Descartes' Quantity of Motion: "New Age" Holism meets the Cartesian Conservation Principle

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Abstract (WORD COUNT: 91)

This essay explores various problematical aspects of Descartes' conservation principle for the quantity of motion (size times speed), particularly its largely neglected "dual role" as a measure of both durational motion and instantaneous "tendencies towards motion". Overall, an underlying non-local, or "holistic", element of quantity of motion (largely derived from his statics) will be revealed as central to a full understanding of the conservation principle's conceptual development and intended operation; and this insight can be of use in responding to some of the recent and traditional criticisms of Descartes' physics.

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While it is commonplace to praise Descartes for having brought conservation principles to the forefront of natural philosophy, his particular development or use of a conservation principle of moving bodies has generally received a less enthusiastic welcome. That is, even though the fruitfulness of the conservation "idea" was quickly perceived (and can be said to largely justify, say, C. Truesdell's judgment that "[Descartes' physics] is the beginning of theory in the modern sense"[1]), it is nevertheless also true that Descartes' handling of his conserved "quantity of motion", or size × speed, was soon challenged by a host of natural philosophers, both Cartesian and non-Cartesian.[2] Much of the early criticism centered upon the apparent incompatibility of the Cartesian conservation principle, and its attendant natural laws, with the basic operation or structure of Descartes' plenum (i.e., matter-filled universe). This line of critical analysis is still pursued by many commentators; such as, A. Nelson, who has charged that the very concepts of Cartesian matter and motion preclude the successful establishment of Descartes' third natural law, and by implication, the conservation principle.[3]

One of these puzzles, to be more precise, is the apparent dichotomy in Descartes' handling of bodily motion. On the one hand, he stipulates that motion is merely the reciprocal translation of a body from the vicinity of its contiguous neighbors (Pr II 25)[4], while, on the other hand, also insisting that God maintains an unchanging measure of motion in the universe (i.e., the conservation principle). But, how do these two descriptions, or hypotheses, interrelate? On the first hypothesis, the complexity of the entire range of plenum activity seems to be reduced to, and explained by, the *local* level of a relational change of "place". In contrast, the second hypothesis explains the local motions and collisions of bodies from the *global* perspective of a quasi-"property" conferred upon the plenum via God's "immutability" (Pr II 36): i.e., QM is a consequence of God's "sustaining act". In short, one approach is local, reductivist, and kinematic; while the other is global and metaphysical (or, non-reductive and non-kinematic). This underlying tension in Descartes' system can be discerned in the work of various commentators on Cartesian motion, although it is often not openly or deliberately acknowledged. In an influential essay by R. J. Blackwell, for instance, it is observed that "everything in the [Cartesian] universe other than extension is due to local motion."[5] In this same work, nevertheless, he concludes that "for Descartes the word 'conservation' [in conservation principle] refers more to something in God (i.e., immutability) than to anything properly inherent in the structure of the material universe." (223) Once again, how can motion be an entirely local affair, but also such that "if there were no God, the total quantity of motion may well not be conserved in the Cartesian universe"? (234)

Blackwell's analysis is typical of the work on Cartesian motion, and nicely demonstrates the subtle ontological and epistemological issues invariably raised by such investigations, both with respect to corporeal substance and God's sustaining act. In this essay, we will attempt to come to grips with at least one aspect of this complex metaphysical problem: the local versus the global, or reductivist versus non-reductivist, nature of Cartesian motion. Despite being long favored by commentators, as well as seemingly endorsed by numerous passages in Descartes' corpus, it will be argued that the local, reductivist reading just cannot make sense of the entire Cartesian conception of motion. A global, non-reductive, and somewhat "holistic" element pervades Descartes' thinking and approach--and the conservation of quantity of motion is the key to unlocking this unacknowledged aspect of his theory. In particular, Descartes' early work on statics will be of central importance for our investigation, since the quasi-holistic character of a system of bodies held in an equilibrium of forces, such as weights on a pulley or balance, retained a pervasive influence on his later approach to the problem of motion and force in the plenum. The blending of this statics outlook with his kinematical set of natural laws inevitably generated numerous difficulties and inconsistencies for Descartes' system, as will be in evidence throughout this essay. Nevertheless, acknowledging the dichotomous nature of Descartes' conservation principle for the quantity of motion will be shown to be oddly useful in the debates surrounding some of the alleged difficulties with Cartesian natural philosophy. (Henceforth, 'QM' will designate quantity of motion, and not the general notion of a conservation principle, which can take any number of forms different from "size times speed".)

Our investigation will bring together many seemingly disparate issues. In section 1, we will examine the origins and structure of Descartes' universally conserved motive quantity. This analysis will help to clarify the ontological status and unifying importance of QM in his universe, as well as its dual instantaneous/durational and dual static/dynamic character. In section 2, the conservation law will be put to work in order to resolve a series of alleged difficulties with Cartesian matter in its plenum setting. The aforementioned interrelated themes of reduction and holism will be a principal component of this investigation, and will be shown to play an important part in the relationship between Descartes' problematic thesis of local, relative motion and his conserved force.

1. Quantity of Motion in Cartesian Natural Philosophy

As D. Garber has carefully documented, Descartes' conservation principle only reached its canonical formulation, in his *Principles of Philosophy* (1644), after a long process of development and gestation. In his earlier, suppressed *The World* (1633), Descartes claims that God "always conserves [in all of matter] the same amount of [motion]". (AT XI 43) But it is not clear, at least in this work, that QM (size × speed) is the "motion" conserved.[6] More than likely, the source of Descartes' interest in conserved properties of motion was his friend and mentor, I. Beeckman, who formulated an early version of the momentum law, or size × velocity.[7] Regardless of the source of his inspiration, however, Descartes' references to a conserved QM become more explicit by the late 1630's (e.g., see AT II 543), culminating in an unambiguous presentation in the *Principles:*

> [Motion] has a certain and determinate quantity, which we can understand easily to be able to remain always the same in the whole universe, even though it may change in its individual parts. That is why we might think that when one part of matter moves twice as fast as another which is twice as large, there is the same amount of motion in the smaller as in the larger. (Pr II 36)

Of course, in this treatise, the conservation principle forms the foundation of the three laws of nature, which describe the basic motions of material bodies in the Cartesian plenum. QM is particularly conspicuous in the third natural law, and the accompanying seven collision rules which constitute its specific instances, since this law dictates that impacting bodies either retain all their motion or transfer as much motion as they lose (and thus conserve QM; Pr II 40). Descartes justifies the conservation principle through an appeal to God's "immutability", as he dubs it (Pr II 37). He states: "It seems obvious to me that it is none other than God himself, who created in the beginning both motion and rest in the whole of matter, and now preserves through his normal concourse alone the same amount of motion and rest as he placed in it at that time." (Pr II 36)

Yet, to a host of Cartesian scholars reared on the "a priori" epistemology of the *Meditations,* the form of Descartes' conservation law (for QM) must appear to be rather unmotivated and problematic. Whereas other Cartesian hypotheses could conceivably have a rational foundation based on God's immutability (albeit improbably; e.g., extension as the sole attribute of matter, Pr II 11, and even the existence of a conserved property of motion), it is not at all obvious, in the a priori sense, why the totality of plenum motion must equal the size $\times$ speed of all plenum bodies. Why this particular quantity? Why not "size + speed", or "size $\times$ speed^2"? In short, Descartes' conservation principle incorporates two key assumptions which demand justification: first, the choice of simples or primitives ("size", "speed"), and second, the mathematical relationship that ties the primitives together (product function, "$\times$"). As for the domain of primitives, Descartes reckons size and speed to be "modes" of extended substance (Pr II 27); which, in the *Principles* at least, are defined as the particular ways in which a corporeal body manifests its extension. Size and speed thus appear to fit into the Cartesian ontological scheme, since they are a natural correlate of his a priori identification of extension as matter's essential attribute. There are problems with this selection, however. Not only is Descartes' concept of "size" notoriously difficult to interpret, but other presumably basic properties of Cartesian matter and motion are excluded without explanation, such as the direction of motion (or, approximately, "determination").[8] A similar puzzle surrounds the choice of the product relationship, $\times$, to interrelate his simples. On the whole, it does not appear that any a priori justification can be given, based on God's immutability, for privileging the product function over the range of all possible quantitative relationships (e.g., addition, subtraction, division, etc.). This dilemma is nicely summarized by S. Nadler: "It is plausible to claim that [God's] immutability implies *some* conservation law. But that it implies the conservation of the total quantity of motion in the universe is questionable."[9]

