**Huygens' Center-of-Mass Space-Time Reference Frame: Constructing a Cartesian Dynamics in The Wake of Newton's *"De gravitatione"* Argument**

(Word Count 99)

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**Abstract**

This paper explores the possibility of constructing a Cartesian space-time that can resolve the dilemma posed by a famous argument from Newton's early essay, *De gravitatione.* In particular, Huygens' concept of a center-of-mass reference frame is utilized in an attempt to reconcile Descartes' relationalist theory of space and motion with the both the Cartesian analysis of bodily impact and conservation law for quantity of motion. After presenting a modern formulation of a Cartesian space-time employing Huygens' frames, a series of Newtonian counter-replies are developed in order to estimate the viability of this relationalist project.

**Huygens' Center-of-Mass Space-Time Reference Frame: Constructing a Cartesian Dynamics in The Wake of Newton's *"De gravitatione"* Argument**

(Word Count: 10,100)

EDWARD SLOWIK

In his early paper *De gravitatione,* Newton presents a devastating argument against Descartes' use of the concept of rectilinear uniform motion within the framework of a relational theory of space and time. Newton argues, essentially, that a theory which regards motion as merely the relations among material bodies (deemed a "relational" theory) is not equipped with the necessary structure to delineate the non-accelerating straight-line motions (inertial motions) necessary for the Cartesian natural laws. Given the reluctance of modern-day relationalists to embrace Descartes' coupling of these apparently incompatible notions, one might conclude that Newton's argument reveals a serious defect within Cartesian physics, an obstacle that the adherents of Descartes' natural philosophy cannot possibly overcome. Yet, must a Cartesian accept this conclusion? Is it possible, more specifically, to develop a theory of "dynamics" (which studies the motions of bodies under the action of forces) that successfully meets Newton's challenge while remaining true to the main tenets of Descartes' physics as put forth in the *Principles of Philosophy,* his most important scientific work? Such a task is somewhat complicated by the very nature of Cartesian "dynamics", moreover, which may be a misnomer given his decidedly kinematic approach to bodily interactions, such as collisions (where "kinematic" is defined as the study of motion exclusive of forces).

Overall, this paper will examine one possible means of achieving the goal of a consistent Cartesian physics. We will develop a theory, originally suggested by Christiaan Huygens, which seeks to reconcile Descartes' conservation law ("quantity of motion") with an accurate Cartesian analysis of bodily impact by invoking a class of preferred reference frames: the center-of-mass of a system of colliding bodies. From the perspective of the center-of-mass reference frame, bodies move inertially while retaining their quantity of motion (size times speed) in accordance with Descartes' hypotheses of impact (albeit some revisions). If Descartes' conservation law, which is based upon his relational theory of motion, can be thus reconciled with his collision hypotheses (via Huygens' frames), then Cartesian science will have gone a long way towards resolving the difficulties imposed by Newton's *De gravitatione* argument. Consequently, this paper will detail the strengths and weaknesses a suitably modified and updated Cartesian space-time; a project that, to the best of my knowledge, has never been undertaken at length. As will be discussed in section 2, the product of our marriage of Huygens' frames and Descartes' physics will be thoroughly Cartesian, and not Huygensian, since many of Huygens' scientific views differed quite drastically from Descartes'. (To resolve any later confusion concerning terminology, the word "dynamics" as applied to either Descartes' or Huygens' physics will simply define their respective theories of bodily interaction, principally collisions, without regard to the purported kinematic/dynamic nature of their approaches.)

1. Motion in The Seventeenth Century: Descartes and Newton

*1.1. Descartes' Theory of Motion.* In the *Principles of Philosophy* (published in 1644), Descartes defines "external place" as the surface of the material bodies contiguous with a given object, while motion *"is the transfer of one piece of matter or of one body, from the vicinity of those bodies immediately contiguous to it [or, roughly, external place] and considered at rest, into the vicinity of others."* (Pr II 25)[[1]](#endnote-1) In Descartes' plenum universe (i.e., a world completely packed with matter), relationalism enters the picture in the following manner: provided Descartes' definition of "external place" as the surface of the containing bodies, which seems to be identical to the "neighborhood" of the contained body (see, Pr II 25-28), any attempt to regard the surrounding bodies "at rest" must amount to a purely arbitrary stipulation, since "we cannot conceive of the body AB being transported from the vicinity of the body CD without also understanding that the body CD is transported from the vicinity of the body AB, and that exactly the same force and action is required for the one transference as for the other." (Pr II 29) He is thus led to conclude "that all the real and positive properties which are in moving bodies, and by virtue of which we say they move, are also found in those [bodies] contiguous to them, even though we consider the second group to be at rest." (Pr II 30) Following Earman (1989, 12), we can define this relationalist conception of motion as follows:

R1: All motion is the relative motion of bodies, and consequently, space-time does not have, and cannot have, structures that support "absolute" quantities of motion.

So far, Descartes' theory more or less follows the standard course of a Scholastic relationalist theory of motion. Yet, in both the *Principles of Philosophy* and his earlier *The World,* Descartes advocates a series of laws on the nature of motion that not only appear to contradict this relational view, but which provided Newton with the model for what was eventually to become a focal point of his own laws of motion: "all movement is, of itself, along straight lines; and consequently, bodies which are moving in a circle always tend to move away from the center of the circle which they are describing." (Pr II 39) According to (R1), the phenomena of "motion" and "rest" are only meaningful when presented relative to an arbitrary body or reference frame. Thus, a moving body's "absolute" or "actual" determination of motion, i.e., its individual speed, velocity, etc., is just not possible due to the conflicting values that any given motion will be assigned relative to these differing perspectives. Likewise, it is not possible, or meaningful, to attempt to ascertain the "unique" path or trajectory of a body in an (R1) relational space-time. Since trajectories are determined relative to each observer, and all observers are in relative motion (which, as above, cannot be determined individually), any endeavor to establish the unique path of a particular moving body will result in a host of contradictory measurements, none of which can lay claim to the "actual" path of the body. Consequently, Descartes' second law of motion (quoted above) would appear to run afoul of his espoused relationalism; as do many of the collision rules that he developed to explicate his third natural law (which concerns the conservation of the total "quantity of motion," or product of size and the scalar quantity speed, , of all material bodies).[[2]](#endnote-2)

