MUST SYNONYMOUS PREDICATES BE COEXTENSIVE?

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In answering this question it is best to distinguish two cases. In one case two predicates belong to two distinct languages. Here I think a persuasive and straight-forward argument shows they might be synonymous but not coextensive. In the second case the predicates belong to the same language. Here the issue is more involved; but a reasonable case can be made for the same conclusion.

I

Consider a first-order language *S*, in Polish notation, whose vocabulary is made up of constants

N, A, Π, ε

and variables

x, y, z, ...

and whose domain of discourse is some set of sets of which the axioms of Zermelo-Fraenkel hold. Let *MS* be a fragment of English suitable for a Tarski-style definition of truth for *S*.

Following Tarski we state in a metametalanguage what each constant of *S* means in *MS*:

(1) 'N' means 'it is not the case that'
(2) 'A' means 'or'
(3) 'Π' means 'for all'
(4) 'ε' means 'is a member of'

On the basis of (1)-(4) plus the syntax of *S* and *MS*, the *MS* translation of each *S* sentence is determined. For example, the *S* sentence

(5) ΠxNεxx

is translated in *MS* by

(6) Nothing is a member of itself.

I am supposing *MS* is suitable for a Tarski-style definition of truth for *S*. This means that *MS*'s domain of discourse must include all infinite sequences of members of *S*'s domain of discourse (among other things). This, in turn, implies that the extension of 'ε' in *S* is *not* the same as the extension 'is a member of' in *MS*. But, by (4), the meaning of 'ε' in *S* *is* the same as the meaning of 'is a member of' in *MS*. So, unless there is something wrong with (4), it is clearly possible for two predicates belonging to different languages to have both the same meaning and different extensions.

I have followed the same procedure Tarski followed in determining metalinguistic translations: the metalinguistic translation of each object language sentence is determined by matching up each constant in the object language with a constant in the metalanguage, and stipulating that the one constant means the same as the other. The sentences (1)-(4) are stipulations. They confer meaning on the constants of *S*. So (4) cannot be said to be false. It might be rejected on other grounds, such as leading to a contradiction or some other incoherency. But I can see no such grounds for rejecting (1)-(4). So, I conclude, 'ε' and 'is a member of' are synonymous but are not coextensive.

Other examples reinforce the conclusion. Let *K* and *H* be two languages whose distinct (possibly overlapping) domains each consist of persons. Might there not be predicates F_1 and F_2 belonging to language *K* and *H*, respectively, each of which mean, respectively in *K* and *H*, the same as 'is the father of' means English? Clearly F_1 and F_2 need not be coextensive.

This raises the question: Can there be two synonymous, non-coextensive predicates belonging to *one* language with a single domain of discourse? I now consider this.

II

Saul Kripke suggests that in general a homophonic truth theory may be produced from a non-homophonic one by expanding the vocabulary of the original metalanguage (so as to include the vocabulary of the object language) and adding certain biconditionals as axioms:

> ... let me mention a more or less mechanical way in which a non-homophonic truth theory can be made homophonic. First extend the metalanguage so that it contains the object language. Next, add to the old truth theory as axioms all statements of the form $\phi \equiv \phi'$, where ϕ is in the object language and ϕ' is its translation into the metalanguage. Then, since $T(\overline{\phi}) \equiv \phi'$ followed from the old axioms, $T\ (\overline{\phi}) \equiv \phi$ follows from the new ones. ([1], p. 358)

By reflecting upon Kripke's ideas, I hope to show that it is possible for synonymous predicates to have different extensions even if they belong to one language with a single domain of discourse.

I shall follow Kripke in his use of the bar: $\overline{\phi}$ is the designation in the metalanguage of the object language sentence ϕ. I also shall understand Kripke to be presupposing that the object language and metalanguage do not share any expressions. (Otherwise, chaos is possible. Suppose, for example, the sign of disjunction in the object language is the same as the sign for conjunction in the metalanguage.)

Let *L* be the same as *S*, described above, except the domain is made up of sets such that no set in the domain is a member of any set in the domain. Now consider a semantical metalanguage *ML* whose domain includes *S*'s domain as a subset plus all infinite sequences of elements of *S*'s domain. I shall suppose that the stipulations (1)-(4), which match the constants of *S* with synonyms in *MS*, apply to *L* and *ML* as well. I shall also suppose *ML* contains Greek letters 'α', 'β', etc. which range over the elements of *L*'s domain. Finally, suppose an adequate, Tarski-style definition of truth for *L* is formulated in *ML*. Then among *ML*'s theorems will be this biconditional:

(7) $T\ \overline{\Pi x\ \Pi y\ N\varepsilon xy} \equiv$ for all α, and for all β, it is not the case that α is a member of β.

