Wittgenstein's picture theory in the linguistic Copenhagen interpretation of dualistic idealism

Shiro Ishikawa[1] [January 05, 2021]

Abstract
Recently, we have proposed quantum language (or the linguistic Copenhagen interpretation of quantum mechanics). Quantum languages describe both classical and quantum systems and therefore have great power to solve almost all philosophical problems. Thus, we believe that quantum language can be regarded as the language of science. Therefore, it makes sense to study Wittgenstein's picture theory within the framework of quantum language, since Wittgenstein's language (i.e., the language that he supposed, but didn't define in his book "Tractatus Logico-Philosophicus") may be a particular subclass of quantum language. In this paper, we show that a class of binary projective measurements in classical quantum language has a logical structure. And thus, the proposition that Wittgenstein studied in his book can be regarded as a binary projective measurement in classical quantum language. Therefore, we conclude that Wittgenstein's language is realized as the central part of classical quantum language. If so, we think that his picture theory should be praised not only from a philosophical point of view but also from a scientific point of view.

Key phrases: Wittgenstein's picture theory, Quantum Language, Linguistic Copenhagen Interpretation,

CONTENTS

1 Introduction

This preprint is written to reinforce my recent papers:

- Ref, [27]: Ishikawa, S. (2020) Wittgenstein's picture theory in the quantum mechanical worldview JQIS,Vol. 10, No.4 ,104-125 DOI:10.4236/jqis.2020.104007
  `https://www.scirp.org/journal/paperinformation.aspx?paperid=106233`: PDF download free

- Ref, [26]: Ishikawa, S. (2020) Sections 4.3 and 11.6 in "History of Western Philosophy from the quantum theoretical point of view", Research Report (Department of mathematics, Keio university, Yokohama) [Ver.3] (KSTS-RR-20/001, 296 pages)
  `http://www.math.keio.ac.jp/academic/research_pdf/report/2020/20001.pdf`: download free

[1]Department of Mathematics, Faculty of Science and Technology, Keio University,
3-14-1, Hiyoshi, Kouhoku-ku Yokohama, Japan. homepage: `http://www.math.keio.ac.jp/~ishikawa/indexe.html`

in which I proposed the quantum linguistic understanding of Wittgenstein's picture theory (which is the main theory in Wittgenstein's book "Tractatus Logico-Philosophicus" (in short, "TLP"; ref. [32])).

TLP is surely one of the most important philosophical books of the 20th century. However, there are various opinions about his picture theory. For example, some people have even declared his picture theory to be nonsensical(*cf.* refs. [1, 28]). Therefore, I don't think the evaluation of his theory is yet settled from the scientific point of view. Wittgenstein said, in the (6.53)-th sentence of TLP, that

> [a](6.53): *The correct method in philosophy is to say nothing but what can be said (i.e., propositions of the natural sciences).*

Therefore, he must have been thinking about the language of science to answer "what is a scientific proposition?" If so, his picture theory may be rewritable in scientific terms (and not philosophical terms).

For almost 30+ years, I've been studying quantum language (abbreviated and cited as QL) (or, "measurement theory", "the linguistic Copenhagen interpretation of quantum mechanics", "the linguistic Copenhagen interpretation of dualistic idealism", "the quantum mechanical worldview"), which was proposed by myself (*cf.* refs. [8–15]).

I think that the location of quantum language in the history of world-description:

Figure 0 [The location of quantum language in the history of world-description (*cf.* ref. [16, 25])]

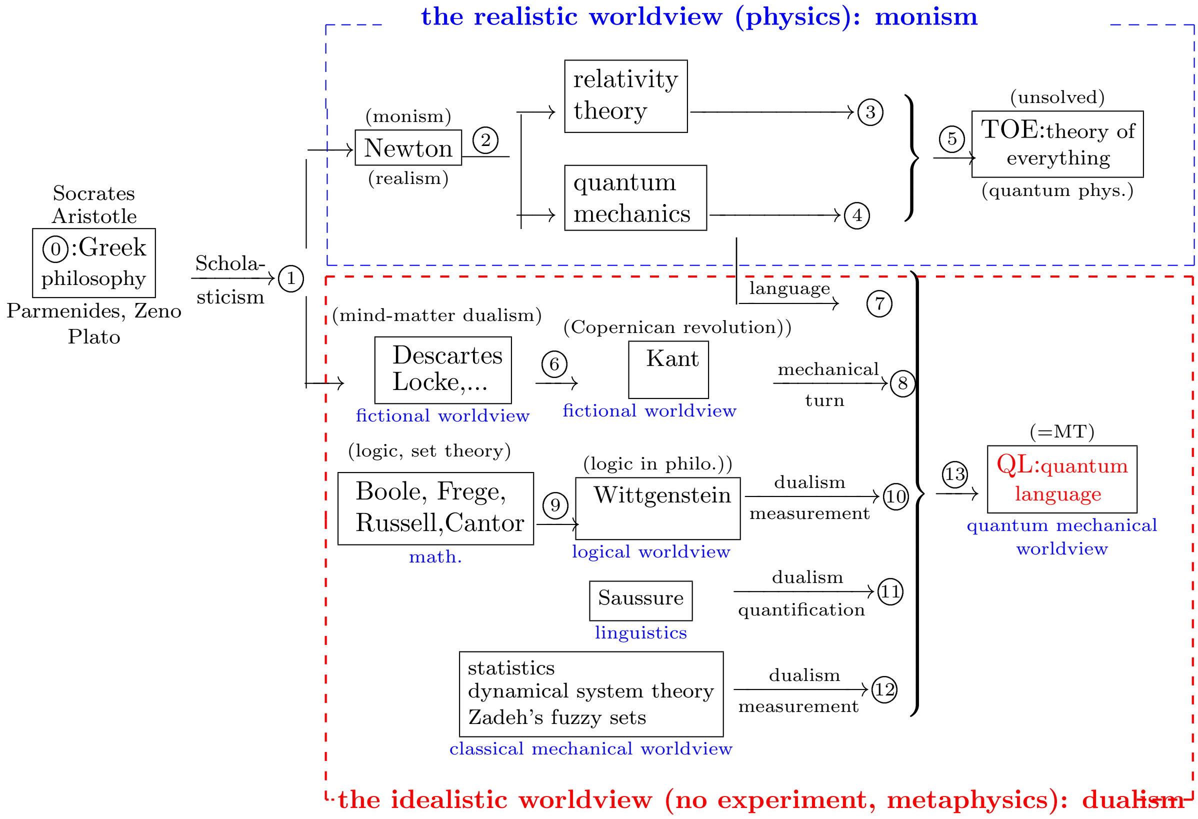


Figure 0: The history of the world-descriptions

(Philosophy (≈dualistic idealism) has progressed towards QL (ref. [26]))

As seen in the above figure, roughly speaking, QL has five aspects such as

[b] $\begin{cases} \text{7: the linguistic turn of quantum mechanics (\textit{cf.} refs. [15, 20, 21]),} \\ \text{8: the scientific turn of Descartes=Kant philosophy (\textit{cf.} refs. [16, 22, 23]),} \\ \text{10: the measurement theoretical aspect of logic (\textit{cf.} ref. [24, 27]),} \\ \text{11: the quantitative turn of Saussure's linguistics (\textit{cf.} Sec.11.5 in ref. [26]),} \\ \text{12: the dualistic turn of statistics (\textit{cf.} refs. [11, 14, 17–19, 24]) ,} \end{cases}$

where, you can think of "dualism = measurement theory (= dualism consisting of a measurer and a measurement object", "idealism = metaphysics (= a discipline dealing with things that cannot be settled by experiments)".

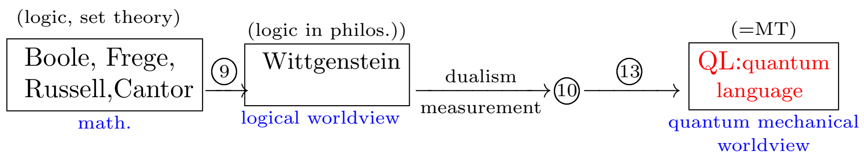
As shown in Figure 0, note that

[c] QL is a kind of language of science such as statistics, Newtonian mechanics, the theory of relativity and so on

(where Newtonian mechanics, the theory of relativity are realistic, QL and statistics are idealistic). If the above [b] is true, it must solve almost all philosophical unsolved problems (e.g., Zeno's paradox, Hume's problem of induction, Hempel's raven problem, etc.) as mentioned in Sec. 6 later.

Wittgenstein's picture theory in TLP is philosophical and unscientific. However I think that attempts to understand this theory from a scientific point of view must not be stopped. Do not stop research, concluding that it is scientific nonsense.

In Sec. 4, I propose my understanding of Wittgenstein's picture theory in the framework of QL. This is reasonable since both Wittgenstein's picture theory and QL are closely related to propositions of science. That is, we devote ourselves to [a; 10]. i.e.,

[d] 

in Figure 0

where [mathematical logic] $\xrightarrow{9}$ [philosophical logic] $\xrightarrow{10,13}$ [scientific logic]. That is, we consider that Wittgenstein was the first philosopher to try to make a clear distinction between mathematical logic and scientific logic. And this clarification is completed in QL (*cf.* Sec. 3).

In section 2, I review measurement theory (=QL), which is composed from Axiom 1(Measurement), Axiom 2 (Causality) and the linguistic Copenhagen interpretation. In section 3, I discuss "Why does logic arise in science ?", and assert that a class of binary projective measurements in classical quantum language has a logical structure, i.e.,

[e] "Logic comes from measurement". (i.e., dualistic idealism is hidden behind Wittgenstein's picture theory)

(where it should be noted that I am not concerned with logic in mathematica but logic in science).

In section 4, I study Wittgenstein's picture theory, which is constructed on inspiration from the results obtained in section 3. After all, I conclude that the proposition Wittgenstein studied in his TLP can be regarded as a binary projective measurement in classical quantum language. Therefore, we think that Wittgenstein's language (i.e., the language that he supposed, but didn't define in TLP) is realized by a class of binary projective measurements in classical quantum language.
That is, I show that:

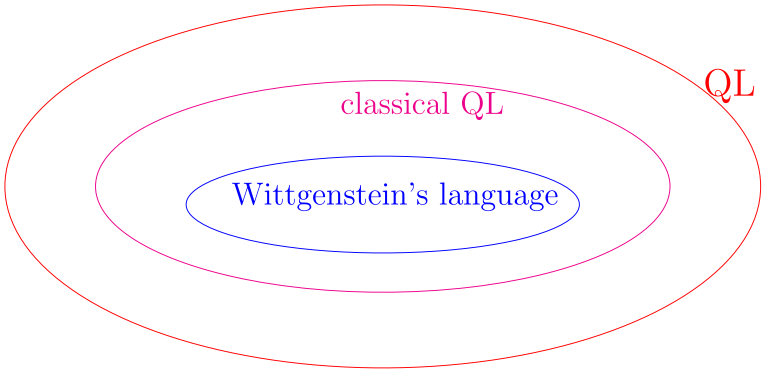


Figure 1: Wittgenstein's language(in which scientific logic holds)
= a class of binary projective measurements in classical quantum language

In addition, in Sec. 6, I add the list of my solutions of philosophical unsolved problems in QL. From the scientific point of view, I think that there is only one philosophical unsolved problem:

[e] Complete the scientific dualistic idealism!

This is realized by QL as I have shown in Figure 0 before. And so, the other philosophical unsolved problems concerning dualistic idealism, which stem from an inadequate understanding of our dualistic idealism, should be clarified in QL. And thus, I conclude that Wittgenstein's picture theory should be located in the central part of science.

2 Review: Quantum language (= QL = Measurement theory)

In this section, we shall review quantum language (i.e., the linguistic Copenhagen interpretation of quantum mechanics, or measurement theory), which has the following form:

$$\underset{(= \text{measurement theory})}{\boxed{\boxed{\text{Quantum language}}}} = \underset{(\text{Axiom 1})}{\boxed{\text{measurement}}} + \underset{(\text{Axiom 2})}{\boxed{\text{causality}}} + \underset{(\text{how to use Axioms 1 and 2})}{\boxed{\text{linguistic (Copenhagen) interpretation}}} \tag{1}$$

My all results concerning QL are summarized in Research report (Keio University): [25, 26]. Also, the sections 4.3 and 11.6 in ref. [26] should be regarded as the preprint of this paper.

2.1 Mathematical Preparations

Consider an operator algebra $B(H)$ (i.e., an operator algebra composed of all bounded linear operators on a Hilbert space H with the norm $\|F\|_{B(H)} = \sup_{\|u\|_H=1} \|Fu\|_H$, [29]. [30], [34].), and consider the pair

$[\mathcal{A},\mathcal{N}]_{B(H)}$ (or, the triplet $[\mathcal{A} \subseteq \mathcal{N} \subseteq B(H)]$), called a *basic structure*. Here, $\mathcal{A}(\subseteq B(H))$ is a C^*-algebra, and $\mathcal{N}$ ($\mathcal{A} \subseteq \mathcal{N} \subseteq B(H)$) is a particular C^*-algebra (called a W^*-algebra) such that $\mathcal{N}$ is the weak closure of $\mathcal{A}$ in $B(H)$.

