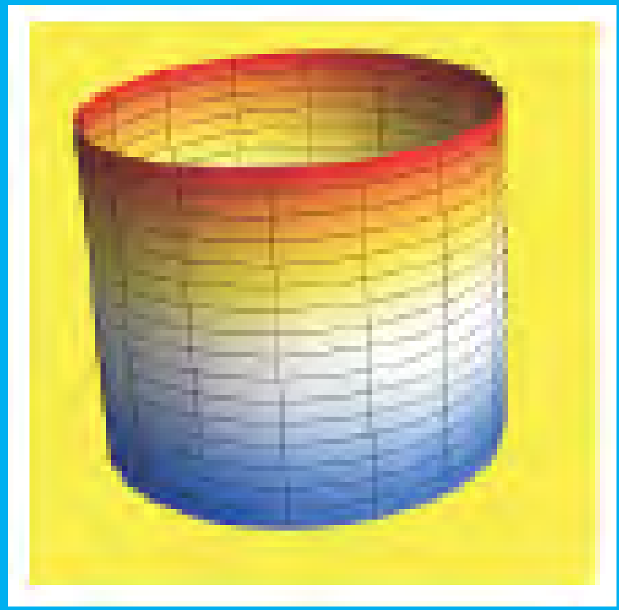

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Florentin Smarandache

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This volum includes 37 papers of mathematics or applied mathematics written by the author alone or in collaboration with the following co-authors: Cătălin Barbu, Mihály Bencze, Octavian Cira, Marian Niţu, Ion Pătraşcu, Mircea E. Şelariu, Rajan Alex, Xingsen Li, Tudor Păroiu, Luige Vlădăreanu, Victor Vlădăreanu, Ştefan Vlăduţescu, Yingjie Tian, Mohd Anasri, Lucian Căpitanu, Valeri Kroumov, Kimihiro Okuyama, Gabriela Tonţ, A. A. Adewara, Manoj K. Chaudhary, Mukesh Kumar, Sachin Malik, Alka Mittal, Neetish Sharma, Rakesh K. Shukla, Ashish K. Singh, Jayant Singh, Rajesh Singh,V.V. Singh, Hansraj Yadav, Amit Bhaghel, Dipti Chauhan, V. Christianto, Priti Singh, and Dmitri Rabounski.

They were written during the years 2010-2014, about the hyperbolic Menelaus theorem in the Poincare disc of hyperbolic geometry, and the Menelaus theorem for quadrilaterals in hyperbolic geometry, about some properties of the harmonic quadrilateral related to triangle simedians and to Apollonius circles, about Luhn prime numbers, and also about the correspondences of the eccentric mathematics of cardinal and integral functions and centric mathematics, or ordinary mathematics; there are some notes on Crittenden and Vanden Eynden's conjecture, or on new transformations, previously non-existent in traditional mathematics, that we call centric mathematics (CM), but that became possible due to the new born eccentric mathematics, and, implicitly, to the supermathematics (SM); also, about extenics, in general, and extension innovation model and knowledge management, in particular, about advanced methods for solving contradictory problems of hybrid position-force control of the movement of walking robots by applying a 2D Extension Set, or about the notion of point-set position indicator and that of point-two sets position indicator, and the navigation of mobile robots in non-stationary and nonstructured environments; about applications in statistics, such as estimators based on geometric and harmonic mean for estimating population mean using information; about Godel's incompleteness theorem(s) and plausible implications to artificial intelligence/life and human mind, and many more.

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Florentin Smarandache: *Collected Papers*, Vol. IV (100 Collected Papers of Sciences). Multispace & Multistructure. Neutrosophic Transdisciplinarity (Hanko, Finland, 2010) http://fs.gallup.unm.edu/MultispaceMultistructure.pdf

MATHEMATICS

THE HYPERBOLIC MENELAUS THEOREM IN THE POINCARE´ DISC MODEL OF HYPERBOLIC GEOMETRY

FLORENTIN SMARANDACHE, CĂTĂLIN BARBU

Abstract. In this note, we present the hyperbolic Menelaus theorem in the Poincaré disc of hyperbolic geometry.

Keywords and phrases: hyperbolic geometry, hyperbolic triangle, gyrovector.

2000 Mathematics Subject Classification: 30F45, 20N99, 51B10, 51M10.

1. Introduction

Hyperbolic Geometry appeared in the first half of the 19^{th} century as an attempt to understand Euclid's axiomatic basis of Geometry. It is also known as a type of non-Euclidean Geometry, being in many respects similar to Euclidean Geometry. Hyperbolic Geometry includes similar concepts as distance and angle. Both these geometries have many results in common but many are different.

There are known many models for Hyperbolic Geometry, such as: Poincaré disc model, Poincaré half-plane, Klein model, Einstein relativistic velocity model, etc. The hyperbolic geometry is a non-euclidian geometry. Menelaus of Alexandria was a Greek mathematician and astronomer, the first to recognize geodesics on a curved surface as natural analogs of straight lines. Here, in this study, we present a proof of Menelaus's theorem in the Poincaré disc model of hyperbolic geometry.

The well-known Menelaus theorem states that if l is a line not through any vertex of a triangle ABC such that l meets BC in D, CA in E, and AB in F, then $\frac{DB}{DC}\cdot\frac{EC}{EA}\cdot\frac{FA}{FB}=1$ [1]. This result has a simple statement but it is of great interest. We just mention here few different proofs given by A. Johnson [2], N.A. Court [3], C. Coşniţă [4], A. Ungar [5]. F. Smarandache (1983) has generalized the Theorem of Menelaus for any polygon with $n\geq 4$ sides as follows: If a line l intersects the n-gon $A_1A_2...A_n$ sides $A_1A_2, A_2A_3, ...,$ and A_nA_1 respectively in the points $M_1, M_2, ...,$ and M_n, then $\frac{M_1A_1}{M_1A_2}\cdot\frac{M_2A_2}{M_2A_3}\cdot...\cdot\frac{M_nA_n}{M_nA_1}=1$ [6].

We begin with the recall of some basic geometric notions and properties in the Poincaré disc. Let D denote the unit disc in the complex z-plane, i.e.

$$D=\{z\in\mathbb{C}:|z|<1\}.$$

The most general Möbius transformation of D is

$$z\rightarrow e^{i\theta}\frac{z_0+z}{1+\overline{z_0}z}=e^{i\theta}(z_0\oplus z),$$

which induces the Möbius addition $\oplus$ in D, allowing the Möbius transformation of the disc to be viewed as a Möbius left gyro-translation

$$z\rightarrow z_0\oplus z=\frac{z_0+z}{1+\overline{z_0}z}$$

followed by a rotation. Here $\theta\in\mathbb{R}$ is a real number, $z, z_0\in D$, and $\overline{z_0}$ is the complex conjugate of z_0. Let $Aut(D,\oplus)$ be the automorphism group of the grupoid $(D,\oplus)$. If we define

$$gyr: D\times D\rightarrow Aut(D,\oplus), gyr[a,b]=\frac{a\oplus b}{b\oplus a}=\frac{1+a\overline{b}}{1+\overline{a}b},$$

then is true gyro-commutative law

$$a\oplus b=gyr[a,b](b\oplus a).$$

A gyro-vector space $(G,\oplus,\otimes)$ is a gyro-commutative gyro-group $(G,\oplus)$ that obeys the following axioms:

(1) $gyr[\mathbf{u},\mathbf{v}]\mathbf{a}\cdot gyr[\mathbf{u},\mathbf{v}]\mathbf{b}=\mathbf{a}\cdot\mathbf{b}$ for all points $\mathbf{a},\mathbf{b},\mathbf{u},\mathbf{v}\in G$.

(2) G admits a scalar multiplication, $\otimes$, possessing the following properties. For all real numbers $r, r_1, r_2\in\mathbb{R}$ and all points $\mathbf{a}\in G$:

(G1) $1\otimes\mathbf{a}=\mathbf{a}$

(G2) $(r_1+r_2)\otimes\mathbf{a}=r_1\otimes\mathbf{a}\oplus r_2\otimes\mathbf{a}$

(G3) $(r_1r_2)\otimes\mathbf{a}=r_1\otimes(r_2\otimes\mathbf{a})$

(G4) $\dfrac{|r| \otimes \mathbf{a}}{\|r \otimes \mathbf{a}\|} = \dfrac{\mathbf{a}}{\|\mathbf{a}\|}$

(G5) $gyr[\mathbf{u},\mathbf{v}](r \otimes \mathbf{a}) = r \otimes gyr[\mathbf{u},\mathbf{v}]\mathbf{a}$

(G6) $gyr[r_1 \otimes \mathbf{v}, r_1 \otimes \mathbf{v}] = 1$

(3) Real vector space structure $(\|G\|, \oplus, \otimes)$ for the set $\|G\|$ of one-dimensional "vectors"

$$\|G\| = \{\pm \|\mathbf{a}\| : \mathbf{a} \in G\} \subset \mathbb{R}$$

with vector addition $\oplus$ and scalar multiplication $\otimes$, such that for all $r \in \mathbb{R}$ and $\mathbf{a}, \mathbf{b} \in G$,

(G7) $\|r \otimes \mathbf{a}\| = |r| \otimes \|\mathbf{a}\|$

(G8) $\|\mathbf{a} \oplus \mathbf{b}\| \leq \|\mathbf{a}\| \oplus \|\mathbf{b}\|$.

Theorem 1 (The law of gyrosines in Möbius gyrovector spaces). *Let ABC be a gyrotriangle in a Möbius gyrovector space $(V_s, \oplus, \otimes)$ with vertices $A, B, C \in V_s$, sides $\mathbf{a}, \mathbf{b}, \mathbf{c} \in \mathbf{V}_s$, and side gyrolengths $a, b, c \in (-s, s)$, $\mathbf{a} = \ominus B \oplus C$, $\mathbf{b} = \ominus C \oplus A$, $\mathbf{c} = \ominus A \oplus B$, $a = \|\mathbf{a}\|$, $b = \|\mathbf{b}\|$, $c = \|\mathbf{c}\|$, and with gyroangles α, β, and γ at the vertices A, B, and C. Then*

$$\frac{a_\gamma}{\sin\alpha} = \frac{b_\gamma}{\sin\beta} = \frac{c_\gamma}{\sin\gamma},$$

where $v_\gamma = \dfrac{v}{1 - \frac{v^2}{s^2}}$ [7, p. 267].

Definition 2 The hyperbolic distance function in D is defined by the equation

$$d(a,b) = |a \ominus b| = \left|\frac{a-b}{1-\overline{a}b}\right|.$$

Here, $a \ominus b = a \oplus (-b)$, for $a, b \in D$.

For further details we refer to the recent book of A.Ungar [5].

2. Main results

In this section, we prove the Menelaus's theorem in the Poincaré disc model of hyperbolic geometry.

Theorem 3 (The Menelaus's Theorem for Hyperbolic Gyrotriangle). *If l is a gyroline not through any vertex of an gyrotriangle ABC such that l meets BC in D, CA in E, and AB in F, then*

$$\frac{(AF)_\gamma}{(BF)_\gamma} \cdot \frac{(BD)_\gamma}{(CD)_\gamma} \cdot \frac{(CE)_\gamma}{(AE)_\gamma} = 1.$$

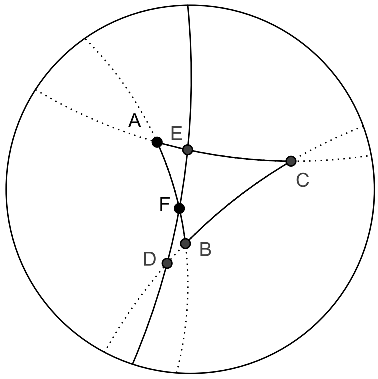

Figure 1

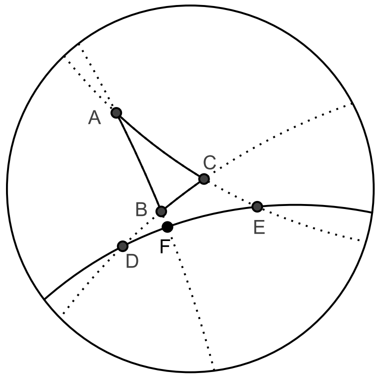

Figure 2

Proof. In function of the position of the gyroline l intersect internally a side of ABC triangle and the other two externally (See Figure 1), or the line l intersect all three sides externally (See Figure 2).

If we consider the first case, the law of gyrosines (See Theorem 1), gives for the gyrotriangles AEF, BFD, and CDE, respectively

$$\frac{(AE)_\gamma}{(AF)_\gamma} = \frac{\sin \widehat{AFE}}{\sin \widehat{AEF}}, \tag{1}$$

$$\frac{(BF)_\gamma}{(BD)_\gamma} = \frac{\sin \widehat{FDB}}{\sin \widehat{DFB}}, \tag{2}$$

and

$$\frac{(CD)_\gamma}{(CE)_\gamma} = \frac{\sin \widehat{DEC}}{\sin \widehat{EDC}}, \tag{3}$$

where $\sin \widehat{AFE} = \sin \widehat{DFB}$, $\sin \widehat{EDC} = \sin \widehat{FDB}$, and $\sin \widehat{AEF} = \sin \widehat{DEC}$, since gyroangles $\widehat{AEF}$ and $\widehat{DEC}$ are suplementary. Hence, by (1), (2) and (3), we have

$$\frac{(AE)_\gamma}{(AF)_\gamma} \cdot \frac{(BF)_\gamma}{(BD)_\gamma} \cdot \frac{(CD)_\gamma}{(CE)_\gamma} = \frac{\sin \widehat{AFE}}{\sin \widehat{AEF}} \cdot \frac{\sin \widehat{FDB}}{\sin \widehat{DFB}} \cdot \frac{\sin \widehat{DEC}}{\sin \widehat{EDC}} = 1, \tag{4}$$

the conclusion follows. The second case is treated similar to the first. ∎

Naturally, one may wonder whether the converse of the Menelaus theorem exists.

Theorem 4 (Converse of Menelaus's Theorem for Hyperbolic Gyrotriangle). *If D lies on the gyroline BC, E on CA, and F on AB such that*

$$\frac{(AF)_\gamma}{(BF)_\gamma} \cdot \frac{(BD)_\gamma}{(CD)_\gamma} \cdot \frac{(CE)_\gamma}{(AE)_\gamma} = 1, \tag{5}$$

then D, E, and F are collinear.

Proof. Relabelling if necessary, we may assume that the gyropoint D lies beyond B on BC. If E lies between C and A, then the gyroline ED cuts the gyroside AB, at F' say. Applying Menelaus's theorem to the gyrotriangle ABC and the gyroline $E - F' - D$, we get

$$\frac{(AF')_\gamma}{(BF')_\gamma} \cdot \frac{(BD)_\gamma}{(CD)_\gamma} \cdot \frac{(CE)_\gamma}{(AE)_\gamma} = 1. \tag{6}$$

From (5) and (6), we get $\frac{(AF)_\gamma}{(BF)_\gamma} = \frac{(AF')_\gamma}{(BF')_\gamma}$. This equation holds for $F = F'$. Indeed, if we take $x := |\ominus A \oplus F'|$ and $c := |\ominus A \oplus B|$, then we get $c \ominus x = |\ominus F' \oplus B|$. For $x \in (-1, 1)$ define

$$f(x) = \frac{x}{1 - x^2} : \frac{c \ominus x}{1 - (c \ominus x)^2}. \tag{7}$$

Because $c \ominus x = \frac{c - x}{1 - cx}$, then $f(x) = \frac{x(1 - c^2)}{(c - x)(1 - cx)}$. Since the following equality holds

$$f(x) - f(y) = \frac{c(1 - c^2)(1 - xy)}{(c - x)(1 - cx)(c - y)(1 - cy)}(x - y), \tag{8}$$

we get $f(x)$ is an injective function and this implies $F = F'$, so D, E, F are collinear.

There are still two possible cases. The first is if we suppose that the gyropoint F lies on the gyroside AB, then the gyrolines DF cuts the gyrosegment AC in the gyropoint E'. The second possibility is that E is not on the gyroside AC, E lies beyond C. Then DE cuts the gyroline AB in the gyropoint F'. In each case a similar application of Menelaus gives the result. ∎

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A NEW PROOF OF MENELAUS'S THEOREM OF HYPERBOLIC QUADRILATERALS IN THE POINCARÉ MODEL OF HYPERBOLIC GEOMETRY

CATALIN BARBU and FLORENTIN SMARANDACHE

ABSTRACT. In this study, we present a proof of the Menelaus theorem for quadrilaterals in hyperbolic geometry, and a proof for the transversal theorem for triangles

2000 Mathematics Subject Classification: 51K05, 51M10

Key words: hyperbolic geometry, hyperbolic quadrilateral, Menelaus theorem, the transversal theorem, gyrovector

1. Introduction

Hyperbolic geometry appeared in the first half of the 19^{th} century as an attempt to understand Euclid's axiomatic basis of geometry. It is also known as a type of non-euclidean geometry, being in many respects similar to euclidean geometry. Hyperbolic geometry includes similar concepts as distance and angle. Both these geometries have many results in common but many are different. Several useful models of hyperbolic geometry are studied in the literature as, for instance, the Poincaré disc and ball models, the Poincaré half-plane model, and the Beltrami-Klein disc and ball models [3] etc. Following [6] and [7] and earlier discoveries, the Beltrami-Klein model is also known as the Einstein relativistic velocity model. Menelaus of Alexandria was a Greek mathematician and astronomer, the first to recognize geodesics on a curved surface as natural analogs of straight lines. The well-known Menelaus theorem states that if l is a line not through any vertex of a triangle ABC such that l meets BC in D, CA in E, and AB in F, then $\frac{DB}{DC} \cdot \frac{EC}{EA} \cdot \frac{FA}{FB} = 1$ [2]. Here, in this study, we give hyperbolic version of Menelaus theorem for quadrilaterals in the Poincaré disc model. Also, we will give a reciprocal hyperbolic version of this theorem. In [1] has been given proof of this theorem, but to use Klein's model of hyperbolic geometry.

We begin with the recall of some basic geometric notions and properties in the Poincaré disc. Let D denote the unit disc in the complex z - plane, i.e.

$$D = \{z \in \mathbb{C} : |z| < 1\}.$$

The most general Möbius transformation of D is

$$z \to e^{i\theta} \frac{z_0 + z}{1 + \overline{z_0} z} = e^{i\theta}(z_0 \oplus z),$$

which induces the Möbius addition $\oplus$ in D, allowing the Möbius transformation of the disc to be viewed as a Möbius left gyro-translation

$$z \to z_0 \oplus z = \frac{z_0 + z}{1 + \overline{z_0} z}$$

followed by a rotation. Here $\theta \in \mathbb{R}$ is a real number, $z, z_0 \in D$, and $\overline{z_0}$ is the complex conjugate of z_0. Let $Aut(D, \oplus)$ be the automorphism group of the grupoid $(D, \oplus)$. If we define

$$gyr : D \times D \to Aut(D, \oplus), gyr[a, b] = \frac{a \oplus b}{b \oplus a} = \frac{1 + a\bar{b}}{1 + \bar{a}b},$$

then is true gyro-commutative law

$$a \oplus b = gyr[a, b](b \oplus a).$$

A gyro-vector space $(G, \oplus, \otimes)$ is a gyro-commutative gyro-group $(G, \oplus)$ that obeys the following axioms:

(1) $gyr[\mathbf{u}, \mathbf{v}]\mathbf{a} \cdot gyr[\mathbf{u}, \mathbf{v}]\mathbf{b} = \mathbf{a} \cdot \mathbf{b}$ for all points $\mathbf{a}, \mathbf{b}, \mathbf{u}, \mathbf{v} \in G$.

(2) G admits a scalar multiplication, $\otimes$, possessing the following properties. For all real numbers $r, r_1, r_2 \in \mathbb{R}$ and all points $\mathbf{a} \in G$:

(G1) $1 \otimes \mathbf{a} = \mathbf{a}$

(G2) $(r_1 + r_2) \otimes \mathbf{a} = r_1 \otimes \mathbf{a} \oplus r_2 \otimes \mathbf{a}$

(G3) $(r_1 r_2) \otimes \mathbf{a} = r_1 \otimes (r_2 \otimes \mathbf{a})$

(G4) $\frac{|r| \otimes \mathbf{a}}{\|r \otimes \mathbf{a}\|} = \frac{\mathbf{a}}{\|\mathbf{a}\|}$

(G5) $gyr[\mathbf{u}, \mathbf{v}](r \otimes \mathbf{a}) = r \otimes gyr[\mathbf{u}, \mathbf{v}]\mathbf{a}$

(G6) $gyr[r_1 \otimes \mathbf{v}, r_1 \otimes \mathbf{v}] = 1$

(3) Real vector space structure $(\|G\|, \oplus, \otimes)$ for the set $\|G\|$ of one-dimensional "vectors"

$$\|G\| = \{\pm \|\mathbf{a}\| : \mathbf{a} \in G\} \subset \mathbb{R}$$

with vector addition $\oplus$ and scalar multiplication $\otimes$, such that for all $r \in \mathbb{R}$ and $\mathbf{a}, \mathbf{b} \in G$,

(G7) $\|r \otimes \mathbf{a}\| = |r| \otimes \|\mathbf{a}\|$

(G8) $\|\mathbf{a} \oplus \mathbf{b}\| \le \|\mathbf{a}\| \oplus \|\mathbf{b}\|$

Definition 1. *The hyperbolic distance function in D is defined by the equation*

$$d(a, b) = |a \ominus b| = \left| \frac{a - b}{1 - \bar{a}b} \right|.$$

Here, $a \ominus b = a \oplus (-b)$, for $a, b \in D$.

For further details we refer to the recent book of A.Ungar [7].

Theorem 2. (***The Menelaus's Theorem for Hyperbolic Gyrotriangle***). *Let ABC be a gyrotriangle in a Möbius gyrovector space $(V_s, \oplus, \otimes)$ with vertices $A, B, C \in V_s$, sides $\mathbf{a}, \mathbf{b}, \mathbf{c} \in \mathbf{V}_s$, and side gyrolengths $a, b, c \in (-s, s)$, $\mathbf{a} = \ominus B \oplus C$, $\mathbf{b} = \ominus C \oplus A$, $\mathbf{c} = \ominus A \oplus B$, $a = \|\mathbf{a}\|$, $b = \|\mathbf{b}\|$, $c = \|\mathbf{c}\|$, and with gyroangles α, β, and γ at the vertices A, B, and C. If l is a gyroline not through any vertex of an gyrotriangle ABC such that l meets BC in D, CA in E, and AB in F, then*

$$\frac{(AF)_\gamma}{(BF)_\gamma} \cdot \frac{(BD)_\gamma}{(CD)_\gamma} \cdot \frac{(CE)_\gamma}{(AE)_\gamma} = 1.$$

where $v_\gamma = \frac{v}{1 - \frac{v^2}{s^2}}$ [6].

2. Main results

In this section, we prove Menelaus's theorem for hyperbolic quadrilateral.

Theorem 3. (***The Menelaus's Theorem for Gyroquadrilateral***). *If l is a gyroline not through any vertex of a gyroquadrilateral $ABCD$ such that l meets AB in X, BC in Y, CD in Z, and DA in W, then*

$$\frac{(AX)_\gamma}{(BX)_\gamma} \cdot \frac{(BY)_\gamma}{(CY)_\gamma} \cdot \frac{(CZ)_\gamma}{(DZ)_\gamma} \cdot \frac{(DW)_\gamma}{(AW)_\gamma} = 1. \tag{1}$$

Proof. Let T be the intersection point of the gyroline DB and the gyroline XYZ (See Figure 1). If we use a theorem 2 in the gyrotriangles ABD and BCD respectively, then

$$\frac{(AX)_\gamma}{(BX)_\gamma} \cdot \frac{(BT)_\gamma}{(DT)_\gamma} \cdot \frac{(DW)_\gamma}{(AW)_\gamma} = 1 \tag{2}$$

and

$$\frac{(DT)_\gamma}{(BT)_\gamma} \cdot \frac{(CZ)_\gamma}{(DZ)_\gamma} \cdot \frac{(BY)_\gamma}{(CY)_\gamma} = 1. \tag{3}$$

Multiplying relations (2) and (3) member with member, we obtain

$$\frac{(AX)_\gamma}{(BX)_\gamma} \cdot \frac{(BY)_\gamma}{(CY)_\gamma} \cdot \frac{(CZ)_\gamma}{(DZ)_\gamma} \cdot \frac{(DW)_\gamma}{(AW)_\gamma} = 1.$$

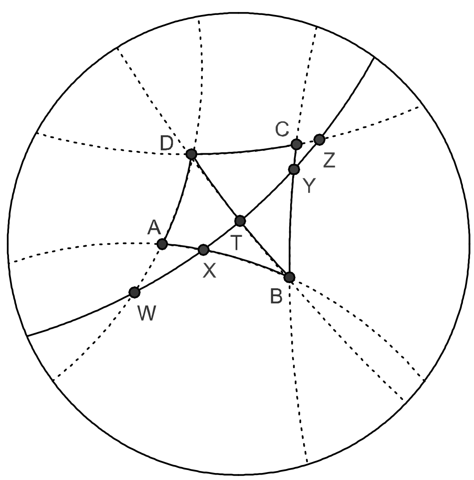

Figure 1

Naturally, one may wonder whether the converse of Menelaus theorem for hyperbolic quadrilateral exists. Indeed, a partially converse theorem does exist as we show in the following theorem.

Theorem 4. (***Converse of Menelaus's Theorem for Gyroquadrilateral***). *Let $ABCD$ be a gyroquadrilateral. Let the points X, Y, Z, and W be located on the gyrolines AB, BC, CD, and DA respectively. If three of four gyropoints X, Y, Z, W are collinear and*

$$\frac{(AX)_\gamma}{(BX)_\gamma} \cdot \frac{(BY)_\gamma}{(CY)_\gamma} \cdot \frac{(CZ)_\gamma}{(DZ)_\gamma} \cdot \frac{(DW)_\gamma}{(AW)_\gamma} = 1,$$

then all four gyropoints are collinear.

Proof. Let the points W, X, Z are collinear, and gyroline WXZ cuts gyroline BC, at Y' say. Using the already proven equality (1), we obtain

$$\frac{(AX)_\gamma}{(BX)_\gamma} \cdot \frac{(BY')_\gamma}{(CY')_\gamma} \cdot \frac{(CZ)_\gamma}{(DZ)_\gamma} \cdot \frac{(DW)_\gamma}{(AW)_\gamma} = 1,$$

then we get

$$\frac{(BY)_\gamma}{(CY)_\gamma} = \frac{(BY')_\gamma}{(CY')_\gamma}. \tag{4}$$

This equation holds for $Y = Y'$. Indeed, if we take $x := |\ominus B \oplus Y'|$ and $b := |\ominus B \oplus C|$, then we get $b \ominus x = |\ominus Y' \oplus C|$. For $x \in (-1, 1)$ define

$$f(x) = \frac{x}{1 - x^2} : \frac{b \ominus x}{1 - (b \ominus x)^2}. \tag{5}$$

Because $b \ominus x = \frac{b-x}{1-bx}$, then $f(x) = \frac{x(1-b^2)}{(b-x)(1-bx)}$. Since the following equality holds

$$f(x) - f(y) = \frac{b(1-b^2)(1-xy)}{(b-x)(1-bx)(b-y)(1-by)}(x - y), \tag{6}$$

we get $f(x)$ is an injective function and this implies $Y = Y'$, so W, X, Z, and Y are collinear. ■

We have thus obtained in (1) the following

Theorem 5. (***Transversal theorem for gyrotriangles***). *Let D be on gyroside BC, and l is a gyroline not through any vertex of a gyrotriangle ABC such that l meets AB in M, AC in N, and AD in P, then*

$$\frac{(BD)_\gamma}{(CD)_\gamma} \cdot \frac{(CA)_\gamma}{(NA)_\gamma} \cdot \frac{(NP)_\gamma}{(MP)_\gamma} \cdot \frac{(MA)_\gamma}{(BA)_\gamma} = 1. \tag{7}$$

Proof. If we use a theorem 2 for gyroquadrilateral $BCNM$ and collinear gyropoints D, A, P, and A (See Figure 2), we obtain the conclusion.

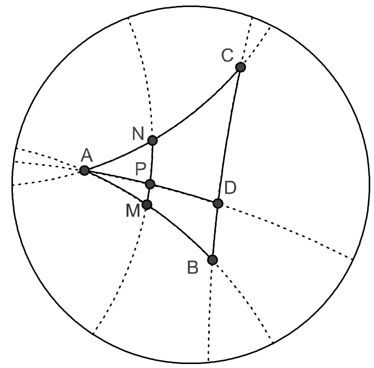

Figure 2

■

The Einstein relativistic velocity model is another model of hyperbolic geometry. Many of the theorems of Euclidean geometry are relatively similar form in the Poincaré model, Menelaus's theorem for hyperbolic gyroquadrilateral and the transversal theorem for gyrotriangle are an examples in this respect. In the Euclidean limit of large s, $s \to \infty$, gamma factor v_γ reduces to v, so that the gyroinequalities (1) and (7) reduces to the

$$\frac{AX}{BX} \cdot \frac{BY}{CY} \cdot \frac{CZ}{DZ} \cdot \frac{DW}{AW} = 1$$

and

$$\frac{BD}{CD} \cdot \frac{CA}{NA} \cdot \frac{NP}{MP} \cdot \frac{MA}{BA} = 1,$$

in Euclidean geometry. We observe that the previous equalities are identical with the equalities of theorems of euclidian geometry.

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SOME PROPERTIES OF THE HARMONIC QUADRILATERAL

ION PATRASCU and FLORENTIN SMARANDACHE

Abstract: In this article, we review some properties of the harmonic quadrilateral related to triangle simedians and to Apollonius circles.

Definition 1. A convex circumscribable quadrilateral $ABCD$ having the property $AB \cdot CD = BC \cdot AD$ is called **harmonic quadrilateral**.

Definition 2. A **triangle simedian** is the isogonal cevian of a triangle median.

Proposition 1. In the triangle ABC, the cevian AA_1, $A_1 \in (BC)$ is a simedian if and only if $\frac{BA_1}{A_1C} = \left(\frac{AB}{AC}\right)^2$. For **Proof** of this property, see *infra*.

Proposition 2. In an harmonic quadrilateral, the diagonals are simedians of the triangles determined by two consecutive sides of a quadrilateral with its diagonal.

Proof. Let $ABCD$ be an harmonic quadrilateral and $\{K\} = AC \cap BD$ (see *Fig. 1*). We prove that BK is simedian in the triangle ABC.

From the similarity of the triangles ABK and DCK, we find that:

$$\frac{AB}{DC} = \frac{AK}{DK} = \frac{BK}{CK} \ (1).$$

From the similarity of the triangles BCK şi ADK, we conclude that:

$$\frac{BC}{AD} = \frac{CK}{DK} = \frac{BK}{AK} \ (2).$$

From the relations (1) and (2), by division, it follows that:

$$\frac{AB}{BC} \cdot \frac{AD}{DC} = \frac{AK}{CK} \ (3).$$

But $ABCD$ is an harmonic quadrilateral; consequently,

$$\frac{AB}{BC} = \frac{AD}{DC};$$

substituting this relation in (3), it follows that:

$$\left(\frac{AB}{BC}\right)^2 = \frac{AK}{CK};$$

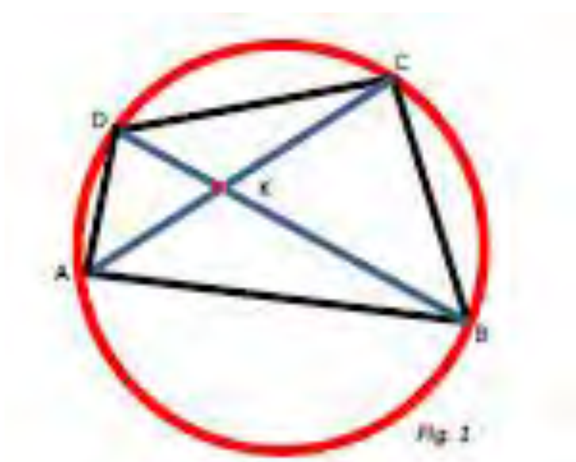


Fig. 1

as shown by Proposition 1, BK is a simedian in the triangle ABC. Similarly, it can be shown that AK is a simedian in the triangle ABD, that CK is a simedian in the triangle BCD, and that DK is a simedian in the triangle ADC.

Remark 1. The converse of the Proposition 2 is proved similarly, i.e.:

Proposition 3. If in a convex circumscribable quadrilateral a diagonal is a simedian in the triangle formed by the other diagonal with two consecutive sides of the quadrilateral, then the quadrilateral is an harmonic quadrilateral.

Remark 2. From Propositions 2 and 3 above, it results a simple way to build an harmonic quadrilateral. In a circle, let a triangle ABC be considered; we construct the simedian of *A*, be it AK, and we denote by *D* the intersection of the simedian AK with the circle. The quadrilateral $ABCD$ is an harmonic quadrilateral.

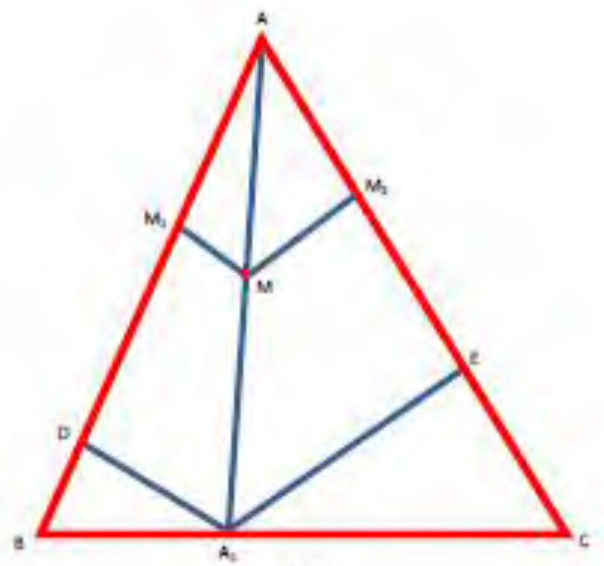


Fig. 2

Proposition 4. In a triangle ABC, the points of the simedian of *A* are situated at proportional lengths to the sides AB and AC.

Proof. We have the simedian AA_1 in the triangle ABC (see *Fig. 2*). We denote by D and E the projections of A_1 on $AB,$ and AC respectively.
We get:

$$\frac{BA_1}{CA_1} = \frac{Area_\Delta(ABA_1)}{Area_\Delta(ACA_1)} = \frac{AB \cdot A_1D}{AC \cdot A_1E}.$$

Moreover, from Proposition 1, we know that

$$\frac{BA_1}{A_1C} = \left(\frac{AB}{AC}\right)^2.$$

Substituting in the previous relation, we obtain that:

$$\frac{A_1D}{A_1E} = \frac{AB}{AC}.$$

On the other hand, $DA_1 = AA_1$. From BAA_1 and $A_1E = AA_1 \cdot sin\widehat{CAA_1}$, hence:

$$\frac{A_1D}{A_1E} = \frac{sin\widehat{BAA_1}}{sin\widehat{CAA_1}} = \frac{AB}{AC} \quad (4).$$

If *M* is a point on the simedian and MM_1 and MM_2 are its projections on AB, and AC respectively, we have:

$$MM_1 = AM \cdot sin\widehat{BAA_1}, \qquad MM_2 = AM \cdot sin\widehat{CAA_1},$$

hence:

$$\frac{MM_1}{MM_2} = \frac{\sin\widehat{BAA_1}}{\sin\widehat{CAA_1}}.$$

Taking (4) into account, we obtain that:

$$\frac{MM_1}{MM_2} = \frac{AB}{AC}.$$

Remark 3. The converse of the property in the statement above is valid, meaning that, if M is a point inside a triangle, its distances to two sides are proportional to the lengths of these sides. The point belongs to the simedian of the triangle having the vertex joint to the two sides.

Proposition 5. In an harmonic quadrilateral, the point of intersection of the diagonals is located towards the sides of the quadrilateral to proportional distances to the length of these sides.
The **Proof** of this Proposition relies on Propositions 2 and 4.

Proposition 6 (R. Tucker). The point of intersection of the diagonals of an harmonic quadrilateral minimizes the sum of squares of distances from a point inside the quadrilateral to the quadrilateral sides.

Proof. Let $ABCD$ be an harmonic quadrilateral and M any point within. We denote by x, y, z, u the distances of M to the AB, BC, CD, DA sides of lenghts $a, b, c,$ and d (see *Fig. 3*).
Let S be the $ABCD$ quadrilateral area.
We have:

$$ax + by + cz + du = 2S.$$

This is true for x, y, z, u and a, b, c, d real numbers.
Following Cauchy-Buniakowski-Schwarz Inequality, we get:

$$(a^2 + b^2 + c^2 + d^2)(x^2 + y^2 + z^2 + u$$

and it is obvious that:

$$x^2 + y^2 + z^2 + u^2 \geq \frac{4S^2}{a^2 + b^2 + c^2 + d}$$

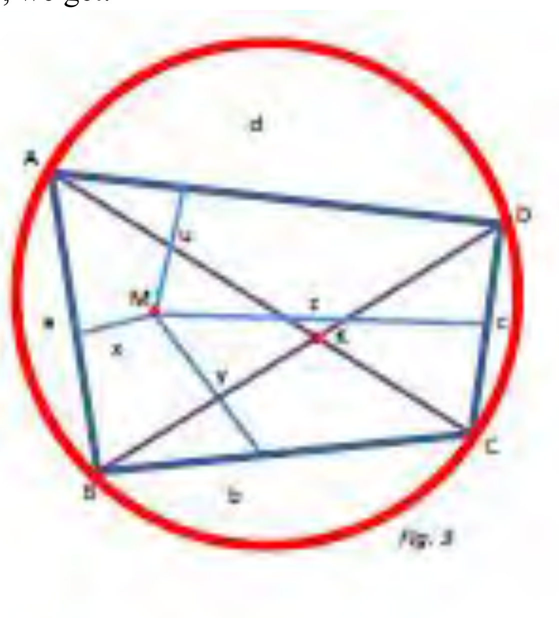
We note that the minimum sum of squared distances is:

$$\frac{4S^2}{a^2 + b^2 + c^2 + d^2} = const.$$

In Cauchy-Buniakowski-Schwarz Inequality, the equality occurs if:

$$\frac{x}{a} = \frac{y}{b} = \frac{z}{c} = \frac{u}{d}.$$

Since $\{K\} = AC \cap BD$ is the only point with this property, it ensues that $M = K$, so K has the property of the minimum in the statement.



Definition 3. We call external simedian of ABC triangle a cevian AA_1' corresponding to the vertex A, where A_1' is the harmonic conjugate of the point A_1 – simedian's foot from A relative to points B and C.

