# Interval-based Dynamics of Loose Talk

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**Abstract**

Carter (2021) argued that while most simple positive numerical sentences are literally false, they can communicate true contents because relevance has a weakening effect on their literal contents. This paper presents a challenge for his account by considering entailments between the imprecise contents of numerical sentences and the imprecise contents of comparatives. I argue that while Carter's weakening mechanism can generate the imprecise contents of plain comparatives such as 'A is taller than B', it cannot generate the imprecise contents of comparatives that quantify the difference between their arguments, such as 'A is exactly n times as tall as B'. I then propose an alternative theory on which intervals serve as both thick degrees on a scale and denotations of numerical expressions. I argue that the alternative theory can account for the imprecise contents of both forms of comparatives as well as the data that motivate Carter's theory.

## 1 Introduction

There are two views on how the imprecise interpretations of numerical sentences come about. On the first view, numerical expressions denote maximally finegrained degrees on a scale. Since no one is 160 cm tall on the dot, and no pizza is exactly 12 inch in diameter, simple positive sentences such as 'Muggsy Bogues is 160 cm tall' and 'This pizza is 12 inch in diameter' are literally false. We normally accept those sentences as true because there are no relevant differences between their literal contents and their true imprecise contents. Call this *the degree view*.[1]

On the second view, numerical expressions denote intervals on a scale. Accordingly, to say that Muggsy Bogues is 160 cm tall is not to say that his height is identical to the point 160.0 cm on the scale of heights, but to say that it falls into an interval centered at that point. This view holds that standard of precision is a primitive contextual parameter,[2] and that numerical expressions denote wider intervals at low standards of precision than they do at high standards of precision. On this view, imprecise interpretations arise because the hearer reasons that the standard of precision is not the absolute highest based on their assumption that the speaker is truthful and cooperative. For example, since 'Muggsy Bogues is 160 cm tall' most likely expresses a falsehood if the standard of precision is the absolute highest, the hearer reasons that the standard of precision is not so high in order to preserve their assumption that the speaker of the sentence is truthful and cooperative. Call this *the interval view*.[3]

It is often argued that the degree view is more adequate than the interval view because while the degree view can explain the infelicity of the sentences below by analyzing them as

[1]See Lasersohn (1999) for an influential version of the degree view.

[2]For a precursor to this view, see Lewis (1979).

[3]For views similar to the interval view, see Sauerland and Stateva (2011), Krifka (2007), Solt (2014), Gyamathy (2017), Sandgren and Bolinger (2020), and Armstrong (2023).

contradictions, the interval view cannot:

(1) #Although Mary arrived at 3 pm, she didn't arrive until 3:02 pm. (Lasersohn 1999:p.535)

(2) #Chicago is 800 miles from New York, but New York is not more than 796 miles from Chicago. (Carter 2021:p.173)

(3) #Although Mary is 180 cm tall, she is taller than 180.2 cm.

Consider (1) as a representative of this class of sentences, which we call *concessive numerical sentences* (CNS).[4] On the degree view, (1) says that Mary's time of arrival is identical to 3:00 pm but greater than or equal to 3:02 pm, which is clearly contradictory. The interval view, on the other hand, predicts that the sentence is consistent: If '3 pm' denotes the interval [2:55 pm, 3:05 pm), the sentence's conjuncts can both be true.

I shall argue that both the degree view and the interval view need to account for the infelicity of CNSs pragmatically, and so the degree view does not have an immediate advantage over the interval view.

But before that, I will motivate an implementation of the interval view by comparing it with a sophisticated version of the degree view recently developed by Carter (2021). Carter's theory belongs to a family of theories that attempt to use relevance-relations — or related constructs such as subject matters and questions under discussion — to generate imprecise communicated contents out of precise literal contents (Yablo 2014; Klecha 2018; Hoek 2018). While we focus on Carter's theory for concreteness, I submit that our objections to his theory, if successful, would apply equally to its kin.

On Carter's view, since numerical expressions literally denote maximally finegrained degrees, most simple positive numerical sentences are literally false; but since relevance has a weakening effect on those sentences' literal contents, those sentences can communicate true contents. Carter implements his theory using a two-layered semantics, on which sentences are assigned both their precise literal contents and their potentially imprecise communicated contents.

In contrast to Carter's semantics, the semantics I propose is single-layered: Since numerical expressions, on the interval view, denote intervals, my semantics will do away with Carter's literal contents, and directly encode Carter's communicated contents into sentences' meanings.

I argue that my semantics is more adequate than Carter's because it can account for both of the following entailments:

(4) a. Mary and Peter are both 180 cm tall. $\vDash$
    b. Mary is not taller than Peter.

(5) a. Mary is 180 cm tall and Peter is 90 cm tall. $\vDash$
    b. Mary is exactly twice as tall as Peter.

We'll focus on the imprecise readings of the sentences above, on which Mary and Peter need not be an exact number of cm tall in order for the sentences to communicate something true. I argue that while there is a strategy by which Carter can account for the first entailment, that strategy cannot be extended to account for the second entailment. What's missing in it is a general mechanism that rounds off Mary's and Peter's heights before their difference is quantified. Such mechanism, I argue, is available to the interval view.

[4]Dinges (2021) observes that CNSs are strikingly similar to concessive knowledge attributions (Rysiew 2001).

The discussion below is structured as follows. We first discuss Carter's theory of imprecision (§2). I then argue that comparatives present a challenge for Carter's theory (§3). After that, I present a novel interval-based theory of imprecision and argue that it can more adequately account for the imprecise contents of comparatives (§4). I then argue that both the degree view and the interval view need to explain the infelicity of CNSs pragmatically, and provide one such explanation that is compatible with my interval-based theory (§5). §6 concludes.

## 2 Carter on imprecision

Carter's theory of imprecision has two main components. The first component is a formal representation of what counts as (ir)relevant differences between states of affairs. The second component systematically assigns to sentences their communicated contents based on the literal contents of simple positive sentences and a contextually given relevance-relation.

Beginning with the first component, information about the world that is mutually accepted by the interlocutors is represented on Carter's theory by a set of possible worlds, which we call the *context set*. To represent relevance, Carter posits contextually given reflexive and symmetric relations over the logical space, each of which holds between two worlds just in case the worlds do not differ in ways that are relevant to the aims of conversation. To simplify our presentation of Carter's theory below, we will strengthen those relations into equivalence relations. This simplification is harmless because every relevance-relation we consider in this paper is most plausibly transitive.[5] Since an equivalence relation over the logical space partitions it into cells, here we have an elegant way of representing relevance: There is no relevant difference between two worlds just in case they are cellmates. For example, if the interlocutors are interested in knowing when Mary arrives to the nearest 10 minutes, the worlds in which Mary's time of arrival falls into [8:55 pm, 9:05 pm) are cellmates, and so are the worlds in which Mary's time of arrival falls into [9:05 pm, 9:15 pm), and so on.

We turn now to how communicated contents are systematically assigned to different sentence types based on the literal contents of simple positive sentences and a contextually given relevance-relation. Consider (6) as an example of a simple positive sentence:

(6) Mary arrives at 9 pm.

Suppose (6) is uttered in a context where the interlocutors are interested in knowing when Mary arrives to the nearest 10 minutes. The contextually given relation is a partition over the logical space such that two worlds are cellmates just in case Mary's times of arrival in them, rounded to the nearest 10 minutes, are the same. The figure below illustrates this equivalence relation:

(7)

| | |
|---|---|
| 8:50 pm | 9:00 pm |
| 9:10 pm | 9:20 pm |

The top left cell contains the worlds in which Mary's time of arrival falls into [8:45 pm, 8:55 pm); the top right cell contains the worlds in which Mary's time of arrival falls into [8:55 pm, 9:05 pm), and so on. The rectangle enclosing the four cells represents a subset of the logical space, which, besides containing the worlds shown on the figure, contains worlds in which Mary's time of arrival falls into other intervals.

