THE BIG FOUR

THEIR INTERDEPENDENCE AND LIMITATIONS

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ABSTRACT

Four intuitions are recurrent and influential in theories about conditionals: the Ramsey's test, the Adams' Thesis, the Equation, and the robustness requirement. For simplicity's sake, I call these intuitions 'the big four'. My aim is to show that: (1) the big four are interdependent; (2) they express our inferential dispositions to employ a conditional on a *modus ponens;* (3) the disposition to employ conditionals on a *modus ponens* doesn't have the epistemic significance that is usually attributed to it, since the acceptability or truth conditions of a conditional is not necessarily associated with its employability on a *modus ponens*.

1. THE BIG FOUR ARE A TIGHT BUNCH

The following principles have been influential in conditional theory:

Ramsey's Test (RT): we accept $A \rightarrow B$[1] if, and only if, after the hypothetical addition of A to our belief system, and after making the required adjustments to maintain consistency without modifying the hypothetical belief in A, we would be willing to accept B[2].

Adams' Thesis (AT): $\mathrm{As}(A \rightarrow B) = \Pr(B|A) = \Pr(A\&B)|\Pr(A)$, provided that $\Pr(A) > 0$[3].

The Equation (TE): $\Pr(A \rightarrow B) = \Pr(B|A) = \Pr(A\&B)|\Pr(A)$, provided that $\Pr(A) > 0$[4].

Robustness Requirement (RR): $A \rightarrow B$ is acceptable when $A \supset B$ is robust with respect to A, i.e., when $\Pr(A \supset B)$ is high and would remain high after learning that A[5].

AT should be read as 'The assertibility of $A \rightarrow B$ is measured by the conditional probability of B given A'. It is arguable that AT and TE are synonymous, since the degree in which we are justified in asserting a proposition P is measured by the probability we attribute to P. In fact, in his later writings Adams presented his thesis in terms of probability rather than assertibility[6]. The only reason why AT and TE are taken to be different is that Adams didn't believe that conditionals have truth-conditions. The problem, however, is that AT becomes unintelligible

[1] I will use '$\rightarrow$' for indicative conditionals, '$\supset$' for the material implication and the capital letters A, B, C…. for propositional variables. The symbols and variables quoted will be modified to ensure that the notation remains uniform.

[2] Stalnaker (1968: 102). This is the modified and more widely discussed formulation of the test. The original idea and formulation can be found in Ramsey (1929: 143).

[3] Adams (1965: 172).

[4] Jeffrey (1964: 702–3).

[5] Jackson (1987: 28). Jackson's notion of robustness is inspired on Skyrms' notion of resilience. See Skyrms (1975).

[6] Adams (1975; 1988).

in case conditionals have no truth-value. If the probability of $A \rightarrow B$ is not the probability of $A \rightarrow B$ being true, what else should it be?

RR states that $A \rightarrow B$ is acceptable when $A \supset B$ is robust with respect to A, i.e., when $\Pr(A \supset B)$ is high and would remain high after learning that A. But since $A \supset B$ is logically equivalent to $\neg A \vee B$, the probability of $A \supset B$ given A is equal to the probability of $\neg A \vee B$ given A, which on its turn is tantamount to the probability of B given A[7]. Thus, RR is satisfied when AT is satisfied. Since we already accepted that AT and TE are one and the same, RR, AT and TE are equivalent.

If the probability of $A \supset B$ is high and would remain high after learning that A, I would be willing to infer B after learning that A, but this is precisely what would happen if we would be willing to infer B after the hypothetical addition of A to our belief system. Thus, RR and RT are linked.

The big four share the same counter-intuitive aspects. Suppose '*A*' and '*B*' stand for necessarily true propositions, let's say, 'two plus two equals four' and 'five is an odd number' respectively. In such scenario, the probability I assign to B given A is high simply in virtue of the probability of B being high. The fact that the propositions lack mutual relevance does not affect the conditional probability. Since $\Pr(B|A)$ is high, the probability of 'If two plus two equals four, then five is an odd number' must be high according to the equation, and acceptable according to AT. This also implies that RT is satisfied: if I add 'two plus two equals four' to my belief system, I would be willing to infer that 'five is an odd number' simply because I have already accepted that five is an odd number. In this case, RR is also satisfied, since the probability that 'five is an odd number' is high and would remain high after learning that 'two plus two equals four'. In this case, I would be willing to employ 'If two plus two equals four, then five is an odd number' in a *modus ponens*, trivially. If learn that 'two plus two equals four', I cannot avoid inferring that 'five is an odd number', since I already accept that 'five is an odd number'.