Remark 4. In *Fig. 4,* the cevian AA_1 is an internal simedian, and AA_1' is an external simedian.
We have:

$$\frac{A_1B}{A_1C} = \frac{A_1'B}{A_1'C}.$$

In view of Proposition 1, we get that:

$$\frac{A_1'B}{A_1'C} = \left(\frac{AB}{AC}\right)^2.$$

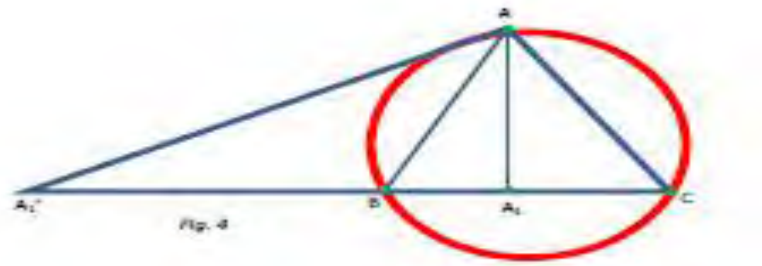


Fig. 4

Proposition 7. The tangents taken to the extremes of a diagonal of a circle circumscribed to the harmonic quadrilateral intersect on the other diagonal.

Proof. Let P be the intersection of a tangent taken in D to the circle circumscribed to the harmonic quadrilateral $ABCD$ with AC (see *Fig. 5*). Since triangles PDC and PAD are alike, we conclude that:

$$\frac{PD}{PA} = \frac{PC}{PD} = \frac{DC}{AD} \quad (5).$$

From relations (5), we find that:

$$\frac{PA}{PC} = \left(\frac{AD}{DC}\right)^2 \quad (6).$$

This relationship indicates that P is the harmonic conjugate of *K* with respect to *A* and *C*, so DP is an external simedian from *D* of the triangle ADC.
Similarly, if we denote by P' the intersection of the tangent taken in *B* to the circle circumscribed with AC, we get:

$$\frac{P'A}{P'C} = \left(\frac{BA}{BC}\right)^2 \quad (7).$$

From (6) and (7), as well as from the properties of the harmonic quadrilateral, we know that:

$$\frac{AB}{BC} = \frac{AD}{DC},$$

which means that:

$$\frac{PA}{PC} = \frac{P'A}{P'C},$$

hence $P = P'$.
Similarly, it is shown that the tangents taken to *A* and *C* intersect at point *Q* located on the diagonal BD.

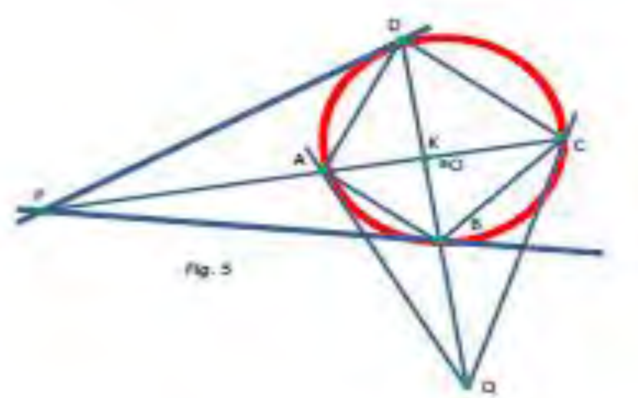


Fig. 5

Remark 5. a. The points P and Q are the diagonal poles of BD and AC in relation to the circle circumscribed to the quadrilateral.
b. From the previous Proposition, it follows that in a triangle the internal simedian of an angle is consecutive to the external simedians of the other two angles.

Proposition 8. Let $ABCD$ be an harmonic quadrilateral inscribed in the circle of center O and let P and Q be the intersections of the tangents taken in B and D, respectively in A and C to the circle circumscribed to the quadrilateral. If $\{K\} = AC \cap BD$, then the orthocenter of triangle PKQ is O.

Proof. From the properties of tangents taken from a point to a circle, we conclude that $PO \perp BD$ and $QO \perp AC$. These relations show that in the triangle PKQ, PO and QO are heights, so O is the orthocenter of this triangle.

Definition 4. The Apollonius circle related to the vertex A of the triangle ABC is the circle built on the segment $[DE]$ in diameter, where D and E are the feet of the internal, respectively ,external bisectors taken from A to the triangle ABC.

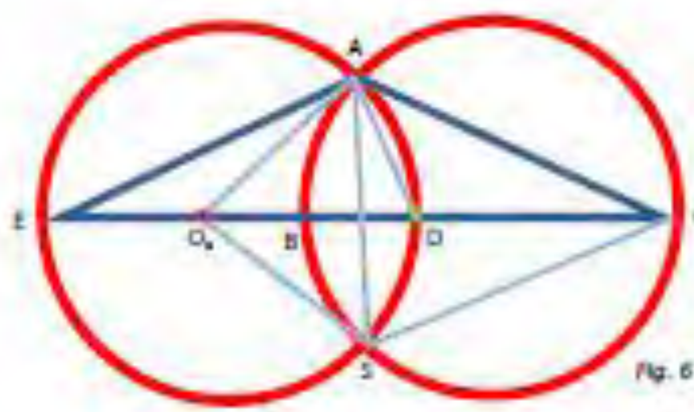


Fig. 6

Remark 6. If the triangle ABC is isosceles with $AB = AC$, the Apollonius circle corresponding to vertex A is not defined.

Proposition 9. The Apollonius circle relative to the vertex A of the triangle ABC has as center the feet of the external simedian taken from A.

Proof. Let O_a be the intersection of the external simedian of the triangle ABC with BC (see *Fig. 6*).
Assuming that $m(\hat{B}) > m(\hat{C})$, we find that $m(\widehat{EAB}) = \frac{1}{2}[m(\hat{B}) + m(\hat{C})]$.
O_a being a tangent, we find that $m(\widehat{O_aAB}) = m(\hat{C})$.
Withal, $m(EAO_a) = \frac{1}{2}[m(\hat{B}) - m(\hat{C})]$ and $m(AEO_a) = \frac{1}{2}[m(\hat{B}) - m(\hat{C})]$.
It results that:
$$OaE = OaA\,;$$
onward, EAD being a right angled triangle, we obtain:
$$OaA = OaD,$$
hence O_a is the center of Apollonius circle corresponding to the vertex A.

Proposition 10. Apollonius circle relative to the vertex A of triangle ABC cuts the circle circumscribed to the triangle following the internal simedian taken from A.

Proof. Let S be the second point of intersection of Apollonius circles relative to vertex A and the circle circumscribing the triangle ABC.

Because O_aA is tangent to the circle circumscribed in A, it results, for reasons of symmetry, that O_aS will be tangent in *S* to the circumscribed circle.
For triangle ACS, O_aA and O_aS are external simedians; it results that CO_a is internal simedian in the triangle ACS, furthermore, it results that the quadrilateral $ABSC$ is an harmonic quadrilateral.
Consequently, AS is the internal simedian of the triangle ABC and the property is proven.
Remark 7. From this, in view of *Fig. 5*, it results that the circle of center *Q* passing through *A* and *C* is an Apollonius circle relative to the vertex *A* for the triangle ABD.
This circle (of center *Q* and radius *QC*) is also an Apollonius circle relative to the vertex *C* of the triangle BCD.
Similarly, the Apollonius circles corresponding to vertexes *B* and *D* and to the triangles ABC, and ADC respectively, coincide; we can formulate the following:
Proposition 11. In an harmonic quadrilateral, the Apollonius circles - associated with the vertexes of a diagonal and to the triangles determined by those vertexes to the other diagonal - coincide.
Radical axis of the Apollonius circles is the right determined by the center of the circle circumscribed to the harmonic quadrilateral and by the intersection of its diagonals.
Proof. Referring to *Fig. 5*, we observe that the power of *O* towards the Apollonius circles relative to vertexes *B* and *C* of triangles ABC and BCU is:
$OB^2 = OC^2$.
So *O* belongs to the radical axis of the circles.
We also have $KA \cdot KC = KB \cdot KD$, relatives indicating that the point *K* has equal powers towards the highlighted Apollonius circles.

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NON-CONGRUENT TRIANGLES WITH EQUAL PERIMETERS AND ARIAS

ION PATRASCU and FLORENTIN SMARANDACHE

In [1] Professor I. Ivănescu from Craiova has proposed the following

Open problem

Construct, using a ruler and a compass, two non-congruent triangles, which have equal perimeters and arias.

In preparation for the proof of this problem we recall several notions and we prove a Lemma.

Definition

An A-ex-inscribed circle to a given triangle ABC is the tangent circle to the side (BC) and to the extended sides (AB), (AC).

The center of the A-ex-inscribed triangle is the intersection of the external bisectors of the angles B and C, which we note it with I_a and its radius with r_a.

Observation 1.

To a given triangle correspond three ex-inscribed circles. In figure 1 we represent the A-ex-inscribed circle to triangle ABC.

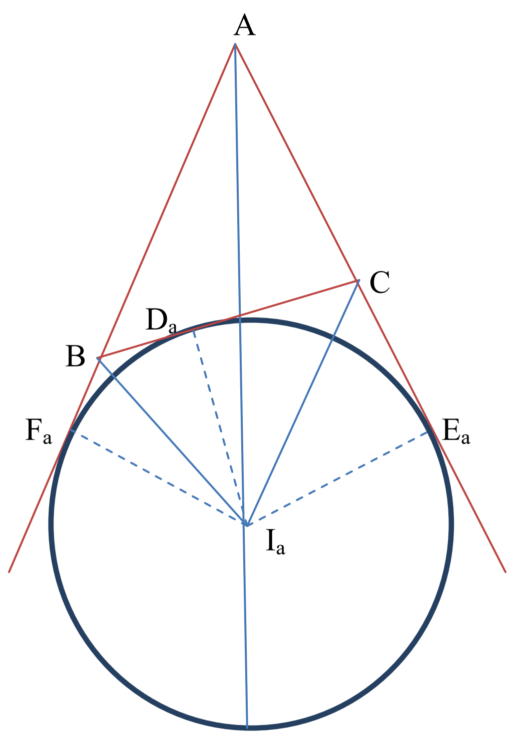

Fig. 1

Lemma 1

The length of the tangent constructed from one of the triangle's vertexes to the corresponding ex-inscribed circle is equal with the triangle's semi-perimeter.

Proof

Let D_a, E_a, F_a the points of contact of the A-ex-inscribed triangle with (BC), AC, AB. We have $AE_a = AF_a$, $BD_a = BF_a$, $CD_a = CE_a$ (the tangents constructed from a point to a circle are congruent). We note $BD_a = x$, $CD_a = y$ and we observe that $AE_a = AC + CE_a$, therefore $AE_a = b + y$, $AF_a = AB + BF_a$, it results that $AF_a = c + x$. We resolve the system:

$$\begin{cases} x + y = a \\ x + c = y + b \end{cases}$$

and we obtain

$$x = \frac{1}{2}(a + b - c)$$

$$y = \frac{1}{2}(a + c - b)$$

Taking into consideration that the semi-perimeter $p = \frac{1}{2}(a + b + c)$ we have $x = p - c;\ y = p - b$, and we obtain that $AF_a = AE_a = p$ thus the lemma is proved.

The proof of the open problem

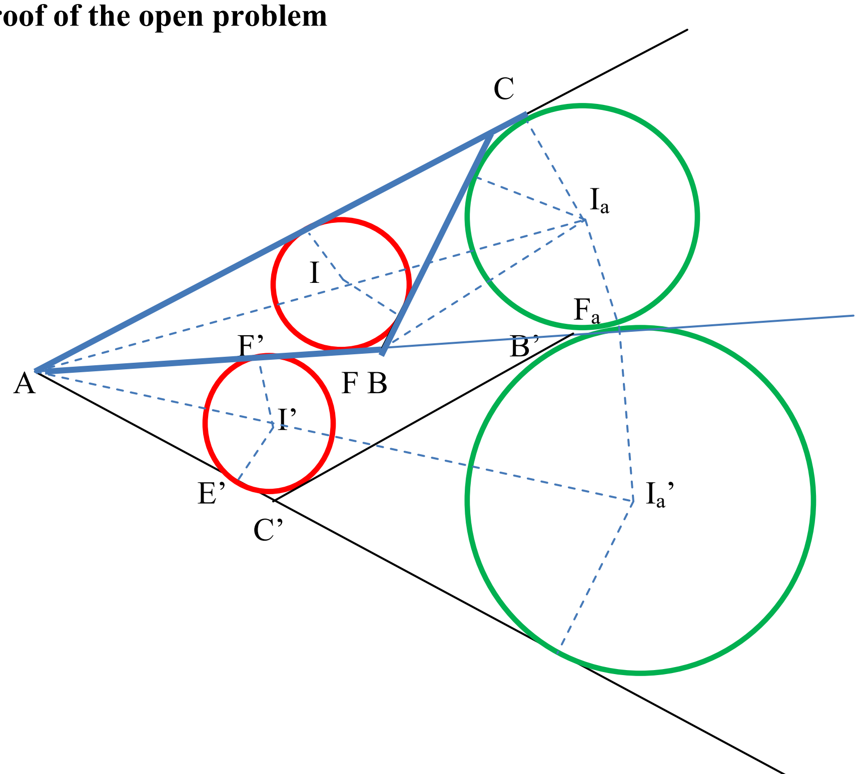

Fig.2

Let ABC a given triangle. We construct $\mathcal{C}(I,r)$ its inscribed circle and $\mathcal{C}(I_a,r_a)$ its A-ex-inscribed circle, see figure 2. In conformity with the Lemma we have that $AF_a = p$ - the semi-perimeter of triangle ABC.

We construct the point $F' \in (AF)$ and the circle of radius r tangent in F' to AB, that is $\mathcal{C}(I',r)$. It is easy to justify that angle *F'AI'* > angle *FAI* and therefore angle *F'AE'* > angle *A* (we noted *E'* the contact point with the circle $\mathcal{C}(I',r)$ of the tangent constructed from A). We note $I_a^{'}$ the intersection point of the lines AI', I_aF_a.

We construct the circle $\mathcal{C}(I_a^{'}I_a^{'}F_a)$ and then the internal common tangent to this circle and to the circle $\mathcal{C}(I',r)$; we note B',C' the intersections of this tangent with AB respectively with AE'. From these constructions it result that the circle $\mathcal{C}(I',r)$ is inscribed in the triangle $AB'C'$ and the circle $\mathcal{C}(I_a^{'}I_a^{'}F_a)$ ex-inscribed to this triangle.

The Lemma states that the semi-perimeter of the triangle $AB'C'$ is equal with AF_a therefore it is equal to p - the semi-perimeter of triangle ABC.

On the other side the inscribed circles in the triangles ABC and $AB'C'$ are congruent. Because the aria S of the triangle ABC is given by the formula $S = p \cdot r$, we obtain that also the aria of triangle $AB'C'$ is equal with S.

The constructions listed above can be executed with a ruler and a compass without difficulty, and the triangles ABC and $AB'C'$ are not congruent.

Indeed, our constructions are such that the angle *B'AC'* is greater than angle *BAC*. Also we can choose F' on (AF) such that $F'AI'$ is different of $\frac{1}{2}\mathrm{C}$ and of $\frac{1}{2}\mathrm{B}$. In this way the angle A of the triangle $AB'C'$ is not congruent with any angle of the triangle ABC.

Observation 2

We practically proved much more than the proposed problem asked, because we showed that for any given triangle ABC we can construct another triangle which will have the same aria and the same perimeter with the given triangle without being congruent with it.

Observation 3

In [2] the authors find two isosceles triangles in the conditions of the hypothesis.

Note

The authors thank to Professor Ştefan Brânzan from the National College "Fraţii Buzeşti" – Craiova for his suggestions, which made possible the enrichment of this article.

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ANOTHER PROOF OF THE I. PATRASCU'S THEOREM

FLORENTIN SMARANDACHE

Abstract. In this note the author presents a new proof for the theorem of **I. Pătraşcu**.
Keywords: median, symmedian, Brocard′s points.
MSC 2010: 97G40.

In [**1**], **Ion Pătraşcu** proves the following

Theorem. *The Brocard′s point of an isosceles triangle is the intersection of a median and the symmedian constructed from the another vertex of the triangle′s base, and reciprocal.*

We′ll provide below a different proof of this theorem than the proof given in [**1**] and [**2**].

We′ll recall the following definitions:
Definition 1. The symmetric cevian of the triangle′s median in rapport to the bisector constructed from the same vertex is called the triangle′s symmedian.
Definition 2. The points Ω, Ω' from the plane of the triangle ABC with the property $\widehat{\Omega BA} \equiv \widehat{\Omega AC} \equiv \widehat{\Omega CB}$, respectively $\widehat{\Omega' AB} \equiv \widehat{\Omega' BC} \equiv \widehat{\Omega' CA}$, are called the Brocard′s points of the given triangle.

Remark. In an arbitrary triangle there exist two Brocard′s points.

Proof of the Theorem. Let ABC an isosceles triangle, $AB = AC$, and Ω the Brocard′s point, therefore $\widehat{\Omega BA} \equiv \widehat{\Omega AC} \equiv \widehat{\Omega CB} = \omega$.

We′ll construct the circumscribed circle to the triangle $B\Omega C$. Having $\widehat{\Omega BA} \equiv \widehat{\Omega CB}$ and $\widehat{\Omega CA} \equiv \widehat{\Omega BC}$, it results that this circle is tangent in B, respectively in C to the sides AB, respectively AC.

We note M the intersection point of the line $B\Omega$ with AC and with N the intersection point of the lines $C\Omega$ and AB. From the similarity of the triangles ABM, ΩAM, we obtain

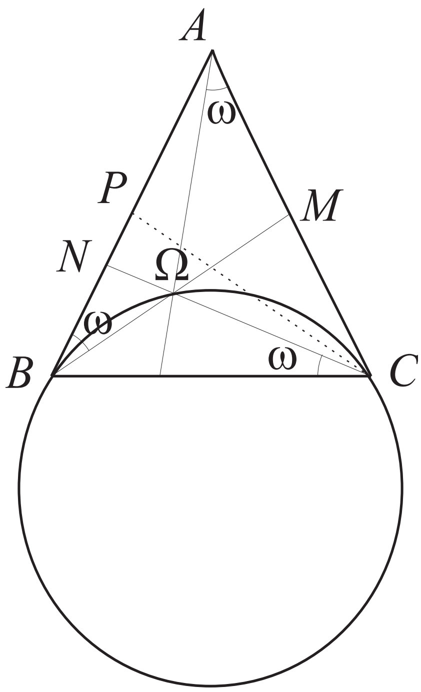
$$MB \cdot M\Omega = AM^2. \tag{1}$$

Considering the power of the point M in rapport to the constructed circle, we obtain

$$MB \cdot M\Omega = MC^2. \tag{2}$$

From the relations (1) and (2) it results that $AM = MC$, therefore, BM is a median.

If CP is the median from C of the triangle, then from the congruency of the triangles ABM, ACP we find that $\widehat{ACP} \equiv \widehat{ABM} = \omega$. It results that the cevian CN is a symmedian and the direct theorem is proved.



We'll prove the reciprocal of this theorem. In the triangle ABC is known that the median BM and the symmedian CN intersect in the Brocard's point Ω. We'll construct the circumscribed circle to the triangle $B\Omega C$. We observe that because

$$\widehat{\Omega BA} \equiv \widehat{\Omega CB}, \tag{3}$$

this circle is tangent in B to the side AB. From the similarity of the triangles ABM, ΩAM it results $AM^2 = MB \cdot M\Omega$. But $AM = MC$, it results that $MC^2 = MB \cdot M\Omega$. This relation shows that the line AC is tangent in C to the circumscribed circle to the triangle $B\Omega C$, therefore

$$\widehat{\Omega BC} \equiv \widehat{\Omega CA}. \tag{4}$$

By adding up relations (3) and (4) side by side, we obtain $\widehat{ABC} \equiv \widehat{ACB}$, consequently, the triangle ABC is an isosceles triangle.

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Published in "Recreaţii Matematice", Iaşi, No. 2, 114-115, 2013.

LUHN PRIME NUMBERS

OCTAVIAN CIRA and FLORENTIN SMARANDACHE

ABSTRACT. The first prime number with the special property that its addition with its reversal gives as result a prime number too is 229. The prime numbers with this property will be called *Luhn prime numbers*. In this article we intend to present a performing algorithm for determining the *Luhn prime numbers*. Using the presented algorithm all the 50598 *Luhn prime numbers* have been, for p prime smaller than $2 \cdot 10^7$.

1. INTRODUCTION

The number 229 is the smallest prime number that added with his reverse gives as result a prime number, too. As $1151 = 229 + 922$ is prime.

The first that noted this special property the number 229 has, was Norman Luhn (after 9 February 1999), on the *Prime Curios* website [2]. The prime numbers with this property will be later called *Luhn prime numbers*.

In the *Whats Special About This Number?* list [3], a list that contains all the numbers between 1 and 9999; beside the number 229 is mentioned that his most important property is that, adding with his reversal the resulting number is prime too.

The *On-Line Encyclopedia of Integer Sequences*, [6, A061783], presents a list 1000 *Luhn prime numbers*. We owe this list to Harry J. Smith, since 28 July 2009. On the same website it is mentioned that Harvey P. Dale published on 27 November 2010 a list that contains 3000 *Luhn prime numbers* and Bruno Berselli published on 5 August 2013 a list that contains 2400 *Luhn prime numbers*.

2. SMARANDACHE'S FUNCTION

The function $\mu : \mathbb{N}^* \to \mathbb{N}^*$, $\mu(n) = m$, where m is the smallest natural number with the property that $n \mid m!$ (or $m!$ is a multiple of n) is know in the specialty literature as Smarandache's function, [7, 8]. The values resulting from $n = 1, 2, \ldots, 18$ are: 1, 2, 3, 4, 5, 3, 7, 4, 6, 5, 11, 4, 13, 7, 5, 6, 17, 6. These values were obtained with an algorithm that results from μ's definition. The program using this algorithm cannot be used for $n \geq 19$ because the numbers $19!$, $20!, \ldots$ are numbers which exceed the 17 decimal digits limit and the classic computing model (without the arbitrary precisions arithmetic [10]) will generate errors due to the way numbers are represented in the computers memory.

3. Kempner's algorithm

Kempner created an algorithm to calculate $\mu(n)$ using classical factorization $n = p_1^{p_1} \cdot p_2^{p_2} \cdots p_s^{p_s}$, prime number and the generalized numeration base $(\alpha_i)_{[p_i]}$, for $i = \overline{1,s}$, [4]. Partial solutions to the algorithm for $\mu(n)$'s calculation have been given earlier by Lucas and Neuberg, [9].

Remark 3.1. If $n \in \mathbb{N}^*$, n can be decomposed in a product of prime numbers $n = p_1^{\alpha_1} \cdot p_2^{\alpha_2} \cdots p_s^{\alpha_s}$, were p_i are prime numbers so that $p_1 < p_2 < \ldots < p_s$, and $s \geq 1$, thus Kempner's algorithm for calculating the μ function is.

$$\mu(n) = \max\left\{p_1 \cdot \left({\alpha_1}_{[p_1]}\right)_{(p_1)},\ p_2 \cdot \left({\alpha_2}_{[p_2]}\right)_{(p_2)},\ \ldots,\ p_s \cdot \left({\alpha_s}_{[p_s]}\right)_{(p_s)}\right\},$$

where by $\left(\alpha_{[p]}\right)_{(p)}$ we understand that α is *"written"* in the numeration base p (noted $\alpha_{[p]}$) and it is *"read"* in the p numeration base (noted $\beta_{(p)}$, were $\beta = \alpha_{[p]}$), [8, p. 39].

4. Programs

The list of prime numbers was generated by a program that uses the Sieve of Eratosthenes the linear version of Pritchard, [5], which is the fastest algorithm to generate prime numbers until the limit of L, if $L \leq 10^8$. The list of prime numbers until to $2 \cdot 10^7$ is generated in about 5 seconds. For the limit $L > 10^8$ the fastest algorithm for generating the prime numbers is the Sieve of Atkin, [1].

Program 4.1. The Program for the Sieve of Eratosthenes, the linear version of Pritchard using minimal memory space is:

$$CEPbm(L) := \left| \begin{array}{l} \lambda \leftarrow floor\left(\frac{L}{2}\right) \\ for\ k \in 1..\lambda \\ \quad is\_prime_k \leftarrow 1 \\ prime \leftarrow (2\ 3\ 5\ 7)^{\mathrm{T}} \\ i \leftarrow last(prime) + 1 \end{array} \right.$$

$$
\begin{array}{|l}
for\ j \in 4, 7..\lambda \\
\quad is\_prime_j \leftarrow 0 \\
k \leftarrow 3 \\
s \leftarrow (prime_{k-1})^2 \\
t \leftarrow (prime_k)^2 \\
while\ t \leq L \\
\quad \begin{array}{|l}
for\ j \in t, t + 2 \cdot prime_k..L \\
\quad is\_prime_{\frac{j-1}{2}} \leftarrow 0 \\
for\ j \in s + 2, s + 4..t - 2 \\
\quad if\ is\_prime_{\frac{j-1}{2}} = 1 \\
\qquad \begin{array}{|l} prime_i \leftarrow j \\ i \leftarrow i + 1 \end{array} \\
s \leftarrow t \\
k \leftarrow k + 1 \\
t \leftarrow (prime_k)^2
\end{array} \\
for\ j \in s + 2, s + 4..L \\
\quad if\ is\_prime_{\frac{j-1}{2}} = 1 \\
\qquad \begin{array}{|l} prime_i \leftarrow j \\ i \leftarrow i + 1 \end{array} \\
return\ prime
\end{array}
$$

Program 4.2. The factorization program of a natural number; this program uses the vector p representing prime numbers, generated with the Sieve of Eratosthenes. The Sieve of Eratosthenes is called upon trough the following sequence:

$$L := 2 \cdot 10^7 \quad t_0 = time(0) \quad p := CEPbm(L) \quad t_1 = time(1)$$

$$(t_1 - t_0)s = 5.064s \quad last(p) = 1270607 \quad p_{last(p)} = 19999999$$

$$
Fa(m) := \begin{array}{|l}
return\ (\text{"}m = \text{"}\ m\ \text{"} > ca\ ultimul\ p^{2}\text{"})\ if\ m > (p_{last(p)})^2 \\
j \leftarrow 1 \\
k \leftarrow 0 \\
f \leftarrow (1\ 1) \\
while\ m \geq p_j \\
\quad \begin{array}{|l}
if\ \ \mod(m, p_j) = 0 \\
\quad \begin{array}{|l} k \leftarrow k + 1 \\ m \leftarrow \dfrac{m}{p_j} \end{array} \\
otherwise \\
\quad \begin{array}{|l} f \leftarrow stack[f, (p_j, k)]\ if\ k > 0 \\ j \leftarrow j + 1 \\ k \leftarrow 0 \end{array}
\end{array}
\end{array}
$$

$$\left| \begin{array}{l} f \leftarrow stack[f, (p_j, k)]\ if\ k > 0 \\ return\ submatrix(f, 2, rows(f), 1, 2) \end{array} \right.$$

We presented the Kempner's algorithm using Mathcad programs required for the algorithm.

Program 4.3. The function counting all the digits in the p base of numeration in which is n.

$$ncb(n,p) := \left| \begin{array}{l} return\ ceil(\log(n,p))\ if\ n > 1 \\ return\ 1\ otherwise \end{array} \right.$$

Where the $ceil(\cdot)$ Mathcad function represents the upper non-decimal number.

Program 4.4. The program intended to generate the p generalized base of numeration (noted by Smarandache $[p]$) for a number with m digits.

$$a(p,m) := \left| \begin{array}{l} for\ i \in 1..m \\ \quad a_i \leftarrow \dfrac{p^i - 1}{p - 1} \\ return\ a \end{array} \right.$$

Program 4.5. The program intended to generate for the p base of numeration (noted by Smarandache (p)) to write the α number.

$$b(\alpha,p) := \left| \begin{array}{l} return\ (1)\ if\ p = 1 \\ for\ i \in 1..ncb(\alpha,p) \\ \quad b_i \leftarrow p^{i-1} \\ return\ b \end{array} \right.$$

Program 4.6. Program that determines the digits of the generalized base of numeration $[p]$ for the number n.

$$Nbg(n,p) := \left| \begin{array}{l} m \leftarrow ncb(n,p) \\ a \leftarrow a(p,m) \\ return\ (1)\ if\ m{=}0 \\ for\ i \in m..1 \\ \quad \left| \begin{array}{l} c_i \leftarrow trunc\left(\dfrac{n}{a_i}\right) \\ n \leftarrow \mod(n, a_i) \end{array} \right. \\ return\ c \end{array} \right.$$

Program 4.7. Program for Smarandache's function.

$$\mu(n) := \left| \begin{array}{l} return\ "Err.\ n\ nu\ este\ intreg"\ if\ n \neq trunc(n) \\ return\ "Err.\ n < 1"\ if\ n < 1 \\ return\ (1)\ if\ n{=}1 \\ f \leftarrow Fa(n) \end{array} \right.$$

$$\left| \begin{array}{l} p \leftarrow f^{\langle 1 \rangle} \\ \alpha \leftarrow f^{\langle 2 \rangle} \\ for\ k = 1..rows(p) \\ \quad \eta_k \leftarrow p_k \cdot Nbg(\alpha_k, p_k) \cdot b(\alpha_k, p_k) \\ return\ \max(\eta) \end{array} \right.$$

This program calls the $Fa(n)$ factorization with prime numbers. The program uses the Smarandache's 3.1 Remark – about the Kempner algorithm. The $\mu.prn$ file generation is done once. The reading of this generated file in Mathcad's documents results in a great time–save.

Program 4.8. Program with which the file $\mu.prn$ is generated

$$VF\mu(N) := \left| \begin{array}{l} \mu_1 \leftarrow 1 \\ for\ n \in 2..N \\ \quad \mu_n \leftarrow \mu(n) \\ return\ \mu \end{array} \right.$$

This program calls the 4.7 program for calculating the value of the μ function. The sequence of the $\mu.prn$ file generation is:

$$t_0 := time(0) \;\; WRITEPRN("\mu.prn") := VF\mu(2 \cdot 10^7) \;\; t_1 := time(1)$$

$$(t_1 - t_0)sec = "5:17:32.625"hhmmss$$

Smarandache's function is important because it characterizes prime numbers – through the following fundamental property:

Teorema 4.9. *Let be p an integer > 4, than p is prime number if and only if $\mu(p) = p$.*

Proof. See [8, p. 31]. □

Hence, the fixed points of this function are prime numbers (to which is added 4). Due to this property the function is used as primality test.

Program 4.10. Program for testing μ's primality. This program returns the 0 value if the number is not prime number and the 1 value if the number is a prime. The file $\mu.prn$ will be read and it will be assigned to the μ vector.

$$ORIGIN := 1 \quad \mu := READPRN(" \ldots \backslash \mu.prn")$$

$$Tp\mu(n) := \left| \begin{array}{l} return\ "Err.\ n < 1\ sau\ n \notin \mathbb{Z}"\ if\ n < 1 \vee n \neq trunc(n) \\ if\ n > 4 \\ \quad \left| return\ 0\ if\ \mu_n \neq n \right. \end{array} \right.$$

$$\begin{array}{l} \quad \left| \; \left| return\ 1\ otherwise \right. \right. \\ \quad \left| otherwise \right. \\ \quad \left| \quad \left| return\ 0\ if\ n{=}1 \vee n{=}4 \right. \right. \\ \quad \left| \quad \left| return\ 1\ otherwise \right. \right. \end{array}$$

Program 4.11. Program that provides the reveres of the given m number.

$$R(m) := \left| \begin{array}{l} n \leftarrow floor(\log(m)) \\ x \leftarrow m \cdot 10^{-n} \\ for\ k \in 1..n \\ \quad \left| \begin{array}{l} c_k \leftarrow trunc(x) \\ x \leftarrow (x - c_k) \cdot 10 \end{array} \right. \\ c_{n+1} \leftarrow round(x) \\ Rm \leftarrow 0 \\ for\ k \in n+1..2 \\ \quad Rm \leftarrow (Rm + c_k) \cdot 10 \\ return\ Rm + c_1 \end{array} \right.$$

Program 4.12. Search program for the *Luhn prime numbers*.

$$PLuhn(L) := \left| \begin{array}{l} n \leftarrow last(p) \\ S \leftarrow (229) \\ k \leftarrow 51 \\ while\ p_k \le L \\ \quad \left| \begin{array}{l} N \leftarrow R(p_k) + p_k \\ S \leftarrow stack(S, p_k)\ if\ Tp\mu(N) = 1 \\ k \leftarrow k + 1 \end{array} \right. \\ return\ S \end{array} \right.$$

The initialization of the S stack is done with the vector that contains the number 229. The variable k has the initial value of 51 because the index of the 229 prime number is 50, so that the search for the *Luhn prime numbers* will begin with $p_{51} = 233$.

5. List of prime numbers Luhn

We present a partial list of the 50598 *Luhn prime numbers* smaller than $2 \cdot 10^7$ (the first 321 and the last 120):
229 239 241 257 269 271 277 281 439 443 463 467 479 499 613 641 653 661 673 677 683 691 811 823 839 863 881 20011 20029 20047 20051 20101 20161 20201 20249 20269 20347 20389 20399 20441 20477 20479 20507 20521 20611 20627 20717 20759 20809 20879 20887 20897 20981 21001 21019 21089 21157 21169 21211 21377 21379 21419 21467 21491 21521 21529 21559 21569 21577 21601 21611 21617 21647 21661 21701 21727 21751 21767 21817 21841 21851 21859 21881 21961 21991 22027

22031 22039 22079 22091 22147 22159 22171 22229 22247 22291 22367
22369 22397 22409 22469 22481 22501 22511 22549 22567 22571 22637
22651 22669 22699 22717 22739 22741 22807 22859 22871 22877 22961
23017 23021 23029 23081 23087 23099 23131 23189 23197 23279 23357
23369 23417 23447 23459 23497 23509 23539 23549 23557 23561 23627
23689 23747 23761 23831 23857 23879 23899 23971 24007 24019 24071
24077 24091 24121 24151 24179 24181 24229 24359 24379 24407 24419
24439 24481 24499 24517 24547 24551 24631 24799 24821 24847 24851
24889 24979 24989 25031 25057 25097 25111 25117 25121 25169 25171
25189 25219 25261 25339 25349 25367 25409 25439 25469 25471 25537
25541 25621 25639 25741 25799 25801 25819 25841 25847 25931 25939
25951 25969 26021 26107 26111 26119 26161 26189 26209 26249 26251
26339 26357 26417 26459 26479 26489 26591 26627 26681 26701 26717
26731 26801 26849 26921 26959 26981 27011 27059 27061 27077 27109
27179 27239 27241 27271 27277 27281 27329 27407 27409 27431 27449
27457 27479 27481 27509 27581 27617 27691 27779 27791 27809 27817
27827 27901 27919 28001 28019 28027 28031 28051 28111 28229 28307
28309 28319 28409 28439 28447 28571 28597 28607 28661 28697 28711
28751 28759 28807 28817 28879 28901 28909 28921 28949 28961 28979
29009 29017 29021 29027 29101 29129 29131 29137 29167 29191 29221
29251 29327 29389 29411 29429 29437 29501 29587 29629 29671 29741
29759 29819 29867 29989 . . .
8990143 8990209 8990353 8990441 8990563 8990791 8990843 8990881
8990929 8990981 8991163 8991223 8991371 8991379 8991431 8991529
8991553 8991613 8991743 8991989 8992069 8992091 8992121 8992153
8992189 8992199 8992229 8992259 8992283 8992483 8992493 8992549
8992561 8992631 8992861 8992993 8993071 8993249 8993363 8993401
8993419 8993443 8993489 8993563 8993723 8993749 8993773 8993861
8993921 8993951 8994091 8994109 8994121 8994169 8994299 8994463
8994473 8994563 8994613 8994721 8994731 8994859 8994871 8994943
8995003 8995069 8995111 8995451 8995513 8995751 8995841 8995939
8996041 8996131 8996401 8996521 8996543 8996651 8996681 8996759
8996831 8996833 8996843 8996863 8996903 8997059 8997083 8997101
8997463 8997529 8997553 8997671 8997701 8997871 8997889 8997931
8997943 8997979 8998159 8998261 8998333 8998373 8998411 8998643
8998709 8998813 8998919 8999099 8999161 8999183 8999219 8999311
8999323 8999339 8999383 8999651 8999671 8999761 8999899 8999981

6. Conclusions

The list of all *Luhn prime numbers*, that totalized 50598 numbers, was determined within a time span of 54 seconds, on an Intel processor of 2.20 GHz.

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CARDINAL FUNCTIONS AND INTEGRAL FUNCTIONS

MIRCEA E. ŞELARIU, FLORENTIN SMARANDACHE and MARIAN NIŢU

Abstract. This paper presents the correspondences of the eccentric mathematics of cardinal and integral functions and centric mathematics, or ordinary mathematics. Centric functions will also be presented in the introductory section, because they are, although widely used in undulatory physics, little known.

In centric mathematics, cardinal sine and cosine are defined as well as the integrals. Both circular and hyperbolic ones. In eccentric mathematics, all these central functions multiplies from one to infinity, due to the infinity of possible choices where to place a point. This point is called eccenter $S(s, \varepsilon)$ which lies in the plane of unit circle $\mathrm{UC}(O, R = 1)$ or of the equilateral unity hyperbola $\mathrm{HU}(O, a = 1, b = 1)$. Additionally, in eccentric mathematics there are series of other important special functions, as $aex\theta$, $bex\theta$, $dex\theta$, $rex\theta$, etc. If we divide them by the argument θ, they can also become cardinal eccentric circular functions, whose primitives automatically become integral eccentric circular functions.

All supermatematics eccentric circular functions (SFM-EC) can be of variable excentric θ, which are continuous functions in linear numerical eccentricity domain $s \in [-1, 1]$, or of centric variable α, which are continuous for any value of s. This means that $s \in [-\infty, +\infty]$.

Keywords and phrases: C-Circular , CC- C centric, CE- C Eccentric, CEL-C Elevated, CEX-C Exotic, F-Function, FMC-F Centric Mathematics, M- Matemathics, MC-M Centric, ME-M Excentric, S-Super, SM- S Matematics, FSM-F Supermatematics FSM-CE- FSM Eccentric Circulars, FSM-CEL- FSM-C Elevated, FSM-CEC- FSM-CE- Cardinals, FSM-CELC-FSM-CEL Cardinals

(2010) Mathematics Subject Classification: 32A17

1. INTRODUCTION: CENTRIC CARDINAL SINE FUNCTION

According to any standard dictionary, the word "cardinal" is synonymous with "principal", "essential", "fundamental".