[5] If the interlocutors are interested in knowing *how many stars there are give or take ten*, then the contextually given relation cannot be transitive because while there being 1000 (1010) stars is not relevantly different from there being 1010 (1020) stars, there being 1000 stars is relevantly different from 1020 stars. See Yablo (2014:p.36).

The literal content of (6) is the proposition that Mary arrives at 9:00:00 pm. It is a set of worlds that occupies a tiny region in the top right cell and does not intersect with other cells. Since every world in that cell is not relevantly different from the worlds in that tiny region, the communicated content of (6) is most naturally identified with the proposition the cell stands for, that is, the proposition that Mary's time of arrival falls into [8:55 pm, 9:05 pm). In general, given a contextually given partition over the logical space, the communicated content of a simple positive sentence is the union of the cells that have a non-empty intersection with that sentence's literal content.[6]

Now consider a sentence which is just like (6) except that it is modified by the adverb 'exactly':

(8) Mary arrives at exactly 9 pm.

Since '9 pm' denotes the moment 9:00:00 pm, 'exactly 9 pm' can only denote the same moment, and so the literal content of 'exactly' is the identity function, and (6) and (8) share the same literal content. Relative to the partition shown in (7), which we call $R_1$, the following procedure determines (8)'s communicated content: The adverb 'exactly' first modifies $R_1$ so that two worlds $w_1$ and $w_2$ are related just in case everyone in the domain of discourse is such that the time they arrive at $w_1$ is the same as the time they arrive at $w_2$ (Carter 2021: p.194; fn.19). Call this modified relation $R_1^e$. For simplicity, let Mary be the only person in the domain of discourse. The communicated content of (8) is identified with the communicated content of 'Mary arrives at 9 pm' relative to $R_1^e$. This procedure delivers the desirable result that the communicated content of (8) relative to $R_1$ is the proposition that Mary arrives at 9:00:00 pm, which is stronger than the communicated content of (6).[7]

We turn now to negations. Consider the negation of (6):

(9) Mary does not arrive at 9 pm.

Assume that the contextually given partition remains to be $R_1$. Relative to $R_1$, the communicated content of (9) is the difference between the logical space and the communicated content of (6). So the communicated content of (9) relative to $R_1$ is the proposition that Mary's time of arrival does not fall into [8:55 pm, 9:05 pm). Since the literal content of (9) is the (very weak) proposition that Mary's time of arrival is not identical to 9:00:00 pm, Carter's theory correctly predicts that the communicated content of (9) relative to $R_1$ is appropriately stronger than its literal content.[8]

Finally, consider the following conjunction:

(10) (a) Mary arrives at 9 pm and (b) Peter arrives at 10 pm.

Since (10) concerns two individuals' times of arrival, it won't do to use the same equivalence relation we have been considering to induce its imprecise content. So let's suppose that the contextually given partition is now such that two worlds are cellmates just in case Mary's times of arrival in those worlds, rounded to the nearest 10 minutes, are the same, and so are Peter's times of arrival in those worlds. The figure below illustrates this equivalence relation:

[6] See Kriz (2016), Klecha (2018), and Hoek (2018) for additional applications of this weakening operation.

[7] Lasersohn (1999:pp.528-529) observes that 'exactly' can be used loosely. But notice that Carter's analysis of 'exactly' comes with the undesirable side-effect that (8) can never have an imprecise communicated content.

[8] As Carter (2021) and Hoek (2018) observe, Lasersohn's (1999) theory fails to predict this fact.

(11)

| | |
|---|---|
| Mary: 8:50 pm<br>Peter: 9:50 pm | Mary: 9:00 pm<br>Peter: 9:50 pm |
| Mary: 8:50 pm<br>Peter: 10:00 pm | Mary: 9:00 pm<br>Peter: 10:00 pm |

Call this partition $R_2$. The top left cell contains the worlds in which Mary's time of arrival falls into [8:45 pm, 8:55 pm) and Peter's falls into [9:45 pm, 9:55 pm); the top right cell contains the worlds in which Mary's time of arrival falls into [8:55 pm, 9:05 pm) and Peter's falls into [9:45 pm, 9:55 pm), and so on. The literal content of (10-a) intersects with both the top right cell and the bottom right cell, and so the union of those cells is a subset of that sentence's communicated content relative to $R_2$. Similarly, since the literal content of (10-b) intersects with both the bottom left cell and the bottom right cell, the union of those cells is a subset of that sentence's communicated content relative to $R_2$. Naturally, the communicated content of (10) relative to $R_2$ is the intersection of the communicated contents of (10-a) and (10-b) relative to the same relation, which is the proposition represented by the bottom right cell.[9]

## 3 Speaking loosely with comparatives

As we've seen, Carter's theory works by using a contextually given relevance-relation to weaken the literal contents of simple positive sentences. This section presents a challenge for Carter's weakening mechanism by considering the imprecise contents of comparatives.[10]

Consider the following entailment:

(4-a) Mary and Peter are both 180 cm tall. $\vDash$

(4-b) Mary is not taller than Peter.

Suppose that what is relevant is how tall Mary and Peter are to the nearest cm, and that while Mary is taller than Peter by a nanometer, their heights, rounded to the nearest cm, are the same. Given these suppositions, (4-a) communicates something true, and so does (4-b). Since (4-a) can communicate something true despite its literal falsity, what (4-b) communicates is weaker than what it literally says. So, what the plain comparative 'Mary is taller than Peter' communicates is stronger than what it literally says. This fact is theoretically interesting: Can Carter's weakening mechanism generate this strengthening effect?

As it turns out, there is a strategy for generating this strengthening effect. But I argue that the main disadvantage of that strategy is that it cannot be extended to account for the intuitive imprecise contents of comparatives that quantify the difference between their arguments, such as 'Mary is exactly twice as tall as Peter'. Before we proceed to the latter task, let's consider how the strengthening strategy works.

[9]Carter encodes sentences' possible communicated contents into their context change potentials. Here is how we can convert our presentation of Carter's theory above into an assignment of CCPs to sentences. A context $\langle C, R\rangle$, on Carter's theory, consists of a context set $C$ and a relevance-relation $R$. Given a context $\langle C, R\rangle$, the CCP of a simple positive sentence updates the context by mapping $C$ to the intersection between $C$ and the sentence's communicated content relative to $R$ (leaving R unmodified); the CCP of a simple positive sentence modified by 'exactly' updates the context by mapping $R$ to the most finegrained relation (e.g. $R_e$) and $C$ to the intersection between $C$ and the sentence's communicated content relative to the modified relation; the CCP of $\neg\phi$ updates the context by eliminating from $C$ those worlds that fall into the communicated content of $\phi$ relative to $R$ (leaving $R$ unmodified); $\phi \wedge \psi$ updates the context by first updating the context with $\phi$ (which, if it contains 'exactly', may modify both the context set and the relevance-relation) and then updating the modified context with $\psi$.

[10]For discussions on how the contents of comparatives are sensitive to standard of precision, see Schwarzschild & Wilkinson (2002:§8), Bale (2008:p.37), and van Rooij (2011:p.160).

