Formalization of dialectical logic, Separation theory of truth. Logic of cellular automata.

Abstract:

By separating the general concept of truth into syntactic truth and semantic truth, this article proposes a new theory of truth to explain several paradoxes like the Liar paradox, Card paradox, Curry's paradox, etc. By revealing the relationship between syntactic /semantic truth and being-nothing-becoming which are the core concepts of dialectical logic, it is able to formalize dialectical logic. It also provides a logical basis for complexity theory by transferring all reasoning into a directed (cyclic/acyclic) graph which explains both paradoxical and paradox-free reasoning. The different structure between cyclic graphs and acyclic graphs is the key to understanding paradoxes. By explaining the immanent between logic, paradox, and cellular automata, it also illustrates dialectical logic as ontology.

Keywords: Theory of truth, the solution to paradoxes, Formalization of dialectical logic, logic of cellular automata

Introduction:

Hegel's dialectical logic has never been recognized by logicians, who dismiss it as error, confusion, and self-contradictory gibberish. This is because people have not found suitable examples for dialectical logic. In the field of logical research, another subject that is also full of contradictions and confusion is the paradox. Paradoxes are dismissed as errors of no realistic significance. However, Paradox and dialectical logic are each the key to understanding the other. This paper will formalize dialectical logic through the analysis of paradox and illustrate the cause and reality of paradox through the example of cellular automata. In fact, under the ontological perspective of formalized dialectical logic, the structure of paradox is the ontological basis of the consciousness and theory of complexity that have puzzled people for a long time.

Analysis:

The proposition "This proposition is false" has troubled logicians for a long time[1]. When people think about this proposition, their judgment is an infinite cycle. Since this sentence contains so few concepts, we can conclude that the problem is in the concept of truth. The reason why self-referential paradox is difficult is that people consider them from an objectified perspective, but the inference is actually a process. Compared with "this proposition is false", "this proposition is true" seems to be self-consistent but implies the same problem[2]. The comparison between the two propositions is enlightening and contains the answer to the paradox. Hegel put forward dialectical logic in his book *Logic of Science*, which almost solved these problems[3], but he didn't clarify how dialectical logic works. This article will clarify how the concept of truth/falsity under classical logic leads to paradoxes, and how dialectical logic reinterprets them.

The concept of truth itself contains paradox because it contains both the concept of the

dynamic relationship between values and the concept of static truth values. They can and should be divided. When people talk about "this proposition is true", first, people feel that there is an object, which is the true value. Second, people are comparing that value with a new value from the sentence again, and the conclusion of this comparison produces a true value again. So "this proposition is true" is a logically valid and satisfying sentence. However, when this sentence is "This proposition is false", the values produced at first and at third are different, so people fall into inconsistency, which means we should realize that the process of generating values should be divided from the static values. Thus truth should not be regarded as a unified concept but as two concepts at completely different levels. Paradoxes are caused by the mixing of these two levels. The first meaning of truth is the truth value, the Boolean value 1, which is a pure symbol without any further semantics, and the falsity is the Boolean value 0, which is different from the truth and also doesn't have any further semantics. We call it syntactic truth/falsity. The second meaning of truth is the process of values matching each other. When we say "P is true ", the semantic truth of that proposition is in the process from the syntactic value of P and the syntactic value of truth. It only describes the relationship between them, when the two are the same we call it semantic truth else semantic falsity. The most difficult part of noticing semantic truth/falsity is that after it is realized, it will be immediately turned into syntactic truth/falsity. When people get a semantic truth/falsity from the comparison between two syntactic values, they subconsciously generate a syntactic value. However, the relationship between signifier and signified is an arbitrary relationship, there is no reason to get syntactic truth rather than syntactic falsity in this situation. That causes people to subconsciously combine two different concepts of truth/falsity and eventually leads to paradoxes. The semantic truth/falsity part is like a jump without taking off or landing which cannot be considered by the objectified perspective. Semantic truth/falsity refers to a process from one state to another so it doesn't follow the law of identity. In Hegel's *Logic of Science,* they are the concepts of pure being (Sein), pure nothing (Nichts), and becoming (Werden). People are confused