*1.2. Newton's "De gravitatione" Argument.* In the manuscript entitled *De gravitatione,* Newton assails this inconsistency in Descartes' theory in a fascinating argument that, despite its critical intent, also reveals the underpinnings of Newton's own spatiotemporal views. He states:

I say that thence it follows that a moving body has no determinate velocity and no definite line in which it moves. And, what is worse, that the velocity of a body moving without resistance cannot be said to be uniform, nor the line said to be straight in which its motion is accomplished. . . . But that this may be clear, it is first of all to be shown that when a certain motion is finished it is impossible, according to Descartes, to assign a place in which the body was at the beginning of the motion; And the reason is that according to Descartes the place cannot be defined or assigned except by the position of the surrounding bodies, and after the completion of a certain motion the position of the surrounding bodies no longer stays the same as it was before. . . . Truly there are no bodies in the world whose relative positions remain unchanged with the passage of time, and certainly none which do not move in the Cartesian sense: that is, which are neither transported from the vicinity of contiguous bodies nor are parts of other bodies so transferred. . . .

Now as it is impossible to pick out the place in which a motion began, for this place no longer exists after the motion is completed, so the space passed over, having no beginning, can have no length; and hence, since velocity depends upon the distance passed over in a given time, it follows that the moving body can have no velocity, just as I wished to prove at first. Moreover, what was said of the beginning of the space passed over should be applied to all indeterminate points too; and thus as the space has no beginning nor indeterminate parts it follows that there was no space passed over and thus no determinate motion, which was my second point. It follows indubitably that Cartesian motion is not motion, for it has no velocity, no definition, and there is no space or distance traversed by it. So it is necessary that the definition of places, and hence of local motion, be referred to some motionless thing such as extension alone or space in so far as it is seen to be truly distinct from bodies. (Newton 1962, 129-131)

Newton puts forth a number of important claims in this crucial passage. In order to better reveal its basic assumptions and form, it would be best to analyze his argument by detailing each important step. In what follows, premises (**1**) through (**5**) are all assumptions:

(**1**) Descartes' law of inertial motion: All bodies tend to remain at rest or move in rectilinear paths at uniform velocity. (Pr II 37-39) This is a conjunction of Descartes' first and second laws of motion.

(**2**) Descartes' relational theory of place and motion, (R1): This premise entails that all places and motions are determined relative to other contiguous bodies.

(**3**) Descartes' plenum theory of matter: All of space is filled with material bodies. This premise is not essential.

(**4**) All the material inhabitants of the universe constantly alter their relative positions (relative to one another).

(**5**) Both straight line motion and velocity (which is described as *distance* divided by time) require a temporally fixed path of determinate length: That is, in order to determine a body's velocity and line of motion, the places successively occupied by the moving body, which together provide a definite length, must remain unaltered over time.

(**6**) From (2) through (5): Straight line motion and velocity cannot be determined in the Cartesian universe. More explicitly, due to the continuous motion and scattering of the contiguous bodies responsible for defining relative place, the trajectory or path of a moving object can exhibit no well defined length and, thus, no well defined velocity.

(**7**) Contradiction from (1) and (6).

(**8**) Conclusion: Premise (2) must be false.

Of course, following the logic, it is perfectly consistent to claim that the contradiction results from premise (**5**): that is, (**2**) is true and (**5**) incorrect. Newton resolutely concludes, however, that the only viable means of coherently defining the velocity and trajectory of a moving body is through the adoption of "absolute" space and time (which, in the *De gravitatione,* "approaches more nearly to the nature of substance", 132). In short, he believes (**2**) must be replaced with: (**2a**) All spatial positions or places are temporally fixed and determinate. In order to determine the length of a spatial path, argues Newton, one must be able to accurately trace across time the initial spatial position from which the motion commenced, as well as the intermediate places through which the motion persists. If this cannot be accomplished, then the length of the spatial path -- and, consequently, the velocity and direction of motion--cannot be meaningfully expressed.[[3]](#endnote-3)

2. Huygens and The Center-of-Mass Reference Frames

*2.1. Towards a Cartesian Response.* Needless to say, a Cartesian bent on resurrecting Descartes' dynamical theory cannot accept the verdict of Newton's argument. It is not merely sufficient for the Cartesian to reject premise (**5**), however: if they intend to thwart Newton's allegations, it is crucial that they procure some means of delineating the inertial from the non-inertial motions of bodies. In fact, translated into its modern equivalent, Newton's argument simply amounts to the observation that Descartes' theory lacks a suitable class of reference frames required to determine inertial motion. (In this context, "reference frame" denotes a three-dimensional Cartesian coordinate system from which a hypothetical observer could conduct calculations of bodily motion.) Hence, the Cartesian must replace the absolute spatial positions that serve Newton as a means of ascertaining inertial trajectories (as developed in premise (**5**)), with a set of reference frames that provide the same function but which can be defined and employed without violating the tenets of Descartes' relationalism. Given the constraints of Descartes' dynamical theory, locating a class of such frames will not be an easy task. Specifically, the Cartesian must establish a coherent process of measuring the motions of bodies that will harmonize Descartes' relational theory of space and time with (i) his views on the interactions of bodies, especially his seven collision rules, and (ii) his conservation law, which holds that the quantity of motion of all bodies in the universe is conserved. ("[God] always maintains in [the universe] an equal quantity of motion." Pr II 36)

Fortunately for the Cartesian, one possible method of correcting Descartes' natural philosophy seems immediately attractive. As noted, Newton's argument relies on the fact that the constant flux of the Cartesian plenum will preclude a "fixed" material reference frame (i.e., a frame fixed to some material occupant of the plenum) from determining straight-line uniform motion. Yet, Newton does not seem to envision the possibility of locating the reference frame alongside the moving or colliding bodies. By invoking frames in this manner, the uniform speeds and the straight-line paths of the two interacting objects (assuming we have such a case) are determined relative to a unique perspective or viewpoint that, in turn, is defined relative to the moving bodies. In other words, it would seem that we could always locate a reference frame from whose perspective both bodies move inertially before and after their collision (but possibly not during the collision, of course). Therefore, since the bodies provide, what we may call, their own "relatively-defined inertial frame," the need for a separately fixed material frame, or an absolute spatial position, is conveniently circumvented. Although a collection of such systems will not normally agree on the exact value of the quantity of motion with regard to one another, this fact does not necessarily preclude the possibility of a developing a consistent Cartesian conservation law. That is, the scope of this law will be limited to each of these relatively-defined inertial frames, since the total quantity of motion can only be preserved from a single perspective inside each system of colliding bodies. This is a price a Cartesian may willingly decide to pay, nonetheless, for such a strategy would seem to reconcile conditions (i) and (ii) above with a relational theory of space and time. (We shall postpone considering some of the Newtonian objections to this form of hypothesis, as well as the type of relationalism and space-time this scheme presupposes, until section 3 and 4.) Finally, these frames must be located with, or "tied" to, the material occupants of the space-time, so as not to invoke any form of absolute determination of motion, contra (R1), which a frame not linked to a material body would seem to entail.