If Descartes did not procure QM based on any sort of a priori reflection, on what grounds did he select this particular quantity (and from where did it originate)? The most plausible explanation is to assume that Descartes simply borrowed this quantity, which was well-known to his predecessors in both statics and impetus theory, and generalized it to cover all material interactions in his plenum. Descartes probably knew the many Scholastic theories grouped under the general title "impetus" from his early Jesuit training in natural philosophy. As a forerunner to the concept of inertial motion, and Descartes' own laws of motion, impetus was conceived as a force or quasi-property causally responsible for the "violent" motion of naturally resting terrestrial bodies (i.e., naturally at rest when confined to the surface of the earth). This force was often measured by the speed or velocity of a body and its quantity of matter, as in the case of John Buridan.[10] In statics, further quantities bearing a close resemblance to QM were the various dynamical renditions of the "law of the lever", later dubbed in their mature form, the principle of "virtual work". Statics is the branch of mechanics that examines the forces of bodies in equilibrium, such as weights suspended in a pulley or balance. Descartes was well-acquainted with the theoretical aspects of these types of simple machines, as is evident in several letters from the late 1630s. For example:

> To raise a 100-pound weight to a height of one foot twice is the same as raising a 200-pound weight to a height of one foot, and also the same as raising a 100-pound weight to the height of two feet. From this it clearly follows that . . . the force required to support it at a certain position and prevent it from falling, is to be measured at the beginning of the motion which the power supporting it must provide if it is going to raise it or if it is going to accompany its falls. (AT II 229)

The measure of Descartes' rendition of the law of the lever incorporates only very small displacements (along with weight), thus rendering it roughly analogous to the later virtual work principle: "Note that I say 'begin to fall', and not simply 'fall', since it is only the beginning of the fall [of the weights] that we need to consider."[11] (AT II 233) Following A. Gabbey, we will label Descartes' treatment of the lever law, the "General Statical Principle", or GSP.[12] The similarities between QM and the GSP are quite evident, as are their differences. Despite the obvious parallel use of the product function, the GSP's elements are weight and infinitesimal displacement (where "infinitesimal" simply denotes "minute" or "small", and not its later mathematical usage). QM, on the other hand, employs size and non-instantaneous speed (Pr II 39; see below).

Nevertheless, the differences between QM and the GSP are not as great as they might at first seem. In one sense, of course, QM is non-instantaneous; i.e., it includes the Cartesian concept of speed, which is only manifest over a non-instantaneous temporal period (as best exemplified in the collision rules, Pr II 46-52). Yet, QM is also closely tied to the instantaneous property[13] which we can loosely entitle, "tendency" (*tendere,* Pr II 39), although the *Principles* employs several designations interchangeably in depicting this notion (e.g., "striving", *conatus,* Pr III 56; "first preparation for motion", *prima praeparatio ad motum,* Pr III 63). Tendencies are involved or implicated in the (non-instantaneous) motions of bodies, and thus are carefully coupled to the states of affairs presiding at a single instant. He states: "Of course, no motion is accomplished in an instant; but it is obvious that every moving body, at any moment in the course of its motion, is determined (*determinatum*) to persevere in its motion in some direction along a straight line, ..." (Pr II 39) Given the context of this passage, Descartes seems to imply that a tendency has a determination (which roughly corresponds to the direction of motion, as above), for he also states in Article 39 that "each part of matter, . . . tends to persevere in its motion along only straight lines. . . ."[14] Furthermore, in discussing the motions of stars, he applies the name "agitation" (*agitation*) to signify the "force", or measure of the force, of these tendencies: "Once moved, gold, lead, or other metals retain more agitation, or force (*vis*) to persevere in their motion, than pieces of wood or rocks do of the same size and shape." (Pr III 122). The descriptions of the agitation force in this part of the *Principles* exactly match his earlier definition (in Part II) of a moving body's QM, which is a measurement of its "force to persevere (*perseverare*) in its motion, i.e., in motion at the same speed in the same direction" (Pr II 43). QM is hence a gauge of agitation force, while the agitation force, in turn, is a measurement of the instantaneous tendencies towards motion exhibited by Cartesian bodies.[15] To summarize, QM would appear to serve two primary functions: first, it is a measure of non-instantaneous size × speed; and second, it is a measure of instantaneous tendencies towards motion. With regards to instantaneous quantities in general, there is thus a close parallel between QM and the GSP.

In fact, bearing in mind that Cartesian motion does not take place at the level of instants, it becomes difficult to draw a sharp distinction between instantaneous tendencies towards motion, as measured by QM, and infinitesimal displacement, as incorporated in the GSP. If instantaneous tendencies towards motion are not "real" motions, in Descartes' sense, should we consider them to be more like an infinitesimal displacement (change of place) than a motion? We are venturing into uncharted waters, here, since there is ostensibly nothing in the Cartesian corpus to resolve such debates (although suggestions will be offered below). Descartes never fully addresses the many issues and problems that pertain to infinitesimals, and his *Géométrie* eschews all reference to them.[16] As mentioned, it is an open debate as to whether he conceived instants of time as absolutely instantaneous or including some temporal duration. Therefore, the relationship between tendencies and minute displacements remains mysterious, if yet somehow closely connected given both their infinitesimal nature and the close connections between motion, displacement, and time.

As for gravity or heaviness (*gravitas*), however, it can be stated with complete assurance that it is closely related to the Cartesian notion of size. The motion of bodies towards the center of the earth is a direct function, or consequence, of bodily volume, surface area, and quantity of second and/or third element matter; i.e., size. It is only in Parts III and IV of the *Principles* (and not in the presentation of the natural laws in Part II), that the complex interplay of these quantities is fully disclosed, along with their relationship to the natural (inertial) motions of bodies.[17] All in all, gravity is simply the process whereby slow moving terrestrial bodies are "pushed down", in a contact-mechanical fashion, by the faster moving plenum particles which rise to take their place. The greater "force" of motion of the plenum particles--i.e., their tendency "to persevere in" straight line uniform motion, as measured by QM--is the sole cause of the descent of terrestrial bodies, as well as the increasing velocity, or acceleration, of their free-fall over time (due to the incremental build-up of each separate "push"). Shape, volume, surface area, etc., play a role in determining the force of these bodily motion, consequently, since Cartesian matter is ultimately pure extension. In short, weight is as closely linked to size and speed, as instantaneous tendencies and QM are related to infinitesimal displacements and the GSP.

The moral of the story, thus far, is that Descartes largely based his conservation principle of QM on his earlier work in statics and the GSP, with an additional contribution possibly stemming from his knowledge of Scholastic impetus theories (especially those which employ QM-like quantities).[18] This assessment of the genesis of Descartes' ideas is not particularly unique or controversial, it should be noted, for similar conclusions have been advanced by previous commentators (see footnote 15). Westfall, for one, has commented that:

> size times velocity [or speed], derived from the law of the lever, bedeviled mechanical discussions throughout the century as scientists attempted to apply to real motions a quantity valid for virtual velocities [weight times infinitesimal motion]. Descartes also employed the concept as the force of a body's motion, and through that concept the statics of simple machines continued to dominate dynamics. (Westfall, ibid., 75)

Needless to say, such estimations are merely conjectural due to the lack of any overt reference in Descartes' work as to the source of his QM. But the "weight" of the evidence based on Descartes' scholastic training and his earlier written work strongly favors a GSP/QM correspondence, as Westfall suggests.

