Modality and Validity in the Logic of John Buridan

by

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ABSTRACT

What makes a valid argument valid? Generally speaking, in a valid argument, if the premisses are true, then the conclusion must necessarily also be true. But on its own, this doesn't tell us all that much. What is truth? And what is necessity? In what follows, I consider answers to these questions proposed by the fourteenth century logician John Buridan († ca. 1358). My main claim is that Buridan's logic is downstream from his metaphysics. Accordingly, I treat his metaphysical discussions as the key to his logic. As has been often noted, Buridan's metaphysics are radically anti-realist about universals, though I think the depth and scope of his anti-realism has often been papered over by his recent commentators. Buridan constructs his logic on an amazingly spartan ontology, and this accomplishment is overdue for reconsideration.

To the foregoing questions: truth is a feature of propositions, and of propositions only—not of proposition-like states of affairs, or anything like that. It is a function of the reference or supposition (*suppositio*) of their terms, which depends on predication in a propositional context. And necessary truth is grounded in causation: a predication is necessary if it cannot be falsified by any power, natural or supernatural, without

annihilating what its terms stand for. “Socrates is a human”, for instance, can only be falsified by annihilating Socrates, and so it is a necessary truth.

These considerations provide an ample theoretical basis for a thorough examination of Buridan’s modal logic. This is what the thesis culminates with. In the final chapter, I set forth some novel and surprising findings, chief of which is this: Buridan’s modal syntax and semantics are nothing like Kripke’s—and indeed are *incompatible* with them. This has significant implications not only for modal logic and metaphysics, but for how we think about medieval logic and philosophy more generally. Throughout the thesis, I advocate a methodology of emphasising the *differences*, rather than the similarities, between past and present thought. Buridan is wildly unlike what we’re used to, and we should let him speak for himself.

ACKNOWLEDGMENTS

you may ask yourself, well, how did I get here?

—David Byrne[1]

First of all, I want to thank Peter King, my supervisor, for his help and encouragement, and for his patience and help in hammering out the problems discussed here. I owe special thanks, too, to Martin Pickavé, who has encouraged me from my very first days at the University of Toronto, and proved an indefatigable source of helpful comments and corrections. I am grateful for the guidance of my committee, Deborah Black, and Nate Charlow, who has suggested many ways to link medieval discussions up with current debates in logic and the philosophy of language. Special thanks to Mark Kingwell, whose advice on my presentations and writing have been immensely helpful, and has helped me to kick my habit of burying the lede.

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[1] Talking Heads, "Once in a Lifetime", track no. 4 on *Remain in Light*, Brian Eno, 1980.

I owe a special thanks to the audience at the University of Canterbury in Christchurch, where I delivered a lecture on Buridan, and especially to Jack Copeland and Douglas Campbell, with whom I discussed Buridan's modal logic, semantics, and ontology, and thanks to whom Appendix B (on Buridanian *possibilia*) and Chapter 5 are significantly improved. I am also grateful for Stephen Read's input on Chapters 1 and 2.

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One final and general acknowledgement: it seems to be in the nature of academic writing that we more often mention each other's work when we disagree with it than when we agree.[2] This thesis is no exception, but I wish to acknowledge at the outset how much I owe to all the scholars I cite here, however much I disagree with them. Indeed I owe them an immense debt of gratitude. Were it not for their work—both commentaries, and editions and translations—writing the present thesis would have been an impossible task. So though I often disagree with what's been said, I more often agree—even if the agreements don't come up as much.

Errors, I am sure, will have crept in, despite the best efforts of everyone involved. I take responsibility for them all. Worse yet, there are many others whom I should have thanked here, but forgot to add. To them I say: I have no doubt I will think of you the moment I formally submit this thesis. And once I do, I will never forget your contributions—believe me: guilt is a powerful mnemonic.

[2] This observation was made by W.K.C. Guthrie, and it has been in the back of my mind since I came across it in *A History of Greek Philosophy, Vol II: The Presocratic Tradition from Parmenides to Democritus* (Cambridge: Cambridge UP, 1969), xvi.

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LIST OF FIGURES

Figures are numbered by chapter, and by their order within the chapter.

There are no figures in Chapters 1 or 2.

NOTATION

I developed the following notation for Buridan's modal logic, which I set out in Chapter 5.

$\vdash$	entailment
$\nvdash$	non-entailment
a, b	terms
$\bar{a}$, $\bar{b}$	negated terms a, b (*not-a*, *not-b*)
$\underline{a}$, $\underline{b}$	terms with non-empty extensions (in A-/I-type propositions)
$\langle a \rangle$, $\langle b \rangle$	ampliated terms
A	Universal affirmative ($\underline{a}\mathrm{A}\underline{b}$ = "every a is b")
E	Universal negative ($a\mathrm{E}b$ = "no a is b")
I	Particular affirmative ($\underline{a}\mathrm{I}\underline{b}$ = "some a is b")
O	Particular negative ($a\mathrm{O}b$ = "some a is not b")

Modal adverbs (which modify a copula C as *e.g.* $C_{\Box}$):

$\Box$	Necessarily
$\Diamond$	Possibly
$\triangledown$	Contingently

ABBREVIATIONS

IN REFERENCES TO JOHN BURIDAN'S WORKS

QAPo — *Quaestiones super libros* Analyticorum Posteriorum. Ed. H. Hubien. Unpublished.

QAPr — *Quaestiones super libros* Analyticorum Priorum. Ed. H. Hubien. Unpublished.

EQC — *Ioannis Buridani Expositio et quaestiones in Aristotelis* De caelo. Ed. Benoît Patar. Louvain: Editions Peeters, 1996

QE — *Questiones elencorum*. Ed. Ria van der Lecq and H.A.G. Braakhuis. Nijmegen: Ingenium, 1994.

QLP — *Quaestiones longe super librum* Perihermeneias. Ed. Ria van der Lecq. Nijmegen: Ingenium Publishers, 1983.

QM — *Quaestiones in Aristotelis* Metaphysicam*: Kommentar zur Aristotelischen Metaphysik*. Paris, 1518; reprint, Frankfurt–am–Main: Minerva, 1964.

QNE — *Quaestiones in decem libros* Ethicorum *Aristotelis ad Nichomachum*. Paris, 1513; reprint, Frankfurt–am–Main: Minerva, 1968.

TC — *Iohannis Buridani Tractatus de Consequentiis*. Ed. Hubert Hubien. Louvian: Publications Universitaires, 1976.

NB items under discussion are lettered and numbered by chapter. I use three letters: P (for *proposition*), A (for *argument*), and S (for *schema*, i.e. formal presentations which incorporate variables). So you'll see, *e.g.*

P1) *Homo est animal*

Which is the first proposition of the chapter it appears in.

Introduction

τίς κα, τίς ποκα ῥᾴ ἄλλω νόον ἀνδρὸς ἐνίσποι;

—Alcman[3]

For the General Reader

Logic is the study of correct argumentation. And in fact there are historians of logic. I am one of them. I study logic in the fourteenth century—focussing on John Buridan († ca. 1358), the finest logician of that century—as well as in the twentieth and our twenty-first. During these centuries in particular, logic has been the focus of significant interest and research. At other times, logic and its development have fallen by the wayside. Why that happened is hard to tell. At any rate, the fact that logic is a going concern is good news for me, since it allows me to justify my historical work to a relatively receptive modern audience. It's pretty easy to explain my project to—and attract attention from—modern logicians and philosophers of logic and language. A modern analytic philosopher with no knowledge of Latin or medieval thought can still recognise in Buridan's writings interests and concerns akin to his or her own. For instance, Buridan is concerned with the structure and makeup of propositions, truth conditions for propositions, and with the behaviour of claims qualified by **modes**[4] like *necessary* ("triangles *necessarily* have three sides") and *possible* ("it is *possible* that it will rain tomorrow"), and so on. These topics are easier

[3] Who can ever speak the mind of someone else? (frag. 104).
[4] Sorry to throw a bold term at you, but I'm going to bring up modal logic (that is, logic that deals with modes) again in a minute, so it's worth flagging now.

sells than more obscure and unfamiliar concerns in medieval philosophy, like the proverbial angels dancing on points of needles.[5]

Still, a lot has changed since Buridan's day. If there is one general trend in the field since Buridan, it's the drift away from real-world application of logic and toward mathematical abstraction. Here's what I mean: medieval thinkers tend to see logic as a toolkit for dealing with arguments as they happen in the real world—which is to say, in a natural language, like Latin or English (as opposed to *artificial* languages, like the language of modern logic, or programming languages).[6] As a result, medieval thinkers are much more concerned with the ways language works—and with the many ways it can pull tricks on us. For example:

	This dog is a father (*i.e.* this dog has puppies)
	This dog is yours
Therefore	This dog is your father.

Clearly, something is going wrong here. But seeing *what* goes wrong in little toy cases like this one can also tell us a lot about what's going on logically under the hood, so to speak. And it arms us against less anodyne (and less obvious) versions of the same fallacy. Logic, for the medievals, helps us sort these things out. Thus for medieval thinkers the goals of logic are practical. As a result, medieval logic incorporates many subjects that we now assign to philosophy of language, linguistics, and even rhetoric.

Modern logical research, for its part, has come to be closely aligned with math and, more recently, computer science. Logics (which are not only plural, but *legion*) are typically well-defined mathematical objects, whose properties the logician studies. Seen in this way, logic is not a tool for winnowing out good arguments from bad. A logic is,

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[5] Which, by the way, don't appear in any known medieval source: angels dancing on the points of needles seems to be a later topic, and probably a parody. Anyway, since angels don't take up space, the obvious answer to this question ("How many angels can dance on the point of a needle?") is *all of them*. For a discussion, see Chris Martin's "Angels and Needles", *Notes and Queries* (2016) 63 (3): 374-5.

[6] Or, where the Middle Ages are concerned, at least *Middle* English. As the hilarious and pedantic Oxford student in Chaucer's *Canterbury Tales* makes clear, students took styles of disputation they learned in Latin and applied them to disputation in English. And why wouldn't they?

rather, an artificial symbolic language, plus an interpretation of the language and/or rules governing the operations of that language. Such objects are typically treated as static and abstract. An analogy here might help: if the medievals see logic as a hammer—a tool for a specific category of jobs—the moderns see a hammer as a set of hammerings, *i.e.* as a collection of all the nails it drives or can drive.

There is much to be said for the mathematical methods and approach. Modern logicians have given sharp edges to logical notions that, formerly, were somewhat murky and qualitative. They have given us fascinating insights into the foundations of mathematics. They have, moreover, made great strides in computer science. These developments are nothing to sneeze at.

But there are also hazards to be aware of. I see two. The first is that we run the risk of ignoring the natural-language component of logic, and the role it plays in everyday reasoning. Whereas the medievals see logic as a tool for the everyday reasoning, our deemphasis of this role of logic (or our demotion of it to *informal* reasoning) amounts to a retreat into the high mountains of mathematical abstraction. By doing so, we abandon the lowlands and plains of ordinary language and day-to-day reasoning. But the lowlands are prime real estate, susceptible to takeover by unscrupulous rhetoricians. Medieval logic, with its more secular considerations and goals, provides a salubrious reminder that logic should also deal with ordinary arguments. We don't need to abandon the mountains—nor should we—but we need to hold the plains.

The second major drawback is that the modern mathematical approach gets overused in historical analyses of medieval texts. The basic intuition seems to be that, if thinkers like Buridan are so similar to us, and if the modern mathematical approach has shown itself to be such a powerful and versatile tool, then this tool can be used to analyse Buridan, too. This is all well and good, provided we take care to understand the texts on their own terms, first and foremost. If we don't, our approach will severely misconstrue the subject matter. Failure along these lines is analogous to using modern power tools to restore or reconstruct a cathedral, without first giving careful thought to its materials and internal structure. The outcome, predictably, is distortion, often beyond recognition.

Unfortunately, this is quite common in the secondary literature on medieval logic. Fortunately, the texts themselves—unlike a damaged cathedral—survive this distortion, and can still speak for themselves.

Usually, this process of distortion is carried out in three moves: (i) make a loose analogy between the approach of a medieval logician like Buridan or Ockham and a modern logician like Frege or Kripke. Next (ii), grind the medieval logician's insights through the symbolic machinery of modern logic, to produce a paper full of impressive formulas, which ostensibly explain these insights (exhaustively). Finally (iii), give some general conclusions, and a promissory note by way of an apology: Buridan's modal logic is Lewis's System 5 (S5), Buridan's notion of conditionals is like Frege's, etc. So maybe Buridan can still help us with S5/conditionals/etc. after all! QED.

This approach is bad for medievalists, bad for medievals, and bad for moderns. It's bad for medievalists because it construes our decision to study the medievals as a waste of time. Why bother learning Latin and poring over dusty old tomes, when the all modal logic you need is available in an online PDF of Lewis & Langford's (1932) *Symbolic Logic*, available at the unbeatable price of $0?

It's bad for medievals because their claims and worries get misconstrued: it muffles their unique voices, and papers over their characteristic discoveries. As I will show, there really *are* insights to be gleaned from medieval texts. But they're to be found precisely in the places medieval thinkers are *unlike* us—and these unfamiliar aspects of medieval thought are precisely the ones that should receive special attention.[7] The fact that they agree with us on the problems, but disagree on the solutions provides alternatives to our sometimes quite entrenched ways of doing things—a thought I'll return to in a moment.

Finally, this approach is bad for moderns, because it makes modern logic out to be contained somehow in their medieval forebears. Why did modern logicians bother writing all these papers and books, when all they needed was a grasp of paleography and a passable facility with Scholastic Latin? Hence this approach downplays the very real accomplishments of modern logic. Modern logic is far more than a mere continuation of its

[7] I discuss this methodology, which I have adopted and adapted from Henry Butterfield, in greater detail in the conclusion. In fact, skipping ahead to the conclusion before reading the thesis itself isn't a bad idea.

medieval counterpart, and we recognise this by viewing both in their proper contexts. In short, if we don't know what the medievals are about, we misunderstand ourselves.

So if this approach is so bad, why do we use it? Because it's good for apologetics: reading modern logic into medieval texts justifies the study of the latter—or at least it *appears* to. But I submit that the *real* reason to study medieval logic is because it gives us a chance to treat these temporally distant writers as our colleagues. Medieval logicians are not thinkers who say exactly the same things we do, but thinkers who (often enough) agree with us on the problems, but disagree on the solutions. They are, to borrow (and adapt) John Edensor Littlewood's memorable phrase, "fellows of another college".[8]

A lot of our modern solutions and approaches are quite different from their medieval analogues. And these solutions and approaches have gone without much defense for a very long time: they are more often stipulated than argued for. This is to be expected, since the mathematical way of doing logic has carried the day, and depends heavily on stipulation. Probably it will in the future, too: I don't see medieval logic as a rival fit to replace its modern counterpart. But it can supplement it in places, and it can stimulate debate, too. All this is good for philosophical business.

So where does all this leave you, as a general reader? Well, if you've gotten this far, you probably have a general interest in arguments, philosophy, or history. If it's arguments you're after, there's plenty to be had. As I say, medievals care deeply about ordinary, day-to-day arguments; (excitingly) they present techniques for spotting errors in your opponents' arguments, and (somewhat embarrassingly) for concealing the flaws in your own.

Or, if it's philosophy that interests you, you'll find plenty of questions here: What is a proposition? What makes a good argument good, and a bad argument bad—and how can

[8] As reported by G.H. Hardy, *A Mathematician's Apology* (Cambridge: Cambridge UP, 1941), 12. To give context: Littlewood and Hardy are speaking of the Greek mathematicians, who are not (they say) "clever schoolboys or 'scholarship candidates', but 'Fellows of another college'."

we recognise good and bad arguments when we see them? What does it mean to say that something *must necessarily* be true? And what is the meaning of the word *is*?[9]

Or, if it's more general history that interests you, be apprised that the study of logic was a major component of medieval university life. All students, whether bound for careers in law, medicine, political administration, or ecclesiastical office, spent their first year studying logic. In no time before or since has logic occupied such a central place in the general university curriculum. This had a profound effect on medieval life: references—oblique and direct—to Scholastic logic are laced throughout many historical texts. It would be well to know them when you see them. And while this thesis is not a general guide, it *does* present a considerable portion of the logic and philosophy of language of the most influential Scholastic logician: John Buridan.

The present thesis is about John Buridan's work on the foundations of logic, and can be seen as one long answer to the question, "What makes an argument valid?" It accordingly deals with subjects that are at the heart of Buridan's logical enterprise. The answer to this question, broadly speaking, is that an argument is valid when its concluding sentence or *consequent* follows necessarily from its premiss(es) or antecedent(s), as it does in the following:

	Every human is an animal	(antecedent)
	Socrates is a human	(antecedent)
therefore	Socrates is an animal	(consequent)[10]

Clearly, the consequent follows from the antecedent here, and necessarily at that. Why necessarily? Buridan's answer is that there is nothing that can make the antecedents true,

[9] Probably this semantic question has taken up more airtime than all the others combined. This is largely thanks to the efforts of W.J. Clinton, whose famous rumination ("It depends what the meaning of *is* is") rivals "I think, therefore I am" in style and richness.

[10] Regarding "Humans are animals" as a self-evident truth sometimes strikes moderns as surprising. But in fact this is a stock example of just such a truth, and it appears all over the place in fourteenth century textbooks. In general, logic textbooks are always in need of standard examples, obvious and necessary truths. Medievals tend to use things like "Man is an animal" and "God exists", whereas their modern counterparts tend toward mathematical or analytical truths: "Triangles are three-sided", "2+2=4", "Bachelors are unmarried males", and so forth.

but falsify the consequents. There is no force or causal power, on earth or in heaven, that can make the antecedents true and the consequent false. Granted, you or I could make the second antecedent false—by, for example, serving up Socrates a hemlock cocktail. And the first antecedent could be made false by the annihilation of all humans (perish the thought). But either of these falsifications would render the consequent false as well. And that's fine: what makes this argument valid is not that the consequent *has* to be true unconditionally. It just has to be true *if* the antecedents are.

Of course, accounting for what's just been presented here leads to all kinds of deeper questions, which this thesis attempts to answer. Here's the whole thesis at a gallop: (i), what *is* an argument like the foregoing? Answer: a kind of sentence or proposition. And (ii), what makes them hold? A: they can't be falsified, in the way just discussed. Now it seems that the above argument holds in virtue of its structure or form, so that we could replace the terms with other terms, and still get a valid argument. And this prompts questions (iii) and (iv), to wit: (iii) what special terms do they contain (like *if* or *therefore*) that tip us off about their structure? A: logical particles, which share a bunch of traits in common, but which we select more or less arbitrarily. And (iv), what even *is* argument structure? A: a somewhat arbitrary way of generalising arguments. Finally (v) what happens when we qualify sentences in arguments with modes like *necessarily* or *possibly*? A: it comes down to causation: what's possible is doable, what's necessary isn't undoable, by any cause whatsoever. These five questions correspond to the five chapters of the thesis.

That's about all I have to say (apart from everything below), so let me close this brief, general introduction by thanking the reader for their interest. If any questions, comments, or criticisms come up, or if you find any typos, I hope you won't hesitate to contact me, at boaz.schuman@mail.utoronto.ca.

Happy reading!

For Logicians, Philosophers, and Historians of Logic

The present thesis is on Buridan's notion of logical consequence. In a way, it can be seen as a long commentary on the *Tractatus de Consequentiis* I.3, where Buridan sets out his definition of logical consequence. Broadly, my claim is that Buridan's metaphysics drives his logic. Part of what makes this so interesting is that Buridan's ontology is so sparse: he is, quite possibly, the most committed and radical nominalist ever to work on logic. Accordingly, he has to make do without a number of things like propositions and propositional content (of the Fregean sort, which is to say *real* in the sense of *realism*), sets of sentences, and even relations among non-existent sentences—which is to say, sentences no one happens to be actually thinking *right now*. So Buridan has to make do with astonishingly little. Sometimes this looks, *prima facie* at least, a bit like deep-sea diving with only a snorkel; other times, it more closely resembles a trapeze act with no net. In any case, it's never boring.

In a moment, I will give an overview of the chapters, but first I want to say a few things about the methodology I've adopted here, namely of emphasising the *differences*, rather than the similarities, between Buridan's logic and philosophy of logic, and our own, largely Fregean one.

Buridan is hard. In part, this is because he has his own technical and very medieval vocabulary, which at times can strike us moderns as bizarre and counterintuitive. Yet Buridan also strangely familiar. Often, he seems to share many of our modern logical concerns: he is worried about the foundations of logic and especially logical consequence. For instance, he defines logical form by appealing to uniform substitution of non-logical terms, and accordingly takes interest in the features and roles of logically constant ones. He moreover has a fully developed and sophisticated modal logic, which he builds up from his own bespoke semantics for modal propositions.

But if the differences are difficult, the similarities are often even worse: they can be very misleading, because often they are superficial. Buridan's account of logical consequence is *metaphysical*—quite unlike the later semantic and syntactic Tarskian accounts. He places formally valid arguments on equal footing with informally (or *materially*) valid ones—which might make us wonder what the role of *formal* logic is in the first place. He is surprisingly unconcerned about the admitted overlap between logical and non-logical terms—where we moderns demand (or at least hope for) a strict and even perhaps *principled* divide. His modal semantics deal with possible *objects* (not worlds), whose modal properties are grounded in causation (not consistency). And his modal semantics *cannot* be modelled in the Kripkean frames we have grown so accustomed to.

All this, I submit, is good news. So far, many of Buridan's commentators have attempted to draw interest to Buridan by placing him under the banner of this or that modern school or development: to cast him as a proto-Tarskian or proto-Kripkean, perhaps to attract more (and more general) readers, in order to fascinate them with Buridan. In many ways, this is a mistake, not only in philological or critical terms, but even as tactical apologetic. Why think that such a proto-philosophical-celebrity should have anything new to tell us, after over a century of intense research in logic and analytic philosophy of language? If Buridan's modal system really *is* just $\mathsf{S5}$ or T, why bother reading Buridan at all?

What's more, Buridan is *fascinating*.[11] But this is not for reasons many commentators have supposed. Not, that is, because he is familiar to us modern analytic philosophy types, but precisely the opposite: because he is unlike anything most of us have experienced. Buridan's differences, as I argue throughout the present dissertation, are what's really exciting about his work. They suggest alternatives to our familiar modern ways of doing things which, if we are not going to adopt, we at least have to repel in a principled way. And they point to shortcomings in modern methods and concepts that we might otherwise have ignored.

Here is a teaser-trailer example of the point I want to make. I mentioned just above that Buridan's modal logic makes use of possible objects, whose modal properties are grounded in causation. This approach is very different from Kripke's use of possible

[11] I realise that being told *by* a PhD student that the subject of his or her *own* dissertation is "fascinating" is, frankly, uncompelling. Like, *really* uncompelling—only slightly more compelling than the bizarre enthusiasm displayed by TV-advert actors for household detergents. But hear me out.

worlds. Following Kripke, we would nowadays say that a truth is necessary if it is true in all possible worlds. But Kripkean necessity is difficult to parse: what does it mean to say that a sentence holds in all worlds? And further, what does this tell us about necessity? Take for instance the following:

1) Triangles necessarily have three sides.

Since it is true, (1) should hold in all possible worlds: every possible world is a world in which the sentence "triangles are three-sided" is true. Yet this doesn't tell us much about *why* this fact is necessary. (And we might also wonder: what's the ontic status of triangles in all these worlds, anyway? Are they just triangular objects, or propositions about triangles, or what?). Further, it presupposes that necessity and possibility are not qualitatively different, but merely *quantitatively.* But is necessity really only *quantitatively* different from possibility?

Conversely, Buridan just says that something is necessary if there is no causal power to make it otherwise without annihilation. So (1) is true because no triangle can have a side added or subtracted—at least without destroying it, *qua* triangle. This seems to bring us closer to the unchangeable or instrumental ways we often think of necessity: not *being* necessary, but being necessary *as* (or *for*). A triangle is unchangeable *as* a three-sided plane figure (or, three sides are necessary *for* a figure to be triangular)—and not because (1) just holds in every world, *pace* Kripke.

So there's plenty of interesting stuff to be gleaned from Buridan. But first, two quick clarifications.

Two Quick Clarifications

I've encountered a lot of misunderstandings in discussions with colleagues (which often go unnoticed, only to be detected rather late). To address these, I want to take a moment to clarify the following facts about Buridan's logic. These facts might seem odd or even confusing to a modern audience, at least at first blush, but they can be sufficiently addressed in a brief treatment like this one.

Existential Requirements

Buridan's existential requirements are not the same as ours. Because modern logic has dispensed with the copula—the linking verb of any proposition: *is* or *are*, or their equivalents—it is forced to read universal propositions as conditionals, and particulars as conjunctions. Hence we ~~indoctrinate~~ teach students of elementary logic to read propositions like

	"Every F is G"	
as	$\forall x\ (Fx \rightarrow Gx)$	("for all *x*, if *x* is an F then *x* is a G")
and	"Some F is not G"	
as	$\exists x\ (Fx \land {\sim}Gx)$	("for some *x*, *x* is both an F and not a G")

The thing is, the universal with an embedded conditional, like the first, can be true if there are no Fs at all; and a negative existential particular like the second presupposes that there *is* an F, but one which is not a G. So we can have vacuous truth for universal propositions, and existential requirements for particulars, negatives included.

Not so on Buridan's system, where affirmative propositions, both particular ("Some F is a G") and universal ("Every F is G") *have existential import*; and where negative propositions, both particular ("Some F is not G") and universal ("No F is G"), do not have existential import, and therefore can be vacuously true.[12] This is a crucial difference between Buridan's logic and its modern classical counterpart, and will become especially important in the discussion of Buridan's modal logic (Chapter 5). I'll clarify this fact wherever it's relevant, but it's worth noting right at the outset.

Propositions

Propositions, in Buridan's logic, are roughly similar to what we now call *sentences*: they exist as particulars in the mind, not as types outside of it. I have, however, stuck with the English translation of "proposition" for the Latin *propositio* throughout. I do this in part because I want to highlight the difference between Buridan's approach and our own modern ways of thinking about propositions, and because alternative translations of *propositio* (e.g. "statement", "assertion") are not obviously any better.

Propositions in Buridan are a funny thing. For starters, they are always asserted—something I discuss in detail in Chapter 2: there is nothing like Fregean assertion being added on (and taken off) in Buridan's semantics. Further still, Buridanian propositions are radically conventional, so that even a barrel hoop, hung outside a tavern, can derivatively be called a proposition, if it is meant to signify the proposition "Wine is sold here!" to passers-by (Buridan's example). Finally, propositions are individual mental acts, and so they are always tokens, never types.

This final point is worth lingering on here, because it produces one of the most significant differences between Buridan's thought and our own. Buridan is a radical anti-realist about universals, and this applies to propositions, too: there are no propositional universals. The existence of Buridanian propositions is, therefore, contingent on their being *actually thought* (or, derivatively, spoken or written). So Buridan has to stipulate that an inference presupposes the existence of the propositions involved in it. Buridan's anti-realism also introduces a whole host of questions related to propositional

[12] This is not to say that Buridan can't express what this latter negative particular does, existential requirements and all. He'll just have to read them as "Some F is a *non*-G". For a discussion of this reading, see Gyula Klima, "Existence and Reference in Medieval Logic", *New Essays in Free Logic*, ed. Alexander Hieke and Edgar Morscher (Dordrecht: Kluwer, 2001), 200-205.

complexity—a modern concern, to be sure, but one that can be thought about in Buridanian terms (as I do below, in Ch. 1, §1.2.4 and Ch. 2, §1.3). For how can we have propositions of arbitrary length, if they have to be thought in order to exist?

In sum, then, we have to bear in mind that Buridanian propositions are token acts, more like dice throws or bike rides, than the abstract propositions of Frege. And we also have to bear in mind that all affirmative propositions, even universal ones, have existential import, whereas all negative propositions, even particular ones, don't.

With these things in mind, I'll conclude this introduction with one of those ubiquitous *in chapter* n, *I show that* φ ... introductory summaries that many readers rightly mostly just skip.

One of Those Mostly-Skippable Chapter-by-Chapter Outlines

Here is a list of the chapters of my thesis. The header sentence (in larger font) sums up the main finding of the chapter, and the following text (in smaller font) gives detail and context.

Chapter 1: What are *Consequentiae?*

Buridan conflates conditionals with inferences, and that's OK.

When Buridan gives his definition of *consequentia* in *Tractatus de Consequentiis* I.3, he claims that conditionals are *consequentiae*, different inasmuch as *if* (*si*) signifies that what follows it is the antecedent, whereas *therefore* (*ergo*) signifies that what follows it is the consequent.

This is all well and good syntactically, but we need more: semantically and pragmatically, we nowadays distinguish conditionals, whose propositional parts are *un*asserted, from inferences, whose parts *are* asserted. This is sometimes called the force-content distinction, and is commonly attributed to Frege. For his part, Buridan can account for the difference without Frege's distinction—and without realism about content. In a slogan: Buridan gets the goal of force-conent, but without the content. Instead, for Buridan all and only propositions are asserted, so the unasserted components of conditionals aren't really propositions at all. So conditionals are different from inferences inasmuch as they don't assert that their parts are true. But they are similar to them in that, unlike *e.g.* conjunctions they are non-commutative: order, in both conditionals and inferences, matters. This is the more general notion of *following* which underwrites Buridan's conflation of conditionals with inferences. Hence we start this chapter with a syntactic notion of consequence, before moving on to a semantic one.

Chapter 2: What Makes *Consequentiae* Valid?

The modal notion that undergirds Buridanian validity is *causal*.

This chapter examines Buridan's semantic account of logical consequence in the *Tractatus de Consequentiis* I.3. There, Buridan sets out three requirements for the relation of entailment. According to him, φ entails ψ just in case:

1. Both φ and ψ are formulated *simultaneously* (the Simultaneous-Formation (SF) Requirement)
2. The truth of φ and ψ is assessed in terms of *signification* (the Signification Requirement), and
3. It *cannot be* that φ is true while ψ is false (the Modal Requirement).

Chapter 2 addresses each of these in turn. The SF-Requirement faces difficulty, because Buridan is a divisibilist about time: for him, time is dense-in-itself, in the sense that no interval is so small that it cannot be further subdivided. But I think there's a solution to the problems faced by Buridan, which doesn't resort to pragmatics, as do some recent accounts of Buridan (especially those of Ernesto Perini-Santos).

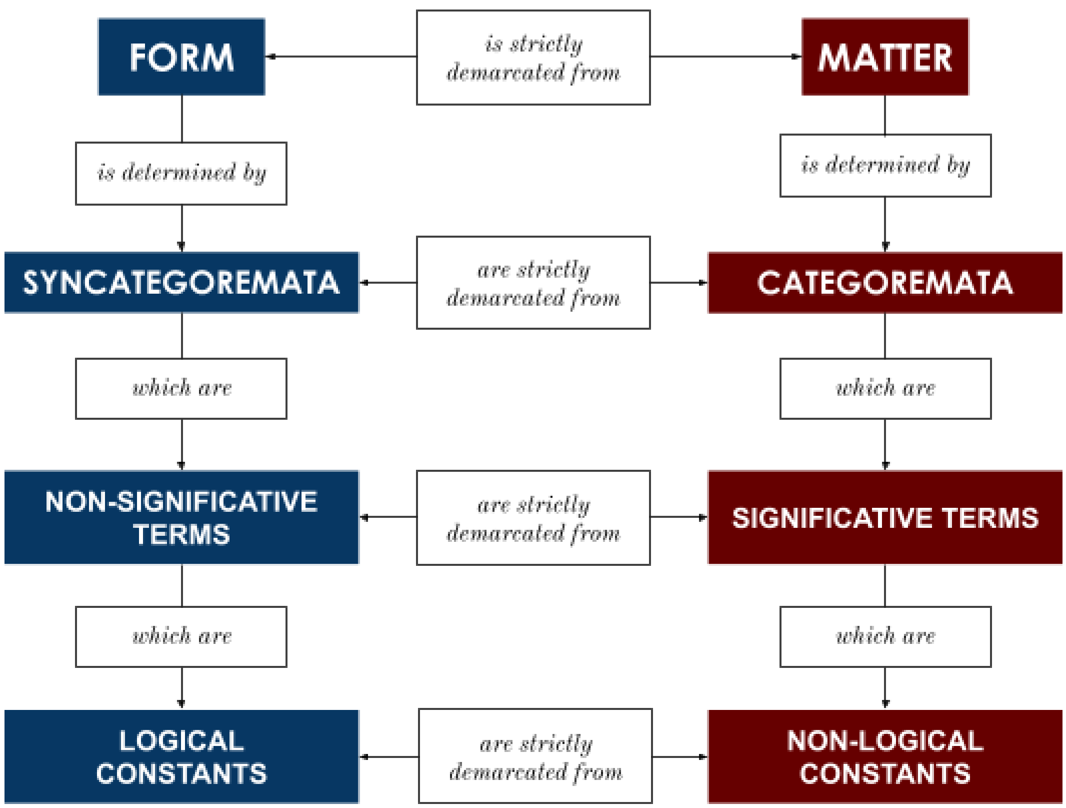
The Signification Requirement allows us to avoid having to endorse invalid consequences on the basis of self-falsifying propositions that are always false but not *necessarily* false. Buridan's example is "No proposition is negative", which describes a way the world could be, but is negative and so self-falsifying. Arthur Prior famously called this the distinction between "The Possibly-True and the Possible", and I adopt this terminology (though I disagree somewhat with Prior). Buridan has also received criticism on his stance on propositional signification, notably from Gyula Klima and David Kaplan, who think it conflicts with his nominalism. But this criticism can be addressed, as I show.

The Modal Requirement is in many ways the most interesting. Buridan has two discussions of propositional modality, which I contrast. The two are reconcilable, at least on the question of logical necessity. In brief: φ entails ψ just in case no causal power can make φ true and ψ false. This metaphysical definition of logical consequence has significant effects downstream, and is quite different from the Tarskian semantic and syntactic definitions. Unlike Tarski, Buridan doesn't define consequence in terms of substitution, and so (i) logical consequence is prior to the division of logical constants from logical variables, and (ii) Buridan can treat both material and formal validity. These two aspects of Buridan's logic are taken up in the next two chapters.

Chapter 3: Types of *Consequentiae* I: Categoremata and Syncategoremata

Syncategoremata and logical constants come (fully) apart.

It is commonly supposed that the medievals had an easier time accounting for the distinction between logical constants and variables (and therefore between logical form and logical matter), because they had a tidy division of terms into stand-alone significative terms (*categoremata* or *categoremes*) and terms that only signify in combination with others (*syncategoremata* or *syncategoremes*).[13] I call this the Clean Divide View, and boil it down to the following:



[13] I use *syncategoreme*, *syncategorema*, and *syncategorematic term* interchangeably; and likewise *categoreme*, *categorema*, and *categorematic term*.

It turns out the above picture is false in just about every way it can be: (i) the demarcations are not strict; (ii) syncategoremes include many terms that are not logical constants; (iii) many logical constants are not (pure) syncategoremes; and (iv) the distinction between logical form and matter is at least sometimes up to convention. So we have misread the syn/categorematic divide, thinking it was a logical form/matter divide, and supposing that it was as important for the medievals as it is for us. But it isn't. Which brings us to:

Chapter 4: Types of *Consequentiae* II: Form and Matter

The form-matter distinction is also conventional.

And it is downstream from the modal notion at play. Since validity depends on causation, Buridan has to treat materially valid *consequentiae*, like

1) A donkey is running
∴ An animal is running

—as just as valid as their formal counterparts, like

2) A donkey is running
Every donkey is an animal
∴ An animal is running.

Thus Buridan's account of logical consequence is very different from the modern account of Tarski, where a well-defined class of logical constants is the lynch-pin of both the semantic and syntactic accounts given by Tarski. This gives Buridan a much broader class of valid arguments, and renders the business of making logic formal an epistemic consideration: (1) is no less valid than (2), but its validity is only *evident* (*evidens*) when it is reduced to (2). In short, the fact that Buridan's logic is downstream from his metaphysics leads him to a view of logical form that is in some respects parallel to Tarski's, but runs in the opposite direction.

There remains the curious business of *ut nunc* consequences—those which hold only 'as of now', like:

3) Gerard is with Buridan
∴ Gerard is on the rue du Fouarre

A consequence like (3) will hold on the assumption that Buridan is also on the rue du F. Modern commentators tend to see *ut nunc* as a sort of derivative class of logical consequences, valid 'by courtesy' only, perhaps akin to the strictly truth-function material implication of *Principia Mathematica*. But Buridan himself calls them necessary. And they are necessary, in a restricted sense, given Buridan's views on the necessity of the present.

Chapter 5: *Consequentiae* in Buridan's Modal Logic

Buridan's modal logic isn't—and can't be—one of possible worlds.

Back in the '80s, G.E. Hughes suggested that Buridan was "implicitly" working with a semantics of possible worlds, which could be teased out in a Kripke-style possible-worlds semantics. At the time, Hughes speculated that Buridan's system most closely resembled the modern system T. At present, the general consensus favours not T but S5.[14] Still, the general claim that Buridan is working with possible worlds, or at least that he can be analysed in terms of them. continues to guide present research projects in Buridan's modal syntax and semantics.

[14] As noted in a survey of the literature by Henrik Lagerlund in "Medieval Theories of the Syllogism", *The Stanford Encyclopedia of Philosophy* (Summer 2021 Edition), ed. Edward N. Zalta, §8.

But Buridan's modal logic is not operating on a semantics of possible worlds, but one of possible *objects*, for which the terms in modal propositions stand. After showing how the extensions of terms can be stretched to include non-existent *possibilia* as well as *actualia*—a process Buridan calls *ampliation*, I turn to Buridan's derived rules. Two of these rules stand out. They are:

> **Rule 3**: no proposition about necessity entails another about actuality
>
> **Rule 4**: affirmative propositions about actuality entail propositions about possibility.

The rationale for Rule 3 is twofold: (i) on Buridan's logic, affirmative assertorics have existential import, and (ii) we can make necessity statements about non-existent *possibilia*. Hence the following argument is invalid, by Rule 3:

1) All dodoes are necessarily birds
∴ All dodoes are (actually existing) birds.

Yet all *actualia* are *possibilia*, and so the following argument *is* valid, by Rule 4:

2) Some donkeys are running
∴ Some donkeys are possibly running.

It turns out that Rule 3 and Rule 4 cannot be captured in any normal modal logic constructed on Kripke frames: we need our accessibility relation R to be irreflexive in order to rule out (1); but we need it to be reflexive in order to get (2). So not only is Buridan *not* dealing with possible worlds semantics. His system is actually *incompatible* with the way we ordinarily construct normal modal logics like T or S5. Hence Buridan is working with something completely different, which is interesting in its own right: first because it provides an alternative to the modern view, and second because it enjoys advantages the modern view doesn't.

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$\therefore$ *this page is not blank*

Chapter 1

What are *Consequentiae*?

Inferences or Conditionals—or Both?

> "Logicians will appreciate what mayhem would be committed
> in translating any modern logical treatise if one failed to distinguish
> between a true conditional proposition and a valid argument."
>
> —Benson Mates[1]

Arguments happen. But whether they work as they purport to is not always clear. It is generally agreed that some do, and some do not. The foundational task of logic, then, is to winnow out the good arguments from the bad. If we're to do this in a principled way, we need a thoroughgoing notion of what it means to follow *logically*. For the medievals, this question boils down to what counts as a valid logical consequence or *consequentia*—from *consequor*, 'to follow'. The present thesis is about the finest treatment of this notion in the Scholastic tradition: that of John Buridan († ca. 1358). The present chapter kicks off the project by asking, first, just what is a *consequentia*?

In modern logic, we typically distinguish conditionals from arguments or inferences, and for good reasons. Contrast for example the following:

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[1] *Stoic Logic* (Berkeley: University of California Press, 1953), 90.

C1) If Beatrix killed Bill, then Bill is dead.

A1) Beatrix killed Bill;

Therefore, Bill is dead.

Clearly, (C1) and (A1) differ: in (A1), the constituent parts ("Beatrix killed Bill", etc.) are asserted as true, whereas in (C1) they are not. This is why, intuitively, these statements say very different things about Bill's life and Beatrix's guilt. Suppose for instance we believe Beatrix is innocent. Even so, we would still readily agree to (C1); whereas if someone put forth (A1), we would demur. To agree to (A1) is to agree, *inter alia*, that Beatrix *in fact* killed Bill. Conversely, to agree with (C1) is not to commit oneself to any such claim.

We accordingly call the relationship between the propositions in a conditional *implication*, which we contrast with *inference* in arguments. This is an important logical distinction, which we ignore at our peril. Accordingly, commentators on Buridan have been justifiably alarmed to find that he seems to conflate conditionals and arguments—and in the passage from the *Tractatus de Consequentiis* in which he gives his *definition* of *consequentiae*, no less.[2] Buridan tells us that:

[2] For instance, Ivan Boh remarks that "medieval logicians disconcertingly use the single notion consequence to cover [conditional implication, entailment, and inference]". See his "Consequences", *The Cambridge History of Later Medieval Philosophy* ed. Norman Kretzmann, Anthony Kenny, and Jan Pinborg. (Cambridge: Cambridge UP, 1982), 300. Peter King argues that *consequentiae* just *are* inferences, and that the apparent conflation of conditionals and inferences is just a matter of occasional sloppiness on Buridan's part. True enough, Buridan is mostly interested in inferences, and more frequently uses language of validity (*consequentia valet/tenet/est bona*) appropriate to inferences than language of truth (*consequentia est vera*) appropriate to conditionals. See King's "Consequence as Inference", *Medieval Formal Logic*, ed. Mikko Yrjönsuuri. (Dordrecht: Springer, 2001), 117-45. And some, like Stephen Read, have opted to translate *consequentia* as 'inference', and then just treat the medieval accounts of things that look like inferences

(C_{Def}) I say that propositions are divided into hypothetical [*hypotheticae*] propositions, and categorical ones. And a consequence is a hypothetical proposition: for it is made up of multiple propositions, joined together by the term [*dictio*] 'if' [*si*] or the term 'therefore' [*ergo*], or an equivalent one. These terms indicate [*designant*] that one of the propositions joined by them follows from the other.[3]

It appears, then, that Buridan's notion of *consequentiae* covers the relations of both inference and implication. In (C_{Def})'s syntactic analysis of *consequentiae*, the terms *if* and *therefore*, and their equivalents, work in the same way: both are syncategorematic terms, which make single hypotheticals out of multiple categoricals.

Thus, *if* and *therefore* bind multiple propositions together to produce a *consequentia*—literally a 'following'. That is, they bind propositions in such a way that their order matters: one proposition is antecedent, the other is consequent. But there is more to say about it than that: in a conditional, the term *if* apparently overrides the assertive force of the conjoined categoricals, whereas in an argument or inference, the term *therefore* does not. Hence the individual propositions of a conditional like (C1) are not

(such as syllogisms and conversions) to the exclusion of conditionals. See Stephen Read, "Inferences", *The Cambridge History of Medieval Philosophy* (Vol. I), ed. Robert Pasnau and Christina van Dyke (Cambridge: Cambridge UP, 2010), 173. I will take up these approaches and claims in further detail in section 3, below, once we have examined Buridan.

[3] "[...] dico quod propositio dividitur in propositionem categoricam et hypotheticam. Consequentia autem est propositio hypothetica; constituta enim est ex pluribus propositionibus coniunctis per hanc dictionem 'si' vel per hanc dictionem 'ergo' aut aequivalentem. Dictae enim dictiones designant quod propositionum per eas coniunctarum una sequitur ad aliam." (*TC* I.3.7-12).

asserted as true, whereas those of an argument like (A1) *are*. This is what we would like Buridan to say here. But he does not.

Instead, when Buridan goes on to discuss the difference between the two terms, he seems to have another distinction altogether in mind. There, he divides them along syntactic lines, not semantic or pragmatic ones:

> the two terms differ in the following respect: the term *if* indicates [*designat*] that the proposition immediately following it is the antecedent, and that the other is the consequent; whereas the term *therefore* indicates [*designat*] the opposite.[4]

This much is clear: the syntactic difference between *if* and *therefore* is that the proposition following the term *if* is the antecedent, whereas that following *therefore* is the consequent. So although these two terms play a similar role in joining propositions as antecedent and consequent, they do so in different orders.

In the *Summulae de Fallaciis*, Buridan discusses this syntactic distinction between *if* and *therefore*, and notes a further difference. He tells us that:

> In a conditional we use the conjunction 'if', whereas in an argument [*argumentum*] we use the conjunction 'therefore'. Further, [...] in a conditional, the conjunction ['if'] is attached to the antecedent, whether the antecedent is placed before or after the consequent, as in
>
> > If a donkey flies, a donkey has wings
>
> and

[4] "Et in hoc differunt quia haec dictio 'si' designat quod propositio sequens eam immediate sit antecedens et alia sit consequens, sed haec dictio 'ergo' designat econverso" (*TC* I.3.12-15).

> A donkey has wings, if a donkey flies.
>
> But in an argument, the conjunction ['therefore'] is attached to the consequent, as in
>
> a human is capable of laughter
>
> therefore an animal is capable of laughter.[5]

This passage adds two noteworthy things to the definition of *consequentiae* (C_{Def}). First, Buridan seems to think that *if* is the syncategorematic term that binds antecedent and consequent together into a conditional, whereas *therefore* binds them into an argument. So at very least, conditionals are identified with the syncategorematic term *if*, and arguments with *therefore*: *if* combines multiple propositions to form a conditional like (C1), whereas *therefore* makes them into an argument like (A1). This is good news. To see why, consider how bad it would be for the present discussion if, for example, Buridan suggested conditionals could be bound by *therefore*. The results would be so disastrous, we might just consider putting the *Summulae* and the *Tractatus* down.

Second, Buridan here distinguishes *if* and *therefore* in terms of the propositions, antecedent and consequent, to which they are bound: *if* binds to the antecedent, *therefore* to the consequent. What Buridan has in mind here (and in *TC* I.3, cited above) is a basic rule for translation of natural language conditionals into their logical form, familiar from any modern introductory textbook in logic. For both "If p then q" and "q, if p" are to be

[5] "Est autem antecedens cui apponitur haec coniunctio 'si' in propositione condicionali et alia est consequens, sed in argumento consequens est cui apponitur haec coniunctio 'ergo' et alia est antecedens [...] in condicionali coniunctio apponitur antecedenti, sive praeponatur antecedens consequenti sive postponatur, ut 'si asinus volat, asinus habet alas' et 'asinus habet alas si asinus uolat', in argumento autem coniunctio apponitur consequenti, ut 'homo est risibilis; ergo animal est risibile'." (*Summulae* 7.4.5).

rendered (p → q), since the sentence immediately following the subordinating conjunction 'if' is to be read as the antecedent of the conditional.

Therefore, of course, does not work this way: it attaches strictly to the consequent of an argument. And there is only one translation of "p ⊢ q" back into a natural language like English—namely as "p, therefore q", never as *"therefore q, p". Since *therefore* binds to the consequent, it always comes after the antecedent in an argument.

All this is well and good. At very least, *if* and *therefore* are here divided according to their different syntactic behaviour. And it is heartening that, in the above passage, Buridan does not conflate conditionals and arguments: he tells us that *if* is the characteristic particle of a conditional proposition, and *therefore* of an argument. So it seems reasonable to conclude he holds that conditionals and arguments are in *some* way different.

But this syntactic distinction is not enough: we would like Buridan to tell us more about the difference between *if* and *therefore* in terms of the semantic and pragmatic differences between the propositions used in conditionals and arguments, respectively. More specifically, we would like to know whether Buridan has a notion of the force-content distinction, which we now typically associate with Frege (and sometimes even call the *Frege point*).[6] On this distinction, the antecedent (and consequent) propositions of arguments like (A1) and their corresponding conditionals like (C1) have the same content, but they differ in force: in (A1) the antecedent and consequent are

[6] This coinage is due to Peter Geach. See his "Assertion", *Logic Matters* (Basil: Blackwell, 1972), 255, and the discussion below (esp. §2.2).

asserted, but in (C1) they are not. Frege proposes to solve the problem with this distinction.

In a nutshell, here is how the Fregean solution works: sentences can express the same content in very different ways. Consider for instance the following examples, which Frege gives at the outset of his *Begriffschrift* (§ I.3):

P1) At Platea the Greeks defeated the Persians.[7]

P2) At Platea the Persians were defeated by the Greeks.[8]

These two sentences have the same content, as does the following question:

P3) Did the Greeks defeat the Persians at Platea?

And the following optative:

P4) If only the Greeks defeated the Persians at Platea!

In virtue of their reference, (P1)-(P4) have the same content: whether or not they are asserted as true is wholly removed from the content they express. Thus (P1) and (P2) are asserted, (P3) and (P4) are not. In grasping the content of (P1)-(P4), we are merely apprehending something, not necessarily judging that it is true. Thus a sentence's content

[7] "Bei Plataeae siegten die Griechen über die Perser." Frege. *Begriffschrift: Eine der arithmetischen nachgebildete Formelsprach des reinen Denkens*. Haale: Verlag von Luis Nebert, 1879.
[8] "Bei Plataeae wurden die Perser von den Griechen besiegt" (*ibid.*)

in no way presupposes anything about assertive force (though force presupposes content: as Eike-Henner Kluge succinctly puts it, "I cannot judge what I cannot grasp").[9]

Frege's generalises his observation in the *Begriffschrift* about optatives, interrogatives, etc. like (P1)-(P4), so theat they extend to the constituent parts of conditionals and arguments as well. In his later essay "Negation" ("Die Verneinung"; 1918-19), Frege offers the following analysis of a conditional:

> The thought that the following sentence contains
>
> > 'If the accused was at Rome at the time of the deed, he did not commit the murder'
>
> may be taken as true by someone who does not know whether the accused was at Rome at the time of the deed, nor if he committed the murder. Of the two component thoughts contained in the whole, neither the antecedent nor the consequent is being uttered assertively when the whole judgment is being presented as true.[10]

Accordingly, for the Fregeans, the sentences that make up a conditional like (C1) above, and the sentences that make up a corresponding argument like (A1), will have the same *content*, but differ in *force*: in (A1) the sentences have assertive force superadded to them, whereas the constituent sentences of (C1) do not.

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[9] Eike-Henner Kluge, *The Metaphysics of Gottlob Frege* (Boston: Martinus Nijhoff, 1980), 203.

[10] "Der in dem Satze 'Wenn der Angeklagte zur Zeit der Tat in Rom gewesen ist, hat er den Mord nicht begangen' enthaltene Gedanke kann als wahr anerkannt werden von einem, der nicht weiß, ob der Angeklagte zur Zeit der Tat in Rom gewesen ist und ob er den Mord begangen hat. Von den beiden in den Ganzen enthaltenen Teilgedanken wird weder die Bedingung noch die Folge mit behauptender Kraft ausgesprochen, wenn das Ganze als wahr hingestellt wird."
Frege, Gottlob, "Die Verneinung", *Logische Untersuchungen*, ed. Gunther Patzig (Gottingen: Vandenhoeck & Ruprecht, 2003 [1966]), 66. Frege, "Assertion", *Gottlob Frege: Collected Papers on Mathematics, Logic, and Philosophy*, ed. Brian McGuinness, trans. Max Black, V.H. Dudman, Peter Geach, Hans Kaal, E.-H. W. Kluge, Brian McGuinness, and R.H. Stoothoff (Basil: Blackwell, 1984), 375.

This is how the problem gets solved in Frege's philosophy of language. But there is a price to pay, and that price is ontological extravagance. It depends upon what, at least to the Buridanian, looks like profligate realism about propositional content: over and above the sentences we are thinking or uttering, there is the higher level of *content*, that is real and independent of them. For Buridan a proposition *just does not exist* unless it exists in thought (a fact I mentioned in the introduction and take up in detail in the next chapter). For Frege, thought or spoken sentences express propositions, which latter do not depend on the former for their existence. Fregean propositions are real abstract objects. But this imposition of another layer of propositional *contents*, which do not depend for their survival on anyone thinking or speaking them, is too high a price to pay—at least for the Buridanian. To adopt Frege's propositional semantics would be to abandon Buridan's entire, strictly nominalist metaphysical project. And we ain't gonna give up ground so easily.

So, in what follows, I put forth Buridan's solution as an alternative to the Fregean one. As we will soon see, Buridan is well aware of the problem that motivated Frege's force-content distinction. Buridan's solution, however, is significantly different from Frege's. Most importantly, it does not depend on realism about such abstract entities as propositional contents in order to get the job done. Briefly put: Buridan does the work of Frege's force-content, but without the content.

To see how this works, let's build our way up: let's see just what hypothetical propositions *are* in Buridan's account.

1. What are *Propositiones Hypotheticae*?

For Buridan, hypothetical propositions (*propositiones hypotheticae*) comprise a rather large array of propositions, including conjunctions and disjunctions, as well as arguments (*argumenta*) and conditionals (*propositiones conditionales*). What all such propositions have in common is that they contain multiple categorical propositions—or, to be precise, they contain multiple expressions equiform with what *would* be propositions if they were set forth on their own, as we will see. So to see what a hypothetical proposition is, we first have to define what a categorical proposition is, so we can see what it means to contain several such. Along the way, we'll see how assertion works in Buridan, the better to understand how he deals with the problem posed by the 'Frege point'.

1.1. What Categoricals Are

In this section of the present chapter, I set out a definition of the most basic proposition in Buridan's logic (and of Aristotelian logic in general): categoricals.

The *genus* to which categorical propositions belong is *expression* (*oratio*). An expression is an utterance made up of multiple parts that are significative independent of each other (*separatae*).[11] Expressions are thus distinguished from stand-alone terms like nouns and verbs, which cannot be boiled down to more basic significative parts: to take Buridan's example, the noun *Dominus* (Lord) denotes only one thing: it is not made of

[11] "Oratio est vox significativa ad placitum cuius partes separatae aliquid significant" (*Summulae* 1.2.3).

separately significative parts. If we were to break it down further, the individual parts would not mean anything—or by chance they might mean something so different that they cannot be called its constituents, as *Do* (“I give”) and *Minus* (“Less”) show in the case of *Dominus*. An expression, on the other hand, has separately significative proper parts: for example, “Running Socrates” or “There is a sound in the clouds” have parts that clearly retain signification, even when taken on their own.[12]

The *differentia*, which sets propositions apart from other expressions, is that propositions are true or false.[13] What makes them capable of being true or false is that they make an assertion. And what makes them assertive is their inclusion of a *copula*: a predicative verb like ‘is’ or ‘are’ as their main syncategorematic part (rather than, say, a sign of disjunction; much much more on the copula in Chapter 3, §2.1.1, and on scope in §1.2 of this chapter). A categorical proposition thus comprises at least the following three parts: a subject and a predicate, as well as a copula.[14] Categoricals’ inclusion of this latter part distinguishes them syntactically and semantically from combinations where the subject and predicate are not separated by a copula: the so-called unseparated combinations (*complexiones indistantes*)—*un*separated because their terms have no mediating copula. These accordingly “produce an incomplete sense in the mind of one who hears them”, since they do not say anything true or false.[15]

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[12] What about compound nouns? Buridan considers the example of *paterfamilias*, which he concludes is an expression (*oratio*) to the logician, and a noun (*nomen*) to the grammarian (*Summulae* 1.2.1). In short, then, classification depends on context. Since our context is logical, we’ll follow Buridan in treating all compound nouns—from *airplane* to *Vergangenheitsbewältigung*—as expressions.
[13] *Summulae* 1.3.1.
[14] *TC* I.8.412-3: “propositio categorica debet habere subiectum et praedicatum et copulam”. Cf. what Buridan says in the *Summulae de Propositionibus*: “ipsa [propositio categorica] est cuius partes principales sunt subiectum et predicatum” (*Summulae* 1.3.2).
[15] “imperfectum sensum generat in animo auditoris” (*Summulae* 1.2.3). Cf. *In Aristotelis* De Anima *Expositio et Quaestiones* III.15.

For example, contrast the following unseparated combination (UC1) with its propositional cousin, (P5):

UC1) A white horse

P5) A horse *is* white

Because it does not contain a verbal component, (UC1) does not make an assertion. It is, in grammatical terms, a sentence fragment, apt to serve as a subject, predicate or subordinate clause, but not as a stand-alone proposition. On the other hand, (P5) *does* say something complete, and therefore, unlike (UC1), is apt to be true or false. For this reason, it is a proposition.[16]

Yet not all categorical propositions take this subject-copula-predicate form (which I'll henceforth call SCP-form). Or, to be more precise, not all propositions bear the SCP-form on their sleeves. There are lots of full-fledged propositions that contain only two components, or (in certain languages, including Latin), only one. Here are some examples from Buridan:

P6) Somebody runs (*homo currit*).[17]

P7) It's thundering (*tonat*).[18]

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[16] "propositio est oratio verum vel falsum significans" (*Summulae* 1.3.1).
[17] *Summulae* 1.3.2.
[18] *TC* I.8.413-15. Stephen Read, in his translation of Buridan's *TC*, translates this as "Thunder" (p.85). The English noun "Thunder" (which corresponds to the Latin *tonitruum*), however, has no propositional content in English (or in Latin). Unfortunately, there is no way to exactly mirror the single-term Latin proposition expressed by *tonat* in English, so I have opted for a full proposition, admittedly with multiple terms.

In such cases, the SCP form is implicit: for (P6), we can make the form explicit by dividing the verb (*currit*) into a copula (*est*, "is") and the verb's participial form (*currens*, "running"), to get "is running" (*est currens*). Things are similar with propositions like (P7), which Buridan calls "propositions implicit in a single verb".[19] Thus we can cash (P7) in for the equivalent proposition "A sound is made in the clouds" (*sonus factus est in nube*), which has explicit SCP-form.[20]

Buridan calls this "resolving [propositions] so that the subject, predicate, and copula are explicit."[21] Similarly, we now ~~indoctrinate~~ teach students to analyse universal affirmatives as conditional propositions in the scope of a universal quantifier, so that

P8) Every dalmatian is a dog

is to be translated and symbolised as

P8′) For every x, if x is a dalmatian, then x is a dog.

$\forall x(\text{Dalmatian}(x) \rightarrow \text{Dog}(x))$

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[19] Propositiones "in quibus totum est in uno verbo implicitum" (*TC* I.8.567-8).
[20] *TC* I.8.570-1.
[21] "Curemus [...] quomodo sint solvendae [propositiones] ut explicite appareant subiectum, praedicatum, et copula" (*TC* I.8.564-6).

It seems reasonable to assume that, in Buridan's time, students would likewise be expected to render implicit categoricals in SCP-form, by recognising and making explicit their implied subjects, copulae, and predicates.

Thus, categoricals are defined as propositions containing a single predication of a one subject by one predicate, through the mediation of one copula—or, at least, propositions which implicitly contain these elements, and are therefore reducible to SCP-form. Now if categoricals are distinguished by their composition of one subject, copula and predicate, and if hypotheticals are just propositions that contain multiple categoricals, then we might be tempted to define hypothetical propositions as those which contain two (or more) subjects, copulae and predicates, and call it a day. This approach would make distinguishing the two classes of propositions straightforward and mechanical. But things are not so simple.

1.2. What Hypotheticals Are

According to Buridan, there are many propositions that have multiple subjects, copulae and predicates, and that even contain syncategorematic terms associated with hypotheticals (*if*, *or*, and the like), but that nevertheless remain categorical. And even more surprisingly, the apparent categoricals that make up a hypothetical proposition aren't really categoricals at all: Buridan repeatedly states that hypotheticals *do not* contain other propositions, but that their proposition-like parts are equiform with what *would* be propositions if they were put forth (*propositae*) on their own.

So propositional complexity is no guarantee of hypothetical-hood; and, in any case, hypotheticals aren't made up of multiple propositions. Let's look at each of these claims in turn.

1.2.1. What Hypotheticals Aren't

As Buridan notes, we cannot mechanically divide categoricals and hypotheticals by the number of subjects, predicates and copulae they contain. Many propositions with multiple such terms remain categorical, though they are not obviously of SCP-form. Buridan gives the following examples:

P9) Someone who is pale is coloured.[22]

P10) A donkey, if it flies, has wings.[23]

P11) The one reading and disputing is a master or a bachelor.[24]

Buridan gives instructions for cashing out propositions like (P9)-(P11) in SCP-form. (I have added braces around the subjects and predicates to make these easier to read):

P9′)【Someone who is pale】is 【coloured】

[22] "Homo qui est albus est coloratus" (*Summulae* 1.3.2, 1.7.1).
[23] "Asinus, si volat, habet pennas" (*Summulae* 1.7.1)
[24] "Legens et disputans est magister vel baccalaureus" (*Summulae* 1.3.2).

P10′) 【A donkey, if it flies】 is 【a thing-having-wings】

P11′) 【The one lecturing and disputing】is【a master or bachelor】

A little reworking, then, is enough to bring these troublesome propositions into SCP-form, and thereby to show that they are categoricals in disguise. Still, we need a thoroughgoing definition of categoricity for propositions: we need to know *why* the transformation of (P9)-(P11) into (P9′)-(P11′) is warranted.

In section of the *Summulae de Propositionibus* in which the division of propositions into categoricals and hypotheticals is presented (1.3.2), Buridan tells us that the correct way to define categorical propositions is as follows:

CP_{Def}) A categorical proposition is one that has a subject, predicate, and copula as its *principal* parts (*partes principales*).[25]

At least (P11), then, counts as categorical because it has only one copula. But (P10) is a little more difficult, supposing we cash out the verb 'flies' as 'is a flying thing', the way we did with runs (*currit*) in (P6), above. And (P9) just has *two* copulae. But what makes (P9) and (P10) categorical is that there is only *one predicate predicated of one*

[25] "Categorica est illa quae habet subiectum et praedicatum et copulam principales partes sui" (1.3.2; van der Lecq, p. 29).

Note that van der Lecq puts "...subiectum et praedicatum *tamquam* principales...", but lists "...et copulam..." as a variant. I follow the translation of Gyula Klima (and the light of natural reason) in taking this variant as the primary meaning. If it weren't, Buridan would be giving the parts of a *complexio indistans* as the sufficient constituents of a proposition. And that would be absurd.

subject—that is, there is only one predicate with proposition-wide scope. Likewise (P11), in which:

> the whole phrase 'the one lecturing or disputing' is a single subject, although hypothetical, namely, conjunctive, and the whole phrase 'master or a bachelor' is in the same way a single predicate, although disjunctive.[26]

Conversely, a hypothetical proposition contains multiple predicates, but none of them is predicated of the whole—that is, none of them is a principal part (*pars principalis*):

> HP_{Def}) A hypothetical proposition is one that has several subjects, several predicates, and several copulae, but none of these is predicated of the rest by means of a single copula.[27]

Thus, since in (P9) there is just one predicate—namely "coloured"—predicated of the whole, it is a single categorical proposition—that is, as a single predication. The case is similar with (P10), which likewise has just one predicate: "a thing-having-wings", predicated by the mediation of a single copula, of the subject "A donkey, if it flies".

So much, then, for what hypotheticals are not. From the definition of hypothetical propositions (HP_{Def}), we can surmise that the principal part of a hypothetical proposition is something other than a single copula. It is, rather, a syncategorematic term like *if*,

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[26] "Hoc totum 'legens et disputans' est unum subiectum, licet hypotheticum puta copulativum, et hoc totum scilicet 'magister vel baccalaureus' est unicum praedicatum consimiliter, licet sit disiunctum" (*Summulae* 1.3.2; van der Lecq, p.32, ll.1-4)

[27] "propositio hypothetica est propositio habens plura subiecta et plura praedicata et plures copulas, cuius nullum est praedicatum dictum mediante una copula de totali resuduo" (*Summulae* 1.7.1).

therefore, *and*, or *or*, which is the most principal part—in more modern terms, it is the particle which takes proposition-wide scope.[28]

Importantly, the term *if* overrides the propositional force of the individual categorical propositions involved in the hypothetical, so that technically speaking these categorical-like expressions are not really propositions at all. To see why, recall that the *differentia* of the species *proposition* from the genus *expression* is that propositions say something true or false, and so they are distinguished from unseparated combinations (*complexiones indistantes*) by the fact that they assert something. But the assertive force of the copulae in the antecedent and consequent of a conditional is overridden by the main connective, so that they are not asserted. Accordingly, since the antecedent and the consequent of a conditional do not, as parts of a conditional, say something true or false, these would-be propositions are not propositions at all. We will see how this works in the following section.

Thus Buridan tells us that "a hypothetical proposition properly speaking does not contain several propositions".[29] This presents a promising solution to the problem that sent us down this path in the first place—namely, the apparent conflation of conditionals with arguments in Buridan's logic: if the constituent parts of conditionals are not propositions at all, they lack propositional force. So Buridan *does* have a framework on which to accommodate the 'Frege point'. This is the idea I am going to pursue. In the next section, I examine how these proposition-like expressions come to make up a hypothetical proposition.

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[28] As Peter King remarks (in "Consequence", 119), this is "a medieval version of our notion of the connective of widest scope".
[29] "loquendo proprie in propositio hypothetica non continentur plures propositiones" (*Summuae* 1.7.1).

1.2.2. What are Hypotheticals Made From?

It is puzzling that a true conditional proposition can comprise what look like false categorical ones. Take for instance the following:

P12) God does not exist or humans are animals.[30]

P13) If a donkey flies, then it has wings.[31]

The two propositions related by (P13)—namely, "a donkey flies" and "a donkey has wings"—are, of course, false, as is the first disjunct of (P12). But this does not render the whole proposition false in either case. Rather, (P12) and (P13) are both true, in spite of their false constituent parts.

Buridan finds it perplexing that any true proposition, in particular a true conditional, should be made up of false parts. For instance, in the course of a discussion brought up by (P12), he tells us "it would be absurd to say that a true proposition has false principal parts."[32] And when he considers conditionals like (P13), he expresses his amazement at this apparent feature of hypotheticals in no uncertain terms:

[30] "Deus non est vel homo est anima;" (*Summulae* 1.7.1).
[31] "Si asinus volat, asinus habet pennas" (*Summulae* 1.7.1).
[32] "Tamen absurdum esset dicere quod propositio vera haberet partes suas principales falsas" (*Summulae* 1.7.1).

> It is rather amazing [*bene mirabile*] that a true proposition has two false main parts, and no true parts at all, and that someone saying it cannot be accused of uttering a falsehood and of lying.[33]

The solution, according to Buridan, is to say that the constituent parts of hypothetical propositions aren't really propositions at all—though they *would* be were they formulated on their own:

> When it is said that a hypothetical proposition is one that contains two categorical propositions, this is not strictly speaking true But it is true in the sense that a hypothetical proposition contains two predicates, two subjects and two copulae, and that each of these predicates is predicated of one subject by one copula. But the aggregate of one predicate, one subject, and one copula is not a proposition, but part of one—though such an utterance [*vox*], were it taken on its own, would indeed be a categorical proposition.[34]

Thus the constituent parts of a hypothetical are not propositions at all, though they look for all the world like them: they contain all the formal parts characteristic of propositions, and yet they do not assert anything in the context of a hypothetical.[35]

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[33] "est bene mirabile quod aliqua propositio est vera cuius duae partes principales sunt falsae et quae nullam partem habet veram, et quod dicens eam non potest argui de falsitate neque de mendacio" (*Summulae* 1.7.3).

[34] "quando dicitur 'propositio hypothetica est quae habet duas categoricas', hoc proprie loquendo non est verum, sed ad istum sensum quod propositio hypothetica continet duo praedicata et duo subiecta et duas copulas, et quod utrumque predicatorum mediante una illarum copularum dicitur de undo illorum subiectorum sed aggregatum ex uno subiecto et uno praedicato et sua copula non est una propositio, sed pars unius propositionis, licet talis vox, si esset separatim sumpta, esset bene una categorica." (*Summulae* 1.7.1)

[35] The same is true of any utterance that does not make an assertion, *e.g.* a question, an imperative, or an optative. Hence for Buridan, Frege's phrase "propositional question" is a contradiction in terms. (For use of this term in Frege, see his "Negation", 373).

In a hypothetical, there is no main copula with proposition-wide scope. Rather, the principal part of a hypothetical is a syncategorematic term like *if*. This principal part does not interfere with the predicative power of the copulae in the constituent 'propositions': the predications going on in them are real, and so they are not non-predicative aggregations of terms like their so-called unseparated combinations (*complexiones indistantes*). What this principal part does is deprive them of their assertive force, so that they are not really propositions at all. Therefore, the problem presented by true hypothetical propositions—like (P12) and (P13)—with false constituent parts is only an apparent one, since the parts themselves are not really propositions at all, but only *apparent* propositions.

By now, it is clear that Buridan has the stuff to build a pretty seaworthy distinction parallel to Frege's disctinction of force and content. Motivated by the worry that true hypothetical propositions can be made up entirely of false parts, he makes two moves. First, he claims that certain syncategorematic terms, when they are serving as the main connective (*pars princeps*) of a proposition, rendering it hypothetical, themselves override the proposition-hood of the constituent parts of that hypothetical. And, second, since all and only propositions are asserted, those constituent parts are neither asserted nor propositional—though they *are* predicative.

But before we get to the force-content distinction, there is a final puzzle about hypotheticals and their parts that needs working out. Some hypothetical propositions, like conjunctions, *do* assert both their parts. Are the constituent parts of conjunctions also not propositions? And if so, why doesn't their propositional assertiveness up and vanish, the way it does in disjunctions and conditionals?

1.2.3. Conjunctions and their Conjuncts

The problems of assertion introduced by propositions like (P12) and (P13) are unique to *if* and *or*, and it would be wrong to paint all syncategorematic terms with the same brush. After all, conjunctions depend for their truth on the truth of (all of) their parts: if we were to swap out *and* for *or* in the true proposition (P12), we would get the following false proposition:

P12′) God does not exist *and* humans are animals.

The problem is that, although it is enough for the falsity of conjunctive propositions that *one* of their conjuncts be false, we seem to be in a difficult position to explain this falsity, given the fact that we have already deprived them of their proposition-hood, so that they can't be false (or true) at all.

Briefly put: we have solved the problem of true disjunctive and conditional proposition with false parts, by denying that the parts themselves are propositions at all. But by so doing, we're in a difficult spot to account for the falsehood of conjunctive hypotheticals with false parts. How do the false parts render them false, if they are not really propositions, and so not eligible for falsehood?

We might put this slightly differently, in terms of the logical schema of simplification, namely:

S1) $\varphi \wedge \psi$

$\therefore$ φ

Both in theory and in practice, Buridan endorses all the usual intuitions about conjunctions and schemata like (S1): he tells us that a conjunctive proposition contains two or more categoricals; that its truth requires that all of them be true; and that for its falsity it is enough that one of them be false.[36] So Buridanian conjunction is the familiar sort.

But how Buridan can *account* for conjunction and schemata like (S1) is an altogether different question, and a puzzling one.[37] How can we simplify conjunctions, when this involves pulling propositions out of hypotheticals that are, in the context of the hypothetical, not propositions *per se*? What kind of operation is going on here? Is Buridan even *entitled* to use this language of truth and falsity to discuss parts of a hypothetical which are, themselves, not categorical propositions at all?

[36] "requiritur ad veritatem eius quod omnes essent verae, et sufficeret ad falsitatem eius quod una esset falsa" (*Summulae* 1.7.4).

[37] And, to be sure, Buridan *does* endorse simplification: though it is not one of his axioms he sets out in *TC* I.8, he endorses it ("ad copulativam sequitur quaelibet eius pars") and makes use of it in his proof that from a formal contradiction, anything follows formally (*TC* I.8.7.167-181). Here is a symbolic version of his proof (simplification appears on lines 2 and 4):

1. $p \wedge \sim p$ (assumption)
2. p (1, simplification)
3. $p \vee q$ (2, addition)
4. $\sim p$ (1, simplification)
5. q (3, 4 modus tollendo ponens; QED)

I can think of three possible solutions: we could (i) admit that things other than propositions can be true. Or (ii) we could admit that some hypotheticals contain propositions, and others do not. That is, we could hold that only the apparent categoricals in disjunctive (conditional, etc.) propositions lose their proposition-hood, whereas those in conjunctive propositions remain full-fledged propositions. This approach is tantamount to distinguishing two types of hypotheticals: those containing multiple full-fledged propositions, and those containing multiple proposition-like expressions. Or (iii) we could accept that the apparent categorical propositions involved in conjunctions are not really propositions at all, and so are not true (or false); rather, the whole hypothetical says something about its constituent parts. Conjunctions, on this final approach, do not have assertive parts, though they assert *that* their parts are true.

The worst of these is (i), since it forces us to abandon the *differentia* for propositions: it is, after all, their characteristic feature of making assertions that separates them from other expressions (*orationes*), as we saw at the outset of this chapter. Recall that

P5) A horse *is* white

makes an assertion, whereas unseparated combinations (*complexiones indistantes*) like

UC1) A white horse

do not. Conflating the two, as (i) would have us do, is too high a price to pay. And there is no evidence for this approach in Buridan, who keeps the two strictly separate.

Things are not much better with (ii): Buridan is adamant that conjunctions are single hypothetical propositions, which can be characterised in terms of necessary (or possible) truth (or falsehood) independent of their constituent parts. Buridan is clear about this in his discussion, in the *Tractatus de Consequentiis*, of the rule that no impossible proposition follows from a possible one.[38] There, he considers the following apparent counterexample to this rule: a syllogism with possible premises, but an impossible conclusion:

A2)	Everything running is a horse	(possible)
	Every human is running	(possible)
∴	Every human is a horse.[39]	(impossible)

The solution, says Buridan, is that there is really only *one* antecedent in this syllogism: a conjunction of the two antecedent premisses: “Everything running is a horse and every human is running.”[40] Since this copulative proposition is impossible, it is no problem that

[38] “Quinta conclusio est: Impossibile est [...] ex possibili [sequi] impossibile” (*TC* I.8.96-7).

[39] “Tamen contra hanc quintam conclusionem obicitur sophistice. Qua haec est bona consequentia syllogistica:

	Omne currens est equus
	Omne homo est currens
Ergo	Omnis homo est equus.

Et tamen utraque premissarum est possibilis, cum conclusio sit impossibilis.” (*TC* I.8.130-3).

[40] “Solutio: Neutra illarum premissarum est totale antecedens ad dictam conclusionem. Immo antecedens est una copulativa ex illis duabus praemissis composita, scilicet haec ‘omne currens est equus et omnis homo est currens’.” (*TC* I.8.134-7). Cf. *QQ in An Pr* I.30: “in syllogismo neutra premissarum est totale antecedens, immo totale antecedens est una copulativa constituta ex duabus praemissis”.

it leads to an impossible conclusion: the apparent move from possible to impossible in (A2) is therefore an illusion.

The details of this method need not detain us here. What's at stake for present purposes is that the proposed solution (ii) leaves the conjuncts of conjunctive propositions intact: unlike the disjuncts of a disjunctive proposition, they retain their proposition-hood. But if that were so, we would not be in a position to discuss changes in their truth conditions in the context of a conjunction—changes like the one we see when we treat the antecedents of (A2) as a single conjunctive proposition. But that is precisely what we need for Buridan's solution here, where two possible propositions become an impossible one, because of their mutual incompossibility. If we leave the propositions in a conjunction independent, as solution (ii) suggests, we are in a bad position to account for this.[41]

This leaves us with (iii): we just admit that all our talk about the truth of the categoricals that make up a hypothetical is a sort of shorthand, as it is with the apparent categoricals in a disjunction or conditional. On this approach, even the conjuncts of a conjunctive proposition are not strictly speaking stand-alone categoricals, nor are they the sorts of things that can be true while embedded in a hypothetical proposition. The conjuncts of a conjunction themselves are not propositions, though they *would be* if they were formulated on their own. The apparent assertive force of the individual propositions in a conjunctive proposition is really just a matter of the semantics of the conjunctive particle (the *and* or *but* or equivalent). The conjunction, then, does not have individual assertive (propositional) parts, but instead asserts *that* its parts are true—or would be if

[41] And don't get me started on what will happen to the so-called De Morgan Laws, whereby *e.g.* a conjunction like $(\sim\varphi \land \sim\psi)$ is equivalent to a disjunction $\sim(\varphi \lor \psi)$.

they were formulated on their own. A conjunction, then, is not its parts, but it is *about* them.

This solution seems well within the bounds of Buridan's logic. And even Geach can't complain here, either, since he considers a very similar proposal for the treatment of conjunctions. Although he ultimately rejects it, he explicitly bases his rejection on arbitrary preference. Turning from *if* to *and*, Geach asks:

> Would he [sc. Frege] say 'and' meant something different in an asserted conjunctive proposition? Probably he would say in that case that the assertoric force attached not to 'and' but to the clauses it joined. *Such a position, however, is clearly arbitrary.*[42]

Notice, however, that there is an ambiguity here: at what level does the assertion take place? At the level of the (constituent) propositions, or at the level of the particle joining them? Geach apparently prefers the former, but for the Buridanian the answer will have to be the latter: the assertion of a conjunction is an assertion *about* its constituent would-be propositions. Therefore, the place to look for the 'force' of the utterance is at the level of the main logical operator, in this case the conjunction.

In any case, the Buridanian needn't worry about what goes on in the logical operation of simplification (*i.e.* schemata like S1, above) on this approach. It is both textually and rationally defensible. Textually since, as I mentioned, Buridan uses simplification throughout his philosophical *oeuvre*. For example, in the *Tractatus de Consequentiis* (I.8, 7th conclusion) he states in passing that "from a conjunction, each of

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[42] Geach, *Reference*, 51 (emphasis added).

its parts follows".[43] He immediately makes use of it in his proof that $(\varphi \wedge {\sim}\varphi)$ formally entails χ.[44] And he catalogues and later endorses it in the *Summulae de Locis Dialecticis* (6.4.18):

> Some say there are other loci from the concomitants of substance, namely, one that goes from the conjunctive whole to its part [...] for example, "Socrates runs and Plato disputes; therefore, Socrates runs".[45]

So there can be no doubt that Buridan endorses the propositional inference rule we now call *simplification*.

Simplification is also rationally defensible on a Buridanian framework. Just because the antecedent contains no categoricals, strictly speaking, doesn't mean it can't *entail* a categorical, as it does in the example from the *Summulae de Locis Dialecticis* that Buridan considers here. For, when we simplify a conjunction, we 'cash out' one of the conjuncts by forming a proposition equiform with it. But this is not just pulling a conjunct out of a pair or list of categoricals. It is, rather, a matter of forming a stand-alone proposition equiform with one of the conjuncts of the antecedent. Hence the modern formalisation of (S1) is, considered from a Buridanian standpoint, slightly misleading. For if we write it out as follows, we beg the question:

S1) $\varphi \wedge \psi$

$\therefore$ φ

[43] "Ad copulativam sequitur quaelibet eius pars" (*TC* I.8, 7th conclusion; ll.172-3).
[44] See note 43, above.
[45] "Aliqui dicunt alios esse locos a concomitantibus substantiam, scilicet a toto copulato ad suam partem [...] ut Socrates currit et Plato disputat
$\therefore$ Socrates currit" (6.4.18; Green-Pedersen, p.83)

Why is this circular? Because in a symbolisation like (S1), the consequent (φ) is exactly the same as it is in the antecedent. But the Buridanian answer does not allow that the embedded expression equiform with φ, but embedded in the antecedent, is itself the same as the φ in the consequent. A Buridanian notation would, therefore, have to distinguish the non-propositional embedded expression φ in the antecedent from its stand-alone propositional cousin in the consequent. Hence the Fregean notation presupposes a Fregean account of what's going on in the logical operation of simplification (or *modus ponens*, or any other logical operation, for that matter): in any such operation, a proposition is just being 'pulled out' of a hypothetical.

Now I am not sure this is a much of a criticism of Frege: after all, our notation(s) should be allowed to presuppose at least some things about our logic(s). But it *is* a salubrious reminder of how theoretically freighted a symbolic notation can be (for all its apparent abstractness and content-neutrality). And it is one of a thousand reasons why we should develop our own notation when working with Buridan, rather than adopting or even adapting the Fregeans'. I'll take up some of the remaining thousand-minus-one reasons in Chapter 5.

The foregoing observations thus give us a clearer picture of what, according to the Buridanian, goes on when we perform the logical operation of simplification: a proposition is not 'pulled out' of a hypothetical. Rather, a new one is formulated. In fact, this account of the logical operation of simplification seems to fall out of the anti-realist requirement—to be considered in Chapter 2, §2 of the present dissertation—that the antecedents and consequents of a valid *consequentia* be formulated at the same time (*simul formatae*). A conjunction may entail each of its conjuncts, but there is no actual inference unless we form it ourselves. This is what we do when, from φ and ψ, we infer φ. So *entailment* of a proposition does not presuppose *containment* of it: a conjunction need not contain its conjuncts *qua* propositions in order to entail them.

This metalinguistic solution faces two important problems, however. First, if a conditional is metalinguistic, then it makes claims about specific linguistic items that are bound to their language. But if this is the correct analysis, it should pass Church's Translation Test—that is, it should allow translation from one language to another.[46] But, if we follow Buridan, a translation of a conditional will really be a translation of statements about the corresponding statements in the original language. For instance, consider again the following conditional:

P13) If a donkey flies, a donkey has wings.

This is a translation of the following proposition in Buridan:

P13′) *si asinus volat, asinus habet pennas*.[47]

But if (P13′) is metalinguistic, the way I have suggested, its translation doesn't correspond to (P13) at all, but rather to (P13″):

[46] See Alonso Church, "On Carnap's Analysis of Sentences of Assertion and Belief", *Analysis* 10 (1950): 97-99.
[47] *Summulae* 1.7.1.

P13″) If the Latin phrase corresponding to the English ‘a donkey flies’ is true, then so is the (again, Latin) phrase corresponding to ‘a donkey has wings’).

The metalinguistic view of hypotheticals, which I've here set out, seems to commit us to (P13″) as the correct translation of (P13′), rather than (the obviously correct translation) (P13). But this is absurd. Therefore, the metalinguistic approach to conditionals (and other hypotheticals, like conjunctions) is untenable.[48]

The foregoing problem, however, is bound to spoken natural languages like English and Latin, which on Buridan's view differ in virtue of convention. Yet Buridan follows Aristotle's divisions of language in *De Interpretatione* (I.16^{a}3-6). According to Aristotle, spoken language is subordinated to *passiones animae*, which Buridan takes to be a mental language—a fact I noted in the introduction, and which I explore at length in Chapter 3 (§1.2.1). Two facts about mental language stand out for our present problem: (i) for Buridan, mental language is primary: when we are speaking about logical operations, we are first and foremost speaking about mental ones. Further (ii), mental language is not conventional: the concepts to which Latin and English (or any other natural language) utterances are subordinated do not differ among linguistic communities. Therefore, to speak of translation in *mental* language is to commit something of a category error—as though one were speaking of translating images into text.

[48] I owe the gist of this objection in particular to a conversation with Fred Kroon.

For this reason, I don't think Church's Translation Test poses a special problem for the Buridanian metalinguistic account of hypotheticals I have here set out to defend. Though, again, I am extrapolating from what is in the texts: Buridan does not offer this analysis, though I think it fits with his view quite nicely. Therefore, if the metalinguistic analysis *does* face difficulty, it is my fault and not Buridan's.

The second problem is that the metalinguistic account seems at odds with Buridan's treatment of categoricals. Why aren't these metalinguistic, too? It can't be just because we don't *need* them to be: that would make the characterisation of hypotheticals as metalinguistic look embarrassingly *ad hoc*. But in fact there is good reason not to characterise categoricals as metalinguistic. If we did, then categoricals would be about their parts—namely, their terms. But then we would conflate the (very necessary) distinction between *personal* and *material* supposition. Here's why: certain statements make claims *about* terms, but don't use them. This is what we do when, for example, we say "*Socrates* is trisyllabic". *Socrates* in this proposition stands in material supposition for the name of Socrates—that is, for a linguistic item. If on the other hand, we said "Socrates is Greek", the subject term *Socrates* would have personal supposition, since it stands for something, not for itself as a term. But if we make categoricals metalinguistic, we lose this necessary distinction. So let's not.

Beyond the foregoing criticisms, there is a further puzzle for the Buridanian: what can Buridan's strict anti-realist and tokenist account of propositions tell us about propositional complexity?

1.2.4. Complexity Complicated

So far, we have been considering hypothetical propositions with what we now call complexity 0—that is, with hypothetical propositions containing two expressions bound by a single conjunction like *if* or *and*. We have not yet considered propositions which contain multiple hypotheticals as their constituent parts. Let's do that now.

What degree of complexity does Buridan's logic allow? There are two ways to approach this question: the first builds on what Buridan says about the way propositions behave: what are the most complex propositions allowable in principle on a Buridanian syntax? The second approach is from the ontological (and psychological) angle: given Buridan's anti-realism, how big can a proposition in thought get before, so to speak, it falls apart under its own weight?[49]

Let's begin with the first approach. Three facts about Buridan's logic are pertinent to this question. The first fact, as we saw earlier in this chapter (§1.2.3), is that for Buridan the premises of a syllogism form a single conjunctive proposition. Recall the following syllogism:

A2) Everything running is a horse

[49] It's worth noting that at one point (*QAPos* I.8), Buridan states in passing that "if there were a thousand subordinate conclusions, in such a way that the second were demonstrated through the first, and the third through the second, and so on for the rest, then the demonstration of the hundredth conclusion would contain within itself by way of conclusion all the demonstrations of the preceding conclusions, and so that demonstration would indeed be a composite syllogism". ("si sint mille conclusiones subordinatae sic quod secunda demonstretur per primam, et tertia per secundam, et sic deinceps, tunc demonstratio centesimae conclusionis continebit in se per modum conclusionis omnes demonstrationes conclusionum praecedentium, et ita illa demonstratio erit bene syllogismus compositus"). This strongly suggests Buridan can happily countenance extremely long propositions. But it remains to be seen *how* he can in this way, in a way that is consistent with his syntax and his metaphysics.

Every human is running

∴ Every human is a horse.[50]

Buridan insists we treat the premisses of syllogisms as single conjunctions.[51] That way, the possible but incompossible premisses of (A2) amount to a single, impossible proposition, entailing another impossible one, namely the conclusion.

Thus Buridan's solution involves treating the antecedent of (A2) as a conjunction. And this will likewise hold of the hypothetical equivalent of (A2)—that is, of a syllogism whose main operator is an *if* and not a *therefore*. Such a hypothetical syllogism will take the following propositional form: $((\varphi \land \psi) \rightarrow \chi)$. Such a syllogism will have one embedded conjunction, and will therefore have degree 1 complexity.

The second fact is that Buridan at least seems to place no limits on conjunctive strings of propositions; indeed, it is his usual way of giving the truth conditions for universal and particular propositions:

> An indefinite proposition has the import of a disjunctive one. So for example 'a man runs' is equivalent to 'Socrates runs or Plato runs' and so on for the rest [...and] a universal proposition has to have the import of a conjunction,

[50] "Tamen contra hanc quintam conclusionem obicitur sophistice. Quia haec est bona consequentia syllogistica:

Omne currens est equus
Omne homo est currens
Ergo Omnis homo est equus.

Et tamen utraque premissarum est possibilis, cum conclusio sit impossibilis." (*TC* I.8.130-3).

[51] As Buridan tells us in the *QQ in An Pr* (I.30), "in syllogismo neutra premissarum est totale antecedens, immo totale antecedens est una copulativa constituta ex duabus praemissis".

> for ‘every man runs’ is equivalent to ‘Socrates runs and Plato runs’, and so on for the other men.[52]

This is familiar stuff, and it is easy to find a similar statement in a modern logical textbook about the behaviour of the quantifiers in Classical FOL, namely that for ‘∀’ and ‘∃’, ranging over a domain with *n* elements,

	$\forall x(\mathrm{F}x)$
is equivalent to	$\mathrm{F}(c_1) \wedge \mathrm{F}(c_2) \ldots \wedge \mathrm{F}(c_n)$.
Ditto for	$\exists x(\mathrm{F}x)$
and	$\mathrm{F}(c_1) \vee \mathrm{F}(c_2) \ldots \vee \mathrm{F}(c_n)$.[53]

For Buridan, as for modern predicate logic, it seems that, in principle, there is no limit to the number of propositions that can make up a conjunctive or disjunctive string.[54]

The third fact is that Buridan allows hypothetical syllogisms, and with universal premisses, to boot. In the *Summulae de Demonstrationibus* 8.7.2 (“On Categorical and

[52] “Indefinita habet vim disiunctivae, ut ‘homo currit’ valet istam ‘Socrates currit vel Plato currit’ et sic de aliis [...et] universalis debet habere modum copulationis. Haec enim ‘omnis homo currit’ valet istam ‘Socrates currit et Plato currit’ et sic de singulis aliis hominibus.” (*Summulae de Suppositionibus* 4.2.6). Cf. *idem* 4.3.5.

[53] I have adopted this notation for simplicity’s sake, and I am aware that I would have to put this differently for an infinite or even nonenumerable domain. Also, I’m taking for granted that each thing in the domain has a constant, a thought at which some logicians have balked, especially for infinite or nondenumerable domains. For instance, Boolos, Burgess and Jeffrey remark that “to allow a ‘language’ with a nondenumerable set of symbols would involve a considerable stretching of the concept [*sc* of language].” See their *Computability and Logic*, 4th ed. (Cambridge: Cambridge UP, 2002), 116.

[54] A word of caution: I do not mean to say here that universal (or particular) propositions *actually are* conjunctive (or disjunctive) strings. If they were, then every proposition would be hypothetical—something Buridan rejects. Categorical propositions like these are, merely, to be *expounded* as conjunctive strings, with which they are equivalent.

Hypothetical Demonstrations”), Buridan gives us the following schema as an example of the form of a hypothetical syllogism:

S2) If every B is A, then every C is A

If every C is A, then every D is A

∴ If every B is A, then every D is A.[55]

At their simplest, such hypothetical syllogisms will take the following form (where the propositional variables represent, of course, universal affirmatives):

HS1) $((\varphi \rightarrow \psi) \land (\psi \rightarrow \chi)) \rightarrow (\varphi \rightarrow \chi)$

This formula has complexity 3, so we’re getting somewhere.

But we can get a lot further if we combine these three facts as follows: take a hypothetical syllogism in the mood Barbara, and conjoin its premisses on the basis of the first fact considered above, as we did with (HS1). Now cash out its constituent universal propositions as conjunctive strings (on the basis of the second fact), *comme ça*:

HS2) $(\varphi_1 \land \varphi_2 \ldots \land \varphi_n) \rightarrow (\psi_1 \land \psi_2 \ldots \land \psi_n)$

$(\psi_1 \land \psi_2 \ldots \land \psi_n) \rightarrow (\chi_1 \land \chi_2 \ldots \land \chi_n)$

[55] “Fit ergo demonstratio condicionalis ut dictum est vel ex ambabus condicionalibus, ut:

Si omne B est A, omne C est A, et

Si omne C est A, omne D est A,

∴ Si omne B est A, omne D est A.”

$$\therefore \quad (\varphi_1 \wedge \varphi_2 \ldots \wedge\varphi_n) \rightarrow (\chi_1 \wedge \chi_2 \ldots \wedge\chi_n)$$

The only limitation to this unfolding complexity is the number of items in the extension of the terms of our universal propositions: as a matter of contingency, some universal propositions will correspond with longer conjunctive strings of singular propositions than others. For example, a universal proposition like "Every atom is an object" will give us a lot more propositions to concatenate as a conjunctive string than will a universal proposition that has a subject term with a much more limited extension, like "Every Fabergé egg is expensive". But this limitation is no limitation at all—or at least not a syntactic one.

