How to Embed Epistemic Modals without Violating Modus Tollens

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Abstract

Epistemic modals in consequent place of indicative conditionals give rise to apparent counterexamples to *Modus Ponens* and *Modus Tollens*. Familiar assumptions of familiar truth conditional theories of modality facilitate a prima facie explanation—viz., that the target cases harbor epistemic modal equivocations. However, these explanations go too far. For they foster other predictions of equivocation in places where in fact there are no equivocations. It is argued here that the key to the solution is to drop the assumption that modal claims are inherently relational (i.e., that they express a logical relation between a prejacent and a premise-set) in favor of a view that treats them as inherently quantificational. In particular it is suggested that modals are mass noun descriptions of information. We demonstrate how this approach unlocks the equivocation problem.[1]

[1]The majority of this paper was written during my time at the Jean Nicod Institute in Paris. Thanks to them for having me and to Saint Louis University for the research leave.

1 Counterexamples to MP and MT

Epistemic modals embedded in consequent place of indicative conditionals cause interesting trouble for *Modus Ponens* (MP). Consider the *zebra argument*, consisting of the following three sentences evaluated relative to a context c:

(Zebra MP)

[The relevant subjects are not sure whether the thing before them is a zebra or a cleverly painted statue.]

1. "If that is a zebra, then [in view of what we know] it must be an animal."
2. "It is a zebra." [not known or asserted by our subjects]

Therefore,

3. "[In view of what we know] It must be an animal." ###

The '[in view of what we know]' is meant to indicate that an epistemic reading of the modal is salient, but implicit, in c. However, the outcome is the same when our epistemic modifier is explicit—namely, the surface structure avows *modus ponens* while an available epistemic reading is invalid. The invalid reading supposes that the knowledge of the relevant subjects does not support the thing's being an animal rather than a statue. Perhaps they have reason to believe that a statue scenario is not remote. Then sentences 3 is false. And that valuation is entirely compatible with both the thing being a zebra unbeknownst to them (making sentence 2 true), and it being epistemically necessary (for them) that it is an animal given the supposition that it is a zebra (making sentence 1 true).

The necessary connection between being a zebra and being an animal is irrelevant to the phenomenon. The following example demonstrates the same apparent breakdown in the logic without the necessary connection:[2]

(Reliable Stalker)

[We know that Ralph is a reliable stalker and that he is currently reliably and tirelessly following Mary wherever she goes.]

(a) "If Ralph is at the party then [in view of what we know] Mary must be at the party."

(b) "Ralph is at the party." [although, we do not know this.]

(c) "[In view of what we know] Mary must be at the party"

[2] Thanks to **** for the example.

A potential utterance of 1 by either of us is true. 2 is true unbeknownst to us. And a potential utterance of 3 may nonetheless be false, since the above valuations do not suppose that our knowledge supports Mary's being at the party.

The phenomenon appears to be special to the epistemic reading. For go ahead and accommodate a bouletic or teleological reading of the embedded modal, respectively—e.g.,

- "If Ralph is at the party, then [in view of our wish to see Mary and Ralph together again] Mary must go to the party." or
- "If Ralph is at the party, then [in view of our goal of getting Mary and Ralph to talk soon] Mary must go to the party."

Then notice that an invalid reading of the corresponding argument is not available. In the bouletic and teleological versions of (Reliable Stalker), the conclusion is true. That is, in view of our wishes/goals, "Mary must go" is true, whether we know that Ralph is there or not. The bouletic and teleological versions do not carry invalid readings, most likely because the truth of the non-modal minor premise is sufficient to alter the default semantic core of the embedded modal.[3] By contrast, with the epistemic version, the truth of the minor premise is not sufficient to affect the default core of the embedded epistemic modal. Knowledge (or some other epistemic state, but not mere truth) of the minor premise is minimally required to affect the default semantic core of the embedded epistemic modal (and in a way that generates a reading of the embedded modal that matches the modal in the conclusion).

A relevant aspect of the problem is that the apparent MP-failures can readily be turned into apparent failures of *Modus Tollens* (MT):

(Zebra MT)

1. "If that is a zebra then [in view of what we know] it must be an animal."
2. "But that thing may not be an animal."

Therefore,

3. "It is not a zebra." ###

The surface structure avows MT, while an available epistemic reading allows for counterexample. If Speaker A uttered the premises, and the hearer, Speaker B, responded with the conclusion, "Oh, so it is not a zebra after all!", then Speaker A would be right to deny ever having claimed so much.

(Zebra MT), unlike (Zebra MP), helps itself to a certain relation between epistemic 'may' and epistemic 'must'. And one would be quite right to worry whether the full duality between 'must' and 'may' is allowed. However, even those who treat 'must', but

[3] I have in mind here the default premise-set or modal base, of which much more will be said later.

not 'may', as an *evidential marker*, and claim that 'must'—in addition to whatever else it does—brands the source of relevant information in some way, e.g., as non-visual and non-testimonial, should not balk at the direction of the inference that we employ. After all, "It may be that not-ϕ, [*in view of what we know via any means*]" obviously entails "It is not the case that it must be that ϕ [*in view of what we know via non-visual and non-testimonial means*]". The half of the duality principle that we employ is then not the trouble-maker. For this reason we care to treat the prima facie MP-failures and the prima facie MT-failures as essentially the same problem of epistemic embedding.

Other examples of apparent MT-failure involve probability operators:[4]

(Fair Die)

[A fair die has been rolled. We do not know the outcome, but we know the die is fair. Somebody argues:]

1. "If the die landed even, then it likely landed either 2 or 4."
2. "It isn't likely that the die landed either 2 or 4."

Therefore,

3. "The die landed odd." ###

We know that each of the six possible outcomes are equally likely. Hence, the subjective probability that the die landed 2 or 4 is only 1/3, and so, our utterance of sentence 2 is true. The conditional probability of the die landing 2 or 4, given that it landed even, is 2/3. We match our credence to this. Hence, our utterance of sentence 1 is true. But none of this rules out the die having actually landed even.

The centerpiece of our contribution to the discussion is one other aspect of this phenomenon: some clearly valid arguments share a surface structure with some of the invalid ones that we have considered. Consider the *coin argument*:

(Coin)

[A game is being played with a fair coin and a trick coin that always land Heads. B does not know which coin was just flipped, but knows that A has privileged access to this information. In particular, B knows that A knows which coin was flipped.]

1. A: "Yes, if that coin is the trick coin then [in view of what we know] it must land Heads. But, trust me, that coin may land Tails."
2. B. "Ah, so it is the fair coin."
3. A: "Right!"

[4] Cf. Yalcin (2012: 1001).

A natural reading (perhaps the most salient with the help of our backstory) is a valid instance of *modus tollens*.[5] So a solution to our embedded puzzle about the apparent MP- and MT-failures, should also speak to these (and other) structurally identical MP and MT success stories. That is the focus of our present investigation: *how to explain the invalidity of apparent MP- and MT-failures without over-generating the predictions of invalidity to cases like (Coin).*

In Section 2 we demonstrate just how tragic it would be to accept the validity of (Zebra MP), (Zebra MT), (Reliable Stalker), or (Fair Die). Such acceptance would collapse significant epistemic modal differences. Section 3 demonstrates the extensional inadequacy of wide-scoping the epistemic operators. Section 4 evaluates Kratzer's uniform treatment of epistemic modality and indicative conditionality. In particular it shows that her approach predicts invalidity in the right places—like in (Zebra), but only at the cost of predicting invalidity in some wrong places—like in (Coin). Sections 5 and 6 examine variations on the Kratzerian theme, including "double modalization" strategies (viz., those that treat our target operator as nested under an independent conditional that is itself a modal operator), and non-standard inheritance functions (viz., those that promote a nonstandard contribution from the antecedent to the modal base of the embedded operator). Neither variation solves the problem. Each variation either predicts the wrong truth values for our conditionals, harbors terrible paradoxes of implication, or over-generates predictions of invalidity. The paper concludes, in the final section, with a prescription for those who wish to retain a truth conditional analysis of epistemic modality—viz., deny that an (epistemic) modal expresses a two-place relation between a proposition and a premise-set. We argue that if (epistemic) modals express quantification, and in particular if they express mass noun descriptions, then we can explain the embedding phenomena. The modals-as-descriptions approach predicts the outcomes outlined above.

