Assertion, Rejection, and Semantic Universals

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Abstract. Natural language contains simple lexical items for some but not all Boolean operators. English, for example, contains conjunction *and*, disjunction *or*, negated disjunction *nor*, but no word to express negated conjunction **nand* nor any other Boolean connective. Natural language grammar can be described by a logic that expresses what the lexicon can express by its primitives, and the rest compositionally. Such logic for propositional connectives is described here as a bilateral extension of update semantics. The basic intuition is that a context can be updated by assertion or by rejection, and by one or multiple propositions at once. These distinctions suffice to characterize the logic of the lexicon.

Keywords: Semantic universals · Dynamic semantics · Update semantics · Assertion · Rejection

1 Semantic Universals

Semantic universals are generalizations pertaining to semantics that hold across natural languages [3,7]. A universal typically concerns what may or may not be found in the lexicon of natural languages. For example, while many languages contain simple words for conjunction $\wedge$ (*and*), disjunction $\vee$ (*or*), and negated disjunction nor (*nor*), no natural language contains a word for negated conjunction nand (the Sheffer Stroke) [13]. Such concept is of course expressible, but only compositionally, e.g. by 'not both ... and ...'. The same may be said of other connectives, such as exclusive disjunction, the bi-conditional, and so on.

The existence of lexical gaps calls for an explanation. The explanation developed in this paper has to do with structural (logical) constraints on information-transmission in a conversational setting.[1] Following [14], a bilateral system of updates is defined in which $\wedge$, $\vee$, and `nor`, can be straightforwardly expressed,

[1] A competing hypothesis is based on the possibility of expressing missing operators by means of scalar reasoning [6,12,15,22]. For a criticism of this approach, see [14].

This work has received funding from the European Research Council (ERC) under the European Union's Horizon 2020 research and innovation program (Grant Agreement No. 758540) within the project *EXPRESS: From the Expression of Disagreement to New Foundations for Expressivist Semantics.*

S. Ghosh and T. Icard (Eds.): LORI 2021, LNCS 13039, pp. 183–191, 2021.
https://doi.org/10.1007/978-3-030-88708-7_15

but that lacks nand and all other connectives. Accordingly, semantic universals may depend on cognitive asymmetries in the dynamic processing of logically structured information.

2 Bilateral Updates

Standard semantics does not appear to offer a reason why $\wedge, \vee,$ nor should be lexicalizable instead of other functions of the same type. Dynamic semantics might offer a reason. Dynamic semantics has been designed for, and applied to, the study of dynamic phenomena, such as presupposition projection and default reasoning [5,11,23]. I do not discuss these application here. Instead, I'll apply a version of dynamic semantics to the study of lexical gaps in the domain of binary truth-conditional connectives.[2]

2.1 Outline of the Story

Following [14], two update systems are defined below. The first, system **U**, encodes $\wedge, \vee,$ nor, but no other truth-function. The second, **H**, delivers classical logic and is obtained by adding to **U** the operation of complementation (i.e., classical negation). **U** captures the logic of the lexicon. All the logical distinctions available in classical logic can be made, as in natural language, with the added complexity that comes with system **H** and the use of classical negation in combination with the lexical material already definable in **U**.

Of course, formal systems can be put together arbitrarily. In what sense is this an explanation of lexical gaps? Here's the story, in outline. There are only so many ways information can be conveyed in speech. Some general constraints on communication include the distinction between assertion and rejection, and between updating with one or more sentences. These constraints are independently plausible, and they suffice to characterize the lexicalizable binary truth-conditional connectives, as shown in **U**. All Boolean distinctions can be made by extending **U** to **H**. The extension involves additional complexity, however, in that one more primitive (complement negation) is required. As shown in [4], learnability considerations on language evolution favor the lexicalization of operators codified by the simpler **U** (see [20,21] for similar considerations).

Update semantics allows to formalize the two conceptual primitives this work is based on: the distinction between assertion and rejection, and the possibility of updating a context with information from multiple sentences. Insofar as these primitives are characteristic of human communication (and, presumably, they are), then the logic of bilateral updates is justified by general and well-motivated assumptions about how communication works.

[2] In fact, the story holds for any many-place connectives with more than one argument. The focus on two-place connectives is for ease of discussion.