Thus on the basis of the three foregoing syntactic facts about Buridan's logic, we can create some very complex formulae indeed. And in so doing, we have hewed very closely to what Buridan *actually says*—though, to be clear, there is no evidence that Buridan is actually interested in propositional complexity of the same sort we think of now. On the contrary, Buridan has no notion of recursive definition, and so does not have the conceptual framework to motivate such interest. On the approach just outlined, however, there seems to be no limit in principle to propositional complexity in Buridan's logic.[56]

[56] Recently, Robert Pasnau hosted a panel in which various answers were presented to the question, "What idea would you bring back to the Middle Ages?" —that is, if you had a time machine, or some way of contacting the people alive then. I submit that the notion of *recursion* would make an excellent candidate, but I will not defend this selection here.

The second approach is more austere than the first. From the foregoing considerations about conjunctive strings and hypothetical syllogisms, we might be tempted to conclude that Buridan can countenance propositions of arbitrary complexity, the way modern logic can. However I do not think this is so: modern logic (with the well-known exception of its Intuitionist cousin) countenances realism about propositions; Buridan, famously, will not. For Buridan, recall, a proposition does not exist unless it is actually being thought. In the *Summulae de Propositionibus*, Buridan claims that for one who does not read Hebrew, the letters signify nothing, since the expressions they designate are not known. He goes on to tell us that:

> an inscription is said to be an expression only because it signifies a spoken expression, and a spoken expression is said to be an expression only because it signifies a mental expression.[57]

So if we want to construct a massive proposition, it won't obviously do just to write it all down, forgetting it as we go. Hence, in order to assess the upper limit for complexity in Buridan's system, we have to ask how many propositions can be entertained by a reasonably intelligent human all at once. And so the ontological question of what degree of complexity his logic allows will come down to a psychological one.

Thinking in this way, we straightaway come up with a negative response to our question: it is clear that (at least for any sublunar intellect) arbitrary complexity will not be possible.[58] There of course remains to be given an affirmative answer about *what*

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[57] "scriptura non dicitur 'oratio' nisi quia significat orationem vocalem, nec vocalis dicitur 'oratio' nisi quia significat orationem mentalem" (*Summulae* 1.1.6).

[58] Of course, God could entertain every possible proposition all at once. But Buridan doesn't address this, and clearly it would undermine his programmatic nominalism about linguistic items: things like propositions are mental acts, not ideas in the mind of God.

degree of complexity is allowable, and this problem can be approached in two ways: first to ask how many propositions can be entertained at one time; and second, to ask how expansive the present can be—that is, we can ask how expansive the *at once* (in which multiple propositions are being entertained) is.

I have relatively little to say about the first, since I have neither the ethics approval nor the funding (nor the interest nor the qualifications) to conduct the requisite empirical study into how many propositions the Average Joe (or even the Above-Average Joe) can entertain at once. For now, therefore, this will have to remain an open question.

We can motivate the second question by picturing the following scenario: suppose I reason through a series of propositions (φ_1, φ_2, φ_3, φ_4... φ_n). But by the time I conclude φ_n ("It wasn't the airplanes—it was beauty killed the beast"), I have completely forgotten (φ_1, φ_2, φ_3, and φ_4). Where have these propositions gone? They are the bases of one long train of thought, so in a certain sense they're part of something that exists into the present; and it would be a very hard thing to draw the line where a process of reasoning recedes so far as to lose its end in Oblivion. But on the other hand, these propositions no longer exist in the present, narrowly defined. So how many propositions exist when I reach my conclusion?

This question will be taken up in greater detail in the next chapter (2, §2.1), where I discuss the simultaneous-formulation requirement for valid *consequentiae*. Though *prima facie* this seems to place a very severe limit on the number of propositions that can be entertained, since the present is only so long, and our memory only so great, I think there is reason to be more optimistic. This is because Buridan's semantics allows for a greatly (indeed, indefinitely) expanded present tense, to accommodate the very differently-sized slices of time picked out by present-tensed statements like “The Raptors are winning” (a very narrow present) on one hand, and “Whales are mammals” (a very broad present) on the other. As we will see, then, this limitation to the present need not be so severe as it initially appears.

With these considerations settled, let’s examine in detail Buridan’s solution to the problems that motivated Frege’s force-content distinction, and then contrast Buridan’s solution with Frege’s.

2. Force, Content, and All That

To address the problems we’ve been considering, Gottlob Frege distinguishes propositional *force* and *content*. On this distinction, one proposition can have the same content, but different force. For example:

P14) It’s raining

P14′) It’s raining?

The difference between (P14) and (P14′) is that the former has assertive force, whereas the latter has only interrogative force. Even so, both (P14) and (P14′) have the same propositional content, namely, that it's raining. Similarly, if (P14) were to serve as the antecedent of a conditional, such as the following, it would lose its assertive force:

P15) If it's raining, the game is cancelled

One who puts forth (P15) entertains its constituent propositions without committing herself to their truth—that is, without asserting them. According to Frege, the propositions embedded in (P15) have the same content as they would have if they were asserted on their own. But in (P15), they lack assertive force.

Now it is an uncontroversial fact about John Buridan's life that he did not read Gottlob Frege. This is correct. And following Geach, it is widely held that Frege was the discoverer of the distinction between force and content. This, as we will see, is not entirely correct: Buridan is explicitly working with a similar distinction, though—as I stated at the outset—Buridan's distinction comes with considerably less ontological baggage. By Ockham's Razor, this is a significant advantage. And (as it turns out), the Buridanian approach enjoys some further, semantic advantages that the Fregean one does not. In what follows, I first set out the distinction as it appears in Buridan, before showing how it differs from the Fregean account.

2.1. John Buridan

As we have already seen, Buridan thinks that hypothetical propositions do not really contain categorical ones: stand-alone categorical propositions are true or false, but when expressions equiform with them appear in a hypothetical proposition, the expressions themselves are not the sort of thing that can be true or false. And since all and only propositions are true or false, those embedded expressions are not propositions at all. In this way, the syncategorematic term that binds the expressions that make up a hypothetical cancels out what I have been calling their proposition-hood (a process we'll look at more closely in connection with syncategoremes in Chapter 3, §2.1.3).

Recall the problem that set us down this path: in his definition of consequence (C_{Def}), cited above, Buridan tells us that a consequence comprises two propositions, an antecedent and a consequent, by means of the term *if* or *therefore*.[59] But when he goes on to discuss the difference between these syncategorematic terms, he does so in *syntactic* terms, rather than the semantic or pragmatic ones we would like: he tells us that *if* binds to the antecedent, *therefore* to the consequent. But he does not say that the propositions in a conditional are unasserted, whereas those of an inference or argument are. What we want is something like the Fregean distinction between force and content.

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[59] *Caveat lector*: if Buridan were being more careful, he would say *expression* (*oratio*), not *proposition* (*propositio*), as we saw in §1.2.2, above. But Buridan often lapses into talking about the propositions contained in a hypothetical, using *proposition* as shorthand for "a proposition-like expression that would be a proposition if it were formulated on its own". In this, I am content to follow him—*quia nomina significant ad placitum*. Granted, there remain puzzles about what relates stand-alone propositions to their non-propositional counterparts in hypotheticals, and these will become especially acute in sophisms like "If this sentence is false, then this whole conditional is true" (I owe this sophism to a suggestion from Calvin Normore). But this is well beyond the present discussion, and best discussed in a future chapter I plan to write on virtual implication for a monograph based on the thesis.

Now from what we've seen in § 1 of the present chapter, we have in place the foundation for a distinction between force and content that allows us to separate conditional sentences from inferences or arguments. In the *TC* and the treatment of hypothetical propositions in the *Summulae de Propositionibus*, Buridan does not make this distinction. But elsewhere he apparently does. There are two places in particular that look especially promising.

The first is Buridan's definition of the syllogism in the *Summulae de Syllogismis*. There, Buridan provides a syntactic definition of syllogisms that echoes his syntactic definition of consequences, (C_{Def}):

> Although a syllogism is made up of several expressions, it is nevertheless a single hypothetical proposition, connecting the conclusion with the premises through the conjunction 'therefore'. Further, it can be reduced to the species of conditional propositions, for just as a conditional is one consequence, so too is a syllogism [...] Strictly speaking, however, a syllogism has an additional feature in comparison to a conditional, in that *a syllogism posits the premises assertively* [assertive], *whereas a conditional does not assert them.*[60]

So a syllogism sets forth the premises *assertively*—that is, in such a way that it asserts that they are true—whereas conditionals lack this assertion. This gets us most of the way

[60] "Licet syllogismus sit compositus ex pluribus orationibus, tamen est una propositio hypothetica coniungens conclusionem cum praemissis per hanc coniunctionem 'ergo'. Et potest reduci ad speciem propositionum condicionalium, quia sicut condicionalis est una consequentia, ita et syllogismus [...] tamen syllogismus proprie addit super condicionalem, quia ponit assertive praemissas, condicionalis autem non asserit eas" (*Summulae* 5.1.3, 10-18; emphasis added). *Cf. Summulae* 1.7.6: "A syllogism differs from a conditional proposition..."

to the distinction we would like to draw between conditionals and arguments. But it doesn't get us all the way there: there are three discrepancies between the above distinction and the general distinctions I would like to draw between arguments on one hand, and conditionals on the other. I arrange them in order of seriousness, from least to most.

First, Buridan here only says that a syllogism asserts the *premises*, but not the conclusion. It seems reasonable to suppose that a syllogism asserts the conclusions it implies, too. But it would be much better for us if Buridan actually said so.

Second, it would be useful for our present purpose if Buridan said something here that allowed us to link these semantic and pragmatic remarks—namely about putting forward premises (as well as conclusions) assertively—back to his *syntactic* remarks about the syncategorematic terms *if* and *therefore*. Specifically, we would like him to say that an *if*-hypothetical does not assert the truth of the expressions it binds, whereas a *therefore*-hypothetical does. If he said so, it would allow us to link the *de Syllogismis* discussion up tidily with his definition (C_{Def}) of *consequentia* in the *de Consequentiis* I.3.

Third, Buridan here distinguishes conditionals from *syllogisms*—which are merely a species of argument. But we would like to draw a distinction between conditionals and arguments generally, including formal conversions (such as $a\mathrm{I}b \vdash b\mathrm{I}a$) and even informally valid arguments. On a strict reading of the text, this extrapolation is not warranted, since syllogisms have many properties which they do not share with other, single-premise arguments.

The foregoing passage from *De Syllogismis* does not give us these three *desiderata*. What it *does* give us is a distinction between expressions put forth assertively (*assertive*), on one hand, and non-assertively, on the other. Conditionals, characteristically, do the latter. But to get the distinction just the way we want it, we need more textual support.

Fortunately, there is a passage that allows us to tie it all together. It appears in Buridan's discussion of the fallacy of the consequent in the *Summulae de Fallaciis*. For context: the fallacy of the consequent comprises those conditional fallacies of denying the antecedent, and affirming the consequent—that is, fallacious arguments of the following schemata:

S3)	$\varphi \rightarrow \psi$	
	$\sim\varphi$	
∴	$\sim\psi$	(denying the antecedent)
S4)	$\varphi \rightarrow \psi$	
	ψ	
∴	φ	(affirming the consequent)

In Peter of Spain's text, on which Buridan is commenting, this section includes a brief syntactic definition of the antecedent and consequent of a *consequentia*, along the lines of the one Buridan sets out in the *de Consequentiis*:

> The antecedent is that to which the conjunction 'if' is added in a conditional proposition, whereas the other proposition is the consequent. But in an

> argument, the consequent is that to which the conjunction 'therefore' is added, and the other proposition is the antecedent.[61]

Thus what divides conditionals from arguments is the main connective: in conditionals, it is an *if*; in arguments, it is a *therefore*. To this, Buridan adds that although all *consequentiae* are single hypothetical propositions, they are to be divided into conditionals (*propositiones conditionales*) and arguments (*argumenta*), in virtue of the assertions they make about their constituent expressions:

> There are two kinds of consequence. The first of these is a conditional proposition, which asserts neither the antecedent nor the consequent (*e.g.* "If a donkey flies, then it has wings"), but asserts only that the latter follows from the former. Such a consequence is therefore not an argument, since it does not conclude to anything. The other kind of consequence is an argument [...] and this asserts the antecedent, and from this it assertively [*assertive*] infers the consequent. In a conditional we use the conjunction *if*, whereas in an argument we use the conjunction *therefore*.[62]

Thus Buridan retains the syntactic division between conditionals and arguments he suggests in *TC* I.3 and discusses in the *Summulae de Syllogismis* passage cited above. But

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[61] "Est autem antecedens cui apponitur haec coniunctio 'si' in propositione condicionali et alia est consequens, sed in argumento consequens est cui apponitur haec coniunctio 'ergo' et alia est antecedens." (*Summulae* 7.4.5).

[62] "Deinde notat duplicem esse consequentiam, scilicet unam quae est propositio condicionalis, et illa nec asserit antecedens nec asserit consequens (ut 'si asinus volat, asinus habet alas'), sed solum asserit quod hoc sequitur ad illud. Et ideo talis consequentia non est argumentum; nihil enim concludit. Alia consequentia est argumentum [...] quae asserit antecedens et ob hoc infert assertive consequens. In condicionali autem utimur hac coniunctione 'si' et in argumento hac coniunctione 'ergo'." (*Summulae* 7.4.5).

he adds a parallel pragmatic and semantic distinction: arguments put forth their constituent parts assertively (*assertive*), whereas conditionals do not.

This is good news. And it has prompted Gyula Klima to treat Buridan's treatment as a sort of force-content distinction *avant la lettre*. As Klima tells us at the outset of his "John Buridan and the Force-Content Distinction", "For his part, Buridan was fully aware of the 'Frege point', without being aware of Frege."[63] Later on in the paper, however, Klima tempers this point, clarifying that for Buridan, the proposition-like expressions involved in a hypothetical are not really propositions at all—as we saw in §1.2.2 of the present chapter:

> for Buridan the assertive force of a proposition is not something that is added to a *per se* unasserted proposition, but rather it is something that belongs to a proposition *per se* when it is propounded.[64]

This is spot-on (if, in light of the title of the paper, a bit of a bait-and-switch). But here I want to make a stronger claim: this difference between Buridan on one hand, and Geach and Frege on the other, is no quibble. On the contrary, it has significant impacts downstream. Further still, there are good reasons to prefer Buridan's approach. In the following section, I examine the Frege-Geach view, before examining three problems it faces—problems that Buridan's account dodges completely.

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[63] Gyula Klima, "John Buridan and the Force-Content Distinction", *Medieval Theories on Assertive and Non-Assertive Language* ed. Alfonso Maierú and Luisa Valente (Rome: Leo S. Olschki Editore, 2004) 415. Klima says much the same in his "Consequence", *The Cambridge Companion to Medieval Logic* (Cambridge: Cambridge UP, 2016), 319ff.

[64] Klima, "John Buridan and the Force-Content Distinction", 422.

2.2. Gottlob Frege and Peter Geach on Assertion

In his *Logic Matters*, Geach sets out Frege's distinction between propositional force and content:

> A thought may have just the same content whether you assent to its truth or not; a proposition may occur in discourse now asserted, now unasserted, and yet be recognizably the same proposition. [...] I shall call this point about assertion *the Frege point*, after the logician who was the first (as far as I know) to make the point clearly and emphatically.[65]

Hence a proposition can be entertained without being asserted, for example in a question, or in a disjunction or conditional like the ones we considered in § 1.2, above. In fact, Geach motivates this distinction with examples practically identical to the ones John Buridan considered some six centuries earlier (and even makes use of the Latin disjunctive particles *aut* and *vel*):

> Now even if the proposition represented by "*p vel q*" or by "*p aut q*" is itself taken to be an asserted proposition, "*p*" will not be asserted in this context, and neither will "*q*"; so if we say that the truth value of the whole proposition is determined by the truth values of the disjuncts, we are

[65] Peter Geach, "Assertion", *Logic Matters* (Oxford: Basil Blackwell, 1973), 254-5; cited by Gyula Klima, "John Buridan and the Force-Content Distinction", *Medieval Theories On Assertive and Non-Assertive Language, Acts of the 14th European Symposium on Medieval Logic and Semantics*, ed. A. Maierú, and L. Valente (Rome: Olschki, 2004), 415.

> committed to recognizing that the disjuncts have truth values independently of being actually asserted.[66]

By now, this is familiar stuff: a disjunction does not assert the individual disjuncts, though it *is* an assertion that (at least) one of them is true. But whether *p* or *q* appear in a disjunctive sentence, or on their own, has no bearing on their *content*, says Geach: even when they are unasserted, they retain the same subject matter. Thus the disjuncts in the hypothetical proposition

> P16) Socrates is sitting or he is standing

have the same content as their asserted counterparts (namely, "Socrates is sitting", etc.), though they differ in force.

So much for the agreement between Frege (and Geach) and Buridan. The two accounts disagree on the propositional status of these unasserted parts. For Buridan, as we have seen, the parts of a hypothetical proposition are emphatically *not* propositions themselves, but expressions that *would* be propositions if they were put forth on their own. For Geach, they just *are* propositions, which haven't had force applied to them. Hence Geach tells us that:

> When I use the term 'proposition' [...] I mean a form of words in which something is propounded, put forward for consideration; it is surely clear

[66] Geach, "Assertion", 258. Geach clarifies that his use of Latin *vel* and *aut* instead of English *or* is "to dodge the idiotic but seemingly perennial discussion as to the proper use of 'or' in ordinary language" (258). From this, we can infer that Latin is no ordinary language.

> that what is then put forward neither is *ipso facto* asserted nor gets altered in its content by being asserted.[67]

Thus, assertion is something that gets *added on* to propositions in certain contexts, and taken off in others. This produces considerable problems for the Fregean view.

2.2.1. Assertion is Not a Sticker

First, there is a mystery for the Frege-Geach account that is actually predicted by Buridan's, namely the lack of an expression of assertion in much of natural language. For instance, note that the following proposition is clearly assertive:

P17) Socrates is running

A proposition like (P17) is true or false: uttered by someone who knows Socrates is sitting, it is a lie. And since (P17) is asserted, adding to it *it is true that* does nothing to alter its assertive force:

P17′) *It is true that* Socrates is running

In intuitive terms: one who knows that Socrates is *not* running and utters (P17) cannot beg off on the charge of lying simply because they uttered (P17) and not (P17′). So in terms of assertion, the clause *it is true that* adds nothing to (P17), because (P17) on its

[67] Geach, "Assertion", 255.

own is already asserted. Indeed, Dummett acknowledges this point in his discussion of assertion:

> We do not succeed in asserting anything by adding to a sentence without the judgment-stroke the words 'is true', or by prefixing the words 'it is true that': we merely get another sentence which expresses a thought and has the same truth-value as before.[68]

So *it is true that* doesn't add anything, at least in terms of assertion. What's needed, according to Dummett, is the trademark Fregean judgment stroke or assertion sign ('⊦'). This sign signifies that the proposition (or propositional variable) following it is being asserted. Thus '⊦*φ*' is to be read as the assertion of '*φ*', which on its own stands for itself unasserted.

Yet there remains the puzzle that there is no expression in natural language that is consistently used to add assertive force: that is, there is no linguistic assertion 'sticker'.[69] If it's so necessary, why don't natural languages have it? They have negation, as well as conditionals and other hypotheticals, after all, and the corresponding syncategorematic terms (*not*; *if*, *or*, etc.). For the Buridanian, this is no mystery: by default, stand-alone propositions just *have* assertion. But for the Fregean, it is a source of bewilderment. As Geach himself admits,

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[68] *Frege*, 316.

[69] And even if there were a natural language assertion sticker, it isn't clear how it would even work pragmatically: as Dummett points out, if there *were* any such marker, actors on stage would be using it, too. See his *Frege*, p. 311.

> There have been a number of attempts to treat some expressions of ordinary language as carrying with it the assertoric force. I think these attempts all miscarry [… and] that is why Frege had to devise a special sign.[70]

Geach's 'special sign' is, of course, the assertion sign or content-judgment stroke, invented by Frege. This invention was not born of necessity: propositions characteristically *have* assertive force, as the example of (P17) and (P17′) shows. Thus the absence of any linguistic item regularly used to denote assertion—that is, a natural language equivalent of Frege's '⊦'—is actually *predicted* by Buridan's view, which takes assertion to be the characteristic feature of propositions, rather than an after-market linguistic particle stuck on to them. This predictive feature of Buridan's view is the first reason to prefer his account.

2.2.2. Kicking About

A second advantage to the Buridanian account is this: it allows us to say more clearly what hypotheticals are *about*. Recall that the syncategorematic term *if* or *therefore* has the role of *designating* (*designans*) that one expression is the antecedent, the other is the consequent.[71] So hypotheticals aren't about what their constituent parts are about. Rather, they're about their constituent parts. Here is an example:

> P18) If it's raining, we won't play ball.

[70] Geach, "Assertion" 262.
[71] "Dictae enim dictiones [sc. 'si' et 'ergo'] designant quod propositionum per eas coniunctarum una sequitur ad aliam" (*TC* I.3.11-12)

What is this conditional about? For Buridan, it's a hypothetical proposition whose principal part is the syncategorematic term *if*, which asserts that the consequent ("we won't play ball") follows from the antecedent ("it's raining"). Recall what Buridan says in (C_{Def}): that the syncategorematic term *if* designates (*designat*) that the consequent of a conditional follows from the antecedent. The assertion here, for Buridan, is about the relationship between the two.

What can Geach say? If (P18) is made up of flesh-and-blood propositions, it seems to be a statement about rain, about us, about baseball, about cancellations (or things we won't do), etc. This is weird. And it gets weirder when we apply this line of reasoning to disjunctives, like the following:

P19) The shadow is a horse's, or anyway it looks like a horse's.

What is a proposition like this one about? It seems the Fregean will have to say that it is about shadows, horses, and things that look like horses and their shadows. The disjunct propositions are real propositions, minus the force of asserted ones, and so they are about the same things as their asserted counterparts. Buridan can skirt this problem by demoting the propositions involved in a hypothetical to expressions. What a disjunctive proposition like (P19) is about is its constituent expressions. What it asserts is that (at least) one of them is true—or, to be precise, that at least one proposition equivalent with its constituent parts, were it formed as a stand-alone proposition, would be true. So if we take Buridan's approach, we can set aside problems posed by the subject matter of hypotheticals. Briefly put: hypotheticals are about their parts; they are not about what their parts are about.

In sum: for Buridan, conditionals are not the same as arguments, but both are *consequentiae*, a species of hypothetical proposition. What makes them *consequentiae* is that they say something about the expressions they contain, namely that one follows (*sequitur ad*) the other. That is, unlike conjunctions or disjunctions, they are non-commutative: for them, order matters. What distinguishes them from arguments is that arguments additionally assert that their constituent parts are true, whereas conditionals do not. That Buridan can readily explain what conditionals and other hypotheticals are about is the second advantage to his account.

2.2.3. A Team of Propositions is Not a Proposition

A third reason to prefer Buridan is that he dodges a problem raised by John Stuart Mill (and noted by Geach). In *A System of Logic* Mill discusses the distinction between simple and complex propositions, and tells us that:

> At first sight this division has the air of an absurdity; a solemn distinction of things into one and more than one; as if we were to divide horses into single horses and teams of horses. And it is true that what is called a complex (or compound) proposition is often not a proposition at all, but several

> propositions, held together by a conjunction. Such, for example, is this: "Cæsar is dead, and Brutus is alive": or even this, "Cæsar is dead, but Brutus is alive". There are here two distinct assertions; and we might as well call a street a complex house, as these two propositions a complex proposition.[72]

There is something reminiscent of the fallacy of composition here: when we call something made up of propositions a *proposition*, we are taking a name for the individual parts and applying it to the composite. In Geach's succinct formulation:

> If you recognize conjunctive propositions as a kind of proposition, you may as well say, as Mill remarked, that a team of horses is a kind of horse, or a street a kind of house.[73]

And Mill (and Geach) are right: it *is* odd to distinguish propositions in terms of their containment of multiple propositions and single ones, and then to call them all propositions—as it is odd to refer to a street as a kind of house, etc.

But Buridan avoids this problem altogether: hypothetical propositions are not distinguished by their composition of several propositions. Rather, they are propositions built up from non-propositional expressions. It is the several expressions that combine to make a hypothetical proposition, even though those expressions *would be* propositions if they were formulated on their own. So we don't get caught in this puzzle Mill introduces. But the Fregean, who wishes to say that hypotheticals are distinguished by their containment of multiple full-fledged propositions, does.

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[72] John Stuart Mill, *A System of Logic*, 8th ed. (New York: Harper & Brothers, 1888), 69.
[73] Geach, "Assertion", 259. Geach does not cite Mill's text, but it seems he has this one in mind.

2.3. One Small Step for Frege…

We can now assess the claims—which are legion—that Frege is doing something altogether new. In spite of the differences in solution, isn't Buridan at least thinking about the same *problem*? And is Frege's solution the quasi-scientific discovery it is often hailed to be? As I'll explain, I think the answers to these questions are *yes* and *no*, respectively.

Still, the Fregean has grounds to criticise Buridan's approach, and in two ways I want to consider. The first of these criticisms is that Buridan's talk of the truth (or falsity) of the propositions that make up a hypothetical is impermissible, since they aren't really propositions at all. The second is that Buridan cannot account for the cancellation of assertion without countenancing propositions whose assertive force is subtracted (or added).

2.3.1. Is Force-Content a Discovery, or an Invention?

It is remarkable that dyed-in-the-wool Fregeans tend to treat the distinction between force and content as a *discovery*, rather than an *invention*. At least *prima facie*, this approach takes the distinction not as one solution to an old problem, but as *the* solution to a problem that had hitherto gone unrecognised. Here, for instance, is a recent (2018) and representative statement, from Robert May and Richard Kimberly Heck:

> The language of affirmation and denial is not only quaint but misplaced, as Frege himself would eventually come to realize. This is essentially what Peter Geach (1965: 449) famously called "the Frege point." It is closely

> connected with what Frege himself called "the dissociation of assertoric force from the predicate" and regarded as one of his most important *discoveries* [...] if one asserts a conditional, then it is only the conditional as a whole that is affirmed; one need neither deny its antecedent nor affirm its consequent.[74]

Thus the Frege point is construed as a *discovery*. This is the first move. (If you find the above passage confusing, by the way, I'm with you. It's ambiguous: do May and Heck mean to say that Frege himself *regarded* the Point as an important discovery? Or do they mean that the Point *is regarded* as such—*i.e.* by common consensus? In what follows, I am reading it as the latter, which is true).

The second move involves a basic conflation: discovery of the problem just *is* the discovery of the solution: a distinction between force and content. Once our eyes have been opened to this distinction, we realise that not all predications are assertions. Geach paints the 'discovery' in dramatic terms:

> A moment's consideration ought to have shown that [...] '*P*' may be predicated of *S* in an *if* or a *then* clause, or in a clause of a disjunction, without the speaker's being in the least committed to affirming that *S* is *P*. Yet it took the genius of the young Frege to dissolve the monstrous and unholy union that previous logicians had made between the import of a predicate and the assertoric force of a sentence.[75]

[74] Robert May and Richard Kimberly Heck, "Truth in Frege", *The Oxford Handbook of Truth*, ed. Michael Glanzberg (Oxford: Oxford UP, 2018), 203. Emphasis added.
[75] Peter Geach, *Reference and Generality* (Ithaca, NY: Cornell UP, 1962), 51.

Thus in Geach's view, logicians before Frege failed to notice that, although there were real-live predications going on in the constituent parts of hypothetical propositions, those propositions were not being asserted at all. It took Frege's *genius* to see through this unholy *monstrosity*: to discover the non-assertive predications that had been right before our eyes all along!

This is how Frege's force-content distinction has been marketed as *the* solution to the problems considered at the outset of this chapter—and the first (and last) of its kind. In the view of May and Heck (and doubtless of Geach), this fact about assertion is a *bona fide* discovery—perhaps even akin to Archimedes' discovery of the principle of displacement, or Newton's of the force of gravity. But is it?

Now there are two ways to think of this putative discovery, as we have seen in the Fregeans just cited: (i) as the first discovery of a problem, and (ii) as the discovery of—not *a*, but—*the* solution. Geach, in the passage cited, seems to have both (i) and (ii) in mind. But we can straightaway rule out (i). We have seen from the foregoing that Buridan is well aware of the problems that motivated the force-content distinction in Frege.

Indeed, Buridan didn't discover this problem, either: as Chris Martin has shown, it has a history that goes back at least as far as the twelfth century, with Abaelard's distinction between the propositional content (*dictum*) on one hand, and the act of proposing or asserting it (*proponere*), on the other.[76] Thus Abaelard can (and does) distinguish the constituent propositions of a conditional, which are not asserted, from their

[76] Christopher J. Martin, "*De Interpretatione* 5-8: Aristotle, Boethius, and Abaelard on Propositionality, Negation, and the Foundations of Logic", *Methods and Methodologies: Aristotelian Logic East and West, 500-1500*, ed. Margaret Cameron and John Marenbon (Leiden: Brill, 2011), 224.

asserted stand-alone counterparts.[77] It would hardly be surprising if awareness of the problem were older than Abelard, as well, though this is not the place to trace its history. At any rate, we have seen that the assertion problem for the constituent parts of a hypothetical was well known centuries before Frege, and plausible solutions had been proposed well before him. This is not to say that Frege was not a brilliant logician, nor to downplay the very real advancements he made. It is just to point out that he is not the singular genius Geach and other Fregeans take him to be, and to note that many advancements are as old as the hills.

At any rate, in the face of all this, the Fregean can still make a case for (ii): namely, that Frege discovered the one and only *correct* way to solve the problem. If such a case were successful, it would naturally be devastating for Buridan's view as an alternative. For suppose there really is only one solution, and Frege was the first to hit upon it. Then any alternative should get no more attention than the Ptolemaic theory of epicycles does in modern astronomy: an historic curiosity of genuine antiquarian interest, to be sure, but no competitor in the marketplace of ideas. After all, no astronomer would treat Ptolemaic astronomy as an attractive alternative to heliocentrism, or even look for an expert in Ptolemaic astronomy to sharpen their wits against. Why then would a modern logician or philosopher of language treat Buridan's approach any differently?

The weakness of this absolute claim is, however, that it—like all absolute claims—is exceedingly brittle. If an alternative to the Fregean view is at least *plausible*, then the Fregean claim cracks. If so, then the force-content distinction is no more a

[77] *ibid.*

discovery than the electric space heater: a tool for a job—and a pretty effective one at that—but certainly not the only way to get the job done. The stakes, therefore, are high: it falls to the Fregean to find a flaw in the Buridanian approach outlined above, and to declare the flaw insoluble.

2.3.2. The Fregean Strikes Back

Here is an important flaw, which the Fregean can take advantage of. As we have seen, Buridan demotes the constituent parts of hypothetical propositions, so that they are mere utterances (*orationes*), and not propositions *per se*. In this, the criticism runs, Buridan goes too far: if he throws out propositions, then how can he account for the different truth values of the constituent parts of a hypothetical? For example, how can Buridan say that, for a disjunctive to be true, at least one of its disjunct propositions has to be true? After all, if the disjuncts aren't propositions, and only propositions are true or false, then how can we speak of these disjuncts as being true or false?

Every kind of hypothetical proposition presents us with a similar problem: in order to account for the truth of a hypothetical, we need to take into consideration the truth (or falsity) of its constituents. But in Buridan's view, these constituents can't be true or false. So it seems we cannot account for the truth or falsity of hypotheticals at all. Only the Fregean, who decouples assertion from truth, can account for the truth (or falsity) of these unasserted constituent propositions.

Here is how Buridan can respond. Granted, the constituent parts of hypotheticals are neither true nor false. Nor are they propositions. But we can—and often *do*—still speak of them as propositions, and as being true or false, since they are identical in form with what *would* be a proposition (and therefore would be true or false) if it were formulated on its own. Hence in virtue of a logical particle like *or* or *if*, a hypothetical proposition makes statements *about* propositions (as we saw in §2.2.3, above). And a hypothetical proposition does not need to be made up of the propositions it makes a statement about. Rather, it is enough that the hypothetical contain utterances (*orationes*) equiform with them. These utterances are identified with the propositions the hypothetical is about, and it is by this equiformity that we may recognise them when they are put forth on their own.

Admittedly, then, speaking of the truth of a hypothetical's constituent propositions is merely a *façon de parler*—in the same way, in fact, that speaking of them as propositions is: not technically correct, but a sort of shorthand. But just because a proposition makes statements about other propositions doesn't mean it has to contain them itself.

Thus this Fregean criticism misses the mark, and in two ways: (i) by taking the Buridanian language of truth and falsity (and even proposition-hood) of hypothetical constituents literally, rather than as a conventional abbreviation; and (ii) by supposing that a proposition has to be made up of the actual things it makes a statement about, in order to make a statement about them. But both assumptions are based on misunderstandings.

The second Fregean criticism is based on cancellation of assertion. For instance, suppose that Socrates is sitting still, and I utter the following:

P20) Socrates is moving…

You disagree, and so you contradict me with the following proposition:

P21) No he isn't.

But I counter this by turning the apparent stand-alone proposition (P20) into the antecedent of a conditional by appending the clause "…*if* he's running", by which addition I turn (P20) into the following full conditional:

P22) Socrates is moving *if he's running*.[78]

You were right to contradict my first statement, but in the shift of (P20) from a stand-alone proposition to the antecedent of (P22), the original assertion is being cancelled. Hence it is the same proposition, the Fregean will claim, with the same content, but appears at one time asserted, and at another time unasserted. The Buridanian, this objection goes, cannot account for this removal (or a corresponding addition) of assertion, since there is no Buridanian framework for unasserted propositions.

[78] Chris Martin suggested this to me in a conversation.

What can the Buridanian say about this objection? Let's look at what happens at the outset of the proposed case, where I first utter the following expression:

P20) Socrates is moving...

You, as my interlocutor, take this to be a full-fledged proposition. Now either it is just such a full-fledged proposition, or it isn't. If it is a proposition, then I am not entitled to turn it into the antecedent of a conditional as I do, since the antecedent of a conditional is not a proposition. Hence if (P20) is a proposition, then what I am doing when I form (P22) is re-forming an equiform, non-propositional expression as its antecedent. Hence I am making two moves: advancing a proposition, and then advancing a conditional with a constituent equiform with the original proposition. But then (P20) and the antecedent of (P22) will not be one expression with assertion now added, now taken away. They will, rather, be two distinct expressions: one a proposition, the other not.

If, on the other hand, (P20) is not a proposition, then it is the antecedent of a well-formed conditional—albeit one that I am taking my time to utter. In that case, there is only one expression at play here, which never was a proposition. If with (P21) you are contradicting anything at all, it is a mental proposition you yourself formulated, equiform with (P20), which you wrongly take to be stand-alone. Indeed, in the *Sophismata* (VIII, 5), Buridan considers a similar case, in which Plato says "It isn't the case that a human is a donkey", and Socrates hears only the expression "...a human is a donkey".[79] But this

[79] The propositions at play here are "*Nullus homo est asinus*" and "*Homo est asinus*". I had to expand these somewhat to make the English natural: Latin has no articles, and so the latter is well-formed. But of course *"Human is a donkey" would not be. (See *Sophismata* VIII, 5; Scott, 130-1, 40v-41r).

latter utterance is not a proposition at all, but an expression that forms a proper part of a proposition. But only propositions can be true or false; so if there is a false proposition here at all, it is one Socrates mistakenly mentally formulated on the basis of a fragment of Plato's spoken proposition.

Hence between (P20) and (P22), one of two things is going on: either (i) I am being sneaky, and there are really two discrete expressions at play in the shift from (P20) to (P22). One of these is a proposition, and the other is a non-propositional conditional antecedent. Or (ii) you are mistaken, and there is only one expression that never was a proposition at all, but is only the antecedent of a conditional.

In case (ii), either (a) your contradictory statement (P21) misses the mark, since there was no proposition at play to be denied at all; or (b) if your contradiction hits a target, it hits the wrong one, namely a proposition you have formulated with the aim of contradicting it—a proposition that is equiform with (P19), but not identical with it.

In any case, what is at play here is different but equiform expressions, and not a single propositional expression that is having assertion now added, now removed. On the Buridanian view, what is really at play here is multiple equiform expressions that are being confused (or, in case (i), deliberately conflated). Such cases do not, therefore, call for a whole framework of propositional assertion as their sole solution. The token-based Buridanian propositional semantics can furnish a solution as well, without taking recourse to abstract propositional content. And I am confident that any such case, where assertion is putatively cancelled, can be given a Buridanian solution along the lines of the one just furnished.

Let me close this section by recalling how Frege's force-content distinction has been marketed as *the* solution to the problems discussed above (§1). As I noted at the outset of the present section (§2.3), the Fregean claim is absolute: Frege did not invent a new solution to an old problem; rather, he discovered a *new problem*, and at once discovered *the* solution.

Of course, Frege did not discover this problem—neither, for that matter, did Buridan: it is already present in Abaelard.[80] So claims about Frege's discovery of the problem are false. And, as I remarked, if a solution alternative to Frege's can be shown to be at least *plausible*, it can deal the *coup de grace* to claims about the absoluteness of Frege's solution—claims whose absoluteness renders them brittle. In the face of such claims, I do not have to show that Buridan's solution is *the* solution. Rather, if it is at least *plausible*, it is enough to cast doubt on the absoluteness of the Fregean claim: Frege did not discover *the* solution, but furnished *a* solution—one with its own drawbacks, and with attractive competitors.

Let's set the Fregean critics aside, and conclude this chapter by considering how Buridan's approach has been analysed by his more sympathetic commentators.

[80] For a discussion, see Peter King, "Peter Abelard", *The Stanford Encyclopedia of Philosophy* (Fall 2018 Edition), ed. Edward N. Zalta, §3.

3. *Quod Sequitur*: Assessing the Secondary Literature

In this closing section of the present chapter, I do two things. First, I return to the commentators on Buridan (and those commenting on medieval logic in general) that I mentioned at the outset.[81] These commentators have been (justifiably) alarmed and even embarrassed to find Buridan and other medieval logicians apparently conflating conditionals and arguments under the general heading *consequentia*. As we will see, this embarrassment has prompted commentators to downplay the role of conditionals in theories of *consequentiae*. Moody (1953) and King (1985) are two notable exceptions, though King later (2001) changes his mind. But there is no cause for (sustained) embarrassment or alarm: Buridan does in fact distinguish conditionals from arguments, as we have seen. Therefore, to over-emphasise the place of arguments under the heading *consequentiae*—that is, to do so to the exclusion of conditionals—is to distort the texts, and needlessly at that.

Second, I ask: is Buridan *right* to group conditionals and arguments under the heading *consequentia*? I think so, and I think his reasons for doing so allow us to make some general remarks about the nature of his logical project.

[81] With one important exception: I have postponed discussion of Catarina Dutilh Novaes's thorough and sophisticated analysis of Buridan's notion of logical consequence to Chapter 5 (§3.3). I do this because her analysis deals with Buridan in terms of possible worlds, which I deal with at length in that chapter, whereas the present chapter is mainly about the conflation of conditionals and inferences which has troubled so many commentators on Buridan.

3.1. Inferences: the Talk of the Town

To begin, here's a brief overview of what commentators and interpreters of Buridan (and of medieval logic more generally) make of this conflation. In *Truth and Consequence in Medieval Logic* (1953), E. A. Moody notes that medieval logicians used *consequentia* to cover both conditionals and arguments. He tells us that:

> The term 'consequence' derived in Latin from the verb 'to follow' (*sequi* or *consequi*) [...] In the medieval Latin tradition this generic conception of consequence or of 'following' was of course retained; but the term 'consequence' came to be used technically to designate sentences of conditional form, such as are true or necessary, or at least such as "claim" by their form to be true or necessary. The later medieval logicians tended to regard all forms of valid deduction, including the syllogism, as forms of 'consequence' and therefore as equivalent to conditional propositions. In this way the entire theory of deduction was organized as a development of the rules governing the validity of conditional sentences.[82]

Moody is right to see *consequentia* as a technical term that is general enough to apply to the relationship between propositions both in conditionals and in arguments. Interestingly, he here claims that the notion of *conditional consequence* was prior to that of argumentative consequence. This is not the place to discuss this claim in detail; but if it is true, then the extent to which Buridan tends to think of arguments as *consequentiae*—and

[82] E.A. Moody, *Truth and Consequence in Medieval Logic* (North-Holland: Amsterdam, 1953), 64-5.

to use arguments as paradigmatic cases of *consequentiae*—shows how far he is from the earlier notions of consequence, which apparently treat conditionals as primary.

In his "Consequences" (1982), Ivan Boh likewise begins with the remark that *consequentia* etymologically "suggests a following along". This is precisely right, and I will return to this observation in a moment. Boh goes on to list three relationships between propositions: conditional *implication*; *entailment*, the relationship between two propositions where one cannot be true without the other; and *inference*, the action of deriving one proposition from another. As we will see in Chapter 2, Buridan's anti-realism entails a conflation of the notions of inference and entailment, since there is no relationship of *consequentia* between two propositions unless they are actually formulated—that is, there is no relation of inference unless there is a relation of entailment. So the problem here is the apparent conflation of arguments and conditionals under the term *consequentia*. This is a term which, as Boh remarks, Buridan and other medieval logicians "disconcertingly use [...] to cover all three of these relationships among propositions".[83] But as we have seen, Buridan does distinguish conditionals from arguments in semantic and pragmatic terms, analogous to Fregean views on assertion and content. Syntactically, however, conditionals and arguments are much the same: in both conditionals and arguments, one sentence follows from another.

In the extensive introduction to his (1985) *Jean Buridan's Logic*, Peter King notes that:

[83] Boh, "Consequences", 300.

> Buridan's theory of *consequences* covers material which modern logic treats under the separate headings of a theory of conditionals and the rules of inference.[84]

King adds that "this need not be an error", and notes that the motivation for distinguishing conditionals and inferences:

> is based on philosophical analysis of ordinary language: it captures in a formal way the difference between assertions which do not require a commitment to the truth of the first statement. One is so committed when using the inferential form, one is not so committed when using the conditional form.[85]

As we have seen, it is very important to keep this distinction in mind. But, as King points out, "there is no overwhelming reason to distinguish the cases as being of different kinds rather than as species of a single genus".[86] This is the view I take here; what characterises the genus in question is a more general notion of *following*, as suggested by Moody's discussion of the etymology of the term *sequor*. I will return to this notion of following in the next section.

In a subsequent (2001) paper, King reverses his initial analysis, arguing that the notion of *consequentia* just *is* a theory of inference, and claiming that "mediaeval logicians not only recognized a difference between implication and inference but found them not to overlap at all".[87] King's case hinges on both negative and positive textual evidence. The

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[84] Peter King, *Jean Buridan's Logic: The Treatise on Supposition, The Treatise on Consequences* (Boston: Reidel, 1985), 59.
[85] King, *Jean Buridan's Logic*, 60.
[86] *ibidem.*
[87] "Consequence", 120.

negative case is the almost complete absence of any treatment of *conditional* and *consequentia* as synonymous. This is right in the examples, but as we saw Buridan's definition explicitly treats conditionals as a species of consequence, and in general it is unsafe to let the examples override the definition.

King's positive case has two prongs: first, medieval logicians actively contrast conditionals and arguments, as we saw in the passages from Buridan cited above. And second, medieval logicians tend to use different language for conditionals and arguments: the former are true (*vera*) or false (*falsa*), whereas the latter hold (*tenet*), are valid (*valet*) or legitimate (*bona*). Both of these are correct. But they are not enough to establish that Buridan's (or any other) notion of *consequentia* is one of inference only.

Here is how King characterises the modern distinction between inferences and conditionals: "conditionals make statements whereas inferences do things with statements".[88] Thus, conditionals belong to the object language, whereas inferences belong to the metalanguage: a distinction we bake into our notation by denoting the former with the arrow '→', and the latter with the turnstile '⊢'. But the medievals, as King notes, have no such distinction between object- and metalanguage (nor for that matter do they have such symbolic apparatus to mark them off).[89] Rather, medievals like Buridan treat conditionals as well as arguments as quasi-metalinguistic (albeit *avant la lettre*): conditionals and arguments contain as their main connective (*pars principalis*) a syncategorematic term that indicates (*designat*) that the consequent follows from the

[88] *idem*, 118.
[89] *idem*, 125. Cf. King's claim in his (1985) *Jean Buridan's Logic* that Buridan "does not distingish an object-language from a metalanguage, so it would be difficult for him to arrive at precisely our distinction between conditionals and rules of inference" (p.59).

antecedent. Thus they make statements *about* their constituent parts, and on these (syntactic) grounds, conditionals and arguments are much the same.

In his (2010) "Inferences", Stephen Read similarly claims that "The term *consequentia* translates most readily as 'inference'", and goes on to discuss inferences, setting aside conditionals altogether.[90] In this, he follows Paul Vincent Spade, who opts for 'inference' for *consequentia* in his (2005) translation of Walter Burley's *De Puritate Artis Logicae*.[91]

Glossing Read's (2010) and the (2001) article by King, Catarina Dutilh Novaes (2016) says that:

> it seems fair to say that, even though analyses of conditionals are often in the background (as is especially obvious in Boethius and Abelard, and in analyses of the syncategorematic term 'si', 'if'), *the main focus* of medieval theories of consequence tends to be the logical relations between sentential/propositional components [...] essentially (though not entirely) in the spirit of modern accounts of the notion of logical consequence.[92]

This seems right: as the textual evidence furnished by King shows, the majority of ink spilled over *consequentiae* in Buridan's logical works (not to mention those of other medieval authors) is over arguments, not conditionals.[93]

Yet—as Klima points out in passing in his (2016), and as we have seen in the present chapter—theories of consequence really *are* about both conditionals and

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[90] Read, "Inferences", 173.
[91] Walter Burley, *On the Purity of the Art of Logic*, trans. Paul Vincent Spade (New Haven: Yale UP, 2000).
[92] Catarina Dutilh Novaes, "Medieval Theories of Consequence", *The Stanford Encyclopedia of Philosophy*, (Fall 2016 Edition), ed. Edward Zalta, § 1.2. Emphasis added.
[93] King, "Consequence", 122-3.

arguments.[94] We therefore have to be careful not to take our claim about what theories of consequence are about too far: true, Buridan focusses primarily on arguments. Nevertheless, it does not follow that conditionals lack full membership in the *genus* of *consequentia*. Granted, medieval theorists of *consequentiae* have more to say about inferences than conditionals. Thus theories of *consequentiae* are *more* theories of arguments than of conditionals, in the way that zoology is *more* about dolphins than damselflies, since the former attract so much more research and attention than the latter. But from this, it does not follow that dolphins are more appropriate to biological research than damselflies. Nor does it follow that arguments are more appropriately called *consequentiae* than conditionals.

3.2. A More General Notion of Following Logically

All commentators agree on the etymology and basic meaning of *consequentia*—from (*con*)*sequi*, 'to follow'. In a *consequentia*, then one proposition follows from another. I submit that the relevant contrast for *consequentia*, considered as a class, is with other hypothetical propositions (*propositiones hypotheticae*) for which this is *not* the case. Recall that these include not only conditionals, but conjunctions, disjunctions, etc. for Buridan. If we look at other types of hypotheticals, we find they share one syntactic trait in common, which they do not share with *consequentiae*: *commutativity*. Order does not matter with propositions of the following forms:

[94] Klima, "Consequence", 319.

φ or ψ (which entails ψ or φ)

φ and ψ (which entails ψ and φ)

But of course with *consequentiae*, order matters, because 'followings' are not reversible. Note the corresponding non-commutativity of '$\vdash$' and '$\rightarrow$':

$\varphi \rightarrow \psi$ (does *not* entail $\psi \rightarrow \varphi$)

$\varphi \vdash \psi$ (does *not* entail $\psi \vdash \varphi$)

This is the basic sense of *consequentia*: in a 'following', order is crucial.

This is why, in (C_{Def}), Buridan sets out a syntactic definition in the first place: we have to distinguish hypotheticals where order matters from those where order does not, *before* we can get into the semantics of the expressions that make up those hypotheticals. That latter distinction *does* come, later on, as one that is posterior. First and foremost, *consequentiae* are defined in syntactic terms, as distinct from other hypotheticals. Accordingly, Buridan is then careful to distinguish them on further syntactic grounds: the characteristic syncategorematic terms of conditionals and arguments—*if* and *therefore*, respectively—impose different orders on their constituent expressions.[95] This is why we get the rather surprising syntactic distinction between the two in the *Tractatus de Consequentiis* passage (C_{Def}), which defines *consequentiae*, rather than the semantic one we hoped for (and got later on). This is therefore no lapse on Buridan's part: for him,

[95] As we saw at the outset of this chapter: "φ if ψ" is equivalent to ($\psi \rightarrow \varphi$), whereas "φ therefore ψ" is equivalent to ($\varphi \vdash \psi$).

consequentiae are to be defined syntactically, first and foremost, so as to set them apart from other, commutative, hypotheticals.[96]

Thus the Scholastic Latin term *consequentia* and our modern English term 'consequence' are, to put it mildly, *faux amis du traducteur*. The Latin term is broader, whereas the English derivative seems to have undergone semantic narrowing. *Consequentiae* are not just consequences. To take the additional step of translating *consequentia* as 'inference' across the board, as Stephen Read and Paul Spade do, is to pave over the versatility of this Latin term, and so to commit the very mayhem warned against by Benson Mates.[97]

[96] What are we to say about *reductio* proofs, where the premises are not all asserted? This is a difficult problem, both for Buridan and for Frege, and I am now working on a project that looks at it in both thinkers. Buridan briefly acknowledges this puzzle in a reply to an objection (*QAPr* II.12, ad. 3). These false premises, he says, are 'assumed' (*assumuntur*), but not asserted. Still, it is difficult to see how this is meant to work on his semantics for propositions.

[97] The epigraph to the present chapter: "Logicians will appreciate what mayhem would be committed in translating any modern logical treatise if one failed to distinguish between a true conditional proposition and a valid argument." (Mates, *Stoic Logic*, 90).

Chapter 2

What Makes *Consequentiae* Valid?

ἀνάγκῃ δ᾽ οὐδὲ θεοὶ μάχονται

—Simonides of Ceos[98]

Having seen in the previous chapter what *consequentiae* are, we can now consider the conditions under which they hold (or fail to hold). This is the foundational task of all logic, past, present and future. At the heart of this enterprise is a notion of *following logically*, to which we make implicit appeal in any argument. Take for instance the following arguments:

A1)	If donkeys fly, then donkeys have wings
	Donkeys fly
Therefore	Donkeys have wings.
A2)	If we followed set rules, then we'd be no better than machines
	But we *don't* follow set rules
Therefore	We can't be machines.[99]

[98] "Against necessity, not even the gods make war." (Cited by Plato, *Protagoras*, 345d).
[99] I adapted this from a sophistic argument Alan Turing presents in his "Computing Machinery and Intelligence", *Mind* 59, 236 (1950): 433-60. The full argument runs as follows: "If each man had a definite

Both (A1) and (A2) present a line of reasoning whereby true propositions are supposed to be generated from others. But these arguments differ in a crucial respect: (A1) is valid, whereas (A2) is not. Intuitively, this means that if we accept the claims in (A1) that come before the *therefore*, then we cannot reject the one that follows it, on pain of self-contradiction or logical inconsistency. Conversely, we might well agree with the claims made before the *therefore* in (A2), and nevertheless consistently reject its conclusion. Thus (A1) is valid, because its premisses guarantee the truth of the conclusion; and since the premisses of (A2) do not, (A2) is invalid.

Slightly more formally, we might say that when one proposition (a consequent) follows from another (an antecedent), it is impossible for the antecedent to be true, and the consequent false. In this way, the truth of the antecedent(s) guarantees or *necessitates* the truth of the consequent. Our task is to define this commonsense notion rigorously. We want to give a principled account of what separates wheat like (A1) from chaff like (A2).

Before I get into the details of Buridan's account, let me remark on its historical situation. There have been times when research into the foundations of logic has seemed especially urgent, and in those times the subject has received considerable study and debate. The fourteenth century, in which Buridan was writing, is one such time; the twentieth and our early twenty-first, another. In the twentieth century, research on logical consequence was kicked off by Alfred Tarski's seminal (1936) paper, "On the Concept of Logical Consequence". There, Tarski attempts to give a more rigorous account of the

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set of rules of conduct by which he regulated his life he would be no better than a machine. But there are no such rules, so men cannot be machines" (452).

intuitive notion of the consequence relation, which is "close in essentials" to the common concept.[100]

In the fourteenth century, the topic of logical consequence likewise suddenly emerged as an independent topic of debate, and whole questions, book chapters, and even stand-alone *tractatus* were devoted to it.[101] The crowning achievement of this enterprise in the fourteenth century is John Buridan's *Tractatus de Consequentiis*. Unlike most other medieval authors, who give catalogues of valid arguments but seem unconcerned with the completeness of their approach, Buridan sets out to "reduce arguments to their first causes (*primae causae*) in virtue of which they hold".[102] From these, he derives the rules of his logic.

In addition to this likeness in subject matter, there is a noteworthy conceptual parallel between our time and Buridan's. Like his twentieth century counterparts, Buridan is concerned with providing a more rigorous definition of the *commonsense* notion of logical consequence. That a medieval logician should take this approach is not surprising: medieval logic is frequently concerned with the practicalities of arguments in natural language in a way that modern logic is not.[103] Accordingly, an analysis of the commonsense notion of following does not seem at all out of place in the Middle Ages.

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[100] Etchemendy, *The Concept of Logical Consequence* (Cambridge, Mass.: Harvard UP, 1990), 1-2 (emphasis added). Of course this generalisation, like all generalisations, is too simple: as Etchemendy observes, Tarski relied very heavily on the definitions of earlier thinkers like Bolzano, Padoa, Bernays, Hilbert and Ackerman, and Gödel (7).

[101] Why this sudden emergence of books and chapters *de consequentiis*? This remains a bit of a mystery. Attempts to reduce this development to the study of Aristotelian Topics (such as Otto Bird (1961), and Eleonore Stump (1982)) have failed. See Niels Jørgen Green-Pedersen (1984), 270ff.

[102] "In hoc libro vellem tractare de consequentiis, tradendo sicut possem causas eorum" (*TA* I.1.7-8).

[103] Consider for example the study of fallacies and *sophismata*, a cornerstone of medieval logic with no modern counterpart.

It is more surprising, however, to see just such a commonsense analysis at work in the thought of a modern logician like Tarski, for whom the subject matter of logic is not natural language arguments, but rigorously defined objects. Yet, on this topic, Tarski and Buridan stand nearly shoulder-to-shoulder: as Tarski's stated aim makes clear (and as subsequent commentators like Etchemendy and McKeon are eager to point out),[104] what lies at the foundation of logic is an *intuitive* notion.

Hence the modern project to account for logical consequence has not been, as Tarski dryly remarks, "a matter of arbitrary decision on the part of this or that investigator".[105] Instead, it deals with a hazier, intuitive notion of what it means for one statement to follow logically from another—quite unlike many other modern pursuits in logic and mathematics, *e.g.* the study of arbitrarily well-defined objects like groups or real closed fields, or of modal systems like S4.[106] In these latter pursuits, it makes little or no sense to ask what the intuitive notion at play is, or how closely our formalisation approximates the commonsense idea. Not so the concept of logical consequence, which then was and now is fundamentally an intuitive one. Thus between the Middle and Modern Ages, the subject matter of research into the foundations and nature of logical consequence has changed relatively little: we are still in pursuit of the intuitive idea.

Yet Buridan's account is significantly different from the model theoretic or deductive approaches outlined by Tarski. Buridan's approach is, rather, a metaphysical one, which turns on falsifiability, which is a matter of causation, as we will see: briefly put,

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[104] See Etchemendy, *Concept*, 2; Matthew McKeon, *The Concept of Logical Consequence* (New York: Peter Lang, 2010), 19.
[105] Tarski, "On the Concept of Logical Consequence", *Logic, Semantics, Metamathematics* (Indianapolis: Hackett, 1983), 409.
[106] Etchemendy, *Concept*, 2.

if φ implies ψ, then there is no causal power capable of making φ true and ψ false. Let's look at this approach in greater detail.[107]

We have already met Buridan's definition of consequence in Chapter 1: a consequence, as we saw, is a kind of hypothetical proposition—specifically a non-commutative one: an argument or a conditional. Let's look again at the passage that sent us down this path:

> C_{Def}) Consequence can be described [*describi*] in the following way: a consequence is a hypothetical proposition (*propositio hypothetica*), made up of an antecedent and a consequent, indicating (*designans*) that the antecedent is antecedent, and that the consequent is consequent; and this indication comes about through the word (*dictio*) "if" (*si*) or "therefore" (*ergo*), or an equivalent.[108]

Thus the consequence relation holds between multiple propositions.[109] Those propositions are defined correlatively, as *antecedent* and *consequent*.[110] Let's begin our inquiry with Buridan's definition of these terms:

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[107] Buridan *does* however endorse a substitutional notion of logical consequence, roughly analogous to Tarski's. We will see what Buridan has to say about substitution in Chapter 3 and Chapter 4.

[108] "Consequentia autem potest describi sic: consequentia est propositio hypothetica ex antecedente et consequente composita, designans antecedens esse antecedens et consequens esse consequens; haec autem designatio fit per hanc dictionem 'si' vel per hanc dictionem 'ergo' aut aequivalentem" (I.3.60-4).

[109] To be perfectly precise, these are not stand-alone propositions, as we have seen in the preceding chapter: they are, rather, equiform with would-be stand-alone propositions. But Buridan often lapses into speaking of them as though they were multiple propositions, doubtless for brevity's sake. I follow him in this.

[110] "Antecedens autem et consequens relative dicuntur ad invicem; ideo per invicem describi debent." *TC* I.3.26-7. Note also that, "if one proposition is 'antecedent' to another, that means that the other does indeed follow from it", as Stephen Read points out in "The Medieval Theory of Consequence" (*Synthese* 187 (2012), 904).

A/C$_{Def}$) One proposition is antecedent to another which is related to it (*se habet ad illam*) in such a way that it is **impossible** that things should be as the former **signifies**, and not be as the latter signifies, when they are formulated at the **same time** (*simul propositis*).[111]

In (A/C$_{Def}$), there are three requirements that merit closer attention, which I have made bold in the text above. In what follows, I will address these in reverse order, starting with the last.

First, the propositions involved in a consequence must be simultaneously formulated (*simul formatae*). I'll henceforth call this the SF Requirement.[112] This requirement stems from Buridan's anti-realism about universals, which imposes two very severe restrictions on propositions' existence: (i) they exist only as particular thoughts, not as types, and so (ii) they do not exist unless they are being thought (or, in a derivative sense, spoken or written).

To borrow Gyula Klima's handy phrase, Buridan's propositional semantics is *token-based*—a term I will use here to describe restrictions (i) and (ii) on propositional existence and types.[113] The SF Requirement introduces some interesting problems, which

[111] "Illa propositio est antecedens ad aliam quae sic se habet ad illam quod impossibile est qualitercumque significat sic esse quin qualitercumque illa alia significat sic sit ipsis simul propositis" (*TC* I.3.48-51; emphasis added).

[112] It may be worried that the SF requirement entails that no consequences can be about non-actual things, including future contingents and unrealised possibilities. But Buridan distinguishes the time *at which* a proposition is true from the time *for which* it is true, as we will see in §1.2, below; and he has a whole semantics for unrealised possibilities, as we'll see when we turn to his semantics of ampliation in Chapter 5.

[113] See Gyula Klima, "Logical Validity in a Token-based, Semantically Closed Logic", *John Buridan* (Oxford: Oxford UP, 2009), 210-33.

we will consider in §1.2, below. Buridan anticipates these problems, and furnishes solutions of his own. This is good news, and not just in the obvious way: as we will see, these solutions cast new light on the question of arbitrary propositional complexity, considered above (Chapter 1, §1.2.4).

The second requirement in (A/C_{Def}) is that things have to be as the propositions in question *signify*—not just that the propositions be *true* (something we'll see in the discussion of Prior in §2.1, below). This clause is a vital component of Buridan's account (as we will see in §2 below), since it invalidates the following sophism:

A3) No proposition is negative
∴ No donkey runs.[114]

Clearly, there is something wrong with (A3): things could be as the antecedent describes, without being as the consequent does. But if we speak only of the truth of a proposition, rather than things being as it signifies, then it seems we have to consider (A3) valid. After all, the antecedent is a negative proposition which, since it claims there are no negative propositions, falsifies itself: the antecedent of (A3) can therefore never be true whenever it is formulated. So it is not possible for the antecedent to be true while the consequent is false, provided they are formulated at the same time.

Of course, a situation in which no negative propositions exist is entirely possible—God could, after all, just annihilate them all. Then the antecedent of (A3) would be true. But supposing that all negative propositions *were* eradicated, this dearth of

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[114] "Nulla propositio est negativa; ergo nullus asinus currit" (*TC* I.3.40).

negative propositions wouldn't stop every donkey from being at rest. That is, we could have a situation in which there were no negative propositions, and in which all the donkeys were at rest. In such a case, things would be as the antecedent signifies, but not as the consequent does. Thus it is entirely possible for things to be as the antecedent of (A3) signifies, without being as the consequent signifies. Arthur Prior famously formulated this as a distinction between the *possible* and the *possibly-true*; in §2.1, below, I'll examine this distinction.

But Buridan's adoption of the Signification Requirement, as I'll call it, leads to an important worry. As we will see (in §2.2), Gyula Klima and David Kaplan think that Buridan's rejection of arguments like (A3) presupposes more than a token-based semantics allows. But this worry not insurmountable.

The third requirement—and in many respects the most interesting one—is the Modal Requirement: a valid logical consequence follows of *necessity*. I deal with this in §3, below, but here is a brief overview. For Buridan, modality, construed in terms of causal powers, underwrites logical consequence: to say that a consequence is holds necessarily is to say that there is no power on Earth (or in Heaven) that can make things to be the way the antecedent signifies, but not the way the consequent does. We will see this in our analysis of Buridan's account of necessity in the *Summulae de Demonstrationibus* and the *Quaestiones in* Analytica Priora, in §3. For Buridan, this modal notion is more foundational even than formality as a criterion for validity—with the result that Buridan has to expand *consequence* to cover so-called *material* validity as well as formal validity (as we will see later on in Chapter 4). But first, the SF Requirement

1. The Simultaneous Formulation Requirement

As we've already noted, Buridanian propositions are not types, but tokens: that is, they are not abstract objects, but concrete and individual acts of thinking. Propositions are, therefore, contingent on our thinking them. Accordingly, they have limited life-spans: if a proposition is not now being thought (or, in a derivative sense, spoken or written), then it quite literally does not exist.

It's natural, then, to require that the propositions involved in a consequence actually coexist. If they didn't, there would be no *relata* for the consequence to relate. It is for this reason that Buridan first presents, and then rejects, the following definition of logical consequence:

> Many say therefore that one of a pair of propositions is antecedent to the other when it is not possible for the one to be true without the other's being true [*illa alia non existente vera*].[115]

Those who endorse this account leave out any requirement that the propositions involved actually *exist*. Who are the many saying this? Presumably, they're those thinkers who are realists about propositions. Such thinkers are more plentiful among modern than medieval logicians. But one prominent medieval realist, contemporary with Buridan (ca. 1300-1358), is Walter Burley (ca. 1275-1344), who defines consequence as follows:

> The first rule of logical consequence is this: in every good simple inference, the antecedent cannot be true without the consequent. Accordingly, if in

[115] "Dicunt ergo multi quod propositionum duarum illa est antecedens ad aliam quam impossibile est esse veram illa alia non existente vera" (*TC* I.3.34-5).

> some posited possible case the antecedent could be true without the consequent, then the consequence is not good.[116]

For a realist like Burley, it's natural to leave out talk about propositional existence: all that matters is the *truth* of the propositions involved in an inference, since propositions exist independent of our thinking them.

Of course, Buridan cannot take this approach to propositions without completely undermining his anti-realist metaphysics. So for him, definitions like Walter Burley's are not *wrong* but "deficient or incomplete".[117] For, as Buridan says,

> It is possible for the antecedent to be true while the consequent is not, *indeed where the consequent does not exist at all*.[118]

This is so because a proposition has to exist in order to be true (or false). And so, in a case in which a conclusion is unformulated, it is possible to have a true premiss and a conclusion that fails to be true. Therefore, the premisses and conclusion have to *exist*, and exist at the same time, in order to be involved in a logical consequence. Accordingly, Buridan adds to (A/C_{Def}) the supplementary clause stipulating the SF requirement: the propositions involved in a consequence must be formulated simultaneously.

This SF Requirement gives us two advantages, which themselves produce two difficulties. I'll consider these in turn. As we will see, Buridan's solutions to these two

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[116] "Prima regula consequentiarum est illa: in omni consequentia bona simplici antecedens non potest esse verum sine consequente. Et ideo, si in aliquo casu possibili posito posset antecedens esse verum sine consequente, non fuit consequentia bona". Walter Burley, *De Puritate Artis Logicae*, ed. Philotheus Boehner (St Bonaventure, NY: The Franciscan Institute, 1951), 1, ll. 23-7.

[117] "Sed haec descriptio deficit vel est incompleta" (*TC* I.3.32).

[118] "Et tamen possibile est primam esse veram secunda non existente vera, immo secunda non existente" (*TC* I.3.27-9, emphasis added).

difficulties cast new light on the subject of propositional complexity, introduced above (Chapter 1, §1.2.4).

First, the advantages.

1.1. Two Advantages: Propositional Existence, Shifting Circumstances

The first advantage is mainly one for anti-realism. For an anti-realist, a consequence does not exist unless it exists in thought. Accordingly, there are many propositions that might well imply others, but don't, simply because their would-be consequents just do not exist. This happens when we formulate a proposition but do not arrive at what it implies. Imagine, for instance, I were to formulate the following proposition:

P1) The interior angles of any triangle add up to 180°

This proposition can serve as the premise of a valid inference, like the following:

A4) The interior angles of any triangle add up to 180°
∴ Each interior angle of an equilateral triangle is 60°

But suppose I am distracted, drained, or dull, and formulate only (P1), without arriving at the conclusion of (A4). For a token-based semantics, there is no consequence here, since all the parts of the would-be inference (A4) are not simultaneously present. But the SF

Requirement stipulates that they have to exist to take part in a valid consequence—in a slogan, *simultaneous formulation entails existence.*

In this way, the SF Requirement allows us to rule out would-be but nonexistent arguments, like the inference from (P1) to the conclusion of (A4), when this conclusion goes unformulated. A bare proposition like (P1) has no status as an argument in an anti-realist framework, whatever it might imply. Granted, (A4) *would* be valid if its premises and conclusions were formulated at the same time. But for an anti-realist like Buridan, any account of consequence that leaves out the requirement that the propositions involved in it *actually exist* is deficient, as we saw above. Thus the first advantage of the SF requirement is that it allows us to focus our discussion only on those propositions that actually exist. We do not, therefore, have to go chasing down and accounting for spooky non-existent would-be premisses and conclusions. If they don't exist, they're not our concern.

A second and greater advantage is that this requirement allows us to rule out invalid arguments whose premisses and conclusions can be true across time, but cannot be true all at once. Circumstances change, and so incompatible propositions can be true at different times. For instance, consider the following pair:

P2) It's nighttime

P3) It's day

These propositions take turns being true and false. But they are never both true at the same time, and so if they were formulated at once, one would be true, and the other false. I cannot formulate (P2), wait, and then formulate (P3), and call this move from (P2) to (P3) a valid inference. Interrupting one's utterance between the two is not enough to render an inference from (P2) to (P3) valid.

Hence we need to take into consideration the time of utterance of the propositions involved in a consequence. If we ignore the time(s) in which propositions in a putative argument are formulated, then a true premise could be antecedent to a false conclusion, provided enough time elapses between the formulation of the two. For instance, suppose it is nighttime, and I utter the following proposition:

P2) It is nighttime

I intend to arrive at the following (valid) consequence:

A5) It is nighttime
∴ It is dark

But I dally. And before I get around to formulating the consequent of (A5), day breaks. By now the consequent has become false. Still, this says nothing about the validity of (A5): clearly, if it were formulated all at once, it would be valid. The SF Requirement thus ensures that the validity of arguments like (A5) gets preserved, even though the conditions they describe are subject to diachronic change. So the SF Requirement allows us to rule out invalid diachronic inferences, like (P2) to (P3), and to countenance valid but interrupted inferences, like (A5). What matters when we formulate a consequence is the time at which it is uttered, and which delineates our circumstances of evaluation.

Now, on to the difficulties.

1.2. Two Difficulties: Propositional Existence, Shifting Circumstances

If the SF Requirement is applied too strictly, the advantages outlined above turn sour. The first advantage—that the SF Requirement limits the scope of our discussion to existent propositions—runs into the problem that thinking or saying anything takes time. Once I've formulated the antecedent, the consequent doesn't exist yet. But once I formulate the consequent, it seems the antecedent no longer exists.

Similarly, the second advantage—that the SF requirement rules out changes in circumstances—runs into the problem that reasoning also takes time, and conditions can change while we're in the middle of an inference. So how much time do we get to infer the premisses from the conclusion?

Call these the *ontological problem* and the *shifting-sands problem* for the SF Requirement, respectively. Let's take the ontological problem first. Since propositions are not formulated all at once, the constitutive parts of a proposition do not strictly speaking all exist at the same time. This is a problem both for categoricals and for hypotheticals, as Buridan observes in his *QAPr*.[119] He discusses the former in detail in *Sophismata* (7,

[119] *QAPr* I.3, obj.4.

soph. 1), and as we will see the problems are much the same. The sophism in the latter is "No spoken proposition is true", the argument for which runs as follows:

> There is never a proposition without a subject or without a predicate; therefore, a proposition never exists unless both its subject and predicate do; but they never exist, since when the subject exists, the predicate doesn't yet, and when the predicate does, the subject doesn't any more, but has ceased to exist. Therefore, etc.[120]

What holds for spoken categoricals holds likewise for hypotheticals, both in speech and in discursive thought, as can readily be seen by replacing *subject* and *predicate* with *premiss* and *conclusion* throughout in the above text. And indeed, we find a parallel (if terser) objection in the *QAPr*, making a parallel point about syllogisms, a species of hypothetical. A syllogism, the objection runs, does not have simultaneously existing parts:

> When a conclusion is reached, already the premisses no longer exist, but are gone. And therefore, the conclusion does not follow because the premisses *are*, but rather because they *were*.[121]

Thus, categoricals and hypotheticals alike have to meet an SF Requirement. But strictly speaking, neither can: we move from subject to predicate or from premiss to conclusion in speech and, at least for hypotheticals, in thought as well. And since this move takes time, not all the constituent parts of any proposition will exist at once.

[120] "Numquam est propositio sine subiecto vel etiam sine praedicato. Ideo numquam est nisi quando subiectum et praedicatum eius sunt. Sed numquam sunt, quia quando subiectum est, praedicatum nondum est, et quando est praedicatum, tunc amplius subiectum non est, sed transivit; ergo etc." (*Sophismata* 7, soph. 1; 34v, Scott 113).

[121] "quando conclusio concluditur, iam praemissae non amplius sunt, sed transiverunt; et ideo conclusio non sequitur ex eo quod praemissae sunt, licet forte sequatur ex eo quod fuerunt." (*QAPr* I.3, obj. 4).

Such a result is, of course, intolerable. And indeed, when Arthur Prior comes to this conclusion, he surmises that the SF requirement is "a somewhat unrealistic provision".[122] But Buridan has a solution, which turns on his divisibilism about time: since there is no basic, indivisible unit of time to make up an instantaneous present, what we take as the present is a matter of more or less arbitrary convention. Therefore, when we speak in the present tense, we can take, for the present, as much time as we like. Thus we can expand the present, so that it is broad enough to encapsulate both subject and predicate, in the case of a categorical, or premiss and conclusion, in the case of a hypothetical. (Or we can narrow it, if we want to speak—as we have been—of the subject term of a proposition no longer existing once the predicate does). This is the solution he settles on in the *Sophismata*, and the one we will ultimately adopt here.

Yet this solution is not the only one available. It is noteworthy that the solution in the *QAPr* is different—and also that it faces considerable difficulties that the *Sophismata* one does not. In the *QAPr*, Buridan admits that, strictly speaking, the premisses and conclusion of a syllogism do not exist at the same time. Nevertheless, the two form one syllogism with simultaneously existing parts, because they are formulated in *succession*:

> I grant that in the precise time in which the spoken conclusion exists, the premisses no longer do. Still, as concerns the total time in which the conclusion and the premisses are formulated, I say that the premisses and conclusion *do* exist—not, however, because of their simultaneous presence, but because of their *succession*.[123]

[122] Prior, "Fugitive Truth", 7.
[123] "concedo quod in praeciso tempore in quo conclusio vocalis est praemissae non sunt. Tamen in totali tempore in quo conclusio et praemissae formantur, ego dico quod praemissae et conclusio sunt, non tamen

There is a clear parallel between the *QAPr* objection about the syllogisms and their parts, on one hand, and the *Sophismata* sophism about spoken categoricals and their parts, on the other. Buridan notes this parallel in the *QAPr*, and gives a brief rebuttal of the latter sophism in his response:

> And from this point the sophist argues likewise that you cannot say anything true. For, when you put forth the subject of the proposition, the predicate doesn't exist yet, and therefore there is no proposition. And when you put forth the predicate, the subject no longer exists, and therefore there is no proposition. Therefore, your proposition never exists, and therefore it is never true. Thus, you cannot say anything true.[124]

The solution, argues Buridan, is to take the subject and predicate as unified by their succession, as he did with the syllogism's premisses and conclusion:

> Solution: I grant that your proposition does not exist in precisely the same time as the subject; neither does it exist in precisely the same time as the predicate. But the proposition *does* exist in the total time made up of the time of the subject and the time of the predicate. And therefore it is true in that time taken together, and its truth is on account of *succession*—as is its existence.[125]

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secundum permanentiam simul, sed secundum successionem. Unde per istum modum argueret sophista quod tu non posses mihi verum dicere." (*QAPr* I.3 ad 4)

[124] "Unde per istum modum argueret sophista quod tu non posses mihi verum dicere: quia quando tu profers subiectum propositionis, praedicatum nondum est, ideo non est ibi propositio; et quando tu profers praedicatum, subiectum non amplius est, ideo etiam non est propositio; ergo numquam est tua propositio; ideo numquam est vera, et sic tu non potes verum dicere." (*ibid.*)

[125] "Solutio: concedo quod tua propositio non est in praeciso tempore subiecti, nec in praeciso tempore praedicati; sed ipsa est secundum successionem in totali tempore composito ex tempore subiecti et tempore praedicati; ideo etiam in illo totali tempore est vera, et est eius veritas secundum successionem, sicut esse eius" (*ibid.*; emphasis added).

Hence since the times of formulation for the subject (or premisses) and for the predicate (or conclusion) are contiguous, the categorical (or hypothetical) proposition can be treated as one continuous whole, and analysed as such.

This solution faces a significant difficulty, however: what happens when the sequence of formulations for the subject (or premiss) and predicate (or conclusion) gets interrupted? If that happens, succession is lost. Consider for instance the following bit of dialogue from Shakespeare's *All's Well That Ends Well*:

> Helena: [...] Now shall he—
> I know not what he shall. God send him well!—
> The court's a learning-place, and he is one—
> Parolles: What one, i' faith?
> Helena: That I wish well. 'Tis pity—
> Parolles: What's a pity?
> Helena: That wishing well had not a body in't,
> Which might be felt.[126]

The effect is comic, the phenomenon commonplace: in Shakespearean courts and academic conferences alike, interruptions like these happen all the time. The problem is, Parolles' interruptions of Helena's successive utterances means that these utterances are not really united by succession (*secundum successionem*) at all. Parolles' questions break up the

[126] William Shakespeare, *All's Well That Ends Well*, ed. William Craig (London: Oxford UP, 1966 [1905]), I.i.191-8.

succession of Helena's utterances, and so there are conversational and temporal gaps between these utterances. Therefore, these utterances are not really one by succession at all. Do we therefore have to say that such interrupted propositions do not count as unified at all, since the times their parts take up are not contiguous?[127]

Probably not. Some time between the writing of the *Questions on the Prior Analytics* and the *Sophismata*, Buridan seems to have changed his mind about the solution.[128] Perhaps the problem just considered is what motivated the change. In any case, when we turn back to the *Sophismata* passage with which we began, we find a different solution:

> But you object that when the subject exists, the predicate doesn't. And I grant this. But it is also true that when the predicate exists, the subject does too, given that the term *when* [*quando*] is here taken indefinitely [*indefinite*] for some time, for instance for the hour in which both the subject and predicate exist.[129]

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[127] This is related to a more general ontological problem of identity across time, which Buridan deals with in his *Quaestiones super octo libros Physicorum Aristotelis* I.10, especially as pertains to identity of persons, as well as of things like rivers, whose flow not only alters their physical makeup, but is subject to interruption. For a lively and interesting discussion, see Olaf Pluta, "Buridan's Theory of Identity", *The Metaphysics and Natural Philosophy of John Buridan*, ed. J.M.M.H. Thijssen and Jazk Zupko (Leiden: Brill, 2001), 49-63.

[128] The claim that Buridan changed his mind depends on the sequence of composition of the works. It is generally agreed that the *Sophismata* is a later work. In light of the succession problem just considered, and in the absence of any further evidence, it is fair to assume that the difference between these two works is the product of a change of mind: Buridan's later, more mature presentation in the *Sophismata* does not face this succession problem, and might well have been adopted in light of the same or similar challenges to the *QAPr* account. Whether philosophical views generally mature and improve with the age of their author is another question altogether, which will not be taken up here.

[129] "Sed tu obicis quod quano subiectum est praedicatum nondum est, concedo. Sed etiam verum est quod quando subiectum est praedicatum etiam est, quia hic accipitur iste terminus 'quando' indefinite. Modo in aliquo tempore, scilicet in ista hora tam subjectum quam praedicatum sunt" (*Sophismata* 7, soph. 1; 34v, Scott 114).

Hence the parts of a proposition, categorical or hypothetical, need merely to be formulated in the present, which is not strictly delimited. Hence we can take an hour as the life-span of a proposition, if need be. And this is plenty of time to formulate a proposition, hypothetical or otherwise. What matters in the *Sophismata* solution, then, is not that the subject (or premiss) and predicate (or conclusion) are formulated at the same time, considered in an arbitrarily narrow way. Nor does the *Sophismata* posit that they be formulated in strictly successive and therefore contiguous instants of time, as the *QAPr* does. Rather, what matters is in the *Sophismata* account that they are formulated within some indefinite time, taken as the present. This seems more reasonable, and so it's the view I'll adopt and explore.

More generally, we might ask: When it comes to delimiting this indefinite present, how much time do we get? The short answer is, "as much as we want":

> But then you ask, 'how long is the present time, since there is no such thing as an indivisible instant? [...] And I say that it isn't determined for us how much time we ought to use as the present, but we may use as much as we like. For we call this year the present, and this day the present, and this hour as the present.[130]

The present is elastic, and so we have all the time we need to move from subject to predicate, or from antecedent to consequent. Apparently, then, the span of the present just depends on speaker intention—as we will see in Buridan's solution to the second problem.

[130] "Sed tu quaeres quantum est ergo tempus praesens, cum non sit instans indivisibile [...] Et ego dico non est nobis determinatum quantum sit tempus praesens quo debemus uti tamquam praesente. Sed licet nobis uti quanto volumus, vocamus enim istum annum praesentem et hanc diem praesentem et hanc horam praesentem" (*Sophismata* 7, soph. 1; 34v, Scott 113).

The second problem is not about the existence of the propositions involved in a consequence, but about the ways the contextual background can shift in the middle of a single unified round of ratiocination. I call this the *shifting-sands problem*. Such shifts happen all the time. Indeed in some circumstances, a shift is virtually guaranteed, since propositions can take longer to formulate than the things they are about. Consider the following example, (slightly) adapted from Arthur Prior:

A6) Eclipse is now just past the winning post
∴ Eclipse wins the Triple Crown.[131]

Just the premise of (A6) will take longer to formulate than the event it describes. So, *a fortiori*, will the whole consequence. Hence, to borrow Prior's handy distinction, it can't be a matter of the time *in* which the antecedent and consequent of (A6) are true. Rather, it has to be a matter of the time *for* which they are true. But it is still not clear what determines the time *for which* the propositions are true.

This problem has received three treatments in the literature: Arthur Prior's (1968), Ernesto Perini-Santos' (2008), and Calvin Normore's (2012). These accounts are more or less at odds, as we will see in a moment. Yet it's worth noting that all three writers agree that grammatical form does *not* give us any reliable information about the scope of tense. This must be correct: nothing about present-tensed verbs themselves tells

[131] Arthur Prior, "Fugitive Truth", *Analysis* 29, 1 (October 1968), 5.

us about how much time is being talked about. For instance, contrast the following propositions:

P4) The time is noon

P5) The earth is spherical

Both (P4) and (P5) have the same grammatical form: “The S is P”. But the present of these present-tensed propositions is clearly very different: (P4) is much more limited than (P5). And most of the time, (P4) is false; but (P5) stays true. Hence there is nothing about the grammar of these two propositions that tells us how extended the present tense they correspond with is. How, then, do we know?

The first treatment of the problem of the scope of the present tense is Arthur Prior’s “Fugitive Truth” (1968).[132] Prior gives us the Eclipse example, discussed already, and attributes the problem to Anthony Kenney—though he notes that Buridan is aware of it, and that he discusses it in the *Sophismata*.[133]

At one point, Prior suggests that the solution to this problem is a matter of convention: we just have to pin the time of evaluation (i) to the duration of an utterance, or (ii) to some point in it. In this way, a proposition will be true if what it describes remains true (i) throughout the duration of the proposition’s formulation, or (ii) at some point in that formulation.[134]

[132] Arthur Prior, “Fugitive Truth”, *Analysis* 29, 1 (1968), 5-8.
[133] Prior, “Fugitive Truth”, 5, n.2.
[134] “It is clear”, he says, “that we need to make our conventions a little more explicit” (6).

But Prior is lukewarm in his endorsement of this solution, and rightly so. Requiring (i), that what a proposition describes be true throughout the duration of its formulation, will rule out propositions like the antecedent of the Eclipse example (A6).

To address this, we might be tempted to just shift the tense of the proposition in question to the past tense: a proposition like the antecedent of (A6) is not *now* true, but *was* true. But, as Prior points out, we can't skirt this problem by changing the tense. To see why, look at what happens when we do this with the antecedent of an argument like (A6), so that we formulate it as (P6′) instead of (P6):

P6) Eclipse is just past the winning post

P6′) Eclipse *was* just past the winning post

The problem is, how do we read (P6′)? Prior thinks we should cash it out as an embedded proposition (of the form "it was the case that φ"). But then we get the following:

P6″) *It was the case that* Eclipse is just past the winning post

The problem is, the truth of this proposition depends on the truth of (P6)—that is, on it having been the case that (P6) was true. So we wind up with our original problem: we can't account for the truth of (P6′) without relying on the truth of the embedded proposition (P6). But we resorted to the past-tense (P6′) precisely to *avoid* the problems associated with the present-tense (P6). So we are right back where we started: our fix is really no fix at all. And in the end, Prior throws up his hands: there is, he tells us, "a genuine difficulty, which I do not know how to solve".[135]

The second view is presented by Calvin Normore in his "Buridanian Possibilities" (2012).[136] Normore thinks propositions' subject matter at least *sometimes* determines the scope of their present tense. For example, consider the following proposition (from Normore):

P7) Summer days are longer than winter days.[137]

Now (P7) is true. But in order for (P7) to be true, we have to have a present tense broad enough to include at least some summer days, and some winter ones as well. If the present tense were any narrower than that, the term *summer days* or the term *winter days* (or both) would have an empty extension, and so (P7) would be false. Hence Normore concludes, because of what (P7) deals with, we can roughly gauge the scope of its present tense: "On Buridan's view, we can *sometimes* read off the utterance itself something about which time is being taken as the present".[138]

It's hard to know what to make of this summer-winter days example. It is Normore's own, and appears nowhere in Buridan. For one thing, it's not at all obvious

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[135] Prior, "Fugitive Truth", 7.
[136] Calvin Normore, "Buridanian Possibilities", *Logic and Language in the Middle Ages: A Volume in Honour of Sten Ebbesen*, ed. Jakob Leth Fink, Heine Hansen and Ana María Mora-Márquez (Leiden: Brill, 2012), 389-402.
[137] Normore, "Possibilities", 392 (emphasis added).
[138] *Ibid.*; emphasis added.

that (P7) needs to include in its present just *some* summer and *some* winter days, as Normore suggests. Rather, the claim made by (P7) is general enough that it needs to refer not just to a few summer days and winter days, but all of them. After all, formally similar comparisons like "Humans are taller than giraffes" can likewise be true if we are implicitly referring only to baby giraffes and professional basketball players. But this seems wrong: general statements require general classes of reference, as a general rule.

But there is a further and more serious problem with this example. It's not at all obvious that we need to have a present broad enough to include summer and winter days in order to compare the two. Rather, it seems that what we would make reference to to support such a claim is just an appeal to astronomical facts about the shape of Earth's orbit around the sun and the tilt of its axis. These facts are sufficient to explain the difference in daylight between summer and winter days, and do not require us to expand the present tense to summer and winter. In any case, better to stick with one of Buridan's examples of lawlike statements, which take all time as their present. But Normore's point is well-taken: the content of a proposition will not reliably tell us the breadth of the present. So we can only *sometimes* read off the utterance what it takes to be the present. So the problem remains.

The third approach to this problem in Buridan is that of Ernesto Perini-Santos, in his "John Buridan on the Bearer of Logical Relations".[139] Perini-Santos finds Prior's conventions-based approach, set out above, *ad hoc*—though, in all fairness, Prior seems to

[139] Ernesto Perini-Santos, "John Buridan on the Bearer of Logical Relations", *Logica universalis* 2 (2008), 59-70.

think so, too.[140] Instead, Perini-Santos turns to Buridan's approach to a similar problem in the *Sophismata*, where he discusses the sophism "No one can contradict my proposition". Here is Buridan's presentation of this sophism:

> If I say 'Socrates runs', you do not know what I am going to say until I have finished speaking. And therefore you do not know what to say in order to contradict me until I have finished speaking. And so you cannot contradict me, since your proposition will not be formulated at the same time as mine, which is what a contradiction requires.[141]

Hence there is an SF Requirement for contradictory propositions, and in the strict sense no two propositions can meet it. So no one can contradict my proposition.

Buridan's solution to this sophism is that, in the case of the logical relation of contradiction, it is enough for the speaker to bind or refer (*referre*) the main verb of their proposition to the main verb of the one they want to contradict.[142] Hence, propositions need not be formulated *at* the same time; rather, they must be formulated *for* the same time. This allows you to deny what I just asserted. And what determines the time for which they are true is speaker intention:

> Contradictory propositions have to be about the same subjects, the same predicates, and they have to be about the same circumstances. Therefore, it should be said that contradictories should have the same subject, the same

[140] As Perini-Santos notes, "Prior himself seems to realize how hard it is to accept conventions associated with each occurrence of each sentence" (65).
[141] "Si ego dico 'Sortes currit', tu ignoras quam propositionem debeam dicere donec locutus sum. Ideo nescis mihi contradicere donec locutus sum: et tunc contradicere non potes, quia non eodem tempore, quod requiritur ad contradictionem." (*Sophismata* VII.2; 35r).
[142] "Debeo referre verbum propositionis meae ad idem tempus ad quod referebas verbum propositionis tuae ita quod intentio sit pro eodem tempore negare pro quo tu affirmabas et econverso" (VII.2; 35r)

> predicate, both in utterance, and in intention as well. Therefore, in intention, I have to bind [*referre*] the verb of my proposition to the same time to which you bound the verb of yours. In that way, my intention will be to deny [something] about the same time as that for which you affirmed [something], or vice versa, even though that time existed at the same time as your proposition, but not mine.[143]

Accordingly, the intention to bind (*referre*) the present tense of my proposition to that of the proposition I wish to contradict is vital: it is this intention to bind that allows the present tense of two contradictories, formulated at different times, to be about the same time. On this reading of Buridan, so far Perini-Santos and I agree.

Further still, in Perini-Santos' view, this intention to bind underwrites *all* logical relations—as indeed the title of the paper makes clear. So for him, what is going on in the *Sophismata* account of contradictory propositions is the same as what's going on with the consequence relation as well. Accordingly, in his treatment of the Square of Opposition, he tells us, "one should not lose sight of the fact that, for Buridan at least, one is talking about statements made with certain intentions".[144] Thus Perini-Santos extrapolates from his observations about Buridan's view on contradictions: what holds of contradictories, Perini-Santos thinks, holds for all logically related propositions. He then surmises that Buridan's propositional semantics is "Austinian-like", since (on his reading) Buridan holds

[143] "contradictoriae debent esse de eodem subiecto et de eodem praedicato et consomilibus circumstantiis. Ideo dicendum est quod propositiones contradictoriae debent esse de eodem subiecto et de eodem praedicato secundum vocem et etiam secundum intentionem. Ideo secundum intentionem debeo referre verbum propositionis meae ad idem tempus ad quod referebas verbum propositionis tuae ita quod intentio sit pro eodem tempore negare pro quo tu affirmabas aut econverso, licet illud tempus coexisteret propositioni tuae et non meae" (*Sophismata* VII.2, 35r).

[144] *Idem*, 67.

that statements, not sentence-types or even sentence tokens along with facts about their context, determine truth.[145, 146]

For me, this final extrapolation is a bridge too far. Granted, the contradiction relation *does* hold between multiple propositions. And, as Buridan points out, in order to contradict I have to intend to bind (*referre*) the present-tense of my proposition to that of your proposition. But it is not at all obvious that observations about contradictories apply generally to all logical relations, including consequence.

First, as we have already seen in Chapter 1, consequences are not strictly speaking multiple propositions at all, but single ones, bound by a syncategorematic particle (*if*, *therefore*). This particle binds the two propositions into one, which will have a single, unified time to which it extends. So hypotheticals like conditionals and arguments bind their constituent parts by means of a *formal* component, and need not rely on speaker intention. Contrast this with contradictory pairs, which as a matter of syntax lack a binding particle, and are indeed stand-alone propositions. Thus the diagonal lines on the Square of Opposition are not composite propositions, the way a hypothetical proposition is. Accordingly, new pragmatic rules need to be trotted out to justify the unity of the former, but not of the latter. Accordingly, there needs to be something to bind these pairs of stand-alone propositions, and intention does the trick. But this does not entail that all logical relations among propositions rely on intention in the way suggested. Rather, the role of binding or referring the present tense of one expression to another is just done by a

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[145] "Logical Relations", 66f. This sets Perini-Santos at odds with Klima, by the way, for whom Buridan's semantics is, as we've said, *token based*. But he doesn't cite Klima or seem to notice.
[146] It is also a little odd that Perini-Santos is so eager to describe Buridan as an *Austinian*. I don't want to get into nitty-gritty metaphysics of time here, but given our ordinary intuitions about the direction of time's flow, wouldn't it make more sense to describe Austin as a *Buridanian*?

term like *if* or *therefore*, when it serves as the principal part of a hypothetical proposition. As Buridan says in (C_{Def}), such a particle has the role of indicating (*designans*) that a consequent follows from an antecedent. It is simpler to see the binding of the present tenses of these as part of the indicating work done by the syncategorematic particle, rather than by speaker intention.