2 Epistemic Modal Collapse

If we defy intuition and accept the validity of the questionable inference forms, then we collapse useful epistemic differences. The collapse flows from the theoremhood of sentences of the following form:

(*) "If ϕ then it must be that ϕ", where 'must' is read epistemically.[6]

[5] There is also an invalid reading—one where the speaker makes salient that the listener is the relevant practical deliberator, and so, by 'may' means something like "for all you the listener know, it may be that...". But I have padded the dialog with cues that move us away from that reading. Thanks to **** for warning that this reading is nearby.

[6] A discussion and generalization of the theorem (as it pertains to deontic modality) is found in Anette Frank's dissertation (1996: 53). For our purposes: 'If ϕ then $M(\psi)$' is true, whenever ϕ necessitates ψ and 'M' is epistemic 'must' or 'might'. Endorsements of the epistemic version of principle appear in Zvolenszky (2006: 167-168) and Geurts (2004: 10-11).

We find that this principle generates the factivity of epistemic 'might'. That is, whenever an utterance of 'MIGHT(ϕ)' is true, it follows that 'ϕ' is true is well. Here is the result at a glance, followed by an explanation of the deduction:

$$\dfrac{\dfrac{}{\neg\phi \rightarrow MUST(\neg\phi)}\,(*) \qquad \dfrac{MIGHT(\phi)}{\neg MUST(\neg\phi)}\,(\text{Dual})}{\dfrac{\neg\neg\phi}{\phi}}\,(\text{MT})$$

We suppose at the top of the right branch of the tree that it might be that ϕ. Then, by (the non-controversial half of) the duality principle, it follows that it is not the case that it must be that not-ϕ. The left branch begins with an instance of our theorem (*): if not-ϕ then it must be that not-ϕ. We let '$\rightarrow$' stand for the natural language indicative conditional. Modus Tollens appears to license, $\neg\neg\phi$, which classically, gives us ϕ. Hence, if it might be that ϕ then ϕ. Only truths are epistemically possible! The converse is standardly treated as trivial: all truths are epistemically possible.[7] In sum, epistemic possibility collapses into truth.

The analogous result, inverting occurrences of 'must' and 'might' shows that "MUST(ϕ)" implies "ϕ". The converse is a trivial consequence of (*), "$\phi \rightarrow MUST(\phi)$". Hence, even epistemic necessity collapses into truth!

If the collapse results are valid and (*) does not have counterexamples, then familiar epistemic modal differences are actually illusory. They highlight the urgency with which the embedding problem needs treatment. Since the results turn on the same controversial inference forms as those in the previous section (and nothing else of much controversy), we treat them as one more symptom of the same underlying embedding problem.

With conventional wisdom we grant the validity of MP and MT. Furthermore, we do not grant the validity of the questionable inferences from Section 1, lest we foreclose on significant epistemic modal distinctions. A solution to the embedding puzzle will then deny that those inferences are instances of MP and MT. We now examine theories that seem naturally positioned to do that.

3 Wide-Scoping

Wide-scoping says that our so-called "embedded" epistemic modals actually scope out over a material conditional. So the indicative conditional with a surface structure that embeds the modals as we have alluded:

$$\phi \rightarrow MUST(\psi)$$

[7]Even if there are some notions of epistemic possibility that are not entailed by truth (e.g., "possible for all Jake falsely believes"), the argument is general enough to be a problem for substantial notions of epistemic possibility that are entailed by truth.

is said actually to have the following form:

$MUST(\phi \to \psi)$.

This would explain the invalidity of our target arguments. After all, on this reading, the so called "MP versions" of the arguments actually take this form:

1. $MUST(\phi \to \psi)$
2. ϕ

   Therefore,

3. $MUST(\psi)$ ###

And the so called "MT versions" of the argument take this form:

1. $MUST(\phi \to \psi)$
2. $\neg MUST(\psi)$

   Therefore,

3. $\neg\phi$ ###

These inference forms are clearly not instances of MP or MT. Moreoever, they are clearly invalid when 'MUST' and '$\to$' are treated as independent operators, and '$\to$' is read as the material conditional and 'MUST' expresses something like entailment or support for the compliment in view of relevant epistemic states. The "MP versions" of the argument at best entail ψ, and the "MT versions" at best entail '$\neg MUST(\phi)$'. Hence, the wide-scope reading seems to explain the invalidity of the original arguments, and we short-circuit the threat to basic logic. The same explanation blocks our use of MT in the collapse results.

The problem with the wide-scope explanation is that it supposes that our puzzle conditionals say something true in circumstances where the epistemic operator applies to the consequent (or the negation of the antecedent) unrestrictedly. But that is just not what is being expressed by those natural language conditionals. For instance, suppose that we know the die is weighted to favor even, hence that the die probably has landed 2, 4 or 6. Then, given the wide-scope reading of the indicative, the following is predicted to be true: "If the die landed odd, then it probably landed 2, 4 or 6." But that conditional is clearly false.[8]

Here is a variation replacing 'probably' with 'must'. Suppose I know that Fred bet handsomely on 4, and now I see that he is elated and cheering after the roll of the die. I

[8] The argument is adapted from Lewis (1975: 10), where it is argued that adverbs of quantification, such as 'often' in "Often, if it is raining my roof leaks", do not have wide scope. For otherwise it would express something true, if my roof leaks often but never during rain. And the sentence does not express something that should be true under those circumstances.

say, "The die must have landed on 4." Wide-scopers then commit me to the truth of this false claim: "If the die landed odd, then it must have landed on 4." The wide-scope reading of that sentence, viz., $MUST(\phi \rightarrow \psi)$, after all, follows from the natural reading of what I said, viz., $MUST(\psi)$. That is because $\neg\phi \vee \psi$ is the material conditional reading, and happens to be supported by what is known, if ψ is supported by what is known.[9]

In this section the wide-scoper advocated a material conditional under the epistemic operator, and so, is stuck with the paradoxes of implication associated with true consequents or false antecedents. However, such paradoxes are an excellent reason to give up on the material conditional reading of the indicative. We will hereafter reject the material reading, or any other reading of the indicative that prescribes vacuous truth values in the face of contingent matters of fact.

Moreover, even if one is not bothered by the paradoxes, wide-scoping will disappoint. For if wide-scoping is always correct, then the invalidity it predicts for (Zebra MT) will mistakenly be predicted for (Coin). If wide-scoping is not always correct, then a story is owed about when it applies and when it does not, and why.

The ultimate solution to our problem may appear ambitious. For to resolve the above set of problems we seem to need an adequate theory of the indicative, together with an adequate theory of epistemic modals, together with an adequate theory of how these operators interact in our target environments. In the next section we examine a widely endorsed version of that tripartite story, which is developed in Angelika Kratzer (1977; 1981; 1986; 2012) and cited as a generalization of work found in David Lewis (1973; 1975).

