John Buridan on Logical Consequence

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Abstract: If an argument is valid, it is impossible for its premises to be true, and its conclusion false. But how should we understand these notions of truth and impossibility? Here, I present the answers given by John Buridan (ca. 1300-60), showing (i) how he understands truth in his anti-realist metaphysics, and (ii) how he understands modality in connection with causal powers. In short: if an argument exists and is valid, there does not exist a power capable of making the premises true and, at the same time, making the conclusion false.

Some arguments hold, and some arguments don't. For example, contrast the following:

1)	Every man's donkey is running;	2)	This dog is a father;
	Socrates is a man;		This dog is yours;
∴	Socrates' donkey is running.[1]	∴	This dog is your father.[2]

[1] "cuiuslibet hominis asinus currit, Socrates est homo; ergo Socratis asinus currit". John Buridan, *Summulae de Suppositionibus* (4.2.6). For a study of the logic of these non-normal forms which incorporate oblique terms, see Terence Parsons' *Articulating Medieval Logic* (Oxford: Oxford UP, 2014). For an analysis of these forms in connection with Buridan's concept of logical consequence, see Stephen Read's "John Buridan's Theory of Consequence and His Octagons of Opposition", *Around and Beyond the Square of Opposition*, ed. Jean-Yves Béziau and Dale Jacquette (Basel: Springer, 2012), 93–110, and his "Non-Normal Propositions in Buridan's Logic", *Formal Approaches and Natural Language in Medieval Logic*, ed. Laurent Cesalli et al. (Barcelona: Brepols, 2016), 453-68, as well as Boaz Faraday Schuman, "Multiple Generality in Scholastic Logic", *Oxford Studies in Medieval Philosophy* 10 (2022): 195-262.

[2] "iste canis est pater et est tuus; ergo est pater tuus". John Buridan, *Sophismata* ch. 4, 7th sophism. To say that a dog is a father is merely to say that it has puppies, as Gyula Klima helpfully clarifies in a footnote to his translation. See his *Summulae de Dialectica* (New Haven: Yale UP, 2001), 889, n.94.

Clearly, there is something wrong with (2): it is invalid. On the other hand, (1) is valid. What makes (2) invalid is that it is possible for the premises to be true while the conclusion is false—as can be seen from the fact that, whereas the premises are possible, the conclusion is absurd. In contrast, in a valid argument like (1), the truth of the premises *necessitates* the truth of the conclusion. That is, if the premises are true, then necessarily the conclusion is true as well. This, in general terms, is the concept of logical consequence.

John Buridan (ca. 1300-60) provides his main analysis of this concept in his *Tractatus de Consequentiis* (*TC*), 1.3. Yet he does not explain there in detail what he thinks the underlying notion of necessity is.[3] Indeed, E.A. Moody, writing in the 1950s—before many of the editions and translations we have today were available—remarks that this question is an obscure one.[4] This paper presents my answer to this question. I first examine the *TC* discussion, and then show how it can be elaborated on by appealing to what Buridan says about necessity in the *Quaestiones in Analytica Priora* (*QAPr*) 1.25, and in the *Summulae de Demonstationibus* 8.6.3. As we will see, Buridan cashes out necessity in causal terms: broadly speaking, if an argument is valid, there is no power—not even God—capable of rendering the premises true and the conclusion false.[5]

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[3] Angel D'Ors flags this absence of an account of the modal notion at play in his "*Ex impossibili quodlibet sequitur* (Jean Buridan)". *Argumentationstheorie*. Ed. Klaus Jacobi (Leiden: Brill, 1993), 204-5. There, he claims in passing that "the term *impossible* appears in all its multiple meanings". There are many ways of reading this claim—set out by d'Ors—and some of these are true. But Calvin Normore disagrees with the strong claim that the modal notions at play are equivocal, in his "*Ex impossibili quodlibet sequitur* (Angel d'Ors)", *Vivarium* 53 (2015), 362*ff*. Normore is right: it is univocal. This is one of the guiding assumptions of §2, below.

[4] E.A. Moody, *Truth and Consequence in Medieval Logic* (Amsterdam: North-Holland Publishing Co., 1953), 78. More recently, several commentators have undertaken to account for this underlying modality. For example, Catarina Dutilh Novaes undertakes to give a possible-worlds interpretation of Buridan's notion of logical consequence, in her *Formalizing Medieval Logical Theories* (Dordrecht: Springer, 2007), but with "no extra metaphysical assumptions" (90). To the extent that her project is successful in this regard, it differs from the present one, which is all about those metaphysical assumptions.

[5] This is the causal reading noted by Gyula Klima, "Consequences", *The Cambridge Companion to Medieval Logic*, ed. Catarina Dutilh Novaes and Stephen Read (Cambridge: Cambridge UP, 2016), 323.

1. Validity in the *Tractatus de Consequentiis*

Buridan's ontology is programmatically sparse. So are the ontological commitments of his logic. In *TC* 1.3, he presupposes two anti-realist doctrines. Since we will frequently have to refer back to these in what follows, it's worth setting them down right now:

Doctrine 1: Propositions[6] do not exist except as token singular, occurrent thoughts: for example, unless someone is actually thinking "Donkeys are animals", that proposition does not exist.

Doctrine 2: There are no such things as the propositional states of affairs, or complexely signifiables (*complexe significabilia*)—that is, propositionally-structured extra-mental significates.[7] Instead, propositional truth depends on the meaning of the proposition's terms.[8]

In *TC* 1.3, Buridan he adopts the following procedure: first give a proposal, then present an objection to it, and then modify the proposal accordingly.[9] We will follow him point-by-point, considering along the way some discussions in the recent literature.

