Path Semantics for Indicative Conditionals

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Extended version, featuring an extra technical appendix.

1 Introduction

My topic is the semantics and the logic of indicative conditionals, i.e. sentences like (1).

(1) If Frida took the exam, she passed.

The literature on conditionals contains two appealing views, each of which seems well-supported by the evidence. The first is the *selectional* view. On this view, indicative conditionals operate by selecting a single possibility, which is used to evaluate the consequent. This insight is at the basis of Stalnaker's (1968; 1981; 1984) semantics for conditionals, on which (1) is true just in case, in the unique closest world where Frida takes the exam, she passes. Selectional theories are desirable for many reasons. A prominent one is that they validate a conditional analog of the principle of Excluded Middle, commonly called 'Conditional Excluded Middle' (CEM). Historically, CEM has been controversial, but at this stage it is widely accepted.

The second view is the *informational* view. On this view, all sentences belonging to epistemic discourse, including indicative conditionals, don't express propositions. Rather, these sentences impose constraints on information states of speakers (see Veltman 1985, 1996; Yalcin 2007, 2012; Gillies 2004, 2009, among others). On informational theories, (1) is analyzed as imposing a constraint on an information state. In particular, it imposes the constraint that all Frida-taking-the-exam worlds in the information state are worlds where she passes. The core argument for the informational view is that it vindicates a number of logical principles that appear correct, but are invalidated by classical truth-conditional semantics. These principles concern both bare indicative conditionals like (1), and epistemic *might*-conditionals.

Both views are supported by strong arguments. Unfortunately, though, they are incompatible on their standard formulations. On the selectional theory, conditionals pick out a single antecedent world. On standard informational theories, conditionals work as universal quantifiers over worlds in an information state. This difference seems insurmountable, hence it appears that we have to choose between mutually exclusive options.

But the options are not exclusive. This paper develops a theory of the semantics and assertibility of conditionals that is both selectional and informational. On the one hand, the consequent of a conditional is invariably evaluated at a single possibility. On the other, conditionals operate by imposing constraints on information states, construed again as sets of world. Accordingly, the theory vindicates the signature inferences of both selectional and informational theories, including CEM and principles concerning the interplay between bare conditionals and *might*-conditionals.

The central notion of the semantics is the notion of a *path*. A path is simply a sequence of worlds from an information state i, without repetitions. All sentences are evaluated relative to a path; in particular, a bare conditional like (1) updates the path with the information expressed by the antecedent, and evaluates the consequent at the first world in the updated path. At the same time, the semantics exploits a notion of support at an information state, defined as truth at all paths that are constructed out of an information state.

Besides covering a wide array of data and vindicating a plausible set of inferences, path semantics also has plausible consequences for the assertion of conditionals. Surprisingly, asserting an indicative conditional in natural language produces the same effects as asserting a material conditional: *If* A, *then* C and $\mathsf{A} \supset \mathsf{C}$ are update-equivalent. This fact is hard to explain on pretty much all standard theories of conditionals, but it automatically falls out of an account of assertion in path semantics.

I proceed as follows. In §2, I introduce a tension between two logical principles, each of which can be used to motivate the selectional and the informational view. §§3-4 give an intuitive presentation of the view. §5 states the formal semantics, §6 defines consequence, and §7 discusses how update and assertion work in the new framework. §8 includes some discussion of the system, and two appendices include formal refinements and proofs.

Throughout the paper, I stick to indicative conditionals, but both the puzzle and (with adaptations) the solution may be extend to counterfactuals.

2 A classical puzzle

I start from a classical puzzle about conditional logic, dating back to the debate between Stalnaker (1968, 1981) and Lewis (1973a, 1973b). The puzzle is that conditionals seem to be subject to two plausible, but jointly incompatible constraints.

2.1 Conditional Excluded Middle

The first principle at stake is:

Conditional Excluded Middle. (CEM) $\vDash (\mathsf{A} > \mathsf{C}) \vee (\mathsf{A} > \neg\mathsf{C})$

The literature includes a growing battery of arguments for CEM. The one that is most often cited revolves around scope: indicative conditionals seem scopeless with respect to logical operators. I.e., importing and exporting these operators inside and outside the consequent makes no difference to meaning. For reasons of space, here I only discuss negation, but the evidence for scopelessness includes the interactions between conditionals and quantifiers (Higginbotham 1986, von Fintel & Iatridou 2002), the adverb *only* (von Fintel 1997), and comparative constructions (Korzukhin 2014).[1]

To start, observe that the sentences in (2) sound equivalent.

(2) a. It's not the case that, if Frida took the exam, she passed.
b. If Frida took the exam, she didn't pass.

The phenomenon persists with expressions that lexicalize negation, like *doubt* ($\approx$ *believe not*) and *fail* ($\approx$ *not pass*).

(3) a. I doubt that, if Frida took the exam, she passed.
b. I believe that, if Frida took the exam, she failed.

The lack of semantic interaction with negation and related expressions is perfectly expected on a theory that vindicates CEM. Given minimal background principles, CEM entails:[2]

Negation Swap. $\neg(\mathsf{A} > \mathsf{C}) \dashv\vDash \mathsf{A} > \neg\mathsf{C}$

Conversely, the lack of semantic scope of negation is unexpected on theories on which CEM fails, e.g. theories that treat conditionals as universal quantifiers.

The case for CEM and selectional theories goes beyond the compositional interactions of conditionals. A different reason is that selectional theories are particularly well-suited to vindicate intuitions about the probabilities of conditionals. Virtually all semantic theories that try to vindicate the link between probabilities of conditionals and conditional probabilities adopt a selectional semantics.[3] I don't have the space to pursue the interaction between conditionals and probability here. Let me just point out that path semantics is put

[1] As *[name omitted]* points out, the semantics I develop in this paper needs further additions to accommodate Korzukhin's data.

[2] The right-to-left direction of **Negation Swap** is uncontroversial. Here is how CEM yields a proof of the left-to-right-one:

i.	$\neg(\mathsf{A} > \mathsf{C})$	Assumption
ii.	$(\mathsf{A} > \mathsf{C}) \vee (\mathsf{A} > \neg\mathsf{C})$	CEM
iii.	$\mathsf{A} > \neg\mathsf{C}$	(i, ii, Disjunctive Syllogism)

[3] See, among many, Van Fraassen 1976; Kaufmann 2009, 2015; Bradley 2012; Bacon 2015.

to use in a sister paper to vindicate bridge principles about conditionals and probability, including the claim that probabilities of conditionals are conditional probabilities (see *reference omitted*).

2.2 *If* and *might*

The second principle at stake states the incompatibility of $A > \neg C$ and $A > \Diamond C$.

If-Might Contradiction. (IMC) $(A > \neg C) \wedge (A > \Diamond C) \vDash \bot$

The evidence for IMC is straightforward. Pairs of conditionals of the relevant forms generate inconsistencies, both in categorical contexts and under supposition.[4]

(4) # If Maria passed, Frida didn't pass; but, even if Maria passed, it might be that Frida passed.

(5) # Suppose that, if Maria passed, Frida didn't pass, and that, if Maria passed, it might be that Frida passed.

Notice that IMC should be kept distinct from Duality:

Duality. $\vDash (A > \Diamond C) \leftrightarrow (\neg(A > \neg C))$

Several classical frameworks (e.g., Lewis 1973a Kratzer 1981b, Kratzer 2012) make IMC and Duality equivalent. But, as I will point out, the two can come apart.

2.3 Collapse

Given a classical notion of consequence, CEM and IMC together entail the equivalence of $A > C$ and $A > \Diamond C$. The direction $A > C \vDash A > \Diamond C$ is uncontroversial; as for the other direction:

i.	$A > \Diamond C$	Assumption
ii.	$\quad A > \neg C$	Supposition for conditional proof
iii.	$\quad A > \neg C \wedge A > \Diamond C$	(i, ii, $\wedge$-Introduction)
iv.	$\quad \bot$	(iii, IMC)
v.	$\neg(A > \neg C)$	(ii-iv, Reductio)
vi.	$A > C$	(v, CEM, Disjunctive syllogism)

This result is unacceptable. In response, classical theories drop one of CEM and IMC. Famously, Stalnaker (1968; 1981; 1984) endorses CEM and rejects

[4] Stalnaker (1981, 1984) suggests that we reanalyze *might*-conditionals as involving an epistemic modal scoping over a conditional. I don't have the space to discuss this proposal in detail; let me just notice that it is incompatible with the widely accepted view that *if*-clauses can work as semantic restrictors of modals (see Kratzer 1986, 2012 for a classical defense of this view).

IMC. Most other theorists, starting from Lewis in his work on on counterfactuals (1973a; 1973b), reject CEM. (For possible worlds theories of indicatives that reject CEM, see Kratzer 1981b, 2012, Nolan 2003, Gillies 2004; 2009.).

Here is a quick overview of the two standard ways out of the puzzle. Stalnaker's semantics is based on the idea that conditional antecedents select a single world to evaluate the consequent. To do this, Stalnaker relativizes interpretation to world and a selection function, i.e. a function that maps the pair of a world and an antecedent to a selected world.[5]

Using the standard double bracket notation $[\![\cdot]\!]$ for the interpretation function, this is the Stalnaker meaning for the conditional:

(6) $[\![\mathsf{A} > \mathsf{C}]\!]^{w,s}$ is true iff $[\![\mathsf{A} > \mathsf{C}]\!]^{s(w,\mathsf{A}),s}$ is true iff C is true at $s(w,\mathsf{A})$

This semantics vindicates CEM because the consequent of a conditional $\mathsf{A} > \mathsf{C}$ is invariably evaluated at a single world, i.e. the selected A-world. Each world will make exactly one of C and $\neg\mathsf{C}$ true, hence one of $\mathsf{A} > \mathsf{C}$ and $\mathsf{A} > \neg\mathsf{C}$ will be true no matter what. Conversely, IMC fails. The details depend on one's semantics for *might*, but the basic reason is that $\mathsf{A} > \neg\mathsf{C}$ and $\mathsf{A} > \Diamond\mathsf{C}$ can both be true when the domain of *might* includes worlds other than the A-world selected by $\mathsf{A} > \mathsf{C}$.

The other route treats conditionals as universal quantifiers. Theories of this sort vary widely; here I use a toy informational semantics based on Yalcin's (2007) semantics for epistemic modals, since this will help me set up my own proposal. On this semantics, conditionals manipulate an information state parameter.

(7) $[\![\mathsf{A} > \mathsf{C}]\!]^{w,i}$ is true iff $\forall w' \in i$, $[\![\mathsf{C}]\!]^{w',i}$

On the assumption that *might* is an existential quantifier over information states, this semantics makes $\mathsf{A} > \neg\mathsf{C}$ and $\mathsf{A} > \Diamond\mathsf{C}$ straightforwardly inconsistent, thus vindicating IMC. In addition, it captures a series of desirable principles that are invalidated by classical truth-conditional semantics (see §6.3). But CEM is invalid: since conditionals are universal quantifiers, there are cases where neither $\mathsf{A} > \mathsf{C}$ nor $\mathsf{A} > \neg\mathsf{C}$ is true or accepted.

