second comes from having true singular propositions about entities that do not presently exist, but did or will persist across time. To use an example: even through Caesar no longer exists, it is true that Caesar crossed the Rubicon and that Caesar conquered Gaul. We can, in general, ask why it is that two different singular propositions are about the same thing. And a typical answer would be: because singular propositions have their subjects as constituents, two singular propositions are about the same thing if they have the same subject-constituent. Presentists can't say this about wholly past or wholly future entities, which they say don't exist. And there is no clear, uncontroversial alternative for the presentist to invoke to answer the question.

After raising these issues, I will argue that the tense-primitive presentist can solve both by adopting a counterpart-theoretic account of persistence through time. According to the counterpart theorist, the entities of ordinary language are instantaneous, time-bound entities (stages). It is still true that entities persist over time. But this is not because they are identical to entities that exist at other times. it is because they stand in counterpart relations to objects that exist at other times. David Lewis introduced the counterpart theory as a theory of *de re* modality in his realist system of concrete possible worlds, and Ted Sider has argued that the four-dimensionalist has an easier time with various philosophical puzzles about persistence through time when adopting it. By showing that the presentist also has tidy solutions to some difficult problems about persistence if she adopts a counterpart theory, I provide a similar argument for the presentist to adopt it.

A brief note on terminology. I will use an italicized F and P for the standard Priorean tense operators, meaning 'it was that' and 'it will that' respectively. I will use an italicized S as a generic span operator. When I mean to talk about, rather than assert, a proposition, I will place it in brackets <like this>.

3.2 Temporary Intrinsics

The problem of temporary intrinsics (or of intrinsic change) arises from the common sense observation that things change their intrinsic properties.[1] Consider the following story:

> PAINTED EGG: there was an egg that someone painted blue. They named the egg Eggl, and Eggl was blue. Then along came someone else. They pained the egg red. When they painted the egg red, they renamed the egg Huevo. Then Huevo was red.

We can collect a few facts from the COLORED EGG story. We know that Eggl is blue, Huevo is red, and Eggl is identical to Huevo. We also know that nothing is both blue and red. Combining these facts with a standard principle governing the logic of identity - if x is identical to y, then every property of x is a property of y and *vice versa* - we get the following argument, which is an instance of the problem of temporary intrinsics.

1. Huevo is identical to Eggl
2. nothing is both Red and Blue
3. Huevo is red
4. Eggl is blue
5. If x is identical to y, then any property of x is a property of y
6. Eggl is Red and Eggl is Blue (likewise Huevo)

[1] Lewis [1986] first brought the problem to light.

Presentists (and other theorists for whom merely past and future entities do not, in fact, have the properties that they did or will have) seem to have a neat way to avoid the problem. Because what is true, in an absolute and unrestricted sense, can change, they do not allow all of the premises to be true together. At first, when Eggl is Blue, it is not true that Huevo is Red. It *will* be true that Huevo is Red. But that is in the future, and things will be different then. What is true is: <Eggl is Blue> and F <Huevo is Red>. But this is no contradiction, for <Huevo is Red> does not follow from F <Huevo is Red>. Soon enough, time passes and now <Huevo is Red> is true, but <Eggl is Blue> no longer is. It is true that P <Eggl is Blue>, but <Eggl is Blue> does not follow from P <Eggl is Blue> and so what is true is now <Huevo is Red> and P <Eggl is Blue.> Since at no time are all of the premises true together, there is no contradiction.

This presentist solution is not available to eternalists or others for whom concrete future times and objects exist with the properties that they have at those times. For an eternalist, <Eggl is Blue> is not merely true at one time. It is true simpliciter. When we survey all of the things that exist (past, present, future), we find Eggl among the things that are Blue. Likewise, <Huevo is Red> is not merely true at one time. It is true simpliciter. Among all of the things that exist (past. present, future), we find Huevo among the Red things. And yet this cannot be, for none of the Red things are Blue things, yet Eggl = Huevo.

Unfortunately, this tidy solution doesn't hold up when presentism has been given the resources to solve other problems.

The presentist uses Priorean tense logic as her regimented language for talking about time. Her eternalist opponent uses a two-sorted first order quantificational logic, with quantification over both objects and times. It is well-known that quantified tense logic is expressively inferior to first order logic with quantification over times. There are things we can say by quantifying over times that we cannot say using quantified tense logic. This is even true if we restrict ourselves to models where time is linear, and allow the presentist not only the basic Priorean tense operators ('it was that,' usually synbolized P, and 'it will be that,' usually symbolized F, and their duals) but Hans

Kamp's 'since' and 'until' operators, which make their language propositionally but not first-order expressively complete.[2]

There has been some confusion in recent literature over what it is exactly the presentist cannot say. David Lewis [2004] suggests that the issue comes from sentences like KINGS, or more generally from numerical quantification:

KINGS: there have been two Kings of England named Charles.

In order to formalize sentences like KINGS some presentists have opted for what are known as *span operators*.[3] The formal semantics of span operators can get tricky, but the basic function is fairly intuitive: span operators allow us to talk about 'chunks' of past times all together. In particular, they allow us to simulate the eternalist quantification over times in the relevant span. For example, suppose we wish to talk about the events of the first world war. We could introduce a span operator, 'during the war,' and use it to simulate eternalist quantification over 1914-18. Thus, if we wish to say'There have been at least five battles at Ypres,' instead of iterating combinations of 'past' operators and quantifiers over and over, we can use our span operator 'during the war.' Span operators are a natural tool for the presentist. Like regular tense operators, Span operators block ontological commitment. Just as regular (slice) tense operators allow us to talk about past and future objects without committing to their existence, span operators let us talk about chunks of past and future time, and anything they did/will contain, without committing ourselves to any additional ontology.

However, KINGS is not actually inexpressible in a tense logic using only slice operators. As Lewis originally noted, it can be done using a combination of nested tense operators and quantifiers. But Lewis thought that giving the general case of 'there have been n many non-simultaeneous Fs,' and in particular the infinite case, would prove

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[2]Kamp's proof may be found in Kamp [1968]. The incompleteness proof may be found in Gabbay [1981], For further discussion and summary see Hodkinson and Reynolds [2006], pp. 693-6.

[3]Lewis [2004] suggests this course but raises some doubts for it, while Brogaard [2007] and Bourne [2007] embrace it.

too great a challenge. But recent work has shown that it can be done.[4]

Nevertheless, Gabbay's theorems hold. Presentism does face an expressivity gap. The kinds of things that it cannot say, it turns out, are the sorts that happen when we need to assert the existence of identical first-order structures at different times. To use an example from Hodkinson and Reynolds:

COUPLES: Everyone married on one day is divorced on a different day

Here we find a sentence that presentists can't capture. But it looks like they should want to. The addition of span operators should allow them to capture it - we simply need a span operator that includes both times in question.

Unfortunately, span operators bring the problem of temporary intrinsics back. Under the scope of a span operator, we can truthfully discuss a thing existing at multiple times, including times at which it has inconsistent intrinsic properties. This resurrects the problem. We will use 'during S' as a generic span operator, and then restate the argument using it. For instance, perhaps S is the span encompassing the year 2018, during which PAINTED EGG took place.

1* During S(Huevo is identical to Eggl)

2* During S(nothing is both Blue and Red)

3* During S(Huevo is Red)

4* During S(Eggl is Blue)

5* During S(If x is identical to y, then any property of x is a property of y)

6* During S(Eggl is Red and Eggl is Blue (likewise Huevo))

The presentist solution to the problem no longer works. Both Eggl and Huevo exist with their conflicting color properties during the span, and consequently both premises 2 and 3 are true together.

[4] Tallent and Ingram [Forthcoming] work through this kind of example in detail.

But wait. The conclusion is not obviously a contradiction. Consider another case, using our 'during the war' operator. The United States was neutral from 1914-1917, then joined the war for 1917-1918. So <during the war, the United States was neutral> is true, and <during the war, the United States was a combatant> is true. Hence: <during the war, the United States was neutral and a combatant> is also true. One is tempted to say: there is no contradiction here. Why? The United States was neutral 1914-1917, and it was a combatant 1917-1918. It was never neutral and a combatant during the war. This tempting response does not work. As David Lewis wrote, when introducing the original puzzle:

> It is not a solution to say just how commonplace and indubitable it is that we have different shapes at different times. To say that is only to insist - rightly - that it must be possible somehow.[5]

Likewise in the case of our span operator. Of course you can have different diplomatic statuses at different times during the war. The question is how. There is a domain of objects that exist during the war, and those objects have properties. We had better not say that one and the same object has incompatible properties, and we had better allow that during the war the United States was neutral and that during the war it was a combatant. And we cannot say, as presentists do in the original case, that it is because <the United States is neutral> is never true alongside <the United States is a combatant> During the war, they both are.

Whether or not 6^* is a contradiction will depend on the resolution of an unsettled issue in the semantics of span operators. Without getting into the technical details, here's why I think it's a problem. One way of thinking about the sentences of a formal language is as instructions about how to build a model. Sentences like 'Fa' tell us to ensure that our model has something in its domain of discourse than answers to 'a' and goes into the extension of the predicate F. Sentences like '$\forall x Fx \rightarrow \neg Gx$' tell us to ensure that none of the objects that go into the extension of F should go into the extension of G. We can then say that a set of sentences is consistent iff they give us

[5] Lewis [1986], 205.

instructions that can in principle all be followed together.

So the sentences in $1^* - 6^*$ are instructions on how to build a model of the world during S, the span of times represented by the span operator. 2^* tells us that the Fs and the Gs are disjoint. Then 3^* tells us about an object, x, that is among the Fs while 4^* tells us of an object, y, that is among the Gs. 1^* tells us that they are identical, and 5^* tells us that if they are identical, then they share their properties. These lead to 6^*, which tells us that this object, named by both x and y, belongs in both the F and the G extensions. But that contradicts 2^*. It doesn't seem like fact that they are under the scope of the span operator helps. Generally, a contradiction remains a contradiction when under the scope of a tense operator. So why should 6^* be any different?

The presentist solution to the original problem doesn't work for the version using span operators. Lewis offers two other solutions to the problem. I will now argue that neither is satisfactory for our new problem. This leaves the presentist in a bind: she can solve the expressivity problem with span operators, but then she needs a solution to the new temporary intrinsics problem.

Lewis did not exhaust the space of solutions to his problem, so I will not claim that what follows is an exhaustive exploration of how presentists with span operators can solve the problem. But they are suggestive of the kinds of issues that will inevitably emerge, and that should be enough to motivate exploring a solution that I know will work. First, I will explain the solutions in brief. Then I will show how they cause trouble when combined with presentist span operators.

The first: temporal parts. Just as objects are extended in space, we can think of (persisting) objects as extended in time. In fact, temporal parts give us an entire theory of persistence. How is it that I persist from yesterday to today? By having a part located at yesterday, and another located at today. The entirety of me is not located at any one point on the timeline, but covers a whole chunk of times, stretching from my birth to my death. I can then change my properties from time to time by having parts at earlier times with some properties and at later times with others. So when I move from sitting to standing, what is really going on is this: the part of me at the first time was sitting, full stop; the part of me at the second time is standing, full stop. The whole of me is neither sitting nor standing, but has parts which do both. Again, thinking about the spatial case can be helpful. My desk changes shape from cylindrical to rectangular as you go up the legs to the workspace. How? By having some parts, the legs, that are cylinders and by having another part, the workspace, that is rectangular. The whole of the desk is neither a cylinder nor a rectangle, but has parts that are either. According to temporal parts theory, I change shape over time much as my desk changes shape going up. By having different parts with different properties.

The second: time-relative properties. Instead of having the simple, monadic, intrinsic properties we think they do, objects have more complicated properties that involve times. There are two ways to implement this strategy.