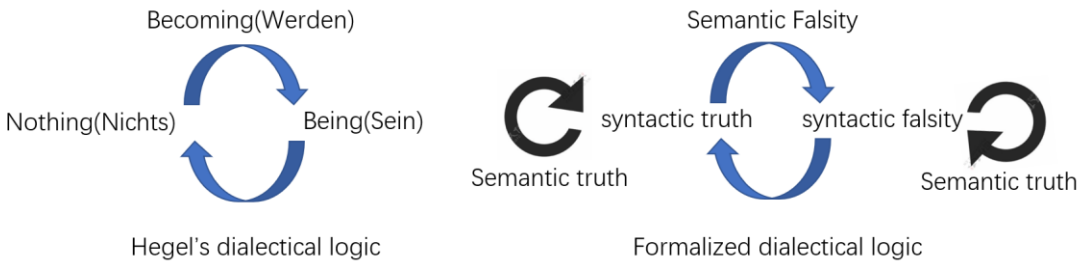

Fig1: Hegel's dialectical logic and formalized dialectical logic[4]

Semantic truth/falsity does not follow the law of identity, but if it is written in text, people will subconsciously transfer it into syntactic truth/falsity, so it must be represented in a graph. Vertexes and edges are able to represent these two parts. Let's start with the new liar paradoxes "this statement is syntactic false" and "this statement is semantic false". The original Liar paradox is the mix of two levels. In general, people always expect a syntactic conclusion, a proposition ought to be syntactic true or false. It works in general reasoning because they have a final conclusion so the semantic part is ignored. For multi-propositions, semantics is built between syntactic values of propositions. However, semantics should be independent of syntactic values.

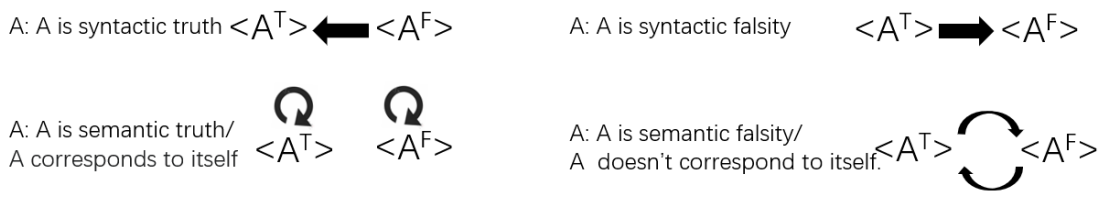

Fig2: Liar's Paradox after separation

Theory:

Logic has two parts: logical objects(syntactic) and assertion constraints(semantic), logical objects are atomic propositions or variables, which can assign truth values (syntactic true/false), For n logical objects, there will be 2^n possible states, which are all vertices of a graph. The states of logic objects are represented like $<A^T, B^F>$ which means A is syntactic true and B is syntactic false. That state is the current belief of the thinker. The assertion constraint is the relationship between different states, which is expressed as the directed edge between states. For example, $A \rightarrow B$ negates the state $<A^T, B^F>$ and reaches $<A^T, B^T>$. "A is syntactic true" means the directed edges start at states which have$<A^F>$ and end at corresponding states which have$<A^T>$, they are axioms in classic logic. Now we can transform all logical reasoning into a corresponding directed graph, and the specific structure of the graph is the key to understanding the difference between general propositions and paradox.

