STANISŁAW LEŚNIEWSKI'S LOGICAL SYSTEMS[1]

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Stanisław Leśniewski's interests were, for the most part, more philosophical than mathematical. Prior to taking his doctorate at Jan Kazimierz University in Lvov, Leśniewski had spent time at several Continental universities, apparently becoming relatively attached to the philosophy of one of his teachers, Hans Cornelius,[2] to the chapters of John Stuart Mill's *System of Logic* that dealt specifically with semantics, and, in general, to studies of general grammar and philosophy of language.[3] In these several early interests are already to be found the roots of the work that was to occupy Leśniewski's life: a search for a definitive doctrine of what sorts of things there are in the world, or better, of what language must be like *if it is adequately and efficiently to represent the world*.[4]

Upon his return to Poland, Leśniewski enrolled in the Jan Kazimierz University, where he came under the influence of Kazimierz Twardowski and Jan Łukasiewicz. Twardowski has been called "the father of Polish philosophy" because of his tremendous influence upon Leśniewski, Łukasiewicz, Tadeusz Kotarbiński and so many of the great Polish philosophers. Like Cornelius, Twardowski had himself been heavily influenced by Franz Brentano. *Un*like Cornelius, however, who probably was influenced by Brentano through Carl Stumpf – a colleague of Cornelius's at Munich, and himself a student of Brentano's – Twardowski had studied under Brentano. Thus, the move from

[1] I am grateful to Judson Webb and Paul Sagal for inspiration in this as in many other projects.

[2] Cornelius was a member of the so-called 'Austrian school' of psychologists and philosophers. The distinguishing characteristic of this school was its advocacy of the 'act psychology' of Franz Brentano, as opposed to the 'content psychology' of Wilhelm Wundt. Brentano had emphasized the doctrine that psychical phenomena are to be thought of as *acts*. When one sees a color, the color *itself* is not mental; it is the *seeing* – the *act* – that is mental. The act always, however, *implies an object* or *refers to a content*. Cornelius himself was only partially committed to this view, and contributed toward a partial analysis of the alleged 'acts' in terms of *complex contents*, an analysis that was later to lead (in the work of Friedrich Schumann) to a compromise of sorts between the act and content schools, but one that was framed in Wundtian terms (see [Boring 1957], 356-361 and 439-447). The influence of Brentano upon Cornelius was great, however.

[3] See [Kotarbiński 1967], 4-5; [Luschei 1962], 18-19; [Ajdukiewicz 1935], 153-154.

[4] See [Kearns 1967], 61-63; [Kotarbiński 1967], 5.

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Cornelius to Twardowski was not an unusual one (or an uncomfortable one) for Leśniewski to make.[5]

Jan Łukasiewicz, the other of Leśniewski's mentors at Lvov, had developed a strong interest in mathematics and mathematical logic. Perhaps Twardowski had stimulated that interest through his lectures on "algebra-like logic", delivered when Łukasiewicz had been his student, but the younger philosopher quickly surpassed his teacher in developing an understanding of and a facility with the new material. It was Łukasiewicz who had "discovered, for Poland, the gold mine of mathematical logic", and Leśniewski first became acquainted with the subject while studying under Łukasiewicz.[6]

At this early stage in his career, Leśniewski was extremely suspicious of the construction of artificial *symbolic* languages as means to his ends. This may seem paradoxical, given Kotarbiński's description of him as "Devoured by a passion for an absolute exactness of statement".[7] But, in fact, Leśniewski found the language of mathematical logic to be "nebulous and equivocal". For example, he once nearly used up all the letters of the alphabet in listing the several possible interpretations of some thesis that was apparently permitted or suggested by the commentaries in *Principia Mathematica*. Further, he argued that the equivocal use of terms such as 'true', 'false', and 'implies' both to express and to describe propositions or relations of propositional arguments made it unclear whether theses were *in* or *about* the system in question.[8]

Perhaps the strongest suspicion that Leśniewski had of the use of symbolism was that it would lead too easily to a departure from intuitive truths about the world and about language. Bertrand Russell's 1901 discovery of a paradox, apparently derivable from Georg Cantor's 'intuitive' characterization of set could only strengthen that suspicion.[9] In order to avoid making such a departure himself,[10] Leśniewski began his work by trying to reconstruct language *in terms of colloquial Polish*, rather than in a formal language with technical notation. But the process led to rather unwieldy and confusing constructions, and it was not long before it became necessary to make some compromise with

