Qualitative Grounds

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Abstract

Suppose that all non-qualitative facts are grounded in qualitative facts. I argue that this view naturally comes with a picture in which trans-world identity is indeterminate. But this in turn leads to either pervasive indeterminacy in the non-qualitative, or else contingency in what facts about modality and possible worlds are determinate.

1   Ground and Necessity

Shamik Dasgupta argues that we shouldn't think there are any fundamental facts about particular individuals: these would be undetectable danglers, redundant to our scientific explanations (2009; 2014; forthcoming; see also 2011; 2013). Rather, we should hold that all facts about particular individuals are *grounded* in what the world is like qualitatively.[1] All non-qualitative facts hold *in virtue of* qualitative facts. He calls this "qualitativism". (Other names for the view are "generalism",

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[1]"Fundamentally speaking, I claim, there are no such things as individuals" (Dasgupta 2009, 2). "Let *qualitativism* be the … view that the most fundamental facts are qualitative facts and that they are sufficient to ground (or at least derivatively ground) the individualistic facts. … I favor qualitativism. Very briefly, my reason is that if individualism were true then the individualistic facts of our world would lie beyond our epistemic ken" (Dasgupta 2014, 6). Compare these earlier statements (though not endorsements) of the view. "[D]oes thisness hold a place beside suchness as a fundamental feature of reality?" (Adams 1979, 5). "[M]etaphysical anti-Haecceitism is a doctrine concerning the identity of individuals. It states that the identity of individuals—or, at least, of certain individuals—is to be explained in terms of their purely qualitative features or in terms of their qualitative relationships to other individuals" (Fine 2005, 30). See also Turner (forthcoming) for critical discussion.

As a point of biography I should note that Dasgupta reports in personal communication that he has changed his mind about the Qualitative Grounds thesis, for different reasons from those I discuss here.

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"structuralism", or "metaphysical anti-haecceitism".) I'll call it the **Qualitative Grounds** thesis. I find this thesis intriguing, but I don't entirely understand it. In this paper I strive to get a clearer view of what it really involves.

One reason for investigating qualitativism is to illuminate a cluster of related philosophical issues. For instance, many philosophers are "qualitativists" (or "structuralists" of some kind) not about all individuals whatsoever, but just about special kinds of individuals, such as fundamental particles, numbers, or regions of space-time. Other views aren't really about individuals in any natural sense, but still have a similar structure. Are there facts about particular quantities of mass like *being one kilogram* which are grounded in structural facts about mass relationships like *being twice as massive*? Are there "quidditistic" facts about particular physical properties like *being an electron* which are grounded in "Ramseyan" facts about their pattern of instantiation? I'll focus on individuals, since this is in many ways the simplest case, but some lessons should carry over to these other applications.

The word "ground" is used in different ways by different authors. Some people use the "grounding project" as a general label for the task of making sense of the status of ordinary claims as they relate to some more "metaphysically perspicuous" description of the world (e.g. Sider 2008, sec. 2; Turner 2016, sec. 1.2). But some philosophers, including Dasgupta, use "grounding" in a narrower sense. (See Fine 2001; Schaffer 2009; Rosen 2010; Koslicki 2012.) In this sense, grounding is supposed to be an especially tight sort of *metaphysical explanation*. If P in virtue of Q, then while P is a genuine fact, and a *distinct* fact from Q (since no fact holds in virtue of itself), in another important sense there is nothing more to the fact that P than Q. When we take our metaphysical inventory, the grounded doesn't count as "extra", over and above its ground. We can't understand "in virtue of" by giving a reductive analysis in other terms, but we can illuminate it by way of its connections to other notions. A key connection is to metaphysical possibility: whatever *grounds* a fact *settles* it (Dasgupta 2014, 4).

Grounds Settle. If P in virtue of Q, then Q entails P.

(As usual, by "Q entails P" I mean that the material conditional if-Q-then-P is metaphysically necessary: that is, it's metaphysically impossible that Q-and-not-P.) If P doesn't require anything more beyond Q, then whenever Q holds, so does P.

Qualitative Grounds and Grounds Settle together have the consequence that all facts are *entailed* by qualitative facts. Suppose Q^+ is **qualitatively complete**: it's a purely qualitative matter whether Q^+, and for any purely qualitative Q, either Q^+ entails Q or Q^+ entails not-Q. To put it another way, Q^+ is a possible conjunction of

all qualitative facts (or else Q^+ is impossible).[2] Then we can put this consequence like this:

Qualitative Supervenience. If Q^+ is qualitatively complete and Q^+ and P, then Q^+ entails P.

(Here P and Q^+ are schematic sentence letters.)[3] The *necessity* of Qualitative Supervenience amounts to a version of "Anti-Haecceitism"—in a slogan, there can be no difference without qualitative difference.[4]

Robert Adams (1979, 22) gave an important argument against Qualitative Supervenience. In the next section I'll present Adams' argument. Dasgupta considers several related modal arguments, and provides materials for a reply. In the remainder of the paper, I'll explore the consequences of Dasgupta's reply.

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[2]In this paper I'll make the simplifying assumption that it is *not contingent* whether Q^+ is qualitatively complete. This follows from the more basic assumptions that (1) it is not contingent what qualitative propositions there are, and (2) it is not contingent what entailments hold between qualitative propositions. Assumption (2) follows from the stronger assumption I'll introduce later on: the correctness of the modal logic S5.

[3]If we help ourselves to certain logical resources, we can make the derivation of Qualitative Supervenience from Qualitative Grounds more explicit. First, a point of clarification. The "in virtue of" operator is usually taken to be *plural* in its right-hand argument. A further complication is that Dasgupta holds that the "in virtue of" operator is plural in its left-hand argument as well. (This is so that, e.g., facts about the places of objects in a structure can be collectively grounded, without any one of them being grounded on its own.) He uses this feature to rebut certain modal arguments against qualitativism (2014, sec. 11). So in general, Qualitative Grounds says:

> If it's a non-qualitative fact that P, then are some $R_1, R_2, \ldots$ and some qualitative facts $Q_1, Q_2, \ldots$, such that $P, R_1, R_2, \ldots$ in virtue of $Q_1, Q_2, \ldots$.

Grounds Settle is similarly generalized:

> If $P, R_1, R_2, \ldots$ in virtue of $Q_1, Q_2, \ldots$ then (Q_1 and Q_2 and …) entails (P and R_1 and R_2 and …)

Suppose Q^+ is qualitatively complete, P is non-qualitative, and Q^+ and P. Then let $R_1, R_2, \ldots$ and $Q_1, Q_2, \ldots$ be as given by Qualitative Grounds. Since $Q_1, Q_2, \ldots$ are each qualitative facts, Q^+ entails each of them, and thus Q^+ entails (Q_1 and Q_2 and …). Since entailment is transitive, Q^+ entails (P and R_1 and R_2 and …), so in particular Q^+ entails P. (In the case where P is *qualitative*, the thesis follows instead from the qualitative completeness of Q^+.)

[4]So Dasgupta writes: "Of course it follows from my assumption that the grounded is necessitated by its grounds that qualitativism implies anti-haecceitism (in at least one of its characterizations)." (2014, 7)

The term "Anti-Haecceitism" has unfortunately taken on several different meanings, referring to various doctrines about modality or possible worlds. For discussion see Skow (2007); Russell (2013); and Section 3 of this paper. The meaning here is a modal doctrine, the one Russell (2013) calls "Possibility Anti-Haecceitism".

2 Adams' Argument

There could have been two spheres, qualitatively exactly alike throughout their histories (Black 1952). Adams (1979, 22) claims that in this case either of the two spheres could have been destroyed while the other survived. If one sphere had been destroyed, things would have been qualitatively just the same as if the other sphere had been destroyed instead. This story seems like it is metaphysically possible—but it is incompatible with Qualitative Supervenience. No matter which of the two spheres is destroyed, things would be qualitatively just alike. But there is a non-qualitative difference between the two possibilities, concerning which sphere is destroyed. Since no qualitative facts distinguish between these two possibilities, no qualitative facts can ground the singular truth (in either one of them) about which of the two spheres is destroyed.

Let Q^+ be a complete qualitative description of a world where just one sphere is destroyed. Adams claims:

Possible Symmetry. It is possible for there to be some x such that possibly (Q^+ and x survives), and possibly (Q^+ and x does not survive).

But if it's possible that Q^+ and x does not survive, then Q^+ does not entail that x survives. So, were the particular sphere x to survive, this fact would not be entailed by the complete qualitative description of the world, and thus it would not entailed by *any* qualitative facts. Thus Possible Symmetry contradicts the necessity of Qualitative Supervenience (using some standard modal reasoning; for details see Theorem 1 in Appendix A).

Before we take up Dasgupta's reply, let's consider a simpler reply: why shouldn't the qualitativist simply accept Adams' premise Possible Symmetry, and draw the conclusion that Qualitative Supervenience is only *contingently* true? Consider an analogy to the form of *physicalism* that says that all genuine facts supervene on physical facts. One might hold that physicalism is contingently true: while there are no angels or immaterial souls or qualia, there could have been (e.g. Lewis 1983, 362). Likewise one might hold that while in fact everything is qualitatively grounded, this need not have been the case (cf. Dasgupta 2009, 18).

In the case of physicalism, it would be odd to say just that every *actual* fact is entailed by some physical fact. That seems to say too little. If there are no angels, but there would have been a host of them had you said the proper incantation, that seems pretty bad for the spirit of physicalism. A more natural physicalist view is that, even if it isn't *necessarily* true that every fact is entailed by physical facts, this is at least

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robustly true. There is some "inner sphere" of nearish possibilities, among which there is no difference without a qualitative difference. (For example, Lewis 1983, 364 proposes that these nearish possibilities be characterized by the absence of *alien natural properties*.)

I think something similar goes for qualitativism. Any natural version of the view would say not *just* that every fact is grounded in the qualitative "by accident"—but also that this is *robustly* true. So for Adams' argument to count against qualitativism, it's enough that Possible Symmetry—or some other possibility relevantly like it— count as nearish, and not radically different in its metaphysical structure from the actual world.

The key feature of Adams' story is that it involves *qualitative symmetry-breaking*. Possible Symmetry involves a qualitative symmetry between the two spheres, and in either case where one of the two spheres is destroyed, that qualitative symmetry between them is broken. If such cases are possible at all, they seem to be consistent with the world's metaphysical structure being just as it is. For instance, our best physical theories apparently have models that involve qualitative symmetries and symmetry-breaking, which suggests that there are possible configurations of actual-world physical stuff relevantly like Adams' two spheres. These aren't possibilities that add in alien metaphysical exotica.

Indeed, there are reasons to accept even the stronger claim that the *actual* world involves qualitative symmetry-breaking. I'll gesture at two tentative arguments for this.

Consider the total contents of space-time. Whatever they are like, the universe could have included not just them, but also a second isolated duplicate. In that case, the world would have been qualitatively symmetric—and the actual world is one way that symmetry could have been broken. But if *this* is possible, then wouldn't the *other* way of breaking the symmetry also have been possible? That is, if there had been two grand copies of our actual qualitative world, it would have been possible for *either* copy to have existed alone. So it could have been possible for things to be qualitatively just as they are, but with different individuals existing. So (using the principle of modal logic that what could have been possible is possible) things could have been qualitatively as they are, but non-qualitatively different. If this is right, then we have a parallel argument to Adams', this time for the conclusion that, not only *could* there have been facts which are not entailed by the qualitative facts, but in fact there *are* such facts.