4 Kratzerian Single Modalization

In the opening section of "What 'Must' and 'Can' Must and Can Mean" (1977; 2012: Ch 1), Angelika Kratzer emphasizes that modals (epistemic or otherwise) are "inherently relational". Specifically, they serve to express a logical relation between the prejacent, ϕ, and some contextually determined premise-set, Γ:

> (Relational Theory of Modality)
>
> A modal claim, $M(\phi)$, expresses a logical relation, $LOG(\Gamma, \phi)$, where LOG is determined primarily by the choice of modal expression and Γ is determined by linguistic or extra-linguistic cues in the context of utterance.[10]

[9] Other objections to wide-scoping appear in Yalcin (manu: **). Yalcin argues that the burden is on the wide-scoper to show that the narrow-scope reading is never available. For to get the puzzle off the ground, one needs only some instances of the narrow-scope reading. Yalcin, however, does not argue that any such narrow scope example can be turned into an instance of the embedding puzzle.

[10] There is at least one other moving part emphasized in Kratzer (1981), and elsewhere, namely, the "ordering source". The ordering source is the contextually determined set of premises that is the source of the ordering relation referred to in the corresponding truth conditions. So if the ordering source, Δ, contributes to the semantic value of the modal claim, then our above characterizations should additionally

On this account a must-claim or might-claim is epistemic just in case the premise-set is determined by some epistemic properties, such the speaker's knowledge. 'Must' generally expresses the logical relation of entailment, while 'might' generally expresses compatibility. Accordingly, an epistemic must-claim says, roughly, that the prejacent is entailed by the epistemic premise-set:

(Bare Epistemic Must)

"It *must* be that ϕ", in the epistemic sense, as uttered in context c expresses $ENTAILMENT(\Gamma,\, \phi)$, where Γ is the epistemic premise-set determined in c

and a might-claim says, roughly, that the prejacent is compatible with the epistemic premise-set:

(Bare Epistemic Might)

"It *might* be that ϕ", in the epistemic sense, as uttered in context c expresses $COMPATIBILITY(\Gamma,\, \phi)$, where Γ is the epistemic premise-set determined in c.

The possible worlds truth conditions for epistemic must/might are as follows:

"$M(\phi)$" as uttered in c in the epistemic sense expresses something true at a world w iff every/some (closest-to-w) epistemically accessible ϕ-world is also a ψ-world, where epistemic accessibility is a suitable function of c.

Our entire study will regard only contingent propositions ϕ and ψ. This allows us to ignore difficulties with inconsistent premise sets, and the corresponding complications to Kratzer's relational analysis.

Kratzer (1981; 1986; and 2012) generalizes further, treating indicative conditionality as a special instance of epistemic modality. A *bare indicative conditional* is one that harbors no overt modal operators. On Kratzer's view the bare indicative harbors implicit epistemic modality. Roughly, we are to think of "If ϕ then ψ" as saying, "If ϕ then it must be that ψ", where 'must' is overt and read epistemically.[11] Indeed, the epistemic operator is the dominant operator in the logical form of that claim. The antecedent serves merely to augment the premise-set associated with that operator.

Accordingly, the bare indicative says, roughly, that the consequent is entailed by the set that is the union of the antecedent and the epistemic premise set:

include that element in the first argument place of the logical relation—giving, $LOG(\Gamma \cup \Delta,\, \phi)$. However, this detail can be overlooked for the majority of our discussion.

[11] I say 'roughly' because the view in Kratzer (2012: 98-99) appears to be that the conditional with the overt 'must' (but not necessarily the bare conditional) carries evidentiality—i.e., information about the source of the information in the epistemic premise-set. In particular, the overt 'must' is said to mark the information as having been acquired "indirectly", which among other things means it was not acquired visually. Incidentally, there are counterexamples. Consider, "If you see with your own two eyes that there are mountains on the moon, then there must be mountains on the moon", where 'must' is read epistemically. Clearly, the epistemic claim here is not, "if we see mountains on the moon, then our *non-visual* information supports there being mountains on the moon."

(Bare Indicative Conditional)

A bare indicative, "If ϕ, then ψ", expresses in c the epistemic modal claim, $ENTAILMENT(\Gamma \cup \phi,\, \psi)$, where Γ is the epistemic premise-set determined in c.[12]

Its truth conditions are as follows:

"If ϕ, then ψ", as uttered in c, expresses something true at a world w iff every (closest-to-w) epistemically accessible ϕ-world is also a ψ-world, where epistemic accessibility is a suitable function of c.

What about the conditionals with overt operators in conditional place? On Kratzer's view expressing epistemic 'must' in consequent place serves to make explicit the otherwise implicit modal.[13] For our purposes we may then treat our target conditionals from section 1 has having the same truth conditions as (Bare Indicative Conditional):

(Indicative Conditional, w/ Overt Epistemic 'Must')

A indicative with an over epistemic 'must' in consequent place, "If ϕ, then it must be that ψ", expresses in c the epistemic modal claim, $ENTAILMENT(\Gamma \cup \phi,\, \psi)$, where Γ is the epistemic premise-set determined in c.

The corresponding possible worlds truth conditions are as following:

"If ϕ, then it must be that ψ", as uttered in c, expresses something true at a world w iff every (closest-to-w) epistemically accessible ϕ-world is also a ψ-world, where epistemic accessibility is a suitable function of c.

The apparent failures of MP and MT that were demonstrated in Section 1 each involve both an embedded modal and a corresponding free modal. So we are now in a position to see what the Kratzerian should say about those cases.

> On the relational view, a modal relates a contextually determined premise-set to a prejacent in the following ways. First, a free epistemic modal claim relates the epistemic premise-set Γ (i.e.,, the set of propositions known by the relevant subjects) to the prejacent. Second, when embedded in consequent place, the epistemic modal augments Γ with the antecedent. Hence, in the usual case where the relevant subjects do not know the truth value of the antecedent, the augmented premise-set of the embedded modal will differ from the epistemic premise-set associated with the corresponding free modal. Moreover, for the Kratzerian, the premise-set associated with a modal claim contributes to the

[12]Again, we suppress the ordering source to avoid unnecessary complexity.

[13]More generally, the explicit modal is thought to replace the otherwise implicit modal, even if that explicit operator is not epistemic.

> semantic content of that claim. Hence, in the usual case where the relevant subjects do not know the truth value of the antecedent, there is an equivocation between the free modal claim and corresponding embedded modal claim. The puzzle cases from section 1 are instances of the usual case. Therefore, they involve an equivocation which explains why they are not instances of MP or MT.

The relational analysis thereby nicely predicts an equivocation in the puzzle arguments from Section 1.

Notice as well that principle (*), repeated here,

(*) $\phi \rightarrow MUST(\phi)$
where $\rightarrow$ is the natural language indicative conditional

is a theorem on the analysis we are investigating. That is because, trivially, all the (closest) epistemically accessible ϕ-worlds are ϕ-worlds. Since, the above analysis validates (*), it will have to block the modal collapse results (of Section 2) in some way other than by denying (*). And it does. It predicts an equivocation in the specious applications of MT that appear in the collapse results. The diagnosis is the same as the equivocation to be diagnosed in (Zebra MT). Kratzer's relational analysis then has a unified solution to the embedding problem and the collapse results.

Notice if we simply deny that the premise-set contributes to the content of the modal claim, then we lose the prediction of equivocation. For instance, one may deny the semantic thesis in favor of a view that identifies the premise-set merely as a parameter in the circumstance of evaluation. Such a position will interpret our problematic inferences as truly instances of MP and MT (since the modals will be interpreted univocally), and so, unfortunately will have the consequence that MP and MT have counterexamples. I take the ability of the Kratzerian picture to predict an equivocation where there seems to be one as a point in its favor.