QL (= quantum language = measurement theory) is classified as follows.

$$\text{(A)} \qquad \text{QL} \;= \begin{cases} (\mathrm{A}_1)\text{: quantum QL} \quad (\text{when } \mathcal{A} = \mathcal{C}(H)) \\ \\ (\mathrm{A}_2)\text{: classical QL} \quad (\text{when } \mathcal{A} = C_0(\Omega)) \end{cases}$$

That is, when $\mathcal{A} = \mathcal{C}(H)$, the C^*-algebra composed of all compact operators on a Hilbert space H, the quantum QL (A_1) is also called quantum measurement theory (or, quantum system theory), which can be regarded as the linguistic aspect of quantum mechanics. Also, when $\mathcal{A}$ is commutative (that is, when $\mathcal{A}$ is characterized by $C_0(\Omega)$, the C^*-algebra composed of all continuous complex-valued functions vanishing at infinity on a locally compact Hausdorff space Ω (*cf.* refs. [30], [34])), the classical QL (A_2) is also called classical measurement theory.

Also, note that, when $\mathcal{A} = \mathcal{C}(H)$, i.e., quantum cases,

(i) $\mathcal{A}^* = Tr(H)$ (=trace class), $\mathcal{N} = B(H)$, $\mathcal{N}_* = Tr(H)$ (i.e., pre-dual space), thus, ${}_{Tr(H)}\big(\rho, T\big)_{B(H)} = \mathrm{Tr}_H(\rho T)$ $(\rho \in Tr(H), T \in B(H))$.

Also, when $\mathcal{A} = C_0(\Omega)$, i.e., classical cases,

(ii) $\mathcal{A}^* = \mathcal{M}(\Omega)$(="the space of all signed measures on Ω)", $\mathcal{N} = L^\infty(\Omega,\nu)(\subseteq B(L^2(\Omega,\nu)))$, $\mathcal{N}_* = L^1(\Omega,\nu)$, where ν is some measure on Ω, thus, ${}_{L^1(\Omega,\nu)}\big(\rho, T\big)_{L^\infty(\Omega,\nu)} = \int_\Omega \rho(\omega)T(\omega)\nu(d\omega)$ $(\rho \in L^1(\Omega,\nu), T \in L^\infty(\Omega,\nu))$ (*cf.* ref. [30]).

Let $\mathcal{A}(\subseteq \mathcal{N} \subseteq B(H))$ be a C^*-algebra, and let $\mathcal{A}^*$ be the dual Banach space of $\mathcal{A}$. That is, $\mathcal{A}^*$ $= \{\rho \mid \rho$ is a continuous linear functional on $\mathcal{A}$ $\}$, and the norm $\|\rho\|_{\mathcal{A}^*}$ is defined by $\sup\{|\rho(F)| \mid F \in \mathcal{A}$ such that $\|F\|_{\mathcal{A}}(=\|F\|_{B(H)}) \leq 1\}$. Define the *mixed state* ρ $(\in \mathcal{A}^*)$ such that $\|\rho\|_{\mathcal{A}^*} = 1$ and $\rho(F) \geq 0$ for all $F \in \mathcal{A}$ such that $F \geq 0$. And define the mixed state space $\mathfrak{S}^m(\mathcal{A}^*)$ such that

$$\mathfrak{S}^m(\mathcal{A}^*)=\{\rho \in \mathcal{A}^* \mid \rho \text{ is a mixed state}\}.$$

A mixed state $\rho(\in \mathfrak{S}^m(\mathcal{A}^*))$ is called a *pure state* if it satisfies that "$\rho = \theta\rho_1 + (1-\theta)\rho_2$ for some $\rho_1, \rho_2 \in \mathfrak{S}^m(\mathcal{A}^*)$ and $0 < \theta < 1$" implies "$\rho = \rho_1 = \rho_2$". Put

$$\mathfrak{S}^p(\mathcal{A}^*)=\{\rho \in \mathfrak{S}^m(\mathcal{A}^*) \mid \rho \text{ is a pure state}\},$$

which is called a *state space*. It is well known (*cf.* ref. [30]) that $\mathfrak{S}^p(\mathcal{C}(H)^*) = \{|u\rangle\langle u|$ (i.e., the Dirac notation) $\mid \|u\|_H = 1\}$, and $\mathfrak{S}^p(C_0(\Omega)^*) = \{\delta_{\omega_0} \mid \delta_{\omega_0}$ is a point measure at $\omega_0 \in \Omega\}$, where $\int_\Omega f(\omega)\delta_{\omega_0}(d\omega)$ $= f(\omega_0)$ $(\forall f \in C_0(\Omega))$. The latter implies that $\mathfrak{S}^p(C_0(\Omega)^*)$ can be also identified with Ω (called a *spectrum space* or simply *spectrum*) such as

$$\underset{\text{(state space)}}{\mathfrak{S}^p(C_0(\Omega)^*)} \ni \delta_\omega \leftrightarrow \omega \in \underset{\text{(spectrum)}}{\Omega}$$

Thus, ω and Ω is also called a *state* and *state space* respectively.

For instance, in the above (ii) we must clarify the meaning of the "value" of $f(\omega_0)$ for $f \in L^\infty(\Omega,\nu)$ and $\omega_0 \in \Omega$. An element $f(\in L^\infty(\Omega,\nu))$ is said to be *essentially continuous at* $\omega_0(\in \Omega)$, if there uniquely exists a complex number α such that

- if ρ $(\in L^1(\Omega,\nu),\ \|\rho\|_{L^1(\Omega,\nu)} = 1,\ \ \rho \geq 0$) converges to $\delta_{\omega_0}(\in \mathcal{M}(\Omega))$ in the sense of weak* topology of $\mathcal{M}(\Omega)$, that is,

$$\rho(G) \longrightarrow G(\omega_0)\ \ (\forall G \in C_0(\Omega)(\subseteq L^\infty(\Omega,\nu))),$$

  then $\rho(f)(= \int_\Omega f(\omega)\rho(\omega)\nu(d\omega)$ converges to α.

And the value of $f(\omega_0)$ is defined by the α.

Definition 1. [Observable] According to the noted idea (*cf.* ref. [2], [5]), an *observable* $\mathsf{O} :=(X,\mathcal{F},\ F)$ in $\mathcal{N}$ is defined as follows:

(i) [σ-field] X is a set, $\mathcal{F}(\subseteq 2^X$, the power set of X) is a σ-field of X, that is, "$\Xi_1,\Xi_2,... \in \mathcal{F} \Rightarrow \cup_{n=1}^\infty \Xi_n \in \mathcal{F}$", "$\Xi \in \mathcal{F} \Rightarrow X \setminus \Xi \in \mathcal{F}$".

(ii) [Countable additivity] F is a mapping from $\mathcal{F}$ to $\mathcal{N}$ satisfying: (a): for every $\Xi \in \mathcal{F}$, $F(\Xi)$ is a non-negative element in $\mathcal{N}$ such that $0 \leq F(\Xi) \leq I$, (b): $F(\emptyset) = 0$ and $F(X) = I$, where 0 and I is the 0-element and the identity in $\mathcal{N}$ respectively. (c): for any countable decomposition $\{\Xi_1,\Xi_2,\ldots,\Xi_n,...\}$ of Ξ $\big($i.e., $\Xi,\Xi_n \in \mathcal{F}$ $(n = 1,2,3,...)$, $\cup_{n=1}^\infty \Xi_n = \Xi$, $\Xi_i \cap \Xi_j = \emptyset$ $(i \neq j)\big)$, it holds that $F(\Xi) = \sum_{n=1}^\infty F(\Xi_n)$ in the sense of weak* topology in $\mathcal{N}$.

Let $(Y,\mathcal{G})$ be a measurable space, and let $\Theta : X \to Y$ be a measurable map. Then, $\Theta(\mathsf{O}) :=(Y,\mathcal{G},\ F(\Theta^{-1}(\cdot))$ in $\mathcal{N}$ is also an observable in $\mathcal{N}$ (which is called an image observable). If $F(\Xi) = F(\Xi)^2$ $(\forall \Xi \in \mathcal{F})$, then $\mathsf{O} :=(X,\mathcal{F},\ F)$ in $\mathcal{N}$ is a projective observable $\mathsf{O} :=(X,\mathcal{F},\ F)$ in $\mathcal{N}$ is also called an X-valued observable. I will devote myself to binary valued (i.e., $\{1,0\}$-valued) projective observables in most of the cases in this paper.

2.2 Axiom 1 [Measurement] and Axiom 2 [Causality]

Measurement theory (A) is composed of two axioms (i.e., Axioms 1 and 2) as follows. With any *system* S, a basic structure $[\mathcal{A},\mathcal{N}]_{B(H)}$ can be associated in which the measurement theory (A) of that system can be formulated. A *state* of the system S is represented by an element $\rho(\in \mathfrak{S}^p(\mathcal{A}^*))$ and an *observable* is represented by an observable $\mathsf{O} :=(X,\mathcal{F},F)$ in $\mathcal{N}$. Also, the *measurement of the observable* O *for the system* S *with the state* ρ is denoted by $\mathsf{M}(\mathsf{O},S_{[\rho]})$ $\big($ or more precisely, $\mathsf{M}_\mathcal{N}(\mathsf{O},S_{[\rho]})$, $\mathsf{M}_\mathcal{N}(\mathsf{O} :=(X,\mathcal{F},F),\ S_{[\rho]})$ $\big)$. An observer can obtain a measured value x $(\in X)$ by the measurement $\mathsf{M}(\mathsf{O},S_{[\rho]})$.

The Axiom 1 presented below is a kind of mathematical generalization of Born's probabilistic interpretation of quantum mechanics. And thus, it is a statement without reality.

Now we can present Axiom 1 in the W^*-algebraic formulation as follows.

Axiom 1 [Measurement]. Consider a basic structure $[\mathcal{A} \subseteq \mathcal{N} \subseteq B(H)]$. The probability that a measured value x $(\in X)$ obtained by the measurement $\mathsf{M}_{\mathcal{N}}(\mathsf{O} := (X, \mathcal{F}, F),\ S_{[\rho]})$ belongs to a set $\Xi (\in \mathcal{F})$ is given by $\rho(F(\Xi))$ if $F(\Xi)$ is essentially continuous at $\rho(\in \mathfrak{S}^p(\mathcal{A}^*))$.

Remark 2. Recall that a statement whose truth or falsity is determined is called a proposition. Also, Axiom 1 says that a measured value is determined by a measurement $\mathsf{M}_{\mathcal{N}}(\mathsf{O} := (X, \mathcal{F}, F),\ S_{[\rho]})$. Therefore, there is a point in thinking that "measured value" is a generalization of "truth value". That is, we can regard "measurement" as a kind of generalizations of "proposition". This idea is essential to Sec. 3.

Next, we explain Axiom 2 (which is not used in this paper). Let $[\mathcal{A}_1, \mathcal{N}_1]_{B(H_1)}$ and $[\mathcal{A}_2, \mathcal{N}_2]_{B(H_2)}$ be basic structures. A continuous linear operator $\Phi_{1,2} : \mathcal{N}_2$ (with weak* topology) $\to \mathcal{N}_1$(with weak* topology) is called a *Markov operator,* if it satisfies that (i): $\Phi_{1,2}(F_2) \geq 0$ for any non-negative element F_2 in $\mathcal{N}_2$, (ii): $\Phi_{1,2}(I_2) = I_1$, where I_k is the identity in $\mathcal{N}_k$, $(k = 1, 2)$. In addition to the above (i) and (ii), we assume that $\Phi_{1,2}(\mathcal{A}_2) \subseteq \mathcal{A}_1$ and $\sup\{\|\Phi_{1,2}(F_2)\|_{\mathcal{A}_1} \mid F_2 \in \mathcal{A}_2$ such that $\|F_2\|_{\mathcal{A}_2} \leq 1\} = 1$.

It is clear that the dual operator $\Phi_{1,2}^* : \mathcal{A}_1^* \to \mathcal{A}_2^*$ satisfies that $\Phi_{1,2}^*(\mathfrak{S}^m(\mathcal{A}_1^*)) \subseteq \mathfrak{S}^m(\mathcal{A}_2^*)$. If it holds that $\Phi_{1,2}^*(\mathfrak{S}^p(\mathcal{A}_1^*)) \subseteq \mathfrak{S}^p(\mathcal{A}_2^*)$, the $\Phi_{1,2}$ is said to be deterministic. If it is not deterministic, it is said to be non-deterministic. Also note that, for any observable $\mathsf{O}_2 := (X, \mathcal{F}, F_2)$ in $\mathcal{N}_2$, the $(X, \mathcal{F},\ \Phi_{1,2}F_2)$ is an observable in $\mathcal{N}_1$.

