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On Content[1]

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I

Frege introduces his powerful theory of cognitive content in "Über Sinn und Bedeutung", observing that $\ulcorner a = a \urcorner$ and $\ulcorner a = b \urcorner$ are obviously statements of differing "Erkenntniswerte"—literally "knowledge worth"—since the former is a priori and, following Kant, to be called "analytic" whereas, for many distinct terms a and b, the latter contains an a posteriori, and indeed a very valuable, extension of knowledge. How is this possible? Prolonging the reader's suspense until the final paragraph, Frege explains that the sense of a sentence, i.e. the thought expressed by it, is no less relevant to the purpose of acquiring knowledge than the *Bedeutung*, i.e. the truth value. Since $\ulcorner a = a \urcorner$ and $\ulcorner a = b \urcorner$ express different thoughts when a and b differ in sense, the two sentences will then differ in *Erkenntniswerte*.

The significance of these passages to the history of the philosophy of semantics can hardly be overstated. Yet there remains widespread disagreement concerning their proper interpretation.[2] Perhaps the most natural interpretation sees Frege as proposing that the *Erkenntniswerte* of a sentence be identified with the thought, or proposition, expressed. I have come to believe that this interpretation may be incorrect. Frege may be proposing instead that the *Erkenntniswerte* of a sentence be seen as a special feature of the thought expressed. The crucial wrinkle would be that whereas different thoughts often yield different *Erkenntniswerte* (as with the thoughts expressed by $\ulcorner a = a \urcorner$ and $\ulcorner a = b \urcorner$), they do not do so invariably. The *Erkenntniswerte* of a sentence would emerge as an intermediate semantic value: something that is determined by the thought but not vice versa, something that determines truth value but not vice versa. If we posit *Erkenntniswerte* also for singular terms, the *Erkenntniswerte* of a singular term would likewise be a semantic value more coarsely grained than sense yet more finely grained than reference.

Is there a significant semantic value that is intermediate in this way between cognitive content and extension? One obvious candidate which is widely

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[1] I am indebted to Jan Alnes for pointing out to me that Frege seemed to have in his *Begriffsschrift* a distinct notion of content, which is supposed to be that respect in which logically equivalent sentences are the same. Most of the ideas presented here emerged in the course of discussions with Alnes.

[2] See David Coder, (1974, pp. 339-343); Gregory Currie, (1982, pp. 108-112, especially 110). Rod Bertolet, in "Conventions and Coreferentiality" (unpublished), issues a provocative reply to the interpretation I offered in my (1986, at pp. 51-54).

employed in philosophical semantics is what is sometimes called the "intension", that is, the function from possible worlds to extensions—or what comes to the same thing in the case of sentences, the set of possible worlds with respect to which the sentence is true. (The former is the characteristic function of the latter.) The intension of a sentence yields one interpretation of the oft-used phrase "truth conditions" as the maximal conditions which might have obtained and with respect to which the sentence is true. More than a few philosophers have misidentified the proposition expressed by a sentence with the sentence's intension. Our ordinary attributions of belief and other propositional attitudes leave little option but to differentiate among numerous beliefs sharing the same intension. There are additional reasons to distinguish between a proposition and its corresponding intension, some of essentially the same sort as the reason that Frege cites in distinguishing the *Erkenntniswerte* of $\ulcorner a = a \urcorner$ from that of $\ulcorner a = b \urcorner$. Indeed, it is clear that Frege should not identify the *Erkenntniswerte* of a sentence with its intension. For the sentences "Either Socrates is wise or he is not" and "Socrates is the only person to have developed from *s* and *e*", where "*s*" and "*e*" name the particular gametes from which Socrates sprang, obviously differ in *Erkenntniswerte*. The first is analytic and its cognitive content a priori; the second is synthetic and contains an a posteriori extension of knowledge. Yet it is at least arguable that the two sentences share a common intension. Contingent and purely a posteriori examples of the same phenomenon abound: "Mary is a Capricorn", "Mary is either a Capricorn or an individual retirement account", and "Mary is a Capricorn but not an individual retirement account" all coincide in intension yet clearly differ in *Erkenntniswerte*. To this extent, epistemology is more discriminating than metaphysics.[3]

Whatever Frege meant, or did not mean, in his use of "Erkenntniswerte", if ordinary propositional-attitude attributions are a guide to the individuation of propositions, they also provide important reasons to posit an intervening semantic value that intercedes between the proposition and its intension. Doing so provides the means for solving a philosophical problem noticed by Ali Kazmi.[4] Some propositions have been given special labels. We say such things as "Michael doubts Church's Theorem". Exactly which proposition is Michael being said to doubt by that sentence? Kazmi points out that if propositions are

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[3] It must be acknowledged that in correspondence with Edmund Husserl, Frege offered a version of logical equivalence as his only criterion for two sentences expressing the same thought. See his (1980, pp. 70-71). Cf. Frege's "Compound Thoughts," in his (1984, pp. 390-406). I shall follow other commentators in assuming that Frege's pronouncements in this connection, since they are inconsistent with what seem to be the fundamentals of his theory of *Sinn* and *Bedeutung* (for example, his doctrines that the *Bedeutung* of the "that" clause in "Galileo believed that the Earth moves" is its customary *Sinn*, and that the truth value of a sentence is a function of the *Bedeutungen* of the sentence components), represent a significant theoretical lapse, oversight, or perhaps a very crude oversimplification. Elsewhere Frege acknowledges that certain true arithmetic equations differ in sense—even though, given his logicism, all true theorems of arithmetic are logically equivalent (indeed, logically valid). See for example his (1979, pp. 224-225. See also p. 211).

