Florentin Smarandache
(author and editor)

Collected Papers

(on Neutrosophics, Plithogenics, Hypersoft Set, and other topics)

Volume X

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Volume X

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Introductory Note

This tenth volume of *Collected Papers* includes 86 papers in English and Spanish comprising 972 pages mainly on Neutrosophics, Plithogenics, Hypersoft Set, Hypergraphs, but also on various other topics such as Pandemic, COVID-19, Low-Level Laser Therapy, Neurology Disorders, Phytochemistry, Phytomedicine, Plant Medicine, Mannose-Binding Lectins, Moringa Oleifera uses, DNA Transduction, Information Medicine, Ivermectin, Laser LED Pointer, Deep Learning, Edge Computing, Transfer Learning, Ratio estimators, Mean square error, Impact factor, Journal impact factor, Garfield impact factor, improved impact factor, extended impact factor, total impact factor, Blue ocean shift, hospitality, restaurant industry, customer satisfaction, Software development methodology, intelligent control, real time control systems, hardening process, materials, high-frequency currents, robot simulation, graphical user interface, semantic word representation, sentiment classiffication, and so on, written between 2014-2022 by the author alone or in collaboration with the following 105 co-authors (alphabetically ordered) from 26 countries: Abu Sufian, Ali Hassan, Ali Safaa Sadiq, Anirudha Ghosh, Assia Bakali, Atiqe Ur Rahman, Laura Bogdan, Willem K.M. Brauers, Erick González Caballero, Fausto Cavallaro, Gavrilă Calefariu, T. Chalapathi, Victor Christianto, Mihaela Colhon, Sergiu Boris Cononovici, Mamoni Dhar, Irfan Deli, Rebeca Escobar-Jara, Alexandru Gal, N. Gandotra, Sudipta Gayen, Vassilis C. Gerogiannis, Noel Batista Hernández, Hongnian Yu, Hongbo Wang, Mihaiela Iliescu, F. Nirmala Irudayam, Sripati Jha, Darjan Karabašević, T. Katican, Bakhtawar Ali Khan, Hina Khan, Volodymyr Krasnoholovets, R. Kiran Kumar, Manoranjan Kumar Singh, Ranjan Kumar, M. Lathamaheswari, Yasar Mahmood, Nivetha Martin, Adrian Mărgean, Octavian Melinte, Mingcong Deng, Marcel Migdalovici, Monika Moga, Sana Moin, Mohamed Abdel-Basset, Mohamed Elhoseny, Rehab Mohamed, Mohamed Talea, Kalyan Mondal, Muhammad Aslam, Muhammad Aslam Malik, Muhammad Ihsan, Muhammad Naveed Jafar, Muhammad Rayees Ahmad, Muhammad Saeed, Muhammad Saqlain, Muhammad Shabir, Mujahid Abbas, Mumtaz Ali, Radu I. Munteanu, Ghulam Murtaza, Munazza Naz, Tahsin Oner, Gabrijela Popović, Surapati Pramanik, R. Priya, S.P. Priyadharshini, Midha Qayyum, Quang-Thinh Bui, Shazia Rana, Akbara Rezaei, Jesús Estupiñán Ricardo, Rıdvan Sahin, Saeeda Mirvakili, Said Broumi, A. A. Salama, Flavius Aurelian Sârbu, Ganeshsree Selvachandran, Javid Shabbir, Shio Gai Quek, Son Hoang Le, Florentin Smarandache, Dragiša Stanujkić, S. Sudha, Taha Yasin Ozturk, Zaigham Tahir, The Houw Iong, Ayse Topal, Alptekin Ulutaș, Maikel Yelandi Leyva Vázquez, Rizha Vitania, Luige Vlădăreanu, Victor Vlădăreanu, Ștefan Vlăduțescu, J. Vimala, Dan Valeriu Voinea, Adem Yolcu, Yongfei Feng, Abd El-Nasser H. Zaied, Edmundas Kazimieras Zavadskas.

Florentin Smarandache is professor of mathematics at the University of New Mexico and he published over 300 articles and books. He coined the words "neutrosophy" [(French neutre < Latin neuter, neutral, and Greek sophia, skill/wisdom) means knowledge of neutral thought] and its derivatives: neutrosophic, neutrosophication, neutrosophicator, deneutrosophication, deneutrosophicator, etc. He is the founder and developer of neutrosophic set / logic / probability / statistics etc. In 2006 he introduced the degree of dependence/independence between the neutrosophic components T, I, F.

In 2007 he extended the neutrosophic set to Neutrosophic Overset (when some neutrosophic component is > 1), and to Neutrosophic Underset (when some neutrosophic component is < 0), and to Neutrosophic Offset (when some neutrosophic components are off the interval [0, 1], i.e. some neutrosophic component > 1 and some neutrosophic component < 0). Then, similar extensions to respectively Neutrosophic Over/Under/Off Logic, Measure, Probability, Statistics etc.

Then, introduced the Neutrosophic Tripolar Set and Neutrosophic Multipolar Set, also the Neutrosophic Tripolar Graph and Neutrosophic Multipolar Graph.

He then generalized the Neutrosophic Logic/Set/Probability to Refined Neutrosophic Logic/Set/Probability [2013], where T can be split into subcomponents T1, T2, ..., Tp, and I into I1, I2, ..., Ir, and F into F1, F2, ..., Fs, where p+r+s = n ≥ 1. Even more: T, I, and/or F (or any of their subcomponents Tj ,Ik, and/or Fl) could be countable or uncountable infinite sets.

In 2015 he refined the indeterminacy "I", within the neutrosophic algebraic structures, into different types of indeterminacies (depending on the problem to solve), such as I1, I2, , Ip with integer p ≥ 1, and obtained the refined neutrosophic numbers of the form Np = a+b1I1+b2I2+ +bpIp where a, b1, b2, , bp are real or complex numbers, and a is called the determinate part of Np, while for each k in {1, 2, , p} Ik is called the k-th indeterminate part of Np.

Then consequently he extended the neutrosophic algebraic structures to Refined Neutrosophic Algebraic Structures [or Refined Neutrosophic I-Algebraic Structures] (2015), which are algebraic structures based on sets of the refined neutrosophic numbers a+b1I1+b2I2+ +bpIp.

He introduced the (T, I, F)-Neutrosophic Structures [2015]. In any field of knowledge, each structure is composed from two parts: a space, and a set of axioms (or laws) acting (governing) on it. If the space, or at least one of its axioms (laws), has some indeterminacy, that structure is a (T, I, F)-Neutrosophic Structure. And he extended them to the (T, I, F)-Neutrosophic I-Algebraic Structures [2015], i.e. algebraic structures based on neutrosophic numbers of the form a+bI, but also having indeterminacy related to the structure space (elements which only partially belong to the space, or elements we know nothing if they belong to the space or not) or indeterminacy related to at least an axiom (or law) acting on the structure space. Then he extended them to Refined (T, I, F)-Neutrosophic Refined I-Algebraic Structures.

Also, he proposed an extension of the classical probability and the imprecise probability to the 'neutrosophic probability' [1995], that he defined as a tridimensional vector whose components are real subsets of the non-standard interval [-0, 1+] introduced the neutrosophic measure and neutrosophic integral: http://fs.gallup.unm.edu/NeutrosophicMeasureIntegralProbability.pdf and also extended the classical statistics to neutrosophic statistics: http://fs.gallup.unm.edu/NeutrosophicStatistics.pdf

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NEUTROSOPHICS

Some Types of Neutrosophic Crisp Sets and Neutrosophic Crisp Relations

A.A. Salama, Said Broumi, Florentin Smarandache

A.A. Salama, Said Broumi, Florentin Smarandache (2014). Some Types of Neutrosophic Crisp Sets and Neutrosophic Crisp Relations. *I.J. Information Engineering and Electronic Business*, 9

***Abstract*—**The purpose of this paper is to introduce a new types of crisp sets are called the neutrosophic crisp set with three types 1, 2, 3. After given the fundamental definitions and operations, we obtain several properties, and discussed the relationship between neutrosophic crisp sets and others. Finally, we introduce and study the notion of neutrosophic crisp relations.

***Index Terms*—**Neutrosophic set, neutrosophic crisp sets, neutrosophic crisp relations, generalized neutrosophic set, Intuitionistic neutrosophic Set.

I. Introduction

Since the world is full of indeterminacy, the neutrosophics found their place into contemporary research. The fundamental concepts of neutrosophic set, introduced by Smarandache in [16, 17, 18], and Salama et al. in [4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15], provides a natural foundation for treating mathematically the neutrosophic phenomena which exist pervasively in our real world and for building new branches of neutrosophic mathematics. Neutrosophy has laid the foundation for a whole family of new mathematical theories generalizing both their classical and fuzzy counterparts [1, 2, 3, 19] such as a neutrosophic set theory. In this paper we introduce a new types of crisp sets are called the neutrosophic crisp set with three types 1, 2, 3. After given the fundamental definitions and operations, we obtain several properties, and discussed the relationship between neutrosophic crisp sets and others. Finally, we introduce and study the notion of neutrosophic crisp relations.

The paper unfolds as follows. The next section briefly introduces some definitions related to neutrosophic set theory and some terminologies of neutrosophic crisp set. Section 3 presents new types of neutrosophic crisp sets and studied some of their basic properties. Section 4 presents the concept of neutrosophic crisp relations . Finally we concludes the paper.

II. Preliminaries

We recollect some relevant basic preliminaries, and in particular, the work of Smarandache in [16 , 17, 18], and Salama et al. [4,5]. Smarandache introduced the neutrosophic components T, I, F which represent the membership, indeterminacy, and non-membership values respectively, where $\left]^{-}0, 1^{+}\right[$ is nonstandard unit interval.

Definition 2.1 *[9, 13, 15]*
A neutrosophic crisp set (NCS for short) $A = \langle A_1, A_2, A_3 \rangle$ can be identified to an ordered triple $\langle A_1, A_2, A_3 \rangle$ are subsets on X, and every crisp event in X is obviously an NCS having the form $\langle A_1, A_2, A_3 \rangle$,

Salama et al. constructed the tools for developed neutrosophic crisp set, and introduced the NCS ϕ_N, X_N in X as follows:

1) ϕ_N may be defined as four types:

i)Type1: $\phi_N = \langle \phi, \phi, X \rangle$, or

ii)Type2: $\phi_N = \langle \phi, X, X \rangle$, or

iii)Type3: $\phi_N = \langle \phi, X, \phi \rangle$, or

iv)Type4: $\phi_N = \langle \phi, \phi, \phi \rangle$

2) X_N may be defined as four types

i) Type1: $X_N = \langle X, \phi, \phi \rangle$,

ii) Type2: $X_N = \langle X, X, \phi \rangle$,

iii) Type3: $X_N = \langle X, X, \phi \rangle$,

iv) Type4: $X_N = \langle X, X, X \rangle$,

Definition 2.2 *[9, 13, 15]*

Let $A = \langle A_1, A_2, A_3 \rangle$ a NCE or UNCE on X, then the complement of the set A (A^c, for short) maybe defined as three kinds of complements

(C_1) Type1: $A^c = \langle A^c{}_1, A^c{}_2, A^c{}_3 \rangle$,

(C_2) Type2: $A^c = \langle A_3, A_2, A_1 \rangle$

(C_3) Type3: $A^c = \langle A_3, A^c{}_2, A_1 \rangle$

One can define several relations and operations between NCS as follows:

Definition 2.3 *[9, 13, 15]*

Let X be a non-empty set, and NCSS A and B in the form $A = \langle A_1, A_2, A_3 \rangle$, $B = \langle B_1, B_2, B_3 \rangle$, then we may consider two possible definitions for subsets ($A \subseteq B$)

($A \subseteq B$) may be defined as two types:

1)Type1:

$A \subseteq B \Leftrightarrow A_1 \subseteq B_1, A_2 \subseteq B_2$ and $A_3 \supseteq B_3$ or

2)Type2:

$$A \subseteq B \Leftrightarrow A_1 \subseteq B_1, A_2 \supseteq B_2 \text{ and } A_3 \supseteq B_3$$

Definition 2.4 *[9, 13, 15]*

Let X be a non-empty set, and NCSS A and B in the form $A = \langle A_1, A_2, A_3 \rangle$, $B = \langle B_1, B_2, B_3 \rangle$ are NCSS Then

1) $A \cap B$ may be defined as two types:

i) Type1:

$A \cap B = \langle A_1 \cap B_1, A_2 \cap B_2, A_3 \cup B_3 \rangle$ or

ii) Type2:

$A \cap B = \langle A_1 \cap B_1, A_2 \cup B_2, A_3 \cup B_3 \rangle$

2) $A \cup B$ may be defined as two types:

i) Type 1: $A \cup B = \langle A_1 \cup B_1, A_2 \cap B_2, A_3 \cup B_3 \rangle$

or

ii) Type 2: $A \cup B = \langle A_1 \cup B_1, A_2 \cap B_2, A_3 \cap B_3 \rangle$

Proposition 2.1 *[9, 13, 15]*

Let $\{A_j : j \in J\}$ be arbitrary family of neutrosophic crisp subsets in X, then

1) $\cap A_j$ may be defined two types as :

i)Type1: $\cap A_j = \langle \cap Aj_1, \cap A_{j2}, \cup A_{j3} \rangle$,or

ii)Type2: $\cap A_j = \langle \cap Aj_1, \cup A_{j2}, \cup A_{j3} \rangle$.

2) $\cup A_j$ may be defined two types as :

i)Type1: $\cup A_j = \langle \cup Aj_1, \cap A_{j2}, \cap A_{j3} \rangle$ or

ii)Type2: $\cup A_j = \langle \cup Aj_1, \cup A_{j2}, \cap A_{j3} \rangle$.

III. New Types of Neutrosophic Crisp Sets

We shall now consider some possible definitions for some types of neutrosophic crisp sets

Definition 3.1

Let X be a non-empty fixed sample space. A neutrosophic crisp set (NCS for short) A is an object having the form $A = \langle A_1, A_2, A_3 \rangle$ where A_1, A_2 and A_3 are subsets of X.

Definition 3.2

The object having the form $A = \langle A_1, A_2, A_3 \rangle$ is called

1) (Neutrosophic Crisp Set with Type 1) If satisfying $A_1 \cap A_2 = \phi$, $A_1 \cap A_3 = \phi$ and $A_2 \cap A_3 = \phi$. (NCS-Type1 for short).

2) (Neutrosophic Crisp Set with Type 2) If satisfying $A_1 \cap A_2 = \phi$, $A_1 \cap A_3 = \phi$ and $A_2 \cap A_3 = \phi$ and $A_1 \cup A_2 \cup A_3 = X$. (NCS-Type2 for short).

3) (Neutrosophic Crisp Set with Type 3) If satisfying $A_1 \cap A_2 \cap A_3 = \phi$ and $A_1 \cup A_2 \cup A_3 = X$. (NCS-Type3 for short).

Definition 3.3

1) (Neutrosophic Set [7]): Let X be a non-empty fixed set. A neutrosophic set (NS for short) A is an object having the form $A = \langle x, \mu_A(x), \sigma_A(x), \nu_A(x) \rangle$ where $\mu_A(x), \sigma_A(x)$ and $\nu_A(x)$ which represent the degree of member ship function (namely $\mu_A(x)$), the degree of indeterminacy (namely $\sigma_A(x)$), and the degree of non-member ship (namely $\nu_A(x)$) respectively of each element $x \in X$ to the set A where $0^- \le \mu_A(x), \sigma_A(x), \nu_A(x) \le 1^+$ and $0^- \le \mu_A(x) + \sigma_A(x) + \nu_A(x) \le 3^+$.

2) (Generalized Neutrosophic Set [8]**)**: Let X be a non-empty fixed set. A generalized neutrosophic (GNS for short) set A is an object having the form $A = \langle x, \mu_A(x), \sigma_A(x), \nu_A(x) \rangle$ where $\mu_A(x), \sigma_A(x)$ and $\nu_A(x)$ which represent the degree of member ship function (namely $\mu_A(x)$), the degree of indeterminacy (namely $\sigma_A(x)$), and the degree of non-member ship (namely $\nu_A(x)$) respectively of each element $x \in X$ to the set A where $0^- \le \mu_A(x), \sigma_A(x), \nu_A(x) \le 1^+$ and the functions satisfy the condition $\mu_A(x) \wedge \sigma_A(x) \wedge \nu_A(x) \le 0.5$ and $0^- \le \mu_A(x) + \sigma_A(x) + \nu_A(x) \le 3^+$.

3) (Intuitionistic Neutrosophic Set [16]**)**. Let X be a non-empty fixed set. An intuitionistic neutrosophic set A (INS for short) is an object having the form $A = \langle \mu_A(x), \sigma_A(x), \nu_A(x) \rangle$ where $\mu_A(x), \sigma_A(x)$ and $\nu_A(x)$ which represent the degree of member ship function (namely $\mu_A(x)$), the degree of indeterminacy (namely $\sigma_A(x)$), and the degree of non-member ship (namely $\nu_A(x)$) respectively of each element $x \in X$ to the set A where $0.5 \le \mu_A(x), \sigma_A(x), \nu_A(x)$ and the functions satisfy the condition $\mu_A(x) \wedge \sigma_A(x) \le 0.5$, $\mu_A(x) \wedge \nu_A(x) \le 0.5$, $\sigma_A(x) \wedge \nu_A(x) \le 0.5$, and $^-0 \le \mu_A(x) + \sigma_A(x) + \nu_A(x) \le 2^+$.

A neutrosophic crisp with three types the object $A = \langle A_1, A_2, A_3 \rangle$ can be identified to an ordered triple $\langle A_1, A_2, A_3 \rangle$ are subsets on X, and every crisp set in X is obviously a NCS having the form $\langle A_1, A_2, A_3 \rangle$.Every neutrosophic set $A = \langle \mu_A(x), \sigma_A(x), \nu_A(x) \rangle$ on X is obviously on NS having the form $\langle \mu_A(x), \sigma_A(x), \nu_A(x) \rangle$.

Remark 3.1

1) The neutrosophic set not to be generalized neutrosophic set in general.
2) The generalized neutrosophic set in general not intuitionistic NS but the intuitionistic NS is generalized NS.

Intuitionistic NS $\longrightarrow$ Generalized NS $\longrightarrow$ NS

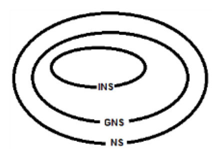

Fig.1: Represents the relation between types of NS

Corollary 3.1

Let X non-empty fixed set and $A = \langle \mu_A(x), \sigma_A(x), \nu_A(x) \rangle$ be INS on X

Then:

1) Type1- A^c of INS be a GNS.
2) Type2- A^c of INS be a INS.
3) Type3- A^c of INS be a GNS.

Proof

Since A INS then $0.5 \le \mu_A(x), \sigma_A(x), \nu_A(x)$, and $\mu_A(x) \wedge \sigma_A(x) \le 0.5, \nu_A(x) \wedge \mu_A(x) \le 0.5$ $\nu_A(x) \wedge \sigma_A(x) \le 0.5$ Implies $\mu^c{}_A(x), \sigma^c{}_A(x), \nu^c{}_A(x) \le 0.5$ then is not to be Type1- A^c INS. On other hand the Type 2- A^c, $A^c = \langle \nu_A(x), \sigma_A(x), \mu_A(x) \rangle$ be INS and Type 3- A^c, $A^c = \langle \nu_A(x), \sigma^c{}_A(x), \mu_A(x) \rangle$ and $\sigma^c{}_A(x) \le 0.5$ implies to $A^c = \langle \nu_A(x), \sigma^c{}_A(x), \mu_A(x) \rangle$ GNS and not to be INS

Example 3.1

Let $X = \{a, b, c\}$, and A, B, C are neutrosophic sets on X,

$A = \langle 0.7,0.9,0.8) \setminus a, (0.6,0.7,0.6) \setminus b, (0.9,0.7,0.8 \setminus c \rangle$,

$B = \langle 0.7,0.9,0.5) \setminus a, (0.6,0.4,0.5) \setminus b, (0.9,0.5,0.8 \setminus c \rangle$

$C = \langle 0.7,0.9,0.5) \setminus a, (0.6,0.8,0.5) \setminus b, (0.9,0.5,0.8 \setminus c \rangle$

By the Definition 3.3 no.3

$\mu_A(x) \wedge \sigma_A(x) \wedge \nu_A(x) \geq 0.5,$ A be not GNS and INS,

$B = \langle 0.7,0.9,0.5) \setminus a, (0.6,0.4,0.5) \setminus b, (0.9,0.5,0.8 \setminus c \rangle$

not INS, where $\sigma_A(b) = 0.4 < 0.5$. Since $\mu_B(x) \wedge \sigma_B(x) \wedge \nu_B(x) \leq 0.5$ then B is a GNS but not INS.

$A^c = \langle 0.3,0.1,0.2) \setminus a, (0.4,0.3,0.4) \setminus b, (0.1,0.3,0.2 \setminus c \rangle$

be a GNS, but not INS.

$B^c = \langle 0.3,0.1,0.5) \setminus a, (0.4,0.6,0.5) \setminus b, (0.1,0.5,0.2 \setminus c \rangle$

be a GNS, but not INS, C be INS and GNS,

$C^c = \langle 0.3,0.1,0.5) \setminus a, (0.4,0.2,0.5) \setminus b, (0.1,0.5,0.2 \setminus c \rangle$

be a GNS but not INS.

Definition 3.4

A neutrosophic crisp set (NCS for short) $A = \langle A_1, A_2, A_3 \rangle$ can be identified to an ordered triple $\langle A_1, A_2, A_3 \rangle$ are subsets on X, and every crisp set in X is obviously an NCS having the form $\langle A_1, A_2, A_3 \rangle$,

Salama et al in [6,13] constructed the tools for developed neutrosophic crisp set, and introduced the NCS ϕ_N, X_N in X as follows:

1) ϕ_N may be defined as four types:

i) Type1: $\phi_N = \langle \phi, \phi, X \rangle$, or

ii)Type2: $\phi_N = \langle \phi, X, X \rangle$, or

iii) Type3: $\phi_N = \langle \phi, X, \phi \rangle$, or

iv) Type4: $\phi_N = \langle \phi, \phi, \phi \rangle$

2) X_N may be defined as four types

i) Type1: $X_N = \langle X, \phi, \phi \rangle$,

ii) Type2: $X_N = \langle X, X, \phi \rangle$,

v) Type3: $X_N = \langle X, X, \phi \rangle$,

vi) Type4: $X_N = \langle X, X, X \rangle$,

Definition 3.5

A NCS-Type1 ϕ_{N_1}, X_{N_1} in X as follows:

1) ϕ_{N1} may be defined as three types:

i) Type1: $\phi_{N_1} = \langle \phi, \phi, X \rangle$, or

ii) Type2: $\phi_{N_1} = \langle \phi, X, \phi \rangle$, or

iii) Type3: $\phi_N = \langle \phi, \phi, \phi \rangle$.

2) X_{N1} may be defined as one type

Type1: $X_{N_1} = \langle X, \phi, \phi \rangle$.

Definition 3.6

A NCS-Type2, ϕ_{N_2}, X_{N2} in X as follows:

1) ϕ_{N2} may be defined as two types:

i) Type1: $\phi_{N_2} = \langle \phi, \phi, X \rangle$, or

ii) Type2: $\phi_{N_2} = \langle \phi, X, \phi \rangle$

2) X_{N2} may be defined as one type

Type1: $X_{N_2} = \langle X, \phi, \phi \rangle$

Definition 3.7

a NCS-Type 3, ϕ_{N3}, X_{N3} in X as follows:

1) ϕ_{N3} may be defined as three types:

i) Type1: $\phi_{N3} = \langle \phi, \phi, X \rangle$, or

ii) Type2: $\phi_{N3} = \langle \phi, X, \phi \rangle$, or

iii) Type3: $\phi_{N3} = \langle \phi, X, X \rangle$.

2) X_{N3} may be defined as three types

i)Type1: $X_{N3} = \langle X, \phi, \phi \rangle$,

ii)Type2: $X_{N3} = \langle X, X, \phi \rangle$,

iii)Type3: $X_{N3} = \langle X, \phi, X \rangle$,

Corollary 3.1

In general

1-Every NCS-Type 1, 2, 3 are NCS.

2-Every NCS-Type 1 not to be NCS-Type2, 3.

3-Every NCS-Type 2 not to be NCS-Type1, 3.

4-Every NCS-Type 3 not to be NCS-Type2, 1, 2.

5-Every crisp set be NCS.

The following Venn diagram represents the relation between NCSs

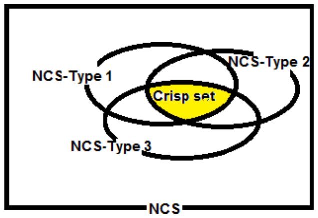

Fig. 2: Venn diagram represents the relation between NCSs

Example 3.2

Let A, B, C, D are NCSs on $X = \{a,b,c,d,e,f\}$, the following types of neutrosophic crisp sets

i) $A = \langle \{a\},\{b\},\{c\}\rangle$ be a NCS-Type 1, but not NCS-Type 2 and Type 3

ii) $B = \langle \{a,b\},\{c,d\},\{f,e\}\rangle$ be a NCS-Type 1, 2, 3

iii) $C = \langle \{a,b,c,d\},\{e\},\{a,b,f\}\rangle$ be a NCS-Type 3 but not NCS-Type 1, 2.

iv) $D = \langle \{a,b,c,d\},\{a,b,c\},\{a,b,d,f\}\rangle$ be a NCS but not NCS-Type 1, 2, 3.

The complement for A, B, C, D may be equals

The complement of A

i)Type 1: $A^c = \langle \{b,c,d,e,f\},\{a,c,d,e,f\},\{a,b,d,e,f\}\rangle$ be a NCS but not NCS—Type1, 2,3

ii)Type 2: $A^c = \langle \{c\},\{b\},\{a\}\rangle$ be a NCS-Type 3 but not NCS—Type1, 2

iii)Type 3: $A^c = \langle \{c\},\{a,c,d,e,f\},\{a\}\rangle$ be a NCS-Type 1 but not NCS—Type 2, 3.

The complement of B may be equals

i)Type 1:

$B^c = \langle \{c,d,e,f\},\{a,b,e,f\},\{a,b,c,d\}\rangle$ be NCS-Type 3 but not NCS-Type 1, 2.

ii) Type 2: $B^c = \langle \{e,f\},\{c,d\},\{a,b\}\rangle$ be NCS-Type 1, 2, 3.

iii)Type 3: $B^c = \langle \{e,f\},\{a,b,e,f\},\{a,b\}\rangle$ be NCS-Type 3, but not NCS-Type 1, 2.

The complement of C may be equals

i)Type 1: $C^c = \langle \{e,f\},\{a,b,c,d,f\},\{c,d,e\}\rangle$.

ii)Type 2: $C^c = \langle \{a,b,f\},\{e\},\{a,b,c,d\}\rangle$,

iii)Type 3:

$$C^c = \langle \{a,b,f\},\{a,b,c,d\},\{a,b,c,d\}\rangle,$$

The complement of D may be equals

i)Type 1: $D^c = \langle \{e,f\},\{d,e,f\},\{c,e\}\rangle$ be NCS-Type 3 but not NCS-Type 1, 2.

ii)Type 2: $D^c = \langle \{a,b,d,f\},\{a,b,c\},\{a,b,c,d\}\rangle$ be NCS-Type 3 but not NCS-Type 1, 2.

iii)Type 3: $D^c = \langle \{a,b,d,f\},\{d,e,f\},\{a,b,c,d\}\rangle$ be NCS-Type 3 but not NCS-Type 1, 2.

Definition 3.8

Let X be a non-empty set, $A = \langle A_1, A_2, A_3\rangle$

1) If A be a NCS-Type1 on X, then the complement of the set A (A^c, for short) maybe defined as one kind of complement Type1: $A^c = \langle A_3, A_2, A_1\rangle$.

2) If A be a NCS-Type 2 on X, then the complement of the set A (A^c, for short) maybe defined as one kind of complement $A^c = \langle A_3, A_2, A_1\rangle$.

3)If A be NCS-Type3 on X, then the complement of the set A (A^c, for short) maybe defined as one kind of complement defined as three kinds of complements

(C_1) Type1: $A^c = \langle A^c{}_1, A^c{}_2, A^c{}_3\rangle$,

(C_2) Type2: $A^c = \langle A_3, A_2, A_1\rangle$

(C_3) Type3: $A^c = \langle A_3, A^c{}_2, A_1\rangle$

Example 3.3

Let $X = \{a,b,c,d,e,f\}$, $A = \langle \{a,b,c,d\},\{e\},\{f\}\rangle$ be a NCS-Type 2, $B = \langle \{a,b,c\},\{\phi\},\{d,e\}\rangle$ be a NCS-Type1., $C = \langle \{a,b\},\{c,d\},\{e,f\}\rangle$ NCS-Type 3, then the complement $A = \langle \{a,b,c,d\},\{e\},\{f\}\rangle$, $A^c = \langle \{f\},\{e\},\{a,b,c,d\}\rangle$ NCS-Type 2, the complement of $B = \langle \{a,b,c\},\{\phi\},\{d,e\}\rangle$, $B^c = \langle \{d,e\},\{\phi\},\{a,b,c\}\rangle$ NCS-Type1. The complement of $C = \langle \{a,b\},\{c,d\},\{e,f\}\rangle$ may be defined as three types:

Type 1: $C^c = \langle \{c,d,e,f\},\{a,b,e,f\},\{a,b,c,d\}\rangle$.

Type 2: $C^c = \langle \{e,f\},\{c,d\},\{a,b\}\rangle$,

Type 3: $C^c = \langle \{e,f\},\{a,b,e,f\},\{a,b\}\rangle$,

Proposition 3.1

Let $\{A_j : j \in J\}$ be arbitrary family of neutrosophic crisp subsets on X, then

1) $\cap A_j$ may be defined two types as :

Type1: $\cap A_j = \left\langle \cap Aj_1, \cap A_{j_2}, \cup A_{j_3} \right\rangle$, or

Type2: $\cap A_j = \left\langle \cap Aj_1, \cup A_{j_2}, \cup A_{j_3} \right\rangle$.

2) $\cup A_j$ may be defined two types as :

Type1: $\cup A_j = \left\langle \cup Aj_1, \cap A_{j_2}, \cap A_{j_3} \right\rangle$ or

Type2: $\cup A_j = \left\langle \cup Aj_1, \cup A_{j_2}, \cap A_{j_3} \right\rangle$.

Definition 3.9

(a) If $B = \langle B_1, B_2, B_3 \rangle$ is a NCS in Y, then the preimage of B under f, denoted by $f^{-1}(B)$, is a NCS in X defined by $f^{-1}(B) = \langle f^{-1}(B_1), f^{-1}(B_2), f^{-1}(B_3) \rangle$.

(b) If $A = \langle A_1, A_2, A_3 \rangle$ is a NCS in X, then the image of A under f, denoted by $f(A)$, is the a NCS in Y defined by $f(A) = \langle f(A_1), f(A_2), f(A_3)^c) \rangle$.

Here we introduce the properties of images and preimages some of which we shall frequently use in the following.

Corollary 3.2

Let A, $\{A_i : i \in J\}$, be a family of NCS in X, and B, $\{B_j : j \in K\}$ NCS in Y, and $f : X \to Y$ a function. Then

(a) $A_1 \subseteq A_2 \Leftrightarrow f(A_1) \subseteq f(A_2)$,

$B_1 \subseteq B_2 \Leftrightarrow f^{-1}(B_1) \subseteq f^{-1}(B_2)$,

(b) $A \subseteq f^{-1}(f(A))$ and if f is injective, then $A = f^{-1}(f(A))$,

(c) $f^{-1}(f(B)) \subseteq B$ and if f is surjective, then $f^{-1}(f(B)) = B$,

(d) $f^{-1}(\cup B_i) = f^{-1}(B_i)$, $f^{-1}(\cap B_i) = \cap f^{-1}(B_i)$,

(e) $f(\cup A_{ii}) = \cup f(A_{ii})$; $f(\cap A_{ii}) \subseteq \cap f(A_{ii})$; and if f is injective, then $f(\cap A_{ii}) = \cap f(A_{ii})$;

(f) $f^{-1}(Y_N) = X_N$, $f^{-1}(\phi_N) = \phi_N$.

(g) $f(\phi_N) = \phi_N$, $f(X_N) = Y_N$, if f is subjective.

Proof

Obvious

IV. Neutrosophic Crisp Relations

Here we give the definition relation on neutrosophic crisp sets and study of its properties.

Let X, Y and Z be three ordinary nonempty sets

Definition 4.1

Let X and Y are two non-empty crisp sets and NCSS A and B in the form $A = \langle A_1, A_2, A_3 \rangle$ on X, $B = \langle B_1, B_2, B_3 \rangle$ on Y. Then

i) The product of two neutrosophic crisp sets A and B is a neutrosophic crisp set $A \times B$ given by $A \times B = \langle A_1 \times B_1, A_2 \times B_2, A_3 \times B_3 \rangle$ on $X \times Y$.

ii) We will call a neutrosophic crisp relation $R \subseteq A \times B$ on the direct product $X \times Y$.

The collection of all neutrosophic crisp relations on $X \times Y$ is denoted as $NCR(X \times Y)$

Definition 4.2

Let R be a neutrosophic crisp relation on $X \times Y$, then the inverse of R is denoted by R^{-1} where $R \subseteq A \times B$ on $X \times Y$ then $R^{-1} \subseteq B \times A$ on $Y \times X$.

Example 4.1

Let $X = \{a, b, c, d\}$, $A = \langle \{a, b\}, \{c\}, \{d\} \rangle$ and $B = \langle \{a\}, \{c\}, \{d, b\} \rangle$ then the product of two neutrosophic crisp sets given by

$A \times B = \langle \{(a,a),(b,a)\}, \{(c,c)\}, \{(d,d),(d,b)\} \rangle$ and

$B \times A = \langle \{(a,a),(a,b)\}, \{(c,c)\}, \{(d,d),(b,d)\} \rangle$, and

$R_1 = \langle \{(a,a)\}, \{(c,c)\}, \{(d,d)\} \rangle$, $R_1 \subseteq A \times B$ on $X \times X$,

$R_2 = \langle \{(a,b)\}, \{(c,c)\}, \{(d,d),(b,d)\} \rangle$

$R_2 \subseteq B \times A$ on $X \times X$.

Example 4.2

From the Example 3.1

$R_1^{-1} = \langle \{(a,a)\}, \{(c,c)\}, \{(d,d)\} \rangle \subseteq B \times A$ and

$R_2^{-1} = \langle \{(b,a)\}, \{(c,c)\}, \{(d,d),(d,b)\} \rangle \subseteq B \times A$.

Example 4.3

Let $X = \{a, b, c, d, e, f\}$,

$A = \langle \{a, b, c, d\}, \{e\}, \{f\} \rangle$,

$D = \langle \{a, b\}, \{e, c\}, \{f, d\} \rangle$ be a NCS-Type 2,

$B=\langle\{a,b,c\},\{\phi\},\{d,e\}\rangle$ be a NCS-Type1.

$C=\langle\{a,b\},\{c,d\},\{e,f\}\rangle$ be a NCS-Type 3. Then

$$A\times D=\left\langle\begin{matrix}\{(a,a),(a,b),(b,a),(b,b),(b,b),(c,a),(c,b)\\ ,(d,a),(d,b)\},\{(e,e),(e,c)\},\{(f,f),(f,d)\}\end{matrix}\right\rangle$$

$$D\times C=\left\langle\begin{matrix}\{(a,a),(a,b),(b,a),(b,b)\},\{(e,c),(e,d),\\ (c,c),(c,d)\},\{(f,e),(f,f),(d,e),(d,f)\}\end{matrix}\right\rangle$$

we can construct many types of relations on products. We can define the operations of neutrosophic crisp relation.

Definition 4.4

Let R and S be two neutrosophic crisp relations between X and Y for every $(x,y)\in X\times Y$ and NCSS A and B in the form $A=\langle A_1,A_2,A_3\rangle$ on X, $B=\langle B_1,B_2,B_3\rangle$ on Y, Then we can defined the following operations

i) $R\subseteq S$ may be defined as two types

a)Type1: $R\subseteq S \Leftrightarrow A_{1_R}\subseteq B_{1_S}$, $A_{2_R}\subseteq B_{2_S}$, $A_{3R}\supseteq B_{3S}$

b)Type2: $R\subseteq S \Leftrightarrow A_{1_R}\subseteq B_{1_S}$, $A_{2R}\supseteq B_{2S}$, $B_{3S}\subseteq A_{3R}$

ii) $R\cup S$ may be defined as two types

a)Type1:

$R\cup S=\langle A_{1R}\cup B_{1S},A_{2R}\cup B_{2S},A_{3R}\cap B_{3S}\rangle$,

b)Type2:

$R\cup S$
$=\langle A_{1R}\cup B_{1S},A_{2R}\cap B_{2S},A_{3R}\cap B_{3S}\rangle$.

iii) $R\cap S$ may be defined as two types

a)Type1: $R\cap S$
$=\langle A_{1R}\cap B_{1S},A_{2R}\cup B_{2S},A_{3R}\cup B_{3S}\rangle$,

b)Type2:

$R\cap S$
$=\langle A_{1R}\cap B_{1S},A_{2R}\cap B_{2S},A_{3R}\cup B_{3S}\rangle$.

Theorem 4.1

Let R, S and Q be three neutrosophic crisp relations between X and Y for every $(x,y)\in X\times Y$, then

i) $R\subseteq S\Rightarrow R^{-1}\subseteq S^{-1}$.

ii) $(R\cup S)^{-1}\Rightarrow R^{-1}\cup S^{-1}$.

iii) $(R\cap S)^{-1}\Rightarrow R^{-1}\cap S^{-1}$.

iv) $(R^{-1})^{-1}=R$.

v) $R\cap(S\cup Q)=(R\cap S)\cup(R\cap Q)$.

vi) $R\cup(S\cap Q)=(R\cup S)\cap(R\cup Q)$.

vii)If $S\subseteq R$, $Q\subseteq R$, then $S\cup Q\subseteq R$

Proof

Clear

Definition 4.5

The neutrosophic crisp relation $I\in NCR(X\times X)$, the neutrosophic crisp relation of identity may be defined as two types

i)Type1: $I=\{<\{A\times A\},\{A\times A\},\phi>\}$

ii)Type2: $I=\{<\{A\times A\},\phi,\phi>\}$

Now we define two composite relations of neutrosophic crisp sets.

Definition 4.6

Let R be a neutrosophic crisp relation in $X\times Y$, and S be a neutrosophic crisp relation in $Y\times Z$. Then the composition of R and S, $R\circ S$ be a neutrosophic crisp relation in $X\times Z$ as a definition may be defined as two types

i)Type1:

$R\circ S\leftrightarrow(R\circ S)(x,z)$
$=\cup\{<\{(A_1\times B_1)_R\cap(A_2\times B_2)_S\}$,
$\{(A_2\times B_2)_R\cap(A_2\times B_2)_S\}$,
$\{(A_3\times B_3)_R\cap(A_3\times B_3)_S\}>$.

ii)Type2 :

$R\circ S\leftrightarrow(R\circ S)(x,z)$
$=\cap\{<\{(A_1\times B_1)_R\cup(A_2\times B_2)_S\}$,
$\{(A_2\times B_2)_R\cup(A_2\times B_2)_S\}$,
$\{(A_3\times B_3)_R\cup(A_3\times B_3)_S\}>$.

Example 4.5

Let $X=\{a,b,c,d\}$, $A=\langle\{a,b\},\{c\},\{d\}\rangle$ and $B=\langle\{a\},\{c\},\{d,b\}\rangle$ then the product of two events given by $A\times B=\langle\{(a,a),(b,a)\},\{(c,c)\},\{(d,d),(d,b)\}\rangle$, and $B\times A=\langle\{(a,a),(a,b)\},\{(c,c)\},\{(d,d),(b,d)\}\rangle$, and

$R_1=\langle\{(a,a)\},\{(c,c)\},\{(d,d)\}\rangle$, $R_1\subseteq A\times B$ on $X\times X$,

$R_2=\langle\{(a,b)\},\{(c,c)\},\{(d,d),(b,d)\}\rangle$

$R_2\subseteq B\times A$ on $X\times X$.

$R_1 \circ R_2 = \cup\langle \{(a,a)\} \cap \{(a,b)\}, \{(c,c)\}, \{(d,d)\}\rangle$
$= \langle \{\phi\}, \{(c,c)\}, \{(d,d)\}\rangle$ and
$I_{A1} = \langle \{(a,a).(a,b).(b.a)\}, \{(a,a).(a,b).(b,a)\}, \{\phi\}\rangle$
$, I_{A2} = \langle \{(a,a).(a,b).(b.a)\}, \{\phi\}, \{\phi\}\rangle$

Theorem 4.2

Let R be a neutrosophic crisp relation in $X \times Y$, and S be a neutrosophic crisp relation in $Y \times Z$ then $(R \circ S)^{-1} = S^{-1} \circ R^{-1}$.

Proof

Let $R \subseteq A \times B$ on $X \times Y$ then $R^{-1} \subseteq B \times A$, $S \subseteq B \times D$ on $Y \times Z$ then $S^{-1} \subseteq D \times B$, from Definition 3.6 and similarly we can $I_{(R\circ S)^{-1}}(x,z) = I_{S^{-1}}(x,z)$ and $I_{R^{-1}}(x,z)$ then $(R \circ S)^{-1} = S^{-1} \circ R^{-1}$.

V. Conclusion

In our work, we have put forward some new types of neutrosophic crisp sets and neutrosophic crisp continuity relations. Some related properties have been established with example. It 's hoped that our work will enhance this study in neutrosophic set theory.

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Neutrosophic Parametrized Soft Set Theory and Its Decision Making

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Abstract: In this work, we present definition of neutrosophic parameterized (NP) soft set and its operations. Then we define NP-aggregation operator to form NP-soft decision making method which allows constructing more efficient decision processes. We also dive an example which shows that they can be successfully applied to problem that contain indeterminacy.

Keywords: Soft set, neutrosophic set, neutrosophic soft set, neutrosophic parameterized soft set, aggregation operator.

1. Introduction

In 1999, Smarandache firstly proposed the theory of neutrosophic set (NS) [28], which is the generalization of the classical sets, conventional fuzzy set [30] and intuitionistic fuzzy set [5]. After Smarandache, neutrosophic sets has been successfully applied to many fields such as;control theory [1], databases [2,3], medical diagnosis problem [4], decision making problem [21], topology [22], and so on.

In 1999 a Russian researcher [27] firstly gave the soft set theory as a general mathematical tool for dealing with uncertainty and vagueness and how soft set theory is free from the parameterization inadequacy syndrome of fuzzy set theory, rough set theory, probability theory. Then, many interesting results of soft set theory have been studied on fuzzy soft sets [8,12,23], on intuitionistic fuzzy soft set theory [14,25], on possibility fuzzy soft set [7], on generalized fuzzy soft sets [26,29], on generalized intuitionistic fuzzy soft [6], on interval-valued intuitionistic fuzzy soft sets [20], on intuitionistic neutrosophic soft set [9], on generalized neutrosophic soft set [10], on fuzzy parameterized soft set theory [17,18], on fuzzy parameterized fuzzy soft set theory [13], on intuitionistic fuzzy parameterized soft set theory [15], on IFP−fuzzy soft set theory [16],on neutrosophic soft set [24].interval-valued neutrosophic soft set [11,19].

In this paper our main objective is to introduce the notion of neutrosophic parameterized soft set which is a generalization of fuzzy parameterized soft set and intuitionistic fuzzy parameterized soft set.The paper is structured as follows. In section 2, we first recall the necessary background on neutrosophic and soft set. In section 3, we give neutrosophic parameterized soft set theoryand their respective properties. In section 4, we present a neutrosophic parameterized aggregation operator. In section 5,a neutrosophic parameterized decision methods is presented with example. Finally we conclude the paper.

2. Preliminaries

Throughout this paper, let U be a universal set and E be the set of all possible parameters under consideration with respect to U, usually, parameters are attributes, characteristics, or properties of objects in U.

We now recall some basic notions of neutrosophic set and soft set. For more details, the reader could refer to [33, 37].

Definition 1.**[37] Let U be a universe of discourse then the neutrosophic set A is an object having the form**

$A = \{< x: \mu_{A(x)}, \nu_{A(x)}, \omega_{A(x)}>, x \in U\}$

where the functions $\mu, \nu, \omega : U \to]^-0,1^+[$ define respectively the degree of membership, the degree of indeterminacy, and the degree of non-membership of the element $x \in X$ to the set A with the condition.

$$^-0 \leq \mu_{A(x)} + \nu_{A(x)} + \omega_{A(x)} \leq 3^+. \quad (1)$$

From philosophical point of view, the neutrosophic set takes the value from real standard or non-standard subsets of $]{-0},1^+[$. So instead of $]^-0,1^+[$ we need to take the interval [0,1] for technical applications, because $]^-0,1^+[$ will be difficult to apply in the real applications such as in scientific and engineering problems.

For two NS,

$A_{NS} = \{<x, \mu_A(x), \nu_A(x), \omega_A(x)> \mid x \in X\}$

and

$B_{NS} = \{<x, \mu_B(x), \nu_B(x), \omega_B(x)> \mid x \in X\}$

Then,

$A_{NS} \subseteq B_{NS}$ if and only if

$\mu_A(x) \leq \mu_B(x), \nu_A(x) \geq \nu_B(x), \omega_A(x) \geq \omega_B(x)$.

$A_{NS} = B_{NS}$ if and only if,

$\mu_A(x) = \mu_B(x), \nu_A(x) = \nu_B(x), \omega_A(x) = \omega_B(x)$ for any $x \in X$.

The complement of A_{NS} is denoted by A_{NS}^o and is defined by

$A_{NS}^o = \{<x, \omega_A(x), 1 - \nu_A(x), \mu_A(x) \mid x \in X\}$

A∩B = {<x, min$\{\mu_A(x), \mu_B(x)\}$,max$\{\nu_A(x), \nu_B(x)\}$, max$\{\omega_A(x), \omega_B(x)\}$>:x ∈ X }

A∪B = {<x, max$\{\mu_A(x), \mu_B(x)\}$,min$\{\nu_A(x), \nu_B(x)\}$, min$\{\omega_A(x), \omega_B(x)\}$>:x ∈ X }

As an illustration, let us consider the following example.

Example 1. Assume that the universe of discourse U= { x_1,x_2,x_3, x_4}. It may be further assumed that the values of x_1, x_2, $x_3$and $x_4$are in [0, 1] Then, A is a neutrosophic set (NS) of U, such that,

A={< x_1, 0.4, 0.6, 0.5>, <x_2, 0.3, 0.4, 0.7>, <x_3, 0.4,0.4, 0.6] >,<x_4, 0.5,0.4, 0.8 >}

Definition 2.[33]

Let U be an initial universe set and E be a set of parameters. Let P(U) denotes the power set of U. Consider a nonempty set A, A ⊂ E. A pair (K, A) is called a soft set over U, where K is a mapping given by K: A → P(U).

As an illustration, let us consider the following example.

Example 2. Suppose that U is the set of houses under consideration, say U= $\{h_1, h_2, h_3, h_4, h_5\}$. Let E be the set of some attributes of such houses, say E = $\{e_1, e_2, e_3, e_4, e_5\}$, where e_1, e_2, e_3, e_4, e_5stand for the attributes “beautiful”, “costly”, “in the green surroundings”, “moderate” and technically, respectively. In this case, to define a soft set means to point out expensive houses, beautiful houses, and so on. For example, the soft set (K, A) that describes the “attractiveness of the houses” in the opinion of a buyer, says Thomas, and may be defined like this:

A= $\{e_1, e_2, e_3, e_4, e_5\}$;

K(e_1) = $\{h_2, h_3, h_5\}$, K(e_2) = $\{h_2, h_4\}$, K(e_3) = $\{h_1\}$, K(e_4) = U, K(e_5) = $\{h_3, h_5\}$.

3. Neutrosophic Parameterized Soft Set Theory

In this section, we define neutrosophic parameterized soft set and their operations.

Definition 3.1. Let U be an initial universe, P (U) be the power set of U,E be a set of all parameters and K be a neutrosophic set over E. Then a neutrosophic parameterized soft sets

Ψ_k =$\{(< x, \mu_K(x), \nu_K(x), \omega_K(x) >, f_K(x)) : x \in E\}$

where μ_K:E → [0, 1], ν_K:E → [0, 1],ω_K:E → [0, 1] and f_K:E →P(U) such that $f_K(x)$ =Φ if $\mu_K(x) = 0$,$\nu_K(x) = 1$ and$\omega_K(x) = 1$.

Here, the function μ_K , $\nu_K$and $\omega_K$called membership function, indeterminacy function and non-membership function of neutrosophic parameterized soft set (NP-soft set), respectively.

Example 3.2. Assume that U= $\{u_1, u_2, u_3\}$ is a universal set and E= $\{x_1, x_2\}$ is a set of parameters.If

K= {<x_1,0.2, 0.3, 0.4> , <x_2,0.3, 0.5, 0.4>}

and

$f_K(x_1)$= {u_2 ,u_3},$f_K(x_2)$=U

Then a neutrosophic parameterized soft set Ψ_k is written by

Ψ_k={(<x_1,0.2, 0.3, 0.4>,{u_2,u_3}),(<x_2,0.3, 0.5, 0.4>,U)}

Definition 3.3. Let $\Psi_k \in$ NP-soft set. if $f_K(x)$ = U , $\mu_K(x)$ =0 , $\nu_K(x) = 1$ and $\omega_K(x)= 1$ all x ∈ E. then Ψ_k is called K-empty NP-soft set, denoted by Ψ_{Φ_k}.

If K=Φ,then the K-emptyNP-soft set is called empty NP-soft set, denoted by Ψ_{Φ}.

Definition 3.4. Let $\Psi_k \in$ NP-soft set. if $f_K(x)$ = U , $\mu_K(x)$ =1 , $\nu_K(x)$ =0and $\omega_K(x)$=0 all x ∈ E. then Ψ_k is called K-universal NP-soft set, denoted by $\Psi_{\tilde{K}}$.

If K= E, then the K-universal NP-soft set is called universal NP-soft set, denoted by $\Psi_{\tilde{E}}$.

Definition 3.5. Ψ_k and Ω_L are two NP-soft set. Then, Ψ_k is NP-subset of Ω_L, denoted by $\Psi_k \sqsubseteq \Omega_L$ if and only if $\mu_K(x) \leq \mu_L(x)$,$\nu_K(x) \geq \nu_L(x)$ and $\omega_K(x) \geq \omega_L(x)$ and $f_K(x) \subseteq g_L(x)$ for all x ∈ E.

Definition 3.6. Ψ_k and Ω_L are two NP-soft set. Then, Ψ_k=Ω_L, if and only if $\Psi_k \sqsubseteq \Omega_L$ and $\Omega_L \sqsubseteq \Psi_k$ for all x ∈ E.

Definition 3.7. Let $\Psi_k \in$ NP-soft set. Then, the complement of Ψ_k, denoted by Ψ_K^c , is defined by

$$\Psi_K^c=\{(< x, \omega_K(x) ,\nu_K(x) ,\mu_K(x) >, f_{K^c}(x)): x \in E\}$$

Where $f_{K^c}(x)$ =U \ $f_K(x)$

Definition 3.8. Let Ψ_k and Ω_L are two NP-soft set. Then ,union of Ψ_k and Ω_L,denoted by $\Psi_k \sqcup \Omega_L$, is defined by

$$\Psi_k \sqcup \Omega_L=\{(< x, \max\{ \mu_K(x), \mu_L(x)\} ,\min\{\nu_K(x),\nu_L(x)\} ,\min\{\omega_K(x), \omega_L(x)\} >, f_{K\cup L}(x)): x \in E \}$$

where $f_{K\cup L}(x) = f_K(x) \cup f_L(x)$.

Definition 3.9. Let Ψ_k and Ω_L are two NP-soft set. Then, intersection of Ψ_K and Ω_L,denoted by $\Psi_K \sqcap \Omega_L$, is defined by

$\Psi_k \sqcap \Omega_L =$
$\{(< x, \min\{\ \mu_K(x), \mu_L(x)\}, \max\{\nu_K(x), \nu_L(x)\}, \max\{\omega_K(x), \omega_L(x)\} >, f_K(x) \cap f_L(x)) : x \in$
$E\}$

where$f_{K\cap L}(x) = f_K(x) \cap f_L(x)$.

Example 3.10. Let$U = \{u_1, u_2, u_3, u_4\}$, $E=\{x_1, x_2, x_3\}$. Then,

$\Psi_K = \{(<x_1, 0.2, 0.3, 0.4>, \{u_1, u_2\}), (<x_2, 0.3, 0.5, 0.4>, \{u_2, u_3\})\}$

$\Omega_L = \{(<x_2, 0.1, 0.2, 0.4>, \{u_3, u_4\}), (<x_3, 0.5, 0.2\ 0.3>, \{u_3\})\}$

Then

$\Psi_K \sqcup \Omega_L = \{(<x_1, 0.2, 0.3, 0.4>, \{u_1, u_2\}), (<x_2, 0.3, 0.2, 0.4>, \{u_2, u_3, u_4\}), (<x_3, 0.5, 0.2\ 0.3>, \{u_3\})\}$

$\Psi_K \sqcap \Omega_L = \{(<x_2, 0.1, 0.5, 0.4>, \{u_3, u_4\})\}$.

$\Psi_K^c = \{(<x_1, 0.4, 0.3, 0.2>, \{u_3, u_4\}), (<x_2, 0.4, 0.5, 0.3>, \{u_1, u_4\})\}$

Remark 3.11.$\Psi_K \sqsubseteq \Omega_L$does not imply that every element of Ψ_K is an element of Ω_L as in the definition of classical subset. For example assume that $U = \{u_1, u_2, u_3, u_4\}$is a universal set of objects and $E = \{x_1, x_2, x_3\}$is a set of all parameters, if NP-soft sets Ψ_K and Ω_L are defined as

$\Psi_K = \{(<x_1, 0.2, 0.3, 0.4>, \{u_1, u_2\}), (<x_2, 0.3, 0.5, 0.4>, \{u_2\})\}$

$\Omega_L = \{(<x_1, 0.1, 0.2, 0.4>, U), (<x_2, 0.5, 0.2\ 0.3>, \{u_1, u_4\})\}$

It can be seen that $\Psi_K \sqsubseteq \Omega_L$, but every element of Ψ_K is not an element ofΩ_L

Proposition 3.12. Let $\Psi_K, \Omega_L \in$ NP-soft set .Then

$\Psi_K \sqsubseteq \Psi_{\tilde{E}}$

$\Psi_{\Phi} \sqsubseteq \Psi_K$

$\Psi_K \sqsubseteq \Psi_K$

Proof .It is clear from Definition 3.3-3.5.

Proposition 3.13. Let Ψ_K, Ω_L and $\Upsilon_M \in$ NP-soft set, Then

$\Psi_K = \Omega_L$ and $\Omega_L = \Upsilon_M \Leftrightarrow \Psi_K = \Upsilon_M$

$\Psi_K \sqsubseteq \Omega_L$and $\Omega_L \sqsubseteq \Psi_K \Leftrightarrow \Psi_K \quad \Omega_L$

$\Psi_K \sqsubseteq \Omega_L$ and $\Omega_L \sqsubseteq \Upsilon_M \Rightarrow \Psi_K \sqsubseteq \Upsilon_M$

Proof .It can be proved by Definition 3.3-3.5

Proposition 3.14 Let $\Psi_K \in$ NP-soft set. Then

$(\Psi_K^c)^c = \Psi_k$

$\Psi_{\emptyset}^c = \Psi_{\hat{E}}$

$\Psi_{E}^c = \Psi_{\emptyset}$

Proof. It is trial.

Proposition 3.15. LetΨ_K,Ω_L and $\Upsilon_M \in$ NP-soft set, Then

$\Psi_K \sqcup \Psi_K = \Psi_K$

$\Psi_K \sqcup \Psi_{\emptyset} = \Psi_K$

$\Psi_K \sqcup \Psi_{\hat{E}} = \Psi_{\hat{E}}$

$\Psi_K \sqcup \Omega_L = \Omega_L \sqcup \Psi_K$

$(\Psi_K \sqcup \Omega_L) \sqcup \Upsilon_M = \Psi_K \sqcup (\Omega_L \sqcup \Upsilon_M)$

Proof.It is clear

Proposition 3.16. Let Ψ_K,Ω_L and $\Upsilon_M \in$ NP-soft set, Then

$\Psi_K \sqcap \Psi_K = \Psi_K$

$\Psi_K \sqcap \Psi_{\emptyset} = \Psi_{\emptyset}$

$\Psi_K \sqcap \Psi_{\hat{E}} = \Psi_K$

$\Psi_K \sqcap \Omega_L = \Omega_L \sqcap \Psi_K$

$(\Psi_K \sqcap \Omega_L) \sqcap \Upsilon_M = \Psi_K \sqcap (\Omega_L \sqcap \Upsilon_M)$

Proof.It is clear

Proposition 3.17. Let Ψ_K,Ω_L and $\Upsilon_M \in$ NP-soft set, Then

$\Psi_K \sqcup (\Omega_L \sqcap \Upsilon_M) = (\Psi_K \sqcup \Omega_L) \sqcap (\Psi_K \sqcup \Upsilon_M)$

$\Psi_K \sqcap (\Omega_L \sqcup \Upsilon_M = (\Psi_K \sqcap \Omega_L) \sqcup (\Psi_K \sqcap \Upsilon_M)$

Proof .It can be proved by definition 3.8 and 3.9

Proposition 3.18. LetΨ_K,Ω_L ∈ NP-soft set, Then

$(\Psi_K \sqcup \Omega_L)^c = \Psi_K^c \sqcap \Omega_L^c$

$(\Psi_K \sqcap \Omega_L)^c = \Psi_K^c \sqcup \Omega_L^c$

Proof. It is clear.

Definition 3.19. LetΨ_K,Ω_L ∈ NP-soft set, Then

OR-product of $\Psi_K$and Ω_L denoted by $\Psi_K \veebar \Omega_L$,is defined as following

$\Psi_K \veebar \Omega_L$={ (<(x,y),(max{$\mu_K(x)$,$\mu_L(y)$},min{$\nu_K(x)$,$\nu_L(x)$},min{$\omega_K(x)$,$\omega_L(y)$}>,$f_{K\cup L}$(x,y)) :x,y ∈ E }

where$f_{K\cup L}(x,y)=f_K(x) \cup f_L(y)$.

AND-product of $\Psi_K$and Ω_L denoted by $\Psi_K \barwedge \Omega_L$is defined as following

$\Psi_K \barwedge \Omega_L$={ (<(x,y),(min{$\mu_K(x)$,$\mu_L(y)$},max{$\nu_K(x)$,$\nu_L(y)$},max{$\omega_K(x)$,$\omega_L(y)$}>,$f_{K\cap L}$(x,y)) :x,y ∈ E }

where$f_{K\cap L}(x,y)=f_K(x) \cap f_L(y)$.

Proposition 3.20. LetΨ_K,Ω_L and Υ_M ∈ NP-soft set, Then

$\Psi_K \barwedge \Psi_{\Phi} = \Psi_{\Phi}$

$(\Psi_K \barwedge \Omega_L) \barwedge \Upsilon_M = \Psi_K \barwedge (\Omega_L \barwedge \Upsilon_M)$

$(\Psi_K \veebar \Omega_L) \veebar \Upsilon_M = \Psi_K \veebar (\Omega_L \veebar \Upsilon_M)$

Proof .It can be proved by definition 3.15

4. NP-aggregation operator

In this section, we define NP-aggregation operator of an NP-soft set to construct a decisionmethod by which approximate functions of a soft set are combined to produce a single neutrosophic set that can be used to evaluate each alternative.

Definition 4.1. $\Psi_K \in$ NP-soft set. Then a NP-aggregation operatorof Ψ_K , denotedby$\Psi_K{}^{agg}$, is definedby

$$\Psi_K{}^{agg}=\{ \ (<u, \mu_K{}^{agg}(u), \nu_K{}^{agg}(u), \omega_K{}^{agg}(u): u \in U \ \}$$

which is a neutrosophic set over U,

where

$\mu_K{}^{agg}: U \rightarrow [0,1]$ $\qquad \mu_K{}^{agg}(u) = \frac{1}{|U|} \sum_{\substack{x \in E \\ ,u \in U}} \mu_K(x)\gamma_{f_{K(x)}}(u)$,

$\nu_K{}^{agg}:: U \rightarrow [0,1]$ $\qquad \nu_K{}^{agg}(u) = \frac{1}{|U|} \sum_{\substack{x \in E \\ ,u \in U}} \nu_K(x)\gamma_{f_{K(x)}}(u)$

and

$\omega_K{}^{agg}:: U \rightarrow [0,1]$ $\qquad \omega_K{}^{agg}(u) = \frac{1}{|U|} \sum_{\substack{x \in E \\ ,u \in U}} \omega_K(x)\gamma_{f_{K(x)}}(u)$

And where

$$\gamma_{f_{K(x)}}(u) = \begin{cases} 1, & x \in f_K(x) \\ 0, & other\ wise \end{cases}$$

$|U|$is the cardinality of U.

Definition 4.2 Let $\Psi_K \in$ NP-soft set and $\Psi_K{}^{agg}$ an aggregation neutrosophic parameterized soft set ,then a reduced fuzzy set of $\Psi_K{}^{agg}$ is a fuzzy set over U denoted by

$$\Psi_K{}^{agg} = \left\{ \frac{\mu_{\Psi_{Kf}}{}^{agg}(u)}{u} : u \in U \right\}$$

where $\mu_{\Psi_{Kf}}{}^{agg}(u)$: $U \rightarrow [0,1]$ *and* $\mu_{\Psi_{Kf}}{}^{agg}(u)=\mu_K{}^{agg}(u)+\nu_K{}^{agg}(u)-\omega_K{}^{agg}(u)$.

NP-Decision Methods

Inspired by the decision making methods regard in [12-19].In this section, we also present NP-decision method to neutrosophic parameterized soft set. Based on definition 4.1 and 4.2 we construct an NP-decision making method by the following algorithm.

Now, we construct a NP-soft decision making method by the following algorithm to produce a decision fuzzy set from a crisp set of the alternatives.

According to the problem, decision maker

i. constructs a feasible Neutrosophic subsets K over the parameters set E,

ii. constructs a NP-soft set Ψ_K over the alternatives set U,

iii. computes the aggregation neutrosophic parameterized soft set $\Psi_K{}^{agg}$ of Ψ_K,

iv. computes the reduced fuzzy set $\mu_{\Psi_{Kf}{}^{agg}(u)}$ of $\Psi_K{}^{agg}$,

v. chooses the element of $\mu_{\Psi_{Kf}{}^{agg}(u)}$ that has maximum membership degree.

Now, we can give an example for the NP-soft decision making method

Example. Assume that a company wants to fill a position. There are four candidates who fill in a form in order to apply formally for the position. There is a decision maker (DM) that is from the department of human resources. He wants to interview the candidates, but it is very difficult to make it all of them. Therefore, by using the NP-soft decision making method, the number of candidates are reduced to a suitable one. Assume that the set of candidates U={u_1, u_2, u_3, u_4} which may be characterized by a set of parameters E={x_1, x_2, x_3} For i=1,2,3 the parameters i stand for experience, computer knowledge and young age, respectively. Now, we can apply the method as follows:

Step i. Assume that DM constructs a feasible neutrosophic subsets K over the parameters set E as follows;

K={<x_1,0.2, 0.3, 0.4>,<x_2,0.3, 0.2, 0.4>,<x_3,0.5, 0.2 0.3>}

Step ii. DM constructs an NP-soft set Ψ_K over the alternatives set U as follows;

Ψ_K={(<x_1,0.2,0.3,0.4>,{u_1, u_2}),(<x_2,0.3,0.2,0.4>,{u_2,u_3,u_4}),(<x_3,0.5,0.2,0.3>,{u_3})}

Step iii. DM computes the aggregation neutrosophic parameterized soft set $\Psi_K{}^{agg}$ of Ψ_K as follows;

$\Psi_K{}^{agg}$={<u_1,0.05, 0.075, 0.1>,<u_2,0.1, 0.125, 0.2>,<u_3,0.2, 0.1 0.175>,<u_4,0.125, 0.05 0.075>}

Step iv .computes the reduced fuzzy set $\mu_{\Psi_{Kf}{}^{agg}(u)}$ of $\Psi_K{}^{agg}$ as follows;

$\mu_{\Psi_{Kf}{}^{agg}(u_1)}$=0.025

$\mu_{\Psi_{Kf}{}^{agg}(u_2)}$=0.025

$\mu_{\Psi_{Kf}{}^{agg}(u_3)}$=0.125

$\mu_{\Psi_{Kf}{}^{agg}(u_4)}$=0.1

Step v . Finally, DM chooses u_3 for the position from $\mu_{\Psi_{Kf}}{}^{agg}{}_{(u)}$ since it has the maximum degree 0.125 among the others.

Conclusion

In this work, we have introduced the concept of neutrosophic parameterized soft set and studied some of its properties. The complement, union and intersection operations have been defined on the neutrosophic parameterized soft set. The definition of NP-aggregation operator is introduced with application of this operation in decision making problems.

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Generalized Interval Neutrosophic Soft Set and its Decision Making Problem

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Abstract – In this work, we introduce the concept of generalized interval neutrosophic soft set and study their operations. Finally, we present an application of generalized interval neutrosophic soft set in decision making problem.

Keywords – *Soft set, neutrosophic set, neutrosophic soft set, decision making*

1. Introduction

Neutrosophic sets, founded by Smarandache [8] has capability to deal with uncertainty, imprecise, incomplete and inconsistent information which exist in real world. Neutrosophic set theory is a powerful tool which generalizes the concept of the classic set, fuzzy set [16], interval-valued fuzzy set [10], intuitionistic fuzzy set [13] interval-valued intuitionistic fuzzy set [14], and so on.

After the pioneering work of Smarandache, Wang [9] introduced the notion of interval neutrosophic set (INS) which is another extension of neutrosophic set. INS can be described by a membership interval, a non-membership interval and indeterminate interval, thus the interval value (INS) has the virtue of complementing NS, which is more flexible and practical than neutrosophic set, and interval neutrosophic set provides a morereasonable mathematical framework to deal with indeterminate and inconsistent information.The theory of neutrosophic sets and their hybrid structures has proven useful in many different fields such as control theory [25], databases [17,18], medical diagnosis problem [3,11], decision making problem [1,2,15,19,23,24,27,28,29,30,31,32,34], physics[7], and etc.
In 1999, a Russian researcher [5] firstly gave the soft set theory as a general mathematical tool for dealing with uncertainty and vagueness. Soft set theory is free from the parameterization inadequacy syndrome of fuzzy set theory, rough set theory, probability theory. Recently, some authors have introduced new mathematical tools by generalizing and extending Molodtsov's classical soft set theory;

fuzzy soft set [22], vague soft set [35], intuitionistic fuzzy soft set [20], interval valued intuitionistic fuzzy set [36].

Similarity, combining neutrosophic set models with other mathematical models has attracted the attention of many researchers: neutrosophic soft set [21], intuitionistic neutrosophic soft set [26], generalized neutrosophic soft set [23], interval neutrosophic soft set [12].

Broumi et al. [33] presented the concept of rough neutrosophic set which is based on a combination of the neutrosophic set and rough set models. Recently, Şahin and Küçük [23] generalized the concept of neutrosophic soft set with a degree of which is attached with the parameterization of fuzzy sets while defining a neutrosophic soft set, and investigated some basic properties of the generalized neutrosophic soft sets.

In this paper our main objective is to extend the concept of generalized neutrosophic soft set introduced by Şahin and Küçük [23] to the case of interval neutrosophic soft set [12].

The paper is structured as follows. In Section 2, we first recall the necessary background on neutrosophic sets,soft set and generalized neutrosophic soft set. The concept of generalized interval neutrosophic soft sets and some of their properties are presented in Section 3.In Section 4, we present an application of generalized interval neutrosophic soft sets in decision making. Finally we conclude the paper.

2. Preliminaries

In this section, we will briefly recall the basic concepts of neutrosophic set,soft sets and generalized neutrosophic soft sets. Let U be an initial universe set of objects and E the set of parameters in relation to objects in U. Parameters are often attributes, characteristics or properties of objects. Let $P(U)$ denote the power set of U and $A \subseteq E$.

2.1 Neutrosophic Sets

Definition 2.1 [8]. Let U be an universe of discourse.The neutrosophic set A is an object having the form $A = \{< x: u_A(x), w_A(x), v_A(x) > : x \in U\}$,where the functions $u, w, v : U \to]0^-, 1^+[$define respectively the degree of membership, the degree of indeterminacy, and the degree of non-membership of the element $x \in U$ to the set A with the condition.

$$0^- \leq u_A(x) + w_A(x) + v_A(x)) \leq 3^+$$

From philosophical point of view, the neutrosophic set takes the value from real standard or non-standard subsets of $]^-0,1^+[$. So instead of $]^-0,1^+[$ we need to take the interval [0,1] for technical applications, because $]^-0,1^+[$will be difficult to apply in the real applicationssuch as in scientific and engineering problems.

Definition 2.2 [8] A neutrosophicset $A$is contained in the other neutrosophic set B, $A \subseteq B$ iff $\inf u_A(x) \leq \inf u_B(x)$, $\sup u_A(x) \leq \sup u_B(x)$, $\inf w_A(x) \geq \inf w_B(x)$, $\sup w_A(x) \geq \sup w_B(x)$and $\inf v_A(x) \geq \inf v_B(x)$, $\sup v_A(x) \geq \sup v_B(x)$ for all $x \in U$.

An INS is an instance of a neutrosophic set, which can be used in real scientific and engineering applications. In the following, we introduce the definition of an INS.

2.2 Interval Neutrosophic Sets

Definition 2.3 [9] Let U be a space of points (objects) and Int[0,1] be the set of all closed subsets of [0,1]. An INS A in U is defined with the form

$$A = \{\langle x, u_A(x), w_A(x), v_A(x)\rangle : x \in U\}$$

where $u_A(x): U \to \text{int}[0,1]$, $w_A(x): U \to \text{int}[0,1]$ and $v_A(x): U \to \text{int}[0,1]$ with $0 \le \sup u_A(x) + \sup w_A(x) + \sup v_A(x) \le 3$ for all $x \in U$. The intervals $u_A(x), w_A(x)$ and $v_A(x)$ denote the truth-membership degree, the indeterminacy-membership degree and the falsity membership degree of $x$to A, respectively.

For convenience,

if let $u_A(x) = [u_A^-(x), u_A^+(x)]$, $w_A(x) = [w_A^-(x), w_A^+(x)]$ and $v(x) = [v_A^-(x), v_A^+(x)]$, then

$$A = \{\langle x, [u_A^-(x), u_A^+(x)], [w_A^-(x), w_A^+(x)], [v_A^-(x), v_A^+(x)]\rangle : x \in U\}$$

with the condition, $0 \le \sup u_A^+(x) + \sup w_A^+(x) + \sup v_A^+(x) \le 3$ for all $x \in U$. Here, we only consider the sub-unitary interval of [0,1]. Therefore, an INS is clearly a neutrosophic set.

Definition 2.4 [9] Let A and B be two interval neutrosophic sets,

$$A = \{\langle x, [u_A^-(x), u_A^+(x)], [w_A^-(x), w_A^+(x)], [v_A^-(x), v_A^+(x)]\rangle : x \in U\}$$
$$B = \{\langle x, [u_B^-(x), u_B^+(x)], [w_B^-(x), w_B^+(x)], [v_B^-(x), v_B^+(x)]\rangle : x \in U\}.$$

Then some operations can be defined as follows:

(1) $A \subseteq B$ iff $u_A^-(x) \le u_B^-(x), u_A^+(x) \le u_B^+(x), w_A^-(x) \ge w_B^-(x), w_A^+(x) \ge w_B^+(x) v_A^-(x) \ge v_B^-(x), v_A^+(x) \ge v_B^+(x)$ for each $x \in U$.
(2) $A = B$iff$A \subseteq B$ and $B \subseteq A$.
(3) $A^c = \{\langle x, [v_A^-(x), v_A^+(x)], [1 - w_A^+(x), 1 - w_A^-(x)], [u_A^-(x), u_A^+(x)]\rangle : x \in U\}$

2.3 Soft Sets

Defnition2.5 [5] A pair (F, A) is called a soft set over, where F is a mapping given by $F : A \to P(U)$. In other words, a soft set over U is a mapping from parameters to the power set of U, and it is not a kind of set in ordinary sense, but a parameterized family of subsets of U. For any parameter$e \in A$, $F(e)$ may be considered as the set of e −approximate elements of the soft set (F, A).

Example 2.6 Suppose that U is the set of houses under consideration, say $U = \{h_1, h_2, \dots, h_5\}$. Let E be the set of some attributes of such houses, say $E = \{e_1, e_2, e_3, e_4\}$, where e_1, e_2, e_3, e_4 stand for the attributes “beautiful”, “costly”, “in the green surroundings” and “moderate”, respectively.
In this case, to define a soft set means to point out expensive houses, beautiful houses, and so on. For example, the soft set (F, A) that describes the “attractiveness of the houses” in the opinion of a buyer, say Thomas, may be defined like this:

$$F(e_1) = \{h_2, h_3, h_5\}, F(e_2) = \{h_2, h_4\}, F(e_4) = \{h_3, h_5\} \text{ for } A = \{e_1, e_2, e_4\}.$$

2.4 Neutrosophic Soft Sets

Definition 2.7 [21] Let $\boldsymbol{U}$ be an initial universe set and $\boldsymbol{A} \subset \boldsymbol{E}$ be a set of parameters. Let NS(U) denotes the set of all neutrosophic subsets of $\boldsymbol{U}$. The collection $(\boldsymbol{F}, \boldsymbol{A})$ is termed to be the neutrosophic soft set over $\boldsymbol{U}$, where **F** is a mapping given by $\boldsymbol{F}: \boldsymbol{A} \rightarrow \boldsymbol{NS}(\boldsymbol{U})$.

Example 2.8 [21] Let U be the set of houses under consideration and E is the set of parameters. Each parameter is a neutrosophic word or sentence involving neutrosophic words. Consider E ={beautiful, wooden, costly, very costly, moderate, green surroundings, in good repair, in bad repair, cheap, expensive}. In this case, to define a neutrosophic soft set means to point out beautiful houses, wooden houses, houses in the green surroundings and so on. Suppose that, there are five houses in the universe U given by $U = \{h_1, h_2, \ldots, h_5\}$ and the set of parameters

$A = \{e_1, e_2, e_3, e_4\}$, where e_1 stands for the parameter `beautiful', $e_2$ stands for the parameter `wooden', e_3 stands for the parameter `costly' and the parameter $e_4$ stands for `moderate'. Then the neutrosophic set (F, A) is defined as follows:

$$(F,A) = \begin{cases} \left(e_1\left\{\dfrac{h_1}{(0.5,0.6,0.3)}, \dfrac{h_2}{(0.4,0.7,0.6)}, \dfrac{h_3}{(0.6,0.2,0.3)}, \dfrac{h_4}{(0.7,0.3,0.2)}, \dfrac{h_5}{(0.8,0.2,0.3)}\right\}\right) \\ \left(e_2\left\{\dfrac{h_1}{(0.6,0.3,0.5)}, \dfrac{h_2}{(0.7,0.4,0.3)}, \dfrac{h_3}{(0.8,0.1,0.2)}, \dfrac{h_4}{(0.7,0.1,0.3)}, \dfrac{h_5}{(0.8,0.3,0.6)}\right\}\right) \\ \left(e_3\left\{\dfrac{h_1}{(0.7,0.4,0.3)}, \dfrac{h_2}{(0.6,0.7,0.2)}, \dfrac{h_3}{(0.7,0.2,0.5)}, \dfrac{h_4}{(0.5,0.2,0.6)}, \dfrac{h_5}{(0.7,0.3,0.4)}\right\}\right) \\ \left(e_4\left\{\dfrac{h_1}{(0.8,0.6,0.4)}, \dfrac{h_2}{(0.7,0.9,0.6)}, \dfrac{h_3}{(0.7,0.6,0.4)}, \dfrac{h_4}{(0.7,0.8,0.6)}, \dfrac{h_5}{(0.9,0.5,0.7)}\right\}\right) \end{cases}$$

2.5 Interval Neutrosophic Soft Sets

Definition 2.9 [12] Let $\boldsymbol{U}$ be an initial universe set and $\boldsymbol{A} \subset \boldsymbol{E}$ be a set of parameters. Let INS(U) denotes the set of all interval neutrosophic subsets of $\boldsymbol{U}$. The collection $(\boldsymbol{F}, \boldsymbol{A})$ is termed to be the interval neutrosophic soft set over $\boldsymbol{U}$, where **F** is a mapping given by $\boldsymbol{F}: \boldsymbol{A} \rightarrow \boldsymbol{INS}(\boldsymbol{U})$.

Example 2.10 [12] Let $\boldsymbol{U} = \{\boldsymbol{x_1}, \boldsymbol{x_2}\}$ be set of houses under consideration and **E** is a set of parameters which is a neutrosophic word. Let **E** be the set of some attributes of such houses, say $\boldsymbol{E} = \{\boldsymbol{e_1}, \boldsymbol{e_2}, \boldsymbol{e_3}, \boldsymbol{e_4}\}$, where $\mathbf{e_1}, \mathbf{e_2}, \mathbf{e_3}, \mathbf{e_4}$ stand for the attributes $\mathbf{e_1}$ = cheap, $\mathbf{e_2}$ = beautiful, $\mathbf{e_3}$ = in the green surroundings, $\mathbf{e_4}$ = costly and $\mathbf{e_5}$ = large, respectively. Then we define the interval neutrosophic soft set **A** as follows:

$$(\boldsymbol{F},\boldsymbol{A}) = \begin{cases} \left(\boldsymbol{e_1}\left\{\dfrac{\boldsymbol{x_1}}{[\mathbf{0.5,0.8}],[\mathbf{0.5,0.9}],[\mathbf{0.2,0.5}]}, \dfrac{\boldsymbol{x_2}}{[\mathbf{0.4,0.8}],[\mathbf{0.2,0.5}],[\mathbf{0.5,0.6}]}\right\}\right) \\ \left(\boldsymbol{e_2}\left\{\dfrac{\boldsymbol{x_1}}{[\mathbf{0.5,0.8}],[\mathbf{0.2,0.8}],[\mathbf{0.3,0.7}]}, \dfrac{\boldsymbol{x_2}}{[\mathbf{0.1,0.9}],[\mathbf{0.6,0.7}],[\mathbf{0.2,0.3}]}\right\}\right) \\ \left(\boldsymbol{e_3}\left\{\dfrac{\boldsymbol{x_1}}{[\mathbf{0.2,0.7}],[\mathbf{0.1,0.5}],[\mathbf{0.5,0.8}]}, \dfrac{\boldsymbol{x_2}}{[\mathbf{0.5,0.7}],[\mathbf{0.1,0.4}],[\mathbf{0.6,0.7}]}\right\}\right) \\ \left(\boldsymbol{e_4}\left\{\dfrac{\boldsymbol{x_1}}{[\mathbf{0.4,0.5}],[\mathbf{0.4,0.9}],[\mathbf{0.4,0.9}]}, \dfrac{\boldsymbol{x_2}}{[\mathbf{0.3,0.4}],[\mathbf{0.6,0.7}],[\mathbf{0.1,0.5}]}\right\}\right) \\ \left(\boldsymbol{e_5}\left\{\dfrac{\boldsymbol{x_1}}{[\mathbf{0.1,0.7}],[\mathbf{0.5,0.6}],[\mathbf{0.1,0.5}]}, \dfrac{\boldsymbol{x_2}}{[\mathbf{0.6,0.7}],[\mathbf{0.2,0.4}],[\mathbf{0.3,0.7}]}\right\}\right) \end{cases}$$

2.6 Generalized Neutrosophic Soft Sets

The concept of generalized neutrosophic soft is defined by Şahin and Küçük [23] as follows:

Definition 2.11 [23] Let U be an intial universe and E be a set of parameters. Let $NS(U)$ be the set of all neutrosophic sets of U. A generalized neutrosophic soft set F^{μ} over U is defined by the set of ordered pairs

$$F^{\mu} = \{(F(e), \mu(e)) : e \in E, F(e) \in N(U), \mu(e) \in [0,1]\},$$

where F is a mapping given by $F: E \to NS(U) \times I$ and μ is a fuzzy set such that $\mu: E \to I = [0,1]$. Here, F^{μ} is a mapping defined by $F^{\mu}: E \to NS(U) \times I$.

For any parameter $e \in E$, $F(e)$ is referred as the neutrosophic value set of parameter e, i.e,

$$F(e) = \{\langle x, u_{F(e)}(x), w_{F(e)}(x), v_{F(e)}(x)\rangle : x \in U\}$$

where u, w, v : U → [0 ,1] are the memberships functions of truth, indeterminacy and falsity respectively of the element $x \in U$. For any $x \in U$ and $e \in E$,

$$0 \le u_{F(e)}(x) + w_{F(e)}(x) + v_{F(e)}(x) \le 3.$$

In fact, F^{μ} is a parameterized family of neutrosophic sets over U, which has the degree of possibility of the approximate value set which is represented by $\mu(e)$ for each parameter e, so F^{μ} can be expressed as follows:

$$F^{\mu}(e) = \left\{\left(\frac{x_1}{F(e)(x_1)}, \frac{x_2}{F(e)(x_2)}, \ldots, \frac{x_n}{F(e)(x_n)}\right), \mu(\mathrm{e})\right\}.$$

Definition 2.12 [4] A binary operation $\otimes: [0,1] \times [0,1] \longrightarrow [0,1]$ is continuous $t-$norm if $\otimes$ satisfies the following conditions:

(1) $\otimes$ is commutative and associative,
(2) $\otimes$ is continuous,
(3) $a \otimes 1 = a, \forall a \in [0,1]$,
(4) $a \otimes b \le c \otimes d$ whenever $a \le c, b \le d$ and $a, b, c, d \in [0,1]$.

Definition 2.13 [4] A binary operation $\oplus: [0,1] \times [0,1] \longrightarrow [0,1]$ is continuous $t-$conorm if $\oplus$ satisfies the following conditions:

(1) $\oplus$ is commutative and associative,
(2) $\oplus$ is continuous,
(3) $a \oplus 0 = a, \forall a \in [0,1]$,
(4) $a \oplus b \le c \oplus d$ whenever $a \le c, b \le d$ and $a, b, c, d \in [0,1]$.

3. Generalized Interval Neutrosophic Soft Set

In this section, we define the generalized interval neutrosophic soft sets and investigate some basic properties.

Definition 3.1. Let U be an initial universe and E be a set of parameters.Suppose that$INS(U)$is the set of all interval neutrosophic sets over$U$ and int$[0,1]$is the set of all closed subsets of $[0,1]$. A generalized interval neutrosophic soft set F^{μ} over U is defined by the set of ordered pairs

$$F^{\mu} = \{(F(e), \mu(e)) : e \in E, F(e) \in INS(U), \mu(e) \in [0,1]\},$$

where F is a mapping given by$F: E \to INS(U) \times I$ and μ is a fuzzy set such that $\mu: E \to I = [0,1]$. Here,$F^{\mu}$is a mapping defined by$F^{\mu}: E \to INS(U) \times I$.

For any parameter $e \in E$,$F(e)$ is referred as the interval neutrosophic value set of parameter e, i.e,

$$F(e) = \{\langle x, u_{F(e)}(x), w_{F(e)}(x), v_{F(e)}(x)\rangle : x \in U\}$$

where $u_{F(e)}, w_{F(e)}, v_{F(e)}: U \to$ int$[0,1]$with the condition

$$0 \le \sup u_{F(e)}(x) + \sup w_{F(e)}(x) + \sup v_{F(e)}(x) \le 3$$

for all $x \in U$.

The intervals $u_{F(e)}(x), w_{F(e)}(x)$ and $v_{F(e)}(x)$are the interval memberships functions of truth, interval indeterminacy and interval falsity of the element $x \in U$, respectively.

For convenience, if let

$$u_{F(e)}(x) = [u^L_{F(e)}(x), u^U_{F(e)}(x)]$$
$$w_{F(e)}(x) = [w^L_{F(e)}(x), w^U_{F(e)}(x)]$$
$$v_{F(e)}(x) = [v^L_{F(e)}(x), v^U_{F(e)}(x)]$$

then

$$F(e) = \{\langle x, [u^L_{F(e)}(x), u^U_{F(e)}(x)], [w^L_{F(e)}(x), w^U_{F(e)}(x)], [v^L_{F(e)}(x), v^U_{F(e)}(x)]\rangle : x \in U\}$$

In fact, $F^{\mu}$is a parameterized family of interval neutrosophic sets on U, which has the degree of possibility of the approximate value set which is represented by $\mu(e)$ for each parameter e, so F^{μ}can be expressed as follows:

$$F^{\mu}(e) = \left\{\left(\frac{x_1}{F(e)(x_1)}, \frac{x_2}{F(e)(x_2)}, \dots\dots, \frac{x_n}{F(e)(x_n)}\right), \mu(\mathrm{e})\right\}$$

Example 3.2. Consider two generalized interval neutrosophic soft set $F^{\mu}$and G^{θ}. Suppose that $U = \{h_1, h_2, h_3\}$ is the set of house and $E = \{e_1, e_2, e_3\}$ is the set of parameters where e_1 =cheap,e_2 =moderate,e_3 =comfortable. Suppose that F^{μ} and $G^{\theta}$are given as follows, respectively:

$$\left\{\begin{array}{l} F^{\mu}(e_1) = \left(\frac{h_1}{([0.2,0.3],[0.3,0.5],[0.2,0.3])}, \frac{h_2}{([0.3,0.4],[0.3,0.4],[0.5,0.6])}, \frac{h_3}{([0.5,0.6],[0.2,0.4],[0.5,0.7])}\right),(0.2) \\ F^{\mu}(e_2) = \left(\frac{h_1}{([0.1,0.4],[0.5,0.6],[0.3,0.4])}, \frac{h_2}{([0.6,0.7],[0.4,0.5],[0.5,0.8])}, \frac{h_3}{([0.2,0.4],[0.3,0.6],[0.6,0.9])}\right),(0.5) \\ F^{\mu}(e_3) = \left(\frac{h_1}{([0.2,0.6],[0.2,0.5],[0.1,0.5])}, \frac{h_2}{([0.3,0.5],[0.3,0.6],[0.4,0.5])}, \frac{h_3}{([0.6,0.8],[0.3,0.4],[0.2,0.3])}\right),(0.6) \end{array}\right\}$$

and

$$\left\{\begin{array}{l} G^{\theta}(e_1) = \left(\frac{h_1}{([0.1,0.2],[0.1,0.2],[0.1,0.2])}, \frac{h_2}{([0.4,0.5],[0.2,0.3],[0.3,0.5])}, \frac{h_3}{([0.6,0.7],[0.1,0.3],[0.2,0.3])}\right),(0.4) \\ G^{\theta}(e_2) = \left(\frac{h_1}{([0.2,0.5],[0.3,0.4],[0.2,0.3])}, \frac{h_2}{([0.7,0.8],[0.3,0.4],[0.4,0.6])}, \frac{h_3}{([0.3,0.6],[0.2,0.5],[0.4,0.6])}\right),(0.7) \\ G^{\theta}(e_3) = \left(\frac{h_1}{([0.3,0.5],[0.1,0.3],[0.1,0.3])}, \frac{h_2}{([0.4,0.5],[0.1,0.5],[0.2,0.3])}, \frac{h_3}{([0.7,0.9],[0.2,0.3],[0.1,0.2])}\right),(0.8) \end{array}\right\}$$

For the purpose of storing a generalized interval neutrosophic soft sets in a computer, we can present it in matrix form. For example, the matrix form ofF^{μ}can be expressed as follows;

$$\begin{pmatrix} ([0.2,0.3],[0.3,0.5],[0.2,0.3]) & ([0.3,0.4],[0.3,0.4],[0.5,0.6]) & ([0.5,0.6],[0.2,0.4],[0.5,0.7]),(0.2) \\ ([0.1,0.4],[0.5,0.6],[0.3,0.4]) & ([0.6,0.7],[0.4,0.5],[0.5,0.8]) & ([0.2,0.4],[0.3,0.6],[0.6,0.9]),(0.5) \\ ([0.2,0.6],[0.2,0.5],[0.1,0.5]) & ([0.3,0.5],[0.3,0.6],[0.4\ 0.5]) & ([0.6,0.8],[0.3,0.4],[0.2,0.3]),(0.6) \end{pmatrix}$$

Definition 3.3. A generalized interval neutrosophic soft setF^{μ}over U is said to be generalized null interval neutrosophic soft set ,denoted by $\emptyset^{\mu}$, if $\emptyset^{\mu}: E \to$IN(U) ×I such that

$\emptyset^{\mu}(e) = \{(F(e), \mu\,(e)\}$, where $F(e) = \{<x, ([0,0],[1,1],[1,1])>\}$ and $\mu\,(e) = 0$ for each $e \in E$ and $x \in U$.

Definition 3.4. A generalized interval neutrosophic soft setF^{μ}over U is said to be generalized absolute interval neutrosophic soft set, denoted by U^{μ}, if $U^{\mu}: E \to IN(U) \times I$such that $U^{\mu}(e) = \{(F(e), \mu\,(e)\}$,where $F(e) = \{<x, ([1,1],[0\,,0],[0,0])>\}and\mu\,(e) = 1$ for each $e \in E$ and $x \in U$.

Definition 3.5. LetF^{μ}be a generalized interval neutrosophic soft set over U, where

$$F^{\mu}\,(\mathrm{e}) = \{(F(e), \mu\,(e)\}$$

and

$$F(e) = \left\{\langle x, \left[u^L_{F(e)}(x), u^U_{F(e)}(x)\right], \left[w^L_{F(e)}(x), w^U_{F(e)}(x)\right], \left[v^L_{F(e)}(x), v^U_{F(e)}(x)\right]\rangle : x \in U\right\}$$

for all $e \in E$. Then, for$e_m \in E$ and $x_n \in U$;

(1) $F^{\star} = [\mathrm{F}^{\star}_L, \mathrm{F}^{\star}_U]$is said to be interval truth membership part of F^{μ}
where$F^{\star} = \{(\mathrm{F}^{\star}{}_{mn}\,(e_m)\,, \mu\,(e_m))\}$ and $F^{\star}{}_{mn}(e_m) = \left\{\langle x_n, \left[u^L_{F(e_m)}(x_n), u^U_{F(e_m)}(x_n)\right]\rangle\right\}$,
(2) $\mathrm{F}^{\wr} = [\mathrm{F}^{\wr}_L, \mathrm{F}^{\wr}_U]$is said to be interval indeterminacy membership part of F^{μ}
where$F^{\wr} = \{F^{\wr}{}_{mn}\,(e_m)\,, \mu\,(e_m)\}$ and$F^{\wr}{}_{mn}(e_m) = \left\{\langle x_n, \left[w^L_{F(e_m)}(x_n), w^U_{F(e_m)}(x_n)\right]\rangle\right\}$,
(3) $\mathrm{F}^{\Delta} = [\mathrm{F}^{\Delta}_L, \mathrm{F}^{\Delta}_U]$is said to be interval falsity membership part of F^{μ}
where$\mathrm{F}^{\Delta} = \{\mathrm{F}^{\Delta}{}_{mn}\,(e_m)\,, \mu\,(e_m)\}$ and $\mathrm{F}^{\Delta}{}_{mn}(e_m) = \left\{\langle x_n, \left[v^L_{F(e_m)}(x_n), v^U_{F(e_m)}(x_n)\right]\rangle\right\}$.

We say that every part of F^{μ} is a component of itself and is denote by $F^{\mu} = (\mathrm{F}^{\star}, \mathrm{F}^{\wr}, \mathrm{F}^{\Delta})$. Then matrix forms of components of F^{μ}in example 3.2 can be expressed as follows:

$$F^{\star}=\begin{pmatrix} ([0.2,0.3],[0.3,0.6],[0.4,0.5]) & (0.1) \\ ([0.2,0.5],[0.3,0.5],[0.4,0.7]) & (0.4) \\ ([0.3,0.4],[0.1,0.3],[0.1,0.4]) & (0.6) \end{pmatrix}$$

$$F^{\wr}=\begin{pmatrix} ([0.2,0.3],[0.3,0.5],[0.2,0.5]) & (0.1) \\ ([0.2,0.5],[0.4,0.8],[0.3,0.8]) & (0.4) \\ ([0.3,0.4],[0.2,0.5],[0.2,0.3]) & (0.6) \end{pmatrix}$$

$$F^{\Delta}=\begin{pmatrix} ([0.2,0.3],[0.2,0.4],[0.2,0.6]) & (0.1) \\ ([0.2,0.5],[0.8,0.9],[0.3,0.4]) & (0.4) \\ ([0.7,0.9],[0.3,0.7],[0.5,0.7]) & (0.6) \end{pmatrix}$$

where

$$F^{\star}{}_{mn}(e_m) = \{\langle x_n, [u^L_{F(e_m)}(x_n), u^U_{F(e_m)}(x_n)]\rangle\}$$
$$F^{\wr}{}_{mn}(e_m) = \{\langle x_n, [w^L_{F(e_m)}(x_n), w^U_{F(e_m)}(x_n)]\rangle\}$$
$$F^{\Delta}{}_{mn}(e_m) = \{\langle x_n, [v^L_{F(e_m)}(x_n), v^U_{F(e_m)}(x_n)]\rangle\}$$

are defined as the interval truth, interval indeterminacy and interval falsity values of n −th element the according to m −th parameter, respectively.

Remark 3.6. Suppose that F^{μ} is a generalizedinterval neutrosophic soft set over U.Then we say that each components ofF^{μ}can be seen as the generalizedinterval valued vague soft set [15]. Also if it is taken $\mu\,(e) = 1$ for all $e \in$ E,the our generalized interval neutrosophic soft set concides with the interval neutrosophic soft set [12].

Definition 3.7. Let U be an universe and E be a of parameters, F^{μ} and G^{θ} be two generalized interval neutrosophic soft sets, we say that F^{μ} is a generalized interval neutrosophic soft subset G^{θ} if

(1) μ is a fuzzy subset of θ,
(2) For $e \in E$, $F(e)$ is an interval neutrosophic subset of$G(e)$, i.e, for all $e_m \in E$ and $m, n \in \wedge$,

$F^{\star}{}_{mn}(e_m) \le G^{\star}{}_{mn}(e_m)$, $F^{\wr}{}_{mn}(e_m) \ge G^{\wr}{}_{mn}(e_m)$ and $F^{\Delta}{}_{mn}(e_m) \ge G^{\Delta}{}_{mn}(e_m)$ where,

$$u^L_{F(e_m)}(x_n) \le u^L_{G(e_m)}(x_n), u^U_{F(e_m)}(x_n) \le u^U_{G(e_m)}(x_n)$$
$$w^L_{F(e_m)}(x_n) \ge w^L_{G(e_m)}(x_n), w^U_{F(e_m)}(x_n) \ge w^U_{G(e_m)}(x_n)$$
$$v^L_{F(e_m)}(x_n) \ge v^L_{G(e_m)}(x_n), v^U_{F(e_m)}(x_n) \ge v^U_{G(e_m)}(x_n)$$

For $x_n \in U$.

We denote this relationship by$F^{\mu} \sqsubseteq G^{\theta}$. Moreover if$G^{\theta}$ is generalized interval neutrosophic soft subset of F^{μ}, thenF^{μ} is called a generalized interval neutrosophic soft superset of G^{θ} this relation is denoted by $F^{\mu} \sqsupseteq G^{\theta}$.

Example 3.8. Consider two generalized interval neutrosophic soft set F^{μ} and G^{θ}.suppose that U= $\{h_1, h_2, h_3]$ is the set of houses and E $= \{e_1, e_2, e_3\}$ is the set of parameters where e_1=cheap,e_2 =moderate,e_3 =comfortable. Suppose that F^{μ} and $G^{\theta}$are given as follows respectively:

$$F^{\mu}(e_1) = \left(\frac{h_1}{([0.1, 0.2], [0.3, 0.5], [0.2, 0.3])}, \frac{h_2}{([0.3, 0.4], [0.3, 0.4], [0.5, 0.6])}, \frac{h_3}{([0.5, 0.6], [0.2, 0.4], [0.5, 0.7])}\right), (0.2)$$
$$F^{\mu}(e_2) = \left(\frac{h_1}{([0.1, 0.4], [0.5, 0.6], [0.3, 0.4])}, \frac{h_2}{([0.6, 0.7], [0.4, 0.5], [0.5, 0.8])}, \frac{h_3}{([0.2, 0.4], [0.3, 0.6], [0.6, 0.9])}\right), (0.5)$$
$$F^{\mu}(e_3) = \left(\frac{h_1}{([0.2, 0.6], [0.2, 0.5], [0.1, 0.5])}, \frac{h_2}{([0.3, 0.5], [0.3, 0.6], [0.4, 0.5])}, \frac{h_3}{([0.6, 0.8], [0.3, 0.4], [0.2, 0.3])}\right), (0.6) \Big\}$$

and

$$G^{\theta}(e_1) = \left(\frac{h_1}{([0.2, 0.3], [0.1, 0.2], [0.1, 0.2])}, \frac{h_2}{([0.4, 0.5], [0.2, 0.3], [0.3, 0.5])}, \frac{h_3}{([0.6, 0.7], [0.1, 0.3], [0.2, 0.3])}\right), (0.4)$$
$$G^{\theta}(e_2) = \left(\frac{h_1}{([0.2, 0.5], [0.3, 0.4], [0.2, 0.3])}, \frac{h_2}{([0.7, 0.8], [0.3, 0.4], [0.4, 0.6])}, \frac{h_3}{([0.3, 0.6], [0.2, 0.5], [0.4, 0.6])}\right), (0.7)$$
$$G^{\theta}(e_3) = \left(\frac{h_1}{([0.3, 0.7], [0.1, 0.3], [0.1, 0.3])}, \frac{h_2}{([0.4, 0.5], [0.1, 0.5], [0.2, 0.3])}, \frac{h_3}{([0.7, 0.9], [0.2, 0.3], [0.1, 0.2])}\right), (0.8) \Big\}$$

Then $F^{\mu}$is a generalized interval neutrosophic soft subset ofG^{θ}, that is$F^{\mu} \sqsubseteq G^{\theta}$.

Definition3.9. The union of two generalized interval neutrosophic soft setsF^{μ}andG^{θ}over U, denoted by $H^{\lambda} = F^{\mu} \sqcup G^{\theta}$ is a generalized interval neutrosophic soft setH^{λ}defined by

$$H^{\lambda} = ([\, H_L^{\star}, H_U^{\star}], [\, H_L^{\wr}, H_U^{\wr}], [\, H_L^{\triangle}, H_U^{\triangle}])$$

where$\lambda\,(e_m) = \mu\,(e_m) \oplus \theta\,(e_m)$,

$$\begin{aligned} H_{L_{mn}}^{\star} &= F_{L_{mn}}^{\star}(e_m) \oplus G_{L_{mn}}^{\star}(e_m) \\ H_{L_{mn}}^{\wr} &= F_{L_{mn}}^{\wr}(e_m) \otimes G_{L_{mn}}^{\wr}(e_m) \\ H_{L_{mn}}^{\triangle} &= F_{L_{mn}}^{\triangle}\,(e_m) \otimes G_{L_{mn}}^{\triangle}(e_m) \end{aligned}$$

and

$$\begin{aligned} H_{U_{mn}}^{\star} &= F_{U_{mn}}^{\star}(e_m) \oplus G_{U_{mn}}^{\star}(e_m) \\ H_{U_{mn}}^{\wr} &= F_{U_{mn}}^{\wr}(e_m) \otimes G_{U_{mn}}^{\wr}(e_m) \\ H_{U_{mn}}^{\triangle} &= F_{U_{mn}}^{\triangle}\,(e_m) \otimes G_{U_{mn}}^{\triangle}(e_m) \end{aligned}$$

for all $e_m \in$ E and $m, n \in \Lambda$.

Definition 3.10. The intersection of two generalized interval neutrosophic soft setsF^{μ} and G^{θ}over U, denoted by $K^{\varepsilon} = F^{\mu} \sqcap G^{\theta}$isa generalized interval neutrosophic soft setK^{ε}defined by

$$K^{\varepsilon} = ([\, K_L^{\star}, K_U^{\star}], [\, K_L^{\wr}, K_U^{\wr}], [\, K_L^{\triangle}, K_U^{\triangle}])$$

where$\varepsilon\,(e_m) = \mu\,(e_m) \otimes \theta\,(e_m)$,

$$\begin{aligned} K_{L_{mn}}^{\star} &= F_{L_{mn}}^{\star}(e_m) \otimes G_{L_{mn}}^{\star}(e_m) \\ K_{L_{mn}}^{\wr} &= F_{L_{mn}}^{\wr}(e_m) \oplus G_{L_{mn}}^{\wr}(e_m) \\ K_{L_{mn}}^{\triangle} &= F_{L_{mn}}^{\triangle}\,(e_m) \oplus G_{L_{mn}}^{\triangle}(e_m) \end{aligned}$$

and

$$\begin{aligned} K_{U_{mn}}^{\star} &= F_{U_{mn}}^{\star}(e_m) \otimes G_{U_{mn}}^{\star}(e_m) \\ K_{U_{mn}}^{\wr} &= F_{U_{mn}}^{\wr}(e_m) \oplus G_{U_{mn}}^{\wr}(e_m) \\ K_{U_{mn}}^{\triangle} &= F_{U_{mn}}^{\triangle}\,(e_m) \oplus G_{U_{mn}}^{\triangle}(e_m) \end{aligned}$$

for all $e_m \in$ E and $m, n \in \Lambda$.

Example 3.11. Let us consider the generalized interval neutrosophic soft sets $F^{\mu} and\ G^{\theta}$ defined in Example 3.2. Suppose that the t-conorm is defined by $\oplus(a,b) = \max\{a,b\}$ and the $t-$norm by $\otimes(a,b) = \min\{a,b\}$ for $a,b \in [0,1]$. Then $H^{\lambda} = F^{\mu} \sqcup G^{\theta}$ is defined as follows:

$$\begin{cases} H(e_1) = \left(\frac{h_1}{([0.2,0.3],[0.1,0.2],[0.1,0.2])}, \frac{h_2}{([0.4,0.5],[0.2,0.3],[0.3,0.5])}, \frac{h_3}{([0.6,0.7],[0.1,0.3],[0.2,0.3])}\right), (0.4) \\ H(e_2) = \left(\frac{h_1}{([0.2,0.5],[0.3,0.4],[0.2,0.3])}, \frac{h_2}{([0.7,0.8],[0.3,0.4],[0.4,0.6])}, \frac{h_3}{([0.3,0.6],[0.2,0.5],[0.4,0.6])}\right), (0.7) \\ H(e_3) = \left(\frac{h_1}{([0.3,0.6],[0.1,0.3],[0.1,0.3])}, \frac{h_2}{([0.4,0.5],[0.1,0.5],[0.2,0.3])}, \frac{h_3}{([0.7,0.9],[0.2,0.3],[0.1,0.2])}\right), (0.8) \end{cases}$$

Example 3.12. Let us consider the generalized interval neutrosophic soft sets $F^{\mu} and\ G^{\theta}$ defined in Example 3.2. Suppose that the $t-$conorm is defined by $\oplus$ (a, b) $= \max\{a,b\}$ and the $t-$norm by $\otimes(a,b) = \min\{a,b\}$ for $a,b \in [0,1]$. Then $K^{\varepsilon} = F^{\mu} \sqcap G^{\theta}$ is defined as follows:

$$\begin{cases} K(e_1) = \left(\frac{h_1}{([0.1,0.2],[0.3,0.5],[0.2,0.3])}, \frac{h_2}{([0.3,0.4],[0.3,0.4],[0.5,0.6])}, \frac{h_3}{([0.5,0.6],[0.2,0.4],[0.5,0.7])}\right), (0.2) \\ K(e_2) = \left(\frac{h_1}{([0.1,0.4],[0.5,0.6],[0.3,0.4])}, \frac{h_2}{([0.6,0.7],[0.4,0.5],[0.5,0.8])}, \frac{h_3}{([0.2,0.4],[0.3,0.6],[0.6,0.9])}\right), (0.5) \\ K(e_3) = \left(\frac{h_1}{([0.2,0.5],[0.2,0.5],[0.1,0.5])}, \frac{h_2}{([0.3,0.5],[0.3,0.6],[0.4,0.5])}, \frac{h_3}{([0.6,0.8],[0.3,0.4],[0.2,0.3])}\right), (0.6) \end{cases}$$

Proposition 3.13. Let F^{μ}, G^{θ} and H^{λ} be three generalized interval neutrosophic soft sets over U. Then

(1) $F^{\mu} \sqcup G^{\theta} = G^{\theta} \sqcup F^{\mu}$,
(2) $F^{\mu} \sqcap G^{\theta} = G^{\theta} \sqcap F^{\mu}$,
(3) $(F^{\mu} \sqcup G^{\theta}) \sqcup H^{\lambda} = F^{\mu} \sqcup (G^{\theta} \sqcup H^{\lambda})$,
(4) $(F^{\mu} \sqcap G^{\theta}) \sqcap H^{\lambda} = F^{\mu} \sqcap (G^{\theta} \sqcap H^{\lambda})$.

Proof. The proofs are trivial.

Proposition 3.14. Let F^{μ}, G^{θ} and H^{λ} be three generalized interval neutrosophic soft sets over U. If we consider the $t-$conorm defined by $\oplus(a,b) = max\{a,b\}$ and the $t-$norm defined by $\otimes(a,b) = min\{a,b\}$ for $a,b \in [0,1]$, then the following relations holds:

(1) $H^{\lambda} \sqcap (F^{\mu} \sqcup G^{\theta}) = (H^{\lambda} \sqcap F^{\mu}) \sqcup (H^{\lambda} \sqcap G^{\theta})$,
(2) $H^{\lambda} \sqcup (F^{\mu} \sqcap G^{\theta}) = (H^{\lambda} \sqcup F^{\mu}) \sqcap (H^{\lambda} \sqcup G^{\theta})$.

Remark 3.15. The relations in above proposition does not hold in general.

Definition 3.16. The complement of a generalized interval neutrosophic soft sets F^{μ} over U, denoted by $F^{\mu(c)}$ is defined by $F^{\mu(c)} = ([\mathrm{F}_L^{\star(c)}, \mathrm{F}_U^{\star(c)}], [\mathrm{F}_L^{\wr(c)}, \mathrm{F}_U^{\wr(c)}], [\mathrm{F}_L^{\Delta(c)}, \mathrm{F}_U^{\Delta(c)}])$ where

$$\mu^{(c)}(e_m) = 1 - \mu(e_m)$$

and

$$\mathrm{F}_{L_{mn}}^{\star(c)} = \mathrm{F}_{L_{mn}}^{\Delta};\ \mathrm{F}_{L_{mn}}^{\wr(c)} = 1 - \mathrm{F}_{U_{mn}}^{\wr};\ \mathrm{F}_{L_{mn}}^{\Delta(c)} = \mathrm{F}_{L_{mn}}^{\star}$$

$$\mathrm{F}^{\star(c)}_{U_{mn}} = \mathrm{F}^{\triangle}_{U_{mn}}\ ;\ \mathrm{F}^{\wr(c)}_{U_{mn}} = 1 - \mathrm{F}^{\wr}_{L_{mn}};\ \mathrm{F}^{\triangle(c)}_{U_{mn}} = \mathrm{F}^{\star}_{U_{mn}}$$

Example 3.17. Consider Example 3.2. Complement of the generalized interval neutrosophic soft set F^{μ} denoted by $F^{\mu(c)}$ is given as follows:

$$\left\{\begin{array}{l} F^{\mu(c)}(e_1) = \left(\frac{h_1}{([0.2, 0.3], [0.5, 0.7], [0.2, 0.3])}, \frac{h_2}{([0.5, 0.6], [0.6, 0.7], [0.3, 0.4])}, \frac{h_3}{([0.5, 0.7], [0.6, 0.8], [0.5, 0.6])}\right), (0.8) \\ F^{\mu(c)}(e_2) = \left(\frac{h_1}{([0.3, 0.4], [0.4, 0.5], [0.1, 0.4])}, \frac{h_2}{([0.5, 0.8], [0.5, 0.6], [0.6, 0.7])}, \frac{h_3}{([0.6, 0.9], [0.4, 0.7], [0.2, 0.4])}\right), (0.5) \\ F^{\mu(c)}(e_3) = \left(\frac{h_1}{([0.1, 0.5], [80.5, 0.5], [0.2, 0.6])}, \frac{h_2}{([0.4, 0.5], [0.4, 0.7], [0.3, 0.5])}, \frac{h_3}{([0.2, 0.3], [0.6, 0.7], [0.6, 0.8])}\right), (0.4) \end{array}\right\}$$

Proposition 3.18. LetF^{μ} and G^{θ} be two generalized interval neutrosophic soft sets over U. Then,

(1) F^{μ} is a generalized interval neutrosophic soft subset of$F^{\mu} \sqcup F^{\mu(c)}$
(2) $F^{\mu} \sqcap F^{\mu(c)}$is a generalized interval neutrosophic soft subset ofF^{μ}.

Proof: It is clear.

Definition 3.19. "And" operation on two generalized interval neutrosophic soft sets F^{μ}andG^{θ} over U,denoted by$\mathrm{H}^{\lambda} = F^{\mu} \wedge G^{\theta}$ is the mapping$\mathrm{H}^{\lambda}: C \to \mathrm{IN(U)} \times$ I defined by

$$\mathrm{H}^{\lambda} = ([\,\mathrm{H}^{\star}_{L}, \mathrm{H}^{\star}_{U}], [\,\mathrm{H}^{\wr}_{L}, \mathrm{H}^{\wr}_{U}], [\,\mathrm{H}^{\triangle}_{L}, \mathrm{H}^{\triangle}_{U}])$$

where$\lambda\,(e_m) = \min(\,\mu\,(e_k), \theta\,(e_h)$ and

$$\begin{array}{c} \mathrm{H}^{\star}_{L}(e_m) = \min\{\mathrm{F}^{\star}_{L}(e_{kn}), \mathrm{G}^{\star}_{L}(e_{hn})\} \\ \mathrm{H}^{\wr}_{L}(e_m) = \max\{\mathrm{F}^{\wr}_{L}(e_{kn}), \mathrm{G}^{\wr}_{L}(e_{hn}) \\ \mathrm{H}^{\triangle}_{L}(e_m) = \max\{\mathrm{F}^{\triangle}_{L}(e_{kn}), \mathrm{G}^{\triangle}_{L}(e_{hn})\} \end{array}$$

and

$$\begin{array}{c} \mathrm{H}^{\star}_{\mathrm{U}}(\mathrm{e_m}) = \min\{\mathrm{F}^{\star}_{\mathrm{U}}(\mathrm{e_{kn}}), \mathrm{G}^{\star}_{\mathrm{U}}(\mathrm{e_{hn}})\} \\ \mathrm{H}^{\wr}_{U}(e_m) = \max\{\mathrm{F}^{\wr}_{U}(e_{kn}), \mathrm{G}^{\wr}_{U}(e_{hn})\} \\ \mathrm{H}^{\triangle}_{U}(e_m) = \max\{\mathrm{F}^{\triangle}_{U}(e_{kn}), \mathrm{G}^{\triangle}_{U}(e_{hn})\} \end{array}$$

for all$e_m = (e_k, e_h) \in C \subseteq E \times E$ and $m, n, k, h \in \Lambda$.

Definition 3.20. "OR" operation on two generalized interval neutrosophic soft sets F^{μ}andG^{θ} over U,denoted by$\mathrm{K}^{\lambda} = F^{\mu} \vee G^{\theta}$ is the mapping$\mathrm{K}^{\varepsilon}: C \to \mathrm{IN(U)} \times$ Idefined by

$$\mathrm{K}^{\varepsilon} = ([\mathrm{K}^{\star}_{L}, \mathrm{K}^{\star}_{U}], [\mathrm{k}^{\wr}_{L}, \mathrm{K}^{\wr}_{U}], [\,\mathrm{K}^{\triangle}_{L}, \mathrm{K}^{\triangle}_{U}])$$

where $\varepsilon\,(e_m)$= max($\mu\,(e_k), \theta\,(e_h)$ and

$$\begin{array}{c} \mathrm{K}^{\star}_{L}(e_m) = \max\{\mathrm{F}^{\star}_{L}(e_{kn}), \mathrm{G}^{\star}_{L}(e_{hn})\} \\ \mathrm{K}^{\wr}_{L}(e_m) = \min\{F^{\wr}_{L}(e_{kn}), G^{\wr}_{L}(e_{hn})\} \\ \mathrm{K}^{\triangle}_{L}(e_m) = \min\{F^{\triangle}_{L}(e_{kn}), G^{\triangle}_{L}(e_{hn})\} \end{array}$$

and

$$\begin{array}{c} \mathrm{K}^{\star}_{\mathrm{U}}(\mathrm{e_m}) = \max\{F^{\star}_{U}(e_{kn}), G^{\star}_{U}(e_{hn})\} \\ \mathrm{K}^{\wr}_{U}(e_m) = \min\{\mathrm{F}^{\wr}_{U}(e_{kn}), \mathrm{G}^{\wr}_{U}(e_{hn})\} \\ \mathrm{K}^{\triangle}_{U}(e_m) = \min\{\mathrm{F}^{\triangle}_{U}(e_{kn}), \mathrm{G}^{\triangle}_{U}(e_{hn})\} \end{array}$$

for all $e_m = (e_k, e_h) \in C \subseteq E \times E$ and $m, n, k, h \in \Lambda$.

Definition 3.21. LetF^{μ}andG^{θ} be two generalizedinterval neutrosophic soft sets over UandC ⊆ E × E , a function $R: C \to$IN(U) ×Idefined by R= $F^{\mu} \wedge G^{\theta}$and $R(e_m, e_h) = F^{\mu}(e_m) \wedge G^{\theta}(e_h)$is said to be a interval neutrosophic relation from F^{μ} to $G^{\theta}$for all $(e_m, e_h) \in C$.

4. Application of Generalized Interval Neutrosophic Soft Set

Now, we illustrate an application of generalized interval neutrosophic soft set in decision making problem.

Example 4.1. Supposethat the universe consists of three machines, that is$U = \{x_1, x_2, x_3\}$ and consider the set of parameters $E = \{e_1, e_2, e_3\}$ which describe their performances according to certain specific task. Assumethat a firm wants to buy one such machine depending on any two of the parameters only. Let there be two observations F^{μ} and $G^{\theta}$by two experts A and B respectively, defined as follows:

$$\begin{cases} F^{\mu}(e_1) = \left(\frac{h_1}{([0.2,0.3],[0.2,0.3],[0.2,0.3])}, \frac{h_2}{([0.3,0.6],[0.3,0.5],[0.2,0.4])}, \frac{h_3}{([0.4,0.5],[0.2,0.5],[0.2,0.6])}\right), (0.2) \\ F^{\mu}(e_2) = \left(\frac{h_1}{([0.2,0.5],[0.2,0.5],[0.2,0.5])}, \frac{h_2}{([0.3,0.5],[0.4,0.8],[0.8,0.9])}, \frac{h_3}{([0.4,0.7],[0.3,0.8],[0.3,0.4])}\right), (0.5) \\ F^{\mu}(e_3) = \left(\frac{h_1}{([0.3,0.4],[0.3,0.4],[0.7,0.9])}, \frac{h_2}{([0.1,0.3],[0.2,0.5],[0.3,0.7])}, \frac{h_3}{([0.1,0.4],[0.2,0.3],[0.5,0.7])}\right), (0.6) \end{cases}$$

$$\begin{cases} G^{\theta}(e_1) = \left(\frac{h_1}{([0.2,0.3],[0.3,0.5],[0.2,0.3])}, \frac{h_2}{([0.3,0.4],[0.3,0.4],[0.5,0.6])}, \frac{h_3}{([0.5,0.6],[0.2,0.4],[0.5,0.7])}\right), (0.3) \\ G^{\theta}(e_2) = \left(\frac{h_1}{([0.1,0.4],[0.5,0.6],[0.3,0.4])}, \frac{h_2}{([0.6,0.7],[0.4,0.5],[0.5,0.8])}, \frac{h_3}{([0.2,0.4],[0.3,0.6],[0.6,0.9])}\right), (0.6) \\ G^{\theta}(e_3) = \left(\frac{h_1}{([0.2,0.6],[0.2,0.5],[0.1,0.5])}, \frac{h_2}{([0.3,0.5],[0.3,0.6],[0.4,0.5])}, \frac{h_3}{([0.6,0.8],[0.3,0.4],[0.2,0.3])}\right), (0.4) \end{cases}$$

To find the "AND" between the two GINSSs, we have $F^{\mu}$and G^{θ},$R = F^{\mu} \wedge G^{\theta}$ where

$$(F^{\mu})^{\star} = \begin{pmatrix} e_1 & ([0.2,0.3],[0.3,0.6],[0.4,0.5]) & (0.2) \\ e_2 & ([0.2,0.5],[0.3,0.5],[0.4,0.7]) & (0.5) \\ e_3 & ([0.3,0.4],[0.1,0.3],[0.1,0.4]) & (0.6) \end{pmatrix}$$

$$(F^{\mu})^{\wr} = \begin{pmatrix} e_1 & ([0.2,0.3],[0.3,0.5],[0.2,0.5]) & (0.2) \\ e_2 & ([0.2,0.5],[0.4,0.8],[0.3,0.8]) & (0.5) \\ e_3 & ([0.3,0.4],[0.2,0.5],[0.2,0.3]) & (0.6) \end{pmatrix}$$

$$(F^{\mu})^{\vartriangle} = \begin{pmatrix} e_1 & ([0.2,0.3],[0.2,0.4],[0.2,0.6]) & (0.2) \\ e_2 & ([0.2,0.5],[0.8,0.9],[0.3,0.4]) & (0.5) \\ e_3 & ([0.7,0.9],[0.3,0.7],[0.5,0.7]) & (0.6) \end{pmatrix}$$

$$(G^{\theta})^{\star} = \begin{pmatrix} e_1 & ([0.2,0.3],[0.3,0.4],[0.5,0.6]) & (0.3) \\ e_2 & ([0.1,0.4],[0.6,0.7],[0.2,0.4]) & (0.6) \\ e_3 & ([0.2,0.6],[0.3,0.5],[0.6,0.8]) & (0.4) \end{pmatrix}$$

$$(G^{\theta})^{\wr} = \begin{pmatrix} e_1 & ([0.3,0.5],[0.3,0.4],[0.2,0.4]) & (0.3) \\ e_2 & ([0.5,0.6],[0.4,0.5],[0.3,0.6]) & (0.6) \\ e_2 & ([0.2,0.5],[0.3,0.6],[0.3,0.4]) & (0.4) \end{pmatrix}$$

$$(G^{\theta})^{\Delta}=\begin{pmatrix} e_1 & ([0.2,0.3],[0.5,0.6],[0.5,0.7]) & (0.3) \\ e_2 & ([0.3,0.4],[0.5,0.8],[0.6,0.9]) & (0.6) \\ e_3 & ([0.1,0.5],[0.4,0.5],[0.2,0.3]) & (0.4) \end{pmatrix}$$

We present the table of three basic component of R, which are interval truth –membership, Interval indeterminacy membership and interval falsity-membership part.To choose the best candidate, we firstly propose the induced interval neutrosophic membership functions by taking the arithmetic average of the end point of the range, and mark the highest numerical grade (underline) in each row of each table. But here, since the last column is the grade of such belongingness of a candidate for each pair of parameters, its not taken into account while making. Then we calculate the score of each component of R by taking the sum of products of these numerical grades with the corresponding values of μ. Next, we calculate the final score by subtracting the score of falsity-membership part of R from the sum of scores of truth-membership part and of indeterminacy membership part of R.The machine with the highestscore is the desired machine by company.

For the interval truth membership function components we have:

$$(F^{\mu})^{\star}=\begin{pmatrix} e_1 & ([0.2,0.3],[0.3,0.6],[0.4,0.5]) & (0.2) \\ e_2 & ([0.2,0.5],[0.3,0.5],[0.4,0.7]) & (0.5) \\ e_3 & ([0.3,0.4],[0.1,0.3],[0.1,0.4]) & (0.6) \end{pmatrix}$$

$$(G^{\theta})^{\star}=\begin{pmatrix} e_1 & ([0.2,0.3],[0.3,0.4],[0.5,0.6]) & (0.3) \\ e_2 & ([0.1,0.4],[0.6,0.7],[0.2,0.4]) & (0.6) \\ e_3 & ([0.2,0.6],[0.3,0.5],[0.6,0.8]) & (0.4) \end{pmatrix}$$

$$(R)^{\star} =$$

$$(R)^{\star}(e_1, e_1) = \left\{\left(\frac{x_1}{[0.2,0.3]}, \frac{x_2}{[0.3,0.4]}, \frac{x_3}{[0.4,0.5]}\right), 0.2\right\}$$

$$(R)^{\star}(e_1, e_2) = \left\{\left(\frac{x_1}{[0.1,0.3]}, \frac{x_2}{[0.3,0.6]}, \frac{x_3}{[0.2,0.5]}\right), 0.2\right\}$$

$$(R)^{\star}(e_1, e_3) = \left\{\left(\frac{x_1}{[0.2,0.3]}, \frac{x_2}{[0.3,0.5]}, \frac{x_3}{[0.2,0.4]}\right), 0.2\right\}$$

$$(R)^{\star}(e_2, e_1) = \left\{\left(\frac{x_1}{[0.2,0.3]}, \frac{x_2}{[0.3,0.4]}, \frac{x_3}{[0.4,0.6]}\right), 0.3\right\}$$

$$(R)^{\star}(e_2, e_2) = \left\{\left(\frac{x_1}{[0.1,0.4]}, \frac{x_2}{[0.3,0.5]}, \frac{x_3}{[0.2,0.4]}\right), 0.5\right\}$$

$$(R)^{\star}(e_2, e_3) = \left\{\left(\frac{x_1}{[0.2,0.5]}, \frac{x_2}{[0.3,0.5]}, \frac{x_3}{[0.4,0.7]}\right), 0.4\right\}$$

$$(R)^{\star}(e_3, e_1) = \left\{\left(\frac{x_1}{[0.2,0.3]}, \frac{x_2}{[0.1,0.3]}, \frac{x_3}{[0.1,0.4]}\right), 0.3\right\}$$

$$(R)^{\star}(e_3, e_2) = \left\{\left(\frac{x_1}{[0.1,0.4]}, \frac{x_2}{[0.1,0.3]}, \frac{x_3}{[0.1,0.4]}\right), 0.6\right\}$$

$$(R)^{\star}(e_3, e_3) = \left\{\left(\frac{x_1}{[0.2,0.4]}, \frac{x_2}{[0.1,0.3]}, \frac{x_3}{[0.1,0.4]}\right), 0.4\right\}$$

	x_1	x_2	x_3	μ
(e_1, e_1)	$[0.2, 0.3]$	$[0.3, 0.4]$	$[0.4, 0.5]$	0.2
(e_1, e_2)	$[0.1, 0.3]$	$[0.3, 0.6]$	$[0.2, 0.5]$	0.2
(e_1, e_3)	$[0.2, 0.3]$	$[0.3, 0.5]$	$[0.2, 0.4]$	0.2
(e_2, e_1)	$[0.2, 0.3]$	$[0.3, 0.4]$	$[0.4, 0.6]$	0.3
(e_2, e_2)	$[0.1, 0.4]$	$[0.3, 0.5]$	$[0.2, 0.4]$	0.5
(e_2, e_3)	$[0.2, 0.5]$	$[0.3, 0.5]$	$[0.4, 0.7]$	0.4
(e_3, e_1)	$[0.2, 0.3]$	$[0.1, 0.3]$	$[0.1, 0.4]$	0.3
(e_3, e_1)	$[0.1, 0.4]$	$[0.1, 0.3]$	$[0.1, 0.4]$	0.6
(e_3, e_2)	$[0.2, 0.4]$	$[0.1, 0.3]$	$[0.1, 0.4]$	0.4

Table 1: Interval truth membership function.

	x_1	x_2	x_3	μ
(e_1, e_1)	0.25	0.35	$\underline{0.45}$	0.2
(e_1, e_2)	0.2	$\underline{0.45}$	0.35	0.2
(e_1, e_3)	0.25	$\underline{0.4}$	0.3	0.2
(e_2, e_1)	0.25	0.35	$\underline{0.5}$	0.3
(e_2, e_2)	0.25	$\underline{0.4}$	0.3	0.5
(e_2, e_3)	0.35	0.4	$\underline{0.55}$	0.4
(e_3, e_1)	$\underline{0.25}$	0.2	$\underline{0.25}$	0.3
(e_3, e_1)	$\underline{0.25}$	0.2	$\underline{0.25}$	0.6
(e_3, e_2)	$\underline{0.3}$	0.2	0.25	0.4

Table 2: Induced interval truth membership function.

The value of representation interval truth membership function $[a, b]$ are obtained using mean value.Then, the scores of interval truth membership function of x_1,x_2 and$x_3$are:

$$S_{(R)^{\star}}(x_1) = (0.25 \times 0.3) + (0.25 \times 0.6) + (0.3 \times 0.4) = \mathbf{0.325}$$
$$S_{(R)^{\star}}(x_2) = (0.45 \times 0.2) + (0.4 \times 0.2) + (0.4 \times 0.5)) = \mathbf{0.37}$$
$$S_{(R)^{\star}}(x_3) = (0.45 \times 0.2) + (0.5 \times 0.3) + (0.55 \times 0.4)) + (0.25 \times 0.3) + (0.25 \times 0.6) = \mathbf{0.685}.$$

For the interval indeterminacy membership function components we have:

$$(F^{\mu})^{\wr} = \begin{pmatrix} ([0.2, 0.3], [0.3, 0.5], [0.2, 0.5]) & (0.2) \\ ([0.2, 0.5], [0.4, 0.8], [0.3, 0.8]) & (0.5) \\ ([0.3, 0.4], [0.2, 0.5], [0.2, 0.3]) & (0.6) \end{pmatrix}$$

$$(G^{\theta})^{\wr} = \begin{pmatrix} ([0.3, 0.5], [0.3, 0.4], [0.2, 0.4]) & (0.3) \\ ([0.5, 0.6], [0.4, 0.5], [0.3, 0.6]) & (0.6) \\ ([0.2, 0.5], [0.3, 0.6], [0.3, 0.4]) & (0.4) \end{pmatrix}$$

$$(R)^{\wr} =$$
$$(R)^{\wr}(e_1, e_1) = \left\{\left(\frac{x_1}{[0.3, 0.5]}, \frac{x_2}{[0.3, 0.5]}, \frac{x_3}{[0.2, 0.5]}\right), 0.3\right\}$$

$$(R)^{\wr}(e_1, e_2) = \left\{\left(\frac{x_1}{[0.5, 0.6]}, \frac{x_2}{[0.4, 0.5]}, \frac{x_3}{[0.3, 0.6]}\right), 0.6\right\}$$

$$(R)^{\wr}(e_1, e_3) = \left\{\left(\frac{x_1}{[0.2, 0.5]}, \frac{x_2}{[0.3, 0.6]}, \frac{x_3}{[0.3, 0.5]}\right), 0.4\right\}$$

$$(R)^{\wr}(e_2, e_1) = \left\{\left(\frac{x_1}{[0.3, 0.5]}, \frac{x_2}{[0.4, 0.8]}, \frac{x_3}{[0.3, 0.8]}\right), 0.5\right\}$$

$$(R)^{\wr}(e_2, e_2) = \left\{\left(\frac{x_1}{[0.5, 0.6]}, \frac{x_2}{[0.4, 0.8]}, \frac{x_3}{[0.3, 0.8]}\right), 0.6\right\}$$

$$(R)^{\wr}(e_2, e_3) = \left\{\left(\frac{x_1}{[0.2, 0.5]}, \frac{x_2}{[0.4, 0.8]}, \frac{x_3}{[0.3, 0.8]}\right), 0.5\right\}$$

$$(R)^{\wr}(e_3, e_1) = \left\{\left(\frac{x_1}{[0.3, 05]}, \frac{x_2}{[0.3, 0.5]}, \frac{x_3}{[0.2, 0.4]}\right), 0.6\right\}$$

$$(R)^{\wr}(e_3, e_2) = \left\{\left(\frac{x_1}{[0.5, 0.6]}, \frac{x_2}{[0.4, 0.5]}, \frac{x_3}{[0.3, 0.6]}\right), 0.6\right\}$$

$$(R)^{\wr}(e_3, e_3) = \left\{\left(\frac{x_1}{[03, 0.5]}, \frac{x_2}{[0.3, 0.6]}, \frac{x_3}{[0.3, 0.4]}\right), 0.6\right\}$$

	x_1	x_2	x_3	μ
(e_1, e_1)	$[0.3, 0.5]$	$[0.3, 0.5]$	$[0.2, 0.5]$	0.3
(e_1, e_2)	$[0.5, 0.6]$	$[0.4, 0.5]$	$[0.3, 0.6]$	0.6
(e_1, e_3)	$[0.2, 0.5]$	$[0.3, 0.6]$	$[0.3, 0.5]$	0.4
(e_2, e_1)	$[0.3, 0.5]$	$[0.4, 0.8]$	$[0.3, 0.8]$	0.5
(e_2, e_2)	$[0.5, 0.6]$	$[0.4, 0.8]$	$[0.3, 0.8]$	0.6
(e_2, e_3)	$[0.2, 0.5]$	$[0.4, 0.8]$	$[0.3, 0.8]$	0.5
(e_3, e_1)	$[0.3, 05]$	$[0.3, 0.5]$	$[0.2, 0.4]$	0.6
(e_3, e_1)	$[0.5, 0.6]$	$[0.4, 0.5]$	$[0.3, 0.6]$	0.6
(e_3, e_2)	$[0.3, 0.5]$	$[0.3, 0.6]$	$[0.3, 0.4]$	0.6

Table 3: Interval indeterminacy membership function

	x_1	x_2	x_3	μ
(e_1, e_1)	0.4	0.4	0.35	0.3
(e_1, e_2)	0.55	0.45	0.45	0.6
(e_1, e_3)	0.35	0.45	0.4	0.4
(e_2, e_1)	0.4	0.6	0.55	0.5
(e_2, e_2)	0.55	0.6	0.55	0.6
(e_2, e_3)	0.35	0.6	0.55	0.5
(e_3, e_1)	0.4	0.4	0.3	0.6
(e_3, e_1)	0.55	0.45	0.45	0.6
(e_3, e_2)	0.4	0.45	0.35	0.6

Table 4: Induced interval indeterminacy membership function

The value of representation interval indeterminacy membership function$[a, b]$ are obtained using mean value. Then, the scores of interval indeterminacy membership function of x_1, x_2 and$x_3$are:

$$S_{(R)^\wr}(x_1) = (0.4 \times 0.3) + (0.55 \times 0.6) + (0.4 \times 0.6) + (0.55 \times 0.6) = \mathbf{1.02}$$

$$S_{(R)^\wr}(x_2) = (0.4 \times 0.3) + (0.45 \times 0.4) + (0.6 \times 0.5) + (0.6 \times 0.6) + (0.6 \times 0.5) + (0.4 \times 0.60) + (0.45 \times 0.6) + = \mathbf{1.77}$$

$$S_{I(R)^\wr}(x_2) = \mathbf{0}.$$

For the interval indeterminacy membership function components we have:

$$(F^{\mu})^{\Delta} = \begin{pmatrix} ([0.2, 0.3], [0.2, 0.4], [0.2, 0.6]) & (0.2) \\ ([0.2, 0.5], [0.8, 0.9], [0.3, 0.4]) & (0.5) \\ ([0.7, 0.9], [0.3, 0.7], [0.5, 0.7]) & (0.6) \end{pmatrix}$$

$$(G^{\theta})^{\Delta} = \begin{pmatrix} ([0.2, 0.3], [0.5, 0.6], [0.5, 0.7]) & (0.3) \\ ([0.3, 0.4], [0.5, 0.8], [0.6, 0.9]) & (0.6) \\ ([0.1, 0.5], [0.4, 0.5], [0.2, 0.3]) & (0.4) \end{pmatrix}$$

$$(R)^{\Delta} =$$

$$(R)^{\Delta}(e_1, e_1) = \left\{\left(\frac{x_1}{[0.2, 0.3]}, \frac{x_2}{[0.5, 0.6]}, \frac{x_3}{[0.5, 0.7]}\right), 0.3\right\}$$

$$(R)^{\Delta}(e_1, e_2) = \left\{\left(\frac{x_1}{[0.3, 0.4]}, \frac{x_2}{[0.5, 0.8]}, \frac{x_3}{[0.6, 0.9]}\right), 0.6\right\}$$

$$(R)^{\Delta}(e_1, e_3) = \left\{\left(\frac{x_1}{[0.2, 0.5]}, \frac{x_2}{[0.4, 0.5]}, \frac{x_3}{[0.2, 0.6]}\right), 0.4\right\}$$

$$(R)^{\Delta}(e_2, e_1) = \left\{\left(\frac{x_1}{[0.2, 0.5]}, \frac{x_2}{[0.8, 0.9]}, \frac{x_3}{[0.5, 0.7]}\right), 0.5\right\}$$

$$(R)^{\Delta}(e_2, e_2) = \left\{\left(\frac{x_1}{[0.3, 0.5]}, \frac{x_2}{[0.8, 0.9]}, \frac{x_3}{[0.6, 0.9]}\right), 0.6\right\}$$

$$(R)^{\Delta}(e_2, e_3) = \left\{\left(\frac{x_1}{[0.2, 0.5]}, \frac{x_2}{[0.8, 0.9]}, \frac{x_3}{[0.3, 0.4]}\right), 0.5\right\}$$

$$(R)^{\Delta}(e_3, e_1) = \left\{\left(\frac{x_1}{[0.7, 0.9]}, \frac{x_2}{[0.5, 0.7]}, \frac{x_3}{[0.5, 0.7]}\right), 0.6\right\}$$

$$(R)^{\Delta}(e_3, e_2) = \left\{\left(\frac{x_1}{[0.7, 0.9]}, \frac{x_2}{[0.5, 0.8]}, \frac{x_3}{[0.6, 0.9]}\right), 0.6\right\}$$

$$(R)^{\Delta}(e_3, e_3) = \left\{\left(\frac{x_1}{[0.7, 0.9]}, \frac{x_2}{[0.4, 0.7]}, \frac{x_3}{[0.5, 0.7]}\right), 0.6\right\}$$

	x_1	x_2	x_3	μ
(e_1, e_1)	$[0.2, 0.3]$	$[0.5, 0.6]$	$[0.5, 0.7]$	0.3
(e_1, e_2)	$[0.3, 0.4]$	$[0.5, 0.8]$	$[0.6, 0.9]$	0.6
(e_1, e_3)	$[0.2, 0.5]$	$[0.4, 0.5]$	$[0.2, 0.6]$	0.4
(e_2, e_1)	$[0.2, 0.5]$	$[0.8, 0.9]$	$[0.5, 0.7]$	0.5
(e_2, e_2)	$[0.3, 0.5]$	$[0.8, 0.9]$	$[0.6, 0.9]$	0.6
(e_2, e_3)	$[0.2, 0.5]$	$[0.8, 0.9]$	$[0.3, 0.4]$	0.5
(e_3, e_1)	$[0.7, 0.9]$	$[0.5, 0.7]$	$[0.5, 0.7]$	0.6
(e_3, e_1)	$[0.7, 0.9]$	$[0.5, 0.8]$	$[0.6, 0.9]$	0.6
(e_3, e_2)	$[0.7, 0.9]$	$[0.4, 0.7]$	$[0.5, 0.7]$	0.6

Table 5: Interval falsity membership function.

	x_1	x_2	x_3	μ
(e_1, e_1)	0.25	0.55	0.6	0.3
(e_1, e_2)	0.35	0.43	0.75	0.6
(e_1, e_3)	0.35	0.45	0.4	0.4
(e_2, e_1)	0.35	0.85	0.6	0.5
(e_2, e_2)	0.4	0.85	0.75	0.6
(e_2, e_3)	0.35	0.85	0.35	0.5
(e_3, e_1)	0.8	0.6	0.6	0.6
(e_3, e_1)	0.8	0.43	0.75	0.6
(e_3, e_2)	0.8	0.55	0.6	0.6

Table 6: Induced interval falsity membership function.

The value of representation interval falsity membership function $[a, b]$ are obtained using mean value. Then, the scores of interval falsity membership function of x_1, x_2 and $x_3$are:

$$S_{(R)^\triangle}(x_1) = (0.8 \times 0.6) + (0.8 \times 0.6) + (0.8 \times 0.6) = \mathbf{1.44}$$
$$S_{(R)^\triangle}(x_2) = (0.45 \times 0.4) + (0.85 \times 0.5) + (0.85 \times 0.6) + (0.85 \times 0.5) = \mathbf{1.54}$$
$$S_{(R)^\triangle}(x_3) = (0.6 \times 0.3) + (0.75 \times 0.6) = \mathbf{0.63}.$$

Thus, we conclude the problem by calculating final score, using the following formula:

$$\mathrm{S}(x_\mathrm{i}) = \mathrm{S}_{(\mathrm{R})^\star}(x_\mathrm{i}) + \mathrm{S}_{(\mathrm{R})^\wr}(x_\mathrm{i}) - \mathrm{S}_{(\mathrm{R})^\triangle}(x_\mathrm{i})$$

so,

$$\mathrm{S}(x_1) = 0.325 + 1.02 - 1.44 = -0.095$$

$$\mathrm{S}(x_2) = 0.37 + 1.77 - 1.54 = 0.6$$

$$\mathrm{S}(x_3) = 0.685 + 0 - 0.63 = 0.055.$$

Then the optimal selection for Mr.X is the x_2.

Table 1, Table 3 and Table 5 present the truth–membership function, indeterminacy-membership function and falsity-membership function in generalized interval neutrosophic soft set respectively.

5. Conclusions

This paper can be viewed as a continuation of the study of Sahin and Küçük [23]. We extended the generalized neutrosophic soft set to the case of interval valued neutrosophic soft set and also gave the application of GINSS in dealing with some decision making problems. In future work, will study another type of generalized interval neutrosophic soft set where the degree of possibility are interval.

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Soft Neutrosophic Left Almost Semigroup

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In this paper we have extended neutrosophic LA-semigroup, neutrosophic sub LA-semigroup, neutrosophic ideals, neutosophic prime ideals, neutrosophic semiprime ideals, neutrosophic strong irreducible ideals to soft neutrosophic LA-semigroup,soft neutosophic sub LA-semigroup,soft neutrosophic ideals,soft neutrosophic prime ideals,soft neutrosophic semiprime ideals and soft strong irreducible neutrosophic ideals respectively. We have found some new notions related to the strong or pure part of neutrosophy and we give explaination with necessary illustrative examples. We have also given rigorous theorems and propositions. The notion of soft neutrosophic homomorphism is presented at the end.

Keywords: Neutrosophic LA-semigroup, neutrosophic sub LA-semigroup, neutrosophic ideal, soft LA-semigroup, soft LA-subsemigroup, soft ideal, soft neutrosophic LA-semigroup, soft sub-neutrosophic LA-semigroup, soft neutrosophic ideal.

1. INTRODUCTION

In 1995, Florentin Smarandache introduced the concept of neutrosophy. In neutrosophic logiceach proposition is approximated to have the percentage of truth in a subset T, the percentage of indeterminacy in a subset I, and the percentage of falsity in a subset F, so that this neutrosophic logic is called an extension of fuzzy logic. In fact neutrosophic set is the generalization of classical sets,conventional fuzzy set$[1]$, intuitionistic fuzzy set $[2]$ and interval valued fuzzy set$[3]$.This mathematical tool is used to handle problems like imprecise,indeterminacy and inconsistent data etc. By utilizing neutrosophic theory, Vasantha Kandasamy and Florentin Smarandache introduced neutrosophic algebraic structures in$[11]$. Some of them are neutrosophic fields, neutrosophic vector spaces, neutrosophic groups, neutrosophic bigroups, neutrosophic N-groups, neutrosophic semigroups, neutrosophic bisemigroups, neutrosophic N-semigroup, neutrosophic loops, neutrosophic biloops, neutrosophic N-loops, neutrosophic groupoids, and neutrosophic bigroupoids and so on.

Neutrosophic LA-semigroup is already introduced. It is basically a midway algebraic structure between neutrosophic groupoid and commutative neutrosophic semigroups. This is in fact a generalization of neutrosophic semigroup theory. In neutrosophic LA-semigroup we have two basic types of notions and they are traditional notions as well as strong or pure neutrosophic notions. It is also an extension of LA-semigroup and involves the origin of neutralities orindeterminacy factor in LA-semigroup structure. This is a rich structure because of the indeterminacy's presence in all the corresponding notions of LA-semigroup and this property makes the differences between approaches of an LA-semigroup and a neutrosophic LA-semigroup. Molodstov introduced the concept of soft set theory which is free from the problems of parameterization inadequacy.

In his paper [11], he presented the fundamental results of new theory and successfully applied it into several directions such as smoothness of functions, game theory, operations research, Riemann-integration, Perron integration, theory of probability. After getting a high attention of researchers, soft set theory is applied in many fields successfully and so as in the field of LA-semigroup theory. A soft LA-semigroup means the parameterized collection of sub LA-semigroup over an LA-semigroup. It is more general concept than the concept of LA-semigroup.

We have further generalized this idea by adding neutrosophy and extended operations of soft set theory. In this paper we introduced the basic concepts of soft neutrosophic LA-semigroup. In the proceeding section we define soft neutrosophic LA-semigroup and characterized with some of their properties. Soft neutrosophic ideal over a neutrosophic LA-semigroup and soft neutrosophic ideal of a neutrosophic LA-semigroup is given in the further sections and studied some of their related results. In the last section, the concept of soft homomorphism of a soft LA-semigroup is extended to soft neutrosophic homomorphism of soft neutrosophic LA-semigroup.

2. PRELIMINARIES

2.1. Definition 1

Let $(S,*)$ be an LA-semigroup and let $\langle S \cup I \rangle = \{a+bI : a,b \in S\}$. The neutrosophic LA-semigroup is generated by S and I under $*$ denoted as $N(S) = \{\langle S \cup I \rangle, *\}$, where I is called theneutrosophic element with property $I^2 = I$ .For an integer n ,$n+I$ and nI are neutrosophic elements and $0.I = 0 \,.\, I^{-1}$, the inverse of I is not defined and hence does not exist.Similarly we can define neutrosophic RA-semigroup on the same lines.

Definition2 Let $N(S)$ be a neutrosophic LA-semigroup and $N(H)$ be a proper subset of $N(S)$. Then $N(H)$ is called a neutrosophic sub LA-semigroup if $N(H)$ itself is a neutrosophic LA-semigroup under the operation of $N(S)$.

Definition3A neutrosophic sub LA-semigroup $N(H)$ is called strong neutrosophic sub LA-semigroup or pure neutrosophic sub LA-semigroup if all the elements of $N(H)$ are neutrosophic elements.

Definition4 Let $N(S)$ be a neutrosophic LA-semigroup and $N(K)$ be a subset of $N(S)$. Then $N(K)$ is called Left (right) neutrosophic ideal of $N(S)$ if $N(S)N(K) \subseteq N(K)$, { $N(K)N(S) \subseteq N(K)$ }.If $N(K)$ is both left and right neutrosophic ideal, then $N(K)$ is called a two sided neutrosophic ideal or simply a neutrosophic ideal.

Definition5A neutorophic ideal $N(P)$ of a neutrosophic LA-semigroup $N(S)$ with left identity e is called prime neutrosophic ideal if $N(A)N(B) \subseteq N(P)$ implies either $N(A) \subseteq N(P)$ or $N(B) \subseteq N(P)$, where $N(A), N(B)$ are neutrosophic ideals of $N(S)$.

Definition 6A neutrosophic LA-semigroup $N(S)$ is called fully prime neutrosophic LA-semigroup if all of its neutrosophic ideals are prime neutrosophic ideals.

Definition7A neutrosophic ideal $N(P)$ is called semiprime neutrosophic ideal if $N(T).N(T) \subseteq N(P)$ implies $N(T) \subseteq N(P)$ for any neutrosophic ideal $N(T)$ of $N(S)$.

Definition8A neutrosophic LA-semigroup $N(S)$ is called fully semiprime neutrosophic LA-semigroup if every neutrosophic ideal of $N(S)$ is semiprime neutrosophic ideal.

Definition9 A neutrosophic ideal $N(R)$ of a neutrosophic LA-semigroup $N(S)$ is called strongly irreducible neutrosophic ideal if for any neutrosophic ideals $N(H), N(K)$ of $N(S)$ $N(H) \cap N(K) \subseteq N(R)$ implies $N(H) \subseteq N(R)$ or $N(K) \subseteq N(R)$.

Definition 10 Let S, T be two LA-semigroups and $\phi: S \to T$ be a mapping from S to T. Let $N(S)$ and $N(T)$ be the corresponding neutrosophic LA-semigroups of S and T respectively. Let $\theta: N(S) \to N(T)$ be another mapping from $N(S)$ to $N(T)$. Then θ is called neutrosophic homomorphis if ϕ is homomorphism from S to T.

2.2 Soft Sets

Throughout this subsection U refers to an initial universe, E is a set of parameters, $P(U)$ is the power set of U, and $A \subset E$. Molodtsov [12] defined the soft set in the following manner:

Definition 11 A pair (F, A) is called a soft set over U where F is a mapping given by

$F : A \to P(U)$.

In other words, a soft set over U is a parameterized family of subsets of the universe U. For $a \in A$, $F(a)$ may be considered as the set of a -elements of the soft set (F, A), or as the set of a-approximate elements of the soft set.

Definition 12 For two soft sets (F, A) and (H, B) over U, (F, A) is called a soft subset of (H, B) if

1) $A \subseteq B$ and
2) $F(a) \subseteq H(a)$, for all $a \in A$.

This relationship is denoted by $(F, A) \subset (H, B)$. Similarly (F, A) is called a soft superset of (H, B) if (H, B) is a soft subset of (F, A) which is denoted by $(F, A) \supset (H, B)$.

Definition 13 Let (F, A) and (G, B) be two soft sets over a common universe U such that $A \cap B \neq \phi$. Then their restricted intersection is denoted by $(F, A) \cap_R (G, B) = (H, C)$ where (H, C) is defined as $H(c) = F(c) \cap G(c)$ for all $c \in C = A \cap B$.

Definition 14 The extended intersection of two soft sets (F, A) and (G, B) over a common universe U is the soft set (H, C), where $C = A \cup B$, and for all $c \in C$, $H(c)$ is defined as

$$H(c) = \begin{cases} F(c) & \text{if c} \in A - B \\ G(c) & \text{if c} \in B - A \\ F(c) \cap G(c) & \text{if c} \in A \cap B. \end{cases}$$

We write $(F, A) \cap_\varepsilon (G, B) = (H, C)$.

Definition 15 The restricted union of two soft sets (F, A) and (G, B) over a common universe U is the soft set (H, C), where $C = A \cup B$, and for all $c \in C$, $H(c)$ is defined as the soft set $(H, C) = (F, A) \cup_R (G, B)$ where $C = A \cap B$ and $H(c) = F(c) \cup G(c)$ for all $c \in C$.

Definition 16 The extended union of two soft sets (F,A) and (G,B) over a common universe U is the soft set (H,C), where $C = A \cup B$, and for all $c \in C$, $H(c)$ is defined as $H(c) = \begin{cases} F(c) & \text{if } c \in A - B \\ G(c) & \text{if } c \in B - A \\ F(c) \cup G(c) & \text{if } c \in A \cap B. \end{cases}$

We write $(F,A) \cup_{\varepsilon} (G,B) = (H,C)$.

2.3 Soft LA-semigroup

Definition 17 The restricted product (H,C) of two soft sets (F,A) and (G,B) over an LA-semigroup S is dfined as the soft set $(H,C) = (F,A) \odot (G,B)$, where $H(c) = F(c)G(c)$ for all $c \in C = A \cap B$.

Definition 18 A soft set (F,A) over S is called soft LA-semigroup over S if $(F,A) \odot (F,A) \subseteq (F,A)$.

Definition19 A soft LA-semigroup (F,A) over S is said to be soft LA-semigroup with left identity e if $F(a) \neq \phi$ is a sub LA-semigroup with left identiy e ,where e is the left identity of S for all $a \in A$.

Definition 20 Let (F,A) and (G,B) be two soft LA-semigroups over S. Then the operation $*$ for soft sets is
defined as $(F,A) * (G,B) = (H, A \times B)$, where $H(a,b) = F(a)G(b)$ for all $a \in A$, $b \in B$ and $A \times B$ is the Cartesian product of A, B.

Definition 21A soft set (F,A) over an LA-semigroup S is called a soft left (right) ideal over S if $\widetilde{A}_S \odot (F,A) \subseteq (F,A), \left((F,A) \odot \widetilde{A}_S \subseteq (F,A)\right)$ where $\widetilde{A}_S$ is the absolute soft LA-semigroup over S.

Definition 22 Let (F,A) and (G,B) be two soft LA-semigroups over S. Then the Cartesian product is defined
as $(F,A) \times (G,B) = (H, A \times B)$, where $H(a,b) = F(a) \times G(b)$ for all $a \in A$ and $b \in B.$

Definition 23 Let (G,B) be a soft subset of (F,A) over S. Then $(G,B)$is called a soft ideal of (F,A), if $G(b)$is an ideal of $F(b)$ for all $b \in B$.

3. SOFT NEUTROSOPHIC LA-SEMIGROUPS

The definition of soft neutrosophic LA-semigroup isintroduced in this section and we also examine some of their properties.Throughout this section $N(S)$will dnote a neutrosophic LA-semigroup unless stated otherwise.

Definition 24 Let (F,A) be a soft set over $N(S)$. Then (F,A) over $N(S)$ is called soft neutrosophic LA-semigroup if $(F,A) \odot (F,A) \subseteq (F,A)$.

Proposition 1 A soft set (F,A) over $N(S)$ is a soft neutrosophic LA-semigroup if and only if $\phi \neq F(a)$ is a
neutrosophic sub LA-semigroup of $N(S)$ for all $a \in A$.

Example 1 Let $N(S) = \{1,2,3,4,1I,2I,3I,4I\}$ be a neutrosophic LA-semigroup with the following table.

*	1	2	3	4	1I	2I	3I	4I
1	1	4	2	3	1I	4I	2I	3I
2	3	2	4	1	3I	2I	4I	1I
3	4	1	3	2	4I	1I	3I	2I
4	2	3	1	4	2I	3I	1I	4I
1I	1I	4I	2I	3I	1I	4I	2I	3I
2I	3I	2I	4I	1I	3I	2I	4I	1I
3I	4I	1I	3I	2I	4I	1I	3I	2I
4I	2I	3I	1I	4I	2I	3I	1I	4I

Let (F,A) be a soft set over $N(S)$. Then clearly (F,A) is a soft neutrosophic LA-semigroup over $N(S)$,
where

$$F(a_1)=\{1,1I\},\ F(a_2)=\{2,2I\},$$

$$F(a_3)=\{3,3I\},\ F(a_4)=\{4,4I\}.$$

Theorem 1A soft LA-semigroup over an LA-semigroup S is contained in a soft neutrosophic LA-semigroup over
$N(S)$.

Proposition 2 Let (F,A) and (H,B) be two soft neutronsophic LA-semigroup over $N(S)$. Then

1) Their extended intersection $(F,A)\cap_{\varepsilon}(H,B)$ is a soft neutrosophic LA-semigroup over $N(S)$
2) Their restricted intersection $(F,A)\cap_{R}(H,B)$ is also soft neutrosophic LA-semigroup over $N(S)$.

Remark 1 Let (F,A) and (H,B) be two soft neutrosophic LA-semigroup over $N(S)$. Then

1) Their extended union $(F,A)\cup_{\varepsilon}(H,B)$ is not a soft neutrosophic LA-semigroup over $N(S)$.
2) Their restricted union$(F,A)\cup_{R}(H,B)$ is nota soft neutrosophic LA-semigroup over $N(S)$.

Proposition 3 Let (F,A) and (G,B) be two soft neutrosophic LA-semigroup over $N(S)$. Then $(F,A)\wedge(H,B)$ is also soft neutrosophic LA-semigroup if it is non-empty.

Proposition 4 Let (F,A) and (G,B) be two soft neutrosophic LA-semigroup over the neutosophic LA-semigroup $N(S)$. If $A\cap B=\phi$ Then their extended union $(F,A)\cup_{\varepsilon}(G,B)$ is a softneutrosophic LA-semigroup over $N(S)$.

Definition 25 A soft neutrosophic LA-semigroup (F,A) over $N(S)$ is said to be a soft neutosophic LA-semigroup with left identity e if for all $a\in A$, the parameterized set $F(a)$ is aneutrosophic sub LA-semigroup with left identity e where e is the left identity of $N(S)$.

Lemma 1 Let (F,A) be a soft neutrosophic LA-semigroup with left identity e over $N(S)$, then

$$(F,A)\odot(F,A)=(F,A).$$

Proposition 5 Let (F,A) and (G,B) be two soft neutronsophic LA-semigroups over $N(S)$. Then the cartesian product of (F,A) and (G,B) is also soft neutrosophic LA-semigroup over $N(S)$.

Definition 26 A soft neutosophic LA-semigroup (F,A) over $N(S)$ is called soft strong neutrosophic LA-semigroup or soft pure neutrosophic LA-semigroup if each $F(a)$ is a strong or pure neutrosophic sub LA-semigroup for all $a\in A$.

Theorem 2 All soft strong neutrosophic LA-semigroups or pure neutrosophic LA-semigroups are trivially soft neutrosophic LA-semigroups but the converse is not true in general.

Definition 28 Let (F,A) be a soft neutrosophic LA-semigroup over $N(S)$. Then (F,A) is called an absolute soft neutrosophic LA-semigroup if $F(a)=N(S)$ for all $a\in A$. We denote it by $\widetilde{A}_{N(S)}$.

Definition 29 Let (F,A) and (G,B) be two soft neutrosophic LA-semigroup over $N(S)$. Then (G,B) is soft sub neutrosophic LA-semigroup of (F,A), if

1) $B\subseteq A$, and
2) $H(b)$ is a neutrosophic sub LA-semigroup of $F(b)$, for all $b\in B$.

Theorem 3 Every soft LA-semigroup over S is a soft sub neutrosophic LA-semigroup of a soft neutrosophic LA-
semigroup over $N(S)$.

Definition 30 Let (G,B) be a soft sub-neutrosophic LA-smigroup of a soft neutrosophic LA-semigroup (F,A) over $N(S)$. Then (G,B) is said to be soft strong or pure sub-neutrosophic LA-semigroup of (F,A) if each $G(b)$is strong or pure neutrosophic sub LA-semigroup of $F(b)$, for all $b\in B$.

Theorem 4 A soft neutrosophic LA-semigroup (F,A)over $N(S)$ can have soft sub LA-semigroups, soft sub-neutrosophic LA-semigroups and soft strong or pure sub-neutrosophic LA-semigroups.

Theorem 5 If (F,A) over $N(S)$is a soft strong or pureneutrosophic LA-semigroup, then every soft sub-neutrosophic LA-semigroup of (F,A) is a soft strong or pure sub-neutrosophic LA-semigroup.

4. SOFT NEUTROSOPHIC IDEALS OVER A NEUTROSOPHIC LA-SEMIGROUP

Definition 31A soft set (F,A) over a neutrosophic LA-semigroup $N(S)$ is called a soft neutrosophic left (right) ideal over $N(S)$ if $\widetilde{A}_{N(S)}\odot(F,A)\subseteq(F,A),\left((F,A)\odot\widetilde{A}_{N(S)}\subseteq(F,A)\right)$ where $\widetilde{A}_{N(S)}$ is the absolute soft neutrosophic LA-semigroup over $N(S)$. A soft set (F,A) over $N(S)$ is a soft neutrosophic ideal if it is soft neutrosophic left ideal as well as soft neutrosophic right ideal over $N(S)$.

Proposition 5 Let (F,A) be a soft set over $N(S)$. Then (F,A) is a soft neutrosophic ideal over $N(S)$ if and only if $F(a)\neq\phi$ is a neutrosophic ideal of $N(S)$, for all $a\in A$.

Proposition 6 Let (F,A) and (G,B) be two soft neutrosophic ideals over $N(S)$. Then

1) Their restricted union $(F,A)\cup_R(G,B)$ is a soft neutrosophic ideal over $N(S)$.

2) Their restricted intersection $(F,A)\cap_R(G,B)$ is a soft neutrosophic ideal over $N(S)$.
3) Their extended union $(F,A)\cup_\varepsilon(G,B)$ is also a soft neutrosophic ideal over $N(S)$.
4) Their extended intersection $(F,A)\cap_\varepsilon(G,B)$ is asoft neutrosophic ideal over $N(S)$.

Proposition 7 Let (F,A) and (G,B) be two soft neutrosophic ideals over $N(S)$. Then

1. Their OR operation $(F,A)\vee(G,B)$ is a soft neutrosophic ideal over $N(S)$.
2. Their AND operation $(F,A)\wedge(G,B)$ is a soft neutrosophic ideal over $N(S)$.

Proposition 8 Let (F,A) and (G,B) be two soft neutrosophic ideals over $N(S)$, where $N(S)$ is a neutrosophic LA-semigroup with left identity e. Then $(F,A)*(G,B)=(H,A\times B)$ is also a soft neutrosophic ideal over $N(S)$.

Proposition 9 Let (F,A) and (G,B) be two soft neutrosophic ideals over $N(S)$ and $N(T)$.Then the cartesian product $(F,A)\times(G,B)$ is a soft neutrosophic ideal over $N(S)\times N(T)$.

Definition 32A soft neutrosophic ideal (F,A) over $N(S)$ is called soft strong or pure neutrosophic ideal over $N(S)$ if $F(a)$ is a strong or pure neutrosophicideal of $N(S)$, for all $a\in A$.

Theorem 6 All soft strong or pure neutrosophic ideals over $N(S)$ are trivially soft neutrosophic ideals but the converse is not true.

Proposition 8 Let (F,A) and (G,B) be two soft strong or pure neutrosophic ideals over $N(S)$. Then

1) Their restricted union $(F,A)\cup_R(G,B)$ is a soft strong or pure neutrosophic ideal over $N(S)$.
2) Their restricted intersection $(F,A)\cap_R(G,B)$ is a soft strong or pure neutrosophic ideal over $N(S)$.
3) Their extended union $(F,A)\cup_\varepsilon(G,B)$ is also a soft strong or pure neutrosophic ideal over $N(S)$.
4) Their extended intersection $(F,A)\cap_\varepsilon(G,B)$ is a soft strong or pure neutrosophic ideal over $N(S)$.

Proposition 9 Let (F,A) and (G,B) be two soft strong or pure neutrosophic ideals over $N(S)$. Then

1) Their OR operation $(F,A)\vee(G,B)$ is a soft strong or pure neutrosophic ideal over $N(S)$.
2) Their AND operation $(F,A)\wedge(G,B)$ is a soft strong or pure neutrosophic ideal over $N(S)$.

Proposition 10 Let (F,A) and (G,B) be two soft strong or pure neutrosophic ideals over $N(S)$, where $N(S)$ is a neutrosophic LA-semigroup with left identity e. Then $(F,A)*(G,B)=(H,A\times B)$ is also a softstrong or pure neutrosophic ideal over $N(S)$.

Proposition 11 Let (F,A) and (G,B) be two soft strong or pure neutrosophic ideals over $N(S)$ and $N(T)$ respectively. Then the cartesian product $(F,A)\times(G,B)$ is a soft strong or pure neutrosophic ideal over $N(S)\times N(T)$.

5. SOFT NEUTROSOPHIC IDEAL OF SOFT NEUTROSOPHIC LA-SEMIGROUP

Definition 33 Let (F,A) and (G,B) be a soft neutrosophic LA-semigroups over $N(S)$. Then (G,B) is soft neutrosophic ideal of (F,A), if

1) $B \subseteq A$, and
2) $H(b)$ is a neutrosophic ideal of $F(b)$, for all $b \in B$.

Proposition 12 If $(F^{'},A^{'})$ and $(G^{'},B^{'})$ are soft neutrosophic ideals of soft neutrosophic LA-semigroup (F,A) and (G,B) over neutrosophic LA-semigroups $N(S)$ and $N(T)$ respectively.
Then $(F^{'},A^{'})\times(G^{'},B^{'})$ is a soft neutrosophic ideal of soft neutrosophic LA-semigroup $(F,A)\times(G,B)$ over $N(S)\times N(T)$.

Theorem 17 Let (F,A) be a soft neutrosophic LA-semigroup over $N(S)$ and $\{(H_j,B_j): j\in J\}$ be a non-empty family of soft neutrosophic sub LA-semigroups of (F,A). Then

1) $\bigcap_{R\, j\in J}(H_j,B_j)$ is a soft neutrosophic sub LA-semigroup of (F,A).
2) $\wedge_{R\, j\in J}(H_j,B_j)$ is a soft neutrosophic sub LA-semigroup of (F,A).
3) $\bigcup_{\varepsilon\, j\in J}(H_j,B_j)$ is a soft neutrosophic sub LA-semigroup of (F,A) if $B_j \cap B_k = \phi$ for all $j,k \in J$

Theorem 8 Let (F,A) be a soft neutrosophic LA-semigroup over $N(S)$ and $\{(H_j,B_j): j\in J\}$ be a non-empty family of soft neutrosophic ideals of (F,A). Then

1) $\bigcap_{R\, j\in J}(H_j,B_j)$ is a soft neutrosophic ideal of (F,A).
2) $\bigwedge_{j\in J}(H_j,B_j)$ is a soft neutrosophic ideal of (F,A).
3) $\bigcup_{\varepsilon\, j\in J}(H_j,B_j)$ is a soft neutrosophic ideal of (F,A).
4) $\bigvee_{j\in J}(H_j,B_j)$ is a soft neutrosophic ideal of(F,A).

Proposition 13 Let (F,A) be a soft neutrosophic LAsemigroup with left identity e over $N(S)$ and (G,B) be a soft neutrosophic right ideal of (F,A). Then (G,B) is also soft neutrosophic left ideal of (F,A).

Lemma 2 Let (F,A) be a soft neutrosophic LA-semigroup with left identity e over $N(S)$ and (G,B) be a soft neutrosophic right ideal of (F,A). Then $(G,B)\odot(G,B)$ is a soft neutrosophic ideal of (F,A).

Definition 34 A soft neutrosophic ideal (G,B) of a soft neutrosophic LA-semigroup (F,A) is called soft strong or pure neutrosophic ideal if $G(b)$ is a strong or pure neutrosophic ideal of $F(b)$ for all $b \in B$.

Theorem 9 Every soft strong or pure neutrosophic ideal of a soft neutrosophic LA-semigroup is trivially a soft
neutrosophic ideal but the converse is not true.

Definition 35A soft neutrosophic ideal (G,B) of a soft neutrosophic LA-semigroup (F,A) over $N(S)$ is
called soft prime neutrosophic ideal if $(H,C)\odot(J,D)\subseteq(G,B)$ implies either $(H,C)\subseteq(G,B)$ or $(J,D)\subseteq(G,B)$ for soft neutosophic ideals (H,C) and (J,D) of (F,A).

Definition 36A soft neutrosophic LA-semigroup (F,A) over $N(S)$ is called soft fully prime neutrosophic LA-semigroup if all the soft neutrosophic ideals of (F,A) are soft prime neutrosophic ideals.

Definition37 A soft neutrosophic ideal (G,B) of a soft neutrosophic LA-semigroup (F,A) over $N(S)$ is called soft semiprime neutrosophic ideal if $(H,C)\odot(H,C)\subseteq(G,B)$ implies that $(H,C)\subseteq(G,B)$ for any soft neutrosophic ideal $(H,C)$of (F,A).

Definition38A soft neutrosophic LA-semigroup (F,A) over $N(S)$ is called soft fully semiprime neutrosophic LA-semigroup if all the soft neutrosophic ideals of (F,A) are soft semiprime neutrosophic ideals.

Definition39A soft neutrosophic ideal (G,B) of a soft neutrosophic LA-semigroup (F,A) over $N(S)$ is called soft strongly irreducible neutrosophic ideal if $(H,C)\cap_R(J,D)\subseteq(G,B)$ implies either $(H,C)\subseteq(G,B)$ or $(J,D)\subseteq(G,B)$for soft neutrosophic ideals (H,C) and (J,D) of (F,A).

6. SOFT NEUTROSOPHIC HOMOMORPHISM

Definition 40 Let (F,A) and (G,B) be two soft neutrosophic LA-semigroups over $N(S)$ and $N(T)$ respectively. Let $f:N(S)\to N(T)$ and $g:A\to B$ be two mappings. Then $(f,g):(F,A)\to(G,B)$ is called soft neutrosophic homomorphism, if

1) f is a neutrosophic homomorphism from $N(S)$ onto $N(T)$.
2) g is a maping from A onto B.
3) $f(F(a))=G(g(a))$ for all $a\in A$.

If f is a neutrosophic isomorphism from $N(S)$ to $N(T)$ and g is one to one mapping from A onto B. Then (f,g) is called soft neutrosophic isomorphism from (F,A) to (G,B).

CONCLUSION

The literature shows us that soft LA-semigroup is a general framework than LA-semigroup but in this paper we can see that there exista more general structure which we call soft neutrosophic LA-semigroup.A soft LA-semigroup becomes soft sub-neutrosophic LA-semigroup of the corresponding soft neutrosophic LA-semigroup.Soft neutrosophic LA-semigroup points out the indeterminacy factor involved in soft LA-semigroup.Soft neutrosophic LA-semigroup can be characterized by soft neutrosophic ideals over a soft neutrosophic LA-semigroup. We can also extend soft homomorphism of soft LA-semigroup to soft neutrosophic homomorphism of soft neutrosophic LA-semigroup. It is also mentioned here that there is still a space to much more work in this field and explorations of further results has still to be done, this is just a beginning.

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Lower and Upper Soft Interval Valued Neutrosophic Rough Approximations of An IVNSS-Relation

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Abstract: In this paper, we extend the lower and upper soft interval valued intuitionstic fuzzy rough approximations of IVIFS –relations proposed by Anjan et al. to the case of interval valued neutrosophic soft set relation(IVNSS-relation for short)

Keywords: Interval valued neutrosophic soft , Interval valued neutrosophic soft set relation

0. Introduction

This paper is an attempt to extend the concept of interval valued intuitionistic fuzzy soft relation (IVIFSS-relations) introduced by A. Mukherjee et al [45]to IVNSS relation .

The organization of this paper is as follow: In section 2, we briefly present some basic definitions and preliminary results are given which will be used in the rest of the paper. In section 3, relation interval neutrosophic soft relation is presented. In section 4 various type of interval valued neutrosophic soft relations. In section 5, we concludes the paper

1. Preliminaries

Throughout this paper, let U be a universal set and E be the set of all possible parameters under consideration with respect to U, usually, parameters are attributes, characteristics, or properties of objects in U. We now recall some basic notions of neutrosophic set, interval neutrosophic set, soft set, neutrosophic soft set and interval neutrosophic soft set.

Definition 2.1.

Let U be an universe of discourse then the neutrosophic set A is an object having the form A= $\{< x: \mu_{A(x)}, \nu_{A(x)}, \omega_{A(x)} >, x \in U\}$, where the functions $\mu, \nu, \omega : U \to]^-0,1^+[$ define respectively the degree of membership , the degree of indeterminacy, and the degree of non-membership of the element x ∈ X to the set A with the condition.

$$^-0 \leq \mu_{A(x)} + \nu_{A(x)} + \omega_{A(x)} \leq 3^+.$$

From philosophical point of view, the neutrosophic set takes the value from real standard or non-standard subsets of $]^{-}0,1^{+}[$.so instead of $]^{-}0,1^{+}[$ we need to take the interval [0,1] for technical applications, because $]^{-}0,1^{+}[$will be difficult to apply in the real applications such as in scientific and engineering problems.

Definition 2.2. A neutrosophic set A is contained in another neutrosophic set B i.e. $A \subseteq B$ if $\forall x \in U$, $\mu_A(x) \le \mu_B(x)$, $\nu_A(x) \ge \nu_B(x)$, $\omega_A(x) \ge \omega_B(x)$.

Definition 2.3. Let X be a space of points (objects) with generic elements in X denoted by x. An interval valued neutrosophic set (for short IVNS) A in X is characterized by truth-membership function $\mu_A(x)$, indeteminacy-membership function $\nu_A(x)$ and falsity-membership function $\omega_A(x)$. For each point **x in X, we have that** $\mu_A(x), \nu_A(x), \omega_A(x) \in [0,1]$ **.**

For two IVNS , A_{IVNS} ={ <x , $[\mu_A^L(x), \mu_A^U(x)]$, $[\nu_A^L(x), \nu_A^U(x)]$, $[\omega_A^L(x), \omega_A^U(x)]$ > | $x \in X$ }
And B_{IVNS} ={ <x , $[\mu_B^L(x), \mu_B^U(x)]$, $[\nu_B^L(x), \nu_B^U(x)]$, $[\omega_B^L(x), \omega_B^U(x)]$> | $x \in X$ } the two relations are defined as follows:
(1) $A_{IVNS} \subseteq B_{IVNS}$ if and only if $\mu_A^L(x) \le \mu_B^L(x)$,$\mu_A^U(x) \le \mu_B^U(x)$,$\nu_A^L(x) \ge \nu_B^L(x)$, $\omega_A^U(x) \ge \omega_B^U(x)$, $\omega_A^L(x) \ge \omega_B^L(x)$, $\omega_A^U(x) \ge \omega_B^U(x)$
(2) $A_{IVNS} = B_{IVNS}$ if and only if , $\mu_A(x) = \mu_B(x)$, $\nu_A(x) = \nu_B(x)$, $\omega_A(x) = \omega_B(x)$ for any $x \in X$

As an illustration ,let us consider the following example.

Example 2.4. Assume that the universe of discourse U={x_1,x_2,x_3},where x_1 characterizes the capability, x2 characterizes the trustworthiness and x3 indicates the prices of the objects. It may be further assumed that the values of x_1, x_2 and x_3 are in [0,1] and they are obtained from some questionnaires of some experts. The experts may impose their opinion in three components viz. the degree of goodness,
the degree of indeterminacy and that of poorness to explain the characteristics of the objects. Suppose A is an interval neutrosophic set (INS) of U, such that,
A = {< x_1,[0.3 0.4],[0.5 0.6],[0.4 0.5] >,< x_2, ,[0.1 0.2],[0.3 0.4],[0.6 0.7]>,< x_3, [0.2 0.4],[0.4 0.5],[0.4 0.6] >}, where the degree of goodness of capability is 0.3, degree of indeterminacy of capability is 0.5 and degree of falsity of capability is 0.4 etc.

Definition 2.5.
Let U be an initial universe set and E be a set of parameters. Let P(U) denotes the power set of U. Consider a nonempty set A, $A \subset E$. A pair (K, A) is called a soft set over U, where K is a mapping given by $K : A \to P(U)$.
As an illustration, let us consider the following example.

Example 2.6 .
Suppose that U is the set of houses under consideration, say U = {h_1, h_2, . . ., h_5}. Let E be the set of some attributes of such houses, say E = {e_1, e_2, . . ., e_8}, where e_1, e_2, . . ., e_8 stand for the attributes “beautiful”, “costly”, “in the green surroundings’”, “moderate”, respectively.
In this case, to define a soft set means to point out expensive houses, beautiful houses, and so on. For example, the soft set (K,A) that describes the “attractiveness of the houses” in the opinion of a buyer, say Thomas, may be defined like this:
A={e_1,e_2,e_3,e_4,e_5};
K(e_1) = {h_2, h_3, h_5}, K(e_2) = {h_2, h_4}, K(e_3) = {h_1}, K(e_4) = U, K(e_5) = {h_3, h_5}.

Definition 2.7 .
Let U be an initial universe set and $A \subset E$ be a set of parameters. Let IVNS(U) denotes the

set of all interval neutrosophic subsets of U. The collection (K,A) is termed to be the soft interval neutrosophic set over U, where F is a mapping given by K : A → IVNS(U).
The interval neutrosophic soft set defined over an universe is denoted by INSS.
To illustrate let us consider the following example:
Let U be the set of houses under consideration and E is the set of parameters (or qualities). Each parameter is a interval neutrosophic word or sentence involving interval neutrosophic words. Consider E = { beautiful, costly, in the green surroundings, moderate, expensive }. In this case, to define a interval neutrosophic soft set means to point out beautiful houses, costly houses, and so on. Suppose that, there are five houses in the universe U given by, U = $\{h_1,h_2,h_3,h_4,h_5\}$ and the set of parameters A = $\{e_1,e_2,e_3,e_4\}$, where each e_i is a specific criterion for houses:
e_1 stands for ‘beautiful’,
e_2 stands for ‘costly’,
e_3 stands for ‘in the green surroundings’,
e_4 stands for ‘moderate’,
Suppose that,

K(beautiful)={< h_1,[0.5, 0.6], [0.6, 0.7], [0.3, 0.4]>,< h_2,[0.4, 0.5], [0.7 ,0.8], [0.2, 0.3] >, < h_3,[0.6, 0.7],[0.2 ,0.3],[0.3, 0.5] >,< h_4,[0.7 ,0.8],[0.3, 0.4],[0.2, 0.4] >,< h_5,[0.8, 0.4] ,[0.2 ,0.6],[0.3, 0.4] >}.K(costly)={< b_1,[0.5, 0.6], [0.6, 0.7], [0.3, 0.4]>,< h_2,[0.4, 0.5], [0.7 ,0.8], [0.2, 0.3] >, < h_3,[0.6, 0.7],[0.2 ,0.3],[0.3, 0.5] >,< h_4,[0.7 ,0.8],[0.3, 0.4],[0.2, 0.4] >,< h_5,[0.8, 0.4] ,[0.2 ,0.6],[0.3, 0.4] >}.
K(in the green surroundings)= {< h_1,[0.5, 0.6], [0.6, 0.7], [0.3, 0.4]>,< b_2,[0.4, 0.5], [0.7 ,0.8], [0.2, 0.3] >, < h_3,[0.6, 0.7],[0.2 ,0.3],[0.3, 0.5] >,< h_4,[0.7 ,0.8],[0.3, 0.4],[0.2, 0.4] >,< h_5,[0.8, 0.4] ,[0.2 ,0.6],[0.3, 0.4] >}.K(moderate)={< h_1,[0.5, 0.6], [0.6, 0.7], [0.3, 0.4]>,< h_2,[0.4, 0.5], [0.7 ,0.8], [0.2, 0.3] >, < h_3,[0.6, 0.7],[0.2 ,0.3],[0.3, 0.5] >,< h_4,[0.7 ,0.8],[0.3, 0.4],[0.2, 0.4] >,< h_5,[0.8, 0.4] ,[0.2 ,0.6],[0.3, 0.4] >}.

Definition 2.8.
Let U be an initial universe and (F,A) and (G,B) be two interval valued neutrosophic soft set . Then a relation between them is defined as a pair (H, AxB), where H is mapping given by H: AxB→IVNS(U). This is called an interval valued neutrosophic soft sets relation (IVNSS-relation for short).the collection of relations on interval valued neutrosophic soft sets on Ax Bover U is denoted by $\sigma_U(A\text{x } B)$.

Defintion 2.9. Let P, Q ∈ $\sigma_U(Ax\ B)$ and the ordre of their relational matrices are same. Then P ⊆ Q if H $(e_{j,}e_j)$ ⊆ J $(e_{j,}e_j)$ for $(e_{j,}e_j)$ ∈ A x B where P=(H, A x B) and Q = (J, A x B)
Example:
P

U	(e_1, e_2)	(e_1, e_4)	(e_3, e_2)	(e_3, e_4)
h_1	([0.2, 0.3],[0.2, 0.3],[0.4, 0.5])	([0.4, 0.6],[0.7, 0.8],[0.1,0.4])	([0.4, 0.6],[0.7, 0.8],[0.1,0.4])	([0.4, 0.6],[0.7, 0.8],[0.1,0.4])
h_2	([0.6, 0.8],[0.3, 0.4],[0.1,0.7])	([1, 1],[0, 0],[0, 0])	([0.1, 0.5],[0.4, 0.7],[0.5,0.6])	([0.1, 0.5],[0.4, 0.7],[0.5,0.6])
h_3	([0.3, 0.6],[0.2, 0.7],[0.3,0.4])	([0.4, 0.7],[0.1, 0.3],[0.2,0.4])	([1, 1],[0, 0],[0, 0])	([0.4, 0.7],[0.1, 0.3],[0.2,0.4])
h_4	([0.6, 0.7],[0.3, 0.4],[0.2,0.4])	([0.3, 0.4],[0.7, 0.9],[0.1,0.2])	([0.3, 0.4],[0.7, 0.9],[0.1,0.2])	([1, 1],[0, 0],[0, 0])

Q

U	(e_1, e_2)	(e_1, e_4)	(e_3, e_2)	(e_3, e_4)
h_1	([0.3, 0.4],[0, 0],[0, 0])	([0.4, 0.6],[0.7, 0.8],[0.1,0.4])	([0.4, 0.6],[0.7, 0.8],[0.1,0.4])	([0.4, 0.6],[0.7, 0.8],[0.1,0.4])
h_2	([0.6, 0.8],[0.3, 0.4],[0.1,0.7])	([1, 1],[0, 0],[0, 0])	([0.1, 0.5],[0.4, 0.7],[0.5,0.6])	([0.1, 0.5],[0.4, 0.7],[0.5,0.6])

h_3	([0.3, 0.6],[0.2, 0.7],[0.3,0.4])	([0.4, 0.7],[0.1, 0.3],[0.2,0.4])	([1, 1],[0, 0],[0, 0])	([0.4, 0.7],[0.1, 0.3],[0.2,0.4])
h_4	([0.6, 0.7],[0.3, 0.4],[0.2,0.4])	([0.3, 0.4],[0.7, 0.9],[0.1,0.2])	([0.3, 0.4],[0.7, 0.9],[0.1,0.2])	([1, 1],[0, 0],[0, 0])

Definition 2.10**.**
Let U be an initial universe and (F, A) and (G, B) be two interval valued neutrosophic soft sets. Then a null relation between them is denoted by 0_U and is defined as 0_U =(H_O , A xB) where H_O $(e_{i,}e_j)$={<h_k , [0, 0],[1, 1],[1, 1]>; h_k ∈ U} for $(e_{i,}e_j)$ ∈ A xB.
Example. Consider the interval valued neutrosophic soft sets (F, A) and (G, B). Then a null relation between them is given by

U	(e_1 ,e_2)	(e_1 ,e_4)	(e_3 ,e_2)	(e_3 ,e_4)
h_1	([0, 0],[1, 1],[1, 1])	([0, 0],[1, 1],[1, 1])	([0, 0],[1, 1],[1, 1])	([0, 0],[1, 1],[1, 1])
h_2	([0, 0],[1, 1],[1, 1])	([0, 0],[1, 1],[1, 1])	([0, 0],[1, 1],[1, 1])	([0, 0],[1, 1],[1, 1])
h_3	([0, 0],[1, 1],[1, 1])	([0, 0],[1, 1],[1, 1])	([0, 0],[1, 1],[1, 1])	([0, 0],[1, 1],[1, 1])
h_4	([0, 0],[1, 1],[1, 1])	([0, 0],[1, 1],[1, 1])	([0, 0],[1, 1],[1, 1])	([0, 0],[1, 1],[1, 1])

Remark. It can be easily seen that P ∪ 0_U =P and P ∩ 0_U =0_U for any P ∈ $\sigma_U(Ax\ B)$
Definition 2.11.
Let U be an initial universe and (F, A) and (G, B) be two interval valued neutrosophic soft sets. Then an absolute relation between them is denoted by I_U and is defined as I_U =(H_I , A xB) where H_I $(e_{i,}e_j)$={<h_k , [1, 1],[0, 0],[0, 0]>; h_k ∈ U} for $(e_{i,}e_j)$ ∈ A xB.

U	(e_1 ,e_2)	(e_1 ,e_4)	(e_3 ,e_2)	(e_3 ,e_4)
h_1	([1, 1],[0, 0],[0, 0])	([1, 1],[0, 0],[0, 0])	([1, 1],[0, 0],[0, 0])	([1, 1],[0, 0],[0, 0])
h_2	([1, 1],[0, 0],[0, 0])	([1, 1],[0, 0],[0, 0])	([1, 1],[0, 0],[0, 0])	([1, 1],[0, 0],[0, 0])
h_3	([1, 1],[0, 0],[0, 0])	([1, 1],[0, 0],[0, 0])	([1, 1],[0, 0],[0, 0])	([1, 1],[0, 0],[0, 0])
h_4	([1, 1],[0, 0],[0, 0])	([1, 1],[0, 0],[0, 0])	([1, 1],[0, 0],[0, 0])	([1, 1],[0, 0],[0, 0])

Defintion.2.12 Let P ∈ $\sigma_U(Ax\ B)$, P= (H, AxB) ,Q = (J, AxB) and the order of their relational matrices are same.Then we define

(i) P ∪ Q= (H ∘J, AxB) where H∘ J :AxB →IVNS(U) is defined as (H ∘J)($e_{i,}e_j$)= H($e_{i,}e_j$) ∨ J($e_{j,}e_j$) for ($e_{i,}e_j$) ∈ A x B, where ∨ denotes the interval valued neutrosophic union.

(ii) P ∩ Q= (H ■J, AxB) where H■J :AxB →IVNS(U) is defined as (H■J)($e_{i,}e_j$)= H($e_{i,}e_j$) ∧ J($e_{i,}e_j$) for ($e_{j,}e_j$) ∈ A x B, where ∧ denotes the interval valued neutrosophic intersection

(iii) P^c= (~H, AxB) , where ~H :AxB →IVNS(U) is defined as ~H($e_{i,}e_j$)=[H($e_{i,}e_j$)] ^{c} for ($e_{i,}e_j$) ∈ A x B, where c denotes the interval valued neutrosophic complement.

Defintion.2.13.
Let R be an equivalence relation on the universal set U. Then the pair (U, R) is called a Pawlak approximation space. An equivalence class of R containing x will be denoted by $[x]_R$. Now for X ⊆ U, the lower and upper approximation of X with respect to (U, R) are denoted by respectively R ∗X and R* X and are defined by

$R_*X=\{x\in U: [x]_R \subseteq X\}$,
$R^*X=\{ x\in U: [x]_R \cap X \neq\}$.
Now if $R_*X = R^* X$, then X is called definable; otherwise X is called a rough set.

3-Lower and upper soft interval valued neutrosophic rough approximations of an IVNSS-relation

Defntion 3.1 .Let R $\in \sigma_U(A\text{x } A)$ and R=(H, Ax A). Let Θ=(f,B) be an interval valued neutrosophic soft set over U and S= (U, Θ) be the soft interval valued neutrosophic approximation space. Then the lower and upper soft interval valued neutrosophic rough approximations of R with respect to S are denoted by $\mathrm{Lwr_S}$(R)and $\mathrm{Upr_S}$(R) respectively, which are IVNSS- relations over AxB in U given by:

$\mathrm{Lwr_S}$(R)= (J, A xB) and $\mathrm{Upr_S}$(R) =(K, A xB)

J(e_i, e_k**) ={<x,** $[\wedge_{e_j\in A}(\inf \mu_{H(e_i,e_j)}(x) \wedge \inf \mu_{f(e_k)}(x)), \wedge_{e_j\in A}(\sup \mu_{H(e_i,e_j)}(x) \wedge \sup \mu_{f(e_k)}(x))]$,

$[\wedge_{e_j\in A}(\inf \nu_{H(e_i,e_j)}(x) \vee \inf \nu_{f(e_k)}(x)), \wedge_{e_j\in A}(\sup \nu_{H(e_i,e_j)}(x) \vee \sup \nu_{f(e_k)}(x))]$,
$[\wedge_{e_j\in A}(\inf \omega_{H(e_i,e_j)}(x) \vee \inf \omega_{f(e_k)}(x)), \wedge_{e_j\in A}(\sup \omega_{H(e_i,e_j)}(x) \vee \sup \omega_{f(e_k)}(x))]$:x ∈ U}.

K(e_i, e_k**) ={<x,** $[\wedge_{e_i\in A}(\inf \mu_{H(e_i,e_j)}(x) \vee \inf \mu_{f(e_k)}(x)), \wedge_{e_j\in A}(\sup \mu_{H(e_i,e_j)}(x) \vee \sup \mu_{f(e_k)}(x))]$,

$[\wedge_{e_j\in A}(\inf \nu_{H(e_i,e_j)}(x) \wedge \inf \nu_{f(e_k)}(x)), \wedge_{e_j\in A}(\sup \nu_{H(e_i,e_j)}(x) \wedge \sup \nu_{f(e_k)}(x))]$,
$[\wedge_{e_j\in A}(\inf \omega_{H(e_i,e_j)}(x) \wedge \inf \omega_{f(e_k)}(x)), \wedge_{e_j\in A}(\sup \omega_{H(e_i,e_j)}(x) \wedge \sup \omega_{f(e_k)}(x))]$:x ∈ U}.

For $e_i \in A$, $e_K \in B$

Theorem 3.2. Let be an interval valued neutrosophic soft over U and S = (U,Θ) be the soft approximation space. Let R_1 , $R_2 \in \sigma_U(A\text{x } A)$ and R_1=(G,Ax A) and R_2=(H,Ax A).Then

(i) $\mathrm{Lwr_S}(0_U)= 0_U$
(ii) $\mathrm{Lwr_S}(1_U)= 1_U$
(iii) $\boldsymbol{R_1} \subseteq \boldsymbol{R_2} \Longrightarrow \mathrm{Lwr_S}(\boldsymbol{R_1}) \subseteq \mathrm{Lwr_S}(\boldsymbol{R_2})$
(iv) $\boldsymbol{R_1} \subseteq \boldsymbol{R_2} \Longrightarrow \mathrm{Upr_S}(\boldsymbol{R_1}) \subseteq \mathrm{Upr_S}(\boldsymbol{R_2}$
(v) $\mathrm{Lwr_S}(\boldsymbol{R_1} \cap \boldsymbol{R_2}) \subseteq \mathrm{Lwr_S}(\boldsymbol{R_1}) \cap \mathrm{Lwr_S}(\boldsymbol{R_2})$
(vi) $\mathrm{Upr_S}(\boldsymbol{R_1} \cap \boldsymbol{R_2}) \subseteq \mathrm{Upr_S}(\boldsymbol{R_1}) \cap \mathrm{Upr_S}(\boldsymbol{R_2})$
(vii) $\mathrm{Lwr_S}(\boldsymbol{R_1}) \cup \mathrm{Lwr_S}(\boldsymbol{R_2}) \subseteq \mathrm{Lwr_S}(\boldsymbol{R_1} \cup \boldsymbol{R_2})$
(viii) $\mathrm{Upr_S}(\boldsymbol{R_1}) \cup \mathrm{Upr_S}(\boldsymbol{R_2}) \subseteq \mathrm{Upr_S}(\boldsymbol{R_1} \cup \boldsymbol{R_2})$

Proof. (i) –(iv) are straight forward.
Let Lwrs($R_1 \cap R_2$) =(S, Ax B).Then for $(\boldsymbol{e_i}, \boldsymbol{e_k}) \in$ A xB , we have

S($\boldsymbol{e_i}, \boldsymbol{e_k}$) ={<x, $[\wedge_{e_j\in A}(\inf \mu_{G\circ H(e_i,e_j)}(x) \wedge \inf \mu_{f(e_k)}(x)), \wedge_{e_j\in A}(\sup \mu_{G\circ H(e_i,e_j)}(x) \wedge \sup \mu_{f(e_k)}(x))]$,
$[\wedge_{e_j\in A}(\inf \nu_{G\circ H(e_i,e_j)}(x) \vee \inf \nu_{f(e_k)}(x)), \wedge_{e_j\in A}(\sup \nu_{G\circ H(e_i,e_j)}(x) \vee \sup \nu_{f(e_k)}(x))]$,

$[\wedge_{e_j\in A}(\inf \omega_{G\circ H(e_i,e_j)}(x) \vee \inf \omega_{f(e_k)}(x)), \wedge_{e_j\in A}(\sup \omega_{G\circ H(e_i,e_j)}(x) \vee \sup \omega_{f(e_k)}(x))]$:x ∈ U}

$=\{<x, [\wedge_{e_j \in A}(\min(\inf \mu_{G(e_i, e_j)}(x), \inf \mu_{H(e_i, e_j)}(x)) \wedge \inf \mu_{f(e_k)}(x))$
$, \wedge_{e_j \in A}(\min(\sup \mu_{G(e_i, e_j)}(x), \sup \mu_{H(e_i, e_j)}(x)) \wedge \sup \mu_{f(e_k)}(x))],$

$[\wedge_{e_j \in A}(\max(\inf \nu_{G(e_i, e_j)}(x), \inf \nu_{H(e_i, e_j)}(x)) \vee \inf \nu_{f(e_k)}(x))$
$, \wedge_{e_j \in A}(\max(\sup \nu_{G(e_i, e_j)}(x), \sup \nu_{H(e_i, e_j)}(x)) \vee \sup \nu_{f(e_k)}(x))],$

$[\wedge_{e_j \in A}(\max(\inf \omega_{G(e_i, e_j)}(x), \inf \omega_{H(e_i, e_j)}(x)) \vee \inf \omega_{f(e_k)}(x))$
$, \wedge_{e_j \in A}(\max(\sup \omega_{G(e_i, e_j)}(x), \sup \omega_{H(e_i, e_j)}(x)) \vee \sup \omega_{f(e_k)}(x))] : x \in U\}$

Also for $\mathrm{Lwr}_S(\boldsymbol{R_1}) \cap \mathrm{Lwr}_S(\boldsymbol{R_2})$ =(Z,A x B) and $(\boldsymbol{e_i}, \boldsymbol{e_K}) \in A \times B$,we have ,
$Z(\boldsymbol{e_i}, \boldsymbol{e_K}) = \{<x, [\,\mathrm{Min}(\wedge_{e_j \in A}(\inf \mu_{G(e_i, e_j)}(x) \wedge \inf \mu_{f(e_k)}(x)), \wedge_{e_j \in A}(\inf \mu_{H(e_i, e_j)}(x) \wedge \inf \mu_{f(e_k)}(x))), \mathrm{Min}(\wedge_{e_j \in A}(\sup \mu_{G(e_i, e_j)}(x) \wedge \sup \mu_{f(e_k)}(x)), \wedge_{e_j \in A}(\sup \mu_{H(e_i, e_j)}(x) \wedge \sup \mu_{f(e_k)}(x)))],$
$[\mathrm{Max}(\wedge_{e_j \in A}(\inf \nu_{G(e_i, e_j)}(x) \vee \inf \nu_{f(e_k)}(x)), \wedge_{e_j \in A}(\inf \nu_{H(e_i, e_j)}(x) \vee \inf \nu_{f(e_k)}(x))),$
$\mathrm{Max}(\wedge_{e_j \in A}(\sup \nu_{G(e_i, e_j)}(x) \vee \sup \nu_{f(e_k)}(x)), \wedge_{e_j \in A}(\sup \nu_{H(e_i, e_j)}(x) \vee \sup \nu_{f(e_k)}(x)))],$

$[\mathrm{Max}(\wedge_{e_j \in A}(\inf \omega_{G(e_i, e_j)}(x) \vee \inf \omega_{f(e_k)}(x)), \wedge_{e_j \in A}(\inf \omega_{H(e_i, e_j)}(x) \vee \inf \omega_{f(e_k)}(x))),$
$\mathrm{Max}(\wedge_{e_j \in A}(\sup \omega_{G(e_i, e_j)}(x) \vee \sup \omega_{f(e_k)}(x)), \wedge_{e_j \in A}(\sup \omega_{H(e_i, e_j)}(x) \vee \sup \omega_{f(e_k)}(x)))] : x \in U\}$

Now since $\min(\inf \mu_{G(e_i, e_j)}, \inf \mu_{H(e_i, e_j)}(x)) \leq \inf \mu_{G(e_i, e_j)}(x)$ and
$\min(\inf \mu_{G(e_i, e_j)}, \inf \mu_{H(e_i, e_j)}(x)) \leq \inf \mu_{H(e_i, e_j)}(x)$ we have
$\wedge_{e_j \in A}(\min(\inf \mu_{G(e_i, e_j)}(x), \inf \mu_{H(e_i, e_j)}(x)) \wedge \inf \mu_{f(e_k)}(x)) \leq \mathrm{Min}(\wedge_{e_j \in A}(\inf \mu_{G(e_i, e_j)}(x) \wedge \inf \mu_{f(e_k)}(x)), \wedge_{e_j \in A}(\inf \mu_{H(e_i, e_j)}(x) \wedge \inf \mu_{f(e_k)}(x)))$.
Similarly we can get
$\wedge_{e_j \in A}(\min(\sup \mu_{G(e_i, e_j)}(x), \sup \mu_{H(e_i, e_j)}(x)) \wedge \sup \mu_{f(e_k)}(x)) \leq \mathrm{Min}(\wedge_{e_j \in A}(\sup \mu_{G(e_i, e_j)}(x) \wedge \sup \mu_{f(e_k)}(x)), \wedge_{e_j \in A}(\sup \mu_{H(e_i, e_j)}(x) \wedge \sup \mu_{f(e_k)}(x)))$.

Again as $\max(\inf \nu_{G(e_i, e_j)}, \inf \nu_{H(e_i, e_j)}(x)) \geq \inf \nu_{G(e_i, e_j)}(x)$,and
$\max(\inf \nu_{G(e_i, e_j)}, \inf \nu_{H(e_i, e_j)}(x)) \geq \inf \nu_{H(e_i, e_j)}(x)$

we have

$\wedge_{e_j \in A}(\max(\inf \nu_{G(e_i, e_j)}(x), \inf \nu_{H(e_i, e_j)}(x)) \vee \inf \nu_{f(e_k)}(x)) \geq \mathrm{Max}(\wedge_{e_j \in A}(\inf \nu_{G(e_i, e_j)}(x) \vee \inf \nu_{f(e_k)}(x)), \wedge_{e_j \in A}(\inf \nu_{H(e_i, e_j)}(x) \vee \inf \nu_{f(e_k)}(x)))$.
Similarly we can get

$\wedge_{e_j \in A}(\max(\sup \nu_{G(e_i, e_j)}(x), \sup \nu_{H(e_i, e_j)}(x)) \vee \sup \nu_{f(e_k)}(x)) \geq \mathrm{Max}(\wedge_{e_j \in A}(\sup \nu_{G(e_i, e_j)}(x) \vee \sup \nu_{f(e_k)}(x)), \wedge_{e_j \in A}(\sup \nu_{H(e_i, e_j)}(x) \vee \sup \nu_{f(e_k)}(x)))$.
Again as $\max(\inf \omega_{G(e_i, e_j)}, \inf \omega_{H(e_i, e_j)}(x)) \geq \inf \omega_{G(e_i, e_j)}(x)$,and
$\max(\inf \omega_{G(e_i, e_j)}, \inf \omega_{H(e_i, e_j)}(x)) \geq \inf \omega_{H(e_i, e_j)}(x)$

we have

$\wedge_{e_j\in A}(\max(\inf\omega_{\mathbf{G}(e_i,e_j)}(x),\inf\omega_{\mathbf{H}(e_i,e_j)}(x))\vee\inf\omega_{\mathbf{f}(e_k)}(x))\geq \text{Max}\,(\wedge_{e_j\in A}(\inf\omega_{\mathbf{G}(e_i,e_j)}(x)\vee\inf\omega_{\mathbf{f}(e_k)}(x)),\ \wedge_{e_j\in A}(\inf\omega_{\mathbf{H}(e_i,e_j)}(x)\vee\inf\omega_{\mathbf{f}(e_k)}(x)))$.

Similarly we can get

$\wedge_{e_j\in A}(\max(\sup\omega_{\mathbf{G}(e_i,e_j)}(x),\sup\omega_{\mathbf{H}(e_i,e_j)}(x))\vee\sup\omega_{\mathbf{f}(e_k)}(x))\geq \text{Max}\,(\wedge_{e_j\in A}(\sup\omega_{\mathbf{G}(e_i,e_j)}(x)\vee\sup\omega_{\mathbf{f}(e_k)}(x)),\ \wedge_{e_j\in A}(\sup\omega_{\mathbf{H}(e_i,e_j)}(x)\vee\sup\omega_{\mathbf{f}(e_k)}(x)))$.

Consequently, $\text{Lwr}_S\,(\boldsymbol{R_1}\cap\boldsymbol{R_2})\subseteq\text{Lwr}_S\,(\boldsymbol{R_1})\cap\text{Lwr}_S\,(\boldsymbol{R_2})$

(vi) Proof is similar to (v)

(vii) Let $\text{Lwr}_S\,(\boldsymbol{R_1}\cup\boldsymbol{R_2})$ **=(S, A xB).Then for** $(\boldsymbol{e_i},\boldsymbol{e_k})\in$ A xB , we have

$S(\boldsymbol{e_i},\boldsymbol{e_k})=\{<x,\ [\wedge_{e_j\in A}(\inf\mu_{\mathbf{G}\blacksquare\mathbf{H}(e_i,e_j)}(x)\wedge\inf\mu_{\mathbf{f}(e_k)}(x)),\wedge_{e_j\in A}(\sup\mu_{\mathbf{G}\blacksquare\mathbf{H}(e_i,e_j)}(x)\wedge\sup\mu_{\mathbf{f}(e_k)}(x))],$

$[\wedge_{e_j\in A}(\inf\nu_{\mathbf{G}\blacksquare\mathbf{H}(e_i,e_j)}(x)\vee\inf\nu_{\mathbf{f}(e_k)}(x)),\wedge_{e_j\in A}(\inf\nu_{\mathbf{G}\blacksquare\mathbf{H}(e_i,e_j)}(x)\vee\inf\nu_{\mathbf{f}(e_k)}(x))],$

$[\wedge_{e_j\in A}(\inf\omega_{\mathbf{G}\blacksquare\mathbf{H}(e_i,e_j)}(x)\vee\inf\omega_{\mathbf{f}(e_k)}(x)),\wedge_{e_j\in A}(\inf\omega_{\mathbf{G}\blacksquare\mathbf{H}(e_i,e_j)}(x)\vee\inf\omega_{\mathbf{f}(e_k)}(x))]:x\in U\}$

$=\{<x,\ [\wedge_{e_j\in A}(\max(\inf\mu_{\mathbf{G}(e_i,e_j)}(x),\inf\mu_{\mathbf{H}(e_i,e_j)}(x))\wedge\inf\mu_{\mathbf{f}(e_k)}(x)),\wedge_{e_j\in A}(\max(\sup\mu_{\mathbf{G}(e_i,e_j)}(x),\sup\mu_{\mathbf{H}(e_i,e_j)}(x))\wedge\sup\mu_{\mathbf{f}(e_k)}(x))],$

$[\wedge_{e_j\in A}(\min(\inf\nu_{\mathbf{G}(e_i,e_j)}(x),\inf\nu_{\mathbf{H}(e_i,e_j)}(x))\vee\inf\nu_{\mathbf{f}(e_k)}(x)),\wedge_{e_j\in A}(\min(\sup\nu_{\mathbf{G}(e_i,e_j)}(x),\sup\nu_{\mathbf{H}(e_i,e_j)}(x))\vee\sup\nu_{\mathbf{f}(e_k)}(x))],$

$[\wedge_{e_j\in A}(\min(\inf\omega_{\mathbf{G}(e_i,e_j)}(x),\inf\omega_{\mathbf{H}(e_i,e_j)}(x))\vee\inf\omega_{\mathbf{f}(e_k)}(x)),\wedge_{e_j\in A}(\min(\sup\omega_{\mathbf{G}(e_i,e_j)}(x),\sup\omega_{\mathbf{H}(e_i,e_j)}(x))\vee\sup\omega_{\mathbf{f}(e_k)}(x))]:x\in U\}$

Also for $\text{Lwrs}(\boldsymbol{R_1})\cup\text{Lwrs}(\boldsymbol{R_2})$ **= (Z, AxB) and** $(\boldsymbol{e_i},\boldsymbol{e_k})\in$ A xB ,we have ,

$Z\,(\boldsymbol{e_i},\boldsymbol{e_K})=\{<x,\ [\,\text{Max}\,(\wedge_{e_j\in A}(\inf\mu_{\mathbf{G}(e_i,e_j)}(x)\wedge\inf\mu_{\mathbf{f}(e_k)}(x)),\ \wedge_{e_j\in A}(\inf\mu_{\mathbf{H}(e_i,e_j)}(x)\wedge\inf\mu_{\mathbf{f}(e_k)}(x))),\ \text{Max}(\wedge_{e_j\in A}(\sup\mu_{\mathbf{G}(e_i,e_j)}(x)\wedge\sup\mu_{\mathbf{f}(e_k)}(x)),\ \wedge_{e_j\in A}(\sup\mu_{\mathbf{H}(e_i,e_j)}(x)\wedge\sup\mu_{\mathbf{f}(e_k)}(x)))],$

$[\text{Min}\,(\wedge_{e_j\in A}(\inf\nu_{\mathbf{G}(e_i,e_j)}(x)\vee\inf\nu_{\mathbf{f}(e_k)}(x)),\ \wedge_{e_j\in A}(\inf\nu_{\mathbf{H}(e_i,e_j)}(x)\vee\inf\nu_{\mathbf{f}(e_k)}(x))),\ \text{Min}(\wedge_{e_j\in A}(\sup\nu_{\mathbf{G}(e_i,e_j)}(x)\vee\sup\nu_{\mathbf{f}(e_k)}(x)),\ \wedge_{e_j\in A}(\sup\nu_{\mathbf{H}(e_i,e_j)}(x)\vee\sup\nu_{\mathbf{f}(e_k)}(x)))],$

$[\text{Min}\,(\wedge_{e_j\in A}(\inf\omega_{\mathbf{G}(e_i,e_j)}(x)\vee\inf\omega_{\mathbf{f}(e_k)}(x)),\ \wedge_{e_j\in A}(\inf\omega_{\mathbf{H}(e_i,e_j)}(x)\vee\inf\omega_{\mathbf{f}(e_k)}(x))),\ \text{Min}(\wedge_{e_j\in A}(\sup\omega_{\mathbf{G}(e_i,e_j)}(x)\vee\sup\omega_{\mathbf{f}(e_k)}(x)),\ \wedge_{e_j\in A}(\sup\omega_{\mathbf{H}(e_i,e_j)}(x)\vee\sup\omega_{\mathbf{f}(e_k)}(x)))]:x\in U\}$

Now since $\max(\inf\mu_{\mathbf{G}(e_i,e_j)},\inf\mu_{\mathbf{H}(e_i,e_j)}(x))\geq\inf\mu_{\mathbf{G}(e_i,e_j)}(x)$ and $\max(\inf\mu_{\mathbf{G}(e_i,e_j)},\inf\mu_{\mathbf{H}(e_i,e_j)}(x))\geq\inf\mu_{\mathbf{H}(e_i,e_j)}(x)$ we have

$\wedge_{e_j\in A}(\max(\inf\mu_{\mathbf{G}(e_i,e_j)}(x),\inf\mu_{\mathbf{H}(e_i,e_j)}(x))\wedge\inf\mu_{\mathbf{f}(e_k)}(x))\geq\max\,(\wedge_{e_j\in A}(\inf\mu_{\mathbf{G}(e_i,e_j)}(x)\wedge\inf\mu_{\mathbf{f}(e_k)}(x)),\ \wedge_{e_j\in A}(\inf\mu_{\mathbf{H}(e_i,e_j)}(x)\wedge\inf\mu_{\mathbf{f}(e_k)}(x)))$.

Similarly we can get

$\wedge_{e_j\in A}(\max(\sup\mu_{\mathbf{G}(e_i,e_j)}(x),\sup\mu_{\mathbf{H}(e_i,e_j)}(x))\wedge\sup\mu_{\mathbf{f}(e_k)}(x))\geq\max(\wedge_{e_j\in A}(\sup\mu_{\mathbf{G}(e_i,e_j)}(x)\wedge\sup\mu_{\mathbf{f}(e_k)}(x)),\ \wedge_{e_j\in A}(\sup\mu_{\mathbf{H}(e_i,e_j)}(x)\wedge\sup\mu_{\mathbf{f}(e_k)}(x)))$.

Again as $\min(\inf \nu_{\mathbf{G}(e_i, e_j)}, \inf \nu_{\mathbf{H}(e_i, e_j)}(x)) \leq \inf \nu_{\mathbf{G}(e_i, e_j)}(x)$,and $\min(\inf \nu_{\mathbf{G}(e_i, e_j)}, \inf \nu_{\mathbf{H}(e_i, e_j)}(x)) \leq \inf \nu_{\mathbf{H}(e_i, e_j)}(x)$

we have

$\wedge_{e_j \in A}(\min(\inf \nu_{\mathbf{G}(e_i, e_j)}(x), \inf \nu_{\mathbf{H}(e_i, e_j)}(x)) \vee \inf \nu_{\mathbf{f}(e_k)}(x)) \leq \mathrm{Min}\,(\wedge_{e_j \in A}(\inf \nu_{\mathbf{G}(e_i, e_j)}(x) \vee \inf \nu_{\mathbf{f}(e_k)}(x)),\ \wedge_{e_j \in A}(\inf \nu_{\mathbf{H}(e_i, e_j)}(x) \vee \inf \nu_{\mathbf{f}(e_k)}(x)))$.

Similarly we can get

$\wedge_{e_j \in A}(\min(\sup \nu_{\mathbf{G}(e_i, e_j)}(x), \sup \nu_{\mathbf{H}(e_i, e_j)}(x)) \vee \sup \nu_{\mathbf{f}(e_k)}(x)) \leq \mathrm{Min}\,(\wedge_{e_j \in A}(\sup \nu_{\mathbf{G}(e_i, e_j)}(x) \vee \sup \nu_{\mathbf{f}(e_k)}(x)),\ \wedge_{e_j \in A}(\sup \nu_{\mathbf{H}(e_i, e_j)}(x) \vee \sup \nu_{\mathbf{f}(e_k)}(x)))$.

Again as $\min(\inf \omega_{\mathbf{G}(e_i, e_j)}, \inf \omega_{\mathbf{H}(e_i, e_j)}(x)) \leq \inf \omega_{\mathbf{G}(e_i, e_j)}(x)$,and $\min(\inf \omega_{\mathbf{G}(e_i, e_j)}, \inf \omega_{\mathbf{H}(e_i, e_j)}(x)) \leq \inf \omega_{\mathbf{H}(e_i, e_j)}(x)$

we have

$\wedge_{e_j \in A}(\min(\inf \omega_{\mathbf{G}(e_i, e_j)}(x), \inf \omega_{\mathbf{H}(e_i, e_j)}(x)) \vee \inf \omega_{\mathbf{f}(e_k)}(x)) \leq \mathrm{Min}\,(\wedge_{e_j \in A}(\inf \omega_{\mathbf{G}(e_i, e_j)}(x) \vee \inf \omega_{\mathbf{f}(e_k)}(x)),\ \wedge_{e_j \in A}(\inf \omega_{\mathbf{H}(e_i, e_j)}(x) \vee \inf \omega_{\mathbf{f}(e_k)}(x)))$.

Similarly we can get

$\wedge_{e_j \in A}(\min(\sup \omega_{\mathbf{G}(e_i, e_j)}(x), \sup \omega_{\mathbf{H}(e_i, e_j)}(x)) \vee \sup \omega_{\mathbf{f}(e_k)}(x)) \leq \mathrm{Min}\,(\wedge_{e_j \in A}(\sup \omega_{\mathbf{G}(e_i, e_j)}(x) \vee \sup \omega_{\mathbf{f}(e_k)}(x)),\ \wedge_{e_j \in A}(\sup \omega_{\mathbf{H}(e_i, e_j)}(x) \vee \sup \omega_{\mathbf{f}(e_k)}(x)))$.

Consequently $\mathrm{Lwr}_S(\boldsymbol{R_1}) \cup \mathrm{Lwr}_S(\boldsymbol{R_2}) \subseteq \mathrm{Lwrs}(\boldsymbol{R_1} \cap \boldsymbol{R_2})$

(vii) Proof is similar to (vii).

Conclusion

In the present paper we extend the concept of Lower and upper soft interval valued intuitionstic fuzzy rough approximations of an IVIFSS-relation to the case IVNSS and investigated some of their properties.

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Rough Neutrosophic Sets

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Abstract. Both neutrosophic sets theory and rough sets theory are emerging as power-ful tool for managing uncertainty, indeterminate, incomplete and imprecise information. In this paper we develop an hybrid structure called *rough neutrosophic sets* and studied their properties.

Keywords: Rough set, rough neutrosophic set.

1. Introduction

In 1982, Pawlak [1] introduced the concept of rough set (RS), as a formal tool for modeling and processing incomplete information in information systems. There are two basic elements in rough set theory, crisp set and equivalence relation, which constitute the mathematical basis of RSs. The basic idea of rough set is based upon the approximation of sets by a pair of sets known as the lower approximation and the upper approximation of a set. Here, the lower and upper approximation operators are based on equivalence relation. After Pawlak, there has been many models built upon different aspect, i.e., universe, relations, object, operators by many scholars [2], [3], [4], [5], [6], [7]. Various notions that combine rough sets and fuzzy sets, vague set and intuitionistic fuzzy sets are introduced, such as rough fuzzy sets, fuzzy rough sets, generalize fuzzy rough, intuitionistic fuzzy rough sets, rough intuitionistic fuzzy sets, rough vagues sets. The theory of rough sets is based upon the classification mechanism, from which the classification can be viewed as an equivalence relation and knowledge blocks induced by it be a partition on universe.

One of the interesting generalizations of the theory of fuzzy sets and intuitionistic fuzzy sets is the theory of neutrosophic sets introduced by F. Smarandache [8], [9]. Neutrosophic sets described by three functions: a membership function indeterminacy function and a non-membership function that are independently related. The theory of neutrosophic set have achieved great success in various areas such as medical diagnosis [10], database [11], [12], topology [13], image processing [14], [15], [16], and decision making problem [17]. While the neutrosophic set is a powerful tool to deal with indeterminate and inconsistent data, the theory of rough sets is a powerful mathematical tool to deal with incompleteness.

Neutrosophic sets and rough sets are two different topics, none conflicts the other. Recently many researchers applied the notion of neutrosophic sets to relations, group theory, ring theory, soft set theory [23], [24], [25], [26], [27], [28], [29], [30], [31], [32] and so on. The main objective of this study was to introduce a new hybrid intelligent structure called *rough neutrosophic sets*. The significance of introducing hybrid set structures is that the computational techniques based on any one of theses structures alone will not always yield the best results but a fusion of two or more of them can often give better results.

The rest of this paper is organized as follows. Some preliminary concepts required in our work are briefly recalled in Section 2. In Section 3, the concept of rough neutrosophic sets is investigated. Section 4 concludes the paper.

2. Preliminaries

In this section we present some preliminaries which will be useful to our work in the next section. For more details the reader may refer to [1], [8], [9].

Definition 2.1. [8] Let X be an universe of discourse, with a generic element in X denoted by x, the neutrosophic (NS) set is an object having the form

$$A = \{\langle x : \mu_A(x), \nu_A(x), \omega_A(x)\rangle \,,\ x \in X\},$$

where the functions $\mu, \nu, \omega : X \to]^-0, 1^+[$ define respectively the degree of membership (or Truth), the degree of indeterminacy, and the degree of non-membership (or Falsehood) of the element $x \in X$ to the set A with the condition

$$^-0 \leq \mu_A(x) + \nu_A(x) + \omega_A(x) \leq 3^+. \tag{1}$$

From a philosophical point of view, the neutrosophic set takes the value from real standard or non-standard subsets of $]^-0, 1^+[$. So, instead of $]^-0, 1^+[$ we need to take the interval $[0, 1]$ for technical applications, because $]^-0, 1^+[$ will be difficult to apply in the real applications such as in scientific and engineering problems.

For two NS,

$$A = \{\langle x, \mu_A(x), \nu_A(x), \omega_A(x)\rangle \mid x \in X\} \text{ and}$$
$$B = \{\langle x, \mu_B(x), \nu_B(x), \omega_B(x)\rangle \mid x \in X\},$$

the relations are defined as follows:

(i) $A \subseteq B$ if and only if $\mu_A(x) \le \mu_B(x)$, $\nu_A(x) \ge \mu_B(x)$, $\omega_A(x) \ge \omega_B(x)$,

(ii) $A = B$ if and only if $\mu_A(x) = \mu_B(x)$, $\nu_A(x) = \mu_B(x)$, $\omega_A(x) = \omega_B(x)$,

(iii) $A \cap B = \{\langle x, \min(\mu_A(x), \mu_B(x)), \max(\nu_A(x), \nu_B(x)), \max(\omega_A(x), \omega_B(x))\rangle \mid x \in X\}$,

(iv) $A \cup B = \{\langle x, \max(\mu_A(x), \mu_B(x)), \min(\nu_A(x), \nu_B(x)), \min(\omega_A(x), \omega_B(x))\rangle \mid x \in X\}$,

(v) $A^C = \{\langle x, \omega_A(x), 1 - \nu_A(x), \mu_A(x)\rangle \mid x \in X\}$

(vi) $0_n = (0, 1, 1)$ and $1_n = (1, 0, 0)$.

As an illustration, let us consider the following example.

Example 2.2. Assume that the universe of discourse $U = \{x_1, x_2, x_3\}$, where x_1 characterizes the capability, x_2 characterizes the trustworthiness and x_3 indicates the prices of the objects. It may be further assumed that the values of x_1, x_2 and x_3 are in $[0, 1]$ and they are obtained from some questionnaires of some experts. The experts may impose their opinion in three components viz. the degree of goodness, the degree of indeterminacy and that of poorness to explain the characteristics of the objects. Suppose A is a neutrosophic set (NS) of U, such that,

$$A = \{\langle x_1, (0.3, 0.5, 0.6)\rangle, \langle x_2, (0.3, 0.2, 0.3)\rangle, \langle x_3, (0.3, 0.5, 0.6)\rangle\},$$

where the degree of goodness of capability is 0.3, degree of indeterminacy of capability is 0.5 and degree of falsity of capability is 0.6 etc.

Definition 2.3. [1] Let U be any non-empty set. Suppose R is an equivalence relation over U. For any non-null subset X of U, the sets

$$A_1(x) = \{x : [x]_R \subseteq X\} \quad \text{and} \quad A_2(x) = \{x : [x]_R \cap X \neq \emptyset\}$$

are called the lower approximation and upper approximation, respectively of X, where the pair $S = (U, R)$ is called an approximation space. This equivalent relation R is called indiscernibility relation.

The pair $A(X) = (A_1(x), A_2(x))$ is called the rough set of X in S. Here $[x]_R$ denotes the equivalence class of R containing x.

Definition 2.4. [1] Let $A = (A_1, A_2)$ and $B = (B_1, B_2)$ be two rough sets in the approximation space $S = (U, R)$. Then,

$$A \cup B = (A_1 \cup B_1, A_2 \cup B_2),$$
$$A \cap B = (A_1 \cap B_1, A_2 \cap B_2),$$
$$A \subseteq B \text{ if } A \cap B = A,$$
$$\sim A = \{U - A_2, U - A_1\}.$$

3. Rough neutrosophic sets

In this section we introduce the notion of rough neutrosophic sets by combining both rough sets and nuetrosophic sets. and some operations viz. union, intersection, inclusion and equalities over them. Rough neutrosophic set are the generalization of rough fuzzy sets [2] and rough intuitionistic fuzzy sets [22].

Definition 3.1. Let U be a non-null set and R be an equivalence relation on U. Let F be neutrosophic set in U with the membership function μ_F, indeterminacy function ν_F and non-membership function ω_F. The lower and the upper approximations of F in the approximation (U, R) denoted by $\underline{N}(F)$ and $\overline{N}(F)$ are respectively defined as follows:

$$\underline{N}(F) = \{< x, \mu_{\underline{N}(F)}(x), \nu_{\underline{N}(F)}(x), \omega_{\underline{N}(F)}(x) >|\ y \in [x]_R,\ x \in U\},$$
$$\overline{N}(F)) = \{< x, \mu_{\overline{N}(F)}(x), \nu_{\overline{N}(F)}(x), \omega_{\overline{N}(F)}(x) >|\ y \in [x]_R,\ x \in U\},$$

where:

$$\mu_{\underline{N}(F)}(x) = \bigwedge_{y \in [x]_R} \mu_F(y),\ \nu_{\underline{N}(F)}(x) = \bigvee_{y \in [x]_R} \nu_F(y),\ \omega_{\underline{N}(F)}(x) = \bigvee_{y \in [x]_R} \omega_F(y),$$
$$\mu_{\overline{N}(F)}(x) = \bigvee_{y \in [x]_R} \mu_F(y),\ \nu_{\overline{N}(F)}(x) = \bigwedge_{y \in [x]_R} \nu_F(y),\ \omega_{\overline{N}(F)}(x) = \bigwedge_{y \in [x]_R} \omega_F(y).$$

So

$$0 \leq \mu_{\overline{N}(F)}(x) + \nu_{\overline{N}(F)}(x) + \omega_{\overline{N}(F)}(x) \leq 3$$

and

$$\mu_{\underline{N}(F)}(x) + \nu_{\underline{N}(F)}(x) + \omega_{\underline{N}(F)}(x) \leq 3,$$

where "$\vee$" and "$\wedge$" mean "max" and "min" operators respectively, $\mu_F(x)$, $\nu_F(y)$ and $\omega_F(y)$ are the membership, indeterminacy and non-membership of y with respect to F. It is easy to see that $\overline{N}(F)$ and $\underline{N}(F)$ are two neutrosophic sets in U, thus the NS mappings $\overline{N}, \underline{N} : N(U \to N(U)$ are, respectively, referred to as the upper and lower rough NS approximation operators, and the pair $(\underline{N}(F), \overline{N}(F))$ is called the rough neutrosophic set in (U, R).

From the above definition, we can see that $\underline{N}(F)$ and $\overline{N}(F)$ have constant membership on the equivalence classes of U, if $\underline{N}(F) = \overline{N}(F)$; i.e.,

$$\mu_{\underline{N}(F)} = \mu_{\overline{N}(F)},$$
$$\nu_{\underline{N}(F)} = \nu_{\overline{N}(F)},$$
$$\omega_{\underline{N}(F)} = \omega_{\overline{N}(F)}.$$

For any $x \in U$, we call F a definable neutrosophic set in the approximation (U, R). It is easily to be proved that Zero O_N neutrosophic set and unit neutrosophic sets 1_N are definable neutrosophic sets. Let us consider a simple example in the following.

Example 3.2. Let $U = \{p_1, p_2, p_3, p_4, p_5, p_6, p_7, p_8\}$ be the universe of discourse. Let R be an equivalence relation its partition of U is given by

$$U/R = \{\{p_1, p_4\}, \{p_2, p_3, p_6\}, \{p_5\}, \{p_7, p_8\}\}.$$

Let

$$N(F)=\{(p_1, (0.2, 0.3, 0.4), (p_4, (0.3, 0.5, 0.4)), (p_5, (0.4, 0.6, 0.2)), (p_7, (0.1, 0.3, 0.5))\}$$

be a neutrosophic set of U. By Definition 3.1, we obtain:

$$\begin{aligned} \underline{N}(F) &= \{(p_1, (0.2, 0.5, 0.4)), (p_4, (0.2, 0.5, 0.4)), (p_5, (0.4, 0.6, 0.2))\}; \\ \overline{N}(F) &= \{(p_1, (0.2, 0.3, 0.4)), (p_4, (0.2, 0.3, 0.4)), (p_5, (0.4, 0.6, 0.2)), \\ &\qquad (p_7, (0.1, 0.3, 0.5)), (p_8, (0.1, 0.3, 0.5))\}. \end{aligned}$$

For another neutrosophic sets

$$N(G) = \{(p_1, (0.2, 0.3, 0.4)), (p_4, (0.2, 0.3, 0.4)), (p_5, (0.4, 0.6, 0.2))\}.$$

The lower approximation and upper approximation of $N(G)$ are calculated as

$$\underline{N}(G) = \{(p_1, (0.2, 0.3, 0.4)), (p_4, (0.2, 0.3, 0.4)), (p_5, (0.4, 0.6, 0.2))\};$$
$$\overline{N}(G) = \{(p_1, (0.2, 0.3, 0.4)), (p_4, (0.2, 0.3, 0.4)), (p_5, (0.4, 0.6, 0.2))\}.$$

Obviously $\underline{N}(G) = \overline{N}(G)$ is a definable neutrosophic set in the approximation space (U, R).

Definition 3.3. If $N(F) = (\underline{N}(F), \overline{N}(F))$ is a rough neutrosophic set in (U, R), the rough complement of $N(F)$ is the rough neutrosophic set denoted $\sim N(F) = (\underline{N}(F)^c, \overline{N}(F)^c)$, where $\underline{N}(F)^c, \overline{N}(F)^c$ are the complements of neutrosophic sets $\underline{N}(F)$ and $\overline{N}(F)$, respectively,

$$\underline{N}(F)^c = \{< x, \omega_{\underline{N}(F)}, 1 - \nu_{\underline{N}(F)}(x), \mu_{\underline{N}(F)}(x) >| \ x \in U\},$$

and

$$\overline{N}(F)^c = \{< x, \omega_{\overline{N}(F)}, 1 - \nu_{\overline{N}(F)}(x), \mu_{\overline{N}(F)}(x) >| \ x \in U\}.$$

Definition 3.4. If $N(F_1)$ and $N(F_2)$ are two rough neutrosophic set of the neutrosophic sets F_1 and F_2 respectively in U, then we define the following:

(i) $N(F_1) = N(F_2)$ iff $\underline{N}(F_1) = \underline{N}(F_2)$ and $\overline{N}(F_1) = \overline{N}(F_2)$.

(ii) $N(F_1) \subseteq N(F_2)$ iff $\underline{N}(F_1) \subseteq \underline{N}(F_2)$ and $\overline{N}(F_1) \subseteq \overline{N}(F_2)$.

(iii) $N(F_1) \cup N(F_2) = \left\langle \underline{N}(F_1) \cup \underline{N}(F_2), \overline{N}(F_1) \cup \overline{N}(F_2) \right\rangle$.

(iv) $N(F_1) \cap N(F_2) = \left\langle \underline{N}(F_1) \cap \underline{N}(F_2), \overline{N}(F_1) \cap \overline{N}(F_2) \right\rangle$.

(v) $N(F_1) + N(F_2) = \left\langle \underline{N}(F_1) + \underline{N}(F_2), \overline{N}(F_1) + \overline{N}(F_2) \right\rangle$.

(vi) $N(F_1) \cdot N(F_2) = \left\langle \underline{N}(F_1) \cdot \underline{N}(F_2), \overline{N}(F_1) \cdot \overline{N}(F_2) \right\rangle$.

If N, M, L are rough neutrosophic set in (U, R), then the results in the following proposition are straightforward from definitions.

Proposition 3.5.

(i) $\sim N(\sim N) = N$

(ii) $N \cup M = M \cup N,\ N \cap M = M \cap N$

(iii) $(N \cup M) \cup L = N \cup (M \cup L)$ and $(N \cap M) \cap L = N \cap (M \cap L)$

(iv) $(N \cup M) \cap L = (N \cup M) \cap (N \cup L)$ and $(N \cap M) \cup L = (N \cap M) \cup (N \cap L)$.

De Morgan 's Laws are satisfied for neutrosophic sets:

Proposition 3.6.

(i) $\sim (N(F_1) \cup N(F_2)) = (\sim N(F_1)) \cap (\sim N(F_2))$

(ii) $\sim (N(F_1) \cap N(F_2)) = (\sim N(F_1)) \cup (\sim N(F_2))$.

Proof. (i) $(N(F_1) \cup N(F_2)) = \sim (\{\underline{N}(F_1) \cup \underline{N}(F_2)\}, \{\overline{N}(F_1) \cup \overline{N}(F_2)\}) = (\sim \{\underline{N}(F_1) \cup \underline{N}(F_2)\}, \sim \{\overline{N}(F_1) \cup \overline{N}(F_2)\}) = (\{\underline{N}(F_1) \cup \underline{N}(F_2)\}^c, \{\overline{N}(F_1) \cup \overline{N}(F_2)\}^c)$ $= (\sim \{\underline{N}(F_1) \cap \underline{N}(F_2)\}, \sim \{\overline{N}(F_1) \cap \overline{N}(F_2)\}) = (\sim N(F_1)) \cap (\sim N(F_2))$.

(ii) Similar to the proof of (i).

Proposition 3.7. *If F_1 and F_2 are two neutrosophic sets in U such that $F_1 \subseteq F_2$, then $N(F_1) \subseteq N(F_2)$*

(i) $N(F_1 \cup F_2) \supseteq N(F_1) \cup N(F_2)$,

(ii) $N(F_1 \cap F_2) \subseteq N(F_1) \cap N(F_2)$.

Proof.

$$\begin{aligned}
\mu_{\underline{N}(F_1\cup F_2)}(x) &= \inf\{\mu_{F_1\cup F_2)}(x) \mid x \in X_i\} \\
&= \inf(\max\{\mu_{F_1}(x), \mu_{F_2}(x) \mid x \in X_i\}) \\
&\geq \max\{\inf\{\mu_{F_1}(x) \mid x \in X_i\}, \inf\{\mu_{F_2}(x) \mid x \in X_i\}\} \\
&= \max\{\mu_{\underline{N}(F_1)}(x_i), \mu_{\underline{N}(F_2)}(x_i)\} \\
&= \mu_{\underline{N}(F_1)} \cup \mu_{\underline{N}(F_2)}(x_i).
\end{aligned}$$

Similarly,

$$\nu_{\underline{N}(F_1\cup F_2)}(x_i) \leq (\nu_{\underline{N}(F_1)} \cup \nu_{\underline{N}(F_2)})(x_i)$$
$$\omega_{\underline{N}(F_1\cup F_2)}(x_i) \leq (\omega_{\underline{N}(F_1)} \cup \omega_{\underline{N}(F_2)})(x_i)$$

Thus,

$$\underline{N}(F_1 \cup F_2) \supseteq \underline{N}(F_1) \cup \underline{N}(F_2).$$

We can also see that

$$\overline{N}(F_1 \cup F_2) = \overline{N}(F_1) \cup \overline{N}(F_2).$$

Hence,

$$N(F_1 \cup F_2) \supseteq N(F_1) \cup N(F_2).$$

(ii) The proof of (ii) is similar to the proof of (i).

Proposition 3.8.

(i) $N(F) = \sim \overline{N}(\sim F)$

(ii) $\overline{N}(F) = \sim \underline{N}(\sim F)$

(iii) $\underline{N}(F) \supseteq \overline{N}(F)$.

Proof. According to Definition 3.1, we can obtain

(i) $F = \{\langle x, \mu_F(x), \nu_F(x), \omega_F(x)\rangle \mid x \in X\}$

$\sim F = \{\langle x, \omega_F(x), 1 - \nu_F(x), \mu_F(x)\rangle \mid |x \in X\}$

$$\overline{N}(\sim F) = \left\{\left\langle x, \omega_{\overline{N}(\sim F)}(x), 1 - \nu_{\overline{N}(\sim F)}(x), \mu_{\overline{N}(\sim F)}(x)\right\rangle \mid y \in [x]_R,\ x \in U\right\}$$

$$\begin{aligned}
\sim \overline{N}(\sim F) &= \left\{\left\langle x, \mu_{\overline{N}(\sim F)}(x), 1{-}(1{-}\nu_{\overline{N}(\sim F)}(x)), \omega_{\overline{N}(\sim F)}(x)\right\rangle |y \in [x]_R,\ x \in U\right\} \\
&= \left\{\left\langle x, \mu_{\overline{N}(\sim F)}(x), \nu_{\overline{N}(\sim F)}(x), \omega_{\overline{N}(\sim F)}(x)\right\rangle \mid y \in [x]_R,\ x \in U\right\}
\end{aligned}$$

where

$$\mu_{\overline{N}(\sim F)}(x) = \bigwedge_{y\in[x]_R} \mu_F(y),\ \nu_{\overline{N}(\sim F)}(x) = \bigvee_{y\in[x]_R} \nu_F(y),\ \omega_{\overline{N}(\sim F)}(x) = \bigvee_{y\in[x]_R} \omega_F(y).$$

Hence $\underline{N}(F) =\sim \overline{N}(\sim F)$.

(ii) The proof is similar to the proof of (i).

(iii) For any $y \in \underline{N}(F)$, we can have

$\mu_{\underline{N}(F)}(x) = \bigwedge_{y\in[x]_R} \mu_F(y) \leq \bigvee_{y\in[x]_R} \mu_F(y)$, $\nu_{\underline{N}(F)}(x) = \bigvee_{y\in[x]_R} \nu_F(y) \geq \bigwedge_{y\in[x]_R} \nu_F(y)$

and $\omega_{\underline{N}(F)}(x) = \bigvee_{y\in[x]_R} \omega_F(y) \geq \bigwedge_{y\in[x]_R} \omega_F(y)$.

Hence $\underline{N}(F) \subseteq \overline{N}(F)$.

4. Conclusion

In this paper we have defined the notion of rough neutrosophic sets. We have also studied some properties on them and proved some propositions. The concept combines two different theories which are rough sets theory and neutrosophic theory. While neutrosophic set theory is mainly concerned with, indeterminate and inconsistent information, rough set theory is with incompleteness; but both the theories deal with imprecision. Consequently, by the way they are defined, it is clear that rough neutrosophic sets can be utilized for dealing with both of indeterminacy and incompleteness.

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Bipolar Neutrosophic Sets and Their Application Based on Multi-Criteria Decision Making Problems

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Abstract— **In this paper, we introduce concept of bipolar neutrosophic set and its some operations. Also, we propose score, certainty and accuracy functions to compare the bipolar neutrosophic sets. Then, we develop the bipolar neutrosophic weighted average operator (A_w) and bipolar neutrosophic weighted geometric operator (G_w) to aggregate the bipolar neutrosophic information. Furthermore, based on the (A_w) and (G_w) operators and the score, certainty and accuracy functions, we develop a bipolar neutrosophic multiple criteria decision-making approach, in which the evaluation values of alternatives on the attributes take the form of bipolar neutrosophic numbers to select the most desirable one(s). Finally, a numerical example of the method was given to demonstrate the application and effectiveness of the developed method.**

Index Terms— **Neutrosophic set, bipolar neutrosophic set, average operator, geometric operator, score, certainty and accuracy functions, multi-criteria decision making.**

I. Introduction

To handle with imprecision and uncertainty, concept of fuzzy sets and intuitionistic fuzzy sets originally introduced by Zadeh [26] and Atanassov [1], respectively. Then, Smarandache [17] proposed concept of neutrosophic set which is generalization of fuzzy set theory and intuitionistic fuzzy sets. These sets models have been studied by many authors; on application [4-6,10-12,15,16], theory [18-20,21-25,27,28], and so on.

Bosc and Pivert [2] said that "Bipolarity refers to the propensity of the human mind to reason and make decisions on the basis of positive and negative effects. Positive information states what is possible, satisfactory, permitted, desired, or considered as being acceptable. On the other hand, negative statements express what is impossible, rejected, or forbidden. Negative preferences correspond to constraints, since they specify which values or objects have to be rejected (i.e., those that do not satisfy the constraints), while positive preferences correspond to wishes, as they specify which objects are more desirable than others (i.e., satisfy user wishes) without rejecting those that do not meet the wishes." Therefore, Lee [8,9] introduced the concept of bipolar fuzzy sets which is an generalization of the fuzzy sets. Recently, bipolar fuzzy models have been studied by many authors on algebraic structures such as; Chen et. al. [3] studied of m-polar fuzzy set and illustrates how many concepts have been defined based on bipolar fuzzy sets. Then, they examined many results which are related to these concepts can be generalized to the case of m-polar fuzzy sets. They also proposed numerical examples to show how to apply m-polar fuzzy sets in real world problems. Bosc and Pivert [2] introduced a study is called bipolar fuzzy relations where each tuple is associated with a pair of satisfaction degrees. Manemaran and Chellappa [14] gave some applications of bipolar fuzzy sets in groups are called the bipolar fuzzy groups, fuzzy d-ideals of groups under (T-S) norm. They investigate some related properties of the groups and introduced relations between a bipolar fuzzy group and bipolar fuzzy d-ideals. Majumder [13] proposed bipolar valued fuzzy subsemigroup, bipolar valued fuzzy bi-ideal, bipolar valued fuzzy (1,2)- ideal and bipolar valued fuzzy ideal. Zhou and Li [29] introduced a new framework of bipolar fuzzy subsemirings and bipolar fuzzy ideals which is a generalization of fuzzy subsemirings and bipolar fuzzy ideals in semirings and and bipolar fuzzy ideals, respectively, and related properties are examined by the authors.

In this paper, we introduced the concept of bipolar neutrosophic sets which is an extension of the fuzzy sets, bipolar fuzzy sets, intuitionistic fuzzy sets and neutrosophic sets. Also, we give some operations and operators on the bipolar neutrosophic sets. The operations and operators generalize the operations and operators of fuzzy sets, bipolar fuzzy sets, intuitionistic fuzzy sets and neutrosophic sets which have been previously proposed. Therefore, in section 2, we introduce concept of bipolar neutrosophic set and its some operations including the score, certainty and accuracy functions to compare the bipolar neutrosophic sets. In the same section, we also develop the bipolar neutrosophic weighted average operator (A_w) and bipolar neutrosophic weighted geometric operator (G_w)

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operator to aggregate the bipolar neutrosophic information. In section 3, based on the (A_w) and (G_w) operators and the score, certainty and accuracy functions, we develop a bipolar neutrosophic multiple criteria decision-making approach, in which the evaluation values of alternatives on the attributes take the form of bipolar neutrosophic numbers to select the most desirable one(s) and give a numerical example of the to demonstrate the application and effectiveness of the developed method. In last section, we conclude the paper.

II. BASIC AND FUNDAMENTAL CONCEPTS

In this section, we give some concepts related to neutrosophic sets and bipolar sets.

Definition 2.1. [17] Let X be a universe of discourse. Then a neutrosophic set is defined as:

$$A = \{\langle x, F_A(x), T_A(x), I_A(x)\rangle : x \in X\},$$

which is characterized by a truth-membership function $T_A: X \to]0^-, 1^+[$, an indeterminacy-membership function $I_A: X \to]0^-, 1^+[$ and a falsity-membership function $F_A: X \to]0^-, 1^+[$. There is not restriction on the sum of $T_A(x)$, $I_A(x)$ and $F_A(x)$, so $0^- \leq \sup T_A(x) \leq \sup I_A(x) \leq \sup F_A(x) \leq 3^+$. For application in real scientific and engineering areas, Wang et al.[18] proposed the concept of an single valued neutrosophic set as follows;

Definition 2.1. [18]Let X be a universe of discourse. Then a single valued neutrosophic set is defined as:

$$A_{NS} = \{\langle x, F_A(x), T_A(x), I_A(x)\rangle : x \in X\},$$

which is characterized by a truth-membership function $T_A: X \to [0,1]$, an indeterminacy-membership function $I_A: X \to [0,1]$ and a falsity-membership function $F_A: X \to [0,1]$.

There is not restriction on the sum of $T_A(x)$, $I_A(x)$ and $F_A(x)$, so $0 \leq \sup T_A(x) \leq \sup I_A(x) \leq \sup F_A(x) \leq 3$.

Set- theoretic operations, for two single valued neutrosophic set.

A_{NS}= {<x, $T_A(x)$, $I_A(x)$, $F_A(x)$> | x ∈ X } and B_{NS}= {<x, $T_B(x)$, $I_B(x)$, $F_B(x)$> | x ∈ X } are given as;

1. The subset; $A_{NS} \subseteq B_{NS}$ if and only if $T_A(x) \leq T_B(x), I_A(x) \geq I_B(x)$, $F_A(x) \geq F_B(x)$.
2. $A_{NS} = B_{NS}$ if and only if , $T_A(x) = T_B(x)$, $I_A(x) = I_B(x)$, $F_A(x) = F_B(x)$ for anyx ∈ X.
3. The complement of A_{NS} is denoted by A^o_{NS} and is defined by A^o_{NS}= {<x, $F_A(x), 1 - I_A(x), T_A(x)$| x ∈ X }
4. The intersection $A_{NS} \cap B_{NS}$ = {<x,min $\{T_A(x), T_B(x)\}$, max $\{I_A(x), I_B(x)\}$, max$\{F_A(x), F_B(x)\}$>:x ∈ X }
5. The union $A_{NS} \cup B_{NS}$= {<x,max $\{T_A(x), T_B(x)\}$, min $\{I_A(x), I_B(x)\}$, min$\{F_A(x), F_B(x)\}$>:x ∈ X }

A single valued neutrosophic number is denoted by $\widetilde{A} = \langle T, I, F\rangle$ for convenience.

Definition 2.2. [15] Let $\widetilde{A}_1 = \langle T_1, I_1, F_1 \rangle$ and $\widetilde{A}_2 = \langle T_2, I_2, F_2 \rangle$ be two single valued neutrosophic number. Then, the operations for NNs are defined as below;

i. $\lambda\widetilde{A} = \langle 1 - (1 - T_1)^\lambda, I_1^\lambda, F_1^\lambda \rangle$
ii. $\widetilde{A}_1^\lambda = \langle T_1^\lambda, 1 - (1 - I_1)^\lambda, 1 - (1 - F_1)^\lambda \rangle$
iii. $\widetilde{A}_1 + \widetilde{A}_2 = \langle T_1 + T_2 - T_1T_2, I_1I_2, F_1F_2 \rangle$
iv. $\widetilde{A}_1.\widetilde{A}_2 = \langle T_1T_2, I_1 + I_2 - I_1I_2, F_1 + F_2 - F_1F_2 \rangle$

where $\lambda > 0$.

Definition 2.3. [15] Let $\widetilde{A}_1 = \langle T_1, I_1, F_1 \rangle$ be a single valued neutrosophic number. Then, the score function $s(\widetilde{A}_1)$, accuracy function $a(\widetilde{A}_1)$ and certainty function $c(\widetilde{A}_1)$ of an SNN are defined as follows:

i. $s(\widetilde{A}_1) = (T_1 + 1 - I_1 + 1 - F_1)/3$;
ii. $a(\widetilde{A}_1) = T_1 - F_1$;
iii. $c(\widetilde{A}_1) = T_1$

Definition 2.4. [15] Let $\widetilde{A}_1 = \langle T_1, I_1, F_1 \rangle$ and $\widetilde{A}_2 = \langle T_2, I_2, F_2 \rangle$ be two single valued neutrosophic number. The comparison method can be defined as follows:

i. if $s(\widetilde{A}_1) > s(\widetilde{A}_2)$, then $\widetilde{A}_1$ is greater than $\widetilde{A}_2$, that is, $\widetilde{A}_1$ is superior to $\widetilde{A}_2$, denoted by $\widetilde{A}_1 > \widetilde{A}_2$
ii. if $s(\widetilde{A}_1) = s(\widetilde{A}_2)$ and $a(\widetilde{A}_1) > a(\widetilde{A}_2)$, then $\widetilde{A}_1$ is greater than $\widetilde{A}_2$, that is, $\widetilde{A}_1$ is superior to $\widetilde{A}_2$, denoted by $\widetilde{A}_1 < \widetilde{A}_2$;
iii. if $s(\widetilde{A}_1) = s(\widetilde{A}_2)$, $a(\widetilde{A}_1) = a(\widetilde{A}_2)$ and $c(\widetilde{A}_1) > c(\widetilde{A}_2)$, then $\widetilde{A}_1$ is greater than $\widetilde{A}_2$, that is, $\widetilde{A}_1$ is superior to $\widetilde{A}_2$, denoted by $\widetilde{A}_1 > \widetilde{A}_2$;
iv. if $s(\widetilde{A}_1) = s(\widetilde{A}_2)$, $a(\widetilde{A}_1) = a(\widetilde{A}_2)$ and $c(\widetilde{A}_1) = c(\widetilde{A}_2)$, then $\widetilde{A}_1$ is equal to $\widetilde{A}_2$, that is, $\widetilde{A}_1$ is indifferent to $\widetilde{A}_2$, denoted by $\widetilde{A}_1 = \widetilde{A}_2$.

Definition 2.4. [6,14] Let X be a non-empty set. Then, a bipolar-valued fuzzy set, denoted by A_{BF}, is difined as;

$$A_{BF} = \{\langle x, \mu_B^+(x), \mu_B^-(x)\rangle : x \in X\}$$

where $\mu_B^+: X \to [0,1]$ and $\mu_B^-: X \to [0,1]$. The positive membership degree $\mu_B^+(x)$ denotes the satisfaction degree of an element x to the property corresponding to A_{BF} and the negative membership degree $\mu_B^-(x)$ denotes the satisfaction degree of x to some implicit counter property of A_{BF}.

III. BIPOLAR NEUTROSOPHIC SEY

In this section, we introduce concept of bipolar neutrosophic set and its some operations including the score, certainty and accuracy functions to compare the bipolar neutrosophic sets. We also develop the bipolar neutrosophic weighted average operator (A_w) and bipolar neutrosophic weighted geometric operator (G_w) operator to aggregate the bipolar neutrosophic information. Some of it is quoted from [2,6,8,9,14,17,18,20,24,26].

Definition 3.1. A bipolar neutrosophic set A in X is defined as an object of the form

$$A=\left\{\left\langle x,T^{+}(x),I^{+}(x),F^{+}(x),T^{-}(x),I^{-}(x),F^{-}(x)\right\rangle : x\in X\right\},$$

where $T^{+},I^{+},F^{+}:X\to[1,0]$ and $T^{-},I^{-},F^{-}:X\to[-1,0]$.

The positive membership degree $T^{+}(x),I^{+}(x),F^{+}(x)$ denotes the truth membership, indeterminate membership and false membership of an element $x\in X$ corresponding to a bipolar neutrosophic set A and the negative membership degree $T^{-}(x),I^{-}(x),F^{-}(x)$ denotes the truth membership, indeterminate membership and false membership of an element $x\in X$ to some implicit counter-property corresponding to a bipolar neutrosophic set A.

Example 3.2. Let $X=\{x_1,x_2,x_3\}$. Then

$$A=\left\{\begin{array}{l}\langle x_1,0.5,0.3,0.1,-0.6,-0.4,-0.01\rangle,\\ \langle x_2,0.3,0.2,0.7,-0.02,-0.003,-0.5\rangle,\\ \langle x_3,0.8,0.05,0.4,-0.1,-0.5,-0.06\rangle\end{array}\right\}$$

is a bipolar neutrosophic subset of X.

Theorem 3.4. A bipolar neutrosophic set is the generalization of a bipolar fuzzy set.

Proof: Suppose that X is a bipolar neutrosophic set. Then by setting the positive components I^{+}, F^{+} equals to zero as well as the negative components I^{-}, F^{-} equals to zero reduces the bipolar neutrosophic set to bipolar fuzzy set.

Definition 3.5. Let $A_1=\langle x,T_1^{+}(x),I_1^{+}(x),F_1^{+}(x),T_1^{-}(x),I_1^{-}(x),F_1^{-}(x)\rangle$ and $A_2=\langle x,T_2^{+}(x),I_2^{+}(x),F_2^{+}(x),T_2^{-}(x),I_2^{-}(x),F_2^{-}(x)\rangle$ be two bipolar neutrosophic sets. Then $A_1\subseteq A_2$ if and only if

$$T^{+}_1(x)\le T_2^{+}(x)\ I^{+}_1(x)\le I_2^{+}(x),\ F^{+}_1(x)\ge F_2^{+}(x),$$

and

$$T^{-}_1(x)\ge T_2^{-}(x),\ I^{-}_1(x)\ge I^{-}_2(x),\ F_1^{-}(x)\le F_2^{-}(x)$$

for all $x\in X$.

Definition 3.6. Let $A_1=\langle x,T_1^{+}(x),I_1^{+}(x),F_1^{+}(x),T_1^{-}(x),I_1^{-}(x),F_1^{-}(x)\rangle$ and $A_2=\langle x,T_2^{+}(x),I_2^{+}(x),F_2^{+}(x),T_2^{-}(x),I_2^{-}(x),F_2^{-}(x)\rangle$ be two bipolar neutrosophic set. Then $A_1=A_2$ if and only if

$$T^{+}_1(x)=T_2^{+}(x),\ I^{+}_1(x)=I_2^{+}(x),\ F^{+}_1(x)=F_2^{+}(x),$$

and

$$T^{-}_1(x)=T_2^{-}(x),\ I^{-}_1(x)=I^{-}_2(x),\ F_1^{-}(x)=F_2^{-}(x)$$

for all $x\in X$.

Definition 3.7. Let $A_1=\langle x,T_1^{+}(x),I_1^{+}(x),F_1^{+}(x),T_1^{-}(x),I_1^{-}(x),F_1^{-}(x)\rangle$ and $A_2=\langle x,T_2^{+}(x),I_2^{+}(x),F_2^{+}(x),T_2^{-}(x),I_2^{-}(x),F_2^{-}(x)\rangle$ be two bipolar neutrosophic set. Then their union is defined as:

$$(A_1\cup A_2)(x)=\left(\begin{array}{l}\max(T_1^{+}(x),T_2^{+}(x)),\dfrac{I_1^{+}(x)+I_2^{+}(x)}{2},\min((F_1^{+}(x),F_2^{+}(x)),\\ \min(\mathrm{T}_1^{-}(x),T_2^{+}(x)),\dfrac{I_1^{-}(x)+I_2^{-}(x)}{2},\max((F_1^{-}(x),F_2^{-}(x))\end{array}\right)$$

for all $x\in X$.

Example 3.8. Let $X=\{x_1,x_2,x_3\}$. Then

$$A_1=\left\{\begin{array}{l}\langle x_1,0.5,0.3,0.1,-0.6,-0.4,-0.01\rangle,\\ \langle x_2,0.3,0.2,0.7,-0.02,-0.003,-0.5\rangle,\\ \langle x_3,0.8,0.05,0.4,-0.1,-0.5,-0.06\rangle\end{array}\right\}$$

and

$$A_2=\left\{\begin{array}{l}\langle x_1,0.4,0.6,0.3,-0.3,-0.5,-0.1\rangle,\\ \langle x_2,0.5,0.1,0.4,-0.2,-0.3,-0.3\rangle,\\ \langle x_3,0.2,0.5,0.6,-0.4,-0.6,-0.7\rangle\end{array}\right\}$$

are two bipolar neutrosophic sets in X.

Then their union is given as follows:

$$A_1\cup A_2=\left\{\begin{array}{l}\langle x_1,0.5,0.45,0.1,-0.6,-0.5,-0.1\rangle,\\ \langle x_2,0.5,0.15,0.7,-0.2,-0.1515,-0.5\rangle,\\ \langle x_3,0.8,0.47,0.6,-0.4,-0.55,-0.7\rangle\end{array}\right\}$$

Definition 3.9. Let $A_1=\langle x,T_1^{+}(x),I_1^{+}(x),F_1^{+}(x),T_1^{-}(x),I_1^{-}(x),F_1^{-}(x)\rangle$ and $A_2=\langle x,T_2^{+}(x),I_2^{+}(x),F_2^{+}(x),T_2^{-}(x),I_2^{-}(x),F_2^{-}(x)\rangle$ be two bipolar neutrosophic set. Then their intersection is defined as:

$$(A_1\cap A_2)(x)=\left(\begin{array}{l}\min(T_1^{+}(x),T_2^{+}(x)),\dfrac{I_1^{+}(x)+I_2^{+}(x)}{2},\max((F_1^{+}(x),F_2^{+}(x)),\\ \max(\mathrm{T}_1^{-}(x),T_2^{+}(x)),\dfrac{I_1^{-}(x)+I_2^{-}(x)}{2},\min((F_1^{-}(x),F_2^{-}(x))\end{array}\right)$$

for all $x\in X$.

Definition 3.10. Let

$$A=\left\{\left\langle x,T^{+}(x),I^{+}(x),F^{+}(x),T^{-}(x),I^{-}(x),F^{-}(x)\right\rangle : x\in X\right\}$$

be a bipolar neutrosophic set in X. Then the complement of A is denoted by A^{c} and is defined by

$$T_{A^c}{}^{+}(x)=\{1^{+}\}-T_A{}^{+}(x),\ I_{A^c}{}^{+}(x)=\{1^{+}\}-I_A{}^{+}(x),$$

$$F_{A^c}{}^{+}(x)=\{1^{+}\}-F_A{}^{+}(x)$$

and

$$T_{A^c}{}^{-}(x)=\{1^{-}\}-T_A{}^{-}(x),\ I_{A^c}{}^{-}(x)=\{1^{-}\}-I_A{}^{-}(x),$$

$$F_{A^c}{}^{-}(x)=\{1^{-}\}-F_A{}^{-}(x),$$

for all $x\in X$.

Example 3.11. Let $X=\{x_1,x_2,x_3\}$. Then

$$A=\left\{\begin{array}{l}\langle x_1,0.5,0.3,0.1,-0.6,-0.4,-0.01\rangle,\\ \langle x_2,0.3,0.2,0.7,-0.02,-0.003,-0.5\rangle,\\ \langle x_3,0.8,0.05,0.4,-0.1,-0.5,-0.06\rangle\end{array}\right\}$$

is a bipolar neutrosophic set in X. Then the complement of A is given as follows:

$$A^c = \begin{Bmatrix} \langle x_1, 0.5, 0.7, 0.9, -0.4, -0.6, -0.99 \rangle, \\ \langle x_2, 0.7, 0.8, 0.3, -0.08, -0.997, -0.5 \rangle, \\ \langle x_3, 0.2, 0.95, 0.6, -0.9, -0.5, -0.94 \rangle \end{Bmatrix}.$$

We will denote the set of all the bipolar neutrosophic sets (NBSs) in X by Q. A bipolar neutrosophic number (NBN) is denoted by $\tilde{a} = \langle T^+, I^+, F^+, T^-, I^-, F^- \rangle$ for convenience.

Definition 3.12. Let $\tilde{a}_1 = \langle T_1^+, I_1^+, F_1^+, T_1^-, I_1^-, F_1^- \rangle$ and $\tilde{a}_2 = \langle T_2^+, I_2^+, F_2^+, T_2^-, I_2^-, F_2^- \rangle$ be two bipolar neutrosophic number . Then the operations for NNs are defined as below;

i. $\lambda\tilde{a}_1 = \langle 1 - (1 - T_1^+)^\lambda, (I_1^+)^\lambda, (F_1^+)^\lambda, -(-T_1^-)^\lambda, -(-I_1^-)^\lambda, -(1 - (1 - (-F_1^-))^\lambda) \rangle$

ii. $\tilde{a}_1^\lambda = \langle (T_1^+)^\lambda, 1 - (1 - I_1^+)^\lambda, 1 - (1 - F_1^+)^\lambda, -(1 - (1 - (-T_1^-))^\lambda), -(-I_1^-)^\lambda, -(-F_1^-)^\lambda \rangle$

iii. $\tilde{a}_1 + \tilde{a}_2 = \langle T_1^+ + T_2^+ - T_1^+T_2^+, I_1^+I_2^+, F_1^+F_2^+, -T_1^-T_2^-, -(-I_1^- - I_2^- - I_1^-I_2^-), -(-F_1^- - F_2^- - F_1^-F_2^-) \rangle$

iv. $\tilde{a}_1 . \tilde{a}_2 = \langle T_1^+T_2^+, I_1^+ + I_2^+ - I_1^+I_2^+, F_1^+ + F_2^+ - F_1^+F_2^+, -(-T_1^- - T_2^- - T_2^-T_2^-), -I_1^-I_2^-, -F_1^-F_2^- \rangle$

where $\lambda > 0$.

Definition 3.14. Let $\tilde{a}_1 = \langle T_1^+, I_1^+, F_1^+, T_1^-, I_1^-, F_1^- \rangle$ be a bipolar neutrosophic number. Then, the score function $s(\tilde{a}_1)$, accuracy function $a(\tilde{a}_1)$ and certainty function $c(\tilde{a}_1)$ of an NBN are defined as follows:

i. $\tilde{s}(\tilde{a}_1) = (T_1^+ + 1 - I_1^+ + 1 - F_1^+ + 1 + T_1^- - I_1^- - F_1^-)/6$

ii. $\tilde{a}(\tilde{a}_1) = T_1^+ - F_1^+ + T_1^- - F_1^-$

iii. $\tilde{c}(\tilde{a}_1) = T_1^+ - F_1^-$

Definition 3.15. $\tilde{a}_1 = \langle T_1^+, I_1^+, F_1^+, T_1^-, I_1^-, F_1^- \rangle$ and $\tilde{a}_2 = \langle T_2^+, I_2^+, F_2^+, T_2^-, I_2^-, F_2^- \rangle$ be two bipolar neutrosophic number. The comparison method can be defined as follows:

i. if $\tilde{s}(\tilde{a}_1) > \tilde{s}(\tilde{a}_2)$, then $\tilde{a}_1$ is greater than $\tilde{a}_2$, that is, $\tilde{a}_1$ is superior to $\tilde{a}_2$, denoted by $a_1 > \tilde{a}_2$

ii. $\tilde{s}(\tilde{a}_1) = \tilde{s}(\tilde{a}_2)$ and $\tilde{a}(\tilde{a}_1) > \tilde{a}(\tilde{a}_2)$, then $\tilde{a}_1$ is greater than $\tilde{a}_2$, that is, $\tilde{a}_1$ is superior to $\tilde{a}_2$, denoted by $\tilde{a}_1 < \tilde{a}_2$;

iii. if $\tilde{s}(\tilde{a}_1) = \tilde{s}(\tilde{a}_2)$, $\tilde{a}(\tilde{a}_1) = \tilde{a}(\tilde{a}_1)$ and $\tilde{c}(\tilde{a}_1) > \tilde{c}(\tilde{a}_2)$, then $\tilde{a}_1$ is greater than $\tilde{a}_2$, that is, $\tilde{a}_1$ is superior to $\tilde{a}_2$, denoted by $\tilde{a}_1 > \tilde{a}_2$;

iv. if $\tilde{s}(\tilde{a}_1) = \tilde{s}(\tilde{a}_2)$, $\tilde{a}(\tilde{a}_1) = \tilde{a}(\tilde{a}_2))$ and $\tilde{c}(\tilde{a}_1) = \tilde{c}(\tilde{a}_2)$, then $\tilde{a}_1$ is equal to $\tilde{a}_2$, that is, $\tilde{a}_1$ is indifferent to $\tilde{a}_2$, denoted by $\tilde{a}_1 = \tilde{a}_2$.

Based on the study given in [15,20] we define some weighted aggregation operators related to bipolar neutrosophic sets as follows;

Definition 3.16. Let $\tilde{a}_j = \langle T_j^+, I_j^+, F_j^+, T_j^-, I_j^-, F_j^- \rangle$ $(j = 1,2, \dots, n)$ be a family of bipolar neutrosophic numbers. A mapping $A_\omega: Q_n \to Q$ *is called* bipolar neutrosophic weighted average operator if it satisfies

$A_w(\tilde{a}_1, \tilde{a}_2, \dots, \tilde{a}_n) = \sum_{j=1}^n \omega_j \tilde{a}_j = \langle 1 - \prod_{j=1}^n (1 - T_j^+)^{\omega_j}, \prod_{j=1}^n I_j^{+\omega_j}, \prod_{j=1}^n F_j^{+\omega_j}, -\prod_{j=1}^n (-T_j^-)^{\omega_j}, -(1 - \prod_{j=1}^n (1 - (-I_j^-))^{\omega_j}), -(1 - \prod_{j=1}^n (1 - (-F_j^-))^{\omega_j}) \rangle$

where ω_j is the weight of $\tilde{a}_j$ $(j = 1,2, \dots, n)$, $\omega_j \in [0,1]$ and $\sum_{j=1}^n \omega_j = 1$.

Based on the study given in [15,20] we give the theorem related to bipolar neutrosophic sets as follows;

Theorem 3.17. Let $\tilde{a}_j = \langle T_j^+, I_j^+, F_j^+, T_j^-, I_j^-, F_j^- \rangle$ $(j = 1,2, \dots, n)$ be a family of bipolar neutrosophic numbers. Then,

i. If $\tilde{a}_j = \tilde{a}$ for all $j = 1,2, \dots, n$ then, $A_w(\tilde{a}_1, \tilde{a}_2, \dots, \tilde{a}_n) = \tilde{a}$

ii. $min_{j=1,2,\dots,n} \{\tilde{a}_j\} \le A_w(\tilde{a}_1, \tilde{a}_2, \dots, \tilde{a}_n) \le max_{j=1,2,\dots,n} \{\tilde{a}_j\}$

iii. If $\tilde{a}_j \le \tilde{a}_j^*$ for all $j = 1,2, \dots, n$ then,

$A_w(\tilde{a}_1, \tilde{a}_2, \dots, \tilde{a}_n) \le A_w(\tilde{a}_1^*, \tilde{a}_2^*, \dots, \tilde{a}_n^*)$.

Based on the study given in [15,20] we define some weighted aggregation operators related to bipolar neutrosophic sets as follows;

Definition 3.18. Let $\tilde{a}_j = \langle T_j^+, I_j^+, F_j^+, T_j^-, I_j^-, F_j^- \rangle$ $(j = 1,2, \dots, n)$ be a family of bipolar neutrosophic numbers. A mapping $G_\omega: Q_n \to Q$ *is called* bipolar neutrosophic weighted geometric operator if it satisfies

$G_w(\tilde{a}_1, \tilde{a}_2, \dots, \tilde{a}_n) = \prod_{j=1}^n \tilde{a}_j^{\omega_j} = \langle \prod_{j=1}^n T_j^{+\omega_j}, 1 - \prod_{j=1}^n (1 - I_j^+)^{\omega_j}, 1 - \prod_{j=1}^n (1 - F_j^+)^{\omega_j}, -(1 - \prod_{j=1}^n (1 - (-T_j^-))^{\omega_j}, -\prod_{j=1}^n (-I_j^-)^{\omega_j}, -\prod_{j=1}^n (-F_j^{-\omega_j}) \rangle$

where ω_j is the weight of $\tilde{a}_j$ $(j = 1,2, \dots, n)$, $\omega_j \in [0,1]$ and $\sum_{j=1} \omega_j = 1$.

Based on the study given in [15,20] we give the theorem related to bipolar neutrosophic sets as follows;

Theorem 3.19. Let $\tilde{a}_j = \langle T_j^+, I_j^+, F_j^+, T_j^-, I_j^-, F_j^- \rangle$ $(j = 1,2, \dots, n)$ be a family of bipolar neutrosophic numbers. Then,

i. If $\tilde{a}_j = \tilde{a}$ for all $j = 1,2, \dots, n$ then, $G_w(\tilde{a}_1, \tilde{a}_2, \dots, \tilde{a}_n) = \tilde{a}$

ii. $min_{j=1,2,\dots,n}\{\tilde{a}_j\} \leq G_w(\tilde{a}_1, \tilde{a}_2, \dots, \tilde{a}_n) \leq max_{j=1,2,\dots,n}\{\tilde{a}_j\}$

iii. If $\tilde{a}_j \leq \tilde{a}_j^*$ for all $j = 1,2, \dots, n$ then,

$G_w(\tilde{a}_1, \tilde{a}_2, \dots, \tilde{a}_n) \leq G_w(\tilde{a}_1^*, \tilde{a}_2^*, \dots, \tilde{a}_n^*)$

Note that the aggregation results are still NBNs.

IV. NBN-DECISION MAKING METHOD

In this section, we develop an approach based on the A_w (or G_w) operator and the above ranking method to deal with multiple criteria decision making problems with bipolar neutrosophic information.

Suppose that $A = \{A_1, A_2, \dots, A_m\}$ and $C = \{C_1, C_2, \dots, C_n\}$ is the set of alternatives and criterions or attributes, respectively. Let $\omega = (\omega_1, \omega_2, \dots, \omega_n)^T$ be the weight vector of attributes, such that $\sum_{j=1}^{n} \omega_j = 1$, $\omega_j \geq 0$ $(j = 1,2, \dots, n)$ and ω_j refers to the weight of attribute C_j. An alternative on criterions is evaluated by the decision maker, and the evaluation values are represented by the form of bipolar neutrosophic numbers. Assume that $(\tilde{a}_{ij})_{m\times n} = \left(\langle T^+_{ij}, I^+_{ij}, F^+_{ij}, T^-_{ij}, I^-_{ij}, F^-_{ij}\rangle\right)_{m\times n}$ is the decision matrix provided by the decision maker; $\tilde{a}_{ij}$ is a bipolar neutrosophic number for alternative A_i associated with the criterions C_j. We have the conditions $T^+_{ij}, I^+_{ij}, F^+_{ij}, T^-_{ij}, I^-_{ij}$ and $F^-_{ij} \in [0,1]$ such that $0 \leq T^+_{ij} + I^+_{ij} + F^+_{ij} - T^-_{ij} - I^-_{ij} - F^-_{ij} \leq 6$ for $i = 1,2, \dots, m$ and $j = 1,2, \dots, n$.

Now, we can develop an algorithm as follows;

Algorithm

Step1. Construct the decision matrix provided by the decision maker as; $(\tilde{a}_{ij})_{m\times n} = \left(\langle T^+_{ij}, I^+_{ij}, F^+_{ij}, T^-_{ij}, I^-_{ij}, F^-_{ij}\rangle\right)_{m\times n}$.

Step 2. Compute $\tilde{a}_i = A_w(\tilde{a}_{i1}, \tilde{a}_{i2}, \dots, \tilde{a}_{in})$ (or $G_w(\tilde{a}_{i1}, \tilde{a}_{i2}, \dots, \tilde{a}_{in})$) for each $i = 1,2, \dots, m$.

Step 3. Calculate the score values of $\tilde{s}(\tilde{a}_1)$ $(i = 1,2, \dots, m.)$ for the collective overall bipolar neutrosophic number of $\tilde{a}_i$ $(i = 1,2, \dots, m.)$

Step 4. Rank all the software systems of $\tilde{a}_i$ $(i = 1,2, \dots, m.)$ according to the score values.

Now, we give a numerical example as follows;

Example 4.1. Let us consider decision making problem adapted from Xu and Cia [20]. A customer who intends to buy a car. Four types of cars (alternatives) $A_i (i = 1,2,3,4)$ are available. The customer takes into account four attributes to evaluate the alternatives; C_1 =Fuel economy; C_2 =Aerod; C_3 =Comfort; C_4 =Safety and use the bipolar neutrosophic values to evaluate the four possible alternatives $A_i (i = 1,2,3,4)$ under the above four attributes. Also, the weight vector of the attributes $C_j (j = 1,2,3,4)$ is $\omega = (\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{8})^T$. Then,

Algorithm

Step1. Construct the decision matrix provided by the customer as;

Table 1: Decision matrix given by customer

	C_1	C_2	C_3	C_4
A_1	$\langle 0.5,0.7,0.2,-0.7,-0.3,-0.6\rangle$	$\langle 0.4,0.4,0.5,-0.7,-0.8,-0.4\rangle$	$\langle 0.7,0.7,0.5,-0.8,-0.7,-0.6\rangle$	$\langle 0.1,0.5,0.7,-0.5,-0.2,-0.8\rangle$
A_2	$\langle 0.9,0.7,0.5,-0.7,-0.7,-0.1\rangle$	$\langle 0.7,0.6,0.8,-0.7,-0.5,-0.1\rangle$	$\langle 0.9,0.4,0.6,-0.1,-0.7,-0.5\rangle$	$\langle 0.5,0.2,0.7,-0.5,-0.1,-0.9\rangle$
A_3	$\langle 0.3,0.4,0.2,-0.6,-0.3,-0.7\rangle$	$\langle 0.2,0.2,0.2,-0.4,-0.7,-0.4\rangle$	$\langle 0.9,0.5,0.5,-0.6,-0.5,-0.2\rangle$	$\langle 0.7,0.5,0.3,-0.4,-0.2,-0.2\rangle$
A_4	$\langle 0.9,0.7,0.2,-0.8,-0.6,-0.1\rangle$	$\langle 0.3,0.5,0.2,-0.5,-0.5,-0.2\rangle$	$\langle 0.5,0.4,0.5,-0.1,-0.7,-0.2\rangle$	$\langle 0.4,0.2,0.8,-0.5,-0.5,-0.6\rangle$

Step 2. Compute $\tilde{a}_i = A_w(\tilde{a}_{i1}, \tilde{a}_{i2}, \tilde{a}_{i3}, \tilde{a}_{i4})$ for each $i = 1,2,3,4$ as;

$\tilde{a}_1$=$\langle 0.471,0.583,0.329,-0.682,-0.531,-0.594\rangle$

$\tilde{a}_2$=$\langle 0.839,0.536,0.600,-0.526,-0.608,-0.364\rangle$

$\tilde{a}_3$=$\langle 0.489,0.355,0.235,-0.515,-0.447,-0.544\rangle$

$\tilde{a}_4$=$\langle 0.751,0.513,0.266,-0.517,-0.580,-0.221\rangle$.

Step 3. Calculate the score values of $\tilde{s}(\tilde{a}_1)$ $(i = 1,2,3,4)$ for the collective overall bipolar neutrosophic number of $\tilde{a}_i$ $(i = 1,2, \dots, m.)$ as;$\tilde{s}(\tilde{a}_1)$=0.50

$\tilde{s}(\tilde{a}_2)$=0.52

$\tilde{s}(\tilde{a}_3)$=0.56

$\tilde{s}(\tilde{a}_4)$=0.54.

Step 4. Rank all the software systems of A_i $(i = 1,2,3,4.)$ according to the score values as;

$$A_3 \succ A_4 \succ A_2 \succ A_1$$

and thus A_3 is the most desirable alternative.

V. CONCLUSION

This paper presented a bipolar neutrosophic set and its score, certainty and accuracy functions. Then, the A_w and G_w operators were proposed to aggregate the bipolar neutrosophic information. Furthermore, based on the A_w and G_w operators and the score, certainty and accuracy functions, we have developed a bipolar neutrosophic multiple criteria decision-making approach, in which the evaluation values of alternatives on the attributes take the form of bipolar neutrosophic numbers. The A_w and G_w operators are utilized to aggregate the bipolar neutrosophic information corresponding to each alternative to obtain the collective overall values of the alternatives, and then the alternatives are ranked according to the values of the score, certainty and accuracy functions to select the most desirable one(s). Finally, a numerical example of the method was given to demonstrate the application and effectiveness of the developed method.

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Neutrosophic social structures specificities

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Abstract

This paper is an extension of "(t, i, f)-Neutrosophic Structures" applicability , where were introduced for the first time a new type of structures, called (t, i, f)-Neutrosophic Structures, presented from a neutrosophic logic perspective.

In any field of knowledge, each structure is composed from two parts: a space, and a set of axioms (or laws) acting (governing) on it. If the space, or at least one of its axioms (laws), has some indeterminacy of the form (t, i, f) * (1, 0, 0), that structure is a (t, i, f)-Neutrosophic Structure. If the structure is applied to social environment, we have (t, i, f)- Neutrosophic Social Structures.

The (t, i, f)- Neutrosophic Social Structures [based on the components t = truth, i = numerical indeterminacy, f = falsehood] are exponential remodeled in social space from the perspective of social actor.

The social structure allows an infinite freedom of opinion, that is, everybody believes what he wants. The neutrosophic effervescence of social space is more powerful than of scientific environment for the case of natural sciences.

Keywords: Neutrosophy, neutrosophic structures, neutrosophic social structures, social science

1 Introduction

The specifics of indeterminacy, of the hesitation between truth and false in social space is given by the fact that this uncertainty is not just a status of variables, but a status of the epistemic subject.

Therefore, in the social environment the indeterminacy is raised of two: that we have a first neutrosophic indetermination specific any epistemic object, but we additionally have an indeterminacy induced by the epistemic subject. To natural entropy it is added exponentially an entropy generated by the people's perceptions variability. Man is the most important entropy inductor. The society, the first of all is not the issue of true and false, but it is the issue of opinion and belief compliance. Social structures so, are double and exponential neutrosophic articulated: indeterminacy also introduce the epistemic object, and epistemic subject, in addition the respect of each other's opinion in society makes that the probability of indeterminacy to increase with every opinion.

Any uncertainty is an uncertainty of creativity. The superior minds have uncertainties, the mediocre one have indecision. In fact, the uncertainty involves a decision in terms of unpredictability. As it is known, Immanuel Kant postulated intelligence as the ability to bear the uncertainty: the more ability to bear the uncertainty is greater, the higher the intelligence is. Uncertainty is inextricably bound by a decision: there is not uncertainty without a thinking direction of estimation, prediction, forecasting, alternative future type. When we are talking about neutrosophic social structures, we have to take into account that the social structure is not a homogeneous and uniform construction. Its uni-plan appearance is the result of a correct conjecture on horizontal dimension. However, on the vertical dimension of social structure are identifiable three levels of the social mechanism of interaction-communication presented as network. The first level is the individual one, of the actor and the relationships he has with other actors individually. The second level is that of structure / structures of which the actor belongs (family, group, clique, clan etc.). Finally, the third level is the social network as an integer as a whole. The social structure is configured as a whole what comprises and crosses the individuals relational (Vlăduțescu, 2013).

2 Arguments for Neutrosophical Social Structures

In specialty literature, T. L. Duncan and J. S. Semura emphasize that „the uncertainty about the detailed state of a system cannot decrease over time – uncertainty increases or stay the same". This conducts to enunciate a principle of "information loss" : "No process can result in a net gain of information" (Duncan & Semura, 2007, p. 1771), and uncertainty reducing: „The construct entails obtaining greater quality, decreases dimensionality, and reduced uncertainty" (Blasch, 2005, p. 5).

In the same context, Tom T. Mitchell, asserts "the information gain would be made on two issues: entropy and uncertainty, reduction of entropy is associated with uncertainty reduction" (Mitchell, 1997). Similarly, E. Blasch correlates uncertainty reduction with entropy: "uncertainty reduction: gain knowledge from entropy" (Blasch, 2005, p. 18). E. Blasch considers that in information fusion, „all the methods are based on the simple idea: uncertainty reduction" (Blasch, 2005, p. 13).

In the structure plan, in fact we deal with two components: the part (actor and micro-social structure of membership) and the whole (the social network). As part, the actor is defined through role and the concrete relationships they develop with other actors. On the systemic- abstract dimension, the actor appears as a way of meeting, arrival, "departure" of some connections, bridges, links. On the other hand, as node are also shaped the sub-structures of the individual actor belonging (organizations, associations, groups, etc.). Depending on the relations between the actors, we have to deal with casual acquaintances, buddies, friends, relatives, business partners, members of the interest groups (cliques, clans, cliques, factions, etc.) (Vlăduțescu, 2012).

These relationships are reflected on systemic plan as weak or strong connections. The sub-structures, on the other hand, appear as sub-sets of nodes that are in the strength connection. Within the social structure the actors develop among them interdependencies and constraints subsumed to some ideas, objective values, financial exchanges, specific relations of friendship, enmity, hatred, violence, trade etc. As a whole, the social structure appears as the panel of nodes and connections that represent abstract actors and relevant relations between them. The main elements of a social structure are the actor and his relationships (Vlăduțescu, 2012).

The agent has a decisive role in „structuring of social relations". Anthony Giddens suggests an analysis procedure of social relationships on two dimensions: a) „a syntagmatic dimension, the patterning of social relations in time-space" and b) „a paradigmatic dimension, involving a virtual order of modes of structuring recursively implicated" (Giddens, 1984, p. 17).

From interpersonal interactions result an impersonal structure. About the mode how appears such as structure, John Levi Martin shows: „Structure emerges, perhaps, out of unstructured interactions quite like the emergence of crystalline structure in a seeming fluid" (Martin, 2009, p. 3). It is nameable in this context that Georg Simmel saw the "systems" and the "super-individual-organizations" as „immediate interactions that occur among men" and „have become crystallized (...) as autonomons phenomena" (Simmel, 1950, p. 10). In his opinion, „society, as its life, is constantly being realized, always signifies that individuals are connected by mutual influence and determination" (Simmel, 1950, p. 10). Therefore, in society the systems-organizations permanently crystallize, being the result of instant interactions of individuals connected by influence and determination relationships.

3 (t, i, f)-Neutrosophic Social Structures

In general, each structure is composed from: a space, endowed with a set of axioms (or laws) acting (governing) on it. If the space, or at least one of its axioms, has some numerical indeterminacy of the form (t, i, f) ≠ (1, 0, 0), we consider it as a (t, i, f)-Neutrosophic Social Structure.

Indeterminacy with respect to the space is referred to some elements that partially belong [i.e. with a neutrosophic value (t, i, f) ≠ (1, 0, 0)] to the space, or their appurtenance to the space is unknown.

An axiom (or law) which deals with numerical indeterminacy is called neutrosophic axiom (or law).

We introduce these structures to social structures because in the real world we do not always know exactly or completely the space we work in; and because the axioms (or laws) are not always well defined on this space, or may have indeterminacies when applying them.

Elements of a group/set/space of a social structure:

Type 1 -individual; Type 2 -group, family, click...; Type 3 -social network;

3.1. Numerical Indeterminacy (or Degree of Indeterminacy), which has the form (t, i, f) ≠ (1, 0, 0), where t, i, f are numbers, intervals, or subsets included in the unit interval [0, 1], and it is the base for the (t, i, f)-Neutrosophic Social Structures.

3.2 Indeterminate Space (due to Unknown Element).

Let the set (space) be NH = {4, 6, 7, 9, a}, where the set NH has an unknown element "a", therefore the whole space has some degree of indeterminacy. Neutrosophically, we write a(0, 1, 0), which means the element a is 100% unknown.

Example

We establish a space structure NS = {e1, e2, e3, z}

The established a relation for elements of the space. According to this releation the neutrosophic social structure looks like: e1(1, 0, 0); e2(1, 0, 0); e3(1, 0, 0); z(0, 1, 0)

The element “z” does not belong to this space, it is unknown, it does not observe the law that decide the appurtenance to group/space, this element is an uncertainty 100%.

3.3 Indeterminate Space (due to Partially Known Element).

Given the set M = {3, 4, 9(0.7, 0.1, 0.3)}, we have two elements 3 and 4 which surely belong to M, and one writes them neutrosophically as 3(1, 0, 0) and 4(1, 0, 0), while the third element 9 belongs only partially (70%) to M, its appurtenance to M is indeterminate (10%), and does not belong to M (in a percentage of 30%).

Example 1

We build the space L = {e1, e2, e3, e4}

We establish the relation/law of the structure, opinion about assertion: “In Bucharest the sky is overcast, it’s raining”.

Element	Place	neutrosophic structure	status
e1	Bucharest	(1, 0, 0)	is certainty 100%
e2	Bucharest	(1, 0, 0)	is certainty 100%
e3	Brasov	(0.7, 0.3, 0.2)	is partially certainty 70%
e4	Iasi	(0, 0.8, 0.1)	is uncertainty 80%, this element does not belong to this space/set

Any other new element of space can be inducer of uncertainty if he is not from Bucharest, he is entropy generator, increase the uncertainty.

Example 2

We establish relation/law: Observing the Law of Moses

We establish the T, I, F as neutrosophic status

T : Stone throwing sinful woman to respect the Law of Moses, the woman dies, Jesus is not the Savior of the world;

F : Do not throw the stone; it is not observed the Law of Moses, the woman survives, Jesus breaks the laws;

I : To throw the stone in sinful woman, the first man without sin; the woman is not punished according to the Law of Moses; Jesus is the Savior; But who is without sin?

We define a space M = {a1, a2, a3} composed of three elements a1, a2, a3.

The neutrosophic structure looks like: a1 (0.8, 0, 0); a2(0.2,0.83,0.12); a3 (0.2,0.4,0.85) and the relation/law was mentioned above.

Element a1 partially appurtenances to the space M, 80%.

Elements a2 and a3 do not belong to the defined space because; a2 has 83% indeterminacy comparing 20% true and a3 has 40% indeterminacy and 85% false.

4 Conclusion

Social structures comply essentially with the neutrosophy rules, it is observed the idea of neutrosophy behavior, these structures fall into states (t, i, f) of neutrosophy, they have a multiple spectrum structures, the structure elements are inducing entropy producing uncertainty. A space with an item, it means an opinion, another element induces another opinion, another element in turn induces another opinion, and so on. The opinion of each element of the structure must be respected. In this way it builds a neutrosophic social structure. The result is a very large socio-neutrosophic structure that is intended to be filtered, evaluated, analyzed by scientific algorithms.

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On Strong Interval Valued Neutrosophic Graphs

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Said Broumi, Mohamed Talea, Assia Bakali, Florentin Smarandache (2016). On Strong Interval Valued Neutrosophic Graphs. *Critical Review* XII, 49-71

Abstract

In this paper, we discuss a subclass of interval valued neutrosophic graphs called strong interval valued neutrosophic graphs, which were introduced by Broumi et al. [41]. The operations of Cartesian product, composition, union and join of two strong interval valued neutrosophic graphs are defined. Some propositions involving strong interval valued neutrosophic graphs are stated and proved.

Keyword

Single valued neutrosophic graph, Interval valued neutrosophic graph, Strong interval valued neutrosophic graph, Cartesian product, Composition, Union, Join.

1 Introduction

Neutrosophic set proposed by Smarandache [13, 14] is a powerful tool to deal with incomplete, indeterminate and inconsistent information in real world. It is a generalization of the theory of fuzzy set [30], intuitionistic fuzzy sets [27, 29], interval-valued fuzzy sets [22] and interval-valued intuitionistic fuzzy sets [28]. The neutrosophic set is characterized by a truth-membership degree (t), an indeterminacy-membership degree (i) and a falsity-membership degree (f) independently, which are within the real standard or nonstandard unit interval $]^{-}0, 1^{+}[$. Therefore, if their range is restrained within the real standard unit interval [0, 1], the neutrosophic set is easily applied to engineering problems. For this purpose, Smarandache [48] and Wang et al. [17] introduced the concept of a single valued neutrosophic set (SVNS) as a subclass of the neutrosophic set. The same authors introduced the notion of interval valued neutrosophic sets [18] as subclass of neutrosophic sets in which the value of truth-membership, indeterminacy-membership and falsity-membership degrees are intervals of numbers instead of the real numbers. Recently, the concept of single valued neutrosophic set and interval valued neutrosophic sets have been applied in a wide variety of fields including computer science, enginnering, mathematics, medicine and economics [3, 4, 5, 6, 16, 19, 20, 21, 23, 24, 25, 26, 32, 34, 35, 36, 37, 38, 43].

Lots of works on fuzzy graphs and intuitionistic fuzzy graphs [7, 8, 9, 31, 33] have been carried out and all of them have considered the vertex sets and edge sets as fuzzy and /or intuitionistic fuzzy sets. But, when the relations between nodes (or vertices) in problems are indeterminate, the fuzzy graphs and intuitionistic fuzzy graphs fail.

For this purpose, Smarandache [10, 11] defined four main categories of neutrosophic graphs. Two are based on literal indeterminacy (I), called I-edge neutrosophic graph and I-vertex neutrosophic graph; these concepts are studied deeply and has gained popularity among the researchers due to their applications via real world problems [1, 12, 15, 44, 45, 46]. The two others graphs are based on (t, i, f) components and are called (t, i, f)-edge neutrosophic graph and (t, i, f)-vertex neutrosophic graph; these concepts are not developed at all. Later on, Broumi et al. [40] introduced a third neutrosophic graph model combining the (t, i, f)-edge and and the (t, i, f)-vertex neutrosophic graph, and investigated some of their properties. The third neutrosophic graph model is called single valued neutrosophic graph (SVNG for short).

The single valued neutrosophic graph is the generalization of fuzzy graph and intuitionistic fuzzy graph. The same authors [39] introduced neighborhood degree of a vertex and closed neighborhood degree of vertex in single valued neutrosophic graph as a generalization of neighborhood degree of a vertex and closed neighborhood degree of vertex in fuzzy graph and intuitionistic fuzzy graph. Broumi et al. [41] introduced the concept of interval valued neutrosophic graph, which is a generalization of fuzzy graph, intuitionistic fuzzy graph, interval valued fuzzy graph, interval valued intuitionistic fuzzy graph and single valued neutrosophic graph. Also, Broumi et al. [42] studied some operations on interval valued neutrosophic graphs.

In this paper, motivated by the operations on (crisp) graphs, such as Cartesian product, composition, union and join, we define the operations of Cartesian product, composition, union and join on strong interval valued neutrosophic graphs and investigate some of their properties.

2 Preliminaries

In this section, we mainly recall some notions related to neutrosophic sets, single valued neutrosophic sets, interval valued neutrosophic sets, fuzzy graph and intuitionistic fuzzy graph, interval valued intuitionstic fuzzy graph and interval valued neutrosophic graph, relevant to the present work.

See especially [2, 7, 8, 13, 17, 40, 41] for further details and background.

Definition 2.1 [13]

Let X be a space of points (objects) with generic elements in X denoted by x; then the neutrosophic set A (NS A) is an object having the form A = {< x: $T_A(x)$, $I_A(x)$, $F_A(x)$>, x ∈ X}, where the functions T, I, F: X→]−0,1+[define respectively the a truth-membership function, an indeterminacy-membership function, and a falsity-membership function of the element x ∈ X to the set A with the condition:

$$^{-}0 \leq T_A(x)+ I_A(x)+ F_A(x) \leq 3^{+}. \quad (1)$$

The functions $T_A(x)$, $I_A(x)$ and $F_A(x)$ are real standard or nonstandard subsets of]−0,1+[.

Since it is difficult to apply NSs to practical problems, Smarandache [48] and Wang et al. [16] introduced the concept of a SVNS, which is an instance of a NS and can be used in real scientific and engineering applications.

Definition 2.2 [17]

Let X be a space of points (objects) with generic elements in X denoted by x. A single valued neutrosophic set A (SVNS A) is characterized by truth-membership function $T_A(x)$, an indeterminacy-membership function $I_A(x)$, and a falsity-membership function $F_A(x)$.

For each point x in X $T_A(x)$, $I_A(x)$, $F_A(x)$ ∈ [0, 1].

A SVNS A can be written as —

$$A = \{< x: T_A(x), I_A(x), F_A(x)>, x \in X\}. \quad (2)$$

Definition 2.3 [7]

A fuzzy graph is a pair of functions G = (σ, μ) where σ is a fuzzy subset of a non empty set V and μ is a symmetric fuzzy relation on σ. i.e σ : V → [0,1] and μ: VxV→[0,1] such that μ(uv) ≤ σ(u) ⋀ σ(v) for all u, v ∈ V where *uv* denotes the edge between *u* and *v* and σ(u) ⋀ σ(v) denotes the minimum of σ(u) and σ(v). σ is called the fuzzy vertex set of V and μ is called the fuzzy edge set of E.

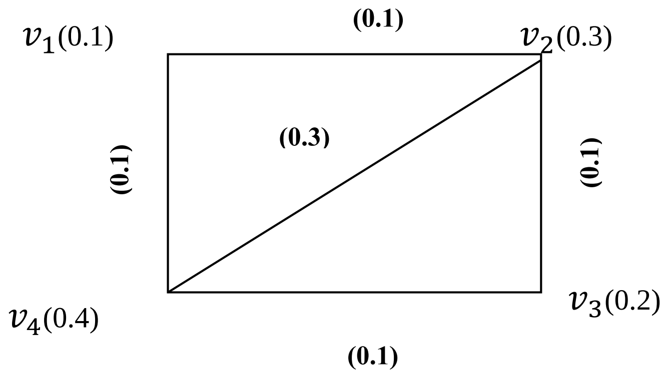

Figure 1. Fuzzy Graph

Definition 2.4 [7]

The fuzzy subgraph H = (τ, ρ) is called a fuzzy subgraph of G = (σ, μ), if τ(u) ≤ σ(u) for all u ∈ V and ρ(u, v) ≤ μ(u, v) for all u, v ∈ V.

Definition 2.5 [8]

An intuitionistic fuzzy graph is of the form G = (V, E), where

i. V={v_1, v_2, ..., v_n} such that μ_1: V→ [0,1] and γ_1: V → [0,1] denote the degree of membership and nonmembership of the element v_i ∈ V, respectively, and 0 ≤ $\mu_1(v_i) + \gamma_1(v_i)$) ≤ 1 for every v_i ∈ V, (i = 1, 2, ..., n),

ii. E ⊆ V x V where μ_2: VxV→[0,1] and γ_2: VxV→ [0,1] are such that $\mu_2(v_i, v_j) \le \min [\mu_1(v_i), \mu_1(v_j)]$ and $\gamma_2(v_i, v_j) \ge \max [\gamma_1(v_i), \gamma_1(v_j)]$ and $0 \le \mu_2(v_i, v_j) + \gamma_2(v_i, v_j) \le 1$ for every (v_i, v_j) ∈ E, (i, j = 1,2, ..., n)

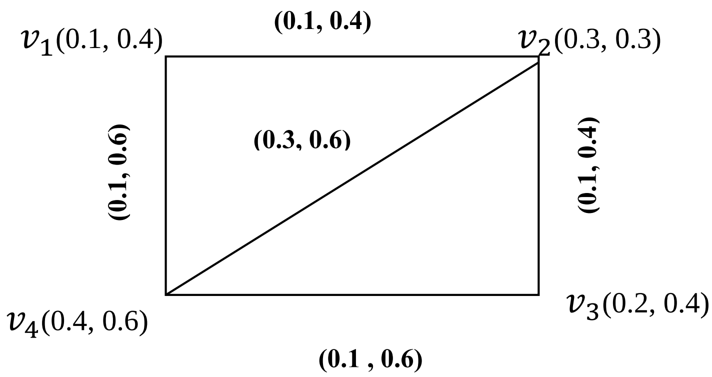

Figure 2. Intuitionistic Fuzzy Graph

Definition 2.6 [40]

Let A = (T_A, I_A, F_A) and B = (T_B, I_B, F_B) be single valued neutrosophic sets on a set X. If A = (T_A, I_A, F_A) is a single valued neutrosophic relation on a set X, then A =(T_A, I_A, F_A) is called a single valued neutrosophic relation on B = (T_B, I_B, F_B) if

$T_B(x, y) \leq \min(T_A(x), T_A(y))$

$I_B(x, y) \geq \max(I_A(x), I_A(y))$ and

$F_B(x, y) \geq \max(F_A x), F_A(y))$

for all x, y ∈ X.
A single valued neutrosophic relation A on X is called symmetric if $T_A(x, y) = T_A(y, x)$, $I_A(x, y) = I_A(y, x)$, $F_A(x, y) = F_A(y, x)$ and $T_B(x, y) = T_B(y, x)$, $I_B(x, y) = I_B(y, x)$ and $F_B(x, y) = F_B(y, x)$, for all x, y ∈ X.

Definition 2.7 [2]

An interval valued intuitionistic fuzzy graph with underlying set V is defined to be a pair G = (A, B), where

1) The functions $M_A : V \to D\ [0, 1]$ and $N_A : V \to D\ [0, 1]$ denote the degree of membership and non membership of the element x ∈ V, respectively, such that 0 such that $0 \leq M_A(x) + N_A(x) \leq 1$ for all x ∈ V.

2) The functions $M_B : E \subseteq V \times V \to D\ [0, 1]$ and $N_B : E \subseteq V \times V \to D\ [0, 1]$ are defined by

$M_{BL}(x, y)) \leq \min\ (M_{AL}(x), M_{AL}(y))$ and $N_{BL}(x, y)) \geq \max\ (N_{AL}(x), N_{AL}(y))$,

$M_{BU}(x, y)) \leq \min\ (M_{AU}(x), M_{AU}(y))$ and $N_{BU}(x, y)) \geq \max\ (N_{AU}(x), N_{AU}(y))$,

such that

$0 \leq M_{BU}(x, y)) + N_{BU}(x, y)) \leq 1$,

for all (x, y) ∈ E.

Definition 2.8 [41]

By an interval-valued neutrosophic graph of a graph G^* = (V, E) we mean a pair G = (A, B), where A = <[T_{AL}, T_{AU}], [I_{AL}, I_{AU}], [F_{AL}, F_{AU}]> is an interval-valued neutrosophic set on V and B =< [T_{BL}, T_{BU}], [I_{BL}, I_{BU}], [F_{BL}, F_{BU}]> is an interval-valued neutrosophic relation on E satisfies the following conditions:

1. V= $\{v_1, v_2, \ldots, v_n\}$ such that T_{AL}:V → [0, 1], T_{AU}:V → [0, 1], I_{AL}:V → [0, 1], I_{AU}:V→[0, 1] and F_{AL}:V→[0, 1], F_{AU}:V→[0, 1] denote the degree of truth-membership, the degree of indeterminacy-membership and falsity-membership of the element y ∈ V, respectively, and

$0 \leq T_A(v_i) + I_A(v_i) + F_A(v_i) \leq 3$ for all v_i ∈ V (I = 1, 2, …, n).

2. The functions T_{BL}:V x V →[0, 1], T_{BU}:V x V →[0, 1], I_{BL}:V x V →[0, 1], I_{BU}:V x V →[0, 1] and F_{BL}:V x V →[0,1], F_{BU}:V x V →[0, 1] are such that

$$T_{BL}(\{v_i, v_j\}) \leq \min\,[T_{AL}(v_i), T_{AL}(v_j)],$$

$$T_{BU}(\{v_i, v_j\}) \leq \min\,[T_{AU}(v_i), T_{AU}(v_j)],$$

$$I_{BL}(\{v_i, v_j\}) \geq \max[I_{BL}(v_i), I_{BL}(v_j)],$$

$$I_{BU}(\{v_i, v_j\}) \geq \max[I_{BU}(v_i), I_{BU}(v_j)],$$

$$\mathrm{F_{BL}}(\{\mathrm{v_i}, \mathrm{v_j}\}) \geq \max[\mathrm{F_{BL}}(\mathrm{v_i}), \mathrm{F_{BL}}(\mathrm{v_j})],$$

$$\mathrm{F_{BU}}(\{\mathrm{v_i}, \mathrm{v_j}\}) \geq \max[\mathrm{F_{BU}}(\mathrm{v_i}), \mathrm{F_{BU}}(\mathrm{v_j})],$$

denote the degree of truth-membership, indeterminacy-membership and falsity-membership of the edge $(v_i, v_j) \in$ E respectively, where

$$0 \leq T_B(\{v_i, v_j\}) + I_B(\{v_i, v_j\}) + F_B(\{v_i, v_j\}) \leq 3,$$

for all $\{v_i, v_j\} \in$ E (i, j = 1, 2,..., n).

We call A the interval valued neutrosophic vertex set of V, B the interval valued neutrosophic edge set of E, respectively. Note that B is a symmetric interval valued neutrosophic relation on A. We use the notation (v_i, v_j) for an element of E. Thus, G = (A, B) is a interval valued neutrosophic graph of G^*= (V, E) if —

$$T_{BL}(v_i, v_j) \leq \min\,[T_{AL}(v_i), T_{AL}(v_j)],$$

$$T_{BU}(v_i, v_j) \leq \min\,[T_{AU}(v_i), T_{AU}(v_j)],$$

$$I_{BL}(v_i, v_j) \geq \max[I_{BL}(v_i), I_{BL}(v_j)],$$

$$I_{BU}(v_i, v_j) \geq \max[I_{BU}(v_i), I_{BU}(v_j)],$$

$$F_{BL}(v_i, v_j) \geq \max[F_{BL}(v_i), F_{BL}(v_j)],$$

$$F_{BU}(v_i, v_j) \geq \max[F_{BU}(v_i), F_{BU}(v_j)], \quad \text{for all } (v_i, v_j) \in \text{E}.$$

Hereafter, we use the notation xy for (x, y) an element of E.

3 Strong Interval Valued Neutrosophic Graph

Throught this paper, we denote G^* = (V, E) a crisp graph, and G = (A, B) an interval valued neutrosophic graph.

Definition 3.1

An interval valued neutrosophic graph G = (A, B) is called strong interval valued neutrosophic graph if

$$T_{BL}(xy) = \min\,(T_{AL}(x), T_{AL}(y)),\ I_{BL}(xy) = \max\,(I_{AL}(x), I_{AL}(y)) \text{ and}$$
$$F_{BL}(xy) = \max\,(F_{AL}(x), F_{AL}(y))$$

$T_{BU}(xy)$= min $(T_{AU}(x), T_{AU}(y))$, $I_{BU}(xy)$ = max $(I_{AU}(x), I_{AU}(y))$ and $F_{BU}(xy)$ =max $(F_{AU}(x), F_{AU}(y))$ such that

$0 \le T_{BU}(x,y)) + I_{BU}(x,y)) + F_{BU}(x,y)) \le 3$, for all $x, y \in$ E.

Example 3.2

Figure 1 is an example for IVNG, G=(A, B) defined on a graph G^* = (V, E) such that V = {x, y, z}, E = {xy, yz, zx}, A is an interval valued neutrosophic set of V.

A={<x, [0.5, 0.7], [0.2, 0.3], [0.1, 0.3]>,<y, [0.6, 0.7], [0.2, 0.4], [0.1, 0.3]>, <z, [0.4, 0.6], [0.1, 0.3], [0.2, 0.4],>},

B={< xy, [0.3, 0.6], [0.2, 0.4], [0.2, 0.4]>, <yz, [0.3, 0.5], [0.2, 0.3], [0.2, 0.4]>, <xz, [0.3, 0.5], [0.1, 0.5], [0.2, 0.4]>}.

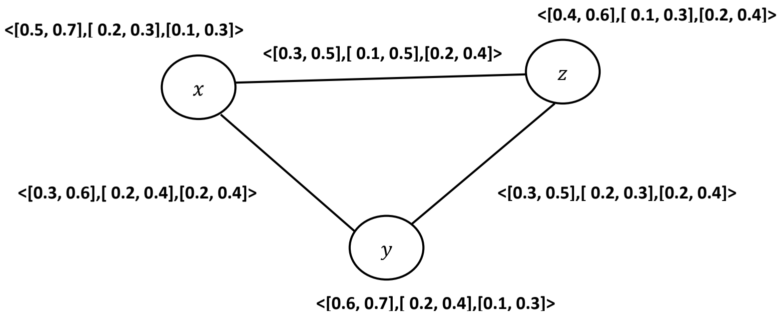

Figure 3. Interval valued neutrosophic graph

Example 3.2

Figure 2 is a SIVNG G = (A, B), where

A={<x, [0.5, 0.7], [0.1, 0.4], [0.1, 0.3]>, <y, [0.6, 0.7],[0.2, 0.3], [0.1, 0.3]>, <z, [0.4, 0.6],[0.2, 0.3], [0.2, 0.4],>},

B={< xy, [0,5, 0.7], [0.2 0.4], [0.1, 0.3]>, <yz, [0.4, 0.6],[0.2, 0.3], [0.2, 0.4]>, <xz, [0.4, 0.6], [0.2, 0.4], [0.2, 0.4]>}

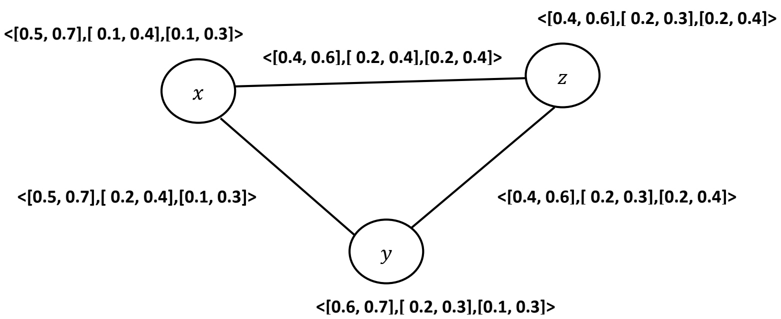

Figure 4. Strong Interval valued neutrosophic graph.

Proposition 3.3

A strong interval valued neutrosophic graph is the generalization of strong interval valued fuzzy graph.

Proof

Suppose G=(V, E) is a strong interval valued neutrosophic graph. Then, by setting the indeterminacy-membership and falsity-membership values of vertex set and edge set equals to zero, the strong interval valued neutrosophic graph is reduced to strong interval valued fuzzy graph.

Proposition 3.4

A strong interval valued neutrosophic graph is the generalization of strong interval valued intuitionistic fuzzy graph.

Proof

Suppose G=(V, E) is a strong interval valued neutrosophic graph. Then by setting the indeterminacy-membership values of vertex set and edge set equals to zero reduces the strong interval valued neutrosophic graph to strong interval valued intuitionistic fuzzy graph.

Proposition 3.5

A strong interval valued neutrosophic graph is the generalization of strong intuitionistic fuzzy graph.

Proof

Suppose G=(V, E) is a strong interval valued neutrosophic graph. Then by setting the indeterminacy-membership, upper truth-membership and upper falsity-membership values of vertex set and edge set equals to zero reduces the strong interval valued neutrosophic graph to strong intuitionistic fuzzy graph.

Proposition 3.6

A strong interval valued neutrosophic graph is the generalization of strong single neutrosophic graph.

Proof

Suppose G = (V, E) is a strong interval valued neutrosophic graph. Then by setting the upper truth-membership equals lower truth-membership, upper indeterminacy-membership equals lower indeterminacy-membership and upper falsity-membership equals lower falsity-membership values of vertex set and edge set reduces the strong interval valued neutrosophic graph to strong single valued neutrosophic graph.

Definition 3.7

Let A_1 and A_2 be interval-valued neutrosophic subsets of V_1 and V_2 respectively. Let B_1 and B_2 interval-valued neutrosophic subsets of E_1 and E_2 respectively. The Cartesian product of two SIVNGs G_1 and G_2 is denoted by $G_1 \times G_2 = (A_1 \times A_2$, $B_1 \times B_2$) and is defined as follows:

1) $(T_{A_1L} \times T_{A_2L})\ (x_1, x_2) = \min\ (T_{A_1L}(x_1), T_{A_2L}(x_2))$
$(T_{A_1U} \times T_{A_2U})\ (x_1, x_2) = \min\ (T_{A_1U}(x_1), T_{A_2U}(x_2))$
$(I_{A_1L} \times I_{A_2L})\ (x_1, x_2) = \max\ (I_{A_1L}(x_1), I_{A_2L}(x_2))$
$(I_{A_1U} \times I_{A_2U})\ (x_1, x_2) = \max\ (I_{A_1U}(x_1), I_{A_2U}(x_2))$
$(F_{A_1L} \times F_{A_2L})\ (x_1, x_2) = \max\ (F_{A_1L}(x_1), F_{A_2L}(x_2))$
$(F_{A_1U} \times F_{A_2U})\ (x_1, x_2) = \max\ (F_{A_1U}(x_1), F_{A_2U}(x_2))$ for all $(x_1, x_2) \in V$

2) $(T_{B_1L} \times T_{B_2L})\ ((x, x_2)(x, y_2)) = \min\ (T_{A_1L}(x), T_{B_2L}(x_2y_2))$
$(T_{B_1U} \times T_{B_2U})\ ((x, x_2)(x, y_2)) = \min\ (T_{A_1U}(x), T_{B_2U}(x_2y_2))$
$(I_{B_1L} \times I_{B_2L})\ ((x, x_2)(x, y_2)) = \max\ (I_{A_1L}(x), I_{B_2L}(x_2y_2))$
$(I_{B_1U} \times I_{B_2U})\ ((x, x_2)(x, y_2)) = \max\ (I_{A_1U}(x), I_{B_2U}(x_2y_2))$
$(F_{B_1L} \times F_{B_2L})\ ((x, x_2)\ (x, y_2)) = \max\ (F_{A_1L}(x), F_{B_2L}(x_2y_2))$
$(F_{B_1U} \times F_{B_2U})\ ((x, x_2)(x, y_2)) = \max(F_{A_1U}(x), F_{B_2U}(x_2y_2))$ $\forall$ x $\in$ V_1 and $\forall\ x_2y_2 \in E_2$

3) $(T_{B_1L} \times T_{B_2L})\ ((x_1, z)\ (y_1, z)) = \min\ (T_{B_1L}(x_1y_1), T_{A_2L}(z))$
$(T_{B_1U} \times T_{B_2U})\ ((x_1, z)\ (y_1, z)) = \min\ (T_{B_1U}(x_1y_1), T_{A_2U}(z))$
$(\mathrm{I}_{B_1L} \times \mathrm{I}_{B_2L})\ ((x_1, z)\ (y_1, z)) = \max\ (\mathrm{I}_{B_1L}(x_1y_1), \mathrm{I}_{A_2L}(z))$
$(I_{B_1U} \times I_{B_2U})\ ((x_1, z)\ (y_1, z)) = \max\ (I_{B_1U}(x_1y_1), I_{A_2U}(z))$
$(F_{B_1L} \times F_{B_2L})\ ((x_1, z)\ (y_1, z)) = \max\ (F_{B_1L}(x_1y_1), F_{A_2L}(z))$
$(F_{B_1U} \times F_{B_2U})\ ((x_1, z)\ (y_1, z)) = \max\ (F_{B_1U}(x_1y_1), F_{A_2U}(z))$ $\forall$ z $\in V_2$ and $\forall\ x_1y_1 \in E_1$

Proposition 3.7

If G_1 and G_2 are the strong interval valued neutrosophic graphs, then the cartesian product G_1x G_2 is a strong interval valued neutrosophic graph.

Proof

Let G_1 and G_2 are SIVNGs, there exist $x_i, y_i \in E_i$, i= 1, 2 such that

$T_{B_iL}(x_i, y_i) = \min\ (T_{A_iL}(x_i), T_{A_iL}(y_i))$, i =1, 2.

$T_{B_iU}(x_i, y_i)$ = min $(T_{A_iU}(x_i), T_{A_iU}(y_i))$, i =1, 2.
$I_{B_iL}(x_i, y_i)$ = max $(I_{A_iL}(x_i), I_{A_iL}(y_i))$, i =1, 2.
$I_{B_iU}(x_i, y_i)$ = max $(I_{A_iU}(x_i), I_{A_iU}(y_i))$, i =1, 2.
$F_{B_iL}(x_i, y_i)$ = max $(F_{A_iL}(x_i), F_{A_iL}(y_i))$, i =1, 2.
$F_{B_iU}(x_i, y_i)$ = max $(F_{A_iU}(x_i), F_{A_iU}(y_i))$, i =1, 2.

Let E= $\{(x, x_2)\ (x, y_2)\ /x \in V_1, x_2y_2 \in E_2\} \cup \{(x_1, z)\ (y_1, z)\ /z \in V_2, x_1y_1 \in E_1\}$.

Consider, $(x, x_2)\ (x, y_2) \in E$, we have

$(T_{B_1L} \times T_{B_2L})\ ((x, x_2)\ (x, y_2))$ = min $(T_{A_1L}(x),\ T_{B_2L}(x_2y_2))$

= min $(T_{A_1L}(x), T_{A_2L}(x_2), T_{A_2L}(y_2))$

Similarly,

$(T_{B_1U} \times T_{B_2U})\ ((x, x_2)\ (x, y_2))$ = min $(T_{A_1U}(x), T_{B_2U}(x_2y_2))$

= min $(T_{A_1U}(x), T_{A_2U}(x_2), T_{A_2U}(y_2))$

$(T_{A_1L} \times T_{A_2L})\ (x_1, x_2)$ = min $(T_{A_1L}(x_1), T_{A_2L}(x_2))$

$(T_{A_1U} \times T_{A_2U})\ (x_1, x_2)$ = min $(T_{A_1U}(x_1), T_{A_2U}(x_2))$

$(T_{A_1L} \times T_{A_2L})\ (x_1, y_2)$ = min $(T_{A_1L}(x_1), T_{A_2L}(y_2))$

$(T_{A_1U} \times T_{A_2U})\ (x_1, y_2)$ = min $(T_{A_1U}(x_1), T_{A_2U}(y_2))$

Min $((T_{A_1U} \times T_{A_2U})\ (x, x_2), (T_{A_1U} \times T_{A_2U})\ (x, y_2))$

= min (min $(T_{A_1U}(x), T_{A_2U}(x_2))$, min $(T_{A_1U}(x), T_{A_2U}(y_2))$)

= min $(T_{A_1U}(x), T_{A_2U}(x_2), T_{A_1U}(y_2))$

Hence

$(T_{B_1L} \times T_{B_2L})\ ((x, x_2)\ (x, y_2))$ = min $((T_{A_1L} \times T_{A_2L})\ (x, x_2), (T_{A_1L} \times T_{A_2L})\ (x, y_2))$

$(T_{B_1U} \times T_{B_2U})\ ((x, x_2)\ (x, y_2))$ = min $((T_{A_1U} \times T_{A_2U})\ (x, x_2), (T_{A_1U} \times T_{A_2U})\ (x, y_2))$.

Similarly, we can show that —

$(I_{B_1L} \times I_{B_2L})\ ((x, x_2)\ (x, y_2))$ = max $((I_{A_1L} \times I_{A_2L})\ (x, x_2), (I_{A_1L} \times I_{A_2L})\ (x, y_2))$

$(I_{B_1U} \times I_{B_2U})\ ((x, x_2)\ (x, y_2))$ = max $((I_{A_1U} \times I_{A_2U})\ (x, x_2), (I_{A_1U} \times I_{A_2U})\ (x, y_2))$.

And also

$(F_{B_1L} \times F_{B_2L})$ $((x, x_2)$ $(x, y_2))$ = max $((F_{A_1L} \times F_{A_2L})$ (x, x_2), $(F_{A_1L} \times F_{A_2L})$ $(x, y_2))$

$(F_{B_1U} \times F_{B_2U})$ $((x, x_2)$ $(x, y_2))$ = max $((F_{A_1U} \times F_{A_2U})$ (x, x_2), $(F_{A_1U} \times F_{A_2U})$ $(x, y_2))$.

Hence, G_1 x G_2 strong interval valued neutrosophic graph. This completes the proof.

Proposition 3.8

If G_1 x G_2 is strong interval valued neutrosophic graph, then at least G_1 or G_2 must be strong.

Proof

Let G_1 and G_2 be no strong interval valued neutrosophic graphs; there exists $x_i, y_i \in E_i$, I = 1, 2, such that

$T_{B_iL}(x_i, y_i) <$ min $(T_{A_iL}(x_i), T_{A_iL}(y_i))$, i =1, 2.
$T_{B_iU}(x_i, y_i) <$ min $(T_{A_iU}(x_i), T_{A_iU}(y_i))$, i =1, 2.
$I_{B_iL}(x_i, y_i) >$ max $(I_{A_iL}(x_i), I_{A_iL}(y_i))$, i =1, 2.
$I_{B_iU}(x_i, y_i) >$ max $(I_{A_iU}(x_i), I_{A_iU}(y_i))$, i =1, 2.
$F_{B_iL}(x_i, y_i) >$ max $(F_{A_iL}(x_i), F_{A_iL}(y_i))$, i =1, 2.
F_{B_iU} $(x_i, y_i) >$ max $(F_{A_iU}(x_i), F_{A_iU}(y_i))$, i =1, 2.

Let E= $\{(x, x_2)$ (x, y_2) $/x \in V_1, x_2y_2 \in E_2\} \cup \{(x_1, z)$ (y_1, z) $/z \in V_2, x_1y_1 \in E_1\}$

Consider, (x, x_2) $(x, y_2) \in E$, we have

$(T_{B_1L} \times T_{B_2L})$ $((x, x_2)$ $(x, y_2))$ = min $(T_{A_1L}(x),$ $T_{B_2L}(x_2y_2))$

$<$ min $(T_{A_1L}(x), T_{A_2L}(x_2), T_{A_2L}(y_2))$.

Similarly —

$(T_{B_1U} \times T_{B_2U})$ $((x, x_2)$ $(x, y_2))$ = min $(T_{A_1U}(x), T_{B_2U}(x_2y_2))$

$<$ min $(T_{A_1U}(x), T_{A_2U}(x_2), T_{A_2U}(y_2))$

$(T_{A_1L} \times T_{A_2L})$ (x_1, x_2) = min $(T_{A_1L}(x_1), T_{A_2L}(x_2))$

$(T_{A_1U} \times T_{A_2U})$ (x_1, x_2) = min $(T_{A_1U}(x_1), T_{A_2U}(x_2))$

$(T_{A_1L} \times T_{A_2L})$ (x_1, y_2) = min $(T_{A_1L}(x_1), T_{A_2L}(y_2))$

$(T_{A_1U} \times T_{A_2U})$ (x_1, y_2) = min $(T_{A_1U}(x_1), T_{A_2U}(y_2))$

min $((T_{A_1U} \times T_{A_2U})$ (x, x_2), $(T_{A_1U} \times T_{A_2U})$ $(x, y_2))$

= min (min $(T_{A_1U}(x), T_{A_2U}(x_2))$, min $(T_{A_1U}(x), T_{A_2U}(y_2)))$

= min $(T_{A_1U}(x), T_{A_2U}(x_2), T_{A_1U}(y_2))$.

Hence

$(T_{B_1L} \times T_{B_2L})$ $((x, x_2)$ $(x, y_2)) <$ min $((T_{A_1L} \times T_{A_2L})$ (x, x_2), $(T_{A_1L} \times T_{A_2L})$ $(x, y_2))$,

$(T_{B_1U} \times T_{B_2U})$ $((x, x_2)$ $(x, y_2)) <$ min $((T_{A_1U} \times T_{A_2U})$ (x, x_2), $(T_{A_1U} \times T_{A_2U})$ $(x, y_2))$.

Similarly, we can show that

$(I_{B_1L} \times I_{B_2L})$ $((x, x_2)$ $(x, y_2)) >$ max $((I_{A_1L} \times I_{A_2L})$ (x, x_2), $(I_{A_1L} \times I_{A_2L})$ $(x, y_2))$,

$(I_{B_1U} \times I_{B_2U})$ $((x, x_2)$ $(x, y_2)) >$ max $((I_{A_1U} \times I_{A_2U})$ (x, x_2), $(I_{A_1U} \times I_{A_2U})$ $(x, y_2))$.

And also

$(F_{B_1L} \times F_{B_2L})$ $((x, x_2)$ $(x, y_2)) >$ max $((F_{A_1L} \times F_{A_2L})$ (x, x_2), $(F_{A_1L} \times F_{A_2L})$ $(x, y_2))$,

$(F_{B_1U} \times F_{B_2U})$ $((x, x_2)$ $(x, y_2)) >$ max $((F_{A_1U} \times F_{A_2U})$ (x, x_2), $(F_{A_1U} \times F_{A_2U})$ $(x, y_2))$.

Hence, G_1 x G_2 is not strong interval valued neutrosophic graph, which is a contradiction. This completes the proof.

Remark 3.9

If G_1 is a SIVNG and G_2 is not a SIVNG, then G_1 x G_2 is need not be an SIVNG.

Example 3.10

Let G_1= (A_1,B_1) be a SIVNG, where A_1= {< a, [0.6, 0.7], [0.2, 0.5], [0.1, 0.3]> ,< b, [0.6, 0.7], [0.2, 0.5], [0.1, 0.3]>} and B_1= {< ab, [0.6, 0.7], [0.2, 0.5], [0.1, 0.3]>}

G_1

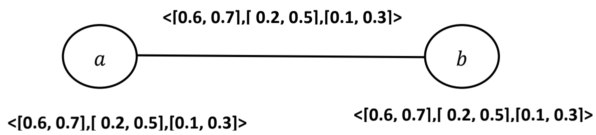

Figure 5. Interval valued neutrosophic G_1.

G_2= (A_2,B_2) is not a SIVNG, where A_2= {< c, [0.4, 0.6], [0.2, 0.4], [0.1, 0.3]>, < d, [0.4, 0.6], [0.1,0.3], [0.2, 0.4] >} and B_2= < cd, [0.3,0.5], [0.1, 0.2], [0.3, 0.5]>}.

G_2

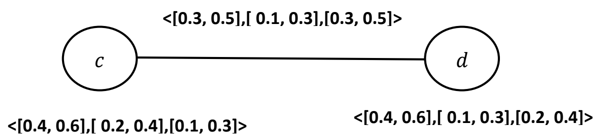

Figure 6. Interval valued neutrosophic G_2.

G_1x G_2= (A_1x A_2, B_1x B_2) is not a SIVNG, where

A_1x A_2= {< (a, c), [0.4, 0.6],[0.2, 0.3], [0.2, 0.4]>, < (a, d), [0.4, 0.6], [0.2, 0.3], [0.2, 0.4]>, < (b, c), [0.4, 0.6], [0.2, 0.6], [0.2, 0.4]>, < (b, d), [0.4, 0.6], [0.3, 0.4], [0.2, 0.4]>},

B_1x B_2= {< ((a, c), (a, d)), [0.3, 0.5], [0.3, 0.5], [0.3, 0.5] >, < ((a, c), (b, c)), [0.4, 0.6], [0.1, 0.4], [0.3, 0.4] >, < ((b, c), (b, d)), [0.4, 0.6], [0.2, 0.4], [0.2, 0.4] >, < ((a, d), (b, d)),[0.4, 0.6], [0.2, 0.4], [0.2, 0.4] >}. In this example, G_1 is a SIVNG and $G_2$is not a SIVNG, then G_1x $G_2$is not a SIVNG.

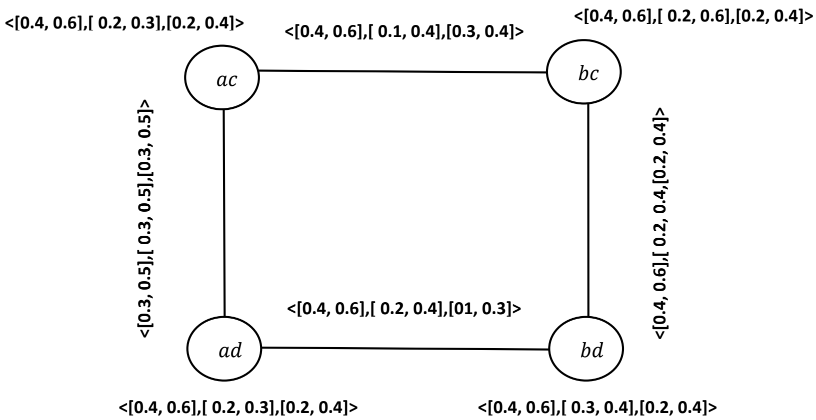

Figure 7. Cartesian product G_1x G_2

Example 3.11

Let G_1= (A_1, B_1) be a SIVNG, where A_1= {< a, [0.4, 0.6], [0.2, 0.4], [0.1, 0.3] >, < b, [0.4, 0.6], [0.2, 0.4], [0.1, 0.3]>} and B_1={< ab, [0.4, 0.6], [0.2, 0.4], [0.1, 0.3] >,

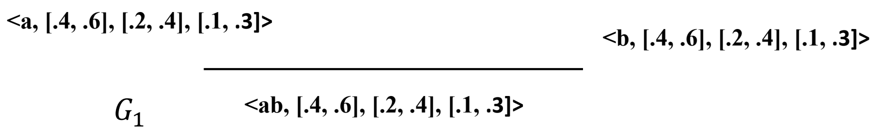

Figure 8. Interval valued neutrosophic G_1.

G_2= (A_2, B_2) is not a SIVIFG, where A_2= {< c, [0.6, 0.7], [0.1, 0.3], [0.1, 0.3] >, < d, [0.6, 0.7], [0.1, 0.3], [0.2, 0.4] >} and B_2= {< cd, [0.5, 0.6], [0.2, 0.4], [0.2, 0.4]>},

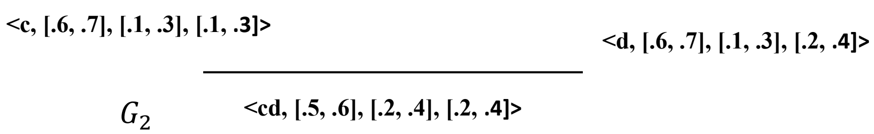

Figure 9. Interval valued neutrosophic G_2.

G_1x G_2= (A_1x A_2, B_1x B_2) is a SIVNG, where

A_1x A_2= {< (a, c), [0.4, 0.6], [0.2, 0.4], [0.1, 0.3] >, < (a, d), [0.4, 0.6], [0.2, 0.4], [0.2, 0.4] >, < (b, c), [0.4, 0.6], [0.2, 0.4], [0.1, 0.3] >, < (b, d), [0.4, 0.6], [0.2, 0.4], [0.2, 0.4] >} and

B_1x B_2= {< ((a, c), (a, d)), [0.4, 0.6], [0.2, 0.4], [0.2, 0.4] >, < ((a, c), (b, c)), [0.4, 0.6], [0.2, 0.4], [0.1, 0.3] >, < ((b, c), (b, d)), [0.4, 0.6], [0.2, 0.4], [0.2, 0.4] >, < ((a, d), (b, d)), [0.4, 0.6], [0.2, 0.4], [0.2, 0.4]>}. In this example, $G_1$is a SIVNG and $G_2$is not a SIVNG, then G_1x $G_2$is a SIVNG.

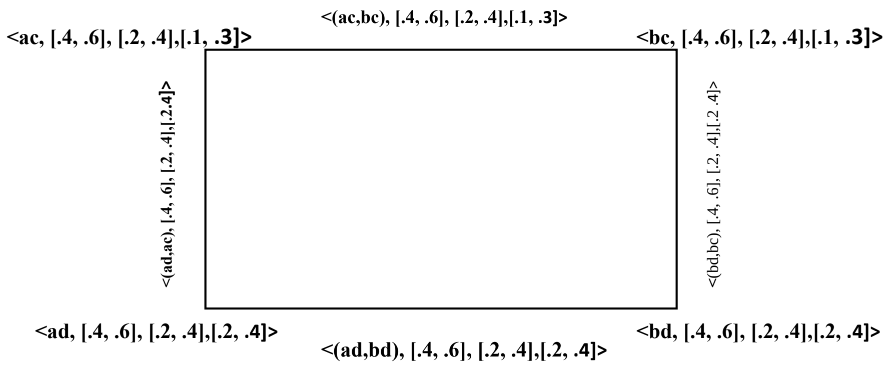

Figure 10. Cartesian product

Proposition 3.12

Let G_1be a strong interval valued neutrosophic graph. Then for any interval valued neutrosophic graph G_2,G_1x G_2 is strong interval valued neutrosophic graph iff

$T_{A_1L}(x_1) \leq T_{B_1L}(x_2y_2)$, $I_{A_1L}(x_1) \geq I_{B_1L}(x_2y_2)$ and $F_{A_1L}(x_1) \geq F_{B_1L}(x_2y_2)$,

$T_{A_1U}(x_1) \leq T_{B_1U}(x_2y_2)$, $I_{A_1U}(x_1) \geq I_{B_1U}(x_2y_2)$ and $F_{A_1U}(x_1) \geq F_{B_1U}(x_2y_2)$, $\forall\, x_1 \in V_1$,$x_2y_2 \in E_2$.

Definition 3.13

Let $A_1$and A_2be interval valued neutrosophic subsets of $V_1$and V_2respectively. Let $B_1$and B_2 interval-valued neutrosophic subsets of E_1 and E_2 respectively. The *composition* of two strong interval valued neutrosophic graphs G_1 and $G_2$is denoted by $G_1[\,G_2]$ = ($A_1 \circ A_2$, $B_1 \circ B_2$) and is defined as follows

1) $(T_{A_1L} \circ T_{A_2L})\,(x_1, x_2)$ = min $(T_{A_1L}(x_1), T_{A_2L}(x_2))$
   $(T_{A_1U} \circ T_{A_2U})\,(x_1, x_2)$ = min $(T_{A_1U}(x_1), T_{A_2U}(x_2))$
   $(I_{A_1L} \circ I_{A_2L})\,(x_1, x_2)$ = max $(I_{A_1L}(x_1), I_{A_2L}(x_2))$
   $(I_{A_1U} \circ I_{A_2U})\,(x_1, x_2)$ = max $(I_{A_1U}(x_1), I_{A_2U}(x_2))$

$(F_{A_1L} \circ F_{A_2L})$ (x_1, x_2) = max $(F_{A_1L}(x_1), F_{A_2L}(x_2))$
$(F_{A_1U} \circ F_{A_2U})$ (x_1, x_2) = max $(F_{A_1U}(x_1), F_{A_2U}(x_2))$ $\forall x_1 \in V_1, x_2 \in V_2$

2) $(T_{B_1L} \circ T_{B_2L})$ $((x, x_2)(x, y_2))$ = min $(T_{A_1L}(x), T_{B_2L}(x_2y_2))$
$(T_{B_1U} \circ T_{B_2U})$ $((x, x_2)(x, y_2))$ = min $(T_{A_1U}(x), T_{B_2U}(x_2y_2))$
$(I_{B_1L} \circ I_{B_2L})$ $((x, x_2)(x, y_2))$ = max $(I_{A_1L}(x), I_{B_2L}(x_2y_2))$
$(I_{B_1U} \circ I_{B_2U})$ $((x, x_2)(x, y_2))$ = max $(I_{A_1U}(x), I_{B_2U}(x_2y_2))$
$(F_{B_1L} \circ F_{B_2L})$ $((x, x_2)$ $(x, y_2))$ = max $(F_{A_1L}(x), F_{B_2L}(x_2y_2))$
$(F_{B_1U} \circ F_{B_2U})$ $((x, x_2)(x, y_2))$ = max $(F_{A_1U}(x), F_{B_2U}(x_2y_2))$ $\forall x \in V_1, \forall x_2y_2 \in E_2$

3) $(T_{B_1L} \circ T_{B_2L})$ $((x_1, z)$ $(y_1, z))$ = min $(T_{B_1L}(x_1y_1), T_{A_2L}(z))$
$(T_{B_1U} \circ T_{B_2U})$ $((x_1, z)$ $(y_1, z))$ = min $(T_{B_1U}(x_1y_1), T_{A_2U}(z))$
$(I_{B_1L} \circ I_{B_2L})$ $((x_1, z)$ $(y_1, z))$ = max $(I_{B_1L}(x_1y_1), I_{A_2L}(z))$
$(I_{B_1U} \circ I_{B_2U})$ $((x_1, z)$ $(y_1, z))$ = max $(I_{B_1U}(x_1y_1), I_{A_2U}(z))$
$(F_{B_1L} \circ F_{B_2L})$ $((x_1, z)$ $(y_1, z))$ = max $(F_{B_1L}(x_1y_1), F_{A_2L}(z))$
$(F_{B_1U} \circ F_{B_2U})$ $((x_1, z)$ $(y_1, z))$ = max $(F_{B_1U}(x_1y_1), F_{A_2U}(z))$ $\forall z \in V_2, \forall x_1y_1 \in E_1$

4) $(T_{B_1L} \circ T_{B_2L})$ $((x_1, x_2)$ $(y_1, y_2))$ = min $(T_{A_2L}(x_2), T_{A_2L}(y_2), T_{B_1L}(x_1y_1))$
$(T_{B_1U} \circ T_{B_2U})$ $((x_1, x_2)$ $(y_1, y_2))$ = min $(T_{A_2U}(x_2), T_{A_2U}(y_2), T_{B_1U}(x_1y_1))$
$(I_{B_1L} \circ I_{B_2L})$ $((x_1, x_2)$ $(y_1, y_2))$ = max$(I_{A_2L}(x_2), I_{A_2L}(y_2), I_{B_1L}(x_1y_1))$
$(I_{B_1U} \circ I_{B_2U})$ $((x_1, x_2)$ $(y_1, y_2))$= max $(I_{A_2U}(x_2), I_{A_2U}(y_2), I_{B_1U}(x_1y_1))$
$(F_{B_1L} \circ F_{B_2L})$ $((x_1, x_2)$ $(y_1, y_2))$ = max$(F_{A_2L}(x_2), F_{A_2L}(y_2), F_{B_1L}(x_1y_1))$
$(F_{B_1U} \circ F_{B_2U})$ $((x_1, x_2)$ $(y_1, y_2))$ = max $(F_{A_2U}(x_2), F_{A_2U}(y_2), F_{B_1U}(x_1y_1))$
$\forall (x_1, x_2)(y_1, y_2) \in E^0$ -E , where E^0 = E $\cup$ $\{(x_1, x_2)(y_1, y_2) | x_1y_1 \in E_1, x_2 \neq y_2\}$.

The following propositions are stated without their proof.

Proposition 3.14

If G_1 and G_2 are the strong interval valued neutrosophic graphs, then the composition $G_1[G_2]$ is a strong interval valued neutrosophic graph.

Proposition 3.15

If $G_1[G_2]$ is strong interval valued neutrosophic graphs, then at least composition G_1 or G_2 must be strong.

Example 3.16

Let G_1= (A_1,B_1) be a SIVNG, where A_1= {< a, [0.6, 0.7], [0.2, 0.3], [0.1, 0.3], > ,< b, [0.6, 0.7], [0.2, 0.3], [0.1, 0.3] >} and B_1= {< ab, [0.6, 0.7], [0.2, 0.3], [0.1, 0.3]>}.

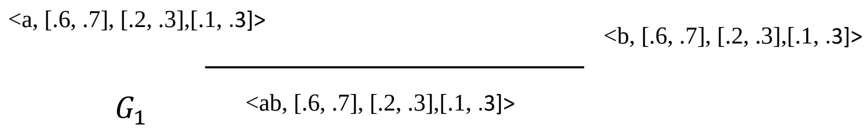

Figure 11. Interval valued neutrosophic G_1.

G_2= (A_2,B_2) is not a SIVNG, where A_2= {< c, [0.4, 0.6], [0.2, 0.4], [0.1, 0.3] >, < d, [0.4, 0.6], [0.2, 0.4], [0.1, 0.3] >} and B_2=<cd, [0.3, 0.5], [0.2, 0.5], [0.3, 0.5]>}.

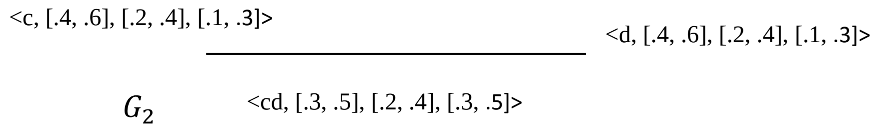

Figure 12. Interval valued neutrosophic G_2.

$G_1[G_2]$= (A_1oA_2, B_1oB_2) is not a SIVNG, where

A_1oA_2= {< (a,c), [0.4, 0.6], [0.2, 0.4], [0.1, 0.3] >, < (a,d), [0.4, 0.6], [0.2, 0.4], [0.1, 0.3] >, < (b, c), [0.4, 0.6], [0.2, 0.4], [0.1, 0.3] >, < (b, d), [0.4, 0.6], [0.2, 0.4], [0.1, 0.3] >},

B_1oB_2= {< ((a, c), (a, d)), [0.3, 0.5], [0.2, 0.4], [0.3, 0.5] >, < ((a, c), (b, c)), [0.4, 0.6], [0.2, 0.4], [0.1, 0.3]>, < ((b, c), (b, d)),[0.3, 0.5], [0.2, 0.4], [0.3, 0.5] >, < ((a, d), (b, d)), [0.4, 0.6], [0.2, 0.4], [0.1, 0.3] >, < ((a, c), (b, d)), [0.4, 0.6], [0.2, 0.4], [0.1, 0.3] >, < ((a, d), (b, c)), [0.4, 0.6], [0.2, 0.4], [0.1, 0.3] >}. In this example, G_1 is a SIVNG and $G_2$is not a SIVNG, then $G_1[G_2]$ is not a SIVNG.

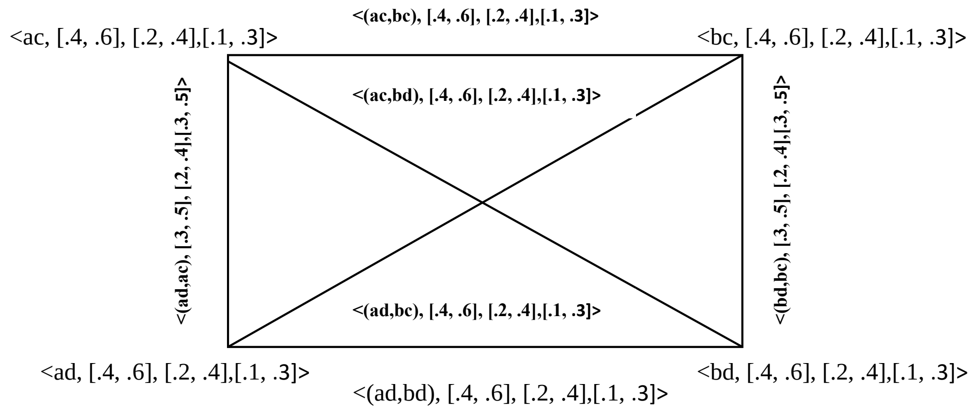

Figure 13. Composition

Example 3.17

Let G_1= (A_1,B_1) be a SIVNG, where A_1= {< a, [0.4, 0.6], [0.2, 0.4], [0.2, 0.4] >, < b, [0.4, 0.6], [0.2, 0.4], [0.2, 0.4] >}and B_1={<ab, [0.4, 0.6], [0.2, 0.4], [0.2, 0.4] >}.

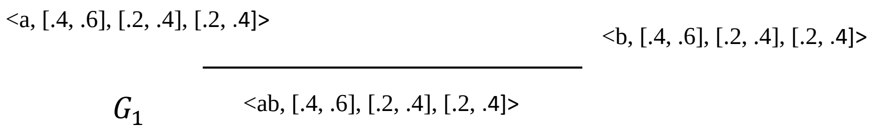

Figure 14. Interval valued neutrosophic G_1.

G_2= (A_2,B_2) is not a SIVNG, where A_2= {< c, [0.6, 0.7], [0.1, 0.3], [0.1, 0.3] >, <d, [0.6, 0.7], [0.2, 0.4], [0.1, 0.3] >} and B_2={< c d, [0.5, 0.6], [0.2, 0.4], [0.2, 0.4] >}.

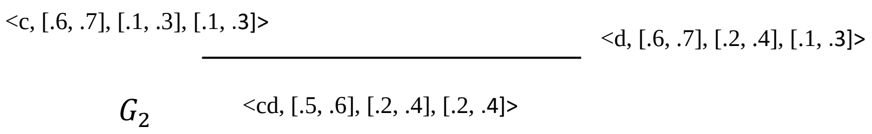

Figure 15. Interval valued neutrosophic G_2.

$G_1[G_2]$ = (A_1oA_2, B_1 oB_2) is a SIVNG, where

A_1oA_2= {< (a, c), [0.4, 0.6], [0.2, 0.4], [0.2, 0.4] >, < (a, d), [0.4, 0.6], [0.2, 0.4], [0.2, 0.4] >, < (b, c), [0.4, 0.6], [0.2, 0.4], [0.2, 0.4] >, < (b, d), [0.4, 0.6], [0.2, 0.4],[0.2, 0.4] >} and

B_1 oB_2= {< ((a, c), (a, d)), [0.4, 0.6], [0.2, 0.4] [0.2, 0.4] >, < ((a, c), (b, c)), [0.4, 0.6], [0.2, 0.4], [0.2, 0.4] >, < ((b, c), (b, d)), [0.4, 0.6], [0,2, 0.4] [0.2, 0.4] >, < ((a, d), (b, d)), [0.4, 0.6], [0.2, 0.4], [0.2, 0.4] >, < ((a, c), (b, d)), [0.4, 0.6], [0.2, 0.4] [0.2, 0.4]>, < ((a, d), (b, c)), [0.4, 0.6], [0.2, 0.4], [0.2, 0.4]>}. In this example, $G_1$is an SIVIFG and $G_2$is not a SIVNG, then $G_1[G_2]$ is a SIVNG.

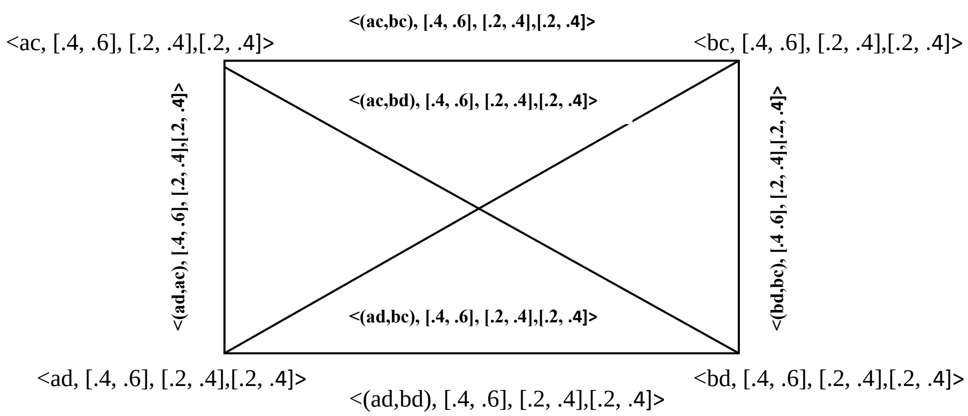

Figure 16. Composition of G_1 and G_2.

Proposition 3.18

Let G_1be a strong interval valued neutrosophic graph. Then for any interval valued neutrosophic graph G_2,$G_1[G_2]$ is strong interval valued neutrosophic graph iff —

$T_{A_1L}(x_1) \leq T_{B_1L}(x_2y_2)$, $I_{A_1L}(x_1) \geq I_{B_1L}(x_2y_2)$ and $F_{A_1L}(x_1) \geq F_{B_1L}(x_2y_2)$,

$T_{A_1U}(x_1) \leq T_{B_1U}(x_2y_2)$, $I_{A_1U}(x_1) \geq I_{B_1U}(x_2y_2)$ and $F_{A_1U}(x_1) \geq F_{B_1U}(x_2y_2)$, $\forall\, x_1 \in V_1$,$x_2y_2 \in E_2$.

Definition 3.19

Let $\mathrm{A}_1$and A_2 be interval valued neutrosophic subsets of $\mathrm{V}_1$and V_2 respectively. Let $\mathrm{B}_1$and B_1 interval valued neutrosophic subsets of E_1 and E_2 respectively. The join of two strong interval valued neutrosophic graphs G_1 and G_2 is denoted by $\mathrm{G}_1 + \mathrm{G}_2 = (\mathrm{A}_1 + \mathrm{A}_2, \mathrm{B}_1 + \mathrm{B}_2)$ and is defined as follows

1) $(T_{A_1L} + T_{A_2L})(x) = \begin{cases} (T_{A_1L} \cup T_{A_2L})(x) & \text{if } x \in V_1 \cup V_2 \\ T_{A_1L}(x) & \text{if } x \in V_1 \\ T_{A_2L}(x) & \text{if } x \in V_2 \end{cases}$

$$(T_{A_1U} + T_{A_2U})(x) = \begin{cases} (T_{A_1U} \cup T_{A_2U})(x) & \text{if } x \in V_1 \cup V_2 \\ T_{A_1U}(x) & \text{if } x \in V_1 \\ T_{A_2U}(x) & \text{if } x \in V_2 \end{cases}$$

$$(I_{A_1L} + I_{A_2L})(x) = \begin{cases} (I_{A_1L} \cap I_{A_2L})(x) & \text{if } x \in V_1 \cup V_2 \\ I_{A_1L}(x) & \text{if } x \in V_1 \\ I_{A_2L}(x) & \text{if } x \in V_2 \end{cases}$$

$$(I_{A_1U} + I_{A_2U})(x) = \begin{cases} (I_{A_1U} \cap I_{A_2U})(x) & \text{if } x \in V_1 \cup V_2 \\ I_{A_1U}(x) & \text{if } x \in V_1 \\ I_{A_2U}(x) & \text{if } x \in V_2 \end{cases}$$

$$(F_{A_1L} + F_{A_2L})(x) = \begin{cases} (F_{A_1L} \cap F_{A_2L})(x) & \text{if } x \in V_1 \cup V_2 \\ F_{A_1L}(x) & \text{if } x \in V_1 \\ F_{A_2L}(x) & \text{if } x \in V_2 \end{cases}$$

$$(F_{A_1U} + F_{A_2U})(x) = \begin{cases} (F_{A_1U} \cap F_{A_2U})(x) & \text{if } x \in V_1 \cup V_2 \\ F_{A_1U}(x) & \text{if } x \in V_1 \\ F_{A_2U}(x) & \text{if } x \in V_2 \end{cases}$$

2) $(T_{B_1L} + T_{B_2L})(xy) = \begin{cases} (T_{B_1L} \cup T_{B_2L})(\mathrm{xy}) & \text{if } xy \in E_1 \cup E_2 \\ T_{B_1L}(xy) & \text{if } xy \in E_1 \\ T_{B_2L}(xy) & \text{if } xy \in E_2 \end{cases}$

$$(T_{B_1U} + T_{B_2U})(xy) = \begin{cases} (T_{B_1U} \cup T_{B_2U})(\mathrm{xy}) & \text{if } xy \in E_1 \cup E_2 \\ T_{B_1U}(xy) & \text{if } xy \in E_1 \\ T_{B_2U}(xy) & \text{if } xy \in E_2 \end{cases}$$

$$(I_{B_1L} + I_{B_2L})\,(xy) = \begin{cases} (I_{B_1L} \cap I_{B_2L})(\mathrm{xy}) & \text{if } xy \in E_1 \cup E_2 \\ I_{B_1L}(xy) & \text{if } xy \in E_1 \\ I_{B_2L}(xy) & \text{if } xy \in E_2 \end{cases}$$

$$(I_{B_1U} + I_{B_2U})\,(xy) = \begin{cases} (I_{B_1U} \cap I_{B_2U})(\mathrm{xy}) & \text{if } xy \in E_1 \cup E_2 \\ I_{B_1U}(xy) & \text{if } xy \in E_1 \\ I_{B_2U}(xy) & \text{if } xy \in E_2 \end{cases}$$

$$(F_{B_1L} + F_{B_2L})\,(xy) = \begin{cases} (F_{B_1L} \cap F_{B_2L})(\mathrm{xy}) & \text{if } xy \in E_1 \cup E_2 \\ F_{B_1L}(xy) & \text{if } xy \in E_1 \\ F_{B_2L}(xy) & \text{if } xy \in E_2 \end{cases}$$

$$(F_{B_1U} + F_{B_2U})\,(x\ y) = \begin{cases} (F_{B_1U} \cap F_{B_2U})(\mathrm{xy}) & \text{if } xy \in E_1 \cup E_2 \\ F_{B_1U}(xy) & \text{if } xy \in E_1 \\ F_{B_2U}(xy) & \text{if } xy \in E_2 \end{cases}$$

3) $(T_{B_1L} + T_{B_2L})\,(x\ \mathrm{y}) = \min\,(T_{B_1L}(x), T_{B_2L}(x))$
$(T_{B_1U} + T_{B_2U})\,(x\mathrm{y}) = \min\,(T_{B_1U}(x), T_{B_2U}(x))$
$(I_{B_1L} + I_{B_2L})\,(x\mathrm{y}) = \max\,(I_{B_1L}(x), I_{B_2L}(x))$
$(I_{B_1U} + I_{B_2U})\,(x\ \mathrm{y}) = \max\,(I_{B_1U}(x), I_{B_2U}(x)$
$(F_{B_1L} + F_{B_2L})\,(x\mathrm{y}) = \max\,(F_{B_1L}(x), F_{B_2L}(x))$

$(\mathrm{F}_{\mathrm{B_1U}} + \mathrm{F}_{\mathrm{B_2U}})$ (xy) = max $(\mathrm{F}_{\mathrm{B_1U}}(\mathrm{x}), \mathrm{F}_{\mathrm{B_2U}}(\mathrm{x}))$ if xy $\in$ E′, where E′ is the set of all edges joining the nodes of V_1 and V_2 and where we assume $\mathrm{V}_1 \cap \mathrm{V}_2 = \emptyset$.

4 Conclusion

Interval valued neutrosophic set is a generalization of fuzzy set and intuitionistic fuzzy set, interval valued fuzzy set, interval valued intuitionstic fuzzy set and single valued neutrosophic set. Interval valued neutrosophic model gives more precisions, flexibility and compatibility to the system as compared to the classical, fuzzy, intuitionistic fuzzy and single valued neutrosophic models. In this paper, we have discussed a subclass of interval valued neutrosophic graph called strong interval valued neutrosophic graph, and we have introduced some operations, such as Cartesian product, composition and join of two strong interval valued neutrosophic graph, with proofs. In future studies, we plan to extend our research to regular interval valued neutrosophic graphs and irregular interval valued neutrosophic graphs.

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Rough Neutrosophic Hyper-complex set and its Application to Multi-attribute Decision Making

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Abstract

This paper presents multi-attribute decision making based on rough neutrosophic hyper-complex sets with rough neutrosophic hyper-complex attribute values. The concept of neutrosophic hyper-complex set is a powerful mathematical tool to deal with incomplete, indeterminate and inconsistent information. We extend the concept of neutrosophic hyper-complex set to rough neutrosophic hyper-complex set. The ratings of all alternatives have been expressed in terms of the upper and lower approximations and the pair of neutrosophic hyper-complex sets which are characterized by two hyper-complex functions and an indeterminacy component. We also define cosine function based on rough neutrosophic hyper-complex set to determine the degree of similarity between rough neutrosophic hyper-complex sets. We establish new decision making approach based on rough neutrosphic hyper-complex set. Finally, a numerical example has been furnished to demonstrate the applicability of the proposed approach.

Keyword

Neutrosophic set, Rough neutrosophic set, Rough neutrosophic hyper-complex set, Cosine function, Decision making.

1. Introduction

The concept of rough neutrosophic set has been introduced by Broumi et al. [1, 2]. It has been derived as a combination of the concepts of rough set proposed by Z. Pawlak [3] and neutrosophic set introduced by F. Smarandache [4, 5]. Rough sets and neutrosophic sets are both capable of dealing with partial information and uncertainty. To deal with real world problems, Wang et al. [6] introduced single valued netrosophic sets (SVNSs).

Recently, Mondal and Pramanik proposed a few decision making models in rough neutrosophic environment. Mondal and Pramanik [7] applied the concept of grey relational analysis to rough neutrosophic multi-attribte decision making problems. Pramanik and Mondal [8] studied cosine similarity measure of rough neutrosophic sets and its application in medical diagnosis. Mondal and Pramanik [9] proposed multi attribute decision making approach using rough accuracy score function. Pramanik and Mondal [10] also proposed cotangent similarity measure under rough neutrosophic sets. The same authors [11] further studied some similarity measures namely Dice similarity measure [12] and Jaccard similarity measure [12] in rough neutrosophic environment.

Rough neutrosophic hyper-complex set is the generalization of rough neutrosophic set [1, 2] and neutrosophic hyper-complex sets [13]. S. Olariu [14] introduced the concept of hyper-complex number and studied some of its properties. Mandal and Basu [15] studied hyper-complex similarity measure for SVNS and its application in decision making. Mondal and Pramanik [16] studied tri-complex rough neutrosophic similarity measure and presented an application in multi-attribute decision making.

In this paper, we have defined rough neutrosophic hyper-complex set and rough neutrosophic hyper-complex cosine function (RNHCF). We have also proposed a multi-attribute decision making approach in rough neutrosophic hyper-complex environment.

Rest of the paper is organized in the following way. Section 2 presents preliminaries of neutrosophic sets, single valued neutrosophic sets and some basic ideas of hyper-complex sets. Section 3 gives the definition of rough neutrosophic hyper-complex sets. Section 4 gives the definition of rough neutrosophic hyper-complex cosine function. Section 5 is devoted to present multi attribute decision-making method based on rough neutrosophic hyper-complex cosine function. Section 6 presents a numerical example of the proposed approach. Finally section 7 presents concluding remarks and scope of future research.

2. Neutrosophic Preliminaries

Neutrosophic set is derived from neutrosophy [4].

2.1 Neutrosophic set

Definition 2.1[4, 5]

Let U be a universe of discourse. Then a neutrosophic set A can be presented in the form:

$$A = \{< x:T_A(x), I_A(x), F_A(x)>, x \in U\}, \quad (1)$$

where the functions T, I, F: U→ $]^-0,1^+[$ represent respectively the degree of membership, the degree of indeterminacy, and the degree of non-membership of the element $x \in U$ to the set Asatisfying the following the condition.

$$^-0 \le \sup T_A(x) + \sup I_A(x) + \sup F_A(x) \le 3^+ \quad (2)$$

Wang et al. [6] mentioned that the neutrosophic set assumes the values from the real standard or non-standard subsets of $]^-0, 1^+[$ based on philosophical point of view. So instead of $]^-0, 1^+[$ Wang et al. [6] consider the interval [0, 1] for technical applications, because $]^-0, 1^+[$ is difficult to apply in the real applications such as scientific and engineering problems. For two netrosophic sets (NSs),

$$A_{NS} = \{<x: T_A(x), I_A(x), F_A(x)> \mid x \in X\} \quad (3)$$

And

$$B_{NS} = \{< x, T_B(x), I_B(x), F_B(x)> \mid x \in X\}, \quad (4)$$

the two relations are defined as follows:

(1) $A_{NS} \subseteq B_{NS}$ if and only if $T_A(x) \le T_B(x)$, $I_A(x) \ge I_B(x)$, $F_A(x) \ge F_B(x)$

(2) $A_{NS} = B_{NS}$ if and only if $T_A(x) = T_B(x)$, $I_A(x) = I_B(x)$, $F_A(x) = F_B(x)$

2. 2 Single valued neutrosophic sets (SVNS)

Definition 2.2 [6]

Assume that X is a space of points (objects) with generic elements in X denoted by x. A SVNS A in X is characterized by a truth-membership function $T_A(x)$, an indeterminacy-membership function $I_A(x)$, and a falsity membership function $F_A(x)$, for each point x in X, $T_A(x)$, $I_A(x)$, $F_A(x) \in [0, 1]$. When X is continuous, a SVNS A can be written as follows:

$$A = \int_x \frac{<T_A(x), I_A(x), F_A(x)>}{x} : x \in X . \tag{5}$$

When X is discrete, a SVNS A can be written as:

$$A = \Sigma_{i=1}^{n} \frac{<T_A(x_i), I_A(x_i), F_A(x_i)>}{x_i} : x_i \in X \tag{6}$$

For two SVNSs ,

$$A_{SVNS} = \{<x: T_A(x), I_A(x), F_A(x)> \mid x \in X\} \tag{7}$$

and

$$B_{SVNS} = \{<x, T_B(x), I_B(x), F_B(x)> \mid x \in X\}, \tag{8}$$

the two relations are defined as follows:

(i) $A_{SVNS} \subseteq B_{SVNS}$ if and only if $T_A(x) \le T_B(x)$, $I_A(x) \ge I_B(x)$, $F_A(x) \ge F_B(x)$

(ii) $A_{SVNS} = B_{SVNS}$ if and only if $T_A(x) = T_Q(x)$, $I_A(x) = I_B(x)$, $F_A(x) = F_B(x)$ for any $x \in X$

2.3. Basic concept of Hyper-complex number of dimension n [13]

The hyper-complex number of dimension n (or n-complex number) was defined by S. Olariu [13] as a number of the form:

$$u = h_0 x_o + h_1 x_1 + h_2 x_2 + \ldots + h_{n-1} x_{n-1}$$
$$= h_0 x_o + h_1 x_1 + h_2 x_2 + \ldots + h_{n-1} x_{n-1} \tag{9}$$

where $n \ge 2$, and the variables $x_0, x_1, x_2, \ldots, x_{n-1}$ are real numbers, while $h_1, h_2, \ldots, h_{n-1}$ are the complex units, $h_o = 1$, and they are multiplied as follows:

$$h_j h_k = h_{j+k} \text{ if } 0 \le j+k \le n-1, \text{ and } h_j h_k = h_{j+k-n} \text{ if } n \le j+k \le 2n-2. \tag{10}$$

The above complex unit multiplication formulas can be written in a simpler form as:

$$h_j h_k = h_{j+k} \pmod n \tag{11}$$

where mod n means modulo n. For example, if n = 5, then

$$h_3 h_4 = h_{3+4} (\mathrm{mod}\ 5) = h_7 (\mathrm{mod} 5) = h_2. \tag{12}$$

The formula(11) allows us to multiply many complex units at once, as follows:

$$h_{j1} h_{j2} \ldots h_{jp} = h_{j1+j2+\ldots+jp} \pmod n, \text{ for } p \ge 1. \tag{13}$$

The Neutrosophic hyper-complex number of dimension n [12] which is a number and it can be written of the form:

$$u+vI \tag{14}$$

where u and v are n-complex numbers and I is the indeterminacy.

3. Rough Neutrosophic Hyper-complex Set in Dimension n

Definition 3.1

Let Z be a non-null set and R be an equivalence relation on Z. Let A be a neutrosophic hyper-complex set of dimension n (or neutrosophic n-complex number), and its elements of the form u+vI, where u and v are n-complex numbers and I is the indeterminacy. The lower and the upper approximations of A in the approximation space (Z, R) denoted by $\underline{N}(A)$ and $\overline{N}(A)$ are respectively defined as follows:

$$\underline{N}(A)=\left\langle < x,[u+vI]_{\underline{N}(A)}(x) > / z \in [x]_R , x \in Z\right\rangle \tag{15}$$

$$\overline{N}(A)=\left\langle < x,[u+vI]_{\overline{N}(A)}(x) > / z \in [x]_R , x \in Z\right\rangle \tag{16}$$

where,

$$[u+vI]_{\underline{N}(A)}(x) = \wedge_z \in [x]_R\, [u+vI]_A(z), \tag{17}$$

$$[u+vI]_{\overline{N}(A)}(x) = \vee_z \in [x]_R\, [u+vI]_A(z) \tag{18}$$

So, $[u+vI]_{\underline{N}(A)}(x)$ and $[u+vI]_{\overline{N}(A)}(x)$ are neutrosophic hyper-complex numbers of dimension n. Here $\vee$ and $\wedge$ denote 'max' and 'min' operators respectively. $[u+vI]_A(z)$ and $[u+vI]_A(z)$ are the neutrosophic hyper-complex sets of dimension n of z with respect to A. $\underline{N}(A)$ and $\overline{N}(A)$ are two neutrosophic hyper-complex sets of dimension n in Z.

Thus, NS mappings $\underline{N}$, $\overline{N}$: N(Z) $\rightarrow$ N(Z) are respectively referred to as the lower and upper rough neutrosophic hyper-complex approximation operators, and the pair $(\underline{N}(A), \overline{N}(A))$ is called the rough neutrosophic hyper-complex set in (Z, R).

Based on the above mentioned definition, it is observed that $\underline{N}(A)$ and $\overline{N}(A)$ have constant membership on the equivalence clases of R, if $\underline{N}(A) = \overline{N}(A)$; i.e. $[u+vI]_{\underline{N}(A)}(x)$ = $[u+vI]_{\overline{N}(A)}(x)$.

Definition 3.2

Let N(A) = ($\underline{N}(A), \overline{N}(A)$) be a rough neutrosophic hyper-complex set in (Z, R). The rough complement of N(A) is denoted by ~ N(A) = $(\underline{N}(A)^c, \overline{N}(A)^c)$, where $\underline{N}(A)^c$ and $\overline{N}(A)^c$ are the complements of neutrosophic hyper-complex set of $\underline{N}(A)$ and $\overline{N}(A)$ respectively.

$$\underline{N}(A)^c=\left\langle < x,[u+v(1-I)]_{\underline{N}(A)}(x) > /, x \in Z\right\rangle, \tag{19}$$

and

$$\overline{N}(A)^c=\left\langle < x,[u+v(1-I)]_{\overline{N}(A)}(x) > /, x \in Z\right\rangle \tag{20}$$

Definition 3.3

Let N(A) and N(B) are two rough neutrosophic hyper-complex sets respectively in Z, then the following definitions hold:

$$N(A)=N(B) \Leftrightarrow \underline{N}(A)=\underline{N}(B) \wedge \overline{N}(A)=\overline{N}(B) \tag{21}$$

$$N(A) \subseteq N(B) \Leftrightarrow \underline{N}(A) \subseteq \underline{N}(B) \ \wedge \ \overline{N}(A) \subseteq \overline{N}(B) \tag{22}$$

$$N(A) \cup N(B) = < \underline{N}(A) \cup \underline{N}(B),\ \overline{N}(A) \cup \overline{N}(B) > \tag{23}$$

$$N(A) \cap N(B) = < \underline{N}(A) \cap \underline{N}(B),\ \overline{N}(A) \cap \overline{N}(B) > \tag{24}$$

If A, B, C are the rough neutrosophic hyper-complex sets in (Z, R), then the following propositions are stated from definitions

Proposition 1

I. $\sim(\sim A) = A$ (25)

II. $\underline{N}(A) \subseteq \overline{N}(B)$ (26)

III. $<\sim \left(\underline{N}(A) \cup \underline{N}(B)\right) = \sim \left(\underline{N}(A)\right) \cap \sim \left(\underline{N}(B)\right)$ (27)

IV. $<\sim \left(\underline{N}(A) \cap \underline{N}(B)\right) = \sim \left(\underline{N}(A)\right) \cup \sim \left(\underline{N}(B)\right)$ (28)

V. $<\sim \left(\overline{N}(A) \cup \overline{N}(B)\right) = \sim \left(\overline{N}(A)\right) \cap \sim \left(\overline{N}(B)\right)$ (29)

VI. $<\sim \left(\overline{N}(A) \cap \overline{N}(B)\right) = \sim \left(\overline{N}(A)\right) \cup \sim \left(\overline{N}(B)\right)$ (30)

Proofs I:

If N(A) = $[\underline{N}(A), \overline{N}(A)]$ is a rough neutrosophic hyper-complex set in (Z, R), the complement of N(A) is the rough neutrosophic hyper-complex set defined as follows.

$$\underline{N}(A)^c = \left\langle < x, [u + v(1 - I)]_{\underline{N}(A)}(x) > / , x \in Z \right\rangle, \tag{31}$$

and

$$\overline{N}(A)^c = \left\langle < x, [u + v(1 - I)]_{\overline{N}(A)}(x) > / , x \in Z \right\rangle \tag{32}$$

From these definitions, we can write:

$$\sim(\sim A) = A. \tag{33}$$

Proof II:

The lower and the upper approximations of A in the approximation space (Z, R) denoted by $\underline{N}(A)$ and $\overline{N}(A)$ are respectively defined as follows:

$$\underline{N}(A)^c = \left\langle < x, [u + v(1 - I)]_{\underline{N}(A)}(x) > / , x \in Z \right\rangle, \tag{34}$$

and

$$\overline{N}(A)^c = \left\langle < x, [u + v(1 - I)]_{\overline{N}(A)}(x) > / , x \in Z \right\rangle \tag{35}$$

where,

$$[u + vI]_{\underline{N}(A)}(x) = \wedge_z \in [x]_R\ [u + vI]_A(z), \tag{36}$$

$$[u + vI]_{\overline{N}(A)}(x) = \vee_z \in [x]_R\ [u + vI]_A(z) \tag{37}$$

So,

$$\underline{N}(A) \subseteq \overline{N}(A) \tag{38}$$

Proof III:

Consider:

$$x \in \sim\left(\underline{N}(A) \cup \underline{N}(B)\right)$$

$$\Rightarrow x \in \sim \underline{N}(A) \text{ and } x \in \sim \underline{N}(B)$$

$$\Rightarrow x \in \sim\left(\underline{N}(A)\right) \cap \sim\left(\underline{N}(B)\right)$$

$$\Rightarrow x \in \sim\left(\underline{N}(A)\right) \cap \sim\left(\underline{N}(B)\right) \tag{39}$$

$$\Rightarrow \sim\left(\underline{N}(A) \cup \underline{N}(B)\right) \subseteq \sim\left(\left(\underline{N}(A)\right) \cap \sim\left(\underline{N}(B)\right)\right).$$

Again, consider:

$$y \in \sim\left(\left(\underline{N}(A)\right) \cap \sim\left(\underline{N}(B)\right)\right)$$

$$\Rightarrow y \in \sim \underline{N}(A) \text{ or } y \in \sim \underline{N}(B)$$

$$\Rightarrow y \in \Rightarrow \sim\left(\underline{N}(A) \cup \underline{N}(B)\right)$$

$$\Rightarrow \sim\left(\underline{N}(A) \cup \underline{N}(B)\right) \supseteq \sim\left(\left(\underline{N}(A)\right) \cap \sim\left(\underline{N}(B)\right)\right). \tag{40}$$

Hence,

$$\sim\left(\underline{N}(A) \cup \underline{N}(B)\right) = \sim\left(\left(\underline{N}(A)\right) \cap \sim\left(\underline{N}(B)\right)\right) \tag{41}$$

Proof IV:

Consider:

$$x \in \sim\left(\underline{N}(A) \cap \underline{N}(B)\right)$$

$$\Rightarrow x \in \sim \underline{N}(A) \text{ or } x \in \sim \underline{N}(B)$$

$$\Rightarrow x \in \sim\left(\underline{N}(A)\right) \cup \sim\left(\underline{N}(B)\right)$$

$$\Rightarrow x \in \sim\left(\underline{N}(A)\right) \cup \sim\left(\underline{N}(B)\right)$$

$$\Rightarrow \sim\left(\underline{N}(A) \cap \underline{N}(B)\right) \subseteq \sim\left(\left(\underline{N}(A)\right) \cup \sim\left(\underline{N}(B)\right)\right) \tag{42}$$

Again, consider:

$$y \in \sim\left(\left(\underline{N}(A)\right) \cup \sim\left(\underline{N}(B)\right)\right)$$

$$\Rightarrow y \in \sim \underline{N}(A) \text{ and } y \in \sim \underline{N}(B)$$

$$\Rightarrow y \in \sim\left(\underline{N}(A) \cap \underline{N}(B)\right)$$

$$\Rightarrow \sim\left(\underline{N}(A) \cap \underline{N}(B)\right) \supseteq \sim\left(\left(\underline{N}(A)\right) \cup \sim\left(\underline{N}(B)\right)\right) \tag{43}$$

Hence,

$$\sim\left(\underline{N}(A)\cap\underline{N}(B)\right)=\sim\left(\left(\underline{N}(A)\right)\cup\sim\left(\underline{N}(B)\right)\right) \quad (44)$$

Proof V:

Consider:

$$x\in\sim\left(\overline{N}(A)\cup\overline{N}(B)\right)$$

$$\Rightarrow x\in\sim\overline{N}(A) \text{ and } x\in\sim\overline{N}(B)$$

$$\Rightarrow x\in\sim\left(\overline{N}(A)\right)\cap\sim\left(\overline{N}(B)\right)$$

$$\Rightarrow x\in\sim\left(\overline{N}(A)\right)\cap\sim\left(\overline{N}(B)\right)$$

$$\Rightarrow\sim\left(\overline{N}(A)\cup\overline{N}(B)\right)\subseteq\sim\left(\left(\overline{N}(A)\right)\cap\sim\left(\overline{N}(B)\right)\right) \quad (45)$$

Again, consider:

$$y\in\sim\left(\left(\overline{N}(A)\right)\cap\sim\left(\overline{N}(B)\right)\right)$$

$$\Rightarrow y\in\sim\overline{N}(A) \text{ or } y\in\sim\overline{N}(B)$$

$$\Rightarrow y\in\sim\left(\overline{N}(A)\cup\overline{N}(B)\right)$$

$$\Rightarrow\sim\left(\overline{N}(A)\cup\overline{N}(B)\right)\supseteq\sim\left(\left(\overline{N}(A)\right)\cap\sim\left(\overline{N}(B)\right)\right) \quad (46)$$

Hence,

$$\sim\left(\overline{N}(A)\cup\overline{N}(B)\right)=\sim\left(\left(\overline{N}(A)\right)\cap\sim\left(\overline{N}(B)\right)\right) \quad (47)$$

Proof VI:

Consider:

$$x\in\sim\left(\overline{N}(A)\cap\overline{N}(B)\right)$$

$$\Rightarrow x\in\sim\overline{N}(A) \text{ or } x\in\sim\overline{N}(B)$$

$$\Rightarrow x\in\sim\left(\overline{N}(A)\right)\cup\sim\left(\overline{N}(B)\right)$$

$$\Rightarrow x\in\sim\left(\overline{N}(A)\right)\cup\sim\left(\overline{N}(B)\right)$$

$$\Rightarrow\sim\left(\overline{N}(A)\cap\overline{N}(B)\right)\subseteq\sim\left(\left(\overline{N}(A)\right)\cup\sim\left(\overline{N}(B)\right)\right) \quad (48)$$

Again, consider:

$$y\in\sim\left(\left(\overline{N}(A)\right)\cup\sim\left(\overline{N}(B)\right)\right)$$

$$\Rightarrow y\in\sim\overline{N}(A) \text{ and } y\in\sim\overline{N}(B)$$

$$\Rightarrow y\in\sim\left(\overline{N}(A)\right)\cap\sim\left(\overline{N}(B)\right)$$

$$\Rightarrow\sim\left(\overline{N}(A)\cap\overline{N}(B)\right)\supseteq\sim\left(\left(\overline{N}(A)\right)\cup\sim\left(\overline{N}(B)\right)\right) \quad (49)$$

Hence,

$$\sim\left(\overline{N}(A)\cap\overline{N}(B)\right)=\sim\left(\left(\overline{N}(A)\right)\cup\sim\left(\overline{N}(B)\right)\right) \quad (50)$$

Proposition 2:

I. $\sim[N(A)\cup N(B)]=(\sim N(A))\cap(\sim N(B))$ (51)

II. $\sim[N(A)\cap N(B)]=(\sim N(A))\cup(\sim N(B))$ (52)

Proof I:

$\sim[N(A)\cup N(B)]$

$=\ \sim<\underline{N}(A)\cup\underline{N}(B),\ \overline{N}(A)\cup\overline{N}(B)>$

$=\ <\sim\left(\underline{N}(P)\cap\underline{N}(Q)\right),\ \sim\left(\overline{N}(P)\cap\overline{N}(Q)\right)>$

$=(\sim N(A))\cap(\sim N(B))$ (53)

Proof II:

$\sim[N(A)\cap N(B)]$

$=\ \sim<\underline{N}(A)\cap\underline{N}(B),\ \overline{N}(A)\cap\overline{N}(B)>$

$=\ <\sim\left(\underline{N}(A)\cup\underline{N}(B)\right),\ \sim\left(\overline{N}(A)\cup\overline{N}(B)\right)>$

$=(\sim N(A))\cup(\sim N(B))$ (54)

4. Rough neutrosophic hyper-complex cosine function (RNHCF)

The cosine similarity measure is calculated as the inner product of two vectors divided by the product of their lengths. It is the cosine of the angle between the vector representations of two rough neutrosophic hyper-complex sets. The cosine similarity measure is a fundamental measure used in information technology. Now, a new cosine function between rough neutrosophic hyper-complex sets is proposed as follows.

Definition 4.1

Assume that there are two rough neutrosophic hyper-complex sets

$$A=\left\langle [u+vI]_{\underline{N}(A)}(x),[u+vI]_{\overline{N}(A)}(x)\right\rangle \quad (55)$$

and

$$B=\left\langle [u+vI]_{\underline{N}(B)}(x),[u+vI]_{\overline{N}(B)}(x)\right\rangle \quad (56)$$

in $X=\{x_1,x_2,\ldots,x_n\}$.

Then rough neutrosophic hyper-complex cosine function between two sets A and B is defined as follows:

$C_{RNHCF}(A,B)=$

$$\frac{1}{n}\sum_{i=1}^{n}\frac{\Delta u_A(x_i).\Delta u_B(x_i)+\Delta v_A(x_i).\Delta v_B(x_i)+\Delta I_A(x_i).\Delta I_B(x_i)}{\sqrt{(\Delta u_A(x_i))^2+(\Delta v_A(x_i))^2+(\Delta I_A(x_i))^2}\sqrt{(\Delta u_B(x_i))^2+(\Delta v_B(x_i))^2+(\Delta I_B(x_i))^2}} \quad (57)$$

where,

$$\Delta u_A(x_i)=0.5.\left|u_{\underline{N}(A)(x_i)}+u_{\overline{N}(A)(x_i)}\right|, \quad (58)$$

$$\Delta u_B(x_i)=0.5.\left|u_{\underline{N}(B)(x_i)}+u_{\overline{N}(B)(x_i)}\right|, \quad (59)$$

$$\Delta v_A(x_i) = 0.5.\left| v_{\underline{N}(A)(x_i)} + v_{\overline{N}(A)(x_i)} \right|, \tag{60}$$

$$\Delta v_B(x_i) = 0.5.\left| v_{\underline{N}(B)(x_i)} + v_{\overline{N}(B)(x_i)} \right|, \tag{61}$$

$$\Delta I_A(x_i) = 0.5.\left| I_{\underline{N}(A)(x_i)} + I_{\overline{N}(A)(x_i)} \right|, \tag{62}$$

$$\Delta I_B(x_i) = 0.5.\left| I_{\underline{N}(B)(x_i)} + I_{\overline{N}(B)(x_i)} \right|. \tag{63}$$

Proposition 3:

Let A and B be rough neutrosophic sets, then:

I. $0 \le C_{RNHCF}(A, B) \le 1$ (64)

II. $C_{RNHCF}(A, B) = C_{RNHCF}(B, A)$ (65)

III. $C_{RNHCF}(A, B) = 1$, if and only if A = B (66)

IV. If C is a RNHCF in Y and $A \subset B \subset C$ then, $C_{RNHCF}(A, C) \le C_{RNHCF}(A, B)$, and $C_{RNHCF}(A, C) \le C_{RNHCF}(B,C)$. (67)

Proofs :

I. It is obvious because all positive values of cosine function are within 0 and 1

II. It is obvious that the proposition is true.

III. When A = B, then obviously $C_{RNHCF}(A, B) = 1$. On the other hand if $C_{RNHCF}(A, B) =1$ then, $\Delta T_A(x_i) = \Delta T_B(x_i)$, $\Delta I_A(x_i) = \Delta I_B(x_i)$, $\Delta F_A(x_i) = \Delta F_B(x_i)$ ie,

This implies that A = B.

IV. If $A \subset B \subset C$, then we can write

$$u_{\underline{N}(A)}(x_i) \le u_{\underline{N}(B)}(x_i) \le u_{\underline{N}(C)}(x_i), \tag{68}$$

$$u_{\overline{N}(A)}(x_i) \le u_{\overline{N}(B)}(x_i) \le u_{\overline{N}(C)}(x_i), \tag{69}$$

$$v_{\underline{N}(A)}(x_i) \le v_{\underline{N}(B)}(x_i) \le v_{\underline{N}(C)}(x_i), \tag{70}$$

$$v_{\overline{N}(A)}(x_i) \le v_{\overline{N}(B)}(x_i) \le v_{\overline{N}(C)}(x_i), \tag{71}$$

$$I_{\underline{N}(A)}(x_i) \ge I_{\underline{N}(B)}(x_i) \ge I_{\underline{N}(C)}(x_i), \tag{72}$$

$$I_{\overline{N}(A)}(x_i) \ge I_{\overline{N}(B)}(x_i) \ge I_{\overline{N}(C)}(x_i) \tag{73}$$

The cosine function is decreasing function within the interval $\left[0, \frac{\pi}{2}\right]$. Hence we can write $C_{RNHCF}(A, C) \le C_{RNHCF}(A, B)$, and $C_{RNHCF}(A, C) \le C_{RNHCF}(B, C)$.

If we consider the weight of each element x_i, a weighted rough neutrosophic hyper-complex cosine function (WRNHCF) between two sets A and B can be defined as follows:

$$C_{WRNHCF}(A, B) =$$

$$\sum_{i=1}^{n} W_i \frac{\Delta u_A(x_i).\Delta u_B(x_i) + \Delta v_A(x_i).\Delta v_B(x_i) + \Delta I_A(x_i).\Delta I_B(x_i)}{\sqrt{(\Delta u_A(x_i))^2 + (\Delta v_A(x_i))^2 + (\Delta I_A(x_i))^2}\sqrt{(\Delta u_B(x_i))^2 + (\Delta v_B(x_i))^2 + (\Delta I_B(x_i))^2}} \tag{74}$$

where,

$$\Delta u_A(x_i) = 0.5.\left| u_{\underline{N}(A)(x_i)} + u_{\overline{N}(A)(x_i)} \right|, \tag{75}$$

$$\Delta u_B(x_i) = 0.5.\left| u_{\underline{N}(B)(x_i)} + u_{\overline{N}(B)(x_i)} \right|, \tag{76}$$

$$\Delta v_A(x_i) = 0.5.\left| v_{\underline{N}(A)(x_i)} + v_{\overline{N}(A)(x_i)} \right|, \tag{77}$$

$$\Delta v_B(x_i) = 0.5.\left| v_{\underline{N}(B)(x_i)} + v_{\overline{N}(B)(x_i)} \right|, \tag{78}$$

$$\Delta I_A(x_i) = 0.5.\left| I_{\underline{N}(A)(x_i)} + I_{\overline{N}(A)(x_i)} \right|, \tag{79}$$

$$\Delta I_B(x_i) = 0.5.\left| I_{\underline{N}(B)(x_i)} + I_{\overline{N}(B)(x_i)} \right|. \tag{80}$$

$W_i \in [0,1]$, i = 1, 2,..., n and $\sum_{i=1}^{n} W_i = 1$.

If we take $W_i = \frac{1}{n}$, i = 1, 2,..., n, then:

$$C_{WRNHCF}(A, B) = C_{RNHCF}(A, B) \tag{81}$$

The weighted rough neutrosophic hyper-complex cosine function (WRNHCF) between two rough neutrosophic hyper-complex sets A and B also satisfies the following properties:

I. $0 \le C_{WRNHCF}(A, B) \le 1$ (82)

II. $C_{WRNHCF}(A, B) = C_{WRNHCF}(B, A)$ (83)

III. $C_{WRNHCF}(A, B) = 1$, if and only if A = B (84)

IV. If C is a WRNHCF in Y and $A \subset B \subset C$ then, $C_{WRNHCF}(A, C) \le C_{WRNHCF}(A, B)$, and $C_{WRNHCF}(A, C) \le C_{WRNHCF}(B, C)$ (85)

5. Decision making procedure based on rough hyper-complex neutrosophic function

In this section, we apply rough neutrosophic hyper-complex cosine function to the multi-attribute decision making problem. Let $A_1, A_2, \dots, A_m$ be a set of alternatives and $C_1, C_2, \dots, C_n$ be a set of attributes.

The proposed multi attribute decision making approach is described using the following steps.

Step1: **Construction of the decision matrix with rough neutrosophic hyper-complex numbers**

The decision maker considers a decision matrix with respect to m alternatives and n attributes in terms of rough neutrosophic hyper-complex numbers as follows.

Table1: Rough neutrosophic hyper-complex decision matrix

$$DM = \left\langle \underline{dm}_{ij}, \overline{dm}_{ij} \right\rangle_{m\times n} =$$

$$\begin{array}{c|cccc}
 & C_1 & C_2 & \cdots & C_n \\
\hline
A_1 & \left\langle \underline{dm}_{11}, \overline{dm}_{11} \right\rangle & \left\langle \underline{dm}_{12}, \overline{dm}_{12} \right\rangle & \dots & \left\langle \underline{dm}_{1n}, \overline{dm}_{1n} \right\rangle \\
A_2 & \left\langle \underline{dm}_{21}, \overline{dm}_{21} \right\rangle & \left\langle \underline{dm}_{22}, \overline{dm}_{22} \right\rangle & \dots & \left\langle \underline{dm}_{2n}, \overline{dm}_{2n} \right\rangle \\
. & \dots & \dots & \dots & \dots \\
. & \dots & \dots & \dots & \dots \\
A_m & \left\langle \underline{dm}_{m1}, \overline{dm}_{m1} \right\rangle & \left\langle \underline{dm}_{m2}, \overline{dm}_{m2} \right\rangle & \dots & \left\langle \underline{dm}_{mn}, \overline{dm}_{mn} \right\rangle
\end{array} \tag{86}$$

Here $\left\langle \underline{dm}_{ij}, \overline{dm}_{ij} \right\rangle$ is the rough neutrosophic hyper-complex number according to the i-th alternative and the j-th attribute.