In centric mathematics (CM), or ordinary mathematics, cardinal is, on the one hand, a number equal to a number of finite aggregate, called the power of the aggregate, and on the other hand, known as the sine cardinal $sinc(x)$ or cosine cardinal $cosc(x)$, is a special function defined by the centric circular function (CCF). $sin(x)$ and $cos(x)$ are commonly used in undulatory physics (see Figure 1) and whose graph, the graph of cardinal sine, which is called as "Mexican hat" (sombrero) because of its shape (see Figure 2).

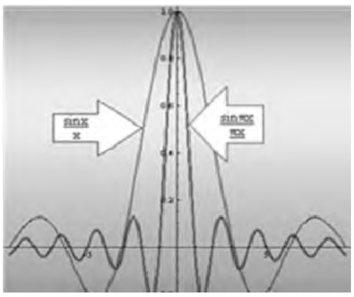
Note that $sinc(x)$ cardinal sine function is given in the speciality literature, in three variants

(1)
$$sinc(x) = \begin{cases} 1, & \text{for } x = 0 \\ \dfrac{sin(x)}{x}, & \text{for } x \in [-\infty, +\infty] \setminus 0 \end{cases}$$
$$= \frac{sin(x)}{x} = 1 - \frac{x^2}{6} + \frac{x^4}{120} - \frac{x^{76}}{5040} + \frac{x^8}{362880} + 0[x]^{11}$$
$$= \sum_{n=0}^{+\infty} \frac{(-1)^n x^{2n}}{(2n+1)!} \rightarrow sinc(\frac{\pi}{2}) = \frac{2}{\pi},$$
$$\frac{d(sinc(x))}{dx} = \frac{cos(x)}{x} - \frac{sin(x)}{x^2} = cosc(x) - \frac{sinc(x)}{x},$$

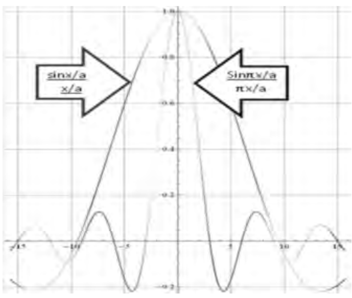
(2)
$$sinc(x) = \frac{sin(\pi x)}{\pi x}$$

(3)
$$sinc_a(x) = \frac{sin(\frac{\pi x}{a})}{\frac{\pi x}{a}}$$

It is a special function because its primitive, called sine integral and denoted $Si(x)$



Centric circular cardinal sine functions



Modified centric circular cardinal sine functions

Figure 1: The graphs of centric circular functions cardinal sine, in 2D, as known in literature

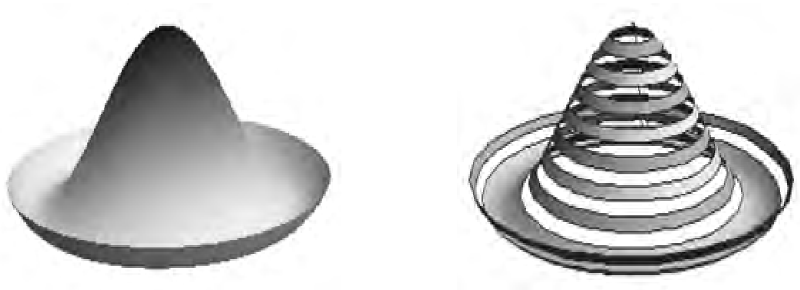


Figure 2: Cardinal sine function in 3D Mexican hat (sombrero)

$$(4)\qquad \begin{aligned} Si(x) &= \int_0^x \frac{sin(t)}{t}dt = \int_0^x sinc(t)\cdot dt \\ &= x - \frac{x^3}{18} + \frac{x^5}{600}\frac{x^7}{35280} + \frac{x^9}{3265920} + 0[x]^{11} \\ &= x - \frac{x^8}{3.3!} + \frac{x^5}{5.5!} - \frac{x7}{7.7!} + \cdots - \ldots \\ &= \sum_{n=0}^{+\infty} \frac{(-1)^n x^{2n}}{(2n+1)^2(2n)!}, \quad \forall x \in \mathbb{R} \end{aligned}$$

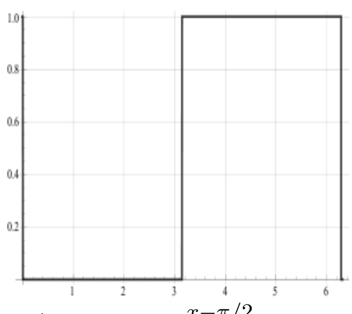
can not be expressed exactly by elementary functions, but only by expansion of power series, as shown in equation (4). Therefore, its derivative is

$$(5)\qquad \forall x \in \mathbb{R},\ Si^{'}(x) = \frac{d(Si(x))}{dx} = \frac{sin(x)}{x} = sinc(x),$$

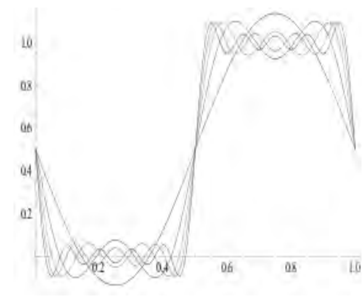
an integral sine function $Si(x)$, that satisfies the differential equation

$$(6)\qquad x\cdot f^{'''}(x) + 2f^{''}(x) + x\cdot f^{\ '}(x) = 0 \rightarrow f(x) = Si(x).$$

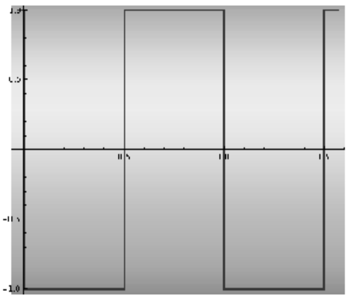
The Gibbs phenomenon appears at the approximation of the square with a continuous and differentiable Fourier series (Figure 3 right ►). This operation could be substitute with the circular eccentric supermathematics functions (CE-SMF), because the eccentric derivative function of eccentric variable θ can express exactly this rectangular function (Figure 3 ▲ top) or square (Figure 3 ▼ below) as shown on their graphs (Figure 3 ◄ left).



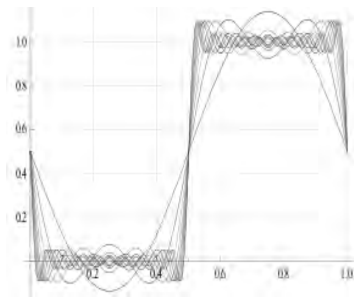
$1 - \cos\frac{x-\pi/2}{\sqrt{1-\sin(x-\pi/2)^2}},$
$\{x, -\pi, 2.01\pi\}$



$\frac{1}{2} - 4x\sum \mathrm{Si}\,nc[2\pi(2k-1)x],$
$\{k, n\}\{n, 5\}\{x, 0, 1\}$



$\frac{1}{2}dex[(\theta, \frac{\pi}{2}), S(1, 0)]$



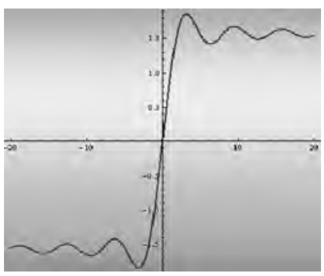
Gibbs phenomenon for a square wave with $n = 5$ and $n = 10$

Figure 3: Comparison between the square function, eccentric derivative and its approximation by **Fourier** serial expansion

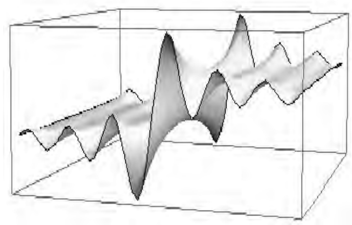
Integral sine function (4) can be approximated with sufficient accuracy. The maximum difference is less than 1%, except the area near the origin. By the CE-SMF eccentric amplitude of eccentric variable θ

$$F(\theta) = 1.57\ aex[\theta, S(0.6, 0)], \tag{7}$$

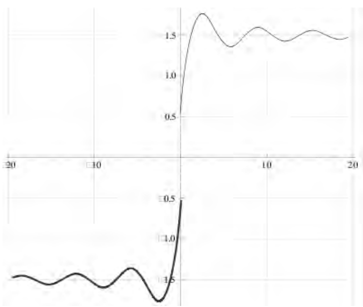
as shown on the graph on Figure 5.



$Sin\int x, \{x, -20, 20\}$



$\mathrm{Sin}\int(x + Iy)$
$\{x, -20, 20\}, \{y, -3, 3\}$



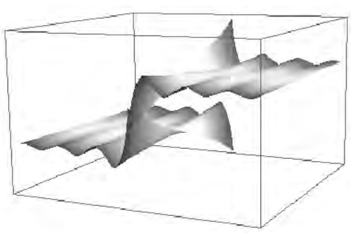


Figure 4: The graph of integral sine function $\mathrm{Si}(x)$ ▲ compared with the graph CE-SMF Eccentric amplitude $1,57aex[\theta, S(0,6;0)]$ of eccentric variable θ▼

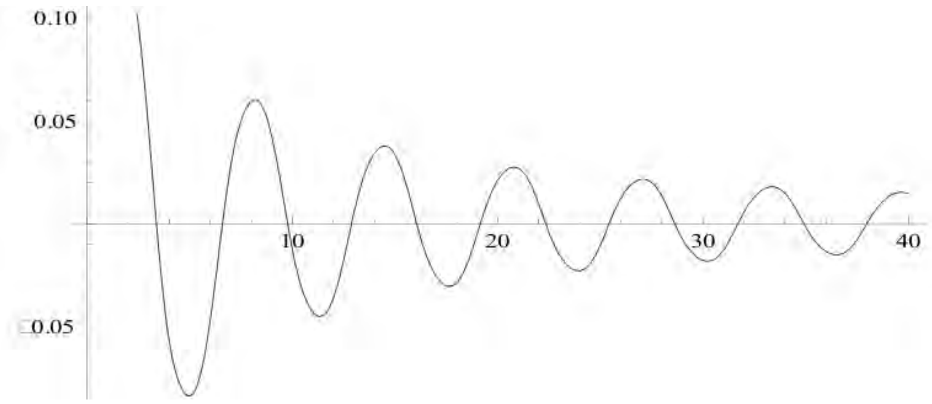


Figure 5: The difference between integral sine and CE-SMF eccentric amplitude $F(\theta) = 1,5aex[\theta, S(0,6;0]$ of eccentric variable θ

2. ECCENTRIC CIRCULAR SUPERMATHEMATICS CARDINAL FUNCTIONS, CARDINAL ECCENTRIC SINE (ECC-SMF)

Like all other supermathematics functions (SMF),they may be eccentric (ECC-SMF), elevated (ELC-SMF) and exotic (CEX-SMF), of eccentric variable θ, of centric variable $\alpha_{1,2}$ of main determination, of index 1, or secondary determination of index 2. At the passage from centric circular domain to the eccentric one, by positioning of the eccenter $S(s, \varepsilon)$ in any point in the plane of the unit circle, all supermathematics functions multiply from one to infinity. It means that in CM there exists each unique function for a certain type. In EM there are infinitely many such functions, and for $s = 0$ one will get the centric function. In other words, any supermathematics function contains both the eccentric and the centric ones.

Notations $sexc(x)$ and respectively, $Sexc(x)$ are not standard in the literature and thus will be defined in three variants by the relations:

$$(8) \qquad sexc(x) = \frac{sex(x)}{x} = \frac{sex[\theta, S(s,s)]}{\theta}$$

of eccentric variable θ and

$$(8') \qquad Sexc(x) = \frac{Sex(x)}{x} = \frac{Sex[\alpha, S(s,s)]}{\alpha}$$

of eccentric variable α.

$$(9) \qquad sexc(x) = \frac{sex(\pi x)}{\pi x},$$

of eccentric variable θ, noted also by $sexc_\pi(x)$ and

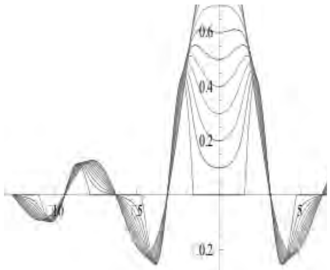
$$(9') \qquad Sexc(x) = \frac{sex(\pi x)}{\pi x} = \frac{Sex[\alpha, S(s,s)]}{\alpha},$$

of eccentric variable α, noted also by $Sexc_\pi(x)$

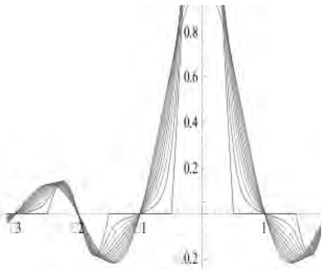
$$(10) \qquad sexc_a(x) = \frac{sex\frac{\pi x}{a}}{\frac{\pi x}{a}} = \frac{sex\frac{\pi\theta}{\theta}}{\frac{\pi\theta}{\theta}},$$

of eccentric variable θ, with the graphs from Figure 6 and Figure 7.

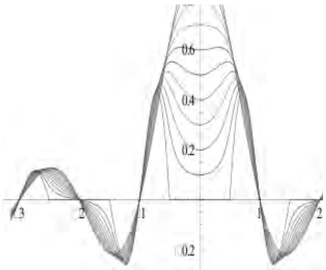
(10')
$$Sex_a(x) = \frac{Sex\frac{\pi x}{a}}{\frac{\pi x}{a}} = \frac{Sex\frac{\pi a}{a}}{\frac{\pi a}{a}}$$



$Sin\frac{\theta - ArcSin[s\ Sin(\theta)]}{\theta}$
$\{s, 0, +1\}, \{\theta, \pi, 4\pi\}$

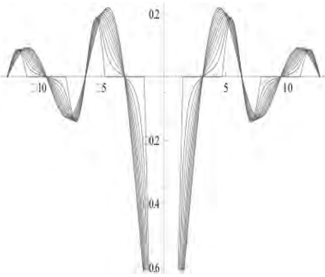


$Sin\frac{\pi\theta - ArcSin[s\ Sin(\pi\theta)]}{\theta}$
$\{s, -1, 0\}, \{\theta, -\pi, \pi\}$

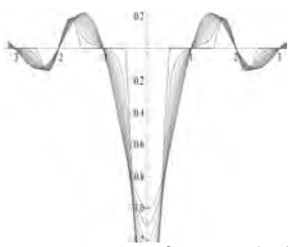


$Sin\frac{\pi\theta - ArcSin[s\ Sin(\pi\theta)]}{\theta}$
$\{s, 0, 1\}, \{\theta, -\pi, \pi\}$

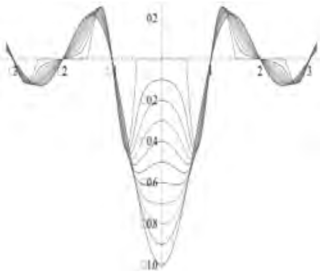
Figure 6: The ECCC-SMF graphs $sexc_1[\theta, S(s, \varepsilon)]$ of eccentric variable θ



$Sin\frac{\theta + ArcSin[s\ Sin(\theta)] - \pi}{\theta}$
$\{s, 0, 1\}, \{\theta, -4\pi, 4\pi\}$



$Sin\frac{\pi\theta - ArcSin[0.1s\ Sin(\pi\theta)] - \pi}{\theta}$
$\{s, -10, 0\}, \{\theta, -\pi, \pi\}$



$Sin\frac{\pi\theta - ArcSin[0.1s\ Sin\pi(\theta)] - \pi}{\theta}$
$\{s, 0.1, 0\}, \{\theta, -\pi, \pi\}$

Figure 7: Graphs ECCC-SMF $sexc_2[\theta, S(s, \varepsilon)]$, eccentric variable θ

3. ECCENTRIC CIRCULAR SUPERMATHEMATICS FUNCTIONS CARDINAL ELEVATED SINE AND COSINE (ECC- SMF-CEL)

Supermathematical elevated circular functions (ELC-SMF), elevated sine $sel(\theta)$ and elevated cosine $cel(\theta)$, is the projection of the fazor/vector

$$\vec{r} = rex(\theta) \cdot rad(\theta) = rex[\theta, S(s, \varepsilon)] \cdot rad(\theta)$$

on the two coordinate axis X_S and Y_S respectively, with the origin in the eccenter $S(s, \varepsilon)$, the axis parallel with the axis x and y which originate in $O(0, 0)$.

If the eccentric cosine and sine are the coordinates of the point $W(x, y)$, by the origin $O(0, 0)$ of the intersection of the straight line $d = d + \cup\ d\hat{a}e$“, revolving around the point $S(s, \varepsilon)$, the elevated cosine and sine are the same coordinates to the eccenter $S(s, \varepsilon)$; ie, considering the origin of the coordinate straight rectangular axes XSY /as landmark in $S(s, \varepsilon)$. Therefore, the relations between these functions are as follows:

$$(11) \qquad \begin{cases} x = cex(\theta) = X + s \cdot cos(\varepsilon) = cel(\theta) + s \cdot cos(\varepsilon) \\ y = Y + s \cdot sin(\varepsilon) = sex(\theta) = sel(\theta) + s \cdot sin(\varepsilon) \end{cases}$$

Thus, for $\varepsilon = 0$, ie S eccenter S located on the axis $x > 0, sex(\theta) = sel(\theta)$, and for $\varepsilon = \frac{\pi}{2}, cex(\theta) = cel(\theta)$, as shown on Figure 8.

On Figure 8 were represented simultaneously the elevated $cel(\theta)$ and the $sel(\theta)$ graphics functions, but also graphs of $cex(\theta)$ functions, respectively, for comparison and revealing $sex(\theta)$ elevation Eccentricity of the functions is the same, of $s = 0.4$, with the attached drawing and $sel(\theta)$ are $\varepsilon = \frac{\pi}{2}$, and $cel(\theta)$ has $\varepsilon = 0$.

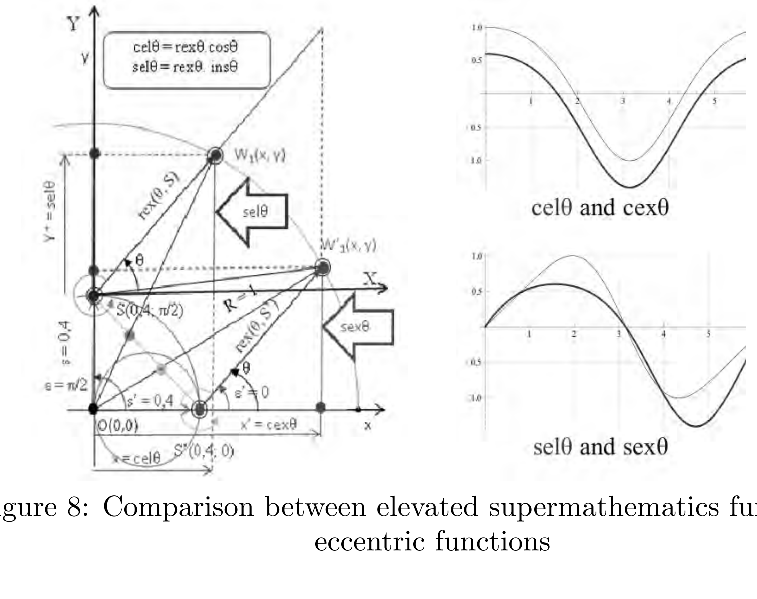


Figure 8: Comparison between elevated supermathematics function and eccentric functions

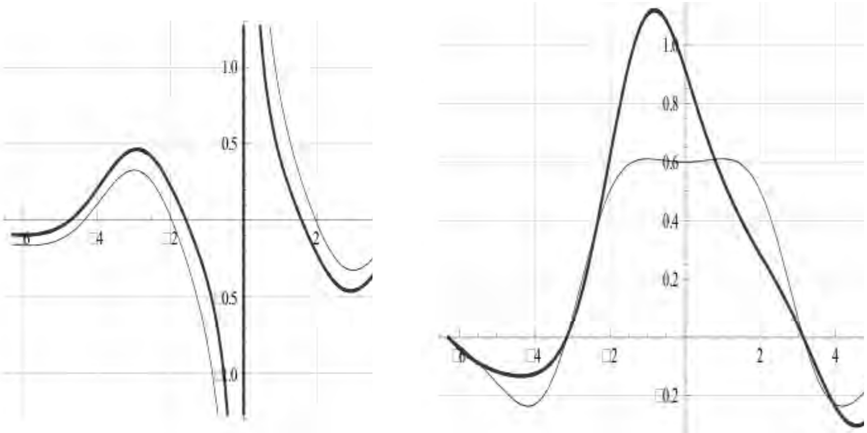


Figure 9: Elevated supermathematics function and cardinal eccentric functions $celc(x)$ ◄ and $selc(x)$ ► of $s = 0.4$

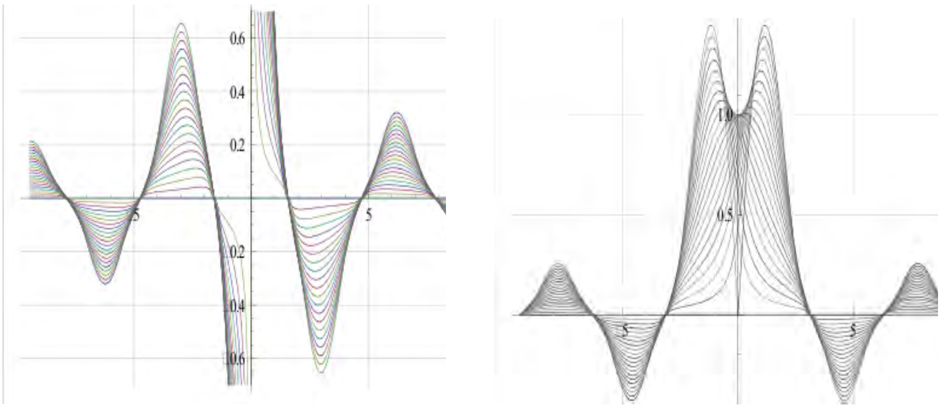


Figure 10: Cardinal eccentric elevated supermathematics function $celc(x)$ ◄ and $selc(x)$ ►

Elevate functions (11) divided by θ become cosine functions and cardinal elevated sine, denoted $celc(\theta) = [\theta, S]$ and $selc(\theta) = [\theta, S]$, given by the equations

$$(12) \qquad \begin{cases} X = celc(\theta) = celc[\theta, S(s,\varepsilon)] = cexc(\theta) - \dfrac{s \cdot cos(s)}{\theta} \\ Y = selc(\theta) = selc[\theta, S(s,\varepsilon)] = sexc(\theta) - \dfrac{s \cdot sin(s)}{\theta} \end{cases}$$

with the graphs on Figure 9 and Figure 10.

4. NEW SUPERMATHEMATICS CARDINAL ECCENTRIC CIRCULAR FUNCTIONS (ECCC-SMF)

The functions that will be introduced in this section are unknown in mathematics literature. These functions are centrics and cardinal functions or integrals. They are supermathematics eccentric functions amplitude, beta, radial, eccentric derivative of eccentric variable [1], [2], [3], [4], [6], [7] cardinals and cardinal cvadrilobe functions [5].

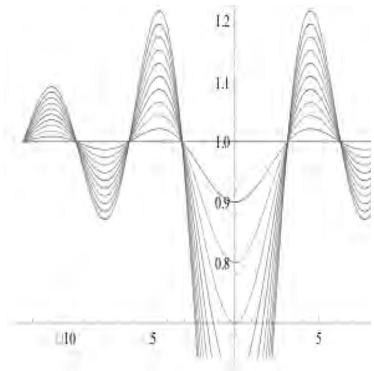
Eccentric amplitude function cardinal $aex(\theta)$, denoted as

$$(x) = aex[\theta, S(s, \varepsilon)], x \equiv \theta,$$

is expressed in

$$aexc(\theta) = \frac{aex(\theta}{\theta} = \frac{aex[\theta, S(s,s)]}{\theta} = \frac{\theta - arcsin[s\ sin(\theta - s]}{\theta} \tag{13}$$

and the graphs from Figure 11.



$\frac{\theta - Sin(\theta)}{\theta}, \{s, 0, 1\}, \{\theta, -4\pi, +4\pi\}$

Figure 11: The graph of cardinal eccentric circular supermathematics function $aexc(\theta)$

The beta cardinal eccentric function will be

$$bexc(\theta) = \frac{bex(\theta)}{\theta} = \frac{bex[\theta S(s,s)]}{\theta} = \frac{arcsin[s\ sin(\theta - s)]}{\theta}, \tag{14}$$

with the graphs from Figure 12.

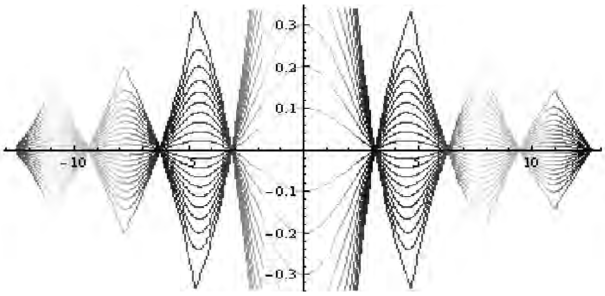


Figure 12: The graph of cardinal eccentric circular supermathematics function $bexc(\theta)$ $(\{s, -1, 1\}, \{\theta, -4\pi, 4\pi\})$

The cardinal eccentric function of eccentric variable θ is expressed by

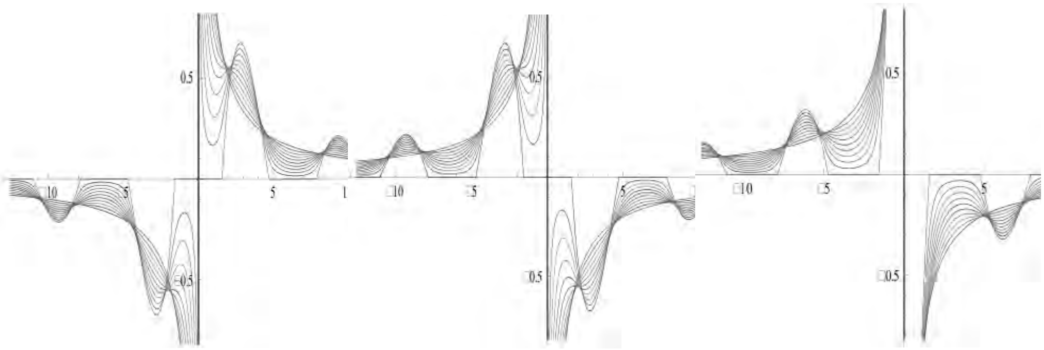
$$(15) \qquad rexc_{1,2}(\theta) = \frac{rex(\theta)}{\theta}$$

$$= \frac{rex[\theta, S(s,s)]}{\theta} = \frac{-s\ cos(\theta - s) \pm \sqrt{1 - s^2 sin(\theta - s)}}{\theta}$$

and the graphs from Figure 13, and the same function, but of centric variable α is expressed by

$$(16) \qquad Rexc(\alpha_{1,2}) = \frac{Rex(\alpha_{1,2})}{\alpha_{1,2}}$$

$$= \frac{Rex[\alpha_{1,2} S(s,s)]}{\alpha_{1,2}} = \frac{\pm\sqrt{1 + s^2 - 2s\ cos(\alpha_{1,2} - s)}}{\alpha_{1,2}}$$



$\frac{-sCos(\theta)+\sqrt{1-[s\,\mathrm{Si}\,n(\theta)]^2}}{\theta}$, $\{s, 0, 1\}, \{\theta, -4\pi, 4\pi\}$ $\qquad$ $\frac{-sCos(\theta)-\sqrt{1-[s\,\mathrm{Si}\,n(\theta)]^2}}{\theta}$, $\{s, -1, 0\}, \{\theta, -4\pi, 4\pi\}$ $\qquad$ $\frac{-sCos(\theta)-\sqrt{1-[s\,\mathrm{Si}\,n(\theta)]^2}}{\theta}$, $\{s, 0, 1\}, \{\theta, -4\pi, 4\pi\}$

Figure 13: The graph of cardinal eccentric circular supermathematical function $rexc_{1,2}(\theta)$

And the graphs for $Rexc(\alpha 1)$, from Figure 14.

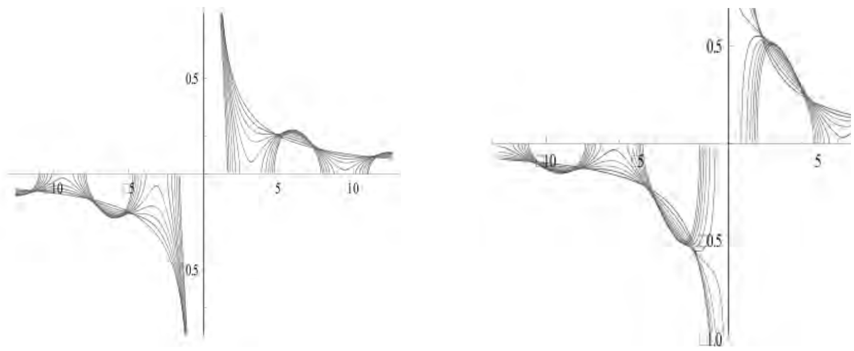


Figure 14: The graph of cardinal eccentric radial circular supermathematics function $Rexc(\theta)$

An eccentric circular supermathematics function with large applications, representing the function of transmitting speeds and/or the turning speeds of all known planar mechanisms is the derived eccentric $dex_{1,2}(\theta)$ and $Dex(\alpha_{1,2})$, functions that by dividing/reporting with arguments θ and, respectively, α

lead to corresponding cardinal functions, denoted $dexc_{1,2}(\theta)$, respectively $Dexc(\alpha_{1,2})$ and expressions

(17) $$dexc_{1,2}(\theta) = \frac{dex_{1,2}(\theta)}{\theta} = \frac{dex_{1,2}[\theta, S(s,s)]}{\theta} = \frac{1 - \frac{s \cdot cos(\theta - \varepsilon)}{1 - s^2 sin^2(\theta - \varepsilon)}}{\theta}$$

(18)
$$Dexc(\alpha_{1,2}) = \frac{Dex(\alpha_{1,2})}{\alpha_{1,2}} = \frac{Dex\{\alpha[\alpha_{1,2} S(s,s)]\}}{\alpha_{1,2}} = \frac{\sqrt{1 + s^2 - 2s \cdot cos(\alpha_{1,2} - s)}}{\alpha_{1,2}}$$

the graphs on Figure 15.

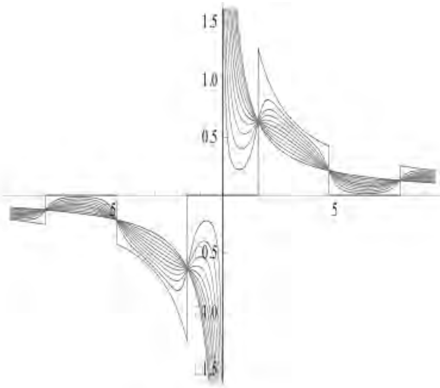


Figure 15: The graph of supermathematical cardinal eccentric radial circular function $dexc_1(\theta)$

Because $Dex(\alpha_{1,2}) = \frac{1}{dex_{1,2}(\theta)}$ results that $Dexc(\alpha_{1,2}) = \frac{1}{dexc_{1,2}(\theta)} siq(\theta)$ and $coq(\theta)$ are also cvadrilobe functions, dividing by their arguments lead to cardinal cvadrilobe functions $siqc(\theta)$ and $coqc(\theta)$ obtaining with the expressions

(19) $$coqc(\theta) = \frac{coq(\theta)}{\theta} = \frac{coq[\theta S(s,s)]}{\theta} = \frac{cos(\theta - s)}{\theta\sqrt{1 - s^2 sin^2(\theta - s)}}$$

(20) $$siqc(\theta) = \frac{siq(\theta)}{\theta} = \frac{siq[\theta S(s,s)]}{\theta} = \frac{sin(\theta - s)}{\theta\sqrt{1 - s^2 cos^2(\theta - s)}}$$

the graphs on Figure 16.

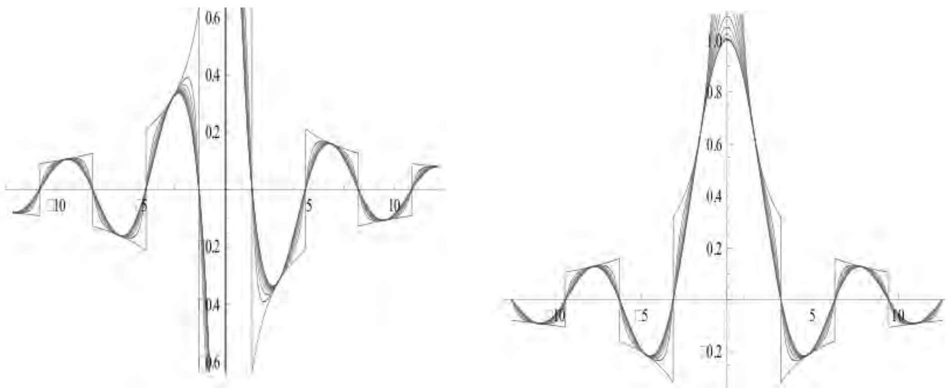


Figure 16: The graph of supermathematics cardinal cvadrilobe function $ceqc(\theta)$ ◄ and $siqc(\theta)$ ►

It is known that, by definite integrating of cardinal centric and eccentric functions in the field of supermathematics, we obtain the corresponding integral functions.

Such integral supermathematics functions are presented below. For zero eccentricity, they degenerate into the centric integral functions. Otherwise they belong to the new eccentric mathematics.

5. ECCENTRIC SINE INTEGRAL FUNCTIONS

Are obtained by integrating eccentric cardinal sine functions (13) and are

$$sie(x) = \int_0^x sexc(\theta) \cdot d\theta \tag{21}$$

with the graphs on Figure 17 for the ones with the eccentric variable $x \equiv \theta$.

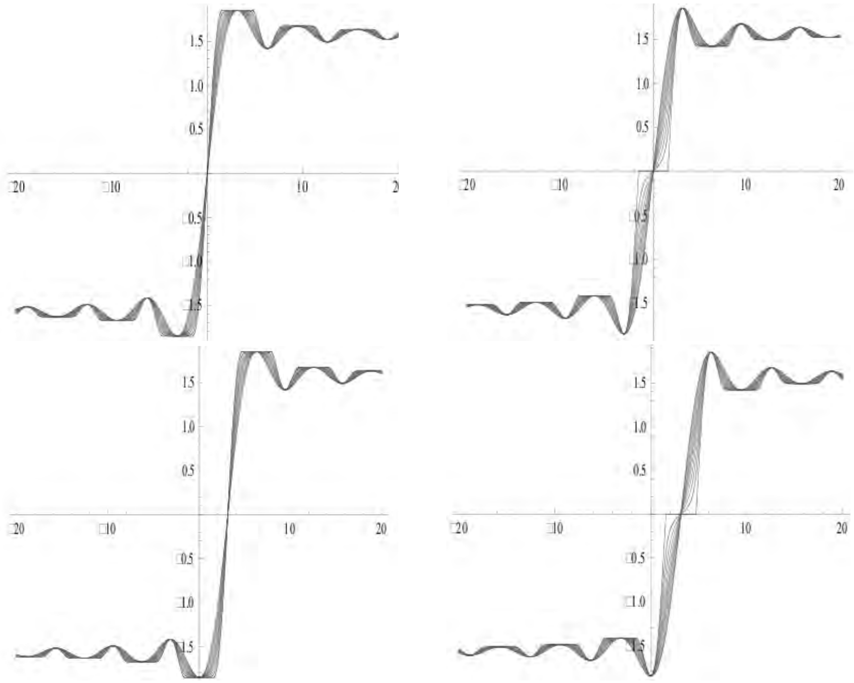


Figure 17: The graph of eccentric integral sine function $sie_1(x)$▲ and $sie_2(x)$▼

Unlike the corresponding centric functions, which is denoted $Sie(x)$, the eccentric integral sine of eccentric variable was noted $sie(x)$, without the capital S, which will be assigned according to the convention only for the ECCC-SMF of centric variable. The eccentric integral sine function of centric variable, noted $Sie(x)$ is obtained by integrating the cardinal eccentric sine of the eccentric circular supermathematics function, with centric variable

$$Sexc(x) = Sexc[\alpha, S(s, \varepsilon)], \tag{22}$$

thus

$$Sie(x) = \int_0^x \frac{Sex[\alpha, S(s, \varepsilon]}{\alpha}, \tag{23}$$

with the graphs from Figure 18.

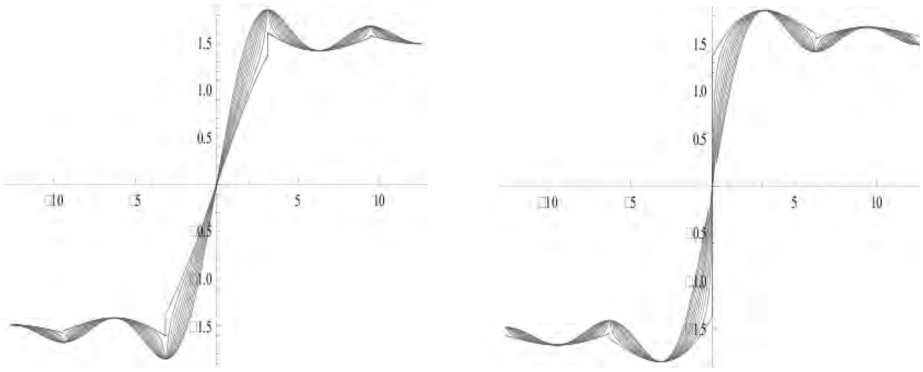


Figure 18: The graph of eccentric integral sine function $sie_2(x)$

6. C O N C L U S I O N

The paper highlighted the possibility of indefinite multiplication of cardinal and integral functions from the centric mathematics domain in the eccentric mathematics's or of supermathematics's which is a reunion of the two mathematics. Supermathematics, cardinal and integral functions were also introduced with correspondences in centric mathematics, a series new cardinal functions that have no corresponding centric mathematics.

The applications of the new supermathematics cardinal and eccentric functions certainly will not leave themselves too much expected.

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ABOUT AN IDENTITY AND ITS APPLICATIONS

MIHALY BENCZE and FLORENTIN SMARANDACHE

Theorem 1. If $x, y \in C$ then $2\left(x^2+y^2\right)^3-(x+y)^3\left(x^3+y^3\right)=(x-y)^4\left(x^2+xy+y^2\right)$.

Proof. With elementary calculus.

Application 1.1. If $x, y \in C$ then

$$\left(2\left(x^2+y^2\right)^3-(x+y)^3\left(x^3+y^3\right)\right)\left(2\left(x^2+y^2\right)^3-(x-y)^3\left(x^3-y^3\right)\right)=\left(x^2-y^2\right)^4\left(x^4+x^2y^2+y^4\right)$$

Proof. In Theorem 1 we replace $y \to -y$, etc.