2.2 Definitions

For each sentence ϕ, I assume two update functions $[^+\phi]$ and $[^-\phi]$: the positive and negative update potentials of ϕ, for assertion and rejection. Moreover, I assume that context can be modified by the use of a single sentence, $c[^\pm\phi]$, or by multiple sentences at once, $c[^\pm\phi_1, \dots, ^\pm\phi_n]$. One can also update with multiple sentences one by one consecutively, $(c[^\pm\phi_1])\dots[^\pm\phi_n]$, but this generates order effects, since a new context is defined after each update. For example, consider updating c by the assertion of ϕ_1 and ϕ_2. If order matters, two contexts will be defined, namely $c[^+\phi_1]$ first, and then $(c[^+\phi_1])[^+\phi_2]$. If order does not matter, only one context will be defined, namely the result of simultaneously adding to c the information carried by ϕ_1 and the information carried by ϕ_2, $c[^+\phi_1, ^+\phi_2]$. I begin with definitions.

Definition 1. *An update system $\langle L, W, \cdot[\cdot], [\![\cdot]\!]^{c,g}\rangle$ is a language L, a set of indices W, an update function $\cdot[\cdot]$, and an interpretation function $[\![\cdot]\!]^{c,g}$ relative to a context and variable assignment. L is built recursively on the following signature:*

$$L := p \mid \top \mid \bot \mid \neg\phi \mid \phi \wedge \phi \mid \phi \vee \phi$$

The set of indices W can be modeled as the set of possible worlds (type *st*). A context c is a subset of W, typically non-empty. The interpretation function $[\![\cdot]\!]^{c,g}$ maps sentences of L to classical propositions. Superscripts are henceforth omitted, since interpretation is standard and there are no bound variables in L.

An update function $\cdot[^\pm\cdot]$ takes two arguments: a context (an object of type *st*), and a non-empty set of (static) propositions (type s, st) decorated by a force sign ($+$ or $-$). The force sign determines the relation between the context and the set of propositions. Thus, assertion and rejection of atomic sentences are:

$$c[^+p] = c \cap \{[\![p]\!]\} \qquad c[^-p] = c \backslash \{[\![p]\!]\}$$

Thus assertion is intersection (as in Stalnaker [19]), and rejection is subtraction (as Stalnaker is naturally extended). For updates with single atomic sentences, this simplifies: $c[^+p] = c \cap [\![p]\!]$, and $c[^-p] = c \backslash [\![p]\!]$.

To study updates we need a notion of validity. Dynamic validity captures the intuitive idea that an argument is valid just in case asserting the premises supports the assertion of the conclusion. A sentence ϕ is supported in context c just in case c is the result of a positive update of c by ϕ. Following [23] and [24], validity is then defined in terms of support.[3]

Definition 2. *Support.* For every c, $c \Vdash \phi$ iff $c[^+\phi] = c$.

[3] In fact, these definitions are slightly simpler, since the language under consideration does not contain "genuinely dynamic" operators, such as modals or conditionals, which are sometimes argued to deserve special treatment [9,25]. The relations of support and validity (and therefore equivalence too) are relative to an update system, but I will not use subscripts for simplicity. The context will make clear which system is under consideration.

Definition 3. *Validity.* $\phi_1, \dots, \phi_n \vDash \psi$ iff for every c, $c[^+\phi_1]\dots[^+\phi_n] \Vdash \psi$.

Finally, equivalence $\phi_1 \Leftrightarrow \phi_2$ is defined as entailment in both directions (and so, sameness of update). The benchmark of a classical update semantics is Stalnaker's Rule of Assertion [19]: the result of updating a context c via the assertion of a sentence ϕ is the intersection of c with the static meaning of ϕ. Stalnaker's Rule will be used to test for the adequacy of an update.

Definition 4. An update is adequate iff for every sentence ϕ and context c, $c[^+\phi] = c \cap [\![\phi]\!]$.

As noted above, Stalnaker's Rule holds for atoms. Consequently, atomic updates are adequate. I now introduce an update system, **U**, that characterizes the logic of natural language lexicon. In particular, **U** lacks the expressive power to capture the non-lexicalizable connectives.