It first proposes that instead of treating 'Mary is taller than Peter' as a simple positive sentence, we analyze it as the following existentially quantified formula (Here 'tall' stands for a 2-place relation symbol whose interpretation holds between an individual $x$ and a degree $d$ just in case $x$ is at least $d$-tall.):[11]

(12) $\exists d(tall(m,d) \wedge \neg tall(p,d))$

It then proposes that we compute the proposition communicated by (12) by taking the union of the propositions communicated by its instances; and that the propositions communicated by (12)'s instances are generated by a contextually given relevance-relation analogous to $R_2$ (illustrated by figure (11) above), which relates two worlds just in case Mary's heights in them, rounded to the nearest cm, are the same, and so are Peter's heights in them. Call this relevance-relation $R_2^{cm}$.

To see how this strategy produces the desired strengthening effect, let us consider the following instance of (12) (Here we assume that, for every possible height in cm, there is a constant in the language denoting that precise degree):

(13) $tall(m, 180\,cm) \wedge \neg tall(p, 180\,cm)$
Gloss: Mary is at least 180.0 cm tall and Peter is less than 180.0 cm tall.

The literal content of $tall(m, 180\,cm)$ is the proposition that Mary is at least 180.0 cm tall. This content is weakened by $R_2^{cm}$ so that the communicated content of $tall(m, 180\,cm)$ is the proposition that Mary's height, rounded to the nearest cm, is at least 180 cm. The communicated content of $tall(p, 180\,cm)$ is computed in a similar way. Since the communicated content of $\neg tall(p, 180\,cm)$ is the complement of that of $tall(p, 180\,cm)$ relative to the set of worlds, it is the proposition that Peter's height, rounded to the nearest cm, is below 180 cm. With a moment's reflection, we can see that every instance of (12) communicates that Mary's height rounded to the nearest cm, is higher than Peter's. Since the communicated content of (12) is the union of the communicated contents of its instances, the communicated content of (12) is, as desired, that Mary is taller than Peter, with their heights rounded to the nearest cm.

By analyzing comparatives as existential quantifications over maximally finegrained degrees and computing the communicated contents of quantified formulas off of their instances' communicated contents that are induced by a well-chosen relevance-relation, this strategy accomplishes the feat of strengthening the contents of plain comparatives. The main disadvantage of this strategy, however, is that it cannot generate the intuitive imprecise contents of comparatives that quantify the difference between their arguments. Consider the following:

(5-a) Mary is 180 cm tall and Peter is 90 cm tall.

(5-b) Mary is exactly twice as tall as Peter.

Suppose, as above, that the relevant measurement unit is cm. There is an entailment from the communicated content of (5-a) to the communicated content of (5-b): If Mary's and Peter's heights, rounded to the nearest cm, are respectively 180 cm and 90 cm, then Mary's height

[11] The interpretation of predicate logic assumed here is inspired by Carter's static semantics (2021:p.16). A model $\langle D, W, I \rangle$ consists of a domain of discourse $D$, which contains individuals as well as (maximally finegrained) degrees. $I$ maps every n-place function symbol to a function from $W$ to a n-place function from $D^n$ to $D$, and every n-place relation symbol to a function from $W$ to a n-place relation over $D$, and every constant to an object in $D$. Let $w_g$ be a denotation function based on world $w$ and variable assignment $g$. If $t$ is a constant, then $w_g(t) = I(t)$. If $t$ is a variable, then $w_g(t) = g(t)$. Given a term $f^n(t_1, ..., t_n)$, which consists of a n-place function symbol and n terms, $w_g(f^n(t_1, ..., t_n)) = I(f^n)(w)(w_g(t_1), ..., w_g(t_n))$. Suppose $\beta$ is a n-place relation symbol and $t_i$ a term. The literal content of $\beta(t_1, ..., t_n)$, relative to assignment $g$, is defined as $\{w | \langle w_g(t_1), ..., w_g(t_n) \rangle \in I(\beta)(w)\}$.

rounded to the nearest cm, is exactly two times that of Peter's, with 'exactly two' understood maximally precisely. Notice that since 'exactly' in (5-b) only regulates the slack of the multiplier 'twice', the standard of precision that applies to 'exactly twice' is independent of the standard of precision that applies to Mary's and Peter's heights, which we assume remain constant as we transition from (5-a) to (5-b). [12]

Since it takes some arithmetic — and a certain choice of measurement unit — to confirm the truth of the imprecise readings of comparatives such as (5-b), we call these comparatives *mathy comparatives*. Mathy comparatives are interesting because they have two independent potential sites of imprecision: the magnitudes to be compared and the ratio between them. That imprecision is switched on in one site does not mean that it must also be in the other.

Let's consider in detail why mathy comparatives present a serious challenge to the strategy under consideration. Assume that degrees are real numbers so that we can do arithmetic with them.[13] It appears that the literal content of (5-b) is best captured by the analysis below, which says that Mary's precise height is exactly two times that of Peter's precise height:

(14) $\exists d_1 \exists d_2 (d_1 = height(m) \wedge d_2 = height(p) \wedge d_1 = 2 \times d_2)$
Gloss: There exists degrees $d_1$ and $d_2$ such that $d_1$ and $d_2$ are respectively Peter's and Mary's heights and $d_1$ equals two times $d_2$.

But this analysis makes wrong predictions about the communicated content of (5-b). To see this, assume that the contextually given relevance-relation remains to be $R_2^{cm}$. Consider the instance based on '181 cm' ($d_1$) and '90.5 cm' ($d_2$). The third conjunct of the instance (i.e. $181\,cm = 2 \times 90.5\,cm$) is necessarily true (and so its literal content is the set of worlds). The literal content of the first conjunct of the resulting instance (i.e. $181\,cm = height(m)$) is the set of worlds in which Mary is exactly 181.0 cm tall. This content is weakened by $R_2^{cm}$ into *Mary's height, rounded to the nearest cm, is 181 cm*. Similarly, the content of the second conjunct of the resulting instance is weakened into *Peter's height, rounded to the nearest cm, is 91 cm*. However, if Mary's and Peter's rounded heights are 181 cm and 91 cm respectively, then on the maximally precise reading of 'exactly', Mary is not exactly twice as tall as Peter.

In the next section, we'll develop an interval-based semantics that accounts for the imprecise contents of comparatives, both plain and mathy. But before that, we'll consider and respond to an important argument that challenges our claim that there is an entailment from the communicated content of (5-a) to the communicated content of (5-b).

The argument begins with the following two plausible assumptions about the properties of 'roughly' (Note: (C1) - (C3) below are supposed to hold only in contexts where $A$ is not exactly $n$

[12]To see the intuition behind this analysis, consider the following examples in the wild where comparisons are intended to be made between rounded values and 'exactly $n$ times' is intended to be interpreted maximally precisely:

(i) In 2021, the federal budget gives only $128 million to USCIS. By contrast, exactly 100 times as much ($12.8 billion) is going to ICE ($7.97B) and Border Patrol ($4.87B). (`https://www.neveragainaction.com/end-deportation`)

(ii) Less than 1 parent in 10 (8 percent) thinks the quality of American family life has improved since they were growing up. Exactly 8 times as many (64 percent) say that family life has declined. (`https://iasculture.org/research/publications/culture-american-families-executive-report`)

Notice that the total budget given to ICE and Border Patrol has been rounded off from 12.84B to 12.8B. Had the author intended 'exactly' to be read loosely, there would have been no point in presenting the rounded figure of '12.8 billion'. In the second example, the use of rounded percentage points seems deliberate: It allows the author to claim truly that the ratio between the two numbers is exactly 8.