Arguments, with which we began, are a type of consequences (*consequentiae*). Consequences, in turn, belong to what we may, loosely, term molecular propositions

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[6] I am here using "proposition" to translate *propositio*, even though the propositions we will be talking about here are little like the abstract entities of Frege and his followers. Caveat lector.

[7] The latter are associated with Gregory of Rimini and Adam Wodeham, and appear to be Buridan's target in his *Questions on Aristotle's "Metaphysics"* VI, 7. For a discussion of these theories in their own right, see Gabriel Nuchelmans, *Theories of the Proposition* (Amsterdam: North-Holland Publishing Co, 1973), 227*ff*.

[8] The meaning in question here is, primarily, the supposition of the terms, as Stephen Read notes in "The Medieval Theory of Consequence". Synthese 187 (2012), 903. I can't go into supposition theory now, however, since it would take the present study too far afield.

[9] Angel d'Ors, in *Ex impossibili*, criticizes Buridan on these grounds, saying that "there is also a need for a reason which, with respect to the definition accepted, guarantees not only that no counterexamples have been found, but also that none will be found in the future" (202). But I take this approach of proposal—objection—modification to be just a feature of Buridan's didactic method.

(*propositiones hypotheticae*).[10] Such propositions contain multiple propositional parts, as opposed to categoricals—like those of the traditional Square of Opposition, ("Some/every **S** is/isn't **P**")—which don't. Accordingly, Buridan tells us that:

> A consequence is a molecular proposition (*propositio hypothetica*), since it is made up of multiple propositions joined together by the term *if* or *therefore*, or an equivalent term. Such terms designate that one of the propositions that they join together follows from the other.[11]

As a molecular proposition, a consequence joins together multiple propositions.[12] In this respect, it is akin to other molecular propositions that bind together multiple propositions by means of terms like *and*, *or*, etc. Where consequences differ from these is that they present their constituent propositions as *following*: in a consequence, one proposition follows from another. This, indeed, is bound up in the etymology of *consequentia*, from *con-* ("with") and *sequi* ("to follow")—as commentators have frequently pointed out.[13]

Hence, when it comes to consequence, order matters: "P, therefore Q" is in no way equivalent with "Q, therefore P". In contrast, terms *and* and *or* and the like are—to borrow a bit of modern terminology—commutative. At least in terms of truth value, it makes no difference whether "P and Q" or "Q and P". The complexes made up with such terms as their main connectives are not, therefore, consequences.

And yet there are important differences between *if* and *therefore*. This point is worth lingering on for a moment, because Buridan's apparent conflation of the two in the

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[10] An important caveat: in contrasting categorical with molecular propositions, I do not wish to suggest that categoricals for Buridan and other Scholastic logicians are akin to the atomic propositions of Frege and his followers. For Frege, an atomic proposition has only two parts: a concept and an object, corresponding to the linguistic predicate and subject. There is, in such propositions, no strictly logical terms at all. For Buridan, even simple propositions like "S is P" contain a logical or syncategorematic element, namely the predicative verb *is*. There are, then, strictly speaking, no atomic propositions of the Fregean sort here in Buridan at all. A discussion of a related point, in connection with quantification in Leibnizian term logic, is made by Fred Sommers, *The Logic of Natural Language* (Oxford: Clarendon Press, 1982), 12*ff*.

[11] "Consequentia autem est propositio hypothetica; constituta enim est ex pluribus propositionibus coniunctis per hanc dictionem 'si' vel per hanc dictionem 'ergo' aut aequivalentem. Dictae enim dictiones designant quod propositionum per eas coniunctarum una sequatur ad aliam".

[12] NB strictly speaking, Buridan does not think that these propositional parts are themselves fully-fledged propositions, since (at least in the case of conditionals and disjunctions), they are not asserted—whereas all propositions are. Buridan discusses this in *Summulae de Propositionibus* 1.7.1 and 1.7.3.

[13] For example, E.A. Moody tells us that "The term 'consequence' [is] derived in Latin from the verb 'to follow' (*sequi* or *consequi*)" in *Truth and Consequence*, 64.

foregoing passage has seemed, at least to some of his commentators, embarrassing. Nor is this embarrassment with Buridan alone: as Ivan Boh remarks, medieval logicians in general "disconcertingly" use the notion of consequence to cover many very different things, including conditionals and arguments.[14] Yet there is plainly a huge difference between the following:

3)	*If* Socrates is a donkey	4)	Socrates is a donkey
then	Socrates is an irrational animal	*therefore*	Socrates is an irrational animal

We would all grant (3); but anyone who put forth (4) would be guilty of asserting two (obvious) falsehoods. The difference is that the parts of (4) are asserted, whereas those of (3) are not.

Against this apparent conflation, Peter King argues that, by *consequentia*, what Buridan really means is *inference*.[15] As King points out, in the overwhelming majority of his discussions, Buridan does seem to have inferences, rather than conditionals, in mind. This fact is reflected in his frequent use of *valet* ("is valid") and related terms appropriate to arguments in his descriptions of consequences, rather than the language of truth and falsity more appropriate to conditionals.