Granting this point should not lead one to conclude, however, that the GSP is somehow foundationally prior to QM as regards Descartes' overall ontological and epistemological scheme. He may have decided to base QM on his experience with the GSP, but the three laws of nature, including the conservation principle, remain the "a priori" cornerstone of his entire natural philosophy. The mechanics of simple machines and the GSP are derivative principles on this plan, as is revealed with notable clarity in his September 12, 1638 letter to Mersenne:

> Many people often confuse consideration of space [distance] with that of time or speed. . . . For it is not the difference in speed that makes it such that one of these weights has to be twice the other [with respect to a balance where the arm of the smaller weight is twice the length of the heavier weight], but the difference of space [i.e., distance through which the weights rise or descend]. . . . If I had wished to link considerations of speed to that of space, I would have had to attribute three dimensions to the force, instead of the two I had attributed, in order to exclude [speed]. . . . For it is not possible to say anything good and sound with respect to speed, without first having explained what weight [gravity] really is, and simultaneously the whole system of the world. (AT II 353-355)

In other words, whereas the force measured by the GSP mandates two "dimensions", weight and distance, the force of bodily motion under the influence of gravity (in raising bodies or in free-fall) requires the added dimension of speed. To adequately account for gravity, he continues, it is necessary to explain first his entire vortex system (Part III and IV of the *Principles*); which, respectively, is grounded upon the Cartesian natural laws (in Part II). The bodily forces involved in gravity, and the corresponding dimensions of weight, distance, and speed, thus depend (in a roundabout way) on QM. In essence, this conceptual scheme places QM at the ground level of Cartesian physics, since the GSP relies on the explanation of weight that only QM, via the vortex theory, can provide.[19]

This assessment of the relative positioning of Descartes' physical principles has been duly noted by A. Gabbey, whose work in this area of Cartesian science has set new standards of scholarship. Gabbey draws a further inference, however, which we will need to address before proceeding to the next section. He states:

> The GSP, Descartes' version of the work principle, is not a *lex* or *principium physicae* in the sense of being irreducible to other than metaphysical considerations. It is a lawlike empirical rule that explains the operation of devices under the inviolate influence of gravity. For Descartes the irreducible *leges physicae* are centrally the three laws of nature set out in Part II of the *principles,* and they are necessarily true of any divinely created possible world containing *res extensae* in motion. On the other hand, the GSP is contingently the case for the actual world God has chosen to create. (1993, ibid., 320)

Gabbey seems to believe that, unlike the natural laws, the GSP is not one of Descartes' "eternal truths" (or, at the least, one of those truths or features of the world that must remain the same in all the worlds God could create).[20] This estimation of the ontological and/or epistemological status of the GSP is not warranted by the evidence, I would argue. Although it is clearly the case that the natural laws possess some form of necessity, Descartes' paltry references regarding the underlying principles of simple machines simply do not rule out the possibility that the GSP is likewise an invariant truth of all divinely created possible worlds. If Descartes would have finished the project started in the *Principles,* and included a comprehensive treatment of mechanics (as confessed in his preface to the French edition, AT IXB 14), this may have decided the matter one way or the other. As it stands, however, there seems as much reason to judge the GSP "a necessary feature of all God's possible worlds" as there is to rule it out.

In fact, given its close kinship to QM, and his analysis of the dimensions of force, it might be more plausible to rank the GSP as a feature of Descartes' world equal in necessity to QM and the natural laws. It would seem odd, for instance, that in moving from the three dimensions of the bodily force of gravity--a system fixed by the necessity of the natural laws--we arrived at a two-dimensional statics level whose corresponding force is "contingently the case for the actual world God has chosen to create". Put briefly, Descartes' statics seems invariably bound to his analysis of gravity; with the latter being, presumably, a necessary consequence of his natural laws. Moreover, even if a different vortex system were compatible with the same natural laws (which might precipitate a different account of gravity and its three dimensions), it is still hard to envision how Cartesian statics could vary independently of Cartesian gravity: as a sort of two-dimensional restriction, or special case, of three-dimensional gravity, there is good reason to believe that what is true at the level of three will also be true at the level of two.[21]

2. Quantity of Motion in the Cartesian Plenum.

In the previous section, two distinct aspects of Descartes' QM were discerned: the measure of a body's non-instantaneous, durational, size × speed; and the measure of a body's instantaneous, non-durational, tendency towards straight-line uniform motion. In the present section, we will explore in more depth how QM's dual nature can be exploited to counter a range of perceived difficulties with the Cartesian conservation principle (and QM itself). In particular, section 2.1 will provide a lengthy examination of A. Nelson's critique of the conservation principle; while sections 2.2 and 2.3 will explore the difficulty of upholding the principle given Descartes' theory of relational motion. The main burden of our attempts to resolve some of QM's problems will be carried by, for lack of a better word, a "holistic" component of QM. Yet, as will become evident, the advantages that accrue to a holistic interpretation of QM (in meeting these challenges) cannot ultimately overcome the basic inconsistency of Descartes' dichotomous local-kinematic/global-dynamic conception of his conservation principle.

2.1 Micro-Chaos. Idealization in Cartesian physics, or its failure, forms the backdrop of Nelson's foray into the vagaries of Descartes' collision theory. Among other things, Nelson contends that the stipulation in the *Principles* for perfectly hard bodies, and for motions unaffected by plenum crowding (Pr II 45), are not sufficient in themselves to permit the successful application of Descartes' natural laws. In order to demonstrate how the holistic properties of QM can be of service in countering Nelson's allegations, it will be necessary to provide first a careful presentation of the many concepts involved. For our purposes, the third natural law, and its accompanying seven collision rules, will be of primary concern, since this law prescribes the conservation of QM for each particular bodily collision (Pr II 40). Nelson argues, with some justification, that it is probably only at the smallest level of Cartesian material substance, the first and second elements, that actual observation of plenum interactions would conform to the predictions of the natural laws. (See, Pr III 52, for the tripartite classification of Descartes' material elements.) At this minute scale, Cartesian elements are not only "harder" than the bodies they comprise (where "hardness" is defined as the lack of relative motion among the constituent parts, 378), but the numerous channels and pores in large bodies probably facilitate their motion more readily, thus guaranteeing a closer approximation to the isolation criterion.

Unfortunately, Descartes' rejection of atomism (Pr II 20), and his belief that first element particles can be "indefinitely small" (Pr III 52), would seem to preclude the realization of perfectly hard material substances, or, to use Nelson's term, "impenetranium". This is only the beginning of the difficulties, however. Any attempt to apply the natural laws to, say, a closed loop of circling bodies bounded by a channel of hard matter (a favorable idealization in its own right), will be inevitably undermined by the complex motions of the first element particles, a process entitled, "micro-chaos".

> Real channels are to some degree soft, and generally in some kind of motion [i.e., they are not impenetranium]. This means that many of the tiny particles of the first element that are being scraped or shorn off the bodies in the circuit [i.e., closed ring of circling matter] will be continually seeping out of the channel forming tiny new circuits of their own. Now none of these tiny circuits will be encased in perfectly uniform channels of impenetranium, so further even smaller particles will be generated some of which will seep out of their channels and so on *ad indefinitum.* There never is a smallest level at which angels could use the laws exactly to calculate how bodies will collide.