[13]I make this assumption in the context of this discussion because it is neutral between the degree view and the interval view and it readily allows both views to account for the truth conditions of the precise readings of such comparatives as (5-b) and 'Mary is 90 cm taller than Peter'. This discussion is not the place to settle the ontological status of degrees of heights — i.e. whether they are real numbers or merely isomorphic to real numbers, and whether they are reducible to equivalent classes of individuals.

tall, with 'exactly' understood in the maximally precise way):

(C1) For any numeral *n*, the literal content of 'A is roughly *n* tall' coincides with the communicated content of 'A is *n* tall'.[14]

(C2) For any numeral *n*, the communicated content of 'A is roughly *n* tall' coincides with the sentence's literal content.[15]

It follows from (C1) and (C2) that the following holds:

(C3) The communicated content of 'Mary is roughly 180 cm tall' is identical to that of 'Mary is 180 cm tall'. Likewise for 'Peter is roughly 90 cm tall' and 'Peter is 90 cm tall'.

From (C3), it follows that the communicated content of 'Mary is roughly 180 cm tall and Peter is roughly 90 cm tall' is identical to that of (5-a), and so the following entailment holds (at the level of communicated content) just in case the entailment from (5-a) to (5-b) holds (at the level of communicated content):

(15) a. Mary is roughly 180 cm tall and Peter is roughly 90 cm tall.
b. Mary is exactly twice as tall as Peter.

But since it is obvious that the entailment above does not hold, it follows that there is no entailment from (5-a) to (5-b) either.

In response, while I agree that (15-a) does not entail (15-b), I argue that this objection fails because, firstly, (C3) is less plausible than it seems and, secondly, (C1) cannot be forced upon the interval view.

It is important to observe that, on the degree view, the fact that a sentence has a literal content that is (i) false or contradictory[16], or (ii) under-informative (consider the negations of simple positive sentences), or (iii) not relevant in the sense we discussed in §2, is not a strike against uttering the sentence, insofar as the speaker can communicate a true, informative, and relevant content by uttering the sentence. So, on the degree view, it is at the level of communicated content that utterances are evaluated for compliance with Quality, Quantity, and Relevance.[17] If so, then in contexts where Mary's precise height is close enough to 180.0 cm without being identical to that degree, the fact that (16-a) below can have a true, informative, and relevant literal content but (16-b) cannot does not count in favor of (16-a).

(16) a. Mary is roughly 180 cm tall.
b. Mary is 180 cm tall.

Since it is fair to assume that inserting 'roughly' into (16-b) incurs a processing cost, other things being equal, Manner ('be brief') favors (16-b) over (16-a). So the supposition that (C3) is true immediately leads to a problem: If (16-a) and (16-b) have the same communicated content —

[14] This principle can be extrapolated from Carter's remark that '[a] LT-weakener assimilates the literal content [of the clause that results from combining it with its argument] to the communicated content [of its argument]' (Carter 2021:p.4). Lasersohn (1999:§6.2) may appear to endorse the same principle. But notice that Lasersohn does not say that the literal content of 'roughly *n*' coincides with the communicated content of *n*. He only says that, compared to the literal content of *n*, the literal content of 'roughly *n*' is more similar to the communicated content of *n*. See also Russell's (2021:§1.3.1) discussion of Lasersohn's view.

[15] This principle can be extrapolated from Carter's analysis that 'the literal contents and communicated contents of $(1_<)$-$(3_<)$ coincide.' (Carter 2021:p.4) $(1_<)$-$(3_<)$ are sentences of the form 'A is roughly/ effectively/ about *n*'.

[16] Notice that Carter's $(1^{\&}_{\neg})$ - $(3^{\&}_{\neg})$ are felicitous while having a contradictory literal content. (Carter 2021:p.4)

[17] See Klecha (2018) for a detailed example of how this style of pragmatic reasoning is implemented.

and so are ranked equally by Quality, Quantity, and Relevance — and Manner favors (16-b) over (16-a), it is difficult to see how (16-a) can ever be rationally uttered.[18]

While it is not clear how a theorist who endorses (C1) and (C2) may solve this problem, it is important to see that this problem cannot be forced upon the interval view. Setting aside cases of implicature and presupposition accommodation where what is communicated goes beyond what is said (i.e. literal content), the interval view holds that what we communicate by uttering numerical sentences is identical to what we say by uttering those sentences. For example, if a speaker of (16-b) says that Mary's height is 180 cm to the nearest cm, then they (16-b) communicates that very same content. Now that we have clarified how what is said/ literal content relates to what is communicated on the interval view, let me sketch a viable way of denying (C1).[19] The interval view can liken 'roughly' to the modal operator $\Diamond$: (16-a) is true at a world $w$ just in case $w$ is related to (or 'sees') a world such that it is just like $w$ except that '180 cm' denotes a slightly wider interval than it does in $w$, and Mary's precise height falls into that interval.[20] (Which worlds see which worlds may vary from context to context and can be accommodated on demand, within reason.) Just as $\Diamond p$ can be true at a world without $p$ being true in that same world, 'A is roughly $n$ tall' can be true without 'A is $n$ tall' being true. So, in general, what is said by 'A is roughly $n$ tall' is not identical to what is said/ communicated by 'A is $n$ tall'. And the reason why (16-a) can rationally be chosen over the (16-b) is because (16-a) is potentially weaker and so less likely to violate Quality.

## 4 Interval-based dynamics of loose talk

In this section, we develop an interval-based dynamic semantics that accounts for the entailments from (4-a) to (4-b) and from (5-a) to (5-b) and respond to three worries about our semantics.

The main reason why I implement the interval view with a dynamic semantics is that numerical sentences can not only describe what the world is like but also inform interlocutors how loosely the numerical expressions contained in them are used. For example, suppose we know that Mary and Peter are respectively 177.5 cm tall and 179.5 cm tall, and only someone who is 180 cm tall or above can join our local basketball team. If the coach declares that Peter is 180 cm tall but Mary is not, we can learn that the prevailing standard of precision is such that 179.5 cm can be rounded off as 180 cm but 177.5 cm cannot. Similarly, suppose we know that the masses of helium and hydrogen are 4.002602 amu and 1.00794 amu, and a scientist says in a popular science article that helium is four times as heavy as hydrogen. Instead of accusing the scientist of getting the math wrong, we should charitably reason that had the scientist intended 'four' to be understood maximally precisely (i.e. exactly four), the scientist must have rounded off the elements' masses to the nearest amu or the nearest 0.1 amu; and that had the scientist intended to not round off the elements' masses, he must have intended 'four' to be understood imprecisely.

Since these uses of numerical sentences are strikingly similar to the metalinguistic uses of vague predicates, Barker's (2002) analysis of the latter is a natural starting point of our semantics. According to Barker's analysis, every possibility under consideration in a context,

[18] With (16-a) in competition with (16-b), on the assumption that 'roughly 180 cm', like '180 cm', is a constituent, (16-a) appears to violate Carter's condition (II.) (2021:p.21) because '180 cm' appears on more coarsenings than 'roughly 180 cm'.

[19] On the assumption that the literal content of 'A is n' coincides with its communicated content, both Sauerland and Stateva (2011:p.128) and Solt (2014:pp.527-528) would deny (C1).

[20] The intuition behind this analysis is that 'roughly' accesses a slightly lower standard of precision without changing the prevailing standard of precision. Consider 'Mary is roughly 180 cm tall and Peter is 91 cm tall'. If 'roughly' lowers the prevailing standard of precision, '91 cm' has to be read just as loosely as 'roughly 180 cm', which seems implausible.

besides containing a possible state of the world, supplies for 'tall' a possible threshold. The meaning of 'Mary is tall' is a *context change potential* (CCP) that updates a context by eliminating from it every possibility where Mary's height fails to exceed the possibility's threshold. When the sentence is used purely metalinguistically, any arbitrary possible state of the world under consideration prior to update — call it $w$ — remains to be under consideration after update, but possibilities that contain $w$ and supply for 'tall' a threshold that is at least as high as Mary's height in $w$ are eliminated.