Further still, it is noteworthy that Buridan's examples in the *Sophismata* passage in question are focussed on second-person interactions: *you* make a proposition, and *I* want to contradict it, or vice versa. Because I aim to contradict you, I have to intend to refer the main verb of my proposition to the main verb of yours, so that they deal with the exact same time. Otherwise, I fail to contradict you. Conversely, the sort of *consequentia* Buridan typically has in mind is a single expression (*oratio*) put forth by a single speaker. Consider for instance the following:

A5)	It is nighttime
Therefore	It is dark

When a single speaker puts forth (A5), what is going on with the verbs? Surely our speaker will intend to bind the verb *is* to some time or other; but then does the speaker need to intend to bind the second *is* to the same time as the first? No: all that's needed is a binding particle. And we have one such with *therefore*, which makes the two expressions into one. So no fancy footwork with intentions is needed here.

To sum up: Perini-Santos's analysis of contradictory pairs of propositions is spot-on. But his extrapolation from them to all logical relations among propositions—and in particular to consequences—is unwarranted: Buridan is no Austinian, and neither is Austin a Buridanian in this regard. This is not to say that intention plays no role in consequences: the scope of the present tense of the horserace example (A6) is clearly different from this one:

A7) Earth is a planet
∴ Earth is a celestial body

But apart from determining the scope of the consequence *qua* single, hypothetical proposition, intention is not necessary to account for the logical relation between the propositions involved in it: we do not need intention to do any binding of the antecedent to the consequent, over and above what the logical particle *if* or *therefore* does. As Buridan says in (C_{Def}), it's the job of this particle to *designate* (*designare*) that the consequent follows from the antecedent. How could it do that, without binding their tenses in the appropriate way as well? Hence I see no reason to rely so heavily on the pragmatics of speaker intention, when hypothetical propositions—unlike pairs of contradictory propositions—can do it syntactically, by way of handy binding syncategorematic terms like *therefore* and *so* that do the job automatically.

To return to our initial question: what does it mean to say that the propositions involved in a consequence must be formulated at the same time? All that is required is that they actually exist as parts of the same, single hypothetical proposition, bound syntactically by a particle like *if* or *therefore*. Hence to this aspect of the consequence relation, the relevant discussion is not *Sophismata* 7, soph.2, but the preceding sophism ("No spoken proposition is true"; 7, soph. 1), which we have just examined.

Now there remain two puzzles to address before we close this section. Both of these were suggested to me by Peter King, and pick up on the discussion of Perini-Santos. The first puzzle is, when we are unsure about the scope of the present, which takes the driver's seat—semantics, or pragmatics? That is, do you figure out the scope of the present, then see whether a given proposition is true, or do you see whether it's true, and determine the scope of the present thereby? It seems we face a bit of a chicken-or-egg puzzle.

I think the solution is to do a bit of both: see, that is, what scope of the present would make a proposition true, and if there is any, opt for that one. The goal, then, is to be as charitable as possible in interpreting propositions, assuming (within reason) that the scope-of-present that would make them true is the actual scope of their present tense. And this is consistent with Buridan's general program: as he remarks in passing in the *Summulae de Suppositionibus*,

> Where an author seems to have set forth a proposition in a true sense, although not in a way that is strictly speaking true, to deny that proposition would be peevish and unfair [*dyscolum et protervum*].[147]

[147] "Videtur ergo mihi omnino quod ubi apparet auctorem posuisse aliquam proposisitionem ad aliquem sensum verum, licet non secundum propriam locutionem, negare simpliciter propositionem esset esse dyscolum et proteruum." (*Summulae* 4.3.2).

Jack Zupko has an interesting treatment of this aspect of Buridan's philosophy of language, which he sees as corrective to the disputational method of refuting literal but uncharitable interpretations (which method was apparently widespread at the University of Paris in the salad days of supposition theory). See Jack Zupko, *John Buridan: Portrait of a Fourteenth-Century Arts Master* (Notre Dame: Notre Dame UP, 2003), 17-21.

In short: when we are faced with a proposition that might be true or false, we should choose the interpretation that makes it true. This, I think, can be generalised to our selection of a present tense: if there is a scope of the present that makes a proposition true, we should take that as our present.

With the foregoing considerations in mind—as well as those of Chapter 1, §2.3—we can consider and resolve a final problem for the SF-requirement. Here is the problem: what happens when an inscribed premise, and a conclusion derived from it, are separated by a large temporal gap? This is the second problem suggested to me by Peter King. Here is an example: suppose an historian comes across a letter written by a Victorian nobleman, Mr. M—, credibly confessing to having committed the crimes of Jack the Ripper. Having read Mr. M—'s letter, our historian concludes:

P8) So Mr. M— is Jack the Ripper!

The concluding (P8) follows from M—'s confession. But all the propositions involved in this inference are not formulated at the same time: the confession of Mr. M— antedates the historian's conclusion by over a century. Examples like this one, with a written antecedent, and a spoken or thought consequent formulated much later, are fairly common. Indeed, without them, history as a discipline and graffiti as a practice, *inter alia*, would be at a total loss. But do *consequentiae* with such far flung propositions prove troublesome for the SF Requirement?

I don't think so. We have two Buridanian solutions available. The first of these picks up on our preceding discussion. Here's how it works: in the historian's case, the syncategorematic term *so* (which is roughly equivalent to *therefore*) binds (P9) as consequent to the confessions of Mr. M— as antecedents, making them into one big hypothetical proposition. Therefore, the present-tense of the historian's utterance (P9) is bound to the present tense of Mr. M—'s confessions. And that is all we need to meet the SF Requirement.

The second Buridanian solution follows the line of reasoning discussed earlier (Chapter 1, §2.3), *à propos* of the problem of cancelling assertion. When the historian reads the confessions of Mr. M—, the historian mentally re-forms them, and then derives the conclusion (P8). If so, the existence of the premisses isn't a concern: propositions equiform to the original premisses just get formulated in the present in the mind of the historian, along with the conclusion drawn from them. Here, the elastic present tense will still have to come into play, extending the present tense of the *is* in (P8) back far enough to include the Victorian age. But the existence of the propositions involved is, in any case, no problem.

Thus there are two Buridanian ways of addressing problems like the historian example: one static, and one dynamic. The static way shifts or expands the present of the conclusion, so that it is bound to the present of the premisses—that is, to the present of Mr. M—'s confessions. The dynamic way has the historian reforming the confessions as propositions in the present, and shifting the time *for* which (rather than *in* which) they are true back to the time of the original utterances.

Let me wrap up this section with the following conclusions: (i) the parts of hypothetical propositions need not all exist together in order for the whole proposition to be said to exist at once. And (ii) these propositions do not need to be true *in* the time they describe, but *for* it; it is enough that the antecedent and the consequent apply to the same slice of time, however thick it may be. Buridan's use of an elastic present makes (i) and (ii) made consistent with the SF Requirement: the present can be as large as we need to bundle together a sequence of propositions into one string, and it can be shifted or stretched so that the present-tensed propositions can be true *for* times in which they are not fully formulated. This double role of the elastic present has important implications for the question of arbitrary propositional length, introduced in Chapter 1 (§1.2.4), above.

Let's look at those implications now.

1.3. Long Long Long Chains of Reasoning

In the preceding chapter (§1.2.4), we asked what degree of propositional complexity Buridan's syntax allows—taking care to note that arbitrary complexity of this sort is a modern concern, stemming largely from interest in recursive definition and inductive proof, which is not on Buridan's mind at all. That section of the previous chapter, along with the present one, are not at all historical, but purely philosophical and speculative. Still, it seems Buridan *could* countenance quite complex propositions, especially given his view that universals are to be expounded as concatenated conjunctive strings of singular propositions. For example, we saw that:

P9) Every human runs

is to be expounded as:

P9′) Socrates runs and Plato runs and Robert runs and…[148]

This fact is important for what follows. A second fact is that Buridan considers hypothetical syllogisms with universal propositions of the following schematic form:

S1) If every B is A, then every C is A
If every C is A, then every D is A
∴ If every B is A, then every D is A.[149]

Combining these two facts, we should be able to generate behemoth hypothetical propositions on the following schema:

[148] "Indefinita habet vim disiunctivae, ut 'homo currit' valet istam 'Socrates currit vel Plato currit' et sic de aliis […et] universalis debet habere modum copulationis. Haec enim 'omnis homo currit' valet istam 'Socrates currit et Plato currit' et sic de singulis aliis hominibus." (*Summulae de Suppositionibus* 4.2.6). Cf. *idem* 4.3.5.

[149] "Fit ergo demonstratio condicionalis ut dictum est vel ex ambabus condicionalibus, ut:
Si omne B est A, omne C est A, et
Si omne C est A, omne D est A,
∴ Si omne B est A, omne D est A." (*Summulae de Demonstrationibus* 8.7.2)

S2) $(\varphi_1 \wedge \varphi_2 \ldots \wedge\varphi_n) \Rightarrow (\psi_1 \wedge \psi_2 \ldots \wedge \psi_n)$

$(\psi_1 \wedge \psi_2 \ldots \wedge\psi_n) \Rightarrow (\chi_1 \wedge \chi_2 \ldots \wedge \chi_n)$

$\therefore$ $(\varphi_1 \wedge \varphi_2 \ldots \wedge\varphi_n) \Rightarrow (\chi_1 \wedge \chi_2 \ldots \wedge \chi_n)$

This is what the syntax allows. But now we have added Buridan's sparse metaphysics into the mix, and so the question of propositional complexity runs up against the problem of Buridan's anti-realism about propositions: the propositions of a consequence have to *exist*. And the existence of these propositions depends on whether we are actually thinking them. But presumably, somewhere in the ellipses of (S2), our mental capacities will run out. So (S2) is, perhaps, too big, and probably will fall apart.

Let's dwell on this for a moment. This limitation is not merely a problem for our modern concern with propositions of arbitrary arbitrary complexity. It is a problem for Buridan's own framework, considered on its own lights, too. After all, our day-to-day reasoning can be very long. The discursive nature of reasoning—at least for us sub-lunar intellects—means that our reasoning happens in a sequence, and that we are not always entertaining all the propositions involved in a string. But then what happens when some of the propositions involved in a line of reasoning vanish—that is, what happens to the whole chain when we are no longer thinking a part of it? Presumably this happens all the time. For example, suppose I reason as follows (where the ellipses marks a string of forgotten and therefore non-existent propositions):

A8) King Kong sure seems to like Fay Wray. . .

. . .

. . .

. . .

∴ No, it wasn't the airplanes—it was beauty killed the beast.[150]

A sizeable portion of our string of propositions, here represented by ellipses, no longer exists. So is (A8) a single, unified consequence at all?

Yes. As we saw above (§1.2), Buridan thinks a line of reasoning can retain its integrity even when the propositions involved in it do not exist in the narrow present. So (A8) is one single consequence, which just takes up a much larger present. Allowing this expansion of the present does not undermine the SF Requirement: as we already saw, since the present is indefinite, we can take as much of it as we like.[151]

We can therefore solve the problem posed by very long lines of reasoning like (A8) and arguments of the form of (S2), using the same technique Buridan used to solve the problem of fleeting propositions in the narrow present: these propositions are still contained in an elastic present, which can stretch to accommodate them all. The fact, then, that the propositions no longer exist at this very moment does not mean that they do not exist as one long string, strung across a more expansive present.

So much for the problem of simultaneous formulation in the present. There is, however, a second problem for very complex propositions: the problem of shifting circumstances. This was the second problem discussed in the preceding section, and it applies to simple as well as complex propositions. Recall the horserace example:

[150] *King Kong*, directed by Merian C. Cooper and Ernest B. Schoedsack (1933; Culver City, CA: Warner Brothers Home Video, 2007), DVD.
[151] "Et ego dico non est nobis determinatum quantum sit tempus praesens quo debemus uti tamquam praesente. Sed licet nobis uti quanto volumus, vocamus enim istum annum praesentem et hanc diem praesentem et hanc horam praesentem" (*Sophismata* 7, soph. 1; 34v, Scott 113).

A6) Eclipse is now just past the winning post

∴ Eclipse wins the Triple Crown.[152]

Be apprised: horseraces conclude very rapidly. So, by the time I'm done forming the antecedent of (A6)—much less the conclusion, too—Eclipse will be well past the winning post. In this way, the circumstances (A6) describes shift too quickly for its antecedent and consequent to be true at the same time as the circumstances themselves.

This problem for relatively simple and brief inferences will apply, *a fortiori*, to more complex and therefore longer ones. For example, consider the following case in *Gravity's Rainbow*:

> He started on a mammoth work entitled *Things That Can Happen in European Politics*. Begin, of course, with England. 'First', he wrote, 'Bereshith, as it were: Ramsay MacDonald can die.' By the time he went through resulting party alignments and possible permutations of cabinet posts, Ramsay MacDonald *had* died. 'Never make it', he found himself muttering at the beginning of each day's work. 'It's changing out from under me. Oh dodgy, very dodgy.'[153]

Circumstances shift. And, as in Pynchon's example, they can even outpace our reasoning. Generally speaking, the longer the string of propositions, the more likely it is that the contextual ground will shift before the string gets concluded.

[152] Adapted from Prior, "Fugitive Truth", 5.
[153] Thomas Pynchon, *Gravity's Rainbow* (New York: Penguin, 1987 [1973]), 77.

This seems to pose a problem for, *inter alia*, Buridan's view that universal propositions are to be expounded as conjunctive strings. For instance, we would expound (P9) as (P9′):

P9 Every human runs

P9′) Socrates runs and Plato runs and Robert runs and...[154]

We can state or think the universal (P9) relatively quickly. But working through (P9′) from beginning to end is going to take a while. Worse still, what happens when Robert or Plato runs out of steam and sits down—something which will undoubtedly happen before we can conclude the string of conjunctions? And if we can't get a string of conjunctions like (P9′) together, how will we ever construct a mammoth hypothetical syllogism with the form of (S2)?

Again, the elastic present comes to the rescue. As the Eclipse example (A6) shows, we can fix the present tense as the moment when we began to utter the antecedent—or even after that. But the present can be taken indefinitely (*indefinite*), as Buridan repeatedly says, so there is no limit to how far we can stretch it for our needs. Therefore, there is no reason in principle that we can't fix as the present the moment I began to utter the beginning of a long concatenation of conjunctions like (P9′), and assess the truth conditions of that conjunctive proposition *for* that time.

[154] "Indefinita habet vim disiunctivae, ut 'homo currit' valet istam 'Socrates currit vel Plato currit' et sic de aliis [...et] universalis debet habere modum copulationis. Haec enim 'omnis homo currit' valet istam 'Socrates currit et Plato currit' et sic de singulis aliis hominibus." (*Summulae de Suppositionibus* 4.2.6). Cf. *idem* 4.3.5.

Hence even if Socrates and Robert stop running before I finish the string of conjunctions (P9′), the conjunctive proposition's truth value depends on the conditions at the time the proposition is about. The same will hold true of much more elaborate propositions, like any hypotheticals that have the same form as the massive (S2).

Now here is a worry: is the elastic present *too* elastic? Consider the following proposition:

P10) Socrates is running and Socrates is sitting

Suppose that, while I utter the first conjunct, Socrates is running; but once I utter the word *and*, Socrates sits down. In such a case, can we stretch the elastic present to make (P10) true—*i.e.*, to cover the whole series of events? I don't think so: notice that a conjunction like (P10) is commutative: it has exactly the same truth conditions no matter what the order of the embedded expressions. Accordingly, (P10) is logically equivalent to the following:

P10′) Socrates is sitting and Socrates is running

This suggests that the slice of time carved out for any conjunctive proposition in the present tense—however large that slice may be—has to be *unified*, and so the constituent parts have to both be true (or false) for that time.

Things are a bit trickier with non-commutative hypotheticals, which can—and often *do*—describe things that happen diachronically. For instance, consider the following argument, suggested to me by Peter King:

A9)	Caesar was a soldier
Therefore	Caesar is a veteran

What is the (presumably unified) present tense that (A9) takes up? We want to say that the *therefore* here binds the time of the antecedent and the consequent, but is it unified? I think so. But then if we take all this time as the present, the past tense *was* in the antecedent will look odd, since it will imply that some time can be both past and present. Now in *QM* IV.15 ("Can Two Contradictory Propositions be Simultaneously True?"), Buridan categorically denies that, in such an expanded present, there can also be a past (*praeteritum*) or a future (*futurum*):

> every part of the present time is present, and that nothing present is past. For no present time is future, since all future time does not yet exist. And all past time has already gone by and no longer exists. Therefore, it follows that no part of present time is past or future, since every part of the present exists at present.[155]

This makes good horse sense, since otherwise we would have to admit that the past and future were present. Even so, the present can be said to have a prior and posterior part:

[155] "temporis praesentis quaelibet pars est praesens, et quod nullum praesens est praeteritum. Nullum etiam tempus praesens est futurum, quoniam omne tempus futurum nondum est. Et omne tempus praeteritum iam transivit et non amplius est. Ideo sequitur quod nulla pars temporis praesentis est praeterita vel futura, cum quaelibet sit praesens." (*QM* IV.15, fol.25r, a).

> I take it that, as it was said earlier, ‘to be changed’ is correctly expounded as to be one way prior and another way posterior, which does not make use of the terms *past* and *future* [...] thus it follows that the present time is divisible into a prior and a posterior part. The posterior part exists, and the prior part exists as well, and consequently any of these is the present.[156]

Hence although there is no past or future in the present, it does not follow that we can’t talk about changes, like Caesar’s going from a soldier to a veteran. We just have to take care that we do not import any past or future into the present by our use of tenses. Thus I think we can express the valid pattern of reasoning in (A9) by altering the tense of the antecedent to the present perfect. We might also supply the suppressed premise, to wit:

A9′)	Caesar has been a soldier
	Whoever has been a soldier is a veteran
Therefore	Caesar is a veteran

The truth conditions for the antecedent of (A9) are the same as that for the first antecedent in (A9′). Yet the tense is rooted in the present, so to speak, though it has a perfect aspect. This way, we can express the pattern of inference in (A9) by extending our present, without implying by the tenses we use that any present can have a past or future, and therefore that the past can be present.

[156] “ego suppono quod sicut dicebatur quod mutari exponatur proprie per se habere aliter et aliter prius et posterius, et non per verbum ‘praeteritum’ vel ‘futurum’ [...] ita sequitur quod praesens tempus divisbilis in partem priorem et posteriorem; pars posterior est et pars prior est et per consequens quaelibet earum est praesens” (*QM* IV.15, fol.25r, a).

Resuming our main line of reasoning, then, we can see that in principle there is no limit to how much time we take for the present, and so there appears to be no limit to propositional complexity, either on ontological or on contextual grounds. Granted, not all the parts of such propositions will exist in a narrow present; but then neither do the parts of ordinary hypothetical propositions of complexity 0, like $(\varphi \rightarrow \psi)$. So this is not a special problem for very large propositions. Therefore, the SF Requirement need not rule out arbitrarily complex propositions. And any grounds on which it would rule them out—impermanence of existence of their constituent parts, or inconstant truth conditions—would be sufficient to rule out all hypotheticals, and even simple categoricals, in one fell swoop. We certainly won't do that.

The discussion of context and context shift brings us to the subject of truth conditions, which is the next of the three criteria set out in (A/C_{Def}).

2. The Signification Requirement

Definition (A/C_{Def}) countenances a notion of propositional truth that we need to examine in greater detail. Let's do that now. As I already noted at the outset, Buridan is careful to say that consequence is not about propositional *truth*, but about propositional *signification*. If it were about truth merely, then contingent but self-falsifying propositions like "no proposition is negative", although not *impossible*, would entail any proposition whatsoever, the way impossible ones do. So the following would be a valid consequence:

A3) No proposition is negative
∴ No donkey is running.[157]

As Buridan is quick to point out, (A3) does not have a valid contrapositive:

A3′) Some donkey runs
∴ Some proposition is negative.[158]

So there is something wrong with (A3); and the way to rule it out is to refine (A/C_{Def}), so that it deals with propositional signification, not propositional truth.

David Kaplan and Gyula Klima worry about Buridan's claim that valid *consequentiae* have valid contrapositives, as we will see. They take Buridan's claim to be that any valid consequence, *by definition*, has to have a valid contrapositive. But such a contrapositive can't be valid unless it exists. And most of the time, these contrapositives don't exist, because they go unformulated. So this stipulation seems to undermine Buridan's token-based account of propositions, and to put Buridan's logic at odds with his metaphysics. Fortunately, there is a solution, as we'll soon see.

But first, let's look at what it means for a proposition to *signify* something true.

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[157] "Nulla propositio est negativa; ergo nullus asinus currit" (*TC* I.3.40).
[158] "Quidam asinus currit; ergo quaedam propositio est affirmativa" (*TC* I.3.40).

2.1. The Possibly-True and the Possible

As Gyula Klima rightly observes, Buridan's definition of *antecedent* and *consequent* (A/C_{Def}) differs in a significant way from the intuitive definition of consequence found in most modern textbooks on logic.[159] Modern treatments tend to deal with validity in terms of the truth of the propositions involved. Take for instance the following modern definition, from Matthew McKeon's *The Concept of Logical Consequence* (2010), which is representative of the current (which is to say Tarskian) approach:

> MC_{Def}) X logically follows from K only if it is necessarily *true* that if all the sentences in K are *true*, then X is true.[160]

In contrast with (MC_{Def}), Buridan is not concerned with the truth of propositions so much as he is with things being as they describe. And this is a conspicuous difference in the language of (A/C_{Def}), compared with its modern counterpart, (MC_{Def}): the latter deals with *truth*, and the former with *signification*.[161] We saw that Buridan's emphasis on signification stems from his worry about arguments like (A3), which seem valid because of

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[159] *John Buridan* (New York: Oxford, 2009), 210.
[160] Matthew McKeon, *The Concept of Logical Consequence* (New York: Peter Lang, 2010), 6; emphasis added. This statement of the notion is broadly representative of what one usually finds in introductory chapters on the subject.
[161] To a reader accustomed to Tarski, this difference may seem like no difference at all. But just you wait! (If you want a preview, here it is: Buridanian truth is a matter of the meanings of terms, and not of interpretations in a model. Accordingly, Buridan will treat a whole slew of arguments that do not survive uniform substitution as valid—indeed, *just as* valid—as formally valid ones which do survive uniform substitution. This is the focus of Chapter 4.

their self-falsifying premisses. But since it is possible for things to be as the antecedent of (A3) *signifies* without things being as the consequent does, (A3) is not valid.

How does this contrast between propositional truth and signification work? Arthur Prior gives a quite clear and concise treatment of the Buridanian theory of propositional truth and signification in his (1969) “The Possibly-True and the Possible”.[162] There, Prior analyses Buridan’s approach to self-falsifying propositions such as:

P11) No proposition is negative

Prior imagines a set of sheets of paper, some of which have propositions written on them, and some of which don’t.[163] These sheets are relatively simple stand-ins for contexts. In the example, the truth of a proposition can be assessed in one of two ways: relative to the sheet on which it appears, or relative to another sheet.

Now there are some propositions that will be true *in* the context of any sheet they are written on. For example:

P12) Some proposition is affirmative

[162] “The Possibly-True and the Possible”, *Mind* (78), 1969, 481-92.
[163] Prior calls these *sentences*, not *propositions*. This usage is well in keeping with the modern language of sentences as tokens and propositions as types, which we saw in connection with the Fregean account of propositional content in the preceding chapter. But Buridan’s semantics is token-based, and he calls them *propositions* (*propositiones*). In this, I follow Buridan. This will only become slightly confusing when I have to cite Prior’s use of the term *sentences*. It should be borne in mind that these terms—*sentence* in Prior, *proposition* in Buridan—are equivalent.

An affirmative like (P12) will always be true on the sheet it is written on, because as an affirmative proposition, it renders itself true. Conversely, a proposition like (P11) will never be true of any sheet it is written on, because it falsifies itself: there is indeed a negative proposition on such a sheet, namely (P11).

Yet while (P11) can't be true of any sheet it's written on, this fact does not imply that (P11) can't be true *of* some other sheet. A necessary (though not sufficient) condition of this truth *of* another sheet will be that (P11) is not written on it. A sufficient condition will be that there are *no* negative propositions written on it. So (P11) will, for instance, be true *of* any blank sheet, or of a sheet that contains only (P9), or of a sheet that contains only affirmative propositions. Conversely, (P12) will be true of any sheet it is written on, but it needn't be true of all sheets. Assuming there are blank sheets, (P12) will not be true of them.

Hence (P11) describes a sheet that is *possible*; but, in the context of any sheet it is written on, (P11) is not *possibly-true*. Similarly, (P12) is *necessarily-true*, but it is not *necessary*. As Prior tells us,

> The important point is to notice that for a sentence S on a sheet X to be 'possible' in virtue of what is on [sheet] Y, the sentence does not itself have to be on Y.[164]

So propositions can be true *of* sheets that they themselves do not appear on. Accordingly, self-falsifying propositions can nevertheless be true *of* contexts in which they are not present, because the situations they describe are possible. Correspondingly, self-satisfying

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[164] Prior, "Possibly-True", 86.

propositions like (P12) might be called *necessary* but not *necessarily true*. A necessarily true proposition will always be true in the context of any sheet it is written on; but it will not be true of every sheet. Indeed there will be plenty of sheets *of* which a proposition like (P12) is not true: blank ones, ones with only (P8) written on them, or ones with only negative propositions written on them. Contrast this with a proposition that is *necessary*—for instance, “Every sheet is either written on or blank”, which will be true of every sheet.

Prior’s sheets are a useful heuristic, but we should not take them too literally. There are two ways Prior’s analysis of Buridan goes awry. First, for Buridan, propositions are only ever *tokens*, never types. But by speaking of multiple occurrences of a single proposition like (P12) on different sheets, we end up treating these occurrences as tokens of a single type, namely (P12). In effect, then, such talk allows individual propositions to have trans-sheet identity, and so Buridan starts to look deceptively like a medieval Saul Kripke. Better to follow Buridan’s lead, and speak of multiple tokens with maximally similar (*consimilis*) form. This is a relatively easy fix, but one that should be borne in mind.

Second, and more importantly, for Buridan propositional truth is not a matter of states of affairs, but of ways that the significative terms in a proposition (*i.e.* the subject and the predicate) stand for the things in their extension. Prior’s sheets look uncomfortably like truthmaking states of affairs—or even like possible worlds. But Buridan emphatically rejects proposition-like complex states of affairs in his *Quaestiones in Metaphysicam Aristotelis* VI, Questions 7 and 8—a discussion I’ll set out in detail in

Chapter 5 (§2.1). And Buridan's account of modality does not turn on anything like possible worlds, as we will see *infra* in the present Chapter (§3), and in greater detail in Chapter 5 (§§3-5).

Thus we are better off treating Prior's account not as an analysis, but as a heuristic. So long as we take care not to let him lead us on, we are safe between Arthur Prior's sheets. Turning back to (A/C_{Def}), then, we can see that it is a proposition's truth *of* and not *in* situations that Buridan has in mind—what matters is not the sheet the antecedent and consequent are true (or false) *in*, but the sheet they are true (or false) *of*. Thus in (A/C_{Def}), Buridan does not say that an antecedent cannot be *true* while its consequent is false. Rather, he says that it cannot be that things be as the antecedent *signifies* without being as the consequent does.

What's the use of this distinction? Again, Buridan is concerned about sophisms like the following:

A3) No proposition is negative
∴ No donkey is running.[165]

It is not possible for the antecedent of (A3) to be true while the consequent is false, since the antecedent is just not possibly-true: any time it is formulated, it falsifies itself. As Buridan observes, there is good reason to think (A3) is not valid, since it does not have a valid contrapositive:

[165] "Nulla propositio est negativa; ergo nullus asinus currit" (*TC* I.3.40).

A3′) Some donkey runs

∴ Some proposition is negative.[166]

Of course, (A3′) is invalid: it is entirely possible for a donkey to be running, and at the same time for all the negative propositions to be annihilated.

Hence although the antecedent of (A3) cannot ever be true, it does describe a possibly-true state of affairs—namely, one in which there happen to be no negative propositions. It is therefore *possible* but not *possibly-true*, since it describes a possible state of affairs of which it can never be part.[167]

It is for these reasons that Buridan's requirement in (A/C_{Def}) is not that it be impossible that the antecedent be *true* while the consequent is false, but that it is be impossible that things be *as the antecedent says they are* but not as the consequent does. And on these grounds, (A3) fails, since it is possibly-true that no negative proposition exists and, at the same time, that a donkey is running.

[166] "Quidam asinus currit; ergo quaedam propositio est affirmativa" (*TC* I.3.40).
[167] "The Possibly-True", 481. Buridan also has a similar discussion in *Sophismata* 8, sophism 10, where the context is one in which there are only four propositions, two true and two false, and so one cannot make the true claim "there are as many true propositions as false ones" without adding a proposition to the mix and therefore tipping the balance 3-2 for true propositions. Even so, it is true that there are as many true as false propositions in that case. Buridan thinks we can only make such a true claim in the past tense: "there *were* as many true propositions as false ones".

2.2. A Criticism from Klima and Kaplan

To close this section on propositional truth and signification, I want to address a criticism of Buridan that appears in Klima (2008), who attributes it to Kaplan.[168] As we've seen, Buridan is aware that an account of consequence that appeals to the truth of the propositions involved—a definition like McKeon's informal account, (MC_{Def})—renders (A3) valid, even though it has an invalid contrapositive. This is what motivates Buridan to modify his final account, (A/C_{Def}), to depend on signification rather than truth.

But new solutions often bring new problems. The Kaplan-Klima worry is this: Buridan's anti-realism about universals (often called his *nominalism*) is apparently undermined by his appeal to contraposition as a criterion for valid consequence. After all, It is quite possible to think, speak, or write (A3) without thinking, etc., (A3′). So (A3) might well exist while its contrapositive, (A3′), does not. As Klima observes:

> given Buridan's token-based conception of propositions, that intuitively clear definition [sc. (A/C_{Def})] immediately invalidates the rule of contraposition, since the existence of the propositions of a consequence is independent from the existence of their negations occurring in the contrapositive.[169]

The concern, I take it, is this: (A/C_{Def}) requires that the propositions involved in a valid consequence be simultaneously formulated. We've been calling this the *SF Requirement*. But then Buridan's treatment of (A3), and its contrapositive (A3′), seems to suggest that all valid consequences actually have valid contrapositives. This could be taken to imply

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[168] *Buridan*, 317, n.8.
[169] Klima, Gyula. "Consequences of a Closed, Token-Based Semantics" (*History and Philosophy of Logic* 25 (May 2004), 95-110), 99.

that the definition of consequence requires valid consequences to have valid contrapositives. Call this the *Contrapositive Requirement*. But simultaneous formulation implies existence: if the propositions are to be formulated simultaneously, then they have to be formulated, full stop. Yet I might well formulate a consequence like (A3), without at the same time formulating its contrapositive, (A3′). So, it seems, the SF Requirement and the Contrapositive Requirement are at odds. Indeed, the latter seems to undermine the former altogether.

Arguing on Buridan's behalf, Klima suggests that by bivalence, any situation that renders the conclusion false will render the premises false. Hence if the contrapositive of an invalid consequence like (A3) *were* formulated, it *would* be false. But Klima goes on to worry that, in a situation in which all four propositions involved in (A3) and its contrapositive (A3′) *were* formulated at once, then the antecedent of (A3′) would be true, in virtue of the existence of the antecedent of (A3). Here, for reference, are the antecedent of (A3) and the consequent of (A3′):

P13)	No proposition is negative	(A3)
P14)	Some proposition is negative	(A10)

If both (A3) and (A3′) were formulated at once, then (P13) and (P14), would all exist at once, too. But then the distinction between the possibly-true and the possible goes out the window. Consequences have to be evaluated with respect to what they signify in their contexts. And (presumably) both (A3) and its contrapositive (A3′) have to exist in the same context. Therefore, so do their troublesome parts, (P13) and (P14). But then (P13) and (P14) get tangled up together again, because the existence of (P13) renders (P14) true. And so it has to be that the consequent of (A3′) is always true, in virtue of the antecedent of its contrapositive. So the Contrapositive Requirement can't allow us to rule out (A3), either! Thus, as Klima recognises, this putative solution not only undermines Buridan's anti-realism, but also puts us right back where we started.

Fortunately, there is another way to avoid this problem: simply to drop the Contrapositive Stipulation altogether. What warrants this move? In Buridan's account of consequence, the employment of contraposition in the lead-up to (A/C_{Def}) is limited to his refutation of definitions that depend on *truth*, rather than on signification.[170] But contraposition plays no role in his final definition (A/C_{Def}). That is, there is no Contrapositive Requirement baked into Buridan's definition of valid consequence (A/C_{Def}). Rather, Buridan thinks the modification of *truth* to *signification* in (A/C_{Def}) is enough to rule out troublesome arguments with invalid contrapositives, like (A3) and (A3′). If this modification is enough, why would he also add to (A/C_{Def}) the requirement that valid consequences have valid contrapositives?

And indeed, there is textual evidence support for this approach. For Buridan thinks contraposition is distinct from (A/C_{Def}), and derivable from it. Contraposition just *is* one of his derived rules, set out later on in *TC* I.8. So there is no reason to take it to be a part of

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[170] Recall the modern definition of logical consequence (MC_{Def}), which we plucked from McKeon above: X logically follows from K only if it is necessarily true that if all the sentences in K are true, then X is true.

While this might look like a truth-requiring (rather than signification-requiring) formulation of the definition of logical consequence, it is not. For here *truth* is relative to a model, and necessity is a matter of truth in all models. But certainly there is a model of "No proposition is negative" that does not model "A donkey runs". So (MC_{Def}), like (A/C_{Def}), requires something like signification as well.

his definition. Thus the SF Requirement and the Signification Requirement, set out in the present section, are integral parts of definition (A/C$_{Def}$). But the Contrapositive Requirement is not. So there is really no tension here, so long as we get the order between (A/C$_{Def}$) and the Law of Contraposition right: the latter depends on the former, not *vice-versa*.

To bolster this point, let's briefly run through Buridan's derivation of the Law of Contraposition from (A/C$_{Def}$). In doing so, we'll see that (A/C$_{Def}$) is prior to the rule of contraposition, and in no way depends on it—indeed, to suppose otherwise is to beg the question. Contraposition is, rather, derived. Here is Buridan's derivation in *TC* I.8:

> This [sc. Law of Contraposition] is often put in the following way: "every consequence is a good one in which, from the opposite of the consequent, the opposite of the antecedent follows." But strictly speaking this begs the question [...] since at the outset it posits that there is a valid consequence here, with both an antecedent and a consequent.[171]

The problem, then, seems to be that the rule of contraposition, stated this way, presupposes that we already have a valid consequence, comprising antecedent and consequent, to use it on. But then if contraposition is a requirement for finding valid consequences, how can we even get started?

Instead, Buridan offers the following rather terse proof of the rule of contraposition, which appeals to (A/C$_{Def}$):

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[171] "Secunda pars huius conclusionis solet poni sub tali forma: 'Omnis conseqentia est bona quando ex opposito consequentis sequitur oppositum antecedentis'. Sed hunc modum non posui, quia esset petitio principii de virtute sermonis. Sermo enim ille ponit quod sit ibi consequentia et consequens et antecedens, ideo bona consequentia". (*TA* I.8.3.61-5).

> Suppose B follows from A. Then we say that not-A follows from not-B. For either not-B does or can stand with A [or not], based on the foregoing. But it is necessary that, when A holds, B does. Therefore, B and not-B hold at once, which is impossible.

Let's expand on this for clarity's sake. Here is a consequence which is valid by (A/C_{Def}):

A10) Socrates is running
∴ Socrates is moving

Now, consider the antecedent and consequent of (A10) as separate propositions:

P15) Socrates is running

P16) Socrates is moving

Either it is possible for (P16) to be false at the same time as (P15), or it is not possible. If the former is the case, then the contradictory of the consequent, namely:

P17) Socrates isn't moving

is true. But if it is possible for (P17) to be true while (P16) is also true, then (P16) doesn't follow from (P15) after all, by (A/C_{Def})—otherwise, it would be possible for the antecedent to hold without the consequent, or even for two contradictories to be true at the same time. Therefore, if a consequence holds, then so does its contrapositive. Here, then, we have textual evidence that Buridan thinks the Law of Contraposition depends on (A/C_{Def}), and not the other way around. So there is no need to posit really existing contrapositives for all valid *consequentiae*.

Thus Buridan is not saying that a valid consequence has to have a valid contrapositive *in order to be valid*. Rather, his claim is simply that any valid consequence will have a valid contrapositive, should that contrapositive be formulated, as a result of what is stipulated in (A/C_{Def}). His invocation of contraposition is just a means to show that (A3), which is invalid, *would* be valid according to the suggested definition. This use of contraposition, therefore, is not a supporting component of (A/C_{Def}).

It is still possible to object that, in ruling out such a definition, Buridan is helping himself to a rule he has not yet derived from (A/C_{Def}), and that he has no right to do so. Now neither Klima nor Kaplan object to Buridan on these grounds. And I don't think this objection amounts to much, either. After all, Buridan has been offering *arguments* for (A/C_{Def}) too, and these arguments tacitly appeal to an intuitive notion of validity that, at this point in *TC*, is yet to be defined. Indeed, things have to be this way: if we need a definition of validity before we can offer arguments for this or that definition, then we can't even get started arguing about proposed definitions of validity in the first place. So our whole project will never get off the ground.

Reasoning in this way leads to all kinds of absurdities. If we likewise needed a definition of *expression* (*oratio*) and *proposition* (*propositio*) before we made use of expressions and propositions, we could never have gotten started with the preceding chapter—or with discussions of logic in general! But these circles are vicious, and their underlying demand unreasonable: we do not need definitions of things in order to use them to discuss possible ways of defining them. And baking the Contrapositive Requirement into (A/C_{Def}) would indeed beg the question—as Buridan himself remarks in the passage cited above. But this is not what Buridan is up to. His use of contrapositives in making the case for adding the Signification Requirement to (A/C_{Def}) therefore neither begs the question nor undermines his anti-realism about types.

3. The Modal Requirement

Now given the central role that necessity plays in (A/C_{Def}), we might expect Buridan to offer a more detailed account of it in the *Tractatus de Consequentiis*. But he doesn't, and so we have to cast about for one. Buridan has two separate treatments of propositional modality: one in *Quaestiones in* Analytica Priora (*QAPr*), and the other in the *Summulae de Demonstrationibus*. In both cases, the modal status of a proposition depends on whether it is falsifiable and, if so, by what sort of cause. In brief: a proposition is necessary if there is no cause—including God—that can render it false. In the present section, I explain what this means, and apply it to (A/C_{Def}).

Before I begin, let me make two remarks. First remark: surprisingly, Buridan's modal semantics have received relatively little attention in the literature. At the time of writing, there are just two stand-alone treatments: Simo Knuuttila's "Necessities in Buridan's Natural Philosophy" (2001), and Calvin Normore's "Buridanian Possibilities" (2013). Both of these are concerned with necessity in Buridan's physics and metaphysics, and less so with the role of necessity in his propositional semantics, which is what we're dealing with here. The kind of necessity Buridan is concerned with is mainly *propositional*: what's at stake is necessary propositions—propositions, that is, that cannot fail to be true.[172]

Second remark: in the present discussion, I am not dealing with modal propositions. These will be covered in Chapter 5. Instead, I am here concerned with the modal properties an assertoric proposition may have. To clarify: the contrast is between propositions *about* necessity (*de necessario*), which are modified by *necesse* or some other mode, and propositions that just *are* necessary (*necessariae*). For example, contrast the following two propositions:

P18) A human is necessarily a donkey.

P19) Humans are animals.

Here, (P18) is a modal proposition, albeit a false one, because it is modified by a mode (*necessarily*). Conversely, (P19) is true, and necessarily so, but is not a modal: it is, rather, an assertoric proposition that is necessarily true. In a slogan: modality for us is a matter of truth and falsifiability, not of syntax. At least for our present discussion: here we're talking about propositions of this latter sort, not the former. We'll take up the former in Chapter 5.

[172] We might wonder whether logical necessity is in any way special, or is just a matter of metaphysics. Buridan himself contrasts truth of a proposition with truth of a thing—namely, as a transcendental property of God, which he mentions in order to set aside in his *Questiones in Metaphysicam* (VI.6, 37v c-d). Hence he seems to have in mind a distinction between properties of objects and of propositions, and to fix his attention on the latter.

In what follows, I look at Buridan's two discussions of this subject: in his *Questions on the Prior Analytics* (I.25), and in his *Summulae de Demonstrationibus* (8.6.3). I begin with the former, since the *QAPr* is in some respects a less developed work, and probably an earlier one.[173] Since Buridan gives a less elaborate formulation of propositional modality in the *QAPr* than in the *Summulae de Demonstrationibus*, and since the latter elaborates on the former, I begin with the former.

3.1. The Modal Scale in the *Questions in Analytica Priora*

A necessary proposition is one which, provided it exists, cannot be rendered false. Here are two examples:

P20) Humans are animals

P21) Triangles are three-sided

Granted, a proposition like (P20) or (P21) doesn't have to be *true*. Such propositions could, after all, be unformulated; and if they were unformulated, they would fail to be true. But any time (P20) or (P21) *is* formulated, it has to be true: it is a necessary truth that triangles have three sides, and that humans are animals.

[173] Note however that Buridan apparently refers to the *Summulae* in the *QAPr*, for example in (I.19, co.), which could be taken as a sign that the former is earlier. But things are not so simple: many of these references are clearly to the *Summulae Logicales* of Peter of Spain, on which Buridan's own *Summulae* is a commentary. This is clearly the case when he refers to the *auctor Summularum* in the third person (*e.g.* in *QAPr* I.12, arg. 6-7, and I.24, co.).

But then a problem arises: what if God were to annihilate all the humans, and then formulate a proposition like (P20)? For an anti-realist like Buridan, there is no abstract entity that a term like *humans* signifies, since the only humans are those that actually exist. Therefore, since the subject term of (P20) would not stand for anything, (P20) would be false. So, it seems, (P20) is not necessary after all—and is, in fact, contingent.

It gets worse: the propositions of geometry are about objects with magnitude. But God could annihilate everything that had a magnitude. And if God were to do that, even geometrical propositions like (P21) would be false:

> If this were so, then no proposition of geometry would be necessary, since God can annihilate all magnitudes, just as God can annihilate all humans. And from this, it would further follow that geometry would not be a science, which everyone would regard as false and inaccurate [*inconveniens*].[174]

Thus if susceptibility to divine falsifiability were sufficient to undermine the necessity of a proposition, then the number of truly necessary propositions would be very few indeed. A proposition like the following would still count as necessary:

P22) God exists

But other propositions, like the true propositions of geometry, which we typically treat paradigmatically necessary propositions, would fail. The propositions of geometry are contingent on the existence of things that need not exist, and therefore are contingent

[174] "si hic obstaret, nulla propositio geometrica esset necessaria, cum deus ita possit annihilare omnes magnitudines, sicut omnes homines. Et tunc ultra sequeretur quod geometria non esset scientia, quod reputatur ab omnibus falsum et inconveniens" (*QAPr* I.25, obj.3).

themselves. But scientific knowledge can only be of necessary truths—as Buridan elsewhere and repeatedly states, "strictly speaking, there is no science of contingent propositions".[175] Therefore, even geometry won't count as a science, since its truths are contingent.

We can't abide this conclusion. And, fortunately, we don't have to. The solution is to weaken the requirement for necessity, by introducing a distinction between necessary propositions of different ranks or grades (*gradus*). Here in the *QAPr*, Buridan introduces three such grades.

The first and highest grade of necessity is that which comes about (*provenit*) simply. Once formulated, a simply necessary proposition cannot be falsified under any circumstances:

> Simple necessity comes about on account of the fact that, when the proposition has been formulated, it is impossible that, at any time (*aliquando*), the subject and predicate do not stand for the same thing. Or it is impossible for things to be other than the proposition signifies, when it is put forth as a simple categorical.[176]

Buridan's example of a proposition of this sort is (P22): even God cannot destroy God, and so it is impossible that propositions like "God exists" could ever be falsified, any time they are formulated.

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[175] "non est scientia proprie de propositionibus contingentibus" (*Summulae de Demonstrationibus* 8.6.2; de Rijk, p.137, l.13)

[176] "Necessitas simpliciter ex eo provenit quod impossibile est quod aliquando subiectum et praedicatum non supponant pro eodem in propositione formata vel quod impossibile est aliter esse quam propositio significat secundum sensum simpliciter categoricum" (*QAPr* I.25, co.).

But Buridan explicitly denies that propositions like (P20) and (P21) are necessary *simpliciter*. These propositions fail to meet the high standard of simple necessity because they can be falsified by a supernatural power (*potentia supernaturalis*)—whereas simply necessary propositions like (P22) cannot. Propositions like (P20) and (P21) *are*, however, necessary, assuming the constancy of the thing(s) their terms stand for. This is the second level of necessity Buridan sets out in the *QAPr*, which he calls necessity *de quando* (a phrase that roughly means 'just when'):

> But *de quando* necessity comes about if, whenever the subject and predicate stand for anything, they stand for the same thing [...] And in this way, I say that the following is necessary: 'humans are animals', or likewise 'horses are animals'. And what's more, even 'roses are flowers' is necessary, even if there are no roses right now.[177]

Buridan gives the following examples of propositions that are necessary *de quando*, even though their subject terms stand for fleeting things:

P23) Thunder is a sound in the clouds.[178]

P24) A lunar eclipse is an interruption of the light of the sun.[179]

What is important here is that whenever the subject term stands for something, the predicate term stands for the same thing, too. For instance, (P23) is necessary because

[177] "Sed necessitas de quando ex hoc provenit quod oportet subiectum et praedicatum quandocumque supponunt pro aliquo supponere pro eodem. Et sic dico quod haec est necessaria: 'homo est animal', vel etiam 'equus est animal'. Immo etiam haec est necessaria 'rosa est flos', licet modo nulla sit rosa" (*QAPr* I.25, co.).
[178] "Tonitruum est sonus factus in nubibus" (*QAPr* I.25, co.).
[179] "Eclipsis lunae est defectus luminis a sole" (*QAPr* I.25, co.).

whenever there was, is, or will be thunder, there was, is, or will be a sound in the clouds. The same is true of the terms in (P20) and (P21), and other such propositions of the natural sciences. Thus, a proposition like (P20) is to be expounded with the following, temporal sense (*sensus*):

P20′) Whenever there was, is, or will be a human,
there was, is, or will be an animal.[180]

Hence it is not necessary for the (necessary) truth of (P20) and its expanded version (P20′) that their subject terms stand for anything. Rather, it is sufficient that, whenever they do, their predicate terms stand for the same thing.

But does the subject term of a proposition like (P20) *have* to stand for something that did, does, or will exist at some time, in order for (P20) to be true? The temporal language of (P20′) seems to require just this. Buridan recognises this requirement, and goes on to add that:

> for such propositions, it is necessary that their subject stand for something at some time or other. And in this way, the following is false: "A vacuum is a place", since it is false that "A vacuum, whenever it was, is, or will be, is a place". And it seems to me that this is the sense in which we use natural supposition in the demonstrative sciences.[181]

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[180] "Et est sensus talium propositionum quod necesse est equum, quandocumque est, fuit vel erit, esse, fuisse vel fore animal" (*QAPr* I.25, co.).

[181] "Et tales temporales requirunt quod subiecta earum aliquando supponant pro aliquo; ideo sic ista non est necessaria 'vacuum est locus', eo quod haec est falsa 'uacuum quandocumque est, fuit vel erit, tunc est, fuit

A true proposition which is Grade-II necessary will thus have to have terms that stand for something that exists at some time or other.[182]

In this way, necessary propositions of natural sciences like (P20)-(P24) are distinguished from false propositions about impossible objects—objects like the vacuum in Aristotelian physics. Accordingly, the following proposition is not necessary at Grade II, since it describes an object that cannot—and therefore, hasn't, doesn't, and won't—exist:

P25) A vacuum is a place.[183]

But a proposition like (P25) *can* be necessarily true at Grade III: the level of conditional necessity (*necessitas conditionalis*). If we read (P25) in a conditional sense (*sensus*), we can produce the following hypothetical proposition that will always be true:

P25′) A vacuum, if it exists, is a place.[184]

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vel erit locus'. Et videtur mihi quod secundum istum sensum ponitur suppositio naturalis in demonstrativis" (*QAPr* I.25, co.).

182 Buridan's claim here that the subject term of a necessary proposition has to stand for something at some time or another might seem to commit him to something like a principle of plenitude. This would be unfortunate, since Buridan elsewhere repeatedly denies that all possible things must exist at some time or another. For example, in his *Questions on De Caelo* I.23, Buridan claims "This wine can be vinegar" is true, even though it never will be, because you are about to drink it. So elsewhere Buridan explicitly denies the principle of plenitude: many possibilities never come about. But this is not what's at stake here: the above proposition about wine is a modal; we are here dealing not with modal propositions but with propositions that are necessarily true (cf. the beginning of §2.3, above). So Buridan is free to say that a necessary assertoric like (P24) has a subject term that stands for something at some time or other, without putting in jeopardy his position on unrealised *possibilia*.

183 "Vacuum est locus" (*QAPr* I.25, co.).

184 "Vacuum, si est, est locus" (*QAPr* I.25, co.).

A conditional like (P25′) is necessary, since its antecedent is always false: there can never be a vacuum, and so it can never be the case that the antecedent is true and the consequent is false. And this is the sort of necessity that we find on this lowest level of Buridan's modal scale in *QAPr*.

How are these levels interrelated? Any proposition that is necessary at any level will be necessary in any lower level(s), assuming there are any. For instance, as we noted, the following proposition is necessary at Grade I:

P22) God exists

And if we read (P22) in the temporal sense of Grade II, it will likewise be true:

P22′) Whenever God was, is, or will be,
there was, is, or will be a being.

Likewise for the conditional sense of (P22):

P22″) God, if He exists, is a being.[185]

[185] This proposition and (P22′) before it are my examples, not Buridan's. Truth be told, it is not immediately clear how we ought to make predications of ScP-form out of propositions like "God is", in which the copula (*est*, "is") is absolute or second-adjacent (*secundum adiacens*), rather than predicative or third adjacent (*tertium adiacens*). But if we recall that, for Buridan, subject-copula-predicate form is canonical, and further that a sentence like "God exists" is equivalent with the canonical "God is a being" (*deus est ens*), we can see how to get a predicate (*ens*, "being") with which to construct the whole conditional (6′). For Buridan's discussion of the different positions of copula, and the reduction of sentences of the form "A *est*" to the canonical "A *est ens*", see the *Summulae de Propositionibus* (1.3.2).

Similarly, we might read a proposition that is necessary *de quando*—that is, at Grade II of the scheme set out in the *QAPr*—as a true conditional:

P20′) Humans, if they exist, are animals.

Of course, this trickle-down necessity won't work in reverse: no merely Grade-III necessary proposition is Grade-II necessary, and the same holds, *mutatis mutandis* for Grade-II propositions and Grade-I necessity.

Now it may seem that there is little difference between conditional necessity and necessity *de quando*. We might be tempted to read *de quando* necessities in a conditional way—for example, as saying that *if* humans exist, then humans are animals. But for Buridan, *de quando* necessities are *per se*: they deal with something essential about natural kinds, which are taken to exist at some time or another. In order to read *de quando* necessities correctly, then, we have to be careful not to import a bare conditional into their analysis, since on its own, a conditional reading does not commit us to the existence of the subject term itself.

To sum up, we can represent the modal scale of Buridan's *Questions on the Prior Analytics* as follows:

Grade I: simple necessity

e.g. "God exists"

Sense: simple assertoric: "S is P" = "S is P"

Grade II: *de quando* necessity

e.g. “A horse is an animal”

Sense: temporal: “S is P” = “whenever there was, is, or will be an S, there was, is, or will be a P”

Grade III: conditional necessity

e.g. “A vacuum is a place”

Sense: conditional: “S is P” = “S, if it exists, is P”

As we will see, Buridan’s account in the *Summulae de Demonstrationibus* is quite similar.

3.2. The Modal Scale in the *Summulae de Demonstrationibus*

In the *QAPr* discussion of necessity which we just considered, there is a background notion of falsifiability, which depends on causes and the effects they can produce. There is a power (namely, the Almighty) capable of annihilating all magnitudes; and from this fact, it follows that the propositions of geometry are falsifiable. Accordingly, we have had to modify our notion of necessity: the propositions of geometry and the natural sciences are necessary, but not simply so. Rather, they are necessary *de quando*—that is, assuming the thing(s) their subject terms stand for exist.

In the *Summulae de Demonstrationibus*, Buridan is more explicit about the role of causal powers in determining the modal status of a proposition. The context of this discussion is predications, which are divided into *per se* and *per accidens*. A *per se* predication affirms a predicate dealing with something essential to a subject of the subject itself. For example:

P26) A horse is an animal

Since such predications can never be rendered false, they are necessary. Contrast these with *per accidens* predications, which affirm something accidental of a subject. For example:

P27) Socrates is bearded

True as a proposition like (P27) may be, what it affirms of Socrates is only accidental, not essential. Since (P27) can be falsified, it is contingent. This modal aspect of *per se* predications is what interests us here.[186] Buridan sums up the contrast thus:

> a proposition is called *per se* because it is necessary [...] and *per accidens* because it is contingent".[187]

Since *per se* predications are at issue, and since *per se* predications are necessary, it follows that there are as many types of *per se* predications as there are types of necessity:

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[186] Of course, Buridan has a great deal more to say about *per se* predication than merely that it is necessary. But this discussion would not help us with the modal aspect of (A/C_{Def}), which is our present concern. For more on *per se* predications, see *Summulae* (8.6.2-3).

[187]"Propositio dicatur *per se* quia necessaria, et [...] *per accidens* quia contingents" (*Summulae* 8.6.2; De Rijk, 134, ll.9-10).

> Further grades might be posited on account of the fact that a *per se* proposition has to be necessary, and that there are diverse grades of necessity, and accordingly, diverse grades of perseity.[188]

What follows are four grades of necessity. (Here, I will list these as 1-4, using Arabic numerals, so as to avoid confusion with Grades I-III necessity of the *QAPr*). Unsurprisingly, Grade 1 in the *Summulae* is identical with Grade I in the *QAPr*:

> The first grade of necessity occurs when it is not possible by any power to falsify the proposition (while its signification remains the same), nor can things be otherwise than it says.[189]

Buridan does not give any examples of a Grade-1 necessary proposition here. But on the basis of what he says about the lack of causal powers capable of falsifying a Grade-1 necessary proposition, he must have in mind propositions like (P22), which we considered above:

> P22) God exists

As we saw above, propositions like (P22) cannot be falsified by *any* cause, and so are simply necessary.

We don't need to spend much time on the parenthetical requirement that Grade-1 necessary propositions have to retain the same signification in order to be true. I'll just

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[188] "Et adhuc possunt poni alii gradus, ex eo quod oportet propositionem per se esse necessariam, quia sunt diversi gradus necessitas et, secundum hoc, et perseitas" (*Summulae* 8.6.3; de Rijk 141, ll.14-15).

[189] "Est enim primus gradus necessitatis quia per nullam potentiam possibile est propositionem falsificari, stante significatione, vel aliter se habere quam significat" (*Summulae* 8.6.3; de Rijk 141, ll.15-16).

remark in passing that this requirement falls out of Buridan's radical conventionalism about language, whereby signification can change by convention or by stipulation. For example, Buridan elsewhere considers a sophism arguing that a proposition like "You will be a donkey" can be true, in a case in which *donkey* is imposed by convention (*ad placitum*) to signify something else. But this is a shift in signification of spoken terms, not of meaning. Consider the following case:

> A spoken utterance like 'A human is a donkey' can be true, positing that, by a deluge or by divine power, the whole of the Latin language is lost, because all who knew Latin are destroyed, and then a new generation following them impose by convention [*ad placitum*] the utterance *human* to signify the same as that utterance signifies to us now; and they impose *donkey* to signify the same as our *animal*.[190]

Conventionalism, along with tokenism, jointly give us this result: tokenism insists that there are no proposition *types*, but only tokens: in modern terms, there are no timeless propositions expressed by sentences, but only the sentences themselves. And conventionalism insists that linguistic items are, by and large, arbitrary: they mean whatever we as a linguistic community want them to mean.[191] Therefore, there are cases in which the spoken proposition "A human is a donkey" can be true. But it is only true

[190] "Secundum vocem 'Homo est asinus' potest esse vera, scilicet ponendo quod per diluvium vel per voluntatem divinam, totum idioma latinum sit perditum, eo quod omnes ipsam scientes sunt corrupti. Et tunc novi supervenientes imponant ad placitum suum istam vocem 'homo' significare idem sicut nunc ipsa significat, et ista vox 'asinus' imponatur ad significandum tantum quantum ista vox 'animal' nobis significat" (*Sophismata* VI, concl. 1; Scott, 103; 31r).

[191] I say "by and large" because, as Buridan (following Peter of Spain) recognises, there are terms that signify but not by convention—the barking of dogs, for instance, or the moans of the sick. (*Summulae* 1.1.4).

because it is subordinated to a true mental proposition, namely “A human is an animal”. Therefore, when we assess the truth of propositions, we have to take their signification as fixed. Otherwise, a spoken token of (P22) could be subordinated to a false mental proposition, and would then be false. Still, assuming (P22) retains the same terms with the same significations as it does for us now, it cannot be that (P22) is falsifiable by any power, natural or supernatural.

One step lower, Grade-2 necessary propositions are falsifiable by a divine power, but not by any natural one:

> Another grade occurs when it is impossible that [a proposition] be falsified, or when it is impossible for things to be otherwise by natural powers, and yet it is possible to falsify it supernaturally or miraculously. For example, ‘The heavens are moving’, ‘The heavens are spherical’, ‘the earth is a sphere’ and ‘Any place is filled’.[192]

As Gyula Klima notes, propositions like these are necessities of Aristotelian physics.[193] To us, they may not seem like examples of things that can only be falsified by a supernatural power, and not a natural one. It is indeed somewhat more difficult to cook up examples of this sort that appeal to modern ways of thinking: granted, no natural power could make the Milky Way into a cube, but this is because it is a gargantuan task, and not because it is qualitatively different from fashioning a lump of clay into a cube. Still, we might give examples of nomologically necessary but logically contingent propositions. For example:

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[192] “Alius gradus est quia impossibile est eam falsificari vel aliter se habere per potentias naturales, licet sit possibile supernaturaliter vel miraculose, ut ‘caelum movetur’, ‘caelum est sphaericum’, mundus est sphaericus’, ‘locus est plenus’” (*Summulae* 8.6.3; de Rijk 141, ll.18-21).

[193] Klima, *Summulae*, 733, n.236.

P28) Nothing is generated out of nothing

P29) Gravity is an attractive force

Propositions like (P28) and (P29) cannot be made true by any natural power, and therefore they are nomologically necessary. But God could generate a donkey out of thin air, and thereby falsify (P28); and God could reverse gravity, so that bodies with mass repel rather than attract each other—thereby falsifying (P29). Any such miracles would not entail a logical contradiction. Therefore, like the propositions of Aristotelian physics considered in Buridan's context, they provide examples of necessary propositions at Grade 2: falsifiable by divine intervention, but not by any natural power.[194]

Notice that the falsification of a Grade-2 necessary proposition does not involve the annihilation of the thing(s) the proposition is about: God could, to take Buridan's example, stop the movement of the cosmos, or form it into a cube. But presumably, doing so would not destroy it. This absence of annihilation sets Grade 2 necessity here apart from the Grade II necessity of the *QAPr*, considered above. And this also sets it apart from the Grade 3 necessity of the *Summulae de Demonstrationibus*, which Buridan describes as follows:

[194] It's noteworthy that Buridan seems to think the annihilation of an entire natural kind is not naturally, but only divinely possible. For instance, in *TC* IV.1 (ll.56-65, pp.112-3) he tells us that Aristotle held that propositions like (P20), "Humans are animals", were necessary because of the eternity of the world; "but it is true that it is not possible by any natural cause, though it is indeed possible by a supernatural miracle, that there should at some time be no horses, no earth, no fire". This seems odd, though perhaps Buridan is thinking of the annihilation of *everything*, not merely the annihilation of one natural kind. Or, perhaps, he's less mindful of the human capacity to annihilate on a grand scale—a blissful ignorance unavailable to us in this century.

> A third grade occurs with the assumption of the constancy of the subject; for instance, 'A lunar eclipse takes place because of the interposition of the earth between the sun and moon', 'Socrates is a human', and 'Socrates is capable of laughter'. These are said to be necessary in this way because it is necessary for Socrates, whenever [*quandocumque*] he exists, to be a human being capable of laughter. And it is necessary that, whenever there is a lunar eclipse, it occurs because of the interposition of the earth, etc.[195]

Here, as with the Grade II necessity of the *QAPr*, the language is temporal: whenever the thing or things that the subject term stands for exist, a Grade-3 necessary proposition is necessarily true. In fact, Buridan uses the same lunar eclipse example in Grade 3 of the *de Demonstrationibus* and Grade II of the *QAPr*.

There seems, however, to be an important difference between the Grade II of *QAPr* and the Grade 3 the *de Demonstrationibus*, at least as far as the examples are concerned. Recall that in the *QAPr*, Grade-II necessary propositions include the following:

P20) Humans are animals.

P21) All triangles are three-sided.

In the *Summulae*, Buridan gives us the following propositions:

[195] "Tertius gradus est ex suppositione constanti subiecti, ut 'lunae eclipssis est propter interpositionem terrae inter solem et lunam', 'Socrates est homo', 'Socrates est risibilis'. Hae enim dicuntur necessariae sic quia necesse est quandocumque est Socrates, ipsum esse honinem risibilem, et necesse est quandocumque est eclipsis lunae, ipsam esse propter interpositionem terrae, etc." (*Summulae* 8.6.3; de Rijk 141, ll.22-6).

P28) Socrates is a human.[196]

P29) Socrates is capable of laughter.[197]

Like (P20) and (P21), (P28) and (P29) are true assuming the constancy of their subject terms. Yet (P21) is falsifiable only by a divine power, whereas (P28) and (P29) can be falsified by a natural cause (hemlock for one), which would annihilate Socrates.[198]

Still, Grade 3 and Grade II necessity seem to be based on the same idea: a proposition can be necessary, and still falsifiable. But such a proposition is only falsifiable by the annihilation or destruction of the thing(s) the proposition is about. This feature of Grade 3 necessity sets it apart from Grade 2. A proposition necessary at Grade 2 can be falsified by a divine cause, but that cause need not be destructive. For example, consider the Grade-2 necessary propositions Buridan gives in the text cited above:

P30) The heavens move.[199]

P31) The heavens are spherical.[200]

Now no natural power can make the heavens into a motionless cube. But God can, and can thereby falsify (P30) and (P31), without annihilating anything. Conversely, God cannot

[196] "Socrates est homo" (*Summulae* 8.6.3; de Rijk 141, l.23)
[197] "Socrates est risibilis" (*Summulae* 8.6.3; de Rijk 141, ll.23-4)
[198] Whether (P20) can only be falsified by a divine power is another matter. Usually, when Buridan gives examples of all of a natural kind being annihilated, he posits a divine cause (for instance in his *QAM* VI.7). But presumably this is overkill: we humans are natural causes, and we could just take it upon ourselves to eradicate horses, as we did with dodoes. Still, we can't eradicate magnitudes, and so the point I am making here about the examples from these two texts stands.
[199] "caelum movetur" (*Summulae* 8.6.3; de Rijk 141, l.20)
[200] "caelum est sphaericum" (*Summulae* 8.6.3; de Rijk 141, l.20)

falsify propositions like (P20) and (P21), (P28) and (P29) without destroying the things these propositions are about. And *a fortiori*, no natural cause can falsify (P28) or (P29) without annihilating what these propositions are about.

Hence Buridan seems less concerned with the difference between divine and natural power at Grade 3 in his *de Demonstrationibus* presentation of the modal scale: what is important here is just that there is no way of falsifying a Grade-3 necessary proposition without destroying the thing(s) it is about. If we insisted on drawing a distinction between divine and natural falsifiability at Grade 3, we would see this level of necessity bifurcate: some propositions, like (P21), are about things that can be annihilated only by a supernatural cause. Others, like (P31), are about things that can be annihilated by a natural cause. Still, I find no evidence in the text that Buridan has such a distinction in mind: what is relevant is just that Grade-3 necessary propositions cannot be falsified without the annihilation of the thing(s) they are about.

As I said at the outset, Buridan seems much more concerned with causal powers in the *Summulae de Demonstrationibus* modal scale than in the *QAPr* one. This concern with causal power extends to Grade 4, which Buridan describes as follows:

> Further still, there is a fourth mode, which involves restriction. For ‘possible’ is sometimes predicated broadly, in relation to the past, present, and future; and sometimes it is predicated restrictively, in relation to the present or the future, in accordance with what was said at the end of the first book of *De Caelo* [I.12, 283^b13ff]—namely that there is no capacity or power to alter the past (*i.e.* on what has been done) but only on what is or

> will be [...]—the same goes for *necessary* and *impossible*, which are either predicated with restriction, or broadly.[201]

Since there is no power over the past, Grade 4 necessity is the necessity of once contingent but now unchangeable things, which there is no longer any power to change. Similarly, Grade 4 impossibility attaches to propositions that were once possible, but now no longer can be the case. Here is Buridan's example:

P32) Aristotle walks.[202]

According to Buridan, a proposition like (P32) is possible in the broad sense, because Aristotle could at some time walk; but in the restricted sense, it is impossible, since Aristotle does not exist, and therefore cannot walk any longer.

Similarly, Buridan thinks the following proposition is necessary in the restricted sense, since Aristotle does not now exist:

P33) Aristotle neither does nor will walk.[203]

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[201] "Adhuc est quartus modus, secundum restrictionem. Nam sicut 'possibile' aliquando ample dicitur, in ordine ad tempus praesens, praeteritum et futurum, et aliquando restrictive, in ordine ad praesens vel futurum, iuxta quod dicitur in fine primi *de Caelo* quod non est virtus sive potestas ad praeteritum, scilicet quod factum esse, sed eius quidem quod est esse vel futurum esse [...] ita etiam 'necesse' et 'impossibile' dicuntur secundum restrictionem vel ample " (*Summulae* 8.6.3; de Rijk 142, ll.1-7)

[202] "Aristoteles ambulat" (*Summulae* 8.6.2; de Rijk 142, l.9)

[203] "Aristoteles non ambulat nec ambulabit" (*Summulae* 8.6.2; de Rijk 142, l.16).

Here again, what makes (P33) necessary is that it deals with a past that is now gone, over which there is no causal power. Accordingly, in the restricted sense, (P33) is necessary: since Aristotle no longer exists, he cannot walk.

To sum up, we can represent the modal scale of Buridan's *Summulae de Demonstrationibus* as follows:

Grade 1: simple necessity

e.g. "God exists"

Unfalsifiable by any power, including divine

Grade 2: nomological necessity

e.g. "The heavens move"

Unfalsifiable by any natural power
Falsifiable by divine power

Grade 3: *de quando* necessity

e.g. "Socrates is a human"
"Socrates is capable of laughter"

Falsifiable by divine or natural power, but only by the annihilation of the subject term

Grade 4: necessity by restriction

e.g. "Aristotle walked"

Once contingent, but now unfalsifiable by any power, including divine

With the foregoing considerations in mind, it remains to be asked: Which kind of necessity is at play in Buridan's definition of consequence (A/C_{Def})?

3.3. Applying the Modal Scale to Buridan's Definition of *Consequentia*

All the foregoing examples of propositions, necessary at the various grades, are categoricals. But consequences are, by definition, hypotheticals (*propositiones hypotheticae*), in the sense that they are made up of multiple proposition-like parts, as we saw in the preceding chapter (§ 1). So it remains to extend this modal notion to hypotheticals. Before I show how this should be done, I should say why I think it *can* be done. Are we right to extend Buridan's claims about necessity and causality from categorical propositions to their hypothetical counterparts?

Yes. Buridan himself makes the connection explicitly: in *QAPr* I.19-20, he asks whether the moods of first-figure syllogisms hold in virtue of their form (*gratia formae*)—that is, whether they are formally valid. In Chapter 4, we will see in greater detail what formal validity is. For now, it is sufficient to note that a formally valid argument holds in all substitutions of its non-logical terms, and therefore can be represented schematically. These are distinct from simple materially valid arguments, which hold in virtue of the meaning of their non-logical terms, and so do not survive substitution.[204] Here is one such:

[204] Buridan recognises two types of materially valid consequences: those that hold simply, and meet the modal requirement; and those that just happen to have consequents that are not false while their antecedents are true. These latter consequences he calls 'as of now' (*ut nunc*). I will deal with them in the next chapter. For now, I will use the term *materially valid* to mean just the former, simply materially valid consequences,

A11) A human runs
∴ An animal runs.[205]

Although (A11) is valid, we can readily cook up a formally identical argument that fails:

A12) A horse walks
∴ Wood walks.[206]

Hence although (A11) is valid—indeed, it is a valid topical inference from genus to species—it is not valid in virtue of its form.

The formally valid schemata Buridan is concerned with in *QAPr* (I.19) are two moods of the first figure, Barbara (S3) and Celarent (S4):

S3) Every A is B
Every B is C
∴ Every A is C

S4) No A is B
Every B is C
∴ No A is C

as contrasted with formally valid ones, since what is at stake here is a modal requirement that is not at play with *ut nunc* consequences.

[205] "Homo currit; ergo animal currit" (*TC* I.4, l.13).

[206] "Equus ambulat; ergo lignum ambulat"(*TC* I.4, l.14).

These seem to be prime candidates for formal validity. But, as Buridan points out, Barbara does not hold in all substitution instances; in divine terms, particularly, it fails:

A13)	Every God is the Father;	(true)
	Every Son is God;	(true)
∴	Every Son is the Father.[207]	(false)

Granted that each member of the Trinity is a distinct person, and yet fully God, the premisses of (A13) are true, and the conclusion is false. Therefore, it seems, syllogisms in the mood Barbara are not valid in virtue of their form.[208]

Likewise, we can cook up a miraculous counterexample to Celarent (S3), though Buridan's example is a bit more involved: as Buridan acknowledges, "it's harder to come up with counterexamples to the other moods".[209] Still, it is possible. Suppose that every

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[207] "Item ponamus quod nullus sit pater vel filius nisi deus; tunc arguitur sic:

omnis deus est pater
omnis filius est deus
ergo omnis filius est pater.

Praemissa sunt verae secundum casum positum, et conclusio est falsa" (*QAPr* I.19 obj. 6).

It might seem odd to see a sign of quantity like 'every' (*omnis*) applied to an apparently singular term like 'God' (*deus*). But for Buridan, *deus* is a common term, not a singular one: for the term is a species term, "since although on the part of the thing signified it is unfit [*repugnat*] that the term should stand for more than one thing—still, by its mode of signification or by its mode of imposition, the term is not unfit to stand for multiple things [...] For those who know the mode of signification and imposition of this term *God* can imagine, according to its imposition, that there are multiple gods. But if there were multiple gods similar to that God who exists, then this term 'god' would moreover stand for each of them, and without a new imposition. And something similar goes on with these terms: 'sun', 'moon', 'earth', and the like" (*Summulae* 2.3.5; de Rijk, pp.33-4, ll.37-43).

[208] This example is not unique to Buridan, and is actually much earlier: Abaelard discusses it in his *Theologia Christiana*, and the *quod est* solution proposed by Buridan is actually due to the *Sentences* commentary of Adam Wodeham (ca. 1295-1358). See Simo Knuuttila, "Trinitarian Theology", *Encyclopedia of Medieval Philosophy*, ed. Henrik Lagerlund (New York: Springer, 2011), 1335-7. I'll discuss the *quod est* locution as a reference-fixing designator in just a moment.

[209] "difficilius est instare contra alios modos" (*QAPr* I.20, co.).

human being is white, and that no white thing (*album*) will be produced in a given house. From this, it would follow that no white person would be produced in that house, either. And therefore, the following syllogism appears to be valid:

A14) No white thing will be produced in this house
Every person is white
∴ No person is going to be produced in this house.[210]

Still, there is a divine counterexample: suppose, says Buridan, God were to produce Ethiopians in the house, *ex nihilo*. Before God does this, the premisses are both true, but the conclusion is false. Therefore, Celarent is not valid in virtue of its form, either.

Buridan's solution to both need not detain us long here: Buridan just restricts the reference of the terms involved in syllogisms like (A13) and (A14) with a 'that is' (*quod est*) locution. If we append this locution to the subject terms in the above syllogisms, then at least one of the premisses will be falsified, and so the syllogism will be valid. Here are the modified syllogisms:

A13′) Everything that is God is the Father; (false?)[211]

[210] "Tunc arguitur sic:
nullum album est generandum in hac domo
omnis homo est albus
ergo nullus homo est generandus in hac domo" (*QAPr* I.20, co.).

[211] This seems to be the false premiss, though Buridan hedges somewhat: he tells us that "Some say that [...] the major premiss is false" ("aliqui dicunt [...] maiorem esse falsam"), but that anyway we should "go ask the theologians about it" ("Quomodo sit de omnibus istis dicendum petatis a theologis"; *QAPr* I.19, co.). This is not out of the ordinary: as an arts master, Buridan frequently skirts theological questions, which he regards as out of his wheelhouse—and out of his pay grade. Anyway we, who are not exposed to the hazards of doing theology at the fourteenth century University of Paris, may speculate freely on the truth of the major premiss. It looks false to me.

	Everything that is the Son is God;	(true)
∴	Everything that is the Son is the Father.[212]	(false)

A14′)	Nothing that is white will be produced in this house	(true)
	Everything that is a person is white	(true)
∴	Nothing that is a person will be produced, etc.[213]	(true)

What produced the problem in the first place was so-called ampliative terms (*termini ampliativi*), which shift the tense or reference of the terms in the propositions—terms like 'will be produced', and the like. Hence by restricting the subject terms of the premisses, we can render at least one of the premisses false, and thereby render the syllogism valid. We will deal more with these ampliative terms, as they apply to modal contexts, in Chapter 5. In (A13′), the syllogism is made valid by falsifying one of the premises; in (A14′), the syllogism is made valid by rendering the conclusion true: although something which will be a person will be produced in this house, nothing which is now a person will be produced, etc.

For our present purpose, the foregoing examples show two things. First, Buridan extends the causal notion of falsification of categoricals, set out in the above modal scales, to cover the invalidation of hypotheticals, too. What makes (A13) and (A14) invalid is that there is a cause capable of rendering the premisses true and the conclusion false. So

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[212] "Et tunc pono istam conclusionem quod primus modus primae figurae est formalis sub isto modo loquendi:
omne *quod est* B est A
omne *quod est* C est B
ergo omne *quod est* C est A" (*QAPr* I.19, co.; emphasis added).

[213] Buridan advises the *quod est* approach for (A14) as well, though he does not set it out explicitly. Its reconstruction is simple, and need not detain us here: it can be done exactly the way we did with (A13), following the schema in the immediately foregoing footnote.

we are not wrong to take a causal notion to be what underwrites the modal requirement in (A/C_{Def}). Therefore, the modal requirement must be that of one or more grades in the scales set out in *QAPr* and the *Summulae*.

Second, these examples show that the modal requirement for formal validity is Grade I/1: an argument is formally valid just in case it cannot be rendered invalid by a divine causal power—and, *a fortiori*, by any power at all. After all, if God can render a syllogism like (A13) or (A14) invalid, then it is not formally valid, in Buridan's view.

Still, it remains to be seen what modal notion is at play with materially valid consequences: although the necessity of formal validity holds at Grade I/1, we should not *eo ipso* infer that this same level of necessity underwrites materially valid consequences, too.

For a long time, I thought that materially valid arguments must hold at some lower grade on the scale—perhaps Grade 2 or II/3. But this is not Buridan's view. Rather, there is direct (which is to say *textual*) and indirect (which is to say *rational*) evidence that Buridan thinks that both materially and formally valid consequences meet the same modal criterion.

First, the indirect evidence: if there are different necessities underwriting formal and material validity, then *consequence* is equivocal. For if the modal requirement in (A/C_{Def}) bifurcates in this way, then the necessity of deduction would differ depending on the type of consequence involved. This seems especially odd given what Buridan says about the way materially valid consequences like enthymemes can be reduced (*reducere*) to formal ones. For instance, the following argument is materially valid if the minor premiss is left out of its formulation, as in (A15); and it is formally valid if this premiss is added in, as in (A15′):

A15) A human runs

∴ An animal runs.

′

A15′) A human runs

(Every human is an animal)

∴ An animal runs.[214]

Buridan says such reductions of enthymemes like (A15) to syllogisms like (A15′) as make the inferences *evident*:

> It seems to me that no materially valid consequence is inferentially evident (*evidens*), except by its reduction to a formally valid one.[215]

That is, the validity of (A15′) is more obvious than that of (A15), since the suppressed minor premiss of the latter is explicitly included in the former. But the language of *making evident* is epistemic, and comes from the literature on demonstrations. It is not modal or metaphysical. Indeed, there is no textual evidence that Buridan thinks a deduction like (A15) undergoes a shift in its modal status with its expansion as (A15′).

Second, and more importantly, Buridan frequently says that formal and material consequences meet the *same* modal requirement. For instance, in the *Summulae de*

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[214] "si dico 'homo currit; ergo animal currit', probabo consequentiam per hoc quod omnis homo est animal" (*TC* I.4, ll.19-20).

[215] "Et videtur mihi quod nulla consequentia materialis est evidens in inferendo nisi per reductionem eius ad formalem" (*TC* I.4, ll. 14-16).

Propositionibus (1.6.1), Buridan characterises the distinction between these types of consequence as follows:

> Since we have now come to the difference between material and formal consequence, it remains to be seen in what ways they are alike, and in what ways they differ. And *they are like in that it is impossible for the antecedent to be true while the consequent is false*. But they differ in that a consequence is called formal if propositions of similar form, formulated with any terms whatsoever, would likewise be valid.[216]

Hence materially and formally valid consequences meet the same modal criterion—a point I will return to in Chapter 4 (§2.1). Since the necessity of formally valid consequences is Grade I/1, so, too is the necessity of materially valid consequences. And, since Grade I/1 does not change in the two scales (as we saw in §3.1 and §3.2 above), we can take deductive necessity to be the same in the *QAPr* and in the *Summulae*. Thus a valid consequence meets the modal requirement just in case even God could not make the antecedent(s) true and the consequent false.

We have seen under what conditions *consequentiae* hold: when they meet the modal, signification, and simultaneous formulation requirements set out in (A/C$_{Def}$). And we have now come to the subject of their division into formally and materially valid *consequentia*.

[216] "Et quia nunc locutum est de consequentia materiali et formali, videndum est quomodo conveniant et differant. Conveniunt enim in hoc quod impossibile est antecedens esse verum consequente existente falso. Sed differunt quia consequentia formalis vocatur, si ex quibuscumque terminis formaretur propositiones similis formae, valeret consequentia" (*Summulae* 1.6.1; van der Lecq 60, ll. 18-22; emphasis added)

In the next two chapters, I look at propositional matter and form, and explain what this distinction hinges on.

Chapter 3

Types of *Consequentiae* I

Categoremata and *Syncategoremata*

Words are pegs to hang ideas on.
—Henry Ward Beecher

Let thy words be few.
—Ecclesiastes 5.2

Immediately following his definition of *consequentiae* in *TC* I.3, Buridan divides them into two types: those that are formally valid, and those that are materially valid (*TC* I.4). Some valid consequences hold in virtue of their logical structure or form, while others are valid, but not in a structural or formal way. For instance, contrast the following:

A1) All horses are mammals
Some quadrupeds are horses
∴ Some quadrupeds are mammals

A2) No horses are humans

∴ All horses are incapable of laughter

In both cases, it cannot be that the antecedents are true and the conclusion false. But in (A1), the argument from premisses to conclusion holds because it is in a valid form (namely, the syllogistic mood Datisi); (A2), on the other hand, holds in virtue of the meaning of the non-logical terms—terms like *horse* and *human*—which render the conclusion necessarily true. Following Buridan, we may call arguments like (A1) *formally* valid, and arguments like (A2) *materially* valid. The purpose of this and the next chapter is to explain this distinction, which Buridan characterises as follows:

> A consequence is called *formal* which holds in all [substitutions of] terms, so long as it retains the same form [...] whereas a material consequence is one which [...] does not hold in all terms, while retaining the same form.[217]

As an example of a formal consequence, Buridan gives the following schema:

S1) Something that is B is A
∴ Something that is A is B.[218]

So long as we uniformly replace the schematic terms A and B with significative ones, we will never encounter a substitution instance in which the antecedent of (S1) is true, and the consequent is false. Therefore, (S1) is valid in virtue of its logical form, which depends at least in part on the terms we keep constant in substitution instances.

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[217] "Consequenta *formalis* vocatur quae in omnibus terminis valet retenta forma consimili [...] Sed consequentia materialis est [...] quae non tenet in omnibus terminis, forma consimili retenta" (*TC* I.4.5-13).
[218] "Quod est B est A; ergo quod est A est B" (*Summulae* 1.6.1; van der Lecq p. 60, l.24).

Conversely, materially valid consequences hold merely in virtue of their significative, non-logical terms. For this reason, such consequences cannot be expressed schematically, like (S1): any example of a materially valid consequence must be an argument with real, referential terms, not schematic letters. To take Buridan's example:

A3) A human is running
∴ An animal is running.[219]

It is impossible for the antecedent of (A3) to be true, and the consequent false, since any human is (necessarily) an animal. But consider what happens when we give a uniform substitution of the non-logical terms of (A3)—namely *human*, *animal*, and *running*—but keep its logical form intact:

A4) A horse is walking
∴ Wood is walking.[220]

Clearly, (A4) is invalid, though it shares its form with (A3). The form both (A3) and (A4) share may be represented schematically as follows:

S2) A is B
∴ C is B

[219] "Homo currit; ergo animal currit" (*TC* I.4.13).
[220] "Equus ambilat; ergo lignum ambulat" (*TC* I.4.14).

A schema like (S2) will be valid in some terms, but invalid in others. Since (S2) does not remain valid across all substitution instances, the argument (A3) does not hold in virtue of its form. The validity of an argument like (A3) thus depends not on the logical constants or the structure of the propositions involved, but on the meanings of its non-logical terms, which Buridan calls the *matter* of a *consequentia*.

Hence the central notion that divides material from formal consequence is *substitution*. And to determine what gets substituted, we have to posit a distinction between non-logical—and therefore substitutable—terms, on one hand, and logical constants, on the other. Accordingly, the present chapter lays the groundwork for the form-matter distinction by examining the distinction between logical and non-logical terms. At that point, we'll have laid a solid foundation for distinction between logical form and matter, to be discussed in Chapter 4.

The distinction between logical constants and inconstants[221] looks a good deal like the distinction between syncategorematic and categorematic terms. And indeed, it depends on this distinction in important ways. Roughly, syncategorematic terms (or *syncategoremes*) are those terms that have no signification on their own but, unlike their categorematic counterparts, signify only in combination with other terms. Easy examples of syncategoremes are terms like *or*, *not* and *is*. Conversely, pure categoremes are those terms that, taken on their own, signify things. So *horse, human, wood*, and other such terms are categorematic. Thus it seems the distinction between formal and material validity depends heavily on the distinction between logical constants and inconstants; this

[221] Sorry for this term. I'm not keen on it either. But it happens that *inconstants* is a much better term than *variable*, which is much too theoretically freighted to use in the present context. For what it's worth, the term only comes up five more times here.

latter distinction, in turn, depends on the distinction between syncategorematic and categorematic terms. And the distinction between syncategorematic and categorematic terms will, finally, depend on signification.

Indeed, it is commonly assumed that the distinction between propositional matter and form is *reducible* to the distinction between categorematic and syncategorematic terms, respectively; and that this distinction is *reducible* to signification: categorematic terms have it, and syncategorematic terms do not, full stop. In the literature on medieval logic and in historically-minded philosophy of logic and language, these assumptions are broadly taken for granted. Such assumptions find perhaps their clearest and most succinct articulation in John MacFarlane's *SEP* article, "Logical Constants":

> The most venerable approach to demarcating the logical constants identifies them with the language's syncategorematic signs: signs that signify nothing by themselves, but serve to indicate how independently meaningful terms are combined [...] In this framework, words divide naturally into those that can be used as subjects or predicates ("categorematic" words) and those whose function is to indicate the relation between subject and predicate or between two distinct subject-predicate propositions ("syncategorematic" words) [...] The syncategorematic words were naturally seen as indicating the structure or form of the proposition, while the categorematic words supplied its "matter."[222]

[222] John MacFarlane, "Logical Constants", *Stanford Encyclopedia of Philosophy* (Winter 2017 Edition), ed. Edward N. Zalta, §1.

Similarly, Gyula Klima holds that syncategoremata have a characteristically *logical* function, though he does not adopt MacFarlane's reductionist line:

> syncategorematic terms [...] are imposed to exercise the *logical functions* of modifying the semantic functions of categorematic terms with which they are construed.[223]

Hence logical matter depends on categoremata, which refer to things, whereas logical form is determined by syncategoremata, which function logically and do not refer to things outside the mind. These two categories are therefore taken to be "mutually exclusive and jointly exhaustive", in Norman Kretzmann's phrase.[224] We can boil this view—which I will subsequently refer to as *the Clean Divide View*—down to the following claims:

Claim I: Syncategorematic and categorematic terms are strictly demarcated.

Claim II: Syncategorematic terms can be identified with the class of logical constants; categorematic terms with the class of nonlogical constants.

Claim III: Syncategorematic terms do not refer to anything outside the mind; categorematic terms invariably do.

Claim IV: Syncategorematic terms alone determine propositional form; categorematic terms alone pertain to propositional matter.

[223] Gyula Klima, "Syncategoremata", *Encyclopedia of Language and Linguistics* (2nd ed.), ed. Keith Brown (Amsterdam: Elsevier, 2006), 355. Emphasis added.
[224] Norman Kretzmann, "Syncategoremata, Exponibilia, Sophismata". *The Cambridge History of Later Medieval Philosophy*. Ed. Norman Kretzmann, Anthony Kenny, and Jan Pinborg. (Cambridge: Cambridge UP, 1982), 211.

Here is a diagram to clarify these relations:

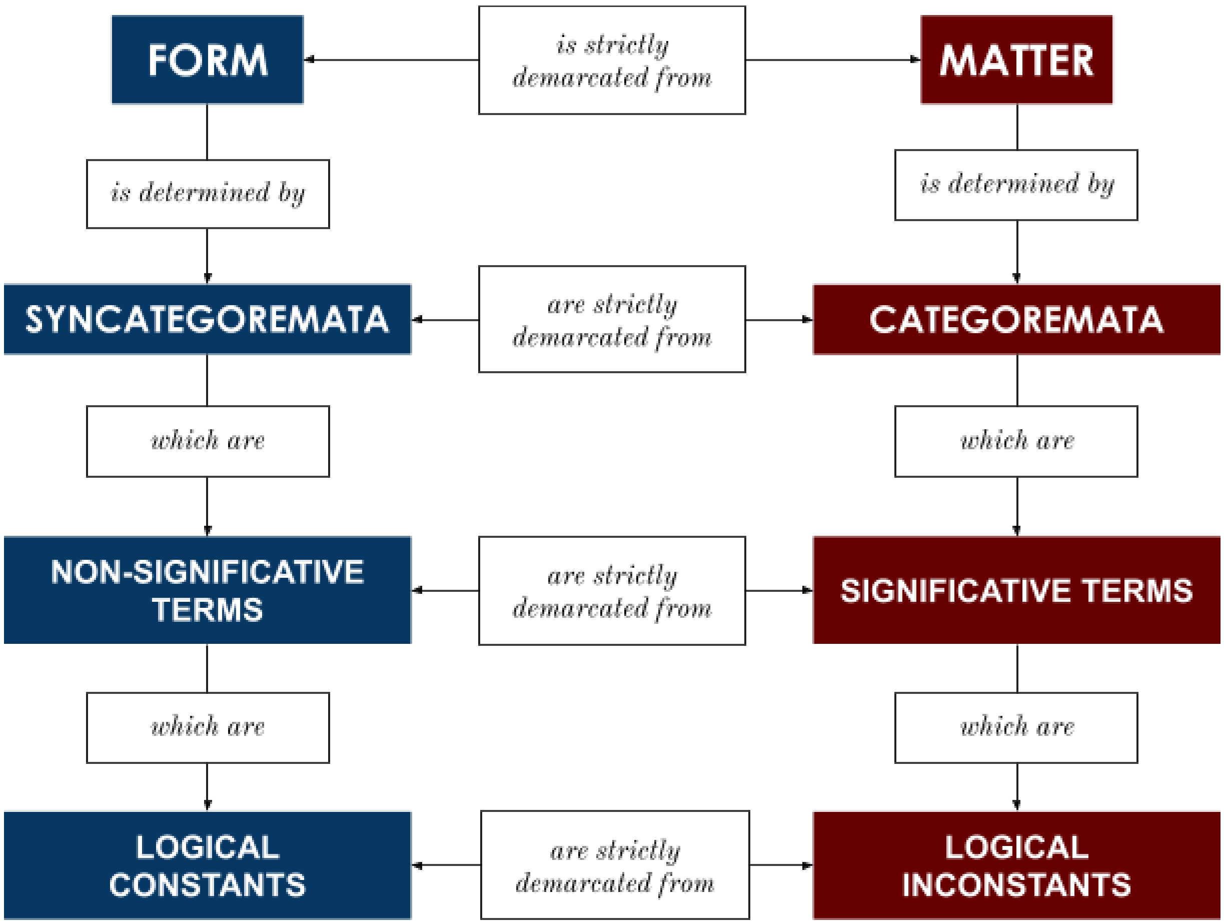

Fig. 3.1:
The Clean Divide View on form, syncategorematic terms, and the like.

To be fair, the Clean Divide View comes in two versions: a stronger and a weaker. MacFarlane provides a nice articulation of the stronger view, on which the clean divide just *is* what's going on in medieval logic. Alternatively, we might endorse a weaker view, on which the Clean Divide is a nice shorthand for avoiding theoretical nitty-gritty —nitty-gritty that is the focus of this chapter. A proponent of the weaker view might acknowledge that it is probably technically false, but that it provides a convenient shorthand in the discussion of other subjects in medieval logic. This seems to be the view of Gyula Klima. To me, this weaker view is perfectly fine, so long as its proponents are willing to admit that (i) the view is false, and (ii) that reconsideration of Claims I-IV is a worthwhile project—one that will give deeper insight into how Buridan's logic (and medieval logic more generally) actually *works*. And, as can be readily seen, anyone who rejects (i) and (ii) winds up back in the strong version of the Clean Divide View.

In any case, the present chapter presents a sustained case against this Clean Divide View, weak and strong. As we will see, the above distinctions—form vs. matter, constant vs. inconstant, syncategoreme vs. categoreme—are not so clear-cut, and their interrelations not so tidy, as has been commonly supposed. In brief: Claims II-IV are false, or at least in need of considerable modification. Claim I is completely false, and unsalvageable. In what follows, I give five lemmata, each of which undermines at least one of Claims I-IV. I conclude this chapter with a reassessment of the Clean Divide View.

Let's begin with categoremata, which are more clearly demarcated than their syncategorematic counterparts.

1. Categoremata: All Things to All People

In the *Summulae de Suppositionibus* (4.2.3), Buridan separates terms into pure categoremata, pure syncategoremata, and mixed. Here we find the following criteria for pure categoreme-hood:

> Terms are called *purely categorematic* when [i] they signify not only the mental concepts they signify immediately, but also the things conceived by those concepts. And [ii] such terms can be subjects or predicates by themselves [*per se*], and they include no purely syncategorematic elements, *e.g.*: *person*, *stone*, *whiteness*, *white thing*, and other terms like these.[225]

Thus purely categorematic terms are distinguished by their meaning, and also by the role they play in well-formed propositions: Buridan sums this up by saying that "terms are called categoremata on account of their predication or their signification".[226] Hence we find here both (i) a semantic and (ii) a syntactic criterion for categoreme-hood: (i) semantically, pure categorematic terms signify something conceived by means of a concept. And (ii) syntactically, categoremata are suited either to serving as subjects or predicates in a well-formed proposition. In what follows, I look at each of these, and consider a problem

[225] "Dicuntur autem pure categorematicae, quia non solum significant conceptus quos immediate significant, sed etiam res illis conceptibus conceptas. Et sunt per se praedicabiles vel subicibiles, et nullum purum syncategorema includunt, ut 'homo', 'lapis', 'albedo', 'album', et huiusmodi." (*Summulae* 4.2.3; van der Lecq, p.18, ll.18-21).