[5] Precisely, a selection function is a function $s : W \times \mathcal{P}(W) \mapsto W$ that satisfies four conditions (I use 'A' to denote schematically both sentences and the proposition they express):

i. if $[\![\mathsf{A}]\!]$ is non-empty, $s(w,\mathsf{A}) \in [\![\mathsf{A}]\!]$
(**Inclusion:** the selected world must make true the antecedent, if at all possible.)

ii. if $s(w,\mathsf{A}) = \lambda$, then $[\![\mathsf{A}]\!] = \varnothing$ (where λ is the absurd world, where every sentence is true)
(**Absurdity-as-last-resort**: λ is selected only if no possible world can be selected.)

iii. if $w \in [\![\mathsf{A}]\!]$, then $s(w,\mathsf{A}) = w$
(**Centering**: if the world of evaluation makes the antecedent true, it is the selected world.)

iv. for all A, A′: if $s(w,\mathsf{A}) \in [\![\mathsf{A}']\!]$ and $s(w,\mathsf{A}') \in [\![\mathsf{A}]\!]$, then $s(w,\mathsf{A}) \in [\![\mathsf{A}']\!] = s(w,\mathsf{A}') \in [\![\mathsf{A}]\!]$
(**Consistency of selection**: the selection must be consistent for all choice of antecedents.)

So both selectional and informational semantics, as they are standardly set up, run into empirical costs, since they have to give up a plausible principle.

2.4 Homogeneity

The semantics literature contains an attempt at solving the puzzle by appealing to **homogeneity**—i.e., roughly, a definedness requirement on certain kinds of sentences.[6] Before presenting my account, it's helpful to show that this attempt is unsatisfactory.[7]

To get a handle on homogeneity, consider a sentence involving a plural definite description, like (8):

(8) The girls are in the library.

(8) is true if all the girls are in the library, false if none of them are, and infelicitous if some are and some aren't (Fodor 1970). In the latter case, as Manuel Križ puts it, "the natural answer [to (8)] is neither *yes* nor *no*, but *well*" (2015: 6).

(9) a. The girls are in the library.
b. #Yes, /# No, /Well, some of them are.

Homogeneity is standardly analyzed as a definedness requirements on sentences including definite phrases. *The* Fs *are* G is defined just in case either all Fs are G, or no Fs are G. (The precise analysis of this kind of undefinedness is controversial, but this is unimportant for us.[8]) Several theorists have suggested that conditionals should be analyzed in analogy with plural definite descriptions (see e.g. von Fintel 1997, Bittner 2001, Schein 2003, Schlenker 2004, Križ 2015). On this analysis, conditionals are subject to the following definedness condition:

Homogeneity Requirement (HR)
A > C is true or false at w only if: either all closest A-worlds to w are C-worlds, or all closest A-worlds to w are not-C-worlds

To investigate how homogeneity affects complex sentences, we need a theory of projection. Homogeneity projection is a controversial matter.[9] Fortunately, though, there is broad agreement about some specific data points, in particular about how homogeneity projects under Boolean connectives. This

[6] See von Fintel 1997, Schein 2003, Schlenker 2004, Križ 2015 for endorsements of this strategy; see also Cariani & Goldstein forthcoming for an attempt at using homogeneity to let conditionals vindicate classically incompatible inferences.

[7] This section builds, in part, on ongoing experimental work with *[names omitted]*.

[8] Traditional analysis treat it as a kind of presupposition, but Križ 2015 provides excellent arguments, based on projection data, to the effect that homogeneity behaves differently.

[9] For some recent work, see Križ 2015, Križ & Chemla 2015

will be enough to show that the properties of conditionals we're interested in are not generated via homogeneity.

According to most theories of homogeneity, homogeneity projection for Boolean compounds is value-functional: whether negations, conjunctions, and disjunctions are true, false, or undefined depends solely on whether their constituents are true, false, or undefined. In particular, projection is taken to follow the Strong Kleene truth tables. Using '*' to mark indeterminate truth values, these truth tables are:

$\neg A$

1	0
*	*
0	1

$A \wedge B$

	1	*	0
1	1	*	0
*	*	*	0
0	0	0	0

$A \vee B$

	1	*	0
1	1	1	1
*	1	*	*
0	1	*	0

The Kleene truth-tables are supplemented by a definition of validity that allows for undefinedness. The central idea, which is due to von Fintel (1999) and builds on Strawson's classical work on presupposition, is that an argument is valid iff, whenever the premises are both defined and true, the conclusion is also defined and true.

Exploiting this definition of validity, we can introduce a semantics for conditionals that vindicates both CEM and IMC. The key idea is to treat conditionals, on a par with definites, as universal quantifiers with a homogeneity requirement. So the truth conditions we obtain are, schematically:

(10) $[\![A > C]\!]$ = defined iff either all A-worlds are C worlds, or none of them are; true iff all all A-worlds are C worlds

On the semantics in (10), CEM is valid whenever $[\![A > C]\!]$ is defined. Moreover, if we define *might*-conditionals as duals of the conditional in (10), we also predict IMC. So this appears to give us a way out of the puzzle.[10]

But this theory is still unsatisfactory. Even if we can define consequence in a way that CEM is valid, some instances of CEM are undefined. In his discussion of homogeneity in plural descriptions, Križ notices exactly this point.

> In [Strong Kleene], on the other hand, the excluded middle can never be false, but it can be undefined (if p has the truth value #) and so it isn't a tautology. (Križ 2015)

As he points out, this seems a good prediction for definites.

> This strikes me as correct for natural language as well, where not every excluded middle statement is unquestionably true.

[10]The collapse proof in §2.3 is blocked because the Strawson-inspired notion of validity is not transitive. See Cariani & Goldstein forthcoming for discussion.

(11) A: Adam either read the books or he didn't read them.
B: Well, what if he read half of the books? (Križ 2015)

Križ's judgment can be sharpened if we consider not only EM, but also **Contradiction** and **Noncontradiction**, listed below.

Contradiction. $\mathsf{A} \wedge \neg\mathsf{A} \vDash \bot$ **Noncontradiction.** $\vDash \neg(\mathsf{A} \wedge \neg\mathsf{A})$

An account of homogeneity projection based on Strong Kleene predicts that these formulas can have undefined instances. And in all these cases, this verdict seems plausible. Suppose that Adam read half of the books; we seem to get an unclear/undefinedness verdict.

Adam either read the books or he didn't.	$\mathsf{A} \vee \neg\mathsf{A}$	??
Adam read the books and he didn't.	$\mathsf{A} \wedge \neg\mathsf{A}$	??
It's not the case that Adam read the books and didn't.	$\neg(\mathsf{A} \wedge \neg\mathsf{A})$	??

Crucially, conditionals behave differently. Consider the conditional counterparts of the relevant inferences:

Conditional Excluded Middle. (CEM) $\vDash (\mathsf{A} > \mathsf{C}) \vee (\mathsf{A} > \neg\mathsf{C})$
Conditional Contradiction. (CC) $(\mathsf{A} > \mathsf{C}) \wedge (\mathsf{A} > \neg\mathsf{C}) \vDash \bot$
Conditional Noncontradiction. (CNC) $\vDash \neg((\mathsf{A} > \mathsf{C}) \wedge (\mathsf{A} > \neg\mathsf{C}))$

For all the relevant formulas, we have crisp judgments about every instance. Consider a simple example:

Context. Perhaps a coin was tossed. In the case that it was tossed, it might have landed heads and it might have landed tails.

If it was flipped, it landed heads, or, if it was flipped, it landed tails.	**CEM**	TRUE
If it was flipped, it landed heads, and, if it was flipped, it landed tails.	**CC**	FALSE
It's not the case that: if it was flipped, it landed heads, or, if it was flipped, it landed tails.	**CNC**	TRUE

Instances of logical truths and logical falsities involving conditionals don't appear to have undefined instances. This marks a difference with respect to definites. Whatever nonclassicality affects conditionals, it is different from homogeneity.

There are several other arguments one could raise against the homogeneity account of conditionals.[11] But my focus in this paper is on building a positive

[11] For a generalization of the arguments from connectives, see the experimental results in *references omitted*. Considerations from probability, similar to the ones raised by Cariani & Santorio 2018 against homogeneity theories of *will*, can also be used against homogeneity accounts of CEM. See also Cremers, Križ and Chemla (2017) for experimental findings to the effect that probability judgments pattern in very different ways for cases of homogeneity failure and for conditionals in contexts where not all antecedent worlds verify the consequent.

alternative view, so I won't pursue this here. Let me just emphasize one point: I am *not* arguing that homogeneity is completely absent from the semantics of conditionals. My own semantics makes some very restricted use of homogeneity, for some complex antecedents. My point is that homogeneity cannot be invoked as a blanket strategy to reconcile CEM and IMC.

3 The analogy with informational semantics

I start by outlining an analogy between the puzzle in §2 and a puzzle about epistemic modals discussed by Yalcin (2007). Yalcin points out that $\neg A$ and $\Diamond A$ seem inconsistent by the lights of some plausible tests. In particular, their conjunctions are infelicitous both when asserted by themselves and in embeddings:

(12) # It's not raining and it might be raining.

(13) # Suppose that it's not raining and it might be raining.

(14) # If it's raining and it might not be raining...

From a logical point of view, Yalcin's puzzle is triggered by the tension between the principle suggested by (12)–(14) and the requirement that $\Diamond A$ should not be veridical, i.e. that it should not entail A.

Epistemic Contradiction. $\neg A \wedge \Diamond A \vDash \bot$

Nonfactivity of Epistemic Modality. $\Diamond A \nvDash A$

Both principles seem plausible, yet they are classically inconsistent.[12] Yalcin's solution is to move to a nonclassical semantics, which generates a nonclassical notion of consequence. The resulting framework, which builds extensively on Veltman's update semantics (1996), can accommodate both principles.

There is an obvious analogy between Yalcin's puzzle and the puzzle in §2. The latter is generated by the tension between three plausible, but classically inconsistent principles.

Conditional Excluded Middle. (CEM) $\vDash (A > C) \vee (A > \neg C)$

If-Might Contradiction. (IMC) $(A > \neg C) \wedge (A > \Diamond C) \vDash \bot$

Nonfactivity of Might-Conditionals. (NMC) $A > \Diamond C \nvDash A > C$

[12] Proof:

i.	$\Diamond A$	Assumption
ii.	$\quad \neg A$	Supposition for conditional proof
iii.	$\quad \neg A \wedge \Diamond A$	(i, ii, $\wedge$-Introduction)
iv.	$\quad \bot$	(iii, Epistemic Contradiction)
v.	$\neg\neg A$	(ii-iv, Reductio)
vi.	A	(v, propositional logic)

The analogy with Yalcin's puzzle is glaring. IMC and NMC are conditional counterparts of, respectively, Yalcin's Epistemic Contradiction and Nonfactivity principles. In the nonconditional case, the two principles are sufficient to generate inconsistency. In the conditional case, we also need CEM to draw a crucial inference from $\neg(\mathsf{A} > \mathsf{C})$ to $(\mathsf{A} > \neg\mathsf{C})$. But, aside from the extra assumption, the situation is parallel.[13]

I suggest that we build on this analogy. The classical debate about CEM and *might*-conditionals should be reframed. On the usual construal, that debate is about the quantificational force of conditionals: it concerns whether we should treat conditionals as selectional, in the style of Stalnaker, or as universals, in the style of Kratzer and many others. This debates simply assumes a classical notion of consequence in the background. I suggest rather that the tension between CEM and IMC points to a basic tension for classical semantics and logics for conditionals. Within a classical framework, this tension can be overcome only at the price of leaving some data unexplained. This motivates exploring nonclassical options, in particular options that build on informational frameworks.