The second argument depends on how some issues in philosophy of physics turn out (for discussion see e.g. Castellani 2003; see also Brighouse 1997). There are certain symmetry transformations that can be applied to solutions of the basic laws

of physics—for instance, symmetries of the state space for fundamental particles, or symmetries of the space-time manifold—for which we might accept the following two claims:

(1) The situations described by solutions which are related by one these transformations differ with respect to the configuration of certain physical structure, but differ in no *qualitative* respect.

(2) Our world is described by a solution that is related to others by such transformations.

We might also accept this principle about how the formalism relates to metaphysical possibility:

(3) If there is a solution to the basic laws of physics describing a situation in which *P*, then possibly *P*.

If those three claims are right, then again we can run a version of the modal argument with the conclusion that Qualitative Supervenience is *actually* false.

(I'll note in passing that, to my mind, considerations about what the laws of physics permit provide the most serious reasons to take seriously possibilities like Adams'—not brute modal intuition.)

It is a bit speculative whether we really live in a universe with physical laws that make these premises plausible, but at any rate I doubt the qualitativist wants their view held hostage to certain answers to the *physical* questions. (Compare Dasgupta's remarks on the possibility of qualitative indiscernibles 2009, sec. 2.) Of course, I don't expect qualitativists to accept at face value the claims about *metaphysical possibility* that these arguments rely on. Rather, I expect them to make the same sort of moves as I'll consider below in response to Adams' Possible Symmetry. These moves work just as well for possible qualitative symmetry-breaking as they do for actual qualitative symmetry-breaking. So I don't think that this difference is where the action is. In light of this, I'll suppose for simplicity that the qualitativist accepts the *necessity* of Qualitative Supervenience.

3   From Impossibility to Indeterminacy

Dasgupta suggests that modal arguments like these equivocate. He distinguishes between "strict possibility" and "loose possibility", and he explains this distinction

by appeal to counterpart theory (see Lewis 1986, ch. 4, esp. pp. 232–3).[5] His picture (as I understand it) is that possible worlds are purely qualitative patterns; so there is no immediate sense to be made of possible worlds that differ haecceitistically: for instance, there aren't two possible worlds, one in which one sphere is destroyed, and another in which the other is. There is just one qualitative pattern where a sphere is destroyed and another survives. But we can still make sense of a "loose" sense in which there are two different possibilities. There are two different ways of assigning a surviving counterpart and a destroyed counterpart to the two symmetric spheres. So it is loosely possible that one sphere survives, and also loosely possible that the other survives instead. Possible Symmetry is true, when understood in in this loose sense—and Qualitative Supervenience is false. (Of course, for the qualitativist this talk about particular counterpart spheres will also ultimately have to be explained in terms of qualitative patterns.)

In contrast to this loose notion of possibility

> … is the notion that quantifies over what I called "possible worlds" earlier and might be called "strict" possibility. This is the notion on which the earlier necessitation principle is true, that if a fact Y holds in virtue of some facts, the Xs, then every possible world in which the Xs obtain is also a possible world in which Y obtains. (Dasgupta 2013, 120)

So the *strict* modal notion is the one for which the Grounds Settle principle is true—and thus Dasgupta affirms the *strict* sense of Qualitative Supervenience.[6]

I'm not entirely sure what strict necessity is. It has something to do with what arises from the nature of the world's fundamental constituents. Whatever it is, I will assume that it lives up to its name: it is a metaphysical modality, which obeys standard principles of modal logic. I'll assume that strict necessity and strict possibility are

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[5]Dasgupta makes this distinction most explicitly in a different context, dealing with modal objections to a "comparativist" view about quantities like mass. But he clearly intends the lessons to carry over to individuals. (See especially 2013, sec. 3; 2014, sec. 10).

[6]I should note that Dasgupta is more cautious about this link in later work:

> Having distinguished these senses of possibility the question arises as to which notion of possibility makes [Grounds Settle] true. Is it the fundamental sense ['strict' in the 2013 article] or the reduced sense? And if the latter (context sensitive) notion, what are the relevant contexts? This is a deep question, but it is beyond the scope of the current paper so I will not try to settle it here. (2014, fn. 45).

I'm not sure why he is reluctant here to affirm his earlier claim. But if the "strict" (or "fundamental") sense *isn't* the one connected to Grounds Settle, then distinguishing these two sorts of possibility wouldn't be helpful for responding to the modal argument—since the response we're considering here is precisely that the argument equivocates between "strict" and "loose" possibility, and Possible Symmetry is true in the loose sense.

dual to one another, and I'll understand the cognates "impossible", "compatible", and "entails" in the usual ways for strict modality.[7] I'll assume furthermore that these strict modalities obey the modal logic S5. It is not obvious that Dasgupta should be happy to accept this—especially in light of some of the consequences I'll explore.[8] Some of the principles of S5 are controversial for other reasons as well (e.g., Salmon 1989). But I think S5 makes a good baseline assumption: as we'll see, there are quite a few other moving parts we'll need to keep track of, so it's good to try to keep this part relatively simple.

Moreover, these logical assumptions fit very naturally with the idea that strict possibility "quantifies over … 'possible worlds'" (as Dasgupta says in the passage quoted above). For short, I'll say "*P*-world" as an abbreviation for "possible world in which *P* obtains" (or "possible world at which *P*"). I understand Dasgupta to be affirming familiar "Leibnizian biconditionals" for strict modalities:

Possible Worlds. *P* is strictly necessary iff every world is a *P*-world.
      P is strictly possible iff some world is a *P*-world.
      P strictly entails *Q* iff every *P*-world is a *Q*-world.
      P is strictly compatible with *Q* iff some *P*-world is a *Q*-world.

(Moreover, I'll take these principles to be necessary.) It is coherent to hold that it is (strictly) contingent what possible worlds there are—that different possible worlds have different "perspectives" on what possible worlds are like. One might hold that *W* is a *P*-world, but there is also a world *V* according to which *W* is *not* a *P*-world. But there is nothing in what Dasgupta says that suggests this is his picture. Rather, this is a natural and simple assumption (for arbitrary worlds *V* and *W*):

Stable Worlds. *W* is a *P*-world iff at *V*, *W* is a *P*-world.

Given this and Possible Worlds, the characteristic axiom of S5 follows:[9]

(4) If *P* is strictly possible, then it is strictly necessary that *P* is strictly possible.

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[7]To be explicit: *P* is strictly impossible iff not-*P* is strictly necessary; *P* is strictly possible iff *P* is not strictly possible; *P* and *Q* are strictly compatible iff (*P* and *Q*) is strictly possible; *P* strictly entails *Q* iff *P* is not strictly compatible with not-*Q*.

[8]Indeed, in personal communication Dasgupta reports doubts about whether the modal axiom T holds for strict possibility: if *P*, then it is strictly possible that *P*. We will see below why this matters.

[9]If *P* is strictly possible, then there is a *P*-world *V*. Given Stable Worlds, at every world, *V* is a *P*-world. Thus at every world, there is a *P*-world, and by Possible Worlds, it is strictly necessary that there is a *P*-world. Then by Possible Worlds again, it is strictly necessary that *P* is strictly possible.

If strict possibility doesn't obey S5, then more needs to be said to make it clear how it is supposed to work.

To avoid equivocation, in what follows I'll focus on the strict modal notion: the one linked to qualitative possible worlds, to the Grounds Settle principle, and to Qualitative Supervenience. When I use modal words without qualification, I intend them to be understood in the "strict" sense, whatever that is.

Other philosophers have endorsed this picture, in broad outline: a space of qualitative possible worlds, with "loose" notions of possibility and necessity explained using counterparts. But the defender of Qualitative Supervenience fills in this outline in a distinctive way. If Qualitative Supervenience is to be true in any sense, this requires that there is a notion of necessity other than the loose one. Moreover, this must be a "strict" notion of even *de re* necessity: otherwise the strict sense of Qualitative Supervenience—which says de re facts are necessitated by qualitative facts—would not even be intelligible. To defend Qualitative Supervenience, then, requires parting ways with other "Anti-Haecceitists" who invoke a similar background picture—including the de re modal skeptic (such as Quine 1976; see Fine 2005 for discussion), or the Lewisian "cheap haecceitist" (at least as presented by Russell 2013). The key question—a question which does not arise for these other views—is how "strict" modalities are supposed to interact with the non-qualitative. Dasgupta's remarks on this aspect of his account are brief and indirect, so I'll be a bit out on an interpetive limb.[10] But this issue is central to understanding Qualitative Grounds. Until I understand the sense in which non-qualitative facts are *entailed* by qualitative facts, I don't really grasp the sense in which the non-qualitative requires nothing more than the qualitative—that is, the sense in which the non-qualitative is *grounded* in the qualitative.

Dasgupta recognizes the strict possibility of qualitatively symmetric spheres (see

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[10]In personal communication, Dasgupta tells me that he now thinks the defender of Qualitative Grounds simply ought to give up Qualitative Supervenience, and with it the thesis that Grounds Settle; it should be replaced with some other thesis in a similar spirit that does not require the intelligibility of *strict* de re possibility or necessity. For instance, one might hold that no two worlds differ with respect to individualistic facts without differing with respect to qualitative facts, where possible worlds are understood in some way which is *not* tied directly to strict necessity and possibility by the Possible Worlds principle above. This idea deserves attention (for one way of developing it, see Russell 2013); but it is also a serious departure from the standard view of grounding (in the narrow sense), which is supposed to strengthen the notion of necessary connection. (For dissent, though, see Skiles 2015.) Moreover—as I noted in footnote 6—if the qualitativist retreats from even the *strict* reading of Grounds Settle, it is unclear what dialectical point is served by distinguishing "strict" and "loose" senses of possibility: for if Grounds Settle is not true in either sense, then not even the strict sense of Qualitative Symmetry is at odds with Qualitative Grounds. My hunch is that while there is a view around here worth taking seriously, the idiom of grounding is not especially helpful for articulating it. In any case, in this paper I focus on how things go with Grounds Settle, rather than without it.

2014, sec. 11), and he also recognizes the strict possibility of a world where one of two previously symmetric spheres is destroyed. He has no objection to either of the underlying *qualitative* stories here. (Indeed, he argues in general that "it would be somewhat perverse if [qualitativism] put additional constraints on what sorts of general facts can hold" 2009, 49.) That much is fine. But Possible Symmetry is a claim about de re possibility. What exactly should we say about it? If Dasgupta's diagnosis of equivocation is to help as a response to the original modal argument, the uniformly strict reading of Possible Symmetry must not be true. Accordingly, Dasgupta says that the defender of Qualitative Grounds "should concede that she can make no sense of the *strict* possibility" of the other sphere being destroyed (adapted to this setting from 2013, 14).

Let's consider things from the perspective of the symmetric world: to simplify the presentation, we'll suppose for now that this is how things actually are. There are two symmetric spheres. Moreover, the qualitative description Q^+, according to which a sphere is spontaneously destroyed, is strictly possible. Let x be one of the two symmetric spheres. Then the necessity of Qualitative Supervenience implies:

(5) It is not both strictly possible that (Q^+ and x survives) and strictly possible that (Q^+ and x does not survive).