Put differently, the indefinite divisibility of plenum matter, which Descartes sanctions as a real process (in order to fill-in the vacuities between larger elements, Pr II 34), undermines the application of natural laws *in principle.* No matter what scale or level of Cartesian matter one considers, the natural laws cannot be successfully applied because the material debris or scrapings that naturally result from the bodily motions at level 1--and which form an even smaller layer of particles at level 2--will escape the application of the laws at level 1 (and so on for level 2, and level 3, etc.). "Micro-chaos quite clearly entails that exact calculations of what is happening to any object at the micro level is utterly impossible", and hence "Descartes' laws are every bit as inapplicable at the micro level, even in principle, as they are at the macro-level." (382-383)

Before venturing to discuss how this critique affects the conservation of QM, a few closely related issues need to be discussed. First of all, it is assumed throughout Nelson's article that Descartes' concept of "perfect hardness" (where the terms *solidus* and *durus* are used interchangeably in the *Principles*) only pertains to the degree of relative motion among a body's constituent particles. Based on this interpretation, and Descartes' admission that "all the variety in matter . . . depends on motion" (Pr II 23), he infers that all material bodies must have some internal motion (if not, then gold would be identical to wood, for example). Yet, as recently argued (in Slowik 1996, reference in footnote 8), there is good reason to view Descartes' perfect hardness idealization more as a means of obtaining a simplified measure of a body's quantity of second/third element matter to total volume than as a stipulation dealing exclusively with the relative motions of particles (or worse, an elastic/inelastic impact property). The evidence of the entire *Principles,* and not just Part II, strongly favors this "quantity of matter to volume" reading of perfect hardness, especially if the theory of stellar motion detailed in Part III, Articles 120-125, is taken into account.[22] Therefore, Nelson's contention that "given Hardness, . . . , it is conceptually impossible that [Descartes'] laws apply to any bodies not consisting of impenetranium" (379) would seem to miss the point of the perfect hardness idealization. This criterion is aimed at getting a manageable grasp on that most unmanageable of Cartesian notions, "size", as is further evidenced by his stated rationale for the idealizations: i.e., "Our calculations [of QM and the natural laws] would be more *easy* if . . . ." (Pr II 45, emphasis added) Descartes does not argue that perfectly hard macroscopic bodies *do* exist; in fact, he seems to suggest that there are no such macro-scopic bodies (Pr IV 132). Rather, immediately *after* defining QM, and *prior* to laying out the collision rules, he argues that the calculation of QM would be made easier *if* the bodies were perfectly hard. The hardness and isolation (from plenum interference) criteria are idealizations in the following sense: as macro level bodies approach perfect hardness, the calculations of QM will become simpler and more accurate; and the less their motions are impeded by plenum crowding, the closer their impact behavior approximates the predictions of the seven collision rules. This reading of the hardness and isolation idealizations would seem to be born out by his admission, just after submitting the collision rules, that "it is very difficult to perform the calculations [due to the interference of other bodies]. . . ." (Pr II 53)

One might insist, however, that such idealizations simply cannot be met in Descartes' plenum, and hence the natural laws remain inapplicable to any real, non-idealized situation. This claim may be partially true, but it is hard to see how this problem is unique to Cartesian physics. This last response leads us naturally to our second point concerning Nelson's critique: In short, how do Descartes' idealizations differ in kind from, say, the Newtonian utilization of "point particles" (which Nelson reckons to be a successful instance of idealization in physics)? He states, "for ordinary bodies, we can calculate a pointlike center of mass that enables us to treat ordinary bodies as if they were themselves point masses." (380) The correctness of this Newtonian idealization is grounded on the following thesis: "Since Newton, it has been characteristic of good idealizations that they permit calculations that hold approximately for real systems, and that the calculations become more exact the more closely the real system resembles the idealization." (387) Yet, if our earlier analysis of Cartesian idealization is correct, then this is exactly what Descartes is attempting to do. Not only is a perfectly hard and isolated Cartesian body an effort to approximate a real system (such that "the calculations become more exact the more closely the real system resembles the idealization"), but it seems to be an idealization exactly akin to the equally-unrealizable Newtonian point mass.

Some of the confusion may stem from the fact that Descartes's isolation criterion seems so alien in comparison to the historical examples often exhibited as "correct" idealizations in Newtonian mechanics; namely, the use of point masses to model complex gravitational systems, such as the solar system. Of course, the development of theoretical mechanics, and the success of Newton's gravitational theory, did largely depend on the point mass approach to the interactions of planetary bodies. But, it is also true that the equally-profitable study of the mechanics of continuously extended materials, i.e., continuum mechanics, employed a very different set of idealizations to a more humble (but no less difficult) range of material phenomena, such as elastic solids and fluid flow. The mathematical models of these phenomena often invoke a criterion of "ideal materials" in an endeavor to get a workable grasp on the extremely complex interplay of material forces and factors. For instance, the interaction of mechanical and thermal processes are often minimized, or simply neglected, in order to focus attention exclusively on the statical and mechanical features of the system. Such idealization, moreover, are not point-like, but extend over an entire body or material. In an analogous sense, Descartes' appeal to perfect hardness and isolation can be seen as a crude forerunner to the types of idealization that concern continuum mechanical theorists. As in the modern theory, Descartes is striving to minimize the number of complicating factors involved in the behavior of extended material bodies. Neglecting thermal processes is thus the analogue of neglecting plenum interference: both are impossible per se, but they can approximate the behavior of bodies reasonably well, and thereby provide useful information on their respective systems.

The analogy between Descartes' plenum physics and continuum mechanics should not be taken to extremes, of course, since they differ on matters of more than just historical origin. Yet, there is a vague Cartesian-esque quality to many of the concepts in the modern theory: for example, an elementary textbook comments that "observed macroscopic behavior [of bodies] is usually explained by disregarding molecular [or particle] considerations and, instead, by assuming the material to be continuously distributed throughout its volume and to completely fill the space it occupies."[23] Correlating spatial volume with indefinitely-divisible matter is quite Cartesian, needless to say, but one has to stretch the interpretation of this passage to encompass Descartes decidedly corpuscularian thesis of the three material elements. Even if we concede this last point, there is a sense in which Descartes does disregard the individual, local behavior of bodies (and elements) in favor of a "holistic" treatment of plenum phenomena, much like the continuum theorist's continuously distributed matter. We have already mentioned, in passing, one instance of this type of thinking: i.e., the quasi-miraculous manner in which the first element particles fill in the gaps left in the wake of larger, moving elements. It is hard to see how this global feature of plenum activity could be derived exclusively from the local behavior of individual first element particles, especially when it is recalled that their behavior is governed by the natural laws alone (i.e., rectilinear uniform motion that conserves QM). This problem would seem to have caught Nelson's attention as well, for he offers it as an alternative interpretation of his micro-chaos argument (382-383).

How can Cartesian bodies harmonize their collective motions so effectively when their individual motions are explained *only* by the three Cartesian natural laws? In the absence of any alternatives, the likely response to this question is to invoke Descartes' conservation principle of QM. In other words, the synchronized motions of the first element particles are merely the result, or consequence, of the conservation of QM at each moment. This non-local, or holistic, property of the conservation principle will figure prominently throughout the remainder of our investigation; and, now that we have prepared the context, we may proceed to demonstrate its potential with regard to Nelson's (first) critique of plenum motion.

All told, the success of Nelson's dilemma relies on the indefinite divisibility of Cartesian matter: since we can never arrive at a smallest level of particles, we can never guarantee that both the hardness and isolation idealizations are being met, and thus that the natural laws are being upheld. Nevertheless, QM is not only conserved in the plenum as a whole (Pr II 36), but also in each individual collision (e.g., the third natural law: "if a body collides with another which is stronger than itself, it does not lose any of its motion, but if it collides with a weaker body, the quantity of motion it loses is equal to the amount it imparts to the other body", Pr II 40). Accordingly, QM should be conserved for any given portion of the plenum, as long as the isolated portion is bordered by bodies, or a body, larger than the bodies contained within that portion. This condition might seem odd, but it follows naturally from the third law, and the fourth collision rule (Pr II 40, 49); which holds that a smaller, moving body rebounds without losing any speed after impact with a larger, stationary body. (It will be assumed throughout that the larger body is at rest.) With a larger body lining the isolated section of the plenum, all the collisions against that border by the smaller bodies located within the section will rebound without losing QM. Therefore, QM will be conserved by the totality of bodily interactions within that plenum section. This is the crucial step, for it guarantees (via the conservation principle) that there exists a fixed or determinate value of QM within the plenum portion (but only as long as the bordering body remains in place). Returning to Nelson's argument, although it is true that there is an ever shrinking level of Cartesian particles to examine, the conservation principle ensures that the values obtained from the measurement of QM will *approach a fixed value* with each descending level of particles we measure. That is, if one adds the measure of QM of each decreasing level of particles to the total QM of all the previously measured levels, then one will inexorably draw closer to a limiting numerical value of QM for the entire material contents of the plenum portion. Mathematically, this could be represented as an infinite series that approaches a limiting sum: e.g., $1 + 1/2 + 1/4 + \ldots + (1/2)^n + \ldots = 2$. As in the mathematical case, one never reaches a last member of the series (or a last layer of Cartesian matter), but the series (and plenum portion) have a limiting sum, nevertheless. This interpretation might draw textual support from the fact that Descartes believed the plenum (=space) to be "indefinitely" extended (Pr II 21), despite having a fixed, and presumably finite, value of QM. Descartes' theory is not unique in this appeal to infinite processes, furthermore. In short, the indefinite divisibility of Cartesian matter is no more a problem for Descartes than are the infinite, and equally unintuitive, consequences of many other scientific theories; e.g., Newton's theory of gravitation, where a single body is subject to the gravitational attraction of a possibly limitless number of bodies distributed through his infinitely extended space.