*2.2. Huygens and Cartesianism.* One of the most successful attempt to rehabilitate Cartesian dynamics, along the lines suggested in section 2.1, can be attributed to Descartes' younger contemporary, Christiaan Huygens. Huygens' scientific instincts, to a large extent, were entrenched in the amalgam of relationalism and vortex mechanics championed in Descartes' *Principles of Philosophy.* Like Descartes, he was inclined towards the incorporation of space-time relationalism[[4]](#endnote-4) with a contact-mechanical physics; i.e., with a theory that limits the interactions of material objects to direct body-to-body impact, and which denies the existence of action-at-a-distance forces, such as gravity.[[5]](#endnote-5) His debt to Cartesian natural philosophy is obvious in the succeeding passage:

To discover a cause of weight that is intelligible, it is necessary to investigate how weight can come about, while assuming the existence only of bodies made of one common matter in which one admits no quality or inclination to approach each other but solely different sizes, figures, and motions[[6]](#endnote-6)

Even though he embraced many of his concepts, Huygens' work on mechanics far outdistanced the achievements of Descartes. Besides formulating several additional conservation laws of greater scope than quantity of motion -- namely, the momentum law, , and *vis viva,* or (which roughly corresponds to the kinetic energy law) -- Huygens' was also the first to provide a quantitative treatment of centrifugal force (see section 3.1 and note 15).

To unproblematically label Huygens a "Cartesian" would be somewhat disingenuous, however, for the details of his physics differed from Descartes' in several major respects. Most notably, Huygens felt it necessary to admit the existence of infinitely hard, impenetrable atoms separated by small empty spaces (vacuum), as well as a non-instantaneous velocity of light. As H. A. M. Snelders remarks: "In this acceptance of atoms and the intercorpuscular vacuum Huygens must be regarded as a Gassendian."[[7]](#endnote-7) Despite these differences, Huygens invariably used Descartes as his starting point when theorizing about natural philosophy, as is evident in his investigation of bodily collisions (to be discussed below).

*2.3. The Center-of-Mass Reference Frames.* In attempting to correct Descartes analysis of impact, Huygens selected as his starting point the first Cartesian collision rule, the only hypothesis in the entire set of seven actually verified through experimentation. This rule governs the collision of two equally sized bodies moving at the same speed (in opposite directions along the same line). Descartes asserts that both bodies will rebound along their initial path in the opposite direction "without having lost any of their speed," (Pr II 46) and thus conserve their total quantity of motion. Recognizing the importance of the first rule, Huygens sought to remedy the deficiencies in the Cartesian impact theory by utilizing an identical analysis for the remaining six cases treated in the *Principles.* By extending its scope over the collisions of all bodily sizes and speeds, the first rule can guarantee the conservation of quantity of motion for all interactions.

Huygens' search was motivated in large part by his understanding of the Principle of Galilean Relativity, which maintains that the state of inertial motion, or uniform speed in a given direction, does not affect the outcomes of physical processes. He realized that a single collision may appear, from one viewpoint, as a collision between a uniformly moving and a stationary object, but from a different perspective, as the collision of two uniformly moving bodies. (His famous example involves an experiment with colliding spheres conducted on board a boat, which is sailing down river, as viewed from both the boat and the distant shore; *Oeuvres Complètes,* vol. 16, 31.) Consequently, the state of inertial motion of the bodies does not affect the outcome of the collision. With this in mind, Huygens endeavored to locate a frame of reference that would permit all bodily collisions to be viewed as a species of Descartes' first rule; i.e., where both bodies preserve their initial speeds (and hence quantity of motion) after rebound. Nevertheless, the first collision rule only treats the interactions of equally sized bodies, a limitation of scope that greatly complicates Huygens' task. Although the relative speeds of bodies (i.e., their individual speed components) can be changed by simply adopting a different relatively non-accelerating frame, such transformations will not alter their relative sizes.

However, with the discovery of a colliding system's center-of-mass reference frame, which we will label a CM frame, Huygens found a means of generalizing Descartes' first collision rule to cover the interactions of various sized bodies. To grasp the significance of this concept, let us examine the case of two colliding bodies, labeled  and , and assume that the latter has twice the size of the former, and that they both approach at the same speed.[[8]](#endnote-8) If  is separated three feet from , and we place a Cartesian coordinate system at a position one foot from , then the products of their size and distance will be equal (2=2). Our presentation can be more precise, if  and  designate the respective coordinate positions of  and , then the center-of-mass between these two bodies is the point where

 or  (2.1)

(and the absolute values of these quantities are assumed throughout). In other words, viewed relative to this frame, the products of the size and position of our two bodies are equal. To maintain this perspective throughout the entire bodily interaction, consequently, it will be necessary to determine or locate the center-of-mass frame at each successive instant. On an absolute conception of space and time, the frame will thus need to constantly alter its position between the two colliding bodies to preserve the center-of-mass viewpoint (whereas, if equal-sized bodies approach at the same speed, the frame will remain stationary). However, since absolute states of motion are not sanctioned by the relationalist theory, the only verdict a Cartesian can provide is that, within the colliding system, a CM reference frame is continually specifiable, and that this frame may or may not be in a state of relative motion with respect to other bodies or reference frames. Given this judgment, the relationalist can claim that equation (2.1) is satisfied throughout a collision without having to classify the reference frame's individual state of motion relative to any outside systems.