[5] See [Luschei 1962], 18; [Boring 1957], 99, 441, 445; [Ajdukiewicz 1935], 153-154. Because of Brentano's direct influence upon Twardowski, plus his indirect influence upon Leśniewski (through Stumpf and Cornelius), it would not be entirely improper to think of Brentano as a kind of 'uncle' of Polish philosophy, if Twardowski was its father. The influence of Brentano is not limited, of course, to the Polish school. Among his students were not only Twardowski and Stumpf, but such other luminaries as Edmund Husserl and Sigmund Freud. These names would serve only to begin a long list of names of the great men in philosophy and psychology who came under Brentano's influence.

[6] See [Ajdukiewicz 1935], 153-154; [Kotarbiński 1967], 1-2.

[7] [Kotarbiński 1967], 4.

[8] See [Luschei 1962], 19.

[9] See [Kearns 1967], 63.

[10] Leśniewski appears to have been fonder of *some* of Cantor's intuitions about sets than of others. See [Luschei 1962], 70.

formalization. Leśniewski's changing attitude is admirably described in the following passage from Eugene Luschei's *Logical Systems of Leśniewski*:

> Between 1915 and 1923 he gradually transformed his style of presentation and abandoned his "stubborn effort logically to subjugate colloquial language and bend it to theoretical ends for which it was never created". For colloquial languages are historical and practical compromise formations, which have to serve many divergent or even competitive purposes, and so remain flexible and universal in tendency. Since they must be sufficiently plastic approximately to express what is expressible in any other language, they must contain terms to express their own semantics. But study of semantic antinomies convinced Leśniewski that in any 'universal' language, semantically closed wholly to incorporate its own semantics, the laws of classical logic cannot consistently hold... Philosophers unaccustomed to deductive methods tend to disparage formalized languages as artificial, in contrast to the natural languages of everyday speech. But Leśniewski believed that rigorous application of classical laws to the semantics of colloquial languages would inevitably lead to confusions and contradictions, which could be overcome only by resolving the ambiguities of colloquial languages and reconstructing them in hierarchical strata, each stage enriched by metalinguistic semantical terms introduced to describe preceding stages of construction... – in short, only by defining their structures and approximating them to formalized languages. And such a logical reform of colloquial languages would be an undertaking not only thankless but doomed to frustration. For the rigor of definition and investigation depends on precise specification of structure, whereas colloquial languages are too vague and ambiguous closely to approximate any structurally determinate model. Colloquial languages and exact logic, both useful, are made incompatible by attempting to subjugate either to the other's purposes instead of treating them as complementary. For the useful vagueness and the ambiguity that make colloquial languages versatile, practical, and associatively rich all-purpose media of expression unfit them for rigorous deductive investigation and analysis.
> So, after talking with Leon Chwistek in 1920, Leśniewski decided to use logical symbolism in his future theoretical work, having concluded that as a scientific instrument it is simpler, more exact, and less conducive to misinterpretation.[11]

Still, Leśniewski subjugated the formalism of his new systems to his intuitions about language and about the world. John Kearns tells us that

> In his formal systems, Leśniewski does not think that he is proposing a language or some languages. Instead he regards himself as presenting the outlines for all languages used to talk about the world. In describing the world, physicists will use different terms than chemists, but these terms can be fitted in (introduced) to

[11] [Luschei 1967], 20-21.

> Leśniewski's formal systems. Leśniewski's formal systems constitute a basis for language used to talk about the world, for these systems enable us to recognize (and describe) those entities which are genuinely constituents of the world without tempting us to admit unreal or fictitious entities.[12]

Leśniewski's transition from colloquial language to formalism was a very cautious one. At first, symbolism was only used as a convenient tool for abbreviating the complex theses and making them more comfortable to work with. The actual deductions of the new theses was still performed in accord with Leśniewski's own "logical intuitions", rather than according to some strictly codified system of mathematical logic. Only much later did he undertake the project of symbolizing and codifying those intuitions themselves.[13] Thus *Mereology* – the "theory of parts" that Leśniewski felt would serve Cantor's original use of 'set' better than did Russell's theory of types or formalistic axiomatizations of set theory like Zermelo's – was formulated first. The purely logical equipment upon which Mereology was founded – Leśniewski's logical intuitions – were formalized later, in what Leśniewski called *Ontology* and *Protothetic*.[14]