4 Path semantics: overview

On standard informational semantics, modal sentences like *must* A or *might* A are evaluated as true or false relative to an information state. *must* A is true at i if all the worlds in i verify A, and *might* A is true at i if at least one world in i verifies A. In this framework, conditionals are standardly treated as involving universal quantification over worlds.

The semantics that I propose, path semantics, treats conditionals as selectional. Conditionals involve no quantification over worlds. Rather, the antecedent, in combination with a point of evaluation, determines a single world, which is used to evaluate the consequent.

The key notion of the semantics is the notion of a **path**. A path is simply a sequence of worlds with no repetitions. For clarity, a path looks like this:

$$\langle w_i, w_k, w_l, \ldots, w_n \rangle$$

As is standard, sentences are evaluated as true or false relative to a point of evaluation. In standard semantics for modality, points of evaluations are sequences of coordinates that include a world, plus some parameters dedicated

[13] *[Name omitted]* points out that the collapse of *might*-conditionals onto bare conditionals can be obtained simply via NMC and the nonconditional principle of Epistemic Contradiction ($\neg\mathsf{A} \wedge \Diamond\mathsf{A} \vDash \bot$); the proof is a minor variant of the proofs in §2. Of course, it is still interesting that there is a version of the puzzle that does not rely on Epistemic Contradiction. (Among other things, this puzzle generalizes to counterfactual modality, for which conjunctions of the form $\neg\mathsf{A} \wedge \Diamond\mathsf{A}$ are perfectly acceptable.)

to the interpretation of modals: e.g. a selection function in Stalnaker's framework (1968; 1981; 1984), or a pair of a modal base and an ordering source in Kratzer's (1986; 2012). In path semantics, a path is the only element in a point of evaluation. Hence all sentences are assigned a truth value relative to a path.

For the case of conditionals, the antecedent is used to update the path of evaluation, and the consequent is evaluated at the updated path. If the antecedent is a nonmodal sentence, updating a path simply means removing all the non-antecedent worlds from the sequence. For example, if A is true at all and only even-numbered worlds, we get:

$$[\![\mathsf{A} > \mathsf{C}]\!]^{\langle w_1, w_2, w_3, w_4 \ldots \rangle} = [\![\mathsf{C}]\!]^{\langle w_1, w_2, w_3, w_4 \ldots \rangle + \mathsf{A}} = [\![\mathsf{C}]\!]^{\langle w_2, w_4, \ldots \rangle}$$

Path semantics is reminiscent of Stalnaker's original selectional semantics. But there are some key differences in the treatment of content and assertion. Let me illustrate them.

As I stated in §2, Stalnaker evaluates conditionals relative to a world and a selection function, i.e. a function mapping a world of evaluation and a conditional antecedent to a selected world. Crucially, Stalnaker's framework adopts a contextualist metasemantics. It assumes that context of utterance determines the value of the world of evaluation and the selection function (with some amount of indeterminacy; see Stalnaker 1981, 1984). As a result, in Stalnaker's semantics we are able to assign truth conditions for conditionals, at a context:

$$\mathsf{A} > \mathsf{C} \text{ is true at } c \quad \text{iff} \quad [\![\mathsf{A} > \mathsf{C}]\!]^{w_c, s_c} \text{ is true} \quad \text{iff} \quad s_c(w_c, \mathsf{A}) \text{ is a } \mathsf{C}\text{-world}$$

Stalnaker claims that, both for indicatives and for counterfactuals, the selection function captures an informal notion of closeness or similarity (though indicative similarity differs from counterfactual similarity). The selected world is the closest world to the world of the evaluation that makes the antecedent true.[14]

In sum: traditional selectional semantics assumes (i) that the context of utterance fixes a value for the selection function, and (ii) that selection functions capture an informal notion of closeness or similarity. I reject both (i) and (ii). A context of utterance does not determine a path of evaluation (not even allowing for some indeterminacy), and paths are not intended to capture a similarity ranking of worlds.

What do paths model, then, and how do we select certain paths rather than others when we evaluate concrete utterances? The answer is that what paths are relevant for evaluating particular utterances is determined indirectly, via information states.

We start from the idea, typical of informational systems, that sentences are evaluated directly at an information state (construed simply as a set of

[14]Stalnaker doesn't explain in detail what indicative closeness consists in, though there are attempts at capturing related. notions in the literature. See in particular Kratzer's 1981a remarks on stereotypical ordering sources for epistemic modals.

worlds). In particular, sentences may be supported or not supported at an information state. For an example, consider an information state i that involves three worlds. Suppose that the numerical subscripts represent the outcome of the toss of a fair die:

$$i : \quad \{w_1, w_2, w_3\}$$

Simple factual sentences are supported at an information state just in case they are true at all worlds in the information state. *might*- and *must*-claims impose global constraints on information states: hence e.g. *might* A is true at an information state just in case the information state contains at least an A-world. Hence i supports the sentences in (15), but not the sentences in (16).

(15) a. The die didn't land on 6.
b. The die landed on 1, 2, 3, or 4.
c. The die might have landed on 1.

(16) a. The die landed on 6.
b. The die landed on 1 or 2.
c. The die might have landed on 4.

Now, every information state i determines a set of paths, i.e. all the paths that include all and only the worlds in i. I call this set $\textsc{path}(i)$. For example, i determines six paths:

$$\langle w_1, w_2, w_3 \rangle$$
$$\langle w_1, w_3, w_2 \rangle$$
$$\langle w_2, w_3, w_1 \rangle$$
$$\langle w_2, w_1, w_3 \rangle$$
$$\langle w_3, w_1, w_2 \rangle$$
$$\langle w_3, w_2, w_1 \rangle$$

A conditional is supported, or accepted, at an information state i just in case it is true at all paths in $\textsc{path}(i)$. For example, i supports (17), but not (18).

(17) If the die landed even, it landed on two.

(18) If the die landed odd, it landed on one.

The basic intuition here is that the evaluation of a conditional at an information state proceeds in a similar way to the evaluation of a nonmodal assertion, like (19).

(19) The die landed on 1, 2, or 3.

To evaluate (19) at i, we check whether it is true at each world in i. We proceed similarly for conditionals. Only, worlds alone are insufficient, since to assign a value to the consequent we need an antecedent-verifying world. Hence we use sequences of worlds: we go through the sequence one world at a time, until we encounter an antecedent-verifying world. But, for nonmodal sentences as for conditionals, and differently from the case of modal claims, support is determined by evaluating a sentence pointwise.

In summary: the notion of a path does not connect directly with metasemantic notions like context and content. Contexts don't fix a path of evaluation for a conditional, and the content of an assertion does not depend on fixing a path (as it happens for parameters that receive a contextualist treatment, like e.g. a threshold for gradable adjectives like *tall*). In addition, as I explained, paths don't capture a notion of similarity or closeness. Paths are formal devices internal to the compositional semantics. The theory makes contact with metasemantic notions at the level of information states.

By endorsing this system, we get the best of both the selectional and the informational view. On the one hand, we vindicate the idea that conditionals select a possibility, and validate the related logical principle of Conditional Excluded Middle. On the other, we vindicate plausible principles about the interplay of bare conditionals and *might*-conditionals, including IMC.

There is a further perk of the account. A lot of attention has been paid to the semantics of conditionals, but theories of the assertion of conditionals are less developed. In §7, I show how path semantics can be combined with a simple and plausible theory of context update. Among other things, the theory correctly predicts that, despite substantial differences in their semantics, indicative conditionals produce the same update as the corresponding material conditionals.[15]

5 Path semantics

This section gives a statement of the semantics (to be refined in Appendix 1).For convenience, I restrict consideration to models with a finite set of worlds. This is a simplification, but it is harmless and allows me to focus on more central issues.

5.1 Basic notions

A **path** is simply a sequence of possible worlds with no repetitions. Paths can be put in a many-to-one relation with information states, construed as sets of worlds. Each path p determines exactly one information state, i.e. the information state that includes all and only the worlds in p. Each information state i determines a set of paths, i.e. all the sequences with no repetitions that include all and only the worlds in i.

[15] A different framework that achieves somewhat similar results is in Mandelkern 2019. I put off a comparison between my system and Mandelkern's to a different occasion.

To state the system precisely, it is helpful to introduce two relations between paths. First, p' is a **subsequence** of p ($p' \leq p$) just in case every world in p' is in p, and the worlds in p' appear in the same order as in p. For example: the path in (20-b) is a subsequence of the path in (20-a), while the path in (20-c) is not.

(20) a. $\langle w_2, w_5, w_8, w_{14}, w_{45}, w_{89} \rangle$
b. $\langle w_2, w_8, w_{14}, w_{89} \rangle$
c. $\langle w_2, w_{89}, w_8 \rangle$

Second, p' is a **permutation** of p ($p' * p$) just in case p' and p contain the exact same worlds, though possibly in a different order.

Here is a formal statement of all relevant notions:

1. A **path** p is a sequence of worlds without replacement.
2. $p' \leq p$ (p' is a **subsequence** of p) iff whenever w occurs earlier in p' than v, w occurs earlier in p than v.
3. $p' * p$ (p' is a **permutation** of p) iff p and p' consist of the same worlds, ordered potentially differently.

Let me also add two pieces of notation: I use 'p_i' to denote the i-th world in a path. For example, if we let $p = \langle w_3, w_6, w_9 \rangle$, $p_1 = w_3$ and $p_2 = w_6$. Also, as noted in §4, I use $\textsc{path}(i)$ to denote the set of paths generated by i.

In the compositional semantics, the interpretation of a sentence is relativized to only one parameter, namely a path parameter. This may seem a major change with respect to standard informational systems, which use information states, or pairs of a world and an information state $\langle w, i \rangle$, as points of evaluation. But, as I explain in §5.5, given a path we can always recover a pair of a world and an information state (though not *vice versa*, since paths contain strictly more information). Hence the semantics could be rewritten as a simple generalization of informational semantics, on which interpretation is relativize to a triple of a world, an index, and an extra parameter.[16] I choose this route just for simplicity.

5.2 Semantics

I state a semantics for a propositional language involving atomic sentences, Boolean connectives, epistemic modals, and conditionals. All sentences are evaluated relative to a path. This evaluation procedure is supplemented with a notion of update, which plays a key role in evaluating conditionals.