King's argument, and the general complaint of Ivan Boh, are motivated by concerns about assertion. But—as we have seen—Buridan's distinction between consequences and other kinds of molecular propositions is a matter of *syntax*: *consequentiae* as "followings", so to speak, are characteristically non-commutative: "If P then Q" says something very different from "If Q then P". Other propositions, like conjunctions and disjunctions, are

[14] Ivan Boh, "Consequences", *The Cambridge History of Later Medieval Philosophy*, ed. Norman Kretzmann, Anthony Kenny, and Jan Pinborg. (Cambridge: Cambridge UP, 1982), 300.
[15] Peter King, "Consequence as Inference", *Medieval Formal Logic*, ed. Mikko Yrjönsuuri. (Dordrecht: Springer, 2001), 117-45.

commutative: "P or Q" is, logically, the same as "Q or P". Since their order doesn't matter, they are not "followings".[16]

Even so, Buridan does have the framework to distinguish (3) and (4) in terms of what we now call propositional force. And he makes this distinction explicitly.[17] Therefore, as impressed as I am by the clarity of King's argument, and the wide range of texts he draws upon to support his claims, I do not think that it is necessary or desirable to cash *consequentiae* out exclusively in terms of inference.

Let's return to reading the *TC*. Following the foregoing passage, Buridan tells us that the concept of consequence should be understood in terms of the relation between the propositions involved.[18] These propositions are respectively labelled *antecedent* (*antecedens*) and *consequent* (*consequens*). Buridan's account of logical consequence turns on the definition of these terms:[19]

> *antecedent* and *consequent* are so called relative to one another. Therefore, they have to be described in relative terms. Many accordingly say that, of a pair of propositions, one is antecedent to another if it is impossible for the first to be true without the other's being true; and a proposition is consequent to another if it is impossible for it not to be true when the other

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[16] Are all non-commutative molecular propositions *consequentiae*? Buridan doesn't say. In *Summulae* 1.7.2, we are given a list of the species of molecular proposition, which includes causal propositions of the form "P because Q". Such propositions are, clearly, non commutative. And while he does not outright say that causal propositions are consequences, he does speak of their antecedents and consequents in *Summulae* 1.7.8.

[17] For a discussion of this point, see Gyula Klima, "John Buridan and the Force-Content Distinction", *Medieval Theories On Assertive and Non-Assertive Language*, eds. A. Maierú and L. Valente (Rome: Olschki, 2004), 415-27.

[18] Here, I am skipping two brief points that Buridan makes in passing. First, Buridan notes that *if* attaches to the antecedent, whereas *therefore* attaches to the consequent. This is related to the point we make in elementary logic classes, when we teach students to symbolise both "if P then Q" and "P if Q" as (Q → P). Second, Buridan observes that some writers have argued over whether only a *true* (*vera*) consequence should be called a consequence, so that false ones are not consequences at all. Buridan dismisses this debate, as he does with many quibbles, on the grounds that terms signify by convention (*ad placitum*). He is here, he tells us, concerned only with the true ones.

[19] Buridan's language here is of description, even though the heading of *TC* 1.3 is "de *diffinitione* consequentiae" (emphasis mine). Probably this is just because consequences are not substances, and so strictly speaking there is no definition of them. So in speaking of definitions of such items as consequences, antecedents, and so on, we are speaking loosely.

> is true.[20]

What the many say here is intuitively plausible. Indeed, it closely resembles formulations in more recent logical works as well. Compare for instance Matthew McKeon's informal statement of the concept of logical consequence:

> One sentence is said to be a logical consequence of a set of sentences, if and only if, in virtue of logic alone, it is impossible for the sentences in the set to be all true without the other sentence being true as well.[21]

But, says Buridan, this initial formulation—and, he would presumably argue, McKeon's as well—is insufficient. It fails by Doctrine 1, by presupposing propositional existence. After all, a non-existent proposition is not true, as Buridan points out:

> But the foregoing description falls short, or is incomplete, because the following is a valid (*bona*) consequence: 'Every man runs, therefore some man runs'. And yet it is possible for the first to be true without the second being true—as indeed happens when the second doesn't exist.[22]

It is possible to have a proposition in mind without thinking of another one that follows from it. This happens all the time, as can be readily observed in political discussions. In such cases, there is no consequence. Therefore, the proposed account falls short.

Even so, this objection is a relatively minor one: to address it, all we need to do is to include in our account that the propositions involved in a consequence must actually exist. This is precisely what Buridan does in his second attempt:

> Therefore, some say that the aforementioned description of consequence ought to be fleshed out as follows: a proposition is antecedent to another when it is not possible for the first to be true while the other is false,

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[20] "Antecedens autem et consequens relative dicuntur ad invicem; ideo per invicem describi debent. Dicunt ergo multi quod propositionum duarum illa est antecedens ad aliam quam impossibile est esse veram illa alia non existente vera et illa est consequens ad reliquam quam impossibile est non esse veram reliqua existente vera, ita quod omnis propositio ad omnem aliam propositionem est antecedens quam impossibile est esse veram illa alia non existente vera."

[21] Matthew McKeon, *The Concept of Logical Consequence* (Berlin: Peter Lang, 2010), 1.

[22] "Sed haec descriptio deficit vel est incompleta, quia hic est bona consequentia: 'Omnis homo currit; ergo aliquis homo currit' et tamen possibile est primam esse veram secunda non existente vera, immo secunda non existente".