[226] "Categorema dicitur a praedicanto vel a significando" (*Summulae* 4.2.3; van der Lecq, p.19, ll.4-5).

The *vel* here is odd: the initial presentation of the criterion is as a single conjunction (see the *et* in the footnote immediately above); but in Buridan's discussion, he subsequently turns it into a disjunction. So the strong claim, that any categoreme will meet both the semantic and the syntactic criteria, gets weakened: at minimum, it will meet one or the other (but perhaps not both).

for Buridan's account of signification. I then turn to syncategorematic terms (§ 2), and then to mixed (*i.e. impure*) categoremata and syncategoremata (§ 3).

1.1 The Syntactic Criterion

I'll start with the syntactic criterion (ii), since it is much simpler than the semantic criterion—but also coarser-grained, as we will see. Following the text cited above, we can sum up the syntactic criterion for categoremes as follows:

> Syn_C) A term is categorematic just in case it can serve as subject or predicate of a well-formed proposition.

We saw already in Chapter 1 (§ 1.1) that the most basic categorical proposition is a copula flanked by two terms, such as:

> P1) Socrates is running
> P2) Socrates isn't sitting

By (Syn_C), categorematic terms are those that can serve as the subject or predicate of a categorical proposition like (P1) or (P2).[227] This will comprise all those terms that have signification, such as *Socrates* and *running*, since any term that is apt to stand for

[227] Note however that the *Summulae*'s syntactic criterion is much older: it goes back at least as far as Priscian (*Institutiones* 9, 2, 54.5; cited by Kretzmann p.211 n.3). As Gyula Klima notes, this distinction is at least implicit in Aristotle's discussion of the copula which "signifies nothing, but co-signifies some combination" (*De Interpretatione* 16^b24-5). See Klima, "Syncategoremata", 353.

something is by definition apt to stand as a term in a categorical proposition, and vice versa.

But (Syn_C) also casts a wide enough net to catch terms that are not simply categorematic, but mixed. Terms like *non-being* and *vacuum* (i.e. *place not filled with a body*) are apt to serve as subjects or predicates in well-formed propositions like the following:

P3) Non-being is incomprehensible
P4) A vacuum is impossible

I will deal with these terms in greater detail below (§1.2.3), where I assess Buridan's account of non-significative categorematic terms. For now, it is sufficient to observe that (Syn_C) does not allow us to distinguish between pure categoremes, like the subjects and predicates of (P1) and (P2), from mixed terms, like the subjects and predicates of (P3) and (P4)—namely, *non-being* and the like. Therefore, as we will see, the primary and most important criterion for pure categoreme-hood is not the syntactic one, (Syn_C), but the semantic one, to be considered presently. Although (Syn_C) catches all and only categorematic terms, it tells us nothing about the finer-grained distinction between pure categoremata and their mixed counterparts. As Buridan himself notes, these mixed terms behave syntactically in the same way as pure categorematic terms, even though they imply a syncategorematic element, like negation:

> there are many predicable utterances that are not purely categorematic, because they imply negations—terms like *nobody*, *nothing*, and negative terms.[228]

Hence we need the semantic criterion to separate the purely categorematic terms from their mixed cousins, particularly since—as we will see later on (§ 3), there are mixed syncategoremes as well as mixed categoremes.

1.2. The Semantic Criterion

By contrast, the semantic criterion set out in the *Summulae* (4.2.3) passage cited above introduces many more complexities than its syntactic counterpart. Here is the semantic criterion for categoremes:

> Sem_C) A term is categorematic just in case it signifies something *ad extra*.

This calls for two important clarifications. First, there is an ambiguity to be resolved about what, for logical analysis, the terms of language *are*: Buridan distinguishes three levels of language: written, spoken, and mental. We will be chiefly concerned with the mental. On the mental level, categorematic terms are simple concepts that signify things beyond (*ad extra*) the mental concepts to which they are subordinated (a feature of Buridan's

[228] "Multae enim sunt dictiones praedicabiles quae non sunt pure categorematicae, quia implicant negationes, ut 'nemo', 'nihil', et termini privativi" (*Summulae* 4.2.3, van der Lecq, p.19).

approach I'll address in the next section). Second, then, we need to say something about what signification *ad extra* is.

1.2.1 The Three Levels of Language

Frequently, we think and speak of terms (and the propositions made out of them) as spoken utterances; and indeed this is the way Buridan frequently discusses them. But as Buridan points out, terms are strictly speaking *concepts*, to which spoken (and written) words are subordinated by convention (*ad placitum*). Thus in the *Summulae de Propositionibus* (1.6.1), Buridan distinguishes three levels of language—consciously following Aristotle's discussion in *De Interpretatione* (I.16^{a}3-6):

> It must be recognised that expressions [*orationes*], and terms [*termini*] or words [*dictiones*], can be distinguished in three ways, as is touched upon in the first chapter of *On Interpretation*: namely as mental, spoken, and written [...] the terms of a mental expression are simple concepts which the mind combines or divides. And just as simple concepts are designated for us by means of simple utterances called *words* [*dictiones*], so also we designate a combination of concepts by means of a combination of words.[229]

So the terms of a mental proposition are the simple concepts out of which the mental proposition is made. For the logician (as opposed to the grammarian), this mental level of

[229] "Sciendum est ergo quod triplex potest distingui oratio et triplex terminus vel dictio, prout tangitur in principio libri Peri hermeneias, scilicet mentalis, vocalis, et scripta [...] Sicut etiam conceptus simplices designantur nobis per voces incomplexas quas vocamus 'dictiones', ita complexionem conceptuum designamus per complexionem dictionum" (*Summulae* 1.6.1; van der Lecq p.16, ll.4-11).

language is where the rubber hits the road: it is *mental* complexity, not grammatical complexity, that makes an utterance a term or a proposition. Thus the complexity of a spoken utterance does not determine its corresponding mental complexity: incomplex spoken expressions can designate complex mental ones, and likewise complex utterances can designate simple concepts.

Buridan's logical works are full of examples where a simple spoken term is subordinated to something complex on the mental level, and vice-versa. A simple spoken term like *Iliad* could correspond with a complex mental expression—for example, if it were taken to signify the whole Trojan story.[230] In such a case, it would be a complex proposition for the logician (though it would still be just a word for the grammarian). Indeed, even a simple barrel hoop hanging outside a tavern can be subordinated to a full mental proposition, if the barrel hoop is taken by convention to mean "Wine sold here!"[231]

Conversely, a complex spoken utterance could be subordinated to a simple mental concept—for example, if we took "A human runs" to mean the same thing as the categorematic term *stone*, then the grammatically complex phrase would, on the mental level, correspond to a simple concept.[232] Thus mental complexity on one hand, and vocal or semiotic complexity on the other, are completely independent.[233]

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[230] "Sed non esset inconveniens quod apud logicum vocaretur oratio significativa, ut si hoc nomen 'Ilias' imponeretur ad significandum aequivalenter 'historiae Troianae" (*Summulae* 1.6.1; van der Lecq p.17, ll.2-4).

[231] "Sic enim circulus pendens ante tabernam est signum impositum ad significandum ad placitum, cuius interpretatio est haec propositio: 'vinum venditur in haec domo'." (*Summulae* 8.2.3; de Rijk, p.33, ll.13-15).

[232] "Si enim haec tota vox 'homo currit' esset imposita ad significandum simplicem lapidem, sicut haec vox 'lapis', tunc 'homo currit' non esset oratio vocalis, sed simplex dictio, sicut 'lapis'." (*Summulae* 1.6.1; van der Lecq, p.16, ll.14-17).

[233] It is worth noting that this primacy of mental language is a crucial difference between Buridan and Ockham, for whom spoken terms can signify directly. Buridan would insist that spoken or written terms only signify by the mediation of concepts, which is why (to use his example) unknown languages, while

Hence in our semantic account of categorematic terms in Buridan, we are concerned with the simple mental concepts to which the terms are subordinated—that is, we are concerned with categorematic terms in *mental* language, rather than in its spoken or written counterparts. Thus in what follows, by *categorematic* (or *syncategorematic*) *term*, I mean to refer primarily to terms in the *mental* language. For brevity's sake, I will generally omit the modifier *mental*, unless it is required to avoid ambiguity.

1.2.2. Simple Concepts as Categorematic Terms

For (Sem_{C}), the most important characteristic of pure categorematic terms in mental language is that they signify something beyond themselves (*ad extra*). In the *Summulae de Suppositionibus* (4.2.3), where Buridan presents the distinction between categorematic and syncategorematic terms, he tells us that:

> Any word [*dictio*] that can be used in a proposition is itself imposed to signify something, *i.e.* a concept of the mind [...] but further, some spoken words [*voces*] are imposed to signify in themselves, and beyond these concepts, they signify the things [*res*] that are conceived by these concepts. In this way, the term *donkey* signifies donkeys, the term *whiteness* whiteness, and the terms *tomorrow* and *today* signify a certain time.[234]

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significative, do not signify for the non-fluent hearer (see *Summulae* 1.1.6, where Buridan's example is written Hebrew).

[234] "Oportet scire quod omnis dictio quae potest intrare propositionem est per se imposita ad aliquam significationem, scilicet ad significandum aliquis mentis conceptum [...] sed ultra aliquae voces impositae sunt ad significandum per se ultra illos conceptus res conceptas illis conceptibus, ut iste terminus 'asinus' asinos, iste terminus 'albedo' albedinem, iste terminus 'hodie' vel 'cras' tempus aliquod" (*Summulae* 4.2.3, van der Lecq, p.19; ll.12-20). I have here opted for *albedo* (in the apparatus) rather than *albus* (in van der Lecq's

Hence the spoken terms subordinated to simple, categorematic concepts signify not only those concepts but, mediately, the things those concepts signify, namely the objects with which they correspond. For example, the term *horse* does not just signify the concept *horse* but, beyond that, horses.

Accordingly, since only the categorematic terms of a proposition have signification, contradictory propositions will nevertheless signify the same object(s) *ad extra*—although they will say different things about the object in question, at least as far as it is conceived in the mind (*apud mentem*). Buridan's example is the following pair of propositions:

P5) God is God

P6) God isn't God

According to Buridan, the only significative terms in the above propositions are the instances of the term *God*. Therefore, the two propositions "do not signify anything *ad extra* other than—or anything more or less than—the term *God* signifies".[235]

But all this talk of signification *ad extra* brings up an important question: what about apparently categorematic terms with empty extensions—*i.e.* terms that signify

text) since, as we will see (in §3.1 below), Buridan does not think unsubstantivised adjectives have signification.

[235] "Unde istae propositiones 'Deus est Deus', 'Deus non est Deus' nihil omnino aliud plus aut minus significant ad extra quam iste terminus 'Deus'." (*Summulae* 4.2.3, van der Lecq, p.19). Cf. *Sophismata* 1.1, ad 3: "Ista autem 'Deus est Deus' nihil ad extra significat nisi Deum" (Scott, p.32, 6r), and *QAPo* I.9, co: "nihil plus vel minus, vel nihil etiam aliud, significo vel intelligo ad extra dicendo 'Deus est Deus' quam dicendo 'Deus non est Deus'". Cf. also Buridan's discussion and rejection of proposition-like complex signifiable extra-mental things in *QM* VI.7-8: as Buridan observes, if (P5) signifies anything but God, it introduces complexity into the thing signified—something which, by definition, is simple.

non-existent objects? Terms like *nothing* and *nobody* signify nothing *ad extra*, and so do not seem to meet (Sem_C). Yet they can serve as the subjects or predicates of well-formed propositions, and so meet (Syn_C). So do (Sem_C) and (Syn_C) come apart?

1.2.3. Signification and the Problem of Non-Existent Objects

Now so far, discussing terms which have signification *ad extra*, I have avoided the translation of *ad extra* as "outside the mind". This English phrase is the conventional way to translate *ad extra* in Buridan's texts (contrasted, as I did a moment ago, with *apud mentem*, "inside the mind"). But this translation is—if we take the terms in the English phrase literally—misleading. For there are many terms that are truly categorematic, but which do not signify any real extra-mental object, because no such extra-mental object exists.

As we've seen, such nonexistent objects pose an apparent problem for (Sem_C). Granted, it is characteristic of (spoken) categorematic terms that they signify objects beyond the concepts they are subordinated to. But then what about terms which signify concepts for which there is no corresponding extra-mental item, like *dodoes* in the following proposition?

P7) Dodoes are extinct

Since dodoes are extinct, the subject term of (P7) does not have any extra-mental signification. But *dodo* still looks and acts like a categorematic term. And we wouldn't want to say that it is not categorematic just because it has no signification. After all, its lack of signification is merely a matter of contingency. And *dodo* meets (Syn_C), since it can serve as the predicate or subject of propositions like (P7). How then can we claim that *dodo* is categorematic according to (Sem_C)?

It gets worse. Some apparently categorematic terms have no signification *by definition*. Such terms signify nothing, not as a matter of contingency (since dodoes could well have made it), but by necessity. Take for instance the terms *nothing* and *chimaera* in the following proposition:

P8) Nothing is a chimaera

Since both *nothing* and *chimaera* are apt to stand in subject (or predicate) position, as in (P8), they meet (Syn_C). So *nothing* and *chimaera* should, it seems, be categorematic terms, though by definition they signify nothing *ad extra*. There are many terms just like *nothing* and *chimaera* which, by contingency or necessity, have no signification beyond the mind, but are doubtless categorematic, at least by (Syn_C). Ideally, (Syn_C) and (Sem_C) should coincide. And so terms like *nothing* and *chimaera* should be categorematic by (Sem_C), too.

Indeed, if the distinction between categorematic and syncategorematic terms were just that the former picked out objects—either *apud mentem* or *ad extra*—whereas the latter did not (the way the Clean Divide View outlined above maintains—see especially Claim III), then I should be able to turn a categorematic term into a syncategorematic one—and therefore a logical constant—simply by annihilating all the objects in its extension. For example, if syncategorematic terms *qua* logical constants were just those terms with no extra-mental reference, then the term *dodo* would have been apotheosised to the status of a logical constant at the very moment dodoes became extinct. But this would be absurd.

There is a Buridanian solution. The framework for it is set out in the *Questiones Longe super Librum Perihermeneias* (*QLP*) I.2, where Buridan asks, "Does every name [*nomen*] signify something?" The short answer is *yes*: by definition a name is a conventionally significative utterance.[236] Therefore, even terms like *dodo* or *nothing* which have empty extensions are significative, in virtue of the fact that they are names (*nomina*).

Here's how the solution works. There are two ways a term can fail to signify an actually existing object: either because its object is possible, but doesn't exist as a matter of contingency (like dodoes); or because its object doesn't exist, because it is impossible (like the usual suspects: chimaeras, nothing, the vacuum, and so forth). Let's look at each of these in turn, beginning with contingently non-existent objects like dodoes.

In the *QLP*, Buridan considers a case in which we are speaking of roses in the past tense. In his example, we saw roses last year, but now there are none (suppose, for instance, our conversation takes place during wintertime):

[236] "nomen est vox significativa ad placitum" (*QLP* I.2, van der Lecq, p.8, l.11; Cf. *Summulae* 1.2.1, "De Nomine").

> Suppose for example you and I saw many roses at the same time last year. Accordingly, if I ask you, 'The roses we saw last year were red, weren't they?', you'll reply 'Indeed', since you know it's true. But you wouldn't know such a thing unless you thought of those roses.[237]

Hence we are able to think and speak of things that do not exist as a matter of contingency, like the roses of yesteryear. Accordingly, our concepts of such absent things can serve as categorematic terms in propositions, just as our concepts of presently existing things can.

Just as we can speak of past things that no longer exist, we can likewise speak of future things that do not exist yet. For instance, consider the following proposition:

P9) The Antichrist will preach

Since the Antichrist (presumably) does not exist yet, the subject term of (P9) does not signify any existing object, and so neither can it stand for one. But what is going on under the semantic hood? What, that is, do terms like *the Antichrist* in (P9), and *rose* in a wintertime conversation stand for, and how?

The solution has two steps. The first is to establish that simple terms are timeless. The second is to show that, in the context of a tensed or modal proposition, such terms have their extension determined by a process called *ampliation*, so that they can stand for past or future, or even possible non-existent things. Let's see how this works in detail.

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[237] "Verbi gratia, tu et ego simul anno praeterito vidimus multas rosas rubras. Si ergo ego peto a te, 'nonne rosae quas vidimus erant rubre?', tu dicis quod 'immo', quod scis esse verum Quod tamen non scires si non intelligeres istas rosas" (*QLP* I.2, van der Lecq pp.12-13, ll.37-3).

According to Buridan, when we grasp simple concepts like *rose*, we do so in a way that does not take time into consideration—that is, in a way that is indifferent to time:

> We can comprehend a thing (*res*) without any difference of time; and we can understand past and future things just like present ones. For this reason we can give names to things in such a way that they are signified without any difference of time. For this is how names signify.[238]

Here, Buridan is following the lead of Aristotle in *De Interpretatione* (I.2-3), where the distinction between nouns and verbs is precisely that the former do not signify time, whereas the latter do. Therefore, because these former types of terms do not signify any time, they can be used to refer to non-existent past or future things.[239]

Buridan gives a rather simple proof of the timelessness of names (*nomina*) in his *Quaestiones super Decem Libros* Ethicorum *Aristotelis ad Nicomachum* (*QNE*) VI.6 ("*Utrum omne scibile sit aeternum*?"—"Whether every knowable thing is eternal?"). There, he notes a name like *Caesar* can stand in future- or past-tensed propositions like the following:

P10) Caesar will be

P11) Caesar was.[240]

[238] "possumus intelligere res sine differentia temporis, et intelligere futuras sicut praesentes. Propter hoc etiam possumus imponere vocem ad significandum sine differentia temporis. Sic enim nomina significant." (*QLP* I.2, van der Lecq p. 15).

[239] *De Interpretatione* I.2-3 (16^a17-16^b25); Cf. also *Summulae* 1.2.1, where Buridan tells us that the phrase "signify without time" is added to the definition of nouns "so as to provide a distinction from verbs", which signify time (van der Lecq, p.19, ll.5-6).

[240] "Caesar fuit, Caesar erit" (*QNE* VI.6, f.122, v., d).

But if the name *Caesar* signified a time, then how could it stand in differently-tensed propositions like (P10) and (P11)? If it did, then it couldn't. But it can. Therefore, a term like *Caesar* does not signify any time at all.

By way of contrast, consider more complex terms that *do* signify a time, for instance *tomorrow* and *today*. If the name *Caesar* were bound to a time the way these terms are, then (P10) and (P11) would have an air of absurdity about them, the way the following propositions do:

P12) *Yesterday will be

P13) *Tomorrow was

But (P10) and (P11) do not express anything absurd, because—unlike the complex terms *tomorrow* and *today*—*Caesar* signifies Caesar in a way that is indifferent to time. Simply put, attaching tense to simple categorematic terms like *Caesar* does no violence to them, because they are tense-less.

Accordingly, a simple concept like *Caesar* can have its extension shifted backward or forward in time in a tensed proposition, or even to *possibilia* that never existed and never will in the context of a modal proposition, in a process called *ampliation* (*ampliatio*). We will see more about this process in Chapter 5, especially as it pertains to modal propositions. For now, it is enough to observe that simple concepts like *Caesar* (or *rose* in wintertime), although they do not signify anything actually existing, are nevertheless comprehensible as terms signifying something that was (or will be) in the world. Existence is therefore not a necessary condition for signification *ad extra*. And this is the solution for all contingently non-existent objects—that is, for objects like dodoes and the Antichrist, that did exist or will exist, but at present do not.[241] Hence the clause about signification in (Sem_C) should be read as dealing with signification in the *general sense*, and not as a matter of what's actually out there in the world.[242]

So much for contingently non-existent objects like dodoes and historical figures. What about the signification of terms whose objects *cannot* exist? Again, these meet (Syn_C): terms like *nothing* and *chimaera* are apt to stand as subjects or predicates of propositions. But when it comes to (Sem_C), terms like these are slightly more complicated than their cousins *Caesar* and *rose*.

According to Buridan, terms like these that cannot signify are complex concepts, which owe their impossibility either (i) to the fact that they combine, often in one term, incompossible categorematic concepts, or (ii) because they imply, in addition to their categorematic concept, a syncategorematic element.

The standard example of (i) is the hapless Chimaera. If there is a concept corresponding to *chimaera* at all, it is one of several combined animal parts—a "composite animal", in Sten Ebbesen's phrase.[243] Buridan tells us in the *QLP* that by *chimaera* we

[241] Note that Buridan is only thinking of past- and future-tensed propositions, not modals, which have much larger extensions than their assertoric counterparts. Hence all of Buridan's examples involve things that existed, but now do not; or which, as a matter of necessity, will exist at some time, like the Antichrist.
[242] Indeed, it's worth noting that even the term *conceptus* looks categorematic, at least by the syntactic criterion, though it by definition exists in the mind.
[243] Ebbesen, "The Chimaera's Diary", 115.

mean "an animal with the head of a lion, the body of an ox, and the tail of a snake".[244] Alternatively, in his *Quaestiones super Tres Libros* De Anima *Aristotelis* (*QDA*) a chimaera is a "complex combination of the concepts *tail of a fish* and *torso of a virgin*"—apparently a sort of mermaid.[245] Or it is made from "the body of a man, the head of an ox, and the tail of a dragon", as he says in his *Quaestiones super libros* Analyticorum Posteriorum.[246] Or it is, according to the *Summulae de Demonstrationibus* (8.2.3), merely "an animal that is made up of parts out of which it is impossible for any animal to be composed".[247] This final description is the most perspicuous, even if it's the least fun, since it hits upon the reason for the chimaera's impossibility—and therefore the reason for its usefulness in logic texts: the chimaera is a creature made of incompossible parts, like Frege's square circle (*viereckiger Kreis*) and wooden iron (*hölzerne Eisen*).[248]

Importantly, all these definitions of *chimaera* are complex: whatever concept *chimaera* corresponds to is not a simple one, like *Caesar* or *rose*, but a complex one which is made up of—and can be resolved into—simple components.[249] The simple components out of which such complex concepts are formed are simple concepts, and so they can be resolved into these simple concepts once again.

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[244] "chimaera est compositum ex capite leonis et corpore bovis et cauda draconis" (*QLP* I.2, van der Lecq, p.11).
[245] "conceptus corresopendens huic termino 'chimaera' est complexus ex conceptu ventris virginis et ex conceptu caudae piscis" (*QDA* III.15, p.486).
[246] "[...] ex corpore hominis, et capite bovis, et cauda draconis" (*QAPo* I.9, co).
[247] "Animal compositum ex membris ex quibus impossibile est aliquod animal componi" (*Summulae de Demonstrationibus* 8.2.3, de Rijk 33, ll. 23-4). Buridan gives a similar phrasing in *QAPo* I.9, co.
[248] Frege, *Die Grundlagen der Arithmetik* (Breslau [Wrocław]: Wilhelm Koebner, 1884), 87.
[249] Buridan uses the term *chimaera* as a standard example of a term whose corresponding object is impossible—even though it's not obvious that the combinations themselves are impossible, in the sense that they couldn't be produced by God. As Ebbesen notes ("The Chimaera's Diary", 141), so long as we are willing to allow that there are complex terms with necessarily empty extensions, we can treat *chimaera* as one such.

According to Buridan, complex concepts like the one with which *chimaera* corresponds are produced by non-propositional combinations of terms—that is, combinations of terms without the addition of a mediating copula like *is* or *are*.[250] Recall that, in the *Summulae de Propositionibus* (1.2.3), Buridan distinguishes propositions, where the terms are separated by a mediating copula, from 'unseparated combinations' (*complexiones indistantes*), which do not assert anything (as we noted in § 1.1 of Chapter 1, above). Recall, further, that for the logician, what is at stake is not grammatical complexity, but *conceptual* complexity (as we noted in § 1.2.1 of the present chapter). Now in spite of its grammatical simplicity, a term like *chimaera* cashes out as "lion-headed, ox-bodied, snake-tailed animal". Therefore, *chimaera* belongs in the genus of non-propositional but nevertheless complex expressions (*orationes*), like "pale man" and "running horse".

Hence each of the terms that go into *chimaera* signifies something, though their combination describes something impossible. And so, taken all together, they signify nothing. As Buridan tells us,

> it often happens that, although both of the simple concepts [of a complex concept] stand for something, nevertheless their combination stands for nothing [...] For example, the term *rational* stands for something, and so does the term *donkey*. Nevertheless, the combination of the two, *rational donkey*, stands for nothing, since there is no such thing as a rational

[250] "Saepe conceptus simplices conplectntur sibi simul in intellectu, non solum mediante copula verbali, quomodo fit enuntiatio, sed etiam aliquando sine copula verbali per modum determinationis et determinabilis, ut dicendo 'homo albus' vel 'animal rationale'." (*QLP* I.2, van der Lecq p.10).

donkey.[251]

Chimaera thus works just the same as *rational donkey*: all the significative and simple concepts from which *chimaera* is made (*lion head*, *ox body*, and *snake tail*—or, if it's more to your liking, *fish tail* and *virgin torso*) themselves *do* have signification, and pick out things that do (or did, or will) exist in the world. But their combination is impossible. Hence at the mental level *chimaera* is made up of significative parts, and therefore it meets (Sem_C) requirement of signification *ad extra*, even if it does not signify any composition *ad extra*.

As I noted at the beginning of the present discussion, a complex term's necessary failure to signify can come about in two ways: either because (i) its constituent terms do not stand for the same thing(s), or because (ii) it implies a (negative) syncategorematic term:

> [i] In the first place, if there is no single thing for which the terms stand, and they are combined in an *affirmative way*, then the combination stands for nothing. [...] And [ii] in the second way, if the things for which the simple terms stand are not disparate but identical, and the combination is done in a *negative way*, since—because the terms *human* and *capable of laughter*, taken simply, stand for the same things—it follows that the combination *human* ***not*** *capable of laughter* stands for nothing.[252]

[251] "Modo contingit saepe quod licet uterque conceptuum simplicium pro aliquo supponat, tamen complexum ex eis pro nullo supponit [...] Verbi gratia, ille terminus 'rationale' pro aliquo supponit, et similiter ille terminus 'asinus'; et tamen illud complexus 'asinus rationalis' pro nullo supponit" (*QLP* I.2, van der Lecq, p. 10).

[252] "Primo, si non est idem pro quo illi termini incomplexi supponunt et fiat complexio modo affirmativo, tunc complexum pro nullo supponit [...] Secundo modo etiam, si non sint diversa sed idem pro quibus termini illi incomplexi supponunt, et fiat complexio modo negativo, ut quia idem est pro quibus isti termini 'homo' et

As we saw, impossible combinations like *chimaera* are produced in the first, affirmative way. In the second, negative way, the signifying terms of a complex categorematic term stand for something, but because of the inclusion of the term-negation *not-* (or its equivalent), the combination stands for nothing. Combinations of this second sort include terms like *vacuum*:

> A place not filled with any body is called a *vacuum* [...] and accordingly, to this term *vacuum* there corresponds a complex concept made up of these: one of them corresponds with the term *place*, and the other with the term *filled with a body*. And these two concepts stand for the same thing, because a place is the same thing as something filled with a body, as Aristotle says [*Physics* IV.6 213ᵃ13-15]. Therefore, because these two terms are combined in a negative way—since, as has been said, a vacuum is a place not filled with a body—that combination stands for nothing.[253]

Every place is a place filled with a body, since in Aristotelian physics, a vacuum is impossible. And so if the combined terms—*place* and *filled with a body*—necessarily stand for the same thing, their negation necessarily stands for nothing.[254]

'risibilis' supponunt incomplexo modo sumpti, ideo illud complexum 'homo non risibilis' pro nullo supponit" (*QLP* I.2, van der Lecq, p.10, ll.29-33; emphasis added).

[253] "Vacuum enim dicitur 'locus non repletus corpore' [...] Modo praedicte orationi, et per consequens huic nomini 'vacuum', correspondet conceptus complexus et conceptibus illorum quorum unus correspondet huic voci 'locus', et alius huic voci 'repletus corpore'. Et pro eodem supponunt illi duo conceptus, quia idem est locus et repletus corpore, cum omnis locus sit repletus corpore, secundum Aristotilem. Ideo, quia complectuntur modo negativo, quia dicitur 'locus non repletus corpore', illud complexum pro nullo supponit. Et per consequens hoc nomen vacuum pro nullo supponit" (*QLP* I.2, van der Lecq p.11, ll.4-13).

[254] Notice that standing for nothing is not the same as failing to signify: certain complex terms will necessarily stand for nothing, as we can see from the examples of *chimaera* and *vacuum*. But others, like Russell's famous *present king of France*, fail to signify only as a matter of contingency: there's nothing inherently self-contradictory about this latter phrase.

In like fashion, a term like *being* stands for all things; and therefore, when it is negated, it stands for nothing. And so we have a solution to the problem presented by *nothing* (*nihil*): for, according to Buridan, this is a complex concept, implying a simple categorematic *thing* (*aliquid*), and a syncatgorematic *not-* (*non*).[255] In a moment, we'll see in greater detail what makes a syncategoreme syncategorematic. For the present discussion, however, it is sufficient merely to stipulate that *non-* is syncategorematic, and operates on a term like *being*, which signifies everything, so as to make it signify nothing. Accordingly, on the verbal level, it has to be expounded as a complex combination; and the categorematic element of that combination, the *something* (*aliquid*), itself *does* have signification.

Thus there are categorematic terms that meet ($\mathrm{Syn}_{\mathrm{C}}$) but do not straightforwardly meet ($\mathrm{Sem}_{\mathrm{C}}$), because they contain a syncategorematic element. And here we get our first lemma in the case against the Clean Divide View, outlined above:

> **Lemma 3.1**: Categorematic terms can contain a syncategorematic element.

This gives us reason to doubt the first claim of the Clean Divide View:

> **Claim I**: Syncategorematic and categorematic terms are strictly demarcated.

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[255] "De illa dictione 'nihil', dico quod ipsa implicat simul hanc dictionem 'non' et hanc dictionem 'aliquid'." (*QLP* I.2, van der Lecq, p.14, ll.35-7). Notice that Buridan uses language of *implication*, not of *containment*. Doubtless this is a consequence of his nominalism: a term can be cashed out, but that doesn't mean it contains the things it is cashed out for. Doubtless we should follow him, at least in our exposition.

Further still, there are categorematic terms with no extra-mental signification, in the strict sense. And this gives us our second lemma, which undermines the second clause of Claim III:

> **Lemma 3.2**: Categorematic terms do not always signify something outside the mind.

> **Claim III**: Syncategorematic terms do not refer to anything outside the mind; categorematic terms invariably do.

(As we will see in §3.2, below, the first clause of Claim III is false, too).

To wrap up our discussion of categoremes: there are two criteria for categoreme-hood: (Syn_C), whereby a categorematic term must be apt to serve as a subject or as a predicate in a well-formed proposition; and (Sem_C), whereby a categorematic term signifies something *ad extra*, or anyway can be cashed out into at least one term with *ad extra* signification.

Hence providing a straightforward and perspicuous translation of the phrase *ad extra* is very difficult. And to take it, with no comment, to mean signification "outside the mind" is misleading—at least so far as the literal meaning of the English phrase is concerned. Worse still, the definite article in "outside the mind" suggests that there is one mind in question—a categoreme signifies, not outside *a* mind, but *the* mind. Whose mind, we might wonder, is The Mind?

A more accurate, if more cumbersome, translation would perhaps be signification "beyond a concept to which the term in question is subordinated". But in a true Buridanian spirit, we may keep the phrase "outside the mind" *ad placitum* for brevity's sake, so long as we are careful not to take it too literally.

With these things in mind, we can turn to pure *syncategoremata*.

2. Syncategoremata

A typical treatment of the intuitive notion of syncategorematic terms begins by giving a list of them. The usual suspects include *is*, *not*, *or*, *every*, *and*, and so forth. Consider for example Joke Spruyt's account:

> The words labelled syncategorematic came to include the following: the verb 'is' used as a *tertium adiacens* (i.e., as part of an attribution), the negation 'not,' the modal adverbs 'necessarily' and 'contingently,' [...] the consecutives 'if ' (*si*), 'unless' (*nisi*), and 'but that' (*quin*), the copulatives (like 'and'), the disjunctive 'or' (*vel*, *aut*) the adverbs 'whether' (*an*) and the verbs 'begins' (*incipit*) and 'ceases' (*desinit*).[256]

Beyond listing a few terms, we might try to give a complete list, as for instance Gyula Klima does in his "Latin as a Formal Language". There, Klima tells us that "For semantic

[256] Joke Spruyt, "Syncategoremata", *The Encyclopedia of Medieval Philosophy*, ed. Henrik Lagerlund (Dordrecht: Springer, 2011), 1244.

purposes we shall have to distinguish between categorematic (CAT) and syncategorematic (SYNC) terms", and gives us the following:

$$\text{SYNC:=Sig[ns of quantity]} \cup \text{Cop[ulae]} \cup \text{Conj[unctions]}.$$[257]

This is all well and good for setting up a formalisation. But we can say more in defense of such a list: why assume that it is *exhaustive*? Further still, even if such a list *were* exhaustive, it would still amount to a mere list. A list tells us nothing about what these syncategorematic words *are*, and what is characteristic of them. Why, we might wonder, should we group them together at all? What special property do these classes of words—signs of quantity like *some* and *every*, copulae like *is* and *isn't*, and propositional conjunctions like *if* and *or*—share in common?

It is frequently said that these syncategorematic words are distinguished from their categorematic counterparts by their lack of signification *ad extra*. This is correct. As Buridan remarks in a passage to which I shall frequently refer:

> terms are called purely syncategorematic, however, which do not signify anything apart from the concepts that they immediately signify (except perhaps those things that are signified by the [categorematic] terms with which they are joined), and these are terms like *not*, *and*, *or*, *therefore*, *every*, and the so forth.[258]

[257] Gyula Klima, "Latin as a Formal Language: Outlines of a Buridanian Semantics", *Cahiers de l'Institut du Moyen Âge Grec et Latin* 61 (1991), 83.

[258] "Dicuntur autem pure syncategorematicae, quia praeter conceptus quos immediate significant, nihil significant, nisi forte ea quae termini quibus adiungitur significant, ut istae dictiones 'non', 'et', 'vel', 'ergo', 'omnis', et huiusmodi" (*Summulae* 4.2.3; van der Lecq, p.18, ll.14-17)

This passage presents the semantic criterion for syncategoreme-hood, which I'll call (Sem_S). According to this criterion, what sets the *syncategoremata* apart is that, on their own, they lack signification *ad extra*. They can only obtain such signification in combination with other, categorematic, terms. Indeed, this function is displayed by their shallow etymology: Buridan is quick to point out that terms are called syncategorematic "from the Greek *syn-*, which is the same as the Latin *con-* [with-], so that they are 'significative with something else', so to speak".[259]

On the basis of these two facts—that the logical particles are syncategorematic, and that syncategorematic terms are non-referring—it is commonly inferred that the class of logical particles like the ones listed above just *is* the same as the class of syncategorematic terms: that is, granted that all logical particles are syncategorematic, it is assumed that all syncategorematic terms are logical particles. So if a term is a logical constant, it has no signification; and if a term has no signification, it is a logical constant. This identification of syncategoremes with logical constants is summed up in Claim II of the Clean Divide View:

Claim II: Syncategorematic terms can be identified with the class of logical constants; categorematic terms with the class of nonlogical constants.

[259] "Et dicitur a 'syn' graece, quod est 'con' latine, quasi 'cum alio significativa'." (*Summulae de Suppositionibus* 4.2.3, van der Lecq, p.19). Cf. Peter of Spain's *Syncategoreumata*: "et dicitur syncategoreuma a 'syn', quod est 'con', et 'categoreuma', quod est 'praedicativum' vel 'significativum', suqsi: 'consignificativum'." (*Syncategoreumata*, introduction, 2; de Rijk, p.40).

Accordingly, it seems that this semantically well-defined class of syncategorematic terms just gives us our logical particles up front. If so, the problem of distinguishing logical constants like *and* and *not* from non-logical terms like *donkey* and *rose*, which has attracted so much attention in the twentieth and twenty-first centuries, is a modern problem—and a problem that would make no sense on a medieval framework. Things were, it seems, so much simpler back then.

This conclusion, as we will see, is incorrect, and for two reasons. First, the most important syncategorematic term in Scholastic (which is to say term-) logic—namely, the copula—signifies time, a fact Buridan explicitly acknowledges. Therefore, the copula is *not* purely syncategorematic by (Sem_S), since it has signification *ad extra*. And second, because there are many non-significative terms with no special logical significance—adverbs like *quickly* or unsubstantivised adjectives like *blue*. As we will see, terms like *quickly* and *blue* do not meet (Syn_C) or (Sem_C), and *do* seem to meet (Sem_S). Yet they get no special treatment in our analysis of the (logically interesting) fragment of natural language. Nor should they. Thus having signification *ad extra* is not sufficient grounds for removal from the logical-constant guest-list; and lacking reference *ad extra* doesn't get you on it, either. The classes of syncategoremes and logical constants are therefore not identical, contrary to Claim II.

But first, let's look at the logical constants—that is, all and only those syncategoremes we want (Sem_S) to net. In the following section, I discuss what these terms share in common, and present in passing an argument that, at least as far as propositional form is concerned, the list presented here is exhaustive.

2.1 The (One and Only) Semantic Criterion

Whereas Buridan presents us with both syntactic and semantic criteria for categoreme-hood, he gives us only a semantic criterion for syncategoreme-hood, and no syntactic one: syncategorematic terms have no signification *ad extra* or even *apud mentem* but are instead subordinated to a complexive concept, as we saw just a moment ago.[260] As we did with (Syn_C) and (Sem_C), we can state it briefly as follows:

> Sem_S) A term is syncategorematic just in case it does not signify anything on its own.

But as we will see, (Sem_S) nets a whole lot more than the logical constants.

Let's begin with the constants. In the *Summulae de Suppositionibus* (4.2.3, "On the Division of Incomplex Utterances into Categorematic, Syncategorematic, and Mixed Terms"), Buridan groups the relevant syncategoremes under (i) signs of quantity (*signa*), (ii) the copulae, and (iii) familiar connectives like *and* and *or*:

[260] However later in the *Summulae* he tells us that purely syncategorematic terms cannot serve as subjects or predicates (*Summulae de Suppositionibus* 4.2.6). But this is not presented as a criterion for syncategoreme-hood; rather, it seems it is a rule about the syntactic function of syncategoremes, which is not uniquely identifying.

> [i] signs of quantity signify only how the vocal terms—and their corresponding mental terms—stand for things [*supponant*], and signify nothing beyond this. And likewise, [ii] the copulae *is* and *isn't* signify different ways [*modi*] of combining mental terms in forming mental propositions [...] And so also [iii] these utterances, *and*, *or*, *if*, *therefore*, and others of the sort, designate complexive concepts in the mind, which are made up of multiple propositions or multiple terms taken at once; and they signify nothing else *ad extra*.[261]

Hence syncategorematic terms operate on categorematic terms by (i) altering what they stand for in the context of a proposition, as terms like *every* and *some* do; (ii) combining terms to make a proposition, as the copulae *is* and *isn't* do. Or, as conjunctions, they can act both on terms and on propositions, by (iii) combining them into complex terms ("the one disputing *and* teaching") or complex propositions ("an animal runs *if* a human runs"), the way conjunctions like *and* and *if* do. In a moment, we'll look at each of these in turn.

Can we be satisfied that this list of types of logical constants presented in the *Summulae de Suppositionibus* is exhaustive? There is evidence elsewhere in the *Summulae* that Buridan thinks so, at least where *propositional* form is concerned.[262] In

[261] "signa solum significant quomodo termini vocales et mentales supponant, nihil ultra significando. Et etiam illae copulae 'est' et 'non est' significant diversos modos complectendi terminos mentales in formando propositiones mentales [...] Et ita etiam istae dictiones 'et', 'vel', 'si', 'ergo' et huiusmodi designant conceptus complexos plurium propositionum simul vel terminorum in mente, et nihil alterius ad extra" (*Summulae de Suppositionibus* 4.2.3, van der Lecq, p.20).

[262] It may come as a surprise that *terms*, too, have form and matter: this mostly gets discussed in Buridan's treatment of the fallacy of accent, where he notes that *mora* ("delay") and *mōra* ("blackberries"—*i.e.* pl. of *mōrum*) have the same matter but differ in form (*Summulae* 7.3.8). (English doesn't typically distinguish vowel length, but cf. a minimal pair like the verb *sew* as pronounced in Standard American English and the vernacular emphatic *soooo* as in *e.g.* "I'm *sooo* hungry!"). Still, since form and matter are predicated correlatively, and since categorematic terms account for propositional matter, we needn't deal with the form of terms for our present discussion.

the *Summulae de Propositionibus*, Buridan tells us there are exactly three things to be found out about any proposition:

> There are three things that can be asked about a proposition in itself, namely [1] what sort it is [*quae*], [2] what quality it is [*qualis*], and [3] what quantity it is [*quanta*]. The question [1] 'What sort?' pertains to the substance [*substantia*] of a proposition, and the response is 'categorical' or 'hypothetical'. The question [2] 'What quality?' pertains to the quality [*qualitas*] of a proposition, and the response is 'affirmative' or 'negative'. And the question [3] 'What is its quantity?' pertains to the quantity [*quantitas*] of a proposition, and the response is 'universal', 'particular', 'indefinite', or 'singular'.[263]

Thus propositions are distinguished, without reference to their content, by substance, quality, and quantity. Generally speaking, each of these categories has its own distinctive logical terms: signs of quantity (i) pertain to the quantity (3) of a proposition; copulae (ii) pertain to quality (2); and connectives like *and* and *or* (iii) pertain to substance (1). Hence the two lists map onto one another; and since the latter list is presented as a complete list of the things that can be asked about a proposition, the former list of logical constants should be complete as well.

Let's look at each of these features of a proposition, and their corresponding constants, in turn. I'll begin with *quality*, which is a function of the copulae. Quality is the

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[263] "Triplex est quaesitivum per quod quaerimus de ipsa propositione, scilicet 'quae', 'qualis', et 'quanta'. 'Quae' quaerit de substantia propositionis, ideo respondetur 'categorica' vel 'hypothetica'; 'qualis' quaerit de qualitate propositionis, ideo responditur 'affirmativa' vel 'negativa'; et 'quanta' quaerit de quantitate propositionis, ideo respondetur 'universalis', 'particularis', 'indefinita', vel 'singularis'." (*Summulae* 1.3.7; van der Lecq, p.41, ll.13-19).

most foundational of the lot, since the copulae are the only type of syncategorematic term that must be present in all propositions (as we saw in chapter 1, §1.1). Once we have examined quality, we can turn to propositional *quantity* and its distinctive signs. Lastly, we will look at the syncategorematic terms that pertain to propositional *substance*—that is, those propositional operators like *if* and *and* and *or*.

2.1.1 *Qualitas*: Copulae

The copula is the primary syncategorematic part of any categorical proposition. Indeed, Buridan goes so far as to call it the formal element (*formale*) of a categorical, or even its "most formal" part (*pars formalior*).[264] It is the one indispensable syncategorematic term in all well-formed propositions—that is, propositions of ScP-form.

Copulae, like the propositions in which they serve as the formal part, have one of two qualities: affirmative or negative. In the *Summulae de Propositionibus* (1.3.6), Buridan distinguishes the two propositional qualities as follows:

> Some categorical propositions are affirmative, and others are negative. An affirmative proposition is one in which a predicate is affirmed of a subject, as *e.g.* in "Someone runs". A negative proposition is one in which a predicate is denied of a subject, as *e.g.* in "Someone doesn't run"[265]

[264] "Cum ergo formale propositionis sit copula [...]" (*Summulae* 1.3.6, van der Lecq, p.40). "[...] penes eius [sc. propositionis] formaliorem partem, id est copulam [...]" (*Summulae* 1.3.4; van der Lecq, p.36, ll.11-12).

[265] "Item, propositionum categoricam alia affirmativa, alia negativa. Affirmativa est in qua praedicatum affirmatur de subiecto, ut 'homo currit'. Negativa est in qua praedicatum negatur de subiecto, ut 'homo non currit'." (*Summulae* 1.3.6).

In a categorical proposition, affirmation or negation is a function of the copula. Therefore, the quality of the copula as main connective (*pars principalis*) determines that proposition's quality.

When it comes to the latter, negative copulae, Buridan is very careful to distinguish negation that, in his terms, "falls on the copula" (*cadat super copulam*)—from negation with narrower scope, such as term negation. The mere presence of a negative particle is not sufficient to render a proposition negative, or else the following affirmative would be a negative:

P14) Socrates is a non-donkey

Since the negation in (P14) is narrow-scope—that is, it falls on the predicate *donkey* and not on the copula—(P14) is an affirmative proposition. Accordingly, (P14) has very different existential requirements and truth conditions from a corresponding proposition with negation that affects the copula:

P15) Socrates isn't a donkey.[266]

As we have already seen, affirmative propositions on Buridan's logic have existential import. So, as an affirmative proposition, (P14) is only true if Socrates exists, and is a

[266] Of course, we can use term negation to express particular negatives with existential requirements, akin to the modern $\exists x(Fx \,\&\, \sim Gx)$—as Gyula Klima points out in "Existence and Reference in Medieval Logic", *New Essays in Free Logic*, ed. Alexander Hieke and Edgar Morscher (Dordrecht: Kluwer, 2001), 3ff. But standardly, a particular negative (*i.e.* O-type) proposition on Buridan's logic has no existential requirements.

non-donkey. The negation in (P14) acts not on the copula, but only on one of the terms. Conversely, negative propositions do *not* have existential import, and so a negative proposition like (P15) will be true if Socrates doesn't exist at all. Thus negation renders a categorical proposition negative just when it acts on the copula.[267]

Because of the primacy of mental language, the copulae owe their characteristic jobs of affirming and negating to their subordination to a mental act. Affirmation is an act of combining, and negation one of dividing. In the *Summulae de Suppositionibus* (4.2.3), Buridan tells us that:

> The copulae *is* and *isn't* signify different ways of combining mental terms, in order to formulate mental propositions. And these different ways of combining mental terms are complex concepts.[268]

Thus a copula like *is* combines two terms to make an affirmative assertion. And the copula *isn't* does the opposite, by *dividing* the terms to which it applies, to formulate a negative assertion. Thus there are two different copulae, and not merely a negation of one or another. Though *is not* (*non est*) might look like it has two components, *is* and *not*, it is subordinated to a unified complexive concept very different from the complexive concept *is*.

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[267] Similarly (as we saw in chapter 1, §1.2.1) the mere presence of a conjunction is not enough to render a proposition hypothetical, or else the following categorical would be hypothetical: "The one lecturing and disputing is a master or a bachelor." What matters here, as ever, is the nature of the principal part. Only when the principal part is a negative or affirmative copula can the proposition be called affirmative or negative.

Notice also that something similar happens, in Latin at least, with the volitional verbs *volo* ('to wish') and *nolo* ('to wish against'). A cognate of the latter survives in fossilised form, as the *nilly* in the English phrase 'willy nilly'. Latin also has a verb *malo* ('to prefer'), the preceding nasal consonant of which is linked with *magis* ('more'). I have not thought hard about preference and negation. It seems to me there must be a connection there. Though I am not sure whether *malo* is reducible to *velo* and negation alone.

[268] "Et etiam illae copulae 'est' et 'non est' significant diversos modos complectendi terminos mentales in formando propositiones mentales, et illi modi complectendi sunt conceptus complexivi" (*Summulae* 4.2.3; van der Lecq, p.20, ll.4-7)

Therefore, because of the primacy of mental language in Buridan's logic (discussed in §1.2.1, above) *is not* is a standalone term, irreducible to *is*.

Now suppose we treat negation as acting on the whole proposition, rather than the copula. Then we will have to treat negative propositions as having a negation, not a copula, as their principal part. Thus negative propositions will be proposition-like expressions, modified by a (unary) operator, analogous to the proposition-like expressions modified by binary operators like *if* and *or*. But then we face two difficulties: (i) such expressions are hypotheticals; and (ii) such expressions do not have the same structure as their contradictory affirmative counterparts. The problem with (i) is that we lose the notion of a copula as the assertion-making formal part of a proposition. If the copula is no longer the uniquely assertive formal part, we are hard pressed to distinguish negations, which assert something, from *e.g.* disjunctions, which do not.

The problem with (ii) is that if we push propositional-scope negation as opposed to copula negation, then many of the rules governing logical relations on term-logic go completely to pieces. For example, the Square of Opposition will have at its right-hand nodes propositions with significantly different—which is to say, hypothetical—structure opposed to categorical affirmatives. In that case, it will only be possible to contradict a categorical affirmative by positing a hypothetical. And that runs contrary to the opposition in terms of propositional quality that a logician like Buridan takes to be the backbone of the Square.

Thus the Buridanian account of negation is markedly different from the way we were taught to think of propositional negation in our elementary logic courses. For

Fregean logic, there is no copula at all, and so a negation acts on a whole proposition *qua* subject-predicate combination. As we will see (in Chapter 4, §1.4), this poses a significant problem for Buridan's account of formal schemata for hypothetical propositions.

For now, I want to consider two significant Fregean criticisms of this account of the copulae. The first is about this difference in negation, and the second is about the ambiguity of the copula in general. According to the first criticism, the sharp division of affirmative from negative copulae, on the grounds that they are subordinated to irreducibly different mental acts, has a hard time accounting for double-negation. As Frege memorably remarks in his *Logische Untersuchungen*, if this view were correct, then "Negation would thus be like a sword that could heal on again the limbs it had cut off".[269] That is, if we are to preserve double negation (and Buridan surely holds the principle that *duplex negatio affirmat*), we have to explain why two acts of division amount to an act of recombination.

This criticism is often taken to be devastating, though I think rests on a misunderstanding. As we saw in Chapter 1 §1.1, Buridan does not think categoricals are mere combinations of terms: there is a significant difference between the unseparated combination (*complexio indistans*) "running Socrates" (*Sortes currens*) and the propositional separated combination (*complexio distans*) "Socrates is running" (*Sortes est currens*). The former, though a combination, does not make an assertion, whereas the latter does. The reason for this, according to Buridan, is that the latter contains a copula, whereas the former does not. The copula is responsible for assertion. We seem to keep

[269] Gottlob Frege, *Logische Untersuchungen*, 148-9.

returning to the fact that Frege and his disciples reject the notion that there is any logical content to the copula. Instead, they will display the difference between these two expressions as "Running(socrates)" and "⊢Running(socrates)", respectively.

But what does the copula do here, given that we already have a combination in the expression "running Socrates"? This is a difficult question. At present, I do not know what to do about it. I have been thinking about a solution based on Irad Kimhi's discussion of assertion in terms of pointing.[270] On such an account, asserting that Socrates is running would be something like pointing or turning toward the combination "running Socrates", and denial of it would be pointing or turning away, toward its separation ("non-running Socrates"). Now it is easier to see why two acts of pointing or turning away become one act of pointing or turning: if, for example, I am walking to the beach, then I turn back, and then turn back again, I will be headed once again to the beach.

This reading might seem a bit tendentious. But it is worth noting that assertion and denial as a turning towards and away finds support in the language of *De Interpretatione* 6. Here is Kimhi's reading:

> An affirmation, Aristotle says, is 'a proposition asserting something toward something [*kata tinos*]'; a denial is 'a proposition asserting something away from something [*apo tinos*]' (17a25). The difference between affirmation and denial is thus a difference in the direction of assertion.[271]

Now I do not find any evidence in Buridan that he thinks of assertion in these spatial metaphors. But the Fregean criticism is of Aristotelian logic generally, not just of

[270] Irad Kimhi, *Thinking and Being* (Cambridge, MA: Harvard UP, 2018), 87.
[271] Kimhi, *Thinking and Being*, 87.

Buridan. And it appears there is an Aristotelian solution, though I'll admit I am still not entirely sure how or what to think of it. At any rate, full development of this idea would take us pretty far afield, so I will set the problem aside for now.

The second important Fregean line of criticism misses the mark, though in an interesting way, and can be more easily addressed. The charge is not that the copula doesn't do *enough*, but that it does *too much*, and is therefore equivocal. The gist of this criticism is that the copula *is* applies to more than one logical relation, namely, that of identity, and that of set membership. We owe this latter criticism to Peter Geach, who tells us in *Reference and Generality:*

> It will [...] be a problem whether the relation expressed by the copula is always the same; logicians of our time commonly suppose that the copula may express either class membership or class inclusion [...] But it is quite wrong to say that 'is' means different relations in [a] 'Socrates is an animal' and in [b] 'Every man is an animal'; there is the same unambiguous expression 'is an animal' in both, and the propositions differ in just the same way as 'Socrates can laugh' and 'Every man can laugh', where there is no copula to be ambiguous.[272]

Geach makes the same points in *Logic Matters*, and in *A History of the Corruptions of Logic*.[273] Here there are three objections: (i) first, that *is* is ambiguous, so that it is doing the work both of '∈' in the statement that *s* is an element of set A, i.e.:

[272] Peter Geach, *Reference and Generality*, 61-2.
[273] Peter Geach, *Logic Matters* (Oxford: Blackwell, 1972), 53-4. Cf. also Peter Geach, *A History of the Corruption of Logic* (Leeds: Leeds UP, 1968), 14-16.

a) $s \in \mathrm{A}$

and also of '⊆' in the statement that set M is a subset of A, i.e.:

b) $\mathrm{M} \subseteq \mathrm{A}$.

Objection (ii) is that the phrase 'is an animal' is just the same in both propositions Geach considers—that is, there is one predicate that has a double-role. And lastly, (iii) Geach objects that there are well-formed propositions, which are not ambiguous, and do not have a copula at all.

Let's take these in reverse order, since (iii) is the easiest objection to address. We already saw (Chapter 1, §1.1) that, for Buridan, all categorical propositions have subject-copula-predicate (ScP)-form, even if this form sometimes needs teasing out. Fortunately, English *can* functions the exact same way Latin *potest* does, and Buridan gives a straightforward treatment of *can* (*potest*) in terms of the copula *is* (*est*):

> One might wonder, in a proposition like [...] "Someone can run" [...] what the subject, predicate and copula are. [...] But concerning this proposition, "Someone can run", we should say the same thing as we do about assertorics [*propositiones de inesse*, *i.e.* non-modals]: so that, in this proposition, "Someone runs", in order to separate out the subject, predicate and copula, we should analyse [*resolvere*] this verb 'runs' as 'is running' [...]

> And then, in this proposition "Someone can be running", *someone* is the subject, and *running* is the predicate, and *possibly-is* is the copula.[274]

Thus Buridan has a relatively straightforward solution for propositions, like "Someone can run", that apparently lack a copula: the verb *can* is interchangeable with the modal copula *possibly-is*.[275] And indeed, this is one of the simpler examples of propositional analysis Buridan considers in the *Summulae*—much simpler, say, than "A man a donkey sees".[276]

Hence to criticise Scholastic logic as incapable of accounting for the apparent lack of a copula in a proposition like "Socrates can laugh" is to misunderstand (or overlook) one of its most elementary doctrines. As I suggested in Chapter 1, based on the examples Buridan gives in his logic texts, students studying logic at the medieval universities were probably given exercises in reducing troublesome propositions to standard logical (which is to say ScP) form. We do the same thing when we have our students in symbolic logic classes render (P16) symbolically as (P16′):

P16) Every dalmatian is a dog

P16′) ∀x(Dalmatian(x) → Dog(x)).

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[274] "potest dubitari de aliquibus, ut de ista [...] homo 'potest currere' [...] quid in eius [sic] sit subiectum et praedicatum et copula [...] Sed de ista, 'homo potest currere' dicendum est sicut de illis de inesse: ut in ista propositione, 'homo potest currere', oportet ad sumendum distincte subiectum et praedicatum et copulam resolvere hoc verbum 'currit' in hoc 'est currens' [...] Et tunc in ista 'homo potest esse currens' 'homo' est subiectum et 'currens' praedicatum et 'potest esse' est copula" (*Summulae de Propositionibus* 1.8.3; van der Lecq, 86).

[275] Though when Buridan *does* give *possibile est* as a single copula, he very frequently uses an accusative-infinitive construction. For example, in *TC* II.2, he cashes out "homo potest currere" as "hominem possibile est currere" (Hubien p.57, ll.18-19).

[276] "Hominem asinus videt" (*Summulae* 4.2.6; van der Lecq, p.24, l.9).

I am not here assessing the relative merits of the Scholastic or modern symbolic approach. But it is surprising to see Geach, who professes familiarity with Buridan's thought, presenting this point as though it never occurred to Buridan, and as though Scholastic logic has no recourse for analysing an important modal verb like *can*. It can, and it does. And in fact it seems to have been part of the elementary logical curriculum of Buridan's day. So argument (iii) goes nowhere.

Things are not much better with (ii). It is not at all clear why Geach thinks the medievals have to treat "is an animal" as the same predicate in the following two propositions:

P17) Socrates is an animal

P18) Every human is an animal.

A quick review of later-medieval theories of the properties of terms is enough to show that Scholastic logicians like Buridan will approach "is an animal" in propositions like (P17) and (P18) in two steps: (α) first, separate out the copula, *which is not an integral part of the predicate*; and (β) second, look at the mode of supposition of the predicate *animal* in the context of the whole proposition. In (P17), *animal* has determinate supposition, since it there stands for a single thing (namely, the animal that Socrates is). In (P18), *animal* has non-distributed, con-fused or fused-together (*confusa*) supposition, since it stands for a bunch of things all taken together (namely, all the animals that all the humans are).[277]

[277] This is not a chapter (or a thesis) on medieval semantics of terms. But I should say something quickly about the modes of supposition: to say that *animal* is **non-distributed** is to contrast it with distributed terms, like *bird* in "Every bird lays eggs". If *animal* were distributed in (P19), then every man would be every

With this latter sort of supposition, it allows descent to singulars in a disjunctive predicate: "For every human, s/he is this animal, or this animal...", and so on. So the predicate in (P18) is very different from that of (P17).

The importance of step (α) in the analysis of "is an animal" can scarcely be overstated here: by skipping it, Geach is forced to treat "is an animal" as a single, predicative unit. Of course, Fregeans just have bare predicates like "is an animal", since they reject the copula, which they regard as logically irrelevant to predication. But why import this restriction into Scholastic logic, for which the copula—as we have seen—is the chief formal part of any categorical proposition? By forcing a medieval theory to treat predicates in a Fregean way, and then criticising the theory on the grounds that it fails to differentiate "is an animal" in (P18) and (P19), Geach straightforwardly begs the question. This probably occurs because Geach is too committed to his 'two-name' theory of predication, which renders him insensitive to these distinctions.

These considerations set us up to address objection (i). We can just admit that the spoken copula *is* really does express a different kind of relation in (P17) and (P18), and therefore corresponds with different mental operations. In the former, it expresses a kind of identity, so that we might reword this proposition as follows to make it more logically perspicuous:

P17′) Socrates is the same thing as one specific animal.

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animal, which is false. To say that *animal* in (P19) is **con-fused** is to say that it does not have determinate supposition, the way *animal* (and *Socrates*) do in (P19).

Similarly, we could treat (P19) in the following way:

P18′) The set of humans is a subset of the set of animals.

Fortunately for us, Buridan is quite happy to say that the verbal copula *is* is ambiguous: it can be used to express membership of a class in another, as it does in (P18); or it can be used, to borrow Sten Ebbsen's phrase, as a "catachrestic way of saying 'the same as'."[278] At times, Buridan cashes out *is* as 'the same as', for instance in his discussion of the causes of propositional truth in *Sophismata* 2, concl. 12. There, he tells us that "'A is B' signifies the same as 'A is the same thing as B'"—at least where A is a singular term.[279] What matters is that an affirmative proposition is true just in case the subject and predicate stand for the same thing *or things*.[280]

Hence Buridan seems quite content to admit that *is* is equivocal at least at the *spoken* level: it is a single verbal utterance that is conventionally subordinated to several kinds of relations. Of course, what matters for Buridan is not the spoken term, but the mental operation it signifies, since the mental operation is where the rubber hits the road (as we saw in §1.2.1, above). So Geach's argument misses its target: it is correct on the verbal level, where *is* is equivocal; but not on the mental level where the *is* of singular propositions is very different from the *is* of universal ones. And in fact, on the verbal level

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[278] Sten Ebbsen, "The Chimaera's Diary", 139.
[279] "Idem enim significant 'A esse B' sicut 'A esse idem quod B'." (*Sophismata* 2, 12th concl.; Scott, p.43, 9v).
[280] "omnis propositio particularis affirmativa vera ex eo est vera quia subiectum et praedicatum supponunt pro eodem vel eisdem. Et omnis universalis affirmativa vera ex eo est vera quia subiectum et praedicatum pro quocumque vel pro quibuscumque subiectum supponit, pro eodem vel pro eisdem praedicatum supponit" (*Sophismata* 2, 14th concl.; Scott, p.44, 10r).

it is even more ambiguous than Geach supposes, as we will see in §1.3.2, below. But no matter: so long as we are careful to distinguish the different mental operations corresponding to each, we can keep the same verb for them as a matter of convention (*ad placitum*). The ambiguity of spoken *is* does not entail a conceptual ambiguity.[281] And so this Fregean criticism misses the mark.[282]

2.1.2 *Quantitas*: Signs of Quantity

We saw in §1.2.2 that categorematic terms signify the same extension *ad extra* no matter the type of proposition they appear in. For example, in the following propositions, the terms *human* and *animal* signify the same things (namely, all humans and all animals) *ad extra*, only in different ways:

P19) No human is an animal

P20) Some human is not an animal

[281] More recently this same point has been made by W.J. Clinton, who perhaps has done more than any popular philosopher to bring the ambiguity of the verbal copula to the popular consciousness: much, after all, "depends upon what the meaning of the word *is* is". Clinton is, however, still perhaps conflating verbal ambiguity with ambiguity at the mental level. Still, unlike Geach, Clinton at least does not profess to be knowledgeable about Buridan, and therefore can escape criticism as one who aspires but fails.

[282] Could we approach this in another way? Stanisław Leśniewski has constructed a logic (called Ontology) with an operator ('ε'), which is meant to function the way *esti* does in Greek (which, conveniently, works like Latin *est*). A subsequent discussion of this system and its applicability to Ockham took place in the *Notre Dame Journal of Formal Logic* between Desmond Paul Henry (1964) and John Trentman (1966). But I am not sure whether this will help us defend Buridan against Geach on the grounds of ambiguity, since at least the semantic analysis of "*a* ε *b*" will differ depending whether the terms *a*, *b* are common or discrete—a fact both Trentman and Henry note.

Of course, the propositions (P19) and (P20) say very different—indeed, contradictory—things: one is a universal affirmative, and the other is a particular negative. What determines this is their signs of quantity (*signa quantitatis*): terms like *all*, *some*, and the like. Thus the categorematic terms in (P19) and (P20) have, over and above their signification, is *supposition*—i.e. the way they stand for things in their propositional context.

We are not here concerned with the semantics of supposition, but only with the way syncategorematic signs of quantity contribute to logical form. To that end, it is sufficient to note that terms like *some* and *every* have no signification *ad extra* unless they are combined with a categorematic term. That is, there is nothing that was, is, will or can be an every or a some, the way there can be—and are—things like humans, donkeys, and so forth. Therefore, by (Sem_S), signs of quantity are shoo-ins for syncategoreme-hood: on their own, they have no signification *ad extra*.

How many types of such syncategorematic terms are there? For Buridan, as an Aristotelian logician, standard categoricals come in four flavours: universals, particulars, singulars, and indefinites.[283] Any sign of quantity will, by definition, belong to exactly one of these groups. For the first two groups, it is relatively easy just to list such terms, which is precisely what Buridan does. We'll start with these. Singulars and indefinites are a bit

[283] Of course, Buridan also analyses non-standard propositions with quantified predicates, like "Every donkey of some man runs" (See *Summulae de Propositionibus* 1.5, and especially the helpful reconstruction of Buridan's Magna Figura on pp. 44-5 of Klima's translation of the *Summulae*). But these need not concern us here, since we are worried about synategorematic signs of quantity and the way they pertain to form, not with propositional semantics. Therefore, we can focus on the signs themselves, as they appear in standard propositions, and note that the form of such multiply quantified propositions is different from their standard counterparts, precisely because of their inclusion of an additional quantificational term. After all, there are no such terms that can modify predicates only; and so we need not analyse such non-standard propositions to discover any new kind of syncategorematic term.

more tricky, as we will see in a moment, since they do not come with explicit signs of quantity.[284]

Universals are, unsurprisingly, those propositions that come with a universal sign, like *all* (*omnis*) or *no* (*nullus*). In a universal proposition, such a term modifies a subject that is a common term—that is, a term apt to stand for more than one thing, as opposed to a singular term, *i.e.* a proper noun or a common term modified by a demonstrative, like *this man*.[285] Here is an example of a universal proposition:

P21) All humans are animals

Universal signs of quantity include, as Buridan tells us in the *Summulae de Propositionibus* (1.3.5), *every* (*omnis*), *no* (*nullus*), *nothing* (*nihil*), *whichever* (*quilibet*), *any* (*quicumque*), *both* (*uterque*), *neither* (*neuter*), etc.[286] Later on, in the *Summulae de Suppositionibus* (4.3.7.1), Buridan adds to this list temporal quantificational terms like *whenever* (*quandocumque*) and *always* (*semper*), as well as universal spatial terms like

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[284] What about terms like *many*, *few*, or *almost all*? Buridan, like most other logicians before and since (with the conspicuous exception of J.E.J. Altham's *The Logic of Plurality* (London: Methuen, 1971)), leaves these by the wayside. But, on the basis of Buridan's criterion for syncategoreme-hood, it seems we'll have to include them, however grudgingly. This is all I'll say about them for the present project.

[285] "Propositio universalis est illa in qua subicitur terminus communis signo universali determinatus, ut 'nullus homo currit'. Terminus communis est qui aptus est praedicari de pluribus, ut 'homo' de Socrate et de Platone" (*Summulae de Propositionibus* 1.3.5; van der Lecq, p.37, ll.18-21).

[286] "Signa universalia sunt haec: 'omnis', 'nullus', 'nihil', 'quilibet', 'quicumque', 'uterque', 'neuter', etc." (*Summulae* 1.3.5; p.37, ll.22-3). Note that, while Buridan does not deal with the semantics of terms like *uterque* and *neuter*, these terms do appear in his more expansive logical diagrams—both the Square of Opposition and the Magnae Figurae presenting irregular forms—in the Vatican MS Pal.Lat. 994, ff.7r and 6r.

everywhere (*ubicumque*), and so on.[287] These terms are all alike in that they distribute the term to which they are applied.

Particular propositions are those with a common term as subject, modified by a particular sign of quantity.[288] For example:

P22) Some human is running

Particular signs of quantity, include *some* (*aliquis*), *someone* (*quidam*), *another* (*alter*), *the last* (*reliquus*), etc.[289]

Singular propositions come with no sign of quantity, but have in their subject position a term that is apt to stand for one thing only.[290] According to Buridan, such a term is either a name or a common term modified by a demonstrative like *this* (*hoc*) or *that* (*illud*):

P23) Socrates is running

P24) This human is running

[287] "Signa autem universalia [...] sunt ut [...] 'ubicumque', 'quandocumque', 'semper', 'perpetue', 'aeternaliter'" (*Summulae* 4.3.7.1; van der Lecq, p. 52, ll. 9-11).
[288] "Propositio particularis est illa in qua subicitur terminus communis signo particulari determinatus, ut 'quidam homo currit'." (*Summulae* 1.3.5; van der Lecq, pp.37-8, ll.23-2).
[289] "Signa particularia sunt haec: ' aliquis', 'quidam', 'alter', 'reliquus', et similia." (*Summulae* 1.3.5; van der Lecq, p.38, ll.2-3).
[290] "Propositio singularis est illa in qua subicitur terminus discretus sive singularis vel terminus communis cum pronomine demonstrativo, ut 'Socrates currit' vel 'iste homo currit'." (*Summulae* 1.3.5; van der Lecq, p.8, ll.5-7).

Now a singular term is always going to be categorematic, by both the syntactic and the semantic criteria (set out in §1.1 and §1.2, above). Such terms are apt to serve as subjects or predicates in well-formed categorical propositions, and have signification *ad extra*. So there are no syncategorematic terms in a singular proposition, beyond the copula, which we have already dealt with.

Indefinite propositions propositions are those with a common term as subject, unmodified by any sign of quantity.[291] These are the sort of propositions that, in English, we prefix with the indefinite article *A(n)*, *e.g.*

P25) A human is running

Such propositions are distinct from the foregoing, since unlike universals or particulars, they come with no sign of quantity; but unlike singular terms, they have a bare common term as their subjects—not a name, or a common term modified by a demonstrative pronoun. Here, too, there seems to be little that is interesting in terms of syncategoremata, since there are no signs of quantity whatsoever. From the foregoing, we can conclude that all the signs of quantity fall into two groups: particular signs, and universal ones, which we have listed above. Singular and indefinite propositions, on the other hand, have no signs of quantity, and so they have only one syncategorematic term: the copula, which we have dealt with already. Hence we have to add particular and universal signs to our list.

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[291] "Propositio indefinita est illa in qua subicitur terminus communis sine signo, ut 'homo currit'." (*Summulae* 1.3.5; van der Lecq, pp.38, ll.3-5).

The present discussion, however brief, gives us enough information to dispel a false notion about what signs of quantity (*signa quantitatis*) are: they are emphatically *not* the quantifiers of modern predicate logic. Rather, they behave more like natural language determiners. That is, they are not the logical operators of Classical FOL—'∀' and '∃'—that range over variables—like x and y—of open formulae—like $(\mathrm{F}(x) \rightarrow \mathrm{G}(x))$.[292] Rather, they are modifications of nouns in subject (and occasionally predicate) position.[293] This allows term-logic to treat the following proposition as categorical:

P21) All humans are animals

Since the term *All* does not range over the whole proposition, but only modifies the subject term *humans*, *All* is not the main connective of (P22). Rather, the copula *are* is. Conversely, modern predicate logic, which depends on quantifiers, will have to analyse (P22) as a hypothetical, namely:

P21′) $\forall x(\mathrm{Human}(x) \rightarrow \mathrm{Animal}(x))$

Which we read as:

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[292] This is not to say that modern logic more generally has adopted the account of FOL in this respect. For instance, Belnap has provided a form of restricted quantification, in which quantifiers range only over the items that answer to noun. See Nuel D. Belnap, "Restricted Quantification and Conditional Assertion", *Truth, Syntax and Modality*, ed. Hughes Leblanc (London: North Holland Publishing Co., 1973), 48-75.

[293] For a detailed refutation of this view as applied to Ockham's theory of supposition, see Gareth Matthews, "*Suppositio* and Quantification in Ockham", *Nous* 7 (1) (1983): 13-24.

P21″) For every x, if x is a human, then x is an animal.

Of course, (P21′) and (P21′) are hypotheticals, with very different truth conditions from (P21)—most importantly, (P21) has existential import, whereas the other two don't. Hence there is no notion of quantification over variables at this stage in the development of logic, and there is accordingly no need for medievals to read (P21) as a hypothetical like (P21′).

We therefore have to distinguish signs of quantity (*signa quantitatis*) in the Latin of Scholastic logic from modern quantifiers, though it is tempting to group them together. For instance, the *Encyclopedia of Medieval Philosophy* edited by Henrik Lagerlund includes an entry ("Quantification") that appears by its title to conflate the two. The author of this entry, Catarina Dutilh Novaes, tells us that:

> The modes of personal supposition were meant to codify the quantificational behavior of what we now refer to as quantifier expressions, and what the medievals referred to as syncategorematic terms.[294]

But quantifier expressions in modern logic behave very differently from the determiners of medieval logic, as we have seen. Dutilh Novaes is aware of this, and notes the anachronism:

> The phrase ''medieval theories of quantification'' is, properly speaking, an anachronism [...] to the point that this approximation may even be

[294] Catarina Dutilh Novaes, "Quantification", *Encyclopedia of Medieval Philosophy*, ed. Henrik Lagerlund (Dordrecht: Springer, 2010), 1093. Let me note in passing that the last clause of this passage contains a misleading generality: as we have seen, *syncategoremata* are the *genus* of signs of quantity, not identifiable with them. Analogously, one might say "We call these chickens, but the medievals referred to them as animals (*animales*)".

> unwarranted [...] Nevertheless, their treatments of such phenomena are often insightful and sophisticated, justifying thus that we consider them from the viewpoint of modern theories of quantification, but provided that the term ‘‘quantification’’ be understood very broadly.[295]

Now the charge of anachronism is a relatively minor one: After all, writing the Pythagorean Theorem algebraically as $a^2 + b^2 = c^2$ is anachronistic, too. But the problem is deeper: if we use modern quantification to understand medieval signs of quantity, we run the risk of severely distorting the latter, in a way that algebraic notation does not distort the Pythagorean Theorem. Rather, if there is a theoretical framework within which to assess medieval theories of signs of quantity, it is theoretical work on determiners in natural language.[296] Hence Dutilh Novaes is right to be wary of anachronism here, and we should be, too.

Let’s move on, turning to the last category of syncategorematic terms: those that determine propositional substance (*substantia*).

[295] Dutilh Novaes, “Quantification”, 1092.

[296] And in fact the error Dutilh Novaes warns against is not an uncommon one: for example, check out David Benovac’s “A History of Quantification” in the *Handbook of the History of Logic* ed. Dov Gabbay.

2.1.3 *Substantia*: Propositional Connectives—and Modal Copulae

As we saw above (§ 1.2.1), Buridan thinks that propositional substance (*substantia*) divides hypotheticals from categoricals. Recall that, in the *Summulae de Propositionibus*(1.3.7), dealing with the types of propositions, Buridan tells us the following:

> The question 'What sort?' pertains to the substance [*substantia*] of a proposition, and the response is 'categorical' or 'hypothetical'.[297]

But in fact propositional *substantia* is twofold: in certain passages, Buridan takes it to divide hypotheticals from categoricals; but in other passages, he takes it to divide assertoric propositions from modal ones. Elsewhere in the *Summulae* (1.3.4), Buridan tells us that categorical propositions come in two types of substance: modal (*propositiones modales*) and assertoric (*propositiones de inesse*):

> Concerning the division of categorical propositions with respect to their substance: some categorical propositions are assertoric [*de inesse*], and others concern a mode, or are modal [*de modo, sive modalis*]. An assertoric categorical is one about the simple inherence of the predicate along with [*cum*] the subject, as for example "Someone runs" or "'Someone runs' is possible". A modal categorical is one which is about a modified inherence of

[297] "'Quae' quaerit de substantia propositionis, ideo respondetur 'categorica' vel 'hypothetica'." (*Summulae* 1.3.7; van der Lecq, p.41, ll.15-6).

> the subject along with the predicate, as for example "Humans are necessarily animals" and "The Antichrist can be a human".[298]

Hence there are two sorts of propositional substance: (i) that which distinguishes hypothetical propositions from categorical ones; and (ii) that which distinguishes assertoric categoricals from modal ones. Here is a diagram to clarify:

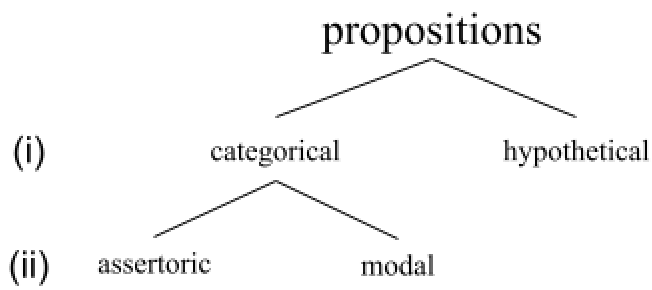

Fig. 3.2: the varieties of propositional substance

To each of these pertains a special class of syncategoremes. Let's work our way up, beginning with the syncategorematic terms that pertain to assertorics and modals (ii), before turning to the connectives that bind expressions into hypotheticals (i).

The modal-assertoric distinction is familiar one: categorical propositions make basic assertions, and do not distinguish between truths like the following:

P26) This text is black

[298] "De divisione propositionis catgoricae penes eius substantiam: propositionum categoricarum alia de inesse, alia de modo sive modalis. Categorica de inesse est illa quae est de simplici inhaerentia subiecti cum praedicato, ut 'homo est animal', 'hominem esse animal est possibile'. Categorica de modo est illa quae est de inhaerentia modificata subiecti cum praedicato, ut 'hominem necesse est esse animal', 'Antichristus potest esse homo'." (*Summulae* 1.3.4; van der Lecq, p.36, ll.1-6).

P27) Triangles have three sides

Modal propositions, on the other hand, say something more about how things *can* or *must* be, and so they allow us to distinguish between *e.g.*

P26′) This text is *contingently* black

and P27′) Triangles *necessarily* have three sides.

It is true that this text is black, but only contingently so: it is not impossible for it to be another colour, and still be recognisably the same text. Conversely, it is necessarily true that triangles are three-sided: nothing can make a triangle otherwise, without making it into something that is no longer a triangle. This intuitive distinction is well-known in modern analytic philosophy, and so there is little need to say much to motivate it right now.

Importantly for our present discussion of syncategorematic terms, in the above passage, Buridan is clear that *de dicto* modals—which Buridan calls *composite* (*compositae*)—are not really modals at all, but assertorics with a modal subject or predicate. Hence for Buridan, the following proposition is not really modal at all:

P28) ‘Someone runs’ is possible

The reason for this is that (P28), although it has a modal predicate, still has as its principal part an *assertoric* copula, *is*, which is unmodified by a modal adverb like *possibly* or *necessarily*. In this way, "if a [modal] determination concerns the subject or the predicate, then it does not render the proposition modal."[299] As Buridan goes on to say,

> Rather, even propositions in which the terms *possible, impossible, necessary, contingent*, *true* and *false* occur, if these terms are subjects or predicates, should be regarded as assertoric, since in them the predicate is predicated of the subject by the mediation of the copula *is* taken simply, without any determination of it.[300]

Where the mode is merely the subject or predicate, its modal determination leaves the assertoric copula intact. And since the copula is the principal part of a categorical, the whole categorical will be assertoric. So there is no special class of copula for *de dicto* or *divided* modals: they just make use of ordinary, assertoric copulae.

For Buridan, then, a categorical proposition is modal just in case a mode applies to the copula itself, making the proposition *de re*—which Buridan calls *divided* (*divisa*). As he tells us later on in the *Summulae de Propositionibus*, such modals have a modified copula: "the mode", he says, "is essential and intrinsic to the copula".[301] Thus modes work

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[299] "Et si determinatio se teneat a parte subiecti vel praedicati, non reddit propositionem modalem" (*Summulae* 1.8.2, van der Lecq, p.83).
[300] "Immo etiam propositiones in quibus ponuntur isti termini 'possibile', 'impossibile', 'necessarium', 'contingens', 'verum', 'falsum', si isti termini fiant subiecta vel praedicata, reputandae sunt de inesse, quia in eis praedicatum dicitur de subiecto mediante ista copula 'est' simpliciter accepta sine eius determinatione" (*Summulae* 1.8.2, van der Lecq, p.84).
[301] "notandum est quod modus est de essentia et de intrinsecitate copulae" (*Summulae* 1.8.3, van der Lecq, p.86).

much like negation which, in Buridan's characteristic phrase, "falls on the copula" (*cadat super copulam*).

In the *Summulae* (1.8.3), Buridan gives a series of arguments for the claim that the mode acts directly on the copula. For our present purposes, the most interesting of these is the syntactic one: briefly, if the mode were not bound to the copula, then propositional conversions would not be able to take place, since what stays fixed in these is the copula. Buridan's example is the following simple conversion:

A5) Socrates can be a runner
∴ A runner can be Socrates

Since conversions like this one involve swapping the place of subject and predicate, what remains—the fulcrum, so to speak, around which the conversion rotates—is the copula. And this fulcrum, here, is the whole phrase *can be* (cashed out as *possibly-is*). So modal propositions have a unique set of characteristic copulae.[302]

Hence for Buridan, a mode is a special determination of the copula. This means that we can read modals as categorical, rather than hypothetical: we can take modal propositions like (P26′) and (P27′) as having ScP-form—rather than as covert conditionals (or conjunctions) with forms like $\forall x(\mathrm{F}x \rightarrow \mathrm{G}x)$ (or $\exists x[\mathrm{F}(x) \wedge \mathrm{G}(x)]$), under the scope of operators like '$\Box$' (or '$\Diamond$'), which quantify across possible worlds, the way modern modal logic does. This will become very important in Chapter 5, when we turn to Buridan's derived rules for modal consequences.

[302] Cf. also *Summulae* 5.6.1

For now, we will have to add the modal copulae to our list of syncategoremata. These copulae will include modals both of affirmative and of negative quality, modified by each of the alethic modes Buridan lists in the *Summulae* (1.3.2) passage cited at the outset of this discussion—namely, *possible, impossible, necessary,* and *contingent,* as well as *true* and *false*.

With this in mind, let's turn to the other, more general sort of substance (level (i) in Fig.3.2, above), which comprises not modals and assertorics, but categoricals and hypotheticals. *Substantia*, in this second and broader sense, divides categoricals from hypotheticals. We have explored these two categories in detail in chapter 1. In sum, a categorical proposition has a copula as its principal part: that is, a predicative verb like *isn't* or *contingently-is*. Conversely, a hypothetical is a combination of multiple categorical-like expressions (*orationes*), bound together by a logical particle like *and* or *if*. In a hypothetical, the principal part is one of these, rather than a copula.

According to Buridan, there are six species of categoricals: conditionals, conjunctions, disjunctions, causals, and temporal and local propositions. Accordingly, there should correspond six kinds of hypothetical-making syncategoreme: to conditionals, *if* (*si*), or an equivalent term; to conjunctions, *and* (*et*) or its equivalent; likewise for disjunctions and *or* (*vel, sive, aut*, etc.), causals and *because* (*quia*, *quod*, *quare*, etc.), temporals and *when* (*quando*), and locals and *where* (*ubi*).

In fact, Buridan puts some thought into simplifying his list. For, as he notes:

> If these latter types, namely local and temporal, are treated as distinct kinds of hypotheticals, then the same should likewise hold for the other categories—for instance in the category of quantity, [*e.g.*] 'Socrates is as large as Plato'.[303]

If we treat the categories of time and place as a distinctive class of hypotheticals, to which there corresponds a distinctive class of syncategorematic terms, we will likewise have to do the same for the other Aristotelian categories like quantity, substance, and relation. If we did so, our list of syncategorematic terms would get quite long.

Fortunately, however, Buridan can treat all such category-specific hypothetical propositions as conjunctions. Thus Buridan is able to reduce the above list of types of hypotheticals to four (namely, conditional, conjunctive, disjunctive, and causal propositions). Here is how he reduces temporals and locals to conjunctions:

P29) Socrates read when (*quando*) Plato disputed.[304]

P29′) Socrates read at some time, and at the same time Plato disputed.[305]

P30) Socrates is where (*ubi*) Plato is.[306]

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[303] "si illae, scilicet species temporalis et localis debeant poni species distinctae hypotheticarum, tunc ita debet fieri de aliis praedicamentis, ut de quantitate, ut 'Socrates est tantus quantus Plato est'" (*Summulae de Propositionibus* 1.7.2; van der Lecq, p.73, ll.13-15).

[304] "Socrates legebat quando Plato disputabat" (*Summulae* 1.7.2; van der Lecq, p.73, l.11).

[305] "Socrates legebat aliquando, et tunc Plato disputabat" (*Summulae* 1.7.2; van der Lecq, p.73, ll.11-12). Note that this proposition is translated in Klima's *Summulae* as "Socrates lectured at some time *and then* Plato disputed" (p.60, emphasis added). Translating *et tunc* as *and then* in this way is tempting, but it makes these actions seem successive, when here they are simultaneous—if they weren't, (P28) and (P28′) would not have the same truth conditions.

[306] "Socrates est ubi Plato est" (*Summulae* 1.7.2; van der Lecq, p.73, l.9).

P30′) Socrates is somewhere, and Plato is there too.[307]

It is enough to note that these pairs, (P29) and (P29′), and (P30) and (P30′), have the same truth conditions. Therefore, there is no need to posit distinct classes of temporal and local hypotheticals and corresponding syncategorematic terms.

This reduces our list of hypothetical syncategoremes to the three truth-functional operators, plus the causal *because* (*quia*). Now we might wonder where *therefore* (*igitur*, *ergo*, etc.) belongs. Buridan notes this absence in Peter of Spain's text, and discusses the irreducibility of *therefore* to (i) *if* on one hand, and (ii) *because* on the other. *Therefore* is not reducible to *if* for the reasons set out in Chapter 1, above: conditionals do not put forth their constituent parts assertively (*assertive*), whereas arguments do. And *therefore* can bind two propositions together that are not causally related. So reducing *because* to *therefore* will make Buridan's logic relevantistic: inferences *ex impossibili*, which Buridan endorses (and which meet the modal requirement set out in (D3), above), will not always hold with *because* the way they do with *therefore*. Contrast, *e.g.*, the following:

A6) A donkey runs
Therefore God exists

A7) God exists
Because A donkey runs.

[307] "Socrates est alicubi et ibi Plato est" (*Summulae* 1.7.2; van der Lecq, p.73, l.10).

Hence *because* and *therefore* operate very differently, and so we can not reduce one to the other, the way we did with *where* (*ubi*) and *and*. So it seems we will have to add *therefore* to our list of hypothetical syncategorematic terms, bringing the total number to five: conditional, conjunctive, disjunctive, causal, and inferential.

From the foregoing, it looks like our logical particles will be reducible to the following:

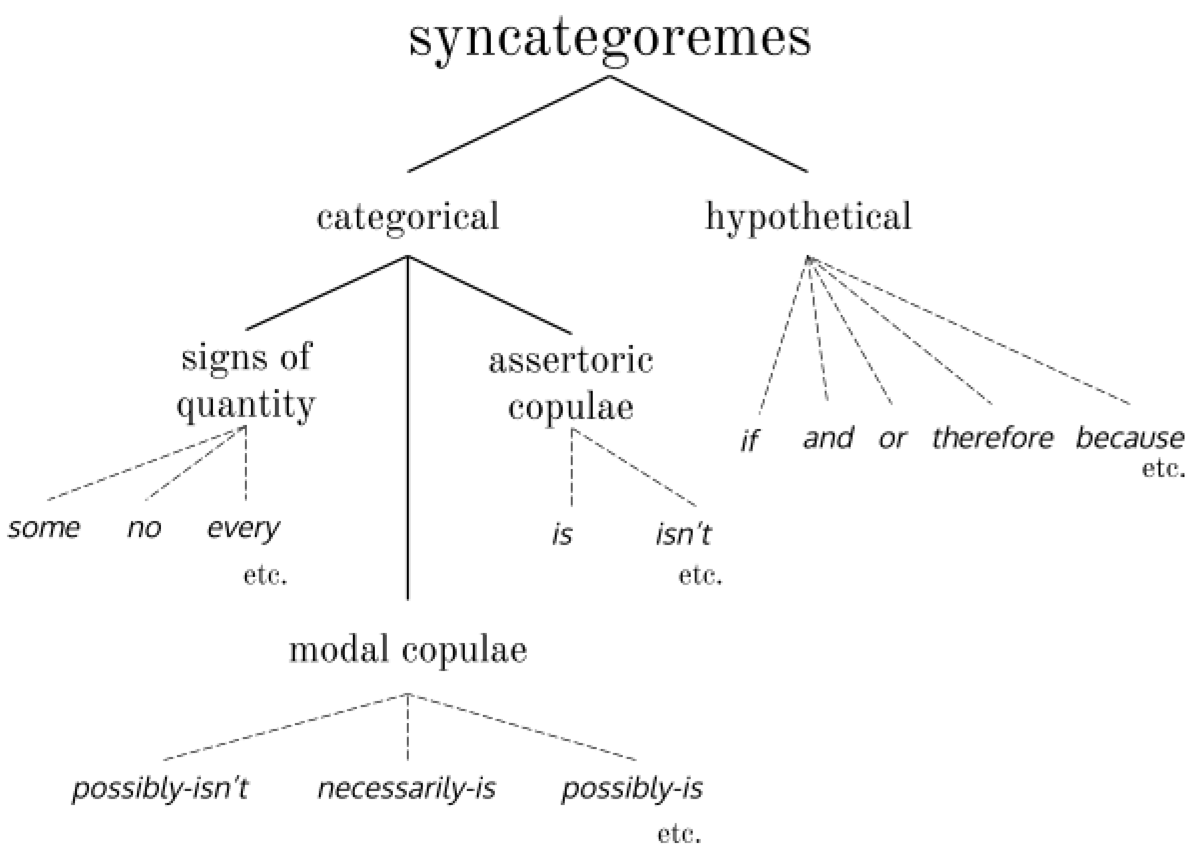

Fig. 3.3: the varieties of syncategorematic terms.[308]

[308] Here *etc.* denotes both (i) additional categoremes under each heading that are not listed for simplicity's sake (*e.g. contingently-is* under the modal copulae), as well as (ii) terms equivalent to the ones listed here, like *but*, which operates truth-functionally the way *and* does.

Here, at last, is an enumeration of the syncategorematic terms we find logically interesting, furnished in accordance with (Sem_S).

On the basis of our discussion in Chapter 2 (§3.1), we should add to this the "that which is" (*quod est*) locution Buridan adopts from Adam Wodeham, and puts to use in resolving certain sophisms.[309] Adding this term to the above list is no great problem: *quod est* taken as a unit does not signify anything on its own, but in combination with other significative categoremata. So it meets (Sem_S). Likewise for other miscellaneous but logically interesting particles mentioned here and there: hypothetical connectives like *unless* (*nisi*); inchoative verbs like *begins* (*incipit*) and *ceases to be* (*desinit*)—which inchoatives are actually *mixed*, since they co-signifiy time (more on mixed terms in a moment); and the like.[310] But we can just tack these on to the end of our list (under the *etc.* of the hypothetical branch), and thereby complete it.[311] Doing so is perfectly principled: though these terms don't get stand-alone treatment, the way the copulae or propositional connectives do, they meet Buridan's (Sem_S). So they are syncategoremata, too.

But does (Sem_S) give us only (and all) we were looking for? There are two reasons to think not.

[309] Knuuttila, "Trinitarian Theology", 1337.

[310] Presumably at least some of these were given independent treatment in earlier *syncategoremata* literature, but by Buridan's day had been absorbed into writing on the *sophismata*. See Fabienne Pironet and Joke Spruyt, "Sophismata" *The Stanford Encyclopedia of Philosophy*, (Winter 2015 Edition), ed. E. N. Zalta.

[311] This approach of attaching a few extra terms and phrases to our logical vocabulary, while perhaps a bit *ad hoc*, hasn't been discarded by modern logic, even Classical Logic. Cf. *e.g.* Bertrand Russell's treatment of the phrase *such that* in *The Principles of Mathematics* (VII, §80): Russell ultimately regards *such that* as irreducible and therefore *sui generis*.

