## A Theory of Bondage

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# A Theory of Bondage 

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## I.

Let $A$ be an assignment of values to variables on which Marlon Brando is the value of ' $x$ ' and Shirley MacLaine is the value of ' $y$ '. In classical semantics, the open formula ("open sentence")
(1) $\quad(\exists x)(y$ is a sister of $x)$
is true under our value-assignment $A$ if and only if there is some element or other $i$ of the universe over which the variables range such that
(2) $y$ is a sister of $x$
is true under the value-assignment $A^{\prime} x_{i}^{\prime}$, a variant of $A$ that assigns $i$ instead of Brando as value for ' $x$ ' and is otherwise the same as $A$ (and so assigns Shirley MacLaine as value for ' $y$ '). In Tarski's terminology, $A$ satisfies (1) if and only if some modified value-assignment $A^{4} x_{i}^{\prime}$ of the sort specified satisfies (2). Assigning Warren Beatty as value for ' $x$ ' does the trick.

This simple example demonstrates a fact not often recognized: the quantifier phrase ' $(\exists x)$ ' is nonextensional. This follows from the fact that it is not truth-functional. Under the original value-assignment $A$,

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' $(\exists x)(x \neq x)$ ' is every bit as false as its matrix, ' $x \neq x$ ', yet ( 1 ) is true even though its matrix is false. The nonextensionality of a quantifier phrase is a surprising but trivial consequence of the way the quantifier works with a variable. The truth-value of (2) under $A$-and, for that matter, also the designatum of ' $x$ ' under $A$-are irrelevant to the truth-value of (1) under $A$. What matters are the designata of ' $x$ ' and ' $y$ ', and therewith the truth-value of (2), under modified value-assignments $A^{\prime} x_{i}^{\prime}$. The original value-assignment $A$ does not satisfy (2), but the value-assignment $A^{\prime x} x^{\prime}$ Beaty does, and that is sufficient for $A$ to qualify as satisfying (1). We achieve satisfaction by offering Brando's role to Beatty.

Under $A$, ' $x$ ' designates Brando and ' $y$ ' designates MacLaine. The variables ' $x$ ' and ' $y$ ' both occur in (1). The original value-assignment $A$ satisfies (1), although the particular value of ' $x$ ' under $A$ and the particular truth-value of (2) under $A$ do not matter in the slightest. When evaluating (1) under $A$, MacLaine is present, whereas Brando is nowhere on the set. Under $A$, (1) makes no mention of Brando. He has nothing to do with the success of (2) under $A^{\prime} x^{\prime}{ }^{\prime}$ beaty. Why does he still receive billing? More to the point, how is it that, under $A$, (1) makes no mention of Brando even though ' $x$ ', which occurs twice therein, designates Brando?

Frege admonished that one should never ask for the designatum or content of an expression in isolation, but only in the context of a sentence. This is his celebrated Context Principle. ${ }^{1}$ Extrapolating from Frege's prohibition, we should not inquire after the designatum of ' $x$ ' under $A$. Instead we should inquire after the designatum of the second ' $x$ ' in (1)—as distinct, for example, from the ' $x$ ' in (2). If ever there was a case in which Frege's Context Principle has straightforward application, this is it: the bound variable. So let us follow Frege's considered advice and ask: If ' $x$ ' as it occurs in (1) does not designate Brando under $A$, what exactly is the ' $x$ ' in (1) doing? Likewise, what is the extension of (2), under $A$, as (2) occurs in (1)?

Classical Tarski semantics does not specify what the second ' $x$ ' in (1) designates under the original assignment. This is because the second ' $x$ ' in (1) is not the variable ' $x$ ', which designates Brando under $A$. It is a bound occurrence of ' $x$ ', which does not. Classical semantics imputes

1. Gottlob Frege, The Foundations of Arithmetic: A Logico-Mathematical Enquiry into the Concept of Number, trans. J. L. Austin (Evanston, IL.: Northwestern University Press, 1968), x, 71, 73.

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semantic extensions to expressions (under assignments of values to variables), not to their occurrences in formulae. Classical semantics does not abide by the Context Principle. But Frege's admonition has a point. One reason for departing from classical semantics-and one possible motivation for the Context Principle-is the desire for universal principles of extensionality for designation and of compositionality for semantic content. (According to extensionality, the extension of a compound expression is a function of the extensions of its meaningful components, including the designata of the component designators. According to compositionality, the semantic content of a compound expression is a function of the contents of the meaningful components.) Even more important is our intuition concerning what is actually being mentioned in a particular context. Consider, for example, the following fallacious inference:

> In 1999, the president of the United States was a Democrat. The president of the United States = George W. Bush.
> Therefore, in 1999, George W. Bush was a Democrat.

The invalidity is partially explained by noting that whereas the definite description in the second premise designates Bush, there is no mention of Bush in the first premise. The argument's two occurrences of the phrase 'the president of the United States' thus do not designate the same thing. Though perhaps incomplete, the explanation is intuitive, even satisfying.

The Context Principle is not a blanket injunction against assigning semantic values to expressions simpliciter. Frege regarded the attributing of semantic values to expressions as legitimate only to the extent that such attribution is derivative from semantic attribution to the occurrence of those expressions in sentences. One need not adopt Frege's attitude in order to make perfectly good sense of attributing a semantic content and a designatum to an expression-occurrence. From the perspective of classical semantics, semantic attribution to occurrences may be regarded as derivative from the metalinguistic $T$-sentences (and similar metatheorems) derived from basic semantic principles. According to Frege, whereas 'Ortcutt is a spy' customarily designates a truth-value, the occurrence in 'Ralph believes that Ortcutt is a spy' instead designates a proposition (Gedanke). Similarly we may choose to say that whereas 'the president of the United States' customarily designates Bush, its occurrence in the major premise above instead designates the function that assigns to any time $t$, the person who is president of the United States
at $t$. The semantic value of the description that bears on the truth-value of the sentence is not Bush, but this function. ${ }^{2}$

My primary objective in what follows is to sketch a proper and natural way of doing quantificational semantics on expression-occurrences. I do this, in part, in the hope of warding off confusion that has resulted from doing occurrence-based quantificational semantics in improper or unnatural ways. In the closing sections below, I apply the occurrencebased semantic apparatus to two separate, seemingly unrelated, contemporary controversies. I do this not because one must adopt occurrencebased semantics in order to obtain the right results in connection with those controversies. On the contrary, in both cases classical expressionbased semantics suffices both to obtain and to justify those results, while occurrence-based semantics supplements the case. I do this, rather, because the two controversies are in fact closely related to one another: each is fueled almost entirely by the same pernicious misconception in occurrence-based semantics. In both cases, one needs to get a handle on occurrence-based semantics to see clearly what went wrong on the wrong side of the controversy, and hence in order to provide a full and definitive response.

At least as important-quite apart from, and independently of, these particular controversies-occurrence-based semantics illuminates. It upholds intuitions about what is actually mentioned, or at least about what is not actually mentioned, in such sentences as 'The temperature is rising' (Barbara Partee ${ }^{3}$ ) and 'In 1999, the president of the United States was a Democrat'. It reveals thereby what is right about the analysis of the fallacy mentioned two paragraphs up. As Frege knew, occurrence-based semantics reveals that there is a sense in which principles of extensionality and compositionality are upheld, at least in spirit (perhaps even in letter), despite the presence of nonextensional devices (for example, modal operators, temporal operators, 'believes that', or quotation). More important for my present purpose, occurrence-based semantics illuminates just what is going on when a quantifier binds a variable. Properly
2. Hence I reject Donald Davidson's appeal to "pre-Fregean semantic innocence," which I believe is based largely (though not entirely) on a confusion between expressionbased and occurrence-based semantics. See the closing paragraph of his "On Saying That," Synthese 19 (1968-69): 130-46. (See note 20 below.)
3. See by way of comparison Richard Montague's treatment of this sentence in his "The Proper Treatment of Quantification in Ordinary English," in Approaches to Natural Language: Proceedings of the 1970 Stanford Workshop on Grammar and Semantics, ed. J. Hintikka, J. Moravcsik, and P. Suppes (Dordrecht: D. Reidel, 1973), 221-42, at 240.
executed, occurrence-based quantificational semantics directly contradicts prevailing views about bound variables and pronouns. Occurrencebased quantificational semantics also reveals that principles of extensionality and compositionality are upheld with regard to the binding of variables despite the nonextensionality (strictly speaking) of quantifier phrases. It reveals how Frege could have accommodated variable-binding and, more important, how he should have done so. It also reveals how Fregean functions from objects to truth-values ("Begriffe"), and even Russellian functions from objects to singular propositions ("propositional functions"), emerge from constructions involving bound variables. Even if the Context Principle is wrong and classical expression-based semantics is the "right" or preferred way to do semantics-as I believeit remains that expression-based semantics is, as Frege insisted, a less discriminating by-product, or subtheory, of occurrence-based semantics. This alone justifies the present investigation.

Important Cautionary Note: Throughout this essay, I distinguish very sharply between an expression (for example, the variable ' $x$ ') occurring in a sentence or formula and the occurrence itself (e.g., the second occurrence of ' $x$ ' in (1)). Equivalently, I draw a sharp distinction between ascribing certain semantic attributes to an expression (of a particular language or semantic system) per se and ascribing those attributes to the expression "as it occurs in," or relative to a particular position in a larger expression (for example, a sentence) or stretch of discourse. ${ }^{4}$ It is essential in what follows that the reader be ever vigilant, paying extremely close attention to the distinction between expressions themselves and their occurrences. Many philosophers of language who think, habitually and almost instinctively, in terms of expression-occurrences and their semantic values-especially
4. An expression-occurrence standing within a formula must not be confused with a token of the expression, such as an inscription or an utterance. A token is a physical embodiment, physical event, or other physical manifestation of the expression (type). An occurrence of an expression is, like the expression itself, an abstract entity. For most purposes, an expression occurrence may be regarded as the expression together with a position that the expression occupies within a larger sequence of expressions.

In contemporary philosophy of language, it has become a common practice to attribute semantic values neither to expressions themselves nor to their occurrences but to expression-utterances. I regard this speech-act-centered conception of semantics a giant leap backward, lamentable in the extreme. See my "Two Conceptions of Semantics," in Semantics versus Pragmatics, ed. Zoltán Gendler Szabó (Oxford: Clarendon, 2005), 317-28, reprinted in my forthcoming Content, Cognition, and Communication (Oxford University Press).

Fregean and linguistics-oriented philosophers-habitually and almost instinctively reinterpret remarks explicitly about expressions occurring in a sentence as concerning not the expressions but their occurrences. Nearly everyone who thinks about expressions at all typically has at least some inclinations of this sort. How many letters are there in the name 'Nathan'? The reader with even the slightest inclination to give the incorrect answer 'six' is implored to remain on the alert and to make every effort in what follows to let intellect overcome inclination, instinct, and habit; else much of what is said will inevitably be seriously misunderstood. ${ }^{5}$

## II.

I assume the classically defined notion of semantic extension in what follows. Context Principle enthusiasts may take this to be Frege's notion of default or customary Bedeutung. In developing an occurrence-based semantics of variable-binding, I take my cue from Frege's theory of indirect (oblique, ungerade) contexts.