> supposing that the propositions in question are formulated at the same time.[23]

As long as we stipulate that the propositions involved must exist, the first objection does not get off the ground.[24]

How are we to understand this requirement of simultaneous existence in dialogical settings? In particular, what happens when, for instance, I speak the premise, and you draw a conclusion from it? Does the above stipulation mean that the consequence must exist, in its entirety, in (at least) one mind? My own answer is *Yes*, and I am pretty sure Buridan's would be too.[25]

Here's why. Suppose I say "Socrates is a donkey", and you reply, "So Socrates is an irrational animal." Here, we have what looks like a clear example of a single consequence shared part-wise across multiple minds. But, I submit, what is actually going on is this: the full consequence exists in duplicate in your mind; if I understand your conclusion, the full consequence exists in my mind, too. In other words, when you hear me say "Socrates is a donkey", that proposition exists in your mind, along with the conclusion.

I believe this can be shown. Buridan occasionally uses Hebrew as an example of a significative language that does not mean anything for his audience, because they do not know the language.[26] In that vein, we can tweak our example, making it a cross-linguistic one between (bilingual) speakers:

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[23] "Et ideo aliqui dicunt dictam descriptionem debere suppleri sic: Illa propositio est antecedens ad aliam propositionem quam impossibile est esse veram illa alia non existente vera illis simul formatis"

[24] This is not to say that there are no deeper problems lurking here. For example, what does it mean for multiple propositions to be formulated simultaneously? And how long a gap can there be between the first and the second before the first no longer really exists? What about long lines of reasoning, which may last many days or even years, where it is not at all clear that (or even *how*) all the propositions involved can simultaneously exist?

[25] It's worth noting that the case just described is, for a medieval logician like Buridan, *primary* in an importaant sense: logic is dialectic, and dialectic is between interlocutors. This point is made in connection with the title of Buridan's opus, the *Summulae de Dialectica*, by Catarina Dutilh Novaes, *The Dialogical Roots of Deduction* (Cambridge: Cambridge UP, 2021), 139. In a way, then, this fact was right in front of our eyes all along!

[26] For example, in *Summulae de Dialectica* 1.1.6.

4′) סוקרטס הוא חמור

∴ Socrates is an irrational animal

For anyone who does not understand the antecedent, (4′) is not a consequence.[27] The reason for this is simple: you have to understand a proposition in order to make an inference from it. And you have to have a proposition in your mind in order to understand it. QED.[28]

Let's return to reading the *TC*. Once he has formulated this second attempt, Buridan turns to a deeper and more difficult objection than the first: what about propositions which, while possible, are self-falsifying?

> But I still say that this description is not correct (*bona*), since the following is not a valid consequence: 'No proposition is negative; therefore no donkey is running'. And yet, if we keep to the aforementioned description, we will have to grant that this consequence is a valid one. Therefore, etc.[29]

Here, Buridan makes two claims: (i) that the consequence under consideration here is not valid; and (ii) that it should be valid, according to the proposed account of logical consequence under consideration. From these two claims, it follows that the proposed account is incorrect.

In support of (i), he points out that this putative consequence does not have a valid contrapositive:

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[27] My apologies to readers who do know Hebrew, for whom this example—at least for the purposes I want to put it to—won't work. I leave to them the exercise of cooking up another example, if they care to.

[28] I wish to express my gratitude to the reviewer who brought up this point, which helped me to make explicit and defend something I had—and should not have—taken for granted. Thanks, whoever you are!

[29] "Sed adhuc dico quod haec descriptio non est bona, quia hic non est bona consequentia: 'Nulla propositio est negativa; ergo nullus asinus currit' et tamen secundum dictam descriptionem oporteret eam concedere esse bonam; ergo etc."

> Proof of the first premise: the opposite of the antecedent does not follow from the opposite of the consequent. That is, it does not follow that 'Some donkey runs, therefore some proposition is negative'.[30]

Given Doctrine 1, it is possible for there to be no negative propositions in existence, and therefore for the conclusion to be false. And yet in such a case, a donkey may be running, and so the antecedent can be true. Thus, the contrapositive of our putative consequence is false. Therefore, the above description of consequence is not correct.

Buridan's use of contraposition here to winnow out a competing alternative account of logical consequence has struck some commentators as dubious. David Kaplan, as reported by Gyula Klima, has argued that Buridan is here giving contraposition as a further requirement for logical consequence.[31] We may state this requirement as follows:

> **CPos:** A consequence is valid iff it has a valid contrapositive.

In practical terms, CPos means that valid consequences will have to come in pairs: both the initial consequence, and its contrapositive. Yet if it is possible to think of a proposition without thinking of another that follows from it—as we observed above—*a fortiori* it is possible to think of a consequence without thinking of its contrapositive. We do so all the time. But if this contrapositive does not exist, and yet is at play in our account of logical consequence, then CPos appears to violate Doctrine 1, by relying on non-existent propositions. If so, it's bad news for Buridan.

Faced with this puzzle, Klima offers the following solution: CPos holds if we assume bivalence, and if we assume that all the propositions involved in the consequence and its contrapositive exist. This latter assumption does the heavy lifting. Here is how

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[30] "Primam praemissam probo. Quia ex opposito consequentis non sequitur oppositum antecedentis; non enim sequitur: 'Quidam asinus currit; ergo quaedam propositio est negativa'. Secunda autem praemissa manifesta est. Quia primam, scilicet quae designatur esse antecedens, impossibile est esse veram; ergo impossibile est ipsam esse veram alia non existente vera"

[31] Gyula Klima, *John Burian* (Oxford: Oxford UP, 2009), 211-16. The attribution to Kaplan is on p. 317, n.8.