In other words, at least one of these is strictly impossible. It's tempting to say that *both* are strictly impossible. But given standard modal logic, this is not an option. It's a theorem of even the quite weak modal system K that, for any P, if Q^+ is possible, then Q^+ is compatible with at least one of P or not-P.[11] Indeed, more generally, it's tempting to understand some of Dasgupta's claims about not being able to "make sense of" de re possibilities as saying that there *aren't* any strict de re possibilities—or in other words, that all non-qualitative propositions are strictly impossible. But this is ruled out for the same reason. If P is non-qualitative, then if anything is strictly possible at all, at least one of P or $\neg P$ is also strictly possible. So there are at least some non-qualitative strict possibilities.

Qualitative Supervenience requires that there are not two destruction possibilities, qualitatively alike, which differ concerning whether it is x that survives. Modal logic requires that there is at least one such possibility. So there must be exactly one possibility. Either Q^+ strictly necessitates that x survives, or Q^+ strictly necessitates that x does not survive. This is surprising de re determinism. What in the world's symmetric qualitative structure could account for this determinism concerning one

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[11] If both alternatives are impossible, then $\Box(Q^+ \to \neg P)$ and also $\Box(Q^+ \to P)$, which together imply $\Box\neg Q^+$ using K (since the logical truth $(Q^+ \to \neg P) \to (Q^+ \to P) \to \neg Q^+$ is necessary).

particular sphere—that *it* must be destroyed, if any sphere is? It seems strange for the qualitativist to admit the strict possibility of this non-qualitative scenario, but it seems just as strange (or maybe even stranger) for the qualitativist to admit the strict *necessity* of this non-qualitative scenario—which is what Qualitative Supervenience requires.

Here is a way to to make sense of this odd situation. Dasgupta's intuitive thought was that strict possibility doesn't really apply to these non-qualitative matters—that the qualitativist "can make no sense" of the strict possibility of a particular sphere's destruction. It is tempting to articulate this thought by denying that either sphere's destruction is strictly possible—and also denying that their survival is strictly possible—and also denying that either of these is strictly necessary. Denying all of these would amount to giving up central conditions on possibility and necessity. But there is an alternative way of articulating the intuitive thought. Perhaps when Dasgupta says one "can make no sense" of a certain strict possibility, he intends to *reject* the claim that it is possible, without *affirming* the claim that it is impossible. Here's a way to spell out this sort of rejection: for each sphere, there is *no determinate fact of the matter* whether its survival is strictly possible or not.

What does "no determinate fact of the matter" mean? This is hard to explain—but as I'll discuss below, Dasgupta, at least, is already committed to the intelligibility of such a notion, since he uses it in his explanation of what possible worlds are like (though "determinate" is my word, not his). He doesn't say much about it, but I'll offer my own suggestions.

Some questions don't have answers which are settled by the world's structure; any choice of answer would be arbitrary. Philosophers, mathematicians, and scientists often come up against this—they say "it makes no sense to ask" certain questions, or that they are "meaningless" or "nonsense"—despite the fact that these questions evidently are made up of meaningful words put together grammatically, and they are intelligible enough that we might at least *wonder* what their answers are. Is two an element of seven? Which inertially moving objects are at rest? Which space-like separated events are simultaneous? Is aleph-nought odd? Is "heavier" a comparison of weight or mass? I'm not saying these questions have no answers, but there is some sort of "factual defectiveness" about them (as Field 1994 puts it; see also Magidor 2013).

Another important (and more directly relevant) example of this sort of defectiveness arises in David Kaplan's exposition of *Anti-Haecceitism* (1975, 723). This term has several meanings (a different one came up in Section 2)—but here's how Kaplan introduced it:

> … Anti-Haecceitism holds that for entities of distinct possible worlds there is

no notion of trans-world being. … Each, in his own setting, may be clothed in attributes which cause them to *resemble* one another closely. But there is no metaphysical reality of sameness or difference which underlies the clothes.

Kaplan clearly distinguishes Anti-Haecceitism from an alternative view, which says that individuals are world-bound:

Although the Anti-Haecceitist may seem to assert that no possible individual exists in more than one possible world, that view is properly reserved for the Haecceitist who holds to an unusually rigid brand of metaphysical determinism.

So when Kaplan says "there is no notion of trans-world being", he is not saying that individuals in distinct worlds are *not* identical to one another. Rather, he is saying that the question of whether they are identical does not even make metaphysical sense—"there is no metaphysical reality of sameness or difference". In short, I take the view to be that there is *no determinate fact of the matter* whether things in different possible worlds are identical or not.

It's natural to understand Dasgupta as advocating a view like Kaplan's Anti-Haecceitism. (This is in addition to the other sort of Anti-Haecceitism that he explicitly endorses: Qualitative Supervenience.) Possible worlds consist just in patterns of qualitative facts. If W is a Q^+-world—a world where one sphere is destroyed and another survives—then for any particular sphere x in the *symmetric* world, "there is no fact of the matter as to whether W is a world in which [x survives]" (adapted to this setting from 2013, 116; see also 2014, 26).[12] I think

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[12]I should address some interpretive difficulties: while I think that this "no fact of the matter" claim is Dasgupta's position on possible worlds, this isn't entirely clear from the text. (Thanks to Meghan Page for pressing me on this.)

The first complication, which I already noted, is that I am transposing these principles from Dasgupta's discussion of the possibility of mass-doubling, rather than haecceitistic switching. It is clear that Dasgupta intends the cases to be analogous.

The second complication is that the "no fact of the matter" passage I just quoted comes from Dasgupta's presentation of an *objection* to his view. He goes on to say that this objection uses "an incorrect model of how a possible world represents my laptop's mass" (pp. 9–10). But I think that the "no fact of the matter" claim is not the part of the objection that Dasgupta rejects, but rather a part that he accepts. The key point of the reply Dasgupta offers is to distinguish between "strict" possibility, which is tied to possible worlds, and "loose" possibility, which is tied to counterparts. The objector's "incorrect model" neglects the counterpart side of the story. But Dasgupta accepts the objector's proposal as correct when it comes to *strict* possibility and possible worlds. Indeed, at the end of his discussion, Dasgupta reiterates that the defender of his favored view "can concede that she can make no sense of a uniformly doubled world," (p. 119) and in a parallel passage, "the structuralist should concede that in the fundamental sense of possibility, the objections under discussion are well taken" (2014, 24).

the picture is that there is no determinately correct way to identify individuals in a qualitative possible world: so while it is determinate that W is a world where *some* sphere survives, there is no fact of the matter which sphere it is.

If the question of whether P has this "defective" status—if "there is no fact of the matter" whether P—then I'll say it is *indeterminate* whether P. (Cognates: it is *determinate whether P* iff it is not indeterminate whether P. It is *determinate that P* iff it is determinate whether P, and also P.) To explore the consequences of this view, I'll make some working assumptions for reasoning with indeterminacy; they are modest, but not uncontroversial.

First, I'll take classical logic for granted. This is disputed. For instance, Field (2003) argues that if there is no fact of the matter whether P, then one ought not accept P-or-not-P.[13] I don't know whether this is right, but it's still worth working out the consequences of a classical approach. One reason is just that non-classical logic is hard. There are some well-known non-classical logics on offer (like those of Kleene and Łukasiewicz), but they face serious technical obstacles when it comes to conditionals, determinacy operators, restricted quantification, and modality. A serious non-classical theory of Qualitative Grounds would have engage with these tricky details, and it isn't clear how they're going to work out. It makes sense to explore how things go with the logic we have, and not just fret about the logic we don't have. The other reason is that, despite what Field says, classical logic might really be the right way to reason about indeterminacy after all, as many others hold.

I'll also assume a fairly standard principle about the logic of determinacy: any determinate consequence of a determinate fact is also a determinate fact:

Transfer of Determinacy. If it is determinate that (if P then Q), and it is determinate that P, then it is determinate that Q.

In various places I also take some simple logical facts to be determinate. Perhaps more controversially, I'll also assume that the principles of *modal* logic are determinate, as well as the principles Possible Worlds and Stable Worlds. Finally, I'll suppose that the metaphysical principle Qualitative Supervenience is not just supposed to be true, but *determinately* true—and indeed, determinately necessary. (Details of the background logic are given in Appendix A.) Again, this is not to say there is no principled way to reject these premises. But they make a natural starting place for our investigation of this Anti-Haeccetist (in Kaplan's sense) development of Qualitative Grounds.

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[13]In conversation Dasgupta has suggested to me that he also thinks a non-classical approach might be the way to go here.

Here is the proposal for how to understand the claim that (from the perspective of the symmetric world) we "can make no sense of the strict possibility" of a particular sphere surviving.

Indeterminate Possibility. For each sphere x, it is indeterminate whether it is strictly possible that (Q^+ and x survives).

I think this a pretty good way to put a better face on the odd de re determinism that follows from Qualitative Supervenience. We can say that *either* Q^+ strictly entails that x survives, *or* Q^+ strictly entails that x doesn't survive, but it is indeterminate which. The qualitative structure of worlds need not arbitrate between the two options.

As a point of comparison, recall the supervaluationist's standard line about vagueness: it's determinate that *some* member of the sorites series is the smallest heap, but it's *indeterminate which one it is* (e.g. Keefe 2000, 185). Stalnaker (1981) takes a similar line on counterfactual conditionals: his view says that, for a symmetric sphere x, it's determinate that *either*, if one sphere had been destroyed, it would have been x, *or* if one sphere were destroyed, it wouldn't have been x. But it's indeterminate *which* of these two counterfactual conditionals is true. The view I'm exploring says that indeterminacy applies not just to how things *would* have been, but also to how they *could* have been.

Given natural assumptions, the Kaplanian view that trans-world identity is indeterminate implies Indeterminate Possibility. Consider the Symmetry-Breaking World W—a world at which the qualitative story Q^+ obtains. Let's suppose, furthermore:

(6) Determinately, W is a Q^+-world.

Then the determinate necessity of Qualitative Supervenience and Possible Worlds together tell us that "W is a P-world", "Q^+ is compatible with P" and "Q^+ entails P" are determinately equivalent.[14] Transfer of Determinacy then implies that if any of these is indeterminate, so are the others. The Kaplanian idea, which I take Dasgupta to endorse, says that in the Symmetric World, for any sphere x,

(7) It is indeterminate whether at W, x survives.

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[14]By the determinacy of Possible Worlds, $\Box(Q^+ \to P)$ determinately implies that every Q^+-world is a P-world. Since it's determinate that W is a Q^+-world, this determinately implies that W is a P-world. This in turn determinately implies that some Q^+-world is a P-world, and thus (by the determinacy of Possible Worlds) Q^+ is compatible with P. By the determinate necessity of Qualitative Supervenience, this in turn determinately implies that Q^+ entails P, completing the circle.

So Indeterminate Possibility follows.

This is how things look from the Symmetric World: for a given sphere x, determinately the Symmetry-Breaking World W is either a world where x survives or a world where x doesn't survive, and accordingly it's determinate that at least one of these is possible. But there's no fact of the matter which it is: there is no fact of the matter whether W is a world where x survives, and accordingly there is no fact of the matter whether it is *possible* for x to be the surviving sphere.

Indeterminate Possibility seems like a plausible way of reconciling apparent possibilites like Adams' with Qualitative Supervenience. The idea is that, while there aren't really two different possibilities that differ merely in which sphere is destroyed, neither possibility is determinately ruled out. Insofar as Possible Symmetry seems true to us, this is because we are mistakenly taking what is *not determinately impossible* to be straightforwardly possible, in the strict sense.[15]

4 Infectious Indeterminacy

Indeterminate Possibility is a narrow thesis, about some specific possibilities in a qualitatively symmetric world. In this section I'll examine a way of generalizing this thesis to apply to *all* strict de re possibilities:

Qualitative World. If it's not qualitative whether P, it's indeterminate whether W is a P-world.