I use the square brackets '$[\![\cdot]\!]$' notation for the interpretation function. As anticipated, I relativize interpretation to only one parameter, namely a path

[16] The extra parameter might be characterized in various ways; one option would be to use a linear ordering on the worlds in the information state.

parameter (represented as 'p'). I also assume a background model $\langle W, V\rangle$, with W a set of worlds and V a valuation function mapping pairs of an atomic sentence and a world to $\{0,1\}$. Borrowing a formal device from Stalnaker 1968, I assume that W includes the absurd world λ, i.e. a world such that $V(\lambda,\mathsf{A}) = 1$, for any A.[17]

Atomics:

$$[\![\mathsf{A}]\!]^p = 1 \text{ iff } \begin{cases} V(p_1,\mathsf{A}) = 1, \text{ if } p \text{ is non-empty;} \\ V(\lambda,\mathsf{A}) = 1, \text{ otherwise.} \end{cases}$$

Connectives:

$$\begin{aligned} [\![\neg\mathsf{A}]\!]^p &= 1 \text{ iff } [\![\mathsf{A}]\!]^p = 0 \\ [\![\mathsf{A}\vee\mathsf{B}]\!]^p &= 1 \text{ iff } [\![\mathsf{A}]\!]^{w,i,p} = 1 \text{ or } [\![\mathsf{B}]\!]^p = 1 \\ [\![\mathsf{A}\wedge\mathsf{B}]\!]^p &= 1 \text{ iff } [\![\mathsf{A}]\!]^{w,i,p} = 1 \text{ and } [\![\mathsf{B}]\!]^p = 1 \end{aligned}$$

Modals:

$$\begin{aligned} [\![\Diamond\mathsf{A}]\!]^p &= 1 \text{ iff for some } p' * p,\ [\![\mathsf{A}]\!]^{p'} = 1 \\ [\![\Box\mathsf{A}]\!]^p &= 1 \text{ iff for all } p' * p,\ [\![\mathsf{A}]\!]^{p'} = 1 \end{aligned}$$

So far, path semantics is entirely conservative with respect to standard informational semantics. Nonmodal and nonconditional sentences are invariably evaluated at the first world in a path. Hence, for the fragment of the language that does not include modals and conditionals, path semantics boils down to ordinary possible worlds semantics. *might*- and *must*-claims are accepted at an information state in path semantics just in case they are accepted at an information state in ordinary informational semantics. (See below for a notion of acceptance at an information state.)

The nonconservative element in path semantics concerns conditionals. Conditionals are evaluated at a path by updating the path with the antecedent, and then evaluating the consequent at the updated path. The notion of update raises some technical issues and will require elaboration later on (in Appendix 1). For now, I use a simplified definition, which yields correct predictions for the great majority of ordinary conditionals, including all data in §2.

Path update (temporary).
$p+\mathsf{A}$ (the **update** of p with A) is the largest member of the following set: $\{p' \leq p \mid \forall p'' \text{ if } p'' * p' \text{ then } p'' \in [\![\mathsf{A}]\!]\}$

I.e., the update of p with A is the largest subsequence of p such that all of its permutations make A true. (In a lot of cases, there is going to be such a largest subsequence; for the other cases, the generalization in Appendix 1 will be useful.)

At this point, we define truth at a path for conditionals in terms of update.

[17] The latter assumption just makes some features of the logic smoother. More realistic treatment of indicative conditionals will assume that conditionals include a presupposition to the effect that the antecedent is possible (see Stalnaker 1975).

Conditionals: $[\![\mathsf{A} > \mathsf{C}]\!]^{p} \;=\; 1 \text{ iff } [\![\mathsf{C}]\!]^{p+\mathsf{A}} = 1$

In the next section, I go through a number of examples to show how the semantics works.

5.3 Examples

Consider a sample path: ('M' and 'F' stand for the propositions that Maria passed and that Frida passed):

$$p: \quad \langle w_{\bar{\text{M}}\bar{\text{F}}}, w_{\text{M}\bar{\text{F}}}, w_{\bar{\text{M}}\text{F}}, w_{\text{MF}} \rangle$$

I show how a number of examples get evaluated at p.

Nonmodal and nonconditional sentences. Nonmodal sentences are invariably evaluated at the first world in a path. As a result, the semantics of nonmodal sentences is fully classical. Here are some sentences that are true at p.

(21) a. Maria didn't pass.
b. Maria passed or Frida didn't pass.
c. Neither Maria nor Frida passed.

Modal sentences. ◇A and □A are true at a path just in case, respectively, some and all permutations of the path make true A. This means that, when A itself is nonmodal, ◇A and □A true at a path just in case, respectively, some and all the world in the path are A-worlds.[18] For illustration, p makes true:

(22) a. It might be that Maria passed.
b. It might be that Frida didn't pass.
c. It might be that Maria didn't pass and Frida did.

Update. The update of a path with A, recall, is the largest subsequence of the path such that every permutation of it makes A true. For the case of nonmodal and nonconditional sentences, this means simply that the update of a path with A is the subsequence of the path including all and only the A-worlds. For an example, below is the update of p with (23).

(23) Maria or Frida passed, or both did.

$$p + (23): \quad \langle w_{\text{M}\bar{\text{F}}}, w_{\bar{\text{M}}\text{F}}, w_{\text{MF}} \rangle$$

[18] This also means that the logic of epistemic modality is S5 and that sentences involving nested modalities are equivalent to sentences involving merely the innermost modal; e.g., ◇□◇A is equivalent to ◇A. Hence this fragment of the system is equivalent to the fragment in Yalcin 2007.

Updating with $\Box$A has identical effects to updating with A.[19] Hence $p + (24)$ is again the subsequence of p that includes all and only the odd worlds.

(24) It must be that Maria or Frida passed, or that both did.

p + (24): $\langle w_{\mathrm{M}\bar{\mathrm{F}}}, w_{\bar{\mathrm{M}}\mathrm{F}}, w_{\mathrm{MF}} \rangle$

Updating with A > C has identical effects to updating with the material conditional A $\supset$ C. So e.g. the update of p with (25) is below.

(25) If Maria passed, Frida didn't

p + (25): $\langle w_{\bar{\mathrm{M}}\bar{\mathrm{F}}}, w_{\mathrm{M}\bar{\mathrm{F}}}, w_{\bar{\mathrm{M}}\mathrm{F}} \rangle$

Conditionals. Conditional antecedents update the path of evaluation; the consequent is evaluated at the updated path. Consider:

(26) If Maria passed, Frida passed.

We first update the path of evaluation with the antecedent of (26). Since the antecedent is a descriptive claim, we merely remove all worlds incompatible with it.

$$\langle w_{\bar{\mathrm{M}}\bar{\mathrm{F}}}, w_{\mathrm{M}\bar{\mathrm{F}}}, w_{\bar{\mathrm{M}}\mathrm{F}}, w_{\mathrm{MF}} \rangle \quad \Rightarrow \quad \langle w_{\mathrm{M}\bar{\mathrm{F}}}, w_{\mathrm{MF}} \rangle$$

Then we evaluate the consequent at the updated path. Since, at the update of p with *Maria passed*, the first world is a world where Frida did pass, the whole conditional is false at p. Notice the key point that guarantees the validity of CEM: nonmodal consequents are always evaluated at a single world.

5.4 Acceptance and support

The foregoing covers the compositional semantics. But it is not enough to determine whether a sentence is accepted in an actual context. To determine this, we need a notion of **support** at an information state. Quite naturally, we say that A is supported at information state i just in case A is true at all paths generated from i.

$i \vDash$ A (i **supports** A) iff, for all p in $\textsc{path}(i)$, $[\![\mathsf{A}]\!]^p = 1$

It is easy to check that support works as in standard informational system. In particular: *might* A is supported at i iff i contains an A-worlds; *must* A is supported at i iff i contains only A-worlds; A > C is supported at i iff every world in i is either not an A-world, or a C-world.

The notion of support tells us what sentences are accepted at an information state. But it doesn't tell us how the assertion of a sentence updates an information state. I take up this question in §7.

[19] Despite the compositional differences between the two; to see them, just embed A and $\Box$A under negation.

5.5 Comparison with standard informational systems

Path semantics relativizes interpretation to a single parameter, namely a path. So it may seem that it departs in a major way from other informational systems, where interpretation is relativized to an information state, or an information state and a world. But we can easily retrieve the values of more traditional parameters from a path.

For concreteness, I take as a benchmark Yalcin's system. On this system, all sentences are evaluated with respect to a pair of a world and an information state $\langle w, i\rangle$. As the semantic clauses illustrate, factual sentences like (27-a) are sensitive to the world parameter, though not the information state parameter; conversely for modal sentences like (28-a).

(27) a. It's raining.
 b. $[\![\text{It's raining}]\!]^{\langle w,i\rangle} = 1$ iff it is raining at w

(28) a. It might be raining.
 b. $[\![\text{It might be raining}]\!]^{\langle w,i\rangle} = 1$ iff $\exists w' \in i$: $[\![\text{it's raining}]\!]^{\langle w',i\rangle} = 1$

To recover a $\langle w, i\rangle$ pair from a path p, we let w be p_1, i.e. the first world of the path, and we let i be the union set of the path, i.e. the set containing all the worlds appearing at any point in the path. So path semantics is in continuity with existing informational systems.

There is, however, one point of divergence. Standard informational treatment of conditionals[20] take them to be sensitive only to an information state, similarly to modal sentences and differently from factual ones. Conversely, on path semantics conditionals are sensitive to a choice of world of evaluation. Recall that we identified the world of evaluation with the first world in the path of evaluation. But the first world in a path can be sufficient to settle whether the conditional is true or false at the relevant path. In particular, if A is true at p_1, then $\mathsf{A} > \mathsf{C}$ is true at p iff C is true at p_1. Hence conditionals, while they are not on a par with factual statements like (27-a), turn out to be sensitive also to the world of evaluation, i.e. the parameter we use to evaluate factual sentences.

The world-sensitivity of conditionals is at the basis of a further feature of path semantics. The so-called centering principles, which link conditionals to descriptive statements, turn out to be semantically valid (on both the notions of validity defined in §6).

Strong Centering. $\mathsf{A} \wedge \mathsf{C} \vDash \mathsf{A} > \mathsf{C}$
Weak Centering. $\mathsf{A} > \mathsf{C} \vDash \mathsf{A} \supset \mathsf{C}$
(for nonmodal and nonconditional A, C)

I give a formal definition of validity in the next section. But the reason why centering principles are valid should be intuitively clear. If the first world of a path validates A and C, the conditional $\mathsf{A} > \mathsf{C}$ is true on the basis of that first

[20] See e.g. Gillies 2004, Gillies 2009.

world; and if $\mathsf{A} > \mathsf{C}$ is true at p, it has to be that either A is false at p_1, or else C is true at p_1 (or both).

The full validity of both centering principles marks a departure from standard informational systems. On the latter, Strong Centering turns out to be invalid on the classical notion of entailment (though it is informationally valid). Hence standard informational systems countenance points of evaluation where $\mathsf{A} \wedge \mathsf{C}$ is true and $\mathsf{A} > \mathsf{C}$ is false, while path semantics does not.