Returning to Huygens' analysis, if we were to examine a collision from the CM frame, and employ Descartes' conservation law of quantity of motion,  (where  and  signify the pre-collision inertial motion of  and , and  and denote their post-collision inertial motion) then,  and  (because, as viewed from that frame, both bodies merely reverse their direction after the collision). By substituting these results into Descartes' conservation law and simplifying both sides, we discover

. (2.2)

An analysis of (2.1) and (2.2) reveals an important fact about the CM frame: Huygens deduces that "if a larger body A strikes a smaller body B, but the velocity of B is to the velocity of A reciprocally as the magnitude [size] A to B, then each will rebound with the same speed with which it came." (*Oeuvres Complètes,* vol. 16, 92) As viewed from the origin of that reference frame, where the bodies preserve their initial speeds after rebound, the ratio of their speeds is reciprocal to the ratio of their sizes.[[9]](#endnote-9)

With the disclosure of the CM frame, Huygens had thus found a relational means of conserving Descartes' quantity of motion in all types of collisions; but he had to reject most of Descartes' collision rules in the process, a realization that prompted him to assert: "If this [the CM frame] is granted, everything can be demonstrated. Descartes is forced to grant it however." (*Oeuvres Complètes,* vol. 16, 96) What Descartes is forced to grant is that six of his collision rules have been discarded, or refuted, by Huygens' CM hypothesis. When one body strikes another, irrespective of their size and speed, an observer situated at the CM perspective will perceive both bodies recoil in the opposite direction while retaining their initial speeds. In short, *relative* to this frame, all collisions are essentially the same. Moreover, it is important to remember that the total quantity of motion--defined as size times speed--is only faithfully conserved from the CM location: despite the fact that this frame will register the same amount both before and after the collision, other relatively non-accelerating (inertial) systems will generally reach different conclusions. From the perspective of most of these outside frames, the interaction of the bodies will fall within the scope of all seven Cartesian collision rules, which, as noted, are generally faulty. Since six of these seven rules will not provide accurate predictions, the total value of the quantity of motion will therefore not remain a conserved quantity across all such relatively non-accelerating frames.

3. The Newtonian Reply

Given our discussion in section 1, how would a substantivalist, who accepts the conclusion of Newton's *De gravitatione* argument, respond to the CM frame proposal? Besides invoking Newton's notorious "rotating bucket" experiment,[[10]](#endnote-10) there are a number of objections that can be raised, some quite substantial.

*3.1. Do CM frames violate Relationalism?* First and foremost, all substantivalists will liken Huygens' frames to a covert reconstruction of the actual relationships and physical laws that obtain in absolute space. A Newtonian may ask: provided the relational equivalence of all frames in assessing motion, defined as (R1) above, why is a special class of coordinate systems (i.e., CM) singled out by Huygens' conservation law? The fact that the conservation laws do not hold from all perspectives, or even all inertial perspectives, will thus be interpreted as support for the substantivalist view (although no substantivalist of the seventeenth century appears to have presented this argument). In essence, they will insist that certain (inertial) reference frames are privileged due to the "embedding" of the laws of nature in the structure of absolute space and time, a fact that Huygens can never adequately explain due to the equivalence of all relational perspectives in determining motion. The space-time "structure" that identifies these inertial trajectories for the substantivalist (in modern Newtonian, or Neo-Newtonian, formulations) is the familiar covariant derivative, often labeled  (or, , in the coordinate frame).[[11]](#endnote-11)

Although it is not clear that the relationalist is committed to accepting some form of absolute space, the force of the substantivalists allegations would nevertheless seem to demand a closer inspection of the particular brand of relationalism utilized by the CM frames: for, if only a special class of frames can uphold the conservation law, then the Cartesian space-time would seem to possess a capacity, or structure, over and above the relations among bodies, in direct violation of (R1). Yet, since the Cartesian space-time we are considering is a member of the class of Leibnizian space-times, which only possess a Euclidean spatial metric on the planes of simultaneity, as well as a time metric (see, Earman 1989, chap. 2), it is not the case that our CM frame method is built upon the much stronger  structure of Newtonian space-time. Rather, it might be possible to use the Cartesian natural laws *themselves* as a sort of "bootstrapping" technique of locating the frames that conserve the desired Cartesian conserved quantities. On this explanation, consequently, it is the sparse structure of the Cartesian space-time in conjunction with the Cartesian natural laws that pick out the CM frames. Overall, the CM frames would appear to play a role similar to that of the "fixed stars" in Mach's attempt to resolve Newton's "bucket" experiment. In order to account for the centrifugal effects of the water's rotation, Mach postulated that the force effects were not due to the water's acceleration relative to absolute space, but a result of its acceleration relative to the fixed stars (see note 10). Mach's strategy, which (at least hypothetically) eliminates the need for absolute space, is succinctly summarized by Sklar: "where Newton fails, [Mach] argues, is in his attempt to show that no material object could be the proper reference frames for the absolute accelerations to be relative to."[[12]](#endnote-12) Mach's fixed stars and the CM frames both constitute a relationalist means of choosing a class of materially-based inertial frames from which to explicate material phenomena (rotation and collisions, respectively)--but, the relational space-time structures are weaker than  because they cannot ascertain the inertial continuation of the privileged material-based frames independent of the particular physical laws, either Descartes' or Newton's, coupled to those worlds.

Since this strategy marks a departure from most interpretations of relationalism, we can label this variant of (R1):

R1\*: All motion is the relative motion of bodies, and consequently, space-time does not have, and cannot have, structures that support "absolute" quantities of motion. But, a privileged class of reference frames may be adapted to the invariant quantities of motion of the space-time as long as it does not entail a fixed, absolute space-time structure.