Step2: **Determination of the weights of the attributes**

Assume that the weight of the attribute C_j (j = 1, 2, … , n) considered by the decision-maker be w_j (j = 1, 2, … , n) such that $\forall w_j \in [0, 1]$ (j = 1, 2, ..., n) and $\sum_{j=1}^{n} w_j = 1$.

Step 3: **Determination of the benefit type attribute and cost type attribute**

Generally, the evaluation of attributes can be categorized into two types: benefit attribute and cost attribute. Let K be a set of benefit attributes and M be a set of cost attributes. In the proposed decision-making approach, an ideal alternative can be identified by using a maximum operator for the benefit attribute and a minimum operator for the cost attribute to determine the best value of each criterion among all alternatives. Therefore, we define an ideal alternative as follows.

$$A^* = \{C_1^*, C_2^*, \dots, C_m^*\}. \tag{87}$$

Benefit attribute:

$$C_j^* = \left[\max_i u_{C_j}{}^{(A_i)}, \max_i v_{C_j}{}^{(A_i)}, \min_i I_{C_j}{}^{(A_i)} \right] \tag{88}$$

Cost attribute:

$$C_j^* = \left[\min_i T_{C_j}{}^{(A_i)}, \min_i I_{C_j}{}^{(A_i)}, \max_i F_{C_j}{}^{(A_i)} \right] \tag{89}$$

where,

$$u_{C_j}{}^{(A_i)} = 0.5. \left| \left(u_{C_j}\right)_{\underline{N}(A_i)} + \left(u_{C_j}\right)_{\overline{N}(A_i)} \right|, \tag{90}$$

$$v_{C_j}{}^{(A_i)} = 0.5. \left| \left(v_{C_j}\right)_{\underline{N}(A_i)} + \left(v_{C_j}\right)_{\overline{N}(A_i)} \right|, \tag{91}$$

and

$$I_{C_j}{}^{(A_i)} = 0.5. \left| \left(I_{C_j}\right)_{\underline{N}(A_i)} + \left(I_{C_j}\right)_{\overline{N}(A_i)} \right|. \tag{92}$$

Step4: **Determination of the over all weighted rough hyper-complex neutrosophic cosine function (WRNHCF)of the alternatives**

Weighted rough neutrosophic hyper-complex cosine function is given as follows.

$$C_{WRNHCF}(A, B) = \sum_{j=1}^{n} W_j C_{WRNHCF}(A, B) \tag{93}$$

Step5: **Ranking the alternatives**

Using the weighted rough hyper-complex neutrosophic cosine function between each alternative and the ideal alternative, the ranking order of all alternatives can be determined and the best alternative can be easily selected with the highest similarity value.

Step6: End

6. Numerical Example

Assume that a decision maker (an adult man/woman who eligible to marrage) intends to select the most suitable life partner for arrange marrage from the three initially chosen candidates (S_1, S_2, S_3) by considering five attributes namely: physical and mental health C_1, education and job C_2, management power C_3, family background C_4, risk factor C_5. Based on the proposed approach discussed in section 5, the considered problem has been solved using the following steps:

Step1: **Construction of the decision matrix with rough neutrosophic hyper-complex numbers**

The decision maker considers a decision matrix with respect to three alternatives and five attributes in terms of rough neutrosophic hyper-complex numbers shown in the Table 2.

Table2. Decision matrix with rough neutrosophic hyper-complex number

$$DM = \left\langle \underline{dm}_{ij}, \overline{dm}_{ij} \right\rangle_{3\times 5} =$$

$$\begin{array}{c|ccccc}
 & C_1 & C_2 & C_3 & C_4 & C_5 \\
\hline
A_1 & \left\langle \begin{array}{l}(i+0.6(1+i)),\\(2i+0.4(2+i))\end{array}\right\rangle & \left\langle \begin{array}{l}((1+i)+0.65(2i)),\\((1+2i)+0.55(3i))\end{array}\right\rangle & \left\langle \begin{array}{l}((1+i)+0.4(2+i)),\\((1+2i)+0.2(2+3i))\end{array}\right\rangle & \left\langle \begin{array}{l}(4i+0.55(1+i)),\\((4+i)+0.45(2+i))\end{array}\right\rangle & \left\langle \begin{array}{l}(3i+0.78(2+3i)),\\((1+3i)+0.72(3+3i))\end{array}\right\rangle \\
A_2 & \left\langle \begin{array}{l}(i+0.6(1+2i)),\\(3i+0.5(1+3i))\end{array}\right\rangle & \left\langle \begin{array}{l}((1+i)+0.55(i)),\\((1+2i)+0.45(3i))\end{array}\right\rangle & \left\langle \begin{array}{l}(2i+0.3(2+i)),\\((2+i)+0.2(1+3i))\end{array}\right\rangle & \left\langle \begin{array}{l}(i+0.52(2+3i)),\\(2i+0.48(4+3i))\end{array}\right\rangle & \left\langle \begin{array}{l}((1+i)+0.82(2+i)),\\(2i+0.78(4+3i))\end{array}\right\rangle \\
A_3 & \left\langle \begin{array}{l}(2i+0.5(1+i)),\\(3i+0.4(1+3i))\end{array}\right\rangle & \left\langle \begin{array}{l}((2+i)+0.69(5i)),\\((2+i)+0.51(6i))\end{array}\right\rangle & \left\langle \begin{array}{l}(i+0.6(1+i)),\\(2i+0.4(3+2i))\end{array}\right\rangle & \left\langle \begin{array}{l}((1+i)+0.48(3+4i)),\\((1+2i)+0.42(5+3i))\end{array}\right\rangle & \left\langle \begin{array}{l}((1+i)+0.9(i)),\\((1+2i)+0.7(2+3i))\end{array}\right\rangle
\end{array} \quad (94)$$

Where, $i=\sqrt{-1}$

Step 2: Determination of the weights of the attributes

The weight vectors considered by the decision maker are 0.25, 0.20, 0.25, 0.10, and 0.20 respectively.

Step 3: Determination of the benefit attribute and cost attribute

Here four benefit type attributes are C_1, C_2, C_3, C_4 and one cost type attribute is C_5. Using equations (12) and (13) we calculate A^* as follows.

A^* = [(5.00, 2.69, 0.45), (4.47, 5.50, 0.50), (3.60, 2.83, 0.25), (6.40, 5.30, 0.45), (3.16, 2.24, 0.80)]

Step 4: Determination of the over all weighted rough hyper-complex neutrosophic similarity function (WRHNSF)of the alternatives

We calculate weighted rough neutrosophic hyper-complex similarity values as follows.

$S_{WRHCF}(A_1, A^*) = 0.9622$

$S_{WRHCF}(A_2, A^*) = 0.9404$

$S_{WRHCF}(A_3, A^*) = 0.9942$

Step 5: Ranking the alternatives
Ranking of the alternatives is prepared based on the descending order of similarity measures. Highest value reflects the best alternative.
Here,

$$S_{WRHCF}(A_3, A^*) \succ S_{WRHCF}(A_1, A^*) \succ S_{WRHCF}(A_2, A^*) \quad (95)$$

Hence, the decision maker must choose the candidate A_3 as the best alternative for arrange marriage.
Step 6: End

7 Conclusion

In this paper, we have proposed rough neutrosophic hyper-complex set and rough neutrosophic hyper-complex cosine function and proved some of their basic properties. We have also proposed rough neutrosophic hyper-complex similarity measure based multi-attribute decision making approach. We have presented an application, namely selection of best candidate for arrange marriage for indian context. The concept presented in this paper can be applied for other multiple attribute decision making problems in rough neutrosophic hyper-complex environment.

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Fundamentos de la lógica y los conjuntos neutrosóficos y su papel en la inteligencia artificial

Fundamentals of neutrosophic logic and sets and their role in artificial intelligence

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Florentin Smarandache, Maikel Leyva-Vázquez (2018). Fundamentos de la lógica y los conjuntos neutrosóficos y su papel en la inteligencia artificial (Fundamentals of neutrosophic logic and sets and their role in artificial intelligence). *Neutrosophic Computing and Machine Learning* 1, 16-21

Abstract: Neutrosophy is a new branch of philosophy which studies the origin, nature and scope of neutralities. This has formed the basis for a series of mathematical theories that generalize the classical and fuzzy theories such as the neutrosophic sets and the neutrosophic logic. In the paper, the fundamental concepts related to neutrosophy and its antecedents are presented. Additionally, fundamental concepts of artificial intelligence will be defined and how neutrosophy has come to strengthen this discipline.

Keywords: neutrosophy, neutrosophic logic, neutrosophic sets, artificial intelligence.

1. Introducción

La neutrosofía es una nueva rama de la filosofía [1] la cual estudia el origen, naturaleza y alcance de las neutralidades, así como sus interacciones con diferentes espectros ideacionales: (A) es una idea, proposición, teoría, evento, concepto o entidad; anti (A) es el opuesto de (A); y (neut-A) significa ni (A) ni anti (A), es decir, la neutralidad entre los dos extremos [2]. Etimológicamente neutron-sofía [Frances neutre < Latin neuter, neutral, y griego sophia, conocimiento] significa conocimiento de los pensamiento neutrales y comenzó en 1995.

Su teoría fundamental afirma que toda idea < A > tiende a ser neutralizada, disminuida, balaceada por <noA> las ideas (no solo <antiA> como Hegel planteó)- como un estado de equilibrio.

<noA> = lo que no es <A>,
<antiA> = lo opuesto a <A>, y
<neutA> = los que no es <A> ni <antiA>.

En su forma clásica <A>, <neutA>, <antiA> son disjuntos de dos en dos.

Como en varios casos los límites entre conceptos son vagos a imprecisas, es posible que <A>, <neutA>, <antiA> (y <nonA> por supuesto) tengan partes comunes dos en dos también.

Esta teoría ha constituido la base para la lógica neutrosófica [3], los conjuntos neutrosófica [4], la probabilidad neutrosófica , y la estadística neutrosófica y multiples aplicaciones prácticas [5].

Antecedentes

El conjunto difuso (FS por sus siglas en inglés) fue introducido por L. Zadeh [6] en 1965, planteando que cada elemento tiene un grado de pertenencia . la teoría clásica de conjuntos, añadiendo una función de pertenencia [7].

La función de pertenencia o inclusión $\mu_a(t)$ indica el grado n en que la variable t está incluida en el concepto representado por la etiqueta A [8]. Para la definición de estas funciones de pertenencia se utilizan convenientemente ciertas familias de funciones, por coincidir con el significado lingüístico de las etiquetas más utilizadas. Las más utilizadas con mayor frecuencia son triangular, trapezoidal y gaussiana (Figura 1).

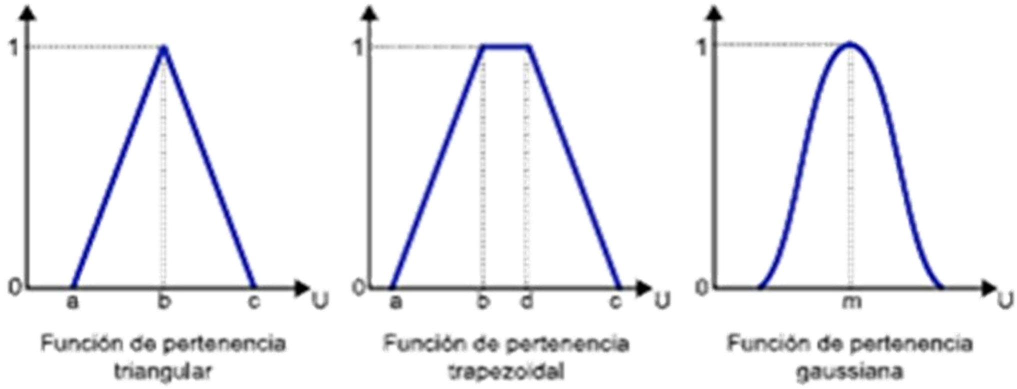

Figura 1 Representación gráfica de las funciones de pertenencia triangular, trapezoidal y gaussiana [9].

El conjunto difuso intuicionista (IFS por sus siglas en inglés) en un universo X fue introducido por K. Atanassov [10] como una generalización de los FS, donde además del grado de pertenencia $\mu_A(x) \in [0,1]$ de cada elemento x a un conjunto A se consideró un grado de no pertenencia $\nu_A(x) \in [0,1]$, pero tal que para $x \in X, \mu_A(x) + \nu_A(x) \leq 1$..

Otro antecedente lo encontramos Belnap [11] definió una lógica de cuatro valores, con verdadero (T), falso (F), desconocida (U) y contradictorio (C). Utilizó bi-retículo donde los cuatro componentes estaban interrelacionados.

Se propuso término "neutrosófico" porque "neutrosófico" proviene etimológicamente de la "neutrosofía", que significa conocimiento del pensamiento neutro, y este tercer / neutral representa la distinción principal, es decir, la parte neutra / indeterminada / desconocida (además de la "verdad" / "pertenencia" y "falsedad" Componentes de "no pertenencia" que aparecen en la lógica borrosa / conjunto). NL es una generalización de la lógica difusa de Zadeh (LD), y especialmente de la lógica difusa intuitiva (LDI) de Atanassov, y de otras lógicas.

Conceptos Fundamentales

Sea U ser un universo de discurso, y M un conjunto incluido en U. Un elemento x de U se anota con respecto al conjunto M como x (T, I, F) y pertenece a M de la siguiente manera: es t % verdadero en el conjunto, i% indeterminado (desconocido s) en el conjunto, y f% falso, donde t varía en T, i varía en I y f varía en F. Estáticamente T, I, F son subconjuntos, pero dinámicamente T, I, F son funciones / operadores que dependen de muchos parámetros conocidos o desconocidos.

Los conjuntos neutrosóficos generalizan el conjunto difuso (especialmente el conjunto difuso e intuicionista), el conjunto paraconsistente, el conjunto intuitivo, etc.

Consideremos el intervalo de unidades no estándar $]-0, 1+[$, con bordes izquierdo y derecho vagos, imprecisos; Sea T, I, F los subconjuntos estándar o no estándar de $]-0, 1+[$;

La Lógica Neutrosófica (LN) [3] es una lógica en la que cada proposición es T% verdadera, I% indeterminada, y F% falsa;

-0 <= inf T + inf I + inf F <= sup T + sup I + sup F <= 3+;

T, I, F no son intervalos necesarios, sino cualquier conjunto (intervalos discretos, continuos, abiertos o cerrados o semi-abiertos / semi-cerrados, intersecciones o uniones de los conjuntos anteriores, etc.);

Ejemplo: la proposición P está entre 30-40% o 45-50% verdadera, 20% indeterminada y 60% o entre 66-70%

falsa (según diversos analizadores o parámetros);

El componente I, la indeterminación, se puede dividir en más subcomponentes para captar mejor la información vaga con la que trabajamos y, por ejemplo, podemos obtener respuestas más precisas a los Sistemas de Respuestas a Preguntas iniciadas por Zadeh [12].

En la lógica de cuatro valores de Belnap [11], la indeterminación se dividió en Incertidumbre (U) y Contradicción (C), pero estaban interrelacionadas.

Con respecto a la lógica difusa intuicionista En la LN no hay restricciones en T, I, F, mientras que en LDI la suma de componentes (o sus límites superiores) = 1; así la LN puede caracterizar la información incompleta (suma < 1), información paraconsistente (suma > 1).

3. Neutrosofía y números SVN

La neutrosofía como ya fue abordado fue propuesta por y Smarandache [13] para el tratamiento de la neutralidades. Esta ha formado las bases para una serie de teorías matemáticas que generalizan las teorías clásicas y difusas tales como los conjuntos neutrosóficos y la lógica neutrosófica[14] .

La definición original de valor de verdad en la lógica neutrosófica es mostrado a continuación [15]:

sean $N = \{(T, I, F) : T, I, F \subseteq [0, 1]\}n$, una valuación neutrosófica es un mapeo de un grupo de fórmulas proposicionales a N , esto es que por cada sentencia p tenemos:

$$v\,(p) = (T, I, F) \qquad (1.1)$$

Con el propósito facilitar la aplicación práctica a problema de la toma de decisiones y de la ingeniería se realizó la propuesta los conjuntos neutrosóficos de valor único [16] (SVNS por sus siglas en inglés) los cuales permiten el empleo de variable lingüísticas [17] lo que aumenta la interpretabilidad en los modelos de recomendación y el empleo de la indeterminación.

Sea X un universo de discurso. Un SVNS A sobre X es un objeto de la forma.

$$A = \{\langle x, u_A(x), r_A(x), v_A(x)\rangle : x \in X\}d \qquad (2)$$

donde $u_A(x): X \rightarrow [0,1]$, $r_A(x),: X \rightarrow [0,1]$ y $v_A(x): X \rightarrow [0,1]$ con $0 \leq u_A(x) + r_A(x) + v_A(x): \leq 3$ para todo $x \in X$. El intervalo $u_A(x)$, $r_A(x)$ y $v_A(x)$ denotan las membrecías a verdadero, indeterminado y falso de x en A, respectivamente. Por cuestiones de conveniencia un número SVN será expresado como $A = (a, b, c)$, donde $a, b, c \in [0,1]$, y $+ b + c \leq 3$. Los números SVN han presentado múltiples aplicaciones en el campo de la Inteligencia Artificial

4. Inteligencia Artificial y la neutrosofía

La Inteligencia Artificial (IA) ha llegado más allá de la ciencia ficción, hoy en día es parte de nuestra vida cotidiana, desde el uso de un asistente personal virtual para organizar nuestra agenda, hasta que nuestros teléfonos sugieran canciones que nos pueden gustar. Más allá de facilitar nuestras vidas, los sistemas inteligentes nos están ayudando a resolver algunos de los mayores desafíos del mundo: tratar enfermedades crónicas, luchar contra el cambio climático y anticipar las amenazas meteorológicas. AI es una de las tecnologías más estratégicas del siglo XXI y con su llegada se crearán numerosos puestos de trabajo, pero otros desaparecerán y la mayoría sufrirá transformaciones [18].

Una definición de Inteligencia Artificial se propone en [19] como la ciencia, que busca la comprensión profunda de la Inteligencia. La definición de esta capacidad, la comprensión de sus límites y alcances, así como su caracterización constituyen un problema de alta complejidad.

Las áreas fundamentales de la Inteligencia Artificial son las siguientes [20]:

Representación del conocimiento y razonamiento

Aprendizaje automático

Procesamiento del lenguaje natural

Visión por computadoras

Robótica

Reconocimiento automático del habla

El Test de Turing[21] es uno de los criterios de vida mental más debatidos y polémicos desde el punto de vista filosófico relacionado a la Inteligencia Artificial. Turing plantea que, si la máquina logra convencer a los jueces humanos, resulta justificado creer que es inteligente y pensante, debido a su capacidad para suplantar a humanos mediante comportamiento lingüístico [22] .

Un elemento importante en la Inteligencia Artificial es el aprendizaje automático. El aprendizaje automático es una rama de la Inteligencia Artificial que tiene como objetivo lograr que las computadoras aprendan. Existen 5 paradigmas fundamentales de la aprendizaje automático [23]:

Algoritmos evolutivos,

Conexionismo y redes neuronales,

Simbolismo,

Redes bayesianas,

Razonamiento por analogía

Otra área de gran importancia y actualidad para la Inteligencia Artificial son los agentes conversacionales. Existen dos tipos de agentes conversacionales fundamentales, los llamados chatbot y los agentes virtuales [24].

Los agentes conversacionales responden a guiones predeterminado de dialogo y los agentes virtuales responden a preguntas más complejas adicionalmente los primeros son distribuidos fundamentalmente por aplicaciones de mensajería. Los chatbots por su parte pueden ser definidos como robots que interactúa con usuarios a través de un chat simulando ser un operador o una persona en tiempo real, excelentes para optimizar la experiencia del usuario, gestionar pedidos y resolver sus necesidades [25]. Un agente virtual por su parte es un asistente personal inteligente que puede realizar tareas u ofrecer servicios a un individuo generalmente controlados mediante la voz [26].

Otra area de relevancia es la lógica difusa y la representacion de la incertidumbre y su empleo para representar sistemas complejos [27]. Los modelos causales son herramientas empleados para la ayuda a la toma de decisones [28, 29].

la causalidad desde un punto de vista computacional, requiere de modelos causales imprecisos que contemplen la incertidumbre [30]. La teoría de los conjuntos difusos o borrosos fue introducida por Zadeh [31] ofreeciendo un marco adecuado en el tratamiento de la causalidad imperfecta, haciendo uso de la vaguedad. Para la expresión del grado de causalidad entre conceptos se pueden emplear expresiones lingüísticas como "negativamente fuerte", "positivamente fuerte", "negativamente débil", "positivamente débil", etc.[32, 33]. Los mapas cognitivos difusos[34] es una técnica creada por Kosko como una extensión de los mapas cognitivos utilizando lógica borrosa [35] los cuales son empleados para el razonamiento causal y la representacion y análisis de modelos mentales [36]. Daveport [37] plantea la necesidad de que los agentes inteligentes construyan modelos mentales incluso de situaciones ficticias.

Es en este campo de la representación de la incertidumbre en que la neutrosofía ha realizado aportes fundamentales a

la IA. Como ya fue planteada la lógica neutrosófica es una generalización de la lógica difusa basada en el concepto de neutrosofía [38, 39]. Una matriz neutrosófica, por su parte, es una matriz donde los elementos $a = (a_{ij})$ han sido reemplazados por elementos en $\langle R \cup I \rangle$, donde $\langle R \cup I \rangle$ es un anillo neutrosófica entero [40].

Un grafo neutrosófico es un grafo en el cual al menos un arco es un arco neutrosófico [41]. La matriz de adyacencia neutrosófica Los bordes significan: 0 = no hay conexión entre nudos, 1 = conexión entre nudos, I = conexión indeterminada (desconocida si es o si no). Tales nociones no se utilizan en la teoría difusa, un ejemplo de muestra a continuación:

$$\begin{bmatrix} 0 & 1 & I & 0 & I \\ 1 & 0 & I & 0 & 0 \\ I & I & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 \\ I & 0 & 1 & 1 & 0 \end{bmatrix}$$

Si la indeterminación es introducida en un mapa cognitivo [42]entonces es llamado un mapa cognitivo neutrosófico, el cual resulta especialmente útil en la representación del conocimiento causal [38, 43].

Conclusiones

La neutrosofía es una nueva rama de la filosofía la cual estudia el origen, naturaleza y alcance de las neutralidades. Esta ha formado las bases para una serie de teorías matemáticas que generalizan las teorías clásicas y difusas tales como los conjuntos neutrosóficos y la lógica neutrosófica En el trabajo se presentaron los conceptos fundamentales relacionados con la neutrosofía y sus antecedentes. Adicionalmente se definieron conceptos fundamentales de la inteligencia artificial y cómo la neutrosofía ha venido a fortalecer esta disciplina.

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Modelos mentales y mapas cognitivos neutrosóficos
Mental models and neutrosophic cognitive maps

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Maikel Leyva-Vázquez, Rebeca Escobar-Jara, Florentin Smarandache (2018). Modelos mentales y mapas cognitivos neutrosóficos (Mental models and neutrosophic cognitive maps. *Neutrosophic Computing and Machine Learning* 1, 22-28

Abstract:

In this work, elements related to mental models elicitation and analysis are addressed through causal models. Issues related to the need to include indeterminacy in causal relationships through neutrophic cognitive maps are discussed. A proposal for static analysis in neutrosophic cognitive maps is presented. The following activities are included in the proposal: Calculate, measures of centrality, Classify nodes, De-neutrosification, and Ranking nodes. As future works, the incorporation of new metrics of centrality in neutrosophic cognitive maps is proposed. The inclusion of scenario analysis to the proposal is another area of future work.

Keywords: mental models, fuzzy cognitive maps, neutrosophic cognitive maps, static analysis in NCM

1 Introducción

Los modelos mentales son representaciones internas de una realidad externa de cada individuo [1, 2]. Esto, quiere decir, que, de la misma realidad externa, cada individuo puede tener variadas representaciones internas. Estas representaciones son modeladas frecuentemente mediante representaciones causales en presencia de incertidumbre [3].

Los modelos causales son herramientas cada vez más empleadas, para la comprensión y análisis de los sistemas complejos [4, 5]. Para considerar la causalidad desde un punto de vista computacional, se requiere la obtención de modelos causales imprecisos que tomen en consideración la incertidumbre [6]. El razonamiento causal es útil en la toma de decisiones por ser natural y fácil de entender y ser convincente porque explica el por qué se llega a una conclusión particular [7].

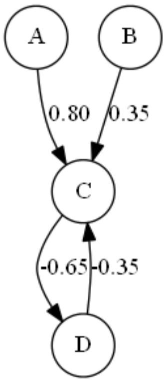
Para considerar la causalidad desde un punto de vista computacional, se requiere la obtención de modelos causales imprecisos empleando grafos dirigidos [6] . En este sentido existen dos técnicas de soft computing para la inferencia causal: redes bayesianas (RB) y mapas cognitivos difusos (MCD) [8]. Los MCD. Estos proveen esquemas más realistas para la representación del conocimiento brindando la posibilidad de representar ciclos y modelar la vaguedad [9].

2.Mapas Cognitivos Difusos (MCD)

Actualmente ha surgido la necesidad de plantear la causalidad en términos de lógica difusa ofreciendo esta un marco adecuado para tratar con la causalidad imperfecta. La teoría de los conjuntos difusos o borrosos fue introducida por Zadeh[11] en el año 1965. Esta parte de la teoría clásica de conjuntos, añadiendo una función de pertenencia [12].

Una función de pertenencia o inclusión $\mu_a(t)$ indica el grado n en que la variable t está incluida en el concepto representado por la etiqueta A [13]. Para la definición de estas funciones de pertenencia se utilizan convenientemente ciertas familias, por coincidir con el significado lingüístico de las etiquetas más utilizadas. Las más frecuentes son triangular, trapezoidal y gaussiana (Figura 1.7).

Los MCD (Figura 1.6) son una técnica desarrollada por Kosko como una extensión de los mapas cognitivos [14] permitiendo describir la fortaleza de la relación mediante el empleo de valores difusos en el intervalo [-1,1]. Constituyen una estructura de grafo difuso dirigido e incluyen la retroalimentación para representar causalidad [8]. La matriz de adyacencia se obtiene a partir de los valores asignados a los arcos (Figura 1).



	A	B	C	D
A	0	0	0.8	0
B	0	0	0.35	0
C	0	0	0	-0.65
D	0	0	-0.35	0

Figura 1 Mapa cognitivo difuso y su correspondiente matriz de adyacencia [15].

En los MCD existen tres posibles tipos de relaciones causales entre conceptos: causalidad positiva, causalidad negativa o la no existencia de relaciones.

–Causalidad positiva ($W_{ij} > 0$): Indica una causalidad positiva entre los conceptos C_i y C_j, es decir, el incremento (disminución) en el valor de C_i lleva al incremento (disminución) en el valor de C_j.

Causalidad negativa ($W_{ij} < 0$): Indica una causalidad negativa entre los conceptos C_i y C_j, es decir, el incremento (disminución) en el valor de C_i lleva la disminución (incremento) en el valor de C_j.

La no existencia de relaciones ($W_{ij} = 0$): Indica la no existencia de relación causal entre C_i y C_j.

Por otra parte el análisis dinámico se centra en el análisis de escenarios y orientado a metas [15]. Permite al usuario realizar observaciones y conclusiones adicionales no disponibles mediante el simple análisis estático. Está basado en un modelo de ejecución que calcula los niveles de activación en iteraciones sucesivas de los distintos conceptos. Esta simulación requiere adicionalmente la definición de los valores iniciales para cada concepto en un vector inicial [16].

Los valores de los conceptos son calculados en cada paso de la simulación forma siguiente:

$$A_i^{(t+1)} = f\left(A_i^{(t)} + \sum_{j=1}^{N} A_j^{(t)} \cdot w_{ji}\right) \quad (1)$$

donde $A_i^{(t+1)}$ es el valor del concepto C_i en el paso $t + 1$ de la simulación, $A_j^{(t)}$ es el valor del concepto C_j en el paso t de la simulación, w_{ji}es el peso de la conexión que va del concepto C_j al concepto C_i y $f(\cdot)$ es la función de activación [17]. Las principales funciones de activación reportadas en la literatura son la sigmoide y la tangente hiperbólica [17]. Estas funciones emplean un valor lambda (λ) para definirla pendiente [18]. De acuerdo al vector de entrada, el MCD convergerá a uno de los siguientes estados: punto fijo, ciclo límite o atractor caótico [19].

Los MCD han sido empleados para la toma de decisión en grupo debido a las facilidades que brinda para la agregación de modelos causales provenientes de múltiples expertos [20, 21]. Cuando participa un conjunto de expertos (k), la matriz de adyacencia del MCD colectivo se calcula de la siguiente forma:

$$E = \mu(E_1, E_2, \ldots, E_k) \quad (2)$$

siendo por lo general el operador μ la media aritmética[22] o la media aritmética ponderada.

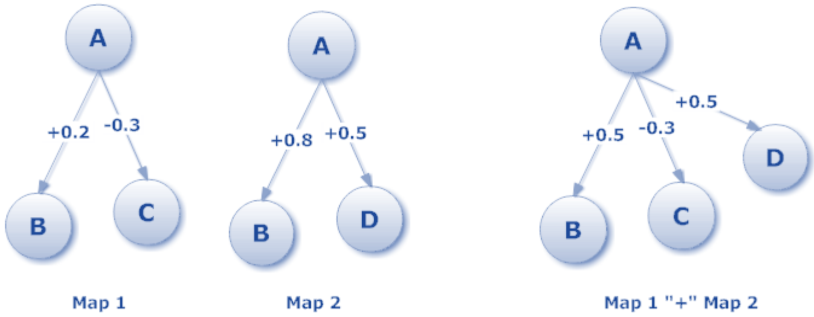


Figura 2. Agregación de MCD[23]

La agregación de MCD resulta especialmente útil debido a la importancia que presenta integrar conocimientos de diferentes expertos con modelos mentales diversos permitiendo la construcción de modelos mentales colectivos[24, 25].

En el proceso de agregación de los mapas cognitivos difusos se emplea fundamentalmente los operadores media y media ponderada (WA por sus siglas en inglés). Un operador WA tiene asociado un vector de pesosV, con $v_i \in [0,1]$ y $\sum_1^n v_i = 1$, teniendo la siguiente forma:

$$WA(a_1, .., a_n) = \sum_{i=1}^{n} v_i a_i \quad (3)$$

donde v_i representa la importancia/relevancia de la fuente de datos a_i.

Si se introduce un valor de credibilidad o fiabilidad de las fuentes se mejora este proceso realizando la agregación mediante la WA [22, 26] para la asignación de pesos se recomienda el empleo del proceso de Jerarquía Analítica (AHP por sus siglas en inglés).

Esta agregación de conocimiento permite mejorar la fiabilidad del modelo final, el cual es menos susceptible a creencias potencialmente erróneas de los expertos individuales [16]. Resulta especialmente útil además debido a la importancia que presenta integrar conocimientos de diferentes expertos con modelos mentales diversos [24]. Sin embargo, esta agregación de conocimiento es muy sensible a la presencia de valores atípicos, errores y valoraciones prejuiciadas [15, 27]. Es criterio de la autora de la investigación que este aspecto debe ser abordado desde nuevos enfoques que vayan más allá de la agregación de información mediante externos al modelo.

3 Mapas Cognitivos Neutrosóficos

La lógica neutrosófica es una generalización de la lógica difusa basada en el concepto de neutrosofía [28, 29]. Una matriz neutrosófica, por su parte, es una matriz donde los elementos $a = (a_{ij})$ han sido reemplazados por elementos en $\langle R \cup I \rangle$, donde $\langle R \cup I \rangle$ es un anillo neutrosófica entero [30]. Un grafo neutrosófico es un grafo en el cual al menos un arco es un arco neutrosófico [31].

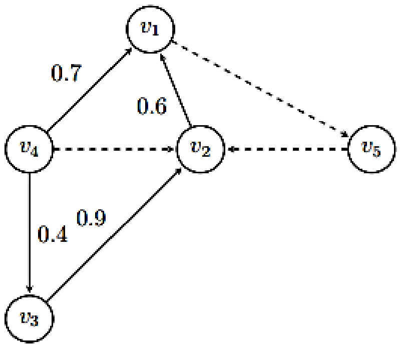


Figura. 3 Ejemplo MCN.

Si la indeterminación es introducida en un mapa cognitivo [32]entonces es llamado un mapa cognitivo neutrosófico, el cual resulta especialmente útil en la representación del conocimiento causal al permitir la representación y análisis de la indeterminación [28, 33].

Análisis estático en MCN

El análisis estático en MCN se centra en la selección de los conceptos que juegan un papel más importante en el sistema modelado [34]. Se realiza a partir de la matriz de adyacencia tomando en consideración el valor absoluto de los pesos [35]. A continuación, se muestra el proceso

Figura 4: Proceso propuesto.

Las siguientes medidas se emplean en el modelo propuesto basado en los valor absolutos de la matriz de adyacencia [16]: Outdegree $od(v_i)$ es la suma de las filas en la matriz de adyacencia neutrosófica. Refleja la fortaleza de las relaciones (c_{ij}) saliente de la variable.

$$od(v_i) = \sum_{i=1}^{N} c_{ij} \qquad (4)$$

Indegree $id(v_i)$ es la suma de las columnas Refleja la Fortaleza de las relaciones (c_{ij}) saliente de la variable.

$$id(v_i) = \sum_{i=1}^{N} c_{ji} \qquad (5)$$

Centralidad total (total degree $td(v_i)$), es la suma del indegree y el outdegree de la variable.

$$td(v_i) = od(v_i) + id(v_i) \qquad (6)$$

En este caso se representa la relación entre las competencias en este caso un subconjunto de las llamadas competencias transversales de los estudiantes de sistemas [36].

Competencia	Descripción
c_1	Grado de capacidad para la resolución de los problemas matemáticos
c_2	Grado de comprensión y dominio de los conceptos básicos sobre las leyes de la informática
c_3	Grado de conocimientos sobre el uso y programación de los ordenadores
c_4	Grado de capacidad para resolver problemas dentro de su área de estudio
c_5	Grado motivación por el logro profesional y para afrontar nuevos retos,

Tabla 1. Competencias analizadas

El NCM se desarrolla mediante la captura de del conocimiento. La matriz de adyacencia neutrosófica generada se muestra en la Tabla 2.

0	0.7	0.4	I	0
0	0	0.9	0.7	0
0	0	0	0.9	0
0	0.5	0	0	0.9
0	I	0	0.7	0

Tabla 2: Matriz de adyacencia.

Las medidas de centralidad calculadas son mostradas a continuación. :

c_1	1.1+I
c_2	1.6
c_3	0.9
c_4	1.4
c_5	0.7+I

Tabla 3: Outdegree

c_1	0
c_2	1.2+I
c_3	1.3
c_4	2.3+I
c_5	0.9

Table 4: Indegree

c_1	1.1+I
c_2	2.8+I
c_3	1.9
c_4	3.7+I
c_5	1.6+I

Table 5: Total degree

Los nodos se clasifican de acuerdo con las siguientes reglas:

- Las variables transmisoras tienen outdegree positivo o indeterminada, y cero indegree.
- Las variables receptoras tienen una indegree indeterminado o positivo, y cero outdegree.
- Las variables ordinarias tienen un grado de indegree y outdegree distinto de cero. A continuación se clasifican los nodos

Nodo	Transmisor	Receptor	Ordinaria
c_1	X		
c_2			X
c_3			X
c_4			X
c_5			X

Table 6: Clasificación de los nodos

Un análisis estático en NCM [37] el cual da como resultado inicialmente número neutrosóficos de la forma (a+bI, donde I = indeterminación) [38]. E por ello que se requiere un procesos de-neutrosificación tal como fue propuesto por Salmerón and Smarandache [39]. I ∈[0,1] es reemplazado por sus valores máximos y mínimos.

c_1	[1.1, 2.1]
c_2	[3.7, 5.7]
c_3	2.18
c_4	[3.4, 4.4]
c_5	[1.6, 2.6]
c_6	[2.2, 3.2]

Tabla 7: De-neutroficación

Finalmente se trabaja con la media de los valores extremos para obtener un único valor [40] .

$$\lambda([a_1, a_2]) = \frac{a_1 + a_2}{2} \qquad (7)$$

entonces

$$A \succ B \Leftrightarrow \frac{a_1 + a_2}{2} > \frac{b_1 + b_2}{2} \quad (3.8)$$

c_1	1.6
c_2	4,7
c_3	2.18
c_4	3,9
c_5	2,1
c_6	2.7

Table 8. Media de los valores extremos

A partir de estos valores numéricos se obtiene el siguiente orden $c_2 \succ c_4 \succ c_6 \succ c_3 \succ c_5 \succ c_1$

En este caso la competencia más importante es:" Comprensión y dominio de los conceptos básicos sobre las leyes de la informática".

Conclusiones

En el presente trabajo se abordaron aspectos relacionados con los modelos mentales mediante modelos causales. Se trataron aspectos relacionados con la necesidad de incluir la indeterminación en las relaciones causales mediante mapas cognitivos neutrosófico. Se presentó una propuesta para el análisis estático en mapas cognitivos neutrosóficos. Se incluyeron las siguientes actividades: Calcular, medidas de centralidad, Clasificar nodos, De-neutrosificación. Ordenar por importancia los nodos.
Como trabajos futuros se plantea la incorporación de nuevas métricas de centralidad en mapas cognitivos neutrosóficos. La inclusión del análisis de escenarios a la propuesta es otra área de trabajo futuro.

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Modelo de Recomendación Basado en Conocimiento y Números SVN

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Abstract

Recommendation models are useful in the decision-making process that allow the user a set of options that are expected to meet their expectations. Recommendation models are useful in the decision-making process that offer the user a set of options that are expected to meet their SVN expectations to express linguistic terms.
Kewords: recommeder systems, SVN numbers, decision-making.

1-Introducción

Los modelos de recomendación son útiles en el proceso de toma de decisiones ya que proporcionan al usuario un conjunto de opciones que se espera satisfagan sus expectativas [1].
En el presente trabajo se propone un modelo de recomendación basado en conocimiento utilizando el de números neutrosóficos de valor único (SVN por sus siglas en inglés) permitiendo la utilización de variables lingüísticas [2, 3].
Sea X un universo de discurso. Un SVNS A sobre X es un objeto con la siguiente forma.

$$\{\langle x, u_A(x), r_A(x), v_A(x)\rangle : x \in X\}d \quad (1)$$

donde $u_A(x): X \to [0,1]$, $r_A(x), : X \to [0,1]$ y $v_A(x): X \to [0,1]$ con $0 \leq u_A(x) + r_A(x) \quad (x) : \leq 3$ para todo $x \in X$.
El intervalo $u_A(x)$, (x) y $v_A(x)$ representa las membrecías a verdadero, indeterminado y falso de x en A, respectivamente. Por cuestiones de conveniencia un número SVN será expresado como $A = (a, b, c)$, donde a, b, $c \in [0,1]$, y $+ b + c \leq 3$. Los números SVN han presentado múltiples aplicaciones en el campo de la toma de decisiones en general y en los sistemas de recomendación en particular.
Adicionalmente se abordarán algunos aspectos relacionados con las herramientas sugeridas para el desarrollo de modelos computacionales en el lenguaje Python.

2 Modelos de Recomendación

Partiendo de la información que recojan estos modelos y de los algoritmos utilizados para generar las recomendaciones se puede distinguir las siguientes técnicas [1, 2]:

- **Modelos de recomendación colaborativa:** Agregan las valoraciones o recomendaciones de los objetos, identifican los gustos comunes de los usuarios basándose en sus valoraciones y generan una nueva recomendación teniendo en cuenta las comparaciones entre usuarios.
- **Modelos de recomendación basada en contenido:** Aprende de un perfil de intereses de los usuarios basándose en las características presentes en los objetos que el usuario ha seleccionado.
- **Modelos de recomendación basada en conocimiento:** Intentan sugerir objetos haciendo inferencias sobre las necesidades de un usuario y sus preferencias, apoyados fundamentalmente en el razonamiento basado en casos.
- **Modelos de recomendación basados en utilidad:** Estos se basan en la construcción de funciones de utilidad. El perfil del usuario lo constituye una función de utilidad, las ventajas de las recomendaciones basadas en utilidad, es poder trabajar con atributos no relacionados directamente con los productos.
- **Modelos de recomendación híbridos:** Individualmente las técnicas presentan algunas limitaciones o problemas. Para solucionar estas deficiencias se ha planteado la hibridación de distintas técnicas de recomendación. Se plantea que existe hibridación cuando se combinan dos o más técnicas de recomendación con el objetivo de obtener mejores resultados que, si se utilizara estas técnicas de forma independiente.

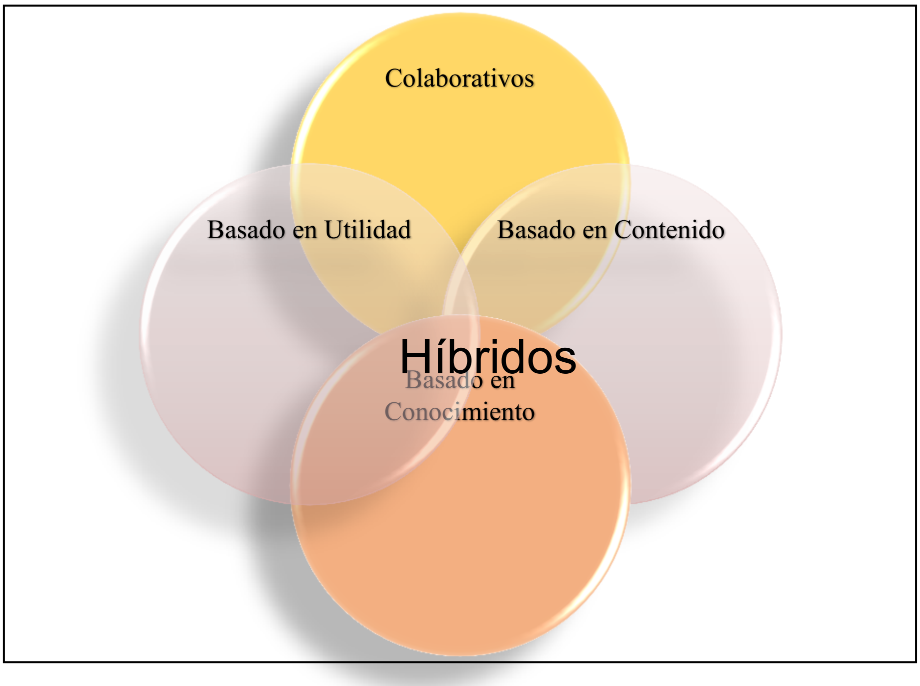

Figura 3.1 Diagrama de Venn de los modelos de recomendación.

Los modelos de recomendación basada en conocimiento realizan sugerencias haciendo inferencias sobre las necesidades del usuario y sus preferencias [1-3]. El enfoque basado en conocimiento se distingue en el sentido que usan conocimiento sobre cómo un objeto en particular puede satisfacer las necesidades del usuario, y por lo tanto tiene la capacidad de razonar sobre la relación entre una necesidad y la posible recomendación que se mostrará. Se basan en la construcción de perfiles de usuarios como una estructura de conocimiento que apoye la inferencia la cual puede ser enriquecida con la utilización de expresiones que emplea lenguaje natural [2, 4]. En [5] el que se propone un modelo de recomendación que hace uso de las redes sociales y la neutrosofía para el campo del e-learning pero no puede ser clasificado en el campo de los sistemas de recomendación basados en conocimiento.

3. Modelo Propuesto

A continuación, se presenta el flujo de trabajo. Está basado fundamentalmente en la propuesta de Cordón [2, 6] para sistemas de recomendación basados en conocimiento permitiendo representar términos lingüísticos y la indeterminación mediante números SVN [7, 8].

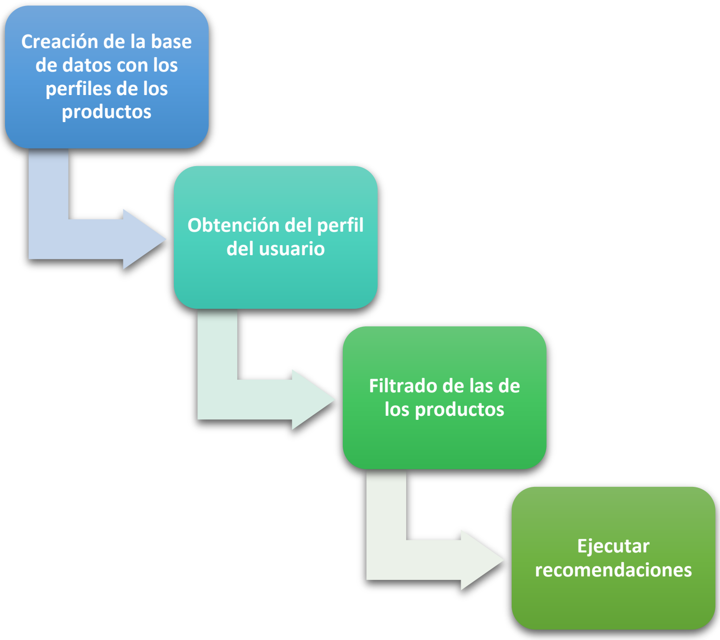

Fig. 1. Figura 3.2. Modelo propuesto

La descripción detallada de cada una de sus actividades y del modelo matemático que soporta la propuesta es presentada a continuación.

Creación de la base de datos con los perfiles de los productos

Cada una de los productos a_i serán descritas por un conjunto de características que conformarán el perfil de los productos.

$$\{c_1, \dots, c_k, \dots, c_l\} \quad (.2)$$

Para la obtención de la base de datos de los productos, el perfil de los usuarios es obtenido mediante números neutrosóficos de valor único (SVN por sus siglas en inglés) [9, 10].

Sea $A^* = (A_1^*, A_2^*, \ldots, A_n^*)$ sea un vector de números SVN tal que $A_j^* = (a_j^*, b_j^*, c_j^*)$ j=(1,2, … , n) y $B_i = (B_{i1}, B_{i2}, \ldots, B_{im})$ (i = 1,2, … , m) sean m vectores de n SVN números tal que y = (a_{ij}, b_{ij}, c_{ij}) (i = 1,2, … , m), (j = 1,2, … , n) entonces la distancia euclidiana es definida como. Las B_i y resulta [10]:

$$d_i = \left(\frac{1}{3}\sum_{j=1}^{n}\left\{\left(|a_{ij}-a_j^*|\right)^2+\left(|b_{ij}-b_j^*|\right)^2+\left(|c_{ij}-c_j^*|\right)^2\right\}\right)^{\frac{1}{2}} \quad (3)$$

(i = 1,2, … , m)

A partir de esta distancia euclidiana se puede definir una medida de similitud [11].

En la medida en que la alternativa sea más cercana al perfil del usuario (s_i) mayor será la similitud, permitiendo establecer un orden entre alternativas [12].

Este perfil puede ser obtenido de forma directa a partir de expertos:

$$= \{v_1^j, \ldots, v_k^j, \ldots v_l^j\}, j = 1, \ldots n \quad (4)$$

Las valoraciones de las características del producto, a_j, serán expresadas utilizando la escala lingüística S, donde $S = \{s_1, \ldots, s_g\}$ es el conjunto de término lingüísticos definidos para evaluar las características c_k utilizando los números SVN. Para esto los términos lingüísticos a emplear son definidos.

Una vez descrito el conjunto de productos

$$= \{a_1, \ldots, a_j, \ldots, a_n\} \quad (5)$$

Estos se guardan en una base de datos.

Obtención del perfil del usuario

En esta actividad se obtiene la información del usuario sobre las preferencias de estos, almacenándose en un perfil:

$$\{p_1^e, \ldots, p_k^e, \ldots, p_l^e\} \quad (6)$$

Dicho perfil estará integrado por un conjunto de atributos:

$$\{c_1^e, \ldots, c_k^e, \ldots, c_l^e\} \quad (7)$$

Donde c

Este puede ser obtenido mediante ejemplo o mediante el llamado enfoque conversacional o mediante ejemplos los cuales pueden ser adaptados [13].

Filtrado de los productos

En esta actividad se filtran los productos de acuerdo al perfil del usuario para encontrar cuáles son las más adecuadas para este.

Con este propósito es calculada la similitud entre perfil de usuario, P_e y cada producto a_j registrado en la base de datos. Para el cálculo de la similitud total se emplea la siguiente expresión:

$$\left(\left(\frac{1}{3}\sum_{j=1}^{n}\left\{\left(|a_{ij}-a_j^*|\right)^2+\left(|b_{ij}-b_j^*|\right)^2+\left(|c_{ij}-c_j^*|\right)^2\right\}\right)^{\frac{1}{2}}\right) \quad (8)$$

La función S calcula la similitud entre los valores de los atributos del perfil de usuario y la de los productos, a_j [14].

Ejecutar recomendaciones

Una vez calculada la similitud entre el perfil del usuario en la base de datos y cada uno de los productos se ordenan de acuerdo a la similitud obtenida, representado por el siguiente vector de similitud.

$$= (d_1, \dots, d_n) \quad (9)$$

Los mejores serán aquellos, que mejor satisfagan las necesidades del perfil del usuario es decir con mayor similitud.

4. Ejemplo Demostrativo

A continuación se presenta un ejemplo demostrativo basado en [15], supongamos una base de datos:

$$\{a_1, a_2, a_3, a_4, a_5\}$$

Descrito por el conjunto de atributos

$$\{c_1, c_2, c_3, c_{4,}, c_5\}$$

Los atributos se valorarán en la siguiente escala lingüística (Tabla 1). Estas valoraciones serán almacenadas por el sistema en una base de datos.

Tabla 1: Términos lingüísticos empleados [10].

Término lingüístico	Números SVN
Extremadamente buena(EB)	(1,0,0)
Muy muy buena (MMB)	(0.9, 0.1, 0.1)
Muy buena (MB)	(0.8,0,15,0.20)
Buena(B)	(0.70,0.25,0.30)
Medianamente buena (MDB)	(0.60,0.35,0.40)
Media(M)	(0.50,0.50,0.50)
Medianamente mala (MDM)	(0.40,0.65,0.60)
Mala (MA)	(0.30,0.75,0.70)
Muy mala (MM)	(0.20,0.85,0.80)
Muy muy mala (MMM)	(0.10,0.90,0.90)
Extremadamente mala (EM)	(0,1,1)

La vista de la base de datos utilizado en este ejemplo, la podemos ver en la Tabla 2.

Tabla 2: Base de datos de productos.

	MDB	M	MMB	B
	B	MD	MB	M
	MMB	M	M	B
	M	B	MMB	B

Si un usuario $_e$, desea recibir las recomendaciones del sistema deberá proveer información al mismo expresando sus preferencias. En este caso:

$$\{\mathrm{MDB}, \mathrm{MB}, \mathrm{MMB}, \mathrm{MB}\}$$

El siguiente paso en nuestro ejemplo es el cálculo de la similitud entre el perfil de usuario y los productos almacenados en la base de datos.

Tabla 1: Similitud entre los productos y el perfil de usuario

0.44	0.76	0.42	0.84

En la fase de recomendación se recomendará aquellos productos que más se acerquen al perfil del usuario. Un ordenamiento de los productos basado en esta comparación sería el siguiente.

$$\{a_4, a_2, a_1, a_3\}$$

En caso de que el sistema recomendara los dos productos más cercanos, estas serían las recomendaciones:

$$a_4, a$$

Con este ejemplo queda demostrada la aplicabilidad de la propuesta.

Conclusiones

En este trabajo se presentó un modelo de recomendación de productos siguiendo el enfoque basado en conocimiento. El mismo que se basa en el empleo de los números SVN para expresar términos lingüísticos.

Trabajos futuros estarán relacionados con la creación de la base de datos a partir de múltiples expertos, así como la obtención de los pesos de las características utilizando valoraciones en grupo. Adicionalmente se trabajará en la inclusión de modelos de agregación más complejos, así como la hibridación con otros modelos de recomendación.

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Lógica neutrosófica refinada n-valuada y sus aplicaciones a la física

n-Valued Refined Neutrosophic Logic and Its Applications to Physics

Florentin Smarandache

Florentin Smarandache (2018). Lógica neutrosófica refinada n valuada y sus aplicaciones a la física (n-Valued Refined Neutrosophic Logic and Its Applications to Physics). *Neutrosophic Computing and Machine Learning* 2, 3-8

Abstract: In this paper we present a short history of logics: from particular cases of 2-symbol or numerical valued logic to the general case of n-symbol or numerical valued logic. We show generalizations of 2-valued Boolean logic to fuzzy logic, also from the Kleene's and Lukasiewicz' 3-symbol valued logics or Belnap's *4*-symbol valued logic to the most general *n-symbol or numerical valued refined neutrosophic logic*. Examples of applications of neutrosophic logic to physics are listed in the last section. Similar generalizations can be done for *n-Valued Refined Neutrosophic Set*, and respectively

Keywords: n-symbol valued logic.Neutrosophic Logic, física neutrosófica, física paradoxista

1 Lógica de 2 valores

1.1 Lógica valuada en dos símbolos

En la filosofía china: Yin y Yang (o Feminidad y Masculinidad) se representan como contrarios::

Fig. 1: Ying y Yang

También en la lógica clásica o booleana, se tienen dos valores: verdad T y falsedad F.

1.2 Lógica de dos valores numéricos

También es la lógica clásica o booleana, se tiene dos valores numéricos: verdad 1 y falsedad 0. Más general es la lógica difusa, donde la verdad (T) y la falsedad (F) pueden ser cualquier número en [0, 1] tal que T + F = 1.
Aún más general, T y F pueden ser subconjuntos de [0,1].

2 Lógica de tres valores

2.1 Lógicas trivalente con tres símbolos

1. *Lógica de Lukasiewicz* : Verdadero, Falso, y Posible.
2. *Lógica de Kleene*: Verdadero, Falso, Desconocido (o Indefinido).
3. Filosofía China extendida a: *Yin, Yang,* y *Neutro* (o Feminidad, Masculinidad, y Neutralidad)- como en la neutrosofía.

4. La filosofía neutrosófica surgió de la neutralidad entre varias filosofías. Conectada con la exténica (Prof. Cai Wen, 1983), y el paradoxismo (F. Smarandache, 1980). La neutrosofía es una nueva rama de la filosofía que estudia el origen naturaleza y alcance de las neutralidades. Esta teoría considera cualquier noción o idea A junto a su opuesto o negación AntiA y el espectro de neutralidades neutA entre ellas (nociones o ideas que no soportan ni a A ni antiA) . NeutA y AntiA juntas se les conoce con noA . La neutrosofía es una generalización de la dialéctica de Hegel (esta solo se basa en A y antiA). De acuerdo a esta teoría toda idea A tiende a ser neutralizada y balanceada por antiA y noA como un estado de equilibrio. De una forma clásica A, neutA y antiA son disjuntos dos por dos. Sin embargo en la mayoría de los casos lo límites entre ellos resultan vagos e imprecisos. La neutrosofía es la base de todas las teorías neutrosóficas con múltiples aplicaciones a la ingeniería (especialmente en la ingeniería de software y la fusión de la información), medicina, militares, aeroespaciales, cibernética y física.

2.2 Lógica numericamente valudas de tres valores

1. *Lógica de Kleene*: Verdadero (*1*), Falso (*0*), Desconocido (o Indefinido) (*1/2*), y utiliza —min” para ∧ , —max” para ∨ , y —1-” para la negación.
2. Más general resulta la lógica [Smarandache, 1995], donde la verdad (*T*) , la falsedad (*F*) y la indeterminación (*I*) pueden ser números en el intervalo *[0, 1]*, entonces : $0 \leq T + I + F \leq 3$.

3 Lógica de cuatro valores

3.1 Lógica de valuda en cuatro símboles Lógica

1. *Lógica de Belnap*: Verdadero (*T*), Falso (*F*), Desconocido(*U*) , y Contradicción (*C*), donde *T*, *F*, U, *C* son símbolo. A continuación la tabla del operador de conjunción de Belnap,

	F	U	C	T

F	F	F	F	F
U	F	U	F	U
C	F	F	C	C
T	F	U	C	T

Restrigda a *T,F,U*, y a *T,F,C*,los conectores d ela lógica de Belnap coincide con las conectivas lógicas de la lógica de Kleene.

2. Sea *G* = Ignorancia. Se puede proponer la siguiente lógica de cuatro símbolos: *(T, F, U, G)*, y *(T, F, C, G)*.
3. *Realidad Absoluta-Relativa 2-, 3-, 4-, 5-, Lógica Valuada en 6 Símbolos* [Smarandache, 1995]. Sea verdadero en todos los mundos posibles (de acuerdo a la definición de Leibniz), sea verdadero en al menos uno de los mundos posibles pero no en los otros, y de forma similar sea indeterminado en todos los mundos posibles , sea indeterminado en al menos uno de los mundos y no en otros; adicionalmente sea falso en todos los mundos posibles pero no todos los mundos, sea falso en al menos uno pero no en todos. los mundos posibles, entonces podemos formar varias lógicas Absolutas-Relativas 2-, 3-, 4-, 5-, o lógica valuada en 6 símbolos solo tomando combinaciones de estos símbolos. O A lógica valuada en 6 símbolos

3.2 Lógica de 4 Valores Numéricos

La indeterminación I se refina (divide) como U = Desconocida, y C = contradicción. T, F, U, C son subconjuntos de [0, 1], en lugar de símbolos; Esta lógica generaliza la lógica de Belnap ya que uno obtiene un grado de verdad, un grado de falsedad, un grado de desconocimiento y un grado de contradicción..

4 Lógica de 4 valores

Lógica neutrosófica valorada en cinco símbolos [Smarandache, 1995]: la indeterminación I se refina (divide) como U = Desconocido, C = contradicción y G = ignorancia; donde los símbolos representan:

T = verdad;

F = falsedad;

U = ni T ni F (indefinido);

C = T $\wedge$F , involucra la Exténica;

G = T $\vee$F

. Si T, F, U, C, G son subconjuntos de [0, 1], entonces obtenemos: una lógica neutrosófica de cinco valores numéricos.

5 Lógica de n valores

1. La lógica neutrosófica de n valores simbólicos [Smarandache, 1995]. En general:

T se puede dividir en muchos tipos de verdades: , I en muchos tipos de nes: y F en muchos tipos de falsedades: d $_s$onde todos son enteros y

Todos los subcomponentes, son símbolos de , para todos { } { } e { }.

2. *La lógica neutrosófica refinada de n-valor numérico.* De la misma manera, pero todos los subcomponentes , no son símbolos, sino subconjuntos de [0,1], para todos { } { } e { }. Si todas las fuentes de información que proporcionan valores neutrosóficos por separado para un subcomponente específico fuentes independientes, entonces en el caso general consideramos que cada uno de los subcomponentes T_j, I_k, F_l, es independiente con respecto a los demás y está en el conjunto no estándar] [. Por lo tanto, tenemos un total para los subcomponentes $_l$, que:

$\sum_{j=1}^{p} T_j + \sum_{k=1}^{r} I_k + \sum$

Donde n, por supuesto, como arriba. Si hay algunas fuentes dependientes (o, respectivamente, algunos subcomponentes dependientes), podemos tratar esos subcomponentes dependientes juntos.

6. Distinción entre física neutrosófica y física paradoxista

En primer lugar se realiza un distinción entre la física neutrosófica y al física paradoxista

1. **Física Nuetrosófica**

Sea A una entidad física (es decir, concepto, noción, objeto, espacio, campo, idea, ley, propiedad, estado, atributo,teorema, teoría, etc.), antiA sea lo opuesto a A, y neutA sea su neutral (es decir, ni A ni antiA, sino en el medio).

La Física Neutrosófica es una mezcla de dos o tres de estas entidades A, antiA y neutA que se mantienen juntas.

Por lo tanto, podemos tener campos neutrosóficos y objetos neutrosóficos, estados neutrosóficos, etc.2. **Paradoxist Physics**

La Física Neutrosófica es una extensión de la Física Paradoxista, ya que la Física Paradoxista es una combinación de contradictorios físicos A y antiA solo que se mantienen unidos, sin referirse a su neutralidad neutA. La física paradójica describe las colecciones de objetos o estados que se caracterizan individualmente por propiedades contradictorias, o que se caracterizan ni por una propiedad ni por el opuesto de esa propiedad, o están compuestos de subelementos contradictorios. Tales objetos o estados se llaman entidades paradojas.

Estos dominios de investigación se establecieron en 1995 en el marco de la neutrosofía, lógica / conjunto / probabilidad / estadística neutrosóficas.

10 Lógica Neutrosófica N-valuada Refinada Aplicada a la Física

Hay muchos casos en los campos científicos (y también humanísticos) en los que dos o tres de estos elementos A, antiA y neutA coexisten simultáneamente.

Varios ejemplos de entidades paradójicas y neutrosóficas:

• los aniones en dos dimensiones espaciales son partículas de spin arbitrarias que no son ni bosones (integerspin) ni fermions (giro de medio entero);

• entre los posibles candidatos de Dark Matter, puede haber partículas exóticas que no sean fermentos de Dirac ni de Majorana;

• mercurio (Hg) es un estado que no es líquido ni sólido en condiciones normales a temperatura ambiente;

• los materiales no magnéticos no son ni ferromagnéticos ni antiferromagnéticos;

• quark gluon plasma (QGP) es una fase formada por quarks casi libres y gluones que no se comporta como un plasma convencional ni como un líquido ordinario;
• no relacionado, que está formado por la materia y la antimateria que se unen (F. Smarandache, 2004);
• kaon neutral, que es un compuesto pión y anti-pión (R. M. Santilli, 1978) y por lo tanto una forma de desapego;
• Métodos neutrosóficos en general relatividad (D. Rabounski, F. Smarandache, L. Borissova, 2005);
• modelo cosmológico neutrosófico (D. Rabounski, L. Borissova, 2011);
• gravitación neutrosófica (D. Rabounski);
• superposición cuántica y en general cuántica de estados;
• los semiconductores no son conductores ni aisladores;
• los componentes ópticos semi-transparentes no son ni opacos ni perfectamente transparentes a la luz;
• los estados cuánticos son metaestables (ni perfectamente estables ni inestables);
• doblete de fotones de neutrinos (E. Goldfain);
• el "multiplete" de partículas elementales es una especie de "campo neutrosófico" con dos o más valores (E. Goldfain, 2011);
• Un "campo de neutrosofía" se puede generalizar al de los operadores cuya acción es selectiva. El efecto del campo neutrosophic es de alguna manera equivalente con el “túnel” de la física de los sólidos, o con la “ruptura espontánea de simetría” (SSB) en la que hay una simetría interna que se rompe por una selección particular del estado de vacío (E. Goldfain). Etc.

Conclusiones

Muchos tipos de lógicas se han presentado arriba. Para la lógica más general, la lógica neutrosófica refinada n-valorada.Se hacen generalizaciones similares para el conjunto neutrosófico refinado n-valorado y la probabilidad neutrosófica refinada n-valorada

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Computación neutrosófica mediante Sympy
Neutrosophic Computing with Sympy

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Maikel Leyva-Vázquez, Florentin Smarandache (2018). Computación neutrosófica mediante Sympy (Neutrosophic Computing with Sympy). *Neutrosophic Computing and Machine Learning* 3, 16-21

Abstract:

In this article the concept of neutrosophic number is presented. Jupyter through Google Colaboratory is introduced for calculations. The Sympy library is used to perform the process of neutrosophic computation. Systems of linear neutrosóficas equations are solved by means of the symbolic computation in python. A case study was developed for the determination of vehicular traffic with indeterminacy. As future works are the development of new applications in different areas of engineering and science

Keywords: neutrosophic computing, sympy, google colaboratory, neutrosophic number.

1. Introducción

Neutrosofía significa conocimiento del pensamiento neutro, y este tercer / neutral representa la distinción principal, es decir, la parte neutra / indeterminada / desconocida (además de la "verdad" / "pertenencia" y "falsedad" Componentes de "no pertenencia" que aparecen en la lógica borrosa / conjunto). La lógica neutrosófica (LN) es una generalización de la lógica difusa de Zadeh (LD), y especialmente de la lógica difusa intuitiva (LDI) de Atanassov, y de otras lógicas multivaluadas (Figura 1) [1].

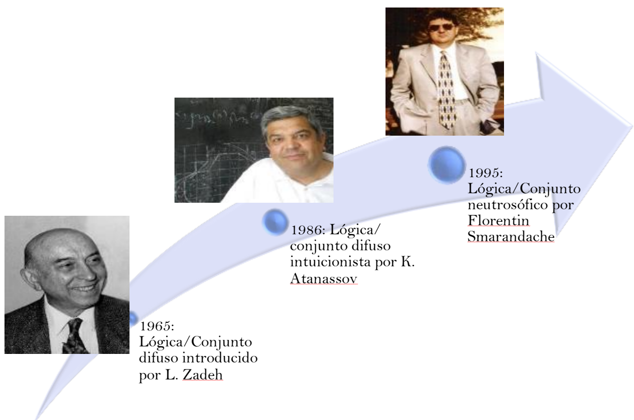

Figura 1. Neutrosofía y sus antecedentes fundamentales [1].

Sea U ser un universo de discurso, y M un conjunto incluido en U. Un elemento x de U se anota con respecto al conjunto M como x (T, I, F) y pertenece a M de la siguiente manera: es t % verdadero en el conjunto, i% indeterminado (desconocido s) en el conjunto, y f% falso, donde t varía en T, i varía en I y f varía en F. Estáticamente T, I, F son subconjuntos, pero dinámicamente T, I, F son funciones / operadores que dependen de muchos parámetros conocidos o desconocidos [2, 3].

Los conjuntos neutrosóficos generalizan el conjunto difuso (especialmente el conjunto difuso e intuicionista), el conjunto paraconsistente, el conjunto intuitivo y otros. Permite manejar un mayor número de situaciones que se dan en la realidad [4].

2. Preliminares

Un número es estadístico neutrosófica es una número de la siguiente forma [5]:

$$N = d + i \quad (1)$$

Donde d es la parte determinada e i es la parte indeterminada [6]. Por ejemplo s: a=5 +1 si $i \in [5, 5.4]$ el número es equivalente a $a \in [5, 5.4]$.

Una matriz neutrosófica, por su parte, es una matriz donde los elementos $a = (a_{ij})$ han sido reemplazados por elementos en $\langle R \cup I \rangle$, donde $\langle R \cup I \rangle$ es un anillo neutrosófica entero [7].

Un grafo neutrosófico, es un grafo en el cual al menos un arco es un arco neutrosófico [8]. La matriz de adyacencia neutrosófica. Los arcos significan: 0 = no hay conexión entre nodos, 1 = conexión entre nudos, I = conexión indeterminada (desconocida si es o si no). Tales nociones no se utilizan en la teoría difusa, un ejemplo de muestra a continuación:

$$\begin{matrix} 0 & 0 & I \\ I & 0 & 1 \\ 1 & 0 & 0 \end{matrix}$$

En el transcurso de presente libro se abordará implementaciones prácticas de la propuesta. Google Colaboratory es una aplicación web que permite crear y compartir documentos que contienen código, fuente, ecuaciones, visualizaciones y texto explicativo tal como se muestra.

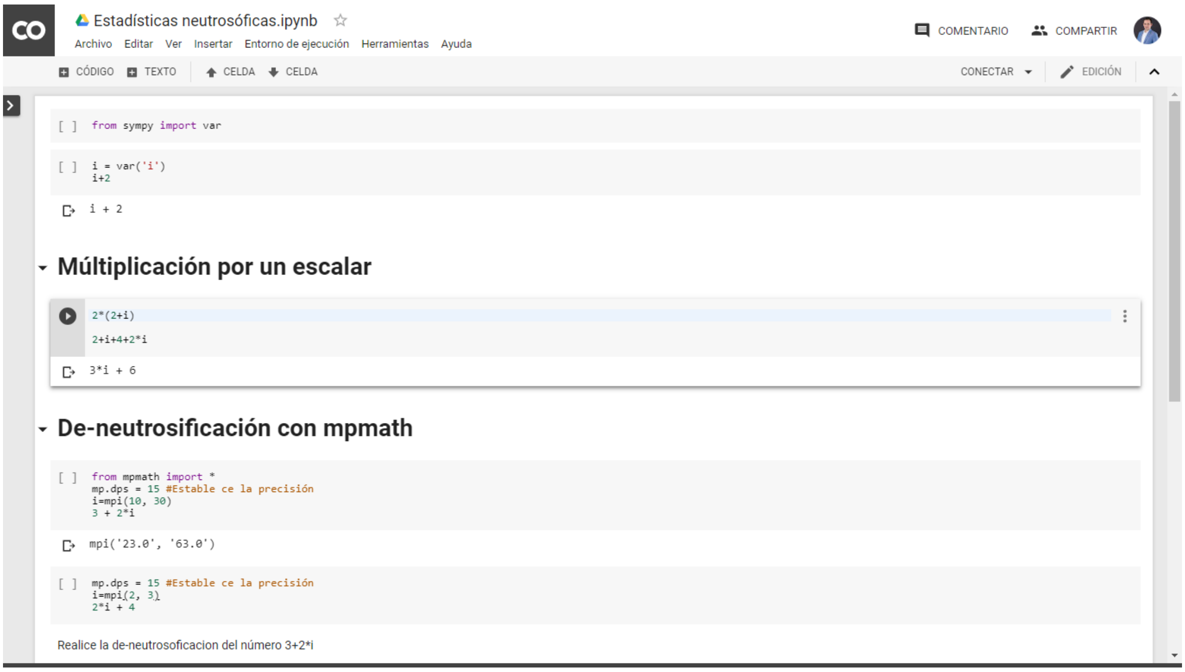

Figura 2. Google Colaboratory

Jupyter permite interactuar con varios lenguajes de programación, en este caso se utiliza Python, un lenguaje de programación bastante sencillo y poderoso, con acceso a una gran variedad de librerías útiles.

3. Computación neutrosófica y Sympy

Para el trabajo computacional con números neutrosóficos en el lenguaje python se puede emplear SymPy . SymPy es una biblioteca escrita en lenguaje Python con el propósito de reunir todas las características de un sistema de álgebra computacional, ser fácilmente extensible y mantener el código de la forma más simple posible [9].

Es por ello que se requiere un procesos de-neutrosificación [10]. I $\in[0,1]$ es reemplazado por sus valores máximos y mínimos . Para la de-neutrosificación es necesario el trabajo con aritmética intervalar.

En este caso trabajamos con la librería mpmath y con el tipo mpi [11]. El tipo mpi maneja los intervalos un par de valores mpf. La aritmética en intervalos utiliza un redondeo conservador de modo que, si un intervalo se interpreta como un intervalo de incertidumbre numérica para un número fijo, cualquier secuencia de operaciones de intervalo producirá un intervalo que contenga el resultado de aplicar la misma secuencia de operaciones al número exacto.

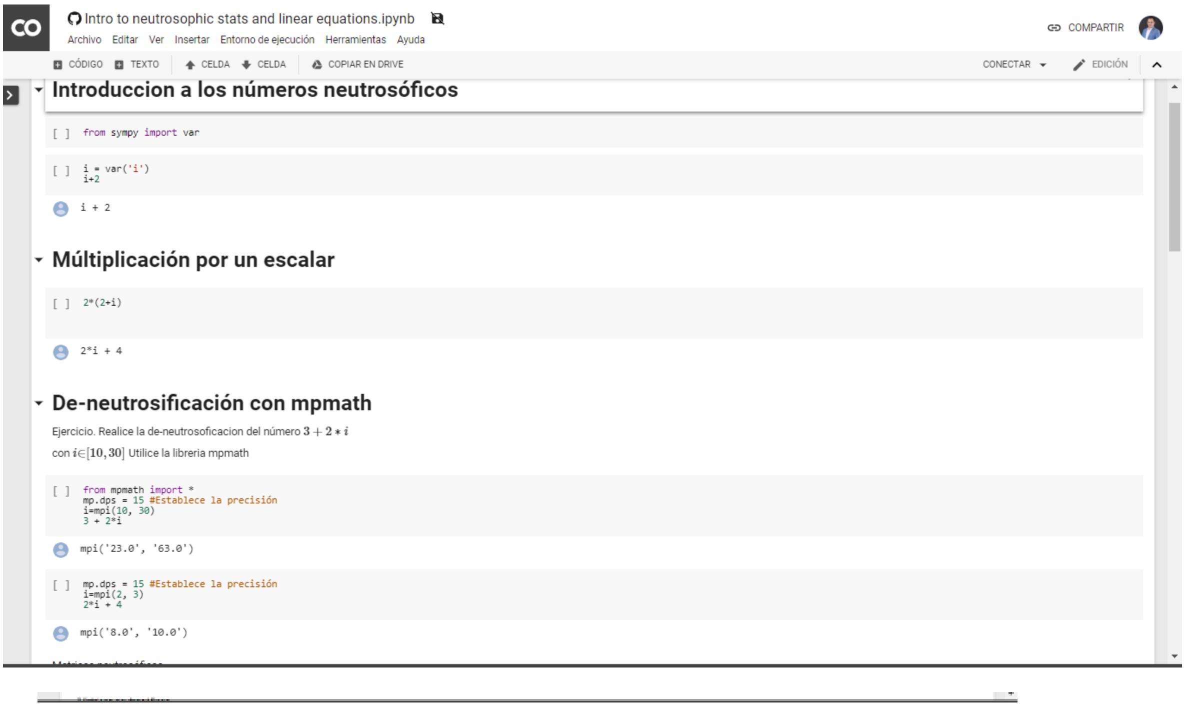

Figura 3. Trabajo con números neutrosóficos

En este caso se pueden resolver sistemas de ecuaciones lineales neutrosóficas[12].

Por ejemplo el sistema de ecuaciones:

$$x + 4y = 2 + i \qquad (2)$$

$$-2x + y = 14 + i \qquad (3)$$

Este caso es resuelto de la siguiente forma

Figura 4. Indetreminación en sistemas de ecuaciones lineales

Un sistema de ecuacuiones lineales podemso determinar el flujo del tráfico en distintas intercepciones.

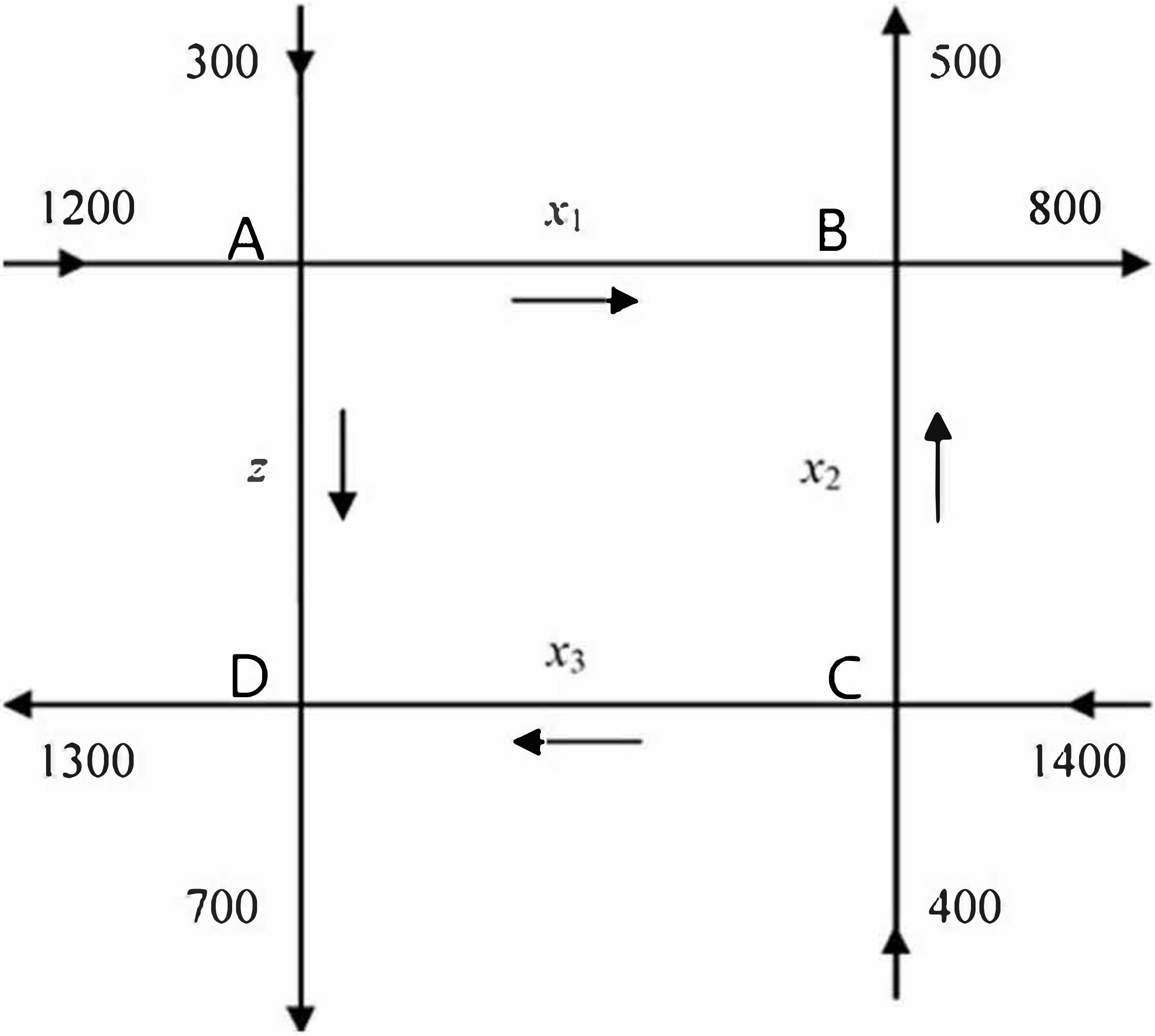

Figura 3. Flujo vehicular [12].

En cada intercepción el flujo de salida debe ser igual al flujo de entrada.
Intercepción A: $1500 = x_1 + z$
Intercepción B: $1300 = x_1 + x_2$
Intercepción C: $1800 = x_2 + x_3$
Intercepción D: $2000 = x_3 + z$
Si $z=400$
Entonces el sistema de ecuaciones queda de la siguiente forma

$$x_1 = 1100$$
$$x_1 + x_2 = 1300$$
$$x_2 + 2x_3 = 3400$$

La solución para este sistema es la siguiente:

$$x_1 = 1100$$
$$x_2 = 200$$

$$x_3 = 300$$

Figura 4. Solcucion del del flujo vehicular con indeterminación.

En el caso de Z=400+I.
Entonces el sistema de ecuaciones queda de la siguiente forma

$$x_1 = 1100 - I$$
$$x_1 + x_2 = 1300$$
$$x_1 + 2x_3 = 3400 - I$$

La solución para este sistema es la siguiente:

$$x_1 = 1100\text{-}i$$
$$x_2 = 200\text{+}i$$
$$x_3 = 1600 + i$$

4. Conclusiones

En este articulo se presentó el concepto de número neutrosófico. Se introduce la herramienta jupyter mediente google colaboratory . Se emplea la libreria Sympy para realizar el proceso de computacion neutrosófica.
Se resuelven sistemas de ecuaciones lineales neutrosóficas mediante la computación simbólica en python. Se desarrolala un estudio de caso para la determinación dle trafico vehicular con indetreminación. Como trabajos futuros se encuentran el desarrollo de nuevas aplicaciones en distintas áreas de la ingeniería y la ciencia . Otras áres de trabajos futuras se encuantran en el desarrollo de nuevas herramientas para la computación neutorsófica.

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Operadores con conjunto neutrosóficos de valor único Oversets, Undersets y Offset

Florentin Smarandache

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Resumen: Neutrosophic Over-/Under-/Off-Set and Logic were defined for the first time in 1995 and published in 2007. During 1995-2016 was presented them to various national and international conferences and seminars. These new notions are totally different from other sets/logics/probabilities. We extended the neutrosophic set respectively to Neutrosophic Overset {when some neutrosophic component is > 1}, to Neutrosophic Underset {when some neutrosophic component is < 0}, and to Neutrosophic Offset {when some neutrosophic components are off the interval [0, 1], i.e. some neutrosophic component > 1 and other neutrosophic component < 0}. This is no surprise since our real-world has numerous examples and applications of over-/under-/off-neutrosophic components. ***Palabras clave.*** desbordado neutrosophic, underset neutrosophic, neutrosophic offset, neutrosophic sobre la lógica, neutrosophic bajo la lógica, neutrosophic off lógica, neutrosophic sobre la probabilidad, neutrosophic bajo probabilidad, neutrosophic de probabilidad, más de miembros (grado de pertenencia> 1), bajo de miembros (grado de pertenencia <0) , (grado de pertenencia fuera del intervalo [0, 1]) off-membership.

1. Introducción

En los conjuntos y teorías lógica clásicas, en el conjunto y lógica difusa, y en conjunto difuso intuicionista y la lógica, el grado de pertenencia y el grado de no pertenencia tienen que pertenecer a, o ser incluidos en, el intervalo [0, 1]. Del mismo modo, en la probabilidad clásica y en la probabilidad imprecisa la probabilidad de un evento tiene que pertenecer a, o respectivamente ser incluidos en, el intervalo [0, 1].

Sin embargo, hemos observado y presentado a muchas conferencias y seminarios en todo el mundo { [12] - [33]} y publicado {véase [1] [8]} que en el mundo real hay muchos casos en los que el grado de la afiliación es superior a 1. El conjunto, que tiene elementos que puedan ser miembros de más de 1, lo llamamos desbordado (overset).

Incluso peor, observamos los elementos que puedan ser miembros con respecto a un conjunto es inferior a 0, y lo llamamos Underset.

En general, un conjunto que tiene elementos en cuyos miembros es superior a 1 y los elementos de cuyos miembros es inferior a 0, lo llamamos Offset (es decir, no son elementos cuyos miembros están fuera (encima y por debajo) el intervalo [0, 1]).

2. Ejemplo de sobrememnbresía-submembresía

En una empresa dado un patrón de tiempo completo trabaja 40 horas por semana. Vamos a considerar el último período de una semana.

Helen trabajaba a tiempo parcial, a tan sólo 30 horas, y las otras 10 horas que estuvo ausente sin el pago; por lo tanto, su grado de pertenencia era 30/40 = 0,75 <1.

John trabajado a tiempo completo, 40 horas, por lo que tuvo el grado de pertenencia 40/40 = 1, con respecto a esta empresa.

Pero George trabajó tiempo extra de 5 horas, por lo que su grado de pertenencia era (40 + 5) / 40 = 45/40 = 1.125> 1. Por lo tanto, tenemos que hacer una distinción entre los empleados que trabajan horas extras, y los que trabajan a tiempo completo o parcial -hora. Es por eso que tenemos que asociar un grado de pertenencia estrictamente mayor que 1 para los trabajadores de tiempo extra.

Ahora, otro empleado, Jane, estaba ausente sin sueldo para toda la semana, por lo que su grado de pertenencia era 0/40 = 0.

Sin embargo, Richard, que también fue contratado como a tiempo completo, no sólo no vino a trabajar la semana pasada en absoluto (0 horas trabajadas), pero se produjo, por el arranque accidental de un incendio devastador, mucho daño a la compañía, que se estimó en un valor medio de su salario (es decir, como lo habría conseguido por trabajar 20 horas que semana). Por lo tanto, su grado de pertenencia tiene que ser menor que la de Jane (Jane ya produjo ningún daño). De ahí, el grado de pertenencia de Richard, con respecto a esta empresa, era - 20/40 =

- 0,50 <0.

En consecuencia, tenemos que hacer una distinción entre los empleados que producen daños, y los que producen beneficio, o producir daños ni ningún beneficio a la sociedad.

Por lo tanto, los grados de pertenencia> 1 y <0 son reales en nuestro mundo, así que tenemos que tomarlos en consideración. (Smarandache, 2007).

3. Definición de overset neutrosófico de un solo valor

Sea U un universo de discurso y el conjunto neutrosophic A1 $1 \subset$ U.

Sea T (x), I (x), F (x) las funciones que describen los grados de pertenencia, indeterminado-miembros, y no pertenencia respectivamente, de un elemento genérico x $\in$ U, con respecto al conjunto neutrosophic A1:

T(X), yo(X), F(X) : T$\rightarrow [0,\Omega]$

donde 0 <1 <Ω, y Ω se llama sobre límite(overlimit).

Un solo valor Neutrosophic Overset A1 se define como: A1 = {(x,
T (x), I (x), F (x)>), x $\in$ U},

Los operadoresoverset de un solo valor neutrosóficos, Neutrosophic Undersets y Neutrosophic compensaciones

tal que existe al menos un elemento en A1 que tiene al menos un componente neutrosophic que es> 1, y ningún elemento tiene componentes neutrosophic que son <0.

Por ejemplo: A1 = {(x1, <1,3, 0,5, 0,1>), (x2, <0,2, 1,1, 0,2>)}, ya que T (x1) = 1,3> 1,

I (x2) = 1.1> 0, y ningún componente neutrosophic es <0.

También O2 = {(a, <0,3, -0,1, 1,1>)}, ya que (a) = - 0,1 <0 y F (a) = 1.1> 1.

4. Definición de underset neutrosófico de un solo valor

Sea U un universo de discurso y el conjunto neutrosophic $A_2 \subset$ U.

Sea T (x), I (x), F (x) las funciones que describen los grados de pertenencia, indeterminado-miembros, y no pertenencia respectivamente, de un elemento genérico x $\in$ U, con respecto al conjunto A_2 neutrosófico:

T(X), yo(X), F(X) : T € Ψ [1]

donde Ψ <0 <1, y Ψ se denomina underlimit.

A NeutrosophicUndersetA2is de un solo valor definido como: A2 = {(x, <T (x), I (x), F (x)>), x ∈ U},

tal que existe al menos un elemento en A2 que tiene al menos un componente neutrosophic que es <0, y ningún elemento tiene componentes neutrosophic que son> 1.

Por ejemplo: A2 = {(x1, <-0.4, 0.5, 0.3>), (x2, <0,2, 0,5, -0,2>)}, ya que T (x1) = -0,4 <0, F (x2) = -0,2 <0, y ningún componente neutrosófico> 1.

5. Definición offset de valor único

Sea U un universo de discurso y el conjunto neutrosophic A3 ⊂ U.

Sea T (x), I (x), F (x) las funciones que describen los grados de pertenencia, indeterminado-miembros, y no pertenencia respectivamente, de un elemento genérico x ∈ U, con respecto a la A3 conjunto:

T(X), yo(X), F(X) : T →[Ψ, Ω]

donde Ψ <0 <1 <Ω, y Ψ se llama underlimit, mientras Ω se llama overlimit. Un Offst neutrosófico A_3 de un solo valor define como:

A_3 = {(x, <T (x), I (x), F (x)>), x ∈ U},

de tal manera que existen algunos elementos en A_3 que tienen al menos un componente neutrosophic que es> 1, y al menos otro componente neutrosophic que es <0.

Para ejemplos: A3 = {(x1, <1,2, 0,4, 0,1>), (x2, <0,2, 0,3, -0,7>)}, ya que T (x1) = 1,2> 1 y F (x2) = -0,7 <0.

También, B3 = {(a, <0,3, -0,1, 1,1>)}, ya que (a) = - 0,1 <0 y F (a) = 1.1> 1.

6. Operadores neutrosóficos overset/underset/offset

Sea U un universo de discurso y A = {(x, <T_A (x), I_A (x), F_A (x)>), x ∈ U} y

y B = {(x, <T_B (x), I_B (x), F_B (x)>), x ∈ U} sean dos overset/underset/offset de valor único

I_A (x), F_a(X), T_B (x), I_B (x), F_B (x): U → [Ψ, Ω]

donde Ψ ≤ 0 <1 ≤ Ω, y Ψ se llama underlimit, mientras Ω se llama overlimit.

Tomamos el ≤ signo de desigualdad en lugar de <en ambos extremos anteriores, con el fin de comprender los tres casos: desbordado {cuando Ψ = 0, y 1 < Ω }, underset {cuando Ψ <0, y 1 = Ω }, y offset {cuando Ψ <0, y 1 < Ω }.

Unión neutrosófica Overset / Underset / Offset.

Entonces A∪B = {(x, <max {T_A (x), la tuberculosis (x)}, min {I_A (x), I_B (x)}, min {F_A (x), F_B (x)}>) , x∈ U}

Intercepción neutrosófica Overset / Underset / Offset.

Entonces A∩B = {(x, <min {T_A (x), la tuberculosis (x)}, max {I_A (x), I_B (x)}, max {F_A (x), F_B (x)}>) , x∈ U}

Complemente neutrosófico Overset / Underset / Offset.

El complemento del conjunto A es neutrosóficoc

C (A) = {(x, <F_A (x), $\Psi + \Omega - I_A$ (x), T_A (x)>), x ∈ T}.

Conclusiones

Los grados de membresía más de 1 (sobrepertenecia), o por debajo de 0 (bajo pertenencia) son parte de nuestro mundo real, por lo que merecen un estudio más en el futuro. Estos presentan muchas aplicaciones en la tecnología, las ciencias sociales, la economía y así sucesivamente que los lectores pueden estar interesados en explorar.

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Neutrosophic Invertible Graphs of Neutrosophic Rings

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ABSTRACT

Let $N(R, \mathrm{I})$ be a Neutrosophic ring of a finite commutative classical ring R with non-zero identity. Then the Neutrosophic invertible graph of $N(R, \mathrm{I})$, denoted by $\mathcal{I}_G(N(R, \mathrm{I}))$ and defined as an undirected simple graph whose vertex set is $N(R, \mathrm{I})$ and two vertices $a+b\mathrm{I}$ and $c+d\mathrm{I}$ are adjacent in $\mathcal{I}_G(N(R, \mathrm{I}))$ if and only $a+b\mathrm{I}$ is different from $-(c+d\mathrm{I})$ which is equivalent to $c+d\mathrm{I}$ is different from $-(a+b\mathrm{I})$. We begin by considering some properties of the self and mutual additive inverse elements of finite Neutrosophic rings. We then proceed to determine several properties of Neutrosophic invertible graphs and we obtain an interrelation between classical rings, Neutrosophic rings and their Neutrosophic invertible graphs.

KEYWORDS: Classical ring, Neutrosophic ring, Neutrosophic invertible graphs, Neutrosophic Isomorphism,self and additive inverse elements.

1. INTRODUCTION

The investigation of simple undirected graphs associated to finite algebraic structures, namely, rings and fields which are very important in the theory of algebraic graphs. In recent years the interplay between Neutrosophic algebraic structure and graph structure is studied by few researchers. For such kind of study, researchers define a Neutrosophic graph whose vertices are set of elements of a Neutrosophic algebraic structure and edges are defined with respect to a well-defined condition on the pre-defined vertex set. Kandasami and Smarandache (2006) introduced the notion and structure of the Neutrosophic graphs. Also, the authors Kandasami and Smarandache (2006) and Kandasamy, Ilanthenral, & Smarandache (2015) studied the notion and structure of the Neutrosophic graphs of several finite algebraic structures and exhibited them with various examples. Later, Chalapathi and Kiran (2017a) introduced another Neutrosophic graph of a finite group and this work was specifically concerned with finite Neutrosophic multiplicative groups only.

Throughout this paper, we will write $N(R, \mathrm{I})$ be a finite Neutrosophic commutative ring with identity 1 and indeterminacy I. For this Neutrosophic algebraic structure, we denote $S\left(N(R, \mathrm{I})\right)$ and $M\left(N(R, \mathrm{I})\right)$ be the set of self and respectively mutual additive Neutrosophic inverse elements. We may construct a new type of graphs associated with Neutrosophic rings. Our primary goal is to introduce Neutrosophic invertible graphs of finite rings and to study properties of these graphs. Further, we determine the diameter of Neutrosophic invertible graphs and introduce an isomorphic relation between classical rings, Neutrosophic rings and their invertible graphs.

2. BASIC PROPERTIES OF NEUTROSOPHIC RINGS

In this section, for all terminology and notations in graph theory, classical ring theory and Neutrosophic ring theory, we refer (Vitaly & Voloshin, 2009), (Lanski, 2004). and (Agboola, Akinola, & Oyebola. (2011); Agboola, Adeleke, & Akinleye, 2012) respectively. Chalapathi and Kiran (2017b) introduced and studied self and mutual additive inverse elements of finite Neutrosophic rings and illustrated them with few examples in different cases and proposed various results regarding the characterization of the Neutrosophic rings with identity $1 \neq 0$. We will restate some of the results as follows (Chalapathi & Kiran, 2017a; 2017b).

Definition 2.1. Let $\left(R, +, \cdot\right)$ be a finite ring. The set $N(R, I) = \langle R \cup I \rangle = \{a + bI : a, b \in R\}$ is called a Neutrosophic finite ring generated by R and I, where I is the Neutrosophic element with the properties $I^2 = I$, $0I = 0$, $I + I = 2I$ and I^{-1} does not exist.

Theorem 2.2. Let R be a finite ring with unity. Then $S(R) = R$ if and only if $S\left(N(R, \mathrm{I})\right) = N(R, \mathrm{I})$.

Theorem 2.3. Let R be a finite Boolean ring with unity. Then $S(R) = R$ and $S\left(N(R, \mathrm{I})\right) = N(R, \mathrm{I})$.

Theorem2.4. Let R and R' be two finite commutative rings with unity. If $R \cong R'$, then $S\left(N(R, \mathrm{I})\right) \cong S\left(N(R', \mathrm{I})\right)$.

Theorem 2.5. Let R and R' be two finite commutative rings with unity. Then $R \cong R'$ if and only if $N(R, \mathrm{I}) \cong N(R', \mathrm{I})$.

Theorem 2.6. Let R be a finite Boolean ring with unity and $|R| > 1$. Then $4 \leq |N(R, \mathrm{I})| \leq |R|^2$

Proof. Since $R = \{0\}$ if and only if $N(R, \mathrm{I}) = \{0\}$. It is clear that $R \neq \{0\}$ implies that $|R| > 1$. Suppose $|R| = 2$. Then, obviously, $R \cong Z_2$. This implies that $N(R, \mathrm{I}) = N(Z_2, \mathrm{I})$

$=\{0, 1, \mathrm{I}, 1+\mathrm{I}\}$, and hence $|N(R, \mathrm{I})| = 4$. It is one extremity of the inequality. For another extremity of the inequality, we set $R^*\mathrm{I} = \{a\mathrm{I}: a \in \mathrm{R}^*\}$, $\mathrm{R}^* + \mathrm{R}^*\mathrm{I} = \{a+b\mathrm{I}: a, b \in \mathrm{R}^*\}$ where $R^* = R - \{0\}$. These sets imply that R, $R^*\mathrm{I}$ and $\mathrm{R}^* + \mathrm{R}^*\mathrm{I}$ are mutually non-empty disjoint subsets of $N(R, \mathrm{I})$. Thus, $N(R, \mathrm{I}) = R \cup R^*\mathrm{I} \cup (R^* + R^*\mathrm{I})$, and clearly the cardinality of $N(R, \mathrm{I})$ is $|N(R, \mathrm{I})| = |R| + |R^*\mathrm{I}| + |R^* + R^*\mathrm{I}| = |R| + (|R|-1) + (|R|-1)^2 = |R|^2$.

Theorem 2.7. For any finite ring R with $|R| > 1$, we have $N(R, \mathrm{I})$ is the disjoint union of $S\left(N(R, \mathrm{I})\right)$ and $M\left(N(R, \mathrm{I})\right)$.

Proof. By the definition of self and mutual additive inverse elements of the Neutrosophic ring,

$$S\left(N(R, \mathrm{I})\right) = \{a+b\mathrm{I}:\ 2a=0, 2b=0\}$$

and $M\left(N(R, \mathrm{I})\right) = \{c+d\mathrm{I}:\ 2c \neq 0, 2d \neq 0\}$.

Clearly, $S\left(N(R, \mathrm{I})\right) \cap M\left(N(R, \mathrm{I})\right) = \phi$, and thus $S\left(N(R, \mathrm{I})\right) \cup M\left(N(R, \mathrm{I})\right) = N(R, \mathrm{I})$.

3. NEUTROSOPHIC INVERTIBLE GRAPHS

In this section, we introduced Neutrosophic invertible graphs and characterized its structural concepts.

Definition3.1. Let R be a finite commutative ring with identity $1 \neq 0$. A graph with its vertex set as $N(R, \mathrm{I})$ and two distinct vertices $a+b\mathrm{I}$ and $c+d\mathrm{I}$ are adjacent if and only $a+b\mathrm{I}$ is different from $-(c+d\mathrm{I})$ which is equivalent to $c+d\mathrm{I}$ is different from $-(a+b\mathrm{I})$ and we denote it by $\mathcal{J}_{\mathrm{G}}$ (N(R, I)).

The following theorem is a consequence of the Definition [3.1].

Theorem3.2. For each $N(R, \mathrm{I}) \neq \{0\}$, there exist Neutrosophic invertible graph $\mathcal{J}_{\mathrm{G}}(\mathrm{N}(R, \mathrm{I}))$.

Further, the aim of this section is to show how Neutrosophic algebraic representation of some philosophical concepts and some real world problems in the society can be modified to the study of algebraic Neutrosophic graphs. So, we shall investigate some important concrete properties of Neutrosophic invertible graphs, and also establish results of these graphs, which we required in the subsequent sections.

We begin with the algebraic graph theoretical properties of $\mathcal{J}_{\mathrm{G}}$ (N(R, I)), $|R| > 1$. Note that $|R| > 1$ if and only if $4 \leq |N(R, \mathrm{I})| \leq |R|^2$.

Theorem 3.3. The Neutrosophic invertible graph $\mathcal{J}_{\mathrm{G}}$ (N(R, I)) is connected.

Proof. Since $0+0\mathrm{I} \in S\left(N(R, \mathrm{I})\right)$ for any $N(R, \mathrm{I})$, $|N(R, \mathrm{I})| \geq 4$. So, $(a+b\mathrm{I})+(0+0\mathrm{I}) = a+b\mathrm{I} \neq 0+0\mathrm{I}$, for any non-zero element in $a+b\mathrm{I}$ in $S\left(N(R, \mathrm{I})\right)$.This implies that the vertex $0+0\mathrm{I}$ is adjacent with remaining all the vertices in $\mathcal{I}_G(\mathrm{N}(R,\mathrm{I}))$. It is clear that there is a pathbetween the vertices $0+0\mathrm{I}$ and $a+b\mathrm{I}$ in $\mathcal{I}_G\left(\mathrm{N}(R,\mathrm{I})\right)$.Hence $\mathcal{I}_G\left(\mathrm{N}(R,\mathrm{I})\right)$is connected.

The next few results provide a characterization for all Neutrosophic rings whose invertible graphs are complete.

Theorem3.4. The Neutrosophic invertible graph $\mathcal{I}_G(\mathrm{N}(R,\mathrm{I}))$ is complete if and only if $S\left(N(R, \mathrm{I})\right) = N(R, \mathrm{I})$.

Proof. Necessity. Suppose that $\mathcal{I}_G\left(\mathrm{N}(R,\mathrm{I})\right)$is complete. Then any two vertices $a+b\mathrm{I}$ and $c+d\mathrm{I}$ are adjacent in $\mathcal{I}_G\left(\mathrm{N}(R,\mathrm{I})\right)$. Consequently,

$(a+b\mathrm{I})+(c+d\mathrm{I}) \neq 0+0\mathrm{I} \Rightarrow 2(a+b\mathrm{I}) = 0$ and $2(c+d\mathrm{I}) = 0$

$$\Rightarrow a+b\mathrm{I},\ \mathrm{c}+d\mathrm{I} \in S\left(N(R, \mathrm{I})\right).$$

This implies that each and every element in $N(R, \mathrm{I})$ is an element of $S\left(N(R, \mathrm{I})\right)$. This shows that $N(R, \mathrm{I}) \subseteq S\left(N(R, \mathrm{I})\right)$. Further, by the Theorem [4.2] (Chalapathi & Kiran, 2017b), $S\left(N(R, \mathrm{I})\right)$ is a Neutrosophic subring of $N(R, \mathrm{I})$. So, $S\left(N(R, \mathrm{I})\right) \subseteq N(R, \mathrm{I})$. Hence, $S\left(N(R, \mathrm{I})\right) = N(R, \mathrm{I})$.

Sufficient. Let $S\left(N(R, \mathrm{I})\right) = N(R, \mathrm{I})$. Then we have to prove that $\mathcal{I}_G(\mathrm{N}(R,\mathrm{I}))$ is complete. Suppose $\mathcal{I}_G(\mathrm{N}(R,\mathrm{I}))$ is not complete. Then there exist at least two vertices $a'+b'\mathrm{I}$ and $c'+d'\mathrm{I}$ in $N(R, \mathrm{I})$ such that $(a'+b'\mathrm{I})+(c'+d'\mathrm{I})=0+0\mathrm{I}$. Therefore,

$a'+b'\mathrm{I} = -(c'+d'\mathrm{I}) \Rightarrow a'+b'\mathrm{I},\ c'+d'\mathrm{I} \in M\left(N(R, \mathrm{I})\right)$

$\Rightarrow a'+b'\mathrm{I},\ c'+d'\mathrm{I} \notin S\left(N(R, \mathrm{I})\right)$, by the Theorem [2.7]

$\Rightarrow S\left(N(R, \mathrm{I})\right) \neq N(R, \mathrm{I})$, this is a contradiction to our hypothesis, and hence $\mathcal{I}_G(\mathrm{N}(R,\mathrm{I}))$ is complete.

Corollary3.5. The Neutrosophic invertible graph of $N(R, \mathrm{I})$ is complete if and only if $N(R, \mathrm{I})$ is a finite Neutrosophic Boolean ring.

Proof. In view of the Theorem [2.5] and Theorem [3.4], $N(R, \mathrm{I})$ is a Neutrosophic Boolean ring if and only if $S\left(N(R, \mathrm{I})\right) = N(R, \mathrm{I})$ if and only if $\mathcal{I}_G(\mathrm{N}(R,\mathrm{I}))$ is complete.

Corollary 3.6. For $n \geq 1$, $\mathcal{I}_G(\mathrm{N}(Z_2^n,\mathrm{I}))$ is complete.

Proof. Since $\mathrm{N}(Z_2^n,\mathrm{I})$ is a Neutrosophic Boolean ring with 2^{2n} elements; $(0, 0,..., 0)$, $(1, 0,..., 0)$,..., $(1, 1,..., 1)$, $(\mathrm{I}, 0,..., 0)$,..., $(\mathrm{I}, \mathrm{I},..., \mathrm{I})$. Clearly, it is the vertex set of the graph $\mathcal{J}_{\mathrm{G}}(\mathrm{N}(Z_2^n,\mathrm{I}))$, and the sum of any two vertices in $\mathcal{J}_{\mathrm{G}}(\mathrm{N}(Z_2^n,\mathrm{I}))$ is non-zero. This implies that $S\left(N(Z_2{}^n, \mathrm{I})\right) = N(Z_2{}^n, \mathrm{I})$. So, by the Theorem [3.4], $\mathcal{J}_{\mathrm{G}}(\mathrm{N}(Z_2^n,\mathrm{I}))$ is complete.

Example3.7. By the definition of Neutrosophic ring, the Neutrosophic ring of Gaussian integers $N(Z_2[i], \mathrm{I})$ of modulo 2 is defined as $\{0, 1, i, 1+i, \mathrm{I}, i\mathrm{I}, (1+i)\mathrm{I}, 1+\mathrm{I}, i+\mathrm{I}, (1+i)+\mathrm{I}, (1+i)+i\mathrm{I}, 1+i\mathrm{I}, i+i\mathrm{I}, i+(1+i)\mathrm{I}, 1+(1+i)\mathrm{I}, (1+i)+(1+i)\mathrm{I}\}$. The Neutrosophic invertible graph of $N(Z_2[i], \mathrm{I})$ is a complete graph because $S\left(N(Z_2[i], \mathrm{I})\right) = N(Z_2[i], \mathrm{I})$, but it is not a Neutrosophic Boolean ring, since $(i+\mathrm{I})^2 \neq (i+\mathrm{I})$, where $i^2 = -1$ and $\mathrm{I}^2 = \mathrm{I}$.

The Example [3.7] explains that the completeness property of the Neutrosophic invertible graph depends on the $S\left(N(R, \mathrm{I})\right) = N(R, \mathrm{I})$, but not the Boolean property.

Theorem 3.8. The graph $\mathcal{J}_{\mathrm{G}}(\mathrm{N}(R,\mathrm{I}))$ is not complete if and only if $S\left(N(R, \mathrm{I})\right) \neq N(R, \mathrm{I})$.

Proof. Follows from the Theorem [3.4].

Theorem3.9. Let p be an odd prime. Then, the Neutrosophic invertible graph of a Neutrosophic field of order p^{2n} is never complete.

Proof. Let $\pi(x)$ be an irreducible polynomial of degree n over the classical field Z_p. Then, the Neutrosophic field of order p^{2n} is isomorphic to $N\left(\frac{Z_p[x]}{\langle \pi(x)\rangle}, \mathrm{I}\right)$. Now to show that its invertible graph is never complete. For this let $u = \frac{p-1}{2}x + \frac{p-1}{2}x\mathrm{I}$, $v = \frac{p+1}{2}x + \frac{p+1}{2}x\mathrm{I}$ be two vertices in $N\left(\frac{Z_p[x]}{\langle \pi(x)\rangle}, \mathrm{I}\right)$, then clearly, $u+v = px + px\mathrm{I} \equiv 0 (\mathrm{mod}\, \mathrm{p})$. This means that u and v are not adjacent. Hence the proof.

Again we recall that the result $4 \leq |N(R, \mathrm{I})| \leq |R|^2$ for each $|R| > 1$. So the immediate results ensures that the Neutrosophic invertible graph has at least one $3-$ cycle when $|N(R, \mathrm{I})| \geq 4$.

Theorem3.10. Let $|N(R, \mathrm{I})| \geq 4$. Then, $\mathcal{J}_{\mathrm{G}}(\mathrm{N}(R,\mathrm{I}))$ has at least one cycle of length 3.

Proof. Let $N(R, \mathrm{I})$ be a finite Neutrosophic ring with $1 \neq 0$ and $|N(R, \mathrm{I})| = 4$. Then clearly $N(R, \mathrm{I}) \cong N(Z_2, \mathrm{I})$, and its invertible graph has a cycle $1-\mathrm{I}-(1+\mathrm{I})-1$ of length 3 because

$1 \ \mathrm{I} \neq +0$, $\mathrm{I}(+1+\mathrm{I}) \neq 0$ and $(1+\mathrm{I}) \ 1 \neq +0$ so in this case the result is true. Now consider $|N(R, \mathrm{I})| > 4$. Then there exist the following two cases.

Case. (i) Suppose $S(N(R, \mathrm{I})) = N(R, \mathrm{I})$. Then, by the Theorem [3.4], the result is trivial.

Case. (ii) Suppose $S(N(R, \mathrm{I})) \neq N(R, \mathrm{I})$. There is at least one element $s + t\mathrm{I}$ in $S(N(R, \mathrm{I}))$ and $m + n\mathrm{I}$ in $M(N(R, \mathrm{I}))$ such that $(s+t\mathrm{I}) + (m+n\mathrm{I}) \neq 0$. It is clear that there is a cycle $0 - (s+t\mathrm{I}) - (m+n\mathrm{I}) - 0$ of length 3 in $\mathcal{J}_{\mathrm{G}}(\mathrm{N}(R, \mathrm{I}))$.

In the area of graph theory, a simple graph G is bipartite if its vertex set $V(G)$ can be partitioned into two disjoint subsets V_1 and V_2 such that no vertices both in V_1 or both in V_2 are connected. In 1931, the Kőnig's theorem provided by KőnigDénes (Dénes, 1931), it describes the relation between bipartite graph and its odd cycles.

Theorem 3.11. A simple graph is bipartite if and only if it does not have an odd length cycle.

Now we are in a position to determine precisely when $\mathcal{J}_{\mathrm{G}}(\mathrm{N}(R, \mathrm{I}))$ is bipartite or not. Note that $N(R, \mathrm{I}) \cong N(Z_2, \mathrm{I})$ if and only if the graph $\mathcal{J}_{\mathrm{G}}(\mathrm{N}(Z_2, \mathrm{I}))$ is isomorphic to the complete graph K_4 of order 4 . It is clear that the following result is hold in view of the Theorem [3.10].

Theorem3.12. Every Neutrosophic invertible graph is never a bipartite graph.

Already we proved that the graph $\mathcal{J}_{\mathrm{G}}(\mathrm{N}(R, \mathrm{I}))$ is connected for any finite Neutrosophic ring $N(R, \mathrm{I})$. Therefore, $\mathcal{J}_{\mathrm{G}}(\mathrm{N}(R, \mathrm{I}))$ has a diameter. Now, we immediate compute the diameter of $\mathcal{J}_{\mathrm{G}}(\mathrm{N}(R, \mathrm{I}))$ for any $N(R, \mathrm{I})$ such that $4 \leq |N(R, \mathrm{I})| \leq |R|^2$.

Theorem 3.13. The diameter of $\mathcal{J}_{\mathrm{G}}(\mathrm{N}(R, \mathrm{I}))$ is at most 2 .

Proof. Let $N(R, \mathrm{I})$ be a finite Neutrosophic ring with unity 1 and indeterminacy I . Then we consider the following two cases for finding diameter of $\mathcal{J}_{\mathrm{G}}(\mathrm{N}(R, \mathrm{I}))$. Note that,

$$\mathrm{diam}(\mathcal{J}_{\mathrm{G}}(\mathrm{N}(R, \mathrm{I}))) = \min \{d(u, v) : u, v \in N(R, \mathrm{I})\},$$

where $d(u, v)$ is the length of the shortest path between the vertices u and v .

Case. (i) Suppose $S(N(R, \mathrm{I})) = N(R, \mathrm{I})$. Then, by the Theorem [3.4], $\mathcal{J}_{\mathrm{G}}(\mathrm{N}(R, \mathrm{I}))$ is complete, so in this case $\mathrm{diam}(\mathcal{J}_{\mathrm{G}}(\mathrm{N}(R, \mathrm{I}))) = 1$.

Case. (ii) Suppose $S(N(R, \mathrm{I})) \neq N(R, \mathrm{I})$ .Then, by the Theorem [3.8], $\mathcal{J}_{\mathrm{G}}(\mathrm{N}(R, \mathrm{I}))$ is never a complete graph. Therefore, $\mathrm{diam}(\mathcal{J}_{\mathrm{G}}(\mathrm{N}(R, \mathrm{I}))) \neq 1$. This implies that $\mathrm{diam}(\mathcal{J}_{\mathrm{G}}(\mathrm{N}(R, \mathrm{I}))) >$

1. So, there exist a path $(s+t\mathrm{I})-0-(m+n\mathrm{I})$ in $\mathcal{J}_{\mathrm{G}}(\mathrm{N}(R,\mathrm{I}))$, which is smallest. Therefore, $d\left(s+t\mathrm{I},\ m+n\mathrm{I}\right)=2$, this implies that $\operatorname{diam}(\mathcal{J}_{\mathrm{G}}(\mathrm{N}(R,\mathrm{I})))=2$.

From case (i) and (ii) we conclude that the diameter of $\mathcal{J}_{\mathrm{G}}(\mathrm{N}(R,\mathrm{I}))$ is at most 2.

4.ISOMORPHIC PROPERTIES OF NEUTROSOPHIC INVERTIBLE GRAPHS

In this section, we compute an interrelation between classical rings, their Neutrosophic rings and their Neutrosophic invertible graphs. Refer the definitions of isomorphism of two classical rings, two Neutrosophic rings and two simple graphs from (Chalapathi &Kiran, (2017b).

Theorem 4.1. Let R and R' be two finite rings with unities. Then the following implications holds.

$$R\cong R'\Rightarrow N(R,\,\mathrm{I})\cong N(R',\,\mathrm{I})\Rightarrow \mathcal{J}_{\mathrm{G}}(\mathrm{N}(R,\mathrm{I}))\cong \mathcal{J}_{\mathrm{G}}(\mathrm{N}(R',\mathrm{I})).$$

Proof. The implication $R\cong R'\Rightarrow N(R,\,\mathrm{I})\cong N(R',\,\mathrm{I})$ follows from Theorem [2.4]. To complete the proof, it is enough to show that the second implication of the result. For any finite rings R and R', suppose $N(R,\,\mathrm{I})\cong N(R',\,\mathrm{I})$. Then by the definition of Neutrosophic isomorphism, there exist a bijection f from $N(R,\,\mathrm{I})$ onto $N(R',\,\mathrm{I})$ such that $R\cong R'$ and $f(\mathrm{I})=\mathrm{I}$ where $\mathrm{I}^2=\mathrm{I}$.Now to show that $\mathcal{J}_{\mathrm{G}}(\mathrm{N}(R,\mathrm{I}))\cong \mathcal{J}_{\mathrm{G}}(\mathrm{N}(R',\mathrm{I}))$. For this we define a map $\varphi:\mathcal{J}_{\mathrm{G}}(\mathrm{N}(R,\mathrm{I}))\to \mathcal{J}_{\mathrm{G}}(\mathrm{N}(R',\mathrm{I}))$as

$$\text{(i).}\ \varphi(a+b\mathrm{I})=f(a+b\mathrm{I})\ \text{and}$$

$$\text{(ii).}\ \varphi\big((a+b\mathrm{I},\ \mathrm{c+dI})\big)=\big(f(a+b\mathrm{I}),\ f(\mathrm{c+dI})\big).$$

Trivially, φ is a bijection since f is bijection. Further, we claim that each edge of$\mathcal{J}_{\mathrm{G}}(\mathrm{N}(R,\mathrm{I}))$ with end vertices $a+b\mathrm{I}$ and $c+d\mathrm{I}$ is mapped to an edge in $\mathcal{J}_{\mathrm{G}}(\mathrm{N}(R',\mathrm{I}))$ with end vertices $f(a+b\mathrm{I})$ and $f(\mathrm{c+dI})$. So, we have

$$\big(a+b\mathrm{I},\ c+d\mathrm{I}\big)\in E\left(\mathcal{J}_{\mathrm{G}}\big(\mathrm{N}(R,\mathrm{I})\big)\right)\Leftrightarrow (a+b\mathrm{I})+(c+d\mathrm{I})\neq 0\Leftrightarrow \varphi\big((a+b\mathrm{I})+(c+d\mathrm{I})\big)\neq\varphi(0)$$

$$\Leftrightarrow \varphi\big((a+c)+(b+d)\mathrm{I}\big)\neq 0\Leftrightarrow f\left((a+c)+(b+d)\mathrm{I}\right)\neq 0\Leftrightarrow f((a+c))+f((b+d)\mathrm{I})\neq 0$$

$$\Leftrightarrow f(a)+f(c)+f(b)\mathrm{I}+f(d)\mathrm{I}\neq 0\Leftrightarrow \big(f(a)+f(b)\mathrm{I}\big)+\big(f(c)+f(d)\mathrm{I}\big)\neq 0$$

$$\Leftrightarrow f(a+b\mathrm{I})+f(c+d\mathrm{I})\neq 0\Leftrightarrow \big(f(a+b\mathrm{I}),\ f(c+d\mathrm{I})\big)\in E(\mathcal{J}_{\mathrm{G}}(\mathrm{N}(R',\mathrm{I}))).$$

Similarly we can show that φ maps non-adjacent vertices in $\mathcal{J}_{\mathrm{G}}\big(\mathrm{N}(R,\mathrm{I})\big)$to non-adjacent vertices in $\mathcal{J}_{\mathrm{G}}(\mathrm{N}(R',\mathrm{I}))$. Thus, φ is a graph isomorphism from $\mathcal{J}_{\mathrm{G}}(\mathrm{N}(R,\mathrm{I}))$ onto $\mathcal{J}_{\mathrm{G}}(\mathrm{N}(R',\mathrm{I}))$, and hence $\mathcal{J}_{\mathrm{G}}(\mathrm{N}(R,\mathrm{I}))\cong \mathcal{J}_{\mathrm{G}}(\mathrm{N}(R',\mathrm{I}))$.

By the Theorem [2.4], two classical rings are isomorphic, so their Neutrosophic rings are isomorphic and consequently their Neutrosophic invertible graphs are also isomorphic, but converse of these implication, in general, not true. The next results provide such a class. First we state the following results due to isomorphism of two simple graphs. The proof of the following results is essentially contained in Bondy and Murty (2008).

Theorem 4.2. Two simple graphs G and G' are isomorphic if and only if their complement graphs $\overline{G}$ and $\overline{G'}$.

Recall from Mullen and Panario (2013) that F_{p^n} is a field of order p^n and Z_{p^n} is a com mutative ring of order p^n, where p is a prime and $n>1$. Note that F_{p^n} is not isomorphic to Z_{p^n} because the characteristic of F_{p^n} is p and the characteristic of Z_{p^n} is p^n.

Theorem 4.3. Let $p > 2$ be a prime. Then the Neutrosophic invertible graphs of order p^{2n} are isomorphic.

Proof. For each odd prime p, we have $N(F_{p^n}, \mathrm{I})$ is a Neutrosophic field of modulo p. $N(Z_{p^n}, \mathrm{I})$ is a Neutrosophic commutative ring of modulo p^n, clearly these Neutrosophic rings not isomorphic. Now it remains to show that the graphs $\mathcal{I}_{\mathrm{G}}\left(\mathrm{N}\left(F_{p^n}, \mathrm{I}\right)\right)$ and $\mathcal{I}_{\mathrm{G}}\left(\mathrm{N}\left(Z_{p^n}, \mathrm{I}\right)\right)$ are isomorphic. For this we shall show that their complement graphs are isomorphic. By the definition of complement graph, $\overline{\mathcal{I}_{\mathrm{G}}}\left(\mathrm{N}\left(F_{p^n}, \mathrm{I}\right)\right) = \left|M\left(N(F_{p^n}, \mathrm{I})\right)\right| K_2 \cup \left|S\left(N(F_{p^n}, \mathrm{I})\right)\right| K_1$ $= \left(\dfrac{p^{2n}-1}{2}\right) K_2 \cup K_1 \cong \overline{\mathcal{I}_{\mathrm{G}}}\left(\mathrm{N}\left(Z_{p^n}, \mathrm{I}\right)\right)$, so due to Theorem [4.2], we get the required result.

Corollary 4.4. For each $n>1$, the Neutrosophic invertible graphs of order 2^{2n} are isomorphic.

Proof. Follows from $\mathcal{I}_{\mathrm{G}}\left(\mathrm{N}(F_{2^n}, \mathrm{I})\right) \cong \left|S\left(N(F_{2^n}, \mathrm{I})\right)\right| K_1 \cong 2^{2n}\ K_1 \cong N_{2n} \cong \mathcal{I}_{\mathrm{G}}\left(\mathrm{N}(Z_{2^n}, \mathrm{I})\right)$, where N_{2n} is totally disconnected graph of order 2^{2n}. It is clear that $F_{2^n} \ncong Z_{2^n}$ and $N(F_{p^n}, \mathrm{I}) \ncong N(Z_{p^n}, \mathrm{I})$ but their Neutrosophic invertible graphs are isomorphic.

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Grado de dependencia e independencia de los (sub) componentes de Conjuntos Borrosos y Neutrosóficos

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Florentin Smarandache (2019). Grado de dependencia e independencia de los (sub) componentes de Conjuntos Borrosos y Neutrosóficos. *Neutrosophic Computing and Machine Learning* 5, 1-6

Resumen. La introducción del grado de dependencia (y en consecuencia el grado de independencia) entre los componentes del conjunto difuso, y también entre los componentes del conjunto neutrosófico, se introduce por primera vez en la quinta edición del libro de Neutrosofía en el año 2006, basado en los elementos descritos en dicha edición del libro, se comienza a conocer conceptos de conjuntos neutrosóficos de los componentes borrosos así como los grados de dependencia e independencia, Por tal motivo el objetivo del presente trabajo es extender el conjunto neutrosófico refinado, teniendo en cuenta la grado de dependencia o independencia de los subcomponentes que integran los conjuntos borrosos y neutrosóficos.

Palabras claves: Neutrosofía, conjunto neutrosófico, conjunto difuso, grado de dependencia e independencia de los subcomponentes.

1 Refinado de Conjuntos Neutrosóficos

Comenzamos con la definición más general, el de una n-valorado refinado de un conjunto neutrosófico el cual es representado por •.

Un elemento • desde • pertenece al conjunto tal y como se muestra en 1.

$$(\bullet\ 1,\bullet\ 2,\ldots,\bullet\bullet;\bullet\ 1,\bullet\ 2,\ldots,\bullet\bullet;\bullet\ 1,\bullet\ 2,\ldots,\bullet\bullet)\in\bullet \tag{1}$$

Dónde:

$\bullet,\bullet,\bullet\geq\ 1$ son enteros, $y\ \bullet\ +\ \bullet\ +\ \bullet=\bullet\geq\ 3$,

y

$$\bullet\ 1,\bullet\ 2,\ldots,\bullet\bullet;\bullet\ 1,\bullet\ 2,\ldots,\bullet\bullet;\bullet\ 1,\bullet\ 2,\ldots,\bullet\bullet \tag{2}$$

Ellos son respectivamente, grados sub-miembros, grados sub-indeterminación, y grados sub-no pertenencia de elemento X con respeto al valor refinado de los conjunto neutrosóficos A. Y por lo tanto, se tiene a n como (sub) componentes. Al considerar todos ellos como números nítidos en el intervalo [0, 1], se obtiene según el caso general que se muestra a continuación.

1.1 Caso general

En la expresión 3 se muestran las variables del componente CRISP.

$$\bullet\ 1,\bullet\ 2,\ldots,\bullet\bullet\in\ [0,1] \tag{3}$$

Si todos ellos son 100% independiente de dos en dos, entonces se suman como se muestra en la expresión 4.

$$0\ \leq\bullet\ 1\ +\bullet\ 2+\ldots+\ \bullet\bullet\leq\bullet \tag{4}$$

Pero si todos ellos son 100% dependientes (totalmente interconectado), entonces:

$$0 \leq \bullet\ 1 + \bullet\ 2 + \ldots + \bullet \bullet \leq 1 \qquad (5)$$

Cuando algunos de ellos son parcialmente dependientes y parcialmente independientes, entonces

$$\bullet\ 1 + \bullet\ 2 + \ldots + \bullet \bullet \in (1, \bullet) \qquad (6)$$

Por ejemplo, si • 1 y • 2 son 100% dependientes, entonces

$$0 \leq \bullet\ 1 + \bullet\ 2 \leq 1 \qquad (7)$$

Mientras que otras variables • 3, ..., • • son 100% independientes unos de otros y también con respecto a • 1 y • 2, entonces;

$$0 \leq \bullet\_3 + \cdots + \bullet\_\bullet \leq \bullet - 2 \qquad (8)$$

así

$$0 \quad \leq \bullet\ 1 + \bullet\ 2 + \bullet\ 3 + \cdots + \bullet \bullet \leq \bullet - 1 \qquad (9)$$

2 Conjunto Borroso

Si • y • constituyen la composición y, respectivamente, la no pertenencia de un elemento • (•, •) con respecto a un conjunto difuso •, dónde •, • son números nítidas en [0, 1].
Si • y • son dependientes el uno del otro 100%, entonces uno tiene como en la teoría de conjuntos difusos clásica.

$$0 \leq \bullet + \bullet \leq 1 \qquad (10)$$

Pero si • y • son 100% independientes entre sí (que se define ahora por primera vez en el dominio de la lógica fuzzy setand), entonces

$$0 \leq \bullet + \bullet \leq 2 \qquad (11)$$

Si Consideramos que la suma • + • = 1, y la información sobre los componentes se ha completado, entonces; y • + • <1, si la información acerca de los componentes es incompleta. Similar, para • + • = 2 con el fin de obtener información completa, y • + • <2 para obtener información incompleta. Por tanto, para obtener información completa sobre el T y F, se tiene:

$$\bullet + \bullet \in [1,2] \qquad (12)$$

Grado de dependencia y Grado de independencia de dos componentes

En general según [1], la suma de dos componentes x e y, que varían en el intervalo unitario [0, 1] es información (suma> 1), o la información completa (suma = 1)

$$0 \leq \bullet + \bullet \leq 2 - {}^{\circ} \bullet (\bullet, \bullet) \qquad (13)$$

Dónde:

° • (•, •) es el grado de dependencia entre X y Y.

Y

2 - ° • (•, •), es el grado de independencia entre X y Y.

Por tanto; ° • (•,•) ∈ [0,1]

Y es cero cuando X y Y son 100% independiente, y 1 cuando X y Y son 100% dependiente.

Si los tres componentes T, I, F son dependientes, a continuación, de manera similar uno deja espacio para la información incompleta (suma <1), o la información completa (suma = 1).
Los componentes dependientes están atados juntos. Tres fuentes que proporcionan información sobre T, I y F, respectivamente, son independientes si ellos no pueden comunicarse entre sí y no se influyen mutuamente.

Por lo tanto, max {T + I + F} es 1 (cuando el grado de independencia es cero) y 3 (cuando el grado de independencia es 1). En general, si T y F son • % dependiente [y por consiguiente (100 - •) % independiente], tal como se muestra en la ecuación 14.

$$0 \leq \bullet + \bullet \leq 2 - \bullet / 100 \quad (14)$$

3 Ejemplo de Set Fuzzy con Parcialmente Componentes dependientes y parcialmente independientes

Un ejemplo lo es, si • y • son 75% (= 0,75) dependiente, entonces esto se muestra como en la ecuación 15

$$0 \leq \bullet + \bullet \leq 2 - 0{,}75 = 1{,}25 \quad (15)$$

4 Conjunto Neutrosófico

El conjunto neutrosófico (NS) es un marco general para la unificación de muchos conjuntos existentes, tales como conjuntos difusos (sobre todo de conjuntos difusos intuicionista), conjuntos paraconsistentes, conjunto intuicionista, etc. La idea principal de NS es la caracterización de cada declaración de valor en un espacio 3D-neutrosófico, donde cada dimensión del espacio es representada respectivamente.

El número de miembros / verdad (T), la no pertenencia / falsedad (F), y la indeterminación con respecto a la pertenencia / no pertenencia (I) de la declaración bajo consideración, en donde T, I, F son subconjuntos estándar o no estándar reales de $]-0, 1+[$, con no necesariamente ninguna conexión entre ellos.

Para las propuestas de ingeniería de software se utiliza el intervalo de la unidad clásica. Para conjunto neutrosófico de un solo valor, la suma de los componentes $(T + I + F)$

$$0 \leq T + I + F \leq 3 \quad (16)$$

Cuando los tres componentes son independientes; T, I, F, de manera similar se deja el espacio para la información incompleta (suma <1), o la información completa (suma = 1). Los componentes dependientes están atados juntos. Tres fuentes que proporcionan información sobre T, I y F, respectivamente, son independientes si ellos no pueden comunicarse entre sí y no se influyen mutuamente.

Por lo tanto, $max\ \{T + I + F\}$ es igual 1 (cuando el grado de independencia es cero) y 3 (cuando el grado de independencia es 1)

5 Ejemplos de Set neutrosófico con componentes parcialmente dependientes e independientes parcialmente

A través del $max\ \{T + I + F\}$ se puede obtener cualquier valor en (1, 3).

Por ejemplo:

a. Supongamos que T y F son 30% dependiente y 70% independiente. Por lo tanto, $T + F \leq 2 - 0{,}3 =$ 1.7), mientras que I y F son 60% dependiente y 40% independiente, por lo tanto;

$$I + F \leq 2 - 0{,}6 = 1{,}4 \tag{17}$$

Entonces;

$max\,\{T + I + F\} = 2{,}4$ y se produce para $T = 1, I = 0.7, F = 0{,}7$.

b. Supongamos que T e I son 100% dependiente, pero I y F son 100% independiente. Por lo tanto, $T + I \leq 1\, e\, i + F \leq 2$, entonces $T + I + F \leq 2$.

6 Más de refinado del conjunto neutrosófico

El Set o conjunto neutrosófico refinado, según [4], se introdujo por primera vez en 2013. En este conjunto el componente neutrosófico (T) se divide en los subcomponentes ($T1,\ T2$),

$$0 \leq T + I + F \leq 2 \tag{18}$$

Cuando dos componentes son dependientes, mientras que el tercero es independiente de ellos; como por ejemplo: ..., T pág.) que representan tipos de verdades (o sub-verdades), el componente neutrosophic (I) se divide en los subcomponentes (I1, yo2, …, YOr) que representa tipos de indeterminaciones (o sub-indeterminaciones), y los componentes neutrosóficos (F) se dividen en el subcomponentes (F1, F2, ..., Ms) que representan tipos de mentiras (o sub-falsedades), de tal manera que p, r, s son enteros ≥ 1 y $p + r + s = n \geq 4$.

Entonces;

$0 \leq T + I + F \leq 1$, cuando los tres componentes son dependientes.

a. Cuando tres o dos de los componentes T, I, F son independientes, la información que se obtiene es incompleta, (suma <1), paraconsistentes y contradictorio

b. Cuando $n = 3$, se obtiene el conjunto no refinado neutrosófico para todo $tj,\ yok,\ y\,Flll$ que son subcomponentes delsubconjuntos de [0, 1].

Al considerar el caso de conjunto neutrosófico de valor único refinado, es decir, cuando todos los subcomponentes n son números nítidas entre [0,1]. La suma de todos los subcomponentes es:

$$S \bullet \mathop{\bullet}_{1}^{pag} T_j \bullet \mathop{\bullet}_{1}^{r} yo_k \bullet \mathop{\bullet}_{1}^{s} F_l \tag{19}$$

Cuando todos los subcomponentes son independientes de dos en dos, ellos se representan como se muestra en 19.

$$0 \leq S \leq n \tag{20}$$

c. Si metro es un subcomponente igual a 100% dependiente, entonces; 2 ≤ m ≤ n, sin importar si se encuentran entre $T\,j,\ yo\,k,\ F\,lllll$ o mezclado, esto se muestra a través de la expresión 21.

$$0 \leq S \leq n - m1 \quad (21)$$

Cuando se tiene $S = n - m + 1$, la información está completa, mientras que $S < n - m + 1$, la información es incompleta.

7 Ejemplos de refinado de conjuntos neutrosóficos con componentes parcialmente dependientes y parcialmente independientes.

a. Supongamos que T se divide en $T1,\ T2,\ T3,$ y yo no se divide, mientras que F se divide en $F1$ y $F2$. Por lo tanto, se tiene:

b.

$$\{T1,\ T2,\ T3;\ YO;\ F1,\ F2\} \quad (22)$$

Lo que representa un total de 6 subcomponentes. Si los 6 componentes son 100 % independiente de dos en dos, entonces:

$$0 \leq T1 + T2 + T3 + I + F1 + F2 \leq 6 \quad (23)$$

c. Supongamos que los subcomponentes $T1$, $T2$, $y\ F1$ son 100% dependiente de todos juntos, mientras que los otros son totalmente independientes de dos en dos e independiente de T1, T2, F1, Por lo tanto:

$$0 \leq T1 + T2 + F1 \leq 1 \quad (24)$$

De dónde:

$$0 \leq T1 + T2 + T3 + I + F1 + F2 \leq 6 - 3 + 1 = 4 \quad (25)$$

Entonces; se obtiene la igualdad a 4 cuando la información está completa, o estrictamente menor que 4 cuando la información es incompleta.

d. Supongamos que en otro caso que $T1$ y I son 20% ° dependientes, o $D\ (t1,\ I)$ = 20%, mientras que los otros de manera similar totalmente independientes de dos en dos e independientes de $T1$ y I, por lo tanto:

$$0 \leq T1 + I \leq 2 - 0.2 = 1.8 \quad (26)$$

Donde:

$$0 \leq T1 + T2 + T3 + I + F1 + F2 \leq 1{,}8 + 4 = 5{,}8 \quad (27)$$

Desde:

$$0 \leq T2 + T3 + F1 + F2 \leq 4 \quad (28)$$

Del mismo modo, a la derecha tiene la igualdad para la información completa, y la desigualdad estricta de información incompleta.

Conclusiones

Se introduce por primera vez el grado de dependencia / independencia entre los componentes del conjunto difuso y conjunto neutrosófico, a través de ejemplos sencillos sobre la gama de la suma de los componentes, y la manera de representar los grados de dependencia y la independencia de los componentes, el objetivo se detalló con profundidad. Por otra parte, se extendió el conjunto refinado neutrosófico, teniendo en cuenta el grado de dependencia o independencia de subcomponentes.

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Tripleta de estructura Neutrosófica y Tripleta de estructura Neutrosófica extendida

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Resumen. En el presente estudio se realiza una revisión de las tripletas de estructura neutrosófica y tripleta de estructura neutrosófica extendida, con el fin de introducir nuevos conceptos a emplear en trabajos futuros.

Palabras Claves: Tripleta de estructura neutrosófica, tripleta de estructura neutrosófica extendida.

1 Introducción

Las tripletas neutrosófica [1, 2, 3, 10] son introducidas por el profesor Florentín Smarandache y Mumtaz Alí en 2014 - 2016, los trabajos relacionados se encuentran en el sitio http://fs.unm.edu/NeutrosophicTriplets.htm. En el año 2016, son extendidas, dichas tripletas neutrosóficas a [4, 5, 8] por el referido autor, trabajos relacionados se encuentran el sitio http://fs.unm.edu/NeutrosophicTriplets.htm

Sea U un universo de discurso y (N, *) un conjunto incluido en el, dotado de una ley binaria bien definida *

1.1. Definición de tripleta de estructura neutrosófica (NT)

Una tripleta neutrosófica es un objeto de la forma $< x, neut(x), anti(x) >, for\ x \in N$, donde $neut(x) \in N$ es el neutral de x, diferente del elemento unitario algebraico clásico si existe, de modo que:

$$x * neut(x) = neut(x) * x = x \quad (1)$$

y $anti(x) \in N$ es el opuesto de x de modo que:

$$x * anti(x) = anti(x) * x = neut(x) \quad (2)$$

Por lo general, un elemento x puede tener más elemento neutros y más anti neutros.

1.2. Definición de Tripleta de estructura Neutrosófica extendida (NET)

Una tripleta de estructura neutrosófica extendida es definido como se definió en el epígrafe 1.1, la tripleta de estructura neutrosófica (NT), pero en este caso el neutro de x, es denotado como $e^{anti's}(x)$, también puede ser igual al elemento unitario algebraico clásico (si existe). Por lo tanto, se libera la restricción, diferente del elemento unitario algebraico clásico, si existe.

Como consecuencia, el "opuesto extendido" de x , denotado por $e^{anti's}(x)$, también puede ser igual al elemento clásico inverso de un grupo clásico. Por lo tanto, una tripleta de estructura neutrosófica extendida es un objeto de la forma, $< x, e^{neut's}(x), e^{anti's}(x) >$, para $x \in N$, donde; $e^{neut's}(x) \in N$ es el neutro extendido de x, que puede ser igual o diferente del elemento unitario algebraico clásico si lo hay, de manera que:

$$x * e^{neut's}(x) = e^{neut's}(x) * x = x \quad (3)$$

y $anti(x) \in N$ es la extensión opuesta de x de manera que:

$$x * e^{anti's}(x) = e^{anti's}(x) * x = e^{neut's}(x) \quad (4)$$

En general, para cada $x \in N$ existen muchos $e^{neut's}$ y $e^{anti's}$.

1.3. Definición del conjunto de tripletas neutrosóficas (Fuerte) (NTSS)

El conjunto N se llama tripleta neutrosófica (fuerte) establecido si para cualquier $x \in N$ existe $neut(x) \in N$ y $anti(x) \in N$.

1.4. Definición del conjunto de tripletas neutrosóficas extendidas (Fuerte) (NETSS)

El conjunto N se llama tripleta extendida neutrosófica (Fuerte) establecido si para cualquier $x \in N$ existe $e^{neut}(x) \in N$ y $e^{anti}(x) \in N$.

1.5. Definición del conjunto débil de tripleta neutrosófica (NTWS)

El conjunto N se denomina conjunto débil de tripleta neutrosófica si para cualquier $x \in N$ existe una tripleta neutrosófica extendida $< y, e^{neut}(y), e^{anti}(y) >$ incluido en N, de manera que $x = y$ or $x = e^{neut}(y)$ or $x = e^{anti}(y)$.

1.6. Teorema 1

a) Un conjunto fuerte de tripletas neutrosóficas también es un conjunto débil de tripletes neutrosóficas, pero no a la inversa.
b) b) Un conjunto fuerte de tripletas extendidas neutrosóficas también es un conjunto débil de tripletas extendidas neutrosóficas, pero no a la inversa.

1.7. Definición de grupo de tripletas neutrosóficas (Fuerte) (NTG)

Sea $(N,*)$ un conjunto de tripletas neutrosóficas (Fuerte). Entonces $(N,*)$ se llama grupo de tripletas neutrosóficas (fuertes), si satisface los siguientes axiomas clásicos:

1) Si $(N,*)$ está bien definido, es decir, para cada $x, y \in N$, se tiene $x * y \in N$.
2) Si *(N, *)* es asociativa, es decir, para cada $x, y, z \in N$, se tiene $x*(y*z) = (x*y)*z$.

NTG, en general, no es un grupo en la forma clásica, porque puede no tener un elemento unitario clásico, ni elementos inversos clásicos.

Se considera, que los neutros neutrosóficos reemplazan el elemento unitario clásico, y los opuestos neutrosóficos reemplazan los elementos inversos clásicos.

1.8. Definición de grupo de tripletas neutrosóficas extendidas (Fuerte) (NETG)

Sea $(N,*)$ un conjunto de tripletas neutrosóficas extendidas (Fuerte). Entonces $(N,*)$ se llama grupo de tripletas neutrosóficas extendidas (Fuerte), si satisfice los siguientes axiomas clásicos:

1) Si $(N,*)$ es bien definido, es decir para cada $x, y \in N$ se tiene $x * y \in N$.
2) Si $(N,*)$ es asociativa, es decir, para cada y $x, y, z \in N$ se tiene $x * (y * z) = (x * y) * z$.

NETG, en general, no es un grupo en la forma clásica, porque puede no tener un elemento unitario clásico, ni elementos inversos clásicos. Consideramos que los neutros extendidos neutrosóficos reemplazan el elemento unitario clásico, y los opuestos extendidos neutrosóficos reemplazan a los elementos inversos clásicos. En el caso de que NETG incluya un grupo clásico, entonces NETG enriquece la estructura de un grupo clásico, ya que puede haber elementos con neutrales más extendidos y más opuestos extendidos.

1.9. Definición de anillo triplete neutrosófico (NTR)

1) El anillo triplete neutrosófico es un conjunto dotado de dos leyes binarias $(N,*,\#)$, tal que:

a) $(N,*)$ es un grupo de tripletas neutrosóficas conmutativas (fuerte).

Lo que significa que:

- N es un conjunto de tripletas neutrosóficas (fuertes) con respecto a la ley * (es decir, si x pertenece a N , entonces $neut^*(x)$ y $anti^*(x)$, definidos con respecto a la ley $*$, también pertenecen a N). Usamos las notaciones $neut^*(.)$ y respectivamente $anti^*$ (.) para significar; con respecto a la ley *; que está bien definida, como asociativa y conmutativa en N; (como en el sentido clásico).

b) (N, #) es un conjunto tal que la ley # sobre N está bien definida y es asociativa; (como en el sentido clásico).

c) La ley # es distributiva con respecto a la ley *; (como en el sentido clásico).

1.10. Definición de tripleta de anillo extendido neutrosófico (NETR)

1) El anillo triplete extendido neutrosÃ³fico es un conjunto dotado de dos leyes binarias $(N,*,\#)$, tal que:

a) $(N,*)$ es un grupo de triplete extendido neutrosófico conmutativo. Lo que significa que:

- N es un conjunto de tripletes neutrosóficas extendidas con respecto a la ley * (es decir, si x pertenece a N, entonces $e^{neut*}(x)$ y e^{anti*} (x), definido con respecto a la ley *, también pertenecen a N).
- la ley * está bien definida, asociativa y conmutativa en N; (como en el sentido clásico).

b) $(N,\#)$ es un conjunto tal que la ley # sobre N está bien definida y es asociativa; (como en el sentido clásico).

c) La ley # es distributiva con respecto a la ley * (como en el sentido clásico).

1.11. Observaciones sobre el anillo de tripletas neutrosóficas

1) El anillo de tripletas neutrosóficas se define en los pasos del anillo clásico. Las únicas dos distinciones son que:

- Se sustituye el elemento unitario clásico con respecto a la ley *; por $neut\ *\ (x)$ con respecto a la ley * para cada x en N en la NTR.
- Del mismo modo, el elemento inverso clásico de un elemento x en N , con respecto a la ley *, se sustituye por el $anti\ *$ (x) con respecto a la ley * en N.

2) Un anillo de tripletas neutrosóficas, en general, es diferente al anillo clásico.

1.12. Observaciones sobre el anillo triplete extendido neutrosófico:

1) Del mismo modo, el anillo de tripleta neutrosófico extinguido se define en los pasos de las únicas dos distinciones son que:

- El elemento unitario clásico con respecto a la ley * se extiende a $e^{neut*}(x)$ con respecto a la ley * para cada x en N en el NETR.
- Del mismo modo, el elemento inverso clásico de un elemento x en N, con respecto a la ley *, se extiende a e^{anti*} (x), con respecto a la ley * en N.

2) Un anillo de tripleta extendido neutrosÃ³fico, en general, es diferente de un anillo clásico.

1.13. Definición de anillo de tripleta neutrosófico hibrido (HNTR)

El anillo triplete neutrosófico híbrido es un conjunto N dotado de dos leyes binarios (N, *, #), tales que:

a) $(N,*)$ es un grupo tripletas neutrosóficas conmutativas (fuerte); lo que significa que:

- N es un conjunto fuerte de tripletas neutrosóficas con respecto a la ley * (es decir, si x pertenece a N, entonces $neut * (x)$ y $anti * (x)$, definidos con respecto a la ley *, también pertenecen a N).
- La ley * está bien definida, es asociativa y conmutativa en N (como en el sentido clásico).

b) $(N,\#)$ es un conjunto fuerte de tripletas neutrosóficas con respecto a la ley # (es decir, si x pertenece a N, entonces $neut\ \#\ (x)$ y $anti\ \#\ (x)$, definidos con respecto a la ley #, también pertenecen a N).

- la ley # está bien definida y no asociativa en N (como en el sentido clásico).

c) La ley # es distributiva con respecto a la ley * (como en el sentido clásico).

1.14. Definición de anillo de tripleta extendido neutrosófico hibrido (HNETR)

El anillo de tripleta extendido neutrosófico híbrido es un conjunto N dotado de dos leyes binarias (N, *, #), de manera que:

a) (N, *) es un grupo tripletico neutrosófico extendido (fuerte); lo que significa que:

- N es un conjunto fuerte de tripleta extendida neutrosófica con respecto a la ley * (es decir, si x pertenece a N, entonces $e^{neut*}(x)$ y e^{anti*} (x), definidos con respecto a la ley *, también pertenecen a N).
- La ley * está bien definida, es asociativa y conmutativa en N (como en el sentido clásico).

b) $(N,\#)$ es un conjunto fuerte de tripletas neutrosóficas extendido con respecto a la ley # (es decir, si x pertenece a N, entonces $e^{neut}\ \#\ (x)$ y $e^{anti}\ \#\ (x)$, definidos con respecto a la ley #, también pertenecen a N).

- La ley # está bien definida y no es asociativa en N (como en el sentido clásico).

c) La ley # es distributiva con respecto a la ley * (como en el sentido clásico).

1.15. Observaciones sobre el anillo híbrido de tripletas neutrosóficas

a) Un anillo de tripletas neutrosóficas híbrido es un campo $(N,*,\#)$ desde el cual se ha eliminado la asociatividad de la segunda ley #.
b) O bien, el anillo de tripletas neutrosófico híbrido es un conjunto $(N,*,\#)$, tal que $(N,*)$ es un grupo de tripletes neutrosóficas conmutativas, y $(N,\#)$ es un bucle de tripletas neutrosóficas, y la ley # es distributiva con respecto a la ley * (como en el sentido clásico).

1.16. Comentarios sobre el anillo híbrido de tripletas neutrosófica extendido

a) Un anillo de tripletas extendido neutrosófico hibrido es un campo $(N,*,\#)$ del cual se elimina la asociatividad de la segunda ley #.
b) O, el anillo de tripletas extendido neutrosófico híbrido es un conjunto $(N,*,\#)$, tal que $(N,*)$ es un grupo de tripletas extendidas neutrosóficas conmutativas, y $(N,\#)$ es un bucle de tripletas extendidas neutrosóficas, y la ley # es distributiva con respecto a la ley * (como en el sentido clásico).

1.17. Definición de campo de tripletas neutrosóficas (NTF)

El campo de tripletas neutrsóficas es un conjunto dotado de dos leyes binarias $(N,*,\#)$, tal que:

a) $(N,*)$ es un grupo de tripletas neutrsóficas conmutativa; lo que significa que:
- N es un conjunto de tripletes neutrsóficas con respecto a la ley *, (es decir, si x pertenece a N , entonces neut * (x) y anti * (x) , definidos con respecto a la ley *, también ambos pertenecen a N).
- La ley * está bien definida, asociativa y conmutativa en N; (como en el sentido clásico).

b) $(N,\#)$ es un grupo tripletas neutrosóficas; lo que significa que:

- M es un conjunto de tripletas neutrosóficas con respecto a la ley # (es decir, si x pertenece a N, entonces $neut\ \#\ (x)$ y anti # (x) , definidos con respecto a la ley #, también ambos pertenecen a N).
- La ley # está bien definida y asociativa en N; (como en el sentido clásico);

c) La ley # es distributiva con respecto a la ley * (como en el sentido clásico).

1.18. Definición de campo de tripletas extendidas neutrosóficas (NETF)

El campo tripletas extendidas neutrsóficas es un conjunto dotado de dos leyes binarias (N, *, #), tal que:

a) (N, *) es un grupo de triplete extendido neutrsófico conmutativo; lo que significa que:

- N es un conjunto de tripletas neutrsóficas extendidas con respecto a la ley * (es decir, si x pertenece a N , entonces e neut * (x) y e anti * (x) , definido con respecto a la ley *, también ambos pertenecen a N).
- La ley * está bien definida, asociativa y conmutativa en N (como en el sentido clásico).

b) (N, #) es un grupo de tripletas extendidas neutrsóficas, lo que significa que:
c) N es un conjunto de tripletas neutrosóficas con respecto a la ley # (es decir, si x pertenece a N , entonces e neut # (x) y e anti # (x) , definido con respecto a la ley #, también ambos pertenecen a N).

- La ley # está bien definida y asociativa en N (como en el sentido clásico).

d) La ley # es distributiva con respecto a la ley *, (como en el sentido clásico).

1.19. Observaciones sobre el campo de tripletas neutrosóficas

El campo de tripletas neutrsoficas se define en los pasos del campo clásicos, las únicas distinciones son que:

1) El elemento unitario clásico con respecto a la primera ley * se extiende a e neut * (x) con respecto a la primera ley * para cada x en N en el NTF.

   - Del mismo modo, el elemento inverso clÃ¡sico de un elemento x en N, con respecto a la primera ley *, se extiende a e anti * (x) con respecto a la primera ley * law * in N ; ley * en N ;
   - y el elemento de unidad clÃ¡sico con respecto a la segunda ley # se extiende a e neut # (x) con respecto a la segunda ley # para cada x en N en el NTF;
   - Del mismo modo, el elemento inverso clÃ¡sico de un elemento x en N , con respecto a la segunda ley #, se extiende a e anti # (x) con respecto a la segunda ley # en N.

2) Un campo de tripletas neutrosóficas, en general, es diferente de un campo clásico.

1.20. Campo de tripletas neutrosóficas hibridas de tipo 1 (HNTF1)

El campo de tripletas neutrosóficas hibridas de tipo 1 (HNTF1), es un conjunto N dotado de dos leyes * y # tales que:

1. $(N,*)$ es un grupo de tripletas neutrosóficas conmutativas
2. $(N,\#)$ es un grupo clásico
3. La ley # es distributiva sobre la ley *.

1.21. Campo de tripletas neutrosóficas extendidas híbrido de tipo 1 (HNETF1)

Es un conjunto N dotado de dos leyes * y # tales que:

1. $(N,*)$ es un grupo tripletas neutrosóficas extendidas conmutativas
2. (N, #) es un grupo clásico
3. La ley # es distributiva sobre la ley *

1.22. Campo de tripletas neutrosóficas híbridas de tipo 2 (HNTF2)

Es un conjunto N dotado de dos leyes * y # tales que:

1. (N, *) es un grupo conmutativo clásico.
2. (N, #) es un grupo tripletas neutrosóficas
3. La ley # es distributiva sobre la ley *.

1.23. Campo de tripletas neutrosóficas híbrido de tipo 2 (HNETF2)

Es un conjunto N dotado de dos leyes * y # tales que:

1. (N, *) es un grupo conmutativo clásico
2. (N, #) es un grupo de tripletas extendidas neutrosóficas
3. La ley # es distributiva sobre la ley *.

1.24. Aplicaciones de las estructuras de tripletas neutrosóficas (NTS) y estructuras de tripletas extendidas neutrosóficas (NETS).

Este nuevo campo de estructuras tripletas neutrs;oficas y tripletas extendido neutrsóficas posee importancia porque a través de ellas es posible reflejar aspectos de la vida cotidiana que requieren ser evaluados, dada la forma en que se expresan, es decir se exprean de forma cualitativa. Con el uso de las tripletas neutrsóficas y las tripletas neutrsóficas extendidas es posible tomar decisión sobre tripletas de resultados, opiniones, entre otras. Ejemplo de como se presentan son: (amigo, neutral, enemigo), (particular positiva, particular neutral, particular negativo), (Si, incierto, no), (pro, neutral, contra), (victoria, empate, derrota), (tomar una desición, indeciso, no tomar una decision), (aceptar, pendiente, rechazar), en general (, ,) como en la neutrosofia, que es una nueva rama de la filosófia que generaliza la dialectica.

Referencias

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[9] F. Smarandache, Hybrid Neutrosophic Triplet Ring in Physical Structures, Annual Meeting of the APS Four Corners Section, Fort Collins, CO, USA, & Bulletin of the American Physical Society, October 20–21, 2017; http://meetings.aps.org/Meeting/4CF17/Session/G1.33
[10] F. Smarandache, Neutrosophic Perspectives: Triplets, Duplets, Multisets, Hybrid Operators, Modal Logic, Hedge Algebras. And Applications. Pons Editions, Bruxelles, second edition, 323 p., 2017; http://fs.unm.edu/NeutrosophicPerspectives-ed2.pdf

Historia de las Teorías Neutrosóficas y sus Aplicaciones (actualizado)

Florentín Smarandache

Florentín Smarandache (2019). Historia de las Teorías Neutrosóficas y sus Aplicaciones (actualizado). *Neutrosophic Computing and Machine Learning 8*, 53-57

1. Introducción

En 1965 Zadeh introdujo el *grado de pertenencia/verdad* (T) y definió el conjunto difuso.

En 1986 Atanassov introdujo el *grado de no pertenencia/falsedad* (F) y definió el conjunto difuso intuicionista.

Smarandache introdujo el ***grado de indeterminación/neutralidad*** (I) como componente independiente en 1995 (publicado en 1998) y definió el conjunto neutrosófico en tres componentes:
(T, I, F) = (Verdad, Indeterminación, Falsedad), donde en general T, I, F son subconjuntos del intervalo [0, 1]; en particular, T, I, F pueden ser intervalos, conjuntos vacilantes o valores únicos; véase
F. Smarandache, *"Neutrosofía/Probabilidad, lógica y conjunto neutrosóficos", Proquest Michigan, EE.UU.*, 1998,
https://arxiv.org/ftp/math/papers/0101/0101228.pdf
http://fs.unm.edu/eBook-Neutrosophics6.pdf ;
revisado en Zentralblatt fuer Mathematik (Berlín, Alemania): https://zbmath.org/?q=an:01273000
y citado por Denis Howe en *The Free Online Dictionary of Computing*, Inglaterra, 1999.
La lógica y el conjunto neutrosóficos son generalizaciones de la lógica y el conjunto difusos, clásicos e intuicionistas:
https://arxiv.org/ftp/math/papers/0404/0404520.pdf
https://arxiv.org/ftp/math/papers/0303/0303009.pdf

Lógica, probabilidad y conjunto neutrosóficos no estándar (1998, 2019)
https://arxiv.org/ftp/arxiv/papers/1903/1903.04558.pdf

1.1. Etimología

Las palabras "neutrosofía" y "neutrosófico" fueron acuñadas/inventadas por F. Smarandache en su libro de 1998.

La Neutrosofía: es una rama de la filosofía, introducida por F. Smarandache en 1980, que estudia el origen, la naturaleza y el alcance de las neutralidades, así como sus interacciones con diferentes espectros ideacionales. La Neutrosofía considera una proposición, teoría, evento, concepto o entidad <A> en relación con su opuesto <antiA>, y con su neutral <neutA>.
La neutrosofía (como dinámica de los opuestos y sus neutrales) es una extensión de la dialéctica (que es la dinámica de los opuestos solamente).
La neutrosofía es la base de la lógica neutrosófica, la probabilidad neutrosófica, el conjunto neutrosófico y la Estadística neutrosófica.
https://arxiv.org/ftp/math/papers/0010/0010099.pdf

La Lógica Neutrosófica es un marco general para la unificación de muchas lógicas existentes, como la lógica difusa (especialmente la lógica difusa intuicionista), la lógica paraconsistente, la lógica intuicionista, etc. La idea esencial de la Lógica Neutrosófica es caracterizar cada declaración lógica en un Espacio 3D-Neutrosófico, donde cada dimensión del espacio representa, respectivamente, la verdad (T), la falsedad (F) y la indeterminación (I) de la declaración bajo consideración, donde T, I, F son subconjuntos reales estándar o no estándar de $]^-0, 1^+[$ sin necesariamente ninguna conexión entre ellos.

Para propuestas de ingeniería de software se puede usar el intervalo de unidad clásico [0, 1].

1.2. Grados de dependencia e independencia entre Componentes Neutrosóficos

T, I, F son *componentes independientes*, dejando espacio para información incompleta (cuando su suma superior <1), información paraconsistente y contradictoria (cuando la suma superior > 1), o información completa (suma de los componentes = 1).
Para propuestas de ingeniería de software se utiliza el intervalo de unidad clásico [0, 1].
Para la lógica neutrosófica de valor único, la suma de los componentes es:

$0 \leq t + i + f \leq 3$ cuando los tres componentes son independientes;
$0 \leq t + i + f \leq 2$ cuando dos componentes son dependientes, mientras que el tercero es independiente de ellos;
$0 \leq t + i + f \leq 1$ cuando los tres componentes son dependientes.

Cuando tres o dos de los componentes T, I, F son independientes, se deja espacio para información incompleta (suma < 1), información paraconsistente y contradictoria (suma > 1), o información completa (suma = 1).

Si los tres componentes T, I, F son dependientes, entonces similarmente uno deja espacio para información incompleta (suma < 1), o información completa (suma = 1).

En general, la suma de dos componentes x e y que varían en el intervalo unitario [0, 1] es:
$0 \leq x + y \leq 2 - d°(x, y)$, donde $d°(x, y)$ es el grado de dependencia entre x e y, mientras que $d°(x, y)$ es el grado de independencia entre x e y.

https://doi.org/10.5281/zenodo.571359
http://fs.unm.edu/NSS/DegreeOfDependenceAndIndependence.pdf

En 2013 Smarandache *refinó el conjunto neutrosófico a n componentes*:
$(T_1, T_2, ...; I_1, I_2, ...; F_1, F_2, ...)$;
https://arxiv.org/ftp/arxiv/papers/1407/1407.1041.pdf
http://fs.unm.edu/n-ValuedNeutrosophicLogic-PiP.pdf.

1.3. Los libros y documentos más importantes sobre el Avance de la Neutrosofía

1995-1998 - *Smarandache generaliza la dialéctica a la neutrosofía;*
introduce la lógica/la probabilidad/la estadística y el conjunto neutrosóficos;
introduce el conjunto neutrosófico de valor único (pp. 7-8);
https://arxiv.org/ftp/math/papers/0101/0101228.pdf (cuarta edición)
http://fs.unm.edu/eBook-Neutrosophics6.pdf (edición en línea)

2002 - Introducción de casos límite de conjuntos/probabilidades/estadística/lógica, tales como:

- **Conjunto neutrosófico intuicionista (distinto del conjunto difuso intuicionista), conjunto neutrosófico paraconsistente, conjunto neutrosófico falibilista, conjunto neutrosófico paradoxista, conjunto neutrosófico pseudo-paradoxista, conjunto neutrosófico tautológico, conjunto neutrosófico nihilista, conjunto neutrosófico dialetista, conjunto neutrosófico trivialista;**
- **Estadística y probabilidades neutrosóficas intuicionistas, estadística y probabilidades neutrosóficas paraconsistentes, estadística y probabilidades neutrosóficas falibilistas, estadística y probabilidades neutrosóficas paradoxistas, estadística y probabilidades neutrosóficas pseudo-paradoxistas, estadística y probabilidades neutrosóficas tautológicas, estadística y probabilidades neutrosóficas nihilistas, estadística y probabilidades neutrosóficas dialetistas, y estadística y probabilidades neutrosóficas trivialistas;**
- **Lógica neutrosófica paradoxista (o paradoxismo), lógica neutrosófica pseudo-paradoxista (o pseudo-paradoxismo neutrosófico), lógica neutrosófica tautológica (o tautologismo neutrosófico):**

https://arxiv.org/ftp/math/papers/0301/0301340.pdf
http://fs.unm.edu/DefinitionsDerivedFromNeutrosophics.pdf

2003 - *Introducción por Kandasamy y Smarandache de*
Los Números Neutrosóficos *(a + bI, donde I = indeterminación, I ^ 2 = I),*
Las Estructuras Algebraicas I-Neutrosóficas
y ***Los Mapas Cognitivos Neutrosóficos***
https://arxiv.org/ftp/math/papers/0311/0311063.pdf
http://fs.unm.edu/NCMs.pdf

2005 - *Introducción de* ***La Lógica/el Conjunto Neutrosófico de Intervalo***
https://arxiv.org/pdf/cs/0505014.pdf
http://fs.unm.edu/INSL.pdf

2006 - Introducción del ***Grado de dependencia y grado de independencia entre los componentes neutrosóficos*** *T, I, F*
http://fs.unm.edu/eBook-Neutrosophics6.pdf (p. 92)
http://fs.unm.edu/NSS/DegreeOfDependenceAndIndependence.pdf

2007 - El Conjunto Neutrosófico se extendió [Smarandache, 2007] a ***Sobreconjunto Neutrosófico*** (cuando algún componente neutrosófico es > 1), ya que observó que, por ejemplo, un empleado que trabaja horas extras merece un grado de pertenencia > 1, con respecto a un empleado que solo trabaja regularmente a tiempo completo y cuyo grado de pertenencia = 1;
y a ***Bajoconjunto Neutrosófico*** (cuando algún componente neutrosófico es < 0), ya que, por ejemplo, un empleado que hace más daño que beneficio a su compañía merece un grado de pertenencia < 0, con respecto a un empleado que produce beneficios para la compañía y tiene el grado de pertenencia > 0;
y a ***Fueraconjunto neutrosófico*** (cuando algunos componentes neutrosóficos están fuera del intervalo [0, 1], es decir, algún componente neutrosófico > 1 y algún componente neutrosófico < 0).
Luego, de manera similar, la Lógica/Medida/Probabilidad y Estadística Neutrosóficas, etc. se extendieron respectivamente a *Sobre-, Bajo-, Fuera-Lógica, Medida, Probabilidad, Estadística,* etc.
https://arxiv.org/ftp/arxiv/papers/1607/1607.00234.pdf
http://fs.unm.edu/NeutrosophicOversetUndersetOffset.pdf
http://fs.unm.edu/SVNeutrosophicOverset-JMI.pdf
http://fs.unm.edu/IV-Neutrosophic-Overset-Underset-Offset.pdf

2007 – Smarandache *introdujo el*
Conjunto tripolar Neutrosófico y ***el Conjunto Multipolar Neutrosófico***
y consecuentemente
El Grafo tripolar neutrosófico y ***el Grafo multipolar neutrosófico***
http://fs.unm.edu/eBook-Neutrosophics6.pdf (p. 93)
http://fs.unm.edu/IFS-generalized.pdf

2009 - *Introducción de* ***la N-norma y la N-conorma***
https://arxiv.org/ftp/arxiv/papers/0901/0901.1289.pdf
http://fs.unm.edu/N-normN-conorm.pdf

2013 - *Desarrollo de* ***la Medida Neutrosófica*** *y* ***La Probabilidad Neutrosófica***
(posibilidad de que ocurra un evento, posibilidad indeterminada de ocurrencia, posibilidad de que el evento no ocurra)
https://arxiv.org/ftp/arxiv/papers/1311/1311.7139.pdf
http://fs.unm.edu/NeutrosophicMeasureIntegralProbability.pdf

2013 - Smarandache refinó ***Los componentes neutrosóficos*** *(T, I, F) como* $(T_1, T_2, \ldots; I_1, I_2, \ldots; F_1, F_2, \ldots)$
http://fs.unm.edu/n-ValuedNeutrosophicLogic-PiP.pdf

2014 - *Introducción de la* ***Ley del Medio Múltiple Incluido***
(<A>; <neut1A>, <neut2A>,…; < antiA >)
http://fs.unm.edu/LawIncludedMultiple-Middle.pdf

2014 - *Desarrollo de* ***La Estadística Neutrosófica*** *(La indeterminación se introduce en la estadística clásica con respecto a la muestra/población, o con respecto a los individuos que pertenecen solo parcialmente a una muestra/población)*
https://arxiv.org/ftp/arxiv/papers/1406/1406.2000.pdf
http://fs.unm.edu/NeutrosophicStatistics.pdf

2015 - *Introducción del* ***Precálculo Neutrosófico y el Cálculo Neutrosófico***
https://arxiv.org/ftp/arxiv/papers/1509/1509.07723.pdf
http://fs.unm.edu/NeutrosophicPrecalculusCalculus.pdf

2015 - ***Los Números Neutrosóficos Refinados*** *($a + b_1 I_1 + b_2 I_2 + \ldots + B_n I_n$), donde $I_1, I_2, \ldots, I_n$ Son las subindeterminaciones de la indeterminación I;*
2015 - *(t, i, f) -grafos neutrosóficos;*
2015 - *Tesis-Antítesis- Neutrotesis y Neutrosíntesis, Sistema Axiomático Neutrosófico, Sistemas dinámicos neutrosóficos, lógica neutrosófica simbólica, (t, i , f) -Estructuras neutrosóficas, Estructuras I-Neutrosóficas, Indeterminación Literal Refinada, Estructuras Algebraicas Neutrosóficas Cuádruples, Ley De Multiplicación de Subindeterminaciones*:
https://arxiv.org/ftp/arxiv/papers/1512/1512.00047.pdf
http://fs.unm.edu/SymbolicNeutrosophicTheory.pdf

2015 - Introducción de las **Subindeterminaciones** de la forma $(I_0)^n = k / 0$, para $k \in \{0, 1, 2,\ldots, n-1\}$, en el anillo de enteros de módulo Z_n - llamadas indeterminaciones neutrosóficas naturales (Vasantha-Smarandache)
http://fs.unm.edu/MODNeutrosophicNumbers.pdf

2015 - Introducción del **Topología y Conjuntos neutrosóficos clásicos** (Salama - Smarandache)
http://fs.unm.edu/NeutrosophicCrispSetTheory.pdf

2016 - Introducción de **los Multiconjuntos Neutrosóficos** (como generalización de los multiconjuntos clásicos)
http://fs.unm.edu/NeutrosophicMultisets.htm

2016 - Introducción de **las Estructuras de Tripletes Neutrosóficos** y estructuras de tripletes neutrosóficos refinados de m valores [Smarandache - Ali]
http://fs.unm.edu/NeutrosophicTriplets.htm

2016 - Introducción de **las Estructuras de Dobletes Neutrosóficos**
http://fs.unm.edu/NeutrosophicDuplets.htm

2017 - En biología Smarandache introdujo la **Teoría de la Evolución Neutrosófica: Grados de Evolución, Indeterminación o Neutralidad** e Involución
http://fs.unm.edu/neutrosophic-evolution-PP-49-13.pdf

2017 - Introducción por F. Smarandache de **la Plitogenia** (como generalización de la dialéctica y la neutrosofía), y **la Lógica/la Probabilidad/la Estadística y el Conjunto Plitogénicos** (como generalización de la lógica/la probabilidad/la estadística y el conjunto neutrosóficos, difusos e intuicionistas)
https://arxiv.org/ftp/arxiv/papers/1808/1808.03948.pdf
http://fs.unm.edu/Plithogeny.pdf

2018 - Introducción a la **Psicología Neutrosófica** (Neutropsique, Memoria Neutrosófica Refinada: consciente, aconsciente, inconsciente, Personalidad Neutropsíquica, Eros/Aoristos/Thanatos, Personalidad Neutropsíquica Clásica)
http://fs.unm.edu/NeutropsychicPersonality-ed3.pdf

2019 - Introducción a la **Sociología Neutrosófica** (Neutrosociología) [concepto neutrosófico, o *concepto (T, I, F)*, es un concepto que es *T% verdad, I%* indeterminado, y *F%* falso]
http://fs.unm.edu/Neutrosociology.pdf

1.4. Aplicaciones en:

Inteligencia Artificial, Sistemas de Información, Informática, Cibernética, Métodos Teóricos, Estructuras Algebraicas Matemáticas, Matemática Aplicada, Automatización, Sistemas de Control, Datos Grandes, Ingeniería, Eléctrica, Electrónica, Filosofía, Ciencias Sociales, Psicología, Biología, Biomédica, Ingeniería, Informática Médica, Investigación de operaciones, Ciencias de la administración, Ciencia de imágenes, Tecnología fotográfica, Instrumentos, Instrumentación, Física, Óptica, Economía, Mecánica, Neurociencias, Radiología Nuclear, Medicina, Imágenes médicas, Aplicaciones interdisciplinarias, Ciencias multidisciplinares, etc.
[Xindong Peng y Jingguo Dai, *Un análisis bibliométrico de conjuntos neutrosóficos: revisión de dos décadas desde 1998 hasta 2017*, Artificial Intelligence Review, Springer, 18 de agosto de 2018; http://fs.unm.edu/BibliometricNeutrosophy.pdf]

La revista internacional **Conjuntos y Sistemas Neutrosóficos** (NSS) salió a la luz en 2013 y está indexada por Scopus, Web of Science (ESCI), DOAJ, Index Copernicus, Redalyc - Universidad Autónoma del Estado de México (Iberoamérica), Publons, CNKI, Google Scholar, Chinese Baidu Scholar, etc. (http://fs.unm.edu/NSS/).
Envíe los documentos sobre lógica/probabilidad/estadística/conjunto neutrosóficos y sus aplicaciones al editor en jefe: smarand@unm.edu.

1.5. Enciclopedia de Investigadores Neutrosóficos

Los autores que han publicado o presentado artículos sobre Neutrosofía y no están incluidos en la *Enciclopedia de Investigadores Neutrosóficos* (ENR), vols. 1, 2 y 3,
http://fs.unm.edu/EncyclopediaNeutrosophicResearchers.pdf
http://fs.unm.edu/EncyclopediaNeutrosophicResearchers2.pdf
http://fs.unm.edu/EncyclopediaNeutrosophicResearchers3.pdf
pueden enviar su CV, foto y lista de publicaciones neutrosóficas a smarand@unm.edu para ser incluidos en el cuarto volumen de ENR.

Traducido por: Ing. Osmanys Perez Peña, Asociación Económica Internacional Arcos-Bouygues Batiment International, Holguín, Cuba

On Neutrosophic Offuninorms

Erick González Caballero, Florentin Smarandache, Maikel Leyva Vázquez

Erick González Caballero, Florentin Smarandache, Maikel Leyva Vázquez (2019). On Neutrosophic Offuninorms. *Symmetry* 11, 1136. DOI: 10.3390/sym11091136

Abstract: Uninorms comprise an important kind of operator in fuzzy theory. They are obtained from the generalization of the t-norm and t-conorm axiomatic. Uninorms are theoretically remarkable, and furthermore, they have a wide range of applications. For that reason, when fuzzy sets have been generalized to others—e.g., intuitionistic fuzzy sets, interval-valued fuzzy sets, interval-valued intuitionistic fuzzy sets, or neutrosophic sets—then uninorm generalizations have emerged in those novel frameworks. Neutrosophic sets contain the notion of indeterminacy—which is caused by unknown, contradictory, and paradoxical information—and thus, it includes, aside from the membership and non-membership functions, an indeterminate-membership function. Also, the relationship among them does not satisfy any restriction. Along this line of generalizations, this paper aims to extend uninorms to the framework of neutrosophic offsets, which are called neutrosophic offuninorms. Offsets are neutrosophic sets such that their domains exceed the scope of the interval [0,1]. In the present paper, the definition, properties, and application areas of this new concept are provided. It is necessary to emphasize that the neutrosophic offuninorms are feasible for application in several fields, as we illustrate in this paper.

Keywords: neutrosophic offset; uninorm; neutrosophic offuninorm; neutrosophic offnorm; neutrosophic offconorm; implicator; prospector; n-person cooperative game

1. Introduction

Uninorms extend the t-norm and t-conorm axiomatic in fuzzy theory. They retain the axioms of commutativity, associativity, and monotony. Alternatively, they generalize the boundary condition, where the neutral element is any number lying in [0,1]. Thus, t-norm and t-conorm are special cases of uninorms, t-norms have 1 as their neutral element and the neutral element of t-conorms is 0, see [1–3].

Uninorms are theoretically important, and moreover they have also been used as operators in several areas of application; for example, in image processing, to aggregate group decision criteria, among others, see [4–8]. An exhaustive search on uninorm applications made by the authors of this paper yielded more than six hundred scientific articles that have been written in the last five years devoted to this subject.

Rudas et al. in [9] report that uninorms have been applied in diverse applications ranging, e.g., from defining Gross Domestic Product index in economics, to fusing sequences of DNA and RNA or combining information on taxonomies or dendograms in biology, and in the fusion of data provided by sensors of robotics in data mining, and in knowledge-based and intelligent systems. Particularly, they offer many examples in Decision Making, Utility Theory, Fuzzy Inference Systems, Multisensor Data Fusion, network aggregation in sensor networks, image approximation, hardware implementation of parametric operations, in Fuzzy Systems, and as software tools for aggregation problems.

Depaire et al. in [10] proposed a new approach to apply uninorms in Importance Performance Analysis, which is a useful technique to evaluate elements in marketing programs. They proved that their approach was superior when compared with regression and that it matched well with the customer satisfaction theory.

A very recent paper written by Modley et al. in [11] applied uninorms in the market basket analysis. Also, Appel et al. proposed a method based on cross-ratio uninorms as a mechanism to aggregate in the Sentiment Analysis; see [12].

Kamiset al. in [13] implement a geo-uninorm operator in a consensus model. They utilized them to derive a consistently based preference relation from a given reciprocal preference relation. Whereas, Wu et al. in [14] and Ureña et al. in [15] applied uninorms in trust propagation and aggregation methods for group decision making in a social network.

Bordignon and Gomide in [16], introduce a learning approach to train uninorm-based hybrid neural networks using extreme learning concepts. According to them, uninorms bring flexibility and generality to fuzzy neuron models. Wang in [17] and Yang in [18] applied uninorms as a basis to define logics.

Other areas of application can be consulted in González-Hidalgo et al. [19] where uninorms were utilized in edge detection of image processing, in fuzzy morphological associative memories (see [20]), and was also applied in time series prediction.

It is well-known that the minimum is the biggest t-norm and the maximum is the lowest t-conorm, thus they are not compensatory operators; whereas uninorms compensate when the truth values are situated on both sides of the neutral element. The compensation property could be the key factor in the wide range of uninorm applicability, mainly in decision making. Zimmermann experimentally proved in [21], many years before the introduction of uninorms, that often human beings do not make decisions interpreting AND like a t-norm and OR like a t-conorm, but that compensatory operators are more adequate to model human aggregations to signify AND and OR in some situations. The use of means as aggregators to define membership functions can be seen in [22]. However, when the aggregated values are situated on one side with respect to the neutral element, then uninorms operate either like a t-norm or a t-conorm.

Uninorms have been extended to other theories more general than fuzzy logic, due to their applicability. Let us mention intuitionistic fuzzy sets, interval-valued fuzzy sets, and interval-valued intuitionistic fuzzy sets; where the generalizations consist of the inclusion of an independent non-membership function or an interval-valued membership function, or both [23]. They have also been generalized as multi-polar aggregators in [24].

Following this trend, the authors of this paper defined the neutrosophic uninorms, such that the uninorms were extended to the neutrosophy framework [25]. Neutrosophy is the philosophical discipline that studies theories, entities, objects, phenomena, among others, related to neutrality [26]. In particular, neutrosophic sets contain three independent functions, namely, a membership function, a non-membership function, and additionally, an indeterminate-membership function. The last one represents what is unknown, contradictory, and paradoxical. Furthermore, these elements can be intervals.

In addition, the relationship among these three functions has no restriction, contrary to the intuitionist fuzzy sets, which must fulfill the constraint that the sum of the membership truth value with the non-membership truth value of an element to the set does not exceed the unit.

Neutrosophy theory has been used in a wide spectrum of applications such as in image processing, decision making, clustering, among others [27–30]. Therefore, it is not difficult to appreciate the applicability of neutrosophic uninorms.

More recently, other concepts have been defined within the neutrosophy framework, which further generalizes the traditional membership functions, including the axiomatic in probability theory.

They are the undersets, oversets, and offsets, where the basic idea is that negative truth values or truth values greater than 1 are permitted in the calculus [31].

A recurring example in literature is that concerning employment, where the truth value of a worker's effectiveness is measured in working hours. Those workers who have met all of their working hours established for the week will be an effectiveness truth value of 1, those workers who have only partially met their working hours have a truth value between 0 and 1, and other workers who have not attended work throughout the week have the truth value of 0. In addition, those who have performed voluntary overtime after meeting their established hours have a truth value greater than 1, and finally, the workers who have not attended work throughout the week and, moreover, have caused losses to the company, must have a negative truth value.

Other examples take into consideration the relationship between two variables or more, where a negative value represents that they are inversely related, whereas a direct relationship is represented by positive values [31].

The aim of this paper is to extend for the first time the theory of uninorms to the offsets framework—we call them neutrosophic offuninorms—in such a way that they are a generalization of both n-offnorms and n-offconorms equivalently, as fuzzy uninorms generalize both t-norms and t-conorms.

In this paper, definitions and also properties of neutrosophic offuninorms will be given. Additionally, we will emphasize the relationship between these new operators and the aggregation functions used in the well-known medical expert system MYCIN [32], as well as define logical implicators in offset fields and solve voting cooperative games.

In particular, the association of the proposed theory with the aggregation functions used in MYCIN supports the hypothesis that neutrosophic offuninorms are more than an interesting theoretical approach. Historically, within the fuzzy logic framework, some authors have accepted the idea of extending the uninorms domain to [a, b], in order to include the aggregation functions used in MYCIN, [33,34]. This proposal is an important precedent for this investigation because uninorms were there adapted to offsets in the fuzzy theory context. The relationship between uninorms and the PROSPECTOR operator, as well as their application, can be consulted in [35], where they were used in e-arning.

Authors in [33,34] also emphasize that this generalization has important practical advantages because it allows us to naturally apply uninorms in fields like Artificial Neural Networks and Cognitive Maps. These elements certainly suggest that the proposed theory can be applied in fields like Artificial Neural Networks based on neutrosophic sets and in neutrosophic cognitive maps, [36,37].

Let us observe that when uninorms have been extended to other domains they have preserved the property of compensation. Further, we shall prove that offuninorms are not the exception; consequently, the applicability of offuninorms is practically guaranteed. In the discussion section, we insist on this aspect and the advantages that offuninorms have over other generalizations.

This paper is divided as follows. It begins with a preliminary section where concepts such as neutrosophic sets, neutrosophic offsets, neutrosophic uninorms, among other useful aspects, are discussed in order to develop the content of this article. The section on neutrosophic offuninorms is devoted to exposing definitions and properties of these novel operators. Next, the applications section is where the three possible areas of application of this theory are explained. We then finish with the sections of discussion and conclusions.

2. Preliminaries

This section contains the main definitions necessary to develop the theory proposed in this paper. We begin with Definitions 1 and 2, which introduce the neutrosophic sets. These sets are characterized by an independent indeterminacy-membership function that models the unknown, contradictions, inconsistencies in information and so on. Additionally, we have the classic membership and non-membership functions, which are not necessarily dependent on each other.

Definition 1. *Let X be a space of points (objects), with a generic element in X denoted by x. A Neutrosophic Set A in X is characterized by a truth-membership function $T_A(x)$, an indeterminacy-membership function $I_A(x)$, and a falsity-membership function $F_A(x)$. $T_A(x)$, $I_A(x)$, and $F_A(x)$ are real standard or nonstandard subsets of $]^{-}0, 1^{+}[$. There is no restriction on the sum of $T_A(x)$, $I_A(x)$, and $F_A(x)$, thus, $^{-}0 \le \inf T_A(x) + \inf I_A(x) + \inf F_A(x) \le \sup T_A(x) + \sup I_A(x) + \sup F_A(x) \le 3^{+}$ (see [26]).*

The neutrosophic sets are useful in their nonstandard form only in philosophy, in order to make a distinction between absolute truth (truth in all possible worlds—according to Leibniz) and relative truth (truth in at least one world), but not in technical applications, thus the *Single-Valued Neutrosophic Sets* are defined, see Definition 2.

Definition 2. *Let X be a space of points (objects), with a generic element in X denoted by x. A Single-Valued Neutrosophic Set A in X is characterized by a truth-membership function $T_A(x)$, an indeterminacy-membership function $I_A(x)$, and a falsity-membership function $F_A(x)$. $T_A(x)$, $I_A(x)$, and $F_A(x)$ are elements of [0,1]. There is no restriction on the sum of $T_A(x)$, $I_A(x)$, and $F_A(x)$, thus, $0 \le T_A(x)+I_A(x) + F_A(x) \le 3$ (see [38]).*

The domain of the single-valued neutrosophic sets does not surpass the limits of the interval [0,1]. This is a classical condition imposed in previous theories such as probability and fuzzy sets. Despite the past, Smarandache in 2007 proposed the membership >1 and <0 and illustrated this proposal; see [39] (pp. 92–93) and the example given in the introduction of this paper. In the following, the *Single-Valued Neutrosophic Oversets*, *Single-Valued Neutrosophic Undersets*, and *Single-Valued Neutrosophic Offsets* are formally defined.

Definition 3. *Let X be a universe of discourse and the neutrosophic set $A_1 \subset X$. Let T(x), I(x), F(x) be the functions that describe the degree of membership, indeterminate-membership, and non-membership respectively, of a generic element $x \in X$, with respect to the neutrosophic set A_1:*

T, I, F: $X \to [0, \Omega]$, where $\Omega > 1$ is called overlimit, T(x), I(x), F(x)$\in [0, \Omega]$. A Single-Valued Neutrosophic Overset A_1 is defined as $A_1 = \{(x, T(x),\ I(x),\ F(x)), x \in X\}$, such that there exists at least one element in A_1 that has at least one neutrosophic component that is bigger than 1, and no element has neutrosophic components that are smaller than 0 (see [31]).

Definition 4. *Let X be a universe of discourse and the neutrosophic set $A_2 \subset X$. Let T(x), I(x), F(x) be the functions that describe the degree of membership, indeterminate-membership, and non-membership, respectively, of a generic element $x \in X$, with respect to the neutrosophic set A_2:*

T, I, F: $X \to [\Psi, 1]$, where $\Psi < 0$ is called underlimit, T(x), I(x), F(x)$\in [\Psi, 1]$. A Single-Valued Neutrosophic Underset A_2 is defined as $A_2 = \{(x, T(x),\ I(x),\ F(x)), x \in X\}$, such that there exists at least one element in A_2 that has at least one neutrosophic component that is smaller than 0, and no element has neutrosophic components that are bigger than 1 (see [31]).

Definition 5. *Let X be a universe of discourse and the neutrosophic set $A_3 \subset X$. Let T(x), I(x), F(x) be the functions that describe the degree of membership, indeterminate-membership, and non-membership respectively, of a generic element $x \in X$, with respect to the neutrosophic set A_3:*

T, I, F: $X \to [\Psi, \Omega]$, where $\Psi < 0 < 1 < \Omega$, Ψ is called underlimit, while Ω is called overlimit, T(x), I(x), F(x)$\in [\Psi, \Omega]$. A Single-Valued Neutrosophic Offset A_3 is defined as $A_3 = \{(x, T(x),\ I(x),\ F(x)), x \in X\}$, such that there exists at least one element in A_3 that has at least one neutrosophic component that is bigger than 1, and at least another neutrosophic component that is smaller than 0 (see [31]).

Let us note that the oversets, undersets, and offsets cover the three possible cases to characterize. Now, the logical operations over these kinds of sets have to be redefined, in view that the classical ones cannot always be straightforwardly extended to these domains. This is the case of complement given by Smarandache in [31], whereas the union and intersection definitions do not change with respect to those of single-valued neutrosophic sets. This is summarized below:

Let X be a universe of discourse, $A = \{(x, \langle T_A(x), I_A(x), F_A(x)\rangle), x \in X\}$ and $B = \{(x, \langle T_B(x), I_B(x), F_B(x)\rangle), x \in X\}$ be two single-valued neutrosophic oversets/undersets/offsets.

$T_A, I_A, F_A, T_B, I_B, F_B$: $X \to [\Psi, \Omega]$, where $\Psi \le 0 < 1 \le \Omega$, Ψ is the underlimit, whilst Ω is the overlimit, $T_A(x), I_A(x), F_A(x), T_B(x), I_B(x), F_B(x) \in [\Psi, \Omega]$. Let us remark that the three cases are here comprised, viz., overset when $\Psi = 0$ and $\Omega > 1$, underset when $\Psi < 0$ and $\Omega = 1$, and offset when $\Psi < 0$ and $\Omega > 1$.

Then, the main operators are defined as follows:

$A \cup B = \{(x, \langle \max(T_A(x), T_B(x)),\ \min(I_A(x), I_B(x)),\ \min(F_A(x), F_B(x))\rangle), x \in X\}$ is the union.

$A \cap B = \{(x, \langle \min(T_A(x), T_B(x)),\ \max(I_A(x), I_B(x)),\ \max(F_A(x), F_B(x))\rangle), x \in X\}$ is the intersection,

$C(A) = \{(x, \langle F_A(x),\ \Psi + \Omega - I_A(x), T_A(x)\rangle), x \in X\}$ is the neutrosophic complement of the neutrosophic set.

Let us remark that when $\Psi = 0$ and $\Omega = 1$, the precedent operators convert in the classical ones. With regard to logical operators, e.g., n-norms and n-conorms, their redefinitions in the offsets framework are not so evident. Below, definitions of *offnegation*, *neutrosophic component n-offnorm*, and *neutrosophic component n-offconorm* are provided.

One *offnegation* can be defined as in Equation (1).

$$\overset{\neg}{O}\langle T,\ I,\ F\rangle = \langle\ F,\ \Psi_I + \Omega_I - I,\ T\rangle \tag{1}$$

Definition 6. *Let c be a neutrosophic component (T_O, I_O or F_O). c: $M_O \to [\Psi, \Omega]$, where $\Psi \le 0$ and $\Omega \ge 1$. The neutrosophic component n-offnorm $N_O^n : [\Psi, \Omega]^2 \to [\Psi, \Omega]$ satisfies the following conditions for any elements x, y, and z $\in M_O$:*

i. $N_O^n(c(x),\ \Psi) = \Psi$, $N_O^n(c(x),\ \Omega) = c(x)$ *(Overbounding Conditions),*
ii. $N_O^n(c(x),\ c(y)) = N_O^n(c(y),\ c(x))$ *(Commutativity),*
iii. *If* $c(x) \le c(y)$ *then* $N_O^n(c(x),\ c(z)) \le N_O^n(c(y),\ c(z))$ *(Monotonicity),*
iv. $N_O^n\big(N_O^n(c(x),\ c(y)), c(z)\big) = N_O^n\big(c(x),\ N_O^n(c(y),\ c(z))\big)$ *(Associativity).*

To simplify the notation, sometimes we use $\langle T_1,\ I_1, F_1\rangle \underset{O}{\wedge} \langle T_2,\ I_2, F_2\rangle = \langle T_1 \underset{O}{\wedge} T_2,\ I_1 \underset{O}{\vee} I_2, F_1 \underset{O}{\vee} F_2\rangle$ instead of $N_O^n(\cdot,\cdot)$.

Let us remark that the definition of the neutrosophic component n-offnorm is valid for every one of the components, thus, we have to apply it three times. Also, Definition 6 contains the definition of n-norm when $\Psi = 0$ and $\Omega = 1$.

Proposition 1. *Let $N_O^n(\cdot,\cdot)$ be a neutrosophic component n-offnorm, then, for any elements x, y $\in M_O$ we have $N_O^n(c(x),\ c(y)) \le min(c(x),\ c(y))$.*

Proof. Because of the monotonicity of the neutrosophic component n-offnorm and one of the overbounding conditions, we have $N_O^n(c(x),\ c(y)) \le N_O^n(c(x),\ \Omega) = c(x)$, hence $N_O^n(c(x),\ c(y)) \le c(x)$ and similarly $N_O^n(c(x),\ c(y)) \le c(y)$ can be proved, therefore, $N_O^n(c(x),\ c(y)) \le \min(c(x),\ c(y))$. □

See that Proposition 1 maintains this property of the n-norms. Likewise to the definition of the neutrosophic component n-offnorm, in Definition 7 it is described the *neutrosophic component n-offconorm.*

Definition 7. *Let c be a neutrosophic component (T_O, I_O or F_O). c: $M_O \rightarrow [\Psi, \Omega]$, where $\Psi \leq 0$ and $\Omega \geq 1$. The neutrosophic component n-offconorm $N_O^{co}: [\Psi, \Omega]^2 \rightarrow [\Psi, \Omega]$ satisfies the following conditions for any elements x, y, and z $\in M_O$:*

i. $N_O^{co}(c(x), \Omega) = \Omega, N_O^{co}(c(x), \Psi) = c(x)$ *(Overbounding Conditions),*
ii. $N_O^{co}(c(x), c(y)) = N_O^{co}(c(y), c(x))$ *(Commutativity),*
iii. *If* $c(x) \leq c(y)$ *then* $N_O^{co}(c(x), c(z)) \leq N_O^{co}(c(y), c(z))$ *(Monotonicity),*
iv. $N_O^{co}\left(N_O^{co}(c(x), c(y)), c(z)\right) = N_O^{co}\left(c(x), N_O^{co}(c(y), c(z))\right)$ *(Associativity).*

To simplify the notation sometimes we use $\langle T_1, I_1, F_1\rangle \underset{O}{\vee} \langle T_2, I_2, F_2\rangle = \langle T_1 \underset{O}{\vee} T_2, I_1 \underset{O}{\wedge} I_2, F_1 \underset{O}{\wedge} F_2\rangle$ instead of $N_O^{co}(\cdot,\cdot)$.

Proposition 2. *Let $N_O^{co}(\cdot,\cdot)$ be a neutrosophic component n-offconorm, then, for any elements x, y$\in M_O$ we have $N_O^{co}(c(x), c(y)) \geq max(c(x), c(y))$.*

Proof. The proof is equivalent to the proof of Proposition 1. □

In this paper, we use the notion of lattice, based on the poset denoted by $\leq_O$, where $\langle T_1, I_1, F_1\rangle \leq_O \langle T_2, I_2, F_2\rangle$ if and only if $T_2 \geq T_1$, $I_2 \leq I_1$ and $F_2 \leq F_1$, where the infimum and the supremum of the set are $\langle \Psi, \Omega, \Omega\rangle$ and $\langle \Omega, \Psi, \Psi\rangle$, respectively.

One property that is preserved of n-norms is that the minimum is the biggest neutrosophic component n-offnorm for T_O, as it is demonstrated in Proposition 1. Proposition 2 proved that the maximum is the smallest neutrosophic component n-offconorm for I_O and F_O when we consider $\leq_O$.

Evidently, the minimum is a neutrosophic component n-offnorm and the maximum is a neutrosophic component n-offconorm; see Example 1.

Example 1. *An example of a pair offAND/offOR is, $c(x) \underset{ZO}{\wedge} c(y) = min(c(x), c(y))$ and $c(x) \underset{ZO}{\vee} c(y) = max(c(x), c(y))$, respectively.*

Example 2. *A pair of offAND/offOR is, $c(x) \underset{LO}{\wedge} c(y) = max(\Psi, c(x) + c(y) - \Omega)$ and $c(x) \underset{LO}{\vee} c(y) = min(\Omega, c(x) + c(y))$, respectively.*

Example 2 extends the Łukasiewicz t-norm and t-conorm to the neutrosophic offsets. Let us remark that the simple product t-norm and its dual t-conorm cannot be extended to this new domain.

Finally, we recall the definition of neutrosophic uninorms that appeared in [25], see Definition 8.

Definition 8. *A neutrosophic uninorm $\boldsymbol{U}_N$ is a commutative, increasing, and associative mapping, $U_N: (]^-0, 1^+[\times]^-0, 1^+[\times]^-0, 1^+[)^2 \rightarrow]^-0, 1^+[\times]^-0, 1^+[\times]^-0, 1^+[$, such that $U_N\left(x\langle T_x, I_x, F_x\rangle, y\langle T_y, I_y, F_y\rangle\right) = \langle U_N T(x,y), U_N I(x,y), U_N F(x,y)\rangle$, where $\boldsymbol{U}_N T$ means the degree of membership, $\boldsymbol{U}_N I$ the degree of indeterminacy, and $\boldsymbol{U}_N F$ the degree of non-membership of both x and y. Additionally, there exists a neutral element $e \in]^-0, 1^+[\times]^-0, 1^+[\times]^-0, 1^+[$, where $\forall x \in]^-0, 1^+[\times]^-0, 1^+[\times]^-0, 1^+[$, $U_N(e,x) = x$.*

Let us observe that this definition can be restricted to single-valued neutrosophic sets. Neutrosophic uninorms generalize n-norms, n-conorms, uninorms in *L**-fuzzy set theory, and fuzzy uninorms.

3. On Neutrosophic Offuninorms

This section contains the core of the present paper. It is devoted to exposing the definitions and properties of the neutrosophic offuninorms.

Definition 9. *Let c be a neutrosophic component (T_O, I_O or F_O). c: $M_O \to [\Psi, \Omega]$, where $\Psi \leq 0$ and $\Omega \geq 1$. The neutrosophic component n-offuninorm $N^u_O : [\Psi, \Omega]^2 \to [\Psi, \Omega]$ satisfies the following conditions for any elements x, y, and z $\in M_O$:*

i. *There exists $c(e) \in M_O$, such that $N^u_O(c(x), c(e)) = c(x)$ (Identity),*
ii. *$N^u_O(c(x), c(y)) = N^u_O(c(y), c(x))$ (Commutativity),*
iii. *If $c(x) \leq c(y)$ then $N^u_O(c(x), c(z)) \leq N^u_O(c(y), c(z))$ (Monotonicity),*
iv. *$N^u_O\left(N^u_O(c(x), c(y)), c(z)\right) = N^u_O\left(c(x), N^u_O(c(y), c(z))\right)$ (Associativity).*

The definition of a neutrosophic uninorm is an especial case of neutrosophic offuninorm when $\Psi = 0$ and $\Omega = 1$ (see Definition 8) and, additionally, we are dealing with single-valued neutrosophic sets.

It is easy to prove that the neutral element *e* is unique.

Let *c* be a neutrosophic component (T_O, I_O or F_O). *c*: $M_O \to [\Psi, \Omega]$, where $\Psi \leq 0$ and $\Omega \geq 1$. Let us define four useful functions, $\varphi_1 : [\Psi, c(e)] \to [\Psi, \Omega]$, $\varphi_1^{-1} : [\Psi, \Omega] \to [\Psi, c(e)]$, $\varphi_2 : [c(e), \Omega] \to [\Psi, \Omega]$, and $\varphi_2^{-1} : [\Psi, \Omega] \to [c(e), \Omega]$, defined in Equations (2)–(5), respectively.

$$\varphi_1(c(x)) = \left(\frac{\Omega - \Psi}{c(e) - \Psi}\right)(c(x) - \Psi) + \Psi \tag{2}$$

$$\varphi_1^{-1}(c(x)) = \left(\frac{c(e) - \Psi}{\Omega - \Psi}\right)(c(x) - \Psi) + \Psi \tag{3}$$

$$\varphi_2(c(x)) = \left(\frac{\Omega - \Psi}{\Omega - c(e)}\right)(c(x) - c(e)) + \Psi \tag{4}$$

$$\varphi_2^{-1}(c(x)) = \left(\frac{\Omega - c(e)}{\Omega - \Psi}\right)(c(x) - \Psi) + c(e) \tag{5}$$

where, the superscript −1 means it is an inverse mapping. If the condition $c(e) \in (\Psi, \Omega)$ is fulfilled, then the degenerate cases $\Omega = \Psi$, $c(e) = \Psi$ and $c(e) = \Omega$ are excluded. Therefore, $\varphi_1(c(x))$ and $\varphi_2(c(x))$ are well-defined non-constant linear functions. Thus, they are bijective and have inverse mappings defined in Equations (3) and (5), respectively, in the sense that for $c(x) \in [\Psi, \Omega]$, then $\varphi_1\left(\varphi_1^{-1}(c(x))\right) = c(x)$ and $\varphi_2\left(\varphi_2^{-1}(c(x))\right) = c(x)$. Whereas, for $c(x) \in [\Psi, c(e)]$, we have $\varphi_1^{-1}(\varphi_1(c(x))) = c(x)$ and for $c(x) \in [c(e), \Omega]$, $\varphi_2^{-1}(\varphi_2(c(x))) = c(x)$. These properties can be easily verified. Also, it is trivial that they are non-decreasing mappings.

Additionally, let $U_C, U_D : [\Psi, \Omega]^2 \to [\Psi, \Omega]$ be two operators defined by Equations (6) and (7), respectively,

$$U_C(c(x), c(y)) = \begin{cases} \varphi_1^{-1}\left(\varphi_1(c(x)) \underset{O}{\wedge} \varphi_1(c(y))\right), & \text{if } c(x), c(y) \in [\Psi, c(e)] \\ \varphi_2^{-1}\left(\varphi_2(c(x)) \underset{O}{\vee} \varphi_2(c(y))\right), & \text{if } c(x), c(y) \in [c(e), \Omega] \\ \min(c(x), c(y)), & \text{otherwise} \end{cases} \tag{6}$$

$$\mathrm{U_D}(c(x),c(y))=\begin{cases}\varphi_1^{-1}\left(\varphi_1(c(x))\ \hat{O}\ \varphi_1(c(y))\right), \text{ if } c(x),c(y)\in[\Psi,\ c(e)]\\ \varphi_2^{-1}\left(\varphi_2(c(x))\ \overset{\vee}{O}\ \varphi_2(c(y))\right), \text{ if } c(x),c(y)\in[c(e),\ \Omega]\\ \max(c(x),c(y)), \text{ otherwise}\end{cases} \tag{7}$$

where, $\hat{O}$ denotes a neutrosophic component n-offnorm and $\overset{\vee}{O}$ denotes a neutrosophic component n-offconorm.

Lemma 1. *Let c be a neutrosophic component (T_O, I_O or F_O). c: $M_O\to[\Psi, \Omega]$, where $\Psi\leq 0$ and $\Omega\geq 1$. Given $\hat{O}$ a neutrosophic component n-offnorm and $\overset{\vee}{O}$ a neutrosophic component n-offconorm, let us consider $U_C(c(x),c(y))$ and $U_D(c(x),c(y))$ the operators defined in Equations (6) and (7) for $c(e)\in(\Psi,\ \Omega)$. They are commutative, non-decreasing, and c(e) is the neutral element.*

Proof.

i. Commutativity is evidently satisfied due to the commutativity of $\hat{O}$, $\overset{\vee}{O}$, *min*, and *max*.

ii. $\varphi_1(\cdot)$, $\varphi_1^{-1}(\cdot)$, $\varphi_2(\cdot)$, $\varphi_2^{-1}(\cdot)$, $\hat{O}$, $\overset{\vee}{O}$, *min* and *max* are non-decreasing mappings, thus both $\mathrm{U_C}(\cdot,\ \cdot)$ and $\mathrm{U_D}(\cdot,\ \cdot)$ satisfy monotonicity.

iii. To prove *c(e)* is the neutral element, we have two cases, which are the following:

- If $c(x)\in[\Psi,\ c(e)]$, then, $\mathrm{U_C}(c(e),c(x))=\mathrm{U_D}(c(e),c(x))=\varphi_1^{-1}\left(\varphi_1(c(e))\ \hat{O}\ \varphi_1(c(x))\right)=\varphi_1^{-1}\left(\Omega\ \hat{O}\ \varphi_1(c(x))\right)=\varphi_1^{-1}(\varphi_1(c(x)))=\ c(x)$.
- If $c(x)\in[c(e),\ \Omega]$, then $\mathrm{U_C}(c(e),c(x))=\mathrm{U_D}(c(e),c(x))=\varphi_2^{-1}\left(\varphi_2(c(e))\ \overset{\vee}{O}\ \varphi_2(c(x))\right)\varphi_2^{-1}\left(\Psi\ \overset{\vee}{O}\ \varphi_2(c(x))\right)=\varphi_2^{-1}(\varphi_2(c(x)))=\ c(x)$.

Therefore, identity is satisfied. □

Lemma 2. *Let c be a neutrosophic component (T_O, I_O, or F_O). c: $M_O\to[\Psi, \Omega]$, where $\Psi\leq 0$ and $\Omega\geq 1$. Given $\hat{O}$ a neutrosophic component n-offnorm and $\overset{\vee}{O}$ a neutrosophic component n-offconorm, let us consider $U_C(c(x),c(y))$ and $U_D(c(x),c(y))$ the operators defined in Equations (6) and (7) for $c(e)\in(\Psi,\ \Omega)$. They are associative.*

Proof. Four cases are possible:

i. Let $c(x),c(y),\ c(z)\in[\Psi,\ c(e)]$, then $\mathrm{U_C}(\mathrm{U_C}(c(x),c(y)),c(z))=\varphi_1^{-1}\left(\varphi_1\left(\varphi_1^{-1}\left(\varphi_1(c(x))\ \hat{O}\ \varphi_1(c(y))\right)\right)\ \hat{O}\ \varphi_1(c(z))\right)=\varphi_1^{-1}\left(\left[\varphi_1(c(x))\ \hat{O}\ \varphi_1(c(y))\right]\ \hat{O}\ \varphi_1(c(z))\right)=\varphi_1^{-1}\left(\varphi_1(c(x))\ \hat{O}\ \left[\varphi_1(c(y))\ \hat{O}\ \varphi_1(c(z))\right]\right)=\varphi_1^{-1}\left(\varphi_1(c(x))\ \hat{O}\ \left[\varphi_1\left(\varphi_1^{-1}\left[\varphi_1(c(y))\ \hat{O}\ \varphi_1(c(z))\right]\right)\right]\right)=\varphi_1^{-1}\left(\varphi_1(c(x))\ \hat{O}\ \varphi_1(\mathrm{U_C}(c(y),c(z)))\right)=\mathrm{U_C}(c(x),\mathrm{U_C}(c(y),c(z)))$.

ii. Let $c(x),c(y),\ c(z)\in[c(e),\ \Omega]$, $\mathrm{U_C}(\mathrm{U_C}(c(x),c(y)),c(z))=\varphi_2^{-1}\left(\varphi_2\left(\varphi_2^{-1}\left(\varphi_2(c(x))\ \overset{\vee}{O}\ \varphi_2(c(y))\right)\right)\ \overset{\vee}{O}\ \varphi_2(c(z))\right)=\varphi_2^{-1}\left(\left[\varphi_2(c(x))\ \overset{\vee}{O}\ \varphi_2(c(y))\right]\ \overset{\vee}{O}\ \varphi_2(c(z))\right)=$

$\varphi_2^{-1}\left(\varphi_2(c(x)) \underset{O}{\vee} \left[\varphi_2(c(y)) \underset{O}{\vee} \varphi_2(c(z))\right]\right) = \varphi_2^{-1}\left(\varphi_2(c(x)) \underset{O}{\vee} \left[\varphi_2\left(\varphi_2^{-1}\left[\varphi_2(c(y)) \underset{O}{\vee} \varphi_2(c(z))\right]\right]\right)\right) =$ $\varphi_2^{-1}\left(\varphi_2(c(x)) \underset{O}{\vee} \varphi_2(U_C(c(y), c(z)))\right) = U_C(c(x), U_C(c(y), c(z)))$. These proofs are also valid for U_D.

iii. Let $c(x), c(y) \in [\Psi,\ c(e)]$ and $c(z) \in [c(e),\ \Omega]$, $U_C(U_C(c(x), c(y)), c(z)) = \min(U_C(c(x), c(y)), c(z)) = U_C(c(x), c(y))$. Also, we have $U_C(c(x), U_C(c(y), c(z))) = U_C(c(x),\ \min(c(y), c(z))) = U_C(c(x), c(y))$, then, it is associative.

iv. Let $c(x), c(y) \in [c(e),\ \Omega]$ and $c(z) \in [\Psi,\ c(e)]$, then $U_C(U_C(c(x), c(y)), c(z)) = \min(U_C(c(x), c(y)), c(z)) = c(z)$. In addition, $U_C(c(x), (U_C(c(y), c(z)))) = U_C(c(x),\ \min(c(y), c(z))) = U_C(c(x), c(z)) = \min(c(x), c(z)) = c(z)$.

Thus, U_C satisfies the associativity.

Similarly, associativity of U_D can be proved.

Let us remark that we applied the properties, $c(x) \underset{O}{\wedge} c(y) \leq \min(c(x),\ c(y))$ and $c(x) \underset{O}{\vee} c(y) \geq \max(c(x),\ c(y))$, as well as $U_C(c(x), c(y)) \leq c(e)$ if $c(x),\ c(y) \in [\Psi,\ c(e)]$ and $U_C(c(x), c(y)) \geq c(e)$ if $c(x),\ c(y) \in [c(e),\ \Omega]$. □

Proposition 3. *Let c be a neutrosophic component (T_O, I_O, or F_O). c: $M_O \rightarrow [\Psi, \Omega]$, where $\Psi \leq 0$ and $\Omega \geq 1$. Given $\underset{O}{\wedge}$ a neutrosophic component n-offnorm and $\underset{O}{\vee}$ a neutrosophic component n-offconorm, let us consider $U_C(c(x), c(y))$ and $U_D(c(x), c(y))$ the operators defined in Equations 6 and 7 for $c(e) \in (\Psi,\ \Omega)$. Then, $U_C(c(x), c(y))$ and $U_D(c(x), c(y))$ are neutrosophic component n-offuninorms and they satisfy the conditions $U_C(\Psi,\ \Omega) = \Psi$ and $U_D(\Psi,\ \Omega) = \Omega$,* i.e., U_C is a conjunctive neutrosophic component n-offuninorm, and U_D is a disjunctive neutrosophic component n-offuninorm.

Proof. Since Lemma 1, they are commutative, non-decreasing operators, and *c(e)* is the neutral element. Since Lemma 2, they are associative operators. Moreover, it is easy to verify that $U_C(\Psi,\ \Omega) = \Psi$ and $U_D(\Psi,\ \Omega) = \Omega$. □

Example 3. *Two neutrosophic component n-offuninorms can be defined as:*

$$U_{ZC}(c(x), c(y)) = \begin{cases} \varphi_1^{-1}\left(\varphi_1(c(x)) \underset{ZO}{\wedge} \varphi_1(c(y))\right), \text{ if } c(x), c(y) \in [\Psi,\ c(e)] \\ \varphi_2^{-1}\left(\varphi_2(c(x)) \underset{ZO}{\vee} \varphi_2(c(y))\right), \text{ if } c(x), c(y) \in [c(e),\ \Omega] \\ \min(c(x), c(y)), \text{ otherwise} \end{cases}$$

$$U_{ZD}(c(x), c(y)) = \begin{cases} \varphi_1^{-1}\left(\varphi_1(c(x)) \underset{ZO}{\wedge} \varphi_1(c(y))\right), \text{ if } c(x), c(y) \in [\Psi,\ c(e)] \\ \varphi_2^{-1}\left(\varphi_2(c(x)) \underset{ZO}{\vee} \varphi_2(c(y))\right), \text{ if } c(x), c(y) \in [c(e),\ \Omega] \\ \max(c(x), c(y)), \text{ otherwise} \end{cases}$$

where $\underset{ZO}{\wedge}$ and $\underset{ZO}{\vee}$ were defined in the Example 1; $c(e) \in (\Psi, \Omega)$.

Then two examples of n-offuninorms are: $U_1(\langle T_1, I_1, F_1\rangle, \langle T_2, I_2, F_2\rangle) = \langle U_{ZC}(T_1, T_2), U_{ZD}(I_1, I_2), U_{ZD}(F_1, F_2)\rangle$ and $U_2(\langle T_1, I_1, F_1\rangle, \langle T_2, I_2, F_2\rangle) = \langle U_{ZD}(T_1, T_2), U_{ZC}(I_1, I_2), U_{ZC}(F_1, F_2)\rangle$.

They satisfy $U_1(\langle \Psi,\ \Omega,\ \Omega\rangle, \langle \Omega, \Psi, \Psi\rangle) = \langle \Psi,\ \Omega, \Omega\rangle$ and $U_2(\langle \Psi,\ \Omega,\ \Omega\rangle, \langle \Omega, \Psi, \Psi\rangle) = \langle \Omega,\ \Psi, \Psi\rangle$.

Example 4. *Two neutrosophic component n-offuninorms can be defined as*

$$U_{LC}(c(x),c(y)) = \begin{cases} \varphi_1^{-1}\Big(\varphi_1(c(x)) \underset{LO}{\wedge} \varphi_1(c(y))\Big), \text{ if } c(x),c(y) \in [\Psi,\ c(e)] \\ \varphi_2^{-1}\Big(\varphi_2(c(x)) \underset{LO}{\vee} \varphi_2(c(y))\Big), \text{ if } c(x),c(y) \in [c(e),\ \Omega] \\ \min(c(x),c(y)), \text{ otherwise} \end{cases}$$

$$U_{LD}(c(x),c(y)) = \begin{cases} \varphi_1^{-1}\Big(\varphi_1(c(x)) \underset{LO}{\wedge} \varphi_1(c(y))\Big), \text{ if } c(x),c(y) \in [\Psi,\ c(e)] \\ \varphi_2^{-1}\Big(\varphi_2(c(x)) \underset{LO}{\vee} \varphi_2(c(y))\Big), \text{ if } c(x),c(y) \in [c(e),\ \Omega] \\ \max(c(x),c(y)), \text{ otherwise} \end{cases}$$

where $\underset{LO}{\wedge}$ *and* $\underset{LO}{\vee}$ *were defined in the Example 2; c(e)∈(Ψ, Ω).*

Now, two examples of n-offuninorms are: $U_3(\langle T_1,I_1,F_1\rangle,\langle T_2,I_2,F_2\rangle) = \langle U_{LC}(T_1,T_2),U_{LD}(I_1,I_2),U_{LD}(F_1,F_2)\rangle$ and $U_4(\langle T_1,I_1,F_1\rangle,\langle T_2,I_2,F_2\rangle) = \langle U_{LD}(T_1,T_2),U_{LC}(I_1,I_2),U_{LC}(F_1,F_2)\rangle$.

They satisfy, $U_3(\langle \Psi,\ \Omega,\ \Omega\rangle,\langle \Omega,\Psi,\Psi\rangle) = \langle \Psi,\ \Omega,\Omega\rangle$ and $U_4(\langle \Psi,\Omega,\Omega\rangle,\langle \Omega,\Psi,\Psi\rangle) = \langle \Omega,\Psi,\Psi\rangle$.

Remark 1. *The neutrosophic components n-offuninorms defined by Equations (6) and (7) are idempotent, i.e.,* $N_O^n(c(x),c(x)) = c(x)$, *if and only if they are defined from idempotent neutrosophic component n-offnorms and n-offconorms. Moreover, they are Archimedean, i.e., they satisfy both,* $N_O^u(c(x),c(x)) <_O c(x)$ *when* $\Psi < c(x) < c(e)$ *and* $c(x) <_O N_O^u(c(x),c(x))$ *when* $c(e) < c(x) < \Omega$, *if and only if the neutrosophic component n-offnorm and n-offconorm are Archimedean. Let us observe that* $<_O$ *is the order* $<$ *defined in the real line when c(x) is* $T_O(x)$ *and it is* $>$ *when c(x) is* $I_O(x)$ *or* $F_O(x)$.

Proposition 4. *Let c be a neutrosophic component (*T_O, I_O *or* F_O*). c:* $M_O \to [\Psi, \Omega]$, *where* $\Psi < 0$ *and* $\Omega > 1$, *and let a neutrosophic component n-offuninorm* $N_O^u : [\Psi,\ \Omega]^2 \to [\Psi,\ \Omega]$. *Then, for every x, y ∈* M_O, *a neutrosophic component n-offnorm and a neutrosophic component n-offconorm are defined by Equations (8) and (9).*

$$c(x) \underset{UO}{\wedge} c(y) = \varphi_1\Big(N_O^u\big(\varphi_1^{-1}(c\ (x)), \varphi_1^{-1}(c(y))\big)\Big) \tag{8}$$

$$c(x) \underset{UO}{\vee} c(y) = \varphi_2\Big(N_O^u\big(\varphi_2^{-1}(c(x)), \varphi_2^{-1}(c(y))\big)\Big) \tag{9}$$

Proof. Evidently, both operators are commutative, since N_O^u is. Also, it is non-decreasing since N_O^u and the functions in Equations (2)–(5) are. They are associative because of the associativity of N_O^u.

It is easy to verify that the overbounding conditions $\Omega \underset{UO}{\wedge} c(y) = c(y)$ and $\Psi \underset{UO}{\vee} c(y) = c(y)$ are also satisfied.

Additionally, we have $\Psi \underset{UO}{\wedge} c(y) = \varphi_1\Big(N_O^u\big(\varphi_1^{-1}(\Psi), \varphi_1^{-1}(c(y))\big)\Big) = \varphi_1\Big(N_O^u\big(\Psi, \varphi_1^{-1}(c(y))\big)\Big) \le \varphi_1\Big(N_O^u(\Psi, c(e))\Big) = \varphi_1(\Psi) = \Psi$, then, $\Psi \underset{UO}{\wedge} c(y) = \Psi$; also, $\Omega \underset{UO}{\vee} c(y) = \varphi_2\Big(N_O^u\big(\varphi_2^{-1}(\Omega), \varphi_2^{-1}(c(y))\big)\Big) = \varphi_2\Big(N_O^u\big(\Omega, \varphi_2^{-1}(c(y))\big)\Big) \ge \varphi_2\Big(N_O^u\big(\Omega, \varphi_2^{-1}(\Psi)\big)\Big) = \varphi_2\Big(N_O^u(\Omega, c(e))\Big) = \varphi_2(\Omega) = \Omega$, then, $\Omega \underset{UO}{\vee} c(y) = \Omega$. □

Proposition 5. *Let (T_O, I_O, or F_O), c_O: $M_O \to [\Psi, \Omega]$ and (T, I, or F), c_N: $MN \to [0,1]$ be a neutrosophic component n-offset and a neutrosophic component, respectively. There exists a bijective mapping such that every neutrosophic component n-offuninormis transformed into a neutrosophic component uninorm and vice versa.*

Proof. Let us define the function $\varphi_3 : [\Psi, \Omega] \to [0, 1]$ and its inverse $\varphi_3^{-1} : [0, 1] \to [\Psi, \Omega]$, expressed in Equations (10) and (11), respectively.

$$\varphi_3(c(x)) = \frac{c(x) - \Psi}{\Omega - \Psi} \tag{10}$$

$$\varphi_3^{-1}(c(x)) = (\Omega - \Psi)c(x) + \Psi \tag{11}$$

Evidently, they are increasing bijective mappings.

If $\hat{\mathrm{U}}_{\mathrm{N}}(\cdot,\cdot)$ is a neutrosophic uninorm, then we can define the neutrosophic component n-offuninorm $\hat{\mathrm{N}}^{\mathrm{u}}_{\mathrm{O}}(\cdot,\cdot)$ as follows:

$$\hat{\mathrm{N}}^{\mathrm{u}}_{\mathrm{O}}(c_O(x), c_O(y)) = \varphi_3^{-1}\Big(\hat{\mathrm{U}}_{\mathrm{N}}(\varphi_3(c_O(x)), \varphi_3(c_O(y)))\Big)$$

Conversely, if we have $\hat{\mathrm{N}}^{\mathrm{u}}_{\mathrm{O}}(\cdot,\cdot)$, we can define $\hat{\mathrm{U}}_{\mathrm{N}}(\cdot,\cdot)$ as follows:

$$\hat{\mathrm{U}}_{\mathrm{N}}(c_N(x), c_N(y)) = \varphi_3\Big(\hat{\mathrm{N}}^{\mathrm{u}}_{\mathrm{O}}\Big(\varphi_3^{-1}(c_N(x)), \varphi_3^{-1}(c_N(y))\Big)\Big)$$

Then, it is easy to prove that $\hat{\mathrm{N}}^{\mathrm{u}}_{\mathrm{O}}(c_O(x), c_O(y))$ is a neutrosophic component n-offuninorm and $\hat{\mathrm{U}}_{\mathrm{N}}(c_N(x), c_N(y))$ is a neutrosophic component uninorm. Moreover, the relationship between the components of their neutral elements $c_O(e_O)$ and $c_N(e_N)$ is $c_N(e_N) = \varphi_3(c_O(e_O))$ and thus $c_O(e_O) = \varphi_3^{-1}(c_N(e_N))$. □

Let us remark that we maintain the definition of inverse mapping that we explained in Equations (3) and (5).

In agreement with Proposition 5, many predefined neutrosophic uninorms can be used to define n-offuninorms. In turn, fuzzy uninorms can be used to define neutrosophic uninorms, thus, it is simply necessary to find examples in the field of fuzzy uninorms; see further Section 4.1. First, let us make reference to some properties of n-offuninorms.

Proposition 6. *Let c be a neutrosophic component (T_O, I_O or F_O). c: $M_O \to [\Psi, \Omega]$, where $\Psi \leq 0$ and $\Omega \geq 1$. Given the neutrosophic component n-offuninorm $N^u_O : [\Psi, \Omega]^2 \to [\Psi, \Omega]$ and the offuninorm $U_O : [\Psi, \Omega]^3 \times [\Psi, \Omega]^3 \to [\Psi, \Omega]^3$ defined from $N^u_O(\cdot, \cdot)$, $U_O(\langle T_O(x), I_O(x), F_O(x)\rangle, \langle T_O(y), I_O(y), F_O(y)\rangle) = \langle N^u_O(T_O(x), T_O(y)), N^u_O(I_O(x), I_O(y)), N^u_O(F_O(x), F_O(y))\rangle$, satisfies the following properties for any $x = \langle T_O(x), I_O(x), F_O(x)\rangle$, denoting $\Psi_O = \langle \Psi, \Omega, \Omega\rangle$ and $\Omega_O = \langle \Omega, \Psi, \Psi\rangle$:*

1. *$U_O(\Psi_O, \Psi_O) = \Psi_O$ and $U_O(\Omega_O, \Omega_O) = \Omega_O$.*
2. *If $c(e) \neq \Psi, \Omega$, then, $U_O(\Psi_O, \Omega_O) = U_O(U_O(\Psi_O, \Omega_O), x)$*
3. *If $c(e) \neq \Psi, \Omega$, then either $U_O(\Psi_O, \Omega_O) = \Psi_O$ or $U_O(\Psi_O, \Omega_O) = \Omega_O$ or $U_O(\Psi_O, \Omega_O)$ is $\leq_O$-incomparable respect to $e = \langle T_O(e), I_O(e), F_O(e)\rangle$.*
4. *If there exists $y = \langle T_O(y), I_O(y), F_O(y)\rangle$, such that either $x \leq_O e \leq_O y$ or $y \leq_O e \leq_O x$, then, $min(x, y) \leq_O U_O(x, y) \leq_O max(x, y)$.*

Proof.

1. Since $\mathrm{N}^{\mathrm{u}}_{\mathrm{O}}(\Psi, c(e)) = \Psi$ and $\mathrm{N}^{\mathrm{u}}_{\mathrm{O}}(\Omega, c(e)) = \Omega$ and considering that $\mathrm{N}^{\mathrm{u}}_{\mathrm{O}}(\Psi, \cdot)$ and $\mathrm{N}^{\mathrm{u}}_{\mathrm{O}}(\Omega, \cdot)$ are non-decreasing, the result is trivial. Then, $\mathrm{U}_{\mathrm{O}}(\Psi_{\mathrm{O}}, \Psi_{\mathrm{O}}) = \Psi_{\mathrm{O}}$ and $\mathrm{U}_{\mathrm{O}}(\Omega_{\mathrm{O}}, \Omega_{\mathrm{O}}) = \Omega_{\mathrm{O}}$.
2. First suppose $c(x) \leq c(e)$, then $\mathrm{N}^{\mathrm{u}}_{\mathrm{O}}(\Psi, c(x)) \leq \mathrm{N}^{\mathrm{u}}_{\mathrm{O}}(\Psi, c(e)) = \Psi$, therefore $\mathrm{N}^{\mathrm{u}}_{\mathrm{O}}(\Psi, c(x)) = \Psi$, thus $\mathrm{N}^{\mathrm{u}}_{\mathrm{O}}(\Psi, \Omega) = \mathrm{N}^{\mathrm{u}}_{\mathrm{O}}\big(\mathrm{N}^{\mathrm{u}}_{\mathrm{O}}(\Psi, c(x)), \Omega\big) = \mathrm{N}^{\mathrm{u}}_{\mathrm{O}}\big(\Omega, \mathrm{N}^{\mathrm{u}}_{\mathrm{O}}(\Psi, c(x))\big) = \mathrm{N}^{\mathrm{u}}_{\mathrm{O}}\big(\mathrm{N}^{\mathrm{u}}_{\mathrm{O}}(\Omega, \Psi), c(x)\big)$

$= \mathrm{N}^{\mathrm{u}}_{\mathrm{O}}\left(\mathrm{N}^{\mathrm{u}}_{\mathrm{O}}(\Psi,\ \Omega), c(x)\right)$. See that we applied the commutativity and associativity of $\mathrm{N}^{\mathrm{u}}_{\mathrm{O}}(\cdot,\ \cdot)$. Now, suppose $c(e) \leq c(x)$, then $\mathrm{N}^{\mathrm{u}}_{\mathrm{O}}(c(x), \Omega) \geq \mathrm{N}^{\mathrm{u}}_{\mathrm{O}}(c(x), \Omega) = \Omega$, therefore, $\mathrm{N}^{\mathrm{u}}_{\mathrm{O}}(c(x),\ \Omega) = \Omega$, and $\mathrm{N}^{\mathrm{u}}_{\mathrm{O}}(\Psi,\ \Omega) = \mathrm{N}^{\mathrm{u}}_{\mathrm{O}}\left(\Psi,\ \mathrm{N}^{\mathrm{u}}_{\mathrm{O}}(c(x), \Omega)\right) = \mathrm{N}^{\mathrm{u}}_{\mathrm{O}}\left(\mathrm{N}^{\mathrm{u}}_{\mathrm{O}}(\Omega, \Psi), c(x)\right)$. Suppose x and $e = T_O(e), I_O(e), F_O(e)$ are $\leq_O$-incomparable, i.e., $x \nleq_O e$ and $e \nleq_O x$. Then,

$$x \underset{O}{\wedge} e = \min(T_O(x),\ T_O(e)), \max(I_O(x), I_O(e)), \max(F_O(x), F_O(e)) \leq_O x \leq_O \max(T_O(x),\ T_O(e)), \min(I_O(x), I_O(e)), \min(F_O(x), F_O(e)) = x \underset{O}{\vee} e$$

Then, according to the previous results we have $\mathrm{U_O}(\Psi_\mathrm{O},\ \Omega_\mathrm{O}) = \mathrm{U_O}\left(\mathrm{U_O}(\Psi_\mathrm{O},\ \Omega_\mathrm{O}), x \underset{O}{\wedge} e\right) = \mathrm{U_O}\left(\mathrm{U_O}(\Psi_\mathrm{O},\ \Omega_\mathrm{O}), x \underset{O}{\vee} e\right)$, thus, for the increasing condition of $\mathrm{U_O}(\cdot,\ \cdot)$ it is satisfied $\mathrm{U_O}(\Psi_\mathrm{O},\ \Omega_\mathrm{O}) = \mathrm{U_O}(\mathrm{U_O}(\Psi_\mathrm{O},\ \Omega_\mathrm{O}), x)$. Then, we proved $\mathrm{U_O}(\Psi_\mathrm{O},\ \Omega_\mathrm{O}) = \mathrm{U_O}(\mathrm{U_O}(\Psi_\mathrm{O},\ \Omega_\mathrm{O}), x)$.

3. Suppose $\mathrm{U_O}(\Psi_\mathrm{O}, \Omega_\mathrm{O})$ is $\leq_O$-comparable respect to e, then, if $\mathrm{U_O}(\Psi_\mathrm{O}, \Omega_\mathrm{O}) \leq_O e$ since the previous proof $\mathrm{U_O}(\Psi_\mathrm{O},\ \Omega_\mathrm{O}) = \mathrm{U_O}(\mathrm{U_O}(\Psi_\mathrm{O},\ \Omega_\mathrm{O}), \Psi_\mathrm{O}) = \Psi_\mathrm{O}$. If $e \leq_O \mathrm{U_O}(\Psi_\mathrm{O}, \Omega_\mathrm{O})$ then $\mathrm{U_O}(\Psi_\mathrm{O},\ \Omega_\mathrm{O}) = \mathrm{U_O}(\mathrm{U_O}(\Psi_\mathrm{O},\ \Omega_\mathrm{O}), \Omega_\mathrm{O}) = \Omega_\mathrm{O}$.
4. Let us assume without loss of generality that $x \leq_O e \leq_O y$, then, $x = \mathrm{U_O}(x, e) \leq_O \mathrm{U_O}(x, y) \leq_O \mathrm{U_O}(e, y) = y$. □

When c_1: $\mathrm{M_O} \to [\Psi_1,\ \Omega_1]$ and c_2: $\mathrm{M_O} \to [\Psi_2,\ \Omega_2]$ are two neutrosophic components, such that $\Psi_1 \neq \Psi_2$ or $\Omega_1 \neq \Omega_2$, satisfying that at least one of Ψ_1 and Ψ_2 is smaller than 0, or at least one of Ω_1 and Ω_2 is bigger than 1, then, a neutrosophic component n-offuninorm aggregates both of them, according to the interpretation we have to obtain.

For example, if c_1: MO→[−1,1] and c2: MO→[0,1], and the first one means the relationship between two variables like the linear regression coefficient and the second one represents a classical probability, if we need to obtain the aggregation in [−1,1] in the framework of variable relationships, then after transforming $c_2 : \mathrm{M_O} \to [0, 1]$ to $\hat{c}_2 : \mathrm{M_O} \to [-1,\ 1]$, we aggregate c_1 and $\hat{c}_2$ using $\mathrm{N}^{\mathrm{u}}_{\mathrm{O}} :\ [-1,\ 1]^2 \to [-1,\ 1]$, only in the case that it makes sense to rescale c_2, otherwise, because [0,1] ⊂[−1,1], we can apply $\mathrm{N}^{\mathrm{u}}_{\mathrm{O}} :\ [-1,\ 1]^2 \to [-1,\ 1]$ over c_1 and c_2.

However, if we need to obtain a classical probabilistic interpretation, then we aggregate c_2: $\mathrm{M_O} \to [0,1]$ and $\hat{c}_1 : \mathrm{M_O} \to [0,\ 1]$, where $\hat{c}_1$ is a transformation obtained from c_1: $\mathrm{M_O} \to [-1, 1]$.

Example 5. *Let us revisit Example 3 with* $U_1 : [-0.7,\ 1.2]^3 \times [-0.7,\ 1.2]^3 \to [-0.7,\ 1.2]^3$ *and neutral element* $e = \langle -0.5,\ 0, 0\rangle$, *defined as* $U_1(\langle T_1, I_1, F_1\rangle, \langle T_2, I_2, F_2\rangle) = \langle U_{ZC}(T_1, T_2), U_{ZD}(I_1, I_2), U_{ZD}(F_1, F_2)\rangle$. Then, we have:

$$\mathrm{U_{LC}}(T_O(x), T_O(y)) = \begin{cases} \max(T_O(x), T_O(y)), \text{ if } T_O(x), T_O(y) \in [-0.5,\ 1.2] \\ \min(T_O(x), T_O(y)), \text{ otherwise} \end{cases}$$

$$\mathrm{U_{LD}}(I_O(x), I_O(y)) = \begin{cases} \min(I_O(x), I_O(y)), \text{ if } I_O(x), I_O(y) \in [-0.7,\ 0] \\ \max(I_O(x), I_O(y)), \text{ otherwise} \end{cases}$$

$$\mathrm{U_{LD}}(F_O(x), F_O(y)) = \begin{cases} \min(F_O(x), F_O(y)), \text{ if } F_O(x), F_O(y) \in [-0.7,\ 0] \\ \max(F_O(x), F_O(y)), \text{ otherwise} \end{cases}$$

Let us aggregate the elements of $A = \{(x_1, \langle 1.2, 0.4, -0.1\rangle), (x_2, \langle 0.2, 0.3, -0.7\rangle)\}$ by using $\mathrm{U}_1(\cdot, \cdot)$, then, $\mathrm{U}_1((x_1, \langle 1.2, 0.4, -0.1\rangle), (x_2, \langle 0.2, 0.3, -0.7\rangle)) = \langle \mathrm{U_{LC}}(1.2, 0.2), \mathrm{U_{LD}}(0.4, 0.3), \mathrm{U_{LD}}(-0.1, -0.7)\rangle = \langle 1.2,\ 0.4, -0.7\rangle$.

4. Applications

In the following, we illustrate the applicability of the present investigation aided by three areas of application.

4.1. N-Offuninorms and MYCIN

Let us start with the parameterized Silvert uninorms, see [40]:

$$u_{N\lambda}(c_N(x), c_N(y)) = \begin{cases} \frac{\lambda c_N(x)c_N(y)}{\lambda c_N(x)c_N(y)+(1-c_N(x))(1-c_N(y))}, \text{ if } (c_N(x),\ c_N(y)) \in [0,\ 1]^2\backslash\{(0,\ 1), (1,\ 0)\} \\ 0, \text{ otherwise} \end{cases}$$

where $\lambda > 0$ and $c_N(e_\lambda) = \frac{1}{\lambda+1}$. To convert this family to the equivalent one defined into $[-1, 1]$ we have to apply the Equations in Proposition 5. Then, it is obtained $u_{O\lambda}(c_O(x), c_O(y)) = \begin{cases} \frac{(\lambda-1)(1+c_O(x)c_O(y))+(\lambda+1)(c_O(x)+c_O(y))}{(\lambda+1)(1+c_O(x)c_O(y))+(\lambda-1)(c_O(x)+c_O(y))}, \text{ if } (c_O(x),\ c_O(y)) \in [-1,\ 1]^2\backslash\{(-1,\ 1), (1,\ -1)\} \\ 0, \text{ otherwise} \end{cases}$ where $c_O(e_\lambda) = \frac{1-\lambda}{1+\lambda}$.

Let us note that $\lim_{\lambda\to 0^+} c_O(e_\lambda) = 1$ and $\lim_{\lambda\to+\infty} c_O(e_\lambda) = -1$. Therefore, the closer λ approximates to 0, the closer $u_{O\lambda}(\cdot,\cdot)$ performs like a neutrosophic component n-offnorm; whereas, the greater λ, the closer $u_{O\lambda}(\cdot,\cdot)$ performs like a neutrosophic component n-offconorm.

An additional consequence of these assertions is that inequalities $0<\lambda_1<\lambda_2$ imply $u_{O\lambda_1}(c_O(x), c_O(y)) < u_{O\lambda_2}(c_O(x), c_O(y))$.

Applying Equations (2)–(5) to the conditions of the present example, the following transformations are obtained:

$\hat{\varphi}_{1\lambda}(c_O(x)) = (1+\lambda)c_O(x) + \lambda$, $\hat{\varphi}^{-1}_{1\lambda}(c_O(x)) = \frac{c_O(x)-\lambda}{1+\lambda}$, $\hat{\varphi}_{2\lambda}(c_O(x)) = \frac{(1+\lambda)c_O(x)-1}{\lambda}$ and $\hat{\varphi}^{-1}_{2\lambda}(c_O(x)) = \frac{\lambda c_O(x)+1}{1+\lambda}$.

Then, a neutrosophic component n-offnorm and a neutrosophic component n-offconorm are defined from Equations (8) and (9), as follows:

$c(x) \underset{\lambda O}{\wedge} c(y) = \hat{\varphi}_{1\lambda}\left(u_{O\lambda}\left(\hat{\varphi}^{-1}_{1\lambda}(c(x)), \hat{\varphi}^{-1}_{1\lambda}(c(y))\right)\right)$ and $c(x) \underset{\lambda O}{\vee} c(y) = \hat{\varphi}_{2\lambda}\left(u_{O\lambda}\left(\hat{\varphi}^{-1}_{2\lambda}(c(x)), \hat{\varphi}^{-1}_{2\lambda}(c(y))\right)\right)$, respectively.

Other properties of $u_{O\lambda}(\cdot,\cdot)$ are the following:

1. $u_{O\lambda}(c_O(x), -c_O(x)) = \begin{cases} \frac{\lambda-1}{1+\lambda}, \text{ if } c_O(x) \in (-1,\ 1) \\ -1, \text{ otherwise} \end{cases}$
2. $u_{O\lambda}(\cdot,\cdot)$ is Archimedean. To prove it, given $c_O(x) < c_O(e_\lambda)$, then $u_{O\lambda}(c_O(x), c_O(x)) \le u_{O\lambda}(c_O(x), c_O(e_\lambda)) = c_O(x)$ and if $c_O(x) > c_O(e_\lambda)$, $u_{O\lambda}(c_O(x), c_O(x)) \ge u_{O\lambda}(c_O(x), c_O(e_\lambda)) = c_O(x)$.

To prove those inequalities are strict, let us suppose the equation $u_{O\lambda}(c_O(x), c_O(x)) = \frac{(\lambda-1)\left(1+c_O^2(x)\right)+2(\lambda+1)c_O(x)}{(\lambda+1)\left(1+c_O^2(x)\right)+2(\lambda-1)c_O(x)} = c_O(x)$ holds, or equivalently $(\lambda-1)\left(1+c_O^2(x)\right) + 2(\lambda+1)c_O(x) = c_O(x)\left[(\lambda+1)\left(1+c_O^2(x)\right)+2(\lambda-1)c_O(x)\right]$, thus, $(\lambda-1)\left(1-c_O^2(x)\right) + (\lambda+1)c_O(x)\left(1-c_O^2(x)\right) = 0$ and finally, $\left(1-c_O^2(x)\right)(\lambda-1+(\lambda+1)c_O(x)) = 0$, hence the solutions are $c_O(x) = \pm 1$ and $c_O(x) = c_O(e_\lambda)$. Then, we conclude it is Archimedean.

A remarkable case is λ = 1, which converts into Equation (12).

$$\mathrm{u}_{O1}(c_O(x), c_O(y)) = \begin{cases} \frac{c_O(x)+c_O(y)}{1+c_O(x)c_O(y)}, \text{ if } (c_O(x),\ c_O(y)) \in [-1,\ 1]^2\backslash\{(-1,\ 1), (1,\ -1)\} \\ -1, \text{ otherwise} \end{cases} \tag{12}$$

$u_{O1}(\cdot,\cdot)$ is the function called PROSPECTOR which aggregates hypothesis values or Certainty Factors (CF) related to MYCIN, the well-known medical Expert System; nevertheless, the function used in MYCIN is undefined for the arguments (−1, 1) and (1, −1), see [32–34]. Summarizing, we can say that PROSPECTOR is a neutrosophic component n-offuninorm, such that $c_O(e_1) = 0$, which is an effective and widely used aggregation operator.

$u_{O1}(\cdot,\cdot)$ means the combination of the CFs of two independent experts about the hypothesis H. CF = –1.0 means expert has 100% evidence against H and CF = 1.0 means he or she has 100% evidence to support H. The smaller the CF, the greater the evidence against H; the larger the CF, the greater the evidence supporting H; whereas evidence with degree close to 0 means a borderline degree of evidence. Here, $u_{O1}(c_O(x), -c_O(x)) = 0$, where $u_{O1}(-1,1) = u_{O1}(1,-1) = -1$ for meaning that the 100% contradiction is assessed as 100% against H. The original $u_{O1}(\cdot,\cdot)$ in [32] accepts they are undefined.

Another function is the *Modified Combining Function* $C(x,y)$, see [34], defined as

$$C(x,y) = \begin{cases} x + y(1-x), & \text{if } \min(x,y) \geq 0 \\ \frac{x+y}{1-\min(|x|,|y|)}, & \text{if } \min(x,y) < 0 < \max(x,y) \\ x + y(1+x), & \text{if } \max(x,y) \leq 0 \end{cases}$$

The components n-offnorm and n-offconorm obtained from the PROSPECTOR are the following: $c_O(x) \overset{\wedge}{_{1O}} c_O(y) = \frac{4(c_O(x)+c_O(y)-2)}{4+(c_O(x)-1)(c_O(y)-1)} + 1$ and $c_O(x) \overset{\vee}{_{1O}} c_O(y) = \frac{4(c_O(x)+c_O(y)+2)}{4+(c_O(x)+1)(c_O(y)+1)} - 1$, respectively, see Figures 1 and 2.

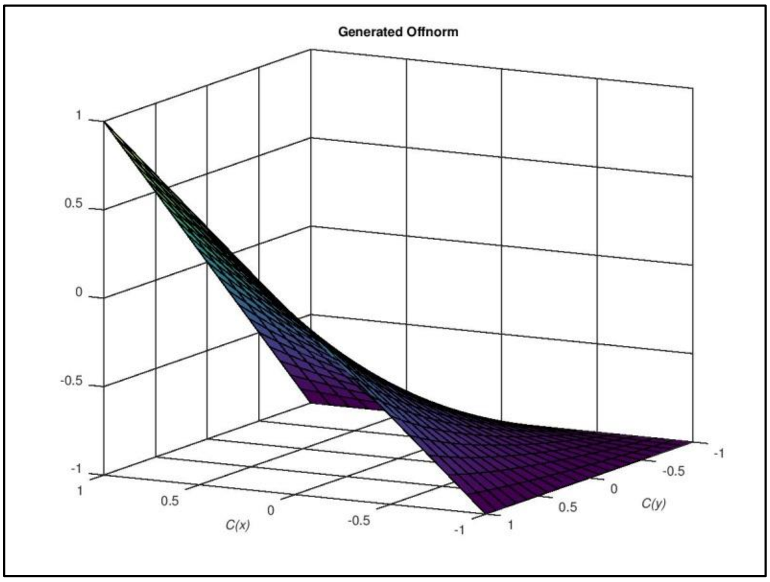

Figure 1. Depiction of the neutrosophic component n-offnorm generated by $u_{O1}(\cdot,\cdot)$.

Hitherto we mostly calculated on neutrosophic components, nevertheless n-offuninorms have to be defined for the three components altogether. For example, given $x, y \in [-1, 1]^3$, $U_{N\lambda}(x,y) = \langle u_{O\lambda_1}(T_O(x), T_O(y)),\ u_{O\lambda_2}(I_O(x), I_O(y)), u_{O\lambda_3}(F_O(x), F_O(y))\rangle$ is an n-offuninorm, which evidently it is not conjunctive, neither is it disjunctive, see that $U_{N\lambda}(\langle -1, 1, 1\rangle, \langle 1, -1, -1\rangle) = \langle -1, -1, -1\rangle$.

Conjunctive and disjunctive neutrosophic component n-offuninorms were illustrated in Example 3; see also Example 5. Example 6 is a hypothetical example to explain the use of this theory in a real-life situation.

Example 6. *Three physicians, denoted by A, B, and C, have to emit a criterion about a patient's disease which suffers from somewhat confusing symptoms. They agree that the Certainty Factor is the better way to express* *their opinions. They use single-valued neutrosophic offsets, instead of a simple CF to increase the accuracy of the criteria.*

After a discussion, they are convinced that it is most likely that the patient has either a thyroid disease or an infectious one. The treatment for each disease is different each other. Therefore, they have two hypotheses; one is H_T which means the patient has thyroid disease and H_I that patient has an infectious disease.

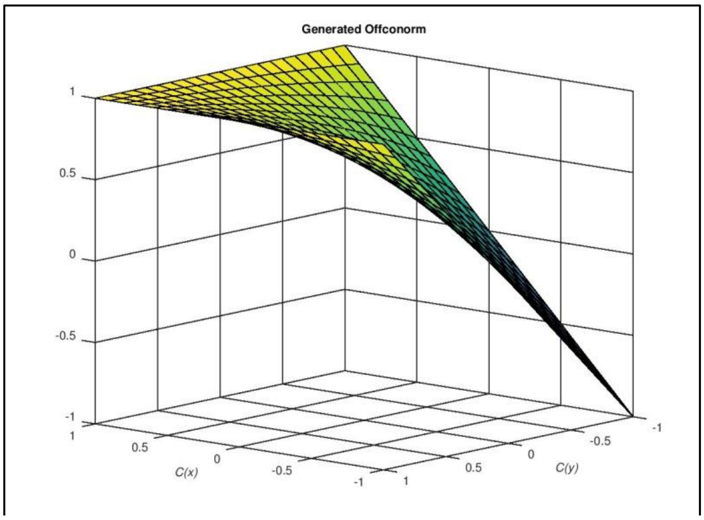

Figure 2. Depiction of the neutrosophic component n-offconorm generated by $u_{O1}(\cdot,\cdot)$.

Physician A thinks that the probability they are dealing with a thyroid disease is A_T = <−0.6, 0.4, 0.6> and that it is an infectious disease is A_I = <0.8, −0.5, −0.8>, thus, A is 60% against H_T and 40% undecided about it; however, A is 80% in favor of H_I and 50% sure about it.

Similarly, we have that B's criteria are, B_T = <−0.1, −0.2,0.1> and B_I = <0.1, 0.8, −0.1>, whereas C's criteria are C_T = <0.7, 0.1, −0.2> and C_I = <−0.6, −0.3, 0.7>.

To decide what is the strongest hypothesis, H_T or H_I, they select the well-known PROSPECTOR function used in MYCIN (see Equation (12)) for each component.

Thus, for H_T we have an aggregated value equal to <0.073684, 0.31064, 0.53043> and for H_I it is <0.46667, 0.23529, −0.32>, therefore, evidently, the infectious disease is the strongest hypothesis, because $\langle 0.073684,\ 0.31064,\ 0.53043\rangle <_O \langle 0.46667,\ 0.23529,\ -0.32\rangle$.

Despite we proved in Proposition 5 that neutrosophic uninorms are mathematically equivalent to offuninorms, it is worthwhile to remark that the reason for using an interval different of [0, 1] is that it could be useful to model real-life problems. The present example is a good one to explain that reason. The advantages arise from the accuracy and compactness of an expert's information. In this example, from an expert's viewpoint, it is easier to express opinions in the scale [−1, 1] with the aforementioned meaning than in the scale [0, 1], which is less clear. Information compactness is given because of only a single offset is semantically equivalent to at least two neutrosophic sets.

Additionally, because of the significance of functions like $u_{O1}(\cdot,\cdot)$ and $C(x,y)$, which were used as aggregation functions in that well-known expert system, some authors have extended the domain of fuzzy uninorms to any interval [a, b], not necessarily restricted to a = 0 and b = 1; see [33,34].

This fact supports the usefulness of the present work, where for the first time the precedent ideas on extending the truth values beyond the scope of [0, 1] naturally associate with the offset concept maintaining the original definitions of the aggregation functions used in MYCIN.

Another powerful reason is the applicability of $u_{O1}(\cdot,\cdot)$ and $C(x,y)$, and hence of the fuzzy uninorms defined in [a, b], as threshold functions of artificial neurons in Artificial Neural Networks, as well as to Fuzzy Cognitive Maps, which are used in fields like decision making, forecasting, and strategic planning [33].

Such applications of uninorms in the fuzzy domain can be explored in the framework of neutrosophy theory, e.g., in Artificial Neural Networks based on neutrosophic sets, in Neutrosophic Cognitive Maps, among others [36,37].

4.2. N-Offuninorms and Implicators

Fuzzy uninorms are used to define implicators (see [41], pp. 151–160). This application was extended to neutrosophic uninorms ([25]). To extend the implication operator in the offuninorm framework, first, we need to consider the notion of offimplication, which has been defined symbolically.

The *Symbolic Neutrosophic Offlogic Operators* or briefly the *Symbolic Neutrosophic Offoperators* extend the Symbolic Neutrosophic Logic Operators, where every one of T, I, F has an under and an over version (see [31], pp. 132–139).

T_O = Over Truth,
T_U = Under Truth;
I_O = Over Indeterminacy,
I_U = Under Indeterminacy;
F_O = Over Falsehood,
F_U = Under Falsehood.

Let $S_N = \{T_O, T, T_U, I_O, I, I_U, F_O, F, F_U\}$ be the set of neutrosophic symbols, an order is defined in S_N as follows: if '<' denotes "more important than", we have the following order, $T_U < I_U < F_U < F < I < T < F_O < I_O < T_O$, where $-\infty < T_U < I_U < F_U < 0$, $0 \leq F < I < T \leq 1$ and $1 < F_O < I_O < T_O < +\infty$; see Figure 3. Let us note that the proposed order is not the unique one, it depends on the decision maker's objective.

Figure 3. Ordered symbolic neutrosophic components in the neutrosophic offlogic.

Let us observe that I is the center of the elements according to <. For every $\alpha \in S_N$, the *symbolic neutrosophic offcomplement* is denoted by $C_O(\alpha)$ and it is defined as the symmetric element respect to the median centered in I, e.g., $C_{SO}(F_O) = F_U$ and $C_{SO}(F) = T$, hence, given $\alpha \in S_N$ its *symbolic neutrosophic offnegation* is $\underset{SO}{\neg}\ \alpha = C_{SO}(\alpha)$.

Additionally, for any $\alpha, \beta \in S_N$ the *symbolic neutrosophic offconjunction* is defined as $\alpha \underset{SO}{\wedge} \beta = \min(\alpha, \beta)$, the *symbolic neutrosophic offdisjunction* is defined as $\alpha \underset{SO}{\vee} \beta = \max(\alpha, \beta)$, whereas the *symbolic neutrosophic offimplication* is defined in Equation (13).

$$\alpha \underset{SO}{\rightarrow} \beta = \left(\underset{SO}{\neg}\ \alpha \right) \underset{SO}{\vee} \beta \tag{13}$$

In this paper, we redefine some of the symbolic neutrosophic offoperators to the continuous quantitative domain. Given $\overline{\alpha} \in [\Psi, \Omega]$, where $\Psi < 0$ or $\Omega > 1$, the *neutrosophic offnegation* is defined by Equation (14).

$$\underset{O}{\neg}\ \overline{\alpha} = \begin{cases} \min\{\Omega, 1 - \overline{\alpha}\}, \text{ if } \overline{\alpha} \leq 0.5 \\ \max\{\Psi, 1 - \overline{\alpha}\}, \text{ if } \overline{\alpha} > 0.5 \end{cases} \tag{14}$$

The *neutrosophic offnegation* satisfies the following properties:

1. It is a non-increasing operator, which extends the classical negation operator in fuzzy logic theory. It is strictly decreasing when $\Omega + \Psi = 1$.
2. It extends the notion of symbolic neutrosophic offnegation because satisfies the following properties:

   2.1 It is centered in 0.5, i.e., $\underset{O}{\neg}\ 0.5 = 0.5$, therefore I = 0.5.

   2.2 If $\overline{\alpha} \in [0,\ 1]$, then $\underset{O}{\neg}\ \overline{\alpha} \in [0,\ 1]$, $\underset{O}{\neg}\ 0 = 1$ and $\underset{O}{\neg}\ 1 = 0$, which is the usual negation operator in fuzzy logic.

   2.3 If $\overline{\alpha} < 0$, then $\underset{O}{\neg}\ \overline{\alpha} \geq 1$. $\underset{O}{\neg}\ \overline{\alpha} = 1$ only when $\Omega = 1$.

   2.4 If $\overline{\alpha} > 1$, then $\underset{O}{\neg}\ \overline{\alpha} \leq 0$. $\underset{O}{\neg}\ \overline{\alpha} = 0$ only when $\Psi = 0$.

   2.5 When $\Omega + \Psi = 1$, we have $\underset{O}{\neg}\ \Psi = \Omega$ and $\underset{O}{\neg}\ \Omega = \Psi$.

3. If $\Omega + \Psi = 1$, then $\underset{O}{\neg}\ \underset{O}{\neg}\ \overline{\alpha} = \overline{\alpha}$, for every $\overline{\alpha} \in [\Psi,\ \Omega]$.

The precedent properties are easy to demonstrate.

Hence, the definition of *offimplication* $\underset{O}{\rightarrow}$: $[\Psi,\ \Omega]^3 \times [\Psi,\ \Omega]^3 \rightarrow [\Psi,\ \Omega]^3$ is defined in Equation (15), for every $\overline{\alpha}, \overline{\beta} \in [\Psi,\ \Omega]^3$.

$$\overline{\alpha} \underset{O}{\rightarrow} \overline{\beta} = \left\langle N_O^{co}\left(\underset{O}{\neg}\ T_O(\overline{\alpha}),\ T_O(\overline{\beta})\right),\ N_O^{n_1}\left(\underset{O}{\neg}\ I_O(\overline{\alpha}),\ I_O(\overline{\beta})\right), N_O^{n_2}\left(\underset{O}{\neg}\ F_O(\overline{\alpha}),\ F_O(\overline{\beta})\right)\right\rangle \tag{15}$$

where, $N_O^{n_i}(\cdot,\ \cdot)$ i = 1, 2 are neutrosophic components n-offnorms, $N_O^{co}(\cdot,\ \cdot)$ is a neutrosophic component n-offconorm, and $\underset{O}{\neg}$ is the offnegation defined in Equation (14).

Equation (15) is generalized by using offuninorms, see Equation (16).

$$\overline{\alpha} \underset{UO}{\rightarrow} \overline{\beta} = \left\langle N_O^{u_1}\left(\underset{O}{\neg}\ T_O(\overline{\alpha}),\ T_O(\overline{\beta})\right),\ N_O^{u_2}\left(\underset{O}{\neg}\ I_O(\overline{\alpha}),\ I_O(\overline{\beta})\right), N_O^{u_3}\left(\underset{O}{\neg}\ F_O(\overline{\alpha}),\ F_O(\overline{\beta})\right)\right\rangle \tag{16}$$

where $N_O^{u_i}(\cdot,\ \cdot)$ for i = 1, 2, and 3 are neutrosophic components n-offuninorms.

Example 7. *One illustrative example of Equation (16) is obtained revisiting Section 4.1, by defining the following neutrosophic component n-offnorm:*

$$u_O(c_O(x), c_O(y)) = \begin{cases} \frac{3(c_O(x)+1)(c_O(y)+1)}{(c_O(x)+1)(c_O(y)+1)+(2-c_O(x))(2-c_O(y))} - 1, & \text{if } (c_O(x),\ c_O(y)) \in [-1,\ 2]^2 \backslash \{(-1,\ 2), (2,\ -1)\} \\ -1, & \text{otherwise} \end{cases}$$

This is the transformation of Silvert uninorms to the domain $[-1, 2]^2$ applying the functions in Equations (10) and (11), and the transformation in Proposition 5. Also, let us take $U_{ZD}(c(x), c(y))$ of Example 3. See that [−1, 2] is symmetric respect to 0.5, and the neutral element is 0.5.

Then, we study the offuninorm defined in the following equation: $U_O(\overline{\alpha}, \overline{\beta}) = \langle U_{ZD}(T_O(\overline{\alpha}), T_O(\overline{\beta})),\ u_O(I_O(\overline{\alpha}), I_O(\overline{\beta})), u_O(F_O(\overline{\alpha}), F_O(\overline{\beta}))\rangle$ for $\overline{\alpha} = \langle T_O(\overline{\alpha}),\ I_O(\overline{\alpha}), F_O(\overline{\alpha})\rangle$ and $\overline{\beta} = \langle T_O(\overline{\beta}),\ I_O(\overline{\beta}), F_O(\overline{\beta})\rangle$ in $[-1, 2]^3$.

Thus, we define the offimplication generated by $U_O(\cdot, \cdot)$ according to Equation (16) as follows:

$$\overline{\alpha} \underset{U_O}{\rightarrow} \overline{\beta} = \langle U_{ZD}\left(\underset{O}{\neg} T_O(\overline{\alpha}),\ T_O(\overline{\beta})\right),\ u_O\left(\underset{O}{\neg} I_O(\overline{\alpha}),\ I_O(\overline{\beta})\right), u_O\left(\underset{O}{\neg} F_O(\overline{\alpha}),\ F_O(\overline{\beta})\right)\rangle.$$

where in this case we have $U_{ZD}\left(T_O(\overline{\alpha}),\ T_O(\overline{\beta})\right) = \begin{cases} \min\left(T_O(\overline{\alpha}),\ T_O(\overline{\beta})\right),\ \text{if } T_O(\overline{\alpha}),\ T_O(\overline{\beta}) \in \left[-1,\ \frac{1}{2}\right] \\ \max\left(T_O(\overline{\alpha}),\ T_O(\overline{\beta})\right),\ \text{otherwise} \end{cases}$,

see Figure 4, and $u_O(\cdot,\cdot)$ models the neutrosophic n-components I_O and F_O, see Figure 5.

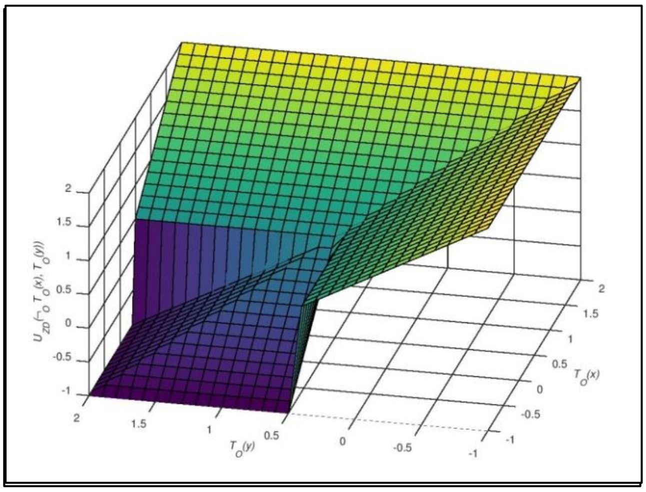

Figure 4. Depiction of the neutrosophic n-offimplication generated by U_{ZD} for T_O.

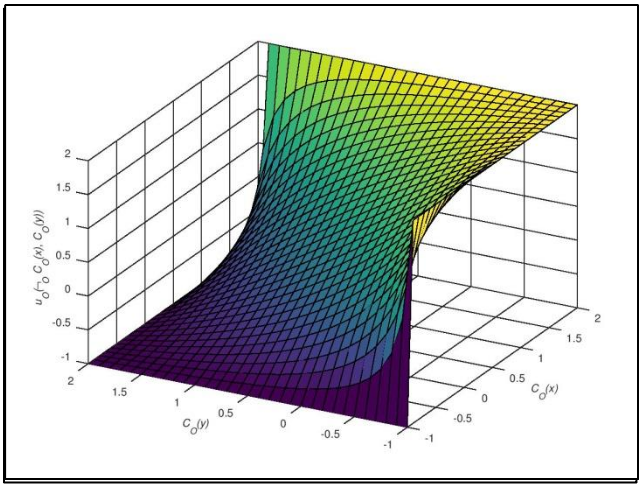

Figure 5. Depiction of the neutrosophic n-offimplication generated by u_O for both, I_O and F_O.

This offimplicator satisfies the overbounding conditions $\langle -1,\ 2,\ 2\rangle \underset{U_O}{\rightarrow} \langle -1,\ 2,\ 2\rangle = \langle -1,\ 2,\ 2\rangle \underset{U_O}{\rightarrow} \langle 2,\ -1,\ -1\rangle = \langle\ 2,\ -1,\ -1\rangle \underset{U_O}{\rightarrow} \langle 2,\ -1,\ -1\rangle = \langle 2,\ -1,\ -1\rangle$, whereas, $\langle 2,\ -1,\ -1\rangle \underset{U_O}{\rightarrow} \langle -1,\ 2,\ 2\rangle = \langle -1,\ 2,\ 2\rangle$.

Also, $\langle 0,\ 1,\ 1\rangle \underset{U_O}{\rightarrow} \langle 0,\ 1,\ 1\rangle = \langle\ 1,\ 0,\ 0\rangle \underset{U_O}{\rightarrow} \langle 1,\ 0,\ 0\rangle = \langle 1,\ 0.5,\ 0.5\rangle$, $\langle 0,\ 1,\ 1\rangle \underset{U_O}{\rightarrow} \langle 1,\ 0,\ 0\rangle = \langle 1,\ -0.4,\ -0.4\rangle$ and $\langle 1,\ 0,\ 0\rangle \underset{U_O}{\rightarrow} \langle 0,\ 1,\ 1\rangle = \langle 0,\ 1.4,\ 1.4\rangle$. Additionally, $\langle 0.5,\ 0.5,\ 0.5\rangle \underset{U_O}{\rightarrow} \langle 0.5,\ 0.5,\ 0.5\rangle = \langle 0.5,\ 0.5,\ 0.5\rangle$ because 0.5 is the neutral element of every neutrosophic component n-offuninorm and $\underset{O}{\neg}\ 0.5 = 0.5$.

It is easy to check that substituting $u_O(\cdot,\cdot)$ by $U_{ZC}(\cdot,\cdot)$ in $\underset{U_O}{\rightarrow}$, we obtain the more classical equations $\langle 0, 1, 1\rangle \underset{U_O}{\rightarrow} \langle 0, 1, 1\rangle = \langle 1, 0, 0\rangle \underset{U_O}{\rightarrow} \langle 1, 0, 0\rangle = \langle 0, 1, 1\rangle \underset{U_O}{\rightarrow} \langle 1, 0, 0\rangle = \langle 1, 0, 0\rangle$ and $\langle 1, 0, 0\rangle \underset{U_O}{\rightarrow} \langle 0, 1, 1\rangle = \langle 0, 1, 1\rangle$.

4.3. N-Offuninorms and Voting Games

The applicability of uninorms to solve group decision problems is evident. However, the use of them as part of a game theory solution is not so obvious. This subsection is devoted to solving voting games based on n-offuninorms.

A *cooperative game with transferable utility* consists of a pair (N,v), where N = {1, 2, . . . ,n} is a non-empty set of *players*,n ∈ ℕ and v: $2^N \rightarrow \mathbb{R}$, i.e., $v(\cdot)$ is a function of the power set of N such that each *coalition* or $S \subseteq$ N is associated with a real number. v is called *characteristic function* and $v(S)$ represents the conjoint payoff of players in S. Additionally, $v(\emptyset) = 0$ (see [42], p. 2).

A *simple game* models voting situations. It is a cooperative game such that for every coalition S, either $v(S) = 0$ or $v(S) = 1$, and $v(\mathrm{N}) = 1$ (see [42], p. 7).

One solution is the Shapley–Shubik index, which is the Shapley value to simple games (see [42], pp. 6–7). The equation of Shapley value is the following:

$$\phi_i(v) = \sum_{S \subseteq N \setminus \{i\}} \frac{|S|!(|N| - |S| - 1)!}{|N|!}[v(S \cup \{i\}) - v(S)] \tag{17}$$

where $|S|$ is the cardinality of coalition S, $|N|$ is the cardinality of the set of players or *grand coalition* and $\phi_i(v)$ is the value assigned to player i in the game.

This is the unique solution which satisfies the following axioms:

- $\sum_{i \in N} \phi_i(v) = v(N)$ (Efficiency),
- If $i, j \in$N are interchangeable in v, then $\phi_i(v) = \phi_j(v)$ (Symmetry),
- If i is such that for every coalition S the equation $v(S \cup \{i\}) = v(S)$ holds, then $\phi_i(v) = 0$ (Dummy),
- Given v and w two games over N, then $\phi_i(v + w) = \phi_i(v) + \phi_i(w)$ (Additivity).

This value is the sum of the terms $[v(S \cup \{i\}) - v(S)]$, which mean the marginal contribution of player i to the coalitions S, multiplied by $\frac{|S|!(|N|-|S|-1)!}{|N|!}$ which is the probability that $|S| - 1$ players precede player i in the game and $|N| - |S|$ players follow him or her. Thus, the Shapley value of i is the expected marginal contribution of i to the game (see [42], p. 7). The result of the Shapley–Shubik index is interpreted as a measure of each player's power.

In the present paper we basically study voting games with some additional features. We call them *voting n-offgames*. A *voting n-offgame* consists in a pair (N, v), where N = {1, 2, . . . , n} is the set of players; the characteristic function v: $2^N \rightarrow \{1, \ldots, 2^n\} \times \{1, \ldots, 2^n\} \times \{1, \ldots, 2^n\}$ is such that for any coalition S we have $v(S) = (k, l, 2^n\text{-}k+1)$ and $v(\emptyset) = (2^n, 2^n, 1)$.

The n-offgame is interpreted in the following way:

1. Experts forecast that voters will rank coalition S in the k^{th} position of their preference, also they cannot decide if S will be ranked in the l^{th} position. The first place or k = 1 corresponds to the preferred coalition of all and so on. Additionally, the n-offgame must satisfy the following rules:
2. Given any two coalitions S_1 and S_2, $S_1 \neq S_2$, we have the first component that both $v(S_1)$ and $v(S_2)$ are different. Thus, every coalition is associated with a unique number in the order of preference.
3. $v(S) = (k, k, 2^n - k+1)$ means experts have no doubt that coalition S will be voted in the k^{th} position.

Let us observe that it is not a simple game. This game can be interpreted as a multicriteria decision-making problem, where its solution is a measure of every player's power in the game according to the forecasted experts' ranking of the coalitions. Each coalition can represent a bloc of political parties.

Shapley value can be the solution to voting n-offgames, in the form given in Equation (18):

$$\phi_i(v) = -\sum_{S\subseteq N\backslash\{i\}} \frac{|S|!(|N|-|S|-1)!}{|N|!}[v(S\cup\{i\}) - v(S)] \tag{18}$$

Let us note that the minus sign in the expression was taken for convenience because the rank we applied is decreasing respect to the coalition´s significance. Additionally, $v(S\cup\{i\}) - v(S)$ is the difference between two 3-tuple values, thus the operation $(k_1, l_1, 2^n - k_1+1)-(k_2, l_2, 2^n - k_2+1) := (k_1 - k_2, l_1 - l_2, k_2 - k_1)$ is defined. Equation (18) means the expected number of places won or lost in voter preference, as predicted by experts.

Apparently, Shapley value cannot be the solution to this problem because $v(\emptyset) \neq 0$ and $v(\cdot)$ is not a game. However, if we take in that $v(S) = (k,l,2^n - k+1)$ in fact represents three games, namely, $v_1(S)=k$, $v_2(S)= l$, and $v_3(S) = 2^n - k+1$, one per component and additionally taking into account they are linear transformations of three games with characteristic functions w_1, w_2, and w_3; where $w_1(S)= 2^n - v_1(S)$, $w_2(S)= 2^n - v_2(S)$, and $w_3(S) = 1 - v_3(S)$, then, the marginal contributions of the three pairs, $w_1(\cdot)$ and $v_1(\cdot)$, $w_2(\cdot)$ and $v_2(\cdot)$, $w_3(\cdot)$ and $v_3(\cdot)$, are the same except for the sign. Thus, these three pairs have the same Shapley value except for the sign and therefore this property is extended to $v(\cdot)$ and $w(\cdot)$.

Shapley value is a rational solution to the game, nevertheless, it can differ from actual human behavior, as Zhang et al. suggested in [43] to model restrictions in game decisions according to the human behavior based on fuzzy uninorms. Therefore, we propose n-offuninorms to explore other behaviors in human decision making by recursively applying an n-offuninorm to every pair of values $\frac{|S|!(|N|-|S|-1)!}{|N|!}[v(S) - v(S\cup\{i\})]$ in the set of $S \subseteq N\backslash\{i\}$.

Here we explore n-offuninorms defined on $[-L, L]$, $L = 2^n - 1$ and with the PROSPECTOR parameterized function with $\lambda > 0$ and neutral element $e = L\left(\frac{1-\lambda}{1+\lambda}\right)$, see Equation (19).

$$U_{O\lambda}(c(x), c(y)) = \varphi_3^{-1}\left(\frac{\lambda\varphi_3(c(x))\varphi_3(c(y))}{\lambda\varphi_3(c(x))\varphi_3(c(y)) + (1-\varphi_3(c(x)))(1-\varphi_3(c(y)))}\right) \tag{19}$$

where $\varphi_3(\cdot)$ and $\varphi_3^{-1}(\cdot)$ are those defined in Equations (10) and (11), respectively, and now they are $\varphi_3(c(x)) = \frac{c(x)+L}{2L}$ and $\varphi_3^{-1}(c(x)) = 2Lc(x) - L$.

Thus the Algorithm for solving voting n-offgames can be described as follows:

Algorithm 1. Algorithm for solving voting n-offgames

1. Given (N, v) a voting n-offgame. Fix $\lambda > 0$.
2. Fix player $i = 1$.
3. Let S_j be the set of coalitions not containing i, and $j = 1, 2, \ldots, 2^{n-1}$. Let us take $a_{i1} = v(S_1)$ and $a_{i2} = v(S_2)$ and calculate $a_{prev} = U_{O\lambda}\left(\frac{|S_1|!(n-|S_1|-1)!}{n!}[v(S_1) - v(S_1\cup\{i\})], \frac{|S_2|!(n-|S_2|-1)!}{n!}[v(S_2) - v(S_2\cup\{i\})]\right)$, fix $j = 3$ and go to step 4.
4. If $j<2^{n-1}$, calculate $a_{curr} = U_{O\lambda}\left(a_{prev}, \frac{|S_j|!(n-|S_j|-1)!}{n!}\left[v\left(S_j\right) - v\left(S_j\cup\{i\}\right)\right]\right)$. $a_{prev} = a_{curr}$ and $j = j + 1$. Repeat this step. Else, if $j = 2^{n-1}$, $\pi_i(v) = a_{curr}$. Go to Step 5.
5. If $i< n$, then $i = i+ 1$ and go to Step 3. Else Finish.

Let us point out that in the precedent algorithm the associativity of n-offuninorms was used. Moreover, the algebraic sum in Shapley value and the n-offuninorms yield to somewhat similar results. Thus, for $U_{o\lambda}(\cdot,\cdot)$ with $\lambda = 1$, we have that $x, y < 0$ imply both $U_{o\lambda}(x,y)<\min(x,y)$ and $x+y< \min(x,y)$, whereas when $x, y > 0$, we have $U_{o\lambda}(x,y)>\max(x,y)$ and $x+y> \max(x,y)$. For x,y satisfying $x\cdot y<0$, then both $U_{o\lambda}(x,y)$ and $x+y$ are compensatory operators, and finally 0 is the neutral element of them. For λ≠ 1 and hence e≠ 0, we obtain other behavioral effects. Let us also recall that $U_{o\lambda}(\cdot,\cdot)$ is a neutrosophic uninorm transformation, which is described as symmetric summation by Silvert in [40].

Example 8. *Let us consider the 3-person voting n-offgame (N, v), where N = {1, 2, 3} and experts predict that coalitions will be ranked according to the positions shown in Table 1.*

Table 1. Position assigned to the coalitions of the 3-person voting n-offgame.

Coalition	Ranking
∅	(8,8,1)
{1}	(3,2,6)
{2}	(4,3,5)
{3}	(7,6,2)
{1,2}	(2,3,7)
{1,3}	(5,6,4)
{2,3}	(6,5,3)
{1,2,3}	(1,1,8)

According to Table 1, the grand coalition N has (1,1,8) as ranking value, i.e., experts think this coalition will undoubtedly be ranked in the first place or k = 1. $v(\emptyset)$ = (8,8,1) because it is axiomatically predetermined, which means that to not negotiate at all is the worst option, whereas $v(\{2,3\})$ = (6,5,3) means this coalition shall be ranked in the sixth place and maybe in the fifth one, but never in the third place.

Thus, to calculate each player´s power according to our approach we have to apply the precedent algorithm. We fixed λ=1 in $U_{O\lambda}$ therefore $c(e)$= 0, which is defined in [−7, 7].

Table 2 contains the detailed calculus of the Shapley value in Equation (18) and the proposed algorithm to resolve the precedent voting n-offgame.

Table 2. Shapley value and n-offuninorm based solutions to the 3-person voting n-offgame. The final values are written in bold font.

Player$i$	SSuch That $i \notin S$	$v(S)$–$v(S\cup\{i\})$	$v(S)$–$v(S\cup\{i\})$ Multiplied by the Probability	Partial Summations of the Shapley Value	Partial Aggregation with U_{o1}
1	∅	(5,6,−5)	(5/3, 2,−5/3)	(5/3,2,−5/3)	(5/3,2,−5/3)
	{2}	(2,0,−2)	(1/3,0,−1/3)	(2,2,−2)	(1.9776,2.0000,−1.9776)
	{3}	(2,0,−2)	(1/3,0,−1/3)	(7/3,2,−7/3)	(2.2802,2.0000,−2.2802)
	{2,3}	(5,4,−5)	(5/3,4/3,−5/3)	**(4,10/3,−4)**	**(3.6628,3.1613,−3.6628)**
2	∅	(4,5,−4)	(4/3,5/3,−4/3)	(4/3,5/3,−4/3)	(4/3,5/3,−4/3)
	{1}	(1,−1,−1)	(1/6,-1/6,−1/6)	(3/2,3/2,−3/2)	(1.4932,1.5086,−1.4932)
	{3}	(1,1,−1)	(1/6,1/6,−1/6)	(5/3,5/3,−5/3)	(1.6515,1.6667,−1.6515)
	{1,3}	(4,5,−4)	(4/3,5/3,−4/3)	**(3,10/3,−3)**	**(2.8565,3.1545,−2.8565)**
3	∅	(1,2,−1)	(1/3,2/3,−1/3)	(1/3,2/3,−1/3)	(1/3,2/3,−1/3)
	{1}	(-2,−4,2)	(-1/3,−2/3,1/3)	(0,0,0)	(0,0,0)
	{2}	(-2,−2,2)	(-1/3,−1/3,1/3)	(-1/3,−1/3,1/3)	(−1/3,−1/3,1/3)
	{1,2}	(1,2,−1)	(1/3,2/3,−1/3)	**(0,1/3,0)**	**(0,0.33485,0)**

According to the results summarized in Table 1, we have that the expected value of places gains by player 1 is 4 with the Shapley value solution and 3.6628 with U_{o1}, whereas the results for player 2 are 3 and 2.8565, respectively, and for player 3 are 0 and 0. Therefore, player 1 is the most powerful of them, followed by player 2 and 3 in this order. Thus, the proposed approach and Shapley value are similar.

Table 3 contains the voting n-offgame solutions comparing U_{o1} with $c(e)$ = 0, $U_{o99/101}$ with $c(e)$ = 7/100 and $U_{o101/99}$ with $c(e)$ = −7/100.

Table 3. Solutions of the 3-person voting n-offgame applying $U_{o\lambda}$ with λ = 1, 99/101 and 101/99, respectively.

Player	Solution with U_{o1}	Solution with $U_{o99/101}$	Solution with $U_{o101/99}$
1	(3.6628,3.1613,−3.6628)	(3.5079,2.9919,−3.8129)	(3.8129,3.3262,−3.5079)
2	(2.8565,3.1545,−2.8565)	(2.6793,2.9849,−3.0293)	(3.0293,3.3196,−2.6793)
3	(0,0.33485,0)	(−0.20994,0.12509,−0.20994)	(0.20994,0.54402,0.20994)

The solutions in Table 3 prove that the greater λ, the greater the solution values. Thus, when λ is increased, its associated solution models more optimistic behavior with respect to the first component, which is compensated with more pessimistic behavior with respect to the third component.

The advantages of the proposed approach are more evident when it is compared with a classical one restricted to {0, 1}. Here we used a semantic represented with natural numbers and we calculated directly on them. In contrast, for applying classical definitions in {0, 1}, we would need to define eight Boolean functions, one per element. What is more, some operations such as marginal contributions, which is an algebraic difference, cannot be directly applied in the logic sense.

In case we would need to extend the approaches to the continuous gradation, then a continuous ranking can be modeled with the identity line $I_d(x) = x$, but in the classical approach, eight memberships functions would have to be considered, where the simplest ones are triangular (see Figure 6). From Figure 6 we can infer that there exists a transformation between both models; however, the proposed model is the simplest one.

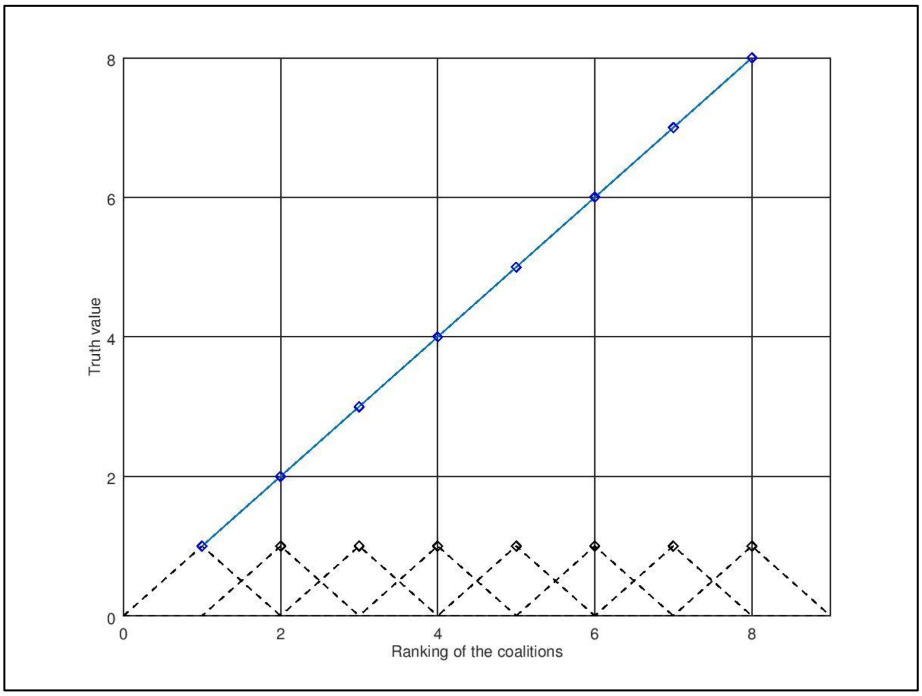

Figure 6. Depiction of two kinds of 3-person game modeling. Classical [0, 1] is represented in dashed lines and triangular membership functions, whereas the solid line represents the solution based on offsets. The points represent the Boolean restrictions.

5. Discussion

Neutrosophic oversets, undersets, and offsets are concepts of a novel and non-conventional theory of uncertainty. Historically, the convention of restricting logic to the interval [0, 1] has dominated fuzzy logic and its generalizations. Possibly this is a legacy of probability and mathematical logic, where, semantically speaking, 0 and 1 have been considered the two extreme opposite sides. Therefore, oversets, undersets, and offsets can be understood as controversial subjects. Nevertheless, Smarandache in [31] illustrates with some examples that such sets, of which their domains surpass the scope of [0, 1], could be useful to represent knowledge in a valid semantic.

This is a recent theory that needs more developing and the scientific community's acknowledgment of its usefulness. One of our aims with this paper is to demonstrate that this theory can be useful. To achieve this end, we introduced the uninorm theory in the neutrosophic offset framework. This union is manifold advantageous, the most evident one being that we have provided a new aggregator operator to these sets. As we mentioned in the introduction, there exists a wide variety of fuzzy uninorm applications, namely, Decision Making [9,14,15], DNA and RNA fusion [9], logic [17], Artificial Neural Networks [16], among others. Uninorm is more flexible than t-norm and t-conorm because it includes the compensatory property in some cases, which is more realistic for modeling human decision making, as was experimentally proved by Zimmermann in [21].

Also, uninorms have enriched other theories when they were generalized to other frameworks. In L^*-fuzzy set theory [23], uninorms also aggregate independent non-membership functions to achieve more precision. Moreover, neutrosophic uninorms aggregate the indeterminate-membership functions [25].

Additionally, some authors have associated uninorms with non-conventional theories. In [33,34] we can find some attempts to extend uninorm domains to an interval [a, b]. The reason is that the PROSPECTOR function related to the MYCIN Expert System is one very important milestone in Artificial Intelligence history. The point is that the PROSPECTOR function is basically a uninorm except it is defined in the interval [−1, 1], thus, we can consider intervals greater than [0, 1]. They have argued that there exist two reasons to maintain the interval [−1, 1]—the first one is the importance of the PROSPECTOR function, the second one is the facility to interchange information among users and decision makers in form of degrees to accept or reject hypotheses.

The second non-conventional approach is the bipolar or Multi-Polar uninorms defined in [24]. The world is (and some people are) is evidently multi-polar; in case of bipolarity they are modeled in [−1, 1]. Especially in [24], we have a multi-polar space consisting of an ordered pair of (k, x), where k∈{1, 2, ... , n} represents a category or class and x∈(0, 1], with the convention **0** = (k, 0) for every category. This is a more complex representation that takes a unique interval [−n, n] where, for x∈[−n, n], the function round(x) represents the category and its fractional part represents the degree of membership to that category. This is a real extension of bipolarity in [−1, 1] to multi-polarity. In [31] (pp. 127, 130) Tripolar offsets and Multi-polar offsets are defined. We illustrated in Example 8 that considering the semantic values belong to {−n,−n+1, ... , 0, 1, ... , n} could be advantageous.

The definition of uninorm-based implicators is not new in literature, they can be seen in [41] (pp. 151–160) for fuzzy uninorms, in [17] it is extended for type 2 fuzzy sets, in [24] for L^*-fuzzy set theory, and in [25] for neutrosophic uninorms. In the present paper, uninorm-based offimplicators are defined, however, we only counted on symbolic offimplication operators (see [31], p. 139). To extend this definition to a continuous framework, we had to extend the symbolic offnegation to a continuous one.

Finally, we preferred to illustrate a voting game solution instead of a group decision method because the relationship of offuninorms with the latter subject is predictable. However, to find any game theory associated with uninorms is uncommon in literature. One remarkable example can be seen in [43], where a behavioral approach has been made to certain kind of games, where uninorms model the humans' restrictions to make the division of gains among the players.

In the present paper, another approach is proposed where an indeterminacy component is taken into account. Also, we proved that modeling with a natural number semantic is simpler than to utilize the classical [0, 1] interval, because of the fact that n membership functions can be substituted by a linear identity function. We basically defined the voting game solution since the Shapley–Shubik index components (see [42], pp. 6–7), where we only changed the algebraic sum by offuninorms. The classical approaches such as the Shapley–Shubik index are interested in a rational and fair solution; nevertheless, many times that does not occur in real negotiations and then behavioral solutions are needed.

6. Conclusions

This paper was devoted to defining for the first time the theory of neutrosophic offuninorms, which is a generalization of both the neutrosophic offnorms and neutrosophic offconorms, where the neutral element lays in the interval [Ψ, Ω]. The properties of these novel operators were proved. Moreover, we defined neutrosophic offuninorms from neutrosophic offnorms and neutrosophic offconorms and vice versa, we also proved their properties. Additionally, we proved the relationship between neutrosophic offuninorms and neutrosophic uninorms.

One of the purposes of this paper is to show the convenience of applying offsets, and to prove that they are not only simple theoretical concepts; furthermore, they are also necessary to define new concepts. This need is demonstrated in this paper by associating offsets with the PROSPECTOR aggregation function, where it is recommendable to extend its domain to the interval [−1, 1]. Some authors in fuzzy logic have suggested the advantages to calculate in the domains [a, b] instead of the classical [0, 1]. Therefore, the use of the idea of the offset in uninorms has some precedence in fuzzy logic.

Additionally, we recommend offsets because they permit more accuracy and compactness. We showed that it is possible to define offimplication operators based on offuninorms. A future direction of this research is to solve problems by using artificial neural networks based on neutrosophic offuninorms, such that neutrosophic offuninorms are utilized as the threshold functions in the neurons or in neutrosophic cognitive maps. For the first time, solutions to cooperative games are defined in the neutrosophic framework—this is an area that it is worthy of development.

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Neutrosofía en Latinoamérica, avances y perspectivas
Neutrosophics in Latin America, advances and perspectives

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Resumen. La neutrosofía ha dado paso a un método de investigación propio al constituir un campo unificado de la lógica para un estudio transdisciplinario que traspasa las fronteras entre las ciencias. En el presente trabajo se analiza el impacto de teoría neutrosófica en Latinoamérica sus principales impulsores y estado de la investigación. Se destaca el aumento de las publicaciones desde la creación de la Asociación Latinoamericana de Ciencia Neutrosóficas. Las áreas más abordadas se encuentran en la interrelación de las ciencias sociales y la neutrosofía presentando resultados destacados en eso ámbitos de la investigación. Como universidad e instituciones más destacadas se encuentran la Universidad Regional Autónoma de los andes en Ecuador y la Universidad de las Ciencias Informáticas en Cuba.

Palabras claves: neutrosofía, Latinoamérica, estudio bibliométrico

Abstract
Neutrosophy has given way to its own research method by constituting a unified field of logic for a transdisciplinary study that crosses the borders between the sciences. This paper analyzes the impact of neutrosophic theory in Latin America, its main drivers and the state of the research. The increase in publications since the creation of the Latin American Association of Neutrosophic Sciences is noteworthy. The most approached areas are found in the interrelation of the social sciences and neutrosophy, presenting outstanding results in these areas of research. The most outstanding university and institutions are the Autonomous Regional University of the Andes in Ecuador and the University of Informatics Sciences in Cuba.

Keywords: neutrosophic, Latin America, bibliometric study

INTRODUCCIÓN

El conjunto Neutrosófico, iniciado por el Prof. Florentin Smarandache, es una herramienta novedosa para caracterizar la información incierta de manera más suficiente y precisa [1]. Es importante destacar que la neutrosófica permite representar la información de forma más completa y real permitiendo abarcar no solo la veracidad o falsedad sino también la ambigüedad, ignorancia, contradicción, neutralidad y saturación[2](Figura 1).

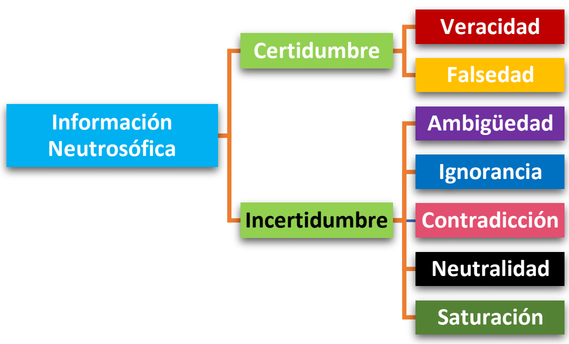

Figura 1. Información neutrosófica

La neutrosofía ha dado paso a un método de investigación propio al constituir un campo unificado de la lógica para un estudio transdisciplinario que traspasa las fronteras entre las ciencias naturales y sociales. Esta ciencia enfrenta los problemas de indeterminación que aparecen universalmente, con vistas a reformar las ciencias actuales, naturales o sociales, con una metodología abierta para promover la innovación.

Extiende también la filosofía y la ciencia al abordar dese la perspectiva de la indeterminación, la contradicción en y la paradoja tal es el caso del teorema de la incompletitud de Gödel planteando que cualquier proposición en cualquier sistema de axioma matemático formal representará la verdad (T), la falsedad (F) y la indeterminación (I) de la declaración bajo consideración[3]. La neutrosofía por tanto establece de una solución única para la existencia de la paradojas en la filosofía [4] .

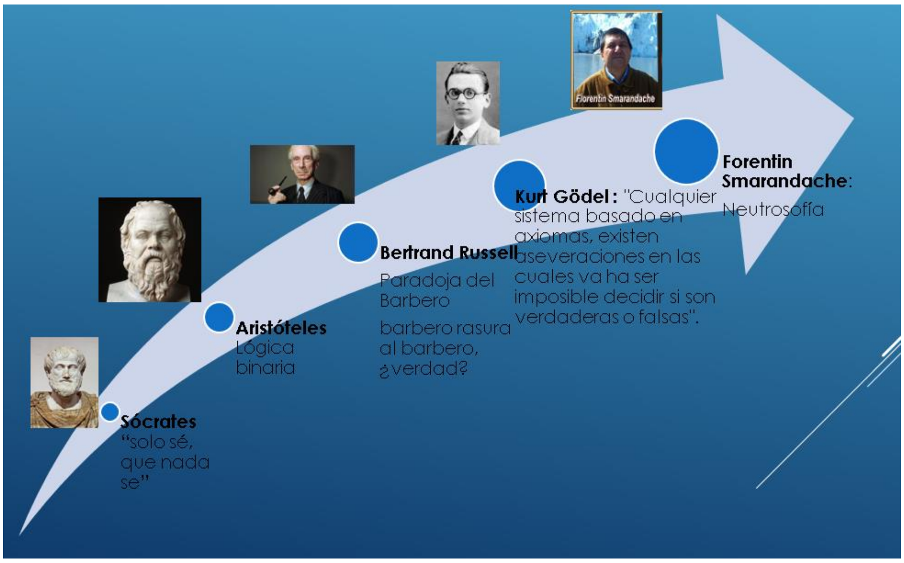

Figura 2. El abordaje de las contradicciones en la filosofía.

El método de investigación neutrosófico es una generalización de la dialéctica de Hegel abordando que la ciencia no solo avanzará tomando en consideración las ideas contrarias sino también las neutrales. Su teoría fundamental afirma que toda idea < A > tiende a ser neutralizada, disminuida, balaceada por las ideas, no solo como Hegel planteó[5]:

<no A> = lo que no es <A>, <antiA> = lo opuesto a <A>, y <neut A>= los que no es ni <A> ni <antiA>.

Los elementos neutrales <neutA> juega un papel en el conflicto entre los opuestos <A> y <antiA>.

Un amplio conjunto de aplicaciones prácticas que han surgido algunas de ellas asociado campos tan disimiles como la psicología y el procesamiento de imágenes (Figura 4)[6]:

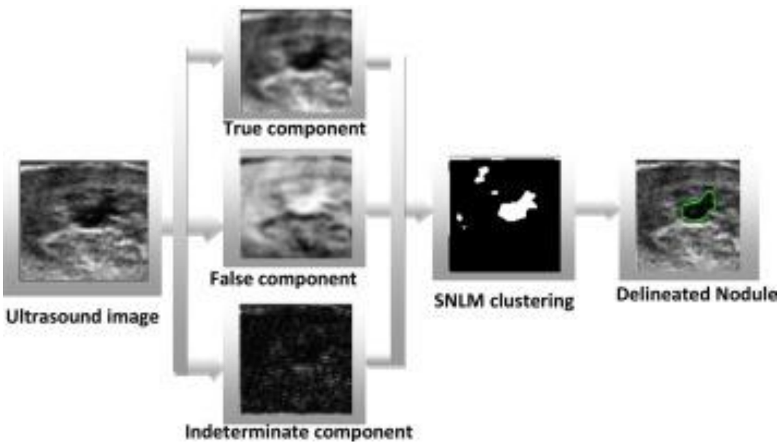

Figura 4. Ejemplo de procesamiento de imágenes neutrosófica
Fuente: [7]

En el presente trabajo se analiza el impacto de teoría neutrosófica en Latinoamérica sus principales impulsores y estado de la investigación.

Materiales y métodos

Se realizó una investigación bibliométrica a las principales bases de datos empelando como apoyo las aplicaciones Publish or Perish[8] y VosViewer[9].

Las revistas analizadas fueron Neutrosophic Sets and Systems(fs.unm.edu/NSSe Investigación Operacional (http://www.invoperacional.uh.cu/index.php/InvOp), /), Neutrosophic Computing and Machine Learning(fs.unm.edu/NCML) , esta última órgano de publicación de la Asociación Latinoamericana de Ciencias Neutrosóficas.

Es de resaltar la importancia de la revista Neutrosophic Sets and Systems (NSS) es una revista académica creada para publicaciones de estudios avanzados en neutrosofía. NSS esta indexados. NSS se encuentra entre las diez principales revistas de sistemas difusos del mundo, según el índice de citas de Google Scholar (Índice h5: 28, mediana h5: 38).

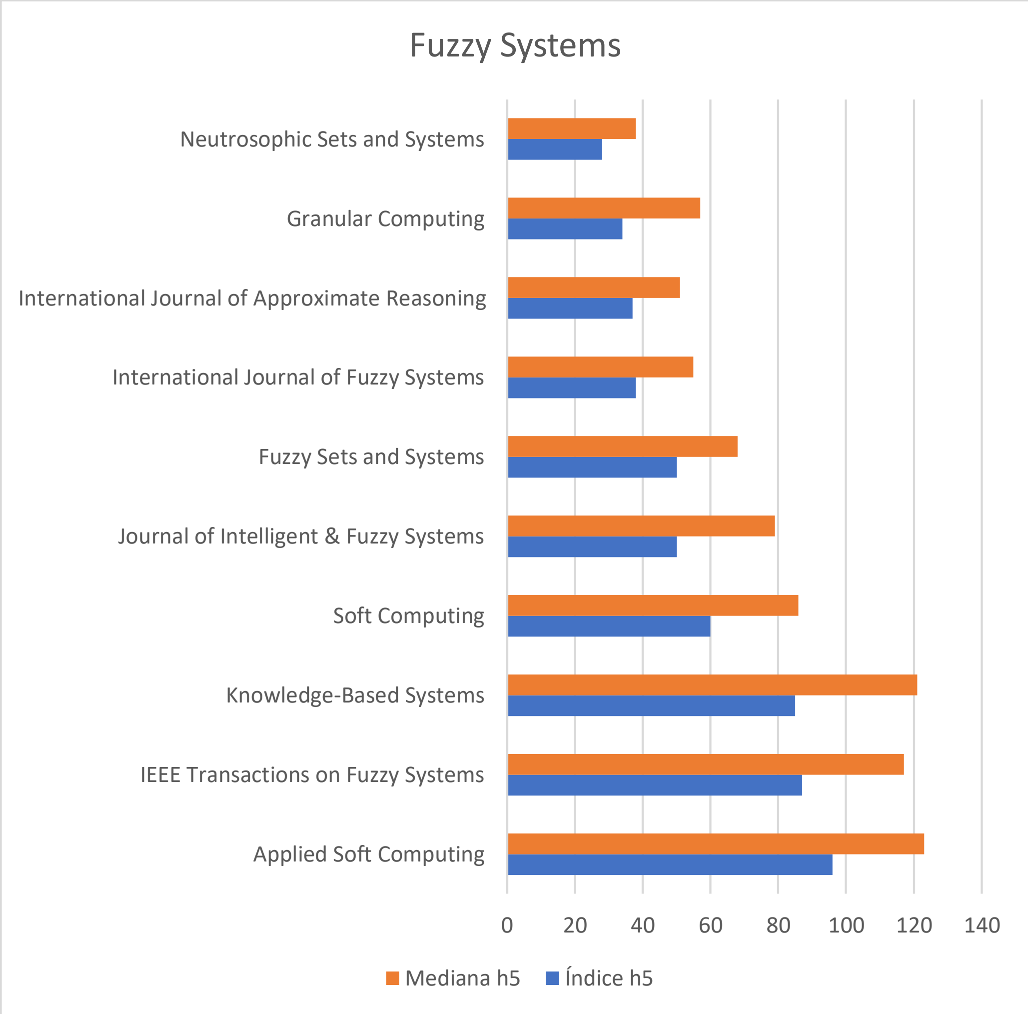

Figura 5. Principales revistas de sistemas difusos
Fuente: Google Scholar citation system (https://scholar.google.com/citations?view_op=top_venues&hl=es&venue=LaeVvxiw2ugJ.2020&vq=eng_fuzzy systems)

Adicionalmente se obtuvo información sobre los libros publicados en la Biblioteca de la Ciencia (http://fs.unm.edu/ScienceLibrary.htm)

DESARROLLO

He existido un avance en la publicación de temas relacionados con la neutrosofía a nivel mundial (Figura 6) fundamentalmente en las áreas de soporte a multicriterio a la toma de decisiones, visión por computadoras y operadores de agregación[2], [10], [11] .

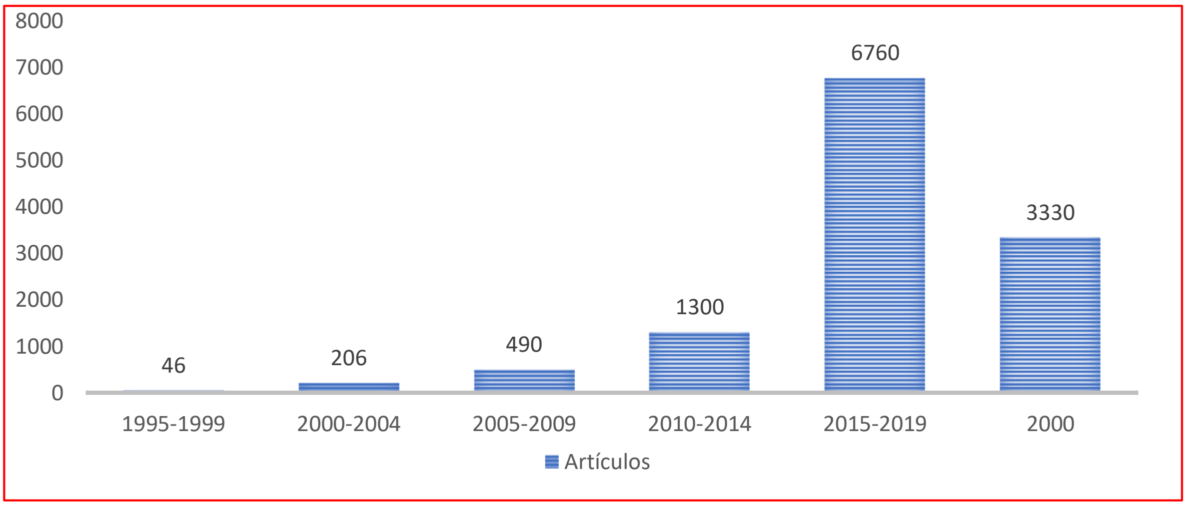

Figura 7. Crecimiento las publicaciones que incluyen el término "neutrosophics"
Fuente: Google Académico

Es de destacar a presencia de 48 patentes en la base de datos patentes de Google (https://patents.google.com/).
En cuanto a los países de la región que reportan una producción científica en la materia se reportan los siguientes (Figura 8).

Figura 8. Países de la región con producción científica en el área de la neutrosofía
Se destacan las producciones científicas de Cuba Universidad de las Ciencias Informáticas y Ecuador Universidad Regional Autónoma de los Andes (UNIANDES) y la Universidad de Guayaquil

Figura 9 Artículos más populares. Actualizado el 11/10/20.
Fuente: https://digitalrepository.unm.edu/nss_journal/topdownloads.html,

Si analizamos la popularidad de artículos de la revista NSS podemos apreciar que 4 de 10 artículos pertenecen a la región.

Se realizó una nube de palabras (Word Cloud) con los términos con las palabras claves de los principales artículos.

Figura 10. Nube de palabras.

Se puede apreciar un abordaje de temas comunes a los más populares en la temática de neutrosofía, pero reforzando el uso en las ciencias sociales y la validación de la investigación[12]. Se nota adicionalmente un aumento del desarrollo de temas neutrosofía con la creación de la Asociación Latinoamericana de Ciencia Neutrosóficas (ALCM) .

Se seleccionó utilizando Publish or Perish los articulo publicados en la revista NSS del periodo 2019-2020 y de ellos los artículos con citas. Arrojando que de los 103 artículos más citados(citas>1) en el periodo de la ALCN de estos artículos 17 corresponden la región de ellos 14 a Uniandes.

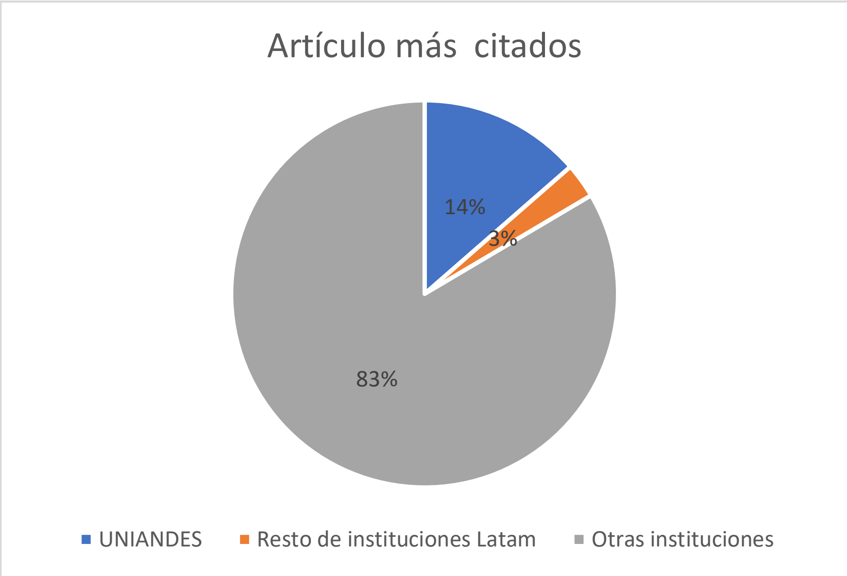

Figura 11. Artículo más Citados
Fuente: Publish or Perish

En cuanto a libros producidos en la región se observa un producción científica sostenida fundamentalmente en temas relacionados a la pedagogía (4 libros) y admiración de empresas(2 libros) de un total de 8. Un análisis basado en la teoría de grafos utilizando el software VOSWIER

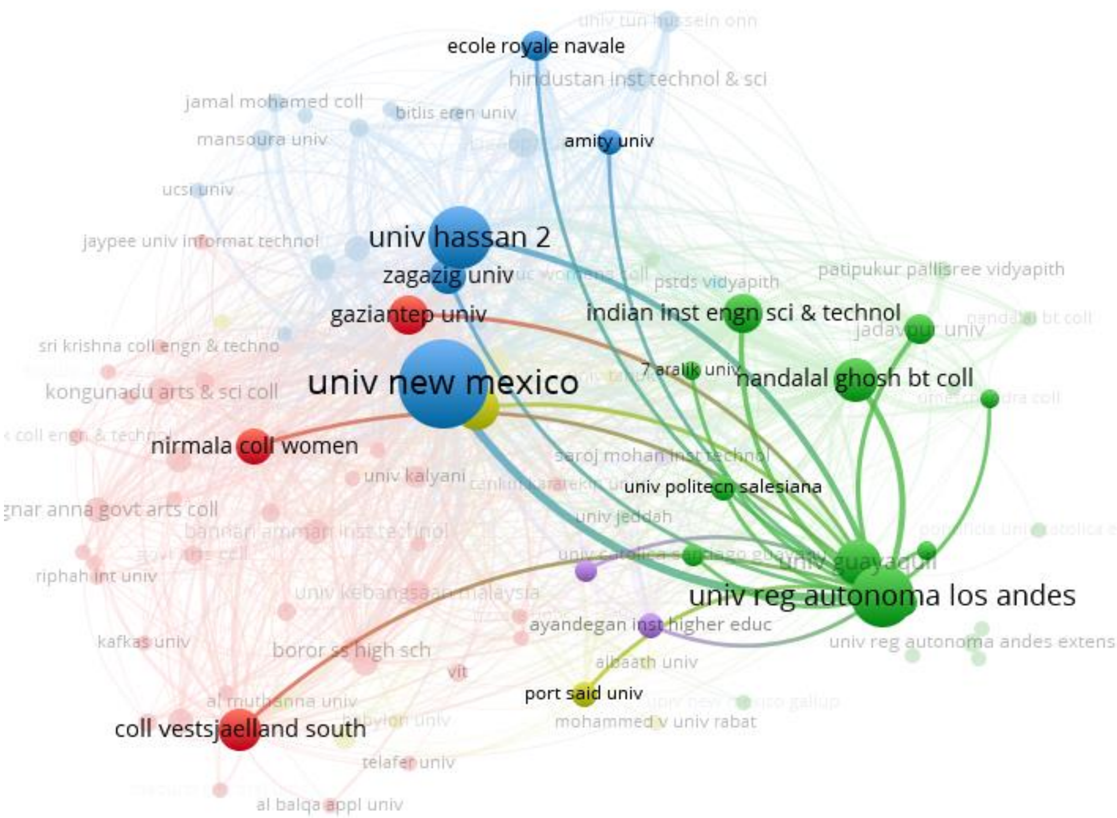

Figura 8. Análisis basado en la teoría de grafos
Fuente: VosViewer

Los resultados muestran que las principales instituciones de región están agrupadas de acuerdos a los temas de investigación destacándose la Universidad Regional Autónoma de los Andes

CONCLUSIONES

En el presente trabajo se analizó el impacto de teoría neutrosófica en Latinoamérica sus principales impulsores y estado de la investigación. Se destaca el aumento de las publicaciones desde la creación de la Asociación Latinoamericana de Ciencia Neutrosófica. Las áreas más abordadas se encuentran en la interrelación de las ciencias sociales y la neutrosofía presentando resultados destacados en eso ámbitos de la investigación. Como universidad e instituciones más destacadas se encuentran la Universidad Regional Autónoma de los andes en Ecuador y la Universidad de las Ciencias Informáticas en Cuba.

Como trabajos futuros se plantea la realización de un análisis Bibliométrico basado en la el análisis de redes complejas para realizar un ranking de investigadores e instituciones.

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Neutrosophic Local Function and Generated Neutrosophic Topology

A.A. Salama, Florentin Smarandache

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Abstract: In this paper we introduce the notion of ideals on neutrosophic set which is considered as a generalization of fuzzy and fuzzy intuitionistic ideals studies in [9,11] , the important topological neutrosophic ideals has been given in [4]. The concept of neutrosophic local function is also introduced for a neutrosophic topological space. These concepts are discussed with a view to find new neutrosophic topology from the original one in [8]. The basic structure, especially a basis for such generated neutrosophic topologies and several relations between different topological neutrosophic ideals and neutrosophic topologies are also studied here. Possible application to GIS topology rules are touched upon.
Keywords: Neutrosophic Set; Intuitionistic Fuzzy Ideal; Fuzzy Ideal; Topological neutrosophic ideal; and Neutrosophic Topology.

1. Introduction

The neutrosophic set concept was introduced by Smarandache [12, 13]. In 2012 neutrosophic sets have been investigated by Hanafy and Salama at el [4, 5, 6, 7, 8, 9, 10]. The fuzzy set was introduced by Zadeh [14] in 1965, where each element had a degree of membership. In 1983 the intuitionstic fuzzy set was introduced by K. Atanassov [1, 2, 3] as a generalization of fuzzy set, where besides the degree of membership and the degree of non- membership of each element. Salama at el [9] defined intuitionistic fuzzy ideal for a set and generalized the concept of fuzzy ideal concepts, first initiated by Sarker [10]. Neutrosophy has laid the foundation for a whole family of new mathematical theories generalizing both their classical and fuzzy counterparts. In this paper we will introduce the definitions of normal neutrosophic set, convex set, the concept of α-cut and topological neutrosophic ideals, which can be discussed as generalization of fuzzy and fuzzy intuitionistic studies.

2. Terminologies

We recollect some relevant basic preliminaries, and in particular, the work of Smarandache in [12, 13], Hanafy and Salama at el. [4, 5, 6, 7, 8, 9, 10].

3. Topological Neutrosophic Ideals [4].

Definition 3.1: Let X is non-empty set and L a non–empty family of NSs. We will call L is a topological neutrosophic ideal (NL for short) on X if

- $A \in L$ and $B \subseteq A \Rightarrow B \in L$ [heredity],
- $A \in L$ and $B \in L \Rightarrow A \vee B \in L$ [Finite additivity].

A topological neutrosophic ideal L is called a σ-topological neutrosophic ideal if

$\{A_j\}_{j \in N} \leq L$, implies $\underset{j \in J}{\vee} A_j \in L$ (countable additivity).

The smallest and largest topological neutrosophic ideals on a non-empty set X are $\{0_N\}$ and NSs on X. Also, $N.L_f$, $N.L_c$ are denoting the topological neutrosophic ideals (NL for short) of neutrosophic subsets having finite and countable support of X respectively. Moreover, if A is a nonempty NS in X, then $\{B \in NS : B \subseteq A\}$ is an NL on X. This is called the principal NL of all NSs of denoted by NL$\langle A \rangle$.

Remark 3.2.

- If $1_N \notin L$, then L is called neutrosophic proper ideal.
- If $1_N \in L$, then L is called neutrosophic improper ideal.
- $O_N \in L$.

Example 3.3.

Any Initiutionistic fuzzy ideal ℓ on X in the sense of Salama is obviously and NL in the form $L = \{A : A = \langle x, \mu_A, \sigma_A, \nu_A \rangle \in \ell\}$.

Example 3.4.

Let $X = \{a, b, c\}$ $A = \langle x, 0.2, 0.5, 0.6 \rangle$, $B = \langle x, 0.5, 0.7, 0.8 \rangle$, and $D = \langle x, 0.5, 0.6, 0.8 \rangle$, then the family $L = \{O_N, A, B, D\}$ of NSs is an NL on X.

Example.3.5

Let $X = \{a, b, c, d, e\}$ and $A = \langle x, \mu_A, \sigma_A, \nu_A \rangle$ given by:

X	$\mu_A(x)$	$\sigma_A(x)$	$\nu_A(x)$
a	0.6	0.4	0.3
b	0.5	0.3	0.3
c	0.4	0.6	0.4
d	0.3	0.8	0.5
e	0.3	0.7	0.6

Then the family $L = \{O_N, A\}$ is an NL on X.

Definition.3.3: Let L1 and L2 be two NL on X. Then L2 is said to be finer than L1 or L1 is coarser than L2 if L1 $\leq$ L2. If also L1 $\neq$ L2. Then L2 is said to be strictly finer than L1 or L1 is strictly coarser than L2. Two NL said to be comparable, if one is finer than the other. The set of all NL on X is ordered by the relation L1 is coarser than L2 this relation is induced the inclusion in NSs. The next Proposition is considered as one of the useful result in this sequel, whose proof is clear.

Proposition.3.1: Let $\{L_j : j \in J\}$ be any non - empty family of topological neutrosophic ideals on a set X. Then $\bigcap_{j\in J} L_j$ and $\bigcup_{j\in J} L_j$ are topological neutrosophic ideal on X, In fact L is the smallest upper bound of the set of the Lj in the ordered set of all topological neutrosophic ideals on X.

Remark.3.2: The topological neutrosophic ideal by the single neutrosophic set O_N is the smallest element of the ordered set of all topological neutrosophic ideals on X.

Proposition.3.3: A neutrosophic set A in topological neutrosophic ideal L on X is a base of L iff every member of L contained in A.

Proof: (Necessity)Suppose A is a base of L. Then clearly every member of L contained in A. (Sufficiency) Suppose the necessary condition holds. Then the set of neutrosophic subset in X contained in A coincides with L by the Definition 4.3.

Proposition.3.4: For a topological neutrosophic ideal L1 with base A, is finer than a fuzzy ideal L2 with base B iff every member of B contained in A.

Proof: Immediate consequence of Definitions

Corollary.3.1: Two topological neutrosophic ideals bases A, B, on X are equivalent iff every member of A, contained in B and via versa.

Theorem.3.1: Let $\eta = \{\langle \mu_j, \sigma_j, \gamma_j \rangle : j \in J\}$ be a non empty collection of neutrosophic subsets of X. Then there exists a topological neutrosophic ideal L (η) = {A $\in$ NSs: A $\subseteq$ $\vee$ Aj} on X for some finite collection {Aj: j = 1,2, ..., n $\subseteq$ η}.

Proof: *Clear.*

Remark.3.3: The topological neutrosophic ideal L (η) defined above is said to be generated by η and η is called sub base of L(η).

Corollary.3.2: Let L1 be an topological neutrosophic ideal on X and A $\in$ NSs, then there is a topological neutrosophic ideal L2 which is finer than L1 and such that A $\in$ L2 if $A \vee B \in L2$ *for each* $B \in L1$.

Corollary.3.3: *Let* $A = \langle x, \mu_A, \sigma_A, \nu_A \rangle \in L_1$ *and* $B = \langle x, \mu_B, \sigma_B, \nu_B \rangle \in L_2$, *where* L_1 *and* L_2 *are topological neutrosophic ideals on the set X. then the neutrosophic set* $A*B = \langle \mu_{A*B}(x), \sigma_{A*B}(x), \nu_{A*B}(x) \rangle \in L_1 \vee L_2$ *on X where* $\mu_{A*B}(x) = \vee\{\mu_A(x) \wedge \mu_B(x) : x \in X\}, \sigma_{A*B}(x)$ *may be* = $\vee\{\sigma_A(x) \wedge \sigma_B(x)\}$ *or* $\wedge\{\sigma_A(x) \vee \sigma_B(x)\}$ *and* $\nu_{A*B}(x) = \wedge\{\nu_A(x) \vee \nu_B(x) : x \in X\}$.

4. Neutrosophic local Functions

Definition.4.1. Let (X, τ) be a neutrosophic topological spaces (NTS for short) and L be neutrsophic ideal (NL, for short) on X. Let A be any NS of X. Then the neutrosophic local function $NA^*(L,\tau)$ of A is the union of all neutrosophic points(NP, for short) $C(\alpha,\beta,\gamma)$ such that if $U \in N(C(\alpha,\beta,\gamma))$ and $NA^*(L,\tau) = \vee\{C(\alpha,\beta,\gamma) \in X : A \wedge U \notin L \text{ for every U nbd of } C(\alpha,\beta,\gamma)\}$, $NA^*(L,\tau)$ is called a neutrosophic local function of A with respect to τ and L which it will be denoted by $NA^*(L,\tau)$, or simply $NA^*(\mathrm{L})$.

Example .4.1. One may easily verify that.

If L= $\{0_N\}$, then $\mathrm{N}\,A^*(L,\tau)=Ncl(A)$, for any neutrosophic set $A\in NSs$ on X.

If $\mathrm{L}=\{\text{all NSs on X}\}$ then $\mathrm{NA}^*(L,\tau)=0_N$, for any $A\in NSs$ on X.

Theorem.4.1. Let (X,τ) be a NTS and L_1,L_2 be two topological neutrosophic ideals on X. Then for any neutrosophic sets A, B of X. then the following statements are verified

i) $A\subseteq B\Rightarrow NA^*(L,\tau)\subseteq NB^*(L,\tau),$

ii) $L_1\subseteq L_2\Rightarrow NA^*(L_2,\tau)\subseteq NA^*(L_1,\tau)$.

iii) $NA^*=Ncl(A^*)\subseteq Ncl(A)$.

iv) $NA^{*^*}\subseteq NA^*$.

v) $N(A\vee B)^*=NA^*\vee NB^*$.,

vi) $N(A\wedge B)^*(L)\leq NA^*(L)\wedge NB^*(L).$

vii) $\ell\in L\Rightarrow N(A\vee\ell)^*=NA^*.$

viii) $NA^*(L,\tau)$ is neutrosophic closed set .

Proof.

i) Since $A\subseteq B$, let $p=C(\alpha,\beta,\gamma)\in NA^*(L_1)$ then $A\wedge U\notin L$ for every $U\in N(p)$. By hypothesis, we get $B\wedge U\notin L$, then $p=C(\alpha,\beta,\gamma)\in NB^*(L_1)$.

ii) Clearly. $L_1\subseteq L_2$ Implies $NA^*(L_2,\tau)\subseteq NA^*(L_1,\tau)$ as there may be other IFSs which belong to L_2 so that for GIFP $p=C(\alpha,\beta,\gamma)\in NA^*$ but $C(\alpha,\beta,\gamma)$ may not be contained in $NA^*(L_2)$.

iii) Since $\{O_N\}\subseteq L$ for any NL on X, therefore by (ii) and Example 3.1, $NA^*(L)\subseteq NA^*(\{O_N\})=Ncl(A)$ for any NS A on X. Suppose $p_1=C_1(\alpha,\beta,\gamma)\in Ncl(NA^*(L_1))$. So for every $U\in N(p_1)$, $NA^*\wedge U\neq O_N$, there exists $p_2=C_2(\alpha,\beta)\in A^*(L_1)\wedge U)$ such that for every V nbd of $p_2\in N(p_2), A\wedge U\notin L.$ Since $U\wedge V\in N(p_2)$ then $A\wedge(U\cap V)\notin L$ which leads to $A\wedge U\notin L$, for every $U\in N(C(\alpha,\beta))$ therefore $p_1=C(\alpha,\beta)\in (A^*(L\ ))$ and so $Ncl(NA^*)\leq NA^*$ While, the other inclusion follows directly. Hence $NA^*=Ncl(NA^*)$.But the inequality $NA^*\leq Ncl(NA^*)$.

iv) The inclusion $NA^*\vee NB^*\leq N(A\vee B)^*$ follows directly by (i). To show the other implication, let $p=C(\alpha,\beta,\gamma)\in N(A\vee B)^*$ then for every $U\in N(p),$ $(A\vee B)\wedge U\notin L, i.e,$ $(A\wedge U)\vee(B\wedge U)\notin L.$ then, we have two cases $A\wedge U\notin L$ and $B\wedge U\in L$ or the converse, this means that exist $U_1,U_2\in N(C(\alpha,\beta,\gamma)$ such that $A\wedge U_1\notin L$, $B\wedge U_1\notin L,$ $A\wedge U_2\notin L$ and $B\wedge U_2\notin L$. Then $A\wedge(U_1\wedge U_2)\in L$ and $B\wedge(U_1\wedge U_2)\in L$ this gives $(A\vee B)\wedge(U_1\wedge U_2)\in L,$ $U_1\wedge U_2\in N(C(\alpha,\beta,\gamma)$ which contradicts the hypothesis. Hence the equality holds in various cases.

v) By (iii), we have $NA^{*^*}=Ncl(NA^*)^*\leq Ncl(NA^*)=NA^*$ let (X,τ) be a GIFTS and L be GIFL on X. Let us define the neutrosophic closure operator $cl^*(A)=A\cup A^*$ for any GIFS A of X. Clearly, let $Ncl^*(A)$ is a neutrosophic operator. Let $N\tau^*(L)$ be NT generated by Ncl^*

.i.e $N\tau^*(L)=\{A: Ncl^*(A^c)=A^c\}$.

Now $L=\{O_N\} \Rightarrow Ncl^*(A)=A\cup NA^* = A\cup Ncl(A)$ for every neutrosophic set A. So, $N\tau^*(\{O_N\})=\tau$. Again $L=\{all$ NSs on X$\}$ $\Rightarrow$ $Ncl^*(A)=A,$ because $NA^*=O_N$, for every neutrosophic set A so $N\tau^*(L)$ is the neutrosophic discrete topology on X. So we can conclude by Theorem 4.1.(ii). $N\tau^*(\{O_N\})=N\tau^*(L)$

i.e. $N\tau \subseteq N\tau^*$, for any neutrosophic ideal L_1 on X. In particular, we have for two topological neutrosophic ideals L_1, and L_2 on X, $L_1\subseteq L_2 \Rightarrow N\tau^*(L_1)\subseteq N\tau^*(L_2)$.

Theorem.4.2. Let τ_1, τ_2 be two neutrosophic topologies on X. Then for any topological neutrosophic ideal L on X, $\tau_1 \le \tau_2$ implies $NA^*(L,\tau_2)\subseteq NA^*(L,\tau_1)$, for every A $\in L$ then $N\tau^*{}_1 \subseteq N\tau^*{}_2$

Proof. Clear.

A basis $N\beta(L,\tau)$ for $N\tau^*(L)$ can be described as follows:

$N\beta(L,\tau)=$ $\{A-B: A\in\tau, B\in L\}$ Then we have the following theorem

Theorem 4.3. $N\beta(L,\tau)=\{A-B: A\in\tau, B\in L\}$ Forms a basis for the generated NT of the NT (X,τ) with topological neutrosophic ideal L on X.

Proof. Straight forward. The relationship between τ and $_N\tau^*$ (L) established throughout the following result which have an immediately proof .

Theorem 4.4. Let τ_1, τ_2 be two neutrosophic topologies on X. Then for any topological neutrosophic ideal L on X, $\tau_1 \subseteq \tau_2$ implies $N\tau^*{}_1 \subseteq N\tau^*{}_2$.

Theorem 4.5: Let (X,τ) be a NTS and L_1, L_2 be two neutrosophic ideals on X. Then for any neutrosophic set A in X, we have

i) $NA^*(L_1\vee L_2,\tau)=NA^*(L_1, N\tau^*(L_1)\ )\wedge NA^*(L_2, N\tau^*(L_2))$

ii) $N\tau^*(L_1\vee L_2)=(N\tau^*(L_1))^*(L_2)\wedge N(\tau^*(L_2))^*(L_1)$

Proof Let $p=C(\alpha,\beta)\notin(L_1\vee L_2,\tau)$, this means that there exists $U_p\in N(P)$ such that $A\wedge U_p\in(L_1\vee L_2)$ i.e. There exists $\ell_1\in L_1$ and $\ell_2\in L_2$ such that $A\wedge U_p\in(\ell_1\vee\ell_2)$ because of the heredity of L$_1$, and assuming $\ell_1\wedge\ell_2=0_N$ .Thus we have $(A\wedge U_p)-\ell_1=\ell_2$ and $(A\wedge U_p)-\ell_2=\ell_1$ therefore $(U_p-\ell_1)\wedge A=\ell_2\in L_2$ and $(U_p-\ell_2)\wedge A=\ell_1\in L_1$. Hence $p=C(\alpha,\beta,\gamma)\notin NA^*(L_2, N\tau^*(L_1))$, or $p=C(\alpha,\beta,\gamma)\notin NA^*(L_1, N\tau^*(L_2))$, because p must belong to either ℓ_1 or ℓ_2 but not to both. This gives $NA^*(L_1\vee L_2,\tau)\ge NA^*(L_1, N\tau^*(L_1)\ )\wedge NA^*(L_2, N\tau^*(L_2))$ .To show the second inclusion, let us assume $p=C(\alpha,\beta,\gamma)\notin NA^*(L_1, N\tau^*(L_2))$. This implies that there exist $U_p\in N(P)$ and $\ell_2\in L_2$ such that $(U_p-\ell_2)\wedge A\in L_1$. By the heredity of L_2, if we assume that $\ell_2\le A$ and define $\ell_1=(U_p-\ell_2)\wedge A$. Then we have $A\wedge U_p\in(\ell_1\vee\ell_2)\in L_1\vee L_2$. Thus, $NA^*(L_1\vee L_2,\tau)\le NA^*(L_1,\tau^*(L_1)\ )\wedge NA^*(L_2, N\tau^*(L_2))$ and similarly, we can get $A^*(L_1\vee L_2,\tau)\le A^*(L_2,\tau^*(L_1)\ )$. This gives the other inclusion, which complete the proof.

Corollary 4.1.Let (X,τ) be a NTS with topological neutrosophic ideal L on X. Then

i) $NA^*(L,\tau)=NA^*(L,\tau^*)$ and $\mathrm{N}\tau^*(L)=N(N\tau^*(L))^*(L)$.

ii) $N\tau^*(L_1\vee L_2)=(N\tau^*(L_1))\ \vee(N\tau^*(L_2))$

Proof. Follows by applying the previous statement.

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Delphi method for evaluating scientific research proposals in a neutrosophic environment

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Abstract. The scientific research proposal is part of the task to be carried out in academic and research institutions around the world. This is a complex decision-making problem, because decision-makers must determine the projects that are appropriate to the subjects addressed by the institution, those projects must be achievable within a reasonable deadline, they must have the financial means and the budget necessary to be carried out, the staff must be sufficiently qualified and an optimum number of personnel must be available to succeed the tasks and not interfere with other research projects. This is a predictive problem, thus, the proposed model is based on Delphi method for evaluating research projects and is supported by neutrosophy. Delphi method is widely applied in the prediction of future events, in this model we introduce the uncertainty and indeterminacy modeled with neutrosophy. As the best of our knowledge, this model is the first one, which applies a neutrosophic Delphi method in the evaluation of scientific research proposals. Finally, a hypothetical case study illustrates the applicability of the method.

Keywords: Research proposal, Delphi method, fuzzy Delphi method, single valued neutrosophic set, single valued triangular neutrosophic number.

1 Introduction

A research proposal is a document that proposes a research project, usually in science or academia, and that usually constitutes a request for sponsorship of such research, see [1-10]. The proposals are evaluated on the cost and potential impact of the research, and on the robustness of the proposed plan to carry it out. Generally, research proposals address several critical points, including the following:

- Which research questions will be addressed and how they will be addressed,
- How much time and expenses will be needed for research,
- What previous research has been done on the subject,
- How research results will be evaluated,
- How research will benefit the sponsoring organization and other parties.

Research proposals could be requested, which means that they are sent in response to a request with specific requirements, such as a request for proposals, or may be unsolicited, which means that they are sent without prior request. Other types of proposals include "pre-proposals", where a letter of intent or documentary summary is sent for review prior to the submission of a full proposal; follow-up proposals, which reiterate an original proposal and its funding requirements to ensure continued funding; and proposals for renewal, which seek the continued sponsorship of a project that would otherwise be terminated.

Academic research proposals are usually written as part of the initial requirements for writing a thesis, research work or dissertation. In general, they follow the same format as a research work, with an introduction, a review of literature, a discussion of the methodology and objectives of research, and a conclusion. This basic structure can vary between projects and between fields, each of which may have its own requirements.

The scientific method is a methodology for obtaining new knowledge, which has historically characterized science, consisting of systematic observation, measurement, experimentation, and the formulation, analysis and modification of hypotheses. Other characteristics of the scientific method are deduction, induction, abduction, prediction, falsifiability, the reproducibility and repeatability of the results, and the peer review. The rules and principles of the scientific method seek to minimize the influence of the subjectivity of the scientist in his\her work, which reinforces the validity of the results, and therefore of the knowledge obtained.

The selection of the most appropriate research topic in the academic or the research institution is not a trivial problem, it needs of the assessment of the topic relevance in the near future, that is, it is a predictive problem. It is also complex, since the decision depends on different factors, some of them depend on the institution's researchers and others are external. This complexity of the problem requires of the experts' opinion on the subject, rather than measuring with objective indicators. The experts are those who can carry out the selection of the most promising projects, which give visibility to the institution and at the same time those projects must be achievable within a reasonable time. Other aspects to be considered are that researchers must have the capacity to attain the selected projects, that there exists the optimal number of scientific personnel working on the project, that the institution must have the necessary financial support to accomplish the research, and the project must be sufficiently relevant such that it can be published in high-impact scientific journals in a relatively short period of time or it results in patents, palpable economic and social results, among others.

Due to the problem complexity and the large number of variables to consider, the proposal selection contains elements of uncertainty and at the also experts could have doubts, ignorance, inconsistencies, among other elements.

This paper aims to propose a model for research proposals selection and evaluation. This model is based on the Delphi method, which is used in predicting future scenarios or events through expert assessment, see [11, 12]. Basically Delphi method is based on the intuitive idea that a group of experts will come to better conclusions than only one of them. The Delphi method consists of applying questionnaires to a group of experts, anonymously, and then each of them gives a response in a first round. The index of agreement between experts is then calculated using a central tendency statistical measure, and if the agreement is not sufficient, a second round is conducted for the experts to reconsider their assessments and so on until sufficient consensus is reached among them. One criticism of the method is that it can converge very slowly and therefore some experts may not continue to collaborate, nevertheless this is a widely used method.

Other authors have extended this method into uncertainty environments, for example fuzzy Delphi includes uncertainty and represents it in form of fuzzy sets, in particular fuzzy numbers are used, see [13-17]. Ishikawa et al. in [18] propose a fuzzy Delphi method where a survey is designed in such a way that a single round is sufficient to perform the calculations. In general, the fuzzy Delphi method has application in several real problems, see [13-17].

Other approaches are based on neutrosophy, which generalizes fuzzy sets, fuzzy intuitionist sets, among others. In the context of neutrosophy Delphi method takes into account the neutrality given by contradictions, ignorance, inconsistencies, among other ones, typical of decision-making. Some papers model fuzzy Delphi method into a neutrosophic framework, see [19, 20]. Abdel-Basset et al. use Delphi method combined with AHP, in a neutrosorphic environment, see [21].

The model proposed in this paper is based on the Delphi method, which helps to select a set of scientific research proposals in a neutrosophic environment. We have not found in the consulted literature the use of Delphi method applied in this topic in a neutrosophic environment. This neutrosophic Delphi method uses single value triangular neutrosophic number, see [22]. The method takes advantage of the possibility of evaluating research proposals in form of linguistic terms, in addition to considering the uncertainty and indeterminacy inherent to neutrosophy frameworks. It is a decision-making model because it allows the evaluation of project alternatives by criteria. In addition, we explicitly set out the minimum criteria that should be considered in conducting evaluations.

The paper consists of the following structure; after this introduction follows Section 2 which contains the concepts necessary to design the model, such as the basic concepts of neutrosophy, its aggregation operators, among others, as well as a brief explanation of the Delphi method. Section 3 describes the proposed model and provides an illustrative case study of the application of the model in a real-life problem. The paper finishes with the conclusions.

2 Basic concepts

This section discusses the concepts and methods to be used throughout this article. Section 2.1. contains a brief explanation of the classic Delphi method, whereas Section 2.2. contains the main concepts of neutrosophy, among them we can find, neutrosophic sets, single valued neutrosophic sets, single valued triangular neutrosophic numbers, aggregation operators for single valued triangular neutrosophic numbers, among other concepts of interest.

2.1 The Delphi method

The Delphi method is a structured communication technique, which is developed as an interactive systematic prediction method, based on a panel of experts, see [11, 12]. It aims to achieve a consensus based on discussion among experts. It is a repetitive process, where its operation is based on the elaboration of a questionnaire to be answered by the experts. Once the information is received, another questionnaire based on the previous one is re-performed to be answered again.

Finally, the study will draw its conclusions from the statistical analysis of the obtained data.

The Delphi as a methodology of forecasting uses expert judgments in technology or social processes considering the responses to a questionnaire to examine the likely guidelines for the development of specific technologies, meta-types of technologies or different processes of social change. The summary of expert judgments (in the forms of quantitative assessments and written comments) are provided as feedback to the experts themselves as parts of a next round of questionnaire. Experts then reassess their views in the light of this information, and a group consensus tends to emerge. The Delphi technique is based on firm concepts to draw conclusions with supported arguments.

Delphi is based on:

- Anonymity of participants.
- Repeatability and controlled feedback.
- Group response in statistical form.

Before starting Delphi, a number of previous tasks are performed, such as:

- Define the context and time horizon in which the forecast on the subject under study is to be made.
- Select the panel of experts and get the commitment to collaboration. People who are elected should not only be very knowledgeable about the subject on which the study is being conducted, but should present a plurality in their approaches. This plurality should avoid the appearance of biases in the information available in the panel.
- Explain to experts what the method is. This is intended to get obtaining reliable forecasts, because the experts are going to know at all times what is the objective of each of the processes required by the methodology.

The core of Delphi technique is a series of questionnaires. The first questionnaire may include general questions. At each later stage, the questions become more specific because they are formed with the answers to the previous questionnaire.

The Delphi technique comprises at least three phases:

1. A questionnaire is sent to a group of experts.
2. A summary of the first phase is prepared.
3. A summary of the second phase is prepared.

Three phases are usually recommended, but more phases can be used, as in the safety management Delphi study.

The number of experts involved can range from just a few to more than 100, depending on the scope of the issue. A range of 15-30 is recommended for a focal issue. As long as experts participate, the costs as well as the coordination required for the technique will also be raised.

2.2 Basic concepts of neutrosophy

Neutrosophy is a branch of philosophy that studies the origin, nature and scope of neutralities, as well as their interactions with different ideological spectra. In mathematics and logic, the most important concept is the neutrosorphic set that generalizes the fuzzy sets of Zadeh and the fuzzy intuitionist sets of Atanassov, in the following these definitions are formally defined.

Definition 1: ([22]) The *Neutrosophic set* N is characterized by three membership functions, which are the truth-membership function T_A, indeterminacy-membership function I_A, and falsity-membership function F_A, where U is the Universe of Discourse and $\forall x \in U, TA(x), IA(x), FA(x) \subseteq]-0, 1+[, and\ -0 \leq inf\ TA(x) + inf\ IA\ (x)\ +\ inf\ FA\ (x) \leq sup\ TA(x) +\ sup\ IA\ (x)\ +\ sup\ FA\ (x) \leq 3+.$

See that according to Definition 3, $T_A(x)$, $I_A(x)$, and $F_A(x)$ are real standard or non-standard subsets of $]-0, 1+[\ and\ hence, TA(x), IA(x)\ and\ FA(x)\ can\ be\ subintervals\ of\ [0, 1]$.

Definition 2: ([22]) The *Single-Valued Neutrosophic Set* (SVNS) N over U is $A = \{< x;\ TA(x), IA(x), FA(x) >: x \in U\}, where\ TA: U \rightarrow [0, 1], IA: U \rightarrow [0, 1], and\ FA: U \rightarrow [0, 1], 0 \leq TA(x)\ + IA(x)\ + FA(x) \leq 3.$

The *Single-Valued Neutrosophic number* (SVNN) is symbolized by $N\ =\ (t, i, f\ )$, such that $0 \leq t, i, f \leq 1$ and $0 \leq t\ +\ i\ +\ f \leq 3$.

Definition 3: ([22]) The *single-valued triangular neutrosophic number* $\tilde{a} = \langle (a_1, a_2. a_3); \alpha_{\tilde{a}}, \beta_{\tilde{a}}, \gamma_{\tilde{a}} \rangle$, is a neutrosophic set on $\mathbb{R}$, whose truth, indeterminacy and falsity membership functions are defined as follows, respectively:

$$T_{\tilde{a}}(x) = \begin{cases} \alpha_{\tilde{a}}\left(\frac{x-a_1}{a_2-a_1}\right), & a_1 \le x \le a_2 \\ \alpha_{\tilde{a}}, & x = a_2 \\ \alpha_{\tilde{a}}\left(\frac{a_3-x}{a_3-a_2}\right), & a_2 < x \le a_3 \\ 0, & \text{otherwise} \end{cases} \tag{1}$$

$$I_{\tilde{a}}(x) = \begin{cases} \dfrac{\left(a_2 - x + \beta_{\tilde{a}}(x - a_1)\right)}{a_2 - a_1}, & a_1 \le x \le a_2 \\ \beta_{\tilde{a}}, & x = a_2 \\ \dfrac{\left(x - a_2 + \beta_{\tilde{a}}(a_3 - x)\right)}{a_3 - a_2}, & a_2 < x \le a_3 \\ 1, & \text{otherwise} \end{cases} \tag{2}$$

$$F_{\tilde{a}}(x) = \begin{cases} \dfrac{\left(a_2 - x + \gamma_{\tilde{a}}(x - a_1)\right)}{a_2 - a_1}, & a_1 \le x \le a_2 \\ \gamma_{\tilde{a}}, & x = a_2 \\ \dfrac{\left(x - a_2 + \gamma_{\tilde{a}}(a_3 - x)\right)}{a_3 - a_2}, & a_2 < x \le a_3 \\ 1, & \text{otherwise} \end{cases} \tag{3}$$

Where $\alpha_{\tilde{a}}, \beta_{\tilde{a}}, \gamma_{\tilde{a}} \in [0,1]$, $a_1, a_2, a_3 \in \mathbb{R}$ and $a_1 \le a_2 \le a_3$.

Definition 4: ([22]) Given $\tilde{a} = \langle (a_1, a_2, a_3); \alpha_{\tilde{a}}, \beta_{\tilde{a}}, \gamma_{\tilde{a}} \rangle$ and $\tilde{b} = \langle (b_1, b_2, b_3); \alpha_{\tilde{b}}, \beta_{\tilde{b}}, \gamma_{\tilde{b}} \rangle$ two single-valued triangular neutrosophic numbers and λ any non null number in the real line. Then, the following operations are defined:

19. Addition: $\tilde{a} + \tilde{b} = \langle (a_1 + b_1, a_2 + b_2, a_3 + b_3); \alpha_{\tilde{a}} \wedge \alpha_{\tilde{b}}, \beta_{\tilde{a}} \vee \beta_{\tilde{b}}, \gamma_{\tilde{a}} \vee \gamma_{\tilde{b}} \rangle$
20. Subtraction: $\tilde{a} - \tilde{b} = \langle (a_1 - b_3, a_2 - b_2, a_3 - b_1); \alpha_{\tilde{a}} \wedge \alpha_{\tilde{b}}, \beta_{\tilde{a}} \vee \beta_{\tilde{b}}, \gamma_{\tilde{a}} \vee \gamma_{\tilde{b}} \rangle$
21. Inversion: $\tilde{a}^{-1} = \langle (a_3^{-1}, a_2^{-1}, a_1^{-1}); \alpha_{\tilde{a}}, \beta_{\tilde{a}}, \gamma_{\tilde{a}} \rangle$, where $a_1, a_2, a_3 \neq 0$.
22. Multiplication by a scalar number:
$$\lambda\tilde{a} = \begin{cases} \langle (\lambda a_1, \lambda a_2, \lambda a_3); \alpha_{\tilde{a}}, \beta_{\tilde{a}}, \gamma_{\tilde{a}} \rangle, & \lambda > 0 \\ \langle (\lambda a_3, \lambda a_2, \lambda a_1); \alpha_{\tilde{a}}, \beta_{\tilde{a}}, \gamma_{\tilde{a}} \rangle, & \lambda < 0 \end{cases}$$
23. Division of two triangular neutrosophic numbers:
$$\frac{\tilde{a}}{\tilde{b}} = \begin{cases} \left\langle \left(\frac{a_1}{b_3}, \frac{a_2}{b_2}, \frac{a_3}{b_1}\right); \alpha_{\tilde{a}} \wedge \alpha_{\tilde{b}}, \beta_{\tilde{a}} \vee \beta_{\tilde{b}}, \gamma_{\tilde{a}} \vee \gamma_{\tilde{b}} \right\rangle, a_3 > 0 \; and \; b_3 > 0 \\ \left\langle \left(\frac{a_3}{b_3}, \frac{a_2}{b_2}, \frac{a_1}{b_1}\right); \alpha_{\tilde{a}} \wedge \alpha_{\tilde{b}}, \beta_{\tilde{a}} \vee \beta_{\tilde{b}}, \gamma_{\tilde{a}} \vee \gamma_{\tilde{b}} \right\rangle, a_3 < 0 \; and \; b_3 > 0 \\ \left\langle \left(\frac{a_3}{b_1}, \frac{a_2}{b_2}, \frac{a_1}{b_3}\right); \alpha_{\tilde{a}} \wedge \alpha_{\tilde{b}}, \beta_{\tilde{a}} \vee \beta_{\tilde{b}}, \gamma_{\tilde{a}} \vee \gamma_{\tilde{b}} \right\rangle, a_3 < 0 \; and \; b_3 < 0 \end{cases}$$
24. Multiplication of two triangular neutrosophic numbers:
$$\tilde{a}\tilde{b} = \begin{cases} \langle (a_1b_1, a_2b_2, a_3b_3); \alpha_{\tilde{a}} \wedge \alpha_{\tilde{b}}, \beta_{\tilde{a}} \vee \beta_{\tilde{b}}, \gamma_{\tilde{a}} \vee \gamma_{\tilde{b}} \rangle, & a_3 > 0 \; and \; b_3 > 0 \\ \langle (a_1b_3, a_2b_2, a_3b_1); \alpha_{\tilde{a}} \wedge \alpha_{\tilde{b}}, \beta_{\tilde{a}} \vee \beta_{\tilde{b}}, \gamma_{\tilde{a}} \vee \gamma_{\tilde{b}} \rangle, & a_3 < 0 \; and \; b_3 > 0 \\ \langle (a_3b_3, a_2b_2, a_1b_1); \alpha_{\tilde{a}} \wedge \alpha_{\tilde{b}}, \beta_{\tilde{a}} \vee \beta_{\tilde{b}}, \gamma_{\tilde{a}} \vee \gamma_{\tilde{b}} \rangle, & a_3 < 0 \; and \; b_3 < 0 \end{cases}$$

Where, $\wedge$ is a t-norm and $\vee$ is a t-conorm.

Let $\tilde{a} = \langle (a_1, a_2, a_3); \alpha_{\tilde{a}}, \beta_{\tilde{a}}, \gamma_{\tilde{a}} \rangle$ be a single valued triangular neutrosophic number, then,

$$S(\tilde{a}) = \frac{1}{8}[a_1 + a_2 + a_3](2 + \alpha_{\tilde{a}} - \beta_{\tilde{a}} - \gamma_{\tilde{a}}) \tag{4}$$

$$\mathbf{A}(\tilde{a}) = \frac{1}{8}[a_1 + a_2 + a_3](2 + \alpha_{\tilde{a}} - \beta_{\tilde{a}} + \gamma_{\tilde{a}}) \tag{5}$$

They are called the score and accuracy degrees of ã, respectively.

Let $\{\widetilde{A}_1, \widetilde{A}_2, \cdots, \widetilde{A}_n\}$ be a set of n SVTNNs, where $\widetilde{A}_j = \langle (a_j, b_j, c_j); \alpha_{\tilde{a}_j}, \beta_{\tilde{a}_j}, \gamma_{\tilde{a}_j} \rangle$ (j = 1, 2, …, n), then the *weighted mean of the SVTNNs* is calculated with the following Equation:

$$\widetilde{A} = \sum_{j=1}^{n} \lambda_j \widetilde{A}_j \tag{6}$$

Where λ_j is the weight of A_j, $\lambda_j \in [0, 1]$ and $\sum_{j=1}^{n} \lambda_j = 1$.

3 The Delphi model in the neutrosophic environment

This section is dedicated to describe the model proposed in this paper. Let us start with two tables, Tables 1 and 2. Table 1 contains the scale for measuring the weights of the criteria and Table 2 summarizes the scale of evaluations associated with the single-valued triangular neutrosophic numbers (SVNTN). We finish with a hypothetical case study.

3.1 The method

Linguistic terms	SVTNN
Extremely unimportant (EU)	$\langle (0,0,1); 0.00, 1.00, 1.00 \rangle$
Not very important (NVI)	$\langle (0,1,3); 0.17, 0.85, 0.83 \rangle$
Not important (NI)	$\langle (1,3,5); 0.33, 0.75, 0.67 \rangle$
Medium (M)	$\langle (3,5,7); 0.50, 0.50, 0.50 \rangle$
Important (I)	$\langle (5,7,9); 0.67, 0.25, 0.33 \rangle$
Very important (VI)	$\langle (7,9,10); 0.83, 0.15, 0.17 \rangle$
Extremely important (EI)	$\langle (9,10,10); 1.00, 0.00, 0.00 \rangle$

Table 1. Importance weight as linguistic variables and their associated SVTNN.

Linguistic term	SVTNN
Very low (VL)	$\langle (0,0,1); 0.00, 1.00, 1.00 \rangle$
Medium low (ML)	$\langle (0,1,3); 0.17, 0.85, 0.83 \rangle$
Low (L)	$\langle (1,3,5); 0.33, 0.75, 0.67 \rangle$
Medium(M)	$\langle (3,5,7); 0.50, 0.50, 0.50 \rangle$
High (H)	$\langle (5,7,9); 0.67, 0.25, 0.33 \rangle$
Medium high (MH)	$\langle (7,9,10); 0.83, 0.15, 0.17 \rangle$
Very high (VH)	$\langle (9,10,10); 0.00, 1.00, 1.00 \rangle$

Table 2: Linguistic terms for evaluations associated with SVTNN.

Let us observe two important aspects, which are the following:

1. The scales shown in Tables 1 and 2 are inspired by the linguistic scales in [16]. SVTNNs are obtained by rescaling the original 0-1 scale to a 0-10 scale. The values $\alpha_{\tilde{a}}, \beta_{\tilde{a}}, \gamma_{\tilde{a}}$ are adapted from another scale appeared in [16].
2. The scale shown in Table 2 was linguistically taken in such a way, because the survey questions asked to experts will be done in the form of probability of events occurrence, which will be evaluated linguistically as Low, Medium low, etc.

Let us observe that the values obtained above are more accurate than fuzzy numbers, because they contain more elements; not only the belongingness, but also the non-belongingness and the indeterminacy.

The algorithm for evaluating research proposals that we offer is as follows:

1. Starts from a subject or group of subjects that are usually investigated in the institution.
2. Experts on the proposed subject or subjects are selected to evaluate the projects and at least one moderator. The experts will be denoted by $E_1, E_2, \dots, E_n$.
   Usually each academic or research institution has a group of specialists who are part of the scientific council that is where the scientific projects of the institution are discussed. This group of people could be used to carry out the evaluations, although external experts are also useful. Experts need not to be in touch with each other, so the moderator must design, implement and process the surveys.
3. The n experts are asked to propose projects that can serve as a research proposal on the basis of the topic identified in the previous point. Each of them proposes at least one, they are called $p_1, p_2, \dots, p_m$.
4. Experts could be asked to identify the criteria they consider for evaluating projects. However we suggest the following criteria:
   C_1: The project is a sufficiently relevant scientific contribution to the subject being investigated over a sufficiently long period of time.
   C_2: The project is scientifically achievable in a sufficiently short time.
   C_3: The institution has sufficient qualified staff to carry out the project.
   C_4: Sufficient personnel are available to conduct the investigation.
   C_5: There are the means and the budget necessary to carry out the research.
   C_6: The desired results will be obtained (publications in high-impact journals, patents, discussion of Master's or doctoral theses, solution of a real-life problem, etc.) in a reasonable time.
   C_7: The project serves as a basis or starting point for another project.
   C_8: The project gives scientific visibility, economic income, prestige, etc. to the institution.
   The survey can contain questions as follows:
   4.1. What do you think is the probability that project *P* will become a sufficiently relevant scientific contribution to the subject under investigation over a sufficiently long period of time?
   4.2. What do you think is the probability that project *P* will be scientifically achievable in a sufficiently short time?
   4.3. What do you think is the probability that the institution will have sufficiently qualified staff to carry out project *P*?
   4.4. What do you think is the probability that the institution will have sufficient available staff to carry out project *P*?
   4.5. What do you think is the probability that the institution will have the means and the budget to carry out project *P*?
   4.6. What do you think is the probability that the desired results of project *P* will be achieved in a reasonable time?
   4.7. What do you think is the probability that project *P* will serve as a basis for another project?
   4.8. What do you think is the probability that project *P* will provide scientific visibility, economic income, prestige, etc. to the institution?
   The answers will be given on the basis of the linguistic scale shown in Table 2.
   Each of the experts assesses the importance of each of the criteria. $\widetilde{w}_{ij}$ shall denote the linguistic value according to Table 1, which expert E_i associates with the criterion C_j (i = 1, 2,..., n; j = 1, 2, ..., 8).

$\mathrm{w}_{ij} = \boldsymbol{A}(\widetilde{w}_{ij})$ (i = 1, 2,..., n; j = 1, 2, ..., 8) is calculated using formula 5. Then they are normalized with respect to each expert, let us use the notation $\mathrm{W}_{ij} = \frac{\mathrm{w}_{ij}}{\sum_{\mathrm{i}=1}^{\mathrm{n}} w_{ij}}$.

5. Each expert E_i (i = 1, 2,..., n) evaluates each project p_k (k = 1, 2,..., m) with respect to criteria C_j (j = 1,2,..,8).
6. For each expert E_i (i = 1, 2,..., n), the evaluation of each project p_k (k = 1, 2,..., m) is obtained by aggregating their values by criterion using the W_{ij} weights in Formula 6. So we have an evaluation of each expert for each project. Let us denote by P_{ik} the evaluation of the k^th^ project by the i^th^ expert in form of the SVTNN associated with the linguistic term in Table 2.
7. It is calculated $\overline{\mathrm{P}}_k = \frac{\sum_{i=1}^{n} \mathrm{P_{ik}}}{n}$ which is the mean of the evaluation of each project for all experts.
8. The *Consensus Indexes* for each project $\mathrm{p_k}$ are calculated with formula $\mathrm{CI_k} = \frac{\sum_{i=1}^{n} |\boldsymbol{A}(\mathrm{P_{ik}}) - \boldsymbol{A}(\overline{\mathrm{P}}_\mathrm{k}))|}{n}$.
9. If $\mathrm{CI_k} \leq 0.2$, see [16], then there exists sufficient expert consensus for all projects and go to Step 11, otherwise there is no consensus and go to point 10.

10. The moderator anonymously informs each of the experts about the results. He\she asks for explanations for each of them, including the weights assigned by them to the criteria and go to a next round. Emphasis is placed on those projects that reached a consensus index of $\mathrm{CI_k} > 0.2$, which reduces the algorithm complexity when concentrating recalculation only on those projects where there was not satisfactory consensus. Next go to point 5.
11. $\widetilde{\mathrm{P}}_k = \boldsymbol{A}(\overline{\mathrm{P}}_k)$ are calculated according to Equation 5. $\widetilde{\mathrm{P}}_k$ is ordered, where projects with higher values are preferred over those with lower values.
    Finish.

3.2 Case Study of case: Comparative analysis

A research group of an academic institution has as its research topic artificial intelligence applied to digital image processing. The institution wishes to work on new projects on the subject so that they become doctoral theses of some members. Supervisors wish to determine which projects could be approved to obtain doctoral theses from a group of members within a maximum of five years. To this end, they decide to apply the method we propose in this article as follows:

1. They decided that the general theses themes should be "artificial intelligence applied to digital image processing".
2. They select one moderator within the institution. The moderator selects the panel of experts on the subject to carry out evaluations. Five experts were selected; let us denote them by E_1, E_2, E_3, E_4, E_5. None of them knows the identity of the others, which is why the moderator keeps in touch with each one via email. Any queries that experts have to make about the institution's data are directly asked to the moderator.
3. The moderator consults them to ask for proposed projects on the subject. These were four; let us call them p_1, p_2, p_3, p_4. This consultation process remains anonymous.
4. The moderator distributes the survey with the questions in Step 4 of the algorithm and asks them to evaluate the importance of each of the given criteria on the linguistic scale in Table 1.
   The results were as follows as in Table 3 for linguistic evaluations and Table 4 for the crisp values of the normalized weights of evaluations in Table 3:

Criteria\Weight given by:	E_1	E_2	E_3	E_4	E_5
C_1	I	VI	I	M	I
C_2	VI	I	I	VI	I
C_3	VI	VI	M	I	VI
C_4	I	VI	I	I	VI
C_5	VI	I	VI	I	I
C_6	M	I	M	I	M
C_7	M	M	M	I	I
C_8	I	I	M	EI	I

Table 3: Importance given by experts to the criteria in form of linguistic terms.

Criteria\Weight given by:	E_1	E_2	E_3	E_4	E_5
C_1	0.122729	0.150978	0.145338	0.076947	0.121720
C_2	0.157475	0.117665	0.145338	0.152047	0.121720
C_3	0.157475	0.150978	0.094375	0.118498	0.156181
C_4	0.122729	0.150978	0.145338	0.118498	0.156181
C_5	0.157475	0.117665	0.186485	0.118498	0.121720
C_6	0.079694	0.117665	0.094375	0.118498	0.121720
C_7	0.079694	0.076406	0.094375	0.118498	0.121720
C_8	0.122729	0.117665	0.094375	0.178516	0.121720

Table 4: Importance given by experts to the criteria in form of normalized crisp values.

5. Each expert evaluates each project on the basis of the criteria. The results are given in Tables 5-9.

Criteria\Project:	p_1	p_2	p_3	P_4
C_1	H	M	L	H
C_2	M	M	VH	M

C_3	L	ML	H	L
C_4	L	VL	L	H
C_5	L	M	H	VL
C_6	M	ML	H	L
C_7	L	M	VH	L
C_8	L	ML	VH	VL

Table 5: Projects evaluated per criterion by Expert 1.

Criteria\Project:	**p_1**	**p_2**	**p_3**	**P_4**
C_1	VH	ML	M	MH
C_2	ML	H	H	H
C_3	L	L	MH	ML
C_4	M	ML	M	MH
C_5	L	H	MH	L
C_6	MH	L	H	VL
C_7	M	VL	H	VL
C_8	M	L	VH	L

Table 6: Projects evaluated per criterion by Expert 2

Criteria\Project:	**p_1**	**p_2**	**p_3**	**P_4**
C_1	M	VL	VL	VH
C_2	L	ML	M	H
C_3	M	L	M	M
C_4	VL	ML	ML	MH
C_5	M	H	MH	L
C_6	M	L	MH	VL
C_7	M	MH	H	VL
C_8	M	VL	H	L

Table 7: Projects evaluated per criterion by Expert 3.

Criteria\Project:	**p_1**	**p_2**	**p_3**	**P_4**
C_1	MH	MH	VL	MH
C_2	L	MH	H	MH
C_3	ML	L	MH	VL
C_4	ML	L	ML	MH
C_5	VL	ML	VH	L
C_6	VL	L	MH	VL
C_7	ML	H	H	ML
C_8	ML	L	MH	VL

Table 8: Projects evaluated per criterion by Expert 4.

Criteria\Project:	**p_1**	**p_2**	**p_3**	**P_4**
C_1	M	ML	ML	VH
C_2	H	H	H	VH
C_3	ML	L	VH	VL
C_4	ML	ML	M	M
C_5	VL	L	M	L
C_6	VL	L	MH	VL
C_7	VL	H	H	VL
C_8	L	VL	H	L

Table 9: Projects evaluated per criterion by Expert 5.

6. Table 10 contains the project evaluation by each expert after aggregating the set of projects, using the criteria weights of Table 4 in form of SVTNN.

Expert\Project	p_1	p_2	p_3	P_4
E_1	$\langle(1.97, 3.97, 5.97); 0.33, 0.75, 0.67\rangle$	$\langle(1.55, 2.95, 4.82); 0.00, 1.00, 1.00\rangle$	$\langle(5.46, 7.10, 8.38); 0.33, 0.75, 0.67\rangle$	$\langle(2.02, 3.46, 5.18); 0.00, 1.00, 1.00\rangle$
E_2	$\langle(3.49, 5.22, 6.80); 0.17, 0.85, 0.83\rangle$	$\langle(1.56, 3.11, 5.03); 0.00, 1.00, 1.00\rangle$	$\langle(5.40, 7.29, 8.78); 0.50, 0.50, 0.50\rangle$	$\langle(2.94, 4.40, 5.90); 0.00, 1.00, 1.00\rangle$
E_3	$\langle(2.27, 3.98, 5.84); 0.00, 1.00, 1.00\rangle$	$\langle(1.78, 3.01, 4.68); 0.00, 1.00, 1.00\rangle$	$\langle(3.63, 5.19, 6.77); 0.00, 1.00, 1.00\rangle$	$\langle(3.62, 5.09, 6.47); 0.00, 1.00, 1.00\rangle$
E_4	$\langle(0.69, 1.68, 3.37); 0.00, 1.00, 1.00\rangle$	$\langle(2.73, 4.61, 6.38); 0.17, 0.85, 0.83\rangle$	$\langle(5.33, 6.94, 8.21); 0.00, 1.00, 1.00\rangle$	$\langle(2.55, 3.60, 4.84); 0.00, 1.00, 1.00\rangle$
E_5	$\langle(1.10, 2.14, 3.82); 0.00, 1.00, 1.00\rangle$	$\langle(1.57, 3.05, 4.93); 0.00, 1.00, 1.00\rangle$	$\langle(4.62, 6.34, 7.95); 0.17, 0.85, 0.83\rangle$	$\langle(2.90, 3.95, 5.10); 0.00, 1.00, 1.00\rangle$

Table 10: Projects evaluated by Experts.

7. From Table 10 it is obtained $\overline{P}_1 = \langle(1.904, 3.398, 5.16); 0.00, 1.00, 1.00\rangle$, $\overline{P}_2 = \langle(1.838, 3.346, 5.168); 0.00, 1.00, 1.00\rangle$, $\overline{P}_3 = \langle(4.888, 6.572, 8.018); 0.00, 1.00, 1.00\rangle$, and $\overline{P}_4 = \langle(2.806, 4.100, 5.498); 0.00, 1.00, 1.00\rangle$. In addition we have $\widetilde{P}_1 = \boldsymbol{A}(\overline{P}_1) = 2.6155$, $\widetilde{P}_2 = \boldsymbol{A}(\overline{P}_2) = 2.5880$, $\widetilde{P}_3 = \boldsymbol{A}(\overline{P}_3) = 4.8695$, and $\widetilde{P}_4 = \boldsymbol{A}(\overline{P}_4) = 3.1010$.
8. Table 11 contains the crisp values by applying the accuracy function to the values in Table 10.

Expert\Project	p_1	p_2	p_3	P_4
E_1	3.3497	2.3300	5.8894	2.6650
E_2	4.1683	2.4250	6.7094	3.3100
E_3	3.0225	2.3675	3.8975	3.7950
E_4	1.4350	3.6872	5.1200	2.7475
E_5	1.7650	2.3875	5.0821	2.9875

Table 11: Projects evaluated by Experts in form of crisp values.

Therefore, the Consensus Indexes for the projects are the following:
$CI_1 = 0.945$, $CI_2 = 0.38824$, $CI_3 = 0.85898$, and $CI_4 = 0.36120$.
9. Let us observe, all of them are greater than 0.2, which means that another round is necessary. Thus, go to the next point.
10. The moderator informs the experts on the results and requests that each one reconsider the weights given to the criteria and evaluations. The process should be repeated in a second round. We must go to Step 5, but we will not repeat this for simplicity. See that if any of the projects had achieved $CI_k \leq 0.2$ it would not be taken into account for the next round and calculations and effort would simplify. Experts should concentrate more on reaching agreement on projects 1 and 3.
11. We obtain $\widetilde{P}_1 = 2.6155$, $\widetilde{P}_2 = 2.5880$, $\widetilde{P}_3 = 4.8695$, and $\widetilde{P}_4 = 3.1010$.
Thus, so far we have $p_3 \succ p_4 \succ p_1 \succ p_2$ and project p_3 is the preferred. Nevertheless, the moderator has to repeat the round.

Conclusion

This paper was devoted to introduce a method of evaluation and selection of scientific research proposals in academic or research institutions. The model is basically a Delphi method in a neutrosorphic environment. This method supports the research project selection according to a group of experts' criteria. The Delphi method ensures that the opinion is agreed among the experts. The neutrosorphic framework offers the advantage of including not only uncertainty, but also indeterminacy in decision-making. Another advantage is that experts carry out evaluations with the help of linguistic scales, which makes the final results more veridical. In addition, the limitation of the classic Delphi method on slow convergence is attenuated, since with this model the projects that are reevaluated in the next round are only those where there was not sufficient consensus. To our knowledge, this is the first time that a model like this is designed which combines the Delphi method in a neutrosophic framework for the solution of this kind of problem. The paper provides the criteria to be followed for measurement, which does not mean that they are not modifiable. A hypothetical case study illustrates how to use the method and demonstrates its usefulness.

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Resolucion de sistemas de ecuaciones lineales neutrosóficas mediante computación simbólica

Solving neutrosophic linear equations systems using symbolic computation

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Maikel Leyva-Vázquez, Florentin Smarandache (2021). Resolucion de sistemas de ecuaciones lineales neutrosóficas mediante computación simbólica (Solving neutrosophic linear equations systems using symbolic computation). *Neutrosophic Computing and Machine Learning* 15, 41-46

Resumen. En este trabajo, aplicamos el concepto de números neutrosóficos para resolver un sistema de ecuaciones lineales neutroficas que utilizan la computación simbólica. Además, utilizamos Jupyter, que es apoyado en el Colaborador de Google para realizar el cómputo simbólico. La biblioteca simbólica de Python se utiliza para realizar el proceso de computación neutrosófica. Los sistemas de ecuaciones lineales neutrosóficas se resuelven a través de la computación simbólica en Python. Se ha desarrollado un estudio de caso para la determinación del tráfico vehicular con indeterminación. Este rey de la computación abre nuevas formas de tratar la indeterminación en problemas del mundo real.

Palabras claves: Computación neutrosófica, sympy, google colaborativo, número neutrosófico, ecuaciones lineales Neutrosóficas.

Abstract. In this paper, we apply the concept of neutrosophic numbers to solve a systems of neutrophic linear equations using symbolic computation. Also, we utilize Jupyter, which is supported in Google Colaboratory for performing symbolic computation. The sympy library of Python is used to perform the process of neutrosophic computation. Systems of neutrosophic linear equations are solved through symbolic computation in Python. A case study was developed for the determination of vehicular traffic with indeterminacy. This king of computation opens new ways to deal with indeterminacy in real-world problems.

Keywords: neutrosophic computing, sympy, google colaboratory, neutrosophic number, neutrosophic linear equations

INTRODUCCIÓN

The word Neutrosophy means knowledge of neutral thought, and this third / neutral represents the main distinction, i.e., the neutral/indeterminate / unknown part (in addition to "truth" / "belonging" and "falsehood" Components of "non-belonging" that appear in the fuzzy logic / set). Neutrosophic logic (NL) is a generalization of Zadeh's fuzzy logic (FL), and especially of Atanassov's intuitionistic fuzzy logic (IFL), and other multi-valued logics, see Figure 1 and [1].

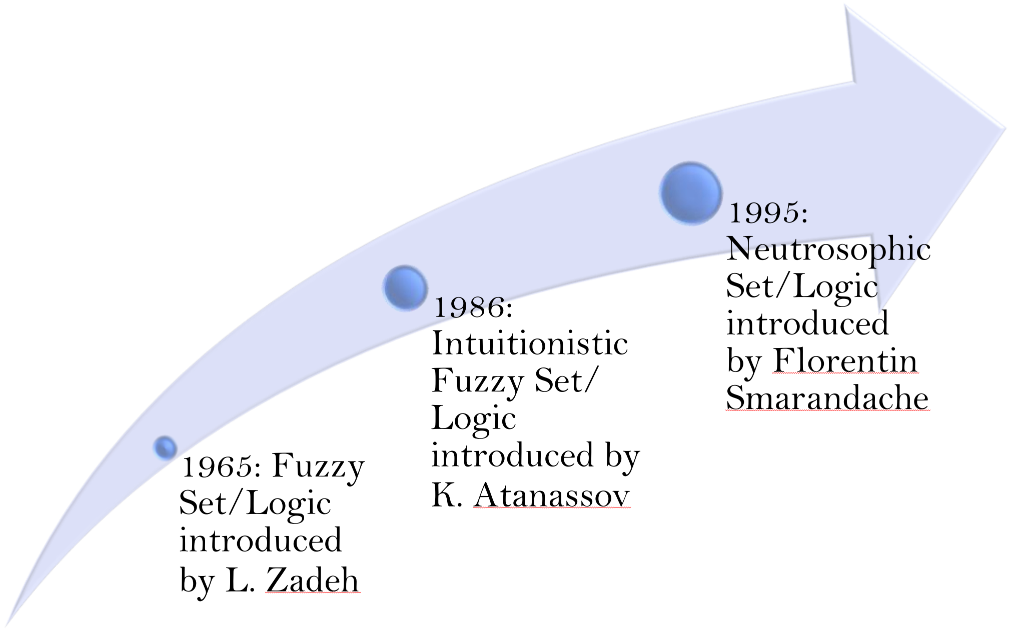

Figure 1. Neutrosophy and its fundamental antecedents ([1]).

Let U be a universe of discourse, and M a set included in U. An element x of U which is denoted with respect to M as x (T, I, F) and belongs to M according to the Neutrosophic Set as follows: it is t % true in the set, i% indeterminate (unknowns) in the set, and f% false, where t varies in T, i varies in I and f varies in F. Statically T, I, F are subsets, but dynamically T, I, F are functions/operators that depend on many known or unknown parameters [2, 3].

Neutrosophic sets generalize the fuzzy sets (especially the fuzzy and fuzzy intuitionist sets), the paraconsistent sets. It allows dealing with a more significant number of situations that occur in real-life [4].

2. PRELIMINARY

A *statistical neutrosophic number* is a number given in the following form ([5]):

$$N=d+i \tag{1}$$

Where d is the *determined part* and i is the *indeterminate part*, see [6]. For example: $a=8.6+i$ if $i\in[3, 3.4]$ the number is equivalent to $a\in [11.6, 12]$.

Additionally, a *neutrosophic matrix* is a matrix such that the elements $a = (a_{ij})$ have been replaced by elements in $\langle R\cup I\rangle$, where $\langle R\cup I\rangle$ is a *neutrosophic ring*, see [7].

A *neutrosophic graph* is a kind of graph in which at least one arc is a *neutrosophic arc*, see [8]. The weights associated with the arcs have the following meanings: 0 = "no connection between nodes", 1 = "connection between nodes", I = "indeterminate connection (unknown if it is or not)". The *neutrosophic adjacency matrix* is an adjacency matrix with at least one neutrosophic arc. Such notions are not used in fuzzy theory, an example of which is shown below:

$$\begin{pmatrix} 0 & 0 & I \\ I & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}$$

Further, we shall describe practical implementations of this approach. *Google Colaboratory* is a web application that allows us to create and share documents containing source codes, equations, visualizations, and illustrative texts, as shown in Figure 2.

Figure 2. Google Colaboratory

Jupyter allows us to interact with several programming languages; in this case, Python is used. Python is a simple and powerful programming language with access to a great variety of useful libraries.

3. NEUTROSOPHIC COMPUTING AND SYMPY

SymPy can be used for computational works with neutrosophic numbers in Python language. It is a library written in Python language with the purpose of bringing together all the features of a computer algebra system, which is easily extensible and maintains the code as simple as possible ([9]).

Here we propose a calculus that is based on interval-valued arithmetic and accordingly, a de-neutrosification process is required [10] to obtain a representative numeric value. Thus, I $\in[0,1]$ is replaced by its maximum and minimum values.

In this case we employ the mpmath library and the mpi type [11]. The mpi type deals with intervals of a pair of mpf values. Interval arithmetic uses conservative rounding, so that, if an interval is interpreted as the numerical uncertainty with respect to a single-valued number, any sequence of interval operations will produce intervals that contain the result of applying the same sequence of operations to a precise number, see Figure 3.

Figure 3. Working with Neutrosophic Numbers

In this case systems of neutrosophic linear equations can be solved, see [12]. For example, given the following system of equations:

$$3x+8y = 1+i \quad (2)$$
$$4x + 7y = 8+i \quad (3)$$

It is solved as follows, see Figure 4:

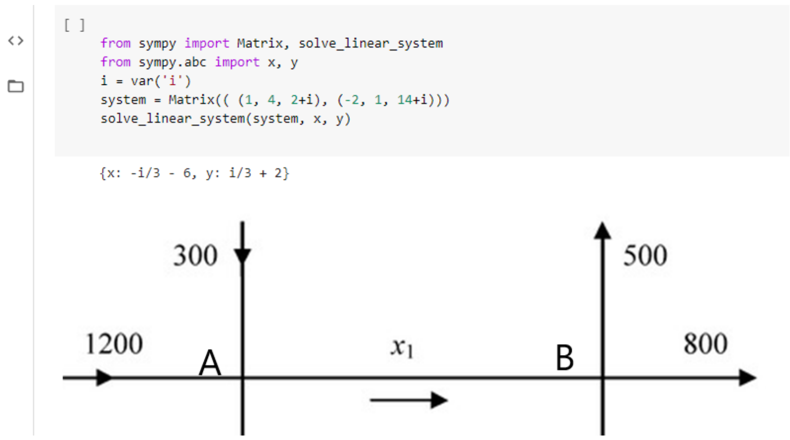

Figure 4. Solution to a system of linear equations with Neutrosophic Numbers, by using Colaboratory.

An example of application in a real-life problem is a system of linear equations that determines the flow of traffic at different intercepts, see Figure 5

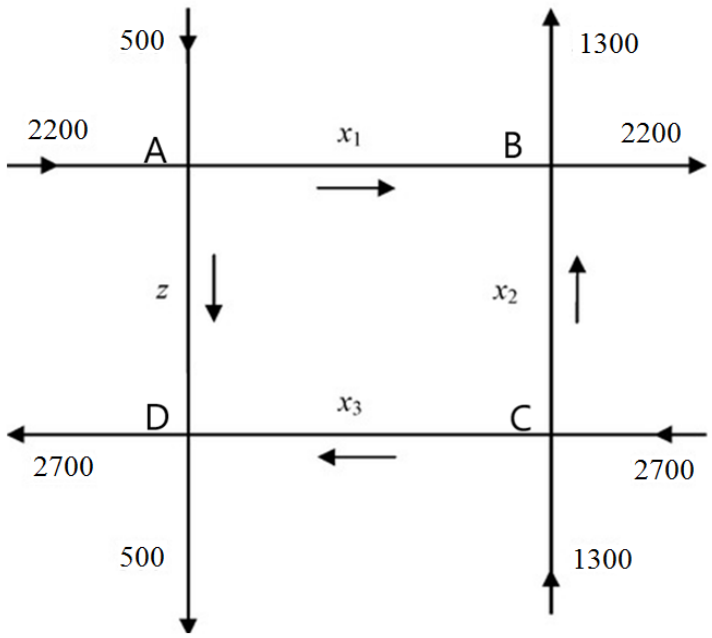

Figure 5. Schematic representation of the Vehicle Flow ([12]).

At each intercept, the outflow must be equal to the inflow.

Intercept A: 2700=x1+z
Intercept B: 3500=x1+x2
Intercept C: 4000=x2+x3
Interception D: 3200=x3+z
If z=500.
Then the system of equations is as follows:
x1=2200
x1+x2=3500
x2+2x3=6700
The solution for this system is the following:
x1=2200
x2=1300
x3=2700

```
from sympy import Matrix, solve_linear_system
from sympy.abc import x, y, z

system = Matrix(( (1,0,0, 1100), (1, 1, 0, 1300), (0, 1, 2,3400)))

solve_linear_system(system, x, y, z)

{x: 1100, y: 200, z: 1600}
```

```
from sympy import Matrix, solve_linear_system
from sympy.abc import x, y, z

system = Matrix(( (1,0,0, 1100-i), (1, 1, 0, 1300), (0, 1, 2,3400-i)))

solve_linear_system(system, x, y, z)

{x: -i + 1100, y: i + 200, z: -i + 1600}
```

Figure 6. A solution of the vehicular flow with indetermination.

In the case of z =500+i.
Then the system of equations is:
x1=2200-i
x1+x2=3500
x1+2x3=6700-i
The solution for this system is:
x1=2200-i
x2=1300+i
x3=2250

This kind of computation allows opening new ways to compute using indeterminacy in different real-world problems [13-19].

CONCLUSIONES

In this paper, the concept of neutrosophic numbers was used to solve linear equations. The tool Jupyter supported by Google Colaboratory is utilized. The Sympy library is applied to perform the neutrosophic computation process. Neutrosophic linear equation systems are solved by symbolic python computation. A case study is developed for the determination of vehicular traffic with indetermination.

Future works include the development of new applications in different areas of engineering and science like neutrosophic statistics and multicriteria. Other areas of future work include the development of new tools for neutrosophic computing.

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Neutrosophic Perspective of Neutrosophic Probability Distributions and its Application

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Abstract

Neutrosophical probability is concerned with inequitable and defective topics and processes. This is a subset of Neutrosophic measures that includes a prediction of an event (as opposed to indeterminacy) as well as a prediction of some unpredictability. When there is no such thing as a non-stochastic occurrence, the Neutrosophic probability is the probability of determining a stochastic process. It is a generalisation of classical probability, which states that the probability of correctly predicting an occurrence is zero. Until now, neutrosophic probability distributions have been derived directly from conventional statistical distributions, with fewer contributions to the determination of the for statistical distribution. We introduced the Poission distribution as a limiting case of the Binomial distribution for the first time in this study, and we also proposed Neutrosophic Exponential Distribution and Uniform Distribution for the first time. With numerical examples, the validity and soundness of the proposed notions were also tested.

Keywords: Neutrosophic Statistics, Poisson,Uniform,Exponential, Probability distribution.

1.Introduction

The classical probability distributions only work with data that is given to them. The study of classical distributions with uncertain values, as well as distribution factors such as time intervals, is aided by this research. Neutrosophic probability distributions are the name given to these distributions. Prof. Florentin Smarandache proposed neutrosophic statistics for the first time in 1995. This is frequently presented as a novel area of philosophy, described as a generalisation of formal logic[1] and an extension of the intuitionistic fuzzy logic that underpins intuitionistic logic[2]. The core notions of Neutrosophic set, as described by smarandach in [3,4,5,6] and salama et al. in [7,8,9,10], provides a new framework for handling indeterminacy data concerns. The indeterminacy data can be numbers, and Neutrosophic numbers are well-defined in [11,12,13,14,15,16].

In this paper, we demonstrate how the Neutrosophic crisp sets theory [17,18] is prone to cater to difficulties that follow classical distributions while still containing data that isn't precisely described. This paves the way for dealing with issues such as whether a specific sort of data can hold all of the information it was created to hold. When classical distributions are extended to be compatible with neutrosophic logic, the parameters of the classical distribution take on indeterminate values. This aids in dealing with any situation that may emerge while working with statistical data, particularly unclear and incorrect data. In 2014, Florentin Smarandache [19,20,21] supplied both the Neutrosophic and Neutrosophic natural distributions.

In this paper, we proposed Poisson distribution as a Limiting case of Binomial distribution ,exponential distribution and uniform distribution on Neutrosophic numbers with illustrative example for the first time.

The remainder of the paper is laid out as follows. Basic definitions have been supplied in section 2 to aid comprehension. In section 3, neutrosophic Poisson distribution is proposed as a limiting case of neutrosophic Binomial distribution. In section 4, one real world problem is solved using the proposed concept. In Section 5 Neutrosophic exponential distribution proposed. In section 6 application problem of Neutrosophic exponential distribution were discussed, In section 7 Neutrosophic Uniform distribution proposed and in section 8 application of Neutrosophic uniform distribution were discussed. In section 9, we conclusion of this work is given.

2.Basics Definition

For a better understanding of the proposed notion, we offered basic concepts of neutrosophic statistical distribution in this part.

Neutrosophic Statistical Distribution:[3]

When we repeat the experiment $n \geq 1$times, the Neutrosophic binomial random variable 'x' is determined as the number of successes. The Neutrosophic binomial probability distribution is also known as the Neutrosophic probability distribution of 'x'.

Neutrosophic Binomial Random Variables: It is marked as 'x' and is determined as the number of successes when we repeat the experiment $n \geq 1$ times.

Neutrosophic Binomial Probability Distribution: Neutrosophic probability distribution is the name given to the Neutrosophic probability distribution of 'x'.

Indeterminacy: It is not constrained by the findings of experiments (Successes and Failures).

Indeterminacy Threshold: It's the amount of trials when the outcome isn't known where $th \in \{0.1.2....n\}$

Let Prob(S) =The probability of success of a given trial

Prob(F) = The probability of a given trial leads to failure, for the two S and F different from indeterminacy.

Prob(I) = The probability of a particular trial results in an indeterminacy.

For example: for $x \in \{0,1,2....n\}, NP = \{T_x, I_x, F_x\}$

Tr_x: Chances of 'x' success, (n-x) failures, and indeterminacy, with the number of indeterminacy equal to or less than the threshold of indeterminacy.

Fal_x: Chances of 'y' success, with $y \neq x$ and $(n-y)$ failures and indeterminacy is less than the indeterminacy threshold.

Ind_x: Indeterminacy probabilities for 'z', when 'z' is strictly bigger than the indeterminacy threshold.

$$Tr_x + Ind_x + Fal_x = \left(\Pr ob\left(S\right) + \Pr ob(I) + \Pr ob(F)\right)^n$$

For complete probability, $\Pr ob(S) + \Pr ob(I) + \Pr ob(F) = 1;$

For incomplete probability, $0 \leq \Pr ob(S) + \Pr ob(I) + \Pr ob(F) < 1;$

For paraconsistent probability, $0 < \Pr ob(S) + \Pr ob(I) + \Pr ob(F) \leq 3;$

Now $$Tr_x = \frac{n!}{x!\left(n-x\right)!}.\Pr ob(S)^x.\sum_{k=0}^{th} C_{n-x}^k \Pr ob(I)^k \Pr ob(F)^{n-x-k}$$

$$Tr_x = \frac{n!}{x!}\Pr ob(S)^x \sum_{k=0}^{th} \frac{\Pr ob(I)^k \Pr ob(F)^{n-x-k}}{k!\left(n-x-k\right)!}$$ …………………..(1)

Equ(1) implies the Truth membership function of Neutrosophic Binomial Distribution.

$$Ind_x = \sum_{z=th+1}^{n} \frac{n!}{z!\left(n-z\right)!}.\Pr ob(I)^z.\left\lfloor \sum_{k=0}^{th} C_{n-z}^k \Pr ob(S)^z \Pr ob(F)^{n-z-k} \right\rfloor$$

$$Ind_x = \sum_{z=th+1}^{n} \frac{n!}{z!\left(n-z\right)!}.\Pr ob(I)^z.\left[\sum_{k=0}^{n-z} \frac{\left(n-z\right)!}{k!\left(n-z-k\right)!} \Pr ob(S)^k \Pr ob(F)^{n-z-k}\right]$$

$$Ind_x = \sum_{z=th+1}^{n} \frac{n!}{z!}.\Pr ob(I)^z.\left\lfloor \sum_{k=0}^{n-z} \frac{\Pr ob(S)^k \Pr ob(F)^{n-z-k}}{k!\left(n-z-k\right)!} \right\rfloor$$

…………………………(2)

Equ(2) implies the Indeterminacy of Neutrosophic Binomial Distribution.

$$Fal_x = \sum_{\substack{y=0 \\ y\neq x}}^{n} T_y = \sum_{\substack{y=0 \\ y\neq x}}^{n} \frac{n!}{y!}\Pr ob(S)^y \left\lfloor \sum_{k=0}^{th} \frac{\Pr ob(I)^k \Pr ob(F)^{n-y-k}}{k!\left(n-y-k\right)!} \right\rfloor$$

…………………………………………………….. (3)

Equ(3) implies the Falsity of Neutrosophic Binomial Distribution.