The problem for the Kratzerian approach is that the mechanisms that generate the equivocation and explain the invalidity in our puzzle cases (e.g., the zebra argument), also generate equivocations in non-puzzle cases (viz., the coin-argument). The salient reading (thanks to the backstory) is a valid instance of modus tollens. The Kratzerian analysis, however, predicts in (Coin) the very same equivocation that is found in (Zebra). For the premise-set of the modal embedded in the conditional, but not the premise-set of the modal in the minor premise, will be augmented by the antecedent. Hence their corresponding premise-sets will not make the same contribution to the content, and an equivocation will be predicted.

A further problem with the Kratzerian approach to the conditional is that indicative conditionals like the coin-conditional, in cases like the coin-circumstances, will give rise to paradoxes of implication. After all, a relevant subject knows the coin is fair. Hence, once the epistemic premise-set is augmented with the antecedent (i.e., the information that the coin is trick), it becomes inconsistent. Inconsistent premise-sets entail everything. Therefore, by (Indicative Conditional, w/ Overt Epistemic 'Must'), it follows that "If the coin is trick, then it must land Tails" is true. And that is the wrong truth value, since the trick coin always lands Heads. If this is right, then the Kratzerian approach to the indicative conditional is not much better than the material conditional approach.[14]

Of course the Kratzerian may reply as follows: typically when the antecedent is incompatible with the epistemic states of the relevant subject, the conditional is read counterfactually and not epistemically. And that is because a norm of indicative conditional assertion is that the speaker does not know the truth value of the antecedent, while a norm of counterfactual assertion is that the speaker knows that the antecedent is false. However, it is not clear how this will help one who embraces a relational theory of the modality. If the point is supposed to be that, with counterfactuals, the modal in consequent place will not absorb content from the antecedent, then yes our consequent will share content with its non-embedded counterpart. This is an attractive outcome for (Coin). However, if the modal in consequent place does not absorb content from the antecedent, then it will be equivalent to its non-embedded counterpart, which is to say the antecedent of the conditional will be superfluous in determining the truth value of the conditional with the embedded modal. For instance, when "The coin must land Heads" is false relative to subject x in the actual coin-circumstances, then that very same sentence will be false relative to x at an arbitrary world. Hence it will be false at all the relevant antecedent-worlds. That is, it will be false at all the relevant worlds where the coin is the trick coin. But then "If it is the trick coin, it must land Heads" is false, even though the trick coin always lands Heads. Alternatively, if the point of going counterfactual is supposed to be that, with counterfactuals, an epistemic modal in consequent place will absorb the epistemic premise-set determined by the subjects' counterparts at closest antecedent-worlds (rather than the premise-set of the relevant subject at the world of evaluation), then we still get trouble for (Coin). That is because the knowledge of the relevant subject in (Coin) is not causally independent of the antecedent. After all, that subject has reliable access to the state of the coin. In the actual circumstance the antecedent is false and the relevant subject knows it. At nearby worlds where the antecedent is true, the counterpart of the relevant subject knows that it is true. But then the premise-set associated with the modal in consequent place of the counterfactual will differ from the premise-set of the modal in the minor premise, and an equivocation is mistakenly predicted for (Coin). So for the Kratzerian it appears not helpful to point to the norms of conditional utterances in an

[14]Notice that it does not help to consider the prejacent against every largest consistent subset of the resulting inconsistent set, as Kratzer (1977; 2012: Ch 1) does. Yes it turns out that not every largest consistent subset entails that the coin lands Tails, and so, "If the coin is trick, then it must land Tails" is correctly predicted false. But this move also incorrectly predicts "If the coin is trick, then it must land *Heads*" to be false. For the not every largest consistent subset of our inconsistent set (recall: comprised of what is known together with the antecedent) will entail that the coin lands Heads. After all, one of those consistent subsets includes the known information that the coin is *fair*. Such a set will not entail that the coin lands Heads.

attempt to block the Kratzerians mistaken predictions of equivocation in (Coin).

The Kratzerian approach has a difficult time treating (Coin) as a proper valid instance of Modus Tollens, even though it correctly diagnoses the equivocation in (Zebra). The next two sections evaluate subtle modifications to the Kratzerian approach.

5 Double Modalization

Double modalization, adapted from Anette Frank (1996), analyzes our target sentences, "If ϕ, then it must that ψ", as involving an implicitly modalized conditional, $\Rightarrow$, with an independent explicit modal, M, nested in consequent place. To evaluate such conditionals, "$\phi \Rightarrow M(\psi)$", we do not merely restrict M's premise-set to (closest epistemically accessible) ϕ-worlds and then check whether the prejacent, ψ, is true throughout. Instead, we go to the (closest epistemically accessible) ϕ-worlds, and then check whether the full modal claim, '$M(\psi)$', is true throughout (relative to some suitably shifted epistemic premise-set).

So relative to which base are we to evaluate "$M(\psi)$"? Is it the premise-set, Γ, comprised of the information states alluded to in the context of use? We begin with that version of the strategy, because it affords the fewest changes in the move from Kratzer single modalization to Frank's double modalization. Then we consider the variation on the strategy that says the base is the set, Γ', comprised of the information states had by the counterparts of those subjects in the relevant nearby antecedent-worlds? We list and consider both positions, after offering a generic epistemic truth condition for bare indicatives:

> (Bare Indicative Conditionals)
>
> an indicative conditional, "$\phi \Rightarrow \psi$", as uttered in context c is true at the world w (of c) just in case 'ψ' is true in all of the worlds, w', (closest to w) that (i) are epistemically accessible from w (as determined by c), and (ii) are ϕ-worlds,
> where ϕ and ψ are contingent.

The generic view is that indicatives are true at a world of evaluation, w, just in case all the closest-to-w epistemically accessible antecedent-worlds are also consequent-worlds, where epistemic accessibility is determined in the context, c, of use. The view retains the idea that indicatives are epistemic modals. We put aside for the moment questions about the logical form and content of such sentences, although it is natural to think of the conditional (pace Kratzer) as taking two propositional arguments. The truth conditions above are an improvement on the material conditional reading, of course, because they do not generally avow vacuous truth in the face of contingently false antecedents or contingently true consequents.

When an epistemic modal overtly appears in the consequent, we may understand the truth conditions for double modalization in one of the following ways:

> (Double Modalization 1)

"$\phi \Rightarrow M(\psi)$", as uttered in context c is true at the world w (of c) just in case '$M(\psi)$' is true in all of the worlds, w', (closest to w) that (i) are epistemically accessible from w (as determined by the epistemic premise-set Γ in c), and (ii) are ϕ-worlds,

where '$M(\psi)$' is true at w' if and only if 'ψ' is true at every world (closest to w') that (a) is a ϕ-world, and (b) is epistemically accessible from w' (as determined by Γ in c).

(Double Modalization 2)

"$\phi \Rightarrow M(\psi)$", as uttered in context c is true at the world w (of c) just in case '$M(\psi)$' is true in all of the worlds, w', (closest to w) that (i) are epistemically accessible from w (as determined by the premise-set Γ delivered in c), and (ii) are ϕ-worlds,

where '$M(\psi)$' is true at w' if and only if 'ψ' is true at every world (closest to w') that (a) is a ϕ-world, and (b) is epistemically accessible from w' (as determined by Γ', where Γ', owing to causal dependencies, results from the shift in the epistemic states of the relevant subjects from w to w').

The difference between these positions lies in the default epistemic base of the embedded modal. In (Double Modalization 1) M automatically involves the epistemic base from the context of utterance—viz., the epistemic premise-set Γ of the relevant subjects in c. The embedded modal claim, $M(\psi)$, then expresses something true (at an arbitrary world), roughly, just in case ψ is true at all the Γ-worlds (i.e., epistemically accessible worlds) where the antecedent, ϕ, is true. By contrast, in (Double Modalization 2), M takes on the epistemic base, Γ', which is the result of allowing any differences that arise in the epistemic states of the relevant subjects once they are deported to (closest) antecedent-worlds. Then the embedded modal claim, $M(\psi)$, expresses something true (at an arbitrary world), roughly, just in case ψ is true at all Γ'-worlds where the antecedent, ϕ, is true.