Application 1.2. If $x \in R$ then

$$(\sin x-\cos x)^4(1+\sin x\cos x)+(\sin x+\cos x)^3\left(\sin^3 x+\cos^3 x\right)=2$$

Proof. In Theorem 1 we replace $x \to \sin x,\ y \to \cos x$

Application 1.3. If $x \in R$ then $2ch^6x-(1+shx)^3\left(1+sh^3x\right)=(1+shx)^4\left(shx+ch^2x\right)$.

Proof. In Theorem 1 we replace $x \to 1,\ y \to shx$

Application 1.4. If $x, y \in C$ $(x \neq \pm y)$ then

$$\frac{2\left(x^2+y^2\right)^3-(x+y)^3\left(x^3+y^3\right)}{(x-y)^4}+\frac{2\left(x^2+y^2\right)^3-(x-y)^3\left(x^3-y^3\right)}{(x+y)^4}=2\left(x^2+y^2\right)$$

Application 1.5. If $x, y \in C$ then

$$\frac{2\left(x^2+y^2\right)^3-(x+y)^3\left(x^3+y^3\right)}{x^2+xy+y^2}+\frac{2\left(x^2+y^2\right)^3-(x-y)^3\left(x^3-y^3\right)}{x^2-xy+y^2}=2\left(x^4+6x^2y^2+y^4\right)$$

Application 1.6. If $x, y \in R$ then $2\left(x^2+y^2\right)^3 \geq (x+y)^3\left(x^3+y^3\right)$.

(See Jószef Sándor, Problem L.667, Matlap, Kolozsvar, 9/2001.)

Proof. See Theorem 1.

Theorem 2. If $x, y, z \in R$ then $3\left(x^2+y^2+z^2\right)^3 \geq (x+y+z)^3\left(x^3+y^3+z^3\right)$.

Proof. With elementary calculus.

Application 2.1. Let $ABCDA_1B_1C_1D_1$ be a rectangle parallelepiped with sides a, b, c and diagonal d. Prove that $3d^6 \geq (a+b+c)^3\left(a^3+b^3+c^3\right)$.

Application 2.2. In any triangle ABC the followings hold:

1) $3\left(p^2-r^2-4Rr\right)^3 \geq 2p^4\left(p^2-3r^2-6Rr\right)$

2) $3\left(p^2-2r^2-8Rr\right)^3 \geq p^4\left(p^2-12Rr\right)$

3) $3\left((4R+r)^2-2p^2\right)^3 \geq (4R+r)^3\left((4R+r)^3-12p^2R\right)$

4) $3\left(8R^2+r^2-p^2\right)^3 \geq (2R-r)^3\left((2R-r)\left((4R+r)^2-3p^2\right)+6Rr^2\right)$

5) $3\left((4R+r)^2-p^2\right)^3 \geq (4R+r)^3\left((4R+r)^3-3p^2(2R+r)\right)$

Proof. In Theorem 2 we take:

$\{x,y,z\} \in$

$$\in\left\{\{a,b,c\};\{p-a,p-b,p-c\};\{r_a,r_b,r_c\};\left\{\sin^2\frac{A}{2},\sin^2\frac{B}{2},\sin^2\frac{C}{2}\right\};\left\{\cos^2\frac{A}{2},\cos^2\frac{B}{2},\cos^2\frac{C}{2}\right\}\right\}.$$

Application 2.3. Let ABC be a rectangle triangle, with sides $a>b>c$ then

$$24a^6 \geq (a+b+c)^3\left(a^3+b^3+c^3\right)$$

Theorem 3. If $x_k>0,\ k=1,2,...,n$, then $n\left(\sum_{k=1}^n x_k^2\right)^3 \geq \left(\sum_{k=1}^n x_k\right)^3 \sum_{k=1}^n x_k^3$.

Application 3.1 The following inequality is true: $\sum_{k=0}^n \left(C_n^k\right)^3 \leq (n+1)\left(\frac{C_{2n}^n}{2}\right)^3$.

Proof. In Theorem 3 we take $x_k=C_n^k,\ k=0,1,2,...,n.$.

Application 3.2. In all tetrahedron $ABCD$ holds:

1) $\dfrac{\left(\sum \frac{1}{h_a^2}\right)^3}{\sum \frac{1}{h_a^3}} \geq \dfrac{4}{r^3}$ 2) $\dfrac{\left(\sum \frac{1}{r_a^2}\right)^3}{\sum \frac{1}{r_a^3}} \geq \dfrac{2}{r^3}$

Proof. In Theorem 3 we take $x_1=\frac{1}{h_a},x_2=\frac{1}{h_b},x_3=\frac{1}{h_c},x_4=\frac{1}{h_d}$ and

$$x_1=\frac{1}{r_a},x_2=\frac{1}{r_b},x_3=\frac{1}{r_c},x_4=\frac{1}{r_d}.$$

Application 3.3. If $S_n^\alpha=\sum_{k=1}^n k^\alpha$ then $n\left(S_n^{2\alpha}\right)^3 \geq \left(S_n^\alpha\right)^3 S_n^{3\alpha}$.

Proof. In Theorem 3 we take $x_k=k^\alpha,\ k=0,1,2,...,n.$.

Application 3.4. If F_k denote Fibonacci numbers, then $\sum_{k=1}^n F_k^3 \leq n\left(\frac{F_nF_{n+1}}{F_{n+2}-1}\right)^3$.

Proof. In Theorem 3 we take $x_k=F_k,\ k=1,2,...,n.$.

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ON CRITTENDEN AND VANDEN EYNDEN'S CONJECTURE

FLORENTIN SMARANDACHE

It is possible to cover all (positive) integers with n geometrical progressions of integers?

Find a necessary and sufficient condition for a general class of positive integer sequences such that, for a fixed n, there are n (distinct) sequences of this class which cover all integers.

Comments:

a) No. Let $a_1,...,a_n$ be respectively the first terms of each geometrical progression, and $q_1,...,q_n$ respectively their ratios. Let p be a prime number different from $a_1,...,a_n, q_1,...,q_n$. Then p does not belong to the union of these n geometrical progressions.

b) For example, the class of progressions $A_f = \{\{a_n\}_{n\geq 1} : a_n = f(a_{n-1},...,a_{n-i})$ for $n \geq i+1$, and $i, a_1, a_2, .. \in N^*\}$ with the property $\exists y \in N^*,\ \forall (x_1,...,x_i) \in N^{*i} : f(x_1,...,x_i) \neq y$. Does it cover all integers?

But, if $\forall y \in N^*,\ \exists (x_1,...,x_i) \in N^{*i} : f(x_1,...,x_i) = y$?

(Generally no.)

This (solved and unsolved) problem remembers Crittenden and Vanden Eynden's conjecture.

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ECCENTRICITY, SPACE BENDING, DIMENSION

MARIAN NIŢU, FLORENTIN SMARANDACHE and MIRCEA E. ŞELARIU

Motto:
The science wouldn't be so good today,
if yesterday we hadn't thought about today.
Grigore C. Moisil

Abstract. The main goal of this paper is to present new transformations, previously non-existent in traditional mathematics, that we call centric mathematics (CM) but that became possible due to the new born eccentric mathematics, and, implicitly, to the supermathematics (SM).

As shown in this work, the new geometric transformations, namely conversion or transfiguration, wipe the boundaries between discrete and continuous geometric forms, showing that the first ones are also continuous, being just apparently discontinuous.

Abbreviations and annotations

C	►	**Circular and Centric**,	**E**	►	**Eccentric and Eccentrics**,
F	►	**Function**,	**M**	►	**Mathematics**,
CE	►	**Circular Eccentric**,	**F CE**	►	**FCE**,
CM	►	**Centric M**,	**EM**	►	**Eccentric M**,
SM	►	**Super M**,	**F CM**	►	**FCM**,
F EM	►	**FEM**,	**F SM**	►	**FSM**

Introduction: conversion or transfiguration

In *linguistics* a **word** is the fundamental unit to communicate a meaning. It can be composed by one or more *morphemes*. Usually, a word is composed by a basic part, named *root*, where one can attach *affixes*. To define some *concepts* and to express the domain where they are available, sometimes more words are needed: two, in our case. In this paper several new concepts are introduced and they are related to *SuperMathematics* (SM).

The principal new idea in this paper is that it introduces a new mathematical transformation with a large significance in the fields of Physics, previously non-existent in the original classical mathematics, named herein as centric mathematics (CM). They became possible thanks to this new mathematics called Eccentric Mathematics (EM) and to the Super Mathematics (SM), which are puts together with (CM) with (EM). The (CM) is now a particular case of a linear numeric eccentricity for $s = 1$ in (SM).

Supermathematical conversion

The concept is the easiest and methodical *idea* which reflects a finite of one or more series of attributes, where these attributes are *essentials*.

The concept is a minimal coherent and usable information, relative to an object, action, property or a defined event.

According to the Explicatory Dictionary, **the conversion** is, among many other definitions/meanings, defined as "changing the nature of an object".

Next, we will talk about this thing, about transforming/changing/converting, previously impossible in the ordinary classic mathematics, now named also CENTRIC (CM), of some forms in others, and that became possible due to the new born mathematics, named ECCENTRIC (EM) and to the new built-in mathematical complements, named temporarily also SUPERMATHEMATICS (SM).

We talk about the *conversion* of a circle into a square, of a sphere into a cube, of a circle into a triangle, of a cone into a pyramid, of a cylinder into a prism, of a circular torus in section and shape into a square torus in section and/or form, etc. (Fig.1).

Supermathematical Conversion (SMC) is an internal pry for the mathematical dictionary enrichment, which consists in building-up of a new denomination, with one or more new terms (two in our case), by assimilating some words from the current language in a specialized domain, as Mathematics, with the intention to name and adequate the new operations that became possible only due to the new born *eccentric mathematics*, and implicity to *supermathematics*. Because previously mentioned conversions could not be made until today in MC, but only in SM, we need to call them as SUPERMATHEMATICAL conversion (SMC).

In [14] the continuous transformation of a circle into a square was named also *eccentric transformation*, because, in that case, the linear numeric eccentricity s varies/grows from 0 to 1, being a slide from centric mathematics domain $MC \to s = 0$ to the eccentric mathematics, $ME(s \neq 0) \to s \in (0, 1]$, where the circular form draws away more and more from the circular form until reaching a perfect square $(s = \pm 1)$.

Eccentric transformation

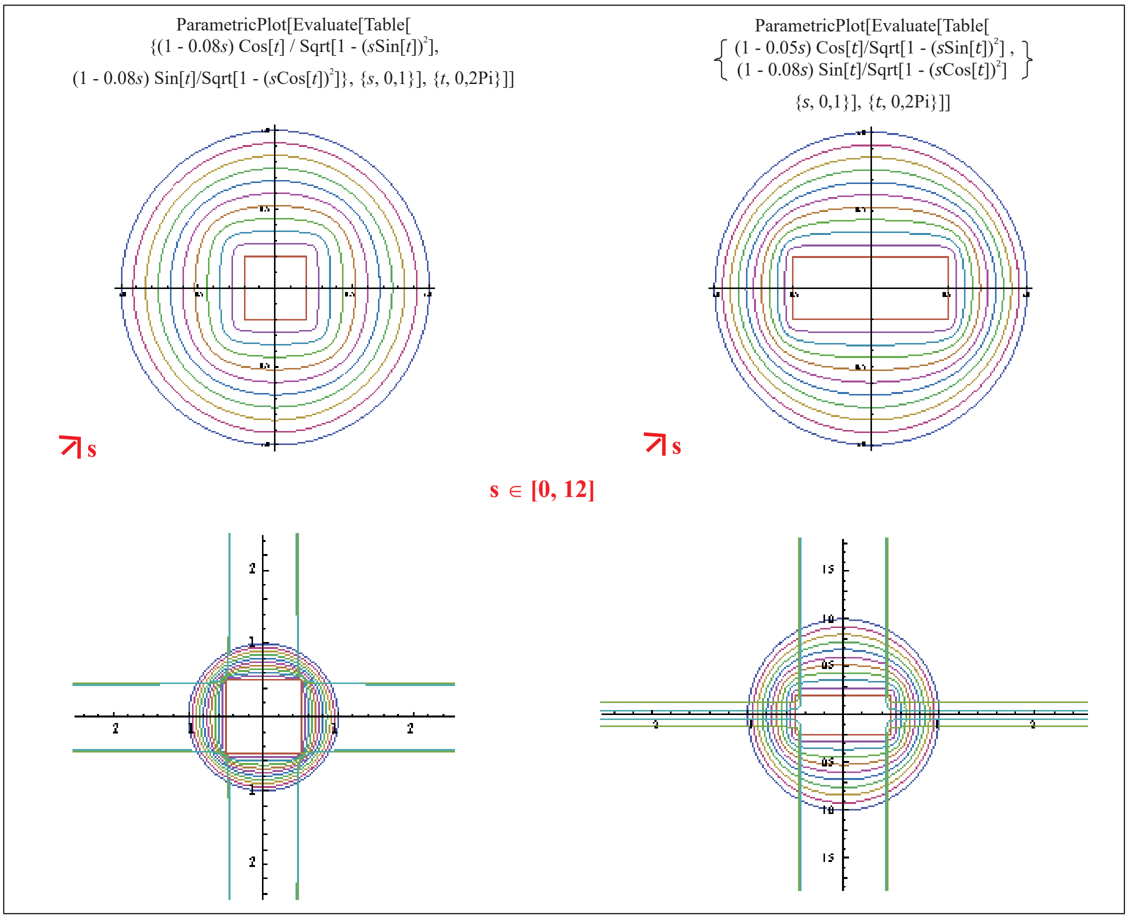

Figure 1: Conversion or transfiguration in 2D of a circle into a square and/or into a rectangle ►ECCENTRIC TRANSFORMATION

In the same work, the reverse transformation, of a square into a circle, was named as *centering transformation*. Same remarks are valid also for transforming a circle into a rectangle and a rectangle into a circle (Fig. 2). Most modern physicists and mathematicians consider that the numbers represent the reality's language. The truth is that the forms are those which generate all physical laws.

Centering transformation

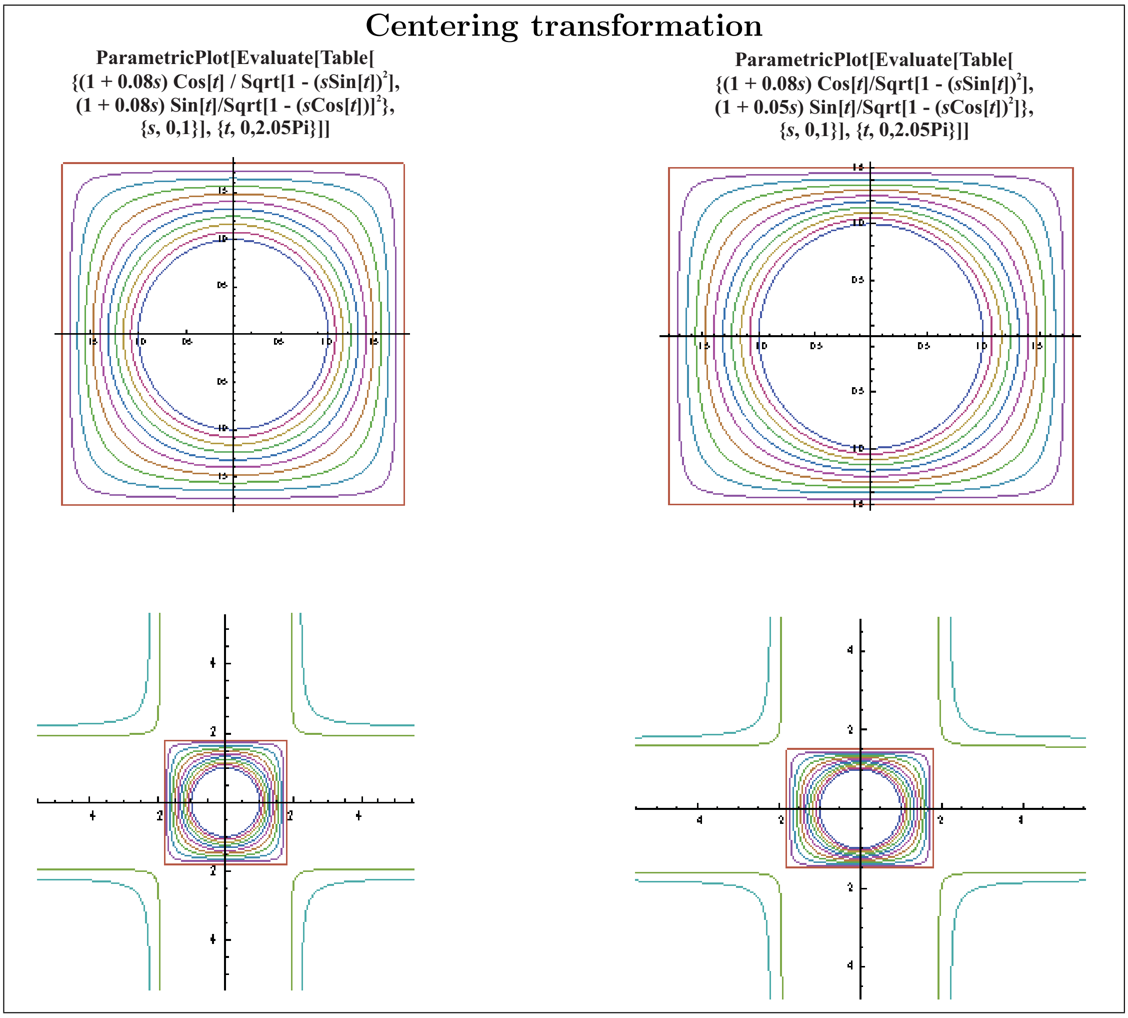

Figure 2: Conversion or transfiguration in 2D of a square and/or a rectangle into a circle ►CENTERING TRANSFORMATION

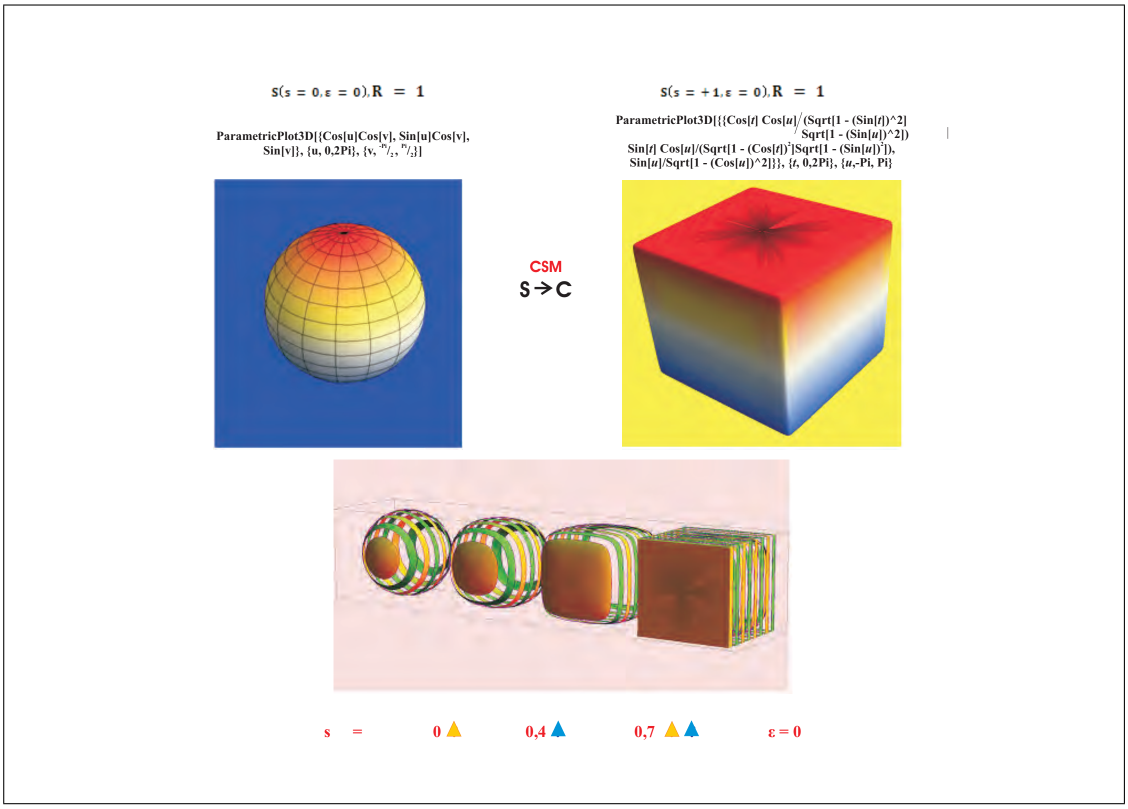

Figure 3: The conversion of a sphere into a cube

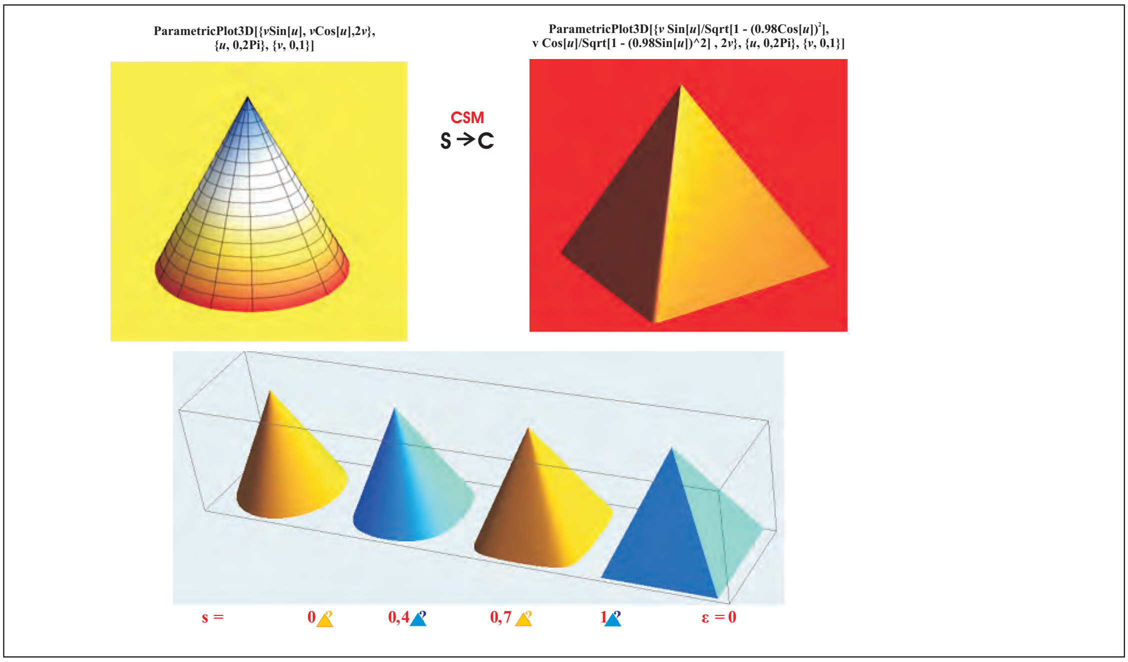

Figure 4: The Conversion of a cone into a pyramid

Look what the famous Romanian physicist Prof. Dr. Liviu Sofonea in *Representative Geometries and Physical Theories*, Ed. Dacia, Cluj-Napoca, p. 24, in 1984, in the chapter named *Mathematical geometry and physical geometry* wrote: In the *centric mathematical geometry* one does what can be done, how can be done, with what can be done, and in supermathematical geometry we can do what must be done, with what must be done, as we will proceed. In the *supermathematical geometry*, between the elements of the 'CM scaffold', one can introduce as many other constructive elements as we want, which will give an infinitely denser scaffold structure, much more durable and, consequently, higher, able to offer an unseen high level and an extremely deep description and gravity.

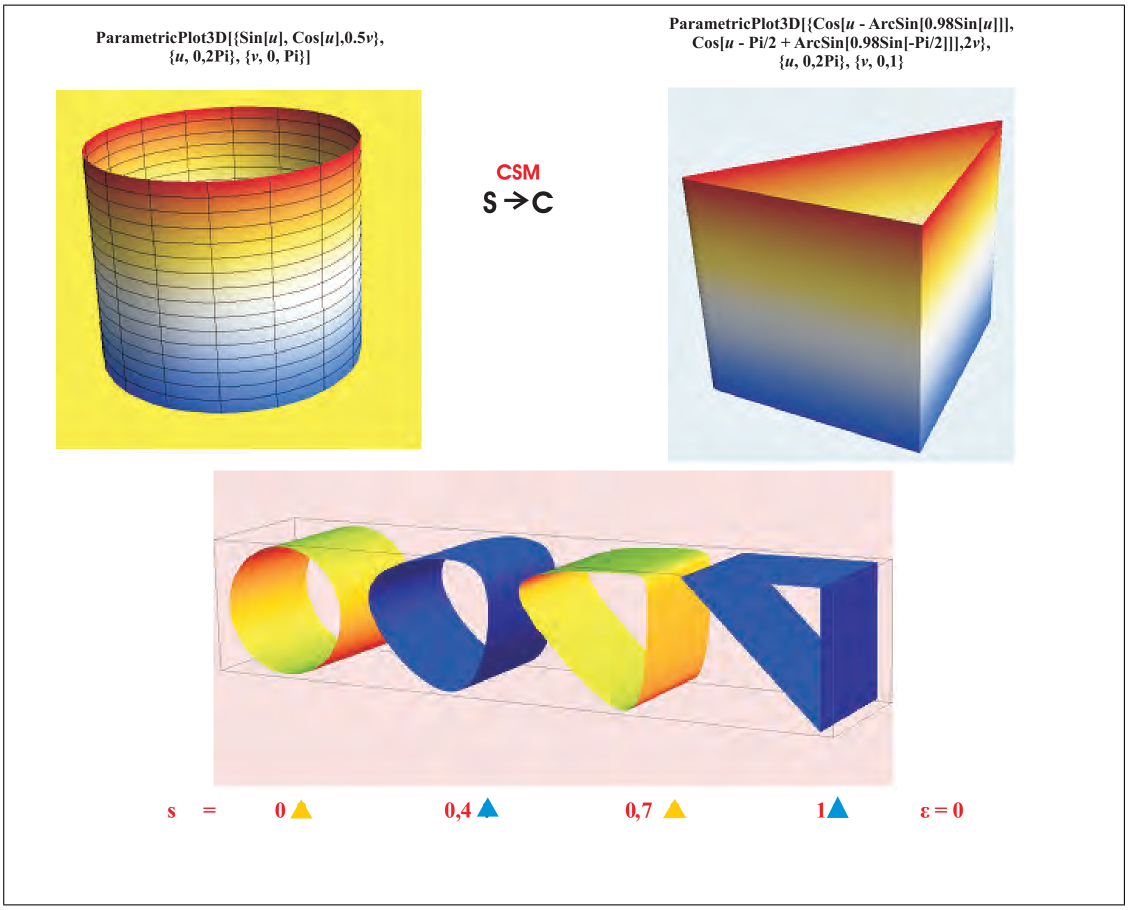

Figure 5: The Conversion or transfiguration of a cylinder into a prism

Figure 6: The conversion or transfiguration of the circular thorus into a square thorus, both in form and in section

The *fundamental principles* of the geometry are, according to their topological dimensions: the *corps* (3), the *line* (2), and the *point* (0).

The *elementary principles* of geometry are the point, the line, the space, the curve, the plane, the geometrical figures (such as the segment, triangle, square, rectangle, rhombus, the polygons, the polyhedrons, etc. the arcs, circle, ellipse, hyperbola, the scroll, the helix, etc.) both in 2D and in 3D spaces.

With the fundamental geometrical elements are defined and built all the forms and geometrical structures of the objects:

- *Discrete (discontinuous) statically forms*, or directly starting from a finite (discrete) set of points statically bonded with lines and planes.
- *Continuous (or dynamical, mechanical) forms*, starting from a single point and considering its motion, therefore the time, and obtaining in this way continuous forms of curves, as trajectories of points or curves, in the plane (2D) or in the space (3D)

Consequently, one has considered, and still is considering, the existence of two geometries: the *geometry of discontinuous*, or discrete geometry, and the *geometry of the continuum*.

As both objects limited by plane surfaces (cube, pyramid, prism), *apparently discontinuous*, as those limited by different kinds of *continuous surfaces* (sphere, cone, cylinder) can be described with the same parametric equations, the first ones for numerical eccentricity $s = \pm 1$ and the last ones for $s = 0$, it results that in SM there exists only one geometry, *the geometry of the continuum*.

In other words, the SM erases the boundaries between continuous and discontinuous, as SM erased the boundaries between linear and nonlinear, between centric and eccentric, between ideal/perfection and real, between circular and hyperbolic, between circular and elliptic, etc.

Between the values of numerical eccentricity of $s = 0$ and $s = \pm 1$, there exist an infinity of values, and for each value, an infinity of geometrical objects, which, each of them has the right to a geometrical existence.

If the geometrical mathematical objects for $s \in [0 \vee \pm 1]$ belong to the centric ordinary mathematics (CM) (circle$\rightarrow$ square, sphere$\rightarrow$ cube, cylinder $\rightarrow$ prism, etc.), those for $s \in (0, \pm 1)$ have forms, equations and denominations unknown in this centric mathematics (CM). They belong to the new mathematics, the eccentric mathematics (EM) and, implicitly, to the supermathematics (SM), which is a reunion of the two mathematics: centric and eccentric, that means SM = MC $\cup$ ME.

Concluding remarks

The principal new idea in this paper is that it introduces a new mathematical transformation with a large significance in the fields of Physics, previously inexistent in the original classical mathematics named here as centric mathematics (CM); and now they became possible thanks to this new mathematics, called Eccentric Mathematics (EM), and to the Super Mathematics (SM), which are put together: (CM) with (EM). The (CM) is now a particular case of a linear numeric eccentricity for $s = 1$ in (SM).

In this paper the authors prove that these new geometric transformations, named Conversions or Transfigures, eliminate the borders between the discrete and continuous forms, showing that the first ones are also continuous but only apparently continuous. They mean: the conversion of a circle in a square, of a sphere in a cube, of a circle in a triangle, of a cone in a pyramid, of a cylinder in a prism, of a torus with circular section in a torus with a square section, etc. Also, they consider this eccentricity in the formation and deformation of the space. The authors claim that all of these transformations are possible because of the eccentricity considered as 4-th up to n-th dimension of the space to complete the usual accepted (x, y, z) dimension. This is the reason why they consider the eccentricity as a dimension of the formation or deformation of the space.

The extension of a three dimensional space to a n-dimensional space became possible if the linear constant eccentricity e and the angle eccentricity *epsilon* which are the polar coordinates of the eccentricity E (*e, epsilon*) became of one or multiple variables considered eccentricities too.

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PROFESSOR SELARIU'S SUPERMATHEMATICS

FLORENTIN SMARANDACHE

ABSTRACT

This article is a brief review of the book "Supermathematics. Bases", Vol. 1 and Vol. 2, 2nd edition, 2012, which represents a new field of research with many applications, initiated by Professor Mircea Eugen Şelariu. His work is unique in the world scientific literature, because it combines centric mathematics with eccentric mathematics.

INTRODUCTION

Supermathematics (SM) is a reunion of the familiar, ordinary mathematics, which was called in this paper centric mathematics (CM), to be distinguished from the new mathematics, called eccentric mathematics (EM). That is **SM = CM ∪ EM**.

For each point in the plane, which can be placed in an eccenter **E (e, ε)**, we can say that there is / there appears a new **EM**. Thus, an infinity of **EM** corresponds to a single **CM**; On the other hand, **CM = SM (e = 0)**;

Thus, **SM** indefinitely multiplies all known circular / trigonometric functions and introduces a host of new circular functions (*aex*, *bex*, *dex*, *rex*, etc.), much more important than the old ones and thereby, finally, indefinitely multiplies all known mathematical entities and introduces several new entities.

It was observed that **CM** is proper to **linear, perfect, ideal** systems and **EM** is proper to **nonlinear, real, imperfect** systems;

Therefore, with the apparition of **SM** the boundary between linear and nonlinear, between the ideal and the real, between perfection and imperfection disappeared;

SM marks out the linear eccentricity ***e*** and the angular one **ε**, the polar coordinates of the eccenter **E (e, ε)** as new dimensions of space: dimensions of its **formation** and **deformation**;

SM could have occured more than 300 years ago, if Euler, when defining trigonometric functions as direct circular functions, hadn't chosen **three superposed points**, which impoverished maths: **Pole E** of a half-line, **the center C** of the trigonometric circle (unit) and the origin **O** (0,0) of a reference point / right rectangular system;

SM occurred when pole **E** was expelled from center and called **eccenter**.

The following functions appear after the possible combination of the three points:

- **CCF centric circular** (**FSM - CC)→** if **C≡ O ≡ E;**
- **FSM eccentric circular (FSM - EC)→** if **C ≡ O≠E;**
- **FSM elevated circular (FSM - ELC)→** if **C ≠O ≡ E;**
- **FSM exotic circular (FSM - EXC)→** if **C≠O ≠E.**

Among the **new entities,** there is also a host of new closed curves, occurring in the **continuous transformation** of the circle into a square (called quadrilobes / cvadrilobes), of the circle into a triangle (trilobes). In 3D, these continuous changes are of sphere into cube, of sphere into prism, of cone into pyramid, etc.

These continuous changes made possible the apparition of new 3D hybrid figures as: sphere-cube, cone-pyramid, pyramid-cone, etc.

In this work, by replacing the circle with a quadrilobe were defined the quadrilobe functions and by replacing it by a trilobe were defined the trilobe functions.

The book introduces new mathematical methods and techniques as well, such as:

- Integration through differential dividing;
- The hybrid analytic-numerical method → Determining K(k) with 15 accurate decimals;
- The method of moments separation → The kinetostatic method, extremely simple and exact, which reduces **d'Alambert method,** requiring the solution of some equations of equilibrium systems, to a simple elementary geometry problem;
- The eccentric circular movement of fixed and mobile point eccenter;
- The rigorous transformation of a polar diagram of pliancy into a circle;
- Solving some vibration systems of nonlinear static elastic features;
- Introduction of quadrilobic / cvadrilobic vibration systems.

DESCRIPTION OF WORK

Ch.1. INTRODUCTION

It is presented a short history of the SUPERMATHEMATICS discovery in connection with the research undertaken by the author at the University of Stuttgart, between 1969 - 1970, at the Institute and Department of Machine Tools of Prof. **Karel Tuffentsammer**, in the group of "Machine Tool Vibration".

Moreover, it is shown that the great mathematician **Leonhard Euler**, in defining trigonometric functions as circular functions, choosing <u>three superposed points</u> [**Origin O (0, 0)**, **circle center**, called at that time <u>trigonometric</u> circle **M** (0, 0), now renamed as <u>unit circle</u> and **the Pole** of a half-line **P(0,0)**] impoverished mathematics from the start. Mathematics itself remained extremely poor, with a single set of periodic functions (sinα, cosα, tanα, cotα, secα, cscα, etc.) and, therefore, generally, with unique mathematical entities (line, circle, square, sphere, cube, elliptic integrals, etc).

Through the mere expulsion of the pole **P** and called, therefore, **eccenter E(e, ε)** for any circle C(O, R) of radius R or marked by S (s, ε) for the unit circle CU(O, 1), for each point on the plane of the unit circle, in which a pole/eccenter S (s, ε) can be placed, a set of circular / trigonometric functions is obtained, called **eccentric**.

They were called **eccenters** because they were expelled from center O.

And on this basis, we obtain an infinite number of new mathematical entities, called **eccentric**, previously non-existent in mathematics (the crook line as an extension / generalization of the line; the eccentric circular or quadrilobes that complement the space between circle and square or, in other words, perform a continuous transformation of the circle into a perfect square, the eccentric sphere, which continually transforms the sphere into a perfect cube, cone-pyramid, sphere-cube, etc.).

The chapter ends with an overview of the main contributions that the new complements in mathematics, collectively called SUPERMATHEMATICS, bring in mathematics, informatics, mechanics, technology and other fields.

Ch. 2. DIVERSIFICATION OF PERIODIC FUNCTIONS

Seizing upon the existence of some "white spots" in mathematics, a number of great mathematicians have tried, in the past as well as today, and managed to partially rectify these shortcomings. Their efforts deserved to be reviewed, along with the discovery of supermathematics, even if they are not of the same broad reach, and some of them were incompletely presented, in a more sketchy way, were shaped by the author to a final form, compatible with mathematical programs.

It is about **Valeriu Alaci**'s quadratic functions and diamond functions, **M. Ovidiu Enulescu's** polygonal functions, **Malvinei Florica Baica** and **Mircea Cârdu**'s transtrigonometric functions, **Eugen Vişa**'s pseudo hyperbolic functions, all mathematics teachers and fellow-citizens with the author.

In the same city, Timişoara, on November 3, 1823, a young engineer officer at Timişoara garrison, **Ianos Bolyai**, (he was then 21), was sending his father, **Farkas Bolyai**, professor of mathematics at the college of Targu-Mures a touching letter. He wrote, among other things: "***From nothing I've created a new world***". It was the world of non-euclidean geometry.

Likewise, through the reunion of the ordinary centric mathematics (**CM**) with the new eccentric mathematics (**EM**) **the supermathematics** was created (**SM = CM ∩ EM**). It infinitely multiplies all **unique** entities of **CM** and, in addition, introduces new mathematical entities previously non-existent (cone-pyramid, sphere-cube, etc.).

In this case, it can be asserted that "**from nothing**" there were created new mathematical entities such as, for example, supermathematical eccentric circular functions (FSM-EC) eccentric amplitude aexθ and Aexα, beta eccentric bexθ and Bexα, radial eccentric rexθ and Rexα, eccentric derivatives dexθ and Dexα, cone-pyramids, square, triangular and other forms of cylinders, etc.

But it can also be asserted that from a single mathematical entity, which exists in **CM**, there were created infinite entities of the same kind in **EM** and, implicitly, in **SM**, or that **SM infinitely multiplies all CM entities**.

The involute functions of **George (Gogu) Constantinescu**, the creator of sonics, are particularly highlighted, the Romanian cosine Corα and the Romanian sine Sirα, which are unfortunately too little known like the inclined trigonometric functions of **Dr. Bihringer**, unfairly forgotten.

Ch.3. ADDITONS AND CORRECT REDEFINITIONS IN CENTRIC MATHEMATICS

Octavian Voinoiu's work, published by Nemira, "**INTRODUCTION IN SIGNADFORASIC MATHEMATICS**" revealed a number of mathematical entities, of first importance, wrongly introduced in mathematics, in centric mathematics (**CM**).