The first is to make monadic properties into relations to times.[6] Instead of being bent at the time I am sitting, where 'being bent' is a simple monadic property, I bear the 'bent-at' relation to the time at which I am sitting. Likewise, instead of having the simple, monadic property of being straight at the time at which I am standing, where ' being straight' is a simple, monadic property, I bear the 'straight-at' relation to the time at which I am standing. Changing shape is then a matter of having different shape relations to different times. But now there is nothing incomatible with my properties. Even when I am sitting, I bear the 'straight-at' relation to the later time at which I am standing, and even when I am standing, I bear the 'bent-at' relation to the earlier time at which I was sitting. having the 'bent-at' relation to t requires that I don't bear any incompatible shape relation to t. But it requires nothing about my shape relations to other times.

The second implementation leaves the properties as simple, monadic properties but changes the instantiation relation.[7] We normally think of instantiation as having two components: the object, and the property that it instantiates. But we could think of it as having an extra component - a time of instantiation. Thus, instead of simply

[6] Lewis [1986] considers and rejects this implementation.

[7] van Inwagen [1990].

instantiating 'being bent' when I am sitting, It is true at all times that I exist that I instantiate-at-t being bent. Likewise, instead of instantiating 'being straight' at the time I am standing, I instantiate-at-t′ being straight at all times that I exist. To change shape on this view is to instatiate-at-t one shape property and to instantiate-at-t′ a different shape property, where t′ is later than t. This too solves the problem; instantiating-at-t being bent requires that I not instantiate-at-t any incompatible shape property, but says nothing about what I instantiate-at-t′.

Neither of these strategies is viable for the presentist with span operators, and for similar reasons: both involve quantification over times that may be innocent while under the scope of a span operator but causes a mash when combined with truths about the present. We will begin with temporal parts. Let's assume that our presentist is a full blown mereological universalist, so she believes in absolutely unrestricted composition. Thus, she would be fine with temporal parts if there were more than one time. Nevertheless, she believes that only one time exists. Consequently, there are no temporally extended objects. Enter the span operator. During the war, more than one time existed. Consequently, during the war there were temporally extended objects. But now our presentist has to say some very odd things, most notably:

ODD: There are no temporally extended objects, but during the war there were temporally extended objects.

It's not incoherent to say ODD. There are no artillery battles in eastern France, but during the war there were artillery battles in eastern France. But it is odd. It's not as if temporally extended objects, like artillery battles, ceased when the war ended. During the next year, there will be temporally extended objects. Nevertheless, no temporally extended objects exist. Again, this is not because we are in a weird moment populated by instantaneous objects. During the next year, I, who exist now, will be a temporally extended object.

The oddness is generated by the presentist's ontologically non-committal span operators. Because only present objects exist, and the present is not temporally extended, temporally extended objects can only be said to exist under the scope of existentially non-committal operators. They're strangers to the presentist ontology, and should be avoided if they can be.

Next we will discuss property-relativizing. The first way to relativize properties is to make them relations to times. The problem here for presentists is fairly straightforward: there is only one time at a time. Consequently, on the plausible assumption that relations require relata, there can only be relations to one time at a time. This makes the first account of change fall apart. Recall that on this view, to change shape is to bear different shape relations to different times. But since, on presentism, there is only ever one concrete time (we'll think about abstract times later) to bear relations to at a time, there would never be any intrinsic change of any sort.

The second way to relativize properties is to relativize instantiation. On this view, instantiation does not merely relate an object to a property. It relates an object to a property and a time. Change is then instantiating-at-different-times different properties. The problem with this way again comes down to the lack of times in the presentist ontology. There is only ever one time, and so, on the plausible assumption that relations can only relate things that exist, instantiation-at-a-time can only relate objects and properties to the present time. Once again, we lose our account of change. At the present, I cannot be both bent-at-t and straight-at-t′. At most one of t and t′ exists, and so I can instantiate-at-a-time properties at at most one of them.

Some presentists may object here that they do allow non-present times into their ontology. They merely make them abstract, or non-concrete. But they exist, and are therefore perfectly eligible as relata in the triadic instantiation relation we are considering.

Unfortunately, this does not resolve all of the problems. Amongst the varieties of change recognized by the presentist there is a very important one that times undergo: change in their A-properties, the change from present to past and from future to present.

This kind of change is poorly served by the three part instantiation relation.

To see the problem, begin with two times. A given time, t, is now present as I am writing. A different time, t′ is present when you are reading what I've written. In a normal presentist theory, what makes t present now is that it instantiates presentness (or whatever grounds presentness) *simpliciter*. What will make t′ present then is the same thing. But in the new theory, t can't instantiate presenteness *simpliciter*. Even if our old dyadic instantiation relation can be defined in terms of triadic instantiation as instantiation-at-all-times, as van Inwagen suggests, t can't instantiate it. Each time is only present at itself, and so only instantiates-at-itself presentness. Instead, t must settle for instantiating-at-itself presentness, instantiating-at-earlier-times pastness, and instantiating-at-later-times futurity. Likewise for t′. This is all it is for t and t′ to change their A-properties.

But now there is no difference between which A properties t and t′ instantiate-at-a-time when they are present, past, or future. Right now, t instantiates-at-t presentness and instantiates-at-t′ pastness, while t′ instantiates-at-t futurity and instantiates-at-t′ presentness. When you read this passage, the situation will be the same. No time will instantiate-at-anything anything different. Every time will instantiate-at-itself presentness, will instantiate-at-later-times pastness, and will instantiate-at-earlier-times futurity. And since no time instantiaties-at-all-times pastness, presentness, or futurity, there will be no instantiation *simpliciter* of any A-properties. The dynamism that an A-theory is supposed to have is gone.

Perhaps the presentist can try to preserve change in A-properties by retaining a primitive two place instantiation relation (e.g. not van Inwagen's one that is defined as instantiating-at-t for all t) alongside the 3-place one. There's a danger here of ideology bloat, but we'll ignore that for now. Keeping a primitive two-place instantiation relation will allow her to retain her old theory of change for A-properties, while analyzing change under the scope of a span operator using the three place instantiation relation. But there's still a problem. The presentist theories of change and of instantiation will now fit poorly together, similarly to how her ontology did when we considered a solution in terms of temporal parts. Under the scope of her span operators, she cannot use the two place instantiation relation for objects that change in their intrinsic properties. That's what the problem of temporary instrinsics is all about. So change under the scope of a span operator will have to be analyzed with the three-place instantiation relation. So she will be comitted to further odd sentences, such as:

ODDER: During the war, no times changed their A-properties. But now, times in the future will become present and the present will become past.

It sounds as if the nature of time itself changed on Armistice Day. But of course it didn't.

This leaves the presentist without a good response to the revived problem of temporary intrinsics. Before we venture a solution, we will see that presentism's woes don't end there. Next, we will explore a different problem for presentists arising from persistence of past or future objects.

3.3 Persistent Non-Existents

The second problem for presentism is generated by several other views in combination with presentism. The first we will call SERIOUS PRESENTISM. Like its modal analog SERIOUS ACTUALISM, SERIOUS PRESENTISM says that non-existent objects cannot bear properties or stand in relations. But while SERIOUS ACTUALISM targets only non-actual objects, SERIOUS PRESENTISM targets merely past and future objects. According to the serious presentist, only present objects bear properties or stand in relations.

Singular propositions are about things directly, not by way of descriptions that the things happen to fulfill or quantified sentences that they happen to witness. Two examples. First: <Caesar crossed the Rubicon>. Second: <John (which we shall provisionally name the first pilot to complete a successful flight in the 23rd century) will take off>. These contrast with nearby propositions, such as: <the man betrayed on

the Ides of March crossed the Rubicon>, <someone or other crossed the Rubicon at a key moment in Roman history>, <someone will complete a successful flight in the 23rd century>, and <the first pilot to complete a successful flight in the 23rd century will take off>. It's tricky to give a hard and fast definition of the difference between singular and non-singular facts, but the canonical statement of a singular fact involves a proper name as its subject, while the canonical statement of a non-singular fact does not.

It's a common view that singular propositions have their subjects as constituents. Serious presentists who accept that there are true singular propositions about merely past or future objects must dissent. I think the dissenters have a strong case,[8] and the problem I am interested in does not require singular propositions to have their subjects as constituents in order to arise, so I will grant them its falsity.

The problem I'm interested in comes about because there are true singular propositions about merely past and future objects at different times in their careers. Caesar not only crossed the Rubicon; he had earlier conquered Gaul. John will not only take off, but he will land. But what makes the propositions <Caesar crossed the Rubicon> and <Caesar conquered Gaul> propositions about the same person? What makes the propositions <John will take off> and <John will land> propositions about the same person?

The usual explanation won't do. The usual explanation of why two singular propositions are about the same object is that they contain the same object as a constituent. But presentist-friendly singular propositions do not have the objects they are about as constituents, so it is not obvious that presentist-friendly singular propositions must have a constituent in common when they are about the same object.

Perhaps we could adapt the usual explanation to use presentist-friendly entities in place of the objects that singular propositions are about. We might, with Alvin Plantinga, help ourselves to individual essences[9], so that singular propositions about

[8] Merricks [2011].

[9] Since there is now some dispute as to whether Plantingan essences capture the concept of essence, see Fine [1994], let it be known that I don't really care if Plantingan essences are proper essences or if they have been misnamed. I am more interested in the entities defined and explored in Plantinga [1974] than I am in something approaching a 'folk' concept of essence.

Caesar feature Caesar's essence as a constituent and singular propositions about John feature John's essence as a constituent.

In order for the strategy of replacing individuals as constituents of singular propositions about them with their essences to work, and so getting an explanation of why different singular propositions can be about the same individual, it must be true that every individual that ever has or will exist has an essence that always exists. Otherwise, the presentist will face all of the same problems that come from making individuals constituents of singular propositions all over again. The point of invoking essences is to invoke an entity that is (a) intimately associated with exactly one object, and (b) never passes into or out of existence.

Recall that a Plantingan essence is a property that a given object would have if it were to exist, and that no other object could possibly have. Following Robert Adams, we can recognize three kinds: thisnesses, qualitative essences, and α-relational essences.[10]

A thisness is the property of being (or of being identical to) a given individual. The thisness of Caesar is the property of being (identical to) Caesar. Whenever Caesar exists, Caesar instantiates this property, and if anything instantiates the property, it is Caesar. A qualitative essence, by contrast, is a property (or conjunction of properties) that are themselves qualitative (roughly: don't make reference to specific individuals in their canonical statement), but could only possibly by possessed by one possible thing. Finally, an α-relational essence is the property of bearing R to $o_1...o_n...$, where $o_1...o_n...$ all exist and R is a qualitative relation. For example (assuming that origins essentialism is true), if we call the particular egg and sperm that combined to produce Caesar S and E, then *being the unique person produced by the union of S and E* is an a α-relational essence of Caesar.[11]

Of these three kinds of essence, two of them can exist uninstantiated. A purely

[10] Adams and Plantinga both mention a fourth: world-indexed properties. But as Adams [1979] notes, there is no reason to believe that there are world-indexed properties that are essences of things that don't exist without there being one of the other kinds as well.

[11] note that while the relation *being the unique person produced by the union of* is a qualitative relation, the property *bearing the relation of being the unique person produced by the union of S and E* is a non-qualitative property.

qualitative essence and an α-relational essence. For example, had Caesar's parents never met, the α-relational essence of Caesar's we discussed still would have, so long as S and E still did. Plantinga is of the opinion that even a thisness could exist uninstantiated, but Adams [1981] disagrees. In this dispute I side with Adams. Those who insist on uninstantiated thisnesses can solve the problem, but at a price I deem too high.