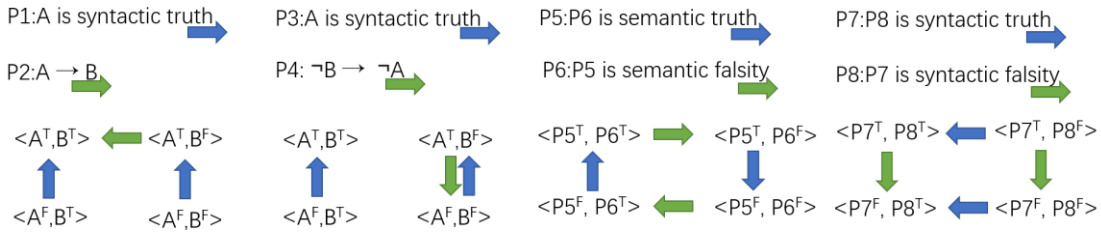

Fig3.Examples of effective reasoning, contrapositive, and card paradox

P1 and P2 constitute the simplest and most reliable inference processes under classical logic. Observing the corresponding inference graph, effective inference means that among all 2^n vertexes, only one vertexes' out-degree is 0, which means any assertion constraint cannot negate this state. At the same time, starting from any vertexes and following the assertion constraints in any sequence, it will eventually reach the only vertex with 0 out-degree, and this vertex is the final result of the reasoning. The law of excluded middle ensures 2^n vertexes total. The law of contradiction ensures only one vertex is the conclusion of reasoning because other vertexes have at least one different syntactic value from it. The law of sufficient reason ensures that whichever vertex the thinker starts at, after finite considering assertion constraints, he will end at the only right state, and it is the conclusion of reasoning. P3 and P4 show that the contrapositive proposition is not exactly the same as the original proposition, but it works in propositional logic because they negate the same state which means their truth values are the same, but their semantics are not.

Considering the law of identity $A = A$ strictly, from the pure syntactic perspective, left A is not identical to right A, their positions are different, which means peoples' belief about identity is based on the concept of A. However, the concept is changing with the subject. The process of thinking itself doesn't follow the law of identity. Many efforts to solve

paradoxes are based on improving the law of contradiction or the law of excluded middle, but it is actually the law of identity that needs to be improved. The key to understanding dialectical logic is to consider change/process/negate/operate/calculate as ontological foundations rather than object/thing/being/value/number.

P5 and P6 constitute the card paradox. If consider P5 and P6 as syntactic statements, they will be like P7 and P8 which are simple and clear. P5 and P6 have the same semantic structure as the liar paradox, the difference is that the card paradox has 2 logical objects and 4 states but the liar paradox has 1 logical object and 2 states. Obviously, we can consider some paradoxes with more logical objects. Rather than avoiding paradoxes, exploring the structure of more complicated paradoxes will lead us to the essence of complexity theory.

Let us consider a classy, obstinate, forgetful old lady who wears only red or blue hats, shirts, and trousers. At the same time, she has many stubborn beliefs about how the colors of clothes should match, but because of her forgetfulness, she can only check whether her clothing conforms to one of these beliefs at a time. She will not synthesize multiple beliefs into a final conclusion through intuition, but only mechanically check and implement each belief.