It's not clear that the defender of Indeterminate Possibility should accept this stronger version. Even so, it will be instructive to explore its consequences, because it turns out that we can also derive similar consequences from more modest premises.

To simplify presentation, in the following let's suppose that the Symmetry-Breaking World W is actual. There is exactly one surviving sphere: let's call it Lucky. Then Qualitative World implies, in particular:

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[15] I'd like to clarify the relationship between the view here under investigation and the version of Anti-Haecceitism presented by Russell (2013). That view, like Dasgupta's, distinguishes between "possible worlds" and "possibilities", and rejects Qualitative Supervenience for "possibilities". It also appeals to indeterminacy (or "non-factuality" as it is called there). But the indeterminacy is not in what is *possible*, but rather in the non-qualitative facts themselves. Furthermore, Russell (2013) does not posit any *modality* corresponding to possible worlds, and thus does not affirm *any* sense of Qualitative Supervenience. So unlike Dasgupta's view, that version is not congenial to Qualitative Grounds, and it *is* congenial to the kind of de re indeterminacy discussed in the next section.

(8) It is indeterminate whether at W: Q^+ and Lucky survives.

Since "W is a P-world" and "Q^+ is compatible with P" are determinately equivalent:

(9) It is indeterminate whether possibly: Q^+ and Lucky survives.

This conclusion is analogous to the one we drew concerning spheres in the Symmetric World. The key difference is that we are now discussing a sphere which actually *does* survive—and this difference matters. Since we have assumed that the modal axiom T—if P, then possibly P—is determinate, then by Transfer of Determinacy, what is determinate is determinately possible. So (9) implies

(10) It is not determinate that Q^+ and Lucky survives.

If we suppose, as we have, that the qualitative fact Q^+ *is* determinate, then

(11) It is not determinate that Lucky survives.

So we can conclude:

Indeterminate Survival. No sphere determinately survives.

The indeterminacy of possible worlds infects de re modality, which in turn infects arbitrary non-qualitative matters. Can the qualitativist accept this?

Our starting point was the Qualitative Grounds thesis: non-qualitative facts are grounded in qualitative facts. But now we have arrived at the view that, in a sense, there *are no* non-qualitative facts: there is no fact of the matter what particular individuals are like (at least in cases of qualitative symmetry-breaking). Call this conclusion the Qualitative Facts thesis. It amounts to a kind of anti-realism about the individualistic: individuals are not just excluded from *fundamental* reality; rather, there are no determinate facts about them at all. (For further discussion of this view, see Russell 2013; MS.) Is this conclusion absurd? Is it at odds with qualitativism?

Recall that one of the reasons for investigating qualitativism was to illuminate other views, including more limited views about special kinds of individuals, such as fundamental particles, regions of space-time, or numbers. When it comes to these applications, I think restricted versions of Qualitative Facts are sober hypotheses. By analogy, it's commonplace to say that there is no fact of the matter concerning

which things are at absolute rest, or which space-like separated events are absolutely simultaneous. I see no obstacle to similar anti-realist positions that say (for example) that there is no fact of the matter concerning what particular space-time regions are like (see Russell 2014; compare Dasgupta 2011).

Other cases of de re indeterminacy along the lines of Indeterminate Survival are harder to accept. Suppose the two spheres in the Symmetry-Breaking World are inhabited. Then it would be very strange for an inhabitant of the surviving sphere to think that *there is no fact of the matter* whether *she* survived (compare Adams 1979, 22; see also van Cleve 1985, 105). Likewise, if we really do live in a world of pervasive broken qualitative symmetry (as I suggested we might in Section 2) then the argument for Indeterminate Survival would also tell us that our own lives are full of de re indeterminacy. There is no determinate fact of the matter whether it's *you* who is alive, rather than some possible duplicate. Maybe we can be brought over to this way of thinking, but it won't be easy.

The question also arises whether Qualitative Grounds is compatible with the absence of non-qualitative facts. To put the concern roughly, if non-qualitative facts are to be grounded in the qualitative, don't there have to *be* such non-qualitative facts? To spell this out a bit, observe that Indeterminate Survival, together with the factivity of grounding

(12) If P in virtue of Q, then P.

implies

(13) For any sphere x, it is not determinate that x survives in virtue of Q.

This conclusion sounds like it conflicts with Qualitative Grounds. But it's not as bad as it sounds! For in fact (13) is compatible with

(14) Determinately, for some sphere x, x survives in virtue of Q.

The scope of "determinately" makes an important difference. Even if it is determinate that everything is grounded in the qualitative, it doesn't immediately follow that it is determinate specifically what grounds what. It might be that there is no fact of the matter whether *this* sphere survives, but it *is* determinate that, whatever it is that survives, its survival is grounded in the world's qualitative pattern.

At least by the letter, Indeterminate Survival (and Qualitative Facts more generally) is compatible with the determinate truth of Qualitative Grounds. Even so, it is

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natural to regard Qualitative Facts and Qualitative Grounds as competing views. Their combination is unnatural. Qualitative Facts says that the non-qualitative has a certain second-rate status: there is no fact of the matter. If this was so, why should we think that they additionally have a *second* second-rate status: being *derivative?* This strikes me as multiplying obscure metaphysical notions beyond necessity.[16]

Consider again the analogies with absolute motion and absolute simultaneity. It's natural to conclude from special relativity that there is no fact of the matter concerning which events are absolutely simultaneous. But it would be odd to conclude that, *in addition*, which events are absolutely simultaneous is grounded in facts about relativistic space-time geometry. Rather, in cases like these, it appears that there is no work for a theory of grounding (parallel to Wilson 2014 about other cases). The work is already done by the theory of "no fact of the matter"—however exactly we fill that theory in.

So it looks like Indeterminate Survival wouldn't be great news for Qualitative Grounds. In this section, we derived Indeterminate Survival from a sweeping generalization of Indeterminate Possibility: the Qualitative World thesis. But on reflection, it really isn't obvious that this generalization fits well with the Anti-Haecceitist picture we've been sketching. Even if we can't determinately identify individuals in different worlds, Qualitative World goes beyond that: it has the upshot that we can't determinately identify individuals in the *same* world, so to speak. If Lucky is the surviving sphere in the Symmetry-Breaking World *W*, then Qualitative World says that *W* isn't determinately a world at which Lucky survives. It isn't especially plausible to pin this strong thesis on the qualitativist.

What I'll show next is that Indeterminate Survival can be derived from Indeterminate Possibility using more modest assumptions. These assumptions are natural, but they are still not obviously true: they involve subtleties about higher-order indeterminacy and contingent indeterminacy. So after presenting the derivation, I'll go on to consider two different models that escape pervasive de re indeterminacy by relaxing those assumptions.

5   Infectious Indeterminacy Redux

As before, let *V* be the Symmetric World and let *W* be the Symmetry-Breaking World. I'll argue that, given two further assumptions, these two claims are incom-

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[16]For what it's worth, Fine (2001) does invoke both a notion of ground and also a notion of "factuality" similar to the notion of determinacy involved here. Fine holds that whatever is grounded in fundamental reality is factual (p. 26). So in Fine's system, if Lucky's survival is non-factual, then it is not *also* grounded in fundamental qualitative facts.

patible:

Trans-World Indeterminacy. At V , for each sphere x , it is indeterminate whether at W x survives.

Intra-World Determinacy. At W , some sphere determinately survives.

Before I introduce the two further assumptions of the argument, I should emphasize a "zeroth" logical assumption about identity: Leibniz's Law.

(15) If $x = y$, then $A(x)$ iff $A(y)$

In particular, I am assuming that Leibniz's Law holds even in the somewhat controversial cases where $A(x)$ includes modal operators or determinacy operators. Given that everything is necessarily and determinately itself—necessarily $x = x$, and determinately $x = x$ —this has two immediate consequences:

(16) If $x = y$, then necessarily $x = y$.
(17) If $x = y$, then determinately $x = y$.

These follow from (15) by instantiating $A(z)$ with "Necessarily $x = z$ " and "Determinately $x = z$ ", respectively (Barcan 1947; Kripke 1971; Evans 1978). Since we are assuming the modal logic S5, we can also derive the necessity of distinctness:

(18) If $x \neq y$, then necessarily $x \neq y$.

But note that we don't have any proof of the *determinacy of distinctness*: for we are not here assuming the analogue of S5 for the logic of determinacy. (More on this below.)

In light of these consequences, you might worry that taking Leibniz's Law for granted already stacks the deck against the indeterminacy of trans-world identity. As we'll see in the next section, this isn't so: on its own, Leibniz's Law is compatible with Trans-World Indeterminacy.

Those preliminaries aside, the first substantive assumption is this principle:

DW. If at W it's determinate that P , then it's determinate that at W , P .

The rough idea is that what's determinate "from the inside" of a possible world is also determinate "from the outside". There are various ways of supporting this principle. Given that "At W, P", "possibly, Q^+ and P", and "Q^+ entails P" are determinately equivalent, we can deduce DW from any of these principles:

(19) If it's determinately necessary that P, then it's necessarily determinately necessary that P.

(20) If it's determinately possible that P, then it's necessarily determinately possible that P.

(21) W is determinately a P-world iff at V, W is determinately a P-world.

These are all ways of extending the idea of Stable Worlds and S5—which was that what P-worlds there are is the same for every world, and thus it isn't contingent what is possible or necessary. The more general idea is that what *determinate P*-worlds there are is the same for every world—and thus it isn't contingent what is *determinately* possible or *determinately* necessary. So the DW principle is a natural way of understanding the general picture of a "fixed" domain of possible worlds. But while it is natural, it isn't inevitable. In Section 6 I'll consider a model where it fails.

As before, let Lucky be the surviving sphere in W. This is the second substantive assumption:

DI. At V, for some sphere x, it's determinate whether x is Lucky.

DI is not an obvious principle. It is related to the determinacy of identity, but the relationship is not entirely straightforward. We can better understand why DI might be reasonable to accept by examining what it would take to give it up. I'll consider three arguments for DI. Each of these arguments deploys a different controversial premise; so giving up DI would involve giving up all three of these controversial premises. (For details see Corollary 1, Corollary 2, and Corollary 3 in Appendix A. Furthermore, in Section 6 I'll discuss how a result very similar to infectious indeterminacy can be derived even without supposing DI.)

The first thought is that, even if "trans-world identity" isn't completely determinate, there are still some constraints. For example, we might think that even just given a qualitative possible world, with no objective way of identifying which individual is which, it's still a determinate fact that I am not a hard-boiled egg in that world. In the case at hand, here's what we might say about the Symmetric World: while it's indeterminate *which* sphere might survive, it's determinate that *some* sphere could

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survive: that is, it's determinate that for some sphere x, it's possible that x survives. Putting this in terms of trans-world identity: while it's indeterminate which one of the symmetric spheres is identical to the surviving sphere in the Symmetry-Breaking World, it's still determinate that *one* of them is. Thus:

(22) At V, determinately, for some sphere x, at W x survives.

Since at W only Lucky survives, if follows that

(23) If (at W x survives) then x is Lucky.