Supporter of **Sophocle**'s principle: "Errare humanum est, perseverare diabolicum", the author considered that, before presenting the new mathematical complements, it is strictly necessary to partially highlight and maybe correct the wrongly introduced entities, existing in **CM**.

In this respect, a simple example is the wrong definition of the sign of a fraction and, as a result, of the tangent as being the ratio tanα = $\frac{sin\alpha}{cos\alpha}$, while the correct definition is tanvα = $\frac{sin\alpha}{Abs[cos\alpha]}$, which has been called **Voinoiu** centric tangent. In this way, the new **FSM-EC Voinoiu** eccentric tangent texvθa could be "ab initio" properly defined, as the ratio between the eccentric sine sexθ and cosine cexθ, that is texvθ =$\frac{\text{sex}\theta}{Abs[\text{cex}\theta]}$.

Moreover, a number of entities that appeared in **EM** and consequently in **SM**, had no equivalents in **CM**. They are the most significant **FSM-EC**, the eccentric radial periodic function **rexθ**, a true "king" function and the eccentric derivative **dexθ,** which expresses alone the second order transfer function or the speed transmission ratio and / or all plane mechanisms turation ratio.

It was determined that the equivalents of these **FSM-EC** in **CM** are the centric radial functions **radα = $e^{i\alpha}$** and the eccentric derivative **derα = $e^{i(\alpha+\pi/2)}$**, which are exactly the **Euler-Cotes** functions or the phasors of centric radial directions, from the center O(0,0), respectively the phasor previously dephased with $\frac{\pi}{2}$ or the tangent phasor to the unit center in point W(α, 1), of polar coordinates, with the pole in origin O(0, 0).

At the end of this chapter, was presented a particularly important and original application on "The rigorous transformation of a polar diagram of pliancy into a circle", which comes to correct the incomplete studies on the most studied oscillating system in the scholarly literature.

Part 1

SUPERMATHEMATICAL ECCENTRIC CIRCULAR FUNCTIONS (FSM-EC)

It is known that in mathematics, the functions may be defined virtually on any closed or open plane curve, as well as direct functions and inverse function.Thus:

- On RIGHT TRIANGLE → Trigonometric functions
- On OBTUSE TRIANGLE →**Bihringer** inclined trigonometric functions
- On TRILOBES → **Şelariu** trilobe functions
- On CIRCLE → **Euler** circular functions
- On ELIPSE → **Jacobi** elliptic functions
- On SQUARE (rotated with $\frac{\pi}{4}$) → **Alaci** quadratic functions
- On RHOMBUS → **Alaci** diamond functions
- On CVADRILOBES → **Şelariu** cvadrilobe functions
- $\begin{cases} \text{On CVADRILOBES}\left(rotated\ with\ \frac{\pi}{4}\right) \rightarrow Transtrigonometric\ functions \\ On\ ASTROIDS \rightarrow Infratrigonometric\ functions \\ On\ SPIRALS\ \rightarrow Spiral\ paratrigonometric\ functions \end{cases}$
  → **Malvina Baica - Mircea Cârdu**
- On POLYGON → **Enulescu** polygonal functions
- On LEMNISCATE → **Marcusevici** lemniscate functions

- On EVOLVENT → **Gogu Constantinescu** involute functions
- On HYPERBOLA ASYMPTOTES → **Eugen Vişa** pseudo-hyperbolic functions
- **On EQUILATERAL HYPERBOLA →** Hyperbolic functions

And there may be other such functions.

In this paper, were presented mainly the supermathematical functions (**FSM**) defined on the circle.

Part 1.1 SUPERMATHEMATICAL ECCENTRIC CIRCULAR FUNCTIONS OF ECCENTRIC VARIABLE

Euler's three superposed points (**Pole S (s, ε)** and the **center** of the unit circle **C (c, φ)** in **origin O** (0, 0) of a fixed point) may be separated in the following three ways, for each way of separation being proper other types of supermathematical functions (**SF**), as follows:

C(0,0) ≡ **O**(0, 0) ≡ **S(0,0) → CCF – Centric Circular Functions**

C(0,0) ≡ **O**(0, 0) ≠ **S(s,ε) → FSM-EC →Supermathematical Functions – Eccentric Circular**

C(c,φ) ≠ **O**(s, ε) ≡ **S(s,ε) → FSM-E$_L$C→ Supermathematical Functions – Elevated Circular**

C(c,φ) ≠**O**(0, 0) ≠**S(s,ε) → FSM-ExC → Supermathematical Functions – Exotic Circular**

All supermathematical functions can be, in their turn, of eccentric variable **θ** and centric variable **α**. The first ones are continuous functions only for an eccenter **S** inside the unit circle / disk, that is a numerical linear eccentricity $s \leq 1$.

The functions of centric variable are continuous for S placed anywhere in the plane of the unit circle, that is for $s \in [0, \infty]$.

By intersecting the unit circle with a line ($\mathbf{d} = \mathbf{d}^+ \cap \mathbf{d}^-$) and not only with the positive half-line ($\mathbf{d}^+$), at the urge of some talented and genuine mathematicians as PhD. **Horst Clep**, **eccentric trigonometry** or **FSM-EC** was brought in agreement with the differential geometry that operates with lines. Therefore, all **FSM-EC** have two determinations: a **main** one, marked by index **1**, or without index, when other determinations are not used and no confusion may occur, resulting from the intersection of the unit circle with the positive half-line $\mathbf{d}^+$ and a **secondary** one, marked by index **2**, resulting from the intersection of the unit circle with the negative half-line $\mathbf{d}^-$.

For eccenter **S**, located outside the unit circle (**s** >1), four determinations appear, of which the intersection of the circle with $\mathbf{d}^+$ generates the first two, by indices **1** and **2**, and the intersection with $\mathbf{d}^-$, for indices 3 and 4, are obtained from the relations for determinations 1, respectively 2, for a variable **θ**, previously dephased with **π** , that is $\boldsymbol{\theta} \rightarrow \boldsymbol{\theta} + \boldsymbol{\pi}$.

In Part 1.1 of this work are mainly presented / approached **FSM-EC** of eccentric variable **θ**, with predilection for the numerical linear eccentricity $\mathbf{s} \leq \mathbf{1}$ and angular eccentricity $\boldsymbol{\varepsilon} = \mathbf{0}$.

There are reviewed and graphically defined, on the unit circle, the main **FSM-EC** which will be subject to a future approach.

Some **FSM-EC** are dependent on origin **O(0,0)** of the reference system / fixed point, while others are independent of it. The description of **FSM-EC** begins in Ch. 4 with a function that is independent of the origin of the polar or rectangular fixed point which underlies the susequent definition and other **FSM-EC**.

Ch. 4 RADIAL ECCENTRIC FUNCTION rex θ AND SOME OF ITS IMPORTANT MATHEMATICAL APPLICATIONS

FSM-EC which the work begins with is the radial eccentric function of eccentric variable **$rex_{1,2}θ$**, the most important periodic function, a true "**king function**", as it was called by PhD. **Octav Em. Gheorghiu**, because it expresses the distance in plane between two points in polar coordinates: **$W_{1,2}$** on the unit circle UC(O, 1), at the intersection with line **d** to the **eccenter S (s, ε)**. Therefore, this function can express by itself the equations of all known plane curves, also called **centric** and of many new curves, which appeared along with **SM**, called **eccentric**.

Note: The expressions of $rex_{1,2}θ$ are the solutions for algebraic equations of 2nd degree that facilitate solving the inequalities of 2nd degree.

Then, there are defined and summarized, with their applications, the following supermathematical functions.

Ch. 5 OTHER MATHEMATICAL AND TECHNICAL APPLICATIONS OF THE RADIAL ECCENTRIC FUNCTION Rex θ

No matter how exact, the determination of a calculus relation of the complete elliptic integral **K(k)** with at least 15 accurate decimals, which led to the new hybrid numerical-analytic methods of calculation (A version of the **Landen method** of the arithmetic-geometric mean is a sheer numerical method, which gives **numerical value**, while **the new method** (let's call it Şelariu) gives a simple analytic calculus relation).

Ch. 6 ECCENTRIC DERIVATIVE FUNCTION dex θ AND SOME MATHEMATICAL AND TECHNICAL APPLICATIONS

The expression of this function is the general expression of the movement ratio (speed, turation) of **ALL** known plane mechanisms.

It expresses the speed of a point on the circle in **eccentric circular motion (ECM)** a generalization of the centric circular motion.

Ch. 7 QUALITY ANALYSIS OF THE PROGRAMMED MOVEMENT WITH SUPERMATHEMATICAL FUNCTIONS

CH. 8 THE METHOD OF FORCES AND MOMENTS SEPARATION

It provides a simple and accurate solution for all mechanical systems required by plane forces or reducible to them (elastostatics) avoiding the need to solve some systems of equilibrium equations using **d'Alambert** method.

The 2nd volume of **"SUPERMATHEMATICS. BASES"** continues with Ch. 12 entitled **"INTERGRALS AND ECCENTRIC ELLIPTIC FUNCTIONS"**. It is preceded by a table regarding **"THE ACTUAL SITUATION OF SUPRMATHEMATICS"** and **"THE LIST OF THE NEW MATHEMATICAL FUNCTIONS INTRODUCED BY THIS WORK"**, those introduced in mathematics, which the author called Centric Mathematics **(CM)** and in mathematics, in general, through the two volumes regarding supermathematics **(SM).** There are presented 60 new symbols for functions, introduced by the author in mathematics, through his work on supermathematics. And there were presented only the main functions, such as eccentric elliptic cosine and sine, **ceex**, **seex**, quadrilobe/(cvadrilobe) cosine and sine, **coq** and **siq**, but not the compound functions, such as tangent, cotangent, secant, cosecant. Yet, **Voinoiu tangent** tanvθ =$\frac{sin\theta}{cos\theta}$, the quadrilobe (cvadrilobe) tangent taqθ = $\frac{siq\theta}{coq\theta}$, etc., the derivative functions, as well as the the derivatives of the mentioned functions are presented.

And only this quantitative observation can reveal a lot of the qualities of this encyclopedic work, which is surprising and unique in the world literature, as it is its name of **SM**, from the moment of publication with this content, in 1978, and with this title, in 1993, as it results from the references attached to this paper.

From the first moment, the reader is impressed by the richness of the explanatory drawings, made with mathematical programs, using exactly **the supermathematical functions FSM** discovered by the author, as well as the numerous charts presenting the familes of new functions described in the work. For their intrinsic beauty, but also to complete the forms of the functions in a family, numerous families of **SM functions** in **3D** are also presented.

Here and now is where to quote **Ioan Ghiocel**, who prefaced the 2nd volume: "Do not wonder when Prof. M. E. Şelariu, under the pressure of inflection and folds of thoughts, brings together words that have not stood alongside from the foundation of the world, such as ***linear viscous damping circle***, ***elevated functions***, ***exotic functions***, ***the line defined as a confluent of the crook line***, etc... !"

If, in the 1st vol., there were introduced particularly the eccentric circular supermathematical functions, abbreviated by the author as **FSM-EC**, of which we mention the functions **aex, bex, dex, cex, sex, rex, tex, ctex,** in the 2nd volume, **Ch. 12**, there were introduced new eccentric elliptic integrals of the first kind and of the second kind, generalizing the centric elliptic integrals, which they may represent, for numerical linear eccentricity **s = 0**, that is if the eccenter **S(s, ε)** overlaps the origin **O(0,0)** of the system of coordinates or of the fixed point **xOy**.

At the same time, there are presented eccentric elliptic, hyperbolic and parabolic functions, in terms of the classical known variables, but also in terms of arc of a unit circle, common tangent to the equilateral hyperbola, unit ellipse and to the parabola, in their peak. Finally, there are presented the centric elliptic, hyperbolic and quadratic functions, in terms of the arc of the unit circle previously mentioned, a unique case in the centric mathematics literature.

The author called them "***functions on cones with common peak***".

Chapter 13 is dedicated to the centric functions and to the eccentric **self-induced** ones, of the form sin[sin[sin[sin[sin[sin[…[sinx]]]]]]]]] or cex[cex[cex[cex[…[cex[θ]]]]]]]] and to the **induced** functions of the form [cos[sin[sin[tan[tan[cos[sin[cos[tan[...sin[x]]]]]] or cex[sex[sex[tex[tex[cex[sin[cos[tex[…sex[θ]]]]]].

There are also presented the derivatives of the induced and self-induced functions, centric and eccentric, as well as the derivatives of the **Voinoiu** centric and eccentric circular functions, initially presented in the first volume, as a necessary correction for the tangent and cotangent functions, wrongly introduced in mathematics, as the great Romanian mathematician **Octavian Voinoiu** demonstrated in his book "**INTRODUCTION IN SIGNADFORASIC MATHEMATICS**".

To make a difference between **Voinoiu** trigonometric functions it was necessary to determine the derivative of the function **Abs[f(x)]**, non-existent derivative in the scholarly literature. The author demonstrates (p. 73) that the derivative of this function is $\frac{d}{dx}[f(x)] = Sign[f(x)]\ \frac{d}{dx}[f(x)]$.

Chapter 14 is dedicated to eccentric hyperbolic functions. First the eccentric hyperbolas are presented and, especially, the eccentric rectangular hyperbola, as well as other centric and eccentric exponential function of the eccentric variable θ and the geometric definition of the centric and eccentric hyperbolic functions. Beside the classical hyperbolic functions, also known in centric mathematics **(CM)** such as cosine - **cexh** -, sine - **sexh** -, tangent - **texh** -, etc. eccentric hyperbolic, there are also presented functions which appeared at the same time with **FSM-EC,** as eccentric hyperbolic amplitude - **aexh** -, eccentric hyperbolic radial - **rexh** -, eccentric hyperbolic derivative - **dexh** - etc.

For the hyperbolic functions there were also presented the elevated hyperbolic cosine (celh) and sine (selh). In the conclusion of this chapter new geometric objects are presented. They are expressed with the help of these functions, newly introduced in mathematics.

Chapter 15 is dedicated to **FSM-EC** of centric variable α, marked by the author with capital letters (Aex, Bex, Cex, Dex, Rex, Sex, Tex, etc.) to be distinguished from those of eccentric variable θ (aex, bex, cex dex, rex, sex, tex, etc.). The chapter begins with the presentation of the explanatory drawings for defining **FSM-EC** in the case of an eccenter **S**(s, ε) placed inside the unit disk, i.e. inside the unit circle, and the case of the eccenter **S** placed outside it is presented separately.

FSM-EC bexθ and Bexα of numerical linear eccentricity s = 1, with their graphics in symmetrical sawteeth, respectively, asymmetric, were named by the author ***Octav Gheorghiu triangular functions*** in memory and honor of PhD **Octav Em. Gheorghiu**, successor of PhD **Alaci Valeriu** to the board of the Department of Mathematics at "Traian Vuia" Polytechnic Institute of Timisoara. Just as, in honor of the mathematician PhD Florentin Smarandache, the step functions, obtained with the help of **FSM-EC,** were called ***Smarandache step functions***.

In this chapter are outlined, without any doubt, the advantages of expressing some special periodic functions, triangular, quadratic, rectangular, step, etc. with the help of **FSM-CE**, which expresses them exactly, and with **FSM-EC** with only two simple terms, compared with their approximate expression by belaying in various series. Here as well, are presented the solutions of an undamped system of variable amplitudes, expressed by bexθ function, of the differential equation $\Delta\ddot{\varphi} + v_0^2 sin\varphi = \varphi_0 v_0^2 sin v_0 t$.

In Figure **15.28** you can find the drawings of the engine skotch yoke and engine slider crank and some **FSM-CE** that can be expressed by these mechanisms.

A new method of integration, which appeared due to **FSM-EC**, is presented in **Chapter 16**.

It is called ***"Method of integration through differential dividing"*** and it is based on dividing the variable θ in variables α and β, according to the **FSM-EC** known relationship: **θ = α + β**, which gives the differential **dθ** the possibility to divide, in its turn, in **dα** and **dβ**, i.e. **dθ = dα + dβ**.

In this way, a series of integrals, solvable by the residue theorem in the complex plane, can be solved directly and much easier, as illustrated through the applications in this chapter. One of the applications is completed together with PhD. Math. Florentin Smarandache and it was previously presented, separately, in an article.

Since at θ = α = 0 and for an angular eccentricity ε = 0, regardless of the numerical linear eccentricity value **s ∈ [-1, 1]** we obtain β = **bexθ** = arcsin [s.sin(θ - ε)] = 0 as well as for θ = α = π, the integration between limits 0 and π as well as between limits 0 and 2π result extremely conveniently. In this respect, the 8 applications presented in the paper are eloquent.

FSM-EC bexθ, described and noted in this chapter as **βsexθ** can also express the solutions of various nonlinear vibrating systems, subject of **Ch.17**.

There are presented the functions **bexθ = βsexθ** and **βcexθ** = arcsin[s.cos (θ - ε)] for an eccenter **S(s** ∈ [-1, +1], ε = 0) or **S(s** ∈ [0, +1] , ε = 0 V π), which is the same thing, as well as their derivatives as their geometric significance (**Fig.17.2**).

Since the wronskian matrix given by the solutions

$\begin{cases} x = \boldsymbol{\beta cex\theta} \\ y = \boldsymbol{\beta sex\theta}, \end{cases}$ is different from zero, it results that the two solutions are linearly independent. The static elastic properties of these vibrating systems and the integral curves in the phase space are also presented.

Chapter 18 is dedicated to the **supermathematical functions** (centric, eccentric, elevated and exotic) on **cones,** as well as on centric cones, depending on the arc of the tangential circle to the peak of cones, and on eccentric cones, like a sort of prelude to **chapter 19**, on **the elliptic supermathematical functions of the arc of the circle**. On this occasion, are defined the **unit ellipses** on x, respectively on y, marked $\mathbf{U_x}$, respectively $\mathbf{U_y}$, so that the projections of the points on axis x, respectively y, inscribes itself in the interval [-1, +1].

Very voluminous, **Chapter 19** covers 42 pages (254...296), where the **supermathematical elliptic functions**, their properties, derivatives and the rotation speed of a point on the unit ellipses are defined. Besides the known elliptic functions in the centric mathematics - cosine cn(u, k) and sine sn(u, k) – are also presented here the new functions, such as eccentric elliptic amplitude, compared with the elliptic function **Jacobi** amplitude or amplitudinus - am(u, k) - and the eccentric elliptic derivative functions according to cosine →**dece**(α, k = s) and to sine →**dese**(α, k = s).

In **figure 19.12**, are presented Jacobi elliptic functions cn, sn dn, not on an ellipse, but on the unit circle, thanks to the new **FSM-EC**. The step elliptic functions were named by the author as **Smarandache** step elliptic functions, noted as **smce**(α, k) and **smse**(α,k), with their graphs presented in **figure 19.13**, along with the graphs of their derivatives.

In **paragraph 19.9** are presented the inter-trigonometric functions, defined on quadrilobes (cvadrilobes), which complement the space between **Alaci Valeriu** square and **Euler** unit circle, as well as the field between **Euler** centric circular functions and **Alaci Valeriu** quadratic trigonometric functions.

It is shown that the new closed curves called quadrilobes (cvadrilobes) by the author are equivalents of a unit 'ellipse' simultaneously on x and y (**Fig.19.19**).

With the help of these quadrilobe (cvadrilobe) functions were defined the continuous transformations of the circle into a perfect square, of the sphere into a perfect cube, as well as of the cone into a perfect pyramid with a square base. Their 3D images are presented in **figure 19.16**, being new (super)mathematical geometric objects.

In **paragraph 19.11** are presented the **supermathematical elliptic functions** as solutions of some nonlinear vibrating systems and **paragraph 19.12** is dedicated to the **elliptic functions of the arc of the circle**.

Paragraphs 19.13 and **19.14** refer to **SM centric hyperbolic functions**, respectively, **SM eccentric hyperbolic functions**, being also presented the cosine, sine and tangent functions and the new functions introduced by the author and called **Voinoiu** hyperbolic tangent.

Entitled **"Wormholes in mathematics"**, **Ch. 20** claims that they can be realised by means of some **hybrid FSM-EC**. In author's opinion, the wormhole would be a possible faster way of connection, between centric circular mathematics and elliptic mathematics, which is the author's lifetime dream, unfortunately not completely realized yet. There are presented two rewardable "breakthroughs": **Neville Theta C** represented exactly by means of **FSM-EC** eccentric cosine **cexθ** (**fig. 20.2, a** and **fig. 20.2, b**) and expressing the **Jacobi Zeta** elliptic function by means of the modified **FSM-EC** sine [bexθ] (**fig. 20.3**).

Paragraph 20.3 presents other **special hybrid mathematical functions**.

Chapter 21 refers to **eccentric analytic trigonometric functions** of real variable (R-analytic **§ 21.2**) and centric **(§ 21.3)**. **Paragraph 21.4** is dedicated to **eccentric analytic circular functions of eccentric variable** dependent on the origin of the reference point (cos, sin, tan, etc.), and **§ 21.5** to those independent of the origin of the coordinate axes system (bex, dex, rex, aex, etc.). **Paragraph 21.10** deals with **double analytic FSM-EC**.

Chapter 22 refers to **FSM-EC of complex variable (C - analytic)** and it is richly illustrated, especially in 3D, as well as **§ 22.3** regarding the various mathematical objects represented by **FSM-EC** and **FSM-AEC**, ending with the mathematical representation of some technical parts and systems.

Instead of afterword, **Ch.23** refers to **"The dark matter of the mathematical universe"** where are presented the eccentric irrational numbers, the **eccentricity** as a **new** hidden **dimension of the space**, the **mathematical hybridization**, the eccentric real numbers and **eccentric trigonometric system**, compared with the centric one, to emphasize the definite advantages of the first, which is a continuous system, while the centric one is discreet. Hence the big advantages of curves and technical surfaces approximation, besides the fact that, along with the appartion of supermathematics, a whole range of surfaces, previously considered non-mathematical, became (super)mathematical surfaces and, therefore, they can be exactly represented using the new functions of **Mircea Eugen Şelariu**'s supermathematics.

CONCLUSION

The innovative force of Professor Mircea Eugen Şelariu's supermathematics recommends it as an internationally valuable theory, which opens new branches of research with lots of applications.

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EXCENTRICITATEA, DIMENSIUNEA DE DEFORMARE A SPAŢIULUI

MARIAN NIŢU, FLORENTIN SMARANDACHE, MIRCEA EUGEN ŞELARIU

Motto: " Ştiinţa nu e bună azi, dacă ieri nu s-a gândit la mâine."
Grigore C. Moisil

0.1. REZUMAT

Ideea centrală a lucrarii este prezentarea unor transformări noi, anterior inexistente în Matematica ordinară, denumită centrică (**MC**), dar, care au devenit posibile graţie apariţiei matematicii excentrice şi, implicit, a supermatematicii.
Aşa cum se demonstrează în cadrul lucrării, noile transformări geometrice, denumite conversi(un)e sau transfigurare, şterg granitele dintre formele geometrice discrete şi cele continuue, demonstrând că primele sunt şi ele continue, fiind doar aparent discontinue.

0.2 ABREVIERI ŞI NOTAŢII

C → Circular şi **Centric, E→ Excentric** şi **Excentrice, F→Funcţie, M→Matematică, Circular Excentric →CE, F CE →FCE, M centrică →MC, M excentrică → ME, Super M →SM, F MC →FMC, F ME→FME, F SM → FSM**

1. INTRODUCERE: CONVERSI(UNE)A sau TRANSFIGURAREA

În lingvistică un **cuvînt** este unitatea fundamentală de comunicare a unui înțeles. El poate să fie compus din unul sau mai multe morfeme. În mod obișnuit un cuvînt se compune dintr-o parte de bază, numită rădăcină, la care se pot atașa afixe. Pentru a defini unele noţiuni şi a exprima domeniul în care sunt valide, sunt necesare, uneri, mai multe cuvinte; două în cazul de faţă:
CONVERSIA (CONVERSIUNEA) SUPERMATEMATICĂ.

Noţiunea este ideea cea mai simplă şi ordonată care reflectă una sau mai multe /(o serie) finită de însuşiri şi obiectele la care aceste însuşiri sunt esenţiale.

Noțiunea este o informație minimală coerentă și utilizabilă, relativ la un obiect, acțiune, proprietate, sau eveniment determinat.

Conform DEX, **CONVERSIUNEA**, printre multe alte definiţii / înţelesuri, o are şi pe aceea de ***"schimbare a naturii, a formei unui lucru"***. In cele ce urmează, tocmai despre aceasta va fi vorba, despre transformare / schimbare / convertirea anterior imposibilă, în matematică ordinară, clasică, denumită acum şi **CENTRICA** (**MC**), a unor forme în altele şi care, a devenit posibilă acum, graţie apariţiei noii matematici, denumită **EXCENTRIUCĂ** (**ME**) şi noilor complemente de matematică, înglobate şi denumite vremelnic / temporar şi **SUPERMATEMATICĂ (SM)**. Ne referim la conversia cercului în pătrat, a sferei în cub, a cercului în triunghi, a conului în piramida, a cilindrului în prismă, a torului circular în secţiune şi ca formă în tor pătrat în secţiune şi/sau formă, ş.m.a. (Fig. **1**).

TRANSFORMAREA EXCENTRICĂ

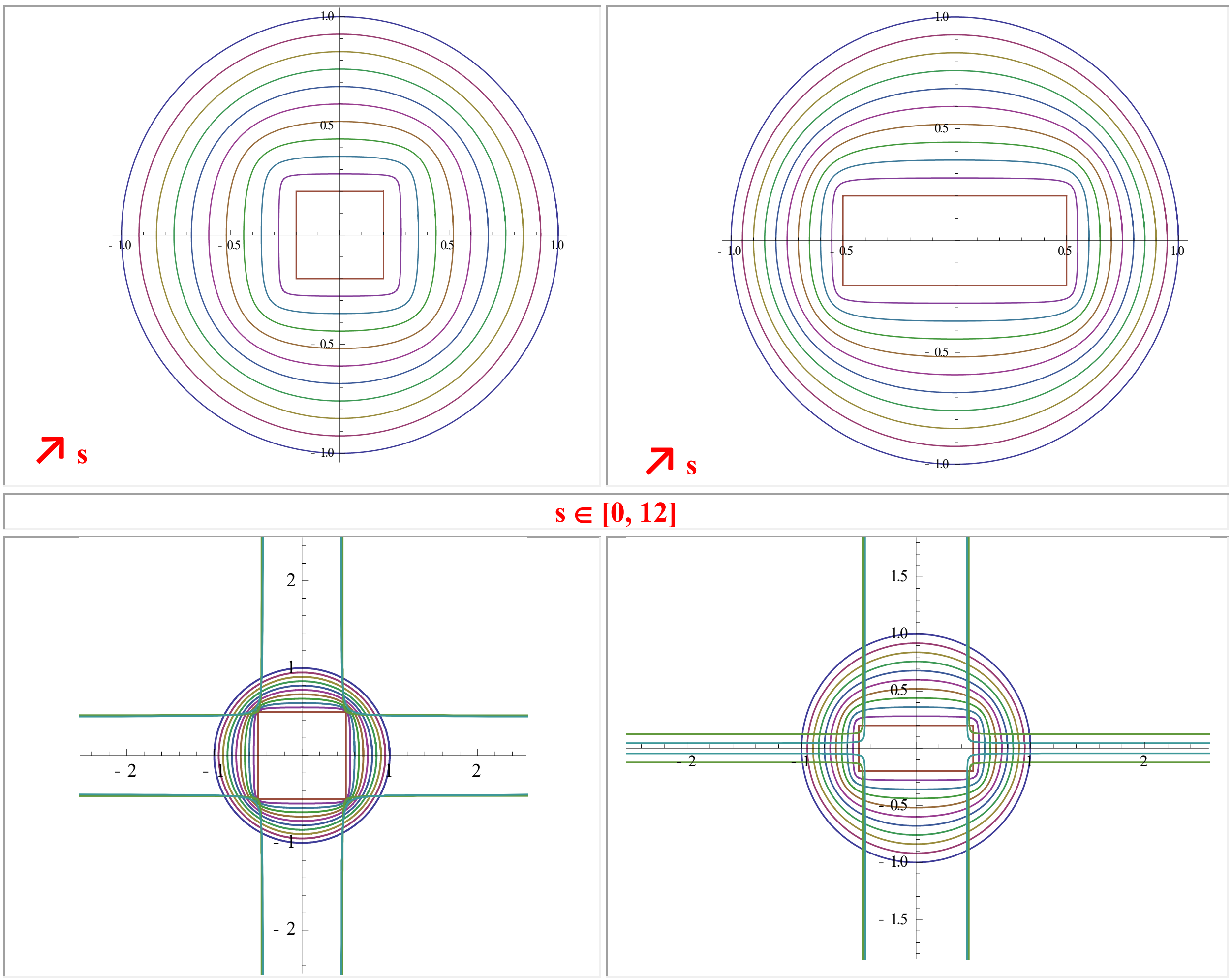


Fig.**1,a** Conversi(une)a sau transfiguraea în **2D**
cecului în **pătrat** şi/sau **dreptunghi** → **TRANSFORMAREA EXCENTRICĂ**

CONVERSI(UNE)A SUPERMATEMATICĂ (CSM) este un mijloc intern de îmbogăţire a vocabularului matematic, care consistă în formarea unei denumiri, cu unul sau mai multe cuvinte noi, cu 2 în cazul de faţă, prin asimilarea unor cuvinte din vorbire curentă într-un domeniu specializat, cum este Matematica, în intenţia de a denumi, mai adecvat, noile operaţii posibile doar graţie apariţiei noii matematici excentrice şi, implicit, a **supermatematicii**. Deoarece, conversiunile anterior amintite, nu au putut fi realizatre/(avea loc), până în prezent, în **MC**, ci în **SM**, suntem nevoiţi s-o denumim **conversie (conversiune) SUPERMATEMATICĂ (CSM).**

TRANSFORMAREA DE CENTRARE

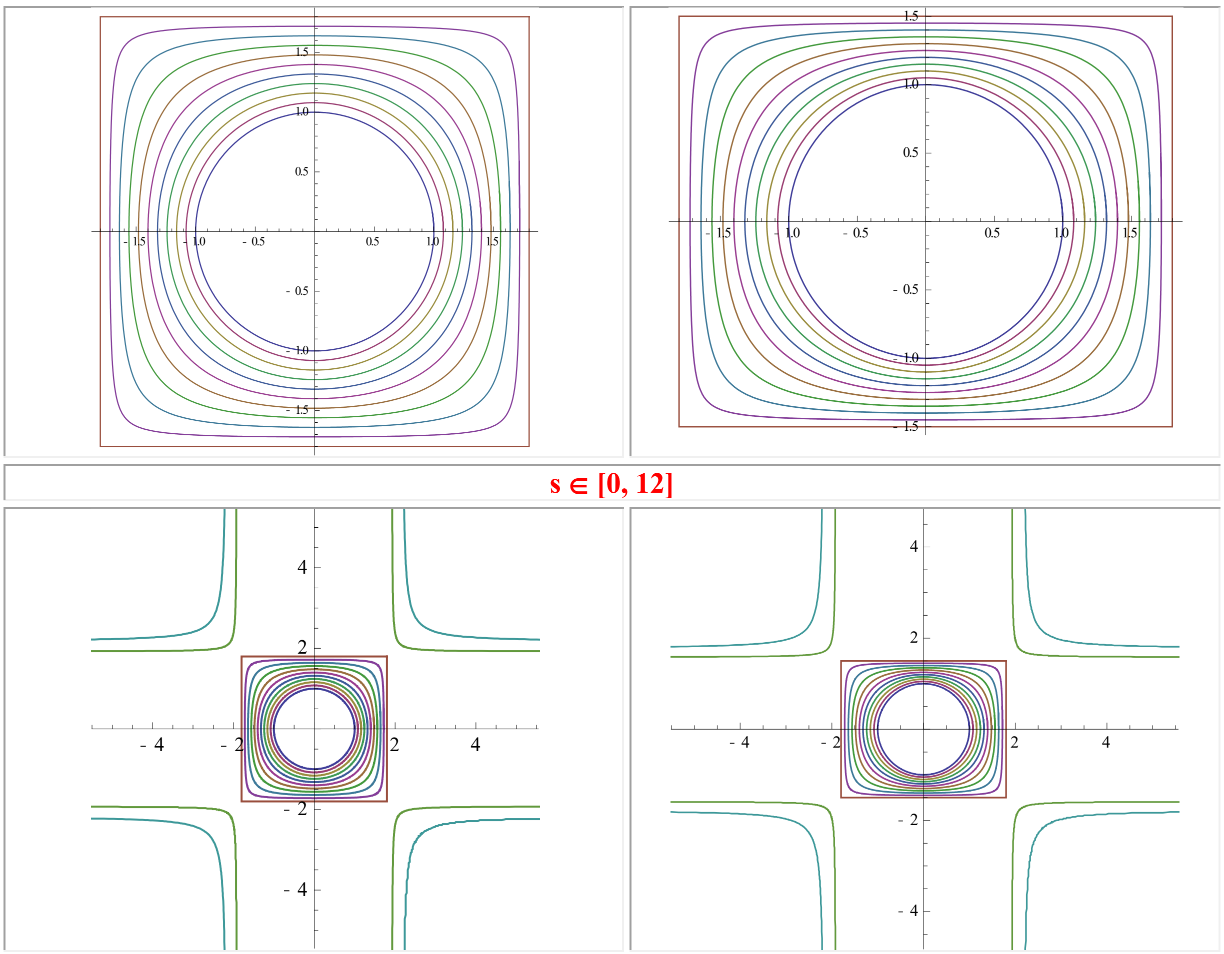


Fig.**1,b** Conversi(une)a sau transfigurarea în **2D**
a pătratului şi/sau a **dreptunghiului în cerc → TRANSFORMAREA DE CENTRARE**

În lucrarea [14], transformarea continuuă a cercului în pătrat a fost denumită şi **transformare excentrică**, deoarece, în acest caz, excentricitatea numerică liniară **s** variază / creşte de la 0 la 1, constituind o trecere din domeniul matematicii centrice, **MC** → s = 0, în cel al matematicii excentrice, **ME** (s ≠ 0) → s ∈ (0,1], prin care forma circulară se îndepartează din ce în ce mai mult de forma de cerc până ce ajunge un pătrat perfect (s = ± 1). În aceeaşi lucrare, transformarea inversă, a pătratului în cerc, a fost denumită **transformare de centrare** din considerente lesne de înţeles. Acesleaşi observaţii sunt valabile şi pentru transformarea cercului în dreptunghi şi a dreptunghiului în cerc (Fig.**1**).

Cei mai mulţi fizicieni şi matematicieni moderni consideră că numerele reprezinta limbajul realităţii. Adevarul este, însă, că formele sunt cele care generează toate legile fizicii.