The variables occurring in (2) occur exclusively free there. Assignments of values to variables are assignments of designata to free occurrences. Under the original assignment $A$, the ' $x$ ' in (2)-that is, the occurrence of ' $x$ ' in (2)—designates Brando, the ' $y$ ' in (2) MacLaine. These are the default or customary designata of the variables ' $x$ ' and ' $y$ ' under $A$, that is, the designata of occurrences in extensional position and not within the scope of a variable-binding operator. ${ }^{6}$ The variables
5. The letters in 'Nathan' are four: ' $A$ ', ' $H$ ', ' $N$ ', and ' $T$ '. Two of these occur twice, making six letter-occurrences in all.

With some trepidation, I follow the common vernacular in speaking of "bound variables" in a sentence where what are mentioned are actually bound occurrences, or of "the initial quantifier of," or "the 'he' in" a sentence, and so on, where what is mentioned is actually an occurrence of a quantifier or the pronoun. I have taken care to see that my usage unambiguously decides each case. For example, there is only one lowercase, italic letter ' $x$ ' and only one English word 'he', but there are infinitely many occurrences of either, so that any talk of "the bound variables" (plural) of a formula containing no variable other than ' $x$ ', or of "the 'he" (with definite article) in a sentence, cannot sensibly concern expressions.
6. Positions within quotation marks and similar devices, including 'believes that', are not extensional. An occurrence of a well-formed expression $\zeta$ is said to be within the scope of an occurrence of a variable-binding-operator phrase ${ }^{\lceil }(\mathbf{B} \alpha){ }^{7}$, where $\mathbf{B}$ is a variable-binding operator and $\alpha$ is a variable, if the latter occurrence is the initial part of an occurrence of a well-formed expression of the form ${ }^{\lceil }(\mathbf{B} \alpha) \phi$, where $\phi$ is a formula and the former occurrence stands within that occurrence of ${ }^{\Gamma}(\mathbf{B} \alpha) \phi$. (It may
have their customary extensions in (2), and (2) is thereby false under $A$. Not all occurrences of variables have their customary extensions. Some occurrences deviate from the default value. On a natural extrapolation from Frege's explicit remarks, the occurrence of ' $x$ ' in 'Ralph believes that $x$ is a spy' has its indirect designatum (ungerade Bedeutung), under a value-assignment, designating its customary or default sense. This is because the ' $x$ ' is within the scope of an occurrence of 'believes that', which induces a semantic shift, whereby expressions take on their indirect designata in lieu of their customary designata. Alonzo Church has developed this idea by considering assignments of customary-sense values ("individual concepts") to variables instead of customary-designatum values. ${ }^{7}$

Fortunately, the matter of indirect designation does not concern us here. Our concern is with the semantics of ordinary bound variables. The outline is the same. The ' $x$ ' in (2) is a free occurrence, and consequently it has its customary extension in (2). But neither occurrence of ' $x$ ' in (1) has its customary extension in (1). This is because both occurrences ("the two bound variables in (1)") are within the scope of an occurrence of a shift-inducing operator.

Quantifiers are variable-binding operators. Like 'believes that', variable-binding operators induce the variables they bind to undergo semantic shift, but a shift of a different sort from intensional or "indirect" (oblique) operators. The occurrences of ' $x$ ' in (1) are no longer in default mode, designating their customary extension. They are in bondage. Classical semantics-the semantics of expressions, as opposed to their occurrences-is the customary semantics of default semantic values: the semantics of free occurrences. Classical semantics is thus the semantics of freedom. Bound variables have their bondage semantics, in many respects analogous to the semantics of indirect occurrences. One could say that the special kind of semantic shift that occurs when a quantifier binds a variable is precisely what variable-binding is.
be assumed that the universal quantifier is ' $\forall$ '-and as a notational convenience is routinely deleted-so that a universal-quantifier phrase written ${ }^{[ }(\alpha){ }^{7}$ is of the form $\left\lceil(\mathbf{B} \alpha){ }^{7}\right.$.)
7. Alonzo Church, "The Need for Abstract Entities in Semantic Analysis," American Academy of Arts and Sciences Proceedings 80 (1951): 100-112. Church does not follow Frege's Context Principle. Church's semantics is on expressions, not on their occurrences. He therefore does not distinguish between designatum and customary designatum, or between sense and customary sense, and has no notion of indirect designatum or indirect sense.

If a free variable has its default or customary extension, which is simply its value under a value-assignment, then what is the extension of a bound variable (of the occurrence, not the variable of which it is an occurrence)? A bound variable ranges over a universe of discourse. It is not that Brando is nowhere on the set. It is that he is part of a cast of thousands. Ranging is not the same thing as designating. The definite description 'the average man', as it occurs in 'The average man sires 2.3 children in his lifetime', does not designate a peculiar biological being that has very peculiar offspring. It ranges over a universe of relatively normal biological beings, each with a definite whole (nonfractional) number of relatively normal offspring. The description does not designate this universe; it ranges over it. Similarly, the bound variable does not designate the universe over which it ranges.

Bound occurrences of different variables of the same sort range over the same universe. Does the variable also designate? A standard view is that free variables (and occurrences of compound designators containing free variables) designate, whereas bound variables do not. An analogous view is generally assumed with regard to natural-language pronouns like 'he': deictic occurrences and some "pronouns of laziness" designate, whereas bound-variable anaphoric occurrences do not. Peter Geach, for example, criticizes "the lazy assumption that pronouns, or phrases containing them, can be disposed of by calling them 'referring expressions' and asking what they refer to. ${ }^{78} \mathrm{He}$ says of anaphoric pronoun-occurrences, "It is simply a prejudice or a blunder to regard such pronouns as needing a reference at all." ${ }^{9}$ Geach's thesis that anaphoric pronoun-occurrences other than pronouns of laziness do not designate is supported by his contention that such pronoun-occurrences are bound variables and his insistence that bound variables do not designate. This attitude (which I once shared) betrays a lack of analytical vision. With regard to the issue of whether anaphoric pronoun-occurrences designate, the prejudice or blunder, I contend, is on Geach's side. He is not alone.

A bound variable has its bondage extension, which is different from the variable's customary extension. In general, an occurrence of a meaningful expression in extensional position and not within the scope of a
8. Peter Geach, "Ryle on Namely-Riders," Analysis 21, no. 3 (1960-61), reprinted in his Logic Matters (Oxford: Basil Blackwell, 1972), 88-92, at 92.
9. Peter Geach, Reference and Generality (Ithaca, NY: Cornell University Press, 1962), at 125-26, and passim.
variable-binding operator has its customary extension under a valueassignment, whereas a bound occurrence has its bondage extension. ${ }^{10}$ The central idea is given by the following principle of identification, analogous to Frege's identification of ungerade Bedeutung with customary sense: The extension of a bound occurrence of an open expression in otherwise extensional position is the function from any potential value of the bound variable to the expression's customary extension under the assignment of that value. It is this function, rather than the extension of the open expression, that bears on the truth-value of sentences in which the open expression occurs bound.

More accurately, the extension of an occurrence depends on the number of variable-binding operators governing it. Let us call the extension, under a value-assignment $s$, of an occurrence of a well-formed expression $\zeta$ within the scope of an occurrence of a variable-binding-operator phrase ${ }^{\lceil }(\mathbf{B} \alpha){ }^{\rceil}$-where $\mathbf{B}$ is a variable-binding operator and $\alpha$ is a vari-able-and not within the scope of any other occurrence of a variable-binding-operator phrase or other nonextensional operator, the bondage extension of $\zeta$ with respect to $\alpha$ under s. Our theory of bondage starts with, and builds upon, the following principle.
$A_{1}$ : The bondage extension of a well-formed (open or closed) expression $\zeta$ with respect to a variable $\alpha$, under a valueassignment $s$, is $(\lambda i)$ [the customary extension of $\zeta$ under $s_{i}^{\alpha}$ ]-that is, the function that maps any element $i$ of the universe over which $\alpha$ ranges to the customary extension of $\zeta$ under the modified value-assignment that assigns $i$ to $\alpha$ and is otherwise the same as $s^{11}$


#### Abstract

10. When a quantifier or other variable-binding operator "quantifies into" an open expression-that is, when an occurrence of the open expression includes a variable occurrence bound by an external quantifier-occurrence, or other variable-binding operator-occurrence-I say that the external quantifier-occurrence, or other variablebinder occurrence, in addition to binding the variable occurrence, also binds the containing open-expression occurrence itself. The effect is that a quantifier (or other vari-able-binder) is said to bind not only variables, but also the open expressions that the quantifier (binder) "quantifies into." Thus the quantifier-occurrence in ' $(\exists x)\left(x^{2}=9\right)$ ' is said to bind not only the two occurrences of ' $x$ ' but also the occurrence of ' $x$ ' and even the occurrence of ' $x^{2}=9$ '. See, by way of comparison, Donald Kalish, Richard Montague, and Gary Mar, Logic: Techniques of Formal Reasoning (1964; 2nd ed., New York: Harcourt Brace Jovanovich, 1980), at 206, 311-12. 11. This function is the customary extension of ${ }^{\lceil }(\lambda \alpha)[\zeta]{ }^{\top}$ under $s$. Compare: The indirect extension of 'Snow is white' is the customary sense-the proposition that snow is white-which is the customary extension of 'that snow is white'. (Frege: "In indi-


The bondage extension of the variable ' $x$ ' with respect to itself is the identity function on the universe over which ' $x$ ' ranges. ${ }^{12}$ Each distinct variable with the same range thus has the same bondage extension, under any given value-assignment, with respect to itself.

Variables are not the only expressions that have bondage extension. Any well-formed expression that has extension does. (See note 10 above.) Occurrences of open formulae bound through an internal variableoccurrence range over a universe of truth-values. (OK, so it is a baby universe.) The bondage extension of a formula is what Frege misleadingly called a concept ('Begriff'), that is, a function from objects to truthvalues. Thus the extension of the occurrence of ' $x$ is bald' in ' $(\exists x)(y$ is a sister of $x \& x$ is bald)', under any particular value-assignment, is the function that maps any bald individual to truth ("the True") and any nonbald individual to falsehood ("the False"). More generally, the bondage extension of a formula $\phi$ with respect to a variable $\alpha$, under a valueassignment $s$, is the characteristic function of the class of objects $i$ from the range of $\alpha$ such that $\phi$ is true under $s_{i}^{\alpha}$. For most purposes, the bondage extension may be identified with this class in lieu of its characteristic function.