This concludes the presentation of path semantics. The next two sections are devoted to defining consequence and update.

6 Defining consequence

6.1 Two notions of consequence

Path semantics allows us to define several notions of consequence. The most obvious one is preservation of truth at a path. On the current proposal, paths rather than world play the role of basic possibilities; hence this is the first, obvious notion to explore.

> **Path consequence.**
> $\mathsf{A}_1, \ldots, \mathsf{A}_n \vDash_{\mathrm{P}} \mathsf{C}$ iff for all paths p such that $[\![\mathsf{A}_1]\!]^p = 1 \ldots, [\![\mathsf{A}_n]\!]^p = 1$, $[\![\mathsf{C}]\!]^p = 1$

Path consequence is the notion that most closely corresponds to classical consequence. As a result, it shares the limitations of the latter. In particular, it does not capture the validity of standard informational inferences, such as e.g. so-called Łukasiewicz's principle. [21]

> **Łukasiewicz's principle.** $\neg\mathsf{A} \vDash \neg\Diamond\mathsf{A}$

As a result, we need to define another notion of consequence. As one might expect, this notion involves preservation of support at an information state. Recall the definition of support:

> $i \vDash \mathsf{A}$ (i **supports** A) iff, for all p in $\textsc{path}(i)$, $[\![\mathsf{A}]\!]^p = 1$

Using this, we define:

> **Path-Informational consequence.**
> $\mathsf{A}_1, ..., \mathsf{A}_n \vDash_{\mathrm{PI}} \mathsf{C}$ iff, for all i such that $i \vDash \mathsf{A}_1, ..., i \vDash \mathsf{A}_n$, $i \vDash \mathsf{C}$.

Path-Informational consequence is the analog, in the current framework, of Veltman's (1996) test-to-test validity, or Yalcin's (2007) informational consequence. Informally, it tracks what follows from an information state that val-

[21] See Santorio 2018 for discussion of the links between reasoning and notions of consequence.

idates certain premises. It is the obvious notion of consequence for assessing consistency and validity for asserted claims in natural language.[22]

6.2 Solving the puzzle

The key result for us is that Path-Informational consequence validates the three logical *desiderata* that we started from. (All proofs are in the appendix.) This solves the puzzle of §2.

Proposition 1. Path-Informational consequence validates CEM, IMC, and NMC.

$$\vDash_{PI} (A > C) \vee (A > \neg C)$$
$$(A > \neg C) \wedge (A > \Diamond C) \vDash_{PI} \bot$$
$$A > \Diamond C \nvDash_{PI} A > C$$

Interestingly, despite validating IMC, PI-consequence doesn't validate Duality.

Proposition 2. Duality fails on PI-consequence.

Duality. $\nvDash_{PI} (A > \Diamond C) \leftrightarrow (\neg(A > \neg C))$

In particular, the left-to-right direction of the entailment fails. To see this, consider an information state $i = \{w_1, w_2, w_3\}$; let A be true at w_1 and w_2, and false at w_3; and let C be true at w_1, and false at w_2 and w_3. Then $A > \Diamond C$ is true at all paths in $\textsc{path}(i)$, and hence is supported at i. , But now consider the path $\langle w_2, w_1, w_3\rangle$. $A > \neg C$ is true relative to this path (since w_2 is an A-world and a $\neg$C-world), hence $\neg(A > \neg C)$ is false relative to it. Hence $\neg(A > \neg C)$ is false at some paths in $\textsc{path}(i)$, hence it's not supported at i.

This failure illustrates a difference between the logic that is generated by Lewis's classical semantics and my system. In Lewis's logic, IMC and Duality are equivalent. On Path-Informational consequence, Duality is stronger than IMC, hence it can still fail even though IMC is valid.

I discuss further features of both Path Consequence and Path-Informational consequence in the appendix.

6.3 Other logical features

Path semantics vindicates a number of other patterns of inferences worth noticing. I mentioned above that both Centering principles are path-valid (and hence also path-informational valid):

Restricted Centering. $A \wedge C \quad \vDash_P \quad A > C \quad \vDash_P \quad A \supset C$
(for nonmodal and nonconditional A, C)

[22] Besides Yalcin 2007, see Bledin 2015 for discussions of informational consequence. Informational consequence is a descendant of Stalnaker's (1975) notion of reasonable inference.

It also vindicates a version of Import-Export (see Kratzer 1986, 2012, as well as McGee 1985). In particular, Import Export is path-valid (and hence path-informational valid).

Restricted Import-Export. $A > (B > C)_P \dashv\vDash_P (A \wedge B) > C$
(for nonmodal and nonconditional A, B)

Finally, it vindicates a number of features that are typical of informational consequence. All are path-informational valid.

Łukasiewicz's principle. $\neg A \vDash_{PI} \neg\Diamond A$

Veridicality. $\Box A \vDash_{PI} A$

Or-to-If. $A \vee C \vDash_{PI} \neg A > C$
(for nonmodal and nonconditional A, C)

Might-If-to-Might-And. $A > \Diamond C_{PI} \dashv\vDash_{PI} \Diamond(A \wedge C)$
(for nonmodal and nonconditional A, C)

In addition, **Might Import-Export** (see Gillies 2020 for discussion) is also valid (both path-valid and path-informational valid).

Might Import-Export. $\Diamond(A > C)_P \dashv\vDash_P A > \Diamond C$

Thus PI-consequence is a genuinely informational notion of consequence, while in addition also vindicating CEM.

7 Assertion and update

The last two sections have stated a compositional semantics and a logic. This is still insufficient to explain the pragmatic effect on conditionals, i.e. how speakers learn and change their belief from the assertion of a conditional. In this section, I show how assertions update information states in path semantics. Among other things, this allows us to predict some surprising properties about the update of conditionals.

7.1 Defining update

It's useful to introduce a simple example, which I will use throughout this section. Take the following scenario:

> **Die.** Sarah tossed a fair, six-sided die; we have no information about the outcome.

We can represent the epistemic state of an agent in **Die** as a set of six worlds, each of which represents a particular outcome.

$$i: \quad \{w_1, w_2, w_3, w_4, w_5, w_6\}$$

The paths generated by i are all the sequence of elements in i; some examples are below.

$$\langle w_1, w_2, w_3, w_4, w_5, w_6\rangle$$
$$\langle w_2, w_1, w_3, w_4, w_5, w_6\rangle$$
$$\langle w_2, w_3, w_1, w_4, w_5, w_6\rangle$$
$$\langle w_6, w_5, w_4, w_3, w_2, w_1\rangle$$
$$\langle w_4, w_2, w_3, w_5, w_1, w_6\rangle$$
$$\ldots$$

I call the whole set $\textsc{path}(i)$ the **epistemic state** of the agent in the **Die** scenario. Information states and epistemic states can be put in one-to-one correspondence, hence here it's irrelevant which of the two we take as primary.[23] For simplicity, here I use information states.

Information state update can be defined in terms of path update. The update of a path with A, recall, is the largest subsequence of the path such that every permutation of it makes A true. We can define the update of an information state with A simply as the set of worlds that are in some path generated by i, updated with A.[24]

Information state update (temporary)
$i + \mathsf{A}$ (the **update** of i with A) is the set:
$\{w : w \in i$ and, for some path p in $\textsc{path}(i)$, w is a member of $p + \mathsf{A}\}$

Like path update, information state update will need refinement (in Appendix 1) to handle update with some complex sentences.

Let us go through an example. Consider again the scenario in **Die**. Suppose that we learn the following:

(29) If the die landed even, it landed on two.

To update i, we consider any path generated by it. For example, suppose we take:

$$p: \quad \langle w_1, w_2, w_3, w_4, w_5, w_6\rangle.$$

We now look for the largest subsequence of p such that all permutations of it make it true. This subsequence is p^+:

[23] *[reference omitted]* combines path semantics with probability theory, with the goal of assigning probabilities to conditionals in a way that vindicates Stalnaker's Thesis. Once we start assigning probabilities to sentences, it becomes clear that epistemic states and paths have a key role in a model of credence, since credences in epistemic modal sentences, including conditionals, are only defined at paths. But, as long as we stay in a qualitative setting, we lose nothing if we take information states as primary.

[24] Since the paths generated by an information state all involve the same worlds, this set is identical to the set of worlds that are in *all* paths generated by i, updated with A.

p^+: $\langle w_1, w_2, w_3, w_5 \rangle$.

(It's easy to check that, if we had taken any other paths, we would have still ruled out the same worlds, i.e. w_4 and w_6.) The update of i with (29) is the set of worlds that include worlds in p^+, i.e.:

(30) $i^+ : \{w_1, w_2, w_3, w_5\}$

This notion of update delivers results that are, in general, entirely plausible. Update with ordinary descriptive sentences is just ordinary intersective update. Update with *might* A leaves i unchanged (thus acting as a test in the sense of Veltman 1996). Updating with *must* A has the same effects as A. The reader is invited to check these cases on their own.

7.2 A puzzle about update

In the remainder of this section, I show how the account of update just outlined solves a puzzle about updating one's epistemic state with conditionals. This will show that path semantics is not only correct compositionally, but also yields plausible consequences about assertion.

Indicative conditionals give rise to a puzzle about update. The puzzle is generated by the fact that indicative and material conditionals are update-equivalent, in the sense that updating with one leads to the same result as updating with the other. This is a surprising fact, which is hard to reconcile with the view that indicative conditionals are strictly stronger than material conditionals.

The puzzle can be set up as a clash between two simple principles. The first principle says that material conditionals don't entail the corresponding indicative conditionals.

Non-Entailment. $\mathsf{A} \supset \mathsf{C} \nvDash \mathsf{A} > \mathsf{C}$

I won't defend **Non-entailment**. I simply assume that, *pace* the defenders of the material conditional analysis, any plausible theory of indicative conditionals vindicates it. Let me just highlight that, in the context of the present paper, the entailment from $\mathsf{A} \supset \mathsf{C}$ to $\mathsf{A} > \mathsf{C}$ would be particularly damaging because of pernicious interactions with CEM.[25]

[25] In fact, to create trouble all we need is a weaker principle than CEM, namely:

Negation Swap. $\neg(\mathsf{A} > \mathsf{C}) \dashv\vDash \mathsf{A} > \neg\mathsf{C}$

Assume moreover that $\neg\neg\mathsf{A}$ is equivalent to A, a restricted principle of Centering, and the fairly uncontroversial principle of Conditional Noncontradiction:

Restricted Centering. $\mathsf{A} > \mathsf{C} \vDash \mathsf{A} \supset \mathsf{C}$ (for nonmodal and nonconditional A, C)
Conditional Noncontradiction (CNC). $(\mathsf{A} > \mathsf{C}) \wedge (\mathsf{A} > \neg\mathsf{C}) \vDash \bot$

Assuming that $\mathsf{A} \supset \mathsf{C}$ entails $\mathsf{A} > \mathsf{C}$ (call this **Entailment**), we prove:

i.	$\mathsf{A} > \neg\mathsf{C}$	Assumption
ii.	$\neg(\mathsf{A} > \mathsf{C})$	(i, Negation Swap)

The second principle concerns the update-equivalence of material and indicative conditionals.[26] Letting '+' denote the update operation, we can state this as:

> **Update Equivalence.** For all i, and for descriptive A, C:
> $i + \mathsf{A} \supset \mathsf{C} = i + \mathsf{A} > \mathsf{C}$.

