Unlimited Possibilities

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Abstract

I distinguish between a metaphysical and a logical reading of Generality Relativism. While the former denies the existence of an absolutely general domain, the latter denies the availability of such a domain. In this paper I argue for the logical thesis but remain neutral in what concerns metaphysics. To motivate Generality Relativism I defend a principle according to which a collection can always be understood as a set-like collection. I then consider a modal version of Generality Relativism and sketch how this version avoids certain revenge problems.

Keywords: absolute generality, Russell's paradox, plural quantification, indefinite extensibility, revenge

1 The incredulous stare

As many philosophers, when trying to explain the questions that occupy my mind, I often end up having to face that infamous look: the incredulous stare of my interlocutor. Personal experience has shown that if the goal is to get the stare in its most expressive format, a safe bet it to reply by saying that I'm worried about absolutely everything. This is almost always perceived as a joke, nevertheless it is quite true for I am in fact worried about everything. If these conversations didn't invariably change their subject, I would begin to provide a better characterization of my worry by following Rayo and Uzquiano (2006) in drawing a distinction between:

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1. The metaphysical question: Is there an all inclusive domain of discourse?

2. The availability question: Is there an all inclusive domain of discourse available to us?

A domain that purports to be all inclusive, must accordingly include absolutely everything in it. To emphasize this even further, if there is an all inclusive domain, then absolutely nothing can lay outside of it. One way to understand the metaphysical question is thus as an inquiry concerning the extension of reality. Does it have limits of some kind? Is reality somehow open-ended or is it completely determined? Although I find these to be meaningful and deeply interesting issues, I will be here mostly concerned with the availability question. At first sight this might appear to be a less exciting topic, but I hope to convince the reader otherwise in the course of the paper. For as the metaphysical issue can be understood as being about the limits of reality, the availability question can be understood as an inquiry on the limits of language. The idea is that just as the alleged existence of an absolutely general domain sets a limit to what there is, the availability of such a domain of discourse would define the limits of what can be talked about.

It seems natural to ask if both questions are not closely intertwined. For instance, if someone denies the existence of an all inclusive domain, it seems that she will also have to deny that such a domain is available to us and contra-positively, if someone defends the availability of such a domain, it seems that she will also have to defend its existence. One suggestion to argue from the availability to the existence of an absolutely general domain would be the following. If there is an absolutely general domain of quantification which is available to us, that thing which is available must somehow exist. The argument would then need to show that the sense in which that thing exists is robust enough to imply an affirmative reply to the metaphysical question. One obvious way to resist the argument would be to deny that robustness and argue that the domain carries no metaphysical weight with it. Someone could perhaps claim that absolute generality is to be understood as a plurality. Since a plurality is to be understood as the mere sum of its elements there seems to be conceptual space for a position that argues for the availability of an absolutely general domain but remains free of a commitment with its metaphysical existence. This position will be considered in more detail further on in the paper, but for now I only wish to notice that the questions above are not intertwined in such a way that replying "yes" (or "no") to one of them, implies a "yes" (or accordingly, a "no") reply to the other. In particular, commitment with the availability of an absolutely general domain of quantification does not imply a commitment with the existence of an absolutely general domain. In the remanding of the paper, we shall use 'generality absolutist' to refer to someone who replies "yes" to the availability question and 'generality relativist' to someone who replies "no."

The plan then is the following. I assume that if an absolutely general domain of quantification is available, then it must be specifiable. That is, when we talk about absolutely everything, we should be able to specify the semantic value of 'absolutely everything' in an unequivocal manner. Presumably, that semantic value takes the form of a collection that somehow contains every object. If the proposed collection fails to contain every object that can be specified or even if its specification leaves open the possibility of there being objects that would not be part of it, the proposed collection cannot be legitimately taken to be absolutely general. I find this to be a reasonable assumption to make. In the absence of such a specification, the claim that an absolutely general domain of quantification is available sounds empty. I then argue that any collection purporting to be absolutely general can be extended. This sort of claim has been accused of being self-defeating but I will try to show how a modal formulation of generality relativism can be shown to avoid such a problem.