Klima proposes to solve the puzzle: suppose that φ, ψ, ∼φ and ∼ψ all exist. Then consider the following case:

∼φ is true, and ∼ψ is false

This case invalidates "∼φ, therefore ∼ψ". By Bivalence, it also makes it the case that:

φ is false, and ψ is true

Which straightforwardly invalidates "ψ, therefore φ"—*i.e.* the contrapositive of "∼φ, therefore ∼ψ". Hence the conditions that invalidate an argument also invalidate its contrapositive. And again, on the assumption that the propositions involve exist, we can straightforwardly accommodate CPos without violating Doctrine 1.

I find this solution elegant, but unnecessary. The worry raised by Kaplan presupposes that Buridan takes CPos to be a part of his definition of logical consequence. But CPos is not part of this definition, here or below. Instead, Buridan is using contraposition to argue against a proposed definition. And in fact, later on Buridan will derive contraposition as one of his rules of inference, which he formulates on the basis of his definition (*TC* I, 8, rule 3). In this sense, CPos as extrinsic to the definition—to borrow an observation from Angel d'Ors, and give it a positive spin.[32] The Law of Contraposition depends on the definition, and not the other way around. Granted, it may seem a bit circular to use a rule one hasn't yet proved, in order to arrive at a definition later used to prove that same rule. But such are the foundations of logic: if we needed a seaworthy account of logical consequence before we could make any arguments, we couldn't even get started arguing about alternative approaches. This problem is not unique to Buridan.

Let's return to reading the *TC*, focusing on the second of the two claims Buridan advances in his rejection of the second attempt at definition. This claim is that (ii) the

[32] Angel d'Ors, "*Ex impossibili*", 203.

second attempt renders valid "No proposition is negative; therefore no donkey is running". *À propos* of this claim, Buridan tells us that:

> The second premise is clear on its own. For it is impossible for the first proposition—that is, the one that is taken to be the antecedent—to be true. Therefore, it is impossible for it to be true while the other is false.[33]

From the text we can posit an important distinction between, to use the words of Arthur Prior, the possible and the possibly-true.[34] The self-falsifying "No proposition is negative" (that is, no negative proposition exists, in the way specified by Doctrine 1) cannot ever be true. It is therefore not *possibly-true*. Even so, what it describes is *possible*. In fact, what it describes was once actual, before the first negative proposition was ever formulated in any human language.[35] And it could become true again: God could just annihilate all negative propositions, if He so chose. But then, if someone were to formulate this proposition again, it would immediately be false, since at least one negative proposition—*i.e.* that proposition itself—would exist.

To elucidate this distinction between the possible and the possibly-true, Prior gives a simple example of sheets of paper, which serve as a sort of stand-in for more complex states of affairs. Propositions written on these sheets are evaluated along two parameters: being true *on* the sheet they are written on, and being true *of* other sheets. Some, like "Some proposition is affirmative", will be true on any sheet they are written on. Yet they need not be true *of* any other sheet—for example, a blank one. They are necessarily-true, but not necessary. Others, like "No proposition is negative", will be true on no sheet, but may well be true *of* some other one, *e.g.* a blank page. They are possible, but not possibly-true.

Thinking along these lines, Prior defines validity in terms of truth *of* sheets rather than truth *on* them:

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[33] "Secunda autem praemissa manifesta est. Quia primam, scilicet quae designatur esse antecedens, impossibile est esse veram; ergo impossibile est ipsam esse veram alia non existente vera."
[34] Arthur Prior, "The Possibly-True and the Possible", *Mind* 78, no. 312 (1969): 481-91. Prior's discussion is in connection with Buridan's *Sophismata*, where Buridan further analyses propositions like "no proposition is negative".
[35] This observation is from Gyula Klima, "Consequences", *The Cambridge Companion to Medieval Logic*, ed. Catarina Dutilh Novaes and Stephen Read (Cambridge: Cambridge UP, 2016), 321.

> a sentence on a sheet may be validly inferred from other sentences on this sheet if and only if there is no sheet (in the set) of which all the premiss-sentences are true but of which the conclusion sentence is false.[36]

This seems to be the right way to think about truth in Buridan's definition of consequence in *TC* 1.3. It also allows us to endorse (5) and reject (6), as Buridan does in his *Sophismata* (8, sophs. 1-2) on which Prior is commenting:

5)	Every proposition is affirmative	6)	No proposition is negative
∴	No proposition is negative	∴	Some proposition is negative

Whereas the premise of (5) is true of any sheet its conclusion is also true of, the same does not hold for (6). Picture two sheets: one with both the premise and the conclusion of (6) written on it, and one with just the conclusion. Then the premise will be true of the latter sheet, whereas the conclusion will not. QED.

Prior's sheets thus provide a useful way of thinking and talking about the notions of modality and truth that Buridan has in mind in his rejection of the second attempt. But we need to tread carefully here: Prior's sheets are, as he tells us at the outset, states of affairs.[37] And we could, he tells us later on, let our sheets be the whole universe.[38] But this sounds dangerously close to a metaphysics that Buridan explicitly rejects, in his discussion of the third attempt:

> Others accordingly define *antecedent* and *consequent* differently, saying that a proposition is antecedent to another which is related to it in such a way that it is impossible for it to be however the first signifies without being however the second one does—supposing that the propositions in question are formulated at the same time. Yet strictly speaking, this description is not

[36] "Possibly-True", 489.
[37] *idem* 481.
[38] *idem* 488.