Plantinga's main use of uninstantited essences was in giving the semantics for his quantified modal logic. Timothy Williamson has raised serious problems for Plantinga about this, although relitigating them would take us too far afield. As Williamson suggests, and Meghan Sullivan argues in more depth, Williamson's views about modality and modal logic can be smoothly transposed to be about time and tense logic. I side with Adams over Plantinga primarily for the reasons Williamson gives, transposed into the temporal case.[12]

Although essences that aren't thisnesses may exist uninstantiated, I will argue that they are not good enough to stand in for objects as constituents of singular propositions. Why? Because there is no guarantee that every object will at every time have an existing essence. And so there is no guarantee, if essences are constituents of singular propositions, that at every time all of the singular propositions that should exist do exist.

I will make this argument by constructing an example: a very simple world where there are times when some past or future objects do not have any existing essences. We begin with a homogenous iron sphere, which we shall name Julius. Next, we will consider a duplicate of Julius, this time named Marc. In our world, Julius and Marc are the only objects that exist. But they do not exist at the same time. Instead, Julius exists first, then is annihilated, and later Marc exists.

In our simple world w_1 we note three blocks of time: t_1, when Julius exists; t_2, when neither Marc nor Julius exists, and t_3, when Marc exists. Because Marc and Julius are qualitative duplicates, neither has a qualitative essence. Because Marc and Julius never

[12] See Sullivan [2012] and Williamson [2013], especially chapters 3-6.

exist at the same time as anything else, neither has an $\alpha - relational$ essence. So the only essences Marc and Julius have are thisnesses. But since thisnesses only exist when instantiated, neither Marc nor Julius has an essence that exists at everytime in their little world. Consequently, if singular propositions have the essences of the things they are about as constituents, the singular propositions about Marc and Julius only exist when Marc and Julius do. A presentist who wishes to have singular propositions exist even when the objects which they are about do not will thus find essences unacceptable as constituents of singular propositions. Consequently, the simple fix to the usual solution to the question of how different singular propositions can be about the same object is unsatisfactory.

Before we move on, we should address a few objections to the case by adding a few wrinkles. First wrinkle: what if Marc or Julius has an α-relational essence involving spacetime itself? So far, I have talked like a substantivalist about time and probably space. But we could do this with a relational spacetime, although we would have to make some modifications. Instead of talking about objects existing "before" or "after" each other, we describe the world thus: Marc exists, and Julius exists, and there is a spatiotemporal relation R between them, so that Rjm is the only true fundamental relational fact in our world. In a relational spacetime, this will not be enough to say whether Marc and Julius exist before, after, or at the same time as each other. But so long as R ensures that Marc and Julius do not overlap, it will then be consistent with all the fundamental facts at our world to say that Marc and Julius exist in different, non-overlapping blocks of time. [13] Second wrinkle: what if we can find α-relational essences for Marc and Julius by using relations to the world they inhabit, which is composed of Marc when Marc exists and Julius when Julius exists? It's a little tricky to find an appropriate relation, since any relation that makes reference to Marc or Julius won't fit the bill. But perhaps we can exploit the fact that Julius is first and Marc is second, with relations like: 'being a world and being composed by x, and never having been composed by anything other than x' to Julius, which the world never bears to

[13] Thanks to Isaac Wilhelm for discussion on this point.

Marc, and 'being a world and being composed by x only after being composed by a homogenous iron sphere that is distinct from x,' which the world bears to Marc and never to Julius.[14] In response, I will deny that there is some thing, 'the world,' over and above the iron sphere that exists and is composed by one sphere at some times and at different times the other. The sphere is all there is; sometimes the one, sometimes the other.[15] So much for the usual explanation. Presentism requires something different.

3.4 A Presentist Counterpart Theory

According to stage theory, objects are timebound (a bit more carefully: the typical referents of names are timebound. A stage theorist is genrally free to believe in temporally extended objects, but her theory of object-identity takes the referents of typical names in ordinary language to be the timebound instantaneous objects and proceeds to give a theory of persistence for them; in our case, temporally extended objects cause problems and stage theory gives us a way to render them useless so we can banish them).[16] They each exist at only one instant. Nevertheless they persist through time, but not by being numerically identical to objects that exist at later times. Instead, they persist through time by having counterparts that exist at later times. There are various ways of spelling out the counterpart relation (things like causal and psychological continuity and objective similarity are important) depending on context, but it depends on qualitative properties. This is important. A goal of David Lewis's modal counterpart theory was to give an account of identity across worlds that did not invoke non-qualitative properties or relations. Likewise, what will make a temporal counterpart theory attractive to us as a solution to the problems I've raised for presentism is its ability to give a theory of persistence through time that does not invoke non-qualitative properties.

Already we can see presentist-friendly elements in the stage theory. It has no use

[14]Adjusting these relations for the relational case is an exercise for the bored and physics-inclined reader.

[15]Thanks to Dean Zimmerman for discussion on this point.

[16]Sider [1996] and Hawley [2001] give in depth expositions.

for temporally extended objects, and it does not invoke non-qualitative properties. Likewise, the presentist's objects do not exist at more than one time, and she will have a hard time with non-qualitative properties for non-present objects. But we will need to tweak standard stage theories a bit to use it as presentists.

A typical stage theory says: object o_n which exists at t_n persists until t_m because it has as counterparts objects $o_n ... o_m$, where at least one of $o_n ... o_m$ exists at each time from t_n to t_m. Thus, while it does not invoke temporally extended objects, it does invoke multiple times and objects existing at different times. This is inconsistent with the presentist ontology. So a presentist stage theory had better find a way to do without them.

The natural presentist approach is to replace times with tense operators, and to confine discussion of existence at other times to sentences within the scope of tense operators. Thus, the presentist might say: object o_n which exists at t_n will exist in the future (did exist in the past) because it will have (has had) as counterparts future (past) objects $o_n ... o_m$. This eliminates any reference to other times, but it still is committed to past and future objects. In order to remain presentist-friendly, we'll need to talk not of past and future objects, but of the kind of objects that will (did) exist: object o_n which exists at t_n will exist in the future (did exist in the past) because there will (did) exist objects $o_n ... o_m$, and these objects will (did) fulfill the conditions of the counterpart relation to o_n.

Now we are only quantifying over non-present objects within the scope of tense operators. But we have talked about the "conditions of the counterpart relation," which bears further elaboration. David Lewis first introduced the counterpart relation as a way of reckoning sameness across (what he took to be concrete) possible worlds. As he says:

> Your counterparts resemble you closely in content and context in important respects. They resemble you more closely than do the other things in their worlds. But they are not really you. For each of them is in his own world, and only you are here in the actual world. Indeed we might say, speaking

> casually, that your counterparts are you in other worlds, that they and you are the same; but this sameness is no more a literal identity than the sameness between you today and you tomorrow...The counterpart relation is a relation of similarity...it is the resultant of similarities and dissimilarities in a multitude of respects, weighted by the importances of the various respects and by the degrees of the similarities.[17]

The basic idea is simple but very powerful. Sometimes we wish to reckon two items in a domain of quantification (what we might wish to call 'bearers of logical quantity' in order to avoid incorrectly referring to them as distinct objects) 'the same' even though they do not share all of their properties. When we do, our goals and context will determine a relation that holds only between items that are the same (in some contexts this will be an equivalence relation, but it need not be[18]; in some contexts this will be numerical identity, but it need not be). Lewis calls the genus of these relations 'counterpart.'

Modality *de re* is not the only question in which counterpart relations have been invoked. Ted Sider has already defended a counterpart theory to explain persistence over time, which he calls a stage theory. Sider's stage theory is set within a four-dimensionalist ontology, which has both temporally extended objects and their temporal parts, along with the eternalist's ontologically egalitarian times. Sider argues that even if there are temporally extended objects available, some of the puzzles about persistence over time (such as Parfit's fission cases) are better explained by making the typical referents of names and objects of ordinary quantification be stages, with persistence across times explained by a counterpart theory. My argument is similar to Sider's: the problems I have introduced are best resolved by a counterpart theory. I will now show how adopting a counterpart theory solves the two problems for presentism I raised in §2 and §3.

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[17]Lewis [1968].

[18]See Sider [2018].

3.4.1 Temporary Intrinsics Revisited

As I have argued, introducing span operators resurrects the problem of temporary intrinsics for presentists. But a counterpart theory makes it go away. The problem of temporary intrinsics depends on a version of Leibniz's Law: x=y iff (Fx iff Fy). If two items in the domain of quantification are the same thing, they have all the same properties. But counterpart theories are designed to provide a sameness relation that is not numerical identity, does not follow Leibniz's Law, and may not even be an equivalence class. Eternalist stage theorists will deny premise 1 in the temporary intrinsics problem (while accepting a replacement for 1 phrased in terms of sameness), while presentist stage theorists will deny premises 1 in the resurrected problem of temporary intrinsics (while accepting a replacement for 1 phrased in terms of sameness). A presentist stage theory solves the new problem of temporary intrinsics straightforwardly and unproblematically.

3.4.2 Persistent Non-Existents Revisted

The second problem we raised for presentism came from entities that do not presently exist, exist(ed) at more than one time, and have true singular facts about them. For simplicity, we will focus on one case: the case of <Caesar crossed the Rubicon> and <Caesar conquered Gaul>. We wish for an explanation of how the fellow who crossed the Rubicon is the same guy who conquered Gaul. We can do this with the counterpart theory.

With each singular proposition we can associate a general one that, instead of using names, uses detailed descriptions of the thing named. Thus, we can pair <Caesar crossed the Rubicon> with <A person with such-and-such description crossed the Rubicon>, where the such and such gives a complete qualitative description of Caesar at the crossing. Likewise, we can pair <Caesar conquered Gaul> with <A person of such-and-such description conquered Gaul>, where the such and such gives a complete qualitative description of Caesar during the conquest. Our counterpart theory will then tell us if the person who conquered Gaul is also the person who crossed the Rubicon. If the pair of descriptions fits the requirements for describing counterparts - which are entirely qualitative - then we can say that they describe the same person. And since each is also a description of Caesar, they each describe Caesar.

This solves the problem. The guy who conquered Gaul is the same one who crossed the Rubicon because the qualitative past tense facts about them underwrite a counterpart relation between them. This solution invokes only presentist-friendly resources, and provides a second demonstration of the rewards counterpart theory has to offer the presentist.

One final note. I have offered a story about what facts in the world explain facts like 'The Caesar who conquered Gaul is the very same person as the Caesar who crossed the Rubicon.' I have not offered a story about the semantic content of < Caesar conquered Gaul > or < Caesar crossed the Rubicon >. I have not, therefore, saddled myself with a descriptivist theory of reference, or with a theory where names are disguised descriptions. What I have said is compatible with different theories of reference. I have offered a theory of how, within a presentist-friendly ontology and ideology, using facts that the presentist already needs to take as fundamental, to ground facts that say when some singular propositions are about the same person. Even when that person does not exist, and so the usual explanation is unavailable.

3.5 Conclusion

To conclude: I have argued that presentism faces two hitherto unsolved problems. The first comes when combining its traditional solution to the problem of intrinsic change with the span operators required for it to answer the expressivity objection levelled against it. The second comes when we ask how it is to ground facts about cross-temporal reference for singular propositions about objects that do not at present exist. The solution to both, I argue, is stage theory. Stage theory can be formulated in a way that is presentist-friendly, and used to dissolve both objections. This recommends it to the presentist.

Chapter 4

Logic and the Open Future

4.1 Introduction

Seated on the deck of his trireme, a Greek admiral ponders: will the enemy emerge from port tomorrow, initiating a great battle on the sea? Or will they stay in port, ensuring a calm day.

Seated in his armchair, Aristotle once puzzled the admiral's predicament. Was there a truth value to the proposition <there will be a sea battle tomorrow>? If so, what was it? Could it change as time passes and events unfold? Aristotle's speculation on this question is a *locus classicus* for the problem of future contingents. The central questions about future contingents ask, of any proposition about the future, whether it has a truth value, and if so whether it could be true.