2 The availability question

A negative reply to the availability question might initially sound surprising. In fact, it is quite tempting to suppose that speakers can easily give a determinate specification of the domain of absolutely everything. For might they not say something like "The maximal domain is to consist of all objects?" When we say things like "Every object is self-identical" we indeed appear to be making an absolutely general claim. However, one reason to doubt this possibility is that the notion of 'object' appears not to lend itself to a determinate specification. For instance, a Meinongian who defends the existence of universes containing non-existent objects may be accused of incurring in conceptual or even factual mistakes. Nevertheless, it would be clearly unfair to accuse him of incompetence with the English language. In accounting for this situation, Glanzberg (2004, p. 549) says that the term ‘object’ is vague and as such, we are not able to decide among its different sharpenings just by appealing to its meaning.[1] The Meinongian, for instance, relies upon some sharpening of the term. The ordinary meaning of ‘object’ is however insufficient to decide whether that particular sharpening is preferable to another, more limited one. Moreover, and this is the crucial point, any attempt to sharpen its the meaning seems to run the risk of hitting upon a notion which is too narrow to really give us absolutely everything.

Glanzberg considers the following strategy to deal with the previous difficulty. Although there is no philosophical consensus concerning the status of meinongian non-existent objects, if we can understand the notion of *object* in a generous enough way to include all the metaphysical objects in the philosophical literature, that would give us an absolutely general domain of quantification. His idea is to specify something like the minimal conditions of objecthood, i.e., the conditions that need to be satisfied by something to qualify as an object. As Glanzberg puts it, equipped with such a definition we would be able to delineate the outer limits of ‘object.’ Although it might turn out that further philosophical investigation leads us to conclude that some of these logical objects have no metaphysical correlate, given a quantifier ranging over all of them, we could rest assured that no object with a metaphysical correlate falls outside of its range.

The task of providing a definition for the notion of ‘object’ might appear to be a non-starter. After all, this notion is so fundamental that it is hard to conceive of a *definiens* which does not presuppose it already. Still, here is an idea. In a well-known passage Russell (1937, p. 43) writes: “Whatever may be an object of thought, or may occur

[1] Notions like ‘vague’ and ‘sharpening’ have undoubtedly become terms of art in the philosophical literature. As such, Glanzberg’s use of these notions in the present discussion might appear somewhat inadequate. For instance, vague terms lend themselves to *sorites* reasoning and there seems to be no reasoning of that kind associated with ‘object’. Nevertheless, and although this appears to be a fair criticism, it does not carry much weight for the claim that Glanzberg is trying to make. His idea seems to be simply that ‘object’ can be said to be vagueness-like insofar as it shares an important feature of typically vague notions: it is by itself insufficient to decide among its potential sharpenings.

in any true or false proposition, or can be counted as one, I call a term. This, then, is the widest word in the philosophical vocabulary." We might then try to understand *logical object* as that to which a singular term refers. The point is not that every object has a name, but rather that in the present context having a name is sufficient to have an object. Accordingly, we might try to specify the domain of quantification of all the objects in the following way: the objects are the referents of singular terms. As we will now see, this does not serve as a specification for an absolutely general domain. The reason is that for any specification of the latter kind, we will be able to run a generalized version of Russell's paradox and name one object that could not belong to the specified domain.

Suppose that we have some specification of a domain. As a result, we can quantify over it. Hence, we can form the class term '$\{x\colon x = x\}$'. Let $y = \{x\colon x = x\}$. Then, by comprehension, there is $z = \{x \in y\colon x \notin x\}$. But $z \in z \leftrightarrow z \notin z$, so we get a contradiction.

In order to block this argument, the generality absolutist might first try to reject the step involving comprehension. Nevertheless, it is not obvious how to justify such a rejection. Generally, we can unequivocally determine whether an object is self-membered. Consequently, and given that the domain is supposed to be absolutely general, it is not immediately obvious why should the comprehension step fail to determine an object. On the other hand, if nothing is rejected in the previous argument the conclusion is clear. By using the most generous notion of 'object' available, we arrived at term that cannot refer to any object in the specified domain. Since having a term is sufficient for having an object, the argument gives us an object that must lie outside of the specified domain.