> true, because it supposes that every true proposition is true on account of the fact that things are as it signifies, which has already been denied.[39]

Here, Buridan falls back on Doctrine 2, set out above. He accordingly rejects the view that propositions, taken as wholes, signify something like a state of affairs. This is the view of, among others, Gregory of Rimini, which Gabriel Nuchelmans nicely sums up as follows:

> The bearers of truth and falsity [...] are not only actually existing propositions and the significates of actually existing propositions, but also states of affairs that are capable of being signified by true or false propositions even if these corresponding propositions do not in fact exist.[40]

Buridan argues quite forcefully against the existence of any such things in his *Questions on Aristotle's "Metaphysics"* 6.7. The details of his argument need not detain us here. For our purposes, it is sufficient to make two brief observations.

First, the notion of truth at play in *TC* 1.3 will have to be cashed out in terms of the semantics of terms, not of propositions taken as significative wholes. Showing how this could be done, in a way that would allow us to reject "No proposition is negative, therefore no donkey is running", would be an interesting undertaking in its own right.[41] It is, however, tangential to the present discussion, and cannot be explored in detail here.

Second, as noted, we need to be careful not to read too much into Prior's exposition, particularly in ontological terms. It is more like Wittgenstein's ladder: once you're up, you leave it behind. Even so, Buridan himself is pretty sanguine about the similar formulation of the third attempt. He adopts it for brevity's sake, acknowledging that it is merely conventional. As he tells us:

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[39] "Ideo alii aliter definiunt, dicentes quod: Illa propositio est antecedens ad aliam quae sic se habet ad illam quod impossibile est qualitercumque ipsa significat sic esse quin qualitercumque illa alia significat sic sit ipsis simul propositis. Tamen adhuc illa descriptio non est vera de virtute sermonis, quia supponit quod omnis propositio vera ex eo sit vera quia qualitercumque significat ita est, quod prius negatum est."

[40] Gabriel Nuchelmans, "The Semantics of Propositions", *The Cambridge History of Later Medieval Philosophy*, eds. Norman Kretzmann, Anthony Kenny, Jan Pinborg, and Eleonore Stump (Cambridge: Cambridge UP, 1982), 204.

[41] Some work in this direction has been recently done by Manuel A. Dahlquist, who focuses on the temporal and indexical aspects of Buridan's definition, in connection with more recent work on contextual hybrid logic. See "Time and Indexicality in Buridan's Concept of Logical Consequence", *History and Philosophy of Logic* 42, no. 4 (2021): 374-97.

> Indeed, we often speak in terms of the first account of consequence, which was clearly disproved, since it has few counterinstances. Yet however we speak of these things, we really intend the sense which has been set out here.[42]

Like Buridan, we may use language that is, in its literal meaning, untrue. Such are Prior's sheets. All we need to be able to do is fill out the conceptual content, when pressed.

In this way, Buridan's language is sometimes a bit like a currency convertible with gold bullion: we need not carry around the gold with us all the time, so long as we can confidently cash our paper money in whenever we have to. It's also, by the way, what can make interpreting Buridan hard: you can't always take him at his (literal) word; you have to hunt for passages where he's being cautious and explicit. Much of the time, he is quite happy to use hollowed out traditional formulations. I suspect that this is partly how his ideas—radical ideas, really—were able to spread so widely.

Our task is now to figure out what Buridan means by his modal language in *TC* 1.3. What does it mean to say that, in a valid consequence, the truth of the premises *necessitates* that of the conclusion? To answer this question, we will have to cast about in his other logical works.

2. Necessity in the *QAPr* and *de Demonstrationibus*

In his *Questions on the "Prior Analytics"* (*QAPr*) 1.25, Buridan asks whether the following proposition is necessary:

7) Humans are animals

[42] "Immo etiam saepe utemur modo loquendi secundum primam descriptionem prius manifeste improbatam, quia ipsa in paucis consequentiis habet instantiam. Tamen quocumque modo loquendi utemur nos intendemus sensum praetactum."

We have it on the authority of Aristotle that (7) expresses a necessary truth.[43] But it is also subject to falsification—albeit in rather remote circumstances—as Buridan points out in the objections:

> Furthermore, if it were said that [7] were not necessary, it would be only because God could annihilate every human being. In that case, no human would exist, and therefore no human would be an animal.[44]

Hence there is a compelling argument to be made that, since it is subject to divine falsification, (7) expresses a contingent truth.

This problem is not isolated: it will even extend to the propositions of geometry, as Buridan is quick to point out:

> But I prove that this should not prevent it [7] from being necessary: for if it did, then no proposition of geometry would be necessary, either, since God can annihilate all magnitudes, just as He can annihilate all people. It would then follow that geometry would not be a science, which everyone would hold is false and unacceptable. And this consequence is clear, because there is no science of anything except what is necessary.[45]

On account of Buridan's anti-realism, the propositions of geometry rely for their truth on the existence of actual magnitudes. If there were no magnitudes, then the things these propositions deal with—angles, shapes, and the like—would not exist, and the propositions themselves would fail to be true. Even the mere fact that God *could* annihilate all magnitudes entails that these propositions are contingent. And because they deal with contingents, they are not apt subject matter for science.

This will not stand. Buridan's solution is to distinguish three types of necessity: simple necessity, necessity 'as-of-when' (*de quando*), and conditional necessity. Simple necessities are relatively straightforward: they are not subject to falsification under any

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[43] *Prior Analytics* I, 9 ($310^{a}31$).