2.2 Relational Motion and QM. Consequently, if we bear in mind the global, or holistic, capacity of the conservation principle to conserve QM over an entire region (and not just locally), then we must reject the claim that the Cartesian natural laws are, in principal, inapplicable. Nelson's inquiry is nevertheless valuable in bringing to light the monumental difficulties that beset the determination of QM given the quirks in the

Cartesian scheme. Foremost among these difficulties is Descartes' handling of relational motion, a subject to which we now turn.

In the *Principles,* Descartes defines "external place" (Pr II 13) as the boundary between an object's surface and the surface of its contiguous neighbors, while motion *"is the transfer of one piece of matter or of one body, from the vicinity of those bodies immediately contiguous to it [roughly, external place] and considered at rest, into the vicinity of others."* (Pr II 25) This account leads to relationalism since, "we cannot conceive of the body AB being transported from the vicinity of the body CD without also understanding that the body CD is transported from the vicinity of the body AB." (Pr II 29) As is well-known, Descartes' natural laws apparently contradict this relational view by attributing individual, determinate states of motion to individual bodies (when only relative motions are allowed). For instance, the second law holds that "all movement is, of itself, along straight lines. . . ." (Pr II 39) Yet, since trajectories are determined relative to observers, and all observers move relatively, there can be no "unique" measurement of a single body's trajectory for a relationalist. Consequently, Descartes' second law appears to violate relationalism, as do many of the collision rules from his third law (which seem to posit "motion" and "rest" as absolute states, as well).[24]

Before turning to the obstacles of conjoining the conservation principle and a relationalist construal of motion, one of the chief advantages of this marriage should be duly acknowledged. It has occasionally been charged that, unless egregious ad hoc reasoning is introduced, Descartes' theory cannot prevent the possible formation of a single, synchronous plenum motion.[25] On this scenario, although QM is conserved, all the plenum occupants have entered into a synchronized motion of identical speed and direction (presumably circular). The fact that Descartes' natural laws cannot rule out this disturbing possible state-of-affairs, which would effectively eliminate all diversity in the universe, is hence regarded as symptomatic of the untenable character of his basic premises. Oddly enough, while the non-durational "tendency" component of QM may be susceptible to this objection, the coupling of the durational measure of QM and relational motion would seem to be immune. Since (non-zero) QM determined over time requires motion, and motion is a change of relative neighborhood, the conservation principle mandates a total value of QM that accrues from variant individual bodily motions, and not, moreover, from a motion of the plenum en masse. In other words, a mass plenum motion is no motion at all, at least for a relationalist of Descartes' stripe. When all bodies move identically in this manner, there is no change of contiguous neighborhood, thereby entailing a total QM value of zero--which is in direct violation of the conservation principle. This line of reasoning is reminiscent of Leibniz' argument against Newton's absolute space, and may constitute one of the few advantages of Descartes' adopted relationalism.[26]

Nevertheless, the overall relationship between Descartes' relationalism and his conservation principle is quite puzzling, prompting Westfall to comment (among others): "For any two bodies, the quantity of motion varies with the frame of reference; if motion is in fact relative, quantity of motion cannot be an absolute value." (1971, 151) Westfall is surely justified in questioning the relational compatibility of the Cartesian natural laws.[27] The problem with this line of inquiry, however, is that it assumes, perhaps anachronistically, that Descartes' QM must correspond to the modern notion of an "invariant" quantity; i.e., as a sort of kinematic property reducible to the individual, durational motions of bodies before and after impact.[28] Put differently, the universal value of QM can be *reduced* to the sum of each individual body's size $\times$ speed (which is why the arbitrariness of relational determinations of motion undermines QM). While this type of thinking may be more typical of later theories, such as Newton's momentum law (size $\times$ velocity), it does not seem to capture fully the multi-faceted Cartesian use of this concept. For Descartes, as disclosed in Section 1, QM has two distinct, if interrelated, personalities: an instantaneous determination of a body's perseverance in straight line uniform motion, and a durational measure of a body's size $\times$ speed. The latter notion may have stemmed from his knowledge of Scholastic impetus theories, but the former concept appears wedded in his statical training. When the force of a body to descend is balanced against that of another, as in simple machines, the resulting quantities or "invariants" of the system take on an infinitesimal or instantaneous form (as in the case of the virtual work principle). What is important to note about these quantities is that they require a system of bodies whose interactions are interconnected or harmonized in a kind of "holistic" fashion. This is not the type of model that is readily derived from experience of the kinematical impact of individual moving bodies, as employed by Newton's, or even Descartes', collision rules. Rather, the arrangement of bodies in a balance will only allow constrained motions of the entire assemblage as a whole: i.e., to analyze the motion of one body, you have to indirectly take into account the movements of all bodies--such that, if one descends, the other rises (at least hypothetically via the law of the lever, and for minute distances or speeds). Consequently, while its exact relationship to relationalism remains inherently problematic, Descartes' instantaneous version of QM implies a system whose overall conserved force cannot be simply reduced to the isolated, individual motions of bodies.

In an ironic twist, the instantaneous tendencies of Cartesian bodies and God's sustaining act are thus similar in their holistic implications. Just as God's role in conserving QM cannot be simply reduced to bodily motion (since God's actions are not mere material phenomena), Descartes' instantaneous version of QM depends on a view of plenum interactions that is equally irreducible to individual bodily motion (since one has to examine the entire interconnected system). Descartes' next crucial step, some might say fatal, is to extend this holistic interpretation of instantaneous tendencies to cover the durational, size × speed, motions of bodies, as well. Much of the analysis of motion in the *Principles* exhibits this statics-based approach. For example, he summarizes at the end of Part III that "for as all bodies in the universe are contiguous and act upon one another, the motion of one body is dependent on the motion of all the others, and therefore varies in numerous ways." (Pr III 157).[29] In a further analogy, this holistic emphasis is presented in a slightly more detailed manner. With respect to a circle of particles, with positions labeled EFGHI spaced along its circumference, he states: "If the air at E is pushed towards F, the air at F will circulate in the direction of GHI and return to E, such that one cannot feel its weight, in the same way that one cannot feel the weight of a rotating wheel if it is balanced perfectly on its axle." (2 June 1631, AT X 205) To envision the motion of a series of plenum particles as involving so little effort, i.e., as if they had no weight, strongly attests to the non-local synchronization or harmony of Cartesian motion (as well as to the continuing influence of simple machines on Descartes' thinking).

This global, as opposed to local, feature of Cartesian natural philosophy is seldom discussed in the literature, although the work of a hand-full of commentators has investigated holistic notions as they pertain to the issue of teleology in general (but not specifically to the problem of QM as addressed here).[30] More typical of the attitude of commentators is a reductionist reading, as powerfully presented by E. Grosholz in commenting upon Cartesian matter and motion: "the whole is not greater than the sum of its parts, and there is no systematic interdependency among the parts, or among the parts and the whole." (ibid., 76) C. Merchant takes a similar line, contrasting the reductionist particle-based approach of the mechanical school with what she takes to be the anti-mechanist, "holistic" outlook of modern ecology. As for the latter, "no element of an interlocking cycle can be removed without the collapse of the cycle. . . . Each particular part is defined by and dependent on the total context. The cycle itself is a dynamic interactive relationship of all its parts, ..."[31] Ironically, this definition of a holistic system seems a more apt description of Descartes' cosmos than of our experience of ecological systems: e.g., the removal of a part of the Cartesian plenum would violate the conservation principle (and may derail the large scale circular motion of bodies, Pr II 33, or lock-up the harmoniously arranged vortex rotations, Pr III 65-66), whereas we know of many plant and animal species whose demise did not appreciably affect the larger ecosystem.