The extension of a doubly bound occurrence of a doubly open expression, like ' $x$ is a sister of $y$ ' or ' $x$ loves $y$ ', must be sensitive to the particular manner in which its internal variables are bound in a particular occurrence. Otherwise ' $(x)(\exists y)(x$ loves $y)$ ' collapses together with ${ }^{\prime}(y)(\exists x)(x$ loves $y)$. How shall this be accomplished?

Let $\alpha$ and $\beta$ be variables, and let $\phi_{(\alpha, \beta)}$ be any formula in which both $\alpha$ and $\beta$ occur free. Suppose an occurrence of $\phi_{(\alpha, \beta)}$ is within the scope of a quantifier-occurrence on $\beta$ that is itself within the scope of quantifier-occurrence on $\alpha$. That is, suppose we are considering a doubly embedding formula of the form
rect discourse the words have their indirect designata [ungerade Bedeutungen], which coincide with what are customarily their senses. In this case then the clause has as its designatum a thought [Gedanke], not a truth-value; its sense is not a thought but is the [customary] sense of the words 'the thought that . . '" [Über Sinn und Bedeutung].)
12. By contrast, though the occurrence of ' $y$ ' in ' $(\exists x) F y$ ' is free, its extension under a value-assignment $s$ is, strictly speaking, not the customary designatum. It is the bondage extension of ' $y$ ' with respect to ' $x$ ', which is the function that maps any element $i$ of the universe over which ' $x$ ' ranges to the customary designatum of ' $y$ ' under the modified value-assignment $s^{\prime} x_{i}$. This is the constant function to the customary designatum, $s\left({ }^{\prime} y\right.$ '), defined over the range of ' $x$ '. For most purposes, this may be replaced with $s\left({ }^{\prime} y\right.$ ') itself.
$(\mathbf{B} \alpha)\left(\ldots(\mathbf{C} \beta)\left[\ldots \phi_{(\alpha, \beta)} \ldots\right] \ldots\right)$
Whereas the occurrence of $\phi_{(\alpha, \beta)}$ still ranges over a universe of truthvalues, it occurs here doubly bound: by $\mathbf{B}$ with respect to $\alpha$ and by $\mathbf{C}$ with respect to $\beta$. We call the extension, under a value-assignment $s$, of an occurrence of a well-formed expression $\zeta$ within the scope of an occurrence of a variable-binding-operator phrase $\left.{ }^{\lceil }(\mathbf{C} \beta)\right)^{7}$, itself within the scope of an occurrence of a variable-binding-operator phrase ${ }^{\lceil }(\mathbf{B} \alpha){ }^{\top}$-where $\mathbf{B}$ and $\mathbf{C}$ are variable-binding operators and $\alpha$ and $\beta$ are variables-but not within the scope of any other occurrence of a nonextensional operator, the double bondage extension of $\zeta$ with respect to $<\alpha, \beta>$ under $s$. Doubly bound occurrences are governed by the following principle.
$A_{2}$ : The double bondage extension of a well-formed (open or closed) expression $\zeta$ with respect to an ordered pair of variables $\langle\alpha, \beta\rangle$, under a value-assignment $s$, is $(\lambda i)(\lambda j)$ [the customary extension of $\zeta$ under $\delta^{\alpha}{ }_{i}^{\beta}$ ]-that is, the function that maps any element $i$ from the range of $\alpha$ to the function that maps any element $j$ from the range of $\beta$ to the customary extension of $\zeta$ under the doubly modified value-assignment that assigns $i$ to $\alpha, j$ to $\beta$, and is otherwise the same as $s^{13}$

This singulary function to singulary functions may be replaced with its corresponding binary function. In the special case where $\zeta$ is a formula $\phi_{(\alpha, \beta)}$, the latter function maps any pair of objects, $i$ and $j$ (from their respective ranges), to the truth-value of $\phi_{(\alpha, \beta)}$ under $s_{i j}^{\alpha \beta}$. For most purposes, we may go further and replace this binary function with the class of ordered pairs that it characterizes.

The double bondage extension of the variable ' $x$ ' with respect to the pair $\langle ' x$ ', ' $y$ '> is not the same as its double bondage extension with respect to the converse pair $\langle ' y$ ', ' $x$ ' . This is just to say that the extension of a bound occurrence of a variable within the scope of a pair of variablebinding operator-occurrences depends on the order of the variablebinding operator-occurrences. Replacing singulary functions to singulary functions with binary functions, the extension of the second ' $x$ ' in ' $(x)(\exists y)(x$ loves $y)$ ' is the binary function, the former of $i$ and $j$, the extension of the second ' $y$ ' (indeed of both occurrences of ' $y$ ') is the binary function, the latter of $i$ and $j$. By contrast, the extension of the second ' $x$ '
13. This function is the customary extension of $\left.{ }^{[ }(\lambda \alpha)(\lambda \beta)[\zeta]\right]^{]}$under $s$.
in ' $(y)(\exists x)(x$ loves $y)$ ' is the function, the latter of $i$ and $j$, the extension of the second ' $y$ ' the function, the former of $i$ and $j .{ }^{14}$

The process iterates. The occurrence of the open formula ' $x$ is positioned between $y$ and $z$ ' in ' $(z)(\exists x)(\exists y)(x$ is positioned between $y$ and $z)$ ' ranges over a universe of truth-values. Its extension is the triple bondage extension with respect to the ordered triple < ' $z$ ', ' $x$ ', ' $y$ '>. The general notion of $n$-fold bondage extension is defined as follows.

> Def. For $n \geq 0$, the $n$-fold bondage extension of a well-formed expression $\zeta$ with respect to an $n$-tuple of variables
> $<\alpha_{1}, \alpha_{2}, \ldots, \alpha_{n}>$, under a value-assignment $s={ }_{\text {def }}$ the extension under $s$ of an occurrence of $\zeta$ within the scope of exactly $n$ occurrences of variable-binding-operator phrases, $\left\lceil\left(\mathbf{B}_{1} \alpha_{1}\right),\left\lceil\left(\mathbf{B}_{2} \alpha_{2}\right), \ldots, \Gamma_{1} \mathbf{B}_{n} \alpha_{n}\right)\right\rceil$, in that order, and not within the scope of any other occurrence of a nonextensional operator.
14. The notion of double bondage extension, and the distinction between it and customary designation, is relevant to resolving Kit Fine's development of Russell's antinomy of the variable, in "The Role of Variables," Journal of Philosophy 100 (2003): 60531. The problem, as Fine poses it, is this: "How is it that any two variables ranging over a given universe have the same semantic role and yet have a different semantic role?" As he develops the problem, the question becomes "How is it that there is no crosscontextual difference in semantic role between the variables ' $x$ ' and ' $y$ ', and yet there is a cross-contextual difference in semantic role between the pair $\left\langle{ }^{\prime} x\right.$ ', ' $y$ ' > and the pair <' $x$ ', ' $x$ '>?" (608). Our theory of bondage provides one possible response to Fine's question. The extension of each of the occurrences of ' $x$ ' in ' $(\exists x)$ ( $x$ loves $x$ ) ' is the bondage extension of ' $x$ ' with respect to itself: the identity function on the universe over which ' $x$ ' ranges. This is equally the extension of the occurrences of ' $y$ ' in ' $(\exists y)$ ( $y$ loves $y$ )'. Here is one sense in which there is no "cross-contextual difference in semantic role" between ' $x$ ' and ' $y$ '. By contrast, the occurrences of ' $x$ ' and ' $y$ ' in ' $(\exists x)(\exists y)(x$ loves $y)$ ', though they range over the same universe, differ in extension. The double bondage extensions of ' $x$ ' and ' $y$ ' with respect to $\langle x$ ' ' ' $y$ '> are neither of them the same as the single bondage extension of ' $x$ ' with respect to itself, or that of ' $y$ ' with respect to itself. Here is one sense in which there is a "cross-contextual difference in semantic role" between the variables in ' $(\exists x)$ ( $\exists y)$ ( $x$ loves $y$ )', on the one hand, and those in ' $(\exists x)(x$ loves $x)^{\prime}$ ' or in ' $(\exists y)$ ( $y$ loves $y$ )', on the other.

The apparent dichotomy here is illusory. The single bondage extension of ' $x$ ' with respect to ' $x$ ' is not the same as that of ' $y$ ' with respect to ' $x$ '. (See note 12.) And although the double bondage extension of ' $x$ ' with respect to $\langle$ ' $x$ ', ' $y$ '> is not the same as that of ' $y$ ' with respect to the same pair $\langle$ ' $x$ ', ' $y$ ' $\rangle$, it is the same as that of ' $y$ ' with respect to the converse pair $\langle$ ' $y$ ', ' $x$ '〉. Furthermore, the double bondage extension of ' $y$ ' with respect to $\left\langle x x^{\prime}\right.$, $y$ ' $\rangle$ is the same as that of ' $x$ ' with respect to the reflexive pair $\langle$ ' $x$ ', ' $x$ '〉.

Identifying the 0 -fold bondage extension with the customary extension, the basic tenet of our theory of bondage may be characterized by the following recursion:
$A_{0}$ : The 0 -fold bondage extension of a well-formed (open or closed) expression $\zeta$ with respect to the 0 -tuple $\leftrightarrow$, under a value-assignment $s$, is the customary extension of $\zeta$ under $s$.
$A_{(n+1)}$ : For $n \geq 0$, the $(n+1)$-fold bondage extension of a wellformed (open or closed) expression $\zeta$ with respect to an ( $n+1$ )-tuple of variables $\left\langle\alpha_{(n+1)}, \ldots, \alpha_{2}, \alpha_{1}>\right.$, under a value-assignment $s$, is ( $\lambda i$ ) [the $n$-fold bondage extension of $\zeta$ with respect to the sub- $n$-tuple obtained by deleting $\alpha(n+1)$ under the value-assignment $s^{\prime}$ that assigns $i$ to $\alpha_{(n+1)}$ and is otherwise the same as $\left.s\right] .{ }^{15}$
This function may be replaced by its corresponding $(n+1)$-ary function. In the special case where $\zeta$ is a formula, the latter function maps an appropriate $(n+1)$-tuple to $\zeta$ 's truth-value under the assignment of those objects as the values of the externally bound variables. For most purposes, we may go further and replace this function with the class of ordered $(n+1)$-tuples that it characterizes. The notions of bondage extension and of double bondage extension, characterized above, fall out as special cases of this recursion. ${ }^{16}$

[^0]There are the makings here of a hierarchy analogous to Frege's hierarchy of indirect senses. Our hierarchy is completely harmless. The ( $n+1$ )-fold bondage extension gives back the $n$-fold bondage extension once the free variables of $\zeta$ have been exhausted. ${ }^{17}$

Consider a concrete example. Suppose the universe over which the variables ' $x$ ' and ' $y$ ' range is the set of people. The occurrence of ' $x$ loves $y^{\prime}$ in ' $(x)(\exists y)(x$ loves $y)$ ' ranges over a universe of truth-values. Its extension is the double bondage extension of ' $x$ loves $y$ ' with respect to $\left\langle^{\prime} x\right.$ ', ' $y$ ' $\rangle$. This is the binary function that maps pairs of people to truth if the first person loves the second, and to falsehood otherwise. The extension of the occurrence of ' $(\exists y)(x$ loves $y)$ ' in ' $(x)(\exists y)(x$ loves $y)$ ' is the bondage extension of ' $(\exists y)(x$ loves $y$ )' with respect to ' $x$ ': the characteristic function of the class of lovers. The sentence is true if and only if this class is universal over the set of people. By contrast, the extension of the occurrence of ' $x$ loves $y$ ' in ' $(y)(\exists x)(x$ loves $y)$ ' is the double bondage extension of ' $x$ loves $y$ ' with respect to $\langle$ ' $y$ ', ' $x$ ' $\rangle$. This is the binary function that maps pairs of people to truth if the second person loves the first, and to falsehood otherwise. The extension of the occurrence of ' $(\exists x)$ ( $x$ loves $y)^{\prime}$ in ' $(y)(\exists x)(x$ loves $y)$ ' is the bondage extension of ${ }^{\prime}(\exists x)(x$ loves $y$ )' with respect to ' $y$ ': the characteristic function of the class of beloveds. The sentence is true if and only if this class is universal over the set of people.