The problem of future contingents has inspired much innovation in formal logic. Lukasiewicz invented multivalent logic after reflecting on Aristotle's discussion of a sea battle, and Prior invented tense logic in his quest for a formal model of his views on future contingents. However, as I shall argue, present frameworks for thinking about the logic of time, in particular as frameworks for formalizing and assessing arguments about the status of future contingents and other debates about the structure of temporal relations, are inadequate. But they do provide a foundation which allows us to give a framework which is adequate, a task that will consume the bulk of this paper. By combining (a version of) Lukasiewicz's trivalent logic with modern tense logic, we can provide a framework for formalizing arguments about time and the future that is neutral on the key questions about the structure of temporal relations.

4.2 Why A Tense Logical Framework?

Formal modeling has several things to offer metaphysics. The first is precision. In some cases, it may be unclear what a view amounts to, whether two views are different, and whether a view is actually coherent. Providing formal models allows us to answer these questions. The demands of the mathematics required to construct a formal model will force as clear a statement of it as can be had, though perhaps at the cost of using technical jargon. The second is clarity. When we have formal models of two views, we can determine whether they are the same view by well-established proof techniques. Whatever formal criterion we accept for theoretical equivalence, we will be able to use it and get a determinate verdict. And if a view can be given a formal model, it passes a minimal test of coherence. This test may be insufficient to establish that a view is really or robustly possible. Possibility may require more than coherence. But coherence is a good place to start.

In the study of temporal relations, we find views endorsed by metaphysicians about which these sorts of questions arise. Lukasiewicz thought that there could be a third truth value for future contingent propositions, interpreted as 'not yet determined.' The coherence of this interpretation has been challenged. In the framework, I will show how to develop his proposal in order to meet that challenge. Prior and others have thought that the future was open, that there are no facts about what contingencies the future holds. There have been debates about whether this is a requirement for the denial of (usually causal or nomic) determinism, and whether it can be made to fit within the standard framework of temporal logic. Since the framework I am giving can be seen as a generalization of the standard framework, we can give this question a qualified affirmative answer. We can also show that having indeterministic laws and having an open future are two different things. In the formalism I introduce, we can give models for open and closed indeterministic futures, showing that debates about determinism vs. indeterminism are different from debates about an open or closed future.

The argument for both of these claims is a kind of pointing-to. The 'not yet determined' intepretation of Lukasiewicz's third truth value is coherent because, given the indicated model structures and semantics, the interpretation makes sense. There is a difference between closed and open futures in indeterministic time because we can clearly delineate different classes of models corresponding to each. The standard framework of tense logic can be generalized to accommodate an open future where future contingents have undetermined truth value because the framework exists and is coherent.

4.3 Foundational Developments: Logic

We begin with a brief of review of relevant developments in formal logics, starting with the trivalent logic of Lukasiewicz, continuing through the basic normal tense logic, and concluding with Arthur Prior's 'Ockhamist' and 'Peircean' semantics for future contingents.

4.3.1 Lukaciewicz

Lukasiewicz read Aristotle as endorsing a view whereby the proposition <there is a sea battle tomorrow> has no truth value until tomorrow arrives and the battle either happens or doesn't. He took this as inspiration to introduce a system of logic with three truth-values rather than the traditional two, which we will lay out below. He intended for his third truth value to be interpreted as 'indeterminate,' so when a Lukasiewicz-valuation assigns it to a formula, we are to interpret that formula's truth-value as 'not yet settled.'[1]

Here is a brief techincal overview of one of the trivalent logics Lukasiewicz came up with, called $\mathfrak{L}_3$[2]. It will serve as the propositional base for the tense logical framework I will develop. This particular logic has several very nice technical features. First, unlike some other trivalent logical systems (e.g. Kleene's strong system) it has tautologies like $P \supset P$. It also has a well developed metalogic; Jerzy Slupecki has proven it to be

[1] Lukasiewicz [1920].

[2] Malinoswki [2009] gives a nice overview, which I more or less follow. Rescher [1969] contains more in-depth discussions

truth-functionally complete, and has provided a sound and complete axiom system. It has the following truth tables:

$P \supset Q$		Q	Q	Q
		1	.5	0
P	1	1	.5	0
P	.5	1	1	.5
P	0	1	1	1

P	$\neg$P
1	0
.5	.5
0	1

We can define the other connectives:

1. $\bot = \neg(\phi \supset \phi)$

2. $P \vee Q = (P \supset Q) \supset Q$

3. $P \wedge Q = \neg(\neg P \vee \neg Q)$

4. $P \leftrightarrow Q = (P \supset Q) \wedge (Q \supset P)$

This yields the following truth tables for $\wedge$ and $\vee$, which retain their classical functional role as MIN and MAX:

$P \wedge Q$		Q	Q	Q
		1	.5	0
P	1	1	.5	0
P	.5	.5	.5	0
P	0	0	0	0

P ∨ Q		Q	Q	Q
		1	.5	0
P	1	1	1	1
P	.5	1	.5	.5
P	0	1	.5	0

In order to achieve truth-functional completeness, we must also add a unary connective **I**, which is true when the formula under its scope is indeterminate and false otherwise. With this addition, the variant system $\mathfrak{L}_3^s$ becomes truth-functionally complete.[3] Given the number of definable truth-functions when there are three truth values, even sticking to connectives that are binary or smaller, this is a salutary result.

P	**I**P
1	0
.5	1
0	0

With $I, \supset$, and $\neg$ as basic connectives, Slupecki has given a sound and complete axiomatization for his modified $\mathfrak{L}_3^s$:

1. $\phi \supset (\psi \supset \phi)$
2. $(\phi \supset \psi) \supset [(\psi \supset \chi) \supset (\phi \supset \chi)]$
3. $(\neg\psi \supset \neg\phi) \supset (\phi \supset \psi)$
4. $[(\phi \supset \neg\phi) \supset \phi] \supset \phi$
5. $I\phi \supset \neg I\phi$
6. $\neg I\phi \supset I\phi$

Prior [1953] regarded the Lukasiewicz system as a good first pass at giving a logic for future contingents. Like Lukasiewicz, Prior thought that a third truth value was

[3]Slupecki [1936].

required in order to adequately characterize propositions like <there will be a sea battle tomorrow>. However, Prior also pointed out one of the prime difficulties facing the Lukasiewicz logic: finding an adequate interpretation of the third truth value. He briefly rehearses what is regarded as the primary obstacle to interpreting it as ‘not yet determined.’

The argument takes a form of a dilemma about the truth-table for $\vee$. In the case where both disjuncts have value .5, if we are using the intended interpretation, we need different truth values depending on which sentences the disjuncts are.

If the disjuncts are contradictories - say, ϕ and $\neg\phi$ - we would like the value to be 1. After all, one of a contradictory pair is true, and even if we don’t know which, the disjunction is settled. Tomorrow either will, or will not, see a sea battle. But if the disjuncts are unrelated, then we want the value to be .5. <There will be a sea battle tomorrow or the day after> is wholly unsettled. Thus, we cannot interpret a trivalent truth-functional connective as ‘not yet settled,’ the argument concludes. For it will be wrong about something. I want to set this argument aside for now, but we will revisit when the rest of the framework is on the table, since it can only provide a satisfactory response to this dilemma once the full semantics have been given.

4.3.2 Prior

Prior is best known as the father of modern tense logic. Inspired by developments in modal logic, including correspondence with a young Saul Kripke,[4] Prior proposed to give a logical framework for thinking about time and tense that supplements traditional extensional logic with tense operators. He also worked out special semantics for operators meant to express his views on future contingents. But first, we will review the ‘basic’ modern tense logic, then we will look at the options Prior provided.

Prior’s goal was to give a formal system for regimenting arguments about time and tense that made the distinction between tensed and tenseless sentences very clear. In English, there is no tenseless sentence. Every verb includes tense (along with other

[4] See Ploug and Ohstrom [2012].

linguistic features). Furthermore, in English, tense is best regimented using quantifiation over times, not using operators.[5] But the study of meaning in English is one of only many projects; Prior's logics are still of interest if we wish to study the logic of temporal relations, which may not perfectly align with tense in English.

We will begin with the basics of Priorean tense logic. The following details are taken without substantial change from Burgess [2009] ch. 2, wherein the reader may find proofs of all key claims.

The syntax of Priorean tense logic is that of propositional logic, supplemented with some tense operators. In the usual base system, there are four such operators: F, P, G, and H. The operator F means 'in the future,' while the operator P means 'in the past.' The operator H means 'throughout the past' or 'hitherto,' while the the operator G means 'throughout the future' or henceforth. Using negation, each of a pair of operators (the two past-directed and the two future-directed) can be defined in terms of the other: $P \leftrightarrow \neg H \neg$; $F \leftrightarrow \neg G \neg$; $G \leftrightarrow \neg F \neg$, and; $H \leftrightarrow \neg P \neg$. In this sense, they are analogous to the $\Box$ and $\Diamond$ of standard modal logic.

Item	Symbol
Propositional Constants	A, B, C, D,...
Boolean Connectives	$\neg, \wedge$
Scope Indicators	), (
Tense Operators	F, P

The model theory for tense logic is like the Kripkean/possible worlds model theory for modal logic: models are ordered triples $\langle T, \prec_t, V \rangle$, where T is a set of times, $\prec_t$ is a temporal accessibility relation, and V is a valuation function. A frame is a class of models that share the same T and $\prec_t$ but have different Vs. Like the possible worlds of modal logic, the members of the set T are points at which sentences are true or false. Instead of truth at a world, the basic notion of truth is the notion of truth at a time. Just as in modal logic the accessibility relation tells us which worlds are possible relative to each other, the accessibility relation in tense logic tells us which times are past and future

[5] Kusumoto [2005] makes this case persuasively

of each other. The valuation function V is a map from time-sentence pairs to truth values, telling us when each sentence of the langauge is true. We can define it - giving our notion of truth at a time - recursively as follows. Since our basic notion of truth is truth at a time in a model, read 'M, t $\vDash \phi$' as 'the sentence ϕ is true at time t in model M,' and 'V$(\phi, t) = n$' as ' the value function in M assigns the sentence ϕ the truth value n at time t.' Because we will be looking at logics with more than two truth values, we will adopt the convention of designating truth values with numbers between 0 and 1, with 0 as the false, 1 as the true, and 0.5 as the third value.

Sentence	Truth-Condition
Atomic ϕ	$M, t \vDash \phi$ iff $\mathrm{V}(\phi, t) = 1$
$\neg\phi$	$M, t \vDash \neg\phi$ iff $\mathrm{V}(\phi, t) = 0$
$\phi \wedge \psi$	$M, t \vDash \phi \,\&\, \psi$ iff $\mathrm{V}(\phi, t) = \mathrm{V}(\psi, t) = 1$
$F\phi$	$M, t \vDash F\phi$ iff $\exists u\, t \prec_t u$ and $\mathrm{V}(\phi, u) = 1$
$P\phi$	$M, t \vDash P\phi$ iff $\exists u\, u \prec_t t$ and $\mathrm{V}(\phi, u) = 1$

This gives rise to a minimal normal tense logic, which can be characterized with the standard system of rules and axioms. This is the logic that you get when assume only that if t is in the past of u, then u is in the future of t. Like Kripke's modal logic K, it is the minimal normal tense logic:

Axioms

i. All propositional tautologies ϕ

ii. $G(\phi \supset \psi) \supset (G\phi \supset G\psi)$

iii. $H(\phi \supset \psi) \supset (H\phi \supset H\psi)$

iv. $\phi \supset GP\phi$

v. $\phi \supset HF\phi$

Inference Rules

vi. $\phi \vdash G\phi$ FUTURE TEMPORAL GENERALIZATION (FTG)

vii. $\phi \vdash H\phi$ PAST TEMPORAL GENERALIZATION (PTG)

viii. $\phi, \phi \supset \psi \vdash \psi$ MODUS PONENS (MP)

Although they are not needed to characterize the logic of the minimal tense structure, the some further inference rules may be derived. I want to highlight one, the rule known as DUAL. It is common to note that operators like F and G and P and H are called duals, as are the truth functions $\vee$ and $\wedge$. There is a derived inference rule in tense logic that takes advantage of these dualities. Unfortunately, an in-depth discussion of duality is beyond our scope, so we shall simply list the duals.