To account for the metalinguistic uses of the numerical sentences we considered, we will add to every possibility (i) a standard of precision that determines the width of the unit intervals on the scale of heights, and (ii) an additional standard of precision that determines how loosely the multiplier (e.g. 'twice') is taken. We assume that the scale of heights is the set of non-negative real numbers, with each real number understood as a maximally finegrained degree on the cm scale.[21] To make standards of precision precise, we define them to be *granularity functions* ($\gamma$) (Sauerland and Stateva 2011; Gyarmathy 2017) satisfying the following conditions:[22]

(17) a. For every degree on the non-negative real number line, $\gamma$ maps it to a convex[23] subset that contains it.
b. The intervals induced by $\gamma$ are pairwise disjoint.
c. The union of the intervals induced by $\gamma$ returns the non-negative real number line.
d. $\gamma$ imposes a *zero interval* on the non-negative real number line: The intervals induced by $\gamma$ are equally spaced after the zero interval. (That is, there do not exist two intervals $[a_1,b_1)$ and $[a_2,b_2)$ after the zero interval such that $|b_1 - a_1| \neq |b_2 - a_2|$.) But the difference between the minimal and the maximal degrees of the zero interval, or the interval's *width*, is half that of other intervals induced by $\gamma$.
e. Every interval has a closed lower bound and an open upper bound.

These conditions guarantee that when the relevant measurement unit is cm, the scale of heights is partitioned into the zero interval [0,0.5) and subsequent intervals with a width of 1, such as [0.5,1.5) and [1.5,2.5). Call intervals other than the zero interval *unit intervals*, and the width of the unit intervals a *granularity*. From now on, we will often collapse the distinction between granularity function and granularity, and refer to a granularity function by the width of the unit intervals it imposes on the non-negative real number line.

We shall piggyback on these same conditions to make available granularities that represent how loosely multipliers such as 'twice' are understood. For example, suppose 'twice' is understood loosely so that number $a$ counts as twice as large as number $b$ just in case $\frac{a}{b}$ falls into the interval [1.9,2.1). We represent that way of understanding 'twice' with a granularity function that maps 2 to [1.9,2.1). For now on, we call granularities dedicated to mapping precise heights to intervals *h-granularities*, and those dedicated to loosening the multipliers *m-granularities*.

Unit intervals have a dual role in our semantics. First, they serve as the denotations of measure phrases such as '180 cm'. Second, they serve as thick degrees (Solt 2014) that the meaning of 'tall' — which we assume is a measure function (Kennedy 2007) — maps to individuals. For example, if the h-granularity at a certain possibility is 1, the meaning of 'tall' maps that possibility and

[21] My choice of measurement unit is arbitrary. But the charge of arbitrariness applies to any choice of measurement unit — including any abstract measurement unit.

[22] The first three constraints are due to Sauerland and Stateva (2011) and Gyarmathy (2017). The last two constraint are introduced to rule out such unnatural intervals as [0.7,1.7) and (0.5,1.5] as potential denotations for '1 cm'.

[23] A set is convex just in case for every two members in the set, every member between those two members is also a member of the set.

Muggsy Bogues to the interval [159.5,160.5).

We implement the dual role of unit intervals by assigning the following meanings to 'n cm' and 'tall':

(18) a. $[\![n \text{ cm}]\!] = \lambda s.s(n)$

Note: $s(n)$ is the image of the real number $n$ under the h-granularity at $s$.

b. $[\![\text{tall}]\!] = \lambda s \lambda x.tall(s)(x)$

Note: Since $[\![\text{tall}]\!]$ is a function from possibilities (of type $i$) to functions from individuals (of type $e$) to thick degrees (of type $D$), it is of type $\langle i, \langle e, D \rangle \rangle$.

The meaning of '$n$ cm' maps a possibility to the interval that is the image of the real number $n$ under the h-granularity at that possibility. The meaning of 'tall' takes a possibility, $s$, and an individual as inputs and returns that individual's thick degree of height at $s$, which we define to be the image of that individual's maximally finegrained degree of height at the world of $s$ under the h-granularity at $s$.

To resolve the type mismatch between the meaning of '$n$ cm' and that of 'tall', I propose that the meaning of '$n$ cm' can be typeshifted by the following rule:

(19) The m-rule:

$m([\![n\text{cm}]\!])$

$= \lambda G_{\langle i,eD \rangle} \lambda x \lambda C.\{s \in C | G(s)(x) = s([\![n\text{cm}]\!])\}$

$= \lambda G_{\langle i,eD \rangle} \lambda x \lambda C.\{s \in C | G(s)(x) = s(n)\}$

This typeshifted meaning takes an adjectival meaning and an individual as inputs and returns a CCP. By making use of this typeshifting rule, we can generate the CCPs of sentences of the form 'A is $n$ cm tall', such as 'Mary is 180 cm tall' (We assume that the meaning of 'is' is an identity function):[24]

(20) $[\![\text{Mary is 180 cm tall}]\!] = \lambda C.\{s \in C | tall(s)(m) = s(180)\}$

We say that someone *counts as n* cm tall at a possibility just in case their precise height at the world of that possibility falls into the image of $n$ under the h-granularity at that possibility. Suppose we accept 'Mary is 180 cm tall' in a context. We eliminate from the context possibilities in which Mary does not count as 180 cm tall.

We define the following meaning for 'er', which contributes to the CCPs of plain comparatives, such as (4-b): (We assume that 'than' denotes the identity function):

(21) a. $[\![\text{er}]\!] = \lambda G_{\langle i,eD \rangle} \lambda y \lambda x \lambda C.\{s \in C | G(s)(x) \rhd G(s)(y)\}$

b. $D_1 \rhd D_2$ iff $\exists d_1 \in D_1 (d_1 \notin D_2 \wedge \forall d_2 \in D_2 (d_1 > d_2))$

(4-b) $[\![\text{Mary is taller than Peter}]\!] = \lambda C.\{s \in C | tall(s)(m) \rhd tall(s)(p)\}$

Comparatives, in our semantics, are interpreted as comparisons between thick degrees. So to say that Mary is taller than Peter is to say that Mary's thick degree of height is bigger than (denoted

[24] Following Kennedy (2007), I assume that the meaning of the positive form is derived from an adjectival meaning and the meaning of a phonologically null morpheme called 'pos'. The meaning of 'pos' is defined as follows:

(i) $[\![\text{pos}]\!] = \lambda G_{\langle i,eD \rangle} \lambda x \lambda C.\{s \in C | G(s)(x) \rhd s(G)\}$

Here $s(G)$ is an interval. Its value is obtained by applying the h-granularity function at $s$ to the result of applying the threshold function at $s$ to adjectival meaning $G$.

by '▷') Peter's.

We are now ready to account for the entailments from (4-a) to (4-b). A set of premises entails a conclusion just in case for any context, the result of updating it successively with the CCPs of the premises equals the result of updating it successively with the CCPs of the premises and the conclusion. To account for this entailment, we will make the simplifying assumption that (4-a) and (4-b) can be analyzed respectively as 'Mary is 180 cm tall and Peter is 180 cm tall' and 'It is not the case that Mary is taller than Peter'; and define the CCPs of '$\phi$ and $\psi$' and 'it is not the case that $\phi$' in the usual way: '$\phi$ and $\psi$' updates a context by updating it first with the CCP of $\phi$ and then with that of $\psi$; 'it is not the case that $\phi$' updates a context by eliminating from it possibilities that survive update by $\phi$. (4-a) entails (4-b) because any context that accepts that both Mary's and Peter's thick degrees of height are identical to an unit interval centered at 180.0 cm must accept that Mary's thick degree of height is not larger than Peter's.