[44] "Item, si poneretur quod non esset necessaria, hoc esset pro tanto quia deus posset annihilare omnem hominem; ideo nullus homo esset, et sic nullus homo esset animal."

[45] "Sed probo quod hoc non debet obstare quin ipsa sit necessaria: quia si hoc obstaret, nulla propositio geometrica esset necessaria, cum deus ita possit annihilare omnes magnitudines, sicut omnes homines. Et tunc ultra sequeretur quod geometria non esset scientia, quod reputatur ab omnibus falsum et inconveniens; et patet haec consequentia per hoc quod scientia non est nisi de necessariis"

circumstances. Presumably these are limited to truths about God, but Buridan doesn't say. Conditional necessities, for their part, are very broad, and are not strictly relevant to our present concerns: they include such items as "A vacuum is a place", under the following conditional reading: "A vacuum, *if it exists*, is a place". Since a vacuum is impossible, the antecedent of this conditional is necessarily false. The conditional, then, is necessarily true.

But most of the necessary propositions of the sciences, including that expressed by (7), are necessarily true *de quando*. This is enough to save (7) and the propositions of geometry, and therefore to preserve the status of the sciences *qua* sciences. Buridan characterises this kind of necessity as follows:

> Necessity *de quando* comes about where, whenever (*quandocumque*) the subject and predicate of a proposition stand for anything whatsoever, they stand for the same thing (I am here speaking of affirmative propositions). And I say that, in this way, 'humans are animals' is necessary, as is 'horses are animals'. So too, 'roses are flowers' is necessary, even there don't happen to be any roses right now. And even if there is no lunar eclipse right now, 'a lunar eclipse is an interruption of the light from the sun' is necessary.[46]

Whereas propositions that are simply necessary are not falsifiable under any circumstances, those that are necessary *de quando* are falsifiable only by annihilation of the things their terms stand for. So long as those things exist, the terms will stand for the same things. This will be important in what follows.

Elsewhere, Buridan presents a similar, though not identical, scale of four types of modality in his discussion of *per se* predication in the *Summulae de Demonstrationibus* 8.6.3. In this latter passage, Buridan's language is more explicitly causal:

> One can posit still more grades of perseity, on account of the fact that a

[46] "Sed necessitas de quando ex hoc provenit quod oportet subiectum et praedicatum quandocumque supponunt pro aliquo supponere pro eodem; et hoc dico in affirmativis. Et sic dico quod haec est necessaria 'homo est animal', vel etiam 'equus est animal'. Immo etiam haec est necessaria 'rosa est flos', licet modo nulla sit rosa. Et quamvis non sit eclipsis lunae, tamen haec est necessaria 'eclipsis lunae est defectus luminis a sole'."

> proposition that is a *per se* predication must also be necessary. For, because there are several grades of necessity, there are also several grades of perseity, as well. The first grade of necessity is where it is not possible, by any power, to falsify the proposition in question (provided the signification of its terms remains the same); nor is it possible for things to be other than it signifies them to be.[47]

As can be readily surmised, the first grade here is identical with the simple necessity of *QAPr* 1.25. It will include those truths that cannot be falsified even by *Deus omnipotens*: that God exists, is a Trinity, and so on. Here again, Buridan does not give us any examples. Presumably, he thinks it is clear enough.

The second grade concerns nomological necessities, like "The cosmos is a sphere". These can be altered, but only by divine power: God could, for instance, make the cosmos into a cube. This would not entail any logical contradiction. Neither—as opposed to the *de quando* necessity of the *QAPr*—would it involve annihilation of the subject. Let's therefore set it aside.[48]

The third grade here is, I submit, identical with the *de quando* necessity of *QAPr*, at least as far as our purposes here are concerned:

> The third grade relies on the assumption that the subject is constant, as in 'A lunar eclipse occurs through the interposition of the earth between the sun and the moon', 'Socrates is a man', and 'Socrates is capable of laughter'. These propositions are said to be necessary in this way because it is necessary that, whenever (*quandocumque*) Socrates *does* exist, he is a man

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[47] "Et adhuc possent poni alii gradus, ex eo quod oportet propositionem per se esse necessariam, quia sunt diversi gradus necessitatis et, secundum hoc, etiam perseitatis. Est enim primus gradus necessitatis quia per nullam potentiam est possibile propositionem falsificari stante significatione, vel aliter habere quam significat".

[48] For Buridan on natural necessity, see Simo Knuuttila, "Natural Necessity in John Buridan", *Studies in Medieval Natural Philosophy*, ed. Stefano Caroti (Florence: L.S. Olschki, 1989): 155-76, and "Necessities in Buridan's Natural Philosophy", *The Metaphysics and Natural Philosophy of John Buridan*, ed. J.M.M.H. Thijssen and Jack Zupko (Leiden: Brill, 2001): 65-76, as well as Guido Alt, "Buridan's Reinterpretation of Natural Possibility and Necessity", *Metaphysics Through Semantics: The Philosophical Recovery of the Medieval Mind: Essays in Honor of Gyula Klima*, ed. Joshua P. Hochschild, Turner C. Nevitt, Adam Wood, and Gábor Borbély (Dordrecht: Springer, 2023): 237-53.