3. Mixed Terms—and Unsought Syncategoremes

There are two problems for (Sem$_S$): it both over- and under-determines the class of logical particles. It over-determines because it foists on us many unwanted, non-logical (which is to say logically uninteresting) syncategorematic terms. And it under-determines because some of the most important logical terms are not themselves purely syncategorematic at all, and therefore do not straightforwardly meet (Sem$_S$). Let's look at each of these in turn.

3.1. Unsought Syncategoremes

Recall that syncategorematic terms are distinguished from categorematic ones by the (semantic) fact that they do not signify anything *ad extra*. As Buridan tells us,

> terms are called purely syncategorematic, however, which do not signify anything apart from the concepts that they immediately signify (except perhaps those things that are signified by the [categorematic] terms with which they are joined), and these are terms like *not, and*, *or*, *therefore*, *every*, and the so forth.[312]

[312] "Dicuntur autem pure syncategorematicae, quia praeter conceptus quos immediate significant, nihil significant, nisi forte ea quae termini quibus adiungitur significant, ut istae dictiones 'non', 'et', 'vel', 'ergo', 'omnis', et huiusmodi" (*Summulae* 4.2.3; van der Lecq, p.18, ll.14-17)

The problem is, there is a whole host of terms like *quickly* and *blue* that straightforwardly meet (Sem_S), and therefore count as full-fledged syncategoremata.

Here is why. According to Buridan, there is a dispute whether unsubstantivised adjectives can stand for (*supponere pro*) anything. In Latin, an adjective can appear substantivised or unsubstantivised. For instance, we can read the adjective *altum* either as *thing that is tall*, or as just a instance of the adjective *tall* in the neuter. Taken as a thing, *altum* is substantivised; taken as a bare adjective, *altum* is not. This is an easier distinction to talk about in English, which does not typically substantivise adjectives (or only does so in stock phrases with definite articles, like *the poor—i.e. people who are poor*). Thus we can speak of a *thing that is tall*, but it makes little sense to talk about a or the *tall*. And thus I will use English examples in what follows.

Peter of Spain's text, which Buridan is commenting on, notes "considerable disagreement among the professors" about unsubstantivised adjectives.[313] It seems they cannot appear as the subjects of propositions, *e.g.* the following:

P32) *Some tall is a tree

But we might be tempted to think unsubstantivised adjectives can serve as predicates of well-formed propositions. For example:

P33) Some tree is tall

[313] "de adiectivo autem non substantivato est bene dubitatio inter doctores" (*Summulae* 4.2.6; van der Lecq, p.23, ll.3-4).

Still, Buridan thinks that the predicate of a proposition like (P33) must be substantivised. As he tells us:

> Concerning unsubstantivised adjectives, I think that they cannot be a subject of a proposition *per se*—that is, without a substantive—because there is grammatical disagreement unless a substantive is implied [*subintelligitur*]. Moreover, I am doubtful whether they can be predicated *per se* in virtue of the implied [*subintellectum*] substantive, since in conversion the subject ought to become the predicate, and the predicate the subject, and an adjective on its own couldn't become a subject unless it were substantivised.[314]

Granted, an unsubstantivised adjective cannot serve as the subject of a well-formed proposition, as we saw a moment ago. But then if propositional conversion is to work, the predicate, even if it is a bare adjective, has to be substantivised as well. Consider for instance simple formal conversion of i-type propositions, which schematically works as follows:

S3)	AiB
∴	BiA

[314] "Sed de adiectivo non substantivato puto quod non possit esse subiectum propositionis per se, scilicet sine substantivo, quia esset incongruitas nisi subintelligeretur substantivum. An autem possit praedicari per se, in virtute substantivi subintellecti, dubito, cum in conversione debeat fieri de subiecto praedicatum et de praedicato subiectum, et adiectivum non potest fieri per se subiectum nisi substantivetur." (*Summulae* 4.2.6; van der Lecq, p.31, ll.11-7).

This conversion can thus take us from (P31) to the following:

> P34) Something tall is a tree

But wait: the subject of (P34) isn't the same as the predicate of (P33): to render it grammatical, we have had to supply a substantive term *something*. But the schema (S3) requires that the subject and predicate be swapped, but remain the same: the rule of conversions, set out in *Summulae* (1.6.1), stipulates just this condition:

> Propositions *sharing both their terms* in the reverse order can be converted in three ways (namely, simply, accidentally, and by contraposition).[315]

Therefore, it seems we have to either (i) rule out the sameness of terms condition, or (ii) rule out simple conversion altogether, or (iii) simply take the substantive *something* to be present and implied (*subintellectum*) in the predicate of (P33) as well. The only palatable option is (iii), and so it is the one Buridan opts for in the text cited above.

What this means is that unsubstantivised adjectives are not apt to serve as subject or predicate, and therefore do not figure into propositions at all. Further still, taken on their own they cannot be taken to signify stand-alone concepts, as Buridan acknowledges in a discussion of adjectives and substantives later on in the *Summulae*:

> just as I cannot formulate a purely syncategorematic concept without a categorematic one (although this can readily happen the other way around), so too I cannot readily form an adjectival concept without a substantive

[315] "Propositionum participantium utroque termino ordine converso triplex est conversio, scilicet simplex, per accidens, et per contrapositionem" (*Summulae* 1.6.1; van der Lecq, p59, ll.1-3).

one.[316]

So for Buridan, adjectives have no conceptual signification in the absence of a corresponding categorematic substantive. But by these lights, they seem to look a good deal like syncategorematic terms—a connection Buridan himself makes in the above text. Accordingly, unsubstantivised adjectives are syncategoremes, at least by the sole semantic criterion ($\mathrm{Sem_S}$) that Buridan gives us. That is, there seems to be no deep semantic difference between a stand-alone adjective like *tall* and a term negation like *non-*. These only take on signification in combination with a categorematic term like *tree* or *human*, to become complex concepts like *tall tree* and *non-human*. On their own, however, they signify nothing. Likewise, ($\mathrm{Sem_S}$) seems to net adverbs like *quickly*, as well as prepositions like *with* (*cum*), *for* (*pro*), *out of* (*ex*) and the like. Indeed, in the earlier syncategorematic literature, such terms were discussed, though by Buridan's day they had fallen out of vogue.[317]

In fact, if we consider the unsubstantivised adjectives of Latin alone, they will vastly outnumber the logical constants. So the overwhelming majority of syncategorematic terms are not logical constants at all.[318] And ($\mathrm{Sem_S}$) gives us no grounds to kick such terms off the guest list, even though we don't like how they behave. Though terms like

[316] "Et est opinandum, ut puto, quod sicut non possum formare conceptum pure syncategorematicum sine categorematico, licet bene e converso, ita etiam non possum bene formare conceptum adiectivum sine conceptu substantivo." (*Summulae* 8.2.4; de Rijk, p.47, ll.17-9).
[317] Aho, "Syncategoremata", 1243.
[318] It's worth noting, however, that which of these particles were singled out for logical analysis shifted over time: in early treatises on syncategoremes, like the one somewhat attributed to Henry of Ghent, we find analysis of such terms as *alone* (*solus*), *whole* (*totum*), *whether* (*an*), and the like. These do not figure prominently in Buridan, although it is noteworthy that an expanded Square of Opposition which appears in one of the MSS of the *Summulae* (Vatican ms. Pal.Lat. 994, fol.6r) displays relations among propositions incorporating *whole* and *part* (*pars*). This is the diagram (fig. 1) reproduced in Read's "John Buridan's Theory of Consequence and His Octagons of Opposition", *Around and Beyond the Square of Opposition*, ed. Jean-Yves Béziau and Dale Jacquette (Basel: Springer, 2012), 94.

tall, *quickly* or *out of* don't operate logically the way *not-* and *and* do, they meet (Sem_S). Hence their logical function—or lack thereof—is not a matter of their status as syncategoremes. We can therefore conclude the following lemma:

> **Lemma 3.3**: (Sem_S) does not uniquely identify the logical constants.

This lemma undermines the second claim of the Clean Divide View:

> **Claim II**: Syncategorematic terms can be identified with the class of logical constants; categorematic terms with the class of nonlogical constants.

Recall that, according to writers like John MacFarlane, the syncategoremes were meant to tell us what we needed to know about logical particles. This was supposed to be an advantage medieval logic enjoyed over its modern counterpart, which has to admit the somewhat arbitrary nature of the class of logical constants. But now it seems the Buridanian account will need an independent standard to exclude certain syncategorematic-looking terms, like unsubstantivised adjectives, from our list of logical constants. Or we will just have to admit we are kicking them out for more or less arbitrary reasons—which is apparently what we're doing.

Now it may be remarked that many adjectives and adverbs nevertheless signify ways that things are, at least when they're in a grammatically appropriate combination with nouns and verbs, respectively. For example, even though *tall* and *quickly* don't stand for things on their own, *tall tree* picks out the tall trees, and *those walking quickly* picks out a subgroup of those walking. Conversely, *non-* in *nonhuman* does not pick out some 'non-ness' in humans.[319]

As I see it, there are two problems with distinguishing *non* from *tall* like this, at least for the present discussion. First, it's a bit *ad hoc*, at least in light of the failure of (Sem$_S$) to distinguish logical from non-logical terms the way we want it to. But this is a pretty minor problem: a ton of philosophy is *ad hoc*, even if we try to cover our tracks once we've gotten the results we want. But the second problem is more serious: what about such obviously negative particles as *un-* in compounds like *unnatural*, which look like they should go with *non-* as term negations? Now granted, it would be odd to speak of *un-* as signifying some 'un-ness', but anyway that's not what it does: generally, it signifies *privation*, an absence of something one would naturally expect.

This, to put it briefly, is why we say something dirty is *unclean* but not that something clean is **undirty*. It's why we can coherently discuss, following 7-Up's 1968 marketing campaign, whether 7-Up is the *Uncola*—more than say, milk or water is.[320] And it's why we identify *un-Christian* behaviour only in Christians: Hindus and Muslims and other non-Christians cannot be said to display *un-Christian* behaviour.[321] So there is something signified *ad extra* by *un-*, even though *un-* and *non-* should be logical cousins.

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[319] I owe the gist of this observation to Calvin Normore (private correspondence, April 19, 2021).

[320] See Laurence R. Horn, "An *Un*-Paper for the Unsyntactician", *Polymorphous Linguistics: Jim McCawley's Legacy*, ed. Salikoko S. Mufwene, Elaine J. Francis, and Rebecca S. Wheeler (Cambridge, MA: The MIT Press, 2005), 329-65. In brief: Horn uses Rosch's prototype semantics and Aristotle's opposition theory to argue that the privation which *un-* picks out is "defined in terms of a marked exception to a general class property" (329).

[321] This example is from John Algeo, "The Voguish Uses of *Non*", *American Speech* 46(2) (1971): 90-1. Cited by Horn, "*Un*-Paper", 335. I'm here setting aside Lewis Carrol's *un-birthday*, in *Through the Looking Glass*. This is clearly a logician's play on this negative particle, and as a deliberate oddity probably bolsters the point I'm making here. See *Through the Looking Glass* (Oxford: Oxford UP, 2009 [1871]), 189.

And we cannot hive off *-un* to solve the problem, the way we tried to do with *-non*. Indeed, it looks like *un-* is both categorematic, by the syntactic criterion ($\mathrm{Syn_C}$), and syncategorematic, by the semantic criterion ($\mathrm{Sem_S}$).

And as we're about to see, there are further terms that play a logical role, even though they themselves are not purely syncategorematic, since they signify or co-signify something *ad extra*. Chief among such terms is the logically indispensable copula.

3.2. Mixed Terms: The Copulae

As we saw (§ 1.2.3), there are mixed terms that imply both a categorematic and a syncategorematic element. The examples we considered there included *vacuum* (cashed out as "place not filled with a body") and *nothing* ("non-being"). These imply, in addition to their categorematic parts, a syncategorematic element—namely, a negative particle. But there are likewise syncategorematic terms that co-signify something *ad extra*, and the most important such term is the copula. The copula always co-signifies time, except under extraordinary (which is to say miraculous) circumstances, as we will see. And, according to Buridan, only in those miraculous circumstances is the copula purely syncategorematic.

As we saw above (§ 2.1.1), Buridan regards the copula as the principal syncategorematic part of a categorical proposition. Indeed, it is impossible to construct a proposition without one (as we saw in Chapter 1, §1.1). But since this indispensable syncategoreme co-signifies time, it does not straightforwardly meet (Sem_S).[322]

Why time? Recall the discussion of signification of terms (§1.2.3, above). There, we saw that purely categorematic terms do not signify any specific time, and so they are not resistant to being used in propositions of any tense. Our examples used the pure categoreme *Caesar*:

P11) Caesar will be

P12) Caesar was.[323]

We contrasted these with the impure categoremes *today* and *tomorrow*. If terms like *Caesar* were bound to some time the way these impure categoremes do, then (P11) or (P12)—or both—would be semantically self-contradictory, like the following:

P13) *Tomorrow was

P14) *Yesterday will be

So pure categoremes like *Caesar* and *rose* do not signify any specific time. Still, many propositions in ordinary use say something about time, be it present, past or future. If they say something about time, but not in virtue of their categorematic constituents, it has to be in virtue of their only other part—their *copula*. And thus the copula co-signifies time.

[322] Granted, it may be thought that *consignification* and signification differ. But then, as we saw above, adjectives and adverbs only consignify, too, even though look like they should be categorematic.
[323] "Caesar fuit, Caesar erit" (*QNE* VI.6, f.122, v., d).

There are two main ways this can be done: a copula like *is* can signify a limited slice of time, or it can signify all time indifferently. For instance, contrast the present time of the following two propositions:

P34) The universe is expanding

P35) The time is noon

A proposition like (P34) covers all time. But a proposition like (P35) is limited to a very narrow present. Accordingly, categorematic terms in propositions can stand for all things they signify, in a way that makes no temporal distinctions—as in (P34). Or they can stand for a proper subset of them at some specific time—as in (P35).

Hence there are two ways a term can stand or supposit for (*supponere pro*) things when it comes to time: 'natural' and 'accidental':

> Supposition is called *natural* when a term supposits indifferently for for all the things for which it can supposit, both present things, as well as past and future ones [...] Supposition is called *accidental* when a term supposits only for present things, or for past and present ones, or for present and future things [...][324]

The terms in (P34) have natural supposition; those in (P35), accidental. Since the time at stake in a proposition is a function of the copula, there are accordingly two kinds of mental

[324] "Suppositio 'naturalis' vocatur secundum quam terminus indifferenter supponit pro omnibus pro quibus potest supponere, tam praesentibus quam praeteritis vel futuris [...] Suppositio 'accidentalis' vocatur secundum quam terminus supponit solum pro praesentibus, vel pro praesentibus et praeteritis, vel pro praesentibus et futuris [...]" (*Summulae* 4.3.4; van der Lecq, p.45, ll.4-9).

copula: temporally restricted, and omnitemporal. But, as Buridan notes, we lack a special syncategorematic term for omnitemporal propositions. Still, the ever-equivocal—and therefore versatile—spoken term *is* can do either job, *ad placitum*:

> We do not, however, have an utterance properly imposed to signify such a mental copula, so we can use the verb 'is' by convention [*ad placitum*] to signify such a copula by which the present time will no more be signified than is the past or future, and indeed no time. And in this way it will be a natural supposition.[325]

Even though we have no term for such a copula, we can just use the spoken term *is* for the copula of atemporal predications, as we use it to cover both temporal and omnitemporal ones. Hence the spoken copula *is* is more ambiguous even than Geach thought (see §3.2.1.1, above): not only can it signify identity or set membership, but it can also consignify a specific time, or every time and therefore no time in particular. But no matter: this ambiguity is limited to the spoken level: at the mental level, the two sorts of copula are completely distinct.

But if *is* can be temporal, and can expand to be omnitemporal, can it contract to nothing? That is, can *is* be *atemporal* as well? Buridan thinks so:

> just as the intellect is able to conceive of man and animal without any distinction of time in the concepts by which 'man' and 'animal' are imposed,

[325] "Sed non habemus vocem proprie impositam ad significandum talem copulam mentalem; ideo, ad placitum, possumus hac uoce 'est' uti ad significandum talem copulam, per quam non significabitur magis tempus praesens quam praeteritum vel futurum, immo nullum, ideo sic erit suppositio naturalis terminorum." (*Summulae* 4.3.4; van der Lecq, p.47, ll.24-7).

> so it is likely that it is able to formulate a complexive concept without any distinction of time.[326]

Thus there is no barrier in principle to our forming a proposition that is truly atemporal. But the conditions under which such a proposition can be formulated have to be extraordinary—which is to say miraculous:

> In fact, perhaps we can show from our faith that we are able to form such mental propositions. For God could preserve all things in rest, without motion (I mean all things other than motion). So let us suppose that God does so. Then nothing would be time, if every time is motion, as Aristotle shows (*Physics* IV).[327] Nevertheless, the souls of the blessed would know and understand by mental propositions that God is good and that they are present to God; and by the copulas of those mental propositions they would not co-understand [*cointelligerent*] time, for they would also know that there is no time, and so they would know that neither they themselves nor God did exist in the present time, and that they did not coexist with the present time either.[328]

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[326] "sicut intellectus potest concipere hominem et animal sine differentia temporis illis conceptibus a quibus imponuntur isti termini 'homo' et 'animal', ita verisimile est quod potest formare conceptum complexivum illorum sine differentia temporis." (*Summulae* 4.3.4; van der Lecq, p.47, ll.19-22).

[327] *Physics* IV.10.

[328] "Immo forte ex fide nostra possumus ostendere quod tales possumus formare propositiones mentales. Quia deus posset omnia conservare in quiete, sine motu (dico omnia alia a motu); ponamus ergo quod ita faciat; tunc nullum esset tempus, si omne tempus est motus, sicut determinat Aristoteles, quarto *Physicorum*, et tamen animae beatae scirent et intelligerent deum esse bonum et ipsas ei assistere, per propositiones mentales; et per copulas illarum propositionum mentalium non cointelligerent tempus, quia etiam scirent nullum tempus esse, et sic scirent se et deum non esse in tempore praesente nec se tempori praesenti coexsistere." (*Summulae* 4.3.4; van der Lecq, p.48, ll.1-8).

Since time is a function of motion, and since all motion can be stopped by an act of divine will, time can be annihilated.[329] And yet it seems that, in such a situation, it would be possible to formulate a mental proposition. But since there would be no time for the copula of such a proposition to consignify, the proposition would be truly atemporal.

And here's the real kicker: according to Buridan, the copula of such a proposition is the only kind of copula that is purely syncategorematic:

> And it appears to me that a spoken copula imposed to signify precisely such a complexive concept *would be purely syncategorematic*. Others, however, which connote a certain time, already share [the characteristics of] categorematic [terms], in that beyond their concept they also signify an external thing conceived besides the things signified by the subject and the predicate, namely, time.[330]

But the case considered above is an extraordinary one indeed. Apparently, the mental formulation of an atemporal—and therefore purely syncategorematic—*is* requires no less than the total annihilation of time.

Conversely, in all ordinary—which is to say, natural (as opposed to supernatural)—contexts, the copula signifies time: either all of it, or a limited portion. But time is something outside the mind. Recall (Sem_S): syncategorematic terms signify nothing *ad extra*. Hence the copula is not purely syncategorematic, in any but the case just

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[329] I suspect the source of this freezing-of-time idea is Scotus' thought experiment in *Quodlibetal Questions* (q.1, a.2), which Scotus uses to argue that time is *not* a function of motion or change. If so, Scotus' modus tollens is Buridan's modus ponens.

[330] "Et videtur mihi quod copula vocalis imposita praecise ad significandum talem conceptum complexivum esset pure syncategorematica; aliae autem, quae connotant certum tempus, iam participant de categoremate, quia praeter conceptum significant etiam rem extra conceptam aliam ab illis quas subiectum et praedicatum significant, scilicet tempus." (*Summulae* 4.3.4; van der Lecq, p.48, ll.8-12; emphasis added).

considered, where God stops time. Therefore, the indispensable formal component of any categorical proposition is not, itself, a purely syncategorematic term. And this is our third lemma:

> **Lemma 3.4**: The copula is not a purely syncategorematic term, because it has signification *ad extra*.

Lemma 3.4 gives us further reason, in addition to Lemmata 3.1-2, to reject Claims I-II. To reiterate:

> **Claim I**: Syncategorematic and categorematic terms are strictly demarcated.
>
> **Claim II**: Syncategorematic terms can be identified with the class of logical constants; categorematic terms with the class of nonlogical constants.

On the basis of Lemma 3, we can also reject Claim III of the Clean Divide View:

> **Claim III**: Syncategorematic terms do not refer to anything outside the mind; categorematic terms do.

As can be seen, under close examination the link between purely syncategorematic terms and the formal components of a proposition starts to come apart. And as we'll see in the next chapter, even the form of a proposition is at least sometimes a matter not of clear-cut logical distinctions, but merely of convention. At any rate, both modern and medieval logic face a similar problem: how can we justify our selection of the logical particles we use, in a way that isn't circular or *ad hoc*?

Chapter 4

Types of *Consequentiae* II

Matter and Form

...what is matter? Never mind.

—Bertrand Russell's Grandmother[331]

With the distinction between logical and non-logical terms now in place, we can turn to the distinction between formally and materially valid consequences. Recall that, for Buridan, formal consequences hold in virtue of their logical structure or *form*, whereas material ones hold in virtue of their significative terms or *matter*. Here, for instance, is a formally valid syllogism in the mood Camestres:

A1) No cats are dogs
All tabbies are cats

[331] Bertrand Russell, *The Autobiography of Bertrand Russell: Vol. II: 1914-1944* (New York: Routledge, 1967 [2000]): 40.

∴ No tabbies are dogs

Because (A1) is formally valid, we can represent it schematically, as follows:

S1) No A is B
Every C is A
∴ No C is B

No matter what categorematic terms we substitute for A, B, and C in (S1), we will never get an invalid argument—that is, we will never get an argument with true premisses and a false conclusion.

Contrast formally valid consequences like (A1) with materially valid ones like the following:

A2) A human is running
∴ An animal is running.[332]

Clearly, if we schematise (A2), we will not get a logical form that holds in all substitution instances. Yet it is impossible that the antecedent of (A2) be true, and the consequent false. So (A2) is valid, though only in virtue of its referential parts: it is, therefore, *materially* valid.

Having presented the above formal-material division, Buridan goes on to add that, of the materially valid consequences, some (like A2) are valid materially and simply

[332] "Homo currit; ergo animal currit" (*TC* I.4.13).

(*simplex*), whereas others are valid materially 'as of now' (*ut nunc*). The difference comes down to modality. Simply materially valid consequences meet the modal requirement set out in (D3), namely that the antecedent cannot be true without the consequent (a claim I'll explore further in §2.1, below). But consequences *ut nunc* do not meet this modal requirement at all. Rather, such consequences hold because they do not have a true antecedent and a false consequent as of now—and, as I am going to argue, because these things *cannot* be otherwise as of now, though they are subject to change in the future.[333] Therefore, such consequences are "not simply speaking (*simpliciter loquendo*) valid", but:

> they are valid as of now (*ut nunc*), since it is impossible for the antecedent to be true and the consequent false, given the way things are as of now (*ut nunc*).[334]

Buridan gives the following as an example of a consequence valid *ut nunc*:

A3) Gerard is with Buridan

∴ Gerard is on the rue de Fouarre.[335]

An argument like (A3) is valid given the circumstances: the rue de Fouarre—"Straw Street", as Gyula Klima helpfully points out in a footnote to his translation of the

[333] Buridan is clear that formal and materially valid consequences meet the *same* modal requirement: in *Summulae* 1.6.1, he tells us that both types of consequence "agree in that it is impossible for their antecedents to be true while their consequents are false." This will become very important in §2.1, below. Conversely, consequences that are valid *ut nunc* do not meet this requirement.

[334] "Aliae vocantur *consequentiae ut nunc*, quae non sunt simpliciter loquendo bonae [...] sed sunt bonae ut nunc, quia impossibile est rebus omnino se habentibus ut nunc se habent antecedens esse verum sine consequente" (*TC* I.4.31-2).

[335] "Gerardus est cum Buridano; ergo ipse est in vico Straminum" (*Summulae* 1.7.3).

Summulae de Dialectica—was where students at the University of Paris took their lectures.[336] Suppose that Buridan, as an arts master at the University of Paris, is there. Under these circumstances, if Gerard is with Buridan, then he will be on the rue de Fouarre, too.

Still, (A3) is not strictly speaking a valid consequence, because it does not meet the modal requirement stipulated in (D3): Buridan could simply get up and go someplace else, and take Gerard with him. Still, given the way things are as of now, it cannot be that the antecedent is true and the consequent is false. Hence *ut nunc* consequences are not strictly speaking (*simpliciter loquendo*) valid at all, since they do not meet the modal requirement for validity that formally and simply-materially valid arguments meet. As we will see (in §4, below), they meet a lower modal notion, which corresponds with Grade 4 of the modal scale set out in the *Summulae de Demonstrationibus* and discussed in Chapter 2, above.

We can thus represent the Buridan's taxonomy of consequences with the following tree:

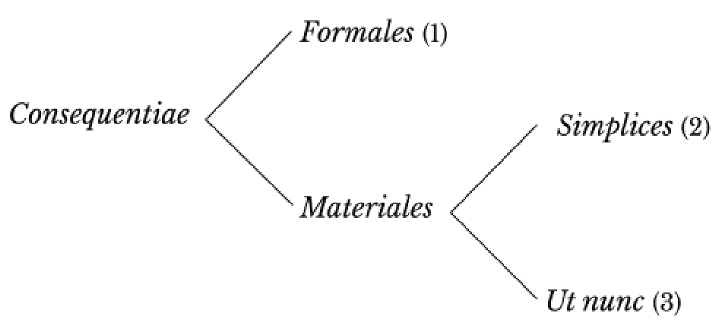

Figure 4.1: Buridan's taxonomy of consequentiae

[336] Klima, *Summulae*, p.62, n.96.

In what follows, I look at each numbered node of the above tree in turn, presenting my conclusions along the way.

1. Formal Validity: A Matter of Form

1.1. What is Propositional Form?

As we've seen, there is a special relationship between propositional form and syncategorematic terms. But there are two more elements to propositional form that are not syncategorematic: order (*ordo*) and relation (*relatio*). In *TC* I.7 ("On the Matter and Form of Propositions"), Buridan gives us the following account of form and the formal-material divide, and we shall repeatedly return to this passage in what follows:

> By [i] the *matter* of a proposition or a consequence, we mean the purely categorematic terms, namely subject and predicate, without the syncategorematic terms which are apposed to them, through which they are joined together or negated, or distributed or drawn out into a mode of supposition. But we say [ii] that all the rest [*totum residuum*] pertains to form. Hence we say that the copulae—both of categorical and hypothetical propositions—pertain to form, as do negations, signs of quantity, and the number both of propositions and of terms, and the **order** [*ordo*] all these have to each other, and the **relations** [*relationes*] of relative terms, and the modes of signification that pertain to the quantity of the proposition in

> question—like discreteness and commonness of terms—and many things that those who are attentive [*diligentes*] will be able to spot if they come up.[337]

In this section, we'll focus on form in (ii); later on, we'll look at matter *qua* combination of categorematic and syncategorematic elements, discussed in (i). Here, I want to give four remarks about (ii), before turning to the meaning of *order* and *relation*, which I have bolded in the above text.

First remark: the requirement that two propositions have the same form just in case they have the same *number* of constituent terms or (proposition-like) expressions introduces a significant problem for formal schemata. How, if number has to be the same, can we allow substitution of more complex propositions in hypotheticals? For instance, consider the following two propositional schemata:

S2) $\varphi \rightarrow \psi$

S3) $((\varphi \vee \chi) \wedge \xi) \rightarrow (\psi \wedge {\sim}\chi)$

Both (S2) and (S3) have the same form, for the sake of our schemata; but they have a very different number of propositions. Is Buridan therefore committed to saying that the

[337] "per 'materiam' propositionis aut consequentiae intelligimus terminos pure categorematicos, scilicet subiecta et praedicata, circumscriptis syncategorematicis sibi appositis, per quae ipsa coniunguntur aut negantur aut distribuuntur vel ad certum modum suppositionis trahuntur; sed ad formam pertinere dicimus totum residuum. Unde copulas tam categoricarum quam hypotheticarum propositionum dicimus ad formam pertinere, et negationes, et signa, et numerum tam propositionum quam terminorum, et ordinem omnium praedictorum ad invicem, et relationes terminorum relativorum, et modos significandi pertinentes ad quantitatem propositionis, ut est discretio et communitas, et multa quae diligentes possum videre si occurrent." (*TC* I.7; Hubien, p.30, ll.8-16; emphasis added). Cf. *Summulae* 1.6.1; van der Lecq, p.61, ll.3-6, which says about the same thing, but is less detailed than the *TC* passage cited here.

two have very different forms? I will address this problem in a dedicated section (§ 1.4) below.

Second remark: from the final clause of this passage, we can surmise that Buridan does not intend to present an exhaustive list of all the things that can pertain to form. Neither, for that matter, did we: recall that, in the preceding chapter, we had to add on and hand-wave at a few final terms like *quod est* and *incipit*. That is, though we presented the main syncategorematic terms, we had to admit that there are always a few further we couldn't add on, or which perhaps we were not attentive enough to recognise. Such terms will pertain to the formal rest (*residuum*) of the proposition. We will, therefore, have to proceed with caution.

Third remark: it is noteworthy that the definition of form here first sets out a well-defined notion of matter, in terms of categorematic terms, and then gives a much hazier notion of form and syncategoremes. This strongly suggests that, in fact, matter and categoremata are much more clearly defined that their counterparts, form and syncategoremata. And it gives the lie to the naive assumption that the medievals have a well-defined set of syncategorematic terms, uniquely furnished by the semantic criterion (Sem_S). If that were so, why not start with form? But for the term logician, matter *qua* categorematic terms is much easier to define; and indeed, there are *two* criteria to define this class, (Syn_C) and (Sem_C), whereas there is only a semantic criterion for the syncategoremes. This is yet another reason to doubt the Clean Divide View set out and refuted in the preceding chapter.

Fourth remark: even purely categorematic terms contribute something to the form of a proposition, as Buridan recognises in the above passage: it pertains to the form of a proposition whether the terms involved in it are *discrete* or *common*. This is not determined by quantificational signs, but is a feature of the terms themselves; compare for instance:

P1) Man is mortal

P2) Socrates is mortal

In (P1), the subject is a common term; in (P2), the subject is discrete. Accordingly, (P1) and (P2) have different form—a fact we implicitly recognise this when we schematise propositions like (P1) and (P2) as follows:

P1′) A is B

P2′) *a* is B

Hence even the types of categorematic terms at play are relevant to form: we cannot substitute a discrete term like *Socrates* for a common term like *man* (or vice-versa) without changing the form of a proposition. Here again, we find that the Clean Divide View hits upon a significant problem: even the categorematic terms, by their type, contribute something to the form, not in virtue of their signification but in virtue of the *way* they stand for things. This gives us our final lemma in the case against the Clean Divide View:

> **Lemma 4.1**: Categorematic type contributes to logical form, and is not always determined by the syncategorematic elements of a proposition.

Recall Claim IV, which is one of the four pillars of the Clean Divide View, and which Lemma 4.1 undermines:

> **Claim IV**: Syncategorematic terms alone determine propositional form; categorematic terms alone pertain to propositional matter.

From the foregoing, it is clear that syncategoremes alone will not tell us everything we need to know about the formal role of categorematic terms in a proposition. When schematising propositions, or substituting terms in them, we will have to attend to the type of term being substituted.

With these things in mind, let's turn to the two aspects of form introduced in the above passage: binding or relation (*relatio*) and arrangement (*ordo*). Relation is what holds between categorematic terms and bound anaphoric pronouns. Compare for example the following propositions:

P3) Some man runs and some man doesn't run.[338]

[338] "Homo currit et homo non currit" (*TC* I.7; Hubien p.31, l.32).

P4) Some man runs and he doesn't run.[339]

As with the former pair of propositions, (P3) and (P4) have very different truth conditions, in any case except one in which there is only one person. If there is one person only, both will have to be false. But if there is more than one person, (P3) can be true. Suppose for instance that Socrates runs and Plato doesn't; then (P3) is true. Still, (P4) will never be true: here the bound anaphoric pronoun has the same supposition as the term for which it stands (namely *some man*). So (P4) makes an impossible claim, namely that one person at one time both runs and does not run. So in order to have the same form, two propositions will have to have the same relation between their significative terms and any bound anaphoric pronouns they contain.

What, on the other hand, is the *order* (*ordo*) of a proposition, and what does order contribute to propositional form? Intuitively, propositions are not just random assortments of terms, judged merely by their contents, the way sets are. Rather, propositions are *structured*. For example, consider the following two propositions, which Buridan contrasts in *Summulae de Locis Dialecticis* (6.4.10):

P5) This homebuilder is a good man.[340]

P6) This man is a good homebuilder.[341]

[339] "Homo currit et ipse non currit" (*TC* I.7; Hubien p.31, l.33).
[340] "Iste domificator est bonus" (*Summulae* 6.4.10; Green Pedersen p.67, ll.9-10).
[341] "Iste est bonus domificator" (*Summulae* 6.4.10; Green Pedersen p.67, l.10).

Clearly, (P5) and (P6) say very different things: (P5) is true of someone who has attained eudaimonia but can't frame a wall to save his life; whereas (P6) is true of an incorrigible akratic who is nevertheless well accomplished in all aspects of homebuilding. But (P5) and (P6) have all the same terms. Order, therefore, matters: propositions are not just jumbles of their constituent terms.[342]

Accordingly, we can tentatively boil propositional form down to (i) the syncategorematic elements that we (more or less arbitrarily) identified as logically interesting in the preceding chapter, (ii) the types of categorematic terms in the proposition, (iii) the relations among the proposition's anaphoric pronouns, and (iv) the order or sequence of terms in the proposition.

Still, Buridan does not seem to take this list as exhaustive: as he says in the passage cited above, there are in addition "many things that those who are attentive will be able to spot if they come up."[343] This claim is critically ambiguous: it is not clear whether Buridan means further *sub-types* or even tokens of things pertaining to form, or whether he thinks there are further irreducible types. If he means sub-types, then this is no great worry: for instance, in the foregoing passage, he mentions assertoric copulae like *is* and *isn't*, but not modal ones like *possibly-is* or *necessarily-isn't*. But the modal copulae

[342] Buridan's example here is a little odd, both for (i) its syntax and (ii) its subject matter. We are used to thinking of Latin word order as more free, but (i) pushes us to think of Latin as a synthetic language, not an analytic one. Yet in Buridan's hands, Latin often looks more analytic: consider for example his rigid rules for word order in multiply-quantified propositions (*Summulae* 1.5). So we can follow him here in taking *bonus* as taking wide-scope when it appears as the predicate, as in (P5). As for (ii), it is noteworthy that the examples given here touch on ethical questions of the unity of the virtues, etc. I do not wish to wade into these here, but only to talk about the syntax. Still, I wish Buridan had picked another example.
[343] "multa quae diligentes possunt videre si occurrant" (*TC* I.7; Hubien, p.30, l.18).

are just another subtype of copula, not a whole other class of things pertaining to form. Similarly, we might give *and* as our conjunctive particle, without mentioning all the other tokens of terms that are truth-functionally equivalent (*although*, *but*, *whereas*, *yet*, and so forth). So if it is just a matter of equivalent tokens, or even unmentioned sub-types, then it poses no great problem at all.

But if there are whole uncharted primitive (and therefore irreducible) types of things pertaining to form, on equal footing with (i)-(iv), then there will remain further work to be done in isolating propositional form. Still, I find no evidence of a further class of terms in Buridan, nor can I think up any on my own. So I leave it to other *diligentes* to point them out to me if they find them.[344]

1.2. Validity by Form: Substitution and the Modal Requirement

Let's tentatively take (i)-(iv) to be the things we keep fixed when we perform uniform substitutions in logically valid schemata. The rest is matter, which can come and go, leaving form intact. Recall that, in such substitutions, we can abstract away from a particular token of an argument to arrive at a type. For instance, we can take an argument like the following:

A4) Something that is a human is an animal

[344] In his (2012) paper "A History of Connexivity", Storrs McCall solicits information on a sample questionnaire, to be distributed among students studying logic for the first time, and leaves his email. I'll follow his lead: I would be delighted to hear of unheard-of formal components of propositions, at boaz.schuman@mail.utoronto.ca.

∴ Something that is an animal is a human.[345]

and abstract from it the following schema:

S4) Something that is B is A
∴ Something that is A is B.[346]

The claim that (A4) is formally valid amounts to the claim that it has the same form as (S4), and that there are no substitution instances of (S4) with true premisses and a false conclusion. Thus in virtue of its fixed formal attributes, (A4) is valid. In this way, the claim that (A4) is a formally valid argument, and that (S4) is a valid formal schema, amounts to a generalisation across all possible substitution instances. As we will see in a moment (§1.3, below), some thinkers (notably Hilary Putnam) think that a nominalist is not entitled to make such generalisations. But this is based on a misunderstanding, which takes root in the ambiguity of the term *nominalism*.

For the remainder of this section, however, I want to discuss the formal validity of arguments like (A4) in light of the modal requirement discussed in the preceding chapter. As we saw in Chapter 2, Buridan thinks formally valid arguments are simply necessary—that is, there is no power, natural or divine, capable of rendering the premisses true and the conclusion false. Now recall that Buridan asks (in *QAPr* I.19) whether the moods of first-figure syllogisms hold in virtue of their form (*gratia*

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[345] "Quod est homo est animal, ergo quod est animal est homo (*Summulae* 1.6.1; van der Lecq, p.60, ll.23-4).
[346] "Quod est B est A; ergo quod est A est B" (*Summulae* 1.6.1; van der Lecq p. 60, l.24).

formae)—that is, whether they are formally valid. If so, then there must be no power capable of rendering the premisses true and the conclusion false, in *any substitution instance whatsoever*. But, as Buridan points out, the syllogistic mood Barbara does not hold in all substitution instances. When we substitute categorematic terms that stand for members of the Trinity, it fails:

A5)	Every God is the Father;	(true)
	Every Son is God;	(true)
∴	Every Son is the Father.[347]	(false)

Since each Person of the Trinity is identical with God, the premisses of (A15) are true. But since the Persons of the Trinity are distinct from each other, the conclusion is false. Therefore, it seems, syllogisms in the mood Barbara are not valid in virtue of their form, since they have at least one counterinstance.

We have already seen how Buridan solves this: if we build a "that is" (*quod est*) locution into the form of syllogisms like (A5), then their formal validity is restored. Here is the modified syllogistic schema:

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[347] "Item ponamus quod nullus sit pater vel filius nisi deus; tunc arguitur sic:

	omnis deus est pater
	omnis filius est deus
ergo	omnis filius est pater.

Praemissa sunt verae secundum casum positum, et conclusio est falsa" (*QAPr* I.19 obj. 6).

S5) Everything *that is* B is A;

Everything *that is* C is B;

∴ Everything *that is* C is A.[348]

If we plug the divine terms back into the schema (S5), we get at least one false premiss, and therefore the conclusion does not move from true premisses to a false conclusion. Barbara is, therefore, saved from the Trinity.[349]

From the discussion of (A5) in the *QAPr* (I.9), reconstructed above, we can infer that the modal requirement at play in formal validity is the *simple* (*i.e.* Grade I/1) necessity of the preceding chapter. We will soon see that materially valid arguments meet the same, simple (*i.e.* Grade I/1) modal requirement as their formal counterparts. But first, we should consider a challenge to nominalism that threatens the Buridanian account of logical form: is the nominalist entitled to talk about logically valid schemata at all? At least in Hilary Putnam's account, the answer is *no*.

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[348] "Et tunc pono istam conclusionem quod primus modus primae figurae est formalis sub isto modo loquendi: omne *quod est* B est A
omne *quod est* C est B
ergo omne *quod est* C est A" (*QAPr* I.19, co.; emphasis added).

[349] Notice that if we think that (S4) is technically not formally valid, whereas (S5) is, then we have to admit that *that is* (*quod est*) is part of the *form* of any valid syllogism. And our admission will become a source of embarrassment when we come to deal with mixed modal syllogisms, since Buridan thinks *quod est* blocks modal ampliation (see Chapter 5, § 1.2, below). But a modal just isn't a modal if it lacks ampliation. So does *quod est* just duck in and out of the forms of the propositions involved in mixed-modal syllogisms, depending whether they're assertoric (in!) or modal (out!)? Apparently so. This looks inelegant—not to mention *ad hoc*. But I know of no better solution.

1.3. Putnam's Challenge

In his (1971) *Philosophy of Logic*, Hilary Putnam presents two problems that might seem troublesome for Buridan. The first problem deals with variables like *S*, *M*, and *P* in the following logically valid schema:

S6) If all *S* are *M* and all *M* are *P*, then all *S* are *P*.[350]

How should we cash (S6) out? Putnam contrasts two approaches: that of the realist, and that of the nominalist. The realist has no scruples about positing abstract entities like classes, and is therefore free to formulate (S6) as follows:

$S6_R$) For all *classes S, M, P:*

If all *S* are *M* and all *M* are *P*, then all *S* are *P*.[351]

The nominalist, however, has such scruples, and is therefore limited to talking about concrete, rather than abstract objects: (S6), for the nominalist, is about concrete things like *words* and *sentences*, rather than abstract things like *classes*. Here, then, is how a nominalist might formulate (S6) while avoiding all talk of abstract entities:

$S6_N$) The following turns into a true *sentence* no matter what *words* or

[350] Hilary Putnam, *Philosophy of Logic* (London: George Allen & Unwin, Ltd., 1971), 11.
[351] Putnam, *Logic*, 10.

> *phrases* of the appropriate kind one may substitute for the letters *S, M, P:*
>
> If all *S* are *M* and all *M* are *P*, then all *S* are *P*.[352]

But what is meant by "word or phrase of the appropriate kind"? It seems here we are talking about possible words and phrases. And, as Putnam is quick to point out, "*possible words and phrases* are no more 'concrete' than classes are".[353] Hence even when we try to avoid abstract things like classes, we end up committed to other abstractions like *possible* words and phrases, which are not so concrete as they may seem *prima facie* in ($S6_N$). Our detour takes us right back where we started.

The second problem is more general than the first: when we speak of substitution instances of *S*, *M*, and *P*—be they classes, words, or whatever—in a valid schema like (S6), how many substitution instances are we talking about? The only reasonable answer seems to be *all of them*. But what is the scope of *all*? If we are committed only to concrete entities, it seems we have to limit ourselves merely to the *actually existing* substitution instances, rather than all possible ones:

> When we speak of *all* substitution-instances [...] we mean all *possible* substitution-instances—not just the ones that happen to 'exist' in the nominalistic sense (as little mounds of ink on paper). To merely say that *those instances of* [S6] *which happen to be written down are true* would not

[352] Putnam, *Logic*, 10-11.
[353] Putnam, *Logic*, 10.

> be to say that [S6] is valid; for it might be that there is a false substitution-instance of [S6] which just does not happen to exist.[354]

If we follow this line, we get all kinds of weird and counterintuitive results: the nominalist approach will make logic out to be empirical and *a posteriori*, rather than rational and *a priori*: the validity of logical schemata will then depend on our not having found counterexamples *so far*. The claim, then, that *modus ponens* is valid will be a tentative one, like the claim that a particular make of aircraft or seatbelt has a perfect safety record. We will never be able to declare with absolute certainty that *modus ponens* is valid: there might be some not-yet-actual and therefore undiscovered counterinstance out there, lurking in the land of *possibilia*. And, conversely, we will be free to endorse some rather dodgy inference schemata, provided no one attempts an invalidating substitution instance. So our endorsement of valid schemata will be tentative; and we will be free likewise to tentatively endorse invalid schemata, too.

Hence we have two problems for nominalism: first, that it ends up committing itself to abstract entities anyway, and so it is self-defeating. And second, that it renders logical findings empirical and tentative. Clearly these problems are intolerable. But are they problems for Buridan?

I don't think so. As can be seen, Putnam's objections are to a brand of nominalism that rejects abstract entities in favour of concrete ones. But Buridan's nominalism is quite ready to countenance abstract things; if anything exists outside the mind, however, it must be *singular*. So the prejudice in favour of the concrete that Putnam identifies as

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[354] Putnam, *Logic*, 13 (emphasis original).

distinctively nominalist is, in fact, nowhere in Buridan. Buridan, for his part, rejects *universals*, not abstract objects—*i.e.*, in Putnam's framing, "non-physical entities".[355]

Hence Putnam takes nominalism to be a rejection of the *abstract* in favour of the *concrete*. But Buridan rejects the *universal* in favour of the *particular*. For Buridan, any talk about universals deals strictly with mental generalisations, not with any real extra-mental universal things. And so for Buridan there is no problem talking about general classes of things, as in ($S6_R$), so long as it is clear that abstractions exist in thought and not in the world (a fact I will return to in a moment). Indeed if Buridan did not allow universals, we could no more generalise about donkeys or mammals than we could about logical schemata like *modus ponens*. But such generalisation is perfectly acceptable on Buridan's semantics and philosophy of mind: the mind has the job of abstracting universals from perceived particulars, and our language allows us to let common terms stand or supposit for (*supponere pro*) such things indifferently. What's important here is that these universals are *mental constructions only*. So long as we say the same about abstract terms like *schema*, *Barbara*, *modus ponens* and the like, we run into no special problems here. So the first problem is not a problem for Buridan.

Likewise, there is no problem talking about unrealised substitution instances, so long as these substitution instances are themselves singular. As we'll see in the next chapter, Buridan is willing to countenance a vast class of unrealised *possibilia* as possible singular objects. We can make universal claims about such objects, including that no uniform substitution of possible terms can invalidate a valid inference schema.

[355] Putnam, *Logic*, 14.

Hence Buridan can countenance the validity of a schema like (S6) as follows:

$S6_B$) For all possible *terms S, M, P:*

If all *S* are *M* and all *M* are *P*, then all *S* are *P*.

This is no more threatening to nominalism than the following claim:

P7) For every possible wine glass *G:*

If *G* were dashed against this rock, *G* would break.

A statement like (P7) tells us how certain possible things operate, in interaction with other objects. A rock breaks all glasses; a valid schema validates all substitution instances. Hence the Buridanian does not need to stick to concrete objects, as Putnam's objection to nominalism assumes; nor does the Buridanian have need any need for real abstract entities.[356] All that is necessary is to posit *possibilia* as singulars that can be quantified over. And this is just what Buridan does. So the second problem is likewise not a problem for Buridan.

And Buridan is not a nominalist in Putnam's sense at all, but a nominalist *qua* anti-realist about universals: when universal terms—terms like *animal* and *plant*—appear in natural language, they are subordinated to general concepts, so that the terms can

[356] Though to be sure, Buridan will allow that there are non-physical entities, such as God and angels.

stand for multiple things. But the things they stand for are merely singulars. As Buridan says, in the *Summulae de Predicabilibus*:

> I say that these terms *universal* and *predicable* are not the same thing, though they are said convertibly, since every predicable is universal, and vice-versa [...] For a term is called *predicable* on account of the fact that it is apt to be predicated of a subject, and *capable of being a subject* [*subicibile*] because it is apt to be the subject for a predicate in a proposition. But the same term is called *universal* on account of the fact that it signifies many things indifferently, and it is fit to stand for [*supponere pro*] many things, as has been said, without regard to serving as a subject or as a predicate. *Nor should we think that a universal term is in the terms subsumed under it*, unless we take 'being in' [*inesse*] for 'to be predicated truly and affirmatively, so that *to be predicated* and *to be in* do not differ, except in speech.[357]

Hence for Buridan, the question of universals comes down to a question of the semantics of universal terms. We should, accordingly, exercise caution when dealing with such universal terms: a term like *animal*, while truly predicated of all animals, nevertheless does not denote something universal *in* them. Thus, as Jack Zupko remarks:

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[357] "dico quod isti termini 'universale' et 'praedicabile' non sunt idem, sed dicuntur idem convertibiliter, sic quod omne praedicabile est universale, et econverso. [...] Nam relative dicuntur isti termini 'praedicatum' et 'subiectum', et isti etiam 'praedicabile' et 'subicibile'. Terminus enim ea ratione dicitur praedicabile qua innatus est praedicari de subiecto, et subicibile ea ratione qua innatus est subici praedicato in propositione. Sed idem terminus dicitur ea ratione universale qua indifferenter significat plura et innatus est supponere pro pluribus, sicut dictum est, sine respectu ad subici vel praedicari. Nec est credendum quod terminus universalis insit terminis sub se contentis, nisi accipiendo 'inesse' pro 'praedicari vere et affirmative', ita quod non different praedicari et inesse nisi secundum vocem." (*Summulae* 2.1.2; De Rijk, p.11-12, ll.9-23; emphasis added).

> For Buridan, nominalism is first and foremost a semantic claim about how to interpret universal signs or words.[358]

It is, in Zupko's apt characterisation, a philosophical and semantic *technique*.[359]

Returning to Putnam, then, we can see that the nominalism that his critique targets is quite different from Buridan's. Buridan rejects not abstract entities, but universal ones. Accordingly, I prefer to characterise Buridan's view as *anti-realist* about universals, rather than as *nominalist*. Granted, *nominalism* is the preferred term in the literature on John Buridan. But it is confusing: as we have seen, disparate views get grouped under the banner of nominalism. Consider, for instance, Hartry Field's characterisation of his nominalist view in *Science Without Numbers*: "Nominalism", he tells us, "is the doctrine that there are no abstract entities"—that is, nominalism posits only concrete things.[360] Such nominalism is clearly different from Buridan's anti-realism about universals, sketched above. Yet both views are often grouped under the heading of *nominalism*, as Rodriguez-Pereyra notes in his *SEP* entry "Nominalism in Metaphysics":

> Nominalism comes in at least two varieties. In one of them it is the rejection of abstract objects; in the other it is the rejection of universals.[361]

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[358] Jack Zupko, *John Buridan: Portrait of a Fourteenth-Century Arts Master* (Notre Dame, IN: University of Notre Dame Press, 2003), 52.
[359] *ibid.*
[360] Hartry Field, *Science Without Numbers: A Defence of Nominalism* (Princeton: Princeton UP, 1980), 1.
[361] Gonzalo Rodriguez-Pereyra, "Nominalism in Metaphysics", *The Stanford Encyclopedia of Philosophy* (Summer 2019 Edition), ed. Edward N. Zalta.

If Buridan's view can be called nominalist at all, it is in the latter, and not the former sense. Better, then, to be precise about what Buridan's view actually *is*, and characterise him as an anti-realist about universals, rather than as a nominalist.[362]

Putnam seems at one point to be aware that his objection against nominalism is half straw-man, and that other brands of nominalism might avoid the problems for the sort of nominalism he has in mind. But he dismisses these other brands. In a passage considering what a contemporary nominalist like Nelson Goodman would say in defense of nominalism, he notes that Goodman takes nominalism to be a restriction to singulars, not to concrete objects. Abstractions, on Goodman's nominalism, are admissible, so long as they are treated as singular—or, presumably, as universals with only mental existence. In this, Goodman looks at least superficially like Buridan, and likewise seems to avoid the problems Putnam sets out. Putnam is aware of this way out for Goodmanian nominalism, and his response is that this brand of nominalism (and, by implication, of Buridan) is both unrepresentative of nominalists in general, and anyway philosophically unmotivated:

> while the view that only physical entities (or 'mental particulars' in an idealistic version of nominalism; or mental particulars and physical things in a dualistic system) alone are real may not be what Goodman intends to defend, it is the view that most people understand by 'nominalism', and there seems little motive for being a nominalist apart from some such

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[362] As Calvin Normore has pointed out to me (personal correspondence, April 19, 2021), the term *nominalism* has the weight of several centuries behind it. I am willing to grant that, although so do many other terms, Latin notwithstanding. Take for instance a roughly contemporary writer in English: Geoffrey Chaucer, for whom *to starve* (*sterven*) is simply to die by any means, not necessarily hunger. Still, I am happy to stick with *nominalism*, provided it's borne in mind what we're talking about. After all, terms signify by convention, as Buridan would say.

view.[363]

The rebuttal, then, is twofold: first, Goodman's nominalism is not the True Nominalism, democratically defined as what most people identify with the view, or as the nominalism most recommended by doctors and pharmacists, or some such. And second, the question of whether anything in addition to singulars exists is simply uninteresting, and so too is Goodman's (and by extension, Buridan's) nominalism.

Considering the first rebuttal, one wonders who these 'most people' are: I can't see a typical man-on-the-street interview about nominalism meeting much success. And anyway, Putnam doesn't tell us *why* he thinks most people hold this view, or even why we should think most people are right. Hence this rebuttal is just a *argumentum ad populum*—with little empirical evidence about the actual beliefs of the *populum* itself, or even the correctness of those beliefs, to boot.

The second rebuttal does not apply to Buridan's thought, and perhaps even highlights a difference between his nominalism and that of Goodman: as I noted above, Buridan countenances many things in his ontology which are singular but non-physical. God's a good example. Here again, then, Putnam's challenge is aimed at a collection of views that are not Buridan's, even if they're described as nominalist.

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[363] Putnam, *Logic*, 16.

1.4. Propositional Complexity and a Problem for Schemata

As I noted at the outset of the present section (§1), Buridan includes *number* in his account of formal equivalence among propositions. Number, here, is the number both of terms in any proposition, and of proposition-like constituents in any hypothetical. And indeed, in at least some cases, this seems well motivated: consider the following pairs of propositions:

P8) Rose is a rose

P9) Rose is a rose is a rose is a rose.[364]

P10) I AM

P11) I AM THAT I AM.[365]

The form of the propositions clearly differs between (P8) and (P9), and (P10) and (P11). It seems that this aspect of form is what Buridan wants to account for with the inclusion of number in *TC* I.7.

But this insistence on number as an indispensable aspect of form also introduces a significant problem for embedding, as I noted above. For instance, consider the following two schemata:

S7) $\varphi \rightarrow \psi$

[364] Gertrude Stein, "Sacred Emily", *Geography and Plays* (Madison, WI: University of Wisconsin Press, 1993 [1922]), 187.
[365] Exodus 3:14.

$\sim\psi$

$\therefore \quad \sim\varphi$

S8) $((\varphi \vee \chi) \wedge \xi) \rightarrow (\psi \wedge \sim\chi)$

$\sim(\psi \wedge \sim\chi)$

$\therefore \quad \sim((\varphi \vee \chi) \wedge \xi)$

Both (S7) and (S8) are instances of *modus tollens*—(S7) is its most basic form, whereas (S8) is more complex. Accordingly, both schemata should have the same form. And yet they do not, by *TC* I.7, since they clearly have a different number of constituent parts. If we follow this line, then, there will not be one single form for *modus tollens*. Rather, there will be as many forms as there are embeddings—and so the cardinality of the set of forms of *modus tollens* (and of any other propositional schema: *modus ponens*, hypothetical syllogism, etc.) will be, by my tally, denumerably infinite.

If true, this will be disastrous. But is it how things stand? Consider again the two conditionals that figure in the above instances of *modus tollens*:

S2) $\varphi \rightarrow \psi$

S3) $((\varphi \vee \chi) \wedge \xi) \rightarrow (\psi \wedge \sim\chi)$

As conditionals, (S2) and (S3) are true just in case their consequents are not false while their antecedents are true. If we want to check, we can make a truth table. But when we assess the truth of the antecedent of (S3), we are assessing the truth of the whole thing—that is, the line in our truth table that will be immediately relevant to our assessment of the truth of the conditional will be the line under the main connective of the antecedent, namely '∧'. So the antecedent of (S3) is assessed as a whole proposition, as is the antecedent of (S2). In this way, then, (S2) and (S3) *do* contain the same number of propositions, at least where the truth of the whole conditional is concerned: both (S2) and (S3) have one antecedent, and one consequent. Because of the formal features of the different antecedents and consequents of the two, their truth will have to be assessed differently: in (S3), we will have to build it up out of more basic constituent parts. But when it comes time to assess the truth of the whole conditional, be it (S2) or (S3). we will be thinking of the truth of just two propositions: the antecedent and the consequent.

That said, Buridan does not provide much discussion of formal schemata for propositional logic at all, though he occasionally makes use of them, and though they figure in at least some of his derived rules for assertorics, set out in *TC* I.8. But Buridan's logic is not fully propositional; and on the whole, he is much more concerned with syllogisms and other structures of term-logic than with propositional schemata like *modus ponens*. Still, the Buridanian must address the problem raised in this section. And I think a solution along the lines I've sketched here is the best way to do that.

We've looked at Buridan's account of logical form; now let's turn to logical matter.

2. Material Validity: A Form of Matter

As we saw at the beginning of the present chapter, materially valid consequences hold in virtue of their significative—which is to say categorematic—parts. Materially valid consequences therefore do not hold across all instances of uniform substitution of these significative parts. Consider again the materially valid consequence examined earlier:

A2) A human is running

∴ An animal is running.[366]

Because of the meanings of its categorematic parts—*human*, *animal*, and *running*—(A3) is a valid consequence. Since every human is (necessarily) an animal, it is impossible for the antecedent to be true, and the consequent false. But we can substitute the categorematic parts of (A3), keeping its form intact, to give an invalid argument:

A6) A horse is walking

∴ Wood is walking.[367]

So although materially valid arguments are valid, their validity is *informal*: it does not depend on their structure or logical particles.

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[366] "Homo currit; ergo animal currit" (*TC* I.4.13).
[367] "Equus ambilat; ergo lignum ambulat" (*TC* I.4.14).

Why bother with material validity at all? Modern logic, for its part, does not—or, if it does, does so in the context of informal arguments, usually discussed at the beginning of 'baby logic' courses, or in introductory courses on critical reasoning ('pre-natal' logic?). But, as we will see, Buridan treats material validity quite seriously, and with good reason: materially valid consequences meet the same modal requirement for validity as their formal counterparts do (—as I argue in §2.1). Accordingly, what separates them from their formal counterparts is not a modal gap but an epistemic one: they are not always evident, and their evidentness can only be guaranteed in all cases by reduction to formally valid *consequentiae* (as I show out in §2.2). This emphasis on the modal rather than formal aspect of validity means that the distinction between form and matter need not be so hard and fast: as we'll see (in §2.3), it is at least sometimes completely arbitrary what we take to be fixed logical form, on one hand, and mutable logical matter on the other.

2.1. Validity by Matter: Meaning and the Unified Modal Requirement (UMR)

For some time it seemed to me that, while formal consequences are *simply* necessary, material consequences are probably necessary at some other (lower) grade—perhaps that of *de quando* necessity. I was wrong.[368] All *consequentiae*—both formal and simply-material (though not *ut nunc*, as I show later on in the chapter)—meet the very

[368] In fact, I presented a paper to this effect at the American Philosophical Association–Pacific in April, 2019. I take it all back now.

same modal requirement, namely, that of simple necessity (Grade I/1). In this section, I sketch the line of reasoning that compelled me to change my mind.

Let's call the claim that there is one single modal requirement that underwrites both types of consequence the *universal modal requirement* (*UMR* for short). I take two steps to establish the UMR: first, I examine textual evidence from Buridan. Second, I argue that abandoning the UMR forces us to take two positions that are, for the Buridanian view, untenable. In a nutshell, if we abandon the UMR, we have to abandon the signification requirement set out in the definition of logical consequence (*TC* I.3). Thus the UMR is an integral part of Buridan's account of logical consequence, and of the formal-material divide. Yet the UMR has significant implications for Buridan's logic and metaphysics, as we will soon see.

2.1.1. Textual Evidence for the UMR

There is both direct and indirect textual evidence for the UMR. Indirectly, we can infer from the order of presentation of *consequentiae* in the *TC* that there is one modal requirement for both: Buridan *first* discusses the modal requirement in his definition of consequence (in *TC* I.3, outlined in the preceding chapter), and *only then* presents its division into material and formal consequence (in *TC* I.4). If formal and material consequence were different sorts of consequence, it would make far more sense to present the division first, and then discuss the different ways these irreducible types of consequence meet their separate modal criteria. It would be misleading to present the division and the definition in the order Buridan gives us: the presentation should be the other way around. But Buridan does no such thing. Nor need he, precisely because of the UMR: there is only one modal requirement for formal and simple-material consequence, and that is why it makes sense to present the modal criterion first, and the division second.

There is also direct textual evidence for the UMR. Perhaps the clearest and best statement of the UMR is in Buridan's discussion of the similarities and differences of formal material consequence in the *Summulae de Propositionibus* (1.6.1). Concerning the similarities between the two, Buridan tells us it comes down to the modal requirement:

> And because we have just now spoken of formal and material consequence, we should investigate the ways in which they are similar, and the ways in which they are different. *And they are similar in this respect: that it is impossible for the antecedent to be true, while the consequent, supposing it exists, is false.*[369]

Buridan's view is clear: there is no difference between the modal requirement for formal validity and that for material validity. Rather, there is one, universal modal requirement for both. And if formal and simple-material consequences agree on the modal requirement, then the modal requirement must be univocal. This is unsurprising, since Buridan's fundamental interest is in truth-preservation, and the UMR is what guarantees that.

But we don't just have to take Buridan's word, direct or indirect, for it. Indeed, there is a better reason to adopt the UMR: if we abandon it, things fall apart.

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[369] "Et quia nunc locutum est de consequentia formali et materiali, uidendum est quo modo conveniant et differant. Conveniunt enim in hoc quod impossibile est antecedens esse verum consequente exsistente falso." (*Summulae* 1.6.1; van der Lecq, p.60, ll.18-20; emphasis added).

2.1.2. Rational Evidence for the UMR

Let's suppose, for *reductio*, that there is no UMR for formal and material consequence. Having established that formal validity holds at the level of simple necessity (Grade I/1), we might for example hold that material validity is necessary by one of the other grades of modality (Grades II-IV or 2-3). Note that the only other grades available to us are *lower*: Grade I/1—unfalsifiability even by God—is the highest on Buridan's scale. There is no stronger notion of hyperintensional containment or relevance, over and above Grade I/1. So if there is no UMR, and material consequence is still necessary in some sense, then it must be at a lower grade on the scale. But if we subtract the UMR, we face a major problem: the signification requirement, set out in the definition of logical consequence in Chapter 2, becomes untenable.

Here is why: Buridan thinks materially valid consequences can be reduced to formal ones. A materially valid consequence, he tells us:

> Is reduced [*reducitur*] to a formal one by the addition of a necessary proposition or propositions. The addition of these to the assumed antecedent renders the consequence formal. So for instance if I say "A human runs; therefore an animal runs", I will prove the consequence through the fact that every human is an animal. For if every human is an animal, and a human runs, then it follows by a formal consequence that an animal runs.[370]

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[370] "Reducitur autem ad formalem per additionem alicuius propositionis necessariae vel aliquarum propositionum necessariarum quarum appositio ad antecedens assumptum reddit consequentiam formalem. Vt si dico 'homo currit; ergo animal currit' probabo consequentiam per hoc quod omnis homo est animal; nam si omnis homo est animal et homo currit, sequitur formali consequentia quod animal currit" (*TC* I.4; Hubien, p. 23, ll.16-22).

I will discuss this passage in greater detail in §2.2, below. For now, we just need the simple example Buridan provides here. Consider the following enthymeme, which is materially valid:

A7) A human runs
∴ An animal runs.[371]

We can turn a materially valid consequence like (A7) into a formally valid one, by the addition of a necessarily true proposition as an additional premiss.

A7′) A human runs
Every human is an animal
∴ An animal runs.

But what is going on when we shift from the simply materially valid (A7) to the formally valid (A7′)? If we reject UMR, then it follows that there is a modal gap between (A7) and (A7′). By making (A7) formal, we effectively leap over this gap. What warrants such a leap? It can only be the added premiss ("Every human is an animal"), which alone accounts for the difference between the two.

So the modal difference between (A7) and (A7′) depends on the addition of a necessarily true premiss. Now in this modal leap, either something changes in the things signified, or something changes in the necessity of the truth of the combined premisses.

[371] "homo currit, ergo animal currit" (*TC* I.4; Hubien, p.23, ll.19-20).

But nothing changes in the things signified—that is, the new premiss does nothing to affect the fact that a running human is a running animal. So the modal leap must depend, not of the things signified, but of the relative necessity of the truth of the premisses: the truth of the premisses in (A7′) *necessitates* the conclusion in a stronger way than the truth of the lone premiss in (A7). That is, beyond merely making the necessary connection obvious or evident (process we'll explore in greater detail in § 2.2, below), the addition of the premiss changes the modal status of the whole consequence.

But if we take this approach, we abandon another supporting pillar of Buridan's definition of *consequentia*: the signification requirement (set out in Chapter 2, § 2, above). The signification requirement is, recall, that a consequence is valid *not* when the premisses can't be true without the conclusion, but when things cannot be as the premisses *signify* without being as the conclusion signifies. Since the premisses change nothing about the way the things signified *are*, it is impossible that the addition of new, consistent premisses should allow us to cross a modal gap like the one we supposed exists between (A7) and (A7′).

Therefore, if we want to keep the signification requirement, we have to admit that the added premiss does not render any change in the things signified, and so there is no modal gap between (A7) and (A7′). And this seems to be precisely Buridan's view: (A7) is not more *necessary* than (A7′), but it can be more *evident*, since it reduces the relationship between the premiss and the conclusion to a recogniseable syllogistic form (namely Disamis). To recognise its validity, then, we can rely on our knowledge of syllogisms, not any prior knowledge of the subject matter and the signification of the terms involved (more on this in §2.2).

In sum, we cannot reject the UMR without abandoning one of the three pillars of the definition of consequence set out in the *Tractatus de Consequentiis* (I.3) and discussed in chapter 2—namely, the signification requirement. This is much too high a price to pay. We should, rather, endorse the UMR: it has ample support, both in the text itself, and in the light of natural reason.

The UMR does, however, have some striking philosophical ramifications for Buridan's metaphysics and logic. Let's look at those now, by way of a conclusion to this section (§3.1).

2.1.3. The Metaphysical and Logical Implications of the UMR

From the foregoing, we can conclude that both formally and simply materially valid consequences meet the *same* modal criterion—namely, Grade I/1. A proposition necessary at Grade I is *simply* necessary, and cannot be falsified by any power, even God. This means that God can no more invalidate a formally valid syllogism in the mood Barbara than a materially valid consequence like the one considered above:

A2) A human is running

∴ An animal is running.[372]

[372] "homo currit, ergo animal currit" (*TC* I.4; Hubien, p.23, ll.19-20).

A consequence like (A2) meets the UMR because God can't separate the essence of being an animal from any human, running or otherwise: to do so would make the human no longer a human—that is, it would annihilate the human's essence. So there is no way to render the antecedent true but the consequent false, and (A2) is therefore a simply materially valid consequence. And that is how (A2) meets the same modal criterion as a formally valid consequence.

So much, then, for essences, which cannot be separated from a subject without annihilating it. But Buridan frequently treats consequences dealing with *propria* or inseparable accidents as materially valid as well. To clarify, I here want to contrast *propria* with essences. An essence accounts for what a thing *is*—that is, what is essential to it. It therefore plays an indispensable rôle in defining the thing: for example, humans are essentially rational animals. *Propria*, on the other hand, are found in all and only the things contained under one species, but are not essential to them. As Buridan tells us, something is called a *proprium* when it

> is present in [*inest*] all and only the things contained under a species, and always, as for instance *capable of laughter* applies to humans.[373]

Capable of laughter [*risibilis*] is the standard example of a *proprium* for humans, as *capable of neighing* [*hinnibilis*] is for horses: all and only humans are capable of laughter, and this is a *proprium* or inseparable accident of them.

[373] "inest omni et soli et semper, ut homini esse risibile" (*Summulae* 2.5.1; de Rijk, p.42, ll.9-10).

What's important here is that Buridan treats these *propria* or inseparable accidents as having the same (modal) footing as essences—that is, they are just as much bound up in their subjects as essences are. For instance, consider the following example from the *Summulae de Locis Dialecticis*:

A8) A human runs
∴ Something capable of laughter runs.[374]

Does the necessity of a consequence that tuns on a *proprium*, like (A8), meet the same modal requirement as one like (A2), that turns on an essence?

It seems so. Recall the modal scale from the *Summulae de Demonstrationibus*, set out in the preceding chapter:

Grade 1: simple necessity

e.g. "God exists"

Unfalsifiable by any power, including divine

Grade 2: nomological necessity

e.g. "The heavens move"

Unfalsifiable by any natural power
Falsifiable by divine power

Grade 3: *de quando* necessity

e.g. "Socrates is a human"
"Socrates is capable of laughter"

[374] "homo currit; ergo risibile currit" (*Summulae de Locis Dialecticis* 6.3.5; Green-Pedersen, p.38, l.5).

Falsifiable by divine or natural power, but only by the annihilation of the subject term

Grade 4: necessity by restriction

e.g. "Aristotle walked"

Once contingent, but now unfalsifiable by any power, including divine

Notice that, under Grade 3, Buridan groups the following two propositions:

P12) Socrates is a human.[375]

P13) Socrates is capable of laughter.[376]

The predicate of (P12) is an essence of Socrates, whereas the predicate of (P13) is a *proprium*. Yet Buridan treats both as modally identical. Thus we cannot make Socrates non-human, or incapable of laughter, without annihilating his nature. Nor, conversely, can we find humanity or capability of laughter in the absence of a human. Modally, the two go hand-in-hand.

Now although categorical propositions like (P12) and (P13) are at Grade 3, when they are used as the antecedent of consequences, the whole consequence *qua* hypothetical proposition is necessary at Grade 1. Here is why: recall the definition of consequence (D3), discussed in the preceding chapter:

[375] "Socrates est homo" (*Summulae* 8.6.3; de Rijk 141, l.23)
[376] "Socrates est risibilis" (*Summulae* 8.6.3; de Rijk 141, ll.23-4)

D3) One proposition is antecedent to another which is related to it (*se habet ad illam*) in such a way that it is impossible that things should be as the former signifies, and not be as the latter signifies, when they are formulated at the same time (*simul propositis*).[377]

Now the only way to separate Socrates' animal-hood or capability of laughter from him is by destroying Socrates himself. But if Socrates does not exist, then any affirmative proposition whose subject term is *Socrates* will be false, since all affirmatives have existential requirement. But then any case in which Socrates is no longer human or capable of laughter is a case in which both the antecedent and the consequent of consequences like the following are false:

A9) Socrates is a human
∴ Socrates is an animal (essence)

A10) Socrates is a human
∴ Socrates is capable of laughter (proprium)

Hence there is no power capable of making the antecedent of (A9) or (A10) true, and the consequent false, since removal of humanity or capability of laughter from Socrates is sufficient to destroy his human nature.

[377] "Illa propositio est antecedens ad aliam quae sic se habet ad illam quod impossibile est qualitercumque significat sic esse quin qualitercumque illa alia significat sic sit ipsis simul propositis" (*TC* I.3.48-51; emphasis added).

Therefore, it is impossible for the antecedent either of (A9) or (A10) to be true, while the consequent is false. Thus both consequences are on equal modal footing. And, at least where hypotheticals are concerned, the modality of Grade 3 collapses into Grade 1. (And, as we'll see in §3, below, something similar holds for Grade 4 and Grade 2). Thus, Buridan treats them as modally and therefore deductively equivalent: there is, on Buridanian logic, no difference between (A9) and (A10), since both of them meet the UMR.

Let's close §2 with a final question: given the UMR, there is no modal gap between formal and material consequence. So what *are* we doing when we reduce material consequences to formal ones?

2.2. Making Material Consequences Formal

As we have seen, rendering a material consequence formally valid, by the addition of a suppressed premiss, does not alter its modal status, per the UMR. But then what changes when we render a material consequence formal? According to Buridan, a material consequence that has been rendered formal has been made *evident*:

> It seems to me, moreover, that no material consequence is **evident** (*evidens*) in inference except by its reduction to a formal one. And a material consequence is reduced to a formal consequence by the addition of a necessary proposition or propositions, whose addition to the antecedent

> renders the consequence formal. For instance, if I say "A human runs, therefore an animal runs", I will prove the consequence by the fact that every human is an animal. For if every human is an animal, and a human runs, then it follows by a formal consequence that an animal runs.[378]

What does it mean to render a consequence evident? The gap between formal and material consequence is not modal, but *epistemic*. A materially valid consequence can only be readily recognised to be valid by someone who has some prior knowledge of the subject matter—that is, by someone who already knows about the things the terms stand for. For instance, consider the following enthymeme, which is by no means evident—at least to the average person on the street:

A11) Every Bouvier is a herder

∴ Some pointer isn't a Bouvier

The premiss guarantees the conclusion, but the relationship between the two is not at all evident to anyone with no prior familiarity with dog types and breeds. We can, however, make the path from the antecedent to the consequent in (A11) evident, by tracing it through the syllogistic moods Camestres and Felapton:

[378] "Et videtur mihi quod nulla consequentia materialis est evidens in inferendo nisi per reductionem eius ad formalem. Reducitur autem ad formalem per additionem alicuius propositionis necessariae vel aliquarum propositionum necessariarum quarum appositio ad antecedens assumptum reddit consequentiam formalem. Ut si dico 'homo currrit; ergo animal currit', probabo consequentiam per hoc quod omnis homo est animal; nam si omnis homo est animal et homo currit, sequitur formali consequentia quod animal currit" (*TC* I.4; Hubien, p.23, ll.14-22; emphasis added).

A11′) Every Bouvier is a herder

No Setter is a herder

∴ No Setter is a Bouvier (Camestres)

Every Setter is a pointer

∴ Some pointer isn't a Bouvier (Felapton)

With (A11′) before us, we don't need any prior knowledge of dog breeds in order to see that the consequent of (A11) follows from its lone antecedent.[379]

Why is this important? According to Buridan, the purpose or end (*finis*) of logic is the acquisition of scientific knowledge through demonstrations.[380] And demonstrations are formally valid[381]—indeed, they have to be, since a materially valid consequence can only be seen to follow when we have prior knowledge of the subject matter. If we needed prior knowledge of the subject matter in order to recognise the validity of an argument whenever we encountered it, then logic would depend upon the sciences. But logic serves the sciences, too. And so we would wind up in a vicious circle. Form, then, serves an epistemic role: it allows us to extend our knowledge into hitherto unknown subjects, where the relations among the things terms stand for is not immediately clear. Seen in this light, syllogistic and the other formal rules (equipollence, conversion, etc.) are a sort of scaffolding that allow us to build our way up from what we already know.

[379] Of course, we cannot tell whether the consequent of (A15′) is *true* on the basis of its form alone, but only that the argument itself is *valid*. The truth of the supplied premisses may still be called into doubt—until, anyway, they are reduced to definitions or first principles. Accordingly, (A15′), while evident, is not yet a demonstration.

[380] *Summulae* 8.1. Cf. also Buridan's preface to the *Summulae*.

[381] *Summulae* 8.4.2.

Marko Malink argues that it is because the Prior Analytics makes explicit all the premises involved in a deduction explicit, whereas the Topics does not, that we identify the birth of formal logic with the former, and not with the latter. In his characterisation:

> for the purposes of the investigation undertaken in the *Prior Analytics*, everything in the premisses and conclusion that is relevant to an argument's counting as a deduction needs to be made explicit by some linguistic expression so that nothing of relevance is left to tacit understanding between speaker and hearer.[382]

This concern is precisely the one I have tried to motivate with the Bouvier syllogism, above. I think it is precisely why Buridan's language about reduction of materially valid to formally valid consequences has a distinctively epistemic flavour: to make an argument formal, we reveal all the elements at play in it. Formal reasoning is perfectly transparent reasoning.

Now it might be objected that there are materially valid consequences which cannot be reduced to formally valid ones, even if the examples considered just now don't make the cut. Many consequences *ex impossibili* or *ad necessarium* are not obviously reducible to formal consequences at all. For instance, consider the following:

A12) God doesn't exist
∴ A stick stands in the corner

A13) A stick stands in the corner

[382] Marko Malink, "The Beginnings of Formal Logic: Deduction in Aristotle's *Prior Analytics* vs. *Topics*", *Phronesis* 60(3) (2015), 284.

∴ God exists

Any argument from an impossible premiss, like (A12), or to a necessary conclusion, like (A13), will hold no matter what the propositions involved are about. Hence there is no relevance between the premisses and conclusion of (A12) and (A13).[383] Accordingly, there will be no readily available middle term(s) to make these materially valid arguments into formally valid ones. So it looks like there are materially valid consequences which are not reducible to formal ones after all.

Not so fast. It turns out Buridan thinks (A12) and (A13) can be rendered formally valid, as well. How? In *TC* I.8, concl. 7, Buridan gives us instructions for making *ex impossibile* arguments like (A12) formal. These hinge on the following rule:

> From any conjunction of two propositions which are mutually contradictory, any conclusion follows, indeed by formal consequence.[384]

Buridan's example is that 'a stick stands in the corner' follows from 'every B is A and some B isn't A'. Here is the proof in outline:

A14) Every B is A and some B isn't A

[383] And here we can reject, in passing, Storrs MacCall's attempt to reduce the history of logic, and especially of fourteenth century logic, to a history of connexivity. Any logic that will countenance arguments like (A17) and (A18) will, of course, not be connexive. See his "A History of Connexivity: Two Thousand Three Hundred Years of Connexive Implication", *Handbook of the History of Logic: Vol. 11: Logic: A History of its Central Concepts*, ed. Dov Gabbay, Francis Jeffry Pelletier, and John Woods. (Waltham, MA: 2012), 415-49.

[384] "Ad omnem propositionem copulativam ex duabus invicem contradictorius constitutam sequi quamlibet aliam, etiam consequentia formali." (*TC* II.8, concl.7; Hubien, p.36, ll.161-3).

	Every B is A	(simplification)[385]
	Every B is A or a stick stands in the corner	(addition)
	Some B isn't A	(simplification)
∴	A stick stands in the corner.[386]	(modus tollendo ponens)

Now (A14) gives us a framework to show that any *ex impossibile* argument, including (A12), is formal. For, as he tells us:

> From the aforementioned conclusion, it is clear that every consequence with an impossible antecedent is reduced to a formal consequence by the addition of a necessary proposition. For, if the antecedent is impossible, then its contradictory is necessary, and with the addition of this necessary proposition the consequence will formally entail anything.[387]

This gives us the recipe to torun (A12) into a formal consequence, as well. All we need to do is add the necessary premise that God exists, and run through an inferential pattern like that of (A14):

[385] I'm here using the modern terms for these operations; Buridan doesn't name them all here, and some of the terms he does use for them will be less familiar to us (e.g. *Locus from Division* for *modus tollendo ponens*). In any case, he explicitly endorses all the rules set out here.
[386] "Probatio. Pono, gratia exempli, quod sequatur: 'Omne B est A et quoddam B non est A; ergo baculus stat in angulo'. Quia sequitur: 'Omne B est A et quoddam B non est A; ergo omne B est A' quia ad copulativam sequitur quaelibet eius pars. Deinde sequitur: 'Omne B est A; ergo omne B est A vel baculus stat in angulo' quia ad quamlibet sequitur ipsamet sub disiunctione ad quamlibet aliam. Tunc ex ista et secunda parte primi antecedentis arguam sic: 'Omne B est A vel baculus stat in angulo; et quoddam B non est A; ergo baculus stat in angulo' (*TC* I.8, concl. 7; Hubien, p. 37, ll.169-77).
[387] "ex dicta conclusione apparet quomodo omnis consequentia ex antecedente impossibili reducatur ad consequentiam formalem per additionem alicuius necessariae. Quia si antecedens est impossibile, suum contradictorium est necessarium, quo sibi addito erit consequentia formalis ad quodlibet, ut dictum est." (*TC* II.8 concl. 7; Hubien, p.37, ll.191-5).

A12′) God doesn't exist and God exists

God doesn't exist or a stick stands in the corner

God exists

∴ A stick stands in the corner

Hence even arguments *ex impossibile*, which often have no clear relation between antecedent and consequent, nevertheless can all be reduced to a formally valid consequence.

How do things stand with arguments whose consequents are necessary, like (A13)? Buridan's account here is more terse:

> One wonders whether every consequence whose consequent is necessary can also be reduced to a formal one, for instance 'a donkey runs, therefore God is just'. And I say that this consequence should be reduced to a consequence with an impossible antecedent, by way of the third conclusion [namely, contraposition]. For it follows that 'no God is just, therefore no donkey runs', and so on.[388]

It thus seems Buridan would have us cash out (A13) as follows, where the first move gets us the contrapositive:

[388] "Sed iuxta hoc aliquis dubitabit quomodo etiam omnis consequentia cuius consequens est necessarium reducatur ad formalem, uerbi gratia: 'asinus currit; ergo deus est iustus'. Dico quod haec consequentia reducetur ad consequentiam de antecedente impossibili, per tertiam conclusionem. Sequitur enim: 'nullus deus est iustus; ergo nullus asinus currit' ergo sequitur etc." (*TC* I.8, concl. 7; Hubien, p.38, ll.207-12).

A13′) A stick stands in the corner, therefore God exists

God doesn’t exist, therefore a stick doesn’t stand in the corner

The consequent of (A13′) is then proved by running it through the same pattern of reasoning as that of (A12′), above:

A13′′) God doesn’t exist

God exists and God doesn’t exist

God doesn’t exist or a stick doesn’t stand in the corner

God exists

∴ A stick stands in the corner

In sum, (A13′′) is the formally valid contrapositive of (A13), by the formally valid (A13′). And there you have it: consequences whose simple material validity depends solely on the necessity of their consequents can nevertheless be reduced to formally valid arguments.

Let me close this section with one final consideration of what can go on in the reduction of consequences, which will anticipate the discussion of consequences which are valid *ut nunc* (§3, below). In most of our examples of simply materially valid consequences, the UMR has held because they dealt with natural necessities. But how are these like what we consider to be nomological necessities nowadays? Take an inference like the following:[389]

[389] This section is mainly in response to a suggestion from Calvin Normore (personal correspondence, April 18, 2021). The example (A15), in particular, is his.

A15) γ is a photon travelling in a vacuum

∴ γ is travelling at nearly 300,000km/second

Now the antecedent of (A15) is nomologically necessary, but not logically necessary: there is no logical contradiction, for instance, in a novel in which γ is a photon that travels at the speed of sound. So the consequent could be falsified, in a way that would leave the antecedent intact. Hence, (A15) does not meet the UMR, though it looks materially valid.

Stranger still, suppose we made (A15) formally valid, by supplying a premise:

A15′) γ is a photon travelling in a vacuum

All photons travelling in a vacuum travel at nearly 300,00km/second

∴ γ is travelling at nearly 300,000km/second

Now it seems that (A15′) *does* meet the UMR. What happened?

For one thing, the natural necessities in Buridan's examples are *per se* and therefore essential: one cannot, on his view, alter a human into a non-animal without annihilating the human. Thus *per se* predications are *de quando* necessary, in a way like the constants of modern physics are not. As for these latter constants, we seem to have two options: either to claim that their falsification entails a contradiction, and so conflate logical with nomological necessity; or admit that they hold at a lower grade of necessity, namely that of *ut nunc*, albeit of a sort which remains constant. I take the latter approach, and I will examine these 'permanent *ut nunc*' arguments in § 3, below.

Before I conclude this section on material consequence, I want to consider one final and remarkable implication of the UMR: since formal and material validity meet the same modal requirement, the difference between them is merely epistemic. Therefore, it is somewhat arbitrary how we distinguish the formal elements from the material ones. This fact, we will see, delivers the *coup de grâce* to the Clean Divide View discussed in the preceding chapter: in at least some instances, it is arbitrary how we divide form and matter in a consequence.

2.3. Matter or Form? Depends How You Slice It

Since formal and material consequences are equally underwritten by the UMR (as we established in §§ 2.1.1-2.1.3, above), formal and material validity on Buridan's logic are much more closely linked than they are in its modern counterpart(s). Accordingly, on Buridan's logic, it is sometimes arbitrary how we divide the two, especially when it comes to borderline cases. This is a significant difference between Buridan's account and modern accounts of validity in the Tarskian mould: the latter depend heavily on a prior notion of logical constants and form, as we will see. But for Buridan, the formal–material (and logical constant–inconstant) divide is more arbitrary, and this arbitrariness does not affect his account of logical consequence. Put briefly: for moderns, the definition (or anyway demarcation) of logical constants is upstream from consequence; for Buridan, it is downstream, and therefore less significant in his whole logical programme.

Nowhere is the relative arbitrariness of the formal–material divide more apparent than in Buridan's discussion of the rôle of term negation in syllogisms in *Summulae de Syllogismis*. The question is, when syllogising with negated terms like *non-animal*, should we treat the negations like *non-* as part of the matter of the syllogism, or as part of its form? For instance, consider the following syllogism:

A16) No non-animal is a human
No non-substance is an animal
∴ No non-substance is a human.[390]

Brief consideration is enough to show that (A16) is valid, since the antecedent propositions cannot be true without the consequent. But do we take term negation to be intrinsically bound to the terms to which it applies, and so substitute the whole term, negation and all, with a single variable? If so, (A16) will have four term variables, and will look like this:

S9) No A is B
No C is D
∴ No C is B

[390] "nullum non animal est homo, nulla non substantia est animal; ergo nulla non substantia est homo" (*Summulae* 5.9.2).

Although (A16) is valid, the schematic form that bundles its term negations in with the terms is not formally valid, as a quick glance at (S9) is enough to show. Here, for example, is an invalid substitution instance of (S9):

A17) No human is a donkey
No animal is a stone
∴ No animal is a donkey

Still, to be sure, the argument itself, (A16) *is* valid—albeit materially so.

Conversely, we can separate the term-negations from the terms, and pack them into the syllogistic form. If so, (A16) will have three term variables, and will look like this:

S10) No non-A is B
No non-C is A
∴ No non-C is B

In contrast with (S9), (S10) *is* formally valid: it holds in all substitution instances of the variables A, B, and C.

The question, then, is how we ought to formalise syllogisms that incorporate negative terms, the way (A16) does: should we bundle up the negation with the terms, producing a materially valid argument that follows a formally invalid schema—as we did with (S9)? Or should we separate out the negation from the terms, and pack it into the syllogism's form, producing a formally valid argument and schema—as we did with

(S10)? Unexpectedly enough, Buridan's response is we can take them to be formal, or material, depending how we carve them up:

> The first question is whether the aforementioned consequences [namely, (S9) and (A17)] are formal, since they hold in all matter—in which there is no ampliation—when the form is retained. And I respond that if you wish to call the subjects and predicates of these syllogisms their *matter*, and to take the rest of them along with their order as pertaining to form, then it is clear that the aforementioned consequences are not formal [...] I say, however, that if we call only the finite terms in syllogisms of this sort their matter, so that we take the infinitising negations and the rest to pertain to the form of these syllogisms [as in (S10)], then syllogisms or consequences of this sort should be called *formal*, because they hold in every matter so long as the syllogistic form is preserved.[391]

Hence what we take to be the form, and what we relegate to matter, is at least sometimes more or less arbitrary. And Buridan seems unconcerned: they *can* be considered formal, but it doesn't matter much either way:

> Thus these consequences can be called *formal*, since when the same number of purely categorematic terms are retained, and with the addition of similar

[391] "Tunc ergo est prima dubitatio utrum praedictae consequentiae sic sint formales quod valeant in omni materia in qua non est ampliatio forma eadem reservata. Et ego respondeo quod si materiam horum syllogismorum tu vis vocare subiecta et praedicata propositionum, et solum residuum cum ordine pertinere ad formam, manifestum est quod praedictae consequentiae non essent formales. [...] Sed ego dico quod si materiam huiusmodi syllogismorum vocamus solum terminos categorematicos finitos, ita quod negationes infinitantes cum residuo et ordine diceremus pertinere ad formam horum syllogismorum, tunc huiusmodi syllogismi, seu consequentiae, deberent dici formales, quia similiter tenerent in omni materia forma syllogismi reservata." (*Summulae de Syllogismis* 5.9.2)

> syncategorematic terms in a similar arrangement [*ordo*], the aforementioned consequences will be valid in the same way, no matter how the purely categorematic terms are changed.[392]

Hence whether we formalise the term-negation in (A16) is not all that important. While Buridan's approach here might seem initially surprising, it is well in keeping with his definition of logical consequence: whether it's formally or materially valid, (A16) meets the UMR. A reduction of (A16) to the formally valid schema (S10) is therefore not undertaken out of concern for the modal status of (A16), but merely its *evidentness*. If a syllogism with negated terms like (A16) is valid but not evident, or if we are in doubt as to its validity, we should reduce it to a formally valid schema like (S10).

From a modern perspective, this discussion in the *Summulae de Syllogismis* is both familiar and surprising. Familiar, because Buridan seems ready to accept that what counts as logically constant, and what on the other hand gets packed into logical matter or inconstants, is at least sometimes arbitrary.[393] Buridan is, then, not so far from thinkers like John MacFarlane and others, who think the selection of logical constants is somewhat arbitrary. And so again, contrary to the Clean Divide View we find in John MacFarlane (which I discussed in Chapter 3), medieval thinkers do not always have a clear-cut distinction between logical constants and inconstants, and form and matter. Here we can see that Buridan does not, and so can treat such distinctions as somewhat arbitrary.

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[392] "Sic ergo possunt vocari consequentiae formales, quia retento eodem numero dictionum pure categorematicarum et additione similium syncategorematum et simili ordine, dictae consequentiae erunt similiter bonae, quantumcumque mutentur termini pure categorematici" (*Summulae de Syllogismis* 5.9.2)

[393] Recall Quine's famous dictum, borrowed from Adolph Meyer: "Where it doesn't itch, don't scratch", *Word and Object* (Cambridge, MA: The MIT Press, 1960), 160

But the above passage is also surprising from a modern perspective, because for Buridan, identifying the form of a given argument is not so all-encompassingly important as it is for us moderns. Formality is, following Tarski, the central notion in the concept of logical consequence. Hence we need a clearly demarcated class of logical constants to account for consequence in either the semantic or the syntactic sense. What follows is a brief account why. But first,

a brief review of Tarski's semantic account of logical consequence: the semantic account of logical consequence depends on a well-demarcated class of logical constants. On this semantic account, a set of sentences A entails a sentence C just in case every model $\mathfrak{M}$ of A is a model of C. A model $\mathfrak{M}$ for a sentence x (∈ X) is a structure ⟨D, I⟩. D is the domain (basically, the set of things under discussion), and I is an interpretation function, which assigns a constant to each element d of the domain D. Suppose D is non-empty. Then consider the following simple formal schema:

S11) $\exists x\,(\mathrm{F}x \wedge \mathrm{G}x)$
∴ $\exists x\mathrm{F}z$

Now suppose that $(\mathrm{F}d \wedge \mathrm{G}d)$ is true of some $d \in \mathrm{D}$. Then $\mathfrak{M} \vDash (\mathrm{F}d \wedge \mathrm{G}d)$. By this fact, and the definition of '∧', it follows that Fd is true on $\mathfrak{M}$, and

> therefore that $\mathfrak{M} \vDash \mathrm{F}d$. But this will hold of any model $\mathfrak{M}$. Hence any model of $(\mathrm{F}d \wedge \mathrm{G}d)$ is a model of $\mathrm{F}d$. And so $\exists x\, (\mathrm{F}x \wedge \mathrm{G}x)$ entails $\exists x \mathrm{F}z$.

The foregoing is a statement of these facts, not a proof of them. The point is that, in the semantic account of logical consequence, we had to appeal to the structure of the propositions involved—specifically, to their logical constants. The foregoing was a relatively simple case; but in any case, semantic validity will depend on logical form.

Second, for the syntactic definition, the same will be even more obviously true. On Tarski's deductive theoretic account of logical consequence, A entails C just in case there is a proof of C from A in a deductive system. And any formal deductive system will be, well, formal: it will rely on some elements of the sentences at play that are recognised as logically constant. So here, too, there is no room for material consequence.