We turn now to accounting for the entailment from (5-a) to (5-b). We define the following meanings for 'twice' and 'exactly' (as in 'exactly twice'), which contribute to the CCP of (5-b) (Note: We assume that 'as' denotes the identity function):

(22) $[\![\text{twice}]\!] = \lambda G_{\langle i,eD \rangle} \lambda y \lambda x \lambda C.\{s \in C | \frac{mid(G(s)(x))}{mid(G(s)(y))} \in m.gran(s)(2))\}$

(23) $[\![\text{Mary is twice as tall as Peter}]\!]$
$= \lambda C.\{s \in C | \frac{mid(tall(s)(m))}{mid(tall(s)(p))} \in m.gran(s)(2)\}$

(24) $[\![\text{exactly}]\!] = \lambda M_{\langle ieD,eeCC \rangle} \lambda G_{\langle i,eD \rangle} \lambda y \lambda x \lambda C.M(G)(y)(x)(C[max_{m.gran}])$
Note: $C[max_{m.gran}]$ is just like $C$ except that every possibility in $C$ has its m-granularity replaced by the granularity function that maps any non-negative real number $n$ to $\{n\}$.

(5-b) $[\![\text{Mary is exactly twice as tall as Peter}]\!]$
$= \lambda C.\{s \in C[max_{m.gran}] | \frac{mid(tall(s)(m))}{mid(tall(s)(p))} \in m.gran(s)(2)\}$

To quantify the difference between thick degrees, we make use of a mid-point function, *mid*, which maps an interval to its mid-point. Since the multiplier 'twice' can be used loosely, we use m-granularity functions to represent how loosely it is understood. So to say that Mary is twice as tall as Peter is to say that the ratio between the mid-point of Mary's thick degree of height and that of Peter's falls into the interval that stands for how loosely 'twice' is understood.

For simplicity, we assume that 'exactly' can only combine with multipliers such as 'twice', and that it changes the context by replacing the m-granularity in every possibility with the maximally finegrained m-granularity. In a more adequate analysis — which must be left for future work — we would want that 'exactly' combines with both multipliers and measure phrases (e.g. 180 cm) and that, like measure phrases and multipliers, it can be used loosely and metalinguistically. But given our provisional analysis of 'exactly', to say that Mary is exactly twice as tall as Peter is to say that the ratio between the mid-point of Mary's thick degree of height and that of Peter's is identical to 2.

We can now easily see that, according to our semantics, (5-a) entails (5-b): Any context that accepts that Mary's (Peter's) thick degree of height is an unit interval centered at 180.0 cm (90.0 cm) must accept that the mid-point of Mary's unit interval, divided by that of Peter's unit interval, is 2.

Before we leave this section, let us consider and respond to three worries about our semantics. The first worry can be articulated in the following way: "Suppose Mary and Peter are respectively

180.8 cm tall and 90.4 cm tall. Intuitively, (5-b) should communicate something true regardless of the granularity involved. However, according to your semantics, when the granularity involves rounding to the nearest cm, Mary's and Peter's heights are respectively 181 cm and 90 cm, and so (5-b) communicates something false, contrary to intuition."

In response, let me explain why I do not share the intuition that, when it is known that Mary and Peter are respectively 180.8 cm tall and 90.4 cm tall, (5-b) always communicates a true content. Recall that, according to our semantics, multiple granularities — for both heights and multipliers — are typically under consideration. Successful communication requires interlocutors' coordination on the granularities under consideration. So it is most natural to hold that what we communicate by uttering a sentence corresponds to the granularities under consideration.[25] Suppose you and I already know that Mary's and Peter's maximally precise heights are 180.8 cm and 90.4 cm and therefore that Mary is exactly twice as tall as Peter for every h-granularity at least as fine as 0.1. If we must ensure that (5-b) be accepted, we should make sure that the h-granularities that are at least as fine as 0.1 are under consideration. We can do so by alerting to our hearers the unusually precise piece of knowledge we have or by convincing our hearers that a h-granularity finer than 0.1 ought to be entertained. However, it is not always practical and wise to do so. Imagine that we are staff members at a local school and we are asked to measure the heights of the students, two of whom being Mary and Peter, using a big ruler with marks that are 1 cm apart. The only salient and relevant h-granularity is 1. But the good news is that we are in a low-stake context: Mary and Peter will not be awarded or punished if (5-b) fails to be accepted. It seems to me there is nothing wrong in playing along with the h-granularity assumed by our colleagues and rejecting (5-b), because in doing so we are not lying: what we reject is that Mary's height to the nearest cm is exactly twice that of Peter's — rather than that Mary is exactly twice as tall as Peter for some h-granularity at least as fine as 0.1 — and because we allow everybody else to go about their business. So, what we do seems rational and cooperative.

The second worry concerns how the degree view's concepts of *literal content* and *communicated content* line up with their counterparts on my interval-based view. To see why this worry is important, consider the measure phrase '160 cm', uttered in a context where the relevant issue is *How tall is Muggsy Bogues to the nearest cm?*. The degree view assigns to '160 cm' the point 160.0 as its literal content and the interval [159.5,160.5) as its communicated content. If my interval-based view assigns to '160 cm' the degenerate interval {160.0} as its literal content — and so my view predicts that 'Muggsy Bogues is 160 cm tall' (hereafter **Muggsy**) is literally false — it is fair to say that my interval-based view is only a degree view in disguise. But if my view assigns to '160 cm' the interval [159.5,160.5) as both its literal content and its communicated content, then my view appears to collapse the helpful distinction between literal content and communicated content.

In response, I deny that the degree view's concepts of literal content and communicated content can be directly mapped onto my view in the ways suggested above. Let's first consider *literal content*. I understand it as a gradable concept. The degenerate interval {160.0} is the most literal content of '160 cm'. The interval [159.5,160.5) is a less literal content. (The boundary between literal and not-literal is vague and context-dependent.) However, that a literal content is more literal than the other literal content does not mean that the former is more legitimate.[26] Nor does it mean that, as charitable interpreters, we ought to attribute to a speaker the more literal content. Unlike the degree view, I take it that speakers are assessed for compliance with pragmatic

[25] I assume that what a sentence communicates in a non-defective context just is what the speaker of the sentence communicates by uttering it.

[26] See Lewis's (1979) response to Unger (1975) on the contents of absolute adjectives. Here I adapt Lewis's move to the numerical case.

principles both at the level of what is said/ literal content and at the level of communicated content. So those principles are already in play in our attribution of literal content(s) to a speaker. In the example at hand, if a speaker utters **Muggsy**, it is more charitable to attribute to them the interval [159.5,160.5) as the operative literal content of '160 cm' because the degenerate interval contributes to a propositional content that violates Quality and fails to satisfy Relevance optimally.

Turning now to *communicated content*, I submit that what the speaker communicates by uttering **Muggsy** is the same as what they say by uttering the sentence. But this does not mean that the distinction between what is said/ literal content and what is communicated disappears on the interval view. As we discussed in §3, what is communicated can go beyond what is said because of well-understood mechanisms such as implicature and presupposition accommodation. For example, we can communicate that not all basketball players are 160 cm tall to the nearest cm by saying that some basketball players are 160 cm tall to the nearest cm,[27] that we have friends by saying that one of our friends is 160 cm tall to the nearest cm. So the interval view does not collapse the literal/ communicated distinction.

The last worry is that our analysis that mathy comparatives have two independent sites of imprecision appears to predict wrongly that the following sentences are acceptable:

(25) a. 11.9 cm is double 6.2 cm.
 b. Anything that is 9.8 cm tall is twice as tall as anything that is 5.1 cm tall.