Of these two, Ontology was developed first. It is perhaps best to be characterized as a logic of names. The name 'ontology', of course, is also applied to a branch of metaphysics, but Leśniewski felt the name to be an appropriate one for this part of his system:

> The medieval conception of a purely metaphysical proposition, as a statement which is true for anything whatsoever, insofar as it is anything at all, is the analogue of the modern conception of a logically true sentence... of the object language...[15]

Ontology is roughly like traditional logic as it has been reconstructed in recent times, and it includes counterparts of predicate calculus, calculus of classes, and calculus of relations including a theory of identity.[16] The genesis of the distinction between Ontology and Mereology will be explored shortly, after a brief characterization of the final – but most basic – part of the system: Protothetic.

Protothetic may be characterized, again rather roughly, as a propositional calculus with quantifiers. It was "the first indefinitely extensible logic of propositions... to be based on the coimplicator as sole undefined constant." Using the coimplicator, Leśniewski was able to base his entire Protothetic upon

[12] [Kearns 1967], 63.
[13] [Luschei 1962], 21.
[14] See [Luschei 1962], 28-29, 31.
[15] Leśniewski, as quoted in [Luschei 1962], 28.
[16] See [Lejewski 1958], 152.

just three relatively simple axioms.[17] Czesław Lejewski has called Protothetic "perhaps the most comprehensive Logic of Propositions which has ever been devised". Lejewski goes on to remark that

> It goes beyond the classical Calculus of Propositions in several respects. It allows for instance for functorial variables for which constant functors of the Calculus of Propositions can be substituted. It provides for the use of the universal quantifier to bind both the propositional and the functional variables. It has a rule of definitions, which enables us to extend at will the variety of semantical categories within the field of the Logic of Propositions, and, in addition, it has a rule of extensionality; but the most significant point about Protothetic is that with its aid we can derive theses which enable us to dispose of the usual rules for operating with the universal quantifier in any deductive theory of lower generality. In the edifice of the possible deductive theories Protothetic forms the very base. It requires no more fundamental theory than itself whereas other deductive theories, not included in it, have to be built on it or on a part of it. This is the case with Ontology.[18]

It is also the case, of course, with Mereology.

I have remarked above that Leśniewski's work was motivated by interest in the development of a definitive doctrine of what language must be like if it is adequately and efficiently to represent the world. I have also noted that phenomena like Russell's paradox served as evidence, for Leśniewski, that formal languages of the Russellian style were *not* adequate and efficient in the important respects. As a matter of fact, it was the study of Russell's paradox, a study that began in 1911, that led to the construction of Mereology (1914-1916).[19]

Leśniewski absolutely rejected the notion that the contradiction derived by Russell was due to the incompatibility of different intuitions. Instead, he placed the blame on the formal systems themselves, and what he took to be their fundamental inadequacy to express intuition in the first place.[20] As a start, Leśniewski rejected Gottlob Frege's early distinction between an individual and the corresponding 'unit set' or 'singular class' as counter to intuition. Further, he claimed to have found a fallacy in Frege's argument that such a distinction was necessary. Thus

> Leśniewski did not require an unintuitive distinction between an *individual* and the totality (i.e., 'collective class') of itself, the totality of individuals identical with itself, any totality of which that individual is sole ingredient element, or ('the distributive class' of) individuals identical with itself. Nor did he resort to

[17] See [Luschei 1962], 39.
[18] [Lejewski 1958], 151-152.
[19] See [Luschei 1962], 28-29.
[20] See [Kearns 1967], 63.

> distinctions between proper and improper objects, membership-eligible elements and ineligible non-elements, ordinary or extraordinary sets or classes, or sets representing classes unrepresented by sets![21]

Leśniewski's own resolution of the paradox was accomplished through a careful analysis of the concept of set or class. He concluded that there were at least two quite distinct ways of interpreting such terminology, and that it was equivocal use of the terms that brought about the apparent paradoxes. On the one hand, Leśniewski pointed to a 'collective use' of class or set terminology: the terms are used collectively when they are used to describe *totalities*, or *collections* of all individuals that have some specified property. Luschei observes that, like "social sets or classes", the collective use of the terms causes them to refer to entities which "literally consist of their ingredient elements". On the other hand, Leśniewski pointed out that this "collective use" was quite different from what he called the "distributive use" of class or set terminology: the terms are used distributively when they are used to *predicate* some specified property of any member element of the corresponding collective class.[22]