[4] In Kazmi (1988). See Richard's (1990, pp. 161, n15, pp. 171-173). Richard defends his theory against Kazmi's problem, in part, by noting that it is a problem that every theory of propositions seems to face.

individuated finely enough, then there is no nonarbitrary way to decide whether Church's Theorem is the proposition that first-order logic is undecidable, or instead the proposition that it is not the case that first-order logic is decidable. It is arguable that the two versions are distinct propositions strictly speaking. On the other hand, both formulations, and others as well, in some sense, clearly contain *the same theorem*. Someone who proffered an unusual formulation of Church's Theorem as a discovery of his own, one that should be placed alongside Church's Theorem in importance, would be properly dismissed as a desperate plagiarist. How can this conflict be resolved?

The general problem here cannot be eliminated satisfactorily by individuating propositions along the lines of their corresponding intensions. Church's Theorem is not the same proposition as Gödel's Completeness Theorem, yet both are true in every possible world. A more discriminating notion is needed. But propositions *qua* objects of belief, doubt, and other propositional attitudes seem to be excessively discriminating. Someone might come to believe that it is not the case that first-order logic is decidable while momentarily suspending judgment whether first-order logic is undecidable. As Mark Richard points out, Kazmi's problem is fuelled by the same forces from propositional-attitude discourse that pressure philosophers to individuate propositions more finely than their intensions.[5]

Let us take another example. Consider Murphy's Law, which is usually formulated: *Whatever can go wrong will*. There are numerous variants: *Whatever might not go right will not*; *Whatever does not go wrong cannot*, etc. All of these formulations share the same intension. More than that, they are logically equivalent. But it is very doubtful that they all express precisely one and the very same proposition. Some of the propositions involve the notion of something's going right, others the notion of something's going wrong. Some involve negation, others do not. Which of these nearly identical propositions is Murphy's Law? Perhaps the proposition expressed by the standard formulation has some special claim to the name, but it cannot be wrong to call the others "Murphy's Law". Each of the formulations, in some sense, contains *the same law*.

To take yet another example, consider Leibniz's Principle of the Identity of Indiscernibles. This principle can be formulated in a number of ways:

> For all things x and y, if x has P iff y has P for all properties P, then $x = y$;[6]
> Any two things are qualitatively different;
> Anything exactly like a given thing is that thing itself;
> Things that are the same in every respect are the very same thing;
> Distinct things are always different from one another in some respect;
> If things do not differ in any respect, then they are one and the same;

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[5] Richard (1990, p. 172).

[6] In order to forestall certain difficulties, it may be assumed here that the properties being quantified over do not include so-called haecceities, i.e. properties of being numerically identical with this or that particular thing. The same (or corresponding) restriction should be made for each of the formulations to be given.

The only thing exactly like a given thing is itself; etc.

Some of these propositions involve the notion of things being exactly alike, while others involve the complementary notion of things being different in some respect. Some involve the notion of two things, others the notion of a single thing. Although they are different propositions, they are intimately related, so much so that no one of them can plausibly be singled out as *the* Principle of the Identity of Indiscernibles, to the exclusion of all the others. All of them qualify equally well.

Are there many different Principles of the Identity of Indiscernibles, each distinct from the others? No; that would miss the point. The solution that I propose lies in the recognition that such things as the Principle of the Identity of Indiscernibles are not themselves propositions. The Identity of Indiscernibles is something generic, something that is common to each of the distinct propositions formulated above. It is… well, the *principle* which is embodied in each of the propositions. A similar point may be made in connection with any number of such items as Goldbach's Conjecture, the Heisenberg Uncertainty Principle, Kepler's Laws of Planetary Motion, the Principle of Sufficient Reason, and a host of ism's (dualism, logicism, behaviourism, pragmatism, logical atomism, functionalism, etc.).

There is something which all sentences logically equivalent to a given sentence share, even when they do not express a single proposition, and which is not shared by logically unrelated sentences. This something is what Frege in his *Begriffsschrift* (§3) called the "conceptual content" (*begrifflichen Inhalt*) of the given sentence—a notion that may have been a precursor to his later notion of *Erkenntniswerte*.[7] To use an overworked and multiply ambiguous term, each of the formulations of the Principle of the Identity of Indiscernibles displayed above contains, in the relevant sense, the same *information*. The operative notion of information cannot be the traditional notion of *cognitive* or *propositional content* (Frege's notion of *Sinn*, Russell's notion of meaning), since the various formulations express different propositions, different thoughts. Nor is it the modal notion of *intension*. We have already seen sentences which are logically unrelated, and which therefore contain different information in the relevant sense, but which share the same intension. I have elsewhere used the phrase "information content"[8] interchangeably with "propositional content". Since the latter expresses a notion that I am here distinguishing from the notion we seek, I shall introduce a different terminology. Let us tentatively call the primary semantic value that a sentence shares with all of its logical equivalents, but does not share with logically unrelated sentences, the *logical content* of the sentence. We may also speak of logical content in connection with singular terms. The key criterial idea is that

[7] Frege's word for his logical formalism, "Begriffsschrift", may be translated as "Conceptual Notation". He says in his preface to *Begriffsschrift* that it was his exclusive interest in "conceptual content" that led to the title of the work.

[8] Compare the explanation of my use of "information" in my 1986, p. 154, n2. I acknowledge there that the use of "information content" as a term for propositional content "constitutes a departure from at least one standard usage, according to which the information content of a sentence is perhaps something like a class of [propositions], closed under logical consequence".

expressions of a given sort have the same logical content if they are logically equivalent.