One may choose to follow Frege in saying that any expression that has an extension designates the extension. For Frege, this entails that any expression-occurrence that has an extension-whether it is the customary extension or a noncustomary extension-is a designator of that extension. Then a bound occurrence of an open expression (such as an individual variable) has its bondage designatum with respect to a variable (on an analogy to Frege's notion of ungerade Bedeutung, or indirect designatum), which is simply the bondage extension. A singly bound variable (the occurrence) would thus designate the identity function on the

[^1]universe over which the variable (the expression) ranges. In the standard and most natural possible-worlds semantics of modality, the range of the individual variables varies from one possible world to the next. (A so-called possibilist, or fixed-universe, modal semantics is an alternative option.) Whereas a free occurrence of ' $x$ ' is a rigid designator under a value-assignment of its value, a singly bound occurrence of ' $x$ ' (on a variable-universe modal semantics) would be regarded as designating identity functions on different universes with respect to different possible worlds. The variable ' $x$ ', which occurs bound in (1), is itself rigid, but its occurrences in (1) (unlike the occurrence of ' $y$ '), insofar as they are designators, are nonrigid.

If one holds with Frege that an expression designates its extension, one may say that the open formula (1) customarily designates truth under $A$. As already noted, our original value-assignment $A$ does not satisfy (2); (2) customarily designates falsehood under A. But falsehood is not what (2) designates as it occurs in (1). Like the occurrences of ' $x$ ' in (1), the occurrence of (2) in (1) is bound, through its occurrence of ' $x$ ', by the initial quantifier occurrence. It therefore ranges over a universe of truth-values. Under $A$, the occurrence of (2) in (1) designates (nonrigidly) the characteristic function of the class of MacLaine's siblings. And (1) designates truth under $A$ as long as (and only as long as) this class is nonempty.

## III.

The foregoing is an outline of a Fregean extensional-semantic theory for both bound and free expression-occurrences. It can be extended into a Fregean theory of sense for bound and free expression-occurrences. To do so in a thoroughgoing Fregean manner, one should follow Church's idea of considering assignments of customary-sense values to variables in lieu of assignments of customary-designatum values.

Russell's intensional-semantic theory avoids this. On a Russellian theory, variables are logically proper names, or directly referential. That is, the semantic content ("meaning") of a variable, under an assignment of values to variables, is simply the variable's designatum (the assigned value) rather than a sense. The content of (2) under $A$ is the false singular proposition about MacLaine and Brando, that she is a sister of his. Suppose that the universe over which ' $x$ ' ranges is the set of people. Then the content of ( 1 ) under $A$ is a somewhat different, more general proposition, having just two components. The first component is the
propositional function that maps anyone $i$ to the singular proposition that MacLaine is a sister of $i$. Or it is the concept (or something similar) corresponding to this, that of having MacLaine as sister. The second component is the content of ' $\exists x$ )..$^{18}$ The proposition so constituted is the singular proposition about MacLaine that she is a sister of someone or other.

This is a theory of semantic content for expressions, not for expres-sion-occurrences. Russellian intensional semantics violates strong compositionality, according to which the semantic content of a compound expression is not only a function of, but is indeed a composite entity whose components are, the semantic contents of the compound expression's meaningful components. The Russellian content of ' $x$ '-of the variable itself-is, in some natural sense, a component of the Russellian content of (2), but it is no part of the Russellian content of (1) even though ' $x$ ' itself is as much a component of (1) as it is of (2). Likewise, the Russellian content of (2) is not a component of the Russellian content of (1).

To satisfy extensionality and compositionality, the notion of a component of a compound expression must be understood to be not an expression but an expression-occurrence. So understood, it is not unreasonable to hope to satisfy compositionality, and even strong compositionality. What we seek is a kind of hybrid Frege-Russellian intensional occurrencebased semantics-a Russellian theory of content that conforms to Frege's Context Principle.

Here is an excessively brief sketch. In Frege-Russellian occurrencebased semantics, what we have been calling "the content of ' $x$ '" under a value-assignment is the customary content of ' $x$ ', that is, the content of its free occurrences (not within quotation marks or the like). Bound variables have their bondage semantics. Suppose again that the universe over which the variables ' $x$ ' and ' $y$ ' range is the set of people. The customary content of the open formula ' $x$ loves $y$ ' under an assignment of values to variables is a singular proposition about the values of ' $x$ ' and ' $y$ '. This proposition is the content of free occurrences of ' $x$ loves $y$ ', not of bound occurrences. The occurrence of ' $x$ loves $y$ ' in ' $(y)(\exists x)(x$ loves $y)$ ' is in bondage, ranging over a universe of singular propositions. Its content, under an assignment $s$ of values to variables, is the double bondage
18. This might be the concept of being a nonempty class, or the second-order propositional function $\Sigma$ that maps any first-order propositional function $F$ to the proposition that $F$ is "sometimes true," that is, that $F$ yields a true proposition for at least one argument, or the corresponding concept, or something similar.
content of ' $x$ loves $y$ ' with respect to < $y$ ', ' $x$ '> under $s$. This is the function that maps a pair of people, $i$ and $j$, to the customary content of ' $x$ loves $y$ ' under the doubly modified value-assignment $s^{\prime} x^{\prime}$ ' $y_{i}$ ' that assigns $j$ as value for ' $x$ ' and $i$ as value for ' $y$ ', and is otherwise the same as $s$-that is, the binary Russellian propositional function ( $\lambda i j$ ) [the singular proposition that $j$ loves $i$. More accurately, the content of the occurrence of ' $x$ loves $y^{\prime}$ in ' $(y)(\exists x)(x$ loves $y)$ ' is the binary-relational concept, being loved by, that corresponds to the double bondage content.

The content of the occurrence of ' $(\exists x)(x$ loves $y)$ ' in ' $(y)(\exists x)(x$ loves $y)$ ', is the bondage content of ' $(\exists x)(x$ loves $y$ )' with respect to ' $y$ '. This is the propositional function ( $\lambda i$ ) [the singular proposition that someone or other loves $i]$. Or rather, the content of the occurrence of ' $(\exists x)(x$ loves $y$ )' in ' $(y)(\exists x)(x$ loves $y)$ ' is the concept corresponding to this propositional function: that of being loved by someone or other. This concept is composed of the content of the occurrence of ' $x$ loves $y$ ' and the customary content of ' $\exists x$ )', the latter being the second-order concept, someone or other. The customary content of ' $(y)(\exists x)(x$ loves $y)$ ' is the proposition composed of the content of the occurrence of ' $(\exists x)(x$ loves $y)$ ' and the customary content of ' $(y)$ ': that everyone is loved.

Similarly, the singular proposition that we have been calling "the content of (2)" under a value-assignment is the customary content of (2), that is, the content of its free occurrences, not of its bound occurrences. The occurrence of (2) in (1) is in bondage, ranging over the universe of singular propositions of the form, MacLaine is a sister of $i$ (that is, the class of propositions $p$ such that for someone $i, p=$ the singular proposition about MacLaine and $i$, that she is a sister of $i$ ). The content under $A$ of the occurrence of (2) in (1) is ( $\lambda i$ ) [the customary content of (2) under the modified value-assignment $A^{\prime}{ }^{x_{i}^{\prime}}$ ]. This is the Russellian propositional function that maps $i$ to the singular proposition that MacLaine is a sister of $i$. Or rather, the content under $A$ of the occurrence of (2) in (1) is the concept corresponding to this propositional function, that of having MacLaine as sister.

Russellian occurrence-based semantics obtains as customary content for (1) under $A$ the same proposition that Russell's expression-based semantics obtains as (1)'s content (simpliciter) under A. Unlike the latter, occurrence-based semantics does this by composition, generating a proposition by combining the semantic contents of the sentence's meaningful components-not the component expressions but the component occurrences.

## IV.

Unlike classical Russell-Tarski expression-based semantics, the FregeRussell occurrence-based semantics sketched above evidently conforms to Frege's Context Principle and to (modestly restricted) principles of extensionality, compositionality, and even strong compositionality. ${ }^{19}$ I should nevertheless strongly advise classical semantics to continue disregarding the Context Principle. This is not because I think it incorrect to attribute semantic values to expression-occurrences. The two approaches, though different, are not intrinsically in conflict. Contrary to the Context Principle, semantics may be done either way. Semantics may even be done both ways simultaneously, assigning semantic values both to expressions and to their occurrences within formulae or other expressions, and without prejudice concerning which is derivative from which. Frege's occurrence-based semantics in fact assigns semantic values both to expressions and their occurrences, even while honoring his Context Principle. His notions of customary designatum, indirect sense, doubly indirect designatum, and the like, are semantic values of the expression itself. The customary designatum is the designatum of the expression's occurrences in "customary" settings, that is, its occurrences that are in extensional position and not within the scope of a variablebinding operator. (See note 15.) And despite its pedigree, the Context Principle is not sacrosanct. Translating the term 'extension' of conventional expression-based semantics into 'customary extension', and so on for the other semantic terms ('designate', 'content', and so forth), occur-rence-based semantics emerges as a conservative extension of conventional expression-based semantics. Occurrence-based semantics may be unorthodox and unconventional, but it is only somewhat unorthodox and only somewhat unconventional. As mentioned, expression-based semantics is its less discriminating by-product.