3 System U

Update functions are obtained by combining the force of an utterance (assertion or rejection), and the contents of the arguments of the main operator in the sentence. Assertion and rejection are defined over sets of formulas.

Definition 5. *Acceptance and Rejection of sets of atomic sentences: Let $\Gamma^+ = \{^+p_1, \dots, ^+p_n\}$ and $\Gamma^- = \{^-p_1, \dots, ^-p_n\}$ be possibly empty sets of atomic assertions and rejections.*

$$c[\Gamma^+] = c \cap \bigcup\{[\![p_i]\!] : p_i \in \Gamma^+\} \qquad c[\Gamma^-] = c \backslash \bigcup\{[\![p_i]\!] : p_i \in \Gamma^-\}$$

Single updates by atomic sentences, as seen above, is now derivable as a special case if the Γs are singletons. Positive and negative updates are then extended to all formulas.

Definition 6. *Positive and Negative Updates in* **U**

$$c[\Gamma] = c[\Gamma^+] \cup c[\Gamma^-]$$
$$c[^+\neg\phi, \Gamma] = c[^-\phi, \Gamma] \qquad c[^-\neg\phi, \Gamma] = c[^+\phi, \Gamma]$$
$$c[^+\phi \wedge \psi, \Gamma] = c[^+\phi][^+\psi] \cup c[\Gamma] \qquad c[^-\phi \wedge \psi, \Gamma] = c[^-\phi][^-\psi] \cup c[\Gamma]$$
$$c[^+\phi \vee \psi, \Gamma] = c[^+\phi, ^+\psi, \Gamma] \qquad c[^-\phi \vee \psi, \Gamma] = c[^-\phi, ^-\psi, \Gamma]$$

An update by a set formulas Γ, some of which are asserted and some rejected, is the union of assertoric and rejective updates. Polar negation $\neg$ switches the polarity of the update from assertion to rejection and *vice versa*. Conjunction is a sequence of updates, first with one conjunct, then with the other. Disjunction simplifies away inside the square brackets. Negative updates are obtained from the positive ones by switching the speech act from assertion to rejection.

The distinction between assertion and rejection gives a natural definition of polar negation, in line with bilateralism in philosophical logic [17,18]. The definition of update by conjunction as sequence of updates is due to Irene Heim

[11]. Update by disjunction reduces to Veltman's update rule for disjunction for the "negation-free" fragment of the language and in **H** [23] (see [14]).

Consider now the behaviour of two special atoms, $\top$ and $\bot$, namely a tautology and a contradiction. A simple remark in connection with the next lemma is that $c[^+\bot] = \varnothing$ while $c[^+\top] = c$. (Proofs are omitted for lack of space.)

Lemma 1. *For any c, $c[^+\top, \Gamma] = c$ and $c[^+\bot, \Gamma] = c[\Gamma]$.*

Conjunction and disjunctions in **U**, together with $\top$ and $\bot$, have two important properties of the meet and join of the underlying algebra.

Theorem 1. The following equivalences hold in **U**:

$$\phi \wedge \bot \Leftrightarrow \bot \qquad\qquad \phi \wedge \top \Leftrightarrow \phi$$
$$\phi \vee \bot \Leftrightarrow \phi \qquad\qquad \phi \vee \top \Leftrightarrow \top$$

3.1 Negative Collapse

It is fairly straightforward to see that conjunction, disjunction, and negated disjunction can be 'encoded' in **U**, with adequacy given by Definition 4.[4]

Definition 7. *A binary truth-conditional connective \$ is encoded in system **U** just in case a sentence $p\$q$, with p and q atomic, is adequate.*

Lemma 2. ***U** encodes $\wedge, \vee$, nor.*

Proof. First, $c[^+p \wedge q] = c[^+p][^+q] = (c \cap \{[\![p]\!]\}) \cap \{[\![q]\!]\}$, which is $c \cap [\![p \wedge q]\!]$. Second, $c[^+p \vee q] = c[^+p, ^+q] = c \cap \{[\![p]\!], [\![q]\!]\}$, which is $c \cap [\![p \vee q]\!]$ since intersection distributes over union. Third, I assume that nor expresses negated disjunction. If so, observe that $c[^+p \text{ nor } q] = c[^-p \vee q] = c[^-p, ^-q] = c \backslash \{[\![p]\!], [\![q]\!]\}$, which is $c \cap [\![p \text{ nor } q]\!]$.