So it is initially quite familiar to see that Buridan is a conventionalist about logical constants, the way we are, too. In fact, Buridan is perhaps *more* of a conventionalist than we are. For this reason, a sense of familiarity might give way to alarm when we see that Buridan is even more conventionalist than we are: determining what the logical constants are, and therefore what logical form is, is dispensable. We do not need it to give an account of logical consequence. Rather, for Buridan, the UMR is key: in both formal and material consequence, it is impossible that the antecedent be true without the consequent. That is, there is no power, divine or otherwise, capable of rendering the antecedent true and the consequent false.

Hence we have to be careful not to overstate the role of form in Buridan's logic. Similarly, we have to be clear that determining what pertains to form is somewhat arbitrary. Even so, it is exciting to note that Buridan, like us, thinks of form in terms of substitution of non-logical terms, and seems therefore to be quite modern. For instance, Tuomo Aho in the *Encyclopedia of Medieval Philosophy* ("Consequences") tells us:

> The idea of logical form was fully developed in Buridan's logic. According to him, the form of a proposition consists of its structure of syncategorematic elements and the distribution of categorematic elements. And as he defines it, ''a consequence is formal if any proposition with similar form would, when stated, be valid.''[394]

This is true, though it does not address the arbitrariness of at least some elements pertaining to form, which we've been considering here. Deciding in favour of counting term negation as form, as in the *Summulae de Syllogismis* (5.9.2), does not determine the syncategorematic status of *non-*. So form is not entirely determined by syncategorematic terms here, either. In a way, then, Buridan is more like us than we have acknowledged up to now, largely because of the attractive picture of the Clean Divide View. In fact, he faces familiar problems of demarcation for non-logical terms.

Buridan's treatment of term-negation in the *Summulae de Syllogismis* further undermines the fourth claim of the Clean Divide View, set out in the preceding chapter:

Claim IV: Syncategorematic terms only determine propositional form; categorematic terms only pertain to propositional matter.

[394] Tuomo Aho, "Consequences, Theory of", *The Encyclopedia of Medieval Philosophy*, ed. Henrik Lagerlund (Dordrecht: Springer, 2011), 232.

Term-negation is, of course, syncategorematic: it signifies nothing on its own. And yet the rôle it plays in determining propositional form is, at least sometimes, ambiguous. This severely undermines Claim IV, which requires a strict demarcation of syncategorematic and categorematic terms, and likewise of propositional form and matter. Let's therefore leave the Clean Divide View, and turn to the final branch of the above taxonomic tree (fig. 4.1, above): consequences that are not strictly speaking (*simpliciter loquendo*) valid, but valid *as of now* (*ut nunc*).

3. Validity *Ut Nunc* and the Necessary Present

The UMR sets a high bar, and much—perhaps even *most*—of our day-to-day reasoning does not meet it. Such reasoning is reasoning nonetheless, and—as we'll soon see—meets its own modal requirement, different from the UMR. Consider for instance the following:

A3) Gerard is with Buridan

∴ Gerard is on the rue de Fouarre.[395]

Buridan calls consequences like (A3) valid *ut nunc* or 'as of now'. Yet (A3) could quite easily be invalidated: Buridan and Gerard could just leave the rue de Fouarre together, rendering thereby the antecedent of (A3) true, and the consequent false.

[395] "Gerardus est cum Buridano; ergo ipse est in vico Straminum" (*Summulae* 1.7.3).

Hence (A3) is not valid by the UMR, and so the antecedent and the consequent of (A3) are not related by the sort of necessity that is at play in formally valid or simply-materially valid arguments. Recall that any argument valid by the UMR cannot be invalidated by any causal power, even God. But (A3) can be invalidated by natural causes, namely John and Gerard. *A fortiori*, (A3) can be invalidated by God, too.

Even so, there is more going on with arguments like (A3) than meets the eye. First, even though (A3) doesn't meet the UMR, there still *is* a modal notion at play, as Buridan's definition of *ut nunc* consequence makes clear:

> D_{UN}) Some material consequences are called *simple consequences* [...] and others are called *ut nunc*. These are not simply speaking [*simpliciter loquendo*] valid, since it is possible for the antecedent to be true while the consequent is not; but they are valid as of now [*ut nunc*], since it is **impossible** for the antecedent to be true and the consequent false, given the way things are as of now [*ut nunc*].[396]

The simple material consequences are those we considered above. Clearly (D_{UN}) countentances a modal notion: in a valid *ut nunc* consequence, it is *impossible* for the antecedent to be true and the consequent false, given the way things are right now. But it

[396] "Consequentiarum materialium quaedam vocantur 'consequentiae simplices' [...] Aliae vocantur 'consequentiae ut nunc', quae non sunt simpliciter loquendo bonae, quia possibile est antecedens esse verum sine consequente, sed sunt bonae ut nunc, quia impossibile est rebus omnino se habentibus ut nunc se habent antecedens esse verum sine consequente" (*TC* I.4.25-32).

cannot be the UMR of simple and formally valid consequences, since it is subject to change. Our chief concern in the present treatment of *ut nunc* consequence will be the clarification of this modal notion. Where should we place it on the modal scales of the *Summulae* and the *QAPr*?

3.1. What's the Modal Notion At Play Here?

Short answer: it is at *Summulae* Grade 4. Recall the scale of necessities from the *Summulae de Demonstrationibus*, which has so far been our guide for thinking about things modal in Buridan:

Grade 1: simple necessity

e.g. "God exists"

Unfalsifiable by any power, including divine

Grade 2: nomological necessity

e.g. "The heavens move"

Unfalsifiable by any natural power
Falsifiable by divine power

Grade 3: *de quando* necessity

e.g. "Socrates is a human"
"Socrates is capable of laughter"

Falsifiable by divine or natural power, but only by the annihilation of the subject term

Grade 4: necessity by restriction

e.g. "Aristotle walked"

Once contingent, but now unfalsifiable by any power, including divine

Right at the outset, we ruled out Grade 1 as the modal notion underpinning *ut nunc*: as (A3) shows, an *ut nunc* consequence can readily be invalidated at some future time by ordinary natural causal powers like Buridan and his pal, and *a fortiori* by God as well. Similarly, an argument like (A3) will not hold at Grade 2, since Buridan and Gerard's location has nothing to do with nomological necessity. Neither will it hold at Grade 3, since invalidation of (A3)—by the falsification of the consequent—does not involve destruction of the things it's about.

This leaves Grade 4. And there is further reason to think that *ut nunc* consequences hold at Grade 4: both *ut nunc* validity and Grade-4 necessity have to do with time. Consider Buridan's example of a once contingent fact that is now Grade-4 necessary:

P14) Aristotle walked

What gives (P14) its necessity is that there is no power over the past, and so there is no way to render (P14) false. Thus, concerning Grade-4 necessary propositions like (P14), Buridan tells us that:

> There is also a fourth degree, which involves restriction [*restrictio*]. For 'possible' is sometimes predicated broadly, in relation to the past, present, and future; and sometimes it is predicated restrictively, in relation to the present or the future, in accordance with what it says at the end of the first book of *de Caelo*[397] (namely that no force or power can act on the past—that is, on that which has occurred, with respect to its having occurred, either to occur or to be going to occur). And the same is true of 'necessary' and 'impossible', which are also predicated either with restriction, or broadly.[398]

Accordingly, Buridan concludes, any present-tensed proposition about Aristotle is now false, though it may once have been possible. But then any contingent facts that have receded into the past are, in this restricted way, necessary: it is true that the Peripatetic walked, and since there is no power over the past, the fact expressed by (P14) is Grade-4 necessary.

A remark on translation: the parenthetical portion of the above text—especially what follows the em-dash—is, to say the least, confusing. It might be tempting to translate it as saying that there is only a power *over that which is or will be*, and not over that which was.[399] But this makes somewhat less sense against the background in *De*

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[397] *De Caelo* I.12 (283^{b}13-14).

[398] "Adhuc est quartus modus, secundum restrictionem. Nam sicut 'possibile' dicitur aliquando ample, in ordine ad omne tempus praesens, praeteritum et futurum, et aliquando restricte, in ordine ad praesens vel futurum, iuxta illud quod dicitur in fine primi *de Caelo* quod non est virtus sive potestas ad praeteritum, scilicet eius quod est factum secundum eius quod est factum esse vel futurum esse; quod enim fuit dicimus quod necesse est fuisse et impossibile est non fuisse. Ita et 'necesse' et 'impossibile' dicuntur secundum restrictionem vel ample." (*Summulae* 8.6.3; de Rijk, p.142, ll.1-7).

[399] For instance, Klima renders this as follows: "...in accordance with what is said at the end of *On the Heavens*—that no force or power can be brought to bear on the past, *i.e.* on that which is done, but only on that which is or will be (for we say that everything that has been necessarily has been, and cannot not have been)" (*Summulae* 8.6.3; Klima, p.733).

Caelo to which Buridan here refers. There, Aristotle discusses questions relating to powers to be otherwise in the past than things in fact were. Such a power does not exist, though consideration of past contingent things introduces a sophistical argument, whereby something which existed (or didn't exist) in the past retains now the capacity for existing (or not existing) in the past—which looks like a power over the past, and which Aristotle wants to reject. Accordingly, in his discussion of this passage of the *De Caelo*, Buridan says in his *EQC* that:

> Aristotle dismisses this objection, saying that there is no power with respect to the past or being in the past, but for being in the present or the future, since if a power now with respect to a year ago should be posited in being, then it will be true that now is a year ago, which is impossible.[400]

This objection, and its solution, is what Buridan apparently has in mind in the parenthetical passage from the *de Demonstrationibus*: namely, that something that was (or wasn't) in the past does not *now* retain a capacity to be (or not to be) in the past, and so there is nothing that has a power over the past. Hence he is not talking about things that occur or are going to occur now, but rather of things that occurred or were going to occur as of then.

Hence Buridan is speaking in the *de Demonstrationibus* and *EQC* passages of the past as though it were the present; yet this does nothing to the lack of contingency of the past. For example, we can take as an example a proposition Buridan gives us in his

[400] "Hoc ergo removet Aristoteles dicens quod non est potentia ad praeteritum vel ad esse in praeterito, sed ad esse in praesenti vel in futuro, quia sic potentia nunc ad annum priorem ponatur in esse, et tunc erit verum dicere quod nunc est annus prior, sed hoc est impossibile." (*EQC* I.4; Patar, pp.87-8, ll.87-90).

discussion of ampliation (a semantic phenomenon to be discussed in the next chapter) in the *TC*:

P15) In Aristotle's time, Averroes was yet to be born.[401]

The past-tensed copula of a proposition like (P15) "binds the predicate to its own [past] time".[402] Critically, the predicate of (P15) is a future passive participle, so we are speaking of the time of Aristotle with respect to things which, at that time, were yet to come about. But this does not mean that such things, like Averroes' birth, are now contingent facts, since they remain in the past, where no causal power can touch them. Thus it is not right to take the *de Demonstrationibus* passage to be about actually present things. Rather, it's about things that were actual (or not), at some earlier time t_0, but no longer have any power over t_0 once they've moved on to a subsequent t_1. And this is why I have translated it this way.

To return to the modal criterion: perhaps the necessity of 'as of now' or *ut nunc* consequences involves a similar sort of restriction. Then the notion of necessity at play there would work like this: suppose, as some medieval thinkers do, that contingency applies only to the future, not the past or present. Then the present is in some sense necessary. So if we restrict ourselves to the present, we can take the (true) antecedent and consequent of a valid *ut nunc* consequent to be necessary in this restricted way. For example, consider again the *ut nunc* consequence (A3):

[401] "Tempore Aristotelis Averroes erat generandus" (*TC* I.6; Hubien, p.30, ll.12-13).
[402] "[...] verbum trahit praedicatum ad tempus suum" (*TC* I.6; Hubien, p.29, l.108).

A3) Gerard is with Buridan

∴ Gerard is on the rue de Fouarre.[403]

Since it is now true that Gerard is with Buridan, and that they are both on the rue de Fouarre, (A3) follows necessarily, albeit restrictedly. And *voilà*: the *ut nunc* modality we've been looking for.

There is, moreover, textual evidence for this account of the modal status of the present. In his *QLP* I.12, Buridan assesses Aristotle's claim in the *De Interpretatione* that the present is, in a restricted sense, necessary—that is, that "What is, necessarily is, when it is".[404] Buridan's answer is *yes*. He tells us that:

> [propositions] about the present have no less determination [*determinatio*] with respect to being true or false than those about the past. But everything that was is determined to having been [*ad fuisse*] in such a way that it is impossible for it not to have been. Therefore, everything that *is* is determined to being in such a way that it is impossible for it not to be. Therefore, when it is, it is of necessity [*de necessitate*].[405]

So the necessity of present-tensed propositions is the same as that of past-tensed ones. So the necessity of (P12) the same as the necessity of the following proposition:

[403] "Gerardus est cum Buridano; ergo ipse est in vico Straminum" (*Summulae* 1.7.3).
[404] *De Interpretatione* 9 (19^a23).
[405] "Item. Non habent minorem determinationem ad verum vel ad falsum illae de presenti quam illae de praeterito. Sed omne quod fuit est sic determinatum ad fuisse quod impossibile est ipsum non fuisse. Ergo omne quod est sic determinatum est ad esse quod impossibile est ipsum non esse. Ideo ipsum de necessitate est quando est." (*QLP* I.12; van der Lecq, p.54, ll.23-7).

P16) You are reading

Granted, in the *future* you could render (P16) false, and at that point it will be true—and necessarily so—as a past-tensed proposition. Still, it is necessary at present, since what is, is of necessity.

Notice however that what Buridan says here in the *QLP* seems to conflict with his claims in the *de Demonstrationibus* about power over the present: there, Buridan seems to think there is power over the present, as we've just seen. But here, Buridan explicitly denies this, claiming instead that only the future is contingent. But I don't think we should weigh both passages equally. The *de Demonstrationibus* passage is about necessity with respect to restriction; the *QLP* passage is specifically about the contingency of the present. Hence where Buridan treats the question of the contingency of the present in particular, he denies that it is contingent. And as we'll see in a moment, there is not sufficient textual evidence to ascribe to him the opposite view—a view that is, in many respects, an outlier in the history of metaphysics.

This, then, is my claim: the modal requirement of (D_{UN}) is Grade 4. After all, *ut nunc* consequences are necessary, and the necessity they countenance is that of the present. But the necessity of the present is no different from that of the past, as we saw in the above passage from *QLP*. And Grade 4 modality explicitly deals with the necessity of the past. Therefore, Grade 4 necessity applies to the present, too. Hence an *ut nunc* consequence like (A3) is necessary, since the present can't be other than it is, and so things can't be as the antecedent signifies without being as the consequent does. Granted, Buridan and Gerard could *in the future* render (A3) invalid. But such contingencies are future-orientied, since for Buridan there are no synchronic contingencies: what is now is of necessity.

This stance on the necessity of the present is nothing new: Buridan shares it with Aristotle, as we've seen, along with a whole host of other thinkers. As Antonie Vos nicely puts it, in answer to the question of whether the present is contingent:

> The chorus that yells '*no*' is very impressive counting as its singing members Plato and Aristotle, Plotinus and Proclus, Avicenna and Averroes, Thomas Aquinas and William of Ockham.[406]

And Buridan, too. Hence the stance Buridan takes on the necessity of the present is hardly distinctively Buridanian: Buridan is, on this point, in good company.

Buridan's stance *does* however set him against the Scotists, for whom—(in)famously—the present is contingent. Now there wouldn't be much to say about Buridan's stance on the present than that, were it not for some confusion in the literature. At least one commentator has taken Buridan to be a Scotist—a conclusion which (as we have seen) would have come as a surprise to Buridan, and would have been disastrous for his logic. Let's see what went wrong.

To clarify the Scotist background: in the *Lectura* (I.39.4), Scotus gives a thought experiment in a case in which a will is created in a single instant, and has the capacity to

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[406] Antonie Vos, "Buridan on Contingency and Free Will", *John Buridan: Master of Arts*, ed. E.P. Bos and H.A. Krop (Nijmegen: Ingenium, 1993), 151. Deborah Black has pointed out to me in conversation that there may be some doubt about whether Avicenna really is in this chorus, though here is not the place to sort that out.

will one of two contraries. The conclusion Scotus draws is that any will, human or supernatural, is capable of willing one of multiple things synchronically—rather than diachronically:

> This logical possibility exists, not in successive acts of the will, but within one instant: for in a single instant in which the will has a one act of willing, the will can have the opposite act of willing within and for that very same instant. For instance, suppose that a will only had existence for one instant, and that in that instant it willed something, and after that it could no longer will or reject anything. Nevertheless, in and for that instant in which the will wills A, it could reject A. For to will in and for that instant does not pertain to the essence of that will, nor is it a natural passion; therefore, it follows that it is merely accidental.[407]

Hence since it is not essential to the will to will one thing or another, the will is capable of willing one thing or its contrary—though of course, not both—within a single instant.

Two things stand out about the *Lectura* passage. First, it apparently countenances indivisible instants of time—something Buridan rejects. Otherwise, the existence of two contingencies in an instant would not entail synchronic contingency, any more than the transition from rain to sunshine within an hour would. For the thought experiment to work, it cannot be that there is any dimension to the instant, within which the created will

[407] "Haec autem possibilitas logica non est secundum quod voluntas habet actus successive, sed in eodem instanti: nam in eodem instanti in quo voluntas habet unum actum volendi, in eodem et pro eodem potest habere oppositum actum volendi, - ut si ponitur quod voluntas tantum habeat esse per unum instans et quod in illo instanti velit aliquid, tunc successive non potest velle et nolle, et tamen pro illo instanti et in illo instanti in quo vult a, potest nolle a, nam velle pro illo instanti et in illo instanti non est de essentia ipsius voluntatis nec est eius passio naturalis; igitur consequitur ipsam per accidens."

could transition from willining one thing to willing another. So synchronic, mutually exclusive contingencies depend on instants, within which no change from one contingency to another is possible. Or anyway, at least the thought experiment designed to show that they exist depends on this. But for Buridan, there is no temporal interval so small it can't be split into smaller intervals. So there is no basic unit of time. Accordingly, Buridan will disagree with Scotus on this point.

Second, the *Lectura* passage commits Scotus to the contingency of the present: unlike the past—which is necessary because unchangeable—the present, like the future, is not determined, but contingent. But Buridan rejects just this view, which he thinks leads to a contradiction:

> Either everything that exists, when it exists, exists of necessity; or something that exists is able not to be, while it still is. Now one of these two disjuncts has to be granted, since they are mutually contradictory. But the second should not be granted. Proof: since this proposition is about possibility, it is not true unless what is propounded to be so were possible. But if it were propounded to be so, then it would not be possible, namely, that "Something which is, is not at the same time that it is".[408]

This passage looks like a direct attack on the Scotist doctrine of synchronic contingency—even if Buridan doesn't mention Scotus by name. For Buridan, the Scotist

[408] "Item. Omne quod est, necesse est esse quando est vel aliquid quod est, possibile est non esse quando est. Oportet enim alteram illarum concedere, quia sunt contradictoriae. Sed secunda non est concedenda. Probo quia, cum ipsa sit de possibili, ipsa non est vera nisi ista quae poneretur in esse esset possibilis. Et tamen si poneretur in esse, non esset possibilis, scilicet ista: 'aliquid quod est, non est quando est'." (*QLP* I.12; de Rijk, p.54, ll.28-33).

position entails that something might both be and not be at the same time—which looks like a clear contradiction. So Buridan is no friend to the Scotist position.

Here then, I part ways with Henrik Lagerlund, who finds evidence in the texts that Buridan endorses the Scotist position—which position Lagerlund identifies with a principle he calls FO (for *freedom of opposition*), and formulates as follows:

> FO) An act of will is free in accordance with freedom of opposition, if in the same instance as the act is performed it is possible, everything else but the act itself being the same, that the act is not performed or that its opposite is performed.[409]

The natural place to look to see whether Buridan endorses FO is in his treatment of the will in his *Quaestiones in decem libros* Ethicorum *Aristotelis ad Nichomachum* (*QNE*). There, Buridan gives us the following example (cited by Lagerlund):

> It is possible for there to be two or more means to the same willed end, through which that end can be attained. And yet these means may be incompossible. For example, one can go from Paris to Avignon either by passing through Lyon or through Duc-le-Roy, each of which has been presented to the will under the appearance of the good. And the will can choose [*acceptare*] for itself either [...], but it cannot determine itself for both at the same time [*simul*], since they are incompossible. And so the will

[409] Henrik Lagerlund, "Buridan's Theory of Free Choice and its Influence", *Emotions and Choice from Boethius to Descartes*, ed. Henrik Lagerlund and Mikko Yrjönsuuri (Dordrecht: Kluwer, 2002), 177.

> can determine itself for either one, without anything else determining it—or it can even determine itself for neither, but remain suspended until it has been determined by reason which road is quicker or better. [410]

So I can choose one of two contingent but incompossible outcomes at t_0, without anything determining the state of my will at a later t_1. This lack of determination of the will distinguishes voluntary agents from involuntary ones, as Buridan tells us (and Lagerlund also cites):

> This is the difference between a voluntary agent and an involuntary one: the voluntary agent can freely assign itself to either of two opposites, all other things remaining the way they are.[411]

Is this Buridan's endorsement of FO? Lagerlund thinks so, and claims of *QNE* III q.1 that "it seems that he here [...] wants to explicate FO".[412]

But FO is too strong. FO makes it seem as though multiple incompossible contingencies exist *synchronically* in the present. Conversely, Buridan seems to have future-oriented incompossible contingencies in mind, not present ones—as is clear in the *QNE* passage just cited: Buridan is imagining a will which is yet to determine which road it will take in the immediate *future*. Thus the example with roads to Paris is a far cry from

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[410] "[...] possibile est ea respectu eiusdem finis voliti duo vel plura esse media per que finis potest attingi incompossibilia tamen, verbi gratia, quod de Parisius ad Avinionem ire vel per Lugdunum vel per Dunonem quorum utrumque presentatur voluntati sub ratione boni, et voluntas quodcumque bonum sibi sub ratione boni praesentatum acceptare potest et non potest illa duo simul acceptare propter incompossibilitatem, ideo libere potest se determinare ad quodlibet illorum absque alio quocumque determinante ipsam—vel etiam potest at neutrum illorum se determinare, sed in suspenso manere donec fuit inquisitum per rationem quae via fuerit expedientior vel melior" (*QNE* III, q.1; 36rb-va); cited by Lagerlund, "Free Choice", 178-9.
[411] "Haec enim est differentia agentis voluntarii et non voluntarii: quia agens voluntarium potest se libere ad utrumque oppositum determinare, ceteris omnibus omnino se habentibus" (*QNE* III, q.1, 37va); cited by Lagerlund, "Free Choice", 178.
[412] Lagerlund, "Free Choice", 177.

the spontaneously-generated-will thought experiment proposed by Scotus in the *Lectura*—which, recall, involves a will that can choose among multiple incompossible contingents in a single, undivided instant. We would have to give a pretty loose reading of Buridan's example to make it fit with Scotus' account of the will and synchronic possibility. And—as I noted above—such a reading is incompatible with Buridan's metaphysics of time, since it hinges on indivisible instants within which to house synchronic incompossible contingencies.

Thus the *QNE* passages just cited furnish no basis for any claim that Buridan takes the present to be contingent the way Scotus does. Quite the contrary. And Buridan's position on the necessity of the present is, judging by the numbers, the default position in the history of philosophy—as Vos notes: Buridan agrees with most thinkers from Aristotle to Aquinas. Indeed, even the examples Buridan offers in the *QNE*, which we have here been discussing, support this reading.

To conclude: we can see what kind of modal notion is at play in (D_{UN}): here, as ever, we are dealing with modality in terms of what is subject to a causal power. A valid *ut nunc* consequence cannot be altered at present, and so is in a restricted sense necessary—that is, it is necessary by Grade 4. So the temporal aspect of *ut nunc* consequence corresponds with its own brand of modality: the lowest rung on the modal scale we've been considering.

Now we might be tempted to think that formal and simply material consequences hold *always*, as opposed to the *sometimes* (and *sometimes not*) of *ut nunc*. This is the approach taken by Jacob Archambault in his (2018). But we should resist this temptation, for three reasons.

First, Archambault's approach ignores the Simultaneous Formulation (SF) Requirement of (D3): a perfectly valid consequence might well go unformulated, and so hold at no time. Still, such a consequence would be valid if it were formulated (see the discussion in Chapter 2, §1, above). Similarly, one might formulate a formally valid consequence, and then forget it, causing it to hold only at *some* time, and not all. Following Archambault, we'll have to call such a consequence *ut nunc*. But this applies to *every* consequence, since sooner or later any given consequence will be forgotten, set aside, or whatever. So either every consequence is *ut nunc*, rendering the distinction between *ut nunc* and other consequences meaningless, or this reading of the temporal language of *ut nunc* is misleading.

Second, the role of time is here downstream from modality—specifically, the modal status of the present—and not the other way around. After all, a consequence valid *ut nunc* might hold for all time, even though there is a cause capable of invalidating it. Witness:

A17) Donkeys exist

∴ Fire is hot

God can perfectly well make fire not hot at any time in the future, but keep donkeys intact. So (A17) is a kind of permanent *ut nunc*, though *ut nunc* all the same. Things would be altogether different if the consequent of (A17) were the proposition "Donkeys are animals", in which case it would be impossible to falsify the consequent without falsifying the antecedent.[413] What is important here is not time, but the modal notion: either the UMR, or the idea of keeping context fixed—*i.e.* the stipulation that no invalidating causal power intervenes.

Third and most importantly, the temporal thrust of Archambault's approach leads directly to disfiguring extrapolations about logical consequence in Buridan. In Archambault's own analysis:

> Upon reflection, it is clear that good consequence cannot here mean what we mean by 'valid' – validity is indifferent to time. Furthermore, 'some time' here must mean 'some, but not every time', since otherwise every simple consequence would also be a good as-of-now consequence, vitiating the exclusivity of the division. Furthermore, the division implies that *there is no kind of consequence that holds at no time.*[414]

If we are to understand this final clause as saying that an invalid argument will not hold no matter when it is formulated, fine. But that doesn't tell us much about validity at all. If, on the other hand, we take the consequences that hold at no time to be invalid, we make a significant mistake: what then do we then say about all the would-be valid consequences that never get formulated, and so do not meet the SF Requirement?

All this temporal talk is, therefore, incorrect. What's at stake is not quantification across time, where some consequences hold at some times and not others. Rather, what's

[413] In fact (A17) is the traditional example of an *accidental* consequence. Boethius characterises it this way, as does Abaelard, who thinks of consequences like (A17) as *temporal*.
[414] Archambault, "Introduction", 212 (emphasis added).

at stake is the immutability of the present—that is, the fact that causal powers are future-oriented only, and can no more act on the present than they can on the past. Otherwise, the temporal aspect of *ut nunc* consequence is nothing but a red herring.

3.2. A Problem for *Ut Nunc*

Now there remains a problem for *ut nunc* consequences. As we saw in Chapter 1 (§1.2), the present for Buridan is elastic. Hence we can take as much time as we need to make our propositions true. We also saw there that Buridan has a kind of principle of charity at play: our default should be to interpret propositions in such a way as to render them true, if possible. With these two things in mind, we should know what to do with arguments like the following:

A18) Adam existed
∴ You are reading this

This argument is valid *ut nunc*, because the past and present are both non-contingent, and so both claims are in a certain sense necessary. Again, you could stop reading this, but that is a future-oriented possibility, not a present one. Examples like this one are not so troubling; but then look what happens when we flip things around:

A19) You are reading this

∴ Adam existed

This is a good deal weirder than (A18), though it is apparently likewise *ut nunc*.[415] At any rate, if we can expand the scope of the present to make (A18) true, it seems we should extend the same courtesy to (A19). What gives?

I think the intuitive oddness of (A19) depends on a feature of colloquial uses of the term *therefore* that is not reflected at the logical level. We often use *therefore* in contexts to suggest a causal connection between statements, and so we take it as a rough synonym of the English term *so*. What makes (A19) look weird, then, is that it seems to imply backwards causation, or a reversed order of what we might expect. But I don't think the intuitive oddness of (A19) is too much of a problem: we can bite this bullet. After all, Buridan distinguishes *ergo* (*therefore*) from *quia* (*because*) in his treatment of hypotheticals in the *Summulae de Propositionibus*. There, we read that:

> A causal proposition is one in which two categorical propositions are joined together by the conjunction *because* (*quia*). The truth of these requires that the antecedent be the cause of the consequent, as for example in 'because the sun is shining on the earth, it is daytime'. Their falsity requires that the antecedent not be the cause of the consequent, as for example in 'because Socrates runs, there is a solar eclipse.[416]

[415] I owe this latter example, and the suggestion which motivates this section, to Calvin Normore (personal communication, April 18, 2021).
[416] "Causalis [propositio] est illa in qua coniunguntur duae categoricae propositiones per hanc coniunctionem 'quia'. Ad veritatem eius requiritur quod antecedens sit causa consequentis, ut 'quia sol lucet super terram, dies est'. Ad falsitatem eius requiritur quod antecedens non sit causa consequentis, ut 'quia Socrates currit, sol eclipsatur'. (*Summulae* 1.7.6).

There can be no doubt that an argument like (A19) is, if read in this causal way, false. So, too, are a good many other valid arguments, like those with impossible premises or necessary conclusions. Furthermore, given certain conventions in natural language, we are inclined to read (A19) in this way. But this is a matter of convention, not logic; however odd, (A19) is *ut nunc*.

I think the fact that we have to bite this bullet can be softened a bit by noticing that similar considerations apply to other particles of natural language which are completely contrary to how we think of their truth conditions in logical contexts. Take for instance the conjunction *and*, whose truth conditions are pretty straightforward. Nevertheless, it has similar causal implications, as L. Jonathan Cohen observes:

> in some cases, the utterance of two sentences conjoined by 'and' asserts more than just the truth of both statements. For example, there is an important difference between what is implied by an assertion, *tout court*, of the sentence
>
> (3) A republic has been declared and the old king has died of a heart attack
>
> and what is implied by an assertion, *tout court*, of the sentence
>
> (4) The old king has died of a heart attack and a republic has been declared
>
> The order of events implied by an assertion of (3) is the converse of that implied by an assertion of (4).[417]

[417] L. Jonathan Cohen, "The Logical Particles of Natural Language", *Pragmatics of Natural Languages*, ed. Yehoshua Bar-Hillel (Dordrecht: D. Reidel, 1971), 54.

I want to go one further here, and say that beyond order of events, there is an implied causal link here as well. An assertion of (3) implies that the declaration of a republic had something to do with the king's death by heart attack. Conversely, an assertion of (4) implies that the king's death is not only prior to the declaration of a republic, but that it provided the necessary preconditions for it. That is, political opportunists took advantage of the king's death to make a declaration, in a way they would not have done had the king not died. So even with *and*, there is an implication of temporal and even causal order. So, too, with *therefore*, which accounts for the weirdness of (A19).

Anyway, it's worth noting that this implication of (A19), like those of (3) and (4) in Cohen, is cancellable in the Gricean sense: for instance, we can assert (3) and then add, without pain of contradiction, "...but I don't mean to say that the declaration caused the king to have a heart attack". And likewise, we can assert (A19), and consistently add, "...but I don't mean to say that your reading this is the cause of Adam's having existed". So the apparent causal element of (A19) is a matter of conversational implicature, not of what's literally said. And problems like this one are not unique to *ut nunc* validity, not unique to *therefore*, and not unique to Buridan.

Speaking of intuitive problems in logic, I want to close this chapter by comparing *ut nunc* consequences with the material implication of *Principia Mathematica*.

3.3. Is Material Implication *Ut Nunc*?

Now that we have a picture of what Buridan thinks about time, modality, and *ut nunc*, we are in a position to address a question I've been wondering about for a good while, namely: how does an *ut nunc* conditional for Buridan compare with the material implication of Russell and Whitehead's (1910-13) *Principia Mathematica*? To clarify, here's

> **a brief review of material implication** (MI): for Russell and Whitehead, a conditional statement like ($\varphi \supset \psi$)—usually read out loud in class as "if phi then psi", thereby confusingly conflating '$\supset$' with some version of English *if*—is to be read as a disjunction, where the antecedent (φ) is negated, thus: ($\sim\varphi \vee \psi$). So ($\varphi \supset \psi$) is true just in case either (φ) is false or (ψ) is true, or both. There are no further semantic requirements.

All this is pretty basic stuff. But it's basic stuff that often strikes first-year students of logic as artificial and weird—it is, to borrow Dorothy Edgington's memorable phrase, "logic's first surprise".[418] There are good reasons for this: all that matters for the truth of a material implication is the truth of its parts, so that there are no relevance conditions of the sort we usually place on conditionals in natural language. For instance, the following will be a perfectly true MI:

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[418] Dorothy Edgington, "Conditionals", *The Blackwell Guide to Philosophical Logic*, ed. Lou Goble (Malden, MA: Blackwell, 2001), 386.

P16) If there's a tornado warning, you're reading this sentence.

This reliance on truth-conditions alone, along with other bizarre and even paradoxical features of material implication, are well known and much discussed.[419] The question for us is, is (P16) an *ut nunc* conditional? Or, to return to the language of arguments, is the corresponding argument—namely,

A20) There is a tornado warning

∴ You are reading this sentence

—valid *ut nunc*?

From the examples of *ut nunc* consequences in Buridan that we've considered so far, we might surmise that there is a relevance condition for antecedent and consequent. Such a condition would stipulate that, in order to be valid *ut nunc*, the antecedent and consequent have to be related in some meaningful way. If so, (A20) won't meet the relevance condition, and so it won't be *ut nunc*. But although Buridan's examples may suggest this conclusion, his formal treatment of the rules for reasoning in *TC* I.8 says otherwise. Having presented the *ex impossibili* and *ad necessarium* rules—namely, that an impossible proposition entails any other, and that any proposition entails a necessary one—Buridan tells us:

[419] A good and historically-conscious treatment is David H. Sanford, *If P then Q: Conditionals and the Foundations of Reasoning* (New York: Routledge, 1989).

> a further conclusion of a similar sort should be added concerning *ut nunc* consequence, namely that from any false proposition, every other proposition follows *ut nunc*, and that any true proposition follows from every other one, and likewise by an *ut nunc* consequence.[420]

This passage tells us two important things about *ut nunc*'s relation with MI: first, there are no relevance conditions between antecedent and consequent in a valid *ut nunc* argument. And second, there is no explicit modal requirement, either.

To begin with the first: all that is at play is the truth or falsity of the propositions involved. Accordingly, (A20) and (P16) are acceptable *ut nunc* consequences, and (P16) is likewise a true material implication.[421] For all that, though, the *ut nunc* consequences you'll encounter in the street have a relevantistic backdrop—as examples like (A20) make clear. And indeed, MI is much the same: although MI is *technically* only concerned with truth, even in its technical applications MI will include some background notion of relevance. Hence in mathematical proofs, for which MI was designed, relevance is not dispensable—or else the following would be a perfectly good proof:

A21) If every number greater than 1 is prime or the product of two primes, then the diagonal of a square is incommensurable with the side;
Every number greater than 1 is prime or the product of two primes;

[420] "Et est notandum quod de consequentia ut nunc modo proportionali ponenda est conclusio, scilicet quod ad omnem propositionem falsam omnis alia sequitur consequentia ut nunc et omnis vera ad omnem aliam sequitur etiam consequentia ut nunc." (*TC* I.8, concl. 2; Hubien, p.32, ll.2124).
[421] Recall, from Chapter 1, that both conditionals and inferences are *consequentiae* in Buridan's terminology.

∴ The diagonal of a square is incommensurable
with the side

Here both the antecedent and the consequent are true. And so the conditional of (A21) seems to be a perfectly good truth-preserving MI, since it can only go from T to T. So here we have a proof that the diagonal of a square is incommensurable with the side.

Hogwash! A pseudo-proof like (A21) doesn't prove this—or of anything else for that matter—on the grounds that the embedded conditional is truth-preserving. When we give a mathematical proof, we expect some thread of *relevance* to run through the whole thing—even though no such relevance relation figures into the definition of the MI itself. The same holds, judging by Buridan's examples, for *UN*. So here, under the heading of relevance conditions, we find a further and interesting similarity between MI and *UN*: the formal definitions of MI and *UN* don't tell us everything we need to know about how they work, in ordinary or even in technical use. Simply put, both *UN* and MI are weak tools, that generally get applied to strong tasks. The notion of truth just isn't the whole story of how *UN* and MI get used, even though that's all it says on the packaging.

Russell was aware of this problem, by the way. But this fact often goes unnoticed, since the paper he addressed it in went unpublished until after his death.[422] Russell distinguishes implication—of the usual MI sort—from inference:

> In the practice of inference, it is plain that something more than implication must be concerned. The reason that proofs are used at all is that we can

[422] Bertrand Russell, "Necessity and Possibility", *Collected Papers: Vol. 4: the Foundations of Logic*. Ed. Alistair Urquhart, and Albert Lewis (New York: Routledge, 1994 [1905]), 507-20

> sometimes perceive that q follows from p, when we should not otherwise know that q is true; while in other cases, 'p implies q' is only to be inferred either from the falsehood of p or from the truth of q. In these other cases, the proposition 'p implies q' serves no practical purpose; it is only when this proposition is used as a means of discovering the truth of q that it is useful.[423]

Still, Russell thinks that this problem can be addressed by the axioms set out in *PM*, which themselves hinge on implication. Indeed, a good deal of the worries about the much discussed puzzles and paradoxes of MI could probably be resolved in this way: by running them through the axioms of *Principia Mathematica*. But unfortunately, Russell's paper went largely unnoticed, and so a lot of ink was spilled over a problem Russell himself was well aware of and attempted to solve.

The second similarity between *UN* and MI is their reliance on *truth*: the above passage from Buridan's *TC* is concerned with *true* propositions, not with ones that are in any way necessary. So Buridan's *ut nunc* does not look like the strict implication of C.I. Lewis, ($\varphi \prec \chi$), which reads into the conditional a modal operator, *i.e.* $\Box(\varphi \rightarrow \chi)$. Here there is no such unrestricted necessity. And here, too, *ut nunc* and MI are alike: MI doesn't explicitly countenance an unrestricted modal notion, either.

Notice, though, that there *is* a (restricted) modal notion underwriting *ut nunc* consequences, as we saw in the preceding section of the present chapter. This might be a key difference between *UN* and the MI: just because *ut nunc* consequence is a matter of

[423] Russell, "Necessity", 515.

the way things happen to be at present, it doesn't mean there's no modality at play—at least for Buridan, as we've seen.

As for the material implication of *Principia Mathematica*, its modal status, if indeed it has any, is unclear. Russell and Whitehead don't tell us much about the underlying metaphysics, so we have nothing to go on from the texts. The answer to this question will clearly come down to whether the present is necessary, and therefore whether MI holds necessarily in some restricted sense of necessity. So the question is whether, to borrow Vos's nice image, Russell and Whitehead are in the enumerated chorus, from Plato and Aristotle up to Aquinas and Ockham. But Russell and Whitehead never ask this question, and so we can't situate them either way. The historical gap here is therefore in the twentieth century, not the fourteenth.

Remember this the next time a logician tells you they "don't do metaphysics", as though such a thing were laudable in itself.

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Chapter 5:

Consequentiae in Buridan's Modal Logic

possibilia sunt apud Deum

—Luke 18.27b

So far, all the consequences we've looked at have involved ordinary assertoric propositions, both categorical and hypothetical. But along the way, we've picked up the conceptual apparatus we need to expound Buridan's modal logic, too. That's what I do here. In Buridan's view, modal propositions differ from ordinary assertoric (*de inesse*) propositions by their inclusion of a mode like 'possible', 'necessary', 'contingent', or 'impossible'.[424] So far, so familiar: in contemporary modal logic(s), we are concerned with precisely these modals, which we typically call *alethic*. So Buridan is interested in the same modal-semantic phenomena we are.

[424] Though as Buridan himself observes, there are many more modes than these: *true* (*verum*) and *false* (*falsum*); as well as epistemic modals such as *known* (*scitum*), *doubted* (*dubitatum*), *believed* (*creditum*), *opined* (*opinatum*) and so forth. Buridan notes that (i) of these latter sort, only *opined* works properly in syllogisms, since it is governed by *dici de omni/nullo*; and (ii) that *scitum* is factive. See the discussion in (*QAPr* I.40), which Buridan ends with the tantalising note that "there are many further conclusions to be set down [about these modals], which those who are diligent [*diligentes*] can themselves consider".

Yet Buridan's account is different from the possible-worlds semantics we learned at our mother's knee. Which latter semantics can be summarised *en passant* as follows:

> **a brief overview of modern modal semantics:** nowadays, modes are typically construed as sentential operators, whose semantics are explicated in terms of quantification over possible worlds. Naturally, there is considerable disagreement about what these worlds *are*. But never mind that for now: the point is that propositions like φ hold in worlds: some propositions in all, some in some, some in none. By these lights, a modal formula like $\Box\varphi$ (where φ is an arbitrary proposition) is typically read "necessarily φ", and cashed out as "in every possible world, φ". In other words, if $\Box\varphi$ is true, then in every maximally-consistent set of sentences, one of them is φ. The other modes are defined similarly, so that $\Diamond\varphi$, "possibly φ", states that φ holds in *some* ($\geq$1) possible world—*i.e.* is a member of at least one set.

This way of reading modal propositions has been standard in logic and the philosophy of language since the seminal work of Saul Kripke.[425]

But the Kripkean way is not the *only* way to construe the semantics of modal propositions. Buridan takes the meaning of *mode* literally: a mode is a *modification* of the predication that goes on in a categorical proposition. Predication (recall from Chapter 1) is a function of the verbal copula, and modes are adverbs. So for a medieval, modes must be

[425] See especially his "A Completeness Theorem in Modal Logic", in *Journal of Symbolic Logic*, 24(1): 1–14 (1959); as well as his 'Semantical Analysis of Modal Logic I: Normal Modal Propositional Calculi', *Zeitschrift für Mathematische Logik und Grundlagen der Mathematik*, 9: 67–96 (1963).

qualifications of the verb (as we saw in Chapter 3, §2.1.3). This fact will be so important in what follows that it is worth reiterating here:

Fact 5.1: for Buridan, modes are a special class of copulae.

We saw some examples of these copulae in Chapter 3, (§2.1.3): they include terms like *possibly-isn't* and *necessarily-is*, and they are irreducible to any further, constituent parts. That is, they are stand-alone logical terms.

The function of the modes is to stretch or ampliate (*ampliat*) the extension of a proposition's subject term. A term thus ampliated in a modal context stands for possible (as well as actual) things. For example, consider an ordinary assertoric:

P1) Some donkey is running

This proposition is true just in case there is at least one actual donkey that is among the currently running things. That is, (P1) is true iff the extension of the subject term (actual donkeys) overlaps with the extension of the predicate term (currently running things). Likewise, the truth of a modal proposition depends on the extensions of its terms, but these extensions are broader than those of their assertoric cousins. Consider:

P2) Some donkey *is-possibly* running

In (P2), the modal copula *is-possibly* ampliates the subject term so that it holds for all possible donkeys, including actual ones. And (P2) is true because the set of possible donkeys overlaps the set of possible running things—that is, because at least one possible donkey can run. Hence Buridan's is a two-term theory of propositions, where modals are modified copulae that ampliate the subject and predicate to range over possible and actual things.

But of course many possible things are not actual. Hence a modal proposition like (P2) can be true even if all actual donkeys are annihilated. For even if there were no actual donkeys, there would still be possible donkeys to talk about, and at least one possible donkey can run. So the truth of (P2) in no way depends on what actually exists, the way the truth of (P1) does.

The foregoing overview of Buridan's modal semantics presents us with three remarkable facts: (i) the modal character of a proposition is determined not by quantity, but by a special sort of copula. This copula *ampliates* or stretches the supposition of the subject term to non-actual possible things. Accordingly (ii) modal propositions are not about possible worlds at all, but possible objects (*possibilia*). And further, (iii) modal propositions, even affirmative ones, do not have existential requirements, the way ordinary affirmative assertorics do, since there is no guarantee that any of the *possibilia* they deal with actually exist.

Thus Buridan has a very different modal semantics (and correspondingly, modal axioms) from the systems we're used to dealing with, especially when it comes to the logical relations between modals and assertorics. I address each of the above facts *seriatim* in the sections below: ampliation in §1, *possibilia* in §2, and Buridan's axioms in §3. Ultimately, as I conclude in §4, Buridan is at odds with the modern account of the semantics of modal propositions in terms of possible worlds.

On to Buridan's modal semantics. And first, by way of clarification, a word on what modals *aren't*. Both for us and for Buridan, modal propositions are so called for syntactic reasons, not semantic ones. Modals are *not* merely propositions that happen to be necessarily true (or possibly true, or whatever); rather, they are propositions that include a modal term. Take necessity: a good number of necessary propositions are simply true assertorics, which happen to be necessary because of their subject matter. And, conversely, many modal propositions—that is, propositions *about* necessity (*de necessario*) by virtue of their inclusion of the modal copula—are just plain false. Consider the following examples:

P3) Humans are animals.[426]

P4) Someone necessarily runs.[427]

According to Buridan, (P3) is necessarily true, but is not a modal, on account of its assertoric syntax: (P3) is unmodified by a mode like *necessarily*. Conversely (P4) *is* a modal, but a false one. Here, we will be talking about modals *de necessario* like (P4), and not about assertorics like (P3) that happen to be necessary. These are distinguished

[426] "Homo est animal" (*TC* II.1; Hubien, p. 56, l.18).
[427] "Homo de necessitate currit" (*TC* II.1; Hubien, p. 56, ll.19-20).

formally from propositions about necessary subject matter like (P3), since the modals have a modal copula as their principal part.

Modal propositions come in two flavours: composite (*compositae*) and divided (*divisae*). The present discussion is about the divided sort. Composites are propositions with an assertoric copula like *is* or *aren't*, flanked by a mode and a proposition (*dictum*). Here is one such:

P5) *That someone runs* is possible.[428]

In (P5) the subject is a phrase (*dictum*) that stands (*i.e.* supposits) for a proposition (*someone runs*); the predicate is a modal term (*possible*)—not, as in our modern view, a sentential operator (more on this in a minute). Formally speaking, then, (P5) is not a full-fledged modal at all, since its copula (*is*) is unmodified by a mode, and its only modal component is its predicate term—not a part of its form, but a part of its matter. Thus, according to Buridan, composite modals are actually just assertorics with a special sort of subject or predicate. Hence he tells us that:

> Sometimes, a mode is a determination of the copula; other times, it's a determination of a term placed in subject- or predicate-position. And if the determination is bound [*se teneat*] to what's in the subject or predicate position, then it does not render the proposition modal.[429]

[428] "Hominem currere est possibile" (*TC* II.2; Hubien, p.57, l.12)

[429] "Modus aliquando est determinatio copulae et aliquando est determinatio alicuius termini, positi a parte subiecti vel a parte praedicati. Et si determinatio se teneat a parte subiecti vel praedicati, non reddit propositionem modalem" (*Summulae* 1.8.2; van der Lecq, p.83, ll5-8).

The subject of (P5), *that a man runs*, is an expression standing for the proposition “a man runs”. So here, a modal predicate is being affirmed of a proposition-like *dictum* by an ordinary assertoric copula.

Now we might wonder what the relation is between a dictum and its corresponding proposition. On the whole, Buridan tells us that the term which is not modal “stands for (*supponat pro*) another proposition, as it does in ‘*that Socrates runs* is contingent’.”[430] However, in a detailed treatment of these terms in the *Summulae de Propositionibus*, Buridan alerts us to the fact that they are equivocal, and can stand either for a thing in the world which the proposition is about, or for a proposition:

> As for modal propositions of this [composite] sort, note that the expression in an infinitive mode (‘that a man runs’) is placed in these, and is usually called the *dictum*. Sometimes, it is taken to have material supposition, in which case it stands for a proposition, so that for instance ‘that a man runs’ stands for the proposition ‘a man runs’. But sometimes, the expression is understood in a significative way. In that case, if it stands for anything, it stands for the thing for which the subject of the *dictum* would stand when determined in such a way, so that for instance ‘for a man to run’ stands for a running man, and ‘for a man to be white’ for a white man.[431]

[430] “soleat talis propositio vocari ‘modalis composita’ si alter terminus supponat pro aliqua propositione, ut ‘Sortes currere est contingens’.” (*Summulae* 1.8.10).

[431] “Circa huius modi propositiones modales, notandum est quod oratio infinitivi modi (ut ‘hominem currere’) posita in huius modi propositionibus, quae solet vocari ‘dictum’, aliquando capitur secundum suppositionem materialem, et tunc supponit pro aliqua propositione (ut ‘hominem currere’ pro tali propositione ‘homo currit’), aliquando sumitur significative, et tunc si supponat pro aliquo, supponit pro re pro qua supponeret subiectum dicti tali determinatione determinatum (ut ‘hominem currere’ pro homine currente et ‘hominem esse album’ pro homine albo).” (*Summulae* 1.8.9.2).

I have followed Gyula Klima's lead here in translating the accusative-infinitive constructions (the *infinitive mode* of the first sentence) as that-clauses in English, when they stand for propositions. Here, the relation is that of material supposition: the phrase 'that a man runs' stands for the proposition 'a man runs', in the same way that '*Socrates*' refers to the name in propositions like "*Socrates* is trisyllabic". Hence it seems that Buridan thinks that the *dictum* of a composite modal stands for a corresponding proposition—indeed any proposition with the same terms, as we'll see in a moment.

First, though, I want to look at the latter portion of this text, where I have opted for *for*-clauses to translate the Latin accusative-infinitive. I do this to reflect the versatility of the Latin accusative-infinitive construction—a versatility Buridan notes, as we'll see in just a moment. It seems to me that the corresponding propositions are just like English propositions like "For Socrates, swimming is fun". If so, then it is easy to see how they stand for the things they are about (in this case, Socrates), and not for propositions ("*that Socrates swims* is fun").

Indeed, later on in the *Summulae de Suppositionibus*, Buridan clarifies these relations in just the way we'd hope for and expect. The passage is worth quoting at length:

> Note, further, concerning material supposition, that if an utterance supposits materially for an utterance, it need not be that it always supposits for itself. For often it supposits for another which is similar or proportional to it. For example, if I say '*that a man is an animal* is true' or '*that a man is a stone* is false', then the subjects of these propositions, which are phrases in the infinitive mode, stand materially for propositions [...] Thus we often use

> such a proposition with an infinitive mode, even if it can often stand personally as well, for instance if I say 'it is good for Socrates to act rightly', or 'to cut is to act', or 'for a man to be white is for a man to be coloured', or 'for God to be is God', and so on.[432]

Hence these Latin accusative-infinitive constructions can be used to stand or supposit materially for propositions, in which case they do not need to be formally identical, but only to have the right relation of similitude or proportionality—apparently, judging by Buridan's examples, inclusion of oblique (*i.e.* accusative) and infinitive forms of the same terms as appear in the nominative and finite forms in the corresponding proposition. Conversely, these constructions can be used to stand 'personally' for objects. I here take Buridan to be treating this former use of them as primary, at least for logical purposes.

Indeed, composite modals will have to be propositions with an accusative-infinitive construction standing, as a term, in material supposition for a proposition. Otherwise, Buridan's derivational rules for composite modals will be incoherent. For instance, here is his example of the first of these rules:

> From 'some proposition *that A is B* is possible', it follows that 'every proposition *that A is B* is possible', and likewise for composite modals about truth and falsity, contingency and necessity.[433]

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[432] "Et est notandum circa hanc suppositionem materialem quod si vox aliqua supponat materialiter pro voce, non oportet tamen quod supponat semper pro se ipsa, sed supponit saepe pro alia simili vel proportionali. Verbi gratia, si dico 'hominem esse animal est verum', vel 'hominem esse lapidem est falsum', subiecta harum propositionum, quae sunt orationes infinitivi modi, supponunt materialiter pro propositionibus. [...] Sic enim consuevimus saepe uti tali propositione infinitivi modi, licet etiam saepe possit supponere personaliter, ut si dico 'Socratem bene agere est sibi bonum', vel 'secare est agere', vel 'hominem esse album est hominem esse coloratum', vel 'deum esse est deus', et sic de aliis." (*Summulae* 4.3.2).

[433] "sequitur 'quaedam propositio *B est A* est possibilis; ergo omnis propositio *B est A* est possibilis', et sic de veritate et falsitate, contingentia et necessitate" (*TC* II.7, 9th concl.; Hubien, p.70, ll.44-6).

If the phrase *that A is B* supposits materially for any instance of the proposition "A is B", then it stands for all of them. Therefore, the universal follows from the singular, as Buridan says. Looking back at our equivocal use of accusative-infinitive, we can see that this is rule is stipulated with the supposition of a phrase for a proposition in mind, and not for the use of the accusative-infinitive to describe an object. To see why, we can pick up the examples Buridan gives us in the *Summulae de Propositionibus*, which we set out with, and run them through this inferential schema:

A1) Some proposition 'that a man runs' is possible

∴ Every proposition 'that a man runs' is possible

A2) For some man to run is possible

∴ For every man to run is possible

Clearly, (A2) is invalid, whereas (A1) is valid by the rule for composite modals just mentioned. Therefore, composite modals are exclusively of the former sort, in spite of the flexibility of the accusative-infinitive construction—they are, that is, proposition with one term standing in material supposition for a proposition, and another term which is modal.

Such are composite modals. It should be noted that the subject here is strictly speaking a *dictum*, not a proposition (*propositio*). Propositions are always asserted, as we saw in Chapter 1. If we recall (P5), we can see that the subject considered on its own, is not asserted:

P5) *That someone runs* is possible.[434]

But then what is a *dictum*? Apparently, it's what's compounded or divided in these modals, and so serves to distinguish them. Buridan's account of *dicta* in the *TC* (II.2) is terse:

> I call the *dictum* all that in a proposition which is put in a proposition apart from its mode, copula, negations, signs of quantity, and other determinations of the mode or the copula.[435]

Notice that the list here looks a good deal like the list of formal components of a proposition that we listed in Chapter 3, above. Subtract them all, and what do you get? Apparently, a subject and a predicate. And again, it is characteristic of the composite modals that they should have a dictum as subject or predicate, paired with a modal term as predicate or subject.

As we've seen, in composite modals, the whole *dictum* serves as the subject or predicate of an assertoric: both the subject (*someone*) and the predicate (*runs*) are bundled up on the side of the subject or the predicate.[436] Buridan contrasts composite modals with their divided counterparts, which split the *dictum*, and which we will mainly be looking at in what follows:

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[434] "hominem currere est possibile" (*TC* II.2; Hubien, p.57, l.12)

[435] "Et voco 'dictum' illud totum quod in propositione ponitur praeter modum et copulam et negationes et signa aut alias determinationes modi vel copulae." (*TC* II.2; Hubien, p.57, ll.9-11).

[436] This is a bit difficult to express in English, since in (P5) the subject also includes a copula. But the Latin equivalent, which Buridan expresses with an accusative + infinitive construction (*hominem currere*) does not; it contains merely a noun and a verb in the infinitive. For present purposes, therefore, please ignore the *is* in the subject of (P5), which is not part of the semantics of such propositions.

> [modal propositions] are called ‘divided’, however, in which part of the *dictum* is the subject, and the other part of the *dictum* is the predicate. And the mode is bound [*se tenet*] to the copula, as a determination of it, as it were.[437]

Hence in a divided modal, the matter—*i.e.* the subject-and-predicate or *dictum*—is split between the subject and predicate. The mode is not part of the subject or predicate at all, but acts on the copula. Buridan accordingly contrasts (P5) with its divided modal counterpart:

P6) Someone is-possibly running.[438]

Unlike (P5), a proposition like (P6) divides the dictum between subject and predicate. And the principal part of (P6) is a modal copula, namely *is-possibly*. Divided modals thus have a modal formal component—namely, a modal copula—as their principal part. As subject and predicate they have ordinary significative terms: here, *someone* and *running*.

In general, then: in any case in which a proposition is the subject (or predicate), and a mode is the predicate (or subject), the whole proposition is classed as a **composite** modal.[439] Such is (P5). On the other hand, in (P6) the mode operates not on the proposition as a whole, but on the copula, the mode is **divided**.[440]

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[437] “Sed ‘divisae’ vocantur in quibus pars dicti subicitur et alia pars praedicatur. Modus autem se tenet ex parte copulae, tamquam eius quaedam determinatio.” (*TC* II.2; Hubien, p.57, ll.16-18).
[438] “homo potest currere” (*TC* II.2; Hubien, p.57, l.18).
[439] “‘Compositae vocantur in quibus modus subicitur et dictum praedicatur, vel econverso” (*DC* II.2.7-8).
[440] “‘Divisae’ vocantur in quibus pars dicti subicitur et alia pars praedicatur. Modus autem se tenet ex parte copulae, tamquam eius quaedam determinatio” (*DC* II.2.16-18).

Hence both the foregoing distinctions turn on the copula, and therefore on form: a proposition *de necessario* has a modal copula, whereas a necessary one is necessary just in virtue of its subject matter. Of modals, a composite modal has an ordinary assertoric copula, and a mode as part of its matter—either in the subject or the predicate, whereas a divided modal has a modal copula as its principal part. In what follows, we are by default dealing with divided modals. Unless stated otherwise, then, any reference to *modal propositions*, *modals*, etc., should be taken as being about these.

One final brief remark to conclude this introduction to the subject of the present chapter: we might wonder how we should classify composite and divided modals *à propos* of our modern distinction between *de dicto* and *re*. At first blush, composite modality looks a lot like our *de dicto*, and divided like our *de re*. Probably the two distinctions (*composite/divided*, and *de re/de dicto*) even share the same roots, as Simo Knuuttilla suggests:

> One example of the prevalence of the traditional use of modal notions can be found in the early medieval *de dicto/de re* analysis of examples such as 'A standing man can sit'. It was commonly stated that the composite (*de dicto*) sense is 'It is possible that a man sits and stands at the same time' and that on this reading the sentence is false. The divided (*de re*) sense is 'A man who is now standing can sit' and on this reading the sentence is true.[441]

Here the sample sentences are indeed *de re* and *de dicto*, and so the distinctions seem to map onto each other well. We might therefore be tempted to treat this as merely a difference in nomenclature: call this the Modal Synonym View (MSV). Indeed, many

[441] Simo Knuuttilla, "Medieval Theories of Modality", in *The Stanford Encyclopedia of Philosophy* (Summer 2017 Edition), ed. Edward N. Zalta.

commentators and translators follow the MSV without much discussion at all: translations of *divisa* and *composita* as *de re* and *de dicto*, often without comment, are relatively common in critical editions and the secondary literature.

The MSV faces problems, however, both for *de re* and *de dicto*. To begin with the former: the modern notion of *de re* modality might be too theoretically freighted for use in the present context as a synonym for *divided*. For, as we have already seen, a divided (*de re*) modal can be true without any corresponding *res*, and this fact alone should give us pause. The reason divided modals can be true *sine re*, as it were, is that the terms of divided modal propositions *ampliate*, so that they stand for things which aren't really *res* at all. In a moment we'll see how this works. Even so, it is sufficient to note this fact in any translation of *divisa* as *de re*.

Yet the claim made by the MSV is less appropriate for *de dicto* and *composita* . The modern notion of *de dicto* modality, as we have seen, hinges on the use of '□' and '◊' as *sentential* operators. But in Buridan's composite modality, the modal terms (*necessarium*, *possibile*) are predicates or subjects. So on the basis of the brief overview of modern and Buridanian modal syntax given in the preceding pages, we can see that the two come apart syntactically: *de dicto* modality in the modern sense is propositional, whereas Buridan's modal logic is one of terms. Granted, we could treat modal terms as predicates in a modern predicate logic as well. But if we do so, what we get is not a modal logic at all, but a non-modal predicate logic. It will remain, then, to explain what this modal predicate signifies. But even if we do so, we've come a long way from the *de dicto* reading that the

MSV first identified with composite modality in Buridan. Syntactically, then, the two distinctions differ, and it is important to keep these facts in mind.

Buridan looks even less familiar when we take a closer look at his modal semantics, for which the guiding notion is not existence in possible worlds, but ampliation of terms to stand for possible objects.[442]

1. Semantics: Supposition and Ampliation

In the preceding section, we saw that divided modals ampliate their terms. Let's see what ampliation is, and how it works. I begin with a primer of Buridan's semantics for terms in *assertoric* propositions, since Buridan builds his modal semantics on this framework. For Buridan, a modal proposition works much like an assertoric, except that its terms refer not merely to actual, but also to possible things. Hence for Buridan, divided modals are really a species of categorical.

[442] Granted, there are modern modal systems based on possible objects to be considered, particularly Fine's (2003) and (2000), and Lambert and van Fraasen's (1970). But I have set these aside for two reasons: first, they would require a chapter of their own, since the present chapter is already quite long, as is the dissertation itself by the standards of the University; and second, because the present literature on Buridan mainly analyses him in terms of possible worlds, and I think it's necessary first to show that why worlds are not suited to the task, before turning to objects. But I will get to these in due time, *Deo volente*.

1.1. Term Supposition and Propositional Truth

For Buridan, there are no proposition-like extra-mental significates that correspond with propositions themselves, and make them true. I want to linger on this point for a moment, because it is crucial to Buridan's view, and often easy to miss. In Buridan's view, whole propositions do not refer to anything like proposition-like states of affairs, with which they share a structural isomorphism. Rather, Buridan thinks that what makes a proposition true is its significative parts, namely the categorematic terms.

Buridan thus explicitly rejects proposition-like states of affairs in his fullest treatment of propositional truth, in *QM,* VI qq. 7-8. Gregory of Rimini (ca. 1300-1358) is a notable (and almost exactly contemporary) proponent of the view that propositional truth relies on extra-mental propositional significates that propositions, being by nature complex, are signifiable in a complex way (*complexe significabilia*). Buridan doesn't name Gregory, but the view he considers and rejects looks a good deal like Gregory's. Here, I'll use Gregory as a foil for Buridan, relying on Nuchelmans' exposition.[443]

According to Gregory, the significate of a (true) proposition is a mind-independent signifiable structure (*significabile*) that the entire proposition signifies in a complex way (*complexe*). This is not merely a thing and its property, but a state of affairs that supervenes on such combinations, an extra-mental *fact* about the world. So the proposition "Socrates is sitting" is true because there is a corresponding fact, namely the fact of *Socrates-being-seated*. Propositions thus complexly signify proposition-like structures in

[443] Gabriel Nuchelmans. *Theories of the Proposition: Ancient and Medieval Conceptions of the Bearers of Truth and Falsity*. London: North Holland Publishing Co., 1973.

the extra-mental world, namely *complexe significabilia*.[444] The two structures at play—the mental (propositional) one, and the extra-mental (worldly) one—share a structural isomorphism. This accounts for the truth of the former.

This is not the place to speculate in detail about what motivated Gregory to adopt this view. I'll limit myself to remarking in passing that his concerns seem to be epistemic. Since these complexes are facts prior to our discovery of them, Gregory has a relatively easy time accommodating the intuition that the fact-hood of facts is independent of our knowledge or discovery of the facts themselves. As Gabriel Nuchelmans observes:

> The bearers of truth and falsity [...] are not only actually existing propositions and the significates of actually existing propositions, but also states of affairs that are capable of being signified by true or false propositions *even if these corresponding propositions do not in fact exist*.[445]

So truth does not depend on the formulation of propositions: it is a prior feature of the state(s) of things.

But Buridan raises significant problems for any view according to which truth requires a composition in the proposition's extra-mental significates. First, propositions aren't bound to actual states of affairs. There are true propositions that don't seem to correspond with any way things are in the actual world. For example, consider the following propositions:

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[444] A term Gregory acquires from the Latin translation of Aristotle's Categories 1^a16 (Nuchelmans 231).

[445] Gabriel Nuchelmans. "Semantics", in *The Cambridge History of Later Medieval Philosophy*, ed. Anthony John Patrick Kenny, Norman Kretzmann, Jan Pinborg, and Eleonore Stump (New York: Cambridge, 1982), 204. Emphasis added.

P7) Julius Caesar's horse cantered well

P8) The Antichrist will preach

P9) Something possible will never be

Any view that takes true propositions to correspond with *complexe significabilia* or states of affairs will be in a tough spot to explain how (P7)-(P9) can be true when their objects are nowhere to be found. Hence for Gregory, it is difficult to say what the state of affairs or *complexe significabilia* are: there is nothing in the world now that answers to the description "Julius Caesar's horse", since Caesar's horse is dead. And while (P8) is true, it does not correspond with how things are, but how they will be, since the Antichrist does not exist.[446] Nor is there anything in the world that corresponds with the non-existent but possible thing described by (P9). To take Buridan's example, consider the vinegar that might be produced by the wine I am about to drink. This wine is possibly vinegar. But since I will drink it before it ever turns to vinegar, that vinegar will never come to be.[447]

Second, propositions are by nature complex. But there are true propositions about what by definition has no complexity. Buridan gives us the following example:

P10) God is God

[446] See *De Consequentiis* I.1.14-27.
[447] See *Ioannis Buridani Quaestiones super libris quattuor* de Caelo et mundo, ed. E.A. Moody (Cambridge: Medieval Academy of America, 1942), I.23.

Here, there is no complexity in the thing signified, but only on the part of the proposition. The thing signified by this proposition can be nothing other than God. And yet God cannot be the significate of the whole proposition, since the proposition is complex, whereas God is not. As Buridan tells us,

> This affirmative proposition, 'God is God', is true. And yet there is no composition in the thing signified, since the thing signified is nothing but God, who is completely simple.[448]

It is not clear how Gregory can account for the truth of a proposition like (P10) without positing complexity in the thing signified—namely, God. But this is intolerable, and so Buridan rejects the view that propositions are true in virtue of their relation to *complexe significabilia*.

So much for what Buridan's view is not. Here is what it is. Consider a universal affirmative proposition:

P11) Every human is an animal

For the truth of a universal affirmative proposition like (P11), Buridan tells us that "it is necessary that its terms stand for [*supponere pro*] the same thing".[449] That is, in order for (P11) to be true, what the subject term (*human*) stands for (i.e. humans) must be *included in* what the predicate term (*animal*) stands for, so that everything the subject stands for, the predicate also stands for.

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[448] "ista affirmativa 'Deus est Deus' est vera. Et tamen in re significata nulla est compositio, quia res significata non est nisi Deus, qui omnino est simplex. Ergo etc." (VI.7.arg.1; fol.38a)
[449] "necesse est quod termini supponant pro eodem"(VI.7.co; fol.38b).

Likewise, for the truth of (P10), it's enough that the subject and predicate (that is, the two instances of the term *God*) stand for the same thing. This allows Buridan to account for the truth of a proposition like (P10) without taking recourse to some extra-mental propositional complexity in the thing signified. The complexity is only a feature of the proposition, and not of the object in question (namely God). The truth of a proposition does not depend on any isomorphism with real-world complexity. Rather, it depends on the ways its significative parts stand for things in the world. Things are likewise with any other affirmative proposition, such as:

(P12) Some animals are humans

Now (P12) is true, since what the subject term (*animals*) stands for overlaps what the predicate term (*humans*) stands for. Hence we can define the truth of universal and particular affirmatives in terms of suppositional inclusion and overlap, respectively. Here is a pair of Venn diagrams to make things clearer:

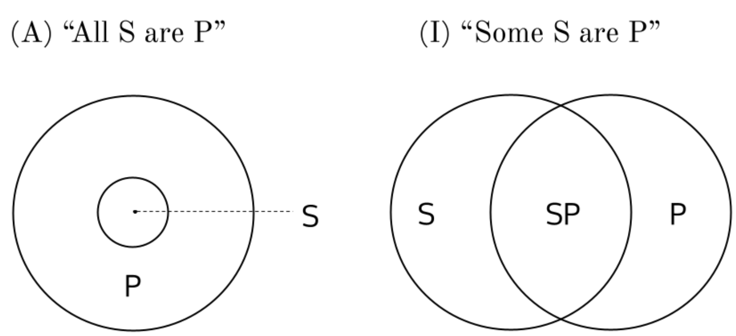

Fig. 5.1:
truth conditions for A-type propositions, and minimal truth conditions for I-type; note that the left figure also makes the corresponding I-type true.

Having thus accounted for the truth of affirmative propositions by means of term supposition, Buridan can rely on bivalence to give a corresponding account of the truth of negative propositions. For, given the necessary connection between affirmative propositions and their negative contradictories, the requirements for supposition of terms are the negative image of the requirements for affirmatives—a connection Buridan is happy to exploit:

> [given] whatever means (*quecumque*), and how many [things] (*quot*) are required for affirmative truth (as far as the part of the things signified is concerned), an absence (*defectus*) of one of them is sufficient for the truth of the contradictory negative (proposition). For otherwise it would not be necessary that, if one were true, the other would be false, and vice-versa.[450]

Hence anything that can make an affirmative false—that is, any failure to meet the truth conditions just set out—is itself a cause of truth of the corresponding negative contradictory. So if an affirmative proposition fails to be true in any way, then whatever accounts for its failure to be true also accounts for the truth of its negative contradictory counterpart.

Accordingly, negative propositions are true just in case their terms *do not* stand for the same thing, just as affirmative propositions are true just in case their terms *do*. For example, the affirmative proposition

[450] "quecumque et quot requiruntur ad veritatem affirmative, quantum est ex parte rerum significatarum, defectus unius illarum sufficit ad veritatem negative contradictorie; quia aliter non esset necesse si una esset vera quod altera esset falsa, et econverso." (*in Metaph.* VI.7.co; fol.38b-c)

P13) Some human is a donkey

is false; and this is because the terms (*human*, *donkey*) fail to supposit for the same thing or things: there is no overlap in supposition between the terms *human* and *donkey*.

Now since (P13) is a particular affirmative, its contradictory is a universal negative, namely:

P14) No human is a donkey

And (P14) is true because its terms, *human* and *donkey*, do not supposit for the same thing(s). Thus an affirmative assertoric fails to be true when the things for which its terms stand or supposit are not the same—conditions under which the corresponding negative contradictory will be true. Hence Buridan tells us that "for the truth of a negative proposition, it is sufficient that the terms do not stand for the same thing."[451]

Importantly, terms will not stand for the same thing if they stand for nothing at all. And so affirmative assertorics have existential import: to be true, they have to be about things that exist. Accordingly, negative assertorics can be vacuously true: the requirement that their subject and predicate terms not supposit for the same thing is met when one or both supposit for *nothing*. Buridan recognises this, and notes that even propositions like the following are true on his propositional semantics:

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[451] "ad veritatem negative sufficit quod termini non supponit pro eodem" (*in Metaph.* VI.7.co; fol.38c; cf. Sophismata II.14)

P15) A chimaera is not a chimaera.[452]

Hence negative propositions, both particular and universal, have no existential requirements. Affirmative propositions, on the other hand, do.[453]

Here is a summary of what we've seen so far, in terms of the traditional A-, E-, I-, and O-types of propositions. The right hand column expresses conditions on the supposition of subject and predicate terms: to say "b is included in a" is just to say that whatever b supposits for, a also supposits for (but not vice versa), and so on. I follow Thom in underlining terms to indicate that their actual extension must be non-empty, and omitting the underlines for proposition-types capable of vacuous truth.

Universal Affirmative (A)		
Every *a* is *b*	iff	*a* exists, and is included in *b* ($\underline{a}$ A $\underline{b}$)
Universal Negative (E)		
No *a* is *b*	iff	a does not exist, or is excluded from *b* (*a* E *b*)
Particular Affirmative (I)		
Some *a* is *b*	iff	*a* exists, and overlaps *b* ($\underline{b}$ I $\underline{a}$)
Particular Negative (O)		
Some *a* is not *b*	iff	*a* does not exist, or is not included in *b* (*a* O *b*)

[452] "chimera non est chimera" (*QM* VI.8; fol. 38c).
[453] Note again however that Buridan does not have propositional negation, but only negative copulae. Hence the problem of double-negation noted in Chapter 3 (§2.1.1).

It can hardly be overstressed that the formal symbols joining a and b, above, are *copulae*. This will become especially important when we go on to consider the modal copulae in a moment.

Where existential requirements are concerned, these propositions behave quite differently from their modern cousins: nowadays, since we generally take universals to be conditionals, and therefore capable of being vacuously true, and particulars to be existentials. Consider the following colour-coded Square of Opposition, where:

i. **B** are the symbolisations on Buridan's term-logic, and **P** are those on modern predicate logic;

ii. blue formulas are those with existential import, whereas red ones are those without; and for both **B** and **P**,

iii. diagonally-opposed formulae are contradictory.[454]

[454] Notice that this Square of Opposition does not come fully equipped: granted, universal affirmatives contradict particular negatives in both **P** and **B**; but other relations, such as subalternation, do not reliably hold here—at least in **P**.

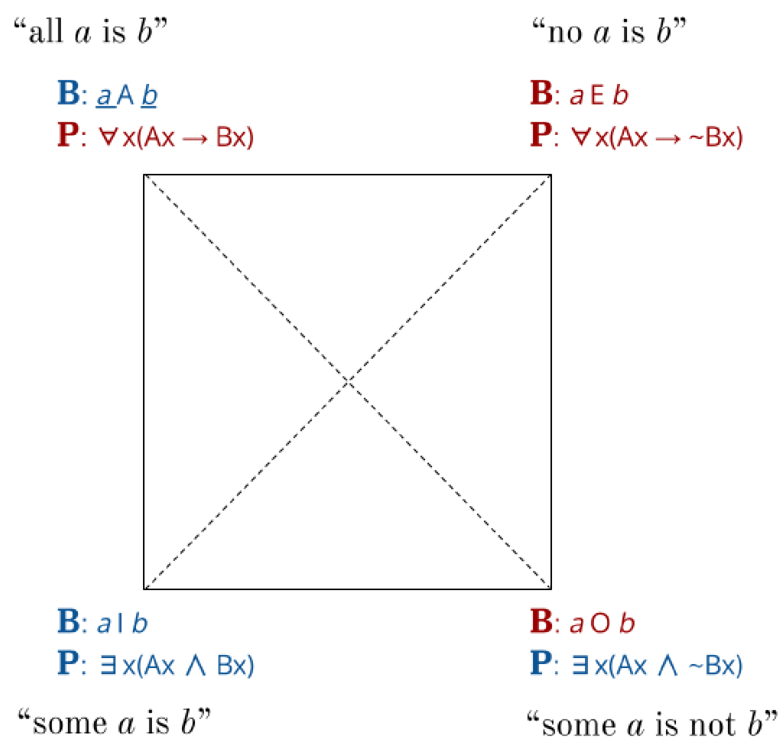

Fig. 5.2. The Square of Opposition

Hence Buridan (and traditional Aristotelian logic in general) group existential requirements around the affirmative (vertical left) axis of the Square of Opposition, unlike Modern Predicate Logic, which groups them around the affirmative (horizontal bottom) axis of the Square. So much for the supposition of terms in ordinary assertorics. Now let's see what happens in contexts that stretch the extensions of subjects and predicates beyond mere actual things.

1.2. Ampliative Contexts and Semantic Stretching

Status is Buridan's technical term for the ordinary range of supposition of a term—that is, the supposition a term has in garden-variety assertorics, concerning which status Buridan tells us in the *Summulae de Suppositionibus* (4.6.1) that:

> First, we have to consider the *status*, according to which a term is said neither to be ampliated nor restricted, and with respect to which status something is sometimes called ampliation, sometimes restriction. Such status can therefore be assigned when a term stands for [*supponit pro*] or appellates precisely for all its significates in the present time.[455]

Thus *status* is a technical term, which, as Gyula Klima helpfully points out in a footnote to his translation of the *Summulae*, "refers to [...] the range of reference that a term has when it is neither ampliated nor restricted".[456] For example, consider the following assertoric:

P16) Every human is running.[457]

Here the *status* of the term *human* is at least the set of all existing donkeys and *running* that of all the things that are now running. By the truth conditions for assertorics set out

[455] "Et oportet primo videre statum secundum quem terminus nec dicitur ampliatus nec restrictus, respectu cuius status aliquando dicitur ampliatio, aliquando restrictio. Status ergo ille potest assignari quando terminus praecise supponit uel appellat pro omnibus suis significatis praesentis temporis" (*Summulae* 4.6.1; van der Lecq, p.89, ll.2-6).
[456] Klima, *Summulae*, p.298, n.95.
[457] "omnis homo currit" (*Summulae* 4.6.1; van der Lecq, p.89, l.7).

in the preceding section, (P16) is true just in case the donkeys are included in the set of running things. So when we assess the truth of present-tensed assertorics like (P16), we're concerned with the items in the *status* of the terms—that is, with the terms' current extension.

But in many contexts, the supposition of a term extends beyond its *status*: tensed, intensional, and modal contexts extend the reference class of a term, so that it includes things beyond what presently exists—that is, things not in the term's *status*. Buridan calls this semantic stretching *ampliation*.

One elegant feature of Buridan's semantics is that ampliation does triple duty: it accounts for the extension of terms in propositions in tensed, intensional, and modal contexts.[458] According to Buridan, all ampliative contexts can be reduced to these three.[459] Here is one example of each, in order:

P17) Aristotle was a Greek

P18) A rose is conceivable

P19) A donkey can run

[458] Notice that Buridan's target phenomenon is the same as what modern linguists call *displacement*, and which likewise includes modals, tensed propositions and statements about tendencies or habits that may not be presently true (*e.g.* "Jane smokes" and "Thunder follows lightning", said on a sunny day when Jane isn't smoking).

[459] *Summulae* 4.6.2. For instance, predicates ending in -ble (Latin *-bilis*) render the copula modal, even though they often appear in what might look like assertoric propositions. So although "A donkey is generable" looks like an ordinary assertoric, it won't fool us: we know to read it as "A donkey is-possibly something generated".

There are many interesting things to be said about tensed and intensional contexts, and propositions about them, like (P17) and (P18). But for now, we are going to focus on modal contexts, and their propositions, like (P19), and leave the others for another day.

In (P19), *donkey* stands not only for actual donkeys, but also possible ones; the same is true, *mutatis mutandis*, for *run* and things that are possible runners. This is why (P18) can be true even in a case where there are no donkeys (or anything else) running, or even no donkeys at all: the modal copula *can* (which Buridan analyses as *is-possibly*),[460] extends or *ampliates* the categorematic terms beyond their *status*, so that they stand for possible objects. Hence Buridan says that:

> A divided modal proposition about possibility has its subject ampliated by the mode following the subject, so that the subject stands not only for the things that are, but for those things that can be, although they aren't.[461]

The mode comes after the subject term, and really is a kind of copula, as we saw in Chapter 3 (§2.1.3). This modal copula changes the extension of the subject term, so that it stands not only for actual things but for possible ones.

Note, by the way, that the extension of terms in modal contexts doesn't shift, but *stretches*: the set of *actualia* is a proper subset of the *possibilia*, since all actual objects are also possible. Put slightly differently: there are no impossible things out there, and so what is actual is also possible. So, since the terms in modal propositions stand for possible things, they also stand for actual ones. When the subject term of a modal proposition like

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[460] *Summulae* 1.8.3.
[461] "propositio divisa de possibili habet subiectum ampliatum per modum sequentem ipsum ad supponendum non solum pro his quae sunt sed etiam pro his quae possum esse quamvis non sint" (*TC* II.4; Hubien, p.58, ll.4-7).

(P18) gets ampliated beyond its status, its new, ampliated extension *includes* the *status*, and so it stands both for possible as well as actual things.

Syntactically, Buridan analyses the subjects of modal propositions as disjunctions. Take for example the following propositional schema:

S1) B can be A.

This proposition should, says Buridan, be analysed as a modal proposition with a disjunctive subject:

S1′) What is or can be B can be A.[462]

Call this approach (i), where the subject is disjunctive, but the whole propositional schema remains categorical because it has a copula as its principal part. Buridan hastens to reject an alternative analysis, which some people endorse (*quod aliqui dicunt*), and which I'll call approach (ii) in what follows. On (ii), (S1) is a disjunction of two modal propositions:

S1″) What is B can be A, or what can be B can be A.[463]

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[462] "Haec propositio 'B potest esse A' aequivalet isti 'quod est vel potest esse B potest esse A'." (*TC* II.4; Hubien, p. 58, ll.9-10).

[463] "Et aliqui dicunt quod ipsa [sc. (P15)] aequivalet university hypotheticae disiunctivae, scilicet isti: 'quod est B potest esse A vel quod potest esse B potest esse A" (*TC* II.4; Hubien, pp.58-9, ll.11-12).

It is worth taking a moment to see why (S1′) of approach (i) and (S1″) of (ii) differ in truth conditions. Consider a corresponding negative proposition (P20), and its analyses, which cash it out on the preceding paradigm:

P20) God-while-creating is able not to be God

P20′) What is or is able to be God-while-creating is able not to be God

P20″) What is God-while-creating is able not to be God, or He who is able to be God-while-creating is able not to be God.[464]

Buridan takes (P20′) to be false: for whatever is or can be God cannot fail to be God. In order to see why this is so, we need merely to remind ourselves of the highest grade of the modal scale considered in Chapter 2: "God exists" is necessarily true, because there is no causal power capable of annihilating God. Therefore, what is God cannot not be God. As Buridan tells us,

> Taking the first approach [(i)], this proposition is false: 'God-while-creating is able not to be God', because its contradictory is true, and indeed holds in any case. For nothing that is or can be God-while-creating is able not to be God. For all and only God is able to be God-while-creating, and God is not able not to be God.[465]

[464] In the translation of (P19)-(P19″), I have cribbed freely from Peter King; see his *John Buridan's Logic* (Boston: Reidel, 1985), 231-2.

[465] "tenendo priam viam haec est falsa 'deus creans potest non esse deus', quia sua contradictoria est vera, etiam quocumque casu posito. Nullum enim quod est deus vel potest esse deus creans potest non esse deus. Nam omnis et solus deus est vel potest esse deus creans et ipse non potest non esse deus." (*TC* II.4; Hibuen, p.59, ll. 14-18).

Accordingly, (P20′) is impossible, and therefore false. And this is the result Buridan wants, since it allows him to say that (P20) is false, which it clearly appears to be.

Buridan's next move is to show that (P20″) can be true—that is, that (P20″) is *possible*. This will establish two things: first, that (P20″) is not equivalent to the necessarily false (P20′), and second, that it is not a correct analysis of the necessarily false (P20). To do this, Buridan notes their different behaviour when their subject terms are empty—that is, when God is not creating, and so what is God-while-creating is actually nothing:

> But if we take the second approach [(ii)], then we have to consider this proposition true: 'God-while-creating is able not to be God' [sc. (P20)], assuming a case in which God is not now creating, because it is equivalent to this disjunctive proposition: 'What is God-while-creating is able not to be God, or He who is able to be God-while-creating is able not to be God' [sc. (P20″)]. Now this disjunctive should be considered true, since its first disjunct is true in the case under consideration, because God-while-creating is nothing, and what is nothing is able not to be God, or indeed God-while-creating is not God, and what is not God is able not to be God.[466]

Imagine, then, a case in which God, who *can* create, has nevertheless closed up shop.

[466] "Sed si teneretur secunda via, oporteret concedere istam: 'deus creans potest non esse deus', posito casu quod deus modo non est creans, quia aequivaleret isti disiunctivae: 'Qui est deus creans potest non esse deus vel qui potest esse deus creans potest non esse deus'. Modo haec disiunctiva est concedenda, quia prima pars eius secundum casum positum est vera, quia deus creans nihil est et quod nihil est potest non esse deus, vel etiam deus creans non est deus et quod non est deus potest non esse deus." (*TC* II.4; Hubien, p.59, ll.18-26).

And since we have established that (P20′) can *never* be true, it can't have been true then, either. But (P20″) itself is, in any such case, true, because its first disjunct, namely "What is God-while-creating can not-be God", is true. The reason seems to be that God-while-creating is nothing, and nothing is able not to be God. Thus, the truth conditions for (P20′) and (P20″) differ, and moreover (P20″) gives us truth conditions we don't want for (P20). Hence the correct propositional schema for analysing (S1) is (S1′), not (S1″).

Let me remark in passing that from Buridan's analysis of modals in terms of hypothetical or complex propositions (*propositiones hypotheticae*), it does not follow that modals are themselves invariably hypothetical. As we saw in Chapter 1, §1.2.4, Buridan analyses categorical propositions in hypothetical terms as well, so that for instance "every man runs" is to be analysed as the conjunctive string "Plato runs, and Socrates runs, and Robert runs..." where each and every man gets named sooner or later. But such analysis does not imply that these categoricals really *are* hypotheticals in themselves. And neither does the disjunctive analysis of modals we're considering here imply that they are covert hypotheticals after all. Granted, they are semantically equivalent with a hypothetical, but they are not hypotheticals by their syntax.

Now that he has established the different truth conditions for propositions like (P20′) and (P20″), we might expect Buridan to say a little more about why he thinks we should prefer (P20′) to (P20″). But Buridan is quite terse on this: he tells us that he prefers this way because it allows him to analyse categorical propositions like (P20) by means of other categoricals like (P20′), rather than by hypothetical propositions like (P20″). As he tells us,

> The reason why I take the first approach, and not the second, is because if I say 'every B can be A', there is one subject and one predicate, and one simply categorical proposition; and the subject is distributed all at once with a single distribution. Therefore, it seems better that it should be expounded by means of a single proposition, and a categorical one at that, with one subject, and one predicate.[467]

So modal propositions remain categorical, even on this disjunctive analysis.[468] This allows Buridan to more readily distinguish categorical modal propositions from hypothetical ones. And, as we will see, this distinction is crucial for any system that distinguishes modality assuming the permanence of the subject (*de quando*) from the conditional (*conditionalis*) sort.

Hence there are both semantic and syntactic reasons to prefer approach (i) to approach (ii): semantic, because (i) gives us the truth conditions we want for propositions like (P20), which (ii) infelicitously renders true. And syntactic, because (i) allows us to keep modals categorical, rather than hypothetical, the way (ii) cashes them out. Thus for

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[467] "Causa autem quare ego teneo primam viam et non secundam est quia si dico 'omne B postest esse A', ibi est unicum subiectum et unicum praedicatum et una propositio simpliciter categorica, et subiectum est simul unica distributione distributum. Ideo melius esse videtur quod exponatur per propositionem unam etiam categoricam, de uno subiecto et uno praedicato" (*TC* II.4; Hubien, pp.59-60, ll.41-6).

[468] Admittedly, Buridan doesn't explicitly *say* here that such categoricals with disjunctive subjects are full-fledged categoricals. But it seems he'll have to treat them that way, as he does propositions like "the one reading and disputing is a master or a bachelor" (*legens et disputans est magister vel bacculaureus*) in *Summulae* 1.3.2, which we considered in Chapter 1, §1.2.1, above. Here, as ever, what matters is what the principal part is; and here, that part is a copula, not a disjunctive particle. Therefore, etc.

Buridan modal propositions correspond with categoricals with disjunctive subjects, whose syntax matches up with the semantics of ampliation.

Now so far we've been talking about propositions *de possibili*, and the ampliation of their terms to *possibilia*, as though they were representative of all modal propositions. But why suppose that what holds for the semantics of propositions *de possibili* likewise holds of those *de necessario*? It accordingly remains to be shown that *all* modal propositions ampliate their terms to *possibilia*. Buridan anticipates this question in the *TC* (II.6, concl. 2). There, he tells us that:

> In every divided modal proposition about necessity [*de necessario*], the subject is ampliated so that it stands for things that can exist. And this conclusion is clearly shown: for otherwise, propositions about necessity would not be equivalent to those about possibility that have a negated mode, since in these latter propositions [sc. those about possibility] it is clearly granted that the subject is ampliated.[469]

Here, then, is how the proof runs: suppose for *reductio* that propositions *de necessario* do *not* ampliate their subject terms to *possibilia*. However, propositions *de possibili* do, as is clear from their subject matter. But propositions *de possibili* have equipollents *de necessario*. So they have equipollents with which they do not share their subject terms. But this flies in the face of the law of equipollence, on which equipollent propositions have

[469] "In omni propositione de necessario divisa subiectum ampliatur ad supponendum pro his quae possunt esse. Haec conclusio manifeste apparet. Quia aliter illae de necessario non aequipollerent illis de possibili habentibus modum negatum, cum in illis de possibili subiectum manifeste concedatur sic ampliari." (*TC* II.6; Hubien, p.63, ll.59-63).

to have the same terms, which in Buridan's semantics means terms with the same supposition. Our assumption, then, is false. And so, Buridan concludes, all modal propositions ampliate their terms to things which can exist—that is, to *possibilia*.

So much for the semantics of terms in modal propositions *de necessario*. It would've been helpful if Buridan gave us a corresponding syntax—that is, a disjunctive reading of propositions *de necessario*, akin to the one he gives us for propositions *de possibili* (rendering (S1) as (S1′), as we saw above). But he doesn't. If he had, it seems he would've cashed them out along the following lines:

S2) B is necessarily A

S2′) What is or can be B *must be* A

What does it mean to say some possible or actual B *must be* A? That is, how are we to understand the copula of (S2) and (S2′)? Recall Buridan's definition of *de quando* or Grade-3 necessity of assertoric propositions (discussed in Chapter 2, §3, above), which we can generalise to propositions *de necessario*:

> A third grade occurs with the assumption of the constancy of the subject; for instance, 'A lunar eclipse takes place because of the interposition of the earth between the sun and moon', 'Socrates is a human', and 'Socrates is capable of laughter'. These are said to be necessary in this way because it is necessary for Socrates, whenever [*quandocumque*] he exists, to be a human

> being capable of laughter. And it is necessary that, whenever there is a lunar eclipse, it occurs because of the interposition of the earth, etc.[470]

Hence an assertoric proposition that is necessary at Grade 3 can be falsified, but only by the annihilation of its subject. For instance, contrast the following propositions:

P21) Some humans are animals

P22) Some humans are bearded

Since, as affirmative propositions, both (P21) and (P22) have existential requirements, they can be rendered false by the annihilation of their subject: if all humans were annihilated, (P21) would become false. This, indeed, is the *only* way to render (P21) false. But we needn't resort to killing in order to render (P22) false: some finite number of shaves will do. The difference, then, between (P21) and (P22) is that (P22) can be falsified without annihilating its subject. But, so long as at least one human exists, (P21) will be true. For any human, then, there is no way to make it false that that human is an animal, except by annihilating that human.

How can we apply these observations about Grade-3 necessary assertorics to true *de necessario* modal propositions? First, notice that everything but God can be annihilated. But then, although these existing things can be annihilated, their annihilation only removes them from the realm of *actual* objects. Such annihilated existents remain possible

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[470] "Tertius gradus est ex suppositione constanti subiecti, ut 'lunae eclipssis est propter interpositionem terrae inter solem et lunam', 'Socrates est homo', 'Socrates est risibilis'. Hae enim dicuntur necessariae sic quia necesse est quandocumque est Socrates, ipsum esse honinem risibilem, et necesse est quandocumque est eclipsis lunae, ipsam esse propter interpositionem terrae, etc." (*Summulae* 8.6.3; de Rijk 141, ll.22-6).

ones. That is, there is no way to so thoroughly annihilate something that it becomes an impossible object. Hence although (P21) will be false if there are no humans, its corresponding proposition *de necessario* won't be:

P23) A human is necessarily an animal

A proposition like (P23) cannot be rendered false by annihilating all humans, since even if there are no actual humans, there will be possible ones; and those possible humans cannot but be animals. And whatever is a possible human—that is, whatever *can* be a human, must be an animal. So (P23) is true.[471]

Returning to our disjunctive reading of the *de necessario* propositional schema (S2), the truth conditions seem to be that no B, actual or possible, can be made other than A—at least without making it no longer B. Consider for instance the following true proposition *de necessario*:

P24) Some human is necessarily an animal

Now cash out (P20) along the lines of (S2′):

P24′) What is or can be a human must be an animal.