The principal reason I nevertheless advocate expression-based semantics over occurrence-based semantics is that the latter inevitably invites serious confusion. It led Frege to his view that each meaningful expression has not only a sense, but an indirect sense, and also a doubly indirect sense, and indeed an entire infinite hierarchy of indirect

[^2]senses. ${ }^{20}$ Occurrence-based semantics has also led to the miscataloging of various terms. In particular, it has led to the misclassification of various noncompound singular terms as nonrigid, and of various compound terms (for example, complex demonstratives and 'that'-clauses in attributions of belief) as restricted quantifiers (often mislabeled generalized quantifiers). Though not Frege's, these errors have been committed by
20. I argue this in "On Indirect Sense and Designation" (unpublished). My attitude resonates to some extent with Rudolf Carnap's in Meaning and Necessity: A Study in Semantics and Modal Logic (1947; 2nd ed., Chicago: University of Chicago Press, 1956), chap. 3, especially secs. 29-32, pp. 124-44. (But see note 2 above.) Carnap calls expressionbased semantics the method of extension and intension, and Frege's occurrence-based semantics the method of the name-relation. Carnap saw Frege's occurrence-based semantics as flowing naturally from his assimilation of semantic extension to "the name-relation" between a singular term and its designatum. See ibid., sec. 28, especially at page 123. (Occurrence-based semantics per se does not require this assimilation. I believe the Context Principle also flows fairly naturally from a "truth-conditional" semantics that does not assimilate extension to designation. I have set out occurrence-based semantics without assuming the assimilation.) A resolute advocate of the expression-based semantic method over Frege's occurrence-based semantics, Carnap points out that the expression-semantic notion of extension and Frege's notion of designation ("nominatum," Bedeutung), though they are very similar, are not to be identified; and likewise the expression-semantic notion of content ("intension") and Frege's notion of sense, though very similar, are not to be identified. "A decisive difference between our method and Frege's consists in the fact that our concepts, in distinction to Frege's, are independent of the context" (125). Still, Carnap noted, the expression-semantic notions of extension and content coincide, respectively, with Frege's notions of customary designatum and sense. (See Carnap's principles 29-1 and 29-2, pp. 125-26.) Carnap advises against doing semantics both ways simultaneously (128-29) and complains that Frege's method led him to postulate an insufficiently explained notion of indirect sense (129) and leads ultimately to Frege's infinite hierarchies (131-32).

Russell had previously blamed the Fregean hierarchy not on occurrence-based semantics, but on the expression-semantic thesis that definite descriptions are singular terms. See note 17. My own view is that the hierarchy discredits neither the Context Principle nor the thesis that definite descriptions are singular terms and is to be traced instead to the union of two fundamental principles of Fregean theory: that any expression-occurrence that has a designatum also has a sense, which is a concept of the designatum; and that the indirect designatum of an expression is the customary sense. See my "On Designating," Mind 114 (2005): 1069-1133, reprinted in my Metaphysics, Mathematics, and Meaning (Oxford: Clarendon, 2005), 286-334; and also "On Indirect Sense and Designation."

There is an analogue to the Fregean hierarchy in Alonzo Church's elegant "Logic of Sense and Denotation" ("LSD"), in Structure, Method, and Meaning: Essays in Honor of Henry M. Sheffer, ed. Paul Henle, Horace M. Kallen, and Susanne K. Langer (New York: Liberal Arts, 1951), 3-24; Noûs (1973): 24-33, 135-56. As Carnap recognizes (132, 13738), however, the hierarchies in $L S D$ are not semantic values of single expressions. They are the senses of infinitely many different expressions.
followers in Frege's footsteps, reinforcing a current quantifier-mania. The misclassifications, and other confusions like them, come about when a philosopher of language fails to distinguish sharply between an expression and its occurrences. ${ }^{21}$

I shall first take up the misclassification of compound terms. This arises when a language philosopher erroneously imputes an open expression's customary semantics to the expression's occurrences in a sentence. I have in mind the recent rash of arguments to the effect that compound terms of a certain grammatical category (for example, 'that'clauses), because they can be quantified into ('Every boy believes that his dad is tougher than every other boys' dad'), cannot be singular terms, or cannot be directly referential singular terms, and should be regarded instead as restricted quantifiers.

The general form of the argument originates with Benson Mates, who employed it as an objection to the Fregean (and Strawsonian/antiRussellian) thesis that definite descriptions are compound singular terms, and that a definite description designates the individual that answers to the description if there is a unique such individual, and designates nothing otherwise, yielding a sentence with no truth-value. ${ }^{22}$
21. Since 1905 it has been illegitimate to presume without argument that definite descriptions are singular terms and not restricted quantifiers-even if it is at least as illegitimate, based largely on intuitions concerning what is mentioned, to presume without argument that definite descriptions are quantifiers and not singular terms. Some of the arguments of Russell and his followers have shaken confidence in the orthodox view that definite descriptions are singular terms. (See my "On Designating.") By contrast, the thesis that demonstratives and 'that'-clauses are singular terms remains quite plausible, also based largely on intuitions concerning what is mentioned, while the rival thesis that they are quantifiers remains enormously implausible. Many of the arguments of Kripke and others that names are not descriptions transfer easily to demonstratives and 'that'-clauses. In particular, that demonstratives are singular terms is common sense, and no persuasive evidence has been adduced that they are quantifiers. Specifically, as will be seen, the general argument presently to be considered provides no evidence whatever concerning demonstratives or 'that'-clauses. (I thank Zoltán Szabó for pressing me to address this. It should not be assumed that he agrees with my assessment.)
22. Benson Mates, "Descriptions and Reference," Foundations of Language 10 (1973): 409-18, at 415. The general form of argument has been employed or endorsed by several others during the past three decades. The following is a chronological partial bibliography: Gareth Evans, "Reference and Contingency," Monist 62 (1979): 161-89, at 169-70; Stephen Neale, Descriptions (Cambridge, MA: MIT Press, 1990), at 56n28; Neale, "Term Limits," in Logic and Language, vol. 7 of Philosophical Perspectives, ed. James E. Tomberlin (Atascadero, CA: Ridgeview, 1993), 89-123, at 107; Jeffrey King, "Are Complex 'That' Phrases Devices of Direct Reference?" Noûs 33 (1999): 155-82, at 157-58,

Although initially plausible, the Fregean thesis apparently falters when a definite description is quantified into, as in:
(3) Every [some/at least one/more than one/exactly one/not one] male soldier overseas misses the only woman waiting for him back home.
If the definite description 'the only woman waiting for him back home' were a singular term, then (3) should not be true-indeed, on the FregeStrawson theory, it should be neither true nor false-if the description has no designatum. But (3) could well be true, Mates argues, even though one cannot assign a designatum to the open definite description 'the only woman waiting for him back home' as occurring in (3), any more "than one can assign a truth-value to 'it is less than 9 ' as occurring in 'If a number is less than 7, then it is less than 9 '. ${ }^{23}$

Let us take a close look at the objection. As Mates notes, the definite description 'the only woman waiting for him back home' occurring in (3) is open. The pronoun 'him' occurring in the description corresponds to a variable bound by an external quantifier. The pronoun may be assigned any one of various soldiers as designatum. If the phrase 'the only woman waiting for him back home' is indeed a singular term, it designates different women under different such assignments. What about the occurrence of the description in (3)? Our theory of bondage demonstrates that Mates overstates the case when he says that one cannot assign anything to the occurrence as its designatum. The occurrence has its bondage extension with respect to 'him', and may be regarded as des-

[^3]ignating the function that assigns to any male the only woman waiting for him back home, if he left exactly one woman waiting for him back home, and assigns nothing otherwise. This much may be said, though: The occurrence of the description in (3) does not designate any particular woman who answers to the description.

Now suppose (3) is true. How does it follow that the description occurring in (3) is not a singular term?

It does not-not without the aid of some additional semantic machinery. What does follow is that if definite descriptions are singular terms, the occurrence of the description in (3) does not designate the description's customary designatum under any particular designatum assignment. But no one ever said that it did. The Fregean thesis is that definite descriptions-the expressions themselves-are singular terms. If one is not careful to distinguish between an expression and its occurrences, one might misconstrue this as the thesis that every occurrence of a definite description designates the object that answers to the description. (Recall the cautionary note in section 1.) But it is well known that Frege, with his doctrine of indirect designation, rejected the latter thesis. For (3) to be true, every male soldier overseas must miss the woman who is value of the function designated by the occurrence of the definite description when that soldier is assigned as argument. As long as the function is defined for every male soldier overseas, this presents no particular problem.

To bridge the gap between the current subconclusion and the Fregean thesis in Mates's crosshairs, the objection tacitly invokes the following semantic theorem:

M: Any sentence $\phi_{\beta}$ [of a restricted class $C$ ], containing an occurrence of a genuine singular term $\beta$ not within the scope of an indirect, intensional, or quotational operator, is true [either true or false] only if that same occurrence of $\beta$ designates the customary designatum of $\beta .{ }^{24}$

Assuming Mates does not misconstrue the Fregean/Strawsonian thesis, his objection assumes ( $M$ ) (or something very much like it) as its major premise, or assumes that his Fregean opponent is committed to it. As we have noted, if the description 'the only woman waiting for

[^4]him back home' is a genuine singular term, its occurrence in (3)-since an external quantifier-occurrence quantifies into it-does not designate the description's customary designatum under a particular designatumassignment. Yet (3) may be true. Given ( $M$ ), it directly follows that the description is not a genuine singular term.

The argument is fallacious. Other versions of Mates's objection are equally fallacious. Those other versions make, or require, semantic assumptions analogous, or otherwise very similar, to ( $M$ ). ${ }^{25}$ What the proponents of the style of argument generally fail to recognize is that, insofar as there are semantic theorems like ( $M$ ) concerning singular terms, there are analogous semantic theorems concerning quantifiers, ${ }^{26}$ as well as other sorts of expressions that have semantic extension. This makes for the possibility of an exactly analogous argument for the conclusion that quantifiers also cannot be quantified into, and therefore definite descriptions (or 'that'-clauses, and so forth) are not quantifiers either, or anything else for that matter. Something has gone very wrong. Restricted quantifiers can be bound by other quantifiers-as, for example, in 'Every male soldier overseas misses some woman waiting for him back home'. For that matter, so can singular terms-witness the case of the individual variable. Somewhere a fatal error has been committed.