The big four also share the same limitations. AT and TE don't provide any answers when A is necessarily false, since the conditional probability of B given A is undetermined[8]. RT can't be applied when A is necessarily false, because I cannot add A hypothetically to my belief system. Finally, RR can't be satisfied when A is necessarily false, because I don't know if the probability of B would remain higher after learning that A because I can't learn that A to begin with. Not surprisingly, I can't employ $A \rightarrow B$ on a *modus ponens*, because A is always false.

One could argue that there is a tension between RT and RR since there is a difference between the degree of belief in B given the hypothetical assumption that A, and the degree of belief in B given the belief that A. Suppose that Mary does not have a high degree of belief that God exists on the hypothesis that her daughter is dangerously ill, but she would become a devote Cristian, if she were to learn that her daughter was dangerously ill. In this example the degree of belief in B given the hypothesis that A is low, so it fails at the RT, but the degree of belief in B given the belief that A is high, i.e., it satisfies the RR[9].

One explanation for this phenomenon is that while there could be a difference between what the epistemic agent expects to be her beliefs given the hypothetical assumption of a fact and her actual beliefs given the occurrence of facts, after the occurrence of the fact the hypothetical conditional probability must catch up with the actual conditional probability, on

[7] Jackson (1987: 28).

[8] Of course, this doesn't mean that we can't modify the principle to surpass these limitations. Ernest Adams himself conceded that the probability of a conditional should be 1 in case the probability of its antecedent is 0. See Adams (1965: 185).

[9] The example is suggested by Stalnaker (1984: 104).

pain of incoherence. In other words, any inconsistencies between RT and RR are naturally dissolved.

2. THE CONNECTION WITH MODUS PONENS

The connection between the big four and *modus ponens* is made obvious by RR, which is built on Jackson's reasoning about the relation between assertion and robustness [10]. Every proposition is robust in relation to itself, since the probability of a proposition will remain high after its own discovery. Thus, the relevant notion of robustness is the one that is relative between two or more propositions. In some cases, the context will indicate in relation to which propositions our assertion is robust, but the context is not always enough. So, Jackson insisted, we need conventions to signal in relation to which propositions are our assertions relatively robust. That's when conditionals come in. They are conventional symbols that signalise the robustness of one proposition, in this case, the consequent, in relation to another, the antecedent:

> What is the point of signalling the robustness of $A \supset B$ with respect to A? The answer lies in the importance of being able to use Modus Ponens. Although '$A \supset B$; A; therefore, B' is certainly valid, there is a difficulty about using it in practice. Suppose my evidence makes $A \supset B$ highly probable, but that I have no evidence concerning A. B is of interest to me, so I set about finding evidence for A if l can. The difficulty is that finding evidence that makes A highly probable is not enough in itself for me to conclude B by Modus Ponens. For the evidence that makes A probable may make $A \supset B$ improbable. Indeed, it is easy to prove from the calculus that, except in special cases of extreme probability, $\Pr(A \supset B|A) < \Pr(A \supset B)$. Normally, on learning A I must *lower* the probability I give ($A \supset B$), so endangering the inference to B. It is thus of particular interest whether or not $A \supset B$'s high probability would be unduly diminished by learning A; that is, it is important whether or not $A \supset B$ is robust with respect to A. In sum, we must distinguish the validity of Modus Ponens from its utility in a situation where I believe $A \supset B$ but do not know A. The robustness of $A \supset B$ relative to A is what is needed to ensure the utility of Modus Ponens in such situations[11].

In other words, we are not only interested in whether $A \rightarrow B$ is highly probable or not, but we are also interested in knowing whether they are inferentially useful or not, and this will happen when $A \supset B$ is robust with respect to A, which also ensures the high conditional probability of B given A, which satisfies AT, TE and RT. Thus, the big four implies that $A \rightarrow B$ is acceptable (or true) when $A \rightarrow B$ is employable in a *modus ponens*.