Definition 3. [Sequential causal operator; Heisenberg picture of causality] Let $(T, \leq)$ be a tree like semi-ordered set such that "$t_1 \leq t_3$ and $t_2 \leq t_3$" implies "$t_1 \leq t_2$ or $t_2 \leq t_1$". The family $\{\Phi_{t_1,t_2} : \mathcal{N}_{t_2} \to \mathcal{N}_{t_1}\}_{(t_1,t_2) \in T^2_{\leqq}}$ is called a **sequential causal operator**, if it satisfies that

(i) For each t $(\in T)$, a basic structure $[\mathcal{A}_t \subseteq \mathcal{N}_t \subseteq B(H_t)]$ is determined.

(ii) For each $(t_1, t_2) \in T^2_{\leqq}$, a causal operator $\Phi_{t_1,t_2} : \mathcal{N}_{t_2} \to \mathcal{N}_{t_1}$ is defined such as $\Phi_{t_1,t_2}\Phi_{t_2,t_3} = \Phi_{t_1,t_3}$ $(\forall(t_1, t_2), \forall(t_2, t_3) \in T^2_{\leqq})$. Here, $\Phi_{t,t} : \mathcal{N}_t \to \mathcal{N}_t$ is the identity operator.

Now we can propose Axiom 2 (i.e., causality). (For details, see ref. [25].)

Axiom 2[Causality]; For each $t(\in T$="tree like semi-ordered set")), consider the basic structure:

$$[\mathcal{A}_t \subseteq \mathcal{N}_t \subseteq B(H_t)]$$

Then, the **chain of causalities** is represented by a **sequential causal operator** $\{\Phi_{t_1,t_2} : \mathcal{N}_{t_2} \to \mathcal{N}_{t_1}\}_{(t_1,t_2) \in T^2_{\leqq}}$.

2.3 The linguistic Copenhagen interpretation (= the manual to use Axioms 1 and 2)

Since so-called Copenhagen interpretation is not firm (*cf.* ref. [6]), we propose the linguistic Copenhagen interpretation in what follows. In the above, Axioms 1 and 2 are kinds of spells, (i.e., incantation, magic words, metaphysical statements), and thus, it is nonsense to verify them experimentally. Therefore, what we should do is not “to understand” but “to use”. After learning Axioms 1 and 2 by rote, we have to improve how to use them through trial and error.

The most important statement in the linguistic Copenhagen interpretation is as follows.

(B) **Only one measurement is permitted. Thus, Axiom 1 can be used only once.** And therefore, the state after a measurement is meaningless since it can not be measured any longer. That is, the state is only one.

Referring ref. [25], we have the following definition:

Definition 4. [(i;a)] Simultaneous observable]: Let $\mathsf{O}_i := (X_i, \mathcal{F}_i,\ F_i)$ $(i = 1, 2, ..., N)$ be commutative projective observables in $\mathcal{N}$. Let $(\times_{i=1}^N X_i, \boxtimes_{i=1}^N \mathcal{F}_i)$ is a product measurable space of $\{(X_i, \mathcal{F}_i)\}_{i=1}^N$. Then, there uniquely exists a projective observable $\times_{i=1}^N \mathsf{O}_i = (\times_{i=1}^N X_i, \boxtimes_{i=1}^N \mathcal{F}_i, \times_{i=1}^N F_i)$ such that

$$[\mathop{\times}_{i=1}^{N} F_i](X_1 \times X_2 \times ... \times X_{j-1} \times \Xi_j \times X_{j+1} \times ... \times X_N) = F_j(\Xi_j) \quad (\forall \Xi_j \in \mathcal{F}_i, j = 1, 2, ..., N)$$

This $\times_{i=1}^N \mathsf{O}_i$ is called a *simultaneous observable* (or, *product observable*) of $\{\mathsf{O}_i \mid i = 1, 2, ..., N\}$. Note that the existence and uniqueness is guaranteed (*cf.*, ref. [25]).

[(i;b)] Simultaneous measurement]: A measurement $\mathsf{M}_{\mathcal{N}}(\ \times_{i=1}^N \mathsf{O}_i = (\times_{i=1}^N X_i, \boxtimes_{i=1}^N \mathcal{F}_i, \times_{i=1}^N F_i)\ , S_{[\rho]})$ is called a simultaneous measurement concerning commutative O_i $(i = 1, 2, ..., N)$ in $\mathcal{N}$.

[(ii; a)] Parallel observable]: Let $\mathsf{O}_i := (X_i, \mathcal{F}_i,\ F_i)$ be a projective observable in $\mathcal{N}_i$ $(i = 1, 2, ..., N)$. Let $(\times_{i=1}^N X_i, \boxtimes_{i=1}^N \mathcal{F}_i)$ is a product measurable space of $\{(X_i, \mathcal{F}_i)\}_{i=1}^N$. Then, there uniquely exists an observable $\bigotimes_{i=1}^N \mathsf{O}_i = (\times_{i=1}^N X_i, \boxtimes_{i=1}^N \mathcal{F}_i, \bigotimes_{i=1}^N F_i)$ in a tensor algebra $\bigotimes_{i=1}^N \mathcal{N}_i$ such that

$$[\bigotimes_{i=1}^{N} F_i](X_1 \times X_2 \times ... \times X_{j-1} \times \Xi_j \times X_{j+1} \times ... \times X_N) = I_1 \otimes I_2 \otimes ... \otimes I_{j-1} \otimes F_i(\Xi_j) \otimes I_{j+1} \otimes ... \otimes I_N$$
$$(\forall \Xi_j \in \mathcal{F}_j, j = 1, 2, ..., N)$$

This $\bigotimes_{i=1}^N \mathsf{O}_i$ is called a *parallel observable* (or, *tensor observable*) of $\{\mathsf{O}_i \mid i = 1, 2, ..., N\}$ in a tensor algebra $\bigotimes_{i=1}^N \mathcal{N}_i$

[(ii; b)] Parallel measurement]: A measurement $\mathsf{M}_{\bigotimes_{i=1}^N \mathcal{N}_i}(\ \bigotimes_{i=1}^N \mathsf{O}_i\ , S_{[\bigotimes_{i=1}^N \rho_i]})$ is called a parallel measurement concerning $\{\ \mathsf{M}_{\mathcal{N}_i}(\mathsf{O}_i := (X_i, \mathcal{F}_i, F_i),\ S_{[\rho_i]})\ \}_{i=1}^N$.

3 Why does logic arise in science ?

It is well-known that logic holds in the class of mathematical propositions. However, it should be noted that it is not guaranteed that logic holds among non-mathematical propositions. Thus, the question ”Why does logic arise in science?” is significant.

From here, we devote ourselves to classical QL (in the classical basic structure $[C_0(\Omega) \subseteq L^\infty(\Omega, \nu) \subseteq B(L^2(\Omega, \nu))]$) and not quantum QL (in the quantum basic structure $[\mathcal{C}(H) \subseteq B(H) \subseteq B(H)]$).

The close relationship between measurement and logic was first discussed in ref. [8]. The argument in this section are regarded as a slight variation of the argument in ref. [8].

3.1 Logic (i.e., $\neg$, $\wedge$, $\vee$, $\rightarrow$) in classical QL

We have the following theorem:

Theorem 5. In a class of binary projective measurements in classical QL, measurement has properties like logic:

In this section we will devote ourselves to the above proof as follows.

Consider a classical basic structure

$$[C_0(\Omega) \subseteq L^\infty(\Omega, \nu) \subseteq B(L^2(\Omega, \nu)]$$

Here, assume that Ω is a locally compact space with a Borel measure ν on Ω such that $\nu(D) > 0$ (for any open set D ($\subseteq \Omega$, $D \neq \emptyset$). Also, without loss of generality, we assume that $\nu(\Omega) = 1$. Consider many tomatoes, that is, roughly speaking, consider T as the set of all tomatoes. Assume that any tomato $t (\in T)$ is represented by a state ω, which is an element of the state space Ω. Thus, we have the map $\widehat{\omega} : T \to \Omega$. That is, the quantitative property of a tomato t is represented by $\widehat{\omega}(t)$. For example, it suffices to consider Ω such that $\Omega \subseteq \mathbb{R}^N$ ($= N$-dimensional real space), where N is sufficiently large natural number (or, $N = \infty$). That is,

$$\Omega \ni \omega = \Big(\omega^{(1)}(= \text{weight}), \omega^{(2)}(= \text{diameter}), \omega^{(3)}(= \text{diameter}), \omega^{(4)}(= \text{color value}),$$
$$\omega^{(5)}(= \text{calorie}), \omega^{(6)}(= \text{sugar content}), ..., \omega^{(N)}(= ...)\Big) \in \mathbb{R}^N$$

Consider a binary projective observable (i.e., $\{1, 0\}$-valued projective observable, or $\{x_1, x_0\}$-valued projective observable) $\mathsf{O} \equiv (X, 2^X, F)$ in $L^\infty(\Omega, \nu)$, where $X = \{1, 0\}$ (or, $X = \{x_1, x_0\}$) and $F(\Xi) = [F(\Xi)]^2$ ($\forall \Xi \in \mathcal{F}$).

Further, as shown in Figure 2 below, we, for the sake of simplicity, assume that $\{\omega \in \Omega \mid [F(\{1\})](\omega) = 1, a.e.\} (\equiv \Gamma)$ is an open set such that

$$\Gamma = [\overline{\Gamma}]^\circ, \qquad \nu(\Omega \setminus (\Gamma \cup [\Gamma^c]^\circ)) = 0$$

where D^c is the complement of D, i.e., $\Omega \setminus D$, $\overline{D}$ ="the closure of D", D° ="the interior of D". Note that we can assume that $\Gamma^c = [\Gamma^c]^\circ$ in the sense of "almost everywhere concerning ν" (i.e., "$\nu(\Gamma^c \setminus [\Gamma^c]^\circ) = 0$") , which will be frequently used without refusal in this paper. The $\{1, 0\}$-valued projective observable $\mathsf{O} \equiv (X, 2^X, F)$ in $L^\infty(\Omega, \nu)$ is also denoted by

$$\mathsf{O}^\Gamma \equiv (X, 2^X, F^\Gamma) \tag{2}$$

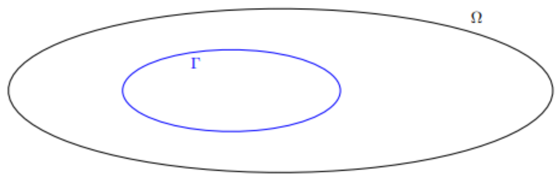


Figure 2: Venn diagram: $\Gamma \subseteq \Omega$

Remark 6. (i): Someone might say that the term "the set of all tomatoes" is as ambiguous as "the set of all dinosaurs". However, for the sake of convenience, here we use the term "the set of all tomatoes". TLP begins with the following sentence:

($\sharp$) 1: *The world is everything that is the case.*
1.1: *The world is the totality of facts, not of things.*

This means that "Consider Ω(=the state space of tomatos), and not T(=the set of all tomatos) !". This problem is the same as that of the Hempel' raven paradox (i.e., "the set of all ravens" leads to contradiction). For further discussion about this, see ref. [24], [26].
(ii): If we want to both tomato's world Ω_1 and apple's world Ω_2, it suffices to start from the product space $\Omega_1 \times \Omega_2$. Thus, in general we consider the world also is represented by a large state space $\widehat{\Omega}$.

Definition 7. [Measured value] Consider a measurement $\mathsf{M}_{L^\infty(\Omega,\nu)}(\mathsf{O}^\Gamma, S_{[\omega]})$, where $\mathsf{O}^\Gamma \equiv (X(=\{1,0\}), 2^X, F^\Gamma)$ is a binary projective observable in $L^\infty(\Omega,\nu)$. Denote the measured value of $\mathsf{M}_{L^\infty(\Omega,\nu)}(\mathsf{O}^\Gamma, S_{[\omega]})$ by $[\mathbf{MV}](\mathsf{M}_{L^\infty(\Omega,\nu)}(\mathsf{O}^\Gamma, S_{[\omega]}))$. Then, we see that

$$[\mathbf{MV}](\mathsf{M}_{L^\infty(\Omega,\nu)}(\mathsf{O}^\Gamma, S_{[\omega]})) = \begin{cases} 1 & (\omega \in \Gamma) \\ 0 & (\omega \notin \Gamma) \end{cases}$$

(with probability 1)

Let $X = \{1,0\}$, Ω and $\widehat{\omega}(t)$ be as before. Put Γ =RD, or Γ =SW in the formula (2) (see Figure 3 below). Consider a binary projective observables $\mathsf{O}^{\mathrm{RD}} \equiv (X(=\{1,0\}), 2^X, F^{\mathrm{RD}})$ and $\mathsf{O}^{\mathrm{SW}} \equiv (X, 2^X, F^{\mathrm{SW}})$ in $L^\infty(\Omega,\nu)$. Consider a measurement $\mathsf{M}_{L^\infty(\Omega,\nu)}(\mathsf{O}^{\mathrm{RD}}, S_{[\widehat{\omega}(t)]})$. That is, we consider that the following three are equivalent (i.e., Axiom 1 (measurement) says that (C_1) $\Leftrightarrow$ (C_2). Also, (C_3) is the expression of (C_1) in ordinary language):

(C_1) A measured value 1 is obtained by the measurement $\mathsf{M}_{L^\infty(\Omega,\nu)}(\mathsf{O}^{\mathrm{RD}}, S_{[\widehat{\omega}(t)]})$ $\big($i.e., $[\mathbf{MV}]\big(\mathsf{M}_{L^\infty(\Omega,\nu)}(\mathsf{O}^{\mathrm{RD}}, S_{[\widehat{\omega}(t)]})) = 1\big)$ (strictly speaking, the probability that a measured value 1 is obtained by the measurement $\mathsf{M}_{L^\infty(\Omega,\nu)}$ $(\mathsf{O}^{\mathrm{RD}}, S_{[\widehat{\omega}(t)]})$ is equal to 1.)