Then (22) implies that at V, some sphere is Lucky. Then the determinacy of identity (which, recall, is a consequence of Leibniz's Law) implies that at V some sphere is *determinately* Lucky. This implies DI.

The second argument for DI uses these two premises:

(24) At V, something is determinately a sphere.

(25) At V, it's determinate whether Lucky is a sphere.

To put (25) another way, either Lucky is one of the two symmetric spheres, or else it's determinate that Lucky is neither sphere. In the first case, there is a sphere x which is Lucky, and so it's determinate that x is Lucky. In the second case, it's determinate that no sphere is Lucky, and since some x is determinately a sphere, it's determinate that x is not Lucky. Either way, we have DI.

We can in turn support (25) with these two premises:

(26) At V, it's determinate whether Lucky is anything at all.

(27) At V, determinately: Lucky is a sphere iff Lucky is anything at all.

The premise (26) is an instance of the general thesis that existence is determinate. David Lewis gives a well-known argument for this thesis. (He says "vague" rather than "indeterminate".)

> Vagueness is semantic indecision. But not all of language is vague. The truth-functional connectives aren't, for instance. Nor are the words for identity and difference, and for the partial identity of overlap. Nor are the idioms of quantification, so long as they are unrestricted. How could any of these be vague? What would be the alternatives between which we haven't chosen? (Lewis 1986, 212)

Lewis is targeting a specific conception of indeterminacy as "semantic indecision". That's not how I've characterized the sort of indeterminacy I've been talking about. (Though nothing I've said is really incompatible with that conception, either: it could be that the sense in which there is no fact of the matter which things are simultaneous is just that our linguistic practice does not settle which of many physically-on-a-par relations is to be called "simultaneity".) But the core argument against indeterminate existence is not essentially tied to this conception. The statement "Lucky is something" can be expressed with just pure first-order logic: $\exists x\, x = y$. (For exposition I've used the name "Lucky", but officially we can do everything with a variable y instead.) But, according to Lewis, each constituent is determinate—as long as the quantifier is unrestricted—and so the entire formula has a determinate extension. So it's determinate whether Lucky is something. (Sider 2003 defends this style of argument, and in particular the claim that there are not multiple candidate extensions for the unrestricted existential quantifier.) Of course, there is much more to be said about whether existence can be indeterminate, but this is not the place.

As for the premise (27), the idea is that, besides the two spheres, there isn't anything else around in the symmetric world which Lucky could plausibly be. This is not entirely uncontroversial. You might think that in V, Lucky is a merely possible sphere—a non-spatiotemporal being which *could* have been a sphere. Or you might think Lucky is an indeterminate sphere—a special sort of object which is distinct from each of the spheres, though not *determinately* distinct from either of them. In either case, Lucky would be something other than a sphere—something exotic.[17]

The third argument for DI is defended by Evans (1978). I mentioned earlier that the necessity and determinacy of identity can be derived from Leibniz's Law, and that the necessity of distinctness can be derived if we assume some additional modal logic: that it isn't contingent what is necessary. Evans' argument assumes an analogous premise for determinacy:

Higher-Order Determinacy. It's determinate whether it's determinate that P.

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[17]For an extended defense of "mere possibilia", see e.g. Williamson (2013). For critical discussion of indeterminate objects, see e.g. McGee (1997). For a defense of them, see e.g. Barnes (2009).

Given this, we can deduce, in addition to the determinacy of *identity*, the determinacy of *distinctness*:[18]

(28) If $x \neq y$, then determinately $x \neq y$.

Both of these together tell us that it's determinate whether $x = y$. So we have DI as a special case: for either sphere x, it's determinate whether x is Lucky.

To sum up, giving up DI involves saying all three of the following things: (i) at the symmetric world V, it is not determinate that either sphere could survive; (ii) if Lucky is the surviving sphere at W, then at V, it is indeterminate whether Lucky is either one of the spheres—and thus either it is indeterminate whether Lucky is anything at all, or else it is not determinately ruled out that Lucky is something other than a sphere; and (iii) there is higher-order indeterminacy. (Alternatively, one might give up DI by rejecting some element of our background logic, such as Leibniz's law.)

Finally, we turn to the main argument. Given DW and DI, we can show that Trans-World Indeterminacy and Intra-World Determinacy are not both true. (For a formalization of this argument, see Theorem 2 in Appendix A.) For simplicity, we'll reason as if the Symmetric World V is actual. If Lucky is the surviving sphere, then Intra-World Determinacy says regarding the Symmetry-Breaking World W:

(29) At W, it's determinate that Lucky survives.

By DW:

(30) It's determinate that at W, Lucky survives.

At W, there is only one surviving sphere. So this holds for arbitrary x:

(31) Determinately, at W: if Lucky survives, then x is Lucky iff x survives.

We can then distribute "at W":

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[18]Determinately: if $x = y$, then determinately $x = y$. So if it's determinately *not* determinate that $x = y$, then it's determinate that $x \neq y$. Furthermore, if $x \neq y$, then it's not determinate that $x = y$, and so by Higher-Order Determinacy it's *determinately* not determinate that $x = y$. Putting the two parts together, if $x \neq y$, then it's determinate that $x \neq y$.

Note that Higher-Order Determinacy is a stronger premise than we really need. The key premise is the analogue of the modal axiom B: if P, then it's determinate that it's not determinate that not P.

(32) Determinately: if (at W, Lucky survives), then (at W, x is Lucky) iff (at W, x survives).

By Transfer of Determinacy, (30) and (32) imply

(33) Determinately, (at W, x is Lucky) iff (at W, x survives).

Since identity is not contingent, "at W, x is Lucky" is equivalent to "x is Lucky". So:

(34) Determinately: x is Lucky iff (at W, x survives).

Then we can conclude (using Transfer of Determinacy again):

(35) If it's determinate whether x is Lucky, then it's determinate whether (at W, x survives).

DI says that at V there is a sphere x for which it *is* determinate whether x is Lucky. So this sphere is such that it's determinate whether at W, it survives. This contradicts Trans-World Indeterminacy.

In short, once again the indeterminacy of cross-world identifications *infects* ordinary "intra-world" de re matters. Starting from the hypothesis that there is no fact of the matter about what individuals are like at other possible worlds, we can deduce that there is no fact of the matter about how particular things actually are.

Here's what's happened so far. The qualitativist accepts a form of Anti-Haecceitism—namely, that all non-qualitative facts are entailed by qualitative facts. This thesis conflicts with the possibility of qualitative symmetry-breaking, as in the case of Adams' spontaneous sphere-destruction. In section 3, I proposed (taking my cue from some remarks of Dasgupta's) that a natural line of response for the qualitativist is to take up a second form of Anti-Haecceitism—namely, that trans-world identity and de re possibility are indeterminate. But in this section we have observed that, given the two further premises DW and DI, along with standard assumptions about the logic of possibility, determinacy, and identity, this version of Qualitative Grounds collapses to the more radical view I called Qualitative Facts—namely, that even *non-modal* singular matters are indeterminate. In section 4, I argued that this is an unhappy consequence for Qualitative Grounds.

So here is the next question for the Anti-Haecceitist qualitativist who accepts this logical framework and wishes to avoid collapse: what do things look like if we give up one of these two further premises DW or DI?

6 Two Models

Keeping Trans-World Indeterminacy and Intra-World Determinacy means giving up at least one of DW or DI. This presents us with two ways to go. The first way involves embracing contingency in the determinate modal facts, and the second way involves embracing higher-order indeterminacy. I'll briefly examine each of them. (For details see Appendix B.)

If DW fails, then—putting things very roughly—there are determinate facts within a world which other worlds can't see from the outside. More generally, different worlds can have different perspectives on what the determinate facts about worlds are, and what the determinate facts about possibility are. I find this a bit dizzying, but we can give a model to at least show that it is consistent.

The model combines familiar Kripke semantics for possibility with familiar supervaluationist semantics for indeterminacy, producing a two-dimensional semantics. Instead of just evaluating sentences at *possible worlds*, we'll evaluate them at *pairs* of a possible world together with a "frame of reference" (or "precisification"). The idea is that, if the truth-value of P varies with the frame of reference within a single possible world, then there is no determinate fact of the matter whether P. To evaluate what is *determinate* at a world-frame pair, we consider what is constant when the frame of reference is shifted and the world is held fixed. To evalute what is *necessary* at a world-frame pair, we consider what is constant when the world is shifted and the frame of reference is held fixed (cf. Einheuser 2006; Russell 2013, 404).

Here's the key feature we need to get a countermodel for DW: which frames of reference are "accessible" from a pair (W, F) should depend not just on the frame F, but also on the world W. A simple example is shown in Figure 1. In this model, there are two worlds V and W (the rows of the diagram) and two frames of reference F_1 and F_2 (the columns). Within each frame of reference, every world is accessible from every other—so the logic of S5 holds (determinately). Holding the possible world fixed, the accessibility relation for determinacy is an equivalence relation—so there is no higher-order indeterminacy (necessarily).

But the *interaction* between the two accessibility relations is non-trivial. Consider things from the perspective of (W, F_1). Here, not only does a survive, but it is *determinate* that a survives, and it is determinately *necessary* that only a survives. But at (W, F_1), the pair (V, F_1) is □-accessible; and at (V, F_1) it is *not* determinately necessary that only a survives. (V, F_2) is Det-accessible from (V, F_1), and at (V, F_2) it is possible that b survives; so at (V, F_1) it is not determinately impossible that b survives.

From the perspective of the Symmetry-Breaking World W, everything is perfectly
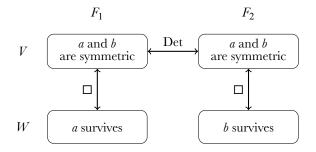
Figure 1: A model of contingently determinate possibility

determinate: it's determinate which thing survives, and it's determinate how things *could* be. But from the perspective of the Symmetric World V, it's indeterminate what world W is like, and it's indeterminate which trans-world identities hold. According to this picture, it isn't just that there is indeterminacy in the modal realm: it's *contingent* what modal indeterminacy there is—or indeed whether there is any at all.

It wasn't at all obvious that this was the intended picture of how strict de re possibility should work. But it looks like the most natural way of developing the combination of Intra-World Determinacy and (contingent) Trans-World Indeterminacy. The alternative response to the argument in Section 5 is to give up DI. This is also consistent, but it doesn't turn out to look very natural.

Recall that DI says that, in the Symmetric World V, for some sphere x it's determinate whether x is Lucky. Giving this up requires affirming that distinctness facts can be indeterminate. Furthermore, it requires rejecting all three of the arguments I sketched in Section 5. So in addition to saying that there can be x and y such that it's indeterminate whether $x = y$, the second model also verifies all three of these things:

- At V, it's not determinate that there is any sphere x such that at W x survives.
- At V, it's indeterminate whether Lucky is a sphere.
- There is higher-order indeterminacy: it's indeterminate which facts are determinate.

Getting higher-order indeterminacy is relatively simple: the accessibility relation for determinacy need not be an equivalence relation. In particular, in our model, there are world-frame pairs such that (W, F_1) is Det-accessible from (W, F_0), but not vice versa (see Figure 2). So we can have P true at (W, F_0)—though not determinately true—but also *determinately false* at (W, F_1).