Where C_u^v means combinations of 'u' elements taken by groups of v elements: $C_u^v = \dfrac{u!}{v!\left(u-v\right)!}$

Score Function of a single valued neutrosophic number:[11]

The score function of the single valued Neutrosophic number $b=\left(t\left(b\right),i\left(b\right),f\left(b\right)\right)$ can be communicated as follows: $S(b)=\dfrac{1+t\left(b\right)-f\left(b\right)}{2}$ for $s(b)\in\left[0,1\right]$

3. Neutrosophic Poisson Distribution (NPD) - Limiting case of Neutrosophic Binomial Distribution (NBD)

In this section, we proposed NPD as a limiting case of NBD under single valued neutrosophic environment.

The Neutrosophic Binomial probability distribution is a law of success, indeterminacy and failure and in a series of 'n' independent trials is defined in Equation (1)-(3)

Neutrosophic Binomial Distribution under limiting case of Poisson distribution when

(i) 'n' is indefinitely large (i.e.,) $n\to\infty$
(ii) P(S),P(I),P(S) is very small when P(I),P(F),P(S) tends to zero.
(iii) $np=\lambda$ (a finite quantity)

By the definition of neutrosophic Binomial distribution truth membership of is given in Equ(1) as follows

$$T_x=\frac{n!}{x!}\Pr ob(S)^x\sum_{k=0}^{th}\frac{\Pr ob(I)^k \Pr ob(F)^{n-x-k}}{k!\left(n-x-k\right)!}$$

The truth membership function of NPD can be deterned using Eqn (1).:

$$Tr_x=\lambda(S)^x\sum_{k=0}^{th}\frac{n\left(n-1\right)\left(n-2\right)..........\left(n-k-x-1\right)}{x!k!}\left(\frac{\lambda(F)}{n}\right)^k\left(1-\frac{\lambda(I)}{n}\right)^{n-x-k}$$

$$=\lambda(S)^x\sum_{k=0}^{th}n^k\frac{\left[\left(1-\frac{1}{n}\right)\left(1-\frac{2}{n}\right)..................\left(1-\frac{\left(k+x-1\right)}{n}\right)\right]}{x!k!}\left(\frac{\lambda(F)}{n}\right)^k\left(1-\frac{\lambda(I)}{n}\right)^{n-x-k}$$

$$=\lambda(S)^x\sum_{k=0}^{th}\frac{\left[\left(1-\frac{1}{n}\right)\left(1-\frac{2}{n}\right)..................\left(1-\frac{\left(k+x-1\right)}{n}\right)\right]}{x!k!}\left(\lambda(F)\right)^k\left(1-\frac{\lambda(I)}{n}\right)^{n-x-k}$$

$$\text{Since}\left[n\to\infty\ \&\lim_{n\to\infty}\left(1-\frac{\lambda}{n}\right)^{n-x-k}=e^{-\lambda}\right]$$

$$Tr_x=\lambda(S)^x\sum_{k=0}^{th}\frac{e^{-\lambda(I)k}\lambda(F)^k}{k!}\qquad\text{..}(4)$$

which is the truth membership of a NPD.

Using Eqn (2) indeterminacy of NPD can be determined as follows:

$$Ind_x = \lambda(I)^z \sum_{k=0}^{th} \frac{n(n-1)(n-2)..........(n-k-z-1)}{x!k!} \left(\frac{\lambda(S)}{n}\right)^k \left(1-\frac{\lambda(F)}{n}\right)^{n-z-k}$$

$$\text{Since} \left[n \to \infty \,\&\, \lim_{n\to\infty} \left(1-\frac{\lambda}{n}\right)^{n-z-k} = e^{-\lambda} \right]$$

$$Ind_x = \sum_{z=th+1}^{n} \lambda(I)^z \sum_{k=0}^{n-z} \frac{e^{-\lambda(F)k} \lambda(S)^k}{k!} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(5)$$

The falsehood membership function of NPD can be defined as follows using Eqn (3).

$$Fal_x = \lambda(S)^y \sum_{k=0}^{th} n^k \frac{\left[\left(1-\frac{1}{n}\right)\left(1-\frac{2}{n}\right)..................\left(1-\frac{(k+y-1)}{n}\right)\right]}{x!k!} \left(\frac{\lambda(F)}{n}\right)^k \left(1-\frac{\lambda(I)}{n}\right)^{n-y-k}$$

$$\text{Since} \left[n \to \infty \,\&\, \lim_{n\to\infty} \left(1-\frac{\lambda}{n}\right)^{n-y-k} = e^{-\lambda} \right]$$

$$Fal_x = \sum_{\substack{y=0\\ y\neq x}}^{n} \lambda(S)^y \sum_{k=0}^{th} \frac{e^{-\lambda(I)k} \lambda(F)^k}{k!} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(6)$$

Therefore the neutrosophic poisson distribution can be determined as

$$Tr_x = \lambda(S)^x \sum_{k=0}^{th} \frac{e^{-\lambda(I)k} \lambda(F)^k}{k!}, \quad I_x = \sum_{z=th+1}^{n} \lambda(I)^z \sum_{k=0}^{n-z} \frac{e^{-\lambda(F)k} \lambda(S)^k}{k!}, \quad F_x = \sum_{\substack{y=0\\ y\neq x}}^{n} \lambda(S)^y \sum_{k=0}^{th} \frac{e^{-\lambda(I)k} \lambda(F)^k}{k!}$$

4. Application of the neutrosophic poisson distribution:

In this section, we used [23] to apply the proposed notion to a real-world problem.

The atoms of a radioactive element disintegrate at random. Every gramme of this element emits between [0.4-0.9] alpha particles per second on average. When the indeterminacy threshold is reached during the next second, the probability of the number of alpha particles emitted from one gramme is exactly two, as shown below using the proposed NPD.

Solution:

Given data: $\lambda(S) = 0.4$; $\lambda(I) = 0.6$; $\lambda(F) = 0.9$

$$x = 2; k = 2$$

From Equ(4) the formula for Neutrosophic truth membership function is given by

$$Tr_x = \lambda(S)^x \sum_{k=0}^{th} \frac{e^{-\lambda(I)k} \lambda(F)^k}{k!}$$

$$= (0.4)^2 \sum_{k=0}^{2} \frac{e^{-0.6k} (0.9)^k}{k!}$$

$$Tr_x = 0.75$$

From Equ(5) the formula for Neutrosophic truth membership function is given by

$$Fal_x = \sum_{\substack{y=0 \\ y \neq x}}^{n} \lambda(S)^y \sum_{k=0}^{th} \frac{e^{-\lambda(I)k} \lambda(F)^k}{k!}$$

$$= (0.4)^2 \sum_{k=0}^{2} \frac{e^{-0.6k} (0.9)^k}{k!}$$

$$Fal_x = 0.52$$

From Equ(4) the formula for Neutrosophic truth membership function is given by

$$Ind_x = \sum_{z=th+1}^{n} \lambda(I)^z \sum_{k=0}^{n-z} \frac{e^{-\lambda(F)k} \lambda(S)^k}{k!}$$

$$= \sum_{z=th+1}^{n} (0.6)^2 \sum_{k=0}^{z} \frac{e^{-0.9k} (0.4)^k}{k!}$$

$$Ind_x = 0.25$$

$$(Tr_x, Fal_x, Ind_x) = (0.75, 0.52, 0.25) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots(7)$$

Therefore from Eqn.(7) we applied all the values in [11] we get,

$$S(b)=\frac{1+0.75-0.52}{2}$$
$$S(b)=0.615$$

As a result, the probability that 1 gm will produce 0.615 alpha particles in the following second is 0.615.

5. The Neutrosophic Exponential Distribution (NED):

In this section, we proposed NED's density function for Truth membership function, Indeteminacy and also for Falsity. Also proposed mean and variance of NED, and their Distribution function.

Definition:5.1

The Neutrosophic Exponential Distribution (NED) is a generalisation of the traditional exponential distribution. It can process any type of data, including non-specific data. The density function of NED is written as follows:

Table 1:

Density function for Neutrosophic Exponential Distribution(NED)		
Truth Membership	Indeteminacy	Falsity
$f_{Neu}\left(T_x(x)\right)=\lambda_{Neu}\left(T_x(x)\right)e^{-\left(\lambda_{Neu}\left(T_x(x)\right)\right)x}$	$f_{Neu}\left(I_x(x)\right)=\lambda_{Neu}\left(I_x(x)\right)e^{-\left(\lambda_{Neu}\left(I_x(x)\right)\right)x}$	$f_{Neu}\left(F_x(x)\right)=\lambda_{Neu}\left(F_x(x)\right)e^{-\left(\lambda_{Neu}\left(F_x(x)\right)\right)x}$

Where 'x' is a Neutrosophic random variable

$\lambda_{Neu}\left(T_x(x)\right)$ - Truth membership function's distribution parameter.

$\lambda_{Neu}\left(I_x(x)\right)$ - Indeterminacy membership function's distribution parameter.

$\lambda_{Neu}\left(F_x(x)\right)$ - Falsity membership function's distribution parameter.

Properties of NED:

Table 2:

	Truth Membership Function	Indeterminacy	Falsity
Expected Value for NED: $E(x)$	$E(T_x)=\frac{1}{\lambda_{Neu}\left(T_x(x)\right)}$	$E(I_x)=\frac{1}{\lambda_{Neu}\left(I_x(x)\right)}$	$E(F_x)=\frac{1}{\lambda_{Neu}\left(F_x(x)\right)}$
Variance for NED: $Var(x)$	$Var(T_x)=\frac{1}{\left(\lambda_{Neu}\left(T_x(x)\right)\right)^2}$	$Var(I_x)=\frac{1}{\left(\lambda_{Neu}\left(I_x(x)\right)\right)^2}$	$Var(F_x)=\frac{1}{\left(\lambda_{Neu}\left(F_x(x)\right)\right)^2}$

Table 3:

	Truth Membership Function	Indeterminacy	Falsity
Distribution Function for NED: $N\left(F\left(T_x, I_x, F_x\right)\right) = NP\left(X \le x\right)$	$\left(1-e^{-\left(\lambda_{Neu}\left(T_x(x)\right)\right)x}\right)$	$\left(1-e^{-\left(\lambda_{Neu}\left(I_x(x)\right)\right)x}\right)$	$\left(1-e^{-\left(\lambda_{Neu}\left(F_x(x)\right)\right)x}\right)$

6. Application of the neutrosophic exponential distribution:
We used the proposed notion in a case study problem from [21] with different intervals in this part.

Consider what amount of time it requires for a bank to end a client assistance's, which follows a dramatic dispersion with a normal of one moment. Compose a thickness work, mean, change, and dissemination work for the time it takes to end a customer's administration, and afterward propose the likelihood of ending a customer's administration in the stretch [1,2] minute.

Assume that x: indicates the time necessary per minute to terminate the client's service.

The average $\dfrac{1}{\lambda\left(T_x(x)\right)} = 1$; $\dfrac{1}{\lambda\left(I_x(x)\right)} = \dfrac{1}{1.5} = 0.6667$; $\dfrac{1}{\lambda\left(F_x(x)\right)} = \dfrac{1}{2} = 0.5$

The probability density function NED:

From table 1:

$$f_{Neu}\left(T_x(x)\right) = \lambda_{Neu}\left(T_x(x)\right)e^{-\left(\lambda_{Neu}\left(T_x(x)\right)\right)x} = 0.3679$$

$$f_{Neu}\left(I_x(x)\right) = \lambda_{Neu}\left(I_x(x)\right)e^{-\left(\lambda_{Neu}\left(I_x(x)\right)\right)x} = 0.3347$$

$$f_{Neu}\left(F_x(x)\right) = \lambda_{Neu}\left(F_x(x)\right)e^{-\left(\lambda_{Neu}\left(F_x(x)\right)\right)x} = 0.2707$$

From Table 2:

$$E\left(T_x\right) = \frac{1}{\lambda_{Neu}\left(T_x(x)\right)} = 1; Var\left(T_x\right) = \frac{1}{\left(\lambda_{Neu}\left(T_x(x)\right)\right)^2} = 1$$

$$E\left(I_x\right) = \frac{1}{\lambda_{Neu}\left(I_x(x)\right)} = 0.6667\,; Var\left(I_x\right) = \frac{1}{\left(\lambda_{Neu}\left(I_x(x)\right)\right)^2} = 0.4444$$

$$E\left(F_x\right) = \frac{1}{\lambda_{Neu}\left(F_x(x)\right)} = 0.5\,; Var\left(F_x\right) = \frac{1}{\left(\lambda_{Neu}\left(F_x(x)\right)\right)^2} = 0.2500$$

There's a chance that the client's service will be terminated in less than 1,0.67,0.5 minute. :

From Table 3

$$Neu\left(F\left(T_x\right)\right) = Neu\left(P\left(X \le x\right)\right) = Neu\left(P\left(X \le 1\right)\right) = \left(1 - e^{-\left(\lambda_{Neu}\left(T_x\left(x\right)\right)\right)x}\right) = 0.63$$

$$Neu\left(F\left(I_x\right)\right) = Neu\left(P\left(X \le x\right)\right) = Neu\left(P\left(X \le 1\right)\right) = \left(1 - e^{-\left(\lambda_{Neu}\left(I_x\left(x\right)\right)\right)x}\right) = 0.4883$$

$$Neu\left(F\left(F_x\right)\right) = Neu\left(P\left(X \le x\right)\right) = Neu\left(P\left(X \le 1\right)\right) = \left(1 - e^{-\left(\lambda_{Neu}\left(F_x\left(x\right)\right)\right)x}\right) = 0.39$$

Probability that the client's service will be terminated in less than a minute:

$$Neu\left(F\left(T_x\right)\right) = Neu\left(P\left(X \le x\right)\right) = \left(1 - e^{-\left(\lambda_{Neu}\left(T_x\left(x\right)\right)\right)x}\right) = \left(1 - e^{-[0.5,1.5]1}\right)$$

That is, the likelihood of terminating a client's service in less than a minute ranges between 0.085 and 0.085 when we use the numbers in [11].

Probability that the client's service will be terminated in less than 1.5 minutes:

$$Neu\left(F\left(I_x\right)\right) = Neu\left(P\left(X \le x\right)\right) = \left(1 - e^{-\left(\lambda_{Neu}\left(I_x\left(x\right)\right)\right)x}\right) = \left(1 - e^{-[0.5,1.5]1.5}\right)$$

When we utilise the numbers in [11], the probability of terminating a client's service in less than 1.5 minutes ranges between 0.2111 and 0.2111.

Probability of terminating a client's service in under 2 minutes:

$$Neu\left(F\left(F_x\right)\right) = Neu\left(P\left(X \le x\right)\right) = \left(1 - e^{-\left(\lambda_{Neu}\left(F_x\left(x\right)\right)\right)x}\right) = \left(1 - e^{-[0.5,1.5]2}\right)$$

When we apply the figures in [11], the probability of terminating a client's service in less than 2 minutes ranges between 0.435 and 0.435.

7. Neutrosophic Uniform Distribution (NUD):

In this section, we proposed NUD’s density function for Truth membership function, Indeteminacy and also for Falsity. Also proposed mean and variance of NUD.

Definition:7.1
The numeric value of a continuous variable is its numeric value. Although X is a conventional Uniform distribution, the distribution parameters an or b, or both, are untrustworthy. For example, 'a' or 'b,' or both,' are sets of two or more components having a< b, and we can use NUD to stand for Truth Membership, Indeterminacy, and Falsity. The following is a NUD definition that has been offered.: $f\left(T_x\right) = \begin{cases} K, & a\left(T_x\right) < b\left(T_x\right) \\ 0 & otherwise \end{cases}$

Since the total probability always unity

$$\int_{a(T_x)}^{b(T_x)} f(T_x)dx = 1$$

$$\Rightarrow \int_{a(T_x)}^{b(T_x)} K dx = 1$$

$$[K]_{a(T_x)}^{b(T_x)} = 1$$

$$K = \frac{1}{b(T_x) - a(T_x)}$$

Density function of NUD for Truth Membership function is given by

$$f_{Neu}(T_x) = \frac{1}{b(T_x) - a(T_x)} \text{ for } a(T_x) < b(T_x) \ldots\ldots\ldots\ldots\ldots\ldots(8)$$

Similiarly we propose NUD for Indeterminacy

$$f_{Neu}(I_x) = \frac{1}{b(I_x) - a(I_x)} \text{ for } a(I_x) < b(I_x) \ldots\ldots\ldots\ldots\ldots\ldots..(9)$$

Also we propose NUD for Falsity function

$$f_{Neu}(F_x) = \frac{1}{b(F_x) - a(F_x)} \text{ for } a(F_x) < b(F_x) \ldots\ldots\ldots\ldots(10)$$

Mean of a NUD:

Mean for NUD for Truth Membership function is given by

$$E(T_x) = \int_{a(T_x)}^{b(T_x)} f(T_x)dx = \frac{b(T_x) - a(T_x)}{2} \ldots\ldots\ldots\ldots\ldots\ldots(11)$$

Mean for NUD for Indeterminacy is given by

$$E(I_x) = \int_{a(T_x)}^{b(T_x)} f(I_x)dx = \frac{b(I_x) - a(I_x)}{2} \ldots\ldots\ldots\ldots\ldots\ldots..(12)$$

Mean for NUD for Falsity is given by

$$E(F_x) = \int_{a(T_x)}^{b(T_x)} f(F_x)dx = \frac{b(F_x) - a(F_x)}{2} \ldots\ldots\ldots\ldots\ldots\ldots..(13)$$

Variance of NUD:

Variance of NUD for Truth Membership function is given by

$$Var(T_x) = \frac{\left(b(T_x) - a(T_x)\right)^2}{12} \text{ ....(14)}$$

Variance of NUD for Indeterminacy is given by

$$Var(I_x) = \frac{\left(b(I_x) - a(I_x)\right)^2}{12} \text{ ....(15)}$$

Variance of NUD for Falsity function is given by

$$Var(F_x) = \frac{\left(b(F_x) - a(F_x)\right)^2}{12} \text{ ....(16)}$$

8. Application of the neutrosophic Uniform distribution:

We used the proposed notion in a case study problem from [21] with different intervals in this part.

The station official explained that assuming 'x; is a variable that denotes a person's waiting time for a passenger's bus (in minutes), the bus arrival time is not mentioned.:

1-the bus arrival time is: either now or in 5 minutes [0,5] or in 15-20 minutes [15,20], then

2- the bus arrival time is: either now or in 5 minutes [0,5], or in 20-25 minutes [20,25], then

3- the bus will arrive in either 5 minutes [0,5] or 25-30 minutes [25,30], depending on when you arrive.

Here $a(T_x) = a(I_x) = a(F_x) = [0,5]$

$$b(T_x) = [15,20];$$
$$b(I_x) = [20,25];$$
$$b(F_x) = [25,30];$$

Then the density

Density function of NUD for Truth Membership function is given by

From Eqn(8)$\Rightarrow$ $f_{Neu}(T_x) = \dfrac{1}{b(T_x) - a(T_x)} = \dfrac{1}{[15,20]-[0,5]} = \dfrac{1}{[10,15]} = [0.067, 0.1]$

Density function of NUD for Indeterminacy is given by

From Eqn(9)$\Rightarrow$ $f_{Neu}(I_x) = \dfrac{1}{b(I_x) - a(I_x)} = \dfrac{1}{[20,25]-[0,5]} = \dfrac{1}{[15,20]} = \dfrac{[0.05, 0.067]}{2} = [0.025, 0.033]$

Density function of NUD for Falsity function is given by

From Eqn(10)$\Rightarrow$ $f_{Neu}(F_x) = \dfrac{1}{b(F_x) - a(F_x)} = \dfrac{1}{[25,30]-[0,5]} = \dfrac{1}{[20,25]} = [0.04, 0.05]$

Mean of the bus arrival time 15-20 minutes is given by

$$E\left(T_x\right)=\frac{b\left(T_x\right)-a\left(T_x\right)}{2}=\frac{[15,20]-[0,5]}{2}=\frac{\left[10,15\right]}{2}=\frac{\left[0.067,0.1\right]}{2}=\left[0.0335,0.05\right]$$

Mean of the bus arrival time 20-25 minutes is provided by

$$E\left(I_x\right)=\frac{b\left(I_x\right)-a\left(I_x\right)}{2}=\frac{[20,25]-[0,5]}{2}=\frac{\left[15,20\right]}{2}=\frac{\left[0.05,0.067\right]}{2}=\left[0.025,0.0335\right]$$

Mean of the bus arrival time 25-30 minutes is provided by

$$E\left(F_x\right)=\frac{b\left(F_x\right)-a\left(F_x\right)}{2}=\frac{[25,30]-[0,5]}{2}=\frac{\left[20,25\right]}{2}=\frac{\left[0.04,0.05\right]}{2}=\left[0.02,0.025\right]$$

Variance of the bus arrival time 15-20 minutes is given by

From Eqn(14)$\Rightarrow$ $Var\left(T_x\right)=\frac{\left(b\left(T_x\right)-a\left(T_x\right)\right)^2}{12}=\frac{\left[0.0335,0.5\right]^2}{12}=\left[0.000093,0.0208\right]$

Variance of the bus arrival time 20-25 minutes is given by

From Eqn(15)$\Rightarrow$ $Var\left(I_x\right)=\frac{\left(b\left(I_x\right)-a\left(I_x\right)\right)^2}{12}=\frac{\left[0.05,0.067\right]^2}{12}=\left[0.000208,0.000374\right]$

Variance of the bus arrival time 25-30 minutes is given by

From Eqn(16)$\Rightarrow$ $Var\left(F_x\right)=\frac{\left(b\left(F_x\right)-a\left(F_x\right)\right)^2}{12}=\frac{\left[0.04,0.05\right]^2}{12}=\left[0.000133,0.000208\right]$

9. Conclusion

Classical probability solely considers determinate data, but neutrosophic probability considers indeterminate data with varying degrees of indeterminacy. Hence in this paper, we proposed many of the standard distribution called Poisson distribution as a limiting case of Binomial distribution,NED,NUD under neutrosophic environment. Also, using the proposed concept probability value has been obtained for a real world problem. In future, probability distributions may be proposed under different neutrosophic environment.

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Introducción a la Super-Hiper-Álgebra y la Super-Hiper-Álgebra Neutrosófica

Introduction to Super-Hyper-Algebra and Neutrosophic Super-Hyper-Algebra

Florentin Smarandache

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Resumen. En este artículo, se revisan los conceptos de Conjunto de enésima Potencia de un Conjunto, Súper-Híper-Operación, Súper-Híper-Axioma, Súper-Híper-Álgebra, y sus correspondientes Súper-Híper-Operación Neutrosófica, Súper-Híper-Axioma Neutrosófico y Súper-Híper-Álgebra Neutrosófica. En general, en cualquier campo del conocimiento, realmente lo que se encuentran son Súper-Híper-Estructuras (o más específicamente Súper-Híper-Estructuras (m, n)).

Palabras clave: Súper-Híper-Operación, Súper-Híper-Álgebra, Súper-Híper-Álgebra Neutrosófica, Súper-Híper-Estructuras

Abstract. In this article, the concepts of Nth Power Set of a Set, Super-Hyper-Oper-Operation, Super-Hyper-Axiom, Super-Hyper-Algebra, and their corresponding Neutrosophic Super-Hyper-Oper-Operation, Neutrosophic Super-Hyper-Axiom and Neutrosophic Super-Hyper-Algebra are reviewed. In general, in any field of knowledge, really what are found are Super-Hyper-Structures (or more specifically Super-Hyper-Structures (m, n)).

Keywords: Super-Hyper-Oper-Operation, Super-Hyper-Algebra, Neutrosophic Super-Hyper-Algebra, Super-Hyper-Structures.

1 Introducción

Se puede recordar a la Súper-Híper-Algebra y Súper-Híper-Algebra Neutrosófica introducidas y desarrolladas por Smarandache [16, 18, 19] entre 2016 y 2022.

1.1 Definición de Híper-Operaciones Clásicas:

Sea U un universo de discurso y H un conjunto no vacío, H ⊂ U. Una **Híper-Operación Binaria Clásica** o_2^* se define de la siguiente manera:

$$o_2^*: H^2 \rightarrow P_*(H), \qquad (1)$$

Donde H es un conjunto continuo o discreto, y $P_*(H)$ es el conjunto potencia de H excluyendo el conjunto vacío $\emptyset$, o expresado de otra manera: $P_*(H) =$ P (H) \ $\{\emptyset\}$.

Una **Híper-Operación m-aria Clásica** o_m^* , se define como:

$$o_m^*: H^m \rightarrow P_*(H), \qquad (2)$$

siendo m un entero, tal que $m \geq 1$. Para m = 1 se obtiene una **Híper-Operación Unaria**

Las **Híper-Estructuras clásicas** son estructuras dotadas de Híper-Operaciones clásicas.

Las Híper-Operaciones clásicas y las Híper-Estructuras clásicas fueron introducidas por F. Marty [12] en 1934.

1.2 Definición de Conjunto de enésima Potencia de un Conjunto:

El conjunto de enésima potencia de un conjunto se introdujo en [16, 18, 19] de la siguiente manera:

$P^n(H)$, como conjunto de enésima potencia del conjunto H, siendo n un entero tal que $n \geq 1$, se define de manera recursiva como:

$$P^2(H) = P\big(P(H)\big),\ P^3(H) = P\big(P^2(H)\big) = P\Big(P\big(P(H)\big)\Big), \dots,$$
$$P^n(H) = P\Big(P^{(n-1)}(H)\Big), \text{ donde } P^o(H) \stackrel{\text{def}}{=} H, \text{ y } P^1(H) \stackrel{\text{def}}{=} P(H).$$

El Conjunto de enésima Potencia de un Conjunto refleja mejor nuestra compleja realidad, ya que un conjunto H (que puede representar un grupo, una sociedad, un país, un continente, etc.) de elementos (tales como: personas, objetos y en general cualquier elemento) se organiza en subconjuntos P (H), y estos subconjuntos se organizan nuevamente en subconjuntos de subconjuntos P (P (H)), y así sucesivamente. Ese es nuestro mundo.

1.3 Híper-Operación Neutrosófica e Híper-Estructuras Neutrosóficas [12]:

En la Híper-Operación clásica y las Híper-Estructuras clásicas, el conjunto vacío ∅ no pertenece al conjunto potencia. Expresado de otra manera, $P_*(H) = P(H) \setminus \{\emptyset\}$.
Sin embargo, en el mundo real nos encontramos con muchas situaciones en las que una Híper-Operación ∘ es indeterminada, por ejemplo $a \circ b = \emptyset$ (desconocido o indefinido),
O parcialmente indeterminado, por ejemplo: $c \circ d = \{[0{,}2,\ 0{,}3], \emptyset\}$.
En nuestra vida cotidiana, hay muchas más operaciones y leyes que tienen algún grado de indeterminación (vaguedad, falta de claridad, desconocimiento, contradicción, etc.), que aquellas que son totalmente determinadas.

Es por eso que en 2016 se ha extendido la Híper-Operación clásica a la Híper-operación Neutrosófica, tomando toda la potencia P(H) (que incluye también el conjunto vacío ∅), en lugar de $P_*(H)$ (que no incluye el conjunto vacío ∅), tal como se detalla a continuación:

1.4 Definición de Híper-Operación Neutrosófica:

Sea U un universo de discurso y H un conjunto no vacío, H⊂ U. Una Híper-Operación Binaria Neutrosófica $\circ_2$ se define de la siguiente manera:

$$o_2: H^2 \to P(H),$$

Donde H es un conjunto discreto o continuo, y P(H) es el conjunto potencia de H que incluye el conjunto vacío ∅.

Una **Híper-operación m-aria Neutrosófica** o_m Se define como:

$$o_2: H^m \to P(H),$$

para m≥1 valor entero. De manera similar, para m = 1 se obtiene una **Híper-operación Unaria Neutrosófica**.

1.5 Híper-estructuras Neutrosóficas:

Una Híper-Estructura Neutrosófica es una estructura dotada de Híper-Operaciones Neutrosóficas.

1.6 Definición de Súper-Híper-Operaciones

Se pueden recordar los conceptos de 2016 de Súper-Híper-Operación, Súper-Híper-Axioma, Súper-Híper-Álgebra y sus correspondientes Súper-Híper-Operaciones Neutrosóficas, Súper-Híper-Axioma Neutrosófico y Súper-Híper-Algebra Neutrosófica [16].

Sea $P_*^n(H)$ el conjunto de enésima potencia del conjunto H, tal que ninguno de P(H), $P^2(H)$,… , $P^n(H)$ contienen el conjunto vacío ∅.

Además, sea $P^n(H)$ el conjunto de enésima potencia del conjunto H, tal que al menos uno de los P(H), $P^2(H)$,… , $P^n(H)$ contienen el conjunto vacío ∅.

Las Súper-Híper-Operaciones son operaciones cuyo codominio es $P_*^n(H)$ y en este caso se tienen **Súper-Híper-Operaciones clásicas**, o $P^n(H)$ y en este caso se tienen **Súper-Híper-Operaciones Neutrosóficas**, siendo n un valor entero y n≥2.

Una Super-Hyper-Operación binaria clásica $o_{(2,n)}^*$ se define de la siguiente manera:

$$o_{(2,n)}^*: H^2 \rightarrow P_*^n(H), \qquad (3)$$

Donde $P_*^n(H)$ es el conjunto de enésima potencia del conjunto H, sin incluir el conjunto vacío.

Ejemplos de súper-híper-operación binaria clásica:

1) Sea H = {a, b} un conjunto discreto finito; entonces su conjunto potencia, sin incluir el conjunto vacío ∅, es:

P(H) ={a, b,{a, b}}, y:

$P^2(H) = P(P(H)) = P(\{a, b, \{a, b\}\}) = \{a, b, \{a, b\}, \{a, \{a, b\}\}, \{b, \{a, b\}\}, \{a, b, \{a, b\}\}\},$

$$o_{(2,2)}^*: H^2 \rightarrow P_*^2(H),$$

$o_{(2,2)}^*$	a	b
a	$\{a, \{a, b\}\}$	$\{b, \{a, b\}\}$
b	a	$\{a, b, \{a, b\}\}$

Tabla 1: Ejemplo 1 de Súper-Híper-Operación Binaria Clásica

2) Sea H = [0, 2] un conjunto continuo.

P(H) = P ([0, 2]) ={A | A⊆[0, 2], A = subconjunto},

P^2(H) = P (P ([0, 2])).

Sean c, d ∈H.

$$o_{(2,2)}^*: H^2 \rightarrow P_*^2(H)$$

$o_{(2,2)}^*$	c	d
c	{[0, 0.5] , [1 , 2]}	{0.7 , 0.9 , 1.8}
d	{2.5}	{(0.3 , 0.6) , {0.4 , 1.9}, 2}

Tabla 2: Ejemplo 2 de Súper-Híper-Operación Binaria Clásica

Súper-Híper-Operación clásica de orden m (o, empleando una denominación más precisa, Súper-Híper-Operación (m, n))

Sea U un universo de discurso y un conjunto no vacío H, H ⊂ U. Entonces:

$$o_{(m,n)}^*: H^m \rightarrow P_*^n(H),$$

Donde m y n son enteros, tales que m, n ≥ 1,

$$H, H^m = H \times H \times \ldots \times H, \text{ (m veces)}$$

Y $P_*^n(H)$ es el conjunto de enésima potencia del conjunto H que incluye el conjunto vacío.

Esta Súper-Híper-Operación es una operación de orden m definida desde el conjunto H hasta el conjunto de enésima potencia del conjunto H.

Súper-Híper-Operación Neutrosófica de orden m (o, empleando una denominación más precisa, **Súper-Híper-Operación Neutrosófica (m, n)**):

Sea U un universo de discurso y un conjunto no vacío H, H ⊂ U, entonces:

$$o_{(m,n)}: H^m \rightarrow P^n(H),$$

Donde m y n son enteros, tales que m, n ≥ 1, Y $P^n(H)$ es el conjunto de enésima potencia del conjunto H que incluye el conjunto vacío.

Súper-Híper-Axioma:

Un Súper-Híper-Axioma clásico o más exactamente un Súper-Híper-Axioma (m, n) es un axioma basado en Súper-Híper-Operaciones clásicas.

De manera similar, un Súper-Híper-Axioma Neutrosófico (o Súper-Híper-Axioma Neutrosófico (m, n)) es un axioma basado en Súper-Híper-Operaciones Neutrosóficas.
Existen:

- **Súper-Híper-Axiomas Fuertes**, cuando el lado izquierdo es igual al lado derecho como en los axiomas que no son de tipo híper,

- y **Súper-Híper-Axiomas Débiles**, cuando la intersección entre el lado izquierdo y el lado derecho no está vacía.

Por ejemplo, se tiene:

- Súper-Híper-Asociación Fuerte, cuando $(x \circ y) \circ z = x \circ (y \circ z)$, para todo $x, y, z \in H^m$, donde
$$o^*_{(m,n)}: H^m \to P^n_*(H)$$
- Y Súper-Híper-Asociación Débil, cuando $[(x \circ y) \circ z] \cap [x \circ (y \circ z)] \neq \emptyset$, para todo $x, y, z \in H^m$

Súper-Híper-Algebra y Súper-Híper-Estructura:

Una Súper-Híper-Algebra o, más exactamente Súper-Híper-Algebra (m−n), es un álgebra que trata con Súper-Híper-Operaciones y Súper-Híper-Axiomas.

Nuevamente, una Súper-Híper-Algebra Neutrosófica (o Súper-Híper-Algebra Neutrosófica (m, n)) es un álgebra que trata con Súper-Híper-Operaciones Neutrosóficas y Súper-Híper-Axiomas Neutrosóficos.

En general, tenemos Súper-Híper-Estructuras (o Súper-Híper-Estructuras (m, n)), y las correspondientes Súper-Híper-Estructuras Neutrosóficas.

Por ejemplo, hay Súper-Híper-Grupos, Súper-Híper-Semigrupos, Súper-Híper-Anillos, Súper-Híper-Espacios-Vectoriales, etc.

1.7 Distinción entre Súper-Híper-Álgebra vs. Súper-Híper-Álgebra Neutrosófica:

i. Si ninguno de los conjuntos potencia $P^k(H)$, $1 \leq k \leq n$, no incluye el conjunto vacío $\emptyset$, entonces se tiene una Súper-Híper-Álgebra de tipo clásico;

ii. Si al menos un conjunto potencia, $P^k(H)$, $1 \leq k \leq n$, incluye el conjunto vacío $\emptyset$, entonces se tiene una Súper-Híper-Álgebra Neutrosófica.

Súper-Híper-Grafo (o Súper-Híper-Grafo-n):

El Súper-Híper-Álgebra se parece al Súper-Híper-Grafo-n [17, 18, 19], introducido por Smarandache en 2019, definido de la siguiente manera:

Definición del Súper-Híper-Grafo-n:

Sea $V = \{v_1, v_2, \ldots, v_m\}$, para $1 \leq m \leq \infty$, un conjunto de vértices, que contiene Vértices Únicos (los clásicos), Vértices Indeterminados (poco claro, vago, parcialmente conocido), y Vértices nulos (totalmente desconocidos, vacíos).

Sea P(V) la potencia del conjunto V, que incluye también al conjunto vacío $\emptyset$.
Entonces sea $P^n(V)$ el n-conjunto potencia del conjunto V, definido de forma recurrente, es decir:
$P(V)$, $P^2(V) = P(P(V))$, $P^3(V) = P(P^2(V)) = P(P(P(V)))$, …,
$P^n(V) = P(P^{(n-1)}(V))$, por $1 \leq n \leq \infty$, donde por definición $P^o(V) \stackrel{\text{def}}{=} V$.

Entonces, el Súper-Híper-Grafo-n (**SHG**-n) es un par ordenado:

$$\text{SHG-n} = (G_n, E_n),$$

Donde $G_n \subseteq P^n(V)$, y $E_n \subseteq P^n(V)$, por $1 \leq n \leq \infty$.
G_n es el conjunto de vértices, y E_n es el conjunto de aristas.

El **conjunto de vértices G_n** contiene los siguientes tipos de vértices:

- Vértices individuales (los clásicos);
- Vértices indeterminados (poco claro, vago, parcialmente desconocido);
- Vértices nulos (totalmente desconocido, vacío); y:
- Súper vértice (o Vértice de Subconjunto), es decir, dos o más (único, indeterminado o nulo) vértices juntos como un grupo (organización).
- Súper vértice-n esa es una colección de muchos vértices tales que al menos uno es un Súper-Vértices (n-1) y todos los demás Súper-Vértices-r en la colección, si los hay, tienen el orden $r \leq n - 1$.

El **conjunto de aristas E_n** contiene los siguientes tipos de aristas:

- Aristas Sencillas (las clásicas);
- Aristas indeterminadas (poco claro, vago, parcialmente desconocido);
- Aristas nulas (totalmente desconocido, vacío); y:
- Híper-arista (conectando tres o más vértices individuales);
- Súper-Arista (conectando dos vértices, siendo al menos uno de ellos un Súper-Vértice);
- Súper-Arista-n (conectando dos vértices, siendo al menos uno un Súper-Vértice-n, y el otro de orden Súper-Vértice-r, con $r \leq n$);
- Súper-Híper-Arista (conectando tres o más vértices, siendo al menos uno un Súper-Vértice);
- Súper-Híper-Arista-n (conectando tres o más vértices, siendo al menos uno un Súper-Vértice-n, y los otros Súper-Vértices-r con $r \leq n$;
- Multi-Aristas (dos o más aristas que conectan los mismos dos vértices);
- Ciclo (y borde que conecta un elemento consigo mismo), y:
- Gráfico dirigido (clásico);
- Gráfico no dirigido (clásico);
- Gráfico dirigido neutrosófico (dirección parcialmente dirigida, parcialmente no dirigida, parcialmente indeterminada).

2 Conclusiones

Se abordó la forma más general de álgebras, denominada Súper-Híper-Algebra (o más precisamente Súper-Híper-Algebra-(m, n)) y la Súper-Híper-Álgebra Neutrosófica, y sus extensiones a Súper-Híper-

Estructuras y Súper-Híper-Álgebra Neutrosóficas en cualquier campo del conocimiento.

Se basan en el Conjunto de enésima Potencias de un Conjunto, que refleja mejor nuestra compleja realidad, ya que un conjunto H (que puede representar un grupo, una sociedad, un país, un continente, etc.) de elementos (tales como: personas, objetos y, en general, cualquier elemento) se organiza en subconjuntos P(H), y estos subconjuntos se organizan nuevamente en subconjuntos de subconjuntos P(P(H)), y así sucesivamente. Ese es nuestro mundo.

Este nuevo campo de súper-Híper-Álgebra puede inspirar a los investigadores a estudiar varios casos particulares interesantes, como súper-Híper-Grupo, Súper-Híper-Semigrupo, Súper-Híper-Grupo, súper-Híper-Anillo, Súper-Híper-Espacio-Vectorial, etc.

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La Neutro-Geometría y la Anti-Geometría como Alternativas y Generalizaciones de las Geometrías no Euclidianas

Neutro-Geometry and Anti-Geometry as Alternatives and Generalizations of Non-Euclidean Geometries

Florentin Smarandache

Florentin Smarandache (2022). La Neutro-Geometría y la Anti-Geometría como Alternativas y Generalizaciones de las Geometrías no Euclidianas (Neurometrics and Anti-Geometry as Alternatives and Generalizations of Non-Euclidean Geometries). *Neutrosophic Computing and Machine Learning* 20, 91-104

Resumen. En este artículo se extiende la Neutro-Álgebra y la Anti-Álgebra a los espacios geométricos, fundando la Neutro/Geometría y Anti-Geometría. Mientras que las Geometrías No-Euclidianas resultaron de la negación total de un axioma específico (Quinto Postulado de Euclides), la Anti-Geometría resulta de la negación total de cualquier axioma o incluso de más axiomas de cualquier sistema axiomático geométrico (Euclidiano, Hilbert, etc.) y de cualquier tipo de geometría como la Geometría (Euclidiana, Proyectiva, Finita, Diferencial, Algebraica, Compleja, Discreta, Computacional, Molecular, Convexa, etc.), y la Neutro-Geometría resulta de la negación parcial de uno o más axiomas [y sin negación total de ningún axioma] de cualquier sistema axiomático geométrico y de cualquier tipo de geometría. Generalmente, en lugar de un Axioma geométrico clásico, se puede tomar cualquier Teorema geométrico clásico de cualquier sistema axiomático y de cualquier tipo de geometría, y transformarlo por Neutrosoficación o Antisoficación en un Neutro-Teorema o Anti-Teorema respectivamente para construir una Neutro-Geometría o Anti-Geometría. Por tanto, la Neutro-Geometría y la Anti-Geometría son respectivamente alternativas y generalizaciones de las Geometrías No Euclidianas. En la segunda parte, se recuerda la evolución desde el Paradoxismo a la Neutrosofía, luego a la Neutro-Álgebra y la Anti-Álgebra, luego a la Neutro-Geometría y la Anti-Geometría, y en general a la Neutro-Estructura y Anti-Estructura que surgen naturalmente en cualquier campo del conocimiento. Al final, se presentan aplicaciones de muchas Neutro-Estructuras en nuestro mundo real.

Palabras clave: Geometrías no euclidianas, Geometría euclidiana, Geometría de Lobachevski-Bolyai-Gauss, Geometría de Riemann, Neutro-Múltiple, Anti-Múltiple, Neutro-Álgebra, Anti-Álgebra, Neutro-Geometría, Anti-Geometría, Neutro-Axioma, Anti-Axioma, Neutro-Teorema, Anti-Teorema, Función parcial, Neutro-Función, Anti-Función, Neutro-Operación, Anti-Operación, Neutro-Atributo, Anti-Atributo, Neutro-Relación, Anti-Relación, Neutro-Estructura, Anti-Estructura.

Abstract. In this paper we extend Neutro-Algebra and Anti-Algebra to geometric spaces, founding Neutro/Geometry and Anti-Geometry. While Non-Euclidean Geometries resulted from the total negation of a specific axiom (Euclid's Fifth Postulate), Anti-Geometry results from the total negation of any axiom or even more axioms of any geometric axiomatic system (Euclidean, Hilbert, etc.) and of any type of geometry such as Geometry (Euclidean, Projective, Finite, Differential, Algebraic, Complex, Discrete, Computational, Molecular, Convex, etc.), and Neutro-Geometry results from the partial negation of one or more axioms [and without total negation of any axiom] of any geometric axiomatic system and of any type of geometry. Generally, instead of a classical geometric Axiom, one can take any classical geometric Theorem of any axiomatic system and of any type of geometry, and transform it by Neutrosophication or Antisofication into a Neutro-Theorem or Anti-Theorem respectively to construct a Neutro-Geometry or Anti-Geometry. Therefore, Neutro-Geometry and Anti-Geometry are respectively alternatives and generalizations of Non-Euclidean Geometries. In the second part, the evolution from Paradoxism to Neutrosophy, then to Neutro-Algebra and Anti-Algebra, then to Neutro-Geometry and Anti-Geometry, and in general to Neutro-Structure and Anti-Structure that arise naturally in any field of knowledge is recalled. At the end, applications of many Neutro-Structures in our real world are presented.

Keywords: Non-Euclidean Geometries, Euclidean Geometry, Lobachevski-Bolyai-Gauss Geometry, Riemannian Geometry, Neutro-Manifold, Anti-Manifold, Neutro-Algebra, Anti-Algebra, Neutro-Geometry, Anti-Geometry, Neutro-Axiom, Anti-Axiom, Neutro-Theorem, Anti-Theorem, Partial Function, Neutro-Function, Anti-Function, Neutro-Operation, Anti-Operation, Neutro-Attribute, Anti-Attribute, Neutro-Relation, Anti-Relation, Neutro-Structure, Anti-Structure.

1 Introducción

En el mundo real, los espacios no son homogéneos, sino mixtos, complejos, incluso ambiguos. Y los elementos que los pueblan y las reglas que actúan sobre ellos no son perfectos, uniformes o completos, sino fragmentarios y dispares, con información poco clara y conflictiva, y no se aplican en el mismo grado a cada elemento. Los perfectos, idealistas, existen sólo en las ciencias teóricas. Vivimos en un multi-espacio dotado de una multi-estructura [35]. Ni los elementos del espacio ni las normas que los gobiernan son igualitarios, todos ellos se caracterizan por grados de diversidad y variación. Los datos y procedimientos indeterminados (vagos, poco claros, incompletos, desconocidos, contradictorios, etc.) nos rodean.

Es por eso que, por ejemplo, los espacios y estructuras algebraicas y geométricas clásicas se extendieron a espacios y estructuras más realistas [1], llamados respectivamente Neutro-Algebra y Anti-Algebra [2019] y respectivamente Neutro-Geometría y Anti-Geometría [1969, 2021], cuyos elementos no necesariamente se comportan igual, mientras que las operaciones y reglas en estos espacios pueden ser solo parcialmente (no totalmente) verdaderas.

Mientras que las Geometrías No Euclidianas resultan de la negación total de un solo axioma específico (Quinto Postulado de Euclides), la Anti-Geometría resulta de la negación total de cualquier axioma e incluso de más axiomas de cualquier sistema axiomático geométrico (los cinco postulados de Euclides, los 20 axiomas de Hilbert, etc.), y el Neutro-Axioma resulta de la negación parcial de uno o más axiomas [y ninguna negación total de ningún axioma] de cualquier sistema axiomático geométrico.

Por lo tanto, la Neutro-Geometría y la Anti-Geometría son respectivamente alternativas y generalizaciones de las geometrías no euclidianas.

En la segunda parte, recordamos la evolución del Paradoxismo a la Neutrosofía, luego a la Neutro-Álgebra y la Anti-Algebra, luego a Neutro-Geometría y Anti-Geometría, y en general a Neutro-Estructura Anti-Estructura que surgen naturalmente en cualquier campo del saber. Al final, presentamos aplicaciones de muchas Neutro-Estructuras en nuestro mundo real.

En un espacio dado, un axioma clásico es totalmente (100%) cierto. Mientras que un Neutro-Axioma es parcialmente verdadero, parcialmente indeterminado y parcialmente falso. Además, un Anti-Axioma es totalmente (100%) falso.

Una Geometría clásica sólo tiene Axiomas totalmente verdaderos. Mientras que una Neutro-Geometría es una geometría que tiene al menos un Neutro-Axioma y ningún Anti-Axioma. Además, una Anti-Geometría es una geometría que tiene al menos un Anti-Axioma.

A continuación se introduce, en la primera parte de este artículo, la construcción de Neutro-Geometría y Anti-Geometría, junto con las geometrías no euclidianas, mientras que en la segunda parte se aborda la evolución del Paradoxismo a la Neutrosofía, y luego a Neutro-Álgebra y la Anti-Álgebra, culminando con la forma más general de Neutro-Estructura y Anti-Estructura en cualquier campo del conocimiento.

Una declaración clásica (100%) verdadera sobre una estructura clásica dada, puede o no ser 100% verdadera en su correspondiente Neutro- Estructura o Anti- Estructura, depende de los procedimientos de neutrosofización o antisofización [1 - 24].

Más adelante, la tripla neutrosófica (Álgebra, Neutro-Álgebra, Anti-Álgebra) se restringió o extendió a todas las triplas de Teorías de Extensión Difusa (TED) de la forma (Álgebra, Neutro-TED-Algebra, Anti-TED-Algebra), donde TED puede ser: Teoría Difusa, Intuicionista Difusa, Inconsistente Difusa intuicionista (Difusa Ternaria), Pitagórica Difusa (Intuicionista Difusa de segundo tipo de Atanassov), Esférica Difusa, n-Híper-Esférica Difusa, Refinada, Neutrosófica, etc.

1.1 Concepto, Neutro-Concepto, Anti-Concepto

Sobre un espacio geométrico dado, un concepto geométrico clásico (como: axioma, postulado, operador, transformación, función, teorema, propiedad, teoría, etc.), se forma la siguiente tripla neutrosófica geométrica:

Concepto (1, 0, 0), *Neutro-Concepto* (T, I, F), *Anti-Concepto* (0, 0, 1),

donde $(T, I, F) \notin \{(1, 0, 0), (0, 0, 1)\}$.

{Por supuesto, considerando solo los tripla Neutrosóficos (Concepto, Neutro-Concepto, Anti-Concepto) eso tiene sentido en nuestra vida cotidiana y en el mundo real.}

Concepto (1, 0, 0) significa que el grado de verdad del concepto es T = 1, I = 0, F = 0, o el Concepto es 100 % verdadero, 0 % indeterminado y 0 % falso en el espacio geométrico dado.

Neutro-Concepto (T, I, F) significa que el concepto es T% verdadero, I% indeterminado y 0% falso en el espacio geométrico dado, con (T, I, F) $\in$ [0, 1], y (T, I, F) $\notin$ {(1, 0, 0), (0, 0, 1)}.

Anti-Concepto (0, 0, 1) significa que T = 0, I = 0 y F = 1, o el Concepto es 0% verdadero, 0% indeterminado, y 100% falso en el espacio geométrico dado.

1.2 Geometría, Neutro-Geometría, Anti-Geometría

Se puede pasar de la tripla neutrosófica (Álgebra, Neutro-Álgebra, Anti-Álgebra) a una tripla neutrosófica similar (Geometría, Neutro-Geometría, Anti-Geometría), de la misma forma.

- Correspondientemente a partir de las estructuras algebraicas, con respecto a las geometrías, se tiene:
- En la Geometría clásica (Euclidiana), en un espacio dado, todos los Conceptos geométricos clásicos son 100% verdaderos (es decir, verdaderos para todos los elementos del espacio).
- Mientras que en una Neutro-Geometría, en un espacio dado, hay al menos un Neutro-Concepto (y ningún Anti-concepto).
- En la Anti-Geometría, en un espacio dado, existe al menos un Anti-Concepto.

1.3. Neutrosoficación Geométrica y Antisoficación Geométrica

De igual forma, en cuanto a las estructuras algebraicas, utilizando el proceso de Neutrosoficación de una estructura geométrica clásica, se produce una Neutro-Geometría; mientras que a través del proceso de Antisoficación de una estructura geométrica clásica se produce una Anti-Geometría.

Sea S un espacio geométrico clásico y <A> un concepto geométrico (como: postulado, axioma, teorema, propiedad, función, transformación, operador, teoría, etc.). El <antiA> es lo opuesto a <A>, mientras que <neutA> (también llamado <neutroA>) es la parte neutra (o indeterminada) entre <A> y <antiA>.

La trisección de Neutrosoficación S en tres subespacios:

- ➢ El primer subespacio, denotado simplemente por <A>, donde el concepto geométrico es totalmente cierto [grado de verdad T = 1]; lo denotamos por Concepto (1, 0, 0).
- ➢ El segundo subespacio, denotado por <neutA>, donde el concepto geométrico es parcialmente verdadero [grado de verdad T], parcialmente indeterminado [grado de indeterminación I] y parcialmente falso [grado de falsedad F], denotado como Neutro-Concepto (T, I, F), donde (V, I, F) $\notin$ {(1, 0, 0), (0, 0, 1)};
- ➢ El tercer subespacio, denotado por <antiA>, donde el concepto geométrico es totalmente falso [grado de falsedad F = 1], indicado por Anti-Concepto (0, 0, 1).

Los tres subespacios pueden o no estar disjuntos, según la aplicación, pero son exhaustivos (su unión es igual a todo el espacio S).

1.4. Geometrías no Euclidianas

1.4.1. La *Geometría de Lobachevsky* (también conocida como *Lobachevsky-Bolyai-Gauss*), y llamada Geometría Hiperbólica, es una Anti-Geometría, porque el Quinto Postulado Euclidiano (en un plano, a través de un punto fuera de una línea, solo se puede dibujar un paralelo a esa línea) se invalida al 100% en el siguiente Anti-Postulado (primera versión): en un plano a través de un punto fuera de una línea, se pueden dibujar infinitas paralelas a esa línea. O sea, (V, I, F) = (0, 0, 1).

1.4.2. La *Geometría de Riemann*, que se llama *Geometría Elíptica*, es también una Anti-Geometría, ya que el Quinto Postulado Euclidiano se invalida al 100% en la siguiente Anti-Postulado (segunda versión): en un lugar, a través de un punto fuera de una línea, no se puede establecer ningún paralelo atraído por esa línea. O sea, (V, I, F) = (0, 0, 1).

1.4.3. Las *Geometrías de Smarandache* (GS) son más complejas [30 – 57]. ¿Por qué este tipo de geometrías mixtas no euclidianas, y en ocasiones parcialmente no euclidianas y parcialmente euclidianas? Porque los espacios geométricos reales no son puros sino híbridos, y las reglas reales no se aplican uniformemente a todos los elementos del espacio, sino que tienen grados de diversidad, aplicándose a algunos conceptos geométricos (punto, línea, plano, superficie, etc.) en un grado menor o mayor.

Del artículo *Pseudo-Manifold Geometries with Applications* [57] del Prof. Dr. Linfan Mao, Universidad de Cornell, Ciudad de Nueva York, EE. UU., 2006, https://arxiv.org/abs/math/0610307:

"Una geometría de Smarandache es una geometría que tiene al menos un axioma negado a la manera de Smarandache (1969), es decir, un axioma se comporta al menos de dos maneras diferentes dentro del mismo espacio, es decir, validado e invalidado, o solo invalidado pero de múltiples maneras distintas y una variedad n de

Smarandache es una variedad n que admite una geometría de Smarandache.

Iseri proporcionó una construcción para la 2-variedad de Smarandache mediante discos triangulares equiláteros en un plano y una forma más general para la 2-variedad de Smarandache en superficies, denominadas geometrías de mapa, presentada por el autor (…).

Sin embargo, pocas observaciones para casos de n ≥ 3 se encuentran en las revistas. Como un tipo de geometrías de Smarandache, en este trabajo se presenta una forma general de construir n pseudo-variedades dimensionales para cualquier número entero n ≥ 2. Los haces de fibras principales y de conexión también se definen en estas variedades. Siguiendo estas construcciones, casi todas las geometrías existentes, como las de la geometría de Euclides, la geometría de Lobachevshy-Bolyai, la geometría de Riemann, la geometría de Weyl, la geometría de Kahler y la geometría de Finsler, etc. son sus sub-geometrías".

Iseri ([34], [39 - 40]) ha construido algunas Variedades de Smarandache (S-variedades) que topológicamente son lineales por partes, y cuyas geodésicas tienen un comportamiento elíptico, euclidiano e hiperbólico. Una geometría GS puede exhibir uno o más tipos de curvaturas negativas, cero o positivas en el mismo espacio dado.

1.4.3.1) Si al menos un axioma es validado (parcialmente verdadero, T > 0) e invalidado (parcialmente falso, F > 0), y ningún otro axioma solo es invalidado (Anti-Axioma), entonces esta primera clase de geometría GS es una Neutro-Geometría.

1.4.3.2) Si al menos un axioma solo se invalida (o F = 1), no importa si los otros axiomas son clásicos o también Neutro-Axiomas o Anti-Axiomas, entonces esta segunda clase de geometría GS es una Anti-Geometría.

1.4.3.3) El modelo de una geometría SG que es una Neutro-Geometría:

Bhattacharya [38] construyó el siguiente modelo GS:

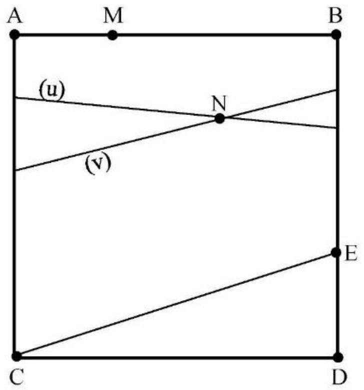

Figura. 1. Modelo de Bhattacharya para la geometría GS como Neutro-Geometría

El espacio geométrico es un cuadrado ABCD, que comprende todos los puntos por dentro y por sus aristas.

"Punto" significa el punto clásico, por ejemplo: A, B, C, D, E, N y M.

"Línea" significa cualquier segmento de línea que conecta dos puntos en los lados cuadrados opuestos AC y BD, por ejemplo: AB, CD, CE, (u) y (v).

Las "líneas paralelas" son líneas que no se cruzan.

Tomemos una línea CE y un punto exterior N a ella. Observamos que hay una infinidad de rectas que pasan por N y paralelas a CE [todas las rectas que pasan por N y entre las rectas (u) y (v) por ejemplo] – el caso hiperbólico.

Además, tomando otro punto exterior, D, no hay una línea paralela que pase por D y sea paralela a CE porque todas las líneas que pasan por D intersecan a CE, el caso elíptico.

Tomando otro punto exterior M ∈ AB, entonces solo tenemos una línea AB paralela a CE, porque solo una línea pasa por el punto M - el caso euclidiano.

En consecuencia, el Quinto Postulado Euclidiano se invalida dos veces, pero también se valida una vez.

Siendo parcialmente hiperbólica no euclidiana, parcialmente elíptica no euclidiana y parcialmente euclidiana, por lo tanto tenemos aquí una GS.

Esta no es una Geometría No-Euclidiana (ya que el Quinto Postulado de Euclides no es totalmente falso, sino sólo parcialmente), pero es una Neutro-Geometría.

Teorema 1.4.3.3.1

Si un enunciado (proposición, teorema, lema, propiedad, algoritmo, etc.) es (totalmente) verdadero (grado de verdad T = 1, grado de indeterminación I = 0 y grado de falsedad F = 0) en la geometría clásica, la declaración puede obtener cualquier valor lógico (es decir, T, I, F pueden ser cualquier valor en [0, 1]) en una Neutro-Geometría o en una Anti-Geometría

Prueba.

El valor lógico que obtiene la declaración en una Neutro- Geometría o en una Anti- Geometría depende de los axiomas clásicos en los que se basa la declaración en la geometría clásica y cómo se comportan estos axiomas en los modelos Neutro-Geometría o Anti-Geometría.

Considerando la siguiente proposición geométrica clásica P (L1, L2, L3) que es 100% cierta:

En un espacio geométrico euclidiano 2D, si dos líneas L1 y L2 son paralelas a la tercera línea L3, entonces también son paralelas (es decir, L1 // L2).

En el Modelo de una geometría GS de Bhattacharya, esta declaración es parcialmente verdadera y parcialmente falsa. Por ejemplo, en la figura 1:

- Grado de verdad: las rectas AB y (u) son paralelas a la recta CE, luego AB es paralela a (u);
- Grado de falsedad: las rectas (u) y (v) son paralelas a la recta CE, pero (u) y (v) no son paralelas ya que se cortan en el punto N.

1.4.3.4) El Modelo de una geometría GS que es una Anti-Geometría

Consideremos el siguiente terreno rectangular PQRS,

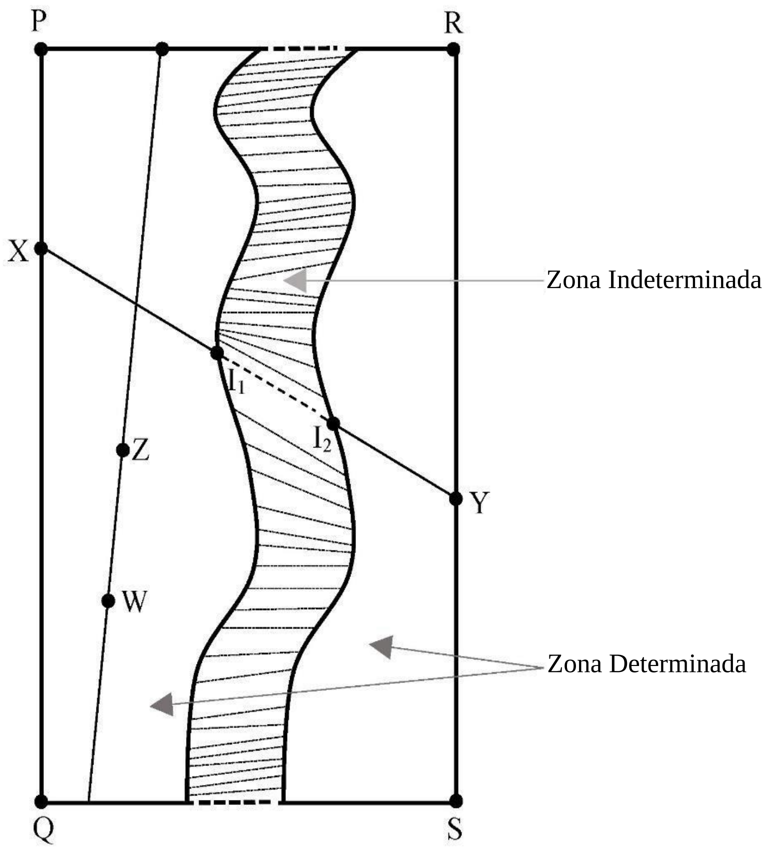

Figura. 2. Modelo para una geometría SG que es una Anti-Geometría

Cuya zona media (sombreada) es una zona indeterminada (un río, con pantano, cañones y sin puente) imposible de cruzar por tierra. Por lo tanto, este pedazo de tierra se compone de una zona determinada y una zona indeterminada (como arriba).

“Punto” significa cualquier punto clásico (usual), por ejemplo: P, Q, R, S, X, Y, Z y W que son puntos conocidos (clásicos) determinados, e I1, I2 que son indeterminados (no conocidos) puntos [en la zona indeterminada].

"Recta" es cualquier segmento de recta que une un punto del lado PQ con un punto del lado RS. Por ejemplo, PR, QS, XY. Sin embargo, estas líneas tienen una parte indeterminada (no conocida, no clara) que es la zona indeterminada. Por otro lado, ZW no es una línea ya que no conecta los lados PQ y RS.

El siguiente axioma geométrico clásico: por dos puntos distintos siempre pasa una sola línea, es totalmente (100%) negado en este modelo de las dos maneras siguientes:

A través de dos puntos distintos, en este modelo dado, no pasa ninguna línea (ver el caso de ZW), o solo pasa una línea parcialmente determinada (ver el caso de XY); por lo tanto, no pasa ninguna línea completamente determinada. Por lo tanto, esta geometría SG es una Anti-Geometría.

1.5. Variedad, Neutro-Variedad, Anti-Variedad

1.5.1. Variedad

La Variedad clásica [29] es un espacio topológico que, en las escalas pequeñas, cerca de cada punto, se parece al Espacio Geométrico clásico (Euclidiano) [es decir, en este espacio sólo hay Axiomas clásicos (totalmente verdaderos)].

O cada punto tiene una vecindad que es homeomorfa a una bola unitaria abierta del Espacio Euclidiano R^n (donde R es el conjunto de los números reales). El homeomorfismo es una función continua y biyectiva cuya inversa también es continua.

"En general, cualquier objeto que sea casi 'plano' en pequeña escala es una variedad" [29].

1.5.2. Neutro-Variedad

La **Neutro-Variedad** es un espacio topológico que, en escalas pequeñas, cerca de cada punto, se parece al Espacio de Neutro-Geometría [es decir, en este espacio hay al menos un Neutro-Axioma (parcialmente verdadero, parcialmente indeterminado y parcialmente falso) y ningún Anti-Axioma].

Por ejemplo, el modelo de Bhattacharya para una geometría GS (Fig. 1) es una Neutro-Variedad, ya que el espacio geométrico ABCD tiene un Neutro-Axioma (es decir, el Quinto Postulado Euclidiano, que es parcialmente verdadero y parcialmente falso) y no tiene Anti-Axioma.

1.5.3. Anti-Variedad

La Anti-Variedad es un espacio topológico que, en las escalas pequeñas, cerca de cada punto, se parece al espacio de la Anti-Geometría [es decir, en este espacio hay al menos un Anti-Axioma (totalmente falso)].

Por ejemplo, el Modelo para una geometría GS (Fig. 2) es una Anti-Variedad, ya que el espacio geométrico PQRS tiene un Anti-Axioma (es decir, por dos puntos distintos siempre pasa una sola línea - lo cual es totalmente falso).

2. Evolución del Paradoxismo a la Neutrosofía luego a Neutro-Álgebra/Anti-Álgebra y ahora a Neutro-Geometría/Anti-Geometría

A continuación se revisan los fundamentos y desarrollos previos que culminaron con la introducción de Neutro-Álgebra y Anti-Álgebra como nuevo campo de investigación, extendido luego a Neutro-Estructura y Anti-Estructura, y ahora particularizado a Neutro-Geometría y Anti-Geometría que son extensiones de las geometrías no euclidianas.

2.1. Del Paradoxismo a la Neutrosofía

El Paradoxismo [58] es un movimiento internacional de ciencia y cultura, fundado por Smarandache en la década de 1980, basado en el uso excesivo de antítesis, oxímoron, contradicciones y paradojas. Durante tres décadas (1980-2020), cientos de autores de decenas de países de todo el mundo contribuyeron con artículos a 15 antologías paradójicas internacionales.

En 1995 extendió la paradoja (basada en opuestos) a una nueva rama de la filosofía llamada Neutrosofía (basada en los opuestos y su neutro) [59], que dio origen a muchas ramas científicas, tales como: lógica neutrosófica, conjunto neutrosófico, probabilidad neutrosófica, estadística neutrosófica, estructuras algebraicas Neutrosóficas, etc. con múltiples aplicaciones en ingeniería, computación ciencia, trabajo administrativo, investigación médica, ciencias sociales, etc.

La Neutrosofía es una extensión de la Dialéctica que se ha derivado de la Filosofía Yin-Yan Chino Antiguo.

2.2. De estructuras algebraicas clásicas a estructuras neutro-algebraicas y estructuras anti-algebraicas

En 2019, Smarandache [1] generalizó las Estructuras Algebraicas clásicas a Estructuras Neutro-Algebraicas (o Neutro-Álgebras) {cuyas operaciones (o leyes) y axiomas (o teoremas) son parcialmente verdaderos, parcialmente indeterminados y parcialmente falsos} como extensiones del Álgebra Parcial, y a Estructuras Anti-Algebraicas (o Anti-Álgebras) {cuyas operaciones (o leyes) y axiomas (o teoremas) son totalmente falsos} y en 2020 siguió desarrollándolas [2, 3, 4].

Generalmente, en lugar de un axioma clásico en un campo de conocimiento, uno puede tomar un teorema clásico en ese campo de conocimiento y transformarlo mediante Neutro-Soficación o Anti-Soficación en un Neutro-Teorema o Anti-Teorema para construir una Neutro-Estructura o Anti-Estructura en ese campo de conocimiento.

Las Neutro-Álgebras y las Anti-Álgebras son un nuevo campo de investigación inspirado en nuestro mundo real. Como se dijo más adelante, también podemos obtener una Neutro-Algebra y Anti-Algebra transformando, en lugar de un Axioma, un Teorema algebraico clásico en un Neutro-Teorema o Anti-Teorema; el proceso se llama Neutro-Soficación o Anti-Soficación respectivamente.

En las estructuras algebraicas clásicas, todas las operaciones están 100% bien definidas y todos los axiomas son 100% ciertos, pero en la vida real, en muchos casos estas restricciones son demasiado duras, ya que en nuestro mundo tenemos cosas que solo verifican parcialmente algunas operaciones o algunas leyes

Al sustituir Concepto con Operación, Axioma, Teorema, Relación, Atributo, Álgebra, Estructura, etc. respectivamente, en lo anterior (Concepto, Neutro-Concepto, Anti-Concepto), obtenemos las siguientes triplas:

2.3. Operación, Neutro-Operación, Anti-Operación

Cuando definimos una operación en un conjunto dado, no significa automáticamente que la operación esté bien definida. Hay tres posibilidades:

1) La operación está bien definida (también llamada internamente definida) para todos los elementos del conjunto [grado de verdad T = 1] (como en las estructuras algebraicas clásicas; esta es una operación clásica). Neutrosóficamente escribimos:Operación (1, 0, 0).
2) La operación si bien definida para algunos elementos [grado de verdad T], indeterminada para otros elementos [grado de indeterminación I], y exteriormente definido para los demás elementos [grado de falsedad F], donde (T, I, F) es diferente de (1, 0, 0) y de (0, 0, 1) (esta es una Neutro-Operación). Neutrosóficamente escribimos: Neutro-Operación (T, I, F).
3) La operación está definida externamente para todos los elementos del conjunto [grado de falsedad F = 1] (esta es una Anti-Operación). Neutrosóficamente escribimos: Anti-Operación (0, 0 ,1).

Una operación * en un conjunto S no vacío dado es en realidad una función de orden n, siendo n un número entero $n \geq 1$, $f : S^n \rightarrow S$.

2.4. Función, Neutro-Función, Anti-Función

Sean U un universo de discurso, A y B dos conjuntos no vacíos incluidos en U, y f una función: $f : A \rightarrow B$

De nuevo, tenemos tres posibilidades:

1) La función está bien definida (también llamada internamente definida) para todos los elementos de su dominio A [grado de verdad T = 1] (esta es una función clásica), es decir, $\forall x \in A, f(x) \in B$. Neutrosóficamente escribimos: Función (1, 0, 0).
2) La función si está bien definida para algunos elementos de su dominio, es decir, $\exists x \in A, f(x) \in B$ [grado de verdad T], indeterminado para otros elementos, es decir, $\exists x \in A, f(x) =$indeterminado [grado de indeterminación I], y definido externamente para los otros elementos, es decir $\exists x \in A, f(x) \notin B$ [grado de falsedad F], donde (T, I, F) es diferente de (1, 0, 0) y de (0, 0, 1). Esta es una Neutro/Función. Neutrosóficamente escribimos: Neutro/Función (T, I, F).
3) La función está definida externamente para todos los elementos de su dominio A [grado de falsedad F = 1] (esta es una Anti/Función), es decir, $\forall x \in A, f(x) \notin B$ (todos los valores de la función están fuera de su codominio B; pueden estar fuera del universo del discurso también). Neutrosóficamente escribimos: Anti-Función (0, 0, 1).

2.5. Neutro-función y Anti-función frente a función parcial

Se prueba que la Neutro-Función y la Anti-Función son extensiones y alternativas de la Función Parcial.

Definición de función parcial [60]

Una función f: A→B a veces se llama una función total, para significar que f(a) está definida para cada a∈A. Si C es cualquier conjunto tal que C⊇A entonces f es también una *función parcial* de C a B.

Claramente, si f es una función de A a B, entonces es una función parcial de A a B, pero una función parcial no necesita definirse para cada elemento de su dominio. El conjunto de elementos de A para los que se define f a veces se denomina dominio de definición.

De otros sitios, la Función Parcial significa: para cualquier a ∈ A se tiene: f(a) ∈ B o f(a) = indefinido.

Comparación

i) "Parcial" se entiende mutuamente cuando existe al menos un elemento a_1∈ A tal que f(a_1) ∈ B, o la Función Parcial está bien definida para al menos un elemento (por lo tanto T > 0).

La Función Parcial no permite el grado bien definido T = 0 (es decir, ningún elemento está bien definido), mientras que la Neutro-Función y la Anti-Función sí lo permiten.

Ejemplo 1.

Consideremos el conjunto de los enteros positivos Z = {1, 2, 3,…}, incluidos en el universo del discurso R, que es el conjunto de los números reales. Definamos la función

$$f_1: Z \to Z, f_1(x) = \frac{x}{0}, para\ todo\ x \in Z$$

Claramente, la función f_1 es 100% indefinida, por lo tanto la indeterminación I = 1, mientras que T =0 y F = 0. Por tanto, f_1 es una Neutro-Función, pero no una Función Parcial.

Ejemplo 2.

Tomemos el conjunto de enteros positivos impares D = {1, 3, 5, …}, incluidos en el universo de discurso R. Definamos la función

$$f_2: D \to D, f_2(x) = \frac{x}{2}, para\ todo\ x \in D$$

La función f_2 está 100% definida externamente, ya que $\frac{x}{2} \notin D$ para todos x∈D. De donde F = 1, T = 0 y I = 0. Por lo tanto, esta es una Anti-Función, pero no una Función parcial.

ii) **La función parcial no detecta todos los tipos de indeterminaciones** que se permiten en una función neutral. Pueden ocurrir indeterminaciones con respecto a: el dominio de la función, el codominio o la relación que conecta los elementos del dominio con los elementos del codominio.

Ejemplo 3.

Consideremos la función g: {1, 2, 3,…, 9, 10, 11} → {12, 13, ⋯, 19}, de quien sólo tienen información vaga y poco clara como se muestra a continuación:

g(1 o 2) = 12, es decir, no estamos seguros si g(1) = 12 o g(2) = 12;

g(3) = 18 o 19, es decir, no estamos seguros si g(3) = 18 o g(3) = 19;

g(4 o 5 o 6) = 13 o 17;

g(7) = desconocido;

g(desconocido) = 14.

Todos los valores anteriores representan el grado de indeterminación de la función (I > 0).

g(10) = 20 que no pertenece al codominio; (definido externamente, o grado de falsedad F > 0);

g(11) = 15 que pertenece al codominio; (definido internamente, o grado de verdad, por lo tanto, T > 0). La Función g es una Neutro-Función (con I > 0, T > 0, F > 0), pero no una Función Parcial ya que este tipo de indeterminaciones no le son propias.

iii) La fracción parcial no captura los valores definidos externamente.

Ejemplo 4.

Sea S = {0, 1, 2, 3} un subconjunto incluido en el conjunto de números racionales Q que sirve como universo de discurso. La función h: S→S, h(x)=$\frac{x}{2}$ es una Neutro/Función, ya que h(0) = 2/0 = indefinido, y h(3) = 2/3 ∉ S (definido exteriormente, 2/3 ∈ Q−S), pero no es una función parcial.

2.6. Axioma, Neutro-Axioma, Anti-Axioma

De manera similar para un axioma, definido en un conjunto dado, dotado de alguna(s) operación(es). Cuando definimos un axioma en un conjunto dado, no significa automáticamente que el axioma sea verdadero para todos los elementos del conjunto. Nuevamente tenemos tres posibilidades:

1) El axioma es verdadero para todos los elementos del conjunto (totalmente verdadero) [grado de verdad T = 1] (como en las estructuras algebraicas clásicas; este es un axioma clásico). Neutrosóficamente escribimos: Axioma (1, 0, 0).
2) El axioma si es verdadero para algunos elementos [grado de verdad T], indeterminado para otros elementos [grado de indeterminación I] y falso para otros elementos [grado de falsedad F], donde (T, I, F) es diferente de (1, 0, 0) y de (0, 0, 1) (esto es Neutro-Axioma). Neutrosóficamente escribimos Neutro-Axioma (T, I, F).
3) El axioma es falso para todos los elementos del conjunto [grado de falsedad F = 1] (esto es Anti-Axioma). Neutrosóficamente escribimos Anti-Axioma (0, 0, 1).

2.7. Teorema, Neutro-Teorema, Anti-Teorema

En cualquier ciencia, un Teorema clásico, definido en un espacio dado, es un enunciado que es 100% verdadero (es decir, verdadero para todos los elementos del espacio). Para probar que un teorema clásico es falso, es suficiente obtener un solo contraejemplo donde el enunciado es falso. Por lo tanto, las ciencias clásicas no dejan lugar a verdad parcial de un teorema (o un enunciado). Pero, en nuestro mundo y en nuestra vida cotidiana, tenemos muchos más ejemplos de declaraciones que son solo parcialmente verdaderas, que declaraciones que son totalmente verdaderas. El Neutro-Teorema y el Anti-Teorema son generalizaciones y alternativas del Teorema clásico en cualquier ciencia.

Consideremos un teorema, establecido en un conjunto dado, dotado de alguna(s) operación(es). Cuando construimos el teorema en un conjunto dado, no significa automáticamente que el teorema es verdadero para todos los elementos del conjunto. Nuevamente tenemos tres posibilidades:

1) El teorema es cierto para todos los elementos del conjunto [totalmente cierto] (como en las estructuras algebraicas clásicas; este es un teorema clásico). Neutrosóficamente escribimos: Teorema (1, 0, 0).
2) El teorema si es verdadero para algunos elementos [grado de verdad T], indeterminado para otros elementos [grado de indeterminación I], y falso para los demás elementos [grado de falsedad F], donde (T, I, F) es diferente de (1, 0, 0) y de (0, 0, 1) (este es un Neutro-Teorema). Neutrosóficamente escribimos: Neutro-Teorema (T, I, F).
3) El teorema es falso para todos los elementos del conjunto (esto es un Anti-Teorema). Neutrosóficamente escribimos: Anti-Teorema (0, 0, 1).

Y lo mismo para (Lema, Neutro-Lema, Anti-Lema), (Consecuencia, Neutro-Consecuencia, Anti-Consecuencia), (Algoritmo, Neutro-Algoritmo, Anti-Algoritmo), (Propiedad, Neutro-Propiedad, Anti-Propiedad), etc.

2.8. Relación, Neutro-Relación, Anti-Relación

1) Una Relación clásica es una relación que es verdadera para todos los elementos del conjunto (grado de verdad T = 1). Neutrosóficamente escribimos Relación (1, 0, 0).
2) Una Neutro-Relación es una relación que es verdadera para algunos de los elementos (grado de verdad T), indeterminada para otros elementos (grado de indeterminación I) y falsa para los otros elementos (grado de falsedad F). Neutrosóficamente escribimos Relación (T, I, F), donde (T, I, F) es diferente de (1, 0, 0) y (0, 0, 1).
3) Una Anti-Relación es una relación que es falsa para todos los elementos (grado de falsedad F = 1). Neutrosóficamente escribimos Relación (0, 0, 1).

2.9. Atributo, Neutro-Atributo, Anti-Atributo

1) Un Atributo clásico es un atributo que es verdadero para todos los elementos del conjunto (grado de verdad T = 1). Neutrosóficamente escribimos Atributo (1, 0, 0).
2) Un Neutro-Atributo es un atributo que es verdadero para algunos de los elementos (grado de verdad T), indeterminado para otros elementos (grado de indeterminación I) y falso para los otros elementos (grado de falsedad F). Neutrosóficamente escribimos Atributo (T, I, F), donde (T, I, F) es diferente de (1, 0, 0) y (0, 0, 1).

3) Un Anti-Atributo es un atributo que es falso para todos los elementos (grado de falsedad F = 1). Neutrosóficamente escribimos Atributo (0, 0, 1).

2.10. Álgebra, Neutro-Álgebra, Anti-Álgebra

1) Una estructura algebraica en la que todas las operaciones están bien definidas y todos los axiomas son totalmente ciertos se llama estructura algebraica clásica (o álgebra).
2) Una estructura algebraica que tiene al menos una Neutro-Operación o un Neutro-Axioma (y ningún Anti-Operación y no Anti-Axioma) se llama Estructura Neutro-Algebraica (o Neutro-Algebra).
3) Una estructura algebraica que tiene al menos una Anti-Operación o un Anti-Axioma se llama Estructura Anti-Algebraica (o Anti-Álgebra).

Por lo tanto, se forma una tripla neutrosófica: <Algebra, Neutro-Algebra, Anti-Algebra>,
Donde “Álgebra” puede ser cualquier estructura algebraica clásica, como: un grupoide, semigrupo, monoide, grupo, grupo conmutativo, anillo, campo, espacio vectorial, BCK-Algebra, BCI-Algebra, etc.

2.11. Álgebra, Neutro$_{TED}$-Algebra, Anti$_{TED}$-Algebra

La tripla neutrosófica (Álgebra, Neutro-Álgebra, Anti-Álgebra) fue más adelante restringida o extendida a todas las teorías difusas y de extensión difusa (TED), formando triplas de la forma: (Álgebra, Neutro$_{TED}$-Algebra, Anti$_{TED}$-Algebra), donde TED puede ser: Teoría Difusa, Intuicionista Difusa, Inconsistente Difusa intuicionista (Difusa Ternaria), Pitagórica Difusa (Intuicionista Difusa de segundo tipo de Atanassov), Esférica Difusa, n-Híper-Esférica Difusa, Refinada, Neutrosófica, etc. A continuación se muestran varios ejemplos.

2.11.1. La tripla intuicionista difusa (Álgebra, Neutro$_{ID}$-Algebra, Anti$_{ID}$-Algebra)

En este caso, "ID" significa "Intuicionista Difusa".

Cuando falta la Indeterminación (I), solo quedan dos componentes, T y F.

1) El Álgebra es la misma que en el entorno neutrosófico, es decir, un Álgebra clásica donde todas las operaciones están totalmente bien definidas y todos los axiomas son totalmente ciertos (T = 1, F = 0).
2) La Neutro$_{ID}$-Algebra significa que al menos una operación o un axioma es parcialmente cierto (grado de verdad T) y parcialmente falso (grado de falsedad parcial F), con $T, F \in [0,1], 0 \leq T + F \leq 1, con\ (T, F) \neq (1,0)$ que representa el axioma clásico, y$(T, F) \neq (1,0)$ que representa el Anti$_{ID}$-Axioma, y sin Anti$_{ID}$-Operación (operación totalmente definida externamente) y sin Anti$_{ID}$-Axioma.
3) La Anti$_{ID}$-Algebra significa que al menos una operación o un axioma es totalmente falso (T = 0, F = 1), sin importar cómo sean las otras operaciones o axiomas.

Por lo tanto, se tienen igualmente las triplas: (Operación, Neutro$_{ID}$-Operación, Anti$_{ID}$-Operación) y (Axioma, Neutro$_{ID}$-Axioma, Anti$_{ID}$-Axioma).

2.11.2. La tripla Difusa (Álgebra, Neutro$_{Difusa}$-Algebra, Anti$_{Difusa}$-Algebra)

Cuando faltan la Indeterminación (I) y la Falsedad (F), sólo queda un componente, T.

1) El Álgebra es la misma que en el entorno neutrosófico, es decir, un Álgebra clásica donde todas las operaciones están totalmente bien definidas y todos los axiomas son totalmente ciertos (T = 1).
2) La Neutro$_{Difusa}$-Algebra significa que al menos una operación o un axioma es parcialmente cierto (grado de verdad T), con T ∈ (0,1), y sin Anti$_{Difusa}$-Operación (operación totalmente definida externamente) y sin Anti$_{Difuso}$-Axioma.
3) La Anti$_{Difusa}$-Algebra significa que al menos una operación o un axioma es totalmente falso (F = 1), sin importar cómo sean las otras operaciones o axiomas.

Por lo tanto, se tienen igualmente las triplas: (Operación, Neutro$_{Difusa}$-Operación, Anti$_{Difusa}$-Operación) y (Axioma, Neutro$_{Difuso}$-Axioma, Anti$_{Difuso}$-Axioma).

2.12. Estructura, Neutro-Estructura, Anti-Estructura en cualquier campo del conocimiento

En general, por Neutro-Soficación, Smarandache extendió cualquier Estructura clásica, en cualquier campo de conocimiento, a una Neutro-Estructura, y por Anti-Soficación a una Anti-Estructura.

i) Una Estructura clásica, en cualquier campo del conocimiento, se compone de: un espacio no vacío, poblado por algunos elementos, y ambos (el espacio y todos los elementos) se caracterizan por unas relación es entre sí (tales como: operaciones, leyes, axiomas, propiedades, funciones, teoremas, lemas, consecuencias, algoritmos, tablas, jerarquías, ecuaciones, desigualdades, etc.), y por sus atributos (tamaño, peso, color, forma, ubicación, etc.).

Por supuesto, a la hora de analizar una estructura, cuenta con respecto a qué relaciones y qué atributos lo hacemos.

ii) Una Neutro-Estructura es una estructura que tiene al menos una Neutro-Relación o un Neutro-Atributo, y ninguna Anti-Relación ni Anti-Atributo.

iii) Una Anti-Estructura es una estructura que tiene al menos una Anti-Relación o un Anti-Atributo.

2.13. Casi todas las Estructuras reales son Neutro-Estructuras

Las Estructuras Clásicas en la ciencia existen principalmente en espacios teóricos, abstractos, perfectos, homogéneos e idealistas, porque en nuestra vida cotidiana casi todas las estructuras son Neutro-Estructuras, ya que no son perfectas ni se aplican a toda la población, y no todos los elementos del espacio tienen las mismas relaciones y los mismos atributos en el mismo grado (no todos los elementos se comportan de la misma manera).

La indeterminación y la parcialidad, respecto del espacio, de sus elementos, de sus relaciones o de sus atributos, no se toman en consideración en las Estructuras Clásicas. Pero nuestro Mundo Real está lleno de estructuras con datos y parcialidades indeterminadas (vagas, poco claras, conflictivas, desconocidas, etc.).

Hay excepciones a casi todas las leyes, y las leyes son percibidas en diferentes grados por diferentes personas.

2.14 Aplicaciones de Neutro-Estructuras en nuestro mundo real

(i) En la sociedad cristiana la ley del matrimonio se define como la unión entre un varón y una mujer (grado de verdad).

Pero, en las últimas décadas, esta ley se ha vuelto menos del 100% cierta, ya que las personas del mismo sexo también podían casarse (grado de falsedad).

Por otro lado, están las personas transgénero (cuyo sexo es indeterminado), y las personas que han cambiado de sexo por procedimientos quirúrgicos, y estas personas (y su matrimonio) no pueden incluirse en las dos primeras categorías (grado de indeterminación).

Por tanto, como tenemos una Neutro-Ley (con respecto a la Ley del Matrimonio) tenemos una Neutro-Estructura Cristiana.

(ii) En India, la ley del matrimonio no es la misma para todos los ciudadanos: los hombres hindúes religiosos pueden casarse con una sola esposa, mientras que los musulmanes pueden casarse con hasta cuatro esposas.

(iii) No siempre la diferencia entre bueno y malo puede ser clara, desde un punto de vista una cosa puede ser buena, mientras que desde otro punto de vista puede ser mala. Hay cosas que son parcialmente buenas, parcialmente neutras y parcialmente malas.

(iv) Las leyes no se aplican por igual a todos los ciudadanos, por lo que son Neutro-Leyes. Algunas leyes se aplican en cierto grado a una categoría de ciudadanos y en diferente grado a otra categoría. Como tal, hay un chiste folclórico estadounidense: ¡Todas las personas nacen iguales, pero algunas personas son más iguales que otras!

- Hay gente poderosa que está por encima de las leyes, y otra gente que se beneficia de la inmunidad respecto de las leyes.
- Por ejemplo, en los tribunales de justicia, las personas privilegiadas se benefician de mejores abogados defensores que las clases bajas, por lo que pueden obtener una sentencia más leve.
- No todos los delincuentes van a la cárcel, sino solo los que son atrapados y se demuestra su culpabilidad en los tribunales de justicia. Ni los criminales que por razón de la locura no pueden ser juzgados y no van a la cárcel ya que no pueden hacer una diferencia entre el bien y el mal.
- Desafortunadamente, incluso personas inocentes fueron y pueden ir a la cárcel debido a veces a errores de jurisdicción...

- La hipocresía y el doble rasero están muy extendidos: ¡alguna regulación se aplica a algunas personas, pero a otras no!

(v) La Ley Antiaborto no se aplica a todas las mujeres embarazadas: el incesto, las violaciones y las mujeres cuya vida corre peligro pueden abortar.

(vi) La Ley de Control de Armas no se aplica a todos los ciudadanos: la policía, el ejército, la seguridad y los cazadores profesionales pueden portar armas. Etc.

Conclusión

En este trabajo se ha extendido las Geometrías No Euclidianas a la Anti-Geometría (un espacio geométrico que tiene al menos un Anti-Axioma) y a la Neutro-Geometría (un espacio geométrico que tiene al menos un Neutro-Axioma y ningún Anti-Axioma) tanto en cualquier sistema axiomático como en cualquier tipo de geometría), de manera similar a Neutro-Algebra y Anti-Algebra. Generalmente, en lugar de un Axioma geométrico, se puede tomar cualquier Teorema geométrico clásico en cualquier sistema axiomático y en cualquier tipo de geometría y transformarlo por Neutro-Soficación o Anti-Soficación en un Neutro-Teorema o Anti-Teorema para construir una Neutro-Geometría o Anti-Geometría respectivamente.

Florentin Smarandache, Neutro-Geometría y Anti-Geometría son alternativas y generalizaciones de las Geometrías No Euclidianas

Un Neutro-Axioma es un axioma que es parcialmente verdadero, parcialmente indeterminado y parcialmente falso en el mismo espacio. Mientras que el Anti-Axioma es un axioma que es totalmente falso en el espacio dado.

Mientras que las Geometrías No Euclidianas resultaron de la negación total de un axioma específico (Quinto Postulado de Euclides), la Anti-Geometría (1969) resultó de la negación total de cualquier axioma e incluso de más axiomas de cualquier sistema axiomático geométrico (Euclidiano, Hilbert, etc.) y de cualquier tipo de geometría como la Geometría (Euclidiana, Proyectiva, Finita, Afín, Diferencial, Algebraica, Compleja, Discreta, Computacional, Molecular, Convexa, etc.), y la Neutro-Geometría resultante de la negación parcial de uno o más axiomas [y ninguna negación total de ningún axioma] de cualquier sistema axiomático geométrico y de cualquier tipo de geometría.

Por tanto, la Neutro-Geometría y la Anti-Geometría son respectivamente alternativas y generalizaciones de las Geometrías No Euclidianas.

En la segunda parte, se analiza la evolución desde el Paradoxismo a la Neutrosofía, luego a la Neutro-Álgebra y Anti-Álgebra, luego a la Neutro-Geometría y Anti-Geometría, y en general a la Neutro-Estructura y Anti-Estructura que surgen naturalmente en cualquier campo del conocimiento.

Al final, presentamos aplicaciones de muchas Neutro-Estructuras en nuestro mundo real.

Más adelante se ha revisado la evolución desde el Paradoxismo a la Neutrosofía, y desde las estructuras algebraicas clásicas a las estructuras Neutro-Álgebra y Anti-Álgebra, y en general a la Neutro-Estructura y Anti-Estructura en cualquier campo del conocimiento. Luego se presentaron muchas aplicaciones de Neutro-Estructuras de la vida cotidiana.

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On neutro-H_v-semigroups

Saeed Mirvakili, Florentin Smarandache, Akbar Rezaei

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Abstract

In this paper, we extend the notion of *Hv*-semigroups to neutro-*Hv*-semigroups and anti-*Hv*-semigroups and investigate many of their properties. We show that these new concepts are different from the classical concept of *Hv*-semigroups by presenting several examples. In general, the neutro-algebras and anti-algebras are generalizations and alternatives of classical algebras. The goal and benefits of our proposed extension of this study is to explore not only the hyperoperations and axioms that are totally true as in previous algebraic hyperstructures, but also the cases when they have degrees of truth, indeterminacy and falsehood. Therefore, we enlarge the field of research.

1Introduction

A hypergroup as a generalization of the notion of a group, was introduced by F. Marty [26] in 1934. Many authors have developed the discussion of hyperstructures and weak hyperstructures, such as P. Corsini [10] and T. Vougiouklis [42]. We can find well-written books for the introduction to hyperstructures, P. Corssini [10], P. Corsini and V. Leoreanu [13], B. Davvaz [14, 15], B. Davvaz and V. Leoreanu-Fotea [17], B. Davvaz and I. Cristea [16]. Another topic which has aroused the interest of several mathematicians, is that one of weak hyperstructure or *Hv*-structure, introduced by T. Vougiouklis [42]. *Hv*-structures are a special kind of hyperstructures, for which the weak associativity holds. Recently, Davvaz and Vougiouklis published a book on *Hv*-structures and their applications [18].

P. Corsini has developed hyperstructure by investigating the relationship between hypergraphs and hypergroups [11]. Corsini and Leoreanu described hypergroups associated with trees and in [12] and [13] some applications of hyperstructures in rough sets were given. hyperalgebraic systems, such as hyperrings, fuzzy hyperideals, fuzzy hypermodules and hyperlattice was introduced by R. Ameri et al. [6–8]. M. Tarnauceanu showed that the set of all subhypergroups of a hypergroup H is not a lattice in general. This is caused mainly by the fact that the intersection fails to be an operation on the set of all subhypergroups [39]. Nowadays, hypergroups have found applications to many subjects of pure and applied mathematics. For example: in geometry, topology, cryptography and coding theory, graphs and hypergraphs, probability theory, binary relations, theory of fuzzy and rough sets and automata theory, physics and also in biological inheritance. Recently, M. AL-Tahan et al. introduced the Corsini hypergroup, topological hypergroupoids and Fuzzy Multi-*Hv*-Ideals [2, 4, 5]. They investigated a necessary and sufficient condition for the productional hypergroup to be a Corsini hypergroup and they characterized all Corsini hypergroups of orders 2 and 3 up to isomorphism [2]. M.K. Sen et al. introduced the notion of hyperset and studied some algebraic structures on it [32] and then G. Chowdhury derived a hypergroup from a hyperset and studied some properties of hyperset in the light of associate hypergroup [9]. Some equivalence relations on a canonical hypergroup to construct a quotient of such hyperstructures were introduced in [31]. S. Hoskova-Mayerova et al. used the fuzzy multisets to introduce the concept of fuzzy multi-hypergroups as a generalization of fuzzy hypergroups and defined the different operations on fuzzy multi-hypergroups and extended the fuzzy hypergroups to fuzzy multi-hypergroups [21]. Recently, D. Heidari et al. considered some classes of semihypergroup such as regular semihypergroup, hypergroups, regular hypergroups and polygroups and investigated their factorization property [20]. Also, V. Vahedi et al. obtained hyperstructures from hyperconics [40] and Mahboob et al. studied hyperideals in semihypergroups [24, 25].

In 2019 and 2020, within the field of neutrosophy, Smarandache [33–35] generalized the classical algebraic Structures to neutro-algebraic structures (or neutro-algebras) {whose operations and axioms are partially true, partially indeterminate, and partially false} as extensions of partial algebra, and to anti-algebraic structures (or anti-algebras) {whose operations and axioms are totally false}. And in general, he extended any classical structure, in no matter what field of knowledge, to a neutro-structure and an anti-structure. These are new fields of research within neutrosophy. Smarandache in [35] revisited the notions of neutro-algebras and anti-algebras, where he studied partial algebras, universal algebras, effect algebras and Boole's partial algebras, and showed that neutro-algebras are generalization of partial algebras. Also, with respect to the classical hypergraph (that contains hyperedges), Smarandache added the supervertices (a group of vertices put all together form a supervertex), in order to form a superhypergraph (SHG). Then he extended the superhypergraph to n-superhypergraph, by extending the power set $P(V)$ to $P^n(V)$ that is the n-power set of the set V (the n-superhypergraph, through its n-SHG-vertices and n-SHG-edges that belong to $P^n(V)$, can the best (so far) to model our complex and sophisticated reality). Further, he extended the classical hyperalgebra to n-ary hyperalgebra and its alternatives n-ary neutro-hyperalgebra and n-ary anti-hyperalgebra [35]. Also, the neutrosophy was used in studying the canonical hypergroups and hyperrings by Agboola and Davvaz [1]. Also AL-Tahan et al. obtained some results in neutrohyperstructures [3]. In this paper, we extend the notion of H_v-semigroups to neutro-H_v-semigroups and anti-H_v-semigroups which are studied and some properties are investigated. We show that these definitions are different from classical definitions by providing several examples. These are particular cases of the classical algebraic structures generalized to neutro-algebraic structures and anti-algebraic structures (Smarandache, 2019).

2 Preliminaries

In this section we recall some basic notions and results regarding hyperstructures.

Definition 1. ([10]) A hypergroupoid $(H, \circ)$ is a non-empty set H together with a map $\circ: H\times H \to P^*(H)$ called (binary) hyperoperation, where $P^*(H)$ denotes the set of all non-empty subsets of H. The image of the pair (x, y) is denoted by $x \circ y$.

If A and B are non-empty subsets of H and $x \in H$, then by $A \circ B$, $A \circ x$, and $x \circ B$ we mean $A\circ B=\bigcup_{a\in A, b\in B} a\circ b$, $A\circ x = A \circ \{ x \}$ and $x \circ B = \{ x \} \circ B$.

Definition 2. ([10]) A hyperoperation on a set H is called associative if it satisfies the associative law:

(**A**) $a \circ (b \circ c) = (a \circ b) \circ c$, for all $a, b, c \in H$.

$(H, \circ)$ is called semihypergroup if the hyperoperation $\circ$ is associative.

Definition 3. ([42]) A hyperoperation $\circ$ on a set H is called weak associative if it satisfies the weak associative law:

(WA) $a \circ (b \circ c) \cap (a \circ b) \circ c \neq \emptyset$, for all $a, b, c \in H$.

$(H, \circ)$ is called H_v-semigroup if the hyperoperation $\circ$ is weak associative.

A hyperoperation $\circ$ on a set H is called commutative if

(C) $a \circ b = b \circ a$, for all $a, b \in H$,

and a hyperoperation $\circ$ on a set H is called weak commutative if

(WC) $a \circ b \cap b \circ a \neq \emptyset$, for all $a, b \in H$.

It clear that every semihypergroup is an H_v-semigroup. Also, every commutative hypergroupoid is a weak commutative hypergroupoid.

Similar to finite semigroups, we can describe the hyperoperation on finite semihypergroups and H_v-semigroups by means Cayley's tables.

Example 1. Let $H = \{a, b, c, d\}$. Define the hyperoperation $(\circ_1)$ on H by the following table.

Table 1

Cayley table for the semihypergroup $(H, \circ_1)$

$\circ_1$	a	b	c	d
a	a	$\{a, b\}$	$\{a, c\}$	$\{a, d\}$
b	a	$\{a, b\}$	$\{a, c\}$	$\{a, d\}$
c	a	b	c	d
d	a	b	c	d

It is not difficult to see that $(H, \circ_1)$ is a semihypergroup.

Example 2. Let $H = \{a, b, c, d\}$. Define the hyperoperation $(\circ_2)$ on H by the following table.

Table 2

Cayley table for the H_v-semigroup $(H, \circ_2)$

$\circ_2$	a	b	c	d
a	a	$\{a, b\}$	$\{a, c\}$	$\{a, d\}$
b	a	$\{a, b\}$	$\{a, c\}$	$\{a, d\}$
c	a	$\{b, c\}$	c	d
d	a	$\{a, b\}$	c	d

It is not difficult to see that the (WA) law is true, but $(d \circ_2 c) \circ_2 b = c \circ_2 b = \{b, c\} \neq \{a, b, c\} = d \circ_2 \{b, c\} = d \circ_2 (c \circ_2 b)$.(d∘2c)∘2b=c∘2b={b,c}≠{a,b,c}=d∘2{b,c}=d∘2(c∘2b).

Then $(H, \circ_2)$ is an H_v-semigroup and is not a semihypergroup.

Example 3. Let H be the unit interval $[0, 1]$. For every $x, y \in H$, we define $x \circ_3 y = [0, \frac{xy}{2}]$x∘3y=[0,xy2]. Then, $(H, \circ_3)$ is a semihypergroup, because for every $x, y, z \in H$, we have $(x \circ_3 y) \circ_3 z = [0, \frac{xy}{2}] \circ_3 z = \bigcup_{u \in [0, \frac{xy}{2}]} u \circ_3 z = \bigcup_{u \in [0, \frac{xy}{2}]} [0, \frac{uz}{2}] = [0, \frac{(\frac{xy}{2})z}{2}] = [0, \frac{x(\frac{yz}{2})}{2}] = \bigcup_{v \in [0, \frac{yz}{2}]} [0, \frac{xv}{2}] = x \circ_3 [0, \frac{yz}{2}] = x \circ_3 (y \circ_3 z)$(x∘3y)∘3z=[0,xy2]∘3z=⋃u∈[0,xy2]u∘3z=⋃u∈[0,xy2][0,uz2]=[0,(xy2)z2]=[0,x(yz2)2]=⋃v∈[0,yz2][0,xv2]=x∘3[0,yz2]=x∘3(y∘3z)

Example 4. Let H be the unit interval $[0, 1]$. For every $x, y \in H$, we define $x \circ_4 y = \{x, \frac{y}{2}, \frac{x}{4}\}$x∘4y={x,y2,x4}. For every $x, y, z \in H$ we have $x \in ((x \circ_4 y) \circ_4 z) \cap (x \circ_4 (y \circ_4 z))$. Then the hyperoperation $\circ_4$ is weak associative and so $(H, \circ_4)$ is an H_v-semigroup, but $(x\circ_4 y)\circ_4 z=\{x,y^2,x^4\}\circ_4 z=\{x,z^2,x^4,y^2,y^8,x^{16}\}$ and $x\circ_4(y\circ_4 z)=x\circ_4\{y,z^2,y^4\}=\{x,y^2,x^4,z^4,y^8\}$. So for some $0 \neq x \in H$, $(x \circ_4 x) \circ_4 x \neq x \circ_4 (x \circ_4 x)$. Then $(H, \circ_4)$ is not a semihypergroup.

3On Neutro-semihypergroups, Anti-semihypergroups, Neutro-H_v-semigroups and Anti-H_v-semigroups

F. Smarandache generalized the classical algebraic structures to the neutro-algebraic structurers and anti-algebraic structures (see Smarandache [33–36]). In this section, we define neutro-semihypergroups, neutro-H_v-semigroups, anti-semihypergroups and anti-H_v-semigroup. Throughout this section, let $P^*(H)=P(H)-\{\emptyset\}$ and $\circ:H\times H\to P^*(U)$ where U is a universe of discourse that contains H. The map $(\circ)$ is called neutro-hyperoperation. If $U=H$ then the neutro-hyperoperation $(\circ)$ is a hyperoperation.

Note that (1,0,0) means that T = 1 (100% true), I = 0, F = 0 and this case corresponds to the classical Hv-semigroup and (0,0,1) means that T = 0, I = 0, F = 1 (100% false) and this corresponds to the anti-Hv-semigroup.

Neutrosophication of an Axiom on a given set *X*, means to split the set *X* into three regions such that:

On one region the axiom is true (we say degree of truth *T* of the axiom), on another region the axiom is indeterminate (we say degree of indeterminacy *I* of the axiom), and on the third region the axiom is false (we say degree of falsehood *F* of the axiom), such that the union of the regions covers the whole set, while the regions may or may not be disjoint, where (*T*, *I*, *F*) is different from (1, 0, 0) and from (0, 0, 1).

Antisophication of an Axiom on a given set *X*, means to have the axiom false on the whole set *X* (we say total degree of falsehood *F* of the axiom), or (0, 0, 1).

Neutrosophication of an operation on a given set *X*, means to split the set *X* into three regions such that on one region the operation is well-defined (or inner-defined) (we say degree of truth *T* of the operation), on another region the operation is indeterminate (we say degree of indeterminacy *I* of the operation), and on the third region the operation is outer-defined (we say degree of falsehood *F* of the operation), such that the union of the regions covers the whole set, while the regions may or may not be disjoint, where (*T*, *I*, *F*) is different from (1, 0, 0) and from (0, 0, 1).

Antisophication of an Operation on a given set *X*, means to have the operation outer-defined on the whole set *X* (we say total degree of falsehood *F* of the axiom), or (0, 0, 1).

Definition 4. (Neutro-hyperoperations and Neutro-semihypergroup)

The neutro-hyperoperation of the hyperoperation (degree of well-defined, degree of indeterminacy, degree of outer-defined)

(NHO) $(\exists\, a, b \in H)$ $(a \circ b \subseteq H)$ {degree of truth T} and $(\exists\, c, d \in H)$ ($c \circ d$ is an indeterminate subset {degree of indeterminacy *I*} and $(\exists\, e, f \in H)$ $(e \circ f \subsetneq H)$ {degree of falsehood *F*}, where (T,I,F) $\notin$ {(1,0,0), (0,0,1)}.

The neutro-hyperaxiom is also characterized by degree of truth, degree of indeterminacy, and degree of falsehood.

Therefore, the neutro-hyperassociativity (NHA) is defined as below:

(NHA) $(\exists\ a, b, c \in H)$ $(a \circ (b \circ c) = (a \circ b) \circ c)$ {degree of truth T},
$(\exists\ d, e, f \in H)$ $(d \circ (e \circ f)$ *or* $(d \circ e) \circ f)$ isindeterminate {degree of indeterminacy I}, and

$(\exists\ g, h, k \in H)$ $(g \circ (h \circ k) \neq (g \circ h) \circ k)$ {degree of falsehood F}, where (T,I,F)$\notin$ {(1, 0, 0) , (0, 0, 1)}.

Definition 5. (Neutro-H_v-semigroup)

The neutro-H_v-semigroup has an H_v-semigroup axiom characterized by degree of truth, degree of indeterminacy, and degree of falsehood, called neutro-weakassociativity (NWA), defined as below:

(NWA) $(\exists\ a, b, c \in H)$ $(a \circ (b \circ c) \cap (a \circ b) \circ c \neq \emptyset)$ {degree of truth T},

$(\exists\ d, e, f \in H)$ $(d \circ (e \circ f)$ or $(d \circ e) \circ f$ are indeterminate {degree on indeterminacy I}, and
$(\exists\ g, h, k \in H)$ $(g \circ (h \circ k) \cap (g \circ h) \circ k = \emptyset)$ {degree of falsehood F}, where (T,I,F) $\notin$ {(1,0,0), (0,0,1)}.

We define neutro-commutative law on $(H, \circ)$ as follows:

(NHC) $(\exists\ a, b \in H)$ $(a \circ b = b \circ a)$ {degree of truth T}, and

$(\exists\ c, d \in H)$ $(c \circ d$ or $d \circ c$ *is indeterminate* {degree of indeterminacy I}, and $(\exists\ e, f \in H)$ $(e \circ f \neq e \circ f)$ {degree of false hood F}), where (T,I,F) $\notin$ {(1,0,0), (0,0,1)}.

Also, we define neutro-weak commutative law on $(H, \circ)$ as follows:

(NHWC) $(\exists\ a, b \in H)$ $(a \circ b \cap b \circ a \neq \emptyset)$ {degree of truth T},

$(\exists\ c, d \in H)$ $(c \circ d$ *or* $d \circ c$, is indeterminate {degree of indeterminacy I}, and
$(\exists\ e, f \in H)$ $(e \circ f \cap f \circ e = \emptyset)$ {degree of falsehood F}), where (T,I,F)$\notin$ {(1, 0, 0) , (0, 0, 1)}.

Now, we define a neutro-hyperalgebraic system (neutro-H_v-algebraic system) $S = H, O, A$, where H is a classical set or a neutrosophic set, O is a set of the hyperoperations or a set of neutro-hyperoperations of the hyperoperations, and A is a set of semihypergroup axioms (H_v-semigroup axioms) or the neutro-semihypergroup axioms (neutro-H_v-semigroup S) of the semihypergroup axioms (H_v-semigroup axioms).

Definition 6. (Anti-hyperoperations and Anti-semihypergroup)

The anti-hyperoperation of the hyperoperation (totally outer-defined) is defined as follows:

(AHO) $(\forall\ x, y \in H)$ $(x \circ y \subsetneq H)$.

The anti-semihypergroup of the semihypergroup axioms (totally false) is defined as follows:

(AA) $(\forall\ x, y, z \in H)$ $(x \circ (y \circ z) \neq (x \circ y) \circ z)$.

Definition 7. (Anti-hyperoperations and Anti-H_v-semigroup)

The anti-hyperoperation of the hyperoperation (totally outer-defined) is defined as follows:

(AHO) $(\forall\ x, y \in H)$ $(x \circ y \subsetneq H)$.

The anti-H_v-semigroup of the H_v-semigroup axioms (totally false) is defined as follows:

(AWA) $(\forall\ x, y, z \in H)$ $((x \circ (y \circ z) \cap (x \circ y) \circ z) = \emptyset)$.

We define anti-commutative law on $(H, \circ)$ as follows:

(AC) $(\forall\ a, b \in H$ with $a \neq b)$ $(a \circ b \neq b \circ a)$.

Also, we define anti-weak commutative law on $(H, \circ)$ as follows:

(AWC) $(\forall\ a, b \in H$ with $a \neq b)$ $(a \circ b \cap b \circ a = \emptyset)$.

Definition 8. A neutro-semihypergroup is an alternative of semihypergroup that has at least an (NHO) or an (NHA) satisfied, with no anti-hyperoperation and no anti-semihypergroup axiom.

Remark 1. Every H_v-semigroup that is not a semihypergroup is a neutro-semihypergroup or an anti-semihypergroup.

Example 5. (i) Let $H=\{a, b, c\}$ and $U=\{a,b,c,d\}$ be a universe of discourse that contains H. We define the neutro-hyperoperation ($\circ_5$) on H with the following Cayley table.

Table 3

Cayley table for the neutro-semihypergroup (H, $\circ_5$)

$\circ_5$	a	b	c
a	a	a	b
b	b	$\{a, b\}$	d
c	b	?	b

Then (H, $\circ_5$) is a neutro-semihypergroup. (NHO) is valid, since $a \circ_5 b \subseteq H$, $b \circ_5 c = \{d\} \subsetneq H$ and $c \circ_5 b =$ indeterminate. Thus, (NHO) holds.

(ii) Let $H=\{a, b, c\}$. Define the hyperoperation ($\circ_6$) on H with the following Cayley table.

Table 4

Cayley table for the neutro-semihypergroup (H, $\circ_6$)

$\circ_6$	a	b	c
a	a	a	a
b	b	$\{a, b\}$	$\{a, b\}$
c	c	$\{b, c\}$	H

Then (H, $\circ_6$) is a neutro-semihypergroup. (NHA) is valid, since
$(b \circ_6 c) \circ_6 a = \{a, b\} \circ_6 a = (a \circ_6 a) \cup (b \circ_6 a) = \{a\} \cup \{b\} = \{a, b\}$
and $b \circ_6 (c \circ_6 a) = b \circ_6 \{c\} = b \circ_6 c = \{a, b\}$. Hence $(b \circ_6 c) \circ_6 a = b \circ_6 (c \circ_6 a)$. Also,
$(b \circ_6 a) \circ_6 c = \{b\} \circ_6 c = b \circ_6 c = \{a, b\}$ and $b \circ_6 (a \circ_6 c) = b \circ_6 \{a\} = b \circ_6 a = \{b\}$, and so
$(b \circ_6 a) \circ_6 c \neq b \circ_6 (a \circ_6 c)$. Thus, (NHA) holds.

Definition 9. An anti-semihypergroup is an alternative of semihypergroup that has at least an (AHO) or an (AA).

Example 6. (i) Let $H=\mathbb{N}$ be the set of natural numbers. Define hyperoperation ($\circ_7$) on $\mathbb{N}$ by $x \circ_7 y = \{\frac{x}{2}, \frac{x}{2}+1, y\}$. Then ($H$, $\circ_7$) is an anti-semihypergroup. (AHO) is valid, since for all $x, y \in \mathbb{N}$, $x \circ_7 y \subsetneq \mathbb{N}$. Thus, (AHO) holds.

(ii) Let $H=\{a, b, c\}$. Define the hyperoperation ($\circ_8$) on H with the following Cayley table.

Table 5

Cayley table for the anti-H_v-semigroup (H, $\circ_8$)

$\circ_8$	a	b	c
a	b	a	b
b	b	a	b
c	b	a	b

Then (H, $\circ_8$) is an anti-semihypergroup. The (AA) law is valid, since for all $x, y, z \in H$, $x \circ_8 (y \circ_8 z) \neq (x \circ_8 y) \circ_8 z$.

Definition 10. A neutro-H_v-semigroup is an alternative of H_v-semigroup that has at least a (NHO) or satisfies (NWA), with no anti-hyperoperation and no anti-H_v-semigroupaxiom.

Example 7. (i) Neutro-semihypergroup $(H, \circ_5)$ in Example 5, is a neutro-H_v-semigroup.

(ii) Let $H =\{ a, b, c, d \}$. Define the hyperoperation $(\circ_9)$ on H with the following Cayley table.

Table 6

Cayley table for the anti-H_v-semigroup $(H, \circ_9)$

$\circ_9$	a	b	c	d
a	H	$\{a, c\}$	$\{a, b\}$	a
b	$\{b, d\}$	$\{a, c\}$	b	a
c	$\{c, d\}$	c	$\{a, b\}$	a
d	d	c	b	a

Then $(H, \circ_9)$ is a neutro-H_v-semigroup but it is not a neutro-semihypergroup. (NWA) is valid, since $(b \circ_9 c) \circ_9 a = b \circ_9 a = \{b, d\}$ and $b \circ_9 (c \circ_9 a) = b \circ_9 \{c, d\} = \{a, b\}$. Hence $(b\circ_9 c) \circ_9 a \cap b \circ_9 (c \circ_9 a) \neq \emptyset$. Also, $(a \circ_9 a) \circ_9 d = H \circ_9 d = \{a\}$ and $a \circ_9 (a \circ_9 d) = b \circ_9 a = \{a, c\}$, and so $(a\circ_9 a) \circ_9 d \cap b \circ_9 (a \circ_9 c) = \emptyset$. Thus, (NWA) holds. Also, we have for every $x, y, z \in H$, $(x \circ_9 y) \circ_9 z \neq x \circ_9 (y \circ_9 z)$ and therefore $(H, \circ_9)$ is not a neutro-semihypergroup.

Definition 11. An anti-H_v-semigroup is an alternative of H_v-semigroup that has at least a (AHO) or (AWA).

Example 8. (i) The anti-semihypergroup $(H, \circ_7)$ in Example 6, is an anti-H_v-semigroup.

(ii) Let $H =\{ a, b, c \}$. Define the hyper operation $(\circ_{10})$ on H with the following Cayley table.

Table 7

Cayley table for the anti-H_v-semigroup $(H, \circ_{10})$

$\circ_{10}$	a	b	c
a	b	a	b
b	b	a	b
c	b	a	b

Then $(H, \circ_{10})$ is an anti-semihypergroup. (AA) law is valid, since for all $x, y, z \in H$, $x \circ_{10} (y \circ_{10} z) \neq (x \circ_{10} y) \circ_{10} z$.

Lemma 1. *Every anti- H_v-semigroup is an anti-semihypergroup.*

The converse of Lemma 1 may not hold. This is because an anti-semihypergroup may be H_v-semigroup or a neutro- H_v-semigroup or an anti- H_v-semigroup.

Example 9. Let $|H| \geqslant 3$ and for every $x, y \in H$ we set $x \circ_{11} y = H - \{y\}$. Then

$(x\circ_{11}y)\circ_{11}z=(H-\{y\})\circ_{11}z=\bigcup_{u\in H-\{y\}}u\circ_{11}z=H-\{z\}$(x∘11y)∘11z=(H-{y})∘11z=⋃u∈H-{y}u∘11z=H-{z}

and

$x\circ_{11}(y\circ_{11}z)=x\circ_{11}(H-\{z\})=\bigcup_{v\in H-\{z\}}x\circ_{11}v=\bigcup_{v\in H-\{z\}}H-\{v\}=H.$x∘11(y∘11z)=x∘11(H-{z})=⋃v∈H-{z}x∘11v=⋃v∈H-{z}H-{v}=H.

Therefore, $(H, \circ_{11})$ is an anti-semihypergroup and an H_v-semigroup.

Example 10. Let $H =\{ a, b \}$ and define the operation $(\circ_{12})$ on $H$with the following Cayley table.

Table 8

Cayley table for the anti-H_v-semigroup $(H, \circ_{12})$

$\circ_{12}$	a	b
a	b	a
b	b	a

Then

Table 9

Test (A) and (WA)

x	y	z	$(x \circ_{12} y) \circ_{12} z$	= or ∩ or Ø	$x \circ_{12} (y \circ_{12} z)$
a	a	a	b	Ø	a
a	a	b	a	Ø	b
a	b	a	b	Ø	a
a	b	b	a	Ø	b
b	a	a	b	Ø	a
b	a	b	a	Ø	b
b	b	a	b	Ø	a
b	b	b	a	Ø	b

Therefore, $(H, \circ_{12})$ is an anti-semihypergroup and an anti-Hv-semigroup.

Example 11. Let $H = \{a, b\}$ and define the hyperoperation $(\circ_{13})$ on H with the following Cayley table.

Table 10

Cayley table for the anti-semihypergroup $(H, \circ_{13})$

$\circ_{13}$	a	b
a	H	a
b	b	a

Then

Table 11

Test (A) and (WA)

x	y	z	$(x \circ_{13} y) \circ_{13} z$	= or ∩ or Ø	$x \circ_{13} (y \circ_{13} z)$
a	a	a	H	∩	a
a	a	b	a	Ø	b
a	b	a	b	∩	H
a	b	b	a	Ø	b
b	a	a	H	∩	a
b	a	b	a	∩	H
b	b	a	b	∩	H
b	b	b	a	∩	H

Therefore, $(H, \circ_{13})$ is an anti-semihypergroup and a neutro-H_v-semigroup.

Lemma 2. *Every neutro-semihypergroup is a neutro-* H_v*-semigroup.*

Proof. A neutro-semihypergroup is endowed with a neutro-associativity, which is also a neutro-weak associativity that characterizes the neutro-H_v-semigroup.

Example 12. Let $H = \{a, b, c\}$ and define the operation $(\circ_{14})$ on H with the following Cayley table.

Table 12

Cayley table for neutro-H_v-semigroup and a neutro-semihypergroup $(H, \circ_{14})$

$\circ_{14}$	a	b	c
a	b	a	a
b	b	a	a
c	a	b	c

We have $(a\circ_{14} a) \circ_{14} a \cap a \circ_{14} (a \circ_{14} a) = \emptyset$ and $(c \circ_{14} c) \circ_{14} c = c \circ_{14} (c \circ_{14} c)$. Therefore, $(H, \circ_{14})$ is a neutro-H_v-semigroup and a neutro-semihypergroup.

Remark 2. From Example 11, we can see that there exists a neutro-H_v-semigroup such that it is an anti-semihypergroup.

Theorem 1. *Let* $(H, \circ)$ *be a weak commutative hypergroupoid. Then it cannot be an anti-* H_v*-semigroup.*

Proof. Let $x \in H$ and $b \in x \circ x$. Then there exists $c \in x \circ b \cap b \circ x$. Therefore
$c\in x\circ b\cap b\circ x\subseteq x\circ(x\circ x)\cap(x\circ x)\circ x.$c∈x∘b∩b∘x⊆x∘(x∘x)∩(x∘x)∘x.

This implies that $x\circ (x \circ x) \cap (x \circ x) \circ x \neq \emptyset$, and so $(H, \circ)$ cannot be an anti-H_v-semigroup.

The next example shows that there exists a weak commutative anti-semihypergroup.

Example 13. Let $(H, \circ_{11})$ be the anti-semihypergroup in the Example 9. Since $|H| \geqslant 3$ then there exists $z \in H$ such that $z \in x \circ_{11} y = H - \{y\}$ and $z \in y \circ_{11} x = H - \{x\}$. Therefore, $(H, \circ_{11})$ is a weak commutative anti-semihypergroup.

Theorem 2. ([30]) *Let* $(H, \circ)$ *be a commutative hypergroupoid. Then it cannot be an anti-semihypergroup.*

Definition 12. ([42]) Let $(H_1, \circ_1)$ and $(H_2, \circ_2)$ be two hypergroupoids. We say that $(\circ_1)$ is less than or equal to $(\circ_2)$, and note $\leqslant$, if and only if there exists $f \in Aut\,(H, \circ_2)$ such that $x \circ_1 y \subseteq f\,(x \circ_2 y)$ for any x, y of H.

From this definition we can deduce the following theorem:

Theorem 3. ([42]) *If a hyperoperation is (WA), then any hyperoperation superior to it and defined on the same set is (WA), too.*

Note that if a hyperoperation is (A), then any hyperoperation superior to it and defined on the same set may not be true in (A) law, but it is true (WA) law.

Example 14. Let $H = \{a, b\}$ and define the operation $(\circ_{15})$ on $H$with the following Cayley table.

Table 13

Cayley table for the Null semigroup $(H, \circ_{15})$

$\circ_{15}$	a	b
a	a	a
b	a	a

$(H, \circ_{15})$ is a semigroup and called Null semigroup. Now, we define the hyperoperation $(\circ_{16})$ on $H$with the following Cayley table.

Table 14

Cayley table for the H_v-semigroup $(H, \circ_{16})$

$\circ_{16}$	a	b
a	a	a

$\circ_{16}$	a	b
b	H	a

Then ($\circ_{15}$) is less than or equal to ($\circ_{16}$) and (H, $\circ_{15}$) is a semihypergroup. It easy to see that (H, $\circ_{16}$) is not a semihypergroup, but it is a neutro-semihypergroup and an H_v-semigroup.

Theorem 4. *If a hyperoperation* ($\circ_2$) *is (AWA) and* ($\circ_1$) *is less than or equal to* ($\circ_2$)*, then* ($\circ_1$) *is (AWA), too.*

Proof. Suppose there exist $x, y, z \in H$ such that $x \circ_2 (y \circ_2 z) \cap (x \circ_2 y) \circ_2 z \neq \emptyset$. Since ($\circ_1$) is less than or equal to ($\circ_2$), then $x \circ_1 (y \circ_1 z) \subseteq x \circ_2 (y \circ_2 z)$, and $(x \circ_2 y) \circ_2 z \subseteq (x \circ_2 y) \circ_2 z$.
Therefore, $x \circ_1 (y \circ_1 z) \cap (x \circ_1 y) \circ_1 z \neq \emptyset$ and this is a contradiction with (AWA) law of hyperoperation ($\circ_1$).

Note that if a hyperoperation ($\circ_2$) is (AA) and ($\circ_1$) is less than or equal to ($\circ_2$), then ($\circ_1$) may not be true in (AA) law or (AWA) law.

Example 15. Let $H = \{a, b, c\}$ and define the hyperoperation ($\circ_{17}$) on $H$with the following Cayley table.
Table 15
Cayley table for the anti-semihypergroup (H, $\circ_{17}$)

$\circ_{17}$	a	b	c
a	$\{b, c\}$	$\{a, c\}$	$\{a, b\}$
b	$\{b, c\}$	$\{a, c\}$	$\{a, b\}$
c	$\{b, c\}$	$\{a, c\}$	$\{a, b\}$

(H, $\circ_{17}$) is an anti-semihypergroup. We define the hyperoperation ($\circ_{18}$) on $H$with the following Cayley table.
Table 16
Cayley table for the neutro-H_v-semigroup (H, $\circ_{18}$)

$\circ_{18}$	a	b	c
a	b	$\{c\}$	$\{b\}$
b	b	$\{a, c\}$	b
c	b	$\{a, c\}$	b

We obtain $(a \circ_{18} a) \circ_{18} a \cap a \circ_{18} (a \circ_{18} a) = \emptyset$ and $(c \circ_{18} c) \circ_{18} c = c \circ_{18} (c \circ_{18} c)$. Therefore, ($H$, $\circ_{18}$) is a neutro-H_v-semigroup and a neutro-semihypergroup.

By the above theorems we have the following:

Theorem 5. *Let* (H, $\circ_1$) *and* (H, $\circ_2$) *be two hypergroupoids and* $\circ_1 \leqslant \circ_2$. *Then*

- **1.** *If* (H, $\circ_1$) *is an* H_v*-semigroup, then* (H, $\circ_2$) *is an* H_v*-semigroup.*
- **2.** *If* (H, $\circ_1$) *is a weak commutative hypergroupoid, then* (H, $\circ_2$) *is a weak commutative hypergroupoid.*
- **3.** *If* (H, $\circ_1$) *is a neutro-* H_v*-semigroup, then* (H, $\circ_2$) *cannot be an anti-* H_v*-semigroup.*
- **4.** *If* (H, $\circ_1$) *is a neutro-weak commutative hypergroupoid, then* (H, $\circ_2$) *cannot be an anti-weak commutative hypergroupoid.*
- **5.** *If* (H, $\circ_2$) *is a neutro-* H_v*-semigroup, then* (H, $\circ_1$) *cannot be an* H_v*-semigroup.*
- **6.** *If* (H, $\circ_2$) *is an anti-weak commutative hypergroupoid, then* (H, $\circ_1$) *cannot be a weak commutative hypergroupoid.*
- **7.** *If* (H, $\circ_2$) *is an anti-* H_v*-semigroup, then* (H, $\circ_1$) *is an anti-* H_v*-semigroup.*

- **8.** *If* $(H, \circ_2)$ *is an anti-weak commutative hypergroupoid, then* $(H, \circ_1)$ *is an anti-weak commutative hypergroupoid.*

Proof.

- **1.** For every $x, y, z \in H$, $x \circ_1 (y \circ_1 z) \subseteq x \circ_2 (y \circ_2 z)$ and $(x \circ_1 y) \circ_1 z \subseteq (x \circ_2 y) \circ_2 z$. So $x \circ_1 (y \circ_1 z) \cap (x \circ_1 y) \circ_1 z \neq \emptyset$ implies that $x \circ_2 (y \circ_2 z) \cap (x \circ_2 y) \circ_2 z \neq \emptyset$. Therefor $(H, \circ_2)$ is an H_v-semigroup.
- **2.** It is straightforward.
- **3.** Let $(H, \circ_1)$ is a neutro-H_v-semigroup, then there exists $x, y, z \in H$ such that $x \circ_1 (y \circ_1 z) \cap (x \circ_1 y) \circ_1 z \neq \emptyset$ and this implies that $x \circ_2 (y \circ_2 z) \cap (x \circ_2 y) \circ_2 z \neq \emptyset$. Therefor $(H, \circ_2)$ cannot be an anti-H_v-semigroup.
- **4.** It proves that similar part 3.
- **5.** It obtains from 1.
- **6.** It is straightforward.
- **7.** For every $x, y, z \in H$, $x \circ_1 (y \circ_1 z) \subseteq x \circ_2 (y \circ_2 z)$ and $(x \circ_1 y) \circ_1 z \subseteq (x \circ_2 y) \circ_2 z$. If for some $x, y, z \in H$, $x \circ_1 (y \circ_1 z) \cap (x \circ_1 y) \circ_1 z \neq \emptyset$ then $x \circ_2 (y \circ_2 z) \cap (x \circ_2 y) \circ_2 z \neq \emptyset$. Therefor $(H, \circ_2)$ is not an anti-H_v-semigroup and this is a contradiction.
- **8.** It is straightforward.

Definition 13. Let $(H, \circ)$ be a hypergroupoid. We call the complement of $(H, \circ)$ is the partial hypergroupoid $(H, *)$ when for all $x, y \in H$, we define $x * y = H - x \circ y$.

Lemma 3. *If* $(H, \circ)$ *is a group and* $|H| \geqslant 2$, *then* $(H, *)$ *is a semihypergroup.*

Proof. For all $x, y \in H$,

$x\star(y\star z)=x\star(H-y\circ z)=\bigcup_{u\in H-y\circ z}x\star u=\bigcup_{u\in H-y\circ z}H-x\circ u=H-\bigcap_{u\in H-y\circ z}x\circ u=H-\emptyset=H$x★(y★z)=x★(H-y∘z)=⋃u∈H-y∘zx★u=⋃u∈H-y∘zH-x∘u=H-⋂u∈H-y∘zx∘u=H-Ø=H

and

$(x\star y)\star z=(H-x\circ y)\star z=\bigcup_{v\in H-x\circ y}v\star z=\bigcup_{v\in H-x\circ y}H-v\circ z=H-\bigcap_{v\in H-x\circ y}v\circ z=H-\emptyset=H$(x★y)★z=(H-x∘y)★z=⋃v∈H-x∘yv★z=⋃v∈H-x∘yH-v∘z=H-⋂v∈H-x∘yv∘z=H-Ø=H

Example 16. Let $(H = \{a, b\}, \circ_{19})$ be the semihypergroup with the following Cayley table.

Table 17
Cayley table for the semihypergroup $(H, \circ_{19})$

$\circ_{19}$	a	b
a	a	b
b	a	b

Then by Example 10, $(\circ_{12}) = (\star)$ and $(H, \star)$ is the anti-semihypergroup and anti-H_v-semigroup.

Example 17. Let $(H = \{a, b\}, \circ_{20})$ be the semihypergroup with the following Cayley table.

Table 18
Cayley table for the semihypergroup $(H, \circ_{20})$

$\circ_{20}$	a	b
a	a	a
b	a	b

Then $(H, \bigstar)$ is a semihypergroup with the following Cayley table.

Table 19

Cayley table for the semihypergroup $(H, \bigstar)$

$\bigstar$	a	b
a	b	b
b	b	a

Example 18. ([14]) The semihypergroup $H =\{ a, b, c \}$ with the following hyperoperation is a semihypergroup.

Table 20

Cayley table for the semihypergroup $(H, \circ_{21})$

$\circ_{21}$	a	b	c
a	b	$\{b, c\}$	$\{b, c\}$
b	$\{b, c\}$	$\{b, c\}$	$\{b, c\}$
c	$\{b, c\}$	$\{b, c\}$	$\{b, c\}$

Then $(H, \bigstar)$ has the following Cayley table.

Table 21

Cayley table for the neutro-semihypergroup $(H, \bigstar)$

$*$	a	b	c
a	$\{a, c\}$	a	a
b	a	a	a
c	a	a	a

We have

$$a\bigstar(a\bigstar a)=a\bigstar\{a,c\}=\{a,c\}=\{a,c\}\bigstar a=(a\bigstar a)\bigstar a$$a★(a★a)=a★{a,c}={a,c}={a,c}★a=(a★a)★a

and

$$b\bigstar(a\bigstar a)=b\bigstar\{a,c\}=a\neq\{a,c\}=b\bigstar a=(b\bigstar a)\bigstar a.$$b★(a★a)=b★{a,c}=a≠{a,c}=b★a=(b★a)★a.

Therefore, $(H, \bigstar)$ is a neutro-semihypergroup.

Theorem 6. *Let $(H, \circ)$ be an H_v-semigroup then for every $x, y \in H$, define $x \bigstar y = H - x \circ y$. Then $(H, \bigstar)$ is a neutro- H_v-semigroup or an anti- H_v-semigroup or an H_v-semigroup.*

Proof. If there exist $x, y \in H$ such that $x \circ y = H$, then (NHO) or (NHO) is valid for partially hyperoperation $(\bigstar)$. Then $(H, \bigstar)$ is a neutro-H_v-semigroup or an anti-H_v-semigroup. So, let for every $x, y \in H$, $x \circ y \neq H$. Then $(\bigstar)$ is a hyperoperation.

Let $(H, \circ)$ be an H_v-semigroup. For every $x, y \in H$, define $x \bigstar y = H - x \circ y$. Then $(H, \bigstar)$ can be a neutro-H_v-semigroup or an anti-H_v-semigroup or an H_v-semigroup.

Example 19. Let $(H, \circ)$ be a total semihypergroup, i.e., for every $x, y \in H$, $x \circ y = H$.
Then $x\bigstar y = H - x \circ y = H - H = \emptyset$, and so $(H, \bigstar)$ is an anti-H_v-semigroup.

Lemma 4. *If for some $x \in H$, $x \in x \circ x$, then $(H, \circ)$ cannot be an anti- H_v-semigroup.*

Proof. Let $x \in x \circ x$. Then $x \in ((x \circ x) \circ x) \cap (x \circ (x \circ x))$, and so $(H, \circ)$ cannot be an anti-H_v-semigroup.

Theorem 7. *Let $(H, \circ)$ be an anti- H_v-semigroup then for every $x, y \in H$. Then $(H, \bigstar)$ cannot be an anti- H_v-semigroup.*

Proof. Let $x \in H$. By Lemma 5, $x \notin x \circ x$, and so $x \in x \bigstar x = H - x \circ x$. Therefore, by Lemma 5, $(H, \bigstar)$ cannot be an anti-H_v-semigroup.

Corollary 1. *Let $(H, \circ)$ be an anti- H_v-semigroup. For every $x, y \in H$, define $x \bigstar y = H - x \circ y$. Then $(H, \bigstar)$ is a neutro- H_v-semigroup or an H_v-semigroup.*

Example 20. Let $(H = \{a, b\}, \circ_{12})$ be the anti-H_v-semigroup with the following Cayley table (see Examples 10 and 17)

Table 22

Cayley table for the anti-H_v-semigroup $(H, \circ_{12})$

$\circ_{12}$	a	b
a	b	a
b	b	a

Then by Example 17, $(\circ_{19}) = (\bigstar)$ and $(H, \bigstar)$ is an H_v-semigroup.

Lemma 6. *Every anti-weak commutative hypergroupoid is an anti-commutative hypergroupoid.*

Theorem 8. *Let $(H_i, \circ)$, where $i \in \Lambda$, be a family of neutro- H_v-semigroups. Then $(\bigcap_{i\in\Lambda} H_i, \circ)$ is a neutro- H_v-semigroup or anti- H_v-semigroup or H_v-semigroup.*

Theorem 9. *Let $(H_i, \circ)$, where $i \in \Lambda$, be a family of anti- H_v-semigroups. Then $(\bigcap_{i\in\Lambda} H_i, \circ)$ is an anti- H_v-semigroup.*

Theorem 10. *Let $(H, \circ)$ be a hypergroupoid and $(G, \circ)$ be an anti- H_v-semigroup. Then $(H \cap G, \circ)$ is an anti- H_v-semigroup.*

Theorem 11. *Let $(H, \circ_H)$ be a neutro- H_v-semigroup and $(G, \circ_G)$ be an anti- H_v-semigroup and $H \cap G = \emptyset$. Define a hyperoperation $\circ$ on $H \cup G$ by:*

$$x \circ y = \begin{cases} x \circ_H y & \text{if } x, y \in H; \\ x \circ_G y & \text{if } x, y \in G; \\ x \circ_H x & \text{if } x \in H, y \in G; \\ x \circ_G x & \text{if } x \in G, y \in H. \end{cases}$$

Then $(H \cup G, \circ)$ is an anti- H_v-semigroup.

Theorem 12. *Let $(H, \circ_H)$ be a neutro- H_v-semigroup and $(G, \circ_G)$ be an anti- H_v-semigroup and $H \cap G = \emptyset$. Define a hyperoperation $\circ$ on $H \cup G$ by:*

$$x \circ y = \begin{cases} x \circ_H y & \text{if } x, y \in H; \\ x \circ_G y & \text{if } x, y \in G; \\ H & x \in H, y \in G; \\ G & x \in G, y \in H. \end{cases}$$

Then $(H \cup G, \circ)$ is a neutro- H_v-semigroup.

Proof. If $x, y, z \in G$ we have $(x \circ_G y) \circ_G z \cap x \circ_G (y \circ_G z) = \emptyset$, and so $(x \circ y) \circ z \cap x \circ (y \circ z) = \emptyset$.
If $x \in H$ and $y \in G$, then $(x \circ y) \circ x = H \circ x = H \circ_H x$ and $x \circ (y \circ x) = x \circ G = H$. Hence $(x \circ y) \circ x = H \circ_H x \subseteq H = x \circ (y \circ x)$ and $(x \circ y) \circ x \cap x \circ (y \circ x) \neq \emptyset$. Therefore, $(H \cup G, \circ)$ is a neutro-H_v-semigroup.

Let $(H_1, \circ_1)$ and $(H_2, \circ_2)$ be two hypergroupoids. Then $(H \times G, *)$ is a hypergroupoid, where $*$ is defined on $H \times G$ by: for any $(x_1, y_1), (x_2, y_2) \in H \times G$

$$(x_1, y_1) * (x_2, y_2) = (x_1 \circ_1 x_2, y_1 \circ_2 y_2).$$

Then we obtain the following table.

Table 23

Semigroup $(H,\times)$

×	SHG	HvSG	NSHG	NHvSG	ASHG	AHvSG
SHG	SHG	HvSG	NSHG	NHvSG	ASHG	AHvSG
HvSG	HvSG	HvSG	NHvSG	NHvSG	ASHG	AHvSG
NSHG	NSHG	NHvSG	NSHG	NHvSG	ASHG	AHvSG
NHvSG	NHvSG	NHvSG	NHvSG	NHvSG	ASHG	AHvSG
ASHG	ASHG	ASHG	ASHG	ASHG	ASHG	AHvSG
AHvSG	AHvSG	AHvSG	AHvSG	AHvSG	AHvSG	AHvSG

Abbreviations: SHG, Semihypergroup; HvSG, H_v-semigroup; NSHG, Neutro-semihypergroup; NHvSG, Neutro-H_v-semigroup; ASHG, Anti-semihypergroup; AHvSG, Anti-H_v-semigroup.

By the above table, we have the following:

Theorem 13. *Set* $H:=\{SHG, HvSG, NSHG$, *NHvSG, ASHG, AHvSG*$\}$. *Then* $(H,\times)$ *is a commutative monoid (i.e. semigroup with identity). SHG is an identity element of* $(H,\times)$ *and AHvSG is a zero element of* $(H,\times)$.

Corollary 2. *Every subset of* H *is a semigroup such that it has an identity and a zero element.*

Theorem 14. *Let* $(H, \circ)$ *be a neutro-* H_v*-semigroup and let* $H_1:=\bigcup_{x\circ y\notin P^*(H)}\{x,y\}$. *If* $H_1\neq \emptyset$, *then* $(H_1, \circ)$ *is a neutro-* H_v*-semigroup or an anti-* H_v*-semigroup.*

Theorem 15. *Let* $(H, \circ)$ *be a neutro-* H_v*-semigroup and let* $H_2:=\bigcup_{x\circ(y\circ z)\neq(x\circ y)\circ z}\{x,y,z\}$. *If* $H_2\neq \emptyset$, *then* $(H_2, \circ)$ *is a neutro-* H_v*-semigroup or an anti-* H_v*-semigroup.*

Suppose that $(H, \circ_1)$ *and* $(G, \circ_2)$ *are two hypergroupoids. A function* $f : H \to G$ *is called a homomorphism if, for all* $a, b \in H$, $f(a \circ_1 b) = f(a) \circ_2 f(b)$ *(see [10] and [14] for details).*

Theorem 16. *Let* $(H, \circ_1)$ *be an* H_v*-semigroup,* $(G, \circ_2)$ *be a neutro-* H_v*-semigroup and* $f : H \to G$ *be a homomorphism. Then* $(f(H), \circ_2)$ *is an* H_v*-semigroup, where* $f(H)=\{ f(h) : h \in H \}$.

Proof. Assume that $(H, \circ_1)$ is an H_v-semigroup and $x, y, z \in f(H)$. Then there exist $h_1, h_2, h_3 \in f(H)$ such that $f(h_1) = x$, $f(h_2) = y$ and $f(h_3) = z$, and there exists $u \in h_1 \circ (h_2 \circ h_3) \cap (h_1 \circ h_2) \circ h_3$. So $x\circ(y\circ z)=f(h_1)\circ(f(h_2)\circ f(h_3))=f(h_1)\circ f(h_2\circ h_3)=f(h_1\circ(h_2\circ h_3))\ni f(u)\in f((h_1\circ h_2)\circ h_3)=f(h_1\circ h_2)\circ f(h_3)=(f(h_1)\circ f(h_2))\circ f(h_3)=(x\circ y)\circ z.$

Therefore, $(f(H), \circ_2)$ is an H_v-semigroup.

4Conclusion

The classical algebraic structures were generalized to neutro-algebraic structurers [*neutro-algebras*] and anti-algebraic structures [*anti-algebras*] in 2019 and 2020 by Smarandache. This generalization was done thanks to the processes of neutrosophication and respectively antisophication of the operations and axioms.

In this paper we defined the neutro-semihypergroups, neutro-H_v-semigroups, anti-semihypergroups and anti-H_v-semigroups, and we proved several of their properties and illustrated the paper with many examples.

For future work, it will be interesting to introduce neutrosophication and antisophication on other hyperstructures. Also, it will be interesting to enumerate neutro-semihypergroups, neutro-H_v-semigroups, anti-semihypergroups and anti-H_v-semigroups of small order.

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The SuperHyperFunction and the Neutrosophic SuperHyperFunction

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Abstract: In this paper, one recalls the general definition of the SuperHyperAlgebra with its SuperHyperOperations and SuperHyperAxioms [2, 6]. Then one introduces for the first time the SuperHyperTopology and especially the SuperHyperFunction and Neutrosophic SuperHyperFunction. One gives a numerical example of a SuperHyperGroup.

Keywords: SuperHyperAlgebra; SuperHyperFunction; Neutrosophic SuperHyperFunction; SuperHyperOperations; SuperHyperAxioms; SuperHyperTopology.

1. System of Sub-Systems of Sub-Sub-Systems and so on

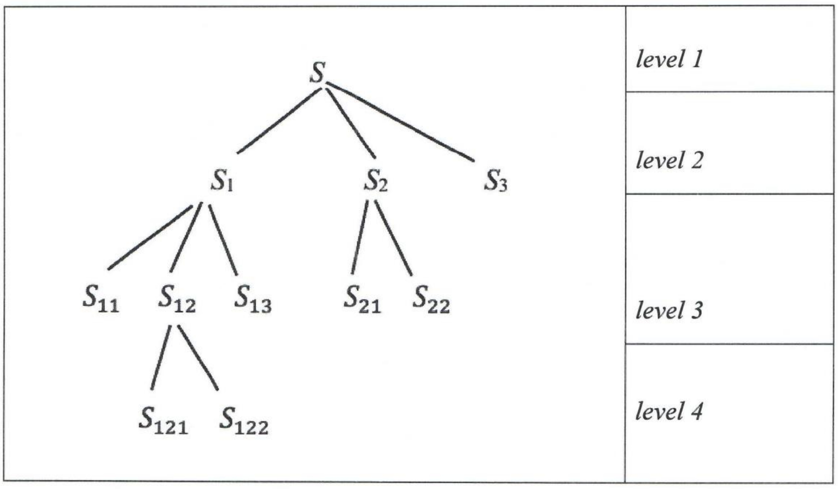
A system may be a set, space, organization, association, team, city, region, country, etc. One considers both: the static and dynamic systems.

With respect to various criteria, such as: political, religious, economic, military, educational, sportive, touristic, industrial, agricultural, etc.,

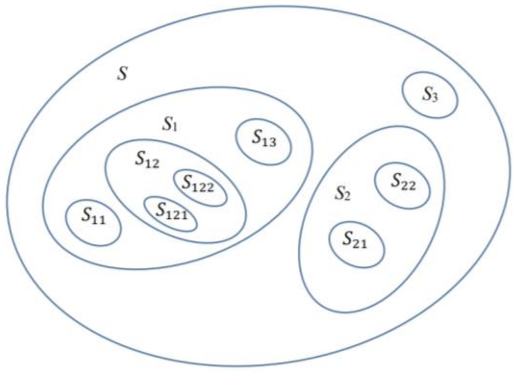
a system S is made up of several sub-systems S_1, S_2, …, S_p, for integer $p \geq 1$; then each sub-system S_i, for $i \in \{1, 2, \dots, p\}$ is composed of many sub-sub-systems S_{i1}, S_{i2}, …, S_{ip_i}, for integer $p_i \geq 1$; then each sub-sub-system S_{ij}, for $j \in \{1, 2, \dots, p_i\}$ is composed sub-sub-sub-systems, S_{ij1}, S_{ij2}, …, S_{ijp_j}, for integer $p_j \geq 1$; and so on.

2. Example 1 of Systems made up of Sub-Sub-Sub-Systems (Level 4)

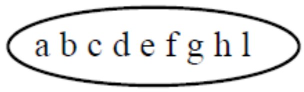
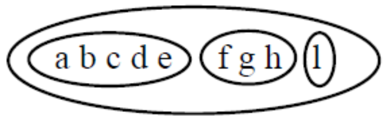
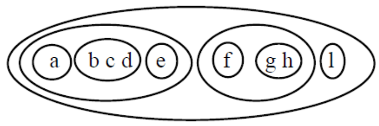
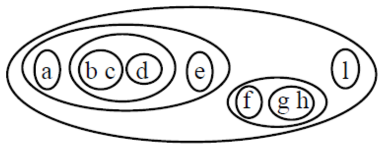
i) Using a *Tree-Graph Representation*, one has:



ii) Using a *Geometric Representation*, one has:



iii) Using an *Algebraic Representation* through pairs of braces { }, one has:

$P^0(S) \stackrel{\text{def}}{=} S = \{a, b, c, d, e, f, g, h, l\}$ 1 level of pairs of braces	 1level of closed curves	*level 1*
$P^1(S) \stackrel{\text{def}}{=} P(S) \ni \{\{a, b, c, d, e\}, \{f, g, h\}, \{l\}\}$ 2 levels of pairs of braces i.e. a pair of braces { } inside, another pair of braces { }, or { … { … } … }	 2 levels of closed curves	*level 2*
$P^2(S) \stackrel{\text{def}}{=} P(P(S))$ $\ni \{\{\{a\}, \{b, c, d\}, \{e\}\}, \{\{f\}, \{g, h\}\}, \{l\}\}$ 3 levels of pairs of braces	 3 levels of closed curves	*level 3*
$P^3(S) \stackrel{\text{def}}{=} P(P^2(S))$ $\ni \{\{\{a\}, \{b, c\}, \{d\}, \{e\}\}, \{\{f\}, \{g, h\}\}, \{l\}\}$ 4 levels of pairs of braces	 4 levels of closed curves	*level 4*

where the symbol "∋" means "contain(s)", it is the opposite of the symbol "∈" (belong(s) to), for example $M \ni x$ means the set M contains the element x, which is equivalent to $x \in M$.

2.1 Remark 1

The pairs of braces { } make a difference on the struture of a set. For example, let's see the distinction between the sets A and B, defined as bellow:

$A = \{a, b, c, d\}$ represents a system (organization) made up of four elements,

while $B = \{\{a, b\}, \{c, d\}\}$ represents a system (organization) made up of two sub-systems and each sub-system made up of two elements. Therefore B has a richer structure, it is a refinement of A.

3. Definition on n^{th}-Power of a Set

The n^{th}-Power of a Set (2016) was introduced by Smarandache in the following way:

$P^n(S)$, as the n^{th}-PowerSet of the Set S, for integer $n \geq 1$, is recursively defined as:

$P^2(S) = P\big(P(S)\big),\ P^3(S) = P\big(P^2(S)\big) = P\Big(P\big(P(S)\big)\Big), \ldots,$

$P^n(S) = P\big(P^{n-1}(S)\big)$, where $P^0(S) \overset{\text{def}}{=} S$, and $P^1(S) \overset{\text{def}}{=} P(S)$.

The n^{th}-Power of a Set better reflects our complex reality, since a set S (that may represent a group, a society, a country, a continent, etc.) of elements (such as: people, objects, and in general aany items) is organized onto subsets $P(S)$, which on their turns are also organized onto subsets of subsets, and so on. That is our world.

In the classical HyperOperation and Classical HyperStructures, the empty set $\emptyset$ does not belong to the power set, or $P_*(S) = P(S) \setminus \{\emptyset\}$.

However, in the real world we encounter many situations when a HyperOperation $\circ$ is:

- *indeterminate*, for example $a \circ b = \emptyset$ (unknown, or undefined),
- or *partially indeterminate*, for example $a \circ b = \{[0.2, 0.3], \emptyset\}$.

In our everyday life, there are many more operations and laws that have some degrees of indeterminacy (vagueness, unclearness, unknowingness, contradiction, etc.), than those that are totally determinate.

That is why is 2016 we have extended the classical HyperOperation to the Neutrosophic HyperOperation, by taking the whole power $P(S)$ (that includes the empty-set $\emptyset$ as well), instead $P_*(S)$ (that does not include the empty-set $\emptyset$), as follows.

3.1 Remark 2

Throughout this paper the definitions, theorems, remarks, examples and applications work for both classical-type and Neutrosophic-type SuperHyper-Algebra and SuperHyper Function.

3.2 Theorem 1

Let S be a discrete finite set of 2 or more elements, and $n \geq 1$ an integer.

Then: $P^0(S) \subset P^1(S) \subset P^2(S) \subset \cdots \subset P^{n-1}(S) \subset P^n(S)$.

Proof

For a discrete finite set $S = \{a_1, a_2, \ldots, a_m\}$ for integer $m \geq 2$ one has:

$P^0(S) \equiv S = \{a_1, a_2, \ldots, a_m\}$.

$P^1(S) = P(S) = \{a_1, a_2, \ldots, a_m; \{a_1, a_2\}, \{a_1, a_2, a_3\}, \ldots, \{a_1, a_2, \ldots, a_m\}\}$,

and cardinal of P(S) is $Card\big(P(S)\big) = C_m^1 + C_m^2 + \cdots + C_m^n = 2^m - 1$,

where C_m^i, $1 \leq i \leq m$, means combinations of m elements taken in groups of i elements.

It is clear that $P^0(S) \subset P^1(S)$.

In general, one computes the set of $P^{k+1}(S)$ by taking the set of the previous $P^k(S) = \{\alpha_1, \alpha_2, \ldots, \alpha_r\}$, where $r = Card\big(P^k(S)\big)$, and making all possible combination of its r elements; but, at the beginning, when one takes the elements only by one, we get just $P^k(S)$, afterwards one takes the elements in group of two, then in groups of three, and so on, and finally all r elements together as a single group.

4. Definition of SuperHyperOperations

We recall our 2016 concepts of SuperHyperOperation, SuperHyperAxiom, SuperHyperAlgebra, and their corresponding Neutrosophic SuperHyperOperation Neutrosophic SuperHyperAxiom and Neutrosophic SuperHyperAlgebra [2].

Let $P_*^n(H)$ be the n$^{\text{th}}$-powerset of the set H such that none of $P(H)$, $P^2(H)$, …, $P^n(H)$ contain the empty set ϕ .

Also, let $P_n(H)$ be the n[th]-powerset of the set H such that at least one of the $P^2(H)$, …, $P^n(H)$ contain the empty set ϕ .

The SuperHyperOperations are operations whose codomain is either $P_*^n(H)$ and in this case one has **classical-type SuperHyperOperations**, or $P^n(H)$ and in this case one has **Neutrosophic SuperHyperOperations**, for integer $n \geq 2$.

4.1 Classical-type m-ary SuperHyperOperation {or more accurate denomination (m, n)-SuperHyperOperation}

Let U be a universe of discourse and a non-empty set H, $H \subset U$. Then:

$$\circ^*_{(m,n)} : H^m \to P_*^n(H)$$

where the integers $m, n \geq 1$,

$$H^m = \underbrace{H \times H \times \ldots \times H}_{m\ times} ,$$

and $P_*^n(H)$ is the n[th]-powerset of the set H that includes the empty-set.

This SuperHyperOperation is a m-ary operation defined from the set H to the n[th]-powerset of the set H.

4.2 Neutrosophic m-ary SuperHyperOperation {or more accurate denomination Neutrosophic (m, n)-SuperHyperOperation}

Let U be a universe of discourse and a non-empty set H, $H \subset U$. Then:

$$\circ_{(m,n)} : H^m \to P^n(H)$$

where the integers $m, n \geq 1; P^n(H)$ - the n-th powerset of the set H that includes the empty-set.

5. SuperHyperAxiom

A **classical-type SuperHyperAxiom** or more accurately a ***(m, n)*-SuperHyperAxiom** is an axiom based on classical-type SuperHyperOperations.

Similarly, a **Neutrosophic SuperHyperAxiom** {or Neutrosphic (m, n)-SuperHyperAxiom} is an axiom based on Neutrosophic SuperHyperOperations.

There are:

- **Strong SuperHyperAxioms**, when the left-hand side is equal to the right-hand side as in non-hyper axioms,
- and **Week SuperHyperAxioms**, when the intersection between the left-hand side and the right-hand side is non-empty.

For examples, one has:

- Strong SuperHyperAssociativity, when $(x \circ y) \circ z = x \circ (y \circ z)$, for all $x, y, z \in H^m$, where the law $\circ^*_{(m,n)} : H^m \to P_*^n(H)$;
- and Week SuperHyperAssociativity, when $[(x \circ y) \circ z] \cap [x \circ (y \circ z)] \neq \phi$, for all $x, y, z \in H^m$.

6. SuperHyperAlgebra and SuperHyperStructure

A **SuperHyperAlgebra** or more accurately ***(m-n)*-SuperHyperAlgebra** is an algebra dealing with SuperHyperOperations and SuperHyperAxioms.

Again, a **Neutrosophic SuperHyperAlgebra** {or Neutrosphic (m, n)-SuperHyperAlgebra} is an algebra dealing with Neutrosophic SuperHyperOperations and Neutrosophic SuperHyperOperations.

In general, we have **SuperHyperStructures** {or *(m-n)*-SuperHyperStructures}, and corresponding **Neutrosophic SuperHyperStructures**.

For example, there are SuperHyperGrupoid, SuperHyperSemigroup, SuperHyperGroup, SuperHyperRing, SuperHyperVectorSpace, etc.

7. Distinction between SuperHyperAlgebra vs. Neutrosophic SuperHyperAlgebra

i. If none of the power sets $P^k(H)$, $1 \leq k \leq n$, do not include the empty set ϕ, then one has a classical-type SuperHyperAlgebra;

ii. If at least one power set, $P^k(H)$, $1 \leq k \leq n$, includes the empty set ϕ, then one has a Neutrosophic SuperHyperAlgebra.

8. Example of SuperHyperGroup

The below $(P^2(S), \#)$ is a commutative *SuperHyperGroup*.

$\#$	$\{a\}$	$\{b\}$	$\{a,b\}$	$\{\{a\},\{a,b\}\}$	$\{\{b\},\{a,b\}\}$	$\{\{a\},\{b\},\{a,b\}\}$
$\{a\}$	$\{a\}$	$\{b\}$	$\{a,b\}$	$\{\{a\},\{a,b\}\}$	$\{\{b\},\{a,b\}\}$	$\{\{a\},\{b\},\{a,b\}\}$
$\{b\}$	$\{b\}$	$\{b\}$	$\{\{a\},\{a,b\}\}$	$\{\{b\},\{a,b\}\}$	$\{\{a\},\{b\},\{a,b\}\}$	$\{a,b\}$
$\{a,b\}$	$\{a,b\}$	$\{\{a\},\{a,b\}\}$	$\{a,b\}$	$\{b\}$	$\{a\}$	$\{\{b\},\{a,b\}\}$
$\{\{a\},\{a,b\}\}$	$\{\{a\},\{a,b\}\}$	$\{\{b\},\{a,b\}\}$	$\{b\}$	$\{\{a\},\{a,b\}\}$	$\{a,b\}$	$\{\{a\},\{a,b\}\}$
$\{\{b\},\{a,b\}\}$	$\{\{b\},\{a,b\}\}$	$\{\{a\},\{b\},\{a,b\}\}$	$\{a\}$	$\{a,b\}$	$\{\{b\},\{a,b\}\}$	$\{b\}$
$\{\{a\},\{b\},\{a,b\}\}$	$\{\{a\},\{b\},\{a,b\}\}$	$\{a,b\}$	$\{\{b\},\{a,b\}\}$	$\{\{a\},\{a,b\}\}$	$\{b\}$	$\{\{a\},\{b\},\{a,b\}\}$

The *SuperHyperLaw* $\#$ is clearly well-defined, according to the above Table. This law is commutative since Table's matrix is symmetric with respect to the main diagonal.

$\{a\}$ is the *SuperHyperNeutral*.

And the *SuperHyperInverse* of an element $x \in P^2(S)$ is itself: $x^{-1} = x$.

8.1 Theorem 2

A law $\#$ that is well-defined, has a neutral element (*e*), and each element x has an inverse x^{-1} on a given set *M*, is also associative.

Proof

We have to prove that, for any *x, y, z* $\in$ *M*, one has:

$(x \# y) \# z = x \# (y \# z)$. (1)

Multiply (1) to the right by z^{-1} (the inverse of z, i.e. $z \# z^{-1} = z^{-1} \# z = e$):

$(x \# y) \# z \# z^{-1} = x \# (y \# z) \# z^{-1}$

(x # zy) # e = x # (y # z) # z^{-1}

x # y = x # (y # z) # z^{-1}.

Multiply to the left by x^{-1}:

x^{-1} # x # y = x^{-1} # x # (y # z) # z^{-1}

e # y = e # (y # z) # z

y = (y # z) # z^{-1}.

Multiply to the right by z:

y # z = (y # z) # z^{-1} # z

y # z = (y # z) # e

y # z = (y # z)

$y \# z = y \# z$, which is true.

9. SuperHyperTopology and Neutrosophic SuperHyperTopology

A topology defined on a SuperHyperAlgebra $(P_*^n(S), \#)$, for integer $n \geq 2$, is called a SuperHyperTopology, and it is formed from SuperHyperSubsets. Similarly for Neutrosophic SuperHyper of $P_*^n(S)$ Topology, where $P_*^n(S)$ is replaced by $P_n(S)$, that includes the empty-set as well.

10. Definition of classical-type Unary HyperFunction (f_H)

Let S be a non-empty set included in a universe of discourse U.

$$f_H: S \to P_*(S)$$

11. Definition of classical-type m-ary HyperFunction (f_H^m)

$$f_H^m: S^m \to P_*(S)$$

where m is an integer ≥ 2, and $P_*(S)$ is the classical powerset of S.

12. Definition of Unary SuperHyperFunction (f_H)

We now introduce for the first time the concept of **SuperHyperFunction** (f_{SH}).

$f_{SH}: S \to P_*^n(S)$, for integer $n \geq 1$, where $P^n(S)$ is the n-th powerset of the set S.

13. Definition of m-ary SuperHyperFunction (f_{SH}^m)

$f_{SH}^m: S^m \to P_*^n((S)$, for integer $m \geq 2$.

14. General Definition of SuperHyperFunction

$f_{SH}^{SH}: P_*^r(S) \to P_*^n(S)$, for integers $r, n \geq 0$.

$f_{SH}: S \to P^n(S)$

$f_{SH}^m: S^m \longrightarrow P^n(S)$

$f_{SH}^{SH}: P_*^r(S) \to P^n(S)$

15. Example 3 and Application of SuperHyperFunctions

$S = \{a, b\}$, a discrete set.

$$P(S) = \{\{a\}, \{b\}, \{a, b\}\}$$

$$P^2(S) = \begin{Bmatrix} \{a\}, \{b\}, \{a, b\} \\ \{\{a\}, \{a, b\}\}, \{\{b\}, \{a, b\}\} \\ \{\{a\}, \{b\}, \{a, b\}\} \end{Bmatrix}$$

$f_{SH}: S \to P^2(S)$

$f_{SH}(x)$ = the system (organization) or { } set that x best belongs to

$f_{SH}(a) = \{a, b\}$

$f_{SH}(b) = \{\{b\}, \{a, b\}\}$

For example, the system $\{b\}$ means that person b is a strong personality and himself alone makes a system.

16. Example 4 and Application of Neutrosophic SuperHyperFunctions

$S = [0, 5]$, a continuous set.

$P_o(S) = \{A, A \text{ is a subset}, A \subseteq [0,5]\}$

$P_o^2(S) = \{A_1, \{A_1, A_2\}, \{A_1, A_2, A_3\}, \dots\}$,

where all A_k are subsets of $[0, 5]$ with index $k \in [0,5]$, therefore one has an uncountable infinite set of subsets of [0, 5].

$f_{SH}: [0, 5] \to P_o^2([0, 5])$

$$f_{SH}(x) = \{[x - 1, x] \cap [0, 5], [x + 1, x + 2] \cap [0, 5]\}$$

For example:

$$f_{SH}(2) = \{[1, 2], [3, 4]\}$$

$$f_{SH}(3.4) = \{[2.4, 3.4], [4.4, 5]\}$$

since $[6.4, 5.4] \cap [0, 5] = [4.4, 4.5]$

$$f_{SH}(0) = \{[0, 0], [1, 2]\} = \{0, [1, 2]\}$$

since $[-1,0] \cap [0,5] = [0,0]$

$$f_{SH}(5) = \{[4,5], \emptyset\}$$

since $[6,7] \cap [0,5] = \emptyset$.

Conclusion

In this paper we recalled the concepts of SuperHyperAlgebra and Neutrosophic HyperSuperAlgebra, and presented an example of SuperHyperGroup. Then, for the first time one introduces and gives examples of SuperHyperFunction and Neutrosophic SuperHyperFunction.

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PLITHOGENICS

Plithogenic Set, an Extension of Crisp, Fuzzy, Intuitionistic Fuzzy, and Neutrosophic Sets - Revisited

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Abstract. In this paper, we introduce the plithogenic set (as generalization of crisp, fuzzy, intuitionistic fuzzy, and neutrosophic sets), which is a set whose elements are characterized by many attributes' values. An attribute value *v* has a corresponding (fuzzy, intuitionistic fuzzy, or neutrosophic) degree of appurtenance d(*x,v*) of the element *x*, to the set *P*, with respect to some given criteria. In order to obtain a better accuracy for the plithogenic aggregation operators in the plithogenic set, and for a more exact inclusion (partial order), a (fuzzy, intuitionistic fuzzy, or neutrosophic) contradiction (dissimilarity) degree is defined between each attribute value and the dominant (most important) attribute value. The plithogenic intersection and union are linear combinations of the fuzzy operators t_{norm} and t_{conorm}, while the plithogenic complement, inclusion (inequality), equality are influenced by the attribute values contradiction (dissimilarity) degrees. This article offers some examples and applications of these new concepts in our everyday life.

Keywords: Plithogeny; Plithogenic Set; Neutrosophic Set; Plithogenic Operators.

1 Informal Definition of Plithogenic Set

Plithogeny is the genesis or origination, creation, formation, development, and evolution of *new entities* from dynamics and organic fusions of contradictory and/or neutrals and/or non-contradictory *multiple old entities*.

While plithogenic means what is pertaining to plithogeny.

A plithogenic set *P* is a set whose elements are characterized by one or more attributes, and each attribute may have many values. Each attribute's value *v* has a corresponding degree of appurtenance d(*x,v*) of the element *x*, to the set *P*, with respect to some given criteria.

In order to obtain a better accuracy for the plithogenic aggregation operators, a contradiction (dissimilarity) degree is defined between each attribute value and the dominant (most important) attribute value.

{However, there are cases when such dominant attribute value may not be taking into consideration or may not exist [therefore it is considered zero by default], or there may be many dominant attribute values. In such cases, either the contradiction degree function is suppressed, or another relationship function between attribute values should be established.}

The plithogenic aggregation operators (intersection, union, complement, inclusion, equality) are based on contradiction degrees between attributes' values, and the first two are linear combinations of the fuzzy operators' t_{norm} and t_{conorm}.

Plithogenic set is a generalization of the crisp set, fuzzy set, intuitionistic fuzzy set, and neutrosophic set, since these four types of sets are characterized by a single attribute value (appurtenance): which has one value (membership) – for the crisp set and fuzzy set, two values (membership, and nonmembership) – for intuitionistic fuzzy set, or three values (membership, nonmembership, and indeterminacy) – for neutrosophic set.

2 Formal Definition of Single (Uni-Dimensional) Attribute Plithogenic Set

Let *U* be a universe of discourse, and *P* a non-empty set of elements, $P \subseteq U$.

2.1 Attribute Value Spectrum

Let $\mathscr{A}$ be a non-empty set of uni-dimensional attributes $\mathscr{A} = \{\alpha_1, \alpha_2, \ldots, \alpha_m\}$, $m \geq 1$; and $\alpha \in \mathscr{A}$ be a given attribute whose spectrum of all possible values (or states) is the non-empty set *S*, where *S* can be a finite discrete set, $S = \{s_1, s_2, \ldots, s_l\}$, $1 \leq l < \infty$, or infinitely countable set $S = \{s_1, s_2, \ldots, s_\infty\}$, or infinitely uncountable (continuum) set $S =]a, b[$, $a < b$, where $]\ldots[$ is any open, semi-open, or closed interval from the set of real numbers or from other general set.

2.2 Attribute Value Range

Let V be a non-empty subset of S, where V is the *range of all attribute's values* needed by the experts for their application. Each element $x \in P$ is characterized by all attribute's values in $V = \{v_1, v_2, \ldots, v_n\}$, for $n \geq 1$.

2.3 Dominant Attribute Value

Into the attribute's value set V, in general, there is a dominant attribute value, which is determined by the experts upon their application. Dominant attribute value means the most important attribute value that the experts are interested in.

{However, there are cases when such dominant attribute value may not be taking into consideration or not exist, or there may be many dominant (important) attribute values - when different approach should be employed.}

2.4 Attribute Value Appurtenance Degree Function

Each attributes value $v \in V$ has a corresponding degree of appurtenance $d(x, v)$ of the element x, to the set P, with respect to some given criteria.

The degree of appurtenance may be: a fuzzy degree of appurtenance, or intuitionistic fuzzy degree of appurtenance, or neutrosophic degree of appurtenance to the plithogenic set.

Therefore, the attribute value appurtenance degree function is:

$$\forall x \in P,\ d: P \times V \rightarrow \mathscr{P}([0, 1]^z), \tag{1}$$

so $d(x, v)$ is a subset of $[0, 1]^z$, and $\mathscr{P}([0, 1]^z)$ is the power set of the $[0, 1]^z$, where $z = 1$ (for fuzzy degree of appurtenance), $z = 2$ (for intuitionistic fuzzy degree of appurtenance), or $z = 3$ (for neutrosophic degree de appurtenance).

2.5 Attribute Value Contradiction (Dissimilarity) Degree Function

Let the cardinal $|V| \geq 1$.

Let $c: V \times V \rightarrow [0, 1]$ be the attribute value contradiction (dissimilarity) degree function (that we introduce now for the first time) between any two attribute values v_1 and v_2, denoted by

$c(v_1, v_2)$, and satisfying the following axioms:
$c(v_1, v_1) = 0$, the contradiction degree between the same attribute values is zero;
$c(v_1, v_2) = c(v_2, v_1)$, commutativity.

For simplicity, we use a fuzzy attribute value contradiction degree function (c as above, that we may denote by c_F in order to distinguish it from the next two), but an intuitionistic attribute value contradiction function ($c_{IF} : V \times V \rightarrow [0, 1]^2$), or more general a neutrosophic attribute value contradiction function ($c_N : V \times V \rightarrow [0, 1]^3$) may be utilized increasing the complexity of calculation but the accuracy as well.

We mostly compute the contradiction degree between *uni-dimensional attribute values*. For *multi-dimensional attribute values* we split them into corresponding uni-dimensional attribute values.

The attribute value contradiction degree function helps the plithogenic aggregation operators, and the plithogenic inclusion (partial order) relationship to obtain a more accurate result.

The attribute value contradiction degree function is designed in each field where plithogenic set is used in accordance with the application to solve. If it is ignored, the aggregations still work, but the result may lose accuracy.

Several examples will be provided into this paper.

Then (P, a, V, d, c) is called a plithogenic set:

● where "P" is a set, "a" is a (multi-dimensional in general) attribute, "V" is the range of the attribute's values, "d" is the degree of appurtenance of each element x's attribute value to the set P with respect to some given criteria ($x \in P$), and "d" stands for "d_F" or "d_{IF}" or "d_N", when dealing with fuzzy degree of appurtenance, intuitionistic fuzzy degree of appurtenance, or neutrosophic degree of appurtenance respectively of an element x to the plithogenic set P;

● and "c" stands for "c_F" or "c_{IF}" or "c_N", when dealing with fuzzy degree of contradiction, intuitionistic fuzzy degree of contradiction, or neutrosophic degree of contradiction between attribute values respectively.

The functions $d(\cdot,\cdot)$ and $c(\cdot,\cdot)$ are defined in accordance with the applications the experts need to solve.

One uses the notation: $x\big(d(x, V)\big)$, where $d(x, V) = \{d(x, v),$ for all $v \in V\}, \forall x \in P$.

2.6 About the Plithogenic Aggregation Set Operators

The attribute value contradiction degree is calculated between each attribute value with respect to the dominant attribute value (denoted v_D) in special, and with respect to other attribute values as well.

The attribute value contradiction degree function c between the attribute's values is used into the definition of plithogenic aggregation operators {*Intersection (AND), Union (OR), Implication* ($\Rightarrow$), *Equivalence* ($\Leftrightarrow$), *Inclusion Relationship (Partial Order, or Partial Inequality)*, and other plithogenic aggregation operators that combine two or more attribute value degrees - that t_{norm} and t_{conorm} act upon}.

Most of the plithogenic aggregation operators are linear combinations of the fuzzy t_{norm} (denoted $\wedge_F$), and fuzzy t_{conorm} (denoted $\vee_F$), but non-linear combinations may as well be constructed.

If one applies the t_{norm} on dominant attribute value denoted by v_D, and the contradiction between v_D and v_2 is $c(v_D, v_2)$, then onto attribute value v_2 one applies:

$$[1 - c(v_D, v_2)]\cdot t_{norm}(v_D, v_2) + c(v_D, v_2)\cdot t_{conorm}(v_D, v_2), \tag{2}$$

Or, by using symbols:

$$[1 - c(v_D, v_2)]\cdot(v_D \wedge_F v_2) + c(v_D, v_2)\cdot(v_D \vee_F v_2). \tag{3}$$

Similarly, if one applies the t_{conorm} on dominant attribute value denoted by v_D, and the contradiction between v_D and v_2 is $c(v_D, v_2)$, then onto attribute value v_2 one applies:

$$[1 - c(v_D, v_2)]\cdot t_{conorm}(v_D, v_2) + c(v_D, v_2)\cdot t_{norm}(v_D, v_2), \tag{4}$$

Or, by using symbols:

$$[1 - c(v_D, v_2)]\cdot(v_D \vee_F v_2) + c(v_D, v_2)\cdot(v_D \wedge_F v_2). \tag{5}$$

3 Plithogenic Set as Generalization of other Sets

Plithogenic set is a generalization of the crisp set, fuzzy set, intuitionistic fuzzy set, and neutrosophic set, since these four types of sets are characterized by a single attribute (*appurtenance*): which has one value (membership) – for the crisp set and for fuzzy set, two values (membership, and nonmembership) – for intuitionistic fuzzy set, or three values (membership, nonmembership, and indeterminacy) – for neutrosophic set.

For examples:

Let U be a universe of discourse, and a non-empty set $P \subseteq U$. Let $x \in P$ be a generic element.

3.1 Crisp (Classical) Set (CCS)

The *attribute* is α = "*appurtenance*";
the *set of attribute values V = {membership, nonmembership}*, with cardinal $|V| = 2$;
the *dominant attribute value = membership*;
the *attribute value appurtenance degree function*:

$$d: P\times V\to\{0, 1\}, \tag{6}$$

$d(x, membership) = 1$, $d(x, nonmembership) = 0$,
and the *attribute value contradiction degree function*:

$$c: V\times V\to\{0, 1\}, \tag{7}$$

$c(membership, membership) = c(nonmembership, nonmembership) = 0$,
$c(membership, nonmembership) = 1$.

3.1.1 Crisp (Classical) Intersection

$$a \wedge b \in \{0, 1\} \tag{8}$$

3.1.2 Crisp (Classical) Union

$$a \vee b \in \{0, 1\} \tag{9}$$

3.1.3 Crisp (Classical) Complement (Negation)

$$\neg a \in \{0, 1\}. \tag{10}$$

3.2 Single-Valued Fuzzy Set (SVFS)

The *attribute* is α = “*appurtenance*”;
the *set of attribute values V = {membership},* whose cardinal |*V*| = 1;
the *dominant attribute value = membership;*
the *appurtenance attribute value degree function:*

$$d: P\times V\to[0, 1], \tag{11}$$

with *d(x, membership)* ∈ [0, 1];
and the *attribute value contradiction degree function*:

$$c: V\times V\to[0, 1], \tag{12}$$

c(membership, membership) = 0.

3.2.1 Fuzzy Intersection

$$a \wedge_F b \in [0, 1] \tag{13}$$

3.2.2 Fuzzy Union

$$a \vee_F b \in [0, 1] \tag{14}$$

3.2.3 Fuzzy Complement (Negation)

$$\neg_F a = 1 - a \in [0, 1]. \tag{15}$$

3.3 Single-Valued Intuitionistic Fuzzy Set (SVIFS)

The *attribute* is α = “*appurtenance*”;
the *set of attribute values V = {membership, nonmembership},* whose cardinal |*V*| = 2;
the *dominant attribute value = membership;*
the *appurtenance attribute value degree function*:

$$d: P\times V\to[0, 1], \tag{16}$$

d(x, membership) ∈ [0, 1], *d(x, nonmembership)* ∈ [0, 1],
with *d(x, membership)* + *d(x, nonmembership)* ≤ 1,
and the *attribute value contradiction degree function*:

$$c: V\times V\to[0, 1], \tag{17}$$

c(membership, membership) = *c(nonmembership, nonmembership)* = 0,
c(membership, nonmembership) = 1,
which means that for SVIFS aggregation operators’ intersection (AND) and union (OR), if one applies the t_{norm} on membership degree, then one has to apply the t_{conorm} on nonmembership degree – and reciprocally.

Therefore:

3.3.1 Intuitionistic Fuzzy Intersection

$$(a_1, a_2) \wedge_{\text{IFS}} (b_1, b_2) = (a_1 \wedge_F b_1, a_2 \vee_F b_2) \tag{18}$$

3.3.2 Intuitionistic Fuzzy Union

$$(a_1, a_2) \vee_{\text{IFS}} (b_1, b_2) = (a_1 \vee_F b_1, a_2 \wedge_F b_2), \tag{19}$$

and

3.3.3 Intuitionistic Fuzzy Complement (Negation)

$$\neg_{\text{IFS}} (a_1, a_2) = (a_2, a_1). \tag{20}$$

where $\wedge_F$ and $\vee_F$ are the fuzzy t_{norm} and fuzzy t_{conorm} respectively.

3.3.4 Intuitionistic Fuzzy Inclusions (Partial Orders)

Simple Intuitionistic Fuzzy Inclusion (the most used by the intuitionistic fuzzy community):

$$(a_1, a_2) \leq_{IFS} (b_1, b_2) \tag{21}$$

iff $a_1 \leq b_1$ and $a_2 \geq b_2$.

Plithogenic (Complete) Intuitionistic Fuzzy Inclusion (that we now introduce for the first time):

$$(a_1, a_2) \leq_P (b_1, b_2) \tag{22}$$

iff $a_1 \leq (1-c_v)\cdot b_1, a_2 \geq (1-c_v)\cdot b_2$,
where $c_v \in [0, 0.5)$ is the contradiction degree between the attribute dominant value and the attribute value v { the last one whose degree of appurtenance with respect to Expert A is (a_1, a_2), while with respect to Expert B is (b_1, b_2) }. If c_v does not exist, we take it by default as equal to zero.

3.4 Single-Valued Neutrosophic Set (SVNS)

The *attribute* is α = "*appurtenance*";
the *set of attribute values V = {membership, indeterminacy, nonmembership}*, whose cardinal $|V| = 3$;
the *dominant attribute value = membership*;
the *attribute value appurtenance degree function*:

$$d: P\times V\to[0, 1], \tag{23}$$

d(x, membership) ∈ [0, 1], *d(x, indeterminacy)* ∈ [0, 1],
d(x, nonmembership) ∈ [0, 1],
with 0 ≤ *d(x, membership)* + *d(x, indeterminacy)* + *d(x, nonmembership)* ≤ 3;
and the *attribute value contradiction degree function*:

$$c: V\times V\to[0, 1], \tag{24}$$

c(membership, membership) = *c(indeterminacy, indeterminacy)* =
c(nonmembership, nonmembership) = 0,
c(membership, nonmembership) = 1,
c(membership, indeterminacy) = *c(nonmembership, indeterminacy)* = 0.5,
which means that for the SVNS aggregation operators (Intersection, Union, Complement etc.), if one applies the t_{norm} on membership, then one has to apply the t_{conorm} on nonmembership {and reciprocally), while on indeterminacy one applies the average of t_{norm} and t_{conorm}, as follows:

3.4.1 Neutrosophic Intersection

Simple Neutrosophic Intersection (the most used by the neutrosophic community):

$$(a_1, a_2, a_3) \wedge_{NS} (b_1, b_2, b_3) = \left(a_1 \wedge_F b_1, a_2 \vee_F b_2, a_3 \vee_F b_3\right) \tag{25}$$

Plithogenic Neutrosophic Intersection:

$$(a_1, a_2, a_3) \wedge_P (b_1, b_2, b_3) = \left(a_1 \wedge_F b_1, \frac{1}{2}\left[\left(a_2 \wedge_F b_2\right)+\left(a_2 \vee_F b_2\right)\right], a_3 \vee_F b_3\right) \tag{26}$$

3.4.2 Neutrosophic Union

Simple Neutrosophic Union (the most used by the neutrosophic community):

$$(a_1, a_2, a_3) \vee_{NS} (b_1, b_2, b_3) = \left(a_1 \vee_F b_1, a_2 \wedge_F b_2, a_3 \wedge_F b_3\right) \tag{27}$$

Plithogenic Neutrosophic Union:

$$(a_1, a_2, a_3) \vee_P (b_1, b_2, b_3) = \left(a_1 \vee_F b_1, \frac{1}{2}\left[\left(a_2 \wedge_F b_2\right)+\left(a_2 \vee_F b_2\right)\right], a_3 \wedge_F b_3\right). \tag{28}$$

In other way, with respect to what one applies on the membership, one applies the opposite on non-membership, while on indeterminacy one applies the average between them.

3.4.3 Neutrosophic Complement (Negation)

$$\neg_{NS} (a_1, a_2, a_3) = (a_3, a_2, a_1). \tag{29}$$

3.4.4 Neutrosophic Inclusions (Partial-Orders)

Simple Neutrosophic Inclusion (the most used by the neutrosophic community):

$$(a_1, a_2, a_3) \leq_{NS} (b_1, b_2, b_3) \tag{30}$$

iff $a_1 \leq b_1$ and $a_2 \geq b_2$, $a_3 \geq b_3$.

Plithogenic Neutrosophic Inclusion (defined now for the first time):
Since the degrees of contradiction are

$$c(a_1, a_2) = c(a_2, a_3) = c(b_1, b_2) = c(b_2, b_3) = 0.5, \tag{31}$$

one applies: $a_2 \geq [1- c(a_1, a_2)]b_2$ or $a_2 \geq (1\text{-}0.5)b_2$ or $a_2 \geq 0.5\cdot b_2$
while

$$c(a_1, a_3) = c(b_1, b_3) = 1 \tag{32}$$

{having $a_1 \leq b_1$ one does the opposite for $a_3 \geq b_3$},
whence

$$(a_1, a_2, a_3) \leq_P (b_1, b_2, b_3) \tag{33}$$

iff $a_1 \leq b_1$ and $a_2 \geq 0.5\cdot b_2$, $a_3 \geq b_3$.

4 Classifications of the Plithogenic Set

4.1 First Classification

4.1.1 Refined Plithogenic Set

If at least one of the attribute's values $v_k \in V$ is split (refined) into two or more attribute sub-values: $v_{k1}, v_{k2}, \ldots \in V$, with the attribute sub-value appurtenance degree function: $d(x, v_{ki}) \in P([0, 1])$, for $i = 1, 2, \ldots,$ then (P_r, α, V, d, c) is called a *Refined Plithogenic Set*, where "*r*" stands for "refined".

4.1.2 Plithogenic Overset / Underset / Offset

If for at least one of the attribute's values $v_k \in V$, of at least one element $x \in P$, has the attribute value appurtenance degree function $d(x, v_k)$ exceeding 1, then (P_o, α, V, d, c) is called a *Plithogenic Overset*, where "*o*" stands for "overset"; but if $d(x, v_k)$ is below 0, then (P_u, α, V, d, c) is called a *Plithogenic Underset*, where "*u*" stands for "underset"; while if $d(x, v_k)$ exceeds 1, and $d(y, s_j)$ is below 0 for the attribute values $v_k, v_j \in V$ that may be the same or different attribute values corresponding to the same element or to two different elements $x, y \in P$, then $(P_{off}, \alpha, V, d, c)$ is called a *Plithogenic Offset*, where "off" stands for "offset" (or plithogenic set that is both overset and underset).

4.1.3 Plithogenic Multiset

A plithogenic set P that has at least an element $x \in P$, which repeats into the set P with the same plithogenic components

$$x(a_1, a_2, \ldots, a_m), x(a_1, a_2, \ldots, a_m) \tag{34}$$

or with different plithogenic components

$$x(a_1, a_2, \ldots, a_m), x(b_1, b_2, \ldots, b_m), \tag{35}$$

then (P_m, α, V, d, c) is called a *Plithogenic Multiset*, where "*m*" stands for "multiset".

4.1.4 Plithogenic Bipolar Set

If $\forall x \in P$, d: $P\times V \rightarrow \{\mathscr{P}([-1, 0]) \times \mathscr{P}([0, 1])\}^z$, then (P_b, α, V, d, c) is called a *Plithogenic Bipolar Set*, since $d(x, v)$, for $v \in V$, associates an appurtenance negative degree (as a subset of $[-1, 0]$) and a positive degree (as a subset of $[0, 1]$) to the value v; where $z = 1$ for fuzzy degree, $z = 2$ for intuitionistic fuzzy degree, and $z = 3$ for neutrosophic fuzzy degree.

4.1.5-6 Plithogenic Tripolar Set & Plitogenic Multipolar Set

Similar definitions for *Plithogenic Tripolar Set* and *Plitogenic Multipolar Set* (extension from *Neutrosophic Tripolar Set* and respectively *Neutrosophic Multipolar Set* {[4], 123-125}.

4.1.7 Plithogenic Complex Set

If, for any $x \in$ P, d: P×V→ $\{\mathscr{P}([0, 1]) \times \mathscr{P}([0, 1])\}^z$, and for any v ∈ V, d($x$, v) is a complex value, i.e. d(x, v) = $M_1 \cdot e^{jM_2}$, where $M_1 \subseteq [0, 1]$ is called *amplitude*, and $M_2 \subseteq [0, 1]$ is called *phase*, and the *appurtenance degree* may be fuzzy ($z = 1$), intuitionistic fuzzy ($z = 2$), or neutrosophic ($z = 3$), then ($P_{com,}$ α, V, d, c) is called a *Plithogenic Complex Set*.

4.2 Second Classification

Upon the values of the appurtenance degree function, one has:

4.2.1 Single-Valued Plithogenic Fuzzy Set

If

$$\forall x \in P,\ d: P\times V\to[0, 1], \tag{36}$$

and $\forall v \in V$, $d(x, v)$ is a single number in $[0, 1]$.

4.2.2 Hesitant Plithogenic Fuzzy Set

If

$$\forall x \in P,\ d: P\times V\to \mathscr{P}([0, 1]), \tag{37}$$

and $\forall v \in V$, $d(x, v)$ is a discrete finite set of the form $\{n_1, n_2, \ldots, n_p\}$, where $1 \leq p < \infty$, included in [0, 1].

4.2.3 Interval-Valued Plithogenic Fuzzy Set

If

$$\forall x \in P,\ d: P\times V\to \mathscr{P}([0, 1]), \tag{38}$$

and $\forall v \in V$, $d(x, v)$ is an (open, semi-open, closed) interval included in [0, 1].

5 Applications and Examples

5.1 Applications of Uni-Dimensional Attribute Plithogenic Single-Valued Fuzzy Set

Let U be a universe of discourse, and a non-empty plithogenic set $P \subseteq U$. Let $x \in P$ be a generic element.
For simplicity, we consider the uni-dimensional attribute and the single-valued fuzzy degree function.

5.1.1 Small Discrete-Set of Attribute-Values

If the *attribute* is "*color*", and we consider only a discrete *set of attribute values V*, formed by the following six pure colors:

V = *{violet, blue, green, yellow, orange, red}*,

the *attribute value appurtenance degree function*:

$$d: P\times V\to[0, 1], \tag{39}$$

$d(x, violet) = v \in [0, 1]$, $d(x, blue) = b \in [0, 1]$, $d(x, green) = g \in [0, 1]$,
$d(x, yellow) = y \in [0, 1]$, $d(x, orange) = o \in [0, 1]$, $d(x, red) = r \in [0, 1]$,

then one has: $x(v, b, g, y, o, r)$, where v, b, g, y, o, r are fuzzy degrees of violet, blue, green, yellow, orange, and red, respectively, of the object x with respect to the set of objects P, where $v, b, g, y, o, r \in [0, 1]$.
The cardinal of the set of attribute values V is 6.
The other colors are blends of these pure colors.

5.1.2 Large Discrete-Set of Attribute-Values

If the *attribute* is still "*color*" and we choose a more refined representation of the color values as:

$x\{d_{390}, d_{391}, \ldots, d_{699}, d_{700}\}$,

measured in nanometers, then we have a discrete finite set of attribute values, whose cardinal is: $700 - 390 + 1 = 311$, where for each $j \in V = \{390, 391, \ldots, 699, 700\}$, d_j represents the degree to which the object x's color, with respect to the set of objects P, is of "j" nanometers per wavelength, with $d_i \in [0, 1]$. A nanometer (nm) is a billionth part of a meter.

5.1.3 Infinitely-Uncountable-Set of Attribute-Values

But if the *attribute* is again “*color*”, then one may choose a continuous representation:

$x(\mathrm{d}([390, 700]))$,

having $V = [390, 700]$ a closed real interval, hence an infinitely uncountable (continuum) set of attribute values. The cardinal of the V is ∞.

For each $j \in [390, 700]$, d_j represents the degree to which the object x’s color, with respect to the set of objects P, is of “j” nanometers per wavelength, with $d_i \in [0, 1]$. And $d([390, 700]) = \{dj, j \in [390, 700]\}$.

The light, ranging from *390* (violet color) to *700* (red color) nanometers per wavelengths is visible to the eye of the human. The cardinal of the set of attribute values V is continuum infinity.

5.2 Example of Uni-Attribute (of 4-Attribute-Values) Plithogenic Single-Valued Fuzzy Set Complement (Negation)

Let’s consider that the attribute “size” that has the following values: *small* (the dominant one), *medium*, *big*, *very big*.

Degrees of contradiction	0	0.50	0.75	1
Attribute values	small	medium	big	very big
Degrees of appurtenance	0.8	0.1	0.3	0.2

Table 1.

5.3 Example of Refinement and Negation of a Uni-Attribute (of 4-Attribute-Values) Plithogenic Single-Valued Fuzzy Set

As a refinement of the above table, let’s add the attribute “bigger” as in the below table.

The opposite (negation) of the attribute value “big”, which is 75% in contradiction with “small”, will be an attribute value which is $1 - 0.75 = 0.25 = 25\%$ in contradiction with “small”, so it will be equal to $\frac{1}{2}[\text{"}small\text{"} + \text{"}medium\text{"}]$. Let’s call it “less medium”, whose degree of appurtenance is 1 – 0.3 = 0.7.

If the attribute “size” has other values, small being dominant value:

Degrees of contradiction	0	**0.14**	**0.25**	0.50	0.75	**0.86**	1
Attribute values	small	**above small (anti-bigger)**	**less medium (anti-big)**	medium	big	**bigger**	very big
Degrees of appurtenance	0.8	**0.6**	**0.7**	0.1	0.3	**0.4**	0.2

Table 2.

The opposite (negation) of “bigger” is 1 - 0.86 = 0.14 = 14% in contradiction degree with the dominant attribute value (“small”), so it is in between “small” and “medium”, we may say it is included into the attribute-value interval *[small, medium]*, much closer to “small” than to “medium”. Let’s call is “above small”, whose degree of appurtenance is 1 – 0.4 = 0.6.

5.4 Example of Multi-Attribute (of 24 Attribute-Values) Plithogenic Fuzzy Set Intersection, Union, and Complement

Let P be a plithogenic set, representing the students from a college. Let $x \in P$ be a generic student that is characterized by three attributes:

- altitude, whose values are {tall, short} $\stackrel{\text{def}}{=} \{a_1, a_2\}$;
- weight, whose values are {obese, fat, medium, thin} $\stackrel{\text{def}}{=} \{w_1, w_2, w_3, w_4\}$;
- hair color, whose values are {blond, reddish, brown} $\stackrel{\text{def}}{=} \{h_1, h_2, h_3\}$.

The multi-attribute of dimension *3* is

$V_3 = \{(a_i, w_j, h_k), \text{for all } 1 \leq i \leq 2, 1 \leq j \leq 4, 1 \leq k \leq 3\}$.

The cardinal of V_3 is $|V_3| = 2 \times 4 \times 3 = 24$.

The *uni-dimensional attribute contradiction degrees* are:
$c(a_1, a_2) = 1$;
$c(w_1, w_2) = \frac{1}{3}$, $c(w_1, w_3) = \frac{2}{3}$, $c(w_1, w_4) = 1$;
$c(h_1, h_2) = 0.5$, $c(h_1, h_3) = 1$.

Dominant attribute values are: a_1, w_1, and h_1 respectively for each corresponding uni-dimensional attribute.
Let's use the fuzzy $t_{norm} =$ a $\wedge_F$ b = ab, and fuzzy $t_{conorm} = a \vee_F b = a + b - ab$.

5.4.1 Tri-Dimensional Plithogenic Single-Valued Fuzzy Set Intersection and Union

Let

$$x_A = \left\{ \begin{matrix} d_A(x, a_i, w_j, h_k), \\ for\ all\ 1 \leq i \leq 2, 1 \leq j \leq 4, 1 \leq k \leq 3 \end{matrix} \right\} \tag{40}$$

and

$$x_B = \left\{ \begin{matrix} d_B(x, a_i, w_j, h_k), \\ for\ all\ 1 \leq i \leq 2, 1 \leq j \leq 4, 1 \leq k \leq 3 \end{matrix} \right\}. \tag{41}$$

Then:

$$x_A(a_i, w_j, h_k) \wedge_P x_B(a_i, w_j, h_k) = \left\{ \begin{array}{l} (1 - c(a_D, a_i)) \cdot \lfloor d_A(x, a_D) \wedge_F d_B(x, a_i) \rfloor \\ + c(a_D, a_i) \cdot [d_A(x, a_D) \vee_F d_B(x, a_i)], 1 \leq i \leq 2; \\ (1 - c(w_D, w_j)) \cdot [d_A(x, w_D) \wedge_F d_B(x, w_j)] \\ + c(w_D, w_j) \cdot [d_A(x, w_D) \vee_F d_B(x, w_j)], 1 \leq j \leq 4; \\ (1 - c(h_D, h_k)) \cdot [d_A(x, h_D) \wedge_F d_B(x, h_k)] \\ + c(h_D, h_k) \cdot [d_A(x, h_D) \vee_F d_B(x, h_k)], 1 \leq k \leq 3. \end{array} \right\} \tag{42}$$

and

$$x_A(a_i, w_j, h_k) \vee_P x_B(a_i, w_j, h_k) = \left\{ \begin{array}{l} (1 - c(a_D, a_i)) \cdot \lfloor d_A(x, a_D) \vee_F d_B(x, a_i) \rfloor \\ + c(a_D, a_i) \cdot [d_A(x, a_D) \wedge_F d_B(x, a_i)], 1 \leq i \leq 2; \\ (1 - c(w_D, w_j)) \cdot [d_A(x, w_D) \vee_F d_B(x, w_j)] \\ + c(w_D, w_j) \cdot [d_A(x, w_D) \wedge_F d_B(x, w_j)], 1 \leq j \leq 4; \\ (1 - c(h_D, h_k)) \cdot [d_A(x, h_D) \vee_F d_B(x, h_k)] \\ + c(h_D, h_k) \cdot [d_A(x, h_D) \wedge_F d_B(x, h_k)], 1 \leq k \leq 3. \end{array} \right\} \tag{43}$$

Let's have

$x_A(d_A(a_1) = 0.8, d_A(w_2) = 0.6, d_A(h_3) = 0.5)$

and

$x_B(d_B(a_1) = 0.4, d_B(w_2) = 0.1, d_B(h_3) = 0.7)$.

We take only one 3-attribute value: (a_1, w_2, h_3), for the other 23 3-attribute values it will be analougsly.

For $x_A \wedge_p x_B$ we calculate for each uni-dimensional attribute separately:

$$[1 - c(a_D, a_1)] \cdot \left[0.8 \wedge_F 0.4\right] + c(a_D, a_1) \cdot \left[0.8 \vee_F 0.4\right] = (1 - 0) \cdot [0.8(0.4)] + 0 \cdot \left[0.8 \vee_F 0.4\right] = 0.32;$$

$$\left[1 - c[w_D, w_2] \cdot \left[0.6 \wedge_F 0.1\right] + c(w_D, w_2) \cdot \left[0.6 \vee_F 0.1\right]\right] = \left(1 - \frac{1}{3}\right)[0.6(0.1)] + \frac{1}{3}[0.6 + 0.1 - 0.6(0.1)]$$

$$= \frac{2}{3}[0.06] + \frac{1}{3}[0.64] = \frac{0.76}{3} \approx 0.25;$$

$$[1-c(h_D,h_3)]\cdot\left[0.5\wedge_F 0.7\right]+c(h_D,h_3)\cdot\left[0.5\vee_F 0.7\right]=[1-1]\cdot[0.5(0.7)]+1\cdot[0.5+0.7-0.5(0.7)]$$
$$=0\cdot[0.35]+0.85=0.85.$$

Whence $x_A\wedge_p x_B(a_1,w_2,h_3)\approx(0.32,0.25,0.85)$.

For $x_A\vee_p x_B$ we do similarly:

$$[1-c(a_D,a_1)]\cdot\left[0.8\vee_F 0.4\right]+c(a_D,a_1)\cdot\left[0.8\wedge_F 0.4\right]=(1-0)\cdot[0.8+0.4-0.8(0.4)]+0\cdot[0.8(0.4)]$$
$$=1\cdot[0.88]+0=0.88;$$

$$\left[1-c[w_D,w_2]\cdot\left[0.6\vee_F 0.1\right]+c(w_D,w_2)\cdot\left[0.6\wedge_F 0.1\right]\right]=\left(1-\frac{1}{3}\right)[0.6+0.1-0.6(0.1)]+\frac{1}{3}[0.6(0.1)]$$
$$=\frac{2}{3}[0.64]+\frac{1}{3}[0.06]=\frac{1.34}{3}\approx 0.44;$$

$$[1-c(h_D,h_3)]\cdot\left[0.5\vee_F 0.7\right]+c(h_D,h_3)\cdot\left[0.5\wedge_F 0.7\right]=[1-1]\cdot[0.5+0.7-0.5(0.7)]+1\cdot[0.5(0.7)]$$
$$=0+0.35=0.35.$$

Whence $x_A\vee_p x_B(a_1,w_2,h_3)\approx(0.88,0.44,0.35)$.

For $\neg_p x_A(a_1,w_2,h_3)=(d_A(a_2)=0.8,d_A(w_3)=0.6,d_A(h_1)=0.5)$, since the opposite of a_1 is a_2, the opposite of w_2 is w_3, and the opposite of h_3 is h_1.

5.5 Another Example of Multi-Attribute (of 5 Attribute-Values) Plithogenic Fuzzy Set Complement and Refined Attribute-Value Set

The 5-attribute values plithogenic fuzzy complement (negation) of

$$x\begin{pmatrix} 0 & 0.50 & 0.75 & 0.86 & 1 \\ \text{small,} & \text{medium,} & \text{big ,} & \text{bigger,} & \text{very big} \\ 0.8 & 0.1 & 0.3 & 0.4 & 0.2 \end{pmatrix}$$

Is:

$$\neg_p x\begin{pmatrix} 1-1 & 1-0.86 & 1-0.75 & 1-0.50 & 1-0 \\ \text{anti}-\text{very big,} & \text{anti}-\text{bigger,} & \text{anti}-\text{big,} & \text{anti}-\text{medium,} & \text{anti}-\text{small} \\ 0.2 & 0.4 & 0.3 & 0.1 & 0.8 \end{pmatrix}$$
$$=\neg_p x\begin{pmatrix} 0 & 0.14 & 0.25 & 0.50 & 1 \\ \text{small,} & \text{anti}-\text{bigger,} & \text{anti}-\text{big,} & \text{medium,} & \text{very big} \\ 0.2 & 0.4 & 0.3 & 0.1 & 0.8 \end{pmatrix}$$
$$=\neg_p x\begin{pmatrix} 0 & 0.14 & 0.25 & 0.50 & 1 \\ \text{small,} & \text{above small,} & \text{below medium,} & \text{medium,} & \text{very big} \\ 0.2 & 0.4 & 0.3 & 0.1 & 0.8 \end{pmatrix}.$$

Therefore, the original attribute-value set

V = {small, medium, big, bigger, very big}

has been partially refined into:

RefinedV = {small, above small, below medium, medium, very big},

where *above small, below medium ∊ [small, medium].*

5.6 Application of Bi-Attribute Plithogenic Single-Valued Set

Let $\mathcal{U}$ be a universe of discourse, and $P\subset\mathcal{U}$ a plithogenic set.

In a plithogenic set P, each element (object) $x\in P$ is characterized by $m\geq 1$ attributes $\alpha_1,\alpha_2,\dots,\alpha_m$, and each attribute α_i, $1\leq i\leq m$, has $r_i\geq 1$ values:

$$V_i=\{v_{i1},v_{i2},\dots,v_{ir_i}\}.$$

Therefore, the element x is characterized by $r=r_1\times r_2\times\dots\times r_m$ attributes' values.

For example, if the attributes are "color" and "height", and their values (required by the application the experts want to do) are:

$Color=\{green,yellow,red\}$

and

$Height = \{tall, medium\}$,

then the object $x \in P$ is characterized by the Cartesian product

$$Color \times Height = \left\{ \begin{matrix} (green, tall), (green, medium), (yellow, tall), \\ (yellow, medium), (red, tall), (red, medium) \end{matrix} \right\}.$$

Let's consider the dominant (i.e. the most important, or reference) value of attribute "*color*" be "*green*", and of attribute "*height*" be "*tall*".

The attribute value contradiction fuzzy degrees are:

$c(green, green) = 0$,
$c(green, yellow) = \frac{1}{3}$,
$c(green, red) = \frac{2}{3}$,
$c(tall, tall) = 0$,
$c(tall, medium) = \frac{1}{2}$.

Suppose we have two experts *A* and *B*. Further on, we consider (fuzzy, intuitionistic fuzzy, or neutrosophic) degrees of appurtenance of each attribute value to the set *P* with respect to experts' criteria.

We consider the single value number fuzzy degrees, for simplicity of the example.

Let v_i be a uni-attribute value and its degree of contradiction with respect to the dominant uni-attribute value v_D be $c(v_D, v_i) \stackrel{\text{def}}{=} c_i$.

Let $d_A(x, v_i)$ be the appurtenance degree of the attribute value v_i of the element x with respect to the set A. And similarly for $d_B(x, v_i)$. Then, we recall the plithogenic aggregation operators with respect to this attribute value v_i that will be employed:

5.6.1 One-Attribute Value Plithogenic Single-Valued Fuzzy Set Intersection

$$d_A(x, v_i) \wedge_p d_B(x, v_i) = (1 - c_i) \cdot [d_A(x, v_i) \wedge_F d_B(x, v_i)] + c_i \cdot [d_A(x, v_i) \vee_F d_B(x, v_i)] \quad (44)$$

5.6.2 One-Attribute Value Plithogenic Single-Valued Fuzzy Set Union

$$d_A(x, v_i) \vee_p d_B(x, v_i) = (1 - c_i) \cdot [d_A(x, v_i) \vee_F d_B(x, v_i)] + c_i \cdot [d_A(x, v_i) \wedge_F d_B(x, v_i)] \quad (45)$$

5.6.3 One Attribute Value Plithogenic Single-Valued Fuzzy Set Complement (Negation)

$$\neg_p v_i = anti(v_i) = (1 - c_i) \cdot v_i \quad (46)$$

$$\neg_p d_A(x, (1 - c_i) v_i) = d_A(x, v_i) \quad (47)$$

5.7 Singe-Valued Fuzzy Set Degrees of Appurtenance

According to Expert A: $d_{\mathrm{A}}: \{green, yellow, red; tall, medium\} \rightarrow [0, 1]$.
One has:

$d_{\mathrm{A}}(green) = 0.6$,
$d_{\mathrm{A}}(yellow) = 0.2$,
$d_{\mathrm{A}}(red) = 0.7$;
$d_{\mathrm{A}}(tall) = 0.8$,
$d_{\mathrm{A}}(medium) = 0.5$.

We summarize as follows:

According to Expert A:

Contradiction Degrees	0	$\frac{1}{3}$	$\frac{2}{3}$		0	$\frac{1}{2}$
Attributes' Values	green	yellow	red		tall	medium
Fuzzy Degrees	0.6	0.2	0.7		0.8	0.5

Table 3.

According to Expert B:

Contradiction Degrees	0	$\frac{1}{3}$	$\frac{2}{3}$		0	$\frac{1}{2}$
Attributes' Values	green	yellow	red		tall	medium
Fuzzy Degrees	0.7	0.4	0.6		0.6	0.4

Table 4.

The element

x{ (green, tall), (green, medium), (yellow, tall), (yellow, medium), (red, tall), (red, medium) } ∈ *P*

with respect to the two experts as above is represented as:

$x_A\{(0.6, 0.8), (0.6, 0.5), (0.2, 0.8), (0.2, 0.5), (0.7, 0.8), (0.7, 0.5)\}$

and

$x_B\{(0.7, 0.6), (0.7, 0.4), (0.4, 0.6), (0.4, 0.4), (0.6, 0.6), (0.6, 0.4)\}$.

In order to find the optimal representation of x, we need to intersect x_A and x_B, each having six duplets. Actually, we separately intersect the corresponding duplets.

In this example, we take the fuzzy t_{norm}: $a \wedge_F b = ab$ and the fuzzy t_{conorm}: $a \vee_F b = a + b - ab$.

5.7.1 Application of Uni-Attribute Value Plithogenic Single-Valued Fuzzy Set Intersection

Let's compute $x_A \wedge_p x_B$.

0 0 0 0 {*degrees of contradictions*}

$$(0.6, 0.8) \wedge_p (0.7, 0.6) = (0.6 \wedge_p 0.7, 0.8 \wedge_p 0.6) = (0.6 \cdot 0.7, 0.8 \cdot 0.6) = (0.42, 0.48),$$

where above each duplet we wrote the degrees of contradictions of each attribute value with respect to their correspondent dominant attribute value. Since they were zero, $\wedge_p$ coincided with $\wedge_F$.

{*the first raw below 0 ½ and again 0 ½ represents the contradiction degrees*}

$$\left(\overset{0}{0.6}, \overset{\frac{1}{2}}{0.5}\right) \wedge_p \left(\overset{0}{0.7}, \overset{\frac{1}{2}}{0.4}\right) = (0.6 \wedge_p 0.7, 0.5 \wedge_p 0.4) = (0.6 \cdot 0.7, (1 - 0.5) \cdot [0.5 \wedge_F 0.4] + 0.5 \cdot [0.5 \vee_F 0.4])$$

$$= (0.42, 0.5[0.2] + 0.5[0.5 + 0.4 - 0.5 \cdot 0.4]) = (0.42, 0.45).$$

$$\left(\overset{\frac{1}{3}}{0.2}, \overset{0}{0.8}\right) \wedge_p \left(\overset{\frac{1}{3}}{0.4}, \overset{0}{0.6}\right) = (0.2 \wedge_p 0.4, 0.8 \wedge_p 0.6) = \left(\left\{1 - \frac{1}{3}\right\} \cdot [0.2 \wedge_F 0.4] + \left\{\frac{1}{3}\right\} \cdot [0.2 \vee_F 0.4], 0.8 \cdot 0.6\right)$$

$$\approx (0.23, 0.48).$$

$$\left(\overset{\frac{1}{3}}{0.2}, \overset{\frac{1}{2}}{0.5}\right) \wedge_p \left(\overset{\frac{1}{3}}{0.4}, \overset{\frac{1}{2}}{0.4}\right) = (0.2 \wedge_p 0.4, 0.5 \wedge_p 0.4)$$

(they were computed above)

$$\approx (0.23, 0.45).$$

$$\left(\overset{\frac{2}{3}}{0.7}, \overset{0}{0.8}\right) \wedge_p \left(\overset{\frac{2}{3}}{0.6}, \overset{0}{0.6}\right) = (0.7 \wedge_p 0.8, 0.8 \wedge_p 0.6) = \left(\left\{1 - \frac{2}{3}\right\} \cdot [0.7 \wedge_F 0.6] + \left\{\frac{2}{3}\right\} \cdot [0.7 \vee_F 0.6], 0.48\right)$$

(the second component was computed above)

$$= \left(\frac{1}{3}[0.7 \cdot 0.6] + \frac{2}{3}[0.7 + 0.6 - 0.7 \cdot 0.6], 0.48\right) \approx (0.73, 0.48).$$

And the last duplet:

$$\left(\overset{\frac{2}{3}}{0.7}, \overset{\frac{1}{2}}{0.5}\right) \wedge_p \left(\overset{\frac{2}{3}}{0.6}, \overset{\frac{1}{2}}{0.4}\right) = (0.7 \wedge_p 0.6, 0.5 \wedge_p 0.4)$$

$$\approx (0.73, 0.45)$$

(they were computed above).

Finally:

$$x_A \wedge_p x_B \approx \begin{Bmatrix} (0.42, 0.48), (0.42, 0.45), (0.23, 0.48), (0.23, 0.45), \\ (0.73, 0.48), (0.73, 0.45) \end{Bmatrix},$$

or, after the intersection of the experts' opinions $A/\backslash_P B$, we summarize the result as:

Contradiction Degrees	0	$\frac{1}{3}$	$\frac{2}{3}$		0	$\frac{1}{2}$
Attributes' Values	green	yellow	red		tall	medium
Fuzzy Degrees of Expert A for x	0.6	0.2	0.7		0.8	0.5
Fuzzy Degrees of Expert B for x	0.7	0.4	0.6		0.6	0.4
Fuzzy Degrees of $x_A \wedge_p x_B$	0.42	0.23	0.73		0.48	0.45
Fuzzy Degrees of $x_A \vee_p x_B$	0.88	0.37	0.57		0.92	0.45

Table 5.

5.7.2 Application of Uni-Attribute Value Plithogenic Single-Valued Fuzzy Set Union

We separately compute for each single attribute value:

$$d_A^F(x, green) \vee_p d_B^F(x, green) = 0.6 \vee_p 0.7 = (1-0) \cdot [0.6 \vee_F 0.7] + 0 \cdot [0.6 \wedge_F 0.7]$$
$$= 1 \cdot [0.6 + 0.7 - 0.6 \cdot 0.7] + 0 = 0.88.$$

$$d_A^F(x, yellow) \vee_p d_B^F(x, yellow) = 0.2 \vee_p 0.4 = \left(1 - \frac{1}{3}\right) \cdot [0.2 \vee_F 0.4] + \frac{1}{3} \cdot [0.2 \wedge_F 0.4]$$
$$= \frac{2}{3} \cdot (0.2 + 0.4 - 0.2 \cdot 0.4) + \frac{1}{3}(0.2 \cdot 0.4) \approx 0.37.$$

$$d_A^F(x, red) \vee_p d_B^F(x, red) = 0.7 \vee_p 0.6 = \left\{1 - \frac{2}{3}\right\} \cdot [0.7 \vee_F 0.6] + \frac{2}{3} \cdot [0.7 \wedge_F 0.6]$$
$$= \frac{1}{3} \cdot (0.7 + 0.6 - 0.7 \cdot 0.6) + \frac{2}{3}(0.7 \cdot 0.6) \approx 0.57.$$

$$d_A^F(x, tall) \vee_p d_B^F(x, tall) = 0.8 \vee_p 0.6 = (1-0) \cdot (0.8 + 0.6 - 0.8 \cdot 0.6) + 0 \cdot (0.8 \cdot 0.6) = 0.92.$$

$$d_A^F(x, medium) \vee_p d_B^F(x, medium) = 0.5 \vee_p 0.4 = \frac{1}{2}(0.5 + 0.4 - 0.5 \cdot 0.4) + \frac{1}{2} \cdot (0.5 \cdot 0.4) = 0.45.$$

5.7.3 Properties of Plithogenic Single-Valued Set Operators in Applications

1) When the attribute value contradiction degree with respect to the corresponding dominant attribute value is 0 (zero), one simply use the fuzzy intersection:

$$d_{A \wedge_p B}(x, green) = d_A(x, green) \wedge_F d_B(x, green) = 0.6 \cdot 0.7 = 0.42,$$

and

$$d_{A \wedge_p B}(x, tall) = d_A(x, tall) \wedge_F d_B(x, tall) = 0.8 \cdot 0.6 = 0.48.$$

2) But, if the attribute value contradiction degree with respect to the corresponding dominant attribute value is different from 0 and from 1, the result of the plithogenic intersection is between the results of fuzzy t_{norm} and fuzzy t_{conorm}.

Examples:

$$d_A(x, yellow) \wedge_F d_B(x, yellow) = 0.2 \wedge_F 0.4 = 0.2 \cdot 0.4 = 0.08 \ (t_{norm}),$$
$$d_A(x, yellow) \vee_F d_B(x, yellow) = 0.2 \vee_F 0.4 = 0.2 + 0.4 - 0.2 \cdot 0.4 = 0.52 \ (t_{conorm});$$

while

$$d_A(x, yellow) \wedge_p d_B(x, yellow) = 0.23 \in [0.08, 0.52]$$

{or 0.23 ≈ 0.2266… = (2/3)×0.08 + (1/3)×0.52, i.e. a linear combination of t_{norm} and t_{conorm}}.

Similarly:

$d_A(x, red) \wedge_p d_B(x, red) = 0.7 \wedge_F 0.6 = 0.7 \cdot 0.6 = 0.42$ (t_{norm}),
$d_A(x, red) \vee_p d_B(x, red) = 0.7 \vee_F 0.6 = 0.7 + 0.6 - 0.7 \cdot 0.6 = 0.88$ (t_{conorm});

while

$d_A(x, red) \wedge_p d_B(x, red) = 0.57 \in [0.42, 0.88]$
{linear combination of t_{norm} and t_{conorm}}.

And

$d_A(x, medium) \wedge_F d_B(x, medium) = 0.5 \wedge_F 0.4 = 0.5 \cdot 0.4 = 0.20,$
$d_A(x, medium) \vee_F d_B(x, medium) = 0.5 \vee_F 0.4 = 0.5 + 0.4 - 0.5 \cdot 0.4 = 0.70,$

while

$d_A(x, medium) \wedge_p d_B(x, medium) = 0.45,$

which is just in the middle (because “medium” contradiction degree is $\frac{1}{2}$) of the interval $[0.20, 0.70]$.

Conclusion & Future Research

As generalization of dialectics and neutrosophy, plithogeny will find more use in blending diverse philosophical, ideological, religious, political and social ideas. After the extension of fuzzy set, intuitionistic fuzzy set, and neutrosophic set to the plithogenic set; the extension of classical logic, fuzzy logic, intuitionistic fuzzy logic and neutrosophic logic to plithogenic logic; and the extension of classical probability, imprecise probability, and neutrosophic probability to plithogenic probability [12] – more applications of the plithogenic set/logic/probability/statistics in various fields should follow. The classes of plithogenic implication operators and their corresponding sets of plithogenic rules are to be constructed in this direction. Also, exploration of non-linear combinations of t_{norm} and t_{conorm}, or of other norms and conorms, in constructing of more sophisticated plithogenic set, logic and probabilistic aggregation operators, for a better modeling of real life applications.

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Conjunto plitogénico, una extensión de los conjuntos crisp, difusos, conjuntos difusos intuicionistas y neutrosóficos revisitado

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Resumen. En el presente artículo, introducimos el conjunto plitogénico (como generalización de conjuntos nítidos, borrosos, intuicionistas, borrosos y neutrosóficos), que es un conjunto cuyos elementos se caracterizan por los valores de muchos atrib u-tos. Un valor de atributo v tiene un grado correspondiente (difuso, intuicionista difuso o neutrosófico) de pertenencia d (x, v) del elemento x, al conjunto P, con respecto a algunos criterios dados. Para obtener una mejor precisión para los operadores d e agregación plitogénica en el conjunto plitogénico, y para una inclusión más exacta (orden parcial), se define un grado de con-tradicción (disimilitud difusa, intuicionista difusa o neutrosófica) entre cada atributo del valor y el valor del atributo dominanante (el más importante). La intersección y la unión plitogénicas son combinaciones lineales de los operadores difusos t_{norm} y $t_{co\text{-}norm}$, mientras que el complemento plitogénico, la inclusión (desigualdad), la igualdad está influenciados por los grados de co n-tradicción (disimilitud) de los valores de los atributos. Por tal motivo el objetivo del presente trabajo es ofrecer algunos ejemplos y aplicaciones de los nuevos conceptos que se proponen, para su aplicación en la vida cotidiana.

Palabras Claves: Plitogenia; Conjunto plitogénico; Conjunto neutrosófico; Operadores Plitogénicos.

1 Definición informal de conjunto plitogénico

La plitogenia es la génesis u originación, creación, formación, desarrollo y evolución de nuevas entidades a partir de dinámicas y fusiones orgánicas de entidades antiguas múltiples contradictorias y / o neutrales y / o no contradictorias. Mientras que plitogénico significa lo que pertenece a la plitogenia.

Un conjunto plitogénico P es un conjunto cuyos elementos se caracterizan por uno o más atributos, y cada atributo puede tener muchos valores. El valor v de cada atributo tiene un grado correspondiente de pertenencia $d\ (x, v)$ del elemento x, al conjunto P, con respecto a algunos criterios dados. Con el fin de obtener una mejor precisión para los operadores de agregación plitogénicos, se define un grado de contradicción (disimilitud) entre cada valor de atributo y el valor de atributo dominante (el más importante).

{Sin embargo, hay casos en que tal valor de atributo dominante puede no tomarse en consideración o puede que no exista [por lo tanto, se considera cero de manera predeterminada], o puede haber muchos valores de atributo dominantes. En tales casos, se suprime la función de grado de contradicción o se debe establecer otra función de relación entre los valores de los atributos.}

Los operadores de agregación plitogénicos (intersección, unión, complemento, inclusión, igualdad) se basan en grados de contradicción entre los valores de los atributos, y los dos primeros son combinaciones lineales de los operadores difusos t_{norm} y t_{conorm}.

El conjunto plitogénico es una generalización del conjunto nítido, conjunto difuso, conjunto difuso intuicionista y conjunto neutrosófico, ya que estos cuatro tipos de conjuntos se caracterizan por un único valor de atributo (appurtenance): que tiene un valor (membresía) - para el conjunto nítido y conjunto difuso, dos valores (pertenencia y no pertenencia) - para conjunto difuso intuicionista, o tres valores (pertenencia, no pertenencia e indeterminación) - para conjunto neutrosófico.

2 Definición formal de conjunto plitogénico de atributo único (unidimensional)

Sea U un universo de discurso y P un conjunto de elementos no vacíos, P

2.1 Atributo del espectro de valor

Sea A un conjunto no vacío de atributos unidimensionales A $= \{\alpha 1, \alpha 2, \ldots, \alpha m\}, m \geq 1$; y α ∈ A es un atributo dado cuyo espectro de todos los valores (o estados) posibles es el conjunto no vacío S, donde S puede ser un conjunto discreto finito, S = {s1, s2, ..., sl}, 1 ≤ l < ∞, o conjunto infinitamente contable S = {s1, s2, ..., s∞}, o conjunto infinitamente incontable (continuo) S =] a, b [, a <b, donde] ... [es cualquier abierto, semiabierto , o intervalo cerrado desde el conjunto de números reales o desde otro conjunto general.

2.2 Rango de valor de atributo

Sea V un subconjunto no vacío de S, donde V es el rango de todos los valores de atributos que necesitan los expertos para su aplicación. Cada elemento $x \in P$ se caracteriza por los valores de todos los atributos en $V = \{v1, v2, \ldots, vn\}$, *para* $n \geq 1$.

2.3 Valor de atributo dominante

En el conjunto de valores V del atributo, en general, hay un valor de atributo dominante, que es determinado por los expertos en su aplicación. El valor de atributo dominante significa el valor de atributo más importante que los expertos están interesados.

{Sin embargo, hay casos en que tal valor de atributo dominante puede no tomarse en consideración o no existir, o puede haber muchos valores de atributo dominantes (importantes), cuando se debe emplear un enfoque diferente.}

2.4 Atributo Valor Función de Grado de Mantenimiento

Cada valor de atributos v ∈ V tiene un grado correspondiente de pertenencia d (x, v) del elemento x, al conjunto P, con respecto a algunos criterios dados. El grado de pertenencia puede ser: un grado difuso de pertenencia, o un grado difuso intuicionista, o un grado neutrosófico de pertenencia al conjunto plitogénico.

Por lo tanto, la función de grado de valor de atributo del atributo es:

$$\forall \boldsymbol{x} \in \boldsymbol{P}, \boldsymbol{d}: \boldsymbol{P} \times \boldsymbol{V} \to \boldsymbol{P}\ ([\boldsymbol{0}, \boldsymbol{1}]\boldsymbol{z}) \qquad (1)$$

Por lo que $d\ (x, v)$ es un subconjunto de [0, 1] z, y P ([0, 1] z) es el conjunto de potencias de [0, 1] z, donde z = 1 (para el grado de rendimiento difuso), z = 2 (para el grado de pertenencia difusa intuicionista), o z = 3 (para el grado de dependencia neutrosófica).

2.5 Atributo Valor de contradicción (disimilitud) Grado Función

Sea que el cardinal | V | ≥ 1.

Sea c: V × V → [0, 1] la función de grado de contradicción (disimilitud) del valor de atributo (que introducimos ahora por primera vez) entre cualquiera de los dos valores de atributo v1 y v2, denotado por:

c (v1, v2), y satisfaciendo los siguientes axiomas:

c (v1, v1) = 0, el grado de contradicción entre los mismos valores de atributo es cero;

c (v1, v2) = c (v2, v1), conmutatividad.

Para simplificar, usamos una función de grado de contradicción de valor de atributo difuso (c como arriba, que podemos denotar con c_F para distinguirla de las dos siguientes), pero una función de contradicción de valor de atributo intuicionista (c_{IF}: V × V → [0 , 1] 2), o más general, se puede utilizar una función de contradicción del valor del atributo neutrosófico (c_N: V × V → [0, 1] 3) aumentando la complejidad del cálculo, pero también la precisión.

Principalmente calculamos el grado de contradicción entre los valores de atributos unidimensionales. Para los valores de atributos multidimensionales, los dividimos en valores de atributos unidimensionales correspondientes.

La función de grado de contradicción del valor de atributo ayuda a los operadores de agregación plitogénica, y la relación de inclusión plitogénica (orden parcial) para obtener un resultado más preciso.

La función de grado de contradicción del valor de atributo se diseña en cada campo donde se usa el conjunto plitogénico de acuerdo con la aplicación para resolver. Si se ignora, las agregaciones aún funcionan, pero el resultado puede perder precisión.

Varios ejemplos serán proporcionados en este documento.

Entonces (P, a, V, d, c) se llama un conjunto plitogénico:

- Donde "P" es un conjunto, "a" es un atributo (multidimensional en general), "V" es el rango de los valores del atributo, "d" es el grado de pertenencia del valor de atributo de cada elemento x al conjunto P con respecto a algunos criterios dados ($\boldsymbol{x\ \varepsilon\ P}$), y “d” significa “$d_F$” o “$d_{IF}$” o “$d_N$”, cuando se trata de un grado de aplicación difuso, un grado de aplicación intuitionistic borroso, o un grado de neutrosofia respectivo respectivamente de un elemento $\boldsymbol{\mathcal{X}}$ al conjunto plitogénico $\boldsymbol{P}$;
- "$\boldsymbol{C}$" significa "c_F" o "c_{IF}" o "c_N", cuando se trata de un grado de contradicción difusa, un grado difuso de contradicción intuicionista o un grado de contradicción neutrosófica entre valores de atributo respectivamente. Las funciones d (·, ·) y c (·, ·) se definen de acuerdo con las aplicaciones que los expertos necesitan resolver. De forma general se utiliza la notación: $\boldsymbol{x}\ (\boldsymbol{d}\ (\boldsymbol{x},\boldsymbol{V}))$,

  Donde; $\boldsymbol{d}\ (\boldsymbol{x},\boldsymbol{V}) = \{\boldsymbol{d}\ (\boldsymbol{x},\boldsymbol{v}), \text{para todos los } v \in V\}$, $\forall \boldsymbol{x} \in \boldsymbol{P}$.

2.6 Operadores de conjuntos de agregación plitogénica

El grado de contradicción del valor de atributo se calcula entre cada valor de atributo con respecto al dominante valor de atributo (denotado vD) en especial, y con respecto a otros valores de atributo también

La función de grado de contradicción del valor del atributo c entre los valores del atributo se utiliza en la definición de operadores de agregación plitogénica {Intersección (AND), Unión (OR), Implicación (⇒), Equivalencia (⇔),

Relación de inclusión (orden parcial o desigualdad parcial) y otros operadores de agregación plitogénica que combine dos o más grados de valor de atributo - sobre los que t_{norm} y t_{conorm} actúan sobre}.

La mayoría de los operadores de agregación plitogénica son combinaciones lineales de la borrosa t_{norm} (denotada $\wedge F$), y fuzzy t_{conorm} (denotado $\vee_F$), pero también pueden construirse combinaciones no lineales

Si uno aplica la t_{norm} en el valor de atributo dominante denotado por v_D, y la contradicción entre v_D y v_2 es c (v_D, v_2), luego en el valor de atributo v_2 se aplica:

$$[1 - c(vD, v2)]\cdot tnorm(vD, v2) + c(vD, v2)\cdot tconorm(vD, v2), \quad (2)$$

O, utilizando símbolos:

$$[1 - c(vD, v2)]\cdot(vD\wedge Fv2) + c(vD, v2)\cdot(vD\vee Fv2). \quad (3)$$

De manera similar, si se aplica t_{conorm} en el valor de atributo dominante denotado por v_D, y la contradicción entre D y v_2 es c (v_D, v_2), entonces en el valor de atributo v_2 se aplica uno:

$$[1 - c(vD, v2)]\cdot tconorm(vD, v2) + c(vD, v2)\cdot tnorm(vD, v2), \quad (4)$$

O, utilizando símbolos:

$$[1 - c(vD, v2)]\cdot(vD\vee Fv2) + c(vD, v2)\cdot(vD\wedge Fv2). \quad (5)$$

3 Conjunto plitogénico como generalización de otros conjuntos

El conjunto plitogénico es una generalización de los conjunto nítidos, de los conjuntos difusos, de los conjuntos difusos intuicionistas y de los conjuntos neutrosófico, dado que estos cuatro tipos de conjuntos se caracterizan por contener un atributo único (*appurtenance*): que tiene un valor (*membresía*): para el conjunto nítido y para conjunto difuso, dos valores (pertenencia y no pertenencia) - para conjunto difuso intuicionista, o tres valores (pertenencia, no pertenencia e indeterminación) - para conjunto neutrosófico.

Por ejemplo:

Sea U un universo de discurso del conjunto no vacío P y también, un conjunto $x \in P$ que forma un elemento genérico.

3.1 Conjunto Crisp (Clásico) (CCS)

El atributo es α = "accesorio";

el conjunto de valores de atributo V = {membresía, no membresía}, con cardinal | V | = 2;

el valor del atributo dominante = membresía;

La función de grado de atributo de valor de atributo:

$$d: P\times V\rightarrow \{0, 1\}, \quad (6)$$

d (x, membresía) = 1, d (x, no membresía) = 0,
y la función de grado de contradicción de valor de atributo:

$$c: V\times V\rightarrow \{0, 1\}, \quad (7)$$

c (membresía, membresía) = c (no membresía, no membresía) = 0, c (membresía, no membresía) = 1.

3.1.1 Intersección Crisp (Clásica)

$a \wedge b \in \{0, 1\}$ (8)

3.1.2 Unión Crisp (Clásica)

$a \vee b \in \{0, 1\}$ (9)

3.1.3 Complemento Crisp (clásico) (negación)

$\neg a \in \{0, 1\}$. (10)

3.2 *Conjunto difuso de valor único (SVFS)*

El atributo α = “mantenimiento”;
el conjunto de valores de atributo V = {membresía}, cuyo cardinal | V | = 1; el valor del atributo dominante = membresía;

La función de grado de valor de atributo de mantenimiento:

$d: P \times V \rightarrow [0, 1]$, (11)
con d (x, membresía) $\in [0, 1]$;

La función de grado de contradicción de valor de atributo:

$c: V \times V \rightarrow [0, 1]$, (12)

c (membresía, membresía) = 0.

3.2.1 Intersección difusa

$a \wedge_F b \, \varepsilon \, [0, 1]$, (13)

3.2.2 Unión difusa

$a \vee_F b \, \varepsilon \, [0, 1]$, (14)

3.2.3 Complemento difuso (negación)

$\neg_F a = 1 - a \in [0, 1]$. (15)

3.3 *Conjunto difuso intuicionista de valor único (SVIFS)*

El atributo es α = "accesorio"; el conjunto de valores de atributo V = {membresía, no membresía}, cuyo cardinal | V | = 2; el valor del atributo dominante = membresía; la función de grado de valor de atributo de mantenimiento:

$d: P \times V \rightarrow [0, 1]$, (16)

d (x, pertenencia) ∈ [0, 1], d (x, no pertenencia) ∈ [0, 1],

con d (x, membresía) + d (x, no membresía) ≤ 1,

y la función de grado de contradicción de valor de atributo:

$c: V \times V \rightarrow [0, 1]$, (17)

c (membresía, membresía) = c (no membresía, no membresía) = 0,
c (membresía, no membresía) = 1,

lo que significa que para la intersección de los operadores de agregación de SVIFS (AND) y unión (OR), si se aplica el t_{norm} en grado de membresía, entonces uno tiene que aplicar t_{conorm} en grado de no membresía - y recíprocamente.

Por tanto:

3.3.1 *Intersección difusa intuicionista*

$$(a1, a2) \wedge IFS\ (b1, b2) = (a1 \wedge F\ b1, a2 \vee F\ b2) \tag{18}$$

3.3.2 *Unión difusa intuicionista*

$$(\boldsymbol{a1}, \boldsymbol{a2}) \vee \boldsymbol{IFS}\ (\boldsymbol{b1}, \boldsymbol{b2}) = (\boldsymbol{a1} \vee \boldsymbol{F}\ \boldsymbol{b1}, \boldsymbol{a2} \wedge \boldsymbol{F}\ \boldsymbol{b2} \tag{19}$$

3.3.3 *Complemento difuso intuicionista (negación)*

$$\neg\ IFS\ (a1, a2) = (a2, a1), \tag{20}$$

donde $\wedge$ F y $\vee$ F; son la configuración difusa y la configuración difusa respectivamente.

3.3.4 *Inclusiones difusas intuicionistas (órdenes parciales)*

Inclusión difusa intuicionista simple (la más utilizada por la comunidad difusa intuicionista), se define como:

$$(\mathrm{a1}, \mathrm{a2}) \leq \mathrm{IFS}\ (\mathrm{b1}, \mathrm{b2}) \tag{21}$$

Si y solo si $\mathrm{a1} \leq \mathrm{b1}$ and $\mathrm{a2} \geq \mathrm{b2}$.

La inclusión borrosa intuicionista plitogénica (completa) (presentada por primera vez) se define como:

$$(\mathrm{a1}, \mathrm{a2}) \leq \mathrm{P}\ (\mathrm{b1}, \mathrm{b2}) \tag{22}$$

Si y solo si $\mathrm{a1} \leq (1\text{-}c_v).\ b_1$, $\mathrm{a2} \geq (1\text{-}c_v).\ b_2$.

donde; cv ε [0, 0.5) es el grado de contradicción entre el valor del atributo dominante y el valor del atributo v {el último cuyo grado de pertenencia con respecto al Experto A es (a1, a2), mientras que con respecto al Experto B es (b1, b2)}. Si c_v no existe, lo tomamos por defecto como igual a cero.

3.4 Conjunto Neutrosófico de Valor Único (SVNS)

El atributo es α = " accesorio ";
el conjunto de valores de atributo V = {membresía, indeterminación, no pertenencia} , cuyo cardinal | V | = 3; el valor del atributo dominante = membresía;
La función de grado de atributo de valor de atributo

$$d: P \times V \rightarrow [0, 1], \tag{23}$$

d (x, pertenencia) ∈ [0, 1], d (x, indeterminación) ∈ [0, 1],
d (x, no pertenencia) ∈ [0, 1],
con 0 ≤ d (x, membresía) + d (x, indeterminación) + d (x, no pertenencia) ≤ 3;

y la función de grado de contradicción de valor de atributo:

c: V × V → [0, 1], *(24)*

c (membresía, membresía) = c (indeterminación, indeterminación) =

c $(a_1, a_2, a_3) \wedge_P (b_1, b_2, b_3) =$ (no miembro, no miembro) = 0,

c $\left(a_1 \wedge_F b_1, \frac{1}{2}\left[(a_2 \wedge_F b_2) + (a_2 \vee_F b_2)\right], a_3 \vee_F b_3\right)$ (membresía, no membresía) = 1,

c (membresía, indeterminación) = c (no membresía, indeterminación) = 0.5,

Lo que significa que para los operadores de agregación SVNS (Intersección, Unión, Complemento, etc.), si se aplica el t_{norm} sobre la membresía, entonces uno tiene que aplicar el t_{conorm} en la no pertenencia {y recíprocamente), mientras que en la indeterminación se aplica el promedio de t_{norm} y t_{conorm}, como sigue:

3.4.1 *Intersección neutrosófica*

Intersección neutrosófica simple (la más utilizada por la comunidad neutrosófica):

$$(a_1, a_2, a_3) \wedge_{NS} (b_1, b_2, b_3) = (a_1 \wedge_F b_1, a_2 \vee_F b_2, a_3 \vee_F b_3)$$

Intersección Neutrosófica Plitogénica:

$$(a_1, a_2, a_3) \vee_{NS} (b_1, b_2, b_3) = (a_1 \vee_F b_1, a_2 \wedge_F b_2, a_3 \wedge_F b_3)$$

3.4.2 *Unión neutrosófica*

Unión Neutrosófica simple (la más utilizada por la comunidad neutrosófica):

$$(a_1, a_2, a_3) \vee_P (b_1, b_2, b_3) = \left(a_1 \vee_F b_1, \frac{1}{2}\left[(a_2 \wedge_F b_2) + (a_2 \vee_F b_2)\right], a_3 \wedge_F b_3\right). \quad (27)$$

De otra manera, con respecto a lo que uno aplica en la membresía, uno aplica lo contrario en la no membresía,

Mientras que en la indeterminación se aplica el promedio entre ellos.

3.1.1 *Complemento Neutrosófico (Negación)*

$\neg_{NS}$ (a1, a2, a3) = (a3, a2, a1). *(29)*

3.1.2 *Inclusiones Neutrosóficas (Pedidos Parciales)*

Inclusión neutrosófica simple (la más utilizada por la comunidad neutrosófica):

$(a1, a2, a3) \leq NS\ (b1, b2, b3)$ *(30)*

iff a1 ≤ b1 and a2 ≥ b2, a3 ≥ b3.

Inclusión neutrosófica plitogénica (definida por primera vez):

Cómo los grados de contradicción son:

$c(a1, a2$ $c(a2, a3$ $c(b1, b2$ $c(b2, b3$,
(31)

Se aplica: a2 ≥ [1- c(a1, a2)]b2 or a2 ≥ (1-0.5)b2 or a2 ≥ 0.5·b2

Mientras;

c(a1, a3) = c(b1, b3) = 1 *(32)*

Dónde:

(a1, a2, a3) ≤P $(b1{,}2{,}b3)$ *(33)*

iff a1 ≤ b1 and a2 ≥ 0.5·b2, a3 ≥ b3.

4 *Clasificaciones del conjunto plitogénico*

4.1 *Primera clasificación*

4.1.1 *Conjunto Plitogénico Refinado*

Si al menos uno de los valores del atributo vk $\in V$ se divide (refina) en dos o más subvalores de atributo: vk1, vk2,… $\in V$, con la función de grado de categoría de subvalor de atributo: d (x, vki) P ([0, 1]), para i = 1, 2,... entonces (Pr, α, V, d, c) se llama un Conjunto Plitogénico Refinado, donde "r" significa "refinado".

4.1.2 Plitogénico Overset / Underset / Offset

Si para al menos uno de los valores del atributo vk ∈ V, de al menos un elemento x ∈ P, tiene el valor del atributo función de grado de mantenimiento d (x, vk) que excede de 1, entonces (Po, α, V, d, c) se denomina sobreexplotación plitogénica, donde "o" significa "sobreexplotación"; pero si d (x, vk) está por debajo de 0, entonces (Pu, α, V, d, c) se llama un arsenal plitogénico, donde "u" significa "underset"; mientras que si d (x, vk) excede de 1, y d (y, sj) está por debajo de 0 para los valores de atributo vk, vj ∈ V, que pueden ser valores de atributo iguales o diferentes correspondientes al mismo elemento o a dos elementos diferentes x , y ∈ P, entonces (Poff, α, V, d, c) se denomina Desviación Plitogénica, donde "off" significa "compensación" (o conjunto plitogénico que se sobrepasa y subraya).

4.1.3 Multiset plitogénico

Un conjunto plitogénico P que tiene al menos un elemento, que se repite en el conjunto P con los mismos componentes plitogénicos

$$(a1,a2,\ldots,am),\ x(a1,a2,\ldots,am) \tag{34}$$

o con diferentes componentes plitogénicos

$$(a1,a2,\ldots,am),\ x(b1,b2,\ldots,bm), \tag{35}$$

entonces se denomina Multiset plitogénico, donde "m" significa "multiset".

4.1.4 Conjunto bipolar plitogénico

If ∀x ∈ P, d: P×V→ {P([-1, 0]) × P([0, 1])}^{z}, entonces (Pb, α, V, d, c) se denomina conjunto bipolar plitogénico, ya que d (x, v), para v ∈ V, asocia un grado negativo de pertenencia (como un subconjunto de [-1, 0]) y un grado positivo (como un subconjunto de [0, 1]) al valor v; donde z = 1 para grado difuso, z = 2 para grado difuso intuicionista, y z = 3 para grado difuso neutrosófico.

4.1.5-6 Conjunto tripolar plitogénico y conjunto multipolar plitogénico

Definiciones similares para el Conjunto Tripolar Plitogénico y el Conjunto Multipolar Plitogénico (extensión del Conjunto Tripolar Neutrosófico y el Conjunto Multipolar Neutrosófico respectivamente [[4], 123-125}.

4.1.6 Conjunto de complejos plitogénicos

Si, para cualquier x∈ P, d: P × V → {P ([0, 1]) × P ([0, 1])} z, y para cualquier v ∈ V, d (x, v) es un valor complejo, es decir, d (x, v) = M1 · $ejM2$, donde M1 ⊆ [0, 1] se llama amplitud, y M2 ⊆ [0, 1] se llama fase, y el grado de mantenimiento puede ser borroso (z = 1), intuitionistic fuzzy (z = 2), o neutrosophic (z = 3), entonces (Pcom, α, V, d, c) se denomina Conjunto de complejos plitogénicos.

4.2 Segunda clasificación

Sobre los valores de la función de grado de mantenimiento, se tiene:

4.2.1 Conjunto difuso plitogénico de un solo valor

Si

$$\forall x \in P,\ d: P \times V \rightarrow [0, 1], \tag{36}$$

y ∀ v ∈ V, d (x, v) es un número único en [0, 1].

4.2.2 Conjunto difuso plitogénico Hesitante

Si

$$\forall x \in P,\ d: P\times V\rightarrow P([0, 1]), \tag{37}$$

y ∀ v ∈ V, d (x, v) sea un conjunto finito discreto de la forma {n1, n2, …, np}, donde 1≤ p <∞, incluido en el intervalo [0, 1].

4.2.3 Conjunto difuso plitogénico de valor intermedio

Si
$\forall x \in$ P, d: P×V→ P ([0, 1]), (38)
y ∀v ∈ V, d (x, v) sea un (conjunto abierto, semi-abierto, cerrado) incluido en el intervalo [0, 1].

5 Aplicaciones y ejemplos

5.1 Aplicaciones de Atributo Unidimensional Plitogénico de valor único del conjunto borroso

Sea U un universo del conjunto plitogénico no vacío de P ⊆ U. Y x ∈ P un elemento genérico. Para simplicidad, se considera el atributo uni - dimensional y la función de simple – valor fuzzy.

5.1.1 Pequeño conjunto discreto de valores-atributo

Si el atributo es "color", y considerando sólo el conjunto de valores discreto del atributo v, formado por los seis colores puros, entonces;
V = *{violeta, azul, verde, amarillo, naranja, rojo},*
El valor del atributo adjunto del grado de la función es:

$$d: P\times V \to [0, 1], \tag{39}$$

$$d(x, violeta) = v \in [0, 1], d(x, azul) = b \in [0, 1], d(x, verde) = g \in [0, 1],$$
$$d(x, amarillo) = y \in [0, 1], d(x, naranja) = o \in [0, 1], d(x, rojo) = r \in [0, 1],$$

entonces se tiene: x (v, b, g, y, o, r), donde v, b, g, y, o, r son grados fuzzy de violeta, azul, verde, amarillo, naranja, y rojo, respectivamente, del objeto x con respecto al conjunto de objetos P, donde v, b, g, y, o, r ε [0, 1].
El cardinal del conjunto de valores de los atributos V is 6.
Los otros colores son mezclas de los colores puros.

5.1.2 Grande conjunto discreto de Valores – Atributos

Si para el atributo "color" se escoge una representación más refinada de los valores como:

$X \{d_{390}, d_{391}, \ldots, d_{699}, d_{700}\}$,

medido en nanómetros, se tiene un conjunto finito de valores de atributos, cuyo cardinal es: 700 - 390 + 1 = 311, donde para cada j ∈ V = {390, 391, ..., 699, 700}, dj representa el grado para lo cual el color del objeto x, con respecto al conjunto de objetos P, es de "j" nanómetros por longitud de onda, con di ε [0, 1]. Un nanómetro (nm) es las mil millonésimas parte de un metro.

5.1.3 Conjunto infinito de valores de atributo

Si el atributo es nuevamente "color", entonces se puede elegir una representación continua de la siguiente forma:

x (d ([390,700])),

Teniendo V = [390,700] un intervalo real cerrado, por lo tanto, el conjunto de valores de atributo es infinitamente incontable (continuo). El cardenal de la V es ∞.
Para cada j ε [390,700], dj representa el grado en que el color del objeto x, con respecto al conjunto de objetos P, es de "j" nanómetros por longitud de onda, con di ε [0, 1]. Y d ([390,700]) = {dj, j ε [390,700]}.
La luz, que va desde 390 (color violeta) a 700 (color rojo) nanómetros por longitudes de onda es visible para el ojo humano. El cardinal del conjunto de valores de atributo V es infinito continuo.

5.2 Ejemplo de Uni - atributo (de 4 valores de atributo) de conjunto Plitogénico, de conjunto borroso de valor único (Negación).

Considerando que el atributo "tamaño" con los siguientes valores: pequeño (el dominante), medio, grande, muy grande.

Grados de contradicción	0	0.50	0.75	1
Valores de atributo	pequeño	medio	grande	Muy
Grados de accesorio	0.8	0.1	0.3	0.2

Tabla 1.

5.3 Ejemplo de refinamiento y negación de un atributo uni (de valores de 4 atributos) conjunto difuso de valor único plitogénico.

Como un refinamiento de la tabla anterior, agreguemos el atributo "más grande" como se muestra en la siguiente tabla (tabla 2).

entonces se tiene: x (v, b, g, y, o, r), donde v, b, g, y, o, r son grados fuzzy de violeta, azul, verde, amarillo, naranja, y rojo, respectivamente, del objeto x con respecto al conjunto de objetos P, donde v, b, g, y, o, r ε [0, 1].

El cardinal del conjunto de valores de los atributos V is 6.

Los otros colores son mezclas de los colores puros.

El opuesto (negación) del valor de atributo "grande", que es 75% en contradicción con "pequeño", será un valor de atributo que es $1-0.75$ = 0.25 = 25% en contradicción con "pequeño", por lo que será igual a 12 ["pequeño" + "$medio$"]. Llamémoslo "menos medio", cuyo grado de mantenimiento es 1 - 0.3 = 0.7.

Si el atributo "tamaño" tiene otros valores, pequeño es el valor dominante:

Grados de contradicción	0	**0.14**	**0.25**	0.50	0.75	**0.86**	1
Valor del atributo	pequeño	**arriba pequeño (anti- más grande)**	**menos medio (anti- grande)**	medio	grande	**más grande**	muy grande
Grados de accesorio	0.8	**0.6**	**0.7**	0.1	0.3	**0.4**	0.2

Tabla 2.

El opuesto (negación) de "más grande" es 1 - 0.86 = 0.14 = 14% en grado de contradicción con el valor del atributo dominante ("pequeño"), por lo que está entre "pequeño" y "medio", podemos decir que es incluido en el intervalo de valor de atributo [pequeño, medio], mucho más cerca de "pequeño" que de "medio". Llamemos "por encima de pequeño", cuyo grado de mantenimiento es 1 - 0.4 = 0.6.

5.4 Ejemplo de atributo múltiple (de 24 valores de atributo) conjunto de unión difusa plitogénica, unión y complemento

Sea P un conjunto plitogénico, representando a los estudiantes de una universidad, donde $x \in P$ que representa a un estudiante genérico caracterizado por los siguiente tres atributos:

- ✓ altitud, cuyos valores son {alto,corto}$\stackrel{\text{def}}{=}${1,2};
- ✓ peso, cuyos valores son {obeso, gordo, medio, delgado} $\stackrel{\text{def}}{=}$ {1, 2, 3, 4};
- ✓ color del cabello, cuyos valores son {rubio, rojizo, marrón} $\stackrel{\text{def}}{=}$ $\{h1, h2, h3\}$.

El multi-atributo de la dimensión 3 es:

$V3$ = $\{(ai, wj, hk)$, para todos $1 \le i \le 2$, $1 \le j \le 4$, $1 \le k \le 3\}$.
El cardinal de $V3$ es $| V3 |$ = 2 × 4 × 3 = 24.

Los grados de contradicción de atributos unidimensionales son:

$(a1, a2) = 1$;
$(w1, w2) = 13$, $c(w1, w3) = 23$, $c(w1, w4) = 1$;
$(h1, h2) = 0.5$, $c(h1, h3) = 1$.

Los valores de los atributos dominantes son: $a1$, $w1$ y $h1$ respectivamente para cada atributo unidimensional correspondiente. Para ello se utilizan los difusos t_{norm} = a ∧F b = ab, y difuso t_{conorm} = a ∨F b = a + b - ab.

5.4.1 Intersección y unión de conjuntos borrosos de valor único plitogénico tridimensional

Sea

$$x_A = \left\{ \begin{matrix} d_A(x, a_i, w_j, h_k), \\ for\ all\ 1 \le i \le 2, 1 \le j \le 4, 1 \le k \le 3 \end{matrix} \right\} \tag{40}$$

y

$$x_B = \left\{ \begin{matrix} d_B(x, a_i, w_j, h_k), \\ for\ all\ 1 \le i \le 2, 1 \le j \le 4, 1 \le k \le 3 \end{matrix} \right\}. \tag{41}$$

entonces:

$$x_A(a_i, w_j, h_k) \vee_P x_B(a_i, w_j, h_k) = \left\{ \begin{array}{l} (1-c(a_D, a_i)) \cdot [d_A(x, a_D) \vee_F d_B(x, a_i)] \\ +c(a_D, a_i) \cdot [d_A(x, a_D) \wedge_F d_B(x, a_i)], 1 \le i \le 2; \\ (1-c(w_D, w_j)) \cdot [d_A(x, w_D) \vee_F d_B(x, w_j)] \\ +c(w_D, w_j) \cdot [d_A(x, w_D) \wedge_F d_B(x, w_j)], 1 \le j \le 4; \\ (1-c(h_D, h_k)) \cdot [d_A(x, h_D) \vee_F d_B(x, h_k)] \\ +c(h_D, h_k) \cdot [d_A(x, h_D) \wedge_F d_B(x, h_k)], 1 \le k \le 3. \end{array} \right\} \tag{42}$$

$$x_A(a_i, w_j, h_k) \wedge_P x_B(a_i, w_j, h_k) = \left\{ \begin{array}{l} (1-c(a_D, a_i)) \cdot [d_A(x, a_D) \wedge_F d_B(x, a_i)] \\ +c(a_D, a_i) \cdot [d_A(x, a_D) \vee_F d_B(x, a_i)], 1 \le i \le 2; \\ (1-c(w_D, w_j)) \cdot [d_A(x, w_D) \wedge_F d_B(x, w_j)] \\ +c(w_D, w_j) \cdot [d_A(x, w_D) \vee_F d_B(x, w_j)], 1 \le j \le 4; \\ (1-c(h_D, h_k)) \cdot [d_A(x, h_D) \wedge_F d_B(x, h_k)] \\ +c(h_D, h_k) \cdot [d_A(x, h_D) \vee_F d_B(x, h_k)], 1 \le k \le 3. \end{array} \right\} \tag{43}$$

Se tiene

, $dA(w2) = 0.6$,

y

$$xB(dB(a1) = 0.4, dB(w2) = 0.1, dB(h3) = 0.7)$$

Tomando solo un valor de 3 atributos: ($a1$, $w2$, $h3$), para los otros 23 valores de 3 atributos lo que analógicamente será:
Para $X_A \wedge p\, X_B$ se realiza el cálculo por separado, para cada atributo unidimensional, resultando:

$$[1 - c(a_D, a_1)] \cdot \left[0.8 \wedge_F 0.4\right] + c(a_D, a_1) \cdot \left[0.8 \vee_F 0.4\right] = (1 - 0) \cdot [0.8(0.4)] + 0 \cdot \left[0.8 \vee_F 0.4\right] = 0.32;$$

$$\left[1 - c[w_D, w_2] \cdot \left[0.6 \wedge_F 0.1\right] + c(w_D, w_2) \cdot \left[0.6 \vee_F 0.1\right]\right] = \left(1 - \frac{1}{3}\right)[0.6(0.1)] + \frac{1}{3}[0.6 + 0.1 - 0.6(0.1)]$$

$$= \frac{2}{3}[0.06] + \frac{1}{3}[0.64] = \frac{0.76}{3} \approx 0.25;$$

$$[1 - c(h_D, h_3)] \cdot \left[0.5 \wedge_F 0.7\right] + c(h_D, h_3) \cdot \left[0.5 \vee_F 0.7\right] = [1 - 1] \cdot [0.5(0.7)] + 1 \cdot [0.5 + 0.7 - 0.5(0.7)]$$
$$= 0 \cdot [0.35] + 0.85 = 0.85.$$

de dónde $xA \wedge p\, xB(a1, w2, h3) \approx$ (0.32, 0.25, 0.85).

Para $xA \vee p\, xB$ hacemos de manera similar

$$[1 - c(a_D, a_1)] \cdot \left[0.8 \vee_F 0.4\right] + c(a_D, a_1) \cdot \left[0.8 \wedge_F 0.4\right] = (1 - 0) \cdot [0.8 + 0.4 - 0.8(0.4)] + 0 \cdot [0.8(0.4)]$$
$$= 1 \cdot [0.88] + 0 = 0.88;$$
$$\left[1 - c[w_D, w_2] \cdot \left[0.6 \vee_F 0.1\right] + c(w_D, w_2) \cdot \left[0.6 \wedge_F 0.1\right]\right] = \left(1 - \frac{1}{3}\right)[0.6 + 0.1 - 0.6(0.1)] + \frac{1}{3}[0.6(0.1)]$$
$$= \frac{2}{3}[0.64] + \frac{1}{3}[0.06] = \frac{1.34}{3} \approx 0.44;$$
$$[1 - c(h_D, h_3)] \cdot \left[0.5 \vee_F 0.7\right] + c(h_D, h_3) \cdot \left[0.5 \wedge_F 0.7\right] = [1 - 1] \cdot [0.5 + 0.7 - 0.5(0.7)] + 1 \cdot [0.5(0.7)]$$
$$= 0 + 0.35 = 0.35.$$

De donde $xA \vee p\, xB\ (a1, w2, h3) \approx$ (0.88, 0.44, 0.35).
Para $\neg pxA\ (a1, w2, h3) = (dA\ (a2$, $dA\ (w3)$ = 0.6, $dA\ (h1)$ = 0.5), ya que el opuesto de $a1$ es $a2$, el opuesto de $w2$ es $w3$, y el opuesto de $h3$ es $h1$.

5.5 Ejemplo de multiatributo (de 5 valores de atributo) Complemento de conjunto difuso plitogénico y conjunto de valor-atributo refinado

Los valores de 5 atributos del complemento difuso plitogénico (negación) se define como:

$$x\begin{pmatrix} \begin{matrix} 0 \\ \textit{pequeña}, \\ 0.8 \end{matrix} & \begin{matrix} 0.50 \\ \textit{medio}. \\ 0.3 \end{matrix} & \begin{matrix} 0.86 \\ \textit{más grande}, \\ 0.4 \end{matrix} & \begin{matrix} 1 \\ \textit{muy grande} \\ 0.2 \end{matrix} \end{pmatrix}$$

es

$$\neg_p x\begin{pmatrix} \begin{matrix} 1-1 \\ \text{anti - muy grande}, \\ 0.2 \end{matrix} & \begin{matrix} 1-0.86 \\ \text{anti - más grande}, \\ 0.4 \end{matrix} & \begin{matrix} 1-0.75 \\ \text{anti - grande}, \\ 0.3 \end{matrix} & \begin{matrix} 1-0.50 \\ \text{anti - medio}, \\ 0.1 \end{matrix} & \begin{matrix} 1-0 \\ \text{anti - pequeño} \\ 0.8 \end{matrix} \end{pmatrix}$$

$$= \neg_p x\begin{pmatrix} \begin{matrix} 0 \\ \text{pequeño}, \\ 0.2 \end{matrix} & \begin{matrix} 0.14 \\ \text{anti - grande}, \\ 0.4 \end{matrix} & \begin{matrix} 0.25 \\ \text{anti - grande}, \\ 0.3 \end{matrix} & \begin{matrix} 0.50 \\ \text{medio}, \\ 0.1 \end{matrix} & \begin{matrix} 1 \\ \text{muy grande} \\ 0.8 \end{matrix} \end{pmatrix}$$

$$= \neg_p x\begin{pmatrix} \begin{matrix} 0 \\ \text{pequeño}, \\ 0.2 \end{matrix} & \begin{matrix} 0.14 \\ \text{arriba pequeño}, \\ 0.4 \end{matrix} & \begin{matrix} 0.25 \\ \text{por debajo del medio}, \\ 0.3 \end{matrix} & \begin{matrix} 0.50 \\ \text{medio}, \\ 0.1 \end{matrix} & \begin{matrix} 1 \\ \text{muy grande} \\ 0.8 \end{matrix} \end{pmatrix}.$$

Por lo tanto, el conjunto de valor de atributo original

V = {pequeño, mediano, grande, más grande, muy grande}
ha sido parcialmente refinado en:
RefinedV = {small, por encima de small, por debajo de medium, medium, very big},

donde por encima de pequeño, por debajo de medio ε [pequeño, medio].

5.6 Aplicación de un conjunto de valores únicos plitogénicos de atributos múltiples

Sea un universo de discurso y $\mathcal{U} \subset$ un conjunto plitogénico.
En un conjunto plitogénico P, cada elemento (objeto) $x \in P$ se caracteriza por $m \geq 1$ atributos $\alpha 1$, $\alpha 2$, … ,αm y cada atributo αi, $1 \leq i \leq m$, tiene valores de $ri \geq 1$:

$V_i = \{v_{i1}, v_{i2}, \ldots, v_{ir_i}\}$.

Por lo tanto, el elemento x se caracteriza por $r = r_1 x\, r_2 x \ldots x r_m$ que son valores del atributo.
Por ejemplo, si los atributos son "color" y "altura", y sus valores (requeridos por la aplicación, los expertos
querer hacer) son:
$Color$ = {*verde, amarillo, rojo*}
y
Altura = {alto, medio},
entonces el objeto $x \in P$ se caracteriza por el producto cartesiano.

entonces el objeto $x \in P$ se caracteriza por el producto cartesiano.
$Color \times Altura$
$(verde, alto), (verde, medium), (amarillo, alto), (amarillo, medio), (rojo, alto), (rojo, medio)$

Consideremos que el valor dominante (es decir, el más importante o de referencia) del atributo "color" es "verde", y el atributo "altura" es "alto".
Los valores difusos de la contradicción del valor del atributo son:

c (verde, verde) = 0,
c (verde, amarillo) = 1/3,
c (verde, rojo) = 2/3,
c (alto, alto) = 0,
c (alto, medio) = 1/2.

Supongamos que tenemos dos expertos A y B. Más adelante, consideramos (borrosos, intuicionistas difusos o neutrosóficos) los grados de pertenencia de cada valor de atributo al conjunto P con respecto a los criterios de los expertos.

Consideramos el número de valor único en grados difusos, para simplificar el ejemplo.

Sea un valor de atributo único y su grado de contradicción con respecto al valor de atributo único dominante vD sea $c\,(vD, vi) \stackrel{\text{def}}{=} ci$.

Sea (x, vi) el grado de mantenimiento del valor de atributo vi del elemento x con respecto al conjunto A. Y de manera similar para $dB\,(x, vi)$. Luego, recordamos los operadores de agregación plitogénicos con respecto a este valor de atributo vi que se empleará:

5.6.1 Intersección de conjuntos borrosos de valores únicos de valores de un atributo

$$dA(x,vi) \wedge pdB(x,vi) = (1-ci)\cdot[dA(x,vi) \wedge FdB(x,vi)] + ci\cdot[dA(x,vi) \vee FdB(x,vi)] \quad (44)$$

5.6.2 Unión de conjuntos difusos de valor único plitogénico de valor de un atributo

$$dA(x,vi) \vee pdB(x,vi) = (1-ci)\cdot[dA(x,vi) \vee FdB(x,vi)] + ci\cdot[dA(x,vi) \wedge FdB(x,vi) \quad (45)$$

5.6.3 Complemento de conjunto borroso de valor único plitogénico de un valor de atributo (Negación)

$$\neg pvi = anti(vi) = (1-ci)\cdot vi \quad (46)$$
$$\neg pdA(x,(1-ci)vi) = dA(x,vi) \quad (47)$$

5.7 Conjunto Fuzzy de valores simples para establecer grados de mantenimiento

Según el Experto A: dA: {verde, amarillo, $r$ojo; alto, medio} → [0,1].

Se tiene:

dA (verde) = 0.6,
dA (amarillo) = 0.2,
dA (rojo) = 0.7;
dA (alto) = 0.8,
dA (medio) = 0.5.

Resumimos de la siguiente manera:

Según el experto A:

Grados Contradicción	0	1/3	2/3	0	1/2
Valores de los atributos	verde	amarillo	rojo	alto	medio
Grados difusos	0.6	0.2	0.7	0.8	0.5

Tabla 3.

Según el experto B:

Grados de Contradicción	0	1/3	2/3	0	1/2
Valores de los atributos	verde	amarillo	rojo	alto	medio
Grados difusos	0.7	0.4	0.6	0.6	0.4

Tabla 4.

El elemento
x {(verde, alto), (verde, medio), (amarillo, alto), (amarillo, medio), (rojo, alto), (rojo, medio)} $\in P$
con respecto a los dos expertos mencionados anteriormente se representa como:

xA *{(0.6,0.8), (0.6,0.5), (0.2,0.8), (0.2,0.5), (0.7,0.8), (0.7,0.5)}*
y xB *{(0.7,0.6), (0.7,0.4), (0.4,0.6), (0.4,0.4), (0.6,0.6), (0.6,0.4)}.*

Para encontrar la representación óptima de x, necesitamos interceptar xA y xB, cada uno con seis duplas. Actualmente, esta intercepción se realiza por separado de acuerdo a las duplas correspondientes.

En este ejemplo, tomamos el conjunto difuso: $a \wedge Fb = ab$ y el t_{conorm} $difuso$: $a \vee Fb = a + b$ ab

5.7.1 Aplicación de la intersección de conjuntos borrosos de valores únicos y plitogénicos de valor de atributo único

Para calcular $xA \wedge pxB$.

0 0 0 0 $\{grados\ de\ contradicciones\}$
$\wedge\, p(0.7{,}0.6) = (0.6 \wedge p0 \qquad \wedge\, p0$

Donde; sobre cada dupla se escribe el grado de contradicciones de cada valor de atributo con respecto a su valor de atributo dominante correspondiente. Como son cero, $\wedge p$ que coincidió con $\wedge F$.

{el primer valor por debajo de 0 ½ y nuevamente 0 ½ representa los grados de contradicción}

$$\begin{pmatrix} 0 & \frac{1}{2} \\ 0.6, & 0.5 \end{pmatrix} \wedge_p \begin{pmatrix} 0 & \frac{1}{2} \\ 0.7, & 0.4 \end{pmatrix} = (0.6 \wedge_p 0.7, 0.5 \wedge_p 0.4) = (0.6 \cdot 0.7, (1-0.5)\cdot[0.5 \wedge_F 0.4] + 0.5 \cdot [0.5 \vee_F 0.4])$$
$$= (0.42, 0.5[0.2] + 0.5[0.5 + 0.4 - 0.5\cdot 0.4]) = (0.42, 0.45).$$

$$\begin{pmatrix} \frac{1}{3}, & 0 \\ 0.2 & 0.8 \end{pmatrix} \wedge_p \begin{pmatrix} \frac{1}{3}, & 0 \\ 0.4 & 0.6 \end{pmatrix} = (0.2 \wedge_p 0.4, 0.8 \wedge_p 0.6) = \left(\left\{1-\frac{1}{3}\right\}\cdot[0.2 \wedge_F 0.4] + \left\{\frac{1}{3}\right\}\cdot[0.2 \vee_F 0.4], 0.8\cdot 0.6\right)$$
$$\approx (0.23, 0.48).$$

$$\begin{pmatrix} \frac{1}{3}, & \frac{1}{2} \\ 0.2 & 0.5 \end{pmatrix} \wedge_p \begin{pmatrix} \frac{1}{3}, & \frac{1}{2} \\ 0.4 & 0.4 \end{pmatrix} = (0.2 \wedge_p 0.4, 0.5 \wedge_p 0.4)$$

(fueron computados arriba)
$\approx (0.23, 0.45).$

$$\begin{pmatrix} \frac{2}{3}, & 0 \\ 0.7 & 0.8 \end{pmatrix} \wedge_p \begin{pmatrix} \frac{2}{3}, & 0 \\ 0.6 & 0.6 \end{pmatrix} = (0.7 \wedge_p 0.8, 0.8 \wedge_p 0.6) = \left(\left\{1-\frac{2}{3}\right\}\cdot[0.7 \wedge_F 0.6] + \left\{\frac{2}{3}\right\}\cdot[0.7 \vee_F 0.6], 0.48\right)$$

(el segundo componente fue computado arriba)

$$= \left(\frac{1}{3}[0.7\cdot 0.6] + \frac{2}{3}[0.7 + 0.6 - 0.7\cdot 0.6], 0.48\right) \approx (0.73, 0.48).$$

Y la última dupla**:**

$$\begin{pmatrix} \frac{2}{3}, & \frac{1}{2} \\ 0.7 & 0.5 \end{pmatrix} \wedge_p \begin{pmatrix} \frac{2}{3}, & \frac{1}{2} \\ 0.6 & 0.4 \end{pmatrix} = (0.7 \wedge_p 0.6, 0.5 \wedge_p 0.4)$$
$$\approx (0.73, 0.45)$$

(Fueron computados arriba).

Finalmente:

$$x_A \wedge_p x_B \approx \left\{ \begin{matrix} (0.42, 0.48), (0.42, 0.45), (0.23, 0.48), (0.23, 0.45), \\ (0.73, 0.48), (0.73, 0.45) \end{matrix} \right\},$$

Después de la intersección de las opiniones de los expertos A $\wedge P_B$, el resultado se resume como se muestra en la tabla 5.

Grados de contradicción	0	1/3	2/3	0	1/2
Valores de los atributos	verde	amarillo	rojo	alto	medio
Grados difusos de Experto A para x	0.6	0.2	0.7	0.8	0.5
Grados difusos de Experto B para x	0.7	0.4	0.6	0.6	0.4
Grados difusos de $xA \wedge p\ xB$	0.42	0.23	0.73	0.48	0.45

Grados difusos de $xA \vee pxB$	0.88	0.37	0.57	0.92	0.45

Tabla 5.

5.7.2 Aplicación de la unión de conjuntos difusos plitogénicos de un solo atributo

Calculamos por separado para cada valor de atributo único:

$$d_A^F(x, green) \vee_p d_B^F(x, green) = 0.6 \vee_p 0.7 = (1-0)\cdot[0.6 \vee_F 0.7] + 0\cdot[0.6 \wedge_F 0.7]$$
$$= 1\cdot[0.6 + 0.7 - 0.6\cdot 0.7] + 0 = 0.88.$$

$$d_A^F(x, yellow) \vee_p d_B^F(x, yellow) = 0.2 \vee_p 0.4 = \left(1-\frac{1}{3}\right)\cdot[0.2 \vee_F 0.4] + \frac{1}{3}\cdot[0.2 \wedge_F 0.4]$$
$$= \frac{2}{3}\cdot(0.2 + 0.4 - 0.2\cdot 0.4) + \frac{1}{3}(0.2\cdot 0.4) \approx 0.37.$$

$$d_A^F(x, red) \vee_p d_B^F(x, red) = 0.7 \vee_p 0.6 = \left\{1-\frac{2}{3}\right\}\cdot[0.7 \vee_F 0.6] + \frac{2}{3}\cdot[0.7 \wedge_F 0.6]$$
$$= \frac{1}{3}\cdot(0.7 + 0.6 - 0.7\cdot 0.6) + \frac{2}{3}(0.7\cdot 0.6) \approx 0.57.$$

$$d_A^F(x, tall) \vee_p d_B^F(x, tall) = 0.8 \vee_p 0.6 = (1-0)\cdot(0.8 + 0.6 - 0.8\cdot 0.6) + 0\cdot(0.8\cdot 0.6) = 0.92.$$

$$d_A^F(x, medium) \vee_p d_B^F(x, medium) = 0.5 \vee_p 0.4 = \frac{1}{2}(0.5 + 0.4 - 0.5\cdot 0.4) + \frac{1}{2}\cdot(0.5\cdot 0.4) = 0.45.$$

5.7.3 Propiedades de los operadores de conjuntos de valor único plitogénico en aplicaciones

1) Cuando el grado de contradicción del valor del atributo con respecto al valor del atributo dominante correspondiente es 0 (cero), uno simplemente usa la intersección difusa:

$$dA \wedge pB(x, verde) = dA(x, verde) \wedge F\ dB(x, verde) = 0.6 \cdot 0.7 = 0.42,$$

y

$$dA \wedge pB(x, tall) = dA(x, tall) \wedge F\ dB(x, tall) = 0.8 \cdot 0.6 = 0.48.$$

2) Si el grado de contradicción del valor de atributo con respecto al valor de atributo dominante correspondiente es diferente de 0 y de 1, entonces el resultado de la intersección plitogénica se encuentra entre los resultados de fuzzy t_{norm} y fuzzy t_{conorm}.

Ejemplo:

$$dA(x, amarillo) \wedge F\ dB(x, amarillo) = 0.2 \wedge F\ 0.4 = 0.2 \cdot 0.4 = 0.08\ (t_{norm}),$$
$$dA(x, amarillo) \vee F\ dB(x, amarillo) = 0.2 \vee F\ 0.4 = 0.2 + 0.4 - 0.2 \cdot 0.4$$
$$= 0.52\ (t_{conorm});$$

mientras;

$$dA(x, amarillo) \wedge p\ dB(x, amarillo) = 0.23 \in [0.08, 0.52]$$

{0.23 ≈ 0.2266… = (2/3) × 0.08 + (1/3) × 0.52, es decir, poseen una combinación lineal de t_{norm} y t_{conorm}}.

Similar:

$$dA(x, rojo) \wedge pdB(x, rojo) = 0.7 \wedge F\ 0.6 = 0.7 \cdot 0.6 = 0.42\ (t_{norm}),$$
$$dA(x, rojo) \vee\ pdB(x, rojo) = 0.7 \vee F\ 0.6 = 0.7 + 0.6 - 0.7 \cdot 0.6 = 0.88\ (t_{conorm});$$

mientras;

$$dA(x, red) \wedge pdB(x, red) = 0.57 \in [0.42, 0.88]$$

{Combinación lineal de t_{norm} y t_{conorm}}.

And

$$dA(x, medio) \wedge FdB(x, medio) = 0.5 \wedge F0.4 = 0.5 \cdot 0.4 = 0.20,$$
$$dA(x, medio) \vee FdB(x, medio) = 0.5 \vee F0.4 = 0.5 + 0.4 - 0.5 \cdot 0.4 = 0.70,$$

mientras;

$$dA(x, medio) \wedge pdB(x, medio) = 0.45,$$

El valor obtenido se encuentra justo en el medio (porque el grado de contradicción "medio" es 1/2) del intervalo [0.20 , 0.70].

Conclusión e investigación futura

Como generalización de la dialéctica y la neutrosofía, la plitogenia encontrará más uso en la mezcla de diversas ideas filosóficas, ideológicas, religiosas, políticas y sociales. Después de la extensión del conjunto difuso, el conjunto difuso intuicionista y el conjunto neutrosófico al conjunto plitogénico; la extensión de la lógica clásica, la lógica difusa, la lógica difusa intuicionista y la lógica neutrosófica a la lógica plitogénica; y la extensión de la probabilidad clásica, la probabilidad imprecisa y la probabilidad no trosófica a la probabilidad plitogénica [12]: deben seguir más aplicaciones del conjunto / lógica / probabilística / estadística plitogénica en varios campos. Las clases de operadores de implicación plitogénica y sus correspondientes conjuntos de reglas plitogénicas se construirán en esta dirección.

Además, la exploración de combinaciones no lineales de tnorm y tconorm, o de otras normas y conormas, en la construcción de operadores plitogénicos de conjuntos, lógicos y de agregación probabilística más sofisticados, para un mejor modelado de las aplicaciones de la vida real.

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Plithogenic Fuzzy Whole Hypersoft Set, Construction of Operators and their Application in Frequency Matrix Multi Attribute Decision Making Technique

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Abstract: In this paper, initially a matrix representation of Plithogenic Hypersoft Set (PHSS) is introduced and then with the help of this matrix some local operators for Plithogenic Fuzzy Hypersoft set (PFHSS) are developed. These local operators are used to generalize PFHSS to Plithogenic Fuzzy Whole Hypersoft set (PFWHSS). The generalized PFWHSS set is hybridization of Fuzzy Hypersoft set (which represent multiattributes and their subattributes as a combined whole membership i.e. case of having an exterior view of the event) and the Plithogenic Fuzzy Hypersoft set (in which multi attributes and their subattributes are represented with individual memberships case of having interior view). Thus, the speciality of PFWHSS is its presentation of an exterior and interior view of a situation simultaneously. Later, the PFWHSS is employed in development of multi attributes decision making scheme named as Frequency Matrix Multi Attributes Decision making scheme (FMMADMS). This innovative technique is not only simpler than any of the former MADM techniques, but also has a unique capability of dealing mathematically a variety of human mind psychologies at every level that are working in different environments (fuzzy, intuitionistic, neutrosophic, plithogenic). Besides, FMMADMS also provides the percentage authenticity of the final ranking which in itself is a new idea providing a transparent and unbiased ranking. Moreover, the new introduced idea of frequency matrix handles the ranking ties in the best possible way and has an ability to provide the authenticity comparative analysis of previously developed schemes. Lastly, application of this FMMADMS is described as a numerical example for a case of ranking and selecting the best alternative.

Keywords: Plithogenic Hypersoft set, Exterior view, Plithogenic Whole Hypersoft set, Interior view, Frequency Matrix, Multi Attribute Decision making Scheme, Percentage authenticity.

1. Introduction

The theory of uncertainty in mathematics was initially introduced by Zadeh [26] in 1965 named as fuzzy set theory (FST). A fuzzy set is a set where each element of the universe of discourse X has some degree of belongingness in unit closed interval $[0,1]$ in given set A, where A is subset of universal set X with respect to some attribute say M with imposing condition that the sum of membership and non membership is one unlike crisp set where element from the universe either belong to given set A or does not belong to A. In Fuzzy set theory, elements of set are expressed with one quantity i.e. degree of membership. To represent this degree of membership a notation $\mu_A(x) \in [0,1] \forall x \in X$ was used and to represent the degree of nonmembership a notation $\upsilon_A(x) \in [0,1] \forall x \in X$ was used. The members of fuzzy set are represented by using one quantity i.e. the degree of membership $\{x:\mu_A(x)\}$.

Due to the condition $\mu_A(x) + \upsilon_A(x) = 1 \forall x \in X$ imposed by Zadeh the degree of non membership $\upsilon_A(x)$ to A will be $1 - \mu_A(x)$, where $\upsilon_A(x) \in [0,1] \forall x \in X$.

Further generalization of fuzzy set was made by Atanassov [1] in 1986 which are known as Intuitionistic fuzzy set (IFS). In IFS the natural concept of hesitation in human mind was used in assigning a degree of membership in unit closed interval such that sum of degree of membership, degree of non membership and degree of hesitation should be one. The degree of hesitation or indeterminacy was represented by the notation"ι"now the improved condition is $\mu_A(x) + \upsilon_A(x) + \iota_A(x) = 1 \forall x \in X$. The members of IFS are represented by using two quantities $\mu_A(x)$and $\upsilon_A(x)\{x:(\mu_A(x), \upsilon_A(x))\}$. Later, IFS were further generalized by Smarandache [15]. He considered membership $\mu_A(x)$, nonmembership $\upsilon_A(x)$ and indeterminacy $\iota_A(x)$ as independent quantities or functions in the unit cube, representing three axis of the unit cube in non standard unit interval $]0^-, 1^+[$. Smarandache represented the elements of Neutrosophic set (NS) by using three independent quantities and introduced "Neutrosophy"[16-17] as a new branch of philosophy which studies the origin nature,by considering neutrality and opposite and their interactions with different ideational spectra. Mathematically, a NS is represented by $\{x:(\mu_A(x), \upsilon_A(x), \iota_A(x))\}$ with condition $0 \le \mu_A(x) + \upsilon_A(x) + \iota_A(x) \le 3$. The new defined approach of dealing with human mind consciousness in form of Neutrosophic Set is utilized in MCDM and MADM techniques ([2-7],[9], ,[12], [18] ,[25]).

Furthermore, Smarandache[13] has generalized the Soft set to Hypersoft set by transforming the function F of one attribute into a multi attribute function where $a_1, a_2, \dots, a_n$ for $n \ge 1$ be n distinct attributes, whose corresponding attributes values are respectively the set $A_1, A_2, \dots, A_n$ with $A_i \cap A_j = \varphi$ for $i \ne j$ and $i, j \in \{1,2,\dots,n\}$ and assigning a combine membership $\mu_{A_1 \times A_2 \times \dots \times A_N}(x)$, non membership $\upsilon_{A_1 \times A_2 \times \dots \times A_N}(x)$ and Indeterminacy $\iota_{A_1 \times A_2 \times \dots \times A_N}(x)$ $\forall x \in X$ with condition and introduced a hybrids of Crisp/ Fuzzy/ Intuitonistic Fuzzy and Neutrosophic Hypersoft set and then generalized Hypersoft set to Plithogenic Hypersoft set (PHSS) by assigning a separate degree of membership, nonmembership and indeterminacy

$\mu_{A_i}(x), \upsilon_{A_i}(x), \iota_{A_i}(x)$ respectively to each attribute value A_i. Thus a Plithogenic Set, as the generalization of Crisp, Fuzzy, Intuitionistic Fuzzy, Picture Fuzzy and Neutrosophic Set was introduced by F.Smarandache in 2017 [14].

In this paper, we have firstly presented to our reader an entirely new concept of looking at a Plithogenic Hypersoft set in a form of a matrix. This matrix representation is further utilized in the emergence of some new local operators such as disjunction, conjunction and averaging operators for Plithogenic Fuzzy Hyper soft sets (PFHSS). In the second stage, we have utilized these local operators to the define a new idea of a Plithogenic Fuzzy Whole Hypersoft Set (PFWHSS). This new PWHSS not only present a deep insight into a Plithogenic decision making environment but also a broader outlook of a situation which clearly is more generalized and precise approach of modelling human mind capabilities. Moreover, the new PWHSS are employed in development of a multi attribute decision making scheme named as Frequency Matrix Multi Attributes Decision Making Scheme (FMMADMS).

In most MADM techniques, ranking is achieved by generating a comparison of alternatives with ideal and non ideal solution ([8], [19], [20]) etc. Mostly, comparison are made on the basis of distance, inclusion, and similarity measurements etc. These scheme when studied analytically are actually representing fuzzy behavior of human mind. The ideal solution represents membership and the non ideal solution represents nonmembership behavior of fuzzy environment. Besides, the selection of any input information taken from any background (fuzzy, intuitionistic fuzzy, neutrosophic or any other) the use of ideal and non ideal solution in modelling of different MADM schemes actually drives the entire scheme to a fuzzy environment. So the ranking is based on optimist and pessimist human behavior. In this new FMMADMS, the ranking includes the three behavior of human mind, optimist behavior (represented mathematically by using Max operator employed in construction of local operators which are involved in ranking procedure), pessimist behavior (represented mathematically by using Min operator used in designing local disjunction also used in ranking process) and the neutral behavior (represented mathematically by using averaging operator). The final decision is made by combining the three human mind behaviors in a matrix called Frequency Matrix which gives the ultimate ranking of alternatives. The major advantage of the new scheme is its capacity of indulging many human mind behaviors by introducing variety of operators between Min, Max and averaging operators. Thus, generalizing the scheme from neutrosophic to plithogenic modelling environment [14]. Also, in our scheme at its final stage a ratios authenticity of the ranking operators is provided to guarantee the rightfulness of the final decision.

With a brief introduction of our work in Section 1, we have organized the rest of the paper in following sections: Section 2, is a collection of all the necessary preliminaries required for understanding of this work while in Section 3, we have presented the new concept of representing a

Plithogenic Fuzzy Hyper Soft Set in form of a matrix. Moreover, have introduced some new local operators on this set and constructed a whole membership using these local operators. This whole membership over a PFHSS set gives a birds eye view of the entire situation thus driving to new idea of Plithogenic Fuzzy Whole Hyper Soft Set. Furthermore, the newly defined PFWHSS is used in constructing a new MADM technique called Frequency Matrix Multi Attributes Decision making scheme (FMMADMS). In Section 4, a numerical example is presented to elaborate the new scheme while in Section 5 we give the final Conclusion of this work along with some open problems related to this field.

2. Preliminaries

In this section, we will present some basic definitions of soft set, fuzzy soft set, hypersoft set, crisp hypersoft set, fuzzy hypersoft set, plithogenic hypersoft set, plithogenic crisp hypersoft set and plithogenic fuzzy hypersoft set which are useful in development of our literature.

Definition 2.1 [21] **(Soft Set)**

Let U be the initial universe of discourse, and E is a set of parameters or attributes with respect to U. Let $P(U)$ denote the power set of U, and $A \subseteq E$ is a set of attributes. Then pair (F, A), where $F: A \longrightarrow P(U)$ is called **Soft Set** over U. In other words, a soft set (F, A) over U is parameterized family of subset of U. For $e \in A$, $F(e)$ may be considered as set of e elements or e approximate elements

$(F, A) = \{(F(e) \in P(U): e \in E, F(e) = \varphi \text{if} e \notin A\}$.

Definition 2.2 [24] **(Soft subset)**

For two soft set (F, A) and (G, B) over a universe U, we say that (F, A) is a soft subset of (G, B) if

(i) $A \subseteq B$, and

(ii) $\forall e \in A, F(e) \subseteq G(e)$

The set of all soft set over U will be denoted by $S(U)$.

Definition 2.3 [26] **(Fuzzy set)**

Let U be the universe . A fuzzy set X over U is a set defined by a membership function μ_X representing a mapping $\mu_X : U \longrightarrow [0,1]$

The vale of $\mu_X(x)$ for the fuzzy set X is called the membership value of the grade of membership of $x \in U$.The membership value represent the degree of belonging to fuzzy set X. Then a fuzzy set X on U can be represented as follows.

$X = \{(\mu_X(x)/x): x \in U, \mu_X(x) \in [0,1]\}$.

Definition 2.4 [9] **(Fuzzy soft set)**

Let U be the initial universe of discourse, $F(U)$ be all fuzzy set over U. E be the set of all parameters or attributes with respect to U and $A \subseteq E$ is a set of attributes. A fuzzy soft set Γ_A on the universe U is defined by the set of ordered pairs as follows, $\Gamma_A = \{x, \gamma_A(x): x \in E, \gamma_A(x) \in F(U)\}$
where $\gamma_A: E \longrightarrow F(U)$ such that $\gamma_A(x) = \varphi$ if $x \notin A$

$$\gamma_A(x) = \{\mu_{\gamma_A(x)}(u)/u: u \in U, \mu_{\gamma_A(x)}(u) \in [0,1]\}.$$

Definition 2.5 [13] **(Hypersoft set)**

Let U be the initial universe of discourse $P(U)$ the power set of U and $a_1, a_2, \ldots, a_n$ for $n \geq 1$ be n distinct attributes, whose corresponding attributes values are respectively the set $A_1, A_2, \ldots, A_n$ with $A_i \cap A_j = \varphi$ for $i \neq j$ and $i, j \in \{1, 2, \ldots, n\}$.
Then the pair $(F, A_1 \times A_2 \times \ldots \times A_n)$ where, $F: A_1 \times A_2 \times \ldots \times A_n \rightarrow P(U)$.
is called a Hypersoft set over U

Definition 2.6 [13] (Crisp Universe of Discourse)

A Universe of Discourse U_C is called Crisp if $\forall x \in U_C$, $x \in 100\%$ to U_C or membership of x $T(x)$ with respect to A in M is 1 denoted as $x(1)$.

Definition 2.7 [13] (Fuzzy Universe of Discourse)

A Universe of Discourse U_F is called Fuzzy if $\forall x \in U_C$ x partially belongs to U_F or membership of x $T(x) \subseteq [0,1]$ where $T(x)$ may be subset , an interval, a hesitant set, a single value set, etc. denoted as $x(T_x)$.

Definition 2.8 [13] (Plithogenic Universe of Discourse)

A Universe of Discourse U_p over a set V of attributes values, where $V = \{v_1, v_2, \ldots, v_n\}$, $n \geq 1$, is called Plithogenic if $\forall x \in U_p$ x belongs to U_p in the degree $d_x^0(v_i)$ with respect to the attribute value v_i, for all $i \in \{1, 2, \ldots, n\}$. Since the degree of membership may be Crisp, Fuzzy, Intuitionistic Fuzzy, or Neutrosophic, the Plithogenic Universe of discourse may can be Crisp, fuzzy, Intuitionistic fuzzy, or Neutrosophic.

Definition 2.9 [13] (Crisp Hypersoft set)

Let U_C be the initial universe of discourse $P(U_C)$ the power set of U.

Let $a_1, a_2, \ldots, a_n$ for $n \geq 1$ be n distinct attributes, whose corresponding attributes values are respectively the set $A_1, A_2, \ldots, A_n$ with $A_i \cap A_j = \varphi$ for $i \neq j$ and $i, j \in \{1, 2, \ldots, n\}$. Then the pair $(F_C, A_1 \times A_2 \times \ldots \times A_n)$ where $F_C: A_1 \times A_2 \times \ldots \times A_n \rightarrow P(U_C)$, is called Crisp Hypersoft set over U_C.

Definition 2.10 [13] (Fuzzy Hypersoft set)

Let U_F be the initial universe of discourse $P(U_F)$ the power set of U_F.

$a_1, a_2, \ldots, a_n$ for $n \geq 1$ be n distinct attributes whose corresponding attributes values are respectively the set $A_1, A_2, \ldots, A_n$ with $A_i \cap A_j = \varphi$ for $i \neq j$ and $i, j \in \{1, 2, \ldots, n\}$. Then the pair

$(F_F, A_1 \times A_2 \times \ldots \times A_n)$ where $F_F: A_1 \times A_2 \times \ldots \times A_n \rightarrow P(U_F)$, is called Fuzzy Hypersoft set over U_F.

Definition 2.11 [13] (Plithogenic Hypersoft set)

Now instead of assigning combined membership $\mu_{A_1 \times A_2 \times \ldots \times A_N}(x)$ $\forall x \in U_c/\ U_F/U_{IF}/U_N$ for Hyper Soft set if each attribute A_j is assigned an individual membership $\mu_{A_j}(x)$, non membership $\upsilon_{A_j}(x)$ and Indeterminacy $\iota_{A_j}(x)$ $j = 1,2,\ldots,n$ in Crisp/Fuzzy/Intuitionistic Fuzzy and Neutrosophic Hypersoft set then these generalized Crisp/Fuzzy/Intuitionistic Fuzzy and neutrosophic Hypersoft set are called Plithogenic Crisp/ Fuzzy/Intuitionistic Fuzzy and Neutrosophic Hypersoft set.

3. Plithogenic Fuzzy Hyper Soft set, their representation in a Matrix form and generalization to Plithogenic Fuzzy Whole Hypersoft set

In this section, we define initially Crisp Whole Hypersoft set, Fuzzy Whole Hypersoft set, Intuitionistic Fuzzy Whole Hypersoft set, Neutrosophic Whole Hypersoft set.

Definition 3.1 (Plithogenic Crisp/ Fuzzy/ Intuitionistic Fuzzy and neutrosophic Whole Hypersoft set)

Let $U_{pl}(X)$ be the plithogenic universe of discourse and $F: A_1^k \times A_2^k \times \ldots \times A_N^k \rightarrow P(U_{pl})$ where $k = 1,2,3,\ldots,M$ represent Numeric values of attributes A_j for each j, k: and A^krepresent sub attributes of the given attributes, can attain different numeric values. Now if in Plithogenic Crisp/Fuzzy/Intuitionistic Fuzzy/Neutrosophic Hypersoft set all attributes A_j^k have both an individual membership $\mu_{A_j}(x)$, non membership $\upsilon_{A_j}(x)$and indeterminacy $\iota_{A_j}(x)$ where $j = 1,2,\ldots,N$ and a whole combined membership $\mu_{A_1 \times A_2 \times \ldots \times A_N}(x)$ denoted by $\Omega(x)$, non membership $\upsilon_{A_1 \times A_2 \times \ldots \times A_N}(x)$ denoted by $\Phi(x)$ and Indeterminacy $\iota_{A_1 \times A_2 \times \ldots \times A_N}(x)$ denoted by $\Psi(x)$ then these generalized Plithogenic Crisp/Fuzzy/Intuitionistic Fuzzy /Neutrosophic Hypersoft set are called *Plithogenic Crisp/ Fuzzy/Intuitionistic Fuzzy / Neutrosophic Whole Hypersoft set.*

The Plithogenic Whole Hypersoft set is hybridization of Plithogenic Hypersoft set and Hypersoft set. If we are representing our set only with fuzzy memberships say $\mu_{A_j}(x)$ for individual attributes and Fuzzy whole memberships $\mu_{A_1 \times A_2 \times \ldots \times A_N}(x)$ say $\Omega(x)$ for combined attributes then the set under consideration are *Plithogenic Fuzzy Whole Hypersoft set.* Initially the literature is developed only for Plithogenic Fuzzy Hypersoft set and Plithogenic Fuzzy Whole Hypersoft set.

3.1 Plithogenic Fuzzy Whole Hypersoft set and Frequency Matrix Multi Attributes Decision Making Scheme (FMMADMS)

For convenience in dealing with plithogenic hypersoft set the data or informations i.e. memberships will be represented in the form of matrix denoted by C_{ij}^{α} for some combination of numeric values of attributes where α represent the given combination of attributes, i represent rows of matrix with respect to objects x_i, j represents columns of matrix with respect to numeric values of attributes A_j. These matrices will be helpful in construction of local Disjunction, Conjunction and Averaging operators. Furthermore, local constructed operators are used for the development of whole memberships denoted by Ω and then these memberships are used to generalize the Plithogenic Hypersoft Set to Plithogenic Whole Hypersoft Set and in development of a multi attributes decision making scheme named as Frequency Matrix Multi Attributes Decision Making Scheme (FMMADMS). The speciality of these local operators is that they deal within the matrix constructed by using informations or one can say within one combination of attributes which gives interior view of the event. In this section, we shall be dealing with PFHSS only. Later the idea can be generalized to other environments (intuitionistic, neutrosophic, plithogenic) etc. Let us now formally introduce the steps of FMMADMS. In this scheme, the first four steps are related to the matrix construction of PFHSS and their local operators while in the next three steps PFWHSS are developed using these operators and are utilized in defining the local ranking. Moreover, a final ranking is obtained using a frequency matrix. Also, a percentage authenticity is calculated to guarantee the transparency of the process.

Step 1. Decision of universe: Consider universe of discourse $U_{pl} = \{x_i\}$ $i = 1,2,3, \dots, M$ and then $T = \{x_i\} \subset U_{pl}$ where i could be chosen between 1 to M. Here x_i represent the objects under consideration.

Step 2. Defining attributes and mapping: Let $A_1^k, A_2^k, A_3^k, \dots, A_N^k$ be the attributes. Choose some attributes represented by A_j, $j = 1,2,3, \dots, N$ and then assign k some numeric values can be presented by A_j^k where k and j can take values $1,2,3, \dots, N$. The data of the numerical values is based on the decision maker's opinion by using the linguistic scales [[10],[11],[23]]. Define $F: A_1^k \times A_2^k \times A_3^k \times \dots \times A_N^k \to P(U_{pl})$, where F is a mappings from combination of attributes to some subset of power set of U_{pl}.

Step 3. Matrix representation: Write the data or information (Memberships) in the form of a matrix. Let C_{ij}^{α}; $j = 1,2,3, \dots, N$ and $i = 1,2,3, \dots, M$: be the matrix and let α represent the given combination of attributes A_j^k for some j and k.

$$C_{ij}^{\alpha} = \begin{array}{c c} & \begin{array}{ccccc} A_1^k & A_2^k & & & A_N^k \end{array} \\ \begin{array}{c} x_1 \\ x_2 \\ . \\ . \\ . \\ x_M \end{array} & \begin{bmatrix} \mu_{A_1}(x_1) & \mu_{A_2}(x_1) & . & . & . & \mu_{A_N}(x_1) \\ \mu_{A_1}(x_2) & \mu_{A_1}(x_2) & . & . & . & \mu_{A_N}(x_2) \\ . & . & . & . & . & . \\ . & . & . & . & . & . \\ . & . & . & . & . & . \\ \mu_{A_1}(x_M) & \mu_{A_2}(x_M) & . & . & . & \mu_{A_N}(x_M) \end{bmatrix} \end{array} \tag{3.1}$$

Step 4. Construction of Local operators and Global whole memberships: Now by using individual memberships $\mu_j(x_i)$, for $x_i \in T$ and varying j from 1 to N one can develop a combined whole membership, say $\Omega^t(x_i)$ to x_i in T with respect to given combination of attributes by using different operators on rows of matrices of representation C_{ij}^{α} for Construction of local operators. These operators can be represented by taking different integer values of t i.e. $t = 1$ represent local disjunction operator , $t = 2$ represent local conjunction operator and $t = 3$ represent local averaging operator. The following local operators are constructed. Here, we define some local operators for Plithogenic Fuzzy Hypersoft Set. It is observed that the same operators are applicable for Plithogenic Crisp Hypersoft set but as the results are trivial so we will consider here only the case of Plithogenic Fuzzy Hypersoft set

Local Disjunction Operator for Plithogenic Fuzzy Hypersoft Set :

$$\vee_{loc}(F) = \cup\left(C_{ij}^{\alpha}\right) = Max_j\left(C_{ij}^{\alpha}\right) = Max_j\left(\mu_j(x_i)\right) \tag{3.2}$$

(Choose maximum membership from i_{th} row)
Here $\vee_{loc}, \cup$ are representations for local disjunctions operators for F, $\mu_j(x_i)$ is the membership for j_{th} attribute with respect to i_{th} object.

Local Conjunction Operators for Plithogenic Fuzzy Hypersoft Set :

$$\wedge_{loc}(F) = \cap\left(C_{ij}^{\alpha}\right) = Min_j\left(C_{ij}^{\alpha}\right) = Min_j\left(\mu_j(x_i)\right) \tag{3.3}$$

(Choose minimum membership from i_{th} row amongst j columns) and the result will be a column matrix representation three entities. Here $\wedge_{loc}$ are representations for local conjunctions operators for F, $\mu_j(x_i)$ is the membership for j_{th} attribute with respect to i_{th} object.

Local Averaging Operator for Plithogenic Fuzzy Hypersoft Set :

$$\Gamma(F) = \Gamma\left(C_{ij}^{\alpha}\right) = \Sigma_{j=1}^{N} \frac{\mu_j(x_i)}{N} \tag{3.4}$$

Here Γ represent averaging operator for mapping F for α combination of attributes applied on the given matrix of representation C_{ij}^{α} by taking average of memberships for i_{th} row.

Local Compliment for Plithogenic Fuzzy Hypersoft Set :

$$C_{loc}(F) = c\left(C_{ij}^{\alpha}\right) = \begin{Bmatrix} Max_j\left(1-\mu_j(x_i)\right) \\ Min_j\left(1-\mu_j(x_i)\right) \\ \sum_{j=1}^{N} \frac{\left(1-\mu_j(x_i)\right)}{N} \end{Bmatrix} \tag{3.5}$$

Here C_{loc} represent the local compliment for F mapping forα combination of attributes applied over matrix of representation C_{ij}^{α} by taking compliment of memberships for i_{th} row and then choosing either maximum or minimum or taking average of them. By applying Local disjunction, Local conjunction and Local averaging operators (3.2, 3.3, 3.4) to (3.1) one can develop a combined whole membership, denoted by $\Omega_{\alpha}^{t}(x_i)$.

Note: Here we have not used the compliment operator to develop the whole membership. But the choice is open for reader to work with this operator or any other operator of their choice.

Here $\Omega_{\alpha}^{t}(x_i)$ is representation for whole combined membership for i_{th} object withe respect to α combination of attributes in subset of $P(U_{pi})$

$$\Omega_{\alpha}^{1}(x_i) = \cup_j\ (C_{ij}^{\alpha}) = Max_j\left(\mu_j(x_i)\right) \tag{3.6}$$

$\Omega_{\alpha}^{1}(x_i)$ represent the combined (whole) membership for i_{th} object obtained by using disjunction operator ($t = 1$) developed in (3.2).

$$\Omega_{\alpha}^{2}(x_i) = \cap_j\ (C_{ij}^{\alpha}) = Min_j\left(\mu_j(x_i)\right) \tag{3.7}$$

$\Omega_{\alpha}^{2}(x_i)$ represent the combined (whole) membership for i_{th} object obtained by using conjunction operator ($t = 2$) developed in (3.3).

$$\Omega_{\alpha}^{3}(x_i) = \Gamma(C_{ij}^{\alpha}) = \sum_{j=1}^{N} \frac{(\mu_j(x_i))}{N} \tag{3.8}$$

$\Omega_{\alpha}^{3}(x_i)$ represent the combined (whole) membership for i_{th} object obtained by using averaging operator Γ ($t = 3$) developed in (3.4).

We shall use $\Omega_\alpha^1(x_i)$, $\Omega_\alpha^2(x_i)$ and $\Omega_\alpha^3(x_i)$ for three different whole memberships of Plithogenic Fuzzy Whole hypersoft set.

Step 5. Matrix representation of Plithogenic Fuzzy Whole Hypersoft set and initial ranking:

Write the data or information (local individual membership and global whole memberships) in the form of an other matrix denoted by $C_{ij}^{\alpha t}$; $j = 1,2,3,\ldots,N$ and $i = 1,2,3,\ldots,M$ and α represents the given combination of attributes and $t = 1,2,3$ represent the local operators used to get the whole combined memberships where $C_{ij}^{\alpha t}$ is the matrix representation for Plithogenic Fuzzy whole Hypersoft set.

$$C_{ij}^{\alpha t} = \begin{array}{c} \\ x_1 \\ x_2 \\ . \\ . \\ . \\ x_M \end{array} \begin{array}{c} \begin{array}{ccccc} A_1^k & A_2^k & & A_N^k \ldots\ldots & \Omega_\alpha^t \end{array} \\ \begin{bmatrix} \mu_{A_1}(x_1) & \mu_{A_2}(x_1) & . & . & . & \mu_{A_N}(x_1) & \Omega_\alpha^t(x_1) \\ \mu_{A_1}(x_2) & \mu_{A_1}(x_2) & . & . & . & \mu_{A_N}(x_2) & \Omega_\alpha^t(x_2) \\ . & . & . & . & . & . & . \\ . & . & . & . & . & . & . \\ . & . & . & . & . & . & . \\ \mu_{A_1}(x_M) & \mu_{A_2}(x_M) & . & . & . & \mu_{A_N}(x_M) & \Omega_\alpha^t(x_M) \end{bmatrix} \end{array}$$

Where in A_j^k, k takes values with respect to given some α combination and in Ω_α^t and in $C_{ij}^{\alpha t}$while i represent rows of matrix and j represent its columns and $C_{ij}^{\alpha t}$; Plithogenic Fuzzy Whole Hypersoft Matrix (PFWHSM). For $t = 1,2,3$ we shall get three PFWHSM's.

In particular, for a fixed t and for some α combination of attributes $A_j, j = 1,2,3,\ldots,N$ we will get an initial ranking for alternatives $T = \{x_i\}$ under consideration in $C_{ij}^{\alpha t}$ from the last column of $C_{ij}^{\alpha t}$ which is the column of whole membership value Ω_α^t . The first position is assigned to an alternative having highest whole membership $\Omega_\alpha^t(x_i)$ [which is the highest numeric value in last column] and the second position to one having second largest membership and so on. If a tie occurs for the position of alternatives in this initial ranking, it will be removed in final ranking. In this step, by varying $t = 1,2,3$ we shall obtain the three types of initial ranking of our alternatives based on three operators see (3.6,3.7 and 3.8). All of these ranking will be utilized in next stage to get the final ranking of alternatives.

It is worth mentioning here the fact that these initial rankings presents three human mind behaviors for three different choices of operators. To be more specific for $t = 1$ the use of Max operator will provide the choice of optimist behavior of human mind. Similarly for $t = 2$ which represent the use of Min operator one can represent the pessimist behavior of human mind. Furthermore, the choice of $t = 3$ i.e., the use of averaging operator will represent the neutral behavior of human mind. Finally in the next step by using the frequency matrix we will combine the three human mind behaviors to provide the final results of the ranking procedure.

Step 6. Construction of frequency matrix F_{qp} for final ranking:

Finally, we have constructed the frequency matrix of positions F_{qp} from initial ranking where $q = 1,2,\dots M$ is used to represent rows (alternatives) of frequency matrix F_{qp} and $p = 1,2,\dots,M$ is used to represent columns (positions attained by these alternatives) of frequency matrix F_{qp}.

$$F_{qp} = \begin{array}{c} \\ x_1 \\ x_2 \\ \cdot \\ \cdot \\ \cdot \\ x_M \end{array} \begin{array}{c} \begin{array}{cccccc} p_1 & p_2 & \cdot & \cdot & \cdot & p_M \end{array} \\ \begin{bmatrix} f_{11} & f_{12} & \cdot & \cdot & \cdot & f_{1M} \\ f_{21} & f_{22} & \cdot & \cdot & \cdot & f_{2M} \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ f_{M1} & f_{M2} & \cdot & \cdot & \cdot & f_{MM} \end{bmatrix} \end{array}$$

The final frequency matrix F_{qp} of alternatives and positions is a square matrix of order $M \times M$ i.e. number of ordering positions will be equal to the number of alternatives, The selection of first position to any alternative will be made by looking into the first column corresponding to the position 1 i.e. p_1. The alternative having the largest frequency value in this column will be assigned first position. Once first position is decided, the entire row corresponding to this alternative and the first column will be excluded from the process of selection. Next, we shall look into the second column to select the candidate having the largest frequency value to be assigned the second position of ordering. Once done he shall be excluded from the process by excluding his row and the second column from the process. This procedure of selection will continue until all the positions are assigned to the rightful alternative.

In final frequency matrix if two alternatives have the same frequency of position 1 which is a very rare case, then we check their frequency of position 2, the one having higher frequency value in position 2 will be assigned the first position. After this selection the particular alternative and the position 1 will be excluded from selection procedure. Then other competitor will be assigned the second position. In this way all the ties can be fairly handled in this process.

Step 7. Percentage measure of authenticity of ranking: Finally the percentage measure of authenticity can be obtained by using the ratios formula:

Percentage authenticity of p_{th} position for q_{th} alternative $= \frac{\max_q (f_{qp})}{\sum_q f_{qp}} \times 100$, where f_{qp} is the obtained frequency of the p_{th} position for q_{th} alternative and $\sum_q f_{qp}$ is the total frequency of p_{th} position.

4. Numerical Example

Step 1. Decision of universe: let $U = \{x_1, x_2, x_3, x_4, x_5\}$ set of five members of Engineering department and $T = \{x_2, x_3, x_5\} \subset U$ set of three members who have applied for the post of Assistant professor.

Step 2. Defining Attributes and mapping:

Let the attributes be $A_j^k; j = 1,2,3,4$ and k may have any value from 1 to 3

$A_1^k =$ Subject skill area with numeric values, $k = 1,2,3$

$A_1^1 =$ Mathematics, $A_1^2 =$ Physics, $A_1^3 =$ Computer science

$A_2^k =$ Qualification with numeric values, $k = 1,2$

$A_2^1 =$ Higher qualification like Ph.D. or equivalent, $A_2^2 =$ lower qualification like MS or equivalent $A_3^k =$ Teaching experience with numeric values, $k = 1,2$

$A_3^1 =$ Three years or less, $A_3^2 =$ More than three years

$A_4^k =$ Age , with numeric values $k = 1,2,3$

$A_4^1 =$ Age is less than thirty years, $A_4^2 =$ Age is between thirty to forty years $A_4^3 =$Age is greater than forty years

We need to select faculty members.

Let the Function F be given by,

$F: A_1^k \times A_2^k \times A_3^k \times A_4^k \to P(U)$ for $k = 1,1,1,2$ respectively.

We are interested in ranking of these three candidates for the Engineering department with the following criteria.

1. Subject skill area: Mathematics: $k = 1$
2. Qualification: Higher qualification like Ph.D or Equivalent $k = 1$
3. Teaching experience: Three years or less $k = 1$
4. Age: Age required is between thirty to forty years $k = 2$

$F(A_1^1, A_2^1, A_3^1, A_4^2) = \{x_2, x_3, x_5\}$ let we name $A_1^1, A_2^1, A_3^1, A_4^2$ combination as α

With respect to T =$\{x_2, x_3, x_5\}$ have memberships in PFHSS. Consider the memberships of x_2, x_3, x_5 as $\mu_j(x_i)$ for $i = 2,3,5$ and $j = 1$ to 4 in T with respect to α combination of attributes.

$F(\alpha) = F(A_1^1, A_2^1, A_3^1, A_4^2) = \{x_2(0.3,0.6,0.4,0.5), x_3(0.4,0.5,0.3,0.1), x_5(0.6,0.3,0.3,0.7)\}$

Step 3. Matrix representation: Let C_{ij}^{α} represented in 3.1 is the matrix of representation for the combination of attributes α,in PFHSS. Here rows are representing x_2, x_3, x_5 and columns are representing $A_1^k, A_2^k, A_3^k, A_4^k$.

$$C_{ij}^{\alpha} = \begin{array}{c} \\ x_2 \\ x_3 \\ x_5 \end{array} \begin{array}{c} \begin{array}{cccc} A_1^1 & A_2^1 & A_3^1 & A_4^2 \end{array} \\ \begin{bmatrix} 0.3 & 0.6 & 0.4 & 0.5 \\ 0.4 & 0.5 & 0.3 & 0.1 \\ 0.6 & 0.3 & 0.3 & 0.7 \end{bmatrix} \end{array}$$

Step 4. Construction of Local operators and Global whole memberships for PFHSS: By using individual memberships $\mu_j^{\alpha}(x_i)$, for $x_i \in T \subset U$ now with respect to α combination of attributes by fixing $i = 2,3$ and 5 and varying j from 1 to 4 in 3.6, 3.7 and 3.8 one can assign a combined (whole) membership, $\Omega_{\alpha}^{t}(x_i)$ to $x_i \in U$ in T with respect to α combination of attributes by using operators developed in 3.6, 3.7 and 3.8 on rows of matrix of representation C_{ij}^{α}. Using (3.1)

$$\Omega_{\alpha}^{1}(x_i) = \cup\,(\mu_j^{\alpha}(x_i)) = Max_j\left(\mu_j^{\alpha}(x_i)\right)$$

$$\Omega_{\alpha}^{2}(x_i) = \cap\left(\mu_j^{\alpha}(x_i)\right) = Min_j\left(\mu_j^{\alpha}(x_i)\right)$$

$$\Omega_{\alpha}^{3}(x_i) = \Gamma\left(\mu_j^{\alpha}(x_i)\right) = \Sigma_{j=1}^{N}\frac{(\mu_j^{\alpha}(x_i))}{N}$$

This membership is used in Generalization of PFHSS to Plithogenic Fuzzy Whole Hyper Soft set.

$\Omega_{\alpha}^{1}(x_2) = \cup\left(\mu_j^{\alpha}(x_2)\right) = Max_j\left(\mu_j^{\alpha}(x_2)\right) = 0.6$ for $i = 2$ and varying j from 1 to 4

$\Omega_{\alpha}^{1}(x_3) = \cup\left(\mu_j^{\alpha}(x_3)\right) = Max_j\left(\mu_j^{\alpha}(x_3)\right) = 0.5$ for $i = 3$ and varying j from 1 to 4

$\Omega_{\alpha}^{1}(x_5) = \cup\left(\mu_j^{\alpha}(x_5)\right) = Max_j\left(\mu_j^{\alpha}(x_5)\right) = 0.7$ for $i = 5$ and varying j from 1 to 4

$\Omega_{\alpha}^{2}(x_2) = \cap\left(\mu_j^{\alpha}(x_2)\right) = Min_j\left(\mu_j^{\alpha}(x_2)\right) = 0.3$ for $i = 2$ and varying j from 1 to 4

$\Omega_{\alpha}^{2}(x_3) = \cap\left(\mu_j^{\alpha}(x_3)\right) = Min_j\left(\mu_j^{\alpha}(x_3)\right) = 0.1$ for $i = 3$ and varying j from 1 to 4

$\Omega_{\alpha}^{2}(x_5) = \cap\left(\mu_j^{\alpha}(x_5)\right) = Min_j\left(\mu_j^{\alpha}(x_5)\right) = 0.3$ for $i = 5$ and varying j from 1 to 4

$$\Omega_{\alpha}^{3}(x_2) = \Gamma\left(\mu_j^{\alpha}(x_2)\right) = \sum_{j=1}^{4} \frac{\left(\mu_j^{\alpha}(x_2)\right)}{4} = 0.45 \text{ for } i = 2, N = 4 \text{and varying } j \text{ from } 1 \text{ to } 4$$

$$\Omega_{\alpha}^{3}(x_3) = \Gamma\left(\mu_j^{\alpha}(x_3)\right) = \sum_{j=1}^{4} \frac{\left(\mu_j^{\alpha}(x_3)\right)}{4} = 0.325 \text{ for } i = 2, N = 4 \text{ and varying } j \text{ from } 1 \text{ to } 4$$

$$\Omega_{\alpha}^{3}(x_5) = \Gamma\left(\mu_j^{\alpha}(x_5)\right) = \sum_{j=1}^{4} \frac{\left(\mu_j^{\alpha}(x_5)\right)}{4} = 0.45 \text{ for } i = 2, N = 4 \text{ and varying } j \text{ from } 1 \text{ to } 4$$

Step 5 Matrix representation of Plithogenic Fuzzy Whole Hypersoft set and initial ranking:

$$C_{ij}^{\alpha 1} = \begin{array}{c} \\ x_2 \\ x_3 \\ x_5 \end{array} \begin{array}{c} \begin{array}{ccccc} A_1^1 & A_2^1 & A_3^1 & A_4^2 & \Omega_{\alpha}^1 \end{array} \\ \begin{bmatrix} 0.3 & 0.6 & 0.4 & 0.5 & 0.6 \\ 0.4 & 0.5 & 0.3 & 0.1 & 0.5 \\ 0.6 & 0.3 & 0.3 & 0.7 & 0.7 \end{bmatrix} \end{array}$$

For choosing the best one will select the largest value from last column i.e. $x_5 = 0.7$ The initial ranking for $t = 1$, is Position 1: for x_5,Position 2: for x_2 and Position 3: for x_3.

$$C_{ij}^{\alpha 2} = \begin{array}{c} \\ x_2 \\ x_3 \\ x_5 \end{array} \begin{array}{c} \begin{array}{ccccc} A_1^1 & A_2^1 & A_3^1 & A_4^2 & \Omega_{\alpha}^1 \end{array} \\ \begin{bmatrix} 0.3 & 0.6 & 0.4 & 0.5 & 0.3 \\ 0.4 & 0.5 & 0.3 & 0.1 & 0.1 \\ 0.6 & 0.3 & 0.3 & 0.7 & 0.3 \end{bmatrix} \end{array}$$

For choosing the best one will select the largest value from last column i.e. $x_2 = x_5 = 0.3$. The initial ranking for $t = 2$, is Position 1: could be assigned to both the candidates x_5 and x_2. This tie will be removed in final step of ranking.

$$C_{ij}^{\alpha 3} = \begin{array}{c} \\ x_2 \\ x_3 \\ x_5 \end{array} \begin{array}{c} \begin{array}{ccccc} A_1^1 & A_2^1 & A_3^1 & A_4^2 & \Omega_{\alpha}^1 \end{array} \\ \begin{bmatrix} 0.3 & 0.6 & 0.4 & 0.5 & 0.45 \\ 0.4 & 0.5 & 0.3 & 0.1 & 0.325 \\ 0.6 & 0.3 & 0.3 & 0.7 & 0.475 \end{bmatrix} \end{array}$$

For choosing the best one will select the largest value from last column i.e. $x_5 = 0.7$ The initial ranking for $t = 3$, is Position 1: for x_5,Position 2: for x_2 and Position 3: for x_3.

Step 6. Construction of frequency matrix F_{qp} for final ranking: Next we construct a frequency matrix to get the final ranking using the data of step 5.

$$F_{qp} = \begin{array}{c} \\ x_2 \\ x_3 \\ x_5 \end{array}\begin{array}{c} \begin{array}{ccc} p_1 & p_2 & p_3 \end{array} \\ \begin{bmatrix} 1 & 2 & 0 \\ 0 & 0 & 3 \\ 3 & 0 & 0 \end{bmatrix} \end{array}$$

This frequency matrix shows the frequency of getting first position for $x_2$is 1, for $x_3$is 0 and for x_5 is 3, the frequency of getting second position for $x_2$is 2, for $x_3$is 0 and for x_5 is 3,and the frequency of getting third position for $x_2$is 3, for $x_3$is 0 and for x_5 is 0. We can see here the initial ranking for $t = 1$ is $x_5 \rhd x_2 \rhd x_3$, for $t = 2$ is $x_5 = x_2 \rhd x_3$ and ranking for $t = 3$ is $x_5 \rhd x_2 \rhd x_3$ and the final ranking from the frequency matrix F_{qp} is same i.e., $x_5 \rhd x_2 \rhd x_3$ which shows use of frequency matrix increases the authenticity of the ranking and selection of right candidate for the post.

Step 7. Percentage measure of authenticity of ranking:

Percentage authenticity of first position for $x_5 = \frac{\max_q(f_{qp})}{\Sigma_q f_{qp}} = \frac{f_{31}}{\Sigma_q f_{q1}} \times 100 = 75\%$

Percentage authenticity of second position for $x_2 = \frac{f_{12}}{\Sigma_q f_{q2}} \times 100 = 100\%$

Percentage authenticity of third position for $x_3 = \frac{f_{23}}{\Sigma_q f_{q3}} \times 100 = 100\%$

5. Conclusion

A novice idea of matrix representation of Plithogenic Fuzzy Hypersoft Set (PFHSS) is introduced along with construction of their local operators such as conjunction, disjunction and averaging operators. These local operators are utilized in defining a new concept of Plithogenic Fuzzy Whole Hyper Fuzzy Soft Set (PFWHSS). The PFWHSS deals fuzziness of the data or information as a combined vision (external view) in case of combined membership of a combination of attributes and individually (internal view) as a in case of considering individual memberships. Furthermore, an innovative yet simple MADM technique called Frequency Matrix Multi Attributes Decision Making Scheme (FMMADMS) is developed. In this technique, at first stage, we have employed three different PFWHSS to get three initial rankings of alternatives representing decisions made by three different human mind behaviors of being optimist (the case in which whole membership is obtained by using conjunction (Max) operator), pessimist (the case in which whole membership is obtained using disjunction (Min) operator) and the neutral behavior (the case in which whole membership is obtained using averaging operator). In the next stage, we have introduced a new concept of frequency matrix that combines all the three possibilities of human mind behavior to provide with a final ranking decision of alternatives. In many decision making schemes, there are possibilities of ties between ranking alternatives. The use frequency matrix in FMMADMS provides a unique way of handling these ties. It results into a final ranking free of ties. Lastly, the scheme works with a percentage measure to guarantee the authenticity and accuracy of the final ranking. This itself, is entirely a new idea to get to get an authenticity of different ranking schemes which shows that the final decision is transparent and unbiased.

Moreover, this technique is more generalized since it use PFWHSS which deals with not only attributes but also sub attributes at the same time. One of the beauty of this scheme is its simplicity as the user need not to handle with complicated long calculations based operators. Also this new technique have a flexible approach of using wide range of operators that can absorb changes according to the requirement of the provided environment. To be more specific, the selection of three operators represent a neutrosophic behavior which clearly is a special case of plithogenic attitude as mentioned in [14]. Now introducing more operators among these three neutrosophic elemental behaviors (membership, nonmembership, neutrality) we can generalize the model of this scheme in plithogenic environment which may handle more of human mind complexities.

Some of the open problems that could be addressed: This work have vast extensions by developments of some new literature on operators, their properties and applications in different environments like Crisp, Fuzzy, Intuitionistic Fuzzy and Neutrosophic etc. and development of multi attributes decision making techniques in different environments. Moreover, the matrix representation of plithogenic whole hypersoft set opens new dimensions towards development of many operators and MADM techniques.

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Extension of Soft Set to Hypersoft Set, and then to Plithogenic Hypersoft Set

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Abstact. In this paper, we generalize the soft set tothe hypersoft set by transforming the function F into a multi-attribute function. Then we introduce the hybrids of Crisp, Fuzzy, Intuitionistic Fuzzy, Neutrosophic, and Plithogenic Hypersoft Set.

1. Introduction

We generalize the soft set to the **hypersoft set** by transforming the function F into a **multi-argument function**.

Then we make the distinction between the types of Universes of Discourse: **crisp**, **fuzzy**, **intuitionistic fuzzy**, **neutrosophic**, and respectively **plithogenic**.

Similarly, we show that a hypersoft set can be **crisp**, **fuzzy**, **intuitionistic fuzzy**, **neutrosophic**, or **plithogenic**.

A detailed numerical example is presented for all types.

2. Definition of Soft Set [1]

Let $\mathcal{U}$ be a universe of discourse, $\mathcal{P}(\mathcal{U})$ the power set of $\mathcal{U}$, and A a set of attributes. Then, the pair $(F, \mathcal{U})$, where

$$F: A \longrightarrow \mathcal{P}(\mathcal{U}) \quad (1)$$

is called a **Soft Set** over $\mathcal{U}$.

3. Definition of Hypersoft Set

Let $\mathcal{U}$ be a universe of discourse, $\mathcal{P}(\mathcal{U})$ the power set of $\mathcal{U}$.

Let $a_1, a_2, \dots, a_n$, for $n \geq 1$, be n distinct attributes, whose corresponding attribute values are respectively the sets $A_1, A_2, \dots, A_n$, with $A_i \cap A_j = \emptyset$, for $i \neq j$, and $i, j \in \{1, 2, \dots, n\}$.

Then the pair $(F, A_1 \times A_2 \times \dots \times A_n)$, where:

$$F: A_1 \times A_2 \times \dots \times A_n \longrightarrow \mathcal{P}(\mathcal{U}) \quad (2)$$

is called a **Hypersoft Set** over $\mathcal{U}$.

4. Particular case

For $n = 2$, we obtain the Γ–Soft Set [2].

5. Types of Universes of Discourses

5.1. A **Universe of Discourse** $\mathcal{U}_C$ is called **Crisp** if $\forall x \in \mathcal{U}_C$, x belongs 100% to $\mathcal{U}_C$, or x's membership (T_x) with respect to $\mathcal{U}_C$ is 1. Let's denote it $x(1)$.

5.2. A **Universe of Discourse** $\mathcal{U}_F$ is called **Fuzzy** if $\forall x \in \mathcal{U}_c$, x partially belongs to $\mathcal{U}_F$, or $T_x \subseteq [0, 1]$, where T_x may be a subset, an interval, a hesitant set, a single-value, etc. Let's denote it by $x(T_x)$.

5.3. A **Universe of Discourse** $\mathcal{U}_{IF}$ is called **Intuitionistic Fuzzy** if $\forall x \in \mathcal{U}_{IF}$, x partially belongs (T_x) and partially doesn't belong (F_x) to $\mathcal{U}_{IF}$, or $T_x, F_x \subseteq [0,1]$, where T_x and F_x may be subsets, intervals, hesitant sets, single-values, etc. Let's denote it by $x(T_x, F_x)$.

5.4. A **Universe of Discourse** $\mathcal{U}_N$ is called **Neutrosophic** if $\forall x \in \mathcal{U}_N$, x partially belongs (T_x), partially its membership is indeterminate (I_x), and partially it doesn't belong (F_x) to $\mathcal{U}_N$, where $T_x, I_x, F_x \subseteq [0,1]$, may be subsets, intervals, hesitant sets, single-values, etc. Let's denote it by $x(T_x, I_x, F_x)$.

5.5. A **Universe of Discourse** $\mathcal{U}_P$ **over a set** ***V*** **of attributes' values**, where $V = \{v_1, v_2, \dots, v_n\}, n \geq 1$, is called Plithogenic, if $\forall x \in \mathcal{U}_P$, x belongs to $\mathcal{U}_P$ in the degree $d_x^0(v_i)$ with respect to the attribute value v_i, for all $i \in \{1, 2, \dots, n\}$. Since the degree of membership $d_x^0(v_i)$ may be crisp, fuzzy, intuitionistic fuzzy, or neutrosophic, the Plithogenic Universe of Discourse can be Crisp, Fuzzy, Intuitionistic Fuzzy, or respectively Neutrosophic.

Consequently, a Hypersoft Set over a Crisp / Fuzzy / Intuitionistic Fuzzy / Neutrosophic / or Plithogenic Universe of Discourse is respectively called **Crisp / Fuzzy / Intuitionistic Fuzzy / Neutrosophic /** or **Plithogenic Hypersoft Set**.

6. Numerical Example

Let $\mathcal{U} = \{x_1, x_2, x_3, x_4\}$ and a set $\mathcal{M} = \{x_1, x_3\} \subset \mathcal{U}$.

Let the attributes be: $a_1 =$ size, $a_1 =$ color, $a_1 =$ gender, $a_1 =$ nationality, and their attributes' values respectively:

Size = A_1 ={small, medium, tall},

Color = A_2 ={white, yellow, red, black},

Gender = A_3 ={male, female},

Nationality = A_4 ={American, French, Spanish, Italian, Chinese}.

Let the function be:

$$F: A_1 \times A_2 \times A_3 \times A_4 \longrightarrow \mathcal{P}(\mathcal{U}). \quad (3)$$

Let's assume:

$F(\{\text{tall}, \text{white}, \text{female}, \text{Italian}\}) = \{x_1, x_3\}$.

With respect to the set $\mathcal{M}$, one has:

6.1. Crisp Hypersoft Set

$$F(\{\text{tall}, \text{white}, \text{female}, \text{Italian}\}) = \{x_1(1), x_3(1)\}, \quad (4)$$

which means that, with respect to the attributes' values $\{\text{tall}, \text{white}, \text{female}, \text{Italian}\}$ *all together*, x_1 belongs 100% to the set $\mathcal{M}$; similarly x_3.

6.2. Fuzzy Hypersoft Set

$$F(\{\text{tall}, \text{white}, \text{female}, \text{Italian}\}) = \{x_1(0.6), x_3(0.7)\}, \quad (5)$$

which means that, with respect to the attributes' values $\{\text{tall}, \text{white}, \text{female}, \text{Italian}\}$ *all together*, x_1 belongs 60% to the set $\mathcal{M}$; similarly, x_3 belongs 70% to the set $\mathcal{M}$.

6.3. Intuitionistic Fuzzy Hypersoft Set

$$F(\{\text{tall}, \text{white}, \text{female}, \text{Italian}\}) = \{x_1(0.6, 0.1), x_3(0.7, 0.2)\}, \quad (6)$$

which means that, with respect to the attributes' values $\{\text{tall}, \text{white}, \text{female}, \text{Italian}\}$ *all together*, x_1 belongs 60% and 10% it does not belong to the set $\mathcal{M}$; similarly, x_3 belongs 70% and 20% it does not belong to the set $\mathcal{M}$.

6.4. Neutrosophic Hypersoft Set

$$F(\{\text{tall}, \text{white}, \text{female}, \text{Italian}\}) = \{x_1(0.6, 0.2, 0.1), x_3(0.7, 0.3, 0.2)\}, \quad (7)$$

which means that, with respect to the attributes' values $\{\text{tall}, \text{white}, \text{female}, \text{Italian}\}$ all together, x_1 belongs 60% and its indeterminate-belongness is 20% and it doesn't belong 10% to the set $\mathcal{M}$; similarly, x_3 belongs 70% and its indeterminate-belongness is 30% and it doesn't belong 20%.

6.5. Plithogenic Hypersoft Set

$F(\{\text{tall}, \text{white}, \text{female}, \text{Italian}\}) =$

$$\left\{\begin{array}{l} x_1\left(d^0_{x_1}(\text{tall}), d^0_{x_1}(\text{white}), d^0_{x_1}(\text{female}), d^0_{x_1}(\text{Italian})\right), \\ x_2\left(d^0_{x_2}(\text{tall}), d^0_{x_2}(\text{white}), d^0_{x_2}(\text{female}), d^0_{x_2}(\text{Italian})\right) \end{array}\right\}, \quad (8)$$

where $d^0_{x_1}(\alpha)$ means the degree of appurtenance of element x_1 to the set $\mathcal{M}$ with respect to the attribute value α; and similarly $d^0_{x_2}(\alpha)$ means the degree of appurtenance of element x_2 to the set $\mathcal{M}$ with respect to the attribute value α; where $\alpha \in \{\text{tall}, \text{white}, \text{female}, \text{Italian}\}$.

Unlike the Crisp / Fuzzy / Intuitionistic Fuzzy / Neutrosophic Hypersoft Sets [where the degree of appurtenance of an element *x* to the set $\mathcal{M}$ is *with respect to all attribute values* ***tall, white, female, Italian*** *together* (as a whole), therefore a degree of appurtenance *with respect to a set of attribute values*], the Plithogenic Hypersoft Set is a **refinement** of Crisp / Fuzzy / Intuitionistic Fuzzy / Neutrosophic Hypersoft Sets [since the degree of appurtenance of an element *x* to the set $\mathcal{M}$ is with respect to each single attribute value].

But the Plithogenic Hypersoft St is also **combined** with each of the above, since the degree of degree of appurtenance of an element *x* to the set $\mathcal{M}$ with respect to each single attribute value may be: crisp, fuzzy, intuitionistic fuzzy, or neutrosophic.

7. Classification of Plithogenic Hypersoft Sets

7.1. Plithogenic Crisp Hypersoft Set

It is a plithogenic hypersoft set, such that the degree of appurtenance of an element *x* to the set $\mathcal{M}$, with respect to each attribute value, is **crisp**:

$d^0_x(\alpha) = 0$ (nonappurtenance), or 1 (appurtenance).

In our example:

$$F(\{\text{tall}, \text{white}, \text{female}, \text{Italian}\}) = \{x_1(1, 1, 1, 1), x_3(1, 1, 1, 1)\}. \quad (9)$$

7.2. Plithogenic Fuzzy Hypersoft Set

It is a plithogenic hypersoft set, such that the degree of appurtenance of an element x to the set $\mathcal{M}$, with respect to each attribute value, is **fuzzy**:

$d_x^0(\alpha) \in \mathcal{P}([0,1])$, power set of $[0,1]$, where $d_x^0(\cdot)$ may be a subset, an interval, a hesitant set, a single-valued number, etc.

In our example, for a single-valued number:

$$F(\{\text{tall}, \text{white}, \text{female}, \text{Italian}\}) = \{x_1(0.4, 0.7, 0.6, 0.5), x_3(0.8, 0.2, 0.7, 0.7)\}. \quad (10)$$

7.3. Plithogenic Intuitionistic Fuzzy Hypersoft Set

It is a plithogenic hypersoft set, such that the degree of appurtenance of an element x to the set $\mathcal{M}$, with respect to each attribute value, is **intuitionistic fuzzy**:

$d_x^0(\alpha) \in \mathcal{P}([0,1]^2)$, power set of $[0,1]^2$, where similarly $d_x^0(\alpha)$ may be: a Cartesian product of subsets, of intervals, of hesitant sets, of single-valued numbers, etc.

In our example, for single-valued numbers:

$$F(\{\text{tall}, \text{white}, \text{female}, \text{Italian}\}) = \{x_1(0.4,0.3)(0.7,0.2)(0.6, 0.0)(0.5, 0.1), x_3(0.8,0.1)(0.2,0.5)(0.7, 0.0)(0.7, 0.4)\}. \quad (11)$$

7.4. Plithogenic Neutrosophic Hypersoft Set

It is a plithogenic hypersoft set, such that the degree of appurtenance of an element x to the set $\mathcal{M}$, with respect to each attribute value, is **neutrosophic**:

$d_x^0(\alpha) \in \mathcal{P}([0,1]^3)$, power set of $[0,1]^3$, where $d_x^0(\alpha)$ may be: a triple Cartesian product of subsets, of intervals, of hesitant sets, of single-valued numbers, etc.

In our example, for single-valued numbers:

$$F(\{\text{tall}, \text{white}, \text{female}, \text{Italian}\}) =$$
$$\{x_1[(0.4,0.1, 0.3)(0.7, 0.0, 0.2)(0.6, 0.3, 0.0)(0.5, 0.2, 0.1)],$$

$$x_3[(0.8, 0.1, 0.1)(0.2, 0.4, 0.5)(0.7, 0.1, 0.0)(0.7, 0.5, 0.4)]\}. \quad (12)$$

8. Future Research

For all types of plithogenic hypersoft sets, the aggregation operators (union, intersection, complement, inclusion, equality) have to be defined and their properties found.

Applications in various engineering, technical, medical, social science, administrative, decision making and so on, fields of knowledge of these types of plithogenic hypersoft sets should be investigated.

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A Hybrid Plithogenic Decision-Making Approach with Quality Function Deployment for Selecting Supply Chain Sustainability Metrics

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Abstract: Supply chain sustainability has become one of the most attractive decision management topics. There are many articles that have focused on this field presenting many different points of view. This research is centred on the evaluation of supply chain sustainability based on two critical dimensions. The first is the importance of evaluation metrics based on economic, environmental and social aspects, and the second is the degree of difficulty of information gathering. This paper aims to increase the accuracy of the evaluation. The proposed method is a combination of quality function deployment (QFD) with plithogenic aggregation operations. The aggregation operation is applied to aggregate: Firstly, the decision maker's opinions of requirements that are needed to evaluate the supply chain sustainability; secondly, the evaluation metrics based on the requirements; and lastly, the evaluation of information gathering difficulty. To validate the proposed model, this study presented a real world case study of Thailand's sugar industry. The results showed the most preferred and the lowest preferred metrics in order to evaluate the sustainability of the supply chain strategy.

Keywords: supply chain sustainability metrics; plithogeny; aggregation operations; neutrosophic set; quality function deployment

1. Introduction

Supply chain sustainability has been one of the most attractive and dynamic research topics in the domain of supply chain management for a long time. The influence of manufacturing activities to global warming and the consumption of natural resources assisted the researchers in considering the importance of the supply chain operation's sustainability [1]. As a result of increasing competition, globalization, technological growth and huge customer expectations, the sustainable supply chain is a significant goal to each supply chain in every field. The supply chain sustainability can be described as the capability of operating the business with the long term goal of preserving economic, environment and societal welfare [2]. A more general definition of sustainable supply chain could be the management of supply chain activities in order to improve the profitability by taking into consideration the environmental impacts and social aspects. Therefore, supply chain sustainability guarantees success and achievements of the whole supply chain management in the long term. Under the uncertainty component, supply chain sustainability became a more important goal for companies. This explains why measuring supply chain sustainability means to identify possible strategic decisions under various situations [3].

The evaluation of supply chain sustainability is an interesting topic based on metrics in economic, environmental and social scopes. Measuring sustainability of the supply chain guides firms in the direction of risk elimination and standards/guidelines following [4]. Moreover, the advantages of evaluating supply chain (SC) sustainability are reducing costs, increasing competence, supporting competitive advantages and improving operational performance [5]. The challenges of measuring supply chain sustainability are [6]: The managerial and organizational absence of the inter-organizational metrics; the variety of the organization's goals and objectives producing different measures; and the difficulty in non-traditional data gathering that reduce the SC performance.

There are several studies in supply chain sustainability assessment including supply chain sustainability risk and assessment [7], literature reviews [8], multi-objective mathematical models for sustainable supply chain management [9] and decision making models for a sustainable supply chain [10]. Evaluating supply chain sustainability is a multi-criteria decision making (MCDM) problem, therefore the evaluation metrics may be the criteria, and the alternatives may be selected based on these sets of metrics. There are some limitations of SC sustainability studies, such as the fact that the researchers do not consider the difficulty of collecting the information for the metrics that will measure the sustainability. In addition, only a few studies use the linguistic variables to evaluate the metrics, leading to less consideration on the uncertainty or lack of information [11]. Also, there is the matter of the decision maker's priorities and contradiction degree between metrics which leads to less accuracy of results. In the comical industry, Rajeev (2019) proposed a framework to describe the evolution of a sustainable supply chain [12].

In this research, most of these limitations were processed by the proposed MCDM model that assists in metrics selection and the weighting of sustainable supply chain. The proposed model is based on a combination of plithogenic aggregation operations with quality function deployment (QFD). The details of the model have been explained in Section 3.

QFD is one of the most popular techniques to improve quality in order to meet customer expectations. This tool combines all customer needs in every aspect of the product, transforming them into technical requirements so they can meet their expectations [13]. QFD records great results in many fields, such as rating engineering characteristics [14], the design of building structures [15], service level measurements [16], industry development [17], product development [18], or supplier selection problems [19].

Plithogeny refers to the creation, development and progression of new entities from composition of contradictory or non-contradictory multiple old entities [20]. It was introduced by Smarandache in 2017 as a generalization of neutrosophy. A plithogenic set (as a generalization of crisp, fuzzy, intuitionistic fuzzy, and neutrosophic sets) is a set whose elements are characterised by the attribute values. Each attribute value has its contradiction degree values $c(v_j,v_D)$ between v_j and the dominant (most important) attribute value v_D. The contradiction degree between attributes assists the model to gain more accurate results. The plithogenic set, logic, probability and statistics that were also introduced by Smarandache in 2017, are obtained from plithogeny, and they are generalizations of neutrosophic sets, logic, probability and statistics respectively.

The rest of this papers is organized as follows: In Section 2, there is a literature review of sustainable supply chain, quality function deployment, clarification of plithogenic sets, and a recapitulation of neutrosophic sets. Section 3 presents the proposed model to evaluate the sustainable supply chain. In Section 4, a real world case is studied in order to evaluate the proposed model. Section 5 discusses the results of this case. Finally, the conclusion and suggestions for future works end Section 6.

2. Literature Review

2.1. Supply Chain Sustainability

In the supply chain management field, there are many considerations that need to be taken into account to minimize the negative influence of business production to environment and social effects. These considerations pushed for the strategic developments plans for sustainability [11]. Three dimensions of sustainability are considered in the supply chain, derived from customer and stakeholder desires, which are economic, environmental and social aspects to manage raw materials, information and finance flows [21]. Other definitions of supply chain sustainability is the integration of an organization's economic, social and environmental dimensions by coordinating the business process in order to improve the organization's performance in the long term [22]. A more focused definition could be supply chain management strategies and activities concerning social and environmental aspects, correlated to the production, distribution, design and supply of products and services [23]. The evaluation of supply chain sustainability metrics are attributes and requirements used to measure the supply chain performance considering economic, social and environmental features [24]. Table 1 summarizes some of the studies on supply chain sustainability metrics and frameworks.

Table 1. Studies about supply chain sustainability metrics.

Authors	Scope	Methodology	Metrics
Akshay Jadhav, Stuart Orr, Mohsin Malik (2018) [25]	Supply chain orientation (SCO)	Literature review analysis (SEM analysis)	Co2 emission management, community engagement, supplier codes of conduct, waste elimination, energy usage efficiency, water usage efficiency, and recycled materials practices, among others.
Elkafi Hassini, ChiragSurti, CorySearcy (2012) [4]	Developing supply chain sustainability metrics	Literature review	Percent of suppliers, Percent of contracts, Percent of purchase orders, Level of stake-holder trust by category
Yazdani, Morteza, Cengiz Kahraman, Pascale Zarate, and Sezi Cevik Onar (2019) [26]	Ranking of supply chain sustainability indicators	Multi-attribute decision making (QFD and GRA)	Quality, managing environmental systems, supply chain elasticity, business social liability, transportation service situation, and financial constancy.
Qorri, Ardian, Zlatan Mujkić, and Andrzej Kraslawski (2018) [27]	Measuring supply chain sustainability performance	Literature review	Number of contributors, products, geographical encompassing, strategic goals, methods, tools, among others
Searcy, Cory, Shane M. Dixon, and W. Patrick Neumann (2016) [28]	Analysis of performance indicators in supply chain sustainability	Literature review and report analysis	Employees number, profits, supplier estimation, trainingcost, among others
Chen, Rong-Hui, Yuanhsu Lin, and Ming-Lang Tseng [29]	Sustainable development indicators in the structure minerals industry in China	Combines fuzzy set theory, the Delphi method, discrete multi-criteria method	Solid waste, Eco-efficiency, Health and safety, Energy use, Investments, Land use and rehabilitation, among others
Haghighi, S. Motevali, S. A. Torabi, and R. Ghasemi [30]	Evaluation of Sustainable Supply Chain Networks	Data envelopment analysis technique	Time delivery, Supplier rejection rate, Amount of Pollution, Customers' satisfaction, Service quality, among others

2.2. Quality Function Deployment (QFD)

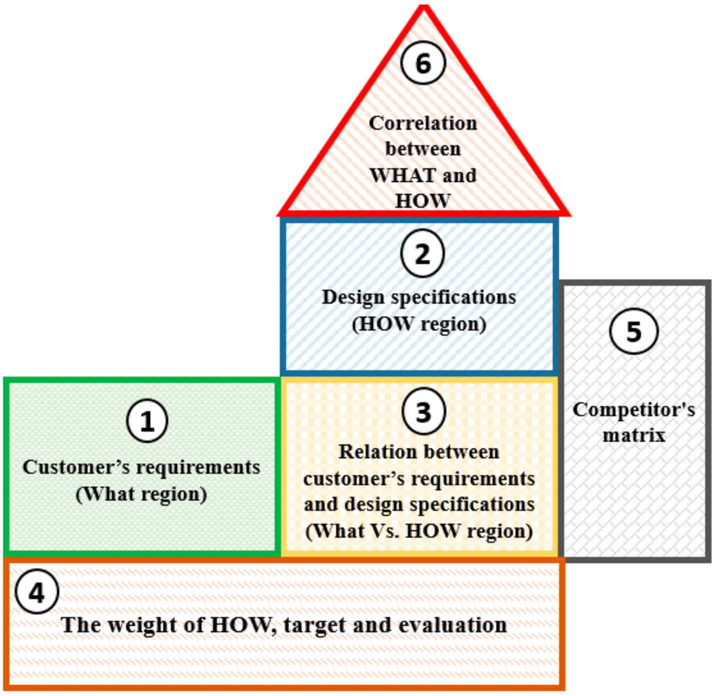
Quality function deployment (QFD) originated in Japan in the 1960s. QFD establishes quality measurement for the improvement and design, rather than just quality control in manufacturing processes [31]. The QFD method is the link that connects the customer voice to the design requirement in order to respond these expectations effectively. As illustrated in Figure 1, the components of QFD are as follows [32]:

- *Area (1)*: The customers' requirements (what region) that consists of two indicators: The customers' requirements and the importance of each of them α_i.
- *Area (2)*: The quality characteristics or design specifications (how region), composed of two parts: The design specifications and the way of development.
- *Area (3)*: The relationship between customer requirements and design specifications (what versus how region) by score $C_{ij} = \{0, 1, 3, \ldots, 9\}$.

- *Area (4)*: This area is a combination of the value of the design specification, the acceptance level of it, and the score

$$S_j = \sum(\alpha_i * C_{ij}) \quad (1)$$

- *Area (5)*: The comparison of the product and competitors, and how much it satisfies customer needs.
- *Area (6)*: The comparison between each design specification, and how much their improvement may affect each other.



There are several studies that combine QFD with other techniques to evaluate supply chain sustainability, such as: A hybrid QFD–ANP approach to design a sustainable maritime supply chain [33]; integration of QFD and grey relational analysis (GRA) in order to solve compound decision making complications [26]; QFD and MCDM techniques in supplier selection problems [34]. Dursun et al. (2018) considered the competition factor in the process of new product development using QFD [35]. A combination of best-worst method (BWM) and QFD was proposed in order to determine the relation between customer requirements and engineering characteristics in Mei et al. (2018) [36].

2.3. Plithogenic Set Characteristics

Plithogeny is the formation, construction, development, germination, and evolution of new entities from combinations of contradictory (dissimilar) or non-contradictory multiple old entities [37]. A plithogenic set (P, A, V, d, c) is a set that includes numerous elements described by a number of attributes $A = \{\alpha_1, \alpha_2, \dots, \alpha_m\}$, $m \geq 1$, which has a values $V = \{v_1, v_2, \dots, v_n\}$, for $n \geq 1$. There are two main features of each attribute's value, V. The first is the appurtenance degree function $d(x,v)$ of the element x, with respect to some given criteria [38]. The contradiction (dissimilarity) degree function $c(v,D)$ is the second one, which is realized between each attribute value and the most important (dominant) one. The contradiction degree function is mainly the key element of the plithogenic aggregation operations (intersection, union, complement, inclusion, and equality) that increase the accuracy of aggregation.

Let A be a non-empty set of uni-dimensional attributes $A = \{\alpha_1, \alpha_2, \dots, \alpha_m\}$, $m \geq 1$, and let $\alpha \in A$ be an attribute with its value spectrum the set S, where S can be defined as a finite discrete set,

$S = \{s_1, s_2, \ldots, s_l\}$, $1 \le l < \infty$, or infinitely countable set $S = \{s_1, s_2, \ldots, s_\infty\}$, or infinitely uncountable (continuum) set $S =]a,b[$, $a < b$, where $] \ldots [$ is any open, semi-open, or closed interval from the set of real numbers or from other general sets [39].

Let V be a non-empty subset of S, where V is the range of all attributes of α's values defined by the experts based on the application, $V = \{v_1, v_2, \ldots, v_n\}$ for $n \ge 1$. In the set V, there is a dominant attribute value which is determined by the experts based on preferences and the nature of the application.

Each attribute value in V has its appurtenance degree $d(x,v)$ with respect to some criteria. The degree of appurtenance may be a fuzzy, or intuitionistic fuzzy, or neutrosophic degree of appurtenance to the plithogenic set. Therefore, the appurtenance degree $d(x,v)$ of attribute value v is:

$$\forall x \in P,\ d\colon P \times V \to \mathrm{P}\,([0, 1]^z), \tag{2}$$

Therefore, $d(x, v)$ is a subset of $[0, 1]^z$, and $\mathrm{P}([0, 1]^z)$ is the power set of $[0, 1]^z$, where z = 1, 2, 3, for fuzzy, intuitionistic fuzzy, and neutrosophic degrees of appurtenance respectively [19].

Let c: $V \times V \to [0, 1]$ be the attribute value contradiction degree function $c(v_1, v_2)$, representing the dissimilarity between two attribute values v_1 and v_2, and satisfying the following axioms:

$c(v_1, v_1) = 0$, contradiction degree between the attribute values and itself is zero.

$c(v_1, v_2) = c(v_2, v_1)$, contradiction degree function can be fuzzy C_F, intuitionistic attribute value contradiction function (C_{IF}: $V \times V \to [0, 1]^2$), or a neutrosophic attribute value contradiction function (C_N: $V \times V \to [0, 1]^3$).

2.4. Neutrosophic Set

Neutrosophy is a new branch of philosophy (generalization of dialectics and Yin Yang Chinese philosophy), introduced by Florentin Smarandache in 1980, which studies the origin, nature, and scope of neutralities, as well as their interactions with different ideational spectra. Neutrosophy is the foundation of neutrosophic logic, neutrosophic probability, neutrosophic sets, and neutrosophic statistics. Neutrosophic set definitions are clearly stated in the following:

Definition 1. *[40] Let X be a universal set of objects, consisting of non-specific elements defined as x. A neutrosophic set $N \subset X$ reflects a set such that each element x from N is characterized by $T_N(x)$–the truth-membership function, $I_N(x)$–the indeterminacy-membership function, and $F_N(x)$–the falsity-membership function. $T_N(x)$, $I_N(x)$ and $F_N(x)$ are subsets of $[0^-, 1^+]$, so the three neutrosophic components are $T_N(x) \in [0^-, 1^+]$, $I_N(x) \in [0^-, 1^+]$ and $F_N(x) \in [0^-, 1^+]$. $I_N(x)$ is depicts uncertainty, indeterminate, unidentified, or error values. The sum of the three components is $0^- \le T_N(x) + I_N(x) + F_N(x) \le 3^+$.*

Definition 2. *[41] Let X be a space of points and $x \in X$. A neutrosophic set N in X is recognized by a truth-membership function $T_N(x)$, an indeterminacy-membership function $I_N(x)$ and a falsity-membership function $F_N(x)$, where $T_N(x)$, $I_N(x)$ and $F_N(x)$ are subsets of]-0, 1+[. $T_N(x)$:X →]-0, 1+[, $I_N(x)$:X →]-0, 1+[and $F_N(x)$:X →]-0,1+[. There is no restriction on the summation of membership functions. Therefore, $0- \le \sup T_N(x) + \sup I_N(x) + \sup F_N(x) \le 3+$.*

Definition 3. *[42] Let $a = \langle (a1, a2, a3); \alpha, \theta, \beta \rangle$ be a single valued triangular neutrosophic set, with truth membership $T_a(x)$, indeterminate membership $I_a(x)$, and falsity membership function $F_a(x)$ as follows:*

$$T_a(x) = \begin{cases} \alpha_a\left(\frac{x-a_1}{a_2-a_1}\right) if\ a_1 \le x \le a_2 \\ \alpha_a \quad if\ \ x = a_2 \\ o \quad otherwise \end{cases} \tag{3}$$

$$I_a(\mathrm{x}) = \begin{cases} \frac{(a_2 - x + \theta_a(x - a_1))}{(a_2 - a_1)} & if\ a_1 \le x \le a_2 \\ \theta_a \quad if\ \ x = a_2 \\ \frac{(x - a_2 + \theta_a(a_3\ - x))}{(a_3 - a_2)} & otherwise \end{cases} \tag{4}$$

$$F_a(x) = \begin{cases} \frac{(a_2 - x + \beta_a(x - a_1))}{(a_2 - a_1)} & if\ a_1 \le x \le a_2 \\ \beta_a \quad if x = a_2 \\ \frac{(x - a_2 + \beta_a(a_3\ - x))}{(a_3 - a_2)} & if\ a_2 < x \le a_3 \\ 1 \quad otherwise \end{cases} \tag{5}$$

where α_a, θ_a, $\beta_a \in [0,1]$. They represent the highest truth membership degree, the lowest indeterminacy membership degree, and the lowest falsity membership degree, respectively.

3. Proposed Model

In this paper, the authors proposed a model to evaluate the supply chain sustainability metrics based on a combination of quality function deployment and plithogenic aggregation operations. This model combines the benefits of the QFD method to link customer needs with design requirements and plithogenic aggregation operator features. The usefulness of this model derives from the plithogenic aggregation operation, because this technique ensures more accurate results and takes into consideration the degree of uncertainty, which is defective in other studies of the same problem. The steps of the proposed model have been explained in detail in this section and it is shown in Figure 2.

❖ **Step 1:** First of all, decision makers (DM) identify a series of requirements to appraise the supply chain sustainability. The most popular requirements of supply chain sustainability evaluation are summarized in Table 2 or the DM can identify other requirements based on their strategy. These requirements must reflect economic, social and environmental features which is called triple bottom line (TPL).

- The decision makers measure the importance of each requirement based on the supply chain strategy using linguistic terms.
- The linguistic scale is defined to describe the assessment of each requirement by the DM. In this model, the scale is suggested as a triangular neutrosophic scale, as shown in Table 3.

❖ **Step 2:** Using plithogenic aggregation operations, the decision maker's opinions are aggregated based on the contradiction degree of each requirement. This step increases the accuracy of results.

- Define contradiction degree c of each requirement with respect to the dominant.
- Plithogenic neutrosophic set intersection is defined as following:

$$\begin{aligned} &((a_{i1}\ , a_{i2}, a_{i3}), 1 \le i \le n)\ \wedge \mathrm{p}\ ((b_{i1}, b_{i2}, b_{i3}), 1 \le i \le n) \\ &= \left(\left(a_{i1}\ \wedge_F\ b_{i1}\ ,\ \tfrac{1}{2}(a_{i2}\ \wedge_F\ b_{i2}) + \tfrac{1}{2}(a_{i2}\ \vee_F\ b_{i2}), a_{i2} \vee_F\ b_{i3}\right)\right),\ 1 \le i \le n. \end{aligned} \tag{6}$$

where $\wedge_F$ and $\vee_F$ are fuzzy t-norm and t-conorm respectively.

- The neutrosophic number is transformed into a crisp number using the following equation:

$$\mathrm{S}\ (\mathrm{a})\ =\ \frac{1}{8}(a_1 + b_1 + c_1) \times (2 + \alpha - \theta - \beta) \tag{7}$$

❖ **Step 3:** In order to find the best requirement considering the set of criteria, the distance of each requirement is found from the best and worst solutions.

- The best (positive) ideal solution S^{+} and worst (negative) ideal solution S^{-} require definition.
- For example, in price requirements, the lowest value is desired (best ideal solution); on the other side, the maximum value is the worst ideal solution. In the opposite of that, in profit requirements, the maximum value is positive and the lowest value is negative.
- The formula of Euclidean distance is used to find the distance of each requirement to the ideal positive and ideal negative solutions, as shown in Equations (8) and (9) [50].

$$D_i^{+} = \left[\sum_{j=1}^{m}\left(V_i - V_j^{+}\right)^2\right]^{0.5} \tag{8}$$

$$D_i^{-} = \left[\sum_{j=1}^{m}\left(V_i - V_j^{-}\right)^2\right]^{0.5} \tag{9}$$

- The superior alternative has the smallest distance from the positive ideal solution S^{+} and the worst alternative has a larger distance from the negative ideal solution S^{-}.

❖ **Step 4:** The performance score of each requirement is found in order to weight each of them based on Equation (10).

$$P_i = \frac{S_i^{-}}{S_i^{+} - S_i^{-}} \tag{10}$$

- The performance score is normalized to find the weight of each requirement that satisfies two constraints which are $0 \le w_i \le 1$ and $\sum w_i = 1$.

❖ **Step 5:** The decision makers define a combination of metrics by considering the requirements selected previously in step 1 and the TPL. Some of the economic metrics are cost reduction, transaction costs, environmental costs, service level, or sales. The environmental metrics are environmental policies, recycling of waste, air pollution emission, solid waste, water consumption, and so on. Finally, the social diminution consists of working conditions, employee satisfaction, government relationships, employee training, and reputation, among others.

- The DMs define the relation between each metric and explain each requirement using linguistic terms as in Table 3.

❖ **Step 6:** Steps 2–4 are repeated on the evaluation metrics. As in Step 2, the plithogenic aggregation operation is used to combine all decision makers' judgments about defined metrics. Then, Equations (8) and (9) are used to establish the distance of every metric from the best and worst solutions. The importance of each metric is determined using the performance score as in Equation (10).

❖ **Step 7:** As proposed in Osiro, Lauro et al. 2018 [11], the limitations of other studies that do not consider the hardness of data gathering of each metric need to be addressed. In this step, the difficulty in regards to three dimensions are evaluated in relation to information accessibility, human resources and time needed for assessment, and other required resources [51].

- The difficulty of assessment metrics data collecting based on three dimensions explained by linguistic variables are evaluated.
- The assessment based on the contradiction degree to obtain accuracy of results are aggregated, and thenits crisp value is found.
- Their performance score of data collecting difficulty based on the distance of best and worst solutions are found.

❖ **Step 8:** In this final step, the goal is to categorize the set of supply chain sustainability metrics.

- The performance degree found in Step 4 (the importance of each metric) and Step 7 (the difficulty of data gathering) using Equation (11) are normalized as proposed in (Osiro, Lauro et al. 2018) [11].

$$v_n = \frac{1}{1 + e^{-\frac{v - \bar{v}}{\sigma_v}}} \tag{11}$$

where v_n is the normalized value, $v - \bar{v}$ is the difference between the value and the mean, and σ_v is the standard deviation.

- This is the result if supply chain sustainability evaluation metrics are categorized according to Figure 3 based on the importance of each metric and its difficulty of data gathering.

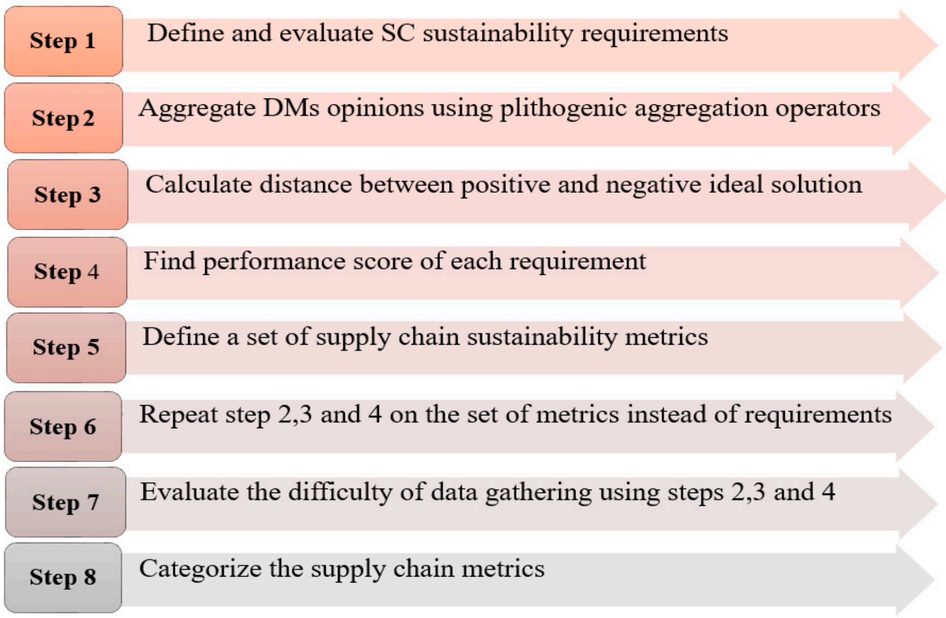


Figure 2. Steps of the proposed model.

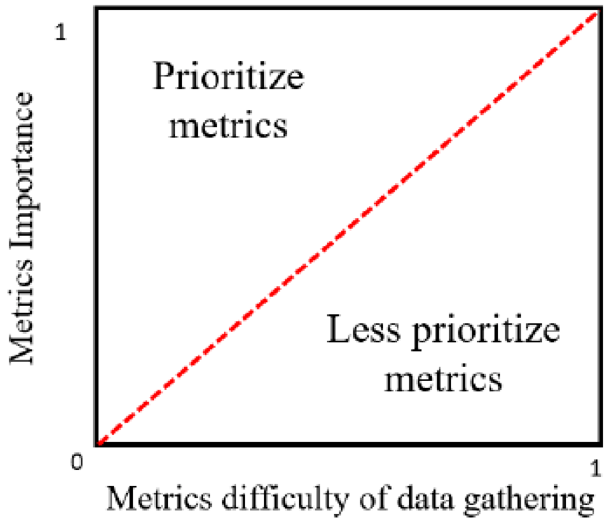


Figure 3. Two dimentions model categorization.

Table 2. Popular requirements of supply chain sustainability evaluations.

	Requirement	Author
1	Cost/profit	Govindan, Kannan, Roohollah Khodaverdi, and Ahmad Jafarian [43]
2	Product quality	Osiro, Lauro, Francisco R. Lima-Junior, and Luiz Cesar R. Carpinetti [44]
3	Environmental influences	Huang, Samuel H., and Harshal Keskar [45]
4	Stability and constancy	Kannan, Devika, et al. [46]
5	Information Technology	Katsikeas, Constantine S., Nicholas G. Paparoidamis, and Eva Katsikea [47]
6	Social aspects	Mani, V., Rajat Agrawal, and Vinay Sharma [48]
7	Delivery accuracy	Chang, Betty, Chih-Wei Chang, and Chih-Hung Wu [49]

Table 3. Linguistic scale.

Linguistic Variable	Triangular Neutrosophic Scale
Nothing (N)	((0.10, 0.30,0.35), 0.1,0.2,0.15)
Very Low (VL)	((0.15,0.25,0.10), 0.6,0.2,0.3)
Low (L)	((0.40,0.35,0.50), 0.6,0.1,0.2)
Medium (M)	(0.65,0.60,0.70),0.8,0.1,0.1)
High (H)	((0.70,0.65,0.80),0.9,0.2,0.1)
Very high (VH)	((0.90,0.85,0.90),0.7,0.2,0.2)
Absolute (A)	((0.95,0.90,0.95),0.9,0.10,0.10)

4. Real World Case Study

In this paper, the proposed model has been illustrated in an application on Thailand's sugar industry in order to measure the overall sustainability of this supply chain (Figure 4). The sugar industry in Thailand is considered one of the most important economic pillars. Thailand is the fourth largest sugar producer and second largest exporter in the world. In this application, four decision makers (DMs) were assisted by their experience in solving such cases to evaluate the sustainability of Thailand's sugar industry. They are experienced in manufacturing (DM_1), quality control (DM_2), finance and purchasing (DM_3), and environmental expert (DM_4). The main goal of this case is to evaluate Thailand's sugar industry supply chain sustainability metrics based on their significance and difficulty degree of data gathering. Initially, the four experts identified a group of seven requirements for Thailand's sugar industry supply chain sustainability evaluation. They are: Profit (R_1), costs (R_2), delivery reliability (R_3), product development (R_4), environmental aspects (R_5), product quality (R_6), health and security (R_7).

- ➢ The requirements by the DMs based on linguistic variables in Table 3 are evaluated as a triangular neutrosophic value. The evaluation is shown in Table 4.
- ➢ As explained in Step 2, the plithogenic aggregation operation is used to combine all decision makers judgments about the requirements based on the contradiction degree of each one as in Table 5.
- ➢ Using Equation (6), the aggregation results are shown in Table 6 and then their crisp value is found using Equation (7).
- ➢ In **Steps 3 and 4**, the distance of positive and negative ideal solutions are found using the Euclidean distance as in Equations (8) and (9). Then, the performance degree is measured as

mentioned in Equation (10) to find the weight vector of the seven requirements as shown in Table 7.

- ➢ The decision makers define a set of supply chain sustainability metrics with respect to economic, environmental, and social dimensions (Table 8). Then, based on linguistic variables in Table 3, the DMs evaluate them as specified in Table 9.
- ➢ Table 10 shows the metrics aggregation using a plithogenic aggregation operator according to each requirement and based on the contradiction degree in Table 11. Then, their crisp values can be found.
- ➢ As in Steps 3 and 4, using the Euclidean distance as in Equations (8) and (9), the distance of positive and negative ideal solutions are found. Then, as mentioned in Equation (10), the performance degree is measured to find the weight vector of the metrics. Their results are shown in Table 12.
- ➢ As proposed in Osiro, Lauro et al. 2018 [11], the limitations of neglecting the difficulty of data gathering of every metric is addressed. In this step, the decision makers evaluate the difficulty in regards to three dimensions: Information accessibility, human resources and time needed for assessments, and other required resources, as shown in Table 13.
- ➢ Then, the decision makers evaluations were aggregated using a plithogenic aggregation equation as shown in Table 14:
- ➢ Equations (8) and (9) were used to find the distance of every metric from the positive ideal solution and negative ideal solution. Then, Equation (10) was used to calculate the performance degree, as shown in the fourth column in Table 15.
- ➢ Finally, the performance score was normalized using Equation (11) that relates to metrics importance and difficulty of data gathering. The normalization results are shown in Table 16.
- ➢ Figure 5 shows the Thailand sugar industry supply chain sustainability metrics distribution categorized in two regions which are the prioritized metrics and less prioritized metrics based on the four decision maker's evaluation.

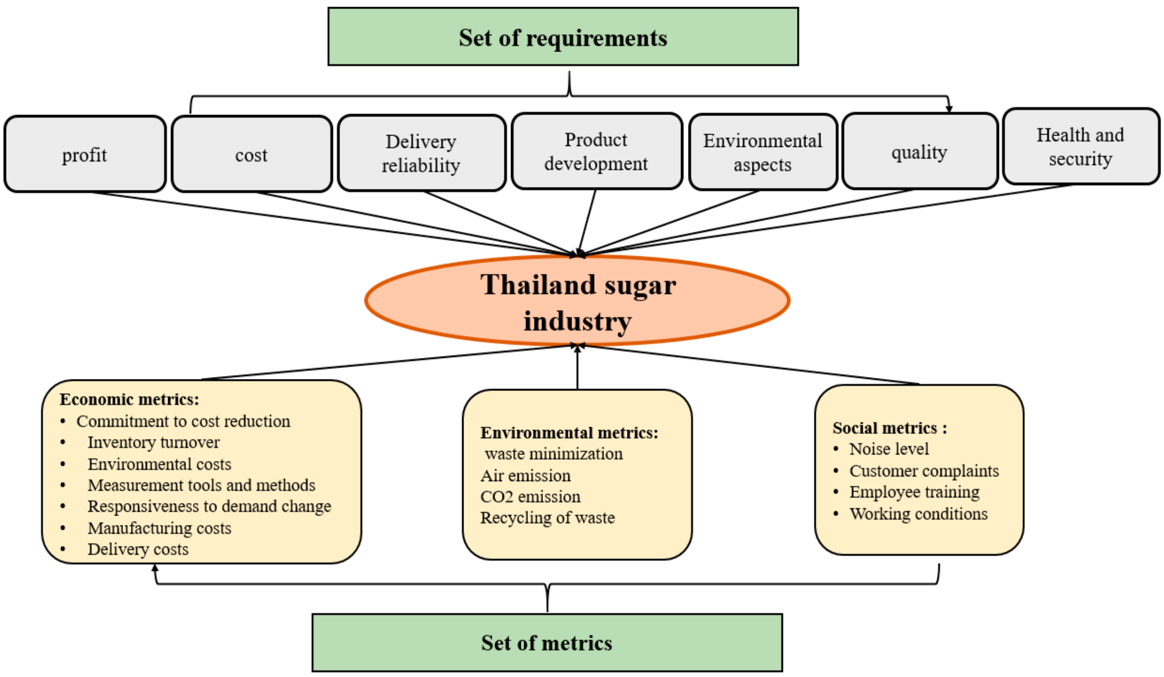


Figure 4. Thailand's sugar industry sustainability requirements and metrics.

Table 4. The evaluation of the requirements by four DMs.

Requirement	DM_1	DM_2	DM_3	DM_4
R_1	VH	H	H	M
R_2	H	VH	M	H
R_3	M	H	H	VH
R_4	H	M	M	H
R_5	H	VH	H	M
R6	H	VH	VH	M
R_7	H	M	H	M

Table 5. The requirements contradiction degree.

Requirement	R_1	R_2	R_3	R_4	R_5	R_6	R_7
Contradiction degree	0	1/7	2/7	3/7	4/7	5/7	6/7

Table 6. The aggregation results of requirements.

Requirement	$DM_1 \wedge_p DM_2 \wedge_p DM_3 \wedge_p DM_4$	Crisp Value
R_1	((0.29,0.69,1),0.45,0.18,0.36)	0.4727
R_2	((0.42,0.69,0.97),0.57,0.18,0.32)	0.1664
R_3	((0.56,0.69,0.91),0.68,0.18,0.24)	0.6130
R_4	((0.61,0.63,0.8),0.6,0.15,0.12)	0.5942
R_5	((0.79,0.69,0.75),0.86,0.18,0.1)	0.7192
R6	((0.92,0.74,0.68),0.9,0.18,0.06)	0.7781
R_7	((0.93,0.63,0.43),0.98,0.15,0.01)	0.7015

Table 7. The weights of requirements based on positive and negative distances.

Requirement	Positive Distance	Negative Distance	Performance Score	Weight	Ranking
R_1	0.2323	0.40872	0.6376	0.1551	4
R_2	0.4674	0.538	0.5351	0.1302	5
R_3	0.0914	0.0208	0.1854	0.0451	7
R_4	0.1048	0.0398	0.2752	0.0669	6
R_5	0.0146	0.0852	0.8537	0.2077	2
R6	0.0736	0.1442	0.6621	0.1611	3
R_7	0.0027	0.0677	0.9616	0.2339	1
total	-	-	4.1107	1	

Table 8. Economic, environmental and social metrics.

Dimension	Metrics
Economic	Commitment to cost reduction (I_1)
	Inventory turnover (I_2)
	Environmental costs (I_3)
	Measurement tools and methods (I_4)
	Responsiveness to demand change (I_5)
	Manufacturing cost (I_6)
	Delivery cost (I_7)
Environmental	Waste minimization(I_8)
	Air emission (I_9)
	CO_2 emission (I_{10})
	Recycling of waste (I_{11})
Social	Noise level (I_{12})
	Customer complaints (I_{13})
	Employee training (I_{14})
	Working conditions (I_{15})

Table 9. The evaluation of metrics.

Metrics	R_1	R_2	R_3	R_4	R_5	R_6	R_7
I_1	VH	A	M	M	L	M	L
I_2	L	M	H	L	L	M	L
I_3	H	VH	L	M	VH	L	H
I_4	M	M	H	VH	H	VH	H
I_5	H	H	VH	L	VL	M	VL
I_6	M	VH	L	VH	VL	VH	VL
I_7	VH	VH	VH	M	VL	H	L
I_8	L	VH	L	L	VH	L	VH
I_9	VL	VL	L	M	VH	L	VH
I_{10}	VL	VL	L	H	VH	M	VH
I_{11}	M	L	L	VL	VH	L	VH
I_{12}	L	L	L	VL	M	L	H
I_{13}	H	L	M	L	M	L	H
I_{14}	H	VH	L	VH	VL	H	H
I_{15}	H	H	M	M	L	H	H

Table 10. The aggregation results of the metrics.

	$R_1 \wedge_p R_2 \wedge_p R_3 \wedge_p R_4$	$R_5 \wedge_p R_6 \wedge_p R_7$	$R_1 \ldots R_7$	CRISP
I_1	((0.36,0.74,1),0.4,0.13,0.42)	((0.1,0.43,0.9),0.29,0.1,0.4)	((0.036,0.59,1),0.12,0.25,0.65)	0.248
I_2	((0.12,0.49,0.96),0.34,0.13,0.5)	((0.15,0.4,0.9),0.3,0.1,0.45)	((0.03,0.5,0.99),0.13,0.1,0.69)	0.2479
I_3	((0.52,0.48,0.87),0.43,0.15,0.39)	((0.6,0.63,0.9),0.5,0.2,0.34)	((0.4,0.6,0.97),0.28,0.17,0.54)	0.3866
I_4	((0.46,0.7,0.94),0.6,0.15,0.32)	((0.6,0.7,0.94),0.7,0.2,0.25)	((0.4,0.7,0.97),0.5,0.18,0.41)	0.4968
I_5	((0.43,0.63,0.92),0.6,0.18,0.3)	((0.1,0.3,0.45),0.5,0.18,0.4)	((0.43,0.5,0.8),0.43,0.18,0.46)	0.3871
I_6	((0.55,0.7,0.87),0.56,0.15,0.3)	((0.2,0.4,0.62),0.5,0.2,0.37)	((0.3,0.6,0.8),0.45,0.18,0.41)	0.3953
I_7	((0.78,0.5,0.87),0.88,0.18,0.24)	((0.3,0.4,0.7),0.7,0.14,0.24)	((0.5,0.45,0.8),0.76,0.16,0.28)	0.5191
I_8	((0.5,0.48,0.64),0.6,0.13,0.2)	((0.75,0.7,0.8),0.7,0.18,0.2)	((0.61,0.6,0.73),0.61,0.16,0.2)	0.5484
I_9	((0.37,0.73,0.32),0.68,0.15,0.21)	((0.8,0.7,0.8),0.7,0.18,0.18)	((0.6,0.73,0.53),0.7,0.17,0.18)	0.5464
I_{10}	((0.45,0.38,0.28),0.77,0.18,0.16)	((0.9,0.8,0.8),0.8,0.18,0.13)	((0.72,0.6,0.48),0.8,0.18,0.12)	0.5607
I_{11}	((0.57,0.4,0.28),0.79,0.2,0.11)	((0.9,0.7,0.85),0.8,0.18,0.1)	((0.8,0.6,0.46),0.84,0.19,0.07)	0.587
I_{12}	((0.57,0.33,0.29),0.8,0.13,0.09)	((0.8,0.6,0.5),0.9,0.15,0.05)	((0.78,0.5,0.28),0.9,0.14,0.04)	0.5168
I_{13}	((0.82,0.49,0.3),0.93,0.13,0.57)	((0.8,0.6,0.5),0.93,0.15,0.2)	((0.9,0.53,0.24),0.97,0.14,0.2)	0.5448
I_{14}	((0.17,0.7,0.46),0.95,0.13,0.02)	((0.8,0.6,0.2),0.97,0.2,0.03)	((0.8,0.6,0.17),0.99,0.2,0.007)	0.5521
I_{15}	((0.97,0.63,0.4),0.99,0.15,0.003)	((0.9,0.6,0.4),0.98,0.2,0.01)	((0.99,0.61,0.14),1,0.17,0.001)	0.6153

Table 11. Contradiction degree of metrics.

Metrics	I_1	I_2	I_3	I_4	I_5	I_6	I_7	I_8	I_9	I_{10}	I_{11}	I_{12}	I_{13}	I_{14}	I_{15}
Contradiction degree	0	$\frac{1}{15}$	$\frac{2}{15}$	$\frac{3}{15}$	$\frac{4}{15}$	$\frac{5}{15}$	$\frac{6}{15}$	$\frac{7}{15}$	$\frac{8}{15}$	$\frac{9}{15}$	$\frac{10}{15}$	$\frac{11}{15}$	$\frac{12}{15}$	$\frac{13}{15}$	$\frac{14}{15}$

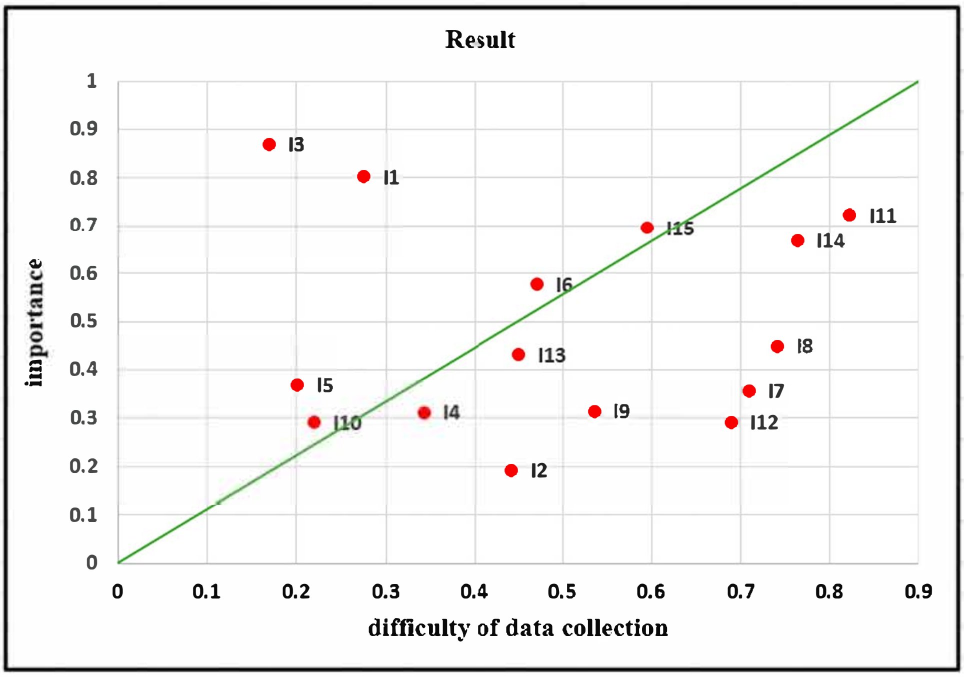


Figure 5. The categorization of Thailand's sugar industry sustainability.

Table 12. The performance score based on positive and negative distances.

Metrics	Positive Distance	Negative Distance	Performance Score
I_1	0.1114	0.5139	**0.8218**
I_2	0.4509	0.1115	0.1983
I_3	0.0272	0.3753	**0.9324**
I_4	0.2651	0.137	0.3407
I_5	0.3748	0.2467	0.3969
I_6	0.264	0.3666	0.5814
I_7	0.3878	0.2428	0.385
I_8	0.2135	0.189	0.4696
I_9	0.4151	0.2155	0.3417
I_{10}	0.4294	0.2012	0.3191
I_{11}	0.1749	0.4557	0.7226
I_{12}	0.3855	0.182	0.3207
I_{13}	0.1854	0.154	0.4537
I_{14}	0.2098	0.4208	0.6673
I_{15}	0.0835	0.2559	**0.754**

Table 13. The evaluation of information gathering difficulty.

Metrics	Information Availability	Human Resource and Time	Additional Resource Required
I_1	H	L	VH
I_2	M	H	H
I_3	H	L	L
I_4	M	H	H
I_5	L	M	L
I_6	H	H	L
I_7	H	L	VH
I_8	M	L	VH
I_9	H	M	L
I_{10}	H	M	L
I_{11}	M	L	VH
I_{12}	H	VH	L
I_{13}	VH	L	L
I_{14}	H	VH	VH
I_{15}	H	H	H

Table 14. The aggregation results of information gathering difficulty.

Metrics	A $\wedge_p$ B $\wedge_p$ C	Crisp Value
I_1	((0.25,0.68,1),0.38,0.18,0.42)	**0.4294**
I_2	((0.37,0.64,0.96),0.69,0.18,0.25)	0.5565
I_3	((0.2,0.4,0.85),0.42,0.13,0.35)	**0.3516**
I_4	((0.47,0.64,0.92),0.75,0.18,0.2)	0.6014
I_5	((0.29,0.42,0.73),0.49,0.1,0.28)	**0.3798**
I_6	((0.43,0.5,0.76),0.66,0.15,0.2)	0.488
I_7	((0.65,0.68,0.83),0.66,0.18,0.22)	0.6102
I_8	((0.7,0.67,0.77),0.68,0.15,0.19)	0.626
I_9	((0.56,0.49,0.6),0.74,0.13,0.14)	0.5094
I_{10}	((0.6,0.49,0.55),0.78,0.13,0.12)	0.506
I_{11}	((0.82,0.67,0.63),0.8,0.15,0.11)	**0.6731**
I_{12}	((0.77,0.55,0.53),0.84,0.15,0.09)	0.6013
I_{13}	((0.78,0.48,0.38),0.83,0.13,0.07)	0.5392
I_{14}	((0.97,0.8,0.72),0.85,0.2,0.04)	**0.8124**
I_{15}	((0.94,0.65,0.55),0.99,0.2,0.01	**0.7437**

Table 15. The performance score of information gathering difficulty.

Metrics	Ideal Positive	Ideal Negative	Performance Degree
I_1	0.7619	0.3594	0.1739
I_2	0.6988	0.6338	0.352
I_3	0.6988	0.3594	0.02197
I_4	0.6988	0.6338	0.2496
I_5	0.6338	0.3594	0.0743
I_6	0.6988	0.3594	0.3789
I_7	0.7619	0.3594	0.6231
I_8	0.7619	0.3594	0.6624
I_9	0.6988	0.3594	0.442
I_{10}	0.6988	0.3594	0.1003
I_{11}	0.7619	0.3594	0.7794
I_{12}	0.7619	0.3594	0.601
I_{13}	0.7619	0.3594	0.4467
I_{14}	0.7619	0.6988	0.6923
I_{15}	0.6988	0.6988	0.5

Table 16. Normalization of importance and data gathering difficulty.

Metrics	Importance	Difficulty of Data Collection
I_1	0.8031	0.2762
I_2	0.1913	0.4434
I_3	0.8711	0.1692
I_4	0.3119	0.3429
I_5	0.3694	0.2018
I_6	0.5763	0.4710
I_7	0.3569	0.7095
I_8	0.4495	0.7418
I_9	0.3129	0.5361
I_{10}	0.2911	0.2197
I_{11}	0.7216	0.8233
I_{12}	0.2926	0.6903
I_{13}	0.4316	0.4509
I_{14}	0.6682	0.7648
I_{15}	0.6958	0.595

5. Results and Discussion

Based on the decision makers evaluations, the prioritize metrics to evaluate Thailand's sugar industry sustainability are: Commitment to cost reduction (I_1), environmental costs (I_3), responsiveness to demand change (I_5), manufacturing costs (I_6), working conditions (I_{15}) and CO_2 emission (I_{10}). In this case, the decision makers were considering the economic aspects more than social or environmental dimensions. The importance of commitment to cost reduction (I_1), environmental costs (I_3) and recycling of wastes (I_{11}) gained the highest importance in value compared to other metrics, as shown in Figure 6. On the other side, recycling of the waste (I_{11}), employee training (I_{14}) and waste minimization (I_8) were the most difficult gathering information metrics. However, the CO_2 emission (I_{10}), environmental costs (I_3) and responsiveness to demand changes (I_5) were the most available information that had the lowest difficulty of data gathering degree as in Figure 7.

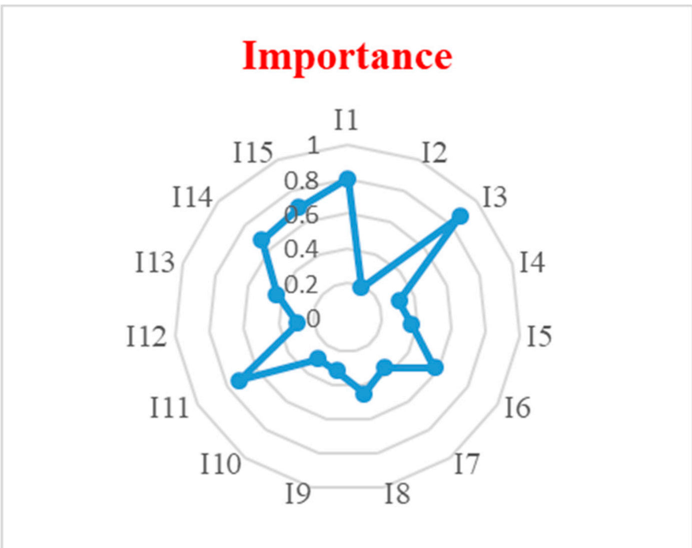


Figure 6. Metrics evaluation based on their importance.