One may develop this worry this way: "Since non-round numerical expressions are used in (25-a) and (25-b), Manner-based reasoning suggests that the relevant h-granuarlities are sufficiently finegrained so that the numerical expressions (e.g. '11.9 cm') do not denote the same intervals denoted by their rounder alternatives (e.g. '12 cm'). But since your semantics allows m-granularities to be coarse-grained while h-granuarlities are finegrained, it wrongly predicts that (25-a) and (25-b) have true readings. For example, (25-a) is predicted to be true when the m-granularity is 0.2 and the h-granularity is 0.1."

My response is twofold. First, I agree that (25-a) and (25-b) are unacceptable if uttered out of the blue. Without prior context, sentences are most naturally understood descriptively rather than metalinguistically. Understood descriptively, (25-a) and (25-b) are unacceptable because they cannot be used to communicate how tall things are (or any empirical fact for that matter), and because if they are intended to describe elementary multiplication facts, we would most naturally read them maximally precisely and so take them to communicate plain contradictions.

Second, while (25-a) and (25-b) are unacceptable in out-of-the-blue contexts, they can have acceptable metalinguistic uses in rich contexts. Imagine that we are factory workers tasked with making pairs of dolls one of which is double the height of the other. We know that 'double' means something imprecise at our assembly line because a 12 cm tall doll and a 6.2 cm tall doll have passed the automated quality control before. Suppose we have just made a 11.9 cm tall doll and a 6.2 cm tall doll, and we will be immediately fired if our dolls do not pass the QC. If we do not want to push our luck, we ought to make sure whether 11.9 cm is double 6.2 cm before passing our dolls to the QC. In this case, an utterance of (25-a), made by someone knowledgeable about the sensitivity of the QC system, would seem perfectly useful and acceptable. (A similar context can be used to justify the metalinguistic use of (25-b). But the context has to be an unusual one where the QC system has in the past applied inconsistent standards to dolls having the same

[27] Notice that, on the degree view, the quantity implicature here is generated based off of the communicated content of 'some basketball players are 160 cm tall'.

heights.)[28]

In the next section, I argue that the degree view has no real advantage over the interval view in explaining the infelicity of CNSs, and provide a pragmatic explanation for the infelicity of (3) that is compatible with the semantics developed in this section.

## 5 The infelicity of concessive numerical sentences

A concessive numerical sentence (CNS) is a conjunction of a numerical sentence with a second sentence (i.e. the concession clause) such that the maximally precise reading of the numerical sentence contradicts with the second sentence. It is often argued that the infelicity of CNSs is evidence against the interval view because while the degree view analyzes them as contradictions, the interval view predicts them to be consistent. But this argument is too quick. Recall that, in order to account for the felicity of literally false sentences such as **Muggsy**, the degree view assigns to those sentences their imprecise communicated contents. It is these communicated contents, rather than their corresponding literal contents, that are added to the common ground. So it is at the level of communicated content that a speaker is assessed for compliance with pragmatic principles. On the degree view, the reason why a speaker can felicitously utter a literally false sentence is because what they communicate by uttering the sentence is true (as well as informative and relevant). So parity suggests that the degree view's explanation of the infelicity of CNSs is incomplete unless it shows that CNSs have contradictory literal contents as well as contradictory communicated contents.[29] This task, however, has not been attempted by any degree theorist.[30]

It may be thought that since contradictory literal content immediately leads to contradictory communicated content, the degree theorist has no additional task to complete. To see why this thought is misguided, let's use Carter's semantics to predict the communicated content of the following CNS:

(3) #(a) Although Mary is 180 cm tall, (b) she is taller than 180.2 cm.

Recall from §2 that the communicated content of a conjunction is the intersection of the communicated contents of its conjuncts. Suppose what is relevant is how tall Mary is to the nearest cm. The literal content of (3-a) is that Mary is 180.0 cm tall. It is weakened by the relevance-relation into *Mary's height, rounded to the nearest cm, is 180 cm*. Suppose we analyze (3-b) as the formula below and apply to it Carter's weakening mechanism.

(26) $\exists d(d > 180.2\,cm \wedge tall(m, d))$

Its communicated content is that *Mary's height, rounded to the nearest cm, is at least 180 cm*, which clearly does not contradict with the communicated content of (3-a).

Unless the degree view abandons the attractive principle that the communicated content of a conjunction is the intersection of the communicated contents of its conjuncts, it cannot offer a purely semantic explanation for the infelicity of CNSs, and so, like the interval view, it must explain the infelicity of CNSs pragmatically.

To show that a pragmatic explanation for the infelicity of CNSs is available to the interval

[28] Thanks to Yunbing Li for suggesting the ideas behind these examples.

[29] Carter notes that the most obvious explanation for the infelicity of CNSs is that 'both their literal and communicated contents are necessarily false.' (2021:p.3)

[30] For example, Lasersohn (1999) has not shown that CNSs always have the empty set as their pragmatic halo.

view, we now focus on (3) and develop a pragmatic explanation for its infelicity that is compatible with our interval-based semantics. We'll complete three preparatory steps before we give the explanation: First, we'll motivate the principles on which our explanation is based. Second, we'll discuss a subtle issue about the truth conditions of (3-b). Third, we'll modify the meaning of '-er' so that (3-b) becomes interpretable in our semantics. (Note: For readability and relevance, 'granularity' is understood as 'h-granularity' (§4) from now on.)

Our explanation for the infelicity of (3) will rely on the following two principles:

(Granularity-Relevance) There are no relevant differences between the degrees in a unit interval.

(Constancy) Without a conventional device that suggests otherwise, what is relevant does not change in the middle of an utterance.

(Granularity-Relevance) is based on the degree view's observation that imprecision goes hand in hand with what is relevant in a conversation. Why does **Muggsy** communicate that Bogues's height, roundest to the nearest cm, is 160 cm? The degree view's explanation is that since what is relevant is how tall Bogues is to the nearest cm, the degree 160.0 cm is not relevantly different from other degrees in the same cm interval, and so the sentence communicates that Bogues's precise height is identical to some degree in that interval. Here we adapt the degree view's observation to our interval-based account. (But notice that since our semantics allows multiple granularities to be under consideration in a context, it in effect allows what is relevant to be indeterminate in a context.)

(Constancy) is a plausible assumption because it does no more than report our expectation that the speaker presents information in an orderly manner.[31] The principle explains why Daisy's response below is inappropriate.

(27) a. Chris: How is your son doing in school?
b. Daisy: He is doing well, and I just published a book.

As much as Chris may be interested in knowing Daisy's publishing success, Daisy cannot change what is relevant in the middle of her utterance without first properly signaling such a change. But notice that (Constancy) allows what is relevant to change in the middle of an utterance in some cases. For example, consider the following variant of the exchange between Chris and Daisy:

(28) a. Chris: How is your son doing in school?
b. Daisy: He is doing well. Aren't you also interested in how I am doing? I just published a book.

With the insertion of a question that introduces a new topic of discussion, Daisy's utterance becomes less jarring.

Besides questions, slack regulators (Lasersohn 1999) such as 'exactly' and 'strictly speaking' are conventional devices for changing what is relevant. As Carter points out, the following sentence is felicitous:

(29) Lena arrived at 9 o' clock, but she did not arrive at 9 o' clock exactly. (Carter 2021:p.171)

[31] See Gyarmathy's Principle 2 (2017:p.493) for a principle in the neighborhood of (Constancy).