These two theories, (DMod1) and (DMod2) for short, come to the same thing in cases where the truth value of the antecedent is logically and causally independent of the epistemic states of the relevant subject(s). For the modal bases, Γ and Γ', are the only places in which the above sets of truth conditions differ. And in circumstances of such logical and causal independence, no changes are reflected in the information states of the relevant subjects when they are deported to nearby antecedent-worlds. After all, given the independence of the epistemic states from the truth value of the antecedent, flipping the truth value of the antecedent at nearby worlds (and changing as little else as possible) will not affect changes in those states. Hence, on either reading, the contribution of the embedded modal is expected to be the same in circumstances of such independence.

Our (Zebra) circumstances and its corresponding conditional

"If that is a zebra then it must be an animal."

involve the independence in question. The relevant subjects do not know whether or not the thing is a zebra. And this epistemic fact is independent of the actual nature of the thing. Ignorance of the truth value of the antecedent obtains at both the world of evaluation and at the closest antecedent-worlds (whatever the truth value happens to be at the world of evaluation). So the operant epistemic base will not differ across the two readings of the zebra-conditional. That is, $\Gamma = \Gamma'$, and the zebra conditional will therefore get the same truth value on (DMod2) that it gets on (DMod1). Importantly, *on either reading the conditional is correctly predicted to be true.* After all, given the identification of Γ with Γ', we happen on either reading to be dealing with only (closest) *zebra*-worlds compatible with what is known at the context of utterance. And, in all of these worlds, the thing is an animal. Hence, the full embedded modal claim (relative to either Γ or Γ') is true at an arbitrary world. Therefore, it is true at all the (closest) antecedent worlds that are epistemically accessible relative to the context of utterance, c, at the world of evaluation, w. So, either DMod strategy handles the zebra-conditional.

How do these strategies fair on the truth-value of the coin-conditional? Repeated,

> "If that coin is the trick coin then it must land Heads."

Recall that the speaker is implied to have special access to the initial state of the coin, and so, knows that the coin is fair. Since the default base, Γ, at the world of evaluation determines only worlds compatible with the subject's knowledge, it only includes fair-coin-worlds. Restricting this set of fair-coin-worlds to worlds where the coin is a trick coin delivers the empty set. But then "The coin must land Heads" is vacuously true at an arbitrary world. Hence, that modal claim is true at all (closest) antecedent-worlds that are epistemically accessible from the world of evaluation, w. *By (DMod 1), it follows that our target conditional is true*—as it should be. However, the truth is vacuous, since the consequent is vacuously true. But then unfortunately, "If the coin is *fair*, then it must land Heads" is predicted to be true as well. Paradox of implication remain.[15]

(DMod 2) does a little better, because it valuates the embedded modal claim as non-vacuously true, at an arbitrary world. For it requires us to evaluate "$M(\psi)$" relative to Γ', which is determined by the epistemic states of the relevant subject at nearby trick-coin-worlds, w'. Since the relevant subject is reliably tracking the state of the coin, at w' she knows that the coin is trick. And since, at w', she also knows that the trick coins always land Heads, it follows that "The coin must land Heads" is true, at an arbitrary world, relative to the epistemic premise-set, Γ'. And this truth value is non-vacuous because the intersection of the set of Γ'-worlds with the set of antecedent- (i.e., trick-coin-)worlds just is

[15]There is a second source of vacuousness that arises for (DMod 1), because the conditional is another independent epistemic modal whose epistemic base set of worlds is empty once it is restricted by the antecedent. In the coin-case, there are no epistemically accessible antecedent-worlds. So, vacuously, the consequent—in this case the embedded modal claim—will be true at all of the epistemically accessible antecedent-worlds.

the set of Γ'-worlds. So, non-vacuously, "The coin must land Heads" is true at the relevant nearby antecedent-worlds. In this respect (DMod 2) is an improvement on (DMod 1).

Nevertheless, like with (DMod 1), the coin-conditional itself ultimately ends up being vacuously true on (DMod 2), because the epistemic reading of the conditional requires us to evaluate the embedded modal claim at all nearby *epistemically accessible* antecedent-worlds. And there are no such worlds, since in the coin-case the relevant subject knows that the antecedent is false.

Whether double modalization predicts an equivocation in the corresponding arguments depends on a number of things. It depends on whether the epistemic base of the embedded modal contributes to the semantic value of that modal. It also depends on whether the augmentation imposed by the antecedent on that epistemic base contributes something to that modal's semantic value. The truth conditions above say nothing about such matters. However, if we charitably read the DMod strategies as contributing to content at both stages, such that differences both in the epistemic base and in the augmentation of that base contribute to relevant differences in content (i.e., differences that would account for fallacies of equivocation), then *the DMod strategies correctly predict the equivocation endemic to the zebra-argument*. After all, in the zebra case, both DMod strategies require the embedded modal to be augmented by an antecedent that is not known. Hence the embedded modal will not be associated with the same premise-set that is associated with its counterpart in the minor premise of the MT-cases (or in the conclusion of the MP-cases). Equivocation correctly predicted!

However, by the same reasoning, the DMod strategies mistakenly predict an equivocation in (Coin). On either version, the epistemic base of the embedded modal will be augmented with the antecedent, while the base of the modal in the minor premise will not. So the fact that (DMod 2) delivers the correct truth values non-vacuously for the embedded modal claim in both zebra- and coin-cases is small consolation. For in the end it over-generates predictions of equivocation. In this respect the double modalization strategies do no better than Kratzer's single modalization at providing a comprehensive solution to our embedding puzzles.

6 Actual Truth Value Inheritance

A novel position discussed in Cian Dorr and John Hawthorne (2010) is that the base of the embedded 'must' (or 'might') inherits information dictated by the *actual* truth value of the antecedent. This sort of high definition inheritance offered by Hawthorne and Dorr, or *HD-Inheritance* for short, says this:

$(HD - Inheritance)$

The modal claim "$M(\psi)$", as it appears in "$\phi \rightarrow M(\psi)$", is true just in case ψ is true in all/some of the epistemically accessible worlds that match actuality with respect to the truth value of ϕ, where our variables ϕ and ψ are non-modal.

HD-Inheritance imports from Kratzer's view the idea that embedded epistemic modals are associated with a restricted set of epistemically accessible worlds, and that the restriction is some function of the antecedent. However, unlike Kratzer's analysis, HD-inheritance restricts the epistemically accessible worlds with the antecedent *or its negation*, depending on which is true at the world of evaluation.

There are a number of ways to fill out the story. For we have said nothing about whether the conditional is to be understood independently of the embedded modal. Nor have we said anything about which epistemic base, Γ or Γ', is to do the inheriting. So we will apply HD-inheritance to each of the three main positions that we have examined in the previous three sections.

First, there is the material conditional analysis, which offers us a non-modal interpretation of the conditional. When HD-inheritance is applied to an epistemic modal embedded under the material conditional, we get the following truth conditions:

> $(Material\ Conditional\ w/\ HD-Inheritance)$
>
> The indicative with the embedded epistemic modal, "$\phi \rightarrow M(\psi)$", as uttered in context c, is true at world w just in case either 'ϕ' is false at w or '$M(\psi)$' is true at w. That is, ... just in case 'ϕ' is false at w, or ψ is true in all/some of the worlds (closest to w) that (i) match w with respect to the truth value of 'ϕ', and (ii) are epistemically accessible from w (as determined by the epistemic premise-set, Γ, associated with c).