However, negated conjunction nand is not encoded in **U**. The most striking property of system **U** is indeed a collapse of negated conjunction, or disjunction of negations, on the conjunction of negations. The latter retains its classical meaning, namely the meaning of *nor*: 'ϕ nor ψ' is true just in case both arguments are false.

Lemma 3. *Negative Collapse: $\neg\phi \vee \neg\psi \Leftrightarrow \neg\phi \wedge \neg\psi$ and $\neg(\phi \wedge \psi) \Leftrightarrow \neg\phi \wedge \neg\psi$.*

In both equivalences, the left-to-right direction is classically valid, but the right-to-left directions are classically repugnant. Both directions are valid in **U**. The two classically repugnant entailments are FC and IC.

$$\neg\phi \vee \neg\psi \vDash \neg\phi \wedge \neg\psi \qquad \text{(FC)}$$
$$\neg(\phi \wedge \psi) \vDash \neg\phi \wedge \neg\psi \qquad \text{(IC)}$$

[4] The definition of 'Encoding' requires that the arguments of a connective be atomic. This is necessary, for failures of a formula to encode its classical interpretation (as it is the case with negated conjunction) obviously percolates upwards.

The former is a form of "Free Choice" inference, in which a disjunction implies a conjunction [1]. The latter is a typical pattern of Informational Conjunction $\otimes$, an operator definable on distributive bilattices [2,8,10]. As a result of FC and IC, the concept `nand` is not encoded in **U**.

Rejecting p and then q (that is, $(c[^-p])[^-q])$) is equivalent to rejecting p and q together (that is, $c[^-p, ^-q]$). This is because the complement of a context with a set of propositions is equivalent to taking iterated complements of the initial context for each proposition. Hence rejected conjunction is equivalent to rejected disjunction. It follows that FC and IC are valid.

Lemma 4. *Validity of* FC *and* IC.

Proof (sketch). By induction on complexity. Consider $c[^+\neg\phi \vee \neg\psi]$. Thus $c[^+\neg\phi, ^+\neg\psi] = c[^-\phi, ^-\psi]$. Moreover, $c[^+\neg\phi \wedge \neg\psi] = c[^+\neg\phi][^+\neg\psi] = c[^-\phi][^-\psi]$. As noted above, $c[^-p, ^-q] = c[^-p][^-q]$, and so FC holds. Consider $c[^+\neg(\phi \wedge \psi)]$. Thus $c[^-\phi \wedge \psi] = c[^-\phi][^-\psi]$, and so we reason as above. Thus IC holds.

Despite the collapse, **U** is not trivial: conjunction and disjunction are distinct, for in general $\phi \vee \psi \nvDash \phi \wedge \psi$. For a countermodel, it suffices to consider a context c that contains two worlds, one in which p is true but q false, and the other in which p is false but q true. Such context supports $p \vee q$ but does not support $p \wedge q$, hence there is no entailment from the former to the latter. Conversely, of course, conjunction does entail disjunction. It may also be checked (but proofs are omitted) that none of the remaining truth-functions is encoded in **U**.[5]

3.2 Further Results

It is apparent based on Lemma 3 that **U** is not classical. However, several classical equivalences hold in **U**. All of the following can be proved by induction.

Double Neg	$\neg\neg\phi \Leftrightarrow \phi$		
Idem $\wedge$	$\phi \wedge \phi \Leftrightarrow \phi$	Assoc $\vee$	$\phi_1 \wedge (\phi_2 \wedge \phi_3) \Leftrightarrow (\phi_1 \wedge \phi_2) \wedge \phi_3$
Idem $\vee$	$\phi \vee \phi \Leftrightarrow \phi$	Assoc $\wedge$	$\phi_1 \vee (\phi_2 \vee \phi_3) \Leftrightarrow (\phi_1 \vee \phi_2) \vee \phi_3$
Symm $\wedge$	$\phi \wedge \psi \Leftrightarrow \psi \wedge \phi$	Distr $\wedge\vee$	$\phi_1 \wedge (\phi_2 \vee \phi_3) \Leftrightarrow (\phi_1 \wedge \phi_2) \vee (\phi_1 \wedge \phi_3)$
Symm $\vee$	$\phi \vee \psi \Leftrightarrow \psi \vee \phi$	Distr $\vee\wedge$	$\phi_1 \vee (\phi_2 \wedge \phi_3) \Leftrightarrow (\phi_1 \vee \phi_2) \wedge (\phi_1 \vee \phi_3)$
DeM 1	$\neg(\phi \vee \psi) \Leftrightarrow \neg\phi \wedge \neg\psi$	DeM 2	$\neg(\phi \wedge \psi) \Leftrightarrow \neg\phi \vee \neg\psi$