This sentence is not a contradiction because 'exactly' shifts what is relevant (§2) so that the second instance of '9 o' clock' is interpreted more precisely than the first stance. In a similar vein, CNSs become less unacceptable if the concession clauses are preceded by 'strictly speaking'.

We turn now to a subtle issue about the truth conditions of (3-b). There are two ways to take the truth conditions of (3-b). The first holds that (3-b) has an granularity-insensitive truth condition. That is, regardless of what the prevailing granularity is, (3-b) is true just in case Mary's precise height exceeds 180.2 cm. The second holds that (3-b) has a granularity-sensitive truth condition. That is, (3-b) is true at a granularity just in case Mary's thick degree of height at that granularity exceeds the interval denoted by '180.2 cm' at that granularity.

We assume that (3-b) at least has the granularity-insensitive truth condition because, with (Granularity-Relevance) and (Constancy) in place, it is problematic to hold that (3-b) only has the granularity-sensitive truth condition. To see this, consider:

(30) a. Emily: Is Mary taller than 180.2 cm?
b. Fred: Of course she is taller than 180.2 cm. She is 200 cm tall.

Suppose (3-b) only has the granularity-sensitive truth condition. Emily's use of '180.2 cm' , which is less round and more costly to utter than '180 cm', suggests that the prevailing granularity is not greater than 0.4. To see this, if the prevailing granularity is greater than 0.4, '180.2 cm' and '180 cm' denote the same interval, and so, on pain of violating Manner, Emily ought to have uttered 'Is Mary 180 cm tall?', which has the same content as (30-a) but is less costly to utter.[32] Now, since Fred doesn't appear to have modified the prevailing granularity with his positive answer to Emily's question, (Granularity-Relevance) and (Constancy) suggest that '200 cm' in (30-b) is interpreted at a granularity not greater than 0.4. However, Fred need not be interpreted as using '200 cm' that precisely: If Mary turns out to be 200.3 cm tall, it seems perverse to insist that Fred communicates a falsehood.

I submit that the reason why '200 cm' in (30-b) can be interpreted at a granularity greater than 0.4 is because (3-b) has a granularity-insensitive truth condition. To implement that truth condition in a bottom-up manner, we'll introduce the following modified meaning for '-er':

(31) $[\![\text{er}]\!] = \lambda G_{\langle i,eD\rangle}\lambda y \lambda x \lambda C.$
a. $\{s \in C | G(s)(x) \rhd G(s)(y)\}$ if $type(y) = e$
b. $\{s \in C | G(s^0)(x) \rhd (s^0)(y)\}$ if $type(y) = \langle i, D\rangle$
Note: $s^0$ is just like $s$ except that the granularity at $s^0$ is zero.

Here we broaden the domain of the argument slot denoted by 'y' so that it can be occupied by either an individual or a function from possibilities to intervals (denoted by a measure phrase). If that argument slot is occupied by an individual, definition (a) applies so that the modified '-er' makes the same contribution to the CCPs of comparatives as the original '-er' (§4). But if that argument slot is occupied by a function from possibilities to intervals, definition (b) applies so that 'Mary is taller than 180.2 cm tall' has the following CCP:

(32) $[\![\text{Mary is taller than 180.2 cm}]\!] = \lambda C.\{s \in C | tall(s^0)(m) \rhd (s^0)(180.2)\}$

A possibility in a context survives update by this meaning just in case Mary's precise height at that possibility exceeds 180.2 cm. Notice that this meaning does not say that the granularities

[32] See Krifka (2007) for a helpful discussion on the relation between non-round numerical expressions and precision standards.

under consideration when (3-b) is considered are converted into the zero granularity. It only says that the interpretation of (3-b) is always precise regardless of which granularities are under consideration.

We are now ready to explain the why (3) is infelicitous. As we discussed, there are two ways to take the truth conditions of (3-b). If (3-b) has the granularity-sensitive truth condition — the statement of which is just like (32) except that $s$ is used in place of $s^0$ — (Granularity-Relevance) and (Constancy) already predict that a speaker cannot utter (3) without contradicting themselves. So we'll focus on the more interesting case where (3-b) has the granularity-insensitive truth condition stated in (32). Recall that (3-a) has the following meaning:

(33) $[\![\text{Mary is 180 cm tall}]\!] = \lambda C.\{s \in C | tall(s)(m) = s(180)\}$

Suppose the context is updated with the meaning above. There are two sorts of possibilities remaining in the context. The first sort is such that the granularities are coarser than 0.4 — so that the interval(s) denoted by '180 cm' extends beyond 180.2 — and Mary counts as 180 cm tall according to those granularities. But if the prevailing standard of precision corresponds to one of those granularities when (3-a) is considered, given (Granularity-Relevance), there are no relevant differences between the degrees in each unit interval. Given (Constancy), there remains no relevant differences between the degrees in each unit interval when (3-b) is considered. In which case, it is decidedly odd for the speaker to propose that we accept that Mary's precise height falls into the interval bounded below by 180.2 cm and above by the upper bound of the unit interval centered at 180.0 cm, because whether Mary's precise height falls into that sub-interval (in addition to its falling into the unit interval) is only relevant at a finer granularity. Since (Granularity-Relevance) and (Constancy) hold at any finer granularity, as long as we restrict ourselves to this sort of possibilities, we cannot preserve our assumption that the speaker observes Relevance and Quantity ('make your contribution as informative as is required') by entertaining a finer granularity.

The second sort of possibilities is such that the granularities are no coarser than 0.4 and Mary counts as 180 cm tall according to those granularities. None of these possibilities can survive update by (3-b). So if the prevailing standard of precision corresponds to one of those granularities, the speaker of (3) essentially proposes to update the context with a contradiction.

On this explanation, CNSs are infelicitous because they are baffling sentences to utter. They are baffling to utter because if the standard of precision is low, the concession clauses violate Relevance and Quantity, and if the hearer attempts to preserve their assumption that the speaker observes Relevance and Quantity by raising the standard of precision, CNSs eventually violate Quality ('do not say what you believe to be false').

# 6 Conclusion

The choice between the degree view and the interval view is not a matter of taste. While Carter's version of the degree view seeks to generate imprecise sentence contents out of precise sentence contents by appealing to relevance, the interval view holds that standard of precision is a primitive determinant of the contents of sub-sentential numerical expressions, and that a sentence's content, be it precise or not, is derived compositionally from the contents of its parts. Above I argued that the infelicity of sentences such as (1) – (3) does not favor the degree view, and showed that the interval view's bottom-up way of generating imprecise contents provides us with a straightforward explanation for the entailments between the imprecise contents of numerical sentences and the imprecise contents of comparatives. By combining the interval view with a dynamic semantics, I also accounted for the metalinguistic uses of measure phrases such as '160 cm' and of multipliers such as 'twice'.

I call sentences such as (1) – (3) 'concessive numerical sentences' because I find them strikingly similar to *concessive knowledge attributions (CKAs)* — sentences of form 'S knows that $p$, but it is possible that $q$ (with $q$ entailing not-$p$)' and their variants. Just as it is often argued that the infelicity of CNSs is evidence for the degree view, it is often argued that the infelicity of CKAs is evidence for *skeptical invariantism (SI)*,[33] the view that 'know' invariably demands the satisfaction of the highest epistemic standard. Both the degree view and SI have appealed to imprecision to account for the felicity of literally false simple positive sentences. Does SI's appeal to imprecision render incomplete their explanation for the infelicity of CKAs in the same way the degree view's appeal to imprecision renders incomplete their explanation for the infelicity of CNSs? Can a pragmatic explanation similar to our explanation for the infelicity of CNSs be given for the infelicity of CKAs? These are issues to be explored in future work.

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[33] See, for example, Dodd (2011) and Lossau (2021).

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