Leśniewski felt that whereas a special 'collective functor' was essential in order to describe any collection or totality as a 'concrete' individual literally composed of one or more individuals, distributive predication required no more than the copula together with other carefully defined functors and constants, all of which properly belonged to logic. The distributive use of class and set terminology was thus dispensable: it could be taken care of in terms of the strictly logical part of the system, and was axiomatically characterized in Ontology. The collective usage is the basis for Mereology.[23]

Given this description of Leśniewski's analysis of the difference between 'distributive' and 'collective' uses of set or class terminology, Leśniewski's resolution of the Russellian paradox can be appreciated. Luschei's account is as follows:

> The original formulation of Russell's paradox (or a protothetical analog) violated Leśniewski's grammar and rules of definition... Translated into the distributive idiom by use of legitimate definitions and meaningful propositions of ontology,

[21] [Luschei 1962], 29-30.

[22] Luschei's characterization of the distributive use of set or class terminology would have the last two words of the sentence read "*distributive* class" ([Luschei 1962], 30). I have been led to alter Luschei's account because, firstly, of the lack of enlightenment provided by a characterization of distributive classes in terms of distributive classes. Secondly, the distinction between distributive and collective use of set or class terminology seems to me to be preserved in my rendering. I take it, therefore, that the words "distributive class," as they occur at line 24 on page 30 of [Luschei 1962], are misprinted, and that they should be corrected to read "collective class".

[23] See [Luschei 1962], 29-31.

> Russell's derivation is no paradox but simply contradicts the supposition in question... Translated into the collective idiom by use of legitimate definitions and meaningful propositions of mereology, it likewise loses the paradoxical appearance of an antimony... and simply contradicts the supposition that the 'collective class' or totality in question exists. For since any individual is the collective class of itself, and is an ingredient element of itself, no individual is a collective class (of individuals) that is not an element of itself, nor *a fortiori* a collective class of such collective classes. So it is simply false (but meaningful to say) that the 'Russellian collective class' is anything at all, even itself, or an element of itself.[24]

In conclusion, it is to be emphasized again that Leśniewski's motive in building his systems was to *formalize intuition*. Kearns remarks that

> In attempting to formalize intuition rather than to devise just any sort of system which "works," Leśniewski is choosing to understand rather than simply to invent.[25]

That is, the construction of the Leśniewskian systems is an examination and elaboration of basic intuitions about the world and about language.

It is difficult, however, to pin down just what it was that Leśniewski was trying to understand – whether it was language or the world. For although intuitions are surely *about* the world, they are themselves linguistic in character: Kearns may be correct in suggesting that

> Leśniewski's intuition is best described as knowledge of how language must be if it is to adequately and efficiently represent the world.[26]

This emphasizes the linguistic element of Leśniewski's work. But might not his intuition be described equally fairly as knowledge of what the *world* must be like, given the distinctive linguistic character of intuitions? In such a formulation, the ontological element of the systems may be seen, along with the justification for Kotarbiński's remark that Leśniewski's Ontology is in fact a "theory of what there is, or general principles of being".[27] Perhaps the best formulation would be that Leśniewski's *Ontology* is a theory of what restrictions *pure logic* places on what can be. This avoids Kearns's objection that the Kotarbiński remark ignores Leśniewski's nominalistic philosophical view, while preserving a kind of ontological characterization of the system. For

[24] [Luschei 1962], 32.
[25] [Kearns 1967], 62-63.
[26] [Kearns 1967], 63.
[27] Kotarbiński, as quoted in [Luschei 1967], 149 and [Kearns 1967], 62; cf. [Lejewski 1958], 152-153 for a similar view of Ontology.

Leśniewski's nominalism enters the scene only in Mereology: Protothetic and Ontology are independent of it.[28]

If this last characterization is correct, then perhaps Leśniewski's great effort can be seen as an attempt to draw the line more carefully between purely linguistic questions and the more strictly ontological ones. The line is still rather fuzzy, as perhaps it must always be, since language has considerable bearing on ontology (in just the same sense in which Protothetic and Ontology have considerable bearing on Mereology). But for all that, the questions *are* to be separated, and Leśniewski's criticism of Russellian-style formalizations of set theory center upon the failure of the latter adequately to make that distinction.

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