To motivate **Update Equivalence**, let me elaborate on the die scenario. As stated, at first you have no information about the outcome of the die toss. As a result, you are uncertain about (31), as well as about (32).

(31) The die landed even.

(32) If the die did not land on two or four, it landed on six.

But now suppose that you learn (31). It appears that you thereby also learn (32). Once you are certain that the die landed even, you cannot help also being certain that, if it didn't land on two or four, it landed on six.

Update Equivalence is related to a widely discussed principle, i.e. so-called Or-to-If. It is a classical observation that any information state that supports $\mathsf{A} \vee \mathsf{B}$, which is equivalent to the material conditional $\neg\mathsf{A} \supset \mathsf{B}$, also supports the conditional $\neg\mathsf{A} > \mathsf{B}$.[27]

> **Or-to-If.** For all i, and for descriptive A, C: if
> $i \vDash \mathsf{A} \supset \mathsf{C}$, then $i \vDash \mathsf{A} > \mathsf{C}$.

Despite the obvious connection, **Update Equivalence** is different from **Or-to-If**. **Or-to-If** is a principle about static properties of information states: any coherent information state at which $\mathsf{A} \supset \mathsf{C}$ is accepted is also an information state at which $\mathsf{A} > \mathsf{C}$ is accepted. Conversely, **Update Equivalence** concerns the evolution of information states as new information comes in.[28]

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iii.	$\mathsf{A} \supset \mathsf{C}$	(Supposition for conditional proof)
iv.	$\mathsf{A} > \mathsf{C}$	(iii, Entailment)
v.	$\perp$	(ii, iv, CNC)
vi.	$\neg(\mathsf{A} \supset \mathsf{C})$	(iii, v, Reductio)
vii.	$\mathsf{A} \wedge \neg\mathsf{C}$	(vi, propositional logic)

Instantiating $\neg\mathsf{C}$ for C and removing the double negations we get:

> **If-to-And.** $\mathsf{A} > \mathsf{C} \vDash \mathsf{A} \wedge \mathsf{C}$ (for nonmodal and nonconditional A, C)

But **If-to-And** is absurd—*If Frida danced*, *Maria danced* doesn't entail that Frida and Maria both danced. Hence we have reason to believe **Non-Entailment**. For extensive arguments against the material conditional analysis, see also Edgington 1995.

[26] It is unclear to me whether a version of Update Equivalence that drops the restriction to descriptive A, Cis motivated. The restricted version is sufficient to make my point.

[27] For classical discussion of Or-to-If, see Stalnaker 1975. For an excellent discussion of Stalnaker's argument, see Cariani 2020.

[28] Also, standard pragmatic accounts of **Or-to-If** can't account for **Update Equivalence**. In particular, the classical pragmatic account of Or-to-If due to Stalnaker 1975, doesn't get off the ground as an account of **Update-Equivalence**. Stalnaker's solution crucially relies on constraints about the selection function for $\mathsf{A} > \mathsf{C}$ once the speakers are in an information state that accepts $\neg\mathsf{A} \vee \mathsf{C}$. But

It is obvious why **Non-Entailment** and **Update Equivalence** are in tension. If $\mathsf{A} > \mathsf{C}$ is stronger than $\mathsf{A} \supset \mathsf{C}$, one would expect that, at least in some cases, learning the latter is not sufficient for learning the former.[29] Yet this is not what happens. Let me now go on to show how path semantics captures this surprising fact.

7.3 Vindicating Update Equivalence

Consider again **Die**. The information state from which we start is:

$$i: \quad \{w_1, w_2, w_3, w_4, w_5, w_6\}$$

To update i with a sentence A, we take any path generated by i and look at the largest subsequence of it such that all of its permutations make A true. Now suppose that we update i in this way with (31) (repeated below):

(31) The die landed even.

As one might expect, we get to:

$$i': \quad \{w_2, w_4, w_6\}$$

But now, consider what happens when we update with (32).

(32) If the die did not land on two or four, it landed on six.

It's easy to check that, by following the same procedure, we get to the same result. Any path that includes one or more of w_1, w_3, and w_5 will have a permutation that makes (32) false. So the update of i with (32) is still i'.

More in general, it can be checked that, given the theory of update stated in this section, **Update Equivalence** holds. Learning a material conditional is equivalent to learning the corresponding indicative, even though the two are not treated as equivalent by the semantics. The fact that we can vindicate this surprising combination of facts is a further strike in favor of path semantics.

these constraints cannot apply *before* an information state is updated with the disjunction. So far as I can see, the only way to account for **Update Equivalence** in a classical truth-conditional framework is to just claim context-dependence. The idea is that, before and after learning (31), the conditional has different meanings. Some usual objections to this strategy apply.

[29] The tension can be turned into full-blown incompatibility if we assume, as classical truth-conditional semantics does, that every sentence denotes a proposition and that update works intersectively.

Classicality. For every i and for every A: (i) the **content** of A ($[\![\mathsf{A}]\!]$) is a set of possible worlds; (ii) $i + \mathsf{A} = i \cap [\![\mathsf{A}]\!]$

Of course, the present account rejects **Classicality**, hence it can make **Update Equivalence** and **Non-Entailment** consistent.

8 Side note: no covert modals in conditionals

Before moving on, I want to notice a further interesting feature of path semantics: bare conditionals involve no covert modality of any sort. Consider again:

(1) If Frida took the exam, she passed.

Standard semantics for indicative conditionals, like Kratzer's (1986, 2012), assume that bare conditionals like (1) systematically involve a covert modal at the level of logical form. So, for Kratzer, the LF of (1) is:

(33) [if Frida took the exam][MUST [Frida passed]]

The presence of this modal is a stipulation. There is no evidence for its presence, aside from the fact that we need it to generate the right truth conditions. Moreover, this modal lacks some of the typical features of overt epistemic modals in natural language. In particular, it is known that overt epistemic modals carry a kind of 'indirectness' component in their meaning, requiring that the speaker has no direct evidence for the proposition expressed by the prejacent (see von Fintel & Gillies 2010 and references therein). For a classical example, consider an utterance of (34) in a context where the a speaker is staring at the rain through a clear window.

(34) #It must be raining.

This indirectness is preserved by conditionals that involve an overt *must*, but not by bare conditionals. Consider a similar scenario:

(35) a. If this water isn't coming from the sprinklers on the roof, it is raining.
b. #If this water isn't coming from the sprinklers on the roof, it must be raining.

So, on theories like Kratzer's, we need to assume that all bare indicative conditionals involve a covert modal, and that at least in some respects this covert modal is unlike overt epistemic modals in natural language.

Conversely, path semantics makes no use of covert modals for sentences like (1). The antecedent *Frida took the exam* updates the path of evaluation, and the consequent is evaluated at the updated path. Since the consequent is nonmodal, it is simply evaluated at the first world of the updated path

$$[\![(1)]\!]^p = [\![\text{Frida passed}]\!]^{p+[\![\text{exam}]\!]} = \text{true iff Frida passed at } [p + [\![\text{exam}]\!]]_1$$

Notice that no syntactic element in the logical form of (1) involves modal quantification, or even a selection function. So there is a clear explanation for why we don't see a modal operator in the structure of bare conditionals: there isn't one.

9 Conclusion

It is common to think that there is a tension between a selectional view of conditionals and an informational one, and that we can't have both. This tension can be traced back to the debate about CEM between Stalnaker and Lewis, at the beginning of modern theorizing about conditionals. After that, much of the debate has focused on choosing which principles to reject. It has simply been taken for granted that we can't have both.

This paper is a possibility proof that we can have our cake and eat it. We can have a theory that is both selectional and informational. Path semantics is such a theory. On path semantics, conditionals have no quantificational force and evaluate their consequent at a single world; at the same time, path semantics vindicates all the signature inferences of informational consequence. Frequently, in semantics we have to make difficult empirical tradeoffs when choosing theories. Luckily, in this case we don't.[30]

[30] *[Acknowledgments omitted.]*

Appendix 1: Refining the system

The notion of path update defined in §5 doesn't cover all cases. This has an impact both on the compositional semantics, and on the theory of update. This section shows how the system can be refined to fix this.

Path update, generalized

Recall the definition of path update:

> **Path update (temporary).**
> $p+\mathsf{A}$ (the **update** of p with A) is the largest member of the following set: $\{p' \leq p \mid \forall p'' \text{ if } p'' * p' \text{ then } p'' \in [\![\mathsf{A}]\!]\}$

In words: the update of a path p with A is the largest subsequence of p such that all of its permutations validate A. The problem is that, for some sentences, we won't be able to find a single largest subsequence that does the job. For an example, take again the **Die** scenario and consider path p and sentence (37) below.

$$p: \quad \langle w_1, w_2, w_3 \rangle$$

(36) The die must have landed on 2 or must have landed on 3.

There are exactly two subsequences of p that make (37) true, i.e. the one-world subsequences $\langle w_2 \rangle$ and $\langle w_3 \rangle$. There is no single largest subsequence with the same feature. The problem occurs whenever we try to update a path that includes both A and B worlds (with A and B incompatible) with a sentence of the form $\Box\mathsf{A} \vee \Box\mathsf{C}$.[31]

To solve this problem, I abandon the idea that, for every path p and every sentence A, we can find a unique update of p with A. Rather, paths may have multiple updates. Let's start by introducing the notion of a **maximal update** of a path, defined in the obvious way as follows.

> **Maximal update of a path.**
> p' is a maximal update of p with respect to A iff:
>
> i p' is a subsequence of p;
> ii for every p'' such that $p'' * p'$, $[\![\mathsf{A}]\!]^{p''} = 1$
> iii There is no $p''' \neq p'$ such that $p''' \leq p'$ and p''' satisfies (i) and (ii).

In words: p' is a maximal update of p wrt A just in case (i) p' is a subsequence of p, (ii) all permutations of p' make true A, and (iii) there is no other path that has p' as its own subsequence that satisfies conditions (i) and (ii).

[31] Thanks to *[name omitted]* for pointing out this problem.

To see an example, go back to the case of $\langle w_1, w_2, w_3 \rangle$ and of sentence (37). The two maximal updates in this case are simply the one-world subsequences $\langle w_2 \rangle$ and $\langle w_3 \rangle$.

The next sections show how the new notion is put to work in a compositional semantics and in the theory of update.

Revised compositional semantics

The notion of update figures in the compositional entries for conditionals, so those entries will be the main component affected. It is at this stage that I reintroduce a very restricted kind of homogeneity, though one of a different kind from the homogeneity discussed in §2.4. The semantics considers *all* the maximal updates of the path of evaluation with respect to the antecedent[32], and assumes that they all behave in a homogenous way. Hence a conditional is defined (at a path) just in case all the ways of updating the path lead to the conditional having the same truth value; it is undefined otherwise.