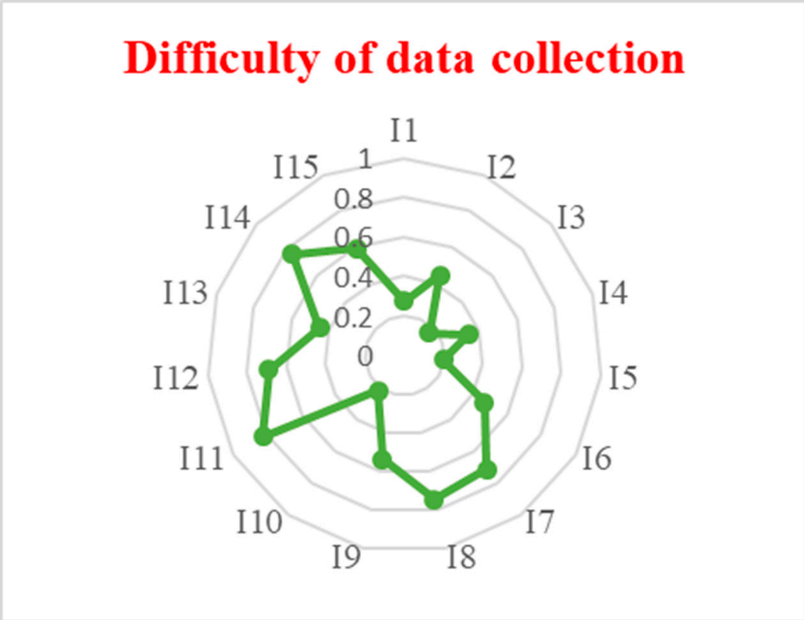


Figure 7. Metrics evaluation based on difficulty of data collection.

As the results of distribution of the Thailand sugar industry sustainability metrics were categorized based on the significance and difficulty of data gathering, this study found that the most preferred metrics were: Environmental cost (I_3), commitment to cost reduction (I_1) and realization to demand change (I_5), respectively, which were all economic metrics. It can be concluded that the economic metrics were more critical than the environmental and social metrics. These results are based on the evaluation of four decision makers, which means that it is not a general result for similar applications.

The main point in this paper is the plithogenic aggregation operation to group the decision maker's opinions in a more accurate manner. The plithogenic aggregation is taking into consideration the contradiction degree that mainly increases the accuracy of the aggregation. This study aggregated the decision maker's assessment of supply chain sustainability requirements, the evaluation of metrics importance, and measuring the difficulty of information gathering. Also, the difficulty of data gathering was not considered in many articles, but the authors found it to be a critical diminution to be measured.

Similar studies on this topic differ based on the model and the nature of the problem. As Ignatius, Joshua, et al. [52] proposed, the ANP-QFD approach which mainly considered environmental indicators in order to ensure a green building structure. On the other hand, Jamalnia, Aboozar, et al. [53] considered economic metrics such as costs, raw material and labour availability to solve a facility location problem using the QFD method. Khodakarami, Mohsen, et al. [54] and Izadikhah, et al. [55] used data envelopment analysis (DEA) for the evaluation of supply chain sustainability by taking into account mostly economic and environmental metrics. Ahmadi, et al. [56] used the best worst method (BWM) to evaluate the social sustainability of supply chain by considering social indicators rather than economic or environmental aspects.

6. Conclusions and Future Works

Due to strict government requests and the huge stress from the public, the consideration of supply chain sustainability was increased [36]. Sustainable development is one of the most significant conditions of saving resources and keeping the supply chain phases operating efficiently [57]. This explains why SSCM became one of the most important competitive strategies in organizations [58]. This study proposed an efficient combination of plithogenic aggregation operations with the quality function deployment method. The advantage of this combination is the improvement of the accuracy of the results while aggregating the assessments of the decision makers. QFD produced great results in supply chain sustainability evaluation. However, it does not consider the depth of data gathering difficulty which loosen the accuracy of the results. The supply chain sustainability evaluation is a critical topic that needs to be studied with high accuracy by considering economic, environmental and social dimensions.

This study observed the proposed combined model in a major real world case study, which was Thailand's sugar industry. Based on the nature of this supply chain strategy, its sustainability requirements was defined and measured by four decision makers. Their opinions were aggregated using plithogenic aggregation operator based on the contradiction degree to maximize the accuracy of the aggregation. In the same way, the measurement metrics was defined based on the previous requirement and included economic, social and environmental dimensions evaluated by the DMs. The results showed that the importance of commitment to cost reduction (I_1), environmental costs (I_3) and recycling of wastes (I_{11}) gained the most important metrics compared to the rest of metrics. Finally, the difficulty of data gathering of these metrics was measured and aggregated. For each dimension of evaluation, the distance of the metrics to the best and worst ideal solutions was calculated to find the importance and the degree of information gathering difficulty. The result of this point was recycling of the waste (I_{11}), employee training (I_{14}) and waste minimization (I_8) were the most difficult gathering information metrics. Moreover, the performance degree of the metrics was intended and normalized to distribute the metrics based on the two defined dimensions. It can be concluded that the most preferred metrics based in both dimensions are environmental cost (I_3), commitment to cost reduction (I_1) and realization to demand change (I_5).

Real contributions of this proposed methodology:

- The main contribution of this proposed model lies in providing accurate results of the decision makers assessments based on the contradiction degree while applying the aggregation.
- The plithogenic aggregation operation allows the DMs to consider several experts opinions in order to maximize the efficiency of the decision making.
- Also, it measures the supply chain sustainability based on two major aspects, the significance of the metrics and its level of difficulty of data gathering, which is really a critical point that affects the evaluations.
- Using a triangular neutrosophic linguistic scale to evaluate the requirement, metrics and information availability improved the level of consideration to uncertainty, because it confirms the best representation by using the three membership degrees positive, negative and boundary areas of decision making.
- The proposed methodology is efficient and has a high accuracy degree in decision making problems, Therefore, it is a great tool that may help firms in their estimation of customer needs in addition to evaluating the supply chain sustainability requirements.

In future research directions, this model may be used in assessment of other supply chain strategies to evaluate their sustainability. In addition, the plithogenic aggregation operators could be combined with other techniques to evaluate the supply chain sustainability. Finally, more evaluation dimensions could be added to the importance and difficulty of information gathering to measure supply chain sustainability.

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Plithogenic Subjective Hyper-Super-Soft Matrices with New Definitions & Local, Global, Universal Subjective Ranking Model

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Abstract

In this paper, we initially introduce a novel type of matrix representation of Plithogenic Crisp/Fuzzy/Intuitionistic/Neutrosophic Hypersoft Set named as **Plithogenic Crisp/Fuzzy/Intuitionistic/Neutrosophic Hypersoft Matrix,** which is generated by multiple parallel sheets of matrices. Furthermore, these parallel sheets are representing parallel universes or parallel realities (a combination of attributes and sub-attributes w.r.t. subjects). We represent cross-sectional cuts of these hyper-soft matrices as parallel sheets (images of the expanded universe). Later, we utilize these Hypersoft matrices to formulate **Plithogenic Subjective Crisp/Fuzzy/Intuitionistic/Neutrosophic Hyper-Super-Soft Matrix.** These matrices are framed by the generalization of Whole Hyper-Soft Set to Subjective Whole Hyper-Soft Set and then their representation in such **hyper-super-soft-matrix (**parallel sheets of matrices) whose elements are matrices. The Hypersoft matrices and hyper-super-soft matrices are tensors of rank three and four, respectively, having three and four indices of variations. Later we provide an application of these Plithogenic Hyper super soft matrices in the form of **Local, Global, Universal Subjective Ranking Model.** The specialty of this model is that it offers precise classification of the universe from micro-universe to macro-universe levels by observing them through several angles of visions in many environments having several ambiguities and hesitation levels. This model provides optimal and neutral values of universes and can compact the expanded universe to a single point in such a way that the compacted universe reflects the cumulative effect of the whole universe. It further offers a transparent ranking by giving a percentage authenticity measure of the ranking. Finally, we provide an application of the model as a numerical example.

Keywords: Plithogenic Hyper-Super-Soft matrices, Sheets of matrices, Expanded Universe, Compacted Universe, Subjective, Local, Global, Universal Ranking,

1.Introduction

The theory of fuzziness in mathematics was initially introduced by Zadeh [7] in 1965 named as fuzzy set theory (FST). As in crisp Set, a member either completely belongs to a set A or completely doesn't belong to the set A which means we assignee membership to set A either 1 or 0 while in a fuzzy set, a doubt of belongingness is addressed as a natural trait of the human mind in decision making. The complete membership reduced according to the doubt of belongingness. We may say a fuzzy set is a set where each element of the universe of discourse X has some degree of membership in the unit closed interval $[0,1]$ in given set A, where A is a subset of universal Set X with respect to an attribute say M with an imposed condition that the sum of membership and non-membership is one unlike crisp Set where the membership is not partial but completely one or completely zero. In fuzzy Set, elements are represented with one variable quantity, i.e., degree of membership noted as $\mu_A(x) \in [0,1]\ \forall x \in X$ and to express the degree of non-membership a notation $v_A(x) \in [0,1]\ \forall x \in X$ was used. $\{x;\mu_A(x)\}$ is the general representation of a fuzzy Set.

Further generalization of fuzzy Set was made by Atanassov [2-3] in 1986, which is known as the Intuitionistic fuzzy set theory (IFS). IFS addresses the doubt on assigning the membership to an element in the previously discussed fuzzy Set. This expanded doubt is the natural trait of the human mind, known as the hesitation factor of IFS. By introducing the hesitation factor in IFS, the sum of opposite membership values was modified. In IFS, the sum of membership, non-membership, and hesitation is one. The degree of hesitation was generally expressed by the notation"ι" now the modified condition is $\mu_A(x) + v_A(x) + \iota_A(x) = 1\ \forall x \in X$. The elements of IFS expressed by using two variable quantities."$\mu_A(x)$"and "$v_A(x)$"$\{x;(\mu_A(x),v_A(x))\}$. Later Intuitionistic fuzzy set theory (IFS) was further modernized by Smarandache [1, 15, 17] in 1995, where he assigned an independent degree to the doubt which was aroused in IFS. He represented membership by $T(x)$, the truth of an event and non-membership by $F(x)$ falsity or opposite to truth and the Indeterminacy $I(x)$ is the neutrality between the truth and its opposite. These three factors are considered as independent factors and represented in a unit cube in the non-standard unit interval $]0^-,1^+[$. Smarandache represented the elements of Standard Neutrosophic Set by using three independent quantities i.e.

$\{x;(T(x),I(x),F(x))\}$ with condition $0 \leq (T(x) + I(x) + F(x) \leq 3$

Soft Set was introduced by Molodtsov [5] in 1999, where he exhibited them as a parametrized family of a subset of the universal set. Soft Set further expanded by Deli, Broumi, Çağman [23][24] in 2015.

Later, Smarandache [16] in 2018 generalized the Soft Set to Hypersoft Set and Plithogenic Hypersoft Set by expanding the function of one attribute into a multi attributes/sub-attribute function. He assigned a combined membership $\mu_{A_1\times A_2\times\ldots\times A_N}(x)$, non-membership $v_{A_1\times A_2\times\ldots\times A_N}(x)$ and indeteriminacy $\iota_{A_1\times A_2\times\ldots\times A_N}(x)\ \ \forall x \in X$ with condition $A_i \cap A_j = \phi$ and introduced hybrids of Crisp/ Fuzzy/ Intuitionistic Fuzzy and Neutrosophic hypersoft set and addressed many open problems of development of new literature and MADM techniques.

We have answered some of the open concerns raised by Smarandache [16] in his article "Extension of Soft set to Hypersoft Set and then to Plithogenic Hypersoft Set" published in 2018, in our previous article, titled "Plithogenic Fuzzy Whole Hyper Soft Set, Construction of Local Operators and their Application in Frequency Matrix Multi-

Attribute Decision Making Technique"[14]. With the help of this Matrix, some local operators for the Plithogenic Fuzzy Hypersoft Set (PFHSS) originated, and their numerical examples constructed. Once these local operators applied to (PFHSS) they gave birth to a new type of Soft Set termed as Plithogenic Fuzzy Whole Hypersoft set (PFWHSS).

Now in this article on its first stage, we represent Plithogenic/Fuzzy/Intuitionistic/Neutrosophic Hyper Soft Set in different and advanced forms of Matrix termed as Plithogenic Crisp/Fuzzy/Intuitionistic/Neutrosophic Hypersoft Matrix (PC/F/I/NHS-Matrix). This special type of Matrix is generated by parallel sheets of matrices representing parallel universes or parallel realities. Then by hybridization of this novel Hyper-soft Matrix and Plithogenic Fuzzy Whole Hyper Soft Set, we generate a more generalized and expanded version of Soft Set and name these new Soft Set, "Plithogenic Fuzzy/Intuitionistic/Neutrosophic Subjective Hyper- Soft Set." To represent these sets, we further generate a more expanded form of Hyper-Soft-Matrix naming as Plithogenic Crisp/Fuzzy/Intuitionistic/Neutrosophic Subjective Hyper-Super-Soft Matrix (PC/F/I/NHSSM) which is a more advance form of Matrix. These matrices are a hybridization of Super-matrices Introduced by Smarandache [8][9] and Hyper matrices [11].

* Now the question arises why we are using hyper-soft and hyper-super-soft matrices for the expression of Plithogenic Hyper-Soft Set and Plithogenic Subjective Hyper-Soft Set? The answer is very simply and truly convincing that is, the Plithogenic universe is so vast and expanded interiorly (like having Fuzzy, Intuitionistic Fuzzy, Picture Fuzzy, Neutrosophic, Pythogorian, Universes with memberships non-memberships and indeterminacies) and exteriorly (dealing with many attributes, sub-attributes and might be sub-sub-attributes regarding many subjects. The expression for such a vastly expanded universe is not possible by using ordinary algebra or matrices. For the repercussion of an expanded universe with vast indeterminate data, we need some new theories like super or hyper-super algebra or hyper-soft and hyper-super-soft matrices.

* This article is an initial draft or effort for initiating and opening a new dimension of expression by using hyper or hyper-super matrices. It is an expanding field, so in this paper, we introduce names and general expressions of these matrices in many environments. A detailed constructed example is presented in Crisp and Fuzzy environments to keep the length of the article within the required limits of the journal. Later, the detailed constructed examples in other suitable environments would be displayed in the form of upcoming articles. For further motivation see [20-26].

In this paper, we discuss some applications of these matrices in the Crisp and fuzzy environment. These Plithogenic Hyper Soft Matrices and Plithogenic Subjective Hyper Super Soft Matrices are tensors of rank three and four, respectively, having three and four indices of variations. The first index represents rows. The second index represents columns. The third index represents parallel sheets of matrices. The fourth index represents several packets of parallel sheets.

In the second stage, we will utilize these special advanced matrices in the development of a ranking model named "Plithogenic Fuzzy/Intuitionistic/Neutrosophic Local, Global, Universal Subjective Ranking model. This subjective ranking model will provide several types of subjective ranking. Initially, it gives Local Subjective Ranking, then expands it to Global Subjective Ranking than further extends it to Universal Subjective ranking.

This model offers decision making in different levels of fuzziness according to the state of mind of decision-makers. We may vary the level of fuzziness by choosing a suitable environment like Fuzzy, Intuitionistic Neutrosophic, or all environments combined in one environment, etc.

This model offers a transparent decision by analyzing the universe through several angles of visions. The choice of aggregation operator provides a different angle of vision, either optimist pessimist or neutral.

In this model, Local Subjective Ranking is associated with a particular angle of vision. Global Subjective Ranking combines several angles of vision to offer a transparent decision while the Universal, Subjective Ranking is through compacting the expanded universe to have an outer look of the universe.

At the final stage, we provide an application of this subjective ranking model with the help of a numerical example.

We aim to establish such data handling structures that can reduce the complexity and long calculations in decision-making techniques. Decisions will often be made understeer and clear uncertainties (i.e., with incomplete and uncertain knowledge). With the help of our "Subjective ranking model," problems of incomplete and uncertain knowledge of Artificial Intelligence can be solved to a greater extent.

We are giving such a Mathematical structure/pattern that would possibly deal with the expanded data and will shrink its size in such a manner that, in a glimpse, the outcome of it will be obvious to the observer.

2. Preliminaries

In this section, we present some basic definitions of soft sets, fuzzy soft sets, hypersoft Set, crisp hypersoft Set, fuzzy hypersoft Set, plithogenic hypersoft Set, plithogenic crisp hypersoft sets and plithogenic fuzzy hypersoft Set, hyper Matrix and super-matrix.

Definition 2.1 [5] (Soft Set)

"Let $\boldsymbol{U}$ be the initial universe of discourse, and $\boldsymbol{E}$ be a set of parameters or attributes with respect to $\boldsymbol{U}$ let $\boldsymbol{P}(\boldsymbol{U})$ denote the power set of $\boldsymbol{U}$, and $\boldsymbol{A} \subseteq \boldsymbol{E}$ is a set of attributes. Then pair $(\boldsymbol{F}, \boldsymbol{A})$, where $\boldsymbol{F}: \boldsymbol{A} \longrightarrow \boldsymbol{P}(\boldsymbol{U})$ is called Soft Set over $\boldsymbol{U}$. For $\boldsymbol{e} \in \boldsymbol{A}$, $\boldsymbol{F}(\boldsymbol{e})$ may be considered as Set of $\boldsymbol{e}$ elements or $\boldsymbol{e}$ approximate elements $(\boldsymbol{F}, \boldsymbol{A}) = \{(\boldsymbol{F}(\boldsymbol{e}) \in \boldsymbol{P}(\boldsymbol{U}): \boldsymbol{e} \in \boldsymbol{E}, \boldsymbol{F}(\boldsymbol{e}) = \boldsymbol{\varphi}$ **if** $\boldsymbol{e} \notin \boldsymbol{A}\}$"

Definition 2.2 [6] (Soft Subset)

"For two soft sets $(\boldsymbol{F}, \boldsymbol{A})$ and $(\boldsymbol{G}, \boldsymbol{B})$ over a universe $\boldsymbol{U}$, we say that $(\boldsymbol{F}, \boldsymbol{A})$ is a soft subset of $(\boldsymbol{G}, \boldsymbol{B})$ if (i) $\boldsymbol{A} \subseteq \boldsymbol{B}$, and (ii) $\forall \boldsymbol{e} \in \boldsymbol{A}, \boldsymbol{F}(\boldsymbol{e}) \subseteq \boldsymbol{G}(\boldsymbol{e})$."

Definition 2.3 [7] (Fuzzy Set)

"Let $\boldsymbol{U}$ be the universe. A fuzzy set $\boldsymbol{X}$ over $\boldsymbol{U}$ is a set defined by a membership function $\boldsymbol{\mu_X}$ representing a mapping $\boldsymbol{\mu_X} : \boldsymbol{U} \longrightarrow [\mathbf{0}, \mathbf{1}]$. The vale of $\boldsymbol{\mu_X}\ (\boldsymbol{x})$ for the fuzzy set $\boldsymbol{X}$ is called the membership value of the grade of membership of $\boldsymbol{x} \in \boldsymbol{U}$. The membership value represents the degree of belonging to fuzzy set $\boldsymbol{X}$. "A fuzzy set $\boldsymbol{X}$ on $\boldsymbol{U}$ can be presented as follows. $\boldsymbol{X} = \{(\boldsymbol{\mu_X}(\boldsymbol{x})/\boldsymbol{x}) : \boldsymbol{x} \in \boldsymbol{U}, \boldsymbol{\mu_X}(\boldsymbol{x}) \in [\mathbf{0}, \mathbf{1}]\}$ "

Definition 2.4 [4] (Fuzzy Soft Set)

"Let $\boldsymbol{U}$ be the initial universe of discourse, $\boldsymbol{F}(\boldsymbol{U})$ be all fuzzy sets over $\boldsymbol{U}$. $\boldsymbol{E}$ be the Set of all parameters or attributes with respect to $\boldsymbol{U}$ and $\boldsymbol{A} \subseteq \boldsymbol{E}$ is a set of attributes. A fuzzy soft set $\boldsymbol{\Gamma_A}$ on the universe $\boldsymbol{U}$ is defined by the Set of ordered pairs as follows, $\boldsymbol{\Gamma_A} = \{\boldsymbol{x}, \boldsymbol{\gamma_A}(\boldsymbol{x}) : \boldsymbol{x} \in \boldsymbol{E}, \boldsymbol{\gamma_A}(\boldsymbol{x}) \in \boldsymbol{F}(\boldsymbol{U})\}$, where $\boldsymbol{\gamma_A} : \boldsymbol{E} \longrightarrow \boldsymbol{F}(\boldsymbol{U})$ such that $\boldsymbol{\gamma_A}(\boldsymbol{x}) = \boldsymbol{\phi}$ if $\boldsymbol{x} \notin \boldsymbol{A}$, $\boldsymbol{\gamma_A}(\boldsymbol{x}) = \{\boldsymbol{\mu_{\gamma_{A(x)}}}(\boldsymbol{u})/\boldsymbol{u}\boldsymbol{:}\boldsymbol{u} \in \boldsymbol{U}, \boldsymbol{\mu_{\gamma_{A(x)}}}(\boldsymbol{u}) \in [\mathbf{0}\mathbf{,}\mathbf{1}]\}$ "

Definition 2.5 [16] (Hypersoft set)

"Let $\boldsymbol{U}$ be the initial universe of discourse $\boldsymbol{P}(\boldsymbol{U})$ the power set of $\boldsymbol{U}$. Let $\boldsymbol{a_1}, \boldsymbol{a_2}, \ldots, \boldsymbol{a_n}$ for $\boldsymbol{n} \geq \mathbf{1}$ be $\boldsymbol{n}$ distinct attributes, whose corresponding attributes values are respectively the sets $\boldsymbol{A_1}, \boldsymbol{A_2}, \ldots, \boldsymbol{A_n}$ with $\boldsymbol{A_i} \cap \boldsymbol{A_j} = \boldsymbol{\varphi}$ for $\boldsymbol{i} \neq \boldsymbol{j}$ and $\boldsymbol{i}, \boldsymbol{j} \in \{\mathbf{1}, \mathbf{2}, \ldots, \boldsymbol{n}\}$. Then the pair $(\boldsymbol{F}, \boldsymbol{A_1} \times \boldsymbol{A} \times \ldots \times \boldsymbol{A_n})$ where, $\boldsymbol{F} : \boldsymbol{A_1} \times \boldsymbol{A} \times \ldots \times \boldsymbol{A_n} \rightarrow \boldsymbol{P}(\boldsymbol{U})$, is called a Hypersoft set over $\boldsymbol{U}$"

Definition 2.6 [16] (Crisp Universe of Discourse)

"A Universe of Discourse $\boldsymbol{U_C}$ is called Crisp if $\forall \boldsymbol{x} \in \boldsymbol{U_c}\ \boldsymbol{x} \in \mathbf{100\%}$ to $\boldsymbol{U_C}$ or membership of $\boldsymbol{x}\ \boldsymbol{T}(\boldsymbol{x})$ with respect to $\boldsymbol{A}$ in $\boldsymbol{M}$ is $\mathbf{1}$ denoted as $\boldsymbol{x}(\mathbf{1})$"

Definition 2.7 [16] (Fuzzy Universe of Discourse)

"A Universe of Discourse $\boldsymbol{U_F}$ is called Fuzzy if $\forall \boldsymbol{x} \in \boldsymbol{U_C}\ \boldsymbol{x}$ partially belongs to $\boldsymbol{U_F}$ or membership of $\boldsymbol{x}\ \boldsymbol{T}(\boldsymbol{x}) \subseteq [\mathbf{0}\mathbf{,}\mathbf{1}]$ where $\boldsymbol{T}(\boldsymbol{x})$ may be a subset, an interval, a hesitant set, a single value set, denoted as $\boldsymbol{x}(\boldsymbol{T_X})$"

Definition 2.8 [16] (Crisp Hypersoft set)

"Let $\boldsymbol{U_c}$ be the initial universe of discourse $\boldsymbol{P}(\boldsymbol{U_c})$ the power set of $\boldsymbol{U}$. let $\boldsymbol{a_1}, \boldsymbol{a_2}, \ldots, \boldsymbol{a_n}$ for $\boldsymbol{n} \geq \mathbf{1}$ be $\boldsymbol{n}$ distinct attributes, whose corresponding attributes values are respectively the sets $\boldsymbol{A_1}, \boldsymbol{A_2}, \ldots, \boldsymbol{A_n}$ with $\boldsymbol{A_i} \cap \boldsymbol{A_j} = \boldsymbol{\varphi}$ for $\boldsymbol{i} \neq \boldsymbol{j}$ and $\boldsymbol{i}, \boldsymbol{j} \in \{\mathbf{1}, \mathbf{2}, \ldots, \boldsymbol{n}\}$ Then the pair $(\boldsymbol{F_c}, \boldsymbol{A_1} \times \boldsymbol{A} \times \ldots \times \boldsymbol{A_n})$ where, $\boldsymbol{F_c} : \boldsymbol{A_1} \times \boldsymbol{A} \times \ldots \times \boldsymbol{A_n} \rightarrow \boldsymbol{P}(\boldsymbol{U_c})$, is called Crisp Hypersoft set over $\boldsymbol{U_c}$."

Definition 2.9 [16] (Fuzzy Hypersoft set)

"Let $\boldsymbol{U_F}$ be the initial universe of discourse $\boldsymbol{P}(\boldsymbol{U_F})$ the power set of $\boldsymbol{U_F}$. Let $\boldsymbol{a_1}, \boldsymbol{a_2}, \ldots, \boldsymbol{a_n}$ for $\boldsymbol{n} \geq \mathbf{1}$ be $\boldsymbol{n}$ distinct attributes, whose corresponding attributes values are respectively the sets $\boldsymbol{A_1}, \boldsymbol{A_2}, \ldots, \boldsymbol{A_n}$ with $\boldsymbol{A_i} \cap \boldsymbol{A_j} = \boldsymbol{\varphi}$ for $\boldsymbol{i} \neq \boldsymbol{j}$ and $\boldsymbol{i}, \boldsymbol{j} \in \{\mathbf{1}, \mathbf{2}, \ldots, \boldsymbol{n}\}$. Then the pair $(\boldsymbol{F_F}, \boldsymbol{A_1}, \boldsymbol{A_2}, \ldots, \boldsymbol{A_n})$ where, $\boldsymbol{F_F} : \boldsymbol{A_1} \times \boldsymbol{A} \times \ldots \times \boldsymbol{A_n} \rightarrow \boldsymbol{P}(\boldsymbol{U_F})$, is called Fuzzy Hypersoft set over $\boldsymbol{U_c}$, Now instead of assigning combined membership $\boldsymbol{\mu_{A_{1\times A_2\times\ldots\times A_N}}}(\boldsymbol{x})\ \forall \boldsymbol{x} \in \boldsymbol{X}$ for Hyper Soft sets if each attribute $\boldsymbol{A_j}$ is assigned an individual membership $\boldsymbol{\mu_{A_j}}(\boldsymbol{x})$, non-membership $\boldsymbol{v_{A_j}}(\boldsymbol{x})$ and Indeteriminacy $\boldsymbol{\iota_{A_j}}(\boldsymbol{x}) \forall \boldsymbol{x} \in \boldsymbol{X}\ \ \boldsymbol{j} = \mathbf{1}, \mathbf{2}, \ldots, \boldsymbol{n}$ in Crisp, Fuzzy, Intuitionistic and Neutrosophic Hypersoft set then these generalized Crisp, Fuzzy, Intuitionistic and Neutrosophic Hypersoft sets are called Plithogenic Crisp, Fuzzy, Intuitionistic Fuzzy and Neutrosophic Hypersoft sets"

Definition 2.10 [8][9] (Super-matrix)

"A Square or rectangular arrangements of numbers in rows and columns are matrices we shall call them as simple matrices while, super-matrix is one whose elements are themselves matrices with elements that can be either scalars or other matrices. $\boldsymbol{a} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$ where $\boldsymbol{a_{11}} = \begin{bmatrix} 2 & -4 \\ 0 & 1 \end{bmatrix}$, $\boldsymbol{a_{12}} = \begin{bmatrix} 0 & 40 \\ 21 & -12 \end{bmatrix}$, $\boldsymbol{a_{21}} = \begin{bmatrix} 3 & -1 \\ 5 & 7 \\ -2 & 9 \end{bmatrix}$, $\boldsymbol{a_{22}} = \begin{bmatrix} 4 & 12 \\ -17 & 6 \\ 3 & 7 \end{bmatrix}$ $\boldsymbol{a}$ is a super-matrix".

Note: The elements of super-matrices are called sub-matrices i.e. $\boldsymbol{a_{11}}, \boldsymbol{a_{12}}, \boldsymbol{a_{21}}, \boldsymbol{a_{22}}$ are sub-matrices of the super-matrix $\boldsymbol{a}$.

in this example, the order of super-matrix $\boldsymbol{a}$ is $\boldsymbol{2 \times 2}$ and order of sub-matrices $\boldsymbol{a_{11}}$ is $\boldsymbol{2 \times 2}$, $\boldsymbol{a_{12}}$ is $\boldsymbol{2 \times 2}$ $\boldsymbol{a_{21}}$ is $\boldsymbol{3 \times 2}$ and order of sub-matrix $\boldsymbol{a_{22}}$ is $\boldsymbol{3 \times 2}$, we can see that the order of super-matrix doesn't tell us about the order of its sub-matrices.

Definition 2.11 [11] (Hyper-matrix)

"For $\boldsymbol{n_1, \ldots, n_d} \in \boldsymbol{N}$, a function $\boldsymbol{f}: (\boldsymbol{n_1}) \times \ldots \times (\boldsymbol{n_d}) \rightarrow \boldsymbol{F}$ is a hyper-matrix, also called an order-d hyper-matrix or d-hyper-matrix. We often just write $\boldsymbol{a_{k_1 \ldots k_d}}$ to denote the value $\boldsymbol{f(k_1, \ldots, k_d)}$ of $\boldsymbol{f}$ at $(\boldsymbol{k_1, \ldots, k_d})$ and think of $\boldsymbol{f}$ (renamed as $\boldsymbol{A}$) as specified by a d-dimensional table of values, writing $\boldsymbol{A} = [\boldsymbol{a_{k_1 \ldots k_d}}]_{k_1, \ldots, k_d}^{n_1, \ldots, n_d}$, A 3-hypermatrix may be conveniently written down on a (2-dimensional) piece of paper as a list of usual matrices, called slices". For example

$$\boldsymbol{A} = [\boldsymbol{a_{ijk}}] = \begin{bmatrix} a_{111} & a_{121} & a_{131} & \cdot & a_{112} & a_{122} & a_{132} \\ a_{211} & a_{221} & a_{231} & \cdot & a_{212} & a_{222} & a_{232} \\ a_{311} & a_{321} & a_{331} & \cdot & a_{312} & a_{322} & a_{332} \end{bmatrix}$$

3. Plithogenic Hyper-Soft Matrices and Plithogenic Subjective Hyper-Super-Soft Matrices

In this section, we develop some literature about the Plithogenic Hypersoft Set in the following order.

1. Introduction to some basic new concepts and definitions relevant to the hypersoft Set.
2. Generalization of plithogenic whole hypersoft Set to plithogenic subjective hypersoft Set in Crisp, Fuzzy, Intuitionistic, and Neutrosophic environments.
3. Generation of new types of Plithogenic Hypersoft matrix and plithogenic Subjective Hyper-Super-Soft matrix in Crisp, Fuzzy, Intuitionistic, and Neutrosophic environments to represent these new type of sets.
4. Development of expanded and compacted versions of plithogenic Subjective Hyper-Super-Soft Matrix in plithogenic environment.
5. Utilization of new types of matrices for the development of a Subjective ranking model. The specialty of this model is that it offers the classification of non-physical phenomena and Plithogenic

Definition 3.1 (Plithogenic Crisp/ Fuzzy,/Intuitionistic/ Neutrosophic/ Whole Hypersoft set)

If Plithogenic Crisp/Fuzzy/Intuitionistic Fuzzy/Neutrosophic Hyper Soft set expressed by both individual memberships $\boldsymbol{\mu_{A_j^k}(x_i)}$ non-membership $\boldsymbol{\upsilon_{A_j^k}(x_i)}$ and Indeterminacy $\boldsymbol{\iota_{A_j^k}(x_i)}$ $\forall \boldsymbol{x_i} \in \boldsymbol{X}$ $\boldsymbol{j = 1,2, \ldots, N}$, $\boldsymbol{i = 1,2, \ldots, M}$

and $k = 1, 2, \ldots L$ for each given attribute and Combined memberships $\Omega_{A_\alpha}(x_i)$, non-membership $\Phi_{A_\alpha}(x_i)$ and indeteriminacy $\Psi_{A_\alpha}(x_i)$ $\forall x_i \in X$ for the given α Combination of attributes/sub-attributes and subjects (α universe), then These Plithogenic Crisp/Fuzzy/Intuitionistic Fuzzy/Neutrosophic Hyper Soft set are called Plithogenic Crisp/Fuzzy/ Intuitionistic Fuzzy/Neutrosophic Whole Hypersoft set.

Definition 3.2 (Plithogenic Crisp/Fuzzy,/Intuitionistic/Neutrosophic Subjective Hypersoft Set)

If Plithogenic Crisp/Fuzzy/Intuitionistic/ Neutrosophic Hypersoft Set is reflected through subjects in such a way that it exhibits both an interior and exterior view of the universe, then, These Plithogenic Crisp/Fuzzy/Intuitionistic/Neutrosophic Hyper-Soft-Set titled, Plithogenic Crisp/Fuzzy/ Intuitionistic/ Neutrosophic Subjective Hypersoft set. The interior view of the universe is reflected through individual memberships $\mu_{A_j^k}(x_i)$ non-memberships $\upsilon_{A_j^k}(x_i)$ and Indeterminacies $\iota_{A_j^k}(x_i)$ $\forall x_i \in X$ $j = 1, 2, \ldots N$, $i = 1, 2, \ldots M$ and $k = 1, 2, \ldots L$ of subjects and the exterior view is reflected through summative memberships $\Omega^k_{A_j}(X)$, non-memberships $\Phi^k_{A_j}(X)$ and indeteriminacy $\Psi^k_{A_j}(X)$ $\forall x_i \in X$ united specifically for all attribute/sub-attributes and exhibited w.r.t subjects.

Note: In Plithogenic Crisp/Fuzzy/ Intuitionistic Fuzzy/ Neutrosophic Subjective Hypersoft set the united membership $\Omega_{A_\alpha}(x_i)$, non-membership $\Phi_{A_\alpha}(x_i)$ and indeteriminacy $\Psi_{A_\alpha}(x_i)$ $\forall x \in X$ are dependent on individual membership $\mu_{A_j^k}(x_i)$, non-membership $\upsilon_{A_j^k}(x_i)$ and Indeterminacy $\iota_{A_j^k}(x_i)$.

To achieve a universe, reflected through its subjects (matter for), some terminologies are introduced and described here.

Definition 3.3 (Local Subjective Operators): The aggregation operators, used to cumulate the memberships $\mu_{A_j^k}(x_i)$ non-memberships $\upsilon_{A_j^k}(x_i)$ and Indeterminacies $\iota_{A_j^k}(x_i)$ Of subjects to achieve a universe reflected through its subjects are termed as local subjective operators.

Definition 3.4 (Local Subjective Ranking): Ranking of subjects under consideration by using a particular aggregation operator is called Local Subjective Ranking. This is the case of the classification of the micro-universe.

Definition 3.5 (Global Subjective Ranking): Ranking of subjects under consideration by using multiple aggregation operators and then obtaining a combined ordering in the form of final ranking is called Global Ranking. This is the case of combining several angles of visions.

Definition 3.6 (Universal Ranking): Ranking of Universes by cumulating effects of attributes and subjects with the help of some local operators is called "Universal Ranking." In Universal Ranking, We converge the whole universe to a single numeric value. Where the combined effect of all attributes and subjects of the universe would be represented by a single numeric value and then writing these converged numeric values in descending order for universal ranking purpose.

Definition 3.7 (Hyper-Soft-Matrix): Let U be the initial universe of discourse $P(U)$ be the powerset of U. Let $A_1^k, A_2^k, \ldots A_n^k$ for $n \geq 1$ be n distinct attributes, $k = 1, 2, \ldots L$ representing attributes values, a function $F: A_1 \times A \times \ldots \times A_n \rightarrow P(U)$ is a hypersoft matrix, also called an order-$M \times N \times L$ hypersoft matrix or d-hypersoft Matrix ($d = 3$), i.e., a matrix representing a hypersoft set is a hyper-soft matrix (HS-Matrix). As we know, all simple $M \times N$ Matrices on real vector space are tensors of rank 2, so the new Hyper-Soft Matrix having three indices of variations are tensors of rank 3. The Hyper-Soft Matrix is obtained by the generalization of ordinary matrices, known as tensors of rank two. $B = [b_{ijk}]$ is an example of Hyper-Soft Matrix where index i give variation on rows j gives a variation on columns, and k gives variation on clusters of rows and columns.

The example and detailed illustration of the Hyper-Soft Matrix is presented in the form of the Plithogenic Crisp/Fuzzy Hypersoft Matrix.

The detailed descriptions and applications of the Plithogenic Intuitionistic and Neutrosophic Hyper-Soft Matrix would be exhibited in upcoming versions of research articles.

Definition 3.8 (Plithogenic Crisp/Fuzzy/Intuitionistic/Neutrosophic Hypersoft Matrix): Plithogenic Crisp/Fuzzy/Intuitionistic/Neutrosophic Hypersoft Set, when represented in the matrix form are called Plithogenic Crisp/Fuzzy/Intuitionistic/Neutrosophic Hyper Soft Matrices, i.e., if $\boldsymbol{B}$ is a hypersoft matrix then $\boldsymbol{B} = [\boldsymbol{b_{ijk}}]$, as $\boldsymbol{b_{ijk}}$ are elements of Matrix and would be expressed in Crisp, Fuzzy, Intuitionistic, Neutrosophic environments with memberships $\boldsymbol{\mu_{A_j^k}(x_i)}$ non-memberships $\boldsymbol{v_{A_j^k}(x_i)}$ and Indeterminacy $\boldsymbol{\iota_{A_j^k}(x_i)}$ $\forall \boldsymbol{x_i} \in \boldsymbol{X}$ $\boldsymbol{j = 1,2,\ldots,N}$, $\boldsymbol{i = 1,2,\ldots,M}$ and $\boldsymbol{k = 1,2,\ldots,L}$.

A general form of Plithogenic Hyper-Soft Matrix in fuzzy environment is expressed as

$$\boldsymbol{B} = \left[\boldsymbol{\mu_{A_j^k}(x_i)}\right]$$

A general form of Plithogenic Hyper-Soft Matrix in Intuitionistic environment is expressed as

$$\boldsymbol{B} = \left[\left(\boldsymbol{\mu_{A_j^k}(x_i)}, \boldsymbol{v_{A_j^k}(x_i)}\right)\right]$$

A general form of Plithogenic Hyper-Soft Matrix in Neutrosophic environment is expressed as

$$\boldsymbol{B} = \left[\left(\boldsymbol{\mu_{A_j^k}(x_i)}, \boldsymbol{\iota_{A_j^k}(x_i)}, \boldsymbol{v_{A_j^k}(x_i)}\right)\right]$$

These matrices described as,

Let $\boldsymbol{U}\,(\boldsymbol{X})$ be the universe of discourse in fuzzy or crisp environment and

$$\boldsymbol{G}: \boldsymbol{A_1^k} \times \boldsymbol{A_2^k} \times \ldots \times \boldsymbol{A_N^k} \rightarrow \boldsymbol{P(U)}$$

The Plithogenic Crisp/Fuzzy Hypersoft Set having individual memberships $\boldsymbol{\mu_{A_j^k}(x_i)}\,\forall \boldsymbol{x_i} \in \boldsymbol{X}$ $\boldsymbol{j = 1,2,\ldots,N}$ vary with respect to each attribute $\boldsymbol{A_j}$ and sub-attribute $\boldsymbol{A_j^k}$ $(\boldsymbol{k = 1,2,3,\ldots,L})$ where $\boldsymbol{k}$ represent Numeric values of attributes called sub-attributes and $\boldsymbol{i = 1,2,\ldots,M}$ are the number of subjects under consideration.

In fact in Plithogenic Crisp/Fuzzy Hyper Soft Matrix we have three types of variations introduced on memberships $\boldsymbol{\mu_{A_j^k}(x_i)}$. First variation of $\boldsymbol{i}$ is with respect to subjects representing $\boldsymbol{M}$ rows of $\boldsymbol{M \times N}$ Matrix. The second variation on $\boldsymbol{j}$ is with respect to attributes representing $\boldsymbol{N}$ columns of $\boldsymbol{M \times N}$ Matrix. A third Variation on $\boldsymbol{k}$ is for sub-attributes and represented in the form of $\boldsymbol{L}$ layers or $\boldsymbol{L}$ level sheets of $\boldsymbol{M \times N}$ Matrix. These level cuts of Hyper Matrix are categorized in three types:

1 Vertical front to back and interior level cuts are $\boldsymbol{M \times N}$ Matrix with $\boldsymbol{L}$ level sheets

2 Vertical left to right and interior level cuts are $\boldsymbol{L \times M}$ Matrix with $\boldsymbol{N}$ level sheets

3 Horizontal upper lower and interior level cuts are $\boldsymbol{L \times N}$ Matrix with $\boldsymbol{M}$ level sheets

Definition 3.9 (Plithogenic Crisp/Fuzzy/Intuitionistic/Neutrosophic Cubical Hypersoft Matrix):

If In Plithogenic Hyper-Soft Matrix $= M = N$ That is, the number of horizontal rows , vertical columns and parallel level cuts are equal, then it is termed a Cubicle Hypersoft Matrix.

Definition 3.10 (Plithogenic Crisp/Fuzzy/Intuitionistic/Neutrosophic Square Rectangular Hypersoft Matrix):

If In plithogenic hypersoft matrix $L \neq M = N$ we shall have an equal number of horizontal rows vertical columns in all sheets, and the number of sheets is different from the number of rows and columns then this hypersoft Matrix is called Square Rectangular Hypersoft Matrices. For example is for the first numeric value of sub-attribute $k = 1$ we get the first sheet in the form of Matrix $[b_{ij1}]$ and for second level of sub-attributes, we get the second sheet of Matrix $[b_{ij2}]$. This procedure will continue until the Lth sheet obtained by taking $k = L$. These all sheets will form a hyper matrix $[b_{ijk}]$

L −Hypermatrix for L numeric values of attributes forming L sheets of $M \times N$ Matrix, and if we take $L = 3$ we will get 3 −Hypersoft Matrix in the form of three $M \times N$ parallel sheets.

If the environment of the plithogenic hypersoft Set is Crisp/Fuzzy/intuitionistic/neutrosophic, then the plithogenic hypersoft Matrix is called "Plithogenic Crisp/fuzzy/Intuitionistic/Neutrosophic hypersoft matrix."

Plithogenic Crisp/Fuzzy Hypersoft Set when represented in a matrix form having memberships $\mu_{A_j^k}(x_i)$, We name it as **Plithogenic Crisp/Fuzzy Hyper-Soft Matrix** is exibited as

Plithogenic Crisp/Fuzzy Hyper-Soft Matrix and level cuts,

$B = [\mu_{A_j^k}(x_i)]$ by fixing k and giving variation to i , j the expanded form of B is obtained as an $M \times N$ matrix, having **L** level cuts. These **L** Level cuts are termed as **Type-1 Level Cuts**.

$$B = \begin{bmatrix} \begin{bmatrix} \mu_{A_1^1}(x_1) & \mu_{A_2^1}(x_1) & \cdots & \mu_{A_N^1}(x_1) \\ \mu_{A_1^1}(x_2) & \mu_{A_2^1}(x_2) & \cdots & \mu_{A_N^1}(x_2) \\ \cdot & \cdot & \cdots & \\ \cdot & \cdot & \cdots & \\ \cdot & \cdot & \cdots & \\ \mu_{A_1^1}(x_M) & \mu_{A_2^1}(x_M) & \cdots & \mu_{A_N^1}(x_M) \end{bmatrix} \\ \begin{bmatrix} \mu_{A_1^2}(x_1) & \mu_{A_2^2}(x_1) & \cdots & \mu_{A_N^2}(x_1) \\ \mu_{A_1^2}(x_2) & \mu_{A_2^2}(x_2) & \cdots & \mu_{A_N^2}(x_2) \\ \cdot & \cdot & \cdots & \\ \cdot & \cdot & \cdots & \\ \cdot & \cdot & \cdots & \\ \mu_{A_1^2}(x_M) & \mu_{A_2^2}(x_M) & \cdots & \mu_{A_N^2}(x_M) \end{bmatrix} \\ \cdot \\ \cdot \\ \cdot \\ \begin{bmatrix} \mu_{A_1^L}(x_1) & \mu_{A_2^L}(x_1) & \cdots & \mu_{A_N^k}(x_1) \\ \mu_{A_1^L}(x_2) & \mu_{A_2^L}(x_2) & \cdots & \mu_{A_N^L}(x_2) \\ \cdot & \cdot & \cdots & \\ \cdot & \cdot & \cdots & \\ \cdot & \cdot & \cdots & \\ \mu_{A_1^L}(x_M) & \mu_{A^L}(x_M) & \cdots & \mu_{A_N^L}(x_M) \end{bmatrix} \end{bmatrix} \qquad (1)$$

Similarly, on the other side, each column of the front sheet with its back columns will form $\boldsymbol{N}$ Level cuts of $\boldsymbol{M} \times \boldsymbol{L}$ matrix termed as **Type-2 Level Cuts**.

These left to right slices are level cuts of type 2, can and be expressed on a two-dimensional page by giving step by step variation to $\boldsymbol{j}$ and expressed as,

$$\boldsymbol{B} = \begin{bmatrix} \begin{bmatrix} \mu_{A_1^1}(x_1) & \mu_{A_1^2}(x_1) & \cdots & \mu_{A_1^L}(x_1) \\ \mu_{A_1^1}(x_2) & \mu_{A_1^2}(x_2) & \cdots & \mu_{A_1^L}(x_2) \\ \vdots & \vdots & \ddots & \\ \mu_{A_1^1}(x_M) & \mu_{A_1^2}(x_M) & \cdots & \mu_{A_1^L}(x_M) \end{bmatrix} \\ \begin{bmatrix} \mu_{A_2^1}(x_1) & \mu_{A_2^2}(x_1) & \cdots & \mu_{A_2^L}(x_1) \\ \mu_{A_2^1}(x_2) & \mu_{A_2^2}(x_2) & \cdots & \mu_{A_2^L}(x_2) \\ \vdots & \vdots & \ddots & \\ \mu_{A_2^1}(x_M) & \mu_{A_2^2}(x_M) & \cdots & \mu_{A_2^L}(x_M) \end{bmatrix} \\ \vdots \\ \begin{bmatrix} \mu_{A_N^1}(x_1) & \mu_{A_N^2}(x_1) & \cdots & \mu_{A_N^L}(x_1) \\ \mu_{A_N^1}(x_2) & \mu_{A_N^2}(x_2) & \cdots & \mu_{A_N^L}(x_2) \\ \vdots & \vdots & \ddots & \\ \mu_{A_N^1}(x_M) & \mu_{A_N^2}(x_M) & \cdots & \mu_{A_N^L}(x_M) \end{bmatrix} \end{bmatrix} \tag{2}$$

Similarly, **Type-3 Level Cuts** are upper lower and central interior sheets. These sheets form $\boldsymbol{M}$ cuts of $\boldsymbol{N} \times \boldsymbol{L}$matrix.

These top to bottom slices are level sheets of type 3 and can be expressed on a two-dimensional page by giving step by step variation to $\boldsymbol{i}$ in $\boldsymbol{B} = \left[\boldsymbol{\mu}_{A_j^k}(\boldsymbol{x_i})\right]$ and described as,

$$\boldsymbol{B} = \begin{bmatrix} \begin{bmatrix} \mu_{A_1^1}(x_1) & \mu_{A_1^2}(x_1) & \cdots & \mu_{A_1^L}(x_1) \\ \mu_{A_2^1}(x_1) & \mu_{A_2^2}(x_1) & \cdots & \mu_{A_2^L}(x_1) \\ \vdots & \vdots & \ddots & \vdots \\ \mu_{A_N^1}(x_1) & \mu_{A_N^2}(x_1) & \cdots & \mu_{A_N^L}(x_1) \end{bmatrix} \\ \begin{bmatrix} \mu_{A_1^1}(x_2) & \mu_{A_1^2}(x_2) & \cdots & \mu_{A_1^L}(x_2) \\ \mu_{A_2^1}(x_2) & \mu_{A_2^2}(x_2) & \cdots & \mu_{A_2^L}(x_2) \\ \vdots & \vdots & \ddots & \vdots \\ \mu_{A_N^1}(x_2) & \mu_{A_N^2}(x_2) & \cdots & \mu_{A_N^L}(x_2) \end{bmatrix} \\ \vdots \\ \begin{bmatrix} \mu_{A_1^1}(x_M) & \mu_{A_1^2}(x_M) & \cdots & \mu_{A_1^L}(x_M) \\ \mu_{A_2^1}(x_M) & \mu_{A_2^2}(x_M) & \cdots & \mu_{A_2^L}(x_M) \\ \vdots & \vdots & \ddots & \vdots \\ \mu_{A_N^1}(x_M) & \mu_{A_N^2}(x_M) & \cdots & \mu_{A_N^L}(x_M) \end{bmatrix} \end{bmatrix} \quad (3)$$

In $\boldsymbol{\mu}_{A_j^k}(\boldsymbol{x}_i)$ $\boldsymbol{i}$ provides row-wise variations of subjects $\boldsymbol{j}$ provides column-wise variations of attributes, and $\boldsymbol{k}$ provides variations of sub-attributes in sheets of matrices. These are hyper matrices $[\boldsymbol{a}_{ijk}]$ In the journal of order $\boldsymbol{M} \times \boldsymbol{N} \times \boldsymbol{L}$

Definition 3.11 (Hyper-Super Matrix): Such a Hyper matrix whose elements are presented in matrix form is titled as Hyper-Super Matrix. These matrices are expressed by using more than two variation indices. These Hyper Super matrices have multiple sheets of ordinary matrices, i.e., = $\begin{bmatrix} [\boldsymbol{a}_{ijk}] & [\boldsymbol{b}_{ijk}] \\ [\boldsymbol{c}_{ijk}] & [\boldsymbol{d}_{ijk}] \end{bmatrix}$, B is an example of a hyper-super matrix. The elements of this Matrix ($[\boldsymbol{a}_{ijk}]$, $[\boldsymbol{b}_{ijk}]$, $[\boldsymbol{c}_{ijk}]$ and $[\boldsymbol{d}_{ijk}]$) are hyper-matrices.

* Tthe literature regarding to Hyper super matrices like operators properties application would be explored in upcoming articles)

A Hyper-Super Matrix, used to express a Plithogenic Subjective Hypersoft Set, is an example of Hyper-Super Matrix. The elements of this Matrix are matrices. These hyper-super matrices have multiple packets of parallel sheets and have more than three variations ($\boldsymbol{d} > \boldsymbol{3}$) .

The examples of HSS-Matrix are **Plithogenic Crisp/Fuzzy/Intuitionistic/Neutrosophic Hyper-Super-Soft matrices**

A detailed illustration of the Plithogenic Crisp/fuzzy Hyper-Super-Soft Matrix is presented here, and the rest of the introduced matrices would be presented in upcoming articles.

3.12 (Plithogenic Crisp/Fuzzy/Intuitionistic/Neutrosophic Subjective-Hyper-Super-Soft Matrices):

If the matrix representation form of PC/FWHSS is in such a way that the last column matrix of each sheet of the hyper Matrix represents the Combined effect of memberships for sub-attributes with respect to each subject under consideration, then these matrices are called Plithogenic Crisp/Fuzzy/Intuitionistic/Neutrosophic Subjective Hyper-Super-Soft Matrices whose last column of cumulative memberships makes another matrix, we can see here the elements of hyper matrices are some matrices, so these matrices are basically hyper super matrices because these are such sheets of matrices whose elements are matrices. For example, if the combined memberships for Plithogenic Crisp/ Fuzzy Subjective Hyper Soft Set are denoted by $\boldsymbol{\Omega}^{t}_{\mathbf{A}^{k}}(\boldsymbol{x_i})$ for $\boldsymbol{x_i}$ subject, combined attributes for $\boldsymbol{k}$th level of sub-attributes denoted by $\mathbf{A}^{\boldsymbol{k}}$ and operator used to cumulate memberships for given attributes denoted by $\boldsymbol{t}$, ($\boldsymbol{t} = \mathbf{1,2,3,4}$) one of the four local previously constructed operators (ref), then $\boldsymbol{B}_{\mathbf{S}_t} = \left[[\boldsymbol{b}_{\boldsymbol{ijk}}][\boldsymbol{b}_{\boldsymbol{ikt}}]\right]$ Is Plithogenic Crisp/ Fuzzy Subjective Hyper super Soft Matrix, which will provide categorization of subjects from last $\boldsymbol{col}$ memberships values. As $\boldsymbol{b}_{\boldsymbol{ijk}}$ the elements of Matrix representing crisp/ fuzzy memberships and $\boldsymbol{b}_{\boldsymbol{ikt}}$ representing cumulative crisp/fuzzy memberships for some subject with respect to all given attributes can be expressed as $\boldsymbol{\Omega}^{t}_{\mathbf{A}^{k}}(\boldsymbol{x_i})$.

More generalized form of $\boldsymbol{B}_{\mathbf{S}_t}$ is. $\boldsymbol{B}_{\mathbf{S}_t} = \left[\left[\boldsymbol{\mu}_{A_j^k}(\boldsymbol{x_i})\right]\left[\boldsymbol{\Omega}^{t}_{\mathbf{A}^{k}}(\boldsymbol{x_i})\right]\right]$ and further expanded form with respect to $\boldsymbol{i}$ and $\boldsymbol{j}$ is,

$$\boldsymbol{B}_{\mathbf{S}_t} = \left[\begin{bmatrix} \boldsymbol{\mu}_{A_1^k}(\boldsymbol{x_1}) & \boldsymbol{\mu}_{A_2^k}(\boldsymbol{x_1}) & \cdots & \boldsymbol{\mu}_{A_N^k}(\boldsymbol{x_1}) \\ \boldsymbol{\mu}_{A_1^k}(\boldsymbol{x_2}) & \boldsymbol{\mu}_{A_2^k}(\boldsymbol{x_2}) & \cdots & \boldsymbol{\mu}_{A_N^k}(\boldsymbol{x_2}) \\ \vdots & \vdots & \ddots & \\ \boldsymbol{\mu}_{A_1^k}(\boldsymbol{x_M}) & \boldsymbol{\mu}_{A_2^k}(\boldsymbol{x_M}) & \cdots & \boldsymbol{\mu}_{A_N^k}(\boldsymbol{x_M}) \end{bmatrix} \begin{bmatrix} \boldsymbol{\Omega}^{t}_{\mathbf{A}^{k}}(\boldsymbol{x_1}) \\ \boldsymbol{\Omega}^{t}_{\mathbf{A}^{k}}(\boldsymbol{x_2}) \\ \vdots \\ \boldsymbol{\Omega}^{t}_{\mathbf{A}^{k}}(\boldsymbol{x_M}) \end{bmatrix}\right] \quad (4)$$

In Plithogenic Crisp/ Fuzzy Subjective Hyper super Soft Matrix, $\boldsymbol{B}_{\mathbf{S}_t}$ we give four type of variations

Example: Different cross-sectional cuts of $\boldsymbol{B}_{\mathbf{S}_t}$ a super hyper matrix will form packets of multiple parallel sheets, i.e., multiple combinations of sheets, which is a more expanded universe.

In $\boldsymbol{B}_{\mathbf{S}_t} = \left[\left[\boldsymbol{\mu}_{A_j^k}(\boldsymbol{x_i})\right]\left[\boldsymbol{\Omega}^{t}_{\mathbf{A}^{k}}(\boldsymbol{x_i})\right]\right]$, Variations of $\boldsymbol{i}$ generates rows, variations of $\boldsymbol{j}$ generates columns, variations of $\boldsymbol{k}$ generates combinations of rows and columns as parallel layers of $\boldsymbol{M} \times \boldsymbol{N}$matrix and variation of $\boldsymbol{t} = \mathbf{1,2,}\boldsymbol{P}$ will provide $\boldsymbol{P}$ set of multiple $\boldsymbol{L}$ parallel layers of matrices.

These sets of combinations of sheets may represent parallel universes, on a two-dimensional sheet of paper the expanded form of general hyper super-matrix

$\boldsymbol{B}_{\mathbf{S}_t} = \left[\left[\boldsymbol{\mu}_{A_j^k}(\boldsymbol{x_i})\right]\left[\boldsymbol{\Omega}^{t}_{\mathbf{A}^{k}}(\boldsymbol{x_i})\right]\right]$ can be represented as

$$\begin{bmatrix}
\begin{bmatrix} \mu_{A_1^1}(x_1) & \mu_{A_2^1}(x_1) & \cdots & \mu_{A_N^1}(x_1) \\ \mu_{A_1^1}(x_2) & \mu_{A_2^1}(x_2) & \cdots & \mu_{A_N^1}(x_2) \\ \vdots & \vdots & & \\ \mu_{A_1^1}(x_M) & \mu_{A_2^1}(x_M) & \cdots & \mu_{A_N^1}(x_M) \end{bmatrix}
\begin{bmatrix} \Omega^1_{A^1}(x_1) \\ \Omega^t_{A^1}(x_2) \\ \vdots \\ \Omega^1_{A^1}(x_M) \end{bmatrix} \\
\begin{bmatrix} \mu_{A_1^2}(x_1) & \mu_{A_2^2}(x_1) & \cdots & \mu_{A_N^2}(x_1) \\ \mu_{A_1^2}(x_2) & \mu_{A_2^2}(x_2) & \cdots & \mu_{A_N^2}(x_2) \\ \vdots & \vdots & & \\ \mu_{A_1^2}(x_M) & \mu_{A_2^2}(x_M) & \cdots & \mu_{A_N^2}(x_M) \end{bmatrix}
\begin{bmatrix} \Omega^1_{A^2}(x_1) \\ \Omega^1_{A^2}(x_2) \\ \vdots \\ \Omega^1_{A^2}(x_M) \end{bmatrix} \\
\vdots \\
\begin{bmatrix} \mu_{A_1^L}(x_1) & \mu_{A_2^L}(x_1) & \cdots & \mu_{A_N^k}(x_1) \\ \mu_{A_1^L}(x_2) & \mu_{A_2^L}(x_2) & \cdots & \mu_{A_N^L}(x_2) \\ \vdots & \vdots & & \\ \mu_{A_1^L}(x_M) & \mu_{A^L}(x_M) & \cdots & \mu_{A_N^L}(x_M) \end{bmatrix}
\begin{bmatrix} \Omega^1_{A^L}(x_1) \\ \Omega^1_{A^L}(x_2) \\ \vdots \\ \Omega^1_{A^L}(x_M) \end{bmatrix} \\
\begin{bmatrix} \mu_{A_1^1}(x_1) & \mu_{A_2^1}(x_1) & \cdots & \mu_{A_N^1}(x_1) \\ \mu_{A_1^1}(x_2) & \mu_{A_2^1}(x_2) & \cdots & \mu_{A_N^1}(x_2) \\ \vdots & \vdots & & \\ \mu_{A_1^1}(x_M) & \mu_{A_2^1}(x_M) & \cdots & \mu_{A_N^1}(x_M) \end{bmatrix}
\begin{bmatrix} \Omega^2_{A^1}(x_1) \\ \Omega^2_{A^1}(x_2) \\ \vdots \\ \Omega^2_{A^1}(x_M) \end{bmatrix} \\
\begin{bmatrix} \mu_{A_1^2}(x_1) & \mu_{A_2^2}(x_1) & \cdots & \mu_{A_N^2}(x_1) \\ \mu_{A_1^2}(x_2) & \mu_{A_2^2}(x_2) & \cdots & \mu_{A_N^2}(x_2) \\ \vdots & \vdots & & \\ \mu_{A_1^2}(x_M) & \mu_{A_2^2}(x_M) & \cdots & \mu_{A_N^2}(x_M) \end{bmatrix}
\begin{bmatrix} \Omega^2_{A^2}(x_1) \\ \Omega^2_{A^2}(x_2) \\ \vdots \\ \Omega^2_{A^2}(x_M) \end{bmatrix} \\
\vdots \\
\begin{bmatrix} \mu_{A_1^L}(x_1) & \mu_{A_2^L}(x_1) & \cdots & \mu_{A_N^k}(x_1) \\ \mu_{A_1^L}(x_2) & \mu_{A_2^L}(x_2) & \cdots & \mu_{A_N^L}(x_2) \\ \vdots & \vdots & & \\ \mu_{A_1^L}(x_M) & \mu_{A^L}(x_M) & \cdots & \mu_{A_N^L}(x_M) \end{bmatrix}
\begin{bmatrix} \Omega^2_{A^L}(x_1) \\ \Omega^2_{A^L}(x_2) \\ \vdots \\ \Omega^2_{A^L}(x_M) \end{bmatrix} \\
\vdots \\
\begin{bmatrix} \mu_{A_1^1}(x_1) & \mu_{A_2^1}(x_1) & \cdots & \mu_{A_N^1}(x_1) \\ \mu_{A_1^1}(x_2) & \mu_{A_2^1}(x_2) & \cdots & \mu_{A_N^1}(x_2) \\ \vdots & \vdots & & \\ \mu_{A_1^1}(x_M) & \mu_{A_2^1}(x_M) & \cdots & \mu_{A_N^1}(x_M) \end{bmatrix}
\begin{bmatrix} \Omega^P_{A^1}(x_1) \\ \Omega^P_{A^1}(x_2) \\ \vdots \\ \Omega^P_{A^1}(x_M) \end{bmatrix} \\
\begin{bmatrix} \mu_{A_1^2}(x_1) & \mu_{A_2^2}(x_1) & \cdots & \mu_{A_N^2}(x_1) \\ \mu_{A_1^2}(x_2) & \mu_{A_2^2}(x_2) & \cdots & \mu_{A_N^2}(x_2) \\ \vdots & \vdots & & \\ \mu_{A_1^2}(x_M) & \mu_{A_2^2}(x_M) & \cdots & \mu_{A_N^2}(x_M) \end{bmatrix}
\begin{bmatrix} \Omega^P_{A^2}(x_1) \\ \Omega^P_{A^2}(x_2) \\ \vdots \\ \Omega^P_{A^2}(x_M) \end{bmatrix} \\
\vdots \\
\begin{bmatrix} \mu_{A_1^L}(x_1) & \mu_{A_2^L}(x_1) & \cdots & \mu_{A_N^k}(x_1) \\ \mu_{A_1^L}(x_2) & \mu_{A_2^L}(x_2) & \cdots & \mu_{A_N^L}(x_2) \\ \vdots & \vdots & & \\ \mu_{A_1^L}(x_M) & \mu_{A^L}(x_M) & \cdots & \mu_{A_N^L}(x_M) \end{bmatrix}
\begin{bmatrix} \Omega^P_{A^L}(x_1) \\ \Omega^P_{A^L}(x_2) \\ \vdots \\ \Omega^P_{A^L}(x_M) \end{bmatrix}
\end{bmatrix} \tag{5}$$

It is observed that the multiple sets of parallel sheets (universes) are achieved by giving step by step variation to the fourth index $\boldsymbol{t}$, which is used to represent several aggregation operators to homogenize attributes established as local operators.

It is interesting to note That if we see the hyper-super-soft-matrix on a two-dimensional screen, then columns of sheets of $\boldsymbol{M} \times \boldsymbol{N} \times \boldsymbol{L}$ Matrix will form a general $\boldsymbol{M} \times \boldsymbol{L}$ matrices.

Greater the numeric value of $\boldsymbol{\Omega}^{t}_{\mathbf{A}^k}(\boldsymbol{x_i})$ better is the $\boldsymbol{x_i}$ subject under consideration. This shows that the whole effect of the Matrix can be visualized by observing the last column. $\left[\left[\boldsymbol{\mu}_{A_j^k}(\boldsymbol{x_i})\right]\left[\boldsymbol{\Omega}^{t}_{\mathbf{A}^k}(\boldsymbol{x_i})\right]\right]$ is representing a Hyper Super Soft Matrix and for subjective categorization with respect to different levels of sub-attributes is obtained by keeping in view a single sheet at a time, to achieve the purpose we shall use one of the sheets of this hyper super-matrix $\left[\left[\boldsymbol{\mu}_{A_j^k}(\boldsymbol{x_i})\right]\left[\boldsymbol{\Omega}^{t}_{\mathbf{A}^k}(\boldsymbol{x_i})\right]\right]$ by fixing $\boldsymbol{k}$ for a specific level of the attribute, and then we will vary " $\boldsymbol{t}$ " which is used to represent a particular aggregation operator, and this operator could be used to cumulate attributes effect at a certain level.

$\boldsymbol{B}_{\mathbf{S}_t} = \left[\left[\boldsymbol{\mu}_{A_j^1}(\boldsymbol{x_i})\right]\left[\boldsymbol{\Omega}^{\mathbf{1}}_{\mathbf{A}^\alpha}(\boldsymbol{x_i})\right]\right]$ will represent a single sheet of hyper super-matrix for $\boldsymbol{\alpha}$ universe at a fixed numeric value of attribute of first level $\boldsymbol{k} = \mathbf{1}$ and using particular operator $\boldsymbol{t} = \mathbf{1}$.

By fixing a combination of attributes, sub-attributes and subjects means considering one of the parallel sheets (one parallel universe) of the hyper-super-soft- Matrix by using Plithogenic Fuzzy Subjective Hyper Super Soft Matrices.

One may decide which subject is superior with respect to all attributes by analyzing the last column of the sheet (case of local Ranking), the subject which corresponds to the greater numeric value in the last column will be considered better, and then we will write them in descending order.

4. Application

Local, Global and Universal Subjective Ranking Model

In this section, we utilize the local operators constructed in an earlier published article ([26]) in formulation of Subjective Ranking model in plithogenic Crisp/Fuzzy environment.

1. The specialty of this model is that it offers the classification of subjects by observing them through several angles of vision.

2. Different angles of visions can be expressed by choosing a suitable local operator

3. This model offers classification in many environments where every environment has its ambiguity and uncertainty level.

4. This Subjective ranking model provides the ranking from micro-universe to the macro universe level.

5. Initially, this model provides the internal Ranking named "Local Subjective ranking" (classification of subjects of micro-universe). In Local Ranking, one perticular local operators is used for the classification purpose.

6. At the next stage, this model offers an external classification of subjects named "Global Subjective Ranking." In Global Ranking, we combine several operators to formulate an expression that reflected through many angles of visions.

7. This model offers the third type of subjective Ranking named "Universal Subjective Ranking (case of macro universe Classification)

8. This model would provide extreme and neutral values of universes that would be helpful to find out the optimal and neutral behaviors of all types of universe micro to macro levels.

9. At the last stage, it provides a measure of the authority of classification by using the frequency matrix.

Initially, we consider the case of plithogenic Crisp/fuzzy Subjective-Hyper-Super-Soft matrices for classification of subjects. Later we may generalize these models in Plithogenic Intuitionistic and Plithogenic Neutrosophic environments. Describe the subjective ranking model in the given steps.

Step 1. Construction of Universe: Consider universe of discourse $\boldsymbol{U} = \{\boldsymbol{x_i}\}$ $\boldsymbol{i} = \mathbf{1,2,3,} \ldots \boldsymbol{P}$. Consider some attributes and subjects needed to be classified where attributes are $\boldsymbol{A_j^k}$ $\boldsymbol{j} = \mathbf{1,2,3,} \ldots \boldsymbol{N}$ and $\boldsymbol{k} = \mathbf{1,2,} \ldots \boldsymbol{L}$ represents numeric values (levels) of $\boldsymbol{A_j}$, and subjects are $\boldsymbol{T} = \{\boldsymbol{x_i}\} \subset \boldsymbol{U}$ where $\boldsymbol{i}$ can take some values from $\mathbf{1}$ to $\boldsymbol{M}$ such that $\boldsymbol{M} \leq \boldsymbol{p}$ and define mappings $\boldsymbol{F}$ and $\boldsymbol{G}$ such that,

$$\boldsymbol{F}: \boldsymbol{A}_1^k \times \boldsymbol{A}_2^k \times \boldsymbol{A}_3^k \times \ldots \times \boldsymbol{A}_N^k \rightarrow \boldsymbol{P}(\boldsymbol{U}) \quad \text{For some fixed } \boldsymbol{k} \text{ (level } \mathbf{1}\text{)}$$

$$\boldsymbol{G}: \boldsymbol{A}_1^k \times \boldsymbol{A}_2^k \times \boldsymbol{A}_3^k \times \ldots \times \boldsymbol{A}_N^k \rightarrow \boldsymbol{P}(\boldsymbol{U}) \quad \text{For some different fixed } \boldsymbol{k} \text{ (level } \mathbf{2}\text{)}$$

Step 2. Plithogenic Crisp/Fuzzy Hyper Soft Matrix: Write the data or information (Memberships) in the form of Plithogenic Fuzzy Hyper Soft Matrix $\boldsymbol{B} = \left[\boldsymbol{\mu}_{A_j^k}(\boldsymbol{x_i})\right]$

Step 3. Plithogenic Crisp/Fuzzy Subjective Hyper-super-Soft Matrix: By using previously constructed local operators formulate Plithogenic Fuzzy Hyper super Soft Matrix

$$\boldsymbol{B}_{\mathbf{S}_t} = \left[\left[\boldsymbol{\mu}_{A_j^k}(\boldsymbol{x_i})\right]\left[\boldsymbol{\Omega}^{\boldsymbol{t}}_{\mathbf{A}^k}(\boldsymbol{x_i})\right]\right].$$

In formulating $\left[\boldsymbol{\Omega}^{\boldsymbol{t}}_{\mathbf{A}^k}(\boldsymbol{x_i})\right]$ we might be handling some favorable, some neutral and some non-favorable attributes, the non-favorable attributes can be handled by replacing their corresponding memberships with standard compliments i.e. $\boldsymbol{\mu}^{\boldsymbol{c}}_{A_j^k}(\boldsymbol{x_i}) = \mathbf{1} - \boldsymbol{\mu}_{A_j^k}(\boldsymbol{x_i})$, While for neutral attributes, the regular memberships can be used.The cumulative memberships $\boldsymbol{\Omega}^{\boldsymbol{t}}_{\mathbf{A}^k}(\boldsymbol{x_i})$ are obtained by uniting regular memberships for favorable and neutral attributes and compliments of memberships for non-favorable attributes by using local operators.

Step 4. Local Ranking: The Local Ranking would be obtained by observing cumulative memberships $\boldsymbol{\Omega}^{\boldsymbol{t}}_{\mathbf{A}^\alpha}(\boldsymbol{x_i})$ of last column of

$$\boldsymbol{B}_{\mathbf{S}_t} = \left[\left[\boldsymbol{\mu}_{A_j^k}(\boldsymbol{x_i})\right]\left[\boldsymbol{\Omega}^{\boldsymbol{t}}_{\mathbf{A}^k}(\boldsymbol{x_i})\right]\right],$$

Higher the membership value better is the subject corresponding to that membership.

At this stage, the classification of all sheets or one selected sheet would be provided according to the requirement. The process can be stopped here if transparent Ranking is achieved.

If there are some ties or ambiguities, would be removed in the next step, and more transparent Ranking can be provided with authenticity measurement.

Step 5. Global Ranking: Final Global Subjective ordering can be provided by using the Frequency matrix.

$$\boldsymbol{F_{ij}} = \begin{array}{c} x_1 \\ x_2 \\ \cdot \\ \cdot \\ \cdot \\ x_M \end{array} \begin{bmatrix} f_{11} & f_{12} & \cdots & f_{1M} \\ f_{21} & f_{22} & \cdots & f_{2M} \\ \cdot & \cdot & \cdots & \\ \cdot & \cdot & \cdots & \\ \cdot & \cdot & \cdots & \\ f_{M1} & f_{M2} & \cdots & f_{MM} \end{bmatrix} \tag{6}$$

In $\boldsymbol{F_{ij}}$ the elements of the first column will represent the frequency of obtaining the first position of given subjects. The elements of the second column will represent the frequency of obtaining the second position, and so on. To find out which subject would attain the first position we observe the entries of the first column of $\boldsymbol{F_{ij}}$ the subject corresponding to the highest value of the first column attains the first position, and then we delete the first position and the subject who have achieved that position. Afterward, for the selection of the second position, add the remaining frequencies of first positions in the frequencies of the second column and then look for the highest frequency in that column.

Once second position is decided to delete column and row of that position correspondingly and continue the procedure until the latest position is assigned

Step 6. Authenticity Measure of Global Ranking: We may check the authenticity in the last step by taking ratios.

percentage authenticity of j $_{th}$ position for i $_{th}$ subject

$$= \frac{\textbf{Highest frequency of } j_{th} \textbf{ position}}{\textbf{Total frequency of } j_{th} \textbf{ position}} \times \mathbf{100} = \frac{\max_i (f_{ij})}{\sum_i (f_{ij})} \times \mathbf{100} \tag{7}$$

Step 7. Universal Ranking: If comparison of universes is required we may provide by combining cumulative memberships of last column for fixed $\boldsymbol{k}$ and $\boldsymbol{t}$ for each universe. The universe having the largest cumulative memberships is considered as a better universe and then writes these cumulative memberships in a descending order.

$$\boldsymbol{\Omega_{k1}} = \max_i \left[\boldsymbol{\Omega}_{\mathrm{A}^k}(\boldsymbol{x_i})\right] \tag{8}$$

$$\boldsymbol{\Omega_{k2}} = \max_i \left[\boldsymbol{\Omega}^2_{\mathrm{A}^k}(\boldsymbol{x_i})\right] \tag{9}$$

$$\boldsymbol{\Omega_{k3}} = \frac{\sum_i \left[\boldsymbol{\Omega}^3_{\mathrm{A}^k}(x_i)\right]}{\boldsymbol{n}} \tag{10}$$

5 Local operators for Construction of Plithogenic crisp/fuzzy Subjective Hyper Super Soft matrices.

These local operators for the plithogenic Crisp/Fuzzy hypersoft Set can be utilized to formulate Plithogenic Crisp/Fuzzy Subjective Hyper Super Soft Matrices.

By using local disjunction, local conjunction and local averaging operators we developed a combined (whole) memberships $\Omega^{t}_{A^{\alpha}}(x_i)$ for Plithogenic Hyper-Soft-Set. By applying local aggregation operators on Crisp/fuzzy Subjective hyper soft matrices $B = \left[\mu_{A_j^k}(x_i)\right]$ the last column of cumulative memberships $\Omega^{t}_{A}(x_i)$ is obtained. To achieve the purpose we use three local operators, $t = 1$ is used for **max**, $t = 2$ is used for **min**, and $t = 3$ is used for averaging operator described as,

$$\cup_i \left(\mu_{A_j^k}(x_i)\right) = \underset{j}{Max}\left(\mu_{A_j^k}(x_i)\right) = \Omega^{1}_{A}(x_i) \qquad (12)$$

$$\cap_i \left(\mu_{A_j^k}(x_i)\right) = \underset{j}{Min}\left(\mu_{A_j^k}(x_i)\right) = \Omega^{2}_{A}(x_i) \qquad (13)$$

$$\Gamma_i \left(\mu_{A_j^k}(x_i)\right) = \frac{\sum_j \left(\mu_{A_j^k}(x_i)\right)}{M} = \Omega^{3}_{A}(x_i) \qquad (14)$$

6 Application of Subjective ranking model in Crisp environment

Numerical Example:

To achieve the purpose, we will develop plithogenic Fuzzy hyper-soft matrix B for two Plithogenic crisp Hyper Soft Sets with α Combination and β Combination of attributes, i.e., for α and β universes.

By choosing different numeric values of k consider α Combination of attributes for F and β Combination of attributes for G .

Step 1. Construction of Universe: Consider U = is a set of five candidates in the Mathematics department. $T = \{\mathbf{Peter}, \mathbf{Ain}, \mathbf{kitty}\} \subset U$ is the Set of three candidates, $T = \{\mathbf{Peter}, \mathbf{Ain}, \mathbf{kitty}\}$ Who have Applied for the selection of the post of Assistant professor are our subjects required to be classified with respect to following A_j^k attributes and sub-attributes.

A_1^k = Subject skill area with numeric values, $k = 1, 2$

A_1^1 = Pure Mathematics, A_1^2 = Applied Mathematics

A_2^k = Qualification with numeric values, $k = 1, 2$

A_2^1 = Qualification like MS or Equivalent, A_2^2 = Higher Qualification like Ph.D. or Equivalent

A_3^k = Teaching experience with numeric values, $k = 1, 2$

A_3^1 = Five years or less, A_3^2 = More than Five years

A_4^k = Age , with numeric values $k = 1, 2$

A_4^1 = Age is less than forty years, A_4^2 = Age is more than forty years

Consider mapping F and G such that,

$F: A_1^k \times A_2^k \times A_3^k \times \dots \times A_N^k \rightarrow P(U)$ (choosing some numeric values of A_j^k, $k \in (1, L)$)

$G: A_1^k \times A_2^k \times A_3^k \times \dots \times A_N^k \rightarrow P(U)$ (choosing some different numeric values of A_j^k $k \in (1, L)$ Let the these candidates are considered as subjects under consideration, and attributes are A_j^k $j = 1, 2, 3, 4$ and $k = 1, 2, 3$

we are looking for a single employ amongst the three candidates, $T = \{\textbf{Peter}, \textbf{Aina}, \textbf{kitty}\} = \{x_1, x_2, x_3\}$, where x_1, x_2, x_3 represent x_i subjects under consideration, initially we consider the first level for $k = 1$ i.e.

1. Subject skill area: Pure Mathematics $j = 1, k = 1$

2. Qualification: Qualification like MS or Equivalent $j = 2, k = 1$

3. Teaching experience: Five years or less $j = 3, k = 1$

4. Age: Age required is less than forty years $j = 4, k = 1$

Let the Function is F is defined as,

$F(A_1^1, A_2^1, A_3^1, A_4^1) = \{x_1, x_2, x_3\}$ let we name $A_1^1, A_2^1, A_3^1, A_4^1$ as α Combination representing the first level for $k = 1$

Let the Function is G is defined as,

$$G: A_1^k \times A_2^k \times A_3^k \times A_4^k \rightarrow P(U)$$

We are looking for another employ amongst these three candidates for the Category of applied mathematics with a different combination of attributes say β Combination for next level ($k = 2$).

1. Subject skill area: Applied Mathematics $j = 1, k = 2$

2. Qualification: Higher Qualification like Ph.D. or equivalent $j = 2, k = 2$

3. Teaching experience: More than five years $j = 3, k = 2$

4. Age: Age is more than forty years $j = 3, k = 2$

$G(A_1^2, A_2^2, A_3^2, A_4^2) = \{x_1, x_2, x_3\}$ let we name $A_1^2, A_2^2, A_3^2, A_4^2$ as β Combination.with respect to. $T = \{x_1, x_2, x_3\}$ Have memberships In PCHSS, PFHSS, which are assigned by decision-makers in crisp environment memberships, are considered as $\mathbf{1}$, or $\mathbf{0}$.

While in a fuzzy environment, memberships are assigned by decision-makers between $\mathbf{0}$ and $\mathbf{1}$ by using five-point scale linguistic chart.

Step 2. Plithogenic Crisp Hyper Soft Matrix:

With respect to $T = \{x_1, x_2, x_3\}$ memberships In PCHSS are,

$$F(A_1^1, A_2^1, A_3^1, A_4^1) = F(\alpha) = \{x_1(1, 1, 1, 1), x_2(1, 1, 1, 1), x_3(1, 1, 1, 1)\}$$

$$G(A_1^2, A_2^2, A_3^2, A_4^2) = F(\beta) = \{x_1(0, 0, 0, 0), x_2(0, 0, 0, 0), x_3(0, 0, 0,)\}$$

The Plithogenic Crisp Hyper Soft matrix $B = \left[\mu_{A_j^k}(x_i)\right]$ 1for $k = 1, j = 1, 2, 3, 4$ and $i = 1, 2, 3$ i.e. α Combination (first level sheet) and $k = 2, j = 1, 2, 3, 4$ and $i = 1, 2, 3$ i.e.β Combination (second level sheet) will be represented as $B = \left[\mu_{A_j^k}(x_i)\right]$

$$B = \begin{bmatrix} \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix} \\ \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \end{bmatrix} \tag{14}$$

In Crisp Hyper Soft matrix $\boldsymbol{B}$ the first level sheet for $\boldsymbol{\alpha}$ Combination of attribute values all memberships are one means the members fulfill the first level requirements fully but for second-level sheet i.e.$\boldsymbol{\beta}$ combination members may fulfill the requirements partially, but the crisp universe don't deal with partial belongingness, so all memberships are zero.

Step 3. Plithogenic Crisp Subjective Hyper-Super-Soft Matrix:

The Plithogenic Crisp Hyper Soft matrix $B_{S_t} = \left[\left[\mu_{A_j^k}(x_i)\right]\left[\Omega^{t}_{A^k}(x_i)\right]\right]$ by using eq. **5** for $= 1,2,3$, $j = 1,2,3,4$ and $k = 1$, i.e., .α Combination (first level sheet), $k = 2$, β Combination (second level sheet) $t = 1$ (disjunction operator) first set of two-level sheets, $t = 2$ (conjunction operator) second Set of two-level sheets will be represented as,

$$B_{S_t} = \begin{bmatrix} \begin{bmatrix} \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix} & \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \\ \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} & \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \end{bmatrix} \\ \begin{bmatrix} \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix} & \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \\ \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} & \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \end{bmatrix} \end{bmatrix} \tag{15}$$

It is obvious from this HSS-Matrix that in Crisp environment B_{St} representing trivial cases where the subjective categorization is not visible because all values in the last column are the same, so we in future Classifications we will use Plithogenic Fuzzy subjective Hyper-soft matrices for subjective categorization. Further, it is observed that the crisp universe case is trivial, where the categorization is not possible. So next steps would not proceed.

Note: In the next levels, we will consider only a fuzzy environment to achieve transparent classification.

Application of Subjective ranking model in Fuzzy environment

Step1. Construction of Universe: To formulate Plithogenic Fuzzy Hyper Soft Matrix Step 1 and Step 2 are the same as already described in the crisp universe case.

Step 2. Plithogenic Fuzzy Hyper-Super-Soft Matrix:

Memberships of $T = \{x_1 x_2 x_3\}$ In PFHSS with respect to given attribute memberships are assigned by the decision-makers by using linguistic five point scale method,[1], [18], [34] [35]

$$F\left(A\ {}^{1}_{1}A\ {}^{1}_{2}A\ {}^{1}_{3}A\ {}^{1}_{4}\right) = F(\alpha) = \{x_1(03,06,04,05\qquad)x\ _2(04,05,03,01\qquad)x\ _3(06,03,03,07\qquad)\}$$

$$G\left(A^{2}_{1}A\ {}^{2}_{2}A\ {}^{2}_{3}A\ {}^{2}_{4}\right) = G(\beta) = \{x_1(05,03,02,06\qquad)x\ _2(06,07,08,04\qquad)x\ _3(07,07,05,09\qquad)\}$$

The Plithogenic Fuzzy Hyper Soft matrix $\boldsymbol{B} = \left[\mu_{A_j^k}(x_i)\right]$ by using eq.1 for $\boldsymbol{i} = \mathbf{1,2,3}$,

$\boldsymbol{j} = \mathbf{1,2,3,4}$ and $\boldsymbol{k} = \mathbf{1}$, i.e.$\boldsymbol{\alpha}$ Combination (first level sheet) , $\boldsymbol{k} = \mathbf{2}$, $\boldsymbol{\beta}$ Combination (second level sheet) will be represented as,

$$\boldsymbol{B} = \begin{bmatrix} \begin{bmatrix} 03 & 06 & 04 & 05 \\ 04 & 05 & 03 & 01 \\ 06 & 03 & 03 & 07 \end{bmatrix} \\ \begin{bmatrix} 05 & 03 & 02 & 06 \\ 06 & 07 & 08 & 04 \\ 07 & 07 & 05 & 09 \end{bmatrix} \end{bmatrix} \tag{16}$$

Step 3. Plithogenic Fuzzy Subjective Hyper Super Soft Matrix:

The Plithogenic Fuzzy Hyper Super Soft matrix $\boldsymbol{B}_{\boldsymbol{S}_t} = \left[\left[\mu_{\ A_j^k}(x_i)\right]\left[\Omega\ {}^{t}_{\mathrm{A}^k}(x_i)\right]\right]$ by using eq. 5, 11, 12, and 13 for $\boldsymbol{i} = \mathbf{1,2,3}$, $\boldsymbol{j} = \mathbf{1,2,3,4}$ and $\boldsymbol{k} = \mathbf{1}$, i.e., $\boldsymbol{\alpha}$ combination (first level sheet), $\boldsymbol{k} = \mathbf{2}$, $\boldsymbol{\beta}$ combination (second level sheet), $\boldsymbol{t} = \mathbf{1}$,(disjunction operator) first set of two-level sheets, $\boldsymbol{t} = \mathbf{2}$,(conjunction operator) second set of two-level sheets, $\boldsymbol{t} = \mathbf{3}$(averaging operator) a third set of two-level sheets will be represented as,

$$\boldsymbol{B}_{\boldsymbol{S}_t} = \begin{bmatrix} \begin{bmatrix} \begin{bmatrix} 03 & 06 & 04 & 05 \\ 04 & 05 & 03 & 01 \\ 06 & 03 & 03 & 07 \end{bmatrix} & \begin{bmatrix} 06 \\ 05 \\ 07 \end{bmatrix} \\ \begin{bmatrix} 05 & 03 & 02 & 06 \\ 06 & 07 & 08 & 04 \\ 07 & 07 & 05 & 09 \end{bmatrix} & \begin{bmatrix} 06 \\ 08 \\ 09 \end{bmatrix} \end{bmatrix} \\ \begin{bmatrix} \begin{bmatrix} 03 & 06 & 04 & 05 \\ 04 & 05 & 03 & 01 \\ 06 & 03 & 03 & 07 \end{bmatrix} & \begin{bmatrix} 03 \\ 01 \\ 03 \end{bmatrix} \\ \begin{bmatrix} 05 & 03 & 02 & 06 \\ 06 & 07 & 08 & 04 \\ 07 & 07 & 05 & 09 \end{bmatrix} & \begin{bmatrix} 02 \\ 04 \\ 05 \end{bmatrix} \end{bmatrix} \\ \begin{bmatrix} \begin{bmatrix} 03 & 06 & 04 & 05 \\ 04 & 05 & 03 & 01 \\ 06 & 03 & 03 & 07 \end{bmatrix} & \begin{bmatrix} 045 \\ 035 \\ 0.475 \end{bmatrix} \\ \begin{bmatrix} 05 & 03 & 02 & 06 \\ 06 & 07 & 08 & 04 \\ 07 & 07 & 05 & 09 \end{bmatrix} & \begin{bmatrix} 04 \\ 0625 \\ 07 \end{bmatrix} \end{bmatrix} \end{bmatrix} \tag{17}$$

Step 4. Local Subjective Ranking:

$\boldsymbol{B}_{\boldsymbol{S}_{1\alpha}}$ In eq. 17 provides the local ordering of subjects for $\boldsymbol{\alpha}$ Combination of attributes or $\boldsymbol{\alpha}$ universe (first level sheet for $\boldsymbol{k} = \mathbf{1}$) by using the first operator ($\boldsymbol{t} = \mathbf{1}$) given in eq. $\mathbf{11}$ as

$$x_3 \succ x_1 \succ x_2 \tag{18}$$

$\boldsymbol{B}_{\boldsymbol{S}_{2\alpha}}$in eq. 17 provides the local ordering of subjects for $\boldsymbol{\alpha}$ Combination of attributes or $\boldsymbol{\alpha}$ universe (first level sheet for $\boldsymbol{k} = \mathbf{1}$) by using second operator ($\boldsymbol{t} = \mathbf{2}$) given in eq. $\mathbf{12}$ as

$$\boldsymbol{x}_3 = \boldsymbol{x}_1 \succ \boldsymbol{x}_2 \qquad (19)$$

which shows a tie between $\boldsymbol{x}_1$ and $\boldsymbol{x}_3$ which can be removed in the final Global Ranking of subjects by using frequency matrix $\boldsymbol{F}_{ij}$

$\boldsymbol{B}_{\boldsymbol{S}_{3\alpha}}$ In eq. 17 provides the local ordering of subjects for $\boldsymbol{\alpha}$ Combination of attributes ($\boldsymbol{\alpha}$ universe) by using a third operator ($\boldsymbol{t} = \mathbf{3}$) given in eq. 13 as

$$\boldsymbol{x}_3 \succ \boldsymbol{x}_1 \succ \boldsymbol{x}_2 \qquad (20)$$

Similarly

$\boldsymbol{B}_{\boldsymbol{S}_{1\beta}}$ in eq. $\mathbf{17}$ provides the local ordering of subjects for $\boldsymbol{\beta}$ Combination of attributes

($\boldsymbol{\beta}$ universe) by using first operator ($\boldsymbol{t} = \mathbf{1}$) given in eq. 11 as

$$\boldsymbol{x}_3 \succ \boldsymbol{x}_2 \succ \boldsymbol{x}_1 \qquad (21)$$

$\boldsymbol{B}_{\boldsymbol{S}_{2\beta}}$ in eq.. 17provides the local ordering of subjects for $\boldsymbol{\beta}$ Combination of attributes

($\boldsymbol{\beta}$ universe) by using the second operator ($\boldsymbol{t} = \mathbf{2}$) given in eq. 12as

$$\boldsymbol{x}_3 \succ \boldsymbol{x}_2 \succ \boldsymbol{x}_1 \qquad (22)$$

$\boldsymbol{B}_{\boldsymbol{S}_{3\beta}}$ in eq.. 17 provides the local ordering of subjects for $\boldsymbol{\beta}$ Combination of attributes

($\boldsymbol{\beta}$ universe) by using the third operator ($\boldsymbol{t} = \mathbf{3}$) given in eq. 13 as

$$\boldsymbol{x}_3 \succ \boldsymbol{x}_2 \succ \boldsymbol{x}_1 \qquad (23)$$

We can observe that the three local ordering of subjects under consideration in $\boldsymbol{\alpha}$ Combination can be provided by using three operators described in eq., 11, 13, 12, to find cumulative memberships for $\boldsymbol{\alpha}$ Combination first, then for $\boldsymbol{\beta}$ Combination and then writing in descending order.

Step 5. Global Subjective Ranking :

Final global ordering of subjects can be provided by using frequency matrix $\boldsymbol{F}_{ij}$described in eq.. 6, $\mathbf{18}$,19, and 20

In the frequency matrix $\boldsymbol{F}^{\alpha}_{i\,j}$which is a square matrix of frequencies positions for first level sheet $\boldsymbol{\alpha}$ Combination the $\boldsymbol{colF}^{\alpha}_{i\,j}$represents frequencies of

positions, i.e., the entries of the first column represents the frequencies of attaining first position by given subjects while $\boldsymbol{rowF}_{ij}$represents subjects.

$$F_{ij}^{\alpha} = \begin{bmatrix} 1 & 2 & 0 \\ 0 & 1 & 2 \\ 3 & 0 & 0 \end{bmatrix}$$

Global Ranking of subjects from F_{ij}^{α} is $x_3 \succ x_1 \succ x_2$ (24)

The frequency matrix for β universe is F_{ij}^{β}

$$F_{ij}^{\beta} = \begin{bmatrix} 0 & 0 & 3 \\ 0 & 3 & 0 \\ 3 & 0 & 0 \end{bmatrix}$$

Global Ranking of subjects from F_{ij}^{β} is $x_3 \succ x_2 \succ x_1$ (25)

We can observe that in both universes x_3 is the superior subject which means that in both Combination of attributes and sub-attributes x_3 is reflecting the best.

Step 6. Authenticity measure of final Ranking:

Percentage authenticity for first level α universe

By using eq. 7 :

Percentage authenticity of first position for $x_3 = 75\%$

Percentage authenticity of first position for $x_2 = 66.67\%$

Percentage authenticity of first position for $x_1 = 100\%$

Percentage authenticity for first level β universe

By using eq. 7 :

Percentage authenticity of first position for $x_3 = 100\%$

Percentage authenticity of first position for $x_2 = 100\%$

Percentage authenticity of first position for $x_1 = 100\%$

Step 7. Final Universal Ranking:

The final ordering of universes provided as.

Maximum Universal Memberships of α and β universes:

taking $k = 1, 2$ for α and β universes and fixing $t = 1$ as described in eq. 8, we can provide maximum universal memberships of all given subjects with respect to attributes,

$$\Omega_{A^1}^{1}(X) = 0.7, \Omega_{A^2}^{1}(X) = 0.9 \qquad (26)$$

Where X representing merged subjects, we can see by using operator $t = 1$, β universe is better than α universe.

Minimum Universal Memberships of $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}$ universes:

Taking $\boldsymbol{k = 1,2}$ for $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}$ universes and fixing $\boldsymbol{t = 2}$ as described in eq.. 9 we can provide minimum universal memberships of all given subjects with respect to attributes,

$$\boldsymbol{\Omega^{2}_{A^1}(X) = 0.1, \Omega^{2}_{A^2}(X) = 0.2} \qquad (27)$$

We can see by using operator $\boldsymbol{t = 2}$, $\boldsymbol{\beta}$ universe is better than $\boldsymbol{\alpha}$ universe.

Average Universal Memberships of $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}$ universes:

similarly, taking $\boldsymbol{k = 1,2}$ for $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}$ universes and fixing $\boldsymbol{t = 3}$ as described in eq.. 10, we can provide average universal memberships of all given subjects with respect to attributes,

$$\boldsymbol{\Omega^{3}_{A^1}(X) = 0.425, \Omega^{3}_{A^2}(X) = 0.575} \qquad (28)$$

we can see by using operator $\boldsymbol{t = 3}$, $\boldsymbol{\beta}$ universe is better than $\boldsymbol{\alpha}$ universe

5. Conclusions

1. We can see from expressions 18, 19, 20 the final order in eq. 24 is the most frequently observed order in all these ranking orders which is also observed same in local ordering of β universe (Eq. 21, 22, 23) and is again same in final global ordering of β universe 25, which shows the final global Ranking is transparent and authentic.

2. expressions 26, 27, 28 provides highest, lowest, and average cumulative memberships of universes.

3. In Universal ordering, it is observed that on the Global Universal level, β universe is better than α universe.

1. Local ordering: we can observe local ordering by using the novel plithogenic hyper-super-soft-matrix and local operators. We can judge the performance of some fixed subjects in a particular universe, i.e., one combination or level sheet. This is the case of the inner classification of the universe.

2. Global ordering: We can provide global ordering by considering the performance of some fixed subjects in multiple universes, i.e., different combinations of sub-attribute or level sheets are combined by using the frequency matrix. This is the case of combining several angles of visions in the final decision.

3. Universal ordering: We can compare these universes by combining the cumulative memberships of the last column for each universe. The universe having the largest cumulative average membership is the better and then can write them by descending order.

4. **Extreme Universal Memberships:** We can also find out extreme values of these universes and can judge these subjects in a grand universe which is made by multiple smaller parallel universes so we can choose the best subject from all universes that is the one who is best in most universes or we can select one reality out of multiple parallel realities these facts are useful in the field of artificial intelligence.

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Plithogenic Cognitive Maps in Decision Making

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Abstract

Plithogenic sets introduced by Smarandache (2018) have disclosed new research vistas and this paper introduces the novel concept of plithogenic cognitive maps (PCM) and its applications in decision making. The new approach of defining instantaneous state neutrosophic vector with the confinement of indeterminacy to (0,1] is proposed to quantify the degree of indeterminacy. The resultant vector is obtained by applying instantaneous state vector through the connection matrix together with plithogenic operators comprising the contradiction degrees. The connection matrix is represented as fuzzy matrix and neutrosophic matrix and the resultant vector is determined by applying plithogenic fuzzy operators and plithogenic neutrosophic operators respectively. The obtained results are highly feasible in making the decision as it incorporates the contradiction degree of the conceptual nodes with respect to the dominant node. This research work will certainly pave the way for developing new approaches in decision making using PCM.

Keywords: Plithogenic set, cognitive maps, plithogenic cognitive maps, confinement of indeterminacy, plithogenic fuzzy operators, plithogenic neutrosophic operators.

1.Introduction

Robert Axelrod [1] developed cognitive maps, a decision making tool primarily used in handling the system of making decisions related to political and social frameworks. A cognitive map is a directed graph with nodes and edges representing the concept variables or factors, and it's causal relationships respectively. The intensity of the relationship between two concepts say Ci and Cj is represented by edge weights eij, where eij∈ {-1,0,1}. The value 1 represents the positive influence of Ci over Cj; 0 represents no influence and -1 represents negative influence. The causal relationship between the nodes is represented as a connection matrix. Cognitive maps have a wide range of applications in diverse fields. Nakamura et al [2] used cognitive maps in decision support systems; Chaib-draa [3] developed multi agent system model using cognitive maps; Klein et al [4] developed cognitive maps in decision makers and other broad spectrum of its applications in student modeling whilst knowledge management are discussed by Alejandro Pena [5]. One of the limitations of cognitive maps is modeling decision making in uncertain environment. The concept of fuzzy sets introduced by Zadeh [6] was integrated with cognitive maps by Kosko [7]. Fuzzy cognitive maps (FCM) introduced by Kosko [7], he handled the aspects of uncertainty and impreciseness. In FCM, the edge weights eij ∈ [-1,1] and the connection matrix has fuzzy values. The comprehensive nature of FCM has several applications such as but not limited to the pattern recognition see Papakostas et al [8],in the medicine see Abdollah et al [9], in large manufacturing system see Chrysostomos et al [10], in the field of decision making on farming scenarios see Asmaa Mourhir et al [11]. Atannsov [12] introduced intuitionistic fuzzy sets that deal with membership, non-membership and hesitancy values. Elpiniki Papageorgiou[13] extended FCM to Intuitionistic FCM models to apply in medical diagnosis and this gained momentum in the domain of FCM. IFCM are the extension of FCM models , that are highly applied in diverse fields. The connection matrix of iFCM models has intuitionistic values. Hajek et al [14-15] extended iFCM models into interval –valued IFCM for stock index forecasting and supplier selection.

Smarandache [16] introduced neutrosophic sets that deal with truth, indeterminacy and falsity membership functions. Neutrosophic sets are applied in various domain of the natural science. Mohamed Bisher Zeina [17] applied neutrosophic parameters in Erlang Service Queuing Model and developed neutrosophic event-based queuing model. Malath [18]studied the integration of neutrosophic thick function. Salma [19] developed online analytical processing operations via neutrosophic systems. Neutrosophy is also extended to explore new algebraic structures and concepts. Agboola [20] proposed the introduction of neutrogroups and neutrorings, Riad et al [21] constructed neutrosophic crisp semi separation axioms in neutrosophic crisp topological spaces. Necati Olgun [22] discussed refined neutrosophic R-module, Ibrahim [23] explored the concepts of n Refined Neutrosophic Vector Spaces. Mohammad Hamidi [24] discussed Neutro – BCK-algebra. Neutrosophic research is gaining momentum and it has wide spectrum of applications in decision making. Abdel-Baset [25] developed a novel neutrosophic approach in green supplier selection and a novel decision making approach was developed to diagnose heart diseases using neutrosophic sets. Interval – valued neutrosophic sets are also used in decision making. Neutrosophic cognitive maps (NCM) introduced by Vasantha Kandasamy [26] has incorporated the concept of indeterminacy into edge weights. NCMs are also applied to diverse decision making scenarios by many social researchers. NCMs are widely applied to analyze the causal relationship between the concepts of decision making problems. Nivetha et al [27] developed decagonal linguistic neutrosophic FCM to analyze the risk factors of lifestyle diseases. Nivetha et al [28] made a case study on the problems faced by entrepreneurs using NCM.

In NCM models, the influence of one factor over another is represented by either -1,0,1,I, where I represents indeterminacy. Let us consider a decision making problem comprising of five factors and the respective 5×5 connection matrix has the values {-1,0,1,I}. The initial state vector X, [X= (10000)] has first of the factors in ON position and other factors in off position. X is passed into connection matrix and the resulting vector Z , [Z = (a1,a2,a3,a4,a5)] is updated using threshold operation by replacing ai by 1 if ai ≥ g and ai by 0 if ai < g, (g is an integer) and ai by I, if ai is not an integer. The process is repeated until two updated resultant vectors obtained are same, which is called as the fixed point or limit cycle. The process ends when the fixed point is obtained. In this NCM procedure suppose the fixed point is (1 0 1 1 I), the inference is the first factor has positive influence on third and fourth factors, no impact on second factor and the fifth factor is indeterminant to it. The existence of indeterminacy in the connection matrix and the fixed point does not give us the complete picture of the decision making, but if indeterminacy is quantified then the decision making will be feasible. To make so, the approach of indeterminacy confinement is introduced in this research work. As the connection matrix is based on expert's opinion, the indeterminacy can also be confined to (0,1] based on the expert's opinion. Also in the instantaneous vector any of the factors are in ON position or combination of factors are in ON position say, X = (1 0 1 01) to see the combined effect of the factors. In this article the factors are kept in indeterminate position and it is confined to give a numerical value. This is a new kind of approach in neutrosophic cognitive maps and the NCMs of this kind can be labeled as novel neutrosophic cognitive maps (NNCM).

The NCM and NNCM can also be extended to plithogenic cognitive maps (PCM). Plithogenic sets introduced by Smarandache [29] are the generalization of crisp sets, fuzzy sets, intuitionistic fuzzy sets and neutrosophic sets. The membership values are mainly used to quantify the qualitative aspects. This principle of quantification of qualitative aspects is used as the underlying principle in the construction of plithogenic sets. The degree of appurtenance and the contradiction degree are the two distinctive aspects of plithogenic sets. The concept of plithogeny is extended to plithogenic hypersoft sets by Smarandache [30]. Plithogenic sets are widely used in decision making. Shazia Rana et al [31] extended plithogenic fuzzy hypersoft set to plithogenic fuzzy whole hypersoft set and developed plithogenic ranking model. Nivetha and Smarandache [32] developed concentric plithogenic hypergraphs. Smarandache [33] developed plithogenic n super hypergraph and a novel decision making approach is proposed by Smarandache and Nivetha [34]. Abdel - Baset [35] framed a hybrid plithogenic decision-making approach with quality function deployment for selecting supply chain sustainability. The compatibility of the plithogenic sets motivated us to incorporate the concept of plithogeny to coginitve maps.

This research work proposes the approach of integrating plithogeny to cognitive maps to develop PCM models as the extension of NNCM, NCM, IFCM and FCM. The PCM model follows the underlying methodology of FCM but it incorporates contradiction degree to the factors of the decision making problem. If any of the factors is in ON position, then it becomes the dominant factor and the contradiction degree of the dominant factor with respect to other factors is considered. The instantaneous vector is passed into connection matric and the resultant vector is obtained by applying plithogenic operators. The resultant vector is updated by using the conventional threshold function. PCM models are classified as cognitive maps if the connection matrix is crisp; fuzzy cognitive maps if the connection matrix has fuzzy values; intuitionistic fuzzy cognitive maps if the connection matrix has intuitionistic values and neutrosophic cognitive maps if the connection matrix has neutrosophic values. Thus the proposed PCM models are the generalization of the earlier forms of FCM models. The incorporation of the contradiction degrees will certainly give us new insights in decision making.

The paper is organized as follows: section 2 presents the outlook of PCM; section 3 consists of the methodology of PCM; section 4 comprises of application of PCM in decision making; section 5 discusses the results and concludes the work.

2. Plithogenic Cognitive Maps

A plithogenic cognitive map is a directed graph consisting of nodes and edges representing the concepts and its causal relationship respectively . The contradiction degree of the nodes with respect to the dominant node is determining the fixed point.

Let P_1, P_2,..P_n denotes n nodes of PCM. The directed edge from P_i to P_j represents the association between the two nodes and the edge weights illustrate the intensity of the association between the nodes. If the edge weight $e_{ij} \in \{-1,0,1\}$ then it is plithogenic crisp cognitive maps; if $e_{ij} \in [-1,1]$ then it is plithogenic fuzzy cognitive maps; if $e_{ij} \in \rho([0,1]^2)$ then it is plithogenic intuitionistic cognitive maps , if $e_{ij} \in \rho([0,1]^3)$ then it is plithogenic neutrosophic cognitive maps. Plithogenic connection matrix or adjacency matrix P(E) = (e_{ij}) represents the relation between the nodes. An instantaneous state vector in PCM of the form A = (a_1, a_2,..a_n) represents the ON-OFF-indeterminate position of the node at an instant of time. If a_i = 1 refer to (ON state); a_i = 0 refer to (OFF state) and a_i = I means (Indeterminate state). In PCM, the indeterminate state I is confined to a value belonging to (0,1], which is the extension of NCM.

Let P1,P2,P3 be three nodes of PCM. Let P1 be in ON position and P2, P3 be in off state, then the node P1 is considered to be dominant. The contradiction degrees of other nodes with respect to dominant node are

P1	P2	P3
0	1/3	2/3

The contradiction degree represents the extent of distinctiveness between the two concepts .The value 1/3 and 2/3 is assigned to P2 or P3 based on the perception of decision makers who have choosen the factors of PCM.

The instantaneous vector X = (1 0 0) is passed into P(E) and the vector which is obtained by applying plithogenic operators is Y. The assumed threshold operation is applied to Y and the resultant vector G is obtained. The recurrence of passing the resultant vector G to P(E) if results in repetition of resultant vectors then the limit cycle of the PCM is obtained and the resultant vector is called as fixed point.

3. Methodology of Plithogenic Cognitive Maps

This section presents the algorithm of obtaining the limit cycle of PCM.

Step 1: The factors P_1, P_2,..P_n or the concepts of the decision making problem are decided based on the expert's opinion.

Step 2: The plithogenic connection matrix P(E) of dimension n (the number of concepts) is obtained from the causal relationship between the concepts.

Step 3: The edge weight e_{ij} may belong to {1,0,1}, [-1,1], $\rho([0,1]^2)$, $\rho([0,1]^3)$. The nature of the edge weights determines the type of plithogenic cognitive maps.

Step 4: To determine the effect of one concept say P1, is kept in ON position and the contradiction degree with respect to other concepts are determined.

Step 5: The instantaneous state vector X = (1 0 0 0 0 …0) is passed into connection matrix and by applying the plithogenic operators, a resultant vector is obtained, and updated by applying the threshold operation by assigning 1 to the values (a_i) greater than k, 0 to the values (a_i) lesser than k, where k - a is a suitable positive integer. In this proposed approach the on position of the concepts is threshold with 1 and the indeterminate position of the concepts will be confined with the value C. The value 0 is assigned to the values lesser than 1 and the value 1 is assigned to the values greater than 1.

The plithogenic operators are defined as

$a \wedge_p b = (1-c).[a \wedge_F b] + c.[a \vee_F b]$, where c represents contradiction degree, $a \wedge_F b$ is the t_{norm} defined by ab and $a \vee_F b$ is the t_{conorm} defined by a + b - ab

The plithogenic new neutrosophic operators are defined as

$$a \wedge_p b = \prec a_1 \wedge_p b_1, \frac{1}{2}\left[(a_2 \wedge_p b_2) + (a_2 \vee_p b_2)\right], a_3 \vee_p b_3 \succ,$$

where $a = (a_1, a_2, a_3)$ and $b = (b_1, b_2, b_3)$, $a \vee_p b = (1-c).[a \vee_p b] + c.[a \wedge_p b]$

Step 6 : The updated vector is passed into P(E) and the process is repeated until the fixed point is arrived. The fixed point is the limit cycle of PCM.

4. Application of Plithogenic Cognitive Maps in Decision Making

This section presents the application of plithogenic cognitive maps in decision making. Let us consider a decision making environment where the expert's opinion is constructed to promote the farming sectors to a progressive phase with their suggestive strategies. The following proposed strategies of the experts are taken as the nodes of the PCM.

P1 Encouraging the reverse migration by helping the socially mobilized groups with credit flow.

P2 Supporting vulnerable farming areas with community driven approach

P3 Promoting Farmer's Productive Organizations as transformative agents

P4 Perpetuating gender equalities to create new opportunities for women

P5 Effective use of modern ICT to connect farmers with extension, market and continuous learning

The causal association between the concepts is represented as linguistic variables and it is quantified by triangular fuzzy numbers and the kind of PCM is plithogenic fuzzy cognitive map which is used to determine the fixed point of the dynamical system.

The plithogenic fuzzy connection matrix with linguistic variables is presented as below

$$\begin{array}{c|ccccc} & P_1 & P_2 & P_3 & P_4 & P_5 \\ \hline P_1 & 0 & M & M & L & L \\ P_2 & L & 0 & H & M & L \\ P_3 & H & H & 0 & M & H \\ P_4 & M & M & L & 0 & M \\ P_5 & H & H & VH & M & 0 \end{array}$$

The linguistic variables are quantified by using triangular fuzzy numbers as in Table 4.1

Table 4.1 Quantification of Linguistic variable

Linguistic Variable	**Triangle Fuzzy Number**	**Crisp value**
Very Low	(0,0.1,0.2)	0.1
Low	(0.2,0.3,0.4)	0.3
Medium	(0.4,0.5,0.6)	0.5
High	(0.6,0.7,0.8)	0.7
Very High	(0.8,0.9,1)	0.9

The modified plithogenic fuzzy connection matrix P(E) is

$$\begin{array}{c|ccccc} & P1 & P2 & P3 & P4 & P5 \\ \hline P1 & 0 & 0.5 & 0.5 & 0.3 & 0.3 \\ P2 & 0.3 & 0 & 0.7 & 0.5 & 0.3 \\ P3 & 0.7 & 0.7 & 0 & 0.5 & 0.7 \\ P4 & 0.5 & 0.5 & 0.3 & 0 & 0.5 \\ P5 & 0.7 & 0.7 & 0.9 & 0.5 & 0 \end{array}$$

The graphical representation of the causal association between the concepts are represented in Fig.4.1

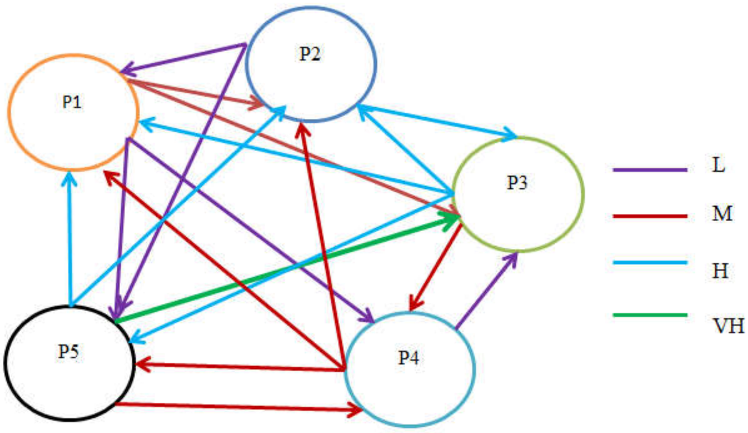

Fig.4.1 Graphical representation of the causal relationship

Case (i) Conventional FCM [7]

Let us consider the conventional approach of FCM without the incorporation of contradiction degree. Let X = (1 0 0 0 0)

X* P(E) = (0 0.5 0.5 0.3 0.3) → (1 0.5 0.5 0.3 0.3) = $X1$

X1 * P(E) = (1.65 2 2.14 1.6 1.25) → (1 1 1 1 1) = X2

X2* P(E) = (2.2 2.4 2.4 1.8 1.8) → ((1 1 1 1 1) = X3

X2 = X3 ------------------------------ (1)

Case (ii) Plithogenic Fuzzy FCM with ON/OFF state of vectors

Let us consider the concept P1 to be in ON position and other factors in off position. The contradiction degrees of the dominant node with respect to other nodes are

P1	P2	P3	P4	P5
0	1/5	2/5	3/5	4/5

Let us consider the instantaneous state vector as X = (1 0 0 0 0)

X *p P(E) = Y , where Y = (a b c d e)

a = Max [1 $\wedge_p$ 0 , 0 $\wedge_p$ 0.3 , 0 $\wedge_p$ 0.7, 0 $\wedge_p$ 0.5, 0 $\wedge_p$ 0.7]

b = Max [1 $\wedge_p$ 0.5 , 0 $\wedge_p$ 0 , 0 $\wedge_p$ 0.7, 0 $\wedge_p$ 0.5, 0 $\wedge_p$ 0.7]

c = Max [1 $\wedge_p$ 0.5 , 0 $\wedge_p$ 0.7 , 0 $\wedge_p$ 0, 0 $\wedge_p$ 0.3, 0 $\wedge_p$ 0.9]

d= Max [1 $\wedge_p$ 0.3 , 0 $\wedge_p$ 0.5 , 0 $\wedge_p$ 0.5, 0 $\wedge_p$ 0, 0 $\wedge_p$ 0.5]

e= Max [1 $\wedge_p$ 0.3 , 0 $\wedge_p$ 0.3 , 0 $\wedge_p$ 0.7, 0 $\wedge_p$ 0.5, 0 $\wedge_p$ 0]

X*p P(E) = (0 0.6 0.7 0.72 0.94) → (1 0.6 0.7 0.72 0.94) = X1

X1 *p P(E) = (0.602 0.6732 0.8588 0.73 0.86) → (1 0.67 0.86 0.73 0.86) = X2

X2 *p P(E) = (0.602 0.6732 0.8588 0.73 0.8868) → (1 0.67 0.86 0.73 0.89) = X3

X3 *p P(E) = (0.623 0.6918 0.8762 0.745 0.8868) → (1 0.69 0.88 0.75 0.89) = X4

X4 *p P(E) = (0.623 0.6918 0.8762 0.745 0.8944) → (1 0.69 0.88 0.75 0.89) = X5

X4 = X5 ----------------------- (2)

Case (iii) Plithogenic Fuzzy FCM with ON/OFF and confined indeterminate I_C state of vectors

Let us consider the concept P1 to be in ON position, P2 be in indeterminate state and other factors in off position. The indeterminate state of the vector here reflects the impact on the concept P2.The contradiction degrees of the dominant node with respect to other nodes are considered as the same.

Let us consider the instantaneous state vector as X = (1 I_C 0 0 0), C = 0.25, the value of indeterminacy is 0.25. i.e X = (1 0.25 0 0 0)

X *p P(E) = (0.075 0.6 0.7 0.72 0.86) → (1 0.25 0.7 0.72 0.86) = X1

X1*p P(E) = (0.602 0.6732 0.8588 0.73 0.86) → (1 0.25 0.86 0.73 0.86) = X2

X2*p P(E) = (0.602 0.673 0.859 0.73 0.887) → (1 0.25 0.86 0.73 0.89) = X3

X3*p P(E) = (0.623 0.6918 0.8762 0.745 0.8868) → (1 0.25 0.88 0.75 0.89) = X4

X4 *p P(E) = (0.623 0.6918 0.8762 0.745 0.8944) → (1 0.25 0.88 0.75 0.89) = X5

X4 = X5 ----------------------- (3)

Let X = (1 I_C 0 0 0), C = 0.5, the value of indeterminacy is 0.5. i.e X = (1 0.5 0 0 0)

The fixed point is (1 0.25 0.88 0.75 0.89) ------------ (4)

Let X = (1 I_C 0 0 0), C = 0.75, the value of indeterminacy is 0.5. i.e X = (1 0.5 0 0 0)

The fixed point is (1 0.75 0.88 0.75 0.89) ------------ (5)

Let X = (1 I_C 0 0 0), C = 0.95, the value of indeterminacy is 0.5. i.e X = (1 0.5 0 0 0)

The fixed point is (1 0.75 0.86 0.78 0.86) ------------ (6)

Eq. (1) states that the concept P1 has influence on the other concepts, but Eq. (2) states the extent of influence of the concept P1 on other concepts which is the added advantage of using contradiction degree. The confined indeterminate state for various values of c results in different fixed points. The confinement of indeterminacy to the values C= 0.25, 0.5,0.75 have same impact on other factors and also it produce the same effect as P2 concept in OFF state, but as the value of indeterminacy is enhanced , slight variation in the impact are found in Eq. (6). This shows that the OFF state and the indeterminate state of concept P2, when the concept P1 being in ON position produce no much difference.

Case (iv). Plithogenic Neutrosophic FCM with new neutrosophic state.

The new neutrosophic instantaneous state vector is of the form ((T_{P1}, I_{P1}, F_{P1}), 0, 0, 0, 0), where the truth (T_{P1}), indeterminacy (I_{P1}) and falsity (F_{P1}) of the concept P1 to be in ON position is expressed. This representation of the ON position of the vector is highly a pragmatic representation. If I_{P1} and F_{P1} are zero then the concept P1 is highly certain to be in ON position. The indeterminate and off state of the concept P1 can be expressed by keeping the values of I_{P1} and F_{P1} to 1 and keeping the other respective set of value to be zero.

The new neutrosophic state vector (X_N) when the On state of the concept P1 is considered.((1,0,0) (0,1,1) (0,1,1) (0,1,1) (0,1,1) (0,1,1)).The plithogenic neutrosophic matrix $P_N(E)$ is

	P1	P2	P3	P4	P5
P1	(0,1,1)	(0.6,0.3,0.4)	(0.6,0.3,0.4)	(0.3,0.4,0.7)	(0.3,0.4,0.7)
P2	(0.3,0.4,0.7)	(0,1,1)	(0.7,0.2,0.2)	(0.6,0.3,0.4)	(0.3,0.4,0.7)
P3	(0.7,0.2,0.2)	(0.7,0.2,0.2)	(0,1,1)	(0.6,0.3,0.4)	(0.7,0.2,0.2)
P4	(0.6,0.3,0.4)	(0.6,0.3,0.4)	(0.3,0.4,0.7)	(0,1,1)	(0.6,0.3,0.4)
P5	(0.7,0.2,0.2)	(0.7,0.2,0.2)	(0.9,0.1,0.1)	(0.6,0.3,0.4)	(0,1,1)

The plithogenic representation of the causal relationship between the concepts is presented in Fig.4.2. The positive sign indicates the positive impacts of the concepts, and it is represented as neutrosophic values in $P_N(E)$.

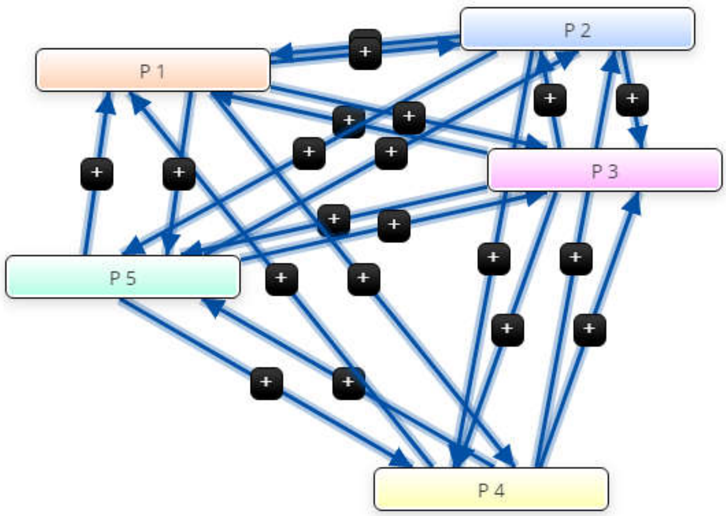

Fig.4.2 Representation of Plithogenic association between the concepts

The plithogenic operators are used to obtain the resultant vector.

Let X_N = ((1,0,0), (0,1,1), (0,1,1), (0,1,1), (0,1,1))

X_N*p P_N (E) = ((0, 0 ,0) (0.68,0.19,0),(0.76,0.18,0) (0.72,0.26,0) (0.86,0.23,0))

$\rightarrow$ ((1, 0 ,0) (0.68,0.19,0),(0.76,0.18,0) (0.72,0.26,0) (0.86,0.23,0)) = X_{N1}

X_{N1}*p P_N (E) = ((0.602, 0.118,0) (0.68,0.1288,0.1088) (0.76,0.1396,0.1632) (0.84,0.1504,0.1632) (0.92 0.16 0.1088)) → ((1, 0 ,0) (0.68,0.1288,0.1088) (0.76,0.1396,0.1632) (0.84,0.1504,0.1632) (0.92 0.16 0.1088)) = X_{N2}

X_{N2}*p P_N (E) = ((0.644 0.15952 0) (0.7104 0.162592 0.1088) (0.7768, 0.165664 0.1632) (0.8432 0.168736 0.1632) (0.92 0.171808 0.1088)) →((1, 0 ,0) (0.7104 0.162592 0.1088) (0.7768, 0.165664 0.1632) (0.8432 0.168736 0.1632) (0.92 0.171808 0.1088)) = X_{N3}

X_{N3}*p P_N (E) = ((0.644 0.1607008 0) (0.7104 0.16448128 0.113664) (0.7768 0.16826176 0.113664) (0.7768 0.16826176 0.170496) (0.8432 0.17204224 0.170496) (0.92 0.17582272 0.113664)) → ((1, 0 ,0) (0.710 0.164 0.114) (0.777 0.168 0.114) (0.777 0.168 0.170) (0.843 0.172 0.170) (0.92 0.176 0.114)) = X_{N4}

By repeating in the same fashion,

X_{N5} = ((1, 0 ,0) (0.71, 0.167,0.114) (0.777, 0.171, 0.17) (0.843, 0.174, 0.17) (0.91 ,0.178, 0.114)

X_{N6} = ((1, 0 ,0) (0.71, 0.167,0.114) (0.777, 0.171 ,0.17) (0.843, 0.174, 0.17) (0.91, 0.178 ,0.114)

$X_{N5} = X_{N6}$

Thus the neutrosophic impact of the concept P1 on other factors is determined. The various kinds of plithogenic cognitive maps are discussed in different cases and in each case, the impact of the concept P1 over the other concepts is determined. In section 4 various cases are discussed and the differences between cognitive maps (CM), fuzzy cognitive maps (FCM), intuitionistic cognitive maps(IFCM) , neutrosophic cognitive maps(NCM) and plithogenic cognitive maps (PCM) based on edge weights (e_{ij}) are presented in Table 4.2.

Table 4.2. Differences beteween CM, FCM, IFCM,NCM and PCM

Cognitive Maps	$e_{ij} \in \{-1,0,1\}$ with no contradiction degree between the concepts
Fuzzy cognitive maps	$e_{ij} \in [-1,1]$ with no contradiction degree between the concepts
Intuitionistic cognitive maps	e_{ij} = (μ, ν)where μ, the membership value and ν , the non-membership value with no contradiction degree between the concepts
Neutrosophic Cognitive Maps	$e_{ij} \in \{-1,0,1, I\}$ with no contradiction degree between the concepts
Plithogenic Cognitive Maps	$e_{ij} \in \{-1,0,1\}$ or $e_{ij} \in [-1,1]$ or $e_{ij} \in \rho([0,1]^2)$ or $e_{ij} \in \rho([0,1]^3)$ with contradiction degree between the concepts

5. Conclusion

This research work proposes the concept of plithogenic cognitive maps and new neutrosophic maps. The integration of contradiction degree with the plithogenic operators is applied to determine the resultant vector. Several kinds of plithogenic cognitive maps are discussed in this article and it is validated with applications in decision making. The proposed plithogenic cognitive maps can be applied in decision making scenario. The state of indeterminacy of the concept is quantified by various confinement values which will certainly assist in making optimal decisions. Plithogenic cognitive maps can be also developed to various representations based on the characterisation of the decision making environment. PCM decision making models can be extended to interval-valued plithogenic cognitive maps and also it can be integrated to multi criteria decision making. The association between the concepts of decision making can be represented in terms of plithogenic hypersoft set. Plithogenic hypergraph can also be integrated with plithogenic cognitive maps to formulate novel and feasible decision making models.

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Plithogenic Cubic Sets

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Abstract

In this article, using the concepts of cubic set and plithogenic set, the ideas of plithogenic fuzzy cubic set, plithogenic intuitionistic fuzzy cubic set, Plithogenic neutrosophic cubic set are introduced and its corresponding internal and external cubic sets are discussed with examples. Primary properties of the Plithogenic neutrosophic cubic sets were also discussed.This concept is extremely suitable for addressing problems involving multiple attribute decision making as this plithogenic neutrosophic set are described by four or more value of attributes and the accuracy of the result is also so precise.

Keywords: Fuzzy set, Intuitionistic fuzzy set, Neutrosophic set, Cubic set, Plithogenic set, Plithogenic fuzzy cubic set, Plithogenic Intuitionistic fuzzy cubic set, Plithogenic neutrosophic cubic set.

1. Introduction

Zadeh implemented Fuzzy Sets[18].In [18] Zadeh gave the perception of a fuzzy set by an interval-valued fuzzy set , i.e. a fuzzy set with an intervalvalued belonging function. In conventional fuzzy concepts,numbers from the interval [0,1] are used to reflect, e.g., the degree of conviction of the expert in various statements.It is always tough for an expert to precisely enumerate their certainty; thus, rather than a particular value, it is more fitting to reflect this certainty degree by an interval or even by a fuzzy set. We obtain an interval valued fuzzy sets which were frequently used in real-life scenario.

Atanassov[2] introduced intuitionistic fuzzy sets(IFS), which is the generalisation of the fuzzy set. In IFS,each element is attached to both belonging and non-belonging grade with the constraint that the sum of these two grades is less than or equal to unity. If available knowledge is not sufficient to identify the inaccuracy of traditional fuzzy sets, IFS architecture can be viewed as an alternative / appropriate solution. Later on IFS were expanded to interval-valued IFS.

Smarandache [9, 10, 11] proposed Neutrosophic sets (NSs), a generalisation of FS and IFS, which is highly helpful for dealing with inadequate, uncertain, and varying data that exists in the real life. NSs are characterised by functions of truth (T), indeterminacy (I) and falsity (F) belonging functions. This concept is very essential in several areas of application since indeterminacy is clearly enumerated and the truth, indeterminacy, and falsity membership functions are independent.

.Wang, Smarandache, Zhang and Sunderraman[17] proposed the definition of the interval valued neutrosophic set (INS) as an extension of the neutrosophic set.The INS could reflect indeterminate, inaccurate, inadequate and unreliable data that occurs in the reality.

Plithogeny is the origin, creation, production and evolution of new objects from the synthesis of conflicting or non-conflicting multiple old objects. Smarandache[12] introduced the plithogenic set as a generalisation of neutrosophy in 2017.

The elements of plithogenic sets are denoted by one or many number of attributes and each of it have several values.Each values of attribute has its respective (fuzzy, intuitionistic fuzzy or neutrosophic) appurtenance degree for the element x (say) to the plithogenic set P (say) with respect to certain constraints. For the first time, Smarandache[12] introduced the contradictory (inconsistency) degree between each value of attribute and the dominant value of attribute which results in getting the better accuracy for the plithogenic aggregation operators(fuzzy, intuitionistic fuzzy or neutrosophic).

Y.B. Jun et al [4] implemented a cubic set which is a combination of a fuzzy set with an interval valued fuzzy set. Internal and external cubic sets were also described and some properties were studied.

Y.B.Jun, Smarandache and Kim [4] introduced Neutrosophic cubic sets and the concept of internal and external for truth, falsity and indeterminacy values.Furthermore they provided so many properties of P (R) -union,P (R) –intersection for internal and external neutrosophic cubic sets.

In this article, using the principles of cubic sets and plithogenic set, we presented the generalisation of plithogenic cubic sets for fuzzy, Intuitionistic fuzzy and neutrosophic sets

2. Preliminaries

Definition 2.1 [15] Let Z be a universe of discourse and the fuzzy set $F = \{\langle z, \mu_f(z)\rangle \mid z \in Z\}$ is described by a belonging function μ_f as , $\mu_f : Z \to [0,1]$.

Definition 2.2 [10] Let Y be a non-empty set. Then, an interval valued fuzzy set B over Y is defined as $B = \{[B^-(y), B^+(y)]/y : y \in Y\}$ where $B^-(x)$ and $B^+(x)$ are stated as the inferior and superior degrees of belonging $y \in Y$ where $0 \le B^-(y) + B^+(y) \le 1$ correspondingly.

Definition 2.3 [2, 3] Let N be a non empty set.The set $A = \{\langle n, \lambda_A, \phi_A \rangle n \in N\}$ is called an intutionistic fuzzy set (in short, IFS) of N where the function $\lambda_A : N \to [0,1], \phi_A : N \to [0,1]$ denotes the membership degree (say $\lambda_A(n)$) and non-membership degree (say $\phi_A(n)$) of each element $n \in N$ to the set A and satisfies the constraint that $0 \le \lambda_A(n) + \phi_A(n) \le 1$.

Definition 2.4 [10] Let Y be a non-empty set. The set $\mathrm{A} = \{< y, M_A(y), N_A(y) > \mid y \in Y\}$ is called an interval valued intuitionistic fuzzy sets (IVIFS) A in Y where the functions $M_A(y) : Y \to [0,1]$ and $N_A(y) : Y \to [0,1]$ denotes the degree of belonging, non-belonging of A respectively. Also $M_A(y) = [M_{AL}(y), M_{AU}(y)]$ and $N_A(y) = [N_{AL}(y), N_{AU}(y)]$, $0 \le M_{AU}(y) + M_{AU}(y) \le 1$ for each $y \in Y$

Definition 2.5 [6] Let N be a non empty set.The set $A = \{\langle n, \lambda_A, \phi_A, \gamma_A \rangle n \in N\}$ is called a neutrosophic set (in short, NS) of N where the function $\lambda_A : N \to [0,1], \phi_A : N \to [0,1]$ and $\gamma_A : N \to [0,1]$ denotes the membership degree (say $\lambda_A(n)$), indeterminacy degree (say $\phi_A(n)$), and non- membership degree (say $\gamma_A(n)$)of each element $n \in N$ to the set A and satisfies the constraint that $0 \le \lambda_A(n) + \phi_A(n) + \gamma_A(n) \le 3$.

Definition 2.6 [1] Let R be a non-empty set. An interval valued neutrosophic set (INS) A in R is described by the functions of the truth-value (A_T), the indeterminacy (A_I) and the falsity-value (A_F) for each point $r \in R$, $A_T(r), A_I(r), A_F(r) \subseteq [0,1]$.

Definition 2.7 [4,5] Let E be a non-empty set. By a cubic set in E ,we construct a set which has the form $\Psi = \{< e, B(e), \mu(e) > | e \in E\}$ in which B is an interval valued fuzzy set (IVFS) in E and μ is a fuzzy set in E.

Definition 2.8 [4] Let E be a non-empty set. If $B^-(e) \le \mu(e) \le B^+(e)$ for all $e \in E$ then the cubic set $\Psi =< B, \mu >$ in E is called an internal cubic set (briefly IPS).

Definition 2.9 [4] Let E be a non-empty set. If $\mu(e) \notin (B^-(e), B^+(e))$ for all $e \in E$ then the cubic set $\Psi =< B, \mu >$ in E is called an external cubic set (briefly ECS)

Definition 2.10 [10] Plithogenic Fuzzy set for an interval valued (IPFS) is defined as $\forall c \in C, d : C \times W \to C([0,1])$, $\forall w \in W$ and $d(c, w)$ is an open, semi-open, closed interval included in [0, 1] and C([0, 1]) is the power set of the unit interval [0,1] (i.e) all subsets of [0,1].

Definition 2.11 [10] A set which has the form $\forall c \in C, d : C \times W \to C([0,1]^2)$ and $\forall w \in W$ is called interval valued plithogenic intuitionistic fuzzy set (IPIFS) where $d(c, w)$ is an open, semi-open, closed for the interval included in $[0,1]$.

Definition 2.12 [10] A set which has the form $\forall c \in C, d : C \times W \to C([0,1]^3)$ and $\forall w \in W$ is called interval valued plithogenic neutrosophic set (IPNS) where $d(c, w)$ is an open, semi-open, closed for the interval included in $[0,1]$.

3. Plithogenic Cubic sets

3.1 Plithogenic fuzzy Cubic sets

Definition 3.1.1 For a non empty set Y, the Plithogenic fuzzy cubic set (PFCS) is defined as $\Lambda = \{< y, B(y), \lambda(y) > | y \in Y\}$ in which B is an interval valued plithogenic fuzzy set in Y and λ is a fuzzy set in Y.

Example 3.1.2 The following values of attribute represents the criteria “Musical instruments” : Piano (the dominant one), guitar, saxophone, violin.

Contradictory Degree	0	0.50	0.75	1
Value of attributes	Piano	Guitar	Saxophone	Violin
Appurtenance Degree $B(y)$	[0.5, 0.6]	[0.2, 0.8]	[0.4, 0.7]	[0.1, 0.3]
$\lambda(y)$	0.3	0.4	0.6	0.2

Definition 3.1.3 For a non-empty set Y, the PFCS $\Lambda =< B, \lambda >$ in Y is called an internal plithogenic fuzzy cubic set (briefly IPFCS) if ${B_{d_i}}^-(y) \le \lambda_i(y) \le {B_{d_i}}^+(y)$ for all $y \in Y$ and d_i represents the contradictory degree and their respective value of attributes.

Example 3.1.4 The following values of attribute represents the criteria “Color” : Yellow(the dominant one), green, orange and red.

Contradictory Degree	0	0.50	0.75	1
Value of attributes	Yellow	Green	Orange	Red

Appurtenance Degree $B(y)$	[0.2, 0.3]	[0.6, 0.8]	[0.3, 0.6]	[0.4, 0.9]
$\lambda(y)$	0.2	0.7	0.5	0.8

Definition 3.1.5 For a non-empty set Y, the PFCS $\Lambda = < B, \lambda >$ in Y is called an external plithogenic fuzzy cubic set (briefly EPFCS) if $\lambda_i(y) \notin (B_{d_i}^{-}(y), B_{d_i}^{+}(y))$ for all $y \in Y$ and and d_i represents the contradictory degree and their respective value of attributes.

Example 3.1.6 The following values of attribute represents the criteria "Subjects" : Mathematics(the dominant one), Physics, Chemistry and Biology

Contradictory Degree	0	0.50	0.75	1
Value of attributes	Mathematics	Physics	Chemistry	Biology
Appurtenance Degree $B(y)$	[0.4, 0.6]	[0.5, 0.8]	[0.2, 0.3]	[0.5, 0.9]
$\lambda(y)$	0.7	0.1	0.6	0.3

Remark 3.1.7 If any one of the attribute value $\lambda_i(y) = y$ for all $y \in Y$ and *i* represents the values of attribute then Λ is neither an IPFCS nor an EPFCS.

3.2 Plithogenic Intuitionistic fuzzy cubic set

Definition 3.2.1 For a non empty set Y, the Plithogenic Intuitionistic fuzzy cubic set (PIFCS) is defined as $\Lambda = \{< y, B(y), \lambda(y) > \mid y \in Y\}$ in which *B* is an interval valued Plithogenic Intuitionistic fuzzy set in Y and λ is a intuitionistic fuzzy set in Y.

Example 3.2.2 The following values of attribute represents the criteria "Mobile phone brands" : Apple (the dominant one), Samsung, Nokia, Lenova.

Contradictory Degree	0	0.50	0.75	1
Value of attributes	Apple	Samsung	Nokia	Lenova
Appurtenance Degree $B(y)$	([0.2, 0.5], [0.3, 0.6])	([0.5, 0.8], [0.1, 0.7])	([0.4, 0.7], [0.2, 0.6])	([0.1, 0.3], [0.4,0.7])
$\lambda(y)$	([0.1,0.7])	([0.2, 0.4])	([0.1, 0.8])	([0.2, 0.5])

Definition 3.2.3 For a non-empty set Y, the PIFCS $\Lambda = < B, \lambda >$ in Y is called an internal plithogenic intuitionistic fuzzy cubic set (briefly IPIFCS) if $B_{d_i}^{-}(y) \le \lambda_i(y) \le B_{d_i}^{+}(y)$ for all $y \in Y$ and d_i represents the contradictory degree and their respective value of attributes.

Example 3.2. The following values of attribute represents the criteria " Proficiency " : Excellent (the dominant one), good, average, poor

Contradictory Degree	0	0.50	0.75	1
Value of attributes	Excellent	Good	Average	Poor
Appurtenance Degree $B(y)$	([0.3, 0.5], [0.1, 0.6])	([0.4, 0.6], [0.5, 0.8])	([0.1, 0.8], [0.7, 0. 8])	([0.2, 0.3], [0.1,0.6])
$\lambda(y)$	([0.4,0.5])	([0.6, 0.7])	([0.2, 0.5])	([0.3, 0.5])

Definition 3.2.5 For a non-empty set Y, the PIFCS $\Lambda = < B, \lambda >$ in Y is called an external plithogenic intuitionistic fuzzy cubic set (briefly EPIFCS) if $\lambda_i(y) \notin (B_{d_i}^{-}(y), B_{d_i}^{+}(y))$ for all $y \in Y$ and d_i represents the contradictory degree and their respective value of attributes.

Example 3.2.6 The following values of attribute represents the criteria " Mode of Transport" : Bus (the dominant one), Car, Lorry, Train

Contradictory Degree	0	0.50	0.75	1
Value of attributes	Bus	Car	Lorry	Train
Appurtenance Degree $B(y)$	([0.4, 0.6], [0.3, 0.5])	([0.5, 0.7], [0.2, 0.4])	([0.2, 0.6], [0.3, 0.7])	([0.3, 0.6], [0.1,0.4])
$\lambda(y)$	([0.1,0.6])	([0.4, 0.5])	([0.7, 0.2])	([0.7, 0.6])

3.3 Plithogenic Neutrosophic cubic set

Definition 3.3.1 Let Ω be an universal set and Y be a non empty set. The structure $\Lambda = \{< y, B(y), \lambda(y) > | \, y \in Y\}$ is said to be Plithogenic Neutrosophic cubic set (PNCS) in Y, where $B = \{(B_{d_i}^{T}(y), B_{d_i}^{I}(y), B_{d_i}^{F}(y))\}$ is an interval valued Plithogenic Neutrosophic set in Y and $\lambda = \{(\lambda_i^{T}(y), \lambda_i^{I}(y), \lambda_i^{F}(y)\}$ is a neutrosophic set in Y.

The pair $\Lambda = < B, \lambda >$ is called plithogenic neutrosophic cubic set over Ω where Λ is a mapping given by $\Lambda : B \to NC(\Omega)$ .The set of all plithogenic neutrosophic cubic sets over Ω will be denoted by P_N^{Ω}

Example 3.3.2 consider the attribute "Size" which has the following values: Small (the dominant one), medium, big, very big.

Contradictory Degree	0	0.50	0.75	1
Value of attributes	Small	Medium	Big	Very big
Appurtenance Degree $B(y)$	([0.3, 0.5], [0.1, 0.8], [0.1, 0.9])	([0.1, 0.3], [0.4, 0.8], [0.4, 0.9])	([0.1, 0.5], [0.2, 0. 6], [0.6, 0.9])	([0.2, 0.6], [0.2,0.5], [0.7, 0.9])
$\lambda(y)$	([0.4,0.5, 0.8])	([0.3, 0.7, 0.4])	([0.2,0.5,0.8])	([0.5,0.2,0.8])

Definition 3.3.3 For a non-empty set Y, the plithogenic neutrosophic cubic set $\Lambda = < B, \lambda >$ in Y is called

(i) truth internal if $B_{d_i}^{-T}(y) \le \lambda_i^{T}(y) \le B_{d_i}^{+T}(y)$ for all $y \in Y$ and d_i represents the contradictory degree and their respective value of attributes. (3.1)
(ii) indeterminacy internal if $B_{d_i}^{-I}(y) \le \lambda_i^{I}(y) \le B_{d_i}^{+I}(y)$ for all $y \in Y$ and d_i represents the contradictory degree and their respective value of attributes. (3.2)
(iii) falsity internal if $B_{d_i}^{-F}(y) \le \lambda_i^{F}(y) \le B_{d_i}^{+F}(y)$ for all $y \in Y$ and d_i represents the contradictory degree and their respective value of attributes. (3.3).

If a plithogenic neutrosophic cubic set in Y satisfies the above equations we conclude that Λ is an internal plithogenic neutrosophic cubic set (IPNCS) in Y.

Example 3.3.4 The following values of attribute represents the criteria "Sports": Volley ball(the dominant one), Basket ball, Cricket, Bat-minton

Contradictory Degree	0	0.50	0.75	1
Value of attributes	Volley ball	Basket ball	Cricket	Bat-minton

Appurtenance Degree $B(y)$	([0.2, 0.5], [0.6, 0.8], [0.1, 0.5])	([0.1, 0.3], [0.4, 0.5], [0.6, 0.9])	([0.5, 0.8], [0.1, 0. 4], [0.6, 0.9])	([0.3, 0.6], [0.1,0.5], [0.7, 0.9])
$\lambda(y)$	([0.3,0.6, 0.4])	([0.2, 0.5, 0.8])	([0.7,0.2, 0.8])	([0.4, 0.3, 0.9])

Definition 3.3.5 For a non-empty set Y, the plithogenic neutrosophic cubic set $\Lambda = < B, \lambda >$ in Y is called

(i) truth external if $\lambda_i^T(y) \notin (B_{d_i}^{-T}(y), B_{d_i}^{+T}(y))$ for all $y \in Y$ and d_i represents the contradictory degree and their respective value of attributes. (3.4)

(ii) indeterminacy external if $\lambda_i^I(y) \notin (B_{d_i}^{-I}(y), B_{d_i}^{+I}(y))$ for all $y \in Y$ and d_i represents the contradictory degree and their respective value of attributes. (3.5)

(ii) falsity external if $\lambda_i^F(y) \notin (B_{d_i}^{-F}(y), B_{d_i}^{+F}(y))$ for all $y \in Y$ and d_i represents the contradictory degree and their respective value of attributes. (3.6)

If a plithogenic neutrosophic cubic set in Y satisfies the above equations, we conclude that Λ is an external plithogenic neutrosophic cubic set (EPNCS) in Y.

Example 3.3.6 The following values of attribute represents the criteria "Seasons" : Spring (the dominant one), Winter, Summer, Autumn

Contradictory Degree	0	0.50	0.75	1
Value of attributes	Spring	Winter	Summer	Autumn
Appurtenance Degree $B(y)$	([0.3, 0.6], [0.7, 0.5], [0.4, 0.8])	([0.1, 0.3], [0.2, 0.3], [0.6, 0.8])	([0.1, 0.4] , [0.2, 0. 6], [0.8, 0.9])	([0.3, 0.5], [0.2,0.7], [0.6, 0.8])
$\lambda(y)$	([0.1,0.2, 0.9])	([0.4, 0.5, 0.9])	([0.7, 0.9, 0.6])	([0.1, 0.8, 0.3])

Theorem 3.3.7 Let Y be a non empty set and $\Lambda = < B, \lambda >$ be a PNCS in Y which is not external. Then there exists $y \in Y$ such that $\lambda_i^T(y) \in (B_{d_i}^{-T}(y), B_{d_i}^{+T}(y))$, $\lambda_i^I(y) \in (B_{d_i}^{-I}(y), B_{d_i}^{+I}(y))$ or $\lambda_i^F(y) \in (B_{d_i}^{-F}(y), B_{d_i}^{+F}(y))$ where d_i represents the contradictory degree and their respective value of attributes..

Proof. Direct proof

Theorem 3.3.8 Let Y be a non empty set and $\Lambda = < B, \lambda >$ be a PNCS in Y. If Λ is both T-internal and T-external, then $(\forall y \in Y)\left(\lambda_i^T(y) \in \{B_{d_i}^{-T}(y) \mid y \in Y\} \cup \{B_{d_i}^{+T}(y) \mid y \in Y\}\right)$ where d_i represents the contradictory degree and their respective value of attributes.

Proof. The conditions (3.1) and (3.4) implies that $B_{d_i}^{-T}(y) \le \lambda_i(y) \le B_{d_i}^{+T}(y)$ and $\lambda_i^T(y) \notin (B_{d_i}^{-T}(y), B_{d_i}^{+T}(y))$ for all $y \in Y$ and i = 1, 2, 3,…n.

Then it indicates that $\lambda_i^T(y) = B_{d_i}^{-T}(y)$ or $\lambda_i^T(y) = B_{d_i}^{+T}(y)$, and hence

$\lambda_i^T(y) \in \{B_{d_i}^{-T}(y) \mid y \in Y\} \cup \{B_{d_i}^{+T}(y) \mid y \in Y\}$ where d_i represents the contradictory degree and their respective value of attributes.

Hence the proof.

Correspondingly the subsequent theorems hold for the indeterminate and falsity values.

Theorem 3.3.9 Let Y be a non empty set and $\Lambda = < B, \lambda >$ be a PNCS in Y. If Λ is both I-internal and I-external, then $(\forall y \in Y)\left(\lambda_i^{\,I}(y) \in \{B_{d_i}^{\,-I}(y) \mid y \in Y\} \cup \{B_{d_i}^{\,+I}(y) \mid y \in Y\}\right)$ where d_i represents the contradictory degree and their respective value of attributes.

Theorem 3.3.10 Let Y be a non empty set and $\Lambda = < B, \lambda >$ be a PNCS in Y. If Λ is both *F*-internal and *F*-external, then $(\forall y \in Y)\left(\lambda_i^{\,F}(y) \in \{B_{d_i}^{\,-F}(y) \mid y \in Y\} \cup \{B_{d_i}^{\,+F}(y) \mid y \in Y\}\right)$ where d_i represents the contradictory degree and their respective value of attributes.

Definition 3.3.11 Let Y be a non empty set, The complement of $\Lambda = < B, \lambda >$ is said to be the PNCS $\Lambda^c = < B^c, \lambda^c >$ where $B^c = \left\{\left(B^c{}_{d_i}{}^T(y), B^c{}_{d_i}{}^I(y), B^c{}_{d_i}{}^F(y)\right)\right\}$ is an interval valued PNCS in Y and $\lambda = \{(\lambda^c{}_i{}^T(y), \lambda^c{}_i{}^I(y), \lambda^c{}_i{}^F(y)\}$ is a neutrosophic set in Y.

Theorem 3.3.12 Let Y be a non empty set and $\Lambda = < B, \lambda >$ be a PNCS in Y. If Λ is both *T*-internal and *T*-external, then the complement $\Lambda^c = < B^c, \lambda^c >$ of $\Lambda = < B, \lambda >$ is an *T*-Internal and *T*-external PNCS in Y.

Proof. Let Y be a non empty set .If $\Lambda = < A, \lambda >$ is an *T*-internal and *T*-external PNCS in Y, then $B_{d_i}^{\,-T}(y) \le \lambda_i(y) \le B_{d_i}^{\,+T}(y)$ and $\lambda_i^{\,T}(y) \notin (B_{d_i}^{\,-T}(y), B_{d_i}^{\,+T}(y))$ for all $y \in Y$ and i = 1, 2, 3,…,n. It follows that $1 - B_{d_i}^{\,-T}(y) \le 1 - \lambda_i(y) \le 1 - B_{d_i}^{\,+T}(y)$ and. $1 - \lambda_i^{\,T}(y) \notin (1 - B_{d_i}^{\,-T}(y), 1 - B_{d_i}^{\,+T}(y))$. Therefore $\Lambda^c = < B^c, \lambda^c >$ is an *T*-Internal and *T*-external PNCS in Y.

Correspondingly the subsequent theorems hold for the indeterminate and falsity values.

Theorem 3.3.13 Let Y be a non empty set and $\Lambda = < B, \lambda >$ be a PNCS in Y. If Λ is both *I*-internal and *I*-external, then the complement $\Lambda^c = < B^c, \lambda^c >$ of $\Lambda = < B, \lambda >$ is an *I*-internal and *I*-external PNCS in Y.

Theorem 3.3.14 Let Y be a non empty set and $\Lambda = < B, \lambda >$ be a PNCS in Y. If Λ is both *F*-internal and *F*-external, then the complement $\Lambda^c = < B^c, \lambda^c >$ of $\Lambda = < B, \lambda >$ is an *F*-internal and *F*-external PNCS in Y.

Definition 3.3.15 Let $\Lambda = < B, \lambda > \in P^{\Omega}{}_N$.

If $B_{d_i}^{\,-T}(y) \le \lambda_i^{\,T}(y) \le B_{d_i}^{\,+T}(y)$, $B_{d_i}^{\,-I}(y) \le \lambda_i^{\,I}(y) \le B_{d_i}^{\,+I}(y)$, $\lambda_i^{\,F}(y) \notin (B_{d_i}^{\,-F}(y), B_{d_i}^{\,+F}(y))$

or

$B_{d_i}^{\,-T}(y) \le \lambda_i^{\,T}(y) \le B_{d_i}^{\,+T}(y)$, $B_{d_i}^{\,-F}(y) \le \lambda_i^{\,F}(y) \le B_{d_i}^{\,+F}(y)$, $\lambda_i^{\,I}(y) \notin (B_{d_i}^{\,-I}(y), B_{d_i}^{\,+I}(y))$

or

$B_{d_i}^{\,-F}(y) \le \lambda_i^{\,F}(y) \le B_{d_i}^{\,+F}(y)$, $B_{d_i}^{\,-I}(y) \le \lambda_i^{\,I}(y) \le B_{d_i}^{\,+I}(y)$, $\lambda_i^{\,T}(y) \notin (B_{d_i}^{\,-T}(y), B_{d_i}^{\,+T}(y))$ for all $y \in Y$ .Then Λ is called $\frac{2}{3}$ IPNCS or $\frac{1}{3}$ EPNCS.

Example 3.3.16 The following values of attribute represents the criteria "Types of humans" : Fun loving (the dominant one), Sensitive, Determined, Serious

Contradictory Degree	0	0.50	0.75	1

Value of attributes	Fun loving	Sensitive	Determined	Serious
Appurtenance Degree $B(y)$	([0.1, 0.6], [0.3, 0.5], [0.4, 0.8])	([0.1, 0.3], [0.2, 0.3], [0.6, 0.8])	([0.1, 0.4] , [0.2, 0. 6], [0.8, 0.9])	([0.3, 0.5], [0.2,0.7], [0.6, 0.8])
$\lambda(y)$	([0.1,0.4, 0.9])	([0.2, 0.5, 0.7])	([0.7, 0.5, 0.8])	([0.3,0.4, 0.3])

Definition 3.3.16 Let $\Lambda = < B, \lambda > \in P^{\Omega}{}_N$.

If $B_{d_i}{}^{-T}(y) \leq \lambda_i{}^T(y) \leq B_{d_i}{}^{+T}(y)$, $\lambda_i{}^I(y) \notin (B_{d_i}{}^{-I}(y), B_{d_i}{}^{+I}(y))$, $\lambda_i{}^F(y) \notin (B_{d_i}{}^{-F}(y), B_{d_i}{}^{+F}(y))$

or

$B_{d_i}{}^{-F}(y) \leq \lambda_i{}^F(y) \leq B_{d_i}{}^{+F}(y)$, $\lambda_i{}^T(y) \notin (B_{d_i}{}^{-T}(y), B_{d_i}{}^{+T}(y))$, $\lambda_i{}^I(y) \notin (B_{d_i}{}^{-I}(y), B_{d_i}{}^{+I}(y))$

or

$B_{d_i}{}^{-I}(y) \leq \lambda_i{}^I(y) \leq B_{d_i}{}^{+I}(y)$, $\lambda_i{}^F(y) \notin (B_{d_i}{}^{-F}(y), B_{d_i}{}^{+F}(y))$, $\lambda_i{}^T(y) \notin (B_{d_i}{}^{-T}(y), B_{d_i}{}^{+T}(y))$ for all $y \in Y$.

Then Λ is called $\frac{1}{3}$ IPNCS or $\frac{2}{3}$ EPNCS.

Example 3.3.17 The following values of attribute represents the criteria " Social Networks " : Whatsapp, Facebook (the dominant one), Instagram, Linkedinn

Contradictory Degree	0	0.50	0.75	1
Value of attributes	Whatsapp	Facebook	Instagram	Linkedin
Appurtenance Degree $B(y)$	([0.2, 0.6], [0.7, 0.5], [0.4, 0.8])	([0.1, 0.3], [0.2, 0.4], [0.6, 0.8])	([0.1, 0.4] , [0.2, 0. 6], [0.8, 0.9])	([0.3, 0.5], [0.2,0.7], [0.6, 0.8])
$\lambda(y)$	([0.3,0.2, 0.9])	([0.4, 0.3, 0.9])	([0.7, 0.2, 0.6])	([0.2, 0.5, 0.4])

Theorem 3.3.14 Let $\Lambda = < B, \lambda > \in P^{\Omega}{}_N$. Then

(i) All IPNCS is a generalisation of ICS.
(ii) All EPNCS is a generalisation of ECS.
(iii) All PNCS is a generalisation of CS.

Proof. Direct proof.

4. Conclusions and future work

In this article,We have introduced the plithogenic fuzzy cubic set, plithogenic intuitionistic fuzzy cubic set, plithogenic neutrosophic cubic sets and their corresponding internal and external cubic sets are defined with examples. Furthermore some of the properties of plithogenic neutrosophic cubic sets are investigated. In the consecutive research, we will study the P-Union, P-Intersection, R-Union, R-Intersection of plithogenic neutrosophic cubic sets.

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Introduction to Plithogenic Hypersoft Subgroup

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Abstract. In this article, some essential aspects of plithogenic hypersoft algebraic structures have been analyzed. Here the notions of plithogenic hypersoft subgroups i.e. plithogenic fuzzy hypersoft subgroup, plithogenic intuitionistic fuzzy hypersoft subgroup, plithogenic neutrosophic hypersoft subgroup have been introduced and studied. For doing that we have redefined the notions of plithogenic crisp hypersoft set, plithogenic fuzzy hypersoft set, plithogenic intuitionistic fuzzy hypersoft set, and plithogenic neutrosophic hypersoft set and also given their graphical illustrations. Furthermore, by introducing function in different plithogenic hypersoft environments, some homomorphic properties of plithogenic hypersoft subgroups have been analyzed.

Keywords: Hypersoft set; Plithogenic set; Plithogenic hypersoft set; Plithogenic hypersoft subgroup

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A LIST OF ABBREVIATIONS

US signifies universal set.

CS signifies crisp set.

FS signifies fuzzy set.

IFS signifies intuitionistic fuzzy set.

NS signifies neutrosophic set.

PS signifies plithogenic set.

SS signifies soft set.

HS signifies hypersoft set.

CHS signifies crisp hypersoft set.

FHS signifies fuzzy hypersoft set.

IFHS signifies intuitionistic fuzzy hypersoft set.

NHS signifies neutrosophic hypersoft set.

PHS signifies plithogenic hypersoft set.

PCHS signifies plithogenic crisp hypersoft set.

PFHS signifies plithogenic fuzzy hypersoft set.

PIFHS signifies plithogenic intuitionistic fuzzy hypersoft set.

PNHS signifies plithogenic neutrosophic hypersoft set.

CG signifies crisp group.

FSG signifies fuzzy subgroup.

IFSG signifies intuitionistic fuzzy subgroup.

NSG signifies neutrosophic subgroup.

DAF signifies degree of appurtenance function.

DCF signifies degree of contradiction function.

PSG signifies plithogenic subgroup.

PCHSG signifies plithogenic crsip hypersoft subgroup.

PFHSG signifies plithogenic fuzzy hypersoft subgroup.

PIFHSG signifies plithogenic intuitionistic fuzzy hypersoft subgroup.

PNHSG signifies plithogenic neutrosophic hypersoft subgroup.

DMP signifies decision making problem.

$\boldsymbol{\rho(U)}$ signifies power set of U.

1. Introduction

FS [1] theory was first initiated by Zadeh to handle uncertain real-life situations more precisely than CSs. Gradually, some other set theories like IFS [2], NS [3], Pythagorean FS [4], PS [5], etc., have emerged. These sets are able to handle ambiguous situations more appropriately than FSs. NS theory was introduced by Smarandache which was generalizations of IFS and FS. He has also introduced neutrosophic probability, measure [6,7] , psychology [8], pre-calculus and calculus [9], etc. Presently, NS theory is vastly used in various pure as well as applied fields. For instance, in medical diagnosis [10, 11], shortest path problem [12–20], DMP [21–26], transportation problem [27, 28], forecasting [29], mobile edge computing [30], abstract algebra [31], pattern recognition problem [32], image segmentation [33], internet of things [34], etc. Another set theory of profound importance is PS theory which is extensively used in handling various uncertain situations. This set theory is more general than CS, FS, IFS, and NS theory. Gradually, plithogenic probability and statistics [35], plithogenic logic [35], etc., have evolved which are generalizations of crisp probability, statistics, and logic. Smarandache has also introduced the notions of plithogenic number, plithogenic measure function, bipolar PS, tripolar PS, multipolar PS, complex PS, refined PS, etc. Presently, PS theory is extensively used in numerous research domains.

The notion of SS [36] theory is another fundamental set theory. Presently, SS theory has become one of the most popular branches in mathematics for its huge areas of applications in various research fields. For instance, nowadays in DMP [37], abstract algebra [38–40], etc., it is widely used. Again, there exist concepts like vague sets [41,42], rough set [43], hard set [44], etc., which are well known for their vast applications in various domains. Gradually, based on SS theory the notions of fuzzy SS [45], intuitionistic SS [46], neutrosophic SS [47] theory, etc., have been introduced by various researchers. In fuzzy abstract algebra, the notions of FSG [48], IFSG [49], NSG [31], etc., have been developed and studied by different mathematicians. SS theory has opened some new windows of opportunities for researchers working not only in applied fields but also in pure fields. As a result, the notions of soft FSG [39], soft IFSG [50], soft NSG [51], etc., were introduced. Later on, Smarandache has proposed the concept of HS [52] theory which is a generalization of SS theory. Also, he has extended and introduced the concept of HS in the plithogenic environment and generalized that further. As a result, a new branch has emerged which can be a fruitful research field for its promising potentials. The following Table 1 contains some significant contributions in SS and PS theory by numerous researchers.

Table 1. Significance and influences of PS & SS theory in various fields.

Author & references	**Year**	**Contributions in various fields**
Majhi et al. [53]	2002	Applied SS theory in a DMP.
Feng et al. [54]	2010	Described an adjustable approach to fuzzy SS based DMP with some examples.
aman [55]	2011	Defined fuzzy soft aggregation operator which allows the construction of more efficient DMP.
Broumi et al. [56]	2014	Defined neutrosophic parameterized SS and neutrosophic parameterized aggregation operator and applied it in DMP.
Broumi et al. [57]	2014	Defined interval-valued neutrosophic parameterized SS a reduction method for it.
Deli et al. [58]	2014	Introduced neutrosophic soft multi-set theory and studied some of its properties.
Deli & Naim [59]	2015	Introduced intuitionistic fuzzy parameterized SS and studied some of its properties.
Smarandache [60]	2018	Introduced physical PS.
Smarandache [61]	2018	Studied aggregation plithogenic operators in physical fields.

continued ...

Author & references	Year	Contributions in various fields
Gayen et al. [62]	2019	Introduced the notions of plithogenic subgroups and studied some of their homomorphic properties.
Abdel-Basset et al. [63]	2019	Described a novel model for evaluation of hospital medical care systems based on PSs.
Abdel-Basset et al. [64]	2019	Described a novel plithogenic TOPSIS-CRITIC model for sustainable supply chain risk management.
Abdel-Basset et al. [65]	2019	Proposed a hybrid plithogenic decision-making approach with quality function deployment.

This Chapter has been systematized as the following: In Section 2, literature reviews of FS, IFS, NS, FSG, IFSG, NSG, PS, PHS, etc., are mentioned. In Section 3, the concepts of PCHS, PFHS, PIFHS, and PNHS have been redefined in a different way and their graphical illustrations have been given. Also, the notions of PFHSG, PIFHSG, and PNHSG have been introduced and further the effects of homomorphism on those notions are studied. Finally, in Section 4, the conclusion is given mentioning some scopes of future researches.

2. **Literature Survey**

In this segment, some important notions like, FS, IFS, NS, FSG, IFSG, NSG, etc., have been discussed. We have also mentioned PS, SS, HS and some aspects of PHS. These notions will play vital roles in developing the concepts of PHSGs.

Definition 2.1. [1] Let U be a CS. A function $\sigma : U \rightarrow [0,1]$ is called a FS.

Definition 2.2. [2] Let U be a CS. An IFS γ of U is written as $\gamma = \{(m, t_\gamma(m), f_\gamma(m)) : m \in U\}$, where $t_\gamma(m)$ and $f_\gamma(m)$ are two FSs of U, which are called the degree of membership and non-membership of any $m \in U$. Here $\forall m \in U$, $t_\gamma(m)$ and $f_\gamma(m)$ satisfy the inequality $0 \leq t_\gamma(m) + f_\gamma(m) \leq 1$.

Definition 2.3. [3] Let U be a CS. A NS η of U is denoted as $\eta = \{(m, t_\eta(m), i_\eta(m), f_\eta(m)) : m \in U\}$, where $t_\eta(m), i_\eta(m), f_\eta(m) : U \rightarrow]^-0, 1^+[$ are the corresponding degree of truth, indeterminacy, and falsity of any $m \in U$. Here $\forall m \in U$ $t_\eta(m)$, $i_\eta(m)$ and $f_\eta(m)$ satisfy the inequality $^-0 \leq t_\eta(m) + i_\eta(m) + f_\eta(m) \leq 3^+$.

2.1. *Fuzzy, Intuitionistic fuzzy & Neutrosophic subgroup*

Definition 2.4. [48] A FS of a CG U is called as a FSG iff $\forall m, u \in U$, the conditions mentioned below are satisfied:

(i) $\alpha(mu) \geq min\{\alpha(m), \alpha(u)\}$

(ii) $\alpha(m^{-1}) \geq \alpha(m)$.

Definition 2.5. [49] An IFS $\gamma = \{(m, t_\gamma(m), f_\gamma(m)) : m \in U\}$ of a CG U is called an IFSG iff $\forall m, u \in U$,

(i) $t_\gamma(mu^{-1}) \geq min\{t_\gamma(m), t_\gamma(u)\}$
(ii) $f_\gamma(mu^{-1}) \leq \max\{f_\gamma(m), f_\gamma(u)\}$.

The set of all the IFSG of U will be denoted as IFSG(U).

Definition 2.6. [31] Let U be a CG and δ be a NS of U. δ is called a NSG of U iff the conditions mentioned below are satisfied:

(i) $\delta(mu) \geq \min\{\delta(m), \delta(u)\}$, i.e. $t_\delta(mu) \geq \min\{t_\delta(m), t_\delta(u)\}$, $i_\delta(mu) \geq \min\{i_\delta(m), i_\delta(u)\}$ and $f_\delta(mu) \leq \max\{f_\delta(m), f_\delta(u)\}$
(ii) $\delta(m^{-1}) \geq \delta(m)$ i.e. $t_\delta(m^{-1}) \geq t_\delta(u)$, $i_\delta(m^{-1}) \geq i_\delta(u)$ and $f_\delta(m^{-1}) \leq f_\delta(u)$.

Theorem 2.1. *[66] Let g be a homomorphism of a CG U_1 into another CG U_2. Then preimage of an IFSG γ of U_2 i.e. $g^{-1}(\gamma)$ is an IFSG of U_1.*

Theorem 2.2. *[66] Let g be a surjective homomorphism of a CG U_1 to another CG U_2. Then the image of an IFSG γ of U_1 i.e. $g(\gamma)$ is an IFSG of U_2.*

Theorem 2.3. *[31] The homomorphic image of any NSG is a NSG.*

Theorem 2.4. *[31] The homomorphic preimage of any NSG is a NSG.*

Some more references in the domains of FSG, IFSG, NSG, etc., which can be helpful to various other researchers are [67–71].

2.2. *Plithogenic set & Plithogenic hypersoft set*

Definition 2.7. [5] Let U be a US and $P \subseteq U$. A PS is denoted as $P_s = (P, \psi, V_\psi, a, c)$, where be an attribute, V_ψ is the respective range of attributes values, $a : P \times V_\psi \rightarrow [0,1]^s$ is the DAF and $c : V_\psi \times V_\psi \rightarrow [0,1]^t$ is the corresponding DCF. Here $s, t \in \{1, 2, 3\}$.

In Definition 2.7, for $s = 1$ and $t = 1$ a will become a FDAF and c will become a FDCF. In general, we consider only FDAF and FDCF. Also, $\forall (u_i, u_j) \in V_\psi \times V_\psi$, c satisfies $c(u_i, u_i) = 0$ and $c(u_i, u_j) = c(u_j, u_i)$.

Definition 2.8. [36] Let U be a US, V_A be a set of attribute values. Then the ordered pair (Γ, U) is called a SS over U, where $\Gamma : V_A \rightarrow \rho(U)$.

Definition 2.9. [52] Let U be a US. Let $r_1, r_2, ..., r_n$ be n attributes and corresponding attribute value sets are respectively $D_1, D_2, ..., D_n$ (where $D_i \cap D_j = \phi$, for $i \neq j$ and $i, j \in \{1, 2, ..., n\}$). Let $V_\psi = D_1 \times D_2 \times \cdots \times D_n$. Then the ordered pair (Γ, V_ψ) is called a HS of U, where $\Gamma : V_\psi \rightarrow \rho(U)$.

Definition 2.10. [72] A US U_C is termed as a crisp US if $\forall u \in U_C$, u fully belongs to U_C i.e. membership of u is 1.

Definition 2.11. [72] A US U_F is termed as a fuzzy US if $\forall u \in U_F$, u partially belongs to U_F i.e. membership of u belonging to $[0, 1]$.

Definition 2.12. [72] A US U_{IF} is termed as an intuitionistic fuzzy US if $\forall u \in U_{IF}$, u partially belongs to U_{IF} and also partially does not belong to U_{IF} i.e. membership of u belonging to $[0, 1] \times [0, 1]$.

Definition 2.13. [72] A US U_N is termed as an neutrosophic US if $\forall u \in U_N$, u has truth belongingness, indeterminacy belongingness, and falsity belongingness to U_N i.e. membership of u belonging to $[0, 1] \times [0, 1] \times [0, 1]$.

Definition 2.14. [72] A US U_P over an attribute value set ψ is termed as a plithogenic US if $\forall u \in U_P$, u belongs to U_P with some degree on the basis of each attribute value. This degree can be crisp, fuzzy, intuitionistic fuzzy, or neutrosophic.

Definition 2.15. [52] Let U_C be a crisp US and $\psi = \{r_1, r_2, ..., r_n\}$ be a set of n attributes with attribute value sets respectively as $D_1, D_2, ..., D_n$ (where $D_i \cap D_j = \phi$ for $i \neq j$ and $i, j \in \{1, 2, ..., n\}$). Also, let $V_\psi = D_1 \times D_2 \times \cdots \times D_n$. Then (Γ, V_ψ), where $\Gamma : V_\psi \rightarrow \rho(U_C)$ is termed as a CHS over U_C.

Definition 2.16. [52] Let U_F be a fuzzy US and $\psi = \{r_1, r_2, ..., r_n\}$ be a set of n attributes with attribute value sets respectively as $D_1, D_2, ..., D_n$ (where $D_i \cap D_j = \phi$ for $i \neq j$ and $i, j \in \{1, 2, ..., n\}$). Also, let $V_\psi = D_1 \times D_2 \times \cdots \times D_n$. Then (Γ, V_ψ), where $\Gamma : V_\psi \rightarrow \rho(U_F)$ is called a FHS over U_F.

Definition 2.17. [52] Let U_{IF} be an intuitionistic fuzzy US and $\psi = \{r_1, r_2, ..., r_n\}$ be a set of n attributes with attribute value sets respectively as $D_1, D_2, ..., D_n$ (where $D_i \cap D_j = \phi$ for $i \neq j$ and $i, j \in \{1, 2, ..., n\}$). Also, let $V_\psi = D_1 \times D_2 \times \cdots \times D_n$. Then (Γ, V_ψ), where $\Gamma : V_\psi \rightarrow \rho(U_{IF})$ is called an IFHS over U_{IF}.

Definition 2.18. [52] Let U_N be a neutrosophic US and $\psi = \{r_1, r_2, ..., r_n\}$ be a set of n attributes with attribute value sets respectively as $D_1, D_2, ..., D_n$ (where $D_i \cap D_j = \phi$ for $i \neq j$ and $i, j \in \{1, 2, ..., n\}$). Also, let $V_\psi = D_1 \times D_2 \times \cdots \times D_n$. Then (Γ, V_ψ), where $\Gamma : V_\psi \rightarrow \rho(U_N)$ is called a NHS over U_N.

Definition 2.19. [52] Let U_P be a plithogenic US and $\psi = \{r_1, r_2, ..., r_n\}$ be a set of n attributes with attribute value sets respectively as $D_1, D_2, ..., D_n$ (where $D_i \cap D_j = \phi$ for $i \neq j$ and $i, j \in \{1, 2, ..., n\}$). Also, let $V_\psi = D_1 \times D_2 \times \cdots \times D_n$. Then (Γ, V_ψ), where $\Gamma : V_\psi \to \rho(U_P)$ is called a PHS over U_P.

Further, depending on someones preferences PHS can be categorized as PCHS, PFHS, PIFHS, and PNHS. In [52], Smarandache has wonderfully introduced and illustrated these categories with proper examples.

In the next section, we have mentioned an equivalent statement of Definition 2.19 and described its categories in a different way. Also, we have given some graphical representations of PCHS, PFHS, PIFHS, and PNHS. Again, we have introduced functions in the environments of PFHS, PIFHS, and PNHS. Furthermore, we have introduced the notions of PFHSG, PIFHSG, and PNHSG and studied their homomorphic characteristics.

3. **Proposed Notions**

As an equivalent statement to Definition 2.19, we can conclude that $\forall M \in$ range(Γ) and $\forall i \in \{1, 2, ..., n\}$, $\exists a_i : M \times D_i \to [0, 1]^s$ ($s = 1, 2$ or 3) such that $\forall (m, d) \in M \times D_i,\ a_i(m, d)$ represent the DAFs of m to the set M on the basis of the attribute value d. Then the pair (Γ, V_ψ) is called a PHS.

So, based on someones requirement one may choose $s = 1, 2$ or 3 and further, depending on these choices PHS can be categorized as PFHS, PIFHS, and PNHS. Also, by defining DAF as $a_i : M \times D_i \to \{0, 1\}$, the notion of PCHS can be introduced. The followings are those aforementioned notions:

Let $\psi = \{r_1, r_2, ..., r_n\}$ be a set of n attributes and corresponding attribute value sets are respectively $D_1, D_2, ..., D_n$ (where $D_i \cap D_j = \phi$, for $i \neq j$ and $i, j \in \{1, 2, ..., n\}$). Let $V_\psi = D_1 \times D_2 \times \cdots \times D_n$ and (Γ, V_ψ) be a HS over U, where $\Gamma : V_\psi \to \rho(U)$.

Definition 3.1. The pair (Γ, V_ψ) is called a PCHS if $\forall M \in$ range(Γ) and $\forall i \in \{1, 2, ..., n\}$ $\exists a_{C_i} : M \times D_i \to \{0, 1\}$ such that $\forall (m, d) \in M \times D_i,\ a_{C_i}(m, d) = 1$.

A set of all the PCHSs over a set U will be denoted as PCHS(U).

Example 3.2. Let a balloon seller has a set $U = \{b_1, b_2, ..., b_{20}\}$ of a total of 20 balloons some which are of different size, color, and cost. Also, let for the aforementioned attributes corresponding attribute value sets are $D_1 = \{$small, medium, large$\}$, $D_2 = \{$red, orange, blue$\}$ and $D_3 = \{$small, medium, large$\}$. Let a person is willing to buy some balloons having the attributes as big, red and expensive. Lets assume (Γ, V_ψ) be a HS over U, where $\Gamma : V_\psi \to \rho(U)$ and $V_\psi = D_1 \times D_2 \times D_3$. Also, let $\Gamma($big, red, expensive$) = \{b_3, b_{10}, b_{12}\}$.

Then corresponding PCHS will be $\Gamma(\text{big, red, expensive}) = \{b_3(1,1,1), b_{10}(1,1,1), b_{12}(1,1,1)\}$. Its graphical representation is shown in Figure 1.

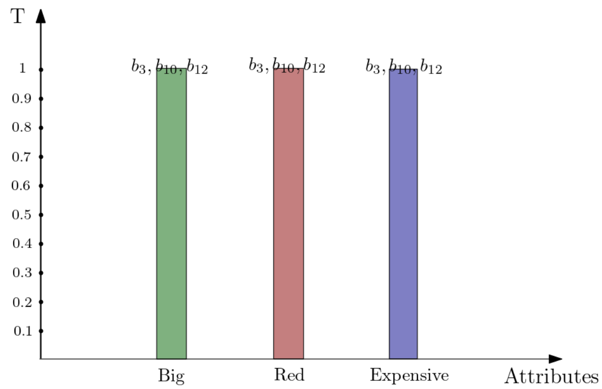

Figure 1. PCHS according to Example 3.2

Definition 3.3. The pair (Γ, V_ψ) is called a PFHS if $\forall M \in \text{range}(\Gamma)$ and $\forall i \in \{1, 2, ..., n\}$, $\exists a_{F_i} : M \times D_i \to [0,1]$ such that $\forall (m,d) \in M \times D_i,\ a_{F_i}(m,d) \in [0,1]$.

A set of all the PFHSs over a set U will be denoted as PFHS(U).

Example 3.4. In Example 3.2 let corresponding PFHS is $\Gamma(\text{big, red, expensive}) = \{b_3(0.75, 0.3, 0.8), b_{10}(0.45, 0.57, 0.2), b_{12}(0.15, 0.57, 0.95)\}$. Its graphical representation is shown in Figure 2.

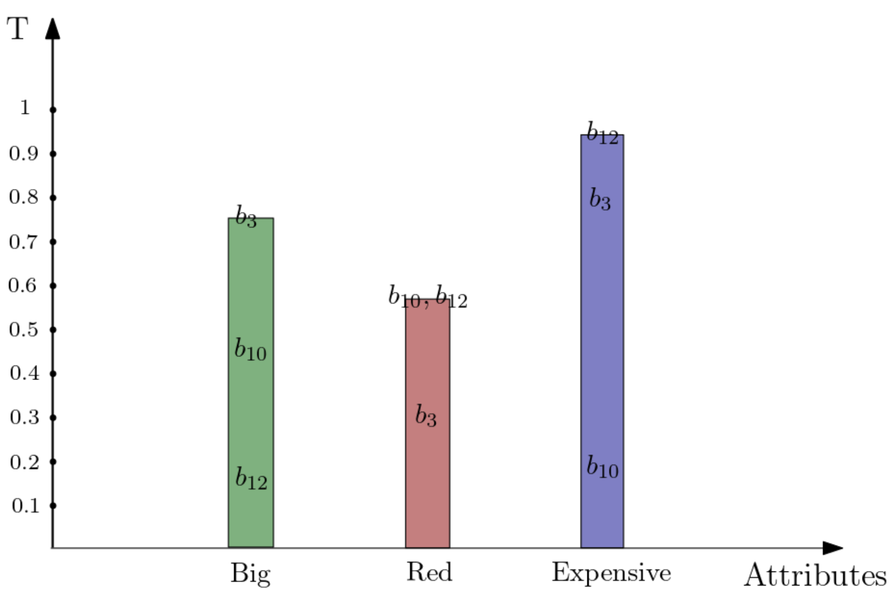

Figure 2. PFHS according to Example 3.4

Definition 3.5. The pair (Γ, V_ψ) is called a PIFHS if $\forall M \in \text{range}(\Gamma)$ and $\forall i \in \{1, 2, ..., n\}$, $\exists a_{IF_i} : M \times D_i \to [0,1] \times [0,1]$ such that $\forall (m, d) \in M \times D_i, a_{F_i}(m, d) \in [0,1] \times [0,1]$.

A set of all the PIFHSs over a set U will be denoted as PIFHS(U).

Example 3.6. In Example 3.2 let corresponding PIFHS is

$$\Gamma(\text{big, red, expensive}) = \left\{ \begin{matrix} b_3(0.87, 0.52, 0.66), b_{10}(0.6, 0.52, 0.2), b_{12}(0.33, 0.2, 0.83) \\ b_3(0.3, 0.4, 0.72), b_{10}(0.5, 0.19, 0.98), b_{12}(1, 0.72, 0.3) \end{matrix} \right\}.$$

Its graphical representation is shown in Figure 3.

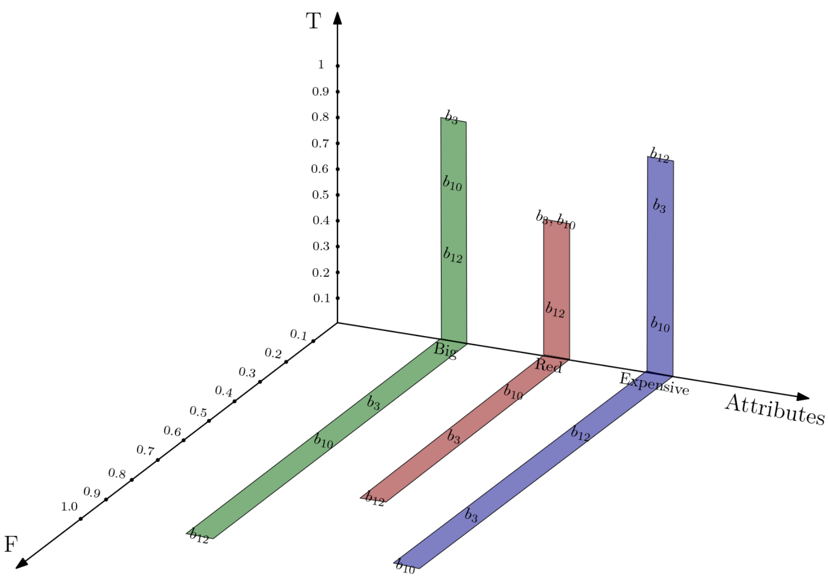

FIGURE 3. PIFHS according to Example 3.6

Definition 3.7. The pair (Γ, V_ψ) is called a PNHS if $\forall M \in \text{range}(\Gamma)$ and $\forall i \in \{1, 2, ..., n\}$, $\exists a_{N_i} : M \times D_i \to [0,1] \times [0,1] \times [0,1]$ such that $\forall (m, d) \in M \times D_i$, $a_{N_i}(m, d) \in [0,1] \times [0,1] \times [0,1]$.

A set of all the PNHSs over a set U will be denoted as PNHS(U).

Example 3.8. In Example 3.2 let corresponding PNHS is

$$\Gamma(\text{big, red, expensive}) = \left\{ \begin{matrix} b_3(0.87, 1, 0.66), b_{10}(0.61, 0.25, 0.2), b_{12}(0.32, 0.7, 0.83) \\ b_3(0.15, 0.72, 0.47), b_{10}(0.77, 0.4, 0.48), b_{12}(0.37, 0.18, 0.2) \\ b_3(0.76, 0.17, 0.29), b_{10}(0.5, 0.71, 0.98), b_{12}(1, 0.35, 0.67) \end{matrix} \right\}.$$

Its graphical representation is shown in Figure 4.

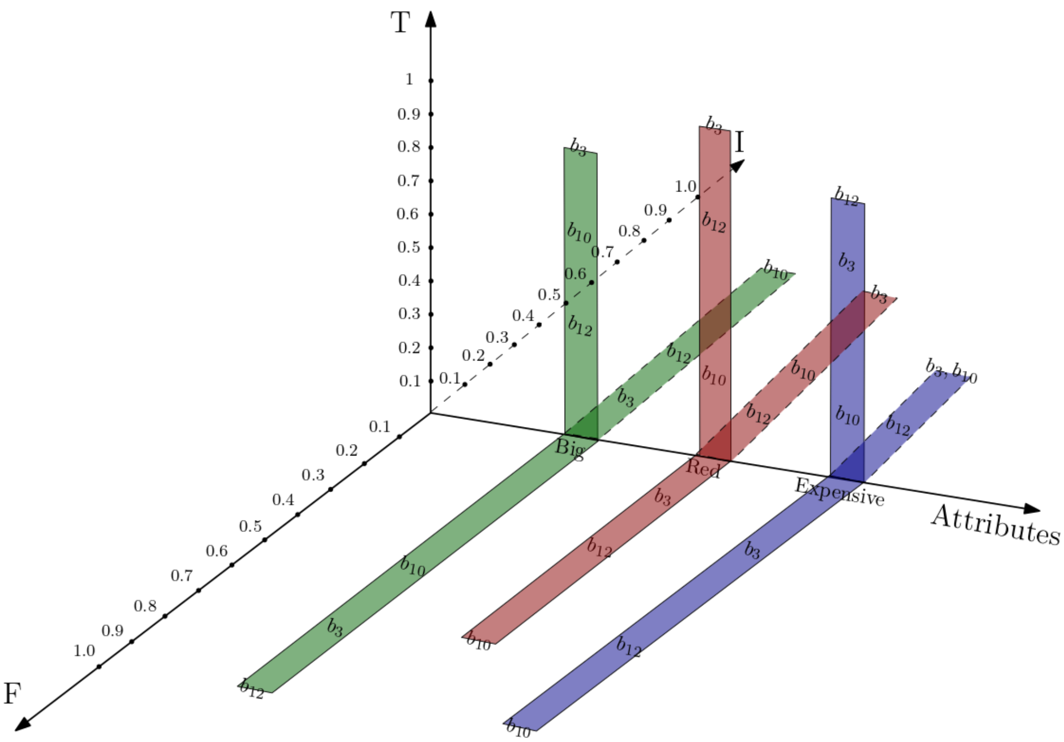

FIGURE 4. PNHS according to Example 3.8

3.1. *Images & Preimages of PFHS, PIFHS & PNHS under a function*

Let U_1 and U_2 be two CSs and $\forall i, j \in \{1, 2, ..., n\}$, D_i and P_j are attribute value sets consisting of some attribute values. Again, let $g_{ij} : U_1 \times D_i \to U_2 \times P_j$ are some functions. Then the followings can be defined:

Definition 3.9. Let $(\Gamma_1, V_\psi^1) \in \text{PFHS}(U_1)$ and $(\Gamma_2, V_\psi^2) \in \text{PFHS}(U_2)$, where $V_\psi^1 = D_1 \times D_2 \times \cdots \times D_n$ and $V_\psi^2 = P_1 \times P_2 \times \cdots \times P_n$. Also, let $\forall M \in \text{range}(\Gamma_1)$, $a_{F_i} : M \times D_i \to [0, 1]$ are the corresponding FDAFs. Again, let $\forall N \in \text{range}(\Gamma_2)$, $b_{F_j} : N \times P_j \to [0, 1]$ are the corresponding FDAFs. Then the images of (Γ_1, V_ψ^1) under the functions $g_{ij} : U_1 \times D_i \to U_2 \times P_j$ are PFHS over U_2 and they are denoted as $g_{ij}(\Gamma_1, V_\psi^1)$, where the corresponding FDAFs are defined as:

$$g_{ij}(a_{F_i})(n, p) = \begin{cases} \max a_{F_i}(m, d) \text{ if } (m, d) \in g_{ij}^{-1}(n, p) \\ 0 \qquad\qquad\qquad \text{otherwise} \end{cases}$$

The preimages of (Γ_2, V_ψ^2) under the functions $g_{ij} : U_1 \times D_i \to U_2 \times P_j$ are PFHSs over U_1, which are denoted as $g_{ij}^{-1}(\Gamma_2, V_\psi^2)$ and the corresponding FDAFs are defined as $g_{ij}^{-1}(b_{F_j})(m, d) = b_{F_j}(g_{ij}(m, d))$.

Definition 3.10. Let $(\Gamma_1, {V_\psi}^1) \in \text{PIFHS}(U_1)$ and $(\Gamma_2, {V_\psi}^2) \in \text{PIFHS}(U_2)$, where ${V_\psi}^1 = D_1 \times D_2 \times \cdots \times D_n$ and ${V_\psi}^2 = P_1 \times P_2 \times \cdots \times P_n$. Also, let $\forall M \in \text{range}(\Gamma_1)$, a_{IF_i}:

$M \times D_i \to [0,1] \times [0,1]$ with $a_{IF_i}(m,d) = \{((m,d), a^T_{IF_i}(m,d), a^F_{IF_i}(m,d)) : (m,d) \in M \times D_i\}$ are the corresponding IFDAFs. Again, let $\forall N \in \text{range}(\Gamma_2)$, $b_{IF_j} : N \times P_j \to [0,1] \times [0,1]$ with $b_{IF_j}(n,p) = \{((n,p), b^T_{IF_j}(n,p), b^F_{IF_j}(n,p)) : (n,p) \in N \times P_j\}$ are the corresponding IFDAFs. Then the images of (Γ_1, V^1_ψ) under the functions $g_{ij} : U_1 \times D_i \to U_2 \times P_j$ are PIFHS over U_2, which are denoted as $g_{ij}(\Gamma_1, V^1_\psi)$ and the corresponding IFDAFs are defined as: $g_{ij}(a_{IF_i})(n,p) = (g_{ij}(a^T_{IF_i})(n,p), g_{ij}(a^F_{IF_i})(n,p))$, where

$$g_{ij}(a^T_{IF_i})(n,p) = \begin{cases} \max a^T_{IF_i}(m,d) \text{ if } (m,d) \in g^{-1}_{ij}(n,p) \\ 0 \qquad\qquad\qquad\quad \text{otherwise} \end{cases}$$

and

$$g_{ij}(a^F_{IF_i})(n,p) = \begin{cases} \min a^F_{IF_i}(m,d) \text{ if } (m,d) \in g^{-1}_{ij}(n,p) \\ 1 \qquad\qquad\qquad\quad \text{otherwise} \end{cases}$$

The preimages of (Γ_2, V^2_ψ) under the functions $g_{ij} : U_1 \times D_i \to U_2 \times P_j$ are PIFHSs over U_1, which are denoted as $g^{-1}_{ij}(\Gamma_2, V^2_\psi)$ and the corresponding IFDAFs are defined as $g^{-1}_{ij}(b_{IF_j})(m,d) = (g^{-1}_{ij}(b^T_{IF_j})(m,d), g^{-1}_{ij}(b^F_{IF_j})(m,d))$, where $g^{-1}_{ij}(b^T_{IF_j})(m,d) = b^T_{IF_j}(g_{ij}(m,d))$ and $g^{-1}_{ij}(b^F_{IF_j})(m,d) = b^F_{IF_j}(g_{ij}(m,d))$

Definition 3.11. Let $(\Gamma_1, V^1_\psi) \in \text{PNHS}(U_1)$ and $(\Gamma_2, V^2_\psi) \in \text{PNHS}(U_2)$, where $V^1_\psi = D_1 \times D_2 \times \cdots \times D_n$ and $V^2_\psi = P_1 \times P_2 \times \cdots \times P_n$. Also, let $\forall M \in \text{range}(\Gamma_1)$, $a_{N_i} : M \times D_i \to [0,1] \times [0,1] \times [0,1]$ with $a_{N_i}(m,d) = \{((m,d), a^T_{N_i}(m,d), a^I_{N_i}(m,d), a^F_{N_i}(m,d)) : (m,d) \in M \times D_i\}$ are the corresponding NDAFs. Again, let $\forall N \in \text{range}(\Gamma_2)$, $b_{N_j} : N \times P_j \to [0,1] \times [0,1] \times [0,1]$ with $b_{N_j}(n,p) = \{((n,p), b^T_{N_i}(n,p), b^I_{N_i}(n,p), b^F_{N_i}(n,p)) : (n,p) \in N \times P_i\}$ are the corresponding NDAFs. Then the images of (Γ_1, V^1_ψ) under the functions $g_{ij} : U_1 \times D_i \to U_2 \times P_j$ are PNHS over U_2, which are denoted as $g_{ij}(\Gamma_1, V^1_\psi)$ and the corresponding NDAFs are defined as: $g_{ij}(a_{N_i})(n,p) = (g_{ij}(a^T_{N_i})(n,p), g_{ij}(a^I_{N_i})(n,p), g_{ij}(a^F_{N_i})(n,p))$, where

$$g_{ij}(a^T_{N_i})(n,p) = \begin{cases} \max a^T_{N_i}(m,d) \text{ if } (m,d) \in g^{-1}_{ij}(n,p) \\ 0 \qquad\qquad\qquad\quad \text{otherwise} \end{cases},$$

$$g_{ij}(a^I_{N_i})(n,p) = \begin{cases} \max a^I_{N_i}(m,d) \text{ if} (m,d) \in g^{-1}_{ij}(n,p) \\ 0 \qquad\qquad\qquad\quad \text{otherwise} \end{cases},$$

and

$$g_{ij}(a^F_{N_i})(n,p) = \begin{cases} \min a^F_{N_i}(m,d) \text{ if } (m,d) \in g^{-1}_{ij}(n,p) \\ 1 \qquad\qquad\qquad\quad \text{otherwise} \end{cases}$$

The preimages of (Γ_2, V_ψ^2) under the functions $g_{ij} : U_1 \times D_i \to U_2 \times P_j$ are PNHS over U_1, which are denoted as $g_{ij}^{-1}(\Gamma_2, V_\psi^2)$ and the corresponding NDAFs are defined as
$g_{ij}^{-1}(b_{N_j})(m,d) = (g_{ij}^{-1}(b_{N_j}^T)(m,d), g_{ij}^{-1}(b_{N_j}^I)(m,d), g_{ij}^{-1}(b_{N_j}^F)(m,d))$, where
$g_{ij}^{-1}(b_{N_j}^T)(m,d) = b_{N_j}^T(g_{ij}(m,d))$, $g_{ij}^{-1}(b_{N_j}^I)(m,d) = b_{N_j}^I(g_{ij}(m,d))$ and
$g_{ij}^{-1}(b_{N_j}^F)(m,d) = b_{N_j}^F(g_{ij}(m,d))$.

In the next segment, we have defined plithogenic hypersoft subgroups in fuzzy, intuitionistic fuzzy, and neutrosophic environments. We have also, analyzed their homomorphic properties.

3.2. *Plithogenic Hypersoft Subgroup*

3.2.1. *Plithogenic Fuzzy Hypersoft Subgroup*

Definition 3.12. Let the pair (Γ, V_ψ) be a PFHS of a CG U, where $V_\psi = D_1 \times D_2 \times \cdots \times D_n$ and $\forall i \in \{1,2,...,n\}$, D_i are CGs. Then (Γ, V_ψ) is called a PFHSG of U if and only if $\forall M \in$ range(Γ), $\forall (m_1,d),(m_2,d') \in M \times D_i$ and $\forall a_{F_i} : M \times D_i \to [0,1]$, the conditions mentioned below are satisfied:

(i) $a_{F_i}((m_1,d)\cdot(m_2,d')) \geq \min\{a_{F_i}(m_1,d), a_{F_i}(m_2,d')\}$ and
(ii) $a_{F_i}(m_1,d)^{-1} \geq a_{F_i}(m_1,d)$.

A set of all PFHSG of a CG U is denoted as PFHSG(U).

Example 3.13. Let $U = \{e,m,u,mu\}$ be the Kleins 4-group and $\psi = \{r_1,r_2\}$ is a set of two attributes and corresponding attribute value sets are respectively, $D_1 = \{1,i,-1,-i\}$ and $D_2 = \{1,w,w^2\}$, which are two cyclic groups. Let $V_\psi = D_1 \times D_2$ and (Γ, V_ψ) be a HS over U, where $\Gamma : V_\psi \to \rho(U)$ such that the range of Γ i.e. R$(\Gamma) = \{\{e,m\},\{e,u\},\{e,mu\}\}$. Let for $M = \{e,m\}$, $a_{F_1} : M \times D_1 \to [0,1]$ is defined in Table 2 and $a_{F_2} : M \times D_2 \to [0,1]$ is defined in Table 3 respectively.

TABLE 2. Membership values of a_{F_1}

a_{F_1}	1	i	-1	$-i$
e	0.4	0.2	0.4	0.2
m	0.2	0.2	0.2	0.2

TABLE 3. Membership values of a_{F_2}

a_{F_2}	1	w	w^2
e	0.8	0.5	0.5
m	0.6	0.5	0.5

Let for $M = \{e,u\}$, $a_{F_1} : M \times D_1 \to [0,1]$ is defined in Table 4 and $a_{F_2} : M \times D_2 \to [0,1]$ is defined in Table 5 respectively.

TABLE 4. Membership values of a_{F_1}

a_{F_1}	1	i	-1	$-i$
e	0.8	0.2	0.7	0.2
u	0.5	0.2	0.5	0.2

TABLE 5. Membership values of a_{F_2}

a_{F_2}	1	w	w^2
e	0.7	0.4	0.4
u	0.3	0.3	0.3

Let for $M = \{e, mu\}$, $a_{F_1} : M \times D_1 \to [0,1]$ is defined in Table 6 and $a_{F_2} : M \times D_2 \to [0,1]$ is defined in Table 7 respectively.

TABLE 6. Membership values of a_{F_2}

a_{F_1}	1	i	-1	$-i$
e	0.9	0.2	0.4	0.2
mu	0.7	0.2	0.7	0.2

TABLE 7. Membership values of a_{F_1}

a_{F_2}	1	w	w^2
e	1	0.3	0.3
mu	0.2	0.2	0.2

Here, for any $M \in \text{range}(\Gamma)$ and $\forall i \in \{1,2\}$, a_{F_i} satisfy Definition 3.12. Hence, $(\Gamma, V_\psi) \in$ PFHSG(U).

Proposition 3.1. *Let U be a CG and $(\Gamma, V_\psi) \in$ PFHSG(U), where $V_\psi = D_1 \times D_2 \times \cdots \times D_n$ and $\forall i \in \{1, 2, ..., n\}$, D_i are CGs. Then for any $M \in$ range(Γ), $\forall (m,d) \in M \times D_i$ and $\forall a_{F_i} : M \times D_i \to [0,1]$, the followings are satisfied:*

(i) $a_{F_i}(e, d_e^i) \geq a_{F_i}(m,d)$, *where e and d_e^i are the neutral elements of U and D_i.*

(ii) $a_{F_i}(m,d)^{-1} = a_{F_i}(m,d)$

Proof. (i) Let e and d_e^i be the neutral elements of U and D_i. Then $\forall (m,d) \in M \times D_i$,

$$\begin{aligned} a_{F_i}(e, d_e^i) &= a_{F_i}((m,d)\cdot(m,d)^{-1}), \\ &\geq \min\{a_{F_i}(m,d), a_{F_i}(m,d)^{-1}\} \text{ (by Definition 3.12)} \\ &\geq \min\{a_{F_i}(m,d), a_{F_i}(m,d)\} \text{ (by Definition 3.12)} \\ &\geq a_{F_i}(m,d) \end{aligned}$$

(ii) Let U be a group and $(\Gamma, V_\psi) \in$ PFHSG(U). Then by Definition 3.12,

$$a_{F_i}(m,d)^{-1} \geq a_{F_i}(m,d) \tag{3.1}$$

Again,

$$\begin{aligned} a_{F_i}(m,d) &= a_{F_i}((m,d)^{-1})^{-1} \\ &\geq a_{F_i}(m,d)^{-1} \end{aligned} \tag{3.2}$$

Hence, from Equation 3.1 and Equation 3.2, $a_{F_i}(m,d)^{-1} = a_{F_i}(m,d)$. □

Proposition 3.2. *Let the pair* (Γ, V_ψ) *be a PFHS of a CG* U*, where* $V_\psi = D_1 \times D_2 \times \cdots \times D_n$ *and* $\forall i \in \{1, 2, ..., n\}$*,* D_i *are CGs. Then* (Γ, V_ψ) *is called a PFHSG of* U *if and only if* $\forall M \in$ *range*(Γ)*,* $\forall (m_1, d), (m_2, d') \in M \times D_i$ *and* $\forall a_{F_i} : M \times D_i \to [0, 1]$*,* $a_{F_i}((m_1, d) \cdot (m_2, d')^{-1}) \geq \min\{a_{F_i}(m_1, d), a_{F_i}(m_2, d')\}$*.*

Proof. Let U be a CG and $(\Gamma, V_\psi) \in$ PFHSG(U). Then by Definition 3.12 and Proposition 3.1

$$\begin{aligned} a_{F_i}((m_1, d) \cdot (m_2, d')^{-1}) &\geq \min\{a_{F_i}(m_1, d), a_{F_i}(m_2, d')^{-1}\} \\ &= \min\{a_{F_i}(m_1, d), a_{F_i}(m_2, d')\} \end{aligned}$$

Conversely, let $a_{F_i}((m_1, d) \cdot (m_2, d')^{-1}) \geq \min\{a_{F_i}(m_1, d), a_{F_i}(m_2, d')\}$. Also, let e and d_e^i be the neutral elements of U and D_i. Then,

$$\begin{aligned} a_{F_i}(m, d)^{-1} &= a_{F_i}((e, d_e^i) \cdot (m, d)^{-1}) \\ &\geq \min\{a_{F_i}(e, d_e^i), a_{F_i}(m, d)\} \\ &= \min\{a_{F_i}((m, d) \cdot (m, d)^{-1}), a_{F_i}(m, d)\} \\ &\geq \min\{a_{F_i}(m, d), a_{F_i}(m, d), a_{F_i}(m, d)\} \\ &= a_{F_i}(m, d) \end{aligned} \tag{3.3}$$

Now,

$$\begin{aligned} a_{F_i}((m_1, d) \cdot (m_2, d')) &= a_{F_i}((m_1, d) \cdot ((m_2, d')^{-1})^{-1}) \\ &\geq \min\{a_{F_i}(m_1, d), a_{F_i}(m_2, d')^{-1}\} \\ &= \min\{a_{F_i}(m_1, d), a_{F_i}(m_2, d')\} \text{ (by Equation 3.3)} \end{aligned} \tag{3.4}$$

Hence, by Equation 3.3 and Equation 3.4, $(\Gamma, V_\psi) \in$ PFHSG(U). □

Proposition 3.3. *Intersection of two PFHSGs is also a PFHSG.*

Theorem 3.4. *The homomorphic image of a PFHSG is a PFHSG.*

Proof. Let U_1 and U_2 be two CGs and $\forall i, j \in \{1, 2, ..., n\}$, D_i and P_j are attribute value sets consisting of some attribute values and let $g_{ij} : U_1 \times D_i \to U_2 \times P_j$ are homomorphisms. Also, let $(\Gamma_1, V_\psi{}^1) \in$ PFHSG(U_1), where $V_\psi{}^1 = D_1 \times D_2 \times \cdots \times D_n$. Again, let $\forall M \in$ range(Γ_1), $a_{F_i} : M \times D_i \to [0, 1]$ are the corresponding FDAFs.
Assuming $(n_1, p_1), (n_2, p_2) \in U_2 \times P_j$, if $g_{ij}^{-1}(n_1, p_1) = \phi$ and $g_{ij}^{-1}(n_2, p_2) = \phi$, then $g_{ij}(\Gamma_1, V_\psi{}^1) \in$ PFHSG(U_2).
Lets assume that $\exists (m_1, d_1), (m_2, d_2) \in U_1 \times D_i$ such that $g_{ij}(m_1, d_1) = (n_1, p_1)$ and

$g_{ij}(m_2, d_2) = (n_2, p_2)$. Then

$$\begin{aligned} g_{ij}(a_{F_i})(n_1,p_1)\cdot(n_2,p_2)^{-1} &= \max_{(n_1,p_1)\cdot(n_2,p_2)^{-1}=g_{ij}(m,d)} a_{F_i}(m,d) \\ &\geq a_{F_i}(m_1,d_1)\cdot(m_2,d_2)^{-1} \\ &\geq \min\{a_{F_i}(m_1,d_1), a_{F_i}(m_2,d_2)\} \text{ (as } (\Gamma_1, V_\psi^1) \in \text{PFHSG}(U_1)) \\ &\geq \min\{\max_{(n_1,p_1)=g_{ij}(m_1,d_1)} a_{F_i}(m_1,d_1), \max_{(n_2,p_2)=g_{ij}(m_2,d_2)} a_{F_i}(m_2,d_2)\} \\ &\geq \min\{g_{ij}(a_{F_i})(n_1,p_1), g_{ij}(a_{F_i})(n_2,p_2)\} \end{aligned}$$

Hence, $g_{ij}(\Gamma_1, V_\psi^1) \in \text{PFHSG}(U_2)$. □

Theorem 3.5. *The homomorphic preimage of a PFHSG is a PFHSG.*

Proof. Let U_1 and U_2 be two CGs and $\forall i, j \in \{1, 2, ..., n\}$, D_i and P_j are attribute value sets consisting of some attribute values and let $g_{ij} : U_1 \times D_i \to U_2 \times P_j$ are homomorphisms. Also, let $(\Gamma_2, V_\psi^2) \in \text{PFHSG}(U_2)$, $V_\psi^2 = P_1 \times P_2 \times \cdots \times P_n$. Again, $\forall N \in \text{range}(\Gamma_2)$, $b_{F_j} : N \times P_j \to [0, 1]$ are the corresponding FDAFs. Lets assume $(m_1, d_1), (m_2, d_2) \in U_1 \times D_i$. As g_{ij} is a homomorphism the followings can be concluded:

$$\begin{aligned} g_{ij}^{-1}(b_{F_i})(m_1,d_1)\cdot(m_2,d_2)^{-1} & \\ &= b_{F_i}(g_{ij}((m_1,d_1)\cdot(m_2,d_2)^{-1})) \\ &= b_{F_i}(g_{ij}(m_1,d_1)\cdot g_{ij}(m_2,d_2)^{-1}) \text{ (As } g_{ij} \text{ is a homomorphism)} \\ &\geq \min\{b_{F_i}(g_{ij}(m_1,d_1)), b_{F_i}(g_{ij}(m_2,d_2))\} \text{ (As } (\Gamma_2, V_\psi^2) \in \text{PFHSG}(U_2)) \\ &\geq \min\{g_{ij}^{-1}(b_{F_i})(m_1,d_1), g_{ij}^{-1}(b_{F_i})(m_2,d_2)\} \end{aligned}$$

Then $g_{ij}^{-1}(\Gamma_2, V_\psi^2) \in \text{PFHSG}(U_1)$. □

3.2.2. *Plithogenic Intuitionistic Fuzzy Hypersoft Subgroup*

Definition 3.14. Let the pair (Γ, V_ψ) be a PIFHS of a CG U, where $V_\psi = D_1 \times D_2 \times \cdots \times D_n$ and $\forall i \in \{1, 2, ..., n\}$, D_i are CGs. Then (Γ, V_ψ) is called a PIFHSG of U if and only if $\forall M \in \text{range}(\Gamma)$, $\forall (m_1, d), (m_2, d') \in M \times D_i$ and $\forall a_{IF_i} : M \times D_i \to [0, 1] \times [0, 1]$ with $a_{IF_i}(m, d) = \{((m, d), a^T_{IF_i}(m, d), a^F_{IF_i}(m, d)) : (m, d) \in M \times D_i\}$, the subsequent conditions are fulfilled:

(i) $a^T_{IF_i}((m_1, d) \cdot (m_2, d')) \geq \min\{a^T_{IF_i}(m_1, d), a^T_{IF_i}(m_2, d')\}$
(ii) $a^T_{IF_i}(m_1, d)^{-1} \geq a^T_{IF_i}(m_1, d)$
(iii) $a^F_{IF_i}((m_1, d) \cdot (m_2, d')) \leq \max\{a^F_{IF_i}(m_1, d), a^F_{IF_i}(m_2, d')\}$
(iv) $a^F_{IF_i}(m_1, d)^{-1} \leq a^F_{IF_i}(m_1, d)$

A set of all PIFHSG of a CG U is denoted as PIFHSG(U).

Example 3.15. Let $U = S_3$ be a CG and $\psi = \{r_1, r_2\}$ is a set of two attributes and corresponding attribute value sets are respectively, $D_1 = A_3$ and $D_2 = S_2$, which are respectively an alternating group of order 3 and a symmetric group of order 2. Let $V_\psi = D_1 \times D_2$ and (Γ, V_ψ) be a HS over U, where $\Gamma : V_\psi \to \rho(U)$ such that the range of Γ i.e. $\mathrm{R}(\Gamma) = \{\{(1),(13)\},\{(1),(23)\}\}$. Let for $M = \{(1),(13)\}$, $a_{IF_1} : M \times D_1 \to [0,1] \times [0,1]$ is defined in Table 8–9 and $a_{IF_2} : M \times D_2 \to [0,1] \times [0,1]$ is defined in Table 10–11 respectively.

TABLE 8. Membership values of a_{IF_1}

$a^T_{IF_1}$	(1)	(123)	(132)
(1)	0.4	0.5	0.5
(13)	0.2	0.2	0.2

TABLE 9. Non-membership values of a_{IF_1}

$a^F_{IF_1}$	(1)	(123)	(132)
(1)	0.4	0.7	0.7
(13)	0.8	0.8	0.8

TABLE 10. Membership values of a_{IF_2}

$a^T_{IF_2}$	(1)	(12)
(1)	0.8	0.4
(13)	0.3	0.3

TABLE 11. Non-membership values of a_{IF_2}

$a^F_{IF_2}$	(1)	(12)
(1)	0.4	0.8
(13)	0.9	0.9

Let for $M = \{(1),(23)\}$ $a_{IF_1} : M \times D_1 \to [0,1] \times [0,1]$ is defined in Table 12–13 and $a_{IF_2} : M \times D_2 \to [0,1] \times [0,1]$ is defined in Table 14–15 respectively.

TABLE 12. Membership values of a_{IF_1}

$a^T_{IF_1}$	(1)	(123)	(132)
(1)	0.6	0.4	0.4
(23)	0.5	0.4	0.4

TABLE 13. Non-membership values of a_{IF_1}

$a^F_{IF_1}$	(1)	(123)	(132)
(1)	0.4	0.7	0.7
(23)	0.6	0.7	0.7

TABLE 14. Membership values of a_{IF_2}

$a^T_{IF_2}$	(1)	(12)
(1)	0.7	0.6
(23)	0.7	0.6

TABLE 15. Non-membership values of a_{IF_2}

$a^F_{IF_2}$	(1)	(12)
(1)	0.5	0.9
(23)	0.8	0.9

Here, for any $M \in \text{range}(\Gamma)$ and $\forall i \in \{1,2\}$, a_{IF_i} satisfy Definition 3.14. Hence, $(\Gamma, V_\psi) \in$ PIFHSG(U).

Proposition 3.6. *Let U be a CG and $(\Gamma, V_\psi) \in PIFHSG(U)$, where $V_\psi = D_1 \times D_2 \times \cdots \times D_n$ and $\forall i \in \{1, 2, ..., n\}$, D_i are CGs. Then for any $M \in range(\Gamma)$ and $\forall (m, d_i) \in M \times D_i$ and $\forall a_{IF_i} : M \times D_i \to [0,1] \times [0,1]$ with $a_{IF_i}(m,d) = \{((m,d), a^T_{IF_i}(m,d), a^F_{IF_i}(m,d)) : (m,d) \in M \times D_i\}$, the subsequent conditions are satisfied:*

(i) $a^T_{IF_i}(e, d^i_e) \geq a^T_{IF_i}(m,d)$, *where e and d^i_e are the neutral elements of U and D_i.*
(ii) $a^T_{IF_i}(m,d)^{-1} = a^T_{IF_i}(m,d)$
(iii) $a^F_{IF_i}(e, d^i_e) \leq a^F_{IF_i}(m,d)$, *where e and d^i_e are the neutral elements of U and D_i.*
(iv) $a^F_{IF_i}(m,d)^{-1} = a^F_{IF_i}(m,d)$

Proof. Here, (i) and (ii) can be easily proved using Proposition 3.1.
(iii) Let e and d^i_e be the neutral elements of U and D_i. Then $\forall (m,d) \in M \times D_i$,

$$\begin{aligned}
a^F_{IF_i}(e, d^i_e) &= a^F_{IF_i}((m,d)\cdot(m,d)^{-1}) \\
&\leq \max\{a^F_{IF_i}(m,d), a^F_{IF_i}(m,d)^{-1}\} \text{ (by Definition 3.14)} \\
&\leq \max\{a^F_{IF_i}(m,d), a^F_{IF_i}(m,d)\} \text{ (by Definition 3.14)} \\
&\leq a^F_{IF_i}(m,d)
\end{aligned}$$

(iv) Let U be a CG and $(\Gamma, V_\psi) \in$ PFHSG(U). Then by Definition 3.14,

$$a^F_{IF_i}(m,d)^{-1} \leq a^F_{IF_i}(m,d) \tag{3.5}$$

Again,

$$\begin{aligned}
a^F_{IF_i}(m,d) &= a^F_{IF_i}(((m,d)^{-1})^{-1} \\
&\leq a^F_{IF_i}(m,d)^{-1}
\end{aligned} \tag{3.6}$$

Hence, by Equation 3.5 and Equation 3.6, $a^F_{IF_i}(m,d)^{-1} = a^F_{IF_i}(m,d)$. □

Proposition 3.7. *Let the pair (Γ, V_ψ) be a PIFHS of a CG U, where $V_\psi = D_1 \times D_2 \times \cdots \times D_n$ and $\forall i \in \{1, 2, ..., n\}$, D_i are CGs. Then (Γ, V_ψ) is called a PIFHSG of U if and only if $\forall M \in range(\Gamma)$, $\forall (m_1, d), (m_2, d') \in M \times D_i$ and $\forall a_{IF_i} : M \times D_i \to [0,1] \times [0,1]$ with $a_{IF_i}(m,d) = \{((m,d), a^T_{IF_i}(m,d),\ a^F_{IF_i}(m,d)) : (m,d) \in M \times D_i\}$, the subsequent conditions are fulfilled:*

(i) $a^T_{IF_i}((m_1,d)\cdot(m_2,d')^{-1}) \geq \min\{a^T_{IF_i}(m_1,d), a^T_{IF_i}(m_2,d')\}$ *and*
(ii) $a^F_{IF_i}((m_1,d)\cdot(m_2,d')^{-1}) \leq \max\{a^F_{IF_i}(m_1,d), a^F_{IF_i}(m_2,d')\}$

Proof. Here, (i) can be proved using Proposition 3.2.
(ii) Let U be a CG and $(\Gamma, V_\psi) \in$ PFHSG(U). Then by Definition 3.14 and Proposition 3.6

$$\begin{aligned}
a^F_{IF_i}((m_1,d)\cdot(m_2,d')^{-1}) &\leq \max\{a^F_{IF_i}(m_1,d), a^F_{IF_i}(m_2,d')^{-1}\} \\
&\leq \max\{a^F_{I}\ (m_1,d), a^F_{I}\ (m_2,d')\}
\end{aligned}$$

Conversely, let $a^F_{IF_i}((m_1,d)\cdot(m_2,d')^{-1}) \le \max\{a^F_{IF_i}(m_1,d), a^F_{IF_i}(m_2,d')\}$. Also, let e and d^i_e be the neutral elements of U and D_i. Then

$$\begin{aligned}
a^F_{IF_i}(m,d)^{-1} &= a^F_{IF_i}((e,d^i_e)\cdot(m,d)^{-1})\\
&\le \max\{a^F_{IF_i}(e,d^i_e), a^F_{IF_i}(m,d)\}\\
&\le \max\{a^F_{IF_i}((m,d)\cdot(m,d)^{-1}), a^F_{IF_i}(m,d)\}\\
&\le \max\{a^F_{IF_i}(m,d), a^F_{IF_i}(m,d), a^F_{IF_i}(m,d)\}\\
&= a^F_{IF_i}(m,d) \qquad (3.7)
\end{aligned}$$

Now,

$$\begin{aligned}
a^F_{IF_i}((m_1,d)\cdot(m_2,d')) &= a^F_{IF_i}((m_1,d)\cdot((m_2,d')^{-1})^{-1})\\
&\le \max\{a^F_{IF_i}(m_1,d), a^F_{IF_i}(m_2,d')^{-1}\}\\
&= \max\{a^F_{IF_i}(m_1,d), a^F_{IF_i}(m_2,d')\} \text{ (by Equation 3.7)} \qquad (3.8)
\end{aligned}$$

Hence, by Equation 3.7 and Equation 3.8, $(\Gamma, V_\psi) \in \text{PFHSG}(U)$. □

Proposition 3.8. *Intersection of two PIFHSGs is also a PIFHSG.*

Theorem 3.9. *The homomorphic image of a PIFHSG is a PIFHSG.*

Proof. Let U_1 and U_2 be two CGs and $\forall i,j \in \{1,2,...,n\}$, D_i and P_j are attribute value sets consisting of some attribute values and let $g_{ij} : U_1 \times D_i \to U_2 \times P_j$ are homomorphisms. Also, let $(\Gamma_1, V^1_\psi) \in \text{PIFHSG}(U_1)$, where $V^1_\psi = D_1 \times D_2 \times \cdots \times D_n$. Again, let $\forall M \in \text{range}(\Gamma_1)$ and $a_{IF_i} : M \times D_i \to [0,1]\times[0,1]$ with $a_{IF_i}(m,d) = \{((m,d), a^T_{IF_i}(m,d), a^F_{IF_i}(m,d)) : (m,d) \in M \times D_i\}$ are the corresponding IFDAFs. Assuming $(n_1,p_1),(n_2,p_2) \in U_2 \times P_j$, if $g^{-1}_{ij}(n_1,p_1) = \phi$ and $g^{-1}_{ij}(n_2,p_2) = \phi$, then $g_{ij}(\Gamma_1, V^1_\psi) \in \text{PIFHSG}(U_2)$. Lets assume that $\exists (m_1,d_1),(m_2,d_2) \in U_1 \times D_i$ such that $g_{ij}(m_1,d_1) = (n_1,p_1)$ and $g_{ij}(m_2,d_2) = (n_2,p_2)$. Then by Theorem 3.4

$$g_{ij}(a^T_{IF_i})(n_1,p_1)\cdot(n_2,p_2)^{-1} \ge \min\{g_{ij}(a^T_{IF_i})(n_1,p_1), g_{ij}(a^T_{IF_i})(n_2,p_2)\}.$$

Again,

$$\begin{aligned}
g_{ij}(a^F_{IF_i})(n_1,p_1)\cdot(n_2,p_2)^{-1} &= \min_{(n_1,p_1)\cdot(n_2,p_2)^{-1}=g_{ij}(m,d)} a^F_{IF_i}(m,d)\\
&\le a^F_{IF_i}(m_1,d_1)\cdot(m_2,d_2)^{-1}\\
&\le \max\{a^F_{IF_i}(m_1,d_1), a^F_{IF_i}(m_2,d_2)\} \text{ (as } (\Gamma_1,V^1_\psi) \in PIFHSG(U_1))\\
&\le \max\{\min_{(n_1,p_1)=g_{ij}(m_1,d_1)} a^F_{IF_i}(m_1,d_1), \min_{(n_2,p_2)=g_{ij}(m_2,d_2)} a^F_{IF_i}(m_2,d_2)\}\\
&\le \max\{g_{ij}(a^F_{I})(n_1,p_1), g_{ij}(a^F_{I})(n_2,p_2)\} \text{ (by Definition 3.10)}
\end{aligned}$$

Hence, $g_{ij}(\Gamma_1, V_\psi^1) \in$ PIFHSG(U_2). □

Theorem 3.10. *The homomorphic preimage of a PIFHSG is a PIFHSG.*

Proof. Let U_1 and U_2 be two CGs and $\forall i, j \in \{1, 2, ..., n\}$, D_i and P_j are attribute value sets consisting of some attribute values and let $g_{ij} : U_1 \times D_i \to U_2 \times P_j$ are homomorphisms. Also, let $(\Gamma_2, V_\psi^2) \in$ PIFHS(U_2), where $V_\psi^2 = P_1 \times P_2 \times \cdots \times P_n$. Again, let $\forall N \in$ range(Γ_2), $b_{IF_j} : N \times P_j \to [0,1] \times [0,1]$ with $b_{IF_j}(n,p) = \{((n,p), b^T_{IF_j}(n,p), b^F_{IF_j}(n,p)) : (n,p) \in N \times P_j\}$ are the corresponding IFDAFs. Lets assume$(m_1, d_1), (m_2, d_2) \in U_1 \times D_i$. Since, g_{ij} is a homomorphism, by Theorem 3.5

$$g_{ij}^{-1}(b^T_{IF_j})(m_1, d_1) \cdot (m_2, d_2)^{-1} \geq \min\{g_{ij}^{-1}(b^T_{IF_j})(m_1, d_1), g_{ij}^{-1}(b^T_{IF_j})(m_2, d_2)\}.$$

Again,

$$\begin{aligned} g_{ij}^{-1}(b^F_{IF_j})(m_1, d_1) \cdot (m_2, d_2)^{-1} &= b^F_{IF_j}(g_{ij}((m_1, d_1) \cdot (m_2, d_2)^{-1})) \\ &= b^F_{IF_j}(g_{ij}(m_1, d_1) \cdot g_{ij}(m_2, d_2)^{-1}) \text{ (As } g_{ij} \text{ is a homomorphism)} \\ &\leq \max\{b^F_{IF_j}(g_{ij}(m_1, d_1)), b^F_{IF_j}(g_{ij}(m_2, d_2))\} \\ &\qquad \text{(As } (\Gamma_2, V_\psi^2) \in PIFHSG(U_2)) \\ &\leq \max\{g_{ij}^{-1}(b^F_{IF_j})(m_1, d_1), g_{ij}^{-1}(b^F_{IF_j})(m_2, d_2)\} \end{aligned}$$

Hence, $g_{ij}^{-1}(\Gamma_2, V_\psi^2) \in$ PIFHSG(U_1). □

3.2.3. *Plithogenic Neutrosophic Hypersoft Subgroup*

Definition 3.16. content.Let the pair (Γ, V_ψ) be a PNHS of a CG U, where $V_\psi = D_1 \times D_2 \times \cdots \times D_n$ and $\forall i \in \{1, 2, ..., n\}$, D_i are CGs. Then (Γ, V_ψ) is called a PNHSG of U if and only if $\forall M \in$ range(Γ), $\forall (m_1, d), (m_2, d') \in M \times D_i$ and $\forall a_{N_i} : M \times D_i \to [0,1] \times [0,1] \times [0,1]$, with $a_{N_i}(m, d) = \{((m, d), a^T_{N_i}(m, d),\ a^I_{N_i}(m, d), a^F_{N_i}(m, d)) : (m, d) \in M \times D_i\}$, the subsequent conditions are fulfilled:

(i) $a^T_{N_i}((m_1, d) \cdot (m_2, d')^{-1}) \geq \min\{a^T_{N_i}(m_1, d), a^T_{N_i}(m_2, d')\}$
(ii) $a^T_{N_i}(m_1, d)^{-1} \geq a^T_{N_i}(m_1, d)$
(iii) $a^I_{N_i}((m_1, d) \cdot (m_2, d')^{-1}) \geq \min\{a^I_{N_i}(m_1, d), a^I_{N_i}(m_2, d')\}$
(iv) $a^I_{N_i}(m_1, d)^{-1} \geq a^I_{N_i}(m_1, d)$
(v) $a^F_{N_i}((m_1, d) \cdot (m_2, d')^{-1}) \leq \max\{a^F_{N_i}(m_1, d), a^F_{N_i}(m_2, d')\}$
(vi) $a^F_{N_i}(m_1, d)^{-1} \leq a^F_{N_i}(m_1, d)$

A set of all PNHSG of a CG U is denoted as PNHSG(U).

Example 3.17. Let $D_6 = \{e, m, u, mu, um, mum\}$ be a dihedral group of order 6 and $\psi = \{r_1, r_2\}$ is a set of two attributes and corresponding attribute value sets are respectively, $D_1 = \{1, w, w^2\}$ and $D_2 = A_3$, which are respectively a cyclic group of order 3 and an alternating group of order 3. Let $V_\psi = D_1 \times D_2$ and (Γ, V_ψ) be a HS over U, where $\Gamma : V_\psi \to \rho(U)$ such that the range of Γ i.e. $\mathrm{R}(\Gamma) = \{\{e, mu, um\}, \{e, mum\}\}$.
Let for $M = \{e, mu\}$, $a_{N_1} : M \times D_1 \to [0,1] \times [0,1] \times [0,1]$ is defined in Table 16–18 and $a_{N_2} : M \times D_2 \to [0,1] \times [0,1] \times [0,1]$ is defined in Table 19–21 respectively.

TABLE 16. Truth values of a_{N_1}

$a^T_{N_1}$	1	w	w^2
e	0.7	0.5	0.5
mu	0.3	0.3	0.3
um	0.3	0.3	0.3

TABLE 17. Indeterminacy values of a_{N_1}

$a^I_{N_1}$	1	w	w^2
e	0.8	0.4	0.4
mu	0.5	0.4	0.4
um	0.5	0.4	0.4

TABLE 18. Falsity values of a_{N_1}

$a^F_{N_1}$	1	w	w^2
e	0.3	0.5	0.5
mu	0.7	0.7	0.7
um	0.7	0.7	0.7

TABLE 19. Truth values of a_{N_2}

$a^T_{N_2}$	(1)	(123)	(132)
e	0.7	0.2	0.2
mu	0.1	0.1	0.1
um	0.1	0.1	0.1

TABLE 20. Indeterminacy values of a_{N_2}

$a^I_{N_2}$	(1)	(123)	(132)
e	0.8	0.5	0.5
mu	0.8	0.5	0.5
um	0.8	0.5	0.5

TABLE 21. Falsity values of a_{N_2}

$a^F_{N_2}$	(1)	(123)	(132)
e	0.3	0.8	0.8
mu	0.9	0.9	0.9
um	0.9	0.9	0.9

Let for $M = \{e, mum\}$, $a_{N_1} : M \times D_1 \to [0,1] \times [0,1] \times [0,1]$ is defined in Table 22–24 and $a_{N_2} : M \times D_2 \to [0,1] \times [0,1] \times [0,1]$ is defined in Table 25–27 respectively.

TABLE 22. Truth values of a_{N_1}

$a^T_{N_1}$	1	w	w^2
e	0.8	0.4	0.4
mum	0.2	0.2	0.2

TABLE 23. Indeterminacy values of a_{N_1}

$a^I_{N_1}$	1	w	w^2
e	0.8	0.6	0.6
mum	0.7	0.6	0.6

Table 24. Falsity values of a_{N_1}

$a^F_{N_1}$	1	w	w^2
e	0.2	0.6	0.6
mum	0.8	0.8	0.8

Table 25. Truth values of a_{N_2}

$a^T_{N_2}$	(1)	(123)	(132)
e	0.9	0.8	0.8
mum	0.9	0.8	0.8

Table 26. Indeterminacy values of a_{N_2}

$a^I_{N_2}$	(1)	(123)	(132)
e	0.5	0.2	0.2
mum	0.1	0.1	0.1

Table 27. Falsity values of a_{N_2}

$a^F_{N_2}$	(1)	(123)	(132)
e	0.1	0.2	0.2
mum	0.1	0.2	0.2

Here, for any $M \in \text{range}(\Gamma)$ and $\forall i \in \{1, 2\}$, a_{N_i} satisfy Definition 3.16. Hence, $(\Gamma, V_\psi) \in$ PNHSG(U).

Proposition 3.11. *Let the pair (Γ, V_ψ) be a PNHS of a CG U, where $V_\psi = D_1 \times D_2 \times \cdots \times D_n$ and $\forall i \in \{1, 2, ..., n\}$, D_i are CGs. Then (Γ, V_ψ) is called a PNHSG of U if and only if $\forall M \in range(\Gamma)$, $\forall (m_1, d), (m_2, d') \in M \times D_i$ and $\forall a_{N_i} : M \times D_i \to [0, 1] \times [0, 1] \times [0, 1]$, with $a_{N_i}(m, d) = \{((m, d), a^T_{N_i}(m, d), a^I_{N_i}(m, d), a^F_{N_i}(m, d)) : (m, d) \in M \times D_i\}$. Then the subsequent conditions are satisfied:*

(i) $a^T_{N_i}(e, d) \geq a^T_{N_i}(m, d)$, *where e is the neutral element of U.*
(ii) $a^T_{N_i}(m, d)^{-1} = a^T_{N_i}(m, d)$
(iii) $a^I_{N_i}(e, d) \geq a^I_{N_i}(m, d)$, *where e is the neutral element of U.*
(iv) $a^I_{N_i}(m, d)^{-1} = a^I_{N_i}(m, d)$
(v) $a^F_{N_i}(e, d) \leq a^F_{N_i}(m, d)$, *where e is the neutral element of U.*
(vi) $a^F_{N_i}(m, d)^{-1} = a^F_{N_i}(m, d)$

Proof. This can be proved using Proposition 3.1 and Proposition 3.6. □

Proposition 3.12. *Let the pair (Γ, V_ψ) be a PNHS of a CG U, where $V_\psi = D_1 \times D_2 \times \cdots \times D_n$ and $\forall i \in \{1, 2, ..., n\}$, D_i are CGs. Then (Γ, V_ψ) is called a PNHSG of U if and only if $\forall M \in range(\Gamma)$, $\forall (m_1, d), (m_2, d') \in M \times D_i$ and $\forall a_{N_i} : M \times D_i \to [0, 1] \times [0, 1] \times [0, 1]$, with $a_{N_i}(m, d) = \{((m, d), a^T_{N_i}(m, d), a^I_{N_i}(m, d), a^F_{N_i}(m, d)) : (m, d) \in M \times D_i\}$. Then the subsequent conditions are fulfilled:*

(i) $a^T_{N_i}((m_1, d) \cdot (m_2, d')^{-1}) \geq \min\{a^T_{N_i}(m_1, d), a^T_{N_i}(m_2, d')\}$
(ii) $a^I_{N_i}((m_1, d) \cdot (m_2, d')^{-1}) \geq \min\{a^I_{N_i}(m_1, d), a^I_{N_i}(m_2, d')\}$
(iii) $a^F_{N_i}((m_1, d) \cdot (m_2, d')^{-1}) \leq \max\{a^F_{N_i}(m_1, d), a^F_{N_i}(m_2, d')\}$

Proof. This can be proved using Proposition 3.2 and Proposition 3.7. □

Proposition 3.13. *Intersection of two PNHSGs is also a PNHSG.*

Theorem 3.14. *The homomorphic image of a PNHSG is a PNHSG.*

Proof. Let U_1 and U_2 be two CGs and $\forall i, j \in \{1, 2, ..., n\}$, D_i and P_j are attribute value sets consisting of some attribute values and let $g_{ij} : U_1 \times D_i \to U_2 \times P_j$ are homomorphisms. Also, let $(\Gamma_1, V_\psi^1) \in$ PNHSG(U_1), where $V_\psi^1 = D_1 \times D_2 \times \cdots \times D_n$. Again, let $\forall M \in$ range(Γ_1), a_{N_i} : $M \times D_i \to [0,1] \times [0,1] \times [0,1]$ with $a_{N_i}(m,d) = \{((m,d), a^T_{N_i}(m,d), a^I_{N_i}(m,d), a^F_{N_i}(m,d)) : (m,d) \in M \times D_i\}$ are the corresponding NDAFs.
Assuming $(n_1,p_1),(n_2,p_2) \in U_2 \times P_j$, if $g_{ij}^{-1}(n_1,p_1) = \phi$ and $g_{ij}^{-1}(n_2,p_2) = \phi$ then $g_{ij}(\Gamma_1, V_\psi^1) \in$ NHSG(U_2). Lets assume that $\exists (m_1,d_1),(m_2,d_2) \in U_1 \times D_i$ such that $g_{ij}(m_1,d_1) = (n_1,p_1)$ and $g_{ij}(m_2,d_2) = (n_2,p_2)$. Then by Theorem 3.4 and Theorem 3.9, we can prove the followings:

$$g_{ij}(a^T_{N_i})(n_1,p_1) \cdot (n_2,p_2)^{-1} \geq \min\{g_{ij}(a^T_{N_i})(n_1,p_1), g_{ij}(a^T_{N_i})(n_2,p_2)\},$$
$$g_{ij}(a^I_{N_i})(n_1,p_1) \cdot (n_2,p_2)^{-1} \geq \min\{g_{ij}(a^I_{N_i})(n_1,p_1), g_{ij}(a^I_{N_i})(n_2,p_2)\},$$

and

$$g_{ij}(a^F_{N_i})(n_1,p_1) \cdot (n_2,p_2)^{-1} \leq \max\{g_{ij}(a^F_{N_i})(n_1,p_1), g_{ij}(a^F_{N_i})(n_2,p_2)\}.$$

Hence, $g_{ij}(\Gamma_1, V_\psi^1) \in$ PNHSG(U_2). □

Theorem 3.15. *The homomorphic preimage of a PNHSG is a PNHSG.*

Proof. Let U_1 and U_2 be two CGs and $\forall i, j \in \{1, 2, ..., n\} D_i$ and P_j are attribute value sets consisting of some attribute values and let $g_{ij} : U_1 \times D_i \to U_2 \times P_j$ are homomorphisms. Also, let $(\Gamma_2, V_\psi^2) \in$ PNHSG(U_2), where $V_\psi^2 = P_1 \times P_2 \times \cdots \times P_n$. Again, let $\forall N \in$ range(Γ_2),b_{N_j} : $N \times P_j \to [0,1] \times [0,1] \times [0,1]$ with $b_{N_j}(n,p) = \{((n,p), b^T_{N_j}(n,p), b^I_{N_j}(n,p), b^F_{N_j}(n,p)) : (n,p) \in N \times P_j\}$ are the corresponding IFDAFs. Lets assume $(m_1,d_1),(m_2,d_2) \in U_1 \times D_i$. Since g_{ij} is a homomorphism by Theorem 3.5 and Theorem 3.10, the followings can be proved:

$$g_{ij}^{-1}(b^T_{N_j})(m_1,d_1) \cdot (m_2,d_2)^{-1} \geq \min\{g_{ij}^{-1}(b^T_{N_j})(m_1,d_1), g_{ij}^{-1}(b^T_{N_j})(m_2,d_2)\},$$
$$g_{ij}^{-1}(b^I_{N_j})(m_1,d_1) \cdot (m_2,d_2)^{-1} \geq \min\{g_{ij}^{-1}(b^I_{N_j})(m_1,d_1), g_{ij}^{-1}(b^I_{N_j})(m_2,d_2)\},$$

and

$$g_{ij}^{-1}(b^F_{N_j})(m_1,d_1) \cdot (m_2,d_2)^{-1} \leq \max\{g_{ij}^{-1}(b^F_{N_j})(m_1,d_1), g_{ij}^{-1}(b^F_{N_j})(m_2,d_2)\}.$$

Hence, $g_{ij}^{-1}(\Gamma_2, V_\psi^2) \in$ PNHSG(U_1). □

4. Conclusions

Hypersoft set theory is more general than soft set theory and it has a huge area of applications. That is why we have adopted and implemented it in plithogenic environment so that we can introduce various algebraic structures. Because of this, the notions of plithogenic hypersoft subgroups have become general than fuzzy, intuitionistic fuzzy, neutrosophic subgroups, and plithogenic subgroups. Again, we have introduced functions in different plithogenic hypersoft environments. Hence, homomorphism can be introduced and its effects on these newly defined plithogenic hypersoft subgroups can be studied. In the future, to extend this study one may introduce general T-norm and T-conorm and further generalize plithogenic hypersoft subgroups. Also, one may extend these notions by introducing different normal versions of plithogenic hypersoft subgroups and by studying the effects of homomorphism on them.

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Entropy Measures for Plithogenic Sets and Applications in Multi-Attribute Decision Making

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Abstract: Plithogenic set is an extension of the crisp set, fuzzy set, intuitionistic fuzzy set, and neutrosophic sets, whose elements are characterized by one or more attributes, and each attribute can assume many values. Each attribute has a corresponding degree of appurtenance of the element to the set with respect to the given criteria. In order to obtain a better accuracy and for a more exact exclusion (partial order), a contradiction or dissimilarity degree is defined between each attribute value and the dominant attribute value. In this paper, entropy measures for plithogenic sets have been introduced. The requirements for any function to be an entropy measure of plithogenic sets are outlined in the axiomatic definition of the plithogenic entropy using the axiomatic requirements of neutrosophic entropy. Several new formulae for the entropy measure of plithogenic sets are also introduced. The newly introduced entropy measures are then applied to a multi-attribute decision making problem related to the selection of locations.

Keywords: neutrosophic set; plithogenic set; fuzzy set; entropy; similarity measure; information measure

1. Introduction

In recent years, there has been numerous authors who gave characterizations of entropy measures on fuzzy sets and their generalizations. Most notably, the majority of them had worked on developing entropy measures on intuitionistic fuzzy sets (IFS). Alongside with their introduction of new ways of entropy measures on IFS, these authors have also given some straightforward examples to show how their entropy measures can be applied to various applications including multi-attribute decision making (MADM) problems [1,2].

In 2016, Zhu and Li [3] gave a new definition for entropy measures on IFS. The new definition was subsequently compared against many other previous definitions of entropy measures on IFS. Montes et al. [4] proposed another new definition for entropy measures on intuitionistic fuzzy sets based on divergence. Both of these research groups [3,4] subsequently demonstrated the applications of their definition of IFS onto MADM problems, and both of them deployed examples of IFS, whose data values were not derived from real-life datasets but were predetermined by the authors to justify their new concepts. On the other hand, Farnoosh et al. [5] also gave their new definition for entropy measures on IFS, but they focused only on discussing its potential application in fault elimination of digital images rather than MADM. Ansari et al. [6] also gave a new definition of entropy measures on IFS in edge detection of digital images. Both research groups [5,6] did not provide examples on how their new definitions for entropy measures on IFS may be applied on MADM.

Some of the definitions of entropy measures defined for IFS were parametric in nature. Gupta et al. [7] defined an entropy measures on IFS, characterized by a parameter α. Meanwhile, Joshi and Kumar [8] independently (with respect to [7]) defined a new entropy measures on IFS, also characterized by a parameter α. An example on MADM was also discussed by Joshi and Kumar [8], once again involving a small, conceptual IFS like those encountered in the work of Zhu and Li [3] as well as Montes et al. [4]. The works by Joshi and Kumar [8] were subsequently followed by Garg et al. [9] who defined an entropy measure on IFS characterized by two parameters: (α, β). Like the previous authors, Garg et al. [9] discussed the application of their proposed entropy measure on MADM using a similar manner. In particular, they compared the effect of different parameters α, β on the results of such decision-making process. Besides, they had also compared the results yielded by the entropy measure on IFS from some other authors. Joshi and Kumar [10] also defined another entropy measure on IFS, following their own previous work on the classical fuzzy sets in [11] and also the work by Garg et al. in [9].

For various generalizations derived from IFS, such as inter-valued intuitionistic fuzzy sets (IVIFS) or generalized intuitionistic fuzzy soft sets (GIFSS), there were also some studies to establish entropy measures on some generalizations, followed by a demonstration on how such entropy measures can be applied to certain MADM problems. Recently, Garg [12] defined an entropy measure for inter-valued intuitionistic fuzzy sets and discussed the application of such entropy measures on solving MADM problems with unknown attribute weights. In 2018, Rashid et al. [13] defined another distance-based entropy measure on the inter-valued intuitionistic fuzzy sets. Again, following the conventions of the previous authors, they clarified the applications of their work on MADM problem using a simple, conceptual small dataset. Selvachandran et al. [14] defined a distance induced intuitionistic entropy for generalized intuitionistic fuzzy soft sets, for which they also clarified the applications of their work on MADM problems using a dataset of the same kind.

As for the Pythagorean fuzzy set (PFS) and its generalizations, an entropy measure was defined by Yang and Hussein in [15]. Thao and Smarandache [16] proposed a new entropy measure for Pythagorean fuzzy sets in 2019. Such new definitions of entropy in [16] discarded the use of natural logarithm as in [15], which is computationally intensive. Such work was subsequently followed by Athira et.al. [17,18], where an entropy measure was given for Pythagorean fuzzy soft sets—a further generalization of Pythagorean fuzzy sets. As for vague set and its generalizations, Feng and Wang [19] defined an entropy measure considering the hesitancy degree. Later, Selvachandran et al. [20] defined an entropy measure on complex vague soft sets. In the ever-going effort of establishing entropy measures for other generalizations of fuzzy sets, Thao and Smarandache [16] and Selvachandran et al. [20] were among the research groups who justified the applicability of their entropy measures using examples on MADM. Likewise, each of those works involved one or several (if more than one example provided in a work) small and conceptual datasets created by the authors themselves.

Besides IFS, PFS, vague sets and all their derivatives, there were also definitions of entropy established on some other generalizations of fuzzy sets in recent years, some came alongside with examples on MADM involving conceptual datasets as well [21]. Wei [22] defined an asymmetrical cross entropy measure for two fuzzy sets, called the fuzzy cross-entropy. Such cross entropy for interval neutrosophic sets was also studied by Sahin in [23]. Ye and Du [21] gave four different new ways entropy measures on interval-valued neutrosophic sets. Sulaiman et al. [24,25] defined entropy measures for interval-valued fuzzy soft sets and multi-aspect fuzzy soft sets. Hu et al. [26] gave an entropy measure for hesitant fuzzy sets. Al-Qudah and Hassan [27] gave an entropy measure for complex multi-fuzzy soft sets. Barukab et al. [28] gave an entropy measure for spherical fuzzy sets. Piasecki [29] gave some remarks and characterizations of entropy measures among fuzzy sets. In 2019, Dass and Tomar [30] further examined the legitimacy of some exponential entropy measures on IFS, such as those defined by Verna and Sharma in [31], Zhang and Jiang in [32], and Mishra in [33]. On the other hand, Kang and Deng [34] outlined the general patterns for which the formula for entropy measures could be formed, thus also applicable for entropy measures to various generalizations of fuzzy sets. Santos et al. [35] and Cao and Lin [36] derived their entropy formulas for data processing based on those for fuzzy entropy with applications in image thresholding and electroencephalogram.

With many entropy measures being defined for various generalizations of fuzzy sets, it calls upon a need to standardize which kind of functions are eligible to be used as entropy measures and which are not. One of the most notable and recent works in this field is accomplished by Majumdar [1], who established an axiomatic definition on the entropy measure on a single-valued neutrosophic set (SVNS). Such an axiomatic definition of entropy measure, once defined for a particular generalization of fuzzy set, serves as an invaluable tool when choosing a new entropy measure for a particular purpose. Moreover, with the establishment of such an axiomatic definition of entropy measure, it motivates researchers to work on deriving a collection of functions, which all qualify themselves to be used as entropy measures, rather than inventing a single standalone function as an entropy measure for a particular scenario.

In 2017, Smarandache [2] firstly established a concept of plithogenic sets, intended to serve as a profound and conclusive generalization from most (if not all) of the previous generalizations from fuzzy sets. This obviously includes the IFS, where most works had been done to establish its great variety of entropy measures. However, Smarandache [2] did not give any definitions on entropy measures for plithogenic sets.

Our work on this paper shall be presented as follows: Firstly, in Section 2, we mention all the prerequisite definitions needed for the establishment of entropy measures for plithogenic sets. We also derive some generalizations of those previous definitions. Such generalizations are necessary to further widen the scope of our investigation on the set of functions that qualifies as entropy measures for plithogenic sets. Then, in Section 3, we first propose new entropy measures for plithogenic sets in which requirements for any function to be an entropy measure of plithogenic sets are outlined in the axiomatic definition. Later in Section 3, several new formulae for the entropy measure of plithogenic sets are also introduced. In Section 4, we will apply a particular example of our entropy measure onto a MADM problem related to the selection of locations.

Due to the complexity and the novelty of plithogetic sets, as well as the scope constraints of this paper, the plithogenic set involved in the demonstration of MADM will be of a small caliber within 150 data values in total. Those data values contained in the plithogenic set example will be also conceptual in nature (only two to three digits per value). Such presentation, although it may be perceived as simple, is in alliance with the common practice of most renowned works done by the previous authors discussed before, whenever a novel way of entropy measure is invented and first applied on a MADM problem. Hence, such a start-up with a small and conceptual dataset does not hinder the justification on the practicability of the proposed notions. Quite the contrary, it enables even the most unfamiliar readers to focus on the procedure of such novel methods of dealing with MADM problems, rather than being overwhelmed by the immense caliber of computation encountered in dealing with up-to-date real-life datasets.

2. Preliminary

Throughout all the following of this article, let U be the universal set.

Definition 1 [1]. *A single valued neutrosophic sets (SVNS) on U is defined to be the collection*

$$A = \{(x, T_A(x), I_A(x), F_A(x)) : x \in U\}$$

where $T_A, I_A, F_A : U \to [0,1]$ *and* $0 \le T_A(x) + I_A(x) + F_A(x) \le 3$.

We denote $SVNS(U)$ *to be the collection of all SVNS on* U.

Majumdar [1] have established the following axiomatic definition for an entropy measure on SVNS.

Definition 2 [1]. *An entropy measure on SVNS is a function* $E_N : SVNS(U) \to [0,\ 1]$ *that satisfies the following axioms for all* $A \in SVNS(U)$:

I. $E_N(A) = 0$ *if* A *is a crisp set i.e.,* $(T_A(x), I_A(x), F_A(x)) \in \{(1,0,0), (0,0,1)\}$ *for all* $x \in U$.

II. $E_N(A) = 1$ *if* $(T_A(x), I_A(x), F_A(x)) = \left(\frac{1}{2}, \frac{1}{2}, \frac{1}{2}\right)$ *for all* $x \in U$.

III. $E_N(A) \ge E_N(B)$ *if* A *is contained in* B *i.e.,* $T_A(x) \le T_B(x)$, $I_A(x) \ge I_B(x)$, $F_A(x) \ge F_B(x)$ *for all* $x \in U$.

IV. $E_N(A) = E_N(A^c)$ *for all* $A \in SVNS(U)$.

In the study of fuzzy entropy, a fuzzy set with membership degree of 0.5 is a very special fuzzy set as it is the fuzzy set with the highest degree of fuzziness. Similarly, in the study of entropy for SVNSs, a SVNS with all its membership degree of 0.5 for all the three membership components is very special as it is the SVNS with the highest degree of uncertainty. Hence, we denote $A_{[\frac{1}{2}]} \in SVNS(U)$ as the SVNS with $(T_A(x), I_A(x), F_A(x)) = \left(\frac{1}{2}, \frac{1}{2}, \frac{1}{2}\right)$ for all $x \in A$. Such axiomatic descriptions in Definition 2 of this paper, defined by Majumdar [1], serve as the cornerstone for establishing similar axiomatic descriptions for the entropy measures on other generalizations of fuzzy sets, which shall certainly include that for plithogenic sets by Smarandache [2].

We however, disagree with (iii) of Definition 2. As an illustrative example, let A be empty, then A has zero entropy because it is of *absolute certainty* that A "does not contain any element". Whereas a superset of A, say $B \in SVNS(U)$, may have higher entropy because it may not be crisp. Thus, we believe that upon establishing (iii) of Definition 2, the authors in [1] concerned only the case where A and B are very close to the entire U. Thus, on the establishment of entropy measures on plithogenic sets in this article, only axioms (i), (ii) and (iv) of Definition 2 will be considered.

Together with axioms (i), (ii), and (iv) of Definition 2, the following two well-established generalizations of functions serve as **our motives of defining the entropies**, which allows different users to customize to their respective needs.

Definition 3 [1]. *Let* $\mathrm{T} : [0,1]^2 \to [0,1]$, *be a function satisfying the following for all* $p, q, r, s \in [0,\ 1]$.

1. $\mathrm{T}(p,q) = \mathrm{T}(q,p)$ (*commutativity*)
2. $\mathrm{T}(p,q) \le \mathrm{T}(r,s)$, *if* $p \le r$ *and* $q \le s$
3. $\mathrm{T}(p, \mathrm{T}(q,r)) = \mathrm{T}(\mathrm{T}(p,q), r)$ (*associativity*)
4. $\mathrm{T}(p,0) = 0$
5. $\mathrm{T}(p,1) = p$

Then T *is said to be a T-norm function.*

Example 1. *"minimum" is a T-norm function.*

Definition 4 [1]. *Let* $\mathrm{S} : [0,1]^2 \to [0,1]$, *be a function satisfying the following for all* $p, q, r, s \in [0,1]$.

1. $\mathrm{S}(p,q) = \mathrm{S}(q,p)$ (*commutativity*)
2. $\mathrm{S}(p,q) \le \mathrm{S}(r,s)$, *if* $p \le r$ *and* $q \le s$

3. $\mathrm{S}(p, \mathrm{S}(q, r)) = \mathrm{S}(\mathrm{S}(p, q), r)$ (*associativity*)
4. $\mathrm{S}(p, 0) = p$
5. $\mathrm{S}(p, 1) = 1$

Then, S *is said to be an S-norm (or a T-conorm) function.*

Example 2. *"maximum" is an S-norm function.*

In the study of fuzzy logic, we also find ourselves seeking functions that measure the central tendencies, as well as a given position of the dataset, besides maximum and minimum. Such measurement often involves more than two entities and those entities can be ordered of otherwise. This is the reason we introduce the concept of *M-type* function and the *S-type* function, defined on all finite (not ordered) sets with entries in $[0, 1]$. Due to the commutativity of S-norm function, S-type function is thus a further generalization of S-norm function as it allows more than two entities. In all the following of this article, let us denote $\Phi_{[0,1]}$ as the collection of all finite sets with entries in $[0, 1]$. To avoid ending up with too many brackets in an expression, it is convenient to denote the image of $\{a_1, a_2, \cdots, a_n\}$ under f as $f(a_1, a_2, \cdots, a_n)$.

Definition 5 [1]**.** *Let* $f : \Phi_{[0,1]} \to [0, 1]$*, be a function satisfying the following:*

(i) $f(0, 0, \cdots, 0) = 0$
(ii) $f(1, 1, \cdots, 1) = 1$

Then f is said to be an M-type function.

Remark 1. *"maximum", "minimum", "mean", "interpolated inclusive median", "interpolated exclusive median", "inclusive first quartile", "exclusive 55th percentile",* $1 - \prod_{k \in K}(1 - k)$ *and* $1 - \sqrt[|K|]{\prod_{k \in K}(1 - k)}$ *are some particular examples of M-type functions.*

Definition 6 [1]**.** *Let* $f : \Phi_{[0,1]} \to [0, 1]$*, be a function satisfying the following:*

(i) $f(0, 0, \cdots, 0) = 0$.
(ii) *If* $K \in \Phi_{[0,1]}$ *contains at least an element* $k > 0$*, then* $f(K) > 0$.
(iii) *If* $K \in \Phi_{[0,1]}$ *contains at least an element* $k = 1$*, then* $f(K) = 1$.
(iv) *For every two sets from* $\Phi_{[0,1]}$ *with the same cardinality:* $K = \{k_1, k_2, \cdots, k_n\}$ *and* $R = \{r_1, r_2, \cdots, r_n\}$*. If* $k_j \geq r_j$ *for all* j*, then* $f(K) \geq f(R)$.

Then f is said to be an S-type function.

Remark 2. *"maximum",* $1 - \prod_{k \in K}(1 - k)$ *and* $1 - \sqrt[|K|]{\prod_{k \in K}(1 - k)}$ *are some particular examples of S-type functions.*

Lemma 1. *If f is an S-type function, then it is also an M-type function.*

Proof. As $\{1, 1, \cdots, 1\}$ contains one element which equals to 1, $f(1, 1, \cdots, 1) = 1$, thus the lemma follows. □

Remark 3. *The converse of this lemma is not true however, as it is obvious that "mean" is an M-type function but not an S-type function.*

All of these definitions and lemmas suffice for the establishment of our entropy measure for plithogenic sets.

3. Proposed Entropy Measure for Plithogenic Sets

In [2], Smarandache introduced the concept of plithogenic set. Such a concept is as given in the following definition.

Definition 7 [2]. *Let U be a universal set. Let $P \subseteq U$. Let A be a set of attributes. For each attribute $a \in A$: Let S_a be the set of all its corresponding attribute values. Take $V_a \subseteq S_a$. Define a function $d_a : P \times V_a \to [0,1]$, called the attribute value appurtenance degree function. Define a function $c_a : V_a \times V_a \to [0,1]$, called the attribute value contradiction (dissimilarity) degree function, which further satisfies:*

(i) $c_a(v,v) = 0$, *for all* $v \in V_a$.
(ii) $c_a(v_1,v_2) = c_a(v_2,v_1)$, *for all* $v_1, v_2 \in V_a$.

Then:

(a) $\mathbf{R} = \langle P, A, V, d, c\rangle$ *is said to form a plithogenic set on* U.
(b) d_a *is said to be the attribute value appurtenance fuzzy degree function (abbr. AFD-function) for* a *in* $\mathbf{R}$, *and* $d_a(x,v)$ *is called the appurtenance fuzzy degree of* x *in* v.
(c) c_a *is said to be the attribute value contradiction (dissimilarity) fuzzy degree function (abbr. CFD-function) for* a *in* $\mathbf{R}$, *and* $c_a(v_1,v_2)$ *is called the contradiction fuzzy degree between* v_1 *and* v_2.

Remark 4. *If $P = U$, $A = \{a_o\}$, $V_{a_o} = \{v_1,v_2,v_3\}$, $c_{a_o}(v_1,v_2) = c_{a_o}(v_2,v_3) = 0.5$ and $c_{a_o}(v_1,v_3) = 1$, then* $\mathbf{R}$ *is reduced to a single valued neutrosophic set (SVNS) on* U.

Remark 5. *If $P = U$, $V_a = \{v_1,v_2\}$ for all $a \in A$, $d_a : P \times V_a \to [0,1]$ is such that $0 \le d_a(x,v_1) + d_a(x,v_2) \le 1$ for all $x \in P$ and for all $a \in A$, $c_a(v_1,v_2) = c_a(v_2,v_1) = 1$ for all $a \in A$, then* $\mathbf{R}$ *is reduced to a generalized intuitionistic fuzzy soft set (GIFSS) on* U.

Remark 6. *If $P = U$, $A = \{a_o\}$, $V_{a_o} = \{u_1,v_1,u_2,v_2\}$, $d_{a_o} : P \times V_{a_o} \to [0,1]$ is such that $0 \le d_{a_o}(x,v_1) + d_{a_o}(x,v_2) \le 1$, $0 \le d_{a_o}(x,u_1) \le d_{a_o}(x,v_1)$ and $0 \le d_{a_o}(x,u_2) \le d_{a_o}(x,v_2)$ all satisfied for all $x \in P$, and $c_{a_o}(u_1,u_2) = c_{a_o}(v_1,v_2) = 1$, then* $\mathbf{R}$ *is reduced to an inter-valued intuitionistic fuzzy set (IVIFS) on* U.

Remark 7. *If $P = U$, $A = \{a_o\}$, $V_{a_o} = \{v_1,v_2\}$, $d_{a_o} : P \times V_{a_o} \to [0,1]$ is such that $0 \le d_{a_o}(x,v_1) + d_{a_o}(x,v_2) \le 1$ for all $x \in P$, and $c_{a_o}(v_1,v_2) = 1$, then* $\mathbf{R}$ *is reduced to an intuitionistic fuzzy set (IFS) on* U.

Remark 8. *If $P = U$, $A = \{a_o\}$, $V_{a_o} = \{v_1,v_2\}$, $d_{a_o} : P \times V_{a_o} \to [0,1]$ is such that $0 \le d_{a_o}(x,v_1)^2 + d_{a_o}(x,v_2)^2 \le 1$ for all $x \in P$, and $c_{a_o}(v_1,v_2) = 1$, then* $\mathbf{R}$ *is reduced to a Pythagorean fuzzy set (PFS) on* U.

Remark 9. *If $P = U$, $A = \{a_o\}$ and $V_{a_o} = \{v_o\}$, then* $\mathbf{R}$ *is reduced to a fuzzy set on* U.

Remark 10. *If $P = U$, $A = \{a_o\}$, $V_{a_o} = \{v_o\}$, and $d_{a_o} : P \times V_{a_o} \to \{0,1\} \subset_{\neq} [0,1]$, then* $\mathbf{R}$ *is reduced to a classical crisp set on* U.

In all the following, the collection of all the plithogenic sets on U shall be denoted as $\mathrm{PLFT}(U)$.

Definition 8. *Let $= \langle P, A, V, d, c\rangle \in \mathrm{PLFT}(U)$. The compliment for* $\mathbf{R}$, *is defined as*

$$\overline{\mathbf{R}} = \left\langle P, A, V, \overline{d}, c \right\rangle,$$

where $\overline{d}_a = 1 - d_a$ for all $a \in A$.

Remark 11. *This definition of compliment follows from page 42 of* [2].

Remark 12. *It is clear that* $\overline{\mathbf{R}} \in \mathrm{PLFT}(U)$ *as well.*

With the all these definitions established, we now proceed to define a way of measurement of entropy for plithogenic sets. In the establishment of such entropy measures, we must let all the AFD-functions $\{d_a : a \in A\}$ and all the CFD-functions $\{c_a : a \in A\}$ to participate in contributing to the overall entropy measures of $= \langle P, A, V, d, c\rangle \in \mathrm{PLFT}(U)$.

We now discuss some common traits of how each element from $\{d_a : a \in A\}$ and $\{c_a : a \in A\}$ shall contribute to the overall entropy measures of $\mathbf{R}$, all of which are firmly rooted in our conventional understanding of entropy as a quantitative measurement for the amount of disorder.

Firstly, on the elements of $\{d_a : a \in A\}$: In accordance with Definition 7, each $d_a(x, v)$ is the appurtenance fuzzy degree of $x \in P$, over the attribute value $v \in V_a$ (V_a in turn belongs to the attribute $a \in A$). Note that $d_a(x, v) = 1$ indicates *absolute certainty* of membership of x in v; whereas $d_a(x, v) = 0$ indicates *absolute certainty* of non-membership of x in v. Hence, any $d_a(x, v)$ satisfying $d_a(x, v) \in \{0, 1\}$ must be regarded as contributing *zero* magnitude to the overall entropy measure of $\mathbf{R}$, as absolute certainty implies zero amount of disorder. On the other hand, $d_a(x, v) = 0.5$ indicates *total uncertainty* of the membership of x in v, as 0.5 is in the middle of 0 and 1. Hence, any $d_a(x, v)$ satisfying $d_a(x, v) = 0.5$ must be regarded as contributing *the greatest* magnitude to the overall entropy measure of $\mathbf{R}$, as total uncertainty implies the highest possible amount of disorder.

Secondly, on the elements of $\{c_a : a \in A\}$: For each attribute $a \in A$, $c_a(v_1, v_2) = 0$ indicates that the attribute values v_1, v_2 are of identical meaning (synonyms) with each other (e.g., "big" and "large"), whereas $c_a(v_1, v_2) = 1$ indicates that the attribute values v_1, v_2 are of opposite meaning to each other (e.g., "big" and "small"). Therefore, in the case of $c_a(v_1, v_2) = 0$ and $\{d_a(x, v_1), d_a(x, v_2)\} = \{0, 1\}$, it implies that x is absolutely certain to be inside one v_i among $\{v_1, v_2\}$, while outside of the other, even though v_1 and v_2 carry identical meaning to each other. Such collection of $\{c_a(v_1, v_2), d_a(x, v_1), d_a(x, v_2)\}$ is therefore of the highest possible amount of disorder, because their combined meaning implies an analogy to the statement of "x is very large and not big" or "x is not large and very big". As a result, such collection of $\{c_a(v_1, v_2), d_a(x, v_1), d_a(x, v_2)\}$ aforementioned must be regarded as contributing *the greatest* magnitude to the overall entropy measure of $\mathbf{R}$. Furthermore, in the case of $c_a(v_1, v_2) = 1$ and $\{d_a(x, v_1), d_a(x, v_2)\} \subset_{\neq} \{0, 1\}$, it implies that x is absolutely certain to be inside both v_1 and v_2 (or outside both v_1 and v_2), even though v_1 and v_2 carry opposite meaning with each other. Likewise, such collection of $\{c_a(v_1, v_2), d_a(x, v_1), d_a(x, v_2)\}$ is of the highest possible amount of disorder, because their combined meaning implies an analogy to the statement of "x something is very big and very small" or "x something is not big and not small". As a result, such a collection of $\{c_a(v_1, v_2), d_a(x, v_1), d_a(x, v_2)\}$ aforementioned must be regarded as contributing *the greatest* magnitude to the overall entropy measure of $\mathbf{R}$ as well.

We now define the three axioms of entropy on plithogenic sets, analogous to the axioms (i), (ii), and (iv) in Definition 2 respectively.

Definition 9. *An entropy measure on plithogenic sets, is a function* $E : \mathrm{PLFT}(U) \to [0, 1]$ *satisfying the following three axioms*

(i) *(analogy to (i) in Definition 2): Let* $\mathbf{R} = \langle P, A, V, d, c\rangle \in \mathrm{PLFT}(U)$ *satisfying the following conditions for all* $(x, v_1, v_2) \in P \times V_a \times V_a$:

   (a) $d_a : P \times V_a \to \{0, 1\}$ *for all* $a \in A$.

   (b) $\{d_a(x, v_1), d_a(x, v_2)\} = \{0, 1\}$ *whenever* $c_a(v_1, v_2) \geq 0.5$.

   (c) $\{d_a(x, v_1), d_a(x, v_2)\} \subset_{\neq} \{0, 1\}$ *whenever* $c_a(v_1, v_2) < 0.5$.

   Then $E(\mathbf{R}) = 0$.

(ii) *(analogy to (ii) in Definition 2). Let* $\mathbf{R} = \langle P, A, V, d, c\rangle \in \mathrm{PLFT}(U)$ *satisfying* $d_a : P \times V_a \to \{0.5\}$ *for all* $a \in A$. *Then* $E(\mathbf{R}) = 1$.

(iii) *(analogy to (iv) in Definition 2). For all* $\mathbf{R} = \langle P, A, V, d, c\rangle \in \mathrm{PLFT}(U)$, $E(\overline{\mathbf{R}}) = E(\mathbf{R})$ *holds.*

The three axioms in Definition 9 thus serve as general rules for which any functions must fulfill to be used as entropy measures on plithogenic sets. However, the existence of such functions satisfying these three axioms needs to be ascertained. To ensure that we do have an abundance of functions satisfying these axioms, we must therefore propose and give characterization to such functions with explicit examples and go to the extent of proving that each one among our proposed examples satisfy all these axioms. Such a procedure of proving the existence of many different entropy functions is indispensable. This is because in practical use, the choices of an entropy measure will fully depend on the type of scenario examined, as well as the amount of computing power available to perform such computations, without jeopardizing the axioms of entropy measures as mentioned. It is only by doing so that users are guaranteed to have plenty of room to customize an entropy measure of plithogenic sets suited for their particular needs. In light of this motivation, a theorem showing a collection of functions satisfying those axioms is presented in this paper.

Theorem 1. *Let* m_1, m_2, m_3 *be any M-type functions. Let* s_1, s_2 *be any S-type functions. Let* Δ *be any function satisfying the following conditions*:

(i) $\Delta(1-c) = \Delta(c)$, $\Delta(0) = \Delta(1) = 0$, $\Delta(0.5) = 1$.
(ii) $\Delta(c)$ *is increasing within [0, 0.5]. In other words,* $\Delta(c_1) \leq \Delta(c_2)$ *whenever* $0 \leq c_1 < c_2 \leq 0.5$.

Let ω *be any function satisfying the following conditions:*

(i) $\omega(x) = 0$ *for all* $x \in [0, 0.5]$, $\omega(1) = 1$.
(ii) $\omega(c)$ *is increasing within [0.5, 1]. In other words,* $\omega(c_1) \leq \omega(c_2)$ *whenever* $0 \leq c_1 < c_2 \leq 1$.

Define $\varepsilon_{\Delta,a} : P \times V_a \to [0,1]$, *where* $\varepsilon_{\Delta,a}(x,v) = \Delta(d_a(x,v))$ *for all* $(x,v) \in P \times V_a$. *Define* $\varphi_{\omega,a} : P \times V_a \times V_a \to [0,1]$, *where:*

$$\varphi_{\omega,a}(x,v_1,v_2) = \omega(1-c_a(v_1,v_2))\cdot\left|d_a(x,v_1) - d_a(x,v_2)\right| + \omega(c_a(v_1,v_2))\cdot\left|d_a(x,v_1) + d_a(x,v_2) - 1\right|$$

for all $(x,v_1,v_2) \in P \times V_a \times V_a$.

Then, any function $E : \mathrm{PLFT}(U) \to [0,\ 1]$, *in the form of*

$$E(\mathbf{R}) = m_3\Big\{m_2\Big\{\ m_1\Big\{s_2\Big\{\varepsilon_{\Delta,a}(x,v),\ s_1\{\varphi_{\omega,a}(x,v,u) : u \in V_a\}\Big\}\Big\} : v \in V_a\Big\} : a \in A\Big\} : x \in P\Big\}$$

for all $\mathbf{R} = \langle P, A, V, d, c\rangle \in \mathrm{PLFT}(U)$, *are all entropy measures on plithogenic sets.*

Proof. + Axiom (i): Taking any arbitrary $u, v \in V_a$, $a \in A$ and $x \in P$.

a. As $d_a(x,v) \in \{0,1\}$, $\varepsilon_{\Delta,a}(x,v) = \Delta(d_a(x,v)) = 0$.
b. Whenever $c_a(v_1,v_2) \geq 0.5$, it follows that $1 - c_a(v_1,v_2) \leq 0.5$, which implies $\omega(1-c_a(v_1,v_2)) = 0$.

Thus, $\varphi_{\omega,a}(x,v_1,v_2) = \omega(c_a(v_1,v_2))\cdot\left|d_a(x,v_1) + d_a(x,v_2) - 1\right|$.

Since $\{d_a(x,v_1), d_a(x,v_2)\} = \{0,1\}$, $d_a(x,v_1) + d_a(x,v_2) - 1 = 0$ follows, which further implies that $\varphi_{\omega,a}(x,v_1,v_2) = 0$.

c. whenever $c_a(v_1,v_2) < 0.5$, it implies $\omega(c_a(v_1,v_2)) = 0$.

Thus, $\varphi_{\omega,a}(x,v_1,v_2) = \omega(1-c_a(v_1,v_2))\cdot\left|d_a(x,v_1) - d_a(x,v_2)\right|$.

Since $\{d_a(x,v_1), d_a(x,v_2)\} \subset_{\neq} \{0,1\}$, $d_a(x,v_1) - d_a(x,v_2) = 0$ follows, which further implies that $\varphi_{\omega,a}(x,v_1,v_2) = 0$.

Hence, $\varphi_{\omega,a}(x,v,u) = \varepsilon_{\Delta,a}(x,v) = 0$ follows for all u, v, a, x.

As a result,

$$\begin{aligned}E(\mathbf{R}) &= m_3\Big\{m_2\Big\{\, m_1\Big\{s_2\Big\{\varepsilon_{\Delta,a}(x,v),\ s_1\{\varphi_{\omega,a}(x,v,u) : u \in V_a\}\,\Big\} : v \in V_a\Big\} : a \in A\Big\} : x \in P\,\Big\} \\ &= m_3\{m_2\{\, m_1\{s_2\{0,\ s_1\{0 : u \in V_a\}\,\} : v \in V_a\} : a \in A\} : x \in P\,\} \\ &= m_3\{m_2\{\, m_1\{s_2\{0,0\} : v \in V_a\} : a \in A\} : x \in P\,\} \\ &= m_3\{m_2\{\, m_1\{0 : v \in V_a\} : a \in A\} : x \in P\,\} = 0.\end{aligned}$$

+ Axiom (ii): Taking any arbitrary $v \in V_a$, $a \in A$ and $x \in P$.
As $d_a : P \times V_a \to \{0.5\}$ for all $a \in A$, we have
$d_a(x,v) = 0.5$ for all v, a, x. This further implies that
$\varepsilon_{\Delta,a}(x,v) = \Delta(d_a(x,v)) = 1$ for all v, a, x.
As a result,

$$\begin{aligned}E(\mathbf{R}) &= m_3\Big\{m_2\Big\{\, m_1\Big\{s_2\Big\{\varepsilon_{\Delta,a}(x,v),\ s_1\{\varphi_{\omega,a}(x,v,u) : u \in V_a\}\Big\} : v \in V_a\Big\} : a \in A\Big\} : x \in P\Big\} \\ &= m_3\{m_2\{\, m_1\{s_2\{1,\ s_1\{\varphi_{\omega,a}(x,v,u) : u \in V_a\}\} : v \in V_a\} : a \in A\} : x \in P\} \\ &= m_3\{m_2\{\, m_1\{1 : v \in V_a\} : a \in A\} : x \in P\} = 1.\end{aligned}$$

+ Axiom (iii): $\overline{d}_a = 1 - d_a$ follows by Definition 8. This will imply the following

(a) $\Delta\big(\overline{d}_a(x,v)\big) = \Delta(1 - d_a(x,v)) = \Delta(d_a(x,v)) = \varepsilon_{\Delta,a}(x,v)$.

(b) First, we have

$$\begin{aligned}\Big|\overline{d}_a(x,v_1) - \overline{d}_a(x,v_2)\Big| &= \Big|(1 - d_a(x,v_1)) - (1 - d_a(x,v_2))\Big| \\ &= \Big|-d_a(x,v_1) + d_a(x,v_2)\Big| \\ &= \Big|d_a(x,v_1) - d_a(x,v_2)\Big|\end{aligned}$$

and

$$\begin{aligned}\Big|\overline{d}_a(x,v_1) + \overline{d}_a(x,v_2) - 1\Big| &= \Big|(1 - d_a(x,v_1)) + (1 - d_a(x,v_2)) - 1\Big| \\ &= \Big|1 - d_a(x,v_1) + 1 - d_a(x,v_2) - 1\Big| \\ &= \Big|1 - d_a(x,v_1) - d_a(x,v_2)\Big| \\ &= \Big|d_a(x,v_1) + d_a(x,v_2) - 1\Big|.\end{aligned}$$

Therefore, it follows that

$$\begin{aligned}&\omega(1 - c_a(v_1,v_2))\cdot\Big|\overline{d}_a(x,v_1) - \overline{d}_a(x,v_2)\Big| + \omega(c_a(v_1,v_2))\cdot\Big|\overline{d}_a(x,v_1) + \overline{d}_a(x,v_2) - 1\Big| \\ &= \omega(1 - c_a(v_1,v_2))\cdot\Big|d_a(x,v_1) - d_a(x,v_2)\Big| + \omega(c_a(v_1,v_2))\cdot\Big|d_a(x,v_1) + d_a(x,v_2) - 1\Big| \\ &= \varphi_{\omega,a}(x,v_1,v_2)\end{aligned}$$

Since

$$E(\mathbf{R}) = m_3\Big\{m_2\Big\{m_1\Big\{s_2\Big\{\varepsilon_{\Delta,a}(x,v), s_1\{\varphi_{\omega,a}(x,v,u) : u \in V_a\}\Big\} :\ v \in V_a\Big\} : a \in A\Big\} :\ x \in P\Big\}$$

$E(\mathbf{R}) = E(\overline{\mathbf{R}})$ now follows. □

Remark 13. *As* $\varepsilon_{\Delta,a}(x,v) = \Delta(d_a(x,v))$ *and*

$$\varphi_{\omega,a}(x,v,u) = \omega(1 - c_a(v,u))\cdot\Big|d_a(x,v) - d_a(x,u)\Big| + \omega(c_a(v,u))\cdot\Big|d_a(x,v) + d_a(x,u) - 1\Big|$$

It follows that

$$E(\mathbf{R}) = m_3\left\{ m_2\left\{ m_1\left\{ s_2\left\{ \Delta(d_a(x,v)),\ s_1\left\{ \begin{array}{c} \omega(1-c_a(v,u))\cdot \\ \left|d_a(x,v)-d_a(x,u)\right|+ \\ \omega(c_a(v,u))\cdot \\ \left|d_a(x,v)+d_a(x,u)-1\right| \end{array} : u \in V_a \right\} \right\} : v \in V_a \right\} : a \in A \right\} : x \in P \right\}$$

Such a version of the formula serves as an even more explicit representation of $E(\mathbf{R})$.

Remark 14. *For instance, the following is one of the many theoretical ways of choosing* $\{m_1, m_2, m_3, s_1, s_2, \Delta, \omega\}$ *to form a particular entropy measure on plithogenic sets.*

(a) $\omega(c) = \begin{cases} 0,\ 0 \le c < \frac{1}{2} \\ 2\left(c-\frac{1}{2}\right),\ \frac{1}{2} \le c \le 1 \end{cases}$ *, for all* $c \in [0,1]$.

(b) $\Delta(c) = \begin{cases} 2c,\ 0 \le c < \frac{1}{2} \\ 2(1-c),\ \frac{1}{2} \le c \le 1 \end{cases}$ *, for all* $c \in [0,1]$.

(c) $s_1(K) = \text{maximum}(K)$, *for all* $K \in \Phi_{[0,1]}$.

(d) $s_2(K) = 1 - \prod_{k \in K}(1-k)$, *for all* $K \in \Phi_{[0,1]}$.

(e) $m_1(K) = \text{mean}(K)$, *for all* $K \in \Phi_{[0,1]}$.

(f) $m_2(K) = \text{median}(K)$, *for all* $K \in \Phi_{[0,1]}$.

(g) $m_3(K) = \text{mode}(K)$, *for all* $K \in \Phi_{[0,1]}$.

In practical applications, however, the choices of $\{m_1, m_2, m_3, s_1, s_2, \Delta, \omega\}$ will depend on the type of scenario examined, as well as the amount of computing power available to perform such computations. Such abundance of choices is a huge advantage, because it allows each user plenty of room of customization suited for their own needs, without jeopardizing the principles of entropy functions.

4. Numerical Example of Plithogenic Sets

In this section, we demonstrate the utility of the proposed entropy functions for plithogenic sets using an illustrative example of a MADM problem involving a property buyer making a decision whether to live in Town P or Town B.

4.1. Attributes and Attributes Values

Three different addresses within Town P are selected: $P = \{p, q, r\}$. Another four different addresses within Town B are selected as well: $B = \{\alpha, \beta, \gamma, \delta\}$. All the seven addresses are investigated by that person based on 3 attributes as follows:

$$A = \left\{ \begin{array}{c} \text{Services near the address } (j),\ \text{Security near the address } (s), \\ \text{Public transport near the address } (t) \end{array} \right\}$$

For each of the 3 attributes, the following attribute values are considered:

$$V_j = \{\text{School}(u_1),\ \text{Bank}(u_2),\ \text{Factory}(u_3),\ \text{Construction Site}(u_4),\ \text{Clinic}(u_5)\}$$
$$V_s = \{\text{Police on Patrol}(v_1),\ \text{Police Station}(v_2),\ \text{CCTV Coverage}(v_3),\ \text{Premise Guards}(v_4)\}$$
$$V_t = \{\text{Bus}(w_1),\ \text{Train}(w_2),\ \text{Taxi}(w_3),\ \text{Grab services}(w_4)\}$$

4.2. Attribute Value Appurtenance Degree Functions

In light of the limitation of one person doing the investigation, there could possibly be some characteristics of Town P left unknown or unsure of. As a result, our example involved in this paper, though small in caliber, shall provide a realistic illustration of such phenomena.

Thus, in our example: Let the attribute value appurtenance degree functions for Town P be given in Tables 1–3 (as deduced by the property buyer).

Table 1. Attribute value appurtenance fuzzy degree function for $j \in A$ on Town P (d_j).

V_j / **Addresses in Town P**	u_1	u_2	u_3	u_4	u_5
p	1.0	1.0	0.0	0.0	1.0
q	0.0	0.0	1.0	0.8	0.0
r	1.0	0.9	0.0	0.3	1.0

Table 2. Attribute value appurtenance fuzzy degree function for $s \in A$ on Town P (d_s).

V_s / **Addresses in Town P**	v_1	v_2	v_3	v_4
p	0.1	1.0	0.9	0.8
q	0.9	0.0	0.8	0.9
r	0.1	1.0	0.8	0.7

Table 3. Attribute value appurtenance fuzzy degree function for $t \in A$ on Town P (d_t).

V_t / **Addresses in Town P**	w_1	w_2	w_3	w_4
p	0.9	0.9	0.9	0.1
q	0.8	0.8	0.1	0.9
r	0.9	1.0	0.1	0.8

For example:

$d_j(p, u_1) = 1.0$ indicates that schools exist near address p in town P.

$d_t(q, w_4) = 0.9$ indicates that Grab services are very likely to exist near address q in town P.

$d_s(r, v_2) = 1.0$ indicates that police stations exist near address r in town P.

Similarly, let the attribute value appurtenance degree functions for Town B be given in Tables 4–6 (as deduced by the property buyer):

Table 4. Attribute value appurtenance fuzzy degree function for $j \in A$ on Town B (h_j).

V_j / **Addresses in Town B**	u_1	u_2	u_3	u_4	u_5
α	0.0	1.0	1.0	0.0	1.0
β	1.0	0.0	1.0	0.8	0.0
γ	0.4	0.5	0.6	0.4	0.6
δ	0.0	0.1	0.1	0.2	0.9

Table 5. Attribute value appurtenance fuzzy degree function for $s \in A$ on Town B (h_s).

V_s / **Addresses in Town B**	v_1	v_2	v_3	v_4
α	0.9	0.8	0.9	0.8
β	0.2	0.1	0.5	0.4
γ	0.8	0.9	0.8	0.5
δ	0.1	0.2	0.6	0.5

Table 6. Attribute value appurtenance fuzzy degree function for $t \in A$ on Town B (h_t).

V_t / **Addresses in Town B**	w_1	w_2	w_3	w_4
α	0.5	0.5	0.3	0.4
β	0.0	0.9	0.9	0.9
γ	0.8	0.0	0.1	0.1
δ	0.9	0.1	0.8	0.9

4.3. Attribute Value Contradiction Degree Functions

Moreover, each of the attributes of a town may be dependent on one another. For example, in a place where schools are built, clinics should be built near to the schools, whereas factories should be built far from the schools. Moreover, the police force should spread their manpower patrolling across the town away from a police station. As a result, our example involved in this paper, though small in caliber, shall provide a realistic illustration of such phenomena as well.

Thus, as an example, let the attribute value contradiction degree functions for the attributes $j, s, t \in A$ be given in Tables 7–9: (as deduced by the property buyer), to be used for both towns.

Table 7. Attribute value contradiction degree functions for $j \in A$ (c_j).

V_j	u_1	u_2	u_3	u_4	u_5
u_1	0.0	0.2	1.0	0.7	0.0
u_2	0.2	0.0	0.9	0.5	0.1
u_3	1.0	0.9	0.0	0.2	0.9
u_4	0.7	0.5	0.2	0.0	0.5
u_5	0.0	0.1	0.9	0.5	0.0

Table 8. Attribute value contradiction degree functions for $s \in A$ (c_s).

V_s	v_1	v_2	v_3	v_4
v_1	0.0	1.0	0.5	0.5
v_2	1.0	0.0	0.5	0.5
v_3	0.5	0.5	0.0	0.1
v_4	0.5	0.5	0.1	0.0

Table 9. Attribute value contradiction degree functions for $t \in A$ (c_t).

V_t	w_1	w_2	w_3	w_4
w_1	0.0	0.3	0.1	0.1
w_2	0.3	0.0	0.0	0.1
w_3	0.1	0.0	0.0	0.9
w_4	0.1	0.1	0.9	0.0

In particular,

$c_j(u_1, u_3) = 1.0$ indicates that schools and factories should not be in the same place, because it is not healthy to the students.

$c_j(u_1, u_5) = 0.0$ indicates that schools and clinics should be available together, so that any student who falls ill can visit the clinic.

$c_s(v_1, v_2) = 1.0$, because it is very inefficient for police to patrol only nearby a police station itself, instead of places of a significant distance to a police station. This also ensures that police force will be present in all places, as either a station or a patrol unit will be present.

$c_t(w_1, w_2) = 0.0$, because all train stations must have buses going to/from it. On the other hand, one must also be able to reach a train station from riding a bus.

$c_t(w_3, w_4) = 0.9$ due to the conflicting nature of the two businesses.

4.4. Two Plithogenic Sets Representing Two Towns

From all attributes of the two towns given, we thus form two plithogenic sets representing each of them

(a) $\mathbf{R} = \langle P, A, V, d, c \rangle$, which describes Town P
(b) $\mathbf{T} = \langle B, A, V, h, c \rangle$, which describes Town B

Intuitively, it is therefore evident that the property buyer should choose Town P over Town B as his living place. One of the many reasons being, in Town P schools and factories are unlikely to appear near one address within the town, whereas in Town B there exist addresses where schools and factories are both nearby (so both schools and factories are near to each other). Moreover, in Town P, the police force is more efficient as they spread their manpower across the town, rather than merely patrolling near their stations and even leaving some addresses unguarded as in Town B. On top of this, there exists places in town where taxi and grab services are near to each other, which can cause conflict or possibly vandalism to each other's property. Town P is thus deemed less "chaotic", whereas Town B is deemed more "chaotic".

As a result, our entropy measure must be able to give Town P as having lower entropy than Town B, under certain choices of $\{m_1, m_2, m_3, s_1, s_2, \Delta, \omega\}$ which are customized for the particular use of the property buyer.

4.5. An Example of Entropy Measure on Two Towns

Choose the following to form $E : \text{PLFT}(U) \rightarrow [0,1]$ in accordance with Theorem 1:

(a) $\omega(c) = \begin{cases} 0, \; 0 \le c < \frac{1}{2} \\ 2\left(c - \frac{1}{2}\right), \; \frac{1}{2} \le c \le 1 \end{cases}$, for all $c \in [0,1]$.

(b) $\Delta(c) = \begin{cases} 2c, \; 0 \le c < \frac{1}{2} \\ 2(1-c), \; \frac{1}{2} \le c \le 1 \end{cases}$, for all $c \in [0,1]$.

(c) $s_1(K) = s_2(K) = 1 - \sqrt[|K|]{\prod_{k \in K}(1-k)}$, for all $K \in \Phi_{[0,1]}$.

(d) $m_1(K) = m_2(K) = m_3(K) = \text{mean}(K)$, for all $K \in \Phi_{[0,1]}$.

Then, by the calculation in accordance with Theorem 1 which is subsequently highlighted in Figure 1.

We have $E(\mathbf{R}) = \frac{0.05541+0.14126+0.25710}{3} = 0.15126$, and $E(\mathbf{T}) = \frac{0.54868+0.43571+0.39926}{3} = 0.46122$.

Town P is concluded to have lower entropy, and, therefore, is less "chaotic", compared to Town B.

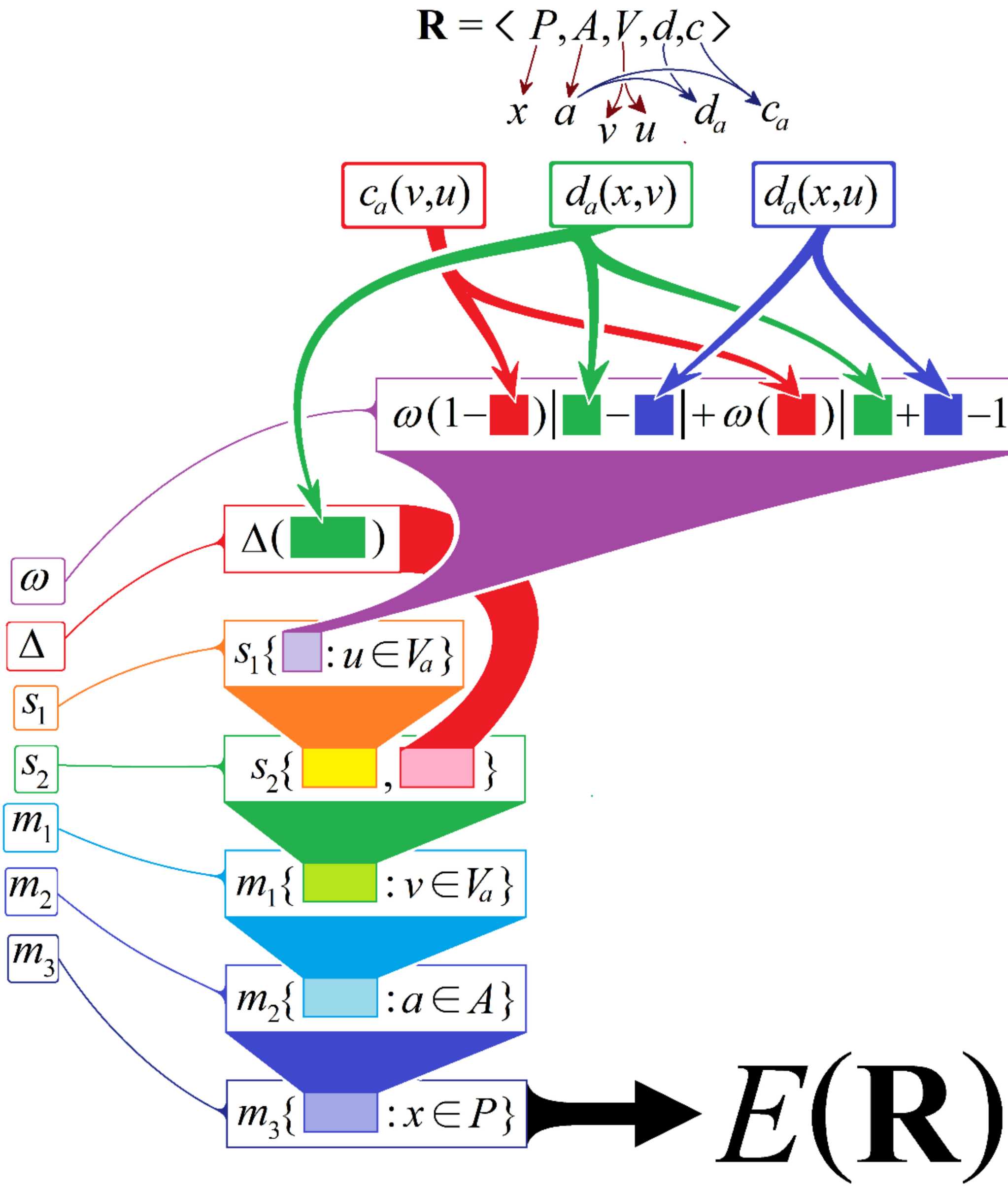

Figure 1. The entire workflow of determining the entropy measure $\mathbf{R}$ of a plithogenic set.

5. Conclusions

The plithogenic set $\mathbf{R} = \langle P, A, V, d, c \rangle$ is an improvement to the neutrosophic model whereby each attribute is characterized by a degree of appurtenance d that describes belongingness to the given criteria, and every pair attribute is characterized by a degree of contradiction c that describes the amount of similarity or opposition between two attributes. In Section 3 of this paper, we have introduced new entropy measures for plithogenic sets $E(\mathbf{R})$. The axiomatic definition of the plithogenic entropy was defined using some of the axiomatic requirements of neutrosophic entropy and some additional conditions. Some formulae for the entropy measure of plithogenic sets have been introduced in Theorem 1 and these formulas have been developed further to satisfy characteristics of plithogenic sets such as satisfying exact exclusion (partial order) and containing a contradiction or dissimilarity degree between each attribute value and the dominant attribute value. The practical application of the proposed plithogenic entropy measures was demonstrated by applying it to a multi-attribute decision making problem related to the selection of locations.

Future works related to the plithogenic entropy include studying more examples of entropy measures for plithogenic sets with structures different from the one mentioned in Theorem 1, and to apply the different types of entropy measure for plithogenic sets onto real life datasets. We are also working on developing entropy measures for other types of plithogenic sets such as plithogenic intuitionistic fuzzy sets and plithogenic neutrosophic sets, and the study of the application of these measures in solving real world problems using real life datasets [36–43].

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Introduction to Plithogenic Logic

Florentin Smarandache

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Abstract: A Plithogenic Logical proposition *P* is a proposition that is characterized by many degrees of truth-values with respect to many corresponding attribute-values (or random variables) that characterize *P*. Each degree of truth-value may be classical, fuzzy, intuitionistic fuzzy, neutrosophic, or other fuzzy extension type logic. At the end, a cumulative truth of *P* is computed.

Keywords: neutrosophic logic, plithogenic logic, plithogenic multi-variate analysis, cumulative truth, plithogenic logic application

1. Introduction

We recall the Plithogenic Logic and explain it in detail by showing a practical application.

A Plithogenic Logical proposition *P* is a proposition that is characterized by many degrees of truth-values with respect to many corresponding attribute-values (or random variables) that characterize *P*.

It is a pluri-logic.

We denote it by $P(V_1, V_2, \ldots, V_n)$, for $n \geq 1$, where $V_1, V_2, \ldots, V_n$ are the random variables that determine, each of them in some degree, the truth-value of *P*.

The variables may be independent one by one, or may have some degree of dependence among some of them. The degrees of independence and dependence of variables determine the plithogenic logic conjunctive operator to be used in the computing of the cumulative truth of *P*.

The random variables may be: classical, fuzzy, intuitionistic fuzzy, indeterminate, neutrosophic, and other types of fuzzy extensions.

$P(V_1) = t_1$ or the truth-value of the proposition *P* with respect to the random variable V_1.

$P(V_2) = t_2$ or the truth-value of the proposition *P* with respect to the random variable V_2.

…

And so on, $P(V_n) = t_n$ or the truth-value of the proposition *P* with respect to the random variable V_n.

The variables $V_1, V_2, \ldots, V_n$ are described by various types of probability distributions, $P(V_1), P(V_2), \ldots, P(V_n)$. The whole proposition *P* is, therefore, characterized by *n* probability distributions, or n sub-truth-values. By combining all of them, we get a *cumulative truth-value* of the logical proposition *P*.

Plithogenic Logic / Set / Probability and Statistics were introduced by Smarandache [1] in 2017 and he further on (2018 -2020) developed them [2-6].

They were applied in many fields by various authors [7-27].

The Plithogenic MultiVariate Analysis used in the Set theory, Probability, and Statistics is now used in Logic, giving birth to the Plithogenic Logic.

Plithogenic MultiVariate Analysis is a generalization of the classical MultiVariate Analysis.

2. Classification of the Plithogenic Logics

Depending on the real-values of $t_1, t_2, \ldots, t_n$, we have:

2.1. Plithogenic Boolean (or Classical) Logic

It occurs when the degrees of truths $t_1, t_2, \ldots, t_n \in \{0,1\}$, where 0 = false, and 1 = true.

2.2. Plithogenic Fuzzy Logic

When the degrees of truths $t_1, t_2, \ldots, t_n$ are included in [0,1], and at least one of them is included in (0, 1), in order to distinguish it from the previous Plithogenic Boolean Logic.
Herein, we have:

2.2.1. Single-Valued Plithogenic Fuzzy Logic, if the degrees of truths $t_1, t_2, \ldots, t_n$ are single (crisp) numbers in [0, 1].

2.2.2. Subset-Valued (such as Interval-Valued, Hesitant-Valued, etc.) Plithogenic Fuzzy Logic, when the degrees of truths $t_1, t_2, \ldots, t_n$ are subsets (intervals, hesitant subsets, etc.) of [0, 1].

2.3. Plithogenic Intuitionistic Fuzzy Logic

When $P(V_j) = (t_j, f_j)$, when t_j, f_j are included in [0,1], $1 \le j \le n$, where t_j is the degree of truth, and f_j is the degree of falsehood of the proposition P, with respect to the variable V_j.
In the same way, we have:

2.3.1. Single-Valued Plithogenic Intuitionistic Fuzzy Logic, when all degrees of truths and falsehoods are single-valued (crisp) numbers in [0, 1].

2.3.2. Subset-Valued Plithogenic Intuitionistic Fuzzy Logic, when all degrees of truths and falsehoods are subset-values included in [0, 1].

2.4. Plithogenic Indeterminate Logic

When the probability distributions of the random variables $V_1, V_2, \ldots, V_n$ are indeterminate (neutrosophic) functions, i.e. functions with vague or unclear arguments and/or values.

2.5. Plithogenic Neutrosophic Logic

When $P(V_j) = (t_j, i_j, f_j)$, with t_j, i_j, f_j included in [0,1], $1 \le j \le n$, where t_j, i_j, f_j are the degrees of truth, indeterminacy, and falsehood respectively of the proposition P with respect to the random variable V_j.
Similarly, we have:

2.5.1. Single-Valued Plithogenic Neutrosophic Logic, when all degrees of truths, indeterminacies, and falsehoods are single-valued (crisp) numbers in [0, 1].

2.5.2. Subset-Valued Plithogenic Neutrosophic Logic, when all degrees of truths, indeterminacies, and falsehoods are subset-values included in [0, 1].

2.6. Plithogenic (other fuzzy extensions) Logic

Where other fuzzy extensions are, as of today: Pythagorean Fuzzy, Picture Fuzzy, Fermatean Fuzzy, Spherical Fuzzy, q-Rung Orthopair Fuzzy, Refined Neutrosophic Logic, and refined any other fuzzy-extension logic, etc.

2.7. Plithogenic Hybrid Logic

When $P(V_1), P(V_2), \ldots, P(V_n)$ are mixed types of the above probability distributions.

3. Applications

3.1. Pluri-Truth Variables

In our everyday life, we rarely have a "simple truth", we mostly deal with "complex truths".

For example:

- You like somebody for something, but dislike him for another thing;
- Or, you like somebody for something in a degree, and for another thing in a different degree;
- Similarly, for hating somebody for something in a degree, and for another thing in a different degree.

An egalitarian society (or system) does not exist in fact in our real world. It is too rigid. The individuals are different, and act differently.

Therefore, in our world, we deal with a "*plitho-logic*" (*plitho* means, in Greek, *many, pluri-*) or "complex logic". And this is best characterized by the *Plithogenic Logic*.

3.2. Types of Random Truth-Variables

The truth depends on many parameters (random variables), not only on a single one, and at the end we need to compute the *cumulative truth* (truth of all truths).

The random variables may be classical (with crisp/exact values), but often in our world they are vague, unclear, only partially known, with indeterminate data.

3.3. Weights of the Truth-Variables

Some truth may weight more than another truth.

For example, you may like somebody for something more than you dislike him/her for another thing.

Or the opposite, you may dislike somebody for something more than you like him/her for another thing.

3.4. Degrees of Subjectivity of the Truth Variables

In the soft sciences, such as: sociology, political science, psychology, linguistics, etc., or in the culture, literature, art, theatre, dance, there exists a significant *degree of subjectivity* in measuring the truth. *It is not beautiful what is beautiful, but it is beautiful what I like myself*, says a Romanian proverb.

4. Generalizations

Plithogenic Logic is a generalization of all previous logics: Boolean, Fuzzy, Intuitionistic Fuzzy, Neutrosophic Logic, and all other fuzzy-extension logics.

It is a MultiVariate Logic, whose truth variables may be in any type of the above logics.

5. Example

Let us consider an ordinary proposition P, defined as below:

$$P = \textit{John loves his city}$$

and let's calculate its complex truth-value.

Of course, lots of attributes (truth-variables) may characterize a city (some of them unknown, other partially known, or approximately known). A complete spectrum of attributes to study is unreachable.

For the sake of simplicity, we consider the below five propositions as 100% independent two by two.

In this example we only chose a few variables V_j, for $1 \le j \le 5$:

V_1 : low / high percentage of COVID-19 virus infected inhabitants;

V_2 : nonviolent / violent;

V_3 : crowded / uncrowded;

V_4 : clean / dirty;

V_5 : quiet / noisy,

A more accurate representation of the proposition P is $P(V_1, V_2, V_3, V_4, V_5)$.

With respect to each variable V_j, the $P(V_j)$ included in [0, 1] has, in general, different truth-values, for $1 \leq j \leq 5$.

Suppose John prefer his city to have (or to be): low percentage of COVID-19 infected inhabitants, non-violent, uncrowded, clean, and quiet.

$P(V_j)$ is the degree in which John loves the city with respect to the way the variable V_j characterizes it.

5.1. Plithogenic Boolean (Classical) Logic

$P(V_1) = 1$

$P(V_2) = 0$

$P(V_3) = 1$

$P(V_4) = 0$

$P(V_5) = 1$

Therefore $P(V_1, V_2, V_3, V_4, V_5) = (1, 0, 1, 0, 1)$, or John loves his city in the following ways:

- in a degree of 100% with respect to variable V_1;
- in a degree of 0% with respect to variable V_2;
- in a degree of 100% with respect to variable V_3;
- in a degree of 0% with respect to variable V_4;
- in a degree of 100% with respect to variable V_5.

The cumulative truth-value will be, in the classical way, the classical conjunction ($\wedge_c$), where c stands for classical:

$$1 \wedge_c 0 \wedge_c 1 \wedge_c 0 \wedge_c 1 = 0,$$

or John likes his city in a cumulative classical degree of 0%!

The classical logic is rough, therefore more refined logics give a better accuracy, as follows.

5.2. Plithogenic Fuzzy Logic

The 100% or 0% truth-variables may not exactly fit John's preferences, but they may be close. For example:

$$P(V_1, V_2, V_3, V_4, V_5) = (0.95, 0.15, 0.80, 0.25, 0.85),$$

which means that John loves his city:

- in a degree of 95% with respect to variable V_1;
- in a degree of 15% with respect to variable V_2;
- in a degree of 80% with respect to variable V_3;
- in a degree of 25% with respect to variable V_4;
- in a degree of 85% with respect to variable V_5.

Using the fuzzy conjunction ($\wedge_F$) min operator, we get:

$$0.95 \wedge_F 0.15 \wedge_F 0.80 \wedge_F 0.25 \wedge_F 0.85 = \min\{0.95, 0.15, 0.80, 0.25, 0.85\} = 0.15$$

or John likes his city in a cumulative fuzzy degree of 15%.

5.3. Plithogenic Intuitionistic Fuzzy Logic

$$P(V_1, V_2, V_3, V_4, V_5) = ((0.80, 0.20), (0.15, 0.70), (0.92, 0.05), (0.10, 0.75), (0.83, 0.07)),$$

which means that John loves his city in a degree of 80%, and dislikes it a degree of 20%, and so on with respect to the other variables.

Using the intuitionistic fuzzy conjunction ($\wedge_{IF}$) min/max operator in order to get the cummulative truth-value, one has:

$$(0.80, 0.20) \wedge_{IF} (0.15, 0.70) \wedge_{IF} (0.92, 0.05) \wedge_{IF} (0.10, 0.75) \wedge_{IF} (0.83, 0.07) =$$
$$= (\min\{0.80, 0.15, 0.92, 0.10, 0.83\}, \max\{0.20, 0.70, 0.05, 0.75, 0.07\}) = (0.10, 0.75),$$

or John likes and dislikes his city in a cumulative intuitionistic fuzzy degree of 10%, and respectively 75%.

5.4. Plithogenic Indeterminate Logic

$$P(V_1, V_2, V_3, V_4, V_5) = (0.80 \text{ or } 0.90, [0.10, 0.15], [0.60, \text{unknown}], > 0.13, 0.79),$$

which means that John loves his city:

- in a degree of 80% or 90% (he is not sure about) with respect to variable V_1;
- in a degree between 10% or 15% with respect to variable V_2;
- in a degree of 60% or greater with respect to variable V_3;
- in a degree greater than 13% (i.e. in the interval (0.13, 1]) with respect to variable V_4;
- in a degree of 79% with respect to variable V_5.

Therefore, the variables provide indeterminate (unclear, vague) values.

Applying the indeterminate conjunction ($\wedge_I$) min operator, we get:

$$\min\{(0.80 \text{ or } 0.90), [0.10, 0.15], [0.60, \text{unknown}), (0.13, 1], 0.79\}= 0.10.$$

5.5. Plithogenic Neutrosophic Logic

$$P(V_1, V_2, V_3, V_4, V_5) = ((0.86, 0.12, 0.54), (0.18, 0.44, 0.72), (0.90, 0.05, 0.05), (0.09, 0.14, 0.82), (0.82, 0.09, 0.14)),$$

which means that John loves the city in a degree of 86%, the degree of indeterminate love is 12%, and the degree of dislike is 54% with respect to the variable V_1, and similarly with respect to the other variables.

Again, using the neutrosophic conjunction ($\wedge_N$) min/max/max operator in order to obtain the cumulative truth-value, one gets:

$$(0.86, 0.12, 0.54) \wedge_N (0.18, 0.44, 0.72) \wedge_N (0.90, 0.05, 0.05) \wedge_N (0.09, 0.14, 0.82) \wedge_N (0.82, 0.09, 0.14) = (\min\{0.86, 0.18, 0.90, 0.09, 0.82\}, \max\{0.12, 0.44, 0.05, 0.14, 0.09\}, \max\{0.54, 0.72, 0.05, 0.82, 0.14\}) = (0.09, 0.44, 0.82),$$

or John loves, is not sure (indeterminate), and dislikes his city with a cumulative neutrosophic degree of 9%, 44%, and 82% respectively.

5. Future Research

To construct the *plithogenic aggregation operators* (such as: intersection, union, negation, implication, etc.) of the variables $V_1, V_2, \ldots, V_n$ all together (cumulative aggregation), in the cases when variables V_i and V_j have some degree of dependence d_{ij} and degree of independence $1- d_{ij}$, with $d_{ij} \in [0, 1]$, for all $i, j \in \{1, 2, \ldots, n\}$, and $n \geq 2$.

6. Conclusions

We showed in this paper that the Plithogenic Logic is the largest possible logic of today. Since we live in a world full of indeterminacy and conflicting data, we have to deal, instead of a *simple truth* with a *complex truth*, where the last one is a cumulative truth resulted from the plithogenic aggregation of many truth-value random variables that characterize an item (or event).

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Introducción a la Lógica Plitogénica
Introduction to Plitogenic Logic

Florentin Smarandache

Florentin Smarandache (2021). Introducción a la Lógica Plitogénica (Introduction to Plitogenic Logic). *Neutrosophic Computing and Machine Learning* 18, 1-6

Resumen: Una proposición lógica plitogénica P es una proposición que se caracteriza por muchos grados de valores de verdad con respecto a muchos valores de atributos correspondientes (o variables aleatorias) que caracterizan a P. Cada grado de valor de verdad puede ser clásico, difuso, intuicionista difuso, neutrosófico u otra lógica de extensión difusa. Al final, se calcula una verdad acumulativa de P.

Palabras claves: lógica neutrosófica, lógica plitogénica, análisis multivariado plitogénico, verdad acumulativa, aplicación lógica plitogénica.

Abstract: A plitogenic logical proposition P is a proposition that is characterized by many degrees of truth values with respect to many corresponding attribute values (or random variables) that characterize P. Each degree of truth value may be classical, fuzzy, intuitionistic fuzzy, neutrosophic, or other fuzzy extension logic. In the end, a cumulative truth of P is computed.

Keywords: neutrosophic logic, plitogenic logic, plitogenic multivariate analysis, cumulative truth, plitogenic logic application.

1 INTRODUCCIÓN

Expliquemos la Lógica Plitogénica en detalle mostrando una aplicación práctica.

Una proposición lógica plitogénica P es una proposición que se caracteriza por muchos grados de valores de verdad con respecto a muchos valores de atributos correspondientes (o variables aleatorias) que caracterizan a P.

Es una pluri-lógica.

Lo denotamos por P (V_1, V_2,…, V_n), para $n \geq 1$, donde V_1, V_2,…, V_n son las variables aleatorias que determinan, cada una de ellas en cierto grado, el valor de verdad de P.

Las variables pueden ser independientes una a una, o pueden tener algún grado de dependencia entre algunas de ellas. Los grados de independencia y dependencia de las variables determinan el operador conjuntivo lógico plitogénico que se utilizará en el cálculo de la verdad acumulada de P.

Las variables aleatorias pueden ser: clásicas, difusas, intuicionistas difusas, indeterminadas, neutrosóficas y otros tipos de extensiones difusas.

$P(V_1) = t_1$ o el valor de verdad de la proposición P con respecto a la variable aleatoria V_1.

$P(V_2) = t_2$ o el valor de verdad de la proposición P con respecto a la variable aleatoria V_2.

...

Y así sucesivamente, $P(V_n) = t_n$ o el valor de verdad de la proposición P con respecto a la variable aleatoria V_n.

Las variables V_1, V_2,…, V_n se describen mediante varios tipos de distribuciones de probabilidad, P(V_1), P(V_2),…, P(V_n). Por lo tanto, toda la proposición P se caracteriza por *n* distribuciones de probabilidad o *n* subvalores de verdad. Al combinarlos todos, obtenemos un *valor de verdad acumulativo* de la proposición lógica P.

Smarandache [1] introdujo la lógica/conjunto/probabilidad y estadística plitogénica en 2017 y en años posteriores (2018-2020) las desarrolló [2-6]. Las cuales fueron aplicadas en numerosos campos y por varios autores [7-27].

El análisis multivariante plitogénico utilizado en la teoría de conjuntos, probabilidades y estadística se utiliza ahora en lógica, dando origen a la lógica plitogénica.

El análisis multivariante plitogénico es una generalización del análisis multivariado clásico.

2 CLASIFICACIÓN DE LAS LÓGICAS PLITOGÉNICAS

Dependiendo de los valores reales de $t_1, t_2, \ldots, t_n$, tenemos:

2.1 Lógica booleana (o clásica) plitogénica

Ocurre cuando los grados de verdades $t_1, t_2, \ldots, t_n \in \{0,1\}$, donde 0 = falso y 1 = verdadero.

2.2 Lógica difusa plitogénica

Cuando los grados de verdades $t_1, t_2, \ldots, t_n$ están incluidos en [0,1], y al menos uno de ellos está incluido en (0, 1), para distinguirlo de la Lógica Booleana Plitogénica anterior.

Aquí tenemos:

2.2.1. Lógica difusa plitogénica de valor único, si los grados de verdades t1, t2,…, tn son números de valor único (nítidos) en [0, 1].

2.2.2. Lógica difusa plitogénica con valores de subconjunto (como valores de intervalo, valores vacilantes, etc.), cuando los grados de verdades t1, t2,…, tn son subconjuntos (intervalos, subconjuntos vacilantes, etc.) de [0, 1].

2.3 Lógica difusa intuicionista plitogénica

Cuando P $(V_j) = (t_j, f_j)$, cuando t_j, f_j se incluyen en [0,1], $1 \leq j \leq n$, donde tj es el grado de verdad y f_j es el grado de falsedad de la proposición P, con respecto a la variable V_j.

De la misma forma, tenemos:

2.3.1. Lógica difusa intuicionista plitogénica de valor único, cuando todos los grados de verdades y falsedades son números de valor único (nítidos) en [0, 1].

2.3.2. Lógica difusa intuicionista plitogénica de valores de subconjuntos, cuando todos los grados de verdades y falsedades son subconjuntos de valores incluidos en [0, 1].

2.4 Lógica indeterminada plitogénica

Cuando las distribuciones de probabilidad de las variables aleatorias $V_1, V_2, \ldots, V_n$ son funciones indeterminadas (neutrosóficas), es decir, funciones con argumentos y/o valores vagos o poco claros.

2.5 Lógica neutrosófica plitogénica

Cuando P (Vj) = (tj, ij, fj), con tj, ij, fj incluidos en [0,1], $1 \leq j \leq n$, donde tj, ij, fj son los grados de verdad, indeterminación y falsedad respectivamente de la proposición P con respecto a la variable aleatoria *Vj*.

Del mismo modo, tenemos:

2.5.1. Lógica neutrosófica plitogénica de valor único, cuando todos los grados de verdades, indeterminaciones y falsedades son números de valor único (nítidos) en [0, 1].

2.5.2. Lógica neutrosófica plitogénica de valores de subconjuntos, cuando todos los grados de verdades, indeterminaciones y falsedades son subconjuntos de valores incluidos en [0, 1].

2.6 Lógica plitogénica (otras extensiones difusas)

Como otras extensiones difusas, podemos mencionar hasta la fecha: difusa Pitagórica, Imagen difusa, Fermateana difusa, Esférica difusa, Ortopar de q-Rung difuso, Lógica neutrosófica difusa refinada y cualquier otra lógica de extensión difusa, etc.

2.7 Lógica híbrida plitogénica

Cuando P (V_1), P (V_2),…, P (V_n) son tipos mixtos de las distribuciones de probabilidad anteriores.

3 APLICACIONES

3.1 Variables de la Pluri-Verdad

En nuestra vida diaria, rara vez tenemos una "verdad simple", en su mayoría tratamos con "verdades complejas".

Por ejemplo:

- Te agrada alguien por algo, pero no te agrada por otra cosa;
- te gusta alguien por algo en un cierto grado y por otra cosa en un grado diferente;
- Del mismo modo, puedes odiar a alguien por algo en un grado y por otra cosa en un grado diferente.

Una sociedad (o sistema) igualitario no existe de hecho en nuestro mundo real. Es un marco demasiado rígido. Todos los individuos son diferentes y por consiguiente actúan de manera diferente.

Por tanto, en nuestro mundo, tratamos con una "plito-lógica" (plitho significa, en griego, pluri o muchos) o "lógica compleja". Y esto se caracteriza mejor por la lógica plitogénica.

3.2 Tipos de variables de verdad aleatorias

La verdad depende de muchos parámetros (variables aleatorias), no de un sólo valor, y al final necesitamos calcular la verdad acumulada (la verdad de todas las verdades).

Las variables aleatorias pueden ser clásicas (con valores nítidos/exactos), pero a menudo en nuestro mundo son vagas, poco claras, solo parcialmente conocidas, con datos indeterminados.

3.3 Pesos de las variables de verdad

Alguna verdad puede pesar más que otra verdad.

Por ejemplo, alguien puede agradarle más por algo de lo que no le agrada por otra cosa.

O al contrario, es posible que alguien no te guste por algo más de lo que te guste por otra cosa.

3.4 Grados de subjetividad de las variables de verdad

En las ciencias blandas, tales como: sociología, ciencias políticas, psicología, lingüística, etc., o en la cultura, literatura, arte, teatro, danza, existe un grado significativo de subjetividad en la medición de la verdad. *No es bello lo que es bello, sino que es bello lo que a mí me gusta*, dice un proverbio rumano.

4 GENERALIZACIONES

La lógica plitogénica es una generalización de todas las lógicas anteriores: booleana, difusa, intuicionista difusa, lógica neutrosófica y todas las demás lógicas de extensión difusa.

Es una Lógica Multivariante, cuyas variables de verdad pueden estar en cualquier tipo de las lógicas anteriores.

5 EJEMPLO

Consideremos una proposición ordinaria P, definida a continuación:

$$P = \text{Juan ama su ciudad}$$

y calculemos su complejo valor de verdad.

Por supuesto, muchos atributos (variables de verdad) pueden caracterizar una ciudad (algunos de ellos desconocidos, otros parcialmente conocidos o aproximadamente conocidos). Un espectro completo de atributos para estudiar es inalcanzable.

En aras de obtener mayor simplicidad, consideramos las cinco proposiciones siguientes como 100% independientes de dos en dos.

En este ejemplo solo elegimos unas pocas variables V_j, para $1 \leq j \leq 5$:

V_1 : porcentaje bajo/alto de habitantes infectados por el virus COVID-19;

V_2 : no violenta/violenta;

V_3 : concurrida/con poca gente;

V_4 : limpia/sucia;

V_5 : tranquila/ruidosa,

Una representación más precisa de la proposición P es P (V_1, V_2, V_3, V_4, V_5).

Con respecto a cada variable V_j, la P (V_j) incluida en [0, 1] tiene, en general, diferentes valores de verdad, para $1 \leq j \leq 5$.

Supongamos que Juan prefiere que su ciudad tenga (o sea): bajo porcentaje de habitantes infectados por COVID-19, no violenta, con poca gente, limpia y tranquila.

P (Vj) es el grado en que Juan ama la ciudad con respecto a la forma en que la variable *Vj* la caracteriza.

5.1. Lógica booleana (clásica) plitogénica

$P(V_1) = 1$

$P(V_2) = 0$

P (V_3) = 1
P (V_4) = 0
P (V_5) = 1

Por lo tanto, P $(V_1, V_2, V_3, V_4, V_5)$ = (1, 0, 1, 0, 1), o Juan ama su ciudad de las siguientes maneras:

- en un grado del 100% con respecto a la variable V_1;
- en un grado del 0% con respecto a la variable V_2;
- en un grado del 100% con respecto a la variable V_3;
- en un grado del 0% con respecto a la variable V_4;
- en un grado del 100% con respecto a la variable V_5.

El valor de verdad acumulativo será, de la manera clásica, la conjunción clásica ($\wedge_c$), donde *c* significa clásico:

$$1,\wedge_c 0 \wedge_c 1 \wedge_c 0 \wedge_c 1 = 0$$

¡o a Juan le gusta su ciudad en un grado clásico acumulativo del 0%!
La lógica clásica es tosca, por lo tanto, las lógicas más refinadas dan una mayor precisión, como sigue.

5.2 Lógica difusa plitogénica

Es posible que las variables de verdad del 100% o del 0% no se ajusten exactamente a las preferencias de Juan, pero pueden estar cerca. Por ejemplo:

P $(V_1, V_2, V_3, V_4, V_5)$ = (0.95, 0.15, 0.80, 0.25, 0.85),

lo que significa que Juan ama su ciudad:

- en un grado del 95% con respecto a la variable V_1;
- en un grado del 15% con respecto a la variable V_2;
- en un grado del 80% con respecto a la variable V_3;
- en un grado del 25% con respecto a la variable V_4;
- en un grado del 85% con respecto a la variable V_5.

Usando el operador min de conjunción difusa ($\wedge_F$), obtenemos:

$0.95 \wedge_F 0.15 \wedge_F 0.80 \wedge_F 0.25 \wedge_F 0.85 = \min\{0.95, 0.15, 0.80, 0.25, 0.85\} = 0.15$

¡o a Juan le gusta su ciudad en un grado difuso acumulativo del 15%!

5.3 Lógica difusa intuicionista plitogénica

P $(V_1, V_2, V_3, V_4, V_5)$ = ((0.80, 0.20), (0.15, 0.70), (0.92, 0.05), (0.10, 0.75), (0.83, 0.07)),

lo que significa que Juan ama su ciudad en un grado del 80%, y no le gusta en un grado del 20%, y así sucesivamente con respecto a las otras variables.

Usando el operador intuicionista de conjunción difusa ($\wedge_{IF}$) min/max para obtener el valor de verdad acumulativo, uno tiene:

$(0.80; 0.20) \wedge_{IF} (0.15; 0.70) \wedge_{IF} (0.92; 0.05) \wedge_{IF} (0.10; 0.75) \wedge_{IF} (0.83; 0.07) =$
$= (\min\{0.80, 0.15, 0.92, 0.10, 0.83\}, \max\{0.20, 0.70, 0.05, 0.75, 0.07\}) = (0.10, 0.75)$,

¡o a Juan le gusta y no le gusta su ciudad en un grado difuso intuicionista acumulativo del 10% y del 75%, respectivamente!

5.4 Lógica indeterminada plitogénica

P $(V_1, V_2, V_3, V_4, V_5)$ = (0,80 o 0,90, [0.10, 0.15], [0.60, desconocido], > 0.13, 0.79),

lo que significa que Juan ama su ciudad:

- en un grado de 80% o 90% (no está seguro) con respecto a la variable V_1;
- en un grado entre el 10% o el 15% con respecto a la variable V_2;
- en un grado del 60% o más con respecto a la variable V_3;
- en un grado superior al 13% (es decir, en el intervalo (0,13, 1]) con respecto a la variable V_4;
- en un grado del 79% con respecto a la variable V_5.

Por lo tanto, las variables proporcionan valores indeterminados (poco claros, vagos).

Aplicando el operador min de conjunción indeterminada ($\wedge_I$), obtenemos:

min {(0.80 o 0.90), [0.10, 0.15], [0.60, desconocido), (0.13, 1], 0.79} = 0,10.

5.5 Lógica neutrosófica plitogénica

P $(V_1, V_2, V_3, V_4, V_5)$ = ((0.86, 0.12, 0.54), (0.18, 0.44, 0.72), (0.90, 0.05, 0.05), (0.09, 0.14, 0.82), (0.82, 0.09, 0.14)),

lo que significa que Juan ama la ciudad en un 86%, el grado de amor indeterminado es del 12% y el grado de desagrado es del 54% con respecto a la variable V_1, y lo mismo con respecto a las demás variables.

Nuevamente, usando el operador de conjunción neutrosófica ($\wedge_N$) min/max/max para calcular el valor de verdad acumulativo, se obtiene:

(0.86; 0.12; 0.54) $\wedge_N$(0.18; 0.44; 0.72)$\wedge_N$ (0.90; 0.05; 0.05)$\wedge_N$ (0.09; 0.14; 0.82)
$\wedge_N$ (0.82, 0.09, 0.14) = (min{0.86, 0.18, 0.90, 0.09, 0.82}, max {0.12, 0.44, 0,05, 0.14, 0.09}, max {0.54, 0.72, 0.05, 0.82, 0.14}) = (0.09, 0.44, 0.82),

¡o Juan ama, no está seguro (indeterminado) y no le gusta su ciudad con un grado neutrosófico acumulativo del 9%, 44% y 82% respectivamente!

6 INVESTIGACIONES FUTURAS

Construir los *operadores de agregación plitogénica* (tales como: intersección, unión, negación, implicación, etc.) de las variables V_1, V_2,…, V_n en conjunto (agregación acumulada), en los casos en que las variables V_i y V_j tengan algún grado de dependencia d_{ij} y grado de independencia 1- d_{ij}, con $d_{ij} \in [0, 1]$, para todo i, j ∈ {1, 2,…, n} y n ≥2.

CONCLUSIONES

Demostramos en este artículo que la lógica plitogénica es la lógica más grande posible de la actualidad. Dado que vivimos en un mundo lleno de indeterminación y datos contradictorios, tenemos que lidiar, en lugar de una verdad simple con una verdad compleja, donde esta última es una verdad acumulativa resultante de la agregación plitogénica, de muchas variables aleatorias de valor de verdad que caracterizan un artículo (o evento).

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A New Decision-Making Model Based on Plithogenic Set for Supplier Selection

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Abstract: Supplier selection is a common and relevant phase to initialize the supply chain processes and ensure its sustainability. The choice of supplier is a multi-criteria decision making (MCDM) to obtain the optimal decision based on a group of criteria. The health care sector faces several types of problems, and one of the most important is selecting an appropriate supplier that fits the desired performance level. The development of service/product quality in health care facilities in a country will improve the quality of the life of its population. This paper proposes an integrated multi-attribute border approximation area comparison (MABAC) based on the best-worst method (BWM), plithogenic set, and rough numbers. BWM is applied to regulate the weight vector of the measures in group decision-making problems with a high level of consistency. For the treatment of uncertainty, a plithogenic set and rough number (RN) are used to improve the accuracy of results. Plithogenic set operations are used to deal with information in the desired manner that handles uncertainty and vagueness. Then, based on the plithogenic aggregation and the results of BWM evaluation, we use MABAC to find the optimal alternative according to defined criteria. To examine the proposed integrated algorithm, an empirical example is produced to select an optimal supplier within five options in the healthcare industry.

Keywords: Supplier selection; rough set theory; MABAC; MCDM; BWM; plithogenic set

1 Introduction

The process of evaluating a set of criteria under a series of constraints to obtain the optimal alternative became popular and significant in many decision-making issues. In MCDM problems, the decision-maker tries to decide the optimal alternative that fulfills most of the criteria considering conditions and constraints. In most current practices, supply chain (SC) managers focus on selecting the proper supplier to improve performance in the supply phase, such as product quality, delivery consistency, and prices. Appropriate supplier selection can significantly increase productivity, meet customer expectations, increase profitability, and reduce the supply costs. But, due to the growth in the number of suppliers, the full range of products available, and the increase of customer expectations, supplier selection became complex, and a real challenge for the supply chain managers, thus it needs to be studied under uncertain environment.

Researchers used different MCDM techniques for selecting the supplier, where the problem of supplier selection among alternatives is presented based on a set of criteria consistent with the nature of the field. It is worth presenting the problem of selecting suppliers in the form of a multi-criteria decision-making problem as the selection criteria differ from one domain to another. Several MCDM methods were used in making proposed models to solve the problem of supplier selection in several areas; for example, a combined model of BWM and fuzzy grey cognitive maps to evaluate green suppliers [1]. In the field of supplier selection to supply chain sustainability, an extension of the Data Envelopment Analysis (DEA) model was suggested [2]. An analytical hierarchy process (AHP), additive ratio assessment (ARAS), and multi-choice goal programming (MCGP) to select a catering supplier is proposed in [3]. A sustainable supplier selection is evaluated with the intuitionistic fuzzy Techniques for Order Preferences by Similarity to Ideal Solution (TOPSIS) method [4].

The supplier selection problem is based on a set of criteria that are defined according to business nature. There are many MCDM methods, such as BWM, that can be used to evaluate these criteria and obtain the optimal alternative. The Best-Worst is a simple pairwise comparison method that shows reliable results in many topics. Gupta (2018) combined BWM with Fuzzy TOPSIS to evaluate green human resource management (GHRM) [5]. The selection of the conceptual design of the products was evaluated using BWM under fuzzy environment [6]. Supply chain sustainability was measured by BWM [7]. In this paper, BWM is used to decide and evaluate the weight of the selection standards that are defined to measure the suppliers.

The aim of this paper is to solve a supplier selection problem according to plithogenic set, which is to aggregate the group decision-makers. To achieve this aim, we propose an integrated plithogenic approach based on BWM to find the weight vector of the criteria and the MABAC method to obtain the optimal alternative.

This paper is structured as follows: a summarized presentation of studies on supplier selection and healthcare industry in Section 2. Section 3 consists of definitions and details of the techniques applied in the research. Section 4 gives the details of the proposed approach to handle the supplier selection problem. Our method is applied to numerous examples, and the results are discussed in Section 5. The conclusion of the research is given in Section 6.

2 Literature Review

Supplier selection is a superior responsibility of SC managers. Supplier selection is considered as MCDM process that involves comparisons among groups of criteria to choose the preferable supplier, providing the highest level of performance to the organization. Appropriate supplier selection can improve the profit, decrease costs, satisfy customer expectations, and stimulate the competitiveness of the organization [8]. That is why the supplier selection process became a critical decision that may influence the whole supply chain performance. In this study, a group of MCDM methods were combined to arrive at the best decision, which is to choose the optimal supplier. The importance of combining these theories is due to the creation of a more accurate model of decisions, where the BWM is characterized by its ability to determine the weights of measurement criteria that relate to selecting the best supplier. MABAC is characterized by the ability to choose the best alternative from the proposed alternatives. This proposed hybrid model was developed under uncertainty environment based on rough numbers and plithogenic set to avoid problems of ambiguity of information that characterizes most decision-making problems. Health care problems are among the most significant problems that need to be studied carefully in decision-making.

The supplier selection can be resumed in three main steps [9]. Firstly, the selection criteria should be chosen, such as product quality, cost, product reliability, or delivery performance. Of course, the set of criteria is not standard for all supplier selection situations. Secondly, the weight of each criterion previously defined should be determined, and the collection of supplier alternatives that may fit the production process should be decided. Finally, an MCDM model should be constructed to assess the alternatives based on the evaluation metrics to find the optimal supplier. The aim of the supplier evaluation process in the supply chain is to recognize the optimal supplier that can provide the accepted quality of product/service and the right capacity at the right stage and right location [10].

Health care products accomplished growth in past years. These products have many standards and guidelines that restrict companies in the supplier selection process. The distribution of health care facilities by the use of GIS and MCDM techniques is studied in [11]. Also, to select the best treatment technology in health-care waste (HCW) management, a model based on MCDM methods was proposed [12]. Weighted average operator and TOPSIS were used to offer a model that assists the hierarchical medical system [13]. An expert system based on the fuzzy set for assessment of health care structures was introduced in [14]. Besides, fuzzy (Quality Function Development) QFD was integrated with ordered weighted averaging (OWA) operator to solve the supplier selection problem in health care facilities [15].

Selecting the optimal supplier process implies a large amount of uncertainty, which requires different methodologies to accurate the results of the selection. There are various types of MCDM techniques that can be involved in supplier selection, but the question is which approach or method to apply. RN is an appropriate tool to deal with uncertainty or to handle subjective judgments of customers, and also to determine the boundary intervals [16]. It helps many MCDM methods to improve efficient results in an uncertain environment. Consequently, the qualitative flexible multiple criteria method was combined with a rough number to disband the trouble of shelter site selection. The assessment of third-party logistics was measured by MABAC and BWM based on the rough number in the research of [17]. Chen et al. (2019) combined a rough number with the fuzzy-DEMATEL and analytical network process method (ANP) to evaluate the product-service system [18].

Plithogeny theory refers to creating, improving, and growth of novel objects from groups of conflicting or non-conflicting old objects [19]. It is considered as a generalization of neutrosophic set theory. Plithogenic set has two features that enhance the importance of its operations. The first feature is the contradiction degree between the set of elements, which improves the plithogenic operation accuracy. This feature compares the dissimilarities between the dominant attributes and the set of attributes. The second feature is the appurtenance degree of the attribute value, which we discuss in detail in the next section.

3 Methods and Definitions

3.1 Plithogenic Set

The three representations of c(v, D) can be fuzzy CF, intuitionistic fuzzy (CIF: $V \times V \to [0,1]^2$), or neutrosophic (CN: $V \times V \to [0,1]^3$). The contradiction degree function used in plithogenic set operators is needed for the Intersection (AND), Union (OR), Implication (=>), Equivalence (⇔), and other plithogenic aggregation operators that combine two or more attribute-value degrees [20].

Definition 1. Let $\tilde{a} = (a1, a2, a3)$ and $\tilde{b} = (b1, b2, b3)$ be two plithogenic sets.

The plithogenic intersection is:

$$((a_{i1}, a_{i2}, a_{i3}), 1 \le i \le n) \wedge_p ((b_{i1}, b_{i2}, b_{i3}), 1 \le i \le n) = \left(\left(a_{i1} \wedge_F b_{i1}, \frac{1}{2}(a_{i2} \wedge_F b_{i2}) + \frac{1}{2}(a_{i2} \vee_F b_{i2}), a_{i2} \vee_F b_{i3}\right)\right), 1 \le i \le n. \quad (1)$$

The plithogenic union is:

$$((a_{i1}\ ,a_{i2},a_{i3}),1\leq i\leq n)\ \vee \mathrm{p}\ ((b_{i1},b_{i2},b_{i3}),1\leq i\leq n) \\ =\left(\left(a_{i1}\vee_F b_{i1},\ \frac{1}{2}(a_{i2}\wedge_F\ \ b_{i2})+\frac{1}{2}(a_{i2}\vee_F b_{i2}),a_{i2}\wedge_F b_{i3}\right)\right),\ 1\leq i\leq n. \tag{2}$$

where

$$a_{i1}\wedge \mathrm{p}\ b_{i1}=\ [1-c(v_D,v_1)].t_{\mathrm{norm}}(v_D,v_1)+c(v_D,v_1).t_{\mathrm{conorm}}(v_D,v_1) \tag{3}$$

$$a_{i1}\vee \mathrm{p}\ b_{i1}=[1-c(v_D,v_1)].t_{\mathrm{conorm}}(v_D,v_1)+c(v_D,v_1).t_{\mathrm{norm}}(v_D,v_1) \tag{4}$$

where, tnorm $= \wedge Fb = ab$, tconorm $a\vee Fb = a+b-ab$

The plithogenic complement (negation) is:

$$\neg((a_{i1}\ ,a_{i2},a_{i3}),1\leq i\leq n)=\quad ((a_{i3}\ ,a_{i2},a_{i1}),1\leq i\leq n) \tag{5}$$

3.2 Rough Number

RN is the approximation of the upper and lower values of the original crisp value. It's an efficient theory in decision making because the decision must be determined by a group of decision-makers rather than a single one. The rough number is inspired by the rough set theory proposed in [21]. Also, RN is regulated by lower and upper bounds vague information. Therefore it can effectively obtain the real decision-makers' expectations and combine them in an accurate manner [22].

Definition 2. Suppose U is the universe containing objects, and there are n classes expressed as $G=\{A_1,\ A_2,\ \ldots,\ A_n\}$ ordered as $A_1<\ A_2<\ldots<A_n$; then, the lower approximation $\underline{Apr}(A_n)$ and the upper approximation $\overline{Apr}(A_n)$ of A_n will be defined as:

$$\underline{Apr}(A_n)=\cup\{Y\in\ U/G(Y)\leq A_n\} \tag{6}$$

$$\overline{Apr}(A_n)=\cup\{Y\in\ U/G(Y)\geq A_n\} \tag{7}$$

Then, A_n can be expressed by RN as $RN(A_n)=\left[\ \underline{RN}(A_n),\overline{RN}(A_n)\right]$, where $\underline{RN}(A_n)$ is the lower limit and $\overline{RN}(A_n)$ is the upper limit, as:

$$\underline{RN}(A_n)=\frac{1}{M_L}\sum G(Y)|Y\in \underline{Apr}(A_n) \tag{8}$$

$$\overline{RN}(A_n)=\frac{1}{M_U}\sum G(Y)|Y\in \overline{Apr}(A_n) \tag{9}$$

where M L and M U are the number of objects contained in $\underline{Apr}(A_n)$ and $\overline{Apr}(A_n)$, respectively. The basic operations of rough numbers were proposed [22].

Definition 3. Let $[a_1]=[\underline{a_1},\overline{a_1}]$ and $[a_2]=[\underline{a_2},\overline{a_2}]$ be two rough numbers, where $\underline{a_1},\overline{a_1},\ \underline{a_2},\overline{a_2}\ >0$ and $\alpha>0$; so:

$$\alpha\times[a_1]=[\alpha\times\underline{a_1},\alpha\times\overline{a_1}] \tag{10}$$

$$[a_1]+[a_2]=\left[\underline{a_1}+\underline{a_2},\overline{a_1}+\overline{a_2}\ \right] \tag{11}$$

$$[a_1]\times[a_2]=\left[\underline{a_1}\times\underline{a_2},\overline{a_1}\times\overline{a_2}\ \right] \tag{12}$$

$$\frac{[a_1]}{[a_2]} = \left[\frac{\underline{a_1}}{\overline{a_2}}, \frac{\overline{a_1}}{\underline{a_2}}\right] \tag{13}$$

Definition 4. Let $s_i = \underline{a_n} + \overline{a_n}$, which is the summation of the upper and lower limits, and $D_i = \underline{a_n} - \overline{a_n}$, which is the subtraction of the lower and upper limits. For $[a_1] = [\underline{a_1}, \overline{a_1}]$ and $[a_2] = [\underline{a_2}, \overline{a_2}]$ two rough numbers,

- If $sum_1 > sum_2$, then $a_1 > a_2$;
- If $sum_1 = sum_2$ and $D_1 = D_2$, then $a_1 = a_2$;
- If $sum_1 = sum_2$ and $D_1 < D_2$, then $a_1 > a_2$.

3.3 Best-Worst Method (BWM)

BWM is an efficient and straightforward pairwise comparison of MCDM problems that compare the most preferred (best) criterion and the least desired (worst) criterion with the rest of the problem criteria [23]. A framework based on the BWM to evaluate the financial performance to compare dynamic analysis and cross-sectional analysis was suggested in [24]. Also, sustainable supplier selection criteria were evaluated using BWM [25]. BWM consists of five steps:

Step 1: The set of criteria A is defined by a committee of decision-makers $k = \{k_1, k_2, \ldots, k_m\}$ based on the scope of the problem as $A = \{c_1, c_2, \ldots, c_n\}$.

Step 2: Determine the Best AB and Worst AW criterion from the set of criteria A according to the decision-maker preferences.

Step 3: Construct the best-to-other vector $A_B = \{a_{B1}, a_{B2}, \ldots a_{Bn}\}$, where a_{Bn} is the preference of criteria n compared by the Best criterion B using a (1–9) scale.

Step 4: Construct the others-to-worst vector $A_w = \{a_{w1}, a_{w2}, \ldots a_{wn}\}$, where a_{wn} is the preference of criteria n compared by the Worst criterion W using a (1–9) scale.

Step 5: Propose the BWM model that evaluates the weight of the criteria w_n:

$$\begin{aligned} &\text{Min } \varepsilon \\ &\text{s.t.} \\ &\left|\frac{w_B}{w_j} - a_{Bj}\right| \leq \varepsilon, \text{ for all j} \\ &\left|\frac{w_j}{w_w} - a_{jW}\right| \leq \varepsilon, \text{ for all j} \\ &\sum_j w_j = 1 \\ &w_j \geq 0, \text{ for all j} \end{aligned} \tag{14}$$

3.4 MABAC Method

MABAC method is a recent MCDM technique that evaluates a set of criteria to find the best alternative. This method compares the relevant distance ideal and anti-ideal solution, and it is valuable when the problem has enormous alternatives or criteria [10].

- Step 1: Construct the decision-making matrix $\tilde{D}$ based on the committee of decision-makers (DMs), which evaluates the alternatives m according to the set of criteria n.

$$\tilde{D} = \left[d_{ij}\right]_{m\times n} = \begin{bmatrix} d_{11} & d_{12} & \dots & d_{1n} \\ d_{21} & d_{22} & \dots & d_{21} \\ \dots & \dots & \dots & \dots \\ d_{m1} & d_{m2} & \dots & d_{mn} \end{bmatrix}_{m\times n} \tag{15}$$

- Step 2: Compute the normalized decision matrix:

$$\tilde{N} = \left[x_{ij}\right]_{m\times n} = \begin{bmatrix} x_{11} & x_{12} & \dots & x_{1n} \\ x_{21} & x_{22} & \dots & x_{21} \\ \dots & \dots & \dots & \dots \\ x_{m1} & x_{m2} & \dots & x_{mn} \end{bmatrix}_{m\times n} \tag{16}$$

where,

$$x_{ij} = \begin{cases} \dfrac{x_{ij} - x_{ij}^{min}}{x_{ij}^{max} - x_{ij}^{min}} \ if\ x_{ij}\ is\ \ benefit\ criteria \\ \dfrac{x_{ij}^{max} - x_{ij}}{x_{ij}^{max} - x_{ij}^{min}} \ if\ x_{ij}\ is\ cost\ criteria \end{cases} \tag{17}$$

- Step 3: Calculate weighted matrix element:

$$V = \left[v_{ij}\right]_{m\times n} = \begin{bmatrix} v_{11} & v_{12} & \dots & v_{1n} \\ v_{21} & v_{22} & \dots & v_{21} \\ \dots & \dots & \dots & \dots \\ v_{m1} & v_{m2} & \dots & v_{mn} \end{bmatrix}_{m\times n}$$

$$= \begin{bmatrix} w_2.(x_{11}+1) & w_2.(x_{12}+1) & \dots & w_n.(x_{1n}+1) \\ w_2.(x_{21}+1) & w_2.(x_{22}+1) & \dots & w_n.(x_{2n}+1) \\ \dots & \dots & \dots & \dots \\ w_2.(x_{m1}+1) & w_2.(x_{m2}+1) & \dots & w_n.(x_{mn}+1) \end{bmatrix}_{m\times n} \tag{18}$$

- Step 4: Determine the border approximation area (BAA) matrix:

$$G[g_n]_{1\times n} = [\,g_1 \quad g_2 \quad \dots \quad g_n\,] \tag{19}$$

$$g_i = \left.\prod_{j=1}^{m} v_{ij}\right)^{1/n} \tag{20}$$

- Step 5: Calculate the distance between each alternative and the BAA:

$$Q = [q_{ij}]_{m\times n} = \begin{bmatrix} q_{11} & q_{12} & \cdots & q_{1n} \\ q_{21} & q_{22} & \cdots & q_{21} \\ \cdots & \cdots & \cdots & \cdots \\ q_{m1} & q_{m2} & \cdots & q_{mn} \end{bmatrix}_{m\times n} = V - G \tag{21}$$

Alternative m may belong to three regions $m \in \{G, G^+, G^-\}$ as Fig. 1 shows:

1. Belong to the BAA (G)
2. Upper approximation area (G+)
3. Lower approximation area (G-)

- Step 6: Rank the alternatives and find the best solution.

$$s_i = \sum_{j=1}^{m} q_{mj} \tag{22}$$

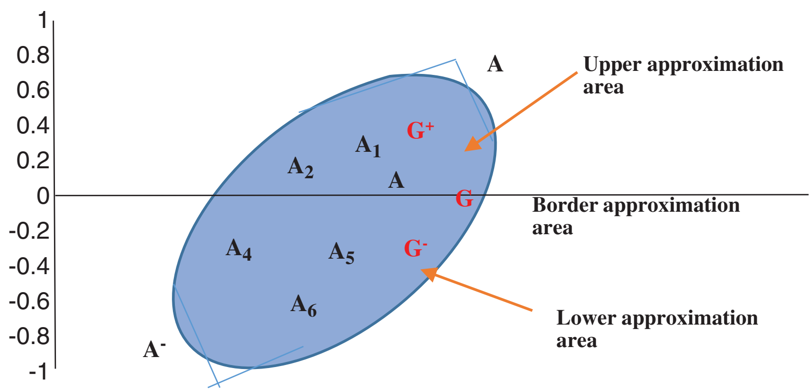

Figure 1: Border approximation area

4 Proposed Approach

This research proposes an integrated approach to disband a supplier selection problem taking into consideration ambiguous information. This approach integrates the features of the four techniques and methods. Firstly, the plithogenic set aggregation features concentrates on providing more accurate aggregation results while considering uncertainty. Secondly, rough numbers consider the higher and lower limits of the information to handle vague information; thirdly, BWM evaluates a set of criteria in MCDM problems; and finally, the MABAC method determines the optimal alternative compared with a group of criteria by measuring the distance of alternatives with the BAA. The value of integrating these methods lies in the development of a more effective decision model. Rough numbers and plithogenic sets support the proposed model to have a more accurate and consistent decision and solve the problem of information ambiguity in an uncertainty environment. The process of the proposed model is summarized in Fig. 2.

- Step 1: A group of DMs with experience in the problem scope must be chosen, $D = \{d_1, d_2, \ldots, d_k\}$, and the set of criteria that control the problem $C = \{c_1, c_2, \ldots, c_n\}$ must be determined, together with the alternatives that need to be compared to find the best one $A = \{a_1, a_2, \ldots, a_m\}$.

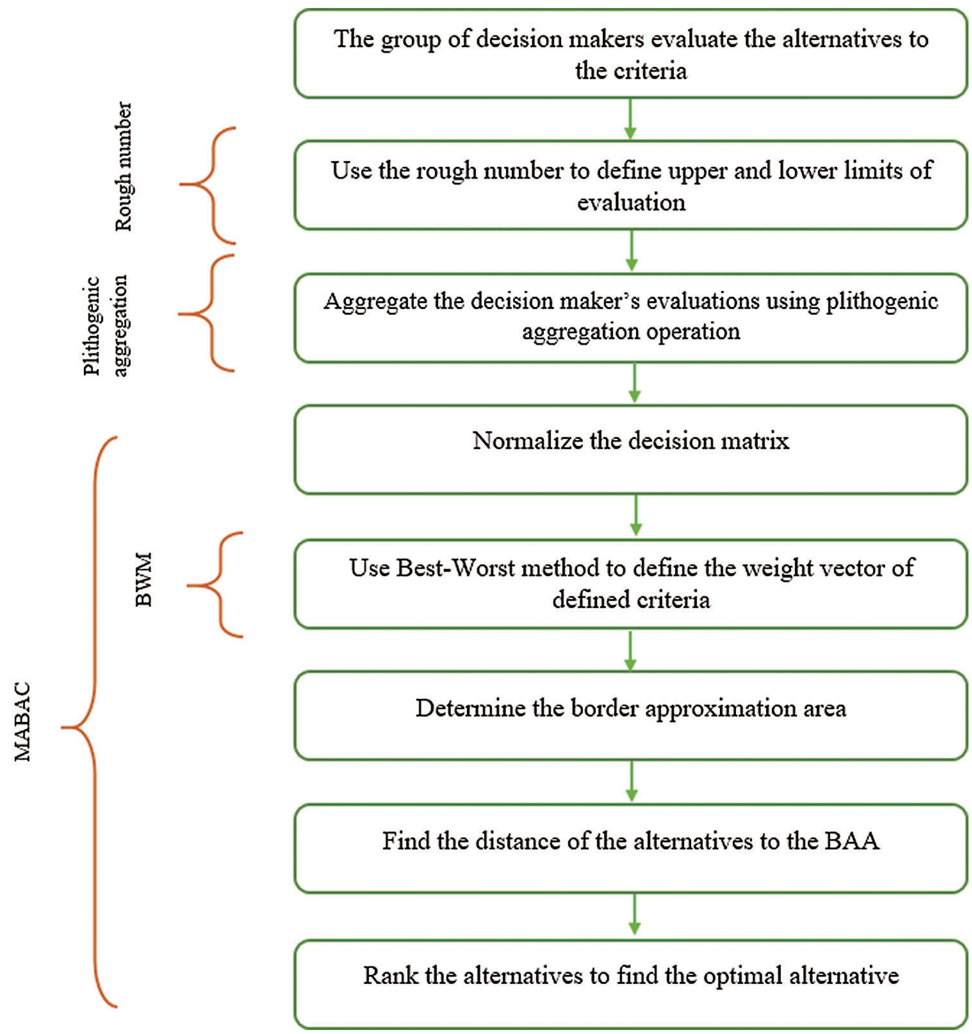

Figure 2: Proposed model's main steps

Table 1: Linguistic scale

Significance Linguistic Variable	Triangular Neutrosophic Scale
Very Low significant (VLS)	((0.10, 0.30,0.35), 0.1,0.2,0.15)
Low significant (LS)	((0.15,0.25,0.10), 0.6,0.2,0.3)
Partially significant (PS)	((0.40,0.35,0.50), 0.6,0.1,0.2)
Equal significant (ES)	(0.65,0.60,0.70),0.8,0.1,0.1)
High significant (HS)	((0.70,0.65,0.80),0.9,0.2,0.1)
Very High significant (VHS)	((0.90,0.85,0.90),0.7,0.2,0.2)
Absolutely significant (AS)	((0.95,0.90,0.95),0.9,0.10,0.10)

- In order to evaluate the alternatives to the criteria by a group of DMs, identify an evaluation scale. In this research, TNN is employed used in a linguistic scale (Tab. 1).
- Step 2: Express the evaluation matrix values as rough numbers to consider the uncertain information and define the upper and lower limits of the evaluation, as explained in detail in Section 3.3.
- Step 3: Aggregate the decision maker's evaluation using plithogenic aggregation operator as discussed in Eqs. (1), (3), (4).
- Define the contradiction degree that establishes the relation between the most preferred (dominant) criterion and other criteria. This feature improves the accuracy of the aggregation operation.
- Step 4: Normalize the decision matrix using Eqs. (16), (17), considering whether the criterion is benefit and cost criteria.
- Convert the neutrosophic evaluation values to crisp values, as in Eq. (23).

$$S(a) = \frac{1}{8}(a_1 + b_1 + c_1) \times (2 + \alpha - \theta - \beta) \tag{23}$$

- Step 5: Find the weighted decision matrix based on BWM.
- Find the best, and the worst criterion according do decision-makers experience or preference
- Construct the best-to-other vector and others-to-worst vector based on the scale [0,1]
- Use the BWM model (24,25) to find the weight vector
- Step 6: Calculate the BAA matrix using Eqs. (19), (20).
- Step 7: Find the distance between the alternatives and BAA matrix to define the alternatives in the upper approximation area, BAA, and lower approximation area, as in Eq. (21).
- Step 8: Using Eq. (22), rank the alternatives.

5 Numerical Application and Results

The proposed approach evaluates a major MCDM problem, which is the supplier selection in the healthcare industry. As a case study, we simulate a supplier selection at a private healthcare firm in Malaysia that owns both a wholesaler and a chain of medical clinics. Wholesaler includes an administrative center and a single warehouse that collects products from five different suppliers. They are seeking to find the best supplier to execute a large supply order.

A committee of four experts who have a long experience in healthcare devices met to help in this decision. They defined a set of nine criteria that must be considered while evaluating the five alternatives. These criteria are:

- product quality (C1),
- supplier expedites emergency orders (C2),
- supplier adequately test new products (C3),
- technology service: problem-solving (C4),
- technology service: responsiveness (C5),
- the supplier provides technical assistance (C6),
- the supplier provides notice of product problems (C7),
- consistency of delivered product (C8), and
- accuracy in filling orders (C9).

The hierarchy of this case is presented in Fig. 3.

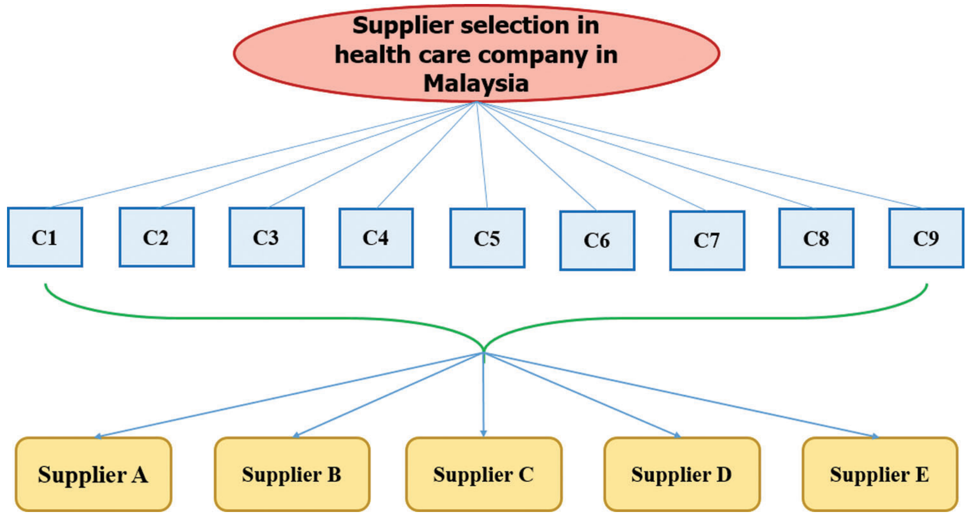

Figure 3: Healthcare industry hierarchy

Applying the proposed integrated approach in the case of a health care company in Malaysia will be as follows:

According to the neutrosophic linguistic scale in Tab. 1, the five suppliers will be evaluated by the group of decision-makers, as in Tab. 2:

Table 2: Evaluation matrix

Alternatives	DM	C1	C2	C3	C4	C5	C6	C7	C8	C9
A1	DM1	HS	ES	ES	HS	PS	ES	ES	PS	PS
	DM2	VHS	HS	VHS	ES	PS	ES	HS	PS	HS
	DM3	AS	VHS	PS	ES	HS	PS	ES	ES	ES
	DM4	HS	ES	HS	ES	ES	ES	PS	PS	ES
A2	DM1	ES	ES	HS	HS	HS	ES	ES	HS	PS
	DM2	HS	ES	VHS	HS	HS	PS	PS	ES	HS
	DM3	HS	VHS	HS	PS	PS	PS	HS	ES	PS
	DM4	VHS	HS	PS	ES	HS	ES	ES	PS	ES
A3	DM1	AS	VHS	VHS	HS	VHS	HS	ES	HS	HS
	DM2	HS	ES	HS	HS	HS	ES	HS	HS	HS
	DM3	AS	VHS	ES	VHS	HS	ES	HS	ES	PS
	DM4	VHS	VHS	VHS	HS	HS	HS	PS	ES	ES
A4	DM1	ES	ES	HS	PS	PS	ES	PS	PS	ES
	DM2	HS	ES	PS	PS	HS	HS	ES	ES	PS
	DM3	ES	HS	HS	HS	ES	ES	HS	PS	PS
	DM4	HS	PS	HS	HS	HS	PS	ES	HS	PS

Table 2 (continued).

Alternatives	DM	C1	C2	C3	C4	C5	C6	C7	C8	C9
A5	DM1	HS	VHS	HS	HS	HS	ES	ES	ES	ES
	DM2	HS	ES	VHS	PS	ES	PS	HS	ES	PS
	DM3	AS	ES	HS	PS	PS	HS	ES	PS	PS
	DM4	ES	VHS	ES	VHS	ES	ES	PS	ES	ES

To handle the uncertainty of information, the evaluation values will be transformed into rough numbers to define the upper and lower limits of the evaluation, as discussed in Section 3.3. The evaluation matrix is represented in rough numbers in Tab. 3. Then, the plithogenic aggregation operator-assisted in combining the assessment of the four decision-makers. The contradiction degree of the criteria was defined to ensure more accurate gathering, as shown in Tab. 4. After converting the evaluation to crisp values as in Eq. (23), the normalized decision matrix must be calculated based on the nature of the criteria, as expressed in Eqs. (16), (17), and the result is presented in Tab. 5.

Table 3: Evaluation matrix by rough numbers

		C1	C2	C3
A1	DM1	[((0.70,0.65,0.80),0.9,0.2,0.1), ((0.81,0.76,0.86),0.85,0.18,0.13)]	[((0.65,0.60,0.70),0.8,0.1,0.1), ((0.73,0.68,0.78),0.8,0.15,0.13)]	[((0.53,0.48,0.6),0.7,0.1,0.5), ((0.75,0.7,0.8),0.8,0.17,0.13)]
	DM2	[((0.77,0.72,0.83),0.83,0.2,0.13), ((0.93,0.88,0.93),0.8,0.15,0.15)]	[((0.67,0.62,0.73),0.83,0.13,0.1), ((0.8,0.75,0.85),0.8,0.2,0.15)]	[((0.66,0.61,0.73),0.75,0.15,0.15), ((0.90,0.85,0.90),0.7,0.2,0.2)]
	DM3	[((0.81,0.76,0.86),0.85,0.18,0.13), ((0.95,0.90,0.95),0.9,0.10,0.10)]	[((0.73,0.68,0.78),0.8,0.15,0.13), ((0.90,0.85,0.90),0.7,0.2,0.2)]	[((0.40,0.35,0.50), 0.6,0.1,0.2), ((0.66,0.61,0.73),0.75,0.15,0.15)]
	DM4	[((0.70,0.65,0.80),0.9,0.2,0.1), ((0.81,0.76,0.86),0.85,0.18,0.13)]	[((0.65,0.60,0.70),0.8,0.1,0.1), ((0.73,0.68,0.78),0.8,0.15,0.13)]	[((0.58,0.53,0.67),0.77,0.13,0.13), ((0.8,0.75,0.85),0.8,0.2,0.15)]
A2	DM1	[((0.65,0.60,0.70),0.8,0.1,0.1), ((0.74,0.69,0.8),0.83,0.18,0.13)]	[((0.65,0.60,0.70),0.8,0.1,0.1), ((0.73,0.68,0.78),0.8,0.15,0.13)]	[((0.6,0.55,0.7),0.8,0.17,0.13), ((0.77,0.72,0.83),0.83,0.2,0.13)]
	DM2	[((0.68,0.63,0.77),0.87,0.17,0.1), ((0.77,0.72,0.83),0.83,0.2,0.13)]	[((0.67,0.62,0.73),0.83,0.13,0.1), ((0.8,0.75,0.85),0.8,0.2,0.15)]	[((0.68,0.63,0.75),0.78,0.18,0.15), ((0.90,0.85,0.90),0.7,0.2,0.2)]
	DM3	[((0.68,0.63,0.77),0.87,0.17,0.1), ((0.77,0.72,0.83),0.83,0.2,0.13)]	[((0.73,0.68,0.78),0.8,0.15,0.13), ((0.90,0.85,0.90),0.7,0.2,0.2)]	[((0.6,0.55,0.7),0.8,0.17,0.13), ((0.77,0.72,0.83),0.83,0.2,0.13)]
	DM4	[((0.74,0.69,0.8),0.83,0.18,0.13), ((0.90,0.85,0.90),0.7,0.2,0.2)]	[((0.65,0.60,0.70),0.8,0.1,0.1), ((0.73,0.68,0.78),0.8,0.15,0.13)]	[((0.40,0.35,0.50), 0.6,0.1,0.2), ((0.68,0.63,0.75),0.78,0.18,0.15)]
A3	DM1	[((0.88,0.83,0.9),0.85,0.15,0.13), ((0.95,0.90,0.95),0.9,0.10,0.10)]	[((0.84,0.79,0.85),0.73,0.18,0.18), ((0.90,0.85,0.90),0.7,0.2,0.2)]	[((0.79,0.74,0.83),0.78,0.18,0.15), ((0.90,0.85,0.90),0.7,0.2,0.2)]
	DM2	[((0.70,0.65,0.80),0.9,0.2,0.1), ((0.88,0.83,0.9),0.85,0.15,0.13)]	[((0.65,0.60,0.70),0.8,0.1,0.1), ((0.84,0.79,0.85),0.73,0.18,0.18)]	[((0.68,0.63,0.75),0.85,0.15,0.1), ((0.83,0.78,0.87),0.77,0.2,0.17)]
	DM3	[((0.88,0.83,0.9),0.85,0.15,0.13), ((0.95,0.90,0.95),0.9,0.10,0.10)]	[((0.84,0.79,0.85),0.73,0.18,0.18), ((0.90,0.85,0.90),0.7,0.2,0.2)]	[((0.65,0.60,0.70),0.8,0.1,0.1), ((0.79,0.74,0.83),0.78,0.18,0.15)]
	DM4	[((0.8,0.75,0.85),0.8,0.2,0.15), ((0.93,0.88,0.93),0.83,0.13,0.13)]	[((0.84,0.79,0.85),0.73,0.18,0.18), ((0.90,0.85,0.90),0.7,0.2,0.2)]	[((0.79,0.74,0.83),0.78,0.18,0.15), ((0.90,0.85,0.90),0.7,0.2,0.2)]

(Continued)

Table 3 (continued).

A4	DM1	[((0.65,0.60,0.70),0.8,0.1,0.1), ((0.68,0.63,0.75),0.85,0.15,0.1)]	[((0.6,0.55,0.65),0.78,0.13,0.13), ((0.70,0.65,0.80),0.9,0.2,0.1)]	[((0.55,0.5,0.65),0.75,0.15,0.15), ((0.70,0.65,0.80),0.9,0.2,0.1)]
	DM2	[((0.68,0.63,0.75),0.85,0.15,0.1), ((0.70,0.65,0.80),0.9,0.2,0.1)]	[((0.57,0.52,0.63),0.73,0.1,0.13), ((0.67,0.62,0.73),0.83,0.13,0.1)]	[((0.40,0.35,0.50), 0.6,0.1,0.2), ((0.55,0.5,0.65),0.75,0.15,0.15)]
	DM3	[((0.65,0.60,0.70),0.8,0.1,0.1), ((0.68,0.63,0.75),0.85,0.15,0.1)]	[((0.40,0.35,0.50), 0.6,0.1,0.2), ((0.6,0.55,0.65),0.78,0.13,0.13)]	[((0.55,0.5,0.65),0.75,0.15,0.15), ((0.70,0.65,0.80),0.9,0.2,0.1)]
	DM4	[((0.68,0.63,0.75),0.85,0.15,0.1), ((0.70,0.65,0.80),0.9,0.2,0.1)]	[((0.57,0.52,0.63),0.73,0.1,0.13), ((0.67,0.62,0.73),0.83,0.13,0.1)]	[((0.55,0.5,0.65),0.75,0.15,0.15), ((0.70,0.65,0.80),0.9,0.2,0.1)]
A5	DM1	[((0.68,0.63,0.77),0.87,0.17,0.1), ((0.78,0.73,0.85),0.9,0.17,0.1)]	[((0.78,0.73,0.8),0.75,0.15,0.15), ((0.90,0.85,0.90),0.7,0.2,0.2)]	[((0.65,0.60,0.70),0.8,0.1,0.1), ((0.74,0.69,0.8),0.83,0.18,0.13)]
	DM2	[((0.68,0.63,0.77),0.87,0.17,0.1), ((0.78,0.73,0.85),0.9,0.17,0.1)]	[((0.65,0.60,0.70),0.8,0.1,0.1), ((0.78,0.73,0.8),0.75,0.15,0.15)]	[((0.68,0.63,0.77),0.87,0.17,0.1), ((0.77,0.72,0.83),0.83,0.2,0.13)]
	DM3	[((0.75,0.7,0.81),0.88,0.15,0.1), ((0.95,0.90,0.95),0.9,0.10,0.10)]	[((0.65,0.60,0.70),0.8,0.1,0.1), ((0.78,0.73,0.8),0.75,0.15,0.15)]	[((0.68,0.63,0.77),0.87,0.17,0.1), ((0.77,0.72,0.83),0.83,0.2,0.13)]
	DM4	[((0.65,0.60,0.70),0.8,0.1,0.1), ((0.75,0.7,0.81),0.88,0.15,0.1)]	[((0.78,0.73,0.8),0.75,0.15,0.15), ((0.90,0.85,0.90),0.7,0.2,0.2)]	[((0.74,0.69,0.8),0.83,0.18,0.13), ((0.90,0.85,0.90),0.7,0.2,0.2)]
		C4	C5	C6
A1	DM1	[((0.66,0.61,0.73),0.83,0.13,0.1), ((0.70,0.65,0.80),0.9,0.2,0.1)]	[((0.40,0.35,0.50), 0.6,0.1,0.2), ((0.54,0.49,0.63),0.73,0.13,0.15)]	[((0.59,0.54,0.65),0.75,0.1,0.13), ((0.65,0.60,0.70),0.8,0.1,0.1)]
	DM2	[((0.65,0.60,0.70),0.8,0.1,0.1), ((0.66,0.61,0.73),0.83,0.13,0.1)]	[((0.40,0.35,0.50), 0.6,0.1,0.2), ((0.54,0.49,0.63),0.73,0.13,0.15)]	[((0.59,0.54,0.65),0.75,0.1,0.13), ((0.65,0.60,0.70),0.8,0.1,0.1)]
	DM3	[((0.65,0.60,0.70),0.8,0.1,0.1), ((0.66,0.61,0.73),0.83,0.13,0.1)]	[((0.54,0.49,0.63),0.73,0.13,0.15), ((0.70,0.65,0.80),0.9,0.2,0.1)]	[((0.40,0.35,0.50), 0.6,0.1,0.2), ((0.59,0.54,0.65),0.75,0.1,0.13)]
	DM4	[((0.65,0.60,0.70),0.8,0.1,0.1), ((0.66,0.61,0.73),0.83,0.13,0.1)]	[((0.45,0.43,0.57),0.65,0.1,0.17), ((0.68,0.63,0.75),0.85,0.15,0.1)]	[((0.59,0.54,0.65),0.75,0.1,0.13), ((0.65,0.60,0.70),0.8,0.1,0.1)]
A2	DM1	[((0.61,0.56,0.7),0.8,0.15,0.13), ((0.70,0.65,0.80),0.9,0.2,0.1)]	[((0.55,0.5,0.65),0.75,0.15,0.15), ((0.70,0.65,0.80),0.9,0.2,0.1)]	[((0.53,0.48,0.6),0.7,0.1,0.15), (0.65,0.60,0.70),0.8,0.1,0.1)]
	DM2	[((0.61,0.56,0.7),0.8,0.15,0.13), ((0.70,0.65,0.80),0.9,0.2,0.1)]	[((0.40,0.35,0.50), 0.6,0.1,0.2), ((0.55,0.5,0.65),0.75,0.15,0.15)]	[((0.40,0.35,0.50), 0.6,0.1,0.2), ((0.53,0.48,0.6),0.7,0.1,0.15)]
	DM3	[((0.40,0.35,0.50), 0.6,0.1,0.2), ((0.61,0.56,0.7),0.8,0.15,0.13)]	[((0.55,0.5,0.65),0.75,0.15,0.15), ((0.70,0.65,0.80),0.9,0.2,0.1)]	[((0.40,0.35,0.50), 0.6,0.1,0.2), ((0.53,0.48,0.6),0.7,0.1,0.15)]
	DM4	[((0.53,0.48,0.6),0.7,0.1,0.15), ((0.68,0.63,0.77),0.87,0.17,0.1)]	[((0.55,0.5,0.65),0.75,0.15,0.15), ((0.70,0.65,0.80),0.9,0.2,0.1)]	[((0.53,0.48,0.6),0.7,0.1,0.15), (0.65,0.60,0.70),0.8,0.1,0.1)]
A3	DM1	[((0.70,0.65,0.80),0.9,0.2,0.1), ((0.75,0.7,0.83),0.85,0.2,0.13)]	[((0.70,0.65,0.80),0.9,0.2,0.1), ((0.75,0.7,0.83),0.85,0.2,0.13)]	[((0.66,0.61,0.73),0.83,0.13,0.1), ((0.70,0.65,0.80),0.9,0.2,0.1)]
	DM2	[((0.70,0.65,0.80),0.9,0.2,0.1), ((0.75,0.7,0.83),0.85,0.2,0.13)]	[((0.70,0.65,0.80),0.9,0.2,0.1), ((0.75,0.7,0.83),0.85,0.2,0.13)]	[((0.65,0.60,0.70),0.8,0.1,0.1), ((0.66,0.61,0.73),0.83,0.13,0.1)]
	DM3	[((0.75,0.7,0.83),0.85,0.2,0.13), ((0.90,0.85,0.90),0.7,0.2,0.2)]	[((0.75,0.7,0.83),0.85,0.2,0.13), ((0.90,0.85,0.90),0.7,0.2,0.2)]	[((0.65,0.60,0.70),0.8,0.1,0.1), ((0.66,0.61,0.73),0.83,0.13,0.1)]
	DM4	[((0.70,0.65,0.80),0.9,0.2,0.1), ((0.75,0.7,0.83),0.85,0.2,0.13)]	[((0.70,0.65,0.80),0.9,0.2,0.1), ((0.75,0.7,0.83),0.85,0.2,0.13)]	[((0.65,0.60,0.70),0.8,0.1,0.1), ((0.66,0.61,0.73),0.83,0.13,0.1)]

Table 3 (continued).

A4	DM1	[((0.53,0.48,0.6),0.7,0.1,0.15), (0.65,0.60,0.70),0.8,0.1,0.1)]	[((0.61,0.56,0.7),0.8,0.15,0.13), ((0.70,0.65,0.80),0.9,0.2,0.1)]	[((0.6,0.55,0.65),0.78,0.13,0.13), ((0.70,0.65,0.80),0.9,0.2,0.1)]
	DM2	[((0.40,0.35,0.50), 0.6,0.1,0.2), ((0.53,0.48,0.6),0.7,0.1,0.15)]	[((0.61,0.56,0.7),0.8,0.15,0.13), ((0.70,0.65,0.80),0.9,0.2,0.1)]	[((0.57,0.52,0.63),0.73,0.1,0.13), ((0.67,0.62,0.73),0.83,0.13,0.1)]
	DM3	[((0.40,0.35,0.50), 0.6,0.1,0.2), ((0.53,0.48,0.6),0.7,0.1,0.15)]	[((0.40,0.35,0.50), 0.6,0.1,0.2), ((0.61,0.56,0.7),0.8,0.15,0.13)]	[((0.40,0.35,0.50), 0.6,0.1,0.2), ((0.6,0.55,0.65),0.78,0.13,0.13)]
	DM4	[((0.53,0.48,0.6),0.7,0.1,0.15), (0.65,0.60,0.70),0.8,0.1,0.1)]	[((0.53,0.48,0.6),0.7,0.1,0.15), ((0.68,0.63,0.77),0.87,0.17,0.1)]	[((0.57,0.52,0.63),0.73,0.1,0.13), ((0.67,0.62,0.73),0.83,0.13,0.1)]
A5	DM1	[((0.5,0.45,0.6),0.7,0.13,0.17), ((0.8,0.75,0.85),0.8,0.2,0.15)]	[((0.6,0.55,0.65),0.78,0.13,0.13), ((0.70,0.65,0.80),0.9,0.2,0.1)]	[((0.6,0.55,0.65),0.78,0.13,0.13), ((0.70,0.65,0.80),0.9,0.2,0.1)]
	DM2	[((0.40,0.35,0.50), 0.6,0.1,0.2), ((0.6,0.55,0.68),0.7,0.15,0.18)]	[((0.57,0.52,0.63),0.73,0.1,0.13), ((0.67,0.62,0.73),0.83,0.13,0.1)]	[((0.57,0.52,0.63),0.73,0.1,0.13), ((0.67,0.62,0.73),0.83,0.13,0.1)]
	DM3	[((0.40,0.35,0.50), 0.6,0.1,0.2), ((0.6,0.55,0.68),0.7,0.15,0.18)]	[((0.40,0.35,0.50), 0.6,0.1,0.2), ((0.6,0.55,0.65),0.78,0.13,0.13)]	[((0.40,0.35,0.50), 0.6,0.1,0.2), ((0.6,0.55,0.65),0.78,0.13,0.13)]
	DM4	[((0.6,0.55,0.68),0.7,0.15,0.18), ((0.90,0.85,0.90),0.7,0.2,0.2)]	[((0.57,0.52,0.63),0.73,0.1,0.13), ((0.67,0.62,0.73),0.83,0.13,0.1)]	[((0.57,0.52,0.63),0.73,0.1,0.13), ((0.67,0.62,0.73),0.83,0.13,0.1)]
		C7	**C8**	**C9**
A1	DM1	[((0.6,0.55,0.65),0.78,0.13,0.13), ((0.70,0.65,0.80),0.9,0.2,0.1)]	[((0.40,0.35,0.50), 0.6,0.1,0.2), ((0.46,0.41,0.55),0.65,0.1,0.18)]	[((0.6,0.55,0.65),0.78,0.13,0.13), ((0.70,0.65,0.80),0.9,0.2,0.1)]
	DM2	[((0.57,0.52,0.63),0.73,0.1,0.13), ((0.67,0.62,0.73),0.83,0.13,0.1)]	[((0.40,0.35,0.50), 0.6,0.1,0.2), ((0.46,0.41,0.55),0.65,0.1,0.18)]	[((0.57,0.52,0.63),0.73,0.1,0.13), ((0.67,0.62,0.73),0.83,0.13,0.1)]
	DM3	[((0.40,0.35,0.50), 0.6,0.1,0.2), ((0.6,0.55,0.65),0.78,0.13,0.13)]	[((0.46,0.41,0.55),0.65,0.1,0.18), (0.65,0.60,0.70),0.8,0.1,0.1)]	[((0.40,0.35,0.50), 0.6,0.1,0.2), ((0.6,0.55,0.65),0.78,0.13,0.13)]
	DM4	[((0.57,0.52,0.63),0.73,0.1,0.13), ((0.67,0.62,0.73),0.83,0.13,0.1)]	[((0.40,0.35,0.50), 0.6,0.1,0.2), ((0.46,0.41,0.55),0.65,0.1,0.18)]	[((0.57,0.52,0.63),0.73,0.1,0.13), ((0.67,0.62,0.73),0.83,0.13,0.1)]
A2	DM1	[((0.6,0.55,0.65),0.78,0.13,0.13), ((0.70,0.65,0.80),0.9,0.2,0.1)]	[((0.6,0.55,0.65),0.78,0.13,0.13), ((0.70,0.65,0.80),0.9,0.2,0.1)]	[((0.40,0.35,0.50), 0.6,0.1,0.2), ((0.54,0.49,0.63),0.73,0.13,0.15)]
	DM2	[((0.57,0.52,0.63),0.73,0.1,0.13), ((0.67,0.62,0.73),0.83,0.13,0.1)]	[((0.57,0.52,0.63),0.73,0.1,0.13), ((0.67,0.62,0.73),0.83,0.13,0.1)]	[((0.40,0.35,0.50), 0.6,0.1,0.2), ((0.54,0.49,0.63),0.73,0.13,0.15)]
	DM3	[((0.40,0.35,0.50), 0.6,0.1,0.2), ((0.6,0.55,0.65),0.78,0.13,0.13)]	[((0.40,0.35,0.50), 0.6,0.1,0.2), ((0.6,0.55,0.65),0.78,0.13,0.13)]	[((0.54,0.49,0.63),0.73,0.13,0.15), ((0.70,0.65,0.80),0.9,0.2,0.1)]
	DM4	[((0.57,0.52,0.63),0.73,0.1,0.13), ((0.67,0.62,0.73),0.83,0.13,0.1)]	[((0.57,0.52,0.63),0.73,0.1,0.13), ((0.67,0.62,0.73),0.83,0.13,0.1)]	[((0.45,0.43,0.57),0.65,0.1,0.17), ((0.68,0.63,0.75),0.85,0.15,0.1)]
A3	DM1	[((0.61,0.56,0.7),0.8,0.15,0.13), ((0.70,0.65,0.80),0.9,0.2,0.1)]	[((0.61,0.56,0.7),0.8,0.15,0.13), ((0.70,0.65,0.80),0.9,0.2,0.1)]	[((0.61,0.56,0.7),0.8,0.15,0.13), ((0.70,0.65,0.80),0.9,0.2,0.1)]
	DM2	[((0.61,0.56,0.7),0.8,0.15,0.13), ((0.70,0.65,0.80),0.9,0.2,0.1)]	[((0.61,0.56,0.7),0.8,0.15,0.13), ((0.70,0.65,0.80),0.9,0.2,0.1)]	[((0.61,0.56,0.7),0.8,0.15,0.13), ((0.70,0.65,0.80),0.9,0.2,0.1)]
	DM3	[((0.40,0.35,0.50), 0.6,0.1,0.2), ((0.61,0.56,0.7),0.8,0.15,0.13)]	[((0.40,0.35,0.50), 0.6,0.1,0.2), ((0.61,0.56,0.7),0.8,0.15,0.13)]	[((0.40,0.35,0.50), 0.6,0.1,0.2), ((0.61,0.56,0.7),0.8,0.15,0.13)]
	DM4	[((0.53,0.48,0.6),0.7,0.1,0.15), ((0.68,0.63,0.77),0.87,0.17,0.1)]	[((0.53,0.48,0.6),0.7,0.1,0.15), ((0.68,0.63,0.77),0.87,0.17,0.1)]	[((0.53,0.48,0.6),0.7,0.1,0.15), ((0.68,0.63,0.77),0.87,0.17,0.1)]

(Continued)

Table 3 (continued).

A4	DM1	[((0.6,0.55,0.65),0.78,0.13,0.13), ((0.70,0.65,0.80),0.9,0.2,0.1)]	[((0.40,0.35,0.50), 0.6,0.1,0.2), ((0.54,0.49,0.63),0.73,0.13,0.15)]	[((0.40,0.35,0.50), 0.6,0.1,0.2), ((0.46,0.41,0.55),0.65,0.1,0.18)]
	DM2	[((0.57,0.52,0.63),0.73,0.1,0.13), ((0.67,0.62,0.73),0.83,0.13,0.1)]	[((0.40,0.35,0.50), 0.6,0.1,0.2), ((0.54,0.49,0.63),0.73,0.13,0.15)]	[((0.40,0.35,0.50), 0.6,0.1,0.2), ((0.46,0.41,0.55),0.65,0.1,0.18)]
	DM3	[((0.40,0.35,0.50), 0.6,0.1,0.2), ((0.6,0.55,0.65),0.78,0.13,0.13)]	[((0.54,0.49,0.63),0.73,0.13,0.15), ((0.70,0.65,0.80),0.9,0.2,0.1)]	[((0.46,0.41,0.55),0.65,0.1,0.18), (0.65,0.60,0.70),0.8,0.1,0.1)]
	DM4	[((0.57,0.52,0.63),0.73,0.1,0.13), ((0.67,0.62,0.73),0.83,0.13,0.1)]	[((0.45,0.43,0.57),0.65,0.1,0.17), ((0.68,0.63,0.75),0.85,0.15,0.1)]	[((0.40,0.35,0.50), 0.6,0.1,0.2), ((0.46,0.41,0.55),0.65,0.1,0.18)]
A5	DM1	[((0.6,0.55,0.65),0.78,0.13,0.13), ((0.70,0.65,0.80),0.9,0.2,0.1)]	[((0.59,0.54,0.65),0.75,0.1,0.13), ((0.65,0.60,0.70),0.8,0.1,0.1)]	[((0.53,0.48,0.6),0.7,0.1,0.15), (0.65,0.60,0.70),0.8,0.1,0.1)]
	DM2	[((0.57,0.52,0.63),0.73,0.1,0.13), ((0.67,0.62,0.73),0.83,0.13,0.1)]	[((0.59,0.54,0.65),0.75,0.1,0.13), ((0.65,0.60,0.70),0.8,0.1,0.1)]	[((0.40,0.35,0.50), 0.6,0.1,0.2), ((0.53,0.48,0.6),0.7,0.1,0.15)]
	DM3	[((0.40,0.35,0.50), 0.6,0.1,0.2), ((0.6,0.55,0.65),0.78,0.13,0.13)]	[((0.40,0.35,0.50), 0.6,0.1,0.2), ((0.59,0.54,0.65),0.75,0.1,0.13)]	[((0.40,0.35,0.50), 0.6,0.1,0.2), ((0.53,0.48,0.6),0.7,0.1,0.15)]
	DM4	[((0.57,0.52,0.63),0.73,0.1,0.13), ((0.67,0.62,0.73),0.83,0.13,0.1)]	[((0.59,0.54,0.65),0.75,0.1,0.13), ((0.65,0.60,0.70),0.8,0.1,0.1)]	[((0.53,0.48,0.6),0.7,0.1,0.15), (0.65,0.60,0.70),0.8,0.1,0.1)]

Table 4: Aggregation result

	C1	C2	C3
A1	[((0.31,0.7,1),0.87,0.2,0.12), ((0.58,0.83,1),0.85,0.15,0.13)]	[((0.29,0.63,0.97),0.81,0.12,0.11), ((0.46,0.74,0.99),0.78,0.18,0.15)]	[((0.23,0.49,0.89),0.72,0.12,0.25), ((0.52,0.73,0.97),0.76,0.18,0.16)]
A2	[((0.22,0.64,1),0.84,0.16,0.11), ((0.39,0.75,0.1),0.8,0.2,0.15)]	[((0.29,0.63,0.97),0.81,0.12,0.11), ((0.46,0.74,0.99),0.78,0.18,0.15)]	[((0.25,0.52,0.91),0.75,0.16,0.15), ((0.53,0.73,0.97),0.79,0.2,0.15)]
A2	[((0.43,0.77,1),0.85,0.18,0.13), ((0.74,0.88,1),0.87,0.12,0.12)]	[((0.47,0.74,0.99),0.75,0.16,0.16), ((0.67,0.84,0.99),0.71,0.2,0.2)]	[((0.44,0.68,0.95),0.8,0.15,0.13), ((0.66,0.81,0.98),0.74,0.2,0.18)]
A4	[((0.2,0.62,0.99),0.83,0.13,0.1), ((0.23,0.64,1),0.88,0.18,0.1)]	[((0.15,0.49,0.93),0.71,0.11,0.15), ((0.27,0.61,0.97),0.84,0.15,0.11)]	[((0.2,0.46,0.88),0.71,0.14,0.16), ((0.36,0.61,0.95),0.86,0.19,0.11)]
A5	[((0.23,0.64,1),0.86,0.15,0.1), ((0.43,0.77,1),0.9,0.15,0.1)]	[((0.34,0.67,0.98),0.78,0.13,0.13), ((0.56,0.79,0.99),0.73,0.18,0.18)]	[((0.39,0.64,0.95),0.84,0.16,0.11), ((0.55,0.75,0.97),0.8,0.2,0.15)]
	C4	C5	C6
A1	[((0.43,0.6,0.88),0.81,0.11,0.1), ((0.46,0.62,0.9),0.85,0.15,0.1)]	[((0.31,0.41,0.69),0.65,0.11,0.18), ((0.48,0.57,0.81),0.8,0.15,0.13)]	[((0.49,0.49,0.66),0.71,0.1,0.15), ((0.59,0.59,0.73),0.79,0.1,0.11)]
A2	[((0.31,0.49,0.82),0.73,0.13,0.15), ((0.46,0.62,0.91),0.87,0.18,0.11)]	[((0.37,0.46,0.74),0.71,0.14,0.16), ((0.53,0.61,0.86),0.86,0.19,0.11)]	[((0.42,0.42,0.6),0.65,0.1,0.18), ((0.54,0.54,0.7),0.75,0.1,0.13)]
A2	[((0.51,0.66,0.93),0.89,0.2,0.11), ((0.61,0.74,0.95),0.81,0.2,0.15)]	[((0.59,0.66,0.89),0.89,0.2,0.11), ((0.69,0.74,0.91),0.81,0.2,0.15)]	[((0.61,0.6,0.75),0.81,0.11,0.1), ((0.63,0.62,0.78),0.85,0.15,0.1)]
A4	[((0.24,0.42,0.77),0.65,0.1,0.18), ((0.36,0.54,0.84),0.75,0.1,0.13)]	[((0.4,0.49,0.75),0.73,0.13,0.15), ((0.54,0.62,0.86),0.87,0.18,0.11)]	[((0.48,0.49,0.65),0.71,0.11,0.15), ((0.61,0.61,0.77),0.84,0.15,0.11)]
A5	[((0.25,0.43,0.78),0.65,0.12,0.19), ((0.52,0.68,0.92),0.73,0.18,0.18)]	[((0.39,0.49,0.73),0.71,0.11,0.15), ((0.53,0.61,0.83),0.84,0.15,0.11)]	[((0.48,0.49,0.65),0.71,0.11,0.15), ((0.61,0.61,0.77),0.84,0.15,0.11)]

Table 4 (continued).