The invariant quantities of motion in our Cartesian space-time are the "relative body speeds (velocities, accelerations)" (more on this in section 4), which fail to qualify as "absolute" since they are only determined relatively among bodies and not to absolute space. Given the invariant quantities of this relational (Leibnizian) space-time (as above), and the Cartesian conservation law, the plan is that the needed CM frames can be chosen without invoking . Overall, this form of relationalism has the advantage of nicely accommodating Descartes' actual treatment of motion, which, as we have seen, violates (R1), alongside his repeated denials that space and time (and hence motion) are anything more than relations among bodies: e.g., "the names 'place' or 'space' do not signify a thing different from the body which is said to be in the place." (Pr II 13; see also, Pr II 8-18) Unfortunately, it is unclear if (R1\*) is generally successful, or even consistent, as there is a sneaking suspicion that an underlying Newtonian structure is being dressed in a relationalist guise. In a sense, the critic may charge that the conjunction of Descartes' natural laws and the invariants of its space-time simply provides the same structure as Newtonian space-time, but merely relabeled to suit relationalist sensibilities. Throughout the remainder of this essay, however, we will assume that (R1\*), and/or (R2), to be discussed below, are relationally consistent.

In its favor, though, the major difficulties for a relationalist maneuver such as (R1\*) would seem to have been historically associated with the phenomenon of rotational motion (Newton's "bucket", again),[[13]](#endnote-13) and not with the sorts of collision hypotheses upon which our CM frames have been constructed. Rotation, in contrast to the type of bodily collisions viewed from a CM frame, presents grave empirical problems for a relationalist (and likewise extols the virtues of -equipped space-times). In the Machian theory described above, the rotation of the fixed stars relative to a fixed bucket is a relationally indistinguishable state-of-affairs from a rotating bucket and fixed stars. Yet, Newton's theory predicts observationally distinguishable effects for these two cases (i.e., the centrifugal force effects), which suggests that Newtonian dynamics is not congenial to Mach's version of (R1\*). Our CM-frame rendition of Cartesian dynamics, however, since it treats a limited class of bodily collisions in a *kinematic* fashion, is thus freed from the potentially disastrous need to explicate the more complex non-inertial phenomena manifest in bodily rotations (or other forms of acceleration, e.g., motion through a non-uniform gravitational field). The kinematic approach to collisions evident in the work of Descartes and Huygens, where bodies merely approach along a straight line at uniform speeds and reverse their direction after impact (while ignoring the processes that occur during impact), thus constitutes a more suitable phenomenon for a marriage of (R1\*) and a set of inertial laws of motion than Mach's similar conjunction of natural laws and (R1\*) for the more dynamically sophisticated behavior of bodily rotation. Mach tried to explain rotation in an equally kinematic fashion, as simply movement relative to a material reference frame, but the dynamic behavior of bodies undergoing these motions breaks the symmetry of this kinematical approach.[[14]](#endnote-14)

Huygens' discovery of the correct quantitative formulation of centrifugal force nicely demonstrates the troublesome questions that such dynamical phenomena can raise for a devoted relationalist. Unlike his conservation laws for quantity of motion, momentum, or *vis viva,* a body's centrifugal force cannot be removed by simply adopting a new reference frame. To put it differently, the quantities conserved in the interactions covered by the above three laws do not hold in all possible frames (i.e., transformations to non-CM frames for quantity of motion, and transformations to non-inertial frames in the case of the other laws, will generally fail to conserve these respective quantities). As Westfall has suggested, and Gabbey's work seems to imply,[[15]](#endnote-15) this fact may have contributed to Huygens' continued interest in Descartes' quantity of motion even after his discovery of the more general momentum and *vis viva* laws, since the "forces" apparently conserved in all these laws could be regarded as a mere manifestation from a particular point of view--and with many possible points of view, there may be different quantities conserved in each different reference frame. However, (assuming Mach was wrong), no matter what perspective one takes with respect to a relative rotational motion among two bodies (i.e., which body is considered to be at rest and which body is rotating), it will still be the case that only one body, the rotating body, will experience the centrifugal force. The existence of this force would seem to strongly imply that at least some quantitative force phenomena are *independent* of the particular reference frame selected to view the behavior of bodies, a conclusion that would seem to challenge the "kinematical" approach to bodily collisions apparent in Huygens' work, such as *De motu corporum ex percussione* (and Descartes' collision rules, Pr II 46-52). Specifically, one cannot eliminate or discharge the unwanted behavior of bodies by merely shifting to a different reference frame.

Finally, a relationalist unsatisfied with the "bootstrapping" process implied in (R1\*) may want to fall back on the relational strategy (R2) instead, which we can define as follows (Earman, 12):

R2: Spatiotemporal relations among bodies are direct; that is, they are not parasitic on relations to a substantival space that underlies bodies.

This form of reasoning is the relationalist's last line of defense, for it is merely the denial of space-time "substantivalism" (i.e., space-time as a substance or entity that is independent of, or can exist apart from, matter). If (R2) is accepted in place of (R1\*), consequently, then the relationalist is ostensibly free to invoke whatever space-time structure is deemed necessary to, in our case, conserve the Cartesian natural laws (possibly even ). As long as the existence of these richer structures are regarded as somehow contingent upon the material occupants of the plenum (however that may be), the relationalist can seemingly lay claim to the Newtonian legacy without the added ontological commitments. But the real question, once again, is whether or not (R2) is a consistent relationalist alternative to (R1), or even (R1\*); a verdict, furthermore, which is still pending (see note 13).

*3.2. The Domain of Phenomena and the CM Frames.* Our second substantivalist counter-reply to the CM frame method is a follow up to the first: in short, if the CM frames are spared the difficult task of determining rotational (accelerated) motions, then this formulation of Cartesian physics will necessarily fail to up hold the conservation law for all material events. If the CM frames are to be used to preserve the quantity of motion in all physical processes, and thus satisfy Descartes' demand for an overall conserved universal quantity, then it will be necessary to view *all material interactions* as a form of collision subsumed under the first collision rule. Nevertheless, as described, Huygens' reconstruction of the first collision rule is confined to a distinct class of material interactions, namely the impact of bodies moving along straight paths at uniform speeds relative to one another. This is all the more problematic when one discovers the substantial role that (non-uniformly) accelerated motions assume in the Cartesian plenum. To give one example, the plenum particles that constitute a large ring of circling matter, or vortex, will momentarily increase their speeds when passing a narrow channel or obstruction situated along their path. In such cases, Descartes insists that "all the inequalities of the spaces [of the path] can be compensated for by corresponding inequalities in the speed of the parts [of the vortex]. . . . Thus, in any given length of time, the same quantity of matter will pass through one section of the circle as through another." (Fr Pr II 33) Given this scenario, one may wonder if it is therefore possible to comprehensively explain all natural phenomena as a form of interaction characterized by uniform pre- and post-impact relative speeds. Since the vortex particles variably accelerate to compensate for obstructions along their path, their random collisions will not normally exhibit the relative constant speeds required to apply effectively the method of the CM frame, and hence conserve quantity of motion. (Once more, there does not appear to be any historical precedents for this argument against the CM frame.)