This position says that the context of utterance, c, determines the default epistemic base set of worlds (i.e., the Γ-worlds), and that the actual truth value of the antecedent restricts this set further. The resulting set is the intersection of the Γ-worlds with the ϕ-worlds or with the not-ϕ-worlds, depending on the truth value of ϕ at w. If ψ is true at some/every (closest) member of that resulting set, then the consequent, '$M(\psi)$', of our material conditional is true. The consequent is false otherwise.

The second position applies HD-Inheritance to the Kratzerian Single Modalization strategy. The resulting position says that the conditional is a single epistemic modal, expressed overtly by the modal in consequent place, and where the epistemic base set of worlds is constrained by the actual truth value of the antecedent (rather than by the antecedent, full stop):

> $(Single\ Modalization\ w/\ HD-Inheritance)$
>
> The indicative with the embedded epistemic modal, "$\phi \rightarrow M(\psi)$", as uttered in context c, is true at world w just in case ψ is true in all/some of the worlds (closest to w) that (i) match w with respect to the truth value of 'ϕ', and (ii) are epistemically accessible from w (as determined by the epistemic premise-set, Γ, associated with c).

The third and fourth analyses emerge when we "double modalize" our target conditionals—that is, when we epistemically modalize the conditional and treat the embedded modal independently. The skeleton of the position is this:

$(Double\ Modalization\ w/\ HD - Inheritance)$

"$\phi \Rightarrow M(\psi)$", as uttered in c, is true at w just in case '$M(\psi)$' is true in all of the worlds, w', (closest to w) that (i) are ϕ-worlds, and (ii) are epistemically accessible from w (as determined by the epistemic premise-set, Γ, associated with c),

where '$M(\psi)$' is true at w' if and only if 'ψ' is true at some/every world (closest to w') that (a) matches w with respect to the truth value of ϕ, and (b) is epistemically accessible from w' (where a story about epistemic accessibility from w' is owed).

"Epistemic accessibility from w'" is unspecified on the most general characterization because there are a number of ways to go, as we noticed in the previous section. One may equate "epistemic accessibility from w'" with epistemic accessibility as determined by c in w, reminiscent of (DMod 1). Or alternatively one may allow "epistemic accessibility from w'" to reflect the differences in epistemic states that obtain at the closest-to-w worlds where ϕ is true. We formulate the corresponding versions of double modalization with HD-inheritance:

$(DMod\ 1\ w/\ HD - Inheritance)$

"$\phi \Rightarrow M(\psi)$", as uttered in c, is true at w just in case '$M(\psi)$' is true in all of the worlds, w', (closest to w) that (i) are ϕ-worlds, and (ii) are epistemically accessible from w (as determined by the epistemic premise-set, Γ, associated with c),

where '$M(\psi)$' is true at w' if and only if 'ψ' is true at some/every world (closest to w') that (a) matches w with respect to the truth value of ϕ, and (b) is epistemically accessible from w' (as determined by the epistemic premise-set, Γ, associated with c).

The second version of the double modalization view with HD-inheritance looks like this:

$(DMod\ 2\ w/\ HD - Inheritance)$

"$\phi \Rightarrow M(\psi)$", as uttered in c, is true at w just in case '$M(\psi)$' is true in all of the worlds, w', (closest to w) that (i) are ϕ-worlds, and (ii) are epistemically accessible from w (as determined by the epistemic premise-set, Γ, associated with c),

where '$M(\psi)$' is true at w' if and only if 'ψ' is true at some/every world (closest to w') that (a) matches w with respect to the truth value of ϕ, and (b) is epistemically accessible from w' (as determined by the epistemic premise-set, Γ', where Γ' results from the shift in the epistemic states of the relevant subjects from w to w').

If Γ is the relevant epistemic state in the world of evaluation w, then we may think of Γ' as the modification that results (if any) at the (closest-to-w) ϕ-worlds, w'. When the determination of Γ at w is causally and logically independent of the truth of ϕ at w, then $\Gamma = \Gamma'$. Recall that when Γ is not independent of whether ϕ is true at w (for instance, because the agents of those Γ-states are reliably monitoring the truth value of ϕ), then a change in the truth value of ϕ from w to w' can affect a difference between Γ and Γ'.

Suppose charitably that advocates of the above theories allow that the relevant premise-set, Γ or Γ', and any augmentations thereon, contribute to a level of semantic content at which equivocations may be properly diagnosed. Then all four HD-Inheritance positions correctly predict the invalidity of the zebra-argument, because each of those positions predict a difference in the premise-set ultimately associated with the embedded modal as compared to the set associated with the modal in the conclusion. Only the premise-set associated with the free modal in the conclusion allows for both antecedent-worlds and negated-antecedent-worlds, since in the zebra-case the relevant subjects are ignorant of the truth value of the antecedent. The base of the embedded modal, by contrast, will allow for only antecedent worlds (or only negated antecedent-worlds)—following the instructions for HD-inheritance.

Moreover, with the exception of (DMod 2 w/ HD-Inheritance), the HD-theories correctly predict no equivocation in the coin-argument. Recall that (Coin) is special in that a relevant subject knows the antecedent is false. So, since the first three HD-theories read the embedded modal as having a base just like the default epistemic base except that it is augmented by the actual truth value of the antecedent, they will predict no difference in the bases of those modals. (DMod 2 w/ HD-Inheritance), by contrast, does not simply take the actual default base and augment it by the actual truth value of the antecedent. Instead, it begins with the base, Γ', determined at nearby (accessible) antecedent-worlds, and then triggers the HD-restriction on that modified premise-set. The result, in the coin-case, is an inconsistent premise-set, since the actual truth value of the antecedent is inconsistent with what is known at nearby trick-coin-worlds, i.e., with the set Γ'. But then, unfortunately, an equivocation will be predicted when we build these premise-sets into the content of the modals.

This feature of the first three HD-theories to predict no equivocation in (Coin) is a notable advance on the non-HD-theories that we discussed earlier. For those theories (viz., Wide-Scoping, Kratzerian Single Modalization, and Double Modalization 1 and 2) all predict the same equivocation in (Coin) that they predict in (Zebra). So HD-Inheritance does well on this front. However, it has terrible trouble when it comes to extensional adequacy and corresponding paradoxes of implication.

When the antecedent of the zebra-conditional is false, most of the HD-theories mistakenly predict a false conditional. For when the antecedent is false, the embedded modal expresses something true just in case the thing is an animal at all the (epistemically accessible) worlds where the thing is *not* a zebra. Of course, some of those worlds are worlds where the thing is a statue and not an animal. So the embedded modal claim is predicted to be false, when in fact the thing is not a zebra. That is not a bad thing for (Material Conditional w/ HD). For since the antecedent is false, the target conditional is true no matter what the truth value of the embedded modal claim.[16] The other HD-theories, by contrast, do not do so well here. On (Single Modalization w/ HD), the falsity of the HD-restricted embedded modal is sufficient for the falsity of the entire conditional. Hence, on that view, the zebra-conditional is false when its antecedent is false. What about double modalizing with HD? The two DMod theories say the truth of the conditional will depend on whether the embedded modal claim is true at all the nearby (epistemically accessible) antecedent worlds. When the antecedent is actually false, the embedded modal claim is false even at closest (epistemically accessible) antecedent-worlds. After all, on either of these two DMod theories, since the epistemic states of the relevant subjects in the zebra-case are independent of the antecedent, the embedded HD-restricted modal claim will be false at nearby antecedent-worlds if it is false at the actual world. Since the HD-restricted modal claim is false at the actual world, and so, at nearby antecedent-worlds, the zebra-conditional is false. Therefore, all our HD-theories, with the exception of (Material Conditional w/ HD-Inheritance), fails to provide the correct truth value for the zebra-conditional when the antecedent is actually false. That is a terrible problem, because the zebra-conditional is true regardless of the actual truth value of the antecedent.