Before getting to the revised entries for conditionals, however, we have to tweak the basic architecture of the semantics, since we are switching from a bivalent to a trivalent system. The interpretation function now maps sentences to one of three truth values: $\{1, 0, \#\}$, with '#' interpreted as 'undefined'. Since indeterminacy is brought in only by conditionals, the clause for atomic sentences remains unchanged. The new clauses for connectives and modals are as follows. (I assume a strong Kleene semantics for connectives.)

$$[\![\neg \mathsf{A}]\!]^p = \begin{cases} 1 \text{ if } [\![\mathsf{A}]\!]^p = 0 \\ 0 \text{ if } [\![\mathsf{A}]\!]^p = 1 \\ \# \text{ otherwise} \end{cases}$$

$$[\![\mathsf{A} \vee \mathsf{B}]\!]^p = \begin{cases} 1 \text{ iff } [\![\mathsf{A}]\!]^p = 1 \text{ or } [\![\mathsf{B}]\!]^p = 1 \\ 0 \text{ iff } [\![\mathsf{A}]\!]^p = 0 \text{ and } [\![\mathsf{B}]\!]^p = 0 \\ \# \text{ otherwise} \end{cases}$$

$$[\![\mathsf{A} \wedge \mathsf{B}]\!]^p = \begin{cases} 1 \text{ iff } [\![\mathsf{A}]\!]^p = 1 \text{ and } [\![\mathsf{B}]\!]^p = 1 \\ 0 \text{ iff } [\![\mathsf{A}]\!]^p = 0 \text{ or } [\![\mathsf{B}]\!]^p = 0 \\ \# \text{ otherwise} \end{cases}$$

$$[\![\Diamond \mathsf{A}]\!]^p = \begin{cases} 1 \text{ if for some } p' * p,\ [\![\mathsf{A}]\!]^{p'} = 1 \\ 0 \text{ if for no } p' * p,\ [\![\mathsf{A}]\!]^{p'} = 1 \\ \# \text{ otherwise} \end{cases}$$

$$[\![\Box \mathsf{A}]\!]^p = \begin{cases} 1 \text{ if for all } p' * p,\ [\![\mathsf{A}]\!]^{p'} = 1 \\ 0 \text{ if for some } p' * p,\ [\![\mathsf{A}]\!]^{p'} = 0 \\ \# \text{ otherwise} \end{cases}$$

[32] For a similar algorithm, see Kolodny & MacFarlane 2010 and Holliday & Icard 2017.

Conditionals quantify over maximal updates of paths. For shorthand, let 'p_{A}' be a variable ranging over maximal updates of p with respect to A. The new entry is:

$$[\![\mathsf{A} > \mathsf{C}]\!]^p \quad = \quad \begin{cases} 1 \text{ iff, for every } p+\mathsf{A},\ [\![\mathsf{C}]\!]^{p+\mathsf{A}} = 1 \\ 0 \text{ iff, for every } p+\mathsf{A},\ [\![\mathsf{C}]\!]^{p+\mathsf{A}} = 0 \\ \# \text{ otherwise} \end{cases}$$

For any nonmodal and nonconditional A, and for all paths p, there is a single maximal update of p with A. Hence, for nonmodal and nonconditional antecedents, the new version of the semantics collapses back into the old version. The only differences in prediction concern certain complex antecedents.

Consequence

Since now we have a notion of undefinedness in the system, we need to update the notion of consequence accordingly. We do so by adding qualifications about definedness to the notions of consequence defined in §6, building on von Fintel's (1999) work on Strawson-entailment. For path consequence, the relevant qualification is that the conclusion is defined:

> **Path consequence (final).**
> $\mathsf{A}_1, \ldots, \mathsf{A}_n \vDash_{\mathrm{P}} \mathsf{C}$ iff, for all paths p such that $[\![\mathsf{A}_1]\!]^p = 1 \ldots, [\![\mathsf{A}_n]\!]^p = 1$:
> if $[\![\mathsf{C}]\!]^p \neq \#$, $[\![\mathsf{C}]\!]^p = 1$

For PI-consequence, the relevant qualification is that the conclusion is defined throughout the relevant information state:

> **Path-Informational consequence (final).**
> $\mathsf{A}_1, ..., \mathsf{A}_n \vDash_{\mathrm{PI}} \mathsf{C}$ iff, for all i such that $i \vDash \mathsf{A}_1, ..., i \vDash \mathsf{A}_n$: if for all $p \in$ PATH(i), $[\![\mathsf{C}]\!]^p \neq \#$, $i \vDash \mathsf{C}$.

Update

Recall the definition of the update of an information state:

> **Information state update (temporary)**
> $i + \mathsf{A}$ (the **update** of i with A) is the set: $\{w : w \in i$ and, for some path p in PATH(i), w is a member of $p + \mathsf{A}\}$

In words, this says that a world w is in the update of i with A just in case for some path p generated by i, w appears in the update of p with A. This needs to be modified to make room, again, for the fact that there might not be a unique update of p wrt A. The natural choice here seems to be to say that a world is in an updated information state as long as it is in *some* maximal update of some path.

Information state update (final)
$i + \mathsf{A}$ (the **update** of i with A) is the set: $\{w : w \in i$ and, for some path p in $\textsc{path}(i)$, for some maximal update of p with A p_{A}, w is a member of $p_{\mathsf{A}}\}$.

For an example, consider once more sentence (37), and suppose that we're updating the information state $i = \{w_1, w_2, w_3\}$

(37) The die must have landed on 2 or must have landed on 3.

The updated information state will involve all the worlds figuring in some maximal update of some path generated by i. It's easy to check that the update of i in this case is $\{w_2, w_3\}$. Again, this appears to be a plausible result.[33]

Appendix 2: Proofs

I give proofs of Propositions 1–3 in the text and of some other results of interest. Some terminology: a sentence A is *nonmodal* iff it contains no modal operators (at any level of embedding), and *nonconditional* iff it contains no conditionals (at any level of embedding). For shorthand, I will write that a world is a member of a path ($w \in p$) iff w appears at some point in p

Finally, one important fact to keep in mind throughout: Path Validity entails Path-Informational validity.

Proposition 1. Path-Informational consequence validates CEM, IMC, and NMC.

$$\vDash_{\text{PI}} (\mathsf{A} > \mathsf{C}) \vee (\mathsf{A} > \neg\mathsf{C})$$

$$(\mathsf{A} > \neg\mathsf{C}) \wedge (\mathsf{A} > \Diamond\mathsf{C}) \vDash_{\text{PI}} \bot$$

$$\mathsf{A} > \Diamond\mathsf{C} \nvDash_{\text{PI}} \mathsf{A} > \mathsf{C}$$

Proof of CEM. CEM is Path-valid. Take an arbitrary path p; assume that $(\mathsf{A} > \mathsf{C}) \vee (\mathsf{A} > \neg\mathsf{C})$ is defined at p, and hence that we have both $[\![\mathsf{A} > \mathsf{C}]\!]^p \neq \#$ and $[\![\mathsf{A} > \neg\mathsf{C}]\!]^p \neq \#$. We have that $[\![(\mathsf{A} > \mathsf{C}) \vee (\mathsf{A} > \neg\mathsf{C})]\!]^p = 1$ iff either, for all $p + \mathsf{A}$, $[\![\mathsf{C}]\!]^{p+\mathsf{A}} = 1$, or, for all $p+\mathsf{A}$, $[\![\mathsf{C}]\!]^{p+\mathsf{A}} = 0$. Given the definedness assumption, this follows from a straightforward induction on C.

Proof of IMC. For *reductio*, suppose that, for some non-empty i: (i) $i \vDash \mathsf{A} > \neg\mathsf{C}$, and (ii) $i \vDash \mathsf{A} > \Diamond\mathsf{C}$.

[33] Readers will have realized that I have chosen to proceed in different ways for the case in which a complex sentence appears in a conditional antecedent, and the case in which it is asserted. In the former case, a homogeneity requirement is in place. In the latter case, we just take the union of all worlds present in all maximal updates. This choice just tracks my intuitive judgments about cases, and could be easily reversed. To settle how to proceed, we might need to get a clearer empirical picture (and possibly some experimental data) about sentences like (37).

Via (i), for all $p \in \textsc{path}(i)$, for all $p + \mathsf{A}$, $[\![\neg \mathsf{C}]\!]^{p+\mathsf{A}} = 1$, i.e. $[\![\mathsf{C}]\!]^{p+\mathsf{A}} = 0$. Via (ii), for all $p \in \textsc{path}(i)$, for all $p + \mathsf{A}$, $[\![\Diamond \mathsf{A}]\!]^{p+\mathsf{A}} = 1$, i.e., there is a path $p' * (p + \mathsf{A})$ such that $[\![\mathsf{C}]\!]^{p'} = 1$. Focus on this latter path p'. Via construction of paths, we know that p' is a member of the following set:

$$S = \{p'': \text{for some } p \in \textsc{path}(i), \text{ for some } p + \mathsf{A}: p'' = p + \mathsf{A}\}$$

But, via (i), $[\![\mathsf{C}]\!]^{p''} = 0$ for all p'' in S. Contradiction.

Proof of NMC. Let $i = \{w_1, w_2\}$, and assume $V(w_1, A) = V(w_2, A) = V(w_1, B) = 1$, and $V(w_2, B) = 0$. Then $i \vDash \mathsf{A} > \Diamond \mathsf{B}$, but $i \nvDash \mathsf{A} > \mathsf{B}$.

Proposition 2. Duality fails on PI-consequence.

Duality. $\nvDash_{\mathrm{PI}} (\mathsf{A} > \Diamond \mathsf{C}) \leftrightarrow (\neg(\mathsf{A} > \neg \mathsf{C}))$

Proof. Same model as the proof of NMC; the left-to-right conditional is false at the path $\langle w_2, w_1 \rangle$.

For some of the next proofs, it is helpful to keep in mind the following two lemmas:

Lemma 1. For any A that is nonmodal and nonconditional, and for all p, p': if $p_1 = p'_1$, then $[\![\mathsf{A}]\!]^p = [\![\mathsf{A}]\!]^{p'}$.

Lemma 2. For any A that is nonmodal and nonconditional, and for all p, there is a unique maximal update of p with A, $p + \mathsf{A}$.

Proof. In both cases, straightforward induction on the semantic clauses.

Proposition 3. Path consequence and Path-Informational consequence validate Centering for conditionals with nonmodal and nonconditional antecedents and consequents.