*3.3. Predictive Scope and the CM Frames.* Our third substantivalist response closely resembles the preceding argument. If the CM frames suffer from a limitation of scope with respect to the types of collisions they can successfully explicate, they seem further restricted by an inability to predict or determine the future states of material bodies after they have departed the frame. Specifically, since a CM frame only measures a given body's quantity of motion during the brief periods spanned by its impact with a second body, one might conclude that it apparently cannot offer any predictions of the future states of this quantity after the bodies have separated and joined in other collisions. Each particular CM frame, consequently, can only track a body's motion, and determine its product of speed and size, for specific finite (infinitesimally small) temporal intervals. If placed within the confines of the Cartesian plenum, *where bodies constantly collide,* the predictive scope of Huygens' frames is subsequently restricted to mere instants. During even the briefest of intervals, a given object will be engaged in a vast number of distinct collisions with a host of adjacent bodies, a situation that would most certainly limit the determination of a body's future motions to an equally short period of time, if not single instants. In addition, there is the further problem of how an infinity of nearby, possibly simultaneous, collisions affects Huygens' method, since such a possibility would seem to greatly complicate, if not hinder, the application of the CM frames.

Summarizing the third substantivalist argument: at best, the CM theory can only provide a measurement of the quantity of motion at each separate or distinct collision, but not continuously over a series of such interactions. A Cartesian would be disinclined to accept this judgment if it entailed a restriction to mere instants, however; for such a confinement of this quantity would appear to conflict with Descartes' analysis of motion. Overall, Descartes envisions motion as a process that necessarily involves a temporal duration, for "no movement is accomplished in an instant." (Pr II 39) Since quantity of motion employs speed, limiting the Cartesian conservation law to single instants would therefore likely raise serious textual objections. In addition, a Newtonian would probably insist that this method of determining motion runs counter to our normal measuring procedures, if not common sense intuition. An object does not require a new reference frame as it approaches every fresh contact; rather, its motion, and hence quantity of motion, can be continuously traced over any given number of collisions. The substantivalist will accordingly interpret our common experience of impact measurements as support for the existence of substantival space, since there must be some mechanism or medium that permits the continual estimation of these physical quantities.

The problems engendered by the third substantivalist counter-argument are not necessarily disastrous to the CM frame project, however. In the next section, we shall demonstrate how a CM frame, when carefully specified, can attempt to overcome the limitations of its inherent lack of predictive scope.

4. Constructing a Center-of-Mass Reference Frame

How should the Cartesian respond to the allegations of the third Newtonian argument? Obviously, if the CM frames are to be retained, the relationalist will need to procure a means of determining and preserving a body's quantity of motion over the course of several collisions. That is, a procedure must be obtained that will permit the coupling or linking of each distinct CM coordinate system (reference frame), and hence allow information on the status of bodies in one frame to be inferred from another frame. The problem, as described above, is based upon the fact that all bodies eventually enter collision frames that are apparently not related to their previous collision system. We can detail the argument as follows: (1) since we are assuming a relational theory of space-time, each center-of-mass coordinate system is in a state of relative motion with respect to all other systems; thus, their exist no meaningful non-relative or individual determinations of a frame's state of motion (which is true regardless of whether we accept an (R1) or (R1\*) construal of the CM project). (2) In order to conserve a body's quantity of motion, the determination of this quantity must be conducted from the CM frame; hence, the particular value assigned to each colliding body is fully dependent upon that coordinate system. Consequently, as is evident from the conjunction of statements (1) and (2), the value of an object's quantity of motion is entirely relative to, and thus only *meaningful in,* its current CM system.

Nevertheless, this argument overlooks a significant fact concerning Huygens' frames: although they do not display individual non-relative states of motion, one can determine the relative speed and distance between two or more frames (as noted above). As a consequence of the space-time structure, a Cartesian can now meaningfully and coherently discuss the "relative speed (or velocity, acceleration, etc.)" among several bodies without violating the tenets of relationalism. These quantities are invariants of our relational (R1), or (R1\*), space-time, and is thus a member of the larger class of Leibnizian space-times. Likewise, all reference frames will calculate an identical spatial difference between bodies, as well as an equal measurement of size. Therefore, despite its sparse appearance, our Cartesian space-time exhibits a number of invariant properties: all frames will observe the same bodily sizes, relative distances, and relative speeds among bodies.

To demonstrate a relationalist means of correlating the CM frames, consider the following example: suppose a material body with size  departs the origin of its CM frame  with a speed  (measured with respect to ), while a second body  exits the center point of its CM frame  with relative speed . Furthermore, assume that  and  approach one another on a collision course directed along the same straight line. Given this scenario, the substantivalist will correctly infer that a third CM frame  is required to measure the quantity of motion of the soon-to-collide  and  (see Figure 1); but, they will also conclude that their respective speeds  and , which are determined relative to , will be different from the previously assigned values  and  (relative to  and ). Due to the relative motion of ,, and , the speeds ascribed to the two moving bodies will not normally be the same. However, although it is true that the value of a body's speed is frame-dependent, this does not exclude the possibility of predicting the magnitude of this quantity in several distinct frames based on the observations conducted from a single frame. More specifically, Cartesian space-time allows an observer located at either  or  to determine the exact values of  and  *in* the CM coordinate system .

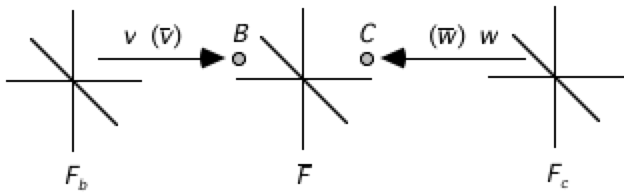


Fig. 1. The objects departing the CM frames  and , with speeds  and  respectively, will collide in the CM frame  (with speeds  and ).