(C_2) $\widehat{\omega}(t) \in \mathrm{RD}(\equiv \{\omega \in \Omega \mid [F^{\mathrm{RD}}(\{1\})](\omega) = 1\})$

(C_3) A tomato t is "red".

Similarly, as shown in Figure 3 below, consider a measurement $\mathsf{M}_{L^\infty(\Omega,\nu)}(\mathsf{O}^{\rm SW}, S_{[\widehat{\omega}(t)]})$. That is, we consider that the following three are equivalent:

(C_1') A measured value 1 is obtained by the measurement $\mathsf{M}_{L^\infty(\Omega,\nu)}(\mathsf{O}^{\rm SW}, S_{[\widehat{\omega}(t)]})$ $\big($i.e., $[\mathbf{MV}]\big(\mathsf{M}_{L^\infty(\Omega,\nu)}(\mathsf{O}^{\rm SW}, S_{[\widehat{\omega}(t)]})) = 1\big)$.

(C_2') $\widehat{\omega}(t) \in \mathrm{SW}(\equiv \{\omega \in \Omega \mid [F^{\rm SW}(\{1\})](\omega) = 1\})$

(C_3') A tomato t is "sweet".

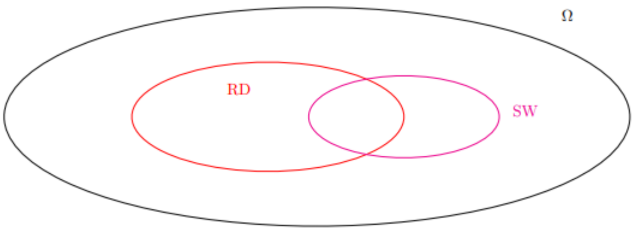


Figure 3: Venn diagram: $\mathrm{RD}, \mathrm{SW} \subseteq \Omega$

[Not]

It is clear that the following four are equivalent:

(D_0) $[\mathbf{MV}]\big(\mathsf{M}_{L^\infty(\Omega,\nu)}(\mathsf{O}^{\rm SW}, S_{[\widehat{\omega}(t)]})\big) = 0$.

(D_1) A measured value 1 is obtained by the measurement $\mathsf{M}_{L^\infty(\Omega,\nu)}(\Theta_\neg(\mathsf{O}^{\rm SW}), S_{[\widehat{\omega}(t)]})$ (which is also denoted by $\neg\mathsf{M}_{L^\infty(\Omega,\nu)}(\mathsf{O}^{\rm SW}, S_{[\widehat{\omega}(t)]})$), where $\Theta_\neg : \{1,0\} \to \{1,0\}$ is defined by $\Theta_\neg(1) = 0$, $\Theta_\neg(0) = 1$ (*cf.* Definition 1; image observable). Thus, $[\mathbf{MV}]\big(\neg\mathsf{M}_{L^\infty(\Omega,\nu)}(\mathsf{O}^{\rm SW}, S_{[\widehat{\omega}(t)]})\big) = 1$.

(D_2) $\widehat{\omega}(t) \in [\overline{\mathrm{SW}}]^c(\equiv \{\omega \in \Omega \mid [F^{\rm SW}(\{0\})](\omega) = 0\})$

(D_3) A tomato t is not "sweet".

[And]

We see that the following four are equivalent:

(E_0) A measured value $(1,1)$ is obtained by the measurement $\mathsf{M}_{L^\infty(\Omega,\nu)}(\mathsf{O}^{\rm SW} \times \mathsf{O}^{\rm RD}, S_{[\widehat{\omega}(t)]})$.

(E_1) A measured value 1 is obtained by the measurement $\mathsf{M}_{L^\infty(\Omega,\nu)}(\Theta_\wedge(\mathsf{O}^{\rm SW} \times \mathsf{O}^{\rm RD}), S_{[\widehat{\omega}(t)]})$ (which is also denoted by $\mathsf{M}_{L^\infty(\Omega,\nu)}(\mathsf{O}^{\rm SW}, S_{[\widehat{\omega}(t)]})\bigwedge\mathsf{M}_{L^\infty(\Omega,\nu)}(\mathsf{O}^{\rm RD}, S_{[\widehat{\omega}(t)]})$), where $\Theta_\wedge : \{1,0\}^2 \to \{1,0\}$ is defined by $\Theta_\wedge(1,1) = 1$, $\Theta_\wedge(1,0) = \Theta_\wedge(0,1) = \Theta_\wedge(0,0) = 0$ (*cf.* Definition 1; image observable).
Thus, $[\mathbf{MV}]\big(\mathsf{M}_{L^\infty(\Omega,\nu)}(\mathsf{O}^{\rm SW}, S_{[\widehat{\omega}(t)]})\bigwedge\mathsf{M}_{L^\infty(\Omega,\nu)}(\mathsf{O}^{\rm RD}, S_{[\widehat{\omega}(t)]})\big) = 1$.

(E_2) $\widehat{\omega}(t) \in \mathrm{SW}(\equiv \{\omega \in \Omega \mid [F^{\rm SW}(\{1\})](\omega) = 1\})\bigcap\mathrm{RD}(\equiv \{\omega \in \Omega \mid [F^{\rm RD}(\{1\})](\omega) = 1\})$

(E_3) A tomato t is "sweet" and "red"

Remark 8. When $\omega_1 \neq \omega_2$, it should be noted that the simbol "$\mathsf{M}_{L^\infty(\Omega,\nu)}(\mathsf{O}^{\rm SW}, S_{[\omega_1]}) \bigwedge \mathsf{M}_{L^\infty(\Omega,\nu)}(\mathsf{O}^{\rm RD}, S_{[\omega_2]})$" is not yet defined. Recall the linguistic Copenhagen interpretation "Only one measurement is permitted". Thus, "there is only one state". Therefore, this should be defined by the parallel $\mathsf{M}_{L^\infty(\Omega,\nu)\otimes L^\infty(\Omega,\nu)}(\Theta_\wedge(\mathsf{O}^{\rm SW} \bigotimes \mathsf{O}^{\rm RD}), S_{[(\omega_1,\omega_2)]})$. More generally, the simbol "$\bigwedge_{\lambda\in\Lambda} \mathsf{M}_{L^\infty(\Omega_\lambda,\nu_\lambda)}(\mathsf{O}^{\Gamma_\lambda}, S_{[\omega_\lambda]})$" is defined by

$$\mathsf{M}_{\bigotimes_{\lambda\in\Lambda} L^\infty(\Omega_\lambda,\nu_\lambda)}(\bigotimes_{\lambda\in\Lambda} \mathsf{O}^{\Gamma_\lambda}, S_{[(\omega_\lambda)_{\lambda\in\Lambda}]})$$

[Or]

We see that the following four are equivalent:

(F_0) A measured value (x_1, x_2) obtained by the measurement $\mathsf{M}_{L^\infty(\Omega,\nu)}(\mathsf{O}^{\rm SW} \times \mathsf{O}^{\rm RD}, S_{[\widehat{\omega}(t)]})$ belongs to $\{(1,1),(1,0),(0,1)\}$

(F_1) A measured value 1 is obtained by the measurement $\mathsf{M}_{L^\infty(\Omega,\nu)}(\Theta_\vee(\mathsf{O}^{\rm SW} \times \mathsf{O}^{\rm RD}), S_{[\widehat{\omega}(t)]})$ (which is also denoted by $\mathsf{M}_{L^\infty(\Omega,\nu)}(\mathsf{O}^{\rm SW}, S_{[\widehat{\omega}(t)]}) \bigvee \mathsf{M}_{L^\infty(\Omega,\nu)}(\mathsf{O}^{\rm RD}, S_{[\widehat{\omega}(t)]})$), where $\Theta_\vee : \{1,0\}^2 \to \{1,0\}$ is defined by $\Theta_\vee(1,1) = \Theta_\vee(1,0) = \Theta_\vee(0,1) = 1$, $\Theta_\vee(0,0) = 0$.
Thus, $[\mathbf{MV}]\big(\mathsf{M}_{L^\infty(\Omega,\nu)}(\mathsf{O}^{\rm SW}, S_{[\widehat{\omega}(t)]}) \bigvee \mathsf{M}_{L^\infty(\Omega,\nu)}(\mathsf{O}^{\rm RD}, S_{[\widehat{\omega}(t)]}) \big) = 1$.

(F_2) $\widehat{\omega}(t) \in \mathrm{SW}(\equiv \{\omega \in \Omega \mid [F^{\rm SW}(\{1\})](\omega) = 1\}) \bigcup \mathrm{RD}(\equiv \{\omega \in \Omega \mid [F^{\rm RD}(\{1\})](\omega) = 1\})$

(F_3) A tomato t is "sweet" or "red"

[Implication]

We see that the following four are equivalent:

(G_0) A measured value (x_1, x_2) obtained by the measurement $\mathsf{M}_{L^\infty(\Omega,\nu)}(\mathsf{O}^{\rm SW} \times \mathsf{O}^{\rm RD}, S_{[\widehat{\omega}(t)]})$ belongs to $\{(1,1),(0,1),(0,0)\}$

(G_1) A measured value 1 is obtained by the measurement $\mathsf{M}_{L^\infty(\Omega,\nu)}(\Theta_\to(\mathsf{O}^{\rm SW} \times \mathsf{O}^{\rm RD}), S_{[\widehat{\omega}(t)]})$ (which is also denoted by $\mathsf{M}_{L^\infty(\Omega,\nu)}(\mathsf{O}^{\rm SW}, S_{[\widehat{\omega}(t)]}) \longrightarrow \mathsf{M}_{L^\infty(\Omega,\nu)}(\mathsf{O}^{\rm RP}, S_{[\widehat{\omega}(t)]})$), where $\Theta_\to : \{1,0\}^2 \to \{1,0\}$ is defined by $\Theta_\to(1,1) = \Theta_\to(0,1) = \Theta_\to(0,0) = 1$, $\Theta_\to(1,0) = 0$. Thus, $[\mathbf{MV}]\big(\mathsf{M}_{L^\infty(\Omega,\nu)}(\mathsf{O}^{\rm SW}, S_{[\widehat{\omega}(t)]}) \longrightarrow \mathsf{M}_{L^\infty(\Omega,\nu)} (\mathsf{O}^{\rm RD}, S_{[\widehat{\omega}(t)]}) \big) = 1$.

(G_2) $\widehat{\omega}(t) \in \mathrm{SW}^c(\equiv \{\omega \in \Omega \mid [F^{\rm SW}(\{1\})](\omega) = 0\}) \cup \mathrm{RD}(\equiv \{\omega \in \Omega \mid [F^{\rm RD}(\{1\})](\omega) = 1\})$

(G_3) A tomato t is not "sweet", or it is "red"

Summing up the above, we have the following theorem:

Theorem 9. We see that for each $\omega \in \Omega$,

(a) $[\mathbf{MV}]\big(\neg \mathsf{M}_{L^\infty(\Omega,\nu)}(\mathsf{O}^\Gamma, S_{[\widehat{\omega}(t)]}) \big) = 1$ -$[\mathbf{MV}]\big(\mathsf{M}_{L^\infty(\Omega,\nu)}(\mathsf{O}^\Gamma, S_{[\widehat{\omega}(t)]}) \big)$

(b) $[\mathbf{MV}]\big(\mathsf{M}_{L^\infty(\Omega,\nu)}(\mathsf{O}^{\Gamma_1}, S_{[\widehat{\omega}(t)]}) \bigwedge \mathsf{M}_{L^\infty(\Omega,\nu)} (\mathsf{O}^{\Gamma_2}, S_{[\widehat{\omega}(t)]}) \big)$
$= \min \Big\{ [\mathbf{MV}]\big(\mathsf{M}_{L^\infty(\Omega,\nu)}(\mathsf{O}^{\Gamma_1}, S_{[\widehat{\omega}(t)]}) \big), [\mathbf{MV}]\big(\mathsf{M}_{L^\infty(\Omega,\nu)} (\mathsf{O}^{\Gamma_2}, S_{[\widehat{\omega}(t)]}) \big) \Big\}$

(c) $\quad [\mathbf{MV}]\big(\mathsf{M}_{L^\infty(\Omega,\nu)}(\mathsf{O}^{\Gamma_1}, S_{[\widehat{\omega}(t)]}) \bigvee \mathsf{M}_{L^\infty(\Omega,\nu)} (\mathsf{O}^{\Gamma_2}, S_{[\widehat{\omega}(t)]}) \big)$
$= \max \Big\{ [\mathbf{MV}]\big(\mathsf{M}_{L^\infty(\Omega,\nu)}(\mathsf{O}^{\Gamma_1}, S_{[\widehat{\omega}(t)]}) \big), [\mathbf{MV}]\big(\mathsf{M}_{L^\infty(\Omega,\nu)} (\mathsf{O}^{\Gamma_2}, S_{[\widehat{\omega}(t)]}) \big) \Big\}$

(d) $\quad [\mathbf{MV}]\big(\mathsf{M}_{L^\infty(\Omega,\nu)}(\mathsf{O}^{\Gamma_1}, S_{[\widehat{\omega}(t)]}) \longrightarrow \mathsf{M}_{L^\infty(\Omega,\nu)} (\mathsf{O}^{\Gamma_2}, S_{[\widehat{\omega}(t)]}) \big)$
$= \max \Big\{ 1 - [\mathbf{MV}]\big(\mathsf{M}_{L^\infty(\Omega,\nu)}(\mathsf{O}^{\Gamma_1}, S_{[\widehat{\omega}(t)]}) \big), [\mathbf{MV}]\big(\mathsf{M}_{L^\infty(\Omega,\nu)} (\mathsf{O}^{\Gamma_2}, S_{[\widehat{\omega}(t)]}) \big) \Big\}$

Therefore, we can expect that a class of binary projective measurements in classical quantum language has a logical structure. In this sense, this class can be regarded as a class of "propositions".