Glanzberg claims that the appeal to class abstracts, or sets does not play any role in the previous argument. According to him, assuming that y is a set or a proper class is not going to cause any substantial changes. It might however be worth considering whether appealing to pluralities might not help the generality absolutist. Perhaps that he will be able to block the Russellian argument above by arguing that the collection of absolutely everything is a plurality. We have briefly considered this idea before, when in the first section we discussed how the metaphysical and the availability question are independent from each other. The idea then was that by taking the collection of absolutely everything to be a plurality, one can talk about it even if the collection itself does not exist (in the sense that it does not carry any metaphysical weight). This was supposed to show that the metaphysical and the availability question are independent of each other. The question now becomes whether the generality absolutist could employ a similar move to block the previous Russellian argument. He could perhaps say that what the paradox reveals is that y fails to refer and that this is all that it takes to block the argument.

3 Strengthened Russell

I will now try to spell out the appeal to pluralities in more detail. This will at first sight offer a positive reply to the availability question. I will however end up arguing that the appeal to pluralities is not satisfactory because it seems to leave the generality absolutist begging the question.

Let us then begin by noticing that although the generality absolutist can deny that y (as used in the last section) refers, it follows from the availability of an absolutely general domain that we can talk about absolutely everything. In particular, we can still talk about the set of all the non-self-membered objects in it. That is, we seem to be able to define $z^* = \{ x \prec y \colon x \notin x \}$. If we have thereby succeeded in defining an object, this needs to be part of absolutely everything and if that is the case, we do in fact end up with a new contradiction. For if z^* refers, we can certainly wonder whether z^* is self-membered and in the context of a proof, this inquiry takes the form of the biconditional $z^* \in z^* \leftrightarrow z^* \notin z^*$.

Perhaps that the generality absolutist can say that what this shows is that z^* also fails to refer. Maybe that he can say that z^* is identical to y. To do this, he only needs to assume that no object belongs to itself, a claim which does not really qualify as being controversial. So, if the absolutist makes this move, the closest we can get to a contradiction is $z^* \prec z^* \leftrightarrow z^* \notin z^*$. So far, so good for the absolutist then.

Let us now consider whether the generality relativist might still fight back at this point with some sort of strengthened Russell. Remember that in the Kripkean truth theory, when we reach the fixed point, the liar sentence cannot be found in the extension nor in the anti-extension of the truth predicate. Nevertheless and although we cannot express it in the language of the theory, it still seems correct to say that the liar sentence is not true. For a sentence to be true, it needs to be in the extension of the truth predicate. Given that the liar cannot be found there, it is not true. But since that is what the liar sentence itself says, we find ourselves back in contradiction. This is the so-called problem of the reinforced liar. Now, in the case at hand, although z^* has no elements, we can still talk about it. Thus, it still seems correct to say that z^* is not self-membered. We are now getting dangerously close to a new contradiction. The argument seems to stop here, however. For it to proceed, z^* would need to be a part of y but since z^* fails to refer, this is simply impossible.

4 Collapse

Appealing to pluralities thus appears to offer the generality absolutist a way out of the Russellian argument. I would however like to suggest that this is a deceiving appearance. To see this, begin by considering a principle which has been recently proposed by Linnebo (2010). In replying to the question "When do some things form a set?", Linnebo argues that nothing is required for some things to form a set. In other words, every plurality of objects collapses into a set. A bit more formally, say that some things *xx form a set y* just in case $\forall u\,(u \prec xx \leftrightarrow u \in y)$, and let $FORM(xx, y)$ abbreviate this claim.[2] Linnebo's principle can then be reformulated as,

(COLLAPSE) $\forall xx \exists y FORM(xx, y)$

Notice that a set is an entity over and above its elements. Let us then say that a collection is *set-like* if it is an object over and above its elements. Pluralities, on the other hand, distinguish themselves from sets in precisely this aspect. That is, a plurality is nothing over and above its elements. The principle that I would now like to propose parallels COLLAPSE in saying that all pluralities collapse into set-like collections. The only difference between the two principles is that mine is compatible with the existence of set-like collections other than sets. I will name this principle COLLAPSE*. The reason behind its extra ontological generosity will soon become clear.