Restricted Centering. $\mathsf{A} \wedge \mathsf{C} \quad \vDash_{\mathrm{P}} \quad \mathsf{A} > \mathsf{C} \quad \vDash_{\mathrm{P}} \quad \mathsf{A} \supset \mathsf{C}$
(for nonmodal and nonconditional A, C)

Proof. For the first entailment: take an arbitrary path p and assume that $[\![\mathsf{A} \wedge \mathsf{C}]\!]^p = 1$, and hence that $[\![\mathsf{A}]\!]^p = 1$ and $[\![\mathsf{C}]\!]^p = 1$. Since A is nonmodal and nonconditional, the first member of $p + \mathsf{A}$ is p_1. But then, via **Lemma 1**, $[\![\mathsf{C}]\!]^{p+\mathsf{A}} = 1$, hence $[\![\mathsf{A} > \mathsf{C}]\!]^p = 1$.

For the second entailment: take an arbitrary path p and assume that $[\![\mathsf{A} > \mathsf{C}]\!]^p = 1$. Now, we reason by cases. Either the first world in $p + A$ is p_1, or it isn't. Suppose it is. Then, since $[\![\mathsf{A} > \mathsf{C}]\!]^p = 1$ and hence $[\![\mathsf{C}]\!]^{p+\mathsf{A}} = 1$, via **Lemma 1** we have also that $[\![\mathsf{C}]\!]^p = 1$. Hence (via the clauses for negation and disjunction) $[\![\mathsf{A} \supset \mathsf{C}]\!]^p = 1$. Suppose now that it isn't. Then, given that A is nonmodal and nonconditional, we know that $[\![\mathsf{A}]\!]^{\langle p_1 \rangle} = 0$, from which it follows that $[\![\mathsf{A} \supset \mathsf{C}]\!]^{\langle p_1 \rangle} = 1$. But then, via **Lemma 1**, $[\![\mathsf{A} \supset \mathsf{C}]\!]^{p} = 1$

Proposition 4. Restricted Import-Export is valid on Path Consequence.[34]

> **Restricted Import-Export.** $\mathsf{A} > (\mathsf{B} > \mathsf{C})_{\mathbf{P}} \dashv\vDash_{\mathbf{P}} (\mathsf{A} \wedge \mathsf{B}) > \mathsf{C}$
> (for nonmodal and nonconditional A, B)

Proof. The result follows from the following fact: for any p, whenever A and B are nonmodal and nonconditional, (1) there are unique updates of p with, respectively, A and then B, and with $\mathsf{A} \wedge \mathsf{B}$, and (2) A $(p + \mathsf{A}) + \mathsf{B} = p + (\mathsf{A} \wedge \mathsf{B})$. (1) Follow from **Lemma 2**. For a proof of (1), suppose for *reductio* that is not the case. Then $(p + \mathsf{A}) + \mathsf{B}$ and $p + (\mathsf{A} \wedge \mathsf{B})$ differ in (at least) one of three ways: (i) they rank two worlds w_i and w_k differently; (ii) there is a w_i s.t. $w_i \in (p + \mathsf{A}) + \mathsf{B}$ but $w_i \notin p + (\mathsf{A} \wedge \mathsf{B})$; (iii) there is a w_i s.t. $w_i \notin (p + \mathsf{A}) + \mathsf{B}$ but $w_i \in p + (\mathsf{A} \wedge \mathsf{B})$. Suppose (i): then at least one of $(p + \mathsf{A}) + \mathsf{B}$ and $p + (\mathsf{A} \wedge \mathsf{B})$ is not a subsequence of p; contradiction. Suppose (ii), and consider the singleton path $\langle w_i \rangle$. Since $\langle w_i \rangle$ shares its first world with some permutation of $(p + \mathsf{A}) + \mathsf{B}$, and since A and B are nonmodal and nonconditional, via **Lemma 1** $[\![\mathsf{A}]\!]^{\langle w_i \rangle} = 1$ and $[\![\mathsf{B}]\!]^{\langle w_i \rangle} = 1$. But then, via the semantic clause for conjunction, $[\![\mathsf{A} \wedge \mathsf{B}]\!]^{\langle w_i \rangle} = 1$. Hence, via Lemma 1 again, any permutation of p starting with w_i makes true $\mathsf{A} \wedge \mathsf{B}$. But then there is a subsequence of p that is larger than $p + (\mathsf{A} \wedge \mathsf{B})$ and such that all its permutations make true $\mathsf{A} \wedge \mathsf{B}$. But, by definition, $p + (\mathsf{A} \wedge \mathsf{B})$ is the maximal subsequence of p such that all its subsequences make true $\mathsf{A} \wedge \mathsf{B}$. Contradiction. (iii) The proof is similar to case (ii).

Proposition 5. Might-Import-Export is valid on Path Consequence.

> **Might Import-Export.** $\Diamond(\mathsf{A} > \mathsf{C})_{\mathbf{P}} \dashv\vDash_{\mathbf{P}} \mathsf{A} > \Diamond \mathsf{C}$

Proof. Left-to-Right: let p be s.t. $[\![\Diamond(\mathsf{A} > \mathsf{C})]\!]^{p} = 1$. For some $p' * p$, $[\![\mathsf{A} > \mathsf{C}]\!]^{p'} = 1$, and hence, for all $p' + \mathsf{A}$, $[\![\mathsf{C}]\!]^{p' + \mathsf{A}} = 1$. Pick an arbitrary path witnessing this universal claim; call it 'p^*'. Assume now that $\mathsf{A} > \Diamond \mathsf{C}$ is defined, and for *reductio* suppose that $[\![\mathsf{A} > \Diamond \mathsf{C}]\!]^{p} = 0$. Then, for all $p + \mathsf{A}$, $[\![\Diamond \mathsf{C}]\!]^{p + \mathsf{A}} = 0$, hence for all $p'' * p + \mathsf{A}$, $[\![\mathsf{C}]\!]^{p''} = 0$. Now, by the construction of paths, p^* is such that, for some $p + \mathsf{A}$, $p^* * (p + \mathsf{A})$. Hence it must be that $[\![\mathsf{C}]\!]^{p^*} = 0$; contradiction.

Right-to-left: the reasoning is analogous.

Proposition 6. Łukasiewicz's principle is valid on Path-Informational Consequence.

> **Łukasiewicz's principle.** $\neg \mathsf{A} \vDash_{\mathbf{PI}} \neg \Diamond \mathsf{A}$

[34] The general version of Import/Export fails for reasons that are analogous to the failure of Import-Export in some classical dynamic systems. See, among others, Gillies 2020 for discussion.

Proof. Suppose $i \vDash \neg \mathsf{A}$, i.e. for all $p \in \textsc{path}(i)$, $[\![\neg \mathsf{A}]\!]^p = 1$. By the clause for '$\neg$', for all $p \in \textsc{path}(i)$, $[\![\mathsf{A}]\!]^p = 0$. Hence, for all $p \in \textsc{path}(i)$, '$\Diamond$', $[\![\Diamond \mathsf{A}]\!]^p = 0$, i.e. $[\![\neg\Diamond \mathsf{A}]\!]^p = 1$.

Proposition 7. Veridicality is valid on Path-Informational Consequence.

> **Veridicality.** $\Box \mathsf{A} \vDash_{\text{PI}} \mathsf{A}$

Proof. Minor variant of the proof of Proposition 6.

Proposition 8. Or-to-If is valid on Path-Informational Consequence.[35]

> **Or-to-If.** $\mathsf{A} \vee \mathsf{C} \vDash_{\text{PI}} \neg \mathsf{A} > \mathsf{C}$ (for nonmodal and nonconditional A, C)

Proof. Suppose $i \vDash \mathsf{A} \vee \mathsf{C}$, hence for all $p \in \textsc{path}(i)$, $[\![\mathsf{A} \vee \mathsf{C}]\!]^p = 1$. Since A and C are nonconditional and nonmodal, via **Lemma 1**, for all for all $p \in \textsc{path}(i)$, $[\![\mathsf{A} \vee \mathsf{C}]\!]^{\langle p_1 \rangle} = 1$, hence $[\![\mathsf{A}]\!]^{\langle p_1 \rangle} = 1$ or $[\![\mathsf{C}]\!]^{\langle p_1 \rangle} = 1$. Assume now that $\neg \mathsf{A} > \mathsf{C}$ is defined at i.

For *reductio*, suppose that $i \nvDash \neg \mathsf{A} > \mathsf{C}$, hence that for some p' in $\textsc{path}(i)$, $[\![\neg \mathsf{A} > \mathsf{C}]\!]^{p'} = 0$. Since $\neg \mathsf{A}$ is nonmodal, via **Lemma 2** there is a unique update $p + \neg \mathsf{A}$ of p'. Via the *reductio* assumption, we know that $[\![\mathsf{C}]\!]^{p' + \neg \mathsf{A}} = 0$; also, we know that $[\![\mathsf{A}]\!]^{p' + \neg \mathsf{A}} = 0$. But, via the construction of $\textsc{path}(i)$, it follows that, for some $p \in \textsc{path}(i)$, $p_1 = [p' + \neg \mathsf{A}]_1$, i.e. $p' + \neg \mathsf{A}$ shares the first world with some path in $\textsc{path}(i)$. Hence, via **Lemma 1** again, for some $p \in \textsc{path}(i)$, $[\![\mathsf{A}]\!]^{\langle p_1 \rangle} = [\![\mathsf{C}]\!]^{\langle p_1 \rangle} = 0$. Contradiction.

Proposition 9. Might-If-to-Might-And is valid on Path-Informational Consequence.

> **Might-If-to-Might-And.** $\mathsf{A} > \Diamond \mathsf{C}_{\text{PI}} \dashv\vDash_{\text{PI}} \Diamond(\mathsf{A} \wedge \mathsf{C})$
> (for nonmodal and nonconditional A, C)

Proof. Left-to-Right: suppose that $i \vDash \mathsf{A} > \Diamond \mathsf{C}$. Then, for all $p \in \textsc{path}(i)$, $[\![\Diamond \mathsf{C}]\!]^{p + \mathsf{A}} = 1$, hence for some $p' * p + \mathsf{A}$, $[\![\mathsf{C}]\!]^{p'} = 1$. Call this latter path p^*. Since A and C are nonmodal and nonconditional, via **Lemma 1**, we know that $[\![\mathsf{A}]\!]^{\langle p*_1 \rangle} = 1$ and $[\![\mathsf{C}]\!]^{\langle p*_1 \rangle} = 1$. But, by the construction of paths, $p*_1$ is the first world in some path in $p \in \textsc{path}(i)$. Hence, for all $p \in \textsc{path}(i)$, there is a $p'' * p$ such that $[\![\mathsf{A} \wedge \mathsf{C}]\!]^{\langle p'' \rangle} = 1$. Hence, for all $p \in \textsc{path}(i)$, $[\![\Diamond(\mathsf{A} \wedge \mathsf{C})]\!]^p = 1$, hence $i \vDash \Diamond(\mathsf{A} \wedge \mathsf{C})$.

Right-to-left: the reasoning is analogous.

[35] It is unclear whether the restriction to nonmodal and nonconditional disjuncts is justified empirically. Intuitions are muddled by the fact that disjunctions of *might*-claims are subject to free choice effects (see Kamp 1973, Kratzer & Shimoyama 2002, among many). In any case, this restriction is not special to path semantics; rather, it is shared with standard informational/dynamic semantics for conditionals.

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