Remark 10. Note that propositional logic (i.e., $\neg$, $\wedge$, $\vee$, $\rightarrow$) and predict logic (i.e., $\neg$, $\wedge$, $\vee$, $\rightarrow$, $\forall$, $\exists$) are essentially the same since $P_1 \wedge P_2 \wedge P_3 \wedge ... = (\forall n)[P_n]$ and $P_1 \vee P_2 \vee P_3 \vee ... = (\exists n)[P_n]$. Thus, this paper does not distinguish propositional logic and predict logic.

3.2 Syllogism

Further, consider a measurement $\mathsf{M}_{L^\infty(\Omega,\nu)}(\mathsf{O}^{\mathrm{RP}}, S_{[\widehat{\omega}(t)]})$. That is, we consider that the following three are equivalent:

(H_1) A measured value 1 is obtained by the measurement $\mathsf{M}_{L^\infty(\Omega,\nu)}(\mathsf{O}^{\mathrm{RP}}, S_{[\widehat{\omega}(t)]})$.

(H_2) $\widehat{\omega}(t) \in \mathrm{RP}(\equiv \{\omega \in \Omega \mid [F^{\mathrm{RP}}(\{1\})](\omega) = 1\})$

(H_3) A tomato t is "ripe".

Theorem 11. [Syllogism]:
Let t be a tomato, and let $\widehat{\omega}(t)(\in \Omega)$ be the state of t. Assume the followings:

(I_0) A measured value (x_1, x_2) obtained by the measurement $\mathsf{M}_{L^\infty(\Omega,\nu)}(\mathsf{O}^{\mathrm{SW}} \times \mathsf{O}^{\mathrm{RP}}, S_{[\widehat{\omega}(t)]})$ belongs to $\{(1,1),(0,1),(0.0)\}$

which is equivalent to

(I_1) $[\mathbf{MV}]\big(\mathsf{M}_{L^\infty(\Omega,\nu)}(\mathsf{O}^{\mathrm{SW}}, S_{[\widehat{\omega}(t)]}) \longrightarrow \mathsf{M}_{L^\infty(\Omega,\nu)}(\mathsf{O}^{\mathrm{RP}}, S_{[\widehat{\omega}(t)]}) \big) = 1$

(I_2) $\widehat{\omega}(t) \notin \mathrm{SW}(\equiv \{\omega \in \Omega \mid [F^{\mathrm{SW}}(\{1\})](\omega) = 1\}) \vee \widehat{\omega}(t) \in \mathrm{RP}(\equiv \{\omega \in \Omega \mid [F^{\mathrm{RP}}(\{1\})](\omega) = 1\})$

(I_3) A tomato t is not "sweet", or it is "ripe".

and

(I'_0) A measured value (x_2, x_3) obtained by the measurement $\mathsf{M}_{L^\infty(\Omega,\nu)}(\mathsf{O}^{\mathrm{RP}} \times \mathsf{O}^{\mathrm{RD}}, S_{[\widehat{\omega}(t)]})$ belongs to $\{(1,1),(0,1),(0.0)\}$

which is equivalent to

(I'_1) $[\mathbf{MV}]\big(\mathsf{M}_{L^\infty(\Omega,\nu)}(\mathsf{O}^{\mathrm{RP}}, S_{[\widehat{\omega}(t)]}) \longrightarrow \mathsf{M}_{L^\infty(\Omega,\nu)}(\mathsf{O}^{\mathrm{RD}}, S_{[\widehat{\omega}(t)]}) \big) = 1$

which is equivalent to

(I'_2) $\widehat{\omega}(t) \notin \mathrm{RP}(\equiv \{\omega \in \Omega \mid [F^{\mathrm{RP}}(\{1\})](\omega) = 1\}) \vee \widehat{\omega}(t) \in \mathrm{RD}(\equiv \{\omega \in \Omega \mid [F^{\mathrm{RD}}(\{1\})](\omega) = 1\})$

(I'_3) A tomato t is not "ripe", or it is "red".

Then the following holds:

(J$_0$) A measured value (x_1, x_3) obtained by the measurement $\mathsf{M}_{L^\infty(\Omega,\nu)}(\mathsf{O}^{\rm SW} \times \mathsf{O}^{\rm RD}, S_{[\widehat{\omega}(t)]})$ belongs to $\{(1,1),(0,1),(0.0)\}$

(J$_1$) $[\mathbf{MV}]\big(\ \mathsf{M}_{L^\infty(\Omega,\nu)}(\mathsf{O}^{\rm SW}, S_{[\widehat{\omega}(t)]}) \longrightarrow \mathsf{M}_{L^\infty(\Omega,\nu)}(\mathsf{O}^{\rm RD}, S_{[\widehat{\omega}(t)]})\ \big) = 1$

(J$_2$) $\widehat{\omega}(t) \notin \mathrm{SW}(\equiv \{\omega \in \Omega \mid [F^{\rm RD}(\{1\})](\omega) = 1\}) \vee \widehat{\omega}(t) \in \mathrm{RD}(\equiv \{\omega \in \Omega \mid [F^{\rm RP}(\{1\})](\omega) = 1\})$

(J$_3$) A tomato t is not "sweet", or it is "red".

[Proof]: Recalling the linguistic Copenhagen interpretation "Only one measurement is permitted" (in Sec. 2.3), we have enough to see the simultaneous observable $\mathsf{O}^{\rm SW} \times \mathsf{O}^{\rm RP} \times \mathsf{O}^{\rm RD}$, which uniquely exists (*cf.* Definition 4 (i)). Thus, we have the measurement $\mathsf{M}_{L^\infty(\Omega,\nu)}(\mathsf{O}^{\rm SW} \times \mathsf{O}^{\rm RP} \times \mathsf{O}^{\rm RD}, S_{[\widehat{\omega}(t)]})$. Let (x_1, x_2, x_3) be the measured value. We easily see that (x_1, x_3) belongs to $\{(1,1),(0,1),(0.0)\}$. Thus, (J) holds.
However, we should add the following. This proof is not self-evident since the existence and uniqueness of the simultaneous observable $\mathsf{O}^{\rm SW} \times \mathsf{O}^{\rm RP} \times \mathsf{O}^{\rm RD}$ is not trivial (*cf.* Definition 4 (i)). Also, see Sec. 5 (i.e., Syllogizm does not always hold in quantum sistems). $\square$

3.3 Elementary measurements

Consider the state space Ω, which is finite (or, countable) with a metric d (i.e., $d(\omega_1, \omega_2) = 1$ $(\omega_1 \neq \omega_2)$, $= 0$ $(\omega_1 = \omega_2)$.

Definition 12. Let λ be any element of Ω. Putting $\Gamma = \{\lambda\}$ in the formula (2), define the elementary binary projective observable $\mathsf{O}^{\{\lambda\}} = (X(=\{1,0\}), 2^X, F^{\{\lambda\}})$ in $L^\infty(\Omega,\nu)$ such that

$$[F^{\{\lambda\}}(\{1\})](\omega) = \left\{ \begin{array}{ll} 1 & \quad (\text{if } \omega = \lambda) \\ 0 & \quad (\text{if } \omega \neq \lambda) \end{array} \right. , \qquad [F^{\{\lambda\}}(\{0\})](\omega) = 1 - [F_{\{\lambda\}}(\{1\})](\omega)$$
$$(\forall \omega \in \Omega)$$

The measurement $\mathsf{M}_{L^\infty(\Omega,\nu)}(\mathsf{O}^{\{\lambda\}}, S_{[\omega]})$ $(\lambda, \omega \in \Omega)$ is called an *elementary measurement.*

It is clear that it holds that

(K$_1$) A measured value 1 is obtained by the elementary measurement $\mathsf{M}_{L^\infty(\Omega,\nu)}(\mathsf{O}^{\{\lambda\}}, S_{[\omega]})$
$\Longleftrightarrow \lambda = \omega$

(K$_2$) A measured value 1 is obtained by the elementary measurement $\mathsf{M}_{L^\infty(\Omega,\nu)}(\mathsf{O}^{\{\lambda\}}, S_{[\omega]})$
$\Longleftrightarrow \lambda \neq \omega$

Under the above preparation, the following theorem is clear

Theorem 13. Let Γ be a subset of Ω. And let $\omega \in \Omega$. Then we see that

$$\mathsf{M}_{L^\infty(\Omega,\nu)}(\mathsf{O}^{\Gamma}, S_{[\omega]}) = \textstyle\bigvee_{\lambda \in \Gamma} \mathsf{M}_{L^\infty(\Omega,\nu)}(\mathsf{O}^{\{\lambda\}}, S_{[\omega]})$$

Remark 14. In this section, I devoted myself to the classical cases. Our arguments in this section are invalid in quantum cases. Consider a binary projective observable $\mathsf{O} = (\{1,0\}, 2^{\{1,0\}}, F)$ in $B(H)$, where $F(\{1\}) = P(=$ projection$)$, $F(\{0\}) = I - P$. Define the state ρ by $|e\rangle\langle e|$ (where $e \in H$ such that $||e||_H = 1$). Axiom 1 say that

- The probability that a measured value 1 $(\in X = \{1,0\})$ is obtained by the measurement $\mathsf{M}(\mathsf{O} :=(X, 2^{\{1,0\}}, F), S_{[\rho]})$ is given by $Tr[\rho F(\{1\})](= ||Pe||_H)$

Thus, if $e \notin PH \cup (1-P)H$ (i.e., in most case) the "probability" belongs to the open interval $(0,1)$. That is, in quantum cases, the probability concept cannot be covered up. Thus, the arguments in this section are invalid in quantum cases.

Remark 15. Many readers may not consider the above theorem to be particularly important. I have the same view. However, this theorem was prepared in preparation for Sec.4 (Theorem 23). Of course, the spirit of expressing complex observables in simple observables is quite important. In quantum language, this spirit is realized by von Neumann's spectral decomposition theorem (*cf.* ref. [29]) and Holevo's theorem (*cf.* ref. [5]), that is, "Any observable can be represented by the product of commutative binary projective observables in QL (i.e., both classical and quantum QL)". In this sense, we say that

($\sharp$) as far as classical QL, a class of binary projective measurements is fundamental.

4 My understanding of Wittgenstein's picture theory

In Wittgenstein's book "Tractatus Logico-Philosophicus" (*cf.* [32]), he studies the following:

(L_1) Assume a certain language L. Then, the following problems are essential:

(i) What is a proposition in the language L? Or, why does logic (i.e., $\neg$, $\wedge$, $\vee$,,...) arise in the language L?

(ii) Everyone knows that complex propositions can be created by logically combining simple propositions. Now, let's think about the opposite. Is there a class of "simplest propositions (or, elementary propositions)" in the language L? Or, can any proposition be constructed from elementary propositions?

I think that the above is quite important. However, Wittgenstein's answer is not sufficient, since he did not answer "What is the language L?"

In the preface of Wittgenstein's book "Tractatus Logico-Philosophicus" (*cf.* [32]), he said that

(L_2) *This book will perhaps only be understood by those who have themselves already thought the thoughts which are expressed in it — or similar thoughts.*

This is a very significant sentence for me. That is because, as I answered in the previous section 3, I already know the answers to question (L_1) before reading this TLP (if the language L = classical QL).

In this section, I propose my understanding of Wittgenstein's picture theory, which is inspired from the arguments in the previous section. The following table will promote the reader's understanding of the arguments in this section.