I mean "logical equivalence" in a fairly common but not overly strict sense, so that, for example, the two sentences "Michael is a husband" and "Michael is male and Michael is married" count as logically equivalent. This use of "logical equivalence" is perhaps nonstandard. The logical content of a sentence, as I intend the phrase, is not something that is strictly dependent on the very words (or other subsentential expressions) involved. Sentences of different languages, even some that do not have the same cognitive content, can share the same logical content relative to those languages. In some sense, the logical content of an expression is actually a feature of the expression's cognitive content, its sense. The operative notion of *logical equivalence* is that of an equivalence relation between propositions, thoughts. We have already seen that sentences with the same logical content may yet differ in the propositions they express. But those propositions must be equivalent. And sentences that express equivalent propositions *ipso facto* share the same logical content.[9]

Some sentences that are arguably logically equivalent in a straightforward sense do not even share the same intension. The two sentences

(1)   Saul Kripke is an anthropologist

and

(2)   Saul Kripke is actually an anthropologist

are equivalent in the modal logic of "actually" (in its indexical sense), but they express propositions, or thoughts, that differ dramatically in intension. The latter proposition, but not the former, is concerned with the goings-on in a specific possible world. Both sentences are false, but where (1) contains a proposition that might have been true, (2) contains a proposition that could not have been true. These two propositions are not related to one another in the same way the various propositions containing Murphy's Law are related.

This goes to the very heart of the difference between the classical, sentential notion of validity and the propositional notion that we are limning. Unlike (1), exactly which proposition (2) contains with respect to a context of its utterance depends crucially on the context. But no matter what the range of possible contexts, with respect to every such context—even possible contexts in which Kripke is an anthropologist—(2) contains a proposition that has the same truth value that the proposition contained in (1) has in the possible world of the context. Given the meaning of "actually", it is no accident, then, that (1) and (2) share a common truth value. It is in this sense that they are logically equivalent. Things are very different with respect to the propositions they contain. Since the proposition contained in (1) could have been true and the proposition contained in (2) could not, it is very much an accident, in the only relevant sense, that they happen to coincide in truth value. If Kripke had gone into anthropology, they would not have.

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[9] This notion of logical equivalence is thus closer to what I have called the derivative notion of equivalence. See my 1986, appendix A, p. 131.

In that case, (2) would have been true, but only because it would have expressed a different proposition altogether. The proposition expressed by (2) with respect to the present, actual context would still have been false. This is as much as to say that whereas "actually" is a logical operator, in the logic of indexicality, its cognitive content with respect to a given context is not a logical operator. This should not be surprising. The first-person pronoun "I" has its own logic, but I am not a logical anything.

Sentences that differ intensionally are not equivalent in their cognitive contents; the propositions they express are not themselves equivalent in the relevant, propositional sense. When such sentences can be called "equivalent", they are equivalent in some other semantic value, one not determined by cognitive content. Sentences (1) and (2), for example, are equivalent in what David Kaplan calls "character", i.e. in their respective functions that assign cognitive contents to contexts of utterance.[10] In the special sense used here, sentences sharing the same logical content must also share the same modal content. The picture is this: Cognitive content determines logical content; logical content determines intension; and intension determines extension for any possible world. But extension does not determine intension; intension does not determine logical content; and logical content does not determine cognitive content. Logical content is, in this sense, something intermediate between cognitive content and intension.

II

Frege's celebrated distinction between *Sinn* and *Bedeutung* is very likely a bifurcation of, and hence a replacement for, his earlier notion in *Begriffsschrift* of conceptual content.[11] My proposal here is to retain three distinct semantic values: cognitive content; extension; logical content.The emphasis of Frege's classic 1892 publication—even its very title—suggests that he saw his two-way distinction as fundamental. On the other hand, his employment of the term "Erkenntniswerte" in addition to "Sinn" and "Bedeutung" might indicate that he had in fact drawn a three-way distinction, of exactly the sort I am proposing here. This would be significant in more than one way. First, if the proposal has merit, it would constitute yet another philosophical discovery of note by the genius from

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[10] This is not to say that the two sentences share a common character. In fact, their separate characters assign distinct propositions to every context. But as has already been said, no matter what the range of possible contexts, the respective characters always assign propositions that coincide in truth value in the possible world of the context in question.

[11] See the editor's preface to *Begriffsschrift* (1967, p. 4). Frege explicitly acknowledges, in his May 24, 1891 letter to Edmund Husserl (1980, p. 63), and more reliably, in the introduction to his *Grundgesetze der Arithmetik* (1964, pp. 6-7), that his distinction between thought and truth value is a bifurcation, as a consequence of distinguishing between *Sinn* and *Bedeutung*, of his earlier notion of judgeable content (*beurtheilbarer Inhalt*)—i.e.; the content of a sentence.

Jena.[12] In addition, if the notion of logical content that I am proffering yields something closer to the correct interpretation of Frege's use of "Erkenntniswerte" in the opening and closing passages of "Uber Sinn und Bedeutung," then the argument in those passages proceeds quite differently—or at least could proceed quite differently—from what many, including myself, have thought.

In *Frege's Puzzle*, I portrayed Frege as arguing that since $\ulcorner a = b \urcorner$ does, but $\ulcorner a = a \urcorner$ does not, contain a valuable extension of knowledge, for some co-referential proper names a and b, the resulting difference in cognitive content between the two sentences in such cases traces to a difference in cognitive content between the names a and b. From this it would follow that so-called Millianism is false, i.e. that the cognitive content of a proper name is not always its bearer. I analysed this argument as relying on a plausible principle of compositionality for thoughts together with what I called "Frege's Law":

> If two sentences S and S' have the same cognitive content, then S contains a valuable extension of knowledge if and only if S' does.