Allowing higher-order indeterminacy makes logical room for indeterminate distinctness. But in order to come up with models, we need something different from standard Kripke-style models. Elizabeth Barnes proposes that the determinacy operator should be interpreted using *counterparts*—and this does the trick (2009; see also Stalnaker 2003).[19] The idea is that when we shift frames of reference, we also shift the values of our individual variables—replacing each individual with a Det-*counterpart*. The reason this trick is helpful is that the function that takes things to their counterparts need not be one-to-one: two distinct things can have the same Det-counterpart, so distinct things need not be *determinately* distinct. (We do require that counterparthood is a *function*, though. We are upholding Leibniz's Law, so while we can tolerate indeterminate distinctness facts, we cannot abide scandalous indeterminate identity facts.)

Here's a model (Figure 2). There are three frames of reference F_0, F_1, F_2. In F_0, everything is world-bound: neither sphere in the symmetric world V is Lucky, the surviving sphere in W. Lucky is just some third thing. But this is not *determinately* the case. (If it were determinate, then both spheres in V would determinately fail to survive at W, which is not what we want.) There is an alternative frame of reference F_1 according to which one of the symmetric spheres in W has the same Det-counterpart as Lucky. And there is yet another frame F_2 where it is the *other* symmetric sphere that shares a Det-counterpart with Lucky. So neither sphere is determinately distinct from Lucky, and neither sphere determinately fails to survive at W.

This shows that Trans-World Indeterminacy and Intra-World Determinacy are consistent with DW, if distinctness and existence can both be indeterminate. But here's an additional odd feature of this model. While it's true at (V, F_0) that no sphere determinately survives at W, this isn't *determinately* true at that pair. At either of the Det-accessible pairs (V, F_1) or (V, F_2), one sphere in particular does determinately survive at W. So Trans-World Indeterminacy is true in this model, but it isn't determinately true. If determinate truth is a norm on assertion, then this means that even were Trans-World Indeterminacy true, it wouldn't be assertible. In this model,

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[19]Note though, that while the idea is the same, the details here differ significantly from Barnes' version. For example, Barnes' proposal turns on context-sensitivity: which things count as "counterparts" of a thing—and thus what properties it counts as having "determinately"—depends on how that thing is picked out. Context-sensitivity plays no role in the version I am presenting. The approach is also different in important details from David Lewis's counterpart theory for possibility (1968; 1986). Lewis's version violates Leibniz's Law for modal predicates: for instance, $x = y \wedge \Diamond Fx \wedge \neg \Diamond Fy$ can be satisfied, basically because the identity pair (a, a) can have a non-identity pair (b, c) as a counterpart-pair. Also, Barnes and Lewis each argue for taking counterpart interpretations metaphysically seriously. In contrast, I'm just using counterparts as a technical device to show that certain claims are logically consistent.
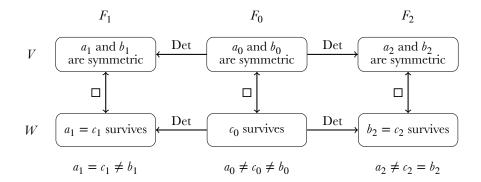
Figure 2: A model of higher-order indeterminacy and indeterminate distinctness

it is *indeterminate* whether trans-world identity is indeterminate!

As it turns out, this isn't an accidental feature of this particular model. We can show that—given DW—there is a conflict between these two claims.

Intra-World Determinate Determinacy. In W, some sphere *determinately* determinately survives.

Determinate Trans-World Indeterminacy. In V, it's *determinate* that for each sphere x, it's indeterminate whether x survives.

Crucially, unlike the original conflict, this argument does not turn on the additional premise DI (though it still uses DW). The proof is presented in Appendix A, Theorem 3. So even though rejecting DI does make Determinate Survival and Indeterminate Possibility consistent with one another, it's an unstable resolution. It can only get us precarious trans-world indeterminacy—too precarious for us to even safely assert that it is really there.

If everything is grounded in the qualitative, then everything supervenes on the qualitative. Then the qualitative facts in a symmetric world metaphysically necessitate one specific way that the qualitative symmetry could be broken, even when both alternatives seem possible. I argued in Section 3 that to put a brave face on this odd position, one can say that there is no determinate fact of the matter *which way* the symmetry must be broken. This fits with a more general view, that trans-world identity and de re modal facts are often indeterminate. The difficulty for this view is that indeterminacy is infectious: de re modal indeterminacy may lead to pervasive de re indeterminacy. To stop the infection from spreading requires giving up principles about the interaction between modality and indeterminacy. These principles are natural, but one can consistently give them up. As I see it, this is the best

hope for making sense of determinate non-qualitative facts with qualitative grounds: trans-world identity and de re modality are *contingently indeterminate*.

A   Logical Matters

This paper—especially the last two sections—used a lot of different machinery, since I have made arguments that turn on subtle interactions between modality, determinacy, identity, and quantification. It is worth being explicit about exactly what this machinery is and how I take it to work.

I'll start by making some things explicit about the background logic. We can regiment the arguments in this paper using a formal language that combines quantification, identity, modal operators, and determinacy operators. The atomic formulas include $S'x$ ("x survives"), $S'y$ ("y survives"), etc., as well as primitive sentences Q_V and Q_W (corresponding to qualitative descriptions of the Symmetric World and the Symmetry-Breaking World, respectively). As usual, formulas are closed under propositional connectives, and if φ is a formula, then so are $\Box\varphi$, Det φ, $\forall x\, \varphi$, $\forall y\, \varphi$, etc. For our purposes, we can read all the quantifiers as restricted to *spheres*. We'll use the following abbreviations:

- $\Diamond\varphi$ abbreviates $\neg\Box\neg\varphi$.
- $\exists x\, \varphi$ abbreviates $\neg\forall x\, \neg\varphi$.
- Ex abbreviates $\exists y(x = y)$ ("x is a sphere").
- $\Delta\varphi$ abbreviates Det $\varphi \vee$ Det $\neg\varphi$ ("it's determinate whether φ").
- V φ abbreviates $\Box(Q_V \to \varphi)$ ("at possible world V, φ").
- W φ abbreviates $\Box(Q_W \to \varphi)$ ("at possible world W, φ").
- Sx abbreviates $\exists y(x = y \wedge \forall z(S'z \leftrightarrow z = y))$ ("x is the only sphere that survives").

A **theory** T is a set of formulas which (a) includes every substitution instance of each tautology in propositional logic, and (b) is closed under the rules of modus ponens, universal generalization, necessitation, and the analogue of necessitation for the determinacy operator. That is, if we use the notation $\vdash \varphi$ to mean that φ is part of the theory T, then we have the following rules:

$$\frac{\vdash \varphi \qquad \vdash \varphi \to \psi}{\vdash \psi} \qquad \frac{\vdash \varphi}{\vdash \forall x\, \varphi} \qquad \frac{\vdash \varphi}{\vdash \Box\varphi} \qquad \frac{\vdash \varphi}{\vdash \text{Det}\, \varphi}$$

The idea is that a theory is supposed to consist not just of truths, but of claims which remain true when prefixed with as many determinacy, necessity, and universal quantifiers as you like. (This is mainly a way to keep notational cruft under

control, so we don't have to keep track of exactly how many "determinately"s and "necessarily"s belong in front of each assumption that we use.)

For any theory T, if Γ is a set of formulas and φ a formula, let $\Gamma \vdash \varphi$ mean that there are some $\psi_1, \ldots, \psi_n \in \Gamma$ such that $\vdash (\psi_1 \wedge \ldots \wedge \psi_n) \to \varphi$. We say Γ is **inconsistent** (in T) iff $\Gamma \vdash \varphi$ for every φ, and we say φ and ψ are **equivalent** (in T) iff $\vdash \varphi \leftrightarrow \psi$, which we abbreviate $\varphi \equiv \psi$. For any theory, we have the usual deduction theorem and cut and weakening rules:

$$\frac{\Gamma, \varphi \vdash \psi}{\Gamma \vdash \varphi \to \psi} \qquad \frac{\Gamma_1 \vdash \varphi \qquad \Gamma_2, \varphi \vdash \psi}{\Gamma_1, \Gamma_2 \vdash \psi} \qquad \frac{\Gamma_1 \vdash \varphi}{\Gamma_1, \Gamma_2 \vdash \varphi}$$

We also have the intersubstitutability of logically equivalent formulas.

The **Core Theory** is the smallest theory that includes every instance of each of the following schemata.

$$\Box(\varphi \to \psi) \to \Box\varphi \to \Box\psi \qquad\qquad \Box\varphi \to \varphi$$
$$\mathrm{Det}(\varphi \to \psi) \to \mathrm{Det}\,\varphi \to \mathrm{Det}\,\psi \qquad\qquad \mathrm{Det}\,\varphi \to \varphi$$
$$\forall x\,(\varphi \to \psi) \to \forall x\,\varphi \to \forall x\,\psi \qquad Ey \to \forall x\,\varphi(x) \to \varphi(y)$$

$$\forall x\,Ex \qquad\qquad \varphi \to \forall x\,\varphi \quad \text{if } x \text{ does not occur free in } \varphi$$
$$x = x \qquad\qquad x = y \to \varphi(x) \to \varphi(y)$$
$$\Diamond\varphi \to \Box\Diamond\varphi$$

The above amounts to the combination of a fairly standard bit of free logic for quantification, the logic S5 for necessity and possibility, and the logic T for determinacy. Note that the logic of necessity also implies the characteristic axiom of S4, $\Box\varphi \to \Box\Box\varphi$.

In what follows, we only consider theories which extend the Core Theory. The following basic logical facts hold for any such theory, and are presented without proof.

Lemma 1. *If Γ is a set of sentences, let $\Box\Gamma$ be the set $\{\Box\psi \mid \psi \in \Gamma\}$, and similarly for other operators. For any extension of the Core Theory, if $\Gamma \vdash \varphi$, then each of these follow:*

$$\Box\Gamma \vdash \Box\varphi \qquad \mathrm{Det}\,\Gamma \vdash \mathrm{Det}\,\varphi \qquad \forall x\,\Gamma \vdash \forall x\,\varphi \qquad \mathrm{W}\,\Gamma \vdash \mathrm{W}\,\varphi$$

Furthermore, for a sentence φ, if $\Gamma, \varphi \vdash \psi$ then each of these follow:

$$\Box\Gamma,\ \Diamond\varphi \vdash \Diamond\psi \qquad \forall x\,\Gamma,\ \exists x\,\varphi \vdash \exists x\,\psi$$

Lemma 2 (Free Logic Facts)**.** *For any extension of the Core Theory:*

(a) $\qquad\qquad\qquad\qquad$ *If x is not free in φ, then* $\quad \exists x\, \varphi \equiv \varphi \equiv \forall x\, \varphi$

(b) $\qquad\qquad\qquad\qquad$ *If $\neg Ex \vdash \varphi$, then* $\quad \Box \forall x\, \varphi \vdash \forall x \Box \varphi$

Since *Sx* abbreviates "*x* is the only thing that survives", *Sx* implies both "*x* is something" and also "nothing other than *x* survives". Thus:

Lemma 3 (Survival Facts)**.** *For any extension of the Core Theory:*

(a) $\qquad\qquad\qquad\qquad\qquad Sx \vdash Ex$

(b) $\qquad\qquad\qquad\qquad\qquad Sx \vdash Sy \leftrightarrow x = y$

Theorem 1. *Possible Symmetry contradicts the necessity of Qualitative Supervenience (see Section 2). In particular, let* **Sym** *and* **Sup** *abbreviate (respectively)*

$$\Diamond(Q_W \wedge Sx) \wedge \Diamond(Q_W \wedge \neg Sx)$$

$$(Q_W \wedge Sx) \to \Box(Q_W \to Sx)$$

For any extension of the Core Theory, $\Diamond \exists x$ **Sym** *and $\Box \forall x$* **Sup** *are inconsistent.*