[2]Notice that unlike second-order variables, plural variables range over the same objects as singular variables do.

I take it that one argument against COLLAPSE* would be the following. Assume that we can use plural quantifiers to talk about absolutely everything. If we collapse this collection into a set-like collection, there is an object that contains every object as an object. Thus, since the collection of absolutely everything contains absolutely everything, that collection would need to contain itself. But this cannot be.[3] Therefore, COLLAPSE* fails in the absolutely general case.

This argument enjoys a certain plausibility. A set-like collection of absolutely everything arising out of COLLAPSE* would be an entity over and above its elements. But if we assume that we already have absolutely everything in the range of quantification, it would be impossible to bring more objects into it. Hence the failure of COLLAPSE*.

Notice however that the previous argument relies on a controversial assumption, namely, that we can quantify over absolutely everything. For the generality relativist this is the very issue under discussion. As such, he is free to accuse the previous argument of begging the question. Simultaneously, the relativist might highlight the plausibility of COLLAPSE*, by noticing some of the instances in which it obtains. If we have a plurality with any finite number of elements, for instance, nothing prevents us from having a set with precisely those elements. Likewise, pluralities with an infinite number of elements might also be collapsed into a set. One controversial case would be that of the collection of all sets. Assuming that there is a set of all sets leads to paradox. Nevertheless, nothing prevents us from collapsing that collection into a proper class. For our present purposes, the crucial thing to be noticed is that although they are not sets, proper classes are still set-like. The proper class that contains all sets is an object over and above its elements.

Our question now is, why should COLLAPSE* fail in the absolutely general case? Saying that this principle must fail because otherwise we would meet a contradiction is hardly an explanation. Contradictions do not have any explanatory power as they stand themselves in need of explanation. Therefore, without a principled reason for the failure of COLLAPSE*, the generality absolutist seems to be left in a question begging position. Appealing to pluralities cannot be his final word in this debate.

[3] Remember that to resist the generalized Russell, the absolutist needs to be committed with the thesis that there are no self-membered objects.

5 Revenge

We have seen how the generality relativist tries to use a version of Russell's paradox to motive a negative reply to the availability question. There is however a difficulty associated with this strategy. The claim that it leads to a self-defeating position can be found in several places in the literature. For instance, David Lewis (1991), Vann McGee (2000) and Timothy Williamson (2003), all put forward similar versions of an argument for this same claim. Essentially, their argument divides the relativist strategy into two stages. In the first stage, the relativist is supposed to come up with one counter-example to an hypothetical case of absolutely general quantification. In the second, the relativist generalizes the conclusion of the first stage, claiming that there is a counter-example for any hypothetical case of absolutely general quantification. A difficulty now arises in the form of a revenge problem. Notice that in order to put forward his last claim, the generality relativist needs to be committed with all the counter-examples. That is, if the relativist is to make a claim about all the hypothetical cases of absolutely general quantification, he needs nothing less than all the counter-examples. But then, how is it that making a claim about all these objects is not to be understood as an absolutely general claim? What other objects could there be? The generality relativist thus seems to contradict himself. While on the one hand he wants to deny absolutely general quantification, on the other he seems to need it in order to put forward his thesis.

In this section I will sketch how is it that Fine's (2006) modal formulation of generality relativism offers a way out of this difficulty. To do this, I begin by reconstructing the previous criticism within the framework proposed by Fine. He says that quantification is always relative to an interpretation and that the Russellian argument is capable of expanding any given interpretation. Generality relativism is then supposed to be understood as a thesis about interpretations of quantification. Following Fine, let I, J, $\ldots$ be variables for interpretations and I_0, J_0, $\ldots$ constants for particular interpretations. Read '$I \subset J$' as '*J (properly) expands I*' and say that '*I is extensible*' if possibly some interpretation extends it, i.e. $\Diamond\exists J\,(I \subset J)$. Generality relativism is then to be expressed in terms of two clauses:

(GR) $\forall I \Diamond \exists J\,(I \subset J)$

$$(\text{GR})^{+} \quad \Box\forall I \Diamond\exists J\,(I \subset J)$$

The revenge problem previously discussed can now be reformulated along the following lines. Consider (GR). This sentence makes a claim about all interpretations. In particular, it says that every interpretation can be expanded. Hence, its intended range of quantification cannot be anything less than all the interpretations. If (GR) leaves some interpretation out its range, the generality relativist would not be making a strong enough claim. In that situation, (GR) would leave open the possibility of there being an absolutely general interpretation outside of its range of quantification. Now, remember that according to Fine, quantification is always relative to some interpretation. Revenge takes place at this point. Since the generality relativist wants to make a claim about all interpretations, the interpretation associated with the quantifier in (GR) cannot be extended. But this reading of the sentence is not coherent with the claim that (GR) is supposed to be making. Namely, that all interpretations are extensible—in particular, the interpretation associated with the universal quantifier in (GR). Therefore, Fine's modal formulation of generality relativism seems to be committed with two irreconcilable claims. One of them presupposes quantification over all interpretation while the other implies that no quantifier ranges over all interpretations.

I now propose a possible way out of this problem. The basic idea is to put the modal vocabulary to work by making the domain of quantification change from world to world. Put differently, my suggestion is to interpret the formulas in our language by means of a variable domain semantics.[4] This semantics allows the range of quantification to change from world to word. In particular, (GR) is always interpreted as making a claim about all the interpretations in some possible world. That is, when we move into some world, (GR) makes a claim about the interpretations in that world and in that world only. Moreover, according to this semantics the range of quantification may vary between two worlds. Consequently, there might be interpretations in one world that do not exist in the other. That is, (GR) might talk about some interpretations in one world and about other interpreta-

[4]For a detailed presentation of the variable domain framework the reader is referred to (Hughes & Cresswell, 1996, p. 3, c. 5). I will restrict myself to an informal discussion of my proposed solution, but hope to present a more detailed account at some other point in the future.

tions in a different world. The idea then is the following. Assume that (GR) is asserted at world w_1. It says that all the interpretations in w_1 are extensible. We can unproblematically assume that none of these interpretations expands the interpretation associated with the universal quantifier in (GR). Eliminate this quantifier to obtain the formula,

$$\Diamond \exists J\,(I_0 \subset J) \qquad\qquad w_1$$

Notice that this formula says that extending I_0 implies a move into some possible world w_2. According to our semantics, we can assume that the range of quantification at w_2 is different from w_1. In particular, the interpretation associated with the universal quantifier in w_1 might exist in w_2. This semantics then allows us to say that the interpretation associated with the universal quantifier in w_1 can be expanded. We cannot say this in w_1 but nothing prevents us from doing so in w_2. Let us now move to w_2 and eliminate the existential quantifier to obtain,

$$(I_0 \subset J_0) \qquad\qquad w_2$$

Finally, notice that by appealing to $(\mathrm{GR})^+$ we can reproduce the same reasoning as before to show how any interpretation in w_2 can be extended. In a nutshell then, my suggestion is to use (GR) to talk about all the interpretations in some world and $(\mathrm{GR})^+$ to talk about all the worlds. Putting the two together, the generality relativist can talk about all interpretations in all worlds. In particular, he can say that they can all be extended.

6 Summing up

A complete account of the problem of absolute generality implies addressing many different issues. I have been here mainly concerned with one, the so-called *availability question*. This is the question of whether a domain of quantification where the variables range over absolutely everything is ever made available to us. I have argued that an appeal to plural quantification cannot be the generality absolutist final world on this issue. Moreover, I considered the problem of how could someone deny the availability of an absolutely general domain. The initial idea was to use a generalized version of Russell's paradox to do the job. Several authors have claimed that this strategy leads the generality relativist to an incoherent position. I then argued that interpreting Fine's modal version of generality relativism in terms of a variable domain semantics opens up a way around this difficulty.

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