	C7	C8	C9
A1	[((0.57,0.49,0.56),0.71,0.11,0.15), ((0.7,0.61,0.7),0.84,0.15,0.11)]	[((0.55,0.37,0.37),0.61,0.1,0.2), ((0.65,0.46,0.45),0.69,0.1,0.16)]	[((0.76,0.49,0.38),0.71,0.11,0.15), ((0.85,0.61,0.53),0.84,0.15,0.11)]
A2	[((0.57,0.49,0.56),0.71,0.11,0.15), ((0.7,0.61,0.7),0.84,0.15,0.11)]	[((0.67,0.49,0.47),0.71,0.11,0.15), ((0.78,0.61,0.61),0.84,0.15,0.11)]	[((0.68,0.41,0.32),0.65,0.11,0.18), ((0.82,0.57,0.5),0.8,0.15,0.13)]
A2	[((0.58,0.49,0.59),0.73,0.13,0.15), ((0.71,0.62,0.74),0.87,0.18,0.11)]	[((0.67,0.49,0.49),0.73,0.13,0.15), ((0.79,0.62,0.66),0.87,0.18,0.11)]	[((0.76,0.49,0.4),0.73,0.13,0.15), ((0.85,0.62,0.59),0.87,0.18,0.11)]
A4	[((0.57,0.49,0.56),0.71,0.11,0.15), ((0.7,0.61,0.7),0.84,0.15,0.11)]	[((0.59,0.41,0.41),0.65,0.11,0.18), ((0.74,0.57,0.58),0.8,0.15,0.13)]	[((0.64,0.37,0.29),0.61,0.1,0.2), ((0.73,0.46,0.36),0.69,0.1,0.16)]
A5	[((0.57,0.49,0.56),0.71,0.11,0.15), ((0.7,0.61,0.7),0.84,0.15,0.11)]	[((0.68,0.49,0.48),0.71,0.1,0.15), ((0.76,0.59,0.56),0.79,0.1,0.11)]	[((0.69,0.42,0.32),0.65,0.1,0.18), ((0.8,0.54,0.43),0.75,0.1,0.13)]

Table 5: Normalized matrix

	C1	C2	C3	C4	C5	C6	C7	C8	C9
A1	[0.48,0.66]	[0.69,0.59]	[0.05,0.53]	[0.52,0.60]	[0.00,0.00]	[0.37,0.56]	[0,0]	[0,0]	[0.76,0.88]
A2	[0.08,0.19]	[0.69,0.59]	[0.26,0.61]	[0.29,0.59]	[0.21,0.21]	[0,0]	[0,0]	[0.94,0.12]	[-2.44,0.64]
A3	[1,1]	[1.00,1]	[1,1]	[1,1]	[1,1]	[1,1]	[1,1]	[1,1.204	[1,1]
A4	[0,0]	[0,0]	[0,0]	[0,0]	[0.315,0.3147]	[0.3304,0.8185]	[0,0]	[0.62,0.97]	[0.0000,0.0000]
A5	[0.2,0.48]	[0.78,0.77]	[0.92,0.75]	[0.01,0.50]	[0.28,0.28]	[0.33,0.82]	[0,0]	[1,1]	[-2.14,0.45]

To evaluate the weights of the criteria, BWM is applied. Decision-makers define the product quality as the most desired criterion and accuracy in filling the order as the least preferred criterion. According to the importance rating scale, best-to-other, and others-to-worst vectors were determined as in Tabs. 6 and 7. After applying the BWM model, the weight vector resulted is presented in Tab. 8. The result of the BWM to weight the set of criteria shows that the product quality (C1) criterion is the highest with weight 0.315, and the lowest one has the accuracy in filling the order (C9) with weight 0.027. The rest of the criteria are arranged as follows: supplier expenditure emergency order (C2) with weight 0.192, technology service: problem-solving (C4) with weight 0.128, supplier adequately test new products (C3) with weight 0.096, technology service: responsiveness (C5) with weight 0.077, the supplier provides notice of product problems (C7) with weight 0.064, the supplier provides technical assistance (C6) with weight 0.055, and consistency of delivered product (C8) with weight 0.048.

Based on the weight vector determined by BWM, the weighted matrix (Tab. 9) was calculated as Eq. (18) shows. According to Eqs. (19), (20), rough BAA can be calculated, as shown in Tab. 10.

Table 6: Best-to-others vector

Best-to-Others	C1	C2	C3	C4	C5	C6	C7	C8	C9
C1	0.1	0.2	0.4	0.3	0.5	0.7	0.6	0.8	0.9

Table 7: Other-to-worst vector

Others-to-Worst	C9
C1	0.9
C2	0.8
C3	0.6
C4	0.7
C5	0.5
C6	0.3
C7	0.4
C8	0.2
C9	0.1

Table 8: Weight vector

	C1	C2	C3	C4	C5	C6	C7	C8	C9
Weights	0.315	0.192	0.096	0.128	0.077	0.055	0.064	0.048	0.027

Table 9: Weighted matrix

	C1	C2	C3	C4	C5	C6	C7	C8	C9
A1	[0.46,0.52]	[0.32,0.31]	[0.1,0.15]	[0.19,0.20]	[0.08,0.08]	[0.08,0.09]	[0.06,0.06]	[0.05,0.05]	[0.05,0.05]
A2	[0.34,0.37]	[0.32,0.31]	[0.1,0.15]	[0.16,0.20]	[0.09,0.09]	[0.06,0.06]	[0.06,0.06]	[0.09,0.09]	[-0.04,0.04]
A3	[0.63,0.63]	[0.38,0.38]	[0.1,0.19]	[0.26,0.26]	[0.15,0.15]	[0.11,0.11]	[0.13,0.13]	[0.09,0.09]	[0.05,0.05]
A4	[0.32,0.32]	[0.19,0.19]	[0.1,0.10]	[0.13,0.13]	[0.10,0.10]	[0.07,0.07]	[0.06,0.06]	[0.08,0.08]	[0.03,0.03]
A5	[0.38,0.47]	[0.34,0.34]	[0.1,0.17]	[0.13,0.19]	[0.10,0.10]	[0.07,0.07]	[0.06,0.06]	[0.10,0.10]	[-0.03,-0.03]

Table 10: BAA matrix

	C1	C2	C3	C4	C5	C6	C7	C8	C9
BAA	[0.4128, 0.4489]	[0.3020, 0.2982]	[0.1329, 0.1476]	[0.169,0.192]	[0.102,0.102]	[0.075, 0.075]	[0.074, 0.074]	[0.079, 0.079]	[0.038, 0.038]

The distance of the alternatives from the rough BBA is obtained using Eq. (21), and the ranking of the alternatives based on the range is calculated according to Eq. (22); the results are expressed in Tab. 11. As Fig. 4 shows, the third alternative is the best for this private healthcare company, while alternative 4 is the least preferred one. By applying the proposed approach in a supplier selection problem in the healthcare industry, the results show that:

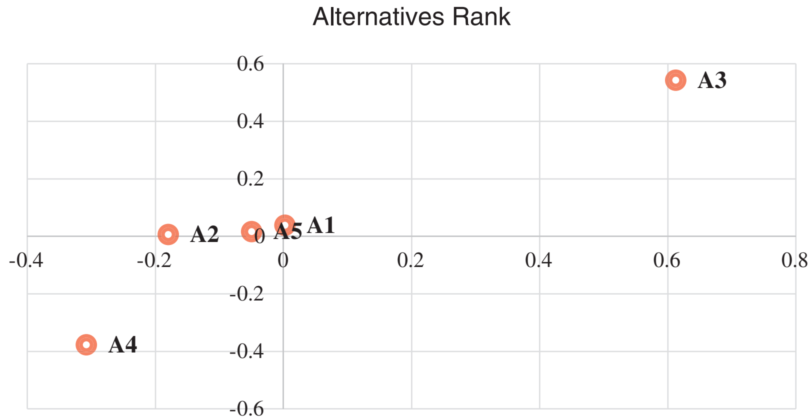

Figure 4: Ranking of the alternatives

Table 11: Evaluating results of alternatives

	C1	C2	C3	C4	C5	C6	C7	C8	C9	Sum	Rank
A1	[0.0472, 0.0711]	[0.0180, 0.0118]	[−0.0319, −0.0006]	[0.026, 0.013]	[−0.025, −0.025]	[0.000, 0.000]	[−0.010, −0.010]	[−0.031, −0.031]	[0.009, 0.009]	[0.0023, 0.0382]	2
A2	[−0.0728, −0.0789]	[0.0180, 0.0118]	[−0.0119, 0.0064]	[−0.004, 0.011]	[−0.009, −0.009]	[−0.020, −0.020]	[−0.010, −0.010]	[0.014, 0.014]	[−0.077, −0.077]	[−0.1798, 0.0061]	4
A3	[0.2172, 0.1811]	[0.0780, 0.0818]	[0.0591, 0.0444]	[0.087, 0.064]	[0.052, 0.052]	[0.035, 0.035]	[0.054, 0.054]	[0.014, 0.014]	[0.016, 0.016]	[0.6123, 0.5422]	1
A4	[−0.0928, −0.1289]	[−0.1120, −0.1082]	[−0.0369, −0.0516]	[−0.041, −0.064]	[−0.001, −0.001]	[−0.002, −0.002]	[−0.010, −0.010]	[−0.001, −0.001]	[−0.011, −0.011]	[−0.3077, −0.3777]	5
A5	[−0.0328, 0.0211]	[0.0380, 0.0418]	[0.0511, 0.0204]	[−0.039, 0.000]	[−0.003, −0.003]	[−0.002, −0.002]	[−0.010, −0.010]	[0.017, 0.017]	[−0.069, −0.069]	[−0.0497, 0.0163]	3

- Decision-makers and experts in the healthcare sector defined nine criteria that control supplier selection decisions among five suppliers. Using the Best-Worst method, we found that the product quality (C1) criterion has the highest weight (0.315), and the lowest one has the accuracy in filling the order (C9) (0.027).
- The order of the weight vector is as follows: C1 > C2 > C4 > C3 > C5 > C7 > C6 > C8 > C9. One of the main features of BWM is the consistency ratio that measures how consistent is the comparison. The consistency ratio is 0.0129, which means that the model is compatible.
- Using the MABAC method, the best supplier for the company based on the previously established set of criteria is the third supplier, and the least preferred is the fourth supplier. The first and the third supplier are in the upper approximation area, while the second, the fourth, and the fifth are in the lower approximation area, as shown in Fig. 4. The ranking of the suppliers on this case study is as follows: A3 > A1 > A5 > A2 > A4. As Fig. 4 shows, the alternative A3 is located at the upper corner of the upper approximation area. followed by the alternative A1 at the lower corner of the area, while the alternative A4 is located at the end of the lower approximation area, so it is the most anti-ideal alternative and at the end of the ranking.
- This study supports health care managers in looking at the problem of selecting the optimal supplier in more detail and taking into account the factor of uncertainty to which the decision-maker may be exposed to in the evaluation processes in a significant way.

6 Conclusions

Supplier selection is a major supply chain problem that has a lot of uncertain information, which leads to a difficult decision-making process. An integrated approach for the manipulation of uncertainty in the supplier selection process based on RNs and plithogenic set theory was applied. Firstly, we discussed the concept of plithogenic set and its aggregation operator that improve the aggregation with high consideration of uncertainty. Then, we considered the transformation of evaluation values into rough numbers to cover the upper and lower evaluation of the decision. Thirdly, the BWM was applied to evaluate and conclude the weight of the criteria. Finally, we used the MABAC method to rank the set of alternate suppliers based on the defined criteria weight that was computed by the BWM.

Our approach provides an exceptional level of consideration of uncertainty. Firstly, the group decision evaluation is presented as rough numbers that observe upper and lower approximation limits of the decision which handle the diversity of DMs' judgments. Secondly, the assessment of the DMs was combined using plithogenic aggregation operation, which considers the contradiction degree to ensure more accurate aggregated results. Thirdly, BWM was applied to identify the weight vector of the criteria, which was considered as a useful and straightforward pairwise comparison method. The consistency ratio of the BWM evaluation was computed to evaluate how consistent is the evaluation. Finally, the MABAC method was applied to assess the set of alternate suppliers using criteria weights resulted from BWM.

Some deficiency of this study is that of not highlighting the priority of decision-makers, in order to get a comprehensive view of the problem and to get the best evaluation that takes into account the weights and priority of decision-makers according to the problem. This study also needs to make comparisons of the results found through other methods. Our recommendation for the further use of this approach is to employ it in decision-making problems from different fields. Moreover, the weight vector of the criteria could be computed by various MCDM methods, instead of BWM. Also, the plithogenic set operators should add an advantage to other decision making approaches.

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COVID-19 Decision-Making Model using Extended Plithogenic Hypersoft Sets with Dual Dominant Attributes

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Abstract

Plithogenic Hypersoft set is the multi argument function with plithogenic universe of discourse and single dominant attribute value. The theory of plithogenic sets deals with the attributes and it is of the form (P, a, V, d, c) characterized by the degree of appurtenance and contradiction. This paper introduces the approach of plithogenic hypersoft sets with two independent dominant attribute values pertaining to each attribute to handle the dual system of decision making. The proposed decision making model is validated with the data of the present COVID -19 pandemic situations. The objective of the model is to rank the patients being identified as asymptotic and affected using Frequency Matrix Multi Attributes Decision making system. Combined plithogenic hypersoft representations of the degree of appurtenance between the patients and the attribute values make the decision-making model more comprehensive and feasible. The developed model can be extended to other decision making environment with various forms of degree of appurtenance.

Keywords: Plithogenic Hypersoft sets, dual Dominant attributes, decision making, pandemic, COVID-19

1.Introduction

Decision-making process is characterized by a sequence of integrated and interconnected activities in arriving at an optimal solution to the decision making problem. The elements of decision-making comprises of alternatives, criteria along with their degree of association. On profound analysis, the deterministic nature of the criterion satisfaction rate by the alternatives do not provide a complete representation of the decision- making problem as it fails to handle uncertainty and impreciseness that exists in reality. The introduction of fuzzy sets by Zadeh [1] plays a significant role in tackling uncertain decision making environment. Fuzzy sets differ from crisp sets in its representation and membership values. The membership values of the crisp set belong to $\{0,1\}$ and the membership values of the fuzzy set belong to $[0,1]$. Fuzzy decision making methods are applied by the researchers to several decision making problems due to the flexibility in representation and resolving of uncertainty. To mention a few, Coroiu [2] applied the strategy of fuzzy decision making in manufacturing systems. Wei et al [3] applied fuzzy decision making tactics in developing new project. Wang et al discussed multi criteria decision making in fuzzy environment.

Soft set theory formulated by Molodstov [4] is yet another domain that predominantly deals with uncertainties and it is widely applied to the field of decision making. The consensus that exists in the objective of fuzzy sets and soft sets has motivated Maji et al [5] to introduce fuzzy soft sets and it has vast applications in various domains of decision-making.

Qinrong Feng et al [6] discussed fuzzy soft sets in group decision making. Muhammad Naveed [7] used fuzzy soft sets in decision making on finding the optimal technique of weight loss. Zhicai Liu [8] developed fuzzy soft set decision making model for determining the ideal solution. Smarandache [9] generalized soft sets to hypersoft sets and fuzzy hypersoft sets and its application in frequency matrix multi attribute decision making technique was discussed by Sagaya Bavia et al. [10].With the introduction of Intuitionistic sets by Atannsov [11], fuzzy soft sets were extended to intutionistic fuzzy soft sets. The intutionistic sets differ from fuzzy sets in its membership value. The former contains both membership and the non-membership values where the latter deals only with membership values. IFSS and intutionistic fuzzy hypersoft sets has wide applications in several decision making scenarios. Cagman, Naim [12] discussed the application of intuitionistic soft sets. Irfan Deli [13] presented the intuitionistic parameterized soft set theory and its applications. Chunqiao et al [14] generalized IFSS and explained its relation with decision making methods on multi attribute.Rana Muhammad [15] extended the method of TOPSIS with the approach of intuitionistic hyper soft set. Babak [16] , Park et al [17], Chetia et al[18] , Das [19] presented the applications of intutionistic fuzzy soft sets in decision making. Smarandache [20-22] introduced Neutrosophic sets which is of comprehensive in nature. Neutrosophic soft sets is an extension of IFSS and the neutrosophic representations consists of truth values, indeterminate values and falsity values and plays a crucial role in expressing the existential nature of the problem and it is an added advantage. Faruk Karaaslan [23] discussed the applications of neutrosophic soft sets in diverse scenarios. Sudan Jha [24] analyzed the stock trends using neutrosophic soft sets. Muhammad Saqlain et al [25] have defined aggregate operators of neutrosophic hypersoft sets and its applications in decision making. Nidhi et al [26] studied multi criteria group decision making in neutrosophic environment. Muhammad Riaz et al[27]discussed the applications of neutrosophic soft rough topology in decision making. Abhijit et al [28] presented neutrosophic approach in multi attribute decision making. Sarwar et al [29], Deli & Broumi [30], Abdel et al [31-32] studied the intervention of the theory of soft neutrosophy in decision making on various scenario. It is very vivid from the literature that researchers have developed decision making models using both the approaches of soft sets and hypersoft sets with various representations of fuzzy, intuitionistic and neutrosophic sets.

Smarandache [33] has given an excellent generalization of the crisp sets, fuzzy sets, intuitionistic fuzzy sets and neutrosophic sets in the name of Plithogenic sets. The concept of plithogeny is gaining momentum in the field of decision making. A plithogenic set of the form (P,a,V,d,c), where P is a set, a, the set of attribute values, V, the attribute range, d, the degree of appurtenance and c the degree of contradiction. The degree of appurtenance and the degree of contradiction of the attribute values are determined with respect to a dominant attribute value. This paper focusses on Plithogenic sets because of its robust nature and are extensively applied in various domains of decision making. Researchers have extended and discussed several concepts in plithogenic environment. To mention a few, Nivetha and Smarandache introduced the concepts of concentric plithogenic hypergraph [34], plithogenic –n-super hypergraph [35], plithogenic cognitive maps [36], plithogenic- sub cognitive maps [37] and discussed its efficiency in decision making.

Plithogenic sets play a significant role in multi-criteria decision making (MCDM). Abdel-Basset et al formulated MCDM models with the intervention of plithogenic sets to make optimal decisions on green supply chain management [38,39], medical diagnosis , IoT [40]. Decision making models with quality function deployment and plithogenic approach was also constructed for making optimal selection of sustainability metrics of supply chain. Plithogenic sets are used predominantly to handle decision making environment involving several attributes and multi attribute values. The hypersoft sets dealing with the utility of multi attribute function in making decisions were further extended to plithogenic hypersoft sets by Smarandache [41] which was extended to plithogenic fuzzy whole hypersoft set by Shazia et al [42] and it was applied to frequency matrix multi attribute technique of making decisions with new development of operators. Nivetha and Smarandache [43] introduced the notion of combined plithogenic hypersoft sets and applied the same approach to frequency matrix in multi attribute decision making. In combined plithogenic hypersoft sets, the degree of appurtenance is a combination of either crisp/fuzzy/intuitionistic/neutrosophic values. Shazia et al [44] introduced the concept of plithogenic subjective hyper-super- soft matrices to rank the alternatives subjectively at local, global and universal levels. Muhammad Rayees [45] et al developed a new MCDM method based on plithogenic hypersoft set with neutrosophic degree of appurtenance. In all of the above decision making models with plithogenic hypersoft sets only one dominant attribute was considered, but in this paper the concept of extended plithogenic hypersoft set is introduced with dual dominant attribute values. The decision making problems with single dominant attribute value helps in handling only one phenomenon, but if dual dominant attribute values are considered, decision making on two different phenomena shall take place simultaneously. The proposed concept is novel and it will certainly enable the decision makers to make decisions based on two distinct aspects as it provides opportunity to lay a special focus on the two entities of dominant attribute values.

The paper is organized as follows: section 2 presents the extended plithogenic hypersoft sets with two dual attribute values; section 3 validates the significance of the extended plithogenic hypersoft sets with application to COVID 19; section 4 discusses the results and the last section concludes the research work.

2. Extended Plithogenic Hypersoft sets

This section presents the need of extending and discussing the extended plithogenic hypersoft sets in decision making by taking the basic preliminaries related to plithogenic hypersoft sets discussed by Smarandache. This section will also discuss extended combined plithogenic hypersoft sets. Let us consider a conventional example of plithogenic hypersoft set with single dominant attribute value.

Let U be the universe of discourse that consists of online teaching tools say $U = \{T_1,T_2,T_3,T_4,T_5\}$ and the set $M =\{T_2,T_4\} \subset U$.

The attributes are a_1 = pricing, a_2 = flexibility, a_3 = Interactive, a_4 = Special Features. The realistic attribute values are A_i (i = 1,2,3,4) corresponding to each attributes a_i are

$A_1 = \{A_1^1, A_1^2, A_1^3\}$={**low**, medium, high}, $A_2 = \{A_2^1, A_2^2, A_2^3\}$={low, medium, **high**}, $A_3 = \{A_3^1, A_3^2, A_3^3\}$= {less, moderate, **high**},

$A_4 = \{A_4^1, A_4^2, A_4^3\}$={minimum, moderate, **maximum**}

If any of the educational institutions is to make a decision on the practice of the online teaching tools, the following conventional considerations are considered with respect to a single dominant attribute value corresponding to each attribute

Let the function be: $G: A_1^1 \times A_2^3 \times A_3^3 \times A_4^3 \longrightarrow P(U)$

Let's assume: G ({low, high, high, maximum}) = $\{T_2,T_4\}$.

The degree of appurtenance states the satisfaction rate of the attribute value by the elements of M and that helps in decision making between the alternatives T_2,T_4.

In the above example only two online tools fulfill the dominant attribute values of each of the attributes. But in decision making we focus not only the best alternatives but also on the worst alternatives so as to make a comprehensive decision making. Now let us consider a situation where we compare all the alternatives with respect to two dominant attribute values so as to take decisions on choosing and rejecting the alternatives.

The attributes are a_1 = pricing, a_2 = flexibility, a_3 = Interactive, a_4 = Special Features. The realistic additional attribute values are A_i (i = 1,2,3,4) corresponding to each attributes a_i are

$A_1 = \{ A_1^1, A_1^2, A_1^3, A_1^4\}$={low, medium, high, **free of cost**}, $A_2 = \{ A_2^1, A_2^2, A_2^3, A_2^4\}$= {low, medium, **high,** nil}, $A_3 = \{ A_3^1, A_3^2, A_3^3, A_3^4\}$={less, more, **high**, nil}, $A_4 = \{ A_4^1, A_4^2, A_4^3, A_4^4\}$= { minimum, moderate, **maximum**, nil}
Let us define a function G_1: $A_1^4 \times A_2^3 \times A_3^3 \times A_4^3 \longrightarrow P(U)$, which considers the attribute values pertains to make decisions on the desirable and feasible online teaching tools.
Let us define another function G_2: $A_1^3 \times A_2^1 \times A_3^1 \times A_4^1 \longrightarrow P(U)$, which considers the attribute values pertains to make decisions on the online teaching tools that are infeasible in nature. $A_1 = \{ A_1^1, A_1^2, A_1^3, A_1^4\}$={low, medium, **high**, free of cost}, $A_2 = \{ A_2^1, A_2^2, A_2^3, A_2^4\}$= {**low**, medium, high, nil}, $A_3 = \{ A_3^1, A_3^2, A_3^3, A_3^4\}$={**less**, more, high, nil}, $A_4 = \{ A_4^1, A_4^2, A_4^3, A_4^4\}$= { **minimum**, moderate, maximum, nil}

Each of the alternatives has two degrees of appurtenance corresponding to feasible and infeasible online teaching tools with respect dual dominant attribute values corresponding to each attribute. The distinctive nature of the attribute values enables us to make optimal decision based on ranking of the alternatives related to two different aspects.
G_1 ({free, high, high, maximum}) = { T_1 (1,0.8,0.7,0.1), T_2 (0.1,0.9,0.4,0.5), T_3 (0.3,0.2,0.5,0.8),T_4 (0.4,0.1,0.5,0.9), T_5 (0.2,0.3,0.4,0.7)}
G_2 ({high, nil, nil, nil}) = { T_1 (0,0.2,0.2,0.9), T_2 (0.9,0.1,0.7,0.2), T_3 (0.8,0.9,0.3,0.1),
T_4 (0.7,0.9,0.6,0.1), T_5 (0.6,0.7,0.9,0.1)}
The extended plithogenic hypersoft set can be discussed in case of combined plithogenic hypersoft set environment as discussed in [29]
G_1 ({free, high, high, maximum}) = { T_1 (1,0.8,(0.7,0.2),0.1), T_2 (0.1,0.9,0.4,(0.5,0.1,0.2)), T_3 (0.3,0.2,0.5,(0.8,0.1)), T_4 (0.4,(0.1,0.7),0.5,(0.9,0.1)), T_5 ((0.2,0.7),0.3,(0.4,0.7),0.7)}
G_2 ({high, nil, nil, nil}) = {T_1 (0,(0.2,0.7),0.2,0.9),T_2(0.9,0.1,0.7,(0.2,0.1,0.7)),
T_3 (0.8,0.9,(0.3,0.6),0.1),T_4(0.7,(0.9,0.1),0.6,0.1),T_5(0.6,(0.7,0.1,0.2),0.9,0.1)}

Thus the plithogenic hypersoft sets that dealt with only one dominant attribute has been extended to plithogenic hypersoft sets with two domination attribute values. This kind of extension plays a significant role in making decisions on both the feasible and infeasible tools of online teaching and also all the alternatives are taken into account. The extended plithogenic hypersoft sets integrated with combined plithogenic hypersoft sets representation is highly pragmatic and will certainly help in making optimal decisions.

3. Application of Extended Plithogenic hypersoft sets in Decision Making

The theoretical development of extended plithogenic hypersoft sets with dual dominant attribute values is validated with a real time data in this section. Presently the entire world is suffering from the consequences of COVID-19. Each nation strengthens its medical emergency to identify the symptomatic and asymptomatic COVID-19 patients to isolate the infected patients to avoid the transmission of the virus. Let U be the set of patients who are to be classified as symptomatic and asymptotic for further treatment. Let the attributes be A_1 = fever, A_2 = body pain, A_3 = Cough, A_4 = Cold, A_5 = Breathing, A_6 = Loss of senses, A_7 = Fatigue
The attribute values are
A_1 = {3-5 days, 1-2 days, no}={A_1^1,A_1^2,A_1^3}
A_2 = {moderate, low, High, nil} = {A_2^1,A_2^2, A_2^3, A_2^4}

A_3 = { moderate, severe, mild, nil}={A_3^1,A_3^2,A_3^3,A_3^4}

A_4 = {5-7 days, 3-4 days, 1-2 days, no}={A_4^1,A_4^2,A_4^3,A_4^4}

A_5 = {Shortness of breath, normal} = {A_5^1,A_5^2}

A_6 = { short term , long term ,nil} = {A_6^1,A_6^2,A_6^3}

A_7 = { severe, less, energetic} = {A_7^1,A_7^2}

There are two dominant attribute values pertaining to each attribute, say for the attribute fever the two dominant attribute values are A_1^1, A_1^3, where A_1^1 is related to symptomatic and A_1^3 is related to asymptomatic and these two dominant attribute values are independent of each other, similarly each attribute has dual dominant attribute values. Table 3.1 presents the dual dominant values of the attributes.

Attribute	**Dominant Attribute value of symptomatic**	**Dominant Attribute value of asymptomatic**
A_1	A_1^1	A_1^3
A_2	A_2^3	A_2^4
A_3	A_3^2	A_3^4
A_4	A_4^1	A_4^4
A_5	A_5^1	A_5^2
A_6	A_6^2	A_6^3
A_7	A_7^1	A_7^2

Let U = { P_1,P_2,P_3,P_4,P_5}

Let us define a function F_1: $A_1^1 \times A_2^3 \times A_3^2 \times A_4^1 \times A_5^1 \times A_6^2 \times A_7^1 \longrightarrow P(U)$ with fuzzy degree of appurtenance and these attribute values are pertaining to the symptomatic patients based on the reliable data source and eminent experts of the field.

F_1({3-5 days, High, severe, 5-7 days, Shortness of breath , long term, severe}) = {P_1(0.4,0.5,0.4,0.3,0.8,0.5,0.3),P_2(0.8,0.6,0.9,0.5,0.7,0.8,0.9),P_3(0.1,0.1,0.2,0.4,0.2,0.3,0.4),P_4(0.6,0.4,0.7,0.9,0.5,0.7 ,0.6),P_5 (0.1,0.2,0.1,0.1,0.2,0.3,0.1)}.

Let us define a function F_2: $A_1^4 \times A_2^4 \times A_3^4 \times A_4^4 \times A_5^4 \times A_6^4 \times A_7^4 \longrightarrow P(U)$ with fuzzy degree of appurtenance and these attribute values are pertaining to the asymptomatic patients based on the reliable data source and eminent experts of the field.

F_2({no,nil,nil,no,normal,nil,energetic})={P_1(0.6,0.6,0.5,0.6,0.7,0.5,0.7),P_2(0.1,0.2,0.1,0.2,0.3,0.2,0.4),P_3(0.8,0.7,0.6, 0.8,0.6,0.7,0.9),P_4(0.2,0.7,0.2,0.2,0.1,0.3,0.2),P_5(0.7,0.8,0.7,0.6,0.7,0.8,0.7)}

The matrix representation based on the symptomatic attribute values is

$$\begin{array}{c} \\ P_1 \\ P_2 \\ P_3 \\ P_4 \\ P_5 \end{array} \begin{array}{c} \begin{array}{ccccccc} A_1^1 & A_2^3 & A_3^2 & A_4^1 & A_5^1 & A_6^2 & A_7^1 \end{array} \\ \begin{pmatrix} 0.4 & 0.5 & 0.4 & 0.3 & 0.8 & 0.5 & 0.3 \\ 0.8 & 0.6 & 0.9 & 0.5 & 0.7 & 0.8 & 0.9 \\ 0.1 & 0.1 & 0.2 & 0.4 & 0.2 & 0.3 & 0.4 \\ 0.6 & 0.4 & 0.7 & 0.9 & 0.5 & 0.7 & 0.6 \\ 0.1 & 0.2 & 0.1 & 0.1 & 0.2 & 0.3 & 0.1 \end{pmatrix} \end{array}$$

The matrix representation based on the asymptomatic attribute values is

$$\begin{array}{c} \\ P_1 \\ P_2 \\ P_3 \\ P_4 \\ P_5 \end{array} \begin{array}{c} \begin{array}{ccccccc} A_1^4 & A_2^4 & A_3^4 & A_4^4 & A_5^4 & A_6^4 & A_7^4 \end{array} \\ \begin{pmatrix} 0.6 & 0.6 & 0.5 & 0.6 & 0.7 & 0.5 & 0.7 \\ 0.1 & 0.2 & 0.1 & 0.2 & 0.3 & 0.2 & 0.4 \\ 0.8 & 0.7 & 0.6 & 0.8 & 0.6 & 0.7 & 0.9 \\ 0.2 & 0.7 & 0.2 & 0.2 & 0.1 & 0.3 & 0.2 \\ 0.7 & 0.8 & 0.7 & 0.6 & 0.7 & 0.8 & 0.7 \end{pmatrix} \end{array}$$

The ranking approach developed by Shazia Rana et al [42] is used to rank the symptomatic and asymptomatic patients. From the matrix representation based on symptomatic attribute values, the frequency matrix F = (f_{qp}) is constructed after applying maximum operator, minimum operator and average operator to each row of the symptomatic matrix representation.

The frequency matrix representing the ranking of symptomatic patients

$$\begin{array}{c} \\ P_1 \\ P_2 \\ P_3 \\ P_4 \\ P_5 \end{array} \begin{array}{c} \begin{array}{ccccc} R_1 & R_2 & R_3 & R_4 & R_5 \end{array} \\ \begin{pmatrix} 0 & 1 & 2 & 0 & 0 \\ 3 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 2 & 0 \\ 1 & 2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2 & 1 \end{pmatrix} \end{array}$$

Table 3.2 comprises of the rankings of symptomatic patients with the authenticity of measures of percentage of p^th^ position for q^th^ alternative is calculated using $\frac{\max_q f_{qp}}{\Sigma_q f_{qp}} \times 100$

Table 3.2 Ranking of Symptomatic Patients

R_1	P_2	75%
R_2	P_4	66.7%
R_3	P_1	66.7%
R_4	P_3	50%

R_5	P_5	100%

The frequency matrix representing the ranking of asymptomatic patients

$$\begin{array}{c} \\ P_1 \\ P_2 \\ P_3 \\ P_4 \\ P_5 \end{array} \begin{array}{c} \begin{array}{ccccc} R_1 & R_2 & R_3 & R_4 & R_5 \end{array} \\ \begin{pmatrix} 0 & 1 & 2 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 \\ 3 & 0 & 0 & 0 & 0 \\ 0 & 0 & 2 & 1 & 0 \\ 1 & 2 & 0 & 0 & 0 \end{pmatrix} \end{array}$$

Table 3.3 comprises of the rankings of asymptomatic patients with the authenticity of measures of percentage.

Table 3.3. Ranking of Asymptomatic Patients

R_1	P_3	75%
R_2	P_5	67%
R_3	P_1	40%
R_4	P_4	50%
R_5	P_2	100%

Let us discuss the significance of two dual dominant attributes with combined plithogenic hypersoft sets representation.

Let $U = \{ P_1,P_2,P_3,P_4,P_5,P_6,P_7\}$

Let us define a function F_1: $A_1^1 \times A_2^3 \times A_3^2 \times A_4^1 \times A_5^1 \times A_6^2 \times A_7^1 \longrightarrow P(U)$ with fuzzy degree of appurtenance and these attribute values are pertaining to the symptomatic patients.

F_1({3-5 days, High, severe, 5-7 days, Shortness of breath , long term, severe})= {P_1(0.3,(0.5,0.6),0.5,(0.3,0.6),0.8,(0.7,0.2),0.9),P_2(0.9,(0.6,0.1,0.2),0.9,0.5,0.4,0.7,(0.5,0.3)), P_3(0.2,0.1,(0.2,0.7),0.4,0.4,(0.7,0.2),0.8),P_4((0.6,(0.2,0.3),0.3,0.8,(0.9,0.1),0.7),P_5 ((0.1,0.8),0.1,0.2,(0.1,0.8),(0.7,0.1,0.2),0.5,0.4),P_6(1,1,0.9,0.3,(0.4,0.3),0.7,0.4),P_7(0.7,0.5,0.6,(0.7,0.2),1,0.8,0.3)}

Let us define a function F_2: $A_1^4 \times A_2^4 \times A_3^4 \times A_4^4 \times A_5^4 \times A_6^4 \times A_7^4 \longrightarrow P(U)$ with fuzzy degree of appurtenance and these attribute values are pertaining to the asymptomatic patients.

F_2({no,nil,nil,no,normal,nil,energetic})={P_1(0.8,0.6,0.8,(0.7,0.2),0.4,0.5,0.3),P_2(0.1,(0.1,0.1,0.8),0.1,0.4,(0.6,0.3),0.4,0.5),P_3(0.8,0.7,0.7,(0.8,0.2),0.2,(0.4,0.3),0.4),P_4(0.2,0.7,0.2,(0.1,0.8),0.7,0.3,0.2),P_5(0.9,0.7,0.7,(0.8,0.1),(0.7,0.2),0.4,0.2),P_6(0,0,0.2,0.8,(0.2,0.4),0.4),P_7(0.2,0.9,0.3,(0.1,0.8),0.7,0.5,0.6)}

The degree of appurtenance is not of same kind as it is in the combined form comprising of crisp, fuzzy, intuitionistic and neutrosophic values; the degree of appurtenance is converted to fuzzy values. The intuitionistic representation of the form (T,F) is converted to fuzzy by using $\frac{T}{T+F}$ [46]. The neutrosophic representation of the form (a,b,c) is converted to fuzzy by using $\frac{1+a-2b-c}{2}$ [47].

The modified matrix representation based on the symptomatic attribute values is

	A_1^1	A_2^3	A_3^2	A_4^1	A_5^1	A_6^2	A_7^1
P_1	0.3	0.45	0.5	0.33	0.8	0.78	0.9
P_2	0.9	0.6	0.9	0.5	0.4	0.7	0.63
P_3	0.2	0.1	0.22	0.4	0.4	0.78	0.8
P_4	0.45	0.3	0.8	0.9	0.8	0.9	0.7
P_5	0.11	0.1	0.2	0.11	0.65	0.5	0.4
P_6	1	1	0.9	0.3	0.57	0.7	0.4
P_7	0.7	0.5	0.6	0.77	1	0.8	0.3

The modified matrix representation based on the asymptomatic attribute values is

	A_1^4	A_2^4	A_3^4	A_4^4	A_5^4	A_6^4	A_7^4
P_1	0.3	0.45	0.5	0.33	0.4	0.5	0.3
P_2	0.9	0.6	0.9	0.5	0.67	0.4	0.5
P_3	0.2	0.1	0.22	0.4	0.2	0.57	0.4
P_4	0.45	0.3	0.8	0.9	0.7	0.3	0.2
P_5	0.11	0.1	0.2	0.11	0.78	0.4	0.2
P_6	1	1	0.9	0.3	0.8	0.34	0.4
P_7	0.7	0.5	0.6	0.77	0.7	0.5	0.6

The frequency matrix representing the ranking of symptomatic patients

	R_1	R_2	R_3	R_4	R_5	R_6	R_7
P_1	1	1	0	0	1	0	0
P_2	2	0	0	1	0	0	0
P_3	0	1	1	0	0	1	0
P_4	1	2	0	0	0	0	0
P_5	0	0	2	0	0	0	1

P_6	2	1	0	0	0	0	0
P_7	0	2	1	0	0	0	0

Table 3.4 comprises of the rankings of symptomatic patients with the authenticity of measures of percentage.

Table 3.4. Ranking of Symptomatic Patients

R_1	P_6	33%
R_2	P_4	29%
R_3	P_7	25%
R_4	P_2	100%
R_5	P_1	100%
R_6	P_3	100%
R_7	P_5	100%

The frequency matrix representing the ranking of asymptomatic patients

$$\begin{array}{c} \\ P_1 \\ P_2 \\ P_3 \\ P_4 \\ P_5 \\ P_6 \\ P7 \end{array} \begin{pmatrix} R_1 & R_2 & R_3 & R_4 & R_5 & R_6 & R7 \\ 0 & 0 & 1 & 0 & 1 & 1 & 0 \\ 1 & 2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 2 & 1 & 0 \\ 1 & 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 1 & 0 & 0 & 0 \end{pmatrix}$$

Table 3.5 comprises of the rankings of asymptomatic patients with the authenticity of measures of percentage.

Table 3.5. Ranking of Asymptomatic Patients

R_1	P_7	33%
R_2	P_2	67%
R_3	P_1	25%
R_4	P_6	25%
R_5	P_5	50%
R_6	P_3	50%
R_7	P_4	100%

4. Discussion

Table 3.2 and 3.3 presents the ranking of symptomatic and asymptomatic patients. The percentage of closer association of a patient P_i (i = 1,2,3,4,5) as symptomatic or asymptomatic helps in classifying the patient as corona infected or not. For instance in table 3.2 the patient P_2 is ranked in first position under symptomatic, but in table 3.3 the same patients occupies fifth rank with 100% of authenticity measure to be asymptomatic. This shows clearly that the patient P2 is symptomatic. In this same fashion each of the patients can be classified as symptomatic or asymptomatic. Thus the position of the patients is determined in both the different cases. The positions of the symptomatic and asymptomatic patients help to classify the patients based on the degree of appurtenance towards the respective attribute values of each attribute. Table 3.4 and 3.5 presents the results using combined plithogenic hypersoft sets representation. The same kind of inferences shall also be made for the results obtained. The decision making based on two dual attributes are highly realistic and it certainly pave way for making comprehensive decisions. The positional values of the patients with respect to the measures of percentage authenticity give us the optimal ranking.

Conclusion

This paper introduces a new approach of extending plithogenic hypersoft set of single dominant attribute value to plithogenic hypersoft sets with two dual dominant attribute values. The proposed approach is discussed in two environments, one with the usual representation of plithogenic hypersoft sets and the other one is the combined plithogenic hypersoft sets. The extended plithogenic hypersoft sets is validated with examples in both the environments. The proposed concept can be validated with degree of appurtenance represented as linguistic variables. This concept can be applied to various domains of decision making. This kind of approach will certainly create a favourable decision making environment. This proposed decision making model can also be discussed under plithogenic hypergraphic as a future direction of this research work.

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New Plithogenic Sub Cognitive Maps Approach with Mediating Effects of Factors in COVID-19 Diagnostic Model

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Abstract

The escalation of COVID-19 curves is high and the researchers worldwide are working on diagnostic models, in the way this article proposes COVID-19 diagnostic model using Plithogenic cognitive maps. This paper introduces the new concept of Plithogenic sub cognitive maps including the mediating effects of the factors. The thirteen study factors are categorized as grouping factors, parametric factors, risks factors and output factor. The effect of one factor over another is measured directly based on neutrosophic triangular representation of expert's opinion and indirectly by computing the mediating factor's effects. This new approach is more realistic in nature as it takes the mediating effects into consideration together with contradiction degree of the factors. The possibility of children, adult and old age with risk factors and parametric factors being infected by corona virus is determined by this diagnostic model.

Keywords: Plithogenic cognitive maps, Sub cognitive maps, Diagnostic model, COVID-19.

1 | Introduction

Robert Axelrod [1] developed cognitive maps, a graphical representation of decision maker's perception towards the problem. The factors of the problem are taken as the nodes and the influence of one factor over the other is represented by directed edges. The edge weights assume either of the values -1, 0, 1. The positive influence of one factor is indicated by 1, no influence by 0 and negative influence by -1. The edge weights in Cognitive maps are crisp in nature and it does not include the partial influence of the factors. Suppose if factor A has some influence on factor B, the edge weight is assumed to be 0 as it lacks the completeness. This limitation of cognitive maps is handled by extending cognitive maps to Fuzzy Cognitive Maps (FCM) by Kosko [2].

In FCM the edge weights take values between the range of values from [-1, 1]. Fuzzy cognitive maps consider both partial and complete influence of the factors. The applications of Fuzzy cognitive maps are extensively investigated by many researchers. Vasantha Kandasamy [3] introduced the method of Combined Disjoint Block Fuzzy Cognitive Maps to study the influence of the factors. In this method the factors are grouped into disjoint blocks and the influence of the factors is determined based on expert's opinion. Combined overlap block fuzzy cognitive maps were also introduced by Vasantha Kandasamy [3] in which the overlapping factors are subjected to investigation. These combined overlap and block FCM are extensively used to design optimal solutions to the decision – making problems on finding the effect of smart phone on children [4], role of happiness in family [5], analysis of environmental education for the next generation [6], reasons of failure of engineering students [7].

The concept of fuzzy introduced by Zadeh [8] was extended to neutrosophic sets by Smarandache [9]. Neutrosophic sets consists of truth, indeterminacy and falsity membership functions and it is significant in handling the notion of indeterminacy in decision – making environment. Neutrosophic representation of expert's opinion in problem solving scenario resolves the mishaps of indeterminacy [10]. Neutrosophic sets are widely used in making feasible decisions. Sudha et al. [11] assessed MCDM problems by the method of TODIM using neutrosophic aggregate weights.

Singh et al. [12] and [13] has used pentagonal neutrosophic numbers to investigate social media linked MCGDM skill. Paul et al. [14] developed a generalized neutrosophic solid transportation model to handle deficit supply. Das and Tripathy [15] developed multi-polar neutrosophic causal modelling for examining the state of institutional culture. Kandasamy and Smarandache introduced Neutrosophic Cognitive Maps (NCM) [16]. In NCM the edge weights are represented as neutrosophic sets. Neutrosophic cognitive maps are used in finding the risk factors of breast cancer [17], situational analysis [18], medical diagnosis [19], selection approach of leaf diagnosis [20]. The different methods of Fuzzy cognitive maps are also discussed in neutrosophic environment. Farahani et al. [21] compared combined block and overlap block NCM and applied the methods to find the hidden patterns and indeterminacies in a case study of Attention-Deficit/Hyperactivity Disorder (ADHD).

Smarandache introduced the concept of plithogeny [22]. Plithogenic sets are the extension of crisp set, fuzzy set, intutionistic and neutrosophic sets and it can be termed as higher order sets. The significance of Plithogenic sets is the incorporation of degree of appurtenance and contradiction degree with respect to the attributes of the decision-making components. Plithogenic sets are widely used in making optimal decisions. Abdel-Basset developed Plithogenic decision making model to solve supply chain problem; constructed Plithogenic MCDM approach for evaluating the financial performance of the manufacturing industries [23].

Concentric Plithogenic hypergraphs, Plithogenic n– super hypergraphs developed by Martin and Smarandache [24] and [25] finds novel applications in decion-making. Plithogenic hypersoft sets introduced by Rana et al. [26] was extended to Plithogenic fuzzy whole hypersoft set by Smarandache [27] and to combined Plithogenic hypersoft sets by Martin and Smarandache [28]. Plithogenic cognitive maps was introduced by Martin and Smarandache [29]. PCM involves contradiction degree in addition to the influence of the factors. Based on this approach of PCM the contradiction degree of the experts was also discussed.

The extension of cognitive models to FCM, NCM and PCM measures only the direct influence of one factor over another, but not the mediating effects of the factors. The factors are grouped as combined or overlapping blocks to study the effects, but the factors are not classified into various groups of factors to study the individual impacts or the combined effects of one factor of a group over the factors of other groups. The connection matrix representing the association between the factors taken for study is fully considered for determining the influence of the factors in ON position over the factors in OFF position.

In this paper the PCM introduced by Nivetha and Florentin is examined profoundly and the concept of Plithogenic sub cognitive maps are introduced to determine the mediating effects of the factors. Also the factors 13 factors taken for study are classified as grouping factors, parametric factors, risks factors and output factor. The proposed approach is modeled to diagnose Covid-19. Several Covid-19 predictive models are framed by researchers.

Ibrahim Yasser et al. [30] has used neutrosophic classifier in developing a framework COVID -19 confrontation. Khalifia [31] used the concept of neutrosophic and deep learning approach to diagnose COVID-19 chext X ray. These models are the forecasting models of COVID -19 are based on neutrosophic sets. A PCM Covid 19 diagnosis model with contradiction degree of expert's opinion is proposed by Broumi. Covid -19 predictive models based on Plithogenic sets are limited and this motivated the authors to explore more in this area of decision-making.

In all these COVID-19 neutrosophic and Plithogenic prediction models the study factors are not categorized and the mediating effects are not studied. But in this paper the mediating effects are considered to determine the cumulative effect of one factor over another together with the respective contradiction degree of the factors with respect to the dominating factor. The proposed approach of PSCM with linguistic representation of impacts based on expert's opinion will pave way for determining the true impacts of the factors. The paper is organized as follows: Section 2 presents the methodology of PSCM of determining mediating effects. Section 3 comprises of the application of the proposed approach to COVID 19 diagnosis model. Section 4 discusses the results and the last section concludes the work.

2| Methodology

2.1| Plithogenic Sub Cognitive Maps with Mediating Effects of the Factors

Plithogenic sub cognitive maps are the subgraphs of Plithogenic cognitive maps that comprises of one of the grouping factors, all the parametric factors, one of the risk factors and the output factor. In general, the factors are categorized based on the nature of the problem, but generally the factors are essentially classified as grouping factors, measuring factors, risk factors and output factors. Measuring factors are the factors that take certain parameters into consideration and sometime they are the describing factors of the grouping factors. The grouping factors are the factors based on which the Plithogenic sub cognitive maps are constructed. The vertex representing the grouping factors in the Plithogenic cognitive maps is the prime vertex in the PSCM.

Let C1, C2, C3, C4, C5, C6, C7 be the factors taken for study to explore the decision-making problem. C1 and C2 are taken as the grouping factors, C3, C4 are taken as the parametric factors, C5, C6 are taken as the risk factors and C7 is the output factor. The intra association between the factors of each kind are not considered, if considered the Plithogenic sub cognitive maps are known as Plithogenic super sub cognitive maps.

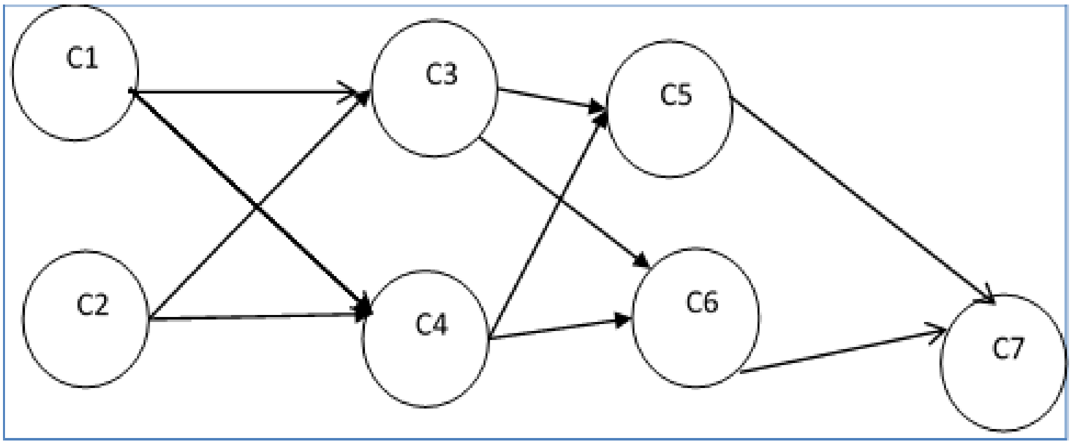


Fig. 1. The Plithogenic cognitive maps.

The Plithogenic cognitive maps represented in the *Fig. 1* presents all the possible inter relationships between the factors, to study the inter association between the grouping factors and the output factor through the mediating effects of the parametric factors and or on the risk factors, the Plithogenic sub cognitive maps are derived from Plithogenic cognitive maps.

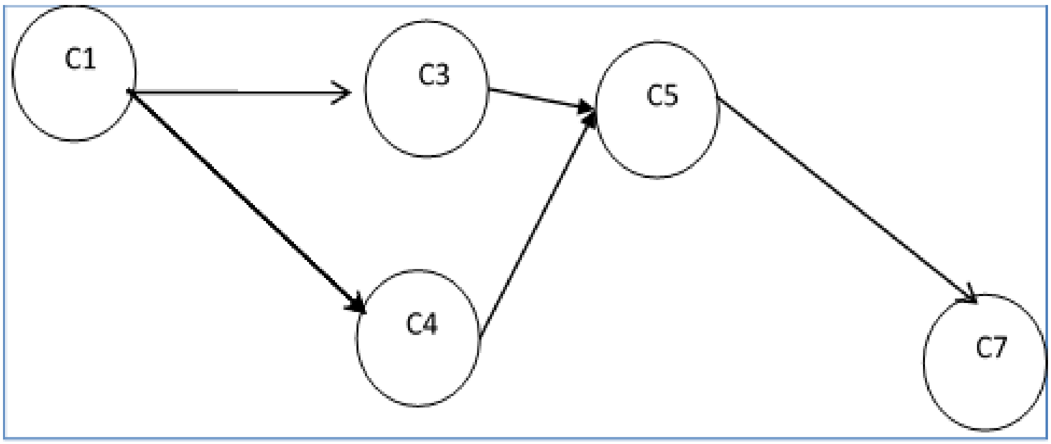
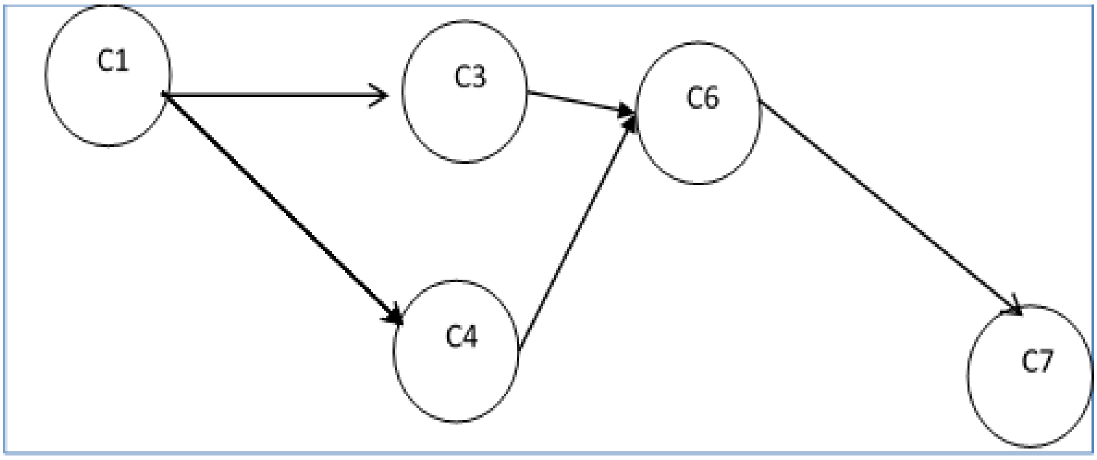
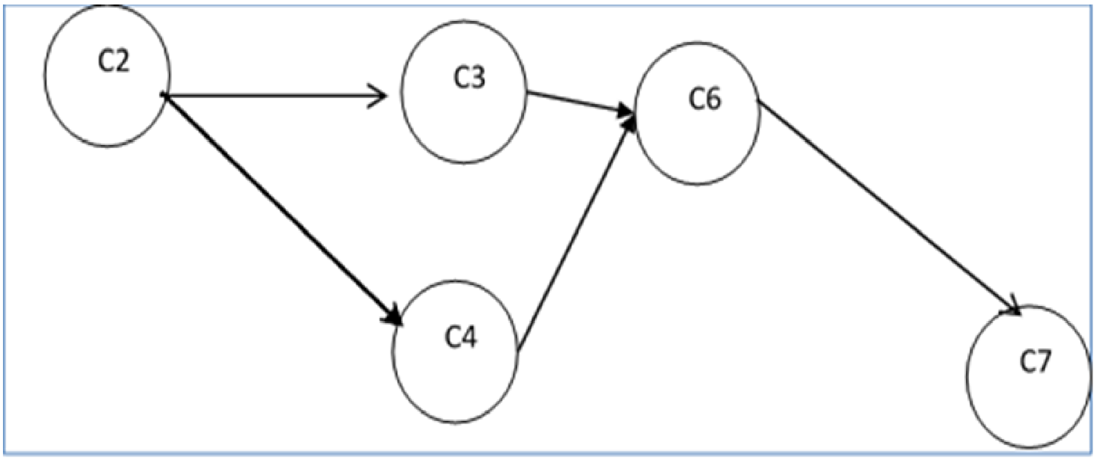


Fig. 2. Prime vertex with inter-association.

In *Fig. 2* the grouping factor C1 is taken as the prime vertex and the inter-association between the parametric factors and C5, one of the risk factors is represented to find the ultimate association between the grouping factor C1 and the output factor C7. The other Plithogenic sub cognitive maps to examine the inter associations between other set of factors can also constructed as in *Fig. 3*.



(a)



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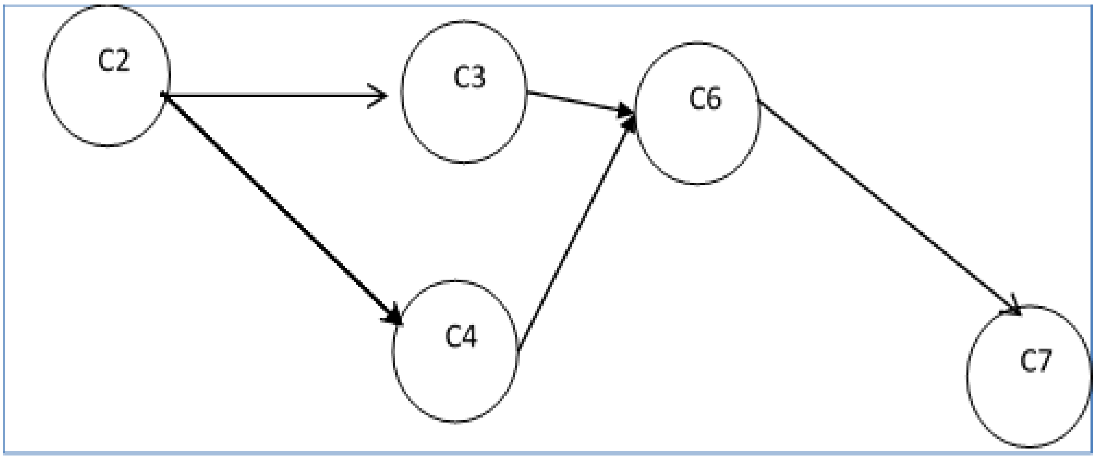


(c)

Fig 3. Plithogenic sub cognitive maps.

In each of the Plithogenic sub cognitive maps, the effects of the mediating factors representing the association between the grouping factor and the output factor are considered.

The steps involved in determining the mediating effects of the factors are presented as follows:

Step 1. The factors of the decision-making problem are categorized as grouping factors, parametric factors, risk factors and output factor.

Step 2. The Plithogenic cognitive maps representing the association between all the factors is represented graphically and the association or the connection matrix is constructed. The connection matrix M comprises of the direct effects and the indirect unknown effects between the factors in terms of linguistic variables. The association between the parametric and the risk factors are known. The levels of association between the grouping factors and the parametric factors are unknown as of certain time, until the real data is collected. The association between the grouping factor and the output factor is determined by finding the cumulative effect i.e. known and the unknown levels of impact are taken to find it. These linguistic representations are quantified using triangular neutrosophic numbers in this method, but it can be quantified by using other kinds of linguistic representations.

Step 3. The Plithogenic sub cognitive maps are constructed to determine the individual association of the grouping factor and that of the output factor. The association matrix M1 presenting the relationships between the vertices of the PSCM is derived from M. The matrix M1 also comprises of the levels of associations and the impacts.

Step 4. The factors presented under each categorization are graded as dominant when it is kept in ON position as in Plithogenic cognitive maps. The contradiction degree of the factors that are considered in PSCM are also taken into account in determining the impact of the factors.

Step 5. The factors in the PSCM are kept in ON position to determine the impact of the ON factors over the output factor and the procedure of finding the fixed point of Plithogenic cognitive maps is adopted.

3| COVID 19 Diagnostic Model

This section comprises of the formulation of COVID 19 diagnostic model. The thirteen factors taken in developing diagnostic model are categorized as grouping factors, parametric factors, risks factors and output factor. The factors C1, C2, C3 are the grouping factors based on the age group vulnerable to Covid-19, C4, C5, C6 are the parametric factors, C7, C8, C9, C10, C11, C12 are the risk factors and C13 is the output factor. The factors are

- *C1 Children,*
- *C2 Adult,*
- *C3 Old age,*
- *C4 Blood sugar level,*
- *C5 Blood pressure level,*
- *C6 State of immune system,*
- *C7 Risk of heart diseases,*
- *C8 Risk of Cancer,*
- *C9 Risk of Chronic kidney disease,*
- *C10 Asthma effect,*
- *C11 Risk of Liver disease,*

- *C12 Risk of Sickle cell disease,*
- *C13 Possibility of being Infected by Corona Virus.*

This model aims in determining the association between grouping factors and the output factor with the intervention of the mediating effects of the parametric and the risk factors. The comprehensive outlook of the association between the factors are presented graphically in *Fig. 4,* and the respective connection matrix M.

The general connection matrix with possible direct effects D (D = H or M or L) and unknown indirect cumulative effects Y & Z. The matrix also comprises of the direct effects of the parametric factors over the risk factors. The effects between the grouping factors and parametric factors are represented as D and it indicates the low or medium or high impacts between the factors. The association between the parametric factors and the risk factors are represented using linguistic variables. The cumulative effects of the grouping factors on the risk factors through the mediating effects of the parametric factors are represented as Y and on the output factor is represented as Z. The connection matrix in the Plithogenic cognitive maps presents the direct associations, but in the constructed connection matrix both the direct and the indirect associations are included to make optimal decisions and this is the distinguishing aspect of this model from the earlier models.

Based on the above description of the model with the direct and indirect effects between the factors of various kinds, the individual examination of one of the grouping factors in relation to the parametric factors and one of the risk factors subjected to the output factor is modelled as follows using the notion of Plithogenic sub cognitive maps. Let us now determine the association between the grouping factor C2 and the output factor C13 with the consideration of the mediating effects. The association refers to the relational impacts between adults and the possibility of being infected by corona. To determine the associational impacts, firstly the relational impacts of the adult [chosen for study] with the parametric factors are initially assigned as linguistic variables on investigation. The levels of the relational impacts are either high or medium or low or sometimes in combination.

Let us consider the factor C2 to be dominant amidst the grouping factors, C4 to be dominant amidst parametric factors and C7 to be the dominant among the risk factors. The associational impact between the adult [possessing the factors C4, C5 and C6 at various levels and the risk factor C7] and the possibility of being infected by Corona Virus is determined as follows. The contradiction degree of the factors is presented.

Table1. Contradiction degree of the factors.

C1	C2	C3
1/3	0	2/3

C4	C5	C6
0	1/3	2/3

C7	C8	C9	C10	C11	C12
0	1/6	2/6	3/6	4/6	5/6

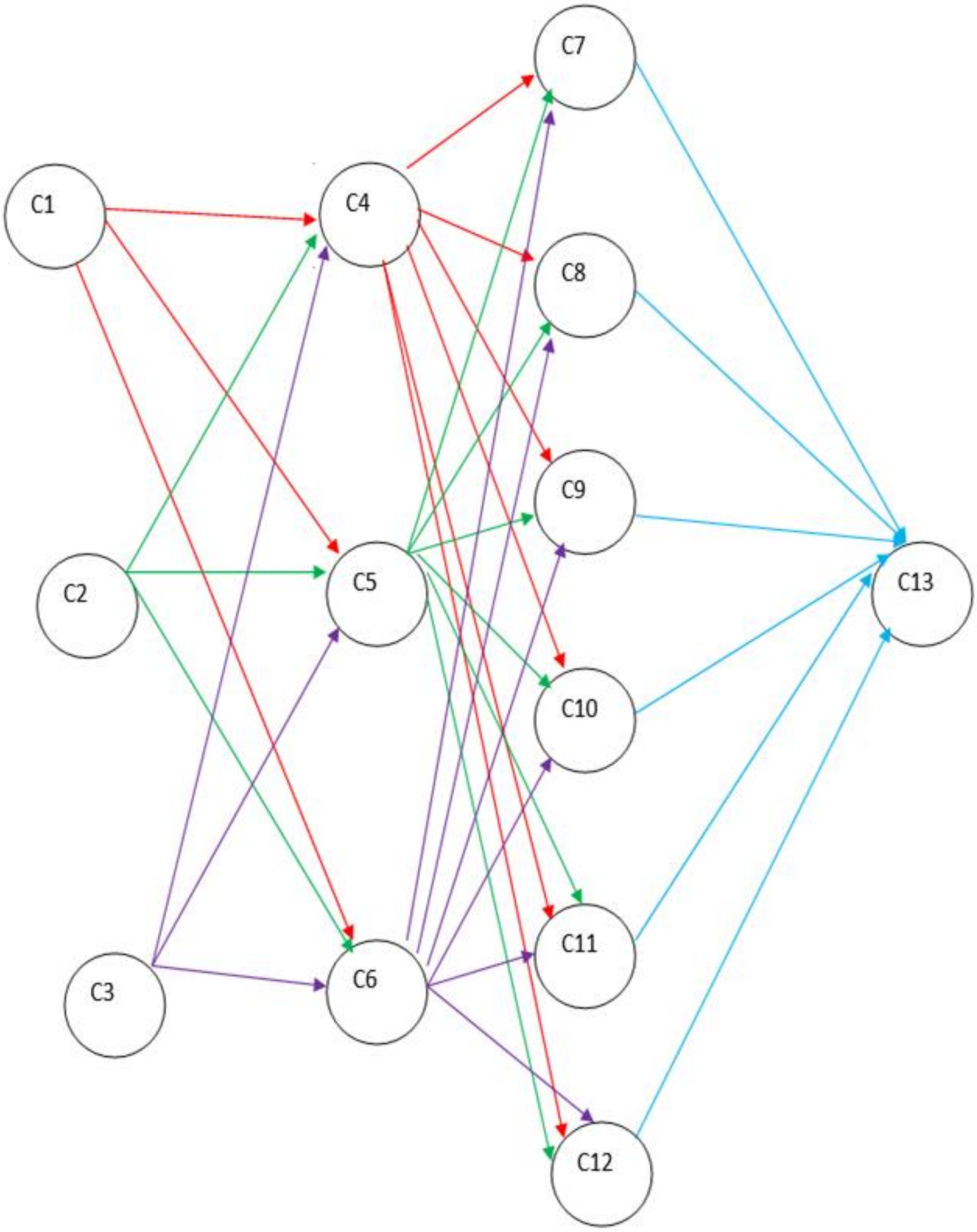


Fig. 4. Association between the factors.

Here D = H/M/L, let us first analyse the case where the relational impacts are high (D=H) and the connection matrix obtained is

$$
\begin{array}{c}
\\ C1 \\ C4 \\ C5 \\ C6 \\ C7 \\ C13
\end{array}
\begin{array}{c}
\begin{array}{cccccc} C1 & C4 & C5 & C6 & C7 & C13 \end{array} \\
\begin{pmatrix}
0 & H & H & H & 2H & 3H \\
0 & 0 & 0 & 0 & H & 2H \\
0 & 0 & 0 & 0 & H & 2H \\
0 & 0 & 0 & 0 & H & 2H \\
0 & 0 & 0 & 0 & 0 & H \\
0 & 0 & 0 & 0 & 0 & 0
\end{pmatrix}
\end{array}
$$

$$
\begin{array}{c}
\\ C1 \\ C4 \\ C5 \\ C6 \\ C7 \\ C13
\end{array}
\begin{array}{c}
\begin{array}{cccccc} C1 & C4 & C5 & C6 & C7 & C13 \end{array} \\
\begin{pmatrix}
0 & D & D & D & Y & Z1 \\
0 & 0 & 0 & 0 & H & Z2 \\
0 & 0 & 0 & 0 & H & Z3 \\
0 & 0 & 0 & 0 & H & Z4 \\
0 & 0 & 0 & 0 & 0 & H \\
0 & 0 & 0 & 0 & 0 & 0
\end{pmatrix}
\end{array}
$$

Fig. 5 represents the respective Plithogenic sub cognitive map and the corresponding connection matrix with known and unknown effects is M1.

The impact Y between C1 and C7 is determined as follows, the relational impact between C1 and C4 is H, the relational impact between C4 and C7 is H, as represented in the general connection matrix and so the cumulative impact between C1 and C7 is 2H, thus the indirect effects Y and Z are determined by taking the cumulative effects between the factors.

Table 2. Quantification of linguistic variable.

Linguistic Variable	Triangular Neutrosophic Representation	Crisp Value
High	(O.1,0.25,0.35)(0.6,0.2,0.3)	0.18
Moderate	(0.35,0.55,0.65)(0.8,0.1,0.2)	.48
Low	(0.65,0.85,0.95)(0.9,0.10.1)	.83

The modified Plithogenic fuzzy connection matrix P(E) is

$$
\begin{array}{c}
\\ C1 \\ C4 \\ C5 \\ C6 \\ C7 \\ C13
\end{array}
\begin{array}{c}
\begin{array}{cccccc} C1 & C4 & C5 & C6 & C7 & C13 \end{array} \\
\begin{pmatrix}
0 & 0.83 & 0.83 & 0.83 & 1.66 & 2.49 \\
0 & 0 & 0 & 0 & .83 & 1.66 \\
0 & 0 & 0 & 0 & .83 & 1.66 \\
0 & 0 & 0 & 0 & .83 & 1.66 \\
0 & 0 & 0 & 0 & 0 & .83 \\
0 & 0 & 0 & 0 & 0 & 0
\end{pmatrix}
\end{array}
$$

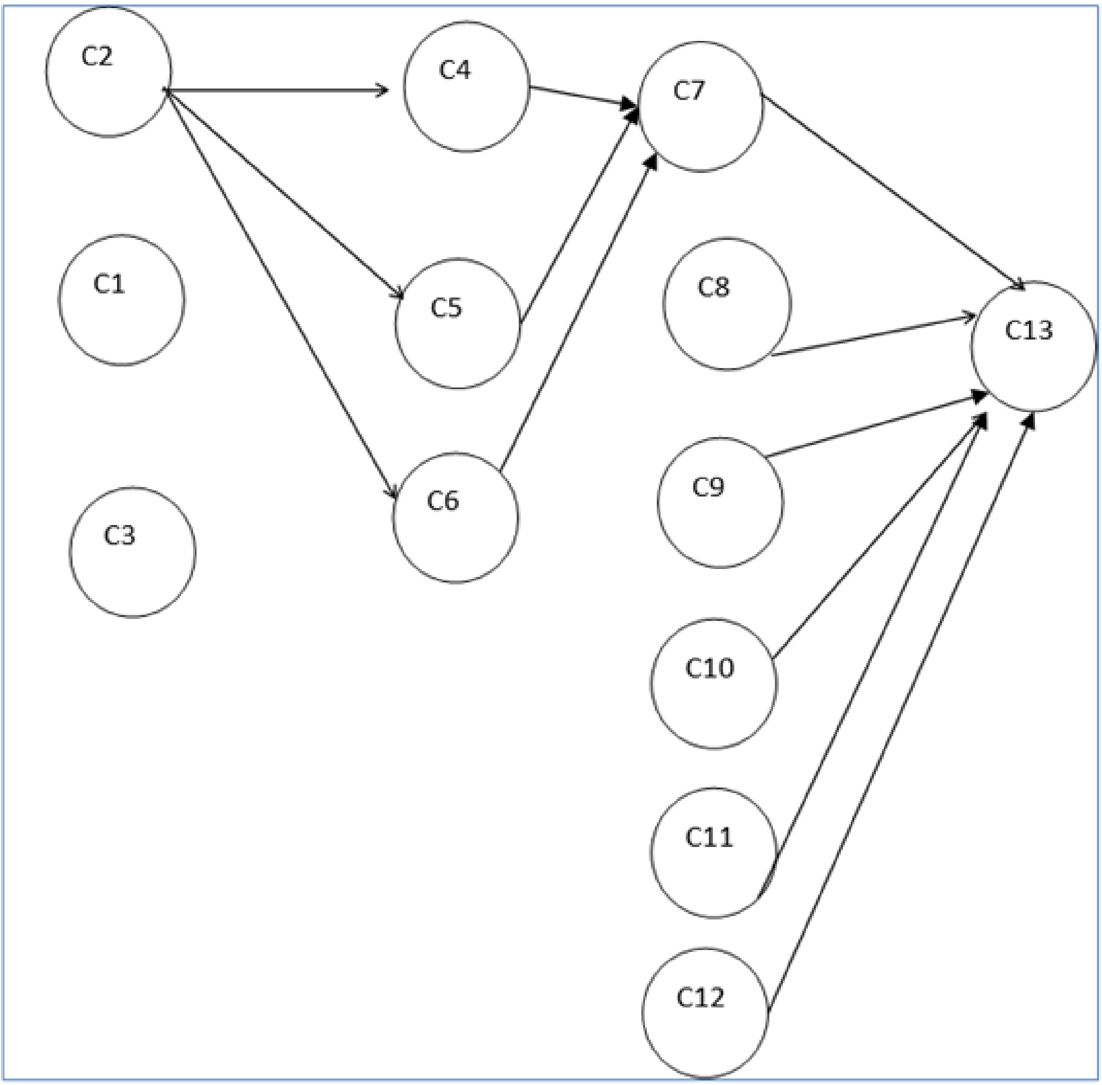


Fig. 5. Respective plithogenic sub cognitive map.

Let us consider the instantaneous state vector as X= (1 1 0 0 1 0)

$X *_p$ P (E) = F where F= (a b c d e f);

a= Max [$1_{\wedge p}$ 0, $1_{\wedge p}$ 0, $0_{\wedge p}$ 0, ,$0_{\wedge p}$ 0, $1_{\wedge p}$ 0, $0_{\wedge p}$ 0,];

b=max [$1_{\wedge p}$ 0.83, $1_{\wedge p}$ 0, $0_{\wedge p}$ 0, ,$0_{\wedge p}$ 0, $1_{\wedge p}$ 0, $0_{\wedge p}$ 0,];

c=max [$1_{\wedge p}$ 0.83, $1_{\wedge p}$ 0, $0_{\wedge p}$ 0, ,$0_{\wedge p}$ 0, $1_{\wedge p}$ 0, $0_{\wedge p}$ 0,];

d=max [$1_{\wedge p}$ 0.83, $1_{\wedge p}$ 0, $0_{\wedge p}$ 0, ,$0_{\wedge p}$ 0, $1_{\wedge p}$ 0, $0_{\wedge p}$ 0,];

e=max [$1_{\wedge p}$ 1.66, $1_{\wedge p}$ 0.83, $0_{\wedge p}$ 0.83, ,$0_{\wedge p}$ 0.83, $1_{\wedge p}$ 0, $0_{\wedge p}$ 0,];

f=max [$1_{\wedge p}$ 2.49, $1_{\wedge p}$ 1.66, $0_{\wedge p}$ 1.66, ,$0_{\wedge p}$ 1.66, $1_{\wedge p}$ .83, $0_{\wedge p}$ 0,];

$X *_p$ P (E)= (0 .83 .8 .94 1.66 2.49)→(1 1 .8 .94 1 1)=X1;

$X1 *_P$ P(E)= ((0 0.83 .89 .94 1.66 2.49) → (1 1 .89 .94 1 1)=X2;

$X2 *_P$ P(E)= (0 0.83 .89 .94 1.66 2.49) → (1 1 .89 .94 1 1) = X3;

X2 = X3.

The fixed point is:

$$(1\ \ 1\ \ .89\ \ .94\ \ 1\ \ 1), \qquad (1)$$

thus obtained represents the influence of the factors in ON position over the other factors.

Let us consider D, the relational impacts to be moderate (i.e.) D= M. The respective connection matrix is as follows:

$$\begin{array}{c} \\ C1 \\ C4 \\ C5 \\ C6 \\ C7 \\ C13 \end{array} \begin{pmatrix} C1 & C4 & C5 & C6 & C7 & C13 \\ 0 & 0.48 & 0.48 & 0.48 & 0.96 & 1.44 \\ 0 & 0 & 0 & 0 & 0.48 & 0.96 \\ 0 & 0 & 0 & 0 & 0.48 & 0.96 \\ 0 & 0 & 0 & 0 & 0.48 & 0.96 \\ 0 & 0 & 0 & 0 & 0 & 0.48 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Let us now consider the instantaneous state vector as X= (1 1 0 0 1 0)

$X *_P$ P(E)=(0 0 .48 .65 .83 0.96 1.44)→(1 1 .65 .83 1 1) = X1;

$X1 *_P$ P(E)=((0 0.48 .65 .83 .96 1.44) → (1 1 .65 .83 1 1) = X2;

X1 = X2.

The fixed point is:

$$(1\ \ 1\ \ .65\ \ .83\ \ 1\ \ 1), \qquad (2)$$

Let us consider D, the relational impacts to be moderate (i.e.) D= L. The respective connection matrix is as follows

$$\begin{array}{c|cccccc} & C1 & C4 & C5 & C6 & C7 & C13 \\ \hline C1 & 0 & 0.18 & 0.18 & 0.18 & 0.36 & 0.54 \\ C4 & 0 & 0 & 0 & 0 & 0.18 & 0.36 \\ C5 & 0 & 0 & 0 & 0 & 0.18 & 0.36 \\ C6 & 0 & 0 & 0 & 0 & 0.18 & 0.36 \\ C7 & 0 & 0 & 0 & 0 & 0 & 0.18 \\ C13 & 0 & 0 & 0 & 0 & 0 & 0 \end{array}$$

$X1 *_P$ $X *_P$ Let us consider the instantaneous state vector as *X=(1 1 0 0 1 0)*

P(E)=(0 0 .18 .45 .73 0.36 .54)→(1 1 .45 .73 1 .54) = X1;

P(E)=((0 .18 .45 .73 .36 .44) → (1 1 .45 .73 1 0.54) = X2;

X1 = X2.

The fixed point is

(1 1 .45 .73 1 0.54), (3)

Eqs. (1)- (3) represents the association of the adult (grouping factor) with the possibility of corona virus (output factor) together with the intervention of the various levels of the mediating effects of the parametric and the risk factor of heart disease.

The same procedure is repeated to analyse the association of the adult (grouping factor) with the possibility of corona virus (output factor) together with the intervention of the various levels of the mediating effects of the parametric and other risk factors. The fixed points thus obtained by considering the grouping factor C2, the output factor C13 , the diverse levels of parametric factors C4, C5, C6 with varying risk factors are tabulated as follows.

Table 3. Diverse levels of parametric factors.

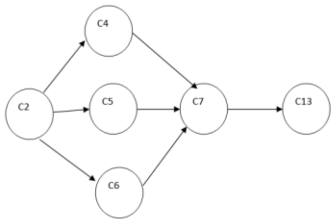
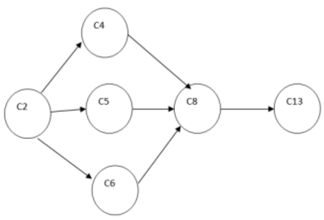
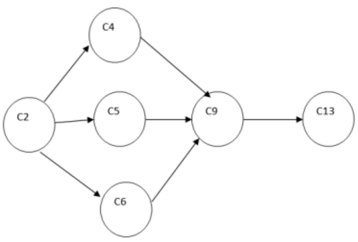
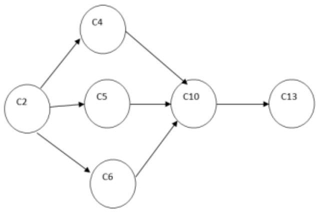
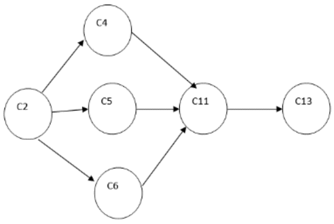
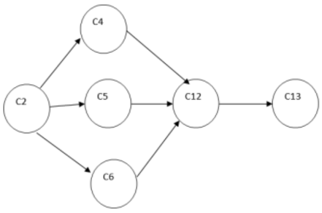
Levels/Risk Factors	High	Moderate	Low
 C7	(1 1 .89 .94 1 1)	(1 1 .65 .83 1 1)	(1 1 .45 .73 1 0.54)
 C8	(1 1 .83 .94 1 1)	(1 1 .65 .83 1 1)	(1 1 .45 .73 1 0.54)
 C9	(1 1 .89 .94 1 1)	(1 1 .65 .83 1 1)	(1 1 .45 .73 1 0.54)

Table 3. (Continued).

Levels/Risk Factors	High	Moderate	Low
 C10	(1 1 .89 .94 1 1)	(1 1 .65 .83 1 1)	(1 1 .45 .73 1 0.54)
 C11	(1 1 .83 .94 1 1)	(1 1 .65 .83 1 1)	(1 1 .45 .73 1 0.54)
 C12	(1 1 .89 .94 1 1)	(1 1 .65 .83 1 1)	(1 1 .45 .73 1 0.54)

It is quite evident that the mediating effects of the risk factors in determining the association of the grouping factor with the output factor by considering various levels of parametric factors yields the same fixed point and thus the association between the grouping factor and the output factor exists and it is invariable with respect to the risk factors, also all the risk factors strongly indicates the association. The same procedure can be applied to determine the association between other grouping factors and the output factor together with the mediating effects of the factors by varying the contradiction degree with respect to the dominant factors

4| Discussion

The proposed diagnostic model provides the platform to take decisions on the possibility of corona virus to the persons of various age group with the various levels of common parametric values and the risk factors. In the first case of taking the relational impacts to be high, the blood sugar level was considered to be dominant as diabetic patients are highly vulnerable to COVID-19 and the other parametric factors are also considered. In addition to it the risk of heart diseases is taken to be dominant. The first case investigates the possibility of corona virus to the adult with high blood sugar level, and with the risk of heart diseases. The same fashion of investigation is done with various levels of the relational impacts and also with the consideration of other risk factors.

5| Conclusion

The paper introduces the concept of Plithogenic sub cognitive maps to design COVID-19 diagnostic model with the mediating effects of the factors. This model can be extended to Plithogenic super sub cognitive maps with the consideration of the intra relationship between the factors. This model assists in making exclusive examination of the possibility of corona virus infection to various age group of people with several mediating factors. This decision-making model will surely disclose novel vistas of constructing many more diagnostic models of this kind. The model can be extended by classifying the factors based on the needs of decision-making problem.

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A short history of fuzzy, intuitionistic fuzzy, neutrosophic and plithogenic sets

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Abstract

Recently, research on uncertainty modeling is progressing rapidly and many essential and breakthrough studies have already been done. There are various ways such as fuzzy, intuitionistic and neutrosophic sets to handle these uncertainties. Although these concepts can handle incomplete information in various real-world issues, they cannot address all types of uncertainty such as indeterminate and inconsistent information. Also, plithogenic sets as a generalization of crisp, fuzzy, intuitionistic fuzzy, and neutrosophic sets, which is a set whose elements are characterized by many attributes' values. In this paper, our aim is to demonstrate and review the history of fuzzy, intuitionistic and neutrosophic sets. For this purpose, we divided the paper as: section 1. History of Fuzzy Sets, section 2. History of Intuitionistic Fuzzy Sets and section 3. History of Neutrosophic Theories and Applications, section 4. History of Plithogenic Sets.

1 History of Fuzzy Sets

In the twentieth century, the problem of representing vagueness in logic, in physics, in linguistics and the questioning of the notion of set caused many first suggestions for the fuzzy set theory. The fuzzy set theory is not a unusual, arbitrary object that abruptly come out of nowhere, yet it clarified the intuitions of some spearheading scientists in the century. The philosophers Charles Peirce[109] and Bertrand Russell[113] thought that vagueness is neglected in mathematical information. There were discussions on the relationships between logic and vagueness in the philosophical literature ([49 76]). It is stated that notions in natural language do not have a apparent class of their properties but they have enlargeable boundaries.[145] To determine vague symbols, the American philosopher Max Black propounded consistency profiles which are the ancestors of fuzzy membership functions.[39] In 1940, H. Weyl gave a generalization of the usual characteristic function by substituting it with a continuous characteristic function.[144] Then Kaplan and Schott proposed a similar generalization of characteristic functions of vague predicates and the fundamental fuzzy set connectives in these works.[82] In 1951, Karl Menger used the term "ensemble flou" which is the French counterpart of fuzzy set for the first-time.[99]

Although the term "fuzzy set" is often worst defined and misunderstood, it is fashinable in scientific areas and daily life. The doctrine of fuzzyiness has been properly defined by M. M. Gupta as "a body of concepts and techniques aimed at providing a systematic framework for dealing with the vagueness and imprecision inherent in human thought processes".[73] Thus, the three basic keywords, the three pillards of this doctrine which form its philosophical basis, are following: thinking, vagueness, and imprecision.[58] Since the activity of thinking is not basically correlated with linguistic expressions,[63] it designates to operations or constructions in the mind, which are in charge of analysis and logical tests. On the other hand, the whole doctrine of fuzziness is founded on vagueness and imprecision, and so, it is handled as properties of language and knowledge. For centuries, scientist have started different enterprises in order to explain these phenomena. Various logical calculus were constructed to the aim of dealing with different aspects of language and information. The fuzzy theory is also one of them. Two different kinds of theories of perception and thought exist.[115] Some suppose that the content of perception is never identical with the perceived entity. However, others assert that this content is identical with the perceived object, and provided that perception is veridical. Identity theories were strongly criticized by B. Russell,[113] who clearly distinguished the properties of words from the properties of things. In this famous paper, he writes that vagueness and precision are characteristics which can only belong to a language and that "apart from representation there can be no such thing as vagueness or precision". He means that vagueness is a characteristic of words, not of things.[58] The aim of fuzzy set theory is to built a formal framework for unfinished and gradual knowledge in natural language. The concepts of ambiguity and imprecision of information can be variously understood by researchers as vagueness, uncertainty, etc. When these concepts are regarded as related for representational systems, everyone agree that language is a typical example.

In the twentieth century, Zadeh introduced the degree of membership/truth (T) in 1965 and defined the fuzzy set.[148] In contrast to classical binary logic, this aims to express thoughts and notions which do not mathematically explain in natural language such as uncertainty and randomness. Thus, Zadeh preferred classes of objects consisting of relative notions in natural language terms such as age, size, height, temperature. However, classical binary logic is not appropriate to analyze such classes where it appears that membership is a gradual notion rather than an all-or-nothing matter. The fuzzy theory summarizes significantly perceptive phenomenas that expound the complexity of the world. Analytical representations of physical phenomena can model natural language but they are sometimes difficult to understand since they have nebulous explanations and is not be obvious for the non-specialist people. Mental representations has uncertainty and vagueness, the lack of specificity of linguistic terms and the lack of well-defined boundaries of the class of preferred objects. In natural language, objects are represented by proper names, properties are expressed by adjectives and nouns, whereas prepositions and verbs tend to express relations between two or more things.[114] In artificial languages, for example, in a language based on first order logic, objects are represented by singular names, whereas properties and relations are represented by predicates.[58]

For years, producing of the devices simulated human thoughts and behaviors is many scientists' dream. Since we have entered the era of information management as soon as the emergence of computers, this dream have started to realize. The developments of science and technology needing detailed knowledge to produce of the devices human thoughts and behaviors and easy attainableness to computers helps to realize the dream. An important problem is to stock and make use of the information in various areas. Because the fuzzy theory attend to this tendency,[56] it has close connection with artificial intelligence.

When Zadeh propounded fuzzy sets, his concerns concentrated on their potential contribution in the fields of model classification,[38] processing and transportation of knowledge, abstraction and summarization.[149] Despite the claims that fuzzy sets were related in these fields emerged in the early sixties, the developments of sciences, technologies and engineering showed that these intuitions were true.

The terms "depressed" and "old" are fuzzy concepts since they cannot be sharply defined. However, people use it to decide. The fuzzy concepts is absolutely contrast to the terms as married, under 25 years old or over 2.5 feet tall. Pure mathematics is interested in classes of objects such as the subset of rational numbers in the set of real numbers. However, when the subset of young people in a given set of people, it may be impossible to decide whether a person is in that subset or not. It is possible to answer it but there can be losing of information when it does. Even though this case has existed from time immemorial, the dominant context is the situation in which statements are exact.

In this day and age, quickly developing technology and the dream of producing devices that mimic human reasoning human thoughts and behaviors which is based on uncertain and imprecise knowledge has drawn many scientists' attention. The theory and applications of fuzzy concepts is largely in the fieds of engineering and applied sciences. The widely usage of fuzzy concepts in technology and science are witnessed by means of the developments of automatic control and expert systems. Although the mathematical elements that form the basis of fuzzy notions have existed for a long time, the rising of its applications has satisfied a motivation for mathematics. Until the rising of fuzzy set theory as a crucial tool in practical applications, there did not exist a force majeure to analyse its mathematics. By virtue of the practical significance of such improvements, studying the mathematical basis of this theory has become important.

In the literature of fuzzy sets, the word "fuzzy" means the word "vague". According to the Oxford English Dictionary, the word "fuzzy" has some various meanings. Unlike any other meaning, "fuzzy" can be used to form a predicate of the form "something is fuzzy". For instance, it can sound normal to say that "a fox is fuzzy" but sound weirdo to say that "old is fuzzy". In this case, the adjective "old" is vague (but not fuzzy in the material sense) since its sense is not set by certain boundaries. However, the word "fuzzy" is applied to predicates and needs to state the gradual nature of some of these words that appear as vague. Additionally, "vagueness" specifies mostly a much larger kind of tabulation for words including ambiguity. Vagueness is interested the meaning of signs in a language and is treated as a certain type of ambiguity. If any notion is not sharply defined, then it is called vague. Since there exists a graceful degradation between situations to which a vague concept fully applies and situations to which it does not never apply, there exist a whole range of situations to which a vague concept fractionally apply.[58] This is called “membership gradience” by Lakoff.[90] It is keynoted that the fuzziness is an main property of the notion of vagueness which is different from uncertainty. Many researchers think that full applicability of a vague notion to a particular situation may be dubious.[112] If a vague concept applies or not to a particular situation, then such doubt can only emerge in the case which is forced to decide. If a proposition involves gradual predicates, then it is called vague. A characteristic property of these propositions is that they can be neither true nor false when they applied to describe a given situation.[49] In a word, they can be stated as a degree. Symbolic forms of these degrees of truth exist in natural languages, for example, “very”, “rather”, “almost no”, “less”, etc. If a person of the class of “depressive people” characterizes “less depressive”, then it is said that the truth level of the statement “this person is depressive” is “less” and this person has a low degree of membership to the class of “depressive people”. These linguistic restrictions apply only to gradual predicates. For example, “married” is not a gradual predicate since the term “less married” does not exist.

There exist two reasons that there are fuzziness and ambiguity in all languages. A first reason is that any language is discrete and the external world seems to be continuous. According to Aristotle, this represents the extensity of ambiguity in languages. Specially, a certain number of words import ostensible continuous numerical scales. For example, let the word ”old” apply to humans. In order to fix an age threshold below which old fully applies and it does not at all is difficult. There exists a conflict between the linguistic representation of the age scale (for example, {old, teen, young, mature}) and its numerical representation (for instance, the real interval [0, 120] years for humans). The range of functions which Zadeh[148] calls membership functions, can involve as many elements as in the age scale and continuous itself. Terms such as old, young, teen, etc. is called fuzzy predicates. A second reason is that natural language overlooks the existence of exceptions. For instance, some of snakes are venomous but other is not since some don’t have venom. Thus, the collection of snakes is not just a set, and also it is a set partially ordered by a relation of compared typicality. Then the membership function is only a partially arbitrary encoding of the partial ordering relation.

Imprecision is also characteristic of language and relates to measurable concepts. These concepts are represented by numbers. This imprecision originates from the fact that any measurement results bounded truth. Then it is possible to mention many correspondences between the results of measurement and real numbers. Chwistek suggested the usage of intervals instead of single numbers[47] and Mellor improved in various direction by Mellor ([97 98]). Imprecision seems in the form of disjunctions in logic. For instance, a proposition “p or q is true” contains imprecision since it is not known whether p is true, q is true, or both. Hence, imprecision is represented by sets as a disjunction (not conjunction) of elements. R. Young attempts to improve a formal theory allowing manipulations with indefinite quantities.[147] This leading work is called interval mathematics. R. Young substituted a variable which supposes severally considered values by a notion of many-valuedness. This notion was stated as a set of values but it considered collectively. S. Lesniewski studied widely on these sets ([91 92]).

Despite imprecision and vagueness mention the contents of a part of knowledge in a language, uncertainty states the ability of an agent to claim whether a proposition holds or not. Uncertainty of Boolean propositions can be gradual as in probability theory where partial doctrine ranges on the unit interval while uncertainty is three-valued in propositional logic, i.e., a proposition is known, its negation is known or both is not known. Even though stating and reasoning these three cases explicitly requires the features of modal logic,[78] the modal possibility has the lack of certainty and certainty reflects logical consistency. However, uncertainty modeled by probability is different from uncertainty modeled by propositional logic. Probability theory often models uncertainty irritating from conflicting, exactly observed, part of knowledge. This situation seems in statistics where a random experiment runs several times an it does not produce the same outcomes. Probability logic is a tool for making inferences from data. Also, Probability logic is used in cases where propositions are either true or false, but the knowledge is imprecise. If the knowledge is fuzzy, then a model of gradual uncertainty is produced since this uncertainty stems from a lack of knowledge. Hence, fuzzy sets causes gradual uncertainty which is different from the one of frequentest probability. This is possibility theory ([58 59]).

In complex real world cases, many types of uncertainty exist. For example, “morality” or “beauty” of each population of humans chosen at random can be researched and so on. These examples are fuzzy concepts that is accurately formulated by fuzzy sets. Each type of uncertainty has its mathematical model. Different mathematical theories are like tools which are advantageous to use than another in a given situation. Some of these theories may be used individually or in conjunction. However, the right combination of mathematical theories should be chosen.

This discussion points out that uncertainty differs from imprecision and vagueness and only result from them. In order to see better the differences between the three notions discussed here, let us consider the following assertions about some car:

- This car is between 10 and 15 years old (pure imprecision).
- This car is very big (imprecision and vagueness).
- This car was probably made in Germany (uncertainty).

In the first case there is a lack of knowledge, due to a lack of ability to measure or to evaluate numerical features. In the second case there is a lack of precise definition of the notion "big" and the modifier "very" indicates a rough degree of "bigness", and the third case expresses uncertainty about a well-defined proposition, perhaps based on statistics.[58]

The originality of fuzzy sets is to catch the idea of partial membership. The membership function of a fuzzy set is a function whose range is an ordered membership set containing more that two values. Thus, a fuzzy set can be thought as a function. Also, the fuzzy set theory are to handle functions as if they were subsets of their domains because such functions are used to state gradual classes. Therefore, the concepts such as intersection, union, complement, inclusion are enlarged in order to combine functions ranging on an ordered membership set. In fuzzy set theory, the union, intersection, inclusion and complementation of functions are committed by taking their point wise maximum, minimum, inequality between functions, and by means of order-reversing automorphism of the membership scale, respectively. Because degrees of membership may be thought as degrees of truth, intersection as conjunction, union as disjunction, complementation as negation and set-inclusion as implication, it seems easily that the fuzzy set theory is closely related to many-valued logics. Moreover, many mathematical notions are extended to fuzzy sets. It is defined special types of fuzzy sets such as convex fuzzy sets, fuzzy numbers, fuzzy intervals and fuzzy relations, and some types of non-classical sets that are different from fuzzy sets.

Gottwald[71] and Ostasiewicz ([106 107]) defined the early developments of fuzzy sets in detailed. Also, Ostasiewicz gave the basic philosophical parts of fuzzy set theory.[105] The first published book on fuzzy sets wrote by Kaufmann in French (1973) and translated into English in 1975.[83] Up to now. Then a mathematical dissertation is followed by Negoita and Ralescu.[100] Dubois and Prade ([60 61]), Kandel,[80] Novak,[102] Yager et al.[146] and Klir and Bo Yuan ([84 85]) are fundamental books on fuzzy sets. Kruse et al. (1994) focused on probabilistic foundations.[89] Gottwald,[72] Lowen[96] and Nguyen and Walker[101] studied on a mathematical structure of fuzzy sets. Zimmerman,[150] Terano, Asai and Sugeno,[141] Klir and Folger,[86] Klir et al[87] and Pedrycz and Gomide[108] investigate methodological problems and applications. Also, the anthology edited by Dubois, Prade and Yager is seen for some basic fuzzy papers of the first twenty five years and other books.[57]

2 History of Intuitionistic Fuzzy Sets

The set theory based on the concept of element membership to sets has proved itself to be one of the most powerful tools of Modern Mathematics and it has allowed to model and to improve other sciences. However, the concept of element membership to a set is a bivalent concept, useded by the values 0 (there is no membership) and 1 (there is membership) and it does not approve other set possibilities in logic.

In today's word, science, technology, biology, social sciences, linguistic, psychology and economics consists of complicated operations and situations for which complete information does not always exist. For these situations, mathematical models are improved to treat various types of systems containing elements of uncertainty. Some of these models are based fuzzy sets. The fuzzy set (FS, briefly) theory introduced by Zadeh regards membership degree and the non-membership degree is the complement of the membership degree. However, this linguistic complement does not provide the logical negation. Since a membership function can be Gaussian, triangular, exponential or any other membership function, and so, there exist a hesitation when a membership function is defined, non-membership degree is less than or equal to the complement of the membership degree. Because different results are obtained with different membership functions.

In 1985, Atanassov introduced the degree of non-membership (T) and falsehood (F) (or non-membership) and defined the intuitionistic fuzzy set (IFS, for short). An intuitionistic fuzzy set may be thought as an alternative approach to describe a fuzzy sets on is not sufficient for the definition of an incomplete notion by means of an ordinary fuzzy set. In the intuitionistic fuzzy set theory, the non-membership degree is not equal to the complement of the membership degree owing to the fact that a lack of information exists when the membership function is defined. In contrast to fuzzy sets, a intuitionistic fuzzy set has two uncertainties called membership and non-membership degrees. Actually, the idea of intuitionistic fuzziness was a coincidence as a mathematical game. Atanassov obtained extension of the ordinary fuzzy set by adding a second degree (degree of non-membership). Since he did not immediately notice properties of intuitionistic fuzzy sets that are different from properties of fuzzy sets, his first studies in this area followed step by step the existing results in fuzzy sets. Some notions of fuzzy sets are not very difficult to extend formally notions of intuitionistic fuzzy sets. However, it is interesting to show that the relevant extension has certain properties which do not exist in the basic concept. Since the method of fuzzification has the idea of intuitionism, the name "intuitionistic fuzzy set" (IFS) is suggested by George Gargov who is Atanassov's former lecturer in the Mathematical Faculty of Sofia University ([6 77]). The intuitionistic fuzzy set is a successful generalization of the fuzzy set. The fuzzy set has already achieved a great success in theoretical researches and practical applications.[94] Therefore, it is expected that the intuitionistic fuzzy set could be used to simulate human decision-making processes and any activities requiring human expertise, experience, and knowledge, which are inevitably imprecise or not totally reliable. Thus, it is completely believed that the intuitionistic fuzzy set has a wide prospect of applications to the fields such as management, economics, business and environment as well as military.[95]

The first example for intuitionistic fuzzy sets is given as follow:[6]

This is a story with Johnny and Mary, characters in many Bulgarian anecdotes. They bought a box of chocolates with 10 pieces inside. Being more nimble, Johnny ate seven of them, while Mary– only two.

One of the candies fell into the floor.

In this moment, a girl friend of Mary came and Mary said, *"We can't treat you with chocolates, because Johnny ate them all"*.

Let us estimate the truth value of this statement at the moment of speaking, i.e., before we have any knowledge of subsequent events.

From classical logic point of view, which uses for estimations the members of the set $\{0, 1\}$, the statement has truth value of 0, since Mary has also taken part in eating the candies and Johnny was not the only one who has eaten them. On the other hand, we are intuitively convinced that the statement is more true than false. Mary greets her guest – statement's truth value is obviously 0.7. However, at the next moment Johnny can take the fallen candy and place it back into the box of treat the guest, preserving the truth value of Mary's statement at 0.7, and falsity 0.3. But he can always take advantage of the distraction and eat the last candy which would result in truth value of 0.8 and falsity of 0.2. In this sense the statement depends to a great extent on Johnny's actions. Therefore, the apparatus of IFSs gives us the most accurate answer to the question: $[0.7, 0.2]$. The degree of uncertainty now is 0.1 and it corresponds to our ignorance of the boundaries of Johnny's gluttony.

In the beginning of the last century, L. Brouwer introduced the notion of the intuitionism.[41] He proposed to the mathematicians to remove Aristoteles' law of excluded middle. He said

An immediate consequence was that for a mathematical assertion the two cases of truth and falsehood, formerly exclusively admitted, were replaced by the following three:

(1) has been proved to be true;

(2) has been proved to be absurd;

(3) has neither been proved to be true nor to be absurd, nor do we know a finite algorithm leading to the statement either that is true or that is absurd".[143]

Thus, if a proposition A exists, then it can be said that either A is true, or A is false, or that it is not known whether A is true or false. Also, the proposition $A \vee \neg A$ is always valid in the first order l ogic. It has truth value "true" (or 1) in Boolean algebras bu this proposition can take value smaller than 1 in the ordinary fuzzy logic of Zadeh and in many-valued logics starting with Lukasiewicz. The same is true for intuitionistic fuzzy sets but the situation happen on semantical and prediction' level. the prediction in Brouwer's sense is fuzzified by clarifying the three possibilities. Thus, Gargov proposed the name "intuitionistic fuzzy set".

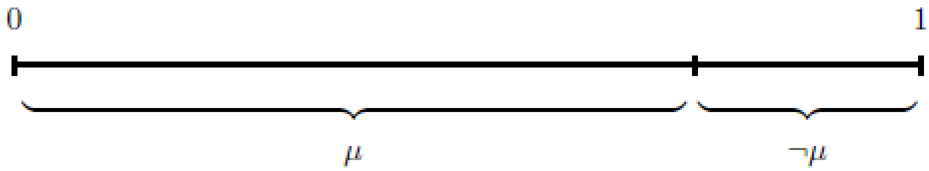
By now, the relations between the intuitionistic fuzzy set theory and Brouwer's intuitionism don't have been researched in detail.

The intuitionistic fuzzy sets allow the definition of operators which are, in a way, similar to the modal operators. Since these operators reduce to identity, they are meaningless in ordinary fuzzy sets.[7] G. Takeuti and S. Titani just attribute a very different meaning in the notion of an "intuitionistic fuzzy sets".[140] Atanassov's first two communications in B ulgarian[7] and English.[8]

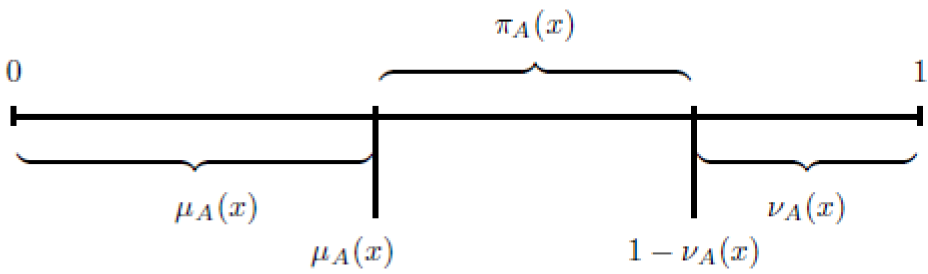
Atanassaov started a discussion on the accuracy of the name of the intuitionistic fuzzy sets.[62] Besides, T. Trifonov and Atanassov built a similar of Takeuti and Titani's research (,[910]). H. Bustince and P. Burillo showed to coincide vague sets with intuitionistic fuzzy sets.[42]

The notion of "interval-valued fuzzy set" (IVFS, shortly) are defined in.[70] Atanassov and Gargov discussed that this notion equals[88] to the intuitionistic fuzzy sets ([11 33]). They showed that each intuitionistic fuzzy set can be stated by an interval-valued fuzzy set and vice versa. Since the intuitionistic fuzzy sets are extensions of the ordinary fuzzy sets, every studies on fuzzy sets may be described in terms of intuitionistic fuzzy sets.

The relations between ordinary fuzzy sets and intuitionistic fuzzy sets are examined from geometrical and probabilistical aspects. Also, some scientists discuss the statement $\mu \vee \neg\mu \leq 1$ in ordinary fuzzy sets as a declaration of the idea of intuitionism. This inequality does not provide the Law of Excluded Middle. However, this is not the situation in a geometrical interpretation. Having in mind that in fuzzy set theory $\neg\mu = 1 - \mu$, it is obtained that the geometrical interpretation as follows:



The situation in the intuitionistic fuzzy sets case is different and as below:



Now, the geometrical sums of both degrees can really be smaller than 1, i.e., the Law of Excluded Middle is not valid. From probabilistic point of view, for case of the ordinary fuzzy sets, if $\mu \& \neg\mu = 0$, then the probability $p(\mu \vee \neg\mu) = p(\mu) + p(\neg\mu) = 1$ as in the geometrical case, while in the case of intuitionistic fuzzy sets, the inequality $p(\mu \vee \neg\mu) \leq 1$ follows that for proper elements of intuitionistic fuzzy sets will be strong. All of these constructions are only on the level of the definition of the set (fuzzy set or intuitionistic fuzzy sets), i.e., not related to the possible operations that can be defined over these sets.[6]

The notion of vague sets is defined.[64] It is proved that the notion of vague sets is equivalent to the notion of intuitionistic fuzzy sets.[42]

A non-probabilistic type of entropy measure is offered for intuitionistic fuzzy sets.[134] Li and Cheng examined similarity measures of intuitionistic fuzzy sets and their application to pattern recognitions.[54]

De et al. studied on Sanchez's approach for medical diagnosis, generalized this concept with the notion of intuitionistic fuzzy set theory and described some operations of intuitionistic fuzzy sets ([50 51]).

Some types of fuzzy connectedness in intuitionistic fuzzy topological spaces are defined.[142] Szmidt and Kacprzyk contemplated the usage of intuitionistic fuzzy sets for constructing soft decision-making models with incomplete information and they offered two solutions about the intuitionistic fuzzy core and the concurrence winner for group decision-making by using intuitionistic fuzzy sets ([135 136 137 138]). Besides, they analyzed distances between intuitionistic fuzzy sets are analyzed.[139] A novel and effective approach to deal with decision-making in medical diagnosis using the composition of intuitionistic fuzzy relations was suggested.[55]

Different theorems for building intuitionistic fuzzy relations on a set with preestablished features are given.[46]

Different forms of fuzzy propositional representations and their relationships are investigated.[48]

However, there has been barely existed researches on multicriteria or multiattribute in discrete decision situations and/or group decision-making using intuitionistic fuzzy sets, which is in fact one of the most crucial fields in decision analysis as in real world decision problems contain multiple criteria and a group of decision makers (,[5294]). Multiattribute decision-making using intuitionistic fuzzy sets is researched, a lot of suitable linear programming models are built to produce optimal weights of attributes and the suitable decision-making methods are suggested.[93]

Moreover, various types of intuitionistic fuzzy sets are introduced ([30 31 32]). Two versions of intuitionistic fuzzy propositional calculus, intuitionistic fuzzy predicate calculus, two versions of intuitionistic fuzzy modal logic and temporal intuitionistic fuzzy logic (TIFL, briefly) were defined and investigated in a series of communications ([33 34 35 65]).

Versions of FORTRAN, C, and PASCAL software packages implementing operations, relations and operators over the intuitionistic fuzzy sets are introduced.[29] $V-$fuzzy Petri nets, reduced $V-$fuzzy generalized nets,[128] intuitionistic fuzzy generalized nets of type I[19] and II ([20 21]) and intuitionistic fuzzy programs[22] have been investigated. A gravity field of many bodies is examined [428]. Intuitionistic fuzzy models of neural networks is improved on the basis of intuitionistic fuzzy sets. Intuitionistic fuzzy expert systems ([23,24,25,110,111]), intuitionistic fuzzy systems,[26] intuitionistic fuzzy PROLOG ([27,28,67]) and intuitionistic fuzzy constraint logic programming ([16,17]) are studied. Some new results on the intuitionistic fuzzy sets theory and its applications were reported to the "Mathematical Foundation of Artificial Intelligence" Seminars held in Sofia in October, 1989 ([18,66,67,74]), March, 1990 ([14,15,30,68,129,133]), June, 1990 ([130,131]), November, 1990 ([13,132]) and October, 1994 ([36,37,43,44,45]). A collection of open questions and problems in the theory of intuitionistic fuzzy sets is presented[13] and discussed.[29]

3 History of Neutrosophic Theories and Applications

The neutrosophic theory that was founded by F. Smarandache in 1998 constitutes a further generalization of fuzzy set, intuitionistic fuzzy set, picture fuzzy set, Pythagorean fuzzy set, spherical fuzzy set, etc. Since then, this logic has been applied in various domains of science and engineering. F. Smarandache introduced the degree of indeterminacy/neutrality (I) as independent component in 1995 (published in 1998) and he defined the neutrosophic set on three components: (T, I, F) = (Truth, Indeterminacy, Falsehood), where in general T, I, F are subsets of the interval $[0, 1]$; in particular T, I, F may be intervals, hesitant sets, or single-values; see
F. Smarandache, Neutrosophy / Neutrosophic probability, set, and logic", Proquest, Michigan, USA, 1998,
https://arxiv.org/ftp/math/papers/0101/0101228.pdf
http://fs.unm.edu/eBook-Neutrosophics6.pdf;
reviewed in Zentralblatt fuer Mathematik (Berlin, Germany):
https://zbmath.org/?q=an:01273000 and cited by Denis Howe in The Free Online Dictionary of Computing, England, 1999.

Neutrosophic Set and Logic are generalizations of classical, fuzzy, and intuitionistic fuzzy set and logic:
https://arxiv.org/ftp/math/papers/0404/0404520.pdf
https://arxiv.org/ftp/math/papers/0303/0303009.pdf

Nonstandard Neutrosophic Logic, Set, Probability (1998, 2019)
https://arxiv.org/ftp/arxiv/papers/1903/1903.04558.pdf

While Neutrosophic Probability and Statistics are generalizations of classical and imprecise probability and statistics.

Etymology.
The words "neutrosophy" and "neutrosophic" were coined/invented by F. Smarandache in his 1998 book.

Neutrosophy: A branch of philosophy, introduced by F. Smarandache in 1980, which studies the origin, nature, and scope of neutralities, as well as their interactions with different idealization spectra. Neutrosophy considers a proposition, theory, event, concept, or entity <A> in relation to its opposite <antiA>, and with their neutral <neutA>. Neutrosophy (as dynamic of opposites and their neutrals) is an extension of the Dialectics (which is the dynamic of opposites only).

Neutrosophy is the basis of neutrosophic logic, neutrosophic probability, neutrosophic set, and neutrosophic statistics.
https://arxiv.org/ftp/math/papers/0010/0010099.pdf

Neutrosophic Logic is a general framework for unification of many existing logics, such as fuzzy logic (especially intuitionistic fuzzy logic), paraconsistent logic, intuitionistic logic, etc. The main idea of NL is to characterize each logical statement in a 3D-Neutrosophic Space, where each dimension of the space represents respectively the truth (T), the falsehood (F), and the indeterminacy (I) of the statement under consideration, where T, I, F are standard or non-standard real subsets of $]^{-}0, 1^{+}[$ with not necessarily any connection between them. For software engineering proposals the classical unit interval $[0, 1]$ may be used.

Degrees of Dependence and Independence between Neutrosophic Components T, I, F are independent components, leaving room for incomplete information (when their superior sum < 1), paraconsistent and contradictory information (when the superior sum > 1), or complete information (sum of components = 1).

For software engineering proposals the classical unit interval $[0, 1]$ is used.
For single valued neutrosophic logic, the sum of the components is:
$0 \leq t + i + f \leq 3$ when all three components are independent;
$0 \leq t + i + f \leq 2$ when two components are dependent, while the third one is independent from them;
$0 \leq t + i + f \leq 1$ when all three components are dependent.

When three or two of the components T, I, F are independent, one leaves room for incomplete information (sum < 1), paraconsistent and contradictory information (sum > 1), or complete information (sum $= 1$).

If all three components T, I, F are dependent, then similarly one leaves room for incomplete information (sum < 1), or complete information (sum $= 1$).

In general, the sum of two components x and y that vary in the unitary interval $[0, 1]$ is: $0 \leq x + y \leq 2 - d^0(x, y)$, where $d^0(x, y)$ is the degree of dependence between x and y, while $d^0(x, y)$ is the degree of independence between x and y.
https://doi.org/10.5281/zenodo.571359
http://fs.unm.edu/NSS/DegreeOfDependenceAndIndependence.pdf

In 2013 Smarandache refined the neutrosophic set to n components:
$(T_1, T_2, \cdots; I_1, I_2, \cdots; F_1, F_2, \cdots)$;
https://arxiv.org/ftp/arxiv/papers/1407/1407.1041.pdf
http://fs.unm.edu/n-ValuedNeutrosophicLogic-PiP.pdf

The Most Important Books and Papers in the Advancement of Neutrosophics 1995-1998 – Smarandache generalized the dialectics to neutrosophy; introduced the neutrosophic set/logic/probability/statistics; introduces the single-valued neutrosophic set (pp. 7-8);
https://arxiv.org/ftp/math/papers/0101/0101228.pdf (fourth edition)
http://fs.unm.edu/eBook-Neutrosophics6.pdf (online edition)

2002 – Introduction of corner cases of sets / probabilities / statistics / logics, such as:

-Neutrosophic intuitionistic set (different from intuitionistic fuzzy set), neutrosophic paraconsistent set, neutrosophic faillibilist set, neutrosophic paradoxist set, neutrosophic pseudo-paradoxist set, neutrosophic tautological set, neutrosophic nihilist set, neutrosophic dialetheist set, neutrosophic trivialist set;

-Neutrosophic intuitionistic probability and statistics, neutrosophic paraconsistent probability and statistics, neutrosophic faillibilist probability and statistics, neutrosophic paradoxist probability and statistics, neutrosophic pseudo-paradoxist probability and statistics, neutrosophic tautological probability and statistics, neutrosophic nihilist probability and statistics, neutrosophic dialetheist probability and statistics,neutrosophic trivialist probability and statistics;

-Neutrosophic paradoxist logic (or paradoxism), neutrosophic pseudo-paradoxist logic (or neutrosophic pseudo-paradoxism), neutrosophic tautological logic (or neutrosophic tautologism):
https://arxiv.org/ftp/math/papers/0301/0301340.pdf
http://fs.unm.edu/DefinitionsDerivedFromNeutrosophics.pdf

2003 – Introduction by Kandasamy and Smarandache of Neutrosophic Numbers ($a + bI$, where I = indeterminacy, $I^2 = I$), I-Neutrosophic Algebraic Structures and Neutrosophic Cognitive Maps
https://arxiv.org/ftp/math/papers/0311/0311063.pdf
http://fs.unm.edu/NCMs.pdf

2005 - Introduction of Interval Neutrosophic Set/Logic
https://arxiv.org/pdf/cs/0505014.pdf
http://fs.unm.edu/INSL.pdf

2006 – Introduction of Degree of Dependence and Degree of Independence between the Neutrosophic Components T, I, F
http://fs.unm.edu/eBook-Neutrosophics6.pdf (p. 92)
http://fs.unm.edu/NSS/DegreeOfDependenceAndIndependence.pdf

2007 – The Neutrosophic Set was extended [Smarandache, 2007] to Neutrosophic Overset (when some neutrosophic component is > 1), since he observed that, for example, an employee working overtime deserves a degree of membership > 1, with respect to an employee that only works regular full-time and whose degree of membership $= 1$; and to Neutrosophic Underset (when some neutrosophic component is < 0), since, for example, an employee making more damage than benefit to his company deserves a degree of membership < 0, with respect to an employee that produces benefit to the company and has the degree of membership > 0; and to and to Neutrosophic Offset (when some neutrosophic components are off the interval $[0, 1]$, i.e. some neutrosophic component > 1 and some neutrosophic component < 0).

Then, similarly, the Neutrosophic Logic/Measure/Probability/Statistics etc. were extended to respectively Neutrosophic Over-/Under-/Off- Logic, Measure, Probability, Statistics etc.
https://arxiv.org/ftp/arxiv/papers/1607/1607.00234.pdf
http://fs.unm.edu/NeutrosophicOversetUndersetOffset.pdf
http://fs.unm.edu/SVNeutrosophicOverset-JMI.pdf
http://fs.unm.edu/IV-Neutrosophic-Overset-Underset-Offset.pdf

2007 – Smarandache introduced the Neutrosophic Tripolar Set and Neutrosophic Multipolar Set and consequently the Neutrosophic Tripolar Graph and Neutrosophic Multipolar Graph

http://fs.unm.edu/eBook-Neutrosophics6.pdf (p. 93)
http://fs.unm.edu/IFS-generalized.pdf

2009 – Introduction of N-norm and N-conorm
https://arxiv.org/ftp/arxiv/papers/0901/0901.1289.pdf
http://fs.unm.edu/N-normN-conorm.pdf

2013 - Development of Neutrosophic Measure and Neutrosophic Probability (chance that an event occurs, indeterminate chance of occurrence, chance that the event does not occur)
https://arxiv.org/ftp/arxiv/papers/1311/1311.7139.pdf
http://fs.unm.edu/NeutrosophicMeasureIntegralProbability.pdf

2013 – Smarandache Refined the Neutrosophic Components (T, I, F) as $(T_1, T_2, \cdots ; I_1, I_2, \cdots ; F_1, F_2, \cdots)$
http://fs.unm.edu/n-ValuedNeutrosophicLogic-PiP.pdf

2014 – Introduction of the Law of Included Multiple Middle $(< A >; < neut1A >, < neut2A >, \cdots , < antiA >)$
http://fs.unm.edu/LawIncludedMultiple-Middle.pdf

2014 - Development of Neutrosophic Statistics (indeterminacy is introduced into classical statistics with respect to the sample/population, or with respect to the individuals that only partially belong to a sample/population)
https://arxiv.org/ftp/arxiv/papers/1406/1406.2000.pdf
http://fs.unm.edu/NeutrosophicStatistics.pdf

2015 - Introduction of Neutrosophic Precalculus and Neutrosophic Calculus
https://arxiv.org/ftp/arxiv/papers/1509/1509.07723.pdf
http://fs.unm.edu/NeutrosophicPrecalculusCalculus.pdf

2015 – Refined Neutrosophic Numbers $(a + b_1I_1 + b_2I_2 + \cdots + b_nI_n)$, where $I_1, I_2, \cdots , I_n$ are subindeterminacies of indeterminacy I;

2015 – (t, i, f)-neutrosophic graphs;

2015 - Thesis-Antithesis-Neutrothesis, and Neutrosynthesis, Neutrosophic Axiomatic System, neutrosophic dynamic systems, symbolic neutrosophic logic, (t, i, f)-Neutrosophic Structures, I-Neutrosophic Structures, Refined Literal Indeterminacy, Quadruple Neutrosophic Algebraic Structures, Multiplication Law of Subindeterminacies:
https://arxiv.org/ftp/arxiv/papers/1512/1512.00047.pdf
http://fs.unm.edu/SymbolicNeutrosophicTheory.pdf

2015 – Introduction of the Subindeterminacies of the form $(I_0)^n = \frac{k}{0}$, for $k \in \{0, 1, 2, \cdots , n - 1\}$, into the ring of modulo integers $\mathbb{Z}_n$ - called natural neutrosophic indeterminacies (Vasantha-Smarandache)
http://fs.unm.edu/MODNeutrosophicNumbers.pdf

2015 – Introduction of Neutrosophic Crisp Set and Topology (Salama & Smarandache)
http://fs.unm.edu/NeutrosophicCrispSetTheory.pdf

2016 – Introduction of Neutrosophic Multisets (as generalization of classical multisets)
http://fs.unm.edu/NeutrosophicMultisets.htm

2016 – Introduction of Neutrosophic Triplet Structures and m-valued refined neutrosophic triplet structures [Smarandache - Ali]
http://fs.unm.edu/NeutrosophicTriplets.htm

2016 – Introduction of Neutrosophic Duplet Structures
http://fs.unm.edu/NeutrosophicDuplets.htm

2017 – 2020 - Neutrosophic Score, Accuracy, and Certainty Functions form a total order relationship on the set of (single-valued, interval-valued, and in general subset-valued) neutrosophic triplets (T, I, F); and these functions are used in MCDM (Multi-Criteria Decision Making):
http://fs.unm.edu/NSS/TheScoreAccuracyAndCertainty1.pdf

2017 - In biology Smarandache introduced the Theory of Neutrosophic Evolution: Degrees of Evolution, Indeterminacy or Neutrality, and Involution
http://fs.unm.edu/neutrosophic-evolution-PP-49-13.pdf

2017 - Introduction by F. Smarandache of Plithogeny (as generalization of Dialectics and Neutrosophy), and Plithogenic Set/Logic/Probability/Statistics (as generalization of fuzzy, intuitionistic fuzzy, neutrosophic set/logic/probability/statistics)
https://arxiv.org/ftp/arxiv/papers/1808/1808.03948.pdf
http://fs.unm.edu/Plithogeny.pdf

2018 – Introduction to Neutrosophic Psychology (Neutropsyche, Refined Neutrosophic Memory: conscious, aconscious, unconscious, Neutropsychic Personality, Eros/ Aoristos/ Thanatos, Neutropsychic Crisp

Personality)
http://fs.unm.edu/NeutropsychicPersonality-ed3.pdf

2019 - Introduction to Neutrosophic Sociology (Neutrosociology) [neutrosophic concept, or (T, I, F)-concept, is a concept that is T
http://fs.unm.edu/Neutrosociology.pdf

2019 & 2020 – Generalization of the classical Algebraic Structures to NeutroAlgebraic Structures (or NeutroAlgebras) [whose operations and axioms are partially true, partially indeterminate, and partially false] and AntiAlgebraic Structures (or AntiAlgebras) [with operations and axioms totally false]:
http://fs.unm.edu/NeutroAlgebra.htm

Applications in: Artificial Intelligence, Information Systems, Computer Science, Cybernetics, Theory Methods, Mathematical Algebraic Structures, Applied Mathematics, Automation, Control Systems, Big Data, Engineering, Electrical, Electronic, Philosophy, Social Science, Psychology, Biology, Biomedical, Engineering, Medical Informatics, Operational Research, Management Science, Imaging Science, Photographic Technology, Instruments, Instrumentation, Physics, Optics, Economics, Mechanics, Neurosciences, Radiology Nuclear, Medicine, Medical Imaging, Interdisciplinary Applications, Multidisciplinary Sciences etc.

[Xindong Peng and Jingguo Dai, A bibliometric analysis of neutrosophic set: two decades review from 1998 to 2017, Artificial Intelligence Review, Springer, 18 August 2018;
http://fs.unm.edu/BibliometricNeutrosophy.pdf]

Neutrosophic Sets and Systems (NSS) international journal started in 2013 and it is indexed by Scopus, Web of Science (ESCI), DOAJ, Index Copernicus, Redalyc - Universidad Autonoma del Estado de Mexico (IberoAmerica), Publons, CNKI, Google Scholar, Chinese Baidu Scholar etc.
http://fs.unm.edu/NSS/
Submit papers on neutrosophic set/logic/probability/statistics and their applications to the editor-in-chief: smarand@unm.edu.

Encyclopedia of Neutrosophic Researchers

The authors who have published or presented papers on neutrosophics and are not included in the Encyclopedia of Neutrosophic Researchers (ENR), vols. 1, 2, 3, and 4,
http://fs.unm.edu/EncyclopediaNeutrosophicResearchers.pdf
http://fs.unm.edu/EncyclopediaNeutrosophicResearchers2.pdf
http://fs.unm.edu/EncyclopediaNeutrosophicResearchers3.pdf
http://fs.unm.edu/EncyclopediaNeutrosophicResearchers4.pdf
are pleased to send their CV, photo, and List of Neutrosophic Publications to smarand@unm.edu in order to be included into the fifth volume of ENR.

4 History of Plithogenic Sets

In 2018, F. Smarandache introduced the plithogenic set (as generalization of crisp, fuzzy, intuitionistic fuzzy, and neutrosophic sets), which is a set whose elements are characterized by many attributes' values. An attribute value v has a corresponding (fuzzy, intuitionistic fuzzy, or neutrosophic) degree of appurtenance d(x,v) of the element x, to the set P, with respect to some given criteria. In order to obtain a better accuracy for the plithogenic aggregation operators in the plithogenic set, and for a more exact inclusion (partial order), a (fuzzy, intuitionistic fuzzy, or neutrosophic) contradiction (dissimilarity) degree is defined between each attribute value and the dominant (most important) attribute value. The plithogenic intersection and union are linear combinations of the fuzzy operators tnorm and tconorm, while the plithogenic complement, inclusion (inequality), equality are influenced by the attribute values contradiction (dissimilarity) degrees.[123]

Plithogeny is the genesis or origination, creation, formation, development, and evolution of new entities from dynamics and organic fusions of contradictory and/or neutrals and/or non-contradictory multiple old entities. While plithogenic means what is pertaining to plithogeny.

A plithogenic set P is a set whose elements are characterized by one or more attributes, and each attribute may have many values. Each attribute's value v has a corresponding degree of appurtenance d(x,v) of the element x, to the set P, with respect to some given criteria.

In order to obtain a better accuracy for the plithogenic aggregation operators, a contradiction (dissimilarity) degree is defined between each attribute value and the dominant (most important) attribute value.

{However, there are cases when such dominant attribute value may not be taking into consideration or may not exist [therefore it is considered zero by default], or there may be many dominant attribute values. In such cases, either the contradiction degree function is suppressed, or another relationship function between attribute values should be established.}

The plithogenic aggregation operators (intersection, union, complement, inclusion, equality) are based on contradiction degrees between attributes' values, and the first two are linear combinations of the fuzzy operators' tnorm and tconorm.

Plithogenic set is a generalization of the crisp set, fuzzy set, intuitionistic fuzzy set, and neutrosophic set, since these four types of sets are characterized by a single attribute value (appurtenance): which has one value (membership) – for the crisp set and fuzzy set, two values (membership, and nonmembership) – for intuitionistic fuzzy set, or three values (membership, nonmembership, and indeterminacy) – for neutrosophic set (see[123]).
http://fs.unm.edu/NSS/PlithogenicSetAnExtensionOfCrisp.pdf

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Prioritization of Logistics Risks with Plithogenic PIPRECIA Methof

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Abstract. Rapidly changing markets, actors, new legal regulations, information and data intensity have increased uncertainty, and as a result, businesses that want to continue operating in the market need to pay more attention to risk criteria. Risk can be explained as unplanned event which affects a business's overall per-formance. Logistics practices that develop and change continuously show a great variety such as weather and road accidents to faults in operations. Logistics risks have important roles in supply chains efficiency as the risks in logistics may adversely affect all parts of the supply chains and lead to decreases in business performances. Multi-criteria decision making methods are commonly used in risk prioritisation. In this study, a newly developed method called Plithogenic PIvot Pairwise RElative Criteria Importance Assessment (PIPRECIA) Method is used to prioritise logistics risks. For identifying weights, data were collected from three experts in the logistics field. Six logistics risks were considered and according to the results of Plithogenic PIPRECIA Transportation-related risk is determined as the most significant risk.

Keywords: Logistics risks · MCDM · PIPRECIA

1 Introduction

Logistics sector has always contained risk in its operations as there were several uncertainties even in the back such as weather, human factors, and safety issues. However, today's logistics sector is quite complicated with rapidly changing markets, actors, new legal regulations, information and data intensity. This complexity has increased uncertainty more than before, and as a result, businesses that want to continue operating in the market need to pay more attention to the risk.

Risk can be explained as unplanned event which affects a business's overall performance. Logistics practices that develop and change continuously show a great variety such as weather and road accidents to faults in operations. Supply chain and logistics are two terms used interchangeably in the literature as logistics risks has an important role in supply chains efficiency as the risks in logistics may adversely affect all parts of the supply chains and lead decreases in business performances. Sustaining stable logistic operations is a requirement for the success of a supply chain.

Risk assessment is a methodology for defining, classifying, and assessing threats. Private and state authorities commonly use risk assessments for decisions related to legislative issues and resource designation decisions. Risk assessment consists two stages; qualitative and quantitative stages. Qualitative stage involves a process of defining, characterizing, and rating risks. Quantitative stage on the other hand involves assessing the risk probability and effects of the risks [1].

As there are many logistics risks to consider, many of which have interconnections, assessment of logistics risks is difficult. Because of the complicated relationships between these risks, prioritizing the risks for mitigation is a difficult task. Multi criteria decision making (MCDM) methods are commonly used in the literature to overcome the uncertainty in prioritizing various conflicting risks therefore a newly developed method called Plithogenic PIvot Pairwise RElative Criteria Importance Assessment (PIPRECIA) method is used to prioritise logistics risks in this study.

This study consists of five sections. In the first section, introduction is presented. In the second section, the studies about the logistics risks have been reviewed. In the third section, methodology has been explained. In the fourth section, research methodology has been applied. In the lest section, conclusion of this study has been presented.

2 Literature Review

Logistics risks have been researched by several studies in the literature. To be thorough in our risk assessment, we conducted a comprehensive literature review to identify all risk factors discussed in previous studies (Table 1).

Various papers used MCDM methods to examine the risks involved in logistics sector. Tüysüz and Kahraman [16] assessed project risks using fuzzy AHP. Sattayaprasert et al. [17] developed a risk assessment model with AHP to evaluate the risks in dangerous goods transportation. Ren [18] assessed fire risks in logistics warehouses located in the cities with fuzzy AHP found that there are four factors that influence the fire risk: warehouse, product, management, and environment. Sari et al. [19] assessed the risks involved in the urban rail with fuzzy AHP. Zhao et al. [20] used the Expectation Maximization Algorithm to derive three key risk factors impacting dangerous goods freight: human factors, equipment and infrastructure, packing and handling. In green logistics, Oztaysi et al. [21] used hesitant fuzzy TOPSIS to assess the risks involved in transforming urban areas. Ilbahar et al. [22] developed a new integrated model consisting Pythagorean Fuzzy Proportional Risk Assessment (PFPRA), Pythagorean fuzzy AHP, to assess the risks related to occupational health and safety. Gul [23] proposed a risk assessment model with fuzzy FAHP for prioritizing evaluation criteria in oil transportation.

Table 1. Review of logistics risks in literature.

Authors	Problems	Risk factors
Tsai [2]	Maritime logistics	Information risk
Jia et al. [3]	Road transportation	Accidents Terrorist attacks
Ambituuni et al. [4]	Road transportation	Accidents
Afenyo et al. [5]	Maritime logistics	Accidents
Park et al. [6]	Global supply chains	Operational risks
Tubis [7]	Road transportation	Operational risks
Ghaleh et al. [8]	Road transportation	Accidents
Huang et al. [9]	3PL logistics	Quality risk
Liu et al. [10]	Maritime logistics	Hazardous good accidents
Ofluoglu et al. [11]	Disaster logistics	Demand Risk Transportation risk Supply risk Interruption Risk Damage Risk
Tumanov [12]	Multimodal transport	Accidents
Mohammadfam et al. [13]	Road transportation	Safety risks Health risks
Ovidi et al. [14]	Railways	Accidents
Zhao et al. [15]	Urban logistics	Accidents

3 Methodology

In this study, the Plithogenic PIPRECIA method is developed to evaluate the logistics risks and to determine the most important logistics risk.

3.1 Neutrosophic Set

$\tilde{k} = (k_1, k_2, k_3); \alpha, \theta, \beta$ is a single valued triangular neutrosophic set including truth membership $T_k(x)$, indeterminate membership $I_k(x)$ and falsity membership function $F_k(x)$ as follows [24]:

$$T_k(x) = \begin{cases} \alpha_k\left(\frac{x-k_1}{k_2-k_1}\right) & if\ k_1 \le x \le k_2 \\ \alpha_k & if\ x = k_2 \\ 0 & otherwise \end{cases} \tag{1}$$

$$I_k(x) = \begin{cases} \left(\frac{k_2-x+\theta_k(x-k_1)}{(k_2-k_1)}\right) & if\ k_1 \le x \le k_2 \\ \theta_k & if\ x = k_2 \\ \left(\frac{x-k_2+\theta_k(k_3-x)}{(k_3-k_2)}\right) & otherwise \end{cases} \tag{2}$$

$$F_k(x) = \begin{cases} \left(\frac{k_2 - x + \beta_k (x - k_1)}{(k_2 - k_1)}\right) & if \;\; k_1 \le x \le k_2 \\ \beta_k & if \;\; x = k_2 \\ \left(\frac{x - k_2 + \beta_k (k_3 - x)}{(k_3 - k_2)}\right) & if \;\; k_2 \le x \le k_3 \\ 1 & otherwise \end{cases} \tag{3}$$

3.2 Plithogenic PIPRECIA

The steps of the Plithogenic PIPRECIA method are explained below.

Step 1: Logistics risks are determined, and decision-makers rank the logistics risks from most important to least important.

Step 2: Commencing with the second criterion, the j th criterion and the $j-1$ th criteria are compared and, in this comparison, they will use plithogenic relative importance ($\tilde{t}_j$) values. These plithogenic values in Table 2 are used for this comparison.

Table 2. Linguistic scale (Adapted from Abdel-Basset et al. [24]).

Linguistic variable	Triangular Neutrosophic Scale (TNS)
Absolutely significant (AS)	((0.95, 0.90, 0.95), 0.90, 0.10, 0.10)
Very strongly significant (VSS)	((0.90, 0.85, 0.90), 0.70, 0.20, 0.20)
Strong significant (STS)	((0.70, 0.65, 0.80), 0.90, 0.20, 0.10)
Equal significant (ES)	((0.65, 0.60, 0.70), 0.80, 0.10, 0.10)
Fairly weakly significant (FWS)	((0.40, 0.35, 0.50), 0.60, 0.10, 0.20)
Weakly significant (WS)	((0.15, 0.25, 0.10), 0.60, 0.20, 0.30)
Very weakly significant (VWS)	((0.10, 0.30, 0.35), 0.10, 0.20, 0.15)

Step 3: A contradiction degree obtains better precision for plithogenic aggregation operations [25], so the contradiction degree is determined between each criterion and the dominant criterion value [26]. Therefore, the contradiction degree ($c : V \times V \rightarrow [0, 1]$) is defined.

Step 4: The judgments of all decision-makers are combined with the following equation.

$$\begin{aligned} &((k_{i1}, k_{i2}, k_{i3}), 1 \le i \le n) \wedge p((m_{i1}, m_{i2}, m_{i3}), 1 \le i \le n) \\ &= \left(k_{i1} \wedge_F m_{i1}, \frac{1}{2}(k_{i2} \wedge_F m_{i2}) + \frac{1}{2}(k_{i2} \vee_F m_{i2}), k_{i3} \vee_F m_{i3}\right), 1 \le i \le n \end{aligned} \tag{4}$$

where $\wedge_F$ and $\vee_F$ indicate the fuzzy t-norm and t-conorm, respectively.

Step 5: The neutrosophic numbers ($\tilde{t}_j$) are transformed into crisp numbers (t_j) as follows:

$$U(k) = \frac{1}{9}(a_1 + b_1 + c_1) \times (2 + \alpha - \theta - \beta) \tag{5}$$

Step 6: The final ranking of the criteria is obtained by combining the criteria rankings of the decision-makers with the geometric mean.

Step 7: k_j coefficient is computed as:

$$k_j = \begin{cases} 1 & j = 1 \\ 2 - t_j & j > 1 \end{cases} \tag{6}$$

Step 8: p_j recalculated weight is computed as:

$$p_j = \begin{cases} 1 & j = 1 \\ \frac{p_{j-1}}{k_j} & j > 1 \end{cases} \tag{7}$$

Step 9: The final weights (w_j) of criteria are obtained as follows:

$$w_j = \frac{p_j}{\sum_{k=1}^{n} p_k} \tag{8}$$

4 Application

In this study, logistics risks are evaluated and these risks are prioritized. Judgments of three experts were obtained for the evaluation regarding the risks. Six logistics risks were identified by the decision of three experts. These six risks are as follows: Transportation-related Risks (TRR), Purchasing-related Risks (PUR), Information-related Risks (INR), Inventory-related Risks (IVR), Packaging-related Risks (PAR), and Organization-related Risks (ORR). Experts have listed these risks according to their importance. The risks rankings of the experts are shown in Table 3.

Table 3. The risks rankings of the experts.

Experts	Risks		
	Exp-1	Exp-2	Exp-3
TRR	1	1	1
PUR	3	2	3
INR	2	4	5
IVR	4	3	2
PAR	6	5	4
ORR	5	6	6

Each expert assigned plithogenic values to each risk, starting with the second risk to compare the risks. The risks comparisons of Expert 1 are shown in Table 4.

Table 4. The risks comparisons of expert 1.

Risks	Rankings	Risks	Linguistic	TNS
TRR	1	TRR	-	-
PUR	3	INR	VWS	((0.10, 0.30, 0.35), 0.10, 0.20, 0.15)
INR	2	PUR	WS	((0.15, 0.25, 0.10), 0.60, 0.20, 0.30)
IVR	4	IVR	ES	((0.65, 0.60, 0.70), 0.80, 0.10, 0.10)
PAR	6	ORR	WS	((0.15, 0.25, 0.10), 0.60, 0.20, 0.30)
ORR	5	PAR	WS	((0.15, 0.25, 0.10), 0.60, 0.20, 0.30)

The contradiction degree of each risk is equally taken as 1/6. Then, the judgments of all decision-makers are combined by using Eq. 4. Aggregated plithogenic values of risks are transformed into crisp numbers by using Eq. 5. Aggregated plithogenic values ($\tilde{t}_j$) of risks and crisp numbers (t_j) are presented in Table 5.

Table 5. Aggregated Plithogenic values of risks and crisp numbers.

Risks	$\tilde{t}_j$	t_j
TRR	-	-
PUR	((0.070, 0.350, 0.629), 0.215, 0.175, 0.375)	0.357
INR	((0.227, 0.513, 0.811), 0.520, 0.125, 0.323)	0.194
IVR	((0.217, 0.413, 0.800), 0.413, 0.100, 0.333)	0.315
PAR	((0.079, 0.425, 0.755), 0.133, 0.200, 0.401)	0.390
ORR	((0.251, 0.550, 0.876), 0.618, 0.200, 0.323)	0.214

The rankings of the risks according to experts are combined with the geometric mean. Then, Eqs. 6–8 are used to determine the weights of logistics risks. The results of plithogenic PIPRECIA are shown in Table 6.

Table 6. The results of Plithogenic PIPRECIA.

Risks	Rankings by geometric mean	t_j	k_j	p_j	w_j
TRR	1	-	1	1	0.423
PUR	2	0.357	1.643	0.609	0.258
IVR	3	0.315	1.685	0.361	0.153
INR	4	0.194	1.806	0.200	0.085
PAR	5	0.390	1.610	0.124	0.052
ORR	6	0.214	1.786	0.069	0.029

According to Table 6, risks are listed from the most important to the least as follows: TRR, PUR, IVR, INR, PAR and ORR.

5 Conclusion

The today's logistics sector is very complex and has a high level of risk. Logistics risk assessment is a difficult task due to several logistic risks to consider and trade-offs. Multi criteria decision making (MCDM) approaches are widely used in the literature to address the difficulty in prioritizing different competing risks. In this analysis, a recently evolved approach called Plithogenic PIPRECIA is used to prioritize logistics risks. The logistics risks are assessed and prioritized in this study based on the opinions of three experts. Experts determined that there were six logistics risks, and these risks were prioritized by Plithogenic PIPRECIA. It has been found that the most important risk is Transportation-related Risks followed by Purchasing-related Risks, Inventory-related Risks, Information-related Risks, Packaging-related Risks, and Organization-related Risks. This method can be applied to different decision-making problems such as supplier selection, location selection in future studies. There are various studies about logistics risk in the literature however Plithogenic PIPRECIA is a new model and therefore there are only two studies in the literature. Therefore, this study contributes to the literature. Plithogenic PIPRECIA model can be used in other areas of decision problems such as location selection, performance evaluation, or machine selection problems in future studies.

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Introduction to Plithogenic Sociogram with Preference Representations by Plithogenic Number

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Abstract

The theory of plithogeny is gaining momentum in recent times as it generalizes the concepts of fuzzy, intuitionistic, neutrosophy and other extended representations of fuzzy sets. The relativity of the comprehensive and accommodative nature of plithogenic sets in dealing with attributes shall be applied to handle the decision–making problems in the field of sociology. This paper introduces the concepts of Plithogenic Sociogram (PS) and Plithogenic Number (PN) where the former is the integration of plithogeny to the sociometric technique of sociogram and the latter is the generalization of fuzzy, intuitionistic and neutrosophic numbers that shall be used in representations of preferences in group dynamics. This research work outlines the conceptual development of these two newly proposed concepts and discusses the merits of the existing theory of similar kind with suitable substantiation. The plithogenic sociogram model encompassing the attributive preferences with plithogenic number representation is also developed to explicate how it can be materialized in the real social field. A conjectural illustration is put forth to analyze the efficiency and the feasibility of the proposed plithogenic sociogram model and its function in decision-making. This paper also throws light on generalized plithogenic number, dominant attribute constrained plithogenic number and combined dominant attribute constrained plithogenic number together with its operations and suitable illustrations.

Keywords: Plithogeny, Plithogenic sociogram, Attributes, Preferences generalized plithogenic number, Dominant attribute constrained plithogenic number, Combined dominant attribute constrained plithogenic number.

1 | Introduction

The social relationship is the resultant of the social interaction between persons and the longevity of their relationship depends on the alikeness in thoughts, behaviour and sometimes the influence of one's attribute over another. The formation of social groups for carrying out group activities is sometimes deliberate but quite natural in any social setting ranging from small schools, organizations to mammoth industries. Should we concern about the strength of the interrelationship between the members of the group? Will making the bond strong between the members benefit the group? The answer is certainly yes, because the extent of functioning as a group with common objectives and the success in goal attainment depends on the coordination and cooperativeness of the members of the group.

Hence, the study of interpersonal relationships in a group, preferably a social group has greater significance in group dynamics. Sociogram developed by Jacob Levy Moreno and it is one of the sociometric techniques that is widely used in the quantitative study on interpersonal relationship [1]. This technique is used to determine the structure of interrelationship in a group setting by determining the order of preferences of the members of the group to work with through a questionnaire. The preferential positions of the members determine the most influential and isolated people of the group and as the result, the decision-makers or the group coordinators can work on enhancing interpersonal relationship and make other alternatives for improving the group efficiency.

Conventional sociogram characterized by crisp preferential ordering, matrix and graphical representations finds several applications in a various social setting. The uncertainty in the order of preferences led to the development of fuzzy sociogram with fuzzy matrix and fuzzy graphical representations and it has made the researchers explore its applicability in determining the interrelationship between the members [2] and [3]. The decision-making environment is characterized not only by uncertainty but also indeterminacy, to handle such circumstances, Abdel-Basset et al. [4] and Smarandache [5] introduced neutrosophic sets which consist of truth values, indeterminacy values and falsity values. Neutrosophic sets are used in decision-making on green supply chain management [6], decision support systems and in many other. Gómez et al. [7] extended fuzzy sociogram to neutrosophic sociogram to incorporate the notion of the existence of indeterminacy in relationships. The preferential ordering is certainly influenced by the indeterminacy that occurs when the members are not sure of certain attributes of others and also they may not sure of their compatibility or suitability to perform a particular task. A hypothetical example was used to illustrate the applicability of the neutrosophic sociogram model to group analysis. On profound analysis over the transition from conventional or the classical sociogram to neutrosophic sociogram, the order of preferences or the preferential ordering is influenced by certainty in the case of classical, uncertainty in the case of fuzzy and indeterminacy in the case of neutrosophic. This fact has led the authors to investigate the factors that influence preferential ordering as it is the deciding factor of the nature of the sociogram. This is the origin of the plithogenic sociogram which encompasses the attributive preferential ordering, i.e order of preference based on the attributives of the members. Before making the order of preferences, in the sociograms of earlier kinds, the activities (such as quiz program, team-based tasks) that require group work are stated first and the members express their preference for working with others, but in the realm, the choice of choosing or giving preference to the members to get involved in activity also depends on the attributes possessed by the members that are essential to make partnership to take part in any particular activity and many times these attributes may be an essential requisite to take part in the activity or the activities may itself demand the same. In such circumstances, the preferential ordering will be characterized not just by stating the members preferred alone but it also carries the additional information on why the members are being preferred and naturally it brings the attributes of the members and the extent to which the members possess in the perception of the choice-maker, i.e the person who makes the preference. The making of choice in preferring a person depending on the attributes has led to the development of plithogenic sociogram and on exploring will certainly yield better results.

Plithogeny is the recently evolved philosophy that deals with the evolution of entities and their attributes. Smarandache [8] introduced plithogenic sets that are widely applied in decision making on sustainability [9], medical decision system model [10] and supply chain management [11]. Plithogenic sets are used in decision making as it is highly embedded with wide-ranging generalization approaches in accommodating crisp, fuzzy, intuitionistic, neutrosophic sets and the other kinds of extended sets. The preferential ordering assumes either crisp, fuzzy or neutrosophic values, but if the preferential ordering presumes linguistic representation then the linguistic variable requires to be quantified using either fuzzy, intuitionistic or neutrosophic numbers. To make such kind of representations more comprehensive, the notion of plithogenic number shall be used. This research work intends to investigate and unveil the plithogenic sociogram with plithogenic number representing the preferential ordering.

The paper is structured into the following sections, Section 2 introduces plithogenic number and discusses their nature; Section 3 describes plithogenic sociogram and its utility in decision making and the last section concludes the work.

2 | Plithogenic Number

Zadeh [12] introduced Fuzzy numbers and their arithmetic operations to characterize uncertainty. A fuzzy number is a fuzzy set if it is a normal fuzzy set with bounded support and alpha cut being a closed interval for every alpha belonging to [0,1]. The fuzzy numbers are the special kind of fuzzy sets used to quantify linguistic variables and it is applied to represent quantities that are uncertain in nature, for instance, the costs parameters, demand are represented as fuzzy numbers. Stefanini et al. [13] and [14] discussed fuzzy numbers, fuzzy arithmetic. Dison Ebinesar [15] presented the different kinds of fuzzy numbers and their properties. Mallak and Bedo [16] described special kinds of fuzzy numbers. Grzegorzewski and Stefanini [17] illustrated the applications of fuzzy numbers. Thus, fuzzy numbers are the simple form of representing uncertainty and are extended to intuitionistic fuzzy numbers which are the next higher or extended form that are extensively applied in decision-making models. Atanassov [18] introduced the concept of intuitionistic sets. Intuitionistic fuzzy numbers are characterized by membership and non-membership values. Mahapatra and Roy [19] briefed the applications of an intuitionistic fuzzy number. Seikh et al. [20] presented the various kinds of intuitionistic fuzzy numbers. Researchers have discussed the different ordering techniques of IFN [21]-[23]. Smarandache [8] extended Intuitionistic sets to neutrosophic sets and discussed the arithmetic operations of neutrosophic numbers. Neutrosophic numbers are the extended or the higher forms of representing uncertainty. Gahlot and Saraswat [24] described single-valued neutrosophic number, Sun et al. [25] elaborated interval-valued neutrosophic number, Karaaslan [26] explored Gaussian neutrosophic number, Chakraborty et al. [27] discussed the applications of Cylindrical neutrosophic single-valued number in networking, decision making. Researchers like Saini et al. [28], El-Hefenawy et al. [29] stated the applications of neutrosophic number in various fields of decision making [30]. Neutrosophic numbers are the extended forms of intuitionistic and fuzzy numbers and neutrosophic numbers can be stated as higher forms or super forms of fuzzy numbers. The defuzzification techniques of the extended higher/super forms of fuzzy numbers to its next sub forms of fuzzy numbers are also discussed by Radhika et al. [31], Mert [32], İrfan and Öztürk [33], and many others. The above discussed forms of fuzzy numbers ranging from simple to higher versions shall be generalized into plithogenic number.

Classical plithogenic set is characterized by (P, a, V, d, c), where P is a set, a is the attribute, V is the set of attribute values, d is the degree of appurtenance stating the extent of elements belonging to P satisfying the attribute values and c is the contradiction degree. In this work, the plithogenic set is newly characterized as (P, A, V A, d, c), where A is a system of attributes and V A is the set of all possible attribute values corresponding to each attribute a in A. The classical characterization is with respect to a single attribute and this newly proposed pertains to the system of attributes. To define plithogenic number, the attributes should also be considered and the plithogenic number can also be differentiated into plithogenic fuzzy number, plithogenic intuitionistic fuzzy number, plithogenic neutrosophic number based on the degree of appurtenance

Let U be a universe of discourse, and a non-empty set M included in U.

Let x be a generic element from M.

Let's consider the attributes $A_1, A_2, \ldots, A_n$, for $n \geq 1$.

The attribute A_1 has the attribute values $A_{11}, A_{12}, \ldots, A_{1m1}$, where $m_1 \geq 1$.

The attribute A_2 has the attribute values $A_{21}, A_{22}, \ldots, A_{2m2}$, where $m_2 \geq 1$.

The attribute An has the attribute values $A_{n1}, A_{n2}, ..., A_{nm}$ where m, $n \geq 1$.

The plithogenic fuzzy number will be of the form

M = {x($A_{11}(t_{11}), A_{12}(t_{12}), ..., A_{1m1}(t_{1m1})$; $A_{21}(t_{21}), A_{22}(t_{22}), ..., A_{2m2}(t_{2m2})$; ... $A_{n1}(t_{n1}), A_{n2}(t_{n2}), A_{nm}(t_{nm})$; with x in U}, where t_{11} is the degree of appurtenance of element x to the set M with respect to the attribute value A_{11}; t_{12} is the degree of appurtenance of element x to the set M with respect to the attribute value A_{12} etc.

The plithogenic intuitionistic fuzzy number will be of the form

M = {x($A_{11}(t_{11}, f_{11}), A_{12}(t_{12}, f_{12}), ..., A_1m_1(t_1m_1, f_1m_1)$; $A_{21}(t_{21}, f_{21}), A_{22}(t_{22}, f_{22}), ..., A_{2m2}(t_{2m2}, f_{2m2})$; ... $A_{n1}(t_{n1}, f_{n1}), A_{n2}(t_{n2}, f_{n2}), A_{nm}(t_{nm}, f_{nm})$; with x in U}, where t_{11} is the degree of appurtenance of element x to the set M with respect to the attribute value A_{11} and f_{11} is the degree of non-appurtenance of element x to the set M with respect to the attribute value A_{11}; t_{12} is the degree of appurtenance of element x to the set M with respect to the attribute value $A_{12}$and f_{12} is the degree of non-appurtenance of element x to the set M with respect to the attribute value A_{12} etc.

The neutrosophic plithogenic set:

M = {x($A_{11}(t_{11}, i_{11}, f_{11}), A_{12}(t_{12}, i_{12}, f_{12}), ..., A_{1m1}(t_{1m1}, i_{1m1}, f_{1m1})$; $A_{21}(t_{21}, i_{21}, f_{21}), A_{22}(t_{22}, i_{22}, f_{22}), ..., A_{2m2}(t_{2m2}, i_{2m2}, f_{2m2})$; ... $A_{n1}(t_{n1}, i_{n1}, f_{n1}), A_{n2}(t_{n2}, i_{n2}, f_{n2}), A_{nm}(t_{nm}, i_{nm}, f_{nm})$; with x in U}, where t_{11} is the degree of appurtenance of element x to the set M with respect to the attribute value A_{11},i_{11} is the degree of indeterminacy of element x to the set M with respect to the attribute value $A_{11}$and f_{11} is the degree of non-appurtenance of element x to the set M with respect to the attribute value A_{11}; t_{12} is the degree of appurtenance of element x to the set M with respect to the attribute value A_{12},i_{12} is the degree of indeterminacy of element x to the set M with respect to the attribute value $A_{12}$and f_{12} is the degree of non-appurtenance of element x to the set M with respect to the attribute value A_{12}; etc.

Example. Let U = { a, b, c, d, e, f}, M = { b, c,e}, A = { a_1,a_2,a_3}, V_{a1} = { A_{11},A_{12},A_{13}}, Va_2 = { A_{21},A_{22}} Va_3 = {$A_{31},A_{32},A_{33},A_{34}$}.

The plithogenic number with fuzzy degree of appurtenance to all the attribute values will be of the form P={b(A11(0.2), A_{12}(0.5), A_{13}(0.6), A_{21}(0.7), A_{22}(0.6), A_{31}(0.5), A_{32}(0.4), A_{33}(0.8), A_{34}(0.9)), c(A11(0.3), A_{12}(0.5), A_{13}(0.6), A_{21}(0.5), A_{22}(0.8), A_{31}(0.9), A_{32}(0.7), A_{33}(0.5), A_{34}(0.6))}This plithogenic number may be termed as generalized plithogenic fuzzy number as it encompasses all the attribute values. From the values of intuitionistic and neutrosophic degrees of appurtenance to all the attribute values the generalized plithogenic intuitionistic and generalized plithogenic neutrosophic numbers can be defined.

2.1 | Dominant Attribute Constrained Plithogenic Number

This section also proposes the concept of dominant attribute constrained plithogenic number and it shall be defined by considering only the dominant attribute values.

Let U = { a, b, c, d, e, f}, M = { b, c,e}, A = { a_1,a_2,a_3}, V_{a1} = { A_{11},A_{12},A_{13}}, Va_2 = {A_{21},A_{22}}, Va_3 = {$A_{31},A_{32},A_{33},A_{34}$}.

Contradiction Degree		0	1/3	2/3	0	½	0	¼	2/4	3/4
Attribute Values		A_{11}	A_{12}	A_{13}	A_{21}	A_{22}	A_{31}	A_{32}	A_{33}	A_{34}
Fuzzy Degree of Appurtenance	b	0.2	0.5	0.6	0.7	0.6	0.5	0.4	0.8	0.9
	c	0.3	0.5	0.6	0.5	0.8	0.9	0.7	0.5	0.6
	e	0.5	0.6	0.4	0.7	0.5	0.6	0.8	0.4	0.5
Intuitionistic Degree of Appurtenance	b	(0.7,0.2)	(0.8,0.1)	(0.4,0.6)	(0.7,0.3)	(0.6,0.2)	(0.7,0.1)	(0.8,0.2)	(0.6,0.3)	(0.5,0.3)
	c	(0.8,0.1)	(0.7,0.3)	(0.5,03)	(0.6,0.2)	(0.7,0.3)	(0.6,0.3)	(0.7,0.2)	(0.8,0.1)	(0.6,0.2)
	e	(0.5,0.3)	(0.8,.01)	(0.7,0.2)	(0.8,0.2)	(0.8,0.1)	(0.6,0.2)	(0.7,0.2)	(0.5,0.3)	(0.6,0.3)
Neutrosophic Degree of Appurtenance	b	(0.7,0.2,0.3)	(0.8,0.1,0.3)	(0.6,0.4,0.5)	(0.7,0.2,0.3)	(0.6,0.2,0.3)	(0.5,0.2,0.4)	(0.5,0.1,0.3)	(0.7,0.2,0.3)	(0.7,0.2,0.2)
	c	(0.5,0.1,0.3)	(0.7,0.2,0.3	(0.5,0.1,0.3)	(0.5,0.2,0.4)	(0.7,0.2,0.3)	(0.6,0.4,0.2)	(0.4,0.1,0.3)	(0.7,0.2,0.2)	(0.8,0.1,0.3)
	e	0.8,0.1,0.3)	(0.5,0.1,0.3)	(0.6,0.4,0.5)	(0.7,0.2,0.3)	(0.5,0.1,0.3)	(0.7,0.2,0.3)	(0.6,0.4,0.5)	(0.5,0.1,0.3)	(0.7,0.2,0.2)

In this example, the attribute values A_{11}, A_{21}, A_{31} are considered to be dominant and the plithogenic number considering the values of degree of appurtenance corresponding only to the dominant attribute values are called as Dominant Attribute Constrained Plithogenic Number**.**

Let P1 = {b ($A_{11}(0.5)$, $A_{21}(0.7)$, $A_{31}(0.8)$) , c ($A_{11}(0.4)$, $A_{21}(0.5)$, $A_{31}(0.6)$), b ($A_{11}(0.4)$, $A_{21}(0.6)$, $A_{31}(0.7)$)} and P2 = {b ($A_{11}(0.6)$, $A_{21}(0.5)$, $A_{31}(0.3)$), c ($A_{11}(0.5)$, $A_{21}(0.2)$, $A_{31}(0.5)$), b ($A_{11}(0.5)$, $A_{21}(0.6)$, $A_{31}(0.8)$)}where P1 and P2 are the Dominant Attribute Constrained plithogenic fuzzy numbers with fuzzy degree of appurtenance with respect to the dominant attribute values.

The union of two Dominant Attribute Constrained plithogenic fuzzy numbers is $P1U_FP2$ is defined as max {a1($A_\alpha(t_\alpha)$, $A_\beta(t_\beta)$, ..., $A_\lambda(t_\lambda)$), a2 ($A_\alpha(t_\alpha)$, A_β (t_β), ..., A_λ (t_λ)),……am($A_\alpha(t_\alpha)$, A_β (t_β), ..., A_λ (t_λ))},

Where A_α , A_β, …. A_λ are the dominant attribute values and t_α , t_β, …. t_λ are the respective fuzzy degree of appurtenance with respective to each elements of M.

$P1U_FP2$={b ($A_{11}(0.6)$, $A_{21}(0.7)$,$A_{31}(0.8)$) , c ($A_{11}(0.5)$, $A_{21}(0.5)$,$A_{31}(0.6)$), b ($A_{11}(0.5)$, $A_{21}(0.6)$,$A_{31}(0.8)$)}.

The intersection of two Dominant Attribute Constrained plithogenic fuzzy numbers is P1 $\cap_F$P2 is defined as min {a1($A_\alpha(t_\alpha)$, A_β (t_β), ..., A_λ (t_λ)), a2 ($A_\alpha(t_\alpha)$, A_β (t_β), ..., A_λ (t_λ)),……am($A_\alpha(t_\alpha)$, A_β (t_β), ..., A_λ (t_λ))}.

P1 $\cap_F$P2= {b (A_{11}(0.5), A_{21}(0.5),A_{31}(0.3)) , c (A_{11}(0.4), A_{21}(0.2),A_{31}(0.5)), b (A_{11}(0.4), A_{21}(0.6),A_{31}(0.7))}.

Let P1 = {b (A_{11}(0.7,0.2), A_{21}(0.8,0.1), A_{31}(0.7,0.1)) , c (A_{11}(0.7,0.3), A_{21}(0.4,0.6), A_{31}(0.5,03)), e (A_{11}(0.6,0.2), A_{21}(0.5,0.3), A_{31}(0.7,0.2)} and P2 = { b (A_{11}(0.6,0.3), A_{21} (0.5,0.3), A_{31}(0.6,0.3)) , c (A_{11}(0.7,0.1), A_{21}(0.5,03), A_{31}(0.7,0.3)), e (A_{11}(0.5,0.3), A_{21}(0.6,0.3), A_{31} (0.8,0.2))} where P1 and P2 are the Dominant Attribute Constrained plithogenic intuitionistic fuzzy numbers with intuitionistic fuzzy degree of appurtenance with respect to the dominant attribute values.

The union of two Dominant Attribute Constrained plithogenic intuitionistic fuzzy numbers is P1U_{IF}P2 is defined as (max {a1($A_\alpha(t_\alpha)$, A_β (t_β), ..., A_λ (t_λ)), a2 ($A_\alpha(t_\alpha)$, A_β (t_β), ..., A_λ (t_λ)),……am($A_\alpha(t_\alpha)$, A_β (t_β), ..., A_λ (t_λ)), min {a1($A_\alpha(f_\alpha)$, A_β (f_β), ..., A_λ (f_λ)), a2 ($A_\alpha(f_\alpha)$, A_β (f_β), ..., A_λ (f_λ)),……am($A_\alpha(f_\alpha)$, A_β (f_β), ..., A_λ (f_λ))}.

P1U_{IF}P2 ={b (A_{11}(0.7,0.2), A_{21}(0.8,0.1), A_{31}(0.7,0.1)) , c (A_{11}(0.7,0.1), A_{21}(0.5,0.3), A_{31}(0.7,03)), e (A_{11}(0.6,0.2), A_{21}(0.6,0.3), A_{31}(0.8,0.2)}.

The intersection of two Dominant Attribute Constrained plithogenic intuitionistic fuzzy numbers is P1$\cap_{IF}$P2 is defined as (min{a1($A_\alpha(t_\alpha)$, A_β (t_β), ..., A_λ (t_λ)), a2 ($A_\alpha(t_\alpha)$, A_β (t_β), ..., A_λ (t_λ)),……am($A_\alpha(t_\alpha)$, A_β (t_β), ..., A_λ (t_λ)), max {a1($A_\alpha(f_\alpha)$, A_β (f_β), ..., A_λ (f_λ)), a2 ($A_\alpha(f_\alpha)$, $A_\beta(f_\beta)$, ..., A_λ (f_λ)),……am($A_\alpha(f_\alpha)$, A_β (f_β), ..., A_λ (f_λ))}.

P1$\cap_{IF}$P2 = {b (A_{11}(0.6,0.3), A_{21}(0.5,0.3), A_{31}(0.6,0.3)) , c (A_{11}(0.7,0.3), A_{21} (0.4,0.6), A_{31}(0.5,03)), e (A_{11}(0.5,0.3), A_{21}(0.5,0.3), A_{31}(0.7,0.2)}.

Let P1 = {b (A_{11}(0.7,0.2,0.3), A_{21}(0.8,0.1,0.3), A_{31}(0.6,0.4,0.5)), c (A_{11}(0.6,0.4,0.2), A_{21}(0.5,0.1,0.3), A_{31} (0.7,0.2,0.2)), e (A_{11}(0.6,0.2,0.1), A_{21}(0.5,0.1,0.3), A_{31}(0.7,0.2,0.3)} and P2 = { b (A_{11}(0.6,0.2,0.3) , A_{21} (0.5,0.2,0.4), A_{31}(0.6,0.4,0.2)),c (A_{11}(0.7,0.2,0.3), A_{21}(0.5,0.2,0.4), A_{31}(0.7,0.2,0.3)), e (A_{11}(0.6,0.4,0.2), A_{21}(0.7,0.2,0.3), A_{31}(0.8,0.1,0.3))} where P1 and P2 are the Dominant Attribute Constrained plithogenic neutrosophic numbers with neutrosophic degree of appurtenance with respect to the dominant attribute values.

The union of two Dominant Attribute Constrained plithogenic neutrosophic numbers is P1U_NP2 is defined as (max {a1($A_\alpha(t_\alpha)$, A_β (t_β), ..., A_λ (t_λ)), a2 ($A_\alpha(t_\alpha)$, A_β (t_β), ..., A_λ (t_λ)),……am($A_\alpha(t_\alpha)$, A_β (t_β), ..., A_λ (t_λ)), max {a1($A_\alpha(I_\alpha)$, A_β (I_β), ..., A_λ (I_λ)), a2 ($A_\alpha(I_\alpha)$, A_β (I_β), ..., A_λ (I_λ)),……am($A_\alpha(I_\alpha)$, A_β (I_β), ..., A_λ (I_λ)),min {a1($A_\alpha(f_\alpha)$, A_β (f_β), ..., A_λ (f_λ)), a2 ($A_\alpha(f_\alpha)$, A_β (f_β), ..., A_λ (f_λ)),……am($A_\alpha(f_\alpha)$, A_β (f_β), ..., A_λ (f_λ))}).

P1U_NP2 = = { b (A_{11}(0.7,0.2,0.3) , A_{21} (0.8,0.2,0.3), A_{31}(0.6,0.4,0.2)),c (A_{11}(0.7,0.4,0.2), A_{21}(0.5,0.2,0.3), A_{31}(0.7,0.2,0.2)), e (A_{11}(0.6,0.4,0.1), A_{21}(0.7,0.2,0.3), A_{31}(0.8,0.2,0.3))}.

The intersection of two Dominant Attribute Constrained plithogenic neutrosophic numbers is P1$\cap_N$P2 is defined as (min {a1($A_\alpha(t_\alpha)$, A_β (t_β), ..., A_λ (t_λ)), a2 ($A_\alpha(t_\alpha)$, A_β (t_β), ..., A_λ (t_λ)),……am($A_\alpha(t_\alpha)$, A_β (t_β), ..., A_λ (t_λ)), max {a1($A_\alpha(I_\alpha)$, A_β (I_β), ..., A_λ (I_λ)), a2 ($A_\alpha(I_\alpha)$, A_β (I_β), ..., A_λ (I_λ)),……am($A_\alpha(I_\alpha)$, A_β (I_β), ..., A_λ (I_λ)),max {a1($A_\alpha(f_\alpha)$, A_β (f_β), ..., A_λ (f_λ)), a2 ($A_\alpha(f_\alpha)$, A_β (f_β), ..., A_λ (f_λ)),……am($A_\alpha(f_\alpha)$, A_β (f_β), ..., A_λ (f_λ))}).

P1$\cap_N$P2 = {b (A_{11}(0.6,0.2,0.3), A_{21}(0.5,0.1,0.4) , A_{31}(0.6,0.4,0.5)), c (A_{11}(0.6,0.4,0.3), A_{21}(0.5,0.2,0.4), A_{31}(0.7,0.2,0.3)), e (A_{11}(0.6,0.4,0.2), A_{21}(0.5,0.2,0.3), A_{31}(0.7,0.2,0.3)}.

2.2 | Combined Dominant Attribute Constrained Plithogenic Number

In Combined Dominant Attribute Constrained Plithogenic Number, the attribute values possess combined degree of appurtenance of the attribute values. For instance

P1 = {b (A_{11}(0.7,0.2), A_{21}(0.8,0.1),A_{31} (0.7,0.1)), c (A_{11}(0.5), A_{21} (0.5),A_{31} (0.3),e (A_{11}(0.6,0.4,0.2), A_{21}(0.5,0.2,0.3), A_{31}(0.7,0.2,0.3)}. In this plithogenic representation, the element b has intuitionistic degree of appurtenance with respect to the attribute values, the element c has fuzzy degree of appurtenance with respect to the attribute values and the element e has neutrosophic degree of appurtenance with respect to the attribute values.

On other hand the combined plithogenic number can also be represented as P1 = {b (A_{11}(0.7,0.2), A_{21} (0.8), A_{31}(0.7,0.1,0.1)) , c (A_{11} (0.5), A_{21}(0.7,0.2) , A_{31} (0.3)),e (A_{11}(0.6,0.4,0.2), A_{21}(0.5,0.2), A_{31} (0.7)} in which the element b has the combination of intuitionistic, fuzzy and neutrosophic degree of appurtenance with respect to the dominant attribute values and the other elements c and e also have a combination of degree of appurtenance.

The union and intersection of combined plithogenic numbers shall be computed after converting the combined degrees of appurtenance into a same degree of appurtenance using 2.1, 2.2 or 2.3

Method I. (Imprecision Membership): Any neutrosophic fuzzy set N_A = (T_A, I_A, F_A) including neutrosophic fuzzy values are transformed into intuitionistic fuzzy values or vague values as η(A) = (T_A, f_A) where f_A is estimated the formula stated below which is called as Impression membership method [34].

$$f_A = \begin{cases} F_A + \frac{[1-F_A-I_A][1-F_A]}{[F_A+I_A]} & \text{if } F_A = 0 \\ F_A + \frac{[1-F_A-I_A][F_A]}{[F_A+I_A]} & \text{if } 0 < F_A \le 0.5. \\ F_A + [1-F_A-I_A]\left[0.5 + \frac{F_A-0.5}{F_A+I_A}\right] & \text{if } 0.5 < F_A \le 1 \end{cases}$$

Method II. (Defuzzification): After Method I (median membership), intuitionistic (vague), fuzzy values of the form η(A)= (T_A, f_A) are transformed into fuzzy set including fuzzy values

as<Δ(A)> = <$\frac{T_A}{[T_A+f_A]}$> [34].

The score function of the intuitionistic set of the form (μ_A, ϑ_A) is μ_A-ϑ_A [34].

Let P1 = {b (A_{11}(0.7,0.2), A_{21}(0.8), A_{31}(0.7,0.1,0.1)) , c (A_{11} (0.5), A_{21}(0.7,0.2) , A_{31} (0.3)),e (A_{11}(0.6,0.4,0.2), A_{21}(0.5,0.2), A_{31}(0.7)} and P2 = {b (A_{11}(0.7), A_{21}(0.5,0.2), A_{31}(0.6)), c (A_{11}(0.5,0.2), A_{21}(0.8), A_{31}(0.2)), e (A_{11}(0.6,0.4), A_{21}(0.5,0.2,0.1), A_{31}(0.5)} be two combined plithogenic number with different degrees of appurtenance and it can be converted to plithogenic number with same degree of appurtenance using the above methods I and II. The modified plithogenic numbers are

P_1'= {b (A_{11}(0.5), A_{21}(0.8), A_{31}(0.58)), c (A_{11}(0.5), A_{21}(0.5), A_{31}(0.3)),e (A_{11}(0.64), A_{21}(0.3), A_{31}(0.7))} and
P_2' = {b (A_{11}(0.7), A_{21}(0.3), A_{31}(0.6)), c (A_{11}(0.3), A_{21}(0.8) , A_{31}(0.2)),e (A_{11}(0.2), A_{21}(0.6), A_{31}(0.5)}.

$P_1' \cup P_2'$ = {b (A_{11} (0.7), A_{21} (0.8), A_{31} (0.6)), c (A_{11} (0.5), A_{21} (0.8) , A_{31} (0.3)),e (A_{11} (0.64), A_{21} (0.6), A_{31} (0.7)}.

$P_1' \cap P_2'$= {b (A_{11}(0.5), A_{21}(0.3), A_{31}(0.58)), c (A_{11}(0.3), A_{21}(0.5) , A_{31}(0.2)),e (A_{11}(0.2), A_{21}(0.6), A_{31}(0.5)}.

3 | Plithogenic Sociogram

In this section, the concept of plithogenic sociogram is discussed with a simple illustration based on the conceptualization of Neutrosophic sociogram developed by Smarandache. A group of members are given a questionnaire to give their choices of preference in partaking as a team with other members based on certain attributives.

Let S = $\{s_1,s_2,s_3,s_4,s_5\}$ be the members interviewed with the following questions. The members are asked to give their preferential choices of teaming with respect to the attributes.

Write your friends with whom you want to work as a team with respect to their

Q_1: Degree of compatibility,

Q_2: Optimistic approaches,

Q_3: Disciplinary Knowledge.

These questions are focusing on the attributive preferential choice making.

The attributes are the degree of compatibility, optimistic approach and disciplinary knowledge. The attribute values of the attributes are as follows

Degree of compatibility = {low (Q_{11}), moderate (Q_{12}), high (Q_{13})}.

Optimistic Approach = {Dispositional (Q_{21}), Unrealistic (Q_{22}), comparative (Q_{23})}.

Disciplinary Knowledge = {Excellent (Q_{31}), good (Q_{32}), average (Q_{33})}.

The preferential choice making of the members with respect to the dominant attributive values say high (Q_{13}), Dispositional (Q_{21}), Excellent (Q_{31}) are presented in the form of Dominant attribute constrained plithogenic number in *Table 1*.

Table 1. Attributive preferential choice-making of the members.

Members	Attributive Preferential Choice-Making
s_1	$\{s_2(Q_{13}(0.5),Q_{21}(0.6),Q_{31}(0.8)),s_4(Q_{13}(0.6),Q_{21}(0.7),Q_{31}(0.8))\}$
s_2	$\{s_1(Q_{13}(0.4),Q_{21}(0.7),Q_{31}(0.6)),s_3(Q_{13}(0.5),Q_{21}(0.6),Q_{31}(0.9)), S_5(Q_{13}(0.3),Q_{21}(0.4),Q_{31}(0.6))\}$
s_3	$\{s_2(Q_{13}(0.5),Q_{21}(0.6),Q_{31}(0.7)),s_4(Q_{13}(0.4),Q_{21}(0.2),Q_{31}(0.5))\}$
s_4	$\{s_1(Q_{13}(0.7),Q_{21}(0.8),Q_{31}(0.6)),s_3(Q_{13}(0.7),Q_{21}(0.5),Q_{31}(0.3))\}$
s_5	$\{s_2(Q_{13}(0.5),Q_{21}(0.6),Q_{31}(0.7)),s_4(Q_{13}(0.5),Q_{21}(0.6),Q_{31}(0.6))\}$

S_1 prefers S_2 with the plithogenic fuzzy degree of appurtenance of 0.5 to high degree of compatibility, 0.6 to dispositional optimistic approach and 0.8 to excellent disciplinary knowledge and similarly the preference to S_4 can also be comprehended with the help of fuzzy degree of appurtenance. The approach of plithogenic sociogram is based on the methodology of neutrosophic sociogram.

The evaluation matrix Mk = (mgh), where mgh assumes the degree of appurtenance (in this case, it is fuzzy) of the member sg selecting sh with respect to the dominant attribute values and when g=h mgh = 0. In neutrosophic sociogram the elements of the evaluation matrix assumes either 0 or 1 based on the number of times a member selects another.

The evaluation matrix M_1 for the dominant attribute value Q_{13} is

	s_1	s_2	s_3	s_4	s_5
s_1	0	0.5	0	0.6	0
s_2	0.4	0	0.5	0	0.3
s_3	0	0.5	0	0.4	0
s_4	0.7	0	0.7	0	0
s_5	0	0.5	0	0.5	0

The evaluation matrix M_2 for the dominant attribute value Q_{21} is

	s_1	s_2	s_3	s_4	s_5
s_1	0	0.6	0	0.7	0
s_2	0.7	0	0.6	0	0.4
s_3	0	0.6	0	0.2	0
s_4	0.8	0	0.5	0	0
s_5	0	0.6	0	0.6	0

The evaluation matrix M_3 for the dominant attribute value Q_{31} is

	s_1	s_2	s_3	s_4	s_5
s_1	0	0.8	0	0.8	0
s_2	0.6	0	0.9	0	0.6
s_3	0	0.7	0	0.5	0
s_4	0.6	0	0.3	0	0
s_5	0	0.7	0	0.6	0

In neutrosophic sociogram each question was given weightage but here in plithogenic sociogram the dominant attributes are given weightage. By considering the weights of the dominant attributes values, the final weighted evaluation matrix is determined by assigning the weights as 0.5, 0.25 and 0.25 to the dominant attribute values high (Q_{13}), Dispositional (Q_{21}) and Excellent (Q_{31}) respectively.

	s_1	s_2	s_3	s_4	s_5
s_1	0	0.56	0	0.69	0
s_2	0.56	0	0.6	0	0.47
s_3	0	0.6	0	0.45	0
s_4	0.69	0	0.45	0	0
s_5	0	0.47	0	0	0

	s_1	s_2	s_3	s_4	s_5
s_1	0	0.6	0	0.675	0
s_2	0.525	0	0.625	0	0.4
s_3	0	0.575	0	0.375	0
s_4	0.7	0	0.55	0	0
s_5	0	0.575	0	0.55	0

The fuzzy amicable degree t_{gh} is calculated by using the formula $\frac{2}{t_{gh}} = \frac{1}{f_{gh}} + \frac{1}{f_{hg}}$, where f_{gh} represents the compatibility existing between the members g and h which means the member g prefers h and it is vice-versa for f_{hg}.

The final scores of the members $s_g(i = 1,2,..5)$ of the group, F (s_g) is determined by $\frac{\Sigma_h t_{gh}}{\Sigma_g \Sigma_h t_{gh}}$.

Table 2. Preferential scores of the members.

s_1	0.225632
s_2	0.294224
s_3	0.189531
s_4	0.205776
s_5	0.084838

Based on the scores as in *Table 2*, it is very vivid that the member s_2 has the maximum score and it represents the significance of the member s_2 in the group and his influencing attributes have made s_2 more preferable, on other hand, the member s_5 has the least score and it shows that the member is not much preferred as the attributes of s_5 may not seems to be influential. This preferential ranking is based on considering plithogenic fuzzy degree of appurtenance. Plithogenic intuitionistic fuzzy, plithogenic neutrosophic degrees of appurtenance and the concept of combined plithogenic shall also be used to represent the attributive preferential choice making.

3.1 | Plithogenic Sociogram in Decision-Making

The approach of plithogenic sociogram shall also be used in decision-making on the alternatives that satisfy the criteria. Let A be the set of alternative methods of food processing say

A= $\{A_1,A_2,A_3,A_4,A_5\}$ and C be the set of criteria or the attributives with attributive values.

C = $\{C_1,C_2,C_3\}$,

C = { cost efficiency, energy efficiency, quality conservation}.

The attribute values are

Cost efficiency ={highly economic (C_{11}), moderately economic (C_{12}), lowly economic (C_{13})},

Energy efficiency = { above 90% (C_{21}), above 70% (C_{22}), above 50% (C_{23})},

Quality conservation = {very good (C_{31}), good (C_{32}), average (C_{33})}.

The comparative attributive preferential choice making over compatibility of the alternatives from expert's point of view with respect to the dominant attribute values highly economic (C_{11}), above 90% (C_{21}) and very good (C_{31}) is presented in the *Table 3*.

Table 3. Alternatives and its compatibility comparison.

Alternatives	Comparative Attributive Preferential Choice-Making over Compatibility Expert-I	Expert-II
A_1	$\{A_3(C_{11}(0.4),C_{21}(0.6),C_{31}(0.8), A_4(C_{11}(0.6),C_{21}(0.6),C_{31}(0.7))\}$	$\{A_2(C_{11}(0.6),C_{21}(0.6),C_{31}(0.8)),A_4(C_{11}(0.7),C_{21}(0.8),C_{31}(0.7))\}$
A_2	$\{A_1(C_{11}(0.5),C_{21}(0.8),C_{31}(0.7)), A_3(C_{11}(0.7),C_{21}(0.5),C_{31}(0.8)),A_4(C_{11}(0.8), C_{21}(0.6),C_{31}(0.7))\}$	$\{A_1(C_{11}(0.6),C_{21}(0.6),C_{31}(0.7)), A_3(C_{11}(0.8),C_{21}(0.6),C_{31}(0.8)),A_5(C_{11}(0.9), C_{21}(0.6),C_{31}(0.7))\}$
A_3	$\{A_4(C_{11}(0.5),C_{21}(0.7),C_{31}(0.9)), A_5(C_{11}(0.6),C_{21}(0.7),C_{31}(0.8))\}$	$\{A_1(C_{11}(0.6),C_{21}(0.7),C_{31}(0.8)), A_2(C_{11}(0.7),C_{21}(0.5),C_{31}(0.8)\}$
A_4	$\{A_2(C_{11}(0.6),C_{21}(0.8),C_{31}(0.8)), A_3(C_{11}(0.6),C_{21}(0.5),C_{31}(0.7))\}$	$\{A_1(C_{11}(0.6),C_{21}(0.7),C_{31}(0.7)), A_3(C_{11}(0.5),C_{21}(0.6),C_{31}(0.8))\}$
A_5	$\{A_3(C_{11}(0.7),C_{21}(0.6),C_{31}(0.7)), A_4(C_{11}(0.5),C_{21}(0.6),C_{31}(0.6))\}$	$\{A_2(C_{11}(0.8),C_{21}(0.6),C_{31}(0.5)), A_4(C_{11}(0.5),C_{21}(0.7),C_{31}(0.8))\}$

With respect to the dominant attribute values, the alternative A_1 is compatible in comparison with the alternatives A_3 and A_4, according to the viewpoint of Expert I and compatible in comparison with the alternatives A_2 and A_4 according to the viewpoint of Expert II.

The weights of the dominant attributes values are considered and the final weighted evaluation matrix is determined by assigning the weights as 0.5, 0.25 and 0.25 to the dominant attribute values highly economic (C_{11}), above 90% (C_{21}) and very good (C_{31}), respectively.

	A_1	A_2	A_3	A_4	A_5
A_1	0	0.325	0.275	0.675	0
A_2	0.625	0	0.7125	0.3625	0.3875
A_3	0.3375	0.3375	0	0.325	0.3
A_4	0.325	0.35	0.6	0	0
A_5	0	0.3375	0.3375	0.5875	0

The amicable degree is presented as in the below

	A_1	A_2	A_3	A_4	A_5
A_1	0	0.428	0.3031	0.439	0
A_2	0.428	0	0.458	0.356	0.361
A_3	0.3031	0.458	0	0.422	0.3176
A_4	0.439	0.356	0.422	0	0
A_5	0	0.361	0.3176	0	0

The score values of the alternatives are presented in *Table 4*.

Table 4. Score values of alternatives.

A_1	0.189662
A_2	0.259831
A_3	0.243249
A_4	0.197264
A_5	0.109994

The alternative A_2 is the most preferred method of food processing based on the satisfaction of the dominant attribute values and in comparison with other alternatives. This plithogenic sociogram is used to determine the most influential member in the group based on the attributives and the most preferred alternative in decision-making.

4 | Conclusion

This paper introduces the concept of generalized plithogenic number, dominant attribute constrained plithogenic number, combined dominant attribute constrained plithogenic number and its utility in plithogenic sociogram. On comparing the proposed plithogenic sociogram with neutrosophic sociogram the former approach is more comprehensive in nature. In neutrosophic sociogram, the questions were deterministic and indeterminate in nature, in the sense, the members are asked to make the selection of their choice with whom they are very sure to take part in a quiz or study and also they are not sure of teaming up for the group activities. The calculation was done separately by considering members of deterministic teaming and later together with the deterministic and indeterminate teaming. Finally, based on the neutrosophic amicable degree, the opportunity of enhancing the relationship between the members, leadership index and potential leadership index was discussed. But in the neutrosophic sociogram, the reasons for preferring and hesitance were not much explored which are very significant to enhance the relationship in future. The calculation of the numerical ranges representing the extent of the relationship shall become more meaningful if the attributes are considered. This is the origin of the plithogenic sociogram in which the choice of the members are based on the attributes and the degree of appurtenance states the nature of their preference. The qualitative nature of the members plays a vital role in decision making on the choice of the members preferred. The score values of the members indicate their preference and significance in the group. The members with the least score can be subjected to counselling and made exposed to other kinds of training programs to enhance their attributes of group dynamics. Thus in the plithogenic sociogram with dominant attribute constrained plithogenic number representing the degree of appurtenance, the attributive preferential choice-making appears to be more realistic and pragmatic in nature. This works on the principle of identifying the attribute deficiency of the members and finds the possibilities of enhancing it to improve the efficiency of teamwork. On enriching the attributes of the members then all the members of the group shall team up with each other without any constraints. The proposed concept shall be extended and employed in decision-making and the illustrations of plithogenic sociogram and plithogenic sociogram in decision making shall be discussed under intuitionistic or neutrosophic degrees of appurtenance.

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HYPERSOFT SETS

Generalization of TOPSIS for Neutrosophic Hypersoft Set using Accuracy Function and its Application

Muhammad Saqlain, Muhammad Saeed, Muhammad Rayees Ahmad, Florentin Smarandache

Muhammad Saqlain, Muhammad Saeed, Muhammad Rayees Ahmad, Florentin Smarandache (2019). Generalization of TOPSIS for Neutrosophic Hypersoft set using Accuracy Function and its Application. *Neutrosophic Sets and Systems* 27, 131-137

Abstract. The purpose of MCDM is to determine the best option amongst all the probable options. Due to linguistic assessments, the traditional crisp techniques are not good to solve MCDM problems. This paper deals with the generalization of TOPSIS for neutrosophic hypersoft set primarily based issues explained in section 3. In section 4, the proposed technique is implemented. The proposed technique is easy to implement, and precise and sensible for fixing the MCDM problem with multiple-valued neutrosophic data. In the end, the applicability of the developed method, the problem of parking on which decision maker has normally vague and imprecise knowledge is used. It seems that the outcomes of these examinations are terrific.

Keywords: Uncertainties, Decision making, FNSS, FNHSS, Linguistic variable, Accuracy Function AF, TOPSIS

1 Introduction

To describe the characteristics people generally use apt values when they come across the decision-making problems. On the other hand, it is observed that in an environment of real decision making we face various complex and alterable factors and for these fuzzy expressions, the decision makers take help from the linguistic evaluations. For instance, the evaluation values are represented with the use of expressions like excellent, v. good, and good by decision makers. Zadeh [15-16] proposed a linguistic variable set to express the evaluation values. The idea of vague linguistic variables and the operational rules were devised by Xu [12]. The level of a linguistic variable just depicts the values of linguistic evaluation of a decision maker, but these can not aptly describe the vague level of decision maker particularly in the environment of linguistic evaluation. This flaw can be taken into account by adjoining the linguistic variables as well as by putting forward its other sets. For example, Ye [13] put forward an interval Neutrosophic linguistic set (INLS) and an interval Neutrosophic linguistic number (INLN); Ye [14] also found a single and multiple valued Neutrosophic linguistic set (SVNLS & MVNLS).

At the primary, soft set theory was planned by a Russian scientist [7] that was used as a standard mathematical mean to come back across the difficulty of hesitant and uncertainty. He additionally argues that however, the same theory of sentimental set is free from the parameterization inadequacy syndrome of fuzzy set theory, rough set theory, and applied mathematics. Neutrosophic set could be a terribly powerful tool to agitate incomplete and indeterminate data planned by F. Smarandache [10] and has attracted the eye of the many students [1], which might offer the credibleness of the given linguistic analysis worth and linguistic set can offer qualitative analysis values. Florentin [11] generalized soft set to hypersoft set by remodeling the function into a multi-attribute function, NHSS (Neutrosophic Hyper Soft Set) is additionally planned in his pioneer work.

[8] applies neutrosophic TOPSIS and AHP to reinforce the normal strategies of personal choice to realize the perfect solutions. To investigate and verify the factors influencing the choice of SCM suppliers, [2] used the neutrosophic set for deciding and analysis technique (DEMATEL). [3] offers a unique approach for estimating the sensible medical devices (SMDs) choice method in an exceedingly cluster deciding (GDM) in an exceedingly obscure call atmosphere. Neutrosophic with TOPSIS approach is applied within the decision-making method to handle the unclearness, incomplete knowledge and therefore the uncertainty, considering the selections criteria within the knowledge collected by the choice manufacturers (DMs) [3]. [4] projected a technique of the ANP method and therefore the VIKOR underneath the neutrosophic atmosphere for managing incomplete info and high order inexactitude. [9] used a neutrosophic soft set to predict FIFA 2018.

The sturdy ranking technique with neutrosophic set [5] to handle practices and performances in green supply chain management (GSCM). [6] projected T2NN, Type 2 neutrosophic number, which might accurately describe real psychological feature info.

In this paper, the generalization of TOPSIS for the neutrosophic hypersoft set is proposed. In the proposed method Fuzzy Neutrosophic Numbers FNNs are converted into crisp by using accuracy function N(A).

2 Preliminaries

Linguistic Set [9]: In a crisp set, an element $\boldsymbol{Y}$ in the universe ℵ is either a member of some crisp set **À** or not. It can be represented mathematically with indicator function: **μ À (У) =** {1, if *У* belongs to **À** and **0**, if *У* doesn't belong to **À**}.

Fuzzy Set [10]: Fuzzy set μ in a universe ℵ is a mapping μ: ℵ → [0,1] which assigns a degree of membership to each element with symbol μ À (*y*) such that μ À (*y*) ϵ [0, 1].
Fuzzy Neutrosophic set: A Fuzzy Neutrosophic set FNs $\boldsymbol{\mathcal{A}}$ over the universe of discourse $\boldsymbol{\mathcal{X}}$ is defined as

$\boldsymbol{\mathcal{A}} = < x, T_{\boldsymbol{\mathcal{A}}}(x), I_{\boldsymbol{\mathcal{A}}}(x), F_{\boldsymbol{\mathcal{A}}}(x) >, \ \ x \in \boldsymbol{\mathcal{X}} \ where \ \ T, F, I: \boldsymbol{\mathcal{X}} \rightarrow [0,1]$ &

$0 \leq T_{\boldsymbol{\mathcal{A}}}(x) + I_{\boldsymbol{\mathcal{A}}}(x) + F_{\boldsymbol{\mathcal{A}}}(x) \leq 3.$

Fuzzy Neutrosophic soft set: Let $\boldsymbol{\mathcal{X}}$ be the initial universal set and Ē be a set of parameters. Consider a non-empty set $\boldsymbol{\mathcal{A}}$, $\boldsymbol{\mathcal{A}} \subset$ Ē. Let Ƥ ($\boldsymbol{\mathcal{X}}$) denote the set of all FNs of $\boldsymbol{\mathcal{X}}$.

Throughout this paper Fuzzy Neutrosophic soft set is denoted by FNS set / FNSS.

3 Algorithm

Let the function be

$$F: P_j \times P_k \times P_l \times \ldots \times P_m \rightarrow P(\boldsymbol{\mathcal{X}}) \text{ , such that } P_q = P_j, P_k, P_l, \ldots, P_m$$

Where

$P_j = p_1, p_2, p_{3,\ldots} p_n \quad 1 \leq j \leq n$

$P_k = p_1, p_2, p_{3,\ldots} p_n \quad 1 \leq k \leq n$

$P_l = p_1, p_2, p_{3,\ldots} p_n \quad 1 \leq l \leq n$

$\vdots$

$P_m = p_1, p_2, p_{3,\ldots} p_n \quad 1 \leq m \leq n$

are multiple valued neutrosophic attributes and $\boldsymbol{\mathcal{X}}$ is a universe of discourse.

Step 1: Construct a matrix of multiple-valued P_q of attributes of order $m \times n$.

$$A = [p_{qr}]_{m \times n}, \qquad 1 \leq q \leq m, \ \ 1 \leq r \leq n$$

Step 2: Fill the column values with zeros if multiple valued attributes are less than equal to n to form a matrix of order $m \times n$ as defined in the below example.

Step 3: Decision makers will assign fuzzy neutrosophic numbers (FNNs) to each multiple valued linguistic variables.

Step 4: Selection of the subset of NHSS.

Step 5: Conversion of fuzzy neutrosophic values of step: 4 into crisp numbers by using accuracy function $A(N)$.

$$A(N) = \left[\frac{P_{ij}}{3}\right]$$

Step 6: Calculate the relative closeness by using the TOPSIS technique of MCDM.

Step 7: Determine the rank of relative closeness by arranging in ascending order.

Remark 1: In step 2, if all values of each tuple of complete row or complete column are null, then eliminate that respective row or column.

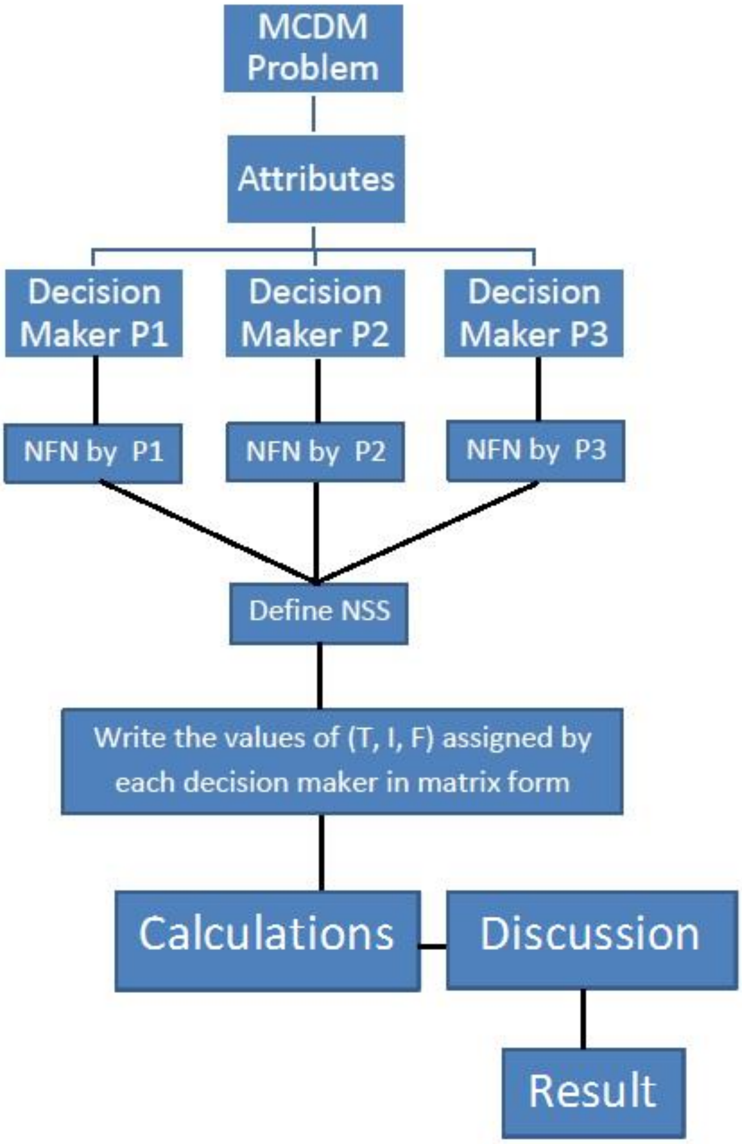

Figure 1: Algorithm design for the proposed technique

We apply the neutrosophic set theory to handle vague data, imprecise knowledge, incomplete information, and linguistic imprecision. The efficiency of the proposed method is evaluated by considering the parking problem as stated below.

The environment of decision making is a multi-criteria decision making surrounded by inconsistency and uncertainty. This paper contributes to supporting the parking problem by integrating a neutrosophic soft set with the technique for order preference by similarity to an ideal solution (TOPSIS) to illustrate an ideal solution amongst different alternatives.

4 Problem Statement

Environmental pollution strongly affects life in cities. The major issue of blockage is due to an excessive number of vehicles in the cities. This causes a major problem in finding a proper place for parking. Therefore, various techniques are implemented to cover this problem. Among them, an application of NHSS (Neutrosophic Hypersoft Set) is used.

In figure 2: there is an elaboration of the trip of a vehicle driver, to his final point. Now he has three numbers of choices to park his vehicle at different distances. So, by Using the NHSS algorithm he will be able to find the nearby spot to stand his vehicle. The driver is intended to go there in the minimum time. This work helped in the Following ways:

- Four Linguistic inputs and an output.
- During his trip how many traffic signals are sensed by the sensor?
- The measure of motor threshold on the way up to final spot is shown by PCU (Parking Car Unit) and
- The Separation between the parking slot and the final point.

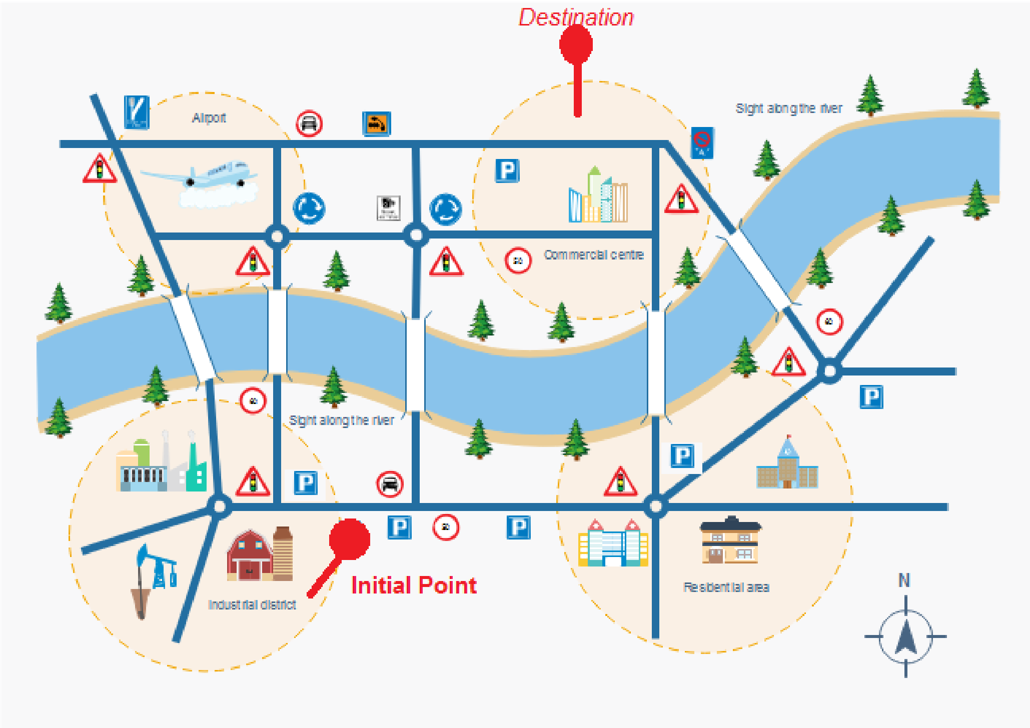

Figure 2: Initial Problem Model

5 Modelling problem into NHSS form

Due to fractional knowledge about the attributes as well as lack of information, mostly the decision makers are observed to be using certain linguistic variables instead of exact values for evaluating characteristics. In such a situation, preference information of alternatives provided by the decision makers may be vague, imprecise, or incomplete.

Sr. #	Linguistic Variable	Code	NFN 1	NFN 2	NFN 3	NFN 4
1	Normal	α	(0.4,0.1,0.0)	(0.3,0.3,0.2)	(0.7,0.2,0.3)	(1.0,1.0,1.0)
2	High	β	(0.3,0.5,0.2)	(0.1,0.1,0.1)	(0.5,0.5,0.3)	(0.5,0.3,0.5)
3	Medium	γ	(0.6,0.6,0.2)	(0.2,0.1,0.1)	(0.6,0.3,0.3)	(0.6,0.4,0.4)
4	Distance i.e., Near	N	(0.2,0.0,0.2)	(0.1,0.2,0.1)	(0.6,0.6,0.1)	(0.4,0.5,0.4)
5	Distance i.e., Far	ב	(0.4,0.4,0.1)	(0.1,0.3,0.4)	(0.1,0.4,0.1)	(0.4,0.2,0.1)
6	No. of Trafic Signal i.e., one	--	(0.5,0.2,0.4)	(0.5,0.4,0.3)	(0.3,0.3,0.3)	(0.6,0.6,0.2)

7	No. of Trafic Signal i.e., two	=	(0.5,0.2,0.1)	(0.5,0.2,0.1)	(0.6,0.5,0.5)	(1.0,1.0,1.0)
8	No. of Trafic Signal i.e., three	≡	(0.4,0.4,0.2)	(0.2,0.1,0.2)	(0.3,0.3,0.1)	(0.4,0.4,0.6)
9	Parking Space i.e., medium	M	(0.3,0.6,0.2)	(0.3,0.6,0.2)	(0.3,0.6,0.2)	(1.0,1.0,1.0)
10	Parking Space i.e., high	H	(0.3,0.1,0.4)	(0.3,0.3,0.3)	(0.1,0.2,0.5)	(0.6,0.6,0.2)

Table 1: Neutrosophic fuzzy number and corresponding linguistic variable.

6 Numerical calculations of problem

Let $F: P_1 \times P_2 \times P_3 \times P_4 \rightarrow \mathcal{P}(\mathcal{X})$, where $\mathcal{X}$ is the universe of discourse, such that

$P_1 = Trafic\ Threshold = \{\alpha, \beta, \gamma\}$

$P_2 = Distance\ of\ destination\ from\ initial\ point = \{N, ב\}$

$P_3 = No.of\ trafic\ lights = \{T_1, T_2, T_3\}$

$P_4 = Distance\ from\ parking\ area\ to\ destination\ point\ = \{M, h\}$

Sr. #	PCU	Distance	No. of traffic signals	Parking space
1	A	N	--	M
2	B	ב	=	h
3	γ		≡	

Table 2: Linguistic variables used in parking problem.

Consider a multiple valued neutrosophic hyper soft set $A = \{P_1, P_3, P_4\}$ such that

$$F(A) = F(\alpha, T_2, M) = \{\alpha(0.4,0.1,0.0), T_2(0.5,0.2,0.1), M(0.3,0.6,0.2), \alpha(0.7,0.3,0.2), T_2(0.6,0.5,0.5), M(0.1,0.5,0.4)$$

$$\alpha(1.0,1.0,1.0), T_2(1.0,1.0,1.0), M(1.0,1.0,1.0)\}$$

Step 1: Construct a matrix of multiple valued Pq of attributes of order $m \times n$.

$$\begin{bmatrix} \alpha & \beta & \gamma \\ N & \Im & \\ T_1 & T_2 & T_3 \\ M & h & \end{bmatrix}$$

Step 2: Fill the column values with zeros if multiple valued attributes are less than equal to n to form a matrix of order $m \times n$ as defined:

$$\begin{bmatrix} \alpha & \beta & \gamma \\ N & \Im & 0 \\ T_1 & T_2 & T_3 \\ M & h & 0 \end{bmatrix}$$

Step 3: The decision makers gives the values to the selected subset i.e. $F(\alpha, T_2, M)$.

$$\begin{bmatrix} (0.4,0.1,0.0) & (0.5,0.2,0.1) & (0.3,0.6,0.2) \\ (0.7,0.2,0.3) & (0.6,0.5,0.5) & (0.3,0.6,0.2) \\ (1.0,1.0,1.0) & (1.0,1.0,1.0) & (1.0,1.0,1.0) \end{bmatrix}$$

Step 4: Conversion of fuzzy neutrosophic values of step 4 into crisp numbers by using accuracy function $A(N)$.

$$\begin{bmatrix} (0.4+0.1+0.0)/3 & (0.5+0.2+0.1)/3 & (0.3+0.6+0.2)/3 \\ (0.7+0.2+0.3)/3 & (0.6+0.5+0.5)/3 & (0.3+0.6+0.2)/3 \\ (1.0+1.0+1.0)/3 & (1.0+1.0+1.0)/3 & (1.0+1.0+1.0)/3 \end{bmatrix}$$

$$\begin{bmatrix} 0.17 & 0.27 & 0.37 \\ 0.4 & 0.53 & 0.37 \\ 1 & 1 & 1 \end{bmatrix}$$

Step 5: Now we will apply the TOPSIS on the resulting matrix.

	A	T_2	M
P_1	0.17	0.40	1
P_3	0.27	0.53	1
P_4	0.37	0.37	1

Table 3: Decision matrix of the parking problem.

Applying the technique of TOPSIS on the above-mentioned matrix obtained in step 5, the following are the results.

Si+	Si-	ci	Rank
0.177459059	0.015787	0.081693	3
0.081871695	0.117439	0.589226	2
0.084195951	0.163743	0.660417	1

Table 4: Results of calculations done by applying TOPSIS technique of MCDM

Graphical representation of the results obtained by applying the TOPSIS technique of MCDM is shown below in figure 3.

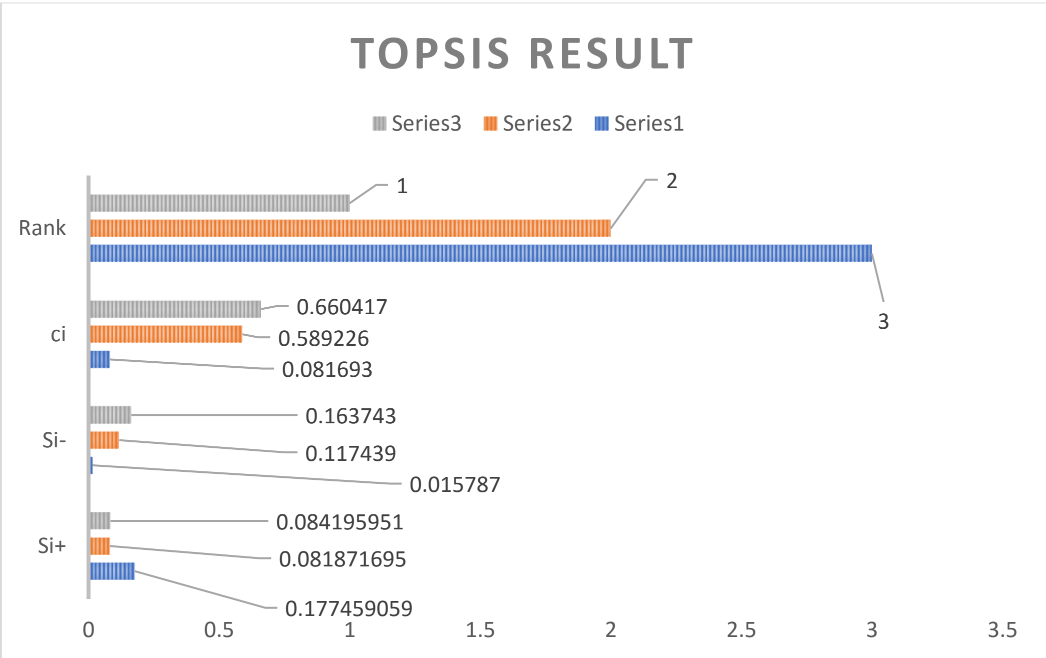

Figure 3: Graphical representation of results done by applying TOPSIS technique of MCDM

In figure 3, $P_1 = series\ 1, P_3 = series\ 2$ and $P_4 = series\ 3$ and the result shows that P_4 is the best alternative for the shortest time to reach the destination for the problem discussed above.

Conclusion

This paper introduces the Generalized Fuzzy TOPSIS by using an accuracy function for NHSS given in [4]. The proposed technique is used to solve a parking problem. Results show that the technique can be implemented to solve the MCDM problem with multiple-valued neutrosophic data in a vague and imprecise environment. In the future, the stability of the proposed technique is to be investigated and the proposed algorithm can be used in neutrosophic set (NS) theory to handle vague data, imprecise knowledge, incomplete information, and linguistic imprecision.

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Aggregate Operators of Neutrosophic Hypersoft Set

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Muhammad Saqlain, Sana Moin, Muhammad Naveed Jafar, Muhammad Saeed, Florentin Smarandache (2020). Aggregate Operators of Neutrosophic Hypersoft Set. Neutrosophic Sets and Systems 32, 294-306

Abstract: Multi-criteria decision making (MCDM) is concerned about organizing and taking care of choice and planning issues including multi-criteria. When attributes are more than one, and further bifurcated, neutrosophic softset environment cannot be used to tackle such type of issues. Therefore, there was a dire need to define a new approach to solve such type of problems, So, for this purpose a new environment namely, Neutrosophic Hypersoft set (NHSS) is defined. This paper includes basics operator's like union, intersection, complement, subset, null set, equal set etc., of Neutrosophic Hypersoft set (NHSS). The validity and the implementation are presented along with suitable examples. For more precision and accuracy, in future, proposed operations will play a vital role is decision-makings like personal selection, management problems and many others.

Keywords: MCDM, Uncertainty, Soft set, Neutrosophic soft set, Hyper soft set.

1. Introduction

The idea of fuzzy sets was presented by Lotfi A. Zadeh in 1965 [1]. From that point the fuzzy sets and fuzzy logic have been connected in numerous genuine issues in questionable and uncertain conditions. The conventional fuzzy sets are based on the membership value or the level of membership value. A few times it might be hard to allot the membership values for fuzzy sets. Therefore, the idea of interval valued fuzzy sets was proposed [2] to catch the uncertainty for membership values. In some genuine issues like real life problems, master framework, conviction framework, data combination, etc., we should consider membership just as the non- membership values for appropriate depiction of an object in questionable and uncertain condition. Neither the fuzzy sets nor the interval valued fuzzy sets is convenient for such a circumstance. Intuitionistic fuzzy sets proposed by Atanassov [3] is convenient for such a circumstance. The intuitionistic fuzzy sets can just deal with the inadequate data considering both the membership and non-membership values. It doesn't deal with the vague and conflicting data which exists in conviction framework.
Smarandache [4] presented the idea of Neutrosophic set which is a scientific apparatus for taking care of issues including uncertain, indeterminacy and conflicting information. Neutrosophic set indicate truth membership value (T), indeterminacy membership value (I) and falsity membership value (F). This idea is significant in numerous application regions since indeterminacy is evaluated exceptionally and the truth membership values, indeterminacy membership values and falsity membership values are independent.

The idea of soft sets was first defined by Molodtsov [5] as a totally new numerical device for taking care of issues with uncertain conditions. He defines a soft set as a parameterized family of

subsets of universal set. Soft sets are useful in various regions including artificial insight, game hypothesis and basic decision-making problems [6] and it serves to define various functions for various parameters and utilize values against defined parameters. These functions help us to oversee various issues and choices throughout everyday life.
In the previous couple of years, the essentials of soft set theory have been considered by different researchers. Maji et al. [7] gives a hypothetical study of soft sets which covers subset and super set of a soft set, equality of soft sets and operations on soft sets, for Example, union, intersection, AND and OR-Operations between different sets. Ali at el. [8] presented new operations in soft set theory which includes restricted union, intersection and difference. Cagman and Enginoglu [9, 10] present soft matrix theory which substantiated itself a very significant measurement in taking care of issues while making various choices. Singh and Onyeozili [11] come up with the research that operations on soft set is equivalent to the corresponding soft matrices. From Molodsov [9, 6, 5, 12] up to present, numerous handy applications identified with soft set theory have been presented and connected in numerous fields of sciences and data innovation.

Maji [13] come up with Neutrosophic soft set portrayed by truth, indeterminacy, and falsity membership values which are autonomous in nature. Neutrosophic soft set can deal with inadequate, uncertain, and inconsistence data, while intuitionistic fuzzy soft set and fuzzy soft set can just deal with partial data.

Smarandache [14] presented a new technique to deal with uncertainty. He generalized the soft to hyper soft set by converting the function into multi-decision function. Smarandache, [15, 16, 17, 18, 19, 20] also discuss the various extension of neutrosophic sets in TOPSIS and MCDM. Saqlain *et.al*. [21] proposed a new algorithm along with a new decision-making environment. Many other novel approaches are also used by many researches [22-39] in decision makings.

1.1 Contribution

Since uncertainty is human sense which for the most part surrounds a man while taking any significant choice. Let's say if we get a chance to pick one best competitor out of numerous applicants, we originally set a few characteristics and choices that what we need in our chose up-and-comer. based on these objectives we choose the best one. To make our decision easy we use different techniques. The purpose of this paper is to overcome the uncertainty problem in more precise way by combing Neutrosophic set with Hypersoft set. This combination will produce a new mathematical tool "Neutrosophic Hypersoft Set" and will play a vital role in future decision-making research.

2.Preliminaries

Definition 2.1: Soft Set

Let ξ be the universal set and € be the set of attributes with respect to ξ. Let P(ξ) be the power set of ξ and Ą ⊆ € . A pair (F, Ą) is called a soft set over ξ and its mapping is given as

$$₣: Ą \to P(\xi)$$

It is also defined as:

$$(₣, Ą) = \{₣(e) \in P(\xi): e \in €, ₣(e) = \emptyset \; if \; e \neq Ą\}$$

Definition 2.2: Neutrosophic Soft Set

Let ξ be the universal set and € be the set of attributes with respect to ξ. Let P(ξ) be the set of Neutrosophic values of ξ and Ą ⊆ € . A pair (F, Ą) is called a Neutrosophic soft set over ξ and its mapping is given as

$$₣: Ą \to P(\xi)$$

Definition 2.3: Hyper Soft Set:

Let ξ be the universal set and $P(\xi)$ be the power set of ξ. Consider $l^1, l^2, l^3 \dots l^n$ for $n \geq 1$, be n well-defined attributes, whose corresponding attributive values are respectively the set $L^1, L^2, L^3 \dots L^n$ with $L^i \cap L^j = \emptyset$, for $i \neq j$ and $i, j \epsilon \{1,2,3 \dots n\}$, then the pair $(₣, L^1 \times L^2 \times L^3 \dots L^n)$ is said to be Hypersoft set over ξ where

$$₣: L^1 \times L^2 \times L^3 \dots L^n \to P(\xi)$$

3. Calculations

Definition 3.1: Neutrosophic Hypersoft Set (NHSS)

Let ξ be the universal set and $P(\xi)$ be the power set of ξ. Consider $l^1, l^2, l^3 \dots l^n$ for $n \geq 1$, be n well-defined attributes, whose corresponding attributive values are respectively the set $L^1, L^2, L^3 \dots L^n$ with $L^i \cap L^j = \emptyset$, for $i \neq j$ and $i, j \epsilon \{1,2,3 \dots n\}$ and their relation $L^1 \times L^2 \times L^3 \dots L^n = \$$, then the pair $(₣, \$)$ is said to be Neutrosophic Hypersoft set (NHSS) over ξ where

$₣: L^1 \times L^2 \times L^3 \dots L^n \to P(\xi)$ and

$₣(L^1 \times L^2 \times L^3 \dots L^n) = \{< x, T(₣(\$)), I(₣(\$)), F(₣(\$)) >, x \in \xi\}$ where T is the membership value of truthiness, I is the membership value of indeterminacy and F is the membership value of falsity such that $T, I, F: \xi \to [0,1]$ also $0 \leq T(₣(\$)) + I(₣(\$)) + F(₣(\$)) \leq 3$.

Example 3.1:

Let ξ be the set of decision makers to decide best mobile phone given as

$$\xi = \{m^1, m^2, m^3, m^4, m^5\}$$

also consider the set of attributes as

$$s^1 = Mobile\ type, s^2 = RAM, s^3 = Sim\ Card, s^4 = Resolution, s^5 = Camera, s^6 = Battery\ Power$$

And their respective attributes are given as

$S^1 = Mobile\ type = \{Iphone, Samsung, Oppo, lenovo\}$

$S^2 = RAM = \{8\ GB, 4\ GB, 6\ GB, 2\ GB\}$

$S^3 = Sim\ Card = \{Single, Dual\}$

$S^4 = Resolution = \{1440 \times 3040\ pixels, 1080 \times 780\ pixels, 2600 \times 4010\ pixels\}$

$S^5 = Camera = \{12\ MP, 10MP, 15MP\}$

$S^6 = Battery\ Power = \{4100\ mAh, 1000\ mAh, 2050\ mAh\}$

Let the function be $₣: S^1 \times S^2 \times S^3 \times S^4 \times S^5 \times S^6 \to P(\xi)$

Below are the tables of their Neutrosophic values

Table 1: Decision maker Neutrosophic values for mobile type

$S^1(Mobile\ type)$	m^1	m^2	m^3	m^4	m^5
Iphone	(0.3, 0.6, 0.7)	(0.7, 0.6, 0.4)	(0.4, 0.5, 0.7)	(0.6, 0.5, 0.3)	(0.5, 0.3, 0.8)
Samsung	(0.7, 0.5, 0.6)	(0.3, 0.2, 0.1)	(0.3, 0.6, 0.2)	(0.8, 0.1, 0.2)	(0.5, 0.4, 0.5)
Oppo	(0.5, 0.2, 0.1)	(0.9, 0.5, 0.3)	(0.9, 0.4, 0.1)	(0.9, 0.3, 0.1)	(0.6, 0.1, 0.2)
Lenovo	(0.5, 0.3, 0.2)	(0.5, 0.2, 0.1)	(0.8, 0.5, 0.2)	(0.6, 0.4, 0.3)	(0.7, 0.4, 0.2)

Table 2: Decision maker Neutrosophic values for RAM

$S^2(RAM)$	m^1	m^2	m^3	m^4	m^5
8 GB	(0.3, 0.4, 0.7)	(0.4, 0.5, 0.7)	(0.5, 0.6, 0.8)	(0.5, 0.3, 0.8)	(0.3, 0.6, 0.7)
4 GB	(0.4, 0.2, 0.5)	(0.3, 0.6, 0.2)	(0.4, 0.7, 0.3)	(0.5, 0.4, 0.5)	(0.7, 0.5, 0.6)
6 GB	(0.7, 0.2, 0.3)	(0.9, 0.4, 0.1)	(0.8, 0.3, 0.2)	(0.6, 0.1, 0.2)	(0.5, 0.2, 0.1)
2 GB	(0.8, 0.2, 0.1)	(0.8, 0.5, 0.2)	(0.9 0.4, 0.1)	(0.7, 0.4, 0.2)	(0.5, 0.3, 0.2)

Table 3: Decision maker Neutrosophic values for sim card

$S^3(Sim\ Card)$	m^1	m^2	m^3	m^4	m^5
Single	(0.6, 0.4, 0.3)	(0.6, 0.5, 0.3)	(0.5, 0.4, 0.3)	(0.7, 0.8, 0.3)	(0.9, 0.2, 0.1)
Dual	(0.8, 0.2, 0.1)	(0.4, 0.8, 0.7)	(0.7, 0.3, 0.2)	(0.3, 0.6, 0.4)	(0.8, 0.4, 0.2)

Table 4: Decision maker Neutrosophic values for resolution

$S^4(Resolution)$	m^1	m^2	m^3	m^4	m^5
1440×3040	(0.7, 0.8, 0.3)	(0.7, 0.5, 0.3)	(0.6, 0.4, 0.3)	(0.5, 0.6, 0.9)	(0.4, 0.5, 0.3)
1080×780	(0.3, 0.6, 0.4)	(0.7, 0.3, 0.2)	(0.8, 0.3, 0.1)	(0.6, 0.4, 0.7)	(0.3, 0.5, 0.8)
2600×4010	(0.5, 0.2, 0.1)	(0.6, 0.3, 0.4)	(0.5, 0.7, 0.2)	(0.9, 0.3, 0.1)	(0.7, 0.4, 0.3)

Table 5: Decision maker Neutrosophic values for camera

$S^5(Camera)$	m^1	m^2	m^3	m^4	m^5
12 MP	(0.6, 0.4, 0.3)	(0.7, 0.8, 0.3)	(0.6, 0.4, 0.3)	(0.4, 0.5, 0.3)	(0.9, 0.2, 0.1)
10 MP	(0.8, 0.3, 0.1)	(0.3, 0.6, 0.4)	(0.8, 0.2, 0.1)	(0.3, 0.5, 0.8)	(0.8, 0.4, 0.2)
15 MP	(0.5, 0.7, 0.2)	(0.5, 0.2, 0.1)	(0.8, 0.5, 0.2)	(0.7, 0.4, 0.3)	(0.7, 0.4, 0.2)

Table 6: Decision maker Neutrosophic values for battery power

$S^6(Battery\ Power)$	m^1	m^2	m^3	m^4	m^5
4100 mAh	(0.7, 0.8, 0.3)	(0.7, 0.6, 0.4)	(0.4, 0.5, 0.7)	(0.9, 0.2, 0.1)	(0.5, 0.3, 0.8)
1000 mAh	(0.3, 0.6, 0.4)	(0.3, 0.2, 0.1)	(0.3, 0.6, 0.2)	(0.8, 0.4, 0.2)	(0.5, 0.4, 0.5)
2050 mAh	(0.5, 0.2, 0.1)	(0.9, 0.5, 0.3)	(0.9, 0.4, 0.1)	(0.7, 0.4, 0.2)	(0.6, 0.1, 0.2)

Neutrosophic Hypersoft set is define as,

$$₣: (S^1 \times S^2 \times S^3 \times S^4 \times S^5 \times S^6) \rightarrow P(\xi)$$

Let's assume $₣(\$) = ₣(samsung, 6\ GB, Dual) = \{m^1, m^4\}$

Then Neutrosophic Hypersoft set of above assumed relation is

$$₣(\$) = ₣(samsung, 6\ GB, Dual\ ) = \{$$
$$< m^1, (samsung\{0.7, 0.5, 0.6\}, 6\ GB\{0.7, 0.2, 0.3\}, Dual\{0.8,0.2,0.1\}) >$$
$$< m^4(samsung\{0.8,0.1,0.2\}, 6\ GB\{0.6, 0.1, 0.2\}, Dual\{0.3, 0.6,0.4\}) >\}$$

Its tabular form is given as

Table 7: Tabular Representation of Neutrosophic Hypersoft Set

$₣(\$) = ₣(\boldsymbol{samsung}, \boldsymbol{6\ GB}, \boldsymbol{Dual}\ )$	$\boldsymbol{m^1}$	$\boldsymbol{m^4}$
Samsung	(0.7,0.5, 0.6)	(0.8, 0.1, 0.2)
6 GB	(0.7, 0.2, 0.3)	(0.6, 0.1, 0.2)
Dual	(0.8, 0.2, 0.1)	(0.3, 0.6, 0.4)

Definition 3.2: Neutrosophic Hypersoft Subset

Let $₣(\$^1)$ and $₣(\$^2)$ be two Neutrosophic Hypersoft set over ξ. Consider $l^1, l^2, l^3 \dots l^n$ for $n \geq 1$, be n well-defined attributes, whose corresponding attributive values are respectively the set $L^1, L^2, L^3 \dots L^n$ with $L^i \cap L^j = \emptyset$, for $i \neq j$ and $i, j \epsilon \{1,2,3 \dots n\}$ and their relation $L^1 \times L^2 \times L^3 \dots L^n = \$$ then $₣(\$^1)$ is the Neutrosophic Hypersoft subset of $₣(\$^2)$ if

$$T(₣(\$^1)) \leq T\big(₣(\$^2)\big)$$
$$I(₣(\$^1)) \leq I\big(₣(\$^2)\big)$$

$$F(₣(\$^1)) \geq F\big(₣(\$^2)\big)$$

Numerical Example of Subset

Consider the two NHSS $₣(\$^1)$ and NHSS $₣(\$^2)$ over the same universe $\xi = \{m^1, m^2, m^3, m^4, m^5\}$. The NHSS $₣(\$) = ₣(samsung, 6\ GB, Dual\ ) = \{m^1, m^4\}$ is the subset of NHSS $₣(\$^2) = ₣(Samsung, 6GB) = \{m^1\}$ if $T(₣(\$^1)) \leq T(₣(\$^2))$, $I(₣(\$^1)) \leq I(₣(\$^2))$, $F(₣(\$^1)) \geq F(₣(\$^2))$. Its tabular form is given below

Table 8: Tabular Representation of NHSS $₣(\$^1)$

$₣(\$^1) = ₣(\boldsymbol{samsung, 6\ GB, Dual}\ )$	$\boldsymbol{m^1}$	$\boldsymbol{m^4}$
Samsung	(0.7,0.5, 0.6)	(0.8, 0.1, 0.2)
6 GB	(0.7, 0.2, 0.3)	(0.6, 0.1, 0.2)
Dual	(0.8, 0.2, 0.1)	(0.3, 0.6, 0.4)

Table 9: Tabular Representation of NHSS $₣(\$^2)$

$₣(\$^2) = ₣(\boldsymbol{samsung, 6\ GB})$	$\boldsymbol{m^1}$
Samsung	(0.9, 0.6, 0.3)
6 GB	(0.8, 0.4, 0.1)

This can also be written as

$$₣(\$^1) \subset ₣(\$^2) = ₣(samsung, 6\ GB, Dual\ ) \subset ₣(samsung, 6\ GB)$$

$$= \begin{Bmatrix} < m^1, (samsung\{0.7, 0.5, 0.6\}, 6\ GB\{0.7, 0.2, 0.3\}, Dual\{0.8,0.2,0.1\}) >, \\ < m^4(samsung\{0.8,0.1,0.2\}, 6\ GB\{0.6, 0.1, 0.2\}, Dual\{0.3, 0.6,0.4\}) > \end{Bmatrix}$$

$$\subset \{< m^1, (samsung\{0.9, 0.6, 0.3\}, 6\ GB\{0.8, 0.4, 0.1\})>\}$$

Here we can see that membership value of Samsung for m^1 in both sets is $(0.7, 0.5, 0.6)$ and $(0.9, 0.6, 0.3)$ which satisfy the Definition of Neutrosophic Hypersoft subset as $0.7 < 0.9, 0.5 < 0.6$, and $0.6 > 0.3$. This shows that $(0.7, 0.5, 0.6) \subset (0.9, 0.6, 0.3)$ and same was the case with the rest of the attributes of NHSS $₣(\$^1)$ and NHSS $₣(\$^2)$.

Definition 3.3: Neutrosophic Equal Hypersoft Set

Let $₣(\$^1)$ and $₣(\$^2)$ be two Neutrosophic Hypersoft set over ξ. Consider $l^1, l^2, l^3 \dots l^n$ for $n \geq 1$, be n well-defined attributes, whose corresponding attributive values are respectively the set $L^1, L^2, L^3 \dots L^n$ with $L^i \cap L^j = \emptyset$, for $i \neq j$ and $i, j \epsilon \{1,2,3 \dots n\}$ and their relation $L^1 \times L^2 \times L^3 \dots L^n = \$$ then $₣(\$^1)$ is the Neutrosophic equal Hypersoft subset of $₣(\$^2)$ if

$$T\big(₣(\$^1)\big) = T\big(₣(\$^2)\big)$$

$$I\big(₣(\$^1)\big) = I\big(₣(\$^2)\big)$$

$$F\big(₣(\$^1)\big) = F\big(₣(\$^2)\big)$$

Numerical Example of Equal Neutrosophic Hypersoft Set

Consider the two NHSS $₣(\$^1)$ and NHSS $₣(\$^2)$ over the same universe $\xi = \{m^1, m^2, m^3, m^4, m^5\}$.

The NHSS $₣(\$^1) = ₣(samsung, 6\ GB, Dual\ ) = \{m^1, m^4\}$ is the equal to NHSS $₣(\$^2) = ₣(samsung, 6\ GB) = \{m^1\}$ if $T(₣(\$^1)) = T(₣(\$^2))$, $I(₣(\$^1)) = I(₣(\$^2))$, $F(₣(\$^1)) = F(₣(\$^2))$. Its tabular form is given below

Table 10: Tabular Representation of NHSS $₣(\$^1)$

$₣(\$^1) = ₣(samsung, 6\ GB, Dual\ )$	m^1	m^4
Samsung	(0.7,0.5, 0.6)	(0.8, 0.1, 0.2)
6 GB	(0.7, 0.2, 0.3)	(0.6, 0.1, 0.2)
Dual	(0.8, 0.2, 0.1)	(0.3, 0.6, 0.4)

Table 11: Tabular Representation of NHSS $₣(\$^2)$

$₣(\$^2) = ₣(\boldsymbol{samsung, 6\ GB})$	$\boldsymbol{m^1}$
Samsung	(0.7,0.5, 0.6)
6 GB	(0.7, 0.2, 0.3)

This can also be written as

$$\big(₣(\$^1) = ₣(\$^2)\big) = \big(₣(samsung, 6\ GB, Dual\ ) = ₣(samsung, 6\ GB)\big)$$
$$= \big((\{< m^1, (samsung\{0.7, 0.5, 0.6\}, 6\ GB\{0.7, 0.2, 0.3\}, Dual\{0.8,0.2,0.1\}) >,$$
$$< m^4(samsung\{0.8,0.1,0.2\}, 6\ GB\{0.6, 0.1, 0.2\}, Dual\{0.3, 0.6,0.4\}) >\}$$
$$= \{< m^1, (samsung\{0.7, 0.5, 0.6\}, 6\ GB\{0.7, 0.2, 0.3\}) >\})\big)$$

Here we can see that membership value of Samsung for m^1 in both sets is $(0.7, 0.5, 0.6)$ and $(0.7, 0.5, 0.6)$ which satisfy the Definition of Neutrosophic Equal Hypersoft set as $0.7 = 0.7, 0.5 = 0.5$ and $0.6 = 0.6$. This shows that $(0.7, 0.5, 0.6) = (0.7, 0.5, 0.6)$ and same was the case with the rest of the attributes of NHSS $₣(\$^1)$ and NHSS $₣(\$^2)$.

Definition 3.4: Null Neutrosophic Hypersoft Set

Let $₣(\$^1)$ be the Neutrosophic Hypersoft set over ξ. Consider $l^1, l^2, l^3 \dots l^n$ for $n \geq 1$, be n well-defined attributes, whose corresponding attributive values are respectively the set $L^1, L^2, L^3 \dots L^n$ with $L^i \cap L^j = \emptyset$, for $i \neq j$ and $i, j \epsilon \{1,2,3 \dots n\}$ and their relation $L^1 \times L^2 \times L^3 \dots L^n = \$$ then $₣(\$^1)$ is Null Neutrosophic Hypersoft set if

$$T\big(₣(\$^1)\big) = 0$$
$$I\big(₣(\$^1)\big) = 0$$
$$F\big(₣(\$^1)\big) = 0$$

Numerical Example of Null Neutrosophic Hypersoft Set

Consider the NHSS $₣(\$^1)$ over the universe $ξ = \{m^1, m^2, m^3, m^4, m^5\}$. The NHSS $₣(\$^1) = ₣(samsung, 6\ GB, Dual\ ) = \{m^1, m^4\}$ is said to be null NHSS if its Neutrosophic values are 0. Its tabular form is given below

Table 12: Tabular Representation of NHSS $₣(\$^1)$

$₣(\$^1) = ₣(samsung, 6\ GB, Dual\ )$	m^1	m^4
Samsung	(0, 0, 0)	(0, 0, 0)
6 GB	(0, 0, 0)	(0, 0, 0)
Dual	(0, 0, 0)	(0, 0, 0)

This can also be written as

$$₣(\$^1) = ₣(samsung, 6\ GB, Dual\ )$$
$$= \{< m^1, (samsung\{0, 0, 0\}, 6\ GB\{0, 0, 0\}, Dual\{0,0,0\}) >,$$
$$< m^4(samsung\{0,0,0\}, 6\ GB\{0, 0, 0\}, Dual\{0, 0,0\}) >\}$$

Definition 3.5: Compliment of Neutrosophic Hypersoft Set

Let $₣(\$^1)$ be the Neutrosophic Hypersoft set over ξ. Consider $l^1, l^2, l^3 \dots l^n$ for $n \geq 1$, be n well-defined attributes, whose corresponding attributive values are respectively the set $L^1, L^2, L^3 \dots L^n$ with $L^i \cap L^j = \emptyset$, for $i \neq j$ and $i, j \epsilon \{1,2,3 \dots n\}$ and their relation $L^1 \times L^2 \times L^3 \dots L^n = \$$ then $₣^c(\$^1)$ is the Compliment of Neutrosophic Hypersoft set of $₣(\$^1)$ if

$$₣^c(\$^1): (\neg L^1 \times \neg L^2 \times \neg L^3 \dots \neg L^n) \rightarrow P(\xi)$$

Such that

$$T^C(₣(\$^1)) = F(₣(\$^1))$$
$$I^C(₣(\$^1)) = I(₣(\$^1))$$
$$F^C(₣(\$^1)) = T(₣(\$^1))$$

Numerical Example of Compliment of NHSS

Consider the NHSS $₣(\$^1)$ over the universe $\xi = \{m^1, m^2, m^3, m^4, m^5\}$. The compliment of NHSS $₣(\$^1) = ₣(samsung, 6\ GB, Dual\ ) = \{m^1, m^4\}$ is given as $T^C(₣(\$^1)) = F(₣(\$^1))$, $I^C(₣(\$^1)) = I(₣(\$^1))$, $F^C(₣(\$^1)) = T(₣(\$^1))$.Its tabular form is given below

Table 13: Tabular Representation of NHSS $₣(\$^1)$

$₣^C(\$^1) = ₣(Not\ samsung, Not\ 6\ GB, Not\ Dual\ )$	m^1	m^4
Not Samsung	(0.6, 0.5, 0.7)	(0.2, 0.1, 0.8)
Not 6 GB	(0.3, 0.2, 0.7)	(0.2, 0.1, 0.6)
Not Dual	(0.1, 0.2, 0.8)	(0.4, 0.6, 0.3)

This can also be written as

$$₣^c(\$^1) = ₣(\ not\ samsung, not\ 6\ GB, not\ Dual\ )$$
$$= \{< m^1, (not\ samsung\{0.6, 0.5, 0.7\}, not\ 6\ GB\{0.3, 0.2, 0.7\}, not\ Dual\{0.1,0.2,0.8\}) >,$$
$$< m^4(not\ samsung\{0.2,0.1,0.8\}, not\ 6\ GB\{0.2, 0.1, 0.6\}, not\ Dual\{0.4, 0.6,0.3\}) >\}$$

Here we can see that membership value of Samsung for m^1 in $₣(\$^1)$ is $(0.7, 0.5, 0.6)$ and its compliment is $(0.6, 0.5, 0.7)$ which satisfy the Definition of compliment of Neutrosophic Hypersoft set. This shows that $(0.6, 0.5, 0.7)$ is the compliment of $(0.7, 0.5, 0.6)$ and same was the case with the rest of the attributes of NHSS $₣(\$^1)$.

Definition 3.6: Union of Two Neutrosophic Hypersoft Set

Let $₣(\$^1)$ and $₣(\$^2)$ be two Neutrosophic Hypersoft set over ξ. Consider $l^1, l^2, l^3 \dots l^n$ for $n \geq 1$, be n well-defined attributes, whose corresponding attributive values are respectively the set $L^1, L^2, L^3 \dots L^n$ with $L^i \cap L^j = \emptyset$, for $i \neq j$ and $i, j \epsilon \{1,2,3 \dots n\}$ and their relation $L^1 \times L^2 \times L^3 \dots L^n = \$$ then $₣(\$^1) \cup ₣(\$^2)$ is given as

$$T\big(₣(\$^1) \cup ₣(\$^2)\big) = \begin{cases} T\big(₣(\$^1)\big) & if\ x \in \$^1 \\ T\big(₣(\$^2)\big) & if\ x \in \$^2 \\ \max\Big(T\big(₣(\$^1)\big), T\big(₣(\$^2)\big)\Big) & if\ x \in \$^1 \cap \$^2 \end{cases}$$

$$I\big(₣(\$^1) \cup ₣(\$^2)\big) = \begin{cases} I\big(₣(\$^1)\big) & if\ x \in \$^1 \\ I\big(₣(\$^2)\big) & if\ x \in \$^2 \\ \frac{\left(I\left(₣(\$^1)\right) + I\left(₣(\$^2)\right)\right)}{2} & if\ x \in \$^1 \cap \$^2 \end{cases}$$

$$F\big(₣(\$^1) \cup ₣(\$^2)\big) = \begin{cases} F\big(₣(\$^1)\big) & if\ x \in \$^1 \\ F\big(₣(\$^2)\big) & if\ x \in \$^2 \\ \min\Big(F\big(₣(\$^1)\big), F\big(₣(\$^2)\big)\Big) & if\ x \in \$^1 \cap \$^2 \end{cases}$$

Numerical Example of Union

Consider the two NHSS $₣(\$^1)$ and NHSS $₣(\$^2)$ over the same universe $\xi = \{m^1, m^2, m^3, m^4, m^5\}$. Tabular representation of NHSS $₣(\$^1) = ₣(samsung, 6\ GB, Dual\ ) = \{m^1, m^4\}$ and NHSS $₣(\$^2) = ₣(samsung, 6\ GB) = \{m^1\}$ is given below,

Table 14: Tabular Representation of NHSS $₣(\$^1)$

$₣(\$^1) = ₣(\boldsymbol{samsung, 6\ GB, Dual}\ )$	$\boldsymbol{m^1}$	$\boldsymbol{m^4}$
Samsung	(0.7,0.5, 0.6)	(0.8, 0.1, 0.2)
6 GB	(0.7, 0.2, 0.3)	(0.6, 0.1, 0.2)
Dual	(0.8, 0.2, 0.1)	(0.3, 0.6, 0.4)

Table 15: Tabular Representation of NHSS $₣(\$^2)$

$₣(\$^2) = ₣(\boldsymbol{samsung, 6\ GB})$	$\boldsymbol{m^1}$
Samsung	(0.9, 0.5, 0.3)
6 GB	(0.8, 0.4, 0.1)

Then the union of above NHSS is given as

Table 16: Union of NHSS $₣(\$^1)$ and NHSS $₣(\$^2)$

$₣(\$^1) \cup ₣(\$^2)$	$\boldsymbol{m^1}$	$\boldsymbol{m^4}$
Samsung	(0.9, 0.5, 0.3)	(0.8, 0.1, 0.2)
6 GB	(0.8, 0.3, 0.1)	(0.6, 0.1, 0.2)
Dual	(0.8, 0.1, 0.0)	(0.3, 0.6, 0.4)

This can also be written as

$$₣(\$^1) \cup ₣(\$^2) = ₣(samsung, 6\ GB, Dual\ ) \cup ₣(samsung, 6\ GB)$$
$$= \{< m^1, (samsung\{0.9, 0.5, 0.3\}, 6\ GB\{0.8, 0.3, 0.1\}, Dual\{0.8,0.1,0.0\}) >,$$
$$< m^4(samsung\{0.8,0.1,0.2\}, 6\ GB\{0.6, 0.1, 0.2\}, Dual\{0.3, 0.6,0.4\}) >\}$$

Definition 3.7: Intersection of Two Neutrosophic Hypersoft Set

Let $₣(\$^1)$ and $₣(\$^2)$ be two Neutrosophic Hypersoft set over ξ. Consider $l^1, l^2, l^3 \ldots l^n$ for $n \geq 1$, be n well-defined attributes, whose corresponding attributive values are respectively the set $L^1, L^2, L^3 \ldots L^n$ with $L^i \cap L^j = \emptyset$, for $i \neq j$ and $i, j \epsilon \{1,2,3 \ldots n\}$ and their relation $L^1 \times L^2 \times L^3 \ldots L^n = \$$ then $₣(\$^1) \cap ₣(\$^2)$ is given as

$$T\big(₣(\$^1) \cap ₣(\$^2)\big) = \begin{cases} T\big(₣(\$^1)\big) & if\ x \in \$^1 \\ T\big(₣(\$^2)\big) & if\ x \in \$^2 \\ \min\Big(T\big(₣(\$^1)\big), T\big(₣(\$^2)\big)\Big) & if\ x \in \$^1 \cap \$^2 \end{cases}$$

$$I\big(₣(\$^1) \cap ₣(\$^2)\big) = \begin{cases} I\big(₣(\$^1)\big) & if\ x \in \$^1 \\ I\big(₣(\$^2)\big) & if\ x \in \$^2 \\ \frac{\left(I\left(₣(\$^1)\right)+I\left(₣(\$^2)\right)\right)}{2} & if\ x \in \$^1 \cap \$^2 \end{cases}$$

$$F\big(₣(\$^1) \cap ₣(\$^2)\big) = \begin{cases} F\big(₣(\$^1)\big) & if\ x \in \$^1 \\ F\big(₣(\$^2)\big) & if\ x \in \$^2 \\ \max\Big(F\big(₣(\$^1)\big), F\big(₣(\$^2)\big)\Big) & if\ x \in \$^1 \cap \$^2 \end{cases}$$

Numerical Example of Intersection

Consider the two NHSS $₣(\$^1)$ and NHSS $₣(\$^2)$ over the same universe $\xi = \{m^1, m^2, m^3, m^4, m^5\}$. Tabular representation of NHSS $₣(\$^1) = ₣(samsung, 6\ GB, Dual\ ) = \{m^1, m^4\}$ and NHSS $₣(\$^2) = ₣(samsung, 6\ GB) = \{m^1\}$ is given below

Table 17: Tabular Representation of NHSS $₣(\$^1)$

$₣(\$^1)$ $= ₣(\boldsymbol{samsung, 6\ GB, Dual}\ )$	$\boldsymbol{m^1}$	$\boldsymbol{m^4}$
Samsung	(0.7,0.5, 0.6)	(0.8, 0.1, 0.2)
6 GB	(0.7, 0.2, 0.3)	(0.6, 0.1, 0.2)
Dual	(0.8, 0.2, 0.1)	(0.3, 0.6, 0.4)

Table 18: Tabular Representation of NHSS $₣(\$^2)$

$₣(\$^2) = ₣(\boldsymbol{samsung, 6\ GB})$	$\boldsymbol{m^1}$
Samsung	(0.9, 0.5, 0.3)
6 GB	(0.8, 0.4, 0.1)

Then the intersection of above NHSS is given as

Table 19: Intersection of NHSS $₣(\$^1)$ and NHSS $₣(\$^2)$

$₣(\$^1) \cap ₣(\$^2)$	$\boldsymbol{m^1}$
Samsung	(0.7, 0.5, 0.6)
6 GB	(0.7, 0.3, 0.3)
Dual	(0.0, 0.1, 0.1)

This can also be written as

$$₣(\$^1) \cap ₣(\$^2) = ₣(samsung, 6\ GB, Dual\ ) \cap ₣(samsung, 6\ GB)$$
$$= \{< m^1, (samsung\{0.7, 0.5, 0.6\}, 6\ GB\{0.7, 0.3, 0.3\}, Dual\{0.0,0.1,0.1\}) >\}$$

Definition 3.8: AND Operation on Two Neutrosophic Hypersoft Set

Let $₣(\$^1)$ and $₣(\$^2)$ be two Neutrosophic Hypersoft set over ξ. Consider $l^1, l^2, l^3 \dots l^n$ for $n \geq 1$, be n well-defined attributes, whose corresponding attributive values are respectively the set $L^1, L^2, L^3 \dots L^n$ with $L^i \cap L^j = \emptyset$, for $i \neq j$ and $i, j \epsilon \{1,2,3 \dots n\}$ and their relation $L^1 \times L^2 \times L^3 \dots L^n = \$$ then $₣(\$^1) \wedge ₣(\$^2) = ₣(\$^1 \times \$^2)$ is given as

$$T(\$^1 \times \$^2) = min\left(T\big(₣(\$^1)\big), T\big(₣(\$^2)\big)\right)$$

$$I(\$^1 \times \$^2) = \frac{\left(I\big(₣(\$^1)\big), I\big(₣(\$^2)\big)\right)}{2}$$

$$F(\$^1 \times \$^2) = max\left(F\big(₣(\$^1)\big), F\big(₣(\$^2)\big)\right)$$

Numerical Example of AND-Operation

Consider the two NHSS $₣(\$^1)$ and NHSS $₣(\$^2)$ over the same universe $\xi = \{m^1, m^2, m^3, m^4, m^5\}$. Tabular representation of NHSS $₣(\$^1) = ₣(samsung, 6\ GB, Dual\ ) = \{m^1, m^4\}$ and NHSS $₣(\$^2) = ₣(samsung, 6\ GB,) = \{m^1\}$ is given below

Table 20: Tabular representation of NHSS $₣(\$^1)$

$₣(\$^1)$ $= ₣(samsung, 6\ GB, Dual\ )$	m^1	m^4
Samsung	(0.7,0.5, 0.6)	(0.8, 0.1, 0.2)
6 GB	(0.7, 0.2, 0.3)	(0.6, 0.1, 0.2)
Dual	(0.8, 0.2, 0.1)	(0.3, 0.6, 0.4)

Table 21: Tabular representation of NHSS $₣(\$^2)$

$₣(\$^2) = ₣(\boldsymbol{samsung, 6\ GB})$	$\boldsymbol{m^1}$
Samsung	(0.9, 0.5, 0.3)
6 GB	(0.8, 0.4, 0.1)

Then the AND Operation of above NHSS is given as

Table 22: AND of NHSS $₣(\$^1)$ and NHSS $₣(\$^2)$

$₣(\$^1) \wedge ₣(\$^2)$	$\boldsymbol{m^1}$	$\boldsymbol{m^4}$
$Samsung \times\ Samsung$	(0.7,0.5,0.6)	(0.0,0.1,0.2)
$Samsung \times\ 6\ GB$	(0.7, 0.45,0.6)	(0.0,0.1,0.2)
$6\ GB\ \times Samsung$	(0.7, 0.35,0.3)	(0.0,0.1,0.2)
$6\ GB \times\ 6\ GB$	(0.7,0.3, 0.3)	(0.0,0,1,0.2)
$Dual\ \times Samsung$	(0.8,0.35,0.3)	(0.0,0.6,0.4)
$Dual \times\ 6\ GB$	(0.8, 0.3, 0.1)	(0.0,0.6,0.4)

Definition 3.9: OR Operation on Two Neutrosophic Hypersoft Set

Let $₣(\$^1)$ and $₣(\$^2)$ be two Neutrosophic Hypersoft set over ξ. Consider $l^1, l^2, l^3 \ldots l^n$ for $n \geq 1$, be n well-defined attributes, whose corresponding attributive values are respectively the set $L^1, L^2, L^3 \ldots L^n$ with $L^i \cap L^j = \emptyset$, for $i \neq j$ and $i, j \epsilon \{1,2,3 \ldots n\}$ and their relation $L^1 \times L^2 \times L^3 \ldots L^n = \$$ then $₣(\$^1) \vee ₣(\$^2) = ₣(\$^1 \times \$^2)$ is given as

$$T(\$^1 \times \$^2) = max\left(T\big(₣(\$^1)\big), T\big(₣(\$^2)\big)\right)$$

$$I(\$^1 \times \$^2) = \frac{\left(I\big(₣(\$^1)\big), I\big(₣(\$^2)\big)\right)}{2}$$

$$F(\$^1 \times \$^2) = min\left(F\big(₣(\$^1)\big), F\big(₣(\$^2)\big)\right)$$

Numerical Example of OR-Operation

Consider the two NHSS $₣(\$^1)$ and NHSS $₣(\$^2)$ over the same universe $ξ = \{m^1, m^2, m^3, m^4, m^5\}$. Tabular representation of NHSS $₣(\$^1) = ₣(samsung, 6\ GB, Dual\ ) = \{m^1, m^4\}$ and NHSS $₣(\$^2) = ₣(samsung, 6\ GB,) = \{m^1\}$ is given below

Table 23: Tabular representation of NHSS $₣(\$^1)$

$₣(\$^1) = ₣(\boldsymbol{samsung, 6\ GB, Dual}\ )$	$\boldsymbol{m^1}$	$\boldsymbol{m^4}$
Samsung	(0.7,0.5, 0.6)	(0.8, 0.1, 0.2)
6 GB	(0.7, 0.2, 0.3)	(0.6, 0.1, 0.2)
Dual	(0.8, 0.2, 0.1)	(0.3, 0.6, 0.4)

$₣(\$^2) = ₣(\boldsymbol{samsung, 6\ GB})$	$\boldsymbol{m^1}$
Samsung	(0.9, 0.5, 0.3)
6 GB	(0.8, 0.4, 0.1)

Table 24: Tabular representation of NHSS $₣(\$^2)$

Then the OR Operation of above NHSS is given as

Table 25: OR of NHSS $₣(\$^1)$ and NHSS $₣(\$^2)$

$₣(\$^1) \vee ₣(\$^2)$	m^1	m^4
$Samsung \times \ Samsung$	(0.9,0.5,0.3)	(0.8,0.1,0.0)
$Samsung \times \ 6\ GB$	(0.8, 0.45,0.1)	(0.8,0.1,0.0)
$6\ GB\ \times Samsung$	(0.9, 0.35,0.3)	(0.6,0.1,0.0)
$6\ GB \times \ 6\ GB$	(0.8,0.3, 0.1)	(0.6,0,1,0.0)
$Dual\ \times Samsung$	(0.9,0.35,0.1)	(0.3,0.6,0.0)
$Dual \times \ 6\ GB$	(0.8, 0.3, 0.1)	(0.3,0.6,0.0)

4. Result Discussion

Decision-making is a complex issue due to vague, imprecise and indeterminate environment specially, when attributes are more than one, and further bifurcated. Neutrosophic softset environment cannot be used to tackle such type of issues. Therefore, there was a dire need to define a new approach to solve such type of problems, So, for this purpose neutrosophic hypersoft set environment is defined along with necessary operations and elaborated with examples.

5. Conclusions

In this paper, operations of Neutrosophic Hypersoft set like union, intersection, compliment, AND OR operations are presented. The validity and implementation of the proposed operations and definitions are verified by presenting suitable example. Neutrosophic hypersoft set NHSS will be a new tool in decision-making problems for suitable selection. In future, many decision-makings like personal selection, office management, industrial equipment and many other problems can be solved with the proposed operations [23]. Properties of Union and Intersection operations, cardinality and functions on NHSS are to be defined in future.

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Basic operations on hypersoft sets and hypersoft point

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Abstract: The aim of this paper is to initiat formal study of hypersoft sets. We first, present basic operations like union, intersection and difference of hypersoft sets; basic ingrediants for topological structures on the collection of hypersoft sets. Moreover we introduce hypersoft points in different envorinments like fuzzy hypersoft set, intuitionistic fuzzy hypersoft set, neutrosophic hypersoft, plithogenic hypersoft set, and give some basic properties of hypersoft points in these envorinments. We expect that this will constitue an appropriate framework of hypersoft functions and the study of hypersoft function spaces. Examples are provided to explain the newly defined concepts.

Keywords: soft set; hypersoft set; set operations on hypersoft sets; hypersoft point; fuzzy hypersoft set; intuitionistic fuzzy hypersoft set; neutrosophic hypersoft; plithogenic hypersoft set.

1. Introduction

Molodtsov [16] defined soft set as a mathematical tool to deal with uncertainties associated with real world problems. Soft set theory has application in decision making, demand analysis, forecasting, information sciences and other disciplines (see for example, [13, 14, 15, 17, 18, 19, 20, 21, 22, 23]). Plithogenic and neutrosophic hypersoft sets theory is being applied successfully in decision making problems (see, [2, 3, 4, 5, 6, 7, 8, 9,10,11,12]).

By definition, a soft set can be identified by a pair (F, A), where F stands for a multivalued function defined on the set of parameters A.

Smarandache [1] extended the notion of a soft set to the hypersoft set by replacing the function F with a multi-argument function defined on the Cartesian product of n different set of parameters. This concept is more flexible than soft set and more suitable in the context of decision making problems.

We expect the notion of hypersoft set will attract the attention of researchers working on soft set theory and its diverse applications. The purpose of this paper is to initiate a formal investigation in this new area of research.

As a first step, we present the basic operations like union, intersection and difference of hypersoft sets. Moreover we introduce hypersoft points and some basic properties of these points which may provide the foundation for the hypersoft functions and hence the hypersoft fixed point theory.

2. Operations on hypersoft sets

In this section, we define basic operations on hypersoft sets. Smarandache defined the hypersoft set in the following manner:

Definition 1 *[1] Let U be a universe of discourse, $P(U)$ the power set U and $E_1, E_2, \ldots, E_n$ the pairwise disjoint sets of parameters. Let A_i be the nonempty subset of E_i for each $i = 1,2,\ldots,n$. A hypersoft set can be identified by the pair $(F, A_1 \times A_2 \times \cdots \times A_n)$, where:*

$$F: A_1 \times A_2 \times \cdots \times A_n \to P(U).$$

For sake of simplicity, we write the symbols $\mathbf{E}$ for $E_1 \times E_2 \times \cdots \times E_n$, $\mathbf{A}$ for $A_1 \times A_2 \times \cdots \times A_n$ and $\boldsymbol{\alpha}$ for an element of the set $\mathbf{A}$. We also suppose that none of the set A_i is empty.

Definition 2 *[1] A hypersoft set;*
on a crisp universe of discourse U_C is called Crisp Hypersoft set (or simply "hypersoft set");
on a fuzzy universe of discourse U_F is called Fuzzy Hypersoft set.
on a Intuitionistic Fuzzy universe of discourse U_{IF} is called Intuitionistic Fuzzy Hypersoft set;
on a Neutrosophic universe of discourse U_N is called Neutrosophic Hypersoft Set;
on a Plithogenic universe of discourse U_P is called Plithogenic Hypersoft Set.

The nature of $F(\boldsymbol{\alpha})$ is determined by the nature of universe of discourse. Therefore $P(U)$ depends upon the nature of universe. We denote $\mathcal{H}(U_*, \mathbf{E})$ by the family of all *-hypersoft sets over $(U_*, \mathbf{E})$, where $*$ can take any value in the set $\{C, F, IF, N, P\}$, where symbols C, F, IF, N, P denote Crisp, Fuzzy, Intuitionistic Fuzzy, Neutrosophic, and Plithogenic sets, respectively.

The following are the basic operations on *-hypersoft set.

Definition 3 *Let U_* be a universe of discourse and $\boldsymbol{A}$ a subset of $\boldsymbol{E}$. Then $(F, \boldsymbol{A})$ is called*

1. *a null *-hypersoft set if for each parameter $\boldsymbol{\alpha} \in \boldsymbol{A}$, $F(\boldsymbol{\alpha})$ is an 0_*. We will denote it by $\Phi_{\boldsymbol{A}}$.*
2. *an absolute *-hypersoft set if for each parameter $\boldsymbol{\alpha} \in \boldsymbol{A}$, $F(\boldsymbol{\alpha}) = U_*$. We will denote it by $\widetilde{U}_{\boldsymbol{A}}$.*

Remark 1 *We consider $0_C = \varnothing$ for empty set, $0_F = \{\frac{x}{0}, x \in U_F\}$ for null fuzzy set, $0_{IF} = \{\frac{x}{<0,1>}, x \in U_{IF}\}$ for null intuitionistic fuzzy set, $0_N = \{\frac{x}{<0,1,1>}, x \in U_N\}$ for null neutrosophic set. However, in case of plithogenic set, we have the following notations:*

- *Null plithgenic crisp set*

$$0_{PC} = \{x(0,0,\ldots,0), \text{for all} x \in U_P\}.$$

- *Universal plithgenic crisp set*

$$1_{PC} = \{x(1,1,\ldots,1), \text{for all} x \in U_P\}.$$

Note that null plithgenic fuzzy set will be same as null plithgenic crisp set and universal plithgenic fuzzy set will be the same as universal plithgenic crisp set.

- *Null plithgenic intuitionistic fuzzy set*

$$0_{PIF} = \{x((0,1),(0,1),\ldots,(0,1)), \text{for all} x \in U_P\}.$$

- *Universal plithgenic intuitionistic fuzzy set*

$$1_{PIF} = \{x((1,0),(1,0),\ldots,(1,0)), \text{for all} x \in U_P\}.$$

- *Null plithgenic neutrosophic set*

$$0_{PN} = \{x((0,1,1),(0,1,1),\ldots,(0,1,1)), \text{for all} x \in U_P\}.$$

- *Universal plithgenic neutrosophic set*

$$1_{PN} = \{x((1,0,0),(1,0,0),\ldots,(1,0,0)), \text{for all} x \in U\}.$$

Definition 4 *Let $(F,\boldsymbol{A})$ and $(G,\boldsymbol{B})$ be two *-hypersoft sets over U_*. Then union of $(F,\boldsymbol{A})$ and $(G,\boldsymbol{B})$ is denoted by $(H,\boldsymbol{C}) = (F,\boldsymbol{A}) \,\tilde{\cup}\, (G,\boldsymbol{B})$ with $\boldsymbol{C} = C_1 \times C_2 \times \cdots \times C_n$, where $C_i = A_i \cup B_i$ for $i = 1,2,\ldots,n$, and H is defined by*

$$H(\boldsymbol{\alpha}) = \begin{cases} F(\boldsymbol{\alpha}), & \text{if} \boldsymbol{\alpha} \in \mathbf{A} - \mathbf{B} \\ G(\boldsymbol{\alpha}), & \text{if} \boldsymbol{\alpha} \in \mathbf{B} - \mathbf{A} \\ F(\boldsymbol{\alpha}) \cup_* G(\boldsymbol{\alpha}), & \text{if} \boldsymbol{\alpha} \in \mathbf{A} \cap \mathbf{B}, \\ 0_*, & \text{else,} \end{cases}$$

where $\boldsymbol{\alpha} = (c_1, c_2, \ldots, c_n) \in C$.

Remark 2 *Note that, in the case of union of two hypersoft sets the set of parameters is a Cartesian product of sets of parameters whereas in the case of union of two soft sets the set of parameter is just the union of sets of parameters.*

Definition 5 *Let $(F,\boldsymbol{A})$ and $(G,\boldsymbol{B})$ be two *-hypersoft sets over U_*. Then intersection of $(F,\boldsymbol{A})$ and $(G,\boldsymbol{B})$ is denoted by $(H,\boldsymbol{C}) = (F,\boldsymbol{A}) \,\tilde{\cap}\, (G,\boldsymbol{B})$, where $\boldsymbol{C} = C_1 \times C_2 \times \cdots \times C_n$ is such that $C_i = A_i \cap B_i$ for $i = 1,2,\ldots,n$ and H is defined as*

$$H(\boldsymbol{\alpha}) = F(\boldsymbol{\alpha}) \cap_* G(\boldsymbol{\alpha}),$$

*where $\boldsymbol{\alpha} = (c_1, c_2, \ldots, c_n) \in \boldsymbol{C}$. If C_i is an empty set for some i, then $(F,\boldsymbol{A}) \,\tilde{\cap}\, (G,\boldsymbol{B})$ is defined to be a null *-hypersoft set.*

Definition 6 *Let $(F,\boldsymbol{A})$ and $(G,\boldsymbol{B})$ be two *-hypersoft sets over U_*. Then $(F,\boldsymbol{A})$ is called a *-hypersoft subset of $(G,\boldsymbol{B})$ if $\boldsymbol{A} \subseteq \boldsymbol{B}$, and $F(\boldsymbol{\alpha}) \subseteq_* G(\boldsymbol{\alpha})$ for all $\boldsymbol{\alpha} \in \boldsymbol{A}$. We denote this by $(F,\boldsymbol{A}) \,\tilde{\subseteq}\, (G,\boldsymbol{B})$. Thus $(F,\boldsymbol{A})$ and $(G,\boldsymbol{B})$ are said to equal if $(F,\boldsymbol{A}) \,\tilde{\subseteq}\, (G,\boldsymbol{B})$ and $(F,\boldsymbol{A}) \,\tilde{\supseteq}\, (G,\boldsymbol{B})$.*

Definition 7 *Let $(F,\boldsymbol{A})$ and $(G,\boldsymbol{B})$ be two *-hypersoft sets over U_*. Then *-hypersoft difference of $(F,\boldsymbol{A})$ and $(G,\boldsymbol{B})$, denoted by $(H,\boldsymbol{C}) = (F,\boldsymbol{A}) \tilde{\setminus} (G,\boldsymbol{B})$, where $\boldsymbol{C} = C_1 \times C_2 \times \cdots \times C_n$ is such that $C_i = A_i \cap B_i$ for $i = 1,2,\ldots,n$, and H is defined by*

$$H(\boldsymbol{\alpha}) = F(\boldsymbol{\alpha}) \setminus_* G(\boldsymbol{\alpha}),$$

where $\boldsymbol{\alpha} = (c_1, c_2, \ldots, c_n) \in \mathbf{C}$. If C_i is an empty set for some i then $(F,\mathbf{A}) \tilde{\setminus} (G,\mathbf{B})$ is defined to be $(F,\mathbf{A})$.

Definition 8 *The complement of a *-hypersoft set $(F,\boldsymbol{A})$ is denoted as $(F,\boldsymbol{A})^c$ and is defined by $(F,\boldsymbol{A})^c = (F^c,\boldsymbol{A})$ where $F^c(\boldsymbol{\alpha})$ is the *-complemet of $F(\boldsymbol{\alpha})$ for each $\boldsymbol{\alpha} \in \boldsymbol{A}$.*

Example 1 Let $\mathrm{U} = \{\mathrm{x}_1, \mathrm{x}_2, \mathrm{x}_3, \mathrm{x}_4\}$. Define the attributes sets by:

$$E_1 = \{a_{11}, a_{12}\}, E_2 = \{a_{21}, a_{22}\}, E_3 = \{a_{31}, a_{32}\}.$$

Suppose that

$A_1 = \{a_{11}, a_{12}\}, A_2 = \{a_{21}, a_{22}\}, A_3 = \{a_{31}\}$, and

$B_1 = \{a_{11}\}, B_2 = \{a_{21}, a_{22}\}, B_3 = \{a_{31}, a_{32}\}$

that is,. $A_i, B_i \subseteq E_i$ for each $i = 1,2,3$.

Let the crisp hypersoft sets $(F, \mathbf{A})$ and $(G, \mathbf{B})$ be defined by

$(F, \mathbf{A}) = \{((a_{11}, a_{21}, a_{31}), \{x_1, x_2\}), ((a_{11}, a_{22}, a_{31}), \{x_2\}),$
$((a_{12}, a_{21}, a_{31}), \{x_3, x_4\}), ((a_{12}, a_{22}, a_{31}), \{x_1, x_4\})\}.$

and

$(G, \mathbf{B}) = \{((a_{11}, a_{21}, a_{31}), \{x_2, x_3\}), ((a_{11}, a_{22}, a_{31}), \{x_2\}),$
$((a_{11}, a_{21}, a_{32}), \{x_1, x_4\}), ((a_{11}, a_{22}, a_{32}), \{x_3, x_4\})\}.$

We have excluded those $\boldsymbol{\alpha} \in \mathbf{A}$ for which $F(\boldsymbol{\alpha})$ is an empty set (similarly for those $\boldsymbol{\beta} \in \mathbf{B}$ for which $G(\boldsymbol{\beta})$ is an empty set).

Then the union and intersections of $(F, \mathbf{A})$ and $(G, \mathbf{B})$ are given by:

$(F, \mathbf{A}) \tilde{\cup} (G, \mathbf{B}) = \{((a_{11}, a_{21}, a_{31}), \{x_1, x_2, x_3\}), ((a_{11}, a_{22}, a_{31}), \{x_2\}),$
$((a_{12}, a_{21}, a_{31}), \{x_3, x_4\}), ((a_{12}, a_{22}, a_{31}), \{x_1, x_4\}),$
$((a_{11}, a_{21}, a_{32}), \{x_1, x_4\}), ((a_{11}, a_{22}, a_{32}), \{x_3, x_4\}),$
$((a_{12}, a_{21}, a_{32}), 0_C), ((a_{12}, a_{22}, a_{32}), 0_C)\};$

and

$$(F, \mathbf{A}) \tilde{\cap} (G, \mathbf{B}) = \{((a_{11}, a_{21}, a_{31}), \{x_2\}), ((a_{11}, a_{22}, a_{31}), \{x_2\})\}.$$

The differences $(F, \mathbf{A}) \tilde{\setminus} (G, \mathbf{B})$ and $(G, \mathbf{B}) \tilde{\setminus} (F, \mathbf{A})$ are the following

$(F, \mathbf{A}) \tilde{\setminus} (G, \mathbf{B}) = \{((a_{11}, a_{21}, a_{31}), \{x_1\}), ((a_{11}, a_{22}, a_{31}), 0_C)\};$
$(G, \mathbf{B}) \tilde{\setminus} (F, \mathbf{A}) = \{((a_{11}, a_{21}, a_{31}), \{x_3\}), ((a_{11}, a_{22}, a_{31}), 0_C)\}.$

Example 2 Let $\mathrm{U} = \{\mathrm{x}_1, \mathrm{x}_2, \mathrm{x}_3, \mathrm{x}_4\}$. Define the attributes sets by:

$$E_1 = \{a_{11}, a_{12}\}, E_2 = \{a_{21}, a_{22}\}, E_3 = \{a_{31}, a_{32}\}.$$

Suppose that

$A_1 = \{a_{11}, a_{12}\}, A_2 = \{a_{21}, a_{22}\}, A_3 = \{a_{31}\}$, and

$B_1 = \{a_{11}\}, B_2 = \{a_{21}, a_{22}\}, B_3 = \{a_{31}, a_{32}\}$

are subsets of E_i for each $i = 1,2,3$, that is,. $A_i, B_i \subseteq E_i$ for each i.

Let the fuzzy hypersoft sets $(F, \mathbf{A})$ and $(G, \mathbf{B})$ be defined by

$(F, \mathbf{A}) = \{((a_{11}, a_{21}, a_{31}), \{\frac{x_1}{0.5}, \frac{x_2}{0.7}\}), ((a_{11}, a_{22}, a_{31}), \{\frac{x_2}{0.3}\}),$
$((a_{12}, a_{21}, a_{31}), \{\frac{x_3}{0.8}, \frac{x_4}{0.9}\}), ((a_{12}, a_{22}, a_{31}), \{\frac{x_1}{0.5}, \frac{x_4}{0.4}\})\}.$

and

$(G, \mathbf{B}) = \{((a_{11}, a_{21}, a_{31}), \{\frac{x_2}{0.2}, \frac{x_3}{0.9}\}), ((a_{11}, a_{22}, a_{31}), \{\frac{x_2}{0.6}\}),$
$((a_{11}, a_{21}, a_{32}), \{\frac{x_1}{0.4}, \frac{x_4}{0.7}\}), ((a_{11}, a_{22}, a_{32}), \{\frac{x_3}{0.2}, \frac{x_4}{0.8}\})\}.$

We have excluded those $\boldsymbol{\alpha} \in \mathbf{A}$ for which $F(\boldsymbol{\alpha})$ is a null fuzzy set (similarly for those $\boldsymbol{\beta} \in \mathbf{B}$ for which $G(\boldsymbol{\beta})$ is a null fuzzy set).

Then the union and intersections of $(F, \mathbf{A})$ and $(G, \mathbf{B})$ are given by:

$(F,\mathbf{A})\ \tilde{\cup}\ (G,\mathbf{B}) = \{((a_{11},a_{21},a_{31}),\{\frac{x_1}{0.5},\frac{x_2}{0.7},\frac{x_3}{0.9}\}),((a_{11},a_{22},a_{31}),\{\frac{x_2}{0.6}\}),$
$((a_{12},a_{21},a_{31}),\{\frac{x_3}{0.8},\frac{x_4}{0.9}\}),((a_{12},a_{22},a_{31}),\{\frac{x_1}{0.5},\frac{x_4}{0.4}\}),$
$((a_{11},a_{21},a_{32}),\{\frac{x_1}{0.4},\frac{x_4}{0.7}\}),((a_{11},a_{22},a_{32}),\{\frac{x_3}{0.2},\frac{x_4}{0.8}\}),$
$((a_{12},a_{21},a_{32}),0_F),((a_{12},a_{22},a_{32}),0_F)\};$

and

$$(F,\mathbf{A})\ \tilde{\cap}\ (G,\mathbf{B}) = \{((a_{11},a_{21},a_{31}),\{\frac{x_2}{0.2}\}),((a_{11},a_{22},a_{31}),\{\frac{x_2}{0.3}\})\}.$$

The differences $(F,\mathbf{A})\tilde{\setminus}(G,\mathbf{B})$ and $(G,\mathbf{B})\tilde{\setminus}(F,\mathbf{A})$ are the following

$(F,\mathbf{A})\tilde{\setminus}(G,\mathbf{B}) = \{((a_{11},a_{21},a_{31}),\{\frac{x_1}{0.5},\frac{x_2}{0.5}\}),((a_{11},a_{22},a_{31}),0_F)\};$
$(G,\mathbf{B})\tilde{\setminus}(F,\mathbf{A}) = \{((a_{11},a_{21},a_{31}),\{\frac{x_3}{0.9}\}),((a_{11},a_{22},a_{31}),\{\frac{x_2}{0.3}\})\}.$

Example 3 Let $\mathrm{U} = \{\mathrm{x}_1,\mathrm{x}_2,\mathrm{x}_3,\mathrm{x}_4\}$. Define the attributes sets by:

$$E_1 = \{a_{11},a_{12}\}, E_2 = \{a_{21},a_{22}\}, E_3 = \{a_{31},a_{32}\}.$$

Suppose that

$A_1 = \{a_{11},a_{12}\}, A_2 = \{a_{21},a_{22}\}, A_3 = \{a_{31}\}$, and
$B_1 = \{a_{11}\}, B_2 = \{a_{21},a_{22}\}, B_3 = \{a_{31},a_{32}\}$

that is,. $A_i, B_i \subseteq E_i$ for each $i = 1,2,3$.

Let the intuitionistic fuzzy hypersoft sets $(F,\mathbf{A})$ and $(G,\mathbf{B})$ be defined by

$$(F,\mathbf{A}) = \{((a_{11},a_{21},a_{31}),\{\frac{x_1}{<0.5,0.3>},\frac{x_2}{<0.7,0.2>}\}),((a_{11},a_{22},a_{31}),\{\frac{x_2}{<0.3,0.5>}\}),$$

$$((a_{12},a_{21},a_{31}),\{\frac{x_3}{<0.8,0.1>},\frac{x_4}{<0.1,0.5>}\}),((a_{12},a_{22},a_{31}),\{\frac{x_1}{<0.5,0.3>},\frac{x_4}{<0.4,0.2>}\})\}.$$

and

$$(G,\mathbf{B}) = \{((a_{11},a_{21},a_{31}),\{\frac{x_2}{<0.2,0.6>},\frac{x_3}{<0.8,0.1>}\}),((a_{11},a_{22},a_{31}),\{\frac{x_2}{<0.6,0.3>}\}),$$

$$((a_{11},a_{21},a_{32}),\{\frac{x_1}{<0.4,0.5>},\frac{x_4}{<0.7,0.2>}\}),((a_{11},a_{22},a_{32}),\{\frac{x_3}{<0.4,0.2>},\frac{x_4}{<0.1,0.8>}\})\}.$$

We have excluded all those $\boldsymbol{\alpha} \in \mathbf{A}$ for which $F(\boldsymbol{\alpha})$ is a null intuitionistic fuzzy set (similarly for those $\boldsymbol{\beta} \in \mathbf{B}$ for which $G(\boldsymbol{\beta})$ is a null intuitionistic fuzzy set).

The union and intersections of $(F,\mathbf{A})$ and $(G,\mathbf{B})$ are given by:

$(F,\mathbf{A})\ \tilde{\cup}\ (G,\mathbf{B})$

$$= \{((a_{11},a_{21},a_{31}),\{\frac{x_1}{<0.5,0.3>},\frac{x_2}{<0.7,0.2>},\frac{x_3}{<0.8,0.1>}\}),((a_{11},a_{22},a_{31}),\{\frac{x_2}{<0.6,0.3>}\}),$$

$$((a_{12},a_{21},a_{31}),\{\frac{x_3}{<0.8,0.1>},\frac{x_4}{<0.1,0.5>}\}),((a_{12},a_{22},a_{31}),\{\frac{x_1}{<0.5,0.3>},\frac{x_4}{<0.4,0.2>}\}),$$

$$((a_{11},a_{21},a_{32}),\{\frac{x_1}{<0.4,0.5>},\frac{x_4}{<0.7,0.2>}\}),((a_{11},a_{22},a_{32}),\{\frac{x_3}{<0.4,0.2>},\frac{x_4}{<0.1,0.8>}\}),$$

$$((a_{12},a_{21},a_{32}),0_{IF}),((a_{12},a_{22},a_{32}),0_{IF})\};$$

and

$$(F,\mathbf{A})\ \tilde{\cap}\ (G,\mathbf{B}) = \{((a_{11},a_{21},a_{31}),\{\frac{x_2}{<0.2,0.6>}\}),((a_{11},a_{22},a_{31}),\{\frac{x_2}{<0.3,0.5>}\})\}.$$

The differences $(F,\mathbf{A})\tilde{\setminus}(G,\mathbf{B})$ and $(G,\mathbf{B})\tilde{\setminus}(F,\mathbf{A})$ are the following

$$(F,\mathbf{A})\tilde{\setminus}(G,\mathbf{B}) = \{((a_{11},a_{21},a_{31}),\{\frac{x_1}{<0.5,0.3>},\frac{x_2}{<0.6,0.2>}\}),((a_{11},a_{22},a_{31}),\{\frac{x_2}{<0.3,0.6>}\})\};$$

$$(G,\mathbf{B})\tilde{\setminus}(F,\mathbf{A}) = \{((a_{11},a_{21},a_{31}),\{\frac{x_2}{<0.2,0.7>},\frac{x_3}{<0.8,0.1>}\}),((a_{11},a_{22},a_{31}),\{\frac{x_2}{<0.5,0.3>}\})\}.$$

Example 4 Let $\mathrm{U} = \{\mathrm{x}_1,\mathrm{x}_2,\mathrm{x}_3,\mathrm{x}_4\}$. Define the attributes sets by:

$$E_1 = \{a_{11},a_{12}\}, E_2 = \{a_{21},a_{22}\}, E_3 = \{a_{31},a_{32}\}.$$

Suppose that

$A_1 = \{a_{11},a_{12}\}, A_2 = \{a_{21},a_{22}\}, A_3 = \{a_{31}\}$, and

$B_1 = \{a_{11}\}, B_2 = \{a_{21},a_{22}\}, B_3 = \{a_{31},a_{32}\}$

that is,. $A_i, B_i \subseteq E_i$ for each $i = 1,2,3$.

Let the neutrosophic hypersoft sets $(F,\mathbf{A})$ and $(G,\mathbf{B})$ be defined by

$$(F,\mathbf{A}) = \{((a_{11},a_{21},a_{31}),\{\frac{x_1}{<0.5,0.2,0.3>},\frac{x_2}{<0.7,0.3,0.2>}\}),((a_{11},a_{22},a_{31}),\{\frac{x_2}{<0.3,0.2,0.5>}\}),$$

$$((a_{12},a_{21},a_{31}),\{\frac{x_3}{<0.8,0.4,0.1>},\frac{x_4}{<0.1,0.5,0.5>}\}),((a_{12},a_{22},a_{31}),\{\frac{x_1}{<0.5,0.2,0.3>},\frac{x_4}{<0.4,0.3,0.2>}\})\}.$$

and

$$(G,\mathbf{B}) = \{((a_{11},a_{21},a_{31}),\{\frac{x_2}{<0.2,0.5,0.6>},\frac{x_3}{<0.8,0.6,0.1>}\}),((a_{11},a_{22},a_{31}),\{\frac{x_2}{<0.6,0.2,0.3>}\}),$$

$$((a_{11},a_{21},a_{32}),\{\frac{x_1}{<0.4,0.3,0.5>},\frac{x_4}{<0.7,0.3,0.2>}\}),((a_{11},a_{22},a_{32}),\{\frac{x_3}{<0.4,0.4,0.2>},\frac{x_4}{<0.1,0.3,0.8>}\})\}.$$

We have excluded those $\boldsymbol{\alpha} \in \mathbf{A}$ for which $F(\boldsymbol{\alpha})$ is a null intuitionistic fuzzy set (similarly for those $\boldsymbol{\beta} \in \mathbf{B}$ for which $G(\boldsymbol{\beta})$ is a null intuitionistic fuzzy set).

The union and intersections of $(F,\mathbf{A})$ and $(G,\mathbf{B})$ are given by:

$$(F,\mathbf{A}) \tilde{\cup} (G,\mathbf{B})$$

$$= \{((a_{11},a_{21},a_{31}),\{\frac{x_1}{<0.5,0.2,0.3>},\frac{x_2}{<0.7,0.3,0.2>},\frac{x_3}{<0.8,0.6,0.1>}\}),((a_{11},a_{22},a_{31}),\{\frac{x_2}{<0.6,0.2,0.3>}\}),$$

$$((a_{12},a_{21},a_{31}),\{\frac{x_3}{<0.8,0.4,0.1>},\frac{x_4}{<0.1,0.5,0.5>}\}),((a_{12},a_{22},a_{31}),\{\frac{x_1}{<0.5,0.2,0.3>},\frac{x_4}{<0.4,0.3,0.2>}\}),$$

$$((a_{11},a_{21},a_{32}),\{\frac{x_1}{<0.4,0.3,0.5>},\frac{x_4}{<0.7,0.3,0.2>}\}),((a_{11},a_{22},a_{32}),\{\frac{x_3}{<0.4,0.4,0.2>},\frac{x_4}{<0.1,0.3,0.8>}\}),$$

$$((a_{12},a_{21},a_{32}),0_N),((a_{12},a_{22},a_{32}),0_N)\};$$

and

$$(F,\mathbf{A}) \tilde{\cap} (G,\mathbf{B})$$

$$= \{((a_{11},a_{21},a_{31}),\{\frac{x_2}{<0.2,0.5,0.6>}\}),((a_{11},a_{22},a_{31}),\{\frac{x_2}{<0.3,0.2,0.5>}\})\}.$$

The differences $(F,\mathbf{A})\tilde{\setminus}(G,\mathbf{B})$ and $(G,\mathbf{B})\tilde{\setminus}(F,\mathbf{A})$ are the following

$$(F,\mathbf{A})\tilde{\setminus}(G,\mathbf{B})$$

$= \{((a_{11}, a_{21}, a_{31}), \{\frac{x_1}{<0.5,0.2,0.3>}, \frac{x_2}{<0.6,0.15,0.2>}\}), ((a_{11}, a_{22}, a_{31}), \{\frac{x_2}{<0.3,0.4,0.6>}\})\};$

$(G, \mathbf{B}) \tilde{\setminus} (F, \mathbf{A})$

$= \{((a_{11}, a_{21}, a_{31}), \{\frac{x_2}{<0.2,0.15,0.7>}, \frac{x_3}{<0.8,0.6,0.1>}\}), ((a_{11}, a_{22}, a_{31}), \{\frac{x_2}{<0.5,0.4,0.3>}\})\}.$

Remark 3 *There are four types of plithogenic hypersoft sets namely: plithogenic crisp hypersoft set, plithogenic fuzzy hypersoft set, plithogenic intuitionistic fuzzy hypersoft set, plithogenic neutrosophic hypersoft set. Here we discuss only plithogenic crisp hypersoft point whereas examples for other types of sets can be constructed in the similar way.*

Example 5 Let $\mathrm{U} = \{\mathrm{x}_1, \mathrm{x}_2, \mathrm{x}_3, \mathrm{x}_4\}$. Define the attributes sets by:

$$E_1 = \{a_{11}, a_{12}\}, E_2 = \{a_{21}, a_{22}\}, E_3 = \{a_{31}, a_{32}\}.$$

Suppose that

$A_1 = \{a_{11}, a_{12}\}, A_2 = \{a_{21}, a_{22}\}, A_3 = \{a_{31}\}$, and

$B_1 = \{a_{11}\}, B_2 = \{a_{21}, a_{22}\}, B_3 = \{a_{31}, a_{32}\}$

that is,. $A_i, B_i \subseteq E_i$ for each $i = 1,2,3$.

Let the plithogenic crisp hypersoft sets $(F, \mathbf{A})$ and $(G, \mathbf{B})$ be defined by

$(F, \mathbf{A}) = \{((a_{11}, a_{21}, a_{31}), \{x_1(1,0,1), x_2(1,1,1)\}), ((a_{11}, a_{22}, a_{31}), \{x_2(0,0,1)\}),$
$((a_{12}, a_{21}, a_{31}), \{x_3(1,1,0), x_4(1,1,1)\}), ((a_{12}, a_{22}, a_{31}), \{x_1(1,0,1), x_4(0,1,0)\})\}.$

and

$(G, \mathbf{B}) = \{((a_{11}, a_{21}, a_{31}), \{x_2(1,1,1), x_3(1,1,0)\}), ((a_{11}, a_{22}, a_{31}), \{x_2(0,1,0)\}),$
$((a_{11}, a_{21}, a_{32}), \{x_1(0,1,1), x_4(1,1,1)\}), ((a_{11}, a_{22}, a_{32}), \{x_3(1,1,1), x_4(1,1,1)\})\}.$

We have excluded all those $\boldsymbol{\alpha} \in \mathbf{A}$ for which $F(\boldsymbol{\alpha})$ is a null plithogenic crisp set (similarly for those $\boldsymbol{\beta} \in \mathbf{B}$ for which $G(\boldsymbol{\beta})$ is a null plithogenic crisp set).

The union and intersections of $(F, \mathbf{A})$ and $(G, \mathbf{B})$ are given by:

$(F, \mathbf{A}) \tilde{\cup} (G, \mathbf{B}) = \{((a_{11}, a_{21}, a_{31}), \{x_1(1,0,1), x_2(1,1,1), x_3(1,1,0)\}),$
$((a_{11}, a_{22}, a_{31}), \{x_2(0,1,1)\}), ((a_{12}, a_{21}, a_{31}), \{x_3(1,1,0), x_4(1,1,1)\}),$
$((a_{12}, a_{22}, a_{31}), \{x_1(1,0,1), x_4(0,1,0)\}),$
$((a_{11}, a_{21}, a_{32}), \{x_1(0,1,1), x_4(1,1,1)\}), ((a_{11}, a_{22}, a_{32}), \{x_3(1,1,1), x_4(1,1,1)\}),$
$((a_{12}, a_{21}, a_{32}), 0_{PC}), ((a_{12}, a_{22}, a_{32}), 0_{PC})\};$

and

$$(F, \mathbf{A}) \tilde{\cap} (G, \mathbf{B}) = \{((a_{11}, a_{21}, a_{31}), \{x_2(1,1,1)\}), ((a_{11}, a_{22}, a_{31}), 0_{PC})\}.$$

The differences $(F, \mathbf{A}) \tilde{\setminus} (G, \mathbf{B})$ and $(G, \mathbf{B}) \tilde{\setminus} (F, \mathbf{A})$ are the following

$(F, \mathbf{A}) \tilde{\setminus} (G, \mathbf{B}) = \{((a_{11}, a_{21}, a_{31}), \{x_1(1,0,1)\}), ((a_{11}, a_{22}, a_{31}), \{x_2(0,0,1)\})\};$
$(G, \mathbf{B}) \tilde{\setminus} (F, \mathbf{A}) = \{((a_{11}, a_{21}, a_{31}), \{x_3(1,1,0)\}), ((a_{11}, a_{22}, a_{31}), \{x_2(0,1,0)\})\}.$

Proposition 1 *Let* $(F, \boldsymbol{A})$ *be a *-hypersoft set over* U_*. *Then the following holds;*

1. $(F, \mathbf{A}) \tilde{\cup} \Phi_{\mathbf{A}} = (F, \mathbf{A});$

2. $(F, \mathbf{A}) \tilde{\cap} \Phi_{\mathbf{A}} = \Phi_{\mathbf{A}}$;
3. $(F, \mathbf{A}) \tilde{\cup} \tilde{U}_{\mathbf{A}} = \tilde{U}_{\mathbf{A}}$;
4. $(F, \mathbf{A}) \tilde{\cap} \tilde{U}_{\mathbf{A}} = (F, \mathbf{A})$;
5. $\tilde{U}_{\mathbf{A}} \tilde{\setminus} (F, \mathbf{A}) = (F, \mathbf{A})^c$;

6. $(F, \mathbf{A}) \tilde{\cup} (F, \mathbf{A})^c = \tilde{U}_{\mathbf{A}}$;
7. $(F, \mathbf{A}) \tilde{\cap} (F, \mathbf{A})^c = \Phi_{\mathbf{A}}$.

Proof. We will prove only (i), (ii) and (v) and proofs of remaining are similar.

(i) By the definition of union, we have

$$(F, \mathbf{A}) \tilde{\cup} \Phi_{\mathbf{A}} = (H, \mathbf{C}),$$

where $\mathbf{C} = \mathbf{A}$ and $H(\boldsymbol{\alpha}) = F(\boldsymbol{\alpha}) \cup_* 0_* = F(\boldsymbol{\alpha})$ for all $\boldsymbol{\alpha} \in \mathbf{C}$. Hence $(H, \mathbf{C}) = (F, \mathbf{A})$.

(ii) By the definition of intersection, we obtain that

$$(F, \mathbf{A}) \tilde{\cap} \Phi_{\mathbf{A}} = (H, \mathbf{C}),$$

where $\mathbf{C} = \mathbf{A}$ and $H(\boldsymbol{\alpha}) = F(\boldsymbol{\alpha}) \cap_* 0_* = 0_*$ for all $\boldsymbol{\alpha} \in \mathbf{C}$. Hence $(H, \mathbf{C}) = \Phi_{\mathbf{A}}$.

(v) By the definition of difference, we get

$$\tilde{U}_{\mathbf{A}} \tilde{\setminus} (F, \mathbf{A}) = (H, \mathbf{C}),$$

where $\mathbf{C} = \mathbf{A}$ and $H(\boldsymbol{\alpha}) = U \setminus_* F(\boldsymbol{\alpha}) = F^c(\boldsymbol{\alpha})$ for all $\boldsymbol{\alpha} \in \mathbf{C}$. Hence $(H, \mathbf{C}) = (F, \mathbf{A})^c$.

3. Hypersoft point

In this section, we define hypersoft point in different frameworks and study some basic properties of such points in each setup.

3.1 Crisp hypersoft point

Definition 9 *Let $\boldsymbol{A} \subseteq \boldsymbol{E}$, $\boldsymbol{\alpha} \in \boldsymbol{A}$, and $x \in U$. A hypersoft set $(F, \boldsymbol{A})$ is said to be a hypersoft point if $F(\boldsymbol{\alpha}')$ is an empty set for every $\boldsymbol{\alpha}' \in \boldsymbol{A} \setminus \{\boldsymbol{\alpha}\}$ and $F(\boldsymbol{\alpha})$ is a singleton set. We will denote hypersoft point $(F, \boldsymbol{A})$ simply by $P^{(\boldsymbol{\alpha}, x)}$.*

Definition 10 *A hypersoft set $(F, \boldsymbol{A})$ is said to be an empty hypersoft point if $F(\boldsymbol{\alpha})$ is an empty set for each $\boldsymbol{\alpha} \in \boldsymbol{A}$. We will denote an empty hypersoft set, corresponding to $\boldsymbol{\alpha}$, by $P^{(\boldsymbol{\alpha}, \emptyset)}$.*

As a matter of fact if $(F, \mathbf{A})$ is a null hypersoft set then for every $\boldsymbol{\alpha} \in \mathbf{A}$ it may be regarded as empty hypersoft set $P^{(\boldsymbol{\alpha}, \emptyset)}$.

Definition 11 *A hypersoft point $P^{(\boldsymbol{\alpha}, x)}$ is said to belong to a hypersoft set $(G, \boldsymbol{A})$ if $P^{(\boldsymbol{\alpha}, x)} \tilde{\subseteq} (G, \boldsymbol{A})$. We write it as $P^{(\boldsymbol{\alpha}, x)} \tilde{\in} (G, \boldsymbol{A})$.*

It is straightforward to check that the hypersoft union of hypersoft points of a hypersoft set $(G, \mathbf{A})$ returns the hypersoft set $(G, \mathbf{A})$, that is,

$$(G, \mathbf{A}) = \tilde{\cup} \{P^{(\boldsymbol{\alpha}, x)} : P^{(\boldsymbol{\alpha}, x)} \tilde{\in} (G, \mathbf{A})\}.$$

We illustrate the above observation through the following example.

Example 6 Let $\mathrm{U} = \{\mathrm{x}_1, \mathrm{x}_2, \mathrm{x}_3, \mathrm{x}_4\}$, and $(\mathrm{F}, \mathbf{A})$ be as given in the example 1. Then the hypersoft points of $(\mathrm{F}, \mathbf{A})$ are the following:

$$P_1^{((a_{11},a_{21},a_{31}),x_1)} = \{((a_{11}, a_{21}, a_{31}), \{x_1\})\};$$
$$P_2^{((a_{11},a_{21},a_{31}),x_2)} = \{((a_{11}, a_{21}, a_{31}), \{x_2\})\};$$
$$P_3^{((a_{11},a_{22},a_{31}),x_2)} = \{((a_{11}, a_{22}, a_{31}), \{x_2\})\};$$
$$P_4^{((a_{12},a_{21},a_{31}),x_3)} = \{((a_{12}, a_{21}, a_{31}), \{x_3\})\};$$
$$P_5^{((a_{12},a_{21},a_{31}),x_4)} = \{((a_{12}, a_{21}, a_{31}), \{x_4\})\};$$
$$P_6^{((a_{12},a_{22},a_{31}),x_1)} = \{((a_{12}, a_{22}, a_{31}), \{x_1\})\};$$
$$P_7^{((a_{12},a_{22},a_{31}),x_4)} = \{((a_{12}, a_{22}, a_{31}), \{x_4\})\}.$$

Moreover

$$(F, \mathbf{A}) = P_1^{((a_{11},a_{21},a_{31}),x_1)} \tilde{\cup} P_2^{((a_{11},a_{21},a_{31}),x_2)} \tilde{\cup} P_3^{((a_{11},a_{22},a_{31}),x_2)}$$
$$\tilde{\cup} P_4^{((a_{12},a_{21},a_{31}),x_3)} \tilde{\cup} P_5^{((a_{12},a_{21},a_{31}),x_4)} \tilde{\cup} P_6^{((a_{12},a_{22},a_{31}),x_1)} \tilde{\cup} P_7^{((a_{12},a_{22},a_{31}),x_4)}.$$

Proposition 2 *Let $(F, \boldsymbol{A}), (F_1, \boldsymbol{A})$ and $(F_2, \boldsymbol{A})$ be hypersoft sets over U. Then the following hold:*

1. *If $(F, \boldsymbol{A})$ is not a null hypersoft set, then $(F, \boldsymbol{A})$ contains at least one nonempty hypersoft point.*
2. *$(F_1, \boldsymbol{A}) \tilde{\subseteq} (F_2, \boldsymbol{A})$ if and only if $P^{(\boldsymbol{\alpha},x)} \tilde{\in} (F_1, \boldsymbol{A})$ implies that $P^{(\boldsymbol{\alpha},x)} \tilde{\in} (F_2, \boldsymbol{A})$.*
3. *$P^{(\boldsymbol{\alpha},x)} \tilde{\in} (F_1, \boldsymbol{A}) \tilde{\cup} (F_2, \boldsymbol{A})$ if and only if $P^{(\boldsymbol{\alpha},x)} \tilde{\in} (F_1, \boldsymbol{A})$ or $P^{(\boldsymbol{\alpha},x)} \tilde{\in} (F_2, \boldsymbol{A})$.*
4. *$P^{(\boldsymbol{\alpha},x)} \tilde{\in} (F_1, \boldsymbol{A}) \tilde{\cap} (F_2, \boldsymbol{A})$ if and only if $P^{(\boldsymbol{\alpha},x)} \tilde{\in} (F_1, \boldsymbol{A})$ and $P^{(\boldsymbol{\alpha},x)} \tilde{\in} (F_2, \boldsymbol{A})$.*
5. *$P^{(\boldsymbol{\alpha},x)} \tilde{\in} (F_1, \boldsymbol{A}) \tilde{\setminus} (F_2, \boldsymbol{A})$ if and only if $P^{(\boldsymbol{\alpha},x)} \tilde{\in} (F_1, \boldsymbol{A})$ and $P^{(\boldsymbol{\alpha},x)} \tilde{\notin} (F_2, \boldsymbol{A})$.*

Proof. We will prove (1), (2) and (3). Proofs of (4) and (5) are similar to that of (3).

(1) Suppose that $(F, \mathbf{A})$ is not a null hypersoft set, that is, $F(\boldsymbol{\alpha}) \neq \varnothing$ for some $\boldsymbol{\alpha} \in \mathbf{A}$. Now if $\boldsymbol{\alpha}_0 \in \mathbf{A}$ is such that $F(\boldsymbol{\alpha}_0) \neq \varnothing$, then for $x \in F(\boldsymbol{\alpha}_0)$, there will be a hypersoft point $P^{(\boldsymbol{\alpha}_0,x)}$ such that $P^{(\boldsymbol{\alpha}_0,x)} \tilde{\in} (F, \mathbf{A})$.

(2) Suppose that $(F_1, \mathbf{A}) \tilde{\subseteq} (F_2, \mathbf{A})$ and $P^{(\boldsymbol{\alpha},x)} \tilde{\in} (F_1, \mathbf{A})$. By the definition 11, we have

$$P^{(\boldsymbol{\alpha},x)} \tilde{\subseteq} (F_1, \mathbf{A}).$$

Thus

$$P^{(\boldsymbol{\alpha},x)} \tilde{\subseteq} (F_1, \mathbf{A}) \tilde{\subseteq} (F_2, \mathbf{A})$$

implies that $P^{(\boldsymbol{\alpha},x)} \tilde{\in} (F_2, \mathbf{A})$.

Conversely suppose that $P^{(\boldsymbol{\alpha},x)} \tilde{\in} (F_1, \mathbf{A})$ which implies that $P^{(\boldsymbol{\alpha},x)} \tilde{\in} (F_2, \mathbf{A})$. By the definition 11, we obtain that

$$P^{(\boldsymbol{\alpha},x)} \tilde{\subseteq} (F_2, \mathbf{A}) \text{forall} P^{(\boldsymbol{\alpha},x)} \tilde{\in} (F_1, \mathbf{A}).$$

Thus we have

$$(F_1, \mathbf{A}) = \tilde{\cup} \{P^{(\boldsymbol{\alpha},x)} : P^{(\boldsymbol{\alpha},x)} \tilde{\in} (G, \mathbf{A})\} \tilde{\subseteq} (F_2, \mathbf{A}).$$

(3) Suppose that $P^{(\boldsymbol{\alpha},x)} \tilde{\in} (F_1, \mathbf{A}) \tilde{\cup} (F_2, \mathbf{A})$. It follows from the definition 11 that

$$P^{(\boldsymbol{\alpha},x)} \tilde{\subseteq} (F_1, \mathbf{A}) \tilde{\cup} (F_2, \mathbf{A}),$$

which implies that $x \in F_1(\boldsymbol{\alpha}) \cup_C F_2(\boldsymbol{\alpha})$. Thus $x \in F_1(\boldsymbol{\alpha})$ or $F_2(\boldsymbol{\alpha})$. Hence we have

$$P^{(\boldsymbol{\alpha},x)} \tilde{\in} (F_1, \mathbf{A}) \text{or} P^{(\boldsymbol{\alpha},x)} \tilde{\in} (F_2, \mathbf{A}).$$

Conversely suppose that $P^{(\boldsymbol{\alpha},x)} \tilde{\in} (F_1, \mathbf{A})$ or $P^{(\boldsymbol{\alpha},x)} \tilde{\in} (F_2, \mathbf{A})$. This implies that $x \in F_1(\boldsymbol{\alpha})$ or $F_2(\boldsymbol{\alpha})$. Thus $x \in F_1(\boldsymbol{\alpha}) \cup_C F_2(\boldsymbol{\alpha})$ and so we have

$$P^{(\boldsymbol{\alpha},x)} \tilde{\subseteq} (F_1, \mathbf{A}) \tilde{\cup} (F_2, \mathbf{A}).$$

3.2 Fuzzy hypersoft point

Definition 12 *Let $\boldsymbol{A} \subseteq \boldsymbol{E}$, $\boldsymbol{\alpha} \in \boldsymbol{A}$, and $x \in U_F$. A fuzzy hypersoft set $(F, \boldsymbol{A})$ is said to be a fuzzy hypersoft point if $F(\boldsymbol{\alpha}')$ is a null fuzzy set for every $\boldsymbol{\alpha}' \in \boldsymbol{A} \setminus \{\boldsymbol{\alpha}\}$ and $F(\boldsymbol{\alpha})(y) = 0$ for all $y \neq x$. We will denote $(F, \boldsymbol{A})$ simply by $FP^{(\boldsymbol{\alpha},x)}$.*

Definition 13 *A fuzzy hypersoft set $(F, \boldsymbol{A})$ is said to be a null fuzzy hypersoft point if $F(\boldsymbol{\alpha})$ is a null fuzzy set for each $\boldsymbol{\alpha} \in \boldsymbol{A}$. We denote a null fuzzy hypersoft set, corresponding to $\boldsymbol{\alpha}$, by $FP^{(\boldsymbol{\alpha},0_F)}$.*

Note that if $(F, \mathbf{A})$ is a null fuzzy hypersoft set then for every $\boldsymbol{\alpha} \in \mathbf{A}$, it can be regarded as null fuzzy hypersoft set $FP^{(\boldsymbol{\alpha},0_F)}$.

Definition 14 *A fuzzy hypersoft point $FP^{(\boldsymbol{\alpha},x)}$ is said to belong to a fuzzy hypersoft set $(G, \boldsymbol{A})$ if $FP^{(\boldsymbol{\alpha},x)} \tilde{\subseteq} (G, \boldsymbol{A})$. We write it as $FP^{(\boldsymbol{\alpha},x)} \tilde{\in} (G, \boldsymbol{A})$.*

It is straightforward to check that the fuzzy hypersoft union of fuzzy hypersoft points of a fuzzy hypersoft set $(G, \mathbf{A})$ returns the fuzzy hypersoft set $(G, \mathbf{A})$, that is,

$$(G, \mathbf{A}) = \tilde{\cup} \{FP^{(\boldsymbol{\alpha},x)} : FP^{(\boldsymbol{\alpha},x)} \tilde{\in} (G, \mathbf{A})\}.$$

We illustrate this observation through the following example.

Example 7 Let $\mathrm{U} = \{\mathrm{x}_1, \mathrm{x}_2, \mathrm{x}_3, \mathrm{x}_4\}$, and $(\mathrm{F}, \mathbf{A})$ be as given in the example 2. Then some of the fuzzy hypersoft points of $(\mathrm{F}, \mathbf{A})$ are given as:

$$FP_1^{((a_{11},a_{21},a_{31}),x_1)} = \left\{((a_{11}, a_{21}, a_{31}), \{\tfrac{x_1}{0.5}\})\right\};$$

$$FP_2^{((a_{11},a_{21},a_{31}),x_1)} = \left\{((a_{11}, a_{21}, a_{31}), \{\tfrac{x_1}{0.2}\})\right\};$$

$$FP_3^{((a_{11},a_{21},a_{31}),x_2)} = \left\{((a_{11}, a_{21}, a_{31}), \{\tfrac{x_2}{0.7}\})\right\};$$

$$FP_4^{((a_{11},a_{22},a_{31}),x_2)} = \left\{((a_{11}, a_{22}, a_{31}), \{\tfrac{x_2}{0.3}\})\right\};$$

$$FP_5^{((a_{12},a_{21},a_{31}),x_3)} = \left\{((a_{12}, a_{21}, a_{31}), \{\tfrac{x_3}{0.8}\})\right\};$$

$$FP_6^{((a_{12},a_{21},a_{31}),x_4)} = \left\{((a_{12}, a_{21}, a_{31}), \{\tfrac{x_4}{0.6}\})\right\};$$

$$FP_7^{((a_{12},a_{21},a_{31}),x_4)} = \left\{((a_{12}, a_{21}, a_{31}), \{\tfrac{x_4}{0.9}\})\right\};$$

$$FP_8^{((a_{12},a_{22},a_{31}),x_1)} = \left\{((a_{12}, a_{22}, a_{31}), \{\tfrac{x_1}{0.5}\})\right\};$$

$$FP_9^{((a_{12},a_{22},a_{31}),x_4)} = \left\{((a_{12}, a_{22}, a_{31}), \{\tfrac{x_4}{0.4}\})\right\}.$$

Moreover we have

$$\begin{aligned}(F, \mathbf{A}) = {} & FP_1^{((a_{11},a_{21},a_{31}),x_1)} \tilde{\cup} FP_2^{((a_{11},a_{21},a_{31}),x_1)} \tilde{\cup} FP_3^{((a_{11},a_{21},a_{31}),x_2)} \\ & \tilde{\cup} FP_4^{((a_{11},a_{22},a_{31}),x_2)} \tilde{\cup} FP_5^{((a_{12},a_{21},a_{31}),x_3)} \tilde{\cup} FP_6^{((a_{12},a_{21},a_{31}),x_4)} \\ & \tilde{\cup} FP_7^{((a_{12},a_{21},a_{31}),x_4)} \tilde{\cup} FP_8^{((a_{12},a_{22},a_{31}),x_1)} \tilde{\cup} FP_9^{((a_{12},a_{22},a_{31}),x_4)}.\end{aligned}$$

Proposition 3 *Let $(F, \boldsymbol{A}), (F_1, \boldsymbol{A})$ and $(F_2, \boldsymbol{A})$ be fuzzy hypersoft sets over U. Then the following hold:*

1. If $(F, \mathbf{A})$ is not a null fuzzy hypersoft set, then $(F, \mathbf{A})$ contains at least one nonnull fuzzy hypersoft point.
2. $(F_1, \mathbf{A}) \tilde{\subseteq} (F_2, \mathbf{A})$ if and only if $FP^{(\boldsymbol{\alpha},x)} \tilde{\in} (F_1, \mathbf{A})$ implies that $FP^{(\boldsymbol{\alpha},x)} \tilde{\in} (F_2, \mathbf{A})$.

3. $FP^{(\alpha,x)} \tilde{\in} (F_1, \mathbf{A}) \tilde{\cup} (F_2, \mathbf{A})$ if and only if $FP^{(\alpha,x)} \tilde{\in} (F_1, \mathbf{A})$ or $FP^{(\alpha,x)} \tilde{\in} (F_2, \mathbf{A})$.
4. $FP^{(\alpha,x)} \tilde{\in} (F_1, \mathbf{A}) \tilde{\cap} (F_2, \mathbf{A})$ if and only if $FP^{(\alpha,x)} \tilde{\in} (F_1, \mathbf{A})$ and $FP^{(\alpha,x)} \tilde{\in} (F_2, \mathbf{A})$.
5. $FP^{(\alpha,x)} \tilde{\in} (F_1, \mathbf{A}) \tilde{\setminus} (F_2, \mathbf{A})$ if and only if $FP^{(\alpha,x)} \tilde{\in} (F_1, \mathbf{A})$ and $FP^{(\alpha,x)} \tilde{\notin} (F_2, \mathbf{A})$.

The proof of above proposition is similar as in the case of crisp hypersoft point.

3.3 Intuitionistic fuzzy hypersoft point

Definition 15 *Let $\boldsymbol{A} \subseteq \boldsymbol{E}$, $\boldsymbol{\alpha} \in \boldsymbol{A}$, and $x \in U_{IF}$. An intuitionistic fuzzy hypersoft set $(F, \boldsymbol{A})$ is said to be an intuitionistic fuzzy hypersoft point if $F(\boldsymbol{\alpha}')$ is a null intuitionistic fuzzy set for every $\boldsymbol{\alpha}' \in \boldsymbol{A} \setminus \{\boldsymbol{\alpha}\}$ and $F(\boldsymbol{\alpha})(y) =< 0,1 >$ for all $y \neq x$. We will denote $(F, \boldsymbol{A})$ simply by $IFP^{(\boldsymbol{\alpha},x)}$.*

Definition 16 *An intuitionistic fuzzy hypersoft set $(F, \boldsymbol{A})$ is said to be a null intuitionistic fuzzy hypersoft point if $F(\boldsymbol{\alpha})$ is a null intuitionistic fuzzy set for each $\boldsymbol{\alpha} \in \boldsymbol{A}$. We will denote a null intuitionistic fuzzy hypersoft set, corresponding to $\boldsymbol{\alpha}$, by $IFP^{(\boldsymbol{\alpha},0_{IF})}$.*

If $(F, \mathbf{A})$ is a null intuitionistic fuzzy hypersoft set, then for every $\boldsymbol{\alpha} \in \mathbf{A}$ it can be regarded as null intuitionistic fuzzy hypersoft set $IFP^{(\boldsymbol{\alpha},0_{IF})}$.

Definition 17 *An intuitionistic fuzzy hypersoft point $IFP^{(\boldsymbol{\alpha},x)}$ is said to belong to an intuitionistic fuzzy hypersoft set $(G, \boldsymbol{A})$ if $IFP^{(\boldsymbol{\alpha},x)} \tilde{\subseteq} (G, \boldsymbol{A})$. We write it as $IFP^{(\boldsymbol{\alpha},x)} \tilde{\in} (G, \boldsymbol{A})$.*

It is straightforward to check that the intuitionistic fuzzy hypersoft union of intuitionistic fuzzy hypersoft points of an intuitionistic fuzzy hypersoft set $(G, \mathbf{A})$ gives the intuitionistic fuzzy hypersoft set $(G, \mathbf{A})$, that is,

$$(G, \mathbf{A}) = \tilde{\cup} \{IFP^{(\boldsymbol{\alpha},x)} : IFP^{(\boldsymbol{\alpha},x)} \tilde{\in} (G, \mathbf{A})\}.$$

We illustrate this observation through the following example.

Example 8 Let $U = \{x_1, x_2, x_3, x_4\}$, and $(F, \mathbf{A})$ be as given in the example 3. Then some of the intuitionistic fuzzy hypersoft points of $(F, \mathbf{A})$ are the following:

$$IFP_1^{((a_{11},a_{21},a_{31}),x_1)} = \left\{((a_{11}, a_{21}, a_{31}), \{\tfrac{x_1}{<0.5,0.3>}\})\right\};$$

$$IFP_2^{((a_{11},a_{21},a_{31}),x_1)} = \left\{((a_{11}, a_{21}, a_{31}), \{\tfrac{x_1}{<0.2,0.3>}\})\right\};$$

$$IFP_3^{((a_{11},a_{21},a_{31}),x_2)} = \left\{((a_{11}, a_{21}, a_{31}), \{\tfrac{x_2}{<0.7,0.2>}\})\right\};$$

$$IFP_4^{((a_{11},a_{22},a_{31}),x_2)} = \left\{((a_{11}, a_{22}, a_{31}), \{\tfrac{x_2}{<0.3,0.5>}\})\right\};$$

$$IFP_5^{((a_{12},a_{21},a_{31}),x_3)} = \left\{((a_{12}, a_{21}, a_{31}), \{\tfrac{x_3}{<0.8,0.1>}\})\right\};$$

$$IFP_6^{((a_{12},a_{21},a_{31}),x_4)} = \left\{((a_{12}, a_{21}, a_{31}), \{\tfrac{x_4}{<0.1,0.6>}\})\right\};$$

$$IFP_7^{(a_{12},a_{21},a_{31}),x_4)} = \left\{((a_{12}, a_{21}, a_{31}), \{\tfrac{x_4}{<0.1,0.5>}\})\right\};$$

$$IFP_8^{((a_{12},a_{22},a_{31}),x_1)} = \left\{((a_{12}, a_{22}, a_{31}), \{\tfrac{x_1}{<0.5,0.3>}\})\right\};$$

$$IFP_9^{((a_{12},a_{22},a_{31}),x_4)} = \left\{((a_{12}, a_{22}, a_{31}), \{\tfrac{x_4}{<0.4,0.2>}\})\right\}.$$

Moreover we have

$$(F, \mathbf{A}) = IFP_1^{((a_{11},a_{21},a_{31}),x_1)} \tilde{\cup}\, IFP_2^{((a_{11},a_{21},a_{31}),x_1)} \tilde{\cup}\, IFP_3^{((a_{11},a_{21},a_{31}),x_2)} \tilde{\cup}\, IFP_4^{((a_{11},a_{22},a_{31}),x_2)} \tilde{\cup}\, IFP_5^{((a_{12},a_{21},a_{31}),x_3)} \tilde{\cup}\, IFP_6^{((a_{12},a_{21},a_{31}),x_4)} \tilde{\cup}\, IFP_7^{((a_{12},a_{21},a_{31}),x_4)} \tilde{\cup}\, IFP_8^{((a_{12},a_{22},a_{31}),x_1)} \tilde{\cup}\, IFP_9^{((a_{12},a_{22},a_{31}),x_4)}.$$

Proposition 4 *Let $(F,\boldsymbol{A}),(F_1,\boldsymbol{A})$ and $(F_2,\boldsymbol{A})$ be intuitionistic fuzzy hypersoft sets over U. Then the following hold:*

1. If $(F,\mathbf{A})$ is not a null intuitionistic fuzzy hypersoft set then $(F,\mathbf{A})$ contains at least one nonnull intuitionistic fuzzy hypersoft point.
2. $(F_1,\boldsymbol{A}) \tilde{\subseteq} (F_2,\boldsymbol{A})$ *if and only if* $IFP^{(\alpha,x)} \tilde{\in} (F_1,\boldsymbol{A})$ *implies that* $IFP^{(\alpha,x)} \tilde{\in} (F_2,\boldsymbol{A})$.
3. $IFP^{(\alpha,x)} \tilde{\in} (F_1,\boldsymbol{A}) \tilde{\cup} (F_2,\boldsymbol{A})$ *if and only if* $IFP^{(\alpha,x)} \tilde{\in} (F_1,\boldsymbol{A})$ *or* $IFP^{(\alpha,x)} \tilde{\in} (F_2,\boldsymbol{A})$.
4. $IFP^{(\alpha,x)} \tilde{\in} (F_1,\boldsymbol{A}) \tilde{\cap} (F_2,\boldsymbol{A})$ *if and only if* $IFP^{(\alpha,x)} \tilde{\in} (F_1,\boldsymbol{A})$ *and* $IFP^{(\alpha,x)} \tilde{\in} (F_2,\boldsymbol{A})$.
5. $IFP^{(\alpha,x)} \tilde{\in} (F_1,\boldsymbol{A}) \tilde{\setminus} (F_2,\boldsymbol{A})$ *if and only if* $IFP^{(\alpha,x)} \tilde{\in} (F_1,\boldsymbol{A})$ *and* $IFP^{(\alpha,x)} \tilde{\notin} (F_2,\boldsymbol{A})$.

The proof of above proposition is similar as in the case of crisp hypersoft point.

3.4 Neutrosophic hypersoft point

Definition 18 *Let $\boldsymbol{A} \subseteq \boldsymbol{E}$ and $\boldsymbol{\alpha} \in \boldsymbol{A}$, $x \in U_N$. A neutrosophic hypersoft set $(F,\boldsymbol{A})$ is said to be a neutrosophic fuzzy hypersoft point if $F(\boldsymbol{\alpha}')$ is a null neutrosophic set for every $\boldsymbol{\alpha}' \in \boldsymbol{A}\setminus\{\boldsymbol{\alpha}\}$ and $F(\boldsymbol{\alpha})(y) =< 0,1,1 >$ for all $y \neq x$. We will denote $(F,\boldsymbol{A})$ simply by $NP^{(\alpha,x)}$.*

Definition 19 *A neutrosophic hypersoft set $(F,\boldsymbol{A})$ is said to be a null neutrosophic hypersoft point if $F(\boldsymbol{\alpha})$ is a null neutrosophic set for each $\boldsymbol{\alpha} \in \boldsymbol{A}$. We will denote a null neutrosophic hypersoft set, corresponding to $\boldsymbol{\alpha}$, by $NP^{(\alpha,0_N)}$.*

Its a matter of fact that if $(F,\mathbf{A})$ is a null neutrosophic hypersoft set then for every $\boldsymbol{\alpha} \in \mathbf{A}$ it can be regarded as null neutrosophic hypersoft set $NP^{(\alpha,0_N)}$.

Definition 20 *A neutrosophic hypersoft point $NP^{(\alpha,x)}$ is said to belong to a neutrosophic hypersoft set $(G,\boldsymbol{A})$ if $NP^{(\alpha,x)} \tilde{\subseteq} (G,\boldsymbol{A})$. We write it as $NP^{(\alpha,x)} \tilde{\in} (G,\boldsymbol{A})$.*

It is straightforward to check that the neutrosophic hypersoft union of neutrosophic hypersoft points of a neutrosophic hypersoft set $(G,\mathbf{A})$ returns the neutrosophic hypersoft set $(G,\mathbf{A})$, that is,

$$(G,\mathbf{A}) = \tilde{\cup}\, \{NP^{(\alpha,x)} : NP^{(\alpha,x)} \tilde{\in} (G,\mathbf{A})\}.$$

We illustrate this observation through the following example.

Example 9 Let $U = \{x_1, x_2, x_3, x_4\}$, and $(F,\mathbf{A})$ be as given in the example 4. Some of the neutrosophic hypersoft points of $(F,\mathbf{A})$ are the following:

$$NP_1^{((a_{11},a_{21},a_{31}),x_1)} = \left\{((a_{11},a_{21},a_{31}), \{\tfrac{x_1}{<0.5,0.2,0.3>}\})\right\};$$

$$NP_2^{((a_{11},a_{21},a_{31}),x_1)} = \left\{((a_{11},a_{21},a_{31}), \{\tfrac{x_1}{<0.2,0.2,0.3>}\})\right\};$$

$$NP_3^{((a_{11},a_{21},a_{31}),x_2)} = \left\{((a_{11},a_{21},a_{31}), \{\tfrac{x_2}{<0.7,0.3,0.2>}\})\right\};$$

$$NP_4^{((a_{11},a_{22},a_{31}),x_2)} = \left\{((a_{11},a_{22},a_{31}), \{\tfrac{x_2}{<0.3,0.2,0.5>}\})\right\};$$

$$NP_5^{((a_{12},a_{21},a_{31}),x_3)} = \left\{((a_{12},a_{21},a_{31}),\{\frac{x_3}{<0.8,0.4,0.1>}\})\right\};$$

$$NP_6^{((a_{12},a_{21},a_{31}),x_4)} = \left\{((a_{12},a_{21},a_{31}),\{\frac{x_4}{<0.1,0.5,0.5>}\})\right\};$$

$$NP_7^{((a_{12},a_{22},a_{31}),x_1)} = \left\{((a_{12},a_{22},a_{31}),\{\frac{x_1}{<0.5,0.2,0.3>}\})\right\};$$

$$NP_8^{((a_{12},a_{22},a_{31}),x_4)} = \left\{((a_{12},a_{22},a_{31}),\{\frac{x_4}{<0.4,0.3,0.2>}\})\right\}.$$

Moreover we have

$$(F,\mathbf{A}) = NP_1^{((a_{11},a_{21},a_{31}),x_1)} \tilde{\cup}\, NP_2^{((a_{11},a_{21},a_{31}),x_1)} \tilde{\cup}\, NP_3^{((a_{11},a_{21},a_{31}),x_2)}$$
$$\tilde{\cup}\, NP_4^{((a_{11},a_{22},a_{31}),x_2)} \tilde{\cup}\, NP_5^{((a_{12},a_{21},a_{31}),x_3)} \tilde{\cup}\, NP_6^{((a_{12},a_{21},a_{31}),x_4)}$$
$$\tilde{\cup}\, NP_7^{((a_{12},a_{22},a_{31}),x_1)} \tilde{\cup}\, NP_8^{((a_{12},a_{22},a_{31}),x_4)}.$$

Proposition 5 *Let $(F,\boldsymbol{A}),(F_1,\boldsymbol{A})$ and $(F_2,\boldsymbol{A})$ be neutrosophic hypersoft sets over U. Then the following hold:*

1. *If $(F,\boldsymbol{A})$ is not a null neutrosophic hypersoft set then $(F,\boldsymbol{A})$ contains at least one nonnull neutrosophic hypersoft point.*
2. *$(F_1,\boldsymbol{A}) \tilde{\subseteq} (F_2,\boldsymbol{A})$ if and only if $NP^{(\boldsymbol{\alpha},x)} \tilde{\in} (F_1,\boldsymbol{A})$ implies that $NP^{(\boldsymbol{\alpha},x)} \tilde{\in} (F_2,\boldsymbol{A})$.*
3. *$NP^{(\boldsymbol{\alpha},x)} \tilde{\in} (F_1,\boldsymbol{A}) \tilde{\cup} (F_2,\boldsymbol{A})$ if and only if $NP^{(\boldsymbol{\alpha},x)} \tilde{\in} (F_1,\boldsymbol{A})$ or $NP^{(\boldsymbol{\alpha},x)} \tilde{\in} (F_2,\boldsymbol{A})$.*
4. *$NP^{(\boldsymbol{\alpha},x)} \tilde{\in} (F_1,\boldsymbol{A}) \tilde{\cap} (F_2,\boldsymbol{A})$ if and only if $NP^{(\boldsymbol{\alpha},x)} \tilde{\in} (F_1,\boldsymbol{A})$ and $NP^{(\boldsymbol{\alpha},x)} \tilde{\in} (F_2,\boldsymbol{A})$.*
5. *$NP^{(\boldsymbol{\alpha},x)} \tilde{\in} (F_1,\boldsymbol{A}) \tilde{\setminus} (F_2,\boldsymbol{A})$ if and only if $NP^{(\boldsymbol{\alpha},x)} \tilde{\in} (F_1,\boldsymbol{A})$ and $NP^{(\boldsymbol{\alpha},x)} \tilde{\notin} (F_2,\boldsymbol{A})$.*

The proof of above proposition is similar as in the case of crisp hypersoft point.

3.5 Plithogenic hypersoft point

There may be four types of plithogenic hypersoft points namely: plithogenic crisp hypersoft point, plithogenic fuzzy hypersoft point, plithogenic intuitionistic fuzzy hypersoft point, plithogenic neutrosophic hypersoft point. But in this section we discuss only plithogenic crisp hypersoft point whereas other concepts and examples can be given in the similar way.

Definition 21 *Let $\boldsymbol{A} \subseteq \boldsymbol{E}$, $\boldsymbol{\alpha} \in \boldsymbol{A}$, and $x \in U_P$. A plithogenic crisp hypersoft set $(F,\boldsymbol{A})$ is said to be a plithogenic crisp hypersoft point if $F(\boldsymbol{\alpha}')$ is a null plithogenic crisp set for every $\boldsymbol{\alpha}' \in \boldsymbol{A}\setminus\{\boldsymbol{\alpha}\}$ and $F(\boldsymbol{\alpha})(y)(\mathbf{0})$ for all $y \neq x$. We will denote $(F,\boldsymbol{A})$ simply by $P_cP^{(\boldsymbol{\alpha},x)}$.*

Definition 22 *A plithogenic crisp hypersoft set $(F,\boldsymbol{A})$ is said to be a null plithogenic crisp hypersoft point if $F(\boldsymbol{\alpha})$ is a null plithogenic crisp set for each $\boldsymbol{\alpha} \in \boldsymbol{A}$. We will denote a null plithogenic crisp hypersoft set, corresponding to $\boldsymbol{\alpha}$, by $P_cP^{(\boldsymbol{\alpha},0_{PC})}$.*

Note that if $(F,\mathbf{A})$ is a null plithogenic crisp hypersoft set, then for every $\boldsymbol{\alpha} \in \mathbf{A}$ it can be regarded as a null plithogenic crisp hypersoft set $P_cP^{(\boldsymbol{\alpha},0_{PC})}$.

Definition 23 *A plithogenic crisp hypersoft point $P_cP^{(\boldsymbol{\alpha},x)}$ is said to belong to a plithogenic crisp hypersoft set $(G,\boldsymbol{A})$ if $P_cP^{(\boldsymbol{\alpha},x)} \tilde{\subseteq} (G,\boldsymbol{A})$. We write it as $P_cP^{(\boldsymbol{\alpha},x)} \tilde{\in} (G,\boldsymbol{A})$.*

It is straightforward to check that the plithogenic crisp hypersoft union of plithogenic crisp hypersoft points of a plithogenic crisp hypersoft set $(G, \mathbf{A})$ gives back the plithogenic crisp hypersoft set $(G, \mathbf{A})$, that is,

$$(G, \mathbf{A}) = \tilde{\cup} \{P_c P^{(\boldsymbol{\alpha}, x)} : P_c P^{(\boldsymbol{\alpha}, x)} \tilde{\in} (G, \mathbf{A})\}.$$

We illustrate this observation through the following example.

Example 10 Let $\mathrm{U} = \{\mathrm{x}_1, \mathrm{x}_2, \mathrm{x}_3, \mathrm{x}_4\}$, and $(\mathrm{F}, \mathbf{A})$ be as given in the example 5. Then some of the plithogenic crisp hypersoft points of $(\mathrm{F}, \mathbf{A})$ are the following:

$$P_c P_1^{((a_{11}, a_{21}, a_{31}), x_1)} = \{((a_{11}, a_{21}, a_{31}), \{x_1(1,0,1)\})\};$$
$$P_c P_2^{((a_{11}, a_{21}, a_{31}), x_2)} = \{((a_{11}, a_{21}, a_{31}), \{x_2(1,1,1)\})\};$$
$$P_c P_3^{((a_{11}, a_{22}, a_{31}), x_2)} = \{((a_{11}, a_{22}, a_{31}), \{x_2(0,0,1)\})\};$$
$$P_c P_4^{((a_{12}, a_{21}, a_{31}), x_3)} = \{((a_{12}, a_{21}, a_{31}), \{x_3(1,1,0)\})\};$$
$$P_c P_5^{((a_{12}, a_{21}, a_{31}), x_4)} = \{((a_{12}, a_{21}, a_{31}), \{x_4(1,1,1)\})\};$$
$$P_c P_6^{((a_{12}, a_{22}, a_{31}), x_1)} = \{((a_{12}, a_{22}, a_{31}), \{x_1(1,0,1)\})\};$$
$$P_c P_7^{((a_{12}, a_{22}, a_{31}), x_4)} = \{((a_{12}, a_{22}, a_{31}), \{x_4(0,1,0)\})\}.$$

Moreover we have

$$(F, \mathbf{A}) = P_c P_1^{((a_{11}, a_{21}, a_{31}), x_1)} \tilde{\cup} P_c P_2^{((a_{11}, a_{21}, a_{31}), x_2)}$$
$$\tilde{\cup} P_c P_3^{((a_{11}, a_{22}, a_{31}), x_2)} \tilde{\cup} P_c P_4^{((a_{12}, a_{21}, a_{31}), x_3)} \tilde{\cup} P_c P_5^{((a_{12}, a_{21}, a_{31}), x_4)}$$
$$\tilde{\cup} P_c P_6^{((a_{12}, a_{22}, a_{31}), x_1)} \tilde{\cup} P_c P_7^{((a_{12}, a_{22}, a_{31}), x_4)}.$$

Proposition 6 *Let* $(F, \boldsymbol{A}), (F_1, \boldsymbol{A})$ *and* $(F_2, \boldsymbol{A})$ *be plithogenic crisp hypersoft sets over* U. *Then the following hold:*

1. *If* $(F, \boldsymbol{A})$ *is not a null plithogenic crisp hypersoft set then* $(F, \boldsymbol{A})$ *contains at least one nonnull plithogenic crisp hypersoft point.*
2. $(F_1, \boldsymbol{A}) \tilde{\subseteq} (F_2, \boldsymbol{A})$ *if and only if* $P_c P^{(\boldsymbol{\alpha}, x)} \tilde{\in} (F_1, \boldsymbol{A})$ *implies that* $P_c P^{(\boldsymbol{\alpha}, x)} \tilde{\in} (F_2, \boldsymbol{A})$.
3. $P_c P^{(\boldsymbol{\alpha}, x)} \tilde{\in} (F_1, \boldsymbol{A}) \tilde{\cup} (F_2, \boldsymbol{A})$ *if and only if* $P_c P^{(\boldsymbol{\alpha}, x)} \tilde{\in} (F_1, \boldsymbol{A})$ *or* $P_c P^{(\boldsymbol{\alpha}, x)} \tilde{\in} (F_2, \boldsymbol{A})$.
4. $P_c P^{(\boldsymbol{\alpha}, x)} \tilde{\in} (F_1, \boldsymbol{A}) \tilde{\cap} (F_2, \boldsymbol{A})$ *if and only if* $P_c P^{(\boldsymbol{\alpha}, x)} \tilde{\in} (F_1, \boldsymbol{A})$ *and* $P_c P^{(\boldsymbol{\alpha}, x)} \tilde{\in} (F_2, \boldsymbol{A})$.
5. $P_c P^{(\boldsymbol{\alpha}, x)} \tilde{\in} (F_1, \boldsymbol{A}) \tilde{\setminus} (F_2, \boldsymbol{A})$ *if and only if* $P_c P^{(\boldsymbol{\alpha}, x)} \tilde{\in} (F_1, \boldsymbol{A})$ *and* $P_c P^{(\boldsymbol{\alpha}, x)} \tilde{\notin} (F_2, \boldsymbol{A})$.

The proof of above proposition is similar as in the case of crisp hypersoft point.

4. Conclusions

In this paper, we have initiated the concept of hypersoft point that will lead to define Cartesian product and then function on *-hypersoft sets. As a future work, one may carry out the study of *-hypersoft topological spaces. Once the functions on *-hypersoft sets are defined, this may lead to the study of fixed point results in this new framework.

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Introduction to Combined Plithogenic Hypersoft Sets

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Abstract: Plithogenic Hypersoft sets was introduced by Florentin Smarandache, who has extended crisp sets, fuzzy sets, intuitionistic sets, neutrosophic sets to plithogenic sets. The plithogenic sets considers the degree of appurtenance of the elements with respect to the attribute system. Smarandache has presented the classification of the plithogenic hypersoft sets and the applications of plithogenic fuzzy whole hypersoft sets in multi attribute decision making. Inspired by these research works, the concept of combined plithogenic hypersoft sets is introduced in this article. The representations of the degree of appurtenance of the elements determines the type of plithogenic hypersoft set, if it takes a combination of values then the new archetype of plithogenic hypersoft sets gets emerged into decision making scenario. This research work is put forth to project the realistic perception of the experts in the construction process of optimal conclusions.

Keywords: Plithogenic hypersoft set, combined plithogenic hypersoft set, decision making, multi attribute system.

1. Introduction

Classical set theory deals with the sets consisting of elements with membership values 0 or 1. The degree of belongingness of an element in a set has been extended to [0,1] by Zadeh [1] in the name of fuzzy sets, which is gaining momentum since its introduction. Sets comprising of quantitative elements can be defined by conventional concepts of sets, but the elements of qualitative nature can be treated only by fuzzy concepts and its membership value states the degree of confidence of its presence in the set. Atanassov [2] investigated on the degree of its absence in the set, by defining non-membership values. This paved way for the intuitionistic fuzzy sets, which consists of degree of membership, non-membership and hesitation. Fuzzy sets and intuitionistic fuzzy sets are extensively applied in decision making process. But still the human perception was not completely reflected in these two kinds of sets. This gap was filled by Florentine Smarandache [3-5] who introduced neutrosophic fuzzy sets, comprising of degree of truth membership, indeterminacy and degree of false membership. Smarandache has represented each of the three function in a more generalized and independent manner. Neutrosophic sets have extensive application in decision making at recent times. Abdel- Basset et al [6-7] has developed neutrosophic decision making models to solve transition difficulties of IoT-based enterprises and to evaluate green supply chain management practices.

Smarandache also extended the classical sets, fuzzy sets, intuitionistic fuzzy sets and neutrosophic fuzzy sets to plithogenic sets which is a quintuple (P, a, V, d, c) consisting of a set P, the attribute a, the range of attribute values V, degree of appurtenance d, and the degree of contradiction c. The nature of d determines the type of plithogenic sets. Smarandache presented an elaborate discussion on the genesis of plithogenic sets in his research work [8]. Abdel-Basset et al [9-11] has developed decision making models with incorporation of plithogenic sets to evaluate green supply chain management practices and intelligent Medical Decision Support Model Based on Soft Computing and IoT was also built; a hybrid plithogenic decision-making approach with quality function deployment for selecting supply chain sustainability metrics was also framed. These plithogenic decision making models are highly robust and feasible.

Molodtsov [12] introduced and applied soft sets in decision making which was extended to fuzzy soft sets predominantly by Maji [13]. The comprehensive outlook of hypersoft sets was made by Smarandache [14] which took the different forms of fuzzy sets in the course of time. Shazia Rana et al [15] in their recent work on application of plithogenic fuzzy whole hypersoft set in multi attribute decision making introduced the matrix representation of plithogenic hypersoft set and plithogenic fuzzy whole hypersoft set which adds to the compatibility of this decision making technique. The validation of the proposed decision making model with a numerical example in this work has inspired to introduce combined plithogenic hypersoft set.

The paper is organized as follows; section 2 presents a brief description of combined plithogenic hypersoft sets; section 3 comprises the application of combined plithogenic hypersoft sets in decision making based on the technique of Shazia Rana et al [15]; section 4 discusses the results and the last section concludes with the future extension of the proposed concept.

2. Combined plithogenic hypersoft sets

This section comprises of the direct narration and representation of the combined plithogenic hypersoft sets based on the preliminaries discussed by Smarandache [14] and Shazia Rana et al [15] to avoid the repetition of the elementary definitions. Smarandache presented the classification of plithogenic hypersoft sets and the categorization was purely based on the nature of degree of appurtenance. Based on his discussion, let us consider the following example to explain the need of combined plithogenic hypersoft sets

Let U be the universe of discourse that consists of pollution mitigation methods say

$U = \{M_1, M_2, M_3, M_4, M_5\}$ and the set $\mathscr{M} = \{M1, M4\} \subset U$.

The attributes are a_1 = Cost efficiency, a_2 = Eco-compatibility, a_3 = Time efficacy, a_4 = Profit yield. If the pollution abatement methods are supposed to fulfill these attributes, then in realistic perspective the possible attribute values are taken as follows,

Cost efficiency = A_1 = {low, medium, high}, Eco-compatibility = A_2 = {very high, high}, Time efficacy = A_3 = {less, more}, Profit yield = A_4 ={maximum, minimum}.

Suppose a manufacturing firm has decided to implement a pollution control method, then the above attributes and its values are considered for making optimal decision with the possible range of values of attributes. By usual consideration,

Let the function be: $G: A_1 \times A_2 \times A_3 \times A_4 \longrightarrow P(U)$

Let's assume: G ({low, high, more, maximum}) = $\{M_1, M_4\}$.

The degree of appurtenance of an element x to the set $\mathcal{M}$, with respect to each attribute value a is d_x^0(a) that is the deciding factor of the nature of plithogenic hypersoft set.

In the context of decision making with the expert's opinion, then d_x^0(a) is the resultant of the expert's perception. Sometimes the expert's outlook may be a combination of certain, fuzzy, intuitionistic and neutrosophic, which is expressed as follows

G({low, high, more, maximum}) = { M_1 (1,0.8,0.7,(0.4,0.5)),

M_4 (1,0.9,(0.8,0.1,0.1),(0.5,0.6)) }.

This is the pragmatic reflection of the person's perception in decision making process and this is the point of origin of combined plithogenic hypersoft sets. Thus a combined plithogenic hypersoft sets is a plithogenic hypersoft set in which the degree of appurtenance of an element x to the set $\mathcal{M}$, with respect to each attribute value is a combination of either crisp, fuzzy, intuitionistic or neutrosophic.

Combined plithogenic hypersoft sets can be classified into completely combined plithogenic hypersoft sets and partially combined plithogenic hypersoft sets based on the nature and combination of values taken by d_x^0(a). In the above stated example G({low, high, more, maximum}) = { M_1 (1,0.8,0.7,(0.4,0.5)), M_4 (1,0.9,(0.8,0.1,0.1),(0.5,0.6))} is a completely combined plithogenic hypersoft sets as $dx0$(a) takes all possible types of values. Suppose G({low, high, more, maximum}) = { M_1 (0.9,0.8,0.7,(0.4,0.5)), M_4 (0.8,0.9,0.6,(0.5,0.6))} then this combined plithogenic hypersoft set is partial in nature as d_x^0(a) takes only a combination of two types of values. Thus a combined plithogenic hypersoft set which is not complete is partial in its nature.

It is very evident that combined plithogenic hypersoft sets are highly rational in nature and it will certainly play a vital role in receiving the expert's opinion, which is very significant in any multi attribute decision making process. Also the need of such new types of plithogenic hypersoft sets are very essential, because in the manufacturing firms and in business sectors the implementation of certain methods and installation of certain mechanisms and machinery may not be characterized by only crisp or fuzzy values with regard to the degree of appurtenance as the possibility aspect has some extent of participation in it. To handle such situations the combined plithogenic hypersoft sets may lend a helping hand to the decision makers.

3. Application of Combined Plithogenic Hypersoft set in Multi Attribute Decision Making

The previous section presented an elaborate portrayal of combined plithogenic hypersoft set, the significant feature is the realistic representation, but it has certain difficulties in computations as the degree of appurtenance varies for each attribute. To handle such crisis, all the values of d_x^0(a) must be similar in nature, i.e. either all the values must be fuzzy values which is the lower level of fuzzy representation or it must be neutrosophic values, the higher level of fuzzy representation.

A manufacturing sector has decided to enhance its production rate by installing new kinds of machinery. The ultimate aim of incorporating such a change in the production mechanism is quality production and customer satisfaction. The market is flooded with several varieties of well equipped, modern machines and since the manufacturing sector makes huge investment, the decision making process takes place in two phases based on the expert's opinion and advice. In the first phase, ten machines were selected by the manufacturing sector and in the next phase five were selected based on the feedback of the users. The decision making problem begins here, as the company has to purchase only three out of five based on the extent of satisfaction of the attributes by these machines.

Let U = { M_1, M_2, M_3, M_4, M_5, M_6, M_7,M_8, M_9, M_{10}} be the university of discourse and set

T = {M_1, M_3, M_6, M_7, M_9} $\subset$ U.

The attribute system is represented as follows A = { (A_1)Maintenance Cost {Maximum in the initial years of utility(A_1^1), Maximum in the latter years of utility(A_1^2)}, (A_2)Reliability {High with additional expenditure(A_2^1), Moderate with no extra expense(A_2^2)}, (A_3)Flexibility {Single task oriented(A_3^1), Multi task oriented(A_3^2)}, (A_4)Durability {Very high in the beginning years of service(A_4^1), High in the latter years of service(A_4^2), }, (A_5)Profitability {Moderate in the initial years(A_5^1), Maximum in the latter years(A_5^2)}}.

The attributes are quite common, but the attribute values are more realistic as it mirror the actual aspects involved in making decision.

Let the function be: G: $A_1^1 \times A_2^2 \times A_3^2 \times A_4^1 \times A_5^2 \longrightarrow$P(U). Based on the Expert's opinion, the degree of appurtenance of the elements with respect to the attribute values is represented as follows

G(A_1^1, A_2^2 , A_3^2 , A_4^1, A_5^2) =

{M1(0.9,(0.7,0.1),0.8,(0.6,0.2),0.5),M3((0.6,0.3),0.5,(0.4,0.1,0.3),0.8,0.7), M6(0.8,0.7,0.6,(0.5,0.2),(0.6,0.1,0.1)),M7((0.7,0.2,0.1),(0.7,0.1),0.9,(0.7,0.2),0.8),M9(1,0.9,0.5,0.8,(0.6,0.1,0.2))}.

The modified lower and higher fuzzy values of the degree of appurtenance of the elements with respect to the attribute values are denoted as G_L(A_1^1, A_2^2 , A_3^2 , A_4^1, A_5^2) and G_H(A_1^1, A_2^2 , A_3^2 , A_4^1, A_5^2)

G_L(A_1^1, A_2^2 , A_3^2 , A_4^1, A_5^2) = {M1(0.9,0.875,0.8,0.75,0.5),M3(0.67,0.5,0.4,0.8,0.7),M6(0.8,0.7,0.6,0.7,0.5), M7(0.67,0.875,0.9,0.78,0.8), M9(1,0.9,0.5,0.8,0.47)}

G_H(A_1^1, A_2^2 , A_3^2 , A_4^1, A_5^2) =

{M1(0.9,0.1,0.1),(0.7,0.2,0.1),(0.8,0.1,0.1),(0.6,0.3,0.2),(0.5,0.2,0.7)),M3((0.6,0.3,0.3), (0.5,0.2,0.7),(0.4,0.1,0.3),(0.8,0.1,0.1),(0.7,0.2,0.1)),M6((0.8,0.1,0.1),(0.7,0.2,0.1),(0.6,0.2,0.3),(0.5,0.3,0.2),(0.6,0.1,0.1)),M7((0.7,0.2,0.1),(0.7,0.1,0.1),(0.9,0.1,0.1),(0.7,0.1,0.2),(0.8,0.1,0.1)),M9((1,0,0),(0.9,0.1,0.1),(0.5,0.2,0.7),(0.8,0.1,0.1),(0.6,0.1,0.2))}

The lower and higher fuzzy values of the degree of appurtenance correspond to single fuzzy value and neutrosophic values. The matrix representation C of the degree of appurtenance of the elements with respect to the attribute values in combined plithogenic hypersoft sets is

$$\begin{array}{c|ccccc} & A_1^1 & A_2^2 & A_3^2 & A_4^1 & A_5^2 \\ \hline M_1 & 0.9 & (0.7,0.1) & 0.8 & (0.6,0.2) & 0.5 \\ M_3 & (0.6,0.3) & 0.5 & (0.4,0.1,0.3) & 0.8 & 0.7 \\ M_6 & 0.8 & 0.7 & 0.6 & (0.5,0.2) & (0.6,0.1,0.1)), \\ M_7 & (0.7,0.2,0.1) & (0.7,0.1) & 0.9 & (0.7,0.2) & 0.8 \\ M_9 & 1 & 0.9 & 0.5 & 0.8 & (0.6,0.1,0.2) \end{array}$$

The intuitionistic and neutrosophic values are transformed to the above fuzzy values by the methods of imprecision and Defuzzification [16]

Method I (Imprecision membership): Any neutrosophic fuzzy set N_A = (T_A ,I_A,F_A) including neutrosophic fuzzy values are transformed into intuitionistic fuzzy values or vague values as η(A) = (T_A , f_A) where f_A is estimated the formula stated below which is called as Impression membership method.

$$f_A = \begin{cases} F_A + \frac{[1-F_A-I_A][1-F_A]}{[F_A+I_A]} & if\ F_A = 0 \\ F_A + \frac{[1-F_A-I_A][F_A]}{[F_A+I_A]} & if\ 0 < F_A \le 0.5 \\ F_A + [1-F_A-I_A]\left[0.5 + \frac{F_A-0.5}{F_A+I_A}\right] & if\ 0.5 < F_A \le 1 \end{cases}$$

Method II (Defuzzification): After Method I (Median membership), intuitionistic (vague),fuzzy values of the form η(A)= (T_A, f_A) are transformed into fuzzy set including fuzzy values as <Δ(A)>= <$\frac{T_A}{[T_A+f_A]}$>.

The matrix representation C_L of the lower fuzzy values of the degree of appurtenance of the elements with respect to the attribute values in combined plithogenic hypersoft sets is

$$\begin{array}{c} \\ M_1 \\ M_3 \\ M_6 \\ M_7 \\ M_9 \end{array} \begin{array}{c} \begin{array}{ccccc} A_1^1 & A_2^2 & A_3^2 & A_4^1 & A_5^2 \end{array} \\ \begin{pmatrix} 0.9 & 0.875 & 0.8 & 0.75 & 0.5 \\ 0.67 & 0.5 & 0.4 & 0.8 & 0.7 \\ 0.8 & 0.7 & 0.6 & 0.7 & 0.5 \\ 0.67 & 0.875 & 0.9 & 0.78 & 0.8 \\ 1 & 0.9 & 0.5 & 0.8 & 0.47 \end{pmatrix} \end{array}$$

By using the procedure of ranking as discussed by Shazia Rana et. al [15] the machines are ranked by considering the values of C_L.

The frequency matrix F_L representing the ranking of the machines is

$$\begin{array}{c} \\ M_1 \\ M_3 \\ M_6 \\ M_7 \\ M_9 \end{array} \begin{array}{c} \begin{array}{ccccc} R_1 & R_2 & R_3 & R_4 & R_5 \end{array} \\ \begin{pmatrix} 1 & 2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 2 \\ 0 & 1 & 0 & 2 & 0 \\ 2 & 0 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \end{pmatrix} \end{array}$$

The percentage measure of authenticity of ranking is presented below in Table 3.1

Table 3.1

R_1	M_7	50%
R_2	M_1	50%
R_3	M_9	50%
R_4	M_6	67%
R_5	M_3	100%

The matrix representation C_H of higher fuzzy values (neutrosophic representations) of the degree of appurtenance of the elements with respect to the attribute values in combined plithogenic hypersoft sets is

$$
\begin{array}{c}
 \\ M_1 \\ M_3 \\ M_6 \\ M_7 \\ M_9
\end{array}
\begin{pmatrix}
A_1^1 & A_2^2 & A_3^2 & A_4^1 & A_5^2 \\
(0.9,0.1,0.1) & (0.7,0.2,0.1) & (0.8,0.1,0.1) & (0.6,0.3,0.2) & (0.5,0.2,0.7) \\
(0.6,0.3,0.3) & (0.5,0.2,0.7) & (0.4,0.1,0.3) & (0.8,0.1,0.1) & (0.7,0.2,0.1) \\
(0.8,0.1,0.1) & (0.7,0.2,0.1) & (0.6,0.2,0.3) & (0.5,0.3,0.2) & (0.6,0.1,0.1) \\
(0.7,0.2,0.1) & (0.7,0.1,0.1) & (0.9,0.1,0.1) & (0.7,0.1,0.2) & (0.8,0.1,0.1) \\
(1,0,0) & (0.9,0.1,0.1) & (0.5,0.2,0.7) & (0.8,0.1,0.1) & (0.6,0.1,0.2)
\end{pmatrix}
$$

To make the ranking of the machines based on the higher values in C_H the score values K(A) of the single valued neutrosophic representations [say A = (a,b,c)] are determined by $K(A) = \frac{1+a-2b-c}{2}$ [17]

$$
\begin{array}{c}
 \\ M_1 \\ M_3 \\ M_6 \\ M_7 \\ M_9
\end{array}
\begin{pmatrix}
A_1^1 & A_2^2 & A_3^2 & A_4^1 & A_5^2 \\
0.8 & 0.6 & 0.75 & 0.4 & 0.2 \\
0.35 & 0.2 & 0.45 & 0.75 & 0.6 \\
0.75 & 0.6 & 0.45 & 0.35 & 0.65 \\
0.6 & 0.7 & 0.8 & 0.65 & 0.75 \\
1 & 0.8 & 0.2 & 0.75 & 0.6
\end{pmatrix}
$$

The frequency matrix F_H representing the ranking of machines is

$$
\begin{array}{c}
 \\ M_1 \\ M_3 \\ M_6 \\ M_7 \\ M_9
\end{array}
\begin{array}{c}
\begin{array}{ccccc} R_1 & R_2 & R_3 & R_4 & R_5 \end{array} \\
\begin{pmatrix}
1 & 0 & 1 & 1 & 0 \\
0 & 0 & 1 & 1 & 1 \\
0 & 1 & 1 & 1 & 0 \\
3 & 0 & 0 & 0 & 0 \\
1 & 1 & 1 & 0 & 0
\end{pmatrix}
\end{array}
$$

The percentage measure of authenticity of ranking is presented below in Table 3.2

Table 3.2

R_1	M_7	60%
R_2	M_9	50%
R_3	M_6	25%
R_4	M_1	33%
R_5	M_3	100%

4. Discussion

The combined plithogenic hypersoft set representations are so deliberate in nature. The resultant of computations in making decisions in two ways is represented in Table 3.1 and 3.2. The machines M_7 and M_3 occupy first and fifth rank respectively in both kinds of representation of degree of appurtenance. Also by making inferences from the table values M_1, M_3 and M_6 can be ranked in second ,third and fourth positions respectively. It is very evident that the transformation of combined attribute values to lower order fuzzy values yields best results in ranking the machines, but still the higher order values will also yield optimum results based on the selection of the score functions. The methods of converting combined attribute value to the values of similar fashion have to be constituted in the upcoming research works to attain feasible solutions to the decision making problems.

5. Conclusions

This research work encompasses the discussion of the new concept of combined plithogenic hypersoft set and its application in multi attribute decision making. Besides these types of appurtenance degrees, others can be used under the plithogenic umbrella, such as: Pythagorean, picture fuzzy, spherical fuzzy, spherical neutrosophic, etc. and even the most general one, refined neutrosophic type of appurtenance degree. The combined plithogenic hypersoft set can be extended to interval-valued combined plithogenic hypersoft sets and it can be converted to simple fuzzy values using score functions. The matrix representations of degree of appurtenance in combined plithogenic hypersoft set has induced the author to extend the proposed theoretical conceptualization to plithogenic concentric hypergraphs, most probably the scope and future research work.

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Development of Hybrids of Hypersoft Set with Complex Fuzzy Set, Complex Intuitionistic Fuzzy Set and Complex Neutrosophic Set

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Abstract. The complex fuzzy soft set and its generalized hybrids are such effective structures which not only minimize the impediments of all complex fuzzy-like structures for dealing uncertainties but also fulfill all the parametric requirements of soft sets. This feature makes it a completely new mathematical tool for solving problems dealing with uncertainties. Smarandache conceptualized hypersoft set as a generalization of soft set as it transforms the single attribute function into a multi-attribute function. This generalization demands an extension of complex fuzzy soft-like structures to hypersoft structure for more precise results. In this study, hybrids of hypersoft set with complex fuzzy set and its generalized structures i.e. complex intuitionistic fuzzy set and complex neutrosophic set, are developed along with illustrative examples to address the demand of literature. Moreover, some of their fundamentals i.e. subset, equal sets, null set, absolute set etc. and theoretic operations i.e. compliment, union, intersection etc. are discussed.

Keywords: Complex fuzzy sets (CF-Sets), soft set, hypersoft set and complex fuzzy hypersoft set.

1. Introduction

Zadeh's theory of fuzzy sets [1] is one of those theories which are considered as mathematical means to tackle many complicated problems involving various uncertainties in different fields of mathematical sciences. But these theories are unable to solve these problems successfully due to the inadequacy of the parametrization tool. This shortcoming is addressed by Molodtsov's soft set theory [2] which is free from all such Impediments and appeared as a new parameterized family of subsets of the universe of discourse. Classical complex analysis is useful in algebraic geometry, number theory, analytic combinatorics and many other branches of mathematical sciences. Ramot et al. [3, 4] introduced the concept of complex fuzzy set (CF-set) to tackle the problems of complex analysis under fuzzy environment. This novel concept used complex-valued state for the membership of its elements. Maji et al. [5] developed and conceptualized fuzzy soft set, a new hybrid of fuzzy set with soft set. They also discussed some of its fundamentals terminologies and operations like equality of two soft sets, subset and super set of a soft set, complement of a soft set, null soft set, absolute soft set, AND, OR etc. in their work. Ça̧ğman et al. [6] extended this concept and discussed some other properties and operations. Nadia [7] developed a new hybrid of complex fuzzy set and soft set. Thirunavukarasu et al. [8] established aggregation properties of complex fuzzy soft set and discussed their applications. Atanassov [9] conceptualized intuitionistic fuzzy sets as generalization of fuzzy set. Alkouri et al. [10] extended this concept and developed complex intuitionistic fuzzy soft set and discussed some of its properties. Kumar et al. [11] further discussed its more properties and calculated its distance measures and entropies. Mumtaz et al. [12] extended neutrosophic set [13] to complex neutrosophic set and discussed its fundamentals, theoretic operations and applications. Broumi et al. [14] conceptualized complex neutrosophic soft set and discussed some of its fundamentals.

In 2018, Smarandache [15] introduced the concept of hypersoft set as a generalization of soft set. In 2020, Saeed et al. [16] extended the concept and discussed the fundamentals of hypersoft set such as hypersoft subset, complement, not hypersoft set, aggregation operators along with hypersoft set relation, sub relation,complement relation, function, matrices and operations on hypersoft matrices.

Having motivation from the work in [6], [8]- [16] and [21], novel hybrids of hypersoft set i.e. complex fuzzy hypersoft set, complex intuitionistic fuzzy hypersoft set and complex neutrosophic hypersoft set, are conceptualized along with their some fundamentals and theoretic operations. This is novel and more generalized work as compared to existing related literature for getting more precise results. Moreover, a comparative discussion is presented on particular cases of such hybrids.

The pattern of rest of the paper is: section 2 reviews the notions of soft sets, complex fuzzy set and relevant definitions used in the proposed work. Section 3, presents complex fuzzy hypersoft set and some of its fundamentals. Section 4, presents complex intuitionistic fuzzy hypersoft set and some of its fundamentals. Section 5, presents complex neutrosophic hypersoft set and some of its fundamentals and then concludes the paper.

2. Preliminaries

Here some existing fundamental concepts regarding fuzzy set, fuzzy soft set and fuzzy hypersoft set are presented along with their structures with complex fuzzy set from literature.

Throughout the paper, $\mathbb{U}$, $P(\mathbb{U})$, $F(\mathbb{U})$, $C(\mathbb{U})$, $C_{Int}(\mathbb{U})$, $C_{Neu}(\mathbb{U})$, $\coprod$ and $\prod$ will present universe of discourse, power set of $\mathbb{U}$, collection of fuzzy sets, collection of complex fuzzy sets, collection of complex intuitionistic fuzzy sets, collection of complex neutrosophic sets, union and intersection respectively.

Definition 2.1. [1]
Suppose a universal set $\mathbb{U}$ and a *fuzzy set* $X \subseteq \mathbb{U}$. The set X will be written as $X = \{(x, \alpha_X(x)) | x \in \mathbb{U}\}$ such that

$$\alpha_X : \mathbb{U} \to [0,1]$$

where $\alpha_X(x)$ describes the membership percentage of $x \in X$.

Definition 2.2. [3]
A *complex fuzzy set* $\mathbb{C}_f$ is of the form

$$\mathbb{C}_f = \left\{(\epsilon, \mu_{\mathbb{C}_f}(\epsilon)) : \epsilon \in \mathbb{U}\right\} = \left\{(\epsilon, r_{\mathbb{C}_f}(\epsilon) e^{i\omega_{\mathbb{C}_f}(\epsilon)}) : \epsilon \in \mathbb{U}\right\}.$$

where $\mu_{\mathbb{C}_f}(\epsilon)$ is a membership function of $\mathbb{C}_f$ with $r_{\mathbb{C}_f}(\epsilon) \in [0,1]$ and $\omega_{\mathbb{C}_f}(\epsilon) \in (0, 2\pi]$ as amplitude and phase terms respectively and $i = \sqrt{-1}$.

Zhang et al. [22] and Buckley [23]- [26] presented fuzzy complex number in different way. However, according to [3], [4], both amplitude and phase terms are captured by fuzzy sets.

Definition 2.3. [2]
A *soft set* $\mathfrak{S}$ over $\mathbb{U}$, is defined as

$$\mathfrak{S} = \{(\epsilon, f_{\mathfrak{S}}(\epsilon)) : \epsilon \in E_1\}$$

where $f_{\mathfrak{S}} : E_1 \to P(\mathbb{U})$. and $E_1 \subseteq E$ (set of parameters).

Definition 2.4. [6]
A *fuzzy soft set* (FS-set) Γ_{E_1} on $\mathbb{U}$, is defined as

$$\Gamma_{E_1} = \{(\epsilon, \gamma_{E_1}(\epsilon)) : \epsilon \in E_1, \gamma_{E_1}(\epsilon) \in F(\mathbb{U})\}$$

where $\gamma_{E_1} : E_1 \to F(\mathbb{U})$ such that $\gamma_{E_1}(\epsilon) = \emptyset$ if $\epsilon \notin E_1$, and for all $\epsilon \in E_1$,

$$\gamma_{E_1}(\epsilon) = \left\{\mu_{\gamma_{E_1}(\epsilon)}(v)/v : v \in \mathbb{U}, \mu_{\gamma_{E_1}(\epsilon)}(v) \in [0,1]\right\}$$

is a fuzzy set over $\mathbb{U}$. Also γ_{E_1} is the approximate function of Γ_{E_1} and the value $\gamma_A(x)$ is a fuzzy set called ϵ-element of FS-set. Note that if $\gamma_{E_1}(\epsilon) = \emptyset$, then $(\epsilon, \gamma_{E_1}(\epsilon)) \notin \Gamma_{E_1}$.

Definition 2.5. [7]
A *complex fuzzy soft set* (CFS-set) χ_{E_1} over $\mathbb{U}$, is defined as

$$\chi_{E_1} = \{(\epsilon, \psi_{E_1}(\epsilon)) : \epsilon \in E_1, \psi_{E_1}(\epsilon) \in C(\mathbb{U})\}.$$

where $\psi_{E_1}: E_1 \to C(\mathbb{U})$ such that $\psi_{E_1}(\epsilon) = \emptyset$ if $\epsilon \notin E_1$ and it is complex fuzzy approximate function of CFS-set χ_{E_1} and its value $\psi_{E_1}(\epsilon)$ is called ϵ-member of CFS-set χ_{E_1} for all $\epsilon \in E_1$. Operations of CFS-sets and CF-sets were defined in [7] and [22] respectively.

Definition 2.6. [27] Let $A = \{(x; \mu_A(x)) : x \in \mathbb{U}\}$ and $B = \{(x; \mu_B(x)) : x \in \mathbb{U}\}$ be two complex fuzzy subsets of $\mathbb{U}$, with membership functions $\mu_A(x) = r_A(x)e^{i\omega_A(x)}$ and $\mu_B(x) = r_B(x)e^{i\omega_B(x)}$, respectively. Then

- A complex fuzzy subset A is said to be a *homogeneous complex fuzzy set* if for all $x, y \in \mathbb{U}$, $r_A(x) \leq r_A(y)$ if and only if $\omega_A(x) \leq \omega_A(y)$
- A complex fuzzy subset A is said to be *homogeneous* with B, if for all $x, y \in \mathbb{U}$, $r_A(x) \leq r_B(y)$ if and only if $\omega_A(x) \leq \omega_B(y)$

Definition 2.7. [10] Let E be a set of attributes with $A \subseteq E$ and $\Psi(a)$ be a CIF-set over $\mathbb{U}$. Then, *complex intuitionistic fuzzy soft set* (CIFS-set) $\xi_A = (\Psi, A)$ over $\mathbb{U}$ is defined as

$$\xi_A = \{(a, \Psi(a)) : a \in A, \Psi(a) \in C_{Int}(\mathbb{U})\}$$

where

$$\Psi : A \to C_{Int}(\mathbb{U}), \qquad \Psi(a) = \emptyset \; if \; a \notin A.$$

is a CIF approximate function of ξ_A and $\Psi(a) = \langle \Psi^T(a), \Psi^F(a) \rangle$.
$\Psi^T(a) = p_T\, e^{i\,\theta_T}$, and $\Psi^F(a) = p_F\, e^{i\,\theta_F}$ are complex-valued membership function, and complex-valued non-membership function of ξ_A respectively and their sum all are lying within the unit circle in the complex plane such that $p_T, p_F \in [0, 1]$ with $0 \leq p_T + p_F \leq 1 (or\ 0 \leq |p_T + p_F| \leq 1)$ and $\theta_T, \theta_F \in (0, 2\pi]$. The value $\Psi(a)$ is called a-member of CIFS-set $\forall a \in A$.

Definition 2.8. [14]
Let E be a set of attributes with $A \subseteq E$ and $\Psi(a)$ be a CN-set over $\mathbb{U}$. Then, *complex neutrosophic soft set* (CNS-set) $\xi_A = (\Psi, A)$ over $\mathbb{U}$ is defined as

$$\xi_A = \{(a, \Psi(a)) : a \in A, \Psi(a) \in C_{Neu}(\mathbb{U})\}$$

where

$$\Psi : A \to C_{Neu}(\mathbb{U}), \qquad \Psi(a) = \emptyset \; if \; a \notin A.$$

is a CN approximate function of ξ_A and $\Psi(a) = \langle \Psi^T(a), \Psi^I(a), \Psi^F(a) \rangle$.
$\Psi^T(a) = p_T\, e^{i\,\theta_T}$, $\Psi^I(a) = p_I\, e^{i\,\theta_I}$ and $\Psi^F(a) = p_F\, e^{i\,\theta_F}$ are complex-valued truth membership function, complex-valued indeterminacy membership function, and complex-valued falsity membership function of ξ_A respectively and their sum all are lying within the unit circle in the complex plane such that $p_T, p_I, p_F \in [0, 1]$ with $^{-}0 \leq p_T + p_I + p_F \leq 3^{+} (or\ 0 \leq |p_T + p_I + p_F| \leq 3)$ and $\theta_T, \theta_I, \theta_F \in (0, 2\pi]$. The value $\Psi(a)$ is called a-member of CNS-set $\forall a \in A$.

For more study about neutrosophic sets see ([28]- [42]).

Definition 2.9. [15]
The pair (H, G) is called a *hypersoft set* over $\mathbb{U}$, where G is the cartesian product of n disjoint sets $H_1, H_2, H_3, ..., H_n$ having attribute values of n distinct attributes $h_1, h_2, h_3, ..., h_n$ respectively and $H : G \to P(\mathbb{U})$.

Definition 2.10. [15]
A hypersoft set over a fuzzy universe of discourse is called *fuzzy hypersoft set.*

For more definitions and operations of hypersoft set, see ([15]- [20]).

3. Complex Hypersoft set(CH-Set) and Complex Fuzzy Hypersoft Set(CFH-Set)

In this section, first we define complex hypersoft set then complex fuzzy hypersoft set is conceptualized with its some fundamentals.

Definition 3.1. Let $\mathbb{C}$ be the set of complex numbers and $P(\mathbb{C})$ be the collection of all non-empty bounded subsets of the set of complex numbers. Let $A_1, A_2, A_3, ..., A_n$ are disjoint sets having attribute values of n distinct attributes $a_1, a_2, a_3, ..., a_n$ respectively for $n \geq 1, A = A_1 \times A_2 \times A_3 \times ..... \times A_n$ then a mapping $\psi : A \to P(\mathbb{C})$ is called a complex hypersoft set. It is denoted by (ψ, A).

Example 3.2. Let $\mathbb{C} = \{2+3i, 1+2i, 3+5i, 4+2i, 3+i\}$ be the set of complex numbers and $E = \{A_1, A_2, A_3\}$ with $A_1 = \{a_{11}, a_{12}\}, A_2 = \{a_{21}, a_{22}\}$ and $A_3 = \{a_{31}, a_{32}\}$ are disjoint set having attribute values then

$$A = \left\{ \begin{array}{l} (a_{11}, a_{21}, a_{31}), (a_{11}, a_{21}, a_{32}), (a_{11}, a_{22}, a_{31}), (a_{11}, a_{22}, a_{32}), \\ (a_{21}, a_{21}, a_{31}), (a_{21}, a_{21}, a_{32}), (a_{21}, a_{22}, a_{31}), (a_{21}, a_{22}, a_{32}) \end{array} \right\}$$

$A = \{x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8\}$, then (ψ, A) can be considered as a complex hypersoft set where

$$(\psi, A) = \left\{ \begin{array}{l} (x_1, \{2+3i, 1+2i\}), (x_2, \{2+3i, 1+2i, 3+5i\}), (x_3, \{4+2i, 1+2i, 3+5i\}), \\ (x_4, \{2+3i, 4+2i, 3+i\}), (x_5, \{3+i, 1+2i\}), (x_6, \{3+i, 2+3i, 3+5i\}), \\ (x_7, \{2+3i, 3+i\}), (x_8, \{4+2i, 3+5i\}), \end{array} \right\}$$

Definition 3.3. Let $A_1, A_2, A_3, ..., A_n$ are disjoint sets having attribute values of n distinct attributes $a_1, a_2, a_3, ..., a_n$ respectively for $n \geq 1, G = A_1 \times A_2 \times A_3 \times ..... \times A_n$ and $\psi(\underline{\epsilon})$ be a CF-set over $\mathbb{U}$ for all $\underline{\epsilon} = (d_1, d_2, d_3, ..., d_n) \in G$ such that $d_1 \in A_1, d_2 \in A_2, d_3 \in A_3, ..., d_n \in A_n$. Then, *complex fuzzy hypersoft set* (CFH-set) χ_G over $\mathbb{U}$ is defined as

$$\chi_G = \{(\underline{\epsilon}, \psi(\underline{\epsilon})) : \underline{\epsilon} \in G, \psi(\underline{\epsilon}) \in C(\mathbb{U})\}$$

where

$$: G \to C(\mathbb{U}), \qquad \psi(\underline{\epsilon}) = \emptyset \ if \ \underline{\epsilon} \notin G.$$

is a CF-approximate function of χ_G and its value $\psi(\underline{\epsilon})$ is called $\underline{\epsilon}$-member of CFH-set $\forall \underline{\epsilon} \in G$.

Example 3.4. Suppose a Department Promotion Committee (DPC) wants to observe(evaluate) the characteristics of some teachers by some defined indicators for departmental promotion. For this purpose, consider a set of teachers as a universe of discourse $\mathbb{U} = \{t_1, t_2, t_3, t_4\}$. The attributes of the teachers under consideration are the set $E = \{A_1, A_2, A_3\}$, where
$A_1 =$ Total experience in years $= \{3, < 10\} = \{e_{11}, e_{12}\}$
$A_2 =$ Total no. of publications$= \{10,\ 10 <\} = \{e_{21}, e_{22}\}$
$A_3 =$ Performance Evaluation Report (PER) remarks $= \{eligible, not\, eligible\} = \{e_{31}, e_{32}\}$
and

$$G = A_1 \times A_2 \times A_3 = \left\{ \begin{array}{l} (e_{11}, e_{21}, e_{31}), (e_{11}, e_{21}, e_{32}), (e_{11}, e_{22}, e_{31}), \\ (e_{11}, e_{22}, e_{32}), (e_{12}, e_{21}, e_{31}), (e_{12}, e_{21}, e_{32}), \\ (e_{12}, e_{22}, e_{31}), (e_{12}, e_{22}, e_{32}) \end{array} \right\} = \{e_1, e_2, e_3, ..., e_8\}$$

Complex fuzzy set $\psi_G(e_1), \psi_G(e_2), ..., \psi_G(e_8)$ are defined as,

$$\psi_G(e_1) = \left\{ \frac{0.4e^{i0.5\pi}}{t_1}, \frac{0.8e^{i0.6\pi}}{t_2}, \frac{0.8e^{i0.8\pi}}{t_3}, \frac{1.0e^{i0.75\pi}}{t_4} \right\},$$

$$\psi_G(e_2) = \left\{ \frac{0.6e^{i0.7\pi}}{t_1}, \frac{0.9e^{i0.9\pi}}{t_2}, \frac{0.7e^{i0.9\pi}}{t_3}, \frac{0.75e^{i0.95\pi}}{t_4} \right\},$$

$$\psi_G(e_3) = \left\{ \frac{0.5e^{i0.6\pi}}{t_1}, \frac{0.8e^{i0.9\pi}}{t_2}, \frac{0.6e^{i0.9\pi}}{t_3}, \frac{0.65e^{i0.95\pi}}{t_4} \right\},$$

$$\psi_G(e_4) = \left\{ \frac{0.3e^{i0.7\pi}}{t_1}, \frac{0.7e^{i0.9\pi}}{t_2}, \frac{0.5e^{i0.9\pi}}{t_3}, \frac{0.75e^{i0.65\pi}}{t_4} \right\},$$

$$\psi_G(e_5) = \left\{ \frac{0.2e^{i0.5\pi}}{t_1}, \frac{0.3e^{i0.8\pi}}{t_2}, \frac{0.8e^{i0.7\pi}}{t_3}, \frac{0.45e^{i0.65\pi}}{t_4} \right\},$$

$$\psi_G(e_6) = \left\{ \frac{0.5e^{i0.9\pi}}{t_1}, \frac{0.3e^{i0.9\pi}}{t_2}, \frac{0.7e^{i0.8\pi}}{t_3}, \frac{0.85e^{i0.95\pi}}{t_4} \right\},$$

$$\psi_G(e_7) = \left\{ \frac{0.6e^{i0.9\pi}}{t_1}, \frac{0.9e^{i0.6\pi}}{t_2}, \frac{0.5e^{i0.6\pi}}{t_3}, \frac{0.85e^{i0.75\pi}}{t_4} \right\},$$

and

$$\psi_G(e_8) = \left\{ \frac{0.8e^{i0.9\pi}}{t_1}, \frac{0.8e^{i0.8\pi}}{t_2}, \frac{0.6e^{i0.8\pi}}{t_3}, \frac{0.65e^{i0.85\pi}}{t_4} \right\}$$

then CFH-set χ_G is written by,

$$\chi_G = \left\{ \begin{array}{l} (e_1, \frac{0.4e^{i0.5\pi}}{t_1}, \frac{0.8e^{i0.6\pi}}{t_2}, \frac{0.8e^{i0.8\pi}}{t_3}, \frac{1.0e^{i0.75\pi}}{t_4}), (e_2, \frac{0.6e^{i0.7\pi}}{t_1}, \frac{0.9e^{i0.9\pi}}{t_2}, \frac{0.7e^{i0.9\pi}}{t_3}, \frac{0.75e^{i0.95\pi}}{t_4}), \\ (e_3, \frac{0.5e^{i0.6\pi}}{t_1}, \frac{0.8e^{i0.9\pi}}{t_2}, \frac{0.6e^{i0.9\pi}}{t_3}, \frac{0.65e^{i0.95\pi}}{t_4}), (e_4, \frac{0.3e^{i0.7\pi}}{t_1}, \frac{0.7e^{i0.9\pi}}{t_2}, \frac{0.5e^{i0.9\pi}}{t_3}, \frac{0.75e^{i0.65\pi}}{t_4}), \\ (e_5, \frac{0.2e^{i0.5\pi}}{t_1}, \frac{0.3e^{i0.8\pi}}{t_2}, \frac{0.8e^{i0.7\pi}}{t_3}, \frac{0.45e^{i0.65\pi}}{t_4}), (e_6, \frac{0.5e^{i0.9\pi}}{t_1}, \frac{0.3e^{i0.9\pi}}{t_2}, \frac{0.7e^{i0.8\pi}}{t_3}, \frac{0.85e^{i0.95\pi}}{t_4}), \\ (e_7, \frac{0.6e^{i0.9\pi}}{t_1}, \frac{0.9e^{i0.6\pi}}{t_2}, \frac{0.5e^{i0.6\pi}}{t_3}, \frac{0.85e^{i0.75\pi}}{t_4}), (e_8, \frac{0.8e^{i0.9\pi}}{t_1}, \frac{0.8e^{i0.8\pi}}{t_2}, \frac{0.6e^{i0.8\pi}}{t_3}, \frac{0.65e^{i0.85\pi}}{t_4}) \end{array} \right\}$$

Definition 3.5. Let $\chi_{G_1} = (\psi_1, G_1)$ and $\chi_{G_2} = (\psi_2, G_2)$ be two CFH-sets over the same $\mathbb{U}$. The set $\chi_{G_1} = (\psi_1, G_1)$ is said to be the *subset* of $\chi_{G_2} = (\psi_2, G_2)$, if

i. $G_1 \subseteq G_2$

ii. $\forall\ \underline{x} \in G_1, \psi_1(\underline{x}) \subseteq \psi_2(\underline{x})$ i.e. $r_{G_1}(\underline{x}) \leq r_{G_2}(\underline{x})$ and $\omega_{G_1}(\underline{x}) \leq \omega_{G_2}(\underline{x})$, where $r_{G_1}(\underline{x})$ and $\omega_{G_1}(\underline{x})$ are amplitude and phase terms of $\psi_1(\underline{x})$, whereas $r_{G_2}(\underline{x})$ and $\omega_{G_2}(\underline{x})$ are amplitude and phase terms of $\psi_2(\underline{x})$.

Definition 3.6. Two CFH-sets $\chi_{G_1} = (\psi_1, G_1)$ and $\chi_{G_2} = (\psi_2, G_2)$ over the same $\mathbb{U}$, are said to be *equal* if

i. $(\psi_1, G_1) \subseteq (\psi_2, G_2)$
ii. $(\psi_2, G_2) \subseteq (\psi_1, G_1)$.

Definition 3.7. Let (ψ, G) be a CFH-set over $\mathbb{U}$.Then

i. (ψ, G) is called a *null CFH-set*, denoted by $(\psi, G)_\Phi$ if $r_G(\underline{x}) = 0$ and $\omega_G(\underline{x}) = 0\pi$ for all $\underline{x} \in G$.
ii. (ψ, G) is called a *absolute CFH-set*, denoted by $(\psi, G)_\Delta$ if $r_G(\underline{x}) = 1$ and $\omega_G(\underline{x}) = 2\pi$ for all $\underline{x} \in G$.

Definition 3.8. Let (ψ_1, G_1) and (ψ_2, G_2) are two CFH-sets over the same universe $\mathbb{U}$.Then

i. A CFH-set (ψ_1, G_1) is called a *homogeneous CFH-set*, denoted by $(\psi_1, G_1)_{Hom}$ if and only if $\psi_1(\underline{x})$ is a homogeneous CF-set for all $\underline{x} \in G_1$.

ii. A CFH-set (ψ_1, G_1) is called a *completely homogeneous CFH-set*, denoted by $(\psi_1, G_1)_{CHom}$ if and only if $\psi_1(\underline{x})$ is a homogeneous with $\psi_1(\underline{y})$ for all $\underline{x}, \underline{y} \in G_1$.

iii. A CFH-set (ψ_1, G_1) is said to be a completely homogeneous CFH-set with (ψ_2, G_2) if and only if $\psi_1(\underline{x})$ is a homogeneous with $\psi_2(\underline{x})$ for all $\underline{x} \in G_1 \prod G_2$.

3.1. *Set Theoretic Operations and Laws on CFH-Sets*

Here some basic set theoretic operations (i.e.complement, union and intersection) and laws (commutative laws, associative laws etc.) are discussed on CFH-sets.

Definition 3.9. The *complement* of CFH-set (ψ, G), denoted by $(\psi, G)^c$ is defined as

$$(\psi, G)^c = \{(\underline{x}, \psi^c(\underline{x})) : \underline{x} \in G, \psi^c(\underline{x}) \in C(\mathbb{U})\}$$

such that the amplitude and phase terms of the membership function $\psi^c(\underline{x})$ are given by $r^c_G(\underline{x}) = 1 - r_G(\underline{x})$ and $\omega^c_G(\underline{x}) = 2\pi - \omega_G(\underline{x})$ respectively.

Proposition 3.10. *Let (ψ, G) be a CFH-set over $\mathbb{U}$.Then $((\psi, G)^c)^c = (\psi, G)$.*

Proof. Since $\psi(\underline{x}) \in C(\mathbb{U})$, therefore (ψ, G) can be written in terms of its amplitude and phase terms as

$$(\psi, G) = \left\{ \left(\underline{x}, r_G(\underline{x}) e^{i\omega_G(\underline{x})} \right) : \underline{x} \in G \right\} \tag{1}$$

Now

$$\psi^c(\underline{x}) = \left\{ \left(\underline{x}, r^c_G(\underline{x}) e^{i\omega^c_G(\underline{x})} \right) : \underline{x} \in G \right\}$$
$$\psi^c(\underline{x}) = \left\{ \left(\underline{x}, (1 - r_G(\underline{x})) e^{i(2\pi - \omega_G(\underline{x}))} \right) : \underline{x} \in G \right\}$$
$$((\psi, G)^c)^c = \left\{ \left(\underline{x}, (1 - r_G(\underline{x}))^c e^{i(2\pi - \omega_G(\underline{x})^c)} \right) : \underline{x} \in G \right\}$$
$$((\psi, G)^c)^c = \left\{ \left(\underline{x}, (1 - (1 - r_G(\underline{x}))) e^{i(2\pi - (2\pi - \omega_G(\underline{x})))} \right) : \underline{x} \in G \right\}$$
$$((\psi, G)^c)^c = \left\{ \left(\underline{x}, r_G(\underline{x}) e^{i\omega_G(\underline{x})} \right) : \underline{x} \in G \right\} \tag{2}$$

from equations (1) and (2), we have $((\psi, G)^c)^c = (\psi, G)$. □

Proposition 3.11. *Let (ψ, G) be a CFH-set over $\mathbb{U}$. Then*

i. $((\psi, G)_\Phi)^c = (\psi, G)_\Delta$

ii. $((\psi, G)_\Delta)^c = (\psi, G)_\Phi$

Definition 3.12. The *intersection* of two CFH-sets (ψ_1, G_1) and (ψ_2, G_2) over the same universe $\mathbb{U}$, denoted by $(\psi_1, G_1) \prod (\psi_2, G_2)$, is the CFH-set (ψ_3, G_3), where $G_3 = G_1 \prod G_2$, and $\psi_3(\underline{x}) = \psi_1(\underline{x}) \prod \psi_2(\underline{x})$ for all $\underline{x} \in G_3$.