Proof. First, $\Box$ **Sup** implies $\neg$ **Sym**. For

$$\Box((Q_W \wedge Sx) \to \Box(Q_W \to Sx))$$

implies

$$\Diamond(Q_W \wedge Sx) \to \Diamond\Box(Q_W \to Sx).$$

By S5, this in turn implies

$$\Diamond(Q_W \wedge Sx) \to \Box(Q_W \to Sx)$$

By duality, we then have

$$\neg(\Diamond(Q_W \wedge Sx) \wedge \Diamond(Q_W \wedge \neg Sx)).$$

Since $\Box$ **Sup** $\vdash \neg$ **Sym**, we can conclude

$$\Box \forall x \Box \text{ **Sup** } \vdash \neg \Diamond \exists x \text{ **Sym** }.$$

The only thing left to do is to get rid of the extra box on the left-hand side; that is, to show that $\Box\forall x\,\mathsf{Sup}$ implies $\Box\forall x\,\Box\,\mathsf{Sup}$. This holds because Sup holds trivially whenever x does not exist, and so Sup satisfies the converse Barcan formula:

$$\Box\forall x\,\mathsf{Sup}\vdash\forall x\,\Box\,\mathsf{Sup} \qquad\qquad\text{Lemma 2b}$$
$$\Box\Box\forall x\,\mathsf{Sup}\vdash\Box\forall x\,\Box\,\mathsf{Sup} \qquad\qquad\text{Lemma 1}$$
$$\Box\forall x\,\mathsf{Sup}\vdash\Box\Box\forall x\,\mathsf{Sup} \qquad\qquad\text{S4}$$
$$\Box\forall x\,\mathsf{Sup}\vdash\Box\forall x\,\Box\,\mathsf{Sup} \qquad\qquad\text{Cut}$$

Putting the two parts together completes the proof. $\qquad\qquad\square$

Lemma 4 (Determinacy Facts). *In any extension of the Core Theory:*

$$\mathsf{Det}(\varphi\leftrightarrow\psi)\vdash\Delta\varphi\leftrightarrow\Delta\psi$$
$$x=y\equiv\mathsf{Det}(x=y)$$
$$\exists x\,\mathsf{Det}\,Ex,\ \mathsf{Det}\,\forall x\,\varphi\vdash\exists x\,\mathsf{Det}\,\varphi$$

The **World Theory** is the smallest theory that extends the Core Theory with each instance of the following two schemata:

$$\Diamond(Q_V\wedge\varphi)\leftrightarrow\Box(Q_V\to\varphi)$$
$$\Diamond(Q_W\wedge\varphi)\leftrightarrow\Box(Q_W\to\varphi)$$

These basically say that Q_V and Q_W are each true at just one world.

Lemma 5 (World Facts). *In any extension of the World Theory:*

(a) $$\mathsf{W}\,\varphi\wedge\mathsf{W}\,\psi\equiv\mathsf{W}(\varphi\wedge\psi)$$
(b) $$\mathsf{W}\,\neg\varphi\equiv\neg\,\mathsf{W}\,\varphi$$
(c) $$\mathsf{W}\,\varphi\equiv\mathsf{V}\,\mathsf{W}\,\varphi$$
(d) $$\vdash\mathsf{W}\,\forall x(\varphi\leftrightarrow\mathsf{W}\,\varphi)$$
(e) $$x=y\equiv\mathsf{W}(x=y)$$

(Similar facts hold for the world V.)

Proof. For part d, note first that since $\Box(Q_W\to\varphi)$ implies $Q_W\to\varphi$ and $Q_W\wedge\varphi$ implies $\Diamond(Q_W\wedge\varphi)$,

$$Q_W\vdash\Box(Q_W\to\varphi)\to\varphi$$
$$Q_W\vdash\varphi\to\Diamond(Q_W\wedge\varphi)$$

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So in the World Theory $Q_W \vdash \varphi \leftrightarrow W \varphi$. Then, since by vacuous quantification $\forall x \, Q_W$ is equivalent to Q_W,

$$\vdash Q_W \to \forall x (\varphi \leftrightarrow W \varphi)$$

$\square$

Lemma 6 (Quantifier Export). *If x is not free in φ, then*

$$W \, \exists y \, (\varphi \wedge V \, \exists x \, \psi) \equiv W \, \exists y \, V \, \exists x \, (W \, \varphi \wedge \psi)$$

Proof.

$$
\begin{aligned}
W(\varphi \wedge V \, \exists x \, \psi) &\equiv W \, \varphi \wedge W \, V \, \exists x \, \psi && \text{Lemma 5a} \\
&\equiv V \, W \, \varphi \wedge V \, \exists x \, \psi && \text{Lemma 5c} \\
&\equiv V(W \, \varphi \wedge \exists x \, \psi) && \text{Lemma 5a} \\
&\equiv V \, \exists x \, (W \, \varphi \wedge \psi) && \text{vacuous quantification}
\end{aligned}
$$

Furthermore, from Lemma 5d we can derive:

$$W \, \exists y \, (\varphi \wedge V \, \exists x \, \psi) \equiv W \, \exists y \, W(\varphi \wedge V \, \exists x \, \psi)$$

Our conclusion then follows by substitution of equivalents. $\square$

In Section 5 I gave an informal argument that, given two principles DW and DI, Trans-World Indeterminacy is inconsistent with Intra-World Determinacy. That is, if at W it's determinate that Lucky survives, then at V for some sphere x it's determinate whether at W x survives. We'll now formalize this argument.

A **DW-theory** is a theory which extends the World Theory with each instance of

(DW) $\qquad\qquad\qquad$ $W \, \text{Det} \, \varphi \to \text{Det} \, W \, \varphi$

The other principle we appealed to in Section 5 was DI, which says: in the Symmetric World, for some sphere x it's determinate whether x is Lucky. In our official version, though, rather than a name "Lucky" we'll use a variable:

(DI(y)) $\qquad\qquad\qquad$ $V \, \exists x \, \Delta \, x = y$

Let TWI ("Trans-World Indeterminacy") and IWD ("Intra-World Determinacy") abbreviate the following:

(TWI) $\qquad\qquad\qquad$ $V \, \forall x \, \neg \Delta \, W \, Sx$

(IWD) $\qquad\qquad\qquad$ $W \, \exists y \, \text{Det} \, Sy$

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Theorem 2. *For any DW-theory, TWI is inconsistent with*

(IWD+DI) $\qquad\qquad$ W $\exists y$ (Det $Sy \wedge$ DI(y))

Proof. By Quantifier Export:

(1) $\qquad\qquad$ IWD+DI $\vdash$ W $\exists y$ V $\exists x$ (W Det $Sy \wedge \Delta\, x = y$)

Since $Sy \vdash Sx \leftrightarrow x = y$, and W($x = y$) is equivalent to $x = y$,

$$\text{W } Sy \vdash \text{W } Sx \leftrightarrow x = y$$
$$\text{Det W } Sy \vdash \text{Det(W } Sx \leftrightarrow x = y) \qquad \text{Lemma 1}$$
$$\text{Det(W } Sx \leftrightarrow x = y) \vdash \Delta \text{ W } Sx \leftrightarrow \Delta\, x = y \qquad \text{Lemma 4}$$
$$\text{W Det } Sy \,\wedge\, \Delta\, x = y \vdash \Delta \text{ W } Sx \qquad\qquad \text{DW, propositional logic}$$
$$\text{W } \exists y \text{ V } \exists x \,(\text{W Det } Sy \wedge \Delta\, x = y) \vdash \text{W } \exists y \text{ V } \exists x \, \Delta \text{ W } Sx \qquad \text{Lemma 1}$$

The right-hand side is equivalent to V $\exists x\, \Delta$ W Sx (since the $\exists y$ is vacuous and W V φ is equivalent to V φ). This in turn implies $\neg$ V $\forall x \neg \Delta$ W Sx (using Lemma 5b). So, putting this together with (1), IWD+DI implies the negation of TWI. $\qquad\square$

In Section 5 I presented three arguments for the premise DI. The next results formalize these arguments. The first argument used the "moderate Haecceitist" premise that—even if it is indeterminate *which* symmetric sphere survives in *W*—it is *some* symmetric sphere that survives in *W*.

(H) $\qquad\qquad\qquad\qquad$ V $\exists x$ W Sx

Lemma 7. *In any extension of the World Theory,*

$$\text{H, } \text{ W } Sy \vdash \text{DI}(y)$$

It follows that H *and* IWD *together imply* IWD+DI.

Proof. Since $Sx, Sy \vdash x = y$ (Lemma 3), and

$$\text{W } x = y \,\equiv\, x = y \,\equiv \text{Det } x = y$$

we then have

$$\text{W } Sx, \text{W } Sy \vdash \Delta x = y$$

Thus, by Lemma 1 and the fact that V $\forall x$ W Sy and W Sy are equivalent:

$$\text{V } \exists x \text{ W } Sx, \text{ W } Sy \vdash \text{V } \exists x \, \Delta\, x = y$$

That proves the first part: H, W $Sy \vdash$ DI(y).

For the second part, we need to apply this general fact to the Lucky sphere in W in particular. Note first that since Det Sy implies Sy, we also have

$$\text{H, W Det } Sy \vdash \text{DI}(y)$$

By Lemma 1 and the fact that H is equivalent to W $\forall y$ H,

$$(2) \qquad\qquad \text{H, W } \exists y \text{ W Det } Sy \vdash \text{W } \exists y(\text{W Det } Sy \wedge \text{DI}(y))$$

Finally, Lemma 5d tells us that

$$\vdash \text{W } \forall y(\text{Det } Sy \leftrightarrow \text{W Det } Sy)$$

This lets us replace each W Det Sy in (2) with Det Sy:

$$\text{H, } \text{ W } \exists y \text{ Det } Sy \vdash \text{W } \exists y(\text{Det } Sy \wedge \text{DI}(y))$$

$\square$

Corollary 1. *In any DW-theory, TWI, IWD, and H are inconsistent.*

The second argument used two premises: in the Symmetric World, there is a determinate sphere; and in the Symmetric World, it's determinate whether Lucky is a sphere.

Lemma 8. *For any extension of the World Theory,*

$$\text{V } \exists x \text{ Det } Ex, \quad \text{V } \Delta Ey \vdash \text{DI}(y)$$

It follows that V $\exists x$ Det Ex *and* W $\exists y$(Det $Sy \wedge$ V ΔEy) *together imply IWD+DI.*

Proof. Since ΔEy means Det $Ey \vee$ Det $\neg Ey$, and Det $Ey \vdash Ey$,

$$(3) \qquad\qquad \Delta Ey \vdash Ey \vee \text{Det } \neg Ey$$

$$(4) \qquad\qquad \Delta Ey \vdash \exists x\, x = y \ \vee \ \text{Det } \forall x\, x \neq y$$

Since $x = y \vdash$ Det $x = y$,

$$\exists x\, x = y \vdash \exists x \text{ Det } x = y$$

Furthermore, by Lemma 4,

$$\exists x \text{ Det } Ex, \ \text{Det } \forall x\, x \neq y \vdash \exists x \text{ Det } x \neq y$$

So in either case,

$$(5) \qquad\qquad \exists x \text{ Det } Ex, \ \Delta Ey \vdash \exists x\, \Delta\, x = y$$

The first part follows by adding V. The second part goes similarly to Lemma 7. $\square$

Corollary 2. *In any DW-theory,* *TWI* *and* $V \exists x$ Det *Ex* *are inconsistent with*

$$W \exists y (\text{Det } Sy \wedge V \Delta Ey)$$

The third argument for DI used the premise that there is no higher-order indeterminacy: it's determinate whether it's determinate that φ.