Most adherents of Millianism would probably reject Frege's Law. I offered a different Millian reply. I objected that whereas Frege's Law is ultimately a special case of Leibniz's Law of the Indiscernibility of Identicals, and as such is unassailable, Frege's innocuous looking minor premise—that for some pair of co-referential proper names a and b, $\ulcorner a = b \urcorner$ contains a valuable extension of knowledge—is unsubstantiated and unsubstantiatable.

Suppose instead that Frege's notion of *Erkenntniswerte* corresponds more closely to logical content than to cognitive content. Then both Frege's Law (so-called) and the minor premise to which I objected become inessential to the argument. Frege may argue for the distinctness of the cognitive contents of $\ulcorner a = a \urcorner$ and $\ulcorner a = b \urcorner$ instead by noting that those two sentences are obviously inequivalent. Certainly it would be correct—and to Frege's purpose in the opening passages of "Uber Sinn und Bedeutung"—to point out that pairs of sentences like

> The point of intersection of lines a and b is the point of intersection of lines a and b

and

> The point of intersection of lines a and b is the point of intersection of lines b and c

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[12] Tyler Burge argues (1979, pp. 398-432) that Frege recognized a three-way distinction, among *Sinn*, *Bedeutung*, and what would be more properly termed "linguistic meaning". The last, a variant of Kaplan's notion of character, is something like a function, or rule, that determines the *Sinn* of a context-sensitive expression (like "today") for any possible context of its use. I remain sceptical of Burge's argument for this interpretation, since Frege's remarks concerning the incompleteness of context-sensitive expressions, taken together with his doctrines concerning the distinction between function and object, seem to render this notion of meaning an idle wheel in Frege's philosophy of semantics. In any event, this notion of meaning, Fregean or not, is quite distinct from my notion of logical content.

differ in logical content.[13] As Frege notes, the first is analytic, the second synthetic. It immediately follows that these two sentences differ in cognitive content, in the thoughts expressed. From this it would follow further—assuming that a definite description is a singular term whose *Bedeutung* is the individual uniquely described (if there is one)—that the cognitive content of a singular term is not its *Bedeutung*. Or at least, not always. One may then argue, by parity of form and function, that the cognitive content of a singular term is never its *Bedeutung* (unless by a certain sort of coincidence, as with the phrase "the cognitive content of the definite description quoted parenthetically in the second-to-the-last sentence of §II of Nathan Salmon's 'On Content'"). This argument calls for a different reply.

III

I have said that the criterial idea of logical content is that two expressions have the same logical content if they are logically equivalent, in a special but fairly standard sense. While this constrains the notion of logical content, it does not yet specify what the logical content of a sentence (or other expression) is. For those of us who share Frege's philosophical scruples pertaining to definitions, this is simply inadequate.[14]

Our question is this: Given that expressions have the same logical content if and only if their cognitive contents are logically equivalent, and given the further constraints that have been laid down on the relevant notion of logical equivalence, what exactly is the logical content of an expression? If laws like Murphy's Law and principles like Leibniz's Principle of the Identity of Indiscernibles are not propositions, then what exactly are they? It is not a trivial matter to provide objects, independently specifiable, to fill the role of logical contents. But the notion of logical content depends for its philosophical legitimacy on our doing just that.

Employing Frege's and Russell's idea in their implementation of logicism, one might take the logical content of a sentence to be the class of sentences logically equivalent to it. One variation on this theme would take the logical content of a sentence to be instead the class of propositions logically equivalent to the cognitive content of the given sentence. Other variations would take instead the class of sentences logically entailed by the given sentence, or the class of propositions logically entailed, in a propositional sense, by the given sentence's cognitive content. These proposals are not strictly circular, as long as the relevant notion of logical equivalence or logical consequence is definable independently of logical content. But there is an obvious sense in which these proposals put the cart before the horse. It is very much like identifying the meaning of an expression with the

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[13] Notice that they probably do not differ in modal intension.

[14] Cf. Frege (1884, §66).

class of the expression's synonyms. Expressions are synonymous *in virtue of* the existence of an object that is their shared meaning, not vice versa.[15]

Indeed, the paradigm equivalence relations are those that are expressible in English by means of some phrase of the form ⌜VP the same NP⌝ ("expresses the same meaning as", "is a sample of the same liquid as", "is an animal of the same species as", etc.). Such phrases may be properly symbolized along the following lines:

$$(\text{the } z)[F(z) \wedge R(x, z)] = (\text{the } z)[F(z) \wedge R(y, z)].$$

This literally asserts an identity between certain intermediate objects, viz. the Fs to which the relata x and y are themselves suitably related by R.[16] In the case of synonymy, "F" symbolizes "meaning" while "R" symbolizes "expresses". Just as synonymy is identity of expressed meaning, so logical equivalence, in the sense sought here, is identity of logical content. Expressions (more accurately, their cognitive contents) are logically equivalent in virtue of having the same logical content, not vice versa. Identification with equivalence classes may have a special justification in the case of numbers, but it sheds no illumination in the present instance.[17] It certainly provides no philosophical foundation for the notion of logical content.

A more promising approach is to look to the definition of logical equivalence in classical model theory for some entity that is shared by all and only those sentences logically equivalent to a given sentence. Equivalence may be defined in terms of validity of argument: A pair of sentences are equivalent if and only if the arguments that take one as premise and the other as conclusion are both valid. Validity of argument, in turn, is understood as the preservation of truth in passing from premises to conclusion, irrespective of the contributions to truth value by the nonlogical components. Equivalent sentences thus emerge as those that share truth value irrespective of the contributions to truth value made by the nonlogical components. The relevant notion of contribution to truth value is represented by a *model*. If models are to be called "interpretations"—as they sometimes are— they are interpretations only in an austere sense. A model provides a semantics for the nonlogical lexicon, but only as much as is required (roughly) to determine mere truth value for any sentence of the language, on the basis of the fixed semantics for the logical lexicon. Thus in the simplest sort of structure for an extensional object language, a model is an assignment both of a domain of individuals over

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[15] Cf. Williamson (1990, pp. 81-82).