[471] Note that from the foregoing, it follows that God cannot render *possibilia* impossible, though He can annihilate any of the *actualia* (apart from Himself). Buridan nowhere discusses this aspect of his modal metaphysics, likely with good reason: placing theoretical restrictions on God's power, especially for a non-theologian like Buridan, is a pretty hazardous thing to do at the fourteenth century University of Paris.

Hence we can give a disjunctive reading of the subject of propositions *de necessario* along the lines of their corresponding propositions *de possibili*. Where the former differ from the latter is in their copulae: *de possibili* just says B *can* be A; whereas *de necessario* claims that B *cannot be other than* A. This will be very important in what follows.

In sum: the subjects of modal propositions ampliate to *possibilia*. What distinguishes propositions *de necessario* from *de possibli* is that the former say of the (ampliated) subject that it *must* be a certain way, and the latter only say that it *can*.

Now we've seen that Buridan takes affirmative assertorics to have existential import. Do affirmative modals have existential import of an analogous sort?

1.2.1. The Intensional Requirement

In his treatment of assertorics, Buridan takes all affirmatives—particular *and* universal—to have existential import; negatives, conversely, have none. This is why universal negatives with subject (or predicate) terms that do not supposit for anything will be vacuously true, for instance:

P25) A chimaera isn't a chimaera.[472] (T)

[472] "chimera non est chimera" (*QM* VI.8; fol. 38c).

Now Buridan adopts the Principle of Bivalence. Therefore, he can take the cause of truth of propositions like (P25) to be—not chimaeras or anything of the sort, but—just whatever accounts for the failure of their contradictory affirmatives. The contradictory of (P25) is:

P26) Every chimaera is a chimaera (F)

Which is false. Hence we really only have to account for the truth of affirmatives, and define the truth of negatives derivatively.

Here, then, is the problem that this treatment of truth for assertorics poses for the truth of modals: what are we to say about the suppositional requirements for the terms in modal propositions? At least, we have a sort of attenuated existential requirement (call it the Intensional Requirement (IR)), defined on the class of *possibilia*:

IR) If the ampliated terms of an affirmative modal proposition fail to stand for any *possibilia*, that affirmative proposition is false—and, accordingly, its negative contradictory is true.

How can a term fail to stand for any *possibilia*? Just if it describes *impossibilia*.

Ampliation, as we've seen, stretches the extension of the terms, so that they can stand for things that don't exist, along with things that do. So though the objects they stand for do not (all) exist, these terms still can stand—or fail to stand—for something.

For instance, here is a proposition where supposition holds, even though the subject term doesn't refer to anything actual:

P27) Varro's Menippean satires are possibly quite funny

Even though its object doesn't exist, (P27) is probably true: at least some of the (now lost) Menippean satires were, by all accounts, real knee-slappers. In any case, compare (P27) with a proposition with ampliated terms that describe *impossibilia*, and therefore stand for nothing:

P28) No round square is possibly a round square. (T)

There are no round squares among the possibilia; and so a negative proposition like (P28) will be (vacuously) true, just like its assertoric counterpart

P28′) No round square is a round square.

But then what do we say about the putatively affirmative equipollent of (P28)? Namely:

P29) Every round square is necessarily not a round square (T)

This looks affirmative. And so, by IR, it should be false, since its term extensions are empty. But, by the law of modal equipollence (whereby *necessarily* is equivalent to *not possibly not*, and so on), it is equivalent to (P28), which is true. So (P29) is true. But why? Judging by its form, (P29) should have existential requirements.

It gets worse: (P28) and its equipollent (P29) will share two contradictories: an affirmative and a negative, both equipollent with each other, and both false. Here they are:

P30) Some round square is possibly a round square (F)

P31) Some round square isn't necessarily not a round square (F)

Now (P30) is a particular affirmative (I-type), and (P31) a particular negative (O-type). But both are false, just as (P28) and (P29) are both true. Yet by the existential requirements analogous to those developed for assertorics, we should expect an O-type like (P29) to be vacuously true, just like its assertoric counterpart:

P29′) Some round square isn't a round square (T)

But (P31) must be false—or else Buridan's whole modal octagon falls apart. Even so, it looks like it has to be true. The situation is grim.

In sum, there is evidently a tension here between two things:

i) modal equipollences, and

ii) IR

We surely don't want to ditch (i), which just about everyone in the history of logic from Ockham to Kripke has unqualifiedly accepted (though with the conspicuous exception of Arthur Prior).[473] Buridan is doubtless among these thinkers—and we'd need industrial grade textual support if we wanted to claim otherwise.

So can we do without (ii)? Buridan's treatment of existential requirements for assertorics set us down this road. After all, extending assertoric semantics to modals here seemed natural enough. But were we right to do so? Consider the schema for a universal affirmative *de possibili*, where *‹a›* and *‹b›* are ampliated *a* and *b*, respectively:

S3) Some *‹a›* is possibly *‹b›*

There are two ways for a proposition constructed on (S3) to be false: either because *‹a›* does not overlap with *‹b›*, or because *‹a›* (or *‹b›*) stands for an impossible object—a chimaera or a round square, or some such.

Now the contradictory form of (S3) is (S4), a particular negative *de necessario*:

[473] Prior argues that in temporal logic, $\Box$ is stronger than, but implies, $\sim\Diamond\sim$. Here's why: if you read '$\Box$' as *always* and '$\Diamond$' as *sometimes*, and then formalise *e.g.* "it either is, has been, or will be the case that someone is flying to the moon" as $\forall x\Box Fx$, then S5 + the usual derivation rules get you the Barcan formula $\Box\forall xFx$—which either commits you to the immortality of Neil Armstrong, or the B-theory of time, on which theory the flow of time is an illusion. Prior's not against the B-theory, but thinks its claims shouldn't be baked into our logic, lest we beg the question. See Prior's (1957) *Time and Modality*.

S4) Every ‹*a*› is necessarily not ‹*b*›

Just like (S3), (S4) can be false in two ways: if the relations between *possibilia* answering to ‹*a*› and ‹*b*› are not as it describes, or if there are no *possibilia* answering to ‹*a*› (or ‹*b*›) whatsoever. That is, it can be false about some class of *possibilia,* or it can be vacuously false. So here, both modals can be vacuously false, because they are about *impossibilia*; or they can be false in virtue of the way they describe 'real' *possibilia.* To clarify: these stand in contrast to *impossibilia*, which are not a class of things which can be referred to, but merely what can't be picked out because it involves an incompossible combination of parts, as we saw in Chapter 3, §1.1.

Now we would expect the equipollents of (S3) and (S4) to behave the same way: that is, to have two ways of coming out false. Here are those equipollents, respectively:

S5) Some ‹*a*› isn't necessarily not ‹*b*› (equipollent of S3)

S6) No ‹*a*› is possibly ‹*b*› (equipollent of S4)

Prima facie, at least, the form of (S5) looks like a particular negative (O-type). So it should be true either if the class of ‹*a*› is not necessarily included in the class of not ‹*b*›, or—by the existential requirement—if ‹*a*› (or ‹*b*›) fail to refer. The first is fine: under the same circumstances, (S3) will be true. Here is a diagram:

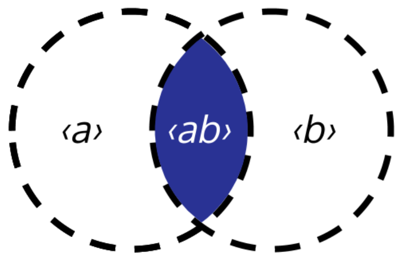

Fig. 5.3: overlapping sets of possibilia.

The dotted lines are meant to display agnosticism about whether the boundaries between ‹a› and ‹b› can be crossed—in contrast with the hard boundaries in fig. 5.4, below.

Both (S3) and (S5) are true in the diagrammed case: (S3) because *‹a›* and *‹b›* overlap; and (S5) because, given the overlap *‹ab›*, at least one *‹a›* is not necessarily not *‹b›*.

But the second way (S5) can be true presents a serious problem: *‹a›* and *‹b›* failing to refer was a condition for the falsity of (S3), which is equipollent with (S5). So a proposition can be false under the same conditions that would render its equipollent true—a clear contradiction.

The situation is no better for (S4) and (S6): both (S4) and (S6) can be true either if every *‹a›* is necessarily excluded from *‹b›*, which we can diagram as follows:

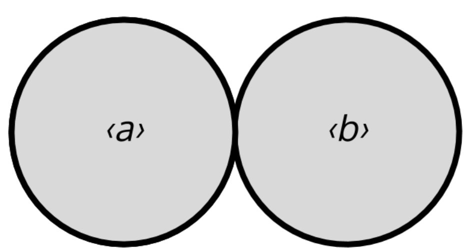

Fig. 5.4: mutually exclusive sets of possibilia

The diagrammed case is sufficient to make (S4) and (S6) true. But as a universal negative, (S6) should further be true if the terms are empty—under which conditions, as we saw earlier, (S4) is false.

It seems, then, (IR) is the source of the problem. Can't we just get rid of it? Notice that (IR) fails only when the ampliated terms *‹a›* or *‹b›* are intensionally empty—that is, when they describe *impossibilia* like chimaeras and round squares. But we want to be able to say of these things that they do not exist as a matter of necessity. So at least some propositions about them, both affirmative and negative, are true—namely, propositions like (P28) and (P29), considered above:

P28)	No round square is possibly a round square	(T)
P29)	Every round square is necessarily not a round square	(T)

Still, we want to be able to make statements like these, so completely ditching any existential requirements won't do. Nor will adopting existential requirements for all modals: if we do that, then every statement about chimaeras and round squares will be false—even statements with contradictory forms. So we'll lose the law of contradictories.

Instead, we need to qualify (IR). Here's how: notice that the rule governing modal equipollences requires the addition of negation where there is none, and the subtraction of negation where it is. There are two places negation can go: the mode, or the dictum (or both, or neither). But then by this rule, any modal with a single negation will convert to another with a single negation; those with no negative terms will convert to those with two, and vice versa. Therefore, we can divide the modal equipollents into two groups: those with one sign of negation in both equipollents, and those with zero or two. Call the first group propositions of *negative quality* (since they will always have at least one negative sign), and the second group propositions of *affirmative quality* (since intuitively, two negatives cancel each other out).

Now consider the list of eight canonical forms for modals given in Buridan's modal octagon (the LHS of the list below), match them with their equipollents (the RHS), and mark them for positive and negative quality. Here, I group contradictory pairs:

1. Every S is necessarily P (+) $\Leftrightarrow$ No S is possibly not P (+)
2. Some S is possibly not P (–) $\Leftrightarrow$ Some S isn't necessarily P (–)

3. Some S is necessarily P (+) $\Leftrightarrow$ Some S isn't possibly not P (+)
4. Every S is possibly not P (–) $\Leftrightarrow$ No S is necessarily P (–)

5. Every S is necessarily not P (–) $\Leftrightarrow$ No S is possibly P (–)
6. Some S is possibly P (+) $\Leftrightarrow$ Some S isn't necessarily not P(+)

7. Some S is necessarily not P (–) $\Leftrightarrow$ Some S isn't possibly P (–)
8. Every S is possibly P (+) $\Leftrightarrow$ No S is necessarily not P (+)

This list points us to our solution. Notice that equipollents all have the same quality, and contradictory pairs all take opposite quality (here denoted by + and –). Now all we need to do is assign attenuated existential requirements only to those propositions with positive quality—that is, we can just redefine (IR) as follows:

> IR′) If the ampliated terms of an affirmative modal proposition **with positive quality** fail to stand for any *possibilia*, that proposition is false—and, accordingly, its negatively valent contradictory is true.

Hence we were right to extend the existential requirements for assertorics to modals; but doing so along merely negative and positive lines was wrong: instead, we should be extending existential requirements along the lines of positive and negative quality.[474]

So much, then, for the subjects and copulae of modal propositions, and the existential requirements of the propositions themselves. Now we just have to see what's going on with the predicates, and then we can get down to brass tacks.

[474] Incidentally, it seems that the above problem for modals will apply to assertorics, too, so long as we adopt the rule that "every S is P" is equivalent to "no S is not P". After all, the latter seems to be negative, and so it should have different existential (and truth) conditions from the affirmative former. Here again, we can solve the problem by assigning quality: both have an even number of negations. Therefore, the equipollent pair has positive quality. And thus the problem is solved.

1.2.2. What About Modal Predicates?

We've seen that Buridan thinks that modal contexts ampliate the *subject* term beyond its *status*. But does the predicate ampliate, too? Although Buridan does not discuss the ampliation of predicates in modal propositions, it might seem that the predicates *must* be ampliated, along with their subjects. To see why, consider the rule governing modal conversions (*TC* II.6, concl. 5):

> Fifth conclusion: from every affirmative proposition *de possibili* there follows another, particular affirmative *de possibili*, though not a universal.[475]

Such modal conversions correspond with the familiar schema governing conversions for assertorics:

S7) Some S is P
∴ Some P is S

As an example of the modal counterpart to (S7), consider the following:

A3) Some donkey is-possibly running
∴ Some runner is-possibly a donkey

[475] "Quinta conclusio est: ad omnem affirmativam de possibili sequi per conversionem in terminis particularem affirmativam de possibili, sed non universalem" (*TC* II.6; Hubien, p.66, ll.172-4).

Yet in order for conversions like (A3) to hold, the terms have to be the same—that is, the S and P of (S7) have to be the same S and P in the antecedent as in the consequent. Now in both the antecedent and the consequent of (A3), the subjects are ampliated. But the subject of each is the other's predicate. So the predicate terms must be ampliated, along with the subjects, in modal contexts.

We can expand our notation to reflect the fact that both subject and predicate terms are likewise ampliated to *possibilia*, and that the modal operator is a special kind of copula. Here, bracketed terms, $\langle a \rangle$, $\langle b \rangle$ are ampliated, and the copulae set out above are updated as modals:, and underlined $\langle \underline{a} \rangle$, $\langle \underline{b} \rangle$ are terms that must meet (IR′):

de necessario

Universal Affirmative

Every b is necessarily a *iff* $\langle \underline{b} \rangle$ ***must be*** *included in* $\langle \underline{a} \rangle$

$(\langle \underline{b} \rangle \subseteq_{\Box} \langle \underline{a} \rangle)$

Particular Affirmative

Some b is necessarily a *iff* $\langle b \rangle$ ***must*** *overlap* $\langle a \rangle$

$(\langle \underline{b} \rangle \smile_{\Box} \langle \underline{a} \rangle)$

Universal Negative

No b is possibly a *iff* $\langle b \rangle$ ***must*** *exclude* $\langle a \rangle$

$(\langle b \rangle \mid_{\Box} \langle a \rangle)$

Particular Negative

Some b is not possibly a *iff* $\langle b \rangle$ ***cannot be*** *included in* $\langle a \rangle$

$(\langle b \rangle \mathrm{O}_{\Box} \langle a \rangle)$

de possibili

Universal Affirmative

Every b is possibly a *iff* *‹b›* ***can be*** *included in ‹a›*
$(\langle \underline{b} \rangle \subseteq_{\Diamond} \langle \underline{a} \rangle)$

Particular Affirmative

Some b is possibly a *iff* *‹b›* ***can*** *overlap ‹a›*
$(\langle \underline{b} \rangle \smile_{\Diamond} \langle \underline{a} \rangle)$

Universal Negative

No b is necessarily a *iff* *‹b›* ***can*** *exclude ‹a›*
$(\langle b \rangle \mid_{\Diamond} \langle a \rangle)$

Particular Negative

Some b is not necessarily a *iff* ‹b› ***can be not*** *included in* ‹a›
$(\langle b \rangle \ \mathrm{O}_{\Diamond} \langle a \rangle)$

Now compare the relationship between (ampliated) subject and predicate in the following two propositional schemata:

S8) $\langle \underline{b} \rangle \subseteq_{\Box} \langle \underline{a} \rangle$

S9) $\langle \underline{b} \rangle \subseteq_{\Diamond} \langle \underline{a} \rangle$

On the face of it, there's no way to represent the difference between these relations with Venn diagrams, the way we did for assertorics above: after all, both assert that the ampliated ‹b›'s are included in the ampliated ‹a›'s. But (S8) says they *have* to be: there is no way to remove *‹b›* from *‹a›*, without rendering it no longer *‹b›*. Conversely, (S9) is agnostic on whether ‹*b*› *has to be* included in ‹*a*›, or whether ‹*b*› just is ‹*a*› as a matter of contingency. All (S9) states is that the class of things that can be ‹a› can include the class of things that can be ‹b›. The modal work being done here is, accordingly, a function of the copula, which tells us about the removability of one class from the other.

Similarly, compare particular affirmatives *de necessario* and *de possibili*:

S10) ‹*b*› $\smile_{\Box}$ ‹*a*›

S11) ‹*b*› $\smile_{\Diamond}$ ‹*a*›

If (S10) is true, some ‹*b*› is necessarily ‹*a*›. So the classes of ‹*b*› and ‹*a*› overlap, but in a way that can't be altered: for at least one ‹*b*›, it is impossible to remove it from ‹*a*› without removing it from ‹*b*›—that is, without making it no longer ‹*b*›. Conversely, if (S11) is true, then ‹*a*› and ‹*b*› overlap, but this is just to say that for at least one ‹*b*›, it *can* be ‹*a*›—not that it *must* be. So again, the modal work being done is a function of the copula, and how it relates the subject and predicate.

Thus accounting for this modally-functioning copula is difficult, especially in terms of the static notions of class inclusion that we learned in elementary set theory, which deals with set membership only in terms of quantity: either the set of all *a* is a subset of that of all *b*, or not—that is, either:

$\{ x \mid x \text{ is a } b\} \subseteq \{y \mid y \text{ is an } a\}$, or

$\{ x \mid x \text{ is a } b\} \not\subseteq \{ y \mid y \text{ is an } a\}$.

But the pairs under consideration, (S8) and (S9), (S10) and (S11), deal with the *removability* of one class from the other—something not handily expressed with our usual notation. What notation, then, should we use? The best contender by far is Paul Thom's, which I have adapted here.

1.2.3. Thom on Ampliation

We saw in Chapter 3 (§2.1.1) that modern (which is to say Fregean) logic and semantics has little interest in the copula. Indeed, Geach follows Frege in claiming that the copula has no logical content whatsoever, and is therefore dispensable. To us, reared as we are in this tradition, Buridan's emphasis on the copula as the principal formal part of an assertoric proposition, and his willingness to pack operations like modality and negation into it, are accordingly quite unfamiliar. It is tempting, then, just to ignore this role of the copula and, when the copula is a modal one, shift the modality elsewhere. One prime candidate seems to be the predicate. So we can shuffle the modality—which, Buridan will insist, belongs to the copula—onto the predicate and, *voilà*, this unfamiliar class of modal copulae is removed from the picture. If nothing strange results from this, then Frege and Geach are right about the copula. If the resulting semantics is irreducibly different from Buridan's, then we have to admit that the copula *does* have some logical content after all.

Paul Thom, in his thorough and exhaustive (2003), supplies a notation which shifts modality from copula to predicate.[476] In this notation, universal affirmatives *de necessario* and *de possibili* are written as follows:

S12) $\underline{a}^{\dagger} \rightarrow b^{*}$

S13) $\underline{a}^{\dagger} \rightarrow b^{\dagger}$

Here '→' denotes class-inclusion, and the superscript dagger ('†') ampliation of a term to *possibilia*. Notice two things. First thing: the '→' is just the ordinary copula, not a modal one: on these lights, we will likewise write a universal assertoric predication as ($\underline{a} \rightarrow b$). What distinguishes ($\underline{a} \rightarrow b$) from the modal (S13) is that its subject and predicate terms are unmodified by a mode. But the copula in both is the same. Second thing: the predicate of (S12) is modified by a superscript asterisk ('*'). This asterisk denotes ampliation of a term to *necessaria*. Hence in contrast with Buridan's account of a universal proposition *de necessario*, which says that all *a*'s are necessarily *b*, (S12) says that every possible-*a* is a *necessary-b*.

This difference is not trivial. For example, see what happens when we take a modal proposition, translate it into (S12), and then translate it back into plain English—for example, by substituting *dodo* and *bird* for *a* and *b*. Whereas such a substitution of terms in the schema (S12) is supposed to say:

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[476] Paul Thom, *Medieval Modal Systems* (Aldershot: Ashgate, 2003).

P32) All (possible) dodoes *are-necessarily* birds

What the retranslation of (S12) *actually* says is:

P33) All possible dodoes are *Necessary Birds*.

We might soften this a bit, and read it as follows:

P33′) All possible-dodoes are things-which-are-necessarily-birds

Still, the reading presented by (33)—and attenuated in (33′)—leaves us with historical, ontic, semantic, and syntactic problems. Let's consider these in order.

1.2.3.1. Historical Questions

We've seen that the mode is a special sort of copula, which ampliates the subject and predicate. This was established here and in Chapter 3 (§2.1.3) as a point on which Buridan's text is clear. But here we might wonder whether the modal copula can be further divided into two elements: an assertoric copula, and a mode of it. If so, then the modal copula might be construed as ampliating the subject, but only so as to produce what would be an assertoric if the subject and predicate stood for *actualia* only, and not *possibilia*. Anyway, surface grammar seems to suggest this: *possibly-is* looks reducible to *possibly* and *is* (and things are similar with the shallow etymology of *potest*, which derives from *potis est*).[477]

But I do not think this can be done, for three reasons: two textual, and one rational. First, as we saw in Chapter 3 (§2.1.3), Buridan's term for what distinguishes modals from their assertoric counterparts is *substantia*, which he regards as a function of the copula. The difference between them is, therefore, not merely accidental, but a matter of substance—that is, of their basic makeup or stuff. Or so the term is generally used in Scholastic contexts.

Second, Buridan uses the same language to describe the interaction between mode and copula as he does for that between negation and copula: both modal adverbs and wide-scope negation 'come down on' the copula (*cadunt super copulam*). We saw this in Chapter 3 as well (§§2.1.1 and 2.1.3). But we also saw there that, in spite of surface grammar, Buridan emphatically denies that the negative copula is further resolvable into negation and an affirmative copula. They are qualitatively different—i.e. they differ in *qualitas*—and irreducible. Things are the same with modal copulae, which differ in *substantia*.

Finally, there is a rational argument to be made that they are not merely assertorics extended to *possibilia*, the way that ordinary assertorics just deal with *actualia*. For if they were, the following two propositions would be basic A-type assertorics with *possibilia* in the extensions of their terms, with little else to tell them apart:

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[477] I owe these suggestions to Calvin Normore (private correspondence, April 21, 2021).

P34) Every donkey is-possibly running

P35) Every donkey is-necessarily running

Here's what a reduction would look like: we would first mark off the terms ampliated to *possibilia*, as follows:

P34′) Every ‹donkey› is-possibly ‹running›

P35′) Every ‹donkey› is-necessarily ‹running›

Now both (P34′) and (P35′) answer to the assertoric form "Every S is P". But beyond that, they say something that their assertoric counterpart cannot: (P34′) tells us that all the donkeys are among all the possible runners, whereas (P35′) tells us both that they are, and that moreover none of them can be removed from the possible runners. This is why (P35′) implies (P34′), but not the other way around. These claims about removability of one class from another are precisely what's absent from ordinary assertorics.

Granted, we can express such irremovability about *actualia* by using a modal with a *quod est* ('that which is') locution. As we've seen (Chapter 2, §3.3; Chapter 3, §2.1.3), this locution blocks the ampliation of a modal proposition, so that its terms stand for *actualia* only. But such a proposition already includes a modal copula, and so is not an assertoric, either.

Hence the mode is a function of the copula, and an irreducible one at that. Moreover, it is not a subject or a predicate, at least in divided modals. Here, then, the copula is being deprived of the job it was historically assigned.

Now *on its own*, this charge is so minor it's hardly worth mentioning—and indeed, even bringing it up would seem a bit nitpicky. Assuming, that is, that it's the only problem. After all, the same charge could be made against the restorers of a medieval mill—of the sort they have in old European castles-*cum*-museums—which now runs off electricity to demonstrate its function, rather than water to grind wheat. Granted, it's no longer working the way it originally did. But if the function is still all there, why quibble?

Thus if the historical problem is the only problem, then at least from the perspective of the logic it doesn't matter all that much. But the historical problem is not the only problem: there is a price to pay for shifting the copula's modal role elsewhere. As we're about to see, it gets paid at all levels: ontic, semantic, and syntactic. We have to analyse these carefully, since they have had a profound influence on the way many commentators have read and undertaken to model Buridan's modal logic.

1.2.3.2. Ontological Questions

This section deals with necessary objects. It shows that on this reading, combined with Buridan's modal ontology, anything—at least, anything belonging to a natural kind—becomes a kind of Necessary Being, apparently at Grade I/1 of the modal scales.

Suppose we read the predicates of universal propositions *de necessario* as b^*—*i.e.* as 'what is necessarily *b*' or even 'necessary-*b*'. Then it seems that, in addition to a whole class of *possibilia*, we'll have to posit not a few *necessaria*. And indeed, any natural kind will have its corresponding class of *necessaria*, since such natural kinds can figure in (true) *per se* and therefore necessary predications. But it remains unclear what the Necessary Bird that every dodo (fantail, magpie, etc.) is. Even so, our modalised predicates call for a whole class of such Necessary Items.

Here is what Thom tells us about the operations of these modal operators, '†' and '*', on terms:

> Sometimes it will be useful to have a way of representing what necessarily, or possibly, or contingently falls under a given term. To do this I will superscribe the symbols '*' '†' and '‡' respectively, respectively to the term-letter. Thus if 'w' stands for what is white, 'w^*' will stand for what is necessarily white, '$w^†$' for what is possibly white, and '$w^‡$' for what is contingently white.[478]

And, lest there be any doubt about the class-inclusion of such terms, Thom gives us the following axioms (where '→' denotes class inclusion, and the axiom numbers are Thom's):

> **Axiom 1.7.** $a^* \rightarrow a$
>
> **Axiom 1.8.** $a \rightarrow a^†$.[479]

[478] Thom, *Modal Systems*, 18.
[479] Thom, *Modal Systems*, 18.

Axiom 1.7 tells us that all the necessary birds are birds; 1.8 tells us that all the birds are possible-birds. Since '→' is transitive, we can combine Axioms 1.7 and 1.8 to get $a^* \rightarrow a \rightarrow a^\dagger$ (and therefore, by Barbara, $a^* \rightarrow a^\dagger$), which we can represent as follows:

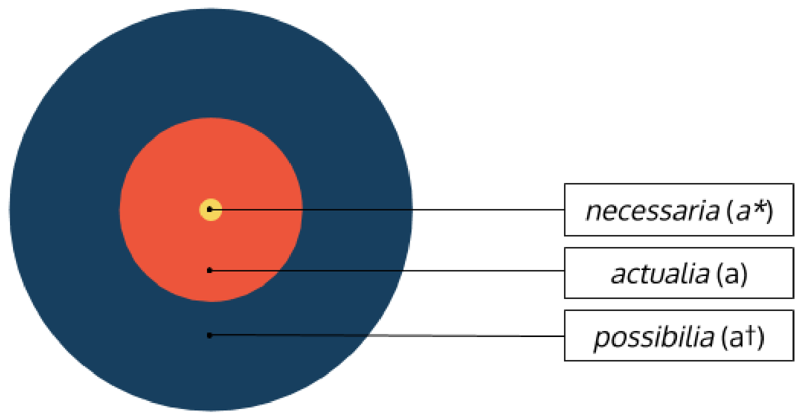

Fig. 5.5: Paul Thom's class-inclusion for universal propositions

Thom's exposition raises an important question: how big is the bullseye of the above Venn diagram? That is, how many *necessaria* are there? We've already seen that Buridan is not opposed to positing necessary items: God is one of these *necessaria*. Indeed, God has to be the only one, or else many things would have Grade 1 modality, and so we would wind up with a polytheistic ontology—including a plurality of indestructible necessary natural kinds. But that's precisely the problem we introduce if we read necessity into the predicate. If we do, it seems we have to posit a whole host of necessary items like birds, planets, and humans. But then we get a much-too-rich modal ontology: something like a Meinongian jungle with indestructible natural—and even artificial—kinds. And then we end up conflating Grade-3 (*de quando*) necessity with Grade 1 (*simplex*), since we have to impute to those kinds a sort of necessary existence.

It may be objected here that this is not what Thom means: rather, Thom means *Bird** on his notation to denote not Necessary Birds, but what is *necessarily* a bird. This much seems certain, but it is hard to see how this is consistent with the notation—as we can see in the shift from the adjective *Necessary* to the adverb *necessarily* in the previous sentence. Modal operators like the superscript '†' and '*' modify terms, as grammatical adjectives do, and not verbs, the way adverbs do.

I want to address a point here in passing. It seems that a single possible object could be picked out by different, mutually incompatible terms: a table is a possibly blue object, and a possibly white one, since it could be painted either colour. But it can't be painted both. So while the ampliated predicate terms (*blue*, *white*) are disjoint, the object remains one.[480] Now it seems that, just as a white table can be green, a green table can be white—at least, at some future time, since as we saw in Chapter 4, §3.1, for Buridan, contingencies are future-oriented. But there cannot be a table which is white-all-over and green-all-over at the same time, and there is no overlap among the disjoint predicates. So it seems that the different possibilities this point raises are *de dicto* and not *de re*, since it is true of a white table that it can be green *de re*, but not *de dicto*. And the ampliated modals dealing with *possibilia* most closely correspond to modern *de re* possibility, as we noted above, at the outset of the present chapter (with the usual caveats about

[480] I owe this suggestion to Calvin Normore (personal correspondence, April 21, 2021).

anachronism and imperfect correspondence). So we need not posit, in this context, green and white tables among the *possibilia*.

Now at least some—though arguably all—of the *possibilia* must have some essence or other. For instance, all dodoes will be Necessary Birds. Any claims about their essences will be necessary, in the sense of being unfalsifiable. Thom is attentive to this underlying Aristotelian essentialism about natural kinds, which he cashes out as follows (where '$\Box$' is just our familiar necessity operator):

> We can define a term '*a*' as being *per se* provided that it is necessary that whatever is *a* is necessarily *a*.
>
> **Definition 1.6.** '*a*' is per se iff $\Box a \rightarrow a^{*}$. 'Horse' is a *per se* term according to Aristotle. 'White' is not a *per se* term, because, even if it happened that all white things were necessarily white, it would still be possible that something white was not necessarily white.[481]

So any *per se* term will be necessarily something—that is, it will have an essence. But by Definition 1.6, even if everything in existence had some necessary property like being white, it would not follow that it was necessary that they be necessarily so. To see why, consider the things that are necessary quadrupeds, and suppose they were the only things that presently existed. Then for everything in existence, it would be necessary for it to be a quadruped. Even so, it would not be necessary that everything existing was a quadruped,

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[481] Thom, *Modal Systems*, 19.

in the sense no biped could possibly exist. God could always make bipeds *ex nihilo*, the way he was able to produce humans in the thought experiment discussed above (Chapter 2, §3.3). Therefore, though everything that exists in such a case would necessarily be a quadruped, it would not be necessary that all things that existed be quadrupeds.

Thom rightly claims, in his exposition of Buridan, that at least *some* things will have their properties necessarily—which is to say, *per se*:

> Buridan makes use of the basic essentialist assumption that some individuals possess some of their properties essentially, and therefore necessarily.[482]

By *some* here I take Thom to mean *some-but-not-all*. This produces a difficult ambiguity, which turns on how we read the scope of *some*. Do some individuals possess all their properties contingently, and not necessarily? If so, then we are allowing the existence of pure *contingentia*: things that only have accidental properties, and no essences whatsoever. Can Buridan countenance any things that have no essences whatsoever—that is, which have no essential properties? Are there any such pure *contingentia* on his ontology? I see no textual evidence to support the claim that there are, though perhaps one could make a case for outliers like mud and hair, following Parmenides in Plato's dialogue.[483] Otherwise, anything belonging to a natural kind will count as a necessary being on this reading. Granted, all things apart from God are contingent as concerns their existence, but from this it does not follow that they have whatever properties they have contingently, in the sense that they could exist without any of them, and are therefore

[482] Thom, *Modal Systems*, 200.
[483] *Parmenides* 130c-d.

essence-free. This is not consistent with Thom's careful treatment of Aristotelian essentialism, and cannot be correct.

Hence the scope of *some* here must be narrower: all things, at least those belonging to a natural kind, possess *some* (*but not all*) of their properties accidentally. But all things possess some (though again, not all) properties essentially, too. But then this introduces a new problem: every one of the *per se possibilia* will be this or that *necessarium*. If so, then no matter what features any possible thing contingently possesses—for instance, the contingent whiteness of a (possible) bird or cloud or star or whatever—those things will still have some essence or other, even if that essence does not include being white: being a necessary animal, a necessary celestial body, or a necessary vapour. If so, the bullseye of our above diagram expands to fill the whole—that is, our above Venn diagram just blends into an undifferentiated modal soup, *comme ça*:

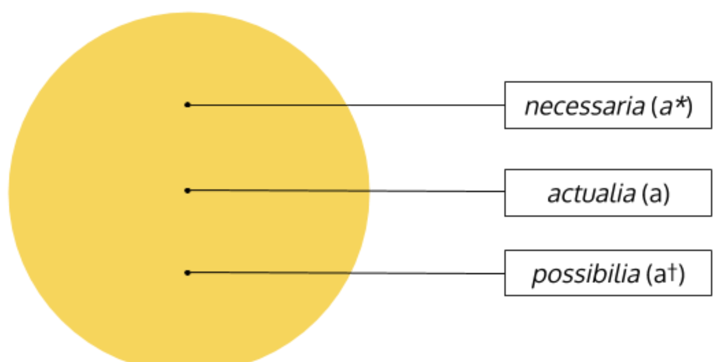

Fig. 5.6: the centre cannot hold.

Hence the *necessaria* include everything belonging to a natural kind, and so our list of necessary—and therefore, by Buridan's modal scales indestructible—items grows very long.

1.2.3.3. Semantic Questions

Let's set aside these metaphysical worries. The next question is whether the foregoing ontic puzzles seep into the semantics; and if so, whether the semantics alter the syntax in an undesirable way. And indeed, there is reason to worry about the existential conditions for terms in propositions about Necessary Birds and the like. In positing the Necessary Bird(s) which every dodo is, we have conflated Grade 3 modality with Grade I/1. But to accommodate the fact that dodoes (and some other *necessaria*) don't exist, we will have to dispense with existential conditions on propositions about *necessaria*: things that can be necessary, but not exist—unless, of course, the *necessarium* in question is God. What are we to say about these other non-existent *necessaria*?

To cut the knot, we could just say that *if* dodoes exist, *then* they're Necessary Birds—and thus cash out Grade3 modality in conditional terms. But then this conditional reading of necessity conflates Grade 3 (*de quando*) with the lower, conditional necessity of *QAPr* (II.25). Conditionally necessary propositions include things like "every four-sided triangle, if it exists, is a plane figure"—propositions that are necessarily true because their subject terms are empty. So to avoid conflation of *simplex* and *de quando*, we've conflated *de quando* with *conditionalis*. Or, to return to the ontic language of the preceding section:

we have avoided the conflation of *necessaria* with *possibilia* by conflating *possibilia* with *impossibilia*.

Now Thom's discussion of the semantics of terms modified by '†' and '*' is brief: it takes up about two pages of his whole exposition, and we have been expounding it very carefully here. But as we have seen, the attribution of modality to predicate terms, rather than to the copula—following Buridan—gives us undesirable results: non-existent *necessaria*, which include all the *possibilia* belonging to a natural kind but which do not exist.[484] Here again, Buridan's approach avoids this problem, since the necessity is a function not of the things under discussion (God notwithstanding), but of the copula: no necessity is imputed *simpliciter* to *things*, on Buridan's modal semantics. In all modal contexts but the theological, we are talking about contingent *possibilia*, not *necessaria*, even though these contingent *possibilia* have necessary attributes.

1.2.3.4. Syntactic Questions

For all these semantic and ontological problems, we can still ask whether Thom's syntax is adequate. Does it get us all and only the inferences we'd like—that is, all and only those that Buridan explicitly endorses in his systematic treatments in *TC*? If so, we can set aside the foregoing ontic and semantic worries. After all, Thom's main concern is syntax; and if his inferential schemata are extensionally adequate, the ontic and semantic worries are arguably irrelevant.

[484] Indeed, beyond dodoes, this will probably also include *possibilia* which belong to non-extinct natural kinds, but which themselves do not exist—for instance, the horses that could be bred, but won't be.

As far as I can see, the relevant chapter of Thom's (2003) accurately sets forth Buridan's modal syllogistic: I have run through all the modal and mixed modal syllogisms Buridan explicitly endorses using Thom's system, and faced no problems. And I have been able to run through the inference rules for divided modals that Buridan presents in *TC* II.6—with one exception: Buridan's first derived rule, which is roughly equivalent to De Morgan's rules for modal propositions. In Buridan's presentation:

> The first conclusion is that from every proposition *de possibili*, there follows by equipollence another proposition *de necessario*, and from every *de necessario* another *de possibili*. And the two are related in such a way that if negation was applied to the mode or the *dictum*, or to both in one proposition, it is not applied in the other proposition; and if there was no negation applied to the one proposition, it is applied to the other, all other things remaining the same.[485]

Here is Buridan's example of one such schema, where negation is absent from the *dictum* of the left hand side (LHS), and applied to that of the right hand side (RHS):

S14) B is necessarily A $\Longleftrightarrow$ B is not possibly not A.[486]

This looks (deceptively) like the following De Morgan rule:

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[485] "Prima conclusio est: ad omnem propositionem de possibili sequi per aequipollentiam aliam de necessario et ad omnem de necessario aliam de possibili, sic se habentes quod si fuerit apposite negatio vel ad modum vel ad dictum vel ad utrumque in una non apponatur ad illud in alia et si non fuerit apposite in una apponatur in alia, aliis manentibus eisdem." (*TC* II.6, concl.2; Hubien, p.61, ll.19-24).
[486] "Istae aequipollent: 'B necesse est esse A' et 'B non possibile est non esse A' (*TC* II.6, concl.2; Hubien, p.61-2, ll.25-6).

S15) $\Box\varphi \iff {\sim}\Diamond{\sim}\varphi$

Of course, (S14) and (S15) aren't equivalent: as we've seen, Buridan does not think of the mode as a sentential operator, and he does not have propositional negation. Rather, as can be easily read off the form, (S14) comes down to the terms.

How are we to represent (S14) on Thom's system? At very least, we'll need to introduce some new operations. Now suppose we want to give a modal equipollence of the following sort:

S16) Every b is necessarily a $\iff$ No b is possibly not a

If we introduce term-negation ('$^{-}$'), we can render (S16) as follows:

S16′) $(b^{\dagger} \rightarrow a^{*}) \iff (b^{\dagger} \mid \bar{a}^{\dagger})$

Here, however, we might wonder: what are we to make of $\bar{a}^{\dagger}$? Simply put, it is the complement of $a^{\dagger}$—i.e. of the complement of the things that are possible-a. What, then, does $\bar{a}^{\dagger}$ denote? We have two options:

i) $\bar{a}^{\dagger}$ is what is possibly **not**-a

ii) $\bar{a}^{\dagger}$ is what is **not**-possibly a

But it cannot be (i), since then the complement of $a^\dagger$ would include what is contingently a—on Thom's notation, $a^\ddagger$—since $a^\ddagger$ is possibly a and possibly not-a. There is thus overlap between $a^\dagger$ and $a^\ddagger$, which Thom's use of the single and double daggers in his notation is likely meant to make clear. Hence reading (i) entails that there is overlap between $a^\dagger$ and its complement, which runs afoul of the definition of complementarity. So we have to go with (ii), and say that $\bar{a}^\dagger$ denotes everything that is not a possible-a. So $\bar{a}^\dagger$ is just all the *impossible*-a. So (S16′) tells us that if the possible-b are contained in the necessary-a, then the possible-b excludes the impossible-a. And this looks right. So the left hand side of (S16′) entails the right hand side.

But since the two are equipollents, we should be able to get back where we started—that is, we should be able to infer the LHS of (S16′) from the RHS. And here things fall apart: the equipollence does not hold. Suppose for instance that b stands for some impossible object(s)—that is, let b be 'four-sided triangles' or 'chimaeras' or whatever. Then the LHS of (S16′) is true, but the RHS is false, since it will not be true of any impossible b that it's necessarily a. So from L to R, (S16′) looks good. Even so, we can't go in the opposite direction, from R to L, they way we would if this were a valid equipollence.

To see why, substitute *chimaera* for b and *animal* for a (the complement of whose *possibilia* class, $\bar{a}^\dagger$, is the impossible animals, whatever those are), which gives us the following substitution instance for the R to L direction of (S16′):

A4)	No chimaera is possibly an impossible animal	(*true*)[487]
∴	Every chimaera is a necessary-animal	(*false*)

So not only do we end up deifying natural kinds, we also drag things like chimaeras up out of metaphysical Oblivion.

In defense of Thom, the problem here seems to be *impossibilia*. Can we avoid the problem by just banishing these, and limiting the domain merely to necessary and possible things only? It turns out we cannot. Consider the following equivalence, which is part of Buridan's rule dealing with modal equipollences:

S17) no b is possibly a $\iff$ every b is necessarily not a.[488]

In our extension of Thom's syntax, we will symbolise this as follows:

S17′) $(b^{\dagger} \mid a^{\dagger}) \iff (b^{\dagger} \rightarrow \bar{a}^{*})$

Suppose $b^{\dagger}$ is not empty, and likewise, neither is $a^{\dagger}$—that is, suppose that there is at least one possible-b and at least one possible-a. Then what (S17′) tells us is that if no possible-b is a possible-a, then every possible-b is an *impossible-a*. So straightaway, some of our *possibilia* get flung out of possible space, and into the land of the chimaera.

[487] How are we to assess the truth conditions for such a tortuous proposition? Well, just treat it as a universal negative which, as we saw, will be true if at least one of its terms fails to stand for anything. Now *chimaera* by definition fails to stand for anything. And so this failure of supposition alone is enough to render the antecedent of (A3) true.
[488] "Istae duae aequipollent: 'B necesse est non esse A' et 'B non possibile est esse A'" (*TC* II.6, concl.1; Hubien, p.2, ll.35-6).

Now let's can plug some (non-empty) terms back into our variables, and run an inference from left to right on the form of (S17′):

A5)	No human is a possible donkey	(T)
∴	Every human is an *impossible-donkey*	(?)

Two things about the conclusion of (A5) stand out. Syntactically, it doesn't look like a modal any more: it looks, rather, like an assertoric with a modalised predicate (*impossible-donkey*). But let's just take it to be equivalent with "Every human is *necessarily* an impossible-donkey"—that is, let's treat it like a universal *de necessario*, and set aside the worry. Semantically, it's still not clear what the predicate term refers to at all: what are impossible donkeys? Are they donkey-like? Can't be, since if they were so, they wouldn't be impossible. Perhaps, then, they are like chimaeras: donkeys combined with features it is impossible for a donkey to have. Let's take it to be so, and set this worry aside, too.

Nevertheless, further results the syntax gives us on the basis of the foregoing are unmanageable, and can't be set aside. To give just one example, take the consequent of (A5) and run it through a mixed modal syllogism Buridan explicitly endorses, namely Darapti LXX:[489]

A6) Every human is necessarily an impossible-donkey

[489] *Summulae de Syllogismis* 5.7.3; Thom also notes this in *Medieval Modal Systems* (179).

Every human is an animal

∴ Some animal is an impossible-donkey

Since the concluding proposition is a particular affirmative, it converts simply:

P36) Some impossible-donkey is an animal

And since (P36) is assertoric, (P36) has existential import: the things its terms stand for must exist. Hence it follows that there actually exist impossible donkeys.

What put us down this road? Our attempt to represent the equipollences Buridan endorses on Thom's syntax. But perhaps it can be said in defense of Thom—whose work I've long admired, and whose clarity and rigour I seek to emulate—that his formal language is canonical, and therefore needs only one way of representing equipollent propositions. After all, at the outset of *TC* II, in a preliminary discussion about negation, Buridan has this to say about doubly-negated modal propositions:

> In some propositions, there is double negation: one negation sign applies to the mode, and the other to the predicate, as in "B is not possibly not A", or "B is not necessarily not A". And I believe these are really affirmatives, since they are equipollent with certain propositions which are clearly affirmative.[490]

[490] "Aliae sunt in quibus ponitur duplex negatio, una ad modum, alia ad praedicatum, ut 'B non possibile est non esse A' vel 'B non necesse est non esse A'. Et credo quae istae secundum veritatem sunt afformativae, quia aequipollent aliquibus manifeste affirmativis" (*TC* II.3.22-5).

Given that doubly-negated propositions are "clearly affirmative", can't we represent them the same way we do their affirmative counterparts in our formal language? But this line of defense introduces three significant problems.

First, it conflates semantics with syntax: just because two propositions have the same truth conditions, it does not follow that we should symbolise them the same way. Indeed, this seems to be one of the central motivating concerns of modern logic as well: for instance, Frege's distinction in "Über Sinn und Bedeutung" would lack motivation if 'a=b' and 'a=a' were merely symbolised as φ.

Second, the modal equipollences are an integral part of Buridan's derived conclusions: they are the second result he presents, and are motivated by his ampliative semantics. Moreover, they play a critical role in his modal syllogistic. Any canonical language which leaves them out is, accordingly, a symbolisation only of a fragment of the language, not the whole thing. If we are to avoid the Fallacy of Composition—taking a part for the whole—then this fact should be explicitly acknowledged.

Third, suppose we go ahead with this suggestion, and symbolise Buridan's account of the modal equipollence expressed by (S17′) as follows:

$$\text{S18)} \quad (b^{\dagger} \mid a^{\dagger}) \iff (b^{\dagger} \mid a^{\dagger})$$

A formula like (S18) is uninformative. But worse, it actually commits a formal fallacy on Buridan's own framework. A defining difference between Aristotelian logic and the Fregean logic which eclipsed it is the status of arguments of the following form:

S19) φ

$\therefore$ φ

The Fregeans don't reject (S19). But Aristotle does: according to Aristotle, (S19) commits a fallacy—namely, that of *petitio principii*.[491] Medieval thinkers follow him on this point, and Buridan is no exception.[492] We have, therefore, no reason to eliminate the equipollences, and every reason to demand them from a formal syntax for Buridan's modal logic.

Thus the analysis of Buridan which shifts modality from the copula to the predicate—to say nothing of treating it as a sentential operator—gives us undesirable results. These results are more difficult to see in the syntax, but they become much more plain when we consider possible worlds semantics running on these syntactic chassis.

1.2.4. Johnston on Ampliation

As it stands, the most detailed and sophisticated analyses of Buridan's modal logic in terms of possible worlds are those of Spencer Johnston. Yet for all their sophistication and subtlety, they face familiar problems. Unlike Thom, whose focus is syntax, Johnston attempts to cash out Buridanian modal *semantics*. But like Thom, he shifts modality off the copula and onto the predicate, and leaving the copula as just the '$\subseteq$' of set

[491] For a representative passage in Aristotle, see *Topics* VIII.13 (162b34-7).
[492] *Summulae de Fallaciis* (7.4.4).

membership. Hence he has to take *e.g.* universal affirmatives *de necessario* ("Every A is necessarily B") as follows:

S20) $\mathrm{M}(w, \mathrm{A}) \subseteq \mathrm{L}(w, \mathrm{B})$ and $\mathrm{M}(w, \mathrm{A}) \neq \varnothing$

Here M and L are the possibility and necessity operators of Polish notation, respectively; and the final clause—the one following the *and*—is just to rule out vacuously-true universal predications *de necessario*—that is, propositions where the subject term is empty.[493] For clarity and elegance, we can reformulate (S19) as follows:

S20′) $\varnothing \subsetneq \Diamond(w, \mathrm{A}) \subseteq \Box(w, \mathrm{B})$

As we'll soon see, this *prima facie* intuitive way of reading Buridan gives us untenable results.

Let's build our way up to (S20′), following what Johnston tells us he's doing. Here's how he presents his model for Buridan's modal semantics:

> A *Buridan Modal Model* is a tuple: $\mathfrak{M} = \langle D, W, R, O, c, v \rangle$ such that:
>
> D and W are non-empty sets. D is the domain of objects and W is a set of worlds.
>
> $\mathrm{R} \subseteq \mathrm{W}^2$ which is reflexive

[493] Spencer Johnston, "A Formal Reconstruction of Buridan's Modal Syllogism", *History and Philosophy of Logic* (36) (2015), 11. I have here adopted '☐' and '◊' for Johnston's 'L' and 'M', respectively. See also Spencer Johnston, "Essentialism, Nominalism, and Modality: The Modal Theories of Robert Kilwardby and John Buridan", PhD diss. (St Andrews, 2015), 107-9.

$$O : W \to \mathcal{P}(D) \text{ such that } O(w) \subseteq D$$
$$v : W \times PRED \to \mathcal{P}(D)$$
$$c : CONS \to D.$$ [494]

Here *PRED* is just the set of (monadic) predicates, and *CONS* the singular terms/objects in the domain.[495] Intuitively, O is just the powerset of the objects in the worlds—which objects, interestingly enough, are not assigned to this or that particular world, and so seem to have something like transworld identity.

On the basis of the foregoing, Johnston defines the following operations (which I've numbered for subsequent reference):

I) $V'(w, \mathrm{P}) = O(w) \cap v(w, \mathrm{P})$

II) $V'(w, {\sim}\mathrm{P}) = D \backslash O(w) \cap v(w, \mathrm{P})$[496]

In sum: the function V' here just gives us the extension of the term P at world w—that is, it gives us all the objects of the domain to which the term P applies at w. And the negated term, ~P, just returns all those elements of the domain in w that are non-P. For example, taking w to be the present world, and P to be *donkey*, V' gives us all those objects under the extension of the term *donkey* that exist in this world. And conversely, $V'(w,$ ~P)—again taking P to be *donkey* and w to be the actual world—gives us all those

[494] Johnston, "Reconstruction", 11. Cf. also "Modal Theories", 207.
[495] Which terms/objects Johnston conflates (see his "Reconstruction", 10). Anyway, these correspond with the singular terms—which for Buridan are all the names plus any common term like *horse* or *human* coupled with a demonstrative like *this* or *that* (*i.e. hic* or *ille*). And these singular terms don't figure in what follows, since we're just looking at the derived rules in *TC* II.6, all of which deal with common terms and not singular ones.
[496] Johnston, "Reconstruction", 11.

elements of the domain that exist in this world and are non-donkeys: horses, icebergs, cities, and so forth.

So far, so (relatively) straightforward. Johnston introduces a variable Q as an abbreviation for either a term or the negation of a term, and goes on to define possible and necessary objects as follows:

III) $\Diamond(w, \mathrm{Q}) = \{\, d \in \mathrm{D} : \text{there is some } z \text{ such that } w\mathrm{R}z \text{ and } d \in V'(z, \mathrm{Q})\}$

IV) $\Box(w, \mathrm{Q}) = \{\, d \in \mathrm{D} : \text{for all } z \text{ if } w\mathrm{R}z \text{ then } d \in V'(z, \mathrm{Q})\}$[497]

In a subsequent (2017) paper on these semantics, Johnston adds the following gloss on (I)-(IV):

> Here the idea is that the operations V', M and L give the extension of a particular term at a particular world. For example, $V'(w, \mathrm{P})$ returns the extension of the predicate for the objects that exist at w while $\mathrm{M}(w, \mathrm{P})$ and $\mathrm{L}(w, \mathrm{P})$ give the set of objects that are possibly (respectively, necessarily) P at w.[498]

Here's how this works: take an object, $d \in \mathrm{D}$, which is in some world w, which our world (call it $w^{@}$) can 'see' on R—that is, that $w^{@}\mathrm{R}w$. Suppose that some predicate P applies to d—that is, that $d \in V'(w, \mathrm{P})$. Since this holds in **at least one** world w such that $w^{@}\mathrm{R}w$, $\Diamond(w^{@}, \mathrm{P})$ is true in our world (and in any other world v such that $v\mathrm{R}w$).

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[497] *ibid.*

[498] Spencer Johnston, "The Modal Octagon and John Buridan's Modal Ontology", *The Square of Opposition: A Cornerstone of Thought*, ed. Jean-Yves Béziau and Gianfranco Basti (Cham, Switzerland: Birkhäuser, 2017), 42.

Now if we replace **at least one** in the preceding sentence with *every*, we get the semantics for $\Box(w, \mathrm{P})$, which holds just in case $d \in V'(w, \mathrm{P})$ for every w. Intuitively, we can say that something is necessarily P just in case it's in every world, and is a P in every world. Take, for instance, our favorite necessary object: God who is a Trinity. In every world, God is; and in every world in which God is, God is a Trinity. So there you have it: a necessary object with a necessary property.

We are about to meet our *necessaria*, including Necessary Dodoes, once again. But first, let me address two ambiguities in this semantic account.

1.2.4.1. Two Ambiguities

There are two things that are left unclear in Johnston's semantics: (i) are we talking about the extensions of predicates in *our* world, or in another world—one, say, where *donkey* means *human*? And (ii) how are the worlds related to each other? At least, if R is reflexive, all the worlds can access or 'see' themselves.[499] Can they see anything else? Let's take these in order.

[499] In what follows, I'll use *see* for *access*. Johnston does not wade into the debate about what this accessibility relation is—whether for instance it is something like conceivability. For present purposes, this seems wise, and I am happy to follow his lead.

Ambiguity 1

Which world's predicates are at stake here? Are we talking about the meaning of P as indexed to whatever world it's in where it applies to an object in that world? Or, when we're saying that *d* is a part of the valuation of P in some (every) world *w*, are we talking about the meaning of P in *our* world, with *our* meanings attached?

Suppose we take the former approach, and just talk about the meaning of P as indexed to this or that *w*, not necessarily our own. But if we do this, we have to grant that the following is possible:

P37) A human is a donkey.[500]

After all, Buridan endorses (P37) in the following *Sophismata* passage:

> It is in our power that a human should be a donkey. This is proved as follows: you and I will debate, and then we can use utterances as we like [*ad placitum nostrum sicut volumus concordare*] with respect to their significations. For in this way, parties to a debate often let *A* signify humans and *B* donkeys. Let's therefore agree that in the present debate, the term *human* should signify what, for others, the term *whiteness* does; and the term *donkey* should signify for us what, for others, the term *colour* does—for we are able to do so. And therefore, by means of synonyms, this proposition 'a human is a donkey' gives us the same concept [*intentio*] as

[500] "Homo est asinus" (*Sophismata* VI, soph.4; Scott, p.108, 33r).

> 'whiteness is a colour' does in others. But the proposition 'whiteness is a colour' is true for others; therefore, 'a human is a donkey' is true for us.[501]

Such a debate, where terms get reassigned, is certainly possible. So for Johnston, it has to take place in a possible world. So there is a possible world in which humans are donkeys, since there is a world w in which $V'(w, \text{Human})$ is the same as $V'(w, \text{Donkey})$. So humans are possibly donkeys.

This can't be. So let's take the latter approach, and say that the predicates only apply in whatever world w the way they do in the actual world. This eliminates the problem brought on by (P37). But it presents a new problem: the debate situation introduced in Buridan's *Sophismata* passage cited just above is not possible. After all, we can't say that terms might signify other than they do, since to say so would be to say that in some possible world they have other meanings. But we just ruled this out. So we have to keep terms fixed, so that they mean just what they do in the actual world. So the *Sophismata* debate, with its reassignment of terms, does not take place in any possible world.

In sum: if we index the meaning of P to the world in which it holds of something, then we have to allow that any well-formed proposition is possible, since there is a world in which its terms, with different meanings, could express a true proposition. Conversely,

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[501] "In potestate nostra est quod homo sit asinus. Probatur quia ego et tu disputabimus, et tunc possumus uti vocibus ad placitum nostrum sicut volumus concordare, quantum ad significationes earum. Sic enim saepe concordant disputantes quod 'A' significet hominem et 'B' asinum. Concordemus ergo quod in hac nostra disputatione, iste terminus 'homo' significet nobis sicut aliis a nobis communiter significat iste terminus 'albedo', et iste terminus 'asinus' significet sicut communiter aliis iste terminus 'color', quia hoc est in potestate nostra. Ergo sinonime se habebit aput nos haec propositio 'Homo est asinus' et eandem intentionem nobis faciet sicut aliis haec propositio 'Albedo est color'. Sed ista propositio 'Albedo est color' est aliis propositio vera. Ergo haec 'Homo est asinus' est nobis vera." (*Sophismata* VI, soph.4; Scott, p.108, 33r).

if we index the meaning of P just to *this* world, we lose Buridan's radical conventionalism about language: we have to admit that term meanings are fixed, and can't alter in different worlds, much less different contexts. So the debate described in the *Sophismata* passage is impossible.

There might be a way between the horns of this dilemma: just admit that spoken terms can mean whatever, but say that the P in the above formulation is a predicate at the *mental* level. In fact, Buridan solves the foregoing sophism on precisely these lines:

> It is impossible for 'A man is a donkey' or another such utterance to be true—that is, when it keeps the very same signification as it has now. For it always designates a false mental proposition, as it does at present, and therefore it is always false.[502]

So Johnston has to take P to be a *mental* predicate, not a spoken one. This introduces an important metaphysical problem, though perhaps not a semantic one: namely, what guarantees the existence of the terms? After all, they won't exist unless they're being thought, *per* Buridan's anti-realism and tokenism about linguistic items like terms and propositions, which forms an important basis of his logic (as we saw with the Simultaneous Formation Requirement in Chapter 2, §2). So to be perfectly perspicuous, we might introduce a conditional into Johnston's formulations of Buridan's modal semantics (here underlined):

$$\text{III}')\ \Diamond(w, \text{Q}) = \{d \in \text{D} : \exists z\ (w\text{R}z)\ \underline{\wedge \exists \text{P}\ (\text{P} \in w \vee \text{P} \in z) \rightarrow}\ d \in V'(z, \text{Q})\}$$

[502] "Impossibile est istam 'Homo est asinus' vel talem secundum vocem esse veram, manente scilicet omnino tali significatione qualem modo habet. Quia semper designaret mentalem falsam, sicut nunc designat, ideo semper esset falsa" (*Sophismata* VI, concl. 2; Scott, p.108, 31r).

This means we have to quantify over predicates, but only as mental-linguistic items, not spoken or written ones. So I am not sure it puts us in the realm of second-order logic. If it did, of course, the attempted completeness proof Johnston presents in his dissertation would fail. But that question is outside the scope of the present study.

On, then, to the second ambiguity, which introduces problems more serious than the first.

Ambiguity 2

How exactly should we characterise the accessibility relation R, here? Johnston has surprisingly little to tell us about the accessibility relation among worlds. Recall that, when he presents R, he gives us only the following:

> $R \subseteq W^2$ which is reflexive.[503]

So R is a subset of all the 2-tuples in the powerset of W. But which subset? At very least, the reflexive one. So at least on R, every world 'sees' itself—that is, $\forall w(w\mathrm{R}w)$.

But it can't be that R is *merely* reflexive, or else the worlds would only 'see' themselves, and so every actual truth would be necessary. Consider for example the actual world, $w^{@}$. Suppose that $d \in \mathrm{V}'(w^{@}, \mathrm{P})$, and that $w^{@}\mathrm{R}w^{@}$ only (that is, that $w^{@}$ can only see itself—which is consistent with mere reflexivity)—then for all w if $w^{@}\mathrm{R}w$ then $d \in V'(w,$

[503] Johnston, "Reconstruction", 11.

P). And this latter clause, recall, is just the definition of $\Box(w, \mathrm{P})$ given to us by (IV). So, on such an R, every actual truth is trivially necessary, since it holds in every world we can see—that is, in all (and only) the actual one. Therefore, everything actual is necessary.

Quod non erat demonstrandum.

Hence it seems there must be more to R than reflexivity—though what more there is, Johnston doesn't say. But in his PhD thesis, which appears to be the basis of the (2014) and (2017) articles, Johnston tells us that "we require that R be an equivalence relationship" and that R is "universal"—that is, that it be transitive and symmetric, as well as reflexive.[504] So instead of expressing R as follows:

$$\mathrm{R} \subseteq \mathrm{W}^2$$

We should really write it like this:

$$\mathrm{R} = \mathrm{W}^2$$

Let's therefore take R on Johnston's reconstruction to be *equivalent*, rather than merely *reflexive*. This solves the present problem, since even if a predicate P holds of something in the actual world, there is no guarantee that it holds in every world the actual world 'sees'. But this solution introduces a whole host of new problems, which fall into two roughly

[504] Spencer Johnston, "Essentialism, Nominalism, and Modality: The Modal Theories of Robert Kilwardby & John Buridan", PhD diss., University of St. Andrews (2015), 107, 109.

overlapping groups: adequacy problems, and problems related to Necessary Birds. The former have to do with the capacity to model all of Buridan's theorems or derived rules. We'll look at these in §3, where I show that no Kripkean possible-worlds semantics can be furnished for Buridan's modal logic.

But first: Necessary Birds and Dodoes.

1.2.4.2. Necessary Birds and Resuscitated Dodoes

The foregoing presentation of the semantics was a bit technical. But the problems that fall out of it are straightforward. As we saw, necessity (possibility) is just existence in every (some) world, and every world sees every other one. Fine, but this introduces two strange things. First, $\Box(w, \mathrm{Q})$ will only hold when some item d is in the valuation of the predicate Q in every world, and so we get necessary beings wherever anything has a necessary or *per se* attribute. Second, any essential self-predication is necessary and therefore actually true. So we can speak dodoes into existence, as it were, just by saying they're all necessarily dodoes. In brief, then, R gives us our Necessary Birds, and it plants them in every world. Since one of these worlds is our own, they get planted here as well, and so dodoes pop back into existence. Let's look at each of these in turn.

Necessary Birds Reappear

—Only this time, they're in the semantics of a semantic account, rather than merely in the semantics read off a syntactic account, as in Thom's. Here's how this happens: recall that when Johnston presents us with the semantics for '$\Box$', he has to make d an element of the valuation of P for every world w. So a predicate term P only picks out a necessary item if some element of every world answers to P.

This is how we get our Necessary Birds. Start with the following proposition:

P38) Dodoes are necessarily birds

Dodoes of course no longer exist in this world, but even so (P38) is true of all the non-existent dodo *possibilia*. Now recall (S20), above:

S20′) $\varnothing \subsetneq \Diamond(w, \text{A}) \subseteq \Box(w, \text{B})$

Let's follow (S20′) and cash out the claim made by (P38) as follows:

$$\varnothing \subsetneq \Diamond(w, \text{Dodo}) \subseteq \Box(w, \text{Bird})$$

This gets further elaborated in the following way:

$$\{d \in \mathrm{D} : \text{there is some } z \text{ such that } w\mathrm{R}z \text{ and } d \in V'(z, \text{Dodo})\}$$

$$\subseteq$$

$$\{d \in \mathrm{D} : \text{for all } z \text{ if } w\mathrm{R}z \text{ then } d \in V'(z, \text{Bird})\}$$

And this just tells us that the possible dodo worlds are a subset of the Necessary Bird worlds. But the Necessary Bird worlds have to be all the worlds, since (i) necessity requires existence at each world w can see, and (ii) w can see every world, since R is universal. So Necessary Birds rear their heads again.

Here is a different way of looking at the same problem: in order for (P38) to be true on the present reading, $\Diamond(w, \text{Dodo})$ can't be empty for every world; and it has to be a subset of $\Box(w, \text{Bird})$—that is, the possible-dodo worlds have to be a subset of the Necessary Bird worlds. Now imagine a possible world—call it w^*—with only dodoes in it. And imagine God removes the dodoes from w^*, and places them in some other world—which is certainly doable since objects are not here indexed to worlds, as we noted above, and so are not barred from finding themselves in this or that world.[505] Or perhaps God just eradicates the dodoes from w^*. Either way, the last of the possible dodoes have been removed, and they were the only birds in w^* to begin with.

Now we can still access a dodo-less world w^* (can't we?), so it's still in our accessibility relation R. And since there are Necessary Birds, it has to be that all worlds we can 'see' on R are Necessary Bird worlds. So w^*, which started out as a world with only dodoes, and ended up losing even those, remains a world in which there are necessary birds. So although we can eradicate the possible birds from w^*, or move all the possible

[505] Which is another problem I won't get into here: by not indexing objects to worlds, Johnston is committed to transworld identity for everything from ferns to desks to salamanders.

birds from w^* into another one, we can't rid w^*—or any world we can see—of its Necessary Birds.

Clearly we're not in Kansas anymore: this takes us a long way from the ordinary propositions *de necessario* that we set out to analyse, like:

P38) All (possible) dodoes *are-necessarily* birds

How did we get here? We are meant to analyse (P33) along the following lines:

S19′) $\varnothing \subsetneq \Diamond(w, \mathrm{A}) \subseteq \Box(w, \mathrm{B})$

The predicate of (S19′) introduces our Necessary Birds, since we have to read it as follows:

$$\Box(w, \mathrm{Bird}) = \{d \in \mathrm{D} : \forall z(w\mathrm{R}z) \rightarrow d \in V'(z, \mathrm{Bird})\}$$

So there are birds in every world we can see. And since we can see every world, there are birds in every world. And so we have to read (P33) as follows: "the possible-dodo worlds are a subset of the necessary-bird worlds, and the possible-dodo worlds are non-empty". This leaves us with a whole forest of weird Meinongian necessary critters—Necessary Dodoes and Necessary Birds and the like, which exist across all possible worlds, including ours.

It gets worse. True propositions *de necessario* with the same term in subject and predicate introduce these objects into the actual world, too, since this world can see itself.

Dodoes Get Resuscitated

It appears we can speak the dodoes back into existence, just by uttering the following:

P39) Dodoes are necessarily dodoes

Again, we have to read this as follows:

P39′) $\varnothing \subsetneq \Diamond(w, \text{Dodoes}) \subseteq \Box(w, \text{Dodoes})$

Since (P34) is true, the predicate gives us Necessary Dodoes, just as it did Necessary Birds.[506] And, just like the Necessary Birds of the previous section, these dodoes have to exist in every world our world can see. And our world can see our world. Therefore, there are objects in our world that answer to the term *dodo*. Therefore, there are actually dodoes.

In sum, we have here been considering the view, which appears in Thom's syntax for Buridan and in Johnston's semantics, that modality can be shifted from the copula to

[506] In defense of the truth of (P34): any quidditative predication is *per se*, and any *per se* predication is necessary. Hence any self-predication of a species-term in a *de necessario* proposition will be true. Buridan touches on this in *QM* (IV.8, fol.18v), where Buridan claims that "a rose is a rose" is quidditative.

the predicate with no alteration to the logic. As we've seen, this approach is untenable, because it posits a vast—and indeed, for natural kinds with essenes, all-encompassing—class of necessary existents. But for Buridan there are no *necessaria*—apart from God. Otherwise, the only classes of things are *possibilia* and what we might call *actualia*, which are a proper subset of the *possibilia*. What's at stake, then, is classes of *possibilia*, and relations among them: what this or that *possibile* can or cannot be. The modal properties of these *possibilia* are themselves grounded in causation: on what they can and cannot be made to be. They are not grounded in anything like the quantification across possible worlds we find in modern modal logic.

With these things in mind, let's turn to Buridan's modal syntax, to see how relations among these classes underwrite—and rule out—inference rules involving divided modals. As we'll see, there's actually no way to model these completely and consistently in any Kripke-style possible-worlds semantics.

2. Syntax: Divide and Rule

In this section, I want to set out symbolically the rules that Buridan gives us for divided modals in *De Consequentiis* II.6. I have adapted Thom's system, which deals with terms ampliated to *possibilia* only (and not to *necessaria*), and which binds modal adverbs to the copulae. Here, then, is that modified notation:

Notation

$\vdash$	entailment
$\nvdash$	non-entailment
a, b	terms
$\bar{a}$, $\bar{b}$	negated terms a, b (*not-a*, *not-b*)
‹a›, ‹b›	ampliated terms
A	Universal affirmative ($\underline{a}$A$\underline{b}$ = "every a is b")
E	Universal negative (aEb = "no a is b")
I	Particular affirmative ($\underline{a}$I$\underline{b}$ = "some a is b")
O	Particular negative (aOb = "some a is not b")

Finally, we have our modal adverbs (which modify a copula C as *e.g.* $C_{\Box}$):

$\Box$	Necessarily
$\Diamond$	Possibly
$\triangledown$	Contingently

A brief remark: for the upcoming discussion, it is crucial to bear in mind that the modes in negative sentences fall under the scope of the negation: ‹a›$E_{\Diamond}$‹b› says that ‹a› is not possibly $\langle b \rangle$, and so is a statement about impossibility (equivalent to $\langle a \rangle \mathrm{A}_{\Box} \langle \bar{b} \rangle$—*i.e.* "All *a* is necessarily not *b*"). If we want to say that *no a* is necessarily *b*, we express it as follows: $\langle a \rangle \mathrm{E}_{\Box} \langle b \rangle$.

Hence any $\mathrm{E}_{\Diamond}$-type sentence is actually about *necessity* and, correspondingly, any $\mathrm{E}_{\Box}$-type is about *possibility* and likewise for $\mathrm{O}_{\Diamond}$ and $\mathrm{O}_{\Box}$). This aspect of the notation—and indeed, of Buridan's own modal semantics—can be a bit tricky to keep in mind. At least, it was for me. And indeed this interaction between modes and negative copulae has already caused some confusion in the literature, where some commentators have conflated "no *a* is necessarily *b*" with "no *a* is possibly *b*".[507] But so long as we read the copula *before* we read the mode (particularly in E- and O-type propositions), we should be able to avoid this error.

With these things in mind, we can turn to Buridan's rules.

[507] See for instance Johnston's (2015) "A Formal Reconstruction of Buridan's Modal Syllogism", p. 6. There, Johnston claims that "no *b* is necessarily *a*" entails that "no *b* is *a*". But "no human is necessarily running" does not entail "no human is running". The confusion here is between $\langle a \rangle \mathrm{E}_{\Diamond} \langle b \rangle$ (which *does* entail $a\mathrm{E}b$) and $\langle a \rangle \mathrm{E}_{\Box} \langle b \rangle$, which does not.

2.1 Buridan's Rules

1. **Equivalences**

> Every modal sentence about necessity has has an equivalent about possibility, and vice-versa; in the equipollences, the negations of modes and terms [*dicta*] are reversed.[508]

This rule is akin to (*though not identifiable with*) our inter-definition of □ and ◊ in propositional modal logic. Based on the canonical forms Buridan gives us in his discussion of modal sentences in the *Summulae de Propositionibus* (especially 1.8.7), we can give the following forms:

i) $\langle b\rangle\ \mathrm{A}_{\Box}\ \langle a\rangle \quad \dashv\vdash \quad \langle b\rangle\ \mathrm{E}_{\Box}\ \langle \bar{a}\rangle$

ii) $\langle b\rangle\ \mathrm{I}_{\Box}\ \langle a\rangle \quad \dashv\vdash \quad \langle b\rangle\ \mathrm{O}_{\Diamond}\ \langle \bar{a}\rangle$

iii) $\langle b\rangle\ \mathrm{E}_{\Box}\ \langle a\rangle \quad \dashv\vdash \quad \langle b\rangle\ \mathrm{A}_{\Diamond}\ \langle \bar{a}\rangle$

iv) $\langle b\rangle\ \mathrm{O}_{\Box}\ \langle a\rangle \quad \dashv\vdash \quad \langle b\rangle\ \mathrm{I}_{\Diamond}\ \langle \bar{a}\rangle$

v) $\langle b\rangle\ \mathrm{A}_{\Diamond}\ \langle a\rangle \quad \dashv\vdash \quad \langle b\rangle\ \mathrm{E}_{\Box}\ \langle \bar{a}\rangle$

vi) $\langle b\rangle\ \mathrm{I}_{\Diamond}\ \langle a\rangle \quad \dashv\vdash \quad \langle b\rangle\ \mathrm{O}_{\Box}\ \langle \bar{a}\rangle$

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[508] "Prima conclusio est: ad omnem propositionem de possibili sequi per aequipollentiam aliam de necessario, et ad omnem de necessario sequi aliam de possibili, sic se habentes quod si fuerit apposita negatio vel ad modum vel ad dictum vel ad utrumque in una, non apponatur ad illud in alia; et si non fuerit apposita in una, apponatur in alia, aliis manentibus eisdem" (II.6.1.19-24).

vii) $\langle b\rangle\ E_{\Diamond}\ \langle a\rangle \quad \dashv\vdash \quad \langle b\rangle\ A_{\Box}\ \langle \bar{a}\rangle$

viii) $\langle b\rangle\ O_{\Diamond}\ \langle a\rangle \quad \dashv\vdash \quad \langle b\rangle\ I_{\Box}\ \langle \bar{a}\rangle$

2. Ampliation

In every modal, including those about necessity, the copula ampliates the terms to *possibilia*.[509]

3. Necessity and Actuality

No sentence about actuality entails one about necessity, and neither does a sentence about necessity entail another about actuality. To this there is one exception: universal negatives sentences about necessity entail universal negative sentences about actuality.[510]

Exception: Negatives about necessity to negatives about actuality

i) $\langle b\rangle\ E_{\Diamond}\ \langle a\rangle \vdash b \mid a$

Rule: Necessity to actuality

ii) $\langle b\rangle\ A_{\Box}\ \langle a\rangle \nvdash \underline{b}\ A\ \underline{a}$

[509] "Secunda consclusio est: in omni propositione de necessario divisa subiectum ampliatur ad supponendum pro his quae possunt esse" (*TC* II.6.2.59-60). We already discussed in §2.1 why this rule holds.
[510] "Tertia conclusio est: ad nullam propositionem de necessario sequi aliquam de inesse vel econverso, praeter quod ad universalem negativam de necessario sequitur universalis negativa de inesse" (*TC* II.6.3.106-8).

iii) $\langle b \rangle\ \mathrm{I}_{\Box}\ \langle a \rangle \nvdash \underline{b}\ \mathrm{I}\ \underline{a}$

iv) $\langle b \rangle\ \mathrm{O}_{\Diamond}\ \langle a \rangle \nvdash b\ \mathrm{O}\ a$

4. Possibility and Actuality

No sentence about possibility entails another about actuality; but an affirmative sentence about actuality entails a corresponding particular sentence about possibility.[511]

Exception: Affirmatives about actuality to affirmatives about possibility

i) $\underline{b}\ \mathrm{A}\ \underline{a} \vdash \langle b \rangle\ \mathrm{I}_{\Diamond}\ \langle a \rangle$

ii) $\underline{b}\ \mathrm{I}\ \underline{a} \vdash \langle b \rangle\ \mathrm{I}_{\Diamond}\ \langle a \rangle$

Rule:

1. Possibility to actuality

iii) $\langle b \rangle\ \mathrm{A}_{\Diamond}\ \langle a \rangle \nvdash \underline{b}\ \mathrm{A}\ \underline{a}$

iv) $\langle b \rangle\ \mathrm{E}_{\Box}\ \langle a \rangle \nvdash b\ \mathrm{E}\ a$

v) $\langle b \rangle\ \mathrm{I}_{\Diamond}\ \langle a \rangle \nvdash \underline{b}\ \mathrm{I}\ \underline{a}$

vi) $\langle b \rangle\ \mathrm{O}_{\Box}\ \langle a \rangle \nvdash b\ \mathrm{O}\ a$

2. Negatives about actuality to negatives about possibility

[511] “Quarta conclusio est: ad nullam propositionem de possibili sequi aliquam de inesse vel econtra, praeter quod ad omnem propositionem affirmativam de inesse sequitur particularis affirmativa de possibili”

vii) $b \text{ E } a \nvdash \langle b \rangle \text{ E}_{\Box} \langle a \rangle$

viii) $b \text{ O } a \nvdash \langle b \rangle \text{ O}_{\Box} \langle a \rangle$

5. **Term Transposition I**

Every affirmative sentence about possibility entails another, particular sentence about possibility with the terms transposed, but not a universal one, [only a particular]. And no negative sentence about possibility entails another sentence about possibility by the transposition of terms.[512]

1. Affirmatives about possibility to affirmatives about possibility

i) $\langle b \rangle \text{ A}_{\Diamond} \langle a \rangle \vdash \langle a \rangle \text{ I}_{\Diamond} \langle b \rangle$

ii) $\langle b \rangle \text{ I}_{\Diamond} \langle a \rangle \vdash \langle a \rangle \text{ I}_{\Diamond} \langle b \rangle$

2. Negatives about possibility to others about possibility

iii) $\langle b \rangle \text{ E}_{\Box} \langle a \rangle \nvdash \langle a \rangle \text{ A}_{\Diamond} \langle b \rangle$

iv) $\langle b \rangle \text{ E}_{\Box} \langle a \rangle \nvdash \langle a \rangle \text{ E}_{\Box} \langle b \rangle$

v) $\langle b \rangle \text{ E}_{\Box} \langle a \rangle \nvdash \langle a \rangle \text{ I}_{\Diamond} \langle b \rangle$

vi) $\langle b \rangle \text{ E}_{\Box} \langle a \rangle \nvdash \langle a \rangle \text{ O}_{\Box} \langle b \rangle$

[512] "Quinta conclusio est: ad omnem affirmativam de possibili sequi per conversionem in terminis particularem affirmativam de possibili, sed non universalem, et ad nullam negativam de possibili sequi per conversionem in terminis aliam de possibili" (*TC* II.6.5.172-5).

vii) $\langle b \rangle\ O_{\Box}\ \langle a \rangle \nvdash \langle a \rangle\ A_{\Diamond}\ \langle b \rangle$

viii) $\langle b \rangle\ O_{\Box}\ \langle a \rangle \nvdash \langle a \rangle\ E_{\Box}\ \langle b \rangle$

ix) $\langle b \rangle\ O_{\Box}\ \langle a \rangle \nvdash \langle a \rangle\ I_{\Diamond}\ \langle b \rangle$

x) $\langle b \rangle\ O_{\Box}\ \langle a \rangle \nvdash \langle a \rangle\ O_{\Box}\ \langle b \rangle$

6. Term Transposition II

No sentence about necessity entails another about necessity with the terms transposed, except universal negatives.[513]

Exception: Universal negatives about necessity

i) $\langle b \rangle\ E_{\Diamond}\ \langle a \rangle \vdash \langle a \rangle\ E_{\Diamond}\ \langle b \rangle$

Rule: All other sentences about necessity:

ii) $\langle b \rangle\ A_{\Box}\ \langle a \rangle \nvdash \langle a \rangle\ A_{\Box}\ \langle b \rangle$

iii) $\langle b \rangle\ I_{\Box}\ \langle a \rangle \nvdash \langle a \rangle\ I_{\Box}\ \langle b \rangle$

iv) $\langle b \rangle\ O_{\Diamond}\ \langle a \rangle \nvdash \langle a \rangle\ O_{\Diamond}\ \langle b \rangle$

[513] "Sexta conclusio est: ad nullam propositione de necessario sequi per conversionem in terminis aliam de necessario, praeter quod universalem negativam sequitur universalis negativa" (*TC* II.6.6.199-201).

7. Contingent Sentences I

On the whole, Buridan appears much less interested in contingency (*contingentia ad utrumlibet*—"contingency on both sides") than he is in necessity and possibility, and his presentation of the derived rules on contingency is relatively terse, and at times a bit scattershot. Accordingly, rules 7 and 8 require more exegesis than the preceding 1-6. The first of these rules about contingency begins with conversion:

> Every sentence about contingency with an affirmed mode converts into the opposite [*i.e.* negative] quality from the affirmative mode, but nothing converting or converted in this way will be from the negative mode.[514]

This is a bit obscure, and calls for clarification. The rule has two parts: one dealing with affirmatives, the other with negatives. For the first, Buridan's example is the following pairs of mutually entailing sentences:

every b is contingently a	$\dashv\vdash$	every b is contingently not a.[515]
some b is contingently a	$\dashv\vdash$	some b is contingently not a.[516]

This gives us the following rules for affirmative sentences about contingency:

[514] "Septima conclusio est: omnem propositionem de contingenti ad utrumlibet habentem modum affirmatum converti in oppositam qualitatem de modo affirmato, sed nullum sic convertens vel conversa fuerit modo negato" (*TC* II.6.7.222-25).
[515] "Sequitur ergo 'omne B contingit esse A; ergo omne B contingit non esse A', et econverso" (*TC* II.6.7.227-8).
[516] "Similiter sequitur 'quoddam B contingit esse A; ergo quoddam B contingit non esse A', et econverso" (*TC* II.6.7.229-30).

i) $\langle b \rangle\ \mathrm{A}_{\triangledown} \langle a \rangle \dashv\vdash \langle b \rangle\ \mathrm{A}_{\triangledown} \langle \bar{a} \rangle$

ii) $\langle b \rangle\ \mathrm{I}_{\triangledown} \langle a \rangle \dashv\vdash \langle b \rangle\ \mathrm{I}_{\triangledown} \langle \bar{a} \rangle$

As for the second part of the rule, which deals with negatives, Buridan notes that from an affirmative about contingency, there does not follow a negative, or vice-versa. Apparently, this is because the mode is in the scope of the negation of a negative sentence. As Buridan tells us:

> 'is contingently' does not entail 'is not contingently'—indeed, these two are opposed. For the following are contraries: 'every *b* is contingently *a*' and 'no *b* is contingently *a*'.[517]

So affirmatives about contingency do not entail negatives about contingency, and negatives about contingency do not entail anything else about contingency. Buridan's presentaton here is terse, and he does not spell out which inferences he thinks are invalid on the pattern of the above two, (i) and (ii). If we make what he says about negatives explicit symbolically, we get the following rules:

iii) $\langle b \rangle\ \mathrm{A}_{\triangledown} \langle a \rangle \nvdash \langle b \rangle\ \mathrm{E}_{\triangledown} \langle \bar{a} \rangle$

iv) $\langle b \rangle\ \mathrm{A}_{\triangledown} \langle a \rangle \nvdash \langle b \rangle\ \mathrm{O}_{\triangledown} \langle \bar{a} \rangle$

v) $\langle b \rangle\ \mathrm{E}_{\triangledown} \langle a \rangle \nvdash \langle b \rangle\ \mathrm{A}_{\triangledown} \langle \bar{a} \rangle$

vi) $\langle b \rangle\ \mathrm{E}_{\triangledown} \langle a \rangle \nvdash \langle b \rangle\ \mathrm{I}_{\triangledown} \langle \bar{a} \rangle$

[517] "Quia non sequitur 'contingit, ergo non contingit', immo est oppositio. Istae enim sunt contrariae 'omne B contigit esse A' et 'nullum B contigit esse A." (*TC* II.6.231-3).

vii) $\langle b \rangle\ \mathrm{I}_{\triangledown} \langle a \rangle \nvdash\ \langle b \rangle\ \mathrm{E}_{\triangledown} \langle \bar{a} \rangle$

viii) $\langle b \rangle\ \mathrm{I}_{\triangledown} \langle a \rangle \nvdash\ \langle b \rangle\ \mathrm{O}_{\triangledown} \langle \bar{a} \rangle$

ix) $\langle b \rangle\ \mathrm{O}_{\triangledown} \langle a \rangle \nvdash\ \langle b \rangle\ \mathrm{A}_{\triangledown} \langle \bar{a} \rangle$

x) $\langle b \rangle\ \mathrm{O}_{\triangledown} \langle a \rangle \nvdash\ \langle b \rangle\ \mathrm{I}_{\triangledown} \langle \bar{a} \rangle$

At the end of this rule, Buridan adds that affirmative sentences about contingency also entail affirmatives and negatives about possibility:

> Moreover, it is clear that, from every sentence about contingency that has an affirmative mode, there follows another about possibility, both affirmative and negative.[518]

Again, Buridan gives us no examples. But what he seems to have in mind are the following:

xi) $\langle b \rangle\ \mathrm{A}_{\triangledown} \langle a \rangle \vdash \langle b \rangle\ \mathrm{A}_{\Diamond} \langle a \rangle$

xii) $\langle b \rangle\ \mathrm{A}_{\triangledown} \langle a \rangle \vdash \langle b \rangle\ \mathrm{E}_{\Box} \langle a \rangle$

xiii) $\langle b \rangle\ \mathrm{I}_{\triangledown} \langle a \rangle \vdash \langle b \rangle\ \mathrm{I}_{\Diamond} \langle a \rangle$

xiv) $\langle b \rangle\ \mathrm{I}_{\triangledown} \langle a \rangle \vdash \langle b \rangle\ \mathrm{O}_{\Box} \langle a \rangle$

From (xi)-(xiv), Buridan tells derives a further rule about statements about contingency and those about necessity—namely that they are incompatible:

[518] "Manifestum est ergo quod ad omnem propositionem habentem modum affirmatum sequitur propositio de possibili tam affirmata quam negativa" (*TC* II.6.7.234-9).

> Hence it is rightly said that contingency is incompatible [*excludit*] with necessity and impossibility. Hence from every affirmative sentence about necessity, there follows a negative about contingency.[519]

This gives us the following rules:

xv) $\langle b\rangle\ \mathrm{A}_{\Box}\ \langle a\rangle \vdash \langle b\rangle\ \mathrm{E}_{\nabla}\ \langle \bar{a}\rangle$

xvi) $\langle b\rangle\ \mathrm{I}_{\Box}\ \langle a\rangle \vdash \langle b\rangle\ \mathrm{O}_{\nabla}\ \langle \bar{a}\rangle$

8. Contingent Sentences II

This rule has two parts, the first of which is that no sentence about contingency entails another with the terms transposed:

> No sentence about contingency can have its terms transposed in another about contingency.[520]

Buridan provides us the following examples of invalid conversions, both affirmative and negative:

> It does not follow that 'God is contingently creating, therefore what is creating is contingently God', because the first is true, and the second is false, since for anything that is creating, it is necessary that it be God. And

[519] "Ideo bene dicitur quod contingens excludit necessarium et impossibile. Unde ad omnem propositionem de necessario habentem modum affirmatum sequitur propositio de contingenti habens modum negatum" (*TC* II.7.236-9, p.68).
[520] "Nullam propositionem de contingenti posse converti in terminis in aliam de contingenti" (*TC* II.8.241-2, p.68)

> similarly, in the negated mode, it does not follow that ‘nothing creating is contingently God, therefore no God is contingently creating’, since the first is true and the second is false.[521]

In short, terms do not transpose in sentences about contingency, the way that certain sentences about possibility and necessity do (see rules 5 and 6, above). If we spell this rule out the way we did with the preceding one, we get the following rules:

i) $\langle b \rangle\ \mathrm{A}_{\triangledown} \langle a \rangle \nvdash \langle a \rangle\ \mathrm{A}_{\triangledown} \langle b \rangle$

ii) $\langle b \rangle\ \mathrm{E}_{\triangledown} \langle a \rangle \nvdash \langle a \rangle\ \mathrm{E}_{\triangledown} \langle b \rangle$

iii) $\langle b \rangle\ \mathrm{I}_{\triangledown} \langle a \rangle \nvdash \langle a \rangle\ \mathrm{I}_{\triangledown} \langle b \rangle$

iv) $\langle b \rangle\ \mathrm{O}_{\triangledown} \langle a \rangle \nvdash \langle a \rangle\ \mathrm{O}_{\triangledown} \langle b \rangle$

The second part of this rule governs conversions of terms from sentences about contingency to corresponding sentences about possibility—a move which *is* warranted. As Buridan tells us:

> Every sentence about contingency that has an affirmative mode can be converted into another about possibility [...] and this is clear from the fact that from every sentence about contingency with an affirmative mode, there

[521] “non sequitur ‘deum contingit esse creantem; ergo creantem contingit esse deum’, quia prima est vera et secunda falsa, cum omnem creantem necesse sit esse deum. Et si affirmativa non convertitur, tunc etiam negativa non convertitur, quia aequipollet vel se mutuo consequuntur. Similiter, de modo negato, non sequitur ‘nullum creantem contingit esse deum; ergo nullum deum contingit esse creantem’, quia prima est vera et secunda falsa” (*TC* II.8.244-50, p.69).

> follows another about possibility, which can be converted into still another about possibility. Therefore, the last follows from the first.[522]

In effect, this rule combines the following sub-rules from 7, above:

7, (xi) $\langle b \rangle\ A_{\nabla} \langle a \rangle \vdash \langle b \rangle\ A_{\Diamond} \langle a \rangle$

7, (xiii) $\langle b \rangle\ I_{\nabla} \langle a \rangle \vdash \langle b \rangle\ I_{\Diamond} \langle a \rangle$

With the following sub-rules from 5, above:

5, (i) $\langle b \rangle\ A_{\Diamond} \langle a \rangle \vdash \langle a \rangle\ I_{\Diamond} \langle b \rangle$

5, (ii) $\langle b \rangle\ I_{\Diamond} \langle a \rangle \vdash \langle a \rangle\ I_{\Diamond} \langle b \rangle$

Hence we get:

$\langle b \rangle\ A_{\nabla} \langle a \rangle \vdash \langle b \rangle\ A_{\Diamond} \langle a \rangle \vdash \langle a \rangle\ I_{\Diamond} \langle b \rangle$ (by 7 (ix), 5 (i)).

$\langle b \rangle\ I_{\nabla} \langle a \rangle \vdash \langle b \rangle\ I_{\Diamond} \langle a \rangle \vdash \langle a \rangle\ I_{\Diamond} \langle b \rangle$ (by 7 (xiii), 5 (ii)

This gives us the following rules, which conclude Rule 8:

(i) $\langle b \rangle\ A_{\nabla} \langle a \rangle \vdash \langle a \rangle\ I_{\Diamond} \langle b \rangle$

[522] "omnem habentem modum affirmatum posse converti in alia de possibili [...et] ex hoc patet quia ad quamlibet de contingenti habentem modum affirmatum sequitur affirmativa de possibili, quae convertitur in aliam de possibili. Ideo, de primum ad ultimum, etc." (*TC* II.6.242-3, 251-4, pp.68, 69).

(ii) $\langle b \rangle\ \mathrm{I}_{\nabla}\ \langle a \rangle \vdash \langle a \rangle\ \mathrm{I}_{\Diamond}\ \langle b \rangle$

Such, then, are Buridan's inference rules for divided modal propositions. The natural next question is, what are we dealing with? That is, what properties do the foregoing rules collectively have?

2.2. What the Rules Rule Out

At the conclusion of his (1989 [1987]) treatment of Buridan's modal logic, Hughes suggests a few future research projects into Buridanian modal semantics, culminating with a Kripke-style analysis:

> A much more elaborate project still would be to try to give a Kripke-style possible worlds semantics for Buridan's modal system and then an axiomatic basis for it. I think this could probably be done, and would be worth doing; but it would take us well into the twentieth century.[523]

This remark, from such a prominent logician, has inspired many research projects into Buridan's modal logic. We can therefore be grateful to him for attracting so much attention to the study of medieval logic, and inspiring so much high-quality research. Nevertheless, Buridan's modal logic has properties that cannot be modelled on *any* Kripkean possible-worlds semantics whatsoever, as we are about to see. For my part, I think this is a very good thing: it gives us all the more reason to study Buridan's modal

[523] Hughes, "The Modal Logic of John Buridan", 108.

logic, because it is so different from what we're used to. Suppose, by way of comparison, that extraterrestrial life exists. Would any biologist hope to find only such life forms elsewhere as we already have on earth?