Formula	Dual
Atomic ϕ	ϕ
$\neg\phi$	$\neg\phi$
$\phi \wedge \psi$	$\phi \vee \psi$
$\phi \vee \psi$	$\phi \wedge \psi$
$\phi \supset \psi$	$\psi^* \supset \phi^*$, where ϕ^*, ψ^* are the duals of ϕ, ψ
$F\phi$	$G\phi$
$P\phi$	$H\phi$
$G\phi$	$F\phi$
$H\phi$	$P\phi$

Note that derived rules of inference preserve theoremhood.

Inference Rules (cont.)

ix. $\phi \vdash \phi^*$, where ϕ^* is obtained from ϕ by replacing every term in ϕ with its dual DUALITY (DUAL)

This minimal logic provides a beginning of a logical framework for studying temporal relations. But as it stands, it is inadequate. There are distinctions in the metaphysics of time that it is incapable of capturing, and views that it rules out as a matter of logic. Before we dwell on its flaws, however, we should review the space of metaphysical views that we would like a logical framework to distinguish between.

4.4 Foundational Development: Metaphysics

Debates in the philosophy of time tend to center around temporal ontology. Presentists, Eternalists, and Changing Block theorists disagree primarily over the ontology of times. Permanentists and Temporaryists disagree primarily over how time interacts with existence. Working, as we are, with a propositional tense logic means that we can remain neutral in these disputes. More precisely: we will be unable to say exactly which model-theoretic constraints differentiate these views. Since these disputes are first and foremost about ontology, they are best addressed in a first order setting.

However, other disputes in the philosophy of time center around the nature of temporal relations - ‘ past of,’ ‘future of,’ and so on. These include the Open vs. Closed past/future debates, the Branching vs. Linear debate, and the Determinist vs. Indeterminist debates. When the dust settles, we will be able to pinpoint exactly where partisans of these views disagree. Here, we give a brief informal characterization of the disputes, since the ability to model all viable combinations of these views is the sign of adequacy.

4.4.1 Open vs. Closed

Open futures are not yet settled. In picture-thought, we might represent this as a tree: although the past forms a settled trunk, starting at the present, there are many ‘branches,’ none of which has any more claim to being the future than any other. Semantically, we might express this by saying, of some non-trivial class of propositions, that neither ‘it will be that ϕ’ nor ‘it will be that $\neg\phi$’ is true. Logically, we might characterize it[6] as the denial of (1) and (2), where F is an operator for ‘in the future,’ and P is an operator for ‘in the past.’

[6] Todd and Rabern [forthcoming] endorse this; McFarlane [2003] denies it

(1) $\phi \rightarrow PF\phi$

(2) $F\phi \vee F\neg\phi$.

There is some linguistic evidence that (2) is true in English.[7] But many views in the metaphysics of time, especially the 'open future' views, will want to deny it.

None of these captures the idea perfectly, and as a result there are two distinct open future views. The *garden of forking paths* view takes its name from the story by Jorge Luis Borges [1941], which features a novel that describes many different possible continuations of its story after each decision point. According to this view, it is not settled at a time which of the many possible unfoldings of the world's history, consistent with the laws of nature, will succeed it. As a result, most sentences containing future-directed operators are neither true nor false.

In contrast, according to the *all-falsist* open future view, every proposition about the future has a classical truth value. False.[8] Of course, the all-falsist will not say that ϕ will never be true just because $F\phi$ is now false. Some of these ϕs might come true, but the future is open, and they also might not. All-falsists want an open future, but they also like bivalence and so embrace a semantics for future contingents that require it. Proponents of the forking paths view and proponents of all-falsism do not disagree about the structure of temporal relations, but about the semantics of future-directed sentences. I will not wade into this debate here; the framework I give is trivalent and aims to accommodate the forking paths view. But a small change in the semantics will instead yield the all-falsist view.

There are many different closed future views, but one in particular is worth mentioning: the 'thin red line' view. According to this view, as in open future views, the non-future directed facts up to a given time + the laws of nature or causal laws are

[7] Copley [2009], McFarlane [2003], Cariani & Santorio [2018].

[8] Todd [2016] offers a defense

consistent with many possible continuations. The world is not deterministic. There is a garden of forking paths. But unlike the open future views, this view sees a special path in the garden. One has the glow of truth - what Belnap and Green [1994] call the 'thin red line.' In effect, this is a further-fact view about which possible continuation of the timeline is the actual future.

A final note: although it is not often defended,[9] an open past is as formally coherent as an open future. We should also be able to model it. Fortunately, we can do so by giving the parts of the model that deal with the past in open past models the same structure as those that deal with the future in open future models.

4.4.2 Deterministic vs. Indeterministic

It's easy to confuse open future views with indeterministic future views. But we shouldn't. The best way of thinking about determinism comes from David Lewis.[10] In seeking to define nomic determinism, Lewis gives the following suggestion: a world is deterministic at t just in case there is a true sentence H which states the history of that world until t, and a true sentence L that states that world's laws of nature, such that H and L jointly entail some sentence describing that world's future in its entirety.[11] We can schematize this to give a general definition of determinism (of a world at a time):

GENERAL LEWISIAN DETERMINISM: A world is ϕ-ish deterministic at t just in case there exists a true sentence H which states the history of the world up to t and a true sentence ϕ such that H and ϕ jointly entail a sentence describing that world's future.

We can then get tidy statements of various determinisms by filling in ϕ. If ϕ is the laws of nature, we get nomic determinism. If ϕ is the causal laws, we get causal determinism. If ϕ is the decree of the gods, we get theological determinism, and so on.

[9] But see Hudson [2014] and Lebens and Goldschmidt [2017].

[10] Lewis [1981].

[11] If we wish to accommodate an open past as well, we can make H instead by the sentence T, which fully describes the present intrinsic state of the world. The Laplacean notion of determinism will consider this sufficient, in company with L, to deduce the world's history.

Assuming that in most discussions of determinism/indeterminism it is nomic or causal determinism that is at stake, we can see how an indeterministic world may yet lack an open future. For why should the failure of two propositions to imply the future imply that the future is not fully determinate? In fact, 'thin red line' views provide the natural example where there can be indeterminism aplenty, but no open future. The fact that this line glows red can be brute. Give H and L as anemic a set of consequences as we like, time still glides along the thin red line. There is a full suite of facts about the future, but determinism is false.

There is, however, a connection between indeterminism and an open future. There cannot be an open future in a deterministic world; we can only have multiple live options for the future if the world is not deterministic.

There is an interesting wrinkle here involving theories of the nature of the laws. Although I have used David Lewis's schematic for determinism, I am not assuming his theory of laws, where laws are the best (simplest and strongest) summary of the fundamental facts. GENERALIZED LEWISIAN DETERMINISM works just as well if laws are necessitation relations between universals, as Armstrong and others think, or something like summaries of powers and liabilities, as neo-Aristotelians prefers. But if the future is open, then either there are not fully determinate Lewisian laws (because we are missing some fundamental facts about the future), or the laws are subject to change, and whether or not the world is deterministic may change. Lewis himself held to a theory of time that excluded an open future, but someone who disagreed with him about the nature of time might still like his theory of laws. Nevertheless, I think it is important to the concept of a law of nature that it does not change. We do not, for instance, discuss theory change in science as laws changing; we discuss it in terms of alleged laws found to be false and new, better ones posited and confirmed. So I am inclined to say that if the future is open then the Lewisian laws are not fully determinate. And if they are not fully determinate, then they do not combiner with facts about the world's history to entail what facts about the future are true. Consequently, the inference from 'open' to 'indeterministic' will stand regardless of which theory of lawhood is true.

4.4.3 Branching vs. Linear

A final important debate over temporal relations comes when we ask about the formal properties of the timeline. According to the classical Newtonian view, time is linear: the ‘future of’ relation is a strict total order. However, developments in physics, in particular the Everettian interpretation of quantum mechanics, have put more options on the table. According to Everett, we solve the measurement problem by giving time a tree structure, with a ‘split’ at every observation, so that all available options obtain in some branch or other. The branching/linear debate cross-cuts the deterministic/indeterministic and the open/closed debates: there are no interesting entailments between them.

4.4.4 The Combinations We Want

So in order to have an adequate framework for tense logics, we need to be able to model all of the consistent combinations of views about temporal relations. We should be able to specify exactly which classes of models correspond to which consistent combination of views. And if a combination of views is inconsistent, we should not be able to assign it to a class of models. Thus, Table 1 tells us what an adequate framework for tense logics should be able to do.

View	Open?	Determinism?	Linear?
1	✓	X	X
2	✓	X	✓
3	X	X	X
4	X	X	✓
5	X	✓	X
6	X	✓	✓

In assessing a framework for tense logic, we require the ability to assign a class of models to each view. Failure to do so is a sign of inadequacy.

4.4.5 Inadequacy of the standard framework

We are now in a position to say why the standard framework is inadequate. In particular, we object to the duality of F 'it will be that' and G 'it will henceforth be that' and of P 'it was that' and H 'it has hitherto been that'. Recall our formal characterization of open future views, the denial of (1) and (2):

(1) $\phi \supset PF\phi$

(2) $F\phi \vee F\neg\phi$

Three observations. First, observation 1: (1) is a consequence of the following axiom (axiom v above) in past-serial frames, i.e. those where any moment is preceded by another.[12]

v. $\phi \supset HF\phi$.

Thus, (1) is a theorem of the Priorean tense logic of many important frames, since past-seriality is a very general property. Observation 2. Consider:

(5) $HG\phi \supset \phi$

In English, (5) says: if it has always been the case that it always will be the case that ϕ, then ϕ. Not only does this sound good in English, this is a desirable theorem for anything like the notion of time we are familiar with. Unfortunately, (5) is equivalent to (1); each is provable from the other by DUAL, making (6) a theorem:

(6) $(HG\phi \supset \phi) \leftrightarrow (\phi \supset PF\phi)$

Observation 3: the sentence (2) is a theorem in future-serial frames, frames where

[12] We can define past-seriality: $\forall x \exists y (y \prec_t x)$, and it ensures that $H\phi \supset P\phi$.

for every time there is at least one future time. So in the standard system, you cannot have both an open future and a timeline with no endpoint (with no time that itself has nothing in its future). But these are entirely separate issues in the metaphysics of time. They should be compatible.

Observations 1-3 are bad for the standard framework. In one case, one of our basic normal axioms implied the falsity of open future views in a wide class of models. In the next case, the falsity of open future views was provably equivalent to a desirable theorem. In the final case, a different commitment of open future views was a theorem of a very broad class of models, ones that intuitively should not exclude open futures. These show that the standard framework requires revision.

4.4.6 Ockhamist Logic

Prior recognized that more would be required than the standard framework in order to address future contingents. In his [1967], he proposed two alternative semantics. The first he called *Ockhamist*, after William of Ockham, as a theory that allowed for an indeterministic temporal relation with a closed future. The second he called *Peircean* after Charles Peirce, meant to have an open future.

The Ockhamist logic was been developed most notably in Thomasson [1970], [1984], Burgess [1980], Zanardo [1985]. and Belnap and Green [1994]. It takes as its base Branching Tree models of tense logic. Branching Tree models earn their name from the special features of their accessibility relations, which possess the following properties:

1. $\forall t \forall u \forall v((t \prec_t u \wedge u \prec_t v) \rightarrow t \prec_t v)$ TRANSITIVITY

2. $\forall t \neg(t \prec_t t)$ IRREFLEXIVITY

3. $\forall t \forall u((\exists v\, t \prec_t v \wedge u \prec_t v) \rightarrow (t \prec_t u \vee u \prec_t t \vee t = u))$ L-CONVERGENCE

4. $\forall t \forall u \exists v(v \prec_t t \wedge v \prec_t u)$ L-CONNECTEDNESS

Properties 1-3 guarantee the right basic ordering. Property 4 guarantees that the only branches on the tree are forward-facing (so that there are no roots), while Property 5 guarantees a trunk: any two elements of the set of times are connected by some (possibly zig-zag) path. We can think of these as together ensuring the unity and fixity of the past.