That someone may be led into thinking that *modus ponens* is central to our theories about conditionals is understandable given the fact that the grammatical form of conditional sentences invite inferences is one of the marked differences of conditionals in relation to categorical propositions. In logic textbooks, the standard example of a conditional sentence is 'If A, then B', and other less usual examples include 'B because A', 'B given A', 'There is no B, unless there is A' and 'Since A, B'. All these forms suggest an inferential passage from A to B. It is also usual to name the subordinate clause, A, as 'antecedent', and the main clause, B, as 'consequent', which naturally predispose us to think that B comes after A. The very name of the sentence in natural language, i.e., 'conditional', suggests that A must be a condition for B, which makes us think that B must be inferable from A. These sentences were also called 'hypotheticals' in the past. The name may be now in disuse, but it was also motivated by the directional and grammatical form of conditional sentences, since the term 'if' apparently

[10] Jackson (1987: 26).
[11] Jackson (1987: 29).

indicates that the antecedent is assumed as a hypothesis used in an inference directed to the consequent.

This inferential passage is also suggested by the symbols used to represent the logical form of conditionals. This happens because our conventions regarding the logical form of conditionals are already imbedded with grammatical induced prejudices, as is attested by the fact that logical symbols used to represent conditional operators ('→', '⊃', '⇒', etc.) point in a direction from A to B.

It is natural then (even though it is ultimately mistaken) to assume that they provide a reliable indicator of conditionals' truth conditions or acceptability conditions.

3. THE LIMITATIONS OF THE BIG FOUR

There is one puzzle that demonstrates in a compelling manner why the acceptance or the truth conditions of a conditional cannot be determined by its employability on a *modus ponens*. RR states that $A \rightarrow B$ is acceptable when $\Pr(A \supset B)$ is high and would remain high after learning that A, which also implies $\Pr(B)$ is high and would remain high after learning that A. Here is another way to define RR: $A \rightarrow B$ is acceptable when $\Pr(B)$ and $\Pr(B|A)$ are both high and close to each other. The following example seems to show these definitions are independent. Suppose I'm certain that I would never know that my wife is deceiving me; she is too smart to get caught. However, because I trust her, I don't believe she is deceiving me. In this case, the conditional probability that I don't know that she is deceiving me given that she is deceiving me is high. Nevertheless, I would not infer that I don't know that she is deceiving me given that I found out that she is deceiving me. In this case, the conditional 'If my wife is deceiving him, I would never know' is not acceptable according to the first definition of robustness, but it is acceptable according to the second definition of robustness[12].

The idea that motivates this counter-example is that in some cases B can be robust in relation to A, but not in relation to the acceptance of A. This is important for our purposes because it shows that a conditional can be acceptable even though it is not employable in a *modus ponens*. Bennett attempts to explain this counter-example by arguing that the speaker will not be willing to employ the conditional in a *modus ponens* but believes that any other person that accepts the conditional would be willing to employ it on a *modus ponens*[13]. This explanation, however, is *ad hoc*.

One immediate answer to this counter-example is that we should stick to the second definition of robustness. After all, since $\Pr(B|A)$ is defined as $\Pr(A\&B)|\Pr(A)$, and given that it is possible to determine the value of the last equation without assuming the truth of A, we could attribute a high value to $\Pr(A\&B)|\Pr(A)$ in the example mentioned above even if the $\Pr(B)$ is zero after learning that A[14]. But this solution seems to be a desperate movie. The reason why the conditional probability is intuitively relevant to our understanding of conditionals is due to the apparent relation between the acceptance of $A \rightarrow B$ and our willingness to infer B given the assumption of A, and not the quotient of $A\&B$ given the probability of A[15].

There is also always a case to be made that conditional probability is primitive[16], i.e., that $\Pr(B|A)$ can't be defined as $\Pr(A\&B)|\Pr(A)$. Intuitively, I can attribute a high probability to B

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[12] The example is from Van Fraassen (1980: 503).
[13] Bennett (2003: 55).
[14] Lewis (1986: 155–6).
[15] Notice that Jackson (2006: 15; 2008: 462) abandoned AT since then, accusing it of being motivated by an illusory intuitive probability we tend to associate with conditionals. This error theory mindset was already present, although in less explicit form, in Jackson (1987: 38-40; 1998).
[16] I'm not alone. Hájek (2003: 315) presents a long list of proponents of this view.

given the assumption of A even if I don't know the probability of A. For instance, I can attribute a high probability to the ceremony being cancelled tomorrow given the assumption that there will be a heavy rainfall, even if I don't know the probability that there will be a heavy rainfall tomorrow. Thus, even if the second definition of robustness seem to work in this case, it will bring even more problems than it solves.