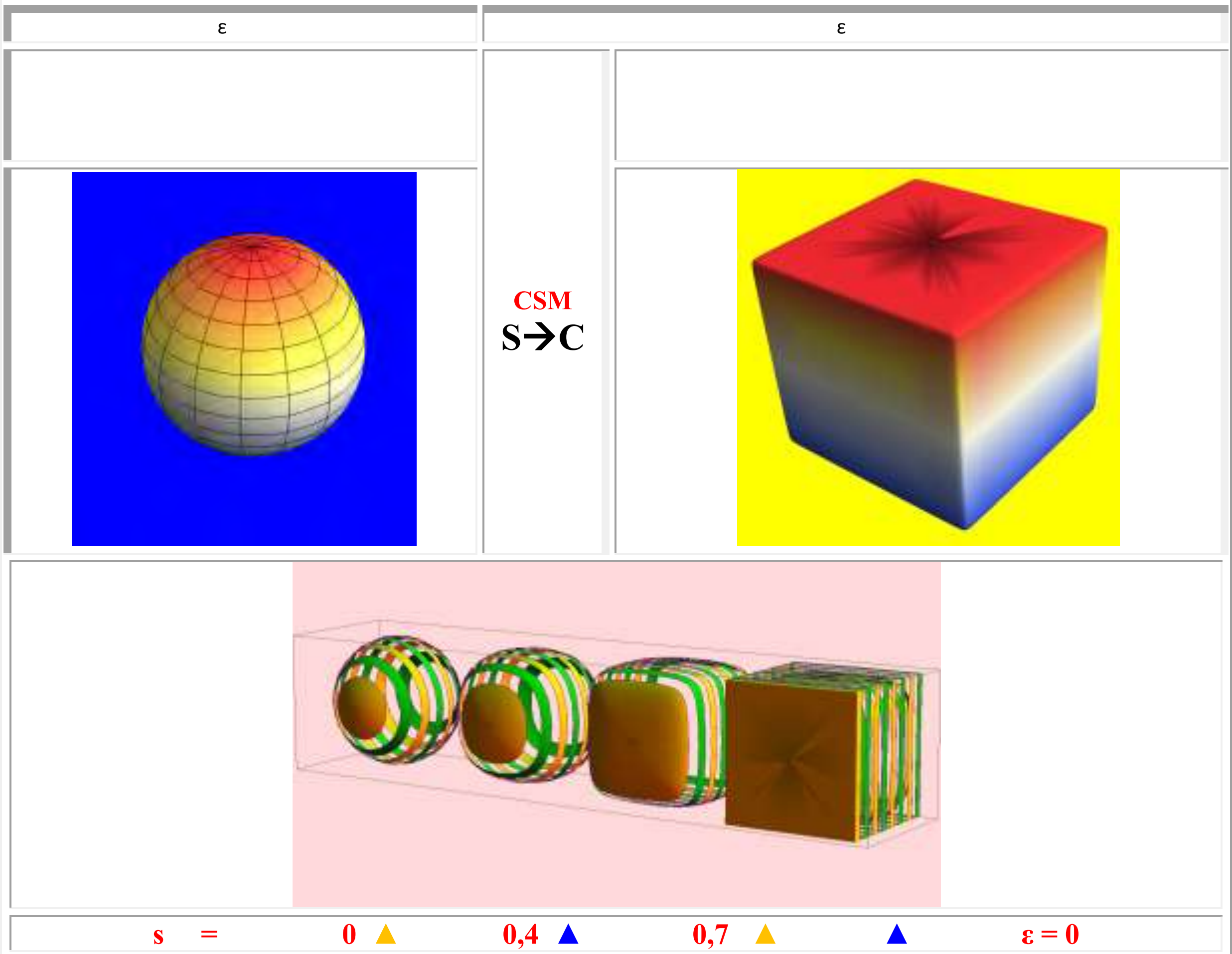


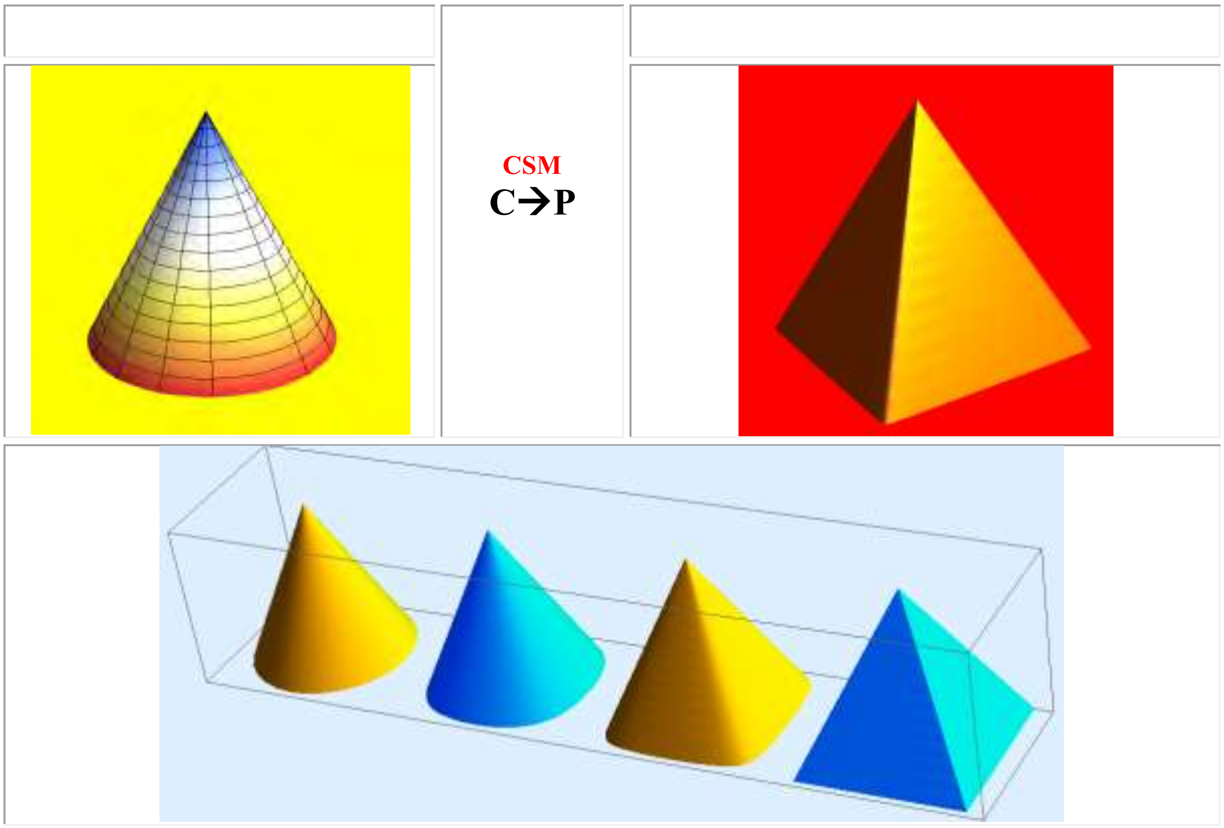
Fig.**2,a** Conversi(une)a sau transfigurarea **sferei** în **cub**

Iată ce scrie reputatul fizician **Prof. Dr. Fiz. Liviu Sofonea** în "**GEOMETRII REPREZENTATIVE ŞI TEORII FIZICE**", Ed.Dacia, Cluj-Napoca, pag. 24, în 1984, în capitolul "**GEOMETRIA MATEMATICĂ** şi **GEOMETRIA FIZICĂ**":

"Prin ***geometrizarea*** căutăm (deliberat şi în mod sui generis) tocmai normele de ordine (fundamentale, detaliate; chiar pe cea supremă, *unica-unificatoare*) gândindu-ne după ordinea pre-stabilită (relativ la demersul *teoretizării fizicii*) din "*lumiile geometrice*" clădite şi mişcate după canoanele disciplinate în stilul *more geometrico* (structură şi derivabilitate logică probată în ***geometric***; unde a reuşit); o extindere în intenţia de a verifica dacă "merge" şi în "***fizic***", iar în masura în care constatăm că avem motive a spune că ea "merge într-adevar", scontăm un câştig metodologico-operant, euristic, dar chiar gnoseologic. Niciodată însă "pre"-normarea ***geometrică*** nu poate "merge" deplin; ea nu poate fi dacât (inerent) parţială, limitată, adesea o simplă trasare de contur, o sugerare, o incitare, o schemă, uneori prea provizorie, dar ne servim de ea ca de o schelă, ca să ne ridicăm, cum putem, spre o cât mai adecvată descriere şi chiar înţelegere"

În **geometria matematică centrică** se face ce se poate, cum se poate, cu ce se poate, iar în **geometria supermatematică** se face ce trebuie, cum trebuie, cu ce trebuie, aşa cum va rezulta în continuare.

În **geometria supermatematică**, între elementele "schelei **MC**" se pot introduce oricât de multe alte elemente constructive, care oferă o structură de "schelă" infinit mai densă şi cu mult mai rezistentă şi, în consecinţă, mult mai înaltă, capabilă să atingă o înălţime nemaiîntâlnită şi o descriere şi înţelegere extrem de profundă.



s = **0** ▲ **0,4** ▲ **0,7** ▲ **1** ▲ **ε = 0**

Fig.**2,b Conversi(une)a** sau transfigurarea **conuli în piramidă**

Noţiunile esenţiale ale geometriei sunt, în funcţie de dimensiunea lor topologică: **corpul** (3), suprafaţa (2), **linia**(1) şi **punctul** (0).

Noţiunile elementare ale geometriei sunt punctul, dreapta, spaţiul, curba, planul, figurile geometrice (segment, triunghi, pătrat, dreptunghi, romb, poligoanele, poliedrele ş.a., arce, cerc, elipsa, hiperbola, spirala, elicea ş.m.a.) în spaţiul 2D cât şi în spaţiul 3D.

Cu elementele geomatrice fundamentale se definesc şi se construiesc toate formele şi structurile geometrice ale obiectelor:

- Forme discrete, sau discontinue, statice, direct, plecând de la o mulţime finită (discretă) de puncte, legându-le static, cu drepte şi plane;
- Forme continue sau dinamice, mecanice, plecând de la un singur punct şi considerând mişcarea acestuia, deci **timpul**, obţinându-se forme continue de curbe, ca traiectorii de puncte, suprafete, ca traiectorii sau urme de curbe, în plan (2D) sau în spaţiul 3D.

În consecinţă, s-a considerat, şi se mai consideră încă, existenţa a două geometrii: geometria discontinuului, sau geometria discretă şi geometria continuului.

Din moment ce, atât obiectele marginite de suprafete plane (cub, piramidă, prisma), aparent discontinue, cât şi cele mărginite de diverse tipuri de suprafeţe continue (sferă, con, cilindru) pot fi descrise cu aceleaşi ecuaţii parametrice, primele pentru excentricitate numerica **s = ± 1** şi cele din urmă pentru **s = 0**, rezultă că în **SM** există o singură geometrie, geometria continuului.

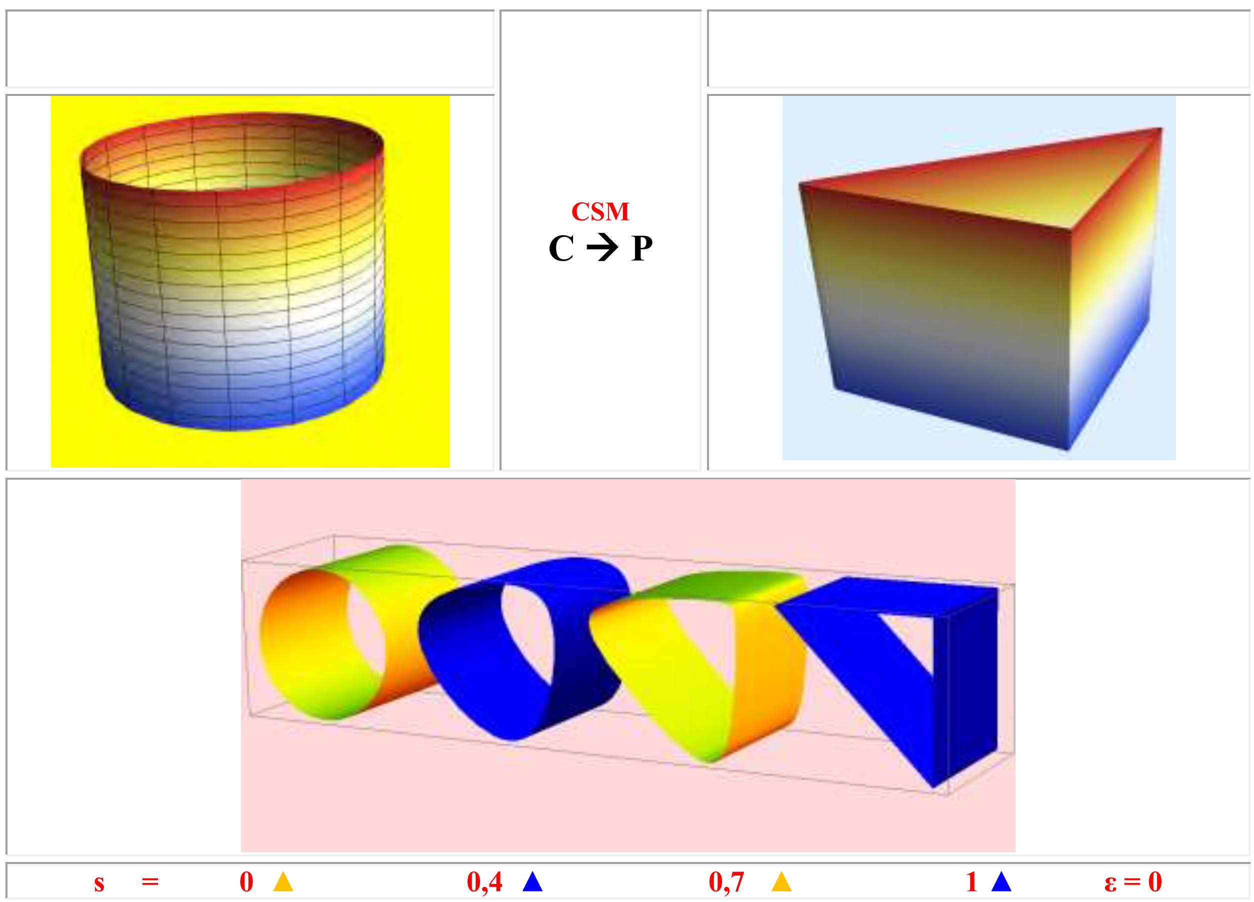


Fig.**2,c Conversiunea** sau transfigurarea **cilindrului** în **prismă**

Altfel spus, **SM** şterge graniţele dintre continuu şi discontinuu, tot aşa cum **SM** a şters graniţele dintre **liniar** şi **neliniar**, dintre **centric** şi excentric, dintre ideal / perfecţiune şi real, dintre circular şi hiperbolic, dintre circular şi eliptic ş.m.a.

Intre valorile excentricităţii numerice de **s = 0** şi **s = ±1**, mai există o infinitate de valori şi, pentru fiecare valoare, o infinitate de obiecte geometrice care, toate, au dreptul la o existenţă geometrică.

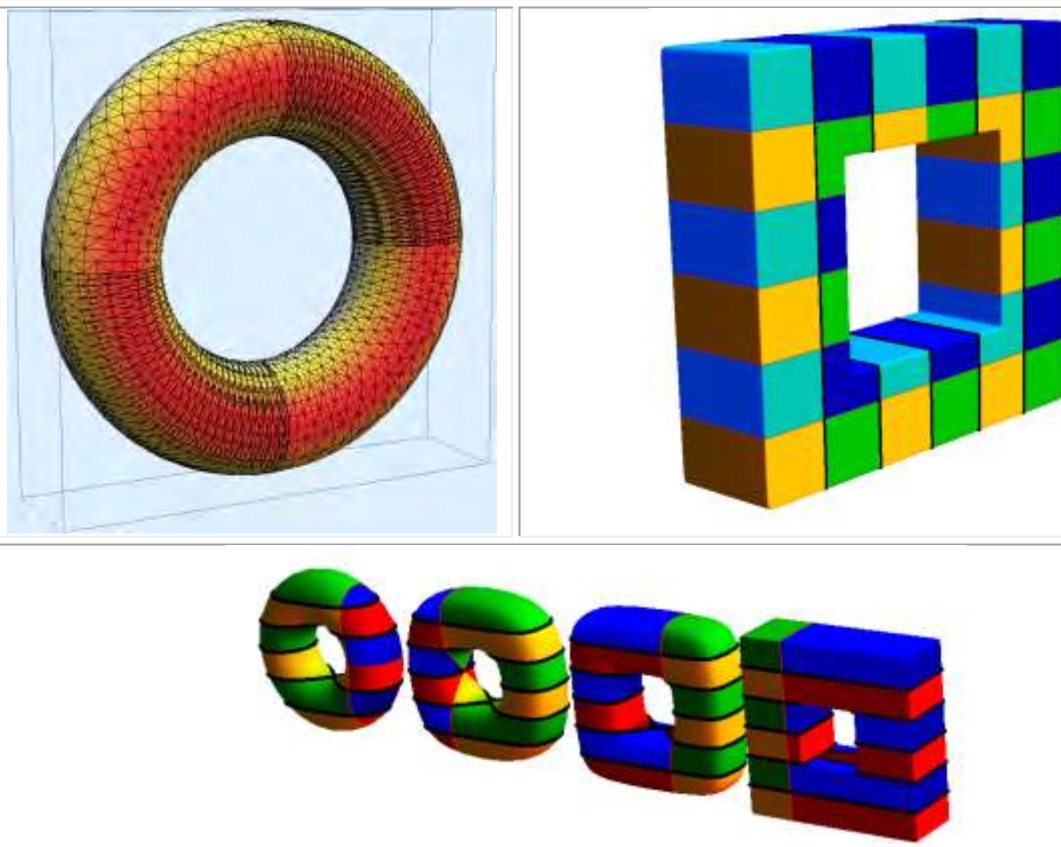
Dacă obiectele matematice geometrice pentru s ∈ [**0** V ±1] aparţin **matematicii centrice** (**MC**), ordinare (cerc→patrat, sferă→cub, cilindru→prisma ş.a.), cele pentru s ∈ (**0 ,** ±1) au forme, ecuaţii şi denumiri necunoscute / inexistente în această **matematică centrică** (**MC**).

Ele aparţin noii matematici, **matematicii excentrice** (**ME**) şi, implicit, **supermatematicii** (**SM**), care este o reuniune a celor două matematici: **centrică** şi **excentrică**, adică **SM** = **MC** ∪ **ME.**

Ştergând graniţele dintre centric şi excentric, **SM** a dizolvat implicit şi granitele dintre **liniar** şi **neliniar**; liniarul fiind apanajul **MC**, iar neliniarul al **ME** şi a introdus o separare între entităţile geometrice centrice şi cele excentrice. Astfel, toate entităţile **matematicii centrice** în 2D au fost denumite

centrice (centrice circulare, centrice pătrate, centrice triunghiulare, centrice eliptice, centrice hiperbolice ş.a.m.d.) iar cele ale **matematicii excentrice** au fost denumite **excentrice** (excentrice circulare, excentrice eliptice, excentrice hiperbolice, excentrice parabolice, excentrice spirale, excentrice cicloidale, ş.a.m.d.).

Dacă entităţile 2D **centrice** pot rămâne la denumirile actuale (cerc, pătrat, elipsă, spirală, ş.a.m.d.) la cele **excentrice** trebuie specificată şi denumirea de **excentrice**. Acelaşi lucru este valabil şi pentru entităţiile 3D: cele **centrice** (sferă, elipsoid, cub, paraboloid ş.a.m.d.) pot purta, în continuare, denumirile vechi, iar celor noi, **excentrice**, e necesar sa li se specifice că sunt **excentrice**. Adică: sferă excentică, elipsoid excentric, cub excentric, paraboloid excentric ş.a.m.d.



s = 0 ▲ **0,4** ▲ **0,7** ▲ **1** ▲ **ε = 0**

Fig.**2,d Conversi(une)a** sau transfigurarea **torului circular**
în **tor pătrat**, atât ca formă cât şi în secţiune

Cu noile funcţii **SM**, precum amplitudine excentrică aex**θ** şi Aexα, de variabilă excentrică **θ** şi, respectiv, centrică α, beta excentrică bex **θ** şi Bexα, radial excentrică rex şi Rex, derivată excentrică dex **θ** şi Dexα, ş.a. care, neavând echivalente în **centric** / (**MC**) nu necesită alte denumiri de precizae / deosebire a domeniului matematic din care fac parte.

Excepţie fac ultimele două **FSM-CE,** rex**α** şi dex**α**, (**θ = α**) cărora li s-a descoperit, ulterior, echivalente în **centric**: funcţiile **radial centrică radα**, care este fazorul direcţie α şi **derivat centrică derα**, care este fazorul direcţiei α + − , fazori reciproc perpendiculari.

HIBRIDAREA ŞI METAMORFOZAREA SUPERMATEMATICĂ CONSECINŢELE ALE NOILOR DIMENSIUNI ALE SPAŢIULUI

Spaţiul este o entitate abstractă care reflectă o formă obiectivă de existenţă a materiei. Apare ca o generalizare şi abstractizare a ansamblului de parametri prin care se realizează **deosebirea între diferite sisteme** ce constituie o stare a universului.

El este o formă obiectivă şi universală a existenței materiei, inseparabilă de materie, care are aspectul unui întreg neîntrerupt cu trei dimensiuni și exprimă ordinea coexistenței obiectelor lumii reale, poziţia, distanţa, mărimea, forma, întinderea lor.

În concluzie, se poate afirma că spaţiul apare ca o sinteză, ca o generalizare şi abstractizare a constatărilor cu privire la o stare, la un moment dat, a universului.

În cadrul mecanicii clasice, noţiunea de spaţiu este aceea a modelului spaţiului euclidian tridimensional (E3) omogen, izotrop, infinit.

Când se discută despre spaţiu, primul gând este îndreptat spre **poziţie**, adică noţiunea de poziţie este direct asociată noţiunii de spaţiu. **Poziţia** este exprimată în raport cu un sistem de referinţă (reper) sau, mai scurt, printr-un sistem de coordonate.

Un obiect tridimensional are în spaţiu E^3 **6 grade de libertate**, constituite din cele **3 translaţii,** pe direcţiile **X**, Y şi Z şi din **3 rotaţii,** în jurul axelor **X**, Y şi Z, notate, respectiv, cu **θ, φ, ψ** în Matematică şi în Mecanică şi cu **A, B** şi **C,** în tehnologie şi în robotică.

Un obiect poate fi "realizat" sau, mai precis, poate fi reprodusă imaginea lui în spaţiul virtual, când apare în 3D, pe ecranul monitorului unui computer, prin folosirea unor programe tehnice (CAD) sau matematice comerciale (MATHEMATICA, MATLAB, MATHCAD, MAPLE, DERIVE, ş,a.) sau speciale, care folosesc **FSM-Excentrice, Elevate** sau/şi **Exotice -** la descrierea obiectelor, cum este **SM-CAD-CAM**.

Prin **modificarea excentricitaţii,** obiectele cunoscute şi formate în domeniul centric al supermatematicii (**SM**), adică, în matematica centrică (**MC**), pot fi deformate în domeniul excentric al **SM**, adică, în matematica excentrică (**ME**) şi transformate iniţial în obiecte hibride, proprii **ME**, ca, apoi, să fie re-transformate în obiecte de alt gen, cunoscute în **MC**. Ca de exemplu, deformarea unui **con** perfect ($s = 0$) în **cono-piramide** [$s \in (0, 1)$] cu baza un pătrat perfect şi vârful conic, care constitue obiectele hibride, situate între con şi piramidă, pâna la transformarea ei într-o **piramidă** perfectă ($s = \pm 1$) cu baza un pătrat perfect (Fig.**3**). Obiectul poate fi realizat în fapt, prin diversele metode de prelucrare mecanice [v. **Mircea Şelariu**, Cap.17 **Dispozitive de prelucrare**, PROIECTAREA DISPOZITIVELOR, EDP, Bucureşti, 1982, coordonator **Sanda-Vasii Roşculeţ**] de **formare** (turnare, sinterizare), **deformare** (la cald şi la rece), **dislocare** (decupare, aşchiere, eroziune, netezire) şi **agregare** (sudare şi lipire).

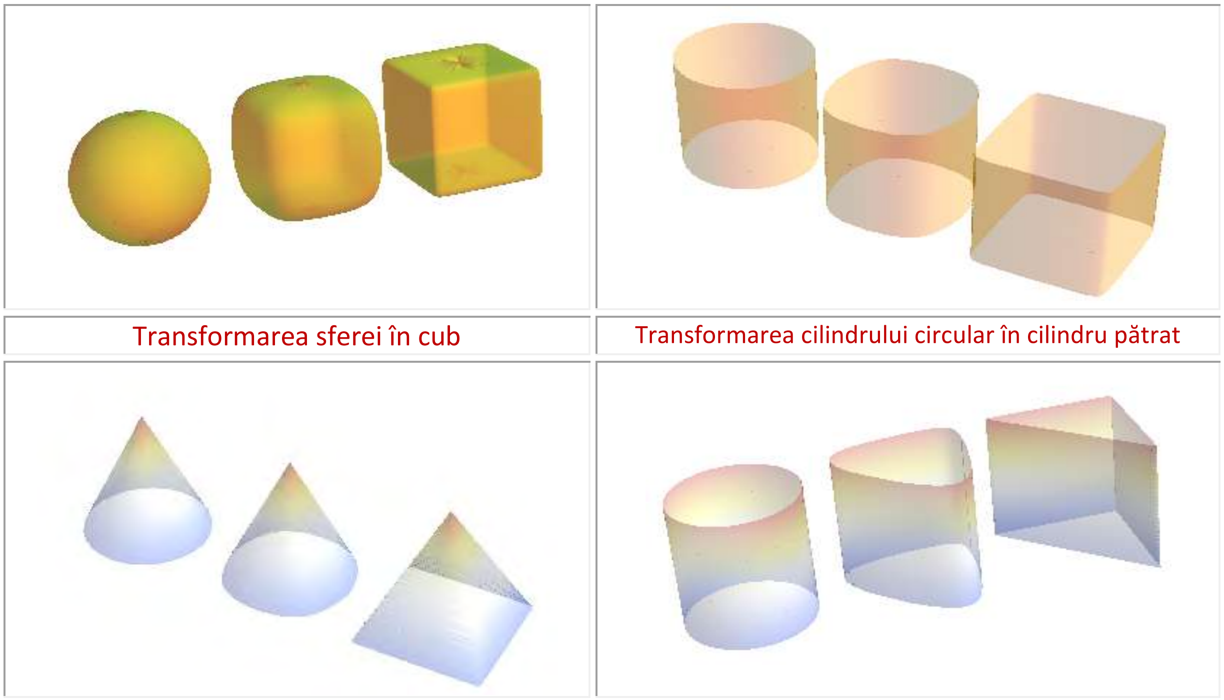
În ambele cazuri, sunt necesare **mişcări** ale sculei şi/sau ale piesei, respectiv, ale spotului luminos care delimitează pe ecran un pixel şi trece de la un pixel la altul.

Mişcarea este strâns legată de spaţiu şi de timp.

Mişcarea mecanică poate fi **de**

- **formare** în timp a corpurilor şi, implicit, a obiectelor ;
- **schimbarea** în timp a **poziţiei** obiectelor, sau a părţior sale, denumite corpuri, în raport cu alte corpuri, alese drept sisteme de referinţă;

schimbarea în timp a **formei** corpurilor şi, implicit, a formei obiectelor, prin **deformarea** lor .



Transformarea sferei în cub | Transformarea cilindrului circular în cilindru pătrat

Transformarea conului în piramidă | Transformarea cilindrului în prismă

Fig.**3** **Metamorfozarea** obiectelor matematice centrice sau **hibridarea matematică**

Spaţiul reflectă raportul de coexistenţă dintre obiecte şi fenomene, sau părţi ale acestora, indicând:

- **întinderea**/mărimea lor, denumită **dimensiune de gabarit;**
- **locul** obiectelor, prin coordonatele **liniare X**, **Y**, **Z**, în spatiul 3D, denumite **dimensiuni de localizare;**
- **orientarea** obiectelor, în spaţiul 3D, prin coordonatele **unghiulare** ψ, φ, θ, sau A, B ,C, denumite dimensiuni de orientare;
- **poziţiile** relative sau distanţele dintre obiecte, denumite **dimensiuni de poziţionare,** dacă se referă la localizarea şi orientarea absolută şi/sau relativă a obiectelor, iar dacă se referă la părţi ale acestora, numite corpuri, atunci sunt denumite **dimensiuni de coordonare;**
- forma obiectelor şi, respectiv, evoluţia fenomenelor, denumite **dimensiuni de formare**, care definesc, totodată, şi ecuaţiile de definire a obectelor;
- deformarea obiectelor şi modificarea evoluţiei fenomenelor, denumite **dimensiuni de deformare** sau **excentricităţi.**
- Ultima dimensiune a spaţiului, **excentricitatea**, făcând posibilă apariţia **matematicii excentrice (ME)** şi realizând trecerea din domeniul matematicii centrice în cel al matematicii excentrice, precum şi saltul de la o singură entitate matematică, existentă în Matematica şi în domeniul **centric**, la o infinitate de entităţi, de acelaşi gen, dar deformate din ce în ce mai pronunţat, odată cu creşterea valorii excentricităţii numerice **s**, până la transformarea lor în alte genuri de obiecte, existente în domeniul centric. Un exemplu, devenit deja clasic, este deformarea continuă a unei sfere până la transformarea ei într-un cub (Fig.**3**), prin utilizarea aceloraşi **dimensiuni de formare** (ecuaţii parametrice), atât pentru sferă cât şi pentru cub, doar excentricitatea

modificându-se: fiind **s** = **e** = 0 pentru sfera de rază R şi **s** = ± 1, sau e = R, pentru cubul de latură L = 2R.

- Pentru **s** ∈ [(-1, 1) \ 0] se obţin **obiecte hibride**, proprii matematicii excentriec (**ME**), anterior inexistente în Matematică, sau, mai precis, în Matematica Centrică (**MC**).
- Aşa cum s-a mai prezentat, **dreapta** este un **spaţiu unidimensional** şi, totodată, în **Supermatematică** (**SM**), o **strâmbă** de excentricitate zero [8].
- Creşterea excentricităţii, de la zero la unu, transformă **linia dreaptă** într-o **linie frântă**, ambele existând şi sunt cunoscute în Matematica **Centrică**, nu şi restul strâmbelor, care sunt proprii Matematicii **Excentrice**, fiind generate de **FSM-CE** amplitudine excentrică. Astfel, dreapta de coeficient unghiular m = tanα = tan$\frac{\pi}{}$ = 1 care trece prin punctul P(2, 3) are ecuaţia

  (1) $y - 3 = x - 2$,

  iar familia de strâmbe, din aceeaşi familie cu dreapta, au ecuaţia

  (2) $y\,[x, S(s, \varepsilon)] - y_0 = m\,\{aex\,[\theta, S(s, \varepsilon)] - x_0\}$,

  (3) $y - y_0 = m\{\theta - \arcsin[s.\sin(\theta - \varepsilon)]\} - x_0$, $m = \tan\alpha$,

  în coordonate excentrice θ şi, în coordonate centrice α, ecuaţia este

  (4) $y[x, S(s, \varepsilon)] - y_0 = m\,(Aex\,[\theta, S(s, \varepsilon)] - x_0)$,

  (5) $y - y_0 = m\,\{\alpha + \arcsin\frac{s.\sin(\propto - \varepsilon)}{} - x_0\}$, $m = \tan\alpha$,

  (6) $y - y_0 = m\,\{\propto \qquad \frac{}{} \qquad {}_0\}$.

- Diferenţa, dintre cele două tipuri de strâmbe, de θ şi de α, este aceea, că cele de θ sunt continue numai pentru excentricitatea numerică din domeniul s ∈ [-1, 1], pe când cele de α sunt continue pentru toate valorile posibile a lui s, adică s ∈ [- ∞ , +∞].
- Linia frântă este cunoscută în Matematica Centrică (**MC**) dar fără să i se cunoască ecuaţiile ei ! Ceea ce nu mai este cazul în **SM** şi, evident, şi în **ME** unde se obţine pentru valoarea **s** = 1 a **excentricităţii numerice s**.
- Un fenomen asemănător metamorfozării matematice, prin care din **MC** un obiect cunoscut trece prin matematica excentrică (**ME**) luând forme hibride şi se reîntoarce în matematica centrică (**MC**), ca un alt tip de obiect (Fig.**3**), este considerat că ar avea loc şi în fizică: din vid apar continuu particule de un anumit tip şi se reîntorc în vidul cosmic. Aceleaşi sau altele ?
- Cosmologia are o teorie ce se aplică întregului Univers, formulată de **Einstein** în 1916: **relativitatea generală**. Ea afirmă că forţa de gravitaţie, ce se exercită asupra obiectelor, acţionează şi asupra structurii spaţiului, care îşi pierde cadrul rigid şi imuabil, devenind maleabil şi curb, în funcţie de materia sau energia pe care le conţine. Adică, **spaţiul se deformează**.
  Continuum-ul spaţiu-timp, al relativităţii generale, nu este conceput fără conţinut, deci nu admite vidul! Cum spunea şi **Einstein** ziariştilor, care îl rugau să le rezume teoria sa: "Înainte, **se credea** că, dacă toate lucrurile ar dispărea din Univers, timpul şi spaţiul ar rămâne, totuşi. În teoria relativităţii, timpul şi spaţiul dispar, odată cu dispariţia celorlalte lucruri din univers."
- Aşa cum s-a mai afirmat, **s** = **e** = 0 este lumea MC a liniarului, a entităţilor perfecte, ideale, în timp ce infinitatea de valori posibile atribuite excentricităţiilor **s** şi **e**, nasc **ME** şi, totodată, lumi ce aparţin realului, lumii imperfecte, tot mai indepărtată de lumea ideală cu cât **s** şi **e** sunt mai îndepărtate de zero.
- Ce se întâmpla dacă **e** = s → 0 ? Lumea reală, ca şi **ME** dispar şi cum lume ideală nu exista, dispare totul !
- Ceea ce susţine teoria autorului din SUPERMATEMATICA. Fundamente, Vol. I, Editura POLITEHNICA, Timisoara, Cap. 1 INTRODUCERE [23],[24] prin care expansiunea universului este un proces de desvoltare a ordinii în haosul absolut, o trecere progresivă a spaţiului haotic în ordine din ce în ce mai pronunţată.

- În concluzie, spaţiul, ca şi timpul, se **formează** şi se **deformează**, adică, **excentricitatea** spaţiului, de o anumită valoare, duce la **formare**a spaţiului, apoi, prin modificare valorii ei, spaţiul se **deformează**/modifică.
- Forma modificată a spaţiului este dependentă de valoarea excentricităţii, care devine o nouă dimensiune a spaţiului: **dimensiunea de deformare**.

Instalarea unei piese de prelucrat (obiect de prelucrat) în spaţiul de lucru a unei maşini-unelte moderne, cu comenzi numerice de conturare (**CNC**), este foarte asemănătoare cu "instalarea " unui obiect matematic în spaţiul euclidian tridimensional R^3. De aceea, vom folosi unele noţiuni din domeniul tehnologic.

În tehnologie, **instalarea** este operaţia premergătoare prelucrării; numai un obiect / piesă instalată poate fi prelucrată. Ea presupune următoarele faze sau operaţii tehnologice, în această succesiune / ordine; numai înfăptuirea unei faze, facând posibilă trecerea la realizarea fazei următoare:

1. **ORIENTAREA,** este acţiunea sau operaţia prin care elementele geometrice ale obiectului, care sunt baze de referinţă tehnologică de orientare, prescurtat baze de orientare (**BO**), primesc o **direcţie** bine determinată, faţă de direcţiile unui sistem de referinţă. În tehnologie, faţă de direcţiile unor mişcări principale şi/sau secundare de lucru, sau/şi faţă de direcţiile mişcărilor de reglare diemensională a sistemului tehnologic.

Drept **baze de orientare (BO)** pot servi :

3) **Un plan** al obiectului, respectiv o suprafaţă plană a piesei, dacă ea există, caz în care, această suprafaţă, determinată de trei puncte de contact dintre obiect şi dispozitiv, este denumită bază de referinţă tehnologică de orientare de **aşezare (BOA)**, sau, pe scurt, **bază de aşezare (BA)**, fiind determinată, teoretic, de cele trei puncte comune de contact ale piesei cu dispozitivul, care are sarcina de a realiza instalarea piese în cadrul maşinii de lucru. Drept **BA**, în principiu, se alege suprafaţa cea mai întinsă a piesei, dacă nu există altfel de condiţii de poziţie, sau de la care suprafaţa rezultată în urma prelucrării are impusă precizia cea mai înaltă, sau condiţii de paralelism cu **BA**.

Punând condiţia păstrării contactului piesă / dispozitiv pe **BA**, obiectul / piesa pierde 3 grade de libertate, dintre care, **o translaţie** pe direcţia, s-o numim **Z**, perependiculară pe **BA** (plan) şi două rotaţii: în jurul axelor **X**, notată în tehnologie cu **A** şi în jurul axei **Y**, notată în tehnologie cu **B.**

Obiectul / piesa se mai poate roti în jurul axei **Z,** rotaţie notată cu **C** şi se poate translata pe **BA** pe direcţiile **X** şi Y păstrând în permanenţă contactul cu **BA**.

De la această suprafaţă se stabileşte, în tehnologie, coordonata **z**, de exemplu, ca distanţă dintre **BOA** şi **baza tehnologică de prelucrare** (**BTP**), sau, pe scurt, **bază de prelucrare** (**BP**), adică planul pe care îl va genera pe piesă scula de prelucrat. Dacă o suprafaţă se prelucrează integral / complet (prin frezare, de exemplu, cu freze de mari dimensiuni, pentru o singură trecere), atunci celelalte coodonate / dimensiuni **y** şi **x** pot fi stabilite cu foarte mare aproximaţie, întrucât ele nu influenţează precizia realizării suprefeţei plane, la distanţa **z** de **BA**, rezultate în urma prelucrării piesei şi denumită **bază tehnologică de prelucrare** (**BTP**), sau, pe scurt, **bază de prelucrare** (**BP**). A cărei cerinţă tehnologică este să fie paralelă cu **BOA** şi să fie situată la distanţa **z** de aceasta. Dimensiunea **z** fiind, în acest caz, o **dimensiune de formare** a piesei, pe de o parte şi **dimensiune de coordonare**, în acelaşi timp, pentru poziţia relativă scula-piesă, iar, d.p.d.v. tehnologic, una dintre **dimensiunile de reglare dimensională** a sistemului tehnologic **MDPS** (**M**aşină-**D**ispozitiv-**P**iesă-**S**culă). Matematic exprimat, două suprafeţe plane situate la distanţa **z**, ca urmare, paralele între ele.

2) O dreaptă aparţinând obiectului, dacă aceasta există, ca axe şi/sau muchii, ca intersecţie de suprafeţe plane în Matematică.

În Tehnologie, muchiile se evită, datorită neregularităţii lor, adică, a abaterilor de la forma geometrică liniară, a semifabricatelor, ca şi a pieselor, în urma prelucrarii semifabricatelor lor.

În Tehnologie, această dreaptă este determinată de cele două puncte de pe o suprafaţă a piesei, alta decât BA, comună piesei şi dispozitivului, care realizează baza de orientare a piesei şi a dispozitivului, ca elemente dedublate, dreaptă denumită bază de orientare de dirijare (**BOD**), sau pe scurt baza de dirijare (**BD**), denumire care derivă din faptul că aceste două elemente, de dirijare, dirijează /ghidează mişcarea obiectului / piesei în vederea localizarii lui, dacă în tot timpul mişcării se menţine contactul piesă-dispozitiv. În acest fel **BD** preia 2 grade de libertate ale obiectului: translaţia pe o direcţie perpendiculară pe dreapta determinată de cele două puncte de contact piesa / dispozitiv, ce materializează **BD**, translaţie pe direcţia **Y**, de exemplu, dacă **BD** este paralelă, întotdeauna, cu **BA** din planul **XOY** şi rotaţia în jurul axei **Z**, notată în tehnologie cu **C**.

Drept **BOD** se alege, în principiu, din motive lesen de înţeles, care vizează precizia de ghidare, suprafaţa cea mai lungă a piesei, dacă nu există alte raţiuni impuse, prin desenul de execuţie al piesei.

De la **BOD** poate fi stabilită / măsurată cota / dimensiunea **y**, paralelă cu **BOA** şi perpendiculară pe **BOD**, ca de exemplu, perpendiculară pe **z**, fiindcă **BOD** este paralelă cu **BOA.**

Astfel, dacă cele două puncte aparţin unei obiect paralelipipedic, mărginit, deci, de suprafeţe plane, şi **BOD** este paralelă cu **BOA**, păstrând contactul piesă / dispozitiv pe cele două baze, printr-o mişcare de translaţie, piesa mai poate fi doar translatată, în dispozitiv, pe direcţia **X**, până când tamponează un **element de localizare**.

1) De la acesta, denumit element de localizare, respectiv **baza tehnologică de localizare** (**BTL**), sau, pe scurt, **baza de localizare** (**BL**) poate fi stabilită coordonata / dimensiunea **x** perpendiculară simultan pe **y** şi **z**. Dar fără să fie coordonate / dimensiuni / segmente concurente într-un punct comun **O(x,y,z)** ca în matematică, decât, dacă **BOD** şi **BTL** coboară la nivelul **BOA** şi, în plus, **BTL** se deplaseaza spre **BOD** şi va fi conţinută şi în ea, ambele urmând să fie conţinute în **BOA**, astfel că, punctul **O(x,y,z)** ca şi **BTL** va fi un vârf al piesei paralelipipedice, conţinut simultan în planul **BOA**, dreapta **BD** în punnctul **BL**, rezultând, în acest caz că **O(x,y,z)** ≡ **BL** .

Dacă, localizarea se realizează printr-o mişcare de translaţie, aşa cum s-a presupus anterior, ea mai poartă denumirea de **localizare prin translaţie (LT)**.

Dacă, localizarea se realizează printr-o mişcare de rotaţie a obiectului, atunci este denumită **localizare prin rotaţie** (**LR**). În acest caz, **BD** poate fi, sau este, deobicei, un plan de simetrie al piesei, de exemplu cilindrice, plan denumit **bază de orientare de semicentrare** (**BOSC**), în cazul unei **semicentrări**, sau o axă a unei suprafeţe de rotaţie (cilindrice sau sferice) a obiectului, denumită **baza de orientare de centrare** (**BOC**) în jurul căreia, obiectul se roteşte, până când, un alt corp al piesei, tamponează elementul de localizare prin rotaţie. Sau, până când un fixator pătrunde într-un orificiu perpendicular pe **BOC** sau într-un canal paralel cu **BOC.**

Obiectele care nu prezintă **elemente / baze de orientare**, cum ar fi sfera în matematică şi bilele de rulment în tehnologie, de exemplu, sunt obiecte neorientabile.