One of the equalities above says that conjunction is symmetric: $\phi \wedge \psi \Leftrightarrow \psi \wedge \phi$. This is shown by establishing that, for any c, $c[^+\phi][^+\psi] = c[^+\psi][^+\phi]$. We can exploit this observation to show that the validity relation in **U** is monotonic.[6]

[5] It can also be shown that the monotonicity properties for $\wedge, \vee$, and `nor` follow from the properties of the update system [14]. Thus the important observation that natural language lexical operators are monotone follows from the logic of speech acts [3].

[6] This also implies that **U** is not "genuinely" dynamic [16]. This should come as no surprise, given that L does not make provision for anaphora, modals, conditionals, or other expressions with dynamic potential. It would be possible to extended **U** to a theory of presupposition projection, but this is left for another occasion.

Theorem 2. If $\Gamma \vDash \phi$ then $\Gamma, \psi \vDash \phi$.

From this it can be shown that Ex Falso ($\Gamma, \phi \wedge \neg\phi \vDash \bot$) and Explosion ($\Gamma, \bot \vDash \phi$) hold in **U**. I conclude by noting that both of the *reductio* rules fail in **U**, constructive and classical. Consider a context in which $\Gamma, p \wedge q \vDash \bot$ but $\Gamma \nvDash \neg(p \wedge q)$. Let c consist of three worlds such that $v(p, w_1) = v(q, w_2) = 1$, and v assigns 0 to all other combinations of atoms and worlds. Then $p \wedge q \vDash \bot$, for $c[^+p][^+q] = \varnothing$. However, $\nvDash \neg(p \wedge q)$. For the latter would require that $c = c[^-p][^-q]$, but $c[^-p][^-q] = \{w_3\}$, whereas $c = \{w_1, w_2, w_3\}$. Thus, constructive *reductio* fails. Classical *reductio* fails by a similar argument.

4 Classicality Strikes Back

A negative update in **U** is not always the complement of the corresponding positive update, whence the failure of classicality. In particular, positive and negative update by disjunction are complements, since $c[^-\phi \vee \psi] = c \backslash c[^+\phi \vee \psi]$, but this is not the case for conjunction. This fact is key for Negative Collapse.

Yet natural language does express negated conjunction. It does so not lexically but compositionally, and so I'll proceed here. In order to recover classical logic from **U**, I now introduce a stronger negation operator besides the polarity-inverting device introduced above (and which is available "for free" so to speak, as soon as we distinguish assertion and rejection): complement negation '$\sim$'. The static semantics of $\sim\phi$ is the same as that of $\neg\phi$: $[\![\sim\phi]\!] = W \backslash [\![\phi]\!]$. It's only the dynamic effect that changes: the assertion of $\sim\phi$ has an effect on context that is complementary to the effect of asserting ϕ.

Definition 8. *Complement Negation.* $c[^+\sim\phi] = c \backslash c[^+\phi]$.

Complement negation is the update rule for negation of classical update systems [11,23]. Following [11], $\sim$ captures the update potential of natural language *not*. In the resulting system **H** (from 'Heim') assertion and rejection of atoms are given by Definition 5 as above, but negative updates may be set aside for semantics, because they are no longer needed.

Definition 9. *Positive Updates in* **H**.

$$c[^+\sim\phi, \Gamma] = c \backslash c[^+\phi] \cup c[\Gamma]$$
$$c[^+\phi \wedge \psi, \Gamma] = c[^+\phi][^+\psi] \cup c[\Gamma] \qquad c[^+\phi \vee \psi, \Gamma] = c[^+\phi, ^+\psi, \Gamma]$$

It may be checked that **H** is fully classical: Stalnaker's Rule hold unrestrictedly (Definition 4), and validity (as in Definition 3) is equivalent to classical consequence (the subset operation in the background model). It follows that all Boolean connectives can be encoded in **H**. Just like in natural language, all classical distinction are now expressible, but only by compositional combinations of $\sim$ (*not*) and other available lexical items as defined in **U**.