They then define a history formally as a maximal $<_t$-chain. Informally, a history is a possible timeline. The collection of histories contains all of the ways of arranging the times in the model into coherent stories of the world. Each history is a path through the tree, and every complete linear path through the tree is a history.

The central insight of the Ockhamist tradition is that we must use histories as well as times to evaluate tense-logical formulae. This tends to be implemented by adding a history as a parameter of evaluation. Instead of defining truth at a point (in a model), truth is defined at a point in a model according to a history. Validity is then no longer preservation of truth at all points in models, but at all points in all histories in all models. Table 4 shows the Belnap-Green semantics for Ockhamist tense logic.

Sentence	Truth-Condition
Atomic ϕ	$M, t, h \vDash \phi$ iff $t \in h$ and $\mathrm{V}(\phi, t) = 1$
$\neg\phi$	$M, t, h \vDash \neg\phi$ iff $t \notin h$ or $t \in h$ and $\mathrm{V}(\phi, t) = 0$
$\phi \wedge \psi$	$M, t, h \vDash \phi \wedge \psi$ iff $t \in h$ and $\mathrm{V}(\phi, t) = \mathrm{V}(\psi, t) = 1$
$F\phi$	$M, t, h \vDash F\phi$ iff $t \in h$ and $\exists u \in h\,(t <_t u)$ and $\mathrm{V}(\phi, u) = 1$
$P\phi$	$M, t, h \vDash P\phi$ iff $t \in h$ and $\exists u \in h\,(u <_t t)$ and $\mathrm{V}(\phi, u) = 1$

Because the Ockhamist tradition simply adds another parameter of evaluation to the formal semantics, it will not substantially change the validities. Familiar villains like (1). (2) and (6) will be theorems of the same kinds of frames that they are in standard tense logics. This is unsurprising, since their proposal no longer allows us to theorize about the future *simpliciter*, but the future-within-a history, and that will behave like a closed future.

Nevertheless, this view represents a step forward. An adequate semantics for tense logics that can model the more exotic views in the metaphysics of time will probably require something like histories in order to define useful operators.

However, the framework is still not up to the task of modeling all six views in Table 1. In particular, the definition of a history rules out branching future views. The tree in their models represents the possibilities for a history; the histories themselves are all linear. Thus, they will have theorems like (7), which is a theorem of the class of linear frames, and will therefore be true at every t, h pair.:

(7) $F\phi \wedge F\psi \supset (F(\phi \wedge F\psi) \vee F(\psi \wedge F\phi) \vee F(\phi \wedge \psi))$

Thus, while the Ockhamist tradition improves on the standard account, it is not yet good enough.

The Peircean logic is a fragment of the Ockhamist logic.[13] However, it does not lose its linear character; the assumption that history does not contain real branches remains, so it will also have (7) as a theorem.[14]

4.5 An Improved Framework for Tense Logic

Now that we've put in place the metaphysical distinctions we need to model and the logical traditions that we will build upon, we now offer a new framework, combining the modified system of trivalent logic from Lukasiewicz with insights from Prior's Ockhamist tradition.

Before we proceed, it will be good to call to mind some helpful terminology. The symbol $\in$ denotes set-membership; we will often encounter it in symbols like $x \in X$ where by convention a lowercase variable ranges over the elements of a set, and the uppercase variable denotes the set itself. When we are dealing with a function *f*, when we see $f(\phi)$, where ϕ is a variable or constant, that means 'the output of f when ϕ is the input.' And when we need to denote the cardinality of a set X, we will write $|X|$, following standard set-theoretic terminology.

In a similar vein, when we are dealing with orderings, like our accessibility relation, it is often useful to employ the concept of a chain. Given a relation $<$, a $<$-chain is a set of things that are totally ordered by $<$. So for any two things, either the first $<$ the

[13] Reynolds [2002].

[14] Zanardo [1990] provides axioms for the Peircean logic from which this may be proven.

second or the second $\prec$ the first. This means that any chain is linear; it has no branches, because by definition two things on different branches of a tree are not related by $\prec$, but both stand in it to a third thing. We say a chain is maximal just in case it cannot be extended. All of its elements stand in the $\prec$ relation to each other, and adding another element to it will mean adding something that does not stand in the $\prec$ relation to at least one of the things already in the chain.

Finally, we will often find bounded quantifiers useful. A bounded quantifier is a quantifier explicitly restricted to some set of objects, and often appears in set-theoretic notation in symbols like '$\forall x \in X$' when x ranges over objects and '$\forall X \in Y$' when X ranges over sets. We will often use the abbreviation 's.t.' for 'such that.'

4.5.1 The Framework

Our syntax is simply the standard syntax of tense logic: proposition letters, Boolean connectives, and the four operators P, F, H, and G. So we begin with models. Frames are now triples $\langle T, \prec_t, \mathfrak{H}\rangle$, and models quadruples $\langle T, \prec_t, \mathfrak{H}, V\rangle$. As before, T is a set of times, $\prec_t$ is an accessibility relation, and V is a sentential valuation function.

A few words about $\prec_t$. Typically, the temporal accessibility relation is meant to model the B-series: it says which states are earlier (later) than which other states. But our interest is not merely in the B-series (because we deny that there need be any such thing); we are interested, rather, in what the B-series could be. Thus, $\prec_t$ is best thought of as telling us which states are possibly earlier (later) than which other states. However, recalling our discussion of openness, indeterminism, and the thin red line in §3.1-3.2, we are not relying on $\prec_t$ to tell us which states are open to be future of which other states. That job will be done by $\mathfrak{H}$. Rather, we should read $t \prec_t t'$ as: 't' is not determined not to be in the future of t.' This is what we officially mean by 't' is future-accessible from t.' Assumptions (even straightforward ones like transitivity) that we make about it will (sometimes partially partially) define classes of frames. In the minimal system, we assume only that if $t \prec_t t'$, then $t \succ_t t$, informally: if t' could have been in the future of t, then t could have been in the past of t'.

The distinctively Ockhamist contribution is $\mathfrak{H}$, which we require in order to define the tense operators and which tells us which futures (pasts) are open from a given time, calling again to mind the difference between openness and indeterminism. $\mathfrak{H}$ is a function from T to its powerset $\mathcal{P}T$. Intuitively, it assigns to each element of T the open world-histories that time could be a part of. The paths in the garden. A history has two parts: a set of states, and their order. Since the order is provided by $\prec_t$, $\mathfrak{H}$ need only tell us the states. In the minimal system, $\mathfrak{H}$ obeys the following constraints:

1. $\forall X \in \mathfrak{H}(t)\, t \in X$ SELF-HISTORICITY

2. $\forall X \in \mathfrak{H}(t)\, x \in X \rightarrow (x \prec_t t \vee t \prec_t x \vee x = t)$ SELF-CONNECTEDNESS

Condition 1 says that when the function $\mathfrak{h}$ assigns the sets of states that make up the timelines that it is open that t is a part of, it makes sure to put tin those sets of states. It ensures that the times are a part of all of their own histories. Condition 2 says that all of the times in the sets of times that the function $\mathfrak{h}$ assigns making up the timelines that it is open that t is a part of are connected to t by the accessibility relation. It ensures that all open histories 'pass through' t.

Note how minimal the minimal models are. They are consistent with open pasts, open futures, branching pasts, branching futures, determinism, and indeterminism. This is exactly what we want in a flexible framework capable of modeling lots of views in the metaphysics of time.

We have already met the tense-free logic I will use. It is the upgraded version of Lukasiewicz's system, $\mathfrak{L}_3^{\mathfrak{s}}$, introduced in §2.1. That leaves only the tense operators. But before we give their semantics, a note. Since the only source of indeterminacy in our system is meant to be from tensed sentences, we will place a constraint on valuations that atomic sentences receive only extreme truth values. Following Prior, we assume that the atoms are entirely free of tense.[15] We want to represent tense with tense operators, not bury in the atoms. This means that the tenseless fragment of the language will be fully classical. The third truth value will only show up when evaluating

[15] Prior [1967].

sentences containing tense operators.

We begin with the classic G and H, which get a similar semantics to the standard account, with an extra clause to account for the third truth value. They may still be interpreted as 'henceforth' and 'hitherto:'

i. $V(t, G\phi) = 1$ iff $\forall v$ s.t. $t \prec_t v$, $V(v, \phi) = 1$

ii. $V(t, G\phi) = .5$ iff $\forall v$ s.t. $t \prec_t v$, $V(v, \phi) > 0$ and $\exists u$ s.t. $t \prec_t u$ and $V(u, \phi) = .5$

iii. $V(t, G\phi) = 0$ iff $\exists v$ s.t. $t \prec_t v$, $V(v, \phi) = 0$

iv. $V(t, H\phi) = 1$ iff $\forall v$ s.t. $v \prec_t t$, $V(v, \phi) = 1$

v. $V(t, H\phi) = .5$ iff $\forall v$ s.t. $v \prec_t t$, $V(v, \phi) > 0$ and $\exists u$ s.t. $u \prec_t t$ and $V(u, \phi) = .5$

vi. $V(t, H\phi) = 0$ iff $\exists v$ s.t. $v \prec_t t$, $V(v, \phi) = 0$

Their duals, $\neg G \neg$ and $\neg H \neg$, may be introduced via metalinguistic abbreviation as $< G >$ and $< H >$, and may be interpreted as 'it is not determined that it won't be' and 'it is not determined that it wasn't,' respectively.

Breaking the duality of F/G and P/H requires making F and G their own primitive operators, and we give them the following semantics.

i. $V(t, F\phi) = 1$ iff $\forall X \in \mathfrak{H}(t) \exists u \in X$ s.t. $t \prec_t u$ and $V(u, \phi) = 1$

ii. $V(t, F\phi) = .5$ iff $\exists X \in \mathfrak{H}(t) \exists u \in X$ s.t. $t \prec_t u$ and $V(u, \phi) > 0$ and $\exists X \in \mathfrak{H}(t) \forall v \in X$ s.t. $t \prec_t v$ and $V(v, \phi) < 1$

iii. $V(t, F\phi) = 0$ iff $\forall X \in \mathfrak{H}(t) \forall u \in X$ s.t. $t \prec_t u$ and $V(u, \phi) = 0$

iv. $V(t, P\phi) = 1$ iff $\forall X \in \mathfrak{H}(t) \exists u \in X$ s.t. $u \prec_t t$ and $V(u, \phi) = 1$

v. $V(t, P\phi) = .5$ iff $\exists X \in \mathfrak{H}(t) \exists u \in X$ s.t. $u \prec_t t$ and $V(u, \phi) > 0$ and $\exists X \in \mathfrak{H}(t) \forall v \in X$ s.t. $v \prec_t t$ and $V(v, \phi) < 1$

vi. $V(t, P\phi) = 0$ iff $\forall X \in \mathfrak{H}(t) \forall u \in X$ s.t. $u \prec_t t$ and $V(u, \phi) = 0$

Once more, we can define the duals $< F >$ and $< P >$ as $\neg F \neg$ and $\neg P \neg$, to be interpreted as 'it is open that it will be' and 'and it is open that it was.'

4.5.2 The Third Truth Value Revisted

We are now in a position to revisit the argument against interpreting the third truth value as 'undetermined.' Recall that the argument was a dilemma. It looked like $\vee$ needed to behave differently depending on which propositions flanked it. In the case of $\phi \vee \neg\phi$, there is a *prima facie* case to make the disjunction true when ϕ is undetermined, on the grounds that once its truth is resolved, it will either be true or false. But in the case of $\phi \vee \psi$, it looked like the disjunction ought to also be undetermined, since there was no guarantee that both ϕ and ψ won't end up false.