Table 16. [Logic in quantum language (Sec. 3) vs. logic in Wittgenstein's picture theory (this section)]

Logic in quantum language (Sec. 3)	Logic in Wittgenstein's theory (Sec.4)
Axiom 1 in Section 2.2 (what is a measurement?)) the linguistic Copenhagen interpretation in Sec. 2.3	Definition 17 (what is a proposition?) Naive set theory (≈ Venn diagram: Fig. 4)
system, particle, object, tomato	object, thing, tomato
state space (state)	logical space (case, fact,, atomic fact)
$[\mathbf{MV}]$, measured value $\{1,0\}$	$[\mathbf{TV}]$, truth value $\{\mathrm{T},\mathrm{F}\}$
classical binary projective measurement $\mathsf{M}_{L^\infty(\Omega,\nu)}(\mathsf{O}^\Gamma \equiv (X(=\{1,0\}),2^X,F^\Gamma),S_{[\widehat{\omega}(t)]})$	proposition $\mathsf{P}_\Omega(\Gamma,S_{[\widehat{\omega}(t)]})$
Theorem 11 (Syllogism in measurements)	Theorem 19 (Syllogism in propositions)
elementary measurement	elementary proposition
Theorem 13 (Remark 15) Elementary measurements are not fundamental	Theorem 23 (Remark 24) Elementary propositions are fundamental

I encourage you to read the following, referring to the above table. If you understand Sec. 3, you should be able to understand this section immediately. Using the above table, we can translate the language of Wittgenstein's theory into quantum language.

♠**Note 1.** In TLP, many important concepts are not clearly defined. In particular, he may use the word "logical space" in a different sense than the above table. However, if "logical space" is the most important concept in Wittgenstein's picture theory, it must mean "state space".

4.1 Logic (i.e., $\neg$, $\wedge$, $\vee$, $\rightarrow$) in my understanding of Wittgenstein's picture theory

The question "What is a proposition?" is easy in mathematics. However, outside of mathematics, this question is not easy. Wittgenstein's purpose is to clarify "proposition" in science.

Let Ω, Γ, $\omega, \widehat{\omega}(t)$, SW, RP, RD, ... be the same as in the previous section. Let us start from the following definition.

Definition 17. [Proposition, Truth value] A pair (Γ,ω) $(\Gamma \subseteq \Omega, \omega \in \Omega)$ is called a proposition in Ω (i.e., "a system with a state ω has Γ-property"), which is denoted by $\mathsf{P}_\Omega(\Gamma,S_{[\omega]})$. Define the truth value of the proposition $\mathsf{P}_\Omega(\Gamma,S_{[\omega]})$ (which is denoted by $[\mathbf{TV}]\big(\ \mathsf{P}_\Omega(\Gamma,S_{[\omega]})\ \big)$) by

$$[\mathbf{TV}]\big(\ \mathsf{P}_\Omega(\Gamma,S_{[\omega]})\ \big) = \begin{cases} \text{T (true)} & (\text{ if } \omega \in \Gamma) \\ \text{F (false)} & (\text{ if } \omega \notin \Gamma) \end{cases}$$

That is, truth values $\{\mathrm{T}, \mathrm{F}\}(=X)$ correspond to measured values $\{1,0\}(=X)$ (or, $X=\{x_1, x_0\}$). Thus, I think that the above is essentially the same as naive set theory (*cf.* ref. [4]).

If $[\mathbf{TV}]\big(\ \mathsf{P}_\Omega(\Gamma, S_{[\widehat{\omega}(t)]})\ \big) =$ "T", we may say that

(M) a tomato t has Γ-property.

As shown in Figure 4 below, consider a proposition $\mathsf{P}_\Omega(\mathrm{RD}, S_{[\widehat{\omega}(t)]})$. Thus, the following three are equivalent (i.e., $(\mathrm{N}_1) \Leftrightarrow (\mathrm{N}_2)$. Also, (N_3) is the expression of (N_1) in ordinary language):

(N_1) $[\mathbf{TV}]\big(\ \mathsf{P}_\Omega(\mathrm{RD}, S_{[\widehat{\omega}(t)]})\ \big) =$ "T".

(N_2) $\widehat{\omega}(t) \in \mathrm{RD}$

(N_3) A tomato t is "red".

Similarly, consider a proposition $\mathsf{P}_\Omega(\mathrm{SW}, S_{[\widehat{\omega}(t)]})$. Thus, we see that the following three are equivalent:

(O_1) $[\mathbf{TV}]\big(\ \mathsf{P}_\Omega(\mathrm{SW}, S_{[\widehat{\omega}(t)]})\ \big) =$ "T".

(O_2) $\widehat{\omega}(t) \in \mathrm{SW}$

(O_3) A tomato t is "sweet".

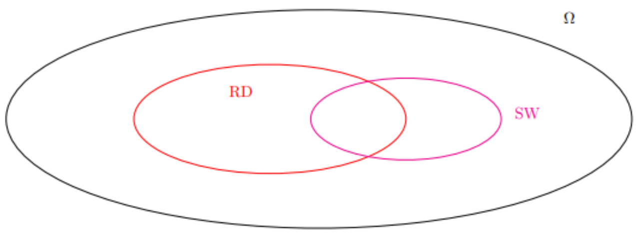


Figure 4 (= Figure 3) Venn diagram: $\mathrm{RD}, \mathrm{SW} \subseteq \Omega$

[Not]

It is clear that the following three are equivalent:

(R_1) $[\mathbf{TV}]\big(\ \mathsf{P}_\Omega(\mathrm{SW}^c, S_{[\widehat{\omega}(t)]})\ \big) =$ "T".

(Here, $\mathsf{P}_\Omega(\mathrm{SW}^c, S_{[\widehat{\omega}(t)]})$ is also denoted by $\neg\mathsf{P}_\Omega(\mathrm{SW}, S_{[\widehat{\omega}(t)]})$), Thus, $[\mathbf{TV}]\big(\ \neg\mathsf{P}_\Omega(\mathrm{SW}, S_{[\widehat{\omega}(t)]})\ \big) =$ "T".

(R_2) $\widehat{\omega}(t) \in [\mathrm{SW}]^c$

(R_3) A tomato t is not "sweet".

[And]

We see that the following three are equivalent:

(S$_1$) A truth value of the proposition $\mathsf{P}_\Omega(\mathrm{SW}\bigcap\mathrm{RD}), S_{[\widehat{\omega}(t)]})$ is equal to "T" (which is also denoted by $\mathsf{P}_\Omega(\mathrm{SW}, S_{[\widehat{\omega}(t)]})\bigwedge\mathsf{P}_\Omega(\mathrm{RD}, S_{[\widehat{\omega}(t)]})$).
Thus, $[\mathbf{TV}]\big(\ \mathsf{P}_\Omega(\mathrm{SW}, S_{[\widehat{\omega}(t)]})\bigwedge\mathsf{P}_\Omega(\mathrm{RD}, S_{[\widehat{\omega}(t)]})\ \big) =$ "T".

(S$_2$) $\widehat{\omega}(t) \in \mathrm{SW}\bigcap\mathrm{RD}$

(S$_3$) A tomato t is "sweet" and "red"

[Or]

We see that the following three are equivalent:

(T$_1$) A truth value of the proposition $\mathsf{P}_\Omega(\Theta_\vee(\mathrm{SW}\bigcup\mathrm{RD}), S_{[\widehat{\omega}(t)]})$
(which is also denoted by $\mathsf{P}_\Omega(\mathrm{SW}, S_{[\widehat{\omega}(t)]})\bigvee\mathsf{P}_\Omega(\mathrm{RD}, S_{[\widehat{\omega}(t)]})$) is equal to "T")
That is, $[\mathbf{TV}]\big(\ \mathsf{P}_\Omega(\mathrm{SW}, S_{[\widehat{\omega}(t)]})\bigvee\mathsf{P}_\Omega(\mathrm{RD}, S_{[\widehat{\omega}(t)]})\ \big) =$ "T".

(T$_2$) $\widehat{\omega}(t) \in \mathrm{SW}\bigcup\mathrm{RD}$

(T$_3$) A tomato t is "sweet" or "red"

[Implication]

We see that the following three are equivalent:

(U$_1$) A truth value of the proposition $\mathsf{P}_\Omega(\mathrm{SW}^c\bigcup\mathrm{RD}, S_{[\widehat{\omega}(t)]})$ (which is also denoted by $\mathsf{P}_\Omega(\mathrm{SW}, S_{[\widehat{\omega}(t)]})\longrightarrow\mathsf{P}_\Omega(\mathrm{RP}, S_{[\widehat{\omega}(t)]})$) is equal to "T",
That is, $[\mathbf{TV}]\big(\ \mathsf{P}_\Omega(\mathrm{SW}, S_{[\widehat{\omega}(t)]})\longrightarrow\mathsf{P}_\Omega(\mathrm{RD}, S_{[\widehat{\omega}(t)]})\ \big) =$ "T".

(U$_2$) $\widehat{\omega}(t) \in \mathrm{SW}^c\bigcup\mathrm{RD}$

(U$_3$) a tomato t is not "sweet", or it is "red"

Remark 18. Note that the above is essentially the same as logical operation (Boolean algebra; $[\neg, \wedge, \vee, \rightarrow]$). However, it should be noted that Definition 17 is essential to the above argument.

4.2 Syllogism

Further, consider a proposition P_Ω (RP, $S_{[\widehat{\omega}(t)]}$). That is, we consider that the following three are equivalent:

(V$_1$) $[\mathbf{TV}]\big(\ \mathsf{P}_\Omega(\mathrm{RP}, S_{[\widehat{\omega}(t)]})\ \big) =$ "T".

(V$_2$) $\widehat{\omega}(t) \in \mathrm{RP}$

(V$_3$) A tomato t is "ripe".

Theorem 19. [Syllogism]:
Let t be a tomato, and let $\widehat{\omega}(t)(\in \Omega)$ be the state of t. Assume the followings:

(W$_1$) $[\mathbf{TV}]\big(\ \mathsf{P}_\Omega(\mathrm{SW}, S_{[\widehat{\omega}(t)]})\longrightarrow\mathsf{P}_\Omega(\mathrm{RP}, S_{[\widehat{\omega}(t)]})\ \big) =$ "T".

which is equivalent to

(W_2) $[\widehat{\omega}(t) \notin \mathrm{SW}] \vee [\widehat{\omega}(t) \in \mathrm{RP}]$

(W_3) A tomato t is not "sweet", or it is "ripe".

And further, assume

(W'_1) $[\mathbf{TV}]\big(\ \mathsf{P}_\Omega(\mathrm{RP}, S_{[\widehat{\omega}(t)]}) \longrightarrow \mathsf{P}_\Omega(\mathrm{RD}, S_{[\widehat{\omega}(t)]})\ \big) = \text{"T"}.$

which is equivalent to

(W'_2) $[\widehat{\omega}(t) \notin \mathrm{RP}] \vee [\widehat{\omega}(t) \in \mathrm{RD}]$

(W'_3) A tomato t is not "ripe", or it is "red".

Then the following holds:

(X_1) $[\mathbf{TV}]\big(\ \mathsf{P}_\Omega(\mathrm{SW}, S_{[\widehat{\omega}(t)]}) \longrightarrow \mathsf{P}_\Omega(\mathrm{RD}, S_{[\widehat{\omega}(t)]})\ \big) = \text{"T"}.$

(X_2) $[\widehat{\omega}(t) \notin \mathrm{SW}] \vee [\widehat{\omega}(t) \in \mathrm{RD}]$

(X_3) A tomato t is not "sweet", or it is "red".

[Proof using Definition 17]
A simple calculation shows that

$$
\begin{aligned}
&[\mathrm{SW}^c \cup \mathrm{RP}] \cap [\mathrm{PR}^c \cup \mathrm{RD}] = [\mathrm{SW}^c \cap \mathrm{PR}^c] \cup [\mathrm{SW}^c \cap \mathrm{RD}] \cup [\mathrm{RP} \cap \mathrm{PR}^c] \cup [\mathrm{RP} \cap \mathrm{RD}] \\
=&[\mathrm{SW}^c \cap \mathrm{PR}^c] \cup [\mathrm{SW}^c \cap \mathrm{RD}] \cup [\mathrm{RP} \cap \mathrm{RD}] \subseteq \mathrm{SW}^c \cup \mathrm{RD}
\end{aligned}
$$

Recalling (W_2) and (W'_2), we immediately see (X_2). and thus, (X_1), (X_3).

□

Remark 20. Note that the proof of this theorem (due to Definition 17) is simple compared to Theorem 11 (using Axiom 1 and the linguistic Copenhagen interpretation in Sec. 2.3). That is, in the proof of Theorem 19 we do not need to check the existence and uniqueness of the simultaneous observable $\mathsf{O}^{\mathrm{SW}} \times \mathsf{O}^{\mathrm{RP}} \times \mathsf{O}^{\mathrm{RD}}$,

The following exercise will promote the reader's understanding of "proposition".