[16] A symbolization closer to the English phrase is given by:

$$R(x, (\text{the } z)[F(z) \wedge R(y, z)]).$$

Given the obvious premises, the two symbolizations are inter-derivable. That the symbolizations define an equivalence relation is also easily proved. For further discussion of the nature of such relations, and especially their involvement with intermediate entities (meanings, liquid substances, etc.), see my (1981, pp. 116-148).

[17] If numerals are taken to be quantifiers, or second-order predicates—as Frege and Russell made plausible—then the Frege-Russell equivalence classes (or their characteristic functions, Frege's "concepts") would seem to be exactly the right entities for numerals to designate.

which the individual variables range and of appropriate extensions to the nonlogical vocabulary for the language, keeping fixed the interpretations of the logical constants, like "not" and "some".[18] Within the framework of model theory, to say that sentences of a given language are *logically equivalent* is to say that they have exactly the same models. The logical content of a sentence might thus be identified with the class of its models, i.e. the class of minimal "interpretations" under which the sentence is true. If we wish to extend this idea to expressions other than sentences, the logical content of an arbitrary well-formed expression E could be identified with the corresponding function that assigns to any model M, the extension (or other contribution to truth value) of E in M. The logical content of a sentence S would thereby become a characteristic function that assigns a truth value—either truth or falsehood—to any model M (viz. the truth value of S in M).

This idea mirrors the idea of modal intension. And as we shall see, logical content and intension, though different, are intimately related. But the models differ from possible worlds in critical respects. We have already seen that there are pairs of sentences that are true with respect to exactly the same possible worlds while differing in their models. We have also seen sentences that share the same models (in the logic of indexicality) while differing in intension. The latter difference points up a feature of the class of a sentence's models that fails to capture the desired notion of logical content.

A further problem with the proposal is that it is confined to a single language. The central idea behind logical content is that a principle like Murphy's Law is not the proposition expressed by the sentence "Whatever can go wrong will", but a coarser type of content shared by that formulation and any of its logical equivalents. This type of content is expressible in any number of different languages. The English and French sentences "Either snow is white, or else snow is white and grass is green" and "La neige est blanche" have the same logical content. We cannot say that they have the same models, however, without drastically modifying the classical notion of a model, which is a notion of an "interpretation" for a single language. Even if this problem can be avoided by considering "models" for two languages combined—interpretations respecting the interlanguage synonymies of French-*cum*-English, for instance—we could not say that "Snow is white" has the same models as both its French and German translations without modifying the notion of a model even further to accommodate combinations of three languages, and so on (or by considering models for theories made up entirely of Carnapian "meaning postulates" for combinations of languages and their consequences). The notion of *logical content* is such that nonsynonymous sentences

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[18] Even in the nonextensional environment of standard modal logic, the models go minimalist, *qua* interpretations, by relativizing extensional semantic values like truth and reference to possible worlds. Such models provide intensions, but not yet cognitive contents, as the contributions to truth value made by expressions lying within the scope of a modal operator. On the other hand, if there are such things as doxastic or epistemic logic, their models are pressed to provide full-blown propositions, or thoughts, as the cognitive contents of sentences that may appear within the scope of a doxastic or epistemic operator.

of arbitrarily many distinct languages may nevertheless share the same logical content; they may contain the same principle, the same law, the same information.

At bottom, the problem with this approach is its focus on sentences and their components, rather than on their cognitive contents. As has already been said, the notion of logical content that we are seeking to clarify is that of a feature of propositions.

IV

Here again, Frege provides at least the beginnings of a possible solution. In the opening paragraph of "Compound Thoughts" he makes the following observation:

> It is astonishing what language can do. With a few syllables it can express an incalculable number of thoughts, so that even if a thought has been expressed by an inhabitant of the Earth for the very first time, a form of words can be found in which it will be understood by someone else to whom it is entirely new. This would not be possible, if we could not distinguish parts in the thought corresponding to the parts of a sentence, so that the structure of the sentence can serve as a picture of the structure of the thought. (1984, p. 390)

If a proposition has essentially a sentence-like structure—with propositional constituents corresponding, at least roughly, to the grammatical components of the sentence for which it is the cognitive content—then propositions no less than sentences are subject to model-theoretic analysis. The central idea is to provide a kind of "semantics" for propositions, like the semantics that is more standardly provided for sentences, by treating the cognitive contents of expressions as the expressions themselves are treated in classical model theory.[19] In the simplest structure, a propositional model would consist of both a nonempty domain of individuals, to serve as the range of the variable-binding operators, and an assignment of extensions, based on the domain of the model, to logically simple propositional constituents that fill the role of cognitive content for nonlogical, unstructured first-order n-adic predicates and for nonlogical, unstructured n-adic functors, for any finite number n (including the limiting case of $n = 0$). If, for example, the cognitive content of a first-order dyadic predicate is assumed to be a binary relation between individuals, then the extension-assignment component of such a propositional model would assign an appropriate extension to every simple, nonlogical binary relation between individuals—not to every simple, nonlogical dyadic predicate like "is to the left of", but to every relation that lacks any logical component, like the relation of being-to-the-left.[20] No separate assignment is made to logical compounds of attributes (e.g. being-to-the-left-of-if-larger-than). As a special case, no separate assignment

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[19] I illustrated this idea in my (1986). The basic idea is described at p. 177 n1. (See also the remarks on pp. 8-9.)