In all fairness, Grosholz' judgment is largely based on the vexed issue of individuation in the plenum (i.e., that bodies are individuated by motion, but motion is a relation among a bodily defined "place", Pr II 25), and Merchant does hint at, if only just (204), a possibly more complex interpretation of Descartes' brand of mechanism. These scholars have raised legitimate concerns, it should be acknowledged, since the potential circularity of Descartes' definitions, and the naive mechanism of some of his biological and physical models, constitute serious obstacles to the success of his program. But, Grosholz and Merchant extend the charge of crass reductionism to the entire Cartesian scheme of natural philosophy, and thus overlook the very tangible "systematic interdependency among the parts" revealed in the workings of QM. In the case of Merchant, for instance, some of the difficulties may lie in her characterization of the general features of Early Modern mechanistic thought, where it is alleged that "nature is made up of modular components or discrete parts connected in a causal nexus that transmitted motion in a temporal sequence from part to part." (228) This is not an accurate description of Descartes' *overall* theory, as should be evident from section 1, for Cartesian bodies interact simultaneously as a connected whole, and not "in a temporal sequence from part to part".[32]

2.3. Kinematics or Dynamics: The Letter to More. Unfortunately, even if we acknowledge the global character of QM, this does not render the conservation principle any more hospitable to relational motion. The critic will surely respond that, even granting an entire synchronized plenum, QM must be determined by the motions of bodies--and motion is a relational change of place, thus raising once more the specter of arbitrary QM measurements. The instantaneous-tendency measure of QM may escape this problem, since as a "virtual" motion it presumably does not necessitate an actual change of place. Yet, the durational-"size × speed" version of QM clearly cannot fall back on a similar line of defense, which raises the difficulty first mentioned in the Introduction: How can motion be both a relational change of place and a conserved quantity? It would be presumptuous to declare that there exists an easy, if not definitive, answer to this major puzzle underlying Cartesian physics. Nonetheless, as argued above, the very question assumes that force, as measured by QM, can be entirely reduced to the sum of the size × speed of individual bodies--and, besides the arguments previously examined, there may exist textual evidence that contradicts this very assumption (at least in one notable, seldom discussed, instance).

In the late correspondence with More, Descartes' relational theory of motion faced one of its only serious documented trials. Provided that translation of a body is merely reciprocal (as change of place), More asked how one should regard the case of wind blowing through a tower window (5 March 1649, AT V 312): Is this motion reciprocal as well, such that the air can be viewed as resting and the tower in motion? Descartes' response reveals much about the interrelationship of kinematic and dynamic factors in Cartesian physics. Rather than answer More's question straightforwardly as a kinematics puzzle, he attempts to resolve the problem by resorting to a different, dynamics-based example. He asks us to picture two men attempting to free a grounded boat; with a man on board pushing against the shore, and the other on shore pushing against the boat:

> If the force [or strength (*vires*)] of the men is identical, the effort of the man on the shore, who is thus connected to the land, contributes no less to the boat's motion that the effort of the man on the boat, who is transported along with it. Therefore it is obvious that the action by which the boat recedes from the shore is equally in the shore as in the boat. (15 April 1649, AT V 346)

Remarkably, Descartes' example employs a dynamics solution to an originally conceived kinematics problem: the reciprocity of the translation of the boat and shore, which is a rather embarrassing kinematical ramification of his espoused relationalism, is reinterpreted as a reciprocity of force between two possible dynamic sources of the motion, namely the push of each man.

One might view this "boat" example, as we will call it, as a sort of unintended conflation of the kinematic and dynamic elements of his theory, which is the suggestion offered by Shea (1991, 323). While this may be true, if we take the passage at face value (i.e., literally construed), then the very kinematic/dynamic distinction in Cartesian physics is seen in a different light, especially as it pertains to relational motion. The reciprocity "process" now appears to be directed at QM, as the measure of force, as much as it is directed at the kinematics of transfer. This realization has important consequences for our investigation, since it demonstrates that the force of motion *cannot be simply reduced* to the kinematical size × speed of individually moving bodies. At the very least, Descartes' "boat" example compels us to consider the force of motion, and thus QM, as ontologically on a par with the kinematics of motion. QM can be measured by size × speed in certain situations, as in the durational interactions of bodies, but QM is far too complex a notion to be fully encompassed as *just* the product of size and speed. Put simply, to regard QM as the mere byproduct of the reciprocity of translation is to envision Cartesian (relational) motion as somehow ontologically/epistemologically complete in itself (or self-evident, prior, etc.)--but Descartes' appeal to the dynamical factor in his "boat" example (to explicate his kinematics) undermines the plausibility of this hierarchical ordering.[33]

3. Conclusion

Upon close examination, Descartes' conserved force of motion, QM, can be seen to harbor an intractable "qualitative" bias, which contrasts with the "quantitative" connotations that accrue from his formula for measuring QM, namely size × speed. Like many other Cartesian concepts, the conservation principle bears the vestigial imprint (if only just barely) of an unwittingly assimilated Scholastic outlook. The qualitative aspects of Descartes' conservation principle have been acknowledged before, principally as regards God's sustaining role (see footnote 30), but it has been the goal of this investigation to disclose some of these details as they pertain to the actual conserved value, QM, and its function in the Cartesian plenum. Not only is the conservation principle more complex and potentially resilient than some of its critics have charged (such as Nelson), but the traditional interpretation of QM as the product of size and durational speed misses much of functional scope of the concept. The instantaneous interconnected-ness of the Cartesian plenum, much like the simple machines his conservation principle may have been modeled upon, defies a classification purely along the lines of "size × speed". The letter to More would likewise appear to rule out any easy reduction of Cartesian dynamics to the kinematics of reciprocal change of place. As for its drawbacks, on the other hand, it would seem that the very Cartesian successes disclosed in our investigation have come at the expense of a conflating of the kinematic and dynamic aspects of Descartes' natural philosophy. And, indeed, a devoted Cartesian may wonder if a literal interpretation of the More letter is too high a price to pay for an effective means of handling the relational motion quandary.

In fact, as hinted above, there is a strange sense in which Descartes' conception and application of his conservation principle resurrects a sort of truncated version of a Scholastic substantial form. Despite his often repeated aversion to positing "mentalistic" substantial forms to *individual* bodies in order to explain their behavior, such as weight (e.g., *Sixth Replies,* AT VII 441-442), Descartes nevertheless invokes a force in the *whole of matter* that possess many of the characteristics of a substantial form: first, it is a quasi-"property" coupled (by God) to a pre-existing material substratum, the world (in this case); second, its teleological-like conservation of QM is irreducible to matter and motion alone (since God's sustenance is required, not to mention the import of the More letter). This "substantial forms" analogy should not be taken to extremes, however, for Descartes categorically rejected any such ontology. In his final letter to More, for instance, he admits that he had been previously disinclined to discuss his views on force out of a "fear of appearing disposed to favor the beliefs of those who regard God as a world-soul conjoined to matter." (July 1649, AT V 404) QM may not be God's soul united with the material world, yet many of the holistic, irreducible, and "mysterious" features of QM nevertheless betray a strong teleological predilection.[34]

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ENDNOTES

[1] C. Truesdell, "Experience, Theory, and Experiment", in *An Idiot's Fugitive Essays on Science* (New York: Springer Verlag, 1984) 6. Truesdell judges Descartes' physics to be a "failed attempt", however.

[2] Among the most notable of these early critiques, and portentous for the development of science, are the works by Huygens (a Cartesian, of sorts), Newton, and Leibniz (both adamantly non-Cartesian). See, C. Huygens, *De motu corporum ex percussione,* in *Oeuvres Complètes,* vol. 16 (La Haye: Société Hollandaise des Sciences, 1950); G. W. Leibniz, "Critical Thoughts on the General Part of the Principles of Descartes," in *Leibniz: Philosophical Papers and Letters,* ed. by L. E. Loemker (Dordrecht: D. Reidel, 1969); I. Newton, *De Gravitatione et aequipondio fluidorum,* in *Unpublished Scientific Papers of Isaac Newton,* trans. and ed. by A. R. Hall and M. B. Hall (Cambridge: Cambridge University Press, 1962). These works also address one of the other frequent objections to Descartes' conservation principle: namely, although it is true that there exists an invariant universal motive force or measure of motion, quantity of motion is not it. This topic will not be a primary concern of this essay, since it is a rather large and separate problem.