Lemma 9. *For any extension of the World Theory, if* $\vdash \Delta$ Det φ *for every* φ, *then* $V \exists x$ Ex *("at V, there is a sphere")) implies* DI(y). *So* ?? *and* *IWD* *together imply* *IWD+DI*.

Proof. Since $x = y$ is equivalent to Det $x = y$, if $\vdash \Delta$ Det $x = y$, then $\vdash \Delta x = y$, and thus:

(6) $\qquad\qquad Ex \vdash \Delta x = y$ $\qquad\qquad\qquad$ Weakening

(7) $\qquad\qquad V \exists x\, Ex \vdash V \exists x \Delta x = y$ $\qquad\qquad$ Lemma 1

$\hfill \square$

Corollary 3. *In any DW-theory such that* $\vdash \Delta$ Det φ *for every* φ, $V \exists x\, Ex$, *TWI, and* *IWD* *are jointly inconsistent.*

In Section 6, I claimed that even if we drop DI, we still have a clash between *determinately* determinate survival and the *determinacy* of trans-world indeterminacy. Here is an explicit proof of this fact.

Theorem 3. *In any DW-theory, the following are inconsistent:*

(8) $\qquad\qquad\qquad\qquad W \exists y$ Det Det Sy

(9) $\qquad\qquad\qquad\qquad V$ Det $\forall x \neg\Delta$ W Sx

(10) $\qquad\qquad\qquad\qquad V \exists x$ Det Ex

Proof. By Lemma 4,

(11) $\qquad\qquad\qquad (9), (10) \vdash V \exists x$ Det $\neg\Delta$ W Sx

Then by similar reasoning to Quantifier Export,

(12) $\qquad$ (8), $V \exists x$ Det $\neg\Delta$ W $Sx \vdash W \exists y\, V \exists x (W$ Det Det $Sy \wedge$ Det $\neg\Delta$ W $Sx)$

Using DW twice,

(13) $\qquad\qquad\qquad W$ Det Det $Sy \vdash$ Det Det W Sy

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Also, Det ¬Δ W *Sx* implies Det ¬ Det W *Sx*, which implies in turn Δ Det W *Sx*.

$$\text{(14)} \qquad\qquad \text{Det } \neg\Delta \text{ W } Sx \vdash \Delta \text{ Det W } Sx$$

Furthermore, since $Sy \vdash Sx \leftrightarrow x = y$, and $\text{W}(x = y)$ and $\text{Det } x = y$ are both equivalent to $x = y$, we also have:

$$\text{(15)} \qquad\qquad \text{Det Det W } Sy \vdash \text{Det}(\text{Det W } Sx \leftrightarrow x = y)$$
$$\text{(16)} \qquad\qquad \text{Det}(\text{Det W } Sx \leftrightarrow x = y) \vdash \Delta \text{ Det W } Sx \leftrightarrow \Delta x = y$$

Furthermore,

$$\text{(17)} \qquad\qquad \text{W Det Det } Sy \vdash \text{W Det } Sy$$

So, piecing together (13), (14), (15), (16), and (17) with a bit of propositional logic,

$$\text{W Det Det } Sy \;\wedge\; \text{Det } \neg\Delta \text{ W } Sx \vdash \text{W Det } Sy \;\wedge\; \Delta\, x = y$$

Adding W ∃*y* V ∃*x* to this, and putting it together with (11) and (12), we conclude

$$\text{(8)}, \text{(9)}, \text{(10)} \vdash \text{W } \exists y \text{ V } \exists x (\text{W Det } Sy \wedge \Delta x = y)$$

By Quantifier Export, the right-hand side is equivalent to

$$\text{W } \exists y (\text{Det } Sy \wedge \text{ V } \exists x \Delta x = y)$$

which is just IWD+DI. Furthermore, (9) straightforwardly implies TWI. So (8), (9), and (10) jointly imply both of these, and by Theorem 2 they are jointly inconsistent. □

B   Models

The next thing I'll do is briefly sketch two models for TWI and IWD. One of these is a countermodel for the principle DW, and the other is a countermodel for the higher-order determinacy principle $\Delta \text{ Det } \varphi$ (as well as the claim that at *V* there is a sphere that survives at *W*).

A **model** consists of the following components:

- A set of **worlds**.
- A set of **reference frames**.
- A set *D* of **individuals**.

- For each world-frame pair, a **domain** of individuals, which is a subset of D.
- For each world-frame pair, and for each n-place predicate, an **extension**, which is a subset of D^n.
- A Det-**accessibility relation** between world-frame pairs. This is required to be reflexive, and to obey the constraint that if (W', F') is Det-accessible from (W, F), then $W = W'$.
- For each pair of reference frames F and F', a Det-**counterpart function** $C_{F,F'} : D \to D$. For any reference frame F, $C_{F,F}$ is the identity function on D.

The constraint on Det-accessibility reflects the idea that the determinacy operator only shifts the frame of reference, and not the world. We could, of course, introduce accessibility relations and counterpart functions for the modal operators as well, but for our purposes there's no need: we can simply suppose that every world is $\Box$-accessible from every other, and $\Box$-counterparthood is just identity.

Here's how we define interpretations. An **evaluation point** (in a model M) is a triple of a world, a reference frame, and an assignment function which takes each individual variable to an individual in D. We define the following relations on evaluation points:

- $(W, F, g) \xrightarrow{\Box} (W', F', g')$ iff $F = F'$ and $g = g'$.

- $(W, F, g) \xrightarrow{\forall x} (W', F', g')$ iff $W = W'$, $F = F'$, g and g' assign the same value to every variable except possibly x, and $g'x$ is in the domain of (W, F).

- $(W, F, g) \xrightarrow{\text{Det}} (W', F', g')$ iff (W', F') is Det-accessible from (W, F), and $g'x = C_{F,F'}(gx)$ for each variable x.

Then we can straightforwardly give an inductive definition of the truth-value of a formula at an evaluation point a.

- $x = y$ is true at (W, F, g) iff $gx = gy$.

- For a primitive n-place predicate P, $Px_1, \ldots, x_n$ is true at (W, F, g) iff $(gx_1, \ldots, gx_n)$ is in the extension of P at W.

- $\varphi \wedge \psi$ is true at a iff φ is true at a and ψ is true at a; similarly for other propositional connectives.

- $\Box\varphi$ is true at a iff φ is true at every b such that $a \xrightarrow{\Box} b$.

- Det φ is true at a iff φ is true at every b such that $a \xrightarrow{\text{Det}} b$.

- $\forall x\, \varphi$ is true at a iff φ is true at every b such that $a \xrightarrow{\forall x} b$.

If φ is true at every evaluation point in a model M, we'll say $M \vDash \varphi$. If $M \vDash \varphi$ for every $\varphi \in \Gamma$, we say M is a **model of** Γ.

Theorem 4 (Soundness). *Every model is a model of the Core Theory.*

Proof. By induction. Verifying Leibniz's Law requires a secondary induction on the complexity of the formula $\varphi(x)$. Most of this is also straightforward, so we'll just work through one inductive step:

$$\text{If} \quad M \vDash x = y \rightarrow \varphi(x) \rightarrow \varphi(y) \quad \text{then} \quad M \vDash x = y \rightarrow \text{Det}\, \varphi(x) \rightarrow \text{Det}\, \varphi(y)$$

Let $a = (W, F, g)$ be any evaluation point in M, and suppose that $x = y$ and Det $\varphi(x)$ are each true at a. Let $b = (W', F', g')$ be any point such that $a \xrightarrow{\text{Det}} b$. Since $x = y$ is true at a, $gx = gy$, and so

$$g'x = C_{F,F'}(gx) = C_{F,F'}(gy) = g'y$$

which means that $x = y$ is also true at b. Also, since Det $\varphi(x)$ is true at a, $\varphi(x)$ is true at b. So, by the inductive hypothesis, $\varphi(y)$ is true at b. Since this holds for any b reachable from a, Det $\varphi(y)$ is true at a. $\qquad\square$

Now we consider the two models described in Section 6. In each of them, we have just two worlds V and W. The primitive sentence Q_V is true at (V, F) for each reference frame F, and no other pairs, and similarly for Q_W. It follows that they are both models of the World Theory.

First, the model of contingently determinate possibility (see Figure 1 in Section 6). We have two frames F_1 and F_2. D contains two individuals a and b. Every counterpart function is the identity function on D. The four world-frame pairs all have the same domain $\{a, b\}$. In (W, F_1) the extension of S includes only a, and in (W, F_2) the extension of S' includes only b. (Elsewhere, the extension of S' is empty.) The pairs (V, F_1) and (V, F_2) are mutually Det-accessible, and the pairs (W, F_1) and (W, F_2) are mutually Det-inaccessible.

Since Det-accessibility is an equivalence relation and counterparthood is just identity, it follows that this is a model of higher-order determinacy and determinate distinctness. But this is not a DW-model. In fact, W Det $S'x$ is true at (V, F_1, g), where

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$gx = a$. But Det $\mathbf{W}\, S'x$ is not true at this evaluation point, since there is another Det-accessible pair (V, F_2) at which $\mathbf{W}\, S'x$ is false of a.

Second, the model of higher-order indeterminacy (see Figure 2 in Section 6). We have three frames F_0, F_1, F_2. There are ten individuals in D:

$$a_0,\ b_0,\ c_0,\ d_0,\quad a_1 = c_1,\ b_1,\ d_1,\quad a_2,\ b_2 = c_2,\ d_2$$

The domains of world-frame pairs are given by

$$(V, F_0) \mapsto \{a_0,\ b_0\}$$
$$(W, F_0) \mapsto \{c_0,\ d_0\}$$
$$(V, F_1) \mapsto \{a_1 = c_1,\ b_1\}$$
$$(W, F_1) \mapsto \{a_1 = c_1,\ d_1\}$$
$$(V, F_2) \mapsto \{a_2,\ b_2 = c_2\}$$
$$(W, F_2) \mapsto \{b_2 = c_2,\ d_2\}$$

The counterpart functions are the obvious ones, where

$$C_{F_0, F_i} = \left\{ \begin{array}{ll} a_0 & \mapsto a_i \\ b_0 & \mapsto b_i \\ c_0 & \mapsto c_i \\ d_0 & \mapsto d_i \end{array} \right\}$$

For each frame F_i, in (W, F_i), the extension of S' includes just c_i. F_1 and F_2 are each accessible from F_0, but not vice versa. This has the consequence that $x \neq y \rightarrow$ Det $x \neq y$ is *false* at a point (W, F_0, g) where $gx = a_0$ and $gy = c_0$. The spheres in (V, F_0) are distinct from the spheres in (W, F_0)—but they are not determinately distinct. In the reference frame F_0, possible individuals are indeterminately world-bound.

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