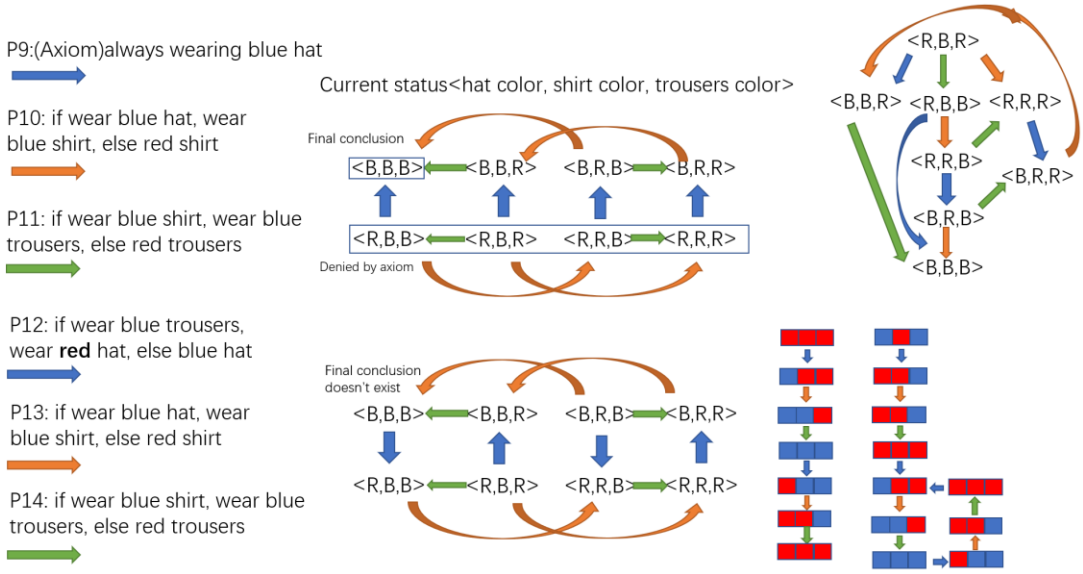

Fig3. Dialectical Interpretations of classical logic and cellular automata

In Fig3, P9, P10, and P11 is an example of classic logic, starting from any state, after considering all the assertion constraints for finite times, the old lady will reach the final state of <B, B, B>, All propositional reasoning under first-order logic can transfer to the corresponding graph and get the final conclusion. For P12, P13, and P14, no matter what color she wears now, she still needs to change. It is obviously a paradox but she can't realize that. However, we can realize this because there are only three logical objects here. When we face the consciousness composed of tens of billions of nerve cells, we are also that old lady. Consciousness is not the directed acyclic graph structure like P9, P10, and P11, because that means a stable endpoint, and people will fall into t+he end point of death-like silence after several periods of neural activation. Consciousness is built on a paradox constructed by tens of billion propositions, which is a directed cyclic graph. Consciousness cannot be explained in analytic philosophy because the tools of formal logic prevent

discussions about the paradoxes on which consciousness is built.

The state transition diagram of Fig3 at the bottom right is intentionally drawn with some colored cubes. It is to remind readers that propositional logic has the same structure as cellular automata[5]. 3 logic objects mean 8 different states so we can observe the cycle, but cellular automata often consider more cells than that. Due to combinatorial explosion, the circularity of cellular automata is ignored. A cellular automaton contains $\boldsymbol{n}$ cells and several rules $\boldsymbol{\phi}$ to determine the next state. States of cells can be transferred into a directed graph of $\boldsymbol{2^n}$ vertexes and $\boldsymbol{\phi}$ is the edges of that graph. A practical example of cellular automata is a search on the graph, starting at the beginning vertex and following some specific order of $\boldsymbol{\phi}$. There are four classes[6] of cellular automata, class 1 is the homogeneous state in which cells stably end up with the same value, and class 2 is stable structures and simple periodic patterns. Class 1 and the stable structure are non-paradoxes logic structures like P9,P10,P11. Simple periodic patterns are the simple paradoxes structure like liar paradox or card paradox. Class 3 and 4 are the complex topology of multiple propositions paradoxes.

After the formalization of dialectical logic, we can explain these paradoxes.

Barber paradox: A barber says he shaves and only shaves those who don't shave themselves. So the paradox here is if he shaves himself, he can't shave himself, if he doesn't shave himself, he should shave himself. People consider this a paradox because they consider the man before and after shaving himself to be the same person. However, the action of shaving changed the man, the true values of "This man shaved himself." of the two men are different. It is a hidden violation of the law of identity.

Unexpected hanging paradox: A judge tells a condemned prisoner that he will be hanged at noon on one weekday in the following week but that the execution will be a surprise to the prisoner. He will not know the day of the hanging until the executioner knocks on his cell door at noon that day. It can't be on Friday because then he will know it and it will not be a surprise. Then he inferences that the surprise hanging cannot be on Thursday either for the same reason, finally, he believes the hanging will not occur at all. He is wrong but where is the mistake? There is a process every day, before and after the last moment that the executioner can show up, the prisoner is not the same person. After that moment, he is a man who knows he will not be hanged that day, but before that moment, he doesn't know whether he will be hanged today. The mistake he makes is that considers himself a consistent man. But he is not the man after the first day's last moment. When the judge says "surprise", it is not for the last day's prisoner, it's for the prisoner now.