Exercise 21. Let $\Gamma \subseteq \Omega$ and $\Lambda \subseteq \Omega$. Then, we have the following question.

- Is the statement "$\Lambda \subseteq \Gamma$" a proposition?

[Answer]:

$$
\begin{aligned}
&\Lambda \subseteq \Gamma \\
\Longleftrightarrow&(\forall \lambda \in \Lambda)[\lambda \in \Gamma] \\
\Longleftrightarrow&(\omega_\lambda)_{\lambda \in \Lambda} \in \mathop{\times}_{\lambda \in \Lambda} \Gamma_\lambda \qquad (\text{where } \omega_\lambda := \lambda\ (\forall \lambda \in \Lambda)),\ \Gamma_\lambda := \Gamma\ (\forall \lambda \in \Lambda)) \\
\Longleftrightarrow&\mathsf{P}_{\Omega^\Lambda}[\mathop{\times}_{\lambda \in \Lambda} \Gamma_\lambda, S_{[(\omega_\lambda)_{\lambda \in \Lambda}]}] \qquad (\text{where } \Omega^\Lambda \text{ is } \Lambda\text{-dimensional product space})
\end{aligned}
$$

Thus, the statement "$\Lambda \subseteq \Gamma$" is a proposition in Ω^Λ. □

4.3 Elementary propositions

Consider the state space Ω, which is finite (or, countable) with a metric d (i.e., $d(\omega_1, \omega_2) = 1$ $(\omega_1 \neq \omega_2)$, $= 0$ $(\omega_1 = \omega_2)$. Further, assume that the Borel measure ν is defined by the point measure, i.e., $\nu(\{\omega\}) = 1$ $(\forall \omega \in \Omega)$.

Definition 22. Let λ be any element of Ω. Putting $\Gamma = \{\lambda\}$, define the proposition $\mathsf{P}_\Omega(\{\lambda\}, S_{[\omega]})$, which is called an elementary proposition.

It is clear that it holds that

- A truth value of the elementary proposition $\mathsf{P}_\Omega(\{\lambda\}, S_{[\omega]})$ is equal to "T"
  $\Longleftrightarrow \lambda = \omega$
- A truth value of the elementary proposition $\mathsf{P}_\Omega(\{\lambda\}, S_{[\omega]})$ is equal to "F"
  $\Longleftrightarrow \lambda \neq \omega$

Theorem 23. Let Γ be a subset of Ω. And let $\omega \in \Omega$. Then we see that

$$\mathsf{P}_\Omega(\Gamma, S_{[\omega]}) = \bigvee_{\lambda \in \Gamma} \mathsf{P}_\Omega(\{\lambda\}, S_{[\omega]})$$

That is, every proposition can be represented by the sum of elementary propositions. This is not trivial since Exercise 21 is not trivial.

[Proof].
We see that

$$\begin{aligned} & \text{The true value of } \mathsf{P}_\Omega(\Gamma, S_{[\omega]}) \text{ is equal to "T"} \\ \Longleftrightarrow & \omega \in \Gamma \\ \Longleftrightarrow & \exists \lambda (\in \Gamma)[\omega \in \{\lambda\}] \\ \Longleftrightarrow & \text{The true value of } \textstyle\bigvee_{\lambda \in \Gamma} \mathsf{P}_\Omega(\{\lambda\}, S_{[\omega]}) \text{ is equal to "T"} \end{aligned}$$

□

Remark 24. I believe that the above is the main assertion in Wittgenstein's picture theory. However many readers may not consider the above theorem to be particularly important. I have the same view. In fact, this theorem holds only in special cases such as Ω is a finite (or more generally, Ω has a discrete topology). Thus, as mentioned in Table 16, I think that Wittgenstein's language is esentially the same as naive set theory ($\approx$ Cantor's set theory, *cf.* ref. [4]).

5 Supplement: Syllogism does not always hold in quantum systems

I was devoted to classical systems in sections 3 and 4, in which logic holds. However, as mentioned in Remark 14, the arguments in Sec. 3 are invalid in quantum systems. Further, I can say as follows.

(Y) syllogism does not always hold in quantum systems

The proof is a slightly modification of the argument about EPR-paradox and Heisenberg's uncertainty relation (*cf* [7], [3]). The (Y) is shown in Sec. 4 of the following paper:

- Ref, [27]: Ishikawa, S. (2020) Wittgenstein's picture theory in the quantum mechanical worldview JQIS,Vol. 10, No.4 ,104-125 DOI:10.4236/jqis.2020.104007 (`https://www.scirp.org/journal/paperinformation.aspx?paperid=106233`)

Thus, in this paper, the proof of the (Y) is omitted.

6 Supplement: Almost all philosophical unsolved problems are solved in QL

6.1 Why are almost all unsolved philosophical problems solved in QL?

In the case of mathematics, it's easy to tell whether an unsolved problem or not. Thus, everyone who solves the famous mathematical unsolved problems (e.g. Fermat's problem, Poincará's problem) is, of course, praised.

On the other hands, the meaning of unsolved problems in philosophy is not simple. For example, speaking of "Wittgenstein's paradox" or "Hume's induction problem", some people are impressed with them even if they are non-sense as philosophical problems. Also, there is no consensus among philosophers whether Zeno's paradox is a paradox. Thus, the meaning of unsolved problems in philosophy is not simple.

I think, from the scientific point of view, that there is only one unsolved philosophical problem:

- Complete the scientific dualistic idealism!

This is realized by QL as I have shown in Figure 0 before. And so, the other philosophical unsolved problems concerning dualistic idealism, which stem from an inadequate understanding of our dualistic idealism, should be clarified in QL.

Thus, there is a reason to consider that QL is located as shown in the following figure (essentially the same as Figure 0):

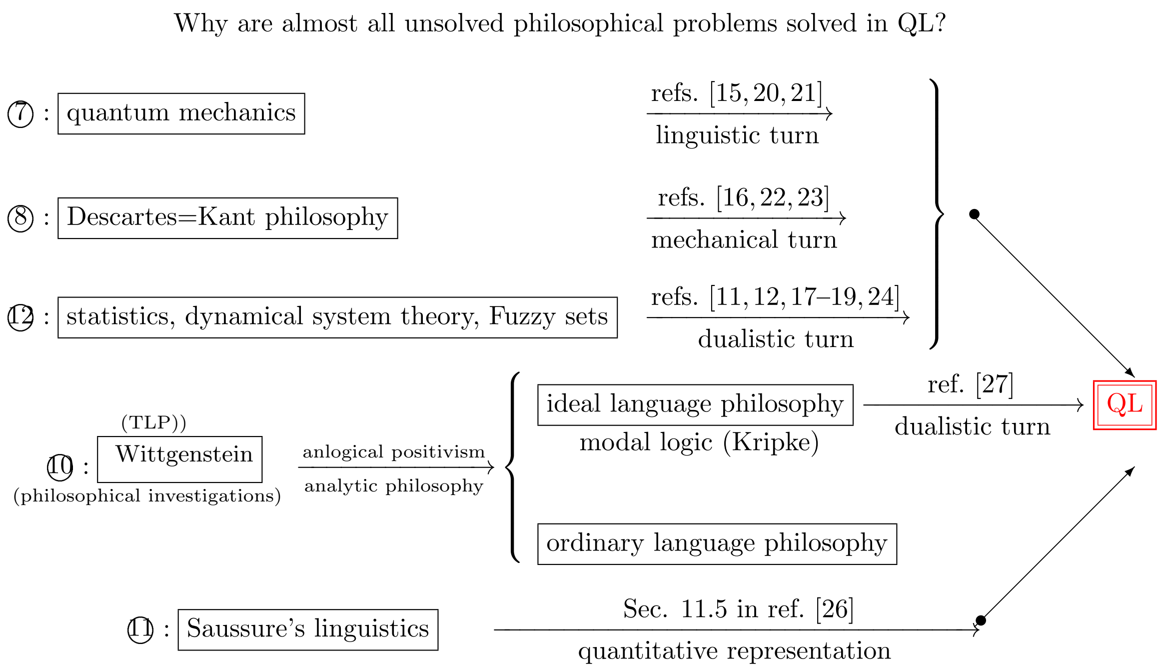


Figure 5 (≈ [b] in Sec. 1 ⊂ Figure 0): QL is the scientific unified theory of five (⑦⑧⑫⑩⑪)
(QL bridges among ⑦⑧⑫⑩⑪, thus almost all unsolved philosophical problems are solved in QL)

6.2 List of quantum linguistic clarifications of open problems

If Figure 5 is true, we can expect that QL has a great power to solve a lot of unsolved problems in quantum mechanics, Descartes=Kant epistemology, statistics, and analytic philosophy. This will be shown as follows.

We think that

($\sharp$) **many unsolved problems raised in the 2500 year history of dualistic idealism can be clarified within the framework of quantum languages.**

My results concerning quantum language are summarized in the following two texts

$(\flat)$ $\begin{cases} (\flat_1)\text{: Ref. [26]: History of western philosophy from the quantum theoretical point of view; [Ver. 3]} \\ (\flat_2)\text{: Ref. [25]: The linguistic Copenhagen interpretation of quantum mechanics: Quantum language [Ver 5]} \end{cases}$

Remark 25. I think that all unsolved problems in $(\flat_1)$ and $(\flat_2)$ have not been solved. That is because they cannot be solved without the scientific dualistic idearism (i.e., quantum language). Some of our next solutions may be disputed (especially those related to Plato's philosophy may have been too aggressive). I hope readers will examine it further. However, even if there are some deficiencies, it's not serious. Because my purpose is to assert the (Z_3) in Sec. 7 (Conclusion).

($\flat_1$) **The list of my answers for scientific unsolved problems:**
ref. [26]; History of Western philosophy in the mechanical worldview Research Report, Dept. Math. Keio University, KSTS/RR-20/001 (2020); 473 p
(`http://www.math.keio.ac.jp/academic/research_pdf/report/2020/20001.pdf`)

- Has philosophy progressed? (the answer is presented throughout [26])
- What is probability (or, measurement, causality) ? (*cf.* Sec. 1.1.1 in [26])
- Zeno paradox (Flying arrow), (*cf.* Sec. 2.4.2 in [26])
- Zeno paradox (Achilles and a tortoise), (*cf.* Sec. 2.4.3 in [26])
- the measurement theoretical understanding of Plato's allegory of the sum , (*cf.* Sec. 3.3.2 in [26])
- Plato's Idea theory≈Zadeh's fuzzy theory(ref. [35])≈Sausuure's linguistic theory (*cf.* Sec. 3.5.3 in [26])
- Syllogism holds in classical systems, but not in quantum systems (*cf.* Sec. 4.3.3 in [26])
- Only the present exists (*cf.* Sec. 6.1.2 in [26])
- What is the problem of universals? (*cf.* Sec. 6.5.1 in [26])
- What is Geocentrism vs. Heliocentrism? After all, the worldviewism (*cf.* Sec. 7.4.2 in [26])
- Two (scientific or non-scientific) interpretations of I think, therefore I am .(*cf.* Sec. 8.2.2 in [26])
- Leibniz-Clark correspondence (i.e., what is space-time?), (*cf.* Sec. 9.3 in [26])
- The problem of qualia (*cf.* Sec. 9.5.1 in [26])
- Brain in a vat argument (*cf.* Sec. 9.5.2 in [26])
- The solution of Hume's problem of induction (*cf.* Sec. 9.7.1 in [26])
- Grue paradox cannot be represented in quantum language (*cf.* Sec. 9.7.2 in [26])
- What is causality? (*cf.* Sec. 10.3 in [26])
- What is Peirce's abduction? (*cf.* Sec. 11.3.1 in [26])
- Five-minute hypothesis (*cf.* Sec. 11.4.1 in [26])
- McTaggart's paradox (*cf.* Sec. 11.4.2 in [26])
- quantitative representation of "Signifier" and "signified" (ref. [31]) (*cf.* Sec. 11.5.3 in [26])
- A scientific understanding of Wittgenstein's picture theory (*cf.* Sec. 11.6.2 in [26])
- Wittgenstein's paradox (*cf.* Sec. 11.6.3 in [26])
- Flagpole problem, (*cf.* Sec. 11.7.1 in [26])
- Hempel's raven paradox (*cf.* Sec. 11.8 in [26])
- the mind-body problem (i.e., How are mind and body connected?), (*cf.* Sec. 11.9.4 in [26])

($\sharp$) Also, for the solutions of unsolved problems in quantum mechanics, statistical mechanics, statistics and probability theory, see ref. [25]). Particularly, I think that the following three are important in physics:

- the discovery of Heisenberg's uncertainty relation (Ref. [7], or Sec. 4.3 in ref. [25])

- The clarification of the projection postulate (i.e., the wavefunction collapse) (ref. [20] , or Sec. 11.2 in ref. [25])
- The measurement theoretical characterizations of equilibrium statistical mechanics (Ref. [19], or Chap.17 in ref. [25])