In every application of which I am aware, the assumed semantic "theorem" is in fact false, and the proponents of the target thesis (for example, that definite descriptions or 'that'-clauses are singular terms) do not endorse it. If $(M)$ were sound, it would establish more generally that the very notion of an occurrence of an open singular term bound ("quantified into") by an external quantifier is semantically incoherent. Despite the objection's popularity, ordinary mathematical notation is
25. The assumed semantic theorem is not generally stated precisely, if it is stated at all. In some applications a somewhat stronger semantic theorem is employed, for example:
$(M+)$ Any sentence $\phi_{\beta}$, of the restricted class $C$, containing an occurrence of a genuine singular term $\beta$ not within the scope of an indirect, intensional, or quotational operator is true if and only if the designatum of that same occurrence of $\beta$ satisfies the formula $\phi_{\alpha}$-where $\phi_{\beta}$ is the result of uniformly substituting occurrences of $\beta$ for the free occurrences in extensional position of a variable $\alpha$ in $\phi_{\alpha}$. (See the appendix.)
26. Thus, for example: Any sentence [of a restricted class C], containing an occurrence of universal generalization ${ }^{\Gamma}(\alpha) \phi_{\alpha}{ }^{7}$ not within the scope of an indirect, intensional, or quotational operator is true only if the extension of that same occurrence of ${ }^{\Gamma}(\alpha) \phi_{\alpha}{ }^{7}$ is truth if the extension of its occurrence of $\phi_{\alpha}$ is the function that assigns truth to everything in the range of the variable $\alpha$, and is falsehood otherwise.
rife with counterexamples to its major premise-for example the ' $x^{2}$ ' in ' $(\exists x)\left(x^{2}=9\right)$ '. The most glaring counterexample is the paradigm of an open designator: the individual variable. To use Mates's own example, if the occurrences of ' $y$ ' in the true sentence ' $(y)(y<7 \supset y<9)$ ' (let this be $\phi_{\beta}$, with $\beta=$ ' $y$ ') designate anything, they designate not the customary designatum of ' $y$ ' under a particular value-assignment, but the bondage extension with respect to ' $y$ ' itself: the identity function on the range of ' $y$ '. Yet the variable ' $y$ ' is a genuine singular term if anything is. ${ }^{27}$ (See the appendix.)

The mistake directly results from imputing the semantic attributes of an expression to its occurrences, including even bound occurrences. The mistaken "theorem" can be corrected, and even generalized:
$M^{\prime}$ : An assignment $s$ of values to variables satisfies a formula $\phi_{\beta}$, of the restricted class $C$, containing a free occurrence of a singular term $\beta$ not within the scope of any nonextensional operator (other than classical variable-binding operators), only if that same occurrence of $\beta$ designates the customary designatum of $\beta$ under s.

This corrected version effectively blocks the objection. ${ }^{28}$ Fregean theory may also countenance a second variation of $(M)$ :
$M^{\prime \prime}:$ Any sentence $\phi_{\beta}$ [of a restricted class C], containing an occurrence
of a genuine singular term $\beta$ not within the scope of any
nonextensional operator (other than classical variable-binding
operators), is either true or false only if that same occurrence of $\beta$
designates.

[^5]As mentioned earlier, according to the occurrence-based semantics sketched above, the occurrence of the open definite description in (3) designates a particular partial function.

It is a trivial matter to extend the theory of bondage from section 2 above to include definite descriptions as singular terms, which, if open, can be quantified into. A definite description ${ }^{\lceil }(1 \alpha) \phi_{\alpha}{ }^{1}$ customarily designates under a value-assignment $s$ the unique object $i$ that is an element of the class characterized by the extension of its occurrence of $\phi_{\alpha}$, if there is a unique such $i$, and customarily designates nothing under $s$ otherwise. A free occurrence of a definite description in extensional position designates the description's customary designatum. The extension of a bound occurrence in otherwise extensional position is then the appropriate bondage extension. ${ }^{29}$ One may consistently add the corrected Mates theorem ( $M^{\prime}$ ) into the mix. On this theory of bondage, quantification into singular terms is not only permitted, it is encouraged.

Saul Kripke has sermonized, "It is important, in discussions of logico-philosophical issues, not to lose sight of basic, elementary distinctions by covering them up with either genuine or apparent technical sophistication. ${ }^{30}$ The distinction between an expression and its occurrences is elementary and fundamental. The Fregean/Strawsonian thesis that Mates aims to refute is that definite descriptions are singular terms. It is no part of the Fregean thesis that every occurrence-even a bound occurrence-of a definite description in otherwise extensional position in a sentence designates the description's customary designatum. The latter thesis is neither Frege's nor Strawson's; it is Strawman's.

There remain significant differences between the Fregean theory sketched above and the Russellian theory that Mates and company prefer. If every male soldier overseas left exactly one woman waiting for him back home, and he does indeed miss her, then contrary to Mates, Frege's theory, no less than Russell's, deems (3) true. If every male soldier overseas left exactly one woman waiting for him back home, but at least one
29. Let a particular $(n+1)$-ary function $f$ from objects to truth values be the $(n+1)$ fold bondage extension of a formula $\phi_{\alpha}$ with respect to a sequence of variables $<\beta_{1}$, $\beta_{2}, \ldots, \beta_{n}, \alpha>$, under a value-assignment $s$. Then the $n$-fold bondage extension of the definite description $\left.{ }^{\Gamma}(\alpha \alpha) \phi_{\alpha}\right\rceil^{\text {w }}$ with respect to $<\beta_{1}, \beta_{2}, \ldots, \beta_{n}>$, under $s$, is the $n$-ary partial function $f_{1}$ that maps $j_{1}, j_{2}, \ldots, j_{n}$ to the unique element $i$ from the range of $\alpha$ such that $f\left(j_{1}, j_{2}, \ldots, j_{n}, i\right)=$ truth, if there is a unique such $i$, and is undefined otherwise.
30. Saul Kripke, "Is There a Problem about Substitutional Quantification?" in Truth and Meaning: Essays in Semantics, ed. Gareth Evans and John Henry McDowell (Oxford: Clarendon, 1976), 325-419, at 408.
male soldier overseas does not miss the woman he left behind, then both Frege and Russell deem (3) false. But suppose at least one male soldier overseas left no woman, or two women, waiting for him back home. On Russell's theory, (3) is false in this third case as well as in the second. On Frege's theory it is not, although it is not true either. This verdict is a straightforward result of ( $M^{\prime}$ ) together with the theory's other semantic principles. The third case, not the first, is the deciding case. To this day, it remains unclear whether the falsity verdicts of Russell's theory, or those of Frege's, are the correct ones.

## V.

Besides the misclassification of various compound terms, there has also occurred a miscataloging of certain directly referential singular terms as nonrigid definite descriptions, again partly as a result of a failure to distinguish sharply between the term and its occurrence. Here the confusion is traceable to a larger confusion between an entire sentence and its occurrence in a discourse. Consider the following discourse fragment:
(4) (i) A comedian composed the musical score for City Lights.
(ii) He was multitalented.

The particular sentence (4ii) is ordinarily regarded as an open formula with a free variable, 'he'. As Geach has noted, the pronoun evidently functions differently as it occurs in (4). Geach takes the pronoun-occurrence to be a variable-occurrence bound by a prenex occurrence of the restricted existential quantifier 'a comedian', as in the following:
(4G) [a $x$ : comedian $(x)]$ ( $x$ composed the musical score for City Lights \& $x$ was multitalented). ${ }^{31}$

Gareth Evans mounted solid evidence against Geach that the scope of 'a comedian' in (4) does not extend beyond (4i), and so the phrase does not bind the 'he' in (4ii)-this despite the fact that the 'he' is anaphoric upon the phrase 'a comedian'. ${ }^{32}$ Following Evans, an anaphoric pronoun-

[^6]occurrence whose grammatical antecedent is a quantifier-occurrence within whose scope that pronoun-occurrence does not stand is often called an E-type pronoun (alternatively, a donkey pronoun because of particular examples originally due to Walter Burley). ${ }^{33}$ The 'he' in (4) appears to be a free occurrence of a closed singular term rather than a bound variable. E-type pronoun-occurrences, according to Evans, are "assigned a reference and their immediate sentential contexts can be evaluated independently for truth and falsehood." Evans takes the 'he' in (4) to be a rigid singular term whose reference is fixed by the description the only comedian who composed the musical score for City Lights'. He thus represents (4) as having the following logical form:
(4E) (i) [a $x$ : comedian $(x)$ ] ( $x$ composed the musical score for City Lights).
(ii) dthat [ [the $y$ : comedian( $y$ )] ( $y$ composed the musical score for City Lights) ] was multitalented.

The bracketed expression in the first sentence is a restricted existential quantifier phrase, which may be read 'a comedian $x$ is such that'. The innermost bracketed expression in the second sentence may be read 'the only comedian $y$ such that'. The full ' dthat'-term-which might be read 'that comedian who composed the musical score for City Lights' (a closed expression) -is alleged to be the formal counterpart of the 'he' in (4ii).

Michael McKinsey, Scott Soames, Stephen Neale, and others argue that the 'he', as it occurs in (4), is not merely codesignative, but synonymous in content, with 'the only comedian who composed the musical score for City Lights'. For although the 'he' in (4) designates Charlie Chaplin with respect to the actual world, (4) may also be evaluated with respect to other possible worlds. Consider a possible world $W$ in which, say, Buster Keaton composed the musical score for Chaplin's classic silent
allows, while the former does not, that a third, nonmultitalented actor also starred in City Lights.) Many, including several critics, have followed Evans in concluding that the pronoun 'they' in the discourse fragment is an occurrence of a closed expression; hence too, by analogy, the pronoun in (4).
33. In the vernacular of theoretical linguistics, the term ' $E$-type pronoun' is used for an anaphoric pronoun-occurrence whose grammatical antecedent is a quantifieroccurrence that does not $c$-command that pronoun-occurrence. Linguists and linguisticsoriented philosophers almost invariably phrase this in terms of a "pronoun" and its antecedent "quantifier," where what are at issue are actually occurrences. (See note 5 above and recall again the cautionary note to which it is appended.)
film. The discourse fragment (4) is true with respect to $W$ if and only if Keaton is a multitalented comedian in W, never mind Chaplin. ${ }^{34}$ With respect to $W$, it is argued, the 'he' in (4) designates Keaton instead of Chaplin, just as the description does. The entire discourse fragment is thus depicted as having the following logical form, in contrast to $(4 E)$ :
(4M) (i) [a $x$ : comedian $(x)$ ] ( $x$ composed the musical score for City Lights).
(ii) [the $y$ : comedian( $y$ )] ( $y$ composed the musical score for City Lights) was multitalented.

The full definite description in (4Mii) is alleged to be the formal counterpart of the 'he' in (4). ${ }^{35}$

The argument is mistaken. That the pronoun 'he' (the expression) is rigid is confirmed by positioning it in the scope of a modal operator-occurrence:

A comedian composed the musical score for City Lights. That he was multitalented is a contingent truth.

The second sentence here does not impute contingency to the fact that whichever comedian composed the music for City Lights was multitalented. (If it did, it would presumably be false.) Instead it expresses something about Chaplin himself: that although in fact multitalented, he might not have been. ${ }^{36}$
34. Insofar as the modal truth-conditions for (4) yield this result, the 'he' does not function in (4) as a demonstrative. By contrast with (4ii), the sentence 'Dthat[the comedian who composed the musical score for City Lights] was multitalented' is true with respect to a context $c$ and a possible world $w$ if and only if the comedian who in the possible world of $c$ (rather than w) composed the musical score for City Lights, was multitalented in $w$.
35. This argument for the pronoun's nonrigidity is McKinsey's, in "Mental Anaphora," Synthese 66 (1986): 159-75, at 161. It is echoed by Scott Soames, in his review of Gareth Evans's Collected Papers, Journal of Philosophy 86 (1989): 141-56, at 145. It is also endorsed by Stephen Neale in "Descriptive Pronouns and Donkey Anaphora," Journal of Philosophy 87 (1990): 113-50, at 130, and again in Descriptions (Cambridge, MA: MIT Press, 1990), 186.
36. See, by way of comparison, my "Demonstrating and Necessity," at 536-37n52. My critique has benefited from discussion with Alan Berger, who realized independently that the arguments of Evans and McKinsey are incorrect. See his Terms and Truth (Cambridge, MA: MIT Press, 2002), 171-78.