The following example can establish this point: due to the invariance of relative speed, assume that the frames ,, both calculate a speed difference between  and  of, say, 6 miles per hour at time . Also, from the invariance of mass and distance relations, suppose our frames measure a spatial difference of 5 feet at , and that the respective sizes of  and  are as 2 to 3 (i.e., ). Given these numbers, it is easy to predict the precise values that  and  will take in , which is the CM frame of the impending collision between  and  (which will transpire at a time later than ). If we recall Descartes' conservation law and (2.1)-(2.2), and apply them to , we arrive at the equation (assuming absolute values)

 (4.1)

where  and  are the respective coordinate positions of  and  in . Thus, since we know that , frames  and  will both determine at , when the invariant spatial difference is 5 feet, that the origin of  should be located at a position where  and  (relative to , of course). In addition, the invariant relative speed is 6 m.p.h.; thus,  and  will both predict that  will assign the speed values  and  (since their ratio is 2/3 and their sum 6). In conclusion, the frames  and  can both determine at time  the exact placement of the origin of , and the speeds that  will gauge for  and . Once we are provided these magnitudes, the quantity of motion as measured from  is obtained by multiplying the relevant speed and mass (or Cartesian size) values: here, we should also note that (2.4)3=(3.6)2 as required by equations (2.2) and the Cartesian conservation law.

Of course, as the bodies draw closer together over time, the values obtained for  from both  and  will change; that is, this method will determine new values at each succeeding instant. Yet, since the bodies in Huygens' frames move at relative uniform speeds before and after impact, so that all the frames are inertial relative to one another, our observers at  and  can easily estimate the precise spatial location where  and  will collide, say, at a time , and the exact location of the CM frame  for each instant from  to . That is, because the bodies move inertial relative to one another, their relative velocity will remain invariant; so,  and  will retain the same value throughout the time period (only their coordinate positions  and  will change in order to preserve the ratio 2/3). Consequently, as was initially desired, we have located a means of connecting or correlating the value of the quantity of motion of a body ( or ) in one frame ( or ) with its quantity of motion in another frame () over a future temporal interval ( to ). Although this relationalist procedure needed both  and  to derive the correct total results, once these values are obtained we can easily infer each object's individual quantity of motion. In short, a Cartesian can now employ Huygens' frames to predict the future coordinate systems that preserve Descartes' quantity of motion, along with the values ascribed to the individual bodies relative to those future frames. By demonstrating an ability to compare quantities across frames, this relationalist strategy would thus appear to overcome at least one important substantivalist objection.

However, it is important to note that this method of conserving Descartes' quantity of motion is not a conservation law in, what we may call, the standard or "classical" sense of the term. The classical conservation laws can track a particle's motion and conserve the desired quantity from the perspective of a *single* (inertial) frame, through employing such conservation principles as momentum or kinetic energy. Yet, our Huygens' system can, at best, only offer predictions on the positions of the future CM frames that will conserve quantity of motion *from the perspective of that future frame.* For example, on Huygens' scheme, a center-of-mass frame, say, , can only predict the location of another CM frame, such as, , that maintains the value of the desired invariant from 's viewpoint:  cannot continuously track a body's motion and calculate the law's invariant quantity over a succession of interactions from the perspective of , as is possible with the standard conservation laws. This is a significant realization, but is not necessarily a major problem for a Cartesian intent on utilizing Huygens' method of conserving quantity of motion. Overall, Descartes' principle that "[God] always maintains an equal quantity [of motion] in the universe." (Pr II 36) is subject to many interpretations; and it is certainly not clear that he had in mind the classical conception of a conservation law. A conservation law that relies on a series of momentary frames defined relative to individual collisions, rather than from a single frame that covers a series of collisions, would seem perfectly compatible with Descartes' request for a relationally-defined conserved quantity of motion.

With respect to this method of linking the CM frames, finally, a substantivalist might raise the objection that this process seems to bestow a sort of global inertial frame upon our Cartesian space-time, and thus constitutes a violation of relationalism by tacitly acknowledging more structure than the relationalist can admit. There are two replies to this argument, which correspond to the two alternative relationalist variants discussed in section 3.1. First, if (R2) is employed, then a global inertial frame would not represent a violation of relationalism, as long as the frame was dependent upon the material occupant of the universe (whatever that may mean). Second, according to (R1\*), or even the original (R1), our method of linking the CM frames does not appear to pose a threat to relationalism since (as we have seen) this process *cannot* track a single particles' motion over an indefinitely long temporal period, a capacity seemingly possessed by "real" global inertial frames. On the whole, given the persistent collisions in Descartes' plenum, the relatively brief life span of each CM frame adapted to the momentary invariants of Cartesian space-time does not seem to admit the possibility of predicting future CM frames for more than a few fleeting instants.

5. Conclusion

We began section 2 with a particular question in mind: Is it possible to harmonize Descartes' relationalism, conservation law, and hypotheses on bodily interactions without assuming the sort of absolute or "fixed" reference frames assumed necessary by Newton's *De gravitatione* argument (as implicated in premise (**5**) above)? After much consideration, the answer is yes, but with serious reservations. Although Huygens' project is handicapped in many areas, the Newtonian must still admit that Descartes' quantity of motion is conserved in collisions viewed from the center-of-mass perspective. More importantly, this conservation law functions within a relational space-time, and thus obviates the need for Newton's absolute space, or at least a substantivalist interpretation of Newtonian space-time, to determine a body's trajectory. Of course, six out of seven of Descartes' impact rules were abandoned along the way, and the form of relationalism advocated by the Cartesian changed from (R1) to either (R1\*) or (R2), but the remaining collision rule serves as the foundation for a working Cartesian dynamics (acknowledging, once again, the ambiguity of "Cartesian dynamics" as a descriptive term). In addition, as remarked in section 3.1, (R1\*), or (R2), appear to be a more accurate reflection of Descartes' actual treatment of motion, regardless of his somewhat deceptive avowal of (R1).