[20] The extension appropriate to a dyadic predicate, or to a binary relation, may be taken

would be made to the binary relation of (numerical) identity, since that relation is the cognitive content of a logical dyadic predicate, the "is" of identity. Identity is a logical relation; its extension therefore remains fixed for all propositional models.

A first-order 0-adic predicate is a sentence. If there are corresponding nonlogical, propositions lacking the structure of an attribute-together-with-its-arguments (e.g., perhaps the proposition that it is raining), a propositional model of this simple type would assign a truth value to any such proposition. A first-order 0-adic functor is an individual constant. If it is assumed that an unstructured individual constant may have a special sort of cognitive content, antecedently recognizable as being unlike the cognitive content of a definite description in its internal composition, then the extension-assignment component of a simple propositional model should also assign to every such cognitive content a single member of the model's domain as the cognitive content's extension in the model. The Millian theory that the cognitive content of a proper name is simply its bearer is the paradigm of the sort of theory on which entities that might serve as cognitive contents for individual constants (viz, the individuals themselves) thus play the same role in propositional models that primitive individual constants play in classical models. The orthodox Fregean theory that all individual constants invariably express a descriptional sense is the paradigm of the contrasting theory. Within the framework of orthodox Fregean theory, the extension of a singular-term sense in a propositional model is not imposed directly and independently by the model's extension-assignment, but as with any complex cognitive content, is determined indirectly and systematically by the extension in the model of the sense constituents. Likewise, the truth value of any standard proposition in a model is determined in a systematic fashion by the extensions in the model of the proposition constituents, chiefly by the application of function to argument. (Compare how the truth value of a structured sentence, or how the extension of a definite description, is fixed in classical model theory.) The general method of propositional models can be applied with respect to any theory of cognitive content (Fregean, Millian, etc.), as long as standard propositions are sufficiently sentence-like in structure to allow for the application of classical model-theoretic techniques.[21]

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to be a set of ordered pairs of elements of the domain over which the variables range, or any variation thereof, e.g. a (possibly partial) function from ordered pairs to truth values.

[21] There is also the possibility of a compromise theory according to which at least some unstructured individual constants, although none of them are Millian terms, have cognitive contents that are significantly unlike the structured senses of definite descriptions—perhaps something like point individual concepts, not comprising the cognitive content of the definite-description operator together with an accompanying property, occurring as separately identifiable constituents. (These would be individual-concept analogues to simple, unstructured propositions.) Although such a theory seems in crucial respects more Fregean than Millian (some argue that Frege in fact held such a theory), the cognitive contents of individual constants are treated the same on such a theory as on the Millian theory.

    The method of propositional models yields distinct notions of logical validity
for arguments consisting of propositions (rather than sentences) and hence also
of validity for propositions taken individually and of logical equivalence between
propositions. A propositional argument is valid, in this sense, if and only if its
conclusion is true in every propositional model in which each of the premises is
true. A proposition is valid if and only if it is the conclusion of a valid premise-
free argument, i.e. if and only if it is true in every propositional model. Proposi-
tions are equivalent if and only if they are true in exactly the same propositional
models.

    This propositional notion of equivalence differs from the corresponding clas-
sical notion of equivalence among sentences, in exactly the ways discussed in §I.
The propositions expressed by (1) and (2) are not equivalent, in the sense just
defined, even though the sentences themselves are equivalent in (more or less) the
classical sense. On the other hand, the proposition that Michael is a husband and
the proposition that Michael is male and Michael is married are equivalent, in the
relevant sense, even though the two sentences are not equivalent in a standard
sense (in the absence of "meaning postulates").

    This points the way to a response to the interpretation suggested in §II for
the argument in the opening passages of "Uber Sinn und Bedeutung." Although
⌜$a = a$⌝ and ⌜$a = b$⌝ are obviously inequivalent sentences when a and b are dis-
tinct terms, it begs the question against Millianism to assume that therefore the
cognitive contents of those sentences are inequivalent thoughts. Millianism
holds that the two sentences share the same logical content, and indeed the
same cognitive content, despite their sentential inequivalence, when the co-ref-
erential terms a and b are both proper names. On my own Millian view, both
sentences are in that case analytic, or true as a consequence of meaning alone.[22]

    On the other side of the coin, the notion of logical content may provide a solu-
tion to the closely related Paradox of Analysis. For it is arguable that philosoph-
ical analyses are unlike synonymy definitions precisely in that the former
typically seek to provide an analysans that shares the same logical content, but
not necessarily the same cognitive content (sense), as the analysandum.[23]

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    [22] I argue for this in my 1986, pp. 131-138, and, on different grounds, in "Relative and
Absolute A Priority" (1992, forthcoming).
    [23] This seems to be true, for example, of the contextual-definition analysis that Russell
gave for sentences involving definite descriptions. In various places he offered any of at
least three different versions of his analysis of "The φ is a ψ"; "Some φ is a ψ, and every
φ is the same"; "There is at least one φ; there is at most one φ; and every φ is a ψ"; and
"There is something x such that something is a φ iff it is x and x is a ψ". In "On Denoting"
he also preferred to eliminate the existential quantifier in terms of negation and universal
quantification. These various versions of his analysis evidently differ in cognitive con-
tent—as Russell himself might have recognized—though, of course, they are always log-
ically equivalent. For a penetrating discussion, see Anderson 1986, pp. 35-43, at 40-42.