As I said at the beginning of the present section, there are two derived rules here that are particularly important for our study of Buridan's system in relation to possible-worlds semantics. These are Rule 3, whereby no proposition *de necessario* entails a corresponding assertoric (with the exception of universal negatives); and Rule 4, whereby any affirmative assertoric entails a corresponding particular proposition *de possibili*. Combined with Buridan's semantics for assertorics, on which all affirmative propositions have existential import, they make consistent modelling on frames impossible. Let's begin with Rule 3. Here is how Buridan puts it:

> No assertoric proposition entails one *de necessario*, and neither does a proposition *de necessario* entail an assertoric. To this there is one exception: universal negatives *de necessario* entail universal negative assertorics.[524]

The rationale for Rule 3 is just that we make statements about necessity concerning *possibilia* that do not exist, but that affirmative propositions—both particular *and* universal—have existential import. Consider again the dodo:

P38) All dodoes are necessarily birds

[524] "Tertia conclusio est: ad nullam propositionem de necessario sequi aliquam de inesse vel econverso, praeter quod ad universalem negativam de necessario sequitur universalis negativa de inesse" (*DC* II.6.3.106-8).

A proposition like (P38) is true, even though there are no actual dodoes. But then universal affirmatives like the following *do* have existential import, and therefore are false:

P40) All dodoes are birds

So (P38) is true, and (P40) is false. By the definition of logical consequence, it accordingly cannot be that (P38) entails (P40).

The exception to this rule is universal negatives *de necessario*, which entail universal negative assertorics. We've represented this schematically as follows:

S21) $\langle b \rangle \; \mathrm{E}_{\Diamond} \; \langle a \rangle \vdash b \; \mathrm{E} \; a$

That is, if no *b* is possibly *a* (i.e. if every *b* is necessarily not *a*), then no *b* is *a*. Here's why (S21) is a valid schema: recall that there are two ways for universal negative assertorics to be true. First, because their subject and predicate terms do not stand for the same thing. Or, second, because one or the other (or both) of these terms fail to stand for anything at all. Here is a proposition that is true in the first way:

P41) No horse is a human

And here is a proposition true in the second way:

P42) No chimaera is a chimaera

So universal negatives are capable of vacuous truth. This is the first fact that underwrites the exception to Rule 3: if no dodo is possibly a human, and moreover no dodoes (or no humans) exist, then the consequent is true, and the consequence is valid.

The second fact is that the *actualia* are a proper subset of the *possibilia*. And if two sets are disjoint, all their subsets will be, too. Here's a diagram to make this clearer:

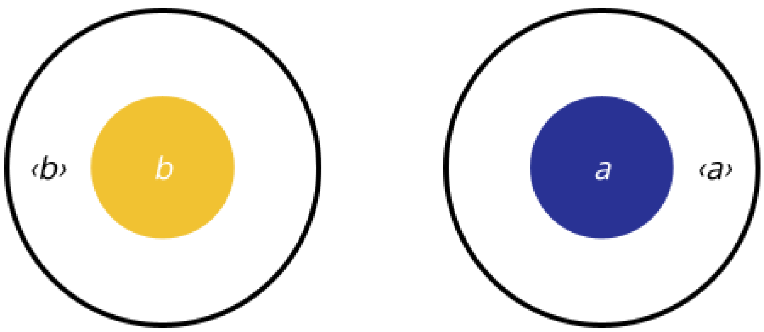

Fig. 5.7: why universal negatives about necessity imply their corresponding assertorics.

(*That is, why* (‹*b*› $\mathrm{E}_{\Diamond}$ ‹*a*›) *entails* (*b* E *a*)).

Hence whether or not there are any actual *a* or *b*, if all of the *a* is necessarily excluded from *b*, any actual *a* will be, too. This is why universal negatives *de necessario* entail universal negative assertorics. And so, from the existential requirements of affirmatives and the lack thereof of negatives, we get Rule 3, and its exception, (S21).

Rule 4 governs the way assertoric propositions are related to their *de possibili* counterparts. Here, the sub-rule we're interested in is that whereby any affirmative assertoric entails an affirmative *de possibili*, for instance an I-type or particular affirmative. For example:

A7) A human is running

∴ A human is possibly running

We can represent inferences like (A7) schematically, as follows:

S22) $\underline{b}\ \mathrm{I}\ \underline{a} \vdash \langle b \rangle\ \mathrm{I}_{\Diamond}\ \langle b \rangle$

What underwrites (A7) and, in general, (S22)? Recall that the *actualia* are a proper subset of the *possibilia*, since of course nothing actual is impossible. And notice that if any subsets overlap, their proper supersets do, too. Here's another diagram to make this clearer:

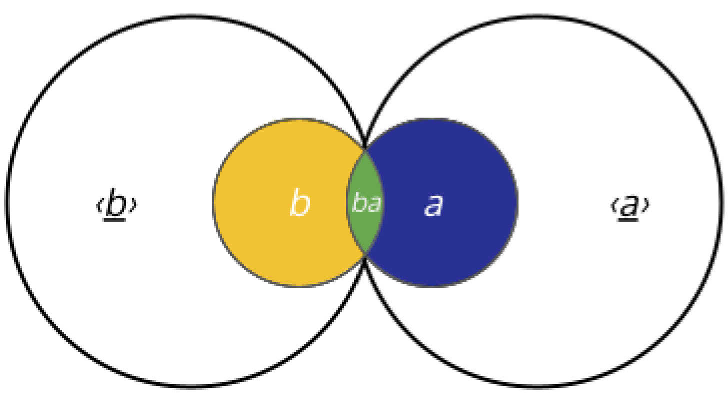

Fig. 5.8: overlapping sets of possibilia and their proper supersets

Thus if the actual *a* overlaps with the actual *b*, and both *a* and *b* are subsets of *‹a›* and *‹b›*, then *‹a›* and *‹b›* will overlap as well. Hence if some (actual) *a* is *b*, some (possible) *‹a›* is *‹b›* as well.

Now Rule 3, which insists that no proposition *de necessario* entails a corresponding assertoric (unless that assertoric is a universal negative) is, at first blush, a bit odd: intuitively, if something is necessary, then it's actual. This is the same intuition that finds its way into modern systems of modal logic as the T axiom, namely:

$$\text{T: } \Box\varphi \vdash \varphi$$

Many familiar systems feature T, including (of course) **T**, all the **M**s, as well as **S4** and **S5**. But Buridan's modal logic conditionally excludes anything like T: no proposition *de necessario* entails a corresponding assertoric, unless it is a universal negative. As we saw, this falls out of Buridan's modal semantics, though it is an intuition not commonly shared by most popular modern systems of modal logic.

The most well-known and thoroughly studied modal logic that excludes T from its axioms is the deontic logic **D** and derived systems like **D45**. We might be tempted, therefore, to go looking for further parallels between Buridan's system and **D**. For what it's worth, I was, and for some time. But agreement between Buridan's system and **D** and its derivatives are few, and their disagreements are far more significant. Any attempt to reconcile **D** with Buridan's system will have to grapple with the negative-universal exception to Rule 3, which undermines any claim of equivalence with **D** or its derivatives: if we interpret the modal operator '☐' as 'it is *obligatory* that', then what Rule 3 says is that whatever is obligatorily not, i.e. *ought not to be*, just *isn't*. But murder, fraud, and jaywalking actually happen (*ut patet*). The only deontic logic Buridan's Rule 3 is suited to, then, is a world in which no one does what they ought not. Frankly, it's hard to see how such a logic merits the title *deontic* at all.

Further still: if there remains any hope of establishing an equivalence between Buridan's system and any of the **D** family, it is dashed by the consideration of the mayhem that would result if '◊' in the above sub-rules of Rule 4 were interpreted as 'it is permissible that':

i) $\underline{b}$ A $\underline{a} \vdash \langle b \rangle\ \mathrm{I}_{\Diamond}\ \langle a \rangle$

ii) $\underline{b}$ I $\underline{a} \vdash \langle b \rangle\ \mathrm{I}_{\Diamond}\ \langle a \rangle$

On the deontic reading of (i) and (ii), everything in the actual world would be permissible, and so the is-ought distinction would fly out the window: the mere existence of something would be enough to establish its permissibility. This can't be true for any deontic logic worth its salt. So much, then, for **D** and its descendants.

The further claims that Buridan's modal logic is not **S5** or **T**—and that it cannot be modelled in Kripke-style possible-worlds semantics—runs contrary to a good deal of what's been said in the literature, by many rightly well-respected and -established scholars and logicians. Their authority should be taken seriously, and considered at length. Even so, I am compelled to make this claim: it's what I think, for reasons I'll outline in just a second. And so far, no one has adduced a compelling argument to the contrary.

On its own, however, the claim that Buridan's modal logic can't be **S5** or **T** is relatively weak: it tells us nothing about what this system is *à propos* of modern modal logics. As I've suggested, I think we ought to make a stronger claim: Buridan's modal logic is incompatible with any modern modal system of the sort we're used to dealing with. There is no way to construct an equivalent system on any such modal propositional or predicate logic. There are two reasons for this: first, because it is impossible to construct an equivalent logic using the toolkit of Kripke-style possible-worlds semantics; and second, because modern predicate logic eschews anything like a copula, and so it cannot distinguish universal (modal) predication from its corresponding conditional. Thus it will conflate Buridan's *de quando* and conditional necessity—that is, it will mix up the modal Grade 3 with Grade 4 on the scale presented in the *Summulae de Demonstrationibus*.

3. Not in Kansas Any More

In this section, I set out three claims: first, we cannot model Buridan's modal logic with Kripke-style conditions on frames. The fragment of Buridan's logic presented above—comprising Rules 3 and 4—while consistent, presupposes conflicting conditions on frames. The conditions on frames we will have to adopt to validate Rule 3 will, invariably, end up invalidating Rule 4—and vice-versa. This is not to say that Buridan's system is inconsistent. On the contrary: his system is simply beyond the limitations of possible-worlds semantics.

Second, we cannot create a system equivalent to Buridan's modal logic with any modern modal *predicate* logic (MMPL). This is because in Buridan's approach, the copula plays an important role that cannot be played by any other constant that MMPL has at its disposal. As a result, any attempt to create a system equivalent to Buridan's using the tools of modern modal predicate logic will distort it completely, by allowing inferences Buridan will not allow, and by invalidating others he allows.

Third, all attempts to account for Buridan's modal logic using the aforementioned modern logical apparatus are doomed to failure—though this has not stopped commentators from trying. Thus I will conclude with an overview of the attempts made in the literature to-date, and an account of how each one fails. With respect to modern modal systems, Buridan's modal logic is *sui generis*.

3.1. Framed-Up Beyond All Recognition

Let's try to make a logic equivalent to Buridan's, using Kripke frames. First, here's a:

> **brief overview of Kripke frames:** the modern approach to modal logic is, at bottom, quantificational: we treat modal operators as quantifiers over possible worlds. To say that φ is necessary (*i.e.* to affirm $\Box\varphi$) is just to say that φ holds in all possible worlds. Accordingly, to say that φ is possible (*i.e.*

to affirm $\Diamond\varphi$) is just to say that φ holds in at least one world. Hence our operators quantify just like the familiar ∀ and ∃ of modern predicate logic.

Following the seminal work of Saul Kripke, philosophers of language and logic began to think about the relation of *accessibility* between worlds. The way of cashing this out that has since become standard takes different sorts of accessibility relations as defined by conditions on frames. A *frame* is just an ordered pair, ⟨W, R⟩, where W is a (nonempty) set of worlds, and R is a binary relation on those worlds. Hence R, sometimes called the *accessibility relation*, just relates two worlds to each other: $w_1 \mathrm{R} w_2$ just says that world w_1 can access or 'see' world w_2. For instance, one condition on a frame allows transitivity: if $w_1 \mathrm{R} w_2$, and $w_2 \mathrm{R} w_3$, then $w_1 \mathrm{R} w_3$—that is, if world w_1 can access world w_2, and w_2 can access w_3, then the world w_1 can access w_3.

The condition on frames that we will consider in what follows is *reflexivity*, on which worlds can access themselves. Here, for instance, is a reflexive Kripke model:

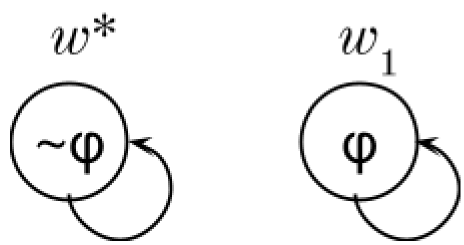

Fig. 5.9: reflexivity on frames.
These worlds can 'see' themselves, but not each other.

In the above model, $\sim\varphi$ is necessary at world w^*, even though $\sim\varphi$ does not hold in all worlds. Indeed, its contradiction, φ, holds at w_1, but no matter: there's no $R^{☆}$ such that $w^*R^{☆}w_1$, and so the status of φ at w_1 is irrelevant to its modal status at w^*.

Let's see whether we can use conditions on frames to construct a propositional modal logic equivalent to Buridan's. The big problem, as we saw, is invalidating the rule T (*i.e.* $\Box\varphi \vdash \varphi$) which (as we saw in §2.1-2) Buridan's system will not unrestrictedly allow. So we'll try invalidating T, by making the actual world (w^*) non-reflexive, so that $\sim(w^*Rw^*)$. That is, we suppose that our own, actual world is not among those it can access.

On this approach, T gets invalidated, just the way we wanted. To see why, consider the following model: take a set of worlds $W = \{w^*, w_1\}$, where w^* is the actual world. Our relation function R will be just the following pair: $\langle w^*, w_1\rangle$. So w^* can see w_2, but no world can see itself, per our restriction on reflexivity. Now, assign φ the value F at w^*, and T at w_1: hence φ will be true in some other world we can access, but it will not be true in our world. Here is a diagram to make this clearer:

Fig. 5.10: a countermodel to M

In this case, φ will be false in the actual world, w^*. But because w^* cannot see itself, and because in every world w^* *can* see (namely, w_1), φ is true, $\Box\varphi$ will be true. But φ will *not* be true in w^*. So T turns out to be invalid, since there is a model in which its antecedent is true, and its consequent is false.

So far, so good. We have ruled out T, which is excluded from Buridan's system by Rule 3. Recall, however, that Rule 3 comes with an exception: it holds in the case of universal negatives *de necessario*:

$$\langle b\rangle \ \mathrm{E}_{\Diamond} \langle a\rangle \vdash b \ \mathrm{E} \ a$$

When it comes to accommodating this exception, we're up a creek, because we just ditched reflexivity in order to invalidate the axiom T. This is partly because propositional logic is not fine-grained enough to distinguish universal negative propositions from other propositions, and so we have to either accept or reject T, not reject it conditionally, as Buridan does.

But it is also because Buridan's rules can't be described in terms of any condition(s) on frames. If, for the sake of invalidating T, we place a non-reflexivity restriction on the accessibility relation, we end up also losing the sub-rules to Rule 4, discussed above—namely, the rule that assertoric affirmatives entail particular affirmatives *de possibili*:

$$\underline{b} \ \mathrm{A} \ \underline{a} \vdash \langle b\rangle \ \mathrm{I}_{\Diamond} \langle a\rangle$$

$\underline{b}$ I $\underline{a} \vdash \langle b \rangle$ $\mathrm{I}_\Diamond \langle a \rangle$

Now these sub-rules look a good deal like (a conditional acceptance of) the following modern axiom:

$$\varphi \vdash \Diamond\varphi$$

But this will not be valid on our non-reflexive condition on frames: since the actual world cannot access itself, the assertoric antecedents will be false. And so any φ will, if it is about the actual world, fail to entail $\Diamond\varphi$. Therefore, if we accommodate Rule 3, we will lose Rule 4.

To clarify why this happens, we can create a countermodel to this axiom by tweaking the one we created to invalidate T above. Just add an accessibility relation from w^* to w_1, so that each world can see the other, but neither can see itself. Here's a diagram:

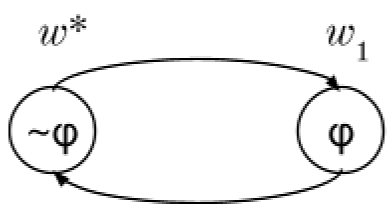

Figure II.4: a countermodel to $\varphi \vdash \Diamond\varphi$

At w_1, φ is true; but at no possible world that w_1 can see is φ true. So $\Diamond\varphi$ is false. So if we limit reflexivity in order to invalidate T, we end up invalidating Buridan's Rule 4: affirmative assertoric propositions entail particular propositions *de possibili*. This is clearly too high a price to pay.

Here is the pickle: if we use frames, we have to decide whether to make our frames reflexive, or irreflexive or non-reflexive. But in either case, we end up invalidating an rule or sub-rule of Buridan's system: if we make them reflexive, we gain Rule 4, but have to admit that any proposition *de necessario* entails a corresponding assertoric. So we lose Rule 3 (minus its exception, which holds in any reflexive model). Conversely, if we make our frames irreflexive or non-reflexive, we accommodate Rule 3 (though not its exception). But then we lose Rule 4, which requires reflexivity in order to be valid. So frames alone can't give us everything we want.

This suggests the following new line of approach: just use the syntax of modern predicate logic (MPL) to do the work of getting us all and only the inferences we want to allow.

This fails, too, but in an interesting way.

3.2. Out of Condition: Modal Predicate Logic

We saw that, if we make R irreflexive or non-reflexive, we end up losing rule 4. So let's make R reflexive, and then try to accommodate Buridan's rules and their exceptions by the way we formalise them in MPL.

Right out of the starting gate, we get some promising results. Notice that universal necessity propositions do not give us corresponding assertoric existential ones:

$$\Box\forall x(Fx \rightarrow Gx) \nvDash \exists x(Fx \ \& \ Gx)$$

Why so? On MPL, universal propositions are read as conditionals, which are vacuously true when their subject terms are empty. Existentials, on the other hand, presuppose the existence of the things they're about. For instance, take the following propositions:

P43) All martians are green

P44) Some martian is green

We can read (P43) as a conditional, as we are encouraged to do by any introductory textbook to first order predicate logic:

P43′) Everything is such that, if it's a martian, then it's green

$\forall x(\text{Martian}(x) \rightarrow \text{Green}(x))$

Correspondingly, we read (P44) as an existential statement:

P44′) There exists something that is both a martian and green

$\exists x(\text{Martian}(x) \land \text{Green}(x))$

The antecedent of the conditional (P43′) is false, since martians (presumably) don't exist. Since, therefore, it cannot be that the antecedent is true while the consequent is false, the whole conditional is true. On the other hand, (P44′) is false, because the extension of the term *martian* is empty. So (P43′) does not entail (P44′). The situation remains unchanged if (P43′) is modified by a modal. Hence we can accommodate Buridan's Rule 3 with the toolkit of MPL.

We can accommodate the exception to this rule, too: that universal negatives *de necessario* entail universal negative assertoric propositions. Here is one such:

P45) No donkey is possibly a human

Translating this into predicate logic, we get:

P45′) $\forall x{\sim}\Diamond(\text{Donkey}(x) \rightarrow \text{Human}(x))$

From which it follows that:

P45′′) $\forall x \sim(\text{Donkey}(x) \rightarrow \text{Human}(x))$

So we can accommodate Buridan's exception to rule 3, that universal negatives *de necessario* entail universal negative assertorics.

Does modern modal predicate logic get us what we want? On the level of syntax, everything seems to be working out pretty well. But recall what we saw about Buridan's modal semantics, and specifically his Modal Scale, which distinguishes two kinds of necessity: (i) *de quando* and (ii) conditional.

De quando (i) necessity holds assuming the constancy of the terms. Consider *e.g.* the following propositions:

P46) Donkeys are animals

P47) Socrates is a human

What makes (P46) and (P47) necessary is that, although they can be falsified—namely by the annihilation of donkeys or Socrates, respectively—they are invariably true when the things their subject terms stand for exist. So although you could falsify (P47) with a hemlock cocktail, you can't make Socrates other than a human while he still exists. Now suppose that we make (P46) modal:

P46′) Donkeys are *necessarily* animals

A modal proposition like (P46′) can, as we've seen, be true even when no donkeys exist, since it's about *possible* donkeys, rather than actual ones. This gives (P46′) a kind of attenuated existential requirement: a proposition like (P46′) has to be about *possibilia* in order to be true. Contrast this with a (false) proposition about *impossibilia*:

P48) Round squares are necessarily plane figures

Since there are no round squares in the class of *possibilia*, (P48) is false.

Why does all this matter? Because a proposition like (P48), while not true at the level of *de quando* (i) necessity, is nevertheless true at the level of conditional (ii)—namely, if we read it as follows:

P48′) Round squares, if they exist, are necessarily plane figures

Since the antecedent of the conditional (P48′) is false, the whole proposition is true. Now Buridan is concerned in his semantics for divided modals with *de quando*, rather than conditional necessity. But the conditional reading forced on us by MPL compels us to read all necessity claims as conditionals.

But we already saw that universal affirmatives, as the conditionals of MPL, can be vacuously true. So if we treat universal modal affirmatives as conditionals, we will conflate conditional necessity with *de quando* necessity. To see why, compare the formalisations of the following (true) propositions:

P49) All donkeys are necessarily mammals

$\forall x(\Diamond \text{Donkey}(x) \rightarrow \Box \text{Mammal}(x))$

P50) All round squares are necessarily shapes

$\forall x(\Diamond \text{RoundSquare}(x) \rightarrow \Box \text{Shape}(x))$

Proposition (P49) is necessary *de quando*, whereas (P50) is only conditionally necessary. But the whole reason we introduced this distinction was to separate vacuously true necessity propositions from those that are true about (some or all) *possibilia*.

Maybe we can fix this by adding an existential quantifier: (P49) will, accordingly, get cashed out like this:

P49′) $\forall x(\Diamond \text{Donkey}(x) \rightarrow \Box \text{Mammal}(x)) \wedge \exists y(\Diamond \text{Donkey}(y))$

This is what Stephen Read does, and at first blush it looks good.[525] This way of symbolising A-type propositions *de necessario* also gets us the subalternate inferential relations we want, since it gets us both the particular proposition *de necessario* and the particular *de possibili*:

[525] Stephen Read, "Non-Contingency Syllogisms in Buridan's *Treatise on Consequences*", in *The Road to Universal Logic*, ed. Arnold Koslow and Arthur Buchsbaum (Baael: Birkhäuser, 2015), 450.

P51) $\exists x(\Diamond \text{Donkey}(x))$

P52) $\exists x(\Diamond \text{Donkey}(x) \land \Box \text{Mammal}(x))$

But problems cloud the horizon: how are we to understand the modal operator in these sentences? If we treat them as quantifiers across possible worlds, then they will get us precisely what we *don't* want:

A8) $\exists x(\Diamond \text{Donkey}(x) \land \Box \text{Mammal}(x))$

$\therefore$ $\exists x(\text{Mammal}(x))$

So if every donkey is necessarily a mammal, then some actual donkey is actually a mammal—something explicitly ruled out by Buridan's Rule 3.

The problem is that the antecedent of (A8) gives us, among other things, the claim that there is something that is necessarily a mammal, and that whatever is necessarily a mammal is a mammal in all worlds. Since our R is reflexive, all worlds include the actual one. And so from the foregoing we can infer that there just *actually is* something that is a mammal. But Buridan's system rules this out: even if all donkeys are annihilated, the following proposition is true:

P53) Some donkey is necessarily an animal

So we're back in our original pickle. The only way to invalidate this is, again, to impose a condition on our frame that rules out reflexivity. But then we lose other axioms, as we saw. So we're stuck either way.

One last ditch effort is this: we could go back and reverse the order of the modal operator and the quantifier, so that the operator was in the scope of the quantifier. We would then re-read (P53) as follows:

P53′) $\exists x \Box(\text{Donkey}(x) \wedge \text{Mammal}(x))$

But then this will likewise be false in an instance in which no such thing as a donkey actually exists. And Buridan wants to allow that we can make necessity statements about non-existent *possibilia*, because, again, his modal logic is not one of possible worlds, but of possible objects. And so we cannot create a system equivalent to his on modern modal predicate logic. Buridan's modal logic is just a whole 'nother kettle of fish.

3.3. What Have We Done? Surveying the Literature

Buridan's modal logic is, as we have seen, one of possible *objects*. It is therefore not one of possible worlds; and, further still, it cannot be reduced to any such modal logic. Still, there have been many suggestions that it is reducible in this way, or that it is a logic of possible worlds. And there have been several attempts to model Buridan's modal logic in this way. It's time to examine these. I will outline them by author, in chronological order of publication. But first, I wish to remind the reader of what I said in the acknowledgements: all of these studies have shaped and guided my thinking on Buridan's modal logic, and have elucidated many things about it for me. Indeed, a study like this one would be impossible without the pioneering work of many the scholars cited here. It also so happens that, in academic writing, we more often cite our disagreements than our agreements, even if the latter outweigh the former many times over.

Chronologically, the first of these is in Peter King's (1985) *Jean Buridan's Logic*, which comprises an extensive study of the *Tractatus de Suppositionibus* and the *Tractatus de Consequentiis*, along with an extensive study. In the chapter on Buridan's divided modal syllogistic, King points out that Buridan's modal logic bears an important syntactic similarity with **S5**. Consider the following modal syllogism in the mood Datisi MLM:

S23) All M is possibly P

<u>Some S is necessarily M</u>

∴ Some S is possibly P

As King points out, "from the premises we can clearly infer that some S is necessarily something which is possibly P".[526] Here it appears that the modes are iterated: S is necessarily possibly P; and from this, we can infer that S is possibly P. King points out the syntactic resemblance this bears to the following axiom of **S5**:

[526] Peter King, *Jean Buridan's Logic: The Treatise on Supposition, The Treatise on Consequences.* (Dordrecht: Reidel, 1985), 81.

$$\Box\Diamond\varphi \vdash \Diamond\varphi$$

Here Buridan does seem to be dealing with iterated modals, which might mean he is dealing with the accessibility relations characteristic of **S5**: symmetry, reflexivity, and transitivity. But what is at stake for Buridan is objects, not worlds, and the reference of terms in ampliated propositions, not the reference of propositions themselves. So it is more accurate to say that Buridan holds of something that is necessarily S that is a possible-P, and so that an S is a possible-P. Thus this *prima facie* syntactic similarity between an axiom of **S5** and a mood valid on Buridan's system is overshadowed by the massive semantic difference. Further, whether we opt for reflexivity or not, we end up invalidating some axioms of Buridan or others. Therefore, the syntactic similarities do not run deep.

G.E. Hughes makes a similar suggestion in his (1989 [1987]) "The Modal Logic of John Buridan". Hughes observes that Buridan's logic is a *term*-logic, and gives a very brief overview of Buridan's semantics for assertorics along the lines of the one I have given in the present chapter (§1.1).[527] But later on, Hughes slips into treating Buridan's logic as a propositional one, suggesting (as we saw above) that Buridan is dealing implicitly with a kind of possible worlds semantics.

Hughes suggests that Buridan's modal system is equivalent to the modern system **T**. He notes the troublesome Rule 3, which rules out inferences from necessity to actuality

[527] Hughes, "Modal Logic of John Buridan", 93.

in all cases but universal negations *de necessario*. But then he offers the following solution, apparently straight from Buridan himself:

> [W]hen necessity propositions are restricted by the 'which is' [*quod est*] insertion before the subject, and thus do not have ampliated subjects, they do entail the corresponding non-modal propositions.[528]

Hughes is right, albeit in a limited way: Buridan does allow '(that) which is' (*quod est*) locutions, which block the ampliation of the subject. For instance, Buridan tells us in the *Summulae de Propositionibus* that restriction on (temporal) ampliation:

> can be achieved by adding in the subject of the [...] proposition the phrase 'which is' [*quod est*], which blocks the ampliation; *e.g.*: 'No dead thing is a man; therefore, nothing which is a man is dead.[529]

In such cases, the *quod est* locution blocks the ampliation of the subject to *possibilia*, so that it only stands for *actualia*. This operation can be performed on ampliative propositions of any sort. For example, consider the following alterations on the ampliative propositions we considered at the outset of § 1.2 of the present chapter:

P17′) *That which is* Aristotle was a Greek

P18′) *That which is* a rose is conceivable

P19′) *That which is* a donkey can run

[528] *Idem*, 101.
[529] *Summulae* 1.6.3.

Here, (P17′) will obviously be false, since at present there is nothing which is Aristotle. The truth of (P18′) and (P19′) will depend on whether there are any *actual* roses or donkeys that are conceivable or capable of running.

Where Hughes goes wrong is to identify this non-ampliative alternative logic with Buridan's logic for divided modals, and then to extend observations about this derivative system to Buridan's modal logic on the whole. The heart of Buridan's system is the ampliative system we have been considering here, to which Buridan devotes nearly all of his attention in his treatment of divided modals in the *TC* and in the relevant sections of the *Summulae*. Probably Hughes's extension of Buridan's modal logic could be more readily modelled with possible worlds than his ampliative one. But it must be stressed that it is an *extension* of the logic, which does not operate on his ampliative modal semantics. These non-ampliative propositions are therefore not really modal propositions at all; or, if they are, they are only modals in a derivative sense.

As I mentioned above, Hughes concludes by suggesting that we undertake to provide a possible worlds semantics for Buridan's modal logic:

> A much more elaborate project still would be to try to give a Kripke-style possible worlds semantics for Buridan's modal system and then an axiomatic basis for it. I think this could probably be done, and would be worth doing; but it would take us well into the twentieth century.[530]

Now lest I seem ungrateful, let me admit that this was what I originally undertook to do in this chapter, before discovering, to my surprise, that such a semantics for Buridan's

[530] Hughes, "Modal Logic of John Buridan", 108.

modal logic is impossible. So the present chapter is a response to Hughes's challenge, albeit one that gives a negative answer.

The first attempt to construct a semantic model for Buridan's modal semantics came one year later, in Gyula Klima's (1988) *Ars Artium*. Klima's semantics is not one of possible worlds *per se*, but of possible situations in a universe of discourse. He does not give a full representation of each of Buridan's modal rules in his syntax, but does claim that it is adequate. In this system, which Klima calls AMPL (for Ampliated Medieval Predicate Logic), modal semantics turns on an sentential ampliaton operator, α, which extends the sentences and, correspondingly, their terms beyond the 'actual situation', a_s. So *a* is possibly *b* if there is a (possible) situation s, in which *a* is *b*; and *a* is necessarily *b* if there is no situation in which *a* is not *b*.[531]

Here again, the problem is that this reading requires any *a* that is necessarily *b* to exist in all situations, including the actual situation. Otherwise, there is a situation in which "*a* is *b*" is false and, likewise, "*a* is necessarily *b*". So we get our necessary dodoes once again, even though they are not a feature of our distinguished actual situation a_s. Or, alternatively, practically all modal claims go out the window. After all, there is at least one situation in which everything but God is annihilated, and since α will ampliate to it, corresponding claims about necessity will be false—a worry which motivated Buridan's restricted *de quando* modality, as we saw above. So while AMPL might work well for simple modality of the sort set out above, Buridan's system of ampliation, which is meant to accommodate necessity 'just when' (*de quando*) will not be compatible with it.

[531] Gyula Klima, *Ars Artium: Essays in Philosophical Semantics, Mediaeval and Modern*. Budapest: Hungarian Academy of Sciences, 1988, p.55-8.

Later on, in his (2001) translation of Buridan's *Summulae de Dialectica*, Klima suggests that Buridan has a possible worlds semantics in mind. The context is Buridan's commentary on Peter of Spain's remark that, with respect to equipollences, negation interacts with signs of quantity in a way analogous to its interaction with modal terms. Peter of Spain, on whose text Buridan's *Summulae* is a commentary, tells us:

> if we treat modes analogously with signs, so that 'necessary' is treated like 'every', 'impossible' like 'no', 'possible' like 'some', and 'possible...not' like 'some...not', then [1] a negation placed after the mode makes it equipollent with its contrary; [2] placed before, it makes it equipollent to its contradictory; and [3] placed both before and after, it makes it equipollent to its subaltern.[532]

We saw how these rules work (§2.1): they are an earlier statement of the basic insight of modal duality: that *possible* is the same as *not impossible*, and so forth. But Klima goes on to make a deeper comparison between the medieval and modern statements of this modal insight:

> What Buridan states here is effectively the gist of the idea of modern possible-worlds semantics, which treats the intensional modal notions analogously to the extensional notions of the quantifiers, in fact treating them as quantifiers over possible worlds or situations. Thus, if s is a variable ranging over possible situations that can serve as indices to formulas, then 'necessary', 'N', can indeed be replaced everywhere by '∀s', and 'possible',

[532] *Summulae* 1.8.7.

> 'M', by '∃s', according to the following rule: if A is a formula, then 'M(A)', 'N(A)', '∃s‹a›s', and '∀s‹a›s' are also formulae, and 'N(A)' ⇔ '∀s‹a›s', and 'M(A)' ⇔ '∃s‹a›s'. From this, the equipollences Buridan states in the text trivially follow on the basis of the definitional rule familiar from quantification theory: '∃x'$=_{df}$~∀~.[533]

In general, this is right: on some level of analysis, Buridan's modal equipollences work just like ~□~ = ◊ and ~◊~ = □. But any comparison will need to be coarse-grained, since the syntax is not the same: Buridan's does not treat negation or modes as sentential operators at all, as we saw in Chapter 3 (§2.1.1). For him, negation is applied to (or removed from) the copula and the *dictum*, not the whole proposition.

Catarina Dutilh Novaes, in her (2007) *Formalizing Medieval Logical Theories*, undertakes to furnish a full possible worlds semantics, in order to flesh out Buridan's idea of logical consequence—though not of Buridan's modal rules themselves, which we have been so far considering in the present chapter.[534] As textual support for this approach to Buridan, she cites his use of *casus* to describe situations in which a proposition would be true if it were formulated. In the foregoing, I have treated this term as most closely aligned with our concept of a *model* or *countermodel*. This is a minor difference, though I want to state my reasons for treating it this way.

First, because in his most careful and extensive treatments of propositional semantics, Buridan explicitly rejects complexly-signifiable states of affairs as the causes of truth of propositions, as we've seen in the present chapter (§1.1). If we are not careful

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[533] *Summulae* 1.8.7.: p.82-3, n. 123.
[534] Catarina Dutilh Novaes, *Formalizing Medieval Logical Theories: Suppositio, Consequentiae, and Obligationes* (Berlin: Springer, 2007), 89ff.

with our translation of *casus*, we run the risk of re-committing him to states of affairs. On the other hand, the possible-worlds approach provides an elegant way to capture the distinction between the possible and the possibly-true, which we examined in Chapter 2, §2.1, above, so there is much to be said for it.

Second, as we've seen (§1.2 of the present chapter, and Chapter 3, §2.1.3), what distinguishes modal propositions from their assertoric cousins is their copula, which ampliates their terms to stand for *possibilia*—singular objects, many of them nonexistent, which have essential properties inalienable by any causal power. This reverses the explanatory relation between modality and causation which we are accustomed to in modern thinking about modality. Nowadays, we tend to cash out causation in terms of possible worlds, rather than the worlds and the modal properties of their inhabitants in terms of causation. Treating *casus* as possible worlds runs the risk of blurring this explanatory shift.

Dutilh Novaes, as we've noted earlier (Chapter 3, §2.1.2), has a keen sense for spotting anachronism where it comes up. Here too, she rightly points out that a possible-worlds semantics is "inappropriate and anachronistic at first sight". Still, she assures us that worlds "will be taken here with no extra metaphysical assumptions".[535] This is a laudable goal, though I am not entirely sure it is possible: Buridan has such an austere anti-realist metaphysics that even the notion of a truth-making state of affairs is untenable for him. And, as I argue in Appendix B, a metaphysics of possible worlds is literally impossible for Buridan—even though, granted, Dutilh Novaes is not taking these

[535] *Idem* 90.

worlds literally. Still, it remains to be seen what the possible worlds reading of Buridan's modal logic stands to give us. At any rate, it will not be a reliable guide to his modal metaphysics, and it is not clear how it can be done without any assumptions whatsoever.

Juan Manuel Campos Benítez, in his (2010) "La conversión modal y el sistema S5 de Lewis", claims that Buridan's modal logic simply is **S5**.[536] Yet the paper also expresses doubt as to whether this can be done with modern syntactic and semantic apparatus:

> Before continuing, I ought to remark that, in this exposition, I am simplifying the medieval theory, treating an abstraction of an important notion that is difficult to interpret: the ampliation of terms that modal operators produce. In effect, a proposition like "A can be B" should be understood as follows: "What is or can be A can be B". I am omitting this analysis for the present, because it would require a special notation to capture the disjunctive terms; this happens when the connective unites not propositions but terms, which is not how contemporary classical logic works. *But furthermore, this approach would make things too complicated, and would not help us in our present aim, namely of showing what relation there is between* **S5** *and modal conversion*.[537]

[536] Juan Manuel Campos Benítez, "La conversión modal y el sistema S5 de Lewis", in *Classica Boliviana: Actas del V Encuentro Boliviano de Estudios Clásicos* (La Paz: Sociedad Boliviano de Estudios Clásicos, 2010), 123-141.

[537] "Debo decir algo antes de continuar. En esta exposición estoy simplificando la doctrina medieval haciendo abstracción de una noción importante y de difícil tratamiento: la ampliación que ejercen los operadores modales. En efecto, en una oración como "A puede ser B" hay que entenderla como "lo que es o puede ser A puede ser B". La omito ahora porque requeriría de una notación especial para capturar los "extremos disyuntados"; esto ocurre cuando la conectiva une no oraciones sino términos, cosa que no hacemos en la lógica ordinaria de nuestros días. Pero además complicaría demasiado las cosas y no nos ayuda para nuestro propósito de mostrar la relación entre S5 y la conversión modal." (*Idem* 133-4, n.11; Emphasis added).

I agree wholeheartedly with this statement: Buridan's modal logic doesn't work like contemporary classical logic, and merits a notation of its own—like the one I have adopted from Thom, and adapted to what we've seen about the modal role of the copula. And Campos Benítez is right to seek to characterise Buridan's modal logic *before* giving a notation for it. Indeed, that's the practice I have followed here. Only I disagree with Campos Benítez's characterisation: Buridan's modal logic isn't **S5**.

Stephen Read, in his (2012) "Non-Contingency Syllogisms in Buridan's *Treatise on Consequences*", undertakes to symbolise Buridan's modal logic using the tools of modern modal predicate logic. But, as we have seen (in §2.2, above), such an approach runs into problems because, contrary to Buridan's system, it validates inferences from propositions *de necessario* to assertorics (propositions *de inesse*). And, worse still, it can only invalidate those inferences by restrictions on frames which, invariably, invalidate other valid inferences of Buridan's modal logic. Read rightly notes that "Buridan, like most other medievals and arguably Aristotle himself, took affirmatives to have existential import".[538] We saw (in §1.1 of the present chapter) that this is so on Buridan's semantics for *assertorics*. But what is good for the goose is not good for the gander here: Buridan explicitly rules out any existential requirements for divided modals.

Spencer Johnston, in his (2015) "A Formal Reconstruction of Buridan's Modal Syllogism" undertakes to give a possible worlds semantics for Buridan's modal logic. We have already seen his approach, and the troubles it faces, in §1.2.4, above. The main problem is that we get Necessary Birds.

[538] *Idem* 451.

Finally, Jack Zupko sagely urges caution about the use of the phrase *possible worlds semantics* to describe the role of ampliation in Buridan's modal logic. In his (2018) revision of the (2002) *Stanford Encyclopedia of Philosophy* entry "John Buridan", he tells us:

> ampliation can be seen as a kind of Buridanian equivalent of possible worlds semantics, though it would be a mistake to regard it as a remarkable anticipation of that twentieth-century doctrine. Buridan's remarks on its theoretical significance are few, and, despite the degree of technical sophistication involved, he probably did not see it as a radical innovation, but as part of his ongoing effort to make existing schemes for checking inferences more practicable.[539]

I agree with Zupko that Buridan's semantics of ampliation is not an anticipation of modern possible-worlds semantics. Indeed, I find no evidence that modern modal logic was constructed on the basis of any deep historical knowledge of what had been done before. As for the novelty of Buridan's approach, I am unsure. Buridan does seem to go far beyond his contemporaries and predecessors with his modal semantics of ampliation, which has no parallel in Aristotle. Whether he is self-consciously innovating is difficult to determine. After all, as a medieval, Buridan is generally less inclined to trumpet the novelty of his approach than his modern counterparts are.

This, then, is how the contemporary literature on Buridan's modal logic now stands. If we may characterise it generally, we can distill some common notions. First, it is

[539] Jack Zupko, "John Buridan", *The Stanford Encyclopedia of Philosophy*, (Fall 2018 Edition), ed. Edward N. Zalta, §4.

generally acknowledged that Buridan's modal logic is not one of possible worlds, though there are superficial (and exciting) similarities, especially on the level of syntax. Second, possible-worlds analyses are broadly regarded as anachronistic. This anachronism is tolerated, however, because third, it is felt that furnishing a possible-worlds semantics for Buridan would elucidate his modal insights. And it is probably also tolerated because it is felt that giving a possible-worlds semantics for Buridan makes good apologetics: it is easier to attract attention to Buridan by discussing him in contemporary terms. I agree with all these sentiments but the third, and I think that the fourth, while probably true, merits reconsideration.

Yet I cannot agree to the third. As I have been at pains to show, I cannot escape the conclusion that this reading of Buridan invariably introduces serious historical, metaphysical, semantic and syntactic distortions. Fundamentally, this seems to be because Classical FOL does not have a flexible enough apparatus to do what the copula does in traditional Aristotelian logic: as we saw, the copula has been completely rejected by the Fregeans, on the grounds that it has no logical content or, when it does, it is equivocal. We saw these reasons as set forth by Geach in Chapter 3, §2.1.1, above. But perhaps this rejection is hasty, and perhaps there are semantic and logical operations which cannot be performed without the copula. If so, this is a finding in itself, though one reached by treating Buridan as *different* from modern logic, not similar. I return to this point in the conclusion.

Finally, I think the fourth needs to be reconsidered. Granted, it is easy to attract attention to Buridan, and to our field more generally, by pitching him as a modern thinker in medieval guise. But we pay a twofold epistemic price: by programmatically finding modern concerns in medieval texts, we misunderstand them, and we misunderstand ourselves. Modern logic has changed the game considerably—sometimes for better, sometimes for worse. By finding proto-modernity in thinkers who are themselves medieval, we downplay what we ourselves have done more recently. I first raised this point in the portion of the introduction addressed to general readers. And given that you've made it this far, I hope by now you will agree.

CONCLUSION

der Text unter der Interpretation verschwand
—Friedrich Nietzsche[540]

Oumk-ek of Hugin, at hann aptr ne komið, Þó siámk meirr of Munin
—Odin[541]

Throughout this thesis, I have advocated against what, to borrow a term from Henry Butterfield, might be called the *Whig approach to medieval logic*. What does a historical Protestant political movement have to do with medieval logic? In his *Whig Interpretation of History*, Butterfield characterises the whig interpretation thus:

> The whig interpretation of history [...] lies in a trick of organisation, an unexamined habit of mind that any historian may fall into. It might be called the historian's 'pathetic fallacy.' It is the result of the practice of abstracting things from their historical context and judging them apart from their context—estimating them and organising the historical story by a system of direct reference to the present.[542]

Like the whig historian, many historians of medieval logic have organised their research so that it is "bound to converge beautifully on the present",[543] by searching for analogues in past texts to developments we now think of as key. Doing so gives a false impression of linear progress, and makes Buridan look like a proto-Tarskian or a proto-Kripkean, rather than the Buridanian he is. Doing so further papers over Buridan's disagreements with us, allowing us to carry on without having to defend the approaches to logic and language

[540] "...the text disappeared under interpretation" (*Beyond Good and Evil*, II.38).
[541] "I fear that Thought will not return, though I am more anxious about Memory". The line is from *Grimnis-mal*, printed in Gudbrand Vigfusson and F. York Powell, eds., *Corpus Poeticum Boreale*, Vol. I (Oxford: Oxford UP, 1883), p.75, ll.11-12.
[542] Henry Butterfield, *The Whig Interpretation of History* (London: G. Bell and Sons, 1931), 31.
[543] *idem*, 12.

that have become so natural to us. Thus, doing so deprives us of awareness of our own historical situation.

So what the whig historian of logic provides amounts to a flattering optical illusion. As it turns out, many of the features of our modern logic(s), especially the Classical Propositional Calculus and Classical First Order Logic are the product of contentious decisions, made in the late nineteenth and early twentieth centuries. What exactly was decided can be brought to light by contrasting modern logic with Buridan's, as I have done here. My method has been to adopt a relatively high tolerance for complexity (as readers of Chapter 3 will be well aware), and to emphasise the differences first, before looking at any similarities. In this, I am following Butterfield's advice:

> It is better to assume unlikeness at first and let any likenesses that subsequently appear take their proper proportions in their proper context.[544]

This, I submit, is the antidote to whig logic. And it is the approach we should adopt in order to give medieval logic its own proper place in the history of the discipline. After all, if we don't properly contextualise medieval logic, how will we contextualise its modern counterpart?

Avoiding the whig-historical approach moreover pays significant dividends. And not only in terms of historical awareness, but even in terms of modern concerns: as we've seen, Buridan's logic highlights the class of materially valid arguments we have been content to ignore, suggests an alternative to the force-content distinction with less ontological baggage, and points to important limitations on Kripkean modal semantics. And this thesis, while (admittedly) long, presents only a fraction of what we might find in Buridan if we abandon the whig methodology. Here be treasure.

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[544] *idem*, 38.

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Appendix A

Descriptions versus Definitions

Recall Buridan's definition of logical consequence from the *Tractatus de Consequentiis* (I.3), in which he tells us:

> A/C_{Def}) Consequence can be described [*describi*] in the following way: a consequence is a hypothetical proposition (*propositio hypothetica*), made up of an antecedent and a consequent, indicating (*designans*) that the antecedent is antecedent, and that the consequent is consequent; and this indication comes about through the word (*dictio*) "if" (*si*) or "therefore" (*ergo*), or an equivalent.[545]

Why, we might wonder, does Buridan here settle merely to *describe* logical consequence in the above passage, rather than give us a definition (*definitio*) of it? Indeed definition (A/C_{Def}) is plucked right from the section of the *Tractatus de Consequentiis* titled "On the

[545] "Consequentia autem potest describi sic: consequentia est propositio hypothetica ex antecedente et consequente composita, designans antecedens esse antecedens et consequens esse consequens; haec autem designatio fit per hanc dictionem 'si' vel per hanc dictionem 'ergo' aut aequivalentem" (I.3.60-4).

Definition (*Definitio*) of Consequences" (*de Definitione Consequentiarum*). So then why, in the section itself, does Buridan seem to hedge? Have we been subjected to a bait-and-switch?

Perhaps it's heartening that elsewhere in *De Consequentiis* Buridan refers back to the formulation in (A/C_{Def}) as a definition (*definitio*).[546] But maybe these are just slips. And it certainly seems more plausible to consider these latter uses of *definitio* as slips, rather than to write off as a slip his terminology in (A/C_{Def}), where Buridan should be at his most cautious. So let's dig deeper: how in Buridan's view do definitions and descriptions differ?

The place to start is the *Summulae de Locis Dialecticis* (6.3.2-3), where Buridan discusses the locus from definition and the locus from description. In the first of these discussions, dealing with the locus from definition, Buridan tells us the following:

> a definition [*definitio*] is an expression signifying what the being of the thing is, in terms of the things that are essential to it.[547]

Hence a definition expresses something about the *essential nature* of the thing in question. Buridan's example is *rational mortal animal* (*animal rationale mortale*) as the definition of *human* (*homo*). Because this expression expresses what is essential to being human, it counts as a definition of it, convertible with the term *human* in any line of reasoning, *e.g.*

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[546] *E.g.* in *TC* I.8.5.101-3, he justifies his rule as follows: "impossibile est tunc antecedens esse verum non existente consequentia vera; *hoc patet per diffinitionem*" (emphasis added).

[547] "Definitio est oratio quid est esse rei significans per essentialia" (6.3.2, Green-Pedersen, p.33 l.12). In fact, Buridan later refers to this formulation as a *descriptio*, which might seem cause for alarm. But this will be resolved in what follows. We have to get a grasp of the contrast between *definitio* and *descriptio*, and so I will put off dealing with the problem posed by this appearance of *descriptio* (in Buridan's account of *definitio*, no less!) until that foundational task is complete.

A1) Socrates is a human

∴ Socrates is a rational mortal animal.

In this way, the locus from definitions turns on the interchangeability of a term with the expression defining it.

The locus from description, on the other hand, interchanges a term with its *description*, rather than its definition. Buridan has less to say about this locus: it apparently works in a way "analogous to the locus from what is defined", and can be reconstructed on the basis of what he says in the preceding *Summulae* passage about the locus from definition.[548] Here are Buridan's examples of descriptions:

> *animal capable of laughter* is said to be a description of *human*, and *curved-nosed* is a description of *snub*.[549]

If the locus from description works the way the locus from definition does, we can give the following argument by analogy with (A1):

A2) Socrates is a human

∴ Socrates is an animal capable of laughter

Of course, *capable of laughter* (*risibilis*) is a *proprium* of *human*—that is, it's an accident present only in humans, and in all of them. Hence *human* and *capable of laughter* will be coextensive. Likewise for *curved-nosed* and *snub*. This fact—that these *propria* are present in all and only the things they are *propria* of—is what underwrites the

[548] "[...] proportionaliter sicut locus a definitione" (6.3.3; Green-Pedersen, p.35, l.3)

[549] "'animal risibile' dicitur descriptio hominis et 'nasus cavus' descriptio simi." (6.3.3; Green-Pedersen, p.35-6, ll.33f).

interchangeability of the description with the thing described. Thus, the definitional expression *rational animal* and the descriptive expression *animal capable of laughter* will both be convertible with *human*, though only the former is a definition.

Thus since the extensions of definitional and descriptive expressions are the same, these expressions are convertible with the things they describe or define. And as we have seen, the distinction between the two loci—one from definition, the other from description—seems to turn on the distinction between propria and essences, respectively.

To return to our problem: why does Buridan give us a mere *description* of consequence in *TC* I.3, whose title promises us a full definition? The short answer is that *consequentiae* are not apt for definition because of their ontological status. To see why, we need to turn to a parallel text: Buridan's *Questions* on the *Prior Analytics* (I.3). There, Buridan asks whether the familiar definition of syllogism from *Prior Analytics* I.1 (24^b19-22) is correct (*bona*). Here is the proposed definition:

S_{Def}) A syllogism is an expression [*oratio*] in which, certain things having been set down, something other than them follows by necessity, on account of the way they are.[550]

[550] "syllogismus est oratio in qua quibusdam positis aliud ab his quae posita sunt de necessitate **sequitur** ex eo quod haec sunt" (*QAP* I.3; emphasis added).

Notice that this is word-for-word identical to the definition in the corresponding passage of the *Prior Analytics* in the *Aristoteles Latinus*, with the sole exception that in Buridan's version, *sequitur* (which I have marked in boldface) replaces *accidit* in the *AL*. I don't know what to make of this—if anything.

According to Buridan, (S_{Def}) isn't really a definition at all, but a *description* (*descriptio*). Buridan seems to think this is so for two reasons. First, because definitions are of substantial terms only, not of connotative or accidental terms. And second, because the *differentiae* here considered are not unique to syllogisms as a kind of expression (*oratio*).

Let's look at each of these in turn. As we saw in the *Summulae de Locis Dialecticis* passage cited above, Buridan thinks definitions are interchangeable with the thing(s) they define. Therefore, says Buridan,

> Properly speaking, the syllogism is not defined here, since a definition has to be convertible with the thing defined, and no syllogism is convertible with this definition.

Now Buridan does not tell us why he thinks this definition is not convertible with a syllogism properly speaking. But we can reconstruct his reasoning as follows: syllogisms are in the final analysis mental acts, and secondarily speech acts. So, too, is definition (S_{Def}) from *Prior Analytics* I.1: the string of words that make up (S_{Def}) is one whole expression. But a syllogism *qua* expression is not convertible with the definition of syllogism *qua* expression. For instance, consider the following:

A3) Socrates is thinking of a syllogism
∴ Socrates is thinking of (S_{Def}), *i.e.* a definition of the syllogism

A4) Plato uttered a syllogism
∴ Plato uttered (S_{Def}), *i.e.* a definition of the syllogism

As examples like (A3) and (A4) make clear, the thought or spoken definition of syllogism is not interchangeable with the thing defined. According to Buridan, what is being defined is not the syllogisms themselves, but the term that denotes them:

> But what is defined here is the term *syllogism*. And so we can grant that the syllogism is here defined, provided we take the term *syllogism* in material supposition.[551]

Hence what is defined here is not a thing, but a term standing for itself: not the syllogism, but the term *syllogism*. To borrow a distinction from modern analytic philosophy, what is being defined is a term that is *mentioned*, not one that is *used*.

Thus, according to Buridan, (S_{Def}) gives an account of a term (*syllogism*) in material supposition. Now only substances can be defined, as Aristotle argues in *Metaphysics* VII.5 (1030^{b}29-35), and Buridan notes here. But then a term in material supposition is not a substance, since words—be they mental or spoken—do not belong in the category of Substance. Therefore, terms standing in material supposition are not suitable subject matter for definitions.

Now Buridan's treatment of this subject are terse, and indeed the text in *QAPr* I.3 looks a bit like a muddled version of Pseudo-Scotus's *Librum Primum* Priorum Analyticorum *Aristoteles Quaestiones* (henceforth *PrAnQQ*; I.5). There, Pseudo-Scotus presents the same question, many of the same objections, and a similar analysis, but with greater detail and cogency. So let's briefly turn to Pseudo-Scotus, to bolster the case we're making about Buridan.

[551] "sed ibi definitur iste terminus 'syllogismus'. Verum est etiam quod potest concedi quod syllogismus ibi definitur, capiendo 'syllogismum' secundum suppositionem materialem" (*QAP* I.3).

Like Buridan, Pseudo-Scotus tells us that there are two sorts of definition: (i) real definition (*quid rei*), and (ii) nominal definition (*quid nominis*).[552] Real definitions come in two flavours: (a) those which account for the essence of a substantial term by means of its genus and differentiae—and which a few lines down Ps.-Scotus calls a *quidditative* definition (*definitio quidditativa*); and (b) those which account for the nature of a connotative term (like *snub*), but have to rely on extraneous terms. The latter have to rely on extraneous terms because they are terms for things that have to inhere in something else: we cannot talk about what it means to be an accident, for example, without making reference to things an accident is an accident of; nor can we speak of matter without making reference to form, and vice versa.[553] Hence these latter real definitions are not stand-alone, the way those of type (a) are.

Type (ii) are nominal definitions, which explicitly express what is meant implicitly by a term. Pseudo-Scotus tells us that:

> these are properly called *descriptions* (*descriptiones*): such descriptions "explicitly express the implicit meaning of a defined term".[554]

These, then, are the type to which (S_{Def}) belongs, as we will see in a moment, because there is overlap between descriptions and nominal definitions.[555] In order to count as a good description, an expression must meet three requirements: it must (α) be convertible

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[552] "Duplex est definitio. Quaedam est definitio *quid rei*, et quaedam *quid nominis*" (*PrAnQQ* I.5, 281b)
[553] "Definitio quid rei est duplex, quia quaedam exprimit quid res est per verum genus et differentias formales, sine explicatione alicuius extraneae conditionis [...] Secunda est definitio *quid rei*, quae non solum exprimit quid res est, sed cum hoc explicat aliquam extraneam conditionem, ut verbi gratia definitio Accidentis, si sit bona, debet exprimere Subiectum, et in definitione Materiae debet explicari Forma, et econtra in definitione Formae, materia" (*PrAnQQ* I.5, 281b)
[554] "Sed definitio *quid nominis* proprie dicitur Descriptio, et est illa *quae exprimit explicite significationem implicitam termini definiti*" (*PrAnQQ* I.5, 281b)
[555] For a discussion of which descriptions might count as nominal definitions, see Ockham's *Summa Logicae* I, especially I.7.

with the thing defined; (β) explicitly express what is signified by the defined term; and (γ) do so without extraneous information (*nugatio*).[556]

Pseudo-Scotus then sweeps through the above categories, showing why (S_{Def}) does not count as (i) (a) and (b), but rather as (ii). A syllogism is not a substance, so it cannot given a type (a) definition—which Ps.-Scotus here calls a *quidditative* definition (*definitio quidditativa*). And though a syllogism is a quality, (S_{Def}) says nothing about what that quality inheres in; therefore, as a *quid rei* definition of type (b), it fails. But it does meet criteria (α), (β) and (γ) of (ii), and therefore it counts as a description (*descriptio*).

Hence if Buridan is following Ps.-Scotus, as he seems to be, then his reason for saying that (S_{Def}) is a description, rather than a definition, is that syllogisms are not apt for definition, because they are not in the category of substance; and because (S_{Def}) makes no reference to the vital extraneous features of a syllogism—which, as a quality, must inhere in something, and therefore be defined in terms of that thing it inheres in. But (S_{Def}) does meet criteria (α)–(γ), as Buridan notes, and therefore it is a perfectly good *description.*

Let's return to Buridan's text. The second problem with (S_{Def}), according to the discussion in the *QAPr* (I.3), is that the *differentiae* it sets out are not unique to syllogisms—that is, they do not identify all syllogisms, and only syllogisms. Thus,

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[556] "Ad talem descriptionem, quod sit bona, sufficiunt tres conditiones. [α] Prima est quod sit *convertibilis cum definito*. [β] Secunda est, quod *exprimat explicite significationem definiti*. [γ] Tertia est quod *sit sine nugatione*" (*PrAnQQ* I.5, 281b).

These three criteria seem to be what Buridan has in mind, by the way, when he tells us, quite tersely, that (D2) is a good description because it: "explains the concept of what's described sufficiently, and without extraneous information [*nugatio*], and because it is convertible with the thing described." Hence it seems Buridan, though he is not so clear on these points, is nevertheless thinking along similar lines to Pseudo-Scotus.

definition (S_{Def}) both under- and over-determines. It under-determines because, as Buridan tells us:

> This definition does not apply to every syllogism, and therefore it is not correct [...] Proof the antecedent: this definition does not apply to enthymemes, since in an enthymeme there are not multiple premisses, but only one; and nevertheless, Aristotle says in book I of the *Prior Analytics* that an enthymeme is a valid syllogism. And therefore, it does not apply to every syllogism.[557]

Hence according to (S_{Def}), enthymemes like the following don't count as syllogisms:

A5) Socrates is a human
∴ Socrates is capable of laughter

The suppressed or unstated premise in (A5) is "All humans are capable of laughter", and if (A5) did include this suppressed premise, it would be a syllogism. But since enthymemes like (A5) have only one premiss, and the account given in *Prior Analytics* I.1 states that syllogisms have multiple premisses, enthymemes like (A5) are not syllogisms. But, later on in the *Prior Analytics*, Aristotle says that they are. Therefore, this putative definition under-determines.

And (S_{Def}) over-determines, too. As Buridan points out, it also extends to such non-syllogistic items as consequences from exponents to what is

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[557] "haec definitio non convenit omni syllogismo; ergo non est bona. Consequentia est de se nota: sed probo antecedens: quia haec definitio non convenit enthymemati: quia in enthymemate non ponuntur quaedam praemissae, sed una tantum; tamen Aristoteles, in primo Posteriorum*, dicit quod enthymema vere est syllogismus; ergo non convenit omni syllogismo" (*QAPr* I.3, arg. 6).

expounded—consequences, that is, that explicitly set out or 'unpack' exponible terms (*exponibilia*). For example:

A6) Socrates is neither animality nor rationality
Socrates is something, and so is animality and rationality
∴ Socrates is something other than animality and rationality.[558]

Any such argument from exponents to what is expounded is not a syllogism; nevertheless, an argument like (A6) still falls under (S_{Def}).

Accordingly, since the (S_{Def}) both under- and over-determines. And, since (S_{Def}) is a definition not of a *substantia* but of a term in material supposition (*syllogism*), it is not apt to be a definition at all. Accordingly, Buridan concludes that it is a mere description (*descriptio*). And, as he points out,

> it is enough that the description be convertible with the thing described, and that it clearly expresses the nominal definition (*quid nominis*) of what's described without any excessive or trivial detail.[559]

Hence Buridan takes (S_{Def}) to be extensionally adequate, and therefore to be convertible with the thing described (sc. the term *syllogismus*).

Now there is a bit of tension here with Buridan's claim, in the main body of his response, that the *differentiae* presented in (S_{Def}) are not unique to one species in the

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[558] "Sortes non est animalitas nec rationalitas, et Sortes est aliquid et animalitas est aliquid et rationalitas est aliquid, ergo Sortes est aliud ab animalitate et rationalitate". I plucked this example from Paul of Venice's *Expositio in VII Librum* Metaphysice *Aristotelis. The Medieval Reception of Book Zeta of Aristotle's Metaphysics*, vol. 2. Ed. Gabrielle Galluzzo (Leiden: Brill, 2013), 200.
[559] "Sed ista est descriptio, ad quam sufficit quod sit convertibilis cum descripto et quod explicite exprimat quid nominis ipsius descripti sine superfluitate vel nugatione" (*QAPr* I.3 co.).

genus *utterance* (*oratio*). This, as we saw, makes room for powerful objections to the extensional adequacy of (S_{Def}), on the grounds that it under- and over-determines. Buridan's proposed solutions are a tad hand-wavy, and need not detain us here long. Briefly, (S_{Def})'s apparent under-generation is not a problem: granted, an enthymeme like (A5) is not a syllogism, strictly speaking. But, as noted, it can be made into one by the addition of another premise.[560] So in a certain, looser sense, it really is a syllogism. And over-generation is similarly not a problem: according to Buridan, arguments from exponents to what is expounded are ruled out by the clause of (S_{Def}) stipulating that something other than the premisses follows: in such an argument, the conclusion is not really distinct from the premisses, at least in a mentalistic sense.[561]

Turning back to (A/C_{Def}), we are now in a position to ask: does either of the above problems apply to the definition of *consequentia* in TC I.3? Not the latter problem, since there is no reason to think Buridan's definition under- or over-determines. Consequence is the most general species of arguments and conditionals syntactically defined, as we saw in the preceding chapter. And (A/C_{Def}) is sufficiently general to cover enthymemes, syllogisms, conversions, equipollences, and so forth. So it does not undergenerate.

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[560] "Et credo etiam quod per hoc differt syllogismus ab illa consequentia quae est ab exponentibus ad expositam, quoniam exponentes non differunt ab exposita nisi secundum vocem, et non secundum intentionem mentalem" (*QAPr* I.3 co.).

[561] "dico quod illa propositio Aristotelis, scilicet quod enthymema est vere syllogismus, est falsa de virtute sermonis; sed ad illum sensum est vera quod ex enthymemate per additionem alterius praemissae fit vere syllogismus" (*QAPr* I.3, ad 6).

Further, (A/C_{Def}) does not give us any *differentiae* which hold of any other species of hypothetical proposition—that is, it does not apply to other types of non-consequence hypotheticals, like conjunctions or disjunctions. So it does not overgenerate.

But there is reason to think that the former problem for (S_{Def}) likewise applies to (A/C_{Def}). After all, consequences, *qua* expressions (*orationes*), are subordinated to mental acts, and so is an expression like (A/C_{Def}). Here again, one mental act is not convertible with another. To see why, recall our examples from the syllogism case considered above, and tweak them for the present purpose:

A3′) Socrates is thinking of a logical consequence
∴ Socrates is thinking of (A/C_{Def})—*i.e.* a definition of logical consequence

A4′) Plato uttered a logical consequence
∴ Plato uttered (A/C_{Def})—*i.e.* a definition of logical consequence

So (A/C_{Def}), like (S_{Def}), does not strictly speaking give us a definition of logical consequence. What it gives us, rather, is a description of logical consequence that is extensionally adequate. So long as we keep this in mind, we needn't worry that (A/C_{Def}) is not a definition, strictly speaking. In fact, to demand a definition of *consequence* is to commit a sort of category error: as a term in material supposition, it is not the sort of thing that can be defined. Still, the extensional adequacy of (A/C_{Def}) means we can treat it as convertible with *consequence*—which is just what I do, without comment, throughout the present thesis.

Appendix B

Buridanian *Possibilia*

In Chapter 5, we saw that the terms of a modal proposition are *ampliated* (*ampliari*) so that they stand not only for actual things, but for possible ones as well.[562] Chapter 5 dealt with the semantics and syntax of these ampliated propositions in Buridan's logic, but there remain some important metaphysical questions. Not least of these is, what *are* these *possibilia*? I'll begin with this question, and conclude that, although the range of *possibilia* is wide, it does not include Lewisian possible worlds. In short, possible worlds are not possible on Buridan's metaphysics. I'll then consider an objection.

The Realm of *Possibilia*

What are Buridanian *possibilia*? In "The Modal Logic of John Buridan", G.E. Hughes ruminates on the question:

> For a long time I was puzzled about what Buridan could mean by talking about possible but non-actual things of a certain kind. Did he mean by 'possible A', I wondered, an actual object which is not in fact A but might have been, or might become, A? My house, e.g., is in this sense a possible green thing because, although it is not in fact green, it could become green by being painted. But this interpretation won't do; for Buridan wants to talk, e.g., about possible horses; and it seems quite clear that he does not believe

[562] "Propositio divisa de possibili habet subiectum ampliatum per modum sequentem ipsum ad supponendum non solum pro his quae sunt sed etiam pro his quae possunt esse quamvis non sunt." (*TC* II.4, ll.1-7).

> that there are, or even could be, things which are not in fact horses but which might become horses.[563]

Hughes is right to think that modality can't be a matter only of powers or capabilities of existing things like horses and houses. Otherwise, we would conflate the class of *actualia* with that of *possibilia*, which is dead wrong. Here are three arguments why: first, as Hughes points out, if modality were just about powers of existing things, we would ascribe some strange capacities to things: the capacity of non-horse items (like fodder or apples) to become horses (more on this in a moment). So just about anything could, possibly, be something else, if you followed it through enough permutations. Even so, it would seem absurd to say that anything is, simply speaking, anything else—for instance that a rag is a possible horse.

Second, if *possibilia* were just the *actualia*, then modal propositions would deal exclusively with existing things. So to say that something is possible would just be to say that some existing object had the capacity to be or do it. But Buridan frequently states that there are non-existent *possibilia*. And rightly so: if the *possibilia* were all and only *actualia*, then Buridan's modal logic would both undergenerate and overgenerate truth for modal propositions. It would undergenerate propositions *de possibili*, because we would have to treat propositions like the

P1) Donkeys are possibly talkers

as false, since no donkey has the capacity to talk. But there is nothing self-contradictory or logically impossible about (P1). Certainly God could make a donkey talk.[564]

Further, Buridan's modal logic would overgenerate true propositions *de necessario*, because we would have to treat as necessary things that could be otherwise, though no mundane power exists to change them. For example,

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[563] G.E. Hughes, "The Modal Logic of John Buridan", in *Atti del Convegno internazionale di storia della logica: la teorie delle modalita*, ed. G. Corsi, C. Mangione, and M. Mugnani (Bologna: Cooperativa Libraria Universitaria Editrice Bologna, 1989), 97.

[564] Consider *e.g.* Baalam's ass in Numbers 22.

P2) The planets' orbits are necessarily elliptical

would be true, since there is no natural power capable of altering the orbits to some other shape. But God could make Jupiter's orbit triangular, if need were, and so falsify (P2).

Third, if *actualia* are all the *possibilia*, then all affirmative possibility statements about creatures *prior* to their creation would be false because vacuous: their subject terms would stand for nothing. But then non-existent *possibilia* would be indistinguishable from *impossibilia* prior to their creation.[565]

In sum, then, Hughes is right to think that the class of *possibilia* is a proper superset of the *actualia*. But Hughes is wrong about what the *possibilia* themselves are. I part ways with him when, in the next few sentences, he analyses such *possibilia* in terms of possible worlds:

> What I want to suggest here, very briefly, is that we might understand what he says in terms of modern 'possible worlds semantics'. Possible world theorists are quite accustomed to talking about possible worlds in which there are more horses than there are in the actual world. And then, if Buridan assures us that by 'Every horse can sleep' he means 'Everything that is or can be a horse can sleep', we could understand this to mean that for everything that is a horse in any possible world, there is a (perhaps other) possible world in which it is asleep. It seems to me, in fact, that in his modal logic he is implicitly working with a kind of possible worlds semantics.[566]

[565] Allow me a moment's digression here: we might wonder, what *can't* God do? That is, what sorts of things would we expect to find in the *impossibilia*? Buridan is relatively silent on this, as one would expect any arts master—*eo ipso* technically unqualified to do theology—at the fourteenth century University of Paris to be on such a theologically fraught question. But we have already seen a couple of illustrative examples of *impossibilia* in his treatment of conditional modality: the vacuum is a good candidate; presumably a four-sided triangle is another. And we saw in Chapter 2 that Buridan thinks the syllogisms, because formally valid, cannot be rendered invalid by a miracle. If they could, they would not be formally valid. Therefore, invalid-Barbara will be one of the *impossibilia* as well. But who can say how large the class of *impossibilia* really is?

[566] G.E. Hughes, "The Modal Logic of John Buridan", 97.

Contra Hughes, it seems to me that in his modal logic Buridan is *not* working, implicitly or otherwise, with possible worlds semantics of any sort. We do not need a plurality of worlds to talk about "more horses than there are in the actual world": what we need is a notion of a power capable of producing (or annihilating) horses at will—that is, we need a power of bringing horses into being.

Worse still: in the penultimate sentence, Hughes hints at something like trans-world identity for horses. Maybe that's the way things are in the worlds of David Lewis, but as we've seen in Chapter 5 it's not the way things are in Buridan's modal semantics. There is no plurality of worlds here: a horse can sleep because it is a kind of thing that has a capacity for sleeping, not because its transworld counterpart is sleeping in another world.

And worst of all, this trans-world identity leaves us in a pickle if we want to explain how, in Buridan's view, things that are actual can become possible but presently non-existent things.

And indeed, Buridan discusses these transmutations in a remarkable passage in his *QM* (IX.5):

> Can a hatchet can come from wool? And it seems so, since anything can come from anything—albeit through several transmutations, as is stated in the first book of the *Physics*.[567] Hence earth can come from wool, and afterwards stone can come from earth, from which can come iron, and from that a hatchet. And likewise, a horse can come from wool, since earth comes from wool [by decomposition], and herbs from the earth, and from those herbs which perhaps a horse will eat there can come horse sperm, and, at length, another horse. And so even a horse can come from wool. And the same holds for all other modes of transmutation.[568]

[567] *Physics* I.7

[568] "[...] utrum ex lana potest fieri securis? Et videtur quod sic, quia ex quolibet potest fieri quodlibet—licet per multas transmutationes, ut habetur primo Physicorum. Unde ex lana potest fieri terra, postea ex terra potest fieri silex, deinde ferrum, deinde securis. Similiter ex eadem lana potest fieri equus, quia ex lana fiet terra, de inde herba, et ex illa herba forte quam equus comedet poterit fieri sperma equi et tandem equus. Et ita etiam ex lana potest fieri equus. Et sic de omnibus aliis modis transmutandi." (*QM* IX.5, fol.58v, b).

So everything is possibly everything else—or so the objection goes. Buridan's solution appeals to the distinction, set out by Aristotle in his *Metaphysics* (VIII/H, 6), between *proximate* and *remote* potencies:

> It should be commonly granted with respect to both proximate and remote potency that everything which will be is able to be, and everything which someone does, that person is able to do that thing. [...] But speaking of 'being able' according to proximate potency, all such things are to be denied. For we should say that a child is not just as powerful as a large man, and that an infant is not yet able to walk, and that a hatchet cannot come from wool.[569]

The error, then, lies in a conflation of proximate potency, like the wool's capacity to decompose into soil, and remote, like the wool's capacity, through a series of permutations, to furnish the matter for a horse. So while wool is possibly soil, wool is not possibly a horse—at least, not in the same way.[570] Buridan goes on to say that, ordinarily, people speak of potency in the proximate sense, and so it seems that that is how we should read claims about possibility in ordinary cases—though in general it is better to qualify a claim than to deny it outright.[571]

What does this example tell us about potencies and *possibilia*? It seems that what grounds the modal properties of an object are the things it can do or be made into. So, to return to Hughes's example, a house is possibly green because, as Buridan tells us, there is a power to make it be that way. Conversely, things like thoughts, forces like gravity, and objects like planetary orbits are not possibly green, since there is no power to make them that way. They're just not the sort of things that can be green.

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[569] "[...] communiter ad propinquam et remotam codedendum est quod omne quod erit potest esse, et omne quod aliquis faciet ipse potest facere illud [...] Sed loquendo de posse secundum potentiam propinquam omnia talia sunt neganda. Diceremus enim quod puer non est aeque potens sicut magnus vir, et quod infans nondum potest ambulare, et ex lana non potest fieri securis." (*QM* IX.5, fol.58r, c).

[570] This restriction on permutations suggests a deeper essentialism that would be interesting to investigate further, in a later project.

[571] "Et ideo breviter credo quod melius et aptius sit in proposito respondere per distinctionem, et secundum unum sensum affirmare et secundum alium negare quam simpliciter affirmare vel negare. Et videtur mihi quod homines magis communiter utuntur illo sensu qui est secundum potentiam propinquam quam illo qui est secundum remotam." (*QM* IX.5, fol.58r, c).

In Chapter 2, we dealt with the necessity of inference in terms of what God can and cannot do; and in Chapter 4, we saw that God cannot invalidate a formally valid syllogism. Since Buridan takes the modals to be dual (so that *possibly* is just *not necessarily not*, and so on), we should define possibility in similar terms: what is possible is just what God *can* do—*i.e.* what it is not impossible for God to do. So let's set this down now:

> *possibilia* $=_{def}$ an object *o* is possible if and only if God can make *o*. And *o* is possibly *F* just in case God can make *o* to be *F*, without annihilating *o*.

This defines a pretty broad class of *possibilia*. Indeed, it is far more reasonable (and far less time-consuming) to talk about the outer bounds of *possibilia* than to enumerate all that the class of *possibilia* contains.[572]

So what *can't* God do? Basically, the only examples we find in Buridan of *impossibilia* are impossible compositions, or transmutations that involve annihilation of the original subject. As an instance of the latter, Similarly, God could make Socrates into a donkey, but doing so would annihilate Socrates *qua* human. As an instance of the former (as we saw in Chapter 3), a chimaera *qua* impossible object is, according to the *Summulae de Demonstrationibus* (8.2.3), merely "an animal made up of parts out of which it is impossible for any animal to be composed".[573] The *impossibilia*, then, are *incompossibilia*: things that cannot be complexes of incompossible attributes, like square triangles and donkey humans.[574]

[572] How would one even *start* such a list? The task would be endless. God could make blue ducks, cloud-sized ice-cream sundaes, even (perhaps) *friendly librarians*...
[573] "Animal compositum ex membris ex quibus impossibile est aliquod animal componi" (*Summulae de Demonstrationibus* 8.2.3, de Rijk 33, ll. 23-4)
[574] Incidentally, I follow Terence Parsons in distinguishing contradictory objects (*e.g.* a non-square square) from what might be called *contrary* ones (*e.g.* a round square). From the text, Buridan clearly has the latter in mind, but would (if asked) judge the former to be *impossibilia* as well, reasoning *a fortiori*. See Terence Parsons, *Nonexistent Objects* (New Haven: Yale UP, 1980), 38-42.

Now if this is so, we might wonder whether possible worlds themselves are among the *possibilia*. The surprising answer is *no*.

Are Possible Worlds Possible?

—Or, to put the question in Buridanian terms: can God create a plurality of possible worlds? First, the argument *pro*: it seems that God can indeed create as many possible worlds as God likes. So long as we conceive of a world as just a cluster of interrelated *possibilia*, there seems to be no barrier in principle to clustering them. Here is why: at least a few—and indeed, probably most—possible objects comprise interrelated possible parts. Consider *e.g.* a possible watch that could be generated, *ex nihilo* or *ex ferro* or whatever, but which does not now exist. Such a possible watch won't be dense and undifferentiated all the way through, like a chunk of *foie gras*, but will have interrelated possible parts—possible gears, possible springs, etc.

Now it would be arbitrary and unprincipled and just plain wrong to place a limit on how large such a possible object could be: if a possible watch can be made the size of a mantle clock, why not a possible watch the size of Manhattan?[575] Likewise, it would be arbitrary to place a limit on their complexity: if a watch the size of Manhattan is permissible, why not also a comparably huge astronomical horologium?

From these considerations, we can distill two principles, namely:

I) *possibilia* can be internally complex, comprising interrelated possible parts; and

II) there is no limit in principle on the size or complexity of such *possibilia*.

[575] You might think, well, but such an object would collapse under its own weight. But that would be to conflate nomological impossibility with logical impossibility, which we certainly won't do.

From (I) and (II), it follows that God could make possible worlds, roughly construed as manifolds of interrelated possible objects.

But now a further consideration: consider a possible object, say a fork: does it make sense to speak of such an object existing outside of a world or manifold? Or does it seem that any such possible object must exist, not isolated from a manifold, but in a possible set of circumstances? Thinking of a fork outside some spatio-temporal manifold seems, if not impossible, at least a little weird. We can conceive of a fork in the absence of other objects—that is, alone in a manifold. But we cannot, it seems, speak meaningfully of a fork that exists out of space. And so, it seems, possible objects only ever inhabit worlds, and so possible-objects metaphysics must, if it is to be coherent, collapse into a metaphysics of possible-worlds.

But let's examine these possible worlds a little more closely. They are either actual, in the sense that God has made them, or they are possible but non-existent, in the sense that God has not, but could. The question is, could God make these worlds to be actual, in some sense, but discrete? Such discreteness will be vital for their status as full-fledged possible worlds: if they are not somehow discrete, they are no more possible worlds than planets in different galaxies are. David Lewis is keenly aware of this need for discreteness, and so tells us of the worlds that:

> The worlds are something like remote planets, except that most of them are much bigger than planets, and they are not remote. Neither are they nearby. They are not any spatial distance whatever from here. They are not far in the past or future, nor for that matter near; they are not at any temporal distance whatever from now.[576]

So possible worlds, if 'actualised' in this way, must not be at any spatio-temporal distance from each other. In a slogan: you can't travel far enough to find yourself in another world. They just aren't interrelated in that way—or any way at all. They are causal isolates.

Suppose (1) that the worlds are actual, in the sense that they've been created by God. Now Lewis says they *don't* interact, but he seems to have in mind a much stronger

[576] David Lewis, *On the Plurality of Worlds* (Oxford: Blackwell, 1986), 2.

claim: that they *can't*, in principle. There are many people in the world I *won't* meet, but in principle *could*. This is very different from our transworld counterparts—yours and mine—whom we not only *won't* meet but *can't*. So the causal isolation of the worlds is more than just a contingent fact. It's a matter of necessity.

Now what does it mean to say such worlds are causal isolates—*i.e.* that they can't interact? Distance won't do the trick, as Lewis makes clear in the passage cited above: worlds are not causally isolated because they're really *really* far away from each other, the way I am isolated from a planet in the galaxy Andromeda. Space is not what separates the worlds.

So why can't these worlds interact? It seems that, as a rule, they exist in different manifolds. But this just pushes the question back: what prevents causal interaction between the manifolds? Bear in mind that, as *possibilia*, these are all produced by (and therefore causally dependent on) God. So it falls to God to keep them from interacting.

Now Calvin Normore has pointed out to me that David Lewis indeed allows counterfactual relations among worlds. If such relations alone exist between worlds in a Buridanian ontology, where one God creates all the worlds, then there is no problem. But Lewis categorically rejects spatio-temporal relations. And he rejects causation as well, apparently on the grounds that causal relations are spatio-temporal ones. Now we have seen that, if one God creates all the worlds, they are causally related by means of their causal dependence on the same prior thing. So they cannot be causally isolated the way Lewis requires.

But set this aside for now: perhaps God can causally isolate the worlds anyway, by stipulating that there is just an impermeable barrier between the manifolds, analogous to the glass plates separating different tanks in a large aquarium, or the walls hiving off different theatres in a cineplex. But now three problems appear.

First, such manifolds won't really be discrete, since they will have been produced by the same source. So although they may be separate, they are not completely causally independent, since they have the same cause. They are, then, causal siblings, even if they never interact.

Second, what happens when two things hit opposite sides of the barrier that separates their respective manifolds? Suppose, for instance, that there is a barrier between manifolds A and B; and *a* and *b*, possible objects in A and B respectively, smack up against the barrier between them—as fish sometimes do in the divided aquariums at Sea World. Then a barrier that prohibits causal interaction between the two worlds, A and B, nevertheless causally interacts with both of them. So the thing separating A and B is, in a causal sense, a member of both. And so the barrier itself crosses between the worlds—but that's precisely what the barrier was invoked to prevent. We can try to block the barrier, say by adding another barrier, so that the two barriers are like parallel sheets of glass in a double-pane window. But then we get a regress: what keeps the barriers themselves apart? What would happen if one barrier collided with whatever separates it from the other?

Third, even if we could separate A and B causally, we couldn't separate them temporally: just as we can speak of one movie in a cineplex beginning halfway through another in another theatre, so we can speak of a universe being half as old as another—that is, as being created *midway along the life cycle* of another universe.

Peter King has argued against me on this point, claiming that the analogy of fictional worlds is incoherent. To take his example, does Bilbo Baggins of *The Lord of the Rings* live before or after Harry Callahan of the *Dirty Harry* films? Either way of arranging them looks strange.

Now fictional entities come with a whole host of problems I'd rather avoid, but I want to resist this counterargument. I think it's coherent to talk about relations and even causal interactions between fictional universes. But first, a distinction: there are two ways to talk about relative time with respect to fictional worlds: as nuclear, the way King does, and as extranuclear. I take it that extranuclear temporal relations are uncontroversial, at least philosophically: we can make good sense of the claim that Sherlock Holmes the character was created before Harry Callahan was. In an extranuclear sense, then, the question whether Bilbo lives before Harry is a question of when these characters were created. And since Bilbo first appeared in Tolkein's (1937) novel *The Hobbit*, whereas

Harry makes his debut in the (1971) film *Dirty Harry*, Bilbo is extranuclear-older than Harry by about three decades.

So if we're rejecting anything like causal or temporal relations between fictional entities or worlds, it has to be nuclear ones we're talking about. But even here, we face trouble: it is relatively uncontroversial in the literature that nuclear properties affect extranuclear ones: if Sherlock were a reclusive stamp collector and not a brilliant detective (nuclear properties, all), then he would not be admired by so many real-world detectives (extranuclear property). But it is all but ignored that extranuclear ones can causally interact with nuclear ones. For instance, if Harry Callahan were not so popular in film (extranuclear), then his character would not have been developed over the course of four major motion pictures, and so he would lack all the (nuclear) properties he gets in these films. For instance, it's only in the last film of the series, *Sudden Impact* (1983), that Harry utters the phrase, "Go ahead, make my day". If Harry had not been popular, this trait (utterer of the above phrase) would never be among Harry's nuclear properties.[577]

But wait: if nuclear properties like being a clever detective can produce extranuclear properties like being famous, and if extranuclear properties like being famous can produce nuclear properties like further character development, then presumably one fictional entity or world can interact with another, by mediation of the real world. *And this happens all the time*: would for instance Ulysses of the (2000) film *O Brother, Where Art Thou?* have his nuclear properties were it not for all the compelling traits of Odysseus in Homer's (ca. 8th cent. BCE) *Odyssey*? No. So Odysseus indirectly causes Ulysses to have certain (nuclear) properties, just as the character of Þórr in the (ca. 13th cent.) *Poetic Edda* indirectly causes Thor in the Marvel films to be a hammer-weilding character.

Examples of this phenomenon abound, because characters that are famous and well-liked (extranuclear) for their nuclear properties spawn spin-offs with distinctive sets of nuclear properties. But it would be quite surprising if the causal order went the other way around: if as a result of Ulysses' singing a hit single in the film resulted in Odysseus

[577] As it happens, I've been discussing this problem in correspondence with Fred Kroon for a while now. I am writing an independent paper on it, because I think there are a lot of inconvenient things about fictional objects that get swept under the rug, and that rug is starting to look pretty lumpy.

composing a bluegrass tune, or if the medieval Icelandic Þórr suddenly teamed up with Captain America and Bucky Barnes. So why this causal asymmetry? Because there is a temporal asymmetry: Þórr's nuclear properties are prior to Thor's, as are Odysseus' to Ulysses'. Even in terms of nuclear relations, fictional worlds exist on a timeline.

Now Buridan follows Aristotle on time: time is a function of change, so that where there is no change, there is no time.[578] But even where there are different causal processes, say in two worlds A and B, it does not entail that A and B are temporally independent. If it did, the distant events in a far-off galaxy would occupy a completely different temporal manifold from ours.[579] But we can't travel so far in three-dimensional space that we enter an altogether separate fourth dimension, completely unconnected with the one we started in.

So it seems there is no way to create causally discrete worlds. But as I noted above, they might not be actual, but merely unactualised and possible. So suppose (2) that they are not actual in the sense that they've been created or produced, but merely possible. But we saw already that such worlds can't be produced. Still, maybe they can exist somehow in unrealised possibility as discrete possible worlds. But then we have to say that something exists in possibility that can never be produced. And this runs contrary to the definition of *possible* that we started with. Recall: what's possible is all (and only) that which can be produced by a causal power—that is, which can be produced without producing a contradiction. And this is a contradiction.

Hence on Buridan's causal definition of possibility, God cannot make possible worlds in the spatio-temporally isolated sense of David Lewis. Granted, they can be very very far apart, or not (broadly speaking) causally interrelated. But then they are no more removed from each other than distant planets are. And we demand more than this from possible worlds: if they're to count as worlds at all, they have to be worlds apart. But this is impossible. So possible worlds, no matter how large or remote, collapse into one world: this one. So possible worlds not only *aren't* part of the picture: they just *can't* be.

[578] *Physics* 4.10 (218b10-11)
[579] Relativity notwithstanding: although we can grant, following Albert E., that time is different depending where you are relative to massive objects, etc., it does not follow that things in another galaxy occupy an altogether different time—or else it would be meaningless to speak of their relativity to our own.

Now maybe this is a problem with Buridan, not with Lewis. But from the passage in Lewis, cited above, one thing is clear: Lewis's doctrine of the separation of worlds, on which worlds stand in no causal, temporal, or spatial relations to one another, is not a metaphysical conclusion. It is, rather, a *stipulation*. My claim here is that Lewisian worlds do not work on Buridan's metaphysics; I am not making the stronger claim that Lewisian worlds are impossible *tout court*. Still, I have my doubts: it seems to me there is a case to be made that this Lewis's doctrine is incoherent. At least, I can't picture these worlds, with no spatial or other relations to each other whatsoever. Perhaps it's a limitation of my intellect, but it's one I share with Kant, who says in his discussion of space that:

> First of all, one can imagine one space only. And when we talk of multiple spaces, we actually mean [proper] parts of one and the same single space. Nor can these parts be prior [*vorhergehen*] to the one and all-encompassing space as components of it (from which its composition is possible), but can only be conceived as within it. *Space is essentially one*...[580]

So perhaps Lewis's stipulation about the separation of worlds is not all it's cracked up to be. For now, however, I am content with this: Lewisian worlds are impossible on Buridanian metaphysics.

But now there remains a lingering doubt for the Buridanian, namely:

Is Buridan's Account Circular?

Apparently it is. Inquisitive minds will want to know: what are we to make of obviously modal terms like *can* in the above definition of *possibilia*? Buridan reads *can* (*potest*) as

[580] "Denn erstlich kann man sich nur einen einigen Raum vorstellen, und wenn man von vielen Räumen redet, so versteht man darunter nur Teile eines und desselben alleinigen Raumes. Diese Teile können auch nicht vor dem einigen allbefassenden Raume gleichsam als dessen Bestandteile (daraus seine Zusammensetzung möglich sei) vorhergehen, sondern nur in ihm gedacht werden. Er ist wesentlich einig[...]". Immanuel Kant, *Kritik der reinen Vernunft* (Hamburg: Verlag von Felix Meiner, 1956 [1781]), 68.

possibly-is. But if we follow Buridan in this, the whole thing looks circular. What can we do?

The Buridanian has two options: we can either show how his definition of possibility is *not* circular, or we can admit that it is. But even if we bite that bullet, we can maintain our position: we can still defend Buridan on the grounds that modality is a primitive notion, any so definition of possibility or necessity will, at some point, become circular. If so, the modern, possible-worlds account of modality should face the same problem. So it remains to show just this. If the modern approach faces the same problem, we can declare a *détente*. But if the circularity of Buridan's account is a trap Buridan could've avoided, then his view is not so attractive. Still, it remains to be seen: is the modern account of possible worlds likewise circular?

The most obvious line of attack is the *possible* in *possible world*. What makes a world possible? When we call these worlds *possible*, we're saying that they're internally consistent. So if we identify our worlds with sets of sentences, and we say that those sentences are maximally *consistent*, then what we mean is that they *can* all hold at the same time. And *voilà*: circularity again.

Now here the defender of the modern notion could appeal to a more foundational notion: that of logical consequence. A set of sentences Γ is consistent just in case it doesn't entail both φ and $\sim\varphi$. So a consistent world is just one which does not entail a contradiction.

Now there are two ways of cashing this out, which correspond to the two Tarskian notions of logical consequence. The first is semantic: Γ, we will say, entails φ just in case every model of Γ is a model of φ. Still, the notion of entailment here depends on models: $\Gamma \vDash \varphi$ just in case for every $\mathfrak{M} \vDash \Gamma$, $\mathfrak{M} \vDash \varphi$. But $\mathfrak{M}$ has to be a consistent model. And what does it mean to be consistent? That all its parts are compossible. Here again, we find a primitive modal notion.

But any would-be defender of the modern view can resort to the syntactic notion, whereby $\Gamma \vdash \varphi$ just in case there is a deductive system D such that $\Gamma \vdash_D \varphi$. This may seem a bit arbitrary—and indeed, our selection of D likely will be. But it's harder to see here what primitive modal notion is at play. A deductive system is, after all, just a set of rules, and while those rules may intuitively appeal to a prior notion of modality, they do not obviously depend on it. So has the modern account done it, and given us an account of possible worlds which does not appeal to any prior notion of modality? If so, the definition of possibility for a possible world *qua* consistent set will be as follows:

$$\mathrm{Con}(\Gamma) \qquad \Leftrightarrow \qquad \Gamma \nvdash_{\mathrm{D}} \varphi \wedge {\sim}\varphi, \text{ for some D.}$$

That is, Γ is a consistent set of sentences iff Γ does not entail both φ and ${\sim}\varphi$ on some deductive system D.

Now it remains to pick our deductive system, D. And not every D is such that, assuming that Γ is inconsistent, $\Gamma \vdash_{\mathrm{D}} \varphi$ for any φ. Granted, if we take D to be classical logic, then this will hold. And classical logic indeed seems like the best fit. If we do so, we won't face much trouble doing metalogic when we're dealing with systems that are weaker than but extensions of classical logic—for instance, intuitionistic or many-valued logics. But what about logics that are not extensions of classical logic, such as connexive logic? Will using classical logic as our metalogic like this—that is, using it to determine which worlds are possible—will we have to say that there is no possible world in which connexive logic holds? It seems so. So we have gained consistency at the price of ruling out as impossible those logics that are not extensions of classical logic. So although there are worlds in which organic life isn't carbon based, or the constants of the physical universe are completely different, there are no worlds in which Aristotle's Thesis—*i.e.* ${\sim}({\sim}\varphi \rightarrow \varphi)$—holds. This is what happens when we give classical logic sole fiat in dictating what our worlds must look like.

Hence modern modal logic, with its appeal to possible worlds, faces a circularity problem akin to Buridan's. The only way around it is to impose arbitrary restrictions on the basis of what classical logic dictates—which opens us up to a whole other set of problems. For classical logic is a jealous god.