Curry's paradox: $\boldsymbol{A:= A\rightarrow B}$ causes $\boldsymbol{B}$ to be true whatever $\boldsymbol{B}$ is. It is caused by mixing two meanings of truth. Consider the inference:

1: $\boldsymbol{A := (A \rightarrow B)}$
2: $\boldsymbol{A \rightarrow A}$
3: $\boldsymbol{A \rightarrow (A \rightarrow B)}$
4: $\boldsymbol{A \rightarrow B}$
5: $\boldsymbol{A}$
6: $\boldsymbol{B}$

$\boldsymbol{A}$ and $\boldsymbol{A\rightarrow B}$ are at different levels, $\boldsymbol{A}$ means $\boldsymbol{A}$ is syntactically true, $\boldsymbol{A\rightarrow B}$ means $\boldsymbol{B}$ semantically corresponds to $\boldsymbol{A}$, it is at the semantic level. The symbol "$\boldsymbol{:=}$" causes the mixing between a

syntactic truth and semantic truth. During replacing $(A \rightarrow B)$ to A, the semantic truth between A and $(A \rightarrow B)$ infect the syntactic truth of A. The formula $A := (A \rightarrow B)$ provide no information about syntactic truth for A or $(A \rightarrow B)$.

No-no paradox[7]:

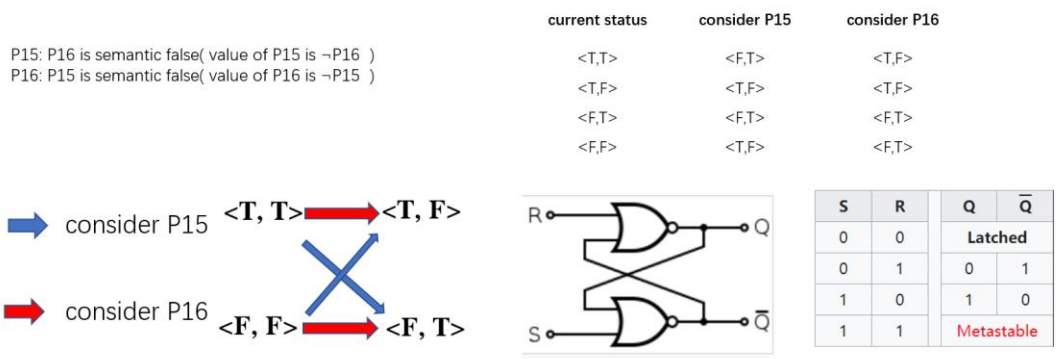

Fig4. no-no paradox

Two propositions that negate each other construct an interesting paradox. If people believe one of them is true, then they are self-consistent. If they are both true or false, they are not self-consistent. It has the same structure and behavior as SR-latch and is the mechanism of memory for both computers and humans.

Conclusion:

Hegel's dialectical logic has long been misunderstood by people as being opposed to formal logic or completely irrelevant to logic in the general sense because of its obscurity and contradictions. Therefore, researchers of formal logic or the entire academic community do not regard it as logic. But in fact, not only does it not conflict with formal logic, but contains formal logic and can further explain paradoxes and provide a theoretical basis for more phenomena. So it is logic and ontology at the same time. By treating change rather than objects as first principles, it is able to resolve the problem of dualism and provide a unifying model of monism. It also answers the question of being. The answer to the question of being is be. It's clearly a syntax error yet no noun can be the answer because the concept of object is generated by subject. Therefore the answer is the most common verb be which is ironically already in the question.

Conflicts of Interest: The author declares no conflicts of interest.

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Senhai Zhou

School of Computer Science and Technology, Harbin Institute of Technology, Shenzhen, China

Correspondence: dfyning@gmail.com