Lest we think that (Material Conditional w/ HD) is an adequate theory, it should not be forgotten that it harbors the familiar paradoxes. So, when the thing is a statue (and not a zebra), then the following claim comes out true: "If that thing is a zebra, then it must not be a zebra." And when the coin is actually fair, the following comes out true: "If the coin is a trick coin, then it must be fair." Both are unfortunate consequences of our best HD-theory.

(Single Modalization with HD) is special in that it predicts the wrong truth value for the coin-conditional. Since the case we considered involves a fair coin, (Single Modalization with HD) says the coin-conditional is true just in case the coin lands Heads in every (epistemically accessible) world where the coin is fair. But in some of the (epistemically accessible) worlds where the coin is fair, the coin does not land Heads. So the coin-conditional is false by the lights of (Single Modalization with HD). DMod strategies with HD do a little better. For no matter what the truth value of the embedded epistemic modal claim the coin-conditional will be true. And that is because in (Coin) there are no epistemically accessible antecedent-worlds, and (DMod 1 with HD) and (DMod 2 with HD) both require, for the conditional to be true, that the consequent be true at all epistemically accessible antecedent-worlds. Accordingly, the correctness of these predications with respect to the truth value of the coin-conditional is vitiated by the vacuousness that generates these predictions.

[16] Dorr and Hawthorne (2012) do take that very conditional to be true, regardless of the truth value of the antecedent. So one may charitably read their conditional as the material conditional.

A central concern for all HD-theories is that we want our conditionals to express what they express without regard for the actual truth value of the antecedent. Otherwise, we are ordinarily blind to the content of such claims to the extent that (i) we are ignorant of the truth value of the antecedent and (ii) the consequent involves an embedded modal. This concern about semantic blindness may be averted if the HD-theorist is happy to build the HD-restriction on the modal base into the index (rather than the content) of evaluation. However, such a position is a non-starter, if predicting an equivocation in the zebra-argument is a desideratum.

Below is the scoreboard. The final row indicates the data to be explained by an adequate theory. A box indicates a place where an incorrect answer is predicted, and where we have been critical about the foregoing approaches:

	Zebra-Conditional True?	Zebra-Argument Equivocates?	Coin-Conditional True?	Coin-Argument Equivocates?	Paradoxes of Implication?
Wide Scoping	YES	YES	YES	YES	YES
Kratzerian Single Mod	YES	YES	YES	YES	YES
Double Mod 1	YES	YES	YES	YES	YES
Double Mod 2	YES	YES	YES	YES	YES
Material Conditional w/ HD	YES	YES	YES	NO	YES
Single Mod w/ HD	NO*	YES	NO	NO	YES
Double Mod 1 w/ HD	NO*	YES	YES	NO	YES
Double Mod 2 w/ HD	NO*	YES	YES	YES	YES
Adequate Theory	YES	YES	YES	NO	NO

'*' indicates that the theory gives the wrong verdict on the truth value of the zebra-conditional, when the antecedent is false (i.e., when the thing is not a zebra)—even if it correctly treats the conditional as true, when the antecedent is true (i.e., when the thing is a zebra).

7 How to Embed Epistemic Modals

A promising way forward emerges when we give up the credo that modals are inherently relational:

> (Relational Account)
>
> A bare modal claim, $M(\phi)$, expresses a logical relation, $LOG(\Gamma, \phi)$, where LOG is determined by the choice of modal expression and Γ is determined by context.

We noticed that the relational view combines nicely with a view about the inheritance of content to handle the equivocation in zebra-arguments:

> (Content Inheritance for Relationalists)
>
> A modal embedded in the right side of a compound, e.g., in
>
> If ϕ, then $M(\psi)$,
>
> inherits content from the left side, resulting in the following claim:
>
> If ϕ, then $LOG(\Gamma \cup \{\phi\}, \phi)$.

This helps to explain the equivocation in the zebra-argument, by guaranteeing that the content of the embedded modal will differ from its unembedded counterpart. For a free modal (or one under negation) the logical relation expressed is that between the prejacent and the epistemically restricted set Γ, while for the embedded modal the relation expressed is that between the prejacent and the union of Γ and the set containing the antecedent.

(Coin), by contrast, does not involve this equivocation. So we will want a principled reason to block (Content Inheritance) in those sorts of cases. However, we learned in the section on double-modalization that merely blocking (Content Inheritance) renders the antecedent superfluous. And that gives rise to paradoxes of implication, even with an independent possible worlds treatment of the conditional. For once the antecedent is superfluous and not relevant to the content and truth value of the embedded modal, the embedded modal claim (relative to a given context of utterance) will have the same truth value at every world. Hence it will have that truth value at every (relevant) antecedent world, no matter what the antecedent looks like.[17]

We should consider a rejection of the relational theory of modality. For toggling (Content Inheritance)—to restrict or fail to restrict a determinate premise-set—is not sufficient to handle the cases. And there is something essentially right about (Content Inheritance)—viz., it makes up a needed semantic difference between the embedded and corresponding

[17] Recall that it does not help to allow the premise-set to shift to whatever information is available to the relevant subject's counterpart at the nearby antecedent-worlds. For such a shift, together with the relational view, incorrectly predicts an equivocation where there is none—viz., in (Coin).

free modal, when there is an equivocation. So there is reason to suspect something essentially wrong with the underlying relational view upon which it is operating.

The proposal here is that *modals are not relational; they are inherently quantificational*—expressing the existence of relevant information standing in the given logical relation to ϕ:

> (Quantificational Account)
>
> A bare modal claim, $M(\phi)$, makes a quantificational claim in virtue of a covert mass noun description, "The information Γ". The quantificational claim says, "The relevant information Γ stands in logical relation LOG to the prejacent ϕ":
>
> [The Γ: Γ is the R-information] $LOG(\Gamma, \phi)$,
> where R is a contextually determined restriction on the domain of the quantifier.

Content Inheritance then requires a slight facelift:

> (Content Inheritance for Quantificationalists)
>
> A modal embedded in the right side of a compound, e.g., in
>
> If ϕ, then $M(\psi)$,
>
> inherits content from the left side, resulting in the following claim:
>
> If ϕ, then [The Γ: Γ is the R-information] $LOG(\Gamma \cup \{\phi\}, \phi)$.

Bare epistemic modals are typically treated as restricted to knowledge. To handle a wider array of cases, we should think about them more generally as restricted to actionable information—i.e., factual information that is affordable to the salient practical deliberator(s) for the purposes of her/their salient deliberation. The quantificational view analyzes bare epistemic modal utterances as follows.

> (Quantificational Account of Epistemic Modals)
>
> Bare epistemic modal claims, $MUST(\phi)$ or $MIGHT(\phi)$, express, respectively, the following quantificational claims:
>
> [The Γ: Γ is the actionable information] $ENTAILMENT(\Gamma, \phi)$
>
> and
>
> [The Γ: Γ is the actionable information] $COMPATIBILITY(\Gamma, \phi)$.

On the quantificational view we see that a must-claim, $MUST(\phi)$, tells us, roughly, "The actionable information entails ϕ".[18] And the epistemic might-claim basically says, "The actionable information is compatible with ϕ".[19]

We prescribe a treatment of conditionals that is independent of the embedded modal. If the presumption of ignorance is satisfied, then read the independent indicative as expressing something that is true, roughly, just in case all the closest epistemically accessible antecedent worlds are consequent worlds. That is, read it epistemically. If the presumption of ignorance is blocked, then read the English indicative conditional as expressing something that is true, roughly, just in case all the closest antecedent-worlds are consequent worlds. That is, read it counterfactually.[20]

Moreover, we take it that (Content Inheritance) for modals is blocked when the presumption of ignorance fails. So, with respect to conditionals, it is typically in play on the epistemic interpretation but not on the counterfactual interpretation. The foregoing understanding of modals, conditionals and content inheritance is sufficient to demonstrate that the quantificational account of the embedded modals can handle the target cases.