($\flat_2$) **The list of my answers for scientific unsolved problems:**
ref. [25]; Linguistic Copenhagen interpretation of quantum mechanics; Quantum Language [Ver 5], Research Report, Dept. Math. Keio University, KSTS/RR-19/003 (2019); 473 p
(`http://www.math.keio.ac.jp/academic/research_pdf/report/2019/19003.pdf`)

- Is QL worth calling a language of science? (the answer is presented throughout [25])
- Kolmogorov's extension theorem in quantum language (Sec.4.1 in ref. [25]) (Sec.4.1 in ref. [25])
- The law of large numbers in quantum language (Sec.4.2 in ref. [25])
- the true discovery of Heisenberg's uncertainty relation (Sec. 4.3 in ref. [25])
- Bell's inequality holds in both classical and quantum systems (Sec. 4.5.2 in ref. [25])
- Measurement theoretical formulation of measurement, inference, control (Sec. 5.2 in ref. [25])
- Monty-Hall problem in quantum language (non-bayesian approach) (Sec.5.5 in ref. [25])
- Two envelope problem in quantum language (non-bayesian approach) (Sec.5.6 in ref. [25])
- Confidence interval and statistical hypothesis test (Chapter 6 in ref. [25])
- Analysis of variance (Chapter 7 in ref. [25])
- Syllogism holds in classical systems, but not in quantum systems (Sec.8.6 and Sec.8.7 in ref. [25])
- Mixed measurement theory (Bayesian measurement theory) (Chap. 9 in ref. [25])
- The measurement theoretical characterization of the wave-function collapse (= projection pustulate) (Sec.11.2 in ref. [25])
- The measurement theoretical characterizations of de Broglie's paradox, quantum Zeno effect, Schrödinger cat, Wigner's friend, Wheeler's delayed choice experiment, Hardy Paradox, quantum eraser (Sec.11.3∼Sec.11.8 in ref. [25])
- The measurement theoretical characterizations of double-slit experiment, Wilson cloud chamber (Sec.12.2, Sec.12.3 in ref. [25])
- The measurement theoretical characterizations of regression analysis (Sec.13.2 in ref. [25])
- The measurement theoretical characterizations of Brownian motion, Zeno's paradox (Sec.14.2 , Sec.14.4 in ref. [25])
- The measurement theoretical characterizations of least-squares method (Chap.15 in ref. [25])
- The measurement theoretical characterizations of Kalman filter (Chap.16 in ref. [25])
- The measurement theoretical characterizations of equilibrium statistical mechanics (Chap.17 in ref. [25])
- The measurement theoretical characterizations of psychological tests (Chap.18 in ref. [25])
- The measurement theoretical characterizations of belief (Chap.19 in ref. [25])

- The mathematical foundation of science (Hempel's raven paradox) (Chap.20 in ref. [25])

7 Conclusion

In Sec. 3, I show that

(Z_1) a class of binary projective measurements in classical QL has a logical structure.

This and Sec. 4 say that

(Z_2) Wittgenstein's language (i.e., the language that he supposed, but didn't define in TLP) is realized by a class of binary projective measurements in classical QL

That is, we see:

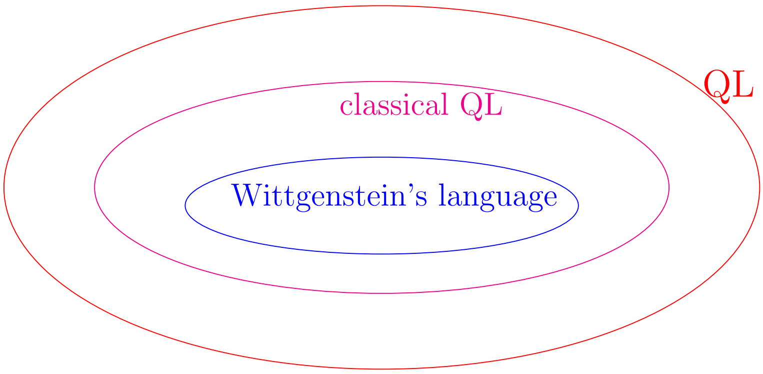


Figure 6 (=Figure 1): Wittgenstein's language(in which scientific logic holds)
= a class of binary projective measurements in classical quantum language

Since classical QL is one of the most important languages of science (such as Newton mechanics, the theory of relativity, statistics, etc. (e.g., see Figure 0 in Sec. 1 and Sec. 6), the (Z_2)(= Figure 6) is the most fundamental in science (*cf.* ($\sharp$) in Remark 15).

Note that logic is derived from Axiom 1 in QL. On the other hand, it should be noted that logic is not derived the principle of Newtonian mechanics. In this sense, I think that Wittgenstein's picture theory is original and not trivial. As seen in [d] in Sec. 1, Wittgenstein was the first philosopher to try to make a clear distinction between mathematical logic and scientific logic. Therefore, I think that his picture theory should be praised not only from a philosophical point of view but also from a scientific point of view.

Lastly, I add the following:

(Z_3) Wittgenstein claimed to have solved all the problems of philosophy in his TLP. Thus, if my proposal (i.e., QL) is an alternative to his picture theory, almost all philosophical problems must be solved under QL. I think that this was completely realized in the lists ($\flat_1$) and ($\flat_2$) in Sec. 6.

I hope that many readers will examine my proposal from various aspects. For more information on my results, see the homepage: (`http://www.math.keio.ac.jp/~ishikawa/indexe.html`).

.

References

[1] Conant, J. and Diamond,C. (2004) On Reading the Tractatus Resolutely,: "On Reading the Tractatus Resolutely: to Meredith Williams and Peter Sullivan" in Max Kölbel and Bernhard Weiss, The Lasting Significance of Wittgenstein's Philosophy, Routledge ,

[2] Davies, E.B.(1976) Quantum theory of open systems, Academic Press

[3] Einstein, A., Podolosky, B. and Rosen, N. (1935) Can quantum-mechanical description of reality be considered completely? Physical Review Ser 2(47) 777–780

[4] Halmos, P. R.(1974) Naive set theory, Princeton, NJ: D. Van Nostrand Company. Reprinted by Springer-Verlag, New York,

[5] Holevo, A.S. (1982) Probabilistic and statistical aspects of quantum theory, North-Holland publishing company (1982)

[6] Howard, D.(2004) Who invented the "Copenhagen Interpretation? A study in mythology, Philosophy of Science, 71 669-682

[7] S. Ishikawa, (1991) Uncertainty relation in simultaneous measurements for arbitrary observables, Rep. Math. Phys., **9**, 257-273,
doi: 10.1016/0034-4877(91)90046-P

[8] Ishikawa,S. (1997) Fuzzy inferences by algebraic method, Fuzzy Sets and Systems 87, 181–200 doi:10.1016/S0165-0114(96)00035-8

[9] Ishikawa,S. (1997) A Quantum Mechanical Approach to Fuzzy Theory, Fuzzy Sets and Systems, Vol. 90, No. 3, 277-306, doi: 10.1016/S0165-0114(96)00114-5

[10] Ishikawa,S. (1998) Fuzzy logic in measurements, Fuzzy Sets and Systems, Vol. 100, 291-300, doi:10.1016/S0165-0114(97)00154-1

[11] Ishikawa, S. et al. (1999) A Dynamical system theoretical approach to Newtonian mechanics, Far east Journal of dynamical systems, Vol. 1, No. 1, 1-24,
(`http://www.pphmj.com/abstract/94.htm`)

[12] Ishikawa,S. (2000) *Statistics in measurements*, Fuzzy sets and systems, Vol. 116, No. 2, 141-154,
doi:10.1016/S0165-0114(98)00280-2

[13] Ishikawa, S. (2006) Mathematical Foundations of Measurement Theory, Keio University Press Inc. 335pages,
(`http://www.keio-up.co.jp/kup/mfomt/`)

[14] Ishikawa, S. (2008) Dynamical systems, measurements, quantitative language and Zeno's paradoxes, Far east Journal of dynamical systems, Vol. 10, No. 3, 277-292,
(`http://www.pphmj.com/abstract/3595.htm`)

[15] Ishikawa, S. (2011) A New Interpretation of Quantum Mechanics, Journal of quantum information science, Vol. 1, No. 2, 35-42, doi: 10.4236/jqis.2011.12005
(`http://www.scirp.org/journal/PaperInformation.aspx?paperID=7610`)

[16] Ishikawa, S. (2012) Quantum Mechanics and the Philosophy of Language: Reconsideration of traditional philosophies, Journal of quantum information science, Vol. 2, No. 1, 2-9,
doi: 10.4236/jqis.2012.21002
(`http://www.scirp.org/journal/PaperInformation.aspx?paperID=18194`)

[17] Ishikawa, S. (2012) A Measurement Theoretical Foundation of Statistics, Applied Mathematics, Vol. 3, No. 3, 283-292, doi: 10.4236/am.2012.33044
(`http://www.scirp.org/journal/PaperInformation.aspx?paperID=18109&`)

[18] Ishikawa, S. (2012) Monty Hall Problem and the Principle of Equal Probability in Measurement Theory, Applied Mathematics, Vol. 3 No. 7, pp. 788-794, doi: 10.4236/am.2012.37117.
(`http://www.scirp.org/journal/PaperInformation.aspx?PaperID=19884`)

[19] Ishikawa, S. (2012) Ergodic Hypothesis and Equilibrium Statistical Mechanics in the Quantum Mechanical World View, World Journal of Mechanics, Vol. 2, No. 2, pp. 125-130. doi: 10.4236/wim.2012.22014.
(`https://www.scirp.org/journal/paperinformation.aspx?paperid=18861`)

[20] Ishikawa, S. (2015) Linguistic interpretation of quantum mechanics; Projection Postulate, Journal of quantum information science, Vol. 5, No.4 , 150-155, DOI: 10.4236/jqis.2015.54017
(http://www.scirp.org/Journal/PaperInformation.aspx?PaperID=62464)

[21] Ishikawa,S., (2017) Bell's inequality should be reconsidered in quantum language , Journal of quantum information science, Vol. 7, No.4 , 140-154,, DOI: 10.4236/jqis.2017.74011
(http://www.scirp.org/Journal/PaperInformation.aspx?PaperID=80813)

[22] Ishikawa,S., (2017) A Final solution to mind-body problem by quantum language, Journal of quantum information science, Vol. 7, No.2 , 48-56, DOI: 10.4236/jqis.2017.72005 (http://www.scirp.org/Journal/PaperInformation.aspx?PaperID=76391)

[23] Ishikawa,S., (2018) Leibniz-Clarke correspondence, Brain in a vat, Five-minute hypothesis, McTaggart's paradox, etc. are clarified in quantum language, Open Journal of Philosophy, Vol. 8, No.5 , 466-480, DOI: 10.4236/ojpp.2018.85032
(https://www.scirp.org/Journal/PaperInformation.aspx?PaperID=87862)

[24] Ishikawa, S., (2019) Philosophy of science for scientists; The probabilistic interpretation of science, Journal of quantum information science, Vol. 9, No.3 , 140-154, DOI: 10.4236/jqis.2019.93007
(https://www.scirp.org/Journal/paperinformation.aspx?paperid=95447)

[25] Ishikawa, S. (2019) Linguistic Copenhagen interpretation of quantum mechanics; Quantum Language, Research Report, Dept. Math. Keio University, [Ver.5]; KSTS/RR-19/003 (2019); 473 p (http://www.math.keio.ac.jp/academic/research_pdf/report/2019/19003.pdf)

[26] Ishikawa, S. (2020) History of Western Philosophy from the quantum theoretical point of view, Research Report (Department of mathematics, Keio university, Yokohama)
[Ver.3] (KSTS-RR-20/001, 296 pages)
(http://www.math.keio.ac.jp/academic/research_pdf/report/2020/20001.pdf)

[27] Ishikawa, S., (2020) Wittgenstein's picture theory in the quantum mechanical worldview, Journal of quantum information science, Vol. 10, No.4 ,104-125 DOI:10.4236/jqis.2020.104007
(https://www.scirp.org/journal/paperinformation.aspx?paperid=106233)

[28] Kremer, Michael (2001) The purpose of Tractarian Nonsense, Noûs, Volume35, Issue 1, Pages 39-73

[29] von Neumann, J. (1932) Mathematical foundations of quantum mechanics Springer Verlag, Berlin

[30] Sakai, S. (1971) C^*-algebras and W^*-algebras, Ergebnisse der Mathematik und ihrer Grenzgebiete (Band 60), Springer-Verlag, Berlin, Heidelberg, New York

[31] Saussure, Ferdinand de(1916) Cours de linguistique générale, publié par Charles BALLY et Albert SÉCHEHAYE, avec la collaboration d ' Albert RIEDLINGER, Payot, Lausanne et Paris

[32] Wittgenstein, L (1921) Tractatus Logico philosophicus, Oxford: Routledge and Kegan Paul,

[33] Wittgenstein, L (1953) Philosophical Investigations, Blackwell Publishing. ISBN 0-631-23127-7

[34] Yosida, K. (1980) Functional analysis, Springer-Verlag (Sixth Edition)

[35] Zadeh, L.A. *Fuzzy sets, Information and Control,* Vol. 8, 338-353 1965