My framework avoids this argument by denying that all instances of $\phi \vee \neg\phi$ are settled, and by locating the cases where the inputs to the $\vee$-function are $\{ .5, .5\}$ among them. By making the tense explicit, and limiting indeterminacy to future-directed sentences, we can show that 'not settled' interpretation of .5 is the appropriate one.

We first note that, when ϕ is atomic or constructed out of atoms and truth-functional connectives, it always has one of the determined truth values (0 or 1). We reserve .5 for formulae with at least one tense operator. This gives us the rule tenseless excluded middle as a validity:

TEM: $\phi \vee \neg\phi$ when ϕ has no tense operators

It is only when considering the future (or past) that we will deal with the third truth value.

In order to get indeterminacy with the G and H operators, the formula under the operator's scope must be indeterminate at at least one of the accessible times and false at none. Thus, once again, for formulae constructed out of atoms, truth-functional connectives, and the G and H operators, a restricted version of excluded middle will hold.

In order to get ground-level indeterminacy, we must use the F and P operators. In order for $F\phi$ (Pϕ) to be indeterminate, two conditions must be fulfilled. First: there must be one history where there is a future (past) time at which ϕ is not false. If ϕ does not contain an F or P operator, this means it must be true. Second: there must be one history according to which there are no future (past) times at which ϕ is true. If ϕ does not contain an F or P operator, this means they are all false. These conditions cannot obtain together when there is only one history, and thus cannot occur in closed frames.

Thus, the only instances where $\phi \vee \neg\phi$ will receive an indeterminate value are cases where the future (past) is open and an F (P) operator appears in ϕ. And this is the exact class of formulae from which intuitive counterexample to excluded middle may be found. Consider the formula $FG\phi$. In a model where every time has in its future a time from which only ϕ-times are future-accessible and a time from which a $\neg\phi$-time is accessible, neither of $FG\phi$ or $\neg FG\phi$ will ever be true or false. In these cases, it is appropriate to consider the disjunction unsettled.

4.5.3 Flexibility and Adequacy

In §3.4, we introduced Table 9 with six views about temporal relations that a good framework for tense logic should be able to define precisely. I will now show which conditions on $\prec_t$ and $\mathfrak{H}$ yield each view.

But before we launch into that, it will be helpful to give a very broad overview of how the pieces of the framework encode information about temporal relations. To do this, there are two primary tools: the accessibility relation $\prec_t$ and the history function $\mathfrak{H}$. The accessibility relations basic job is to tell us which times are possibly past/future of which other times. The $\mathfrak{H}$ function's main job is to tell us which histories remain open. Given a time, $\mathfrak{H}$ tells us which histories containing that time are not closed off by the facts about the past/future. If we are in a closed timeline, $\mathfrak{H}$ returns only one timeline, the one and only actual history. If it returns more than one continuation, then we have an open past or future. To see if a model is open or closed, count the number of timelines $\mathfrak{H}$ returns given a time as input.

If time is deterministic (and therefore closed), $\mathfrak{H}$ simply returns the one timeline that the model contains. To see whether a model has deterministic time, look at how $\prec_t$ and $\mathfrak{H}$ interact. If there is more to the model than the timeline $\mathfrak{H}$ returns, we have indeterministic time.

Finally, to see whether a model has Everett-style branching time or classical linear time, look at the timelines $\mathfrak{H}$ returns. If it returns trees, we have branching time. If it returns lines, we have linear time.

Although this tells us broadly how information is stored in the models, we can say precisely what a model must look like to encode one of the coherent structures for temporal relations we are interested in. The remainder of this section is devoted to giving the exact specifications.

Open, Indeterminstic, Linear Time

In order to model open, indeterministic, linear time, we require a branching accessibility relation with linear histories. Indeterministic time comes in three varieties: indeterministic future, indeterministic past, indeterministic both. Thus, we will give three accessibility relations: one for each. Furthmore. open time comes in three varieties: open future, open past, and open both. So we will give three groups of constrains on $\mathfrak{H}$, one for each. We start with the accessibility relations.

INDETERMINISTIC FUTURE

1. $\forall t \forall u \forall v((t \prec_t u \wedge u \prec_t v) \rightarrow t \prec_t v)$ TRANSITIVITY
2. $\forall t \neg(t \prec_t t)$ IRREFLEXIVITY
3. $\forall t \forall u((\exists v\, t \prec_t v \wedge u \prec_t v) \rightarrow (t \prec_t u \vee u \prec_t t \vee t = u))$ L-CONVERGENCE
4. $\forall t \forall u \exists v(v \prec_t t \wedge v \prec_t u)$ L-CONNECTEDNESS

INDETERMINISTIC PAST

1. $\forall t \forall u \forall v((t \prec_t u \wedge u \prec_t v) \rightarrow t \prec_t v)$ TRANSITIVITY
2. $\forall t \neg(t \prec_t t)$ IRREFLEXIVITY

3. $\forall t \forall u((\exists v\, v \prec_t t \wedge v \prec_t u) \rightarrow (u \prec_t t \vee t \prec_t u \vee t = u))$ R-CONVERGENCE

4. $\forall t \forall u \exists v(t \prec_t v \wedge u \prec_t v)$ R-CONNECTEDNESS

INDETERMINISTIC BOTH

1. $\forall t \forall u \forall v((t \prec_t u \wedge u \prec_t v) \rightarrow t \prec_t v)$ TRANSITIVITY

2. $\forall t \neg(t \prec_t t)$ IRREFLEXIVITY

Indeterministic accessibility relations branch. If the indeterminism is future directed, they branch to the right. If the indeterminism is past directed, they branch to the left. If it goes both ways, as in Hud Hudson's morphing block view, they branch in both directions.

In the conditions we have given, IRREFLEXIVITY and TRANSITIVITY ensure that there are no time loops that times that are in the future (past) of the future (past) are themselves in the future. L-CONNECTEDNESS and L-CONVERGENCE ensure that there is a determined past when we have future indeterminacy, while R-CONNECTEDNESS and R-CONVERGENCE do the same for the future when we have past indeterminacy. We omit them when the indeterminacy goes in both directions.

We now encode both the openness and the linearity with our constraints on the $\mathfrak{H}$ function as follows, recalling that $\mathfrak{h}$ always has the SELF-HISTORICITY and SELF-CONNECTEDNESS properties, by definition:

OPEN LINEAR FUTURES

1. $\forall X\, X \in \mathfrak{H}(t) \rightarrow X$ is a $\prec_t$-chain TIMELINE LINEARITY

2. $|\mathfrak{H}(t)| \geq 2$ OPEN TIMELINE

3. $\forall X, Y \in \mathfrak{H}(t) \forall x \in T((x \prec_t t) \rightarrow (x \in X \leftrightarrow x \in Y))$ SAME PAST

TIMELINE LINEARITY transparently makes all of the open histories linear, by the definition of a chain. OPEN TIMELINE simply says that there are at least two timelines open at t, which we need for the future to be open. SAME PAST ensure that those timelines share a past so that only the future is open.

OPEN LINEAR PASTS

1. $\forall X\ X \in \mathfrak{H}(t) \to X$ is a maximal $\prec_t$-chain — TIMELINE LINEARITY
2. $|\mathfrak{H}(t)| \geq 2$ — OPEN TIMELINE
3. $\forall X, Y \in \mathfrak{H}(t) \forall x \in T((t \prec_t x) \to (x \in X \leftrightarrow x \in Y))$ — SAME FUTURE

The open past $\mathfrak{H}$ is simply the mirror image of the open future $\mathfrak{H}$; this time, the futures are all the same, but the pasts differ.

LINEAR OPEN BOTH

1. $\forall X\ X \in \mathfrak{H}(t) \to X$ is a maximal $\prec_t$-chain — TIMELINE LINEARITY
2. $|\mathfrak{H}(t)| \geq 2$ — OPEN TIMELINE

If both past and future are open, then there are multiple potential histories, but the only thing the timelines must have in common is that they contain t, are linear, and are connected to t. These conditions added to the minimal requirements are sufficient for that.

Open, Indeterministic, Branching Time

As above, we adopt the appropriate accessibility relation for indeterminstic time, depending on whether it is the past, the future, or both that is open. However, now we must allow $\mathfrak{H}$ to output branching timelines. Since we can branch time to the past, the future, or both, we give a version of $\mathfrak{H}$ for each.

OPEN FUTURE BRANCHING HISTORIES

1. $|\mathfrak{H}(t)| \geq 2$ — OPEN TIMELINE
2. $\forall X, Y \in \mathfrak{H}(t) \forall x \in T((x \prec_t t) \to (x \in X \leftrightarrow x \in Y))$ — SAME PAST

3. $\forall X \in \mathfrak{H}(t)\ \forall x \in X \neg(x \prec_t x)$ IRREFLEXIVITY

4. $\forall X \in \mathfrak{H}(t)\ \forall x, y, z \in X((x \prec_t y \wedge y \prec_t z) \to x \prec_t z)$ TRANSITIVITY

5. $\forall X \in \mathfrak{H}(t)\ \forall x, y \in X((\exists z \in X\ x \prec_t z \wedge y \prec_t z) \to (x \prec_t y \vee y \prec_t x \vee x = y))$ L-CONVERGENCE

6. $\forall X \in \mathfrak{H}(t)\ \forall x, y \in X\ \exists z \in X(z \prec_t x \wedge z \prec_t y)$ L-CONNECTEDNESS

As before, OPEN TIMELINE ensures that there is no one future history, no thin red line. SAME PAST makes it the future that is open, while the other conditions guarantee that the branching futures are tree-shaped with the past as the trunk. This gives us the models of time that are indeterministic, open, and branching. Since one of my main complaints about the Ockhamist and Peircean traditions was that they could not deliver this, it is a key point in favor of my framework.

OPEN PAST BRANCHING HISTORIES

1. $|\mathfrak{H}(t)| \geq 2$ OPEN TIMELINE

2. $\forall X, Y \in \mathfrak{H}(t) \forall x \in T((t \prec_t x) \to (x \in X \leftrightarrow x \in Y))$ SAME FUTURE

3. $\forall X \in \mathfrak{H}(t)\ \forall x \in X \neg(x \prec_t x)$ IRREFLEXIVITY

4. $\forall X \in \mathfrak{H}(t)\ \forall x, y, z \in X((x \prec_t y \wedge y \prec_t z) \to x \prec_t z)$ TRANSITIVITY

5. $\forall X \in \mathfrak{H}(t)\ \forall x, y \in X((\exists z \in X\ z \prec_t x \wedge z \prec_t y) \to (y \prec_t x \vee x \prec_t y \vee x = y))$ R-CONVERGENCE

6. $\forall X \in \mathfrak{H}(t)\ \forall x, y \in X\ \exists z \in X(x \prec_t z \wedge y \prec_t z)$ R-CONNECTEDNESS

As before, an open past is simply the mirror of an open future. We have simply changed things so that the potential timelines now share a future instead of a past, and the trees branch toward the past with the future as a stem instead of vice versa.

OPEN BOTH BRANCHING HISTORIES

1. $|\mathfrak{H}(t)| \geq 2$ OPEN TIMELINE

2. $X \in \mathfrak{H}(t) \rightarrow \forall x \in X \neg(x \prec_t x)$ IRREFLEXIVITY

3. $X \in \mathfrak{H}(t) \rightarrow \forall x, y, z \in X((x \prec_t y \wedge y \prec_t z) \rightarrow x \prec_t z)$ TRANSITIVITY

When both past and future are open and branching, all they need have in common is t. The requirement of multiple open histories and some basic ordering assumptions are all we can impose.