5 Conclusion

The logic of **U** captures the expressive power of the lexicon of natural languages, with respect to the connectives. All Boolean distinctions may be recovered by adding a "genuine" negation $\sim$ that takes complements and that, following [11], may well be regarded as the meaning of *not*.

I presented and described in some details the logic of an update system, **U**. Justification for this logic comes from a bilateral account of update semantics, in which sentences can be asserted and rejected, and from a generalization of classical updates to sets of propositions. Assuming that languages tend to lexicalize simpler operators [4,20,21], the lexicon of natural language only includes the connectives encoded in **U**.

References

1. Aloni, M.: Logic and Conversation: The case of Free Choice (2021)
2. Arieli, O., Avron, A.: Reasoning with logical bilattices. J. Log. Lang. Inf. **5**(1), 25–63 (1996). https://doi.org/10.1007/BF00215626
3. Barwise, J., Cooper, R.: Generalized quantifiers and natural language. Linguist. Philos. **4**(2), 159–219 (1981)
4. Carcassi, F., Sbardolini, G.: Assertion, Rejection, and the Evolution of Boolean Operators
5. Dekker, P.: Dynamic Semantics, Studies in Linguistics and Philosophy, vol. 91. Springer, Dordrecht (2012). https://doi.org/10.1007/978-94-007-4869-9
6. Enguehard, É., Spector, B.: Explaining gaps in the logical lexicon of natural languages: a decision-theoretic perspective on the square of Aristotle. Seman. Pragmat. **14**, 5 (2021)
7. von Fintel, K., Matthewson, L.: Universals in semantics. Linguist. Rev. **25**, 139–201 (2008)
8. Fitting, M.: Kleene's logic, generalized. J. Logic Comput. **1**, 797–810 (1990)
9. Gillies, A.S.: Epistemic conditionals and conditional epistemics. Noûs **38**, 585–616 (2004)
10. Ginsberg, M.L.: Multivalued logic: a uniform approach to reasoning in artificial intelligence. Comput. Intell. **4**, 265–316 (1991)
11. Heim, I.: On the projection problem for presuppositions. In: Portner, P., Partee, B.H. (eds.) Formal Semantics - the Essential Readings, pp. 249–260. Blackwell, Oxford (1983)
12. Horn, L.: A Natural History of Negation. University of Chicago Press, London (1989)
13. Horn, L.: On the semantic properties of the logical operators in English. Ph.D. thesis, UCLA (1972)
14. Incurvati, L., Sbardolini, G.: Update Rules and Semantic Universals (ms)
15. Katzir, R., Singh, R.: Constraints on the lexicalization of logical operators. Linguist. Philos. **36**, 1–29 (2013)
16. Rothschild, D., Yalcin, S.: Three notions of dynamicness in language. Linguist. Philos. **39**(4), 333–355 (2016). https://doi.org/10.1007/s10988-016-9188-1
17. Rumfitt, I.: Yes and no. Mind **109**(436), 781–823 (2000). https://doi.org/10.1093/mind/109.436.781

18. Smiley, T.: Rejection. Analysis **56**, 1–9 (1996)
19. Stalnaker, R.: Assertion. Synt. Seman. **9**, 315–332 (1978)
20. Steinert-Threlkeld, S., Szymanik, J.: Learnability and semantic universals. Seman. Pragmat. **12**, 1–35 (2019)
21. Steinert-Threlkeld, S., Szymanik, J.: Ease of learning explains semantic universals. Cognition **195**, 1–10 (2020)
22. Uegaki, W.: "NAND and the communicative efficiency model. x x, x (ms)
23. Veltman, F.: Defaults in update semantics. J. Philos. Logic **25**(3), 221–261 (1996). https://doi.org/10.1007/BF00248150
24. Willer, M.: Dynamics of epistemic modality. Philos. Rev. **122**, 45–92 (2013)
25. Yalcin, S.: Epistemic modals. Mind **116**, 983–1026 (2007)