Though the pronoun 'he' is rigid, so-called laziness occurrences (in addition to bound occurrences) may be nonrigid. The occurrence in (4ii) is not a laziness occurrence.

This does not mean that Evans was right and Geach wrong. The pronoun-occurrence in (4) is more plausibly regarded as a variableoccurrence bound by a restricted quantifier implicit in (4ii), perhaps 'a comedian who composed the musical score for City Lights'. The entire discourse fragment is plausibly regarded as having an underlying logical form more like the following, where items in boldface correspond to explicit elements in the surface form (4):
(4') (i) $[\mathbf{a} x$ : comedian $(x)]$ ( $x$ composed the musical score for City Lights).
(ii) [a $y$ : comedian ( $y$ ); $y$ composed the musical score for City Lights] ( $y$ was multitalented).

The open formula ' $y$ was multitalented' occurring in ( $4^{\prime} i i$ ) makes an explicit appearance in the surface form, as (4ii). The rest of (4'ii) does not. On this analysis, an $E$-type pronoun-occurrence is a species of bound-variable occurrence, as Geach has long maintained. In fact, the conjunction corresponding to $\left(4^{\prime}\right)$ is equivalent to ( $4 G$ ) (and to the second conjunct ( $4^{\prime} i i$ ) alone). Contrary to Geach, however, the anaphora between an E-type pronoun and its antecedent is not the same relation as that between a bound variable and its binding operator. Instead the $E$-type pronoun is bound by an absent operator recoverable from the antecedent.

One important advantage of this analysis over both ( $4 E$ ) and $(4 M)$ is that the mere grammar of (4) does not support an inference to a uniqueness claim of the sort presupposed or otherwise entailed by the use of 'the only comedian that scored the music for City Lights'. Though this may not be obvious with (4) (since typically, if someone scored the musical score for a particular film, then no one else did), it is with the following discourse:

A comedian panned the musical score for City Lights. He was jealous. Another comedian also panned the musical score for City Lights. He wasn't jealous; he was tone-deaf.

Another important difference is that there is no definite description in (4') to be regarded as a formal counterpart of the 'he' in (4). There is no nonrigid designation of Chaplin in (4'). There is no designation at all of Chaplin in (4'), except by the variables ' $x$ ' and ' $y$ ' under appropriate
value-assignments. The rigidity of 'he' suggests that its formal counterpart in ( 4 ') is simply the last occurrence of ' $y$ '. ${ }^{37}$

Recall again the cautionary note of section 1. It is extremely important here to distinguish sharply between the English sentence (4ii) and its occurrence in the discourse-fragment (4). The former is the naturallanguage analogue of an open formula. That is the sentence itself-an expression-whose logical form is given, nearly enough, by ' $y$ was multitalented'. The occurrence of (4ii) in (4) is a horse of a different color. Here the surface form of an occurrence is not a reliable guide to the logical form. The occurrence of (4ii) in (4) corresponds not merely to ' $y$ was multitalented' but to the whole of (4'ii), in which a restricted quantifier binds the open formula. Though superficially an occurrence of an open formula, the underlying logical form is that of a closed sentence, which "can be evaluated independently for truth and falsehood." In effect, the second sentence-occurrence in (4), though syntactically an occurrence
37. By contrast with (4), the two " $E$-type" pronoun-occurrences in "If a man has a home, it is his castle' are more naturally taken as variable-occurrences bound by implicit universal-quantifier occurrences. Compare the account of Berger, Terms and Truth, 159-89, 203-27. The analysis Berger provides for discourse-fragments like (4) looks to be a notational variant of (4'). (Berger has informed me that he is inclined to think it is.)

The anonymous referee for the Philosophical Review worries that although the two $E$-type pronouns in the following discourse are anaphorically linked to each other, on the analysis proposed here they are not co-bound by the same quantifier-occurrence:
(5) (i) I spoke to a philosopher yesterday. (ii) He sides with Geach against Evans.
(iii) He lives in California.

Imagine the referee spoke with only two male philosophers yesterday, one of whom sides with Geach against Evans but does not live in California, the other lives in California but does not side with Geach against Evans. Then (5ii) and (5iii) are not both true.

The worry is misplaced. The underlying logical form of (5) is arguably given by:
(5') (i) [a $x$ : philosopher $(x)]$ (I spoke to $x$ yesterday).
(ii) [a $y$ : philosopher $(y)$; I spoke to $y$ yesterday] ( $y$ sides with Geach against Evans).
(iii) [a $z$ : philosopher $(z)$; I spoke to $z$ yesterday; $z$ sides with Geach against Evans)] ( $z$ lives in California).
On this analysis each of the $E$-type pronouns is a bound variable. Whereas ( $5^{\prime} i i$ ) is true in the envisaged circumstance, ( $5^{\prime} i i i$ ) is false-evidently in conformity with the English sentences they represent. The final occurrence of ' $z$ ' in ( 5 ' $i i i$ ) is indeed cobound with the second-to-last, as it should be, by the initial quantifier phrase ' $[a z]$ '. It is not co-bound with the occurrences of ' $y$ ' in ( $5^{\prime}$ iii). Nor should it be. If the two $E$-type pronouns in (5) were co-bound variables, (5ii) would be an open sentence and, as such, would not have truth-value. (The conjunction corresponding to ( $5^{\prime}$ ) is equivalent to the conjunct ( $5^{\prime} i i i i$ ) alone.)
of (4ii), is semantically an occurrence of (4'ii). One could say that the sentence (4ii) itself is bound in (4), though not by any element of (4i)indeed, not by any element of the surface form of (4). One might even say that the occurrence of (4ii) in (4) is a pro-clause of laziness; although syntactically an occurrence of (4ii), it has the logical form of the whole consisting of (4ii) together with a binding quantifier phrase. The quantifier phrase itself, though invisible, is present behind the scenes. ${ }^{38}$

If the occurrence of ' $y$ was multitalented' in ( 4 'ii) is to be regarded as having an extension, it has the open formula's bondage extension: the function that maps individuals in the range of ' $y$ ' who were multitalented to truth and maps those who were not to falsehood. The whole of ( $4^{\prime} i i$ ) -and hence the occurrence of (4ii) in (4)-is true if and only if the class characterized by this function includes a comedian who composed the musical score for City Lights. As was noted, the occurrence of (4ii) in (4) is thus true with respect to the possible world $W$ if and only if Keaton was multitalented in $W$.

The very fact that the occurrence of (4ii) in (4) has these modal truth-conditions despite the rigidity of 'he' indicates that, contrary to Evans and several of his critics, the 'he' in (4) is not a closed-term occurrence but a bound variable. One can say with some justification that the 'he' in (4)-the occurrence-is a nonrigid designator. But this is not because the occurrence designates Chaplin with respect to one pos-

[^7]sible world and Keaton with respect to another. It does neither. Where it occurs free-as for example in a deictic use (and not as a pronoun of laziness) -'he' is a rigid designator of its customary extension under a des-ignatum-assignment. If the pronoun-occurrence in (4) is to be regarded as designating at all, it designates the pronoun's bondage extension: the identity function on the range of 'he'. Insofar as the occurrence is nonrigid, it is so only because it has its bondage extension, ranging over different universes with respect to different possible worlds.

## Appendix

Jeffrey King, as cited in note 22, applies a version of Mates's objection against the thesis that demonstratives are directly referential singular terms. Quantification into a complex demonstrative is odd at best. Although King assumes it is permissible, almost all his examples involve, or appear to involve, a stylistically altered definite description rather than a genuine demonstrative, for example, 'Every professor cherishes that first publication of his'. (Compare with (3).) Where the phrase 'that first publication of his' occurs as a genuine demonstrative, it should be possible to delete the word 'first' by pointing to the publication in question. But this is problematic with King's example.

The issue is significant, but set it aside. King explicitly aims to establish the conclusion that at least some complex demonstratives (the expressions) are not singular terms at all, let alone directly referential singular terms. His argument employs the following tacit premise: ( $K 1$ ) Any sentence $\phi_{\beta}$ containing a directly referential occurrence of singular term $\beta$ not within the scope of an indirect, intensional, or quotational operator expresses as its semantic content a singular proposition in which the designatum of that same occurrence of $\beta$ occurs as a component. The conclusion King derives using this premise is that bound occurrences of complex demonstratives are not directly referential occurrences, i.e., the occurrence's semantic content is not the expression's customary designatum. Although King evidently believes this refutes the target thesis, strictly speaking the target thesis is perfectly compatible with this conclusion-just as Mates's subconclusion before invoking $(M)$ is compatible with the Fregean thesis that definite descriptions are singular terms. An additional premise is required to validate King's argument against the target thesis: (K2) If a singular term $\beta$ is directly referential, then every occurrence in a sentence of $\beta$ not within the scope of an indirect, intensional, or quotational operator is a directly referential occurrence.

King has confirmed in correspondence that he accepts ( $K 2$ ) as well as ( $K 1$ ). He adds that he believes both are partly stipulative, by virtue of the meaning of 'directly referential'. (He also adds that ( $K 2$ ), because it concerns expressions as well as expression-occurrences, is likely to confuse.) Taken together, ( $K 1$ ) and ( $K 2$ ) yield the direct-reference analogue of Mates's semantic theorem: $(K)$ Any sentence $\phi_{\beta}$ containing an occurrence of a directly referential singular term $\beta$ not within the scope of an indirect, intensional, or quotational operator expresses as its semantic content a singular proposition in which the designatum of that same occurrence of $\beta$ occurs as a component. This theorem may be taken as premise in place of ( $K 1$ ) and ( $K 2$ ).

Jason Stanley has confirmed in correspondence that in his review he interprets King's objection as tacitly invoking ( $K$ ) as a stipulative premise-or alternatively, ( $K 1$ ) and ( $K 2$ ). Stanley (ibid.) maintains that whereas Mates's original argument and others like it fail-essentially on the same grounds argued in the text above-King's variant of Mates's argument is nevertheless decisive against the thesis that demonstratives are directly referential singular terms. Stanley's position is based on his contention that an intensional semantics of content (as opposed to classical, extensional semantics in the style of Tarski) does not relativize content to assignments of values to variables. Contrary to Stanley, however, wherever there is variable binding, the natural method of systematically assigning contents involves doing so under value-assignments. Church's "The Need for Abstract Entities in Semantic Analysis" and the Russellian intensional semantics sketched in section 3 above both do so explicitly. ${ }^{39}$ Mate's argument cannot be made to succeed simply by choosing to speak of the semantic content of a definite-description occurrence and the individual of which that content is a concept, rather than speaking of the occurrence designating the individual.