Definition 3.13. The *difference* between two CFH-sets (ψ_1, G_1) and (ψ_2, G_2) is defined as

$$(\psi_1, G_1) \setminus (\psi_2, G_2) = (\psi_1, G_1) \prod (\psi_2, G_2)^c$$

Definition 3.14. The *union* of two CFH-sets (ψ_1, G_1) and (ψ_2, G_2) over the same universe $\mathbb{U}$, denoted by $(\psi_1, G_1) \coprod (\psi_2, G_2)$, is the CFH-set (ψ_3, G_3), where $G_3 = G_1 \coprod G_2$, and for all $\underline{x} \in G_3$,

$$\psi_3(\underline{x}) = \begin{cases} \psi_1(\underline{x}) & , if\ \underline{x} \in G_1 \backslash G_2 \\ \psi_2(\underline{x}) & , if\ \underline{x} \in G_2 \backslash G_1 \\ \psi_1(\underline{x}) \coprod \psi_2(\underline{x}) & , if\ \underline{x} \in G_1 \prod G_2 \end{cases}$$

Proposition 3.15. *Let (ψ, G) be a CFH-set over $\mathbb{U}$. Then the following results hold true:*

i. $(\psi, G) \coprod (\psi, G)_\Phi = (\psi, G)$

ii. $(\psi, G) \coprod (\psi, G)_\Delta = (\psi, G)_\Delta$

iii. $(\psi, G) \prod (\psi, G)_\Phi = (\psi, G)_\Phi$

iv. $(\psi, G) \prod (\psi, G)_\Delta = (\psi, G)$

v. $(\psi, G)_\Phi \coprod (\psi, G)_\Delta = (\psi, G)_\Delta$

vi. $(\psi, G)_\Phi \prod (\psi, G)_\Delta = (\psi, G)_\Phi$

Proposition 3.16. *Let (ψ_1, G_1), (ψ_2, G_2) and (ψ_3, G_3) are three CFH-sets over the same universe $\mathbb{U}$. Then the following commutative and associative laws hold true:*

i. $(\psi_1, G_1) \prod (\psi_2, G_2) = (\psi_2, G_2) \prod (\psi_1, G_1)$
ii. $(\psi_1, G_1) \coprod (\psi_2, G_2) = (\psi_2, G_2) \coprod (\psi_1, G_1)$
iii. $(\psi_1, G_1) \prod ((\psi_2, G_2) \prod (\psi_3, G_3)) = ((\psi_1, G_1) \prod (\psi_2, G_2)) \prod (\psi_3, G_3)$
iv. $(\psi_1, G_1) \coprod ((\psi_2, G_2) \coprod (\psi_3, G_3)) = ((\psi_1, G_1) \coprod (\psi_2, G_2)) \coprod (\psi_3, G_3)$

Proposition 3.17. *Let (ψ_1, G_1) and (ψ_2, G_2) are two CFH-sets over the same universe $\mathbb{U}$. Then the following De Morganss laws hold true:*

i. $((\psi_1, G_1) \prod (\psi_2, G_2))^c = (\psi_1, G_1)^c \coprod (\psi_2, G_2)^c$
ii. $((\psi_1, G_1) \coprod (\psi_2, G_2))^c = (\psi_1, G_1)^c \prod (\psi_2, G_2)^c$

4. Complex Intuitionistic Fuzzy Hypersoft Set(CIFH-Set)

In this section, fundamental theory of CIFH-set is developed.

Definition 4.1. Let $B_1, B_2, B_3, ..., B_n$ are disjoint sets having attribute values of n distinct attributes $b_1, b_2, b_3, ..., b_n$ respectively for $n \geq 1$, $B = B_1 \times B_2 \times B_3 \times ..... \times B_n$ and $\xi(\underline{\nu})$ be a CIF-set over $\mathbb{U}$ for all $\underline{\nu} = (s_1, s_2, s_3, ..., s_n) \in B$ such that $s_1 \in B_1, s_2 \in B_2, s_3 \in B_3, ..., s_n \in B_n$. Then, *complex intuitionistic fuzzy hypersoft set* (CIFH-set) $\Gamma_B = (\xi, B)$ over $\mathbb{U}$ is defined as

$$\Gamma_B = \{(\underline{\nu}, \xi(\underline{\nu})) : \underline{\nu} \in B, \xi(\underline{\nu}) \in C_{Int}(\mathbb{U})\}$$

where

$$\xi : B \to C_{Int}(\mathbb{U}), \qquad \xi(\underline{\nu}) = \emptyset \ if \ \underline{\nu} \notin B.$$

is a CIF approximate function of Γ_B and $\xi(\underline{\nu}) = \langle \xi^T(\underline{\nu}), \xi^F(\underline{\nu}) \rangle$.
$\xi^T(\underline{\nu}) = \alpha_T \, e^{i\,\beta_T}$ and $\xi^F(\underline{\nu}) = \alpha_F \, e^{i\,\beta_F}$ are complex-valued grade of membership and nonmembership of Γ_B respectively and their sum all are lying within the unit circle in the complex plane such that $\alpha_T, \alpha_F \in [0,1]$ with $0 \leq \alpha_T + \alpha_F \leq 1$ and $\beta_T, \beta_F \in (0, 2\pi]$. The value $\xi(\underline{\nu})$ is called $\underline{\nu}$-member of CIFH-set $\forall \underline{\nu} \in B$.

Example 4.2. Considering example 3.4 with $B = \{e_1, e_2, e_3, ..., e_8\}$, CIF-sets $\xi_B(e_1), \xi_B(e_2), ..., \xi_B(e_8)$ are defined as,

$$\xi_B(e_1) = \left\{ \frac{\langle 0.6, 0.2 \rangle e^{i \langle 0.5, 0.3 \rangle \pi}}{t_1}, \frac{\langle 0.8, 0.1 \rangle e^{i \langle 0.5, 0.3 \rangle \pi}}{t_2}, \frac{\langle 0.6, 0.4 \rangle e^{i \langle 0.7, 0.2 \rangle \pi}}{t_3}, \frac{\langle 0.3, 0.1 \rangle e^{i \langle 0.65, 0.35 \rangle \pi}}{t_4} \right\},$$

$$\xi_B(e_2) = \left\{ \frac{\langle 0.5, 0.2 \rangle e^{i \langle 0.6, 0.3 \rangle \pi}}{t_1}, \frac{\langle 0.8, 0.01 \rangle e^{i \langle 0.8, 0.02] \pi}}{t_2}, \frac{\langle 0.6, 0.2 \rangle] e^{i \langle 0.8, 0.03 \rangle \pi}}{t_3}, \frac{\langle 0.65, 0.25 \rangle e^{i \langle 0.85, 0.05 \rangle \pi}}{t_4} \right\},$$

$$\xi_B(e_3) = \left\{ \frac{\langle 0.4, 0.3 \rangle e^{i \langle 0.5, 0.1 \rangle \pi}}{t_1}, \frac{\langle 0.7, 0.o2 \rangle e^{i \langle 0.8, 0.03 \rangle] \pi}}{t_2}, \frac{\langle 0.5, 0.1 \rangle e^{i \langle 0.9, 0.01 \rangle \pi}}{t_3}, \frac{\langle 0.55, 0.25 \rangle e^{i \langle 0.85, 0.05 \rangle \pi}}{t_4} \right\},$$

$$\xi_B(e_4) = \left\{ \frac{\langle 0.3, 0.1 \rangle e^{i \langle 0.6, 0.1 \rangle \pi}}{t_1}, \frac{\langle 0.6, 0.01 \rangle e^{i \langle 0.8, 0.09 \rangle \pi}}{t_2}, \frac{\langle 0.5, 0.05 \rangle e^{i \langle 0.2, 0.01 \rangle \pi}}{t_3}, \frac{\langle 0.45, 0.25 \rangle e^{i \langle 0.55, 0.15 \rangle \pi}}{t_4} \right\},$$

$$\xi_B(e_5)=\left\{\frac{\langle 0.3,0.2\rangle e^{i\langle 0.4,0.3\rangle\pi}}{t_1},\frac{\langle 0.7,0.1\rangle e^{i\langle 0.7,0.08\rangle\pi}}{t_2},\frac{\langle 0.7,0.01\rangle e^{i\langle 0.6,0.1\rangle\pi}}{t_3},\frac{\langle 0.55,0.05\rangle e^{i\langle 0.45,0.05\rangle\pi}}{t_4}\right\},$$

$$\xi_B(e_6)=\left\{\frac{\langle 0.4,0.01\rangle e^{i\langle 0.5,0.1\rangle\pi}}{t_1},\frac{\langle 0.4,0.1\rangle e^{i\langle 0.8,0.1\rangle\pi}}{t_2},\frac{\langle 0.6,0.070\rangle e^{i\langle 0.7,0.01\rangle\pi}}{t_3},\frac{\langle 0.65,0.05\rangle e^{i\langle 0.85,0.15\rangle\pi}}{t_4}\right\},$$

$$\xi_B(e_7)=\left\{\frac{\langle 0.5,0.09\rangle e^{i\langle 0.8,0.09\rangle\pi}}{t_1},\frac{\langle 0.4,0.09\rangle e^{i\langle 0.5,0.06\rangle\pi}}{t_2},\frac{\langle 0.4,0.05\rangle e^{i\langle 0.5,0.06\rangle\pi}}{t_3},\frac{\langle 0.75,0.15\rangle e^{i\langle 0.65,0.25\rangle\pi}}{t_4}\right\},$$

and

$$\xi_B(e_8)=\left\{\frac{\langle 0.7,0.08\rangle e^{i\langle 0.1,0.09\rangle\pi}}{t_1},\frac{\langle 0.5,0.08\rangle e^{i\langle 0.7,0.02\rangle\pi}}{t_2},\frac{\langle 0.5,0.06\rangle e^{i\langle 0.8,0.03\rangle\pi}}{t_3},\frac{\langle 0.4,0.05\rangle e^{i\langle 0.75,0.15\rangle\pi}}{t_4}\right\}$$

then CIFH-set Γ_B is written by,

$$\Gamma_B=\left\{\begin{array}{l}
\left(e_1,\frac{\langle 0.6,0.2\rangle e^{i\langle 0.5,0.3\rangle\pi}}{t_1},\frac{\langle 0.8,0.1\rangle e^{i\langle 0.5,0.3\rangle\pi}}{t_2},\frac{\langle 0.6,0.4\rangle e^{i\langle 0.7,0.2\rangle\pi}}{t_3},\frac{\langle 0.3,0.1\rangle e^{i\langle 0.65,0.35\rangle\pi}}{t_4}\right),\\
\left(e_2,\frac{\langle 0.5,0.2\rangle e^{i\langle 0.6,0.3\rangle\pi}}{t_1},\frac{\langle 0.8,0.01\rangle e^{i\langle 0.8,0.02\rangle]\pi}}{t_2},\frac{\langle 0.6,0.2\rangle] e^{i\langle 0.8,0.03\rangle\pi}}{t_3},\frac{\langle 0.65,0.25\rangle e^{i\langle 0.85,0.05\rangle\pi}}{t_4}\right),\\
\left(e_3,\frac{\langle 0.4,0.3\rangle e^{i\langle 0.5,0.1\rangle\pi}}{t_1},\frac{\langle 0.7,0.o2\rangle e^{i\langle 0.8,0.03\rangle]\pi}}{t_2},\frac{\langle 0.5,0.1\rangle e^{i\langle 0.9,0.01\rangle\pi}}{t_3},\frac{\langle 0.55,0.25\rangle e^{i\langle 0.85,0.05\rangle\pi}}{t_4}\right),\\
\left(e_4,\frac{\langle 0.3,0.1\rangle e^{i\langle 0.6,0.1\rangle\pi}}{t_1},\frac{\langle 0.6,0.01\rangle e^{i\langle 0.8,0.09\rangle\pi}}{t_2},\frac{\langle 0.5,0.05\rangle e^{i\langle 0.2,0.01\rangle\pi}}{t_3},\frac{\langle 0.45,0.25\rangle e^{i\langle 0.55,0.15\rangle\pi}}{t_4}\right),\\
\left(e_5,\frac{\langle 0.3,0.2\rangle e^{i\langle 0.4,0.3\rangle\pi}}{t_1},\frac{\langle 0.7,0.1\rangle e^{i\langle 0.7,0.08\rangle\pi}}{t_2},\frac{\langle 0.7,0.01\rangle e^{i\langle 0.6,0.1\rangle\pi}}{t_3},\frac{\langle 0.55,0.05\rangle e^{i\langle 0.45,0.05\rangle\pi}}{t_4}\right),\\
\left(e_6,\frac{\langle 0.4,0.01\rangle e^{i\langle 0.5,0.1\rangle\pi}}{t_1},\frac{\langle 0.4,0.1\rangle e^{i\langle 0.8,0.1\rangle\pi}}{t_2},\frac{\langle 0.6,0.070\rangle e^{i\langle 0.7,0.01\rangle\pi}}{t_3},\frac{\langle 0.65,0.05\rangle e^{i\langle 0.85,0.15\rangle\pi}}{t_4}\right),\\
\left(e_7,\frac{\langle 0.5,0.09\rangle e^{i\langle 0.8,0.09\rangle\pi}}{t_1},\frac{\langle 0.4,0.09\rangle e^{i\langle 0.5,0.06\rangle\pi}}{t_2},\frac{\langle 0.4,0.05\rangle e^{i\langle 0.5,0.06\rangle\pi}}{t_3},\frac{\langle 0.75,0.15\rangle e^{i\langle 0.65,0.25\rangle\pi}}{t_4}\right),\\
\left(e_8,\frac{\langle 0.7,0.08\rangle e^{i\langle 0.1,0.09\rangle\pi}}{t_1},\frac{\langle 0.5,0.08\rangle e^{i\langle 0.7,0.02\rangle\pi}}{t_2},\frac{\langle 0.5,0.06\rangle e^{i\langle 0.8,0.03\rangle\pi}}{t_3},\frac{\langle 0.4,0.05\rangle e^{i\langle 0.75,0.15\rangle\pi}}{t_4}\right)
\end{array}\right\}$$

Definition 4.3. Let $\Gamma_{B_1}=(\xi_1,B_1)$ and $\Gamma_{B_2}=(\xi_2,B_2)$ be two CIFH-sets over the same $\mathbb{U}$. The set $\Gamma_{B_1}=(\xi_1,B_1)$ is said to be the *subset* of $\Gamma_{B_2}=(\xi_2,B_2)$, if

i. $B_1\subseteq B_2$

ii. $\forall\ \underline{p}\in B_1,\xi_1(\underline{p})\subseteq\xi_2(\underline{p})$ implies ${\xi^T}_1(\underline{p})\subseteq{\xi^T}_2(\underline{p}),{\xi^F}_1(\underline{p})\subseteq{\xi^F}_2(\underline{p})$ i.e. $\alpha_{TB_1}(\underline{p})\le\alpha_{TB_2}(\underline{p}),\alpha_{FB_1}(\underline{p})\le\alpha_{FB_2}(\underline{p}),\beta_{TB_1}(\underline{p})\le\beta_{TB_2}(\underline{p})$ and $\beta_{FB_1}(\underline{p})\le\beta_{FB_2}(\underline{p})$, where

$\alpha_{TB_1}(\underline{p})$ and $\beta_{TB_1}(\underline{p})$ are amplitude and phase terms of $\xi_1^T(\underline{p})$,
$\alpha_{FB_1}(\underline{p})$ and $\beta_{FB_1}(\underline{p})$ are amplitude and phase terms of $\xi_1^F(\underline{p})$,
$\alpha_{TB_2}(\underline{p})$ and $\beta_{TB_2}(\underline{p})$ are amplitude and phase terms of $\xi_2^T(\underline{p})$, and
$\alpha_{FB_2}(\underline{p})$ and $\beta_{FB_2}(\underline{p})$ are amplitude and phase terms of $\xi_2^F(\underline{p})$.

Definition 4.4. Two CIFH-set $\Gamma_{B_1} = (\xi_1, B_1)$ and $\Gamma_{B_2} = (\xi_2, B_2)$ over the same $\mathbb{U}$, are said to be *equal* if

i. $(\xi_1, B_1) \subseteq (\xi_2, B_2)$
ii. $(\xi_2, B_2) \subseteq (\xi_1, B_1)$.

Definition 4.5. Let (ξ, B) be a CIFH-set over $\mathbb{U}$.Then

i. (ξ, B) is called a *null CIFH-set*, denoted by $(\xi, B)_\Phi$ if $\alpha_{TB}(\underline{p}) = \alpha_{FB}(\underline{p}) = 0$ and $\beta_{TB}(\underline{p}) = \beta_{FB}(\underline{p}) = 0\pi$ for all $\underline{p} \in B$.
ii. (ξ, B) is called a *absolute CIFH-set*, denoted by $(\xi, B)_\Delta$ if $\alpha_{TB}(\underline{p}) = \alpha_{FB}(\underline{p}) = 1$ and $\beta_{TB}(\underline{p}) = \beta_{FB}(\underline{p}) = 2\pi$ for all $\underline{p} \in B$.

Definition 4.6. Let (ξ_1, B_1) and (ξ_2, B_2) are two CIFH-sets over the same universe $\mathbb{U}$.Then

i. A CIFH-set (ξ_1, B_1) is called a *homogeneous CIFH-set*, denoted by $(\xi_1, B_1)_{Hom}$ if and only if $\xi_1(\underline{p})$ is a homogeneous CIF-set for all $\underline{p} \in B_1$.

ii. A CIFH-set (ξ_1, B_1) is called a *completely homogeneous CIFH-set*, denoted by $(\xi_1, B_1)_{CHom}$ if and only if $\xi_1(\underline{p})$ is a homogeneous with $\xi_1(\underline{q})$ for all $\underline{p}, \underline{q} \in B_1$.

iii. A CIFH-set (ξ_1, B_1) is said to be a completely homogeneous CIFH-set with (ξ_2, B_2) if and only if $\xi_1(\underline{p})$ is a homogeneous with $\xi_2(\underline{p})$ for all $\underline{p} \in B_1 \prod B_2$.

4.1. *Set Theoretic Operations and Laws on CIFH-set*

Here some basic set theoretic operations (i.e.complement, union and intersection) and laws (commutative laws, associative laws etc.) are discussed on CFH-set.

Definition 4.7. The *complement* of CIFH-set (ξ, B), denoted by $(\xi, B)^c$ is defined as

$$(\xi, B)^c = \{(\underline{p}, (\xi(\underline{p}))^c) : \underline{p} \in B, (\xi(\underline{p}))^c \in C_{Int}(\mathbb{U})\}$$

such that the amplitude and phase terms of the membership function $(\xi(\underline{p}))^c$ are given by
$(\alpha_{TB}(\underline{p}))^c = 1 - \alpha_{TB}(\underline{p})$
$(\alpha_{FB}(\underline{p}))^c = 1 - \alpha_{FB}(\underline{p})$
and
$(\beta_{TB}(\underline{p}))^c = 2\pi - \beta_{TB}(\underline{p})$,
$(\beta_{FB}(\underline{p}))^c = 2\pi - \beta_{FB}(\underline{p})$ respectively.

Proposition 4.8. *Let* (ξ, B) *be a CIFH-set over* $\mathbb{U}$.*Then* $((\xi, B)^c)^c = (\xi, B)$.

Proof. Since $\xi(\underline{p}) \in C_{Int}(\mathbb{U})$, therefore (ξ, B) can be written in terms of its amplitude and phase terms as

$$(\xi, B) = \left\{ \left(\underline{p}, \left(\alpha_{TB}(\underline{p}) e^{i\beta_{TB}(\underline{p})}, \alpha_{FB}(\underline{p})) e^{i\beta_{FB}(\underline{p})} \right) \right) : \underline{p} \in B \right\} \tag{3}$$

Now

$$(\xi, B)^c(\underline{p}) = \left\{ \left(\underline{p}, \left((\alpha_{TB}(\underline{p}))^c e^{i(\beta_{TB}(\underline{p}))^c}, (\alpha_{FB}(\underline{p}))^c e^{i(\beta_{FB}(\underline{p}))^c} \right) \right) : \underline{p} \in B \right\}$$

$$(\xi, B)^c(\underline{p}) = \left\{ \left(\underline{p}, \left((1 - \alpha_{TB}(\underline{p})) e^{i(2\pi - \beta_{TB}(\underline{p}))}, (1 - \alpha_{FB}(\underline{p})) e^{i(2\pi - \alpha_{FB}(\underline{p}))} \right) \right) : \underline{p} \in B \right\}$$

$$((\xi, B)^c)^c = \left\{ \left(\underline{p}, \left((1 - \alpha_{TB}(\underline{p}))^c e^{i(2\pi - \beta_{TB}(\underline{p}))^c}, (1 - \alpha_{FB}(\underline{p}))^c e^{i(2\pi - \beta_{TB}(\underline{p}))^c} \right) \right) : \underline{p} \in B \right\}$$

$$((\xi, B)^c)^c = \left\{ \left(\underline{p}, \left((1 - (1 - \alpha_{TB}(\underline{p}))) e^{i(2\pi - (2\pi - \beta_{TB}(\underline{p})))}, (1 - (1 - \alpha_{FB}(\underline{p}))) e^{i(2\pi - (2\pi - \beta_{FB}(\underline{p})))} \right) \right) : \underline{p} \in B \right\}$$

$$((\xi, B)^c)^c = \left\{ \left(\underline{p}, \left(\alpha_{TB}(\underline{p}) e^{i\beta_{TB}(\underline{p})}, \alpha_{FB}(\underline{p})) e^{i\beta_{FB}(\underline{p})} \right) \right) : \underline{p} \in B \right\} \tag{4}$$

from equations (3) and (4), we have $((\xi, B)^c)^c = (\xi, B)$. □

Proposition 4.9. *Let (ξ, B) be a CIFH-set over $\mathbb{U}$. Then*

i. $((\xi, B)_\Phi)^c = (\xi, B)_\Delta$
ii. $((\xi, B)_\Delta)^c = (\xi, B)_\Phi$

Definition 4.10. The *intersection* of two CIFH-set (ξ_1, B_1) and (ξ_2, B_2) over the same universe $\mathbb{U}$, denoted by $(\xi_1, B_1) \prod (\xi_2, B_2)$, is the CIFH-set (ξ_3, B_3), where $B_3 = B_1 \coprod B_2$, and for all $\underline{p} \in B_3$,

$$\xi^T{}_3(\underline{p}) = \begin{cases} \alpha_{TB_1}(\underline{p}) e^{i\beta_{TB_1}(\underline{p})} & , if\ \underline{p} \in B_1 \backslash B_2 \\ \alpha_{TB_2}(\underline{p}) e^{i\beta_{TB_2}(\underline{p})} & , if\ \underline{p} \in B_2 \backslash B_1 \\ min(\alpha_{TB_1}(\underline{p}), \alpha_{TB_2}(\underline{p})) e^{i\, min(\beta_{TB_1}(\underline{p}), \beta_{TB_2}(\underline{p}))} & , if\ \underline{p} \in B_1 \prod B_2 \end{cases}$$

and

$$\xi^F{}_3(\underline{p}) = \begin{cases} \alpha_{FB_1}(\underline{p}) e^{i\beta_{FB_1}(\underline{p})} & , if\ \underline{p} \in B_1 \backslash B_2 \\ \alpha_{FB_2}(\underline{p}) e^{i\beta_{FB_2}(\underline{p})} & , if\ \underline{p} \in B_2 \backslash B_1 \\ min(\alpha_{FB_1}(\underline{p}), \alpha_{FB_2}(\underline{p})) e^{i\, min(\beta_{FB_1}(\underline{p}), \beta_{FB_2}(\underline{p}))} & , if\ \underline{p} \in B_1 \prod B_2 \end{cases}$$

Definition 4.11. The *difference* between two CFH-set (ξ_1, B_1) and (ξ_2, B_2) is defined as

$$(\xi_1, B_1) \setminus (\xi_2, B_2) = (\xi_1, B_1) \prod (\xi_2, B_2)^c$$

Definition 4.12. The *union* of two CFH-set (ξ_1, B_1) and (ξ_2, B_2) over the same universe $\mathbb{U}$, denoted by $(\xi_1, B_1) \coprod (\xi_2, B_2)$, is the CFH-set (ξ_3, B_3), where $B_3 = B_1 \coprod B_2$, and for all $\underline{p} \in B_3$,

$$\xi^T{}_3(\underline{p}) = \begin{cases} \alpha_{TB_1}(\underline{p}) e^{i\beta_{TB_1}(\underline{p})} & , if\ \underline{p} \in B_1 \backslash B_2 \\ \alpha_{TB_2}(\underline{p}) e^{i\beta_{TB_2}(\underline{p})} & , if\ \underline{p} \in B_2 \backslash B_1 \\ max(\alpha_{TB_1}(\underline{p}), \alpha_{TB_2}(\underline{p})) e^{i\, max(\beta_{TB_1}(\underline{p}), \beta_{TB_2}(\underline{p}))} & , if\ \underline{p} \in B_1 \prod B_2 \end{cases}$$

and

$$\xi^F{}_3(\underline{p}) = \begin{cases} \alpha_{F B_1}(\underline{p}) e^{i\beta_{F B_1}(\underline{p})} & , if\ \underline{p} \in B_1 \backslash B_2 \\ \alpha_{F B_2}(\underline{p}) e^{i\beta_{F B_2}(\underline{p})} & , if\ \underline{p} \in B_2 \backslash B_1 \\ max(\alpha_{F B_1}(\underline{p}), \alpha_{F B_2}(\underline{p})) e^{i\, max(\beta_{F B_1}(\underline{p}), \beta_{F B_2}(\underline{p}))} & , if\ \underline{p} \in B_1 \prod B_2 \end{cases}$$

Proposition 4.13. *Let (ξ, B) be a CIFH-set over $\mathbb{U}$. Then the following results hold true:*

i. $(\xi, B) \coprod (\xi, B)_\Phi = (\xi, B)$
ii. $(\xi, B) \coprod (\xi, B)_\Delta = (\xi, B)_\Delta$
iii. $(\xi, B) \prod (\xi, B)_\Phi = (\xi, B)_\Phi$
iv. $(\xi, B) \prod (\xi, B)_\Delta = (\xi, B)$
v. $(\xi, B)_\Phi \coprod (\xi, B)_\Delta = (\xi, B)_\Delta$
vi. $(\xi, B)_\Phi \prod (\xi, B)_\Delta = (\xi, B)_\Phi$

Proposition 4.14. *Let (ξ_1, B_1), (ξ_2, B_2) and (ξ_3, B_3) are three CIFH-sets over the same universe $\mathbb{U}$. Then the following commutative and associative laws hold true:*

i. $(\xi_1, B_1) \prod (\xi_2, B_2) = (\xi_2, B_2) \prod (\xi_1, B_1)$
ii. $(\xi_1, B_1) \coprod (\xi_2, B_2) = (\xi_2, B_2) \coprod (\xi_1, B_1)$
iii. $(\xi_1, B_1) \prod ((\xi_2, B_2) \prod (\xi_3, B_3)) = ((\xi_1, B_1) \prod (\xi_2, B_2)) \prod (\xi_3, B_3)$
iv. $(\xi_1, B_1) \coprod ((\xi_2, B_2) \coprod (\xi_3, B_3)) = ((\xi_1, B_1) \coprod (\xi_2, B_2)) \coprod (\xi_3, B_3)$

Proposition 4.15. *Let (ξ_1, B_1) and (ξ_2, B_2) are two CIFH-sets over the same universe $\mathbb{U}$. Then the following De Morganss laws hold true:*

i. $((\xi_1, B_1) \prod (\xi_2, B_2))^c = (\xi_1, B_1)^c \coprod (\xi_2, B_2)^c$
ii. $((\xi_1, B_1) \coprod (\xi_2, B_2))^c = (\xi_1, B_1)^c \prod (\xi_2, B_2)^c$

5. Complex Neutrosophic Hypersoft Set(CNH-Set)

In this section, CNH-set and its some fundamentals are developed.

Definition 5.1. Let $N_1, N_2, N_3, ..., N_n$ are disjoint sets having attribute values of n distinct attributes $n_1, n_2, n_3, ..., n_n$ respectively for $n \geq 1, N = N_1 \times N_2 \times N_3 \times ..... \times N_n$ and $\zeta(\underline{\lambda})$ be a CN-set over $\mathbb{U}$ for all $\underline{\lambda} = (a_1, a_2, a_3, ..., a_n) \in N$ such that $a_1 \in N_1, a_2 \in N_2, a_3 \in N_3, ..., a_n \in N_n$. Then, *complex neutrosophic hypersoft set* (CNH-set) $\Theta_N = (\zeta, N)$ over $\mathbb{U}$ is defined as

$$\Theta_N = \{(\underline{\lambda}, \zeta(\underline{\lambda})) : \underline{\lambda} \in N, \zeta(\underline{\lambda}) \in C_{Neu}(\mathbb{U})\}$$

where

$$\zeta : N \to C_{Neu}(\mathbb{U}), \qquad \zeta(\underline{\lambda}) = \emptyset\ if\ \underline{\lambda} \notin N.$$

is a CN approximate function of Θ_N and $\zeta(\underline{\lambda}) = \langle \zeta^T(\underline{\lambda}), \zeta^I(\underline{\lambda}), \zeta^F(\underline{\lambda}) \rangle$.
$\zeta^T(\underline{\lambda}) = \delta_T\, e^{i\,\eta_T}$, $\zeta^I(\underline{\lambda}) = \delta_I\, e^{i\,\eta_I}$ and $\zeta^F(\underline{\lambda}) = \delta_F\, e^{i\,\eta_F}$ are complex-valued truth membership function, complex-valued indeterminacy membership function, and complex-valued falsity

membership function of Θ_N respectively and their sum all are lying within the unit circle in the complex plane such that $\delta_T, \delta_I, \delta_F \in [0,1]$ with $^-0 \leq \delta_T+\delta_I+\delta_F \leq 3^+ (or\ 0 \leq |\delta_T+\delta_I+\delta_F| \leq 3)$ and $\eta_T, \eta_I, \eta_F \in (0, 2\pi]$. The value $\zeta(\underline{\lambda})$ is called $\underline{\lambda}$-member of CNH-set $\forall \underline{\lambda} \in N$.

Example 5.2. Considering example 3.4 wit $N = \{e_1, e_2, e_3, ..., e_8\}$, CNF-sets $\zeta_N(e_1), \zeta_N(e_2), ..., \zeta_N(e_8)$ are defined as,

$$\zeta_N(e_1) = \left\{ \frac{\langle 0.6,0.1,0.2\rangle e^{i\langle 0.5,0.2,0.3\rangle\pi}}{t_1}, \frac{\langle 0.8,0.3,0.1\rangle e^{i\langle 0.5,0.4,0.3\rangle\pi}}{t_2}, \frac{\langle 0.6,0.5,0.4\rangle e^{i\langle 0.7,0.6,0.2\rangle\pi}}{t_3}, \frac{\langle 0.3,0.7,0.1\rangle e^{i\langle 0.65,0.55,0.35\rangle\pi}}{t_4} \right\},$$

$$\zeta_N(e_2) = \left\{ \frac{\langle 0.5,0.2,0.1\rangle e^{i\langle 0.6,0.3,0.2\rangle\pi}}{t_1}, \frac{\langle 0.8,0.01,0.2\rangle e^{i\langle 0.8,0.02,0.3\rangle\pi}}{t_2}, \frac{\langle 0.6,0.2,0.2\rangle e^{i\langle 0.8,0.03,0.4\rangle\pi}}{t_3}, \frac{\langle 0.65,0.25,0.5\rangle e^{i\langle 0.85,0.05,0.5\rangle\pi}}{t_4} \right\},$$

$$\zeta_N(e_3) = \left\{ \frac{\langle 0.4,0.3,0.3\rangle e^{i\langle 0.5,0.1,0.8\rangle\pi}}{t_1}, \frac{\langle 0.7,0.02,0.3\rangle e^{i\langle 0.8,0.03,0.7\rangle]\pi}}{t_2}, \frac{\langle 0.5,0.1,0.9\rangle e^{i\langle 0.9,0.01,0.7\rangle\pi}}{t_3}, \frac{\langle 0.55,0.25,0.1\rangle e^{i\langle 0.85,0.05,0.4\rangle\pi}}{t_4} \right\},$$

$$\zeta_N(e_4) = \left\{ \frac{\langle 0.3,0.1,0.9\rangle e^{i\langle 0.6,0.1,0.5\rangle\pi}}{t_1}, \frac{\langle 0.6,0.01,0.4\rangle e^{i\langle 0.8,0.09,0.5\rangle\pi}}{t_2}, \frac{\langle 0.5,0.05,0.3\rangle e^{i\langle 0.2,0.01,0.4\rangle\pi}}{t_3}, \frac{\langle 0.45,0.25,0.01\rangle e^{i\langle 0.55,0.15,0.3\rangle\pi}}{t_4} \right\},$$

$$\zeta_N(e_5) = \left\{ \frac{\langle 0.3,0.2,0.1\rangle e^{i\langle 0.4,0.3,0.4\rangle\pi}}{t_1}, \frac{\langle 0.7,0.1,0.5\rangle e^{i\langle 0.7,0.08,0.4\rangle\pi}}{t_2}, \frac{\langle 0.7,0.01,0.4\rangle e^{i\langle 0.6,0.1,0.5\rangle\pi}}{t_3}, \frac{\langle 0.55,0.05,0.4\rangle e^{i\langle 0.45,0.05,0.3\rangle\pi}}{t_4} \right\},$$

$$\zeta_N(e_6) = \left\{ \frac{\langle 0.4,0.01,0.3\rangle e^{i\langle 0.5,0.1,0.4\rangle\pi}}{t_1}, \frac{\langle 0.4,0.1,0.3\rangle e^{i\langle 0.8,0.1,0.3\rangle\pi}}{t_2}, \frac{\langle 0.6,0.070,0.5\rangle e^{i\langle 0.7,0.01,0.1\rangle\pi}}{t_3}, \frac{\langle 0.65,0.05,0.3\rangle e^{i\langle 0.85,0.15,0.4\rangle\pi}}{t_4} \right\},$$

$$\zeta_N(e_7) = \left\{ \frac{\langle 0.5,0.09,0.3\rangle e^{i\langle 0.8,0.09,0.5\rangle\pi}}{t_1}, \frac{\langle 0.4,0.09,0.4\rangle e^{i\langle 0.5,0.06,0.4\rangle\pi}}{t_2}, \frac{\langle 0.4,0.05,0.1\rangle e^{i\langle 0.5,0.06,0.5\rangle\pi}}{t_3}, \frac{\langle 0.75,0.15,0.4\rangle e^{i\langle 0.65,0.25,0.2\rangle\pi}}{t_4} \right\},$$

and

$$\zeta_N(e_8) = \left\{ \frac{\langle 0.7,0.08,0.3\rangle e^{i\langle 0.1,0.09,0.01\rangle\pi}}{t_1}, \frac{\langle 0.5,0.08,0.3\rangle e^{i\langle 0.7,0.02,0.6\rangle\pi}}{t_2}, \frac{\langle 0.5,0.06,0.5\rangle e^{i\langle 0.8,0.03,0.3\rangle\pi}}{t_3}, \frac{\langle 0.4,0.05,0.35\rangle e^{i\langle 0.75,0.15,0.6\rangle\pi}}{t_4} \right\}$$

then CNH-set Θ_N is written by,

$$\Theta_N = \left\{ \begin{array}{l}
\left(e_1, \frac{\langle 0.6,0.1,0.2\rangle e^{i\langle 0.5,0.2,0.3\rangle\pi}}{t_1}, \frac{\langle 0.8,0.3,0.1\rangle e^{i\langle 0.5,0.4,0.3\rangle\pi}}{t_2}, \frac{\langle 0.6,0.5,0.4\rangle e^{i\langle 0.7,0.6,0.2\rangle\pi}}{t_3}, \frac{\langle 0.3,0.7,0.1\rangle e^{i\langle 0.65,0.55,0.35\rangle\pi}}{t_4}\right), \\
\left(e_2, \frac{\langle 0.5,0.2,0.1\rangle e^{i\langle 0.6,0.3,0.2\rangle\pi}}{t_1}, \frac{\langle 0.8,0.01,0.2\rangle e^{i\langle 0.8,0.02,0.3\rangle\pi}}{t_2}, \frac{\langle 0.6,0.2,0.2\rangle] e^{i\langle 0.8,0.03,0.4\rangle\pi}}{t_3}, \frac{\langle 0.65,0.25,0.5\rangle e^{i\langle 0.85,0.05,0.5\rangle\pi}}{t_4}\right), \\
\left(e_3, \frac{\langle 0.4,0.3,0.3\rangle e^{i\langle 0.5,0.1,0.8\rangle\pi}}{t_1}, \frac{\langle 0.7,0.02,0.3\rangle e^{i\langle 0.8,0.03,0.7\rangle]\pi}}{t_2}, \frac{\langle 0.5,0.1,0.9\rangle e^{i\langle 0.9,0.01,0.7\rangle\pi}}{t_3}, \frac{\langle 0.55,0.25,0.1\rangle e^{i\langle 0.85,0.05,0.4\rangle\pi}}{t_4}\right), \\
\left(e_4, \frac{\langle 0.3,0.1,0.9\rangle e^{i\langle 0.6,0.1,0.5\rangle\pi}}{t_1}, \frac{\langle 0.6,0.01,0.4\rangle e^{i\langle 0.8,0.09,0.5\rangle\pi}}{t_2}, \frac{\langle 0.5,0.05,0.3\rangle e^{i\langle 0.2,0.01,0.4\rangle\pi}}{t_3}, \frac{\langle 0.45,0.25,0.01\rangle e^{i\langle 0.55,0.15,0.3\rangle\pi}}{t_4}\right), \\
\left(e_5, \frac{\langle 0.3,0.2,0.1\rangle e^{i\langle 0.4,0.3,0.4\rangle\pi}}{t_1}, \frac{\langle 0.7,0.1,0.5\rangle e^{i\langle 0.7,0.08,0.4\rangle\pi}}{t_2}, \frac{\langle 0.7,0.01,0.4\rangle e^{i\langle 0.6,0.1,0.5\rangle\pi}}{t_3}, \frac{\langle 0.55,0.05,0.4\rangle e^{i\langle 0.45,0.05,0.3\rangle\pi}}{t_4}\right), \\
\left(e_6, \frac{\langle 0.4,0.01,0.3\rangle e^{i\langle 0.5,0.1,0.4\rangle\pi}}{t_1}, \frac{\langle 0.4,0.1,0.3\rangle e^{i\langle 0.8,0.1,0.3\rangle\pi}}{t_2}, \frac{\langle 0.6,0.070,0.5\rangle e^{i\langle 0.7,0.01,0.1\rangle\pi}}{t_3}, \frac{\langle 0.65,0.05,0.3\rangle e^{i\langle 0.85,0.15,0.4\rangle\pi}}{t_4}\right), \\
\left(e_7, \frac{\langle 0.5,0.09,0.3\rangle e^{i\langle 0.8,0.09,0.5\rangle\pi}}{t_1}, \frac{\langle 0.4,0.09,0.4\rangle e^{i\langle 0.5,0.06,0.4\rangle\pi}}{t_2}, \frac{\langle 0.4,0.05,0.1\rangle e^{i\langle 0.5,0.06,0.5\rangle\pi}}{t_3}, \frac{\langle 0.75,0.15,0.4\rangle e^{i\langle 0.65,0.25,0.2\rangle\pi}}{t_4}\right), \\
\left(e_8, \frac{\langle 0.7,0.08,0.3\rangle e^{i\langle 0.1,0.09,0.01\rangle\pi}}{t_1}, \frac{\langle 0.5,0.08,0.3\rangle e^{i\langle 0.7,0.02,0.6\rangle\pi}}{t_2}, \frac{\langle 0.5,0.06,0.5\rangle e^{i\langle 0.8,0.03,0.3\rangle\pi}}{t_3}, \frac{\langle 0.4,0.05,0.35\rangle e^{i\langle 0.75,0.15,0.6\rangle\pi}}{t_4}\right)
\end{array} \right\}$$

Definition 5.3. Let $\Theta_{N_1} = (\zeta_1, N_1)$ and $\Theta_{N_2} = (\zeta_2, N_2)$ be two CNH-sets over the same $\mathbb{U}$. The set $\Theta_{N_1} = (\zeta_1, N_1)$ is said to be the *subset* of $\Theta_{N_2} = (\zeta_2, N_2)$, if

i. $N_1 \subseteq N_2$

ii. $\forall\ \underline{u} \in N_1, \zeta_1(\underline{u}) \subseteq \zeta_2(\underline{u})$ implies $\zeta_1^T(\underline{u}) \subseteq \zeta_2^T(\underline{u}), \zeta_1^I(\underline{u}) \subseteq \zeta_2^I(\underline{u}), \zeta_1^F(\underline{u}) \subseteq \zeta_2^F(\underline{u})$ i.e.
$\delta_{TN_1}(\underline{u}) \leq \delta_{TN_2}(\underline{u})$,
$\delta_{IN_1}(\underline{u}) \leq \delta_{IN_2}(\underline{u})$,
$\delta_{FN_1}(\underline{u}) \leq \delta_{FN_2}(\underline{u})$,
$\eta_{TN_1}(\underline{u}) \leq \eta_{TN_2}(\underline{u})$,
$\eta_{IN_1}(\underline{u}) \leq \eta_{IN_2}(\underline{u})$ and
$\eta_{FN_1}(\underline{u}) \leq \eta_{FN_2}(\underline{u})$,
where
$\delta_{TN_1}(\underline{u})$ and $\eta_{TN_1}(\underline{u})$ are amplitude and phase terms of $\zeta_1^T(\underline{u})$,
$\delta_{IN_1}(\underline{u})$ and $\eta_{IN_1}(\underline{u})$ are amplitude and phase terms of $\zeta_1^I(\underline{u})$,
$\delta_{FN_1}(\underline{u})$ and $\eta_{FN_1}(\underline{u})$ are amplitude and phase terms of $\zeta_1^F(\underline{u})$,
$\delta_{TN_2}(\underline{u})$ and $\eta_{TN_2}(\underline{u})$ are amplitude and phase terms of $\zeta_2^T(\underline{u})$,
$\delta_{IN_2}(\underline{u})$ and $\eta_{IN_2}(\underline{u})$ are amplitude and phase terms of $\zeta_2^I(\underline{u})$, and
$\delta_{FN_2}(\underline{u})$ and $\eta_{FN_2}(\underline{u})$ are amplitude and phase terms of $\zeta_2^F(\underline{u})$.

Definition 5.4. Two CNH-set $\Theta_{N_1} = (\zeta_1, N_1)$ and $\Theta_{N_2} = (\zeta_2, N_2)$ over the same $\mathbb{U}$, are said to be *equal* if

i. $(\zeta_1, N_1) \subseteq (\zeta_2, N_2)$

ii. $(\zeta_2, N_2) \subseteq (\zeta_1, N_1)$.

Definition 5.5. Let (ζ, N) be a CNH-set over $\mathbb{U}$.Then

i. (ζ, N) is called a *null CNH-set*, denoted by $(\zeta, N)_\Phi$ if $\delta_{TN}(\underline{u}) = \delta_{IN}(\underline{u}) = \delta_{FN}(\underline{u}) = 0$ and $\eta_{TN}(\underline{u}) = \eta_{IN}(\underline{u}) = \eta_{FN}(\underline{u}) = 0\pi$ for all $\underline{u} \in B$.

ii. (ζ, N) is called a *absolute CNH-set*, denoted by $(\zeta, N)_\Delta$ if $\delta_{TN}(\underline{u}) = \delta_{IN}(\underline{u}) = \delta_{FN}(\underline{u}) = 1$ and $\eta_{TN}(\underline{u}) = \eta_{IN}(\underline{u}) = \eta_{FN}(\underline{u}) = 2\pi$ for all $\underline{u} \in B$.

Definition 5.6. Let (ζ_1, N_1) and (ζ_2, N_2) are two CNH-sets over the same universe $\mathbb{U}$.Then

i. A CNH-set (ζ_1, N_1) is called a *homogeneous CNH-set*, denoted by $(\zeta_1, N_1)_{Hom}$ if and only if $\zeta_1(\underline{u})$ is a homogeneous CN-set for all $\underline{u} \in N_1$.

ii. A CNH-set (ζ_1, N_1) is called a *completely homogeneous CNH-set*, denoted by $(\zeta_1, N_1)_{CHom}$ if and only if $\zeta_1(\underline{u})$ is a homogeneous with $\zeta_1(\underline{v})$ for all $\underline{u}, \underline{v} \in N_1$.

iii. A CNH-set (ζ_1, N_1) is said to be a completely homogeneous CNH-set with (ζ_2, N_2) if and only if $\zeta_1(u)$ is a homogeneous with $\zeta_2(u)$ for all $\underline{u} \in N_1 \prod N_2$.

5.1. *Set Theoretic Operations and Laws on CNH-set*

Here some basic set theoretic operations (i.e.complement, union and intersection) and laws (commutative laws, associative laws etc.) are discussed on CNH-set.

Definition 5.7. The *complement* of CNH-set (ζ, N), denoted by $(\zeta, N)^c$ is defined as

$$(\zeta, N)^c = \{(\underline{u}, (\zeta(\underline{u}))^c) : \underline{u} \in B, (\zeta(\underline{u}))^c \in C_{Neu}(\mathbb{U})\}$$

such that the amplitude and phase terms of the membership function $(\zeta(\underline{u}))^c$ are given by
$(\delta_{TN}(\underline{u}))^c = \delta_{FN}(\underline{u})$,
$(\delta_{IN}(\underline{u}))^c = 1 - \delta_{IN}(\underline{u})$,
$(\delta_{FN}(\underline{u}))^c = \delta_{TN}(\underline{u})$,
and
$(\eta_{TN}(\underline{u}))^c = 2\pi - \eta_{TN}(\underline{u})$,
$(\eta_{IN}(\underline{u}))^c = 2\pi - \eta_{IN}(\underline{u})$,
$(\eta_{FN}(\underline{u}))^c = 2\pi - \eta_{FN}(\underline{u})$ respectively.

Proposition 5.8. *Let (ζ, N) be a CNH-set over $\mathbb{U}$. Then $((\zeta, N)^c)^c = (\zeta, N)$.*

Proof. Since $\zeta(\underline{u}) \in C_{Neu}(\mathbb{U})$, therefore (ζ, N) can be written in terms of its amplitude and phase terms as

$$(\zeta, N) = \left\{\left(\underline{u}, \left\langle \delta_{TN}(\underline{u})e^{i\eta_{TN}(\underline{u})}, \delta_{IN}(\underline{u})e^{i\eta_{IN}(\underline{u})}, \delta_{FN}(\underline{u}))e^{i\eta_{FN}(\underline{u})}\right\rangle\right) : \underline{u} \in N\right\} \tag{5}$$

Now

$$(\zeta, N)^c(\underline{u}) = \left\{\left(\underline{u}, \left\langle (\delta_{TN}(\underline{u}))^c e^{i(\eta_{TN}(\underline{u}))^c}, (\delta_{IN}(\underline{u}))^c e^{i(\eta_{IN}(\underline{u}))^c}, (\delta_{FN}(\underline{u}))^c e^{i(\eta_{FN}(\underline{u}))^c}\right\rangle\right) : \underline{u} \in N\right\}$$

$$(\zeta, N)^c(\underline{u}) = \left\{\left(\underline{u}, \left\langle (\delta_{FN}(\underline{u}))e^{i(2\pi - \eta_{TN}(\underline{u}))}, (1 - \delta_{IN}(\underline{u}))e^{i(2\pi - \eta_{IN}(\underline{u}))}, (\delta_{TN}(\underline{u}))e^{i(2\pi - \delta_{FN}(\underline{u}))}\right\rangle\right) : \underline{u} \in N\right\}$$

$$((\zeta, N)^c)^c = \left\{\left(\underline{u}, \left\langle (\delta_{FN}(\underline{u}))^c e^{i(2\pi - \eta_{TN}(\underline{u}))^c}, (1 - \delta_{IN}(\underline{u}))^c e^{i(2\pi - \eta_{IN}(\underline{u}))^c}, (\delta_{TN}(\underline{u}))^c e^{i(2\pi - \eta_{TN}(\underline{u}))^c}\right\rangle\right) : \underline{u} \in N\right\}$$

$$((\zeta, N)^c)^c = \left\{\left(\underline{u}, \left\langle \begin{array}{l} \delta_{TN}(\underline{u})e^{i(2\pi - (2\pi - \eta_{TN}(\underline{u})))}, \\ (1 - (1 - \delta_{IN}(\underline{u})))e^{i(2\pi - (2\pi - \eta_{IN}(\underline{u})))}, \\ \delta_{FN}(\underline{u})e^{i(2\pi - (2\pi - \eta_{FN}(\underline{u})))} \end{array}\right\rangle\right) : \underline{u} \in N\right\}$$

$$((\zeta, N)^c)^c = \left\{\left(\underline{u}, \left\langle \delta_{TN}(\underline{u})e^{i\eta_{TN}(\underline{u})}, \delta_{IN}(\underline{u})e^{i\eta_{IN}(\underline{u})}, \delta_{FN}(\underline{u}))e^{i\eta_{FN}(\underline{u})}\right\rangle\right) : \underline{u} \in N\right\} \tag{6}$$

from equations (5) and (6), we have $((\zeta, N)^c)^c = (\zeta, N)$. □

Proposition 5.9. *Let (ζ, N) be a CNH-set over $\mathbb{U}$. Then*

i. $((\zeta, N)_\Phi)^c = (\zeta, N)_\Delta$

ii. $((\zeta, N)_{\Delta})^c = (\zeta, N)_{\Phi}$

Definition 5.10. The *intersection* of two CNH-set (ζ_1, N_1) and (ζ_2, N_2) over the same universe $\mathbb{U}$, denoted by $(\zeta_1, N_1) \prod (\zeta_2, N_2)$, is the CNH-set (ζ_3, N_3), where $N_3 = N_1 \coprod N_2$, and for all $\underline{u} \in N_3$,

$$\zeta^T{}_3(\underline{u}) = \begin{cases} \delta_{TN_1}(\underline{u}) e^{i\eta_{TN_1}(\underline{u})} & , if\ \underline{u} \in N_1 \backslash N_2 \\ \delta_{TN_2}(\underline{u}) e^{i\eta_{TN_2}(\underline{u})} & , if\ \underline{u} \in N_2 \backslash N_1 \\ [\delta_{TN_1}(\underline{u}) \otimes \delta_{TN_2}(\underline{u})] \,.\, e^{i\left[\eta_{TN_1}(\underline{u}) \otimes \eta_{TN_2}(\underline{u})\right]} & , if\ \underline{u} \in N_1 \prod N_2 \end{cases}$$

$$\zeta^I{}_3(\underline{u}) = \begin{cases} \delta_{IN_1}(\underline{u}) e^{i\eta_{IN_1}(\underline{u})} & , if\ \underline{u} \in N_1 \backslash N_2 \\ \delta_{IN_2}(\underline{u}) e^{i\eta_{IN_2}(\underline{u})} & , if\ \underline{u} \in N_2 \backslash N_1 \\ [\delta_{IN_1}(\underline{u}) \otimes \delta_{IN_2}(\underline{u})] \,.\, e^{i\left[\eta_{IN_1}(\underline{u}) \otimes \eta_{IN_2}(\underline{u})\right]} & , if\ \underline{u} \in N_1 \prod N_2 \end{cases}$$

and

$$\zeta^F{}_3(\underline{u}) = \begin{cases} \delta_{FN_1}(\underline{u}) e^{i\eta_{FN_1}(\underline{u})} & , if\ \underline{u} \in N_1 \backslash N_2 \\ \delta_{FN_2}(\underline{u}) e^{i\eta_{FN_2}(\underline{u})} & , if\ \underline{u} \in N_2 \backslash N_1 \\ [\delta_{FN_1}(\underline{u}) \otimes \delta_{FN_2}(\underline{u})] \,.\, e^{i\left[\eta_{FN_1}(\underline{u}) \otimes \eta_{FN_2}(\underline{u})\right]} & , if\ \underline{u} \in N_1 \prod N_2 \end{cases}$$

where $\otimes$ denotes minimum operator.

Definition 5.11. The *difference* between two CNH-set (ζ_1, N_1) and (ζ_2, N_2) is defined as

$$(\zeta_1, N_1) \setminus (\zeta_2, N_2) = (\zeta_1, N_1) \prod (\zeta_2, N_2)^c$$

Definition 5.12. The *union* of two CNH-set (ζ_1, N_1) and (ζ_2, N_2) over the same universe $\mathbb{U}$, denoted by $(\zeta_1, N_1) \coprod (\zeta_2, N_2)$, is the CNH-set (ζ_3, N_3), where $N_3 = N_1 \coprod N_2$, and for all $\underline{u} \in N_3$,

$$\zeta^T{}_3(\underline{u}) = \begin{cases} \delta_{TN_1}(\underline{u}) e^{i\,\eta_{TN_1}(\underline{u})} & , if\ \underline{u} \in N_1 \backslash N_2 \\ \delta_{TN_2}(\underline{u}) e^{i\,\eta_{TN_2}(\underline{u})} & , if\ \underline{u} \in N_2 \backslash N_1 \\ [\delta_{TN_1}(\underline{u}) \oplus \delta_{TN_2}(\underline{u})] \,.\, e^{i\left[\eta_{TN_1}(\underline{u}) \oplus \eta_{TN_2}(\underline{u})\right]} & , if\ \underline{u} \in N_1 \prod N_2 \end{cases}$$

$$\zeta^I{}_3(\underline{u}) = \begin{cases} \delta_{IN_1}(\underline{u}) e^{i\,\eta_{IN_1}(\underline{u})} & , if\ \underline{u} \in N_1 \backslash N_2 \\ \delta_{IN_2}(\underline{u}) e^{i\,\eta_{IN_2}(\underline{u})} & , if\ \underline{u} \in N_2 \backslash N_1 \\ [\delta_{IN_1}(\underline{u}) \oplus \delta_{IN_2}(\underline{u})] \,.\, e^{i\left[\eta_{IN_1}(\underline{u}) \oplus \eta_{IN_2}(\underline{u})\right]} & , if\ \underline{u} \in N_1 \prod N_2 \end{cases}$$

and

$$\zeta^F{}_3(\underline{u}) = \begin{cases} \delta_{FN_1}(\underline{u}) e^{i\,\eta_{FN_1}(\underline{u})} & , if\ \underline{u} \in N_1 \backslash N_2 \\ \delta_{FN_2}(\underline{u}) e^{i\,\eta_{FN_2}(\underline{u})} & , if\ \underline{u} \in N_2 \backslash N_1 \\ [\delta_{FN_1}(\underline{u}) \oplus \delta_{FN_2}(\underline{u})] \,.\, e^{i\left[\eta_{FN_1}(\underline{u}) \oplus \eta_{FN_2}(\underline{u})\right]} & , if\ \underline{u} \in N_1 \prod N_2 \end{cases}$$

where $\oplus$ denotes maximum operator.

Proposition 5.13. *Let (ζ, N) be a CNH-set over $\mathbb{U}$. Then the following results hold true:*

i. $(\zeta, N) \coprod (\zeta, N)_{\Phi} = (\zeta, N)$

ii. $(\zeta, N) \coprod (\zeta, N)_{\Delta} = (\zeta, N)_{\Delta}$

iii. $(\zeta, N) \prod (\zeta, N)_{\Phi} = (\zeta, N)_{\Phi}$
iv. $(\zeta, N) \prod (\zeta, N)_{\Delta} = (\zeta, N)$
v. $(\zeta, N)_{\Phi} \coprod (\zeta, N)_{\Delta} = (\zeta, N)_{\Delta}$
vi. $(\zeta, N)_{\Phi} \prod (\zeta, N)_{\Delta} = (\zeta, N)_{\Phi}$

Proposition 5.14. *Let (ζ_1, N_1), (ζ_2, N_2) and (ζ_3, N_3) are three CNH-sets over the same universe $\mathbb{U}$. Then the following commutative and associative laws hold true:*

i. $(\zeta_1, N_1) \prod (\zeta_2, N_2) = (\zeta_2, N_2) \prod (\zeta_1, N_1)$
ii. $(\zeta_1, N_1) \coprod (\zeta_2, N_2) = (\zeta_2, N_2) \coprod (\zeta_1, N_1)$
iii. $(\zeta_1, N_1) \prod ((\zeta_2, N_2) \prod (\zeta_3, N_3)) = ((\zeta_1, N_1) \prod (\zeta_2, N_2)) \prod (\zeta_3, N_3)$
iv. $(\zeta_1, N_1) \coprod ((\zeta_2, N_2) \coprod (\zeta_3, N_3)) = ((\zeta_1, N_1) \coprod (\zeta_2, N_2)) \coprod (\zeta_3, N_3)$

Proposition 5.15. *Let (ζ_1, N_1) and (ζ_2, N_2) are two CNH-sets over the same universe $\mathbb{U}$. Then the following De Morganss laws hold true:*

i. $((\zeta_1, N_1) \prod (\zeta_2, N_2))^c = (\zeta_1, N_1)^c \coprod (\zeta_2, N_2)^c$
ii. $((\zeta_1, N_1) \coprod (\zeta_2, N_2))^c = (\zeta_1, N_1)^c \prod (\zeta_2, N_2)^c$

Discussion on particular cases of CFH-sets, CIFH-sets and CNH-sets

- If $\zeta(\underline{\lambda}) = \left\langle \zeta^T(\underline{\lambda}), \zeta^I(\underline{\lambda}), \zeta^F(\underline{\lambda}) \right\rangle, {}^{-}0 \leq \delta_T + \delta_I + \delta_F \leq 3^{+} (or\ 0 \leq |\delta_T + \delta_I + \delta_F| \leq 3)$ is replaced by $\zeta(\underline{\lambda}) = \left\langle \zeta^T(\underline{\lambda}), \zeta^F(\underline{\lambda}) \right\rangle, 0 \leq \delta_T + \delta_F \leq 1 (or\ 0 \leq |\delta_T + \delta_F| \leq 1)$ with omission of indeterminacy, then complex neutrosophic hypersoft set reduces to complex intuitionistic fuzzy hypersoft set.

- If $\zeta(\underline{\lambda}) = \left\langle \zeta^T(\underline{\lambda}), \zeta^I(\underline{\lambda}), \zeta^F(\underline{\lambda}) \right\rangle$ is replaced by $\zeta(\underline{\lambda}) = \left\langle \zeta^T(\underline{\lambda}) \right\rangle$ with omission of indeterminacy and falsity, then complex neutrosophic hypersoft set reduces to complex fuzzy hypersoft set.

This concludes that complex fuzzy hypersoft set and complex intuitionistic fuzzy hypersoft set are the particular cases of complex neutrosophic hypersoft set. Since Complex fuzzy hypersoft sets and complex intuitionistic fuzzy hypersoft sets cannot handle imprecise, indeterminate, inconsistent, and incomplete information of periodic nature so to overcome this hurdle, complex neutrosophic hypersoft set is conceptualized.

Conclusion

In this work, new hybrids of hypersoft set i.e. complex fuzzy hypersoft set, complex intuitionistic fuzzy hypersoft set and complex neutrosophic hypersoft set, are conceptualized with their some fundamentals and theoretic operations. Future study may include other hybrids of hypersoft set with interval-valued complex fuzzy set etc., similarity and distance measures, aggregations operators and applications in multi-criteria decision making problems.

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Convex and Concave Hypersoft Sets with Some Properties

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Abstract. Convexity plays an imperative role in optimization, pattern classification and recognition, image processing and many other relating topics in different fields of mathematical sciences like operation research, numerical analysis etc. The concept of soft sets was first formulated by Molodtsov as a completely new mathematical tool for solving problems dealing with uncertainties. Smarandache conceptualized hypersoft set as a generalization of soft set (h_S, E) as it transforms the function h_S into a multi-attribute function h_{HS}. Deli introduced the concept of convexity cum concavity on soft sets to cover above topics under uncertain scenario. In this study, a theoretic and analytical approach is employed to develop a conceptual framework of convexity cum concavity on hypersoft set which is generalized and more effective concept to deal with optimization relating problems. Moreover, some generalized properties like δ-inclusion, intersection and union, are established. The novelty of this work is maintained with the help of illustrative examples and pictorial version first time in literature.

Keywords: Convex Soft Set; Concave Soft Set; hypersoft Set; convex hypersoft set; concave hypersoft set.

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1. **Introduction**

The theories like theory of probability, theory of fuzzy sets, and the interval mathematics, are considered as mathematical means to tackle many Intricate problems involving various uncertainties in different fields of mathematical sciences. These theories have their own complexities which restrain them to solve these problems successfully. The reason for these hurdles is, possibly, the inadequacy of the parametrization tool. A mathematical tool is needed for dealing with uncertainties which should be free of all such Impediments. In 1999, Molodtsov [1] has the honor to introduce the such mathematical tool called soft sets in literature as a new parameterized family of subsets of the universe of discourse. In 2003, Maji et al. [2] extended the concept and introduced some fundamental terminologies and operations like equality of two soft sets, subset and super set of a soft set, complement of a soft set, null soft set, absolute soft set, AND, OR and also the operations of union and intersection. They verified De Morgan's laws and a number of other results too. In 2005, Pei et al. [3] discussed the relationship between soft sets and information systems. They showed the soft sets as a class of special information systems. In 2009, Ali et al. [4] pointed several assertions in previous work of Maji et al. and defined new notions such as the restricted intersection, the restricted union, the restricted difference and the extended intersection of two soft sets. In 2010, 2011, Babitha et al. [5,6] introduced the concepts of soft set relations as a sub soft set of the Cartesian product of the soft sets and also discussed many related concepts such as equivalent soft set relation, partition, composition and function. In 2011, Sezgin et al. [7], Ge et al. [8], Fuli [9] gave some modifications in the work of Maji et al. and also established some new results. Many researchers [10]- [19] developed certain hybrids with soft sets to get more generalized results for implementation in decision making and other related disciplines.

In 2013, Deli [20] defined soft covex and soft concave sets with some properties. In 2016, Majeed [21] investigated some more properties of convex soft sets. She developed the convex hull and the cone of a soft set with their generalized results. In 2018, Salih et al. [22] defined strictly soft convex and strictly soft concave sets and they discussed their properties.

In 2018, Smarandache [23] introduced the concept of hypersoft set and in 2020, M. Saeed et al. [24] extended the concept and discussed the fundamentals of hypersoft set such as hypersoft subset, complement, not hypersoft set, aggregation operators along with hypersoft set relation, sub relation,complement relation, function, matrices and operations on hypersoft matrices.

Convexity is an essential concept in optimization, recognition and classification of certain patterns, processing and decomposition of images, antismatroids, discrete event simulation, duality problems and many other related topics in operation research, mathematical economics, numerical analysis and other mathematical sciences. Deli provided a mathematical tool to tackle all such problems under soft set environment. Hypersoft set theory is more generalized than soft set theory so it's the need of the literature to carve out a conceptual framework for solving such kind of problems under more generalized version i.e. hypersoft set. Therefore, to meet this demand, an abstract and analytical approach is utilized to develop a basic framework of convexity and concavity on hypersoft sets along with some important results. Examples and pictorial version of convexity and concavity on hypersoft sets are presented first time in literature.

The rest of this article is structured as follows: Section 2 recalls some basic definitions and terms from literature to support main results. Section 3 discusses the main results i.e. convex and concave hypersoft sets along with some generalized results. Section 4 concludes the paper and describes future directions. Throughout the paper, G , $J^{\bullet}$, II and $P(\text{II})$, will play the role of R^n, unit interval, universal set and power set respectively.

2. Preliminaries

In this section, some fundamental terms regarding soft set, hypersoft set and their convexity-cum-concavity are presented.

Definition 2.1. [1](**Soft Set**)
Let II be an initial universe set and let E be a set of parameters. A pair (h_S, E) is called a soft set over II, where h_S is a mapping given by $h_S : E \to P(U)$. In other words, a soft set (h_S, E) over II is a parameterized family of subsets of II. For $\omega \in E, h_S(\omega)$ may be considered as the set of ω-elements or ω-approximate elements of the soft set (h_S, E).

Definition 2.2. [2]
Let (f_S, A) and (g_S, B) be two soft sets over a common universe II,

(1) we say that (f_S, A) is a **soft subset** of (g_S, B) denoted by $(f_S, A) \subseteq (g_S, B)$ if
   i $A \subseteq B$, and
   ii $\forall \omega \in A, f_S(\omega)$ and $g_S(\omega)$ are identical approximations.
(2) the **union** of (f_S, A) and (g_S, B), denoted by $(f_S, A) \cup (g_S, B)$, is a soft set (h_S, C), where $C = A \cup B$ and $\omega \in C$,

$$h_S(\omega) = \begin{cases} f_S(\omega), & \omega \in A - B \\ g_S(\omega), & \omega \in B - A \\ f_S(\omega) \cup g_S(\omega), & \omega \in A \cap B \end{cases}$$

(3) the **intersection** of (f_S, A) and (g_S, B) denoted by $(f_S, A) \cap (g_S, B)$, is a soft set (h_S, C), where $C = A \cap B$ and $\omega \in C, h_S(\omega) = f_S(\omega)$ or $g_S(\omega)$ (as both are same set).

Definition 2.3. [2](**Complement of Soft Set**)
The complement of a soft set (h_S, A), denoted by $(h_S, A)^c$, is defined as $(h_S, A)^c = ({h_S}^c, \neg A)$ where

$${h_S}^c : \neg A \to P(\text{II})$$

is a mapping given by

$${h_S}^c(\omega) = \text{II} - h_S(\neg\omega) \forall \omega \in \neg A.$$

Definition 2.4. [23](**Hypersoft Set**)
Let II be a universe of discourse, $P(\text{II})$ the power set of II. Let $a_1, a_2, a_3, ..., a_n$, for $n \geq 1$, be n distinct attributes, whose corresponding attribute values are respectively the sets $A_1, A_2, A_3, ..., A_n$, with $A_i \cap A_j = \emptyset$, for $i \neq j$, and $i, j \in \{1, 2, 3, ..., n\}$. Then the pair

(h_{HS}, G), where $G = A_1 \times A_2 \times A_3 \times ..... \times A_n$ and $h_{HS} : G \to P(\text{II})$ is called a hypersoft Set over II.

Definition 2.5. [24](**Union of Hypersoft Sets**)
Let (Φ, G_1) and (Ψ, G_2) be two hypersoft sets over the same universal set II, then their union $(\Phi, G_1) \cup (\Psi, G_2)$ is hypersoft set (h_{HS}, C), where $C = G_1 \cup G_2$; $G_1 = A_1 \times A_2 \times A_3 \times ..... \times A_n$, $G_2 = B_1 \times B_2 \times B_3 \times ..... \times B_n$ and $\forall\, e \in C$ with

$$h_{HS}(e) = \begin{cases} \Phi(e)\ ,\ e \in G_1 - G_2 \\ \Psi(e)\,,\ e \in G_2 - G_1 \\ \Phi(e) \cup \Psi(e)\,,\ e \in G_2 \cap G_1 \end{cases}$$

Definition 2.6. [24](**Intersection of Hypersoft Sets**)
Let (Φ, G_1) and (Ψ, G_2) be two hypersoft sets over the same universal set II, then their intersection $(\Phi, G_1) \cap (\Psi, G_2)$ is hypersoft set (h_{HS}, C), where $C = G_1 \cap G_2$; where $G_1 = A_1 \times A_2 \times A_3 \times ..... \times A_n$, $G_2 = B_1 \times B_2 \times B_3 \times ..... \times B_n$. and $\forall\, e \in C$ with $h_{HS}(e) = \Phi(e) \cap \Psi(e)$.

For more definition and results regarding hypersoft set, see [24–27].

Definition 2.7. [20](δ**-inclusion**)
The δ-inclusion of a soft set (h_S, Λ) (where $\delta \subseteq \text{II}$) is defined by

$$(h_S, \Lambda)^{\delta} = \{\omega \in \Lambda : h_S(\omega) \supseteq \delta\}$$

Definition 2.8. [20](**Convex Soft Set**)
The soft set (h_S, Λ) on Λ is called a convex soft set if

$$h_S(\epsilon\omega + (1 - \epsilon)\mu) \supseteq h_S(\omega) \cap h_S(\mu)$$

for every $\omega, \mu \in \Lambda$ and $\epsilon \in J^{\bullet}$.

Definition 2.9. [20](**Concave Soft Set**)
The soft set (h_S, Λ) on Λ is called a concave soft set if

$$h_S(\epsilon\omega + (1 - \epsilon)\mu) \subseteq h_S(\omega) \cup h_S(\mu)$$

for every $\omega, \mu \in \Lambda$ and $\epsilon \in J^{\bullet}$.

For more about convex soft, see [20, 21].

3. **Convex and Concave hypersoft sets**

Here convex hypersoft sets and concave hypersoft sets are defined and some important results are proved.

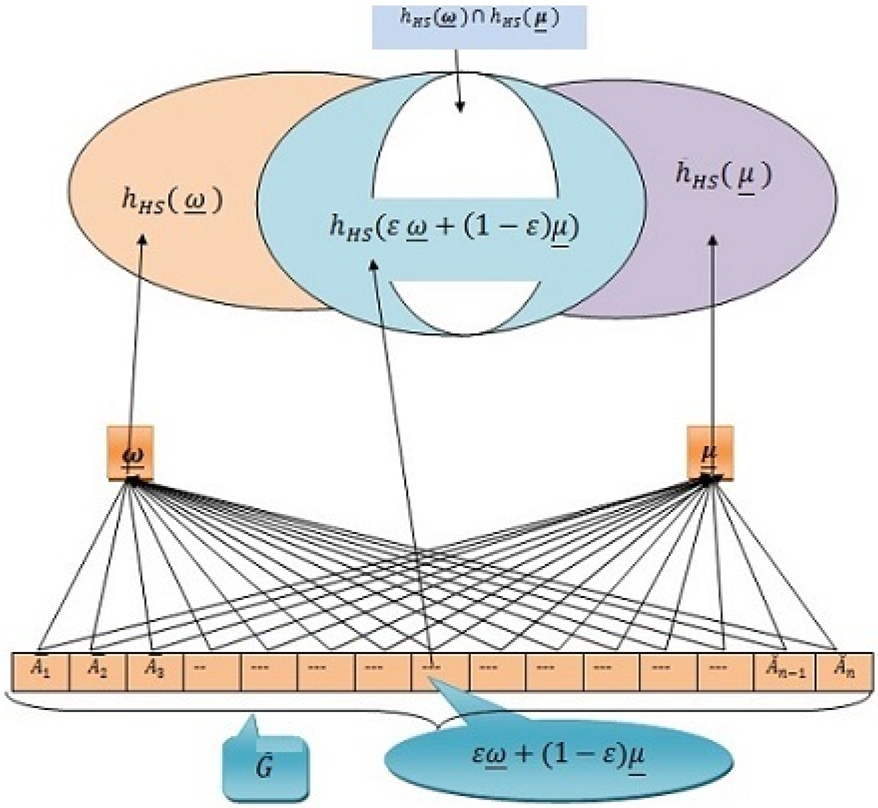

FIGURE 1. Convex hypersoft Set

Definition 3.1. δ-inclusion for hypersoft Set
The $\delta - inclusion$ of a hypersoft set (h_{HS}, G) (where $\delta \subseteq \text{II}$) is defined by

$$(h_{HS}, G)^{\delta} = \{\underline{\omega} \in G : h_{HS}(\underline{\omega}) \supseteq \delta\}$$

Definition 3.2. Convex hypersoft Set
The hypersoft set (h_{HS}, G) is called a convex hypersoft set if

$$h_{HS}\left(\epsilon\underline{\omega} + (1-\epsilon)\,\underline{\mu}\right) \supseteq h_{HS}(\underline{\omega}) \cap h_{HS}\left(\underline{\mu}\right)$$

for every $\underline{\omega}$, $\underline{\mu} \in G$ where, $G = A_1 \times A_2 \times A_3 \times ..... \times A_n$ with $A_i \cap A_j = \emptyset$, for $i \neq j$, and $i, j \in \{1, 2, 3, ..., n\}$; $h_{HS} : G \to P(\text{II})$ and $\epsilon \in J^{\bullet}$.

Example 3.3. Suppose a university wants to observe(evaluate) the characteristics of its teach ers by some defined indicators. For this purpose, consider a set of teachers as a universe of discourse $\text{II} = \{t_1, t_2, t_3, ..., t_{10}\}$. The attributes of the teachers under consideration are the set $\Lambda = \{A_1, A_2, A_3\}$, where
A_1 = Total experience in years
A_2 = Total no. of publications

$A_3 =$ Student's evaluation against each teacher
such that the attributes values against these attributes respectively are the sets given as
$A_1 = \{1year, 2years, 3years, 4years, 5years\}$
$A_2 = \{1, 2, 3, 4, 5\}$
$A_3 = \{Excellent(1), verygood(2), good(3), average(4), bad(5)\}$
For simplicity, we write
$A_1 = \{1, 2, 3, 4, 5\}$
$A_2 = \{1, 2, 3, 4, 5\}$
$A_3 = \{1, 2, 3, 4, 5\}$
The hypersoft set (h_{HS}, G) is a function defined by the mapping $h_{HS} : G \to P(\text{II})$ where $G = A_1 \times A_2 \times A_3$.
Since the cartesian product of $A_1 \times A_2 \times A_3$ is a 3-tuple. we consider $\underline{\omega} = (2, 1, 3)$, then the function becomes $h_{HS}(\underline{\omega}) = h_{HS}(2, 1, 3) = \{t_1, t_5\}$. Also, consider $\underline{\mu} = (3, 2, 2)$, then the function becomes $h_{HS}(\underline{\mu}) = h_{HS}(3, 2, 2) = \{t_1, t_3, t_4\}$
Now

$$h_{HS}(\underline{\omega}) \cap h_{HS}(\underline{\mu}) = h_{HS}(\{2, 1, 3\}) \cap h_{HS}(\{3, 2, 2\}) = \{t_1, t_5\} \cap \{t_1, t_3, t_4\} = \{t_1\} \tag{1}$$

Let $\epsilon = 0.6 \in J^{\bullet}$, then, we have
$\epsilon\underline{\omega} + (1 - \epsilon)\underline{\mu} = 0.6(2, 1, 3) + (10.6)(3, 2, 2) = 0.6(2, 1, 3) + 0.4(3, 2, 2)$
$= (1.2, 0.6, 1.8) + (1.2, 0.8, 0.8) = (1.2 + 1.2, 0.6 + 0.8, 1.8 + 0.8) = (2.4, 1.4, 2.6)$
which is again a 3-tuple. By using the decimal round off property, we get $(2, 1, 3)$

$$h_{HS}(\epsilon\underline{\omega} + (1 - \epsilon)\underline{\mu}) = h_{HS}(2, 1, 3) = \{t_1, t_5\} \tag{2}$$

it is vivid from equations (1) and (2), we have

$$h_{HS}\left(\epsilon\underline{\omega} + (1 - \epsilon)\,\underline{\mu}\right) \supseteq h_{HS}\left(\underline{\omega}\right) \cap h_{HS}\left(\underline{\mu}\right)$$

Theorem 3.4. *$(f_{HS}, S) \cap (g_{HS}, T)$ is a convex hypersoft set when both (f_{HS}, S) and (g_{HS}, T) are convex hypersoft sets.*

Proof. Suppose that $(f_{HS}, S) \cap (g_{HS}, T) = (h_{HS}, G)$ with $G = S \cap T$, for $\underline{\omega}_1$, $\underline{\omega}_2 \in G; \epsilon \in J^{\bullet}$, we have then

$$h_{HS}\left(\epsilon\underline{\omega}_1 + (1 - \epsilon)\underline{\omega}_2\right) = f_{HS}\left(\epsilon\underline{\omega}_1 + (1 - \epsilon)\underline{\omega}_2\right) \cap g_{HS}\left(\epsilon\underline{\omega}_1 + (1 - \epsilon)\underline{\omega}_2\right)$$

As (f_{HS}, S) and (g_{HS}, T) are convex hypersoft sets,

$$f_{HS}\left(\epsilon\underline{\omega}_1 + (1 - \epsilon)\underline{\omega}_2\right) \supseteq f_{HS}\left(\underline{\omega}_1\right) \cap f_{HS}\left(\underline{\omega}_2\right)$$
$$g_{HS}\left(\epsilon\underline{\omega}_1 + (1 - \epsilon)\underline{\omega}_2\right) \supseteq g_{HS}\left(\underline{\omega}_1\right) \cap g_{HS}\left(\underline{\omega}_2\right)$$

which implies

$$h_{HS}\left(\epsilon\underline{\omega}_1+(1-\epsilon)\underline{\omega}_2\right)\supseteq\ \left(f_{HS}\left(\underline{\omega}_1\right)\cap\ f_{HS}\left(\underline{\omega}_2\right)\right)\cap\ \left(g_{HS}\left(\underline{\omega}_1\right)\cap\ g_{HS}\left(\underline{\omega}_2\right)\right)$$

and thus

$$h_{HS}\left(\epsilon\underline{\omega}_1+(1-\epsilon)\underline{\omega}_2\right)\supseteq\ h_{HS}\left(\underline{\omega}_1\right)\cap\ h_{HS}\left(\underline{\omega}_2\right)$$

□

Remark 3.5. If $\left\{(h^i{}_{HS},G_i):i\in\{1,2,3,...\}\right\}$ is any family of convex hypersoft sets, then the intersection $\bigcap_{i\in I}(h^i{}_{HS},G_i)$ is a convex hypersoft set.

Remark 3.6. The union of any family $\left\{(h^i{}_{HS},G_i):i\in\{1,2,3,...\}\right\}$ of convex hypersoft sets is not necessarily a convex hypersoft set.

Theorem 3.7. *(h_{HS},G) is convex hypersoft set iff for every $\epsilon\in J^{\bullet}$ and $\delta\in\ P(\mathrm{II}),(h_{HS},G)^{\delta}$ is convex hypersoft set.*

Proof. Suppose (h_{HS},G) is convex hypersoft set. If $\underline{\omega}$, $\underline{\mu}\in\ G$ and $\delta\in\ P(\mathrm{II})$, then $h_{HS}\left(\underline{\omega}\right)\supseteq\delta$ and $h_{HS}\left(\underline{\mu}\right)\supseteq\delta$, it implies that $h_{HS}\left(\underline{\omega}\right)\cap h_{HS}\left(\underline{\mu}\right)\supseteq\delta$.
So we have,

$$h_{HS}\left(\epsilon\underline{\omega}+(1-\epsilon)\,\underline{\mu}\right)\supseteq h_{HS}\left(\underline{\omega}\right)\cap h_{HS}\left(\underline{\mu}\right)\supseteq\delta$$

$\Rightarrow\quad h_{HS}\left(\epsilon\underline{\omega}+(1-\epsilon)\,\underline{\mu}\right)\supseteq\delta$
thus $(h_{HS},G)^{\delta}$ is convex hypersoft set.
Conversely suppose that $(h_{HS},G)^{\delta}$ is convex hypersoft set for every $\epsilon\in J^{\bullet}$. For $\underline{\omega}$, $\underline{\mu}\in\ G$, $(h_{HS},G)^{\delta}$ is convex hypersoft set with $\delta\ =\ h_{HS}\left(\underline{\omega}\right)\cap\ h_{HS}\left(\underline{\mu}\right)$. Since $h_{HS}\left(\underline{\omega}\right)\ \supseteq\ \delta$ and $h_{HS}\left(\underline{\mu}\right)\supseteq\delta$, we have $\underline{\omega}\in\ (h_{HS},G)^{\delta}$ and $\underline{\mu}\in\ (h_{HS},G)^{\delta}$,
$\Rightarrow\quad \epsilon\underline{\omega}\ +(1-\epsilon)\,\underline{\mu}\in\ (h_{HS},G)^{\delta}$.
Therefore,

$$h_{HS}\left(\epsilon\underline{\omega}+(1-\epsilon)\,\underline{\mu}\right)\supseteq\ \ \delta$$

So

$$h_{HS}\left(\epsilon\underline{\omega}+(1-\epsilon)\,\underline{\mu}\right)\supseteq\ \ h_{HS}\left(\underline{\omega}\right)\cap h_{HS}\left(\underline{\mu}\right),$$

Hence (h_{HS},G) is convex hypersoft set. □

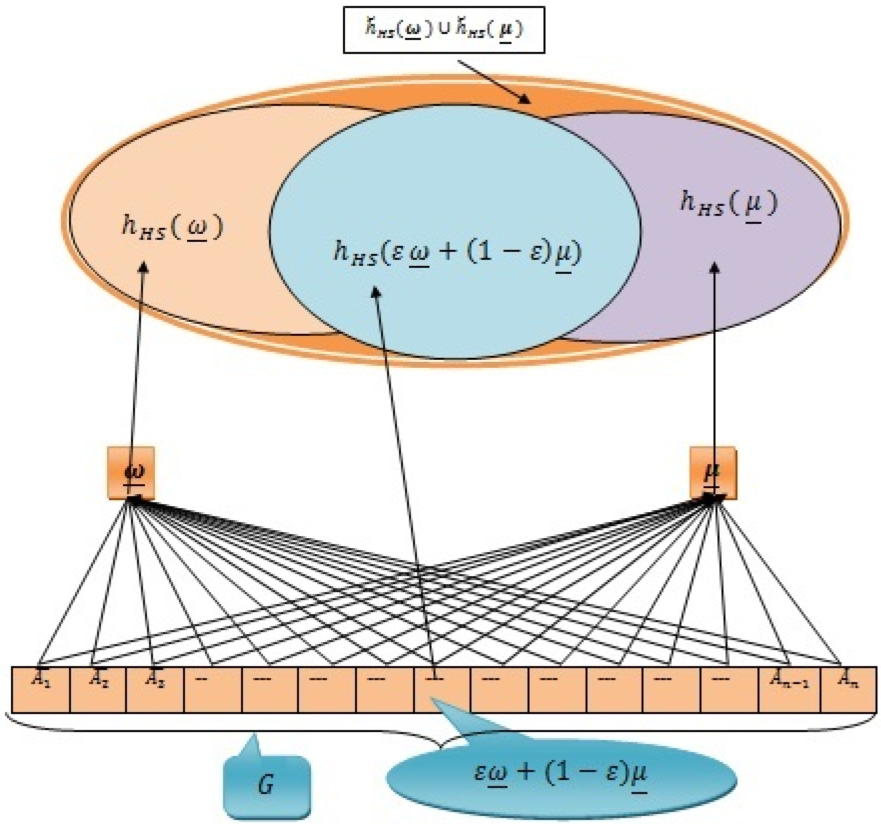

FIGURE 2. Concave hypersoft Set

Definition 3.8. Concave hypersoft Set
The hypersoft set (h_{HS}, G) on Λ is called a *concave hypersoft set* if

$$h_{HS}\left(\epsilon\underline{\omega} + (1-\epsilon)\underline{\mu}\right) \subseteq h_{HS}\left(\underline{\omega}\right) \cup h_{HS}\left(\underline{\mu}\right)$$

for every $\underline{\omega} = (A_1, A_2, A_3, ..., A_n)$, $\underline{\mu} = (B_1, B_2, B_3, ..., B_n) \in G$ where, $G = A_1 \times A_2 \times A_3 \times ..... \times A_n$ with $A_i \cap A_j = \emptyset$, for $i \neq j$, and $i, j \in \{1, 2, 3, ..., n\}$; $h_{HS} : G \to P(\text{II})$ and $\epsilon \in J^{\bullet}$.

Example 3.9. Considering given data in Example 3.3, we have

$$h_{HS}(\underline{\omega}) \cup h_{HS}(\underline{\mu}) = h_{HS}(\{2,1,3\}) \cup h_{HS}(\{3,2,2\}) = \{t_1, t_5\} \cup \{t_1, t_3, t_4\} = \{t_1, t_3, t_4, t_5\} \quad (3)$$

it is vivid from equations (2) and (3), we have

$$h_{HS}\left(\epsilon\underline{\omega} + (1-\epsilon)\underline{\mu}\right) \subseteq h_{HS}\left(\underline{\omega}\right) \cup h_{HS}\left(\underline{\mu}\right)$$

Theorem 3.10. *$(f_{HS}, S) \cup (g_{HS}, T)$ is a concave hypersoft set when both (f_{HS}, S) and (g_{HS}, T) are concave hypersoft sets.*

Proof. Suppose that $(f_{HS}, S) \cup (g_{HS}, T) = (h_{HS}, G)$ with $G = S \cup T$, for $\underline{\omega}_1,\ \underline{\omega}_2 \in\ G; \epsilon \in J^{\bullet}$, we have then

$$h_{HS}\left(\epsilon\underline{\omega}_1 + (1-\epsilon)\underline{\omega}_2\right) =\ f_{HS}\left(\epsilon\underline{\omega}_1 + (1-\epsilon)\underline{\omega}_2\right) \cup\ g_{HS}\left(\epsilon\underline{\omega}_1 + (1-\epsilon)\underline{\omega}_2\right)$$

As (f_{HS}, S) and (g_{HS}, T) are concave hypersoft sets,

$$f_{HS}\left(\epsilon\underline{\omega}_1 + (1-\epsilon)\underline{\omega}_2\right) \subseteq\ f_{HS}\left(\underline{\omega}_1\right) \cup\ f_{HS}\left(\underline{\omega}_2\right)$$

$$g_{HS}\left(\epsilon\underline{\omega}_1 + (1-\epsilon)\underline{\omega}_2\right) \subseteq\ g_{HS}\left(\underline{\omega}_1\right) \cup\ g_{HS}\left(\underline{\omega}_2\right)$$

which implies

$$h_{HS}\left(\epsilon\underline{\omega}_1 + (1-\epsilon)\underline{\omega}_2\right) \subseteq\ \left(f_{HS}\left(\underline{\omega}_1\right) \cup\ f_{HS}\left(\underline{\omega}_2\right)\right) \cup\ \left(g_{HS}\left(\underline{\omega}_1\right) \cup\ g_{HS}\left(\underline{\omega}_2\right)\right)$$

and thus

$$h_{HS}\left(\epsilon\underline{\omega}_1 + (1-\epsilon)\underline{\omega}_2\right) \subseteq\ h_{HS}\left(\underline{\omega}_1\right) \cup\ h_{HS}\left(\underline{\omega}_2\right)$$

□

Remark 3.11. If $\left\{(\breve{h}^i{}_{HS}, G_i) : i \in \{1,2,3,...\}\right\}$ is any family of concave hypersoft sets, then the union $\bigcup_{i\in I}(\breve{h}^i{}_{HS}, G_i)$ is a concave hypersoft set.

Theorem 3.12. *$(f_{HS}, S) \cap (g_{HS}, T)$ is a concave hypersoft set when both (f_{HS}, S) and (g_{HS}, T) are concave hypersoft sets.*

Proof. Suppose that $(f_{HS}, S) \cap (g_{HS}, T) = (h_{HS}, G)$ with $G = S \cap T$, for $\underline{\omega}_1,\ \underline{\omega}_2 \in\ G; \epsilon \in J^{\bullet}$, we have then

$$h_{HS}\left(\epsilon\underline{\omega}_1 + (1-\epsilon)\underline{\omega}_2\right) =\ f_{HS}\left(\epsilon\underline{\omega}_1 + (1-\epsilon)\underline{\omega}_2\right) \cap\ g_{HS}\left(\epsilon\underline{\omega}_1 + (1-\epsilon)\underline{\omega}_2\right)$$

As (f_{HS}, S) and (g_{HS}, T) are concave hypersoft sets,

$$f_{HS}\left(\epsilon\underline{\omega}_1 + (1-\epsilon)\underline{\omega}_2\right) \subseteq\ f_{HS}\left(\underline{\omega}_1\right) \cup\ f_{HS}\left(\underline{\omega}_2\right)$$

$$g_{HS}\left(\epsilon\underline{\omega}_1 + (1-\epsilon)\underline{\omega}_2\right) \subseteq\ g_{HS}\left(\underline{\omega}_1\right) \cup\ g_{HS}\left(\underline{\omega}_2\right)$$

which implies

$$h_{HS}\left(\epsilon\underline{\omega}_1 + (1-\epsilon)\underline{\omega}_2\right) \subseteq\ \left(f_{HS}\left(\underline{\omega}_1\right) \cup\ f_{HS}\left(\underline{\omega}_2\right)\right) \cap\ \left(g_{HS}\left(\underline{\omega}_1\right) \cup\ g_{HS}\left(\underline{\omega}_2\right)\right)$$

and thus

$$h_{HS}\left(\epsilon\underline{\omega}_1 + (1-\epsilon)\underline{\omega}_2\right) \subseteq\ h_{HS}\left(\underline{\omega}_1\right) \cup\ h_{HS}\left(\underline{\omega}_2\right)$$

□

Remark 3.13. The intersection of any family $\left\{(\breve{h}^i{}_{HS}, G_i) : i \in \{1,2,3,...\}\right\}$ of concave hypersoft sets is a concave hypersoft set.

Theorem 3.14. *$(h_{HS}, G)^c$ is a convex hypersoft set when (h_{HS}, G) is a concave hypersoft set.*

Proof. Suppose that for $\underline{\omega}_1, \underline{\omega}_2 \in G,\ \epsilon \in J^\bullet$ and (h_{HS}, G) be concave hypersoft set.
Since (h_{HS}, G) is concave hypersoft set,

$$h_{HS}\left(\epsilon\underline{\omega}_1 + (1-\epsilon)\underline{\omega}_2\right) \subseteq h_{HS}\left(\underline{\omega}_1\right) \cup h_{HS}\left(\underline{\omega}_2\right)$$

or

$$\text{II}\backslash\, h_{HS}\left(\epsilon\underline{\omega}_1 + (1-\epsilon)\underline{\omega}_2\right) \supseteq \text{II}\backslash\left\{h_{HS}\left(\underline{\omega}_1\right) \cup h_{HS}\left(\underline{\omega}_2\right)\right\}$$

If $h_{HS}\left(\underline{\omega}_1\right) \supset h_{HS}\left(\underline{\omega}_2\right)$ then $h_{HS}\left(\underline{\omega}_1\right) \cup h_{HS}\left(\underline{\omega}_2\right) = h_{HS}\left(\underline{\omega}_1\right)$ therefore,

$$\text{II}\backslash\, h_{HS}\left(\epsilon\underline{\omega}_1 + (1-\epsilon)\underline{\omega}_2\right) \supseteq \text{II}\backslash\ h_{HS}\left(\underline{\omega}_1\right). \tag{4}$$

If $h_{HS}\left(\underline{\omega}_1\right) \subset h_{HS}\left(\underline{\omega}_2\right)$ then $h_{HS}\left(\underline{\omega}_1\right) \cup h_{HS}\left(\underline{\omega}_2\right) = h_{HS}\left(\underline{\omega}_2\right)$ therefore,

$$\text{II}\backslash\, h_{HS}\left(\epsilon\underline{\omega}_1 + (1-\epsilon)\underline{\omega}_2\right) \supseteq \text{II}\backslash\ h_{HS}\left(\underline{\omega}_2\right). \tag{5}$$

From (4) and (5) , we have

$$\text{II}\backslash\, h_{HS}\left(\epsilon\underline{\omega}_1 + (1-\epsilon)\underline{\omega}_2\right) \supseteq \left(\text{II}\backslash h_{HS}\left(\underline{\omega}_1\right)\right) \cap \left(\text{II}\backslash h_M\left(\underline{\omega}_2\right)\right).$$

So, $(h_{HS}, G)^c$ is a convex hypersoft set. □

Theorem 3.15. *$(h_{HS}, G)^c$ is a concave hypersoft set when (h_{HS}, G) is a convex hypersoft set.*

Proof. Suppose that for $\underline{\omega}_1, \underline{\omega}_2 \in G,\ \epsilon \in J^\bullet$ and (h_{HS}, G) be convex hypersoft set.
since (h_{HS}, G) is convex hypersoft set,

$$h_{HS}\left(\epsilon\underline{\omega}_1 + (1-\epsilon)\underline{\omega}_2\right) \supseteq h_{HS}\left(\underline{\omega}_1\right) \cap h_{HS}\left(\underline{\omega}_2\right)$$

or

$$\text{II}\backslash\, h_{HS}\left(\epsilon\underline{\omega}_1 + (1-\epsilon)\underline{\omega}_2\right) \subseteq \text{II}\backslash\left\{h_{HS}\left(\underline{\omega}_1\right) \cap h_{HS}\left(\underline{\omega}_2\right)\right\}$$

If $h_{HS}\left(\underline{\omega}_1\right) \supset h_{HS}\left(\underline{\omega}_2\right)$ then $h_{HS}\left(\underline{\omega}_1\right) \cap h_{HS}\left(\underline{\omega}_2\right) = h_{HS}\left(\underline{\omega}_2\right)$ therefore,

$$\text{II}\backslash\, h_{HS}\left(\epsilon\underline{\omega}_1 + (1-\epsilon)\underline{\omega}_2\right) \subseteq \text{II}\backslash\ h_{HS}\left(\underline{\omega}_2\right). \tag{6}$$

If $h_{HS}\left(\underline{\omega}_1\right) \subset h_{HS}\left(\underline{\omega}_2\right)$ then $h_{HS}\left(\underline{\omega}_1\right) \cap h_{HS}\left(\underline{\omega}_2\right) = h_{HS}\left(\underline{\omega}_1\right)$ therefore,

$$\text{II}\backslash\, h_{HS}\left(\epsilon\underline{\omega}_1 + (1-\epsilon)\underline{\omega}_2\right) \subseteq \text{II}\backslash\ h_{HS}\left(\underline{\omega}_1\right). \tag{7}$$

From (6) and (7) , we have

$$\text{II}\backslash\, h_{HS}\left(\epsilon\underline{\omega}_1 + (1-\epsilon)\underline{\omega}_2\right) \supseteq \left(\text{II}\backslash h_{HS}\left(\underline{\omega}_1\right)\right) \cup \left(\text{II}\backslash h_M\left(\underline{\omega}_2\right)\right).$$

So $(h_{HS}, G)^c$ is a concave hypersoft set. □

Theorem 3.16. *(h_{HS}, G) is concave hypersoft set iff for every $\epsilon \in J^\bullet$ and $\delta \in\ P(\text{II}), (h_{HS}, G)^\delta$ is concave hypersoft set.*

Proof. Suppose (h_{HS}, G) is concave hypersoft set. If $\underline{\omega}$, $\underline{\mu} \in G$ and $\delta \in P(\text{II})$, then $h_{HS}(\underline{\omega}) \supseteq \delta$ and $h_{HS}(\underline{\mu}) \supseteq \delta$, it implies that $h_{HS}(\underline{\omega}) \cup h_{HS}(\underline{\mu}) \supseteq \delta$.
So we have,

$$\delta \subseteq h_{HS}(\underline{\omega}) \cap h_{HS}(\underline{\mu}) \subseteq h_{HS}\left(\epsilon\underline{\omega} + (1-\epsilon)\underline{\mu}\right) \subseteq h_{HS}(\underline{\omega}) \cup h_{HS}(\underline{\mu})$$

$\Rightarrow \quad \delta \subseteq h_{HS}\left(\epsilon\underline{\omega} + (1-\epsilon)\underline{\mu}\right)$
thus $(h_{HS}, G)^{\delta}$ is concave hypersoft set.
Conversely suppose that $(h_{HS}, G)^{\delta}$ is concave hypersoft set for every $\epsilon \in J^{\bullet}$. For $\underline{\omega}$, $\underline{\mu} \in G$, $(h_{HS}, G)^{\delta}$ is concave hypersoft set with $\delta = h_{HS}(\underline{\omega}) \cup h_{HS}(\underline{\mu})$. Since $h_{HS}(\underline{\omega}) \subseteq \delta$ and $h_{HS}(\underline{\mu}) \subseteq \delta$, we have $\underline{\omega} \in (h_{HS}, G)^{\delta}$ and $\underline{\mu} \in (h_{HS}, G)^{\delta}$,
$\Rightarrow \quad \epsilon\underline{\omega} + (1-\epsilon)\underline{\mu} \in (h_{HS}, G)^{\delta}$.
Therefore,

$$h_{HS}\left(\epsilon\underline{\omega} + (1-\epsilon)\underline{\mu}\right) \subseteq \delta$$

So

$$h_{HS}\left(\epsilon\underline{\omega} + (1-\epsilon)\underline{\mu}\right) \subseteq h_{HS}(\underline{\omega}) \cup h_{HS}(\underline{\mu}),$$

Hence (h_{HS}, G) is concave hypersoft set. □

4. **Conclusion**

In this study, convexity cum concavity on hypersoft sets, is conceptualized by adopting an abstract and analytical technique. This is novel addition in the literature and may enable the researchers to deal important applications of convexity under hypersoft environment with precise results. Moreover, some important results are established. Future work may include the introduction of strictly and strongly conexity cum concavity, convex hull, convex cone and many other types of convexity like (m,n)-convexity, ϕ-convexity, graded convexity, triangular convexity, concavoconvexity etc. on hypersoft set. It may also include the extension of this work by considering the modified versions of complement, intersection and union as discussed in [2]- [4].

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Intuitionistic Fuzzy Hypersoft Sets

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Abstract. In this paper, a new environment namely, intuitionistic fuzzy hypersoft set (IFHSS) is defined. We introduce some fundamental operators of intuitionistic fuzzy hypersoft sets such as subset, null set, absolute set, complement, union, intersection, equal set etc. Validity and application are presented with appropriate examples. For greater precision and accuracy, in the future, proposed operations in decision making processes such as personal selection, management issues and others will play a vital role.

1. Introduction

The fuzzy set theory identified in 1965 by Zadeh [26] is one of the most popular theories of recent times. Zadeh specified that there is a considerable amount of ambiguity in most real-life situations and physical problems that the classical set theory and its normal mathematical theories centered on such set theory did not give us the necessary knowledge and inferences. Fuzzy set theory has brought a great paradigmatic change in mathematics, but this theory also has some structural difficulties in its nature. Fuzzy set structure is defined with the help of membership function. It is difficult to create a membership function for each event, according to Molodtsov, because creating a membership function is too individual.

Molodtsov [15] introduced the soft set theory in 1999 which he felt was more practical. This theory is a relatively new approach to solving problems involving decision making and uncertainty. The major benefit of soft set theory is that in fuzzy set and other theories it is free from difficulties. Soft set theory has become popular among researchers in a short time and many scientific studies are carried out on this theory every year [8, 16, 17, 27]. Maji et al. [12, 14] proposed an implementation of soft sets focused on parameter reduction to preserve optimum selection artifacts in decision-making problems. Chen [5] investigated a new definition and various applications of the reduction of soft sets to parameters. Pei and Miao [19] have shown that soft sets are a special class of information systems. Kong et al. [9] introduced the reduction and algorithm of soft sets to normal parameters. Zou and Xiao [28] discussed the soft data analysis approach. Aktaş and Çağman [2] defined the algebraic structure of soft set theory.

The intuitionistic fuzzy set (IFS) theory [3, 4] was created by adding a non-membership function to the fuzzy set structure. The non-member function makes IFS more functional in decision-making problems. Maji et al. [10, 13] combined soft set theory with the theory of the intuitionistic fuzzy sets and called intuitionistic fuzzy soft Set (IFSS). The parameterization and hesitancy acquired by IFSS from this mixture facilitates very accurately the description of real-world situations. IFSS is a valuable method for addressing data uncertainty and vagueness. Many scientific paper have shown the suitability of IFSS to issue decision making [7, 11].

Smarandache [21] introduced a new technique to deal with uncertainty. By converting the function into multiple decision functions, he generalized the soft to hypersoft set. Many studies have been done recently using hypersoft set structure [1, 6, 18, 20, 22, 23, 24, 25].

Multi-criteria decision-making (MCDM) is concerned with coordinating and taking care of matters of preference and preparation, including multi-criteria. Intuitionistic fuzzy soft set environments can not be used to solve certain types of problems if attributes are more than one and further bifurcated. Therefore, there was a serious need to identify a new approach to solve such problems, so a new setting, namely the intuitionistic fuzzy hypersoft Sets (IFHSS), is established for this reason. In the present paper, we introduce intuitionistic fuzzy hypersoft set theory. Intuitionistic fuzzy hypersoft set theory is a mixture of IFS theory and the hypersoft set theory. The complement, subset, equal set, "AND", "OR", intersection, union notions are defined on intuitionistic fuzzy hypersoft sets. This paper also is supported by many suitable examples.

2. Preliminary

Definition 1. *[3] An intuitionistic fuzzy set H in U is $H = \{(u, \theta_H(u), \sigma_H(u)) : u \in U\}$, where $\theta_H : U \to [0,1]$, $\sigma_H : U \to [0,1]$ with the condition $0 \leq \theta_H(u) + \sigma_H(u) \leq 1$, $\forall u \in U$. $\theta_H, \sigma_H \in [0,1]$ denote the degree of membership and non-membership of u to H, respectively. The set of all intuitionistic fuzzy sets over U will be denoted by $IFP(U)$.*

Definition 2. *[15] Let U be an initial universe and E be a set of parameters. A pair (H, E) is called a soft set over U, where H is a mapping $H : E \to \mathcal{P}(U)$. In other words, the soft set is a parameterized family of subsets of the set U.*

Definition 3. *[13] Let U be an initial universe and E be a set of parameters. A pair (H, E) is called an intuitionistic fuzzy soft set over U, where H is a mapping given by, $H : E \to IFP(U)$.*

In general, for every $e \in E$, $H(e)$ is an intuitionistic fuzzy set of U and it is called intuitionistic fuzzy value set of parameter e. Clearly, $H(e)$ can be written as a intuitionistic fuzzy set such that $H(e) = \{(u, \theta_H(u), \sigma_H(u)) : u \in U\}$.

Definition 4. *[21] Let U be the universal set and $P(U)$ be the power set of U. Consider $e_1, e_2, e_3, ..., e_n$ for $n \geq 1$, be n well-defined attributes, whose corresponding attribute values are resspectively the sets $E_1, E_2, ..., E_n$ with $E_i \cap E_j = \emptyset$, for $i \neq j$ and $i, j \in \{1, 2, ..., n\}$, then the pair $(H, E_1 \times E_2 \times ... \times E_n)$ is said to be Hypersoft set over U where $H : E_1 \times E_2 \times ... \times E_n \to P(U)$.*

3. Intuitionistic Fuzzy Hypersoft Sets

Definition 5. *Let U be the universal set and $IFP(U)$ be the intuitionistic fuzzy power set of U. Consider $e_1, e_2, e_3, ..., e_n$ for $n \geq 1$, be n well-defined attributes, whose corresponding attribute values are respectively the sets $E_1, E_2, ..., E_n$ with $E_i \cap E_j = \emptyset$, for $i \neq j$ and $i, j \in \{1, 2, ..., n\}$. Let A_i be the nonempty subset of E_i for each $i = 1, 2, ..., n$. An intuitionistic fuzzy hypersoft set defined as the pair $(H, A_1 \times A_2 \times ... \times A_n)$ where; $H : A_1 \times A_2 \times ... \times A_n \to IFP(U)$ and*

$$H(A_1 \times A_2 \times ... \times A_n) = \{< \xi, (\frac{u}{\theta_{H(\xi)}(u), \sigma_{H(\xi)}(u)}) >: u \in U, \xi \in A_1 \times A_2 \times ... \times A_n \subseteq E_1 \times E_2 \times ... \times E_n\}$$

where θ and σ are the membership and non-membership value, respectively such that $0 \leq \theta_{H(\xi)}(u) + \sigma_{H(\xi)}(u) \leq 1$ and $\theta_{H(\xi)}(u), \sigma_{H(\xi)}(u) \in [0, 1]$. For sake of simplicity, we write the symbols Δ for $E_1 \times E_2 \times ... \times E_n$, Ω for $A_1 \times A_2 \times ... \times A_n$ and ξ for an element of the set Ω.

Definition 6. *i) An intutionistic fuzzy hypersoft set (H, Δ) over the universe U is said to be null intuitionistic fuzzy hypersoft set and denoted by $0_{(U_{IFH}, \Delta)}$ if for all $u \in U$ and $\xi \in \Delta$, $\theta_{H(\xi)}(u) = 0$ and $\sigma_{H(\xi)}(u) = 1$.*

ii) An intutionistic fuzzy hypersoft set (H, Δ) over the universe U is said to be absolute intuitionistic fuzzy hypersoft set and denoted by $1_{(U_{IFH}, \Delta)}$ if for all $u \in U$ and $\xi \in \Delta$, $\theta_{H(\xi)}(\alpha) = 1$ and $\sigma_{H(\xi)}(u) = 0$.

Example 7. *Let U be the set of cars given as $U = \{u_1, u_2, u_3\}$ also consider the set of attributes as $E_1 =$ Fuel, $E_2 =$Transmission, $E_3 =$ Color and their respective attributes are given as*

$$\begin{aligned} E_1 &= Fuels = \{Gasoline(\alpha_1),\ Diesel(\alpha_2),\ Electric(\alpha_3)\} \\ E_2 &= Transmissions = \{Automatic(\beta_1),\ Manual(\beta_2)\} \\ E_3 &= Colors = \{Black(\gamma_1),\ Blue(\gamma_2),\ White(\gamma_3)\} \end{aligned}$$

Suppose that

$$A_1 = \{\alpha_3\},\ A_2 = \{\beta_1, \beta_2\},\ A_3 = \{\gamma_1, \gamma_3\}$$

$$B_1 = \{\alpha_1, \alpha_3\},\ B_2 = \{\beta_1\},\ B_3 = \{\gamma_1, \gamma_2\}$$

are subset of E_i for each $i = 1, 2, 3$. Then the intuitionistic fuzzy hypersofts (H, Ω_1) and (G, Ω_2) defined as follows;

$$(H,\Omega_1) = \left\{ \begin{array}{c} <(\alpha_3,\beta_1,\gamma_1),\{\frac{u_1}{(0.6,0.3)},\frac{u_2}{(0.4,0.2)}\}>,<(\alpha_3,\beta_1,\gamma_3),\{\frac{u_2}{(0.2,0.6)},\frac{u_3}{(0.3,0.1)}\}>, \\ <(\alpha_3,\beta_2,\gamma_1),\{\frac{u_1}{(0.7,0.3)},\frac{u_2}{(0.3,0.5)},\frac{u_3}{(0.1,0.6)}\}>,<(\alpha_3,\beta_2,\gamma_3),\{\frac{u_1}{(0.8,0.2)},\frac{u_2}{(0.2,0.4)}\}> \end{array} \right\}$$

$$(G,\Omega_2) = \left\{ \begin{array}{c} <(\alpha_1,\beta_1,\gamma_1),\{\frac{u_1}{(0.3,0.3)},\frac{u_2}{(0.1,0.2)},\frac{u_3}{(0.6,0.4)}\}>,<(\alpha_1,\beta_1,\gamma_2),\{\frac{u_1}{(0.7,0.2)},\frac{u_2}{(0.6,0.1)}\}>, \\ <(\alpha_3,\beta_1,\gamma_1),\{\frac{u_1}{(0.9,0.1)},\frac{u_2}{(0.2,0.7)}\}>,<(\alpha_3,\beta_1,\gamma_2),\{\frac{u_2}{(0.5,0.3)},\frac{u_3}{(0.3,0.7)}\}> \end{array} \right\}$$

Tabular form of these sets are given as follows:

Table 1. Tabular form of IFHSS (H, Ω_1)

(H,Ω_1)	u_1	u_2	u_3
$(\alpha_3,\beta_1,\gamma_1)$	$(0.6,0.3)$	$(0.4,0.2)$	$(0,1)$
$(\alpha_3,\beta_1,\gamma_3)$	$(0,1)$	$(0.2,0.6)$	$(0.3,0.1)$
$(\alpha_3,\beta_2,\gamma_1)$	$(0.7,0.3)$	$(0.3,0.5)$	$(0.1,0.6)$
$(\alpha_3,\beta_2,\gamma_3)$	$(0.8,0.2)$	$(0.2,0.4)$	$(0,1)$

Table 2. Tabular form of IFHSS (G, Ω_2)

(G,Ω_2)	u_1	u_2	u_3
$(\alpha_1,\beta_1,\gamma_1)$	$(0.3,0.3)$	$(0.1,0.2)$	$(0.6,0.4)$
$(\alpha_1,\beta_1,\gamma_2)$	$(0.7,0.2)$	$(0.6,0.1)$	$(0,1)$
$(\alpha_3,\beta_1,\gamma_1)$	$(0.9,0.1)$	$(0.2,0.7)$	$(0,1)$
$(\alpha_3,\beta_1,\gamma_2)$	$(0,1)$	$(0.5,0.3)$	$(0.3,0.7)$

Corollary 8. *It is clear that each intuitionistic fuzzy hypersoft set is also intuitionistic fuzzy soft set. An example of this situation is provided below.*

Example 9. *We consider that Example-7. If we select the parameters from a single attribute set such as E_1 while creating the intuitionistic fuzzy hypersoft set, then the resulting set becomes the intuitionistic fuzzy soft set. Therefore, it is clear that each intuitionistic fuzzy hypersoft set is also intuitionistic fuzzy soft set.That is, the intuitionistic fuzzy hypersoft set structure is the generalized version of the intuitionistic fuzzy soft sets.*

Definition 10. *Let U be an initial universe set and $(H, \Omega_1), (G, \Omega_2)$ be two intuitionistic fuzzy hypersoft sets over the universe U. We say that (H, Ω_1) is an intuitionistic fuzzy hypersoft subset of (G, Ω_2) and denote $(H, \Omega_1) \widetilde{\subseteq} (G, \Omega_2)$ if*

i) $\Omega_1 \subseteq \Omega_2$

ii) For any $\xi \in \Omega_1$, $H(\xi) \subseteq G(\xi)$,

That is for all $u \in U$ and $\xi \in \Omega_1$, $\theta_{H(\xi)}(u) \leq \theta_{G(\xi)}(u)$ and $\sigma_{H(\xi)}(u) \geq \sigma_{G(\xi)}(u)$.

Example 11. *We consider that attributes in Example-7 and Let (H,Ω_1), (G,Ω_2) be two intuitionistic fuzzy hypersoft set over the same universe $U = \{u_1, u_2, u_3\}$. Tabular forms of (H,Ω_1) and (G,Ω_2) are following:*

TABLE 3. Tabular form of IFHSS (H,Ω_1)

(H,Ω_1)	u_1	u_2	u_3
$(\alpha_3,\beta_1,\gamma_1)$	$(0.5, 0.4)$	$(0.1, 0.8)$	$(0.3, 0.4)$
$(\alpha_3,\beta_1,\gamma_3)$	$(0.2, 0.7)$	$(0.3, 0.6)$	$(0.1, 0.6)$
$(\alpha_3,\beta_2,\gamma_1)$	$(0, 1)$	$(0, 1)$	$(0, 1)$
$(\alpha_3,\beta_2,\gamma_3)$	$(0, 1)$	$(0, 1)$	$(0, 1)$

TABLE 4. Tabular form of IFHSS (G,Ω_2)

(G,Ω_2)	u_1	u_2	u_3
$(\alpha_3,\beta_1,\gamma_1)$	$(0.6, 0.1)$	$(0.2, 0.5)$	$(0.7, 0.3)$
$(\alpha_3,\beta_1,\gamma_3)$	$(0.6, 0.4)$	$(0.4, 0.6)$	$(0.7, 0.4)$
$(\alpha_3,\beta_2,\gamma_1)$	$(0, 1)$	$(0, 1)$	$(0, 1)$
$(\alpha_3,\beta_2,\gamma_3)$	$(0.4, 0.3)$	$(0.8, 0.2)$	$(0.6, 0.4)$

It is clear that $(H,\Omega_1)\widetilde{\subseteq}(G,\Omega_2)$. we can also written as

$$
\begin{aligned}
(H,\Omega_1)\widetilde{\subseteq}(G,\Omega_2) &= \left\{ \begin{array}{c} <(\alpha_3,\beta_1,\gamma_1),\{\frac{u_1}{(0.5,0,4)},\frac{u_2}{(0.1,0.8)},\frac{u_3}{(0.3,0.4)}\}>, \\ <(\alpha_3,\beta_1,\gamma_3),\{\frac{u_1}{(0.2,0,7)},\frac{u_2}{(0.3,0.6)},\frac{u_3}{(0.1,0.6)}\}>, \\ <(\alpha_3,\beta_2,\gamma_1),\{\frac{u_1}{(0,1)},\frac{u_2}{(0,1)},\frac{u_3}{(0,1)}\}>, \\ <(\alpha_3,\beta_2,\gamma_3),\{\frac{u_1}{(0,1)},\frac{u_2}{(0,1)},\frac{u_3}{(0,1)}\}> \end{array} \right\} \\
&\widetilde{\subseteq} \left\{ \begin{array}{c} <(\alpha_3,\beta_1,\gamma_1),\{\frac{u_1}{(0.6,0,1)},\frac{u_2}{(0.2,0.5)},\frac{u_3}{(0.7,0.3)}\}>, \\ <(\alpha_3,\beta_1,\gamma_3),\{\frac{u_1}{(0.6,0,4)},\frac{u_2}{(0.4,0.6)},\frac{u_3}{(0.7,0.4)}\}>, \\ <(\alpha_3,\beta_2,\gamma_1),\{\frac{u_1}{(0,1)},\frac{u_2}{(0,1)},\frac{u_3}{(0,1)}\}>, \\ <(\alpha_3,\beta_2,\gamma_3),\{\frac{u_1}{(0.4,0.3)},\frac{u_2}{(0.8,0.2)},\frac{u_3}{(0.6,0.4)}\}> \end{array} \right\}
\end{aligned}
$$

Definition 12. *Let U be an initial universe set and $(H,\Omega_1),(G,\Omega_2)$ be two intuitionistic fuzzy hypersoft sets over the universe U. We say that (H,Ω_1) is an intuitionistic fuzzy hypersoft equal (G,Ω_2) and denote $(H,\Omega_1)\widetilde{=}(G,\Omega_2)$ if for all $u \in U$ and $\xi \in \Delta$, $\theta_{H(\xi)}(u) = \theta_{G(\xi)}(u)$ and $\sigma_{H(\xi)}(u) = \sigma_{G(\xi)}(u)$.*

Theorem 13. *Let U be an initial universe set, $\Omega_1, \Omega_2, \Omega_3 \subseteq \Delta$ and $(H,\Omega_1),(G,\Omega_2)$, (K,Ω_3) be intuitionistic fuzzy hypersoft sets over the universe U. Then,*

i) $(H,\Omega_1)\widetilde{\subseteq}1_{(U_{IFH},\Delta)}$,

ii) $0_{(U_{IFH},\Delta)}\widetilde{\subseteq}(H,\Omega_1)$,

iii) $(H,\Omega_1)\widetilde{\subseteq}(G,\Omega_2)$ *and* $(G,\Omega_2)\widetilde{\subseteq}(K,\Omega_3) \Rightarrow (H,\Omega_1)\widetilde{\subseteq}(K,\Omega_3)$.

Proof. i) $(H,\Omega_1)\tilde{\subseteq}1_{(U_{IFH},\Delta)}$, since $\theta_{H(\xi)}(u)\leq\theta_{H(\Delta)}(u)=1$ and $\sigma_{H(\xi)}(u)\geq\sigma_{H(\Delta)}(u)=0$ for all $\xi\in\Delta$, $u\in U$,

ii)$0_{(U_{IFH},\Delta)}\tilde{\subseteq}(H,\Omega_1)$, since $0=\theta_{\tilde{\emptyset}}(u)\leq\theta_{H(\varepsilon)}(u)$ and $1=\sigma_{\tilde{\emptyset}}(u)\geq\sigma_{H(\xi)}(u)$ for all $\xi\in\Delta$, $u\in U$,

iii) $(H,\Omega_1)\tilde{\subseteq}(G,\Omega_2)\Rightarrow\theta_{H(\xi)}(u)\leq\theta_{G(\xi)}(u)$ and $\sigma_{H(\xi)}(u)\geq\sigma_{H(\xi)}(u)$ for all $\xi\in\Delta$, $u\in U$. Also $(G,\Omega_2)\tilde{\subseteq}(K,\Omega_3)\Rightarrow\theta_{G(\xi)}(u)\leq\theta_{K(\xi)}(u)$ and $\sigma_{G(\xi)}(u)\geq\sigma_{K(\xi)}(u)$ for all $\xi\in\Delta$, $u\in U$. Therefore $\theta_{H(\xi)}(u)\leq\theta_{K(\xi)}(u)$ and $\sigma_{H(\xi)}(u)\geq\sigma_{K(\xi)}(u)$. Thus, we obtain $(H,\Omega_1)\tilde{\tilde{\subseteq}}(K,\Omega_3)$. □

Definition 14. *The complement of intutionistic fuzzy hypersoft set (H,Ω) over the universe U is denoted by $(H,\Omega)^c$ and defined as $(H,\Omega)^c=(H^c,\Omega)$, where $H^c:(E_1\times E_2\times...\times E_n)=\Delta\rightarrow IFP(U)$ and $H^c(\Omega)=(H(\Omega))^c$ for all $\Omega\subseteq\Delta$. Thus if $(H,\Omega)=\{<\xi,(\frac{u}{\theta_{H(\xi)}(u),\sigma_{H(\xi)}(u)})>: u\in U,\xi\in\Omega)\}$, then $(H,\Omega)^c=\{<\xi,(\frac{u}{\sigma_{H(\xi)}(u),\theta_{H(\xi)}(u)})>: u\in U,\xi\in\Omega)\}$.*

Example 15. *According to Example-7, consider the intuitionistic fuzzy hypersoft set (H,Ω) over the universe $U=\{\alpha_1,\alpha_2,\alpha_3,\alpha_4\}$.*

$$(H,\Omega_1)=\left\{\begin{array}{c}<(\alpha_3,\beta_1,\gamma_1),\{\frac{u_1}{(0.6,0.3)},\frac{u_2}{(0.4,0.2)}\}>,<(\alpha_3,\beta_1,\gamma_3),\{\frac{u_2}{(0.2,0.6)},\frac{u_3}{(0.3,0.1)}\}>,\\ <(\alpha_3,\beta_2,\gamma_1),\{\frac{u_1}{(0.7,0.3)},\frac{u_2}{(0.3,0.5)},\frac{u_3}{(0.1,0.6)}\}>,<(\alpha_3,\beta_2,\gamma_3),\{\frac{u_1}{(0.8,0.2)},\frac{u_2}{(0.2,0.4)}\}>\end{array}\right\}$$

Then the complement of (H,Ω) is written as;

$$(H,\Omega_1)^c=\left\{\begin{array}{c}<(\alpha_3,\beta_1,\gamma_1),\{\frac{u_1}{(0.3,0.6)},\frac{u_2}{(0.2,0.4)},\frac{u_3}{(1,0)}\}>,<(\alpha_3,\beta_1,\gamma_3),\{\frac{u_1}{(1,0)},\frac{u_2}{(0.6,0.2)},\frac{u_3}{(0.1,0.3)}\}>,\\ <(\alpha_3,\beta_2,\gamma_1),\{\frac{u_1}{(0.3,0.7)},\frac{u_2}{(0.5,0.3)},\frac{u_3}{(0.6,0.1)}\}>,<(\alpha_3,\beta_2,\gamma_3),\{\frac{u_1}{(0.2,0.8)},\frac{u_2}{(0.4,0.2)}\}>\end{array}\right\}.$$

Theorem 16. *Let (H,Ω) be any intuitionistic fuzzy hypersoft set over the universe U. Then,*

i) $((H,\Omega)^c)^c=(H,\Omega)$

ii) $0^c_{(U_{IFH},\Delta)}=1_{(U_{IFH},\Delta)}$

iii) $1^c_{(U_{IFH},\Delta)}=0_{(U_{IFH},\Delta)}$

Proof. Proofs are trivial. □

Definition 17. *Let U be an initial universe set, $\Omega_1,\Omega_2\subseteq\Delta$ and $(H,\Omega_1),(G,\Omega_2)$ be two intuitionistic fuzzy hypersoft sets over the universe U. The union of (H,Ω_1) and (G,Ω_2) is denoted by $(H,\Omega_1)\tilde{\cup}(G,\Omega_2)=(K,\Omega_3)$ where $\Omega_3=\Omega_1\cup\Omega_2$ and*

$$\theta_{K(\xi)}(u)=\begin{cases}H(\xi) & if\quad \xi\in\Omega_1-\Omega_2\\ G(\xi) & if\quad \xi\in\Omega_2-\Omega_1\\ \max(H(\xi),(G(\xi)) & if\quad \xi\in\Omega_1\cap\Omega_2\end{cases}$$

$$\sigma_{K(\xi)}(u)=\begin{cases}H(\xi) & if\quad \xi\in\Omega_1-\Omega_2\\ G(\xi) & if\quad \xi\in\Omega_2-\Omega_1\\ \min(H(\xi),(G(\xi)) & if\quad \xi\in\Omega_1\cap\Omega_2\end{cases}$$

Theorem 18. *Let U be an initial universe set, $\Omega_1, \Omega_2, \Omega_3 \subseteq \Delta$ and $(H,\Omega_1), (G,\Omega_2)$, (S,Ω_3) be intuitionistic fuzzy hypersoft sets over the universe U. Then;*
i) $(H,\Omega_1)\tilde{\cup}(H,\Omega_1) = (H,\Omega_1)$
ii) $0_{(U_{IFH},\Delta)}\ \tilde{\cup}(H,\Omega_1) = (H,\Omega_1)$
iii) $(H,\Omega_1)\tilde{\cup}\ 1_{(U_{IFH},\Delta)} = 1_{(U_{IFH},\Delta)}$
iv) $(H,\Omega_1)\tilde{\cup}(G,\Omega_2) = (G,\Omega_2)\tilde{\cup}(H,\Omega_1)$
v) $((H,\Omega_1)\tilde{\cup}(G,\Omega_2))\,\tilde{\cup}(S,\Omega_3) = (H,\Omega_1)\tilde{\cup}\,((G,\Omega_2)\tilde{\cup}(S,\Omega_3))$

Proof. Proofs are trivial. □

Definition 19. *Let U be an initial universe set, $\Omega_1, \Omega_2 \subseteq \Delta$ and $(H,\Omega_1), (G,\Omega_2)$ be two intuitionistic fuzzy hypersoft sets over the universe U. The intersection of (H,Ω_1) and (G,Ω_2) is denoted by $(H,\Omega_1)\tilde{\cap}(G,\Omega_2) = (K,\Omega_3)$ where $\Omega_3 = \Omega_1 \cap \Omega_2$,*

$$(K,\Omega_3) = \{<\xi, (\frac{u}{(\min\{\theta_{H(\xi)}(u), \theta_{G(\xi)}(u)\}, \max\{\theta_{H(\xi)}(u), \theta_{G(\xi)}(u)\})}) >: u \in U, \xi \in \Omega)\}$$

Theorem 20. *Let U be an initial universe set, $\Omega_1, \Omega_2, \Omega_3 \subseteq \Delta$ and $(H,\Omega_1), (G,\Omega_2)$, (S,Ω_3) be intuitionistic fuzzy hypersoft sets over the universe U. Then;*
i) $(H,\Omega_1)\tilde{\cap}(H,\Omega_1) = (H,\Omega_1)$
ii) $0_{(U_{IFH},\Delta)}\ \tilde{\cap}(H,\Omega_1) = 0_{(U_{IFH},\Delta)}$
iii) $(H,\Omega_1)\tilde{\cap}1_{(U_{IFH},\Delta)} = (H,\Omega_1)$
iv) $(H,\Omega_1)\tilde{\cap}(G,\Omega_2) = (G,\Omega_2)\tilde{\cap}(H,\Omega_1)$
v) $((H,\Omega_1)\tilde{\cap}(G,\Omega_2))\,\tilde{\cap}(S,\Omega_3) = (H,\Omega_1)\tilde{\cap}\,((G,\Omega_2)\tilde{\cap}(S,\Omega_3))$

Proof. Proofs are trivial. □

Definition 21. *Let U be an initial universe set, $\Omega_1, \Omega_2 \subseteq \Delta$ and $(H,\Omega_1), (G,\Omega_2)$ be two intuitionistic fuzzy hypersoft sets over the universe U. The difference of (H,Ω_1) and (G,Ω_2) is denoted by $(H,\Omega_1)\widetilde{\setminus}(G,\Omega_2) = (K,\Omega_3)$ where*
$(H,\Omega_1)\tilde{\cap}(G,\Omega_2)^c = (K,\Omega_3)$.

Example 22. *We consider that attributes in Example-7 and Let (H,Ω_1), (G,Ω_2) be any two intuitionistic fuzzy hypersoft set over the same universe $U = \{u_1, u_2, u_3\}$. The IFHSS (H,Ω_1) and (G,Ω_2) defined as follows;*

$$(H,\Omega_1) = \left\{ \begin{array}{c} <(\alpha_3,\beta_1,\gamma_1), \{\frac{u_1}{(0.6,0,3)}, \frac{u_2}{(0.4,0.2)}\}>, <(\alpha_3,\beta_1,\gamma_3), \{\frac{u_2}{(0.2,0.6)}, \frac{u_3}{(0.3,0.1)}\}>, \\ <(\alpha_3,\beta_2,\gamma_1), \{\frac{u_1}{(0.7,0.3)}, \frac{u_2}{(0.3,0.5)}, \frac{u_3}{(0.1,0.6)}\}>, <(\alpha_3,\beta_2,\gamma_3), \{\frac{u_1}{(0.8,0.2)}, \frac{u_2}{(0.2,0.4)}\}> \end{array} \right\}$$

$$(G,\Omega_2) = \left\{ \begin{array}{c} <(\alpha_1,\beta_1,\gamma_1), \{\frac{u_1}{(0.3,0,3)}, \frac{u_2}{(0.1,0.2)}, \frac{u_3}{(0.6,0.4)}\}>, <(\alpha_1,\beta_1,\gamma_2), \{\frac{u_1}{(0.7,0.2)}, \frac{u_2}{(0.6,0.1)}\}>, \\ <(\alpha_3,\beta_1,\gamma_1), \{\frac{u_1}{(0.9,0.1)}, \frac{u_2}{(0.2,0.7)}\}>, <(\alpha_3,\beta_1,\gamma_2), \{\frac{u_2}{(0.5,0.3)}, \frac{u_3}{(0.3,0.7)}\}> \end{array} \right\}$$

The union,intersection and difference operator of above IFHSS is written as;

$$(H,\Omega_1)\tilde{\cup}(G,\Omega_2) =$$

$$\left\{ \begin{array}{c} <(\alpha_3,\beta_1,\gamma_1), \{\frac{u_1}{(0.9,0,1)}, \frac{u_2}{(0.4,0.2)}\}>, <(\alpha_3,\beta_1,\gamma_3), \{\frac{u_2}{(0.2,0.6)}, \frac{u_3}{(0.3,0.1)}\}>, \\ <(\alpha_3,\beta_2,\gamma_1), \{\frac{u_1}{(0.7,0.3)}, \frac{u_2}{(0.3,0.5)}, \frac{u_3}{(0.1,0.6)}\}>, <(\alpha_3,\beta_2,\gamma_3), \{\frac{u_1}{(0.8,0.2)}, \frac{u_2}{(0.2,0.4)}\}>, \\ <(\alpha_1,\beta_1,\gamma_1), \{\frac{u_1}{(0.3,0,3)}, \frac{u_2}{(0.1,0.2)}, \frac{u_3}{(0.6,0.4)}\}>, <(\alpha_1,\beta_1,\gamma_2), \{\frac{u_1}{(0.7,0.2)}, \frac{u_2}{(0.6,0.1)}\}> \\ <(\alpha_3,\beta_1,\gamma_2), \{\frac{u_2}{(0.5,0.3)}, \frac{u_3}{(0.3,0.7)}\}> \end{array} \right\}$$

$$(H,\Omega_1)\tilde{\cap}(G,\Omega_2) = \left\{ < (\alpha_3, \beta_1, \gamma_1), \{\frac{u_1}{(0.6, 0, 3)}, \frac{u_2}{(0.2, 0.7)}\} > \right\}$$

and

$$(H,\Omega_1)\tilde{\setminus}(G,\Omega_2) = \left\{ < (\alpha_3, \beta_1, \gamma_1), \{\frac{u_1}{(0.1, 0, 9)}, \frac{u_2}{(0.4, 0.2)}\} > \right\}$$

Theorem 23. *Let U be an initial universe set, $\Omega_1, \Omega_2 \subseteq \Delta$ and $(H,\Omega_1), (G,\Omega_2)$ be two intuitionistic fuzzy hypersoft sets over the universe U. Then De-Morgan Laws are hold.*
i) $((H,\Omega_1)\tilde{\cup}(G,\Omega_2))^c = (H,\Omega_1)^c\tilde{\cap}(G,\Omega_2)^c$
ii)$((H,\Omega_1)\tilde{\cap}(G,\Omega_2))^c = (H,\Omega_1)^c\tilde{\cup}(G,\Omega_2)^c$

Proof. We only prove $((H,\Omega_1)\tilde{\cup}(G,\Omega_2))^c = (H,\Omega_1)^c\tilde{\cap}(G,\Omega_2)^c$. The other properties can be similarly proved. Suppose that $((H,\Omega_1)\tilde{\cup}(G,\Omega_2))^c = (K,\Omega_1 \cup \Omega_2)$ and $(H,\Omega_1)^c\tilde{\cap}(G,\Omega_2)^c = (I,\Omega_1 \cup \Omega_2)$. For any $\xi \in \Omega_1 \cup \Omega_2$, we consider the following cases.

Case 1: $\xi \in \Omega_1 - \Omega_2$. Then $K(\xi) = H^c(\xi) = I(\xi)$.

Case 2: $\xi \in \Omega_2 - \Omega_1$. Then $K(\xi) = G^c(\xi) = I(\xi)$.

Case 3: $\xi \in \Omega_1 \cap \Omega_2$. Then $K(\xi) = (\sigma_{H(\xi)}(u) \cap \sigma_{G(\xi)}(u), \theta_{H(\xi)}(u) \cup \theta_{G(\xi)}(u)) = H^c(\xi) \cap G^c(\xi) = I(\xi)$.

Therefore, K and I are same operators, and so
$((H,\Omega_1)\tilde{\cup}(G,\Omega_2))^c = (H,\Omega_1)^c\tilde{\cap}(G,\Omega_2)^c$. □

Definition 24. *Let U be an initial universe set, $\Omega_1, \Omega_2 \subseteq \Delta$ and $(H,\Omega_1), (G,\Omega_2)$ be two intuitionistic fuzzy hypersoft sets over the universe U. The "AND" operation on them is denoted by $(H,\Omega_1) \wedge (G,\Omega_2) = (K,\Omega_1 \times \Omega_2)$ is given as;*

$$(K,\Omega_1 \times \Omega_2) = \left\{ < (\xi_1, \xi_2), (\frac{u}{\theta_{K(\xi_1,\xi_2)}(u), \sigma_{K(\xi_1,\xi_2)}(u)}) >: u \in U, (\xi_1, \xi_2) \in \Omega_1 \times \Omega_2 \right\}$$

where

$$\begin{aligned} \theta_{K(\xi_1,\xi_2)}(u) &= \min\left\{\theta_{H(\xi_1)}(u), \theta_{G(\xi_2)}(u)\right\} \\ \sigma_{K(\xi_1,\xi_2)}(u) &= \max\left\{\sigma_{H(\xi_1)}(u), \sigma_{G(\xi_2)}(u)\right\} \end{aligned}$$

Definition 25. *Let U be an initial universe set, $\Omega_1, \Omega_2 \subseteq \Delta$ and $(H,\Omega_1), (G,\Omega_2)$ be two intuitionistic fuzzy hypersoft sets over the universe U. The "OR" operation on them is denoted by $(H,\Omega_1) \vee (G,\Omega_2) = (K,\Omega_1 \times \Omega_2)$ is given as;*

$$(K,\Omega_1 \times \Omega_2) = \left\{ < (\xi_1, \xi_2), (\frac{u}{\theta_{K(\xi_1,\xi_2)}(u), \sigma_{K(\xi_1,\xi_2)}(u)}) >: u \in U, (\xi_1, \xi_2) \in \Omega_1 \times \Omega_2 \right\}$$

where

$$\begin{aligned} \theta_{K(\xi_1,\xi_2)}(u) &= \max\left\{\theta_{H(\xi_1)}(u), \theta_{G(\xi_2)}(u)\right\} \\ \sigma_{K(\xi_1,\xi_2)}(u) &= \min\left\{\sigma_{H(\xi_1)}(u), \sigma_{G(\xi_2)}(u)\right\} \end{aligned}$$

Theorem 26. *Let U be an initial universe set, $\Omega_1, \Omega_2, \Omega_3 \subseteq \Delta$ and $(H, \Omega_1), (G, \Omega_2)$, (S, Ω_3) be intuitionistic fuzzy hypersoft sets over the universe U. Then;*
i) $(H, \Omega_1) \vee [(G, \Omega_2) \vee (S, \Omega_3)] = [(H, \Omega_1) \vee (G, \Omega_2)] \vee (S, \Omega_3)$
ii) $(H, \Omega_1) \wedge [(G, \Omega_2) \wedge (S, \Omega_3)] = [(H, \Omega_1) \wedge (G, \Omega_2)] \wedge (S, \Omega_3)$

Proof. Straightforward. □

Theorem 27. *Let U be an initial universe set, $\Omega_1, \Omega_2 \subseteq \Delta$ and $(H, \Omega_1), (G, \Omega_2)$ be two intuitionistic fuzzy hypersoft sets over the universe U. Then;*
i) $[(H, \Omega_1) \vee (G, \Omega_2)]^c = (H, \Omega_1)^c \wedge (G, \Omega_2)^c$
ii) $[(H, \Omega_1) \wedge (G, \Omega_2)]^c = (H, \Omega_1)^c \vee (G, \Omega_2)^c$.

Proof. We only prove (i). The other properties can be similarly proved.
For all $(\xi_1, \xi_2) \in \Omega_1 \times \Omega_2$ and $u \in U$,

$$\begin{aligned}
(H, \Omega_1) \vee (G, \Omega_2) &= \left\{< u, \max\left\{\theta_{H(\xi_1)}(u), \theta_{G(\xi_2)}(u)\right\}, \min\left\{\sigma_{H(\xi_1)}(u), \sigma_{G(\xi_2)}(u)\right\} >\right\}, \\
[(H, \Omega_1) \vee (G, \Omega_2)]^c &= \left\{< u, \min\left\{\sigma_{H(\xi_1)}(u), \sigma_{G(\xi_2)}(u)\right\}, \max\left\{\theta_{H(\xi_1)}(u), \theta_{G(\xi_2)}(u)\right\} >\right\}
\end{aligned}$$

On the other hand,

$$\begin{aligned}
(H, \Omega_1)^c &= \left\{< u, \sigma_{H(\xi_1)}(u), \theta_{H(\xi_1)}(u) >: \xi_1 \in \Omega_1\right\} \\
(G, \Omega_2)^c &= \left\{< u, \sigma_{G(\xi_2)}(u), \theta_{G(\xi_2)}(u) >: \xi_2 \in \Omega_2\right\}
\end{aligned}$$

Then,

$$\begin{aligned}
(H, \Omega_1)^c \wedge (G, \Omega_2)^c &= \left\{\left\{< u, \min\left\{\sigma_{H(\xi_1)}(u), \sigma_{G(\xi_2)}(u)\right\}, \max\left\{\theta_{H(\xi_1)}(u), \theta_{G(\xi_2)}(u)\right\} >\right\}\right\} \\
&= [(H, \Omega_1) \vee (G, \Omega_2)]^c
\end{aligned}$$

Hence, $[(H, \Omega_1) \vee (G, \Omega_2)]^c = (H, \Omega_1)^c \wedge (G, \Omega_2)^c$ is obtained. □

Example 28. *We consider that attributes in Example-7. Then the fuzzy hypersoft sets (H, Ω_1) and (G, Ω_2) defined as follows;*

$$(H, \Omega_1) = \left\{ \begin{array}{c} < (\alpha_3, \beta_1, \gamma_1), \{\frac{u_1}{(0.6,0,3)}, \frac{u_2}{(0.4,0.2)}\} >, < (\alpha_3, \beta_1, \gamma_3), \{\frac{u_2}{(0.2,0.6)}, \frac{u_3}{(0.3,0.1)}\} >, \\ < (\alpha_3, \beta_2, \gamma_1), \{\frac{u_1}{(0.7,0.3)}, \frac{u_2}{(0.3,0.5)}, \frac{u_3}{(0.1,0.6)}\} >, < (\alpha_3, \beta_2, \gamma_3), \{\frac{u_1}{(0.8,0.2)}, \frac{u_2}{(0.2,0.4)}\} > \end{array} \right\}$$

$$(G, \Omega_2) = \left\{ \begin{array}{c} < (\alpha_1, \beta_1, \gamma_1), \{\frac{u_1}{(0.3,0,3)}, \frac{u_2}{(0.1,0.2)}, \frac{u_3}{(0.6,0.4)}\} >, < (\alpha_1, \beta_1, \gamma_2), \{\frac{u_1}{(0.7,0.2)}, \frac{u_2}{(0.6,0.1)}\} >, \\ < (\alpha_3, \beta_1, \gamma_1), \{\frac{u_1}{(0.9,0.1)}, \frac{u_2}{(0.2,0.7)}\} >, < (\alpha_3, \beta_1, \gamma_2), \{\frac{u_2}{(0.5,0.3)}, \frac{u_3}{(0.3,0.7)}\} > \end{array} \right\}$$

Let's assume $(\alpha_3, \beta_1, \gamma_1) = m_1$, $(\alpha_3, \beta_1, \gamma_3) = m_2$, $(\alpha_3, \beta_2, \gamma_1) = m_3$, $(\alpha_3, \beta_2, \gamma_3) = m_4$ in (H, Ω_1) and $(\alpha_1, \beta_1, \gamma_1) = n_1$, $(\alpha_1, \beta_1, \gamma_2) = n_2$, $(\alpha_3, \beta_1, \gamma_1) = n_3$, $(\alpha_3, \beta_1, \gamma_2) = n_4$ in (G, Ω_2) for easier operation. The tabular forms of these sets are as follows.

Then the "AND" and "OR" operations of these sets are given as below.

TABLE 5. Tabular form of FHSS (H,Ω_1)

(H,Ω_1)	u_1	u_2	u_3
m_1	$(0.6, 0, 3)$	$(0.4, 0.2)$	$(0, 1)$
m_2	$(0, 1)$	$(0.2, 0.6)$	$(0.3, 0.1)$
m_3	$(0.7, 0.3)$	$(0.3, 0.5)$	$(0.1, 0.6)$
m_4	$(0.8, 0.2)$	$(0.2, 0.4)$	$(0, 1)$

TABLE 6. Tabular form of FHSS (G,Ω_2)

(G,Ω_2)	u_1	u_2	u_3
n_1	$(0.3, 0, 3)$	$(0.1, 0.2)$	$(0.6, 0.4)$
n_2	$(0.7, 0.2)$	$(0.6, 0.1)$	$(0, 1)$
n_3	$(0.9, 0.1)$	$(0.2, 0.7)$	$(0, 1)$
n_4	$(0, 1)$	$(0.5, 0.3)$	$(0.3, 0.7)$

TABLE 7. Tabular form of FHSS $(H,\Omega_1) \wedge (G,\Omega_2)$

$(H,\Omega_1) \wedge (G,\Omega_2)$	u_1	u_2	u_3
$m_1 \times n_1$	$(0.3, 0.3)$	$(0.1, 0.2)$	$(0, 1)$
$m_1 \times n_2$	$(0.6, 0.3)$	$(0.4, 0.2)$	$(0, 1)$
$m_1 \times n_3$	$(0.6, 0.3)$	$(0.2, 0.7)$	$(0, 1)$
$m_1 \times n_4$	$(0, 1)$	$(0.4, 0.3)$	$(0, 1)$
$m_2 \times n_1$	$(0, 1)$	$(0.1, 0.6)$	$(0.3, 0.4)$
$m_2 \times n_2$	$(0, 1)$	$(0.2, 0.6)$	$(0, 1)$
$m_2 \times n_3$	$(0, 1)$	$(0.2, 0.7)$	$(0, 1)$
$m_2 \times n_4$	$(0, 1)$	$(0.2, 0.6)$	$(0.3, 0.7)$
$m_3 \times n_1$	$(0.3, 0.3)$	$(0.1, 0.5)$	$(0.1, 0.6)$
$m_3 \times n_2$	$(0.7, 0.3)$	$(0.3, 0.5)$	$(0, 1)$
$m_3 \times n_3$	$(0.7, 0.3)$	$(0.2, 0.7)$	$(0, 1)$
$m_3 \times n_4$	$(0, 1)$	$(0.3, 0.5)$	$(0.1, 0.7)$
$m_4 \times n_1$	$(0.3, 0.3)$	$(0.1, 0.4)$	$(0, 1)$
$m_4 \times n_2$	$(0.7, 0.2)$	$(0.2, 0.4)$	$(0, 1)$
$m_4 \times n_3$	$(0.8, 0.2)$	$(0.2, 0.7)$	$(0, 1)$
$m_4 \times n_4$	$(0, 1)$	$(0.2, 0.4)$	$(0, 1)$

4. CONCLUSION

The aim of this paper is to overcome the uncertainty trouble in more particular way by way of combing Intuitionistic fuzzy set with Hypersoft set. Some operations of Intuitionistic Fuzzy Hypersoft set such as subset, equal set, union, intersection, complement, AND, OR operations are presented. By defining these notions, the foundation of the intuitionistic fuzzy hypersoft set structure was built.

TABLE 8. Tabular form of FHSS $(H,\Omega_1) \vee (G,\Omega_2)$

$(H,\Omega_1) \vee (G,\Omega_2)$	u_1	u_2	u_3
$m_1 \times n_1$	$(0.6, 0.3)$	$(0.4, 0.2)$	$(0.6, 0.4)$
$m_1 \times n_2$	$(0.7, 0.2)$	$(0.6, 0.1)$	$(0, 1)$
$m_1 \times n_3$	$(0.9, 0.1)$	$(0.4, 0.2)$	$(0, 1)$
$m_1 \times n_4$	$(0.6, 0.3)$	$(0.5, 0.2)$	$(0.3, 0.7)$
$m_2 \times n_1$	$(0.3, 0.3)$	$(0.2, 0.2)$	$(0.6, 0.1)$
$m_2 \times n_2$	$(0.7, 0.2)$	$(0.6, 0.1)$	$(0.3, 0.1)$
$m_2 \times n_3$	$(0.9, 0.1)$	$(0.2, 0.6)$	$(0.3, 0.1)$
$m_2 \times n_4$	$(0, 1)$	$(0.5, 0.3)$	$(0.3, 0.1)$
$m_3 \times n_1$	$(0.7, 0.3)$	$(0.3, 0.2)$	$(0.6, 0.4)$
$m_3 \times n_2$	$(0.7, 0.2)$	$(0.6, 0.1)$	$(0.1, 0.6)$
$m_3 \times n_3$	$(0.9, 0.1)$	$(0.3, 0.5)$	$(0.1, 0.6)$
$m_3 \times n_4$	$(0.7, 0.3)$	$(0.5, 0.3)$	$(0.3, 0.6)$
$m_4 \times n_1$	$(0.8, 0.2)$	$(0.2, 0.2)$	$(0.6, 0.4)$
$m_4 \times n_2$	$(0.8, 0.2)$	$(0.6, 0.1)$	$(0, 1)$
$m_4 \times n_3$	$(0.9, 0.1)$	$(0.2, 0.4)$	$(0, 1)$
$m_4 \times n_4$	$(0.8, 0.2)$	$(0.5, 0.3)$	$(0.3, 0.7)$

The validity and implementation of the proposed operations and definitions are validated through presenting suitable instance. Matrices, similarity measure, single and multi-valued, interval valued, functions, distance measures, algorithms: score function, VIKOR, TOPSIS, AHP of Intutionistic Fuzzy Hypersoft sets will be future work. We hope that, this study will play a critical function in future decision-making research.

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A theoretical and analytical approach to the conceptual framework of convexity cum concavity on fuzzy hypersoft sets with some generalized properties

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Abstract

Fuzzy hypersoft set (FHS-set) is an effective and flexible model as it not only minimizes the complexities of fuzzy set for dealing uncertainties, but also fulfills the parameterization requirements of soft set and fuzzy soft set. FHS-set is projected to address the limitations of these models regarding the entitlement of multi-argument approximate function. This kind of function maps the sub-parametric tuples to power set of universe. It emphasizes the partitioning of each attribute into its attribute-valued set that is missing in existing soft set-like structures. These features make it a completely new mathematical tool for solving problems dealing with uncertainties. As convexity has an essential function in optimization and control, pattern classification and recognition, image processing and in different fields of operation research, numerical analysis, etc. In order to tackle the various features of classical convexity (concavity) with uncertain environment of multi-argument approximate function, an articulate cum mathematical technique is utilized to develop a theoretical framework of convexity cum concavity on fuzzy hypersoft set which is more generalized and effective concept to deal with optimization relating problems. Moreover, some generalized properties like strictly convex (concave), strongly convex (concave), δ-inclusion and aggregation operations are established. The proposed study is authenticated with the provision of daily-life application based on proposed decision-making algorithm. Lastly, the features of proposed study are compared with the some existing relevant models to show its meritorious impact.

Keywords Convex soft set · Concave soft set · Hypersoft set · Fuzzy hypersoft set · Convex fuzzy hypersoft set · Concave fuzzy hypersoft set

1 Introduction

Zadeh (1965) initiated the concept of fuzzy sets. The theo-ries like theory of probability, theory of fuzzy sets, and the interval mathematics are considered as mathematical means to tackle many intricate problems involving various uncertainties, in different fields of mathematical sciences. These theories have their own complexities which restrain them to solve these problems successfully. The reason for these hur-dles is, possibly, the inadequacy of the parametrization tool. A mathematical tool is needed for dealing with uncertainties which should be free of all such impediments. Molodtsov (1999) has the honor to introduce such mathematical tool called soft sets in the literature as a new parameterized fam-ily of subsets of the universe of discourse. Maji et al. (2003) extended the concept and introduced some fundamental ter-minologies and operations like equality of two soft sets, subset and super set of a soft set, complement of a soft set, null soft set, absolute soft set, AND, OR and also the operations of union and intersection. They verified De Morgan's laws and a number of other results too. They also defined fuzzy soft set in Maji et al. (2001) and successfully applied it in decision mak-ing. Pei and Miao (2005) discussed the relationship between soft sets and information systems. They showed the soft set as a class of special information systems. Ali et al. (2009)

pointed several assertions in previous work of Maji et al. They defined new notions such as the restricted intersection, the restricted union, the restricted difference and the extended intersection of two soft sets. Babitha and Sunil (2010, 2011) introduced the concepts of soft set relation as a sub-soft set of the Cartesian product of the soft sets and also discussed many related concepts such as equivalent soft set relation, partition, composition and function. Sezgin and Atagün (2011), Ge and Yang (2011) and Li (2011) gave some modifications in the work of Maji et al. They also established some new results. Alcantud et al. (2017) employed the theory of valuation fuzzy soft set in decision making to valuate the assets. Feng et al. (2010), Liu et al. (2018) and Zhan and Zhu (2017) combined fuzzy soft set with rough set to tackle the vague information. Guan et al. (2013) discussed the order relations of fuzzy soft set along with some essential properties and applications. Hassan et al. (2017) initiated the gluing model of fuzzy soft expert set and applied it in the prediction of coronary artery disease. Khameneh and Kılıçman (2018) used fuzzy soft sets in three-way decision system for parametric reduction. Paik and Mondal (2020) employed the distance-similarity measures of fuzzy soft sets in decision-making process. Xiao (2018) and Zhang et al. (2020) discussed decision-making applications based on fuzzy soft set and fuzzy soft logic.
Deli (2019) defined convexity cum concavity on soft set and fuzzy soft set. Majeed (2016) investigated some more properties of convex soft sets. She developed the convex hull and the cone of a soft set with their generalized results. Salih and Sabir (2018) defined strictly and strongly convexity cum concavity on soft sets and they discussed their properties.
In some daily-life scenarios, it is necessitated to classify parameters into their respective parametric values in the form of non-overlapping sets. Soft set is inadequate for such scenarios. Smarandache (2018) introduced the concept of hypersoft set to tackle such scenarios with uncertain data. Saeed et al. (2021a) extended the concept and discussed the fundamentals of hypersoft set such as hypersoft subset, complement, not hypersoft set, aggregation operators along with hypersoft set relation, subrelation, complement relation, function, matrices and operations on hypersoft matrices. Abbas et al. (2020) investigated some properties of hypersoft points and hypersoft functions. They applied them to develop hypersoft function spaces. Rahman et al. (2021a, 2020, 2021b) developed hybridized structures of hypersoft set e.g., fuzzy parameterized hypersoft set, hypersoft hybrids with complex sets and bijective hypersoft sets, respectively. Saeed et al. (2021b) developed complex multi-fuzzy hypersoft set and discussed application in multi-criteria decision making based on its entropy and similarity measures. Martin and Smarandache (2020) initiated the concept of combined plithogenic hypersoft sets and discussed their properties with certain aggregations. Kamacı and Saqlain (2021) and Ihsan et al. (2021a, b) combined the theory of hypersoft sets with expert sets to tackle multi-decisive opinions of experts under uncertain environments. Debnath (2021) investigated the various rudiments of fuzzy hypersoft sets with numerical examples. Ahsan et al. (2021) employed an abstract approach to develop a framework of fuzzy hypersoft classes with certain properties.

1.1 Research gap and motivation

In many daily-life decision-making problems, we encounter with some scenarios where each attribute is required to be further classified into its respective attribute-valued set. Some examples of such scenarios are given below:

1. **Recruitment Process**: In this process, decision-makers usually use **qualification**, **age**, **experience** etc., as evaluating attributes. Since different candidates have different ages, qualifications and experiences so it is much pertinent to classify these attributes into their respective attribute values, i.e., ages (20 years, 25 years, etc.), qualifications (Graduate, Undergraduate, etc.) and experiences (5 years, 10 years, etc.).
2. **Product Selection**: In order to select a mobile from a mobile market, we usually prefer **RAM**, **ROM**, **Camera Resolution** etc., for its evaluation. As different mobile models are available with different RAMs, ROMs and Camera Resolutions so it is much better to classify these parameters into their respective sub-parametric valued disjoint sets, i.e., RAM (2 GB, 4 GB, 8 GB, etc.), ROM (32 GB, 64 GB, 128 GB, etc.) and Camera Resolution (5 Mega Pixels, 7 Mega pixels, etc.).
3. **Medical Diagnosis**: In order to diagnose heart diseases in patients, doctors (decision-makers) usually prefer **chest pain type**, **resting blood pressure**, **serum cholesterol** etc., as diagnostic parameters. After keen analysis, it is vivid that these parameters are required to be further partitioning into their sub-parametric values, i.e., chest pain type (typical angina, atypical angina, etc.), resting blood pressure (110 mmHg, 150 mmHg, 180 mmHg, etc.) and serum cholesterol (210 mg/dl, 320 mg/dl, 430 mg/dl, etc.).

In order to tackle such scenarios, hypersoft set is projected which employs the Cartesian product of disjoint attribute-valued sets as domain of approximate function (i.e., multi-argument approximate function). Fuzzy hypersoft set, a hybridized structure of fuzzy set and hypersoft set, assigns a fuzzy membership degree to each element in the universal set. The existing models like fuzzy set, soft set and fuzzy soft set are insufficient to deal uncertainties with such kind of approximate function. The vivid difference of fuzzy soft set and fuzzy hypersoft set is shown in Fig. 1 with the support of product selection decision making.

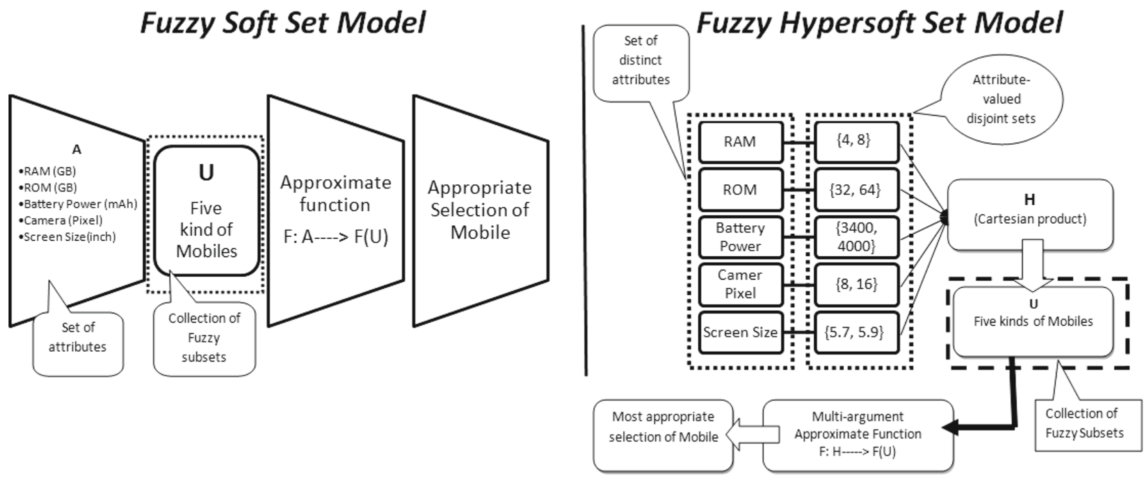

Fig. 1 Comparison of Fuzzy Soft Set and Fuzzy Hypersoft Set

Convexity is an essential concept in optimization, recognition and classification of certain patterns, processing and decomposition of images, discrete event simulation, duality problems and many other related topics in operation research, mathematical economics, numerical analysis and other mathematical sciences. Zadeh introduced the classical convexity under uncertain environment (fuzzy convex set) but it lacked parameterization tool so Deli resolved the problem and translated fuzzy convexity under soft and fuzzy soft set environment and Salih et al. extended the concept with the development of certain variants of convexity like strictly convexity, strongly convexity, etc., under soft set environment. These concepts are not capable to cope the convex optimization relating problems having further partitioning of parameters into sub-parametric values and multi-argument approximate function. Hence, it is the need of the literature to carve out a conceptual framework for solving such kind of problems under more generalized version, i.e., fuzzy hypersoft set. Therefore, to meet this demand, an abstract cum analytical approach is utilized to develop a basic framework of convexity and concavity on fuzzy hypersoft sets along with some important results inspiring from the above literature in general, and from Deli (2019), Salih and Sabir (2018) in specific. Pictorial version and examples of convexity and concavity on fuzzy hypersoft sets are presented first time in the literature.

1.2 Main contributions

The main contributions of this paper are summarized as follows.

1. A novel conceptual framework of classical convexity cum concavity under fuzzy hypersoft set is proposed. It tackles data or information in those convex optimization relating problems which involve uncertain fuzzy values attached with approximate elements. This leads to more precise results as compared to previous frameworks and contributes to make more reliable decisions.
2. In this proposed framework, some classical properties and results are investigated and proved under uncertain environment of fuzzy hypersoft set.
3. This proposed framework is equipped with strict and strong nature of classical convexity (concavity) with the provision of proofs of their essential properties.
4. The proposed framework is further authenticated with the help of proposed algorithm and applied in decision making to solve real-world problem.
5. The advantageous aspects of proposed framework are presented through its comparison with some existing relevant models.
6. Future directions and scope of the proposed work are mentioned with brief description on its implementation.

1.3 Organization of the paper

The rest of this article is structured as follows: In Sect. 2, some basic definitions and terms have been recalled from the literature to support main results. In Sect. refs3, convex and concave fuzzy hypersoft sets have been introduced along with some generalized results. In Sect. 4, strictly and strongly convex (concave) fuzzy hypersoft sets have been introduced along with some generalized results. In Sect. 5, an algorithm is proposed to solve real-life decision-making problem. In Sect. 6, the proposed study is compared with some existing relevant models. In Sect. 7, the paper has been concluded along with future directions and scope. Throughout the paper, G, $J^{\bullet}$, J°, $\sqcup$ and $P(\sqcup)$ will play the role of R^n, unit closed interval, unit open interval, universal set and power set, respectively.

2 Preliminaries

In this section, some fundamental terms regarding fuzzy set, soft set, hypersoft set, fuzzy hypersoft set and their convexity cum concavity are presented.

Definition 1 Zadeh (1965)
Suppose a universal set $\sqcup$ and a *fuzzy set* X is written as $X = \{(x, \alpha_X(x)) | x \in \sqcup\}$ such that

$$\alpha_X : \sqcup \to [0, 1]$$

where $\alpha_X(x)$ describes the membership percentage of $x \in \sqcup$.

Definition 2 Molodtsov (1999)
Let $\sqcup$ be an initial universe set and let E be a set of parameters. A pair (ζ_S, E) is called a *soft set* over $\sqcup$, where ζ_S is a mapping given by $\zeta_S : E \to P(\sqcup)$. In other words, a soft set (ζ_S, E) over $\sqcup$ is a parameterized family of subsets of $\sqcup$. For $\omega \in E$, $\zeta_S(\omega)$ may be considered as the set of ω-elements or ω-approximate elements of the soft set (ζ_S, E).

Definition 3 Maji et al. (2003)
Let (Φ_S, A) and (Ψ_S, B) be two soft sets over a common universe $\sqcup$,

1. we say that (Φ_S, A) is a *soft subset* of (Ψ_S, B) denoted by $(\Phi_S, A) \subseteq (\Psi_S, B)$ if
   i $A \subseteq B$, and
   ii $\forall \omega \in A$, $\Phi_S(\omega)$ and $\Psi_S(\omega)$ are identical approximations.
2. the *union* of (Φ_S, A) and (Ψ_S, B), denoted by $(\Phi_S, A) \cup (\Psi_S, B)$, is a soft set (ζ_S, C), where $C = A \cup B$ and $\omega \in C$,
$$\zeta_S(\omega) = \begin{cases} \Phi_S(\omega), & \omega \in A - B \\ \Psi_S(\omega), & \omega \in B - A \\ \Phi_S(\omega) \cup \Psi_S(\omega), & \omega \in A \cap B \end{cases}$$
3. the *intersection* of (Φ_S, A) and (Ψ_S, B) denoted by $(\Phi_S, A) \cap (\Psi_S, B)$, is a soft set (ζ_S, C), where $C = A \cap B$ and $\omega \in C$, $\zeta_S(\omega) = \Phi_S(\omega)$ or $\Psi_S(\omega)$ (as both are same set).

Definition 4 Maji et al. (2003)
The *complement* of a soft set (ζ_S, A), denoted by $(\zeta_S, A)^c$, is defined as $(\zeta_S, A)^c = ({\zeta_S}^c, \neg A)$ where

$${\zeta_S}^c : \neg A \to P(\sqcup)$$

is a mapping given by

$${\zeta_S}^c(\omega) = \sqcup - \zeta_S(\neg\omega) \forall \omega \in \neg A.$$

Definition 5 Maji et al. (2001)
Let $F(\sqcup)$ be the collection of all fuzzy sets over $\sqcup$; then, a pair (ζ_{FSS}, A) is called a fuzzy soft set over $\sqcup$, where

$$\zeta_{FSS} : A \to F(\sqcup)$$

is a mapping from A into $F(\sqcup)$.

Definition 6 Smarandache (2018)
Let $\sqcup$ be a universe of discourse, $P(\sqcup)$ the power set of $\sqcup$. Let $a_1, a_2, a_3, ..., a_n$, for $n \geq 1$, be n distinct attributes, whose corresponding attribute values are, respectively, the sets $A_1, A_2, A_3, ..., A_n$, with $A_i \cap A_j = \emptyset$, for $i \neq j$, and $i, j \in \{1, 2, 3, ..., n\}$. Then, the pair (ζ, G), where $G = A_1 \times A_2 \times A_3 \times ..... \times A_n$ and $\zeta : G \to P(\sqcup)$, is called a *hypersoft Set* over $\sqcup$.

Definition 7 Saeed et al. (2021a)
Let (Φ, G_1) and (Ψ, G_2) be two hypersoft sets over the same universal set $\sqcup$; then, their *union* $(\Phi, G_1) \cup (\Psi, G_2)$ is hypersoft set (ζ, G_3), where $G_3 = G_1 \cup G_2$; $G_1 = A_1 \times A_2 \times A_3 \times ..... \times A_n$, $G_2 = B_1 \times B_2 \times B_3 \times ..... \times B_n$ and $\forall\, e \in G_3$ with
$$\zeta(e) = \begin{cases} \Phi(e), & e \in G_1 - G_2 \\ \Psi(e), & e \in G_2 - G_1 \\ \Phi(e) \cup \Psi(e), & e \in G_2 \cap G_1 \end{cases}$$

Definition 8 Saeed et al. (2021a)
Let (Φ, G_1) and (Ψ, G_2) be two hypersoft sets over the same universal set $\sqcup$; then, their *intersection* $(\Phi, G_1) \cap (\Psi, G_2)$ is hypersoft set (ζ, G_3), where $G_3 = G_1 \cap G_2$; where $G_1 = A_1 \times A_2 \times A_3 \times ..... \times A_n$, $G_2 = B_1 \times B_2 \times B_3 \times ..... \times B_n$. and $\forall\, e \in G_3$ with $\zeta(e) = \Phi(e) \cap \Psi(e)$.

Definition 9 Smarandache (2018)
A hypersoft set (Φ, G) over a fuzzy universe of discourse is called *fuzzy hypersoft set* and denoted by (Φ_{FHS}, G).

More definitions and examples can be seen from Abbas et al. (2020) and Saeed et al. (2021a).

Definition 10 Deli (2019)
The δ-*inclusion* of a soft set (ζ_S, A) (where $\delta \subseteq \sqcup$) is defined by

$$(\zeta_S, A)^\delta = \{\omega \in A : h_S(\omega) \supseteq \delta\}$$

Definition 11 Deli (2019)
The soft set (ζ_S, A) on A is called a *convex soft set* if

$$\zeta_S(\epsilon\omega + (1 - \epsilon)\mu) \supseteq \zeta_S(\omega) \cap \zeta_S(\mu)$$

for every $\omega, \mu \in A$ and $\epsilon \in J^\bullet$.

Definition 12 Deli (2019)
The soft set (ζ_S, A) on A is called a *concave soft set* if

$$\zeta_S(\epsilon\omega + (1-\epsilon)\mu) \subseteq \zeta_S(\omega) \cup \zeta_S(\mu)$$

for every $\omega, \mu \in A$ and $\epsilon \in J^\bullet$.

Definition 13 Deli (2019)
The fuzzy soft set (ζ_{FS}, B) on B is called a *convex fuzzy soft set* if

$$\zeta_{FS}(\epsilon\omega + (1-\epsilon)\mu) \supseteq \zeta_{FS}(\omega) \cap \zeta_{FS}(\mu)$$

for every $\omega, \mu \in A$ and $\epsilon \in J^\bullet$.

Definition 14 Deli (2019)
The fuzzy soft set (ζ_{FS}, B) on B is called a *concave fuzzy soft set* if

$$\zeta_{FS}(\epsilon\omega + (1-\epsilon)\mu) \subseteq \zeta_{FS}(\omega) \cup \zeta_{FS}(\mu)$$

for every $\omega, \mu \in A$ and $\epsilon \in J^\bullet$.

Definition 15 Salih and Sabir (2018)
The soft set (ζ_S, A) on A is called a *strictly convex soft set* if

$$\zeta_S(\alpha\beta + (1-\alpha)\theta) \supset \zeta_S(\beta) \cap \zeta_S(\theta)$$

for every $\beta, \theta \in A$, $\zeta_S(\beta) \neq \zeta_S(\theta)$ and $\alpha \in J^\circ = (0, 1)$.

Definition 16 Salih and Sabir (2018)
The soft set (ζ_S, A) on A is called a *strictly concave soft set* if

$$\zeta_S(\alpha\beta + (1-\alpha)\theta) \subset \zeta_S(\beta) \cup \zeta_S(\theta)$$

for every $\beta, \theta \in A$, $\zeta_S(\beta) \neq \zeta_S(\theta)$ and $\alpha \in J^\circ$.

Definition 17 Salih and Sabir (2018)
The soft set (ζ_S, A) on A is called a *strongly convex soft set* if

$$\zeta_S(\alpha\beta + (1-\alpha)\theta) \supset \zeta_S(\beta) \cap \zeta_S(\theta)$$

for every $\beta, \theta \in A$, $\beta \neq \theta$ and $\alpha \in J^\circ$.

Definition 18 Salih and Sabir (2018)
The soft set (ζ_S, A) on A is called a *strongly concave soft set* if

$$\zeta_S(\alpha\beta + (1-\alpha)\theta) \subset \zeta_S(\beta) \cup \zeta_S(\theta)$$

for every $\beta, \theta \in A$, $\beta \neq \theta$ and $\alpha \in J^\circ$.

More about convex soft sets can be seen from Deli (2019), Majeed (2016).

3 Convex and concave fuzzy hypersoft sets

Here, convex fuzzy hypersoft sets and concave fuzzy hypersoft sets are defined and some important results are proved.

Definition 19 Let $F(\sqcup)$ be the collection of all fuzzy sets over $\sqcup$. Let $A_1, A_2, A_3, \ldots.., A_n$, for $n \geq 1$, be n distinct attributes, whose corresponding attribute values are, respectively, the sets $A_1, A_2, A_3, \ldots.., A_n$, with $A_i \cap A_j = \emptyset$, for $i \neq j$, and $i, j \in \{1, 2, 3, ..., n\}$. Then, a *fuzzy hypersoft set* (ζ_{FHS}, G) over $\sqcup$ is defined by the set of ordered pairs as follows:

$$(\zeta_{FHS}, G) = \{(\underline{x}, \zeta_{FHS}(\underline{x})) : \underline{x} \in G, \zeta_{FHS}(\underline{x}) \in F(\sqcup)\}$$

where $\zeta_{FHS} : G \to F(\sqcup)$ and for all $\underline{x} \in G = A_1 \times A_2 \times A_3 \times \ldots.. \times A_n$

$$\zeta_{FHS}(\underline{x}) = \{\mu_{\zeta_{FHS}(\underline{x})}(u)/u : u \in \sqcup, \mu_{\zeta_{FHS}(\underline{x})}(u) \in [0, 1]\}$$

is a fuzzy set over $\sqcup$.

Above definition is a modified version of fuzzy hypersoft set given in Smarandache (2018).

Definition 20 The $\delta - inclusion$ of a fuzzy hypersoft set (ζ_{FHS}, G) (where $\delta \subseteq \sqcup$) is defined by

$$(\zeta_{FHS}, G)^\delta = \left\{\underline{\omega} \in G : \zeta_{FHS}\left(\underline{\omega}\right) \supseteq \delta\right\}$$

Definition 21 The fuzzy hypersoft set (ζ_{FHS}, G) on $\sqcup$ is called a *convex fuzzy hypersoft set* if

$$\zeta_{FHS}\left(\epsilon\underline{\omega} + (1-\epsilon)\underline{\mu}\right) \supseteq \zeta_{FHS}\left(\underline{\omega}\right) \cap \zeta_{FHS}\left(\underline{\mu}\right)$$

for every $\underline{\omega}$, $\underline{\mu} \in G$ where $G = A_1 \times A_2 \times A_3 \times \ldots.. \times A_n$ with $A_i \cap A_j = \emptyset$, for $i \neq j$, and $i, j \in \{1, 2, 3, ..., n\}$; $\zeta_{FHS} : G \to F(\sqcup)$ and $\epsilon \in J^\bullet$.

Theorem 1 *$(f_{FHS}, S) \cap (g_{FHS}, T)$ is a convex fuzzy hypersoft set when both (f_{FHS}, S) and (g_{FHS}, T) are convex fuzzy hypersoft sets.*

Proof Suppose that $(f_{FHS}, S) \cap (g_{FHS}, T) = (\zeta_{FHS}, G)$ with $G = S \cap T$, for $\underline{\omega}_1$, $\underline{\omega}_2 \in G$; $\epsilon \in J^\bullet$, we have then

$$\zeta_{FHS}\left(\epsilon\underline{\omega}_1 + (1-\epsilon)\underline{\omega}_2\right) = f_{FHS}\left(\epsilon\underline{\omega}_1 + (1-\epsilon)\underline{\omega}_2\right) \cap g_{FHS}\left(\epsilon\underline{\omega}_1 + (1-\epsilon)\underline{\omega}_2\right)$$

As (f_{FHS}, S) and (g_{FHS}, T) are convex fuzzy hypersoft sets,

$$f_{FHS}\left(\epsilon\underline{\omega}_1 + (1-\epsilon)\underline{\omega}_2\right) \supseteq f_{FHS}\left(\underline{\omega}_1\right) \cap f_{FHS}\left(\underline{\omega}_2\right)$$
$$g_{FHS}\left(\epsilon\underline{\omega}_1 + (1-\epsilon)\underline{\omega}_2\right) \supseteq g_{FHS}\left(\underline{\omega}_1\right) \cap g_{FHS}\left(\underline{\omega}_2\right)$$

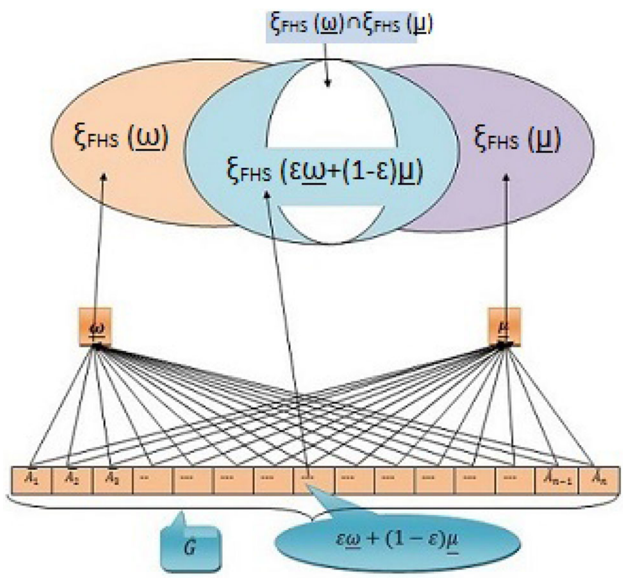

Fig. 2 Convex fuzzy hypersoft Set

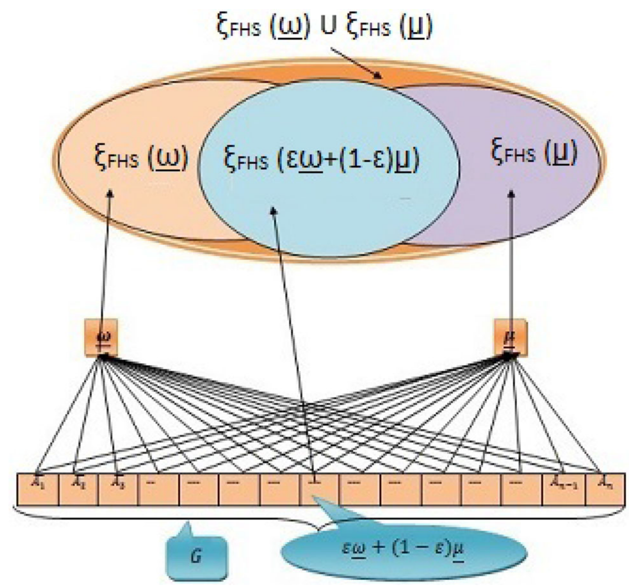

Fig. 3 Concave fuzzy hypersoft Set

which implies

$$\zeta_{FHS}\left(\epsilon\underline{\omega}_1+(1-\epsilon)\underline{\omega}_2\right)\supseteq \left(f_{FHS}\left(\underline{\omega}_1\right)\cap\ f_{FHS}\left(\underline{\omega}_2\right)\right)\cap\left(g_{FHS}\left(\underline{\omega}_1\right)\cap\ g_{FHS}\left(\underline{\omega}_2\right)\right)$$

and thus

$$\zeta_{FHS}\left(\epsilon\underline{\omega}_1+(1-\epsilon)\underline{\omega}_2\right)\supseteq\ \zeta_{FHS}\left(\underline{\omega}_1\right)\cap\ \zeta_{FHS}\left(\underline{\omega}_2\right)$$

□

Remark 1 If $\left\{(h^i{}_{FHS},G_i):i\in\{1,2,3,...\}\right\}$ is any family of convex fuzzy hypersoft sets, then the intersection $\bigcap_{i\in I}(h^i{}_{FHS},G_i)$ is a convex fuzzy hypersoft set.

Remark 2 The union of any family

$$\left\{(h^i{}_{FHS},G_i):i\in\{1,2,3,...\}\right\}$$

of convex fuzzy hypersoft sets is not necessarily a convex fuzzy hypersoft set.

Theorem 2 *(ζ_{FHS},G) is convex fuzzy hypersoft set iff for every $\epsilon\in J^\bullet$ and $\delta\in F(\sqcup)$, $(\zeta_{FHS},G)^\delta$ is convex fuzzy hypersoft set.*

Proof Suppose (ζ_{FHS},G) is convex fuzzy hypersoft set. If $\underline{\omega},\ \underline{\mu}\in G$ and $\delta\in F(\sqcup)$, then $\zeta_{FHS}\left(\underline{\omega}\right)\supseteq\delta$ and $\zeta_{FHS}\left(\underline{\mu}\right)\supseteq\delta$, it implies that $\zeta_{FHS}\left(\underline{\omega}\right)\cap\zeta_{FHS}\left(\underline{\mu}\right)\supseteq\delta$. So we have,

$$\zeta_{FHS}\left(\epsilon\underline{\omega}+(1-\epsilon)\,\underline{\mu}\right)\supseteq\zeta_{FHS}\left(\underline{\omega}\right)\cap\zeta_{FHS}\left(\underline{\mu}\right)\supseteq\delta$$

$$\Rightarrow\quad\zeta_{FHS}\left(\epsilon\underline{\omega}+(1-\epsilon)\,\underline{\mu}\right)\supseteq\delta$$

thus $(\zeta_{FHS},G)^\delta$ is convex fuzzy hypersoft set.
Conversely suppose that $(\zeta_{FHS},G)^\delta$ is convex fuzzy hypersoft set for every $\epsilon\in J^\bullet$. For $\underline{\omega},\ \underline{\mu}\in G$, $(\zeta_{FHS},G)^\delta$ is convex fuzzy hypersoft set with $\delta=\zeta_{FHS}\left(\underline{\omega}\right)\cap\zeta_{FHS}\left(\underline{\mu}\right)$. Since $\zeta_{FHS}\left(\underline{\omega}\right)\supseteq\delta$ and $\zeta_{FHS}\left(\underline{\mu}\right)\supseteq\delta$, we have $\underline{\omega}\in(\zeta_{FHS},G)^\delta$ and $\underline{\mu}\in(\zeta_{FHS},G)^\delta$,
$\Rightarrow\quad\epsilon\underline{\omega}+(1-\epsilon)\,\underline{\mu}\in(\zeta_{FHS},G)^\delta$.
Therefore,

$$\zeta_{FHS}\left(\epsilon\underline{\omega}+(1-\epsilon)\,\underline{\mu}\right)\supseteq\ \delta$$

So

$$\zeta_{FHS}\left(\epsilon\underline{\omega}+(1-\epsilon)\,\underline{\mu}\right)\supseteq\ \zeta_{FHS}\left(\underline{\omega}\right)\cap\zeta_{FHS}\left(\underline{\mu}\right),$$

Hence, (ζ_{FHS},G) is convex fuzzy hypersoft set. □

Definition 22 The fuzzy hypersoft set (ζ_{FHS},G) on $\sqcup$ is called a *concave fuzzy hypersoft set* if

$$\zeta_{FHS}\left(\epsilon\underline{\omega}+(1-\epsilon)\,\underline{\mu}\right)\subseteq\zeta_{FHS}\left(\underline{\omega}\right)\cup\zeta_{FHS}\left(\underline{\mu}\right)$$

for every $\underline{\omega}=(a_1,a_2,a_3,.....,a_n)$, $\underline{\mu}=(b_1,b_2,b_3,.....,b_n)\in G$ where, $G=A_1\times A_2\times A_3\times.....\times A_n$ with $A_i\cap A_j=\emptyset$, for $i\neq j$, and $i,j\in\{1,2,3,...,n\}$; $\zeta_{FHS}:G\to F(\sqcup)$ and $\epsilon\in J^\bullet$.

Example 1 Consider a set of mobiles as a universe of discourse $\sqcup = \{m_1, m_2, m_3, ..., m_5\}$. The attributes of mobiles under consideration is the set $Y = \{y_{11}, y_{12}, y_{13}\}$, where

y_{11} = Company
y_{12} = Resolution of Camera in MP
y_{13} = RAM/Storage in GB

such that the attribute values against these attributes, respectively, are the sets given as

$$Z_{11} = \{C_1, C_2, C_3, C_4, C_5\}$$
$$= \{1, 2, 3, 4, 5\}$$
$$Z_{12} = \{6, 7, 8, 9, 10\}$$
$$Z_{13} = \{1/16, 2/32, 3/64, 4/128, 5/256\}$$

The fuzzy hypersoft set (ζ_{FHS}, G) is a function defined by the mapping $\zeta_{FHS} : G \to F(\sqcup)$ where $G = Z_{11} \times Z_{12} \times Z_{13}$. Consider $\underline{\beta} = (2, 6, 3/64)$, then

$\zeta_{FHS}(\underline{\beta}) = \zeta_{FHS}(2, 6, 3/64) = \{0.01/m_1, 0.05/m_5\}$.
Also, consider $\underline{\theta} = (3, 7, 2/32)$, then
$\zeta_{FHS}(\underline{\theta}) = \zeta_{FHS}(3, 7, 2/32) = \{0.01/m_1, 0.03/m_3, 0.04/m_4\}$

Now,
$\zeta_{FHS}(\underline{\beta}) \cap \zeta_{FHS}(\underline{\theta}) = \zeta_{FHS}(2, 6, 3/64) \cap \zeta_{FHS}(3, 7, 2/32)$
$\zeta_{FHS}(\underline{\beta}) \cap \zeta_{FHS}(\underline{\theta}) = \{0.01/m_1, 0.05/m_5\} \cap \{0.01/m_1, 0.03/m_3, 0.04/m_4\}$

$$\zeta_{FHS}(\underline{\beta}) \cap \zeta_{FHS}(\underline{\theta}) = \{0.01/m_1\} \tag{1}$$

Let $\alpha = 0.1 \in J^{\bullet}$; then, we have
$\alpha\underline{\beta} + (1-\alpha)\underline{\theta} = 0.1(2, 6, 3/64) + (1-0.1)(3, 7, 2/32) = 0.1(2, 6, 3/64) + 0.9(3, 7, 2/32) = (0.2, 0.6, 0.3/64) + (2.7, 6.3, 1.8/32) = (0.2+2.7, 0.6+6.3, 0.3/64+1.8/32) = (2.9, 6.9, 3.9/64)$
By using the decimal round off property, we get $(3, 7, 4/64) = (3, 7, 2/32)$

$$\begin{aligned}\zeta_{FHS}(\alpha\underline{\beta} + (1-\alpha)\underline{\theta}) &= \zeta_{FHS}(3, 7, 2/32)\\ &= \{0.01/m_1, 0.03/m_3, 0.04/m_4\}\end{aligned} \tag{2}$$

it is vivid from equations (1) and (2), we have

$$\zeta_{FHS}\left(\alpha\underline{\beta} + (1-\alpha)\underline{\theta}\right) \supseteq \zeta_{FHS}\left(\underline{\beta}\right) \cap \zeta_{FHS}\left(\underline{\theta}\right)$$

Similarly,
$\zeta_{FHS}(\underline{\beta}) \cup \zeta_{FHS}(\underline{\theta}) = \zeta_{FHS}(2, 6, 3/64) \cup \zeta_{FHS}(3, 7, 2/32)$
$\zeta_{FHS}(\underline{\beta}) \cap \zeta_{FHS}(\underline{\theta}) =$
$\{0.01/m_1, 0.05/m_5\} \cup \{0.01/m_1, 0.03/m_3, 0.04/m_4\}$

$$\begin{aligned}&\zeta_{FHS}(\underline{\beta}) \cap \zeta_{FHS}(\underline{\theta})\\ &= \{0.01/m_1, 0.03/m_3, 0.04/m_4, 0.05/m_5\}\end{aligned} \tag{3}$$

From (2) and (3), we have

$$\zeta_{FHS}\left(\alpha\underline{\beta} + (1-\alpha)\underline{\theta}\right) \subseteq \zeta_{FHS}\left(\underline{\beta}\right) \cup \zeta_{FHS}\left(\underline{\theta}\right)$$

Theorem 3 *$(f_{FHS}, S) \cup (g_{FHS}, T)$ is a concave fuzzy hypersoft set when both (f_{FHS}, S) and (g_{FHS}, T) are concave fuzzy hypersoft sets.*

Proof Suppose that $(f_{FHS}, S) \cup (g_{FHS}, T) = (\zeta_{FHS}, G)$ with $G = S \cup T$, for $\underline{\omega}_1, \underline{\omega}_2 \in G$; $\epsilon \in J^{\bullet}$, we have then

$$\zeta_{FHS}\left(\epsilon\underline{\omega}_1 + (1-\epsilon)\underline{\omega}_2\right) = f_{FHS}\left(\epsilon\underline{\omega}_1 + (1-\epsilon)\underline{\omega}_2\right) \cup g_{FHS}\left(\epsilon\underline{\omega}_1 + (1-\epsilon)\underline{\omega}_2\right)$$

As (f_{FHS}, S) and (g_{FHS}, T) are concave fuzzy hypersoft sets,

$$f_{FHS}\left(\epsilon\underline{\omega}_1 + (1-\epsilon)\underline{\omega}_2\right) \subseteq f_{FHS}\left(\underline{\omega}_1\right) \cup f_{FHS}\left(\underline{\omega}_2\right)$$
$$g_{FHS}\left(\epsilon\underline{\omega}_1 + (1-\epsilon)\underline{\omega}_2\right) \subseteq g_{FHS}\left(\underline{\omega}_1\right) \cup g_{FHS}\left(\underline{\omega}_2\right)$$

which implies

$$\zeta_{FHS}\left(\epsilon\underline{\omega}_1 + (1-\epsilon)\underline{\omega}_2\right) \subseteq \left(f_{FHS}\left(\underline{\omega}_1\right) \cup f_{FHS}\left(\underline{\omega}_2\right)\right) \cup \left(g_{FHS}\left(\underline{\omega}_1\right) \cup g_{FHS}\left(\underline{\omega}_2\right)\right)$$

and thus

$$\zeta_{FHS}\left(\epsilon\underline{\omega}_1 + (1-\epsilon)\underline{\omega}_2\right) \subseteq \zeta_{FHS}\left(\underline{\omega}_1\right) \cup \zeta_{FHS}\left(\underline{\omega}_2\right)$$

□

Remark 3 If $\left\{(\breve{h}^i{}_{FHS}, G_i) : i \in \{1, 2, 3, ...\}\right\}$ is any family of concave fuzzy hypersoft sets, then the union $\bigcup_{i\in I}(\breve{h}^i{}_{FHS}, G_i)$ is a concave fuzzy hypersoft set.

Theorem 4 *$(f_{FHS}, S) \cap (g_{FHS}, T)$ is a concave fuzzy hypersoft set when both (f_{FHS}, S) and (g_{FHS}, T) are concave fuzzy hypersoft sets.*

Proof Suppose that $(f_{FHS}, S) \cap (g_{FHS}, T) = (\zeta_{FHS}, G)$ with $G = S \cap T$, for $\underline{\omega}_1, \underline{\omega}_2 \in G$; $\epsilon \in J^{\bullet}$, we have then

$$\zeta_{FHS}\left(\epsilon\underline{\omega}_1 + (1-\epsilon)\underline{\omega}_2\right) = f_{FHS}\left(\epsilon\underline{\omega}_1 + (1-\epsilon)\underline{\omega}_2\right) \cap g_{FHS}\left(\epsilon\underline{\omega}_1 + (1-\epsilon)\underline{\omega}_2\right)$$

As (f_{FHS}, S) and (g_{FHS}, T) are concave fuzzy hypersoft sets,

$$f_{FHS}\left(\epsilon\underline{\omega}_1 + (1-\epsilon)\underline{\omega}_2\right) \subseteq f_{FHS}\left(\underline{\omega}_1\right) \cup f_{FHS}\left(\underline{\omega}_2\right)$$
$$g_{FHS}\left(\epsilon\underline{\omega}_1 + (1-\epsilon)\underline{\omega}_2\right) \subseteq g_{FHS}\left(\underline{\omega}_1\right) \cup g_{FHS}\left(\underline{\omega}_2\right)$$

which implies

$$\zeta_{FHS}\left(\epsilon\underline{\omega}_1 + (1-\epsilon)\underline{\omega}_2\right) \subseteq \left(f_{FHS}\left(\underline{\omega}_1\right) \cup f_{FHS}\left(\underline{\omega}_2\right)\right) \cap \left(g_{FHS}\left(\underline{\omega}_1\right) \cup g_{FHS}\left(\underline{\omega}_2\right)\right)$$

and thus

$$\zeta_{FHS}\left(\epsilon\underline{\omega}_1 + (1-\epsilon)\underline{\omega}_2\right) \subseteq \zeta_{FHS}\left(\underline{\omega}_1\right) \cup \zeta_{FHS}\left(\underline{\omega}_2\right)$$

□

Remark 4 The intersection of any family

$$\left\{(\breve{h}^i{}_{FHS}, G_i) : i \in \{1, 2, 3, ...\}\right\}$$

of concave fuzzy hypersoft sets is a concave fuzzy hypersoft set.

Theorem 5 *$(\zeta_{FHS}, G)^c$ is a convex fuzzy hypersoft set when (ζ_{FHS}, G) is a concave fuzzy hypersoft set.*

Proof Suppose that for $\underline{\omega}_1, \underline{\omega}_2 \in G$, $\epsilon \in J^\bullet$ and (ζ_{FHS}, G) be concave fuzzy hypersoft set.
Since (ζ_{FHS}, G) is concave fuzzy hypersoft set,

$$\zeta_{FHS}\left(\epsilon\underline{\omega}_1 + (1-\epsilon)\underline{\omega}_2\right) \subseteq \zeta_{FHS}\left(\underline{\omega}_1\right) \cup \zeta_{FHS}\left(\underline{\omega}_2\right)$$

or

$$\sqcup\backslash\, \zeta_{FHS}\left(\epsilon\underline{\omega}_1 + (1-\epsilon)\underline{\omega}_2\right) \supseteq \sqcup\backslash \left\{\zeta_{FHS}\left(\underline{\omega}_1\right) \cup \zeta_{FHS}\left(\underline{\omega}_2\right)\right\}$$

If $\zeta_{FHS}\left(\underline{\omega}_1\right) \supset \zeta_{FHS}\left(\underline{\omega}_2\right)$ then
$\zeta_{FHS}\left(\underline{\omega}_1\right) \cup \zeta_{FHS}\left(\underline{\omega}_2\right) = \zeta_{FHS}\left(\underline{\omega}_1\right)$
Therefore,

$$\sqcup\backslash\, \zeta_{FHS}\left(\epsilon\underline{\omega}_1 + (1-\epsilon)\underline{\omega}_2\right) \supseteq \sqcup\backslash\, \zeta_{FHS}\left(\underline{\omega}_1\right). \tag{4}$$

If $\zeta_{FHS}\left(\underline{\omega}_1\right) \subset \zeta_{FHS}\left(\underline{\omega}_2\right)$ then
$\zeta_{FHS}\left(\underline{\omega}_1\right) \cup \zeta_{FHS}\left(\underline{\omega}_2\right) = \zeta_{FHS}\left(\underline{\omega}_2\right)$
Therefore,

$$\sqcup\backslash\, \zeta_{FHS}\left(\epsilon\underline{\omega}_1 + (1-\epsilon)\underline{\omega}_2\right) \supseteq \sqcup\backslash\, \zeta_{FHS}\left(\underline{\omega}_2\right). \tag{5}$$

From (4) and (5) , we have

$$\sqcup\backslash\, \zeta_{FHS}\left(\epsilon\underline{\omega}_1 + (1-\epsilon)\underline{\omega}_2\right) \supseteq (\sqcup\backslash\zeta_{FHS}\left(\underline{\omega}_1\right)) \cap (\sqcup\backslash\zeta_M\left(\underline{\omega}_2\right)).$$

So, $(\zeta_{FHS}, G)^c$ is a convex fuzzy hypersoft set. □

Theorem 6 *$(\zeta_{FHS}, G)^c$ is a concave fuzzy hypersoft set when (ζ_{FHS}, G) is a convex fuzzy hypersoft set.*

Proof Suppose that for $\underline{\omega}_1, \underline{\omega}_2 \in G$, $\epsilon \in J^\bullet$ and (ζ_{FHS}, G) be convex fuzzy hypersoft set.
since (ζ_{FHS}, G) is convex fuzzy hypersoft set,

$$\zeta_{FHS}\left(\epsilon\underline{\omega}_1 + (1-\epsilon)\underline{\omega}_2\right) \supseteq \zeta_{FHS}\left(\underline{\omega}_1\right) \cap \zeta_{FHS}\left(\underline{\omega}_2\right)$$

or

$$\sqcup\backslash\, \zeta_{FHS}\left(\epsilon\underline{\omega}_1 + (1-\epsilon)\underline{\omega}_2\right) \subseteq \sqcup\backslash \left\{\zeta_{FHS}\left(\underline{\omega}_1\right) \cap \zeta_{FHS}\left(\underline{\omega}_2\right)\right\}$$

If $\zeta_{FHS}\left(\underline{\omega}_1\right) \supset \zeta_{FHS}\left(\underline{\omega}_2\right)$ then
$\zeta_{FHS}\left(\underline{\omega}_1\right) \cap \zeta_{FHS}\left(\underline{\omega}_2\right) = \zeta_{FHS}\left(\underline{\omega}_2\right)$
Therefore,

$$\sqcup\backslash\, \zeta_{FHS}\left(\epsilon\underline{\omega}_1 + (1-\epsilon)\underline{\omega}_2\right) \subseteq \sqcup\backslash\, \zeta_{FHS}\left(\underline{\omega}_2\right). \tag{6}$$

If $\zeta_{FHS}\left(\underline{\omega}_1\right) \subset \zeta_{FHS}\left(\underline{\omega}_2\right)$ then
$\zeta_{FHS}\left(\underline{\omega}_1\right) \cap \zeta_{FHS}\left(\underline{\omega}_2\right) = \zeta_{FHS}\left(\underline{\omega}_1\right)$
Therefore,

$$\sqcup\backslash\, \zeta_{FHS}\left(\epsilon\underline{\omega}_1 + (1-\epsilon)\underline{\omega}_2\right) \subseteq \sqcup\backslash\, \zeta_{FHS}\left(\underline{\omega}_1\right). \tag{7}$$

From (6) and (7) , we have

$$\sqcup\backslash\, \zeta_{FHS}\left(\epsilon\underline{\omega}_1 + (1-\epsilon)\underline{\omega}_2\right) \supseteq (\sqcup\backslash\zeta_{FHS}\left(\underline{\omega}_1\right)) \cup (\sqcup\backslash\zeta_M\left(\underline{\omega}_2\right)).$$

So $(\zeta_{FHS}, G)^c$ is a concave fuzzy hypersoft set. □

Theorem 7 *(ζ_{FHS}, G) is concave fuzzy hypersoft set iff for every $\epsilon \in J^\bullet$ and $\delta \in F(\sqcup)$, $(\zeta_{FHS}, G)^\delta$ is concave fuzzy hypersoft set.*

Proof Suppose (ζ_{FHS}, G) is concave fuzzy hypersoft set. If $\underline{\omega}, \underline{\mu} \in G$ and $\delta \in F(\sqcup)$, then $\zeta_{FHS}\left(\underline{\omega}\right) \supseteq \delta$ and $\zeta_{FHS}\left(\underline{\mu}\right) \supseteq \delta$, it implies that $\zeta_{FHS}\left(\underline{\omega}\right) \cup \zeta_{FHS}\left(\underline{\mu}\right) \supseteq \delta$.
So we have,

$$\delta \subseteq \zeta_{FHS}\left(\underline{\omega}\right) \cap \zeta_{FHS}\left(\underline{\mu}\right) \subseteq \zeta_{FHS}\left(\epsilon\underline{\omega} + (1-\epsilon)\,\underline{\mu}\right) \subseteq \zeta_{FHS}\left(\underline{\omega}\right) \cup \zeta_{FHS}\left(\underline{\mu}\right)$$

$\Rightarrow \quad \delta \subseteq \zeta_{FHS}\left(\epsilon\underline{\omega} + (1-\epsilon)\,\underline{\mu}\right)$
thus $(\zeta_{FHS}, G)^\delta$ is concave fuzzy hypersoft set.
Conversely, suppose that $(\zeta_{FHS}, G)^\delta$ is concave fuzzy hypersoft set for every $\epsilon \in J^\bullet$. For $\underline{\omega}, \underline{\mu} \in G$, $(\zeta_{FHS}, G)^\delta$ is concave fuzzy hypersoft set with $\delta = \zeta_{FHS}\left(\underline{\omega}\right) \cup \zeta_{FHS}\left(\underline{\mu}\right)$.
Since $\zeta_{FHS}\left(\underline{\omega}\right) \subseteq \delta$ and $\zeta_{FHS}\left(\underline{\mu}\right) \subseteq \delta$, we have
$\underline{\omega} \in (\zeta_{FHS}, G)^\delta$ and $\underline{\mu} \in (\zeta_{FHS}, G)^\delta$,
$\Rightarrow \quad \epsilon\underline{\omega} + (1-\epsilon)\,\underline{\mu} \in (\zeta_{FHS}, G)^\delta$.

Therefore,

$$\zeta_{FHS}\left(\epsilon\underline{\omega}+(1-\epsilon)\,\underline{\mu}\right)\subseteq\ \delta$$

So

$$\zeta_{FHS}\left(\epsilon\underline{\omega}+(1-\epsilon)\,\underline{\mu}\right)\subseteq\ \zeta_{FHS}\left(\underline{\omega}\right)\cup\zeta_{FHS}\left(\underline{\mu}\right),$$

Hence, (ζ_{FHS}, G) is concave fuzzy hypersoft set. □

4 Strongly and strictly convex cum concave fuzzy hypersoft sets

Here, strongly and strictly convex (concave) fuzzy hypersoft sets are defined, and some results are generalized with proofs.

Definition 23 The fuzzy hypersoft set (ζ_{FHS}, G) is called a *strongly convex fuzzy hypersoft set* if

$$\zeta_{FHS}\left(\alpha\underline{\beta}+(1-\alpha)\,\underline{\theta}\right)\supset\zeta_{FHS}\left(\underline{\beta}\right)\cap\zeta_{FHS}\left(\underline{\theta}\right)$$

for every

$$\underline{\beta}=(a^{11},a^{12},a^{13},.....,a^{1n})$$
$$\underline{\theta}=(a^{21},a^{22},a^{23},.....,a^{2n})\in G,\ \underline{\beta}\neq\underline{\theta}$$

where $G=A^1\times A^2\times A^3\times.....\times A^n$ with $A^i\cap A^j=\emptyset$, for $i\neq j$, and $i,j\in\{1,2,3,...,n\}$; $\zeta_{FHS}:G\rightarrow P(\sqcup)$ and $\alpha\in J^\circ$.

Definition 24 The fuzzy hypersoft set (ζ_{FHS}, G) on is called a *strongly concave fuzzy hypersoft set* if

$$\zeta_{FHS}\left(\alpha\underline{\beta}+(1-\alpha)\,\underline{\theta}\right)\subset\zeta_{FHS}\left(\underline{\beta}\right)\cup\zeta_{FHS}\left(\underline{\theta}\right)$$

for every $\underline{\beta}$, $\underline{\theta}\in G$, $\underline{\beta}\neq\underline{\theta}$ where, $G=A^1\times A^2\times A^3\times.....\times A^n$ with $A^i\cap A^j=\emptyset$, for $i\neq j$, and $i,j\in\{1,2,3,...,n\}$; $\zeta_{FHS}:G\rightarrow P(\sqcup)$ and $\alpha\in J^\circ$.

Definition 25 The fuzzy hypersoft set (ζ_{FHS}, G) is called a *strictly convex fuzzy hypersoft set* if

$$\zeta_{FHS}\left(\alpha\underline{\beta}+(1-\alpha)\,\underline{\theta}\right)\supset\zeta_{FHS}\left(\underline{\beta}\right)\cap\zeta_{FHS}\left(\underline{\theta}\right)$$

for every $\underline{\beta}$, $\underline{\theta}\in G$ where $G=A^1\times A^2\times A^3\times.....\times A^n$ with $A^i\cap A^j=\emptyset$, for $i\neq j$, and $i,j\in\{1,2,3,...,n\}$; $\zeta_{FHS}:G\rightarrow P(\sqcup)$, $\zeta_{FHS}\left(\underline{\beta}\right)\neq\zeta_{FHS}\left(\underline{\theta}\right)$ and $\alpha\in J^\circ$.

Definition 26 The fuzzy hypersoft set (ζ_{FHS}, G) is called a *strictly concave fuzzy hypersoft set* if

$$\zeta_{FHS}\left(\alpha\underline{\beta}+(1-\alpha)\,\underline{\theta}\right)\subset\zeta_{FHS}\left(\underline{\beta}\right)\cup\zeta_{FHS}\left(\underline{\theta}\right)$$

for every $\underline{\beta}$, $\underline{\theta}\in G$ where, $G=A^1\times A^2\times A^3\times.....\times A^n$ with $A^i\cap A^j=\emptyset$, for $i\neq j$, and $i,j\in\{1,2,3,...,n\}$; $\zeta_{FHS}:G\rightarrow P(\sqcup)$, $\zeta_{FHS}\left(\underline{\beta}\right)\neq\zeta_{FHS}\left(\underline{\theta}\right)$ and $\alpha\in J^\circ$.

Example 2 Suppose a university wants to observe (evaluate) the characteristics of its teachers by some defined indicators. For this purpose, consider a set of teachers as a universe of discourse $\sqcup=\{t_1,t_2,t_3,...,t_{10}\}$. The attributes of the teachers under consideration are the set $A=\{a^{11},a^{12},a^{13}\}$, where

$$\begin{aligned}a^{11}&=\text{Total experience in years}\\a^{12}&=\text{Total no. of publications}\\a^{13}&=\text{Student's evaluation against each teacher}\end{aligned}$$

such that the attribute values against these attributes, respectively, are the sets given as

$$\begin{aligned}A^{11}&=\{1year,2years,3years,4years,5years\}\\A^{12}&=\{1,2,3,4,5\}\\A^{13}&=\left\{\begin{matrix}Excellent(1),verygood(2),good(3),\\average(4),bad(5)\end{matrix}\right\}\end{aligned}$$

For simplicity, we write

$$\begin{aligned}A^{11}&=\{1,2,3,4,5\}\\A^{12}&=\{1,2,3,4,5\}\\A^{13}&=\{1,2,3,4,5\}\end{aligned}$$

The fuzzy hypersoft set (ζ_{FHS}, G) is a function defined by the mapping $\zeta_{FHS}:G\rightarrow F(\sqcup)$ where $G=A^{11}\times A^{12}\times A^{13}$.
Since the elements of $A^{11}\times A^{12}\times A^{13}$ is a 3-tuple, we consider $\underline{\beta}=(2,1,3)$; then, the function becomes $\zeta_{FHS}(\underline{\beta})=\zeta_{FHS}(2,1,3)=\{0.1/t_1,0.5/t_5\}$. Also, consider $\underline{\theta}=(3,2,2)$, then the function becomes
$\zeta_{FHS}(\underline{\theta})=\zeta_{FHS}(3,2,2)=\{0.1/t_1,0.3/t_3,0.4/t_4\}$
Now,
$\zeta_{FHS}(\underline{\beta})\cap\zeta_{FHS}(\underline{\theta})=\zeta_{FHS}(2,1,3)\cap\zeta_{FHS}(3,2,2)$
$\zeta_{FHS}(\underline{\beta})\cap\zeta_{FHS}(\underline{\theta})=\{0.1/t_1,0.5/t_5\}\cap\{0.1/t_1,0.3/t_3,0.4/t_4\}$

$$\zeta_{FHS}(\underline{\beta})\cap\zeta_{FHS}(\underline{\theta})=\{0.1/t_1\}\tag{8}$$

Let $\alpha=0.6\in J^\circ$, then, we have
$\alpha\underline{\beta}+(1-\alpha)\underline{\theta}=0.6(2,1,3)+(1-0.6)(3,2,2)=0.6(2,1,3)+0.4(3,2,2)$
$=(1.2,0.6,1.8)+(1.2,0.8,0.8)=(1.2+1.2,0.6+0.8,1.8+0.8)=(2.4,1.4,2.6)$

which is again a 3-tuple. By using the decimal round off property, we get (2, 1, 3)

$$\zeta_{FHS}(\alpha\underline{\beta} + (1-\alpha)\underline{\theta}) = \zeta_{FHS}(2, 1, 3) = \{0.1/t_1, 0.5/t_5\} \tag{9}$$

It is vivid from equations (8) and (9), we have

$$\zeta_{FHS}\left(\alpha\underline{\beta} + (1-\alpha)\,\underline{\theta}\right) \supset \zeta_{FHS}\left(\underline{\beta}\right) \cap \zeta_{FHS}\left(\underline{\theta}\right)$$

Similarly,
$\zeta_{FHS}(\underline{\beta}) \cup \zeta_{FHS}(\underline{\theta}) = \zeta_{FHS}(2, 1, 3) \cup \zeta_{FHS}(3, 2, 2)$
$\zeta_{FHS}(\underline{\beta}) \cap \zeta_{FHS}(\underline{\theta}) = \{0.1/t_1, 0.5/t_5\} \cup \{0.1/t_1, 0.3/t_3, 0.4/t_4\}$

$$\zeta_{FHS}(\underline{\beta}) \cap \zeta_{FHS}(\underline{\theta}) = \{0.1/t_1, 0.3/t_3, 0.4/t_4, 0.5/t_5\} \tag{10}$$

From (9) and (10), we have

$$\zeta_{FHS}\left(\alpha\underline{\beta} + (1-\alpha)\,\underline{\theta}\right) \subset \zeta_{FHS}\left(\underline{\beta}\right) \cup \zeta_{FHS}\left(\underline{\theta}\right)$$

Theorem 8 *(ζ_{FHS}, G) be a strictly convex fuzzy hypersoft set.If there exists $\alpha \in J^\bullet$, $\forall\, \underline{\beta}, \underline{\theta} \in G$ such that*

$$\zeta_{FHS}\left(\alpha\underline{\beta} + (1-\alpha)\,\underline{\theta}\right) \supseteq \zeta_{FHS}\left(\underline{\beta}\right) \cap \zeta_{FHS}\left(\underline{\theta}\right)$$

then (ζ_{FHS}, G) is a convex fuzzy hypersoft set.

Proof Assume that $\zeta_{FHS}\left(\underline{\beta}\right) \subseteq \zeta_{FHS}\left(\underline{\theta}\right)$ and $\exists\, \underline{\beta}, \underline{\theta} \in G), \alpha_1 \in J^\bullet$ such that

$$\begin{aligned} &\sqcup\backslash\, \zeta_{FHS}\left(\alpha_1\underline{\beta} + (1-\alpha_1)\underline{\theta}\right) \\ &\quad \supseteq \sqcup\backslash \left\{\zeta_{FHS}\left(\underline{\beta}\right) \cap \zeta_{FHS}\left(\underline{\theta}\right)\right\} \end{aligned} \tag{11}$$

If $\zeta_{FHS}\left(\underline{\beta}\right) \subset \zeta_{FHS}\left(\underline{\theta}\right)$, then (11) contradicting (ζ_{FHS}, G) is a strictly convex fuzzy hypersoft set.
If $\zeta_{FHS}\left(\underline{\beta}\right) = \zeta_{FHS}\left(\underline{\theta}\right)$ and $\alpha_1 \in [0, \alpha]$, let $\underline{\nu} = \frac{\alpha_1}{\alpha}(\underline{\beta}) + (1-\frac{\alpha_1}{\alpha})\underline{\theta}$ and $\alpha_2 = (\frac{1}{\alpha} - 1)(\frac{1}{\alpha_1} - 1)^{-1}$. Thus, by hypothesis,

$$\begin{aligned} &\zeta_{FHS}\left(\alpha_1\underline{\beta} + (1-\alpha_1)\underline{\theta}\right) \\ &= \zeta_{FHS}\left(\alpha(\frac{\alpha_1}{\alpha}(\underline{\beta}) + (1-\frac{\alpha_1}{\alpha})\underline{\theta}) + (1-\alpha)\underline{\theta}\right) \end{aligned}$$

$$= \zeta_{FHS}\left(\alpha\underline{\nu} + (1-\alpha)\underline{\theta}\right)$$

$$\zeta_{FHS}\left(\alpha_1\underline{\beta} + (1-\alpha_1)\underline{\theta}\right) \supseteq \zeta_{FHS}\left(\underline{\theta}\right) \cap \zeta_{FHS}\left(\underline{\nu}\right) \tag{12}$$

Now,

$$\zeta_{FHS}\left(\underline{\nu}\right) = \zeta_{FHS}\left(\frac{\alpha_1}{\alpha}(\underline{\beta}) + (1-\frac{\alpha_1}{\alpha})\underline{\theta}\right)$$

$$\zeta_{FHS}\left(\underline{\nu}\right) = \zeta_{FHS}\left(\alpha_3\underline{\beta} + (1-\alpha_3)(\alpha_1\underline{\beta} + (1-\alpha_1)\underline{\theta})\right) \tag{13}$$

From (11), (12) and $\zeta_{FHS}\left(\underline{\beta}\right) = \zeta_{FHS}\left(\underline{\theta}\right)$, it follows that

$$\zeta_{FHS}\left(\alpha_1\underline{\beta} + (1-\alpha_1)\underline{\theta}\right) \supseteq \zeta_{FHS}\left(\underline{\nu}\right) \tag{14}$$

From (11), (13), $\zeta_{FHS}\left(\underline{\beta}\right) = \zeta_{FHS}\left(\underline{\theta}\right)$ and strictly convex fuzzy hypersoft set condition, it follows that

$$\zeta_{FHS}\left(\underline{\nu}\right) \supset \zeta_{FHS}\left(\alpha_1\underline{\beta} + (1-\alpha_1)\underline{\theta}\right) \tag{15}$$

or

$$\sqcup\backslash\zeta_{FHS}\left(\alpha_1\underline{\beta} + (1-\alpha_1)\underline{\theta}\right) \supseteq \sqcup\backslash\zeta_{FHS}\left(\underline{\nu}\right) \tag{16}$$

Hence, (14) and (16) gives contradicts.
If $\zeta_{FHS}\left(\underline{\beta}\right) = \zeta_{FHS}\left(\underline{\theta}\right)$ and $\alpha_1 \in [\alpha, 1]$, let $\underline{\rho} = (\frac{\alpha_1-\alpha}{1-\alpha})\underline{\beta} + (\frac{1-\alpha_1}{1-\alpha})\underline{\theta}$. Thus, by hypothesis,

$$\begin{aligned} &\zeta_{FHS}\left(\alpha_1\underline{\beta} + (1-\alpha_1)\underline{\theta}\right) = \zeta_{FHS}\left(\alpha\underline{\beta} + (1-\alpha)\underline{\rho}\right) \\ &\zeta_{FHS}\left(\alpha_1\underline{\beta} + (1-\alpha_1)\underline{\theta}\right) \supseteq \zeta_{FHS}\left(\underline{\beta}\right) \cap \zeta_{FHS}\left(\underline{\rho}\right) \end{aligned} \tag{17}$$

From (11), (17) and $\zeta_{FHS}\left(\underline{\beta}\right) = \zeta_{FHS}\left(\underline{\theta}\right)$, it follows that

$$\zeta_{FHS}\left(\alpha_1\underline{\beta} + (1-\alpha_1)\underline{\theta}\right) \supseteq \zeta_{FHS}\left(\underline{\rho}\right) \tag{18}$$

On the other hand, $\alpha_1\underline{\beta} + (1-\alpha_1)\,\underline{\theta} = \alpha\underline{\beta} + (1-\alpha)\,\underline{\rho}$ gives

$$\begin{aligned} \underline{\rho} &= \left(\frac{1}{1-\alpha}\right)\left(\alpha_1\underline{\beta} + (1-\alpha_1)\underline{\theta}\right) - \left(\frac{\alpha}{\alpha-1}\right)\underline{\beta} \\ \underline{\rho} &= \left(\frac{1}{1-\alpha}\right)\left(\alpha_1\underline{\beta} + (1-\alpha_1)\,\underline{\theta}\right) \\ &\quad - \left(\frac{\alpha}{\alpha-1}\right)\left(\frac{1}{\alpha_1}\left(\alpha_1\underline{\beta} + (1-\alpha_1)\,\underline{\theta}\right) - \frac{1-\alpha_1}{\alpha_1}\underline{\theta}\right) \\ \underline{\rho} &= \left(\frac{\alpha_1-\alpha}{(1-\alpha)\,\alpha_1}\right)\left(\alpha_1\underline{\beta} + (1-\alpha_1)\,\underline{\theta}\right) \\ &\quad + \left(1 - \frac{\alpha_1-\alpha}{(1-\alpha)\,\alpha_1}\right)\underline{\theta} \end{aligned} \tag{19}$$

From (11), (19), $\zeta_{FHS}\left(\underline{\beta}\right) = \zeta_{FHS}\left(\underline{\theta}\right)$ and strictly convex fuzzy hypersoft set condition, it follows that

$$\sqcup\backslash\zeta_{FHS}\left(\alpha_1\underline{\beta} + (1-\alpha_1)\underline{\theta}\right) \supseteq \sqcup\backslash\zeta_{FHS}\left(\underline{\rho}\right) \tag{20}$$

Hence, (18) and (20) gives a contradict. □

Theorem 9 (ζ_{FHS}, G) *be a strictly concave fuzzy hypersoft set.If there exists* $\alpha \in J^\bullet$, $\forall\, \underline{\beta}, \underline{\theta} \in G$ *such that*

$$\zeta_{FHS}\left(\alpha\underline{\beta} + (1-\alpha)\,\underline{\theta}\right) \subseteq \zeta_{FHS}\left(\underline{\beta}\right) \cup \zeta_{FHS}\left(\underline{\theta}\right)$$

then (ζ_{FHS}, G) *is a concave fuzzy hypersoft set.*

Proof This can easily be proved by following the procedure discussed in Theorem (8). □

Theorem 10 *Let* (ζ_{FHS}, G) *be a convex fuzzy hypersoft set. If there exists* $\alpha \in J^\bullet$, $\forall\, \underline{\beta}, \underline{\theta} \in G$, $\zeta_{FHS}\left(\underline{\beta}\right) \neq \zeta_{FHS}\left(\underline{\theta}\right)$ *such that*

$$\zeta_{FHS}\left(\alpha\underline{\beta} + (1-\alpha)\,\underline{\theta}\right) \supset \zeta_{FHS}\left(\underline{\beta}\right) \cap \zeta_{FHS}\left(\underline{\theta}\right)$$

then (ζ_{FHS}, G) *is a strictly convex fuzzy hypersoft set.*

Proof Assume that $\exists\ \underline{\beta}, \underline{\theta} \in G), \alpha_1 \in J^\bullet$ such that

$$\begin{aligned}&\sqcup\backslash\, \zeta_{FHS}\left(\alpha_1\underline{\beta} + (1-\alpha_1)\underline{\theta}\right)\\ &\quad \supset \sqcup\backslash\left\{\zeta_{FHS}\left(\underline{\beta}\right) \cap \zeta_{FHS}\left(\underline{\theta}\right)\right\}\end{aligned} \tag{21}$$

If $\zeta_{FHS}\left(\underline{\beta}\right) \supset \zeta_{FHS}\left(\underline{\theta}\right)$, then (21) gives

$$\sqcup\backslash\, \zeta_{FHS}\left(\alpha_1\underline{\beta} + (1-\alpha_1)\underline{\theta}\right) \supset \sqcup\backslash\zeta_{FHS}\left(\underline{\theta}\right) \tag{22}$$

On the other hand, from the convex fuzzy hypersoft set condition, we have that

$$\zeta_{FHS}\left(\alpha_1\underline{\beta} + (1-\alpha_1)\underline{\theta}\right) \supseteq \left\{\zeta_{FHS}\left(\underline{\beta}\right) \cap \zeta_{FHS}\left(\underline{\theta}\right)\right\} \tag{23}$$

From (21) and (23), it follows that

$$\zeta_{FHS}\left(\alpha_1\underline{\beta} + (1-\alpha_1)\underline{\theta}\right) = \left\{\zeta_{FHS}\left(\underline{\beta}\right) \cap \zeta_{FHS}\left(\underline{\theta}\right)\right\} \tag{24}$$

which together with $\zeta_{FHS}\left(\underline{\beta}\right) \supset \zeta_{FHS}\left(\underline{\theta}\right)$, getting that

$$\zeta_{FHS}\left(\alpha_1\underline{\beta} + (1-\alpha_1)\underline{\theta}\right) = \zeta_{FHS}\left(\underline{\theta}\right) \tag{25}$$

or

$$\zeta_{FHS}\left(\alpha_1\underline{\beta} + (1-\alpha_1)\underline{\theta}\right) \subset \zeta_{FHS}\left(\underline{\beta}\right) \tag{26}$$

Thus, from (26) and the hypothesis,

$$\begin{aligned}&\zeta_{FHS}\left(\alpha\underline{\beta} + (1-\alpha)(\alpha_1\underline{\beta} + (1-\alpha_1)\underline{\theta})\right) \supset\\ &\zeta_{FHS}\left(\alpha_1\underline{\beta} + (1-\alpha_1)\underline{\theta}\right)\end{aligned} \tag{27}$$

More generally, for $n \in \{1, 2, 3, ...\}$ can easily show that

$$\begin{aligned}&\zeta_{FHS}\left(\alpha^n\underline{\beta} + (1-\alpha^n)(\alpha_1\underline{\beta} + (1-\alpha_1)\underline{\theta})\right) \supset\\ &\zeta_{FHS}\left(\alpha_1\underline{\beta} + (1-\alpha_1)\underline{\theta}\right)\end{aligned} \tag{28}$$

Let $\underline{\nu} = \alpha_2\underline{\beta} + (1-\alpha_2)\underline{\theta}$ where $\alpha_2 = \alpha_1 - \alpha^n\alpha_1 + \alpha^n \in J^\bullet$ for some n . Then, from (28), we see that

$$\begin{aligned}&\zeta_{FHS}\left(\underline{\nu}\right) = \zeta_{FHS}\left(\alpha_2\underline{\beta} + (1-\alpha_2)\underline{\theta}\right)\\ &\zeta_{FHS}\left(\underline{\nu}\right) = \zeta_{FHS}\left(\alpha^n\underline{\beta} + (1-\alpha^n)(\alpha_1\underline{\beta} + (1-\alpha_1)\underline{\theta})\right)\\ &\zeta_{FHS}\left(\nu\right) \supset \zeta_{FHS}\left(\alpha_1\underline{\beta} + (1-\alpha_1)\underline{\theta}\right)\end{aligned} \tag{29}$$

Also, let $\underline{\rho} = \alpha_3\underline{\beta} + (1-\alpha_3)\underline{\theta}$ where $\alpha_3 = \alpha_1 - \alpha^n + \frac{1}{1-\alpha} + \frac{\alpha^n\alpha_1}{1-\alpha} \in J^\bullet$ for some n.
Then,

$$\zeta_{FHS}\left(\alpha_1\underline{\beta} + (1-\alpha_1)\underline{\theta}\right) = \zeta_{FHS}\left(\alpha\underline{\nu} + (1-\alpha)\underline{\rho}\right) \tag{30}$$

Now, if $\zeta_{FHS}\left(\underline{\nu}\right) \subseteq \zeta_{FHS}\left(\underline{\rho}\right)$, then (30) and (ζ_{FHS}, G) is a convex fuzzy hypersoft set implies that

$$\sqcup\backslash\, \zeta_{FHS}\left(\underline{\nu}\right) \supset \sqcup\backslash\zeta_{FHS}\left(\alpha_1\underline{\beta} + (1-\alpha_1)\underline{\theta}\right)$$

this contradicts (29).

If $\sqcup\backslash\, \zeta_{FHS}\left(\underline{\nu}\right) \subseteq \sqcup\backslash\, \zeta_{FHS}\left(\underline{\rho}\right)$, then (30) and the hypothesis of the theorem implies that

$$\begin{aligned}&\zeta_{FHS}\left(\alpha_1\underline{\beta} + (1-\alpha_1)\underline{\theta}\right) \supset \zeta_{FHS}\left(\underline{\nu}\right) \cap \zeta_{FHS}\left(\underline{\rho}\right)\\ &\zeta_{FHS}\left(\alpha_1\underline{\beta} + (1-\alpha_1)\underline{\theta}\right) \supseteq (\zeta_{FHS}\left(\underline{\beta}\right) \cap \zeta_{FHS}\left(\underline{\theta}\right)) \cap\\ &(\zeta_{FHS}\left(\underline{\beta}\right) \cap \zeta_{FHS}\left(\underline{\theta}\right)) = \zeta_{FHS}\left(\underline{\beta}\right)\end{aligned}$$

This contradicts (26). □

Theorem 11 *Let* (ζ_{FHS}, G) *be a concave fuzzy hypersoft set. If there exists* $\alpha \in J^\bullet$, $\forall\, \underline{\beta}, \underline{\theta} \in G$, $\zeta_{FHS}\left(\underline{\beta}\right) \neq \zeta_{FHS}\left(\underline{\theta}\right)$ *such that*

$$\zeta_{FHS}\left(\alpha\underline{\beta} + (1-\alpha)\,\underline{\theta}\right) \subset \zeta_{FHS}\left(\underline{\beta}\right) \cup \zeta_{FHS}\left(\underline{\theta}\right)$$

then (ζ_{FHS}, G) *is a strictly concave fuzzy hypersoft set.*

Proof This can easily be proved by following the procedure discussed in Theorem (10). □

Theorem 12 *Let* (ζ_{FHS}, G) *be a strongly convex fuzzy hypersoft set. If there exists* $\alpha \in J^\bullet$, $\forall\, \underline{\beta}, \underline{\theta} \in G$, *such that*

$$\zeta_{FHS}\left(\alpha\underline{\beta} + (1-\alpha)\,\underline{\theta}\right) \supseteq \zeta_{FHS}\left(\underline{\beta}\right) \cap \zeta_{FHS}\left(\underline{\theta}\right) \tag{31}$$

then (ζ_{FHS}, G) *is a convex fuzzy hypersoft set.*

Proof Assume that $\exists\ \underline{\beta}, \underline{\theta} \in G), \alpha_1 \in J^{\bullet}$ such that

$$\begin{aligned} &\sqcup\backslash\, \zeta_{FHS}\left(\alpha_1\underline{\beta} + (1-\alpha_1)\underline{\theta}\right) \\ &\quad \supseteq \sqcup\backslash \left\{\zeta_{FHS}\left(\underline{\beta}\right) \cap \zeta_{FHS}\left(\underline{\theta}\right)\right\} \end{aligned} \tag{32}$$

If $\underline{\beta} \neq \underline{\theta}$, then (31) contradicting that (ζ_{FHS}, G) is a strongly convex fuzzy hypersoft set.

If $\underline{\beta} = \underline{\theta}$, then choose $\alpha_1 \neq \alpha_2 \in J^{\circ}$ such that $\alpha_1 = \alpha\alpha_2 + (1-\alpha)\alpha_2$.

Let $\underline{\beta} = \underline{\theta} = \alpha_2\underline{\beta} + (1-\alpha_2)\underline{\theta}$. Then, (31) implies that

$$\sqcup\backslash\, \zeta_{FHS}\left(\underline{\beta}\right) \supseteq \sqcup\backslash \left\{\zeta_{FHS}\left(\underline{\beta}\right) \cap \zeta_{FHS}\left(\underline{\theta}\right)\right\} \tag{33}$$

$$\sqcup\backslash\, \zeta_{FHS}\left(\underline{\theta}\right) \supseteq \sqcup\backslash \left\{\zeta_{FHS}\left(\underline{\beta}\right) \cap \zeta_{FHS}\left(\underline{\theta}\right)\right\} \tag{34}$$

According to (31), (33) and (34), we have

$$\begin{aligned} &\zeta_{FHS}\left(\alpha\underline{\beta} + (1-\alpha)\,\underline{\theta}\right) \supseteq \zeta_{FHS}\left(\underline{\beta}\right) \cap \zeta_{FHS}\left(\underline{\theta}\right) \\ &\subset (\zeta_{FHS}\left(\underline{\beta}\right) \cap \zeta_{FHS}\left(\underline{\theta}\right)) \cap (\zeta_{FHS}\left(\underline{\beta}\right) \cap \zeta_{FHS}\left(\underline{\theta}\right)) \\ &= \zeta_{FHS}\left(\underline{\beta}\right) \cap \zeta_{FHS}\left(\underline{\theta}\right) \end{aligned}$$

which contradicts that (ζ_{FHS}, G) is a strongly convex fuzzy hypersoft set. □

Theorem 13 *Let* (ζ_{FHS}, G) *be a strongly concave fuzzy hypersoft set. If there exists* $\alpha \in J^{\bullet}$, $\forall\, \underline{\beta}, \underline{\theta} \in G$, *such that*

$$\zeta_{FHS}\left(\alpha\underline{\beta} + (1-\alpha)\,\underline{\theta}\right) \subseteq \zeta_{FHS}\left(\underline{\beta}\right) \cup \zeta_{FHS}\left(\underline{\theta}\right)$$

then (ζ_{FHS}, G) *is a concave fuzzy hypersoft set.*

Proof This can easily be proved by following the procedure discussed in Theorem (12). □

Theorem 14 *Let* (ζ_{FHS}, G) *be a convex fuzzy hypersoft set. If there exists* $\alpha \in J^{\bullet}$, $\forall\, \underline{\beta}, \underline{\theta} \in G$, $\underline{\beta} \neq \underline{\theta}$, *such that*

$$\zeta_{FHS}\left(\alpha\underline{\beta} + (1-\alpha)\,\underline{\theta}\right) \supset \zeta_{FHS}\left(\underline{\beta}\right) \cap \zeta_{FHS}\left(\underline{\theta}\right)$$

then (ζ_{FHS}, G) *is a strongly convex fuzzy hypersoft set.*

Proof Assume that $\exists\ (\underline{\beta} \neq \underline{\theta})], \underline{\beta}, \underline{\theta} \in G), \alpha_1 \in J^{\bullet}$ such that

$$\begin{aligned} &\sqcup\backslash\, \zeta_{FHS}\left(\alpha_1\underline{\beta} + (1-\alpha_1)\underline{\theta}\right) \\ &\quad \supseteq \sqcup\backslash \left\{\zeta_{FHS}\left(\underline{\beta}\right) \cap \zeta_{FHS}\left(\underline{\theta}\right)\right\} \end{aligned} \tag{35}$$

Thus, from (35) and the convex fuzzy hypersoft set condition, we get that

$$\zeta_{FHS}\left(\alpha_1\underline{\beta} + (1-\alpha_1)\underline{\theta}\right) = \zeta_{FHS}\left(\underline{\beta}\right) \cap \zeta_{FHS}\left(\underline{\theta}\right) \tag{36}$$

Furthermore, it can be easily seen that

$$\alpha\underline{\beta} + (1-\alpha)\underline{\theta} = \alpha_1\underline{\beta} + (1-\alpha_1)\underline{\theta} \tag{37}$$

where both $\underline{\beta}$ and $\underline{\theta}$ are of the form $\underline{\beta} = \alpha_1\underline{\beta} + (1-\alpha_1)\underline{\theta}$ and $\underline{\theta} = \alpha_1\underline{\beta} + (1-\alpha_1)\underline{\theta}$ for choosing $\alpha_1 \in J^{\circ}$

On the other hand, from the convex fuzzy hypersoft set condition and our definition of $\underline{\beta}$ and $\underline{\theta}$, getting

$$\zeta_{FHS}\left(\underline{\beta}\right) \supseteq \zeta_{FHS}\left(\underline{\beta}\right) \cap \zeta_{FHS}\left(\underline{\theta}\right) \tag{38}$$

$$\zeta_{FHS}\left(\underline{\theta}\right) \supseteq \zeta_{FHS}\left(\underline{\beta}\right) \cap \zeta_{FHS}\left(\underline{\theta}\right) \tag{39}$$

Therefore, from (37), (38), (39) and the hypothesis of the theorem, we get that

$$\begin{aligned} &\zeta_{FHS}\left(\alpha_1\underline{\beta} + (1-\alpha_1)\underline{\theta}\right) \\ &\quad = \zeta_{FHS}\left(\alpha\underline{\beta} + (1-\alpha)\underline{\theta}\right) \supset \zeta_{FHS}\left(\underline{\beta}\right) \cap \zeta_{FHS}\left(\underline{\theta}\right) \\ &\quad \supseteq (\zeta_{FHS}\left(\underline{\beta}\right) \cap \zeta_{FHS}\left(\underline{\theta}\right)) \cap (\zeta_{FHS}\left(\underline{\beta}\right) \cap \zeta_{FHS}\left(\underline{\theta}\right)) \\ &\quad = \zeta_{FHS}\left(\underline{\beta}\right) \cap \zeta_{FHS}\left(\underline{\theta}\right) \end{aligned}$$

This contradicts (36). □

Theorem 15 *Let* (ζ_{FHS}, G) *be a concave fuzzy hypersoft set. If there exists* $\alpha \in J^{\bullet}$, $\forall\, \underline{\beta}, \underline{\theta} \in G$, $\underline{\beta} \neq \underline{\theta}$, *such that*

$$\zeta_{FHS}\left(\alpha\underline{\beta} + (1-\alpha)\,\underline{\theta}\right) \subset \zeta_{FHS}\left(\underline{\beta}\right) \cup \zeta_{FHS}\left(\underline{\theta}\right)$$

then (ζ_{FHS}, G) *is a strongly concave fuzzy hypersoft set.*

Proof This can easily be proved by following the procedure discussed in Theorem (14). □

5 Application in decision making

In this section, we first propose an algorithm based on proposed study and then apply it generally to solve a real-world problem through decision making.

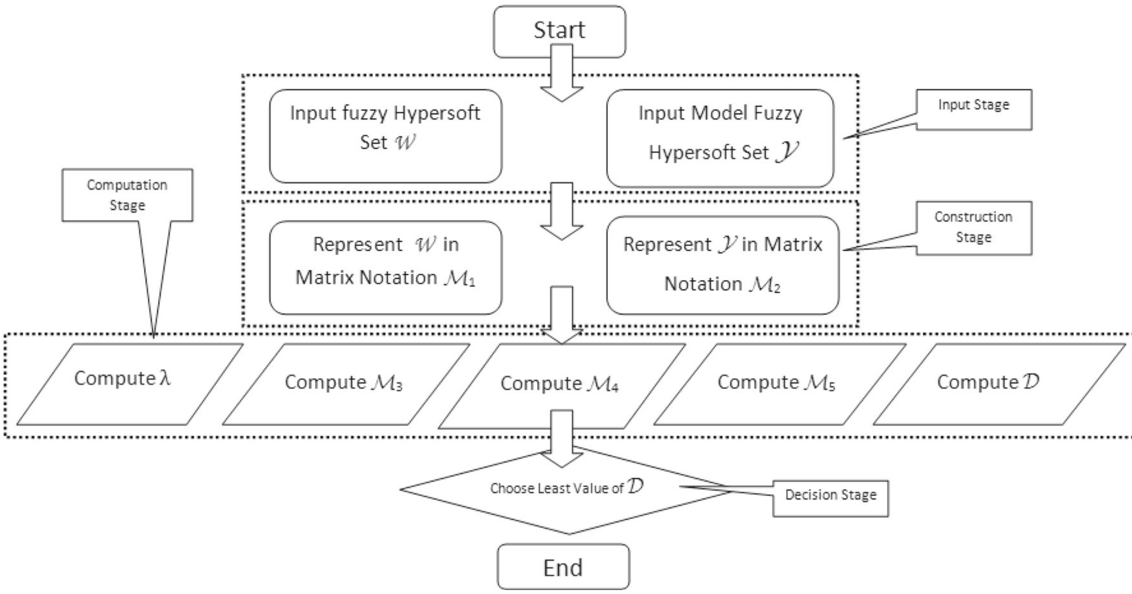

Fig. 4 Flowchart of Proposed Algorithm

Algorithm: Appropriate Selection of Hand-Sanitizer

▷ **Start**
▷ **Input:**
1. Input Fuzzy Hypersoft set $\mathscr{W}$
2. Input Model Fuzzy Hypersoft set $\mathscr{Y}$
▷ **Construction:**
3. Represent $\mathscr{W}$ in matrix notation $\mathscr{M}_1$
4. Represent $\mathscr{Y}$ in matrix notation $\mathscr{M}_2$
▷ **Computation:**
5. Compute λ by taking arithmetic mean of all fuzzy values used in $\mathscr{W}$ and $\mathscr{Y}$
6. Compute $\mathscr{M}_3 = \lambda\mathscr{M}_1 \bigvee (1-\lambda)\mathscr{M}_2$
7. Compute $\mathscr{M}_4 = \lambda\mathscr{M}_1 \bigwedge (1-\lambda)\mathscr{M}_2$
8. Compute $\mathscr{M}_5 = \mathscr{M}_3 \setminus \mathscr{M}_4$
9. Compute Score $\mathscr{D}$ corresponding to each member in the universe.
▷ **Output:**
10. Choose least score as final decision
▷ **End**

The flowchart of above algorithm is shown in Fig. 4.

5.1 Statement of the problem

Professor John is the head of an educational institution. He is very concerned about the health of the students as well as the faculty members of his institution in view of the current coronary epidemic. He wants to buy good and useful Hand Sanitizers for his institution but he is also worried about the non-standard Hand Sanitizers available in the market. So he decides to call for bids from different potential suppliers for this purchase to fulfil the departmental official compliances and to avoid any expected loss. Some suppliers are scrutinized by adopting proper procedure already framed by relevant department. For the sake of satisfaction, he constitutes a committee consisting of some staff members with good procurement experience to evaluate the items (Hand Sanitizers) offered by scrutinized suppliers. The following example elaborates the whole procedure of such evaluation:

Example 3 Suppose there are six kinds of Hand Sanitizer (options) which form the set of discourse $\sqcup = \{\sqcup_1, \sqcup_2\}$ where $\sqcup_1 = \{\mathscr{H}^1, \mathscr{H}^2, \mathscr{H}^3\}$ and $\sqcup_2 = \{\mathscr{H}^4, \mathscr{H}^5, \mathscr{H}^6\}$ are the collections of Hand Sanitizers made by manufacturers X_1 and X_2, respectively. With their mutual consensus, the committee members (experts) agreed on a set of parameters after observing various attributes for this evaluation. The finalized evaluating attributes are : b^1 = Manufacturer, b^2 = Quantity of Ethanol (percentage), b^3 = Quantity of Distilled Water (percentage), b^4 = Quantity of Glycerol (percentage), and b^5 = Quantity of Hydrogen peroxide (percentage). After observing the opinions of various professionals and other relevant sources on the composition of Hand Sanitizers, these attributes are further classified into attribute-valued sets which are given as:

$$B^1 = \{b^{11} = X_1, b^{12} = X_2\}$$
$$B^2 = \{b^{21} = 75.15, b^{22} = 80\}$$

$B^3 = \{b^{31} = 23.425, b^{32} = 18.425\}$
$B^4 = \{b^{41} = 1.30, b^{42} = 1.45\}$
$B^5 = \{b^{51} = 0.125\}$

then $\mathscr{Q} = B^1 \times B^2 \times B^3 \times B^4 \times B^5$
$\mathscr{Q} = \{q^1, q^2, q^3, q^4, ..., q^{16}\}$ where each q^i, $i = 1, 2, ...,$ 16, is a 5-tuple element. For convenience, take
$\mathscr{R} = \{q^1, q^4, q^7, q^9, q^{13}, q^{16}\} \subseteq \mathscr{Q}$.

Input Stage (1-2):
Then, the fuzzy hypersoft set $\mathscr{W}$ and model fuzzy hypersoft set $\mathscr{Y}$ corresponding to adopted $\mathscr{R}$ are constructed as: $\mathscr{W} =$

$$\left\{\begin{array}{l} \left(q^1, \mathscr{H}^1/0.1, \mathscr{H}^2/0.2, \mathscr{H}^3/0.3, \mathscr{H}^4/0.4, \mathscr{H}^5/0.5, \mathscr{H}^6/0.6\right), \\ \left(q^4, \mathscr{H}^1/0.2, \mathscr{H}^2/0.3, \mathscr{H}^3/0.4, \mathscr{H}^4/0.5, \mathscr{H}^5/0.6, \mathscr{H}^6/0.7\right), \\ \left(q^7, \mathscr{H}^1/0.3, \mathscr{H}^2/0.4, \mathscr{H}^3/0.5, \mathscr{H}^4/0.6, \mathscr{H}^5/0.7, \mathscr{H}^6/0.8\right), \\ \left(q^9, \mathscr{H}^1/0.4, \mathscr{H}^2/0.5, \mathscr{H}^3/0.6, \mathscr{H}^4/0.7, \mathscr{H}^5/0.8, \mathscr{H}^6/0.9\right), \\ \left(q^{13}, \mathscr{H}^1/0.5, \mathscr{H}^2/0.6, \mathscr{H}^3/0.7, \mathscr{H}^4/0.8, \mathscr{H}^5/0.9, \mathscr{H}^6/0.1\right), \\ \left(q^{16}, \mathscr{H}^1/0.6, \mathscr{H}^2/0.7, \mathscr{H}^3/0.8, \mathscr{H}^4/0.9, \mathscr{H}^5/0.1, \mathscr{H}^6/0.2\right) \end{array}\right\}$$

and $\mathscr{Y} =$

$$\left\{\begin{array}{l} \left(q^1, \mathscr{H}^1/0.4, \mathscr{H}^2/0.4, \mathscr{H}^3/0.4, \mathscr{H}^4/0.5, \mathscr{H}^5/0.5, \mathscr{H}^6/0.5\right), \\ \left(q^4, \mathscr{H}^1/0.3, \mathscr{H}^2/0.3, \mathscr{H}^3/0.3, \mathscr{H}^4/0.6, \mathscr{H}^5/0.6, \mathscr{H}^6/0.6\right), \\ \left(q^7, \mathscr{H}^1/0.2, \mathscr{H}^2/0.2, \mathscr{H}^3/0.2, \mathscr{H}^4/0.7, \mathscr{H}^5/0.7, \mathscr{H}^6/0.7\right), \\ \left(q^9, \mathscr{H}^1/0.5, \mathscr{H}^2/0.5, \mathscr{H}^3/0.5, \mathscr{H}^4/0.3, \mathscr{H}^5/0.3, \mathscr{H}^6/0.3\right), \\ \left(q^{13}, \mathscr{H}^1/0.1, \mathscr{H}^2/0.1, \mathscr{H}^3/0.1, \mathscr{H}^4/0.8, \mathscr{H}^5/0.8, \mathscr{H}^6/0.8\right), \\ \left(q^{16}, \mathscr{H}^1/0.6, \mathscr{H}^2/0.7, \mathscr{H}^3/0.8, \mathscr{H}^4/0.1, \mathscr{H}^5/0.1, \mathscr{H}^6/0.1\right) \end{array}\right\}$$

Construction Stage (3-4):
The matrix notations $\mathscr{M}_1 = [a_{ij}]_{(6\times 6)}$ and $\mathscr{M}_2 = [b_{ij}]_{(6\times 6)}$ of Fuzzy Hypersoft Sets $\mathscr{W}$ and $\mathscr{Y}$ are presented below where $\mathscr{H}^i$ are arranged in columns and opted attribute-valued tuples q^j are arranged in rows.

$$\mathscr{M}_1 = \begin{bmatrix} 0.1 & 0.2 & 0.3 & 0.4 & 0.5 & 0.6 \\ 0.2 & 0.3 & 0.4 & 0.5 & 0.6 & 0.7 \\ 0.3 & 0.4 & 0.5 & 0.6 & 0.7 & 0.8 \\ 0.4 & 0.5 & 0.6 & 0.7 & 0.8 & 0.9 \\ 0.5 & 0.6 & 0.7 & 0.8 & 0.9 & 0.1 \\ 0.6 & 0.7 & 0.8 & 0.9 & 0.1 & 0.2 \end{bmatrix}$$

$$\mathscr{M}_2 = \begin{bmatrix} 0.4 & 0.4 & 0.4 & 0.5 & 0.5 & 0.5 \\ 0.3 & 0.3 & 0.3 & 0.6 & 0.6 & 0.6 \\ 0.2 & 0.2 & 0.2 & 0.7 & 0.7 & 0.7 \\ 0.5 & 0.5 & 0.5 & 0.3 & 0.3 & 0.3 \\ 0.1 & 0.1 & 0.1 & 0.8 & 0.8 & 0.8 \\ 0.6 & 0.7 & 0.8 & 0.1 & 0.1 & 0.1 \end{bmatrix}$$

Computation Stage (5-9):
By taking arithmetic mean of all fuzzy values of $\mathscr{W}$ and $\mathscr{Y}$, we obtain the value of λ that is 0.4792 (rounded off to 4 decimal places). Now, we have

$$\mathscr{M}_3 = \begin{bmatrix} 0.25624 & 0.30416 & 0.35208 & 0.45208 & 0.5 & 0.54792 \\ 0.25208 & 0.3 & 0.34792 & 0.55208 & 0.6 & 0.3604 \\ 0.289584 & 0.337504 & 0.385424 & 0.65208 & 0.41428 & 0.4604 \\ 0.45208 & 0.5 & 0.54792 & 0.20416 & 0.25208 & 0.3 \\ 0.29168 & 0.3396 & 0.1 & 0.51248 & 0.5604 & 0.60832 \\ 0.6 & 0.41248 & 0.51248 & 0.19584 & 0.24376 & 0.29168 \end{bmatrix}$$

and $\mathscr{M}_4 = \begin{bmatrix} -0.1604 & -0.11248 & -0.06456 & -0.06872 & -0.0208 & 0.02712 \\ -0.0604 & -0.01248 & 0.03544 & -0.07288 & -0.02496 & -0.26456 \\ -0.00206 & 0.045856 & 0.093776 & -0.07704 & -0.31664 & -0.26872 \\ -0.06872 & -0.0208 & 0.02712 & -0.10832 & -0.0604 & -0.01248 \\ 0.18752 & 0.23544 & -0.00416 & -0.3208 & -0.27288 & -0.22496 \\ -0.02496 & -0.31664 & -0.3208 & 0.09168 & 0.1396 & 0.18752 \end{bmatrix}$.

Now, we compute $\mathscr{M}_5$ that is given as

$$\mathscr{M}_5 = \begin{bmatrix} 0.41664 & 0.41664 & 0.41664 & 0.5208 & 0.5208 & 0.5208 \\ 0.31248 & 0.31248 & 0.31248 & 0.62496 & 0.62496 & 0.62496 \\ 0.291648 & 0.291648 & 0.291648 & 0.72912 & 0.72912 & 0.72912 \\ 0.5208 & 0.5208 & 0.5208 & 0.31248 & 0.31248 & 0.31248 \\ 0.10416 & 0.10416 & 0.10416 & 0.83328 & 0.83328 & 0.83328 \\ 0.62496 & 0.72912 & 0.83328 & 0.10416 & 0.10416 & 0.10416 \end{bmatrix}.$$

It is pertinent to mention here that $\bigvee$ and $\bigwedge$ represents the ordinary sum and subtraction of matrices, whereas $\setminus$ denotes ordinary numerical difference. Hence, score values corresponding to each element of $\sqcup$ are given as

$$\mathscr{D} = \left\{ \begin{array}{l} \mathscr{H}^1/2.270688, \mathscr{H}^2/2.374848.\mathscr{H}^3/2.479008, \\ \mathscr{H}^4/3.1248, \mathscr{H}^5/3.1248, \mathscr{H}^6/3.1248 \end{array} \right\}.$$

Out Stage:
10. As $\mathscr{H}^1$ has attained the least score value 2.270688 that means its nature is more convex as compared to other so it is selected as most appropriate product.

6 Comparison analysis

In this section, the proposed study is compared with relevant models. The concepts presented by Deli (2019) and Salih and Sabir (2018) are the most relevant to this proposed study for comparison. Table 1 shows that the proposed study is more flexible as it addresses the insufficiencies of existing structures. As in the proposed study, attributes are observed deeply through entitlement of their respective sub-attributive values, the decision-making process becomes more reliable and authentic to have precise and accurate results while dealing with optimization-relating problems.

7 Conclusion

In this study, convexity cum concavity on fuzzy hypersoft sets is introduced by adopting an abstract cum analytical technique. This is novel addition in the literature and may enable the researchers to deal with important applications of convexity under fuzzy and hypersoft environment with precise results. Moreover, strictly and strongly convexity cum concavity on fuzzy hypersoft sets are also conceptualized along with generalized results. Future directions and scope are stated below:

1. A hypothetical data is used to validate the proposed study in decision making. This may be applied to real-life scenarios by using real data as a case study in medical sciences (diagnostic study), pattern recognition, image processing and operation research (convex optimization).
2. It is convenient to develop convex hull, convex cone and many other types of convexity like (m, n)-convexity, ϕ-convexity, graded convexity, triangular convexity, concavo-convexity, etc., on fuzzy hypersoft set by using proposed study.
3. Special conditions can be applied to the aggregation operations discussed in the proposed study to have more generalized results, e.g., considering the modified ver-

Table 1 Comparison of Proposed Study with Existing Relevant Structures

Authors	Structures	Focus on attributes	Focus on sub-attributive values	Nature of Approximate Function	Fuzzy Membership	Numerical Examples	Real-world Application
Deli (2019)	Convex (Concave) Soft Set	Yes	No	Single-Argument	No	No	No
Deli (2019)	Convex (Concave) Fuzzy Soft Set	Yes	No	Single-Argument	Yes	No	No
Salih and Sabir (2018)	Strictly and Strongly Convex (Concave) Soft Set	Yes	No	Single-Argument	No	No	No
Proposed Study	Convex (Concave) Fuzzy Hypersoft Set	Yes	Yes	Multi-Argument	Yes	Yes	Yes

sions of complement, intersection and union as discussed in Ali et al. (2009) and Maji et al. (2003).

4. This study may further be employed to establish certain mathematical inequalities, e.g., Ostrowski's Inequalities, Hardi's inequalities, etc., by introducing convex and convex hypersoft functions.
5. This may also be applied in algebraic structures like topological spaces, vector spaces, etc., to modify various important results.
6. As the proposed study emphasizes on membership degree, it can further be extended for considering non-membership and indeterminacy grades.

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Multi-Attribute Decision Support Model Based on Bijective Hypersoft Expert Set

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Abstract. Soft set tackles a single set of attributes whereas its extension hypersoft set is projected for dealing attribute-valued disjoint sets corresponding to distinct attributes with entitlement of multi-argument approximate function. In order to furnish soft set-like models with multi-decisive opinions of multi-experts, a new model i.e. soft expert set has been developed but this is inadequate for handling the scenario where partitioning of attributes into their respective attribute-valued sets is necessary. Hence hypersoft expert set has made its place to be developed. This article intends to develop a new type of hypersoft set called bijective hypersoft expert set which is more flexible and effective. After characterization of its essential properties and set-theoretic operations like union, relaxed and restricted AND, a decision-support system is designed which is characterized by new operations such as decision system, reduced decision system, etc. with illustrated examples. The proposed decision-support system is applied in multi-attribute decision-making process to manage a real-life application.

Key Words: Soft set, Soft expert set, Bijective soft set, Hypersoft set, Hypersoft expert set, Bijective hypersoft set, Bijective hypersoft expert set.

1. Introduction

In 1999, Molodtsov [35] introduced the structure of soft set to explain problems of vague data and uncertain environment. After this, Maji et al. [33] presented its some basic properties, operations, laws and used this structure in different fields to explain different situations. In 2005, Pei et al. [37] explained the relationship between soft set and information system. Many other researchers [5, 7, 8, 11, 32, 58] worked on this theory and introduced

some new operations, properties, laws and used them in decision-making problems. Saeed et al. [48] made use of this theory and introduced soft elements and members. Rahman et al. [39, 40] conceptualized m-convexity (m-concavity) and (m, n)-convexity ((m, n)-concavity) on soft sets with some properties. Gong et al. [12] conceptualized new structure of bijective soft set and discussed its essential characteristics. Kamacı et al. [20, 21, 22, 23] presented bijective soft matrix theory and reviewed different operations by making its use in decision making system. They also developed the structures of N-soft set, bipolar N-soft set and studied their applications. Many researchers [1, 2, 3, 9, 10, 26, 27, 36, 38, 64] broadened soft set theory and developed different soft set-like hybrids to make its use in different fields like decision making and information system.
Alkhazalah [4] introduced the concept of soft expert set to solve the problem of different opinions of different experts in a single model. He used this structure in different areas like medical diagnosis and decision making. Ihsan et al. [15, 16] introduced convexity (concavity) on soft expert sets and fuzzy soft expert sets respectively and proved their certain properties. Soft set is used only for single set of attributes whereas hypersoft set, developed by Smarandache [59], is useful for tackling further partitioning of attributes into their respective sub-attribute values in the form of disjoint sets. In 2020, Saeed et al. [49, 50] studied basic properties, operations of hypersoft set and explained with different examples. In 2020, Rahman et al. [41, 42] developed some new structures of hypersoft set like complex fuzzy hypersoft set and introduced the concept of convexity in hypersoft set. In 2021, Rahman et al. [43, 44, 45] presented some new structures like rough, fuzzy parameterized, neutrosophic hypersoft set and used them in decision-making problems. Saeed et al. [51, 52, 53, 54] worked on hypersoft classes, complex multi-fuzzy hypersoft set and explained some new methods of decision making for mappings. They also presented hypersoft graphs, a new class of hypersoft set with some characteristics. Working on hypersoft set, Yolcu et al. [62] gave the idea of fuzzy, intuitionistic fuzzy hypersoft sets and made use of them in decision making. Saqlain et al. [55, 56, 57] made their contributions in hypersoft set by introducing single and multi-valued neutrosophic hypersoft sets and calculated their tangent similarity measures. They described aggregate operators and TOPSIS method for neutrosophic hypersoft sets. Ihsan et al. [17] extended hypersoft set to hypersoft expert set and used it in decision-making problems. Ihsan et al. [18] introduced the structure of fuzzy hypersoft expert set with application in decision making problem. Ali et al. [6] developed the Einstein geometric aggregation operators using a novel complex interval-valued Pythagorean Fuzzy setting with application in green supplier chain management. Riaz et al. [47] worked on decision-making problems and described certain properties of soft multi set topologies with applications in decision-making problems.

1.1. **Research Gap and Motivation.** Following points will explain the research gap and motivations behind the choice of proposed structure:

(1) Gong et al. [12] introduced the concept of bijective soft set and discussed its necessary operations with an application in decision-making problems. After this, Gong et al. [13] extended their work in fuzzy environment and introduced the concept of bijective fuzzy soft set to deal with more uncertain problems. Kumar et al. [28] applied this concept of bijective soft set in classification of medical data. Tiwari et al. [60] used an integrated Shannon entropy and TOPSIS for product

design concept evaluation based on bijective soft set. Gong et al. [14] worked on bijective soft set and used it in mining data from soft set environments and applied it in some fields. Tiwari et al. [61] made a bijective soft set theoretic approach for concept selection in design process. Inbarani et al. [19] introduced the idea of rough bijective soft set and applied in medical field. Kumar et al. [29] gave the idea of improved bijective soft set and proposed a model for the classification of cancer based on gene expression profiles. Kumar et al. [30] discussed the structure of hybrid bijective soft set to construct novel automatic classification system for analysis of ECG signal and decision making purposes. Kumar et al. [31] also applied this structure for the identification of heart valve disease. Kamac et al. [25] introduced the concept of bijective soft matrix and applied for decision-making problems.

(2) It can be seen that the above bijective soft set like models deal with opinion of only single expert. But in real life, there are certain situations where we need different opinions of different experts in one model. To tackle this situation, soft expert set has been developed. However, there are also certain situations when attributes are further classified into their respective attribute-valued disjoint sets. Therefore, there is a need of new structure to handle such situations with multi-decisive opinions under multi-argument soft set like environment. So hypersoft expert set is developed.

(3) Having motivation from the above literature in general and specifically from [12] and [17], a novel structure bijective hypersoft expert set (BHSES) is developed with certain properties. By using the aggregate operations of BHSES, a new decision system is proposed and is used in multi-attribute decision-making problems.

1.2. **Main Contributions.** The following are the main contributions of the proposed study:

(1) Some basic definitions of soft set, soft expert, hypersoft set, bijective soft set, bijective hypersoft set are reviewed from literature.
(2) Theory of bijective hypersoft expert set (i.e., axiomatic properties, set-theoretic operations and laws) is conceptualized with the support of numerical illustrative examples.
(3) An algorithm is proposed and then validated by applying it in decision-making based daily-life problem.
(4) The proposed study is compared with existing relevant models to judge the advantageous aspects of proposed study.
(5) Paper is summarized with description of its scope and future directions to motivate the readers for further extensions.

1.3. **Paper Organization.** The remaining paper is organized as under:
Section 2 recalls some basic definitions and terms from existing literature to support main results. Section 3 describes the theory of bijective hypersoft expert set with the description of its decision support system. Section 4 proposes an algorithm based of BHSES with utilization in daily-life multi-attribute decision-making problem and section 5 summarizes the paper with more future directions.

2. Preliminaries

The following portion describes some basic definitions related to the literature and suggested work. In this article, universe of discourse will be shown by $\mathfrak{Z}$, $\mathfrak{S}$ will be used as an experts set and $\mathfrak{O}$ as an opinions set, $\mathfrak{R} = \mathfrak{P} \times \mathfrak{S} \times \mathfrak{O}$ with $\mathfrak{V} \subseteq \mathfrak{R}$, while $\mathfrak{P}$ a set of parameters. The symbol $\mathcal{P}(\mathfrak{Z})$ will denote the power set of universe of discourse.

Definition 1. [35]
A soft set S is defined by an approximate function $\beta_S : \theta \rightarrow \mathcal{P}(\mathfrak{Z})$ which is defined by approximate elements $\beta_S(\hat{s})$ for all members $\hat{s}$ of θ, a subset of parameters.

Definition 2. [33]
A soft expert set ϖ is characterized by an approximate function $\Phi_\varpi : \mathfrak{H} \rightarrow \mathcal{P}(\mathfrak{Z})$ which is defined by approximate elements $\Phi_\varpi(\hat{v})$ for all members $\hat{v}$ of $\mathfrak{H}$ where $\mathfrak{H} \subseteq \mathfrak{P}$.

Definition 3. [59]
Let $\phi_1, \phi_2, \phi_3, ..., \phi_\pi$, for $\pi \geq 1$, be π distinct attributes having $\mathfrak{H}_1, \mathfrak{H}_2, \mathfrak{H}_3, ..., \mathfrak{H}_\pi$ as their respective attribute-valued sets with $\mathfrak{H}_\alpha \cap \mathfrak{H}_\beta = \emptyset$, for $\alpha \neq \beta$, and $\alpha, \beta \in \{1, 2, 3, ..., \pi\}$. The pair $(\Psi, \mathfrak{G})$ is named as a *hypersoft set* over $\mathfrak{Z}$ where $\mathfrak{G} = \mathfrak{H}_1 \times \mathfrak{H}_2 \times \mathfrak{H}_3 \times ..... \times \mathfrak{H}_\pi$ and $\Psi_\Upsilon : \mathfrak{G} \rightarrow \mathcal{P}(\mathfrak{Z})$ is its multi-argument approximate function characterized by approximate elements $\Psi_\Upsilon(\hat{g})$ for all $\hat{g} \in \mathfrak{G}$.

Definition 4. [12]
A bijective soft set is a soft set (β_S, θ) which satisfies the following conditions:
(i) $\cup_{\epsilon \in \theta} \beta_S(\epsilon) = \mathfrak{Z}$
(ii) $\beta_S(\epsilon_i) \cap \beta_S(\epsilon_j) = \emptyset$, for any two parameters $\epsilon_i, \epsilon_j \in \theta,\ \epsilon_i \neq \epsilon_j$.

For the sake of collection of abstracts cum statistical data and information, scholars design questionnaires having description of parametric statements in the form of questions and then it is circulated to individuals with relevant field of expertise. In this scenario, the approached evaluators for such questionnaires are directed to provide their expert opinions regarding the relevance of certain elements of universal set (topics/areas under consideration for the study) with parameters. After the completion of this data collection, the submissions are further reviewed by internal experts (scholars, supervisors, co-supervisors etc.) who classify the collected data on agree and dis-agree basis and then evaluate with their weightage subject to the condition that two opinions collected from field should not overlap. It is pertinent to mention here that the questions of questionnaires are of parametric nature i.e. each question is parameterized with sub-parametric values. The existing literature is inadequate to provide a uncertain model to tackle such scenario so bijective hypersoft expert set is being characterized in the following section to address the scarcity of literature.

3. Bijective Hypersoft Expert Set (BHSES-set)

The following portion contains the definition of hypersoft expert set with example and the theory of bijective hypersoft expert set.

Definition 5. [46]
Let $\hat{p}_1, \hat{p}_2, \hat{p}_3, ..., \hat{p}_n$, for $n \geq 1$, be n distinct attributes having $\mathcal{P}_1, \mathcal{P}_2, \mathcal{P}_3, ..., \mathcal{P}_n$ as

their respective attribute-valued sets with $\mathcal{P}_i \cap \mathcal{P}_j = \emptyset$, for $i \neq j$. The pair $(\Psi, \mathfrak{G})$ is said to be *bijective hypersoft set* over $\mathfrak{Z}$ where $\mathfrak{G} = \mathfrak{H}_1 \times \mathfrak{H}_2 \times \mathfrak{H}_3 \times ..... \times \mathfrak{H}_\pi$ and $\Psi_\Upsilon : \mathfrak{G} \to \mathcal{P}(\mathfrak{Z})$ is its multi-argument approximate function characterized by approximate elements $\Psi_\Upsilon(\hat{g})$ for all $\hat{g} \in \mathfrak{G}$ which satisfy the following conditions:

(1) $\bigcup_{\hat{g} \in \mathfrak{G}} \Psi_\Upsilon(\hat{g}) = \mathfrak{Z}$

(2) $\Psi_\Upsilon(\hat{g}_i) \cap \Psi_\Upsilon(\hat{g}_j) = \emptyset$, for $\hat{g}_i, \hat{g}_j \in \mathfrak{S}, \hat{g}_i \neq \hat{g}_j$.

Definition 6. [17]
A pair$(\xi, \mathbb{S})$ is known as a *hypersoft expert set* over $\mathfrak{Z}$, where $\xi : \mathbb{S} \to \mathcal{P}(\mathfrak{Z})$ where $\mathcal{P}(\mathfrak{Z})$ is collection of all fuzzy subsets of $\mathfrak{Z}$, $\mathbb{S} \subseteq \mathcal{H} = \mathcal{G} \times \mathcal{D} \times \mathbb{C}$ and $\mathcal{G} = \mathcal{G}_1 \times \mathcal{G}_2 \times \mathcal{G}_3 \times .... \times \mathcal{G}_p$ here $\mathcal{G}_i$ are disjoint attributive-valued sets corresponding to distinct attributes $g_i, i = 1, 2, 3, ..., p$, $\mathcal{D}$ be a set of specialists (operators) and $\mathbb{C}$ be a set of conclusions. For simplicity, $\mathbb{C} = \{0 = disagree, 1 = agree\}$.

Example 3.1. Suppose that Mr. John intends to purchase a mask from a medical store. There are four types of masks available in market forming initial universe $\mathfrak{Z} = \{\mathbb{k}_1, \mathbb{k}_2, \mathbb{k}_3, \mathbb{k}_4\}$. The choice of mask may be carried out by keeping in mind the following attributes i.e. γ_1 = Colour, γ_2 = Size, and γ_3 = Price. Following are the attribute-valued sets corresponding to these attributes are:
$\tau_{\gamma_1} = \{\gamma_{11}, \gamma_{12}\}$
$\tau_{\gamma_2} = \{\gamma_{21}, \gamma_{22}\}$
$\tau_{\gamma_3} = \{\gamma_{31}, \gamma_{32}\}$
then $\tau_\gamma = \tau_{\gamma_1} \times \tau_{\gamma_2} \times \tau_{\gamma_3}$

$$\tau_\gamma = \left\{ \begin{array}{c} (\varpi_1, \{\gamma_{11}, \gamma_{21}, \gamma_{31}\}), (\varpi_2, \{\gamma_{11}, \gamma_{21}, \gamma_{32}\}), \\ (\varpi_3, \{\gamma_{11}, \gamma_{22}, \gamma_{31}\}), (\varpi_4, \{\gamma_{11}, \gamma_{22}, \gamma_{32}\}), \\ (\varpi_5, \{\gamma_{12}, \gamma_{21}, \gamma_{31}\}), (\varpi_6, \{\gamma_{12}, \gamma_{21}, \gamma_{32}\}), \\ (\varpi_7, \{\gamma_{12}, \gamma_{22}, \gamma_{31}\}), (\varpi_8, \{\gamma_{12}, \gamma_{22}, \gamma_{32}\}) \end{array} \right\}$$

Now $\mathfrak{H} = \tau_\gamma \times \mathfrak{D} \times \mathfrak{C}$

$$\mathfrak{H} = \left\{ \begin{array}{l} (\varpi_1, \phi_1, 0), (\varpi_1, \phi_1, 1), (\varpi_1, \phi_2, 0), (\varpi_1, \phi_2, 1), (\varpi_1, \phi_3, 0), (\varpi_1, \phi_3, 1), \\ (\varpi_2, \phi_1, 0), (\varpi_2, \phi_1, 1), (\varpi_2, \phi_2, 0), (\varpi_2, \phi_2, 1), (\varpi_2, \phi_2, 0), (\varpi_2, \phi_3, 1), \\ (\varpi_3, \phi_1, 0), (\varpi_3, \phi_1, 1), (\varpi_3, \phi_2, 0), (\varpi_3, \phi_2, 1), (\varpi_3, \phi_3, 0), (\varpi_3, \phi_3, 1), \\ (\varpi_4, \phi_1, 0), (\varpi_4, \phi_1, 1), (\varpi_4, \phi_2, 0), (\varpi_4, \phi_2, 1), (\varpi_4, \phi_3, 0), (\varpi_4, \phi_3, 1), \\ (\varpi_5, \phi_1, 0), (\varpi_5, \phi_1, 1), (\varpi_5, \phi_2, 0), (\varpi_5, \phi_2, 1), (\varpi_5, \phi_3, 0), (\varpi_5, \phi_3, 1), \\ (\varpi_6, \phi_1, 0), (\varpi_6, \phi_1, 1), (\varpi_6, \phi_2, 0), (\varpi_6, \phi_2, 1), (\varpi_6, \phi_3, 0), (\varpi_6, \phi_3, 1), \\ (\varpi_7, \phi_1, 0), (\varpi_7, \phi_1, 1), (\varpi_7, \phi_2, 0), (\varpi_7, \phi_2, 1), (\varpi_7, \phi_3, 0), (\varpi_7, \phi_3, 1), \\ (\varpi_8, \phi_1, 0), (\varpi_8, \phi_1, 1), (\varpi_8, \phi_2, 0), (\varpi_8, \phi_2, 1), (\varpi_8, \phi_3, 0), (\varpi_8, \phi_3, 1) \end{array} \right\}$$

let

$$\phi_{\mathfrak{W}} = \left\{ \begin{array}{l} (\varpi_1, \phi_1, 0), (\varpi_1, \phi_1, 1), (\varpi_1, \phi_2, 0), (\varpi_1, \phi_2, 1), (\varpi_1, \phi_3, 0), (\varpi_1, \phi_3, 1), \\ (\varpi_2, \phi_1, 0), (\varpi_2, \phi_1, 1), (\varpi_2, \phi_2, 0), (\varpi_2, \phi_2, 1), (\varpi_2, \phi_3, 0), (\varpi_2, \phi_3, 1) \\ (\varpi_3, \phi_1, 0), (\varpi_3, \phi_1, 1), (\varpi_3, \phi_2, 0), (\varpi_3, \phi_2, 1), (\varpi_3, \phi_3, 0), (\varpi_3, \phi_3, 1), \end{array} \right\}$$

be a subset of $\mathfrak{H}$ and ϖ_1= Price ϖ_2= Colour ϖ_3= Reliability
$\mathfrak{D} = \{\phi_1, \phi_2, \phi_3, \}$ be a set of specialists.
Following survey depicts choices of three specialists:

$\pi_1 = \pi(\varpi_1, \phi_1, 1) = \{\Bbbk_2, \Bbbk_4\}$, $\pi_2 = \pi(\varpi_1, \phi_2, 1) = \{\Bbbk_1, \Bbbk_2, \Bbbk_3, \Bbbk_4\}$,
$\pi_3 = \pi(\varpi_1, \phi_3, 1) = \{\Bbbk_3\}$, $\pi_4 = \pi(\varpi_2, \phi_1, 1) = \{\Bbbk_2, \Bbbk_3\}$,
$\pi_5 = \pi(\varpi_2, \phi_2, 1) = \{\Bbbk_4\}$, $\pi_6 = \pi(\varpi_2, \phi_3, 1) = \{\Bbbk_1, \Bbbk_4\}$,
$\pi_7 = \pi(\varpi_3, \phi_1, 1) = \{\Bbbk_1\}$, $\pi_8 = \pi(\varpi_3, \phi_2, 1) = \{\Bbbk_1, \Bbbk_3\}$,
$\pi_9 = \pi(\varpi_3, \phi_3, 1) = \{\Bbbk_2, \Bbbk_3, \Bbbk_4\}$, $\pi_{10} = \pi(\varpi_1, \phi_1, 0) = \{\Bbbk_2\}$,
$\pi_{11} = \pi(\varpi_1, \phi_2, 0) = \{\Bbbk_1, \Bbbk_3, \Bbbk_4\}$, $\pi_{12} = \pi(\varpi_1, \phi_3, 0) = \{\Bbbk_1, \Bbbk_2, \Bbbk_4\}$,
$\pi_{13} = \pi(\varpi_2, \phi_1, 0) = \{\Bbbk_1, \Bbbk_2\}$, $\pi_{14} = \pi(\varpi_2, \phi_2, 0) = \{\Bbbk_1, \Bbbk_2, \Bbbk_3, \Bbbk_4\}$,
$\pi_{15} = \pi(\varpi_2, \phi_3, 0) = \{\Bbbk_1, \Bbbk_2, \Bbbk_4\}$, $\pi_{16} = \pi(\varpi_3, \phi_1, 0) = \{\Bbbk_3, \Bbbk_4\}$,
$\pi_{17} = \pi(\varpi_3, \phi_2, 0) = \{\Bbbk_1, \Bbbk_2\}$, $\pi_{18} = \pi(\varpi_3, \phi_3, 0) = \{\Bbbk_1, \Bbbk_2, \Bbbk_3\}$.

The hypersoft expert set can be written as

$$(\pi, \phi_{\mathfrak{W}}) = \left\{ \begin{array}{l} ((\varpi_1, \phi_1, 1), \{\Bbbk_2, \Bbbk_4\}), ((\varpi_1, \phi_2, 1), \{\Bbbk_1, \Bbbk_2, \Bbbk_3, \Bbbk_4\}), ((\varpi_1, \phi_3, 1), \{\Bbbk_3\}), \\ ((\varpi_2, \phi_1, 1), \{\Bbbk_2, \Bbbk_3\}), ((\varpi_2, \phi_2, 1), \{\Bbbk_4\}), ((\varpi_2, \phi_3, 1), \{\Bbbk_1, \Bbbk_4\}), \\ ((\varpi_3, \phi_1, 1), \{\Bbbk_1\}), ((\varpi_3, \phi_2, 1), \{\Bbbk_1, \Bbbk_3\}), ((\varpi_3, \phi_3, 1), \{\Bbbk_2, \Bbbk_3, \Bbbk_4\}), \\ ((\varpi_1, \phi_1, 0), \{\Bbbk_2\}), ((\varpi_1, \phi_2, 0), \{\Bbbk_1, \Bbbk_3, \Bbbk_4\}), ((\varpi_1, \phi_3, 0), \{\Bbbk_1, \Bbbk_2, \Bbbk_4\}), \\ ((\varpi_2, \phi_1, 0), \{\Bbbk_1, \Bbbk_2, \Bbbk_4\}), ((\varpi_2, \phi_2, 0), \{\Bbbk_1, \Bbbk_2, \Bbbk_3, \Bbbk_4\}), ((\varpi_2, \phi_3, 0), \{\Bbbk_1, \Bbbk_2, \Bbbk_4\}), \\ ((\varpi_3, \phi_1, 0), \{\Bbbk_3, \Bbbk_4\}), ((\varpi_3, \phi_2, 0), \{\Bbbk_1, \Bbbk_2\}), ((\varpi_3, \phi_3, 0), \{\Bbbk_1, \Bbbk_2, \Bbbk_3\}) \end{array} \right\}$$

Following table represents the hypersoft expert set $(\pi, \mathfrak{S})$, TABLE 1, with $\Bbbk_i \in \mathfrak{S}(\varpi_i)$ then $\widehat{\oplus}$ otherwise $\widehat{\otimes}$.

....	$\Bbbk_1$	$\Bbbk_2$	$\Bbbk_3$	$\Bbbk_4$
π_1	$\widehat{\otimes}$	$\widehat{\oplus}$	$\otimes$	$\widehat{\oplus}$
π_2	$\widehat{\oplus}$	$\widehat{\oplus}$	$\widehat{\oplus}$	$\widehat{\oplus}$
π_3	$\otimes$	$\otimes$	$\widehat{\oplus}$	$\otimes$
π_4	$\widehat{\otimes}$	$\widehat{\oplus}$	$\widehat{\oplus}$	$\widehat{\otimes}$
π_5	$\widehat{\otimes}$	$\widehat{\otimes}$	$\widehat{\otimes}$	$\widehat{\oplus}$
π_6	$\widehat{\oplus}$	$\widehat{\otimes}$	$\widehat{\otimes}$	$\widehat{\oplus}$
π_7	$\widehat{\oplus}$	$\widehat{\otimes}$	$\widehat{\otimes}$	$\widehat{\otimes}$
π_8	$\widehat{\oplus}$	$\widehat{\otimes}$	$\widehat{\oplus}$	$\widehat{\otimes}$
π_9	$\widehat{\otimes}$	$\widehat{\oplus}$	$\widehat{\oplus}$	$\widehat{\oplus}$
π_{10}	$\widehat{\otimes}$	$\widehat{\oplus}$	$\widehat{\otimes}$	$\widehat{\otimes}$
π_{11}	$\widehat{\oplus}$	$\widehat{\otimes}$	$\widehat{\oplus}$	$\widehat{\oplus}$
π_{12}	$\widehat{\oplus}$	$\widehat{\oplus}$	$\widehat{\otimes}$	$\widehat{\oplus}$
π_{13}	$\widehat{\oplus}$	$\widehat{\oplus}$	$\widehat{\otimes}$	$\widehat{\otimes}$
π_{14}	$\widehat{\oplus}$	$\widehat{\oplus}$	$\widehat{\oplus}$	$\widehat{\oplus}$
π_{15}	$\widehat{\oplus}$	$\widehat{\oplus}$	$\widehat{\otimes}$	$\widehat{\oplus}$
π_{16}	$\widehat{\otimes}$	$\widehat{\otimes}$	$\widehat{\oplus}$	$\widehat{\oplus}$
π_{17}	$\widehat{\oplus}$	$\widehat{\oplus}$	$\widehat{\otimes}$	$\widehat{\otimes}$
π_{18}	$\widehat{\oplus}$	$\widehat{\oplus}$	$\widehat{\oplus}$	$\widehat{\otimes}$

TABLE 1. $(\pi, \mathcal{S})$ shows the tabular representation of HSES

Definition 7. A hypersoft expert set $(\pi, \mathcal{S})$ is named as a bijective hypersoft expert set if

(1) $\bigcup\limits_{\varpi \in \mathcal{S}} \pi(\varpi) = \mathfrak{Z}$

(2) $\pi(\varpi_i) \cap \pi(\varpi_j) = \emptyset$ for any two $\varpi_i, \varpi_j \in \mathfrak{S}, \varpi_i \neq \varpi_j$

The symbol Υ_{BHSES} denotes the collection of all BHSESs over $\mathfrak{Z}$.

Example 3.2. Considering Example 3.1, we get BHSES

$$(\pi, \mathfrak{S}) = \left\{ \ (\varpi_1, (\Bbbk_2, \Bbbk_4), (\varpi_2, \Bbbk_3), (\varpi_3, (\Bbbk_1) \ \right\}$$

and TABLE 2 shows the tabular form of bijective hypersoft expert set.

....	$\Bbbk_1$	$\Bbbk_2$	$\Bbbk_3$	$\Bbbk_4$
ϖ_1	$\widehat{\otimes}$	$\widehat{\oplus}$	$\widehat{\otimes}$	$\widehat{\oplus}$
ϖ_2	$\widehat{\otimes}$	$\widehat{\otimes}$	$\widehat{\oplus}$	$\widehat{\otimes}$
ϖ_3	$\widehat{\oplus}$	$\widehat{\otimes}$	$\widehat{\otimes}$	$\widehat{\otimes}$
ϖ_4	$\widehat{\oplus}$	$\widehat{\otimes}$	$\widehat{\otimes}$	$\widehat{\otimes}$

TABLE 2. The table form of BHSES $(\pi, \mathfrak{S})$

Definition 8. The operation AND between two hypersoft expert sets $(\pi_1, \mathfrak{S}_1)$ and $(\pi_2, \mathfrak{S}_2)$, shown as $(\pi_1, \mathfrak{S}_1) \bigwedge (\pi_2, \mathfrak{S}_2)$, is a hypersoft expert set $(\pi_3, \mathfrak{S}_3))$ with $\mathfrak{S}_3 = \mathfrak{S}_1 \times \mathfrak{S}_2$ and for $\varpi \in \mathfrak{S}_3$,

$$\pi_3(\varpi) = \pi_1(\varpi) \cap \pi_2(\varpi)$$

Theorem 3.3. *Suppose $(\pi_1, \mathfrak{H}_1)$ and $(\pi_2, \mathfrak{H}_2)$ are BHSESs, then $(\pi_1, \mathfrak{H}_1) \bigwedge (\pi_2, \mathfrak{H}_2)$ is a BHSES.*

Proof. Using definition 7 , we have
$(\pi_1, \mathfrak{H}_1) \wedge (\pi_2, \mathfrak{H}_2) = (\pi_3, \mathfrak{H}_3)$, where $\mathfrak{H}_3 = \mathfrak{H}_1 \times \mathfrak{H}_2$ and $\pi_3(\mathfrak{h}_1, \mathfrak{h}_2) = \pi_1(\mathfrak{h}_1) \cap \pi_2(\mathfrak{h}_2), \ \forall \ (\mathfrak{h}_1, \mathfrak{h}_2) \in \mathfrak{H}_3$.
Consider $\varepsilon \in \mathfrak{H}_3$ is a parameter of $(\pi_3, \mathfrak{H}_3)$ then

$$\pi_3(\varepsilon) = \pi_1 \left(\mathfrak{h}_1\right) \cap \pi_2 \left(\mathfrak{h}_2\right)$$

$\therefore \bigcup\limits_{\varepsilon \in \mathfrak{H}_2} \pi_3(\varepsilon) = \bigcup\limits_{\mathfrak{h}_1 \in \mathfrak{H}_1} \bigcup\limits_{\mathfrak{h}_2 \in \mathfrak{H}_2} \pi_1 \left(\mathfrak{h}_1\right) \cap \pi_2 \left(\mathfrak{h}_2\right) = \bigcup\limits_{\mathfrak{h}_1 \in \mathfrak{H}_1} \pi_1 \left(\mathfrak{h}_1\right) \cap \left(\bigcup\limits_{\mathfrak{h}_2 \in \mathfrak{H}_2} \pi_2 \left(\mathfrak{h}_2\right) \right) = \bigcup\limits_{\mathfrak{h}_1 \in \mathfrak{H}_1} \pi_1 \left(\mathfrak{h}_1\right) \cap \mathfrak{Z} = \mathfrak{Z}$.
Suppose $\varepsilon_i, \varepsilon_j \in \mathfrak{H}_3, \varepsilon_i \neq \varepsilon_j, \varepsilon_i = \alpha_1 \times \beta_1, \alpha_1 \in \mathfrak{H}_1, \beta_1 \in \mathfrak{H}_2, \varepsilon_j = \alpha_2 \times \beta_2, \alpha_2 \in \mathfrak{H}_1, \beta_2 \in \mathfrak{H}_2$. Then

$$\pi_3(\varepsilon_i) \cap \pi_3(\varepsilon_j) = (\pi_1(\alpha_1) \cap \pi_2(\beta_1)) \cap (\pi_1(\alpha_2) \cap \pi_2(\beta_2)) = \emptyset.$$

Hence $(\pi_3, \mathfrak{H}_3) = (\pi_1, \mathfrak{H}_1) \bigwedge (\pi_2, \mathfrak{H}_2)$ is a bijective hypersoft expert set. □

Definition 9. A HSES $(\pi, \mathfrak{S})$ is named as a *null HSES*, shown as $(\pi, \mathfrak{S})_\Phi$, if $\pi(\varpi) = \emptyset$, $\forall$ $\varpi \in \mathfrak{S}$.

Definition 10. The operation union between two HSESs is a HSES $(\pi_3, \mathfrak{S}_3)$ with $\mathfrak{S}_3 = \mathfrak{S}_1 \cup \mathfrak{S}_2$

$$\pi_3(\varpi) = \begin{cases} \pi_1(\varpi) & \varpi \in (\mathfrak{S}_1 \setminus \mathfrak{S}_2) \\ \pi_2(\varpi) & \varpi \in (\mathfrak{S}_2 \setminus \mathfrak{S}_1) \\ \pi_1(\varpi) \cup \pi_2(\varpi) & \varpi \in (\mathfrak{S}_1 \cap \mathfrak{S}_2) \end{cases}$$

for $\varpi \in \mathfrak{S}_3$.

Theorem 3.4. *Suppose $(\pi, \mathfrak{H})$ is a BHSES. Then $(\pi, \mathfrak{H}) \cup (\pi, \mathfrak{H})_\Phi$ is a BHSES.*

Proof. Suppose $(\pi, \mathfrak{H})_\Phi = (\pi_\Phi, \mathfrak{H}_1)$,
by using the definitions of 9 and 10, we get
$(\pi_2, \mathfrak{H}_2) = (\pi, \mathfrak{H}) \cup (\pi_\Phi, \mathfrak{H}_1)$

$$= \begin{cases} \pi(\epsilon) & ; \epsilon \in \mathfrak{H} - \mathfrak{H}_1 \\ \pi_\Phi(\epsilon) = \emptyset & ; \epsilon \in \mathfrak{H}_1 - \mathfrak{H} = (\pi, \mathfrak{H} \cup \mathfrak{H}_1) \\ \pi(\epsilon) \cup \pi_\Phi(\epsilon) = \pi(\epsilon) \cup \emptyset & ; \epsilon \in \pi \cap \pi_1 \end{cases}$$

where $\epsilon \in \mathfrak{H}_2$ and $(\pi, \mathfrak{H}_1) \subset (\pi, \mathfrak{H} \cup \mathfrak{H}_1)$ is a Null hypersoft set, implies
$(\pi_2, \mathfrak{H}_2) = (\pi, \mathfrak{H} \cup \mathfrak{H}_1)$ is a bijective hypersoft expert set over $\mathfrak{Z}$.

□

Definition 11. Suppose $\mathfrak{Z}_1 \subset \mathfrak{Z}$ and $(\pi, \mathfrak{H})$ is a BHSES. The restricted AND operation is written as $(\pi, \mathfrak{H}) \bigwedge_{\mathcal{R}} \mathfrak{Z}_1$, and is described as

$$\bigcup_{\varpi \in \mathfrak{H}} \{\pi(\varpi) : \pi(\varpi) \subseteq \mathfrak{Z}_1\}.$$

Example 3.5. Suppose $\mathfrak{Z} = \{F_1, F_2, F_3, ..., F_6\}$ and $\mathfrak{Z}_1 = \{F_1, F_2, F_3, \}$. If $(\pi, \mathfrak{H}) \in \Upsilon_{BHSES}$ with

$$(\pi, \mathfrak{H}) = \left\{ \ (\varpi_1, \{F_1, F_2\}), (\varpi_2, \{F_3\}), (\varpi_3, \{F_5, F_6\}) \ \right\}$$

then

$$(\pi, \mathfrak{H}) \bigwedge_{\mathcal{R}} \mathfrak{Z}_1 = \{F_1, F_2\} \cup \{F_3\} = \{F_1, F_2, F_3\}$$

Definition 12. Suppose $\mathfrak{Z}_1 \subset \mathfrak{Z}$ and $(\pi, \mathfrak{H})$ are BHSES. The relaxed AND operation is written as

$$\bigcup_{\varpi \in \mathfrak{H}} \{\pi(\varpi) : \pi(\varpi) \cap \mathfrak{Z}_1 \neq \emptyset\}.$$

and it is represented by $(\pi, \mathfrak{H}) \bigwedge_{\mathcal{R}el} \mathfrak{Z}_1$.

Example 3.6. Let $\mathfrak{Z} = \{F_1, F_2, F_3, ..., F_6\}$ and $\mathfrak{Z}_1 = \{F_1, F_2, F_3\}$. Suppose $(\pi, \mathfrak{H})$ is a BHSES with

$$(\pi, \mathfrak{H}) = \left\{ \ (\varpi_1, \{F_1, F_6\}), (\varpi_2, \{F_3, F_5\}), (\varpi_3, \{F_2, F_4\}), \ \right\}$$

then

$$(\pi, \mathfrak{H}) \bigwedge_{\mathcal{R}el} \mathfrak{Z}_1 = \{F_1, F_6\} \cup \{F_3, F_5\} \cup \{F_2, F_4\} = \{F_1, F_2, F_3, F_4, F_5, F_6\} = \mathfrak{Z}$$

Definition 13. Suppose $(\pi, \mathfrak{H})$ is a BHSES, a boundary region of BHSES w.r.t $\mathfrak{Z}_1 \subset \mathfrak{Z}$, written by $(\pi, \mathfrak{H})_\bullet$, is presented as

$$(\pi, \mathfrak{H})_\bullet = \left((\pi, \mathfrak{H}) \bigwedge_{\mathcal{R}el} \mathfrak{Z}_1 \right) \setminus \left((\pi, \mathfrak{H}) \bigwedge_{\mathcal{R}} \mathfrak{Z}_1 \right)$$

Example 3.7. Taking 3.6 for $\mathfrak{Z}$ and $\mathfrak{Z}_1$, we have

$$(\pi, \mathfrak{H}) = \left\{ \ (\varpi_1, \{F_1, F_6\}), (\varpi_2, \{F_3, F_5\}), (\varpi_3, \{F_2, F_4\}) \ \right\}$$

. Now

$$(\pi, \mathfrak{H}) \bigwedge_{\mathcal{R}el} \mathfrak{Z}_1 = \{F_1, F_6\} \cup \{F_3, F_5\} \cup \{F_2, F_4\} = \{F_1, F_2, F_3, F_4, F_5 F_6\}$$

and

$$(\pi, \mathfrak{H}) \bigwedge_{\mathcal{R}} \mathfrak{Z}_1 = \{F_1, F_6\} \cup \{F_3, F_5\} = \{F_1, F_3, F_5, F_6\}$$

therefore

$$(\pi, \mathfrak{H})_{\bullet} = \{F_2, F_6\}$$

Definition 14. Suppose $(\pi_1, \mathfrak{H}_1)$ and $(\pi_2, \mathfrak{H}_2)$ are two BHSES with $\mathfrak{H}_1 \cap \mathfrak{H}_2 = \emptyset$, then $(\pi_1, \mathfrak{H}_1)$ is said to depend on $(\pi_2, \mathfrak{H}_2)$ with degree $\chi \in [0, 1]$, shown by $(\pi_1, \mathfrak{H}_1) \underset{\chi}{\Rightarrow} (\pi_2, \mathfrak{H}_2)$, if

$$\chi = \daleth((\pi_1, \mathfrak{H}_1), (\pi_2, \mathfrak{H}_2)) = \frac{\left| \underset{\varpi \in \mathfrak{H}_2}{\cup} \{(\pi_1, \mathfrak{H}_1) \bigwedge_{\mathcal{R}} \pi_2(\varpi)\} \right|}{|\mathfrak{Z}|}$$

such that $|\cdot| =$ shows the cardinality of a set.
Note:

(i) If $\chi = 1$ so $(\pi_1, \mathfrak{H}_1)$ is full depended on $(\pi_2, \mathfrak{H}_2)$.
(ii) If $\chi = 0$ so $(\pi_1, \mathfrak{H}_1)$ is not depended on $(\pi_2, \mathfrak{H}_2)$.

Example 3.8. Taking 3.6 for $\mathfrak{Z}$, we have

$$(\pi_1, \mathfrak{H}_1) = \left\{ \begin{array}{l} (\varpi_1, \{F_1\}), (\varpi_2, \{F_3\}), (\varpi_3, \{F_6\}), \\ (\varpi_4, \{F_5\}), (\varpi_5, \{F_2\}), (\varpi_6, \{F_4\}) \end{array} \right\}$$

and

$$(\pi_2, \mathfrak{H}_2) = \left\{ \ (\varpi_7, \{F_1, F_2\}), (\varpi_8, \{F_3, F_4\}), (\varpi_9, \{F_5, F_6\}) \ \right\}$$

Now

$$(\pi_1, \mathfrak{H}_1) \bigwedge_{\mathcal{R}} \pi_2(\varpi_7) = \{F_1\} \cup \{F_2\} = \{F_1, F_2\}$$

$$(\pi_1, \mathfrak{H}_1) \bigwedge_{\mathcal{R}} \pi_2(\varpi_8) = \{F_3\} \cup \{F_4\} = \{F_3, F_4\}$$

$$(\pi_1, \mathfrak{H}_1) \bigwedge_{\mathcal{R}} \pi_2(\varpi_9) = \{F_5\}$$

therefore

$$\underset{\varpi \in \mathfrak{H}_2}{\cup} \{(\pi_1, \mathfrak{H}_1) \bigwedge_{\mathcal{R}} \pi_2(\varpi)\} = \{F_1, F_2, F_3, F_4, F_5,\}$$

with

$$\chi = \frac{5}{6} = 0.833$$

Definition 15. Suppose $(\pi, \mathfrak{H})$ and $(\mathfrak{L}, \mathfrak{S})$ are two BHSESs. This $((\pi, \mathfrak{H}), (\mathfrak{L}, \mathfrak{S}), \mathfrak{Z})$ is named as BHSES decision system over $\mathfrak{Z}$, shown by $\mathfrak{D}_{BHE}$, if

(i) $\exists$ a property HSES $(\pi, \mathfrak{H}) = \bigcup\limits_{i=1}^{n} (\pi_i, \mathfrak{H}_i) \; \forall \, (\pi_i, \mathfrak{H}_i) \in \Upsilon_{BHSES}$ with $\mathfrak{H}_i \cap \mathfrak{H}_j = \emptyset, \quad i \neq j$

(ii) $\exists$ a decision HSES $(\mathfrak{L}, \mathfrak{S})$ for which $\mathfrak{S} \cap \mathfrak{H}_i = \emptyset$.

Example 3.9. Consider 3.6 for $\mathfrak{Z}$, we have

$$(\pi_1, \mathfrak{H}_1) = \left\{ \; (\varpi_1, \{F_1\}), (\varpi_2, \{F_2\}), (\varpi_3, \{F_3\}) \; \right\}$$

$$(\pi_2, \mathfrak{H}_2) = \left\{ \; (\varpi_4, \{F_1, F_3\}), (\varpi_5, \{F_2, F_5\}), (\varpi_6, \{F_4, F_6\}) \; \right\}$$

$$(\pi_3, \mathfrak{H}_3) = \left\{ \; (\varpi_7, \{F_1, F_2, F_4\}), (\varpi_8, \{F_3, F_5, F_6\}) \; \right\}$$

and

$$(\mathfrak{L}, \mathfrak{S}) = \left\{ \; (\varpi_{10}, \{F_1, F_3, F_5\}), (\varpi_{11}, \{F_2, F_4\}), (\varpi_{12}, \{F_6\}) \; \right\}$$

therefore

$$\mathfrak{D}_{BHE} = (\bigcup_{i=1}^{3} (\pi_i, \mathfrak{H}_i), (\mathfrak{X}, \mathfrak{S}), \mathfrak{Z})$$

Definition 16. The BHSE dependency between $(\pi_1, \mathfrak{H}_1) \bigwedge (\pi_2, \mathfrak{H}_2) \bigwedge .... \bigwedge (\pi_m, \mathfrak{H}_m)$ and $(\mathfrak{X}, \mathfrak{S})$ is called BHSE decision system dependency of $\mathfrak{D}_{BHE}$ and represented by $\chi = \daleth \left(\bigwedge\limits_{i=1}^{m} (\pi_i, \mathfrak{H}_i), (\mathfrak{X}, \mathfrak{S}) \right)$.

Example 3.10. Considering the $\mathfrak{Z}$ from Example 3.6, Let we have

$$(\pi_1, \mathfrak{H}_1) = \left\{ \; (\varpi_1, \{F_1, F_4, F_6\}), (\varpi_2, \{F_2, F_5\}) \; \right\}$$

$$(\pi_2, \mathfrak{H}_2) = \left\{ \; (\varpi_3, \{F_1, F_5, F_6\}), (\varpi_4, \{F_4\}) \; \right\}$$

$$(\pi_3, \mathfrak{H}_3) = \left\{ \; (\varpi_5, \{F_1, F_2, F_4\}), (\varpi_6, \{F_3, F_5, F_6\}) \; \right\}$$

$$(\mathfrak{X}, \mathfrak{S}) = \left\{ \; (\varpi_7, \{F_1, F_4, F_5, F_6\}), (\varpi_8, \{F_2, F_3\}) \; \right\}$$

then

$(\pi, \mathfrak{H}) = (\pi_1, \mathfrak{H}_1) \bigwedge (\pi_2, \mathfrak{H}_2) \bigwedge (\pi_3, \mathfrak{H}_3) =$

$$\left\{ \begin{array}{l} (\varepsilon_1 = (\varpi_1, \varpi_3, \varpi_5), \{F_1\}), (\varepsilon_2 = (\varpi_1, \varpi_3, \varpi_6), \{F_6\}), (\varepsilon_3 = (\varpi_1, \varpi_3, \varpi_7), \{F_1, F_6\}) \\ (\varepsilon_4 = (\varpi_1, \varpi_4, \varpi_5), \{F_4\}), (\varepsilon_5 = (\varpi_1, \varpi_4, \varpi_6), \emptyset), (\varepsilon_6 = (\varpi_1, \varpi_4, \varpi_7), \{F_4\}) \\ (\varepsilon_7 = (\varpi_2, \varpi_3, \varpi_5), \emptyset), (\varepsilon_8 = (\varpi_2, \varpi_3, \varpi_6), \{F_5\}), (\varepsilon_9 = (\varpi_2, \varpi_3, \varpi_7), \{F_4\}) \\ (\varepsilon_{10} = (\varpi_2, \varpi_4, \varpi_5), \emptyset), (\varepsilon_{11} = (\varpi_2, \varpi_4, \varpi_6), \emptyset), (\varepsilon_{12} = (\varpi_2, \varpi_4, \varpi_7), \emptyset) \\ (\varepsilon_{13} = (\varpi_2, \varpi_4, \varpi_8), \emptyset), (\varepsilon_{14} = (\varpi_1, \varpi_3, \varpi_8), \emptyset) \\ (\varepsilon_{15} = (\varpi_1, \varpi_4, \varpi_8), \emptyset), (\varepsilon_{16} = (\varpi_2, \varpi_3, \varpi_8), \emptyset) \end{array} \right\}.$$

The tabular form of $(\pi_1, \mathfrak{H}_1) \bigwedge (\pi_2, \mathfrak{H}_2) \bigwedge (\pi_3, \mathfrak{H}_3)$ is shown in TABLE 3.

Now

$$\left(\bigwedge_{i=1}^{3} (\pi_i, \mathfrak{H}_i) \bigwedge\nolimits_{\mathcal{R}} (\mathfrak{X}, \mathfrak{S}) \right) = \{F_1, F_4, F_5, F_6\}$$

therefore

$$\chi = \daleth \left(\bigwedge_{i=1}^{3} (\pi_i, \mathfrak{H}_i), (\mathfrak{X}, \mathfrak{S}) \right) = \tfrac{4}{6} = 0.666$$

....	F_1	F_2	F_3	F_4	F_5	F_6
$\pi(\varepsilon_1)$	$\oplus$	$\widehat{\otimes}$	$\widehat{\otimes}$	$\widehat{\otimes}$	$\widehat{\otimes}$	$\widehat{\otimes}$
$\pi(\varepsilon_2)$	$\widehat{\otimes}$	$\widehat{\otimes}$	$\widehat{\otimes}$	$\widehat{\otimes}$	$\widehat{\otimes}$	$\oplus$
$\pi(\varepsilon_3)$	$\oplus$	$\widehat{\otimes}$	$\widehat{\otimes}$	$\widehat{\otimes}$	$\widehat{\otimes}$	$\oplus$
$\pi(\varepsilon_4)$	$\widehat{\otimes}$	$\widehat{\otimes}$	$\widehat{\otimes}$	$\oplus$	$\widehat{\otimes}$	$\widehat{\otimes}$
$\pi(\varepsilon_5)$	$\widehat{\otimes}$	$\widehat{\otimes}$	$\widehat{\otimes}$	$\widehat{\otimes}$	$\widehat{\otimes}$	$\widehat{\otimes}$
$\pi(\varepsilon_6)$	$\widehat{\otimes}$	$\widehat{\otimes}$	$\widehat{\otimes}$	$\oplus$	$\widehat{\otimes}$	$\widehat{\otimes}$
$\pi(\varepsilon_7)$	$\widehat{\otimes}$	$\widehat{\otimes}$	$\widehat{\otimes}$	$\widehat{\otimes}$	$\widehat{\otimes}$	$\widehat{\otimes}$
$\pi(\varepsilon_8)$	$\widehat{\otimes}$	$\widehat{\otimes}$	$\widehat{\otimes}$	$\widehat{\otimes}$	$\oplus$	$\widehat{\otimes}$
$\pi(\varepsilon_9)$	$\widehat{\otimes}$	$\widehat{\otimes}$	$\widehat{\otimes}$	$\oplus$	$\widehat{\otimes}$	$\widehat{\otimes}$
$\pi(\varepsilon_{10})$	$\widehat{\otimes}$	$\widehat{\otimes}$	$\widehat{\otimes}$	$\widehat{\otimes}$	$\widehat{\otimes}$	$\widehat{\otimes}$
$\pi(\varepsilon_{11})$	$\widehat{\otimes}$	$\widehat{\otimes}$	$\widehat{\otimes}$	$\widehat{\otimes}$	$\widehat{\otimes}$	$\widehat{\otimes}$
$\pi(\varepsilon_{12})$	$\widehat{\otimes}$	$\widehat{\otimes}$	$\widehat{\otimes}$	$\widehat{\otimes}$	$\widehat{\otimes}$	$\widehat{\otimes}$

TABLE 3. $(\pi_1, \mathfrak{H}_1) \bigwedge (\pi_2, \mathfrak{H}_2) \bigwedge (\pi_3, \mathfrak{H}_3)$ in table form

Theorem 3.11. *Suppose* $\mathfrak{D}_{BHE} = ((\pi, \mathfrak{H}), (\mathfrak{X}, \mathfrak{S}), \mathfrak{Z})$*, where* $(\pi, \mathfrak{H}) = \bigcup_{i=1}^{n} (\pi_i, \mathfrak{H}_i)$ *and* $(\pi_i, \mathfrak{H}_i) \in \Upsilon_{BHSES}$*. If* $\chi = \daleth\left(\bigwedge_{i=1}^{n} (\pi_i, \mathfrak{H}_i), (\mathfrak{X}, \mathfrak{S})\right)$ *and* $\chi_1 = \daleth\left(\bigwedge_{i=1}^{m} (\pi_i, \mathfrak{H}_i), (\mathfrak{X}, \mathfrak{S})\right)$ *with* $m \le n$ *then* $\chi_1 \le \chi$*.*

Proof. Suppose that $(\mathfrak{P}, \mathfrak{C}) = \bigwedge_{i=1}^{n} (\pi_i, \mathfrak{H}_i), (\mathfrak{J}, \mathcal{K}) = \bigwedge_{i=1}^{m} (\pi_i, \mathfrak{H}_i)$ then we have

$$\chi = \daleth\left(\bigwedge_{i=1}^{n} (\pi_i, \mathfrak{H}_i), (\mathfrak{X}, \mathfrak{S})\right) = \frac{\left|\bigcup_{\varepsilon \in \mathfrak{S}} (\mathfrak{P}, \mathfrak{C}) \bigwedge_{\mathcal{R}} \mathfrak{X}(\varepsilon)\right|}{|\mathfrak{Z}|} = \frac{\left|\bigcup_{\varepsilon \in \mathfrak{S}} \bigcup_{\lambda \in \mathfrak{C}} \{\mathfrak{P}(\lambda) : \mathfrak{P}(\lambda) \subseteq \mathfrak{X}(\varepsilon)\}\right|}{|\mathfrak{Z}|}$$

$$\chi_1 = \daleth\left(\bigwedge_{i=1}^{m} (\pi_i, \mathfrak{H}_i), (\mathfrak{X}, \mathfrak{S})\right) = \frac{\left|\bigcup_{\varepsilon \in \mathfrak{S}} (\mathfrak{J}, \mathcal{K}) \bigwedge_{\mathcal{R}} \mathfrak{X}(\varepsilon)\right|}{|\mathfrak{Z}|} = \frac{\left|\bigcup_{\varepsilon \in \mathfrak{S}} \bigcup_{\lambda \in \mathcal{K}} \{\mathfrak{J}(\lambda) : \mathfrak{J}(\lambda) \subseteq \mathfrak{X}(\varepsilon)\}\right|}{|\mathfrak{Z}|}.$$

From Definition 2.6,

$$\mathfrak{P}(\varepsilon_1, \phi_2, .., \phi_n) = \pi_1(\phi_1) \cap \pi_2(\phi_2) \cap \cdot\cdot \cap \pi_m(\phi_m) \cap \cdot\cdot \cap \pi_n(\phi_n), \forall (\phi_1, \phi_2, .., \phi_n) \in \mathfrak{H}_1 \times \mathfrak{H}_2 \times \cdots \times \mathfrak{H}_n$$

$$\mathfrak{J}(\phi_1, \phi_2, .., \phi_m) = \pi_1(\phi_1) \cap \pi_2(\phi_2) \cap \cdot\cdot \cap \pi_m(\phi_m), \ \forall (\phi_1, \phi_2, .., \phi_m) \in \mathfrak{H}_1 \times \mathfrak{H}_2 \times \cdots \times \mathfrak{H}_m$$

$n > m$

$$\mathfrak{P}(\phi_1, \phi_2, .., \phi_n) \supseteq \mathfrak{J}(\phi_1, \phi_2, .., \phi_m)$$

and

$$\bigcup_{\phi \in \mathfrak{C}} \mathfrak{P}(\phi) = \mathfrak{Z}, \ \bigcup_{\phi \in \mathcal{K}} \mathfrak{J}(\phi) = \mathfrak{Z}.$$

Therefore,

$$\left|\underset{\phi\in\mathfrak{C}}{\cup}\{\mathfrak{P}(\phi):\mathfrak{P}(\phi)\subseteq\mathfrak{S}(\phi)\}\right|\geq\left|\underset{\phi\in\mathcal{K}}{\cup}\{\mathfrak{I}(\phi):\mathfrak{I}(\phi)\subseteq\mathfrak{S}(\phi)\}\right|.$$

$$\daleth\left(\bigwedge_{i=1}^{m}(\pi_i,\mathfrak{H}_i),(\mathfrak{X},\mathfrak{S})\right)\leq\chi.$$

□

Definition 17. Suppose $\mathfrak{D}_{BHE}=((\pi,\mathfrak{H}),(\mathfrak{X},\mathfrak{S}),\mathfrak{Z})$, where $(\pi,\mathfrak{H})=\overset{n}{\underset{i=1}{\cup}}(\pi_i,\mathfrak{H}_i)$ and $\overset{m}{\underset{i=1}{\cup}}(\pi_i,\mathfrak{H}_i)\subset(\pi,\mathfrak{H})$. If $\daleth\left(\bigwedge\limits_{i=1}^{n}(\pi_i,\mathfrak{H}_i),(\mathfrak{X},\mathfrak{S})\right)=\daleth\left(\bigwedge\limits_{i=1}^{m}(\pi_i,\mathfrak{H}_i),(\mathfrak{X},\mathfrak{S})\right)=\chi$ then $\overset{m}{\underset{i=1}{\cup}}(\pi_i,\mathfrak{H}_i)$ is named as a reduct of $\mathfrak{D}_{BHE}$.

Example 3.12. Using 3.6 for $\mathfrak{Z}$ and 3.10 for sets , we see

$$(\pi_1,\mathfrak{H}_1)=\left\{\ (\varpi_1,\{F_1,F_4,F_6\}),(\varpi_2,\{F_2,F_5\})\ \right\}$$
$$(\pi_2,\mathfrak{H}_2)=\left\{\ (\varpi_3,\{F_1,F_5,F_6\}),(\varpi_4,\{F_4\})\ \right\}$$
$$(\pi_3,\mathfrak{H}_3)=\left\{\ (\varpi_5,\{F_1,F_2,F_4\}),(\varpi_6,\{F_3,F_5,F_6\})\ \right\}$$
$$(\mathfrak{X},\mathfrak{S})=\left\{\ (\varpi_7,\{F_1,F_4,F_5,F_6\}),(\varpi_8,\{F_2,F_3\})\ \right\}$$

then

$$(\pi_1,\mathfrak{H}_1)\bigwedge(\pi_2,\mathfrak{H}_2)=\left\{\begin{array}{l}(\phi_1=(\varpi_1,\varpi_3),\{F_1,F_6\}),(\phi_2=(\varpi_1,\varpi_4),\{F_4\}),\\(\phi_3=(\varpi_2,\varpi_3),\{F_5\})\,(\phi_4=(\varpi_2,\varpi_4),\emptyset)\end{array}\right\}.$$

Now

$$\left(\bigwedge_{i=1}^{2}(\pi_i,\mathfrak{H}_i)\bigwedge\nolimits_{\mathcal{R}}(\mathfrak{X},\mathfrak{S})\right)=\{F_1,F_4,F_5,F_6\}$$

therefore

$\chi=\daleth\left(\bigwedge\limits_{i=1}^{2}(\pi_i,\mathfrak{H}_i),(\mathfrak{X},\mathfrak{S})\right)=\frac{4}{6}=0.666$ which is similar to $\daleth\left(\bigwedge\limits_{i=1}^{3}(\pi_i,\mathfrak{H}_i),(\mathfrak{X},\mathfrak{S})\right)$ measured in Example 3.10. So $(\pi_1,\mathfrak{H}_1)\cup(\pi_2,\mathfrak{H}_2)$ is a reduct of $\mathfrak{D}_{BHE}$.

Definition 18. Suppose $\mathfrak{D}_{BHE}=(\overset{n}{\underset{i=1}{\cup}}(\pi_i,\mathfrak{H}_i),(\mathfrak{X},\mathfrak{S}),\mathfrak{Z})$. The significance of BHSES to decision HSES, shown $\varpi((\pi_j,\mathfrak{H}_j),\overset{n}{\underset{i=1}{\cup}}(\pi_i,\mathfrak{H}_i),(\mathfrak{X},\mathcal{G}))$, is presented as

$$\varpi((\pi_j,\mathfrak{H}_j),\overset{n}{\underset{i=1}{\cup}}(\pi_i,\mathfrak{H}_i),(\mathfrak{X},\mathfrak{S}))=\chi-\daleth((\mathfrak{P},\mathfrak{C}),(\mathfrak{X},\mathfrak{S})),$$

where $(\mathfrak{P},\mathfrak{C})=\bigwedge_{i=1}^{n}(\pi_i,\mathfrak{H}_i)(i\neq j)$.

Example 3.13. Taking 3.10, we have

$\chi=\daleth\left(\bigwedge\limits_{i=1}^{3}(\pi_i,\mathfrak{H}_i),(\mathfrak{X},\mathfrak{S})\right)=\frac{4}{6}=0.666$ and

$$(\pi_2,\mathfrak{H}_2)\bigwedge(\pi_3,\mathfrak{H}_3)=\left\{\begin{array}{l}(\phi_1=(\varpi_3,\varpi_5),\{F_1\}),(\phi_2=(\varpi_3,\varpi_6),\{F_5,F_6\}),\\(\phi_4=(\varpi_4,\varpi_5),\{F_4\}),(\phi_5=(\varpi_4,\varpi_6),\{\})\end{array}\right\}.$$

Now
$\left(\bigwedge_{i=2}^{3}(\pi_i,\mathfrak{H}_i)\bigwedge_{\mathcal{R}}(\mathfrak{X},\mathfrak{S})\right)=\{F_1,F_4,F_5,F_6\}$
therefore
$\urcorner\left(\bigwedge_{i=2}^{3}(\pi_i,\mathfrak{H}_i),(\mathfrak{X},\mathfrak{S})\right)=\frac{4}{6}=0.666$ hence

$$\varpi((\pi_1,\mathfrak{H}_1),\tilde{\bigcup}_{i=1}^{3}(\pi_i,\mathfrak{H}_i),(\mathfrak{X},\mathfrak{S}))=\chi-\urcorner\left(\bigwedge_{i=2}^{3}(\pi_i,\mathfrak{H}_i),(\mathfrak{X},\mathfrak{S})\right)=0.666-0.666=0$$

Definition 19. A BHSES $(\mathfrak{P},\mathfrak{C})$ is named as a core BHSES of $\mathfrak{D}_{BHE}$ when it $\in$ reduct of $\mathfrak{D}_{BHE}$.

Definition 20. Suppose $\mathfrak{D}_{BHE}=((\pi,\mathfrak{H}),(\mathfrak{X},\mathfrak{S}),\mathfrak{Z})$, where $(\pi,\mathfrak{H})=\bigcup_{i=1}^{n}(\pi_i,\mathfrak{H}_i)$ and $\bigcup_{i=1}^{m}(\pi_i,\mathfrak{H}_i)\subset(\pi,\mathfrak{H})$ is a reduct of $\mathfrak{D}_{BHE}$. Let $(\mathfrak{P},\mathfrak{C})=\bigwedge_{i=1}^{m}(\pi_i,\mathfrak{H}_i)$. We say

$$if\ \ e_i\ \ ,then\ \ e_j\ \left(\frac{|\mathfrak{P}(e_i)|}{|\mathfrak{X}(e_j)|}\right)$$

a decision rule induced by $\bigcup_{i=1}^{m}(\pi_i,\mathfrak{H}_i)$ where $e_i\in\mathfrak{C}$, $\mathfrak{X}(e_j)\supseteq\mathfrak{P}(e_i)$, $e_j\in\mathfrak{S}$ and $\frac{|\mathfrak{P}(e_i)|}{|\mathfrak{X}(e_j)|}$ presents the coverage proportion rule.

Example 3.14. Taking 3.12, we have

$$(\pi_1,\mathfrak{H}_1)\bigwedge(\pi_2,\mathfrak{H}_2)=\left\{\begin{array}{l}(\phi_1=(\varpi_1,\varpi_3),\{F_1,F_6\}),(\phi_2=(\varpi_1,\varpi_4),\{F_4\}),\\ (\phi_3=(\varpi_2,\varpi_3),\{F_5\})\,(\phi_4=(\varpi_2,\varpi_4),\emptyset)\end{array}\right\}.$$

Now

(i) If ϕ_1 then $\varpi_7(2/4)$
(ii) If ϕ_2 then $\varpi_7(1/4)$
(iii) If ϕ_3 then $\varpi_7(1/4)$
(iv) If ϕ_4 then $\varpi_7(0/4)$

4. An application of Bijective Hypersoft Expert set

This section presents an application of bijective hypersoft expert set to describe the decision rules.

Example 4.1. Suppose that one of the immediate selling organizations wishes to assess eight products from a producer and pick the most appropriate product for it to advertise. Let there are eight products which form the universe of discourse $Q=\{o_1,o_2,...,o_8\}$ with expert set $P=\{p_1,p_2,p_3\}$. Let $E=\{w_1,w_2,w_3,w_4,w_5\}$ be the set of parameters which stand for
w_1 = Price $=\{low=\tilde{\mathfrak{p}}_1,high=\tilde{\mathfrak{p}}_2\}$
w_2 = Effectiveness = $\{more=\tilde{\mathfrak{p}}_3,less=\tilde{\mathfrak{p}}_4\}$
w_3 = Date of expire = $\{ok=\tilde{\mathfrak{p}}_5,notok=\tilde{\mathfrak{p}}_6\}$
w_4 = Utilization = $\{more=\tilde{\mathfrak{p}}_7,less=\tilde{\mathfrak{p}}_8\}$
w_5 = Quality = $\{good=\tilde{\mathfrak{p}}_9,better=\tilde{\mathfrak{p}}_{10}\}$
and then

$\tilde{\mathfrak{p}} = \tilde{\mathfrak{p}}_1 \times \tilde{\mathfrak{p}}_2 \times \tilde{\mathfrak{p}}_3 \times \tilde{\mathfrak{p}}_4 \times \tilde{\mathfrak{p}}_5$

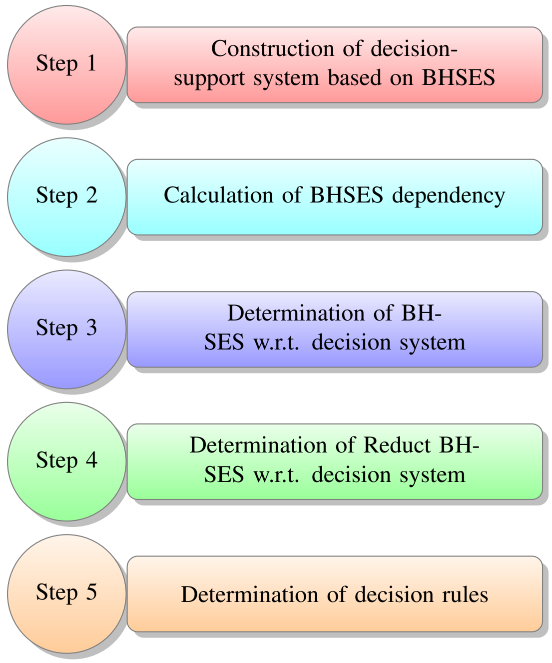
$$\tilde{\mathfrak{p}} = \left\{ \begin{array}{l} (\tilde{\mathfrak{p}}_1, \tilde{\mathfrak{p}}_3, \tilde{\mathfrak{p}}_5, \tilde{\mathfrak{p}}_7, \tilde{\mathfrak{p}}_9), (\tilde{\mathfrak{p}}_1, \tilde{\mathfrak{p}}_3, \tilde{\mathfrak{p}}_5, \tilde{\mathfrak{p}}_7, \tilde{\mathfrak{p}}_{10}), (\tilde{\mathfrak{p}}_1, \tilde{\mathfrak{p}}_3, \tilde{\mathfrak{p}}_5, \tilde{\mathfrak{p}}_8, \tilde{\mathfrak{p}}_9), (\tilde{\mathfrak{p}}_1, \tilde{\mathfrak{p}}_3, \tilde{\mathfrak{p}}_5, \tilde{\mathfrak{p}}_8, \tilde{\mathfrak{p}}_{10}), (\tilde{\mathfrak{p}}_1, \tilde{\mathfrak{p}}_3, \tilde{\mathfrak{p}}_6, \tilde{\mathfrak{p}}_7, \tilde{\mathfrak{p}}_9), \\ (\tilde{\mathfrak{p}}_1, \tilde{\mathfrak{p}}_3, \tilde{\mathfrak{p}}_6, \tilde{\mathfrak{p}}_7, \tilde{\mathfrak{p}}_{10}), (\tilde{\mathfrak{p}}_1, \tilde{\mathfrak{p}}_3, \tilde{\mathfrak{p}}_6, \tilde{\mathfrak{p}}_8, \tilde{\mathfrak{p}}_9), (\tilde{\mathfrak{p}}_1, \tilde{\mathfrak{p}}_3, \tilde{\mathfrak{p}}_6, \tilde{\mathfrak{p}}_8, \tilde{\mathfrak{p}}_{10}), (\tilde{\mathfrak{p}}_1, \tilde{\mathfrak{p}}_4, \tilde{\mathfrak{p}}_5, \tilde{\mathfrak{p}}_7, \tilde{\mathfrak{p}}_9), (\tilde{\mathfrak{p}}_1, \tilde{\mathfrak{p}}_4, \tilde{\mathfrak{p}}_5, \tilde{\mathfrak{p}}_7, \tilde{\mathfrak{p}}_{10}), \\ (\tilde{\mathfrak{p}}_1, \tilde{\mathfrak{p}}_4, \tilde{\mathfrak{p}}_5, \tilde{\mathfrak{p}}_8, \tilde{\mathfrak{p}}_9), (\tilde{\mathfrak{p}}_1, \tilde{\mathfrak{p}}_4, \tilde{\mathfrak{p}}_5, \tilde{\mathfrak{p}}_8, \tilde{\mathfrak{p}}_{10}), (\tilde{\mathfrak{p}}_1, \tilde{\mathfrak{p}}_4, \tilde{\mathfrak{p}}_6, \tilde{\mathfrak{p}}_7, \tilde{\mathfrak{p}}_9), (\tilde{\mathfrak{p}}_1, \tilde{\mathfrak{p}}_4, \tilde{\mathfrak{p}}_6, \tilde{\mathfrak{p}}_7, \tilde{\mathfrak{p}}_{10}), (\tilde{\mathfrak{p}}_1, \tilde{\mathfrak{p}}_4, \tilde{\mathfrak{p}}_6, \tilde{\mathfrak{p}}_8, \tilde{\mathfrak{p}}_9), \\ (\tilde{\mathfrak{p}}_1, \tilde{\mathfrak{p}}_4, \tilde{\mathfrak{p}}_6, \tilde{\mathfrak{p}}_8, \tilde{\mathfrak{p}}_{10}), (\tilde{\mathfrak{p}}_2, \tilde{\mathfrak{p}}_3, \tilde{\mathfrak{p}}_5, \tilde{\mathfrak{p}}_7, \tilde{\mathfrak{p}}_9), (\tilde{\mathfrak{p}}_2, \tilde{\mathfrak{p}}_3, \tilde{\mathfrak{p}}_5, \tilde{\mathfrak{p}}_7, \tilde{\mathfrak{p}}_{10}), (\tilde{\mathfrak{p}}_2, \tilde{\mathfrak{p}}_3, \tilde{\mathfrak{p}}_5, \tilde{\mathfrak{p}}_8, \tilde{\mathfrak{p}}_9), (\tilde{\mathfrak{p}}_2, \tilde{\mathfrak{p}}_3, \tilde{\mathfrak{p}}_5, \tilde{\mathfrak{p}}_8, \tilde{\mathfrak{p}}_{10}), \\ (\tilde{\mathfrak{p}}_2, \tilde{\mathfrak{p}}_3, \tilde{\mathfrak{p}}_6, \tilde{\mathfrak{p}}_7, \tilde{\mathfrak{p}}_9), (\tilde{\mathfrak{p}}_2, \tilde{\mathfrak{p}}_3, \tilde{\mathfrak{p}}_6, \tilde{\mathfrak{p}}_7, \tilde{\mathfrak{p}}_{10}), (\tilde{\mathfrak{p}}_2, \tilde{\mathfrak{p}}_3, \tilde{\mathfrak{p}}_6, \tilde{\mathfrak{p}}_8, \tilde{\mathfrak{p}}_9), (\tilde{\mathfrak{p}}_2, \tilde{\mathfrak{p}}_3, \tilde{\mathfrak{p}}_6, \tilde{\mathfrak{p}}_8, \tilde{\mathfrak{p}}_{10}), (\tilde{\mathfrak{p}}_2, \tilde{\mathfrak{p}}_4, \tilde{\mathfrak{p}}_5, \tilde{\mathfrak{p}}_7, \tilde{\mathfrak{p}}_9), \\ (\tilde{\mathfrak{p}}_2, \tilde{\mathfrak{p}}_4, \tilde{\mathfrak{p}}_5, \tilde{\mathfrak{p}}_7, \tilde{\mathfrak{p}}_{10}), (\tilde{\mathfrak{p}}_2, \tilde{\mathfrak{p}}_4, \tilde{\mathfrak{p}}_5, \tilde{\mathfrak{p}}_8, \tilde{\mathfrak{p}}_9), (\tilde{\mathfrak{p}}_2, \tilde{\mathfrak{p}}_4, \tilde{\mathfrak{p}}_5, \tilde{\mathfrak{p}}_8, \tilde{\mathfrak{p}}_{10}), (\tilde{\mathfrak{p}}_2, \tilde{\mathfrak{p}}_4, \tilde{\mathfrak{p}}_6, \tilde{\mathfrak{p}}_7, \tilde{\mathfrak{p}}_9), (\tilde{\mathfrak{p}}_2, \tilde{\mathfrak{p}}_4, \tilde{\mathfrak{p}}_6, \tilde{\mathfrak{p}}_7, \tilde{\mathfrak{p}}_{10}), \\ (\tilde{\mathfrak{p}}_2, \tilde{\mathfrak{p}}_4, \tilde{\mathfrak{p}}_6, \tilde{\mathfrak{p}}_8, \tilde{\mathfrak{p}}_9), (\tilde{\mathfrak{p}}_2, \tilde{\mathfrak{p}}_4, \tilde{\mathfrak{p}}_6, \tilde{\mathfrak{p}}_8, \tilde{\mathfrak{p}}_{10}) \end{array} \right\}$$

and now take $\mathfrak{K} \subseteq \mathfrak{L}$ as
$\mathfrak{K} = \{\varpi_1 = (\tilde{\mathfrak{p}}_1, \tilde{\mathfrak{p}}_3, \tilde{\mathfrak{p}}_5, \tilde{\mathfrak{p}}_7, \tilde{\mathfrak{p}}_9), \varpi_2 = (\tilde{\mathfrak{p}}_1, \tilde{\mathfrak{p}}_3, \tilde{\mathfrak{p}}_6, \tilde{\mathfrak{p}}_7, \tilde{\mathfrak{p}}_{10}), \varpi_3 = (\tilde{\mathfrak{p}}_1, \tilde{\mathfrak{p}}_4, \tilde{\mathfrak{p}}_6, \tilde{\mathfrak{p}}_8, \tilde{\mathfrak{p}}_9), \varpi_4 = (\tilde{\mathfrak{p}}_2, \tilde{\mathfrak{p}}_3, \tilde{\mathfrak{p}}_6, \tilde{\mathfrak{p}}_8, \tilde{\mathfrak{p}}_9), \varpi_5 = (\tilde{\mathfrak{p}}_2, \tilde{\mathfrak{p}}_4, \tilde{\mathfrak{p}}_6, \tilde{\mathfrak{p}}_7, \tilde{\mathfrak{p}}_{10})\}$
and

$$(\xi, \mathfrak{K}) = \left\{ \begin{array}{l} ((\varpi_1, P_1, 1), \{o_1, o_2, o_3\}), ((\varpi_1, P_2, 1), \{o_1, o_4, o_5, o_8\}), \\ ((\varpi_1, P_3, 1), \{o_1, o_3, o_4, o_5, o_6, o_7, o_8\}), ((\varpi_2, P_1, 1), \{o_1, o_3, o_5, o_8\}), \\ ((\varpi_2, P_2, 1), \{o_1, o_3, o_4, o_5, o_6, o_8\}), ((\varpi_2, P_3, 1), \{o_1, o_2, o_4, o_5, o_6, o_8\}), \\ ((\varpi_3, P_1, 1), \{o_1, o_4, o_5, o_7\}), ((\varpi_3, P_2, 1), \{o_1, o_2, o_5, o_8\}), \\ ((\varpi_3, P_3, 1), \{o_1, o_3, o_5, o_8\}), ((\varpi_4, P_1, 1), \{o_1, o_7, o_8\}), \\ ((\varpi_4, P_2, 1), \{o_1, o_4, o_5, o_8\}), ((\varpi_4, P_3, 1), \{o_1, o_6, o_7, o_8\}), \\ ((\varpi_5, P_1, 1), \{o_1, o_3, o_4, o_5, o_7, o_8\}), ((\varpi_5, P_2, 1), \{o_1, o_4, o_5, o_6, o_8\}), \\ ((\varpi_5, P_3, 1), \{o_1, o_3, o_4, o_5, o_6, o_7, o_8\}), ((\varpi_1, P_1, 0), \{o_1, o_6, o_7, o_8\}), \\ ((\varpi_1, P_2, 0), \{o_2, o_3, o_6, o_7, o_8\}), ((\varpi_1, P_3, 0), \{o_1, o_5\}), \\ ((\varpi_2, P_1, 0), \{o_1, o_2, o_4, o_5, o_6\}), ((\varpi_2, P_2, 0) o_1, o_7\}, \\ ((\varpi_2, P_3, 0), \{o_1, o_3, o_4, o_5, o_6, o_7\}), ((\varpi_3, P_1, 0), \{o_1, o_2, o_6, o_8\}), \\ ((\varpi_3, P_2, 0), \{o_3, o_4, o_6, o_7\}), ((\varpi_3, P_3, 0), \{o_1, o_3, o_4, o_5, o_7\}), \\ ((\varpi_4, P_1, 0), \{o_1, o_2, o_3, o_4, o_5, o_7\}), ((\varpi_4, P_2, 0), \{o_2, o_3, o_6, o_7\}), \\ ((\varpi_4, P_3, 0), \{o_1, o_3, o_4, o_5\}), ((\varpi_5, P_1, 0), \{o_1, o_6\}), \\ ((\varpi_5, P_2, 0), \{o_1, o_2, o_6, o_7\}), ((\varpi_5, P_3, 0), \{o_1, o_4, o_6\}), \end{array} \right\}$$

is a hypersoft expert set.
Here an algorithm of bijective hypersoft expert sets is presented for the establishment of decision rules.

Proposed Algorithm for Optimal Selection of Surgical Mask



Step 1

suppose there are the following bijective hypersoft expert sets

$$(\mathcal{M}_1, \mathcal{N}_1) = \left\{ \ (\varpi_1, \{o_1, o_2, \}) , (\varpi_2, \{o_1, o_3, o_8\}) \ \right\}$$
$$(\mathcal{M}_2, \mathcal{N}_2) = \left\{ \ (\varpi_3, \{o_1, o_5, o_7\}) , (\varpi_4, \{o_4, o_5, o_8\}) \ \right\}$$
$$(\mathcal{M}_3, \mathcal{N}_3) = \left\{ \ (\varpi_5, \{o_1, o_2, o_7\}) , (\varpi_6, \{o_3, o_6\}) , (\varpi_7, \{o_6, o_7\}) \ \right\}$$
$$(\mathfrak{X}, \Theta) = \left\{ \ (\varpi_8, \{o_1, o_4, o_5, o_6\}) , (\varpi_9, \{o_2, o_3, o_7, o_8\}) \ \right\}$$

which form $\mathfrak{D}_{BHE} = (\bigcup_{i=1}^{3} (\mathcal{M}_i, \mathcal{N}_i), (\mathfrak{X}, \Theta), \mathfrak{Z})$ and $\mathcal{N}_i, \Theta \subseteq \mathbb{L}$.

Step 2

Since

$$(\mathcal{M}_1, \mathcal{N}_1) \bigwedge (\mathcal{M}_2, \mathcal{N}_2) = \left\{ \begin{array}{l} (\digamma_1 = (\varpi_1, \varpi_3), \{o_1\}) , (\digamma_2 = (\varpi_1, \varpi_4), \{o_8\}) , \\ (\digamma_3 = (\varpi_2, \varpi_3), \{o_1\}) , (\digamma_4 = (\varpi_2, \varpi_4), \{o_8\}) \end{array} \right\}$$

and

$$(\mathcal{M}_2, \mathcal{N}_2) \bigwedge (\mathcal{M}_3, \mathcal{N}_3) = \left\{ \begin{array}{l} (\digamma_5 = (\varpi_3, \varpi_5), \{o_1, o_7\}) , (\digamma_6 = (\varpi_3, \varpi_6), \emptyset) , (\digamma_7 = (\varpi_3, \varpi_7), \{o_7\}) \\ (\digamma_8 = (\varpi_4, \varpi_5), \emptyset) , (\digamma_9 = (\varpi_4, \varpi_6), \emptyset) , (\digamma_{10} = (\varpi_4, \varpi_7), \emptyset) \end{array} \right\}$$

and

$$(\mathcal{M}_1, \mathcal{N}_1) \bigwedge (\mathcal{M}_3, \mathcal{N}_3) = \left\{ \begin{array}{l} (\digamma_{11} = (\varpi_1, \varpi_5), \{o_1, o_2\}) , (\digamma_{12} = (\varpi_1, \varpi_6), \emptyset) , (\digamma_{13} = (\varpi_1, \varpi_7), \emptyset) \\ (\digamma_{14} = (\varpi_2, \varpi_5), \{o_1\}) , (\digamma_{15} = (\varpi_2, \varpi_6), \{o_3\}) , (\digamma_{16} = (\varpi_2, \varpi_7), \{o_4\}) \end{array} \right\}$$

Now

$\chi_1 = \daleth((\mathcal{M}_1, \mathcal{N}_1), (\mathfrak{X}, \Theta)) = \frac{4}{8} = 0.5$

$\chi_2 = \daleth((\mathcal{M}_2, \mathcal{N}_2), (\mathfrak{X}, \Theta)) = \frac{5}{8} = 0.625$

$\chi_3 = \daleth\left((\mathcal{M}_3, \mathcal{N}_3), (\mathfrak{X}, \Theta)\right) = \frac{5}{8} = 0.625$
$\chi_4 = \daleth\left((\mathcal{M}_1, \mathcal{N}_1) \bigwedge (\mathcal{M}_2, \mathcal{N}_2), (\mathfrak{X}, \Theta)\right) = \frac{2}{8} = 0.5$
$\chi_5 = \daleth\left((\mathcal{M}_2, \mathcal{N}_2) \bigwedge (\mathcal{M}_3, \mathcal{N}_3), (\mathfrak{X}, \Theta)\right) = \frac{3}{8} = 0.375$
$\chi_6 = \daleth\left((\mathcal{M}_1, \mathcal{N}_1) \bigwedge (\mathcal{M}_3, \mathcal{N}_3), (\mathfrak{X}, \Theta)\right) = \frac{4}{8} = 0.5$

Step 3

$(\mathcal{M}_1, \mathcal{N}_1) \bigwedge (\mathcal{M}_2, \mathcal{N}_2) \bigwedge (\mathcal{M}_3, \mathcal{N}_3) =$

$$\left\{ \begin{array}{l} (F_1 = (\varpi_1, \varpi_3, \varpi_5), \{o_1\}), (F_2 = (\varpi_1, \varpi_3, \varpi_6), \emptyset), (F_3 = (\varpi_1, \varpi_3, \varpi_7), \emptyset) \\ (F_4 = (\varpi_1, \varpi_4, \varpi_5), \emptyset), (F_5 = (\varpi_1, \varpi_4, \varpi_6), \emptyset), (F_6 = (\varpi_1, \varpi_4, \varpi_7), \emptyset) \\ (F_7 = (\varpi_2, \varpi_3, \varpi_5), \{o_1\}), (F_8 = (\varpi_2, \varpi_3, \varpi_6), \emptyset), (F_9 = (\varpi_2, \varpi_3, \varpi_7), \{o_3\}) \\ (F_{10} = (\varpi_2, \varpi_4, \varpi_5), \emptyset), (F_{11} = (\varpi_2, \varpi_4, \varpi_6), \emptyset), (F_{12} = (\varpi_2, \varpi_4, \varpi_7), \emptyset) \end{array} \right\}.$$

therefore

$\chi = \daleth\left(\bigwedge_{i=1}^{3} (\mathcal{M}_i, \mathcal{N}_i), (\mathfrak{X}, \Theta)\right) = \frac{2}{8} = 0.5$

Step 4

As

$$\daleth\Big((\mathcal{M}_1, \mathcal{N}_1) \bigwedge (\mathcal{M}_2, \mathcal{N}_2), (\mathfrak{X}, \Theta)\Big) = 0.5 = \daleth\left(\bigwedge_{i=1}^{3} (\mathcal{M}_i, \mathcal{N}_i), (\mathfrak{X}, \Theta)\right)$$

therefore $(\mathcal{M}_1, \mathcal{N}_1) \cup (\mathcal{M}_2, \mathcal{N}_2)$ is a reduct of $\mathfrak{D}_{BHE}$.

Step 5

Since $(\mathcal{M}_1, \mathcal{N}_1) \cup (\mathcal{M}_2, \mathcal{N}_2)$ is a reduct of $\mathfrak{D}_{BHE}$ so, decision rules w.r.t. $\mathfrak{D}_{BHE}$

(i) If F_1 then $\varpi_8(1/4)$
(ii) If F_2 then $\varpi_9(1/4)$
(iii) If F_3 then $\varpi_8(1/4)$
(iv) If F_4 then $\varpi_9(1/4)$

Here ϖ_8 and ϖ_9 have same values, so both are valuable for further evaluation.

4.2. **Comparative study.** In this subsection, we compare our proposed structure with the existing studies.

5. Conclusion

The paper is summarized as under

(1) Axiomatic properties, set-theoretic operations and laws of bijective hypersoft expert set are conceptualized with the support of numerical illustrative examples.
(2) A novel decision-support system is constructed with the help of some special type of aggregation operations like relaxed and restricted AND, dependency etc.
(3) A decision-making based daily-life problem is discussed with the help of an algorithm based on aggregation of bijective hypersoft expert set.
(4) The advantageous aspects of proposed study are judged through comparison with existing relevant models.

Following models may be developed by extending this study:

- Bijective fuzzy hypersoft expert set
- Bijective intuitionistic fuzzy hypersoft expert set
- Bijective Pythagorean fuzzy hypersoft expert set

TABLE 4. Comparison of proposed study with existing relevant models

Authors	Structures	Remarks
H. Kamacı et al. [20]	Bijective soft matrix theory	• Single set of attributes is employed to develop decision system via bijection on matrix theory • Multi-bijective linguistic soft decision system is established
Gong et al. [12]	Bijective soft set	• Single set of attributes is employed to develop decision system
Rahman et al. [46]	Bijective hypersoft set	• Attributes are further classified into disjoint attribute-valued sets • Decision system is developed via employment of multi-argument approximate functions.
Proposed structure	Bijective hypersoft expert set	• Attributes are further classified into disjoint attribute-valued sets • Decision system is developed via employment of multi-argument approximate functions • Multi Decisive opinion is being used to get the required result.

- Bijective picture fuzzy hypersoft expert set
- Bijective neutrosophic hypersoft expert set and many other hybridized structures with their applications in decision-making, optimization and other fields of pure and applied mathematics.

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HYPERGRAPHS

Regular and Totally Regular Interval Valued Neutrosophic Hypergraphs

Ali Hassan, Muhammad Aslam Malik, Florentin Smarandache

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Abstract

In this paper, we define the regular and the totally regular interval valued neutrosophic hypergraphs, and discuss the order and size along with properties of the regular and the totally regular single valued neutrosophic hypergraphs. We extend work to completeness of interval valued neutrosophic hypergraphs.

Keywords

interval valued neutrosophic hypergraphs, regular interval valued neutrosophic hypergraphs, totally regular interval valued neutrosophic hypergraphs.

1 Introduction

Smarandache [8] introduced the notion of neutrosophic sets (NSs) as a generalization of the fuzzy sets [14], intuitionistic fuzzy sets [12], interval valued fuzzy set [11] and interval-valued intuitionistic fuzzy sets [13] theories.

The neutrosophic sets are characterized by a truth-membership function (t), an indeterminacy-membership function (i) and a falsity membership function (f) independently, which are within the real standard or non-standard unit interval $]^{-}0 , 1^{+}[$.

In order to conveniently use NS in real life applications, Smarandache [8] and Wang et al. [9] introduced the concept of the single-valued neutrosophic set (SVNS), a subclass of the neutrosophic sets.

The same authors [10] introduced the concept of the interval valued neutrosophic set (IVNS), which is more precise and flexible than the single valued neutrosophic set.

The IVNS is a generalization of the single valued neutrosophic set, in which the three membership functions are independent and their value belong to the unit interval [0, 1]. More works on single valued neutrosophic sets, interval valued neutrosophic sets and their applications can be found on *http://fs.gallup.unm.edu/NSS/*.

Hypergraph is a graph in which an edge can connect more than two vertices, and can be applied to analyse architecture structures and to represent system partitions. J. Mordesen and P. S. Nasir gave the definitions for fuzzy hypergraphs. R. Parvathy and M. G. Karunambigai's paper introduced the concept of intuitionistic fuzzy hypergraphs and analysed its components. The regular intuitionistic fuzzy hypergraphs and the totally regular intuitionistic fuzzy hypergraphs were introduced by I. Pradeepa and S. Vimala [38].

In this paper, we extend the regularity and the totally regularity on interval valued neutrosophic hypergraphs.

2 Preliminaries

Definition 2.1.

Let X be a space of points (objects) with generic elements in X denoted by x. A single valued neutrosophic set A (SVNS A) is characterized by truth membership function $T_A(x)$, indeterminacy membership function $I_A(x)$ and a falsity membership function $F_A(x)$. For each point $x \in X$; $T_A(x)$, $I_A(x)$, $F_A(x) \in [0, 1]$.

Definition 2.2.

Let X be a space of points (objects) with generic elements in X denoted by x. An interval valued neutrosophic set A (IVNS A) is characterized by truth membership function $T_A(x)$, indeterminacy membership function $I_A(x)$ and a falsity membership function $F_A(x)$. For each point $x \in X$; $T_A(x) = [TL_A(x), TU_A(x)]$, $I_A(x) = [IL_A(x), IU_A(x)]$ and $F_A(x) = [FL_A(x), FU_A(x)]$ are contained in $[0, 1]$.

Definition 2.3.

A hypergraph is an ordered pair $H = (X, E)$, where:

(1) $X = \{x_1, x_2, \ldots, x_n\}$ a finite set of vertices.

(2) $E = \{E_1, E_2, \ldots, E_m\}$ a family of subsets of X.

(3) E_j for $j = 1,2,3,\ldots,m$ and $\cup_j(E_j) = X$.

The set X is called set of vertices and E is the set of edges (or hyperedges).

Definition 2.4.

An interval valued neutrosophic hypergraph is an ordered pair $H = (X, E)$, where:

(1) $X = \{x_1, x_2, \ldots, x_n\}$ a finite set of vertices.

(2) $E = \{ E_1, E_2, ..., E_m \}$ a family of IVNSs of X.

(3) $E_j \neq O = ([0,0], [0,0], [0,0])$ for $j = 1,2,3,...,m$ and $\bigcup_j Supp(E_j) = X$.

The set X is called set of vertices and E is the set of IVN-edges (or IVN-hyperedges).

Example 2.5.

Consider an interval valued neutrosophic hypergraphs $H = (X, E)$, where $X = \{ a, b, c, d \}$ and $E = \{ P, Q, R \}$, defined by:

$P = \{(a, [0.8, 0.9], [0.4, 0.7], [0.2, 0.7]), (b, [0.7, 0.9], [0.5, 0.8], [0.3, 0.9])\}$,

$Q = \{(b, [0.9, 1.0], [0.4, 0.5], [0.8, 1.0]), (c, [0.8, 0.9], [0.4, 0.5], [0.2, 0.7])\}$,

$R = \{(c, [0.1, 0.9], [0.5, 0.7], [0.4, 1.0]), (d, [0.1, 1.0], [0.9, 1.0], [0.5, 0.9])\}$.

Proposition 2.6.

The Interval Valued Neutrosophic Hypergraph (IVNHG) is the generalization of fuzzy hypergraph, intuitionistic fuzzy hypergraphs, interval valued fuzzy hypergraph, interval valued intuitionistic fuzzy hypergraph and single valued neutrosophic hypergraph.

3 Regular and Totally Regular IVNHGs

Definition 3.1.

The open neighbourhood of a vertex x in the interval valued neutrosophic hypergraphs (IVNHGs) is the set of adjacent vertices of x, excluding that vertex, and it is denoted by $N(x)$.

Definition 3.2.

The closed neighbourhood of a vertex x in the interval valued neutrosophic hypergraphs (IVNHGs) is the set of adjacent vertices of x, including that vertex, and it is denoted by $N[x]$.

Example 3.3.

Consider the interval valued neutrosophic hypergraphs $H = (X, E)$, where $X = \{ a, b, c, d, e \}$ and $E = \{ P, Q, R, S \}$, defined by:

$P = \{(a, [0.1, 0.4], [0.2, 0.8], [0.3, 0.9]), (b, [0.4, 0.5], [0.5, 0.6], [0.6, 0.8])\}$,

$Q = \{(c, [0.1, 0.7], [0.2, 0.8], [0.3, 0.9]), (d, [0.4, 0.8], [0.5, 0.9], [0.6, 0.7]),$

(e, [0.7, 0.9], [0.8, 0.9], [0.9, 1.0])},

R = {(b, [0.1, 0.4], [0.2, 0.8], [0.3, 0.9]), (c, [0.4, 0.8], [0.5, 0.9], [0.6, 0.7])},

S = {(a, [0.4, 0.8], [0.5, 0.9], [0.6, 0.7]), (d, [0.1, 0.4], [0.2, 0.8], [0.3, 0.9])}.

Then, the open neighbourhood of a vertex a is b and d.

The closed neigh-bourhood of a vertex b is b, a and c.

Definition 3.4.

Let *H = (X, E)* be an IVNHG; the open neighbourhood degree of a vertex x is denoted and defined by:

$$deg(x) = ([deg_{TL}(\mathrm{x}), deg_{TU}(\mathrm{x})], [deg_{IL}(\mathrm{x}), deg_{IU}(\mathrm{x})], [deg_{FL}(\mathrm{x}), deg_{FU}(\mathrm{x})]), \quad (1)$$

where:

$$deg_{TL}(\mathrm{x}) = \sum_{x \in N(x)} TL_E(x), \quad (2)$$

$$deg_{IL}(\mathrm{x}) = \sum_{x \in N(x)} IL_E(x), \quad (3)$$

$$deg_{FL}(\mathrm{x}) = \sum_{x \in N(x)} FL_E(x), \quad (4)$$

$$deg_{TU}(\mathrm{x}) = \sum_{x \in N(x)} TU_E(x), \quad (5)$$

$$deg_{IU}(\mathrm{x}) = \sum_{x \in N(x)} IU_E(x), \quad (6)$$

$$deg_{FU}(\mathrm{x}) = \sum_{x \in N(x)} FU_E(x). \quad (7)$$

Example 3.5.

Consider the interval valued neutrosophic hypergraphs *H = (X, E),* where *X = {a, b, c, d, e }* and *E = { P, Q, R, S }*, defined by:

P = {(a, [0.1, 0.2], [0.2, 0.3] [0.3, 0.4]), (b, [0.4, 0.5], [0.5, 0.6], [0.6, 0.7])},

Q = {(c, [0.1, 0.2], [0.2, 0.3], [0.3, 0.4]), (d, [0.4, 0.5], [0.5, 0.6], [0.6, 0.7]), (e, [0.7, 0.8], [0.8, 0.9], [0.9, 1.0])},

R = {(b, [0.1, 0.2], [0.2, 0.3], [0.3, 0.4]), (c, [0.4, 0.5], [0.5, 0.6], [0.6, 0.7]},

S = {(a, [0.1, 0.2], [0.2, 0.3], [0.3, 0.4]), (d, [0.4, 0.5], [0.5, 0.6], [0.6, 0.7]}.

Then, the open neighbourhood of a vertex a is b and d.

Therefore, the open neighbourhood degree of a vertex a is *([0.8, 1.0], [1.0, 1.2], [1.2, 1.4]).*

Definition 3.6.

Let $H = (X, E)$ be an IVNHG; the closed neighbourhood degree of a vertex x is denoted and defined by:

$$deg[x] = ([deg_{TL}[\mathrm{x}], deg_{TU}[\mathrm{x}]], [deg_{IL}[\mathrm{x}], deg_{IU}[\mathrm{x}]], [deg_{FL}[\mathrm{x}], deg_{FU}[\mathrm{x}]]), \tag{8}$$

where:

$$deg_{TL}[x] = deg_{TL}(x) + TL_E(x), \tag{9}$$

$$deg_{IL}[x] = deg_{IL}(x) + IL_E(x), \tag{10}$$

$$deg_{FL}[x] = deg_{FL}(x) + FL_E(x), \tag{11}$$

$$deg_{TU}[x] = deg_{TU}(x) + TU_E(x), \tag{12}$$

$$deg_{IU}[x] = deg_{IU}(x) + IU_E(x), \tag{13}$$

$$deg_{FU}[x] = deg_{FU}(x) + FU_E(x). \tag{14}$$

Example 3.7.

Consider the interval valued neutrosophic hypergraphs $H = (X, E)$, where $X = \{ a, b, c, d, e \}$ and $E = \{ P, Q, R, S \}$, defined by:

$P = \{(a, [0.1, 0.2], [0.2, 0.3] [0.3, 0.4]), (b, [0.4, 0.5], [0.5, 0.6], [0.6, 0.7])\}$,

$Q = \{(c, [0.1, 0.2], [0.2, 0.3], [0.3, 0.4]), (d, [0.4, 0.5], [0.5, 0.6], [0.6, 0.7]), (e, [0.7, 0.8], [0.8, 0.9], [0.9, 1.0])\}$,

$R = \{(b, [0.1, 0.2], [0.2, 0.3], [0.3, 0.4]), (c, [0.4, 0.5], [0.5, 0.6], [0.6, 0.7]\}$,

$S = \{(a, [0.1, 0.2], [0.2, 0.3], [0.3, 0.4]), (d, [0.4, 0.5], [0.5, 0.6], [0.6, 0.7]\}$.

The closed neighbourhood of a vertex a is a, b and d.

Hence the closed neighbourhood degree of a vertex $\underline{a}$ is $([0.9, 1.2], [1.2, 1.5], [1.5, 1.8])$.

Definition 3.8.

Let $H = (X , E)$ be an IVNHG; then H is said to be a n-regular IVNHG if all the vertices have the same open neighbourhood degree,

$$n = ([n_1 , n_2], [n_3 , n_4], [n_5 , n_6]). \tag{15}$$

Definition 3.9.

Let $H = (X, E)$ be an IVNHG; then H is said to be a m-totally regular IVNHG if all the vertices have the same closed neighbourhood degree,

$$m = ([m_1 , m_2], [m_3 , m_4], [m_5 , m_6]). \tag{16}$$

Proposition 3.10.

A regular IVNHG is the generalization of regular fuzzy hypergraphs, regular intuitionistic fuzzy hypergraphs, regular interval valued fuzzy hypergraphs and regular interval valued intuitionistic fuzzy hypergraphs.

Proposition 3.11.

A totally regular IVNHG is the generalization of the totally regular fuzzy hypergraphs, totally regular intuitionistic fuzzy hypergraphs, totally regular interval valued fuzzy hypergraphs and totally regular interval valued intuitionistic fuzzy hypergraphs.

Example 3.12.

Consider the interval valued neutrosophic hypergraphs $H = (X, E)$, where $X = \{ a, b, c, d \}$ and $E = \{ P, Q, R, S \}$, defined by:

$P = \{(a, [0.8, 0.9], [0.2, 0.3], [0.3, 0.4]), (b, [0.8, 0.9], [0.2, 0.3], [0.3, 0.4])\}$,

$Q = \{(b, [0.8, 0.9], [0.2, 0.3], [0.3, 0.4]), (c, [0.8, 0.9], [0.2, 0.3], [0.3, 0.4])\}$,

$R = \{(c, [0.8, 0.9], [0.2, 0.3], [0.3, 0.4]), (d, [0.8, 0.9], [0.2, 0.3], [0.3, 0.4])\}$,

$S = \{(d, [0.8, 0.9], [0.2, 0.3], [0.3, 0.4]), (a, [0.8, 0.9], [0.2, 0.3], [0.3, 0.4])\}$.

Here, the open neighbourhood degree of every vertex is $([1.6, 1.8], [0.4, 0.6], [0.6, 0.8])$, hence H is regular IVNHG and the closed neighbourhood degree of every vertex is $([2.4, 2.7], [0.6, 0.9], [0.9, 1.2])$. Hence H is both a regular and a totally regular IVNHG.

Theorem 3.13.

Let $H = (X, E)$ be an IVNHG which is both a regular and a totally regular IVNHG; then E is constant.

Proof.

Suppose H is a n-regular and a m-totally regular IVNHG. Then,

$$deg(x) = n = ([n_1, n_2], [n_3, n_4], [n_5, n_6]), \tag{17}$$

$$deg[x] = m = ([m_1, m_2], [m_3, m_4], [m_5, m_6]), \tag{18}$$

for all $x \in E_i$.

Consider

$$deg[x] = m, \tag{19}$$

hence, by definition,

$$deg(x) + E_i(\mathrm{x}) = m; \tag{20}$$

this implies that

$$E_i(\mathrm{x}) = m - n, \tag{21}$$

for all $x \in E_i$.

Hence E is constant.

Remark 3.14.

The converse of above theorem need not to be true in general.

Example 3.15.

Consider the interval valued neutrosophic hypergraphs $H = (X, E)$, where $X = \{a, b, c, d\}$ and $E = \{P, Q, R, S\}$, defined by:

$P = \{ (a, [0.8, 0.9], [0.2, 0.3], [0.3, 0.4]), (b, [0.8, 0.9], [0.2, 0.3], [0.3, 0.4]) \}$,

$Q = \{ (b, [0.8, 0.9], [0.2, 0.3], [0.3, 0.4]), (d, [0.8, 0.9], [0.2, 0.3], [0.3, 0.4]) \}$,

$R = \{ (c, [0.8, 0.9], [0.2, 0.3], [0.3, 0.4]), (d, [0.8, 0.9], [0.2, 0.3], [0.3, 0.4]) \}$,

$S = \{ (d, [0.8, 0.9], [0.2, 0.3], [0.3, 0.4]), (d, [0.8, 0.9], [0.2, 0.3], [0.3, 0.4]) \}$.

Here E is constant, but $deg(a) = ([1.6, 1.8], [0.4, 0.6], [0.6, 0.8])$ and $deg(d) = ([2.4, 2.7], [0.6, 0.9], [0.9, 1.2])$, i.e $deg(a)$ and $deg(d)$ are not equals, hence H is a not regular IVNHG. Next, $deg[a] = ([2.4, 2.7], [0.6, 0.9], [0.9, 1.2])$ and $deg[d]= ([3.2, 3.6], [0.8, 1.2], [1.2, 1.6])$, hence $deg[a]$ and $deg[d]$ are not equals, hence H is not a totally regular IVNHG.

We conclude that H is neither a regular and nor a totally regular IVNHG.

Theorem 3.16.

Let $H = (X, E)$ be an IVNHG; then E is constant on X if and only if the following are equivalent:

(1) H is a regular IVNHG;

(2) H is a totally regular IVNHG.

Proof.

Suppose $H = (X, E)$ is an IVNHG and E is constant in H, i.e.:

$$E_i(x) = c = ([c_1 , c_2], [c_3 , c_4], [c_5 , c_6]), \tag{22}$$

for all $\mathrm{x} \in E_i$.

Suppose H is a n-regular IVNHG; then

$$deg(x) = n = ([n_1 , n_2], [n_3 , n_4], [n_5 , n_6]), \quad (23)$$

for all x ∈ E_i .

Consider

$$deg[x] = deg(x) +E_i(x) = n + c, \quad (24)$$

for all x ∈ E_i.

Hence, H is a totally regular IVNHG.

Next, suppose that H is a m-totally regular IVNHG; then:

$$deg[x] = m = ([m_1 , m_2], [m_3 , m_4], [m_5 , m_6]), \quad (25)$$

for all x ∈ E_i. i.e.:

$$deg(x) + E_i(x) = m, \quad (26)$$

for all x ∈ E_i.

This implies that

$$deg(x) = m - c, \quad (27)$$

for all x ∈ E_i.

Thus, H is a regular IVNHG, and consequently *(1)* and *(2)* are equivalent.

Conversely.

Assume that *(1)* and *(2)* are equivalent, i.e. H is a regular IVNHG if and only if H is a totally regular IVNHG.

Suppose by contrary that E is not constant, that is E_i(x) and E_i(y) not equals for some x and y in X. Let $H = (X , E)$ be a n-regular IVNHG; then

$$deg(x) = n = ([n_1 , n_2], [n_3 , n_4], [n_5 , n_6]), \quad (28)$$

for all x ∈ E_i.

Consider:

$$deg[x] = deg(x) + E_i(x) = n + E_i(x), \quad (29)$$

$$deg[y] = deg(y) + E_i((y) = n + E_i(y), \quad (30)$$

since E_i(x) and E_i(y) are not equals for some x and y in X, hence *deg[x]* and *deg[y]* are not equals, thus H is not a totally regular IVNHG, which is a contradiction to our assumption.

Next, let H be a totally regular IVNHG, then

$$deg[x] = deg[y]. \quad (31)$$

That is

$$deg(x) + E_i(x) = deg(y) + E_i(y), \quad (32)$$

$$deg(x) - deg(y) = E_i(y) - E_i(x), \quad (33)$$

since RHS of above equation is nonzero, hence LHS of above equation is also nonzero, thus *deg(x)* and *deg(y)* are not equals, so H is not a regular IVNHG, which is again a contradiction to our assumption, thus our supposition was wrong, hence E must be constant, and this completes the proof.

Definition 3.17.

Let $H = (X, E)$ be a regular IVNHG; then the order of an IVNHG H is denoted and defined by:

$$O(H) = ([p, q], [r, s], [t, u]), \tag{34}$$

where

$$p = \sum_{x \in X} TL_{E_i}(x),\ q = \sum_{x \in X} TU_{E_i}(x),\ r = \sum_{x \in X} IL_{E_i}(x), \tag{35}$$

$$s = \sum_{x \in X} IU_{E_i}(x),\ t = \sum_{x \in X} FL_{E_i}(x),\ u = \sum_{x \in X} FU_{E_i}(x), \tag{36}$$

for every $x \in X$, and the size of a regular IVNHG is denoted and defined by:

$$S(H) = \sum_{i=1}^{n}(S_{E_i}), \tag{37}$$

where

$$S(E_i) = ([a, b], [c, d], [e, f]) \tag{38}$$

and

$$a = \sum_{x \in E_i} TL_{E_i}(x),\ b = \sum_{x \in E_i} TU_{E_i}(x),\ c = \sum_{x \in E_i} IL_{E_i}(x) \tag{39}$$

$$d = \sum_{x \in E_i} IU_{E_i}(x),\ e = \sum_{x \in E_i} FL_{E_i}(x),\ f = \sum_{x \in E_i} FU_{E_i}(x). \tag{40}$$

Example 3.18.

Consider the interval valued neutrosophic hypergraphs $H = (X, E)$, where $X = \{a, b, c, d\}$ and $E = \{P, Q, R, S\}$, defined by:

$P = \{ (a, [0.8, 0.9], [0.2, 0.3], [0.3, 0.4]), (b, [0.8, 0.9], [0.2, 0.3], [0.3, 0.4]) \}$,

$Q = \{ (b, [0.8, 0.9], [0.2, 0.3], [0.3, 0.4]), (c, [0.8, 0.9], [0.2, 0.3], [0.3, 0.4]) \}$,

$R = \{ (c, [0.8, 0.9], [0.2, 0.3], [0.3, 0.4]), (d, [0.8, 0.9], [0.2, 0.3], [0.3, 0.4]) \}$,

$S = \{ (d, [0.8, 0.9], [0.2, 0.3], [0.3, 0.4]), (a, [0.8, 0.9], [0.2, 0.3], [0.3, 0.4]) \}$.

Here, the order and the size of H are given, $([3.2, 3.6], [.8, 1.2], [1.2, 1.6])$, and $([6.4, 7.2], [1.6,2.4], [2.4,3.2])$ respectively.

Proposition 3.19.

The size of a n-regular IVNHG $H = (H , E)$ is $\frac{nk}{2}$ where $|X|= k$.

Proposition 3.20.

If $H = (X, E)$ is a m-totally regular IVNHG, then $2S(H) + O(H) = mk$, where $|X| = k$.

Corollary 3.21.

Let $H = (X, E)$ be a n-regular and a m-totally regular IVNHG; then $O(H) = k(m - n)$, where $|X|=k$.

Proposition 3.22.

The dual of a n-regular and a m-totally regular IVNHG $H = (X, E)$ is again a n-regular and a m-totally regular IVNHG.

Definition 3.23.

The interval valued neutrosophic hypergraph (IVNHG) is said to be a complete IVNHG if for every x in X, $N(x) = \{ x : x \text{ in } X\text{-}\{x\} \}$; that is $N(x)$ contains all remaining vertices of X except x.

Example 3.24.

Consider the interval valued neutrosophic hypergraphs $H = (X, E)$, where $X = \{ a, b, c, d \}$ and $E = \{ P, Q, R \}$, defined by:

$$P = \{(a, [0.4, 0.5], [0.6, 0.7], [0.3, 0.4]), (c, [0.8, 0.9], [0.2, 0.3], [0.3, 0.4])\}$$

$$Q = \{(a, [0.8, 1.0], [0.7, 0.9], [0.3, 0.7]), (b, [0.8, 0.9], [0.2, 0.3], [0.1, 0.9]), (d, [0.8, 0.9], [0.2, 0.5], [0.1, 0.9])\}$$

$$R = \{(c, [0.4, 0.6], [0.9, 1.0], [0.9, 1.0]), (d, [0.7, 0.9], [0.2, 0.7], [0.1, 0.7]), (b, [0.4, 0.6], [0.2, 0.7], [0.1, 0.8])\}$$

Here, $N(a) = \{b, c, d\}$, $N(b) = \{a, c, d\}$, $N(c) = \{a, b, d\}$, $N(d) = \{a, b, c\}$. Hence H is a complete IVNHG.

Remark 3.25.

In a complete IVNHG $H = (X, E)$, the cardinality of $N(x)$ is the same for every vertex.

Theorem 3.26.

Every complete IVNHG $H = (X , E)$ is both a regular and a totally regular if E is constant in H.

Proof.

Let $H = (X, E)$ be a complete IVNHG; suppose E is constant in H.

Consequently:

$$E_i(x) = c = ([c_1, c_2], [c_3, c_4], [c_5, c_6]), \quad (41)$$

for all x ∈ E_i; since IVNHG is complete, then by definition for every vertex *x* in *X*, *N(x) = { x : x in X-{x} }*, and the open neighbourhood degree of every vertex is same, that is:

$$deg(x) = n = ([n_1, n_2], [n_3, n_4], [n_5, n_6]), \quad (42)$$

for all x ∈ E_i.
Hence, a complete IVNHG is a regular IVNHG.
Also,

$$deg[x] = deg(x) + E_i(x) = n + c \quad (43)$$

for all x ∈ E_i.
Hence *H* is a totally regular IVNHG.

Remark 3.27.

Every complete IVNHG is totally regular even if *E* is not constant.

Definition 3.28.

An IVNHG is said to be *k*-uniform if all the hyper-edges have the same cardinality.

Example 3.29.

Consider an interval valued neutrosophic hypergraphs *H = (X , E),* where *X = { a, b, c, d }* and *E = { P, Q, R },* defined by:

P = {(a, [0.8, 0.9], [0.4,0.7], [0.2, 0.7]), (b, [0.7, 0.9], [0.5, 0.8], [0.3, 0.9])},

Q = {(b, [0.9, 1.0], [0.4, 0.5], [0.8, 1.0]), (c, [0.8, 0.9], [0.4, 0.5], [0.2, 0.7])},

R = {(c, [0.1, 0.9], [0.5, 0.7], [0.4, 1.0]), (d, [0.1, 1.0], [0.9, 1.0], [0.5, 0.9])}.

4 Conclusion

The theoretical concepts of graphs and hypergraphs are highly used in computer science applications. The interval valued neutrosophic hypergraphs are more flexible than the fuzzy hypergraphs and the intuitionistic fuzzy hypergraphs, the interval valued fuzzy hypergraphs and the interval valued intuitionistic fuzzy hypergraphs. The concept of interval valued neutrosophic hypergraphs can be applied in various areas of engineering and computer science.

In this paper, we defined the regular and the totally regular interval valued neutrosophic hypergraphs.

We plan to extend our research work to the irregular interval valued neutrosophic hypergraphs.

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Isomorphism of Single Valued Neutrosophic Hypergraphs

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Abstract

In this paper, we introduce the homomorphism, weak isomorphism, co-weak isomorphism, and isomorphism of single valued neutrosophic hypergraphs. The properties of order, size and degree of vertices, along with isomorphism, are included. The isomorphism of single valued neutrosophic hypergraphs equivalence relation and of weak isomorphism of single valued neutrosophic hypergraphs partial order relation is also verified.

Keywords

homomorphism, weak-isomorphism, co-weak-isomorphism, isomorphism of single valued neutrosophic hypergraphs.

1 Introduction

The neutrosophic set (NS) was proposed by Smarandache [8] as a generalization of the fuzzy sets [14], intuitionistic fuzzy sets [12], interval valued fuzzy set [11] and interval-valued intuitionistic fuzzy sets [13] theories, and it is a powerful mathematical tool for dealing with incomplete, indeterminate and inconsistent information in the real world. The neutrosophic sets are characterized by a truth-membership function (t), an indeterminacy-membership function (i) and a falsity membership function (f) independently, which are within the real standard or non-standard unit interval $]^-0, 1^+[$. To conveniently use NS in the real-life applications, Wang et al. [9] introduced the single-valued neutrosophic set (SVNS), as a subclass of the neutrosophic sets. The same authors [10] introduced the interval valued neutrosophic set (IVNS), which is even more precise and flexible than the single valued neutrosophic set. The IVNS is a generalization of the single valued neutrosophic set, in which the three membership functions are independent, and their values belong to the unit interval [0, 1]. The hypergraph is a graph in which an edge can connect more than two vertices. Hypergraphs can be applied to analyse architecture structures and to represent system partitions. In this paper, we extend the concept into isomorphism of single valued neutrosophic hypergraphs, and some of their properties are introduced.

2 Preliminaries

Definition 2.1

A hypergraph is an ordered pair $H = (X, E)$, where:

(1) $X = \{x_1, x_2, \dots, x_n\}$ a finite set of vertices;

(2) $E = \{E_1, E_2, \dots, E_m\}$ a family of subsets of X;

(3) E_j are not-empty for $j = 1,2,3, \dots, m$ and $\cup_j (E_j) = X$.

The set X is called set of vertices and E is the set of edges (or hyper-edges).

Definition 2.2

A fuzzy hypergraph H = (X, E) is a pair, where X is a finite set and E is a finite family of non-trivial fuzzy subsets of X, such that $X = \cup_j Supp(E_j)$, $j = 1, 2, 3, \dots, m$.

Remark 2.3

The collection $E = \{E_1, E_2, E_3, \dots, E_m\}$ is the collection of edge sets of H.

Definition 2.4

A fuzzy hypergraph with underlying set X is of the form $H = (X, E, R)$, where $E = \{E_1, E_2, E_3, \dots, E_m\}$ is the collection of fuzzy subsets of X, that is $E_j : X \to [0, 1]$, $j = 1, 2, 3, \dots, m$ and $R : E \to [0, 1]$ is a fuzzy relation on fuzzy subsets E_j, such that:

$$R(x_1, x_2, \dots, x_r) \leq \min(E_j(x_1), \dots, E_j(x_r)), \quad (1)$$

for all $\{x_1, x_2, \dots, x_r\}$ subsets of X.

Definition 2.5

Let X be a space of points (objects) with generic elements in X denoted by x. A single valued neutrosophic set A (SVNS A) is characterized by truth membership function $T_A(x)$, indeterminacy membership function $I_A(x)$, and a falsity membership function $F_A(x)$. For each point x ∈X; $T_A(x)$, $I_A(x)$, $F_A(x) \in [0, 1]$.

Definition 2.6

A single valued neutrosophic hypergraph (SVNHG) is an ordered pair $H = (X, E)$, where:

(1) $X = \{x_1, x_2, \dots, x_n\}$ a finite set of vertices.

(2) $E = \{E_1, E_2, \dots, E_m\}$ a family of SVNSs of X.

(3) $E_j \neq O = (0, 0, 0)$ for $j = 1, 2, 3, \dots, m$ and $\cup_j Supp(E_j) = X$.

The set X is called set of vertices and E is the set of SVN-edges (or SVN-hyper-edges).

Proposition 2.7

The SVNHG is the generalization of the fuzzy hypergraphs and of the intuitionistic fuzzy hypergraphs.

Let be given a SVNHG $H = (X, E, R)$, with underlying set X, where $E = \{E_1, E_2, \dots, E_m\}$ is the collection of non-empty family of SVN subsets of X, and R being SVN's relation on SVN subsets E_j such that:

$$R_T(x_1, x_2, \dots, x_r) \leq \min([T_{E_j}(x_1)], \dots, [T_{E_j}(x_r)]), \quad (2)$$

$$R_I(x_1, x_2, \dots, x_r) \geq \max([I_{E_j}(x_1)], \dots, [I_{E_j}(x_r)]), \quad (3)$$

$$R_F(x_1, x_2, \dots, x_r) \geq \max([F_{E_j}(x_1)], \dots, [F_{E_j}(x_r)]), \quad (4)$$

for all $\{x_1, x_2, \dots, x_r\}$ subsets of X.

Example 2.8

Consider the SVNHG $H = (X, E, R)$ with underlying set $X = \{a, b, c\}$, where $E = \{A, B\}$ and R, which is defined in the *Tables* given below.

H	A	B
a	(0.2,0.3,0.9)	(0.5,0.2,0.7)
b	(0.5,0.5,0.5)	(0.1,0.6,0.4)
c	(0.8,0.8,0.3)	(0.5,0.9,0.8)

R	R_T	R_I	R_F
A	0.2	0.8	0.9
B	0.1	0.9	0.8

By routine calculations, $H = (X, E, R)$ is a SVNHG.

3 Isomorphism of SVNHGs

Definition 3.1

A homomorphism $f: H \to K$ between two SVNHGs $H = (X, E, R)$ and $K = (Y, F, S)$ is a mapping $f: X \to Y$, which satisfies:

$$\min[T_{E_j}(x)] \leq \min[T_{F_j}(f(x))], \quad (5)$$

$$\max[I_{E_j}(x)] \geq \max[I_{F_j}(f(x))], \quad (6)$$

$$\max[F_{E_j}(x)] \geq \max[F_{F_j}(f(x))], \quad (7)$$

for all $x \in X$, and

$$R_T(x_1, x_2, \dots, x_r) \leq S_T(f(x_1), f(x_2), \dots, f(x_r)), \quad (8)$$

$$R_I(x_1, x_2, \dots, x_r) \geq S_I(f(x_1), f(x_2), \dots, f(x_r)), \quad (9)$$

$$R_F(x_1, x_2, \dots, x_r) \geq S_F(f(x_1), f(x_2), \dots, f(x_r)), \quad (10)$$

for all $\{x_1, x_2, \dots, x_r\}$ subsets of X.

Example 3.2

Consider the two SVNHGs $H = (X, E, R)$ and $K = (Y, F, S)$ with underlying sets $X = \{a, b, c\}$ and $Y = \{x, y, z\}$, where $E = \{A, B\}$, $F = \{C, D\}$, R and S, which are defined in the *Tables* given below, and $f: X \to Y$ defined by $f(a)=x$, $f(b)=y$ and $f(c)=z$.

H	A	B
a	(0.2,0.3,0.9)	(0.5,0.2,0.7)
b	(0.5,0.5,0.5)	(0.1,0.6,0.4)
c	(0.8,0.8,0.3)	(0.5,0.9,0.8)

K	C	D
x	(0.3,0.2,0.2)	(0.2,0.1,0.3)
y	(0.2,0.4,0.2)	(0.3,0.2,0.1)
z	(0.5,0.8,0.2)	(0.9, 0.7, 0.1)

R	R_T	R_I	R_F
A	0.2	0.8	0.9
B	0.1	0.9	0.8

S	S_T	S_I	S_F
C	0.2	0.8	0.3
D	0.1	0.7	0.3

By routine calculations, $f: H \to K$ is a homomorphism between H and K.

Definition 3.3

A weak isomorphism $f: H \to K$ between two SVNHGs $H = (X, E, R)$ and $K = (Y, F, S)$ is a bijective mapping $f : X \to Y$, which satisfies f is homomorphism, such that:

$$\min[T_{E_j}(x)] = \min[T_{F_j}(f(x))], \tag{11}$$

$$\max[I_{E_j}(x)] = \max[I_{F_j}(f(x))], \tag{12}$$

$$\max[F_{E_j}(x)] = \max[F_{F_j}(f(x))], \tag{13}$$

for all $x \in X$.

Note

The weak isomorphism between two SVNHGs preserves the weights of vertices.

Example 3.4

Consider the two SVNHGs $H = (X, E, R)$ and $K = (Y, F, S)$ with underlying sets $X = \{a, b, c\}$ and $Y = \{x, y, z\}$, where $E = \{A, B\}$, $F = \{C, D\}$, R and S, which are defined in the *Tables* given below, and *f: X → Y* defined by *f(a)=x, f(b)=y* and *f(c)=z*.

H	A	B
a	(0.2,0.3,0.9)	(0.5,0.2,0.7)
b	(0.5,0.5,0.5)	(0.1,0.6,0.4)
c	(0.8,0.8,0.3)	(0.5,0.9,0.8)

K	C	D
x	(0.2,0.3,0.2)	(0.2,0.1,0.8)
y	(0.2,0.4,0.2)	(0.1,0.6,0.5)
z	(0.5,0.8,0.9)	(0.9,0.9,0.1)

R	R_T	R_I	R_F
A	0.2	0.8	0.9
B	0.1	0.9	0.9

S	S_T	S_I	S_F
C	0.2	0.8	0.9
D	0.1	0.9	0.8

By routine calculations, *f: H → K* is a weak isomorphism between *H* and *K*.

Definition 3.5

A co-weak isomorphism *f: H → K* between two SVNHGs *H = (X, E, R)* and *K = (Y, F, S)* is a bijective mapping *f: X → Y* which satisfies *f* is homomorphism, i.e.:

$$R_T(x_1, x_2, \dots, x_r) = S_T(f(x_1), f(x_2), \dots, f(x_r)), \quad (14)$$

$$R_I(x_1, x_2, \dots, x_r) = S_I(f(x_1), f(x_2), \dots, f(x_r)), \quad (15)$$

$$R_F(x_1, x_2, \dots, x_r) = S_F(f(x_1), f(x_2), \dots, f(x_r)), \quad (16)$$

for all $\{x_1, x_2, \dots, x_r\}$ subsets of *X*.

Note

The co-weak isomorphism between two SVNHGs preserves the weights of edges.

Example 3.6

Consider the two SVNHGs $H = (X, E, R)$ and $K = (Y, F, S)$ with underlying sets $X = \{a, b, c\}$ and $Y = \{x, y, z\}$, where $E = \{A, B\}$,$F = \{C, D\}$, R and S are defined in the *Tables* given below, and $f: X \rightarrow Y$ defined by $f(a)=x$, $f(b)=y$ and $f(c)=z$.

H	A	B
a	(0.2,0.3,0.9)	(0.5,0.2,0.7)
b	(0.5,0.5,0.5)	(0.1,0.6,0.4)
c	(0.8,0.8,0.3)	(0.5,0.9,0.8)

K	C	D
x	(0.3,0.2,0.2)	(0.2,0.1,0.3)
y	(0.2,0.4,0.2)	(0.3,0.2,0.1)
z	(0.5,0.8,0.2)	(0.9, 0.7, 0.1)

R	R_T	R_I	R_F
A	0.2	0.8	0.9
B	0.1	0.9	0.8

S	S_T	S_I	S_F
C	0.2	0.8	0.9
D	0.1	0.9	0.8

By routine calculations, $f: H \rightarrow K$ is a co-weak isomorphism between H and K.

Definition 3.7

An isomorphism *f: H → K* between two SVNHGs *H = (X, E, R)* and *K = (Y, F, S)* is a bijective mapping *f: X → Y*, which satisfies:

$$\min[T_{E_j}(x)] = \min[T_{F_j}(f(x))], \tag{17}$$

$$\max[I_{E_j}(x)] = \max[I_{F_j}(f(x))], \tag{18}$$

$$\max[F_{E_j}(x)] = \max[F_{F_j}(f(x))], \tag{19}$$

for all $x \in X$, and:

$$R_T(x_1, x_2, \dots, x_r) = S_T(f(x_1), f(x_2), \dots, f(x_r)), \tag{20}$$

$$R_I(x_1, x_2, \dots, x_r) = S_I(f(x_1), f(x_2), \dots, f(x_r)), \tag{21}$$

$$R_F(x_1, x_2, \dots, x_r) = S_F(f(x_1), f(x_2), \dots, f(x_r)), \tag{22}$$

for all $\{x_1, x_2, \dots, x_r\}$ subsets of *X.*

Note

The isomorphism between two SVNHGs preserves both the weights of vertices and the weights of edges.

Example 3.8

Consider the two SVNHGs *H = (X, E, R)* and *K = (Y, F, S)* with underlying sets *X = {a, b, c}* and *Y = {x, y, z}*, where *E = {A, B}*, *F = {C, D}*, *R* and *S*, which are defined in the *Tables* given below, and *f: X → Y* defined by, *f(a)=x* , *f(b)=y* and *f(c)=z*.

H	A	B
a	(0.2,0.3,0.7)	(0.5,0.2,0.7)
b	(0.5,0.5,0.5)	(0.1,0.6,0.4)
c	(0.8,0.8,0.3)	(0.5,0.9,0.8)

K	C	D
x	(0.2,0.3,0.2)	(0.2,0.1,0.8)
y	(0.2,0.4,0.2)	(0.1,0.6,0.5)
z	(0.5,0.8,0.7)	(0.9, 0.9, 0.1)

R	R_T	R_I	R_F
A	0.2	0.8	0.9
B	0.0	0.9	0.8

S	S_T	S_I	S_F
C	0.2	0.8	0.9
D	0.0	0.9	0.8

By routine calculations, $f: H \to K$ is an isomorphism between H and K.

Definition 3.9

Let $H = (X, E, R)$ be a SVNHG; then, the *order* of H is denoted and defined by:

$$O(H) = (\sum \min T_{E_j}(x), \sum \max I_{E_j}(x), \qquad (23)$$

and the *size* of H is denoted and defined by:

$$S(H) = (\sum R_T(E_j), \sum R_I(E_j), \sum R_F(E_j)). \qquad (24)$$

Theorem 3.10

Let $H = (X, E, R)$ and $K = (Y, F, S)$ be two SVNHGs, such that H is isomorphic to K.

Then:

(1) $O(H) = O(K)$;

(2) $S(H) = S(K)$.

Proof.

Let $f: H \to K$ be an isomorphism between H and K with underlying sets X and Y respectively.

Then, by definition, we have:

$$\min[T_{E_j}(x)] = \min[T_{F_j}(f(x))], \qquad (25)$$

$$\max[I_{E_j}(x)] = \max[I_{F_j}(f(x))], \qquad (26)$$

$$\max[F_{E_j}(x)] = \max[F_{F_j}(f(x))], \qquad (27)$$

for all $x \in X$, and:

$$R_T(x_1, x_2, \dots, x_r) = S_T(f(x_1), f(x_2), \dots, f(x_r)), \tag{28}$$

$$R_I(x_1, x_2, \dots, x_r) = S_I(f(x_1), f(x_2), \dots, f(x_r)), \tag{29}$$

$$R_F(x_1, x_2, \dots, x_r) = S_F(f(x_1), f(x_2), \dots, f(x_r)), \tag{30}$$

for all $\{x_1, x_2, \dots, x_r\}$ subsets of *X.*

Consider:

$$O_T(H) = \sum \min T_{E_j}(x) = \sum \min T_{F_j}(f(x)) = O_T(K) \tag{31}$$

Similarly, $O_I(H) = O_I(K)$ and $O_F(H) = O_F(K)$, hence *O(H) = O(K).*

Next,

$$S_T(H) = \sum R_T(x_1, x_2, \dots, x_r) = \sum S_T(f(x_1), f(x_2), \dots, f(x_r)) = S_T(K) \tag{32}$$

Similarly, $S_I(H) = S_I(K)$, $S_F(H) = S_F(K)$, hence $S(H) = S(K)$.

Remark 3.11

The converse of the above theorem need not to be true in general.

Example 3.12

Consider the two SVNHGs *H = (X, E, R)* and *K = (Y, F, S)* with underlying sets *X = {a, b, c, d}* and *Y = {w, x, y, z}*, where *E = {A, B}*, *F = {C, D}*, *R* and *S,* which are defined in the *Tables* given below, where *f* is defined by *f(a)=w, f(b)=x, f(c)=y, f(d)=z*.

H	A	B
a	(0.2, 0.5, 0.33)	(0.16,0.5,0.33)
b	(0.0,0.0,0.0)	(0.2,0.5,0.33)
c	(0.33,0.5,0.33)	(0.2,0.5,0.33)
d	(0.5,0.5,0.33)	(0.0,0.0,0.0)

K	C	D
w	(0.2,0.5,0.33)	(0.2,0.5,0.33)
x	(0.16,0.5,0.33)	(0.33,0.5,0.33)
y	(0.33,0.5,0.33)	(0.2,0.5,0.33)
z	(0.5,0.5,0.33)	(0.0,0.0,0.0)

R	R_T	R_I	R_F
A	0.2	0.5	0.33
B	0.16	0.5	0.33

S	S_T	S_I	S_F
C	0.16	0.5	0.33
D	0.2	0.5	0.33

Here, $O(H) = (1.06, 2.0, 1.32) = O(K)$ and $S(H) = (0.36, 1.0, 0.66) = S(K)$, but, by routine calculations, H is not isomorphism to K.

Corollary 3.13

The weak isomorphism between any two SVNHGs preserves the orders.

Remark 3.14

The converse of above corollary need not to be true in general.

Example 3.15

Consider the two SVNHGs $H = (X, E, R)$ and $K = (Y, F, S)$ with underlying sets $X = \{a, b, c, d\}$ and $Y = \{w, x, y, z\}$, where $E = \{A, B\}$, $F = \{C, D\}$, R and S, which are defined in the *Tables* given below, where f is defined by $f(a)=w, f(b)=x, f(c)=y, f(d)=z$.

H	A	B
a	(0.2,0.5,0.3)	(0.14,0.5,0.3)
b	(0.0,0.0,0.0)	(0.2,0.5,0.3)
c	(0.33,0.5,0.3)	(0.16,0.5,0.3)
d	(0.5,0.5,0.3)	(0.0,0.0,0.0)

K	C	D
w	(0.14,0.5,0.3)	(0.16,0.5,0.3)
x	(0.0,0.0,0.0)	(0.16,0.5,0.3)
y	(0.25,0.5,0.3)	(0.2,0.5,0.3)
z	(0.5,0.5,0.3)	(0.0,0.0,0.0)

Here, *O(H)= (1.0, 2.0, 1.2) = O(K),* but, by routine calculations, *H* is not weak isomorphism to *K*.

Corollary 3.16

The co-weak isomorphism between any two SVNHGs preserves sizes.

Remark 3.17

The converse of above corollary need not to be true in general.

Example 3.18

Consider the two SVNHGs *H = (X, E, R)* and *K = (Y, F, S)* with underlying sets *X = {a, b, c, d}* and *Y = {w, x, y, z}*, where *E = {A, B}, F = {C, D},R* and *S* are defined in the *Tables* given below, where *f* is defined by *f(a)=w, f(b)=x, f(c)=y, f(d)=z.*

H	A	B
a	(0.2,0.5,0.3)	(0.14,0.5,0.3)
b	(0.0,0.0,0.0)	(0.16,0.5,0.3)
c	(0.3,0.5,0.3)	(0.2,0.5,0.3)
d	(0.5,0.5,0.3)	(0.0,0.0,0.0)

K	C	D
w	(0.0,0.0,0.0)	(0.2,0.5,0.3)
x	(0.14,0.5,0.3)	(0.25,0.5,0.3)
y	(0.5,0.5,0.3)	(0.2,0.5,0.3)
z	(0.3,0.5,0.3)	(0.0,0.0,0.0)

R	R_T	R_I	R_F
A	0.2	0.5	0.3
B	0.14	0.5	0.3

S	S_T	S_I	S_F
C	0.14	0.5	0.3
D	0.2	0.5	0.3

Here, $S(H) = (0.34,1.0,0.6) = S(K)$, but, by routine calculations, H is not co-weak isomorphism to K.