> and capable of laughter; and it is necessary that, whenever there *is* a lunar eclipse, that it is through the interposition of the earth, etc.[49]

There is ample evidence that the necessity Buridan sets out here is the same as *de quando* necessity. First, there is an overlap in the examples: both the *Summulae* and the *QAPr* passages give lunar eclipses as an example of the sort of things about which propositions can be necessary, even if the lunar eclipses themselves don't exist (that is, are not happening) at present. Second, this necessity relies on the same temporal language: *whenever* (*quandocumque*) the subject term stands for anything at all, these propositions are true. While they are not guaranteed to be true at any time, they will be invariably true whenever what they are about exists. The only way to falsify them is by removing from existence whatever the things they are about. Such is the extent to which causal powers can alter the truths expressed by propositions that are necessary *de quando*.[50]

We are now in a position to give a more precise statement of these two types of necessity—simple and *de quando*—in Buridan. To streamline our definitions, we will take it for granted that (i) the meanings of the terms in the propositions stay fixed, so as to avoid sophistical difficulties;[51] and (ii) that the propositions in question actually exist, so that we do not fall afoul of Doctrine 1. Then:

Necessity$_S$ A proposition φ is simply necessary when no power—not even God—can render it false.

Necessity$_{DQ}$ A proposition φ is necessary *de quando* when it is only falsifiable by

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[49] "tertius gradus est ex suppositione constantiae subiecti, ut 'eclipsis lunae est per interpositionem terrae inter solem et lunam', 'Socrates est homo', 'Socrates est risibilis'. Haec enim dicuntur necessariae sic quia necesse est quandocumque est Socrates ipsum esse hominem et risibilem, et necesse est quandocumque est eclipsis lunae ipsam esse per interpositionem, et caetera." (Emphasis mine).

[50] There is a fourth grade in the *Summulae de Demonstrationibus* (8.6.3): since there is no power over the past, facts about the past are in a certain sense necessary. For a clear and helpful analysis of this kind of necessity, in connection with the other four, as well as a concise historical overview of Buridan's position on this kind of necessity vis-à-vis other medieval thinkers, see Calvin Normore, "Buridanian Possibilities", *Logic and Language in the Middle Ages: A Volume in Honour of Sten Ebbesen* (Leiden: Brill, 2013): 389-402.

[51] That is, difficulties of the sort Buridan tackles in *Sophismata*, ch. 6.

> the removal of its subject matter. So long as the things for which its terms stand exist, it will invariably be true.

The examples we have seen in Buridan's texts of such necessities involve simple predications, rather than molecular propositions like consequences. So we have to extend these notions of necessity to the latter sort of propositions, to complete our account of the underlying notion of necessity in *TC* 1.3. To that end, consider the valid argument with which our exposition began:

1) Every man's donkey is running;
   Socrates is a man;
∴ Socrates' donkey is running.[52]

Suppose that the premises and conclusion of (1) are all true. Apart from simply annihilating the propositions involved, or changing the meanings of the terms, the Almighty—or some other suitable causal power—could falsify one or more of the propositions themselves. Let's focus on the conclusion. There are three relevant ways of making it false: God could (i) halt Socrates' donkey; (ii) make it so that Socrates does not own a donkey; or (iii) kill Socrates (that is, make it so that Socrates is no longer a man).

But notice that each of these moves would falsify one of the premises: (i) and (ii) would falsify the first, and (iii) would falsify the second. More generally, in a valid consequence, any falsification of the conclusion would falsify one (or more) of the premises. Hence if a consequence is valid, no causal power—not even God—can render the conclusion false while keeping all the premises true. This is the underlying notion of validity, to which Buridan appeals in *TC* 1.3.

There is some further textual evidence that this is Buridan's view on the necessity of logical consequence. In *Sophismata* ch. 5, 2nd sophism, Buridan considers the following

[52] "cuiuslibet hominis asinus currit, Socrates est homo; ergo Socratis asinus currit". John Buridan, *Summulae de Suppositionibus* (4.2.6).

apparent counterexample to the syllogistic mood Barbara, with terms for the members of the Trinity:

8)	Every God is the Son;	(T)
	Every Father of God is God;	(T)
∴	Every Father of God is the Son.[53]	(F)

Here we have an apparently invalid consequence. In his treatment of this syllogism, which was apparently the subject of considerable controversy in his day, Buridan is rather circumspect. Yet he suggests that the form of Barbara can be restored by adding a 'that is' (*quod est*) clause to the subjects of the premises and conclusion, which falsifies one of the former—though he does not tell us which one. This syntactic solution to the puzzle posed by (8) gives us the following (valid) consequence:

8′)	Everything that is God is the Son;	(F?)
	Everything that is the Father of God is God;	(F?)
∴	Everything that is the Father of God is the Son.[54]	(F)

This puzzle, and Buridan's treatment of it, corroborate the account of necessity developed in this section: even the Trinity cannot be allowed to render valid any argument with all true premises and a false conclusion. If there is a divine counterexample, the consequence is not valid. In this way, the underlying notion of necessity in *TC* 1.3 is causal, and the causal capacities can be in principle reduced to the powers of the Almighty: what is necessary in this way is what no power, not even an omnipotent one, can undo. Therefore, consequences that meet the criteria set out in *TC* 1.3 are safe even from divine intervention.

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[53] "omnis Deus est Filius, omnis Pater Dei est Deus; ergo omnis Pater est Filius".

[54] "omne quod est Deus est Filius, omne quod est Pater Dei est Deus; ergo omne quod est Pater Dei est Filius". Buridan does not tell us which of the premises he takes to be false.

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