Contrary to both King and Stanley, $(K)$ is not an analytic or stipulative truth. In fact, it has extremely dubious consequences, for example, that variables are not directly referential-assuming that a bound variable, since its semantic content is not the variable's customary designatum, is not a "directly referential occurrence." (This is how both King and Stanley understand the phrase.) More specifically, both (K2) and $(K)$ are evidently falsified by the same paradigm-case as ( $M$ ): bound vari-

[^8]ables. Furthermore, proponents of the direct-reference theory, though they may accept ( $K 1$ ), do not endorse either ( $K 2$ ) or ( $K$ )-again, witness the case of bound variables. Contrary to Stanley, King's argument and Mates's original argument thus evidently fail for the same general reason. ${ }^{40}$

Stanley responds that both ( $K 2$ ) and ( $K$ ) are true despite bound variables because the lower-case letter ' $x$ ' (qua variable) ambiguously represents two distinct expressions: ' $x$ '-bound and ' $x$ '-free. (He maintains that this alleged ambiguity is a corollary of $(K)$.) The bondage extension of a variable is indeed distinct from its customary extension, and one might choose to express this (I believe misleadingly) by saying that the variable is ambiguous, having a bondage reading distinct from its customary or default reading. (It is incorrect to express this by saying that a bound occurrence and a free occurrence of ' $x$ ' are occurrences of different expressions.) Expressing the point in terms of an "ambiguity" between customary and bondage readings, however, is ineffective as a defense of King's objection. The bondage semantics of any open expression deviates from the customary semantics, for example, 'the only woman waiting for him', 'his first publication', and so forth. Insofar as open expressions are deemed ipso facto ambiguous, the thesis that King's argument aims to refute is that demonstratives on their customary readings are directly referential singular terms. The alleged bondage reading is irrelevant.

[^9]
[^0]:    15. This function is the customary extension of $\left.{ }^{[ }\left(\lambda \alpha_{(n+1)}\right)\left(\lambda \alpha_{n}\right) \ldots\left(\lambda \alpha_{1}\right)[\zeta]\right]^{]}$under $s$. The recursion principles, $A_{0}$ and $A_{(n+1)}$, might be taken as axioms. This construal may seem more appropriate for the latter principle than the former, which is plausibly construed instead as a definition of 'customary extension'. (See note 6.) What entity the customary extension of an expression is can be determined by invoking the classical characterization of extension simpliciter.
    16. Let a particular $(n+1)$-ary function $f$ from objects to truth values be the $(n+1)$ fold bondage extension of a formula $\phi_{\alpha}$ with respect to a sequence of variables $<\beta_{1}$, $\beta_{2}, \ldots, \beta_{n}, \alpha>$, under a value-assignment $s$. Then:
    (i) The $n$-fold bondage extension of the universal generalization ${ }^{\lceil }(\alpha) \phi_{\alpha}{ }^{1}$ with respect to $\left\langle\beta_{1}, \beta_{2}, \ldots, \beta_{n}>\right.$, under $s$, is an $n$-ary function $f_{\Pi}$ that maps $j_{1}, j_{2}, \ldots$, $j_{n}$ to truth if every element $i$ from the range of $\alpha$ is such that $f\left(j_{1}, j_{2}, \ldots, j_{n}, i\right)=$ truth, and that maps $j_{1}, j_{2}, \ldots, j_{n}$ to falsehood if at least one element $i$ from the range of $\alpha$ is such that $f\left(j_{1}, j_{2}, \ldots, j_{n}, i\right)=$ falsehood; and
    (ii) The $n$-fold bondage extension of the existential generalization ${ }^{〔}(\exists \alpha) \phi_{\alpha}{ }^{7}$ with respect to $<\beta_{1}, \beta_{2}, \ldots, \beta_{n}>$, under $s$, is an $n$-ary function $f_{\Sigma}$ that maps $j_{1}, j_{2}, \ldots$, $j_{n}$ to truth if at least one element $i$ from the range of $\alpha$ is such that $f\left(j_{1}, j_{2}, \ldots\right.$, $\left.j_{n}, i\right)=$ truth, and that maps $j_{1}, j_{2}, \ldots, j_{n}$ to falsehood if every element $i$ from the range of $\alpha$ is such that $f\left(j_{1}, j_{2}, \ldots, j_{n}, i\right)=$ falsehood.
[^1]:    A universal generalization ${ }^{「}(\alpha) \phi_{\alpha}{ }^{\top}$ is true under a value-assignment $s$ if and only if the class characterized by the extension of its occurrence of $\varphi_{\alpha}$ under $s$ (that is, the bondage extension of $\phi_{\alpha}$ with respect to $\alpha$ under $s$ ) is universal. An existential generalization ${ }^{\Gamma}(\exists \alpha) \phi_{\alpha}{ }^{\top}$ is true under $s$ if and only if the class characterized by the extension of its occurrence of $\phi_{\alpha}$ under $s$ is nonempty.
    17. See notes 11 and 12 above. By contrast, on Frege's hierarchies of multiply indirect extensions, the ( $n+1$ )-fold indirect extension is the $n$-fold indirect sense-which, as Russell noted in his infamous "Gray's Elegy" argument, is a new entity, entirely distinct from the $n$-fold indirect extension.

[^2]:    19. An actual proof that a modestly restricted principle of strong compositionality is satisfied (or falsified) awaits a suitable theory of concepts analogous to ZermeloFrankel set theory.
[^3]:    161-62; Ernest Lepore and Kirk Ludwig, "The Semantics and Pragmatics of Complex Demonstratives," Mind 109 (2000): 200-241, at 205-206, 210-22, and passim; King, Complex Demonstratives (Cambridge, MA: MIT Press, 2001), at xi-xii, 1, 10-11, 20-22; Kent Johnson and Ernest Lepore, "Does Syntax Reveal Semantics? A Case Study of Complex Demonstratives," in Language and Mind, vol. 16 of Philosophical Perspectives, ed. James E. Tomberlin (Atascadero, CA: Ridgeview, 2002), 17-41, at 31; and Jason Stanley, "Review of Jeffrey King, Complex Demonstratives, Philosophical Review 111 (2002): 605-9. See, by way of comparison, my "Being of Two Minds: Belief with Doubt," Noûs 29 (1995): 1-20, at 18n26, and "Demonstrating and Necessity," Philosophical Review 111 (2002): 497-537, at 534-35n47; both reprinted in my Content, Cognition, and Communication.
    23. Before Mates, Geach had drawn a somewhat different conclusion from the same data: that the occurrence of the definite description in (3), since it does not designate, does not "have the role of a definite description." See his "Ryle on Namely-Riders," in Logic Matters, 91-92; also, "Referring Expressions Again," Analysis 24 (1963-64), reprinted in Geach's Logic Matters, 97-102, at 99-100.

[^4]:    24. See note 6. The bracketed material represents variations or restrictions that Mates might have in mind. The restricted class $C$ excludes such problematic sentences as ${ }^{\lceil } \beta$ does not exist ${ }^{\top}$ and things that entail it.
[^5]:    27. Let $\phi_{\alpha}$ in (M+) be the open formula ' $(y)(y<7 \supset x<9)$ ', with $\alpha=$ ' $x$ '. The customary designatum of ' $y$ ' under the assignment of 10 as value does not satisfy it.
    28. There are likewise corrected versions of the more elaborate assumptions mentioned in note 25 above. Thus:
    $M+^{\prime}:$ An assignment s of values to variables satisfies a formula $\phi_{\beta}$, of the restricted class $C$, containing a free occurrence of a genuine singular term $\beta$ not within the scope of any nonextensional operator (other than classical variable-binding operators), if and only if the modified value-assignment s' that assigns the designatum of that same occurrence of $\beta$ under s as value for a variable $\alpha$ and is otherwise the same as $s$, satisfies the formula $\phi_{\alpha}$-where $\phi_{\beta}$ is the result of uniformly substituting free occurrences of $\beta$ for the free occurrences of $\alpha$ in extensional position in $\phi_{\alpha}$.
    Each of these corrected versions effectively blocks the objection.
[^6]:    31. Geach, Reference and Generality, at 129ff; and "Quine's Syntactical Insights," in Logic Matters, at 118-19.
    32. See, by way of comparison, Gareth Evans, "Pronouns, Quantifiers, and Relative Clauses (I)," Canadian Journal of Philosophy 7 (1977): 777-97; "Pronouns," Linguistic Inquiry 11 (1980): 337-62. The analogous discourse fragment-Just two actors starred in City Lights. They were both multitalented-is not equivalent to the quantified generalization 'Just two actors both: starred in City Lights and were multitalented'. (The latter
[^7]:    38. The discourse fragment mentioned in note 32 is plausibly regarded as having an underlying logical form given, nearly enough, by:
    (i) [just two $x: \operatorname{actor}(x)$ ] ( $x$ starred in City Lights).
    (ii) [every $y$ : actor $(y) ; y$ starred in City Lights] ( $y$ was multitalented).

    The occurrence of 'they' corresponds to the final occurrence of ' $y$ '. See the previous note. Consider, in contrast, the discourse fragment:
    (i) A man and a woman starred in City Lights. (ii) The man was multitalented. If this does not entail that only one man starred in City Lights ( . . . 'Another man who also starred in City Lights was not multitalented'), its logical form is arguably given by,
    (i) $[\mathrm{a} x: \operatorname{man}(x)]$ ( $x$ starred in City Lights) and [a $x:$ woman $(x)]$ ( $x$ starred in City Lights).
    (ii) $[\mathrm{a} y: \operatorname{man}(y) ; y$ starred in City Lights] ([the $z: \operatorname{man}(z)](z=y)$ was multitalented).
    It is an interesting question under what circumstances a so-called $E$-type pronoun or similar occurrence is bound by an implicit (typically restricted) universal-quantifier occurrence and under what circumstances it is bound instead by an implicit existentialquantifier occurrence. In many cases, the issue might not be settled unambiguouslyfor example, 'Some senators are liars, but they have redeeming qualities'. It is possible that some $E$-type pronouns (occurrences) are pronouns of laziness rather than bound.

[^8]:    39. See by way of comparison also my Frege's Puzzle (Atascadero, CA: Ridgeview, 1986), at 144-47.
[^9]:    40. See note 28. There is a similarly corrected version of King's (K2): (K2') If a singular term $\beta$ is directly referential, then every free occurrence in a sentence of $\beta$ not within the scope of any nonextensional operator (other than classical variable-binding operators) is a directly referential occurrence. As with the replacement of $(M)$ by ( $M^{\prime \prime}$ ), and ( $M+$ ) by ( $M+{ }^{\prime}$ ), correcting ( $K 2$ ) effectively blocks King's argument.