Definition 3.19

Let $H = (X, E, R)$ be a SVNHG; then the degree of vertex x_i is denoted and defined by:

$$\deg(x_i) = (deg_T(x_i), deg_I(x_i), deg_F(x_i)), \tag{33}$$

where

$$deg_T(x_i) = \sum R_T(x_1, x_2, \dots, x_r), \tag{34}$$

$$deg_I(x_i) = \sum R_I(x_1, x_2, \dots, x_r), \tag{35}$$

$$deg_F(x_i) = \sum R_F(x_1, x_2, \dots, x_r), \tag{36}$$

for $x_i \neq x_r$.

Theorem 3.20

If H and K are two isomorphic SVNHGs, then the degree of their vertices is preserved.

Proof.

Let f: $H \rightarrow K$ be an isomorphism between H and K with underlying sets X and Y respectively; then, by definition, we have

$$\min[T_{E_j}(x)] = \min[T_{F_j}(f(x))], \tag{37}$$

$$\max[I_{E_j}(x)] = \max[I_{F_j}(f(x))], \tag{38}$$

$$\max[F_{E_j}(x)] = \max[F_{F_j}(f(x))], \tag{39}$$

for all $x \in X$, and:

$$R_T(x_1, x_2, \dots, x_r) = S_T(f(x_1), f(x_2), \dots, f(x_r)), \tag{40}$$

$$R_I(x_1, x_2, \dots, x_r) = S_I(f(x_1), f(x_2), \dots, f(x_r)), \tag{41}$$

$$R_F(x_1, x_2, \dots, x_r) = S_F(f(x_1), f(x_2), \dots, f(x_r)), \tag{42}$$

for all $\{x_1, x_2, \dots, x_r\}$ subsets of *X.*

Consider:

$$deg_T(x_i) = \sum R_T(x_1, x_2, \dots, x_r) = \sum S_T(f(x_1), f(x_2), \dots, f(x_r)) = deg_T(f(x_i)). \tag{43}$$

Similarly:

$$deg_I(x_i) = deg_I(f(x_i)), deg_F(x_i) = deg_F(f(x_i)) \tag{44}$$

Hence:

$$deg(x_i) = deg(f(x_i)). \tag{45}$$

Remark 3.21

The converse of the above theorem may not be true in general.

Example 3.22

Consider the two SVNHGs *H = (X, E, R)* and *K = (Y, F, S)* with underlying sets *X = {a, b}* and *Y = {x, y}*, where *E = {A, B}*, *F = {C, D}*, *R* and *S* are defined in the *Tables* given below, where *f* is defined by, *f(a)=x, f(b)=y,* here *deg(a) = (0.8, 1.0, 0.6) = deg(x)* and *deg(b) = (0.45, 1.0, 0.6) = deg(y).*

H	A	B
a	(0.5,0.5,0.3)	(0.3,0.5,0.3)
b	(0.25,0.5,0.3)	(0.2,0.5,0.3)

K	C	D
x	(0.3,0.5,0.3)	(0.5,0.5,0.3)
y	(0.2,0.5,0.3)	(0.25,0.5,0.3)

S	S_T	S_I	S_F
C	0.2	0.5	0.3
D	0.25	0.5	0.3

R	R_T	R_I	R_F
A	0.25	0.5	0.3
B	0.2	0.5	0.3

But H is not isomorphic to K, i.e. H is neither weak isomorphic nor co-weak isomorphic to K.

Theorem 3.23

The isomorphism between SVNHGs is an equivalence relation.

Proof.

Let *H = (X, E, R), K = (Y, F, S)* and *M = (Z, G, W)* be SVNHGs with underlying sets X, Y and *Z*, respectively:

- Reflexive.

Consider the map (identity map) $f: X \to X$ defined as follows: $f(x) = x$ *for all* $x \in X$, since identity map is always bijective and satisfies the conditions:

$$\min[T_{E_j}(x)] = \min[T_{E_j}(f(x))], \tag{46}$$

$$\max[I_{E_j}(x)] = \max[I_{E_j}(f(x))], \tag{47}$$

$$\max[F_{E_j}(x)] = \max[F_{E_j}(f(x))], \tag{48}$$

for all $x \in X$, and:

$$R_T(x_1, x_2, \ldots, x_r) = R_T(f(x_1), f(x_2), \ldots, f(x_r)), \tag{49}$$

$$R_I(x_1, x_2, \ldots, x_r) = R_I(f(x_1), f(x_2), \ldots, f(x_r)), \tag{50}$$

$$R_F(x_1, x_2, \ldots, x_r) = R_F(f(x_1), f(x_2), \ldots, f(x_r)), \tag{51}$$

for all $\{x_1, x_2, \ldots, x_r\}$ subsets of *X.*

Hence *f* is an isomorphism of SVNHG *H* to itself.

- Symmetric.

Let $f: X \to Y$ be an isomorphism of *H* and *K*, then *f* is bijective mapping, defined as $f(x) = y$ for all $x \in X$.

Then, by definition:

$$\min[T_{E_j}(x)] = \min[T_{F_j}(f(x))], \tag{52}$$

$$\max[I_{E_j}(x)] \;= \max[I_{F_j}\big(f(x)\big)], \tag{53}$$

$$\max[F_{E_j}(x)] \;= \max[F_{F_j}\big(f(x)\big)], \tag{54}$$

for all $x \in X$, and:

$$R_T(x_1, x_2, \dots, x_r) \;= S_T(f(x_1)\,, f(x_2), \dots, f(x_r)\,), \tag{55}$$

$$R_I(x_1, x_2, \dots, x_r) \;= S_I(f(x_1)\,, f(x_2), \dots, f(x_r)\,), \tag{56}$$

$$R_F(x_1, x_2, \dots, x_r) \;= \; S_F(f(x_1)\,, f(x_2), \dots, f(x_r)\,), \tag{57}$$

for all $\{\, x_1, x_2, \dots, x_r\}$ subsets of *X.*

Since f is bijective, then we have $f^{-1}(y) = x$ for all $y \in Y$.

Thus, we get:

$$\min[T_{E_j}\left(f^{-1}(y)\right)] \;= \min[T_{F_j}(y)], \tag{58}$$

$$\max[I_{E_j}\left(f^{-1}(y)\right)] \;= \max[I_{F_j}(y)], \tag{59}$$

$$\max[F_{E_j}\left(f^{-1}(y)\right)] \;= \max[F_{F_j}(y)], \tag{60}$$

for all $x \in X$, and:

$$R_T\left(f^{-1}(y_1), f^{-1}(y_2), \dots, f^{-1}(y_r)\right) \;= S_T(y_1\,, y_2, \dots, y_r), \tag{61}$$

$$R_I\left(f^{-1}(y_1), f^{-1}(y_2), \dots, f^{-1}(y_r)\right) \;= S_I(y_1\,, y_2, \dots, y_r), \tag{62}$$

$$R_F\left(f^{-1}(y_1), f^{-1}(y_2), \dots, f^{-1}(y_r)\right) \;= S_F(y_1\,, y_2, \dots, y_r), \tag{63}$$

for all $\{\, y_1, y_2, \dots, y_r\}$ subsets of *Y.*

Hence, we have a bijective map $f^{-1} : Y \to X$, which is an isomorphism from K to H.

- Transitive.

Let $f: X \to Y$ and $g: Y \to Z$ be two isomorphism of SVNHGs of H onto K and K onto M, respectively. Then gof is a bijective mapping from X to Z, where gof is defined as $(gof)(x) = g\big(f(x)\big)$ for all $x \in X$.

Since f is an isomorphism, then, by definition, $f(x) = y$ for all $x \in X$, which satisfies:

$$\min[T_{E_j}(x)] \;= \min[T_{F_j}\big(f(x)\big)]\,, \tag{64}$$

$$\max[I_{E_j}(x)] \;= \max[I_{F_j}\big(f(x)\big)], \tag{65}$$

$$\max[F_{E_j}(x)] \;= \max[F_{F_j}\big(f(x)\big)], \tag{66}$$

for all $x \in X$, and:

$$R_T(x_1, x_2, \dots, x_r) \;=\; S_T(f(x_1)\,, f(x_2), \dots, f(x_r)\,), \tag{67}$$

$$R_I(x_1, x_2, \dots, x_r) \;=S_I(f(x_1)\,, f(x_2), \dots, f(x_r)\,), \tag{68}$$

$$R_F(x_1, x_2, \dots, x_r) \;=\; S_F(f(x_1)\,, f(x_2), \dots, f(x_r)\,), \tag{69}$$

for all $\{\, x_1, x_2, \dots, x_r\}$ subsets of *X.*

Since $g\, : Y\, \rightarrow Z$is an isomorphism, then, by definition, $g(y) = z$ for all $y\; \in Y$, satisfying the conditions:

$$\min[T_{F_j}(y)] \;= \min\left[T_{G_j}\big(g(y)\big)\right], \tag{70}$$

$$\max[I_{F_j}(y)] \;= \max[I_{G_j}\big(g(y)\big)], \tag{71}$$

$$\max[F_{F_j}(y)] \;= \max[F_G\big(g(y)\big)], \tag{72}$$

for all $x\in X$, and:

$$S_T(y_1, y_2, \dots, y_r) \;=\; W_T(g(y_1)\,, g(y_2), \dots, g(y_r)\,), \tag{73}$$

$$S_I(y_1, y_2, \dots, y_r) \;= W_I(g(y_1)\,, g(y_2), \dots, g(y_r)\,), \tag{74}$$

$$S_F(y_1, y_2, \dots, y_r) \;=\; W_F(g(y_1)\,, g(y_2), \dots, g(y_r)\,), \tag{75}$$

for all $\{\, y_1, y_2, \dots, y_r\}$ subsets of *Y.*

Thus, from above equations, we conclude that:

$$\min[T_{E_j}(x)] \;=\; \min[T_{G_j}(g(f(x)))]\,, \tag{76}$$

$$\max[I_{E_j}(x)] \;=\; \max[I_{G_j}(g(f(x)))], \tag{77}$$

$$\max[F_{E_j}(x)] \;=\; \max[F_{G_j}(g(f(x)))], \tag{78}$$

for all $x\in X$, and:

$$R_T(x_1, \dots, x_r) = W_T(g\big(f(x_1)\big), \dots, g\big(f(x_r)\big)), \tag{79}$$

$$R_I(x_1, \dots, x_r) = W_I(g\big(f(x_1)\big), \dots, g\big(f(x_r)\big)), \tag{80}$$

$$R_F(x_1, \dots, x_r) = W_F(g\big(f(x_1)\big), \dots, g\big(f(x_r)\big)), \tag{81}$$

for all $\{\, x_1, x_2, \dots, x_r\}$ subsets of *X.*

Therefore, gof is an isomorphism between *H* and *M*. Hence, the isomorphism between SVNHGs is an equivalence relation.

Theorem 3.24

The weak isomorphism between SVNHGs satisfies the partial order relation.

Proof.

Let *H = (X, E, R), K = (Y, F, S)* and *M = (Z, G, W)* be SVNHGs with underlying sets X, Y and *Z,* respectively.

- Reflexive.

Consider the map (identity map) $f: X \to X$, defined as follows $f(x)=x$ for all $x \in X$, since the identity map is always bijective and satisfies the conditions:

$$\min[T_{E_j}(x)] = \min[T_{E_j}(f(x))], \tag{82}$$

$$\max[I_{E_j}(x)] = \max[I_{E_j}(f(x))], \tag{83}$$

$$\max[F_{E_j}(x)] = \max[F_{E_j}(f(x))], \tag{84}$$

for all $x \in X$, and:

$$R_T(x_1, x_2, \dots, x_r) \leq R_T(f(x_1), f(x_2), \dots, f(x_r)), \tag{85}$$

$$R_I(x_1, x_2, \dots, x_r) \geq R_I(f(x_1), f(x_2), \dots, f(x_r)), \tag{86}$$

$$R_F(x_1, x_2, \dots, x_r) \geq R_F(f(x_1), f(x_2), \dots, f(x_r)), \tag{87}$$

for all $\{x_1, x_2, \dots, x_r\}$ subsets of X.

Hence f is a weak isomorphism of SVNHG H to itself.

- Anti-symmetric.

Let f be a weak isomorphism between H onto K, and g be a weak isomorphic between K and H, that is $f: X \to Y$ is a bijective map defined by $f(x) = y$ for all $x \in X$, satisfying the conditions:

$$\min[T_{E_j}(x)] = \min[T_{F_j}(f(x))], \tag{88}$$

$$\max[I_{E_j}(x)] = \max[I_{F_j}(f(x))], \tag{89}$$

$$\max[F_{E_j}(x)] = \max[F_{F_j}(f(x))], \tag{90}$$

for all $x \in X$, and:

$$R_T(x_1, x_2, \dots, x_r) \leq S_T(f(x_1), f(x_2), \dots, f(x_r)), \tag{91}$$

$$R_I(x_1, x_2, \dots, x_r) \geq S_I(f(x_1), f(x_2), \dots, f(x_r)), \tag{92}$$

$$R_F(x_1, x_2, \dots, x_r) \geq S_F(f(x_1), f(x_2), \dots, f(x_r)), \tag{93}$$

for all $\{x_1, x_2, \dots, x_r\}$ subsets of X.

Since g is also a bijective map $g(y) = x$ for all $y \in Y$ satisfying the conditions:

$$\min[T_{F_j}(y)] = \min[T_{E_j}(g(y))], \tag{95}$$

$$\max[I_{F_j}(y)] = \max[I_{E_j}(g(y))], \tag{96}$$

$$\max[F_{F_j}(y)] = \max[F_{E_j}(g(y))], \tag{97}$$

for all $y \in Y$, and:

$$R_T(y, y_2, \dots, y_r) \ \leq \ S_T(g(y_1), g(y_2), \dots, g(y_r)), \tag{98}$$

$$R_I(y_1, y_2, \dots, y_r) \ \geq S_I(f(y_1), f(y_2), \dots, f(y_r)), \tag{99}$$

$$R_F(y_1, y_2, \dots, y_r) \ \geq \ S_F(f(y_1), f(y_2), \dots, f(y_r)), \tag{100}$$

for all $\{ y_1, y_2, \dots, y_r\}$ subsets of Y.

The above inequalities hold for finite sets X and Y only when H and K SVNHGs have same number of edges and the corresponding edge have same weight, hence H is identical to K.

- Transitive.

Let $f: X \rightarrow Y$ and $g: Y \rightarrow Z$ be two weak isomorphism of SVNHGs of H onto K and K onto M, respectively. Then gof is a bijective mapping from X to Z, where gof is defined as $(gof)(x) = g\big(f(x)\big)$ for all $x \ \in X$.

Since f is a weak isomorphism, then, by definition, $f(x) = y$ for all $x \ \in X$, which satisfies the conditions:

$$\min[T_{E_j}(x)] \ = \min[T_{F_j}\big(f(x)\big)], \tag{101}$$

$$\max[I_{E_j}(x)] \ = \max[I_{F_j}\big(f(x)\big)], \tag{102}$$

$$\max[F_{E_j}(x)] \ = \max[F_{F_j}\big(f(x)\big)], \tag{103}$$

for all $x \in X$, and:

$$R_T(x_1, x_2, \dots, x_r) \ \leq \ S_T(f(x_1), f(x_2), \dots, f(x_r)), \tag{104}$$

$$R_I(x_1, x_2, \dots, x_r) \ \geq S_I(f(x_1), f(x_2), \dots, f(x_r)), \tag{105}$$

$$R_F(x_1, x_2, \dots, x_r) \ \geq \ S_F(f(x_1), f(x_2), \dots, f(x_r)), \tag{106}$$

for all $\{ x_1, x_2, \dots, x_r\}$ subsets of X.

Since $g: Y \rightarrow Z$ is a weak isomorphism, then, by definition, $g(y) = z$ for all $y \ \in Y$ satisfying the conditions:

$$\min[T_{F_j}(y)] \ = \min[T_{G_j}\big(g(y)\big)], \tag{107}$$

$$\max[I_{F_j}(y)] \ = \max[I_{G_j}\big(g(y)\big)], \tag{108}$$

$$\max[F_{F_j}(y)] \ = \max[F_G\big(g(y)\big)], \tag{109}$$

for all $x \in X$, and:

$$S_T(y_1, y_2, \dots, y_r) \leq W_T(g(y_1), g(y_2), \dots, g(y_r)), \tag{110}$$

$$S_I(y_1, y_2, \dots, y_r) \ \geq W_I(g(y_1), g(y_2), \dots, g(y_r)), \tag{111}$$

$$S_F(y_1, y_2, \dots, y_r) \ \geq \ W_F(g(y_1), g(y_2), \dots, g(y_r)), \tag{112}$$

for all $\{ y_1, y_2, \dots, y_r\}$ subsets of Y.

Thus, from above equations, we conclude that:

$$\min[T_{E_j}(x)] = \min[T_{G_j}(g(f(x)))]\,, \tag{113}$$

$$\max[I_{E_j}(x)] = \max[I_{G_j}(g(f(x)))], \tag{114}$$

$$\max[F_{E_j}(x)] = \max[F_{G_j}(g(f(x)))], \tag{115}$$

for all $x\in X$, *and:*

$$R_T(x_1,\dots,x_r) \le W_T(g(f(x_2)),\dots,g(f(x_r))), \tag{116}$$

$$R_I(x_1,\dots,x_r) \ge W_I(g(f(x_2)),\dots,g(f(x_r))), \tag{117}$$

$$R_F(x_1,\dots,x_r) \ge W_F(g(f(x_2)),\dots,g(f(x_r))) \tag{118}$$

for all $\{x_1, x_2, \dots, x_r\}$ subsets of X.

Therefore gof is a weak isomorphism between H and M.

Hence, a weak isomorphism between SVNHGs is a partial order relation.

4 Conclusion

Theoretical concepts of graphs and hypergraphs are highly used by computer science applications. Single valued neutrosophic hypergraphs are more flexible than fuzzy hypergraphs and intuitionistic fuzzy hypergraphs. The concepts of single valued neutrosophic hypergraphs can be applied in various areas of engineering and computer science.

In this paper, the isomorphism between SVNHGs is proved to be an equivalence relation and the weak isomorphism to be a partial order relation. Similarly, it can be proved that a co-weak isomorphism in SVNHGs is a partial order relation.

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Isomorphism of Interval Valued Neutrosophic Hypergraphs

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Abstract

In this paper, we introduce the homomorphism, weak isomorphism, co-weak isomorphism and isomorphism of interval valued neutrosophic hypergraphs. The properties of order, size and degree of vertices, along with isomorphism, are included. The isomorphism of interval valued neutrosophic hypergraphs equivalence relation and weak isomorphism of interval valued neutrosophic hypergraphs partial order relation are also verified.

Keywords

homomorphism, weak-isomorphism, co-weak-isomorphism, isomorphism of interval valued neutrosophic hypergraphs.

1 Introduction

The neutrosophic sets are characterized by a truth-membership function (t), an indeterminacy-membership function (i) and a falsity membership function (f) independently, which are within the real standard or non-standard unit interval $]^-0 , 1^+[$.

Smarandache [8] proposed the notion of neutrosophic set (NS) as a generalization of the fuzzy set [14], intuitionistic fuzzy set [12], interval valued fuzzy set [11] and interval-valued intuitionistic fuzzy set [13] theories.

For convenient use of NS in real life applications, Wang et al. [9] introduced the concept of the single-valued neutrosophic set (SVNS), a subclass of the neutrosophic sets. The same authors [10] introduced the concept of the interval valued neutrosophic set (IVNS), which is more precise and flexible than the single valued neutrosophic set. The IVNS is a generalization of the single valued neutrosophic set, in which the three membership functions are independent and their value belong to the unit interval [0, 1].

More works on single valued neutrosophic sets, interval valued neutrosophic sets and their applications can be found on *http://fs.gallup.unm.edu/NSS/*.

Hypergraph is a graph in which an edge can connect more than two vertices. Hypergraphs can be applied to analyze architecture structures and to represent system partitions. Mordesen and Nasir gave the definitions for fuzzy hyper graphs. Parvathy R. and M. G. Karunambigai's paper introduced the concepts of intuitionistic fuzzy hypergraphs and analyze its components. Radhamani and Radhika introduced the concept of Isomorphism on Fuzzy Hypergraphs.

In this paper, we extend the concept to isomorphism of interval valued neutrosophic hypergraphs, and some of their important properties are introduced.

2 Preliminaries

Definition 2.1

A hypergraph is an ordered pair $H = (X, E)$, where:

(1) $X = \{x_1, x_2, \dots, x_n\}$ is a finite set of vertices.

(2) $E = \{E_1, E_2, \dots, E_m\}$ is a family of subsets of X.

(3) E_j are not-empty for $j = 1, 2, 3, \dots, m$ and $\cup_j (E_j) = X$.

The set X is called set of vertices and E is the set of edges (or hyper-edges).

Definition 2.2

A fuzzy hypergraph $H = (X, E)$ is a pair, where X is a finite set and E is a finite family of non-trivial fuzzy subsets of X, such that $X = \cup_j Supp(E_j)$, $j = 1, 2, 3, \dots, m$.

Remark 2.3

$E = \{E_1, E_2, E_3, \dots, E_m\}$ is the collection of edge set of H.

Definition 2.4

A fuzzy hypergraph with underlying set X is of the form $H = (X, E, R)$, where $E = \{E_1, E_2, E_3, \ldots, E_m\}$ is the collection of fuzzy subsets of X, i.e. $E_j : X \to [0, 1]$, $j = 1, 2, 3, \ldots, m$ and $R : E \to [0, 1]$ is a fuzzy relation on fuzzy subsets E_j, such that:

$$R(x_1, x_2, \ldots, x_r) \leq \min(E_j(x_1), \ldots, E_j(x_r)), \quad (1)$$

for all $\{x_1, x_2, \ldots, x_r\}$ subsets of X.

Definition 2.5

Let X be a space of points (objects) with generic elements in X, which is denoted by x. A single valued neutrosophic set A (SVNS A) is characterized by truth membership function $T_A(x)$, indeterminacy membership function $I_A(x)$ and a falsity membership function $F_A(x)$. For each point x $\in$X; $T_A(x)$, $I_A(x)$,$F_A(x)$ $\in [0, 1]$.

Definition 2.6

A single valued neutrosophic hypergraph is an ordered pair $H = (X, E)$, where:

(1) $X = \{x_1, x_2, \ldots, x_n\}$ is a finite set of vertices.

(2) $E = \{E_1, E_2, \ldots, E_m\}$ is a family of SVNSs of X.

(3)$E_j \neq O = (0, 0, 0)$ for $j = 1, 2, 3, \ldots, m$ and $\bigcup_j Supp(E_j) = X$.

The set X is called set of vertices and E is the set of SVN-edges (or SVN-hyper-edges).

Proposition 2.7

The single valued neutrosophic hypergraph is the generalization of fuzzy hypergraphs and intuitionistic fuzzy hypergraphs.

Note that a given a SVNHG$H = (X, E, R)$ with underlying set X, where $E = \{E_1, E_2, \ldots, E_m\}$ is the collection of non-empty family of SVN subsets of X, and R is SVN relation on SVN subsets E_j, such that:

$$R_T(x_1, x_2, \ldots, x_r) \leq \min([T_{E_j}(x_1)], \ldots, [T_{E_j}(x_r)]), \quad (2)$$

$$R_I(x_1, x_2, \ldots, x_r) \geq \max([I_{E_j}(x_1)], \ldots, [I_{E_j}(x_r)]), \quad (3)$$

$$R_F(x_1, x_2, \ldots, x_r) \geq \max([F_{E_j}(x_1)], \ldots, [F_{E_j}(x_r)]), \quad (4)$$

for all $\{x_1, x_2, \ldots, x_r\}$ subsets of X.

Definition 2.8

Let X be a space of points (objects) with generic elements in X denoted by x. An interval valued neutrosophic set A (IVNS A) is characterized by lower truth membership function $TL_A(x)$, lower indeterminacy membership function $IL_A(x)$, lower falsity membership function $FL_A(x)$, upper truth membership function $TU_A(x)$, upper indeterminacy membership function $IU_A(x)$, upper falsity membership function $FU_A(x)$, for each point x ∈ X; $[TL_A(x), TU_A]$, $[IL_A(x), IU_A(x)]$, $[FL_A(x), FU_A(x)]$ subsets of [0, 1].

Definition 2.9

An interval valued neutrosophic hypergraph is an ordered pair $H = (X, E)$, where:

(1) $X = \{x_1, x_2, \ldots, x_n\}$ be a finite set of vertices.

(2) $E = \{E_1, E_2, \ldots, E_m\}$ be a family of IVNSs of X.

(3) $E_j \neq O = ([0, 0], [0, 0], [0, 0])$ for $j = 1, 2, 3, \ldots, m$ and $\bigcup_j Supp(E_j) = X$.

The set X is called set of vertices and E is the set of IVN-edges (or IVN-hyper-edges).

Note that a given IVNHG $H = (X, E, R)$ with underlying set X, where $E = \{E_1, E_2, \ldots, E_m\}$ is the collection of non-empty family of IVN subsets of X, and R is IVN relation on IVN subsets E_j such that:

$$R_{TL}(x_1, x_2, \ldots, x_r) \leq \min([TL_{E_j}(x_1)], \ldots, [TL_{E_j}(x_r)]), \quad (5)$$

$$R_{IL}(x_1, x_2, \ldots, x_r) \geq \max([IL_{E_j}(x_1)], \ldots, [IL_{E_j}(x_r)]), \quad (6)$$

$$R_{FL}(x_1, x_2, \ldots, x_r) \geq \max([FL_{E_j}(x_1)], \ldots, [FL_{E_j}(x_r)]), \quad (7)$$

$$R_{TU}(x_1, x_2, \ldots, x_r) \leq \min([TU_{E_j}(x_1)], \ldots, [TU_{E_j}(x_r)]), \quad (8)$$

$$R_{IU}(x_1, x_2, \ldots, x_r) \geq \max([IU_{E_j}(x_1)], \ldots, [IU_{E_j}(x_r)]), \quad (9)$$

$$R_{FU}(x_1, x_2, \ldots, x_r) \geq \max([FU_{E_j}(x_1)], \ldots, [FU_{E_j}(x_r)]), \quad (10)$$

for all $\{x_1, x_2, \ldots, x_r\}$ subsets of X.

Proposition 2.10

The interval valued neutrosophic hypergraph is the generalization of fuzzy hypergraphs, intuitionistic fuzzy hypergraphs, interval valued fuzzy hypergraphs and interval valued intuitionistic fuzzy hypergraphs.

Example 2.11

Consider the IVNHG *H = (X, E, R)* with underlying set *X = {a, b, c},* where *E = {A, B}* and *R,* which are defined in the *Tables* given below:

H	A	B
a	([0.5,0.7], [0.2, 0.9], [0.5,0.8])	([0.3,0.5],[0.5,0.6], [0.0,0.1])
b	([0.0,0.0], [0.0,0.0], [0.0,0.0])	([0.1,0.4],[0.3,0.9],[0.9,1.0])
c	([0.2,0.3], [0.1,0.5], [0.4,0.7])	([0.5,0.9],[0.2,0.3],[0.5,0.8])

R	R_T	R_I	R_F
A	[0.1, 0.2]	[0.6, 1.0]	[0.5, 0.9]
B	[0.1, 0.3]	[0.9, 0.9]	[0.9, 1.0]

By routine calculations, *H = (X, E, R)* is IVNHG.

2 Isomorphism of SVNHGs

Definition 3.1

A homomorphism *f: H → K* between two IVNHGs *H = (X, E, R)* and *K = (Y, F, S)* is a mapping *f: X → Y* which satisfies the conditions:

$$\min[TL_{E_j}(x)] \le \min[TL_{F_j}(f(x))], \tag{11}$$

$$\max[IL_{E_j}(x)] \ge \max[IL_{F_j}(f(x))], \tag{12}$$

$$\max[FL_{E_j}(x)] \ge \max[FL_{F_j}(f(x))], \tag{13}$$

$$\min[TU_{E_j}(x)] \le \min[TU_{F_j}(f(x))], \tag{14}$$

$$\max[IU_{E_j}(x)] \ge \max[IU_{F_j}(f(x))], \tag{15}$$

$$\max[FU_{E_j}(x)] \ge \max[FU_{F_j}(f(x))], \text{ for all } x \in X. \tag{16}$$

$$R_{TL}(x_1, x_2, \dots, x_r) \le S_{TL}(f(x_1), f(x_2), \dots, f(x_r)), \tag{17}$$

$$R_{IL}(x_1, x_2, \dots, x_r) \ge S_{IL}(f(x_1), f(x_2), \dots, f(x_r)), \tag{18}$$

$$R_{FL}(x_1, x_2, \dots, x_r) \ge S_{FL}(f(x_1), f(x_2), \dots, f(x_r)), \tag{19}$$

$$R_{TU}(x_1, x_2, \dots, x_r) \le S_{TU}(f(x_1), f(x_2), \dots, f(x_r)), \tag{20}$$

$$R_{IU}(x_1, x_2, \dots, x_r) \ge S_{IU}(f(x_1), f(x_2), \dots, f(x_r)), \tag{21}$$

$$R_{FU}(x_1, x_2, \dots, x_r) \ge S_{FU}(f(x_1), f(x_2), \dots, f(x_r)), \tag{22}$$

for all $\{x_1, x_2, \dots, x_r\}$ subsets of *X.*

Example 3.2

Consider the two IVNHGs *H = (X, E, R)* and *K = (Y, F, S)* with underlying sets *X = {a, b, c}* and *Y = {x, y, z}*, where *E = {A, B}*, *F = {C, D}*, *R* and *S,* which are defined in the *Tables* given below:

H	A	B
a	([0.2,0.3], [0.3,0.4], [0.9,1.0])	([0.5,0.6], [0.2,0.3], [0.7,0.8])
b	([0.5,0.6], [0.5,0.6], [0.5,0.6])	([0.1,0.2], [0.6,0.7], [0.4,0.5])
c	([0.8,0.9], [0.8,0.9], [0.3, 0.4])	([0.5,0.6], [0.9,1.0], [0.8,0.9])
K	C	D
x	([0.3,0.4], [0.2,0.3], [0.2,0.3])	([0.2,0.3], [0.1,0.2], [0.3,0.4])
y	([0.2,0.4], [0.4,0.5], [0.2,0.3])	([0.3,0.4], [0.2,0.3], [0.1,0.2])
z	([0.5,0.6], [0.8,0.9], [0.2, 0.3])	([0.9,0.1], [0.7,0.8], [0.1,0.2])

R	R_T	R_I	R_F
A	[0.2,0.3]	[0.8,0.9]	[0.9,1.0]
B	[0.1,0.2]	[0.9,1.0]	[0.8,0.9]
S	S_T	S_I	S_F
C	[0.2,0.3]	[0.8,0.9]	[0.3,0.4]
D	[0.1,0.2]	[0.7,0.8]	[0.3,0.4]

and *f: X → Y* defined by, *f(a)=x* , *f(b)=y* and *f(c)=z*. Then, by routine calculations, *f: H → K* is a homomorphism between *H* and *K*.

Definition 3.3

A weak isomorphism *f: H → K* between two IVNHGs *H = (X, E, R)* and *K = (Y, F, S)* is a bijective mapping $f : X \to Y$ which satisfies the condition f is homomorphism, such that:

$$\min[TL_{E_j}(x)] = \min[TL_{F_j}(f(x))], \tag{23}$$

$$\max[IL_{E_j}(x)] = \max[IL_{F_j}(f(x))], \tag{24}$$

$$\max[FL_{E_j}(x)] = \max[FL_{F_j}(f(x))], \tag{25}$$

$$\min[TU_{E_j}(x)] = \min[TU_{F_j}(f(x))], \tag{26}$$

$$\max[IU_{E_j}(x)] = \max[IU_{F_j}(f(x))], \tag{27}$$

$$\max[FU_{E_j}(x)] = \max[FU_{F_j}(f(x))], \tag{28}$$

for all $x \in X$.

Note

The weak isomorphism between two IVNHGs preserves the weights of vertices.

Example 3.4

Consider the two IVNHGs *H = (X, E, R)* and *K = (Y, F, S)* with underlying sets *X = {a, b, c}* and *Y={x, y, z}*, where *E = {A, B}*, *F = {C, D}*, *R* and *S*, which are defined in the *Tables* given below:

H	A	B
a	([0.2,0.3], [0.3,0.4], [0.9,1.0])	([0.5,0.6], [0.2,0.3], [0.7,0.8])
b	([0.5,0.6], [0.5,0.6], [0.5,0.6])	([0.1,0.2], [0.6,0.7], [0.4,0.5])
c	([0.8,0.9], [0.8,0.9], [0.3, 0.4])	([0.5,0.6], [0.9,1.0], [0.8,0.9])
K	C	D
x	([0.2,0.3], [0.3,0.4], [0.2,0.3])	([0.2,0.3], [0.1,0.2], [0.8,0.9])
y	([0.2,0.3], [0.4,0.5], [0.2,0.3])	([0.1,0.2], [0.6,0.7], [0.5,0.6])
z	([0.5,0.6], [0.8,0.9], [0.9, 1.0])	([0.9,1.0], [0.9,1.0], [0.1,0.2])

R	R_T	R_I	R_F
A	[0.2,0.3]	[0.8,0.9]	[0.9,1.0]
B	[0.1,0.2]	[0.9,1.0]	[0.9,1.0]
S	S_T	S_I	S_F
C	[0.2,0.3]	[0.8,0.9]	[0.9,1.0]
D	[0.1,0.2]	[0.9,1.0]	[0.8,0.9]

and $f : X \rightarrow Y$ defined by, $f(a)=x$, $f(b)=y$ and $f(c)=z$. Then, by routine calculations, $f: H \rightarrow K$ is a weak isomorphism between *H* and *K*.

Definition 3.5

A co-weak isomorphism $f: H \rightarrow K$ between two IVNHGs *H = (X, E, R)* and *K = (Y, F, S)* is a bijective mapping $f : X \rightarrow Y$ which satisfies the condition *f* is homomorphism, such that:

$$R_{TL}(x_1, x_2, \dots, x_r) = S_{TL}(f(x_1), f(x_2), \dots, f(x_r)), \tag{29}$$

$$R_{IL}(x_1, x_2, \dots, x_r) = S_{IL}(f(x_1), f(x_2), \dots, f(x_r)), \tag{30}$$

$$R_{FL}(x_1, x_2, \dots, x_r) = S_{FL}(f(x_1), f(x_2), \dots, f(x_r)), \tag{31}$$

$$R_{TU}(x_1, x_2, \dots, x_r) = S_{TU}(f(x_1), f(x_2), \dots, f(x_r)), \tag{32}$$

$$R_{IU}(x_1, x_2, \dots, x_r) = S_{IU}(f(x_1), f(x_2), \dots, f(x_r)), \tag{33}$$

$$R_{FU}(x_1, x_2, \dots, x_r) = S_{FU}(f(x_1), f(x_2), \dots, f(x_r)), \quad (34)$$

for all $\{x_1, x_2, \dots, x_r\}$ subsets of *X.*

Note

The co-weak isomorphism between two IVNHGs preserves the weights of edges.

Example 3.6

Consider the two IVNHGs *H = (X, E, R)* and *K = (Y, F, S)* with underlying sets *X = {a, b, c}* and *Y = {x, y, z}*, where *E = {A, B}*, *F = {C, D}*, *R* and *S,* which are defined in the *Tables* given below:

H	A	B
a	([0.2,0.3], [0.3,0.4], [0.9,1.0])	([0.5,0.6], [0.2,0.3], [0.7,0.8])
b	([0.5,0.6], [0.5,0.6], [0.5,0.6])	([0.1,0.2], [0.6,0.7], [0.4,0.5])
c	([0.8,0.9], [0.8,0.9], [0.3, 0.4])	([0.5,0.6], [0.9,1.0], [0.8,0.9])
K	C	D
x	([0.3,0.4], [0.2,0.3], [0.2,0.3])	([0.2,0.3], [0.1,0.2], [0.3,0.4])
y	([0.2,0.3], [0.4,0.5], [0.2,0.3])	([0.3,0.4], [0.2,0.3], [0.1,0.2])
z	([0.5,0.6], [0.8,0.9], [0.2, 0.3])	([0.9,1.0], [0.7,0.8], [0.1,0.2])

R	R_T	R_I	R_F
A	[0.2,0.3]	[0.8,0.9]	[0.9,1.0]
B	[0.1,0.2]	[0.9,1.0]	[0.8,0.9]
S	S_T	S_I	S_F
C	[0.2,0.3]	[0.8,0.9]	[0.9,1.0]
D	[0.1,0.2]	[0.9,1.0]	[0.8,0.9]

and *f : X → Y* defined by, *f(a)=x* , *f(b)=y* and *f(c)=z*. Then, by routine calculations, *f : H → K* is a co-weak isomorphism between *H* and *K*.

Definition 3.7

An isomorphism *f: H → K* between two IVNHGs *H = (X, E, R)* and *K = (Y, F, S)* is a bijective mapping *f : X → Y* which satisfies the conditions:

$$\min[TL_{E_j}(x)] = \min[TL_{F_j}(f(x))], \quad (35)$$

$$\max[IL_{E_j}(x)] = \max[IL_{F_j}(f(x))], \quad (36)$$

$$\max[FL_{E_j}(x)] = \max[FL_{F_j}(f(x))], \quad (37)$$

$$\min[TU_{E_j}(x)] = \min[TU_{F_j}(f(x))], \tag{38}$$

$$\max[IU_{E_j}(x)] = \max[IU_{F_j}(f(x))], \tag{39}$$

$$\max[FU_{E_j}(x)] = \max[FU_{F_j}(f(x))], \tag{40}$$

for all $x \in X$.

$$R_{TL}(x_1, x_2, \dots, x_r) = S_{TL}(f(x_1), f(x_2), \dots, f(x_r)), \tag{41}$$

$$R_{IL}(x_1, x_2, \dots, x_r) = S_{IL}(f(x_1), f(x_2), \dots, f(x_r)), \tag{42}$$

$$R_{FL}(x_1, x_2, \dots, x_r) = S_{FL}(f(x_1), f(x_2), \dots, f(x_r)), \tag{43}$$

$$R_{TU}(x_1, x_2, \dots, x_r) = S_{TU}(f(x_1), f(x_2), \dots, f(x_r)), \tag{44}$$

$$R_{IU}(x_1, x_2, \dots, x_r) = S_{IU}(f(x_1), f(x_2), \dots, f(x_r)), \tag{45}$$

$$R_{FU}(x_1, x_2, \dots, x_r) = S_{FU}(f(x_1), f(x_2), \dots, f(x_r)), \tag{46}$$

for all $\{x_1, x_2, \dots, x_r\}$ subsets of *X.*

Note

The isomorphism between two IVNHGs preserves the both weights of vertices and weights of edges.

Example 3.8

Consider the two IVNHGs *H = (X, E, R)* and *K = (Y, F, S)* with underlying sets *X = {a, b, c}* and *Y = {x, y, z}*, where *E = {A, B}*, *F = {C, D}*, *R* and *S,* which are defined in the *Tables* given below**,**

H	A	B
a	([0.2,0.3], [0.3,0.4], [0.7,0.8])	([0.5,0.6], [0.2,0.3], [0.7,0.8])
b	([0.5,0.6], [0.5,0.6], [0.5,0.6])	([0.1,0.2], [0.6,0.7], [0.4,0.5])
c	([0.8,0.9], [0.8,0.9], [0.3, 0.4])	([0.5,0.6], [0.9,1.0], [0.8,0.9])
K	C	D
x	([0.2,0.3], [0.3,0.4], [0.2,0.3])	([0.2,0.3], [0.1,0.2], [0.8,0.9])
y	([0.2,0.3], [0.4,0.5], [0.2,0.3])	([0.1,0.2], [0.6,0.7], [0.5,0.6])
z	([0.5,0.6], [0.8,0.9], [0.7, 0.8])	([0.9,1.0], [0.9,1.0], [0.1,0.2])

R	R_T	R_I	R_F
A	[0.2,0.3]	[0.8,0.9]	[0.9,1.0]
B	[0.0,0.1]	[0.9,1.0]	[0.8,0.9]
S	S_T	S_I	S_F
C	[0.2,0.3]	[0.8,0.9]	[0.9,1.0]
D	[0.0,0.1]	[0.9,1.0]	[0.8,0.9]

and $f: X \rightarrow Y$ defined by, $f(a)=x$, $f(b)=y$ and $f(c)=z$. Then, by routine calculations, $f: H \rightarrow K$ is a isomorphism between H and K.

Definition 3.9

Let $H = (X, E, R)$ be a IVNHG; then, the order of H, which is denoted and defined by:

$$O(H)= ([\sum \min TL_{E_j}(x), \sum \min TU_{E_j}(x)], [\sum \max IL_{E_j}(x), \sum \max IU_{E_j}(x)], [\sum \max FL_{E_j}(x), \sum \max FU_{E_j}(x)]) \quad (47)$$

and the size of H, which is denoted and defined by:

$$S(H) = ([\sum R_{TL}(E_j), \sum R_{TU}(E_j)], [\sum R_{IL}(E_j), \sum R_{IL}(E_j)], [\sum R_{FL}(E_j), \sum R_{FU}(E_j)]) \quad (48)$$

Theorem 3.10

Let $H = (X, E, R)$ and $K = (Y, F, S)$ be two IVNHGs such that H is isomorphic to K; then:

(1) $O(H) = O(K)$,

(2) $S(H) = S(K)$.

Proof.

Let $f : H \rightarrow K$ be an isomorphism between two IVNHGs H and K with underlying sets X and Y respectively; then, by definition, we have that:

$$\min[TL_{E_j}(x)] = \min[TL_{F_j}(f(x))], \quad (49)$$

$$\max[IL_{E_j}(x)] = \max[IL_{F_j}(f(x))], \quad (50)$$

$$\max[FL_{E_j}(x)] = \max[FL_{F_j}(f(x))], \quad (51)$$

$$\min[TU_{E_j}(x)] = \min[TU_{F_j}(f(x))], \quad (52)$$

$$\max[IU_{E_j}(x)] = \max[IU_{F_j}(f(x))], \quad (53)$$

$$\max[FU_{E_j}(x)] = \max[FU_{F_j}(f(x))], \quad (54)$$

for all $x \in X$.

$$R_{TL}(x_1, x_2, \dots, x_r) = S_{TL}(f(x_1), f(x_2), \dots, f(x_r)), \quad (55)$$

$$R_{IL}(x_1, x_2, \dots, x_r) = S_{IL}(f(x_1), f(x_2), \dots, f(x_r)), \quad (56)$$

$$R_{FL}(x_1, x_2, \dots, x_r) = S_{FL}(f(x_1), f(x_2), \dots, f(x_r)), \quad (57)$$

$$R_{TU}(x_1, x_2, \dots, x_r) = S_{TU}(f(x_1), f(x_2), \dots, f(x_r)), \tag{58}$$

$$R_{IU}(x_1, x_2, \dots, x_r) = S_{IU}(f(x_1), f(x_2), \dots, f(x_r)), \tag{59}$$

$$R_{FU}(x_1, x_2, \dots, x_r) = S_{FU}(f(x_1), f(x_2), \dots, f(x_r)), \tag{60}$$

for all $\{x_1, x_2, \dots, x_r\}$ subsets of X.

Consider:

$$O_{TL}(H) = \sum \min TL_{E_j}(x) = \sum \min TL_{F_j}(f(x)) = O_{TL}(K) \tag{61}$$

$$O_{TU}(H) = \sum \min TU_{E_j}(x) = \sum \min TU_{F_j}(f(x)) = O_{TU}(K) \tag{62}$$

Similarly:

$$O_{IL}(H) = O_{IL}(K) and O_{FL}(H) = O_{FL}(K), \tag{63}$$

$$O_{IU}(H) = O_{IU}(K) and O_{FU}(H) = O_{FU}(K). \tag{64}$$

Hence, $O(H) = O(K)$.

Next,

$$S_{TL}(H) = \sum R_{TL}(x_1, x_2, \dots, x_r)$$

$$= \sum S_{TL}(f(x_1), f(x_2), \dots, f(x_r)) = S_{TL}(K), \tag{65}$$

and similarly:

$$S_{TU}(H) = \sum R_{TU}(x_1, x_2, \dots, x_r)$$

$$= \sum S_{TU}(f(x_1), f(x_2), \dots, f(x_r)) = S_{TU}(K) \tag{66}$$

Similarly,

$$S_{IL}(H) = S_{IL}(K), S_{FL}(H) = S_{FL}(K), \tag{67}$$

$$S_{IU}(H) = S_{IU}(K), S_{FU}(H) = S_{FU}(K), \tag{68}$$

hence $S(H) = S(K)$.

Remark 3.11

The converse of the above theorem needs not to be true in general.

Example 3.12

Consider the two IVNHGs $H = (X, E, R)$ and $K = (Y, F, S)$ with underlying sets $X = \{a, b, c, d\}$ and $Y = \{w, x, y, z\}$, where $E = \{A, B\}$, $F = \{C, D\}$, R and S, which are defined in the *Tables* given below:

H	A	B
a	([0.2, 0.3],[0.5,0.6],[0.33,0.43])	([0.16,0.26],[0.5,0.6],[0.33,0.43])
b	([0.0,0.0], [0.0,0.0], [0.0,0.0])	([0.2,0.3],[0.5,0.6],[0.33,0.43])
c	([0.33,0.43],[0.5,0.6],[0.33,0.43])	([0.2,0.3], [0.5,0.6],[0.33,0.43])
d	([0.5,0.6],[0.5,0.6],[0.33,0.43])	([0.0,0.0], [0.0,0.0], [0.0,0.0])
K	C	D
w	([0.2,0.3],[0.5,0.6],[0.33,0.43])	([0.16,0.26],[0.5,0.6],[0.33,0.43])
x	([0.0,0.0], [0.0,0.0], [0.0,0.0])	([0.2,0.3],[0.5,0.6],[0.33,0.43])
y	([0.33,0.43],[0.5,0.6],[0.33,0.43])	([0.2,0.3], [0.5,0.6],[0.33,0.43])
z	([0.5,0.6],[0.5,0.6],[0.33,0.43])	([0.0,0.0], [0.0,0.0], [0.0,0.0])

R	R_T	R_I	R_F
A	[0.2,0.3]	[0.5,0.6]	[0.33,0.43]
B	[0.16,0.26]	[0.5,0.6]	[0.33,0.43]
S	S_T	S_I	S_F
C	[0.16,0.26]	[0.5,0.6]	[0.33,0.43]
D	[0.2,0.3]	[0.5,0.6]	[0.33,0.43]

where *f* is defined by *f(a)=w, f(b)=x, f(c)=y, f(d)=z*.

Here, *O(H) = ([1.06, 1.46], [2.0, 2.4], [1.32, 1.72]) = O(K)* and *S(H) = ([0.36, 0.56], [1.0, 1.2], [0.66, 0.86]) = S(K)*.

By routine calculations, *H* is not isomorphism to *K*.

Corollary 3.13

The weak isomorphism between any two IVNHGs *H* and *K* preserves the orders.

Remark 3.14

The converse of the above corollary need not to be true in general.

Example 3.15

Consider the two IVNHGs *H = (X, E, R)* and *K = (Y, F, S)* with underlying sets *X = {a, b, c, d}* and *Y = {w, x, y, z}*, where *E = {A, B}*, *F = {C, D}*, *R* and *S,* which are defined in the *Tables* given below, where *f* is defined by *f(a)=w, f(b)=x, f(c)=y, f(d)=z.*

Here *O(H)= ([1.0,1.4], [2.0,2.4], [1.2,1.6]) = O(K).*

By routine calculations, *H* is not weak isomorphism to *K*.

H	A	B
a	([0.2,0.3],[0.5,0.6],[0.3,0.4])	([0.14,0.24],[0.5,0.6],[0.3,0.4])
b	([0.0,0.0], [0.0,0.0], [0.0,0.0])	([0.2,0.3],[0.5,0.6],[0.3,0.4])
c	([0.33,0.43],[0.5,0.6],[0.3,0.4])	([0.16,0.26], [0.5,0.6], [0.3,0.4])
d	([0.5,0.6], [0.5,0.6], [0.3,0.4])	([0.0,0.0], [0.0,0.0], [0.0,0.0])
K	C	D
w	([0.14,0.24],[0.5,0.6],[0.3,0.4])	([0.16,0.26],[0.5,0.6],[0.3,0.4])
x	([0.0,0.0], [0.0,0.0], [0.0,0.0])	([0.16,0.26],[0.5,0.6],[0.3,0.4])
y	([0.33,0.43],[0.5,0.6],[0.33,0.43])	([0.2,0.3], [0.5,0.6],[0.3,0.4])
z	([0.5,0.6],[0.5,0.6],[0.3,0.4])	([0.0,0.0], [0.0,0.0], [0.0,0.0])

Corollary 3.16

The co-weak isomorphism between any two IVNHGs *H* and *K* preserves the sizes.

Remark 3.17

The converse of the above corollary need not to be true in general.

Example 3.18

Consider the two IVNHGs *H = (X, E, R)* and *K = (Y, F, S)* with underlying sets *X = {a, b, c, d}* and *Y = {w, x, y, z}*, where *E = {A, B}*, *F = {C, D}*, *R* and *S*, which are defined in the *Tables* given below, where *f* is defined by, *f(a)=w*, *f(b)=x*, *f(c)=y*, *f(d)=z*. Here *S(H)= ([0.34,0.54], [1.0,1.2], [0.6,0.8]) = S(K)*, but, by routine calculations, *H* is not co-weak isomorphism to *K*.

H	A	B
a	([0.2,0.3],[0.5,0.6],[0.3,0.4])	([0.14,0.24],[0.5,0.6],[0.3,0.4])
b	([0.0,0.0], [0.0,0.0], [0.0,0.0])	([0.16,0.26],[0.5,0.6],[0.3,0.4])
c	([0.3,0.4],[0.5,0.6],[0.3,0.4])	([0.2,0.3], [0.5,0.6], [0.3,0.4])
d	([0.5,0.6], [0.5,0.6], [0.3,0.4])	([0.0,0.0], [0.0,0.0], [0.0,0.0])
K	C	D
w	([0.0,0.0],[0.0,0.0],[0.0,0.0])	([0.2,0.3],[0.5,0.6],[0.3,0.4])
x	([0.14,0.24], [0.5,0.6], [0.3,0.4])	([0.25,0.35],[0.5,0.6],[0.3,0.4])
y	([0.5,0.6], [0.5,0.6], [0.3,0.4])	([0.2,0.3], [0.5,0.6],[0.3,0.4])
z	([0.3,0.4], [0.5,0.6], [0.3,0.4])	([0.0,0.0], [0.0,0.0], [0.0,0.0])

R	R_T	R_I	R_F
A	[0.2,0.3]	[0.5,0.6]	[0.3,0.4]
B	[0.14,0.24]	[0.5,0.6]	[0.3,0.4]
S	S_T	S_I	S_F
C	[0.14,0.24]	[0.5,0.6]	[0.3,0.4]
D	[0.2,0.3]	[0.5,0.6]	[0.3,0.4]

Definition 3.19

Let $H = (X, E, R)$ be a IVNHG; then, the degree of vertex x_i is denoted and defined by:

$$\deg(x_i) = ([deg_{TL}(x_i), deg_{TU}(x_i)], [deg_{IL}(x_i), deg_{IU}(x_i)],$$

$$[deg_{FL}(x_i), deg_{FU}(x_i)]), \quad (69)$$

where

$$deg_{TL}(x_i) = \sum R_{TL}(x_1, x_2, \dots, x_r)\ , \quad (70)$$

$$deg_{IL}(x_i) = \sum R_{IL}(x_1, x_2, \dots, x_r), \quad (71)$$

$$deg_{FL}(x_i) = \sum R_{FL}(x_1, x_2, \dots, x_r), \quad (72)$$

$$deg_{TU}(x_i) = \sum R_{TU}(x_1, x_2, \dots, x_r)\ , \quad (73)$$

$$deg_{IU}(x_i) = \sum R_{IU}(x_1, x_2, \dots, x_r), \quad (74)$$

$$deg_{FU}(x_i) = \sum R_{FU}(x_1, x_2, \dots, x_r), \quad (75)$$

for $x_i \neq x_r$.

Theorem 3.20

If H and K are two isomorphic IVNHGs, then the degree of their vertices are preserved.

Proof.

Let f: $H \to K$ be an isomorphism between two IVNHGs H and K with underlying sets X and Y, respectively. Then, by definition, we have:

$$\min[TL_{E_j}(x)] = \min[TL_{F_j}(f(x))], \quad (75)$$

$$\max[IL_{E_j}(x)] = \max[IL_{F_j}(f(x))], \quad (77)$$

$$\max[FL_{E_j}(x)] = \max[FL_{F_j}(f(x))], \quad (78)$$

$$\min[TU_{E_j}(x)] = \min[TU_{F_j}(f(x))], \quad (79)$$

$$\max[IU_{E_j}(x)] = \max[IU_{F_j}(f(x))], \quad (80)$$

$$\max[FU_{E_j}(x)] = \max[FU_{F_j}(f(x))], \tag{81}$$

for all $x \in X$.

$$R_{TL}(x_1, x_2, \dots, x_r) = S_{TL}(f(x_1), f(x_1), \dots, f(x_r)), \tag{82}$$

$$R_{IL}(x_1, x_2, \dots, x_r) = S_{IL}(f(x_1), f(x_2), \dots, f(x_r)), \tag{83}$$

$$R_{FL}(x_1, x_2, \dots, x_r) = S_{FL}(f(x_1), f(x_2), \dots, f(x_r)), \tag{84}$$

$$R_{TU}(x_1, x_2, \dots, x_r) = S_{TU}(f(x_1), f(x_2), \dots, f(x_r)), \tag{85}$$

$$R_{IU}(x_1, x_2, \dots, x_r) = S_{IU}(f(x_1), f(x_2), \dots, f(x_r)), \tag{86}$$

$$R_{FU}(x_1, x_2, \dots, x_r) = S_{FU}(f(x_1), f(x_2), \dots, f(x_r)), \tag{87}$$

for all $\{x_1, x_2, \dots, x_r\}$ subsets of X.

Consider,

$$deg_{TL}(x_i) = \sum R_{TL}(x_1, x_2, \dots, x_r) = \sum S_{TL}(f(x_1), f(x_2), \dots, f(x_r)) = deg_{TL}(f(x_i)) \tag{88}$$

and similarly:

$$deg_{TU}(x_i) = deg_{TU}(f(x_i)), \tag{89}$$

$$deg_{IL}(x_i) = deg_{IL}(f(x_i)), deg_{FL}(x_i) = deg_{FL}(f(x_i)), \tag{90}$$

$$deg_{IU}(x_i) = deg_{IU}(f(x_i)), deg_{FU}(x_i) = deg_{FU}(f(x_i)). \tag{91}$$

Hence,

$$deg(x_i) = deg(f(x_i)). \tag{92}$$

Remark 3.21

The converse of the above theorem may not be true in general.

Example 3.22

Consider the two IVNHGs *H = (X, E, R)* and *K = (Y, F, S)* with underlying sets *X = {a, b}* and *Y = {x, y}*, where *E = {A, B}, F = {C, D}, R* and *S,* which are defined in the *Tables* given below, where *f* is defined by *f(a)=x, f(b)=y*, where *deg(a) = ([0.8,1.0], [1.0,1.2], [0.6,0.8]) = deg(x)* and *deg(b) = ([0.45,0.65], [1.0,1.2], [0.6,0.8]) = deg(y)*. But *H* is not isomorphic to *K*, i.e. *H* is neither weak isomorphic nor co-weak isomorphic *K*.

H	A	B
a	([0.5,0.6], [0.5,0.6], [0.3,0.4])	([0.3,0.4],[0.5,0.6],[0.3,0.4])
b	([0.25,0.35], [0.5,0.6], [0.3,0.4])	([0.2,0.3], [0.5,0.6], [0.3, 0.4])

K	C	D
x	([0.3,0.4], [0.5,0.6], [0.3,0.4])	([0.5,0.6],[0.5,0.6],[0.3,0.4])
y	([0.2,0.3], [0.5,0.6], [0.3,0.4])	([0.25,0.34], [0.5,0.6], [0.3,0.4])

R	R_T	R_I	R_F
A	[0.25,0.35]	[0.5,0.6]	[0.3,0.4]
B	[0.2,0.3]	[0.5,0.6]	[0.3,0.4]

S	S_T	S_I	S_F
C	[0.2,0.3]	[0.5,0.6]	[0.3,0.4]
D	[0.25,0.35]	[0.5,0.6]	[0.3,0.4]

Theorem 3.23

The isomorphism between IVNHGs is an equivalence relation.

Proof.

Let *H = (X, E, R), K = (Y, F, S)* and *M = (Z, G, W)* be IVNHGs with underlying sets X, Y and *Z*, respectively:

Reflexive.

Consider the map (identity map) $f: X \to X$, defined as follows: $f(x) = x$ for all $x \in X$, since the identity map is always bijective and satisfies the conditions:

$$\min[TL_{E_j}(x)] = \min[TL_{E_j}(f(x))], \tag{93}$$

$$\max[IL_{E_j}(x)] = \max[IL_{E_j}(f(x))], \tag{94}$$

$$\max[FL_{E_j}(x)] = \max[FL_{E_j}(f(x))], \tag{95}$$

$$\min[TU_{E_j}(x)] = \min[TU_{E_j}(f(x))], \tag{96}$$

$$\max[IU_{E_j}(x)] = \max[IU_{E_j}(f(x))], \tag{97}$$

$$\max[FU_{E_j}(x)] = \max[FU_{E_j}(f(x))], \tag{98}$$

for all $x \in X$.

$$R_{TL}(x_1, x_2, \dots, x_r) = R_{TL}(f(x_1), f(x_2), \dots, f(x_r)), \tag{99}$$

$$R_{IL}(x_1, x_2, \dots, x_r) = R_{IL}(f(x_1), f(x_2), \dots, f(x_r)), \tag{100}$$

$$R_{FL}(x_1, x_2, \dots, x_r) = R_{FL}(f(x_1), f(x_2), \dots, f(x_r)), \tag{101}$$

$$R_{TU}(x_1, x_2, \dots, x_r) = R_{TU}(f(x_1), f(x_2), \dots, f(x_r)), \tag{102}$$

$$R_{IU}(x_1, x_2, \dots, x_r) = R_{IU}(f(x_1), f(x_2), \dots, f(x_r)), \tag{103}$$

$$R_{FU}(x_1, x_2, \dots, x_r) = R_{FU}(f(x_1), f(x_2), \dots, f(x_r)), \tag{104}$$

for all $\{x_1, x_2, \dots, x_r\}$ subsets of X.

Hence f is an isomorphism of IVNHG H to itself.

Symmetric.

Let $f: X \to Y$ be an isomorphism of H and K, then f is bijective mapping defined as: $f(x) = y$ for all $x \in X$. Then, by definition:

$$\min[TL_{E_j}(x)] = \min[TL_{F_j}(f(x))], \tag{105}$$

$$\max[IL_{E_j}(x)] = \max[IL_{F_j}(f(x))], \tag{106}$$

$$\max[FL_{E_j}(x)] = \max[FL_{F_j}(f(x))], \tag{107}$$

$$\min[TU_{E_j}(x)] = \min[TU_{F_j}(f(x))], \tag{108}$$

$$\max[IU_{E_j}(x)] = \max[IU_{F_j}(f(x))], \tag{109}$$

$$\max[FU_{E_j}(x)] = \max[FU_{F_j}(f(x))], \tag{110}$$

for all $x \in X$.

$$R_{TL}(x_1, x_2, \dots, x_r) = S_{TL}(f(x_1), f(x_2), \dots, f(x_r)), \tag{111}$$

$$R_{IL}(x_1, x_2, \dots, x_r) = S_{IL}(f(x_1), f(x_2), \dots, f(x_r)), \tag{112}$$

$$R_{FL}(x_1, x_2, \dots, x_r) = S_{FL}(f(x_1), f(x_2), \dots, f(x_r)), \tag{113}$$

$$R_{TU}(x_1, x_2, \dots, x_r) = S_{TU}(f(x_1), f(x_2), \dots, f(x_r)), \tag{114}$$

$$R_{IU}(x_1, x_2, \dots, x_r) = S_{IU}(f(x_1), f(x_2), \dots, f(x_r)), \tag{115}$$

$$R_{FU}(x_1, x_2, \dots, x_r) = S_{FU}(f(x_1), f(x_2), \dots, f(x_r)), \tag{116}$$

for all $\{x_1, x_2, \dots, x_r\}$ subsets of X. Since f is bijective, then we have $f^{-1}(y) = x$ for all $y \in Y$. Thus, we get:

$$\min[TL_{E_j}\left(f^{-1}(y)\right)] = \min[TL_{F_j}(y)], \tag{117}$$

$$\max[IL_{E_j}\left(f^{-1}(y)\right)] = \max[IL_{F_j}(y)], \tag{118}$$

$$\max[FL_{E_j}\left(f^{-1}(y)\right)] = \max[FL_{F_j}(y)], \tag{119}$$

$$\min[TU_{E_j}\left(f^{-1}(y)\right)] = \min[TU_{F_j}(y)], \tag{120}$$

$$\max[IU_{E_j}\left(f^{-1}(y)\right)] = \max[IU_{F_j}(y)], \tag{121}$$

$$\max[FU_{E_j}\left(f^{-1}(y)\right)] = \max[FU_{F_j}(y)], \tag{122}$$

for all $x \in X$.

$$R_{TL}\left(f^{-1}(y_1), f^{-1}(y_2), \dots, f^{-1}(y_r)\right) = S_{TL}(y_1, y_2, \dots, y_r), \quad (123)$$

$$R_{IL}\left(f^{-1}(y_1), f^{-1}(y_2), \dots, f^{-1}(y_r)\right) = S_{IL}(y_1, y_2, \dots, y_r), \quad (124)$$

$$R_{FL}\left(f^{-1}(y_1), f^{-1}(y_2), \dots, f^{-1}(y_r)\right) = S_{FL}(y_1, y_2, \dots, y_r), \quad (125)$$

$$R_{TU}\left(f^{-1}(y_1), f^{-1}(y_2), \dots, f^{-1}(y_r)\right) = S_{TU}(y_1, y_2, \dots, y_r), \quad (126)$$

$$R_{IU}\left(f^{-1}(y_1), f^{-1}(y_2), \dots, f^{-1}(y_r)\right) = S_{IU}(y_1, y_2, \dots, y_r), \quad (127)$$

$$R_{FU}\left(f^{-1}(y_1), f^{-1}(y_2), \dots, f^{-1}(y_r)\right) = S_{FU}(y_1, y_2, \dots, y_r), \quad (128)$$

for all $\{y_1, y_2, \dots, y_r\}$ subsets of Y.

Hence we have a bijective map $f^{-1} : Y \to X$, which is an isomorphism from K to H.

Transitive.

Let $f : X \to Y$ and $g : Y \to Z$ be two isomorphism of IVNHGs of H onto K and K onto M respectively. Then gof is bijective mapping from X to Z, where gof is defined as $(gof)(x) = g(f(x))$ for all $x \in X$.

Since f is isomorphism, then, by definition, $f(x) = y$ for all $x \in X$, which satisfies the conditions:

$$\min[TL_{E_j}(x)] = \min[TL_{F_j}(f(x))], \quad (129)$$

$$\max[IL_{E_j}(x)] = \max[IL_{F_j}(f(x))], \quad (130)$$

$$\max[FL_{E_j}(x)] = \max[FL_{F_j}(f(x))], \quad (131)$$

$$\min[TU_{E_j}(x)] = \min[TU_{F_j}(f(x))], \quad (132)$$

$$\max[IU_{E_j}(x)] = \max[IU_{F_j}(f(x))], \quad (133)$$

$$\max[FU_{E_j}(x)] = \max[FU_{F_j}(f(x))], \quad (134)$$

for all $x \in X$.

$$R_{TL}(x_1, x_2, \dots, x_r) = S_{TL}(f(x_1), f(x_2), \dots, f(x_r)), \quad (135)$$

$$R_{IL}(x_1, x_2, \dots, x_r) = S_{IL}(f(x_1), f(x_2), \dots, f(x_r)), \quad (136)$$

$$R_{FL}(x_1, x_2, \dots, x_r) = S_{FL}(f(x_1), f(x_2), \dots, f(x_r)), \quad (137)$$

$$R_{TU}(x_1, x_2, \dots, x_r) = S_{TU}(f(x_1), f(x_2), \dots, f(x_r)), \quad (138)$$

$$R_{IU}(x_1, x_2, \dots, x_r) = S_{IU}(f(x_1), f(x_2), \dots, f(x_r)), \quad (139)$$

$$R_{FU}(x_1, x_2, \dots, x_r) = S_{FU}(f(x_1), f(x_2), \dots, f(x_r)), \quad (140)$$

for all $\{x_1, x_2, \dots, x_r\}$ subsets of X. Since $g : Y \to Z$ is isomorphism, then by definition $g(y) = z$ for all $y \in Y$ satisfy the conditions:

$$\min[TL_{F_j}(y)] = \min\left[TL_{G_j}\big(g(y)\big)\right], \tag{141}$$

$$\max[IL_{F_j}(y)] = \max[IL_{G_j}\big(g(y)\big)], \tag{142}$$

$$\max[FL_{F_j}(y)] = \max[FL_{G_j}\big(g(y)\big)], \tag{143}$$

$$\min[TU_{F_j}(y)] = \min[TU_{G_j}\big(g(y)\big)], \tag{144}$$

$$\max[IU_{F_j}(y)] = \max[IU_{G_j}\big(g(y)\big)], \tag{145}$$

$$\max[FU_{F_j}(y)] = \max[FU_{G_j}\big(g(y)\big)], \tag{146}$$

for all $x \in X$.

$$S_{TL}(y_1, y_2, \dots, y_r) = W_{TL}(g(y_1), g(y_2), \dots, g(y_r)), \tag{147}$$

$$S_{IL}(y_1, y_2, \dots, y_r) = W_{IL}(g(y_1), g(y_2), \dots, g(y_r)), \tag{148}$$

$$S_{FL}(y_1, y_2, \dots, y_r) = W_{FL}(g(y_1), g(y_2), \dots, g(y_r)), \tag{149}$$

$$S_{TU}(y_1, y_2, \dots, y_r) = W_{TU}(g(y_1), g(y_2), \dots, g(y_r)), \tag{150}$$

$$S_{IU}(y_1, y_2, \dots, y_r) = W_{IU}(g(y_1), g(y_2), \dots, g(y_r)), \tag{151}$$

$$S_{FU}(y_1, y_2, \dots, y_r) = W_{FU}(g(y_1), g(y_2), \dots, g(y_r)), \tag{152}$$

for all $\{y_1, y_2, \dots, y_r\}$ subsets of Y. Thus, from the above equations, we conclude that:

$$\min[TL_{E_j}(x)] = \min\left[TL_{G_j}\Big(g\big(f(x)\big)\Big)\right], \tag{153}$$

$$\max[IL_{E_j}(x)] = \max[IL_{G_j}(g(f(x)))], \tag{154}$$

$$\max[FL_{E_j}(x)] = \max[FL_{G_j}(g(f(x)))], \tag{155}$$

$$\min[TU_{E_j}(x)] = \min\left[TU_{G_j}\Big(g\big(f(x)\big)\Big)\right], \tag{156}$$

$$\max[IU_{E_j}(x)] = \max[IU_{G_j}(g(f(x)))], \tag{157}$$

$$\max[FU_{E_j}(x)] = \max[FU_{G_j}(g(f(x)))], \tag{158}$$

for all $x \in X$.

$$R_{TL}(x_1, \dots, x_r) = W_{TL}(g\big(f(x_1)\big), \dots, g\big(f(x_r)\big)), \tag{159}$$

$$R_{IL}(x_1, \dots, x_r) = W_{IL}(g\big(f(x_1)\big), \dots, g\big(f(x_r)\big)), \tag{160}$$

$$R_{FL}(x_1, \dots, x_r) = W_{FL}(g\big(f(x_1)\big), \dots, g\big(f(x_r)\big)), \tag{161}$$

$$R_{TU}(x_1, \dots, x_r) = W_{TU}(g\big(f(x_1)\big), \dots, g\big(f(x_r)\big)), \tag{162}$$

$$R_{IU}(x_1, \dots, x_r) = W_{IU}(g(f(x_1)), \dots, g(f(x_r))), \tag{163}$$

$$R_{FU}(x_1, \dots, x_r) = W_{FU}(g(f(x_1)), \dots, g(f(x_r))), \tag{164}$$

for all $\{x_1, x_2, \dots, x_r\}$ subsets of *X.*

Therefore, gof is an isomorphism between H and M. Hence, the isomorphism between IVNHGs is an equivalence relation.

Theorem 3.24

The weak isomorphism between IVNHGs satisfies the partial order relation.

Proof.

Let *H = (X, E, R), K = (Y, F, S)* and *M = (Z, G, W)* be IVNHGs with underlying sets X, Y and *Z* respectively,

Reflexive.

Consider the map (identity map) *f: X → X,* defined as follows: *f(x)=x* for all *x* ∈ *X,* since identity map is always bijective and satisfies the conditions:

$$\min[TL_{E_j}(x)] = \min[TL_{E_j}(f(x))], \tag{165}$$

$$\max[IL_{E_j}(x)] = \max[IL_{E_j}(f(x))], \tag{166}$$

$$\max[FL_{E_j}(x)] = \max[FL_{E_j}(f(x))], \tag{167}$$

$$\min[TU_{E_j}(x)] = \min[TU_{E_j}(f(x))], \tag{168}$$

$$\max[IU_{E_j}(x)] = \max[IU_{E_j}(f(x))], \tag{169}$$

$$\max[FU_{E_j}(x)] = \max[FU_{E_j}(f(x))], \tag{170}$$

for all $x \in X$.

$$R_{TL}(x_1, x_2, \dots, x_r) \le R_{TL}(f(x_1), f(x_2), \dots, f(x_r)), \tag{171}$$

$$R_{IL}(x_1, x_2, \dots, x_r) \ge R_{IL}(f(x_1), f(x_2), \dots, f(x_r)), \tag{172}$$

$$R_{FL}(x_1, x_2, \dots, x_r) \ge R_{FL}(f(x_1), f(x_2), \dots, f(x_r)), \tag{173}$$

$$R_{TU}(x_1, x_2, \dots, x_r) \le R_{TU}(f(x_1), f(x_2), \dots, f(x_r)), \tag{174}$$

$$R_{IU}(x_1, x_2, \dots, x_r) \ge R_{IU}(f(x_1), f(x_2), \dots, f(x_r)), \tag{175}$$

$$R_{FU}(x_1, x_2, \dots, x_r) \ge R_{FU}(f(x_1), f(x_2), \dots, f(x_r)), \tag{176}$$

for all $\{x_1, x_2, \dots, x_r\}$ subsets of *X.*

Hence f is a weak isomorphism of IVNHG H to itself.

Anti-symmetric.

Let f be a weak isomorphism between H onto K, and g be weak isomorphic between K and H, i.e. $f: X \to Y$ is a bijective map defined by: $f(x) = y$ for all $x \in X$ satisfying the conditions:

$$\min[TL_{E_j}(x)] = \min[TL_{F_j}(f(x))], \tag{177}$$

$$\max[IL_{E_j}(x)] = \max[IL_{F_j}(f(x))], \tag{178}$$

$$\max[FL_{E_j}(x)] = \max[FL_{F_j}(f(x))], \tag{179}$$

$$\min[TU_{E_j}(x)] = \min[TU_{F_j}(f(x))], \tag{180}$$

$$\max[IU_{E_j}(x)] = \max[IU_{F_j}(f(x))], \tag{181}$$

$$\max[FU_{E_j}(x)] = \max[FU_{F_j}(f(x))], \tag{182}$$

for all $x \in X$.

$$R_{TL}(x_1, x_2, \dots, x_r) \leq S_{TL}(f(x_1), f(x_2), \dots, f(x_r)), \tag{183}$$

$$R_{IL}(x_1, x_2, \dots, x_r) \geq S_{IL}(f(x_1), f(x_2), \dots, f(x_r)), \tag{184}$$

$$R_{FL}(x_1, x_2, \dots, x_r) \geq S_{FL}(f(x_1), f(x_2), \dots, f(x_r)), \tag{185}$$

$$R_{TU}(x_1, x_2, \dots, x_r) \leq S_{TU}(f(x_1), f(x_2), \dots, f(x_r)), \tag{186}$$

$$R_{IU}(x_1, x_2, \dots, x_r) \geq S_{IU}(f(x_1), f(x_2), \dots, f(x_r)), \tag{187}$$

$$R_{FU}(x_1, x_2, \dots, x_r) \geq S_{FU}(f(x_1), f(x_2), \dots, f(x_r)), \tag{188}$$

for all $\{x_1, x_2, \dots, x_r\}$ subsets of X.

Since g is also bijective map $g(y) = x$ for all $y \in Y$ satisfying the conditions:

$$\min[TL_{F_j}(y)] = \min[TL_{E_j}(g(y))], \tag{189}$$

$$\max[IL_{F_j}(y)] = \max[IL_{E_j}(g(y))], \tag{190}$$

$$\max[FL_{F_j}(y)] = \max[FL_{E_j}(g(y))], \tag{191}$$

$$\min[TU_{F_j}(y)] = \min[TU_{E_j}(g(y))], \tag{192}$$

$$\max[IU_{F_j}(y)] = \max[IU_{E_j}(g(y))], \tag{193}$$

$$\max[FU_{F_j}(y)] = \max[FU_{E_j}(g(y))], \tag{194}$$

for all $y \in Y$.

$$R_{TL}(y_1, y_2, \dots, y_r) \leq S_{TL}(g(y_1), g(y_2), \dots, g(y_r)), \tag{195}$$

$$R_{IL}(y_1, y_2, \dots, y_r) \geq S_{IL}(g(y_1), g(y_2), \dots, g(y_r)), \tag{196}$$

$$R_{FL}(y_1, y_2, \dots, y_r) \geq S_{FL}(g(y_1), g(y_2), \dots, g(y_r)), \tag{197}$$

$$R_{TU}(y, y_2, \ldots, y_r) \leq S_{TU}(g(y_1), g(y_2), \ldots, g(y_r)), \quad (198)$$

$$R_{IU}(y_1, y_2, \ldots, y_r) \geq S_{IU}(g(y_1), g(y_2), \ldots, g(y_r)), \quad (199)$$

$$R_{FU}(y_1, y_2, \ldots, y_r) \geq S_{FU}(g(y_1), g(y_2), \ldots, g(y_r)), \quad (200)$$

for all $\{y_1, y_2, \ldots, y_r\}$ subsets of Y.

The above inequalities hold for finite sets X and Y only whenever H and K have the same number of edges, and the corresponding edge have same weights, hence H is identical to K.

Transitive.

Let $f: X \to Y$ and $g: Y \to Z$ be two weak isomorphism of IVNHGs of H onto K and K onto M, respectively. Then gof is bijective mapping from X to Z, where gof is defined as $(gof)(x) = g\big(f(x)\big)$ for all $x \in X$.

Since f is a weak isomorphism, then by definition $f(x) = y$ for all $x \in X$ which satisfies the conditions:

$$\min[TL_{E_j}(x)] = \min[TL_{F_j}\big(f(x)\big)], \quad (201)$$

$$\max[IL_{E_j}(x)] = \max[IL_{F_j}\big(f(x)\big)], \quad (202)$$

$$\max[FL_{E_j}(x)] = \max[FL_{F_j}\big(f(x)\big)], \quad (203)$$

$$\min[TU_{E_j}(x)] = \min[TU_{F_j}\big(f(x)\big)], \quad (204)$$

$$\max[IU_{E_j}(x)] = \max[IU_{F_j}\big(f(x)\big)], \quad (205)$$

$$\max[FU_{E_j}(x)] = \max[FU_{F_j}\big(f(x)\big)], \quad (206)$$

for all $x \in X$.

$$R_{TL}(x_1, x_2, \ldots, x_r) \leq S_{TL}(f(x_1), f(x_2), \ldots, f(x_r)), \quad (207)$$

$$R_{IL}(x_1, x_2, \ldots, x_r) \geq S_{IL}(f(x_1), f(x_2), \ldots, f(x_r)), \quad (208)$$

$$R_{FL}(x_1, x_2, \ldots, x_r) \geq S_{FL}(f(x_1), f(x_2), \ldots, f(x_r)), \quad (209)$$

$$R_{TU}(x_1, x_2, \ldots, x_r) \leq S_{TU}(f(x_1), f(x_2), \ldots, f(x_r)), \quad (210)$$

$$R_{IU}(x_1, x_2, \ldots, x_r) \geq S_{IU}(f(x_1), f(x_2), \ldots, f(x_r)), \quad (211)$$

$$R_{FU}(x_1, x_2, \ldots, x_r) \geq S_{FU}(f(x_1), f(x_2), \ldots, f(x_r)), \quad (212)$$

for all $\{x_1, x_2, \ldots, x_r\}$ subsets of X.

Since $g: Y \to Z$ is a weak isomorphism, then by definition $g(y) = z$ for all $y \in Y$ which satisfies the conditions:

$$\min[TL_{F_j}(y)] = \min[TL_{G_j}\big(g(y)\big)], \quad (213)$$

$$\max[IL_{F_j}(y)] = \max[IL_{G_j}\big(g(y)\big)], \quad (214)$$

$$\max[FL_{F_j}(y)] = \max[FL_G(g(y))], \quad (215)$$

$$\min[TU_{F_j}(y)] = \min[TU_{G_j}(g(y))], \quad (216)$$

$$\max[IU_{F_j}(y)] = \max[IU_{G_j}(g(y))], \quad (217)$$

$$\max[FU_{F_j}(y)] = \max[FU_G(g(y))], \quad (218)$$

for all $x \in X$.

$$S_{TL}(y_1, y_2, \dots, y_r) \leq W_{TL}(g(y_1), g(y_2), \dots, g(y_r)), \quad (219)$$

$$S_{IL}(y_1, y_2, \dots, y_r) \geq W_{IL}(g(y_1), g(y_2), \dots, g(y_r)), \quad (210)$$

$$S_{FL}(y_1, y_2, \dots, y_r) \geq W_{FL}(g(y_1), g(y_2), \dots, g(y_r)), \quad (211)$$

$$S_{TU}(y_1, y_2, \dots, y_r) \leq W_{TU}(g(y_1), g(y_2), \dots, g(y_r)), \quad (212)$$

$$S_{IU}(y_1, y_2, \dots, y_r) \geq W_{IU}(g(y_1), g(y_2), \dots, g(y_r)), \quad (213)$$

$$S_{FU}(y_1, y_2, \dots, y_r) \geq W_{FU}(g(y_1), g(y_2), \dots, g(y_r)), \quad (214)$$

for all $\{y_1, y_2, \dots, y_r\}$ subsets of *Y.*

Thus, from the above equations, we conclude that,

$$\min[TL_{E_j}(x)] = \min[TL_{G_j}(g(f(x)))], \quad (215)$$

$$\max[IL_{E_j}(x)] = \max[IL_{G_j}(g(f(x)))], \quad (216)$$

$$\max[FL_{E_j}(x)] = \max[FL_{G_j}(g(f(x)))], \quad (217)$$

$$\min[TU_{E_j}(x)] = \min[TU_{G_j}(g(f(x)))], \quad (219)$$

$$\max[IU_{E_j}(x)] = \max[IU_{G_j}(g(f(x)))], \quad (220)$$

$$\max[FU_{E_j}(x)] = \max[FU_{G_j}(g(f(x)))], \quad (221)$$

for all $x \in X$.

$$R_{TL}(x_1, \dots, x_r) \leq W_{TL}(g(f(x_1)), \dots, g(f(x_r))), \quad (222)$$

$$R_{IL}(x_1, \dots, x_r) \geq W_{IL}(g(f(x_1)), \dots, g(f(x_r))), \quad (223)$$

$$R_{FL}(x_1, \dots, x_r) \geq W_{FL}(g(f(x_1)), \dots, g(f(x_r))), \quad (224)$$

$$R_{TU}(x_1, \dots, x_r) \leq W_{TU}(g(f(x_1)), \dots, g(f(x_r))), \quad (225)$$

$$R_{IU}(x_1, \dots, x_r) \geq W_{IU}(g(f(x_1)), \dots, g(f(x_r))), \quad (226)$$

$$R_{FU}(x_1, \dots, x_r) \geq W_{FU}(g(f(x_1)), \dots, g(f(x_r))), \quad (227)$$

for all $\{x_1, x_2, \dots, x_r\}$ subsets of *X.*

Therefore, gof is a weak isomorphism between H and M. Hence, the weak isomorphism between IVNHGs is a partial order relation.

4 Conclusion

The concepts of interval valued neutrosophic hypergraphs can be applied in various areas of engineering and computer science. In this paper, the isomorphism between IVNHGs is proved to be an equivalence relation and the weak isomorphism is proved to be a partial order relation. Similarly, it can be proved that the co-weak isomorphism in IVNHGs is a partial order relation.

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Regular Bipolar Single Valued Neutrosophic Hypergraphs

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Muhammad Aslam Malik, Ali Hassan, Said Broumi, Florentin Smarandache (2016). Regular Bipolar Single Valued Neutrosophic Hypergraphs. *Neutrosophic Sets and Systems* 13, 84-89

Abstract. In this paper we define the regular and totally regular bipolar single valued neutrosophic hypergraphs, and discuss the order and size along with properties of regular and totally regular bipolar single valued neutrosophic hypergraphs, we extended work on completeness of bipolar single valued neutrosophic hypergraphs.

Keywords: bipolar single valued neutrosophic hypergraphs, regular bipolar single valued neutrosophic hypergraphs and totally regular bipolar single valued neutrosophic hyper graphs.

1 Introduction

The notion of neutrosophic sets (NSs) was proposed by Smarandache [8] as a generalization of the fuzzy sets [14], intuitionistic fuzzy sets [12], interval valued fuzzy set [11] and interval-valued intuitionistic fuzzy sets [13] theories. The neutrosophic set is a powerful mathematical tool for dealing with incomplete, indeterminate and inconsistent information in real world. The neutrosophic sets are characterized by a truth-membership function (*t*), an indeterminacy-membership function (*i*) and a falsity membership function (*f*) independently, which are within the real standard or nonstandard unit interval $]^-0 , 1^+[$. In order to conveniently use NS in real life applications, Wang et al. [9] introduced the concept of the single-valued neutrosophic set (SVNS), a subclass of the neutrosophic sets. The same authors [10] introduced the concept of the interval valued neutrosophic set (IVNS), which is more precise and flexible than the single valued neutrosophic set. The IVNS is a generalization of the single valued neutrosophic set, in which the three membership functions are independent and their value belong to the unit interval [0, 1]. More works on single valued neutrosophic sets, interval valued neutrosophic sets and their applications can be found on *http://fs.gallup.unm.edu/NSS/*.

Hypergraph is a graph in which an edge can connect more than two vertices, hypergraphs can be applied to analyse architecture structures and to represent system partitions, Mordesen J.N and P.S Nasir gave the definitions for fuzzy hypergraphs. Parvathy. R and M. G. Karunambigai's paper introduced the concepts of Intuitionistic fuzzy hypergraphs and analyse its components, Nagoor Gani. A and Sajith Begum. S defined degree, order and size in intuitionistic fuzzy graphs and extend the properties. Nagoor Gani. A and Latha. R introduced irregular fuzzy graphs and discussed some of its properties.

Regular intuitionistic fuzzy hypergraphs and totally regular intuitionistic fuzzy hypergraphs are introduced by Pradeepa. I and Vimala. S in [0]. In this paper we extend regularity and totally regularity on bipolar single valued neutrosophic hypergraphs.

2 Preliminaries

In this section we discuss the basic concept on neutrosophic set and neutrosophic hyper graphs.

Definition 2.1 Let X be the space of points (objects) with generic elements in X denoted by x. A single valued neutrosophic set A (SVNS A) is characterized by truth membership function $T_A(x)$, indeterminacy membership function $I_A(x)$ and a falsity membership function $F_A(x)$. For each point $x \in X$; $T_A(x), I_A(x), F_A(x) \in [0, 1]$.

Definition 2.2 Let X be a space of points (objects) with generic elements in X denoted by x. A bipolar single valued neutrosophic set A (BSVNS A) is characterized by positive truth membership function $PT_A(x)$, positive indeterminacy membership function $PI_A(x)$ and a positive falsity membership function $PF_A(x)$ and negative truth membership function $NT_A(x)$, negative indeterminacy membership function $NI_A(x)$ and a negative falsity membership function $NF_A(x)$.

For each point $x \in$ X; $PT_A(x)$, $PI_A(x)$, $PF_A(x)$ ∈ [0, 1] and $NT_A(x)$, $NI_A(x)$, $NF_A(x)$ ∈ [-1, 0].

Definition 2.3 Let *A* be a BSVNS on *X* then support of *A* is denoted and defined by

Supp(A) = {x : x ∈X, $PT_A(x) > 0$*,* $PI_A(x) > 0$*,* $PF_A(x) > 0$*,* $NT_A(x) < 0$*,* $NI_A(x) < 0$*,* $NF_A(x) < 0$*}.*

Definition 2.4 A hyper graph is an ordered pair *H = (X, E),* where

(1) $X = \{x_1, x_2, \ldots, x_n\}$ be a finite set of vertices.

(2) $E = \{E_1, E_2, \ldots, E_m\}$ *be* a family of subsets of *X*.

(3) E_j for *j= 1,2,3,...,m* and $\bigcup_j(E_j)= X$.

The set *X* is called set of vertices and *E* is the set of edges (or hyper edges).

Definition 2.5 A bipolar single valued neutrosophic hypergraph is an ordered pair *H = (X, E),* where

(1) $X = \{x_1, x_2, \ldots, x_m\}$ be a finite set of vertices.

(2) $E = \{E_1, E_2, \ldots, E_m\}$ be a family of BSVNSs of *X*.

(3) $E_j \neq O = (0, 0, 0)$ for *j= 1,2,3,...,m* and $\bigcup_j Supp(E_j)= X$.

The set *X* is called set of vertices and *E* is the set of BSVN-edges (or BSVN-hyper edges).

Proposition 2.6 The bipolar single valued neutrosophic hyper graph is the generalization of fuzzy hyper graphs, intuitionistic fuzzy hyper graphs, bipolar fuzzy hyper graphs and single valued neutrosophic hypergraphs.

3 Regular and totally regular BSVNHGs

Definition 3.1 The open neighbourhood of a vertex *x* in bipolar single valued neutrosophic hypergraphs (BSVNHGs) is the set of adjacent vertices of *x*, excluding that vertex and is denoted by *N(x)*.

Definition 3.2 The closed neighbourhood of a vertex *x* in bipolar single valued neutrosophic hypergraphs (BSVNHGs) is the set of adjacent vertices of *x*, including that vertex and is denoted by *N[x]*.

Example 3.3 Consider a bipolar single valued neutrosophic hypergraphs *H = (X, E)* where, *X = {a, b, c, d, e}* and *E = {P, Q, R, S},* which is defined by

P = {(a, 0.1, 0.2, 0.3, -0.4, -0.6 -0.8), (b, 0.4, 0.5, 0.6, -0.4, -0.6 -0.8)}

Q = {(c, 0.1, 0.2, 0.3, -0.4, -0.4 -0.9), (d, 0.4, .5, 0.6, -0.3, -0.5 -0.6), (e, 0.7, 0.8, 0.9, -0.7, -0.9, -0.2)}

R = {(b, 0.1, 0.2, 0.3, -0.2, -0.5, -0.8), (c, 0.4, 0.5, 0.6, -0.9, -0.7 -0.4)}

S = {(a, 0.1, 0.2, 0.3, -0.7, -0.6, -0.9), (d, 0.9, 0.7, 0.6, -0.4, -0.7, -0.9)}

Then the open neighbourhood of a vertex *a* is the *b* and *d*, and closed neighbourhood of a vertex *b* is *b*, *a* and *c*.

Definition 3.4 Let *H = (X, E)* be a BSVNHG, the open neighbourhood degree of a vertex *x,* which is denoted and defined by

deg(x) = (deg_{PT}(x), deg_{PI}(x), deg_{PF}(x), deg_{NT}(x), deg_{NI}(x) , deg_{NF}(x)*)*

where

$$deg_{PT}(x) = \sum_{x\in N(x)} PT_E(x)$$

$$deg_{PI}(x) = \sum_{x\in N(x)} PI_E(x)$$

$$deg_{PF}(x) = \sum_{x\in N(x)} PF_E(x)$$

$$deg_{NT}(x) = \sum_{x\in N(x)} NT_E(x)$$

$$deg_{NI}(x) = \sum_{x\in N(x)} NI_E(x)$$

$$deg_{NF}(x) = \sum_{x\in N(x)} NF_E(x)$$

Example 3.5 Consider a bipolar single valued neutrosophic hypergraphs *H = (X, E)* where, *X = {a, b, c, d, e}* and *E = {P, Q, R, S},* which are defined by

P = {(a, .1, .2, .3, -0.1, -0.2, -0.3), (b, .4, .5, .6, -0.1, -0.2, -0.3)}

Q = {(c, .1, .2, .3, -0.1, -0.2, -0.3), (d, .4, .5, .6, -0.1, -0.2, -0.3), (e, .7, .8, .9, -0.1, -0.2, -0.3)}

R = {(b, .1, .2, .3, -0.1, -0.2, -0.3), (c, .4, .5, .6, -0.1, -0.2, -0.3)}

S = {(a, .1, .2, .3, -0.1, -0.2, -0.3), (d, .4, .5, .6, -0.1, -0.2, -0.3)}

Then the open neighbourhood of a vertex *a* contain *b* and *d* and therefore open neighbourhood degree of a vertex *a* is (.8, 1, 1.2, -0.2, -0.4, -0.6).

Definition 3.6 Let *H = (X, E)* be a BSVNHG, the closed neighbourhood degree of a vertex *x* is denoted and defined by,

deg[x] = (deg_{PT}[x], deg_{PI}[x], deg_{PF}[x], deg_{NT}[x], deg_{NI}[x] , deg_{NF}[x])

which are defined by

$$deg_{PT}[x] = deg_{PT}(x) + PT_E(x)$$
$$deg_{PI}[x] = deg_{PI}(x) + PI_E(x)$$
$$deg_{PF}[x] = deg_{PF}(x) + PF_E(x)$$
$$deg_{NT}[x] = deg_{NT}(x) + NT_E(x)$$
$$deg_{NI}[x] = deg_{NI}(x) + NI_E(x)$$
$$deg_{NF}[x] = deg_{NF}(x) + NF_E(x)$$

Example 3.7 Consider a bipolar single valued neutrosophic hypergraphs *H = (X, E)* where, *X = {a, b, c, d, e}* and *E = {P, Q, R, S},* which is defined by

P = {(a, 0.1, 0.2, 0.3, -0.1, -0.2, -0.3), (b, 0.4, 0.5, 0.6, -0.1, -0.2, -0.3)}

Q = {(c, 0.1, 0.2, 0.3, -0.1, -0.2, -0.3), (d, 0.4, 0.5, 0.6, -0.1, -0.2, -0.3), (e, 0.7, 0.8, 0.9, -0.1, -0.2, -0.3)}

R = {(b, 0.1, 0.2, 0.3, -0.1, -0.2, -0.3), (c, 0.4, 0.5, 0.6, -0.1, -0.2, -0.3)}

S = {(a, 0.1, 0.2, 0.3, -0.1, -0.2, -0.3), (d, 0.4, 0.5, 0.6, -0.1, -0.2, -0.3)}

The closed neighbourhood of a vertex *a* contain *a*, *b* and *d*, hence the closed neighbourhood degree of a vertex *a* is (0.9, .1.2, 1.5, -0.3, -0.6, -0.9).

Definition 3.8 Let *H = (X, E)* be a BSVNHG, then *H* is said to be an *n*-regular BSVNHG if all the vertices have the same open neighbourhood degree $n = (n_1, n_2, n_3, n_4, n_5, n_6)$

Definition 3.9 Let *H = (X, E)* be a BSVNHG, then *H* is said to be *m*-totally regular BSVNHG if all the vertices have the same closed neighbourhood degree $m = (m_1, m_2, m_3, m_4, m_5, m_6)$.

Proposition 3.10 A regular BSVNHG is the generalization of regular fuzzy hypergraphs, regular intuitionistic fuzzy hypergraphs, regular bipolar fuzzy hypergraphs and regular single valued neutrosophic hypergraphs.

Proposition 3.11 A totally regular BSVNHG is the generalization of totally regular fuzzy hypergraphs, totally regular intuitionistic fuzzy hypergraphs, totally regular bipolar fuzzy hypergraphs and totally regular single valued neutrosophic hypergraphs.

Example 3.12 Consider a bipolar single valued neutrosophic hypergraphs *H = (X, E)* where, *X = {a, b, c, d}* and *E = {P, Q, R, S}* which is defined by

P = {(a, 0.8, 0.2, 0.3, -0.1, -0.2, -0.3), (b, 0.8, 0.2, 0.3, -0.1, -0.2, -0.3)}

Q = {(b, 0.8, 0.2, 0.3, -0.1, -0.2, -0.3), (c, 0.8, 0.2, 0.3, -0.1, -0.2, -0.3)}

R = {(c, 0.8, 0.2, 0.3, -0.1, -0.2, -0.3), (d, 0.8, 0.2, 0.3, -0.1, -0.2, -0.3)}

S = {(d, 0.8, 0.2, 0.3, -0.1, -0.2, -0.3), (a, 0.8, 0.2, 0.3, -0.1, -0.2, -0.3)}

Here the open neighbourhood degree of every vertex is *(1.6, 0.4, 0.6, -0.2, -0.4, -0.6)* hence *H* is regular BSVNHG and closed neighbourhood degree of every vertex is *(2.4, 0.6, 0.9, -0.3, -0.6, -0.9),* Hence *H* is both regular and totally regular BSVNHG.

Theorem 3.13 Let *H = (X, E)* be a BSVNHG which is both regular and totally regular BSVNHG then *E* is constant.

Proof: Suppose *H* is an *n*-regular and *m*-totally regular BSVNHG. Then *deg(x)* = *n* = $(n_1, n_2, n_3, n_4, n_5, n_6)$ and *deg[x]* = *m* = $(m_1, m_2, m_3, m_4, m_5, m_6)$ $\forall x \in E_i$. Consider *deg[x]* = *m.* Hence by definition, *deg(x)* + E_i(x) = m this implies E_i(x) = *m* – *n* for all $x \in E_i$. Hence *E* is constant.

Remark 3.14 The converse of above theorem need not to be true in general.

Example 3.15 Consider a bipolar single valued neutrosophic hypergraphs *H = (X, E)* where, *X = {a, b, c, d}* and

E = {P, Q, R, S}, which is defined by

P = {(a, 0.8, 0.2, 0.3, -0.1, -0.2, -0.3), (b, 0.8, 0.2, 0.3, -0.1, -0.2, -0.3)}

Q = {(b, 0.8, 0.2, 0.3, -0.1, -0.2, -0.3), (d, 0.8, 0.2, 0.3, -0.1, -0.2, -0.3)}

R = {(c, 0.8, 0.2, 0.3, -0.1, -0.2, -0.3), (d, 0.8, 0.2, 0.3, -0.1, -0.2, -0.3)}

S = {(d, 0.8, 0.2, 0.3, -0.1, -0.2, -0.3), (a, 0.8, 0.2, 0.3, -0.1, -0.2, -0.3)}

Here *E* is constant but *deg(a) = (1.6, 0.4, 0.6, -0.2, -0.4, -0.6)* and *deg(d) = (2.4, 0.6, 0.9, -0.3, -0.6, -0.9)* i.e deg(a) and deg(d) are not equals hence *H* is not regular BSVNHG. Next *deg[a] = (2.4, 0.6, 0.9, -0.3, -0.6, -0.9)* and *deg[d]= (3.2, 0.8, 1.2, -.4, -0.8, -1.2),* hence deg[a] and deg[d] are not equals hence *H* is not totally regular BSVNHG, Thus that *H* is neither regular and nor totally regular BSVNHG.

Theorem 3.16 Let *H = (X, E)* be a BSVNHG then *E* is constant on *X* if and only if following are equivalent,

(1) *H* is regular BSVNHG.

(2) *H* is totally regular BSVNHG.

Proof: Suppose *H = (X, E)* be a BSVNHG and *E* is constant in *H*, that is $E_i(x) = c = (c_1, c_2, c_3, c_4, c_5, c_6)$ $\forall x \in E_i$. Suppose *H* is *n*-regular BSVNHG, then *deg(x) = n* = (n_1, n_2, n_3, n_4, n_5, n_6) $\forall x \in E_i$, consider *deg[x] = deg(x)* +E_i*(x) = n + c*

$\forall x \in E_i$, hence *H* is totally regular BSVNHG.

Next suppose that *H* is *m*-totally regular BSVNHG, then *deg[x] = m* = (m_1, m_2, m_3, m_4, m_5, m_6) for all $x \in E_i$, that is *deg(x)* + E_i*(x) = m* $\forall x \in E_i$, this implies that *deg(x) = m – c*

$\forall x \in E_i$. Thus *H* is regular BSVNHG, thus *(1)* and *(2)* are equivalent.

Conversely: Assume that *(1)* and *(2)* are equivalent. That is *H* is regular BSVNHG if and only if *H* is totally regular BSVNHG. Suppose contrary *E* is not constant, that is E_i(x) and E_i(y) not equals for some *x* and *y* in *X*. Let *H = (X, E)* be *n*-regular BSVNHG, then *deg(x) = n* = (n_1, n_2, n_3, n_4, n_5, n_6) for all x ∈ E_i. Consider

$$deg[x] = deg(x) + E_i(x) = n + E_i(x)$$

$$deg[y] = deg(y) + E_i(y) = n + E_i(y)$$

Since E_i(x) and E_i(y) are not equals for some *x* and *y* in *X*. Hence *deg[x]* and *deg[y]* are not equals, thus *H* is not totally regular BSVNHG, which contradict to our assumption.

Next let *H* be totally regular BSVNHG, then *deg[x] = deg[y], that is deg(x) +* E_i*(x) = deg(y) +* E_i*(y)* and *deg(x) – deg(y) =* E_i*(y) –* E_i*(x)*, since RHS of last equation is nonzero, hence LHS of above equation is also nonzero, thus *deg(x)* and *deg(y)* are not equals, so *H* is not regular BSVNHG, which is again contradict to our assumption, thus our supposition was wrong, hence *E* must be constant, this completes the proof.

Definition 3.17 Let *H = (X, E)* be a regular BSVNHG, then the order of BSVNHG *H* is denoted and defined by

O(H) = (p, q, r, s, t, u), where $p = \sum_{x \in X} PT_{E_i}(x)$, $q = \sum_{x \in X} PI_{E_i}(x)$, $r = \sum_{x \in X} PF_{E_i}(x)$, $s = \sum_{x \in X} NT_{E_i}(x)$, $t = \sum_{x \in X} NI_{E_i}(x)$, $u = \sum_{x \in X} NF_{E_i}(x)$. For every $x \in X$ and size of regular BSVNHG is denoted and defined by *S(H)* = $\sum_{i=1}^{n}(S_{E_i})$, where *S(E_i) = (a, b, c, d, e, f)* which is defined by

$$a = \sum_{x \in E_i} PT_{E_i}(x)$$

$$b = \sum_{x \in E_i} PI_{E_i}(x)$$

$$c = \sum_{x \in E_i} PF_{E_i}(x)$$

$$d = \sum_{x \in E_i} NT_{E_i}(x)$$

$$e = \sum_{x \in E_i} NI_{E_i}(x)$$

$$f = \sum_{x \in E_i} NF_{E_i}(x)$$

Example 3.18 Consider a bipolar single valued neutrosophic hypergraphs *H = (X, E)* where, *X = {a, b, c, d}* and

E = {P, Q, R, S}, which is defined by

P = {(a, .8, .2, .3, -.1, -.2, -.3), (b, .8, .2, .3, -.1, -.2, -.3)}

Q = {(b, .8, .2, .3, -.1, -.2, -.3), (c, .8, .2, .3, -.1, -.2, -.3)}

R = {(c, .8, .2, .3, -.1, -.2, -.3), (d, .8, .2, .3, -.1, -.2, -.3)}

S = {(d, .8, .2, .3, -.1, -.2, -.3), (a, .8, .2, .3, -.1, -.2, -.3)}

Here order and size of *H* are given (3.2, .8, 1.2, -.4, -.8, -1.2) and (6.4, 1.6, 2.4, -.8, -1.6, -2.4) respectively.

Proposition 3.19 The size of an *n*-regular BSVNHG *H = (H, E)* is *nk/2,* where *|X|= k*.

Proposition 3.20 If *H = (X, E)* be *m*-totally regular BSVNHG then *2S(H) + O(H) = mk*, where *|X|= k*.

Corollary 3.21 Let *H = (X, E)* be a *n*-regular and *m*-totally regular BSVNHG then *O(H) = k(m - n)*, where *|X|=k*.

Proposition 3.22 The dual of *n*-regular and *m*-totally regular BSVNHG *H = (X, E)* is again an *n*-regular and *m*-totally regular BSVNHG.

Definition 3.23 A bipolar single valued neutrosophic hypergraph (BSVNHG) is said to be complete BSVNHG if for every *x* in *X*, *N(x) = {x: x in X-{x}},* that is *N(x)* contains all remaining vertices of *X* except *x*.

Example 3.24 Consider a bipolar single valued neutrosophic hypergraphs *H = (X, E),* where *X = {a, b, c, d}* and *E = {P, Q, R},* which is defined by

P = {(a, 0.4, 0.6, 0.3, -0.5, -0.2, -0.3), (c, 0.8, 0.2, 0.3, -0.1, -0.8, -0.3)}

Q = {(a, 0.8, 0.8, 0.3, -0.1, -0.6, -0.3), (b, 0.8, 0.2, 0.1, -0.1, -0.2, -0.3), (d, 0.8, 0.2, 0.1, -0.1, -0.9, -0.3)}

R = {(c, 0.4, 0.9, 0.9, -0.1, -0.2, -0.3), (d, 0.7, 0.2, 0.1, -0.5, -0.9, -0.3), (b, *0.4, 0.2, 0.1, -0.8, -0.4, -0.2)}.* Here *N(a) = {b, c, d} , N(b) = {a, c, d}, N(c) = {a, b, d}, N(d) = {a, b, c}* hence *H* is complete BSVNHG.

Remark 3.25 In a complete BSVNHG *H = (X, E),* the cardinality of *N(x)* is same for every vertex.

Theorem 3.26 Every complete BSVNHG *H = (X, E)* is both regular and totally regular if *E* is constant in *H*.

Proof: Let *H = (X, E)* be complete BSVNHG, suppose *E* is constant in *H*, so that $E_i(x) = c = (c_1, c_2, c_3, c_4, c_5, c_6)$ $\forall x \in E_i$, since BSVNHG is complete, then by definition for every vertex *x* in *X*, *N(x) = {x: x in X-{x}}*, the open neighbourhood degree of every vertex is same. That is *deg(x) = n* = $(n_1, n_2, n_3, n_4, n_5, n_6)$ $\forall x \in E_i$. Hence complete BSVNHG is regular BSVNHG. Also, *deg[x] = deg(x) +* $E_i(x)$ *= n + c* $\forall x \in E_i$. Hence *H* is totally regular BSVNHG.

Remark 3.27 Every complete BSVNHG is totally regular even if *E* is not constant.

Definition 3.28 A BSVNHG is said to be *k*-uniform if all the hyper edges have same cardinality.

Example 3.29 Consider a bipolar single valued neutrosophic hypergraphs *H = (X, E),* where *X = {a, b, c, d}* and

E = {P, Q, R}, which is defined by

P = {(a, 0.8, 0.4, 0.2,-0.4, -0.6, -0.2), (b, 0.7, 0.5, 0.3, -0.7, -0.1, -0.2)}

Q = {(b, 0.9, 0.4, 0.8, -0.3, -0.2, -0.9), (c, 0.8, 0.4, 0.2, -0.4, -0.3, -0.7)}

R = {(c, 0.8, 0.6, 0.4, -0.3, -0.7, -0.2), (d, 0.8, 0.9, 0.5, -0.4, -0.8, -0.9)}

4 Conclusion

Theoretical concepts of graphs and hypergraphs are utilized by computer science applications. Single valued neutrosophic hypergraphs are more flexible than fuzzy hypergraphs and intuitionistic fuzzy hypergraphs. The concepts of single valued neutrosophic hypergraphs can be applied in various areas of engineering and computer science. In this paper we defined the regular and totally regular bipolar single valued neutrosophic hyper graphs. We plan to extend our research work to irregular and totally irregular on bipolar single valued neutrosophic hyper graphs.

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Regular Single Valued Neutrosophic Hypergraphs

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Abstract. In this paper we define the regular and totally regular single Valued neutrosophic hypergraphs, and discuss the order and size along with properties of regular and totally regular single valued neutrosophic hyper-graphs,we extended work on completeness of single valued neutrosophic hypergraphs.

Keywords: Single valued neutrosophic hypergraphs, regular single valued neutrosophic hypergraphs, totally regular single valued neutrosophic hypergraphs

1 Introduction

The notion of neutrosophic sets (NSs) was proposed by Smarandache [39, 8] as a generalization of the fuzzy sets [14], intuitionistic fuzzy sets [12], interval valued fuzzy set [11] and interval-valued intuitionistic fuzzy sets [13] theories. The neutrosophic set is a powerful mathematical tool for dealing with incomplete, indeterminate and inconsistent information in real world. The neutrosophic sets are characterized by a truth-membership function (*t*), an indeterminacy-membership function (*i*) and a falsity membership function (*f*) independently, which are within the real standard or nonstandard unit interval $]^{-}0\ ,\ 1^{+}[$. Smarandache [39] introduced the concept of the single-valued neutrosophic set (SVNS), mentioned in [40], a subclass of the neutrosophic sets. The same authors [10] introduced the concept of the interval valued neutrosophic set (IVNS), which is more precise and flexible than the single valued neutrosophic set. The IVNS is a generalization of the single valued neutrosophic set, in which the three membership functions are independent and their value belong to the unit interval [0, 1]. More works on single valued neutrosophic sets, interval valued neutrosophic sets and their applications can be found on *http://fs.gallup.unm.edu/NSS/* [38].

Hypergraph is a graph in which an edge can connect more than two vertices, hypergraphs can be applied to analyse architecture structures and to represent system partitions, Mordesen J.N and P.S Nasir gave the definitions for fuzzy hypergraphs. Parvathy.R and M. G. Karunambigai's paper introduced the concepts of Intuitionistic fuzzy hypergraphs and analyse its components, Nagoor Gani.A and Sajith Begum.S defined degree, order and size in intuitionistic fuzzy graphs and extend the properties. Nagoor Gani.A and Latha.R introduced irregular fuzzy graphs and discussed some of its properties.

Regular intuitionistic fuzzy hypergraphs and totally regular intuitionistic fuzzy hypergraphs are introduced by I. Pradeepa and S. Vimala [38] in this paper we will extend regularity and totally regularity on single valued neutrosophic hypergraphs.

2 Preliminaries

Definition 2.1 Let X be a space of points (objects) with generic elements in X denoted by x. A single valued neutrosophic set A (SVNS A) is characterized by truth membership function $T_A(x)$, indeterminacy membership function $I_A(x)$ and a falsity membership function $F_A(x)$. For each point x ∈ X , $T_A(x),\ I_A(x),\ F_A(x) \in [0\ ,1]$.

Definition 2.2 Let A be a SVNS on X then support of A is denoted and defined by,

$Supp(A) = \{ x : x \in X\ ,\ T_A(x) > 0\ ,\ I_A(x) > 0\ ,\ F_A(x) > 0 \}$

Definition 2.3 A hypergraph is an ordered pair H = (X , E) where,

(1) $X = \{ x_1\ ,\ x_2, \ldots.\ ,\ x_n\}$ a finite set of vertices.

(2) $E = \{ E_1,\ E_2\ , \ldots.,\ E_m\}$ a family of subsets of X.

(3) E_j for $j = 1,2,3,\ldots,m$ and $\bigcup_j (E_j) =$ X.

The set X is called set of vertices and E is the set of edges(or hyperedges).

In a hypergraph two or more vertices x_1, $x_2, \ldots, x_n$ are said to be adjacent if there exist an edge E_j which contains those vertices. In a hypergraph two edges E_i and E_j ←for i and j not equals) is said to be adjacent if their intersection is not empty. The size of a hypergraph depends on the cardinality of its hyperedges. We define the size of H as the sum of the cardinalities of its hyperedges. A regular hyper graph is one in which every vertex is contained in k edges for some constant k. In a complete hypergraph the edge set consists of the power set $P(X)$, where X is the set of vertices other than singleton set and empty sets. A hyper graph H is said to be k-uniform if the number of vertices in each hyper edge is k.

Definition 2.4 A single valued neutrosophic hypergraph is an ordered pair H = (X , E) where,

(1) $X = \{ x_1, x_2, \ldots, x_n\}$ a finite set of vertices.

(2) $E = \{ E_1, E_2, \ldots, E_m\}$ a family of SVNSs of X.

(3) $E_j \neq O = (0, 0, 0)$ for $j = 1,2,3,\ldots,m$ and $\bigcup_j Supp(E_j)$= X.

The set X is called set of vertices and E is the set of SVN-edges(or SVN-hyperedges).

Proposition 2.5 Single valued neutrosophic hypergraph is the generalization of fuzzy hypergraphs and intuitionistic fuzzy hypergraphs.

3 Regular and Totally regular SVNHGs

Defination 3.1 The open neighborhood of a vertex x in single valued neutrosophic hypergraphs (SVNHGs) is the set of adjacent vertices of x , excluding that vertex and is denoted by N(x).

Defination 3.2 The closed neighborhood of a vertex x in single valued neutrosophic hypergraphs (SVNHGs) is the set of adjacent vertices of x , including that vertex and is denoted by N[x].

Example 3.3 Consider a single valued neutrosophic hypergraphs H = (X , E) where, $X = \{ a,b,c,d,e \}$ and $E = \{ P,Q,R,S \}$ dfined by

$P = \{ (a, .1, .2, .3) , (b, .4, .5, .6) \}$

$Q = \{ (c, .1, .2, .3) , (d, .4, .5, .6) , (e, .7, .8, .9) \}$

$R = \{ (b, .1, .2, .3) , (c, .4, .5, .6) \}$

$S = \{ (a, .1, .2, .3) , (d, .4, .5, .6) \}$

Then the open neighborhood of a vertex a is b and d, and closed neighborhood of a vertex b is b, a and c.

Definition 3.4 Let H = (X , E) be a SVNHG , the open neighborhood degree of a vertex x is denoted and defined by

$$deg(x) = (deg_T(x) , deg_I(x) , deg_F(x))$$

where,

$$deg_T(x) = \sum_{x \in X} T_E(x)$$

$$deg_I(x) = \sum_{x \in X} I_E(x)$$

$$deg_F(x) = \sum_{x \in X} F_E(x)$$

Example 3.5 Consider a single valued neutrosophic hypergraphs H = (X , E) where, $X = \{ a,b,c,d,e \}$ and $E = \{ P,Q,R,S \}$ defined by

$P = \{ (a, .1, .2, .3) , (b, .4, .5, .6) \}$

$Q = \{ (c, .1, .2, .3) , (d, .4, .5, .6) , (e, .7, .8, .9) \}$

$R = \{ (b, .1, .2, .3) , (c, .4, .5, .6) \}$

$S = \{ (a, .1, .2, .3) , (d, .4, .5, .6) \}$

Then the open neighborhood of a vertex a is b and d, and therefor open neighbourhood degree degree of a vertex a is (.8 , 1, 1.2).

Definition 3.6 Let H=(X , E) be a SVNHG , the closed neighborhood degree of a vertex x is denoted and defined by

$$deg[x] = (deg_T[x], deg_I[x], deg_F[x])$$

where,

$$deg_T[x] = deg_T(x) + T_E(x)$$

$$deg_I[x] = deg_I(x) + I_E(x)$$

$$deg_F[x] = deg_F(x) + F_E(x)$$

Example 3.7 Consider a single valued neutrosophic hypergraphs H = (X , E) where, $X = \{ a,b,c,d,e \}$ and $E=\{ P,Q,R,S \}$ defined by

$P = \{ (a, .1, .2, .3) , (b, .4, .5, .6) \}$

$Q = \{ (c, .1, .2, .3) , (d, .4, .5, .6) , (e, .7, .8, .9) \}$

$R = \{ (b, .1, .2, .3) , (c, .4, .5, .6) \}$

$S = \{ (a, .1, .2, .3) , (d, .4, .5, .6) \}$

The closed neighbourhood of a vertex b is b, a and c, hence the closed neighbourhood degree of a vetex a is (.9 , .1.2 , 1.5).

Definition 3.8 Let H = (X , E) be a SVNHG , then H is said to be n-regular SVNHG if all the vertices have the same open neighbourhood degree n = (n_1 , n_2 , n_3)

Definition 3.9 Let H = (X , E) be a SVNHG , then H is said to be m-totally regular SVNHG if all the vertices have the same closed neighbourhood degree m = (m_1 , m_2 , m_3)

Proposition 3.10 A regular SVNHG is the generalization of regular fuzzy hypergraphs and regular intuitionistic fuzzy hypergraphs.

Proposition 3.11 A totally regular SVNHG is the generalization of totally regular fuzzy hypergraphs and totally regular intuitionistic fuzzy hypergraphs.

Example 3.12 Consider a single valued neutrosophic hypergraphs H = (X , E) where, *X = { a, b, c, d }* and

E = { P, Q, R, S } defined by

P = { (a, .8, .2, .3), (b, .8, .2, .3) }

Q = { (b, .8, .2, .3), (c, .8, .2, .3) }

R = { (c, .8, .2, .3), (d, .8, .2, .3) }

S = { (d, .8, .2, .3), (a, .8, .2, .3) }

Here the open neighbourhood degree of every vertex is (1.6, .4, .6) hence H is regular SVNHG and closed neighbourhood degree of every vertex is (2.4, .6, .9) Hence H is both regular and totally regular SVNHG.

Theorem 3.13 Let H = (X, E) be a SVNHG which is both regular and totally regular SVNHG then E is constant.

Proof: Suppose H is n-regular and m-totally regular SVNHG. Then ,

$$deg(x) = n = (n_1 , n_2 , n_3)$$

$$deg[x] = m = (m_1 , m_2 , m_3)$$

for all x ∈ E_i . consider , deg[x] = m hence by definition, deg(x) + E_i(x) = m this implies E_i(x) = m – n for all x in E_i . hence E is constant.

Remark 3.14 The converse of above theorem need not to be true in general.

Example 3.15 Consider a single valued neutrosophic hypergraphs H = (X , E) where, *X = { a, b, c, d }* and

E = { P, Q, R, S } defined by

P = { (a, .8, .2, .3) , (b, .8, .2, .3) }

Q = { (b, .8, .2, .3) , (d, .8, .2, .3) }

R = { (c, .8, .2, .3) , (d, .8, .2, .3) }

S = { (d, .8, .2, .3) , (d, .8, .2, .3) }

Here E is constant but deg(a) = (1.6, .4, .6) and deg(d) = (2.4, .6, .9) i.e deg(a) and deg(d) are not equals hence H is not regular SVNHG. Next deg[a] = (2.4, .6, .9) and deg[d] = (3.2, .8, 1.2) hence deg[a] and deg[d] are not equals hence H is not totally regular SVNHG , we conclude that H is neither regular and nor totally regular SVNHG.

Theorem 3.16 Let H = (X, E) be a SVNHG then E is costant on X if and only if following are equivalent,

(1) H is regular SVNHG.

(2) H is totally regular SVNHG.

Proof : Suppose H = (X , E) be a SVNHG and E is constant in H, i.e ,

$$E_i(x) = c = (c_1 , c_2 , c_3)$$

For all x ∈ E_i . Suppose H is n-regular SVNHG, then

$$deg(x) = n = (n_1 , n_2 , n_3)$$

for all x ∈ E_i . consider

$$deg[x] = deg(x) + E_i(x) = n + c$$

for all x ∈ E_i , hence H is totally regular SVNHG.

Next suppose that H is m-totally regular SVNHG , then

$$deg[x] = m = (m_1 , m_2 , m_3)$$

for all x ∈ E_i , i.e,

$$deg(x) + E_i (x) = m$$

for all x ∈ E_i , this implies that

$$deg(x) = m – c$$

for all x ∈ E_i , thus H is regular SVNHG, thus (1) and (2) are equivalent.

Conversely : Assume that (1) and (2) are equivalent, i.e H is regular SVNHG if and onl if H is totally regular SVNHG.

Suppose contrary E is not constant , i.e , E_i(x) and E_i(y) not equals for some x and y in X. Let H = (X , E) be n-regular SVNHG , then

$$deg(x) = n = (n_1 , n_2 , n_3)$$

for all x ∈ E_i , consider ,

$deg[x] = deg(x) + E_i(x) = n + E_i(x)$

$deg[y] = deg(y) + E_i(\,(y) = n + E_i(y)$

since E_i(x) and E_i(y) are not equals for some x and y in X , hence deg[x] and deg[y] are not equals, thus H is not totally regular SVNHG , which is contradiction to our assumption.

Next let H be totally regular SVNHG, then deg[x] = deg[y] i.e ,

$deg(x) + E_i\ (x) = deg(y) + E_i\ (y)$

$deg(x) – deg(y) = E_i(y) – E_i(x)$

since RHS of above equation is nonzero, hence LHS of above equation is also nonzero, thus deg(x) and deg(y) are not equals , so H is not regular SVNHG, which is again contradict to our assumption , thus our supposition was wrong , hence E must be constant , this completes the proof.

Definition 3.17 Let H= (X , E) be a regular SVNHG, then the order of SVNHG H is denoted and defined by

O(H) = (p, q, r) , where

$$p = \sum_{x\in X} T_{E_i}(x)$$

$$q = \sum_{x\in X} I_{E_i}(x)$$

$$r = \sum_{x\in X} F_{E_i}(x)$$

For every x ∈ X and size of regular SVNHG is denoted and defined by

$$S(H) = \sum_{i=1}^{n}(S_{E_i})$$

Where $S(E_i)$ = (a , b , c) which is defined by

$$a = \sum_{x\in E_i} T_{E_i}(x)$$

$$b = \sum_{x\in E_i} I_{E_i}(x)$$

$$c = \sum_{x\in E_i} F_{E_i}(x)$$

Example 3.18 Consider a single valued neutrosophic hypergraphs H = (X , E) where, *X = { a, b, c, d }* and

E = { P, Q, R, S } defined by

P = { (a, .8, .2, .3), (b, .8, .2, .3) }

Q = { (b, .8, .2, .3), (c, .8, .2, .3) }

R = { (c, .8, .2, .3), (d, .8, .2, .3) }

S = { (d, .8, .2, .3), (a, .8, .2, .3) }

Here order and size of H are given (3.2, .8, 1.2) and (6.4, 1.6, 2.4) respectively.

Proposition 3.19 The size of n-regular SVNHG H = (H , E) is nk/2 where |X|= k.

Proposition 3.20 If H = (X, E) be m-totally regular SVNHG then *2S(H) + O(H) = mk* , where |X|= k.

Corollary 3.21 Let H = (X,E) be a n-regular and m-totally regular SVNHG then *O(H) = k(m - n)*, where |X|= k.

Proposition 3.22 The dual of n-regular and m-totally regular SVNHG H= (X, E) is again a n-regular and m-totally regular SVNHG.

Definition 3.23 A single valued neutrosophic hypergraph (SVNHG) is said to be complete SVNHG if for every x in X , *N(x) = { x : x in X-{x} }* that is N(x) contains all remaining vertices of X except x.

Example 3.24 Consider a single valued neutrosophic hypergraphs H = (X , E) where, *X = { a, b, c, d }* and

E = { P, Q, R } defined by

P = { (a, .4, .6, .3), (c, .8, .2, .3) }

Q = { (a, .8, .8, .3), (b, .8, .2, .1), (d, .8, .2, .1) }

R = { (c, .4, .9, .9), (d, .7, .2, .1), (b, .4, .2, .1) }

Here N(a) = {b, c, d} , N(b) = {a, c, d}, N(c) = {a, b, d}, N(d) = {a, b, c} hence H is complete SVNHG.

Remark 3.25 In a complete SVNHG H = (X , E) the cardinality of N(x) is same for every vertex .

Theorem 3.26 Every complete SVNHG H = (X , E) is both regular and totally regular if E is constant in H.

Proof : Let H = (X , E) be complete SVNHG, suppose E is constant in H, so

$$E_i(x) = c = (c_1\,,\, c_1\,,\, c_3)$$

For all x ∈ E_i , since SVNHG is complete, then by definition for every vertex x in X , *N(x) = { x : x in X-{x} }* , open neighbourhood degree of every vertex is same. i.e,

$$deg(x) = n = (\,n_1\,,n_2,\, n_3\,)$$

for all x ∈ E_i , Hence complete SVNHG is regular SVNHG.

Also

$$deg[x] = deg(x) + E_i(x) = n + c$$

for all x ∈ E_i . Hence H is totally regular SVNHG.

Remark 3.27 Every complete SVNHG is totally regular even if E is not constant.

Definition 3.28 A SVNHG is said to be k-uniform if all the hyperedges have same cardinality.

Example 3.29 Consider a single valued neutrosophic hypergraphs H = (X , E) where, *X = {a, b, c, d}* and

E = { P, Q, R } defined by

P = { (a, .8, .2, .3), (b, .7, .5, .3) }

Q = { (b, .8, .1, .8), (c, .8, .4, .2) }

R = { (c, .8, .1, .4), (d, .8, .9, .5) }

4 Conclusion

Theoretical concepts of graphs and hypergraphs are highely utilized by computer science applications. Single valued neutrosophic hypergraphs are more flexible than fuzzy hypergraphs and intuitionistic fuzzy hypergraphs. The concepts of single valued neutrosophic hypergraphs can be applied in various areas of engineering and computer science. In this paper we defined the regular and totally regular single valued neutrosophic hypergraphs. We plan to extend our research work to regular and totally regular on Bipolar single valued neutrosophic hypergraphs, regular and totally regular on interval valued neutrosophic hypergraphs, regular and totally regular on Bipolar single valued neutrosophic hypergraphs, irregular and totally irregular on single valued neutrosophic hypergraphs, irregular and totally irregular on bipolar single valued neutrosophic hypergraphs, irregular and totally irregular on interval valued neutrosophic hypergraphs.

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Plithogenic n-Super Hypergraph in Novel Multi-Attribute Decision Making

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Abstract

An optimal decision-making environment demands feasible Multi-Attribute Decision-Making methods. Plithogenic n – Super Hypergraph introduced by Smarandache is a novel concept and it involves many attributes. This article aims to bridge the concept of Plithogenic n-Super Hypergraph in the vicinity of optimal decision making. This research work introduces the novel concepts of enveloping vertex, super enveloping vertex, dominant enveloping vertex, classification of the dominant enveloping vertex (input, intervene, output dominant enveloping vertices), plithogenic connectors. An application of Plithogenic n-super hypergraph in making optimum decisions is discussed under various decision-making scenarios. Several insights are drawn from this research work and will certainly benefit the decision-makers to overcome the challenges in building decisions.

Keywords: Plithogenic n-super hypergraph, decision making, attributes, dominant enveloping vertex.

1.Introduction

It is quite inevitable for each one is taking up the role of decision-maker in their instances of life. Decision making isn't an activity, but a process comprising of many tasks. The desired outcomes of decisions are a success, if it fails then the process has to be revived. The cognitive contribution in choosing the best alternative with the consideration of criteria and criteria weights is not a simple task; it demands sequential steps and scientific approach. The managerial of either a start-up company or a multinational organization must possess the skills of making optimal decisions to make their companies march in the path of victory. The decision-making environment is not deterministic always and it is characterized mostly by uncertainty and impreciseness, to tackle these challenges the decision-makers are moving towards Multi-Criteria Decision Making methods (MCDM) to design optimal solutions.

MCDM has been explored for the past seventy years and it has been broadly divided into MADM (Multi-Attribute Decision Making) and MODM (Multi-Objective Decision Making) [1]. The former helps in the selection of the alternatives based on attribute description and the latter is based on optimization of decision maker's multi objectives. MADM methods are gaining impetus in the decision-making environment as they are highly developed with robust mathematical principles and also these methods prevent small and medium-sized companies in purchasing expensive software or executing erudite systems of the decision process. MADM methods are more operative and the most widely used methods are Analytic Hierarchy Process (AHP) and Analytic Network Process (ANP) introduced by Satty [2]; Decision Making Trial and Evaluation Laboratory (DEMATEL) developed by Tzeng and Huang [3]; The Technique for Order Preferences by Similarity to an Ideal Solution (TOPSIS) method was proposed by Hwang and Yoon; Vlse Kriterijumska Optimizacija I Kompromisno Resenje (VIKOR) method was developed by Tzeng and Huang.

In the above described MADM methods, the major steps involved are (i) formulation of initial decision-making matrix (IDMM) comprising of values representing the degree of fulfilling the criteria by the alternatives. (ii) Normalization of the values in IDMM (iii) Determination of criterion weight (iv) Ranking of alternatives. In these MADM methods, the alternatives are ranked based only on the extent of criteria satisfaction, but the consistency of ranking is not checked as these methods do not provide space for it. The selection of alternatives is based only on attribute satisfaction and it does not consider any other input such as previous data related to the impacts or the effects of these kinds of chosen alternatives. These other inputs are not brought into the decision-making environment and the previous feedback review is also not incorporated into the decision-making environment.

Let us consider the possible situations of exercising decision making in a company, for example in the selection of personnel, methods of production, the extension of product features, the above instances of decision-making situations are not new to companies, as these processes are routine. In making decisions, certainly the managerial will be aware of the desired target to be achieved and will employ his previous experience or the feedback received by him from various sources as inputs in the selection of alternatives. The above-said MADM does not provide space for such kinds of feedback inputs. A comprehensive decision-making environment must comprise of alternative selection based on several inputs such as attribute satisfaction, feedback, and impact of attributes towards the desired output. To overcome such shortcomings, a novel MADM method is introduced in this research work with the integration of Plithogenic – super Hypergraphs introduced by Smarandache [4]. Plithogenic sets introduced by Smarandache [5] are the extension of neutrosophic sets that are characterized by truth, indeterminacy, and false functions. The robust nature of neutrosophic sets inspired several researchers to employ it in diverse fields. Gayathri et al [6] developed multiple attribute group decision making neutrosophic environments with the utilization of Jaccard index measures. Muhammad Naveed Jafar et al [7] used neutrosophic soft matrices with score function to evaluate new technology in Agriculture. Ajay et al [8] developed a single-valued triangular neutrosophic number approach of multi-objective optimization based on simple ratio analysis based on the MCDM method. Luis Andrés Crespo Berti [9] applied a neutrosophic system to tax havens with a criminal approach. Abdel-Basset [10] developed three-way decisions based on neutrosophic sets and AHP-QFD framework for supplier selection problem, also developed a hybrid neutrosophic group ANP-TOPSIS framework for supplier selection problem [11].Plithogenic sets that deal with attributes, degree of appurtenance, and degree of contradiction have been extensively used in decision making with quality function deployment for selecting supply chain sustainability metrics and for evaluating hospital medical care systems by Abdel-Basset et al [12,13]. In these decision making approaches plithogenic aggregation operations are used to make decisions based on the best and worst criteria with decision-makers' opinions as inputs. These methods of decision making focus primarily on evaluation and selection of alternatives based on combining plithogenic aggregation operators and do not provide space for any graphical representation of the relational impacts between the alternatives.

In the proposed MADM each alternative is considered as an object encompassing several attributes. The decision-making environment consists of three kinds of objects namely input, intervention, and output. The alternatives are taken as inputs, desired target as output, and intervene (intermediate) objects are the objects that combine with the input objects. A company always works on target based. Personnel design project and work on it tirelessly to achieve various sets of goals. The project never gets accomplished with the attainment of a single goal but a series of goals. The success of a project is defined in various dimensions. In the proposed MADAM, the selection of alternatives is based on the degree of association between the attributes of inputs and the attributes of outputs independent or dependent on intervening objects. This decision-making approach is more comprehensive than the conventional MADM methods as it incorporates attributes and feedback into the input system. Also, many times the company prefers collaborative works and the effects of combined initiatives are high. Conventional MADM does not provide space for it, but the proposed MADM is designed exclusively for measuring the optimal combination. Also in MADM methods, graphical representations are not made so far to represent alternatives, criteria, and their relationship. In this novel MADM, plithogenic –n super hypergraphs are used to represent the objects as enveloping vertices and the association between the vertices by plithogenic connectors.

The article is structured as follows: Section 2 introduces new concepts used in novel MADM; section 3 presents the application of novel MADM in optimal decision making; section 4 discusses the results and the last section concludes the work.

2. Preliminaries

2.1 Enveloping vertex

A vertex representing an object comprising of attributes and sub-attributes in the graphical representation of a multi attribute decision-making environment.

For instance

Let us consider Personnel (V) as an input object, this input has a vital role in target achievement, the output object. These attributes are like databases.

The attributes like Qualification (V1), Age (V2), Experience (V3) are taken into consideration

Attribute sets = {Qualification, Age, Experience}

Qualification = {Graduation, Graduation with additional degree}

Age = {25-35, 36-45}

Experience = { Local, National ,International }

Local {0-5,6-10}, National{0-3,4-6}, International {0-2,2-5}

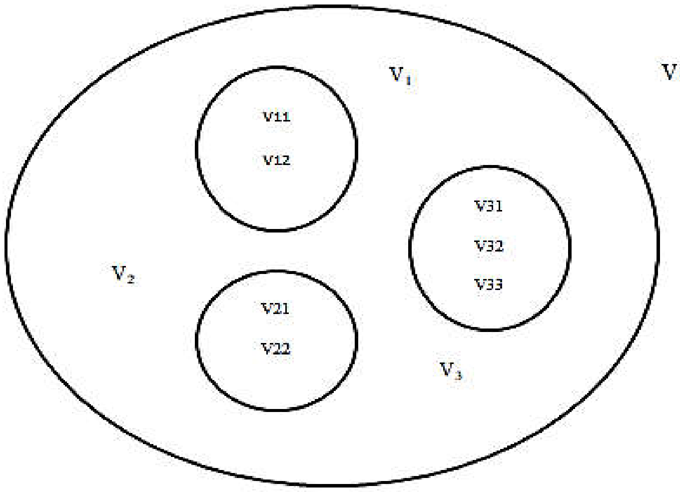

Fig.2.1Enveloping vertex

Thus an enveloping vertex comprises hyperedges, where each hyperedge represents values of the attributes.

2.2 Super Enveloping vertex

An enveloping vertex comprises of Super hyper edges

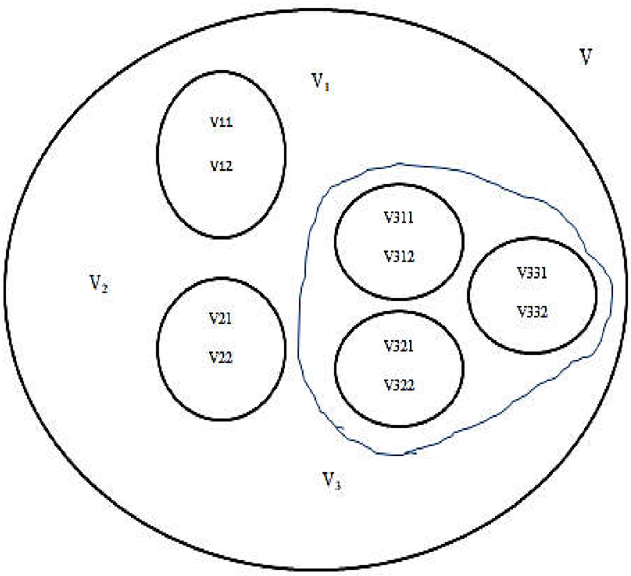

Fig.2.2 Super Enveloping vertex

2.3 Dominant Enveloping Vertex

An enveloping vertex is with dominant attribute values

Attribute sets = {Qualification, Age, Experience}

The dominant attribute values

Qualification = {Graduation, **Graduation with additional degree**}

Age = {25-35, **36-45**}

Experience = { Local, National ,**International** }

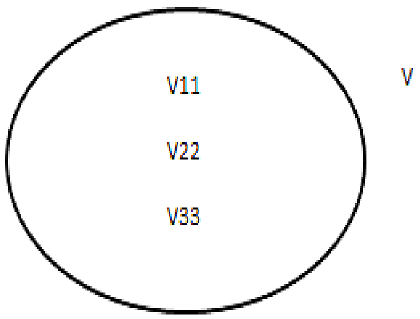

Fig.2.3 Dominant Enveloping Vertex

2.4 Dominant Super Enveloping Vertex

A super enveloping vertex with dominant attribute values

Attribute sets = {Qualification, Age, Experience}

The dominant attribute values

Qualification = {Graduation, **Graduation with additional degree**}

Age = {25-35, **36-45**}

Experience = { Local, National ,**International** }

Local {0-5,6-10}, National{0-3,4-6}, International {**0-2**,**2-5**}

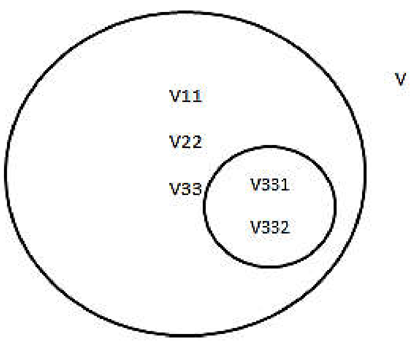

Fig.2.4 Dominant Super Enveloping Vertex

2.5 Classification of Dominant Enveloping Vertex

The dominant enveloping vertex set are classified as input, intervene and output based on the nature of object's representation.

2.6 Plithogenic Connectors

The connectors associate the input enveloping vertex with output enveloping vertex. These connectors associate the effects of input attributes to output attributes and these connectors are weighted by plithogenic weights.

Let us consider the MADM environment with the product as input object, advertising as intervene object and product success as the output object

Product is the input enveloping vertex, advertising as intervening enveloping vertex and product success as the output enveloping vertex.

Input attributes = {Design, Price}

Design = {**creative**, conventional}

Price = {High, **moderate**, low}

Intervene attributes = { Target group, Medium of advertising}

Target group ={**female**, children}

Medium of advertising = { social networks, **media**}

Output Attributes = { Profit, Customer Acquisition, Product Reach}

Profit = { **Expected**, Beyond the target}

Customer Acquisition = { High, **Extremely High**}

Product Reach = { **National**, International}

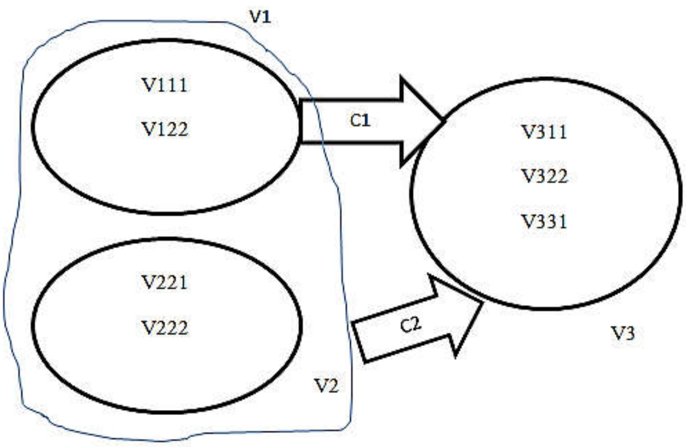

Fig.2.5 Plithogenic Connectors

C1 is the simple plithogenic connector representing the relation between the dominant input attributes to dominant output attributes.

C2 is the combined plithogenic connector representing the relation between the combined dominant input and intervene attributes to dominant output attributes.

Dominant Attribute Relational Matrix Representation

	V311	V322	V331
V111	0.5	0.2	0.3
V122	0.6	0.7	0.8
V111,V221	0.5	0.6	0.4
V111,V222	0.6	0.3	0.8
V122,V221	0.4	0.6	0.8
V122,V222	0.4	0.6	0.7

3. Application of Novel MADM method

3.1 Description of Decision-making Environment

COVID 19 has locked the academic activities to a great extent; the stratagem of Work from Home is employed by the teaching fraternity to engage the learners. One of the biggest challenges to teaching community lies in handling online learning forums and they are badly in need of exposure to the E-learning system of education. To make academicians surpass this task, educational institutions are offering various online courses and organize E-programmes to enhance the professional competency of faculty in partnership with several industries. In this period of the lockdown, the linkage between industries and institutions is getting enhanced in developing countries especially in India. The companies enter institutions as academic partners in establishing virtual laboratories and entertain many online programs in the form of webinars, online courses, and software training programs to handle online classes. The conduct of such programs will certainly contribute to the professional efficiency of faculty. Suppose if an institution decides to conduct any one of the forms of the online program, then it has to decide whether to conduct the program in partnership with industry or independently and also the decision of selecting the kind of online program is based on the feedback acquired from other institutions on the previous organization of such programs. The institution before organizing such programs should decide the component of professional efficiency to be enhanced and determine the contributing factors of the online program towards the same. An optimal solution to this decision-making situation is determined by using the representation of Plithogenic –n Super hypergraph and novel MADM method based on attributes. This decision-making method involves not only the selection process of alternatives based on criteria alike other multi-attribute decision-making methods but it provides space for the selection of alternatives independent or dependent on other alternatives based on their attributes. The outcome of decision making is also considered in the decision-making process. The selection of the alternatives is based on attributes of input objects, intervene objects and output objects.

In this decision-making environment there exist five objects [3 input objects, 1 intervene object and 1 output object] that are represented by enveloping vertices. The input enveloping vertices are Webinars, online courses, training programs on computer languages, intervene enveloping vertex is Industrial partnership and the output enveloping vertex is Professional Efficiency. The description of the attributes of the objects are presented in Table 3.1

Table 3.1 Description of Attributes

Vertex	**Representation**	**Vertex Attributes**		**Vertex Sub Attributes**			
V1	Webinars	V_{11}	Focus	V_{111}	General	V_{1111}	Education
						V_{1112}	Health
						V_{1113}	Psychology
				V_{112}	Specific	V_{1121}	Physics
						V_{1122}	Chemistry
						V_{1123}	Mathematics
						V_{1124}	Engineering
		V_{12}	Resource persons	V_{121}	Local	V_{1211}	within the college
						V_{1212}	neighboring colleges
				V_{122}	National	V_{1221}	AICTE affiliated
						V_{1222}	Non-AICTE affiliated
				V_{123}	International	V_{1231}	Affiliation with the host college
						V_{1232}	Non-affiliation with the host college
		V_{13}	Duration	V_{131}	Day	V_{1311}	One day
						V_{1312}	Two days
						V_{1313}	Three days
				V_{132}	Week	V_{1321}	One
						V_{1322}	Two
		V_{14}	Target Group	V_{141}	Students	V_{1411}	Engineering
						V_{1412}	Non-Engineering

				V_{142}	Research Scholars	V_{1421}	Engineering
						V_{1422}	Non-Engineering
				V_{143}	Academicians	V_{1431}	Engineering
						V_{1432}	Non-Engineering
V2	Online courses	V_{21}	Course nature	V_{211}	Basic	V_{2111}	remembrance
						V_{2112}	understanding
				V_{212}	Moderate	V_{2121}	understanding
						V_{2122}	application
				V_{213}	Advanced	V_{2131}	Analysis
						V_{2132}	Evaluation
		V_{22}	Course Delivery	V_{221}	Synchronous	V_{2211}	Zoom
						V_{2212}	Zoho
						V_{2213}	Examineer
						V_{2214}	Google meet
				V_{222}	Asynchronous	V_{2221}	Google Classroom
						V_{2222}	Youtube upload
		V_{23}	Duration	V_{231}	Day	V_{2311}	One day
						V_{2312}	Two days
						V_{2313}	Three days
				V_{232}	Week	V_{2321}	One
						V_{2322}	Two
		V_{24}	Target Group	V_{241}	Students	V_{2411}	Engineering
						V_{2412}	Non-Engineering
				V_{242}	Research Scholars	V_{2421}	Engineering
						V_{2422}	Non-Engineering
				V_{243}	Academicians	V_{2431}	Engineering
						V_{2432}	Non-Engineering
V3	Training programme on Computer languages	V_{31}	Course nature	V_{311}	Basic	V_{3111}	remembrance
						V_{3112}	understanding
				V_{312}	Moderate	V_{3121}	understanding

						V_{3122}	application
				V_{313}	Advanced	V_{3131}	Analysis
						V_{3132}	Evaluation
		V_{32}	Course Delivery	V_{321}	Synchronous	V_{3211}	Zoom
						V_{3212}	Zoho
						V_{3213}	Examineer
						V_{3214}	Google meet
				V_{322}	Asynchronous	V_{3221}	Google Classroom
						V_{3222}	Youtube upload
		V_{33}	Duration	V_{331}	Day	V_{3311}	One day
						V_{3312}	Two days
						V_{3313}	Three days
				V_{332}	Week	V_{3321}	One
						V_{3322}	Two
		V_{34}	Target Group	V_{341}	Students	V_{3411}	Engineering
						V_{3412}	Non-Engineering
				V_{342}	Research Scholars	V_{3421}	Engineering
						V_{3422}	Non-Engineering
				V_{343}	Academicians	V_{3431}	Engineering
						V_{3432}	Non-Engineering
V4	Industrial Partnership	V_{41}	MOU	V_{411}	Internship	V_{4111}	Merit-based
						V_{4112}	All students
				V_{412}	Placement	V_{4121}	Merit-based
						V_{4122}	All students
		V_{42}	Financial Support	V_{421}	Equipment purchase	V_{4211}	Partial
						V_{4212}	Complete
				V_{422}	Program organization	V_{4221}	Partial
						V_{4222}	Complete
				V_{431}	Knowledge sharing	V_{4311}	Periodic

		V_{43}	Technical Support			V_{4312}	Regular
				V_{432}	Experts Visit	V_{4321}	Periodic
						V_{4322}	Regular
V5	Professional Efficiency	V_{51}	Publications	V_{511}	National	V_{5111}	Scopus
						V_{5112}	ICI
				V_{512}	International	V_{5121}	Scopus
						V_{5122}	ICI
		V_{52}	Pedagogy	V_{521}	Teacher-Centered	V_{5211}	lecture
						V_{5212}	chalk & talk
				V_{522}	Learner-Centered	V_{5221}	Blended
						V_{5222}	ICT
		V_{53}	Content preparation	V_{531}	Own	V_{5311}	original
						V_{5312}	modified
				V_{532}	Experts Visit	V_{5321}	Web sources
						V_{5322}	Youtube
		V_{54}	Course Delivery	V_{541}	OER	V_{5411}	Zoom
						V_{5412}	Zoho
						V_{5413}	Google meet
						V_{5414}	Examineer
				V_{542}	Asynchronous	V_{5421}	Google Classroom
						V_{5422}	Youtube upload

In the above table, the input objects such as webinars, online courses, training programs are represented as the input enveloping vertices V1, V2, and V3 in Fig 3.1,3.2 and 3.3 respectively. The intervening object Industrial Partnership is represented as V4 in Fig 3.4. The output object Professional Efficiency is represented as V5 in Fig 3.5.

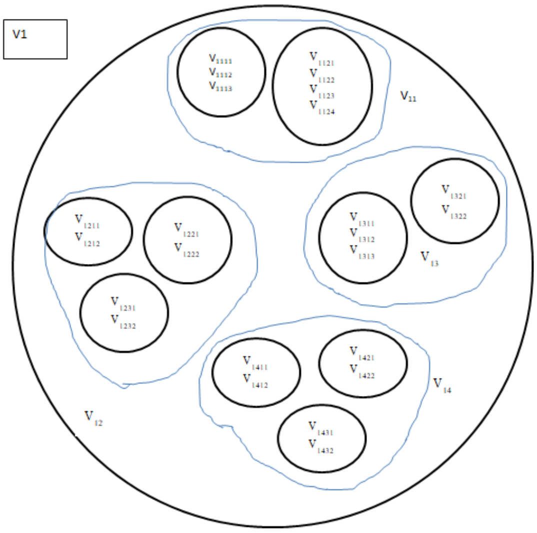

Fig.3.1 Representation of Input Object V1

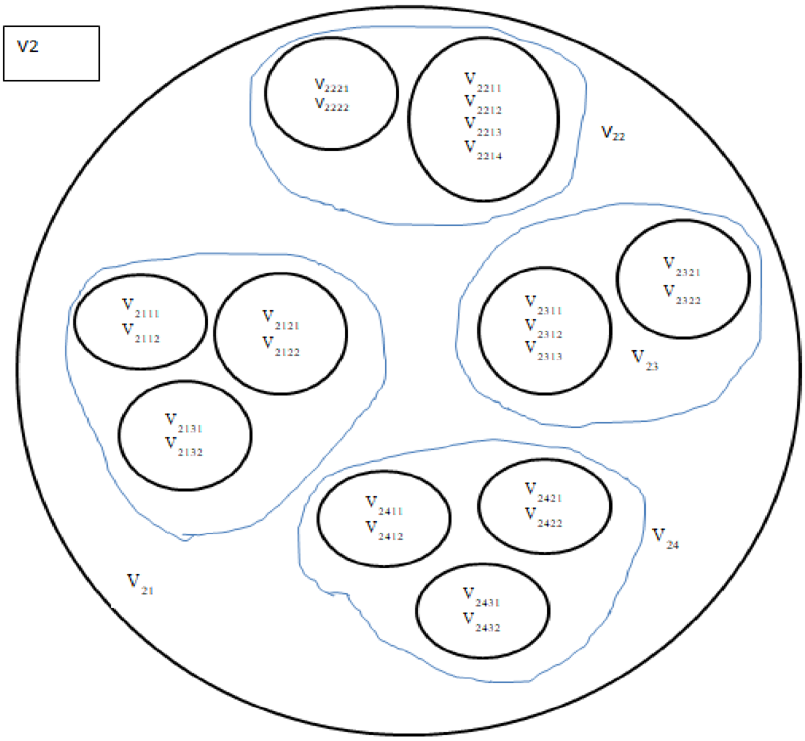

Fig.3.2 Representation of Input Object V2

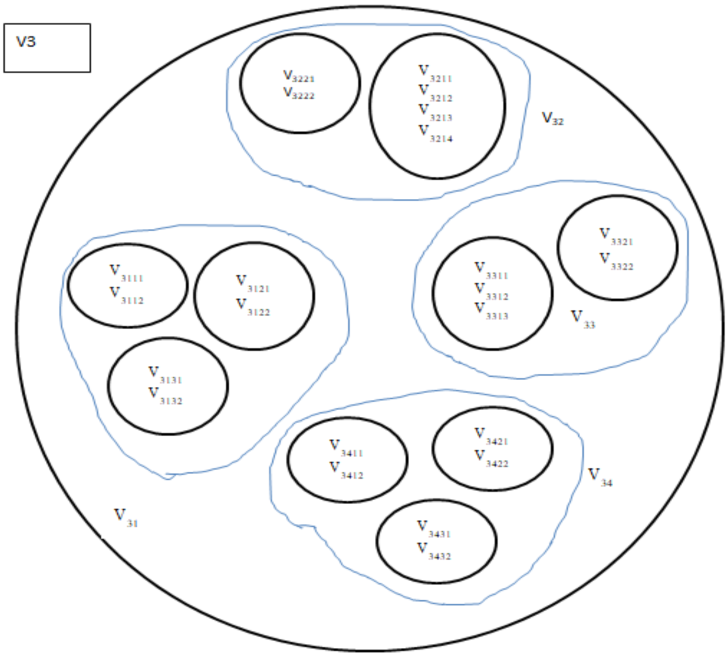

Fig.3.3 Representation of Input Object V3

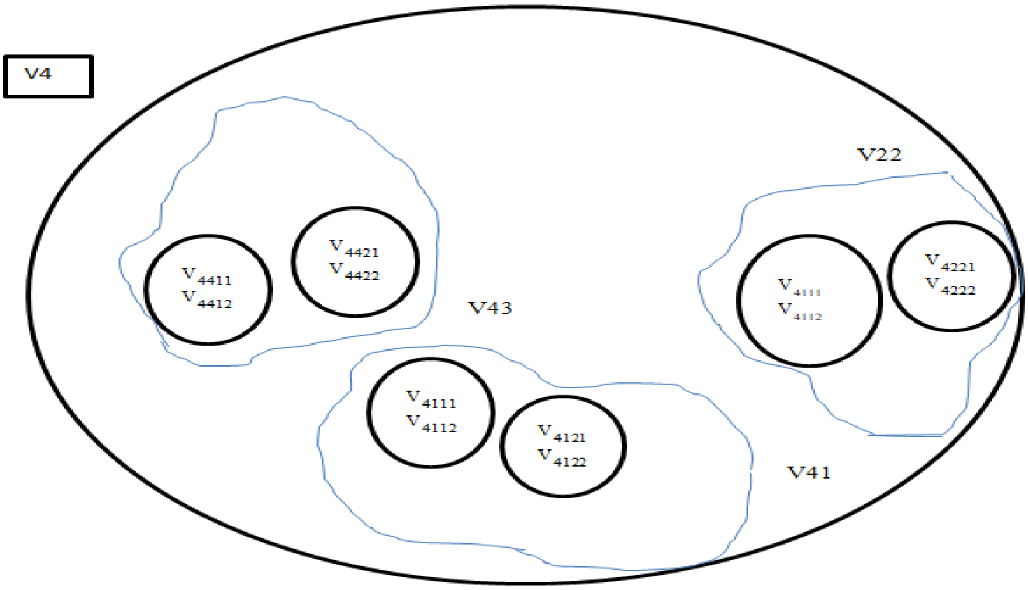

Fig.3.4 Representation of Intervening Object V4

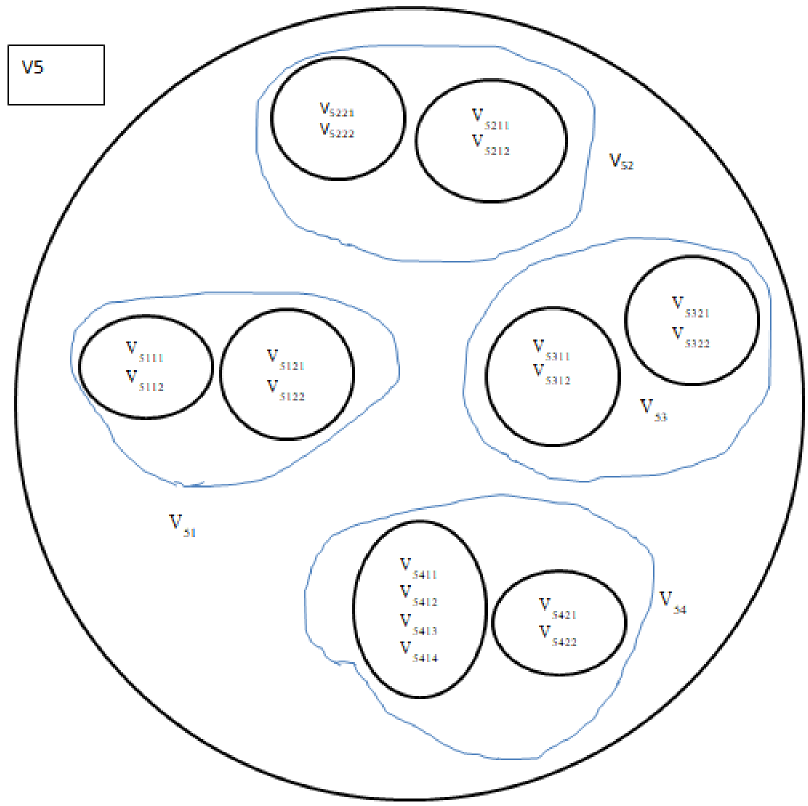

Fig.3.5 Representation of Output Object V5

Each enveloping vertices comprises of many attribute and sub-attribute values. To determine the desired output with and without the combination of input and intervene objects, the dominant attributes are chosen by the decision-makers. The dominant attribute values of the objects are represented in Table 3.2

Table 3.2 Representation of Dominant Attributes

Vertex	Representation	Vertex Attributes		Vertex Sub Attributes			
V1	Webinars	V_{11}	Focus	V_{111}	General	V_{1111}	Education
		V_{12}	Resource persons	V_{122}	National	V_{1221}	AICTE affiliated
		V_{13}	Duration	V_{131}	Day	V_{1312}	Two days
		V_{14}	Target Group	V_{143}	Academicians	V_{1431}	Engineering

V2	Online courses Course Nature,	V_{21}	Course nature	V_{212}	Moderate	V_{2122}	application
		V_{22}	Course Delivery	V_{222}	Asynchronous	V_{2221}	Google Classroom
		V_{23}	Duration	V_{231}	Day	V_{2312}	Two days
		V_{24}	Target Group	V_{243}	Academicians	V_{2431}	Engineering
V3	Training program on Computer languages	V_{31}	Course nature	V_{311}	Moderate	V_{3122}	application
		V_{32}	Course Delivery	V_{321}	Synchronous	V_{3214}	Google meet
		V_{33}	Duration	V_{332}	Week	V_{3321}	One
		V_{34}	Target Group	V_{343}	Academicians	V_{3431}	Engineering
V4	Industrial Partnership	V_{41}	MOU	V_{412}	Placement	V_{4121}	Merit-based
						V_{4122}	All students
		V_{42}	Financial Support	V_{422}	Program organization	V_{4222}	Complete
		V_{43}	Technical Support	V_{432}	Experts Visit	V_{4322}	Regular
						V_{4312}	Regular
V5	Professional Efficiency	V_{51}	Publications	V_{511}	National	V_{5111}	Scopus
		V_{52}	Pedagogy	V_{522}	Learner-Centered	V_{5221}	Blended
		V_{53}	Content preparation	V_{531}	Own	V_{5311}	original
		V_{54}	Course Delivery	V_{542}	Asynchronous	V_{5421}	Google Classroom

The Dominant Enveloping vertices are presented in Fig 3.6

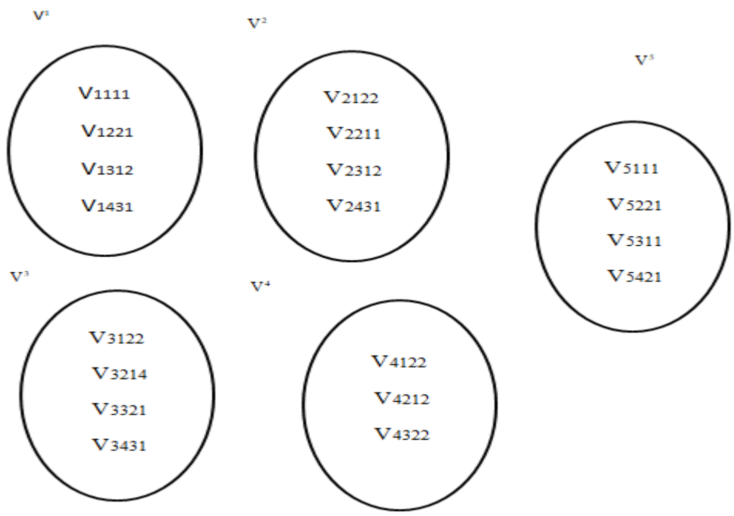

Fig. 3.6 Dominant Enveloping vertices

3.1 Decision-Making Scenario I

The institution is certain of the dominant sub-attributes and makes decisions based on dominant attributes of the input objects. The graphical representation of attribute relation between input dominant enveloping vertices and the output dominating attribute vertex with simple plithogenic fuzzy connectors is presented in Fig 3.7

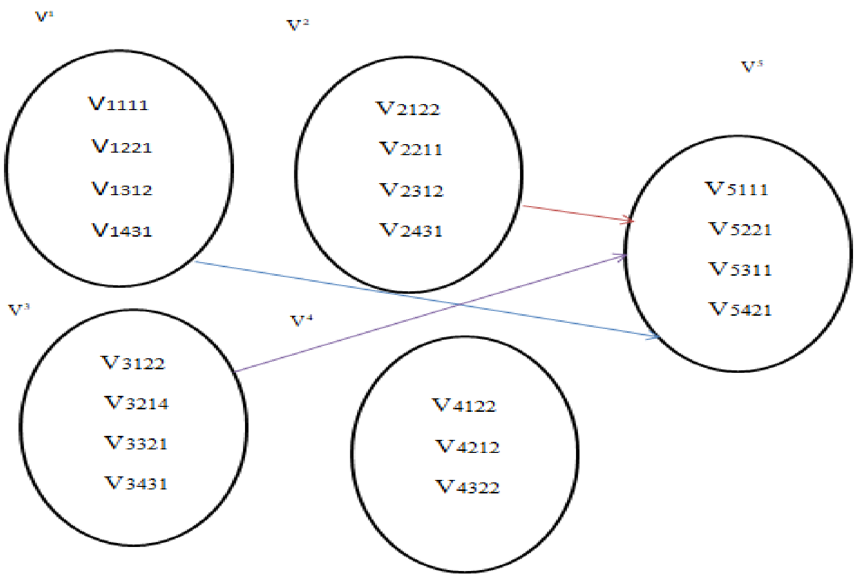

Fig.3.7 Representation of Decision-Making Scenario I

The dominant attribute relational matrix representation between the input objects on the output objects is presented as follows

	V_{5111}	V_{5221}	V_{5311}	V_{5421}
V_{1111}	0.55	0.2	0.5	0.6
V_{1221}	0.8	0.5	0.5	0.8
V_{1312}	0.75	0.6	0.6	0.9
V_{1431}	0.65	0.8	0.1	0.4
V_{2122}	0.5	0.9	0.3	0.6
V_{2211}	0.3	0.5	0.6	0.8
V_{2312}	0.45	0.4	0.4	0.9
V_{2431}	0.6	0.8	0.2	0.4
V_{3122}	0.85	0.2	0.8	0.8
V_{3214}	0.9	0.3	0.9	0.9
V_{3321}	0.5	0.55	0.5	0.4
V_{3431}	0.6	0.8	0.2	0.4

The frequency matrix as discussed by [14] shall be constructed to rank the dominant attributes of input objects contributing to the dominant attribute of the output object. This is a simple decision-making environment as it does not involve the role of an intervening object.

3.2 Decision Making Scenario II

The institution is certain of the dominant sub-attributes and makes decisions based on dominant attributes of the input and intervene objects. The graphical representation of attribute relation between input and intervene dominant enveloping vertices and the output dominating attribute vertex with combined plithogenic fuzzy connectors is presented in Fig.3.8

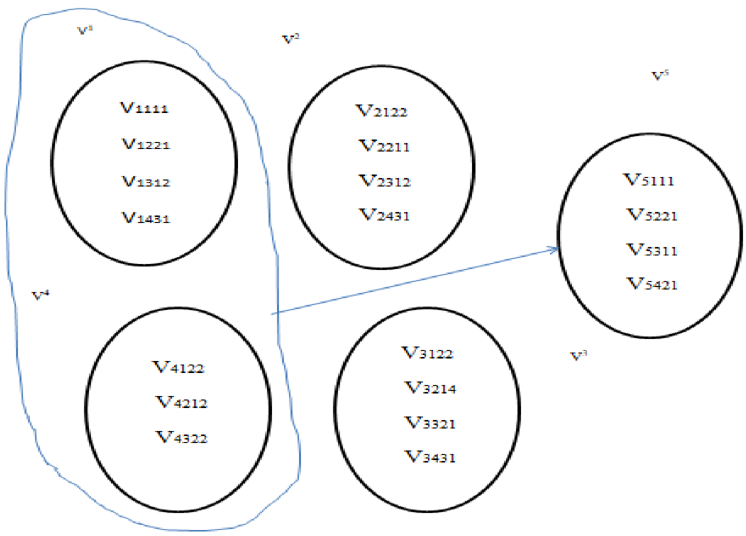

Fig.3.8 Representation of Decision-Making Scenario II

The dominant attribute relational matrix representation between the input and intervene objects on the output objects is presented as follows

	V_{5111}	V_{5221}	V_{5311}	V_{5421}
V_{1111},V_{4122}	0.65	0.6	0.5	0.6
V_{1111},V_{4212}	0.8	0.65	0.7	0.8
V_{1111},V_{4322}	0.56	0.7	0.9	0.6
V_{1221},V_{4122}	0.75	0.8	0.85	0.6
V_{1221},V_{4212}	0.9	0.6	0.95	0.8
V_{1221},V_{4322}	0.6	0.8	0.45	0.9
V_{1312},V_{4122}	0.53	0.7	0.75	0.7
V_{1312},V_{4212}	0.43	0.5	0.6	0.7
V_{1312},V_{4322}	0.5	0.6	0.78	0.7
V_{1431},V_{4122}	0.62	0.85	0.8	0.69
V_{1431},V_{4212}	0.67	0.78	0.7	0.63
V_{1431},V_{4322}	0.6	0.89	0.58	0.7

The frequency matrix shall be constructed to rank the combined dominant attributes of input and intervene objects contributing to the dominant attribute of the output object. This is a little complex decision-making environment as it involves the role of an intervening object. Fig 3.9 presents the graphical representation of it.

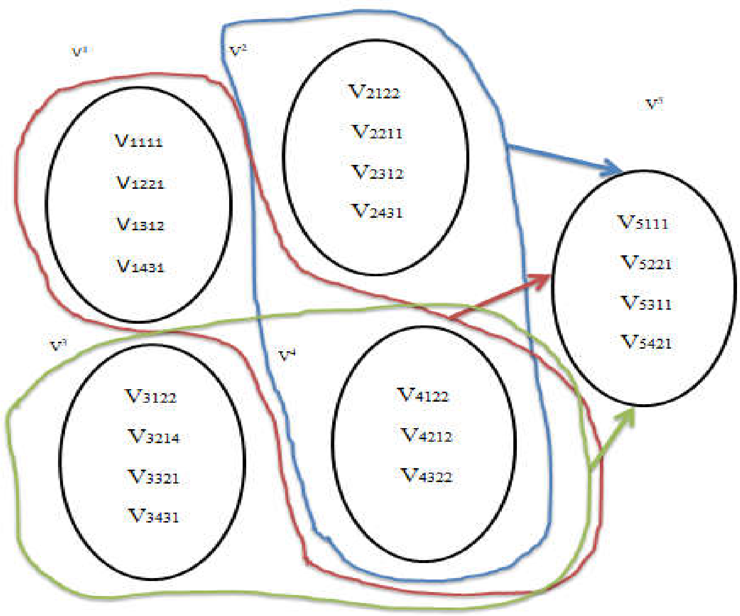

Fig.3.9 Representation of Decision-Making Scenario II with Intervening object

3.3 Decision Making Scenario III

The institution is certain of the dominant sub-attributes. Let us consider a situation, suppose if the institution decides to conduct a webinar with a focus on general, but not able to decide whether to give priority to Education, Health or Psychology, then the decision-making environment becomes more complex. The graphical representation of all sub-attribute relation between input and the output dominating attribute vertex with simple plithogenic fuzzy connectors is presented in Fig. 3.10

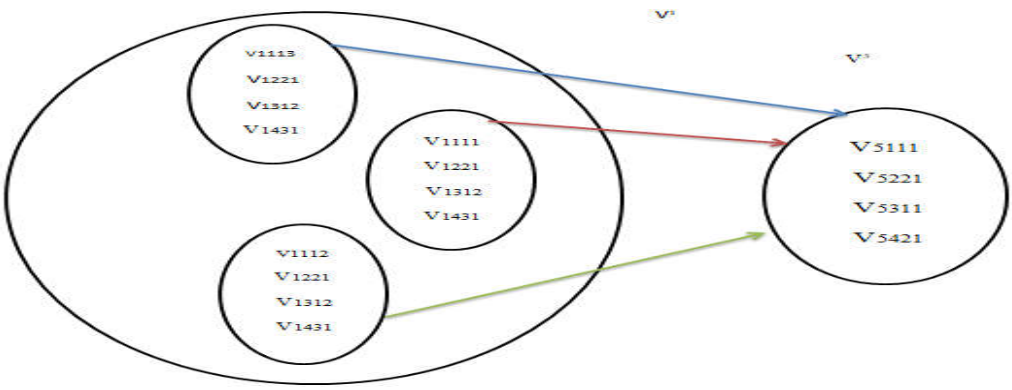

Fig. 3.10 Representation of Decision-Making Scenario III

The dominant attribute relational matrix representation is as follows

	V_{5111}	V_{5221}	V_{5311}	V_{5421}
V_{1111}	0.55	0.2	0.5	0.6
V_{1112}	0.6	0.55	0.7	0.85
V_{1113}	0.5	0.3	0.6	0.6
V_{1221}	0.8	0.4	0.4	0.8
V_{1312}	0.7	0.64	0.6	0.9
V_{1431}	0.6	0.89	0.62	0.4
V_{2122}	0.54	0.9	0.73	0.6
V_{2211}	0.83	0.5	0.6	0.8
V_{2312}	0.4	0.4	0.45	0.9
V_{2431}	0.6	0.8	0.72	0.49
V_{3122}	0.9	0.52	0.8	0.8
V_{3214}	0.95	0.63	0.9	0.9
V_{3321}	0.5	0.8	0.5	0.43
V_{3431}	0.6	0.8	0.52	0.4

3.4 Decision-Making Scenario IV

This decision-making situation is characterized when the institution is uncertain of the dominant sub-attribute values of the input object. Suppose if the institution decides to conduct a webinar with focus on general, but not able to decide whether to give priority to Education, Health or Psychology, In this case, the dominant sub-attribute value is not certain and suppose it wishes to collaborate with the industry then the decision-making environment becomes highly complex. Fig 3.11 presents this graphically

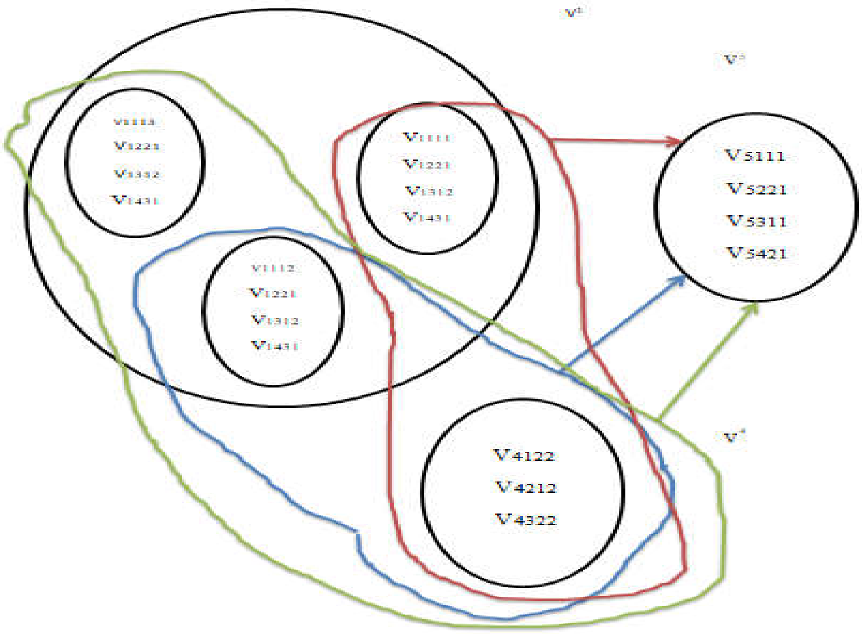

Fig.3.11 Representation of Decision-Making Scenario IV

The dominant attribute relational matrix representation is as follows

	V_{5111}	V_{5221}	V_{5311}	V_{5421}
V_{1111},V_{4122}	0.65	0.6	0.5	0.6
V_{1111},V_{4212}	0.8	0.65	0.7	0.8
V_{1111},V_{4322}	0.56	0.7	0.9	0.6
V_{1112},V_{4122}	0.52	0.68	0.57	0.66
V_{1112},V_{4212}	0.45	0.67	0.72	0.56
V_{1112},V_{4322}	0.56	0.68	0.74	0.69
V_{1113},V_{4122}	0.67	0.79	0.83	0.79
V_{1113},V_{4212}	0.57	0.82	0.74	0.68
V_{1113},V_{4322}	0.5	0.7	0.7	0.6
V_{1221},V_{4122}	0.75	0.8	0.85	0.6
V_{1221},V_{4212}	0.9	0.6	0.95	0.8
V_{1221},V_{4322}	0.6	0.8	0.45	0.9
V_{1312},V_{4122}	0.53	0.7	0.75	0.7
V_{1312},V_{4212}	0.43	0.5	0.6	0.7
V_{1312},V_{4322}	0.5	0.6	0.78	0.7
V_{1431},V_{4122}	0.62	0.85	0.8	0.69
V_{1431},V_{4212}	0.67	0.78	0.7	0.63
V_{1431},V_{4322}	0.6	0.89	0.58	0.7

The ranking of the attributes contributing to Professional efficiency corresponding to each decision-making environment is presented in Table 3.3

Table 3.3 Ranking of the attributes

Decision Making Environment	Ranking of the Attributes contributing to Professional Efficiency
Decision Making Scenario I	$V_{1312} > V_{1221} > V_{3122} > V_{1221} > V_{2122} > V_{2211} > V_{2431} > V_{1111} > V_{2312} > V_{1431} > V_{3431} > V_{3321}$
Decision Making Scenario II	$V_{1221}, V_{4212} > V_{1221}, V_{4322} > V_{1431}, V_{4122} > V_{1111}, V_{4212}, V_{1221}, V_{4122} > V_{1431}, V_{4322} > V_{1312}, V_{4122} > V_{1312}, V_{4322} > V_{1111}, V_{4122} > V_{1312}, V_{4212}$
Decision Making Scenario III	$V_{3214} > V_{2431} > V_{1312} > V_{2122} > V_{3122} > \mathbf{V_{1111}} > V_{2211} > V_{1431} > V_{1221} > V_{3431} > V_{3321} > V_{2312} > \mathbf{V_{1112} > V_{1113}}$
Decision Making Scenario IV	$V_{1221}, V_{4212} > V_{1113}, V_{4122} > V_{1221}, V_{4122} > V_{1431}, V_{4122} > V_{1111}, V_{4212} > V_{1113}, V_{4212} > V_{1431}, V_{4212} > V_{1431}, V_{4322} > V_{1111}, V_{4322} > V_{1221}, V_{4322} > V_{1312}, V_{4122} > V_{1112}, V_{4322} > V_{1312}, V_{4322} > V_{1113}, V_{4322} > V_{1112}, V_{4122} > V_{1112}, V_{4212} > V_{1111}, V_{4122} > V_{1312}, V_{4212}$

4. Discussion

The ranking of the input attribute values of the dominant attributes contributing to the output dominant attribute values shows the significance of the individual contribution of each input attribute value. In the first decision-making scenario, the attribute values of the input object are ranked. In the second decision making a scenario the combined attribute values of input and intervene objects are ranked. This helps in finding the combined effect towards the attainment of the output attribute values. In the third decision-making scenario, the ranking of sub-attribute values are made, in this case, there was a choice to choose between Education (V_{1111}), Health (V_{1112}) or Psychology (V_{1113}), but the preference s should be given to Education based on the ranking. In the fourth decision-making scenario, the combined effects of sub-attribute values along with intervening attribute values are ranked and here also the combined effect of the sub-attribute value, Education is gaining more significance. The above decision-making scenarios were focusing on the effects of one input object and the same can be applied to other input objects and the respective results can be determined. The same method of decision making can be applied to production sectors in strategy selection which considers many attribute values and sub-attribute values and this proposed plithogenic –n superhypergraph MADM can be applied in such decision-making scenario.

5. Conclusion

This article presents the application of plithogenic n-super hypergraph in the context of optimal decision making. This research work introduces many new concepts such as enveloping vertex, dominant enveloping vertex, super enveloping vertex, and plithogenic connectors. This research work creates a new avenue in MADM by providing space for comprehensive decision making. A new approach to ranking the attribute values based on the frequency matrix is initiated. The theoretical description of plithogenic n-super hypergraph is translated into practical application in this research work and this will certainly open new vistas of research. This work can be further extended with various plithogenic sets.

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Concentric Plithogenic Hypergraph based on Plithogenic Hypersoft sets – A Novel Outlook

Nivetha Martin, Florentin Smarandache

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Abstract: Plithogenic Hypersoft sets (PHS) introduced by Smarandache are the extensions of soft sets and hypersoft sets and it was further protracted to plithogenic fuzzy whole Hypersoft set to make it more applicable to multi attribute decision making environment. The fuzzy matrix representation of the plithogenic hypersoft sets lighted the spark of concentric plithogenic hypergraph. This research work lays a platform for presenting the concept of concentric plithogenic hypergraph, a graphical representation of plithogenic hypersoft sets. This paper comprises of the definition, classification of concentric plithogenic hypergraphs, extended hypersoft sets, extended concentric plithogenic hypergraphs and it throws light on its application. Concentric Plithogenic hypergraphs will certainly open the new frontiers of hypergraphs and this will undoubtedly bridge hypersoft sets and hypergraphs.

Keywords: Plithogenic sets; Hypergraph; Plithogenic Hypersoft sets; Concentric Plithogenic Hypergraphs

1. Introduction

The structure of a graph comprises of vertices and edges, has a range of applications in diverse fields. In general an edge in a graph represents the relation between two vertices. Berge [1] extended this basic idea and introduced hypergraph as the generalization of graph. In a hypergraph hyperedge links one or more vertices and they are mainly used to explore configuration of the systems by clustering and segmentation, but to handle the uncertain and imprecise environment; Kaufmann [2] introduced fuzzy hypergraphs. The concept of fuzzy set was introduced by Lofti.A. Zadeh [3]. The fuzzy hypergraph introduced by Kaufmann was later generalized by Hyung and Keon [4] to overcome the limitations of inappropriate representation of fuzzy partition by redefining fuzzy hypergraph and developing many expedient concepts which finds extensive applications in system analysis, circuit clustering and pattern recognition. In a fuzzy hypergraph the hyper edges are fuzzy sets of vertices. Mordeson and Nair [5] have made significant contributions to fuzzy graphs and fuzzy hypergraphs. Parvathi et al [6] extended of intuitionistic fuzzy graphs to intuitionistic fuzzy hypergraph in which (α, β)-cut hypergraph represent intuitionistic fuzzy partition. In an intuitionistic fuzzy hypergraph the hyperedge sets are intuitionistic fuzzy sets of vertices consisting of both membership and non-membership values as Atanssov [7] introduced in Intuitionistic set. Akram and Dudek [8] discussed the properties and applications of intuitionistic fuzzy hypergraph. Akram et al [9] introduced neutrosophic hypergraphs and single valued neutrosophic hypergraphs. Neutrosophic sets introduced by Smarandache [10] deals with truth function, indeterminacy function and falsity function, based on conceptualization of neutrosophic sets, Akram.et al [11] investigated the properties of line graph of neutrosophic hypergraph, dual neutrosophic hypergraph, tempered neutrosophic hypergraph and transversal neutrosophic hypergraph with illustrations. Neutrosophic theory has extensive applications in the domain of decision- making. Abdel-Baset et al [12] introduced a novel neutrosophic approach to assess the green supply chain management practices and the recent research in multi criteria decision- making uses the neutrosophic representations. On other hand Vasantha Kanthasamy et al [13]discussed Plithogenic graph, special type of graphs based on fuzzy intuitionistic and single valued neutrosophic graphs. The characteristics of plithogenic intuitionistic fuzzy graph, plithogenic neutrosophic graphs and plithogenic complex graphs are also examined, but notion of plithogenic hypergraph was not discoursed.

The plithogenic sets introduced by Smarandache [14] deals with attributes and it is extension of crisp, fuzzy, intuitionistic and neutrosophic sets. Plithogenic sets are widely used in multi attribute decision -making systems as it plays a vital role in deriving optimal solutions to the decision-making problems. Abdel-Basset et al [15,16] has framed a novel plithogenic TOPSIS-CRITIC model for sustainable supply chain risk management and formulated a hybrid plithogenic decision-making approach with quality function deployment for selecting supply chain sustainability metrics. These proposed models are highly advantageous, compatible to make decisions as it handles multi attributes or multi criteria environment. The selection process of alternatives based on different attributes containing several attribute values becomes easier in plithogenic representation. Furthermore plithogenic hypersoft introduced by Smarandache [17] also has significant contribution in multi attribute decision-making methods. Molodtsov [18] introduced soft sets and Smarandache generalized to hypersoft set by modifying single attribute function to multi attribute function. The plithogenic hypersoft set, generalization of crisp, fuzzy , intuitionistic and neutrosophic soft sets. Shazia Rana et al [19] extended plithogenic fuzzy hypersoft set to plithogenic fuzzy whole hypersoft set ;developed Frequency Matrix Multi Attributes Decision making scheme to rank the alternatives and proposed a new ranking approach based on frequency matrix in their research work. Plithogenic hypersoft sets are finding new avenue in decision- making and in ranking process.

Nivetha and Pradeepa [20,21] initiated integration of hypergraphs and fuzzy hypergraphs with Fuzzy Cognitive Maps (FCM). Kosko [22] introduced FCMs, directed graphs consisting of nodes and edges which represent the casual factors and its relationship. FCM assumes simple weights such as -1 if factors have negative impact over another, 0 if no impact and 1 for positive impact. Weighted FCM assumes values from [-1,1]. The approach of FCM is analogues to the reasoning and decision -making of human and it facilitates conception of intricate social systems. Peláezand Bowles [23],Miao and Liu [24], Papageorgiou et al [25]have proposed various algorithms and methods to handle various forms of FCM. The nature of weights classifies FCMs as intuitionistic and neutrosophic FCM. One of the most difficult aspects in handling FCM is consideration of large number of study factors. Confinement of the number of inputs is essential to make optimal decision-making and this has to take place step wise. It is helpful to limit the study factors for analyzing their inter impacts, to make so, FCMs with hypergraphic and fuzzy hypergraphic approaches facilitated to formulate student's low academic performance model and assessment model of blended method of teaching. Nivetha and Pradeepa [26] also introduced concentric fuzzy hypergraphs for inclusive decision -making and this kind of hypergraph deals with hyper envelopes instead of hyper edges; examined the properties of concentric fuzzy hypergraph and justified with suitable illustrations and applications. Concentric fuzzy hypergraphs was further extended to concentric neutrosophic hypergraphs to explore the factors causing autoimmune diseases using Fuzzy Cognitive Maps (FCM).

The proposed integrated models of FCM with hypergraphs, fuzzy hypergraphs and concentric fuzzy hypergraphs focus only on the factors based on single criteria. Suppose if the factors are dependent on multi criteria then the above integrated models do not meet the need. This is limitation of the above described integrated models. Concentric plithogenic hypergraphs integrated with FCMs helps to overcome such shortcomings. As the concept of plithogenic hypergraph was not disclosed so far, this research work extends concentric hypergraphic approach of FCM to concentric plithogenic hypergraph with plithogenic hypersoft representation to make optimal decisions by ranking the study factors based on multi attribute. Such representations will he highly pragmatic and it will certainly ease the decision-making process. The frequency matrices ranks the factors represented as plithogenic hypersoft sets and the core factors considered for determining the inter relationship and inter impacts using FCM method. This approach will definitely yield optimal results with simplified computations.

The objectives of this research work are to introduce notion of concentric plithogenic hypergraph; define concentric plithogenic hypergraph based on concentric fuzzy hypergraph and plithogenic graphs; classify concentric plithogenic hypergraph based on degree of appurtenance; widens concentric plithogenic hypergraph to extended concentric plithogenic hypergraph; proposes FCM decision making model integrated with extended concentric plithogenic hypergraphic approach. But this research work concentrates on proposing concentric plithogenic hypergraphs in a more distinct way. With the brief introduction of the research work in section 1, the rest of the paper is organized with construction of concentric plithogenic hypergraph in section 2, extension and classification of concentric plithogenic hypergraphs in section 3, applications of the proposed approach in section 4, discussion of the results in section 5 and the concluding remarks in the section 6.

2. Construction of Concentric Plithogenic Hypergraphs

The concept of concentric fuzzy hypergraphs evolved at times of integrating hypergraphs with Fuzzy Cognitive Maps after integration of hypergraphic and fuzzy hypergraphic approaches. Concentric plithogenic hypergraph is an integration concentric hypergraph with plithogenity. To make it more comprehensive, the following definitions are put forward.

2.1 Hypergraph

A hypergraph H is an ordered pair H = (X,E), where

(i) $X = \{x_1,x_2,..x_n\}$ is a finite set of vertices.

(ii) $E = \{ E_1, E_2,..E_n\}$ is a family of subsets of X and each E_j is a hyper edge.

(iii) $E_j \neq \varphi$, j = 1,2,..3 and $\cup_j E_j = X$

2.2 Concentric Fuzzy Hypergraph

A concentric fuzzy hypergraph $\mathcal{C}_H$ is defined as follows

$\mathcal{C}_H = (X, \mathcal{E})$

X- finite set of vertex set

$\mathcal{E}$- Concentric fuzzy hyper envelope – family of fuzzy sets of X

$\mathcal{E}_j = \{(x_i, \mu_j(x_i))/ \mu_j(x_i) > 0 \text{ and } \forall x_i \in X\}$ j = 1,2,…m

Supp ($\mathcal{E}$) = X = Supp ($\mathcal{E}_j$) $\forall$ j = 1,2,…m

To illustrate concentric fuzzy hypergraph,

let $\mathcal{C}_H = (X, \mathcal{E})$, where

$X = \{x_1,x_2,x_3,x_4\}$

$\mathcal{E}_1 = \{(x_1,0.4), (x_2,0.6), (x_3,0.5),(x_4,0.3)\}$

$\mathcal{E}_2$= {(x1,0.3), (x2,0.8), (x3,0.4),(x4,0.5)}

$\mathcal{E}_3$={(x1,0.6), (x2,0.6), (x3,0.7),(x4,0.4)}

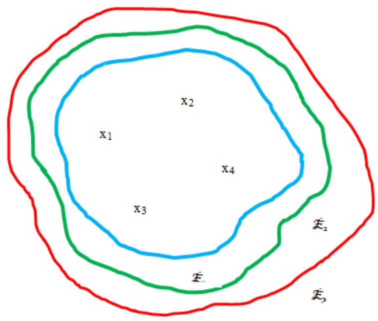

Fig.2.1. Concentric Fuzzy Hypergraph

2.3 Concentric Plithogenic Hypergraph

A hypergraph with non- empty, disjoint hyperedges and plithogenic envelopes is called as concentric plithogenic hypergraph P_{C_H}.

A concentric Plithogenic hypergraph P_{C_H} is defined as follows

P_{C_H} = (X, E, $\mathcal{P}_{\mathcal{E}}$)

- X- Finite vertex set
- E – Hyperedge set
- $\mathcal{P}_{\mathcal{E}}$ - Plithogenic envelope
- $\mathcal{P}_{\mathcal{E}j} = \{(x_{ij}, \mu(x_{ij}))/\ \mu(x_{ij}) > 0, x_{ij} \in E_i\}$
- $E_i \cap E_k = \emptyset$and$UE_i = X$

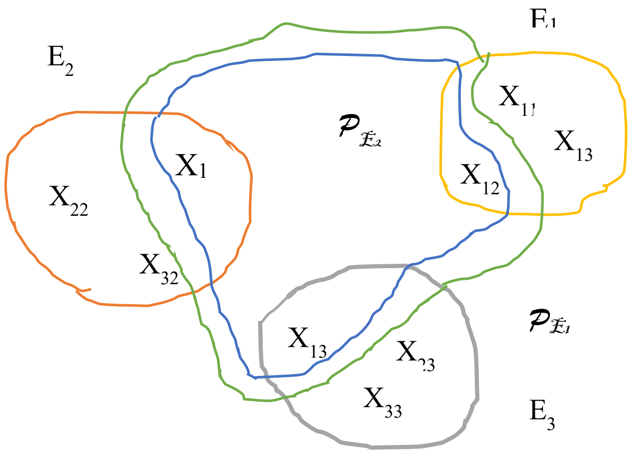

Fig.2.2. Concentric Plithogenic Hypergraph

2.4 Integration of Plithogenic Hypersoft sets and Concentric Plithogenic Hypergraph

Plithogenic hypersoft sets are extensively used in decision making situations. Let us consider the plithogenic hypersoft set presented below.

Let U = { $M_1, M_2, M_3, M_4, M_5, M_6, M_7, M_8, M_9, M_{10}$} be the university of discourse and set T = {M_1, M_3, M_6} $\subset$ U. The attribute system is represented as follows $\mathcal{A}$ = { (A_1)**Maintenance Cost** {Maximum in the initial years of utility(A_1^1), Maximum in the latter years of utility(A_1^2), Moderate throughout (A_1^3)}, (A_2)**Reliability** {High with additional expenditure(A_2^1), Moderate with no extra expense(A_2^2), Moderate with high expense(A_2^3)}, (A_3)**Flexibility** {Single task oriented(A_3^1), Multi task oriented(A_3^2), Dual task oriented(A_3^3) }, (A_4)**Durability** {Very high in the beginning years of service(A_4^1), High in the latter years of service(A_4^2),Moderate (A_4^3) }, (A_5)**Profitability** {Moderate in the initial years(A_5^1), Maximum in the latter years(A_5^2), Moderate throughout the years (A_5^3)}}.

G: $A_1^1 \times A_2^2 \times A_3^2 \times A_4^1 \times A_5^2 \longrightarrow (\mathcal{U})$.

G($A_1^1, A_2^2, A_3^2, A_4^1, A_5^2$) =

{M_1(0.9,0.875,0.8,0.75,0.5),M_3(0.67,0.5,0.4,0.8,0.7),M_6(0.8,0.7,0.6,0.7,0.5)}

In the below graphical representations the attributes A_1 to A_5 are the hyperedges consisting of the vertices (x_{ij}) A_i^j , i = 1,2,3,4,5 and j = 1,2,3. M_1, M_3 and M_6 are the plithogenic envelopes

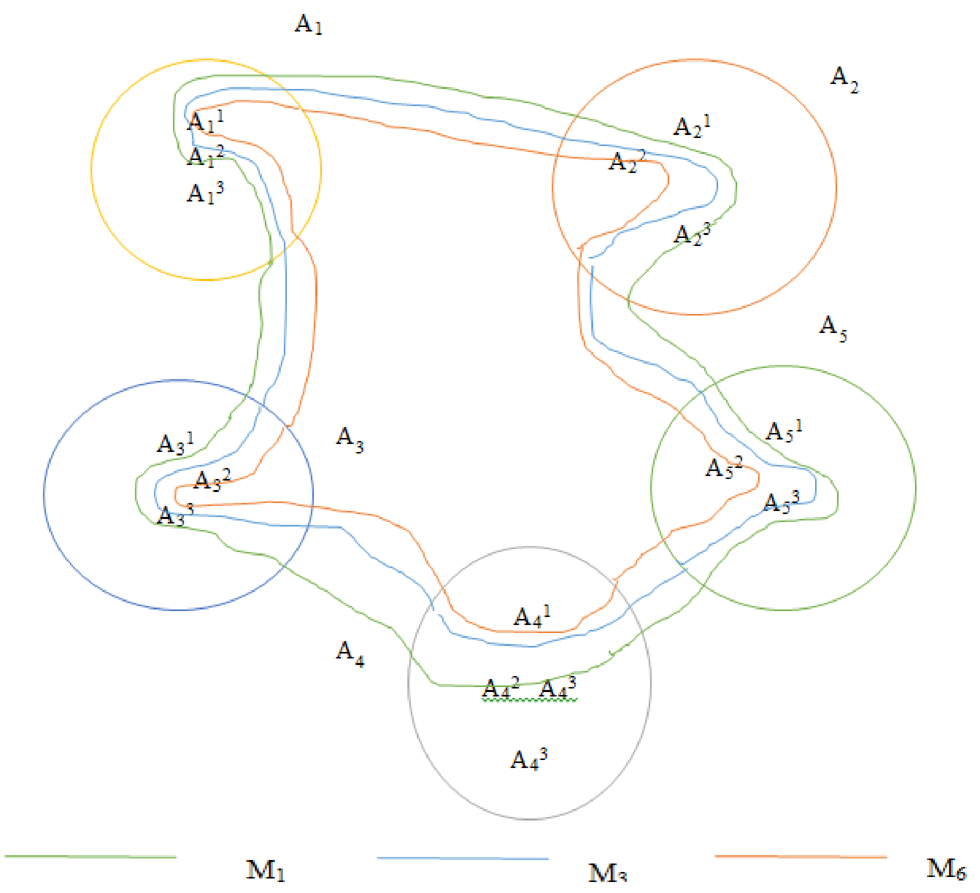

Fig 2.3 Graphical Representation of Plithogenic Hypersoft set by Concentric Plithogenic Hypergraph

To illustrate Concentric Plithogenic Hypergraph P_{C_H} based on Plithogenic Hypersoft sets, Let X= { $A_1^1, A_1^2, A_1^3, A_2^1, A_2^2, A_2^3, A_3^1, A_3^2, A_3^3, A_4^1, A_4^2, A_4^3, A_5^1, A_5^2, A_5^3$}

$E = \{A_1, A_2, A_3, A_4, A_5\}$

$\mathcal{P}_{\mathcal{E}_1} = \{(A_1{}^1, 0.9), (A_2{}^2, 0.875), (A_3{}^2, 0.8), (A_4{}^1, 0.75), (A_5{}^2, 0.5)\}$

$\mathcal{P}_{\mathcal{E}_2} = \{(A_1{}^1, 0.67), (A_2{}^2, 0.5), (A_3{}^2, 0.4), (A_4{}^1, 0.8), (A_5{}^2, 0.7)\}$

$\mathcal{P}_{\mathcal{E}_3} = \{(A_1{}^1, 0.8), (A_2{}^2, 0.7), (A_3{}^2, 0.6), (A_4{}^1, 0.7), (A_5{}^2, 0.5)\}$

The formulation of the notion of concentric plithogenic hypergraph based on plithogenic hypersoft will be more rational. Also the plithogenic hypergraphs and concentric plithogenic hypergraphs can be defined based on plithogenic graphs as in the below fig.2.4 and fig.2.5

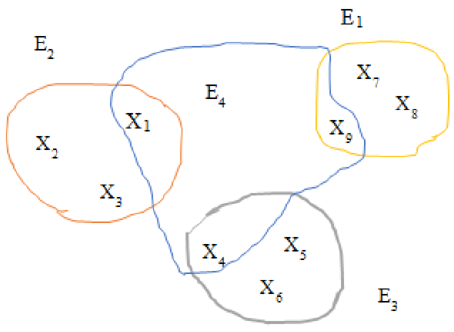

Fig.2.4 Plithogenic hypergraph

To illustrate Plithogenic Hypergraph P_{H^*}, based on Plithogenic graphs,

Let $X = \{x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8, x_9\}$

$E_1 = \{x_7(0.2, 0.3, 0.4), x_8 (0.8, 0.6, 0.3), x_9 (0.4, 0.2, 0.6)\}$

$E_2 = \{x_1(0.1, 0.5, 0.4), x_2 (0.5, 0.6, 0.8), x_3 (0.3, 0.2, 0.7)\}$

$E_3 = \{x_4(0.9, 0.3, 0.5), x_5 (0.2, 0.4, 0.3), x_6(0.6, 0.2, 0.1)\}$

$E_4 = \{x_1(0.2, 0.9, 0.7), x_4 (0.3, 0.7, 0.9), x_9 (0.1, 0.8, 0.6)\}$

X is the vertex set and E_i, i = 1,2,3 are the plithogenic hyperedges of plithogenic hypergraph.

A hypergraph with hyperedges possessing plithogenic weight representations is defined as plithogenic hypergraph otherwise Plithogenic hypergraphs can be defined as hypergraphs with plithogenic hyperedges. A hypergraph with plithogenic hyper envelopes is defined as concentric plithogenic hypergraphs.

Fig.2.5 Concentric Plithogenic hypergraph

To illustrate Concentric Plithogenic Hypergraph P_{CH^*}, based on Plithogenic graphs

Let X = {x_1,x_2,x_3}

$\mathcal{PE}_1$= {(x_1, (0.4,0.2,0.1), (x_2,(0.2,0.3,0.4)), (x_3,(0.4,0.8,0.5))}

$\mathcal{PE}_2$= {(x_1, (0.8,0.5,0.6), (x_2,(0.4,0.3,0.2)), (x_3,(0.7,0.6,0.1))}

$\mathcal{PE}_3$= {(x_1, (0.7,0.6,0.3), (x_2,(0.1,0.5,0.6)), (x_3,(0.4,0.5,0.8))}

X is the vertex set and $\mathcal{PE}_i$,i = 1,2,3 are the plithogenic hyper envelopes of concentric plithogenic hypergraph.

Plithogenic hypergraphs and concentric plithogenic hypergraphs based on plithogenic graphs are distinct from the concentric plithogenic hypergraphs defined based on plithogenic hypersoft sets. The former definition doesn't take the condition of $E_i \cap E_k = \emptyset$. The graphs are called as plithogenic based on their dimension of membership values. In comparison of both kinds of representation, the latter is more feasible in nature as it incorporates the degree of appurtenance and the holistic meaning of plithogeny is reflected.

3. Classification and Extension of Concentric Plithogenic Hypergraphs

The concentric plithogenic hypergraphs is classified into crisp, fuzzy, intuitionistic and neutrosophic based on the values of the degree of appurtenance.

3.1 Crisp Concentric Plithogenic Hypergraphs

Let X= { $A_1^1, A_1^2, A_1^3, A_2^1, A_2^2, A_2^3, A_3^1, A_3^2, A_3^3, A_4^1, A_4^2, A_4^3, A_5^1, A_5^2, A_5^3$}

E = {A_1, A_2,A_3,A_4, A_5}

$\mathcal{P}_{E_1}$ ={(A_1^1,1),(A_2^2,1),(A_3^2,1), (A_4^1,1),(A_5^2,1)}

$\mathcal{P}_{E_2}$ = {(A_1^1,1),(A_2^2,1),(A_3^2,1), (A_4^1,1),(A_5^2,1)}

$\mathcal{P}_{E_3}$ = {(A_1^1,1),(A_2^2,1),(A_3^2,1), (A_4^1,1),(A_5^2,1)}

3.2 Fuzzy Concentric Plithogenic Hypergraphs

Let X= { $A_1^1, A_1^2, A_1^3, A_2^1, A_2^2, A_2^3, A_3^1, A_3^2, A_3^3, A_4^1, A_4^2, A_4^3, A_5^1, A_5^2, A_5^3$}

E = {A_1, A_2,A_3,A_4, A_5}

$\mathcal{P}_{E_1}$ ={(A_1^1,0.9),(A_2^2,0.875),(A_3^2,0.8), (A_4^1,0.75),(A_5^2,0.5)}

$\mathcal{P}_{E_2}$ ={(A_1^1,0.67),(A_2^2,0.5),(A_3^2,0.4), (A_4^1,0.8),(A_5^2,0.7)}

$\mathcal{P}_{E_3}$ = {(A_1^1,0.8),(A_2^2,0.7),(A_3^2,0.6), (A_4^1,0.7),(A_5^2,0.5)}

3.3 Intuitionistic Concentric Plithogenic Hypergraphs

Let X= { $A_1^1, A_1^2, A_1^3, A_2^1, A_2^2, A_2^3, A_3^1, A_3^2, A_3^3, A_4^1, A_4^2, A_4^3, A_5^1, A_5^2, A_5^3$}

E = {A_1, A_2,A_3,A_4, A_5}

$\mathcal{P}_{E_1}$ ={(A_1^1,(0.9,0.1)),(A_2^2,(0.5,0.2)),(A_3^2,(0.8,0.2)), (A_4^1,(0.75,0.5)),(A_5^2,(0.5,0.2))}

$\mathcal{P}_{E_2}$ ={(A_1^1,(0.6,0.7),(A_2^2,(0.7,0.5),(A_3^2,(0.9,0.4), (A_4^1,(0.7,0.1),(A_5^2,(0.7,0.2)}

$\mathcal{P}_{E_3}$ ={(A_1^1,(0.6,0.4),(A_2^2,(0.2,0.58),(A_3^2,(0.19,0.54), (A_4^1,(0.7,0.2),(A_5^2,(0.8,0.2)}

3.4 Neutrosophic Concentric Plithogenic Hypergraphs

Let X= { $A_1^1, A_1^2, A_1^3, A_2^1, A_2^2, A_2^3, A_3^1, A_3^2, A_3^3, A_4^1, A_4^2, A_4^3, A_5^1, A_5^2, A_5^3$}

E = {A_1, A_2,A_3,A_4, A_5}

$\mathcal{P}_{E_1}$ ={(A_1^1,(0.9,0.1,0.2),(A_2^2,(0.8,0.5,0.2),(A_3^2,(0.2,0.2,0.8), (A_4^1,(0.1,0.3,0.75),(A_5^2,(0.4,0.2,0.5)}

$\mathcal{P}_{E_2}$ ={(A_1^1,(0.7,0.1,0.2),(A_2^2,(0.6,0.2,0.1),(A_3^2,(0.3,0.2,0.5), (A_4^1,(0.4,0.3,0.5),(A_5^2,(0.6,0.2,0.3)}

$\mathcal{P}_{E_3}$ ={(A_1^1,(0.7,0.1,0.2),(A_2^2,(0.6,0.4,0.2),(A_3^2,(0.6,0.2,0.3), (A_4^1,(0.5,0.2,0.7),(A_5^2,(0.6,0.1,0.4)}

3.5 Extended Plithogenic Hypersoft sets

Extended plithogenic hypersoft sets comprises of the degree of appurtenance of the elements to the corresponding attributes along with multi expert's opinion. These sets comprise of the opinion of several experts. The representation of such kind of set is presented as follows

Let us consider a situation that the below values are given by two experts for the same example discussed under plithogenic hypersoft sets.

G (A_1^1,A_2^2 , A_3^2 , A_4^1,A_5^2) given by the first expert =
{M_1(0.9,0.875,0.8,0.75,0.5),M_3(0.67,0.5,0.4,0.8,0.7),M_6(0.8,0.7,0.6,0.7,0.5)}

G (A_1^1,A_2^2 , A_3^2 , A_4^1,A_5^2) given by the second expert =
{M_1(0.6,0.875,0.8,0.5,0.5),M_3(0.7,0.6,0.3,0.9,0.7),M_6(0.8,0.7,0.7,0.7,0.6)}

The aggregate representation will be
$\mathcal{G}$ (A_1^1,A_2^2 , A_3^2 , A_4^1,A_5^2) = {M_1{(0.9,0.875,0.8,0.75,0.5), (0.6,0.875,0.8,0.5,0.5)}, M_3 {(0.67,0.5,0.4,0.8,0.7), (0.7,0.6,0.3,0.9,0.7)}, M_6 {(0.8,0.7,0.6,0.7,0.5), (0.8,0.7,0.7,0.7,0.6)}}

The extended concentric plithogenic hypergraphs are based on the extended plithogenic hypersoft sets. The graphical representation is depicted in fig 3.1

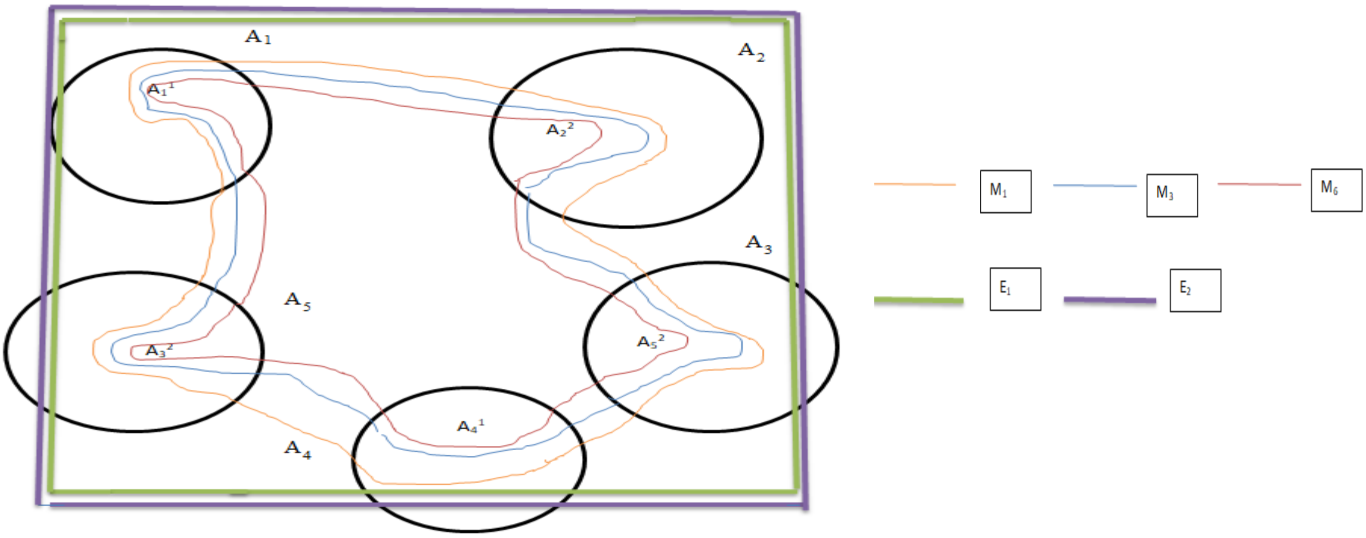

Fig.3.1. Graphical Representation of Extended Concentric Plithogenic Hypergraph

3.6 Extended Concentric Plithogenic Hypergraph

A hypergraph with non- empty, disjoint hyperedges and extended plithogenic envelopes is called as extended concentric plithogenic hypergraph $E_{P_{C_H}}$

An extended concentric Plithogenic hypergraph P_{C_H} is defined as follows

$E_{P_{C_H}}$ = (X, E, $E_{\mathcal{P}\mathcal{E}}$)

- X- finite vertex set
- E- Hyperedge set
- $E\mathcal{P}_{\mathcal{E}}$ =Extended Plithogenic hyper envelope
- $E\mathcal{P}_{\mathcal{E}j}$ = $\left\{ \left(x_{ij}, \mu_{El}(x_{ij})\right) / \mu(x_{ij}) > 0, x_{ij} \in E_i \right\}$
- $E_i \cap E_k = \emptyset$ and $\cup E_i = X$

4. Fuzzy Cognitive Maps integrated with Extended Concentric Plithogenic Hypergraphic Approach.

Fuzzy Cognitive Maps (FCM) is a decision making tools that are predominantly used in finding the cause and effect relationship. Basically FCM is a directed graph consisting of nodes and edges representing the factors of study and its relationship respectively. The adjacency or the connection matrix is the representation of the relationship between the nodes. Let us consider a decision making problem, which comprises of several factors, then the connection matrix will be of higher order which will make the computational process complicated. In order to handle such situations the core factors can be determined by using the approach of extended concentric plithogenic hypergraph.

Let us consider a decision making situation to find inter relational impacts of the factors contributing towards the sales promotion of a manufacturing firm. The promotion of sales generally depends on major attributes of a company such as Customers, Pricing stratagem and marketing strategies. Different companies follow various other aspects to foster their sales promotion, but the above three attributes play predominant roles.

If customer (A_1), pricing stratagem (A_2) and marketing strategies (A_3) are considered as the attribute sets then A_1 = {**Potential**, Impulsive, Novel, Loyal}, A_2 = {Competition- Based, Skimming, **Penetration**, Dynamic}, A_3 = {**Social**, Service, Green, Holistic, Direct} are the attribute values. A company has decided to launch a new product in the market with the focus towards potential customers by following penetration pricing strategy through social marketing, the ultimate target of the company to increase the sales and attain the target within the stipulated time, so it has called the experts to present the aspects the company has to concentrate in deep. The expert's perception is presented as factors.

F_1 The touch of innovation in the entire lifecycle of the product

F_2 Extensive design of the product

F_3 Customer centric approach

F_4 Product improvisation suiting the contemporary needs

F_5 Widening of the distribution channels

F_6 Implementation of Price breaks

F_7 Enriching the portals of communication

F_8 Employment of self- assessment tools

F_9 Placement of suitable personnel

F_{10} Counter actions to the competitors

The extended concentric plithogenic hyper envelopes with linguistic representations in accordance to the expert's opinion are presented below in Table 4.1

Table 4.1. Representation of Expert's opinion

Factors	$E\mathcal{P}_{E_1}$	$E\mathcal{P}_{E_2}$
F_1	(H,M,M)	(M,H,VH)
F_2	(VH,H,H)	(H,H,M)
F_3	(VH,H,VH)	(H,H,H)
F_4	(H,VH,H)	(VH,VH,M)
F_5	(VH,H,VH)	(VH,VH,VH)
F_6	(H,H,L)	(H,VH,M)
F_7	(H,H,VH)	(H,H,VH)
F_8	(H,H,H)	(H,H,VH)
F_9	(VH,VH,H)	(VH,VH,VH)
F_{10}	(H,H,H)	(H,H,H)

The linguistic representations of the experts are quantified using hexagonal fuzzy numbers based on the below values in Table 4.2.

Table 4.2. Hexagonal Quantification of values

Very Low (VL)	(0,0.05,0.1,0.15,0.2,0.25)	0.125
Low (L)	(0.15,0.2,0.25,0.3,0.35,0.4)	0.275
Moderate (M)	(0.3,0.35,0.4,0.45,0.5,0.55)	0.425
High (H)	(0.45,0.5,0.55,0.6,0.65,0.7)	0.575
Very High (VH)	(0.65,0.7,0.75,0.8,0.9,1)	0.8

The quantified representations of the experts are presented in Table 4.3

Table 4.3. Modified representation of Expert's opinion

Factors	$E\mathcal{P}_{E_1}$	$E\mathcal{P}_{E_2}$
F_1	(0.575,0.425,0.425)	(0.425,0.575,0.8)
F_2	(0.8,0.575,0.575)	(0.575,0.575,0.425)
F_3	(0.8,0.575,0.8)	(0.575,0.575,0.575)
F_4	(0.575,0.8,0.575)	(0.8,0.8,0.425)
F_5	(0.8,0.575,0.8)	(0.8,0.8,0.8)
F_6	(0.575,0.575,0.275)	(0.575,0.8,0.425)
F_7	(0.575,0.575,0.8)	(0.575,0.575,0.8)
F_8	(0.575,0.575,0.575)	(0.575,0.575,0.8)
F_9	(0.8,0.8,0.575)	(0.8,0.8,0.8)
F_{10}	(0.575,0.575,0.575)	(0.575,0.575,0.575)

The combined values of each factor are presented in below Table 4.4.

Table 4.4 Combined values of the factors

Factors	EP_E
F_1	(1,1,1.225)
F_2	(1.375,1.15,1)
F_3	(1.375,1.15,1.375)
F_4	(1.375,1.6,1)
F_5	(1.6,1.375,1.6)
F_6	(1.15,1.375,0.7)
F_7	(1.15,1.15,1.6)
F_8	(1.15,1.15,1.375)
F_9	(1.6,1.6,1.375)
F_{10}	(1.15,1.15,1.15)

These factors are to be ranked and the above values corresponding to each factor are represented in matrix form

$$
\begin{array}{c}
\\
\mathbf{F_1}\\ \mathbf{F_2}\\ \mathbf{F_3}\\ \mathbf{F_4}\\ \mathbf{F_5}\\ \mathbf{F_6}\\ \mathbf{F_7}\\ \mathbf{F_8}\\ \mathbf{F_9}\\ \mathbf{F_{10}}
\end{array}
\begin{array}{c}
\begin{array}{ccc} \mathbf{A_1^1} & \mathbf{A_2^3} & \mathbf{A_3^1} \end{array}\\
\begin{pmatrix}
1 & 1 & 1.225\\
1.375 & 1.15 & 1\\
1.375 & 1.15 & 1.375\\
1.375 & 1.6 & 1\\
1.6 & 1.375 & 1.6\\
1.15 & 1.375 & 0.7\\
1.15 & 1.15 & 1.6\\
1.15 & 1.15 & 1.375\\
1.6 & 1.6 & 1.375\\
1.15 & 1.15 & 1.15
\end{pmatrix}
\end{array}
$$

By using the procedure of ranking as discussed by Shazia Rana et.al [19] the factors are ranked

The frequency matrix F representing the ranking of the factors is

$$
\begin{array}{c}
\\
F_1\\ F_2\\ F_3\\ F_4\\ F_5\\ F_6\\ F_7\\ F_8\\ F_9\\ F_{10}
\end{array}
\begin{array}{c}
\begin{array}{cccccccccc} R_1 & R_2 & R_3 & R_4 & R_5 & R_6 & R_7 & R_8 & R_9 & R_{10} \end{array}\\
\begin{pmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 2 & 0\\
0 & 0 & 0 & 0 & 1 & 0 & 2 & 0 & 0 & 0\\
0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0\\
1 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0\\
3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 1\\
1 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 1 & 0 & 1 & 1 & 0 & 0 & 0 & 0\\
3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 1
\end{pmatrix}
\end{array}
$$

Based on the percentage measure of authenticity of ranking of the factors, the following factors F_3, F_4, F_5, F_8, F_9 are considered for analyzing their inter relation using fuzzy cognitive maps (FCM). These factors are taken as C_1, C_2, C_3, C_4, C_5 and by using the procedure of finding the cause and effect relationship, the limit points are presented in Table 4.5.

The core factors considered for the study are

C_1 Customer centric approach

C_2 Product improvisation suiting the contemporary needs

C_3 Widening of the distribution channels

C_4 Employment of self- assessment tools

C_5 Placement of suitable personnel

The connection matrix M representing the degree of association between the core factors and the graphical representation in Fig 4.1 is presented as follows

$$\begin{array}{cc} & \begin{array}{ccccc} C_1 & C_2 & C_3 & C_4 & C_5 \end{array} \\ \begin{array}{c} C_1 \\ C_2 \\ C_3 \\ C_4 \\ C_5 \end{array} & \begin{pmatrix} 0 & 1 & 1 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 & 0 \end{pmatrix} \end{array}$$

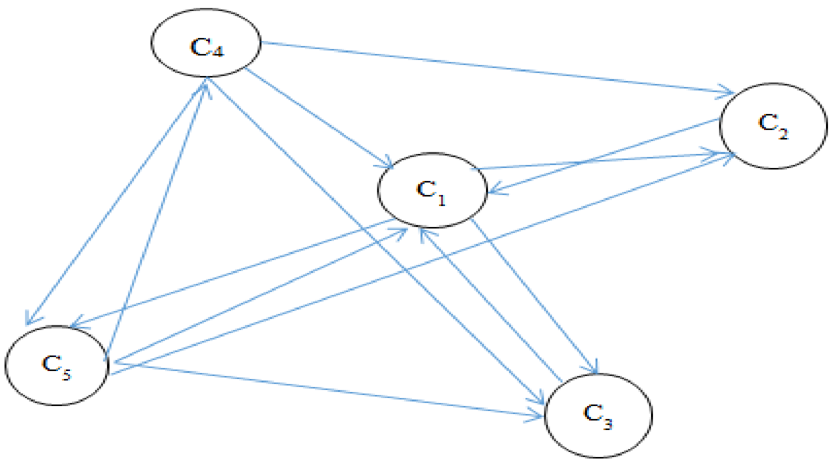

Fig 4.1 Graphical Representation of the association of core factors

Table 4.5. Limit points of the Core Factors

Core Factors in On Position	Limit Points
(10000)	(11111)
(01000)	(11111)
(00100)	(11111)
(00010)	(11111)
(00001)	(11111)

5. Discussion

The integration of FCM with extended concentric plithogenic hypergraphic approach is an innovative effort in minimizing the number of factors considered for studying interrelationship. In the decision making problem as discussed in section 4, the factors which are to be concentrated deeply for sales promotion are considered based on the fulfilment of the three attributes. The consideration of the factors are very specific, but if the company wishes to find the association and impact between the factors using FCM, then the proposed ten factors are to be considered and the computation of the limit points using higher order matrix will be tedious and time consuming, but if the core factors are only considered, the process becomes compatible and the intervention of extended concentric plithogenic hypergraphic approach makes it highly objective and reliable.

6. Conclusion

This research article presents the evolution of the concept of concentric plithogenic hypergraphs based on plithogenic hypersoft sets and plithogenic graphs. This work also introduces extended hypersoft sets and extended concentric plithogenic hypergraphs. The integration of extended concentric plithogenic hypergraphs with fuzzy cognitive maps using linguistic expert's representation is presented with a decision making problem and such integrations are highly advantageous in dealing with several multi attribute factors of study using FCM approach. The linguistic representation of expert's opinion is a new initiative made in this research work and the proposed model of making decisions can be extended using refined plithogenic sets in representations.

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PROMTHEE Plithogenic Pythagorean Hypergraphic Approach in Smart Materials Selection

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Abstract

The production sectors are optimizing its profit with the employment of smart materials at recent times and one of the challenges faced is the selection of smart materials. This article proposes a new decision-making method based on the approach of PROMTHEE plithogenic Pythagorean hypergraph. The efficiency of the proposed method is determined in the selection of smart materials that are significantly utilized in the production processes by the production sectors. In this research work, the decision making on smart material selection environment is characterized by four major influencing factors such as production processes, operational necessities, fiscal constraints and external effects with eighteen sub-factors. The main objective of this research work is to determine the core sub-factors persuading the selection of smart materials based on the five-point scale of expert's opinion represented as Pythagorean neutrosophic number. Out of eighteen sub-factors four significant factors belonging to each of the major factors were identified by this method. The plithogenic hypergraphs with Pythagorean representation are the extension of plithogenic hypergraphs and the novel decision - making method with the integration of PROMTHEE developed in this article will certainly benefit the decision makers on smart material selection. Comparative analysis on the criteria is also made based on Pythagorean neutrosophic sets & neutrosophic sets.

Keywords: Plithogenic sets, Pythagorean Neutrosophic number, Hypergraph, PROMTHEE, Smart materials

1. Introduction

Decision making is a complicated and multi staged process which involves the selection of alternatives subjected to criteria satisfaction. The researchers have proposed various multi-criteria decision making methods (MCDM) such as Analytical hierarchal process, Analytical network process, TOPSIS, ELECTRE, DEMATEL,VIKOR,PROMTHEE and these MCDM methods are applied with reference to the needs. Incomplete descriptive information on the alternatives, multiple qualitative and quantitative criteria are some of the challenges of decision making and it can be handled by outranking methods. The most commonly used outranking method is PROMTHEE developed by Vincke and Brans in 1985. (The Preference Ranking Order Method for Enrichment Evaluation) This method is advantageous as it caters to partial and complete ranking of the alternatives. The compatibility nature of this outranking method has attracted the researchers and it has been extended to design optimal soultions to various MCDM problems and as a strategic tool in planning of natural resources. The method of PROMTHEE was extended to fuzzy PROMTHEE and it is applied in material selection, supply chain management, medical analysis. The subsequent extensions are intutionistic fuzzy PROMTHEE and neutrosophic PROMTHEE in which the criterion alternative satisfaction rate is expressed as intutionistic sets and neutrosophic sets respectively.

Smarandache [1] introduced plithogenic sets as an extension of crisp sets, fuzzy sets, intuitionistic sets and neutrosophic sets. The plithogenic sets are inclusive in nature as it considers the degree of appurtenance and contradiction with respect to the attributes. The process of making feasible decisions encompasses different entities and varied phases. The milieu of decision making is dependent on various factors and the highly significant attributes of the decision-making elements. Plithogenic sets are applied in multi attribute decision making in various perspectives. Abdel Basset et al [2] constructed a new plithogenic decision-making model for evaluating the medical care systems, Grida et al [3] measured the performance of IoT based supply chain. Quek et al [4] used the measures of entropy in the model framed for multi- attribute decision making. Abdel Basset et al [5] developed a combined multi criteria decision-making model to select the manufacturing industries also formulated a hybrid decision making method with deployment function to select the sustainability supply chain metrics. An integrated multi –criteria decision making method was also developed by Abdel Basset et al [6] to evaluate the financial performance of the manufacturing sectors. Smarandache [7] introduced the concept of plithogenic hypersoft sets by extending soft sets to hypersoft sets. Muhammad et al [8] proposed a new multi-criteria decision-making model based on the Plithogenic hypersoft sets. Shazia et al [9] discussed the application of plithogenic whole hypersoft sets in mullti attribute decision making by using the approach of frequency matrix and also developed the approach of Plithogenic Subjective Hyper-Super-Soft Matrices. Nivetha and Smarandache [10] discussed combined plithogenic hypersoft sets and its application in multi attribute decision making. Nivetha and Smarandache [11] developed the approach of concentric plithogenic hypergraphic approach based on plithogenic hypersoft sets and examined its efficiency in multi attribute decision making. Smarandache [12] coined plithogenic n super hypergraph by extending the concepts of hypergraph and n-super hypergraph. Nivetha and Smarandache [13] developed a multi attribute decision making model based on plithogenic n super hypergraph. The multi attribute plithogenic decision making models are highly compatible as it facilitates in making optimal decisions with the considerations of dominant attributes.

In the plithogenic decision making methods the expert's opinion is given significance and it is represented as neutrosophic numbers. The plithogenic aggregation operators are used to determine the aggregate expert's opinion. Jansi et al [14] generalized neutrosophic sets by the concept of Pythagorean neutrosophic set with the condition of $(\alpha(x))^2 + (\beta(x))^2 + (\gamma(x))^2 \leq 2$, where $\alpha(x)$, $\beta(x)$ and $\gamma(x)$ represent the truth, indeterminacy and falsity degrees. The truth and falsity neutrosophic components are dependent in pythagorean neutrosophic sets and the general dependence degree of all the three components is taken as ½ = 0.5. Jansi et al [14] developed the correlation measure between pythagorean neutrosophic sets and sternly substantiated the need of pythagorean neutrosophic sets in the field of medical analysis of diseases with symptoms . Yager [15] introduced a new class of pythagorean sets, which are discussed in different dimensions and are extensively used in varied decision-making scenario.The arguments in favour of pythagorean neutrosophic sets are taken into account and this has motivated to propose multi attribute decision making method with pythagorean plithogenic hypergraphic approach. The plithogenic hypergraph comprises of plithogenic envelopes with generalized representation of membership values, but it is confined to Pythagorean sets in plithogenic Pythagorean hypergraphs in the proposed approach, also the neutrosophic expert's opinion is more comprehensive and if the expert's opinion is of Pythagorean in nature, then this developed approach shall be adopted as the Pythagorean neutrosophic sets are the special case of neutrosophic sets. The decision-making method developed in this paper proposes a two-step processes. In the first phase the criteria for smart material selection is determined based on the pythagorean neutrosophic representation of expert's opinion. In the second phase the alternatives are ranked based on the any of the multi criteria decision making methods with criteria contradiction degree. Lazim Abdullah and Pinxin Goh [16] used the pythagorean representation of expert's opinion to select the feasible solid waste management methods. Carlos Granados [17] presented on Pythagorean Neutrosophic Pre-Open Sets. Ajay and Chellamani [18] discussed about Pythagorean Neutrosophic Fuzzy Graphs. Jansi and Mohana [19] developed the concepts of Pythagorean neutrosophic Subring of a ring.According to Pythagorean researchers, the representation of expert's opinion as pythagorean sets are highly compatible in making decisions. In this research work the method of PROMTHEE is used for ranking the alternatives in the second phase. Also based on the defuzzification method of neutrosophic sets proposed by Solairaju and Shahjahan [17] the method of defuzzifing pythagorean neutrosophic sets is suggested in this article. The expert's opinion based on the representations of pythagorean neutrosophic and neutrosophic expert's opinion with plithogenic operators in criterion selection is compared.

The structure of the remaining paper is as follows: the preliminaries are presented in section 2; the two step phases of multi criterion approach is developed in section 3; the proposed method is applied in smart materials selection in section 4; the results are discussed in section 5 and the last section concludes the work.

2. Preliminaries

This section presents the basic definitions required for the research work.

Definition 2.1 [14]

A Pythagorean set P is of the form $\{(x, A(x), C(x)): x \in X\}, A(x): X \to [0,1], C(x): X \to [0,1]$, where X is the universal set , A(x) & C(x) are the membership & non-membership degrees for each $x \in X$ satisfying the condition of $(A(x))^2 + (C(x))^2 \leq 1$.

Definition 2.2 [14]

A neutrosophic set N is of the form $\{(x, A(x), B(x), C(x)): x \in X\}, A(x): X \to [0,1], B(x): X \to [0,1]$, $C(x): X \to [0,1]$where X is the universal set , A(x) ,B(x)& C(x) are the degrees of membership, indeterminacy and non-membership for each $x \in X$.

Definition 2.3 [14]

A pythagorean neutrosophic set P_N is of the form $\{(x, \alpha(x), \gamma(x), \beta(x)): x \in X\}, \alpha(x): X \to [0,1], \beta(x): X \to [0,1]$, $\gamma(x): X \to [0,1]$where X is the universal set , α(x) , γ (x)& β(x) are the degrees of membership, indeterminacy and non-membership for each $x \in X$ satisfying the condition of $(\alpha(x))^2 + (\gamma(x))^2 + (\beta(x))^2 \leq 2$. Also if $\alpha(x)$ and $\beta(x)$ (where $0 \leq \alpha(x) + \beta(x) \leq 1$) are 100 % dependent then $0 \leq (\alpha(x))^2 + (\beta(x))^2 \leq 1$. If $\gamma(x)$ is independent 100% from $\alpha(x)$ and $\beta(x)$ with $0 \leq \alpha(x) + \gamma(x) + \beta(x) \leq 2$, then $(\alpha(x))^2 + (\gamma(x))^2 + (\beta(x))^2 \leq 2$.

Definition 2.4

A pythagorean neutrosophic set is a special case of neutrosophic set, based on the defuzzification method of Solairaju and Shahjahan [17] the Pythagorean neutrosophic set is transformed to Pythagorean set of the form $<\alpha , f>$, where

$$f = \begin{cases} \gamma^2 + \frac{[1-\gamma^2-\beta^2][1-\gamma^2]}{[\gamma^2+\beta^2]} & if\ \gamma = 0 \\ \gamma^2 + \frac{[1-\gamma^2-\beta^2]\gamma^2]}{[\gamma^2+\beta^2]} & if\ 0 < \gamma^2 \leq 0.5 \\ \gamma^2 + [1-\gamma^2-\beta^2]\left[0.5 + \frac{\gamma^2-0.5}{\gamma^2+\beta^2}\right] & if\ 0.5 < \gamma^2 \leq 1 \end{cases}$$

3. Methodology

This section comprises of the steps involved in the proposed method. The method proposed consists of two phases in which pythagorean plithogenic approach is used in criterion selection in the first phase and in the later phase the method of PROMTHEE is used to rank the alternatives. Preference Ranking Organization method for Enrichment of Evaluations (PROMTHEE) developed Jean-Pierre Brans [18] to make optimal ranking of alternatives. This method is predominantly used in several decision-making environments on resources management in various dimensions. The method of PROMTHEE is extended to fuzzy PROMTHEE by combining the concept of fuzzy in handling uncertain aspects. Fuzzy PROMTHEE is applied to make feasible decisions on supply chain management. Intuitionistic and neutrosophic approaches in PROMTHEE method are the extensions of fuzzy PROMTHEE [19-21]. The representations of the expert's opinion are varied in each of the approaches and at some instances, interval-valued representations are also used. The Pythagorean neutrosophic sets integrated with PROMTHEE is used in making decisions on solid waste management. The linguistic variables stating the expert's opinion are quantified using Pythagorean sets. But to the best of the author's knowledge, plithogenic Pythagorean hypergraph approach is not so far used to make criterion selection. Also the representation of the sub-factors as plithogenic hypergraphs and the selection of the core sub-factors using aggregate plithogenic operators is the noteworthy phenomenon of the method.

Step 1: The decision-making problem is formulated with the selection of criteria and sub-criteria factors based on the expert's opinion ($E_1, E_2, .. E_n$) together with Pythagorean neutrosophic linguistic rating scale.

Step 2: The criterion selection is made by the plithogenic hypergraph approach with the contradiction degree of the sub criteria belonging to each criterion and plithogenic aggregate operators. E1 $\wedge_p$ E2 = $(a_1 \wedge_F a_2, \frac{1}{2}((b_1 \wedge_F b_2) + (b_1 \vee_F b_2)), c_1 \vee_F c_1)$. Based on the defuzzified scores the core criteria are selected.

Step 3: The decision making matrix consisting of the w alternatives together with the contradiction degree and the linguistic expert's opinion on g criteria satisfaction is constructed

Step 4: The aggregate decision-making matrix is determined by using the aggregate plithogenic operators.

Step 5: The normalized decision matrix N = [z_{bh}] is obtained

Where $z_{bh} = (\frac{\alpha_{bh}}{\gamma_h}, \frac{\beta_{bh}}{\gamma_h}, \frac{\gamma_{bh}}{\gamma_h})$ γ_h = max (γ_{bh}) [Benefit criteria]

$z_{bh} = (\frac{\alpha_h}{\alpha_{bh}}, \frac{\alpha_h}{\beta_{bh}}, \frac{\alpha_h}{\gamma_{bh}})$ α_h = min ($\gamma\alpha_{bh}$) [Cost criteria]

Step 6: The weighted normalized matrix S = [j_{bh}] is obtained

Where $j_{bh} = z_{bh} * w_h$ b = 1,2,..w, h = 1,2,.. g

Step 7 : The preference function P (w_i, w_j) is defined as

P (w_i, w_j) = 0 if d (w_i, w_j) ≤ 0 [d (w_i, w_j) denotes the pairwise comparison of the alternatives]

= d (w_i, w_j) if d (w_i, w_j) > 0

Step 8 : The positive $O^+(w_k)$ and negative $O^-(w_k)$ outranking flows is calculated from the aggregate preference values.

Step 9: The net outranking flow N (w_k) = $O^+(w_k)$ - $O^-(w_k)$ is calculated and the alternatives are ranked based on the values

4. Smart Material Selection using the Proposed Method

In this section, the proposed method is applied in criteria selection at first phase and in ranking of the alternatives in the second phase.

Phase I : Criteria Selection using Plithogenic Pythagorean Hypergraphic representation

The selection of the smart materials is based on the four major influencing factors and each factor has certain sub-factors. The criteria is selected from each of the sub-factors belonging to the core factors as represented in Table 4.1 and the expert's linguistic rating scale is given in Table 4.2.

- Production Processes
- Operational Necessities
- Fiscal Constraints
- External Effects

Table 4.1 List of Criteria

CRITERIA	SUB-CRITERIA

Production Processes	P1 Flexibility P2 Adaptability P3 Reliability P4 Ductility P5 Malleability
Operational Necessities	O1 Resistivity O2 Solidity O3 Durability O4 Conductivity
Fiscal Constraints	F1Material Expenditure F2Manpower F3Maintenance F4Machinery F5Screening
External Effects	E1 Compatibility to production environment E2 Affordability by the production sector E3Nature of the material E4Deterioration rate

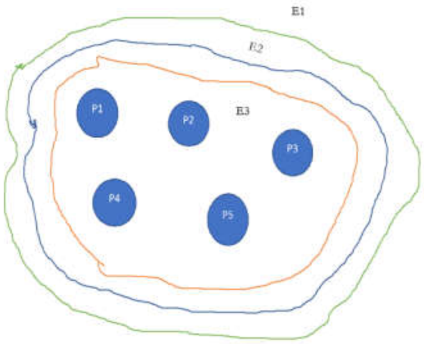
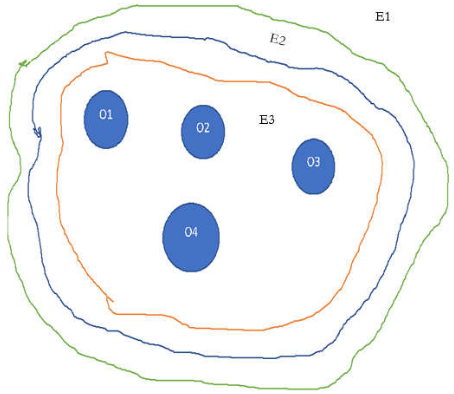
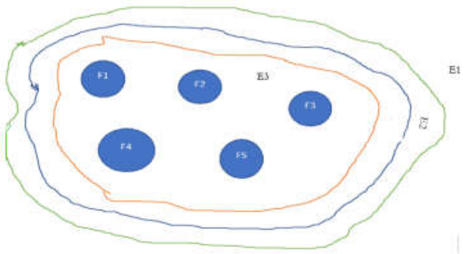
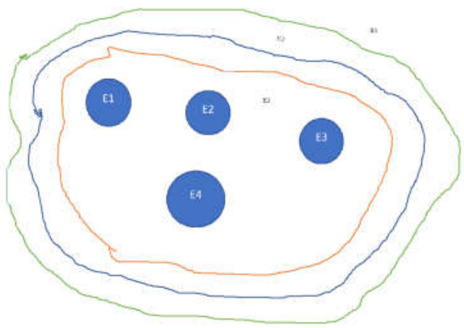
Table 4.2 Linguistic Criteria Significant Rating Scale

Linguistic Variable	**Neutrosophic Representation**
Very Highly Significant (HS)	(0.9,0.1,0.1)
Highly Significant (S)	(0.7,0.1,0.2)
Moderately Significant (MS)	(0.4,0.2,0.8)
Highly Insignificant (IS)	(0.1,0.1,0.9)
Very Highly Insignificant (HIS)	(0,0,0)

The plithogenic hypergraphic representation based on expert's opinion for each of the major criteria is presented in Table 4.3. In the graphical representation, the sub criteria of each criterion is considered as the vertices and each of the hyperedges represents the expert's opinion. The sub criterion weight is computed using plithogenic operators.

Table 4.3 Plithogenic Hypergraphic Representation of Major Criteria

Plithogenic Hypergraphic Representation	**C**	**E1**	**E2**	**E3**	**E1** $\wedge_p$ **E2** $\wedge_p$ **E3**	**Crisp Score**
Production Processes						
	P1	VHS	VHS	HS	(0.997,0.1,0.002)	**0.994**
	P2	HS	VHS	VHS	(0.992,0.1,0.002)	0.984
	P3	MS	MS	HS	(0.892,0.15,0.128)	0.618
	P4	VHS	MS	HS	(0.982,0.125,0.016)	0.964
	P5	HS	VHS	MS	(0.982,0.15,0.016)	0.964

	**P1 > P2 > P3 & P4 > P3**					
Operational Necessities						
	**O1**	HS	VHS	VHS	(0.997,0.4995,0.002)	**0.994**
	O2	VHS	HS	VHS	(0.991,0.495,0.004)	0.984
	O3	HS	MS	HS	(0.838,0.491,0.144)	0.789
	O4	VHS	HS	VHS	(0.997,0.4995,0.002)	0.964
	O1 > O2 > O4 > O3					
Fiscal Constraints						
	**F1**	HS	VHS	VHS	(0.997,0.4995,0.002)	**0.99**
	F2	VHS	MS	HS	(0.982,0.499,0.016)	0.796
	F3	VHS	HS	VHS	(0.997,0.4995,0.002)	0.984
	F4	VHS	VHS	MS	(0.994,0.501,0.008)	0.796
	F5	VHS	HS	HS	(0.991,0.4975,0.004)	0.964
	F1 > F3 > F5 > F3 & F4					
External Effects						
	**E1**	**HS**	**HS**	**VHS**	(0.991,0.4975,0.004)	**0.994**
	E2	**MS**	**HS**	**HS**	(0.946,0.489,0.032)	**0.984**
	E3	**VHS**	**MS**	**HS**	(0.982,0.499,0.016)	**0.796**
	E4	**HS**	**MS**	**VHS**	(0.991,0.4975,0.004)	**0.964**
	E1 > E2 >E4 >E3					

Phase II – Selection of Alternatives using Plithogenic PROMTHEE

Table 4.4 Linguistic Rating Scale by the Experts for Alternatives & Criteria

Criterion Satisfaction of the alternatives Linguistic rate by the Experts		**Criterion vitality linguistic rating scale by the Experts**	
Very Highly Satisfied (VHS)	(0.9,0.1,0.1)	Very Highly Inessential (VHIE)	(0,0,0)

Highly Satisfied (HS)	(0.8,0.2,01)	Highly Inessential (HIE)	(0.1,02,0.9)
Moderately Satisfied (MS)	(0.5,0.1,03)	Moderately Essential (ME)	(0.5,0.3,0.7)
Highly Dissatisfied (HD)	(0.1,0.1,0.9)	Highly Essential (HE)	(0.8,0.1,0.2)
Very Highly Dissatisfied (VHD)	(0,0,0)	Very Highly Essential (VHE)	(0.9,01,0.1)

Let S1, S2, S3, S4, S5 be the smart materials that are taken as the alternatives and the criterion satisfaction of the alternatives by the expert's opinon are presented in Table.4.5.

Table 4.5 Decision Making Matrix based on Expert's opinion

	Linguistic Weight	ME	HE	HE	VHE	HE	HE	VHE	HE	HE	ME	HE	VHE
Smart Materials (Alternatives)	**Contradiction Degree**	**E1**				**E2**				**E3**			
		C1	**C2**	**C3**	**C4**	**C1**	**C2**	**C3**	**C4**	**C1**	**C2**	**C3**	**C4**
S1	0	HS	MS	HS	HD	MS	HD	MS	HS	HS	HS	MS	HD
S2	0.2	MS	HS	HS	HS	MS	MS	MS	HS	MS	VHS	HD	MS
S3	0.4	VHS	HD	HS	VHS	HS	HS	HD	HS	MS	VHS	HS	HS
S4	0.6	HS	VHS	MS	VHS	HS	HS	HD	VHS	HD	HD	HS	HS
S5	0.8	HD	HS	HS	MS	VHS	HS	VHS	MS	HS	MS	HS	HS

Table 4.6 Aggregated Decision matrix

Smart Materials (Alternatives)	**Contradiction Degree**	**C1**	**C2**	**C3**	**C4**
S1	0	(0.32,0.205,0.433)	(0.04,0.17,0.937)	(0.2,0.14,0.559)	(0.008,0.14,0.991)
S2	0.2	(0.275,0.118,0.504)	(0.55,0176,0.716)	(0.15,0.14,0.79)	(0.45,0.20,0.34)
S3	0.4	(0.61,0.19,0.25)	(0.75,0.123,0.22)	(0.7,0.16,022)	(0.85,0.25,0.09)
S4	0.6	(0.72,0.18,0.16)	(0.56,0.14,0.41)	(0.634,0.12,0.26)	(0.86,0.24,0.07)
S5	0.8	(0.87,0.103,0.086)	(0.85,0.13,0.078)	(0.90,0.19,0.05)	(0..82,0.11,0.087)

Table 4.7 Final criterion weights

C1	**C2**	**C3**	**C4**
(0.1,0.233,0.9)	(0.8,0.1,0.2)	(0.8,0.1,0.2)	(0.8,0.1,0.2)

Table 4.8 Normalized decision making matrix

Smart Materials (Alternatives)	**C1**	**C2**	**C3**	**C4**
S1	(0.635,0.407,0.859)	(0.042,0.1814,1)	(0.253,0.177,0.707)	(0.008,0.141,1)
S2	(0.546,0.234,1)	(0.586,0.187,0.764)	(0.189,0.177,1)	(0.45,0.201,0.34)
S3	(1.21,0.37,0.49)	(0.8,0.13,0.234)	(0.88,0.202,0.278)	(0.85,0.25,0.09)
S4	(1.42,0.35,0.31)	(0.597,0.149,0.437)	(0.80,0.151,0.329)	(0.86,0.24,0.07)
S5	(1.72,0.204,0.171)	(0.91,0.139,0.083)	(1.139,0.241,0.063)	(0.82,0.11,0.08)

Table 4.9 Weighted normalized decision making matrix

Smart Materials (Alternatives)	C1	C2	C3	C4
S1	(0.063,0.093,0.773)	(0.034,0.018,0.2)	(0.202,0.017,0.142)	(0.0065,0.0141,0.2)
S2	(0.054,0.05,0.9)	(0.469,0.018,0.152)	(0.15,0.017,0.2)	(0.36,0.02,0.068)
S3	(0.12,0.086,0.44)	(0.64,0.013,0.047)	(0.71,0.02,0.055)	(0.686,0.03,0.018)
S4	(0.14,0.08,0.285)	(0.478,0.015,0.087)	(0.64,0.02,0.065)	(0.69,0.02,0.014)
S5	(0.172,0.05,0.153)	(0.725,0.0134,0.0167)	(0.91,0.024,0.013)	(0.662,0.011,0.0175)

Table 4.10. Ranking of alternatives

Alternatives	Positive Outranking Flow	Negative Outranking Flow	Net Outranking Flow	Ranking of the Alternatives
S1	0.0145	0.9105	-0.896	5
S2	0.00775	0.70425	-0.6965	4
S3	0.63425	0.07925	0.555	2
S4	0.499	0.1315	0.3675	3
S5	0.74625	0.07625	0.67	1

4. Results and Discussion

Among thirteen criteria discussed under four major core factors, the core criteria are determined using the plithgothenic hypergraphic approach with Pythagorean representations. The expert's aggregate opinion is calculated and based on the final crisp score value the following criteria namely Flexibility, Resistivity, Material Expenditure and Compatibility to Production Environment are designated as the core criteria. The alternatives are ranked based on the net flow values. Table is constructed using the expert's opinion on the criterion satisfaction of the alternatives and the vitality of the criteria in smart materials selection. The contradiction degree of the alternatives is taken into account as each of the materials varies from one another in its nature and properties. The alternatives S1,S2,S3,,S5 are ranked as S5 >S3>S4>S2>S1

Conclusion

A new decision-making two phase method with PROMTHEE integrated with plithogenic hypergraphic approach is developed in this article. The efficiency of the proposed method is examined in smart material selection. The Pythagorean neutrosophic representation of the expert's opinion and its defuzzification approach used in this method is a novel initiative to explore the possible ways of quantifying the linguistic rating scales. Plithogenic PROMTHEE method with the contradiction degree of the alternatives is also an added novelty to the proposed decision-making method. The two-phase method of making decisions can further be validated by using other multi criteria decision-making methods in phase two to rank the alternatives, also comparative analysis of the efficiency of integrated plithogenic MCDM methods can be made. This method will definitely benefit the decision makers to make compatible decisions on the alternatives.

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MISCELLANEA

Determining the Duration of R&D Processes through Monte Carlo Simulation

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Monika Moga, Gavrilă Calefariu, Florentin Smarandache, Flavius Aurelian Sârbu, Laura Bogdan (2014). Determining the Duration of R&D Processes through Monte Carlo Simulation. *Applied Mechanics and Materials* 657, 886-890. DOI: 10.4028/www.scientific.net/AMM.657.886

Abstract. The research and development (R&D) processes influence the economic development of a company, because in an industry that is changing fast, firms must continually revise their projects and range of products. Therefore in order to determine the specific costs of research and development activity we have to highlight the efforts and effects of these activities and to calculate some indicators of economic efficiency. In the cost calculation process in R & D we have emphasized the identification of the components of cost and the duration of the R & D processes, as a component of the R&D cost, that is why as a new method we used Monte Carlo simulation. The novelty of the paper is that it focuses on determining components of cost and the duration of the R & D processes in its cost calculation procedure. The originality of this work is the use of Monte Carlo simulation to determine the average length of producing a new product.

Keywords: R&D, R&D cost calculation, R&D process duration, and Monte Carlo simulation.

Introduction

After [1] the R & D function of a firm can be defined as a whole of activities of the industrial organization by which it is conceived and implemented its technical and scientific progress, considering the minimization of deviations between targets and results on the market. Determining the specific costs of R&D activity consist of highlighting the efforts and effects of these activities and the calculating some indicators of economic efficiency. Discrete event simulation is used to simulate situations in which there are identified different events that change the status of the studied system. Events are discrete because it is believed that between two events, the system state does not change. Monte Carlo method is an important component of discrete event simulation to generate random intervals between two successive events, duration of service, etc. [2]. Time points of the events are random. Due to the dynamic nature of the discrete event simulation models, the simulation time is necessary to know the value of the simulated time and it is also a need for a simulated progress of time from one value to another. The purpose and applicability of the simulation is not limited only to assist managerial decision-making processes. Simulation has important uses in: computer aided design, virtual reality systems, for computer game industry, training of professionals (pilots, surgeons) or military applications [3]. Among the objectives of the

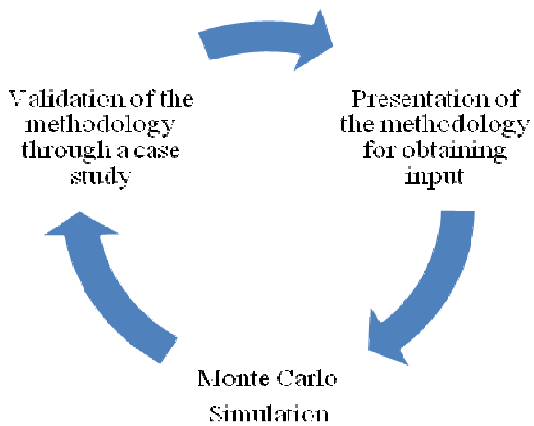

Fig. 1. Stages of the research

paper we list the better sizing of the research team and needs, presentation of the methodology for obtaining the input data, which will be introduced in the Monte-Carlo model, and also validating the methodology through a study case.

Research Stages

Our research referring to the determination of the cost components and duration of a research consists of the stages presented in Fig. 1.

Presentation of the Methodology for Obtaining Input Data. Because of the importance of creativity in the process of R&D appeared the issue of its stimulation. The number practical methods existing in the world today, used to stimulate creativity is of the hundreds. The general classification divides two important methods of stimulating creativity: intuitive methods and analytical methods. Intuitive methods are called so because it is based on the use of intuitive thinking, particular focus on the imagination, freed from the constraints of reality.

The main intuitive methods are: *Brainstorming* can be defined as a way to get in a short time, a large number of ideas from a group of people. *Sinectica* is the rival of brainstorming. *Thinking Hats method*, is based on the interpretation of roles depending on the color of the chosen hat. *Delphi method,* the goal is to obtain guidance, forecasts and solutions to complex problems using a group of experts. *Mind-mapping method* involves building a graphical diagram suggesting how ideas arise from each other [4]. Analytical methods are based on the predominant use of logical thinking in order to stimulate the process of combining the real plan using information directly related to the problem. The main analytical methods are: *Osborn's interrogative method* is a list of 60 questions grouped into 9 categories. *Attribute listing technique*: to get new ideas, identify and list as many of the attributes of a problem, then work, in turn, to each of them. *Morphological analysis*, its principle is to describe analytically and systematically, all solutions of the problem, than choose the best. *Multiple criteria analysis* has 5 stages: establish criteria, determination of the weight of each criterion, identifying all variants, granting a grade N, calculate the product of the N notes and the weighting coefficients. *The "ETECTRE" method*, aims to examine a number (m) of possible options in terms of (n) selection criteria through a simple procedure for estimating the "effect". *PINDAR technique*, is a successful combination of morphological analysis with the criteria analysis

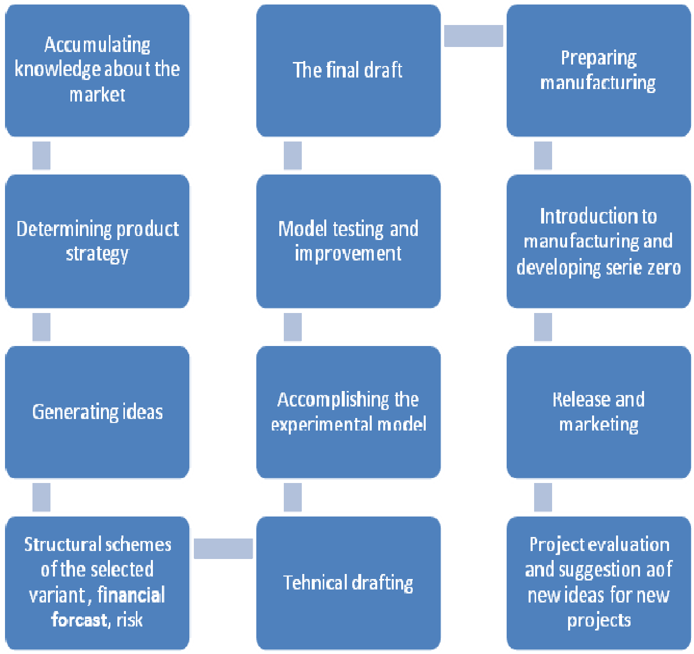

Fig. 2. Stages of product developing

The stages of product development are presented in Fig. 2 after [5].

Cost calculation is made for the first 8 stages of development, including the final draft. Sometimes, for complex projects, after the first eight stages, there is still a prototype development phase. For the input stage in current manufacture, cost issues will be treated in a forthcoming paper. Direct costs are in direct contact with the innovation and development of a new product. Indirect costs are general operating cost of the R & D department. Their highlighting separate from the direct makes sense, especially if the department runs several projects simultaneously.

Table 1. The direct cost of the R&D process

Cost of determining the parameters of innovation	$C_{PI} = E_{MR}+P_P+E_{KH}+E_S$ [unit/project],(1)	E_{MR} is the expenditure with market research; E_P is the expenditure with patents; E_{KH} is the expenditure with the Know –how; E_s is the expenditure with the specialized software.
Cost of materials	$C_m = \sum_{k=1}^{N} n_{ck} \times p_{mk}$ [unit/project],(2)	N is the number of components and subassemblies; n_{ck} is the norm of the material consumption for reference k; P_{mk} is the unit price of the material or of the subassemblies.
Cost of technological acquisition – C_{TA}	$C_{TA} = C_E+C_{TE}+C_{DCT}$ [unit/project],(3)	C_{UT} is the expenditure with equipment; C_{TE} is the expenditure with technological equipment; C_{CDT} is the expenditure with the special dispositive checker tools.
Cost of utilities	$C_U = C_E+C_{TF}+C_{IW}$ [unit/project],(4)	C_E is the expenditure with electricity; C_{TF} is the expenditure with technological fuel; C_{IW} is the expenditure with industrial water used in the manufacture, testing and improvement of the experimental model.
Total costs of directly productive staff	$C_{emp/year} = N_s (S_{d/month}+I_{S/month})T$ [unit/project],(5)	Direct productive staff means who is involved in the 8 stages of development. N_S is the average number of employees involved in the 8 stages; S_d is the average gross hourly salary; I_S is the average monthly charges; T is the total project time in months. This duration is determined by the Monte Carlo method.
Total direct costs	Summing the above elements.	

Table 2. The indirect cost of the R&D process

Expenses for maintenance and repair		$C_{MR/year}$ [unit/project]
Material expenses DCT normal and special		$C_{DCT/year}$ [unit/project]
Energy expenditure, other than that used in the process, including lighting departments, offices etc.	$C_{EE2/year} = (N_{TSP} - N_{TMS}) \cdot p_{UE}\, T$ [unit/project], (6)	N_{TPS} [kW] is the total power used by the entire system of production; N_{TMS} [kW] is the total power used by the manufacturing system; T is the effective time of functioning in one, two or three shifts; p_{UE} [mu / kWh] is the unit price of electricity.
Fuel costs for heating and for preparing domestic hot water	$C_{GM2/year} = (N_{GMH} + N_{GMW}) \cdot p_{UG}\, T$ [unit/project], (7)	N_{GMH} [m^3] is the volume of gas used for heating purposes; N_{GMW} [m^3] is the volume of gas consumed for hot water; p_{UG} [unit/m^3] is the unit price of natural gas.
Annual expenses for amortization of fixed assets	$C_{A/year} = \sum_{i=1}^{q} \frac{C_{MF\,i}}{T_{A\,i}}$ [unit/project], (8)	q is the number of fixed assets of the company; C_{MFi} is the expense recorded with the i [unit] asset purchase including transportation charges, installation, commissioning, and so on; T_{Ai} is the normal duration of operation of the i[years] asset, based on that given in the catalog of normal duration of fixed assets.
Expenses with the indirectly productive employees	C_{eIP} [unit/project]	Calculated similarly as those with the direct productive personnel, taking into consideration the number of people, average salary, and duration.
Total annual indirect costs	Summing the above elements.	

Monte Carlo Simulation. The relative probability and the cumulative probability was calculated according. Than in a two axes coordinate system we presented the durations and the cumulative probabilities. On each interval associated with a duration a vertical bar was built having an equal height to the corresponding cumulative probability of that duration. Next it was generated a set of N random numbers uniformly distributed in the interval [0, 1] using a random number generator. Then these numbers were represented by a point on the horizontal axis, from that point a parallel was led to the vertical axis until it meets the vertical front and the length of the base bar was read. Finally, the average of duration was calculated, its standard deviation, and the confidence interval of the mean.

The Validation of the Methodology through a Case Study. This paper serves on determining the financial projections from step 4 (Fig. 2). It should be also noted that the Monte Carlo method can be used for determining total duration of the development process based on accumulated experiments from firms and other developments that they made.

Table 3. The duration of the operations and the number of experiments

The duration of the operation (xi) months	The number of the experiments(ni)
3.5	1
4	3
4.5	3
5	4
6.5	5
7	2

Table 4. The relative and the cumulated probability

Relative Prob.	Cumulated Prob.
0.06	0.06
0.17	0.22
0.17	0.39
0.22	0.61
0.28	0.89
0.11	1.00

Next it was determined for a company with Monte Carlo simulation the total duration of the development process based on accumulated experiments from the company of middle size from the domain of rubber roller construction with relative low complexity products with a medium size, owning 30% of the local market with the intention of increasing its market share. It should be noted that these values are for incremental innovation. For determining the time required there were carried 18 experiments with the results presented in Table 3.

Because the determination of the average duration with a good accuracy requires a large number of experiments, which are time, energy, materials and not least human resource consuming, we use the Monte Carlo method that can generate simulated observations.

1. The relative and the cumulative probability was calculated:
2. In a two axes coordinate system we presented vertically the durations and horizontally the cumulative probabilities. On each interval associated with a duration a vertical bar was built having the height of the corresponding cumulative probability of that duration.

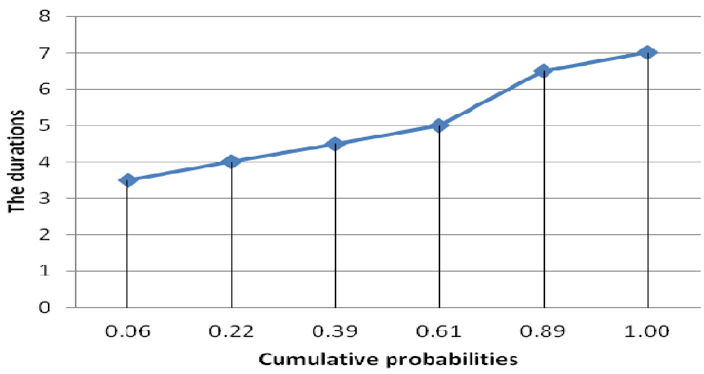

Fig. 3. Representation of the durations and the cumulative probabilities

3. After that it was generated a set of 50 N random numbers distributed in the interval [0, 1] using a random number generator.
4. Each generated number was represented by a point on the horizontal axis, from that point a parallel was led to the vertical axis until it meets the vertical front and the length of the base bar was read. This resulted a series of N simulated durations as in [6,7].
5. Finally, it was calculated the average of duration m = 5.13 months and its standard deviation = 5.09 and the confidence interval of the mean, which is about 5.2 months.

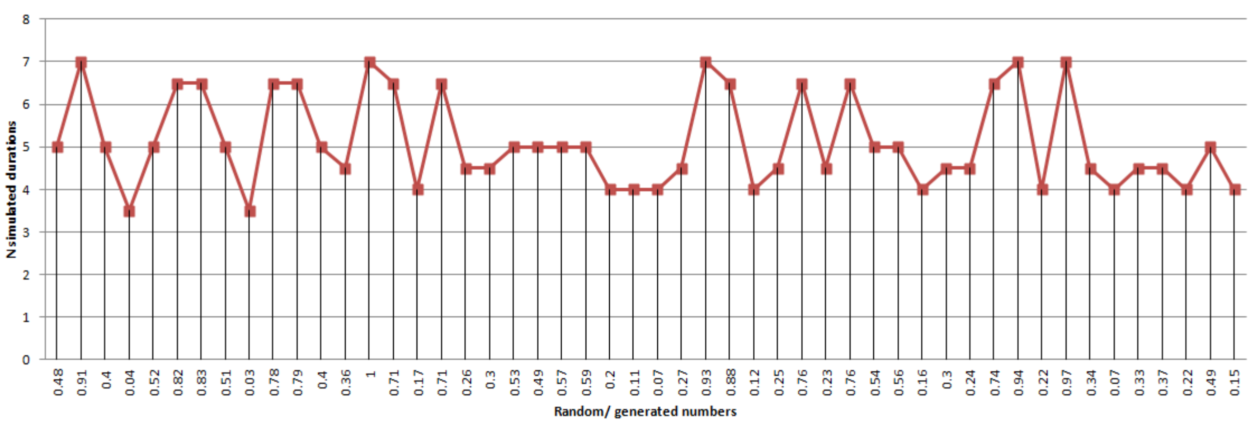

Fig. 4. Representation of the random numbers and N simulated durations

Research Method

In the process of the cost calculation in R & D, the presented study was focused on determining components of cost and the duration of the R & D processes, as an important component of the R&D cost. A new method Monte Carlo simulation was used. Determination of average duration with good accuracy requires a large number of experiments that consumes much time, energy, materials and human resources. Therefore the Monte Carlo method was used for generate simulated observations with the help of cumulative distribution of the duration of realizing a new product and a generator of random numbers uniformly distributed in [0, 1].

Conclusions

The major novelty of the work is that it emphasizes the components of cost and also the duration of R&D processes. The calculation of costs was realized through Monte Carlo simulation. This method can be defined as a random variable modeling method for the calculation of the characteristics of their distributions. Thus by using the Monte Carlo simulation it could be forecast the research cost and the duration of achievement. The validation of the methodology was realized through a case study. In a future work it is possible to discuss a comparative study, applying full method. Then, for different industries or for groups of companies producing similar goods, it is possible to obtain forecast of R&D costs and time. Furthermore, by using the fuzzy matrices it is possible to determine the optimum parameters in R&D.

Acknowledgment

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Towards a Practical Communication Intervention

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Abstract

The study starts from evidence that several communication acts fail, but nobody is called to intervene and nobody thinks of intervening. Examining diffe-rent branches (specialties) of the communication discipline and focusing on four possible practices, by comparison, differentiation, collating and corroboration, the current study brings arguments for a branch of the communication discipline that has as unique practical aim the communicational intervention, the practical, direct and strict application of communication research. Communication, as disci-pline, must create an instrument of intervention. The discipline which studies communication globally (General Communication Science) has developed a strong component of theoretical and practical research of communication phenomena (Applied Communication Research), and within a niche theory (Grounded Prac-tical Theory – Robert T. Craig & Karen Tracy, 1995) took incidentally into account the direct, practical application of communication research. We propose Practical Communication Intervention, as speciality of communication as an academic discipline. Practical Communication Intervention must be a field specialty in the universe of communication.

Keywords: communication, communication research, social intervention, communicational intervention, Practical Communication Intervention (PCI)

Introduction

In general, any intervention in human life has, first of all, a sociological character. Taking into account the fact that one of the main sources of Communication as discipline is constituted by Sociology;

we contend that such an in-strument must be a communicational instrument which can value the sociological experience of social intervention. Wolfgang Donsbach was the president of the International Communication Association (the most important world organization of specialists in the domain of communication). In a study with major impact on the scientific community, “The Identity of Communication Research” (2006), Donsbach shows that: “Communication research has the potential and the duty to focus on research agendas that can help societies and help people to communicate better” (Donsbach, 2006). The specialty from the universe of the discipline of communication that deals with *communication research* (having as a main aim the research of concrete communication phenomena) is Applied Communication Research. Essentially, Applied Communication Research, according to its title, is a practice of *research*, a practice of *inquiry*, and not *application*, or *intervention*. Obviously, *communication research can help societies and help people to communicate better*, but the help is a theoretical one. The practice that can be induced by *communication research* is one of lecture and enforcement of the theoretical human competence. Individuals read theories that are elaborated within *commu-nication research*; they become more cultured and trained, they improve their theoretical knowledge of communication, they impregnate communication with the know-how resulted from *communication research* and consequently, they will communicate better. Nevertheless, *communication research* has an indirect in-fluence on the individual practice of communication. Wolfgang Donsbach ex-plains, further on: “that is, to make up their minds on any issue on a sound basis of evidence and as little influenced by other people or institutions as possible – be it great persuaders in personal communication, the news media, or political or economic powers, in either a national or global context” (Donsbach, 2006). *Communication research* really illuminates and educates the minds, and indi-viduals are better informed and more capable of carrying out better and more successful communication. So, *communication research* indirectly intervenes in the daily communication practice; it intervenes only as preliminary preparation. In fact, pertaining to Applied Communication Research, *communication research*, as it is conceived by Professor Donsbach, remains a component of theoretical practice. We believe that communication must directly intervene so that it makes people communicate better; it must directly intervene in the improvement of basic communication competencies. Consequently, we contend that General Communication Science must develop one component dedicated to *direct commu-nicational intervention*. Anyway, the discipline which studies communication (we will call it General Communication Science) will be one day obliged to extend its competence and will have to optimize its performance by the development of such an instrument of communicational intervention. Such a research direction was founded by Robert T. Craig and Karen Tracy (1995) under the title Grounded Practical Theory. This scientific orientation must be extended and consolidated within the larger domain of Communication. Communication needs a practical component, a component of *application*, of *direct communicational intervention*.

General Communication Science must be able to directly intervene, through programs of support in communication, of improvement of concrete communication relationships. As some people go to psychologists or psychotherapists to be helped in order to relate better with other people, *communicologists* must be called to intervene in situations of communication failure.

The Sources of Communication as Academic Discipline

As a discipline, communication represents ontological, epistemological, methodological and hermeneutical similarities to social sciences such as sociology, psychology, anthropology, political science. Actually, communication has taken over and adapted methods of all these domains-fields. Moreover, the fathers and founders of communication as a social or socio-human domain of study came from the universe/multi-verse of these sciences: Kurt Lewin and Carl I. Hovland were psychologists, Paul F. Lazarsfeld was a sociologist, Harold D. Lasswell was a specialist in political science. Marshall McLuhan and W. L. Schramm had a literature doctorate. Even today's great specialists originate in psychology, such as Charles R. Berger, and D. R. Roskos-Ewoldsen. Robert T. Craig, Judee K. Burgoon and Michael E. Roloff are among the few ones who have originated in communication from the *speech communication* area. Ontologically, the four social sciences (sociology, psychology, anthropology, political science) form socio-scientific sources of Communication. Remarkably, Craig remembers two of these sources as *traditions* of communication (sociology and psychology). (Never forget that, with the help of C. E. Shannon, W. Weaver and N. Wiener, communication has both mathematical and cybernetic perspective as a source of foundation for the communication systems).

On the other hand, communication reveals less ontological, epistemological, methodological and hermeneutical similarities to humanities like rhetoric, persuasion, semiotics, linguistics. In these do we identify humanistic sources of the discipline of Communication. Robert T. Craig (1999) Craig & Muller, 2007) considers semiotics and rhetoric the righteous *traditions* of communication. We would, convergently, say that the discipline of communication has more sources. Some are of a socio-scientific type, others are humanistic. Some are principal, others are secondary. The principal sources of communication as discipline are the eight mentioned already: sociology, psychology, anthropology, political science, rhetoric, persuasion, semiotics, linguistics.

We retain that sociology is the fundamental source of Communication discipline. As a consequence, we admit that, primarily, communication should have more important similarities with sociology. Sources like pragmatics, philosophy, ethics, ecology and so on are appreciated as secondary. Naturally, these sources have influenced its ontological and methodological profile and identity

(Donsbach, 2006). As a consequence, the streams gather together in the river bed of the greatest of them. When two rivers meet, one stays, whereas the other joins it and then disappears: the name is given by the strongest one. Since it has appeared at the confluence and by the radiation of 8-10 sources of knowledge, communication is something different from each of them: the discipline of com-munication is an unpredictable emergence in comparison with its sources. Subseq-uently, communication has the characteristics of a social science (in accordance with the sociological sources or the political science) and the characteristics of a discipline in the category of humanities (according to its linguistic, semiotic, rhetorical and persuasive sources). It is Robert T. Craig who has insisted that the discipline of communication should be viewed as having a practical character, not a scientific one. And we certainly think that communication has a practical character. Due to its preponderantly practical quality, we do consider that commu-nication has a scientific character. As communication has a more practical-appli-cative character than a hermeneutical-interpretative one, it must be seen as belon-ging to social sciences or social-human sciences.

What Kind of Practice is Communication Research?
What Kind of Practice is Communicational Intervention (Application)?

Practice is the source, the criterion and the purpose of theory, of science. Science starts from practice and goes back to the design of practice. Throughout the years, the major stimulating and directive role of theory has been observed. Lewin said: “There is nothing so practical as a good theory” (Lewin, 1951: 169), and Einstein showed: “It is the theory which decides what we can observe” (apud. Heisenberg, 1971: 77). It may be also said that a good practice is the best theory. Anyway, there is no theory without any practical support and no practice without any as small as possible theoretical idea.

Everything that human beings do constitutes practices. Practices are all the activities of the entire world. Not all the practices are identical, and, especially, not all the practices take place at the same level, in the same context and so on. But, above all, all the practices happen between themselves. At the first level of practice, there are the practices as practices. These are practices per se, meaning the practices which besides their own purpose have no more other. Pure practices are pure engaging in activities, they are practices whose purposes are to be just practices. Craig believes that “Practices involve not only engaging in certain activities, but also thinking and talking about activities in particular ways “ (Craig, 2006: 38). So, any practice would imply the activities plus *thinking and talking about activities*. And any practice could have two aspects, *conceptual* and *the-oretical*: “Practices also have a conceptual - sometimes, even, a theoretical -

aspect” (Craig, 2006: 39). On another plan, we can say that there are four types of practices:

(1) practice per se (pure practice or activity practice) or the first practice;

(2) theoretical practice (practice of theorization of activity) or the second practice;

(3) general conceptual practice (practice of thinking abstractly about the theoretical practice and the activity practice) or the third practice, and (4) the fourth practice, the applied practice of practice per se for the efficiency and the theoretical-conceptual value added to the second and the third practice. These ideas are available in the field-universe of communication.

Craig talks about *practice of communication*, *communication practices*, about the fact that “people everywhere are practicing communication” and “communication is a practice” (Craig, 2006: 40-45). He stresses that “communication per se is a practice (a coherent, meaningful set of activities)” and “in our culture” it has developed a “discourse about communication” which “has developed to such

an extent that an academic discipline of communication studies (….): we call communication theory” (Craig, 2006: 41). Not only “is communication a practice”, but “(1) theory is a practice; (2) theory provides ways of interpreting practical, knowledge; and (3) theory is fundamentally normative” (Craig, 2006: 42). In other words, R. T. Craig thinks there would be two practices: one practice meaning “communication per se” and the other practice meaning “communication theory” as a form of “metadiscursive practice”: “I envision communication theory as an open field of discourse engaged with the problems of communication as a social practice, a theoretical meta-discourse that emerges from, extends, and informs practical metadiscourse” (Craig, 1999: 129). Communication is under-stood as discourse and communication theory as “metadiscourse (discourse about discourse)” (Craig, 2006: 41). Thus, communication is practice, meaning dis-course, and communication theory is a different practice: meta-discourse.

We would say that in communication “field”, Craig makes the difference between two kinds of plans and two kinds of practice. In our opinion, there can be delimited not only two plans, but three plans (three envelopes of knowledge) and four types of practice: (a) But before everything, there is the first practice or practice per se (pure practice or activity practice, occurrence-practice) meaning practice of communication, communication practice, communication discourse. In 1999, Craig appositively said that “communication theory” is “theory of this practice” (Craig, 1999: 123); (b) The second practice is theoretical practice (practice of theorization of activity) meaning communication theory, theory of communication or theorizing discourse about communication discourse, metadiscursive practice, and theoretical practice. This practice means the application of some research methods on the concrete communication phenomenon, events. Being an application method, this is an applicative practice with an applied character; (c) The third practice is scientific conceptual practice (practice of thinking abstractly about the other two practices: theoretical practice and activity practice), meaning communication science, science of communication, scientific discourse, meta-theory (meta-meta-discourse). As far as “meta-theory” is concerned, we must remember that Craig, in his capital ontological study “Communication theory as a field” (1999), refers to it together with Muller (2007) alone at the chapter “Metatheory” without any further explanations; (d) The fourth practice is practice of concrete application in initial activity practice for the efficiency of concepts, theories, laws depicted from zetetic practices. The second and the third practices make up zetetic investigation practices.

In our opinion, communication science is a meta-theory, and its method is “meta-analysis” (Courtright, 2013). The whole communication universe forms the perimeter of preoccupation of General Communication Science. As regards the practice and discourse types mentioned above, communication universe pre-sents three envelopes of knowledge. General Communication Science (GCS) is the socio-humanistic theoretical-scientific field that deals with the communication universe. This includes the three practices which form three knowledge-research-application frames: (a) Fundamental Communication Science (FCS) is the first envelop-frame: the constitutive branch of General Communication Science which deals directly and preponderantly with the conceptual basic, fundamental pro-blems of communication. Applied Communication Science or *Applied Commu-nication Research* (ACR) is the grounded-theoretical envelop of communication. “Grounded Practical” Communication Application-Intervention (PCI) is the im-plementation frame of direct practice for the general conceptual theoretical-applicative benefits. At a certain moment, D. French and M. Richards speak about “importance of fundamental Communication Science” (French & Richards, 1994);(b) *Applied Communication Research* is the theoretical-applicative level of Ge-neral Communication Science, the “theorizing”-research level. The mission of *Applied Communication Research* stands for the specialized research of the con-crete phenomenon and events, of communicative (pure) or conducted practices to discover, to separate, to extract, to abstract regularities, dependences, patterns, concepts, categories, theories, laws, principles etc.; (c) Practical Communication Intervention (PCI) is the purely practical component of General Communication Science and deals with putting into practice the results of theoretical and con-ceptual research. Its aim is the efficiency of communication practice. Practical Communication Intervention (PCI) constitutes the concrete use of the theoretical conceptual benefits. In science order, this is the fourth practice: a practice that consists of concrete induction of communication wisdom.

If general methods of Fundamental Communication Science (observation expe-riment research, content analysis, survey research, participatory research and so on) are applied in *Applied Communication Research* to study communicative practices, communication practices and practical practice, Practical Commu-nication Intervention deals with proceedings, procedures and techniques to im-prove concrete and direct communication practice. Communication is created in *Grounded Practical Communication Intervention (Application)*. Phenomena, events, situations and communication practices are produced in comparison with concepts, theories and principles of Fundamental Communication and practical conclusions separated in Practical Communication Intervention (Application). Practical Communication Intervention also includes communication practical knowledge, meaning communicative know-how. In the communication ontology essay (1999), Robert T. Craig talks about “Exploration, Creation, Application” of “communication theory” “applying communication theory by engaging it with practical metadiscourse on communication problems”, “applying communication theory involves engaging the traditions of theoretical metadiscourse with practical metadiscourse on real communication problems. It is in this process of application that the communication theory can be most logically tested in order to establish its relevance and usefulness for guiding the conduct and criticism of practice” (Craig, 1999: 146-152). From our point of view, this is the position of the second practice, Applied Communication Theory-Research, compared to the fourth prac-tice, Practical Communication Intervention, the one in which problems of the first practice are concretely solved, problems of communication per se, but also pro-blems of political communication with masses, public meetings, negotiations, crisis etc. In 1995, Craig and Tracy introduced a research direction named Groun-ded Practical Theory (GPT): “as a rational reconstruction of practices for the purpose of informing further practice and reflection” (Craig & Tracy, 1995: 248). As it is known, a theory is an idea of descriptive, explanatory or normative relevance about a set of practices and concrete phenomena. They view Grounded Practical Theory as an approach with a methodological, conceptual and the-oretical-normative characteristic supposed to solve problems of communication practice in real world. It is followed to identify communication problems and issues of humankind and to find out practical communicational adequate solutions. Finally, Grounded Practical Theory would be either a “theoretical” intervention to solve some communication component conflicts or an intervention to optimize some communication processes. Programmatically, Craig and Tracy keep Groun-ded Practical Theory in the envelop-frame of Applied Communication Research. We think that, if Grounded Practical Theory starts from practice per se, - the first practice-, and selects a theory, it depends on Applied Communication Theory-Research then, as it is a pure research. When, in Grounded Practical Theory, the theory depicted from the examination of the first practice, practice per se, returns to some phenomena of communication practice or even to the starting commu-nicational phenomenon, we believe that we no longer deal with the Applied Communication Theory-Research envelop-frame, but we are in the Practical Communication Intervention envelop-frame. In this respect, we view Grounded Practical Theory as a complex research, having a theoretical practice component, the second practice (which belongs to Applied Communication Theory-Research) and a fourth practice component to improve the practice (which belongs to Practical Communication Intervention). Through Practical Communication Intervention, the circuit practice-science-theory closes by the action of science-theory on pure, natural practice.

Applied Communication Research vs. Practical Communication Intervention; communication research vs. communicational intervention

In the years 1970-1980, communication is institutionalized as an academic discipline. *Applied Communication Research* emerges as a natural development and building element. Among those who gave it a profile of resistance there are included: Hickson (1973), Cissna (1982), Miller, Sunnafrank (1984), Cissna (1987), Pettegrew (1988). Hickson is the one who launched the *Journal of Applied Communication Research* in 1973.

During 1990-2000, *Applied Communication Research* strengthened and developed its knowledge background. It also contributed a) to broaden the ideational theoretical and conceptual thesaurus of Fundamental Communication Science, and b) to demonstrate a consubstantial domain of communication called by us "Grounded Practical" Communication Application-Intervention which was born under the name Grounded Practical Theory (Craig & Tracy, 1995). For this stage of *Applied Communication Research* the contributions of such specialists as O'Hair & Kreps (1990), Cissna (2000), Wood (2000), Keyton (2000), O'Hair (2000) are very important. In the 2000s, *Applied Communication Research* is a knowledge envelop-frame reinforced in communication universe: Vangelisti (2004), Frey & Cissna (2009).

Since its founding in 1970, *Applied Communication Research* has undergone three phases: the initiation (starting) phase, the consolidation phase and the founding-acknowledgment phase as knowledge envelop-frame component of General Communication Science alongside with Fundamental Communication Science and Practical Communication Intervention: (a) Initially, Cissna, won-dering in 1982 "what is Applied Communication Research?", identified the do-main as "inquiry that sought to make a difference in the world through examining some feature of human communication" (Cissna, 1982: I). *Applied Commu-nication Research* was at the beginning and worked on its own profile, sought its place among research approaches in the area of communication; (b) In the con-solidation phase, approximately 20 years after *Applied Communication Research* had been out of question: Cissna found that the domain was a "legitimate and respected area of communication study" (Cissna, 2000: 169). In the field of communication, *Applied Communication Research* was defined as a specific "area"; (c) In the consolidation phase as envelope of knowledge and research framework in the world of communication, *Applied Communication Research* reaches the year 2010. Now, Cissna and Frey state: “The study of real-world communication concerns, issues, and problems is known as Applied Communication Research” (Frey & Cissna, 2009: XXIX). It is a *field of study* in which there happens “the systematic investigation of real-world concerns, issues, and problems to help people better manage them” and which is part of “commu-nication discipline” (Frey & Cissna, 2009: XXIX-XXXIX). The two experts noted that the principles and practices of *Applied Communication Research* were used in several programs: “principles and practices have been employed in some exemplary programs” (Frey & Cissna, 2009: XXXIX).

Programs that are used consciously and in a special way for *principles and practices* are actually, in our opinion, concrete cases of Practical Communication Intervention (Application). In fact, the *principles* belong to Fundamental Science Communication and the *practices* which are referred to are related to Applied Communication Research. We would say that Practical Communication Application-Intervention is one of the *practices* that *have been employed*, meaning some good, acknowledged, tested, practices validated in Applied Communication Research. We see *Grounded Practical Communication Intervention (Application)* as a direct improvement of communication’s work by applying the concepts, theories, and fundamental cogitations of Applied Communication Research. It is a feedback of the theoretical practice; it is theoretical-practical, firstly practical application of the theory. In that general argumentative plan, Professor Craig emphasizes that: “Every theoretical discourse has essential practical aspects, and every practical discourse has essential theoretical aspects. Practices are theorizing to varying degrees, but every practice is theorized to some degree” (Craig, 1996: 461). *Applied Communication Research* produces theories, procedures, processes and techniques that can lead to improved communication. It generates theoretical constructs that have latent applicability. When we use their potential applicability, the approach does not start from *Applied Communication Research* envelop. The practical use of the research constructs resulted from *Applied Communication Research* is achieved in *Practical Communication Intervention* envelop. Clear:

(a) *Applied Communication Research* is the domain of communication research;
(b) *Practical Communication Intervention* is the activity of communication intervention. Within *Applied Communication Research* there are created constructs with *applicability* and within Practical Communication Intervention these con-structs and concepts of Fundamental Science Communication are practically used. *Applicability* and *usability* form the core of both. Argyris believes that both *applicability* and *usability* would lie within the *Applied Communication Research*; he says that “applicability refers to relevance” and defines “usability” “as appli-cable implementing knowledge, to carry into effect, to fully fulfil, to bring to full success whatever the propositions of applicable knowledge assert” (Argyris, 1995: 1).

Part V of *Routledge Handbook of Applied Communication Research* (2009) (edited by Frey and Cissna) entitled *Exemplary Programs of Applied Communication Research* is a model for what *Practical Application Communication Intervention* means the implementation in the social environment of the conclusions resulted from the researches in *Fundamental Communication Science* and *Applied Communication Research*. Hecht and Miller-Day (2009) deal with “Drug Resistance Strategies Project: Using Narrative Theory to Enhance Ado-lescents Communication Competence”; in the article, the *fundamental* concept of communication competence and the *applied* concept of *narrative theory* are relevant. The implementation is denoted by *using*. Wille and Roberto (2009) thematize *Fear Appeals and Public Health: Managing Fear and Creating Hope*. In “global process of change” of behavior (Zamfir, 2013), in „organisational change” (Cojocaru, 2012), in doctor-patient relationship (Cojocaru, Cace & Gavri-lovici, 2013), in “social gradient in health” (Bulgaru-Iliescu, Oprea, Cojocaru & Sandu, 2013) is inevitable a communicational intervention. The clearest “commu-nicational interventions” (Carson, 1977; Lien, 2006), “communication interven-tion” (Lilly *et al*., 2000; Prizant, Wetherby & Rydell, 2000; Goldstein, 2000; Rossetti, 2001) are recorded in crisis situations (see’s Communication Crisis) in medical contexts (Goldstein, 2002; Aldred, Green & Adams, 2004; enescu, 2009; Strunga & Bunaiasu, 2013). Examining the contributions of Palo Alto School at *interactional view* of communication, Carson spoke about “basic themes of psychopathology as communication disorder and psychotherapy as strategic commu-nicational intervention” (Carson, 1977). Rao, Anderson, Inui and Frankel (2007) assert that “communication interventions make a difference in conversations” and that „interventions were characterized by type (e.g., information, modeling, feed-back, and practice), delivery strategy, and overall intensity”. The communicational intervention has some dimensions which must be investigated.

Conclusion

As we have previously demonstrated, communication as an academic discipline has matured: (a) communication has gained a clear self-consciousness; (b) com-munication has become autonomous and lives on its own; (c) various emerging fields of communication have developed a strong and functional coherence, so that set of communication fields has turned into a universe with its own identity and recognizable profile; (d) the influence of disciplines which “labored” and “attended”, Socratically speaking, to the birth of communication as a discipline (Rhetoric, Anthropology, Semiotics, Psychology and Sociology) is not radiant and determinant anymore; (e) communication is mainly developing on its own account. Furthermore, we ascertain that within the communication universe there are structured new areas: such a field is Practical Communication Intervention.

Practical Communication Intervention is an unmediated injection of theory into practice; it must be a direct implementation. By Practical Communication Intervention, the theory-research of communication infuses the practice of communication. With the words of Steve Duck: there shall be "the systematic application of that work in a practical context", "application of insights" (Duck, 2007, p. IX).

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The optimization of intelligent control interfaces using Versatile Intelligent Portable Robot Platform

Victor Vlădăreanu, Radu I. Munteanu, Ali Mumtaz, Florentin Smarandache, Luige Vlădăreanu

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Abstract

The paper presents a VIPRO versatile, intelligent and mobile platform for robots, using an original virtual projection method which involves the representation of modern mobile robots in a 3D virtual environment using a strong robotic simulator, an open architecture system and adaptive networks over the classical control system of the robot, developing intelligent control interfaces The advanced control technologies adapted to the robot environment such as neutrosophic control, robot Extenics control and robot haptic control are used. The obtained results lead to the conclusion that the VIPRO platform is to be integrated on the IT market as a new component alongside the existing ones, allowing a correct evaluation of robot behaviours in hazardous or challenging environments and high level real-time simulation in order to correctly model interactions among the robots and between the robots and the environment.

Keywords: mobile autonomous intelligent robots; real time control; virtual projection method; open architecture system.

1. Introduction

Robots with artificial intelligence and networked remote control by human operators are playing increasingly important roles in hazardous or challenging environments where human lives might be at risk. This imposes the urgent need for the development of autonomous mobile robots which can be controlled remotely and can provide support in case of natural disasters, fires or calamities, dangerous landmine detection activities and other explosive devices (Fig.1a). Following the terrible earthquake from Japan an international project named “RoboCup-Rescue project” which reunites important research teams from all over the world was carried on in order to build rescuing and seeking robots (Hanshi-Awaji Earthquake, Fig.1b) divided in two sub-projects: one for multi-agent simulation and virtual robot, and the second focused on developing the real robot.

Real time control robots with remote network control with human operators' ability play an important part in hazardous and challenging environments where human life may be exposed to great dangers such as support and repair in nuclear contaminated area, fire and earthquake disaster areas. An important amount of research led to the development of different robots with sensing abilities, transport and manipulation of different applications. Developing mobile and remote control autonomous robots, which can help people in order to perform search and rescue operations in nuclear contaminated environments, fires and earthquake disaster areas, is a priority and a complex task[1,2].

a) Military robot for bomb diffusing b) Hanshi-Awaji Earthquake

Fig. 1. Autonomous mobile robots in case of dangerous landmine or natural disasters

The VIPRO platform, presented in paper, brings the virtual robots to the real world and creates an international innovative robot platform, based on virtual projection method by Vladareanu-Munteanu, IMSAR patent[3], which allows developing mechatronic systems of mobile robots in virtual environments and communicating with real robot systems through a high speed interface[4,5]. The result is development of an intelligent, portable, versatile platform VIPRO which allows the improvement of robot motion and stability performances in a virtual and real environment on unstructured and uneven terrains. Using VIPRO platform will allow efficiently building of mobile robots endowed with Robot Neutrosophic Control (RNC), Robot Extenics Control Interface (eHFPC) and Robot Haptic Control (RHC) Interface, etc. in addition to other similar products on the market.

2. The VIPRO Platform Strategies

The VIPRO Platform allows the development of a complete research tool for the robot mechanical modelling of 3D virtual environment, virtual platform for the robot modelling and simulation, planning strategies, robot motion developing, image processing, robot adaptive intelligent control and behaviour based control.

Robotic simulation is essential in developing control[6] and perception algorithms[7] for robotics applications. A 3D virtual platform for mobile robots must correctly simulate the dynamics of the robots and to avoid the objects in the environment, thus allowing for a correct evaluation of robot behaviours in the environment. Moreover, real-time simulation is important in order to correctly model interactions among the robots and between the robots and the environment. Since the simulation accuracy requires significant computing power, it is often necessarily to make the approximations for obtain the robot real time control.

Mechanical modeling of motion. The Robo-Cup version of the NAO (H21 model) has 21 joints, resulting in 21 degrees of freedom (DOF)[8]. See Figure 2 for a complete schematic overview of the NAO robot. The movement of each joint can be described by a rigid body equation[1]. Two different types of contacts can be distinguished. The first is a contact caused by bumping into another rigid body or into the world. The other type of contact is caused by having a joint defined between two rigid bodies.

Motion planning. A Motion Editor for testing, debugging and to realize new motions was developed. For some predefined motions, such as standing up of the robot was used a method based on main-condition. One motion network is defined as a DFSA (Determinist Finite State Automata).

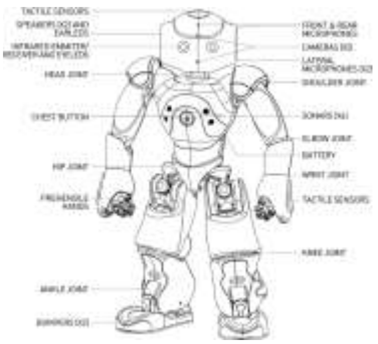

Fig.2 The NAO Robot (Aldebaran Robotics)

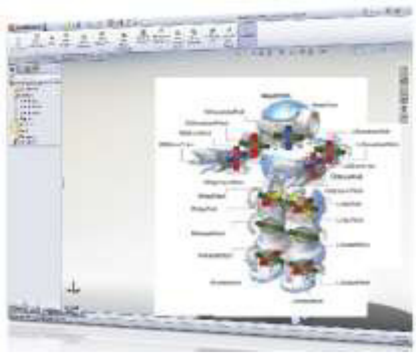

Fig.3 The 3D representation of the NAO robot

Each node describes the state of the robot (actuators) and each line describes the transition between two states (Figure 3) following the occurrence of certain events, such as receiving information from sensors[7,8].

The VIPRO intelligent control interfaces adapted to the surrounding environment were developed as alternative to classic AI, where the intelligent behaviour was built in a top-down manner. Using this concept, the robot used the motion networks and real time control that simulates the artificial intelligence, adding the control complex laws in order to improvement the stability of the mobile autonomous robots[9,10]. For this purpose a strategy was used for dynamic stabilization and balance of the walking robot through applying the patent "Method and device for real time robot control in virtual projection" known as the Vladareanu-Munteanu method. This was integrated in the development of controlling.

In order to build the virtual platform, a software development tool was selected based on analysing the available solutions, for to modeling and respectively simulation of the 3D objects, robots and the environments in which they are moving. Control interface and components integration have been achieved using the web services compatible with SOA (Service Oriented Architecture) architecture and/or REST (Representational State Transfer). The services technology for modelling, control and simulating the robots was implemented successfully in Microsoft Robotics simulator[11,12].

Validation of the robot's mechanical structure and simulation was done using the NAO robot. As seen in Figure 3, the NAO robot has 21 DOF. It has the advantage of being small and the manufacturers deliver it with a control software for the end-user. We have used this robot aiming to model a bipedal robot in 3D virtual platform, to test the interaction between the physical and virtual system and to validate VIPRO performance operation.

The VIPRO has the additional purpose to develop the platform for DHFPC (Dynamic Hybrid Force-Position Control) that will lead to the study of mobility and stability of mobile robots, the 3D design of the simulation environment, controlled through the intelligent control methods: extension theory[13-16] or[18,19], neutrosophic logic (DSmT)[20,21], fuzzy[9,21], neural networks[2], Petri Nets modelled with Markov chains[4,6], methods for dynamic balance control[1,4] or [6], hybrid force-position control method[6,9] or [12].

3. Building of the open architecture system

The architecture of the VIPRO control system, applied in open architecture[2,3] or [5,9], is presented in Figure 4, in which SCMC is the classic control system, which drives the servo-actuators MS1, MSm,-with „m" being the number of the robots degrees of freedom-, and receives signals of TM1-TMm measure.

Also, a number of **m AS** load actuator modules rigidly coupled to the „m” **MS** servo-actuator modules receive control signals from a **MCS** load controller module with the role to ensure the load of the **m MS** servo-actuator modules. An **MCS** load controller module which receives the $\mathbf{X_R^P}$ and $\mathbf{X_R^F}$, position and force reference and a $\mathbf{X_R^S}$ reference signal to generate loads to **MS** servo-actuator modules, from the **ICMF**multi function control interface with the role to ensure the real time control and the load of the „m” **MS** load actuator modules.

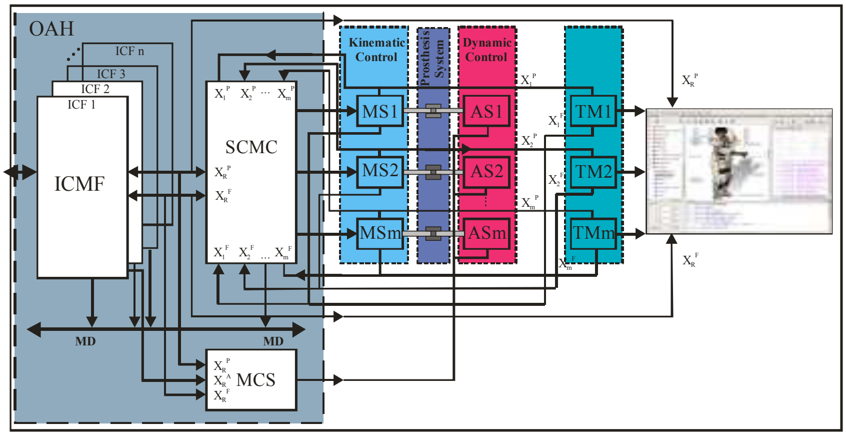

Fig.4. Virtual projection method by Vladareanu-Munteanu applied to VIPRO Platform

The novelty of the virtual projection method by Vladareanu-Munteanu allows a complex control for the dynamic walking robots, real time hybrid control for positioning and joint trajectories of robot legs, the feed-back forces and walking robots dynamic control to increase stability and mobility. The main advantages of the method consists of real-time robot balance control, gait control and predictable motion control providing increased mobility and stability in order to achieve higher performance according to the robot walk developing new technological capabilities of the control systems.

In order to carry out new capabilities for walking robots, such as walking down the slope, by avoiding or passing over the obstacles, it is necessary to develop high-level intelligent algorithms, because the mechanism of walking robots stepping on a road with bumps is a complicated process to understand, being a repetitive process of tilting or unstable motion that can lead to the overthrow of the robot. The chosen method that adapts well to walking robots is the ZMP method[6,9]. A strategy was developed for the dynamic control of walking robot gait using ZMP and inertial information. This, includes pattern generation of compliant walking, real-time ZMP compensation in one phase - support phase, the leg joint damping control, stable stepping control and stepping position control based on angular velocity of the platform.

4. The optimization of intelligent control methods using the VIPRO Platform

The optimization of intelligent control methods allows the mobile autonomous robot to adapt to uneven ground, through real time control, without losing its stability during walking[2,3] or[5]. The VIPRO platform architecture, in correlation with the virtual projection method, was developed in Figure 4. In this article are presented three intelligent control interfaces (I.C.), implemented by using the versatile, intelligent and portable robot platform VIPRO.

4.1. Robot Neutrosophic Control (RNC)

The proposal is to build a module which uses the neutrosophic logic to fusion the information provided by robot’s sensors in order to find the most accurate sensors’ results[9,16] or[20]. These results are later used in robot’s decision-making process. Hybrid position and force control of industrial robots equipped with compliant joints must take into consideration the passive compliance of the system. The generalized area where a robot works can be defined in a constraint space with six degrees of freedom (DOF), with position constrains along the normal force of this area and force constrains along the tangents.

On the basis of these two constrains there the ariables X_C and F_C represent the Cartesian position and the Cartesian force exerted onto the environment. Considering X_C and F_C expressed in specific frame of coordinates, its selection matrices S_x and S_f can be determined, which are diagonal matrices with 0 and 1 diagonal elements, and which satisfy relation: $S_x + S_f = I_d$, where S_x and S_f are methodically deduced from kinematics constrains imposed by the working environment[5,10] or[20].

For the fusion of information received from various sensors, information that can be conflicting in a certain degree, the robot uses the fuzzy and neutrosophic logic or sets[3,20]. In real time a neutrosophic dynamic fusion is used, so an autonomous robot can take a decision at any moment. For the combination of the information we can use the information fusion theories (Dezert-Smarandache Theory, Dempster-Shafer Theory, Smets's Transferable Belief Model) and different fusion rules, among them the Proportional Conflict Redistribution, the Hybrid Rule, etc. We can also use fuzzy logic/set and neutrosophic logic/set for designing a model of combining robot sensor's information using the neutrosophic logic operators (N-norm and N-co-norm, which are generalizations of the fuzzy T-norm and T-co-norm).

Robot neutrosophic control (RNC) by *Vladareanu-Smarandache* method through applying the neutrosophic logic and the Dezert Smarandache Theory (DSmT) represent a new theory which merge the fuzzy theories and information fusion[20].

4.2. Robot Extenics Control Interface

The robot extended control interface (ICEx) integrates the Extenics real time control method through the application of Extenics theory for solving contradictory problems.

Extenics is a science dealing with modeling contradictory and antithetic problems. Extended system control allows the optimization of the control process through the application of Extenics theory, metrics and dependence function in the extended space defined by Extenics logic. The founder of Extenics is Prof. Cai Wen from Guangdong University who in 1983 laid the basis of this theory1[3,14]. Extenics has been researched by numerous other academics, with significant contributions being brought by Prof. Sandru from the Politehnica University Bucharest and Prof. Smarandache from the University of New Mexico, USA, both members in the VIPRO research team. Significant obtained results consist in generalizing from the 1-dimensional case to the n-dimensional case of certain indicators fundamental to Extenics theory in general[15,16] or[18].

The contradictory force-position problem regarding real time control of the robot motion using the VIPRO platform has been designed using **extended hybrid force-position control (eHFPC)** for robots and mechatronic systems developed by Vladareanu et al[5,16,21].

Real time functioning consists of implementing in the principial design of the extended robot control interface (ICEx) in Figure 5. The scientific foundation is based on Extenics theory in defining the extended position distance $\rho(X,X_o)$, the extended force distance $\rho(F_a,X_{Fo})$, the dependence function $K(X, X_o, X_{CR})$ of the current position signal X in relation to the standard positive intervale of the reference position X_o and the transitive positive position interval $\mathbf{X_{CR}}$, respectively the dependence function $K(F_a, X_{Fo}, X_{FCR})$ of the force signal F_a in relation to the standard positive interval of the reference force X_{Fo} and the transitive positive force interval $\mathbf{X_{FCR}}$. Thus, an extended transformation is done through a relation which includes an extended dependence function in the universe of discourse U by using an extended position distance $\rho(X,X_o)$ and an extended force distance $\rho(F_a,X_{Fo})$.

The eHFPC architecture with explicit control using force and position sensors. The robot extended control interface (ICEx) developed by integrating the Extenics real time robot control method (eHFPC), presented in Figure 12, is aimed at solving the contradictory hybrid force-position control problem of the movement of robotic and mechatronic systems.

This is conceptually obtained by replacing the logic values of 0 and 1 in the selection matrices Sx and Sf, depending on the force-position sequences with Cantor logic, with dependence function values using extended distance, followed by the extended domain transformation for position S_{Kx}, respectively for force S_{Kf}, generating a new selection matrix and correlation coefficients for position, respectively for force, accomplishing explicit control in force and in position

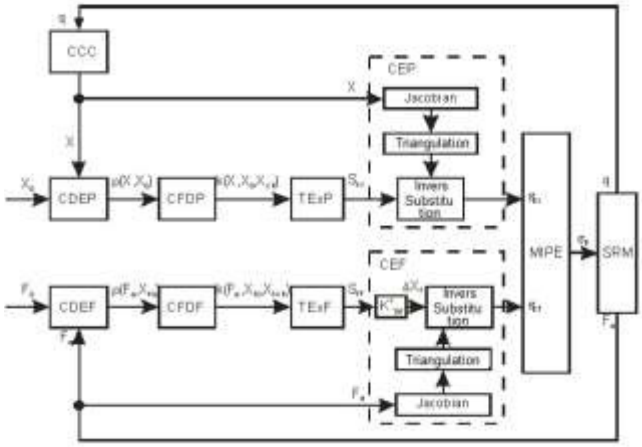

Fig.5. System Architecture eHFPC with Explicit Control

Thus, a module for computing the extended position distance CDEP receives the current position signals X processed by the Carthesian coordinates computation module CCC through direct kinematics for the robotic mechatronic system SRM and, according to the experimentally defined standard positive position reference interval X_o, calculates the extended position distance $\rho(X,X_o)$, which it sends to the position dependence function computational module CFDP. The extended position distance $\rho(X,X_o)$, according to extenics theory, is calculated as the distance from a point, in this case the current position signal X, to an interval, in this case the standard positive reference position interval X_o. The data is similarly processed for the computation of the extended force distance CDEF, which works quasi-simultaneously with the computational module of the extended position distance CDEP.

The implementation methodology of this advance hybrid force-position control method for robotic and mechatronic systems consists of experimental determination of the standard positive DSP domain and the transitive positive DTP domain for each control component, applying the transformation to the force and position signals taking into account their real position in relation to the standard positive DSP domain, resulting in a transformed force and position error representing an optimized function for hybrid force-position control in a metric generated by extended distance and the dependence function from extenics theory. The universe of discourse has been configured so as to allow a negative transition domain DTN, defined by the points for position c_x and d_x, respectively c_f and d_f for force, so that when passing these points the force and position errors will be limited to not allow for controller saturation, with all its negative effects.

4.3. Robot Haptic Control (RHC) Interface

The disadvantages of known robot control solutions for motion and navigation on irregular terrain and uncertain, unstructured or undefined environments, consist of the inability to control the rigidity of the leg sole joints and robot segment joints when detecting unevenness, depending on the robot environment map and do not ensure remote control by the human operator, who can see the robot environment map and simultaneously feel the damping of the robot leg movement, with the aim of generating the haptic Cartesian positions for robot motion adaptation to the actual conditions of the terrain and environment.

The innovative solution developed and patented for haptic robot control[17] allows the robot to “feel” the terrain on which the mobile autonomous robot moves by the **modification in rigidity** of the leg sole joints and of the segment joints when detecting unevenness depending on the rigidity K_{X_c} associated to the leg sole joint position of the robot X_C on the robot environment map. Modifications in rigidity are realized from the time the leg sole touches the terrain until complete contact of the leg sole segment. **The human operator** has the possibility to remotely control the robot movement, through two parameters, one visual and the second haptic, respectively seeing the robot environment map and simultaneously to feel remotely the dampening of the robot leg movement when using the haptic device stick. Depending on the type of manipulation of the haptic device, the human operator generates the haptic Cartesian positions X_{CH} to ensure the robot motion is adapted to the uneven and unstructured terrain in crisis situations or natural disasters where human lives may be at risk.

In order to generate the robot environment map, images are processed from a CCD camera, stabilized for the various robot motion directions. This is done by processing the signals received from a 3D gravitational transducer (TGR3D) and a magnetic compass (TBM), resulting in an interface of the 3D robot environment map with a stable image to the robot movement. Each point in the robot environment map is associated with the rigidity of the robot leg sole joint position X_C, named associated rigidity K_{Xc}. **The movement damping** at contact between the uneven terrain and the robot leg sole is obtained by comuting from position control to force control from the moment the tip or the posterior of the leg sole touches the terrain, depending on the robot walking scheme, until complete contact of the leg sole segment is made.

Changing the robot walking scheme is a biologically inspired technical solution, conforming to human walking, and consists in touching the terrain with the tip of the leg sole and damping movement until complete contact of the leg sole segment, for uneven and unstructured terrain, as opposed to regular walking, which consists of touching the terrain with the posterior of the leg sole and damping movement until complete contact of the leg sole segment is made, for smooth terrain. **Visual robot motion control by the human operator** is achieved through a haptic map of the robot environment, which copies the graphic 3D interface of the robot environment map (HMR) remotely sent through the communication modules.

Haptic control of the robot movement by the human operator is achieved through a haptic device which allows the human operator to feel the damping of the robot leg movement and generates the Cartesian reference positions of the robot movement, called haptic Cartesian positions X^H_{CH}, for adapting robot movement to uneven and unstructured terrain. **The telemetry module (TL)** allows the measurement of the distance to the leg sole segment by using an optic scanning device.

The device for haptic robot control (DCRH) interacts with both the robot (RO), from which it receives the signals generated from the sensors mounted on the robot and runs the actuators on the robot movement axes, as well as with a human operator (OU), who receives visual information from the haptic map of the robot environment, with the aim that the human operator feel, during hand-operated manipulation of the haptic device stick, the reaction force due to the reaction associated to the position of the leg sole joint X_C and the position error due to the terrain contact force ΔX_F.

5. Results and conclusions

The paper presents new concepts and approaches for the development of a versatile intelligent portable robot (VIPRo) platform through an original method of virtual projection, in which the robot motion control is done through intelligent interfaces using Robot Neutrosophic Control (RNC), Robot Extenics Control Interface (eHFPC) and Robot Haptic Control (RHC) Interface. The overall goal is to develop a versatile intelligent portable robot platform by using 3D virtual representation, on a PC with high graphic processing power and advanced programming languages, of robots through mechanical structure modeling, building an open architecture system made of a robot classic control system (with embedded software) and intelligent control interfaces (fuzzy, information fusion, multi-agent, hybrid force position dynamic control, robot neutrosophic control, dynamics and adaptive, robust and iterative learning control, etc.) implemented through IT&C techniques on fast time and high data processing PC server, in order to improve the stability performances and real time motion control.

In comparison to the global level existing solutions and the enormous research done in developing the different types of robots with sensory, transport and manipulation capabilities, the VIPRO platform has the advantage of providing innovative elements related to robot control, such as Neutrosophic Logic or Extenics Theory and also of being an universal, intelligent, portable solution, thus improving the performance of movement and stability in virtual and real environments for mobile autonomous intelligent robots and in particular for search and rescue robots.

The focus is on the three areas which imply creating the virtual environment of the walking robot motion, in each case, with obstacle avoidance, image processing and motion on unstructured terrains by keeping it in a balanced state. The result is a novelty VIPRo prototype platform, which will be able to be marketed in competition with other similar virtual simulation platforms with applications in mechatronics, called virtual instrumentation, CDA, CAM, CAE, Solid Works, etc., which is very reliable in powerful modeling but only in a virtual environment or the MatLab, Simulink, COMSOL, Lab View platforms, which allow extensions for real time data acquisition and signal processing, but none of which allow the design, testing and experiment of intelligent control methods in the absence of the mechanical structure using of the classical robot real time control system.

Acknowledgements

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Robot System Identification using 3D Simulation Component Applied on VIPRO Platform

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***Abstract*. The paper presents automated estimation techniques for robot parameters through system identification, for both PID control and future implementation of intelligent control laws, with the aim of designing the experimental model in a 3D virtual reality for testing and validating control laws in the joints of NAO humanoid robots. After identifying the maximum likelihood model, the PID amplification factors are optimized and introduced into the Unity environment as a script for controlling the joint. The program used for identifying PID parameters for the NAO robot is developed using the virtual reality platform Unity 3D and integrated into the Graphical Station component of the VIPRO Platform for the control of versatile, intelligent, portable robots. The obtained results, validated in the virtual reality environment, have led to the implementation of the PID identification and optimization component on the VIPRO Platform.**

Keywords—intelligent robotic control systems; robotic system identification; modelling system; virtual reality; robot stability

I. INTRODUCTION

The last few years have seen mobile robots gain increased attention in the research community, as well as in the manufacturing industry, resulting in remarkable hardware and software development. Among the applications of great interest for researchers are: dangerous activities such as detection of antipersonnel mines and other explosives, surveillance activities ("Remotec" has developed the Marauder technology which later led to the development of the Andros Mark V robot) and rescue operation in case of calamity.

Following the devastating earthquakes in Japan, an international project has been developed which reunites renowned research teams from all over the world for the design of search and rescue robots, under the banner of the RoboCup – Rescue Project, divided into two sub-projects: multi-agent simulation using a virtual robot and development of a real robot.

Developing remote-controlled, autonomous mobile robots, which can support humans in search and rescue operations in a contaminated nuclear environment, after fires or in calamitous earthquake areas has become a priority and entails a complex challenge. To this end, numerous robot control methods have been developed for moving on uneven and uncertain environments, which allows improvements in robot mobility and stability, through intelligent algorithms: fuzzy logic, extenics, neutrosophy, neural networks, Petri nets with Markov models, hybrid force-position control method, among others.

II. 3D UNITY SIMULATION COMPONENT APPLIED ON VIPRO PLATFORM

Real time, remotely-controlled robots with the capabilities of a human operator have an increasingly important role in hazardous or challenging environments, where human life might be endangered, such as nuclear contamination areas, fires and earthquake zones [1 - 3].

Research in these fields have led to an accumulation of important expertise regarding robot movement in virtual environments, with improvements in navigation, obstacle avoidance, high fidelity environment simulation, etc., but lacking the environment – virtual robot – robot interactions. In this context by developing an innovative platform [4 - 7], the VIPRO Platform has been conceived for brings virtual robots into the real world, mainly consisting in the projection into a virtual environment of the robot mechanical structure, and communicate in real time through a high-speed interface with real robotic control systems, in order to improve performances of the robot control laws. The result is a versatile, intelligent, portable robot platform (VIPRO), which allows improved of the robot motion and stability performance in a virtual and real environment on uneven and unstructured terrain for mobile, autonomous, intelligent robots, such as the NAO robots, or in particular the search and rescue robots RABOT.

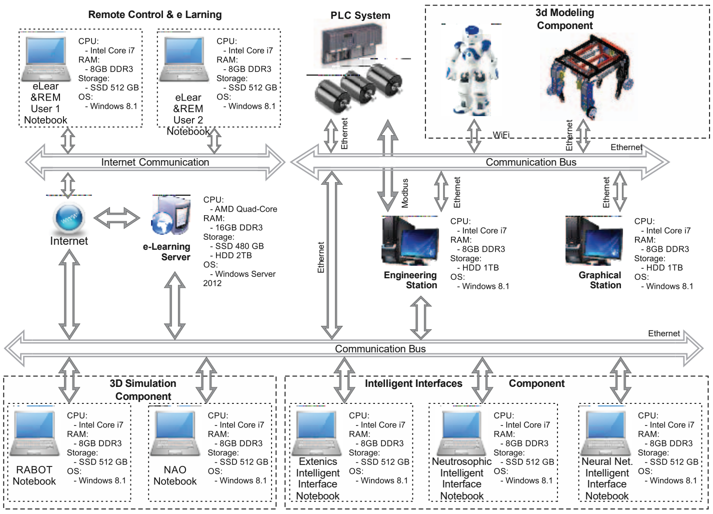

Figure 1. VIPRO Platform Architecture

The VIPRO Platform architecture for modelling and simulation of mobile robots is based on the virtual projection method [6, 8-11], through which robotics and mechatronics systems are developed in a virtual environment.

The technical solution, presented in an open architecture real time control structure (Figure 1), contains the main modules of the VIPRO Platform. The intelligent control interface module uses advanced control strategies adapted to the robot environment such as extended control (extenics) [12 - 14] , neutrosophic control [15 - 18], human adaptive mechatronics, etc., implemented through computational techniques for fast processing and real time communication. The following intelligent control interfaces have been designed and implemented on the VIPRO Platform: neutrosophic robot control interface (ICNs), extended control interface for robot extenics control (ICEx) and the neural network interface (INN) for dynamic hybrid force position control DHFPC [19, 20].

The two main components of the VIPRO Platform are the workstation “Engineering Station” for the PLC classic position control of robot joint actuators and speed control of load actuators, and the “Graphical Station” for the development of a virtual robot environment and virtual reality for robot motion.

The VIPRO Platform has allotted 5 user stations dedicated to modelling the NAO robot using direct and inverse kinematics, modelling the RABOT robot in the Unity development environment, neutrosophic intelligent control (ICN) through the integration of the RNC method, extended control through the extenics method (ICEx) and modelling inverse kinematics in the robot motion control using fuzzy inference systems and neural networks.

For remote control in establishing the e-learning component of the VIPRO Platform, a PC server was integrated to ensure large data traffic for internet communication, with two addition workstations for end-user applications.

The “Engineering Station” component is mainly aimed at integrating the AC500 development environment for programmable automate (PLC) applications, control of the application stand for the virtual projection method on 6 DOF, and testing of the intelligent neutrosophic control (ICNs), extenics control (ICEx) and dynamic hybrid force position control DHFPC interfaces.

After testing, these are integrated in real-time control of a new robot with improved performance and stability of motion through the Graphical Station, as follows: for multi-users through the components of the VIPRO Platform consisting of Remote Control& eLearning User 1, Remote Control& eLearning User 2 or individually through the VIPRO Platform components consisting of the dedicated intelligent interfaces on the Notebook workstations, namely “Extenics Intelligent Interface Notebook”, “Neutrosophic Intelligent Interface Notebook” and “Neural Network Intelligent Interface Notebook”.

Using 3D UNITY Simulation Component Applied to the VIPRO Platform, the paper presents automated estimation techniques for robot system identification.

III. AUTOMATED TECHNIQUES FOR PARAMETER ESTIMATION

A. Identification of PID parameters for the NAO robot

Designing the experimental model in a virtual 3D environment for testing and validation of the PID control law parameters of the robot joints entails an accurate identification of the system model through automated parameter estimation algorithms for both the PID controller, as well as the future implementation of intelligent control laws [21 - 23].

Ensuring the stability of the experimental virtual model, using PID control in the robot joints, requires knowledge of an approximate model of the controlled process, based on which the amplification factors for the parallel PID structure are established. The model is developed through system identification algorithms using known vectors of input and related output data of the unknown system. The Unity 3D environment allows data generation for input references to a virtual robot joint and monitoring its behavior, thus obtaining the required output data. These are used for the parameter estimation of system models applied to the phenomenon. After identifying the maximum likelihood model, the amplification factors for a PID controller structure are optimized and introduced into the Unity environment as a controller script for the robot joint. The robot joint in the Unity environment is treated as a black-box system, without the need to intervene on the development environment's libraries or source code (as relates to understanding or modifying the physics engine).

B. NAO Leg Joint 3D Simulation Data in UNITY

The two sets of data (the input and output vectors, respectively) are exported to an Excel working file in *.csv format, to be further imported into the numerical processing environment. An example is shown in Figure 2.

The data is imported into Matlab using the function xlsread, resulting in a data vector structure used to identify the position controlled system.

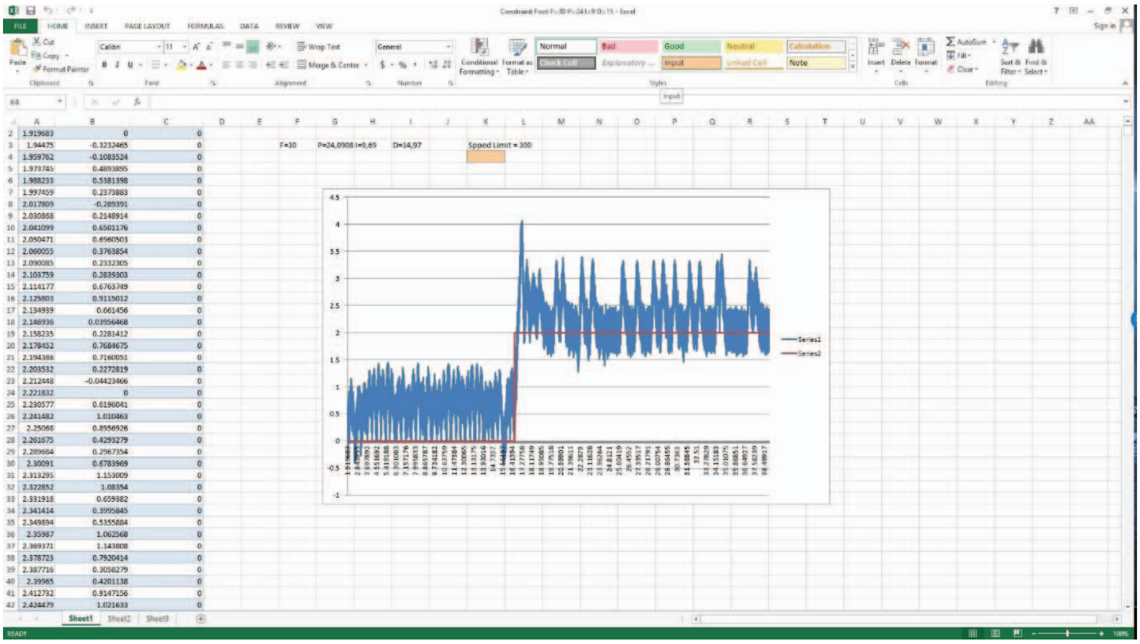

*Figure 2. Data structure exported from Unity to a *.csv file*

With the help of the native xlsread function, the file obtained in the Unity 3D tests can be imported into the Matlab environment for processing.

The result is shown in Figure 3, in which the data is structured as two vectors representing the reference (input variable) and the system output.

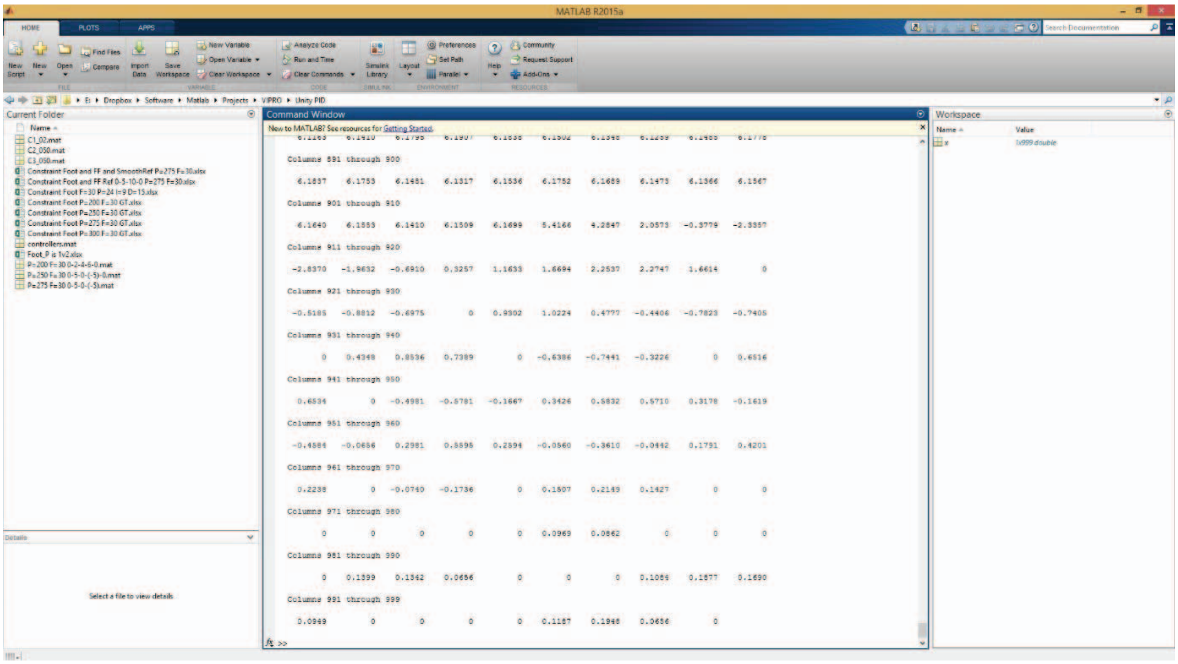

Figure 3. Vector imported into the workspace

C. Data pre-processing

The obtained data is pre-processed, as can be seen in Figure 4, to ease the task of system identification by eliminating nonessential areas and their respective noise. For example, the reference has a null value for the beginning of the test in order to calibrate the system. However, the acquired data shows nonzero values due to the existing noise in the simulation environment and the inexactness assumed in representing the mechanical system.

Figure 4. Pre-processing the obtained data

IV. SYSTEM IDENTIFICATION

System identification using the automated identification function from the Matlab toolbox is tried for a number of various order systems.

After the input data has been brought to a desired form and the possible noise components removed, system identification can begin in earnest. In the respective application in the Matlab programming environment are investigated a number of possible system models, with the program handling the automatic parameter identification.

Given a certain model chosen for representation, the interface optimizes automatically the model parameters using the samples from the input and output vectors (Figure 5).

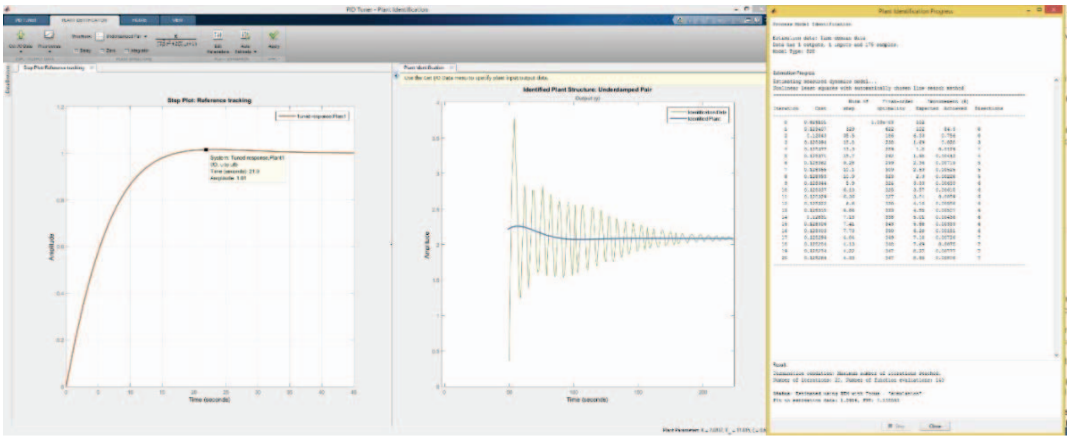

Figure 5. Model identification for two paired poles

A. Control model adaptation

For complex black-box system identification and adaptation to the nonlinear behaviour of the data set, the separate system identification interface is used. The various available options and the parameter optimization algorithms are shown in Figure 6.

B. Control law optimization interface

Optimization of the PID controller in a varied number of options and of the amplification factors for each of these, which control the chosen process, is developed in the virtual environment through an intelligent control interface.

Within the existing interface, the controller type was chosen (P/PI/PD/PID), as well as the desired response for a closed loop system including this controller. The estimated result, seen within the interface in Figure 7, has allowed establishing the desired transitive response, the convergence speed and the direct adjustment of the amplification factors of the three branches of the controller.

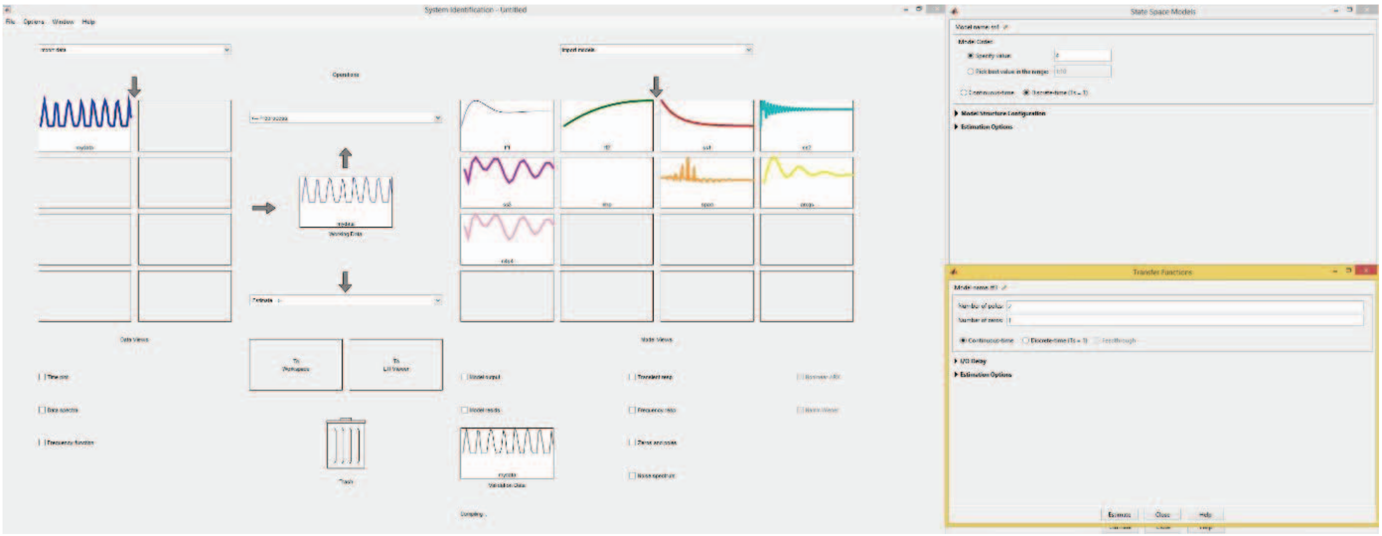

Figure 6. Comprehensive interface for robotic system identification

After the theoretical validation of the model, the obtained data is exported back into the virtual reality 3D environment in Unity, in which the controller behaviour is simulated through a compiled script, which generated the amplifications determined in the Matlab control interface presented above.

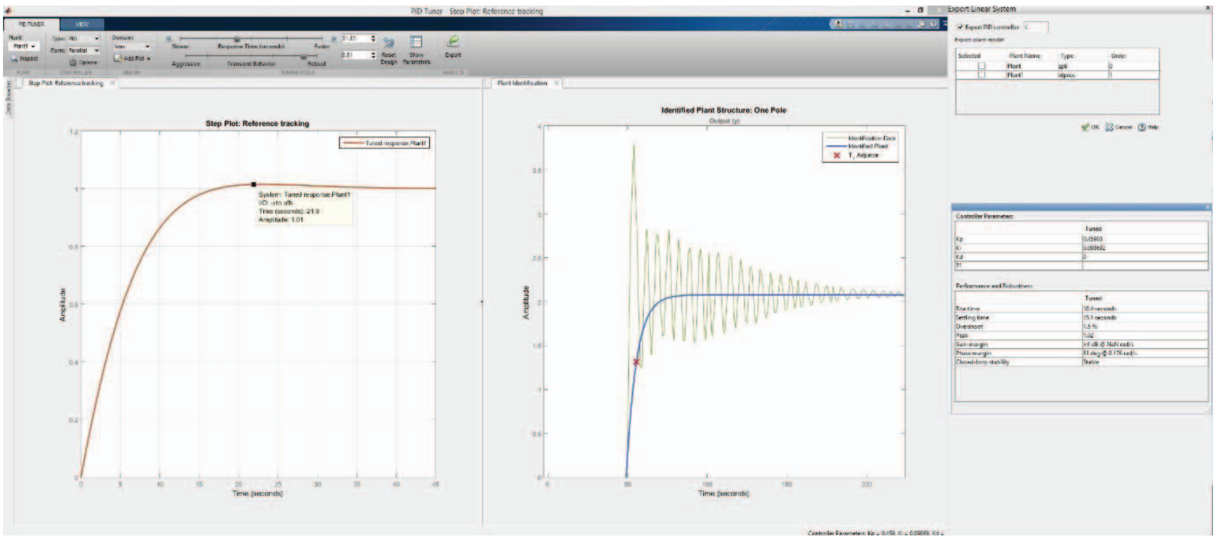

Figure 7. PID answer estimated

V. Results and Conclusions

Identifying the control law parameters for the Nao walking robot, using the 3D simulation component of the virtual reality platform Unity 3D, through automated parameter estimation techniques, for both PID control and future implementations of intelligent control laws, allows an improvement in stability and robot motion control in a virtual reality environment.

By applying the virtual projection method, the improvement in robot performance is transferred from the virtual world of modelling and simulation to the real world of experimental models, representing a powerful experimental validation tool.

The PID parameter identification program for the Nao humanoid robot using the virtual reality platform Unity 3Dand the 3D simulation component are shown in Figure 8, are available for users of the VIPRO platform and accessible from the VIPRO interface, either locally or from a remote location.

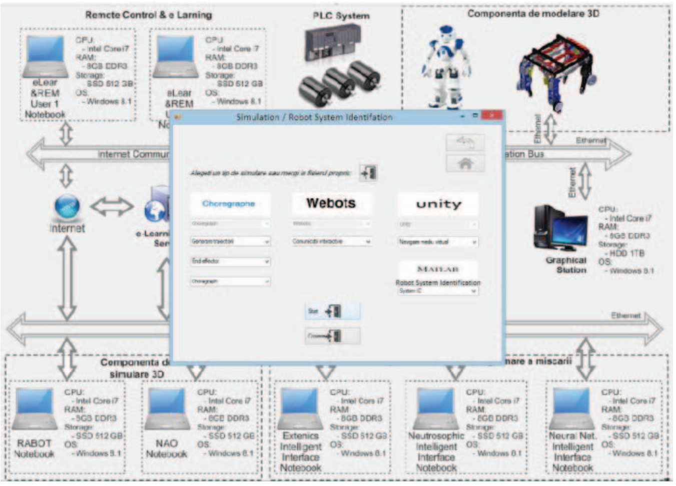

Fig.8. Robot system identification on VIPRO Platform

The obtained results, validated in the virtual reality environment, have led to the implementation on the VIPRO Platform in the 3D environment Unity, of the simulation component for the PID parameter identification for the NAO humanoid robot, with the possibility of extension to the RABOT search and rescue robot.

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What we can do to save humanity in the coming era of global eavesdroppers

(or The social innovation way to solve collective action problem)

Victor Christianto, Florentin Smarandache

Victor Christianto, Florentin Smarandache (2018). What we can do to save humanity in the coming era of global eavesdroppers (or the social innovation way to solve collective action problem), 5 p.

Abstract — **In this paper, we tried to draw a fair assessment on things which will take place soon with the coming era of IoT, 5G technology, global eavesdropping and all that. Nonetheless, we are aware that this article sounds quite gloomy. We are not techno-utopians (read Evvgeny Morozov's WSJ article on digital dictatorship[1]), but we are not techno-pessimists either. Perhaps you can consider us as: "*techno-realists*."[2] This paper was written in the same spirit of Jonathan L. Zittrain's book *The Future of Internet and how to stop it.***

Keywords — wireless technology, network security, mobile internet security, global eavesdropping, digital dictatorship.

I. Introduction

One of the great economists of 20th century, John Maynard Keynes, once remarked: "*Everybody wants to go to heaven, but not too soon*." Surely, it depends much on how you define heaven. If you define heaven as fast internet access anywhere, possibility of tracking everything, and plenty choices of movie channels, then you can expect your dream will be fulfilled soon. Especially considering recent news of 5G technology already in place for several cities in China, and *Digit Act Bill* passed by US Senate since 2016, and smartphones getting cheaper and cheaper each month.(1) So you can get access on everything faster than ever. Some futurists even declare the coming of "abundance" era, accelerated by rapid advancement of technology. But now the hard questions: is that really a heaven for the entire global population? Or, are we running faster to nowhere? Let us consider some real examples on how bad things can happen along the way.

II. A few examples

1. The leak report by Edward Snowden revealed ongoing advanced eavesdropping by NSA on the entire population of US citizens. Although the details are rather complicated, including perhaps a very peculiar software called PRISM, soon it became clear that such a report is not just fake. Another report reveals argument by intelligence community that such an eavesdropping is necessary in order to anticipate terrorism attack. But Snowden criticized effectiveness of massive surveillance on US ordinary people for tracking potential threat; instead he argued that such a massive surveillance only distracts intelligence community from doing real work on tracking potential harmful terrorists. His prediction became confirmed at the time of attack in Boston, and also in other areas – when no surveillance method could anticipate.

2. After Snowden story was forgotten, there is a recent report on the stolen passwords of all Yahoo email users, during 2013-2014. The number is quite staggering, not just 100 millions, not 200 millions, but the whole 3 billions users. Of course, nobody from Yahoo officers would admit whether they were just sloppy with their system, or they allowed a sort of backdoor access on PRISM eavesdropping. Other email service providers remain muted on this topic too.(2)

3. There is also a growing number of research papers discussing potential global eavesdropping on various wireless communication systems, including each and every piece of handheld devices.(3)

4. And with *Digit Act Bill*, we can expect there will be plethora of new kind of surveillance cameras with built-in RF technology.

5. On top of that, Internet of Things will enable remote controlling of devices, such as wireless sensors etc. Of course, official ads that you watch on television and newspapers only tell you the best out of these things, such as monitoring your kids at home while you are working and so on. But as the Murphy Law tells us, "*all things which can possibly go wrong, will go wrong*."(5) A number of dystopian movies like "Eagle Eye" depicts how bad things can go when you are being monitored 24 hours a day, and there is no such thing as privacy anymore. And sort of those things are already put in place or underway.

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1 https://www.wsj.com/articles/SB10001424052748703983004575073911147404540

2 http://www.technorealism.org/

III. WHAT IS GLOBAL EAVESDROPPER

According to Alejandro Proano et al.: (6)

> *Wireless sensor networks (WSNs) have shown great potential in revolutionizing many applications including military surveillance, patient monitoring, agriculture and industrial monitoring, smart buildings, cities, and smart infrastructures. Several of these applications involve the communication of sensitive information that must be protected from unauthorized parties. As an example, consider a military surveillance WSN, deployed to detect physical intrusions in a restricted area. Such a WSN operates as an event-driven network, whereby detection of a physical event (e.g., enemy intrusion) triggers the transmission of a report to a sink.*
>
> *Although the WSN communications could be secured via standard cryptographic methods, the communication patterns alone leak contextual information, which refers to event-related parameters that are inferred without accessing the report contents. Event parameters of interest include: (a) the event location, (b) the occurrence time of the event, (c) the sink location, and (d) the path from the source to the sink. Leakage of contextual information poses a serious threat to the WSN mission and operation. In the military surveillance scenario, the adversary can link the events detected by the WSN to compromised assets. Moreover, he could correlate the sink location with the location of a command center, a team leader, or the gateway. Destroying the area around the sink could have far more detrimental impact than targeting any other area. Similar operational concerns arise in personal applications such as smart homes and body area networks. The WSN communication patterns could be linked to one's activities, whereabouts, medical conditions, and other private information.*

In the above contexts, contextual information can be exposed by *eavesdropping* on over-the-air transmissions and obtaining transmission attributes, such as inter-packet times, packet source and destination IDs, and number and sizes of transmitted packets. (6)

IV. THE BIG PICTURE

In other words, with the coming of IoT, it would mean that we are in the dawn of global eavesdropping. So, what can we do to save our daily life as human being in this planet?

This situation looks really gloomy from each angle, but that will surely happen if we allow corporate-giants take control over each minute of our life – just like in Aldous Huxley's *The brave new world.*[3]

It reminds us to an old story:

> *"There was a guy who one night got into a nightmare, where he live in a country controlled by a terrible dictatorial governor in a province. Many people suffer under that governor. So, he asked himself: "What should I do now? Should I become a rebel, fighting for freedom? Or should I become a liberator, to avoid suffering of those people? Or should I work out my own way up to become a new governor, to replace that cruel bastard? Finally, he came up with a simpler solution: he woke up from his dream. That way he became conscious."*

Perhaps the lesson of the above story is quite similar with a wonderful Italian movie: *Life is beautiful*.[4]

The movie tells a story of an Italian Jewish bookseller called Guido, who just married with Dora. And they got a boy (Giosue). Their happiness was abruptly halted, however, when Guido and Giosue were separated from Dora and taken to a concentration camp. Determined to shelter his son from the horrors of his surroundings, Guido convinced Giosue that their time in the camp is merely a game. He told that in the end his boy will get a prize: a tank. At the end of the movie, Guido did not survive, but his wife and Giosue did. Then a US soldier put him up to a tank, just like what his father promised.

The lesson is that no matter how hard the situation will be, actually we determine our own state of mind. We can choose to be happy, or to be defeated in spirit. We can choose to be human or to be absorbed in the entire system of global eavesdropping. Therefore, let us now consider what our options are.

Here are a few options which you can consider:

1. There are extreme ways of living advocated by technophobia people (Luddism), like cutting off your internet wires, throwing your laptop out of the window, and go to a remote mountain or find the end of the rainbow. We certainly do not advocate that.

2. Going to an exoplanet, a few million light years away from here, is not an option either. Perhaps we should give a decade or more to visionary people like Elon Musk or Jeff Bezos to figure out how we can go there, if it is possible at all.

3. So, for the rest of us, what we can do is to use internet technologies wisely. Update regularly your antivirus software, and change your passwords each 2 months or sooner. And don't use too much free wi-fi in public places, because many people can track you. But if it is okay for you to be monitored by someone else. It is up to you.

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[3] http://www.idph.com.br/conteudos/ebooks/BraveNewWorld.pdf

[4] https://en.wikipedia.org/wiki/Life_Is_Beautiful

4. If you belong to millennial generation, chance is you have become more adept with all these tips. But perhaps you want to do more for society. Our advice is, quoting a word of wisdom for environment activists in 90s: "*Think globally, act locally*." That would mean you should better find a number of friends near you who think likewise, and try to do something good for your community, be it helping orphanage or something like that. We have heard that a number of CEOs only work 3-4 days a week, and they spend the rest of the week to do what they can do for their community.

5. If your small group gets larger and becomes a national movement, then things get interesting. Do not do lobbying to Senate like those big oil companies in order to advance their interests. Instead, you can try to solve Mancur Olson's problem: "*how your group can do collective action at large scale, while the benefits are not so tangible for everyone*" (4). Our hypothesis is: Olson's collective action problem only applies to unconnected society. In a heavily connected society like ours now, we can figure out how to solve this Olson's dilemma, and doing some meaningful collective actions in the internet.[5] For example: there are some initiatives of online crowdfunding, crowdsourcing, and online cooperatives.[6] So, actually you can start to do something good to your community even with a small amount of fund, provided you plan properly and do it collectively.

6. A few hints for IT folks
If you are IT folks, perhaps you can try to do some advanced tips as follows:

> *To mitigate global eavesdropping, Proano et al. proposed traffic normalization methods that regulate the sensor traffic patterns of a subset of sensors that form MCDSs. They developed two algorithms for partitioning the WSN to MCDSs and SS-MCDSs and evaluated their performance via simulations. Compared to prior methods capable of protecting against a global eavesdropper, they showed that limiting the dummy traffic transmissions to MCDS nodes, reduces the communication overhead due to traffic normalization.(6)*

V. THE UTILITARIAN QUESTION: PSYCHOPATHIC TRAITS INSIDE OUR MINDS

By suggesting an option to do collective action, it does not mean we are not aware that each of us has selfish motive. In fact, some of us on top of the ladder of society have inclination to be a psychopath. Let us quote an interesting article by Lindsay Dodgson: (9)

> In the Diagnostic and Statistical Manual of Mental Disorders, or DSM-5, antisocial or psychopathic personality types are defined as having an inflated, grandiose sense of themselves, and a habit of taking advantage of other people. However, it's still a hard disorder to define, as most of us have some psychopathic traits. In fact, some psychologists believe everyone falls on the psychopathy spectrum somewhere.
> On their own, some traits are beneficial to us, such as keeping a cool head, and having charisma. This is why many psychopaths become CEOs, because they can look at the cold, hard facts and make decisions without becoming emotionally involved.
> Still, a number of researchers have attempted to find a way of diagnosing psychopathic behavior. One well-known test for psychopaths is the "*The Hare Psychopathy Checklist*," which analyses how you see yourself and other people.
> The team from Columbia Business School and Cornell Universities gave participants a set of moral dilemmas, and also asked them to complete three personality tests: one for assessing psychopathic traits, one assessing Machiavellian traits, and one assessing whether they believed life was meaningful.
> This was one of the questions they were asked:
>
> > *"A runaway trolley is about to run over and kill five people and you are standing on a footbridge next to a large stranger; your body is too light to stop the train, but if you push the stranger onto the tracks, killing him, you will save the five people. Would you push the man?"*
>
> The team found that those who answered the dilemmas with an "ethic of utilitarianism" — the view which says the morally right action is whichever one produces the best consequence overall — possessed more psychopathic and Machiavellian personality traits. In the above question, if you'd choose to push the man, you have more in common with the people who had psychopathic or Machiavellian traits.
>
> This makes sense when you think about how Machiavelli generally believed *"the ends justifies the means*," and that killing innocent people could be normal and effective in politics, as long as the outcome was for the greater good.(9)

This article seems convince us that we need to become aware on our own tendency of being a psychopath. Moreover, it takes honesty to admit that we are prone to be selfish person…then we can work out to be a better person. But there is a deeper question: if controlling our own motive can be very difficult, then where our society is heading? What are our choices?

[5] An outline of reasoning to support this hypothesis has been written by us, but it is not included here due to space limitation. Interested reader may wish to contact corresponding author.
[6] For example: www.startsomegood.com

VI. WHERE WE ARE HEADING FROM HERE

Now, some of you may ask: by suggesting solution to Olson's collective action problem to save our humanity, where is the article heading? Are we advocating collective society as in old day Marxism hammer? Or are we advocating how to escape from the curse of capitalism's social darwinism?[7]

Yes, normally you read numerous political-economics jargons, e.g. leftist, right wing, centrist left or centrist right and so on.

But it is not our intention to submit another ideological parlance. In fact, these authors are scientist and mathematician, so we are not so inclined to any parlance.

In our opinion, our tendency to cooperate or compete is partly influenced by the culture that we inherit from our ancestors. One of us (VC) once lived for a while in Russia, and he found that many people there are rather cold and distant (of course not all of them, some are friendly). He learned that such a trait is quite common in many countries in Europe. They tend to be individual and keep a distant to each other. In physics term, they are like *fermions*.[8]

There is a developmental psychology hypothesis that suggests that perhaps such a trait correlates to the fact that many children in Europe lack nurtures and human touch from their parents, which make them rather cold and individual. Of course, whether this is true correlation, it should be verified.

On the contrary, most people in Asia are gregariously groupie (except perhaps in some big metropolitans). They tend to spend much time with family and friends, just like many Italians. They attend religious rituals regularly, and so on. In physics term, they are *bosons*. Of course, this sweeping generalization may be oversimplifying.[9]

Therefore, it seems quite natural to us, why Adam Smith wrote a philosophy book suggesting that individual achievement is a key to national welfare (because he was a British which emphasized individualism).[10] It took more than hundred years until mathematicians like John F. Nash, Jr. figured it out that individual pursuit towards their own goals will not lead them to achieve a common goal as society.

That is why, we choose to work out Mancur Olson's theorem, because he is able to condense the complicated game theoretical reasoning (whether one should cooperate or not) into a matter of collective actions.

So, which is better: *to be like fermions or bosons?* Our opinion is: just like in particle physics, both fermions and bosons are required. In the same way, fermion behavior and boson behavior are both needed to advance the quality of life. Fermion people tend to strive toward human progress, while boson people are those who make us alive. Just like an old song: Ebony and Ivory….they make harmony in society.

We hope this paper help us to see that collective actions are what made us a human society.[11] And it seems related to *social innovations* and also social *capital* too, in other words a society with social capital and collective actions will ensure its sustainable future.[12] But this is beyond the scope of this article, let us leave such a discussion to economists.

But this article surely does not offer a bold answer to where we are heading as global community. Do we arrive at the end of history or this is just a beginning to a new era? Let time will tell.

VII. CONCLUDING REMARKS

In this paper, we tried to draw a fair assessment on things which will take place soon with IoT, 5G and all that. Nonetheless, we are aware that this article sounds quite gloomy. We are not techno-utopians (read Evvgeny Morozov's WSJ article on digital dictatorship[13]), but we are not techno-pessimists either. Perhaps you can consider us as:

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[7] See for example Richard Hofstadter: Social Darwinism in American thought. url: http://culturism.us/booksummaries/SocialDarwinismHofst.pdf

[8] While our proposed simplifying analogy of human behaviour, i.e. individualism and collectivism sound not so common. Indeed such cultural psychology research has been reported since Harry C. Triandis et al. See for example: (a) The Self and Social Behavior in Differing Cultural Contexts, Psychological Review, vol. 96 no. 3; (b) Harry C. Triandis and Eunkook M. Suh, CULTURAL INFLUENCES ON PERSONALITY, Annu. Rev. Psychol. 2002. 53:133–60; (c) J. Allik & A. Realo, Individualism-collectivism and social capital, JOURNAL OF CROSS-CULTURAL PSYCHOLOGY, Vol. 35 No. 1, January 2004, 29-49. This last mentioned paper includes a quote from Emile Durkheim: "The question that has been the starting point for our study has been that of the connection between the individual personality and social solidarity. How does it come about that the individual, whilst becoming more autonomous, depends ever more closely upon society? How can he become at the same time more of an individual and yet more linked to society?"

[9] After writing up this article, we found that Sergey Rashkovskiy also wrote a quite similar theme, albeit with a statistical mechanics in mind. The title of his recent paper is: "'Bosons' and 'fermions' in social and economic systems." Here is abstract from his paper: "We analyze social and economic systems with a hierarchical structure and show that for such systems, it is possible to construct thermostatistics, based on the intermediate Gentile statistics. We show that in social and economic hierarchical systems there are elements that obey the Fermi-Dirac statistics and can be called fermions, as well as elements that are approximately subject to Bose-Einstein statistics and can be called bosons. We derive the first and second laws of thermodynamics for the considered economic system and show that such concepts as temperature, pressure and financial potential (which is an analogue of the chemical potential in thermodynamics) that characterize the state of the economic system as a whole, can be introduced for economic systems." Url: https://arxiv.org/ftp/arxiv/papers/1805/1805.05327.pdf

[10] If only Adam Smith was born in Bangkok or Manila, probably he wrote his book in a different way.

[11] In our country, there is a specific word for some people who work together to achieve a common goal: "*gotong royong.*"

[12] Emily Ouma & Awudu Abdulai. Contributions of Social Capital Theory in Predicting Collective Action Behavior among Livestock Keeping Communities in Kenya. url: https://ageconsearch.umn.edu/bitstream/49994/2/Manuscript%20No.%20423_Social%20capital%20theory%20and%20collective%20action.pdf

[13] https://www.wsj.com/articles/SB10001424052748703983004575073911147404540

"*techno-realists*."[14] This paper was written in the same spirit of Jonathan L. Zittrain book's *The Future of Internet and how to stop it.*

Allow us to conclude this message with a short message: "*With the coming era of global eavesdroppers, it is not the end of history (Fukuyama). But it will be the end of humanity as we know it, unless we do something collectively to prevent it to happen*." Thank you.

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url: www.hup.harvard.edu/catalog.php?isbn=9780674537514 "Olson develops a theory of group and organizational behavior that cuts across disciplinary lines and illustrates the theory with empirical and historical studies of particular organizations, examining the extent to which individuals who share a common interest find it in their individual interest to bear the costs of the organizational effort."

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The World Within Us

(Or A Sketch of Consciousness Space Beyond Freudian Mental Model and Implications to Socio-Economics Modeling and Integrative Cancer Therapy)

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Victor Christianto, Florentin Smarandache (2018). The World Within Us (Or A Sketch of Consciousness Space Beyond Freudian Mental Model and Implications to Socio-Economics Modeling and Integrative Cancer Therapy). BAOJ Cancer Research & Therapy 4(3): 057, 13

Abstract

In this paper, we give an outline of an ongoing study to go beyond Freudian mental archetypal model. First, we discuss the essence of nu-merous problems that we suffer in our sophisticated and modernized society. Then we discuss possibility to reintroduce spirit into human consciousness. While we are aware that much remain to be done and we admit that this is only a sketch, we hope that this paper will start a fresh approach of research towards more realistic nonlinear con-sciousness model with wide ranging implications to socio-economic modeling and also integrative cancer therapy. At the last section we also shortly outline plausible method to vindicate our proposed bo-son-fermion model of human society in a physical experiment.

Introduction

One of us (FS) recently published a new book, with title: Neutropsy-chic personality [1]. In this book, FS described possible extension of Freudian mental model: id-ego-superego, using his Neutrosophic Logic theory. He goes on to develop implications of this approach.

Later on, we thought that it would be necessary to push the boundary one step further, by considering a more realistic way to go beyond that classic Freudian mental model, i.e. by reintroducing the spirit into hu-man consciousness model.

We are aware that many researchers have proposed such an extension, especially Italian tradition which was continually developed by stu-dents of Carl Jung, such as Assagioli and Piere Ferrucci, namely the Psychosynthesis movement. See for example [2].

But here we offer a different starting point of mental model, based on Matthew 22, i.e. The Great Commandments. As far as we know, i.e. this is the simplest model of human consciousness, yet it is profoundly inspired by the Bible.

This author adopts a rather relaxed approach to present their ideas, with the hope to stimulate both sides of your brain, in order you can realize on how we as human society badly need thoroughly review the present healthcare especially to socio-psychiatry and also to cancer therapy.

Problem with this Modern Society

Our modernized and highly sophisticated society bring numerous advantages over our ancestors, but it is not without consequences. To summarize, we are running anywhere but we find less and less happi-ness, as it has been pointed long time ago by Albert Einstein.

As per records in Caltech, he once spoke [1]:

"Why does this magnificent applied science, which saves work and makes life easier, bring us so *little happiness?* The simple answer is because we have not yet learned to make sensible use of it. In war, it serves that we may poison and mutilate each other. In peace, it has made our lives hurried and uncertain instead of freeing us in great measure from spiritually exhausting labor. It has made men into the slaves of machinery, who for the most part complete their monotonous long days' work with disgust, and must continually tremble for their poor rations. You will be thinking that the old man sings an ugly song. I do it, however, with a good purpose, in order to point out a consequence. It is not enough that you should understand about applied science in order that you may increase man's blessings.

Concern for man himself and his fate always forms the chief interest of all technical endeavors. Concern for the great unsolved problems of the organization of labor, for the distribution of goods, *in order that the creations of our minds shall be a blessing and not a curse. Never forget this in the midst of your diagrams and equations.*" (italic emphasizes by these authors)

Although that speech was translated from German, but the essence remain relevant even for our today's life as scientists, as Harry Gray once remarked:

"That was Albert Einstein on February 16, 1931, to the Caltech student body, translated by somebody and slightly retranslated by me. -- Obviously, what he said over 40 years ago has relevance to our situation today [1]."

Moreover, we are constantly under pressure in every direction of our life. Perhaps the best sociologist and observer of this heavy burden of life is *Queen*, a British super group from 70-90s era:

Under Pressure

Queen, David Bowie

Mm ba ba de
Um bum ba de
Um bu bu bum da de
Pressure pushing down on me
Pressing down on you no man ask for
Under pressure that brings a building down
Splits a family in two
Puts people on streets
Um ba ba be
Um ba ba be
De day da
Ee day da - that's okay
It's the terror of knowing
What the world is about
Watching some good friends
Screaming 'Let me out'
Pray tomorrow gets me higher
Pressure on people people on streets
Day day de mm hm
Da da da ba ba
Okay
Chippin' around - kick my brains around the floor
These are the days it never rains but it pours
Ee do ba be
Ee da ba ba ba
Um bo bo
Be lap
People on streets - ee da de da de
People on streets - ee da de da de da de da
It's the terror of knowing
What this world is about
Watching some good friends
Screaming 'Let me out'
Pray tomorrow - gets me higher higher high
Pressure on people people on streets
Turned away from it all like a blind man
Sat on a fence but it don't work
Keep coming up with love but it's so slashed and torn
Why - why - why?
Love love love love love
Insanity laughs under pressure we're breaking
Can't we give ourselves one more chance
Why can't we give love that one more chance
Why can't we give love give love give love give love
Give love give love give love give love give love
'Cause love's such an old fashioned word
And love dares you to care for
The people on the (People on streets) edge of the night
And loves (People on streets) dares you to change our way of
Caring about ourselves
This is our last dance
This is our last dance
Under pressure
Under pressure
Pressure

Songwriters: David Bowie / John Deacon / Brian Harold May / Freddie Mercury / Roger Taylor

Under Pressure lyrics © Peermusic Publishing, Sony/ATV Music Publishing LLC

Now the question is: how can we find out the root cause of this problem of modern society?

Allow us to recall what Adam Grant emphasizes: the basic human motives are selfishness and altruism.

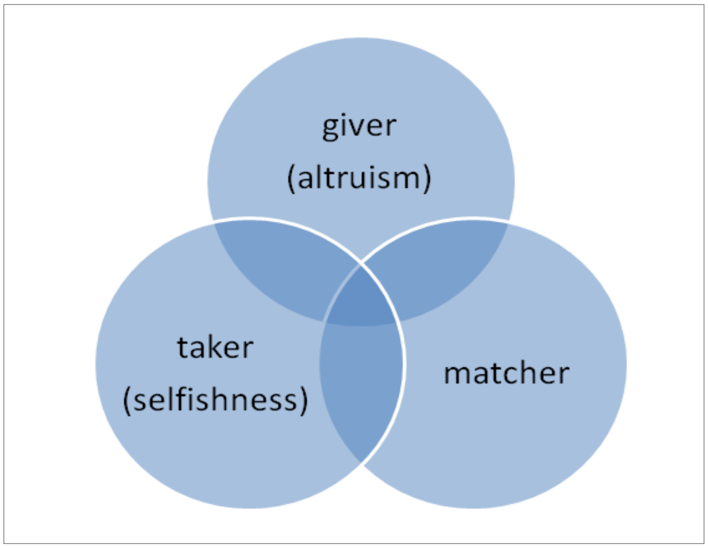


Figure 1: Adam Grant's model of human basic traits

And also we can recall from Genesis 3 that the first fall of our ancestors came from greediness. Now, do you realize: *"How far we have fallen in this modern society, where greed has been hailed as highest virtue?"* Quoting Grekko's remark: *Greed is good.*

"The point is, ladies and gentleman, that greed, for lack of a better word, is good. Greed is right, greed works. Greed clarifies, cuts through, and captures the essence of the evolutionary spirit. Greed, in all of its forms; greed for life, for money, for love, knowledge has marked the upward surge of mankind."

We consider this is the true core of our modern reality, all of us have been consumed and drowning in the ocean of greediness. The real irony is that greediness has eaten us alive, from our childhood until we die. Even if we once die, there are those greed developers who sell a piece of cemetery with high price. They capitalize our bodies, our eyes, our jealousy, our heart, our mind, our consciousness. Literally speaking, we are more or less as walking zombies. We are getting improved at the outside, but we are no more than rotten tomatoes deep inside.

At this point, some may ask: How can we repair such a deep problem of our modern society?

Outline of Reasoning: Toward Pneumatological View of Psychology

We all know that Hebrew's thought on human being is integral, i.e. the whole of body-mind-spirit. But how can we come up with a model of human consciousness based on the Bible?

As a starting point, we choose to begin with Jesus's sayings, instead of using other trivial sources.

Let us begin by the Greatest Commandment

Matthew 22:37-40King James Version (KJV)

37Jesus said unto him, Thou shalt love the Lord thy God with all thy heart, and with all thy soul, and with all thy mind.

38This is the first and great commandment.

39And the second is like unto it, Thou shalt love thy neighbour as thyself.

40On these two commandments hang all the law and the prophets.

Our re-reading of the above commandments lead us to model a Trinitarian dialogue within human self: God, self, and others.

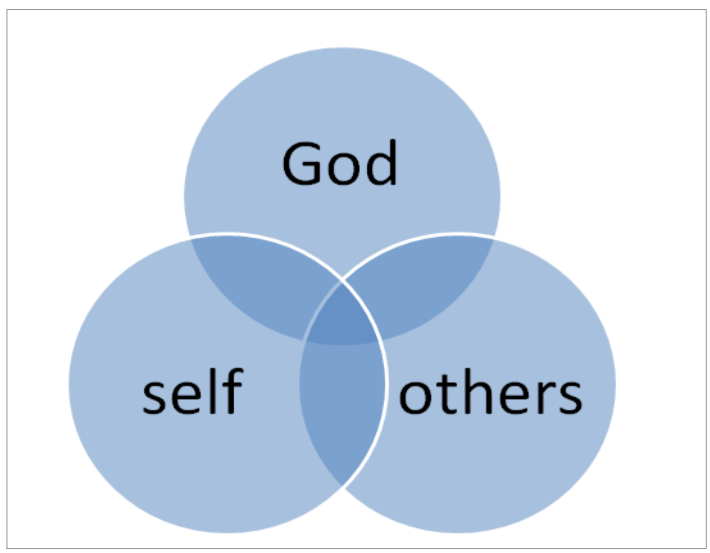


Figure 2: Three directions of human love based on The Greatest Commandments in Matthew 22:37-40.

Comparing with Adam Grant's give and take model of human basic tensions inside our mind. Let us consider parallels, i.e. "taking" reflects selfishness/greediness motive of ego, and "giving" reflects altruism motive of conscience.

In other word, now we have two entities in human consciousness: ego and conscience. There is always deep tension between ego and consciousness, between selfishness and altruism. Along these two poles, we need a third entity which has purpose to ease and being intermediary between these two motives. In this problem, along with Neutrosophic Logic, allow us to submit wholeheartedly that the third entity, is actually no other than "the spirit." (pneuma in Greek, ruach in Hebrew)

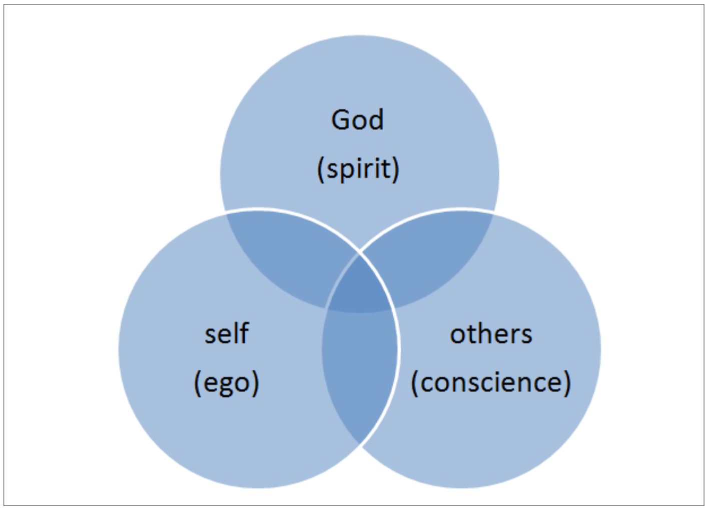


Figure 3: A model of human consciousness based on The Greatest Commandments in Matthew 22:37-40..

The exact role of human spirit is to enlighten both ego and conscience. While some may raise question of what is new here? It seems similar with id-ego-superego model.

No, it is really in contrast with Freud's model which is purely materialistic in origin. The notion of spirit is rejected in freud model, that is why mankind reduces to animals in his model, determined by his/her sexual instinct. And there is no way out of such animal instinct in his model.

Sometimes it is called transpersonal psychology:

*"**Transpersonal psychology** is a sub-field or "school" of **psychology** that integrates the **spiritual** and transcendent aspects of the human experience with the framework of modern **psychology.** It is also possible to **define** it as a "**spiritual psychology**".*

An interesting argument spiritual psychology has been discussed in University of Santa Monica's site:

"If you look up the word "psyche" in the dictionary, you will find "breath, principle of life, Soul." But if you look up "psychology," you will find "the science of mind and behavior." Somehow, in the translation from essence to practice, the most important aspect of "psyche" has been lost. At the University of Santa Monica, we recognize our task as reintegrating the spiritual dimension back into the essence of an authentic psychological inquiry. It is this reintegration that evokes the emergence of a Spiritual Psychology.

Spiritual Psychology is the study and practice of the art and science of Conscious Awakening. To engage in this genre, we must begin by distinguishing the essence of human evolution—what does it mean to evolve? In short, it means learning how to identify, recognize, and navigate successfully within the Context of Spiritual Reality. Practically, it means learning how to surrender—or let go of—anything that disturbs one's peace. It also means sacrificing our illusions of separation. Essentially, this "surrendering" and "sacrificing" is work that can and has been called "healing," which includes healing on the physical, mental, and emotional levels in service to the deeper revelation of who we truly are as Loving, Peaceful, Compassionate, and Joyful beings. We refer to this level of awareness as the Authentic Self."

Of course, there are various approaches of spiritual psychology. But, what is really different in our simpler model?

Two Possible Implications: a) in Socio-Economics Model

In this time we would only discuss the economics implications, based on modelling human identities into two opposites: (a) individualism (we call them: fermions), and (b) collectivism (we call them: bosons).

In a recent paper, we discuss how to solve Mancur Olson's collective action problem [3].

Now, some of you may ask: by suggesting solution to Olson's collective action problem to save our humanity, where is the article heading? Are we advocating collective society as in old day Marxism hammer? Or are we advocating how to escape from the curse of capitalism's social darwinism?

Yes, normally you read numerous political-economics jargons, e.g. leftist, right wing, centrist left or centrist right and so on.

But it is not our intention to submit another ideological parlance. In fact, these authors are scientist and mathematician, so we are not so inclined to any parlance.

In our opinion, our tendency to cooperate or compete is partly influenced by the culture that we inherit from our ancestors. One of us (VC) once lived for a while in Russia, and he found that many people there are rather cold and distant (of course not all of them, some are friendly). He learned that such a trait is quite common in many countries in Europe. They tend to be individual and keep a distant to each other. In physics term, they are like fermions.

There is a developmental psychology hypothesis that suggests that perhaps such a trait correlates to the fact that many children in Europe lack nurtures and human touch from their parents, which make them rather cold and individual. Of course, whether this is true correlation, it should be verified.

On the contrary, most people in Asia are gregariously groupie (except perhaps in big metropolitans). They tend to spend much time with family and friends, just like many Italians. They attend religious rituals regularly, and so on. In physics term, they are bosons. Of course, this sweeping generalization may be oversimplifying.

Therefore, it seems quite natural to us, why Adam Smith wrote a philosophy book suggesting that individual achievement is a key to national welfare (because he was a British which emphasized individualism). It took more than hundred years until mathematicians like John F. Nash, Jr. figured it out that individual pursuit toward their own goals will not lead them to achieve a common goal as society.

That is why, we choose to work out Mancur Olson's theorem, because he is able to condense the complicated game theoretical reasoning (whether one should cooperate or not) into a matter of collective actions.

So, which is better: to be like fermions or bosons? Our opinion is: just like in particle physics, both fermions and bosons are required. In the same way, fermion behavior and boson behavior are both needed to advance the quality of life. Fermion people tend to strive toward human progress, while boson people are those who make us alive. Just like an old song: Ebony and Ivory….they make harmony in society.

We hope this paper helps us to see that collective actions are what made us a human society. And it seems related to social innovations and also social capital too, in other words a society with social capital and collective actions will ensure its sustainable future. But this is beyond the scope of this article, let us leave such a discussion to economists.

So, by introducing this analogy from particle physics theories, we hope to resolve the classic clash between socialism and capitalism, which are no other than a cruel reformulation of the above basic human motives into political struggles, in attempt to put the entire mankind into eternal slavery.

Too many decades have been wasted by numerous countries to fight on these ideologies, but the truth is these opposite ideological poles were crafted in order to trap mankind into eternal struggles.

It needs to be stopped right now.

In Appendix I of this paper, the author gives a reflection on how we should slow down our pace, to become in tune with the speed of love, i.e. the speed of Jesus Christ: 3 mile-an-hour (cf. Kosuke Koyama).

Two possible implications: b) integrative cancer therapy

In this time we would only give a rough sketch of our ideas in cancer therapy, based on the aforementioned: ***Pneumatological approach to psychology.***

In the light of the fact that proper discussion of theology of medicine is quite rare, this section highlights the fundamental problem with modern (Western) medicine. China has taken a step forward by recognizing their cultural heritage called TCM. Of course it must be acknowledged that modern (Western) medicine has been very advanced, but also many problems such as side effects and also many toxic materials due to synthetic materials. It is also well known that chemotherapy has a chance to work at a miserable rate of less than 20%, so it is reasonable to argue that the 21st century requires a conceptual, new approach to treatment.

A few months ago, a respected senior professor of physics in Indonesia, Prof. Dr. Bambang Hidayat, a member of the Indonesian Academy of Sciences, sent an article to a group of academics. In essence he asked: how our response should be to China's recent policies that want to facilitate the practice of treatment based on TCM (traditional Chinese Medicine) in a balanced way.

His concern is certainly understandable, given the current perception of society is that traditional medicine, often referred to as alternative medicine, is usually associated with shamanic practices or strange methods such as turtles, snakes, bruises etc., many of which have not passed any clinical trials.

But there are two important things that we should take note of Xi Jinping's new policy on TCM:

a. This policy starts from realizing that the cost of Western medicine is very expensive, mainly due to clinical trials of humans, so it is quite reasonable that the Chinese government wants to give more balanced attention to the Chinese medicine tradition.

b. Traditional Chinese medicine has grown for no less than 4000 years. However, we shall also note that there are some reports that in Asia, liver cancer can be linked to the use of (excessive) herbal medicines.

Of course this needs further study [4].

Regarding some people's concerns about the removal of clinical trials, it seems the Chinese government is quite cautious, see the following quote:

"Lixing Lao, director of Hong Kong University's School of Chinese Medicine, says that although traditional medicines will no longer need to go through clinical trials, the CFDA will still require remedies to undergo preclinical pharmacological testing and drug-toxicity studies in animals or cells to gain approval" [3].

Certainly it can be expected that the new policy will further strengthen the interest of people to develop and produce drugs based on herbs that have been known to be useful for thousands of years, rather than synthetic (artificial) substances that could potentially not be processed and become toxic [5].

In Indonesia, it is also known a variety of medicinal plants, and there are several apps that provide catalog of such live pharmacies. One of which can be called for example is gendola, which reportedly efficacious for diabetes, cancer, stroke, coronary heart, liver etc. Of course clinical trials are required for this gendola [6].

The Fundamental Problem of Modern Medicine (Western)

There are several scientific authors who express vividly how fundamental the problem with modern (Western) medicine. The fundamental problem is commonly expressed with a *mechanistic* worldview as well as a Cartesian dualism philosophy [7,8].

Sheldrake has revealed that the mechanistic view is actually derived from Neo-Platonic philosophy, so it is not based on biblical teaching [9].
A similar argument was developed by Fritjof Capra in his famous book, *The Turning Point* [10]. In rather similar tone, Christian philosopher *Alvin Plantinga* has written a paper criticizing materialism [8].

Unfortunately, however, the thinking of scientists from such disciplines often fails in the midst of massive dis-information (and advertising) that modern (Western) medicine has managed to address almost all human health problems. Is that true?

Let's take a look at the colonial post-reading of Gen. 2: 7 and some other texts.

The post-colonial reading of Gen. 2: 7

If we glance at Gen. 2: 7, we see at a glance that man is made up of the dust of the ground (*adamah*) which is breathed by the breath of life by God (*nephesh*). Here we can ask, does this text really support the Cartesian dualism view?

We do not think so, because the Hebrew concept of man and life is integral. The bottom line: it is not the spirit trapped in the body (Platonic), but the body is flowing in the ocean of spirit [11]. This means that we must think of as an open possibility for developing an integral treatment approach (Ken Wilber), or perhaps more properly called **"spirit-filled medicine."**

Let's look at three more texts:

a. Gen. 1: 2, "The earth is without form and void, darkness over the deep, and the Spirit of God hovering over the waters." Patterns such as Adam's creation can also be encountered in the creation story of the universe. Earth and the oceans already exist (similar to adamah), but still empty and formless. Then the Spirit of God hovered over it, in the original text "*ruach*" can be interpreted as a strong wind (storm). So we can imagine there is wind/hurricane, then in the storm that God said, and there was the creation of the universe. From a scientific point of view, it is well known in aerodynamics that turbulence can cause sound (turbulence-generated sound). And primordial sound waves are indeed observed by astronomers.

b. Ps. 107: 25, "He said, he raised up a storm that lifted up his waves." The relation between the word (sound) and the storm (turbulence) is interactive.

Which one can cause other. That is, God can speak and then storms, or the Spirit of God causes a storm. Then came the voice.

c. Ezekiel. 37: 7, "Then I prophesy as I am commanded, and as soon as I prophesy, it sounds, indeed, a crackling sound, and the bones meet with one another." In Ezekiel it appears that the story of the creation of Adam is repeated, that the Spirit of God is blowing (storm), then the sound of the dead bones arises.

The conclusion of the three verses above seems to be that man is made up of adamah which is animated by the breath or Spirit of God. He is not matter, more accurately referred to as spirit in matter. Like a popular song around 80s goes: "We are spirits in the material world." See also Amos Yong [11]. Therefore, it is inappropriate to develop only materialistic or Cartesian dualism treatment. We can develop a more integral new approach [7].

The integral view of humanity and spirituality, instead of two-tiered Western view of the world, appears to be more in line with majority of people in underdeveloping countries, especially in Asia and Africa. See for instance the work by Paul Hiebert [7,12].

Among the studies supporting such an integral approach is the view that cells are waves, see the paper from Prof. Luc Montagnier [13,14]. And also our paper on the wave nature of matter, as well as the possibility of developing a wave-based (cancer) treatment [15,16].

Concluding Remarks

In this paper, we give an outline of an ongoing study to go beyond Freudian consciousness model. First, we review a recent book by our colleague, FS. Neutropsychology. Then we discuss possibility to reintroduce spirit into human consciousness. While we are aware that much remain to be done and we admit that this is only a sketch, we hope that this paper will start a fresh approach of research towards Pneumatological view of psychology in a realistic nonlinear consciousness space view.

This short article also highlights the fundamental problem with modern (Western) medicine. China has taken a step forward by recognizing their cultural heritage called TCM. Of course it must be acknowledged that modern (Western) medicine has been very advanced, but also many problems such as side effects and also many toxic materials due to synthetic materials. It is also well known that chemotherapy has a chance to work for less than 20%, so it is reasonable to argue that the 21st century requires a conceptual, new approach to treatment.

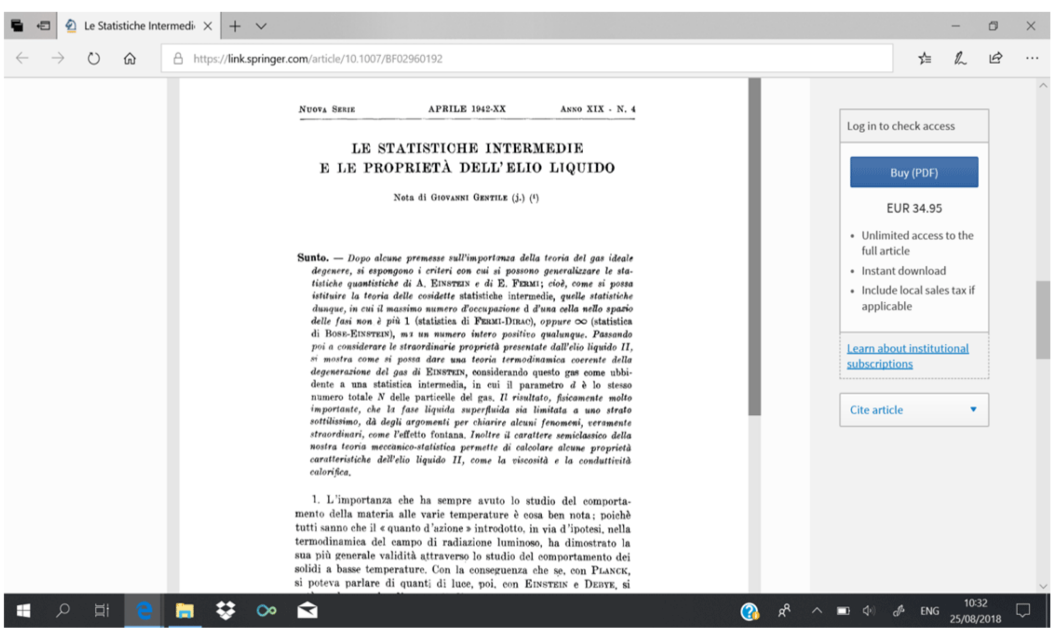
Message to young readers:

We hope this short article may inspire younger generation of physicists and biologists to rethink and renew their approaches to Nature, and perhaps it may also help to generate new theories which will be useful for a better future of mankind.

Postscript

A Short Note on Plausibility of Experimental Vindication of the Proposed Model

These authors just think of plausible vindication of the proposed intermediate state of fermion-boson, which may be called "ferson". It may have a chance to get into real observation at CERN etc. It may be indeed interesting for particle physicists who wish to continue the service period of CERN expensive facilities after discovery of Higgs particle. As the readers may already know, they tried to extend standard model to super symmetry but it failed to come to detectors. Meanwhile, we just read that there are two possible theories which seem correspond to an intermediate statistics we're looking for: (1) any on fractional statistics by Franck Wilczek, which we are not sure, (2) G. Gentile's statistics which predict the existence of "intermediate particle" between fermion and boson, but nobody has identified any experiment with such an intermediate particle so far. So, allow us to suggest interested readers to read and examine Giovani Gentile's original paper in Nuovo Cimento (1941). See [17].



NUOVA SERIE APRILE 1942-XX ANNO XIX - N. 4

LE STATISTICHE INTERMEDIE E LE PROPRIETÀ DELL'ELIO LIQUIDO

Nota di GIOVANNI GENTILE (j.) (1)

Sunto. — *Dopo alcune premesse sull'importanza della teoria del gas ideale degenere, si espongono i criteri con cui si possono generalizzare le statistiche quantistiche di* A. EINSTEIN *e di* E. FERMI; *cioè, come si possa istituire la teoria delle cosidette* statistiche intermedie, *quelle statistiche dunque, in cui il massimo numero d'occupazione* d *d'una cella nello spazio delle fasi non è più* 1 (statistica di FERMI-DIRAC), *oppure* ∞ (statistica di BOSE-EINSTEIN), *ma un numero intero positivo qualunque. Passando poi a considerare le straordinarie proprietà presentate dall'elio liquido II, si mostra come si possa dare una teoria termodinamica coerente della degenerazione del gas di* EINSTEIN, considerando questo gas come ubbidente a una statistica intermedia, in cui il parametro *d* è lo stesso numero totale *N* delle particelle del gas. *Il risultato, fisicamente molto importante, che la fase liquida superfluida sia limitata a uno strato sottilissimo, dà degli argomenti per chiarire alcuni fenomeni, veramente straordinari, come l'*effetto fontana. *Inoltre il carattere semiclassico della nostra teoria meccanico-statistica permette di calcolare alcune proprietà caratteristiche dell'elio liquido II, come la viscosità e la conduttività calorifica.*

1. L'importanza che ha sempre avuto lo studio del comportamento della materia alle varie temperature è cosa ben nota; poichè tutti sanno che il « quanto d'azione » introdotto, in via d'ipotesi, nella termodinamica del campo di radiazione luminoso, ha dimostrato la sua più generale validità attraverso lo studio del comportamento dei solidi a basse temperature. Con la conseguenza che se, con PLANCK, si poteva parlare di quanti di luce, poi, con EINSTEIN e DEBYE, si

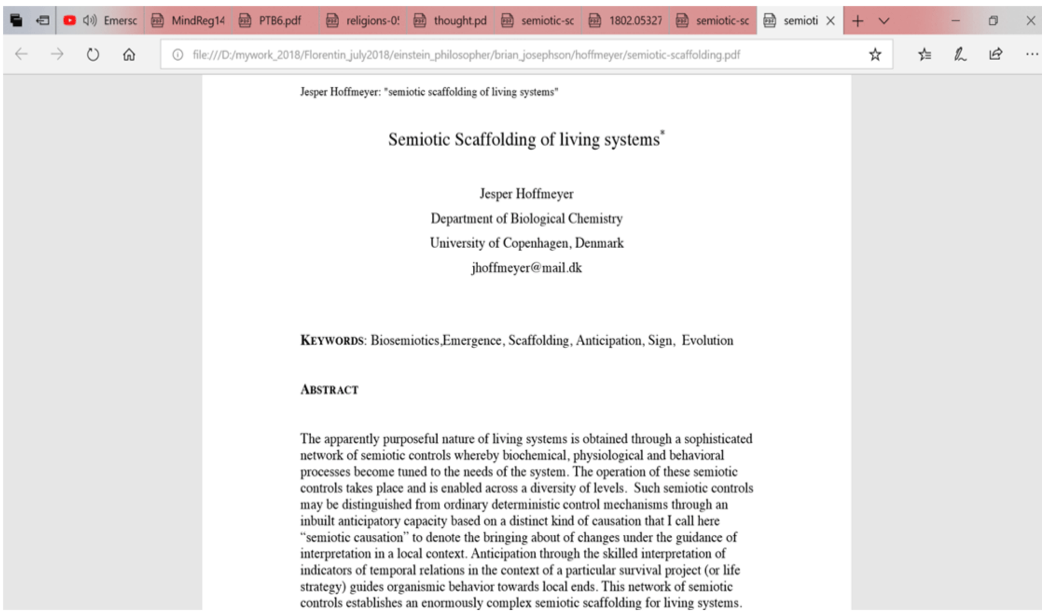
Picture 1: Screenshot of first page of G. Gentile's paper (1941)

We also plan to write up a short speculative paper on this topic, perhaps with title like: "On possible detection of intermediate state of fermion-boson particle from Klein-Bottle physics." But of course, this topic is to be discussed in other paper.

Toward Pneumatological Mind-Matter Interaction

Various models have been proposed to suggest possibility of mind-matter interaction, but mostly fall within QM theory. Other experiments seem to suggest that the effect is quite real, albeit many aspects remain mystery.

There are vast amount of mind-matter interaction models of living systems, from Stuart Hameroff etc's model, Semiotic Scaffolding model of Jesper Hoffmeyer (which Brian Josephson suggests a new term: *Semiophysics*) etc. See for instance [18-20].



Jesper Hoffmeyer: "semiotic scaffolding of living systems"

Semiotic Scaffolding of living systems*

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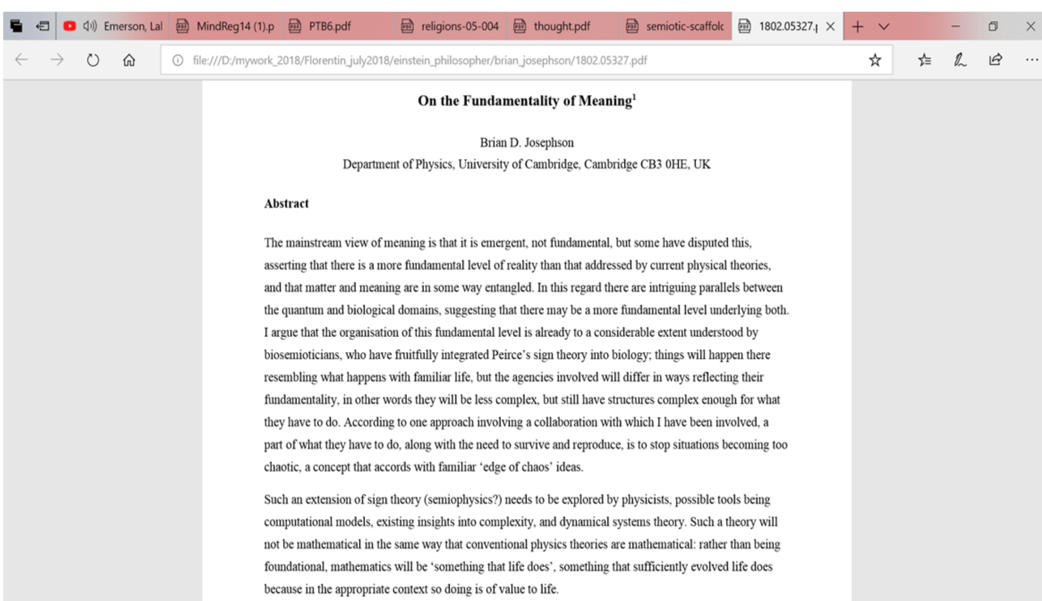
KEYWORDS: Biosemiotics,Emergence, Scaffolding, Anticipation, Sign, Evolution

ABSTRACT

The apparently purposeful nature of living systems is obtained through a sophisticated network of semiotic controls whereby biochemical, physiological and behavioral processes become tuned to the needs of the system. The operation of these semiotic controls takes place and is enabled across a diversity of levels. Such semiotic controls may be distinguished from ordinary deterministic control mechanisms through an inbuilt anticipatory capacity based on a distinct kind of causation that I call here "semiotic causation" to denote the bringing about of changes under the guidance of interpretation in a local context. Anticipation through the skilled interpretation of indicators of temporal relations in the context of a particular survival project (or life strategy) guides organismic behavior towards local ends. This network of semiotic controls establishes an enormously complex semiotic scaffolding for living systems.