V

One may well wonder whether the logical contents we have been seeking can really be classes of propositional models, or functions from propositional models to extensions. The sense of artificiality has two principal sources. It stems partly from erroneously thinking of propositional models on the model of a linguistic interpretation, and partly from a genuine artificiality intrinsic to model-theoretic analysis. Standard models are contrived artifacts, set-theoretic constructs put forward to represent a logico-semantic idea in a mathematically tractable way—in much the same way that the logico-mathematical idea of a function is represented by means of a set of ordered pairs, or the way that an expression is represented by its Gödel number in the arithmetization of syntax. Model theory is the set-theoretization of semantics. If there is an intuitive idea that the model-theoretic definition of validity correctly captures, it is something like the following: Irrespective of what the variables range over, and irrespective of what contributions the simple nonlogical expressions make to truth value, if the premises are true sentences, then so is the conclusion. This is a notion from what may be called the *Pure Theory of Validity.* The relevant notion of *irrespectiveness* is that of a modality—and indeed a logical modality (as opposed to, say, a metaphysical modality), but one relevant to Logic for Sentences. The class of all models represents metalogical space, the totality comprising every *logically possible* range for the variables together with contributions to truth value for the nonlogical lexicon.[24]

The propositional models we are considering are significantly different. They too are representational artifacts, but they are not *semantic* entities in the usual

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[24] I distinguish here between logic and metalogic. Logic proper asserts only such humdrum things as that if it is raining, then it is not the case that it is not raining. By contrast, the notion mathematically represented by the class of models is a metatheoretic notion from the Pure Theory of Validity, roughly that of the entire space of logically possible contributions to truth value for the variables and the nonlogical lexicon. John Etchemendy (1990) presents a sustained criticism of the classical model-theoretic definition of validity. In his central objection, Etchemendy argues by means of examples that the classical definition makes validity dependent on such factors as the minimum size of the universe, as determined by the axiom of infinity or the pair-set axiom of the theory from which the models are imported, and that such factors go beyond mere truths of logic. This objection (or some variant of it) may be warranted against an excessively unsubtle reading of the classical definition. The principal thrust of the critique, however, shows insufficient appreciation of certain facts: First, the invalidity of, for example, "It is raining" is not a truth of logic proper, but one of metalogic. The mere fact that a proposed definition makes validity depend on matters that go beyond logic proper does not make that definition illegitimate, unless those matters also go beyond metalogic. More importantly, the classical definition of validity constitutes a model-theoretic reconstruction—a mathematical representation— of our intuitive concept. The representation itself, as opposed to the idea it represents, is not genuinely a matter of pure metalogic. This might be demonstrated, quite independently of such factors as the minimum size of the universe, by noting that the Pure Theory of Validity for English declares the sentence "It is raining" invalid without strictly entailing the existence of any function (in the set-theoretic sense) that assigns falsehood to it. Models, and their extension assignments, are sets of sets of sets. The Pure Theory of Validity, by contrast, is not concerned with actual sets but with logically possible ranges for the variables together with contributions to truth value for the nonlogical lexicon. The availability

sense. They assign extensions not to bits of language, but directly to such entities as logically simple properties and logically simple relations, and perhaps to individuals and to logically simple propositions. Propositional models indirectly assign extensions also to logical compounds of attributes, ëtc., by means of the fixed extensions of the logical operations. The basic idea of propositional validity is something like the following: Irrespective of what objects there are, and irrespective of which of those objects have which logically simple properties or stand in which logically simple relations to one another, if the propositions contained in the premises are truths, then so is that contained in the conclusion. The relevant notion of irrespectiveness is a modality relevant to Logic for Propositions. The class of all propositional models represents the totality comprising every logically possible ontology together with a cosmology. Propositional models thus emerge as constructs that are more representational of possible worlds than of linguistic interpretations. Correspondingly, logical contents emerge as entities that are very much like intensions; the "genuine" logical content of an expression, if one insists, is something like a function from logically possible worlds to extensions (or to contributions to propositional truth value). A sentence's propositional models may be regarded as representing the sentence's cognitive content's *logical truth conditions*: the maximal conditions which are logically possible and with respect to which the proposition is true. The crucial difference between logical contents and intensions is this: the former are limited not by the laws of metaphysics, but by the more permissive laws of logic. The worlds represented by propositional models include metaphysically impossible worlds, as long as they are *logically* possible.[25]

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of a rich variety of models provides the basis of the suitability of the mathematical representation, in the same way that various facts of arithmetic provide the basis of the suitability of Gödel numbers as surrogates for expressions, though the things represented are strictly distinct from such matters. (Analogously, an excessively unsubtle reading of Tarski's celebrated definition of truth makes it seem as if the nontruth of "Snow is green" depends on the irrelevant ontological question of whether there are set-theoretic sequences. The objection that the definition is therefore illegitimate is misplaced. Compare also the misguided criticism that Gödel's arithmetization of syntax is defective because it makes syntax dependent on arithmetic.) Far from being discredited by its reliance on set theory, the model-theoretic definitioñ of validity can arguably be credited with having sharpened our grasp of the metalogical notion it represents, by revealing the nature of the relevant notion of truth preservation. Clearly, validity does not consist merely in the material conditional fact that the conclusion is true if the premises are true, since many invalid arguments share that feature. But neither is it the criterion of validity that this material conditional should be metaphysically necessary, nor even that it should be knowable a priori. Indeed, neither of these claims is correct about valid arguments in natural language, since the very meanings of the premises and conclusion are a contingent, a posteriori matter. The utility of the classical definition derives, in part, from its indicating that the point of validity is rather that truth is preserved irrespective of the contributions of the variables and the simple nonlogical components.