[3] A. Nelson, "Micro-Chaos and Idealization in Cartesian Physics", *Philosophical Studies,* 77, 1995, 377-391.

[4] I will identify passages from the *Principles* according to the following convention: Article 15, Part II, will be labeled "Pr II 15." Translations are based on the Adam and Tannery edition of the *Oeuvres de Descartes* (Paris: Vrin, 1976), with passages marked, "AT", followed by volume and page number.

[5] R. J. Blackwell, "Descartes' Laws of Motion", *Isis,* 57, 1966, 222.

[6] Descartes does refer to a "quantity of motion" in this section from *The World* (*quantité de mouvemens,* which is strangely plural), but, once again, it is unclear if size × speed is what is meant.

[7] See, I. Beeckman, *Journal Tenu par Isaac Beeckman,* 4 vols. (The Hague: M. Nighoff, 1939-53), vol. III, 129. Although these statements of a conserved quantity date from the late 1630's, he probably had discussed such concepts with Descartes at a much earlier time. In addition, Beeckman's conservation principle is unusual in that he thinks that the impact of his hard bodies would ultimately result in a loss of motion if it were not for the infinite amount of motion God placed in the universe at its creation (which continuously "recharges" the bodies to maintain the same amount). On this theory, see A. Gabbey, "Essay review of W. L. Scott's, *The Conflict Between Atomism and Conservation Theory: 1644-1860," Studies in History and Philosophy of Science,* 3, 1973, 373-385.

[8] See, E. Slowik, "Perfect Solidity: Natural Laws and The Problem of Matter in Descartes' Universe", *History of Philosophy Quarterly,* 13, 1996, 187-204, for an attempt to clarify Descartes' notion of size. See, P. Damerow et al., *Exploring the Limits of Preclassical Mechanics* (New York: Springer-Verlag, 1992) 103-126, and D. Garber, *Descartes' Metaphysical Physics* (Chicago: University of Chicago Press, 1992) 188-193, on Descartes' concept of determination and its status as a mode of corporeal substance.

[9] S. Nadler, "Deduction, Confirmation, and the Laws of Nature in Descartes' *Principia Philosophiae", Journal of the History of Philosophy,* 28, 1990, 366 (Nadler's emphasis). We will not enter into the complex debate on the alleged "necessity" of the laws of nature, i.e., how they resemble, if at all, the other so-called necessary truths of the Cartesian scheme, such as geometry or substance. For more on this complex problem, as well as on the relationship between the conservation principle and a priori truths based on God's immutability, see: C. Normore, "The Necessity in Deduction: Cartesian Inference and its Medieval Background", *Synthese,* 96, 1993, 437-454; B. Dutton, "Indifference, Necessity, and Descartes' Derivation of the Laws of Motion", *Journal of the History of Philosophy,* 34, 1996, 193-212; and, J. Broughton, "Necessity and Physical Laws in Descartes' Philosophy", *Pacific Philosophical Quarterly,* 68, 1987, 205-221. These studies reach similar inconclusive or negative results on the potential a priori grounding of QM.

[10] It us unclear, however, if Buridan utilized the *product* of velocity and quantity of matter for his measure of impetus. See, e.g., M. Clagett, *The Science of Mechanics in Middle Ages* (Madison: University of Wisconsin Press, 1959) chap. 8. Also on impetus theory, see A. Maier, *On the Threshold of Exact Science,* trans. by S. D. Sargent (Philadelphia: U. of Pennsylvania Press, 1982) chap. 4.

[11] Descartes' treatment lacks the mathematical concepts of the calculus which Jean Bernoulli employed in formulating the first "official" virtual work principle, or "virtual speed", as he called it. Here, Descartes' use

of minute displacements almost certainly drew on the similar, earlier work of Guido Ubaldo, for he refers to Ubaldo in the October 11 1638 correspondence to Mersenne (AT II 380-399). For more on Ubaldo and Bernoulli, see, R. Dugas, *A History of Mechanics,* trans. by J. R. Maddux (New York: Dover Publications, 1988) 100-101, and 231-233. On statics in general, see, J. E. Brown, "The Science of Weights", in *Science in the Middle Ages,* ed. by D. C. Lindberg (Chicago: University of Chicago Press, 1978) 179-206. Descartes' QM might have also been influenced by Galileo's concepts of *impeto* and *momento,* since he was generally familiar with Galileo's work; e.g., the letter to Mersenne, 11 October 1638, AT II 379-419. On Galileo's and Descartes' work on statics, see, R. S. Westfall, *The Concept of Force in Newton's Physics* (London: MacDonald, 1971) chaps. 1 & 2. Gabbey notes, ibid., 1973, 383, that "quantity of motion", size speed, may have been a fairly common measure of motion among seventeenth century natural philosophers.

12 A. Gabbey, "Descartes's Physics and Descartes's Mechanics: Chicken and Egg", in *Essays on the Philosophy and Science of René Descartes,* ed. by S. Voss (Oxford: Oxford University Press, 1993).

13 The inconclusive debate on Descartes' understanding of instants, i.e., whether instants are durationless or comprise some length, will not be discussed (since it is a separate problem). See, Garber, ibid., 268-270, for a discussion of the respective sides in this controversy; and, D. Des Chene, *Physiologia: Natural Philosophy in Late Aristotelian and Cartesian Thought* (Ithaca: Cornell University Press, 1996) 280, fn. 33. As used in this essay, 'instant' and 'instantaneous' will allow for either reading, just as long as the corresponding concept of 'beginning to fall' mentioned in his statics correspondence (AT II 229-233) is likewise allowed both interpretations (in order to secure the arguments put forth in this section).

14 Consequently, determination applies to both instantaneous tendencies and non-instantaneous motions (as Garber likewise concludes, ibid., 219-220). It is important to keep these two notions separate, for a misrepresentation of Descartes' views can result if this difference is not heeded. For example, Demarow et al., ibid., 77, fn. 15, appear to believe that determinations are only instantaneous quantities, and QM non-instantaneous, based on the oversight of this distinction, as well as on an oversight of the overall role of tendencies in Cartesian physics.

15 As Westfall, ibid., 61-62, has pointed out, Descartes' use of the term 'agitation' is somewhat ambiguous. Yet, he agrees that it signifies 'momentum' (or QM, if it is a scalar property) in the articles on stellar motions in Part III of the *Principles.* E. J. Aiton also reaches the same conclusion; see, *The Vortex Theory of Planetary Motions* (London: MacDonald, 1972) 63, fn. 83. As regards QM's alternative role as a gauge of instantaneous bodily tendencies, both Garber (ibid., 1992, 208) and T. Prendergast concede this basic

point along lines similar to those advanced above. See, T. Prendergast, "Motion, Action, and Tendency in Descartes' Physics", *Journal of the History of Philosophy,* 13, 1975, 453-462.

16 For Descartes on the concept of infinitesimals, see, P. Mancosu, *Philosophy of Mathematics and Mathematical Practice in the Seventeenth Century* (Oxford: Oxford University Press, 1996) 82-82, 142-143. Mancuso argues convincingly that Descartes attitude towards infinitesimals is more ambiguous than historians have hitherto acknowledged, since he at times made use of them: see the letter to Clerselier, July 1646, AT IV 443.

17 Of course, Descartes' natural laws do not constitute the present-day concept of inertial motion, but his theory is a close forerunner to this notion. See, e.g., A. Gabbey, "Force and Inertia in the Seventeenth Century: Descartes and Newton", in S. Gaukroger, ed., *Descartes: Philosophy, Mathematics, and Physics* (Sussex: Harvester Press, 1980) 230-320. On the baroque concept of "size" in Descartes' natural philosophy, see E. Slowik, reference in footnote 8. See, Pr IV 20-27, for Descartes' analysis of terrestrial gravity.