4. THE BIG FOUR REAL SIGNIFICANCE

The attempt to maintain the relevance of the conditional probability by reducing it to something else only clouds the issue. In fact, there is a simpler explanation for this case: our inferential dispositions are determined by the reasons that lead us to accept the conditional. This explains why the big four result in so many false negatives. For example, some conditionals are accepted only when we are willing to employ the conditional in a *modus tollens* inference, instead of a *modus ponens*. When I accept 'If John's speaking the truth, I'm a Dutchman', I am not willing to infer that I am a Dutchman if it turns out that John was telling the truth: the conditional was asserted under the assumption that the antecedent is false. In this case, I accept $A \rightarrow B$ only when I'm willing to infer $\neg A$ from $\neg B$ in a *modus tollens* inference. Now with this mindset we can understand what really happened in the counter-example. When I accept the conditional 'If my wife is deceiving me, I will never know', I am not willing to infer that I will never know that she is deceiving me if I found out that she is deceiving me after all. In this case, I have good reasons to accept the conditional, but they prevent me from employing the conditional in a *modus ponens* or in a *modus tollens*. This shows that the assumption that *modus ponens* employability determines the truth-conditions of conditionals confuses our inferential dispositions, an epistemic phenomenon indirectly related to our reasons to accept a conditional, with its truth conditions, which is a logical phenomenon that is independent of our inferential dispositions.

Thus, the big four are acceptable only if they are interpreted as follows: a conditional is acceptable if, and only if, our disposition to employ it in an inference is compatible with the reasons that lead us to accept it in the first place. The big four are not particularly impressive in this new form. It is trivially true that in most cases our inferential dispositions are determined by our reasons to accept a conditional, but that is not a universal principle, as the deceiving wife case makes clear.

5. CONCLUDING REMARKS

The essence of the big four lies in the *modus ponens* employability requirement, but this feature is too superficial to work as the foundation of a general theory. Perhaps more importantly, we should consider what are the consequences of the present mindset for theories that are either based or are heavily influenced by the big four. There are reasons to think that this is the case. Let's consider the suppositional view, according to which conditionals that have assertions in their consequents are not propositions, but acts of conditional assertion. The idea is that there is no assertion of $A \rightarrow B$, but an assertion of B given the assumption of A[17]. This hypothesis has both RT and AT as obvious sources of inspiration, so it is not surprising that the suppositional view faces the same limitations. When I assert the conditional 'If John's speaking the truth, I'm a Dutchman', I'm not asserting that I am a Dutchman given the assumption that John is speaking the truth. Another example involves the possible world semantics, according

[17] See, for example, Edgington (1986; 1995).

to which $A \rightarrow B$ is true if, and only if, B is true in the closest A-world[18]. They are also motivated by the RT[19]. It is no wonder that they are also inadequate to explain the truth conditions of Dutch conditionals since in the closest world in which John's speaking the truth, I'm not a Dutchman. Skyrms (2013) also argued that AT and the possible world semantics are associated, which should be another reason to doubt these theories. It seems that many influential theories are compromised by a *modus ponenscentric* view of conditionals associated with the big four.

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[18] Stalnaker (1968: 102). David Lewis (1973) offers a distinct version where $A \rightarrow B$ is true if, and only if, in every possible A-world that is as closest to the actual world as the truth of A allows, B is true. There are many other possible worlds semantics inspired on the same idea—see Davis (1979). In any case, our core objection holds for any possible world semantics employed, since it is directed against the same pre-theoretic intuitions that motivate the different versions.

[19] Stalnaker is pretty clear on this: 'The concept of a *possible world* is just what we need to make this transition [from belief conditions of the Ramsey's test to the truth conditions of a semantics], since a possible world is the ontological analogue of a stock of hypothetical beliefs' (Stalnaker, 1968: 102).