[25] In my 1989, pp. 3-34, I discuss some of the various differences between these other

VI

The method of propositional models has sufficient flexibility to accommodate additional notions of content, notions even less discriminating than logical content but still more discriminating than intension. The solution to Kazmi's problem so far consists in taking Church's Theorem to be not a proposition, but the logical content of, e.g., the sentence "First-order logic is undecidable". But now a more resilient problem arises at the level of logical content. Mathematicians use the label "Church's Theorem" freely in connection with differing formulations. Church's Theorem may be formulated as "The set of deducible sentences of first-order logic is undecidable". But it is also sometimes formulated as "The set of valid sentences of first-order logic is undecidable". Is Church's Theorem the logical content of the former, or is it instead the logical content of the latter? By Gödel's Completeness Theorem, the two are equivalent; so it would seem not to matter which one is selected as *Church's Theorem*. The two formulations, and anything logically equivalent to either, embody the same result. That result is what mathematicians often refer to using the phrase "Church's Theorem". Yet the two formulations differ in logical content as well as cognitive content; the propositions expressed are not logically equivalent. The Completeness Theorem inextricably links the two very different notions of first-order deducibility (derivability from the empty set of premises) and first-order validity. But the Completeness Theorem is a metatheorem; it is not itself a truth of logic.[26]

Church's Theorem, if it is the common result embodied in both of the formulations above, cannot be the logical content of one and not the other. It must therefore be the logical content of neither. If Church's Theorem is a single result contained in both formulations, we should not say that it is a logical content at all. But neither is it a proposition; that, I have argued, is the proper lesson of Kazmi's problem. What kind of thing, then, is Church's Theorem, as the label is (at least sometimes) used by mathematicians?

While the Completeness Theorem that links the two formulations is not a truth of logic, it is a truth of something. It is a proven result from the restricted portion of the meta-theory for first-order logic that includes some proof theory (syntax)

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types of worlds, as these differences bear on the question of what is the correct modal logic.

[26] Interestingly, in the original publication of his result, Church sharply distinguished between the two, pointing out that although the proof of the undecidability of the set of first-order theorems is constructive, the proof of Gödel's Completeness Theorem is non-constructive, thus making the undecidability of the set of first-order valid sentences less certain. See the final two paragraphs of his (1936). At the time of writing, Church might have regarded his theorem as a particular proposition concerning deducibility (or perhaps as a set of various propositions logically equivalent to that proposition), and not the alternative proposition concerning validity. Most mathematicians today would probably not distinguish between the two on these grounds, and would instead regard either as the same mathematical result.

and some model theory (semantics), but nothing, for example, from recursive-function theory proper. Let us call this limited meta-theory "M". One thing the two formulations of Church's Theorem have in common is the set of propositional models that honour the truths of M and with respect to which the sentence in question is true. Either formulation will be false in some nonstandard propositional models for M—those which make extension assignments in such a way that the set assigned to the property of first-order theoremhood is among those in the set assigned to the property of decidability. But the two formulations will not differ in truth value with respect to any propositional model for M, no matter how nonstandard. They are not logically equivalent—they differ in truth value with respect to propositional models that do not respect the truths of M—but they are equivalent *within the framework of M*. We might say, therefore, that while the two formulations differ in logical content, they share the same M-content. That is to say, they have the same M-models; they are true in exactly the same logically possible M-worlds.

We may likewise say that various versions of the Axiom of Choice have the same ZF-content. For they are provably equivalent, once we are allowed to assume the axioms of Zermelo-Fraenkel set theory. If the setting is sufficiently liberal, one might even go so far as to identify the Axiom of Choice with the relevant class of ZF-models. This would be to equate Zorn's Lemma, for example, with the Axiom of Choice in a more thorough way than is customary, assuming the former is also identified with its ZF-content. There is similar variation in the phrase "Church's Theorem", which is perhaps more often used in a restrictive sense on which the formulation in terms of validity is said to be not exactly Church's Theorem, but something equivalent to it by Gödel's Completeness Theorem. (See note 26 above.) I would suggest that this type of variation in usage typically reflects different kinds of content acting as denotations for labels like "Choice" and "Church's Theorem". In short, such labels are often used ambiguously.

One difficulty with the various proposals made here arises from the fact that we use labels like "Murphy's Law" in ascribing propositional attitudes, as if such labels referred to propositions. We say such things as "Mary believes the Identity of Indiscernibles" and "Michael doubts Church's Theorem". It is unclear how best to accommodate this. Perhaps we should construe such remarks in accordance with a Principle of Charity, as meaning that Mary believes some proposition whose logical content is the Identity of Indiscernibles and that Michael doubts some proposition whose M-content is Church's Theorem. Alternatively, perhaps we should liberalize the philosopher's notion of belief and other so-called propositional attitudes. The things we are said to believe or to doubt, in common parlance, are a mixed bag. To be sure, we believe, or fail to believe, propositions; belief of propositions is doubtless the fundamental form of belief. But we are also said to believe, or not to believe, such things as signposts, omens, inscriptions, warning signs, sentences, other people, our own eyes. We are even sometimes said to believe, or not to believe, seemingly stranger things than these ("Michael

could not believe the size of Mary's house"). In addition, we are said to believe, or not to believe, *in* a bewildering array of (alleged) phenomena and other things: magic, divine revelation, ghosts, demonic possession, miracles, destiny, heaven, America, love at first sight. Why not simply accept that logical contents—or more generally, theoretical contents—are also among the nonpropositional things we believe and fail to believe? It seems likely that all of these apparently nonpropositional forms of belief may be reducible in one way or another to belief of propositions.[27] If we harbour propositional attitudes toward the logical contents of propositions, that psychological fact may yield part of the explanation for the historical fact that propositions, or thoughts, are sometimes misidentified with their intensions.

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[27] There might even be reason to hold that "that"-clauses are also used ambiguously, serving as terms typically for propositions, but sometimes for their logical or other theoretical contents. In the absence of compelling evidence, however, this hypothesis should probably be resisted.

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