2. **LOCALIZAREA,** este operaţia sau acţiunea de stabilire a **locul**, în spaţiul euclidian tridimensional E^3, a unui punct **O(x,y,z)** caracteristic al obiectului, ce aparţine unui element de referinţă de orientare al acestuia, de la care se stabilesc coordonatele / dimensiunile liniare **x, y, z** faţă de un sistem de referinţă dat, sau, în tehnologie, faţă de scula de prelucrare.

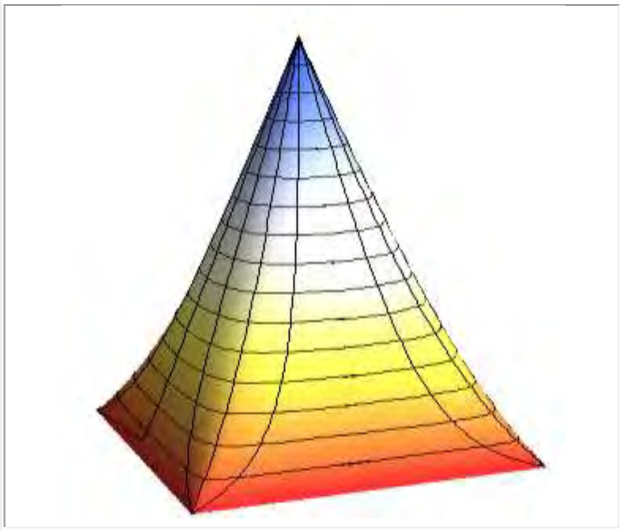
Punctul **O(x,y,z)** al obiectelor **neorientabile** este centrul de simetrie al acestora, iar al pieselor orientabile, ca cele paralelipipedice, în Tehnologie, de exemplu, punctul **O(x,y,z)** este **diseminat** în trei puncte distincte, pentru fiecare coordonată în parte **Ox ⊂ BL** pentru **x** , **Oy ⊂ BD** pentru y şi **Oz ⊂ BA** pentru **z**, aşa cum s-a explicat anterior.

In tehnologie, succesiunea orientare → localizare este obligatorie; numai un obiect orientat poate fi apoi localizat. Ca şi în matematică, dealtfel. Intâi se alege un sistem de referinţă solidar cu obiectul (O, x, y, z) apoi, unul invariant (O, X, Y, Z) ce coincide, iniţial, cu celălalt, în spaţiul 3D sau euclidian tridimensional E^3 şi apoi se operează diverse transformări de translaţii şi / sau de rotaţii.

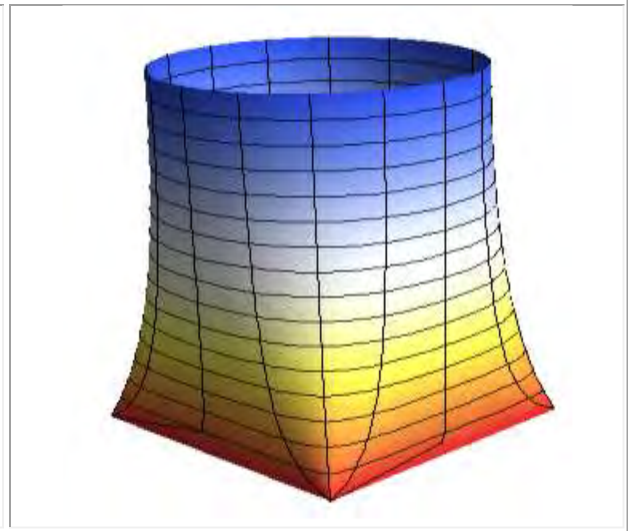
Reuniunea dintre **orientare** şi **localizare** reprezintă cea mai importantă acţiune / operaţie tehnologică, denumită **poziţionare**, adică: **orientarea** ∪ **localizarea** = **poziţionare**

Dacă **poziţionarea** obiectului este realizată / desăvârşită / implinită, atunci, poate fi menţinută poziţia relativă piesă / dispozitiv prin operţia de **fixare** a piesei în dispozitiv.

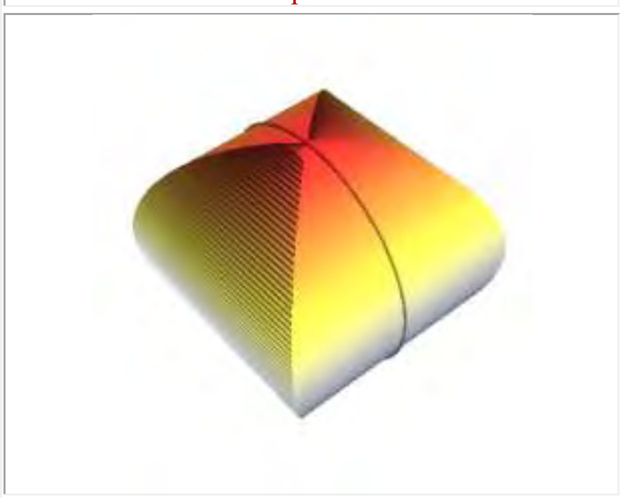
În continuare pot fi stabilite cotele / dimensiunile dintre scula şi piesă, astfel, încât să se obţină piesa la dimensiunile şi preciziile impuse prin desenul de execuţie al piesei. Această operaţie tehnologică este denumita **reglare dimensională.** Cu ea, operaţia de instalare este incheiată şi prelucrarea piesei poate să înceapă.



Conopiramidă



Cilindru C/P



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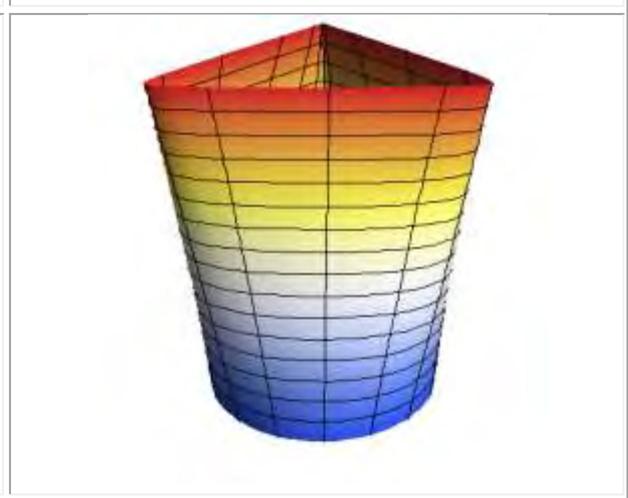

Cilindru C/T

Fig.**4** Obiecte matematice **hibride**

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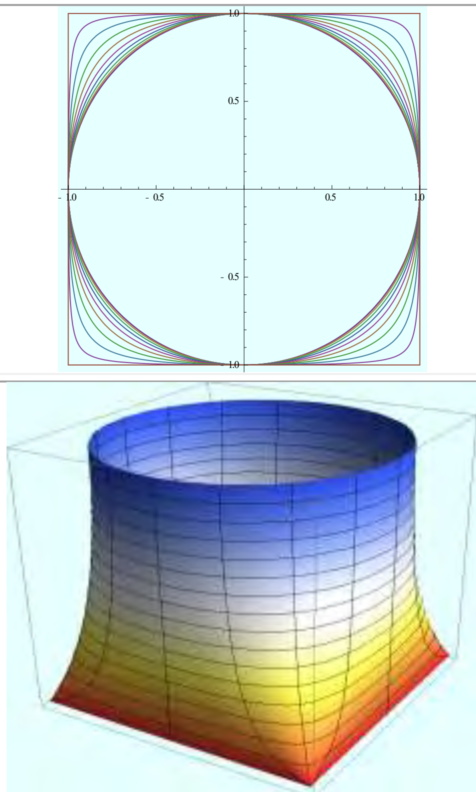
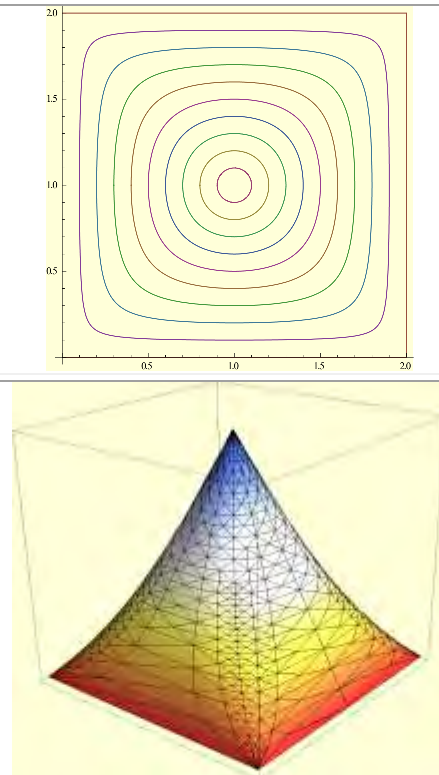
Ca urmare, **istalarea** unui obiect este o reuniune a **poziţionarii** cu **fixarea** şi cu **reglarea dimensională** a sistemului tehnologic, adică: **instalare** = **poziţionare** ∪ **fixare** ∪ **reglare (dimensională)**

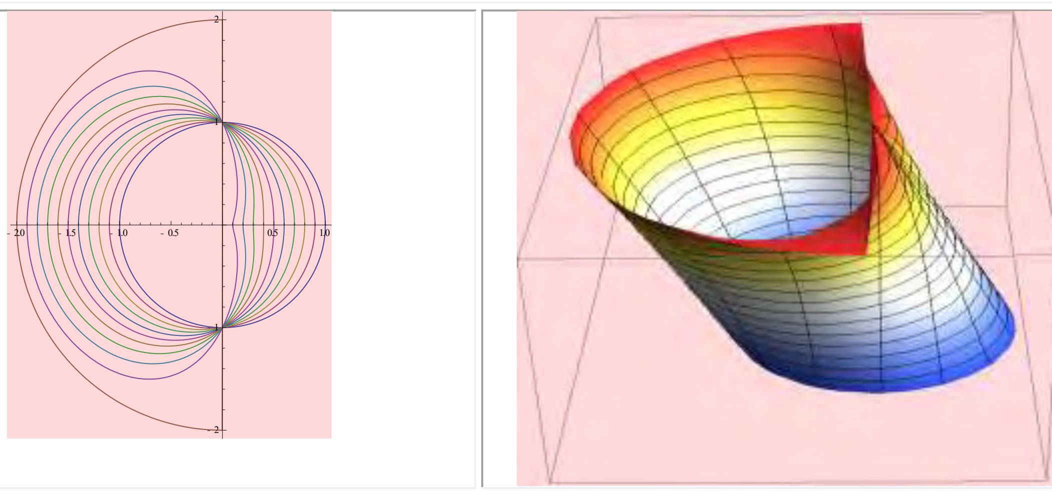
În Tehnologie, **fixarea** se poate realiza prin **forţă** (de fixare) sau prin **formă** (care impiedică deplasarea piesei în timpul preucrării). În Matematică, fixarea se “realizeaza” prin **convenţie**.





a) θ, R = u = 1

b) –



c) $\left\{ y = R.dex\theta.\sin\theta \right.$, $R = 1$; s∈[0,1]

▲ TRANSFORMAREA **CERCULUI C**$_1$[O_C(0, 0), R = 1] ÎN SEMICERCUL **SC [O_{SC}(-1, 0), R = 2]**

Fig.**5** Obiecte matematice **hibride înrudite**

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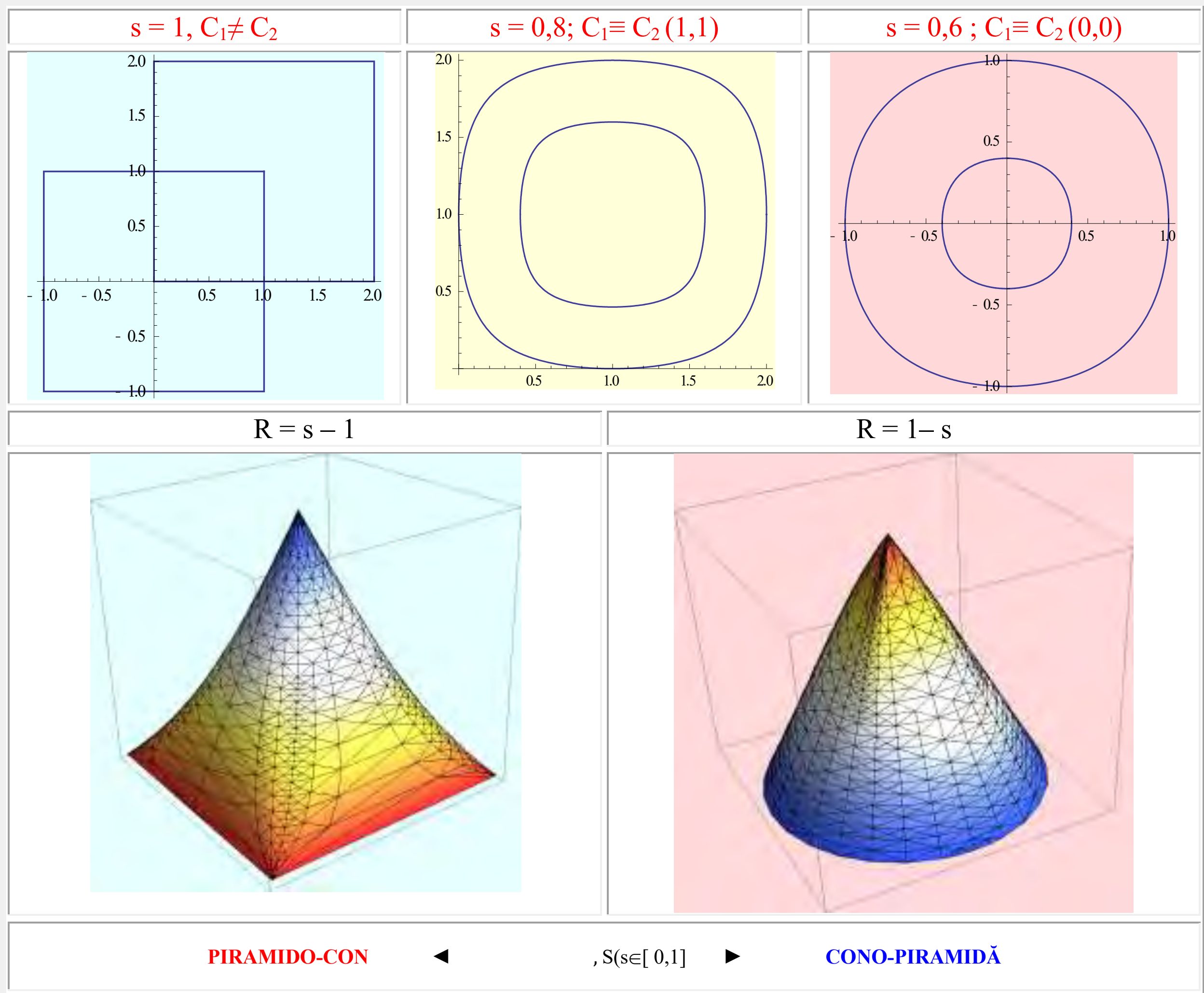

Fig.**6** Deosebirile dintre obiectele matematice **hibride înrudite**

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Zicând că sistemul (**O, x, y, z**) este legat de piesă el nu mai poate fi deplasa relativ faţă de ea (dezlega), ci numai împreună cu obiectul, deci sunt "fixate" unele de altele. Astfel, în Matematică, fixarea obiectelor, faţă de sistemele de referinţa, se subînţelege, sau se realizează de la sine, ea nu mai există, pentru că în Matematică nu există "forţe matematice"; ele fiind proprii Mecanicii, în speţă dinamicii ei şi nici scule de prelucrare, nici diverse dimensiuni de coordonare, de reglare dimensionala, de prelucrare ş.a.

De aceea, în Matematica Centrică (**MC**), există doar 3 dimensiuni liniare **x**, **y**, şi **z** care sunt, totodată, şi dimensiuni de formare a obiectelor 3D, prin ecuaţiile lor parametrice, de exemplu.

Ca urmare, în această Matematica Centrică (**MC**) entităţi ca dreapta, pătratul, cercul, sfera, cubul ş.a. sunt unice, pe când, în Matematica Excentrică (**ME**) şi, implicit în Supermatematică (**SM**), ele sunt multiplicate la infinit prin **hibridare**, hibridare posibilă prin introducerea noii dimensiuni a spaţiului **excentricitatea.**

Hibridarea supermatematică poate fi definită ca procesul matematic de **încrucişare** a două entităţi matematice din **MC** (cercul şi pătratul, sfera şi cubul, **conul** şi **piramida**) şi obţinerea unei noi entităţi supermatematice în **ME** ce nu este cunoscută / inexistentă în MC (de exemplu:**cono-piramidă**).

Prin **metamorfozare** se înţelege trecere continuă de la o entitate oarecare, existentă în **MC**, la o altă entitate, existentă în **MC**, printr-o infinitate de entităţi hibride, proprii doar **ME**. Altfel spus, o transformare a unei entităţi matematice centrice în altă entitate matematică centrică, acţiune devenită posibilă în cadrul **Matematicii Excentrice** (**ME**) prin utilizarea funcţiilor **supermatematice**.

Prin **metamorfozare** se obţin entităţi noi, anterior inexistente în **MC**, denumite **entităţi hibride**, ca şi entităţi **excentrice** sau **supermatematice** (**SM**), pentru a se deosebi de cele **centrice**, şi prin denumire, pentru că, **prin formă**, diferă esenţial.

Primul corp obţinut prin **hibridare matematică** a fost **conopiramida**: un obiect supermatematic cu baza pătrată a unei piramide şi cu vârful unui con circular drept, rezultat din transformarea continuă a pătratului unitate de L = 2 în cercul unitate de R = 1, şi/sau invers (Fig.**4**). Ecuaţiile parametrice ale conopiramidei se obţin din ecuaţiile parametrice ale conului circular drept, în care **FCC** sunt înlocuite/convertite cu funcţiile supermatematice cvadrilobe (**FSM-Q)** corespondente

(7) C P **2**, C **CILIN**

(Fig. **1,** Fig.**3** şi Fig. **5,a**), deoarece **FSM-Q** pot realiza transformarea continua a cercului în pătrat şi invers, la fel ca şi **FSM-CE** derivată excentrică **$dex_{1,2}\theta$**

(8) – –],$pent$ – C $PIRA$ C

(Fig.**4** şi Fig. **5,b şi** Fig. **5,c**).

Relaţiile (7) sunt exprimate cu ajutorul **FSM-Q** cvadrilobe, introduse în Matematică în anul 2005 prin lucrarea [19], cosinus cvadrilob **coqθ** şi sinus cvadrilob **siqθ**.

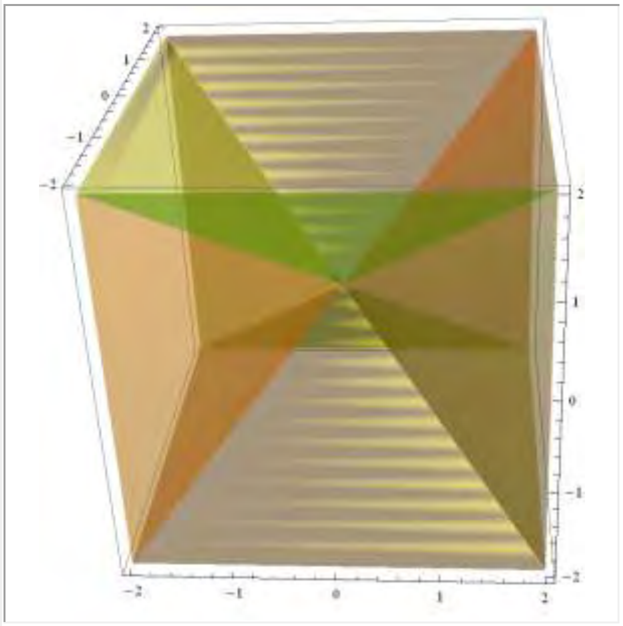
Relaţiile (7) şi (8) exprimă aceleaşi forme, cu observaţiile:

- De **cerc** numai pentru un excentru S(s = 0, ε = 0), cu deosebirea că primul (7) are raza R = 1, iar celălalt (8) are raza R = 0, Fig. **6**, sus ▲;
- De **pătrat** pentru un excentru S (s = 1, ε = 0), de aceleaşi dimensiuni L = 2R, aşa cum se poate constata în figura **6**., dar centrate în puncte diferite; unul este centrat în originea O(0, 0), cel exprimat prin relaţiile (7), iar celălalt este ex-centrat - centrat excentric faţă de originea O(0, 0)- în punctul C(1,1);
- De **cavdrilobă** (nici cerc şi nici pătrat, adică o infinitate de forme hibride, între cerc şi pătrat). Pentru aceeaşi excentricitate numerică **s ∈ (0, 1)**, ce caracterizează domeniul **matematic excentric** (**ME**) ele au aceleaşi forme dar sunt de dimensiuni diferite; primele având dimensiuni mai mari decât cele exprimate cu funcţia dexθ, ceea ce se poate deduce şi din figura **5,b** din 2D.
  Se observă că dimensiunea cvadrilobelor exprimate de relaţia (8) prin dexθ scade cu creşterea excentricităţii.

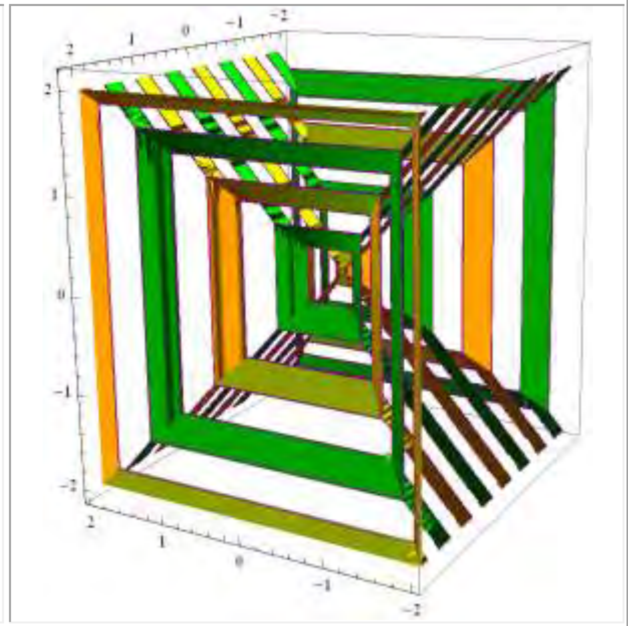
Cubul românesc din figura **7**, "**cel mai uşor cub din lume**", este cubul de volum nul, obţinut din 6 piramide, fără suprafeţele lor de bază pătrate, cu vârful comun în centrul de simetrie al cubului.

În acest caz piramida a fost exprimata de relaţiile (7), prin funcţii cvadrilobe de s = 1.

În concluzie, **supermatematica** oferă multiple posibilităţi de exprimare a diverselor entităţi matematice din **matematica centrica** (**MC**) şi, totodată, o infinitate de entităţi hibride din **matematica excentrică** (**ME**).



CR semitransparent



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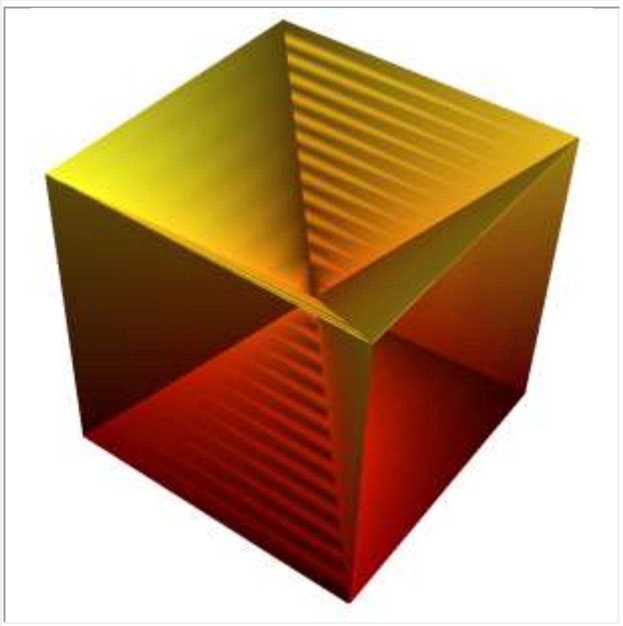
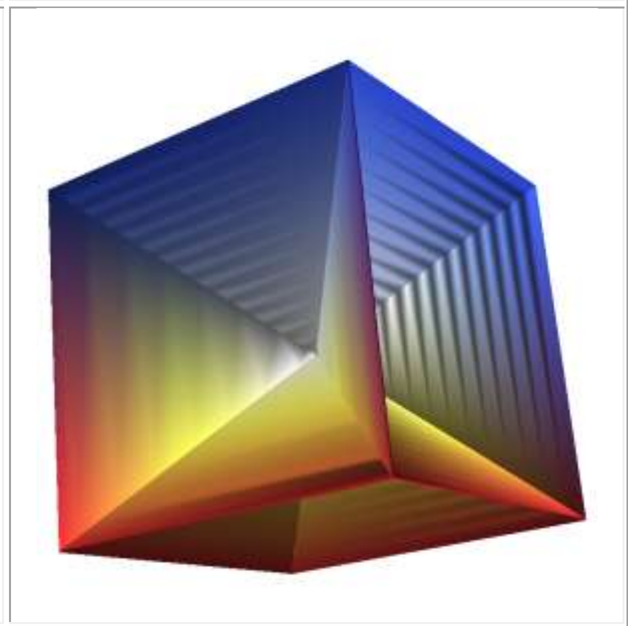


CR opac



CR tricolor

Fig. **7 CUBUL ROMÂNESC (CR)** , cel mai uşor cub din lume, de volum V = 0

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66	Dumitru Bălă	SUPERMATHEMATICAL – ŞELARIU FUNCTIONS BETA ECCENTRIC bexθ SOLUTIONS OF SOME OSCILATORY NON-LINIAR SYSTEMS (SOβ)	Proceedings of the2nd World Congress on Science, Economics and Culture, 25-29 August 2008 New York, paper published in Denbridge Journals, p.27 … 41
67	Şelariu Mircea Eugen Smarandache Florentin Niţu Marian	CARDINAL FUNCTIONS AND INTEGRAL FUNCTIONS	International Journal of Geometry Vol.1 (2012), N0. 1, 5-14

FUNCŢII CARDINALE ŞI FUNCŢII INTEGRALE CIRCULARE EXCENTRICE

MIRCEA EUGEN ŞELARIU, FLORENTIN SMARANDACHE and MARIAN NIŢU

0. REZUMAT

Lucrarea prezintă corespondentele din matematica excentrică ale funcţiilor cardinale şi integrale din matematica centrică, sau matematica ordinară, funcţii centrice prezentate şi în introducerea lucrării, deoarece sunt prea puţin cunoscute, deşi sunt utilizate pe larg în fizica ondulatorie.

În matematica centrică, sunt definite sinusul şi cosinusul cardinal, ca şi cele integrale, atât cele circulare cât şi cele hiperbolice. În matematica excentrică, toate aceste funcţii centrice se multiplică de la unu la infinit, datorită infinităţii de puncte în care poate fi plasat un punct,denumit excentru **S(s, ε),** în planul cercului unitate CU(O,R =1) sau a hiperbolei unitate echilatere HU(O, a = 1, b =1). În plus, în matematica excentrică apar o serie de alte funcţii deosebit de importante, ca aexθ, bexθ, dexθ, rexθ ş.a care, prin împărţirea lor cu argumentul θ, pot să devină şi funcţii circulare excentrice cardinale, ale căror primitive devin automat funcţii circulare excentrice integrale.

Toate funcţiile supermatematice circulare excentrice (**FSM**-**CE**) pot fi de variabilă excentrică θ, care sunt funcţii continue în domeniul excentricităţii numerice liniare $s \in [-1,1]$, sau de variabilă centrică α, care sunt continue pentru oricare valoare a lui s, adică $s \in [-\infty, +\infty]$.

KEYWORDS AND ABBREVIATIONS

C-**Circular** , CC-C **centric,** CE-C **Excentric,** CEL-C **Elevat,** CEX-C **Exotic,** F-**Funcţie,** FMC-F **Matematice centrice,** M- **Matematică,** MC-M **Centrică,** ME-M **Excentrică,** S-**Super,** SM-S **Matematică,** FSM-F **Supermatematice,** FSM-CE-FSM–**Circulare Excentrice**, FSM-CEL-FSM-C **Elevate,** FSM-CEC-FSM-CE-**Cardinale,** FSM-CELC-FSM-CEL **Cardinale**

1. ÎNTRODUCERE : FUNCŢIA SINUS CARDINAL CENTRIC

În dicţionar, cuvântul **cardinal** este sinonim cu principal, esenţial, fundamental. În matematica centrică, sau matematica ordinară, **cardinal** reprezintă, pe de o parte, un număr egal cu numărul membrilor unei mulţimi finite, denumit şi **puterea** mulţimii, iar, pe de altă parte, sub denumirea de **sinus cardinal** (**sinc x**) sau **cosinus cardinal**, (**cosc x**), este o funcţie specială, definită cu ajutorul funcţiei circulare centrice (**FCC**) **sinx** şi, respectiv, **cosx**, utilizate frecvent în fizica ondulatorie (Fig.**1**) şi a cărui grafic, al sinusului cardinal, este denumit, datorită formei lui (Fig.**2**), şi "pălaria mexicană (sombrero)".

Notată sinc x, funcţia sinus cardinal este dată, în literatura de specialitete, în trei variante

(1) $$\text{sinc x} = \begin{cases} 1, & pentru\ x = 0 \\ \frac{sinx}{x}, & pt.x \in [-\infty, +\infty]\backslash 0' \end{cases}$$

$$\frac{\sin x}{x} = 1 - \frac{x^2}{6} + \frac{x^4}{120} - \frac{x^{76}}{5040} + \frac{x^8}{362880} + O[x]^{11} =$$

$$= \sum_{n=0}^{+\infty} \frac{(-1)^n x^{2n}}{(2n+1)!} \rightarrow \text{sinc}\,\frac{\pi}{2} = \frac{2}{\pi}, \quad \frac{d(sinc\,x)}{dx} =$$

$$= \frac{cosx}{x} - \frac{sinx}{x^2} = \text{cosc x} - \frac{sinc\,x}{x},$$

(2) $$\text{sinc x} = \frac{\sin \pi x}{\pi x},$$

(3) $$\text{sinc}_a\text{x} = \frac{sin\frac{\pi x}{a}}{\frac{\pi x}{a}}.$$

Este o **funcţie specială** deoarece primitiva ei, denumită **sinus integral** şi notată Si(x)

(4) $$\forall x \in \mathbb{R}, \quad \text{Si(x)} = \int_0^x \frac{\sin t}{t} dt = \int_0^x \text{sinc t.dt} =$$

$$= x - \frac{x^3}{18} + \frac{x^5}{600} - \frac{x^7}{35280} + \frac{x^9}{3265920} + O[x]^{11} =$$

$$= x - \frac{x^3}{3.3!} + \frac{x^5}{5.5!} - \frac{x^7}{7.7!} + \ldots - \ldots = \sum_{n=0}^{+\infty} \frac{(-1)^n x^{2n}}{(2n+1)^2 (2n)!}$$

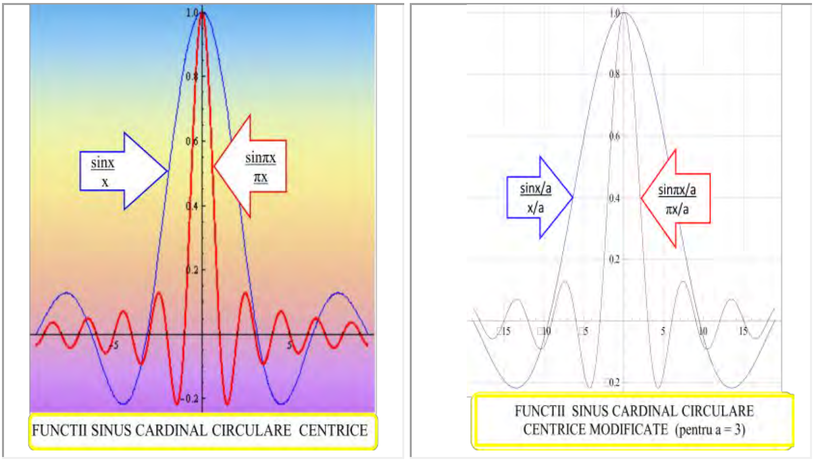


Fig.**1** Graficele funcţiilor circulare centrice sinus cardinal, în 2D, aşa cum sunt cunoscute în literatură

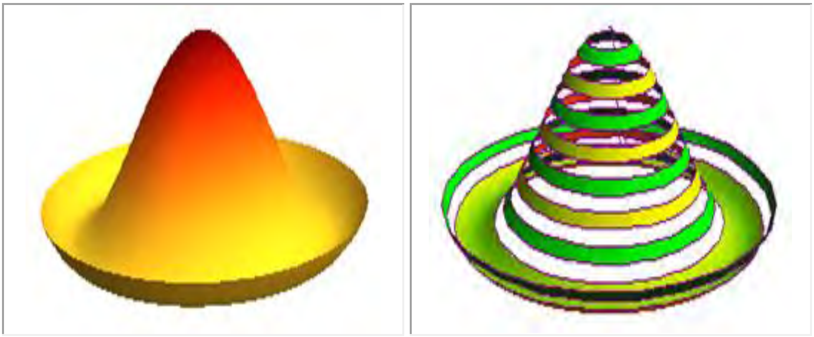
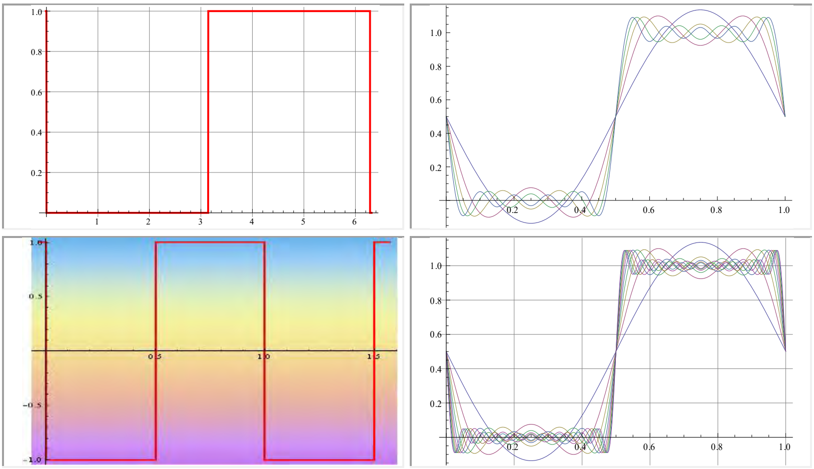


Fig.**2** Funcţia sinus cardinal în 3D sau pălaria mexicană (sombrero) nu poate fi exprimată exact cu ajutorul funcţiilor elementare, ci doar prin dezvoltări în serii de puteri, aşa cum rezultă din relaţia (4).

Plot[1-Cos[x-Pi/2]/Sqrt[1 -Sin[x-Pi/2]^2], {x,-Pi,2Pi}]	Plot[Evaluate[Table[1/2–4xSum[Sinc [2Pi(2 k-1) x],{k,n}],{n, 5}]], {x, 0, 1}]



Unda dreptunghiulară ▲ şi unda pătrată ▼0,5dex[$(\theta - \frac{\pi}{2})$, S(1, 0)]	Fenomenul **Gibbs** pentru o undă pătrată cu n= 5 ▲ şi cu n = 10 ▼

Fig.**3** Comparaţie între funcţia pătrată, derivat excentric şi aproximarea ei prin dezvoltări în serii **Fourier**.

Ca urmare, derivata ei este

(5) $$\forall x \in \mathbb{R},\ Si'(x) = \frac{d(Si\ x)}{dx} = \frac{sinx}{x} = sinc\ x$$

Funcţia **sinus integral** **Si[x]** satisface ecuaţia diferenţială

(6) $$x.f'''_{(x)} + 2f''_{(x)} + x.f'_{(x)} = 0 \ \rightarrow\ \mathrm{f(x) = Si(x)}$$

Fenomenul **Gibbs** apare la aproximarea funcţiei pătrate cu o serie **Fourier** continuă şi diferenţiabilă (Fig.**3** → dreapta), operaţie care nu mai are sens, odată cu descoperirea funcţiilor **supermatematice circulare excentrice** (**FSM-CE**), deoarece funcţia **derivat excentric** de variabilă excentrică θ poate exprima **exact** această funcţie dreptunghiulară (Fig.3 ▲ sus) sau pătrată (Fig,3▼ jos), aşa cum se poate observa în graficele lor (Fig. **3** ◄ stânga).

Plot[SinIntegral[x],{x,-20,20}	Plot3D[Re[SinIntegral[x+Iy]], {x,-20,20},{y,-3,3}

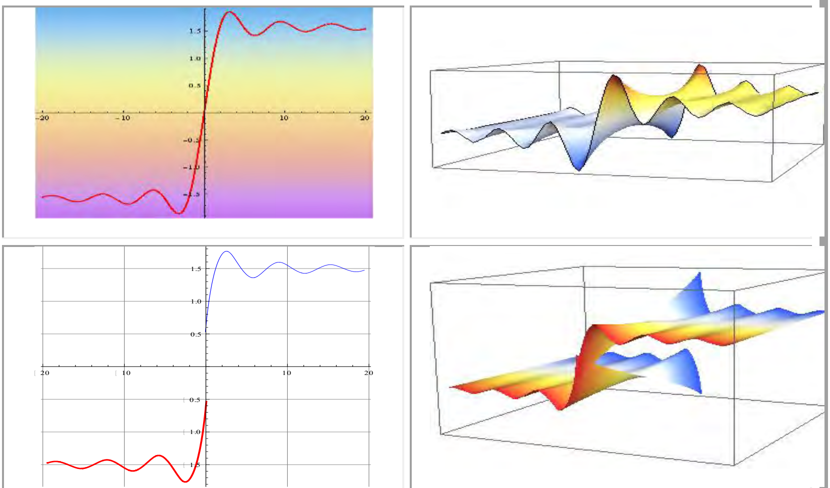


Fig.**4,a** Graficul funcţie sinus integral Si(x) ▲comparativ cu graficul **FSM**-**CE** amplitudine excentrică 1,57 aex[θ, S(0,6; 0)] de variabilă excentrică θ▼

Plot[SinIntegral[x] - (1.57 (x - ArcSin[0.6Sin[x + 0.3Pi]])/x),{x, 0, 40}]

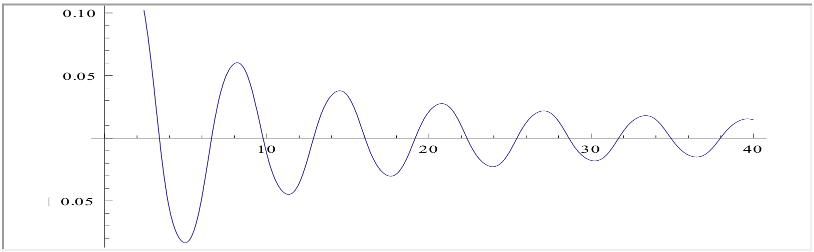


Fig.**4,b** Diferenţa dintre sinus integral şi **FSM**-**CE** amplitudine excentrică F(θ) =1,5 aex[θ, S(0,6; 0)] de variabilă excentrică θ

Funcţia sinus integral (4) poate fi aproximată cu suficienta precizie, cu diferenţe maxime de sub 1 %, cu excepţia zonei din apropierea originii, de **FSM-CE** amplitudine excentrică de variabilă excentrică θ

(6) F(θ) =1,57 aex[θ, S(0,6; 0)], aşa cum rezultă din graficul din figura **4,b**.