The instructions for embedding go like this. When the indicative conditional, "If ϕ then $MUST(\psi)$", is uttered with the presumption of ignorance about the truth value of the antecedent, then the conditional is read epistemically, and the embedded modal inherits content from the antecedent as expected:

> (Embedded Quantificational 'Must' w/ the Norm of Ignorance Satisfied)
>
> The epistemic claim, '$MUST(\phi)$', in "If ϕ then $MUST(\psi)$", expresses the quantificational claim,
>
> [The Γ: Γ is the actionable information] $ENTAILMENT(\Gamma \cup \{\phi\}, \psi)$.

And when the same sentence, "If ϕ then $MUST(\psi)$", is uttered while the presumption of ignorance is blocked, and all else is equal, then the conditional is read counterfactually, and the embedded modal does not inherit content from the antecedent, giving

> (Embedded Quantificational 'Must' w/o the Norm of Ignorance Satisfied)
>
> The epistemic claim, '$MUST(\phi)$', in "If ϕ then $MUST(\psi)$", expresses the quantificational claim,
>
> [The Γ: Γ is the actionable information] $ENTAILMENT(\Gamma, \psi)$.

[18] We are open here to tinkering with the precise restriction and choice of logical relation.

[19] We say, "basically" and "roughly", because some subtleties are overlooked, including an ordering source to account for the closeness relation.

[20] A more ambitious approach would offer a generalization of the "modals as descriptions" account to include conditionals. However, the generalization is not required to resolve the embedding conundrum. For a defense of the position that conditionals are definite descriptions, see Philippe Schlenker (2004).

The account provides us with two readings our target conditionals. The zebra-conditional expresses the first reading, roughly, "If that thing is a zebra, then the actionable information, augmented by the fact that that thing is a zebra, supports the thing's being an animal." It expresses that reading because the zebra-argument, which satisfies the presumption of ignorance, triggers both the epistemic reading of the conditional and the inheritance of content for the embedded modal. Of course the inheritance does not occur in free modals, like the one in the conclusion of (Zebra MP) or the one in the minor premise of (Zebra MT) . So we get a nice account of the equivocation that is present in the zebra-arguments (and in the unexpected collapse results).

The second reading is enjoyed by the coin-conditional. The coin-conditional ultimately expresses the counterfactual claim, "If that were the trick coin, then the actionable information would support its landing heads." Here, the independent conditional is treated like a subjunctive conditional (thanks to the blocking of the presumption of ignorance), and the embedded modal (for the same reason) absorbs no content from the antecedent. It is familiar that a context in which the presumption of ignorance is blocked is one in which a counterfactual reading is available. Such contexts violate a norm of indicative conditional assertion, and satisfy a norm of subjunctive conditional assertion.[21] We are proposing that blocking the presumption will play an additional role, namely the role of shutting down the inheritance of content from the antecedent to the consequent. The result is that in (Coin) the embedded modal and the free modal share a quantificational content, and it is thereby predicted that no equivocation occurs in (Coin). Some of our HD-theories delivered the same prediction, but they all harbored terrible paradoxes.

Does the quantificational account do any better on the paradoxes of implication? It appears so. That is because without the presumption of ignorance about the truth value of the antecedent, the conditional itself is read counterfactually. So knowledge incompatible with the antecedent will not avow a vacuous truth value as it would with an epistemic reading of the conditional. On the counterfactual reading we are required to valuate the quantificational interpretation of the embedded modal claim (roughly, "The actionable information supports the coin's landing Heads") at those nearby worlds where the coin is a trick coin—rather than at those nearby *epistemically accessible* worlds.... We get the correct truth value, since at those worlds the counterpart of the salient deliberator (say, our speaker) knows that the coin is trick and knows that trick coins always land Heads, hence, her actionable information supports the coin's landing Heads. Accordingly, "The actionable information supports the coin landing Heads" is true at nearby antecedent-worlds, and the conditional, in ordinary Lewis-Stalnaker semantics, is assigned True. The comparable conditional, "If the coin is *fair*, then it must land Heads", also gets the correct truth value—viz., False. For the very same quantificational claim is not true at all nearby worlds where the coin is fair. After all, in nearby worlds where the coin is fair (including the actual world), a relevant subject knows that the coin is fair and knows that fair coins do

[21] Cf. Robert Stalnaker (1975) and Kratzer (1979: 135).

not always land Heads. Hence it is false at nearby worlds that the actionable information supports the coin's landing Heads. The upshot is that no vacuous truth values appear to arise for contingent antecedents in coin-like circumstances, even though they did for most HD-inheritance views, and most other strategies that we considered. The quantificational account deals most comprehensively with the set of problems surrounding (Coin).

Where do the motivations for the relational view go wrong? One argument in Kratzer (1977: 341-342; 2012: 7) said to support the relational view goes something like this: since modals with explicit modifiers (e.g., with "for all Jimmy knows") contribute to relational claims (such as, "*In view of what Jimmy knows* it must be that ϕ") and we want a unified account of bare and modified modals, we should infer that the semantic core of the bare modal is a relational claim. However, this sort of argument goes wrong in suggesting that the relevant semantic value of the free relative, "what Jimmy knows", is a set, just because it determines one. "What Jimmy knows" expresses either a descriptive content, "the set of propositions known by Jimmy" or, as we prefer, a mass noun description "the information known by Jimmy". The logic of Kratzer's reasoning then takes us in an altogether different direction than she suggests. We should conclude that a modal, bare or modified, predominantly expresses quantificational rather relational content.

A second piece of data in Kratzer (2012: 12) used to support the relational analysis is that it "has the important consequence that [modal sentences] can express contingent propositions". And that, Kratzer says, is because the truth value depends on the premise-set, and the premise-set may vary from one world (and use) to the next. But this argument blurs different notions of "meaning". The relational claim said to be expressed by must-sentences and might-sentences in context is $ENTAILMENT(\Gamma, \phi)$, where Γ and ϕ are both provided for, and $COMPATIBILITY(\Gamma, \phi)$, where the arguments are provided for, respectively. Such completed claims are not contingent. Kratzer should say that, on the relational analysis, the modal *sentence* or perhaps its Kaplanean character (i.e., its function from contexts to truth values) is contingent. For the character of the modal sentence (and the sentence itself), but not the proposition expressed by that sentence in context (i.e., the Kripkean intension associated with the completed relational claim), underdetermines the premise-set denoted—and so, underdetermines the truth value across worlds (and contexts).

By contrast, understanding modals as descriptions does capture the contingent nature of the modal proposition. For the description may change its denotation across worlds (and contexts) without changing its Kripkean intension. Hence, the truth value of the modal proposition may vary across worlds (and contexts). The modals-as-descriptions approach therefore does not clash with, but rather accommodates, the motivations for the relational theory of modality.

We have argued that the modals-as-descriptions approach succeeds where other truth conditional theories fail—viz., at unlocking the embedding puzzle. The puzzle is to explain the difference between valid and apparently invalid instances of MP and MT, preferably without vacuous truth values arising from contingent matters of fact. We do not countenance actual violations of deeply entrenched logical principles. Instead, we explain the difference in terms of the presence or lack of an epistemic modal equivocation—a difference best accommodated if we treat epistemic modals as expressing a quantificational, rather than relational, content. For only embedded modals that express quantification will interact with (Content Inheritance) to predict the equivocations precisely where they are, and without generating paradoxes of contingent implication. The ability of the modals-as-descriptions approach to unlock the embedding puzzle is strong evidence in its favor.

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