18 The statics background of Descartes' physics may seem to be called into question by S. Gaukroger's investigation of the early hydrostatics tract (of 1618, AT X 67-74). See, S. Gaukroger, *Descartes: An Intellectual Biography* (Oxford: Oxford University Press, 1995) 247. He regards Cartesian physics as more "hydrostatic", as opposed to kinematic (or some other branch of mechanics), given the presence of the many fluid motion metaphors in *The World* and *Principles.* Yet, Descartes' treatment of hydrostatics is decidedly statical, since he tackles the traditional hydrostatic problems (such as fluid pressure depending on height and not area) from the perspective of the instantaneous tendencies of single fluid particles to descend. As W. R. Shea likewise observes, Descartes' hydrostatics approach thus appears to be based on his previous knowledge of statics, which he almost certainly acquired in his earlier Scholastic training. See, W. R. Shea, *The Magic of Numbers and Motion: The Scientific Career of René Descartes* (Canton, Mass.: Science History Publications, 1991) 27-30. If this point is conceded, however, then Gaukroger is probably correct in arguing that Descartes' understanding of hydrostatics largely shaped his later plenum model, since this would explain its latent statical characteristics.

19 See also the letter to Constantin Huygens, March 9 1638, AT II 662. On the topic of "dimensions", see, J. A. Schuster, "Descartes' *Mathesis Universalis: 1619-28,"* in S. Gaukroger, 1980, ibid., 64-69.

20 One again, the status of Descartes' natural laws as regards their "necessity" is much debated: see references listed in footnote 9 for a discussion of the eternal truths and their relationship to necessity in

Descartes' physics. Gabbey's use of Descartes' "possible worlds" description of (some form of) necessity stems from *The World,* AT XI 47.

21 The preceding analysis may be reading too much into Gabbey's assertions, however. He may be implying that three-dimensional gravity and two-dimensional statics are *both* equally contingent aspects of this particular world (since a different vortex system could be imagined). The tenor of his argument, nonetheless, seems to confer, at least to me, a higher level of necessity on three-dimensional gravity than on two-dimensional statics, which is the basis of the disagreement.

22 For example: "On earth, we see that gold, lead, or other metals when moved retain more agitation, or force to continue in their motion, than do pieces of wood or rocks that have the same size and shape; and therefore metals are also believed to be more solid [or hard, *solidus*], or to contain more third element matter and have smaller pores filled with first and second element matter." (Pr III 122) For more on Descartes' "indefinite" notion, see, J. E. MacGuire, "Space, Geometrical Objects and Infinity: Newton and Descartes on Extension", in *Nature Mathematized: Historical and Philosophical Case Studies in Classical Modern Natural Philosophy,* ed. by W. R. Shea (Dordrecht: D. Reidel, 1983).

23 G. E. Mase, *Theory and Problems of Continuum Mechanics* (New York: McGraw-Hill, 1970) 44. For more on the idealizations in continuum mechanics, see, M. Wilson, "There's a Hole and a Bucket, Dear Leibniz", in P. A. French, T. E. Uehling, Jr., H. K. Wettstein, eds., *Midwest Studies in Philosophy Vol. XVIII, Philosophy of Science* (Notre Dame, IN: U. of Notre Dame Press, 1993). It should be noted, however, that Wilson's position on Cartesian physics differs considerably from the views expressed in this essay.

24 As a notable example of the supposed relational incompatibility of the Cartesian impact rules, one needs only to compare the predictions of the fourth and fifth rules. For an in-depth discussion of these problems, see, e.g., Garber, ibid., 240-241.

25 See, A. Gabbey, "The Mechanical Philosophy and its Problems: Mechanical Explanations, Impenetrability, and Perpetual Motion", in *Change and Progress in Modern Science,* ed. by J. C. Pitt (Dordrecht: D. Reidel, 1985) 57; and, E. Grosholz, *Cartesian Method and the Problem of Reduction* (Oxford: Clarendon Press, 1991) 90. One of the earliest versions of this argument appears in Book II of Newton's *Principia.* See, I. Newton, *Mathematical Principals of Natural Philosophy,* trans. by A. Motte and F. Cajori (Berkeley: University of California Press, 1962) 391.

26 It is assumed, here, that the plenum as a whole does not have a "place" (contiguous neighborhood) from which it could move relative to, and thus conserve QM. Many of the Scholastics would have also rejected this possibility, for the place of an indefinitely extended plenum would seem a conceptual contradiction. For more on the historical development of these theories and arguments, including Leibniz's, see, E. Grant, *Much Ado About Nothing: Theories of Space and Vacuum from the Middle Ages to the Scientific Revolution* (Cambridge: Cambridge University Press, 1981).

27 Although it is beyond the bounds of this essay, some commentators would contend that Descartes' theory does not sanction the sort of "reference frame" relationalism seemingly implied in Westfall's allegations: see, D. Garber, ibid., 169-171, and D. Des Chene, ibid., 270-271. Garber's and Des Chene's arguments are mainly correct, but they still do not free Descartes' natural laws from relational contradictions, as argued in, E. Slowik, "Descartes, Space-Time, and Relational Motion", forthcoming in *Philosophy of Science.*

28 "Kinematic" refers to the study of motion in the absence of forces, unlike "dynamics", which does study motion under the action of forces. In addition, "invariant", as used in this context, can mean either; (a) a numerical value which remains the same from the perspective of all frames, but is not necessarily the same numerical value in each frame; or (b), the same as (a), but the numerical value *is* the same in all frames. The type of invariant in classical conservation laws, such as Newton's momentum law, is (a); while the invariant space-time interval of Special Relativity is (b). See, M. Friedman, *Foundations of Space-Time Theories* (Princeton: Princeton University Press, 1983) 56, for more on space-time invariants. Descartes' QM violates either reading, if relationalism is also in play. For example, if in one frame of reference, a small moving body merely reverses its motion (from right to left) after striking a larger resting body, then QM will be conserved as mandated by rule four (Pr II 49; and where the scalar nature of speed is in evidence). But this same collision as viewed from another frame, at rest relative to the smaller moving body, will see both bodies retreat to the left after impact (i.e., this frame, assuming it is inertial, will keep moving to the right)--and this joint motion of both bodies will (in most cases) register an *increase* in QM *after* the collision (thus violating the conservation principle).

29 Descartes' hypothesis of instantaneous circular motions of plenum bodies (Pr II 33) is also relevant to this discussion, needless to say, although it is clear from the context that *circular* motions are necessary, at least in Part II, to avoid the creation of a vacuum. The interconnected-ness of *all types* of motions, however, seems to stem from other sources, as argued above.

30 For some studies of teleology in Cartesian physics, see, J. Collins, *Descartes' Philosophy of Nature* (Oxford: Blackwell, 1971) Part 1; and, P. K. Machamer, "Causality and Explanation in Descartes' Natural

Philosophy", in *Motion and Time, Space and Matter: Interrelations in the History and Philosophy of Science,* ed. by P. K. Machamer and R. G. Turnbull (Columbus: Ohio State University Press, 1976).

31 C. Merchant, *The Death of Nature: Women, Ecology, and the Scientific Revolution.* (New York: HarperCollins Pub., 1980), 293.

32 Merchant also errs, it would seem, in reading overt mechanist overtones in the *Discourse on the method* (ibid., p. 231). From the context of the discussion, however (AT VI 19-20), the a priori certainty and structure of geometry is the more obvious analogy that Descartes is attempting to draw in formulating his methodilogical rules.

33 Finally, it should be mentioned that the conclusions reached in this essay appear to be neutral with respect to the debate over the ontological status of Cartesian force. That is, the present investigation does not seem to be affected if force is construed as *in* bodies--as argued by Gabbey, 1980, ibid., 238; or M. Gueroult, "The Metaphysics and Physics of Force in Descartes", in S. Gaukroger, ibid., 1980, 198--or force just *appears* to be in bodies (through God's sustaining act), as argued by Garber, ibid., 283, and Des Chene, ibid., 327. In short, both views appear compatible with the results obtained above.

34 I would like to thank Calvin Normore, Mark Wilson, Daniel Garber, and Jonathan Bennett for their helpful discussions and comments which led to the formulation of this essay.