Picture 2: Hoffmeyer's first page of his paper: *Semiotic Scaffolding of living systems* [18]



On the Fundamentality of Meaning[1]

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Abstract

The mainstream view of meaning is that it is emergent, not fundamental, but some have disputed this, asserting that there is a more fundamental level of reality than that addressed by current physical theories, and that matter and meaning are in some way entangled. In this regard there are intriguing parallels between the quantum and biological domains, suggesting that there may be a more fundamental level underlying both. I argue that the organisation of this fundamental level is already to a considerable extent understood by biosemioticians, who have fruitfully integrated Peirce's sign theory into biology; things will happen there resembling what happens with familiar life, but the agencies involved will differ in ways reflecting their fundamentality, in other words they will be less complex, but still have structures complex enough for what they have to do. According to one approach involving a collaboration with which I have been involved, a part of what they have to do, along with the need to survive and reproduce, is to stop situations becoming too chaotic, a concept that accords with familiar 'edge of chaos' ideas.

Such an extension of sign theory (semiophysics?) needs to be explored by physicists, possible tools being computational models, existing insights into complexity, and dynamical systems theory. Such a theory will not be mathematical in the same way that conventional physics theories are mathematical: rather than being foundational, mathematics will be 'something that life does', something that sufficiently evolved life does because in the appropriate context so doing is of value to life.

Picture 3: Prof. Brian Josephson's first page of his paper [19]

But we prefer to suggest a simpler model based on the fact observed by Benveniste and also later by Maxim Trushin: there is a kind of antenna or sonic-mediated communication between cells. Therefore, we submit a model of mind-matter interaction by a new term: Pneumatological cymatic mechanism, i.e. by the human voice, soaked in the Holy Spirit, then it may affect the material/environment. Nonetheless, we admit that the exact mechanism of Pneumatological mind-matter interaction remains mystery, and this topic is reserved for future research. What we can say for now is: it seems the effect of mind-matter effect over long distance (more than 150km) has been reported, which suggests that this topic is very interesting for next research [21].

See also our previous papers on theo-cymatic view cosmology, in Part III.

Acknowledgement

This paper is inspired partly by Prof. Adam Grant's deep and insightful book: Give and Take. VC gratefully appreciates Elizabeth and Sujarwo in Jkt. Thanks so much for spending many years of being trustful friends and being sparring partners in such uneasy intellectual endeavor like this. He is also deeply grateful to his parents for showing the way through their life on how to walk in 3-mile-an-hour speed with God. Special thanks to Prof. Dr. Bambang Hidayat, a member of the Indonesian Academy of Sciences, for sending Nature News article on TCM (Appendix II). VC dedicates this paper to Father in Heaven, Jesus Christ, and Holy Spirit for always guiding him through all valley of darkness. Jesus Christ is the Good Shepherd (Ps. 23).

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Appendix I

Three Mile-an-Hour God and the Speed of Love

Victor Christianto, Founder of The Second Coming Institute

Shalom, all brothers and sisters in Jesus Christ. Do you realize that our life can be summarized in one word: faster.

Anything we do, we do that faster and faster. Read fast. Eat fast. Speak fast. Walk fast. Drive fast. Pray fast. And so on.

Sometimes we forget that God want to walk with us at 3 mile an hour speed. As a Christian blogger wrote recently [1]:

John 9:1 says, "As he passed by, Jesus saw a man blind from birth." What if he was driving, running, or in a hurry? Instead, Jesus moved with a pace at which he could "see." He saw the man. He saw his need and he had compassion.

A Japanese theologian named Kosuke Koyama wrote a book called Three Mile an Hour God. In it he wrote:

"Love has its speed. It is a different kind of speed from the technological speed to which we are accustomed. It goes on in the depth of life at 3 miles per hour. It is the speed we walk and therefore the speed the love of God walks."

Jesus walks at the speed of love. He's our 3 mile-an-hour Savior. And he sees you. He sees your secrets and baggage, your pain and fear. He sees death and dung, and still chooses to walk among us. To forgive, to heal, to help.

Would you adjust your pace? Would you slow down so that you can "see"? See God's work in the world. See how you might join in on what He's doing. See the people around you. Know their needs. How can we be unhurried, undistracted, and attentive to the world around us? Go for a walk.

Sit on your front step in the evening.

Redefine how you use electronic devices.

Remove a few unnecessary items from your crowded calendar.

Set aside a few quiet moments every day to read God's word. To commune with him in prayer.

"As he passed by . . ." Jesus sees you. He's your 3 mile-an-hour Savior.

Here is a story of a man who chooses to walk for Jesus, and people whom he met along his walk:

William C. Heller Jr.6/21/2018 05:32:24 pm

This is my brief true story of a time in my life when I took a walk for Jesus. This journey began at highway 55 and Butler Hill road. I began walking up the ramp and praying to God this prayer. God you know I cannot walk to where ever you wish me to go. Would you please send me a ride and the person you desire me to talk to. Half way up the ramp a young man of college age stopped and offered me a ride. He then began to tell me all about his life and the church he attended which is the First Baptist church of Festus.

The next thing he told me is how he was worried about his final exams in college. I told him how I once had to take my exams for my GED and asked God to help me take the test and that help came in the sense of calmness.The next thing I said to this young man was, You go to church, Have you asked God for any help in your life? He looked at me as if he new what to do next. By this time he was pulling off the side of the road right in front of the First Baptist Church which sits on the side of the highway. As I got out of his car he thanked me for my help. there was now a calm about him as well.

I sat on the guard rail for no more than fifteen minutes and began to walk as I prayed once more the same prayer as before. Right away I heard air brakes on a truck behind me and looked back as this man was only a few feet away and motioned for me to get into his truck. I am John Murdock a dairy driver from Madison, Wisconsin. I said my name is William Heller and I am walking for Jesus. He then told me about a young lady he had met on the road the week before doing the same thing. John would ask me about all things he had questions about the Bible. As he made his deliveries for the day and the day ended he invited me to stay with him and he bought me dinner and breakfast. I spent three days with John and he left me off on Highway 75 leading down to Atlanta. His last words were, I going to go home and read my Bible this weekend. This is only a small part of my walk for Jesus. If you like to hear more let your fingers do the walking and write me.

My prayer in this sunday morning (22/7/2018, pk. 7:23)

"Jesus, forgive me for trying to do things faster and faster.

Meanwhile, teach me to learn how to walk and work and talk and pray at a lower speed.

Teach me to meet and greet people whom I see along the walk.

Thank you for Your forgiveness and patience on me. Amen."

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Appendix II

China to Roll Back Regulations for Traditional Medicine Despite Safety Concerns

Article by David Gray from Reuters

Scientists fear plans to abandon clinical trials of centuries-old remedies will put people at risk.

The Chinese government is promoting traditional Chinese medicines as an alternative to expensive Western drugs.

Support for traditional medicine in China goes right to the top. President Xi Jinping has called this type of medicine a "gem" of the country's scientific heritage and promised to give alternative therapies and Western drugs equal government support. Now the country is taking dramatic steps to promote these cures even as researchers raise concerns about such treatments.

From early next year, traditional Chinese medicines may no longer be required to pass safety and efficacy trials in humans in China. Draft regulations announced in October by the China Food and Drug Administration (CFDA) mean traditional medicines can skip such costly and time-consuming trials as long as manufacturers prepare ingredients using essentially the same method as in classic Chinese formulations. The State Administration of Traditional Chinese Medicine and the CFDA will compose a list of the approved methods.

The Chinese government has been forcefully promoting traditional Chinese medicines (TCMs) as an alternative to expensive Western drugs. Doctors of Chinese medicine have welcomed the new policy, saying that it will make it easier for companies who produce such medicines to get drugs approved and make them available to patients. Lixing Lao, director of Hong Kong University's School of Chinese Medicine, says that although traditional medicines will no longer need to go through clinical trials, the CFDA will still require remedies to undergo preclinical pharmacological testing and drug-toxicity studies in animals or cells to gain approval.

Safety Concerns

But scientists say that safety concerns continue to plague the industry, and that minimizing clinical-trial requirements could put more patients at risk. On 23 September, the CFDA recalled batches of two injectable TCMs after about ten people fell ill with fevers and chills.

Less than a month later, on 18 October, researchers in Singapore and Taiwan published a study in Science Translational Medicine linking liver cancer to aristolochic acid, an ingredient widely used in traditional remedies1. Lead author Steven Rozen, a cancer-genomics researcher at Duke-NUS Medical School in Singapore, is convinced that aristolochic acid contributed to the mutations, but says it's harder to determine to what extent it caused the tumors.

Aristolochic acid has also been linked to cancers of the urinary tract and can cause fatal kidney damage 2, 3. Rozen says it is still in common use, despite warnings from the US Food and Drug Administration that it is associated with kidney disease. "It would be a good time to reassess regulations" of aristolochic acid, he says.

Lao sees people take remedies containing aristolochic acid every day, and says it should not cause problems if taken "moderately and to treat diseases" rather than as a regular supplement. He says more research is needed into how to ensure the safe use of the potentially toxic substance. Overall, Lao is not concerned about safety issues with traditional medicines because, "unlike Western drug development, these herbal formulas have been used for hundreds and thousands of years," he says.

But Li Qingchen, a paediatric surgeon at the Harbin Children's Hospital and a well-known critic of TCMs, says the recent recalls of remedies show that current safety measures aren't adequate. He says doctors need to inform the public about some of the dangers associated with traditional medicines, but that most are unwilling to speak out against them. "Few doctors would dare to publicly criticize TCMs," he says. Li thinks that the government's promotion of TCMs will make it harder for scientists to criticize the drugs "because the matter gets escalated to a political level and open discussions become restricted".

Criticism Muted

With strong government support for the alternative medicines industry, Chinese censors have been quick to remove posts from the Internet that question its efficacy. On 23 October, an article on a medical news site that called for closer attention to the risks of aristolochic acid was removed from social media site WeChat. The story had been viewed more than 700,000 times in three days.

Debate over TCMs has been silenced before in China. Last year, a Beijing think tank — the Development Research Center of the State Council — proposed banning the practice of extracting Asiatic black bear bile, another common ingredient in TCMs. The think tank's report questioned the remedy's efficacy and suggested using synthetic alternatives. It was removed from the think tank's website after the Chinese Association of Traditional Chinese Medicine, which supports the development of TCM, called it biased and demanded an apology.

As well as reducing regulations for TCMs, the Chinese government has made it easier to become a doctor of traditional medicine and to open hospitals that use the approach. Since July 2017, students studying traditional medicine no longer need to pass the national medical exams based on Western medicine. Instead, traditional medicine students can attend apprenticeship training and pass a skills test. And practitioners who want to open a clinic no longer need approval from the CFDA. They need only register with the authority.

The government's ultimate goal is to have all Chinese health-care institutions provide a basic level of TCMs by 2020. A roadmap released in February 2016 by the State Council, China's highest administrative body, plans to increase the number of TCM-licensed doctors to 4 per 10,000 people, an increase from less than 3 practitioners per 10,000 people. The government also wants to push TCMs' share of pharmaceutical sales from 26% to 30% by the end of the decade.

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Robot Advanced Intelligent Control developed through Versatile Intelligent Portable Platform

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Abstract— **The paper presents a versatile, intelligent portable robot platform VIPRo, which involves developing intelligent control interfaces by applying advanced control techniques adapted to the robot environment such Robot Neutrosophic Control (RNC), Robot Extenics Control (eHFPC), Robot Haptic Control (RHC), human adaptive mechatronics, implemented by high speed processing IT&C techniques and real time communication for a high amount processing data. An original virtual projection method is applied to SMOOTH firefighting robots through representation of the intelligent mobile robots in a 3D virtual environment using VIP-F^{2}Ro with robotic strong simulator, an open architecture system and adaptive networks over the classical control system of the robot.**

Keywords: VIPRO platform, robot simulation, graphical user interface, reference generation.

I. Introduction

Mobile robots have caught the attention of the research community and the manufacturing industry as well, leading to a great hardware and software developing. Some applications of great interest for researchers are human behaviour in fires and the simulation of the movement of individuals in such hazardous environment [1-3]. Simultaneously, the real time robot control with remote network control having human operators' ability play an important part in hazardous and challenging environments of human life exposed to great dangers such as support and repair in nuclear contaminated area, fire, earthquake or any other disaster area in case of an accident or a terrorist attack involving CBRN materials. [2-3]. A big amount of researches led to the development of different robots with sensing abilities, transport and manipulation of different applications [4-7].

This calls further developing of the mobile and remote control autonomous robots which can help people to perform searching and saving operations, representing a priority and a complex task.

In general, the total impact costs of large incidents are very high, and are much higher that of counter-measures. For biological threats, the indirect economic impact is assessed to be in the range of several billion to tens of billions of US dollars. The countermeasure cost range is much lower, ranging from hundreds of millions to about 10 billion USD. Taking the bio defence programs alone, a few hundred thousand to tens of millions are spent by European countries for a reference year, while the USA invests about 200 million euros.

Intelligent heterogeneous robot networks, remotely controlled by humans, have an increasingly important role in hazardous and challenging environments, where human lives might be at risk [8-10]. This is in fact the challenge of developing autonomous systems perceptive to human requirements and having the ability of continuous learning, adapting and improving in "real world" complex environments, so as to provide support in natural disasters, fires, or other calamities [14-17].

The paper presents a VIPRo versatile, intelligent robot platform, which involves developing intelligent control interfaces by applying advanced control techniques adapted to the robot environment such Robot Neutrosophic Control (RNC), Robot Extenics Control (eHFPC), Robot Haptic Control (RHC), human adaptive mechatronics, etc. An original virtual projection method is applied to SMOOTH firefighting robots, through the development of VIP-F^{2}Ro Platform, which allows representation of the intelligent mobile robots in a 3D virtual environment using a strong robotic simulator, an open architecture system and adaptive networks over the classical control system of the robot.

The VIP-F^{2}Ro Virtual Intelligent Portable platform, is the one designed to acquire the data received from unmanned ground vehicles (UGV), to process and analyse them, to provide feedback. The VIP-F^{2}Ro brings the virtual robots to the real world, wanting to create an innovative robot platform, which will allow to develop mechatronic systems of mobile robots in virtual environments and communicate with real robot systems through a high speed interface.

The obtained results lead to the conclusion that the advanced intelligent robot control methods using ***neutrosophic control***, ***extended control*** (Extenics), ***human adaptive mechatronics***, developed through versatile intelligent portable platform, allow a correct evaluation of robot behaviours in hazardous or challenging environments and improving the robot performances at the interaction with the environment.

II. Versatile Intelligent Portable Platform

The VIP-F^{2}Ro Virtual Intelligent Portable Platform for firefighting robots, is the development of an e-learning and remote-control platform enabling community interested in the topic and long-term plans to further develop research and innovation. This, in fact, is the tool of ensuring the ability of continuously learning, adapting and improving in "real world" complex environments, modeling in real time the information gathered by advanced technologies so as to provide support in "big data" management and development of international clusters able to process the information in an unifying vision.

This way, networking activities will be in good balance with scientific and technical activities contributing equally to advance the project and to achieve the specific objectives mentioned above [18, 20].

To develop new features for the unmanned ground robotic mobile vehicle, like motion on uneven ground, or motion by overcoming or bypassing obstacles, high level intelligent algorithms are required to be developed. This is due to the fact that the motion mechanism is a complex process, and because it is a repetitive process of tilting and unstable movements that sometimes occur on a bumpy road, it will lead the robot to tip over.

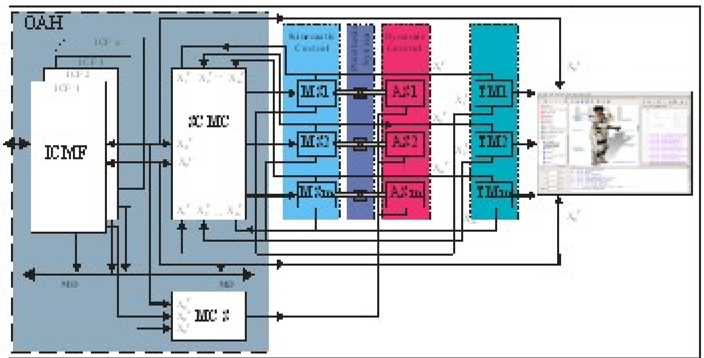

Fig. 1 Virtual projection method by Vladareanu-Munteanu applied to VIP-CBRN Platform

The virtual projection method [19] (Figure 1) tests the performance of dynamic position-force control by integrating dynamic control loops using a Bayesian interface for the sensor network and neutrosophic interface for decision making [9-11]. The CMC classical mechatronic control directly controls servomotors MS1, MSm, where m is the number of the robot's degrees of freedom. These signals are sent to a virtual control interface (VCI), which processes them and generates the necessary signals for graphical representation in 3D on a graphical terminal CGD. Development of an open architecture control system by intergrating ***n*** control functions in addition to those supplied by the CMC mechatronic control system. With the help of these, new control methods can be implemented, such as: contour tracking functions, motion control schemes, control of the centre of gravity , the orientation control through image processing, Bayesian interface for sensor networks, decision making by neutrosophic logic control [11,12]. Priority control, real time control and information exchange management between the n interfaces is ensured by the multifunctional control interface MCI, interconnected through a high speed data bus.

The optimization of intelligent control methods allows the Unmanned Ground robotic mobile Vehicle (UGV) to adapt to environmental scene of in case of the fire investigation, hazardous chemicals detection, fire and rescue threat the firefighter's safety and life, through real time control, without losing its stability during the mission.

For modelling through the adaptive mechatronics methods of the robot implemented by the versatile, intelligent and portable robot VIPRO platform are presented three intelligent control interfaces (ICs). Human adaptive mechatronics are intelligent electrical-mechanical systems that are able to adapt themselves to the human's skill in various environments and providing assistance in improving the skill, and overall operation of the combined human machine system to achieve the improved performance. The VIPO platform architecture, in correlation with the virtual projection method (Figure 1) is developed in Figure 2.

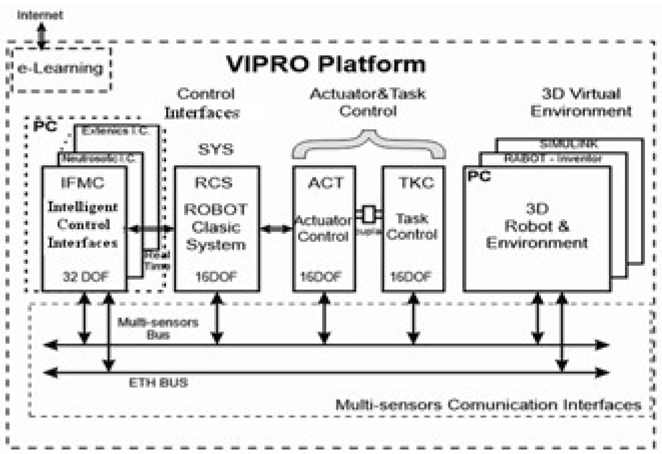

Fig.2 Integration of the VIP-F^{2}Ro Platform in the VIPRO Platform architecture

The results of simulation investigation and identifying the features and parameters of the virtual intelligent platform VIP-F²Ro are obtaining by simulation studies. These will be used to establish the UGV optimal parameters for intelligent interfaces development. VIP Platform allows intensive simulation studies for damping motion, motion compensation, UGV swing amplitude, UGV rotation/advance, motion timing, motion orientation, UGV tilt over, landing position.

The technical solution for the VIP-F²Ro platform contains the intelligence control interface module, which uses advanced control strategies adapted to the robot environment such as extended control – Extenics, neutrosophic control human adaptive mechatronics, etc., implemented through various IT&C techniques, with fast processing and real time communication. This module contains mainly the interface for intelligent neutrosophic control by integrating the RNC (Robot Neutrosophic Control) method [12], known as Vladareanu-Smarandache method, Extended Control Interface through Extenics (ICEx) [10, 13] and Haptic Robot Control Interface (CRH) [9-11].

The control system comprises the proposed intelligent control interfaces: neutrosophic control interface (ICN) which integrates neutrosophic robot control (RNC), extended control interface (ICEx) which integrates extended hybrid force-position control (eHFPC) and the multifunctional control interface (ICM). In addition, the haptic robot control interface (CRH) is designed for movement and navigating on uneven terrain and uncertain environments.

III. Advanced Intelligent Control of the SMOOTH Robot, Throurgh VIP Platform

The new virtual intelligent portable platform of firefighting robots, VIP- F²Ro, is the one designed to acquire the data received from unmanned robotic vehicles, to process and analyse them, to provide feedback. The , VIP- F²Ro brings the virtual robots to the real world, wanting to create an innovative robot platform, which will allow to develop mechatronic systems of mobile robots in virtual environments and communicate with real robot systems through a high speed interface

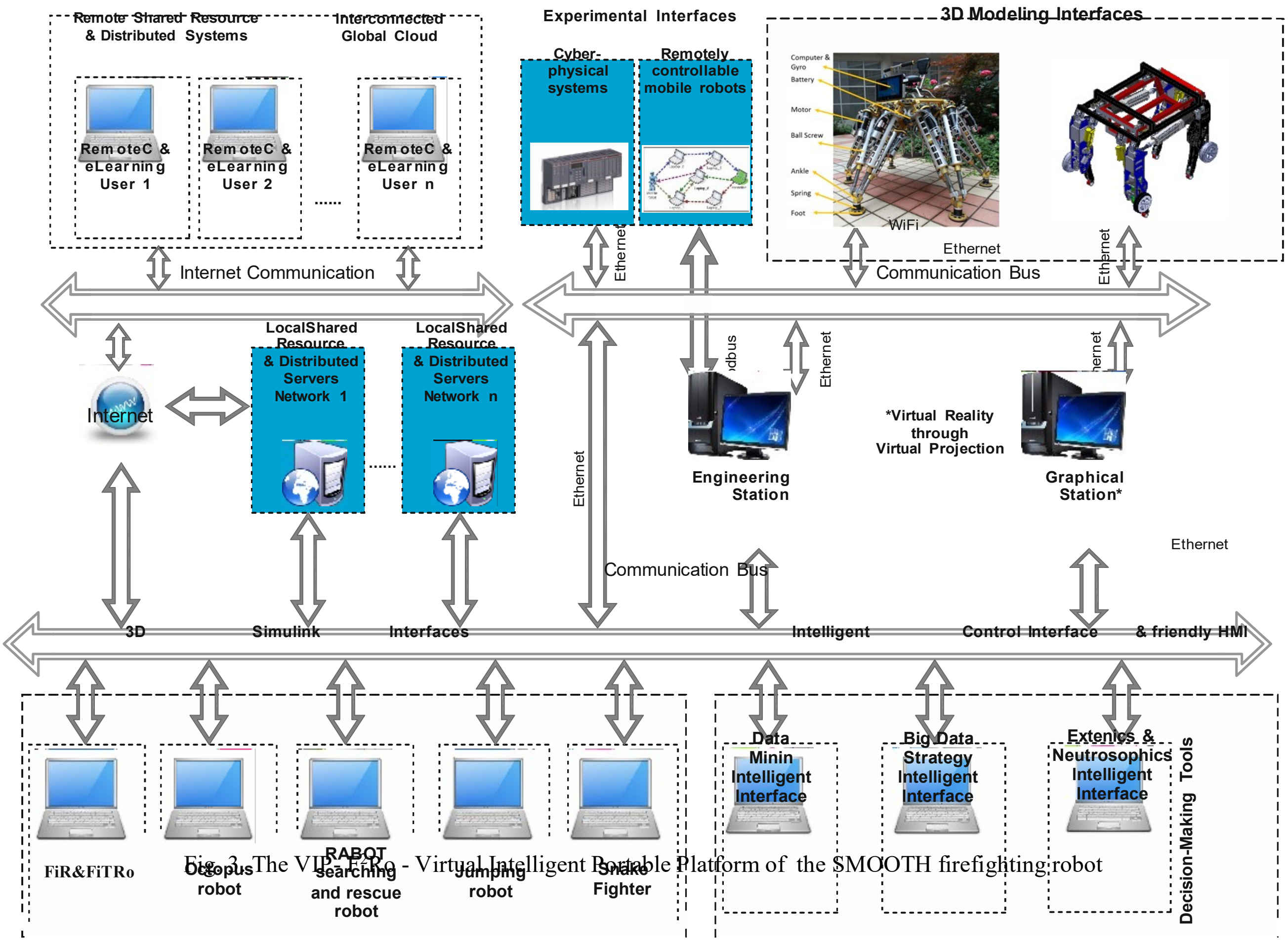

Fig. 3. The VIP- F²Ro - Virtual Intelligent Portable Platform of the SMOOTH firefighting robot

The outputs and predictions from the generated models could be used in multiple ways. In some circumstances, such as fire growth, the results could be sent directly to personnel at the fire ground or to other community services. If the model were to predict that the fire might spread into a portion of a building where toxic compounds are known to be stored, the model could be integrated with a smoke-generation model and a weather model to predict the likely impact on the surrounding community.

That information then would be sent directly to disaster management departments, law enforcement agencies as well as local hospitals to enable planning for a potential evacuation and treatment of victims. In most cases, model outputs and predictions would drive real-time 3D visualization of the fire ground, equipment, and personnel. The ICs would use the display to monitor the evolution of the fire incident and to analyse the potential impact of decisions and actions before issuing any commands to personnel. The visualization then would be recorded for future analysis, lessons learned, and training.

The computational platform VIP-F^2Ro designed in this project will be based on the virtual projection method. VIP-F^2Ro is extendable for integration, testing and experimenting of firefighting environments through building an open architecture system and adaptive networks, combining the expertise of a team of specialists in fire engineering, electronics, mathematics, computer sciences with the expertise of a diverse group of researchers in different fire specialties.

The innovative platform VIP- F^2Ro (Figure 3), developed as open architecture system and adaptive networks integrates Future Internet Systems vision enabling: cyber-physical systems by adaptive networks, intelligent network control systems, human in the loop principles, data mining, big data, intelligent control interfaces, network quality of service, shared resources and distributed server network - remote control and e-learning users by interconnected global clouds. Based on all the above, the challenges and, therefore, expected progress of VIP- F^2Ro are its ability to be interactive, integrated and competitive with advanced scientific research concepts.

The idea is that the robotics mobile unit will go to the safe proximity of the firefighting emergency area, in particular fire and rescue operations such as aircraftIairport rescue, wilderness fire suppression, and search and rescue, including emergency medical services. It can do that as it is equipped with innovative devices that determine the direction and the identification of dangerous clouds and the toxic environment created by combustible materials, their moving direction, nature of agents that contaminate, oxygen deficiency, elevated temperatures, and poisonous atmospheres, provided in safe condition for personnel protection. After the safe stop of robotics mobile unit, there are the correlated actions of unmanned ground and aerial vehicles (VIP- F^2Ro and UAV), all of these coordinated by the virtual intelligent platform, as follows next.

The need to manage all behaviors and interactions is solved by developing a new interface for intelligent control based on advanced control strategies, such as extended control (Extenics), neutrosophic control, human adaptive mechatronics, implemented by high speed processing IT&C techniques in real time communication for a high amount of data processing, including a remote control & e-learning component and an adaptive networked control. This will allow the development of new methodologies, evaluation metrics, test platforms, reproducibility of experiments, novel approaches to academia-industry co-operation, of the products and process innovation and last but not least, an fire engineering network for research and modeling complex of the data for firefighting quick actions, and management of fire and emergency services.

Robotic control is essential in developing control and perception algorithms for robotics applications. A 3D simulator for mobile robots must correctly control the dynamics of the robots and of the objects in the environment. Moreover, real-time control is important in order to correctly model interactions among the robots and between the robots and the environment, so it is often necessarily an approximation to obtain real-time performance.

The innovative firefighting robotic mobile ground vehicle, is sent for support to people, physical evaluation, examination and collection of material I evidence. Some "plus (+)" aspects of this innovative firefighting robot are: high stability and ease of remote control (manoeuvrability) in severe ground topography and I or narrow spaces like pipes; modular structure with, relatively, low costs specific components; ability to work in natural disasters and emergency incidents threatening life and property.

The networked real- time control will be distributed and decentralized using multi-processor devices for fusion control, data reception from transducers mounted on the robot, peripheral devices connected through a wireless LAN for off-line communications and CAN, MODBUS, PROFIBUS or ETHERNET fast communication network for real time control. The VIP- F^2Ro system was designed in a distributed and decentralized structure to enable development of new applications easily and to add new modules for new hardware or software control functions. Moreover, the short time execution will ensure a faster feedback, allowing other programs to be performed in real time as well, like the apprehension force control, objects recognition, making it possible that the control system have a human flexible and friendly interface.

The VIP-F^2Ro Platform develop the intelligent interfaces using Robot Neutrosophic Control (RNC), Robot Extenics Control Interface (eHFPC) and Robot Haptic Control (RHC) Interface for Unmanned Ground robotic mobile Vehicle (UGV) which acts in correlation and interaction with Unmanned Aerial Vehicle (UAV) through implementation of the network mobile robot system over Mobile Ad-hoc Network. The target robot is equipped with a robotic arm to execute various tasks. The relay I observer robot can route network packets between the controller and the target robot. It also produces visual feedback of the target robot to the user at the controlling end.

IV. Haptic Intelligent Control Interfaces

In the recent years haptic interfaces became a reliable solution in order to solve problems which arise when humans interact with the environment. If in the research area of the haptic interaction between human and environment there are important researches, a innovative approach for the interaction between the robot and the environment using haptic interfaces and virtual projection method is presented in this paper. In order to control this interaction we used the Virtual Projection Method where haptic control interfaces of impedance and admittance will be embedded.

For moving of the firefighting robots in uncertain environments, allowing actuation in crisis situations or natural disaster, in which human life is in danger, SMOOTH will develop haptic interfaces that provides the robot spatial orientation and navigation based on that the robot feels the land on which it moves by changing the stiffness of the robot paw joints and of the segments robot joints, using the stiffness associated of the paw joints position X_C on the robot environment map if uneven ground is detected [9-11].

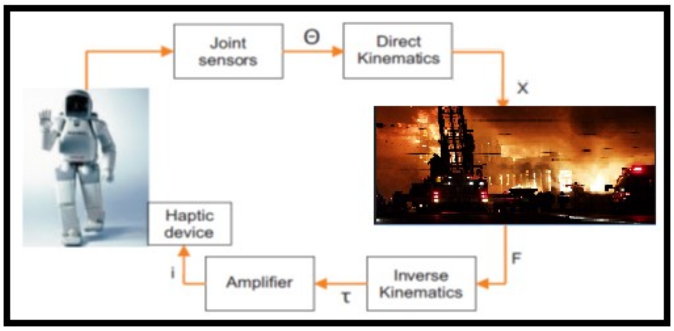

Figure 4. Haptic interfaces for firefighting robots using VIP-F^{2}Ro Platform

This leads to successively change the robot movement scheme and change the position control loop to the force control. Thus, the human operator can remotely control the robot's movement through two parameters, first visual and the second haptic (Figure 4). Respectively, the human operator sees the robot environment map and simultaneously feels damping of the robot leg movement at actuation of the haptic device lever, with the possibility of generating the haptic Cartesian positions X_{CH} for adapting the robot movement, on uneven and unstructured terrain.

Haptic interfaces intend to reproduce or to include the sense of touch through manipulation or, perception of real environments using mechatronic devices and computer control. They consist of a haptic device and a computer for control, which incorporates software that associates input data from the human operator with haptic information rendering. Figure 4 illustrates how haptic interfaces work and the way it will be implemented for controlling firefighting robots using VIP-F^{2}Ro Platform.

The innovative solution developed and patented for haptic robot control allows the robot to "feel" the terrain on which the mobile autonomous robot moves by the **modification in rigidity** of the joints and of the joints segment when detecting unevenness depending on the rigidity K_{Xc} associated to the joint position of the robot X_C on the robot environment map. Modifications in rigidity are realized from the time the joint touches the terrain until complete contact of the joint segment. **The human operator** has the possibility to remotely control the robot movement, through two parameters, one visual and the second haptic, respectively seeing the robot environment map and simultaneously to feel remotely the dampening of the robot joint movement when using the haptic device stick. Depending on the type of manipulation of the haptic device, the human operator generates the haptic Cartesian positions X_{CH} to ensure the robot motion is adapted to the uneven and unstructured terrain in crisis situations or natural disasters where human lives may be at risk.

In order to generate the robot environment map, images are processed from a CCD camera, stabilized for the various robot motion directions. This is done by processing the signals received from a 3D gravitational transducer (TGR3D) and a magnetic compass (TBM), resulting in an interface of the 3D robot environment map with a stable image to the robot movement. Each point in the robot environment map is associated with the rigidity of the robot joint position X_C, named associated rigidity K_{Xc}. **The movement damping** at contact between the uneven terrain and the robot joint is obtained by switching from the position control to the force control from the moment when the tip or the posterior of the joint touches the terrain, depending on the robot motion scheme, until complete contact of the joint segment is made.

Haptic control of the robot movement by the human operator is achieved through a haptic device which allows the human operator to feel the damping of the robot joint movement and generates the Cartesian reference positions of the robot movement, called haptic Cartesian positions X^H_{CH}, for adapting robot movement to uneven and unstructured terrain. **The telemetry module (TL)** allows the measurement of the distance to the joint segment by using an optic scanning device.

The novelty VIP-F^{2}Ro Virtual Intelligent Portable platform for firefighting robots, is competitive with other similar virtual simulation platforms with applications in robotics, called virtual instrumentation, CDA, CAM, CAE, Solid Works, etc., very powerful in modeling but only in a virtual environment, or the MatLab, Simulink, COMSOL, Lab View platforms, which allow extensions for real time data acquisition and signal processing. In addition to these, VIP-F^{2}Ro allow the experimental validation of intelligent control methods by integrating the classical robot real time control system in modelling, design, simulation and testing of the robot motion and stability.

V. Conclusion

Development of 3D dynamic perception and visualization, and human-robot interaction software systems are formidably challenging and accordingly the activities to support software developments and project management processes are of vital importance to this piece of research. Attribute selected techniques can be categorised on the basis of a number of criteria. Dynamic data come from environmental and wearable sensors, mobile robots and radio communications. SMOOTH will therefore develop software systems for real-time data analytics to assess situational awareness, asses risk and improve decision-making by firefighters and ICs. New computational software tools and virtual reality engines are being developed to support both risk and the decisions. The VIP-F^{2}Ro Platform also develop adequate metrics and testing tools to determine the effectiveness and validity.

This is part of a larger effort to completely define a virtual environment for the simulation and testing of mechatronic systems on a remote virtual platform, encompassing all the usual and innovative aspects in the field of Robotics research, from low-level actuator control and mechanism design to intelligent operational strategies and environment configuration modelling. It has the advantage of allowing virtually all manner of testing to be made remotely, with little or no extra configuration cost, while reducing the risk of equipment damage and maintaining the realism and end-result application value that can only come with actual hardware testing. This approach combines the best features of both scientific lines of enquiry, software simulation and direct hardware implementation.

Major outcome of this work is development of an Integrated Safe Smart Robotics Mobile Unit & Virtual Intelligent Platform for Remotely - Controlled Technologies in the fire investigation, hazardous chemicals detection, fire and rescue threat the firefighter's safety and life in emergency situations. Its innovation potential comes from the fact that it integrates, through VIP-F^{2}Ro Platform, both UGV (with innovative robotic arm module) and UAV (with innovative sensors and miniature sensors). This is how it enables intervention in various ground condition (uneven terrain, narrow spaces) where examination by humans may not be possible, or could be severely restricted. It allows searching and rescuing in smart firefighting control, safe operating in highly contaminated radioactive and chemical environments, and to facilitate the decision making with higher efficiency and collecting evidence / data which are further automated processed and generated reports are transmitted to decision centre. Also, prediction and local prognoses on highly contaminated areas are available.

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Application for Position and Load Reference Generation of a Simulated Mechatronic Chain

Victor Vladareanu, Sergiu Boris Cononovici, Marcel Migdalovici, Hongbo Wang, Yongfei Feng, Florentin Smarandache

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***Abstract*—The paper presents the position and load reference generation for a motor stand simulating a mechatronic chain, in this case a three degree of freedom robot leg. The task is accomplished using three PLC controlled motors in position as the robot joint actuators coupled with three controlled in torque, simulating the load at each simulation time-step. The paper briefly discusses the mathematical model and presents the visual interface used in the simulation, which is then to be further integrated into a virtual environment robot control application.**

Keywords: VIPRO platform, robot simulation, graphical user interface, reference generation.

I. Introduction

The walking robots' ability to adapt to uneven terrain makes them very useful in today's intelligent applications, from rescue operations and autonomous firefighting to elderly and disabled care. Walking locomotion ensures leg adaptation to the available environment, avoidance of unsuitable step positions and robot movement adapted to the terrain configuration [1-4].

The walking robot is physically composed of a body containing the elements capable of executing the allocated tasks and the robot legs, generally with three degrees of freedom, which ensure locomotion. The legs are either in a support phase – the phase in which the leg is in contact with the ground, the body is either stationary or moving along the walking route, or in the walking phase – in which the leg is off the ground, executing motion in relation to the robot body with the aim of contacting the ground in a new position. The legs are RRR kinematic chains with active couples [5, 6].

The design of the leg control system requires the existence of a way to simulate the walking process, virtually as well as physically, which is one of the goals of the overall project [7-10]. Numerous intelligent control interfaces based on advanced control strategies, such as neutrosophic control [4,8], extended control (Extenics) [6,9], human adaptive mechatronics [2,3], have been developed.

Uniform and periodic walking is studied on a terrain with possible irregularities. The walking is periodic if similar positions of the same leg, during the mechanical walking, appear at a given time interval. The time interval is a walking cycle and the movement done during it is an elementary step of the robot. Periodic walking allows a relatively simple control and a smooth motion of the robot body. Irregular walking appears in applications on uneven terrain.

This paper is divided as follows: Chapter II will describe the physical model investigated from which load reference generation is obtained, Chapter III will show the visual application interface and describe the immediate action of each of its elements, Chapter IV discusses the expected functionality of the visual interface as a whole, Chapter V delves into consideration regarding the code called underneath the graphical elements and its structure, and Chapter VI sums up the main points of the paper and the integration of the obtained results into the larger intelligent virtual environment platform.

II. Load Reference Generation

Motion with an elementary step corresponds to a walking cycle and transports the characteristic point M of the robot, usually chosen as the centre of mass or the geometric centre or on a vertical axis passing through it. Point M is the origin of a referential attached to the robot. The trajectory of point M is given in relation to a referential attached to a fixed point in the workspace.

Available equipment includes a stand composed of three PLC – controlled actuators, used for simulating the kinematic chain of a robot leg. Another three actuators are physically coupled to them, simulating the resistive loads.

Taking into account that actuating the kinematic chain of the leg does not imply the appearance of new large accelerations, as well as the fact that actuating the elements is done through reducers with large transmission fractions (i.e. n = 100), the resistive moments can be calculated using a kineto – static method. The quasi – static moments are calculated at each sample time.

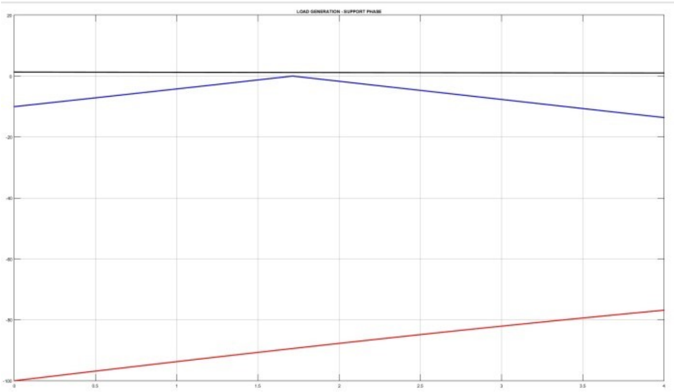

Figure 1. Load profiles for the support phase

In order to control the motors on the simulation stand for the joints of a walking hexapod robot, a calculation and transfer model is developed for the joint control references. The application simulating the robot leg on the motor stand requires knowledge of the position of each joint, as well as the loads they are sustaining. The dataset calculated as the robotic actuator load, simulated by a torque-controlled coupled motor at each simulation time-step, constitutes the control references for the secondary motor set, which replace the load during simulation.

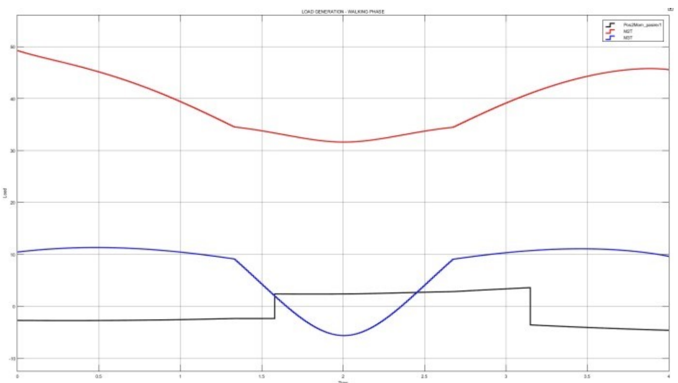

Figure 2. Load profiles for the walking phase

To properly test the mechatronic configuration within the virtual environment interface (thus allowing an unspecialized user to test various assumptions about their model), the position reference and load reference of a specific joint need to be given exactly at each time-step. Therefore, the two reference datasets are sent jointly to the PLC control structure, which synchronizes the application parameters.

For the considered example (an RRR kinematic chain representing the leg of a walking hexapod robot), the results of the mathematical model are shown in the next section. Further details are available in [11]. Using these mathematical equations and a standard set of input values (see discussion on "*Default*" setting), load profiles were obtained for the three joints. These are shown in Figures 1 and 2, for the support phase and walking phase, respectively. The simulation was run for 4 seconds, which would be the time it takes for a complete leg movement. For both figures, the black line represents the load value of the first joint (M_{B1}), the blue line the second joint (M_{B2}) and the red line, the third (M_C).

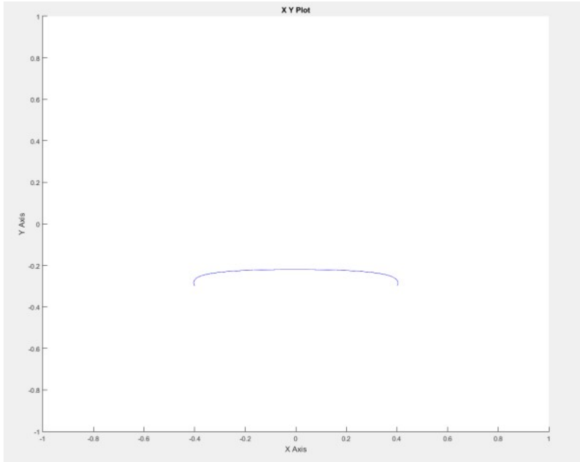

Figure 3. Leg motion during walking phase

The equivalent motion made by the robot leg during the walking phase is shown in Figure 3. It is ellipsoidal in the plane perpendicular to the second and third joints (M_{B2} and M_C).

The calculation of a specific load set, dependant on the input values fed into the calculations previously presented, can then be further sent to the PLC – motor system for actual hardware testing. The entire process can be achieved through a graphical user interface, which allows setting the initial inputs, specifying the simulation parameters and visualizing the obtained results, before feeding them into the hardware actuator system. Under the hood, this graphical user interface calls routines and functions which implement the equations previously discussed in the paper. The final aim is to additionally provide remote access and control capabilities, which is to be achieved in a future version of this implementation.

III. APPLICATION INTERFACE

The mathematical model, functioning and detailed calculations are geared towards implementation as a standalone application, as well as integration into a massive robot control and virtualization platform, such as the VIPRO project [8, 10].

For facilitating the implementation of the proposed model in testing the mechatronic chain, a graphical interface is designed and implemented, aimed at visualizing the working environment of the data sent to the PLC stand. Figure 4 shows the graphical elements and the control buttons.

The upper side of the interface contains three main areas which specify or select the options necessary for simulation and modelling. The mathematical model is described by the following general formulae for the support phase and walking phase, respectively.

Figure 4. Load modelling interface

For the support phase:

$$M_{B1}=M_1=\rho*\mu*q*G*h$$
$$M_{B2}=M_2=q*G*$$
$$M_C=M_3=l_1*\cos(\theta_2)*G*$$

For the walking phase:

$$M_{B1}=M_1=q_{CM}q*a*\sin(\theta_{1*})*(m_1+m_2)$$
$$M_{B2}=M_2=G_1*l_12*c_2+G_2(l_1*c_2+l_22*c_{23})$$
$$M_C=M_3=G_2*c_{23}$$

The upper left corner contains the simulation parameters, based on which the mathematical model is composed. These are:

ρ – (ro) – radius of the friction couple to axis B_l
μ – (u) – the friction coefficient
l – the robot leg characteristic length, based on which all length elements are calculate:

$l_1=2*l$; $l_2=3*l$;$h=1.5*l$
m – robot total mass, assumed to be equally dispersed on the number of legs

All other parameters present in the calculations are obtained based on those described above, or are simulation constants. These simulation constants are exceptions, however, as the model and application are strongly parameterized, in order to allow approaching situations and equipment that is as varied as possible.

For a quick run-through of the application or for check runs, a standard set of values can be loaded by using the „*Default*" button. This will load the following data vector: ρ=0.02; μ=0.1; l=O.2; m=lOO; g=lO; n=6. This button only modifies the existing values in the simulation parameter input fields. As will be mentioned when discussing the application code, the values can be modified until pressing the „*Select*" button, without a model being loaded for processing.

The central part of the application deals with the movement of each joint, for which there can be input the movement limits of the actuated angle (in radians), in the first two fields. The third field is used to specify the resolution of the walking space by inputting the number of steps of the loaded linear space. For each of the angles, there is the possibility of specifying whether its reference point changes, or it is maintained constant, by selecting the marking field to the right of each. For an actively actuated angle, the model will load a linear space of the number of points selected, between the specified limits. For a static angle, the mean of the two limits will be used, as a constant value. This group also allows a quick load of standard values by using its respective „*Default*" button

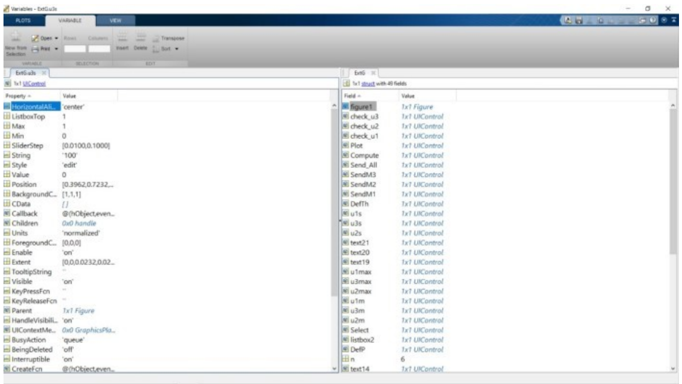

Figure 5. Variables in the workspace

Before running the application, there still needs to be specified the phase that the robot leg is in. The user can select one of the two options, „*Support*" and „*Movement*". As can be seen from the robot's mathematical model, the two situations are completely disjoint, and different calculations are made for each of them. After specifying the input parameters, the model is loaded into the runtime memory by pressing the „*Select*" button. If the application is run with the Matlab environment in the background, a new variable of type structure can be observed in the workspace, which contains all the described parameters. An example of such an occurrence is shown in Figure 5.

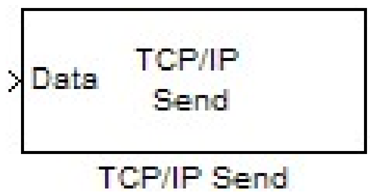

Figure 6. TCP/IP Simulink block

The three buttons above each axis system handle the communication between the load profiles obtained with the interface and another application which controls the PLC behaviour. The integration of the visual interface for load generation into the larger intelligent virtual environment platform is achieved through data transmission on a local area network. The desired outcome is that the various applications would reside on different computers, thereby increasing the available processing power and modularity of the platform. The communication is done through the TCP/IP protocol on an Ethernet connection. Each of the three buttons, as well as the initial test diagrams (see code structure for details), fire off intents to a TCP/IP transmission object in Matlab. The diagrams, running concurrently, simply replaced the final sink for each load profile (usually a Scope block) with a TCP/IP send block, like the one shown in Figure 6 [12].

IV. Application Diagram

The application diagram visually describes the user interaction and is shown in Figure 7. After loading the model at the previous step, the „*Compute*" button starts the calculation of the mathematical model discussed before. The user is notified through a dialog box when the numerical processing is done, at which time the processed data is made available.

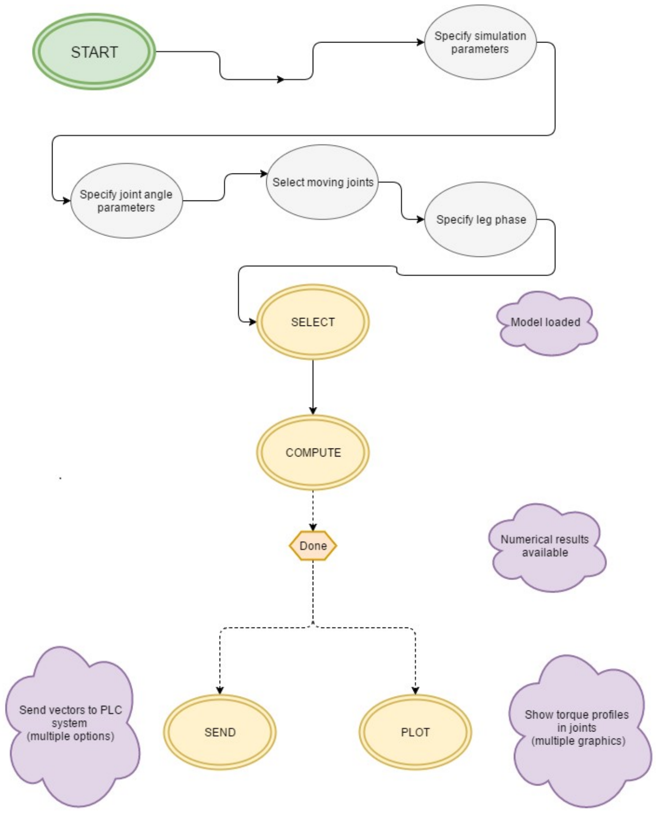

Figure 7. Application usage diagram

The interface now allows the visualization of the obtained data for each of the motor loads by pressing the „*Plot*" button, or sending these to the PLC control system by using the „*Send All*" button or the individual send commands, as desired.

The application diagram shows the expected one-time run-through for an untrained user. The process can be repeated and the parameters changed to obtain different sets of results. However, the application receiving the data from the interface will not be expecting a new set of incoming load or position profiles until its current run simulation is ended, so there is no option to send new data during the PLC runtime. This is by design as a safety feature, setting the destination listening port in the PLC application to only refresh between hardware simulations.

V. Code Structure

In reviewing the code used in the application, the main standout are the Simulink diagrams used for visualization. The application actually uses coded functions written directly in the programming language, which execute the same operations, but obviously run much faster than a visual model.

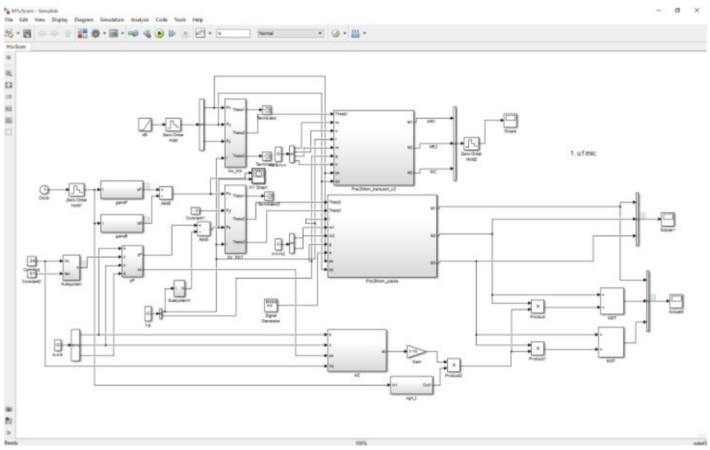

Figure 8. General structure of the load reference application

A diagram containing all of the application components is found in Figure 8. A noteworthy aspect is the multitude of inputs and pre-processing blocks, which help with model parameterization.

As can be seen from the figure, different sets of data are calculated, from different model components, depending on the selected state of the robot leg. In the visual model, these are shown in parallel, but in the application code they are only done once, for the selected state.

A snippet of code is shown in Figure 9. The functions within the graphical user interface are independent, using local variables, according to standard practice. Therefore, the „*handles*" structure is used to pass the saved or generated values between various functions inside the application.

Figures 10 and 11 shows the visual disposition of operations for the support and walking phase, respectively. There can be noted the equations used to calculate the model. The two diagrams are contained in the main subsystems visible in Figure 8.

```
% handles    structure with handles and user data
u1=handles.u1;
u2=handles.u2;
u3=handles.u3;

ro=handles.ro;
u=handles.u;

l1=handles.l*2;
l2=handles.l*3;
h=handles.l*1.5;

Gstar=handles.m*handles.g/(handles.n-1);
px=cos(u1)*cos(u2+u3)*l2+cos(u1)*cos(u2)*l1;
py=sin(u1)*cos(u2+u3)*l1+sin(u1)*cos(u2)*l1;
q=sqrt(px.^2+py.^2);

handles.M1=ro*u*q*Gstar/h;
handles.M2=q*Gstar;
handles.M3=l1*cos(u2)*Gstar;

 % Save the handles structure.
guidata(hObject,handles)
msgbox('Done! ;)')
```

Figure 9. Example code in the callback function of the „Compute" button

The third main subsystem present in Figure 8 is responsible for adjusting the load value due to present acceleration on the Z axis, for a leg in the walking phase. This is done as follows:

$$A_Z = -\frac{b}{c*u} * \frac{\chi^2}{\sqrt{u}} * V_x$$

where $u = c^2 - \chi^2$.

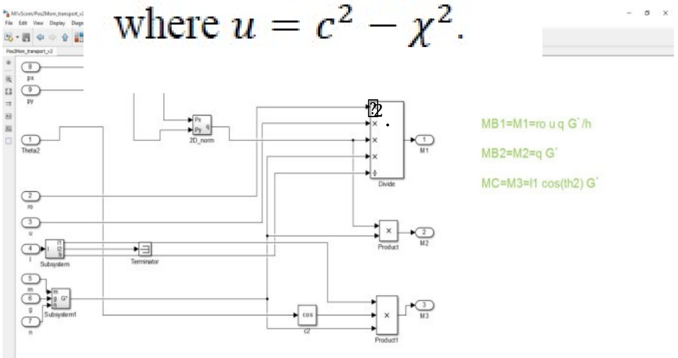

Figure 10. Diagram structure for load calculation in support phase

These are further used in:

$$M_2^{nou} = M_2 + M_2 * \frac{A_Z}{10} * sgn(t)$$

$$M_3^{nou} = M_3 + M_3 * \frac{A_Z}{10} * sgn(t)$$

$$\text{where: } sgn(t) = \begin{cases} 1, 0 \le t \le \frac{T_{sim}}{2} \\ -1, \frac{T_{sim}}{2} \le t \le T_{sim} \end{cases}$$

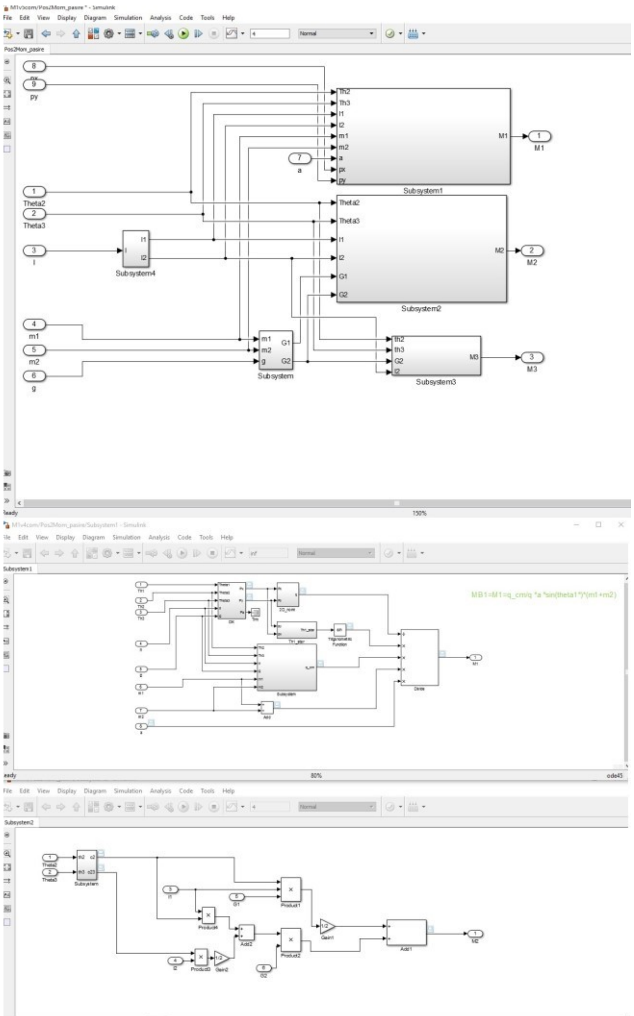

Figure 11. Diagram structure for load calculation in walking phase

The final results look similar to those shown in Chapter II, for the default dataset. As regards the transmission coding, they are n-dimensional vectors, with the timestamp being implicitly calculated from the simulation time-step, which is a known variable set by the user. The diagrams also include Zero-order_Hold blocks to illustrate the concept.

VI. CONCLUSION

The paper presents the design and strategies for modelling a robotic kinematic chain using coupled actuators driven from PLCs on a testing stand. This entails the calculation of dynamic resistive loads within the proposed virtual environment, which are duplicated in coupled actuators controlled in torque. Once this model is fully operational, it can be used to safely test any type of control and motion strategies on a virtual robot system, while maintaining a solid base in concrete hardware implementation. With this final adjustment step, the load profiles can be returned to the interface for plotting or sending to the PLC control component of the platform. As can be seen from the described process of carrying out the modelling and mathematical simulations, the end-result is well parameterized and can be scaled to a wide variety of robotic applications, as well as various environment and kinematic chain situations.

This is part of a larger effort to completely define a virtual environment for the simulation and testing of mechatronic systems on a remote virtual platform, encompassing all the usual and innovative aspects in the field of Robotics research, from low-level actuator control and mechanism design to intelligent operational strategies [10] and environment configuration modelling. It has the advantage of allowing virtually all manner of testing to be made remotely, with little or no extra configuration cost, while reducing the risk of equipment damage and maintaining the realism and end-result application value that can only come with actual hardware testing. This approach combines the best features of both scientific lines of enquiry, software simulation and direct hardware implementation.

ACKNOWLEDGMENT

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Improvement of the Material's Mechanical Characteristics using Intelligent Real Time Control Interfaces in HFC Hardening Process

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Luige Vladareanu, Mihaiela Iliescu, Victor Vladareanu, Alexandru Gal, Octavian Melinte, Florentin Smarandache, Adrian Margean (2018). Improvement of the Material's Mechanical Characteristics using Intelligent Real Time Control Interfaces in HFC Hardening Process. J Fundam Appl Sci. 10(4S), 193-198. DOI: 10.4314/jfas.v10i4s.81

***Abstract*— The paper presents Intelligent Control (IC) Interfaces for real time control of mechatronic systems applied to Hardening Process Control (HPC) in order to improvement of the material's mechanical characteristics. Implementation of IC laws in the intelligent real time control interfaces depends on the particular circumstances of the models characteristics used and the exact definition of optimization problem. The results led to the development of the IC interfaces in real time through Particle Swarm Optimization (PSO) and neural networks (NN) using off-line the regression methods.**

Keywords: intelligent control, real time control systems, hardening process, materials, high-frequency currents

I. INTRODUCTION HARDENING

Induction heating is a quick and precise technique to heat conductive materials without contact [1]. In order to obtain induction heating, equipment is necessary consisting of a source of alternative current and a solenoid to generate the electromagnetic field. The piece to be tempered is positioned inside the solenoid, with the electromagnetic field generating current inside the piece which, in turn, generates heat. Induction can heat pieces to temperatures between 100 and 300 grades Celsius. This is obtained using an electromagnetic field passing through a solenoid, which transfers energy to a working piece, which needs to be heated. When an electric field passes through a conductor, an electromagnetic field is produced around that conductor. This magnetic field produces electric current through a piece positioned in the middle of the solenoid's magnetic field, which in turn produces heat.

Eddy or Foucault currents are induced electric current loops in conductive materials by changing the magnetic field in that conductive material, due to Faraday's induction law. Eddy currents are transmitted in closed loop through the conductors, in perpendicular planes to the magnetic field. The intensity of the current in the conductor is proportional to the intensity of the magnetic field, the loop surface and the current frequency, as well as inversely proportional to the material resistance [2]. The piece to be heated and its material determine the operating frequency of the heat induction system.

It is necessary to use an induction system that ensures a larger frequency spectrum than that necessary for applications. The reason is due to the fact that when an electromagnetic field induces a current into the piece, this is largely transferred to the surface of the piece. When the operating frequency is high, the heated depth is thin. Similarly, when the operating frequency is low, the depth to which the electromagnetic field penetrates is larger. The thickness of the heated portion depends on the temperature, operating frequency and properties of the heated material.

Figure 1. HFC Hardening Process Control (HPC)

During the hardening process control (figure 1) using ***HFC*** (high-frequency currents) a metal part is placed in the electromagnetic field inside a copper tube bended to the shape of the part and the alternating high frequency currents are induced. The currents are pushed out to the part surface by the magnetic current induced inside. Since the induced currents have an extremely high density on the part surface which is being heated, the surface layer is heated quickly. The ***HFC*** induction hardening is characterized by two parameters: by the depth and hardness of the part layer being treated. Induction heaters (HFC apparatus) with the capacity ranging from 40 kV to 160 kVA with the frequency of 20-40 kHz or 40-100 kHz are used to get thin layer in the hardened item. If deeper layers are required, the range of frequencies from 6 to 20 kHz is used. The HFC hardening has proved to be very effective [3].

Key features of induction hardening are fast heating cycles, accurate heating patterns and cores that remain relatively cold and stable. Such characteristics minimize distortion and make heating outcomes extremely repeatable, reducing post-heat processing such as grinding. This is especially true when comparing induction hardening to case carburizing. This is a cost-saving and high-productive way of metal heat treatment that provides a part with high strength and durability.

II. Architecture of the HFC Hardening System Control using Intelligent Control Interfaces

The architecture of the experimental model of the HFC (high-frequency currents) system allows the improvement of mechanical characteristics of the metallic profile by improving the performance of the control function interfaces, specific to the method of virtual projection [4,5,6], used in the real time control of the HFC hardening system. With this aim, there have been conceived and developed intelligent control interfaces using particle swarm optimisation and neural networks. The multi-functional interface (ICFM) specifies the virtual projection method, with decision making using fuzzy logic or neutrosophic logic. Through offline control, applying the methods of polynomial regression and exponential regression in estimating the frequency (FRQ) and power (POW) parameters, the parameters of the control laws for the HFC hardening system are determined. The digital – analogue conversion is done through the DAC module. The results of offline experimentation are used in implementing the versatile intelligent platform VIP, as coefficients applied to the intelligent control interfaces (ICF 1-ICF3) and in control decisions (ICMF) for the real time control of the HFC hardening system. Communication with the VIP platform is done through a software adapted to the research in the remote control communication interface (ICRM). Implementing the HFC hardening system entails applying the remote control methods through sockets with command elements through the VIP server terminals and the CIF equipment's PC system.

The inputs of the CIF command system, considered independent variables in controlling the technological process and determined in designing the CIF experimental model, are:

Metallurgic Parameters:

TMC: Tempering temperature

PCU: Currie point

TCL: Tempering time = f(V_{Motion}, w)

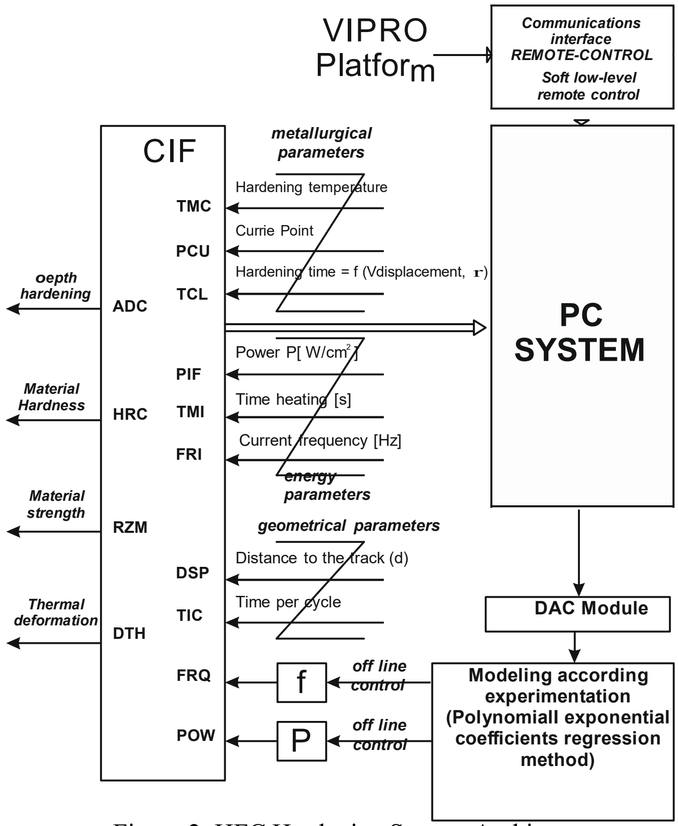


Figure 2. HFC Hardening System Architecture

Energetic Parameters:

PIF: Power P[W/cm^2]

TMI: Heating time [s]

FRI: Current frequency [Hz]

Geometric Parameters:

DSP: Distance to piece (d)

T/C: Time per cycle

The outputs of the command and control system, dependent variables upon the system inputs and the real time control laws of the speed of movement V_{Motion} and angular speed **r** of the metallic profile, determined in designing the experimental model of the CIF, are:

ADC –Tempering depth

HRC – Hardness of the metallic material (Rockwell Scale)

RZM – Resistance of the metallic material

DTH – Thermic deformation

The intelligent control interfaces module uses advanced control strategies adapted to the industrial technological process of induction heating and tempering, applying IT&C techniques with fast processing and real time communication. There were designed, analysed and conceived virtual experimentations for intelligent control interfaces using ***particle swarm optimisation (PSO)*** and ***neural networks*** in decision support systems employing ***fuzzy or neutrosophic logic***, which will be shown in the following chapters.

3. INTELLIGENT CONTROL INTERFACE USING PARTICLE SWARM OPTIMIZATION

Particle Swarm Optimization (PSO) is an evolutionary algorithm gaining more recent interest after being discovered by Kennedy and Eberhart [7, 8]. Instead of using genetics to modify the potential solution, the PSO algorithm has particles traverse a solution space at variable speeds, adjusted depending on their history and that of the swarm. Both of these are known for the entire population. At each instance, the speed of each particle is influenced by these two best solutions found.

The algorithm was first intended to simulate social behaviour, as a representation of swarms of flies or schools of fish. It has since been simplified to a model usable in optimisation problems. [9, 10]. It is a meta-heuristic algorithm, as there are very few assumptions, if any, about the optimisation problem, and it can search large spaces of candidate solutions. However, such algorithms cannot guarantee the optimal solution is found. PSO is not a gradient method, which means that it does not require for the optimisation problem to be differentiable, as do classical optimisation methods. It can be successfully used with partially irregular problems, including noise, time-variable, etc.

The basic version of the algorithm uses a population called swarm, made up of candidate solutions, called particles. These are moved in the search space according to a few simple calculation rules. Their movement is guided by their own best position found in the search space and by the best overall position found by the swarm. When better positions are discovered, these will be used to guide the entire swarm. The process is then repeated, which will lead in the end to a satisfactory solution.

Let f: $R^n \rightarrow R$ be the objective function which is to be minimised. The function takes as candidate solution a vector of real values and produces a real number which indicates the fitness of the given candidate solution. The gradient of f is unknown. The aim is to find a solution a for which $(a) \leq f(b)$ for any b in the search space, which would make it the global minimum. The maximisation can be done using $h = -f$.

Let S be the number of particles in the swarm, each with a position $x_i \in R^n$ in the search space and a speed $v_i \in R^n$. Let p_i be the best known position of the particle i and g the best known position of the entire swarm. Then, for each particle i=1,…,S, the position is initialised with a random distributed vector $x_i \sim (b_{lo}, b_{up})$, where b_{lo} and b_{up} are the upper and lower limits of the search space, the initial particle position is initialised with its best known value, $p_i \rightarrow x_i$. If $(p_i) < f(g)$ the best known position of the swarm is updated: $g \leftarrow p_i$. Then the particle speed is initialised $v_i \sim (-|b_{up} - b_{lo}|, |b_{up} - b_{lo}|)$. Until a termination criterion is met (for example the number of iterations or the solution fitness), for each particle i=1,…,S and for each dimension d=1,…,n, the particle speed is updated to:

$$v_{i,d} \leftarrow \omega\, v_{i,d} + \varphi_p r_p (p_{i,d} - x_{i,d}) + \varphi_g r_g (g_d - x_{i,d})$$

and the particle position is updated to: $x_i \leftarrow x_i + v_i$. If $f(x_i) < f(p_i)$, the particle best known position is updated $p_i \leftarrow x$ and, if $f(p_i) < f(g)$, the best known swarm position is also updated to $g \leftarrow p_i$. Now g contains the best known position of the swarm.

The parameters ω, φp and φg are selected by the user and control the behaviour and functionality of the POS method. The choice of PSO parameters has a significant impact on the performance of the optimisation algorithm. Therefore, the selection of PSO parameters to increase performance has been the subject of much research.

Speed initialisation may require extra inputs to the problem. A simpler alternative is accelerated particle swarm optimisation (APSO), which does not use speeds and may converge quicker in some applications.

Considering a swarm p in a n-dimensional search space, the position vector x(i,k) of each particle i is updated by:

$(i,\ k + 1) = x(i,\ k) + v(i,\ k + 1)$, where k is a temporal pseudo-increment.

The term $(i,\ k + 1)$ is the speed vector, obtained by applying the law: $v(i,\ k + 1) = \omega \bullet v(i,\ k) + v(i,\ k)$, in which the inertial factor ω is a real number and v(i,k) is the stochastic speed vector.

The last term is formed by summing the other two:

$$(i, k) = c1R1(p(i, k) - x(i, k)) + 2R2(p(g, k) - x(i, k)), p(i, k)$$

Representing the best position vector of the particle i, while p(g,k) is the best position vector of the entire swarm (up to moment k).

The vectors (p(i,k) − x(i,k)) and (p(g,k) − x(i,k)) use the amplitudes and directions of the vectors uniting the current particle position x(i,k) with the best particle position p(i,k) or the best swarm position p(g,k). At each iteration, the quality of each particle is evaluated through the fitness function. Each particle retains the best value of the objective function, as well as its position. The interaction between particles allows them to retain the best values of the objective function for the entire swarm over time. The particles can move in a continuous, discrete or mixed domain.

In order to design the ICF2 intelligent control interface, two possible approaches were investigated using particle swarm optimisation through virtual experimentation. The running script for particle swarm optimisation uses the following work parameters:

ac = randlim(100,1,0,12); % tempering depth

tc = randlim(100,1,0,10); % tempering time

pc = randlim(100,1,40,70); % tempering power

the first being the dependent variable.

There are defined the maximum degree to which the independent variables will be combined in a polynomial, n=6, and the fitness function:

```
f=@(x) testfun(x,ac,tc,pc,n)
```

The implementation of PSO used does not require an explicit setting of simulation characteristics, being natively limited to 100 iterations and automatic listing every tenth generation. The model parameters are optimised in 5.79 s. The second version model run obtains the optimised model parameters for the variable Z in 2.2 s. A number of strategies were investigated for the optimisation of HFC Hardening. Their implementation into the intelligent real time control interfaces depends on the specific circumstances of the model characteristics used and the exact definition of the optimisation problem.

IV. INTELLIGENT CONTROL INTERFACE USING NEURAL NETWORKS

Starting from the research done in the field of high frequency tempering, a number of experimentations were developed for the improvement of mechanical characteristics of the used profiles in constructing metal structure buildings through improved performance for the real time control of the HFC Hardening System using neural networks in implementing the Intelligent Real Time Control Interfaces.

To this end, with the aim of testing and simulation through neural networks [11, 12], there were considered the following inputs: - specific power, p, [kW/ cm^2], advance speed, v, [mm/min] and piece angular speed, n, [rot/min], the desired output being the piece hardness (HRC).

The dependence between inputs and outputs is based on a polynomial regression model:

$$HRC = 23 + 25\,p - 0{,}106\,v + 0{,}23n + 0{,}025\,pv - 0{,}125\,pn - 0{,}00126\,nv + 0{,}00075\,pvn$$

In order to train the neural network, a Levenberg-Marquardt algorithm was used, with two layers (hidden layer and output layer) and 10 neurons (figure 3). The considered samples were divided so that 70 % were used for training, 15% for validation and 15% for testing.

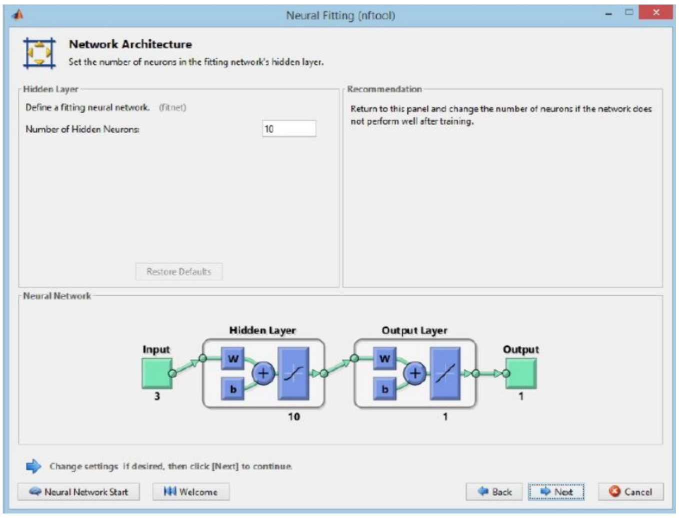


Figure 3. Neural network representation

Figure 4 shows the algorithms used in all stages of the neural network training, the progress in time and the resulting diagrams.

Training the network was done multiple times and was adjusted until the training errors were minimal.

In the case of simulation and testing in Matlab of the HFC Hardening interface, the neural network model can be used as a standalone block in Simulink.

Figure 5 shows the graphics resulting after training, validation and testing the network.

There can be observed that the errors are smaller for training and validation, but larger in testing the network.

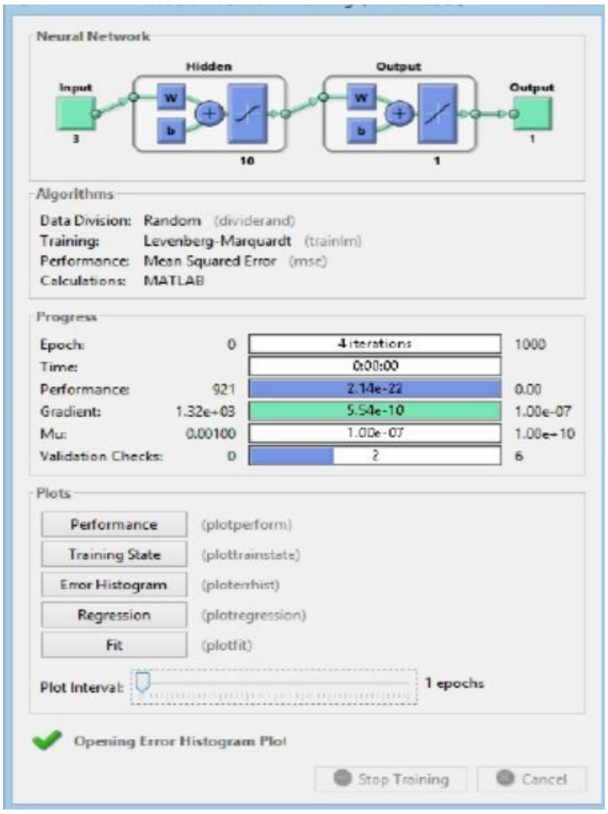


Figure 4. Neural network performance

These larger errors during testing are largely due to the restricted sample considered when training.

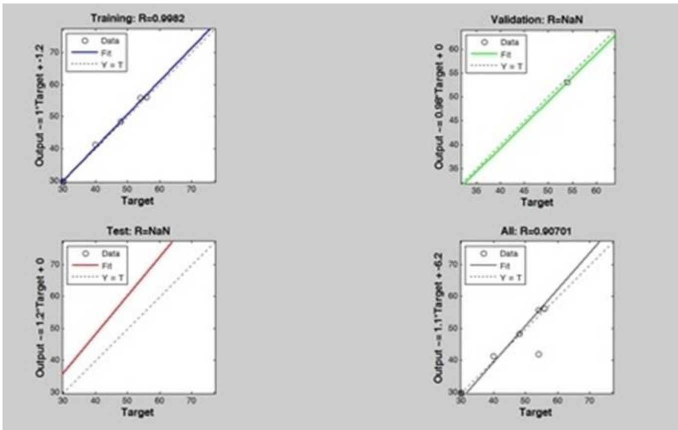


Figure 5. Training the neural networks for HFC Hardening

As the available data increases, the errors in testing will decrease to the normal level of those obtained during training and validation.

V. REGRESSION MODEL FOR SPECIFIC PARAMETERS OF THE HFC HARDENING SYSTEM

With the aim of researching the dependence functions between the CIF process parameters, specific methods of applied statistics are used, namely experiment projection and multiple regression.

If the response of a process or system is influenced by two or more factors, a factorial experiment can be programmed, meaning an experiment where each sample is a possible combination of factor levels.

The effect of a factor is evidenced by the variation of the response values, subject to modifications occurring in its level. This is called a main effect as it refers to the considered inputs (the primary factors).

There are experiments in which the variation in response values, for two different levels of a factor, are not the same at all levels of the other factors, which suggests an interaction between the factors. There are situations where the interactions are significant enough that they mask the main effects, which no longer hold a significant effect on the output value.

The program matrix (experimentation matrix) is defined as being the matrix where the levels of all factors are shown for each sample within the experiment. For ease of calculation, usually, this matrix does not contain real (natural) values of the independent variables studied, but coded (un-dimensional) values obtained from these.

The values, on various variation levels, are in arithmetic or geometric progression (in the latter case they can be brought to arithmetic progression through logarithm). The interdepence relations between the natural value z_j and the coded value are:

$$x_j = 2\frac{z_j - \bar{z}_j}{z_{\max} - z_{\min}} \quad \text{and} \quad z_j = \bar{z}_j + \frac{z_{\max} - z_{\min}}{2} x_j \qquad (1)$$

where: x_j is the coded value of factor j;

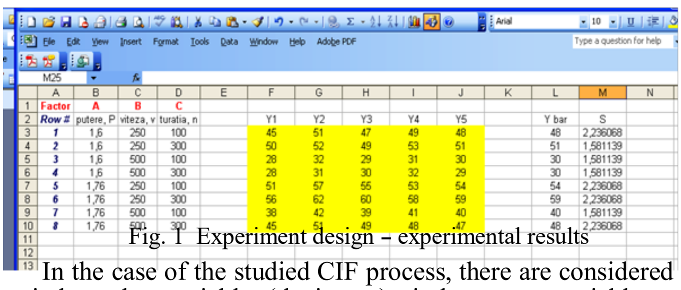
z_j - the natural value of factor j

$\bar{z}_j$ - the mean of the natural values of factor j;

$z_{\min}$; $z_{\max}$ - the minimum and maximum, respectively, of factor j.

Among the most frequently used types of experimental programs are ***factorial programs***, which show both the main effects of the factors, as well as their interactions. If there are k factors, each of these with two variation levels (minimum / maximum), then the complete factorial program (FFD) has 2^k experiences.

A flexible and efficient design for modelling the second order dependencies is the Box-Wilson or the centred composed design (CCD).



	A	B	C	D	E	F	G	H	I	J	K	L	M	N
1	Factor	A	B	C										
2	Row #	putere, P	viteza, v	turatia, n		Y1	Y2	Y3	Y4	Y5		Y bar	S	
3	1	1,6	250	100		45	51	47	49	48		48	2,236068	
4	2	1,6	250	300		50	52	49	53	51		51	1,581139	
5	3	1,6	500	100		28	32	29	31	30		30	1,581139	
6	4	1,6	500	300		28	31	30	32	29		30	1,581139	
7	5	1,76	250	100		51	57	55	53	54		54	2,236068	
8	6	1,76	250	300		56	62	60	58	59		59	2,236068	
9	7	1,76	500	100		38	42	39	41	40		40	1,581139	
10	8	1,76	500	300		45	51	49	48	47		48	2,236068	

Fig. 1 Experiment design – experimental results

In the case of the studied CIF process, there are considered as independent variables (the inputs), zj, the process variables:

- specific power, p, in kW/ cm2 (z1);
- advance speed, v : [mm/min] (z2);
- angular speed, n : [rot/min] (z3).

The desired dependent variable (output) is the hardness of the superficial strata, HRC. The software used for programming the experiments, data processing and obtaining the regression results is the DOE KISS (student free version).

The experimentation matrix (experiment design and experimental results, for 5 values of the replicates), is shown in Figure 1x.

A multiple regression model is applied which contains more than one variable (regressor). The dependent variable, the response Y, can be correlated to k independent variables, through a linear model of multiple regression

$$Y = P_0 + P_1 x_1 + P_2 x_2 + \ldots + P_k x_k + E \qquad (2)$$

where: β_j, $j = 0,1,2,\ldots, k$ are called *regression coefficients*.

A second order model with interactions is considered:

$$Y = P_0 + P_1 x_1 + P_2 x_2 + P_{11} x_1^2 + P_{22} x_2^2 + P_{12} x_1 x_2 + E \qquad (3)$$

then, using the notations:

$P_{11} = P_3$; $P_{22} = P_4$; $P_{12} = P_5$

$x_1^2 = x_3$; $x_2^2 = x_4$; $x_1 x_2 = x_5$

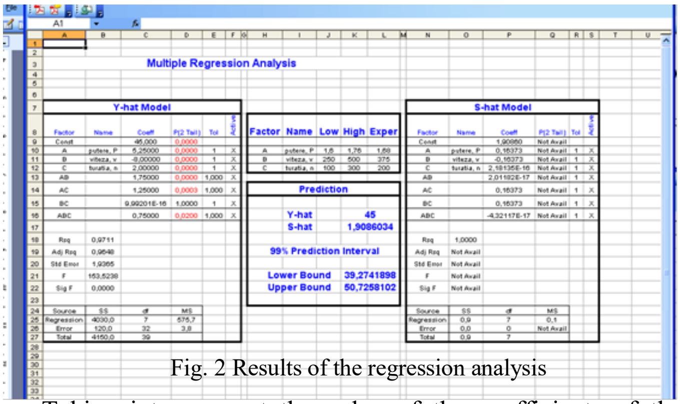
the model becomes linear, a multiple regression, namely:

$$Y = P_0 + P_1 x_1 + P_2 x_2 + P_3 x_3 + P_4 x_4 + P_5 x_5 + E \qquad (4)$$

In order to determine the estimators for the regression model coefficients, the *least squares method* can be used. Thus, there are the observations $(x_{i1}, x_{i2}, \ldots, x_{ik}, y_i)$, where $i = 1,2,\ldots, n$ and $n > k$, such that, for each of them is fulfilled the relation (9-1), namely:

$$y_i = P_0 + P_1 x_{i1} + P_2 x_{i2} + \ldots + P_k x_{ik} + E_i = P_0 + \sum_{j=1}^{k} P_j x_{ij} + E_i \qquad (5)$$

For the studied CIF process, the results of the regression analysis, obtained with the DOE KISS software, are shown in Figure 2x.



Multiple Regression Analysis

Y-hat Model

Factor	Name	Coeff	P(2 Tail)	Tol	Active
Const		46,000	0,0000		
A	putere, P	6,25000	0,0000	1	X
B	viteza, v	-8,00000	0,0000	1	X
C	turatia, n	2,00000	0,0000	1	X
AB		1,75000	0,0000	1,000	X
AC		1,25000	0,0003	1,000	X
BC		9,99201E-16	1,0000	1	X
ABC		0,75000	0,0200	1,000	X

Rsq	0,9711
Adj Rsq	0,9648
Std Error	1,9365
F	153,5238
Sig F	0,0000

Source	SS	df	MS
Regression	4030,0	7	575,7
Error	120,0	32	3,8
Total	4150,0	39	

Factor	Name	Low	High	Exper
A	putere, P	1,6	1,76	1,68
B	viteza, v	250	500	375
C	turatia, n	100	300	200

Prediction

Y-hat	45
S-hat	1,9086034

99% Prediction Interval

Lower Bound	39,2741898
Upper Bound	50,7258102

S-hat Model

Factor	Name	Coeff	P(2 Tail)	Tol	Active
Const		1,90850	Not Avail		
A	putere, P	0,16373	Not Avail	1	X
B	viteza, v	-0,16373	Not Avail	1	X
C	turatia, n	2,18136E-16	Not Avail	1	X
AB		2,01182E-17	Not Avail	1	X
AC		0,16373	Not Avail	1	X
BC		0,16373	Not Avail	1	X
ABC		-4,32117E-17	Not Avail	1	X

Rsq	1,0000
Adj Rsq	Not Avail
Std Error	Not Avail
F	Not Avail
Sig F	Not Avail

Source	SS	df	MS
Regression	0,9	7	0,1
Error	0,0	0	Not Avail
Total	0,9	7	

Fig. 2 Results of the regression analysis

Taking into account the value of the coefficients of the polynomial model with interactions (see Figure 2, Y-hat Model) and the dependence relations between the coded variables and the natural ones (see equation (1)), there is obtained the dependence function between the studied parameters of the CIF process, namely the second order polynomial relation with interactions:

$$HRC = 23 + 25p - 0{,}106v + 0{,}23n + 0{,}025\,pv - 0{,}125\,pn - 0{,}00126\,nv + 0{,}00075\,pvn$$

VI. RESULTS AND CONCLUSIONS

regression method, and achieved intelligent control (IC) interfaces by virtual projection method, through Particle Swarm Optimization (PSO) and neural networks (NN) in which the control decisions (Decision making) use fuzzy logic or logic neutrosophic.

The results obtained from experiments and the multiple regression show the degree of influence on the output variable of each of the independent variables and their respective interactions.

Following the regression analysis results that the polynomial regression model with interactions is adequately adapted to the task at hand. All studied independent variables influence the output variable, the most significant influence being that of the advance speed.

There is a single interaction with insignificant influence on the hardness of the superficial tempered strata (CIF), namely the interaction between the advance speed **v** and the piece angular speed ***n.***

The results obtained outline the development of the ICI modules using advanced control strategies adapted to the induction hardening process, applying ICT techniques with fast processing and real-time communications, which led to improvement of the material's mechanical characteristics.

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On the Efficacy of Moringa Oleifera as Anticancer Treatment: A Literature Survey

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Victor Christianto, Florentin Smarandache (2019). On the Efficacy of Moringa Oleifera as Anticancer Treatment: A Literature Survey. BAOJ Cancer Research & Therapy 5(2): 069, 5

Abstract

Medicinal plants are important elements of indigenous medical system that have persisted in developing countries. Many of the botanical chemo-preventions currently used as potent anticancer agents. However, some important anticancer agents are still extracted from plants because they cannot be synthesized chemically on a commercial scale due to their complex structures that often contain several chiral centers. The aim of this study was to test different extracts from the Moringa oleifera leaves. Previous studies have shown potentially antioxidant, antitumor promoter, anticlastogen and anticarcinogen activities both in vitro and in vivo. Emerging evidence indicates that efficacy of Moringa oleifera in cancer treatment deserves re-examination. This paper is a short literature survey of research in recent years.

Keywords: moringa oleifera, antioxidant, anticancer

Introduction

Moringa Oleifera (MO), aplant from the family Moringa ceaisa major cropin Asia and Africa (they can be found in Himalaya Mountain, and have been used for thousand years in India etc.). MO has been studied for its health properties, attributed to the numerous bioactive components, including vitamins, phenolic acids, flavonoids, isothiocyanates, tannins and saponins, which are present in significant amounts in various components of the plant. Moringa Oleifera leaves are the most widely studied and they have shown to be beneficial in several chronic conditions, including hypercholesterolemia, high blood pressure, diabetes, insulin resistance, non-alcoholic liver disease, cancer and overall inflammation. Mean while, it is known that cancers are the leading causes of morbidity and mortality worldwide, with approximately 14 million new cases and 8.2 million cancer related deaths. The number of new cases is expected to rise by about 70% over the next 2 decades. Among men, the 5 most common sites of cancer diagnosed were lung, prostate, colorectum, stomach, and liver cancer. Among women the 5 most common sites diagnosed were breast, colorectum, lung, cervix, and stomach cancer [7].This paper is a short literature survey of research on MO efficacy as anticancer treatment in recent years.

Identification

[4] "Moringa is a small, fast-growing, drought deciduous tree or shrub that reaches 12m in height at maturity. It has a wide-open, typically umbrella- shaped crown, straight trunk (10-30 cm thick) and a corky, whitish bark. The plant (depending on climate) has leaflets 1-2 cm in diameter and 1.5-2.5 cm in length its leaves are impair pinnate ,rachis 3 to 6 cm long with 2 to 6 pairs of pinnules. Each pinnule has 3 to 5 obovate leaflets that are 1 to 2 cm long. The terminal leaflet is often slightly larger. Its leaflets are quite pale when young, but become richer in color with maturity. Cream-colored flowers emerge in sweet-smelling panicles during periods of drought or water stress when the tree loses its leaves. The pods are triangular in cross-section-30 to 50 cm long and legume-like in appearance. The oily seeds are black and winged. The tree produces a tuberous taproot, which explains its tolerance to drought conditions."

Occurrence throughout the world

[5] "This species is a fast growing soft wood tree that can reach 12 m in height and is indigenous to the Himalayan foothills (northern India Pakistan and Nepal) [2,3]. Its multiple uses and potential attracted the attention of farmers and researchers in past historical eras. Ayurvedic traditional medicine says that Moringa oleifera can prevent 300 diseases and its leaves have been exploited both for preventive and curative purposes [4]. Moreover, a study in the Virudhunagar district of Tamil Nadu India reports Moringa among the species utilized by traditional Siddha healers [5]. Ancient Egyptians used Moringa oleifera oil for its cosmetic value and skin preparation [6]. even if the species never became popular among Greeks and Romans, they were aware of its medical properties [7]. Moringa oleifera has been grown and consumed in its original areas until recently (the 1990s) when a few researchers started to study its potential use in clarifying water treatments, while only later were its nutritional and medical properties "discovered" and the species was spread throughout almost all tropical countries. In 2001, the first international conference on Moringa oleifera was held in Tanzania and since then the number of congresses and studies increased disseminating the information about the incredible properties of Moringa oleifera. Now this species has been dubbed "miracle tree", or "natural gift", or mother's best friend."

[4] "Moringa trees though native in the sub-Himalayan tracts, it is widely cultivated in Africa, Central and South America, Sri Lanka, India, Mexico, Malaysia, Indonesia and the Philippines. According to Muluvi et al (1999), the Moringa tree wide natural spread in the world and introduced to Africa from India where it used as a health supplement and it was originally an ornamental tree in the Sudan, planted during British rule in the alleys along the Nile, public parks, and the gardens of foreigners. It seems likely that the Arab women of Sudan discovered this remarkable clarifier tree. "

Phytochemistry

As Moringa oleifera leaves are most used part of the plant, we review articles concerning Phytochemistry and pharmacological properties of leaves. Several bioactive compounds were recognized in the leaves of Moringa oleifera. They are grouped as vitamins, carotenoids, polyphenol, phenolic acids, flavonoids, alkaloids, glucosinolates, isothiocyanates, tannins, saponins and oxalates and phytates. The amounts of different bioactive compounds found in Moringa oleifera leaves and reported in literature are summarized in following tables.

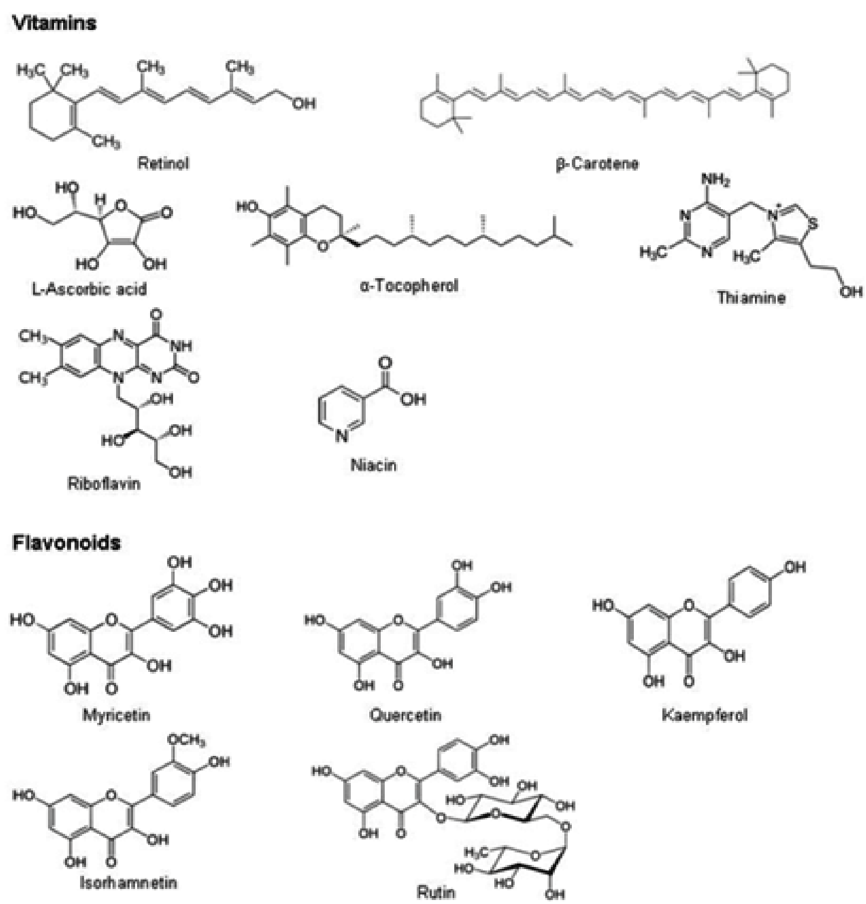

Figure 1: SChemical structures of bioactive compounds in MO. After Leone et al. [5]

Table 1. Vitamins content in *Moringa oleifera* leaves.

Bioactive Compound	Leaves	Value Found in Literature	Value Express as Dry Weight	Drying Method	Extractive Method	Analytical Method	Country	Reference
Vitamins								
Vitamin A	fresh	11,300 IU	45,200 IU		N/A	N/A	India	[14]
	fresh	23,000 IU	92,000 IU [a]		N/A	N/A	Brazil	[52]
Vitamin B1–Thiamine	fresh	0.06 mg/100 g	0.24 mg/100 g		N/A	N/A	India	[14]
	fresh	0.21 mg/100 g	0.84 mg/100 g		N/A	N/A	N/A	[53]
	fresh	0.6 mg/100 g	2.58 mg/100 g		N/A	Microbiological method	India	[54]
	dried	2.64 mg/100 g	2.85 mg/100 g	N/A	N/A	N/A	N/A	[53]
Vitamin B2–Riboflavin	fresh	0.05 mg/100 g	0.20 mg/100 g		N/A	N/A	India	[14]
	fresh	0.05 mg/100 g	0.20 mg/100 g		N/A	N/A	N/A	[53]
	fresh	0.17 mg/100 g	0.726 mg/100 g		N/A	Microbiological method	India	[54]
	dried	20.5 mg/100 g	22.16 mg/100 g	N/A	N/A	N/A	N/A	[53]
Vitamin B3–Niacin	fresh	0.8 mg/100 g	3.20 mg/100 g		N/A	N/A	India	[14]
	fresh	0.8 mg/100 g	3.20 mg/100 g		N/A	N/A	N/A	[53]
	fresh	0.82 mg/100 g	3.5 mg/100 g		N/A	Microbiological method	India	[54]
	dried	8.2 mg/100 g	8.86 mg/100 g	N/A	N/A	N/A	N/A	[53]
Vitamin C–Ascorbic acid	fresh	220 mg/100 g	880 mg/100 g		N/A	N/A	India	[14]
	dried	17.3 mg/100 g	18.7 mg/100 g	N/A	N/A	N/A	N/A	[53]
	dried	92 mg/100 g 140 mg/100 g 56 mg/100 g	92 mg/100 g 140 mg/100 g 56 mg/100 g	Sun-drying for 4 days Shadow-drying for 6 days Oven-drying at 60 °C for 1 h	N/A	AOAC 2004	India	[55]
	dried	38.8 mg/100 g [b]	38.8 mg/100 g [b]	Air-drying	Metaphosphoric acid	Indophenol titration	Pakistan	[56]
	freeze-dried	271 mg/100 g	271 mg/100 g	Freeze-drying	Deionized water	Colorimetric method	Florida, USA	[57]
	freeze-dried	920 mg/100 g 840 mg/100 g 680 mg/100 g	920 mg/100 g 840 mg/100 g 680 mg/100 g	Freeze-drying	6% metaphosphoric acid	Titration against 2,6-dichlorophenolindophenol	Nicaragua India Niger	[58]
Vitamin E–Tocopherol	fresh	9.0 mg/100 g	16.21 mg/100 g		N-hexane + ethyl acetate + BHT	Reverse-phase HPLC	Malaysia	[59]
	dried	113 mg/100 g	122.16 mg/100 g	N/A	N/A	N/A	N/A	[53]
	[illegible]	[illegible]	[illegible]	[illegible]	Microscale saponification and	[illegible]	[illegible]	[illegible]

Table 1: Vitamin content in MO leaves. After Leone et al. [5]

Table 2. Carotenoids content in *Moringa oleifera* leaves.

Bioactive Compound	Leaves	Value Found in Literature	Value Express as Dry Weight	Drying Method	Extractive Method	Analytical Method	Country	Reference
Carotenoids								
β-carotene	fresh	6.63 mg/100 g	33.48 mg/100 g		Acetone–n-hexane	HPLC	Taiwan	[62]
	fresh	6.8 mg/100 g	27.22 mg/100 g		N/A	N/A	N/A	[53]
	dried	36 mg/100 g 39.6 mg/100 g 37.8 mg/100 g	36 mg/100 g 39.6 mg/100 g 37.8 mg/100 g	Sun-drying for 4 days Shadow-drying for 6 days Oven-drying at 60 °C for 1 h	N/A	AOAC 2004	India	[55]
	dried	16.3 mg/100 g	17.62 mg/100 g	N/A	N/A	N/A	N/A	[53]
	dried	18.5 mg/100 g	20.44 mg/100 g	Air-dried under shade	N/A	HPLC	South Africa	[61]
	freeze-dried	66 mg/100 g	66 mg/100 g	Freeze-drying	Acetone	HPLC	Florida, USA	[57]
Lutein	fresh	6.94 mg/100 g	35.05 mg/100 g		Acetone–n-hexane	HPLC	Taiwan	[62]
	freeze-dried	102 mg/100 g	102 mg/100 g	Freeze-drying	Acetone	HPLC	Florida, USA	[57]

Abbreviation: N/A = Not available.

Table 2: Carotenoids content of MO leaves. After Leone et al. [5]

Table 3. Polyphenols content in *Moringa oleifera* leaves.

Bioactive Compound	Leaves	Value Found in Literature	Value Express as Dry Weight	Drying Method	Extractive Method		Analytical Method	Country	Reference
Polyphenols									
Total phenols	dried	4581 mgGAE/100 g [a] 3602 mgGAE/100 g [b]	4581 mgGAE/100 g [a] 3602 mgGAE/100 g [b]	Shade-drying	Water Soxhlet extraction for 18–20 h		Folin-Ciocalteau	India	[63]
	dried	3290 mgGAE/100 g	3290 mgGAE/100 g	N/A	50% MeOH		Folin-Ciocalteau	India	[64]
	dried	2090 mgGAE/100 g	2090 mgGAE/100 g	N/A	50% MeOH, 100% MeOH and water		Folin-Ciocalteau	India	[65]
	dried	10,504 mgGAE/100 g	10,504 mgGAE/100 g	N/A	Water at 80 °C for 2 h		Folin-Ciocalteau	India	[66]
	dried	10,616 mgGAE/100 g [c]	10,616 mgGAE/100 g [c]	Air-drying	80% MeOH		Folin-Ciocalteau	Pakistan	[56]
	dried	10,300 mgGAE/100 g 12,200 mgGAE/100 g 9720 mgGAE/100 g 11,600 mgGAE/100 g	10,300 mgGAE/100 g 12,200 mgGAE/100 g 9720 mgGAE/100 g 11,600 mgGAE/100 g	Air-drying	100% MeOH 80% MeOH 100% EtOH 80% EtOH	Extraction by shaker	Folin-Ciocalteau	Pakistan	[67]
	dried	9630 mgGAE/100 g 10,700 mgGAE/100 g 6160 mgGAE/100 g 8210 mgGAE/100 g	9630 mgGAE/100 g 10,700 mgGAE/100 g 6160 mgGAE/100 g 8210 mgGAE/100 g	Air-drying	100% MeOH 80% MeOH 100% EtOH 80% EtOH	Extraction by reflux	Folin-Ciocalteau	Pakistan	[67]
	dried	2070 mg TAE/100 g	2070 mg TAE/100 g	Air-drying	Acetone/Water (7:3)		Folin-Ciocalteau	India	[68]
	dried	1600 mgTEA/100 g [d] 3400 mgTEA/100 g [e]	1600 mgTEA/100 g [d] 3400 mgTEA/100 g [e]	Air-drying	80% EtOH		Folin-Ciocalteau	Nicaragua	[69]
	dried	5350 mgCAE/100 g 2930 mgCAE/100 g 3710 mgCAE/100 g 3280 mgCAE/100 g 4550 mgCAE/100 g 4460 mgCAE/100 g	5350 mgCAE/100 g 2930 mgCAE/100 g 3710 mgCAE/100 g 3280 mgCAE/100 g 4550 mgCAE/100 g 4460 mgCAE/100 g	Oven-drying at 60 °C for 24 h	Maceration with 70% EtOH Maceration with 50% EtOH Percolation with 70% EtOH Percolation with 50% EtOH Soxhlet extraction with 70% EtOH Soxhlet extraction with 50% EtOH		Folin-Ciocalteau	Thailand	[70]

Table 3: Polyphenols content of MO leaves. After Leone et al. [5]

Anticancer Effects

MO has been studied for its chemo preventive properties and has been shown to inhibit the growth of several human cancer cells. The capacity of MO leaves to protect organisms and cells from oxidative DNA damage, associated with cancer and degenerative diseases, has been reported in several studies. Khalafalla et al. found that the extract of MO leaves inhibited the viability of acute myeloid leukemia, acute lymphoblastic leukemia and hepatocellular carcinoma cells. Several bioactive compounds, including 4-(α-L-rhamnosyloxy) benzyl isothiocyanate, niazimicin and β-sitosterol-3-O-β-D-glucopyranoside present in MO, may be responsible for its anti-cancer properties. MO leaf extract has also been proven to be efficient in pancreatic and breast cancer cells. In pancreatic cells, MO was shown to contain the growth of pancreatic cancer cells, by inhibiting NF-kB signaling as well as increasing the efficacy of chemotherapy, by enhancing the effect of the drug in these cells. In breast cancer cells, the antiproliferative effects of MO were also demonstrated. A recent study by Abd-Rabou et al. evaluated the effects of various extracts from Moringa Oleifera, including leaves and roots, and preparations of nanocomposites of these compounds against HepG, breast MCF7 and colorectal HCT116/Caco2 cells. All these preparations were effective on their cytotoxic impact, as measured by apoptosis. Several animal studies have also confirmed the efficacy of Moringa Oleifera leaves in preventing cancer in rats with hepatic carcinomas induced by diethyl nitrosamine and in suppressing azoxymethane-induced colon carcinogenesis in mice. Alist of some bioactive components present in MO leaves, their postulated actions in the animal model used, their protection against a specific disease and the corresponding reference are presented in Table 1."

Moreover, according to Abdull Razis et al., MO leaves also have anti-inflammatory, antitumor and anticancer effects [2].

Table 4: Bioactive Components in *Moringa Oleifera* and their Positive Effects on Chronic Disease.

Compounds	Postulated Function	Model Used	Disease Protection	References
Flavonoids: Quercitin	Hypolipidemic and anti-diabetic properties	Zucker rat	Diabetes	[36]
	Lower hyperlipidemia	Rabbits	Atherosclerosis	[37,38]
	Decrease expression of DGAT	Guinea Pigs	NAFLD	[80]
	Inhibition of cholesterol esterase and α-glucosidase	In vitro study	Cardiovascular disease and Diabetes	[60]
	Inhibits activation of NF-kB	High fat fed Mice	Cardiovascular disease	[74]
Chlorogenic Acid	Glucose lowering effect	Diabetic rats	Diabetes	[45]
	Cholesterol lowering in plasma and liver	Zucker rat	Cardiovascular disease	[46]
	Decrease expression of CD68, SERBP1c	Guinea pigs	NAFLD	[87]
	Anti-obesity properties	High-fat induced obesity rats	Obesity	[49]
	Inhibit enzymes linked to T2D		Diabetes	[90]
Alkaloids	Cardioprotection	Cardiotoxic-induced rats	Cardiovascular disease	[49]
Tannins	Anti-inflammatory	Rats	Cardiovascular/Cancer	[54]
Isothiocyanates	Decreased expression of inflammatory markers	RAW Macrophages	Cardiovascular disease	[76]
	Reduction in insulin resistance	Mice	Diabetes	[88]
	Inhibition of NF-kB signaling	Cancer breast cells	Cancer	[99]
B-Sitosterol	Decrease cholesterol absorption	High-fat fed rats	Cardiovascular disease	[18]

Abbreviations used: CD68: cluster of differentiation 68; DGAT: diacyl glycerol transferase; NF-kB: nuclear factor-kB; SRBP1c: sterol regulatory binding protein 1c; T2D: type 2 diabetes; NAFLD: non-alcoholic fatty liver disease.

Table 4: Bioactive components in MO. (After Vergara-Jimenez et al. [1])

Hypotheses on MO chemo preventive effects

[3] "We speculated that the chemo preventive effect of bMO arose from fatty acids present in MO which might modulate cell proliferation and/or apoptosis and anti-inflammation which plays an important role in colon carcinogenesis. It has been reported that human colon tumor growth is promoted by oleic acid through mechanisms that comprise an increase in fatty acid oxidation and disturbance of membrane enzymes [4]. In contrast, olive oil, an important source of omega-9 oleic fatty acid, may prevent against the development of colorectal cancer through its influence on secondary bile acid patterns in the colon. Another hypothesis for chemo preventive effect of MO pods may be due to the modulation of detoxification enzyme. It has been shown that MO pods extract has the potential for modulating phase I and II enzymes such as cytochrome b5, cytochrome P450, catalase, glutathioneperoxidase, reductase and S-transferase in mice [5]. Moreover, the diet containing bMO showed potentially anti-clastogenic activity against both direct and indirect-acting clastogens in male ICR mice [6]. In the present study, a potent colon carcinogen, AOM, was used to induce colon carcinogenesis, so bMO in the diet might act via the carcinogenesis processes through metabolic activation [7]."

Concluding Remarks

We have discussed some real positive effects on the use and efficacy of Moringa Oleifera as anticancer treatment. Nonetheless, further studies and procedures to maximize such positive impact of MO as anticancer should be continued.

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Selection of Optimal Software Development Methodology Based on Weighted Aggregated Sum Product Assessment Method

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Abstract: The software development methodology covers the complete software life cycle. It involves the production of quality and reliable software in a systematic, controlled and efficient manner using formal methods for specification, evaluation, analysis and design, implementation, testing, and maintenance. Today, software is used in all domains of education. From primary and secondary schools to higher education institutions are using specialist software packages intended for research. The aim of this manuscript is a selection of the software development methodology based on multiple-criteria decision-making methods. PIPRECIA method is applied for defining the weights of the criteria, whereas WASPAS method is applied for the ranking of alternatives. The application of the proposed approach, as well as its efficiency and effectiveness, are shown in the conducted case study.

Keywords: Software, Software development methodology, MCDM, WASPAS

1. INTRODUCTION

The software development methodology covers the complete software life cycle. It involves the production of quality and reliable software in a systematic, controlled and efficient manner using formal methods for specification, evaluation, analysis and design, implementation, testing, and maintenance. Today, software is used in all domains of education. From primary and secondary schools to higher education institutions are using specialist software packages intended for research.

Manger (2012) states that software product is a collection of computer programs and related documentation, created precisely for the reason of being sold. It can be developed for a specific user (customized product) or generally for the market (generic product). Today's software includes the necessity that he must be of good quality. More specifically, a software product is expected to be characterized by the following quality attributes: a) Maintenance; b) Reliability and security; c) Efficiency; d) Usability.

Thus, the software represents a series of commands that are stored in the computer's memory. It is executed on hardware and is required for proper operation and functioning of a computer system. According to the purpose one of the most common software division is into two groups: a) System software: all programs, software packages, etc. intended for the functioning of a computer system are belonging to system software; b) Application software: all programs, software packages, etc. intended to solve specific problems and tasks of computer system users are belonging to application software (Tomašević, 2012).

The pace of change in the software development industry is still high. People continue to push the boundaries of known techniques and practices to develop as efficient and effective software as possible. Software development lifecycle models and business decision models contribute to controlling product development in different ways.

Software products are among the most complex systems made by man. Therefore their development requires the use of techniques and processes that can be successfully scaled up to very large applications while satisfying demands for size, performance and security, all within the time and budget constraints. The complexity of large software systems is overcome by the use of higher-level abstraction structures, such as software architecture.

Thus, a set of activities that are related to the "initialization, design, realization and sale of software products and managing all the resources that are related to that product" is called software engineering which is of crucial importance to software development (Steward, 1987).

Today, decisions are made daily and are one of the most important elements of management activities. With globalization and increasing business dynamics, changes have been made in the decision-making process, so decision-making has become much more demanding and complex. Multiple-Criteria Decision-Making (MCDM) represents the process of selecting one alternative from a set of available alternatives or, in some cases, ranking alternatives based on a predefined set of specific criteria that most often have different significance. Justification for the application of the MCDM methods is relevant approach to making decisions and the adoption of sustainable solutions (Stanujkić *et al.*, 2019a; Stanujkic *et al.*, 2019b; Karabašević & Maksimović, 2018; Stanujkić *et al.*, 2017b).

Therefore, the main aim of this manuscript is to present an approach for the selection of software development methodology based on MCDM methods. For the defining weights of the criteria, PIPRECIA method is applied, whereas for the purpose of the ranking of alternatives WASPAS method is applied.

2. THEORETICAL BACKGROUND

Nowadays, software engineering is receiving increasing attention in software development. Software engineering is a systematic approach to the development, exploitation, maintenance, and replacement of software products. Software engineering is a technological and management discipline that deals with the systematic production and maintenance of software products, which should be developed on time and at an estimated cost (Mead, 2009).

Characteristics of software engineering are (Buckley, 1987): a) modeling as the basis of design (object-connection model, process model, data model); b) methods of analysis, synthesis, and identification are used; c) division into levels, related to the phases of the software product life cycle and the project phase; d) interdisciplinarity and user involvement; d) project organization; e) validation and verification of results, quality assurance, presentation of results, documentation.

Because of all of the above, software engineering requires the use of both analytical and descriptive tools that have been developed within the computer sciences, along with the rigorous approach that engineering disciplines bring to achieving adequate reliability and security, all through the teamwork of software engineers working in a cohesive environment. At today's level of software engineering development, an organization's ability to handle software development in this way is precisely measurable and ranges from level 1 where software processes are unpredictable to level 5, where software processes are optimized.

The software development model is selected depending on the nature of the project and the application, the technical orientation of the people who will participate in the development, the methods and tools that will be used in the development, the methods of control as well as the products that are required. The primary goal of model creation is to provide software products that meet user requirements.

Depending on the importance of particular stages and activities of software development and the forms of organization and development management, as well as the experience of employees and the nature of the product, there are (Balaji & Murugaiyan, 2012; Martin, 2002):

a) Sequential software development model, the so-called *waterfall*;
b) Iterative and incremental model of software development;
c) Agile development model.

The purpose of the SWEBOK project is to characterize the content of software engineering as a discipline, as well as to differentiate software engineering from other disciplines such as computer science, project management, computer engineering, etc. According to the SWEBOK project, software engineering methods can be divided into three areas: a) heuristic methods relating to methods based on the informal approach; b) formal methods based on a mathematical approach; and c) prototyping methods relating to methods based on different types of prototyping (Antović *et al*., 2008; Stanojević *et al*., 2006).

Each model of the software development process uses a requirement specification as input and a delivered product as an output. Over the years, many such models have been proposed. Below, some of the most popular will be discussed in order to better understand their similarities and differences.

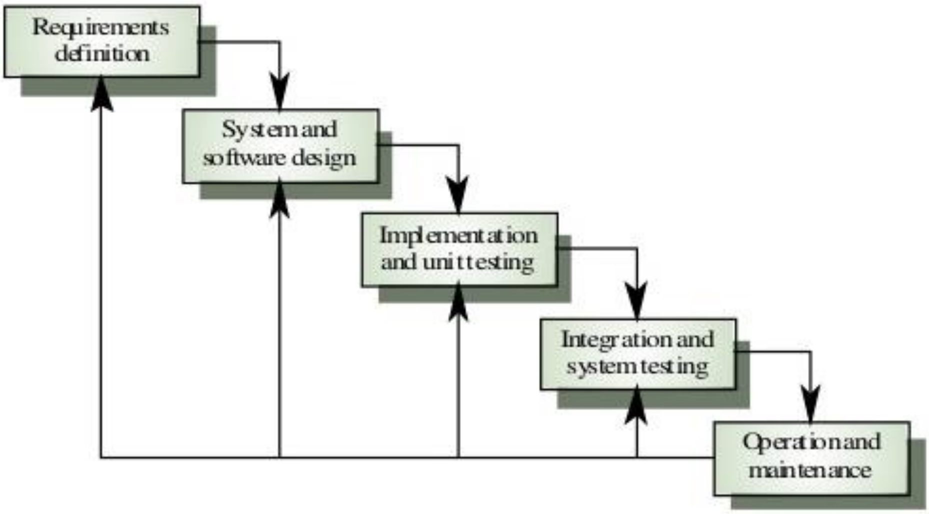
2.1. The waterfall model

One of the first proposed is a waterfall model. The waterfall model implies that it is necessary that one phase of the development must be fully completed before the beginning of the next one (Royce, 1970).

The waterfall model is very useful as an aid in expressing what the software development team needs to do. Its simplicity makes it easy to provide explanations to those unfamiliar with the software development process as it explicitly indicates among the steps necessary to begin the next phase. Many other more complex models represent a "beautified" waterfall model, through the inclusion of feedbacks and activities (Balaji & Murugaiyan, 2012). The biggest problem with the waterfall model is that it does not reflect the actual way in which the code evolves. Except perhaps for the very clear problems, the software is usually developed through a number of iterations. Software is often used to solve a problem that has never been resolved before, or whose solution must be improved to reflect changes that have occurred in the business or work environment (Pfleeger & Atlee, 1998).

Although this model has been used for many years in the production of many quality systems, it does not mean that no problems occur. In recent years, the model has been criticized for its rigidity and inflexible procedure.

Figure 1. *Waterfall model*



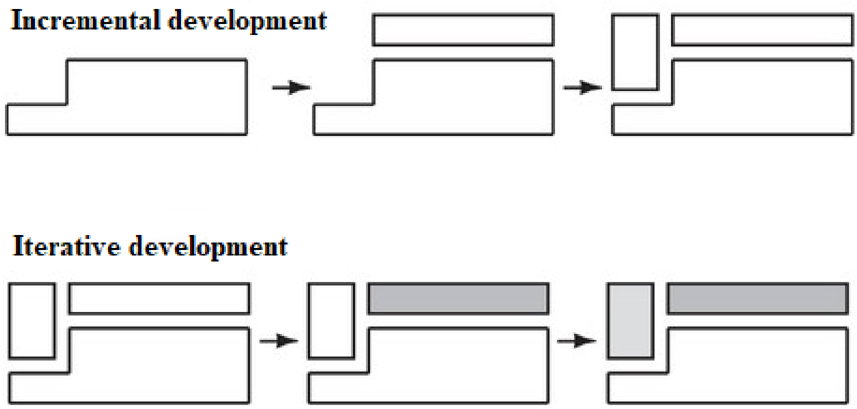
Source: https://www.slideshare.net/Ehtesham17/waterfall-model-in-software-engineering

2.2. The iterative and incremental model

The problems encountered in the waterfall model led to the need for a new method of system development that would provide quick results, which would require less initial information and offer more flexibility.

When iterative development is applied, the project is divided into smaller parts. This allows the development team to demonstrate results early in the process and to receive valuable feedback from system users. Often, each iteration is actually one mini-waterfall with feedback from one phase that provides vital information for the next phase. In iterative development, iteration is delivered immediately, at the very beginning, and then the functions of each subsystem are changed, in each new version. In incremental development, the system as specified in the requirement specification is subdivided into subsystems by functions. Versions are defined initially as small, functional subsystems, and then new features are added to each new version (Larman, & Basili, 2003).

Figure 2. *Incremental and iterative development*



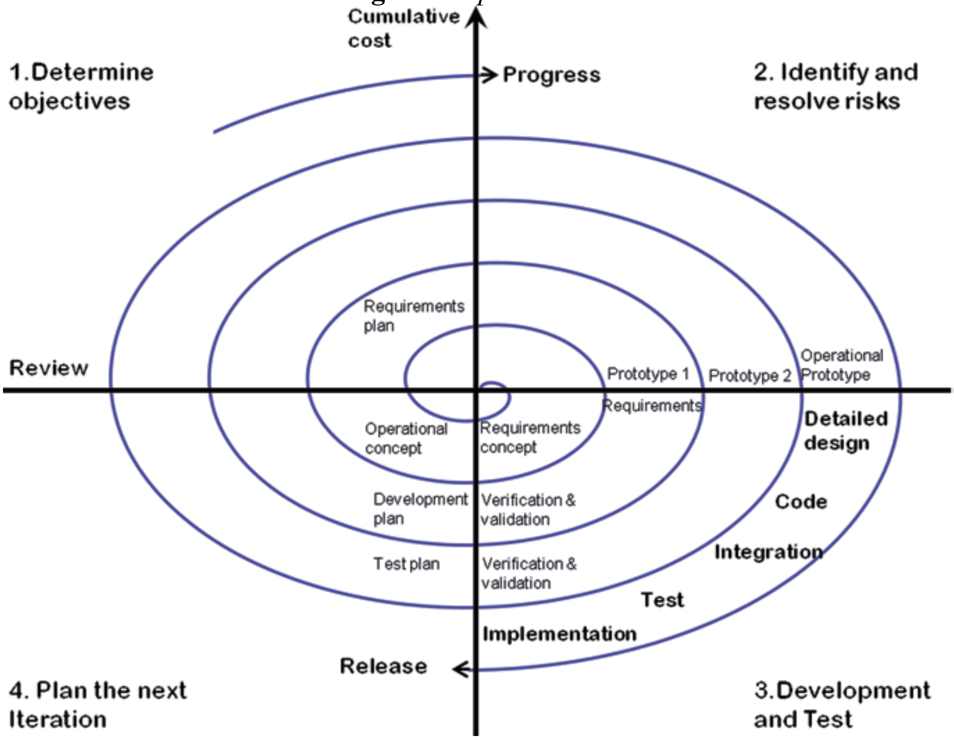
Source: http://www.link-university.com/lekcija/Fazni-razvoj-i-spiralni-model/2620

2.3. The spiral model

Boehm (1988) observed the development of software in light of the risks involved, suggesting that the spiral model can combine development activities with risk management, in order to be smaller and easier to control. This model is designed to include the best features of the waterfall model and introduces a new component - the risk assessment. The term "spiral" is used to describe the process that follows the development of the system.

The spiral model, shown in Figure 3, in some ways is similar to the iterative development shown in Figure 2. Starting with the requirements and initial development plan (including budget, constraints and alternatives in terms of staff, design and development environment), this process introduces a risk assessment step and a prototype alternatives, before producing a "working principles" document, to describe the functioning of the system at a high level of abstraction. From that document, a set of requests is defined and monitored to verify the completeness and consistency of the request. Therefore, the principle of operation is the product of the first iteration, while the requirements are the main product of the second iteration. In the third iteration, system development produces the design, while the fourth iteration enables testing (Fliger *et al*., 2006).

Figure 3. *Spiral model*



Source: Cowley (2014)

In each iteration, the risk analysis identifies different variants in terms of requirements and constraints, while prototyping verifies the feasibility or desirability of selecting the appropriate variant. After identifying the risks, project managers must decide how to eliminate or minimize them. To avoid the risk of choosing interfaces that would prevent productive use of the new system, designers can prototype both interfaces and test them in order to determine which is preferable, or even include both interfaces in the project so users can, after logging in, select interface. Restrictions such as budget and delivery time help in choosing a risk management strategy (Neill & Laplante, 2003).

2.4. The agile methods

Agile methodologies emerged in the late 90s when a group of software engineers concluded that previous approaches and methodologies for software development were not suitable in a turbulent environment and that it was not possible to bind and achieve firm delivery times for software solutions and customer satisfaction. They met and through mutual exchange of opinions came to the basic principles of agile methodologies, which they wrote down in the so-called Agile Manifesto (Jovanović *et al.*, 2016).

Kilibarda *et al*. (2016) find that agile methodologies differ from traditional ones in that they require the development of software products through shorter development cycles. With its completion, it is possible to deliver one piece of a software product to the client and make the necessary changes with it, in order to reach the final result faster and more efficiently.

In the practice of implementing agile methodologies for software product development, a number of methods are proposed and used today. The most famous are Scrum, Extreme Programming (XP), Crystal Clear, DSDM, and more. The Scrum method is one of the most popular and in practice the most used method of agile software development management.

2.4.1 SCRUM methodology

This method is based on the basic principles that characterize the agile approach and is convenient in practice because it is very easy to use. This method suggests that software development work takes place in shorter cycles called sprints, followed by ongoing consultations with the client, and that, after a certain cycle, analysis and review will be carried out and, if necessary, the desired and necessary changes will be made. This includes mandatory meetings before and after each sprint, in order to consider whether everything was done accordingly to the requirements and if it is necessary to introduce some changes. In a particular situation, it is possible to go back and implement a specific sprint according to new requirements (Jovanović *et al*., 2016; Pichler, 2010).

Development cycles - sprints are time intervals that can last one month, usually lasting two or more weeks. The software development team using the SCRUM method has special authority in terms of organizing and operating, as well as the special member or product owner that has certain authorization and responsibilities regarding the work of the development team and delivering the desired results to the client. In addition to team members working on software development, the SCRUM methodology envisages two specific roles related to team operations. These are the product owner and SCRUM master (moderator or mediator) (Jovanović *et al*., 2016).

One of the most commonly used and researched methods is SCRUM, which describes an iterative development process with the gradual delivery of value. SCRUM methodology can only reach its full potential if all elements are well defined with fully dedicated teams.

3. METHODOLOGY

Weighted aggregates sum product assessment (WASPAS) method was developed by Zavadskas *et al*. (2012). The WASPAS method represents a unique combination of two MCDM approaches weighted sum (WS) method and weighted product (WP) method.

In order to cope with a wider range of problems, the WASPAS method has many extensions, such as: WASPAS-G (Zavadskas *et al*., 2015), WASPAS-IFIV (Zavadskas *et al*., 2014), WASPAS-SVNS (Baušys & Juodagalvienė, 2017), WASPAS-IFN (Stanujkic & Karabasevic, 2018); WASPAS-R (Stojic *et al*., 2018).

Also, until now WASPAS is applied for solving the most diverse problems, such as: manufacturing decision making (Chakraborty *et al*., 2014), construction site selection (Turskis *et al*., 2015), personnel selection (Karabasevic *et al*., 2016; Urosevic *et al*., 2016), website selection (Stanujkic & Karabasevic, 2018), and so on.

The computational procedure of WASPAS method can be precisely presented as follows (Karabasevic *et al*., 2016; Urosevic *et al*., 2016):

Step 1. Determine the optimal performance rating for each criterion. In this step, the optimal performance ratings are calculated as follows:

$$x_{0j} = \begin{cases} \max\limits_{i} x_{ij}; & j \in \Omega_{\max} \\ \min\limits_{i} x_{ij}; & j \in \Omega_{\min} \end{cases}, \tag{1}$$

where x_{0j} denotes the optimal performance rating of j-th criterion, $\Omega_{\max}$ denotes the benefit criteria, i.e. the higher the values are, the better it is; and $\Omega_{\min}$ denotes the set of cost criteria, i.e. the lower the values are, the better it is, m denotes number of alternatives; i = 0, 1, ..., m; and n denotes number of criteria, , j = 0, 1, ..., n.

Step 2. Construct the normalized decision matrix. The normalized performance ratings are calculated as follows:

$$r_{ij} = \begin{cases} \dfrac{x_{ij}}{x_{0j}}; & j \in \Omega_{\max} \\ \dfrac{x_{0j}}{x_{ij}}; & j \in \Omega_{\min} \end{cases}, \tag{2}$$

where r_{ij} denotes the normalized performance rating of *i*-th alternative in relation to the *j*-th criterion.

Step 3. Calculate the relative importance of i-th alternative, based on WS method. The relative importance of *i*-th alternative, based on WS method, is calculated as follows:

$$Q_i^{(1)} = \sum_{j=1}^{n} w_j \, r_{ij} \,, \tag{3}$$

where $Q_i^{(1)}$ denotes the relative importance of *i*-th alternative in relation to the *j*-th criterion, based on WS method.

Step 4. Calculate the relative importance of i-th alternative, based on WP method (Madić, 2014). The relative importance of *i*-th alternative, based on WP method, is calculated as follows:

$$Q_i^{(2)} = \prod_{j=1}^{n} r_{ij}^{\;w_j} \tag{4}$$

where $Q_i^{(2)}$ denotes the relative importance of *i*-th alternative in relation to the *j*-th criterion, , based on WP method.

Step 5. Calculate total relative importance, for each alternative. The total relative importance, or more precisely the joint generalized criterion of weighted aggregation of additive and multiplicative methods is calculated as follows:

$$Q_i = 0.5 Q_i^{(1)} + 0.5 Q_i^{(2)} = 0.5 \sum_{j=1}^{n} w_j \, r_{ij} + 0.5 \prod_{j=1}^{n} r_{ij}^{\;w_j} \tag{5}$$

In order to have increased ranking accuracy and effectiveness of the decision making process, in WASPAS method, a more generalized equation for determining the total relative importance of *i*-th alternative is developed as below:

$$Q_i = \lambda Q_i^{(1)} + (1-\lambda) Q_i^{(2)} = \lambda \sum_{j=1}^{n} w_j \, r_{ij} + (1-\lambda) \prod_{j=1}^{n} r_{ij}^{\;w_j} \tag{6}$$

4. CASE STUDY OF THE SELECTION OF SOFTWARE DEVELOPMENT METHODOLOGY

This section will present a case study of the selection of software development methodology based on the use of PIPRECIA and WASPAS methods.

Based on the literature review, alternatives that will be evaluated are as follows: Waterfall methodology – A_1; Iterative and incremental methodology – A_2; Spiral methodology – A_3; and SCRUM methodology – A_4.

Pivot Pairwise Relative Criteria Importance Assessment method (PIPRECIA) method is developed by Stanujkic *et al*. (2017a) and is used for the determination of the weights of the criteria. Based on the research carried by Mahapatra and Goswami (2015), in this manuscript following criteria were determined, namely: Requirement analysis – C_1; Status of the development team – C_2; User's participation – C_3; and Project type and associated risk – C_4.

Step 1. Determination of weights of criteria

Responses and assigned weights of the evaluated criteria obtained from the three Decision Makers (DMs) by applying PIPRECIA method are shown in Table 1-3, whereas in Table 4 are shown group weights.

Table 1. *Weights of the criteria obtained from the first of the three DMs*

	Criteria	s_j	k_j	q_j	w_j
C_1	Requirement analysis		1	1	0.29
C_2	Status of the development team	0.85	1.15	0.87	0.25
C_3	User's participation	0.85	1.15	0.76	0.22
C_4	Project type and associated risk	1.1	0.9	0.84	0.24
				3.47	1.00

Source: Author's calculations

Table 2. *Weights of the criteria obtained from the second of the three DMs*

	Criteria	s_j	k_j	q_j	w_j
C_1	Requirement analysis		1	1	0.28
C_2	Status of the development team	0.89	1.11	0.90	0.26
C_3	User's participation	0.9	1.1	0.82	0.23
C_4	Project type and associated risk	0.98	1.02	0.80	0.23
				3.52	1.00

Source: Author's calculations

Table 3. *Weights of the criteria obtained from the third of the three DMs*

	Criteria	s_j	k_j	q_j	w_j
C_1	Requirement analysis		1	1	0.32
C_2	Status of the development team	0.7	1.3	0.77	0.24
C_3	User's participation	0.9	1.1	0.70	0.22
C_4	Project type and associated risk	1	1	0.70	0.22
				3.17	1.00

Source: Author's calculations

The group weights of the criteria based on the stances of the three DMs are shown in Table 4.

Table 4. *The weights of the criteria obtained from the three DMs*

	Criteria	w_j^1	w_j^2	w_j^3	w_j^*	w_j
C_1	Requirement analysis	0.289	0.284	0.316	0.296	0.296
C_2	Status of the development team	0.251	0.256	0.243	0.250	0.250
C_3	User's participation	0.218	0.232	0.221	0.224	0.224
C_4	Project type and associated risk	0.242	0.228	0.221	0.230	0.230
					0.999	1.000

Source: Author's calculations

Step 2. Ranking of alternatives

Based on the ratings obtained from the three *DMs*, group ratings are calculated as follows:

$$x_{ij} = \left(\prod_{k=1}^{3} x_{ij}^{k} \right)^{1/3}, \tag{7}$$

The group ratings of the four evaluated alternatives obtained from the three DMs are shown in Table 5.

Table 5. *The initial decision-making matrix*

Criteria Alternatives	C_1	C_2	C_3	C_4
w_j	0.296	0.250	0.224	0.230
A_1	3.33	3.00	2.67	2.33
A_2	2.67	2.67	2.67	3.33
A_3	3.67	3.67	3.00	4.00
A_4	5.00	4.67	4.67	4.67

Source: Author's calculations

By applying Eq. (2), a normalized decision matrix has been formed. The normalized decision matrix, as well as the weights of the criteria are shown in Table 6.

Table 6. *The normalized decision matrix and the weight of the criteria*

Criteria Alternatives	C_1	C_2	C_3	C_4
w_j	0.296	0.250	0.224	0.230
A_1	0.67	0.60	0.53	0.47
A_2	0.53	0.53	0.53	0.67
A_3	0.73	0.73	0.60	0.80
A_4	1.00	0.93	0.93	0.93

Source: Author's calculations

The relative importance of the evaluated alternatives, based on weighted sum (WS) method and weighted product (WP) are shown in Table 7.

Table 7. *The relative and total importance of the alternatives*

	$Q_i^{(1)}$	$Q_i^{(2)}$	Q_i	**Rank**
A_1	0.29	0.07	0.36	**3**
A_2	0.28	0.07	0.35	**4**
A_3	0.36	0.09	0.45	**2**
A_4	0.48	0.12	0.60	**1**

Source: Author's calculations

Data from the Table 7 show us that alternative designated as A_4 has the highest total importance in terms of evaluated criteria.

CONCLUSION

The pace of change in the software development industry is still high. People continue to push the limits of known techniques and practices to develop the most efficient and effective software. Software development lifecycle models and business decision models contribute to controlling product development in different ways.

A particular software development model can significantly affect various software product-related issues. If the model fails to fully meet the requirements, it will certainly affect the end product. Often a major reason for the failure of software development is the lack of good methodology or the implementation of inadequate. Also, a common barrier to successful software development is the misunderstanding and failure to meet user requirements. Continuous communication with the client is implied in agile methodologies, and such omissions are much harder to come by. Certainly, the ultimate goal, for both sides, is applicable software.

The proposed PIPRECIA-WASPAS approach has successfully responded to the requirements in terms of selection of the of software development methodology. The conducted case study has proved the applicability, ease of use and effectiveness of the proposed approach. Based on the conducted case study, alternative designated as A_4 has the highest total importance in terms of evaluated criteria. Therefore, SCRUM methodology is the most convenient by the stances of the DMs.

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An Approach to Determining Customer Satisfaction in Traditional Serbian Restaurants

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Dragisa Stanujkic, Darjan Karabasevic, Edmundas Kazimieras Zavadskas, Florentin Smarandache, Fausto Cavallaro (2019). An Approach to Determining Customer Satisfaction in Traditional Serbian Restaurants. Entrepreneurship and Sustainability Issues 6(3), 1127- 1138. DOI: 10.9770/jesi.2019.6.3(5)

Abstract. The aim of this paper is to make a proposal for an easy–to–use approach to the evaluation of customer satisfaction in restaurants. In order to provide a reliable way to collect respondents' real attitudes, an approach based on the use of smaller number of evaluation criteria and interactive questionnaire created in a spreadsheet file is proposed in this paper, whereby an easy-to-understand and simple-to-use procedure is proposed for determining weights of criteria. In addition to the said, the proposed approach applies the simplified SERVQUAL-based approach, for which reason a simplified version of the Weighted Sum Method based on the decision maker's Preferred Levels of Performances is used for the final ranking of the alternatives. The usability of the proposed approach is considered in the case study intended for the evaluation of traditional restaurants in the city of Zajecar.

Keywords: hospitality, restaurant industry, customer satisfaction, PIPRECIA, WS PLP approach

1. Introduction

The Serbian word "kafana" originates from the Turkish word "kahvehane", which means "a place for drinking coffee". Such places have emerged in the Balkan region under the influence of the Ottoman Empire in the 16th century.

Under the influence of different cultures, kafana generated its specificity on the Balkan Peninsula, so that it also became a place where food was consumed and later a place where alcoholic drinks were served. Over time, kafanas have increasingly become and have found their place in the social and cultural life, as well as in business. Nowadays, kafanas continue to be a place where you meet your friends, a place for celebrations, talking about and discussing things and so on. Therefore, kafanas could be denoted as traditional Serbian restaurants. Compared with the other types of restaurants, kafanas have similarities to taverns and pubs, as places of a pleasant ambience.

Certain new trends in the restaurant and food industry, as well as the growing presence of various cuisines, have had an impact on traditional Serbian restaurants. Fortunately, in some parts of Serbia, traditional Serbian restaurants somehow still resist unfortunately unstoppable trends.

In the city of Zajecar, located in eastern Serbia, traditional restaurants are successfully resisting the actual trends and it is still possible for you to find good restaurants, such as: “Dva brata” (“The Two Brothers”), “Gradska Mehana” (The City Meyhane”), “Meda” (“The Bear”), “Roko” (“The Roko”) and so forth.

The factors influencing the satisfaction of restaurants’ customers have been considered in many previous studies. Based on these studies, an approach to the determining of the significance of the relevant factors that influence customer satisfaction is proposed.

The proposed approach also uses the concept of measuring the difference between expectations and perceptions, so it provides an easy identification of the criteria against which customer expectations are not met. Beside all of the above-said, the proposed model can also be used to determine the overall ratings of the considered alternatives, thus making a comparison with competitors.

Based on all of the above-mentioned reasons that have been taken into account, the remaining part of this paper is organized as follows: In Section 2, a review of the relevant research studies is given. After that, in Section 3 and Section 4, the PIPRECIA and the WS PLP methods are considered. In Section 5, an empirical illustration of the evaluation of Serbian traditional restaurants, based on the integrated use of the PIPRECIA and the WS PLP methods, is presented in detail. Finally, the conclusions are given at the end of the paper.

2. Literature Research

Measuring customer satisfaction could be very important in a competitive environment (e.g. Stepaniuk 2018; Raudeliūnienė et al. 2018). For the purpose of determining that, Parasuraman et al. (1988) proposed the Service Quality and Customer Satisfaction (SERVQUAL) model. On the basis of that model, many others more specialized models have been proposed later, such as: WebQual (Loiacono et al. 2002; Parasuraman et al. 2005), eTailQ Wolfinbarger and Gilly (2003), E-RecS-QUAL (Parasuraman et al. 2005), and eTransQual (Bauer et al. 2006).

The SERVQUAL model was used for determining the levels of customer satisfaction in many different areas. As one of these areas, tourism and hospitality can be mentioned. For example: Saleh and Ryan (1991) used SERVQUAL to determine the gap between clients’ and the management’s perceptions in the hotel industry, whereas Devi Juwaheer (2004) explore the tourists' perceptions about hotels in Mauritius by using an adapted SQRVQUAL approach. Further, on the basis of the SERVQUAL model, Tribe and Snaith (1998) proposed the HOLSAT model, adapted for determining tourists’ satisfaction with their holidays.

Besides, a number of other approaches have also been used to determine customer satisfaction in tourism and hospitality industry, such as: Chaturvedi (2017), Lee and Severt (2017), Engeset and Elvekrok (2015), Albayrak and Caber (2015), Chan et al. (2015), Bernini and Cagnone (2014), Battour et al. (2014).

The SERVQUAL model has also been used in the restaurant industry for determining customer satisfaction. As some examples of these studies, the following can be mentioned: Liu at al (2017), Kurian and Muzumdar, (2017), Hanks et al (2017); Bufquin, et al. (2017), Saad Andaleeb and Conway (2006), Heung, et al. (2000), Lee and Hing (1995).

Some other studies have also been dedicated to the restaurant industry. For example: Adam et al. (2015) investigates tourist satisfaction with Ghanaian restaurants based on a factor analysis, and Jung and Yoon (2013) investigate the relationship between employees' satisfaction and customers' satisfaction in a family restaurant.

Dobrovolskienė et al. (2017) state that decision making is crucial to every aspect of business. Multiple-criteria decision-making (MCDM) is a scientific field that has undergone extremely rapid development over the last two decades. Multiple-criteria decision-making considers situations in which the decision-maker must choose one of the alternatives from a set of available alternatives and which are judged on the basis of a number of criteria. This is why MCDM contributes to easier decision-making and adoption of long-term and lasting solutions.

MCDM has also been successfully applied in the hospitality industry. Chou et al. (2008) and Tzeng (2008) used MCDM models for selecting the restaurant location. Yildiz and Yildiz (2015) proposed a model for evaluating customer satisfaction in restaurants, based on the use of the AHP and TOPSIS methods. In their studies: Duarte Alonso et al. (2013), Chi et al. (2013), Kim et al. (2007), Yuksel and Yuksel (2003) and Jack Kivela (1997) investigate the criteria that have an impact on customer preferences and satisfaction.

3. The PIPRECIA Method

The Step-wise Weight Assessment Ratio Analysis (SWARA) method was proposed by Kersuliene et al. (2010). The usability of the SWARA method has been proven in solving many MCDM problems, of which only several are mentioned: Zolfani et al. (2013), Zolfani and Saparauskas (2013), Stanujkic et al. (2017; 2015), Karabasevic et al. (2017), Mardani et al. (2017) and Juodagalviene et al. (2017).

The SWARA method has a certain similarity with the prominent AHP method. The first similarity is that both methods can be used to completely solve MCDM problems or to only determine the weight of the criteria; the second is that both methods are based on the use of pairwise comparisons.

However, the computational procedures of the SWARA and the AHP methods significantly differ from one another. Because of that, the SWARA method has some advantages, as well as some disadvantages, in comparison with the AHP method.

As the main disadvantage of the SWARA method, the fact that its computational procedure does not include a procedure for determining the consistency of pairwise comparisons made can be mentioned. Contrary to that, a significantly lower number of pairwise comparisons required for solving an MCDM problem and for determining criteria weights, too, can be mentioned as an advantage of the SWARA method.

Its requirement that evaluation criteria should be sorted in descending order according to their expected significances, which can prove to be inadequate in some survey cases, can also be mentioned as the weakness of the SWARA method. Therefore, with the aim of extending the use of the SWARA method in the cases where a consensus on the expected significance of the criteria is not easy to reach, Stanujkic et al. (2017) proposed the use of the following equation for the purpose of determining the importance of criteria as follows:

$$s_j = \begin{cases} >1 & \text{when } C_j \succ C_{j-1} \\ 1 & \text{when } C_j = C_{j-1} \\ <1 & \text{when } C_j \prec C_{j-1} \end{cases}. \tag{1}$$

where: s_j denotes the comparative importance of the criterion j, and $C_j \, \Theta \, C_{j-1}$ denotes the significance of the criterion j in relation to the j-1 criterion.

In an extension of the SWARA method, proposed under the name of PIPRECIA, Stanujkic et al. (2017) also mention that a lack an integrated procedure for checking the consistency in the ordinary SWARA method can successfully be compensated for by using Kendall's Tau or Spearman's Rank Correlation Coefficient.

Because of all the foregoing, the PIPRECIA method has been chosen to be used in this approach.

3.1. The Computational Procedure of the PIPRECIA Method

The computational procedure of the PIPRECIA method can be shown as follows:

***Step* 1**. Choose the criteria on the basis of which an evaluation of alternatives will be carried out.

***Step* 2**. Set the value of the relative importance of the criteria by using Eq. (1), starting from the second criterion.

***Step* 3**. Calculate the coefficient k_j for the criterion j as follows:

$$k_j = 2 - s_j. \tag{2}$$

***Step* 4**. Calculate the recalculated weight q_j for the criterion j as follows:

$$q_j = \begin{cases} 1 & \text{if } \quad j = 1 \\ \dfrac{q_{j-1}}{k_j} & \text{when } \; j > 1 \end{cases}. \tag{3}$$

***Step* 5**. Calculate the weights of the criteria as follows:

$$w_j = \frac{q_j}{\sum_{k=1}^{n} q_k}. \tag{4}$$

where w_j denotes the weight of the criterion j.

4. The WS PLP Approach

Based on the Weighted Sum Method (Churchman and Ackoff, 1954, MacCrimon, 1968), Stanujkic and Zavadskas (2015) proposed the Weighted Sum Preferred Levels of Performances (WS PLP) approach.

The simplified computational procedure of the WS PLP approach for solving an MCDM problem that contains the *m* alternatives that are evaluated based on the *n* beneficial criteria (a higher value of the performance rating is desirable) can be shown as follows:

***Step* 1**. Evaluate the alternatives in relation to the selected criteria.

***Step* 2**. Set the preferred performance ratings for each criterion.

***Step* 3**. Calculate the normalized performance ratings of the alternatives as follows:

$$r_{ij} = \frac{x_{ij} - x_{0j}}{x_j^+ - x_j^-}, \tag{5}$$

where: x_{ij} and r_{ij} denote the performance rating and the normalized performance rating of the alternative *i* in relation to the criterion *j*, respectively; x_{0j} denotes the preferred performance rating of the criterion *j*; $x_j^+ = \max_i x_{ij}$, and $x_j^- = \min_i x_{ij}$.

***Step* 4**. Calculate the overall performance rating of the alternatives as follows:

$$S_i = \sum_{j=1}^{n} w_j \, r_{ij}, \tag{6}$$

where S_i denotes the overall performance rating of the alternative *i*, $S_i \in [-1,\ 1]$; w_j is the weight of the criterion *j*.

In the proposed approach, the alternatives whose S_i is greater than or equal to zero make a set of the most appropriate alternatives, out of which one should be selected.

5. A Case Study

In order to determine the preferences of the passionate visitors of Serbian traditional restaurants, a supervised survey has been performed in the city of Zajecar, located in Serbia, near the Romanian and the Bulgarian borders.

In this study, the five previously mentioned restaurants have been evaluated on the basis of the six criteria adopted from Stanujkic et al. (2016):

- C_1 - the interior of the building and the friendly atmosphere,
- C_2 - the helpfulness and friendliness of the staff,
- C_3 - the variety of traditional food and drinks,
- C_4 - the quality and taste of the food and drinks, including the manner of serving,
- C_5 - the appropriate price for the quality of the services provided, and
- C_6 - other.

In the proposed approach the criterion “other” is used to enable personalization.

The survey presented in this study was conducted by e-mail, or more precisely by using an interactive questionnaire created in a spreadsheet file. By using such an approach, the respondents can see the calculated weights of the criteria and can also modify his/her responses if he or she is not satisfied with the obtained results. In addition, by using such an approach, the obtained results can also be presented graphically, which can make easier to understand the procedure used for determining weights of criteria, and thus lead to obtaining more realistic views of the respondents.

The interactive questionnaire was sent to the selected respondents known as the "bohemians" and/or frequent visitors of traditional Serbian restaurants. Out of the approximately 80 sent questionnaires, the 42 of them were returned, out of which only 30 questionnaires were selected as those properly filled in.

The weights of the criteria calculated on the basis of the responses obtained from the two selected respondents are accounted for in Table 1 and Table 2.

Table 1. The weights of the criteria obtained from the first respondent

Criteria		s_j	w_j
C_1	The interior of the building and friendly atmosphere		0.13
C_2	The helpfulness and friendliness of the staff	1.10	0.15
C_3	The variety of traditional food and drinks	1.20	0.19
C_4	The quality and taste of the food and drinks, including the manner of serving	1.05	0.20
C_5	The appropriate price for the quality of the services provided	0.95	0.19
C_6	Other	0.70	0.14

Source: Own calculations

Table 2. The weights of the criteria obtained from the second respondent

Criteria		s_j	w_j
C_1	The interior of the building and friendly atmosphere		0.15
C_2	The helpfulness and friendliness of the staff	1.10	0.17
C_3	The variety of traditional food and drinks	0.90	0.16
C_4	The quality and taste of the food and drinks, including the manner of serving	1.15	0.18
C_5	The appropriate price for the quality of the services provided	0.95	0.17
C_6	Other	0.90	0.16

Source: Own calculations

Some significant descriptive statistical parameters related to the weights of the criteria obtained by the conducted survey are presented in Table 3.

Table 3. The descriptive statistics for the weights of the criteria

Criteria	Min	Max	Range	Mean	Standard Deviation	Variance	Screw	Kurtosis
C_1	0.01	0.17	0.17	0.12	0.05	0.002	-0.84	-0.12
C_2	0.05	0.19	0.15	0.15	0.05	0.002	-0.77	-0.81
C_3	0.03	0.19	0.15	0.14	0.05	0.003	-0.52	-1.13
C_4	0.17	0.37	0.19	**0.23**	0.06	0.003	0.91	-0.27
C_5	0.17	0.35	0.18	**0.22**	0.06	0.003	0.76	-0.65
C_6	0.11	0.23	0.12	0.16	0.03	0.001	0.41	-0.77

Source: Own calculations

According to Table 3, the criteria C_4 and C_5 have significantly higher importance related to the other criteria, i.e. the quality and the taste of the food and the appropriate price are identified as the most significant criteria.

The obtained correlation coefficient between the responses obtained from the respondents and the mean ranges between 0.44 and 0.98.

Criterion C_6 - "other" also has a high weight, which can be interpreted as follows:

- in addition to the criteria C_1 - C_5 there are other criteria that affect satisfaction of restaurant customers, which can be applied in much more sophisticated models, and
- criterion C_6 can successfully substitute many less significant criteria and such enable forming an efficient MCDM models based on the use of a smaller number of criteria.

In addition to the conducted research, the respondents also evaluated the five preselected traditional restaurants by using the five-point Likert Scale. The results obtained from the two of the above-mentioned respondents are accounted for in Tables 4 and 5.

Table 4. The ratings obtained from the first respondent

Criteria \ Alternatives	Expected	Meda	Dva brata	MS	Roko	Nasa kafana	S_i	Rank
C_1	3	4	5	3	4	4	0.61	2
C_2	4	5	5	3	3	3	0.85	1
C_3	4	4	5	3	3	4	-0.09	5
C_4	3	5	5	3	4	3	0.20	3
C_5	3	5	4	2	4	4	0.20	4
C_6	2	3	4	3	3	3	0.61	2

Source: Own calculations

Table 5. The ratings obtained from the second respondent

Criteria \ Alternatives	Expected	Meda	Dva brata	MS	Roko	Nasa kafana	S_i	Rank
C_1	4	4	4	2	2	4	0.32	1
C_2	5	5	4	3	3	4	-0.02	2
C_3	3	4	4	3	3	4	-0.42	4
C_4	5	5	4	4	3	4	-0.51	5
C_5	4	4	4	4	4	3	-0.04	3
C_6	3	4	3	3	3	4	0.32	1

Source: Own calculations

Ranges between the maximum and minimum weights of criteria are also not negligible, as previously shown in Table 3. Therefore, the separate ranking list of considered alternatives has been formed for each respondent, in this approach, by using the WS PLP approach.

In this way, the attitudes of the respondents do not drown into the group attitudes, obtained on the basis of the average weight of and average ratings, and remain clear until the end of the evaluation, where the final ranking of the considered alternative was made based on dominance theory.

The results achieved based on all properly filled questionnaires are shown in Table 6. The appearance of the considered alternative in the first position is given in Column I of Table 6. The appearance of the considered alternatives in the second and the third positions is given in Columns II and III of Table 6.

Table 6. The number of the appearances of the alternatives in different positions

	Number of appearances at positions		
Alternatives	I	II	III
A_1	15	7	3
A_2	12	6	6
A_3	1	1	9
A_4	4	10	7
A_5	0	4	5

Source: Own calculations

According to Column I of Table 5, based on the dominance theory, the best-placed alternative is the alternative labelled as A_1.

In this approach, only the appearances on the first position are used for the determination of the best alternative, or more precisely, the most popular traditional restaurant. The appearances in the second, the third, as well as the other positions, could be used for a further analysis.

The overall ratings, obtained by using WS PLP approach, can also be used for various analysis, especially when it is known that WS PLP approach $S_i < 0$ indicates an alternative where expected customers' satisfaction has not been reached yet.

Conclusions

The main objective of this paper is to determine the most significant criteria that have an influence on customers' satisfaction in traditional Serbian restaurants, as well as weights of these criteria, and propose an easy–to–use approach for the evaluation of customers' satisfaction in restaurants.

For that reason, the newly proposed PIPRECIA method, that is an extension of the SWARA method, is proposed for determining the weight of criteria in order to provide an effective and simple-to-use procedure for gathering the attitudes of the examined respondents that will be as realistic as possible.

The gaps between the expected and the achieved satisfaction obtained based on a set of criteria are used to determine the overall performance of any of the considered alternatives, which is done by applying the WS PLP approach. The final ranking of the alternatives is made by referring to dominance theory.

The approach proposed in this paper has significant similarities to the proven SERVQUAL model or models like that one. However, it is based on the use of a significantly smaller number of evaluation criteria, which could allow the forming of the simplest questionnaires that could be more appropriate when preferences and ratings are collected through conducting surveys with ordinary respondents, i.e. those unprepared in advice for surveying.

The usability of the proposed approach has been verified in the case study on the evaluation of traditional Serbian restaurants. The achieved results confirm the efficiency and usability of the proposed approach for solving similar, as well as numerous other, decision-making problems.

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A Bipolar Fuzzy Extension of the MULTIMOORA Method

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Abstract. The aim of this paper is to make a proposal for a new extension of the MULTIMOORA method extended to deal with bipolar fuzzy sets. Bipolar fuzzy sets are proposed as an extension of classical fuzzy sets in order to enable solving a particular class of decision-making problems. Unlike other extensions of the fuzzy set of theory, bipolar fuzzy sets introduce a positive membership function, which denotes the satisfaction degree of the element x to the property corresponding to the bipolar-valued fuzzy set, and the negative membership function, which denotes the degree of the satisfaction of the element x to some implicit counter-property corresponding to the bipolar-valued fuzzy set. By using single-valued bipolar fuzzy numbers, the MULTIMOORA method can be more efficient for solving some specific problems whose solving requires assessment and prediction. The suitability of the proposed approach is presented through an example.

Key words: bipolar fuzzy set, single-valued bipolar fuzzy number, MULTIMOORA, MCDM.

1. Introduction

The management of very complex systems is the most complex, and therefore the most difficult task of the managers of today's organizations. The effectiveness of the management and managers of an organization depends to a large extent on the quality of the decisions they make on a daily basis.

Decision-making and decisions are the core of managerial activities. Bearing in mind the globalization and, therefore, the dynamics of business doing, all of the above-stated have caused business and the decision-making process to become more demanding. Mak-ing quality decisions requires an ever more extensive preparation, which also involves the consideration of the different aspects of a decision, for the reason of which the decision-making process becomes considerably formalized. Thus, real problems and situations in real life are characterized by a large number of mostly conflicting criteria, whose strict optimization is generally impossible.

When it is necessary to make a decision on choosing one of several potential solutions to a problem, it is desirable to apply one of the models based on multiple-criteria decision-making methods (MCDM). This most often involves the process of selecting one of several alternative solutions, for which certain goals are set. When MCDM is concerned, Greco *et al.* (2010) point out the fact that it is the study of the methods and procedures aimed at making a proposal for solutions in terms of multiple, often conflicting criteria. Hwang and Yoon (1981) states that MCDM is based on the two basic approaches, i.e. on multiple attribute decision-making (MADM), which implies a choice of courses in the presence of multiple, and often conflicting criteria, i.e. a selection of the best alternative from a finite set of possible alternatives. Unlike MADM, in multiple objective decision-making (MODM), the best alternative is that which is formed with multiple goals, based on the continuous variables of the decision with additional constraints.

So, all the problems of today are, in general, multi-criterial, primarily because problems are mainly related to the achievement of the objectives related to a larger num-ber of, usually conflicting, criteria, which is a great approximation to real tasks in decision-making processes (Das *et al.*, 2012; Zavadskas *et al.*, 2014). The increasing ap-plication of the MCDM method to solving various problems has caused an exceptional growth of multi-criteria decision-making as an important field of operational research, especially since 1980 (Aouni *et al.*, 2018; Masri, *et al.*, 2018; Wallenius *et al.*, 2008; Dyer *et al.*, 1992).

Within MADM, some of the methods that have been proposed are: Weighted Sum Model (WSM) (Fishburn, 1967); Simple Additive Weighting (SAW) method (MacCrimon, 1968), Elimination Et Choix Traduisant la REalité (ELECTRE) method (Roy, 1968), DEcision-MAking Trial and Evaluation Laboratory (DEMATEL) method (Gabus and Fontela, 1972), Compromise Programming (CP) method (Zeleny, 1973), Simple Multi Attribute Rating Technique (SMART) (Edwards, 1977), Analytic Hierarchy Process (AHP) method (Saaty, 1978), Technique for Order of Preference by Similarity to Ideal Solution (TOPSIS) method (Hwang and Yoon, 1981), Preference Ranking Organization Method for Enrichment Evaluations (PROMETHEE) method (Brans, 1982), Measuring Attractiveness by a Categorical Based Evaluation Technique (MACBETH) (Bana e Costa and Vansnick, 1994), Complex Proportional Assessment of alternatives (COPRAS) method (Zavadskas *et al.*, 1994), Analytic Network Process (ANP) method (Saaty, 1996), Vlse Kriterijumska Optimizacija i kompromisno Resenje (VIKOR) (Opricovic, 1998), Multi-Objective Optimization on basis of Ratio Analysis (MOORA) method (Brauers and Zavadskas, 2006), Additive Ratio ASsessment (ARAS) method (Zavadskas and Turskis, 2010), Multi-Objective Optimization by Ratio Analysis plus the Full Multiplicative Form (MULTIMOORA) method (Brauers and Zavadskas, 2010a), and so on. While within the MODM methods that have been proposed can be stated: Data envelopment analysis (DEA) method (Charnes *et al.*, 1978), Linear Programming (LP) and Nonlinear Programming (NP) (Luenberger and Ye, 1984), Multi-Objective Programming (MOP) technique (Charnes *et al.*, 1989), Multi-Objective Linear Programming (Ecker and Kouada, 1978), and so on.

The MULTIMOORA method (Brauers and Zavadskas, 2010b) is an important MCDM method that has been applied so far to solve the most diverse problems in the field of economics, management, etc. Basically, the MULTIMOORA method consists of the well-known MOORA method (Brauers and Zavadskas, 2006) and the method of multi-object optimization (the Full Multiplicative Form of Multiple Objects method). Thus, Brauers and Zavadskas (2010a) proposed the updating of the MOORA method by adding a multi-object optimization method which involves maximizing and minimizing useful multiplicative functions (Lazauskas *et al.*, 2015).

As noted above, the MULTIMOORA method was applied in order to solve a variety of problems, such as: using MULTIMOORA for ranking and selecting the best performance appraisal method (Maghsoodi *et al.*, 2018), project critical path selection (Dorfeshan *et al.*, 2018), the selection of the optimal mining method (Liang *et al.*, 2018), pharmacological therapy selection (Eghbali-Zarch *et al.*, 2018), ICT hardware selection (Adali and Işik, 2017), industrial robot selection (Karande *et al.*, 2016), a CNC machine tool evaluation (Sahu *et al.*, 2016), personnel selection (Karabasevic *et al.*, 2015; Baležentis *et al.*, 2012), the economy (Baležentis and Zeng, 2013; Brauers and Zavadskas, 2011a, 2010b; Brauers and Ginevičius, 2010), and so on.

However, most decisions made in the real world are made in an environment in which goals and constraints cannot be precisely expressed due to their complexity; therefore, a problem cannot be displayed exactly in crisp numbers (Bellman and Zadeh, 1970). For such problems, characterized by uncertainty and indeterminacy, it is more appropriate to use values expressed in intervals instead of concrete (crisp) values. In this case, the existing, ordinary MCDM methods are expanded by using the extensions based on fuzzy sets (Zadeh, 1965), intuitionistic fuzzy sets (Atanassov, 1986), and neutrosophic sets (Smarandache, 1999). Accordingly, in order to allow a much wider use of the MULTIMOORA method, some extensions of the MULTIMOORA method have been proposed, some of which are as follows: Brauers *et al.* (2011) proposed a fuzzy extension of the MULTIMOORA method; Baležentis and Zeng (2013) proposed an IVFN extension of the MULTIMOORA method; Baležentis *et al.* (2014) also proposed an IFN extension of the MULTIMOORA method; Stanujkic *et al.* (2015) proposed an extension of the MULTIMOORA method based on the use of interval-valued triangular fuzzy numbers; Zavadskas *et al.* (2015) proposed an IVIF-based extension of the MULTIMOORA method; Hafezalkotob *et al.* (2016) proposed an extension of the MULTIMOORA method based on the use of interval numbers; Stanujkic *et al.* (2017a) proposed a neutrosophic extension of the MULTIMOORA method, and so on.

In addition to the aforementioned extensions of the fuzzy set theory, Zhang (1994) introduced the concept of bipolar fuzzy sets and proposed the usage of the two membership functions that represent membership to a set and membership to a complementary set, thus providing an efficient approach to solving a widely larger number of complex decision-making problems.

Despite an advantage that can be achieved by using bipolar fuzzy logic, they are significantly less used for solving MCDM problems compared to other fuzzy logic extensions. The following examples can be mentioned as some of the really rare usages of BFS for solving MCDM problems: Alghamdi *et al.* (2018) and Akram and Arshad (2018) proposed bipolar fuzzy extensions of TOPSIS and ELECTRE I methods; while Han *et al.* (2018) provide a comprehensive mathematical approach based on the TOPSIS method for improving the accuracy of bipolar disorder clinical diagnosis.

It is also important to note that these are current researches. In addition, the bipolar logic has been considerably used in the neutrosophic set theory, where Uluçay *et al.* (2018), Pramanik *et al.* (018) and Tian *et al.* (2016) can be cited as some of the current researches.

Therefore, in order to enable a wider use of the MULTIMOORA method for solving even a wider range of problems, a bipolar extension of the MULTIMOORA method is proposed in this paper. Accordingly, the paper is structured as follows: in Section 1, the introductory considerations are presented. In Section 2, some basic definitions regarding bipolar fuzzy sets are given. In Section 3, the ordinary MULTIMOORA method is presented, whereas in Section 4, an extension of the MULTIMOORA method based on single-valued bipolar fuzzy numbers is proposed. In Section 5, a numerical example is demonstrated, and finally, the conclusions are given at the end of the paper.

2. The Basic Elements of a Bipolar Fuzzy Set

Definition 1 (*Fuzzy set*, Zadeh, 1965). Let X be a nonempty set, with a generic element in X denoted by x. Then, a fuzzy set A in X is a set of ordered pairs:

$$A = \{\langle x, \mu_A(x)\rangle \big| x \in X\}, \tag{1}$$

where the membership function $\mu_A(x)$ denotes the degree of the membership of the element x to the set A, and $\mu_A(x) \in [0, 1]$.

Definition 2 (*Bipolar fuzzy set*, Lee, 2000). Let X be a nonempty set. Then, a bipolar fuzzy set (BFS) is defined as:

$$A = \{\langle x, \mu_A^+(x), \nu_A^-(x)\rangle \big| x \in X\}, \tag{2}$$

where: the positive membership function $\mu_A^+(x)$ denotes the satisfaction degree of the element x to the property corresponding to the bipolar-valued fuzzy set, and the negative membership function $\nu_A^-(x)$ denotes the degree of the satisfaction of the element x to some implicit counter-property corresponding to the bipolar-valued fuzzy set, respectively; $\mu_A^+(x) : X \to [0, 1]$ and $\nu_A^-(x) : X \to [-1, 0]$.

Definition 3. A single-valued bipolar fuzzy number (SVBFN) $a = \langle a^+, a^- \rangle$ is a special bipolar fuzzy set on the real number set R, whose positive membership and negative membership function are as follows:

$$\mu^+(x) = \begin{cases} 1 & x = a^+, \\ 0 & \text{otherwise}, \end{cases} \tag{3}$$

$$\nu^-(x) = \begin{cases} 1 & x = a^-, \\ 0 & \text{otherwise}, \end{cases} \tag{4}$$

respectively.

Definition 4. Let $a_1 = \langle a_1^+, a_1^- \rangle$ and $a_1 = \langle a_2^+, a_2^- \rangle$ be two SVBFNs, and $\lambda > 0$. Then, the basic operations for these numbers are defined as shown below:

$$a_1 + a_2 = \langle a_1^+ + a_2^+ - a_1^+ a_2^+, -a_1^- a_2^- \rangle, \tag{5}$$

$$a_1 \cdot a_2 = \langle a_1^+ a_2^+, -\big(-a_1^- - a_2^- - a_1^- a_2^-\big) \rangle, \tag{6}$$

$$\lambda a_1 = \langle 1 - \big(1 - a_1^+\big)^\lambda, -\big(-a_1^-\big)^\lambda \rangle, \tag{7}$$

$$a_1^\lambda = \langle \big(a_1^+\big)^\lambda, -\big(1 - \big(1 - \big(-a_1^-\big)\big)^\lambda\big) \rangle. \tag{8}$$

Definition 5. Let $a = \langle a^+, a^- \rangle$ be an SVBFN. Then, the score function $s_{(a)}$ is as follows:

$$s_a = \big(1 + a^+ + a^-\big)/2. \tag{9}$$

Definition 6. Let a_1 and a_2 be two SVBFNs. Then, $a_1 > a_2$ if $s_{a_1} > s_{a_2}$.

Definition 7. Let $a_1 = \langle a_1^+, a_1^- \rangle$ and $a_1 = \langle a_2^+, a_2^- \rangle$ be two SVBFNs. The Hamming distance between a_1 and a_2 is as follows:

$$d_H(a_1, a_2) = \frac{1}{2}\big(\big|a_1^+ - a_2^+\big| + \big|a_1^- - a_2^-\big|\big). \tag{10}$$

Definition 8. Let $a_j = \langle a_j^+, a_j^- \rangle$ be a collection of SVBFNs. The bipolar weighted average operator (A_w) of the n dimensions is a mapping as follows:

$$\begin{aligned} A_w(a_1, a_2, \ldots, a_n) &= \sum_{j=1}^{n} w_j a_j \\ &= \left(1 - \prod_{j=1}^{n} \big(1 - a_j^+\big)^{w_j}, -\left(1 - \prod_{j=1}^{n} \big(1 - \big(-a_j^-\big)\big)^{w_j}\right)\right), \end{aligned} \tag{11}$$

where: w_j is the element j of the weighting vector, $w_j \in [0, 1]$ and $\sum_{j=1}^{n} w_j = 1$.

Definition 9. Let $a_j = \langle a_j^+, a_j^- \rangle$ be a collection of SVBFNs. The bipolar weighted geometric operator (G_w) of the n dimensions is a mapping $G_w : Q_n \to Q$ as follows:

$$G_w(a_1, a_2, \ldots, a_n) = \prod_{j=1}^{n} a_j^{w_j} = \left(\prod_{j=1}^{n} (a_j^+)^{w_j}, -\prod_{j=1}^{n} (-a_j^-)^{w_j} \right). \tag{12}$$

3. The MULTIMOORA Method

Compared to the other MCDM methods, the MULTIMOORA method is characteristic because it combines three approaches, namely: the Ratio System (RS) Approach, the Reference Point (RP) Approach and the Full Multiplicative Form (FMF) Approach, in order to select the most appropriate alternative.

In addition, this method does not calculate and does not use the overall significance for ranking alternatives and selecting the most appropriate one. Instead of using an overall parameter for ranking alternatives, the final ranking order of the considered alternatives, as well as the selection of the most appropriate alternative, is based on the use of the theory of dominance.

For an MCDM problem that includes the m alternatives that should be evaluated on the basis of the n criteria, the computational procedure of the MULTIMOORA can be expressed as follows:

Step 1. *Construct a decision matrix and determine the weights of criteria.*

Step 2. *Calculate a normalized decision matrix*, as follows:

$$r_{ij} = \frac{x_{ij}}{\sqrt{\sum_{i=1}^{n} x_{ij}^2}}, \tag{13}$$

where: r_{ij} denotes the normalized performance of the alternative i with respect to the criterion j, and x_{ij} denotes the performance of the alternative i to the criterion j.

Step 3. *Calculate the overall significance of each alternative*, as follows:

$$y_i = \sum_{j \in \Omega_{\max}} w_j r_{ij} - \sum_{j \in \Omega_{\min}} w_j r_{ij}, \tag{14}$$

where: y_i denotes the overall importance of the alternative i, $\Omega_{\max}$ and $\Omega_{\min}$ denote the sets of the benefit cost criteria, respectively.

Step 4. *Determine the reference point*, as follows:

$$r^* = \{r_1^*, r_2^*, \ldots, r_n^*\} = \left\{ \left(\max_i r_{ij} \,\middle|\, j \in \Omega_{\max} \right), \left(\min_i r_{ij} \,\middle|\, j \in \Omega_{\min} \right) \right\}. \tag{15}$$

Step 5. *Determine the maximal distance between each alternative and the reference point*, as follows:

$$d_i^{\max} = \max_j \left(w_j \left| r_j^* - r_{ij} \right| \right), \tag{16}$$

where: $d_i^{\max}$ denotes the maximal distance of the alternative i to the reference point.

Step 6. *Determine the overall utility of each alternative*, as follows:

$$u_i = \frac{\prod_{j \in \Omega_{\max}} w_j r_{ij}}{\prod_{j \in \Omega_{\min}} w_j r_{ij}}, \tag{17}$$

where: u_i denotes the overall utility of the alternative i.

In particular case, when evaluation is made only on the basis of benefit criteria, Eq. (17) is as follows:

$$u_i = \prod_{j \in \Omega_{\max}} w_j r_{ij}. \tag{18}$$

Step 7. *Determine the final ranking order of the considered alternatives and select the most appropriate one*. In this step, the considered alternatives are ranked based on their:

– overall significance,
– maximal distance to the reference point, and
– overall utility.

As a result of these rankings, the three different ranking lists are formed, representing the rankings based on the RS approach, the RP approach and the FMF approach of the MULTIMOORA method.

The final ranking of the alternatives is based on the dominance theory, i.e. the alternative with the highest number of appearances in the first positions on all ranking lists is the best-ranked alternative.

4. An Extension of the MULTIMOORA Method Based on Single-Valued Bipolar Fuzzy Numbers

For an MCDM problem involving m alternatives and n criteria and K decision-makers, whereby the performances of the alternatives are expressed by using SVBFNs, the calculation procedure of the extended MULTIMOORA method can be expressed as follows:

Step 1. *Evaluate the alternatives in relation to the evaluation criteria*, and do that for each DM. In this step, each DM evaluates the alternatives and forms an evaluation matrix.

In order to provide an easier evaluation, the following Likert scale, shown in Table 1, is proposed for evaluating alternatives in relation to the evaluation criteria.

Table 1
Nine-point Likert scale for expressing degree of satisfaction.

Satisfaction level	Numerical value
Neutral/without attitude	0
Extremely low	1
Very low	2
Low	3
Medium low	4
Medium	5
Medium high	6
High	7
Very high	8
Extremely high	9
Absolute	10

However, the respondents should be introduced that the values listed in Table 1 are only approximative and that they can use any value from the interval [0, 10] and $[-10, 0]$.

After forming initial decision-making matrix, obtained responses should be divided by 10 in order to transform it into the allowed interval $[-1, 1]$. This approach for evaluating alternatives is proposed to avoid the use of vector normalization procedure, used in the ordinary MULTIMOORA method.

Step 2. *Determine the importance of the evaluation criteria*, and do that for each DM. In this step, each DM determines the weights of the criteria by using one of several existing methods for determining the weights of criteria.

Step 3. *Determine the group decision matrix*. In order to transform individual into group preferences, individual evaluation matrices are transformed into group one by applying Eq. (11).

Step 4. *Determine the group weights of the criteria*. In order to transform individual into group preferences with respect to the weights of criteria, the group weights of criteria can be determined as follows:

$$w_j = \sum_{k=1}^{K} w_j^k \tag{19}$$

where: w_j denotes the weight of the criterion j, and w_j^k denotes the weight of the criterion j obtained from the DM k.

After calculating the group evaluation matrix and the group weights of the criteria, all the necessary prerequisites for applying all the three approaches integrated in the MULTIMOORA method are obtained. Based on the approach proposed by Stanujkic *et al.* (2017b), the remainder steps of the proposed approach are as follows:

Step 5. *Determine the significance of the evaluated alternatives based on the RS approach*. This step can be explained through the following sub-steps:

Step 5.1. *Determine the impact of the benefit and cost criteria to the importance of each alternative*, as follows:

$$Y_i^+ = \left(1 - \prod_{j \in \Omega_{max}}^{n} (1 - r_{ij})^{w_j}, -\left(1 - \prod_{j \in \Omega_{\max}}^{n} \left(1 - (-r_{ij})\right)^{w_j}\right), \tag{20}$$

$$Y_i^- = \left(1 - \prod_{j \in \Omega_{min}}^{n} (1 - r_{ij})^{w_j}, -\left(1 - \prod_{j \in \Omega_{\min}}^{n} \left(1 - (-r_{ij})\right)^{w_j}\right), \tag{21}$$

where: Y_i^+ and Y_i^- denote the importance of the alternative i obtained on the basis of the benefit and cost criteria, respectively; Y_i^+ and Y_i^- are SVBFNs.

It is evident that A_w operator is used to calculate the impact of the benefit and cost criteria.

Step. 5.2. *Transform Y_i^+ and Y_i^- into crisp values by using the Score Function*, as follows:

$$y_i^+ = s\left(Y_i^+\right), \tag{22}$$

$$y_i^- = s\left(Y_i^-\right). \tag{23}$$

Step 5.3. *Calculate the overall importance for each alternative*, as follows:

$$y_i = y_i^+ - y_i^-. \tag{24}$$

Step 6. *Determine the significance of the evaluated alternatives based on the RP approach*. This step can be explained through the following sub-steps:

Step 6.1. *Determine the reference point*. The coordinates on the bipolar fuzzy reference point $r^* = \{r_1^*, r_2^*, \ldots, r_n^*\}$ can be determined as follows:

$$r^* = \left\{\left(\left\langle \max_i r_{ij}, \min_i r_{ij}\right\rangle \middle| j \in \Omega_{\max}\right), \left(\left\langle \min_i r_{ij}, \max_i r_{ij}\right\rangle \middle| j \in \Omega_{\min}\right)\right\} \tag{25}$$

where: r_j^* denotes the coordinate j of the reference point.

Step 6.2. *Determine the maximum distance from each alternative to all the coordinates of the reference point*. The maximum distance of each alternative to the reference point can be determined as follows:

$$d_{ij}^{\max} = d_{\max}\left(r_{ij}, r_j^*\right) w_j, \tag{26}$$

where $d_{ij}^{\max}$ denotes the maximum distance of the alternative i to the criterion j determined by Eq. (10).

Step 6.3. *Determine the maximum distance of each alternative*, as follows:

$$d_i^{\max} = \max_j d_{ij}^{\max}, \tag{27}$$

where $d_i^{\max}$ denotes the maximum distance of the alternative i.

Step 7. *Determine the significance of the evaluated alternatives based on* the FMF.

This step can be explained through the following sub-steps:

Step 7.1. *Calculate the utility obtained based on the benefit U_i^+ and cost U_i^- criteria*, for each alternative, as follows:

$$U_i^+ = \left(\prod_{j \in \Omega_{max}}^{n} (r_{ij})^{w_j}, - \prod_{j \in \Omega_{max}}^{n} (-r_{ij})^{w_j} \right), \tag{28}$$

$$U_i^- = \left(\prod_{j \in \Omega_{min}}^{n} (r_{ij})^{w_j}, - \prod_{j \in \Omega_{min}}^{n} (-r_{ij})^{w_j} \right), \tag{29}$$

where U_i^+ and U_i^- are SVBFNs.

Step 7.2. *Transform U_i^+ and U_i^- into crisp values by using the Score Function*, as follows:

$$u_i^+ = s\left(U_i^+\right), \tag{30}$$

$$u_i^- = s\left(U_i^-\right). \tag{31}$$

Step 7.3. *Determine the overall utility for each alternative*, as follows:

$$u_i = \frac{u_i^+}{u_i^-}. \tag{32}$$

In the case when evaluation is made only on the basis of benefit criteria, Eq. (32) is as follows:

$$u_i = u_i^+. \tag{33}$$

Step 8. *Determine the final ranking order of the alternatives*. The final ranking order of the alternatives can be determined as in the case of the ordinary MULTIMOORA method, i.e. based on the dominance theory (Brauers and Zavadskas, 2011b).

In this stage, the alternatives are ranked based on their overall importance, maximum distance to the reference point and overall utility. As a result of that, three ranking lists are formed.

Based on these ranking lists, the final ranking list of the alternatives is formed on the basis of the theory of dominance, i.e. the alternative with the largest number of appearances on the first position in the three ranking lists is the most acceptable.

5. A Numerical Example

In this section, a numerical example of purchasing rental space is considered in order to explain the proposed approach in detail.

Table 2
The ratings obtained from the first of the three DMs.

	C_1		C_2		C_3		C_4		C_5	
	a^+	a^-	a^+	a^-	a^+	a^-	a^+	a^-	a^+	a^-
A_1	7	−2	7	−3	5	−1	7	−5	8	−1
A_2	4	−1	5	−2	4	−2	4	−6	7	−1
A_3	7	−1	3	−1	2	0	2	−1	7	−2
A_4	9	−1	4	−1	3	0	3	−1	6	−1

Table 3
The ratings obtained from the first of the three DMs, in the form of SVBFNs.

	C_1	C_2	C_3	C_4	C_5
A_1	$\langle 0.70, -0.20\rangle$	$\langle 0.70, -0.30\rangle$	$\langle 0.50, -0.10\rangle$	$\langle 0.70, -0.50\rangle$	$\langle 0.80, -0.10\rangle$
A_2	$\langle 0.40, -0.10\rangle$	$\langle 0.50, -0.20\rangle$	$\langle 0.40, -0.20\rangle$	$\langle 0.40, -0.60\rangle$	$\langle 0.70, -0.10\rangle$
A_3	$\langle 0.70, -0.10\rangle$	$\langle 0.30, -0.10\rangle$	$\langle 0.20, 0.00\rangle$	$\langle 0.20, -0.10\rangle$	$\langle 0.70, -0.20\rangle$
A_4	$\langle 0.90, -0.10\rangle$	$\langle 0.40, -0.10\rangle$	$\langle 0.30, 0.00\rangle$	$\langle 0.30, -0.10\rangle$	$\langle 0.60, -0.10\rangle$

Table 4
The ratings obtained from the second of the three DMs, in the form of SVBFNs.

	C_1	C_2	C_3	C_4	C_5
A_1	$\langle 0.70, -0.20\rangle$	$\langle 0.70, -0.50\rangle$	$\langle 0.40, -0.20\rangle$	$\langle 0.70, -0.50\rangle$	$\langle 0.80, -0.10\rangle$
A_2	$\langle 0.60, -0.10\rangle$	$\langle 0.40, -0.60\rangle$	$\langle 0.40, -0.20\rangle$	$\langle 0.40, -0.60\rangle$	$\langle 0.80, -0.10\rangle$
A_3	$\langle 0.80, -0.10\rangle$	$\langle 0.20, -0.10\rangle$	$\langle 0.20, -0.10\rangle$	$\langle 0.20, -0.10\rangle$	$\langle 0.70, -0.10\rangle$
A_4	$\langle 0.90, -0.10\rangle$	$\langle 0.30, -0.10\rangle$	$\langle 0.30, -0.10\rangle$	$\langle 0.30, -0.10\rangle$	$\langle 0.60, -0.10\rangle$

Suppose that a company is planning to start its sales business in a new location, and therefore is looking for a new sales building. After the initial consideration of the available alternatives, four alternatives have been identified as suitable. For this reason, a team of three decision-makers (DMs) was formed with the aim of evaluating suitable alternatives based on the following criteria:

- C_1 – Rental space quality;
- C_2 – Rental space adequacy;
- C_3 – Location quality;
- C_4 – Location distance from the city centre, and
- C_5 – Rental price.

As previously reasoned, in this evaluation the ratings of the alternatives in relation to the criteria are expressed by using SVBFNs.

The ratings obtained from the first of the three DMs are shown in Table 2, as the points of the Likert scale, whereas in Table 3, they are shown in the form of SVBFNs.

The ratings obtained from the second and the third of the three DMs are accounted for in Table 4 and Table 5.

The group decision matrix, calculated by applying Eq. (11), is presented in Table 6.

Table 5
The ratings obtained from the third of the three DMs, in the form of SVBFNs.

	C_1	C_2	C_3	C_4	C_5
A_1	$\langle 0.60, -0.10\rangle$	$\langle 0.90, -0.20\rangle$	$\langle 1.00, 0.00\rangle$	$\langle 1.00, 0.00\rangle$	$\langle 0.80, -0.10\rangle$
A_2	$\langle 0.40, -0.60\rangle$	$\langle 0.40, -0.60\rangle$	$\langle 1.00, -0.40\rangle$	$\langle 1.00, 0.00\rangle$	$\langle 0.80, -0.10\rangle$
A_3	$\langle 0.20, -0.10\rangle$	$\langle 0.90, -0.40\rangle$	$\langle 0.80, -0.30\rangle$	$\langle 0.70, -0.10\rangle$	$\langle 0.70, -0.10\rangle$
A_4	$\langle 0.30, -0.10\rangle$	$\langle 1.00, -0.30\rangle$	$\langle 0.80, -0.20\rangle$	$\langle 0.80, -0.10\rangle$	$\langle 0.60, -0.10\rangle$

Table 6
The group decision-making matrix.

	C_1	C_2	C_3	C_4	C_5
A_1	$\langle 0.67, -0.16\rangle$	$\langle 0.79, -0.32\rangle$	$\langle 1.00, 0.00\rangle$	$\langle 1.00, 0.00\rangle$	$\langle 0.80, -0.10\rangle$
A_2	$\langle 0.47, -0.18\rangle$	$\langle 0.43, -0.42\rangle$	$\langle 1.00, -0.26\rangle$	$\langle 1.00, 0.00\rangle$	$\langle 0.77, -0.10\rangle$
A_3	$\langle 0.64, -0.10\rangle$	$\langle 0.61, -0.16\rangle$	$\langle 0.49, 0.00\rangle$	$\langle 0.41, -0.10\rangle$	$\langle 0.70, -0.13\rangle$
A_4	$\langle 0.81, -0.10\rangle$	$\langle 1.00, -0.15\rangle$	$\langle 0.53, 0.00\rangle$	$\langle 0.53, -0.10\rangle$	$\langle 0.60, -0.10\rangle$

Table 7
The weights of the criteria obtained from the first of the three DMs.

	s_j	k_j	q_j	w_j
C_1		1	1	0.19
C_2	1.2	0.80	1.25	0.23
C_3	0.9	1.10	1.14	0.21
C_4	0.7	1.30	0.87	0.16
C_5	1.2	0.80	1.09	0.20
		5.00	5.35	1.00

Table 8
The group criteria weights.

	w_j^1	w_j^2	w_j^3	w_j
C_1	0.19	0.17	0.19	0.18
C_2	0.23	0.24	0.23	0.24
C_3	0.21	0.22	0.21	0.21
C_4	0.16	0.17	0.16	0.16
C_5	0.20	0.21	0.20	0.21
				1.00

The weights obtained from the first of the three DMs by applying the PIPRECIA method (Stanujkic *et al.*, 2017b) are accounted for in Table 7, while the group weights of the criteria, calculated by applying Eq. (19), are shown in Table 8.

On the basis of the ratings from Table 6 and the weights from Table 8, the overall significance, the maximum distance to the reference point and the overall utility are calculated for each alternative in the next step.

The overall significances, accounted for in Table 9, are calculated by applying Eqs. (20)–(24).

Table 9
The overall significances of the considered alternatives.

	Y_i^+	Y_i^-	y_i^+	y_i^-	y_i	Rank
A_1	$\langle 1.00, -0.11\rangle$	$\langle 1.00, -0.02\rangle$	0.94	0.99	-0.05	3
A_2	$\langle 1.00, -0.20\rangle$	$\langle 1.00, -0.02\rangle$	0.90	0.99	-0.09	4
A_3	$\langle 0.42, -0.06\rangle$	$\langle 0.30, -0.05\rangle$	0.68	0.63	0.05	2
A_4	$\langle 1.00, -0.06\rangle$	$\langle 0.29, -0.04\rangle$	0.97	0.62	0.35	1

Table 10
The reference points.

	$\Omega_{\max}$		$\Omega_{\min}$	
	r^+	r^-	r^+	r^-
r^*	1.00	-0.20	0.29	-0.02

Table 11
The ratings of the alternatives obtained based on the reference point approach.

	C_1	C_2	C_3	C_4	C_5	$d_i^{\max}$	Rank
A_1	0.08	0.16	0.13	0.29	0.10	0.08	4
A_2	0.17	0.28	0.00	0.29	0.09	0.00	1
A_3	0.13	0.33	0.38	0.05	0.06	0.05	3
A_4	0.04	0.14	0.36	0.11	0.00	0.00	1

Table 12
The overall utility of the considered alternatives.

	U_i^+ U_i^-	u_i^+ u_i^-	u_i	Rank		
A_1	$\langle 1.00, -0.11\rangle$	$\langle 1.00, -0.02\rangle$	0.94	0.99	-0.05	3
A_2	$\langle 1.00, -0.20\rangle$	$\langle 1.00, -0.02\rangle$	0.90	0.99	-0.09	4
A_3	$\langle 0.42, -0.06\rangle$	$\langle 0.30, -0.05\rangle$	0.68	0.63	0.05	2
A_4	$\langle 1.00, -0.06\rangle$	$\langle 0.29, -0.04\rangle$	0.97	0.62	0.35	1

After that, the reference point shown in Table 10 is determined by applying Eq. (25).

The maximum distances to the reference point accounted for in Table 11 are determined by applying Eq. (26) and Eq. (27).

The overall utility shown in Table 12 is calculated by applying Eqs. (28)–(32).

Finally, on the basis of the ranking orders shown in Tables 9, 11 and 12, the most appropriate alternative is determined by means of the theory of dominance, as is shown in Table 13.

As can be seen from Table 12, the most appropriate alternative is the alternative denoted as A_4.

Table 13
The final ranking order of the considered alternatives.

	RS	*RP*	*FMP*	Final rank
A_1	3	4	3	3
A_2	4	1	4	4
A_3	2	3	2	2
A_4	1	1	1	1

6. Conclusions

The bipolar fuzzy sets introduced two membership functions, namely the membership function to a set and the membership function to a complementary set.

On the other hand, the MULTIMOORA method is an efficient and already proven multiple-criteria decision-making method, which has been used for solving a number of different decision-making problems so far.

Therefore, an extension of the MULTIMOORA method enabling the use of single-valued bipolar fuzzy numbers is proposed in this article. The usability and efficiency of the proposed extension is successfully demonstrated on the example of the problem of the best location selection.

In the literature, numerous extensions of the MULTIMOORA methods have been proposed with the aim to adapt it for the use of grey system theory, fuzzy set theory, as well as various extensions of fuzzy set theory. Some extensions that enable the use of neutrosophic sets are also proposed. The mentioned extensions aim to exploit the specificities of particular sets for solving certain types of decision-making problems, and thus enable more efficient decision making.

Because of the specificity that bipolar fuzzy sets provide, the proposed expanded MULTIMOORA method can be expected to be acceptable for solving a particular class of complex decision-making problems.

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Entropy of Polysemantic Words for the Same Part of Speech

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ABSTRACT In this paper, a special type of polysemantic words, that is, words with multiple meanings for the same part of speech, are analyzed under the name of neutrosophic words. These words represent the most difficult cases for the disambiguation algorithms as they represent the most ambiguous natural language utterances. For approximate their meanings, we developed a semantic representation framework made by means of concepts from neutrosophic theory and entropy measure in which we incorporate sense related data. We show the advantages of the proposed framework in a sentiment classification task.

INDEX TERMS Neutrosophic sets, semantic word representation, sentiment classification.

I. INTRODUCTION

Every natural language word can have multiple realisations from the part-of-speech point of view, and for each of its possible parts-of-speech, it can have multiple meanings (especially the English words). Each sense creates a ''sub-dimension'' in the word's space determined by the part-of-speech to which it belongs in the given statement. The polysemantic words (words with multiple senses) can be described by several spaces (one space for each possible part-of-speech) and each space can include several subspaces determined by the meanings the word can have. In this manner, every dimension describes a certain facet of the analysed word. It is also true that certain senses are more frequent than others and in this manner they can force a certain facet to be more prominent than others.

We need a comprehensive and unitary study for natural lan-guage words formulated as a Multicriteria Decision Making problem [1] in which uncertainty is inevitably involved due to the subjectivity of humans [2]. It has been shown that different senses of the same word usually imply differ-ent sentiment orientations for the word under analysis. For instance, the word ''good'': in ''good man'' produces a posi-tive utterance while in ''good fight'' indicates a negative state-ment. As a direct consequence we need studies that address both the interaction between word sense disambiguation and sentiment analysis.

These are quite new studies in the lit-erature as the researchers in this area must be intrigued by the usability of sense level information in sentiment analysis. Some researchers take this approach and compute the polarity score for each word sense [3], [4].

The present paper proposes a novel approach for word sentiment classification by extracting a set of semantic data from the SentiWordNet in order to compute a final estimation of the word polarity. SentiWordNet [3], [5] is a well-known freely available lexical resource for sentiment analysis which annotates each sense of a word with three polarity scores. These polarity scores represent the positivity, objectivity and negativity degrees of the annotated word sense ranging from 0 to 1 with their sum up to one. SentiWordNet (SWN) was built on the semantically-oriented WordNet [6], [7], which in its primary form, that is for English language, comprises 155287 words and 117659 senses.

There are two main approaches for sentiment analysis: machine learning and knowledge-based. From the machine learning perspective, the Support Vector Machines (SVM) classification (see, for example, [8], [9]) has the best classification performance for sentiment analysis [10], [11] out-performing both the Naïve Bayes and Maximum entropy classification methods. The knowledge-based methods usually make use of the most common sense of the words and in this manner an improvement of accuracy over the baseline was observed [12]. Also, the overall polarities of different senses in each part-of-speech tag categories are also determined [13]. However, the commonly used n-gram features are not robust enough and show widely varying behaviour across different domains [14].

The method we propose in this paper offers a knowledge-based solution for semantic word representation which targets sentiment classification and makes use of the general concepts of neutrosophic theory and entropy measure. A previous study that applies neutrosophic theory in sentiment analysis domain is given in [15]. In this paper we concentrate our approach by keeping in mind only the most difficult cases for sentiment classification. They are represented by a special class of polysemantic words with different meanings for the same part-of-speech realisation. In the present paper these words are named *neutrosophic words* because their representation involves the core concepts of neutrosophic theory.

With this article we are in line with the neutrosphic word representations firstly proposed in [16] and then refined in [17] in which the SentiWordNet (shortly SWN) sentiment scores are interpreted as truth-fullness degrees. The study proposed in this paper also makes use of the SWN polarity scores of each word's sense, this time in order to determine the overall sentiment score value. The involved computations apply entropy on the words' sentiment scores as a measure of disorder for the words' polarities.

The paper is organised as follows: the Related Works section overviews the most commonly used multi-space representation techniques in neutrosophy. Section III presents the proposed semantic-level representation which treats the words as union of neutrosophic sets. In Section IV we show how this type of representation can be used in conjunction with a sentiment analysis study. Section V exemplifies all the involved theoretical concepts on a study case also providing the obtained results and the last section is dedicated to the conclusions and our future directions.

II. RELATED WORKS

The concept of multi-space was introduced by Smarandache in 1969 [18] by following the idea of hybrid mathematics - especially hybrid geometry [19], [20] for combining different fields into a unifying field [21]–[24].

Let Ω be a universe of discourse and a subset $S \subseteq \Omega$. Let $[0, 1]$ be a closed interval and three subsets $T, I, F \subseteq [0, 1]$. Then, a relationship of an element $x \in S$ with respect to the subset S is $x(T, I, F)$, which means the following: the *confidence set* of x is T, the *indefinite set* of x is I, and the *failing set* of x is F. A set S, together with the corresponding three subsets T, I, F for each element x in S, is said to be a *neutrosophic set* [19], [25].

Let Σ be a set and $A_1, A_2, \ldots, A_k \subseteq \Sigma$. Define $3k$ functions $f_1^z, f_2^z, \ldots, f_k^z$ by $f_i^z : A_i \to [0, 1]$, $1 \le i \le k$, where $z \in \{T, I, F\}$. If we denote by $(A_i; f_i^T, f_i^I, f_i^F)$ the subset A_i together with three functions f_i^T, f_i^I, f_i^F, $1 \le i \le k$, then [19]:

$$\bigcup_{i=1}^{k}(A_i; f_i^T, f_i^I, f_i^F)$$

is a union of neutrosophic sets which are generalizations of classical sets.

Indeed, if we take $f_i^T = 1, f_i^I = f_i^F = 0$ for $i = \overline{1, k}$ we obtain [19]:

$$\bigcup_{i=1}^{k}(A_i; f_i^T, f_i^I, f_i^F) = \bigcup_{i=1}^{k} A_i$$

and correspondingly, for $f_i^T = f_i^I = 0, f_i^F = 1$, $i = \overline{1, k}$ we obtain the complementary sets [19]:

$$\bigcup_{i=1}^{k}(A_i; f_i^T, f_i^I, f_i^F) = \overline{\bigcup_{i=1}^{k} A_i}$$

The appurtenance and non-appurtenance is obtained if there is an integer s such that $f_i^T = 1, f_i^I = f_i^F = 0$, $1 \le i \le s$, but $f_j^T = f_j^I = 0, f_j^F = 1$, $s + 1 \le j \le k$.

$$\bigcup_{i=1}^{k}(A_i; f_i^T, f_i^I, f_i^F) = \bigcup_{i=1}^{s} A_i \cup \overline{\bigcup_{i=s+1}^{k} A_i}$$

The general neutrosophic set is obtained if there is an integer l such that $f_l^T \neq 1$ for $1 \le l \le s$, or $f_l^F \neq 1$ for $s < l \le n$. The resulted union cannot be represented by abstract sets.

III. SEMANTIC-LEVEL REPRESENTATION FOR WORDS

As we have already pointed out in the Introduction section, a word is not a simple data, it can have several (syntactic) attributes and can support more than one semantic interpretations. Metaphorically speaking a word is like a diamond: it can brighter a life or, by contrary, it can cut and destroy. But, from our study point of view, a word is just an entity that can have multiple semantic facets.

As we have already pointed out, a word can have more than one part-of-speech, like the word "good" which can be adjective, noun or adverb and to which we dedicate an extensive study in the Section V. There are programs that can automatically identify the part-of-speech of a certain word in a given context. These programs are named Part-Of-Speech Taggers and for most of the languages their accuracy is quite high (more than 90%).

On contrary, determining the meaning of a polysemous word in a specific context - that is, performing a disambiguation on the word's senses, can be a laborious task. In spite of the great number of existing disambiguation algorithms, the problem of word sense disambiguation remains an open one [26]. For some languages like English the accuracy of the disambiguation algorithms does not overcome 75%.

It is obviously that we need to model indeterminacy in the semantic word representations. This is the reason why, in the present study we choose to model word representations using neutrosophic theory as, in contrast to intuitionistic fuzzy sets and also interval valued intuitionistic fuzzy sets, indeterminacy degree of an element is explicitly expressed by the neutrosophic sets [27]. Moreover, in [29] the authors state that single valued neutrosophic (SVN) set is a better tool to deal with incomplete, inconsistent and indeterminate information than fuzzy set (FS) and intuitionistic FS (IFS). With the present study we are in line with these assumptions continuing also our previous works in which the natural language words are modelled as single-valued neutrosophic sets in order to approximate their ambiguous meaning [16], [17].

In the representation we propose in this paper a word can have multiple dimensions organised on several plans:

- the POS plans are determined by the possible part of speech data of the word
- each POS plan can have several sense units, determined by the possible word's senses under that POS data
- finally, each sense unit is made of some components (sentiment scores) which describe the sense meaning polarity

A. WORDS AS UNION OF NEUTROSOPHIC SETS

The first step in creating a semantic representation is to decide what features to use and how to encode these features. From the features set a word can have, in this study we consider the part-of-speech as the syntactic feature and the word's sense(s) as its semantic interpretation(s).

In what follows, we name semantic facets or simply facets - all the word's data based on which the semantic interpretation can be defined. Using concepts from neutrosophic sets theory [30] we propose the following semantic representation of a word.

Definition 1: The semantic representation of a word by means neutrosophic theory concepts is defined as:

$$w = \bigcup_{i=1}^{k} (sense_i; f_i^T, f_i^I, f_i^F)$$

where:

- k represents the number of senses the word can have
- $f_i^T, f_i^I f_i^F \; : \; Facets \rightarrow [0, 1]$ are the membership functions for the $sense_i$, $i = \overline{1, k}$, such that:
  - f_i^T represents the membership degree,
  - f_i^I represents the indeterminacy degree and
  - f_i^F is the degree of nonmembership degree
- *Facets* set includes all the data that characterise the word from the semantic point of view.

In this assertion, a word becomes a union of neutrosophic sets. For the ith sense of the word w, the membership functions of the word's semantic facets fulfil the following properties:

$$\forall x \in Facets : f_i^T(x) + f_i^I(x) + f_i^F(x) = 1 \tag{1}$$

and if $Facets = \{x_1, \ldots, x_m\}$ then:

$$\sum_{j=1}^{m} f_i^T(x_j) + f_i^I(x_j) + f_i^F(x_j) = m \tag{2}$$

In order to include the information about the part-of-speech data (shortly POS data) we need to refine the representation given in Definition 1. We consider the general case in which a word can have n possible parts of speech $POS_1, \ldots, POS_n$, with $n \geq 1$, and for each part of speech POS_j the word can have k_j senses, $k_j \geq 1$. The representation given in Definition 1 becomes:

$$w = \bigcup_{i=1}^{k_1} (sense_{i;POS_1}; f^T_{i;POS_1}, f^I_{i;POS_1}, f^F_{i;POS_1}) \cup \ldots$$
$$\ldots \cup \bigcup_{i=1}^{k_n} (sense_{i;POS_n}; f^T_{i;POS_n}, f^I_{i;POS_n}, f^F_{i;POS_n}) \tag{3}$$

Using the representation given in Equation 3, the senses corresponding to a certain part of speech POS_j with $j \in \{1, \ldots n\}$, can be obtained as follows:

$$\begin{aligned}(w)_{POS_j} &= w \cap (w)_{POS_j} \\ &= \bigcup_{i=1}^{k_j} (sense_{i;POS_j}; f^T_{i;POS_j}, f^I_{i;POS_j}, f^F_{i;POS_j})\end{aligned} \tag{4}$$

Furthermore, we can apply another filtering on word representation given in Equation 4 if we consider the case in which a specific sense of the word w results to be realised in a given context. Let us note this sense with $sense_{m;POS_j}$ with $m \in \{1, k_j\}$. By applying concepts from neutrosophic sets theory we obtain:

$f^T_{m;POS_j} = 1$, $f^I_{m;POS_j} = f^F_{m;POS_j} = 0$ and $f^T_{l;POS_j} = f^I_{l;POS_j} = 0$, $f^F_{l;POS_j} = 1$ for $l \neq m$, $l, m = \overline{1, k_j}$

which implies:

$$\begin{aligned}(w)_{POS_j} &= \bigcup_{i=1}^{k_j} (sense_{i;POS_j}; f^T_{i;POS_j}, f^I_{i;POS_j}, f^F_{i;POS_j}) \\ &= (sense_{m;POS_j}; 1, 0, 0) \cup \bigcup_{l \neq m} (sense_{l;POS_j}; 0, 0, 1) \\ &= sense_{m;POS_j} \cup \overline{\bigcup_{l \neq m} sense_{l;POS_j}} \\ &= sense_{m;POS_j} = (m{-}th \; sense \; of \; w)_{POS_j}\end{aligned} \tag{5}$$

The representation given in Equation 5 corresponds to the most unambiguous case, more precisely to the situation in which we know both the word's part of speech (noted here with POS_j) and the word sense (noted with $sense_{m;POS_j}$).

But, the problems with natural language processing comes from ambiguity - when we could not identify (using automatic tools) which sense is realised in the given context from the set of the word's possible senses (noted here with $\cup_{i=1}^{k_j} sense_{i;POS_j}$). This ambiguity case is depicted by the general case given in Equation 3.

In what follows we will use a simplified form of Equation 3 in which POS_j data is removed from the annotations sequences corresponding to the senses and membership functions. Thus, Equation 3 becomes:

$$(w)_{POS_j} = \bigcup_{i=1}^{k_j} (sense_i; f_i^T, f_i^I, f_i^F) \tag{6}$$

In the next section we present a method by means of which we can eliminate the "noises" from an ambiguous semantic word representation, more precisely, a representation that includes more than one possible sense. We resolve these issues using Neutrosophic Theory and Entropy measure. Our proposal is described in conjunction with a sentiment analysis study in which the semantic word representation has the form of a three sentiment scores tuple.

IV. WORD SEMANTIC REPRESENTATION WITH SENTIMENT SCORES

Sense discrimination addresses words with multiple senses and is done in conjunction with a particular context in which only one sense is realised. This analysis has a semantic nature and is quite difficult to perform it using automatic tools, especially if the realisation context is poor in information that could filter the correct word meaning from the set of possible ones. In order to exemplify our proposal we choose to interpret the word semantics from a sentiment analysis the point of view. Thus, each sense of a word will be represented using its sentiment scores.

In what follows, let us consider the approach firstly proposed in [16] and then extended in [17] in which a word w is interpreted as a single-value neutrosophic set constructed upon its sentiment scores which describe the word's sense-level polarity information being denoted in what follows with (sc_+, sc_0, sc_-), where:

- sc_+ denotes the word positive score,
- sc_0 represents the word neutral score and
- sc_- stands for the word negative score.

As in [16] and [17] we use SentiWordNet lexical resource for providing the required information for the sentiment scores of the English words.

For a word w with k_j senses under a POS_j part-of-speech realisation, the semantic representation is defined as a union of the tuples: $sense_i = (sc_{+_i}, sc_{0_i}, sc_{-_i})$ with $i \in \{1, \dots, k_j\}$. The Equation 6 becomes:

$$(w)_{POS_j} = \bigcup_{i=1}^{k_j}((sc_{+_i}, sc_{0_i}, sc_{-_i}); f_i^T, f_i^I, f_i^F) \tag{7}$$

with $sc_{+_i}, sc_{0_i}, sc_{-_i} \in [0, 1]$. The semantic representation given in Equation 7 implies that each word's sense will include three facets: the positive, the neutral and the negative one. By preserving the notation where $+$ stands for positive, 0 for neutral and $-$ for negative facet, we take $Facets = \{+, 0, -\}$.

The representation given in Equation 7 can be rewritten as:

$$(w)_{POS_j} = \bigcup_{i=1}^{k_j}((sc_{+_i}, sc_{0_i}, sc_{-_i}); (\{f_i^T(x)\}_{x\in Facets}), (\{f_i^I(x)\}_{x\in Facets}), (\{f_i^F(x)\}_{x\in Facets})) \tag{8}$$

where $f_i^T(x)$, $f_i^I(x)$ and $f_i^F(x)$ represents the membership functions corresponding to the facet x of the ith sense, $x \in Facets$ and $(\{f_i^M(x)\}_{x\in Facets})$ briefly notes the membership functions $\begin{pmatrix} f_i^M(+) \\ f_i^M(0) \\ f_i^M(-) \end{pmatrix}$, $M \in \{T, I, F\}$.

Remark: For the representation given in Equation 8, the default case corresponds to the maximum certainty case where no imprecision occurs which, in terms of membership function is depicted by $f_i^T(\{+ \mid 0 \mid -\}_i) = 1, f_i^I(\{+ \mid 0 \mid -\}_i) = 0, f_i^F(\{+ \mid 0 \mid -\}_i) = 0, i = \overline{1, k_j}$.

We preface the study that addresses the multi-facets words by enumerating the form in which the *one facet words* are represented in our proposal. These words are the extreme cases of our study and every neutrosophic study provides them.

Case 1: If $sc_{+_i} = 1$, $sc_{0_i} = sc_{-_i} = 0$ and $f_i^T(\{+ \mid 0 \mid -\}_i) = 1, f_i^I(\{+ \mid 0 \mid -\}_i) = 0, f_i^F(\{+ \mid 0 \mid -\}_i) = 0$ for every $i = \overline{1, k_j}$ then:

$$(w)_{POS_j} = \bigcup_{i=1}^{k_j}\left((1,0,0); \begin{pmatrix}1\\1\\1\end{pmatrix}, \begin{pmatrix}0\\0\\0\end{pmatrix}, \begin{pmatrix}0\\0\\0\end{pmatrix}\right) = (1,0,0)$$

The interpretation of Case 1 is: for all the senses corresponding to the POS_j part-of-speech the word w is *pure positive*.

Case 2: If $sc_{+_i} = sc_{0_i} = 0$, $sc_{-_i} = 1$ and $f_i^T(\{+ \mid 0 \mid -\}_i) = 1, f_i^I(\{+ \mid 0 \mid -\}_i) = 0, f_i^F(\{+ \mid 0 \mid -\}_i) = 0$ for every $i = \overline{1, k_j}$ then:

$$(w)_{POS_j} = \bigcup_{i=1}^{k_j}\left((0,0,1); \begin{pmatrix}1\\1\\1\end{pmatrix}, \begin{pmatrix}0\\0\\0\end{pmatrix}, \begin{pmatrix}0\\0\\0\end{pmatrix}\right) = (0,0,1)$$

The interpretation of Case 2 is: for all the senses corresponding to the POS_j part-of-speech the word w is *pure negative*.

Case 3: If $sc_{+_i} = sc_{-_i} = 0$, $sc_{0_i} = 1$ and $f_i^T(\{+ \mid 0 \mid -\}_i) = 1, f_i^I(\{+ \mid 0 \mid -\}_i) = 0, f_i^F(\{+ \mid 0 \mid -\}_i) = 0$ for every $i = \overline{1, k_j}$ then:

$$(w)_{POS_j} = \bigcup_{i=1}^{k_j}\left((0,1,0); \begin{pmatrix}1\\1\\1\end{pmatrix}, \begin{pmatrix}0\\0\\0\end{pmatrix}, \begin{pmatrix}0\\0\\0\end{pmatrix}\right) = (0,1,0)$$

The interpretation of Case 3 is: for all the senses corresponding to the POS_j part-of-speech the word w is *pure neutral*.

These three cases correspond to the non-ambiguous words, that is, words with a unique sense (one semantic representation) or similar semantic representations for all of their possible senses.

Since in a natural language there are many words (especially in English) with multiple senses - the *polysemantic words*, in what follows we will concentrate our study only on these words. For the polysemantic words we get different semantic representations that must be resolved by dealing with many degrees of uncertainties. In this case, the simple reunion of their semantic dimensions is a general neutrosophic set that cannot be formalised using abstract set theories. For this reason, in the next definition we introduce the concept of *neutrosophic word* in conjunction with a sentiment analysis.

Definition 2: A *neutrosophic word* is a polysemantic word that under the same part of speech realization has at least two different sentiment polarities which means:

$(\exists (w)_{POS_j}$ with $k_j > 1$ senses$) \wedge (\exists i_1, i_2 \in \{1, \ldots, k_j\}, i_1 \neq i_2$: $sense_{i_1} \neq sense_{i_2})$

Different sense tuples imply different sentiment scores and we obtain:

$(\exists (w)_{POS_j}$ with $k_j > 1$ senses$) \wedge [\exists i_1, i_2 \in \{1, \ldots, k_j\}, i_1 \neq i_2$: $(sc_{+_{i_1}}, sc_{0_{i_1}}, sc_{-_{i_1}}) \neq (sc_{+_{i_2}}, sc_{0_{i_2}}, sc_{-_{i_2}})]$

As a direct consequence, the semantic representation of neutrosophic words is:

$$(w)_{POS_j} = \bigcup_{i \in \{i_1, i_2, \ldots\}} ((sc_{+_i}, sc_{0_i}, sc_{-_i}); (\{f_i^T(x)\}_{x \in Facets}), (\{f_i^I(x)\}_{x \in Facets}), (\{f_i^F(x)\}_{x \in Facets}))$$

with $sc_{+_{i_1}} \neq sc_{+_{i_2}}$ or $sc_{0_{i_1}} \neq sc_{0_{i_2}}$ or $sc_{-_{i_1}} \neq sc_{-_{i_2}}$ and $f_{i_1}^T(\{+ \mid 0 \mid -\}), f_{i_2}^T(\{+ \mid 0 \mid -\}) > 0$, $i_1 \neq i_2$. By the fact that the membership degrees are greater than 0, we obtain for a neutrosophic word *w the necessity of having (at least) two different sentiment representations* for the same $(w)_{POS_j}$.

The neutrosophic theory means from the very beginning dealing with uncertainty. This is also true for the neutrosophic words. These words can be evidenced in case of an imprecise disambiguation mechanism which fails in recognising what sense is realised in the given context even if the part-of-speech data is correctly provided.

In our approach, a neutrosophic word is synonym with a word that has different sense facets and for which we cannot establish a unique semantic representation. For the chosen sentiment analysis exemplification, different sense facets for a word means different sentiment scores tuples.

In the next section we exemplify how the proposed method works. We show that using the neutrosophic sets theory and applying the entropy measure on the word representations we can identify the word's sentiment facet with respect to the given part-of-speech.

A. ENTROPY AS A MEASURE OF UNCERTAINTY FOR THE NEUTROSOPHIC WORDS REPRESENTATIONS

Fuzzy entropy, distance measure and similarity measure are three basic concepts used in fuzzy sets theory [27]. Among them, Entropy is an efficient tool to model uncertainty [28] or, in layman terms, Entropy is a measure of disorder. It can be used in order to measure how disorganised an input values set is by calculating the entropy of their values/labels. Entropy is high if the input values are highly varied and low if many input data have the same value. In mathematical terms, Entropy is defined as the sum of the probability of each input values or labels times the log probability of that label:

$$E(labels) = -\sum_{l \in labels} P(l) log_2 P(l) \tag{9}$$

where $P(l)$ is the frequency probability of the *label* item in the considered data and *labels* denotes the set of possible labels.

From this definition we obtain that labels with low frequency do not affect much the entropy (because $P(l)$ is small). The same result for labels with high frequency as in their case, $log_2 P(l)$ is small. Only when the inputs have wide varieties of labels (and as a direct consequence, these many labels have a medium frequency) the entropy is high because neither $P(l)$ nor $log_2 P(l)$ is small.

Entropy has values between 0 and 1 and high entropy values stand for high levels of disorder or ''low level of purity''. Following this property, we can qualify the uncertainty of the words' semantic nature by applying the entropy measure on their sense representation labels: *the higher the values for entropy measure the higher the level of uncertainty for the analyzed word representations*.

The neutrosophic word is a concept with more than one possible sense for at least one of its possible part-of-speech data. On the other hand, entropy is a measure of uncertainty. Between the possible senses we can have certain similarity degrees and the entropy measure can be used in order to determine how similar or dissimilar these senses are.

The most common manner to unify a set of possible representations into a single one is to consider only the maximum (or the minimum) value or to average the values (in our case, the sentiment scores) as in the following formula:

$$Avg\left(\bigcup_{i=1}^{k_j}(sc_{+_i}, sc_{0_i}, sc_{-_i})\right) = \left(\frac{1}{k_j}\sum_{i=1}^{k_j} sc_{+_i}, \frac{1}{k_j}\sum_{i=1}^{k_j} sc_{0_i}, \frac{1}{k_j}\sum_{i=1}^{k_j} sc_{-_i}\right) \tag{10}$$

where k_j notes the number of senses for the analysed word. But this method of unifying different representation can be trustful only if the averaged values are not very dissimilar with the initial ones.

Example 1: Let us consider a word w with two extreme sentiment scores tuples: (0, 0, 1) and (1, 0, 0). Overall, we obtain two different facets: in the first representation we have a *pure positive word* while in the second we get a *pure negative word*. If we merge these two representation by averaging their sentiment scores values we get (0.5, 0, 0.5) - a representation that could be interpreted as a *neutral word*. Definitely this would be a wrong classification for a strong sentiment word.

We define a bijective mapping for labelling the sentiment score values to a set of three strength degrees, $SD = \{low, medium, high\}$. We obtain $sd : [0, 1] \rightarrow SD$ with:

$$sd(score) = \begin{cases} low, & \text{if } score < 0.4 \\ medium, & \text{if } score \in [0.4, 0.6] \\ high, & \text{if } score > 0.6 \end{cases}$$

Mapping the score values to the *SD* labels we get ''low'' label for a small score, ''medium'' for not a small but also not a high score and ''high'' for a big score. Using these strength degrees we can qualify by means of the entropy measure calculated as in Equation 9 how disorganised the scores values are from the point of view of the sentiment strength. All the involved operations are given in Algorithm 1.

Algorithm 1 Merging Multiple Semantic Representations of a Neutrosophic Word $(w)_{POS_j}$

```
INPUT: ∪_{i=1}^{k_j}(sc_{+i}, sc_{0i}, sc_{-i})
for each x in Facets:
    Entropy(x) ← E(∪_{i=1}^{k_j} sd(sc_{x_i}))
    Avg(x) ← Avg(∪_{i=1}^{k_j} sc_{x_i}) ← (1/k_j) Σ_{i=1}^{k_j} sc_{x_i}
    f^T(x) ← 1 − Entropy(x)
    f^I(x) ← Entropy(x)
    f^F(x) ← 0
endfor
OUTPUT: ∪_{x∈Facets} Avg(x), f^T(x), f^I(x), f^F(x)
```

We can now give the manner in which the multiple representations of a neutrosophic word $(w)_{POS_j}$ can be unified into a unique sentiment representation $Avg(w)_{POS_j}$ based on the values provided by Algorithm 1:

$$
\begin{aligned}
&Avg((w)_{POS_j})\\
&= Avg\left(\bigcup_{i=1}^{k_j}\left((sc_{+_i}, sc_{0_i}, sc_{-_i}); \begin{pmatrix}1\\1\\1\end{pmatrix}, \begin{pmatrix}0\\0\\0\end{pmatrix}, \begin{pmatrix}0\\0\\0\end{pmatrix}\right)\right)\\
&= (\left(Avg(\cup_{i=1}^{k_j} sc_{+_i}), Avg(\cup_{i=1}^{k_j} sc_{0_i}), Avg(\cup_{i=1}^{k_j} sc_{-_i})\right);\\
&\quad (\{f^T(x)\}_{x\in Facets}), (\{f^I(x)\}_{x\in Facets}), (\{f^F(x)\}_{x\in Facets}))
\end{aligned} \tag{11}
$$

In Algorithm 1 we model the degrees of trustfulness for the resulted average scores representation by means of the membership functions, such that $\forall x \in Facets$:

- If the entropy $Entropy(x)$ is small (the minimum value is 0) then the average value $Avg(x)$ can approximate with high degree of certainty the initial word's sentiment scores; in this case the membership function for the facet x is set to a big value (almost 1) as $f^T(x) \leftarrow 1 - Entropy(x)$.
- If the entropy $Entropy(x)$ is high (the maximum value is 1) then the membership function is set to a small value (almost 0) while the indeterminacy degree $f^I(x)$ is set to be equal with the entropy function value.
- For preserving the sum of the membership functions to value 1 (see Equation 1), the nonmembership degree $f^F(x)$ for the facet x is always 0.

For the case given in **Example 1** we obtain that the entropy corresponding to the positive and negative scores is equal to its maximum value: $E(+) = E(-) = 1$, while the entropy for the neutral scores is zero. The resulted average representation can be written as follows:

$$
\begin{aligned}
Avg(w) &= ((Avg(\cup_{i=1}^{2}\ sc_{+_i}), Avg(\cup_{i=1}^{2}\ sc_{0_i}),\\
&\qquad Avg(\cup_{i=1}^{2}\ sc_{-_i})); f^T, f^I, f^F)\\
&= ((0.5, 0, 0.5); f^T, f^I, f^F)\\
&= \left((0.5, 0, 0.5); \begin{pmatrix}0\\1\\0\end{pmatrix}, \begin{pmatrix}1\\0\\1\end{pmatrix}, \begin{pmatrix}0\\0\\0\end{pmatrix}\right)
\end{aligned} \tag{12}
$$

The representation given in Equation 12 tells more about *what the word is not* than about the type the word *is* as we consider **Example 1** only for showing why the simple unification of multiple representations by averaging their values is not always enough. As one can observe, the representation given in Equation 12 tells with maximum certainty that the word is not a neutral word. For the obtained positive and negative scores the indeterminacy membership functions have maximum values, illustrating in this way a maximum indeterminacy degree. This extreme case is quite rarely, being presented only for its theoretical purpose.

In the next section we apply the proposed method on a real data: a neutrosophic word in its all possible parts of speech. With this complex case we show that the method described in this article succeeds in merging multiple and diverse semantic word representations.

V. STUDY CASE

The word "good" appears in WordNet with three different parts of speech (noun, adjective, and adverb) and with many senses for each of its syntactic labels. We consider this word represents a perfect example for the neutrosophic word concept introduced in this paper and for this reason we dedicate the study case to it.

In Table 1 are given all the senses the word "good" can have, grouped upon the part-of-speech data. Each sense is given together with the sentiment scores extracted from SentiWordNet and also with its definition and some examples (as they are given in WordNet).

In Table 2 we gather all the data extracted from SentiWordNet: the word's parts of speech, the three facets given by the corresponding sentiment scores and the distributions among the senses of the sentiment scores. We also give the entropy measures for each word's facet in all the three parts of speech and also the average values of the sentiment scores.

By applying Algorithm 1 on the SentiWordNet scores of the word "good" we obtain the following representations (see also Table 2):

$$
\begin{aligned}
&Avg((good)_{ADJ})\\
&\quad = (\left(Avg(\cup_{i=1}^{21} sc_{+_i}), Avg(\cup_{i=1}^{21} sc_{0_i}), Avg(\cup_{i=1}^{21} sc_{-_i})\right);\\
&f^T_{ADJ}, f^I_{ADJ}, f^F_{ADJ})\\
&\quad = \left((0.61, 0.38, 0); f^T_{ADJ}, f^I_{ADJ}, f^F_{ADJ}\right)\\
&\quad = \left((0.61, 0.38, 0); \begin{pmatrix}0.59\\0.59\\1\end{pmatrix}, \begin{pmatrix}0.41\\0.41\\0\end{pmatrix}, \begin{pmatrix}0\\0\\0\end{pmatrix}\right)
\end{aligned} \tag{13}
$$

$$
\begin{aligned}
&Avg((good)_{NOUN})\\
&\quad = (\left(Avg(\cup_{i=1}^{4} sc_{+_i}), Avg(\cup_{i=1}^{4} sc_{0_i}), Avg(\cup_{i=1}^{4} sc_{-_i})\right);\\
&\quad f^T_{NOUN}, f^I_{NOUN}, f^F_{NOUN})\\
&\quad = \left((0.5, 0.5, 0); f^T_{ADJ}, f^I_{ADJ}, f^F_{ADJ}\right)\\
&\quad = \left((0.5, 0.5, 0); \begin{pmatrix}0.25\\0.25\\1\end{pmatrix}, \begin{pmatrix}0.75\\0.75\\0\end{pmatrix}, \begin{pmatrix}0\\0\\0\end{pmatrix}\right)
\end{aligned} \tag{14}
$$

TABLE 1. **The SentiWordNet data for the word "good".**

POS	Sentiment Scores	Example
Noun	(0.5, 0.5, 0)	benefit; "for your own good"; "what's the good of worrying?"
	(0.875, 0.125, 0)	moral excellence or admirableness; "there is much good to be found in people"
	(0.625, 0.375, 0)	that which is pleasing or valuable or useful; "weigh the good against the bad"; "among the highest goods of all are happiness and self-realization"
	(0, 1, 0)	articles of commerce
ADV	(0.375, 0.625, 0)	in a good or proper or satisfactory manner or to a high standard; "the baby can walk pretty good"
	(0, 1, 0)	completely and absolutely; "he was soundly defeated"; "we beat him good"
ADJ	(0.75, 0.25, 0)	having desirable or positive qualities especially those suitable for a thing specified; "good news from the hospital"; "a good report card"
	(0, 1, 0)	having the normally expected amount; "gives full measure"; "gives good measure"
	(1, 0, 0)	morally admirable
	(1, 0, 0)	deserving of esteem and respect; "ruined the family's good name"
	(0.625, 0.375, 0)	promoting or enhancing well-being; "the experience was good for her"
	(1, 0, 0)	agreeable or pleasing; "we all had a good time"; "good manners"
	(0.75, 0.25, 0)	of moral excellence; "a genuinely good person"
	(0.625, 0.375, 0)	having or showing knowledge and skill and aptitude; "a good mechanic"
	(0.625, 0.375, 0)	thorough; "had a good workout"; "gave the house a good cleaning"
	(0.5, 0.5, 0)	with or in a close or intimate relationship; "a good friend"
	(0.5, 0.5, 0)	financially sound; "a good investment"
	(0.375, 0.625, 0)	most suitable or right for a particular purpose; "a good time to plant tomatoes"
	(0.625, 0.375, 0)	resulting favorably; "it's a good thing that I wasn't there"; "it is good that you stayed"
	(0, 1, 0)	exerting force or influence; "a warranty good for two years"
	(0.625, 0.375, 0)	capable of pleasing; "good looks"
	(0.75, 0.25, 0)	appealing to the mind; "good music"
	(0.75, 0.25, 0)	in excellent physical condition; "good teeth"; "I still have one good leg"
	(0.875, 0.125, 0)	tending to promote physical well-being; beneficial to health; "a good night's sleep"
	(0.5, 0.5, 0)	not forged; "a good dollar bill"
	(0.375, 0.5, 0.125)	not left to spoil; "the meat is still good"
	(0.75, 0.25, 0)	generally admired; "good taste"

$$
\begin{aligned}
&Avg((good)_{ADV})\\
&\quad = \left(\left(Avg(\cup_{i=1}^{2} sc_{+_i}), Avg(\cup_{i=1}^{2} sc_{0_i}), Avg(\cup_{i=1}^{2} sc_{-_i})\right);\right.\\
&\qquad \left. f_{ADV}^{T}, f_{ADV}^{I}, f_{ADV}^{F}\right)\\
&\quad = \left((0.18, 0.81, 0); f_{ADV}^{T}, f_{ADV}^{I}, f_{ADV}^{F}\right)\\
&\quad = \left((0.18, 0.81, 0); \begin{pmatrix}0.59\\0.59\\1\end{pmatrix}, \begin{pmatrix}0.41\\0.41\\0\end{pmatrix}, \begin{pmatrix}0\\0\\0\end{pmatrix}\right) \qquad (15)
\end{aligned}
$$

These results can be interpreted as follows: no matter its part of speech realisation, we can precisely say that the word "good" is NOT a negative word. Two possible facets remain:

TABLE 2. **The semantic representations of the word "good". The negative scores, being not representative (the greatest value is 0.12), are omitted in the listing.**

	Adjective			Noun			Adverb		
	sc_+	sc_0	#senses	sc_+	sc_0	#senses	sc_+	sc_0	#senses
Sent. Scores	0.75	0.25	5	0.5	0.5	1			
	0	1	2				0.37	0.62	1
	1	0	3	0.87	0.12	1			
	0.62	0.37	5						
	0.5	0.5	3	0.62	0.37	1			
	0.37	0.62	1				0	1	1
	0.87	0.12	1	0	1	1			
	0.37	0.5	1						
Entropy	0.41	0.41		0.75	0.75		0	0	
Avg Scores	0.61	0.38		0.5	0.5		0.18	0.81	

the positive and the neutral. From the results obtained in Equations 13 and 15 we can conclude:

- the word "good" as adverb is a neutral word because its neutral average score is 0.81 with $f_{ADV}^{T}(0) = 0.59$, a value that exceeds by far its positive average score (0.18 with $f_{ADV}^{T}(+) = 0.59$)
- the word "good" as adjective is a positive word because its positive average score is 0.61 with $f_{ADJ}^{T}(+) = 0.59$ while the neutral average score is only 0.38, with $f_{ADJ}^{T}(0) = 0.59$

As a noun, we can consider it positive or neutral word, in both cases with high indeterminate degrees: $f_{NOUN}^{I}(+) = f_{NOUN}^{I}(0) = 0.75$, its average positive and neutral scores equal with 0.5 (see Equation 14). This is the case when additional filters taken from the context in which the word occurs must be applied in order to establish the word semantic facet.

VI. CONCLUSION AND FUTURE WORK

As pointed out in [31] each object has a corresponding (fuzzy, intuitionistic fuzzy, or neutrosophic) degree of appurtenance to a set of classification classes, with respect to its attributes' values.

In the present paper we propose a method that determines the appurtenance degrees of the semantic facets of a natural language word based on the entropy measure. We apply the proposed method on a real data: a polysemantic word in its all possible parts of speech. We prove with this complex study case that the method succeeds in merging multiple and diverse semantic word representations by filtering the "noises" through the entropy function values. The proposed method can be improved in case of high entropy values when additional filters must be applied by taken into account the word contextual data. The developing of these additional filters represents the trigger of our future studies.

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A Short Remark on Multipurpose Laser Therapy “Helios” in Ukraine and its Potential Application for Treatment of Neurology Disorders

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Abstract

Low-level laser therapy or sometimes called biophotomodulation has been known for long time for medicine applications. However, a truly multipurpose laser therapy method is very rarely available. Here we introduce a multipurpose laser therapy device in Ukraine, which is capable to take care a multitude of diseases. It is called “*Helios*”, by one of us (VK). We also give a case where a patient who suffered from Covid-19 has been treated successfully until he is recovered to healthy condition. In the last section we also discuss potential future application of *Helios* for other fields, i.e. neurology disorders.

Keywords: *Low-Level Laser Therapy; Helios; Neurology Disorders*

Introduction

From time to time, it is often found useful to come up with a new approach in medicine technologies, in order to seek a new insight from where we can develop and take further steps.

Low-level laser therapy or sometimes called biophotomodulation has been known for long time for medicine applications. However, a truly multipurpose laser therapy method is very rarely available. Here we introduce a multipurpose laser therapy device in Ukraine, which is capable to take care a multitude of diseases. It is called “*Helios*”. We also give a case where a patient suffers from Covid-19 has been treated successfully until he is recovered to healthy condition.

We will start with a simplified description of *Helios* as multipurpose laser treatment device.

Simplified description of *Helios* device

As one of us, Krasnoholovets told a physicist in New York, his laser treatment invention, “*Helios*”, can cure a huge number of illnesses. This apparatus is absolutely unique especially owing to its productivity - up to 500 people per 24 hours.

The following is a message by VK, the inventor of *Helios*, as his own words: “In a last couple of days I sent off about 70 messages to different clinics here in Ukraine - nobody replied. I called the director of the Institute of Epidemiology and Infectious Diseases and then sent a message. The director said that he printed my materials and gave to his colleagues, they would consider. But when? -- In my country all is doing slowly and usually people do not have interest to any new ideas, methods etc. Western Europe also gradually is approaching to a similar tendency. Here is a good example. My partner from Brussels (we have tried to develop different technologies) had a talk at the European Parliament about 15 years ago. He told that Europe should start a program to support small companies that wish to work in the energy sector (grants, cheap credits, developments of new technologies, etc.). Up to now no one company like those were created. 4 years ago my partner was invited to the USA - Florida and 4 other nearest states carried out a conference. At the conference he leaned that in these 5 states during last 10 years there were founded 40 thousands new companies that work in the energy sector.

These bad tendencies gradually touch the USA as well. The world changed dramatically after the collapse of the USSR and the appearance of the internet and mobile connection. People stopped to believe and trust because of the internet and its ocean of free information, spam and rogues. Besides, these new rules like the FDA and similar - they allow the prosperity of only big rich companies that can wait 2 - 4 years before they come to the market.

So, it seems here in Kyiv our team that has two *Helios* lasers, will not be allowed to patients who are infected with the coronaviruses…. The point is that this natural approach is able to very positively influence not only medicine but the whole people community. People have to look at themselves from the natural analytical point of view (not digital!) - all things are mutually connected.

There are a lot of different viruses and one should not concentrate only on the COVID-19. The *Helios* laser does not kill microorganism/virus at all. Our body is able to do this because its cleaners T-cells are universal. But when the body is attacked with infection, the body may not produce the needed value of T-cells owing to a number of reasons. The *Helios* laser helps the body to produce these T-cells (T-lymphocytes) and the T-cells being in a quantity several tens of times large than in the normal state will clean all the cells of the body like a vacuum cleaner. Additional positive functions of the *Helios*: red cells of the blood are also beginning to build up, and also the blood is saturated with oxygen (even in the unfunctional lungs) and the blood acquires properties of superfluidity.

Regarding the so called "Case Studies". Of course we have them in a quantity of about 150. Without such studies it was impossible to obtain a certificate that allows the apparatus to be used in medical practice. The *Helios* was certified in Ukraine, Belarus and Russia. I have here in my computer some copies of certificates and can send them to you (they in Ukrainian, but I can type the translation if the interest will be met from somebody with medical orientation)".

An example of *Helios* treatment for Guillain-Barre syndrome

One of his client was from Brussels and he was going home from a clinic last month - he was diagnosed with COVID-19 and had a 4 days a pneumonia and they used for 4 days the apparatus of artificial breathing. After several days of treatment with *Helios* laser method, he returned home fully recovered.

See the following figure 1-5. In Kyiv, VK treated an English businessman, Peter A. Wollsey (78 y.o.), who was barely alive when he arrived in Kyiv because he had just suffered from Guillain-Barré syndrome (a muscle paralysis of a viral or immune origin).

Figure 1: *Helios apparatus.*

Figure 2: *A patient was being treated with Helios. Professor Yuri Zabulonov, physicist, the inventor of Helios, was sitting at the Helios computer.*

Figure 3: *A patient under laser treatment.*

Figure 4: *A patient is being treated.*

Figure 5: *A side view of Helios treating a patient.*

Figure 6 shows a month later after a few sessions of the *Helios* laser therapy Peter A. Woolsey is water skiing.

Figure 6: *The patient from Belgium has been fully recovered, and he is able to skiing on water.*

Prospect of *Helios* application for neurology disorder treatment

Now, having discussed a bit on two case examples: patients with Covid-19 and Guillain Barre syndrome, in this section we will discuss potential future application in treatment of neurology disorders.

What is photobiomodulation (PBM)?

Photobiomodulation (PBM) is an innovative way to stimulate neuronal activity and improve neurological and psychological conditions. This term describes the use of Red and Near Infrared light to relieve inflammation and pain and tissue death. The neural tissues are exposed to Low Flow Light (LFL) with wavelengths that range from 60 to 100 nanometers (nm), depending on the method of treatment to be used [1].

A pilot study on the use of Photobiomodulation has been reported in [2] and also [3-5].

In reference [2], the authors report which can be paraphrased as follows: "the apparent upgrades in regular day to day existence. 29 Italian patients experienced NIR incitement treatment for 1-month and were tried when this incitement period so as to survey whether there will be a distinction in their impression of subjective disappointments that happen in regular daily existence dependent on the Cognitive Self-Assessment Questionnaire. In spite of the fact that the example is little, the information gathered show that there is an improvement in the apparent personal satisfaction in each neurotic gathering considered".

In reference [3], the authors report deep tissue laser therapy treatment which can be paraphrased as follows: "The impacts of profound tissue laser treatment (DTLT) were surveyed in a randomized, twofold veiled, trick controlled, interventional preliminary. Forty members were randomized (1:1) to get either DTLT or hoax laser treatment (SLT). Notwithstanding the standard-of-care treatment, members got either DTLT or SLT twice week by week for 4weeks and afterward once week by week for 8weeks (a 12-week mediation period). The two medicines were indistinguishable, then again, actually laser discharge was handicapped during SLT. Appraisals for torment, usefulness, serum levels of fiery biomarkers, and personal satisfaction (QOL) were performed at gauge and after the 12-week intercession period. The outcomes from the two medicines were looked at utilizing ANOVA in a pre-test-post-test structure".

And so on….

All in all, these results seem to suggest that there is bright future for potential treatment of various neurologic disorder using laser therapy. See also [6,7].

Concluding Remark

Despite a kind of theoretical work is quite in lacking, what we describe above is hopefully quite stimulating for future investigation. We also discuss some results which seem to suggest that there is bright future for potential treatment of various neurologic disorder using laser therapy.

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Improved, extended, and total impact factor of a journal

Florentin Smarandache

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Abstract In this short paper we recall the (Garfield) Impact Factor of a journal, we improve and extend it, and eventually present the Total Impact Factor that reflects the most accurate impact factor.

Key words Impact factor, Journal impact factor, Garfield impact factor, improved impact factor, extended impact factor, total impact factor.

1 Introduction

The Impact Factor (IF) or Journal Impact Factor (JIF), that is used today, was proposed by Eugene Eli Garfield (1925 –2017), an American linguist and businessman, the founder of the Institute of Scientific Information (ISI), Science Citation Index (SCI), and especially Journal Citation Reports (JCR). Among others the Impact Factor is computed since 1975 only for the journals registered in the database of the Journal Citation Reports (see [1]). We call it the Garfield Impact Factor (GIF) to distinguish it from the three new types of impact factors that we propose now, in order to improve, extend, and totalize the impact factors' formulas for a better accuracy of the citations of articles published in a specified journal.

2 Garfield impact factor

Let us consider a journal J that started in the year Y_1. We want to compute its impact factor in the year Y_2, where $Y_1 < Y_2$, and the calculation is done in the year $Y_2 + 1$.
The Garfield IF of the journal J for the year Y_2 is defined as follows:

$$IF_{Y_2}^{\text{Garfield}}(J) = \frac{C(Y_2,\ Y_2-1) + C(Y_2,\ Y_2-2)}{P(Y_2-1) + P(Y_2-2)}, \tag{2.1}$$

where $C(Y_2,\ Y_2-1)$ means the number of citations during the year Y_2 of the said journal's published articles during the previous year $Y_2 - 1$; $C(Y_2,\ Y_2-2)$ is similarly the number of citations during the year Y_2 of the journal's published articles during the past two years, i.e., $Y_2 - 2$ and $Y_2 - 1$. $P(Y_2-1)$ and $P(Y_2-2)$ represent the number of the journal's published articles during the years $Y_2 - 1$, and $Y_2 - 2$ respectively. $IF_{Y_2}^{\text{Garfield}}(J)$ is calculated for the next year $Y_2 + 1$.

3 Flaws of the Garfield IF

We list the following flaws of the Garfield impact factor:

a) The number of citations of the journal's articles published in the year Y_2 and cited in the same year Y_2 are missed.

b) The journal's published articles taken into consideration are only for the previous two years $Y_2 - 1$ and $Y_2 - 2$, which is superficial.

4 The improved impact factor

The case **a)** is always omitted by IF^{Garfield} that never takes into consideration the citations in the same year in which the articles were published.
An improved and more accurate IF^{Garfield} is:

$$IF_{Y_2}^{\text{Improved}}(J) = \frac{C(Y_2,\ Y_2) + C(Y_2,\ Y_2 - 1) + C(Y_2,\ Y_2 - 2)}{P(Y_2) + P(Y_2 - 1) + P(Y_2 - 2)}, \tag{4.1}$$

by including the citations during year Y_2 of the journal's papers published during the year Y_2. This is, of course, computed in the year $Y_2 + 1$.

5 The extended impact factor

The case **b)** shows the incompleteness of the Garfield IF which we remove by defining the Extended Impact Factor as follows:

$$IF_{Y_2}^{\text{Extended}}(J) = \frac{\sum_{k=Y_1}^{Y_2} C(Y_2,\ k)}{\sum_{k=Y_1}^{Y_2} P(k)}, \tag{5.1}$$

where $C(Y_2, k)$ is the number of citations during the year Y_2 of the journal's published articles during the year k; and $P(k)$ is the number of the journal's published articles during the year k; of course, $k \in \{Y_1, Y_1 + 1,\ Y_1 + 2, \ldots, Y_2\}$.

5.1 Distinctions between the extended impact factor and the Garfield impact factor

The main distinctions with respect to the Garfield Impact Factor are the following:

- IF^{Extended} shows all the citations during the year Y_2 of the journal's all published articles since the starting of the year Y_1, while, IF^{Garfield} shows the citations during the year Y_2 of only previous two years' published articles, therefore IF^{Garfield} is incomplete;
- IF^{Extended} also includes the citations during the year Y_2 of the journal's published articles in the same year Y_2, while, IF^{Garfield} misses it, so IF^{Garfield} is less accurate.

6 The total impact factor

Now we define the best and the most accurate and complete or exact impact factor, i.e, the *Total Impact Factor*, as defined below:

$$IF_{Y_2}^{\text{Total}}(J) = \frac{\sum_{k=Y_1}^{Y_2} C(k,\ [Y_1,\ \ k])}{\sum_{k=Y_1}^{Y_2} P(k)}, \tag{6.1}$$

where $C(k,\ [Y_1,\ k])$ is the number of citations during the year k of the journal's all the published articles during the years $Y_1, Y_1 + 1, \ldots, k$ altogether, where $Y_1 \le k \le Y_2$, and $[Y_1, k] = \{Y_1, Y_1 + 1, \ldots, k\}$; and $P(k)$ is the number of the journal's articles published during the year k.

7 Accuracy relationship of order

Let us consider the relationship of order " $>_a$ ", that means "better accuracy".
Then we have:

$$IF^{\text{Total}} >_a IF^{\text{Extended}} >_a IF^{\text{Improved}} >_a IF^{\text{Garfield}}.$$

8 Numerical example

We present an illustrative example in Table 1.

We read this table on columns, for example:

in the year 2015 the journal (J) has published 20 articles; these articles published in the year 2015 got:

6 citations in the year 2015;

15 citations in the year 2016;

4 citations in the year 2017;

no citations in the year 2018;

and 9 citations in the year 2019;

then, the total number of citations of the articles published in the year 2015 in the journal J is 6+15+4+0+9=34;

and so on;

in the year 2019, the journal (J) published 40 articles, and they got 90 citations in the same year 2019.

Let's use all four impact factor formulas to compute the journal's impact factors for year 2019 (that is computing in the year 2020).

1. Garfield Impact Factor for year 2019:
$$IF_{2019}^{\text{Garfield}}(J) = \frac{C(2019,\ 2018) + C(2019,\ 2017)}{P(2018) + P(2017)} = \frac{16+55}{45+50} = \frac{71}{95} \simeq 0.747.$$

2. Improved Impact Factor for year 2019:
$$IF_{2019}^{\text{Improved}}(J) = \frac{C(2019,\ 2019) + C(2019,\ 2018) + C(2019,\ 2017)}{P(2019) + P(2018) + P(2017)}$$
$$= \frac{90+16+55}{40+45+50} = \frac{161}{135} \simeq 1.193.$$

3. Extended Impact Factor for year 2019:
$$IF_{2019}^{\text{Extended}}(J) = \frac{C(2019,\ 2015) + C(2019,\ 2016) + C(2019,\ 2017) + C(2019,\ 2018) + C(2019,\ 2019)}{P(2015) + P(2016) + P(2017) + P(2018) + P(2019)}$$
$$= \frac{9+11+55+16+90}{20+40+50+45+40} = \frac{181}{195} \simeq 0.928.$$

4. Total Impact Factor for year 2019:
$$IF_{2019}^{\text{Total}}(J) = \frac{34+38+135+28+90}{20+40+50+45+40} = \frac{325}{195} \simeq 1.667.$$

Therefore, according to the accuracy relationship of order $>_a$ we have:

$$1.667 >_a 0.928 >_a 1.1928 >_a 0.747.$$

Whence, the exact (correct, most accurate) impact factor of journal (J) is equal to 1.667.

Table 1: Illustrative table to show the comparative study of the proposed impact factors.

Example						*Journal (J)*									
Year of Publication	2015					2016				2017			2018		2019
Number of published articles	20					40				50			45		40
Year of citations	2015	2016	2017	2018	2019	2016	2017	2018	2019	2017	2018	2019	2018	2019	2019
Number of citations per year	6	15	4	0	9	19	0	8	11	10	70	55	12	16	90
Total number of citations	34					38				135			28		90

9 Conclusion

We have defined for the first time three new types of impact factors of a journal and we designed an accuracy relationship of order. On a numerical example each type of impact factor was computed. Upon each impact factor's formula we clearly have: The Total Impact Factor is more accurate than the Extended Impact Factor, which is more accurate than the Improved Impact Factor, which, in turn is more accurate than the Garfield Impact Factor.

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A Survey on Deep Transfer Learning to Edge Computing for Mitigating the COVID-19 Pandemic

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A B S T R A C T

Global Health sometimes faces pandemics as are currently facing COVID-19 disease. The spreading and infection factors of this disease are very high. A huge number of people from most of the countries are infected within six months from its first report of appearance and it keeps spreading. The required systems are not ready up to some stages for any pandemic; therefore, mitigation with existing capacity becomes necessary. On the other hand, modern-era largely depends on Artificial Intelligence(AI) including Data Science; and Deep Learning(DL) is one of the current flag-bearer of these techniques. It could use to mitigate COVID-19 like pandemics in terms of stop spread, diagnosis of the disease, drug & vaccine discovery, treatment, patient care, and many more. But this DL requires large datasets as well as powerful computing resources. A shortage of reliable datasets of a running pandemic is a common phenomenon. So, Deep Transfer Learning(DTL) would be effective as it learns from one task and could work on another task. In addition, Edge Devices(ED) such as IoT, Webcam, Drone, Intelligent Medical Equipment, Robot, etc. are very useful in a pandemic situation. These types of equipment make the infrastructures sophisticated and automated which helps to cope with an outbreak. But these are equipped with low computing resources, so, applying DL is also a bit challenging; therefore, DTL also would be effective there. This article scholarly studies the potentiality and challenges of these issues. It has described relevant technical backgrounds and reviews of the related recent state-of-the-art. This article also draws a pipeline of DTL over Edge Computing as a future scope to assist the mitigation of any pandemic.

1. Introduction

The COVID-19 is a disease caused by a novel coronavirus called 'SARS-CoV-2'. This virus is transferable from human to human and it's spreading, and infection factors are very high [1,2]. Almost ten million people are infected and over 500 thousands are died within just six months from it's originating, and it is increasing steadily[1]. The World Health Organization(WHO) has declared it a pandemic [3,4]. But this is not the only pandemic human civilization is facing, there are many outbreaks had come in the past or it may come in the future [5,6]. The appropriate drugs, vaccines, infrastructure, etc. are not available up to some stages of any outbreaks. Therefore, mitigate these types of diseases with existing capacity becomes most important in those stages [7,8]. Many researchers from all over the world trying hard to develop such kind of techniques to cope with such challenges [9,10,131].

Modern-era largely depends on Artificial Intelligence(AI) including Data Science; and Deep Learning(DL) is one of the current flag-bearer of these techniques [11]. Therefore, these techniques could also assist to mitigate COVID-19 like pandemics in terms of stop spread, diagnosis of the disease, drug & vaccine discovery, treatment, patient care, and many more [12,13,132]. But to trained this DL, large datasets as well as powerful computing resources are required. For a new pandemic, data insufficiency and it's variation over different geographic regions is a huge problem, so here, Deep Transfer Learning (DTL) would be effective as it learns from one task and could apply in another task after required fine-tuning [14]. On the other hand edge devices such as IoT, Webcam, Drone, Intelligent Medical Equipment, Robot, etc. are very useful in any pandemic situation. These types of equipment make the infrastructures sophisticated and automated which helps to cope with an outbreaks [15]. Though, such devices are equipped with low computing resources which represent the main challenges of Edge Computing(EC) [16]. As a way to overcome this challenge, transfer learning could be a possible way to consolidate the needed computational power and facilitate more efficient EC. Therefore, DTL in edge devices as an EC could be smart techniques to mitigate a new pandemic [17]. This survey article has tried to report all these issues scholarly as potentialities and challenges with relevant technical backgrounds. Here, we also proposed a possible pipeline architecture for future scopes to brings DTL over EC to assists mitigation in any outbreaks.

1.1. Contributions of this Article

Some highlights of the contributions of this article are as follows:

- Presented a systematic study of Deep Learning(DL), Deep Transfer Learning(DTL) and Edge Computing(EC) to mitigate COVID-19.
- Surveyed on existing DL, DTL, EC, and Dataset to mitigate pandemics with potentialities and challenges.
- Drawn a precedent pipeline model of DTL over EC for a future scope to mitigate any outbreaks.
- Given brief analyses and challenges wherever relevant in perspective of COVID-19.

1.2. Organization of the Article

Starting from the introduction in Section 1, the remainder of the article organized follows. Section 2 for technical background whereas review of generic state-of-the-art of DTL in EC in Section 3. Existing computing(DL, DTL, EC & Dataset) to mitigate pandemic in Section 4. A proposed pipeline of DTL in EC to mitigate pandemics in Section 5. Finally, conclusion in Section 6.

2. Technical Backgrounds

The main focus of this article is how DL, DTL, EC, and it's associate could assist to mitigate any pandemics. The possible roles and challenges of these techniques in a pandemic, especially for COVID-19, are mentioned in Section 4. This section has tried to bring an overview and general progress of DL, DTL, and EC in the following three subsections.

2.1. Deep Learning(DL)

Deep learning (DL) (also known as hierarchical learning or deep structured learning) is one of the great inventions for modern-era of Artificial Intelligence (AI) [11]. Until the decade '90s, classical machine learning techniques were largely used for making inferences on data and prediction. Nevertheless it had several drawbacks such as depend on handcrafted features, bounded by human-level accuracy, etc [18]. But in case DL, handcrafted feature engineering is not required rather features are extracted from data during training. In addition, DL can make more accurate classifications and predictions with the help of innovative algorithms, computing power of modern machines, and the availability of Big Datasets [19]. Nowadays, DL methods have been successfully applied for several AI-based medical applications such as Magnetic Resonance Imaging (MRI) images analysis for cancer and diabetes diagnoses, conjunction with biometric characteristics, etc [20].

DL is a kind of learning algorithm or model under the umbrella of AI which is based on Artificial Neural Networks(ANN) [21]. These models are trained using dataset through backpropagation algorithm [22] and a suitable optimizer method [23]. The inherent capacities of such DL model such massive parallelism, non-linearity, and capabilities of feature extraction have made them powerful and widely used [19]. There are several variety of DL algorithm such as Convolutional Neural Networks(CNN) [24,25], Recurrent Neural Networks (RNN) [26], Long Short Term Memory(LSTM) [27], GAN [28], etc. After success of a CNN-based model, called AlexNet [29], many deep learning model has proposed such ZFNet [30], VGGNet [31], GooglNet [32], ResNet [33], DenseNet [34], etc specially for computer vision tasks [35]. In Fig. 1 we try to illustrate a typical methodology of a DL based screening system, where the system uses a DL algorithm (CNN) to predict whether the X-ray images of suspected patient's lung is normal or having viral pneumonia or COVID-19 pneumonia.

In the time of the COVID-19 crisis, when the numbers of infected patients are at a time very high and the disease is still spreading, many research groups are using the DL techniques for screening COVID-19 patients by detection fever temperature, viral and COVID-19 pneumonia, etc. In addition, DL is using or could be used for other purposes such as patient care, detection systematic social distancing violation, etc [13]. As for reference, S. Wang et.al used a CNN based DL for screening COVID-19 patients with an accuracy, sensitivity, and specificity of 89.5%, 87%, and 88% respectively by using their computed tomography (CT) images [36]. Similarly, in another study [37] L. Wang et.al used chest X-ray images for a screening of COVID-19 cases with 83.5% accuracy. The description of such works is in Section 4.1.1.

2.2. Deep Transfer Learning(DTL)

Transfer Learning is a technique that effectively uses knowledge of an already learned model to solve another new task (possibly related or little related) with require of minimal re-training or fine-tuning [38,39]. Since DL requires a massive training data compared to traditional machine learning methods. So, the requirement of a large amount of labeled data is a big problem in solving some critical domain-specific tasks, specifically the applications for the medical domain, where the making of large-scale, high-quality annotated medical datasets is very complex, and expensive [40]. In addition, the usual DL model requires large computing power such as GPU enabled sever, although researchers are trying hard to optimizing it [41,42]. Therefore, Deep Transfer Learning (DTL), a DL based Transfer Learning try to overcome this problem [43]. DTL significantly reduces the demand for training data and training time for a target domain-specific task by choosing a pre-trained model (trained on another large dataset of same target domain) for a fixed feature extractor [44] or for further fine-tuning [45].

Fig. 2 depicting the main steps of the methodology of a Deep Transfer Learning approach, where an untrained model is trained using a benchmark dataset (task-1) for feature extraction. Then that pre-trained model is further used to tackle a new challenge such as the task (task-2) of COVID-19 by just replacing only few last layers in the head of the architecture and required fine-tuning.

So far, many DTL models have been proposed [14]. A few recent are reported and discussed in the article. In a research study [43], Mingsheng Long et.al proposed a joint adaptation network. It learns a transfer network by aligning the joint distributions of multiple domain-specific layers across domains based on a joint maximum mean discrepancy. In another study [46], Yuqing Gao and Khalid M. Mosalam proposed a state-of-the-art transfer learning model based on VGG model [31]. They have used ImagNet [47] dataset for features extractor and their hand label construction images for fine-tuning. Abnormality classification in MR images through DTL proposed in a study [48]. The authors of that study also have used pre-trained ResNet34 model with fine-tuning. In a research practice [49], a DTL for diagnosing faults in target applications without labeling was proposed. Their framework used condition distribution adaptation. *Q*-TRANSFER [50], another DTL framework proposed by Trung V. Phan et.al. To mitigate the dataset insufficiency problem in the context of communication networking, a DTL-based reinforcement learning approach is used.

As the COVID-19 disease spread is terrifying all over the world, screening, quarantine, and providing appropriate treatment to COVID-19 patients has become the first priority in the current scenario. But the global standard diagnostic pathogenic laboratory testing is massive time consuming and more costly with significant false-negative results [51]. At the same time, tests are are hardly to be taken place in the common healthcare centres or hospital due to limited resources and places compared with the high volume of cases at one time. To combat this kind of situation, the researcher from this domain are trying hard to develop some possible DTL models to mitigate this challenges [52,53]. As for example M. Loey et.al in [52] use DTL along with the GAN model on their very limited, only 307 chest X-ray images to test COVID-19 disease

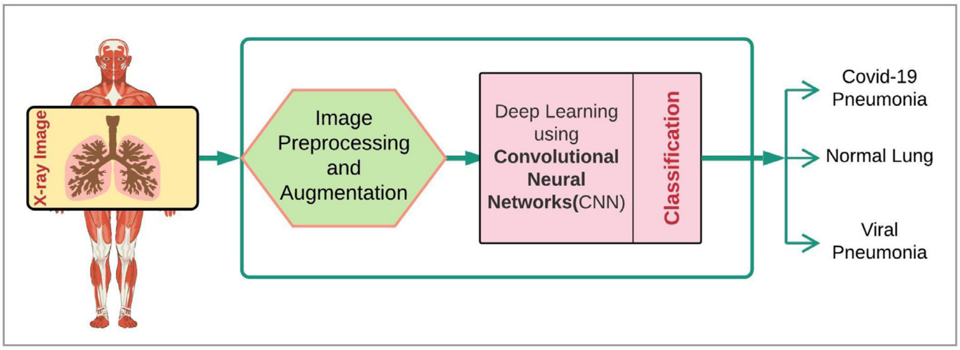

Fig. 1. A block diagram of a Deep learning-based screening system.

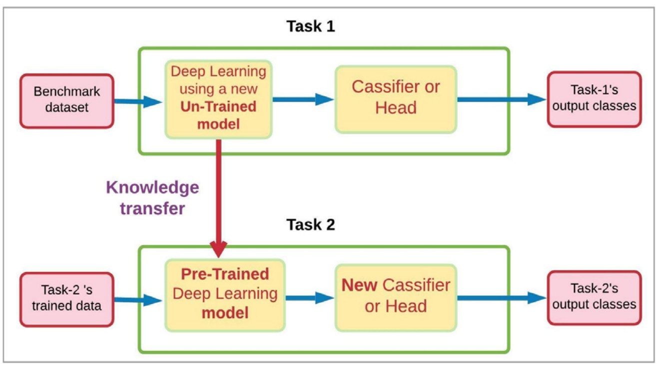

Fig. 2. Block diagram of an example of Deep Transfer Learning.

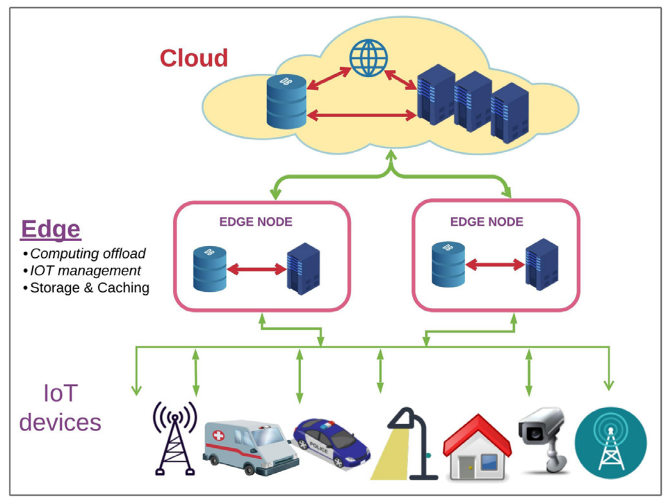

Fig. 3. Hierarchy of Edge, Fog, and Cloud Computing.

based patient chest X-ray. Here, they have three pre-trained state-of-the-art model namely Alexnet [29], GoogleNet [32], and ResNet18 [33]. Among these three pre-trained GoogleNet give the highest accuracy in their experimental studies.

2.3. Edge Computing(EC)

In the era of cloud computing, maximum IT depended organizations in the world rely on very few selected cloud providers for hosting and computation power. The user's data from millions of devices around the world is being delivered to some centralized cloud servers for processing, computation or storing. This data transformation always resulted in extra latency and extra bandwidth consumption [54]. The explosive proliferation of IoT devices along with the requirement of real-time computing power have forced to move the scenario of computing paradigm towards Edge Computing(EC). Therefore, instead of relying on doing all the work at a cloud, it focuses to start the computational process close to the IoT devices or Edge (near to the source of data) in order to reduce the utilized bandwidth and latency [55,56]. Sometime in EC, an additional nearby server called Fog is associated between the cloud and the Edge or IoT devices. It locally stores the copy of densely used data from the cloud and it provides additional functionality to IoT devices to analyze and process their data locally with real-time working capability. Hence only the relevant data from IoT devices is need to transferred to the cloud through the Fog server [57].

In Fig. 3, the hierarchy of a possible framework for EC association with Fog and Cloud computing is illustrated. The data are collected from various IoT devices are being pre-processed before sending by Edge to Fog server for the analysis and computation with the real-time speed (because of the minimal distance between Edge layer and IoT devices and the local database of Fog). While the cloud holds the central control system and it manages the whole database of the system. The database on the cloud is continuously uploaded by the Fog only when it has important data or information.

Although EC is not a new concept but it becomes popular in the last five years in the era of IoT [58,59]. Few recent different type of state-of-the-art of the EC are mentioned in this section as a way to familiarize the reader with the recent development with the era of EC and its potential benefits in mitigating COVID-19 as pandemic. EdgeIoT [60], a study of mobile EC proposed by X. Sun et.al. It is a SDN-based EC work with Fog Computing(FC) [61] to provide computational load locally. In a study [62], F. Wang et.al have proposed a joint offloading strategy of mobile EC and wireless power transfer. This scheme tried to address energy consumption, latency, and access point issues in IoT. In another study [63], Wei Ding et.al propose a field-programmable gate array-based depth-wise separable CNN accelerator to improve the system throughput and performance. They have used double-buffering-based memory channels to handle the data-flow between adjacent layers for mobile EC. On the other hand, G. Premsankar et.al in their case study [64] have discussed how efficient mobile gaming can run through EC. In a study [65], S. Wang et.al have proposed a mobile edge computing with an edge server placement strategy. In their multi-objective constraint optimization-based EC have tried reduced delay between a mobile user and an edge server. In-Edge AI [65], an integrate the deep reinforcement learning techniques and Federated Learning framework with mobile edge systems are proposed by X. Wang et.al. This framework intelligently utilizes the collaboration among devices and edge nodes to exchange the learning parameters for betterment. In another recent study [66], an integrated two key technologies, ETSI and 3GPP are introduced to enhanced slicing capabilities to the edge of the 5G network. In the case of COVID-19 like pandemics, discussion of the possible role of EC is done in Section 4.3.

3. Review of State-of-the-art

Although the whole article is referred and cited current relevant state-of-the-art wherever relevant, this section is dedicated to provide a review on some of the very generic recent state-of-the-art works related to transfer learning approaches over edge computing. As mentioned in Section 2.1, the progress of DL is very fast but when it comes to application in Edge or IoT devices then a huge gap is noticeable [67]. However, researchers are working hard to cope with the challenges, as results in many computing ideas, optimized model, as well as some computing ac- celerator devices, comes in picture [68,69]. Deep Transfer Learning as mentioned in Section 2.2 is one such area that is useful where the size of datasets is not sufficient [43]. This transfer learning is also useful where computing resources are not sufficient such as Edge or IoT devices [70]. Since edge computing becomes popular in the last few years, so, we restricted this review to the last five years with chronological order.

Lorenzo Valerio et.al have studied the trade-off between accuracy and traffic load of computing in edge-based on transfer learning [71]. They have suggested that sometimes the partial model needs to move across edge devices and data will stay at those edge devices and vice-versa. In a study [72], Tingting Hou et.al proposed a transfer learning approach in edge computing for proactive content caching. In their learning based cooperative caching technique they have used a greedy algorithm for solving the problem of cache content placement. On the other hand, Junjue Wang et.al proposed a model [73] for live video analytic through drone using edge computing. They have used a transfer learning approach to formulate a pre-trained model to apply a few aerial view image classification. In another study [74], Ragini Sharma et.al proposed a teacher(large networks) student(small network at edge) model using transfer learning. The applied different transfer learning techniques of teacher-student with considering accuracy and convergence speed.

Qiong Chen et.al used a multitask transfer learning in their work [75]. In their data-driven cooperative task allocation scheme, they have used the concepts of the Knapsack problem to prioritized the tasks before transferring them for use in another task. In a study [76], Wen Sun et.al suggested an edge-cloud framework. Here, pre-trained networks used in their framework that are trained in the cloud. In other work, Rih-Teng Wu et.al proposed an edge computing strategy for autonomous robots [77]. They have used CNN with pruning through the transfer learning technique. In their presented work, pr-trained VGG16 [31] and ResNet18 [33] are used for classification after fine-tuning. Cartel [78], a model of collaborative transfer learning approach for edge computing was proposed by Harshit Daga et.al. Here, they have created a model-sharing environment where a pre-trained model was adapted by each edge according to the needs. In a study [79], Yiqiang Chen et.al proposed a framework using Federated Transfer Learning for Wearable Healthcare (FedHealth). They have first performed data aggregation using federated learning and then created personalized models for each edge using transfer learning. OpenEI [80], an edge intelligence framework that was proposed by Xingzhou Zhang et.al. This framework with lightweight software equips with the edges as well as intelligent computing and data sharing capability.

In a research study [81], Changyang She et.al proposed a reliable low latency communication and edge computing system. They have adopted deep transfer learning in the architecture to fine-tune the pre-trained networks in non-stationary networks. This proposed work was designed for future 6G networks systems. On the other hand, Guangshun Li et.al proposed a task allocation load balancing strategy for edge computing [82]. They have used the concept of transfer learning from cloud to intermediate node to edge. In another study [83], Gary White and Siobhan Clarke have proposed a deep transfer learning-based edge computing for urban intelligent systems. They have also used VGG16 pretrained network at edge devices and experimented to classify Dog vs. Cat images. MobileDA [84], a domain adaption framework in edge computing was proposed by Jianfei Yang et.al. Here, a teacher network was trained in a server and transfer knowledge or feature to student networks was implemented at the edge side. Their model was evaluated and obtained promising results on an IoT-based WiFi gesture recognition scenario. Davy Preuveneers et.al proposed a resource and performance trade-off strategy for a smart environment [85]. They have used a transfer learning model for less training efforts in smart edge devices. In their study, multi-objective optimization also was utilized to optimize the trade-off between computing resources uses and performances.

4. Existing Computing (DL, DTL, EC & Dataset) to Mitigate Pandemic

As mentioned in Section 1, the appropriate drugs, vaccines, infrastructure are not ready up to some stages of any pandemic. Therefore, to cope with challenges existing knowledge, infrastructures, AI-based models could exploit to mitigate such pandemic. This section tried to bring four insights of the discussion topics and their roles in mitigating pandemics. Each of them is systematically discussed with potentiality with recent state-of-the-art and challenges.

4.1. Deep Learning Approaches to Mitigate Pandemic

4.1.1. Potentiality

As described in Section 2.1, Deep Learning(DL) can extract features directly from labeled data. In COVID-19 like pandemic data are new, so, handcrafted feature engineering might be difficult. But for DL, no feature engineering required, so that problem could be solved. The DL can assist in many ways to mitigate COVID-19 like pandemics along with other healthcare issues [86]. Some of them are Testing Sample Classification, Medical Image Understanding, Forecasting, etc [87]. Some recent DL based models have already proposed to cope with pandemics are listed and their main features are highlighted in Table 1.

This table brings some proposed peer-reviewed as well as few promising pre-print works. Table 1 has placed some recent works in upper rows.

4.1.2. Analysis and Challenges

From Table 1 it could be drawn one conclusion that the majority of the works are for assisting radiologists to diagnose diseases. Some of are mentioned forecasting, fake news alert, etc, but more critical parts of this pandemic maybe are addressed by this DL approach. Successfully apply DL in COVID-19 or any running pandemic has three main challenges. The first one is a shortage of reliable datasets. As data collection and validation are a time-consuming process as well as privacy issues also there whereas a pandemic or epidemic comes suddenly. The second one is the variety of data of a pandemic virus. This COVID-19 virus 'SARS-CoV-2' has been mutating itself over different geographic regions, environments, and time [102,103]. Therefore, the pandemic dataset collected from one region may not be work to drawn inference on the pandemic of other regions. The third one high computational resources required for a DL-based model whereas to cope with an outbreak IoT or Edge Device (ED) are useful for many purposes [15]. Though these types of equipments have low computing resources.

In order to overcome such challenges, cleaver implementation of relevant AI strategies is required. For the first two challenges, DTL or few shot learning and GAN [28] could be a possible approach towards possible solutions. DTL has described in Section 4.2 whereas details about GAN are out of the scope of this article. The third challenge could be mitigated using Cloud Computing, Fog Computing, and Edge Computing [104]. However, for Cloud or even Fog Computing latency and data security & privacy could be a problem. Therefore, Edge Computing could be effective for the third challenge, which has described in Section 4.3.

Table 1
Recent DL based works to mitigate pandemics.

Reference of Proposed Works	Dedicated task of a Pandemic	Main Contributions
F. Ucar and D. Korkmaz [88]	Deep Bayes-SqueezeNet based diagnosis of COVID-19 from X-ray images.	• Develop an intelligent diagnosis system for COVID-19 using practical DL networks for medical image processing. • A new decision-making system for COVID-19 with the integration of conventional and state-of-the-art methods for chest X-ray images.
D. Das et al. [133]	Screening chest X-ray images to classify COVID-19 positive or negative using a CNN model.	• Proposed a CNN based model called Truncated Inception Net for the task. • Their works pointed out a good accuracy on different datasets to classify COVID-19 pneumonia, Normal Pneumonia, Tuberculosis, and healthy cases.
C. Butt et al. [89]	Screen coronavirus disease 2019 pneumonia.	• A study that compared multiple CNN models to classify CT samples with COVID-19, influenza viral pneumonia, and no-infection. • Shown an accuracy of 0.996 (95%%CI: 0.989-1.00) for COVID-19 vs Non-COVID-19 cases per CT studies, and calculated a sensitivity: 98.2% and specificity: 92.2%.
H. Mukherjee et al. [134]	A shallow CNN-based automatic COVID-19 cases detected using chest X-rays.	• Proposed a light-weight shallow CNN-tailored model that can detect COVID-19 positive cases in chest X-rays. • They considered different dataset including MERS, SARS, and pointed out a good accuracy on automatic detection of these cases. [135]
S. Hu et al. [90]	COVID-19 Infection Detection and Classification from CT Images.	• A weakly supervised DL for detecting and classifying COVID-19 infection from CT images. • Minimize the requirements of labeling of CT images.
T. Ozturka, et al. [91]	automated detection of COVID-19 cases using raw X-ray.	• DL based COVID-19 vs No-finding as well as COVID-19 vs No-finding vs. usual Pneumonia as binary and multi-classification model. • A combined of YOLO and DarkNet model [92] used as the backbone of the model to achieved a good accuracy.
E. Luz et al. [93]	COVID-19 Patterns Detection in X-ray Images.	• Identification of COVID-19 disease with a model. • A resource efficient model with overall accuracy of 91.4%, COVID-19, sensitivity of 90% and positive prediction of 100% in the dataset from [94].
M. Zhou et al. [95]	Differentiating novel coronavirus and influenza pneumonias.	• An early diagnosis tool that works on chest CT images to differentiate Coronovirus pneumonia and normal Influenza with transferability.
A. Lopez-Rincon et al. [96]	Identification of SARS-CoV-2 from Viral Genome Sequences.	• Interaction between viromics and DL model. • A DL-based model to develop an assisted detection tests for SARS-CoV-2.
O.Gozes et al. [97]	Automated Detection & Patient Monitoring using Deep Learning and CT Image Analysis.	• A model that utilizing 2D-3D DL for clinical understanding. • Proposed a systematic continuous monitoring of COVID-19 patients and their clinical data to make a statistical Corona score of each patient for monitoring their progress.
S. M. Ayyoubzadeh et al. [98]	Predict the incidence of COVID-19 in Iran.	• Data about COVID-19 were mined from the Google Trends website. • Linear regression and LSTM-based models were used to estimate positive COVID-19 cases.
L. li et al. [99]	Fully automatic framework to detect COVID-19 using CT images of chest.	• Developed a DL-based model, COVNet to detect COVID-19 by extracted visual features from chest CT exams. • The collected dataset consisted of 4356 chest CT exams from 3,322 patients from six hospitals.
L. Wang et al. [37]	Open source Chest X-Ray Image dataset and a deep CNN for Detection of COVID-19 Cases.	• Proposed a publicly available COVID-Net, a deep CNN for the detection of COVID-19 cases from CXR images. • COVIDx, an open access chest X-ray(CSR) dataset consisting 13,800 CSR images across 13,725 patient.
S. J. Fong [100]	A forecasting model of COVID-19.	• A Composite Monte-Carlo simulation forecasting model. • A case study of using above simulation through deep learning.
S. Chae [101]	Predicting infectious disease using DL and Big data.	• A study on DL and LSTM-based model over the ARIMA model to predict future infectious diseases. • The proposed model tried to improve existing surveillance systems to detect future infectious diseases.

4.2. Deep Transfer Learning to Mitigate Pandemic

4.2.1. Potentiality

Section 2.2 has described about Deep Transfer Learning (DTL) in general. In this sections, how DTL could help to mitigate COVID-19 like pandemics is described. As mentioned, sufficient datasets of COVID-19 or any running pandemic are difficult to develop in a short period of time. Therefore, to exploit the benefit of DL to cope with COVID-19 or other pandemics are a bit challenging. Therefore, the DTL could be effective in this case. As through DTL, a DL model could be trained using a large scale benchmark or available dataset and learned features could be used in the domain of COVID-19 [53]. Many researchers are trying hard to use this DTL in the domain COVID-19 for many purposes. We have tried to summarize in Table 2 some of the recent state-of-the-art along with their main contribution towards mitigation of pandemics. As the number of peer-reviewed work is limited as this pandemic is new, so this table also has listed some pre-print works, which have tried to introduce some of the contributions in mitigating this current pandemic.

4.2.2. Analysis and Challenges

The DTL does task adaption that is very necessary for analyzing, diagnosing as well as mitigating COVID-19 like pandemics. The number of studies is not many; in addition most of the existing studies and experiments on COVID-19, were applied for chest image analysis as reported in Table 2. Only a few among them are proposed for target drug interaction, cough sound classification, etc. Lots of work could be done to mitigate this pandemic such as Intensive Care Unit(ICU) Monitoring, Patient Care, Hygienic Practice Monitoring, Wearing Personal Protective Equipment(PPE) Monitoring, Monitoring Systematic Social Distancing, Automatic fever detection, rumor detection, economical and social impact, etc. Most of these works could be easier when AI is cooperating and forming such a model along with IoT or ED [13]. Some issues could be solved by EC as described in Section 4.3. Though a better system could be delivered when the most suitable algorithm applied on EC. One possibility of archiving this when DTL implemented alongside with EC which as conceptually describes in Section 5.

Table 2
Recent DTL based works to mitigate pandemics.

Reference of Proposed Works	Dedicated tasks of a pandemic	Main contributions
J. P. Cohen et al. [135]	A severity score prediction model for COVID-19 pneumonia for frontal chest X-ray images using beside tool.	• A DTL model that was pre-trained on large size non-COVID-19 chest X-ray datasets for predicting COVID-19 pneumonia. This study uses a pre-trained model predicts a geographic extent score of range 0-8 with 1.14 MAE and lung opacity score of range 0-6 with 0.78 MAE. • A COVID-19 chest image dataset from a public COVID-19 database were scored retrospectively by three experts.
S. Minaee et al. [105]	Predicting COVID-19 From Chest X-Ray Images.	• DTL methods on a subset of 2000 of 5000 radiograms was used to train four popular CNN, including ResNet18, ResNet50, SqueezeNet, and DenseNet-121, to identify COVID-19 disease. • Evaluated these trained models using remaining 3000 radiograms and achieved a sensitivity rate of 97%(5%), while a specificity rate of 90% (approx).
S. Basu et al. [106]	Screening COVID-19 using Chest X-Ray Images.	• A domain extension transfer learning with pre-trained deep CNN is tuned for classifying four classes: normal, other diseases, pneumonia, and Covid-19. • A 5-fold cross-validation has experimented and overall accuracy measured around 95.3% .
N. E. M Khalifa et al. [107]	An Experimental Case on a limited COVID-19 chest X-Ray dataset.	• A study on neutrosophic and deep transfer learning models on limited COVID-19 chest X-Ray dataset. • They first converted grayscale X-ray images into neutrosophic images then applied pre-trained Alexnet, Googlenet, and Restnet18 to classify four classes: COVID-19, Normal, bacterial, and virus Pneumonia.
B.R. Beck et al. [108]	Predicting commercially available antiviral drugs that may act on SARS-CoV-2.	• DTL-based drug-target interaction model called MT-DTI to recognize commercially available drugs that could act on SARS-CoV-2. • Proposed a list of antiviral drugs identified by this MT-DTI model.
A. Narin et al. [109]	Automatic Detection of COVID-19.	• Three different pre-trained CNN (ResNet50, InceptionV3, and Inception-ResNetV2)-based models for the detection of COVID-19 pneumonia infection using X-ray radiography. • Proposed that the pre-trained ResNet50 has given the best result among these three.
I.D. Apostolopoulos et al. [53]	Evaluation of state-of-the-art CNN architectures through TL over medical image classification for COVID-19.	• Suggested a DTL method with X-ray imaging may extract significant bio-markers related to the COVID-19 disease. • A dataset of 1427 X-ray images consisting of 224 images of Covid-19 disease, 700 images of common bacterial pneumonia, and 504 images of no infection. .
B. Subirana et al. [110]	New crowdsource AI approach to support health care dealing with COVID-19.	• Proposed a transfer learning works on recognition of cough sound records by phone as a diagnostic test for possible COVID-19 positive.
N.E.M. Khalifa et al. [111]	Detection COVID-19 using GAN and TL method.	• A combination of GAN and DTL models for enhancing testing accuracy. • Their ResNet18-based combined model achieved state-of-the-art accuracy in a chest x-ray dataset.

4.3. Edge Computing to Mitigate Pandemic

4.3.1. Potentiality

Edge or IoT devices-based sophisticated equipments such as smart medical equipment, webcam, drone, wearable sensors, etc. are very useful in a pandemic like situations [112]. As mentioned in Section 2.3, edge computing brings the computation to near edge devices. It reduces latency, security & privacy issue, etc. Therefore, this computing paradigm will be very effective to mitigate a pandemic situation [113]. The researchers from all over the world are trying hard to bring this along with other AI techniques to mitigate current COVID-19 pandemic [15,114]. So far only a limited number of studies have investigated the use of EC in obtain an efficient and effective mitigation system of COVID-19. This subsection tried to bring some potentiality and scopes which shall help to mitigate COVID-19 like pandemics. Table 3 has mentioned some EC based studies on COVID-19 and related healthcare.

4.3.2. Analysis and Challenges

The EC works on site, so, many benefits could draw from EC with IoT or ED. Nevertheless as mentioned IoT or ED has limited computing resources. Therefore, to get the benefit of modern AI algorithm such as DL it is still challenging. To cope with these challenges researchers from all over the world are working hard to propose many ideas [17,68,121]. But so far only a few studies on EC in pandemic are proposed in limited areas of application as mentioned in Table 3; this table also mentioned some non-pandemic but related works. Assisting many critical COVID-19 related tasks such as remote sensing-based COVID-19 patient monitoring, Hygienic practice monitoring, systematic social distancing monitoring in a crowded area, etc could be done through EC [13]. This article brings a conceptual model of EC with DTL in Section 5 as a future scope to cope with such challenges.

4.4. Dataset to Mitigate Pandemic

4.4.1. Potentiality

Data is the fuel of a modern computing. Whether it is medical field or retailer market, in every field data are the most precious things. Recent AI techniques are mostly follow data driven approaches [122,123]. DL or DTL based algorithms almost fully depend on the dataset. Therefore, to cope with a pandemic, data is one of the driving forces. For a pandemic as COVID-19, the dataset could be chest X-ray, CT images, pathological images, geographical region based spreading patterns, seasonal behavior of the virus, regional mortality rates, impact on the economy, etc. [124]. In Table 4 some available datasets that are related to COVID-19 like pandemics are mentioned with brief descriptions.

4.4.2. Analysis and Challenges

As mentioned data is the main driving force to which bring the knowledge but it not easily available. Specially COVID-19 or a sudden pandemic or epidemic, gathering data and arrange it in a knowledgeable form are not expected as an easy task. Although for COVID-19, many sectors, organizations are very active as a result many data sources are quickly oriented towards COVID-19 pandemic. Some data sources are listed in Table 5 where COVID-19, as well as other pandemic data are available, so, researchers may use them for many purposes. The main challenges are sufficient datasets especially machine-readable datasets in every affected sector are yet to be available. Therefore, that are the challenges for data-driven AL algorithms or models, hence existing studies on real data and analysis are few. Although some datasets mentioned in Table 4 but most of them are for clinical purposes. As said this novel coronavirus is behaving differently across geographic regions, different environments, etc. Therefore, data of one region may not be effective to enhance knowledge in other regions. Data privacy and security also

Table 3
Recent EC based works to mitigate pandemics.

Reference of Proposed Works	Dedicated task related to a pandemic or healthcare	Main contributions
A. Sufian et al. [13]	EC based model to stop spread COVID-19	• Proposed a method for EC based ICU, Critical Areas monitoring. • This proposed EC method uses DL and Computer Visionfor surveillance.
C. Hegde et al. [115]	An open-source EC for clinical screening system.	• Fever and Cyanosis detection using visible and far-infrared cameras in emergency departments. • This image segmentation-based EC uses open source hardwares.
A. A. Abdellatif et al. [116]	Data and application-specific energy-efficient smart health systems	• An optimizes medical data transmission from edge nodes to the healthcare provider with energy efficiency and quality-of-service. • Managing a heterogeneous wireless network through EC to provide fast emergency response.
A. H. Sudhro et al. [117]	QoS optimization in medical healthcare applications.	• A window-based Rate Control Algorithm to QoS in mobile EC. • A framework for Mobile EC based Medical Applications.
M. Chen et al. [118]	Smart Healthcare System.	• Edge cognitive computing-based smart healthcare mechanism to dynamic resource allocation in healthcare.
P. Pace et al. [119]	Efficient Applications for Healthcare Industry 4.0.	• Proposed BodyEdge, an architecture suited for human-centric applications in context of the emerging healthcare industry. • A tiny mobile client module with EC for better health service.
H. Zhang et al. [120]	Smart Hospitals Using Narrowband-IoT.	• An architecture to connect intelligent things in smart hospitals based on Narrowband IoT. • Smart hospital by connecting intelligent with low latency.

Table 4
Some Datasets of COVID-19 pandemic and related areas.

Name of dataset and Reference	Brief description
COVID-CT-Dataset [125].	• A publicly CT scan dataset consisting of 275 positives for COVID-19 cases.
COVID-19 X-ray image dataset with two different combinations for applying with DTL-based models of different experimental setup. [53]	• One dataset of 1427 X-ray images consisting of 224 images of Covid-19 positive, 700 images of common bacterial pneumonia, and 504 images of no infections. • Another dataset of 1442 X-ray images consisting 224 images of Covid-19 positive, 714 images of common bacterial pneumonia, and 504 images of no infections.
COVID-19 Image Data Collection [94].	• It is hosting of crowdsourcing images that currently contain 123 frontal X-rays images at reporting time.
Chest CT Images [99]	• A dataset consisted of 4356 chest CT exams images from 3,322 patients. • Data are collected from six hospitals of average age is 49 years, among them 1838 were male patients.
Coronavirus Twitter Dataset [126].	• A multilingual COVID-19 Twitter dataset that has been continuously collecting since Jan 22, 2020. • It consists online conversation about COVID-19 to track scientific misinformation, rumors, etc.
COVIDx CXR Dataset [37].	• This large dataset consisting of 13,800 images of chest radiography across 13,725 patients.
Epidemiological COVID-19 data [127].	• Individual-level data from municipal, provincial, and national health reports, as well as additional information from online reports. • All data are geo-coded including where available, including symptoms, key dates, and travel history.
H1N1 Fever Dataset [128].	• Two datasets collected at Narita International Airport during the H1N1 pandemic 2009. • The first dataset only 16 candidates and the second one is 1049 collected using infrared thermal scanners.
Registry data from the 1918-20 pandemic [129].	• A high-quality vital registration data with mortality for the 1918-20 pandemic from all countries.

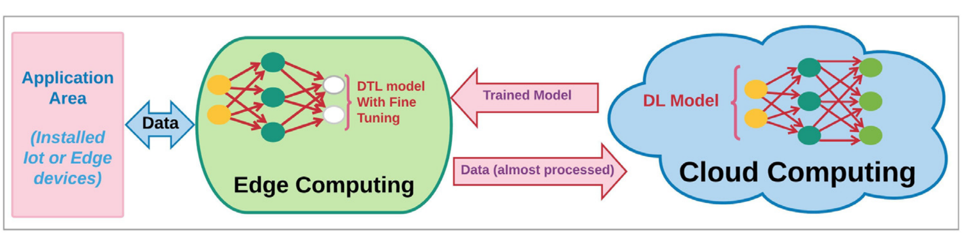

Fig. 4. Proposed pipeline for DTL in EC.

are considered ones of the big issues. To this reason this article suggesting transfer learning approaches to be used in developing models for mitigating COVID-19 like pandemics or epidemics.

5. A Precedent Pipeline of DTL over EC to Mitigate COVID-19

As mentioned in section 4.1.2, the DL has some limitations to cope with the challenges of a pandemic whereas Section 2.2 has described the task adaptability through DTL methods where data shortages are there. Section 2.3 mentioned the potentialities of EC where computing power is low. Therefore, the merging of these three computing models could be more effective in assisting the mitigation of pandemic situations. This combined model, that is, Deep Transfer Learning over Edge Computing(DTL-EC) will take the power of DL through DTL as well as would be applicable in critical sectors by EC to cope with a sudden pandemic. There are some studies that exist in DTL-EC as in [68] and some related work mentioned in Section 3. However, these works are still in general concept or their proposed methods are applicable only to some others application areas. As per literature studies, this idea has not been studied or experimented to mitigate COVID-19 pandemic. This section tried to present a precedent working pipeline of DTL-EC to assist mitigation of pandemic as well as any future pandemic or epidemic if arises.

Table 5
Some Data Sources of COVID-19 pandemic.

Sources and/or Reference	Brief description
World Health Organization(WHO) [3]	• WHO leading this battle by providing each and every possible data and information. • Most of the data are unstructured so it bit challenging to feed into an AI model.
Johns Hopkins University is in the forefront to provide COVID-19 dataset [130] through their portal: https://coronavirus.jhu.edu	• A machine-readable dataset that aggregates relevant data from country-level governmental, academic sources, journalistic, etc. • Some notable COVID-19 dataset are 'county-level time-series', 'healthcare system-related metrics', 'climate', 'transit scores', 'hospital', etc.
University of Oxford dataset regarding COVID-19 at their portal: https://www.bsg.ox.ac.uk/news/coronavirus-research-blavatnik-school.	• Oxford Covid-19 Government Response Tracker (OxCGRT), an index-based data indication which govt. taking what kind of policies are mentioned. • Several policy data of different govt. taken during pandemic including education policy and their impact.
European Union provides an open data portal: https://www.europeandataportal.eu/en/highlights/covid-19.	• Open data and COVID-19: provide many dataset medical data, spreading data, etc. • An Interactive map are provided and by clicking region specific dataset can be downloaded.
European Center for Disease Prevention and Control(ECDC): https://qap.ecdc.europa.eu/public/extensions/COVID-19/COVID-19.html.	• Many datasets about infectious diseases including COVID-19. • Enhanced surveillance dataset including daily update dataset, medical dataset, public health in communicable diseases.
Google: https://google.com/ covid19-map/.	• Different statistical, numeral data including number active cases, number of death, number of recovered. • Provide COVID-19 interactive map in addition dedicated dataset search engine that is also available.
GitHub: https://github.com/open-covid-19/data.	• An open repository where many datasets is stored. • Many research projects stores their data and mentioned links to their article, but they provide a link to see and access the COVID-19 dataset.
Kaggle: https://www.kaggle.com/c/covid19-global-forecasting-week-#.	• An online community of data scientists and machine learning practitioners • Forecasting dataset and other COVID-19, or pandemic dataset available.

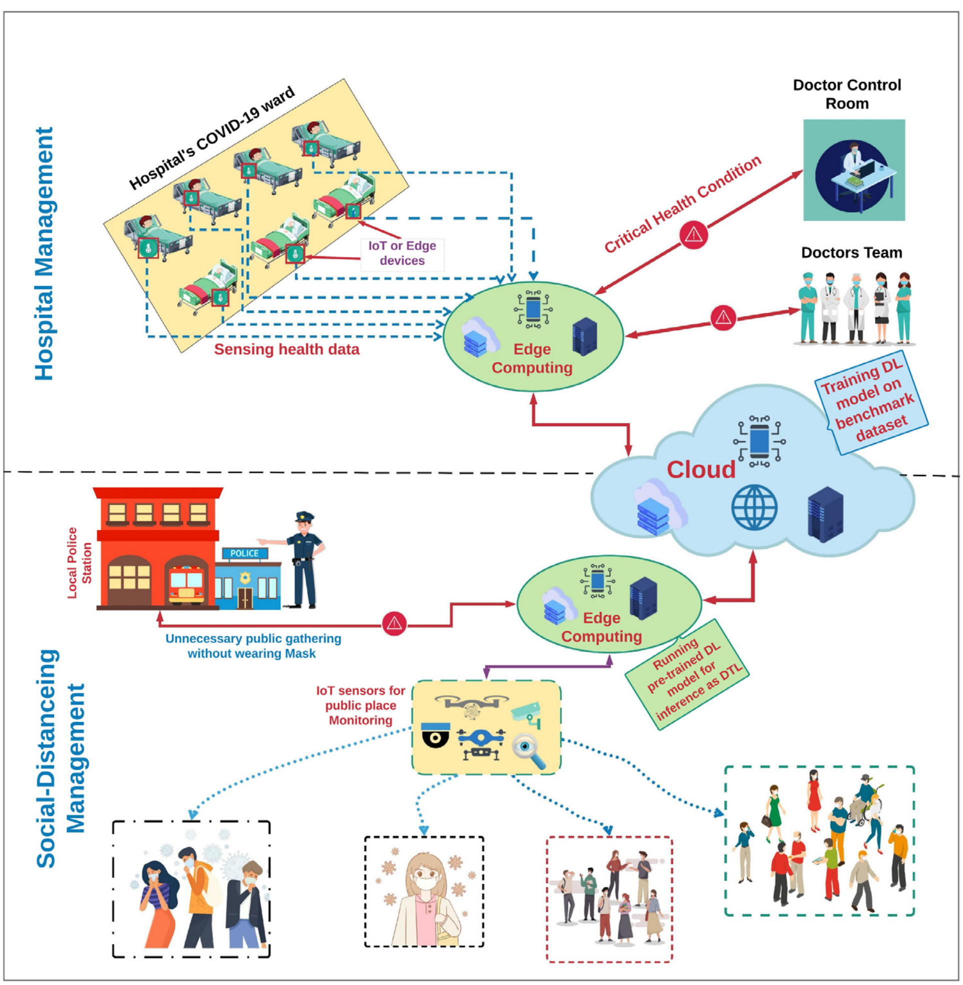

Fig. 5. Framework for Edge Computing.

5.1. Model Description

DTL-EC model could be helpful in the healthcare sector, quarantine center, or other critical areas where an outbreak may arise as well as it may be used for remote health monitoring, elderly care, etc. As in Fig. 4 edge or IoT devices that are set up in those areas may be embedded with EC, and then it could be connected with a cloud server. A state-of-the-art DL model shall train in GPU enabled cloud server by using a benchmark or related available dataset for features extraction. Then a pre-trained model(with extracted weight or features except for classification layer) shall push down to the edge devices. In edge devices required fine-tuning mechanism to be implemented into that model with some real data with ground truth. In this way, the model may ready to work in some critical areas where outbreaks are affected such as hospitals, crowded places, and many more.

In Fig. 5 a typical current COVID-19 outbreaks situation and possible working model are shown. This figure illustrating the proposed framework to tackle the COVID-19 related two situations by using DTL in EC in both COVID-19 patient care and management systematic social distancing. In the first scenario, we may use several healthcare sensors like blood pressure sensors, body temperature sensors, webcam, etc. to sense the data about the running health condition of each patient. Then all of the collected data would be sent to the EC layer where a pre-trained DL based model will be used to process the captured data and making an inference out of it. If the generated report is a critical health condition then an automatic system alert message will be sent with details to the hospital control room and also to all the doctors of associated team. In the second scenario, several public place monitoring sensors (like a drone, CCTV, traffic cameras, etc) could be used to detect unnecessary illegal crowd with or without wearing PPE with help from the DTL-EC-based model. If the model finds any such gathering then an automatic system alert message will be sent with all the details to the nearby authority.

5.2. Future Challenges

This model may be successfully deployed in some critical sectors such as hospitals, airports, markets, emergency service areas, and those areas which are the primary hotshots for spreading pandemics. It could also be used for remote health monitoring or elderly care. The model has to be work on real data to draw the inference. In order to make it successfully deployed, lots of collaborative works need to be done, which may need to adress many challenges. Some challenges could be such as: (i.) At first, IoT or Edge devices need to be connected with each other and a cloud server, hence an optimized sensor networking protocol shall be required. (ii.) EC through DTL need be implemented, for that appropriate pre-trained deep learning based model need be carefully selected after some studies. (iii.) For the transfer learning approach, only EC is not sufficient, while the adoption of EC-Fog-Cloud combined model would be more useful. A deep learning model shall be trained at a cloud server using a benchmark or available related dataset for feature extraction. After that pre-trained model will push down to the edge where limited re-training (or fine-tuning) shall be carried out to orient a few last layers for required inference. So, at least a small task oriented dataset needs to be created. Here, the Fog server could work as a cluster. (iv.) Security and privacy issues of data need to be addressed. This inquires much more attention by researchers in analyzing numerous vulnerabilities that associated with such outbreak due to rumours and fake news. Besides, the privacy of captured data from multiple sources (things in IoT or individuals) will open a new research direction for the near coming future. (v.) A new simulation model may be required for experimental studies. These are a few of the many challenges we can work for.

6. Conclusion

This article has tried to bring potentialities and challenges of Deep Transfer Learning, Edge Computing and their related issues to mitigate COVID-19 pandemic. It has also proposed a conceptual combined model with its scope and future challenges of working at critical sites and real data. As the running pandemic is very new, so, there is a limited number of peer-reviewed studies and experimental results. Therefore, this systematic study article also considered some pre-print studies which are tried to make some contributions in mitigating running pandemic. The running pandemic shall be mitigated but there will be a left over impact on global health, economics, education, etc, so mitigation of this pandemic is necessary to restrict further worsen. Every scientific community of the world needs to think seriously to get prepared to cope with such kind of crisis in a case similar outbreaks appear in the future. This article will definitely assist the research community; especially deep transfer learning and edge computing to work further in developing many tools and applications towards the mitigation of running pandemic or any future pandemic if that arises.

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A Review of the Evidences Showing that Certain Plant Medicines can be Useful for Novel Corona Virus Treatment

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Abstract

Considering growing concerns of the world's authorities on the spreading of novel corona virus (Covid-19), in this paper we review the evidences showing that certain plant medicines can be useful for novel corona virus treatment. Our argument is based on research finding that corona virus has viral envelope glycoproteins. In this regard, Mannose-binding lectins have been shown to be able to break down the shells that surround this class of viruses - which includes COVID-19 virus. Therefore, it seems useful to consider medicinal plants which have Mannose-binding lectins in order to break the glycoproteins envelope of the virus. This is an initial review on this subject, further research in direction as outlined here is recommended.

Keywords: *Phytochemistry; Phytomedicine; Plant Medicine; Mannose-Binding Lectins*

"It is better to light a candle than curse the darkness" - a Chinese proverb[1]

Introduction

The present paper reflects a growing concern on the spreading Covid-19 virus over more than 60 countries to date. The SARS-like coronavirus that appears to have originated in Wuhan, China has now infected thousands of people. According to Worldometers, as of March 4, 2020, there have been 93,191 confirmed cases around the world, 3,203 death cases and 50,984 recovered. The COVID-19 is affecting 80 countries and territories around the world and 1 international conveyance (the Diamond Princess cruise ship harbored in Yokohama, Japan)[2].

The virus was initially named novel coronavirus of 2019 (nCoV-2019 or 2019-nCoV) as of now. This has now been renamed as COVID-19. Sequencing of the virus has determined it to be 75 to 80 percent match to SARS-CoV and more than 85 percent similar to multiple coronaviruses found in bats. SARS stands for severe acute respiratory syndrome. It is also a coronavirus or CoV [1].

[1]Note: some authors attributed this quote to William Watkinson, 1907.

[2]Source: https://www.worldometers.info/coronavirus/

Despite this gloomy picture, there is also news reporting that all corona patients in 3 countries, including Vietnam and Nepal have recovered[3]. Therefore, there is ground to be hopeful that cure does exist.

Research over the past two decades shows that certain herbal medicines can fight the new Wuhan coronavirus contagion. Therefore, in this short review, we discuss some of the evidences showing that certain plant medicines can fight similar viral infections such as SARS, MERS and Ebola and why this can also apply to the Wuhan coronavirus, in particular by virtue of Mannose binding lectins [1].

A preliminary review of basics of respiratory medicine

It has been commonly accepted that Covid-19 belongs to respiratory diseases related to viral pneumonia. Studies show that there are glycoprotein shells covering the corona viruses, which make it difficult to break the virus.

First of all, let us review some basic facts from textbooks of respiratory medicine:

1. Oxford Handbook of Respiratory Medicine wrote regarding viral pneumonia, which can be rephrased as follows: "Viral URTIs are normal, yet commonly self-restricting, and are typically overseen in the network. Viral pneumonia is less normal yet is progressively genuine and typically requires hospitalization. Viral pneumonia in the immunocompetent is uncommon and regularly influences kids or the old; flu strains are the commonest cause in grown-ups. Studies recommend that infections are perceivable in 15 - 30% of patients hospitalized with pneumonia. Infections may cause genuine respiratory disease in the immunocompromised (especially patients with discouraged T-cell work, for example following organ transplantation). CMV is the commonest genuine viral pathogen that influences immunocompromised patients. Flu, parainfluenza, rSV, measles, and adenovirus may likewise cause pneumonia in the immunocompromised, despite the fact that analysis of these infections is troublesome and contamination is generally undetected" [9].

2. Shen Wei Lim in ERS Handbook of Respiratory Medicine wrote on influenza and pandemic which can rephrased as follows: "Flu is profoundly transmissible. Human-to human transmission happens through huge bead spread and direct contact with emissions (or fomites)…. Treatment: There are two principle classes of medication that are dynamic against flu. The M2 particle channel inhibitors, amantadine and rimantadine, are viable against flu A. Be that as it may, their utilization is thwarted by the quick rise of protection from these medications together with a high rate of symptoms. Antibiotics are generally prompted for patients with flu related pneumonia or patients with serious flu disease who are at high danger of creating auxiliary bacterial contaminations. The utilization of corticosteroids in serious flu can't be routinely supported dependent on current information; observational accomplice contemplates led during the 2009 H1N1 pandemic have detailed blended outcomes including expanded mischief" [10].

Therefore, it seems we can conclude that even though there are recommendations for such a viral pneumonia, there is no clear suggestion yet of how the best treatment in a pandemic situation.

Literature Review and Discussions

Review on glycoproteins envelope and mannose-binding lectins

Studies show that there are glycoprotein shells covering the corona viruses, which make it difficult to break the virus.

[3]Source: https://japantoday.com/category/world/all-16-of-vietnam%27s-coronavirus-sufferers-cured

Research tells us, which can be rephrased as follows: The viruses that infect human beings motive a massive global disease burden and produce immense challenge closer to healthcare system. Glycoproteins are one of the principal aspects of human pathogenic viruses. They have been tested to have vital role(s) in contamination and immunity. Concomitantly high titres of antibodies towards these antigenic viral glycoproteins have paved the way for development of novel diagnostics [3].

Stephen Harod Buhner in his book: Herbal Antivirals, argues that it is possible to find plants which can be used for treatment of viral respiratory infections. In page 36 in his book, he wrote, which can be paraphrased as follows: "Plants that decrease the other primary cytokines that the infection animates will likewise help diminish illness seriousness and forestall lung harm. I think the most significant are inhibitors of NF-κB (Chinese senega root, Chinese skullcap, ginger, houttuynia, kudzu, licorice, boneset, astragalus), IL-6 (kudzu, Chinese skullcap, isatis), IL-8 (cordyceps, isatis, Japanese knotweed), RANTES (licorice, isatis), MCP-1 (houttuynia), CXCL10 (boneset), CCL2 (boneset), the ERK pathway (kudzu, Chinese skullcap, cordyceps), the p38 pathway (Chinese skullcap, houttuynia, cordyceps) and the JNK pathway (Chinese skullcap, cordyceps etc)" [11].

Following Buhner's suggestion as mentioned above, in this paper we will also review a number of evidences showing that certain plant medicines can be useful for novel corona virus treatment. Our argument is based on research finding that corona virus has viral envelope glycoproteins. In this regard, Mannose-binding lectins have been shown to penetrate and break down the shells that surround this class of viruses - which includes COVID-19 virus. It is also known that MBL deficiency is responsible for weakened immune system, which may affect pneumonia etc [2].

As Mbae., *et al.* wrote, which can be paraphrased as follows: "Restorative plants have various helpful low sub-atomic weight phytochemicals and macromolecules, for example, polysaccharides and proteins. Lectins are glycan restricting proteins pervasive in the cell and the extracellular surface of every living creature. Plant parts contain lectins with assorted glycan restricting specificities" [16].

Studies have discovered that these Mannose-binding lectins spoil down the glycoprotein shells of the viruses noted above, inclusive of Ebola and SARS. A quantity of animal checks and human telephone laboratory exams have shown that these mannose-binding lectins are successful in halting replication of the virus [1].

Therefore, it seems worthy to consider medicinal plants which have Mannose-binding lectins, in order to break the glycoproteins envelope of the corona virus.

A short review of possible treatment of corona virus by three medicinal plants

Griffithsin red algae

According to Case Adams: Red algae Griffithsin has also verified to be antiviral towards HIV-1 (human immunodeficiency virus), HSV-2 (Herpes simplex virus), HCV (Hepatitis C) and the Ebola virus. What do these viruses have in common? Along with COVID-19, they all have glycoprotein shells around them [1].

Another mannose-binding lectin found to be antiviral against these viruses is the *Scytonema varium* red algae, also called Scytovirin. Another one was found in the *Nostoc ellipsosporum* algae species - called Cyanovirin-N [1].

Other than that, in 2019 France's Institut de Recherche et Développement has tested a number of different species and observed the *Ulva pertusa* algae species and *Oscillatoria agardhii* blue-green algae can be useful to halt the replication of these viruses [1].

Health store availability

Red algae is a supplement that can be purchased in fitness meals shops and online. Most of the commercial dietary supplements labeled pink algae make use of the Gigartina species of purple algae (such as *Gigartina skottsbergii*). This species has been tested towards HSV and HIV in laboratory testing, but not yet on CoVs to date [1].

Licorice root

Adams et al. have also published evidence that licorice root can fight SARS and MERS CoV infections. Studies have found that licorice root extracts were able to reduce SARS and MERS-CoV replication [1].

Besides, in a 2008 study the UK's Luton and Dunstable Hospital NHS Foundation Trust tested licorice root extracts towards a quantity of viruses, such as HIV and SARS. They found that the extract broke down the viral envelope and also boosted immune activity [1].

For mechanisms, researchers stated, which can be rephrased as follows: Mechanisms for antiviral endeavor of licorice root consist of reduced transport to the membrane and sialylation of hepatitis B virus surface antigen, discount of membrane fluidity leading to inhibition of fusion of the viral membrane of HIV 1 with the cell, induction of interferon gamma in T cells, inhibition of phosphorylating enzymes in vesicular stomatitis virus contamination and discount of viral latency [1].

Curcuma and other plant lectins

In this section, allow us to shortly mention *Curcuma zedoaria* rosc. A paper by R. Lobo et al. stated, which can be rephrased as follows: *Curcuma zedoaria* Rosc, additionally regarded as white turmeric, zedoaria or gajutsu, is a perennial rhizomatous herb that belongs to the Zingiberaceae family. The plant is indigenous to Bangladesh, Sri Lanka and India, and is additionally widely cultivated in China, Japan, Brazil, Nepal and Thailand. In India it is recognized by using its countless vernacular names, the most oftentimes used ones being Krachura (Sanskrit), Gandamatsi (Hindi) and Sutha (Bengali). It is used historically for the therapy of menstrual disorders, dyspepsia, vomiting and for cancer. Rural people use the rhizome for its rubefacient, carminative, expectorant, demulcent, diuretic and stimulant houses whilst the root is used in the remedy of flatulence, dyspepsia, cold, cough and fever [5].

Another study by Tipthara., *et al*. has an abstract which can be rephrased as follows: Mannose-binding lectin was once isolated from rhizomes of the medicinal plant *Curcuma zedoaria*. We used extraction with 20 mM phosphate buffer, ammonium sulfate precipitation, ion alternate chromatography on Q-Sepharose, gel filtration chromatography on Superdex 75 and reverse-phase HPLC. The purified lectin yielded a single band on SDS-PAGE that corresponded to a molecular mass of 13 kDa [4].

Therefore, the studies seem to suggest that medicinal plant *Curcuma zedoaria* rosc. may also be found useful for corona virus. Nonetheless, more research is recommended. See also related studies [6,7].

Beside *Curcuma zedoaria* rosc, a 2007 study from Belgium's University of Gent studied plant-derived mannose-binding lectins on SARS (severe acute respiratory syndrome) coronavirus and the feline infectious peritonitis virus (FIPV). The researchers studied known plant lectins from 33 different plants in the laboratory, using infected cells. The researchers wrote, which can be rephrased as follows: Of the 33 flora tested, 15 extracts inhibited the replication of both coronaviruses. Those antiviral lectins have been profitable in inhibiting the replication of the viruses [1].

The 15 coronavirus-inhibiting plants were:

- Amaryllis (*Hippeastrum* hybrid)
- Snowdrop (*Galanthus nivalis*)
- Daffodil (*Narcissus pseudonarcissus*)
- Red spider lily (*Lycoris radiate*)
- Leek (*Allium porrum*)
- Ramsons (*Allium ursinum*)
- Taro (*Colocasia esculenta*)
- Cymbidium orchid (*Cymbidium hybrid*)
- Twayblade (*Listera ovata*)
- Broad-leaved helleborine (*Epipactis helleborine*)
- Tulip (*Tulipa hybrid*)
- Black mulberry tree (*Morus nigra*)
- Stinging nettles (*Urtica dioica*)
- Tobacco plant (*Nicotiana tabacum*).

Last but not least, besides *Curcuma zedoaria* rosc, based on his preclinical study Prof. Chaerul Anwar Nidom from Indonesia stated that curcumin which is contained in various traditional herbs found in many places in Indonesia can also be helpful to protect human body against the cytokine storm which is triggered by corona virus [8].

For other references on Mannose binding lectins in certain plants, see for instance [14-17].

Concluding Remarks

We have reviewed a possible mechanism to break the corona virus envelope with lectins. This is just an early schematic paper, it would need more study to establish which the suggested medicinal plants lectins are the most beneficial for corona virus treatment.

In the last section, we reviewed three medicinal plants which have mannose-binding lectins, and therefore they may be found useful for corona virus treatment. Those three plants are: griffithsin red algae, licorice root, and *Curcuma zedoaria* Rosc. Nonetheless, further investigation is recommended especially to find out the mechanism of the lectins to break down the virus envelope.

This paper is just an initial review on this subject, further research in this direction as outlined here is recommended.

We hope that this short review article can be found useful for policy makers of health in reducing the effect of Covid-19 in many countries.

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Neutrosophic ratio-type estimators for estimating the population mean

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Abstract
All researches, under classical statistics, are based on determinate, crisp data to estimate the mean of the population when auxiliary information is available. Such estimates often are biased. The goal is to find the best estimates for the unknown value of the population mean with minimum mean square error (MSE). The neutrosophic statistics, generalization of classical statistics tackles vague, indeterminate, uncertain information. Thus, for the first time under neutrosophic statistics, to overcome the issues of estimation of the population mean of neutrosophic data, we have developed the neutrosophic ratio-type estimators for estimating the mean of the finite population utilizing auxiliary information. The neutrosophic observation is of the form $Z_N = Z_L + Z_U I_N$ where $I_N \in [I_L, I_U]$, $Z_N \in [Z_l, Z_u]$. The proposed estimators are very helpful to compute results when dealing with ambiguous, vague, and neutrosophic-type data. The results of these estimators are not single-valued but provide an interval form in which our population parameter may have more chance to lie. It increases the efficiency of the estimators, since we have an estimated interval that contains the unknown value of the population mean provided a minimum MSE. The efficiency of the proposed neutrosophic ratio-type estimators is also discussed using neutrosophic data of temperature and also by using simulation. A comparison is also conducted to illustrate the usefulness of Neutrosophic Ratio-type estimators over the classical estimators.

Keywords Neutrosophic · Classical statistics · Ratio estimators · Bias · Mean square error

Introduction

Data in classical statistics are known and formed by crisp numbers. Many authors worked on several estimators for estimating the mean of the finite population in the existence of auxiliary information under classical statistics. “The study suggested that in the presence of high correlation between the study variable and auxiliary variable, we get significantly low sampling error for ratio, instead of taking the study variable only and hence, we may need less sampling for ratio estimation method or the ratio estimation method reduces the sample size providing equal precision [13]”. A detailed discussion on ratio estimation and its properties and examples were present in one study ([14], pp. 150–186). Furthermore, one study discussed the applications of a ratio-type estimator for multivariate k-statistics [23]. More studies and various uses and types of ratio-type estimation techniques developed as time passed. The use of auxiliary information with the coefficient of variation was also studied [22, 28]. The known parameters or other known statistics were used as auxiliary variables by various researchers [18, 24, 25]. The transformations of the auxiliary variables were also studied [31]. The performance of ratio-type estimators was improved when using different types of auxiliary information [24]. One suggested an exponential-type of ratio estimation [8]; others try to improve the exponential-type ratio estimators [32]. One studied the estimation of mean by exponential ratio-type estimators in the presence of non-response [24]. A study proposed that their estimator using complete information is a better version of the exponential ratio-type estimator [19].

"Classical statistics deal with determined data when there is no uncertainty in measurements of the observations. Therefore, we need new methods to deal with the data which are not determined. The fuzzy logic is one solution to tackle data, where we might not have exact measurements of the variable under study. Fuzzy statistics are used to analyze the data having fuzzy, ambiguous, uncertain, or imprecise parameters/ observations, but it ignores the measure of indeterminacy. Whereas, neutrosophic logic is characterized as the generalization of fuzzy logic, and it allows to measure indeterminacy along with determinate part of the observations and used to analyze under vague/ uncertain observations [2, 3]."

Methods under fuzzy logic are being developed rapidly and used widely in the decision-making environment [1, 17]. Further advancement in the fuzzy sets is complex fuzzy sets, and its generalized form is a complex neutrosophic set [21]. A study provided a detailed flow chart of fuzzy sets and their generalizations, along with a discussion on some properties and operations, including interval-valued neutrosophic sets [20].

In decision-making problems, if the fuzzy set fails to handle uncertainty, then the neutrosophic set is a better alternative. Neutrosophic sets are classified into many types. One study presented a trapezoidal bipolar neutrosophic number and its classification for decision-making problems [11]. One study introduced generalized spherical fuzzy numbers and established a detailed analysis scheme and more related methods for multi-criteria group decision making (MCGDM) [16]. Another study suggested arithmetic and geometric operations under pentagonal neutrosophic numbers along with the application based on MCGDM in mobile communication [9]. The neutrosophic numbers are gaining much interest of the researchers as time passes, for instance, an MCGDM scheme was proposed under the cylindrical neutrosophic domain [10].

Thus, in problems when the data have some indeterminacy, neutrosophic statistics are used. Neutrosophic statistics is an extension of classical statistics, used when there is neutrosophy in data or a sample. When observations in the population or the sample are imprecise, indeterminate, and vague, then neutrosophic statistics are applied [29].

Neutrosophic data

Neutrosophic data refer to a data set that is indeterminate to some degree, and neutrosophic statistical methods are used to analyze such data. The sample size, in neutrosophic statistics, may not be known as the exact number [29]. Researchers discussed that neutrosophic statistics are very effective and suitable for applying them in the uncertainty system [2, 30]. In rock engineering, to study the scale effect and anisotropy of joint roughness coefficient, Neutrosophic numbers had been used, which results in a better and effective method to overcome the loss in information giving sufficient fitted functions [12]. New neutrosophic analysis of variance technique presented under neutrosophic data [3]. The area of neutrosophic interval statistics (NIS), neutrosophic applied statistics (NAS), and neutrosophic statistical quality control (NSQC) were developed by [4–7].

Research gap

All previous researches on survey sampling are on the type of data that are determined, certain, and clear. These methods provide a single crisp result, which may have chances of being wrong, over, or underestimated, which is a drawback sometimes. However, in many cases, data are of neutrosophic nature under some circumstances; this is the point, where Neutrosophic statistics is applied, and old classical methods failed. Data of neutrosophic nature are uncertain and ambiguous observations, non-clear arguments, and vague interval values. Thus, the information obtained from experiments or populations may behave as interval-valued neutrosophic numbers (INN). The actual observation, which is indeterminate at the time of collection, was believed to be a value that belongs to that interval. In real life, more indeterminate data are available than the determinate data. Therefore, more neutrosophic statistical techniques are required.

In life, many study variables are available for whom the collection of information is very expensive, especially when the information is ambiguous. Therefore, it will be risky and costly to compute the unknown true value of the population by the old classical methods for indeterminate data. When the study variable and auxiliary variables are of neutrosophic nature, there is no method available to solve the problem using ratio estimation. Thus, a neutrosophic ratio-type estimation method is proposed in this study.

After exhaustive research of the published studies, no research has been found in survey sampling for the ratio-type estimation methods to estimate the unknown population mean in the presence of auxiliary variables under neutrosophic data. This field of statistics is yet to be filled with promising articles. This study is the first step in this area.

Scope of the study

Neutrosophic Statistical analysis helps deal with the data containing a certain amount of indeterminacy or incomplete information. In addition, this method allows for inconsistent beliefs as well. Data collection through some tools might present some observations in a range of uncertain values with the chance of inclusion of an actual measurement in that range. In the case of indeterminacy, classical statistics failed to analyze data. Hence, neutrosophic statistics is applied under the uncertain environment, which is the alternative and generalization of Classical Statistics and more flexible. Considerable researches have been done so far in the field of survey sampling under the Neutrosophy, in which the ratio estimation is still fresh and requires a great deed of attention for the uncertain system of data. For example, if we take measurements of a machine's product (say produces nuts or bolts), it might manufacture items with minor measurement errors or manufacturing errors, and we can accept that product if it lies in the particular range of measurement. In these cases, if we use classical statistics providing a single-valued result will cause lots of loss by rejecting the items even these are usable. Thus, neutrosophic statistics can cover these problems by providing the best estimate of interval results with the least MSE.

Neutrosophic observation

Several types of neutrosophic observations, including quantitative neutrosophic data, were presented, which stated that a number might lie in the interval $[a, b]$ (unknown exactly). [30]. The interval value of neutrosophic numbers can be exhibit in many ways. We have taken neutrosophic interval values as $Z_N = Z_L + Z_U I_N$ with $I_N \epsilon [I_L, I_U]$. Thus, we used notation for our neutrosophic data, which are in the interval form $Z_N \epsilon [a, b]$, where 'a' is lower value and 'b' is the upper value of the neutrosophic data.

First, this study proposed several neutrosophic estimators for estimating the mean of the finite population in the presence of auxiliary information, which is very suitable to overcome the problem of sample indeterminacy.

Terminology

Consider a neutrosophic random sample of size $n_N \in [n_L, n_U]$, which is drawn from a finite population of 'N' units $(T_1, T_2, \ldots, T_N)$. Let $y_N(i)$ is the ith sample observation of our neutrosophic data, which is of the form $y_N(i) \in [y_L, y_U]$ and similarly for auxiliary variable $x_N(i) \in [x_L, x_U]$. Let $\bar{y}_N(i) \in [\bar{y}_L, \bar{y}_u]$ is our neutrosophic variable of interest, and $\bar{x}_N(i) \in [\bar{x}_L, \bar{x}_U]$ is our auxiliary neutrosophic variable which is correlated to our study variable $\bar{y}_N$. In addition, $Y_N \in [Y_L, Y_U]$ and $X_N \in [X_L, X_U]$ are the overall averages of the neutrosophic set of data. $C_{yN} \in [C_{yNL}, C_{yNU}]$ and $C_{xN} \in [C_{xNL}, C_{xNU}]$ are neutrosophic coefficients of variation for Y_N and X_N, respectively. ρ_{xyN} is the neutrosophic correlation between X_N and Y_N (neutrosophic variables). In addition, $\beta_{2(x)N} \in [\beta_{2(x)L}, \beta_{2(x)U}]$ is the neutrosophic coefficient of kurtosis for auxiliary variable X_N. $\bar{e}_{yN} \in [\bar{e}_{yL}, \bar{e}_{yU}]$ and $\bar{e}_{xN} \in [\bar{e}_{xL}, \bar{e}_{yU}]$ are the neutrosophic mean errors. These terms can be computed by the following relations. Similarly, $Bias(\bar{y}_N) \in [Bias_L, Bias_U]$ and MSE, $MSE(\bar{y}_N) \in [MSE_L, MSE_U]$ belong to the neutrosophic sets were also computed for the analysis: $\bar{e}_{yN}(i) = \bar{y}_N(i) - \bar{Y}_N; \bar{e}_{xN}(i) = \bar{x}_N(i) - \bar{X}_N;$

$$E(\bar{e}_{yN}) = E(\bar{e}_{xN}) = 0$$

$$\mathrm{E}\left(\bar{e}_{yN}^2\right) = \theta_N \bar{Y}_N^2 C_{yN}^2; \quad E(\overset{-2}{e}_{xN}) = \theta_N \bar{X}_N^2 C_{xN}^2$$

$$E(\bar{e}_{yN}\bar{e}_{xN}) = \theta_N \bar{X}_N \bar{Y}_N C_{xN} C_{yN} \rho_{xyN}$$

where $\bar{e}_{yN} \in \left[\bar{e}_{yL}, \bar{e}_{yU}\right]; \bar{e}_{xN} \in [\bar{e}_{xL}, \bar{e}_{xU}]; e_{yN}e_{xN} \in \left[\bar{e}_{xL}\bar{e}_{yL}, \bar{e}_{xU}\bar{e}_{yU}\right]$

$$e_{yN}^2 \in \left[e_{yL}^2, e_{yU}^2\right] \; ; e_{xN}^2 \in [e_{xL}^2, e_{xU}^2]$$

$$C_{xN}^2 = \frac{\sigma_{xN}^2}{\bar{X}_N^2}; C_{xN}^2 \in \left[C_{xL}^2, C_{xU}^2\right]; C_{yN}^2 = \frac{\sigma_{yN}^2}{\bar{Y}_N^2}; C_{yN}^2 \in \left[C_{yL}^2, C_{yU}^2\right]$$

$$\rho_{xyN} = \frac{\sigma_{xyN}}{\sigma_{xN}\sigma_{yN}}; \rho_{xyN} \in [\rho_{xyL}, \rho_{xyU}]$$

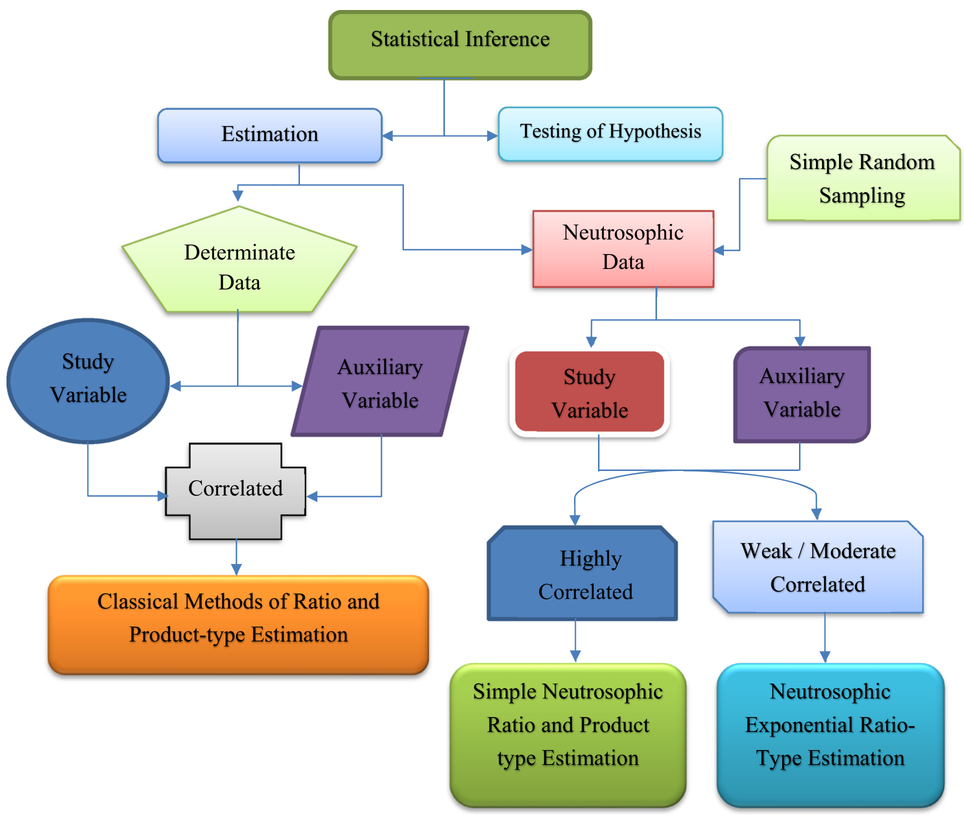
$$\therefore \theta_N = \frac{1 - f_N}{n_N}; \theta_N \in [\theta_L, \theta_U]; n_N \in [n_L, n_U]$$

$$\sigma_{xN}^2 \in \left[\sigma_{xL}^2, \sigma_{xU}^2\right]; \sigma_{yN}^2 \in \left[\sigma_{yL}^2, \sigma_{yU}^2\right]; \sigma_{xyN} \in [\sigma_{xyL}, \sigma_{xyU}]$$

Flow chart

The following flow chart explains the path of using proposed methods under neutrosophic numbers.

where $\bar{y}_{RN} \in [\bar{y}_{RL}, \bar{y}_{RU}]$, $\bar{y}_N \in [\bar{y}_L, \bar{y}_U]$, $\bar{x}_N \in [\bar{x}_L, \bar{x}_U]$, $\bar{X}_N \in \left[\bar{X}_L, \bar{X}_U\right]$, $\bar{Y}_N \in [\bar{Y}_L, \bar{Y}_U]$



Proposed neutrosophic estimators

Here, several existing estimators were transformed into neutrosophic estimators to overcome the problem of data indeterminacy and neutrosophic data.

Neutrosophic ratio estimator

The following is a proposed neutrosophic ratio estimator for estimating the mean of the finite population in the presence of auxiliary variables:

$$\bar{y}_{RN} = \frac{\bar{y}_N}{\bar{x}_N}\bar{X}_N \tag{1}$$

$$\bar{y}_{RN} = \left(\bar{Y}_N + \bar{e}_{yN}\right)\left(1 + \frac{\bar{e}_{xN}}{\bar{X}_N}\right)^{-1} \tag{2}$$

and $\bar{e}_{yN} \in \left[\bar{e}_{yL}, \bar{e}_{yU}\right]; \bar{e}_{xN} \in [\bar{e}_{xL}, \bar{e}_{xU}]$.

The bias and MSE of $\bar{y}_{RN}$ up to first-order approximation are given by

$$\mathrm{Bias}(\bar{y}_{RN}) = \theta_N \bar{Y}_N \left[C_{xN}^2 - C_{xN}C_{yN}\rho_{xyN}\right] \tag{3}$$

$$\mathrm{MSE}(\bar{y}_{RN}) = \theta_N \bar{Y}_N^2 \left[C_{yN}^2 + C_{xN}^2 - 2C_{xN}C_{yN}\rho_{xyN}\right] \tag{4}$$

$where\ \theta_N \in [\theta_L, \theta_U]; n_N \in [n_L, n_U]$

$C_{xN}^2 \in \left[C_{xL}^2, C_{xU}^2\right]$,

$C_{yN}^2 \in \left[C_{yL}^2, C_{yU}^2\right]$, $\rho_{xyN} \in [\rho_{xyL}, \rho_{xyU}]$

Several modified neutrosophic ratio estimators

Motivated by [28], we have developed a modified neutrosophic ratio estimator, where we used the coefficient of variation as an auxiliary variable:

$$\bar{y}_{SDrN} = \bar{y}_N \frac{\bar{X}_N + C_{xN}}{\bar{x}_N + C_{xN}} \tag{5}$$

$$\bar{y}_{SDrN} = \left(\bar{Y}_N + \bar{e}_{yN}\right)\left(1 + \frac{\bar{e}_{xN}}{\bar{X}_N + C_{xN}}\right)^{-1} \tag{6}$$

where $\bar{y}_{SDrN} \in [\bar{y}_{SDrL}, \bar{y}_{SDrU}]$, $\bar{y}_N \in [\bar{y}_L, \bar{y}_U]$, $\bar{x}_N \in [\bar{x}_L, \bar{x}_U]$, $\bar{X}_N \in \left[\bar{X}_L, \bar{X}_U\right]$, $\bar{Y}_N \in [\bar{Y}_L, \bar{Y}_U]$ and $\bar{e}_{yN} \in \left[\bar{e}_{yL}, \bar{e}_{yU}\right]$; $\bar{e}_{xN} \in [\bar{e}_{xL}, \bar{e}_{xU}]$, $C_{xN} \in [C_{xL}, C_{xU}]$

Expressions of bias and MSE of $\bar{y}_{SDrN}$ up to first-order approximation are given as

$$\mathrm{Bias}(\bar{y}_{SDrN}) = \theta_N \bar{Y}_N \left[\left(\frac{\bar{X}_N}{\bar{X}_N + C_{xN}}\right)^2 C_{xN}^2 - \left(\frac{\bar{X}_N}{\bar{X}_N + C_{xN}}\right) C_{xN} C_{yN} \rho_{xyN}\right] \tag{7}$$

$$\mathrm{MSE}(\bar{y}_{SDrN}) = \theta_N \bar{Y}_N^2 \left[C_{yN}^2 + \left(\frac{\bar{X}_N}{\bar{X}_N + C_{xN}}\right)^2 C_{xN}^2 - 2\left(\frac{\bar{X}_N}{\bar{X}_N + C_{xN}}\right) C_{xN} C_{yN} \rho_{xyN}\right] \tag{8}$$

where $\theta_N \in [\theta_L, \theta_U]$; $n_N \in [n_L, n_U]$

$C_{xN}^2 \in \left[C_{xL}^2, C_{xU}^2\right]$, $C_{yN}^2 \in \left[C_{yL}^2, C_{yU}^2\right]$, $\rho_{xyN} \in [\rho_{xyL}, \rho_{xyU}]$

Now, another neutrosophic estimator is suggested, where we have considered the coefficient of kurtosis as an auxiliary variable:

$$\bar{y}_{SKrN} = \bar{y}_N \frac{\bar{X}_N + \beta_{2(x)N}}{\bar{x}_N + \beta_{2(x)N}} \tag{9}$$

$$\bar{y}_{SKrN} = \left(\bar{Y}_N + \bar{e}_{yN}\right)\left(1 + \frac{\bar{e}_{xN}}{\bar{X}_N + \beta_{2(x)N}}\right)^{-1} \tag{10}$$

where $\bar{y}_{SKrN} \in [\bar{y}_{SKrL}, \bar{y}_{SKrU}]$, $\bar{y}_N \in [\bar{y}_L, \bar{y}_U]$, $\bar{x}_N \in [\bar{x}_L, \bar{x}_U]$, $\bar{X}_N \in \left[\bar{X}_L, \bar{X}_U\right]$, $\bar{Y}_N \in [\bar{Y}_L, \bar{Y}_U]$ and $\beta_{2(x)N} \in \left[\beta_{2(x)L}, \beta_{2(x)U}\right]$, $\bar{e}_{yN} \in \left[\bar{e}_{yL}, \bar{e}_{yU}\right]$, $\bar{e}_{xN} \in [\bar{e}_{xL}, \bar{e}_{xU}]$

The bias and MSE of $\bar{y}_{SKrN}$ correct up to first-order approximation are given by

$$\mathrm{Bias}(\bar{y}_{SKrN}) = \theta_N \bar{Y}_N \left[\left(\frac{\bar{X}_N}{\bar{X}_N + \beta_{2(x)N}}\right)^2 C_{xN}^2 - \left(\frac{\bar{X}_N}{\bar{X}_N + \beta_{2(x)N}}\right) C_{xN} C_{yN} \rho_{xyN}\right] \tag{11}$$

$$\mathrm{MSE}(\bar{y}_{SKrN}) = \theta_N \bar{Y}_N^2 \left[C_{yN}^2 + \left(\frac{\bar{X}_N}{\bar{X}_N + \beta_{2(x)N}}\right)^2 C_{xN}^2 - 2\left(\frac{\bar{X}_N}{\bar{X}_N + \beta_{2(x)N}}\right) C_{xN} C_{yN} \rho_{xyN}\right] \tag{12}$$

Motivated by [31], using both coefficient of variation and kurtosis in neutrosophic ratio-type estimator given as

$$\bar{y}_{USrN} = \bar{y}_N \frac{\bar{X}_N \beta_{2(x)N} + C_{xN}}{\bar{x}_N \beta_{2(x)N} + C_{xN}} \tag{13}$$

$$\bar{y}_{USrN} = \left(\bar{Y}_N + \bar{e}_{yN}\right)\left(1 + \frac{\beta_{2(x)N} \bar{e}_{xN}}{\bar{X}_N \beta_{2(x)N} + C_{xN}}\right)^{-1} \tag{14}$$

Where $\bar{y}_{USrN} \in [\bar{y}_{USrL}, \bar{y}_{USrU}]$, $\bar{y}_N \in [\bar{y}_L, \bar{y}_U]$, $\bar{x}_N \in [\bar{x}_L, \bar{x}_U]$, $\bar{X}_N \in \left[\bar{X}_L, \bar{X}_U\right]$, $\bar{Y}_N \in [\bar{Y}_L, \bar{Y}_U]$ and $\beta_{2(x)N} \in \left[\beta_{2(x)L}, \beta_{2(x)U}\right]$, $C_{xN} \in [C_{xL}, C_{xU}]$, $\bar{e}_{yN} \in \left[\bar{e}_{yL}, \bar{e}_{yU}\right]$, $\bar{e}_{xN} \in [\bar{e}_{xL}, \bar{e}_{xU}]$

The bias and MSE of $\bar{y}_{USrN}$ to the first order of approximation are given by

$$\mathrm{Bias}(\bar{y}_{USrN}) = \theta_N \bar{Y}_N \left[\left(\frac{\bar{X}_N \beta_{2(x)N}}{\bar{X}_N \beta_{2(x)N} + C_{xN}}\right)^2 C_{xN}^2 - \left(\frac{\bar{X}_N \beta_{2(x)N}}{\bar{X}_N \beta_{2(x)N} + C_{xN}}\right) C_{xN} C_{yN} \rho_{xyN}\right] \tag{15}$$

$$\mathrm{MSE}\,(\bar{y}_{USrN}) = \theta_N \bar{Y}_N^2 \left[C_{yN}^2 + \left(\frac{\bar{X}_N \beta_{2(x)N}}{\bar{X}_N \beta_{2(x)N} + C_{xN}}\right)^2 C_{xN}^2 - 2\left(\frac{\bar{X}_N \beta_{2(x)N}}{\bar{X}_N \beta_{2(x)N} + C_{xN}}\right) C_{xN} C_{yN} \rho_{xyN}\right] \tag{16}$$

where $\theta_N \in [\theta_L, \theta_U]$; $n_N \in [n_L, n_U]$

$C_{xN}^2 \in \left[C_{xL}^2, C_{xU}^2\right]$, $C_{yN}^2 \in \left[C_{yL}^2, C_{yU}^2\right]$, $\rho_{xyN} \in [\rho_{xyL}, \rho_{xyU}]$.

Neutrosophic exponential estimators

Here, a neutrosophic exponential-type estimator for estimating the mean for a finite population in the presence of auxiliary variables is suggested:

$$\bar{y}_{BTrN} = \bar{y}_N exp\left(\frac{\bar{X}_N - \bar{x}_N}{\bar{X}_N + \bar{x}_N}\right) \tag{17}$$

$$\bar{y}_{BTrN} = \left(\bar{Y}_N + \bar{e}_{yN}\right)exp\left[-\frac{\bar{e}_{xN}}{2\bar{X}_N}\left(1+\frac{\bar{e}_{xN}}{2\bar{X}_N}\right)^{-1}\right] \tag{18}$$

where $\bar{y}_{BTrN} \in [\bar{y}_{BTrL}, \bar{y}_{BTrU}]$, $\bar{y}_N \in [\bar{y}_L, \bar{y}_U]$, $\bar{x}_N \in [\bar{x}_L, \bar{x}_U]$, $\bar{X}_N \in \left[\bar{X}_L, \bar{X}_U\right]$, $\bar{Y}_N \in [\bar{Y}_L, \bar{Y}_U]$ and $\bar{e}_{yN} \in \left[\bar{e}_{yL}, \bar{e}_{yU}\right]$, $\bar{e}_{xN} \in [\bar{e}_{xL}, \bar{e}_{xU}]$.

The bias and MSE of $\bar{y}_{BTrN}$ up to first-order approximation are given by

$$\mathrm{Bias}(\bar{y}_{BTrN}) = \theta_N \bar{Y}_N\left[\frac{3}{8}C_{xN}^2 - \frac{1}{2}C_{xN}C_{yN}\rho_{xyN}\right] \tag{19}$$

$$\mathrm{MSE}(\bar{y}_{BTrN}) = \theta_N \bar{Y}_N^2\left[C_{yN}^2 + \frac{1}{4}C_{xN}^2 - C_{xN}C_{yN}\rho_{xyN}\right] \tag{20}$$

$where\, \theta_N \in [\theta_L, \theta_U]; n_N \in [n_L, n_U]$

$C_{xN}^2 \in \left[C_{xL}^2, C_{xU}^2\right]$, $C_{yN}^2 \in \left[C_{yL}^2, C_{yU}^2\right]$, $\rho_{xyN} \in [\rho_{xyL}, \rho_{xyU}]$

Motivated by ([26], we have developed a new neutrosophic exponential ratio-type estimator for estimating the mean of a finite population:

$$\bar{y}_{RrN} = \bar{y}_N exp\left[\frac{\left(a\bar{X}_N + b\right) - \left(a\bar{x}_N + b\right)}{\left(a\bar{X}_N + b\right) + \left(a\bar{x}_N + b\right)}\right] \tag{21}$$

$$\bar{y}_{RrN} = \left(\bar{Y}_N + \bar{e}_{yN}\right)exp\left[-\frac{ae_{xN}}{2(a\bar{X}_N + b)}\left(1+\frac{a\bar{e}_{xN}}{2(a\bar{X}_N + b)}\right)^{-1}\right] \tag{22}$$

where $\bar{y}_{RrN} \in [\bar{y}_{RrL}, \bar{y}_{RrU}]$, $\bar{y}_N \in [\bar{y}_L, \bar{y}_U]$, $\bar{x}_N \in [\bar{x}_L, \bar{x}_U]$, $\bar{X}_N \in \left[\bar{X}_L, \bar{X}_U\right]$, $\bar{Y}_N \in [\bar{Y}_L, \bar{Y}_U]$ and $\bar{e}_{yN} \in \left[\bar{e}_{yL}, \bar{e}_{yU}\right]$, $\bar{e}_{xN} \in [\bar{e}_{xL}, \bar{e}_{xU}]$

The bias and MSE of $\bar{y}_{RrN}$ up to first-order approximation are given by

$$\mathrm{Bias}(\bar{y}_{RrN}) = \theta_N \bar{Y}_N\left[\frac{3}{8}\left(\frac{\bar{X}_N}{a\bar{X}_N + b}\right)^2 C_{xN}^2 - \frac{1}{2}\left(\frac{\bar{X}_N}{a\bar{X}_N + b}\right)C_{xN}C_{yN}\rho_{xyN}\right] \tag{23}$$

$$\mathrm{MSE}(\bar{y}_{RrN}) = \theta_N \bar{Y}_N^2\left[C_{yN}^2 + \left(\frac{a\bar{X}_N}{2(a\bar{X}_N + b)}\right)^2 C_{xN}^2 - \frac{1}{2}\left(\frac{2a\bar{X}_N}{a\bar{X}_N + b}\right)C_{xN}C_{yN}\rho_{xyN}\right] \tag{24}$$

$where\, \theta_N \in [\theta_L, \theta_U]$, $n_N \in [n_L, n_U]$

$C_{xN}^2 \in \left[C_{xL}^2, C_{xU}^2\right]$,

$C_{yN}^2 \in \left[C_{yL}^2, C_{yU}^2\right]$, $\rho_{xyN} \in [\rho_{xyL}, \rho_{xyU}]$

Neutrosophic generalized exponential-type estimator

Motivated by [19], we have developed a neutrosophic generalized exponential-type estimator for estimating the mean of a finite population:

$$\bar{y}_{\mathrm{KNN}} = \bar{y}_N \exp\left[\alpha\left(\frac{\bar{X}_N^{\frac{1}{h}} - \bar{x}_N^{\frac{1}{h}}}{\bar{X}_N^{\frac{1}{h}} + (a-1)\bar{x}_N^{\frac{1}{h}}}\right)\right] \tag{25}$$

where $\alpha(-\infty < \alpha < \infty)$ and $h(h > 0)$ are two real constants and assumed to be known, and the other constant $a(a \neq 0)$ is supposed to be estimated so that $\bar{y}_{NGEN}$ is optimal and MSE of $\bar{y}_{NGEN}$ is minimum:

$$\bar{y}_{\mathrm{KNN}} = \left(\bar{Y}_N + \bar{e}_{yN}\right)\exp\left[\frac{-\alpha\bar{e}_{xN}}{ah\bar{X}_N}\left(1+\frac{\bar{e}_{xN}}{h\bar{X}_N} - \frac{\bar{e}_{xN}}{ah\bar{X}_N}\right)^{-1}\right] \tag{26}$$

where $\bar{y}_{KNN} \in [\bar{y}_{KNL}, \bar{y}_{KNU}]$, $\bar{y}_N \in [\bar{y}_L, \bar{y}_U]$, $\bar{x}_N \in [\bar{x}_L, \bar{x}_U]$, $\bar{X}_N \in \left[\bar{X}_L, \bar{X}_U\right]$, $\bar{Y}_N \in [\bar{Y}_L, \bar{Y}_U]$ and $\bar{e}_{yN} \in \left[\bar{e}_{yL}, \bar{e}_{yU}\right]$, $\bar{e}_{xN} \in [\bar{e}_{xL}, \bar{e}_{xU}]$

The bias and MSE of $\bar{y}_{\mathrm{NGEN}}$, correct up to first-order approximation are given by

$$\mathrm{Bias}\left(\bar{y}_{\mathrm{KNN}}\right) = \theta_N \bar{Y}_N\left[\frac{\alpha C_{xN}^2}{ah^2} - \frac{\alpha C_{xN}^2}{a^2h^2} + \frac{\alpha^2 C_{xN}^2}{2a^2h^2} - \frac{\alpha C_{xN}C_{yN}\rho_{xyN}}{ah}\right] \tag{27}$$

$$\mathrm{MSE}(\bar{y}_{\mathrm{KNN}}) = \theta_N \bar{Y}_N^2\left[C_{yN}^2 + \frac{\alpha^2 C_{xN}^2}{a^2h^2} - \frac{2\alpha C_{xN}C_{yN}\rho_{xyN}}{ah}\right] \tag{28}$$

$where\, \theta_N \in [\theta_L, \theta_U]; n_N \in [n_L, n_U]$

$C_{xN}^2 \in \left[C_{xL}^2, C_{xU}^2\right]$, $C_{yN}^2 \in \left[C_{yL}^2, C_{yU}^2\right]$, $\rho_{xyN} \in [\rho_{xyL}, \rho_{xyU}]$

To obtain the minimum MSE, we estimate the value of 'a'. From Eq.(28), the optimum value of 'a' is given by

$$\widehat{a} = \frac{\alpha C_{xN}^2}{hC_{xN}C_{yN}\rho_{xyN}} \tag{29}$$

We can write the expression of minimum MSE of $(\bar{y}_{\mathrm{KNN}})$ as follows:

$$\mathrm{MSE}(\bar{y}_{\mathrm{KNN}})_{\min} = \theta_N \bar{Y}_N^2 C_{yN}^2\left(1-\rho_{xyN}^2\right) \tag{30}$$

Table 1 Population's characteristics for single auxiliary variable

Parameters	Source (data): temperature of Lahore, Punjab, Pakistan from the years 2014–2019				
	Population available: $N = 30$ (years); sample taken: $n_N = [6, 6]$ (years)				
	X No. of Year	Y Average temperature (min, max)			
$\bar{X}_{N.}$	$\bar{Y}_{\mathrm{N.}}$	$C_{y\mathrm{N}}$	$C_{x\mathrm{N}}$	$\rho_{xy\mathrm{N}}$	$\beta_{2(x)\mathrm{N}}$
[3.5, 3.5]	[44, 64]	[0.0345, 0.0360]	[0.53, 0.53]	[0.23, 0.40]	[– 1.2, – 1.2]
	[50,72]	[0.0424, 0.0529]		[0.14, 0.18]	
	[58, 80]	[0.0424, 0.0454]		[0.17, 0.43]	
	[69, 94]	[0.0291, 0.0293]		[0.55, 0.60]	
	[77,102]	[0.0187, 0.0282]		[– 0.04, 0.04]	
	[81,103]	[0.0272, 0.0316]		[– 0.23, – 0.08]	
	[80, 95]	[0.0151, 0.0171]		[– 0.60, – 0.49]	
	[80, 95]	[0.0108, 0.0141]		[– 0.59, – 0.53]	
	[77, 94]	[0.0231, 0.0254]		[0.01, 0.09]	
	[68, 90]	[0.0277, 0.0338]		[– 0.39, – 0.20]	
	[55, 79]	[0.0233, 0.0260]		[– 0.55, 0.42]	
	[45, 69]	[0.0341, 0.050]		[– 0.14, – 0.05]	

Numerical study

Empirical study

As it is a new concept and to the best of the authors' knowledge, no work has been done so far on the neutrosophic ratio-type estimators. Therefore, in this case, we compared the MSE of the proposed Eq. (25) neutrosophic estimator with other proposed neutrosophic estimators given in Eqs. (1), (5), (9), (13), (17), and (21) to evaluate which neutrosophic ratio-type estimator performs more efficiently. We have also computed the relative efficiencies of these estimators. In statistics, for an estimator, the minimum MSE is required to be better among the class of estimators. For the numerical study, we have considered real-life indeterminacy interval data of temperature, as the daily temperature is taken as of neutrosophic nature and varies in an interval with vague values. The one reason to take temperature as neutrosophic data is that its value diverges in an interval form, where the value considers to be mention as the reference temperature of the day may be one of the lowest or highest temperatures recorded in a day or any point between them. Data of the past 6 years is noted from the weather websites available/ published online and arranged monthwise (Temperature of Lahore, Punjab, Pakistan from the years 2014—2019) described in Table 1 [15]. This data is obtained from publicly published sources, online available for all, and therefore, no ethical approval is needed. We have taken a sample of 6 years month-wise average temperature of lowest and highest temperature during each month, and X is the codding of the time from 1 to 6 (number of years). Neutrosophic averages of lower and upper limits of the temperatures of each month of 6 years were measured, which are the neutrosophic part of the data in Y corresponding to known X year, where the monthwise total averages for all 6 years are taken as the neutrosophic data. The temperature is taken as neutrosophic data (Temp, $y_N \epsilon [y_L, \; y_U]$), corresponding to time (in years X) as the independent determinate variable. The $\bar{X}_N \in [\bar{X}_L, \; \bar{X}_U]$ is an average of 6 years for which the data are collected, so it is the same value for all lower and upper limits of the corresponding neutrosophic data. C_{xN} is the coefficient of variation and $\beta_{2(x)N}$ is coefficient of kurtosis of the auxiliary variable.

In Table 2, the neutrosophic MSE for the proposed estimators are given for each month of a year. MSE is arranged as upper value and lower value in Table 2 (i.e., $\mathrm{MSE}_N \in [\mathrm{MS}E_U, \; \mathrm{MSE}_L]$). We can see the last column of Table 2 showing $\mathrm{MSE}(\bar{y}_{\mathrm{KNN}})$ is minimum compared to others, which means it is the most efficient estimator for available neutrosophic data.

Table 3 consists of the relative efficiencies of the proposed neutrosophic ratio estimators to $\bar{y}_{\mathrm{KNN}}$. An estimator with the lowest value, i.e., a value less than or equal to 100 in comparison to all other estimators, is considered the most efficient. Here, the estimator is given in Eq. (25), is the most efficient neutrosophic ratio estimator among all, as none of the other columns giving values less than 100.

Simulation

For evaluating efficiencies of the proposed estimators, we used simulated neutrosophic data, such that X_N and Y_N Neu-

Table 2 Mean square errors of proposed neutrosophic ratio-type estimators

$n_N = 6$	MSE($\bar{y}_{\mathrm{RN}}$)	MSE($\bar{y}_{\mathrm{SDrN}}$)	MSE($\bar{y}_{\mathrm{SKrN}}$)	MSE($\bar{y}_{\mathrm{USrN}}$)	MSE($\bar{y}_{\mathrm{BTrN}}$)	MSE($\bar{y}_{RrN}$) ($a = 0$, $b = 0$)	MSE($\bar{y}_{RrN}$) ($a = 1$, $b = 1$)	MSE($\bar{y}_{KNN}$)
Jan	[150.296, 71.321]	[112.297, 101.450]	[353.937, 166.457]	[197.814, 88.118]	[35.958, 17.524]	[553.683, 256.056]	[35.958, 17.524]	[0.599, 0.290]
Feb	[192.530, 92.637]	[144.580, 131.326]	[448.771, 215.414]	[253.324, 121.792]	[47.858, 23.076]	[691.220, 329.540]	[47.858, 23.076]	[1.872, 0.580]
Mar	[230.338, 127.100]	[171.699, 180.555]	[545.481, 296.095]	[304.895, 167.215]	[54.376, 31.492]	[863.572, 454.540]	[54.376, 31.492]	[1.446, 0.794]
Apr	[317.357, 171.925]	[236.550, 247.792]	[751.088, 406.151]	[420.034, 227.393]	[74.578, 40.638]	[1184.408, 638.383]	[74.578, 40.638]	[0.641, 0.382]
May	[400.600, 227.968]	[301.719, 321.283]	[926.224, 527.912]	[525.634, 299.282]	[100.768, 57.203]	[1396.868, 799.390]	[100.768, 57.203]	[0.488, 0.635]
Jun	[414.140, 250.553]	[313.281, 351.374]	[948.458, 577.700]	[541.458, 328.406]	[107.331, 63.642]	[1406.153, 867.376]	[107.331, 63.642]	[1.327, 0.638]
Jul	[357.884, 251.726]	[270.951, 350.643]	[817.730, 577.235]	[467.540, 329.292]	[93.059, 64.787]	[1204.832, 856.433]	[93.059, 64.787]	[0.224, 0.148]
Aug	[355.156, 250.028]	[268.569, 349.121]	[813.637, 574.648]	[464.431, 327.349]	[91.639, 63.907]	[1204.832, 856.203]	[91.639, 63.907]	[0.158, 0.072]
Sep	[335.231, 225.191]	[252.130, 316.944]	[777.626, 520.942]	[440.389, 295.527]	[83.599, 56.635]	[1180.641, 787.001]	[83.599, 56.635]	[0.624, 0.509]
Oct	[317.327, 185.718]	[239.769, 256.450]	[728.557, 422.169]	[415.273, 242.145]	[81.587, 49.153]	[1084.909, 616.826]	[81.587, 49.153]	[0.801, 0.597]
Nov	[247.176, 112.295]	[187.437, 160.553]	[562.761, 263.438]	[322.480, 148.139]	[64.973, 27.096]	[823.728, 408.647]	[64.973, 27.096]	[0.309, 0.228]
Dec	[183.098, 79.570]	[138.390, 111.150]	[420.508, 182.739]	[239.601, 104.137]	[47.401, 20.478]	[629.075, 272.522]	[47.401, 20.478]	[1.568, 0.311]

Table 3 Relative efficiencies of proposed neutrosophic ratio estimators to $\bar{y}_{KNN}$

$n_N = 6$	MSE($\bar{y}_{RN}$)	MSE($\bar{y}_{SDrN}$)	MSE($\bar{y}_{SKrN}$)	MSE($\bar{y}_{USrN}$)	MSE($\bar{y}_{BTrN}$)	MSE($\bar{y}_{RrN}$) ($a = 0$, $b = 0$)	MSE($\bar{y}_{RrN}$) ($a = 1$, $b = 1$)	MSE($\bar{y}_{KNN}$)
Jan	[25091.151, 24,593.448]	[18747.412, 34,982.758]	[59087.979, 57,398.965]	[33024.040, 30,385.517]	[6003.005, 6042.758]	[92434.557, 88,295.172]	[6003.005, 6042.758]	[100, 100]
Feb	[10284.722, 15,971.896]	[7723.290, 22,642.413]	[23972.809, 37,140.344]	[13532.264, 20,998.620]	[2556.517, 3978.620]	[36924.145, 56,817.241]	[2556.517, 3978.620]	[100, 100]
Mar	[15929.322, 16,007.556]	[11874.066, 22,739.924]	[37723.443, 37,291.561]	[21085.408, 21,059.823]	[3760.442, 3966.246]	[59721.438, 57,246.851]	[3760.442, 3966.246]	[100, 100]
Apr	[49509.672, 45,006.544]	[36903.276, 64,867.015]	[117174.415, 106,322.251]	[65527.925, 59,526.963]	[11634.633, 10,638.219]	[184775.039, 167,115.968]	[11634.633, 10,638.219]	[100, 100]
May	[82090.163, 35,900.472]	[61827.663, 50,595.748]	[189800, 83,135.748]	[107711.885, 47,131.023]	[20649.180, 9008.346]	[286243.442, 125,888.189]	[20649.180, 9008.346]	[100, 100]
Jun	[31208.741, 39,271.630]	[23608.214, 55,074.294]	[71473.850, 90,548.589]	[40803.165, 51,474.294]	[8088.244, 9975.235]	[105964.807, 35,952.351]	[8088.244, 9975.235]	[100, 100]
Jul	[159769.642, 170,085.135]	[120960.267, 236,920.945]	[365058.035, 390,023.648]	[208723.214, 222,494.594]	[41544.196, 43775]	[537871.428, 578,670.945]	[41544.196, 43775]	[100, 100]
Aug	[224782.278, 347,261.111]	[169980.379, 484,890.277]	[514960.126, 798,122.222]	[293943.670, 454,651.388]	[57999.367, 88,759.722]	[762551.898, 1,189,170.833]	[57999.367, 88,759.722]	[100, 100]
Sep	[53722.916, 44,241.846]	[40405.448, 62,267.976]	[124619.551, 102,346.169]	[70575.160, 58,060.314]	[13397.275, 11,126.719]	[189205.288, 154,617.092]	[13397.275, 11,126.719]	[100, 100]
Oct	[39616.354, 31,108.542]	[29933.707, 42,956.448]	[90955.930, 70,715.075]	[51844.319, 40,560.301]	[10185.642, 8233.333]	[135444.319, 103,320.938]	[10185.642, 8233.333]	[100, 100]
Nov	[79992.233, 49,252.192]	[60659.223, 70,417.982]	[182123.301, 115,542.982]	[104362.459, 64,973.245]	[21026.860, 11,884.210]	[266578.640, 179,231.140]	[21026.860, 11,884.210]	[100, 100]
Dec	[11677.168, 25,585.209]	[8825.892, 35,739.549]	[26818.112, 58,758.520]	[15280.676, 33,484.565]	[3023.022, 6584.565]	[40119.579, 87,627.652]	[3023.022, 6584.565]	[100, 100]

Table 4 Descriptive statistics for simulation under neutrosophic data

Parameters	Neutrosophic values	Parameters	Neutrosophic values
N_N	[1000, 1000]	σ_{yN}	[12.9, 17.2]
n_N	[20,20]	C_{xN}	[0.03381, 0.03794]
μ_{xN}	[171.2, 180.4]	C_{yN}	[0.1724, 0.206]
μ_{yN}	[76.0, 84.9]	$\beta_{2(x)N}$	[0.2039, 0.06235]
σ_{xN}	[5.8, 6.7]	ρ_{xyN}	[0.02847, 0.01041]

Table 5 Descriptive statistics for simulation under classical data

Parameters	Classical values	Parameters	Classical values
N	1000	σ_y	14.522
n	20	C_x	0.042568
$\bar{X}$	178.246	C_y	0.170319
$\bar{Y}$	82.817	ρ_{xy}	0.024808
σ_x	7.172	$\beta_{2(x)}$	− 0.03362

trosophic random variates (NRV) follows neutrosophic normal distributions (NND), i.e., $X_N \sim NN\left(\mu_{xN}, \sigma^2_{xN}\right); X_N \in (X_L, X_U)$, $\mu_{xN} \in (\mu_{xL}, \mu_{xU})$, $\sigma^2_{xN} \in \left(\sigma^2_{xL}, \sigma^2_{xU}\right)$ and $Y_N \sim N\left(\mu_{yN}, \sigma^2_{yN}\right); Y_N \in (Y_L, Y_U)$, $\mu_{yN} \in \left(\mu_{yL}, \mu_{yU}\right)$, $\sigma^2_{yN} \in \left(\sigma^2_{yL}, \sigma^2_{yU}\right)$.

We took $Y_N \sim NN\left([76.0, 84.9], \left[(12.9)^2, (17.2)^2\right]\right)$, where $\mu_{yN} \in (76.0, 84.9)$, $\sigma_{yN} \in (12.9, 17.2)$, and $X_N \sim NN\left([171.2, 180.4], \left[(5.8)^2, (6.7)^2\right]\right)$, where $\mu_{xN} \in (171.2, 180.4)$, $\sigma_{xN} \in (5.8, 6.7)$ for simulating 1000 normal random variates. Table 4 shows the results of the neutrosophic data used to compare the performance efficiency of the proposed estimators and the traditional estimators under classical statistics. For classical statistics, Table 5 gives the information used to compute results.

Table 6 shows MSE under neutrosophic data and classical data and among all the estimators, $\bar{y}_{KNN}$ is more efficient under neutrosophic data with minimum MSE. When compare neutrosophic results with the classical, we may conclude that in situations, where data are not clear and crisp, instead of relying on a single value in the case of classical estimators, we have an interval to rely on for better results as we can accept the output if it falls in between these values, since we are dealing with uncertain or indeterminate data.

Table 7 includes the percentage relative efficiencies of neutrosophic estimators along with the classical estimator. It is clear from the results that $\bar{y}_{KNN}$ is the most efficient estimator whether it is neutrosophic data; or classical data. Furthermore, the neutrosophic estimators are more efficient than the classical estimators with low percentage relative efficiencies (PREs).

Table 6 Comparison of MSE of estimators

SR. No	Estimator	MSE Neutrosophic	MSE Classical
1	$\bar{y}_{KNN}$	[8.403468, 14.98543]	10.44198
2	$\bar{y}_{RN}$	[8.639845, 15.4381]	10.87109
3	$\bar{y}_{SDrN}$	[8.639736, 15.4379]	10.87085
4	$\bar{y}_{SKrN}$	[8.639187, 15.43777]	10.87029
5	$\bar{y}_{USrN}$	[8.63931, 15.43488]	10.86943
6	$\bar{y}_{BTrN}$	[8.444193, 15.08545]	10.51607
7	$\bar{y}_{RrN}$ $(a = 0, b = 0)$	[8.410288, 14.98705]	10.45423
8	$\bar{y}_{RrN}(a = 1, b = 1)$	[8.443529, 15.08421]	10.51492

Table 7 PREs of estimators (neutrosophic vs classical)

SR. No	Estimator	PREs Neutrosophic	PREs Classical
1	$\bar{y}_{KNN}$	[100, 100]	100
2	$\bar{y}_{RN}$	[102.8128, 103.0207]	104.1094
3	$\bar{y}_{SDrN}$	[102.8115, 103.0194]	104.1072
4	$\bar{y}_{SKrN}$	[102.805, 103.0185]	104.1018
5	$\bar{y}_{USrN}$	[102.8065, 102.9992]	104.0936
6	$\bar{y}_{BTrN}$	[100.4846, 100.6675]	100.7096
7	$\bar{y}_{RrN}$ $(a = 0, b = 0)$	[100.0811, 100.0108]	100.1173
8	$\bar{y}_{RrN}$ $(a = 1, b = 1)$	[100.4767, 100.6592]	100.6985

Discussion

Tables 2 and 3 show the numerical results of MSE and PREs of the proposed neutrosophic ratio estimators for neutrosophic data from the population described in Table 1. It is observed from the indeterminacy interval results from Eq. (25) neutrosophic ratio-type estimator $\bar{y}_{\mathrm{KNN}}$, is highly efficient than the rest of the other proposed estimators under study in this article for the complete data. The indeterminacy interval results also indicate that the estimators $\bar{y}_{BTrN}$ and $\bar{y}_{RrN}\ for\ (a = 1\ and\ b = 1)$ are more efficient than all the other estimators except $\bar{y}_{\mathrm{KNN}}$ for the neutrosophic population that has a moderate and low (regardless of positive or negative direction) correlation between the study variable and the auxiliary variable. The analysis by simulated neutrosophic data also verifies that the estimator $\bar{y}_{\mathrm{KNN}}$ is most efficient, while the estimators $\bar{y}_{BTrN}$ and $\bar{y}_{Rrn}(a = 1, b = 1)$ are precisely equally efficient. $\bar{y}_{RrN}(a = 0, b = 0)$ becomes a simple ratio estimator of mean, so it is better than others after $\bar{y}_{\mathrm{KNN}}$. The simulation results for both neutrosophic data and classical data, when compared, we conclude that the neutrosophic estimators give more reliable and more precise results, especially for unclear/ vague data. Neutrosophic results of MSEs in Table 6 suggested that our proposed neutrosophic estimator $\bar{y}_{KNN}$ with minimum MSE is better than other proposed estimators. PRE of estimator $\bar{y}_{\mathrm{KNN}}$ is also lowest among all results under neutrosophic data and classical data. All the estimators are unbiased (for order one), sufficient and consistent. In addition, the one with minimum variance is more efficient which is $\bar{y}_{\mathrm{KNN}}$ in this study.

Conclusions

The present study aims to use the ratio estimation method under neutrosophic data derived from simple random sampling. The study suggested that neutrosophic ratio-type estimators are more efficient than the classical estimators in the case of indeterminate data. Neutrosophic observations are of a unique form that comprises ambiguous, uncertain, or indeterminate values. The classical ratio estimation method provides single-valued results, which are sometimes not representative, especially in neutrosophic data. Through our proposed neutrosophic ratio-type estimators, we have tried to solve the issue of estimating the mean of the finite population in the case of neutrosophic data. This study is the first step, and a whole new area is open ahead for establishing improved estimators under different types of neutrosophic data under different sampling plans.

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Remark on Neutrosophy Perspective on Blue Ocean Shift

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ABSTRACT

In recent years, there is an alternative scheme in corporate strategy discourse, called Blue Ocean (shift) Strategy by W. Chan Kim and R. Mauborgne (2004). In this paper we offer a new insight based on Neutrosophic Logic perspective, which combines red ocean and blue ocean, while a company moves forward and shift to blue ocean space.

KEYWORDS: Blue ocean shift, Neutrosophic Logic, Brue ocean

INTRODUCTION

In 2005 INSEAD teachers W. Chan Kim and Renée Mauborgne, distributed Blue Ocean Strategy, one of the most effective procedure books ever composed. Selling over 3.6 million duplicates and distributed in 44 dialects, the book has become a precept for any association that is hoping to make new market spaces ready for development. The investigation incorporates examination of more than 150 fruitful new market manifestations, across in excess of 30 businesses, including less effective contenders. What they discovered is that most organizations center around how to beat the opposition in existing business sectors. They portrayed this as a red sea, frequently packed and with contracting overall revenues and restricted development opportunities. (Passos, 2019)

Blue ocean procedure is the concurrent quest for separation and ease to open up another market space and provoke new interest. It is tied in with making and catching uncontested market space, in this manner making the opposition insignificant. It depends on the view that market limits and industry structure are not guaranteed and can be reproduced by the activities and convictions of industry players.[1]

Blue ocean scheme differs appreciably in approaching the industry, as an organization would not accept it as granted. Even the traditional value-low cost trade off can be surpassed, in order to expand the existing market space.

In their Blue Ocean Shift, W. Chan Kim and R. Mauborgne offer some clear and good examples of organizations who have made such a transition to blue ocean (Kim & Mauborgne, 2018). There is the case of an inn network that applied the demonstrated advances plot in the book to break out of the exceptionally serious inn industry – which is 'redder than red'– to make the new market of moderate lavish inns offering five-star comfort at three-star costs. Today it has 90% inhabitance rates, visitor appraisals called it 'magnificent' and 'spectacular' on booking destinations, and portrayed it as the least

[1]https://www.blueoceanstrategy.com/what-is-blue-ocean-strategy/

expenses in the most stylish areas. It is turning out to significant urban communities over the world. The book likewise clarifies how a worldwide, little machine organization with over 100 years of history turned an industry, whose worth was declining by 10% every year, into a high-development one. The organization did that by reclassifying its contribution so much that it permits we all today to make scrumptious French fries with no browning and practically no oil. The aftereffect of its work day: Not just requested develop by 40%, its stock cost lifted by 5 percent (Passos, 2019). Another study on organizations using blue ocean method have been discussed by Saputri et al. 2015 also M. Shafiq (2019) ('Blue Ocean strategy for creating value innovation: A study over Kedai Digital in Yogyakarta, Indonesia', 2015) (Shafiq et al., 2019).

In Eka Saputri et al. (2015) paper, their abstract tells which can be paraphrased as follows:
"Product business has been growing quickly lately particularly in Yogyakarta. This marvel causes business visionaries in stock industry to contend to hold clients, and the majorities of them amazingly decline their cost and cut the edge benefit. Kedai Digital executed Blue Ocean Strategy to make another market and make the opposition insignificant. Worth Innovation is fundamental method of this methodology. In particular, this investigation examines examination of significant worth advancement in "Kedai Digital" Yogyakarta. The target of this investigation is to decide esteem development and recognize esteem driver in "Kedai Digital".

This examination was utilizing blend strategy approach though subjective methodology was embraced by doing inside and out meeting with 12 of chief in 6 organizations in the Merchandise Business in Yogyakarta and quantitative methodology was led by spreading poll to 100 people who are got from the purposive example of "Kedai Digital" clients. Both information investigating was utilized as subjective and quantitative. Examination apparatuses are Kanvas Strategy and Four Framework Analysis."('Blue Ocean strategy for creating value innovation: A study over Kedai Digital in Yogyakarta, Indonesia', 2015)

PROBLEMS OF TRANSITION

While the Blue Ocean Shift book has offered some practical tools to help organizations mapping their position and going toward blue ocean, such a transition or shift to become blue ocean organization is not so easy. In physics term, this process can be called as *transition phase.* In this context, Tantau and Mateesescu offer a bit more realistic pathway that they call: Green Ocean, where a mixture of red ocean space and blue ocean space is allowed while an organizations move gradually toward Blue Ocean. (Tanțău& Mateesescu, 2013)

Such a transition can be seen from Neutrosophic Logic Perspective (Smarandache, 1999), albeit with a bit rather different lingo, i.e. in Neutrosophic Logic it is known (T,I,F) means: degree of truth, indeterminacy, and falsehood. Meanwhile, in green ocean scheme, there are R, I, B: x percent of (R) red ocean, indeterminacy, and y percent of (B) blue ocean.

In the meantime, instead of neutrosophic logic we can use Neutrosophy, since in neutrosophy we have in general <A> and <antiA>, the opposites, and the neutral <neutA>. In this case we take Red = <A> and Blue = <antiA>, while green (or other color in between) as part of <neutA>.

To summarize such an approach, we offer the following table:

Table 1: Neutrosophic Logic perspective to red-blue ocean mixture

Description	Red Ocean	Indeterminacy	Blue Ocean
Analogy with Neutrosophic Logic	Truth	Indeterminacy	Falsehood
Green Ocean	X percentage of red		Y percentage of blue
In Neutrosophy framework	Red <A>	Green <neutA>	Blue <antiA>
Main strategy	Competitive	A mixture	Non-competitive
Porter scheme	Value or low budget trade off	A mixture	Value leap while keeping low budget
Disruptive/non-disruptive pattern	Disruptive innovations		Non-disruptive creations (*value leap*)

CONCLUSION

To simplify the above notions, perhaps we should not call it "green ocean strategy" which only makes it more complicated, but perhaps "*brue*" from a mixture of blue ocean-red ocean strategy. (Perhaps we can call it: *Brue ocean strategy*: from "red in mixture with blue.")

We hope a simple scheme as outlined above can be developed further in the near future.

Note: In connection with the notion of "value leap" above and also non-disruptive creations, we submitted a new paper with title: "Six impossible things before breakfast" to Jurnal Indonesia Maju, June 2021. In that article, we discuss several potential breakthroughs which can be associated with non-disruptive creations. In other words, we don't agree with fundamental assumptions in recent books that major innovations/changes should always be associated with disruptive technologies.

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A Possible Application of DNA Transduction Experiment: Information Medicine for Pedestrians

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Abstract

Light-based therapy has been known for a long time, some records even went back to ancient Egypt time. A modern version is the use of laser therapy for various purposes, and they are now categorized as Laser therapy in general and Low Level Laser therapy (LLLT). But most laser therapy devices are very expensive, and they are only available at large hospitals and clinics. In this article we discuss possible use of LLLT in the form of laser LED pointer for treatment of several illnesses. First of all, we begin with a short review of laser LED pointer for acupuncture and then we discuss its implication to information medicine. We also discuss possible reasoning from DNA transduction experiment, as reported by Luc Montagnier., *et al*. While this idea is not really new, the implication for information medicine with laser LED pointer is quite striking.

Keywords: *DNA Transduction; Information Medicine; Ivermectin; Laser LED Pointer*

Abbreviations

LASER: Term Laser is the acronym for Light Amplification by Stimulated Emission of Radiation. A laser light is monochromatic, collimated, and coherent. A laser is a device that produces such a light; LED: Light-Emitting Diode (a semiconductor diode which glows when a voltage is applied); LLLT: Low Level Laser Therapy. Low level laser therapy (LLLT) is used by some physiotherapists to treat various musculoskeletal condition. LLLT is a non-invasive light source treatment that generates a single wavelength of light. It emits no heat, sound, or vibration. It is also called photobiology or biostimulation

Introduction

Light-based therapy has been known for a long time, some records even went back to ancient Egypt time. A modern version is the use of laser therapy for various purposes and they are now categorized as Laser therapy in general and Low Level Laser therapy (LLLT). In this article we discuss possible use of LLLT in the form of laser LED pointer for treatment of various diseases, which may include "covid-19" patients.

The abbreviation "LASER" represents Light (photons) Amplification by Stimulated Emission of Radiation. Low level laser treatment (LLLT) is the awesome most broadly acknowledged descriptor of the kind of lasers utilized in recovery. The actual instrument is viewed as a "*remedial laser*". LLLT has truly been named a non-warm modality. Non-warm modalities are those actual specialists that don't raise the subcutaneous tissue temperature more noteworthy than 36.5ºC. Consequently, the restorative impacts of LLLT are not related with a warming reaction, yet rather a photochemical reaction. When light (photons) enters the phone, certain atoms called chromophores respond to it, and trigger a photochemical response that prompts attractive physiologic impacts. LLLT is just another type of energy (actual specialist) that can be utilized to make physiological changes in tissue.

For further discussions on Laser therapy/biophotomodulation applications, see for instance [1,2,6,7].

While we don't conduct experiment yet on information medicine use of laser LED therapy, we review some existing literatures on this subject, along with a discussion with a biophysics expert colleague, around two years ago. So, we present this novel alternative treatment, with precaution for users.

History of light-based therapy[1]

Based on archaeology findings, light-based therapy was already in use since around ancient Egypt time. They utilized sunlight to treat several illness.

It has been known for quite a while that light has as a heap of helpful impacts. Since 1400 AD, many major human advancements were utilizing daylight in the therapy of skin infections, including vitiligo, disease, and psoriasis. The Egyptians constructed exceptional light recuperating sanctuaries that pre-owned sun and shaded light for different mending purposes. Alongside the Egyptians, the Assyrians and the Babylonians all rehearsed restorative sun mending, called heliotherapy. Herodotus, a popular Greek doctor, was believed to be the father of heliotherapy and accentuated the significance of sun openness for the restoration of well-being. The Greek city, *Heliopolis*, was known for its recuperating sanctuaries and light rooms, which had windows covered with different shaded materials which were thought to have distinctive mending properties. Many accepted that the red light of the sun added to these remedial impacts.

As time went on, numerous others got inspired by heliotherapy. In 1855, Arnold Rikli, a characteristic healer from Switzerland, created helio-hydroscopic treatment focuses in Bled, Slovenia. People who lived at these focuses lived in extraordinary houses, washed and sun tanned naked. Rikli accepted that the sun, air, and water were the wellsprings of well-being and recuperating. Notwithstanding, it was not until the 1870s that researchers in the field of light treatment turned out to be completely mindful of the recuperating properties of light.

In 1876, Augustus Pleasontan built up a hypothesis that blue light from the sun was useful in the development of plants just as in the wellbeing of people and creatures. His hypothesis depended on his perceptions that plants filled best in the spring when the sky was bluer. He performed probes the development of grapes that were presented to characteristic daylight and blue light. The outcomes showed that grapes presented to blue light developed quicker than those presented to coordinate daylight. In creatures, he found that blue light raised ripeness and expanded the rate of actual development. Pleasanton additionally found that blue light, from either the sun or a fake source, was a successful methods for animating the secretory organs and sensory system in people. He discovered this to be valuable in the treatment of different illnesses, particularly those went with torment. Despite the fact that his hypothesis was rarely completely received in the academic local area, his blue light hypothesis is viewed as the introduction of current chromotherapy.

[1]This section is adapted with paraphrasing from: Microsoft Word - BRIEF HISTORY OF PHOTOTHERAPY.docx (mimhtraining.com).

During the 1990s, there was an expansion in the utilization of low-level light treatment (LLLT). In 1996, Michael Conlan started to research the impacts of close infrared laser light treatment on injury mending. Additionally, during the 1990s, NASA started investigating the impacts of light producing diodes (LEDs) on injury recuperating. Their outcomes show that when presented to LED, there was a 140 - 200% increment in cell development in mouse muscle and skeletal cells. They finished up their results would extraordinarily build the mending season of wounds when this application is applied to people.

A significant achievement throughout the entire existence of PDT happened in 1975 when Thomas Dougherty and partners detailed the complete fix of a tumor following organization of HpD and initiation with red light. Dougherty., *et al*. directed mice 2.5 - 5.0 mg/kg of HpD and presented them to red light for three hours per day, over a five-day time frame. Their outcomes showed that 48% of the mice were restored of their tumor. Following Dougherty's work, in 1976, Kelly and Snell were the first to utilize porphyrin based-PDT in the therapy of bladder malignant growth in people. Their outcomes showed that fluorescence was just seen in threatening and premalignant territories of the tumor, demonstrating that HpD could be utilized in the determination and therapy of bladder malignant growth.

Laser LED therapy for acupuncture

One of our friends, a doctor, told us how laser LED therapy is quite commonly used in his practice of laser-based acupuncture. Several treatments which often use laser acupuncture methods including but not limited to: stroke, removing tatoos, skin rejuvenation, facial/ageing problems etc. There is also hope for treatment of degenerative problems with this method, such as Alzheimer and Parkinson diseases too.

Figure 1: *Laser pointer for acupuncture therapy.*

The key of this method is that laser wavelength should be quite long to interact with cells below the skin (See figure 1 above).

While the above method of laser LED therapy and LLLT are already known, nonetheless there is only few literature discussing the use of laser LED pointer. In the following section, we will discuss how information medicine can be applied for molecule replication.

Information medicine using laser LED pointer

Although medicine treatment using laser LED method is already known, mostly they are based on photonic aspect of laser light. That is why such a method is called biophotonic modulation, photodynamics, or phototherapy.

But less is known on using information medicine in clinical practice.

A biophysics expert told us that there is special way one can use laser LED pointer.

Discussion with a biophysics expert

Around two years ago, we got a discussion via email with a biophysics information expert from Princeton Biotechnology Corporation, namely Dr Robert N. Boyd. Among other things he suggested a simple form of information medicine with laser LED pointer: "I've performed an experiment which has produced very good results. Using this method, the information of any beneficial herb or medication can be copied directly into the body, while the Ambient Intelligence removes all the side effects and after effects of the given substance, so that untoward influences are not present in the body or the psyche. It's like the spin field, only better, because laser pointers are much cheaper than spin field generators".

The experiment is rather simple: Get a laser pointer (I bought a red one). Get some aluminum foil and form a cone of aluminum foil around the light emitting end so that the base of the aluminum cone (the larger end of the cone) widens out where the light comes out. The small end of the cone is wrapped around the barrel of the laser pointer, so you end up with an aluminum cone surrounding the light-emitting end of the laser. Then, select any herb or medication, and place it on the back of your hand. Then place the aluminum-coned laser so that the laser light is aimed directly at the herb or medication.

Turn on the laser. Get the laser, with its cone, as close as possible to the medication, in the vertical sense, and keep it centered on the medication. Keep the laser on that spot for at least 5 minutes. The laser light bounces off the medication, then bounces off the aluminum cone, then radiates into the skin, for as long as the aim is accurate and the laser light is on. The information of that medication is now inside your body, in your blood stream, and so on.

The Ambient Intelligence removes all undesirable effects. It appears that laser-induced medication, results in effects which are better than the original substance. The effects of the laser-induced medication have lasted for hours longer than the effects of the actual medication is expected to last, when the medicine is consumed. No drug interactions can occur, since no drugs are ingested. Overdose seems impossible, since there is no dose of any chemicals".

Possible scenario for information medicine

Let say, there is epidemic in small town in one country in Africa, like Congo or Zimbabwe. The epidemic is spreading fast, and local doctors only got a limited source of medication.

Let say a medication is known, for instance Ivermectin, but supply at those town is very limited, therefore it is needed to replicate quickly Ivermectin in supply.

In that case, the following method can be used:

- Find a laser LED pointer available in the market,
- Put Ivermectin tablet on human skin under treatment,
- Make a cone shape from a aluminum foil,
- Then apply laser LED ray on the tablet, directed at producing exact copy of molecule of the tablet into cells below the skin,
- This method can be considered as information medicine way to inject the tablet to human body. Or it may be referred as the next generation of injection method.
- In that way, the particular medication can be used million times for all people in the town, even the doctor only has one pill of Ivermectin.

Illustration of laser LED pointer

Figure 2: *Laser LED pointer.*

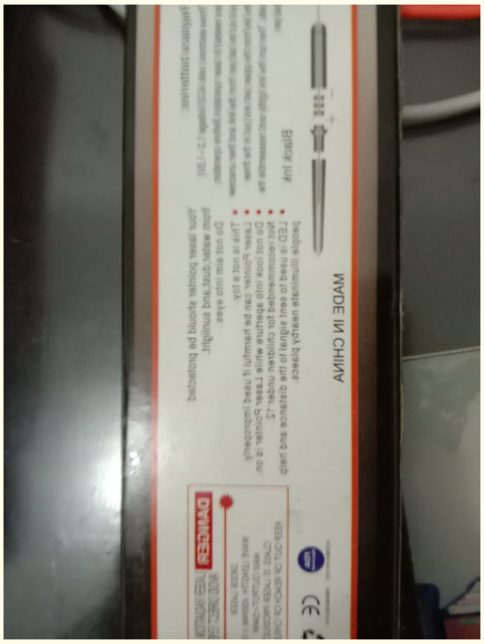

Figure 3: *Laser LED pointer.*

For more reports on low level laser therapy and photomodulation in general, see for instance [8-12].

Outline of reasoning: DNA transduction experiment

Such a proposed new method of injection based on information medicine is based on known DNA transduction experiment, as reported by Prof. Luc Montagnier., *et al* [4].

In their report, they explain how light ray sent through a glass of water comprising DNA molecule can make exact copy of DNA in question to the next glass of water.

The abstract of their report tells us the following [4]: "The experimental conditions by which electromagnetic signals (EMS) of low frequency can be emitted by diluted aqueous solutions of some bacterial and viral DNAs are described. That the recorded EMS and nanostructures induced in water carry the DNA information (sequence) is shown by retrieval of that same DNA by classical PCR amplification using the TAQ polymerase, including both primers and nucleotides. Moreover, such a transduction process has also been observed in living human cells exposed to EMS irradiation. These experiments suggest that coherent long range molecular interaction must be at work in water so to allow the observed features. The quantum field theory analysis of the phenomenon is presented".

That would imply exact replication of DNA molecule is possible, just as advised by the biophysics expert colleague aforementioned above.

See also Luc Montagnier., *et al.*'s other papers [3,5].

Concluding Remarks

Light-based therapy has been known for a long time, some records even went back to ancient Egypt time. A modern version is the use of laser therapy for various purposes, and they are now categorized as Laser therapy in general and Low Level Laser therapy (LLLT). But most laser therapy devices are very expensive, and they are only available at large hospitals and clinics. In this article we discuss possible use of LLLT in the form of laser LED pointer for treatment of several illnesses.

While this short article does not discuss this novel approach of Laser LED pointer therapy in more detail, we hope that what we discuss here make sure that there are enormous potential applications of information medicine. We can expect to move from medicine of scarcity towards medicine of abundance.

While we don't conduct experiment yet on information medicine use of laser LED therapy, we review some existing literatures on this subject, along with a discussion with a biophysics expert colleague, around two years ago. So, we present this novel alternative treatment, with precaution for users.

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