Following Kripke's suggestion we can extend *ML* to contain *L*, and add to the axioms of *ML* such biconditionals as:

(8) $\Pi x\ \Pi y\ N\varepsilon xy \equiv$ for all α, and for all β, it is not the case that α is a member of β.

From (7) and (8) it follows that a theorem of the extended metalanguage ('*ML* + *L*', for short) is this:

(9) $\quad \mathrm{T}\,\overline{\Pi x\ \Pi y\ N\varepsilon xy} \equiv \Pi x\ \Pi y\ N\varepsilon xy$

Kripke does not say that in shifting from *L* to *ML* + *L* the variables of *L* have to be restricted in *ML* + *L* to range over the same entities as they do in *L*. So suppose that in *L* and *ML* + *L* the variables 'x', 'y', 'z', etc. are unrestricted in range. I shall also assume that the axioms of *ML* are true in *ML*.

From these suppositions this follows:

Theorem: If Kripke's suggestion is correct, then 'ε' and 'is a member of' are not coextensive in *ML* + *L*.

Proof: Kripke's construction requires adding each sentence ϕ of *L* to *ML* to form *ML* + *L*. The construction is purely syntactic. So if the axioms of *ML* are true in *ML*, they are also true in *ML* + *L*. Suppose now 'ε' and 'is a member of' are coextensive. Then, since 'x' and 'y' are unrestricted in *ML* + *L*,

(10) $\quad \Pi x\ \Pi y\ N\varepsilon xy \equiv$ nothing is a member of anything

is true in *ML* + *L*. But the right side of (10) is false in *ML* + *L*. So 'Πx Πy Nεxy' is false in *ML* + *L* while being true in *L*. Since (7) is true in *ML* + *L* (it is a theorem of *ML*), the right side of (8) is true in *ML* + *L*. So the axiom (8) is false in *ML* + *L*. Hence, Kripke's suggestion is not correct. For some of the axioms of the homophonic truth theory obtained by his method will be false in that theory.

From stipulations (1)-(4) we have:

Corollary: If Kripke's suggestion is correct, 'ε' and 'is a member of' are synonymous and not coextensive in *ML* + *L*.

Is Kripke's suggestion incorrect? The following is intended as a defense of his suggestion.

By the axioms of *ML*, carried over to *ML* + *L*, for each sentence ϕ of *L* there is a sentence ϕ' of *ML* such that $\mathrm{T}\overline{\phi} \equiv \phi'$ is a theorem of *ML*. Further ϕ' is an *ML* translation of ϕ. We are assuming the axioms of *ML* are true in *ML*. Then they shall also be true in *ML* + *L*. Hence the theorems of *ML* are true in both *ML* and *ML* + *L*. Thus each theorem $\mathrm{T}\overline{\phi} \equiv \phi'$ is true in *ML* + *L*.

Now Kripke says that, for each such ϕ and ϕ', $\phi \equiv \phi'$ is to be an axiom of *ML* + *L*. If this axiom is true in *ML* + *L*, then, since $\mathrm{T}\,\overline{\phi} \equiv \phi'$

is true in $ML+L$, ϕ is true in L iff ϕ is true in $ML+L$. Thus 'Πx Πy Nεxy' cannot be false in $ML+L$ if it is true in L.

How could $\phi \equiv \phi'$ be false in $ML+L$? The axiom determines the truth conditions of ϕ in $ML+L$. By this axiom ϕ gains in $ML+L$ the truth conditions of ϕ' in $ML+L$.

Collectively the axioms $\phi \equiv \phi'$ of $ML+L$ determine the extension of 'ε' in $ML+L$. Its extension is the same as it is in L. By (4), 'ε' and 'is a member of' are synonymous in $ML+L$. By the axioms of $ML+L$ the extension of 'ε' is the restriction of the membership relation to a proper subset of the domain of $ML+L$, viz., the domain of L. The extension of 'is a member of' in $ML+L$, however, is the membership relation defined on the domain of $ML+L$. Thus, 'ε' and 'is a member of' mean the same in $ML+L$ but are not coextensive in $ML+L$.

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REFERENCE

[1] Saul KRIPKE, 'Substitutional Quantification', *Truth and Meaning*, edited by Gareth Evans and John McDowell, Oxford, 1976.