Yet, despite the development of a means of linking the information across distinct CM frames (section 4), there still exists powerful reservations connected with Huygens' scheme: even if we grant the disputed consistency of (R1\*) and/or (R2), the CM reference frames apparently cannot explicate the interactions of (relatively) accelerating bodies. As noted, if one attempts to conserve the total quantity of motion throughout the universe, as mandated in Descartes' *Principles,* then it will be necessary to provide CM frames for the collisions of bodies not exhibiting uniform pre- and post-impact relative speeds, since these interactions are rather prevalent in Descartes' world. Nonetheless, quantity of motion will not normally be conserved from a CM perspective in collisions involving such accelerating bodies. If these frames cannot conserve the Cartesian universal quantity, then the method for linking the CM frames outlined in section 4 will be to no avail.[[16]](#endnote-16)

ENDNOTES

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1. Descartes (1983). Translations are based on Miller and Miller but are checked against the Adam and Tannery edition of the *Oeuvres de Descartes* (1976). I will identify passages according to the standard convention: thus, Article 15, Part II, of the *Principles* will be labeled "Pr II 15." Passages from the French translation of 1647 will be prefaced by "Fr". [↑](#endnote-ref-1)
2. As a notable example of the supposed relational incompatibility of the Cartesian impact rules, one needs only to compare the predictions of the fourth and fifth rules. For an in-depth discussion of these problems, see, e.g., Garber (1992, 240-241). [↑](#endnote-ref-2)
3. Newton presents a series of additional arguments against the Cartesian analysis of motion in *De gravitatione.* However, these latter arguments will not enter into the discussion since our exclusive concern is with Newton's sole argument concerning the possibility of defining Cartesian inertial motion. [↑](#endnote-ref-3)
4. Huygens' relationalism seems to have remained largely (R1), as is evident in the following passage: "there is nothing to distinguish straight motion from rest, and both one and the other are relative. . . ." Huygens (1950, vol. 16, 183), trans. by A. Elzinga (1972, 96). But, the center-of-mass reference frame seems to usher him into the realm of (R1\*) relationalism, as will be discussed below. [↑](#endnote-ref-4)
5. For a discussion of Huygen's role in the history of relational space-time theories, see Stein (1977). Various aspects of Huygens' natural philosophy are examined in Elzinga (1958) and Dugas (1958, chap. 10). [↑](#endnote-ref-5)
6. Huygens (1950, vol. 16, 186), trans. by Westfall (1971, 149). Unless otherwise noted, all following translations will be based on Westfall, but checked against the *Oeuvres Complètes.* [↑](#endnote-ref-6)
7. Snelders (1980, 120). Although it is beyond the scope of this paper to cover this issue in depth, see also, Westman (1980) and Shapiro (1980) for more details of Huygens' physics and its relation to Cartesianism. [↑](#endnote-ref-7)
8. I owe many of the details of the following discussion to Barbour (1989, 473-478). [↑](#endnote-ref-8)
9. Huygens' utilization of the center-of-mass frame owed much to his prior understanding of the principles of statics, the branch of mechanics that deals with bodies under the equilibrium of forces (e.g., the mechanical lever). In balancing two weights, the balance's suspension point is the center-of-mass, or the center-of-gravity, of the two bodies. One of the important principles of statics which Huygens' employed is that "the common centre of gravity of bodies cannot be raised by that motion of the bodies which is generated by the gravity of those bodies themselves." (1950, vol. 16, 57) Based on this fact, as well as Galileo's work on free-fall, Huygens demonstrated that if two falling bodies reach speeds that are inversely proportional to their sizes, their collision must cause them to rebound with their original speeds (assuming they are perfectly "elastic", or "hard"). If not, the center-of-gravity (mass) of the two bodies might reach a height after collision that is greater than the height from which they fell, in direct violation of the above statics principle. (Westfall 1971, 148-159) The various meanings of "perfect hardness" as employed by the early modern natural philosophers is an under-appreciated subject of study among current commentators; but, see Slowik (1996) for an attempt to explicate Descartes' use of this notion. [↑](#endnote-ref-9)
10. In this experiment, Newton allegedly demonstrates that the centrifugal force manifest by the water in a rotating bucket could not have been produced by the motion of the water relative to the sides of the bucket, its containing surface, but only relative to absolute space. See, e.g., Laymon (1978). Examining the precise details of the "bucket experiment" is outside the scope of this paper, since we are mainly concerned with the implications of the sole *De gravitatione* argument presented in section 1 (although the implications of this argument are closely related to the "bucket" experiment). [↑](#endnote-ref-10)
11. See, e.g., Friedman (1983, chapters 2 and 3). [↑](#endnote-ref-11)
12. Sklar (1974, 200). Mach's theory is presented in (1942, 280-286). [↑](#endnote-ref-12)
13. See Earman (1989, chap. 4), for an analysis of the various attempts by relationalists, including Huygens, to explicate circular motion and its dynamic effects; and chapter 6, for an examination of the viability of (R2), to be discussed below. Earman gives a nice exposition of how Huygens, at least in his later years, tried to preserve (R1) and still account for rotation. Furthermore, there is the related problem of whether (R1) is implicitly contained in (R2), but there does not appear to be any overt reason for accepting this entailment relationship. [↑](#endnote-ref-13)
14. Mach seems to suggest that the Newtonian predictions are simply wrong; i.e., that the rotation of the stars relative to the fixed bucket would likewise result in centrifugal effects being experienced by the latter. (see, Mach 1942, 283-284) However, if Mach is correct, so that local inertial effects depend on distant massive bodies, then this fact should be *in principle* verifiable through available experimental or observational evidence (possibly by studying large stellar bodies; see, Sklar 1974, 201)--but, Mach left no hints as to how his theory could be verified by anything less than rotating the entire set of fixed stars. [↑](#endnote-ref-14)
15. Westfall (1971, 156-158); Gabbey (1980, 178-181). Huygens' treatment of centrifugal force can be expressed in the modern formulation, , where  is the radius of the circle (Westfall 1971, 170), although Huygens did not carry his discovery that crucial step further, as Newton did, and postulate the force of gravity required to offset this force--the influence of, or devotion to, the Cartesian vortex was crucial , here--see, e.g., Hall (1976). [↑](#endnote-ref-15)
16. I would like to thank Mark Wilson, Ron Laymon, Calvin Normore, Bob Batterman, and two anonymous referees from *Synthese,* for their helpful comments and suggestions on the earlier drafts of this paper. [↑](#endnote-ref-16)