Closed, Indeterministic, Linear

As before, we encode assumptions about determinism in $\prec_t$, which means we can borrow the same conditions used in the open indeterministic models. Furthermore, with a closed timeline, we need only give one characterization of $\mathfrak{H}$:

CLOSED LINEAR TIMELINES

1. $\forall X\, X \in \mathfrak{H}(t) \rightarrow X$ is a $\prec_t$-chain TIMELINE LINEARITY

2. $|\mathfrak{H}(t)| = 1$ CLOSED TIMELINE

As before, we require that the open histories be $\prec_t$-chains to preserve linearity of the timeline. But his time, we allow $\mathfrak{H}$ to return only one timeline per time: the model's 'thin red line.' This gives us our model of closed, indeterministic, linear futures.

Closed, Indeterministic, Branching

For closed, indeterministic, branching models, we adopt the same accessibility relation as in the open, indeterministic, branching models. These models are like the thin red line, but with a branching structure instead of a linear one; a thin red tree, as it were. And, as before, we must give constraints on $\mathfrak{H}$ suitable to timelines that branch to the past, to the future, or to both.

CLOSED FUTURE-BRANCHING TIMELINES

1. $|\mathfrak{H}(t)| = 1$ CLOSED TIMELINE

2. $\forall X \in \mathfrak{H}(t)\ \forall x \in X \neg(x \prec_t x)$ IRREFLEXIVITY

3. $\forall X \in \mathfrak{H}(t)\ \forall x, y, z \in X((x \prec_t y \wedge y \prec_t z) \rightarrow x \prec_t z)$ TRANSITIVITY

4. $\forall X \in \mathfrak{H}(t)\ \forall x, y \in X((\exists z \in X\ x \prec_t z \wedge y \prec_t z) \rightarrow (x \prec_t y \vee y \prec_t x \vee x = y))$ L-CONVERGENCE

5. $\forall X \in \mathfrak{H}(t)\ \forall x, y \in X\ \exists z \in X(z \prec_t x \wedge z \prec_t y)$ L-CONNECTEDNESS

We can safely eliminate the same-past condition, since CLOSED TIMELINE forces there to be only one timeline in $\mathfrak{H}(t)$. Beyond that, all we need to do is gaurantee that the timeline is a tree that branches to the future, which is what the other conditions do.

CLOSED PAST-BRANCHING TIMELINES

1. $|\mathfrak{H}(t)| = 1$ CLOSED TIMELINE

2. $\forall X \in \mathfrak{H}(t)\ \forall x \in X \neg(x \prec_t x)$ IRREFLEXIVITY

3. $\forall X \in \mathfrak{H}(t)\ \forall x, y, z \in X((x \prec_t y \wedge y \prec_t z) \rightarrow x \prec_t z)$ TRANSITIVITY

4. $\forall X \in \mathfrak{H}(t)\ \forall x, y \in X((\exists z \in X\ z \prec_t x \wedge z \prec_t y) \rightarrow (y \prec_t x \vee x \prec_t y \vee x = y))$ R-CONVERGENCE

5. $\forall X \in \mathfrak{H}(t)\ \forall x, y \in X\ \exists z \in X(x \prec_t z \wedge y \prec_t z)$ R-CONNECTEDNESS

We can safely eliminate the same-future condition, since CLOSED TIMELINE forces there to be only one timeline in $\mathfrak{H}(t)$. Beyond that, all we need to do is gaurantee that the timeline is a tree that branches to the past, which is what the other conditions do.

CLOSED BOTH-BRANCHING INDETERMINISTIC TIMELINE

1. $|\mathfrak{H}(t)| = 1$ CLOSED TIMELINE

2. $\forall X \in \mathfrak{H}(t)\ \forall x \in X \neg(x \prec_t x)$ IRREFLEXIVITY

3. $\forall X \in \mathfrak{H}(t)\ \forall x, y, z \in X((x \prec_t y \wedge y \prec_t z) \rightarrow x \prec_t z)$ TRANSITIVITY

As before, we can give only ordering assumptions on the one available future.

Closed, Deterministic, Linear

Closed, deterministic, linear models are those we find in standard tense logics most often. As a result, it would be best if our system collapsed into the standard system in these models. After laying down the appropriate restrictions on $\prec_t$ and $\mathfrak{H}(t)$, we will see that it does.

DETERMINISTIC LINEAR ACCESSIBILITY

1. $\forall t \forall u \forall v((t \prec_t u \wedge u \prec_t v) \rightarrow t \prec_t v)$ — TRANSITIVITY
2. $\forall t \neg(t \prec_t t)$ — IRREFLEXIVITY
3. $\forall t \forall u(t \prec_t u \vee u \prec_t t \vee t = u)$ — CONNECTEDNESS

For the deterministic linear frames, we use a strict total order. The only possibly past/future states are those in the timeline. This also gives us a fairly specific $\mathfrak{H}$, since there can only be one history:

DETERMINISTIC CLOSED LINEAR TIMELINE

1. $\forall X\ X \in \mathfrak{H}(t) \rightarrow X$ is a maximal $\prec_t$-chain — MAXIMAL TIMELINE LINEARITY
2. $|\mathfrak{H}(t)| = 1$ — CLOSED TIMELINE

   which together with the deterministic constraints on $\prec_t$ imply

3. $\mathfrak{H}(t) = T$ — ONE POSSIBLE HISTORY

Recall that closed linear timelines require that elements of $\mathfrak{H}(t)$ be maximal $\prec_t$-chains. But if $\prec_t$ is a strict total order, then there is only one maximal $\prec_t$-chain: T as ordered by $\prec_t$. Thus, it is the timeline returned by $\mathfrak{H}$ for every time. We can now prove that in the closed, deterministic, linear case our four modalities collapse into the two of standard tense logic, making standard tense logic a special case of my framework.

THEOREM 1: In closed, deterministic, linear frames: $F\phi \leftrightarrow \neg G \neg \phi$

proof:

$\Rightarrow$

Assume that $F\phi$ is true at t. Then, by the semantics for F, there is a u s.t. $t \prec_t u$ at which ϕ is true, which is exactly the truth condition for $\neg G \neg \phi$.

$\Leftarrow$

Assume that $\neg G \neg \phi$ is true at t. Then there is a u s.t. $t \prec_t u$ and at which ϕ is true. But since $\mathfrak{H}(t) = T$, there is an $X \in \mathfrak{H}(t)$ containing a u s.t. $t \prec_t u$ and at which ϕ is true. Furthermore, since $|\mathfrak{H}(t)| = 1$, there are no other elements of $\mathfrak{H}(t)$, and therefore $\forall X \in \mathfrak{H}(t) \exists u$ s.t. $t \prec_t u$ and ϕ is true at u, which were exactly the truth-conditions for $F\phi$.

$\square$

THEOREM 2: In closed, deterministic, linear frames: $P\phi \leftrightarrow \neg H \neg \phi$

proof:

Same as THEOREM 1, *mutatis mutandis* to account for past-directed operators.

Theorems 1-2 show that the logic induced by my framework in closed, deterministic, linear frames is exactly the logic induced by the standard account when $\prec_t$ is a strict total order, a.k.a. the standard tense logic of deterministic linear time.

Closed, Deterministic, Branching

The final frame needed to define the views in Table 1, and thus to demonstrate the framework's adequacy, uses the same branching accessibility relations we have seen before. The difference is in our characterization of $\mathfrak{H}$. As in the linear case, it is maximal. But it can't be a chain, since chains enforce linearity. Instead, we will call it a 'structure,' with the requirement that a $\prec_t$-structure is one in which any two times are connected by some (potentially zig-zag) path:

CLOSED DETERMINISTIC BRANCHING TIME

1. $\forall X\, X \in \mathfrak{H}(t) \rightarrow X$ is a maximal $\prec_t$-structure — BRANCHING DETERMINISM
2. $|\mathfrak{H}(t)| = 1$ — CLOSED TIMELINE

which together with the constraints on $\prec_t$ imply

3. $\mathfrak{H}(t) = T$ — ONE POSSIBLE HISTORY

Likewise, as in the previous section, we can collapse the four modalities into tense logic's standard two.

THEOREM 3: In closed, deterministic, branching frames: $F\phi \leftrightarrow \neg G \neg \phi$

proof: Same as THEOREM 1

THEOREM 4: In closed, deterministic, branching frames: $P\phi \leftrightarrow \neg H \neg \phi$

proof: Same as THEOREM 2

As with closed deterministic linear frames, closed deterministic branching frames yield the same logic as standard branching frames.

Adequacy Achieved

In the preceding, I set forward several tasks. First, I required a good framework for tense logic to be able to model all six combinations of views about the structure of temporal relations in Table 1. As I have shown in §4.3.1-6, my framework is able to do that. But along the way, I have leveled complaints at rival frameworks using specific formulae.

I will now briefly show that those formulae are not theorems in my basic framework. First, recall (1):

(1) $\phi \to PF\phi$

We can give a simple countermodel:

T = {t, u, v}

$\prec_t = v \prec_t t, v \prec_t u$

$\mathfrak{H}(v) = \{\{v, t\}\{v, u\}\}$ and otherwise anything consistent with the constraints

$V(t, \phi) = 1, V(u, \phi) = 0$ and otherwise anything consistent with the constraints

In this model, ϕ is true at t. But $F\phi = .5$ at v, and because v is the only possible past state of t, $PF\phi$ is not true at t. Note that by adding infinitely-many states and putting them in line behind v, we can make the model past-serial while maintaining the failure of (1). As we noted, (1) is a theorem of past-serial models in the standard framework.

Next, recall (2), the principle of future excluded middle:

(2) $F\phi \vee F\neg\phi$

We can use the same countermodel to show that it is not valid. Since ϕ is true in one possibly future state in an open history for v, but false in all states in another open history for v, (2) is false at v and therefore not a theorem.

Next, recall (6):

(6) $(HG\phi \supset \phi) \leftrightarrow (\phi \supset PF\phi)$

We used duality to prove that (6) is a theorem of the minimal standard temporal logic. Any model that breaks the F/G and P/H duality blocks the proof of (6).

Finally, recall (n), true in all linear frames, which I used to complain about the Priorean tradition's use of a history as a parameter of evaluation:

(7) $F\phi \wedge F\psi \supset (F(\phi \wedge F\psi) \vee F(\psi \wedge F\phi) \vee F(\phi \wedge \psi))$

Because my four operators collapse into the standard operators in closed, deterministic frames, we can use closed, deterministic models to show that (n) fails in my framework without worrying about the new semantics I gave to F. Because I have closed, deterministic, branching models, any of those will do to counter (7).

4.6 Conclusion

We began thinking about the problem of future contingents, and set a goal of finding a formal framework for thinking about the logic of temporal relations, with an eye towards one that could be used to explore the logical consequences of an open future. We began by assembling some important pieces from logical developments in the 20th century.

The first piece was Lukasiewicz's trivalent logic. Inspired by Aristotle, Lukasiewicz set out to give a logic where sentences could take, in addition to the classical True and False, the truth value Indeterminate. He produced a mathematically coherent formal system, but one with some difficulties in interpreting the Indeterminate truth value.

The second piece was Aruthr Prior's 'Ockhamist' tense logic. Prior realized that. to give an adequate logical framework for an open future, it was necessary to have not only times that could be in the future in a model, but to also group those times together into possible histories. However, the way this idea has been implemented in the Priorean tradition - using a history as a point of evalution alongside a time within a model - has inadertently excluded another view about the structure of temporal relations that is of interest, namely, branching time.

The framework developed here weaves these two pieces together and finds ways to avoid their inadequacies. In particular, theorems from the Priorean models that excluded perfectly good views in the metaphysics of time are now only theorems when we assume those views to be false, and we have shown how give an interpretation of Lukasiewicz's third truth value as indeterminate.

Nevertheless, there remain open questions. Foremost among these is axiomatization - for the minimal system and, to the extent possible given known restrictions on definability in tense logic, for the classes of frames that correspond to interesting views in the metaphysics of time. I leave this question to be taken up later.

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