(7)

2. FUNCŢII SUPERMATEMATICE CIRCULARE EXCENTRICE CARDINALE. SINUS EXCENTRIC CARDINAL(FSM-CEC)

Ca toate celelalte funcţii supermatematice (**FSM**) ele pot fi excentrice (**FSM-CE**), **elevate** (**FSM-CEL**) şi exotice (**FSM-CEX**), de variabilă excentrică θ, sau de variabilă centrică $\alpha_{1,2}$, de determinare principală, de indice 1, sau de determinare secundară, de indice 2.

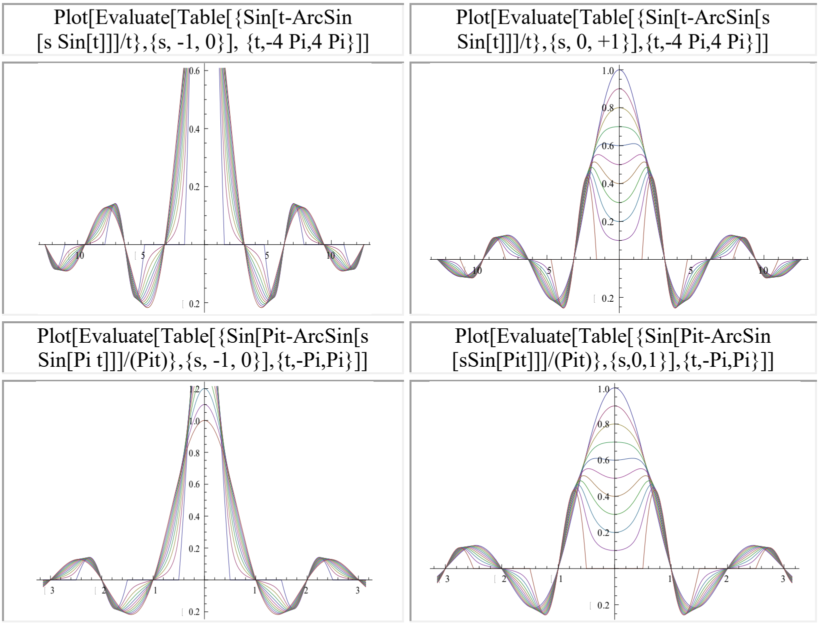


Fig.**5,a** Graficele **FSM-CEC sexc₁ [θ, S(s, ε)], de variabilă excentrică θ**

La trecerea din domeniul circular **centric** în cel **excentric**, prin poziţionarea excentrului S(s, ε) în oricare punct din planul cercului unitate, toate funcţiile supermatematice se multiplică de la unu la infinit, adică, dacă în **MC** există câte o unică funcţie, de un anumit gen, în **ME** există o infinitate de astfel de funcţii, iar pentru **s = 0** se va obţine funcţia centrică. Altfel spus, oricare funcţie supermatematică conţine atât pe cele excentrice, cât şi pe cea centrică.

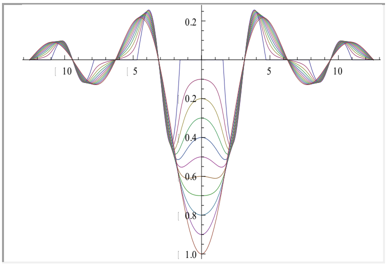
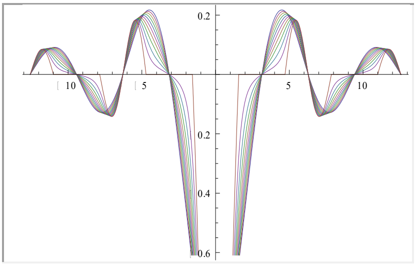
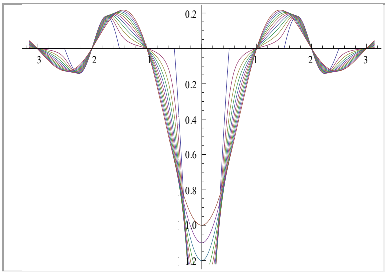
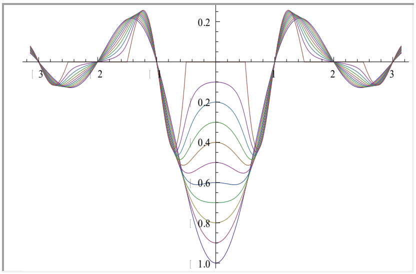
Plot[Evaluate[Table[{Sin[t+ArcSin[s Sin[t]]-Pi]/t},{s,-1,0}],{t,-4Pi, 4Pi}]]	Plot[Evaluate[Table[{Sin[t+ArcSin[s Sin[t]]-Pi]/t},{s,0,1}],{t,-4Pi, 4Pi}]]
	
Plot[Evaluate[Table[{Sin[Pit-ArcSin[s Sin[Pit]]-Pi]/(Pit)},{s,-1,0}],{t,-Pi,Pi}]]	Plot[Evaluate[Table[{Sin[Pit-ArcSin[s Sin[Pit]]-Pi]/(Pit)},{s, 0, 1}],{t,-Pi, Pi}]]
	

Fig.5,b Graficele **FSM-CEC sexc$_2$ [θ, S(s, ε)], de variabilă excentrică θ**

Notată sexc x şi respectiv Sexc x, inexistentă în literatura de specialitate, va fi dată, în cele trei variante, de relaţiile

(8) $\text{sexc } x = \frac{\text{sex}\, x}{x} = \frac{sex\,[\theta, S(s,\varepsilon)]}{\theta}$, de variabilă excentrică θ şi

(8') $\text{Sexc } x = \frac{Sex\; x}{x} = \frac{Sex[\propto, S(s,\varepsilon)]}{\alpha}$, de variabilă centrică α.

(9) sexc x = $\frac{\text{sex}\,\pi x}{\pi x}$, de variabilă excentrică θ,
notata şi prin sexc_π x şi
(9') Sexc x = $\frac{sex\ \pi x}{\pi x} = \frac{Sex[\alpha, S(s,\varepsilon)]}{\alpha}$, de variabilă centrică α, notata şi prin Sexc_π x.
(10) sexc_a x = $\frac{sex\frac{\pi x}{a}}{\frac{\pi x}{a}} = \frac{sex\frac{\pi\theta}{\theta}}{\frac{\pi\theta}{\theta}}$, de variabilă excentrică θ,
cu graficele din figura **5,a** şi
(10') Sexc_a x = $\frac{Sex\frac{\pi x}{a}}{\frac{\pi x}{a}} = \frac{Sex\frac{\pi\alpha}{a}}{\frac{\pi\alpha}{a}}$, de variabilă centrică α,
cu grafuicele din figura **5,b.**

3. FUNCŢIILE SUPERMATEMATICE CIRCULARE EXCENTRICE SINUS ŞI COSINUS ELEVATE CARDINALE (FSM-CELC)

Funcţiile supermatematice circulare elevate (**FSM-CEL**), sinus elevat selθ şi cosinus elevat celθ, reprezintă proiecţia fazorului / vectorului $\vec{r} = rex\theta.rad\theta = rex[\theta, S(s,\varepsilon)]$.radθ pe cele două axe de coordonate X_S şi, respectiv, Y_S cu originea în excentrul S(s, ε), axe paralele cu axele x şi y care au originea în O(0, 0).

Dacă cosinusul şi sinusul excentrice sunt coordonatele punctului W(x,y), faţă de originea **O(0, 0)**, de intersecţie ale dreptei **d** = $d^+ \cup d^-$, turnantă în jurul punctului S(s, ε), cosinusul şi sinusul elevate sunt aceleaşi coordonate faţă de excentrul S(s, ε), adică, considerând originea sistemului de axe de coordonate XSY rectangular drept/reper în S(s, ε). De aceea, între ceste funcţii există relaţiile

(11) $$\begin{cases} x = cex\theta = X + s.cos\varepsilon = cel\theta + s.cos\varepsilon \\ y = Y + s.sin\varepsilon = sex\theta = sel\theta + s.sin\varepsilon \end{cases}$$

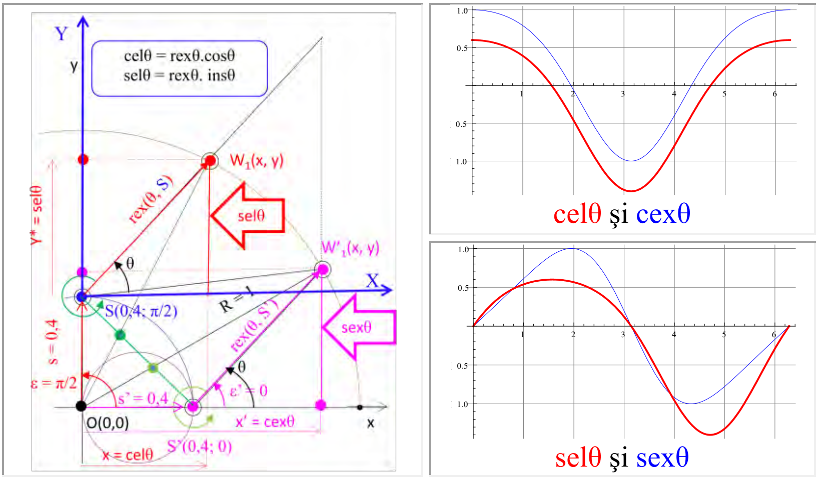


Fig. **6,a** Comparaţie între funcţii supermatematice elevate şi funcţii excentrice

Plot[{Cos[t-ArcSin[0.4Sin[t]]]/t,(-0.4 Cos[t] +Sqrt[1-(0.4Sin[t])^2]) Cos[t]/t},{t,-2 Pi, 2 Pi}]

Plot[{Sin[t-ArcSin[0.4Sin[t]]]/t,(-0.4 Cos[t-Pi/2] +Sqrt[1-(0.4Sin[t-Pi/2])^2]) Sin[t]/t},{t,-2 Pi ,2 Pi}]

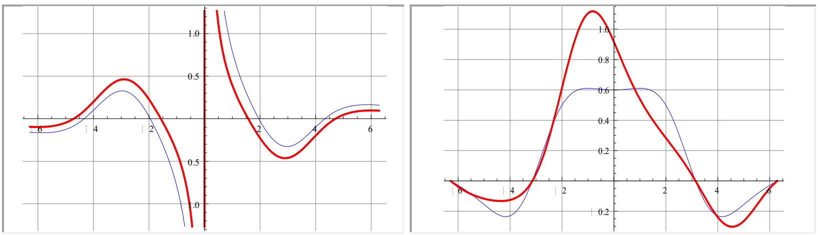


Fig. **6,b** Funcţii supermatematice elevate şi funcţii excentrice cardinale celc(x)◄ şi selc(x) ►de s = 0.4

Din această cauză, pentru ε = 0, adică excentrul S situat pe axa x > 0, sexθ = selθ, iar pentru ε = π/2, cexθ = celθ, aşa cum se poate observa în figura **6,a**. In această figură au fost reprezentate, simultan, graficele funcţiilor elevate celθ şi selθ, dar şi graficele funcţiilor cexθ şi, respectiv, sexθ pentru comparaţie şi pentru relevarea elevaţiei. Excentricitatea funcţiilor este aceeaşi, de s = 0,4, cu cea din schiţa alăturată şi selθ are $\varepsilon = \frac{\pi}{2}$, iar celθ are ε = 0.

Plot[Evaluate[Table[{(-sCos[t]+Sqrt[1- (s Sin[t])^2]) Cos[t]/t}, {s,-1,1}], {t,-3 Pi, 3 Pi}]	Plot[Evaluate[Table[{(-sCos[t-Pi/2] +Sqrt[1-(sSin[t-Pi/2])^2]) Sin[t]/t}, {s,-1,1}],{t,-3 Pi,3 Pi}]

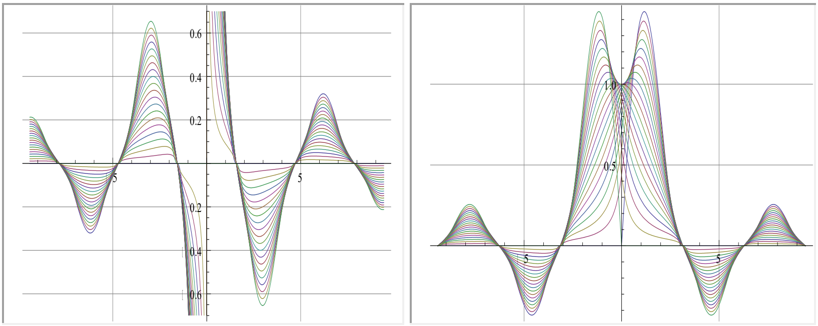


Fig. **6,c** Funcţii supermatematice elevate excentrice **cardinale** celc(x)◄ şi selc(x) ►

Prin impărţire cu θ, funcţiile elvate, date de relaţiile (11), se transformă în funcţii cosinus şi sinus elvate cardinale, notate celcθ = celc[θ,S] şi selcθ = selc[θ,S], date de expresiile

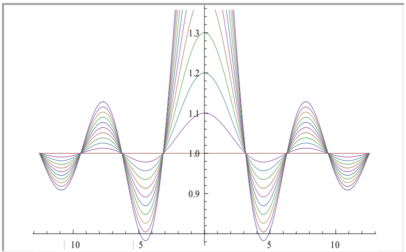
$$(12) \qquad \begin{cases} X = celc\theta = celc[\theta, S(s,\varepsilon)] = cexc\theta - \dfrac{s.cos\varepsilon}{\theta} \\ Y = selc\theta = selc[\theta, S(s,\varepsilon)] = sexc\theta - \dfrac{s.sin\varepsilon}{\theta} \end{cases} \qquad \text{cu}$$

graficele din figura **6,b** şi **6,c**.

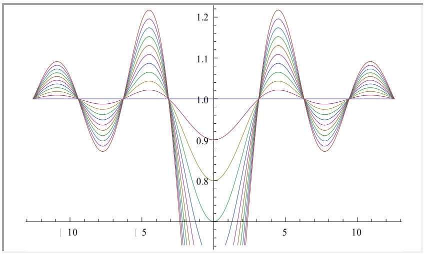
4. FUNCŢII SUPERMATEMATICE CIRCULARE EXCENTRICE CARDINALE (FSM-CEC) NOI

În acest paragraf sunt prezentate funcţii care sunt necunoscute în literatura matematicii centricele, nici ca atare şi nici ca funcţii cardinale sau integrale. Ele sunt funcţiile supermatematice excentrice

Plot[Evaluate[Table[{(t-0.1 s Sin[t])/t}, {s, -10, 0}],{t, -4 Pi, +4 Pi}]]



Plot[Evaluate[Table[{(t – 0.1 s Sin[t])/t}, {s, 0, 10}],{t, -4 Pi, +4 Pi}]]



Plot[Evaluate[Table[{(t – 0.1 s Sin[t])/t}, {s, -10, +10}],{t, -3 Pi, +3 Pi}]]

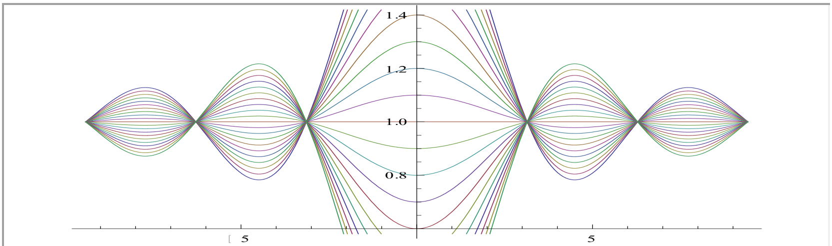


Fig.**7,a** Graficul funcţie supermatematice circulare excentrice cardinală aexc(θ)

Plot[Evaluate[Table[{ArcSin[0.1 s Sin[t]]/t},{s,-10,10}],{t,-4 Pi,4 Pi},ColorFunction->(Hue[2.72 #]&)]]

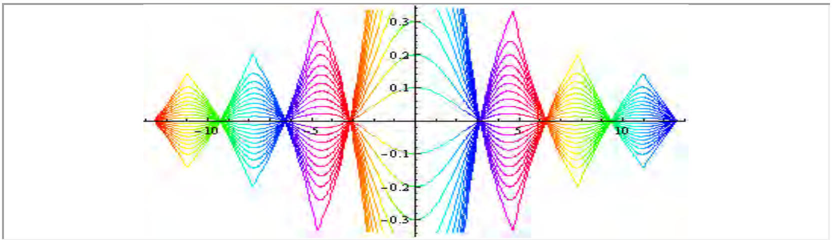


Fig.**7,b** Graficul funcţie supermatematice circulare excentrice cardinală bexc(θ) amplitudine, beta, radial, derivată excentrice de variabilă excentrică [1], [2], [3], [4], [6], [7] **cardinale** precum şi funcţiile cvadrilobe [5] **cardinale**.

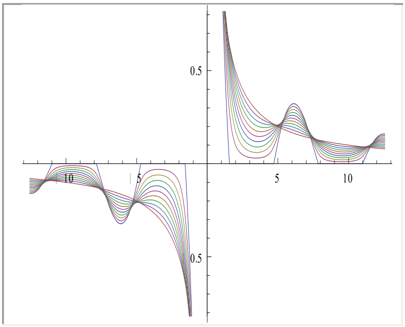
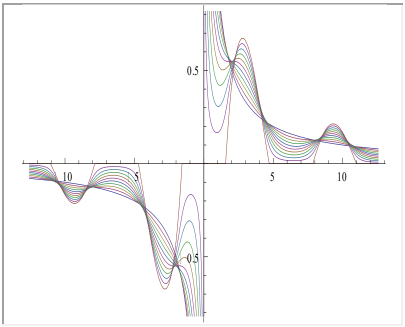
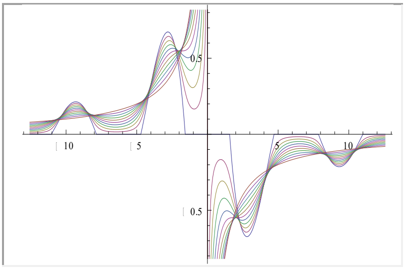
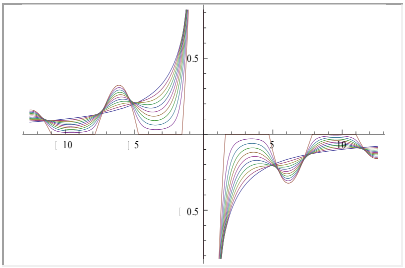
Plot[Evaluate[Table[{(-0.1 s Cos[t] +Sqrt[1-(0.1 s Sin[t])^2])/t},{s,-10,0}],{t,-4 Pi,4 Pi}]]	Plot[Evaluate[Table[{(-0.1 s Cos[t] +Sqrt[1-(0.1 s Sin[t])^2])/t},{s,0,10}],{t,-4 Pi,4 Pi}]]
	
Plot[Evaluate[Table[{(-0.1 s Cos[t] – Sqrt[1-(0.1 s Sin[t])^2])/t},{s,-10,0}],{t,-4 Pi,4 Pi}]]	Plot[Evaluate[Table[{(-0.1 s Cos[t] – Sqrt[1-(0.1 s Sin[t])^2])/t},{s, 0, 10}],{t,-4 Pi,4 Pi}]]
	

Fig.**7,c** Graficul funcţiilor supermatematice circulare excentrice cardinale $rexc_{1,2}$ (θ)

Funcţia amplitudine excentrică aexθ cardinală, notată aexc(x) = aex[θ, S(s, ε)] , x ≡ θ, are expresia

(13) $$aexc\,(\theta) = \frac{aex\theta}{\theta} = \frac{aex[\theta, S(s,\varepsilon)]}{\theta} = \frac{\theta - \arcsin[s \sin(\theta - \varepsilon)]}{\theta}$$

şi graficele din figura **7,a**.

Funcţie beta excentrică cardinală va fi

Plot[Evaluate[Table[{(Sqrt[1-(0.1 s)^2 -0.2 s Cos[t]])/t},{s,-10,0}],{t, -4 Pi, 4 Pi}]]	Plot[Evaluate[Table[{(Sqrt[1-(0.1 s)^2 - 0.2 s Cos[t]])/t},{s,0,10}],{t,-4 Pi,4 Pi}]]

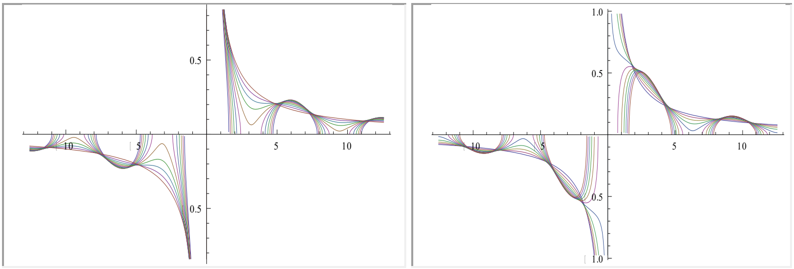
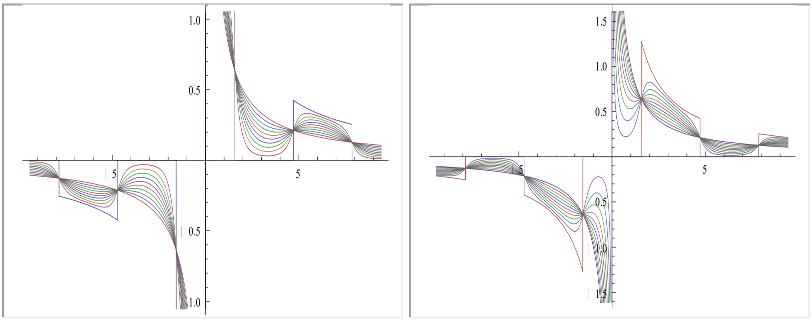


Fig.**7,d** Graficul funcţie supermatematice circulare radial excentrică cardinală Rexc(θ)

Plot[Evaluate[Table[{(1-sCos[t]/Sqrt[1-(s Sin[t])^2])/t},{s,-1,0}],{t,-3 Pi,3 Pi}]]	Plot[Evaluate[Table[{(1-sCos[t]/Sqrt[1-(s Sin[t])^2])/t},{s, 0, 1}],{t,-3 Pi,3 Pi}]]



Plot[Evaluate[Table[{((1-sCos[t])/(1+(s)^2 -2 s Cos[t]))/t},{s,-1,0}],{t,-3Pi,3 Pi}]]	Plot[Evaluate[Table[{((1-sCos[t])/ (1+(s)^2-2sCos[t]))/t},{s,0,10}], {t,-3 Pi,3 Pi}]]

Fig.**8,a** Graficul funcţie supermatematice circulare radial excentrică cardinală $\text{dexc}_1(\theta)$

(14) $\text{bexc}(\theta) = \frac{bex\theta}{\theta} = \frac{bex[\theta,S(S,\varepsilon)]}{\theta} = \frac{\arcsin[s\sin(\theta-\varepsilon)]}{\theta}$,

cu graficele din figura **7,b**.

Funcţia radial excentric cardinal de variabilă excentrică θ are expresia

(15) $\text{rexc}_{1,2}(\theta) = \frac{rex\theta}{\theta} = \frac{rex[\theta, S(s,\varepsilon)]}{\theta} = \frac{-s\cos(\theta-\varepsilon) \pm \sqrt{1-s^2\sin(\theta-\varepsilon)}}{\theta}$ şi graficele din figura **7,c**, iar aceeaşi funcţie, dar de variabilă centrică α are expresia

(16) $\text{Rexc}(\alpha_{1,2}) = \frac{Rex\alpha_{1,2}}{\alpha_{1,2}} = \frac{Rex[\alpha_{1,2}, S(s,\varepsilon)]}{\alpha_{1,2}} = \frac{\pm\sqrt{1+s^2-2s\cos(\alpha_{1,2}-\varepsilon)}}{\alpha_{1,2}}$

şi graficele, pentru $\text{Rexc}(\alpha_1)$, din figura **7.d**.

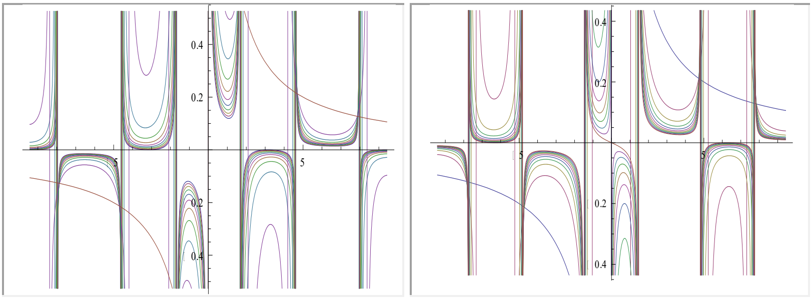


Fig.**8,b** Graficul funcţie supermatematice circulare radial excentrică cardinală $\text{Dexc}(\alpha_1)$

O funcţie supermatematică circulară excentrică cu largi aplicaţii, ea reprezentând funcţia de transmitere a vitezelor şi/sau a turaţiilor tuturor mecanismelor plane cunoscute, este funcţia derivată excentrică $dex_{1,2}\theta$ şi $Dex\alpha_{1,2}$ care prin impărţire / raportarea cu argumentele θ şi, respectiv, α, conduc la funcţiile corespondente cardinale, notate $dexc_{1,2}(\theta)$ şi, respectiv $Dexc(\alpha_{1,2})$ şi de expresii

(17)
$$dexc_{1,2}\theta = \frac{dex_{1,2}\theta}{\theta} = \frac{dex_{1,2}[\theta, S(s,\varepsilon)]}{\theta} = \frac{1-\frac{s.\cos(\theta-\varepsilon)}{\sqrt{1-s^2\sin^2(\theta-\varepsilon)}}}{\theta},$$
$$Dexc\alpha_{1,2} = \frac{Dex\alpha_{1,2}}{\alpha_{1,2}} = \frac{Dex[\alpha_{1,2},\ S(s,\varepsilon)]}{\alpha_{1,2}} = \frac{\pm\sqrt{1+s^2-2s.\cos(\alpha_{1,2}-\varepsilon)}}{\alpha_{1,2}}$$

cu graficele din figura **8**.

Deoarece $\text{Dex}\alpha_{1,2} = \frac{1}{dex_{1,2}\theta}$ rezultă că şi → $\text{Dexc}\alpha_{1,2} = \frac{1}{dexc_{1,2}\theta}$

Funcţiile cvadrilobe siqθ şi coqθ prin împărţirea lor cu argumentul θ, conduc la obţinerea funcţiilor cvadrilobe cardinale siqc θ şi coqc θ de expresii

$$(18)\qquad \begin{cases} coqc\ \theta = \dfrac{coq\theta}{\theta} = \dfrac{coq[\theta, S(s,\varepsilon)]}{\theta} = \dfrac{\cos(\theta-\varepsilon)}{\theta\sqrt{1-s^2 sin^2(\theta-\varepsilon)}} \\ siqc\ \theta = \dfrac{siq\theta}{\theta} = \dfrac{siq[\theta, S(s,\varepsilon)]}{\theta} = \dfrac{\sin(\theta-\varepsilon)}{\theta\sqrt{1-s^2 cos^2(\theta-\varepsilon)}} \end{cases},$$

cu graficele din figura 9.

Plot[Evaluate[Table[{(Cos[t] /Sqrt[1-(0.1 s Sin[t])^2])/t},{s,0,10}],{t,-4 Pi,4 Pi}]]	Plot[Evaluate[Table[{(Sin[t] /Sqrt[1-(0.1 s Cos[t])^2])/t},{s,0,10}],{t,-4 Pi,4 Pi}]]

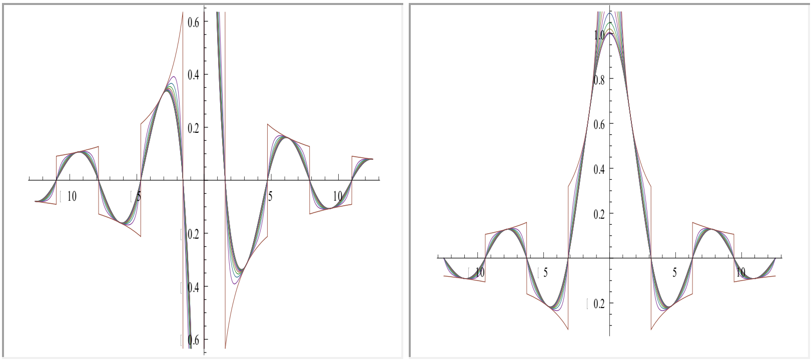


Fig.**9** Graficul funcţie supermatematice cvadrilobe cardinală ceqc (θ) ◄ şi siqc(θ) ►

Se ştie ca, prin integrarea definită a funcţiilor cardinale centrice şi excentrice, într-un cuvânt supermatematice, se obţin funcţiile integrale corespunzătoare.

Astfel de funcţii supermatematice integrale sunt prezentate în continuare. Pentru excentricitate nulă, ele degenerează în funcţii integrale centrice, in rest ele aparţin noii matematici excentrice.

5. FUNCŢII SINUS INTEGRAL EXCENTRICE

Se obţin prin integrarea funcţiilor sinus cardinal excentrice (13) şi sunt

(19) sie x = $\int_0^x sexc\,\theta . d\theta$ cu graficele din figura **10**, pentru cele de variabilă excentrică x $\equiv \theta$.

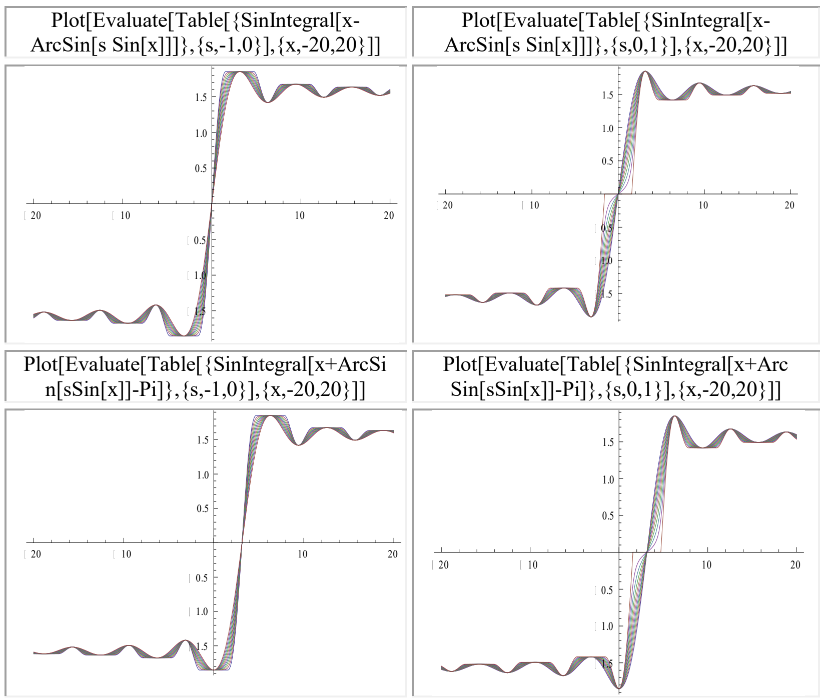


Fig.**10,a** Graficul funcţie sinus integral excentric $sie_1(x)$ ▲ şi $sie_2(x)$ ▼

Spre deosebire de funcţiile centrice corespondente, unde sinusul integral este notat cu Si(x), sinusul integral excentric de variabilă excentrică a fost notat sie(x), fără majuscula S, care se va atribui, conform convenţiei, doar **FSM-CEC** de variabilă centrică.

Funcţia sinus integral excentric de variabilă centrică, notate Sie(x) se obţin prin integrarea funcţiei supermatematice circulare excentrice sinus excentric cardinal de variabilă centrică (14)

(20) Sexc(x) = Sexc[α, S(s, ε)], astfel că ea este

(21) $\quad \text{Sie}(x) = \int_0^x \frac{Sex[\propto, S(s,\varepsilon)]}{\alpha}\, d\alpha$, cu graficele din figura **10,b**.

Plot[Evaluate[Table[SinIntegral[x+ArcTan [sSin[x]/(1-sCos[x])]],{s,-1,0}], {x,-4 Pi,4 Pi}]]	Plot[Evaluate[Table[SinIntegral[x+ArcTan [sSin[x]/(1-sCos[x])]],{s,0,1}], {x,-4 Pi,4 Pi}]]

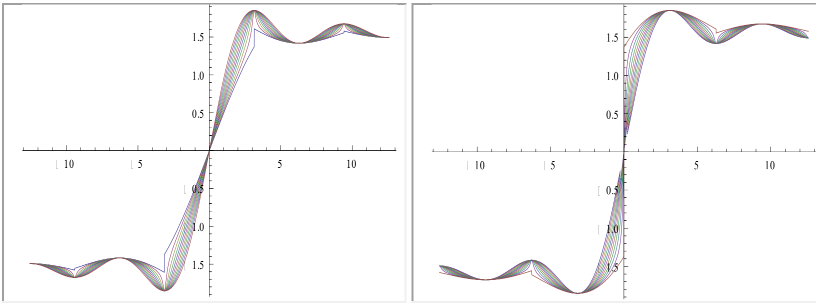


Fig.**10,a** Graficul funcţie sinus integral excentric sie$_1$ (x)

6. CONCLUZII

Lucrarea a scos în evidenţă posibilitatea multiplicării nedefinite a funcţiilor cardinale şi a celor integrale din domeniul matematicii centrice în cel al matematicicii excentrice sau al supermatematicii care constitue o reuniune a celor două matematici.

Totodată, au fost întroduse prin supermatematică, pe lângă funcţiile cardinale şi integrale cu corespondente în matematica centrică, o serie de funcţii cardinale noi ce nu au corespondente în matematica centrică.

Nici aplicaţiile noilor funcţii supermatematice cardinale şi integrale, cu siguranţă, că nu se vor lăsa prea mult aşteptate.

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SUPERMATEMATICA PROFESORULUI ŞELARIU

FLORENTIN SMARANDACHE

ABSTRACT.
Acest articol este o scurtă trecere în revistă a cărţii "SuperMatematica. Fundamente", Vol. 1 şi Vol. 2, ediţia a II-a, 2012, care constituie un domeniu nou de cercetare şi cu multe aplicaţii, iniţiat de profesorul universitar Mircea Eugen Şelariu. Lucrarea sa este unică în literatura mondială, deoarece combină matematica centrică cu matematica excentrică.

INTRODUCERE.
Supermatematica (**SM**) este o reuniune a matematicii cunoscute, ordinare, care în prezenta lucrare a fost denumită **matematică centrică (MC),** pentru a se deosebi de noua matematică, denumită **matematică excentrică** (**ME**). Adică **SM = MC ∪ ME.**
Pentru fiecare punct din plan, în care poate fi plasat un excentru **E(e, ε)**, se poate spune că există / apare o nouă **ME**. Astfel, la o singură **MC** îi corespund o infinitate de **ME;**
Pe de altă parte, **MC = SM(e = 0);**
În consecinţa, **SM** multiplică la infinit toate funcţiile circulare / trigonometrice cunoscute şi introduce o pleiadă de funcţii circulare noi (*aex, bex, dex, rex*, s.a), mult mai importante decât cele vechi şi, prin acestea, în final, multiplică la infinit toate entităţile matematice cunoscute şi introduce multe entităţi noi.
S-a constatat ca **MC** este proprie sistemelor **liniare, perfecte, ideale**, iar **ME** este proprie sitemelor **neliniare, reale, imperfecte**;
Ca urmare, odată cu apariţia **SM** a dispărut graniţa dintre liniar şi neliniar, dintre ideal şi real, dintre perfecţiune şi imperfecţiune;
SM evidenţiază excentricitatea liniara ***e*** şi pe cea unghiulară ***ε***, coordonatele polare ale excentrului **E(*e, ε*)**, ca noi dimensiuni ale spaţiului: dimensiuni de **formare** şi de **deformare** ale acestuia;
SM ar fi putut să apară cu peste 300 de ani în urmă, dacă **Euler**, la definirea funcţiilor trigonometrice ca funcţii circulare directe, n-ar fi ales **trei puncte confundate,** puncte care au sărăcit matematica: **Polul E** al unei semidrepte, **centrul C** al cercului trigonometric (unitate) şi **originea O**(0,0) a unui reper / sistem rectangular drept;
SM a apărut atunci când polul **E** a fost expulzat din centru şi a fost denumit **excentru.**
Din combinarea posibilă a celor trei puncte apar urmatoarele funcţii:

- **FCC circulare centrice** (**FSM - CC**)→ dacă **C≡ O ≡ E;**
- **FSM circulare excentrice (FSM - CE)**→ dacă **C ≡ O ≠E;**
- **FSM circulare elevate (FSM - CEL)**→ dacă **C ≠ O ≡ E;**
- **FSM circulare exotice (FSM - CEX)**→ dacă **C ≠ O ≠ E.**

Dintre **entităţile noi apărute** sunt şi o pleiadă de noi curbe închise, care apar la **transformarea continuă** a cercului în pătrat (denumite **quadrilobe** / **cvadrilobe**), a cercului în triunghi (**trilobe**).

În 3D, aceste transformari continue sunt a **sferei în cub**, a **sferei în prismă**, a conului în piramida ş.m.a.
Aceste transformări continue au facut posibila apariţia unor noi corpuri **3D hibride** ca: sfera-cub, cono-piramida, piramida-con s.m.a.
Prin înlocuirea cercului cu o quadrilobă au fost definite funcţiile quadrilobe, iar prin înlocuirea cu o trilobă au fost definite în lucrare şi funcţiile trilobe.

Totodată, în carte sunt introduse şi metode matematice şi tehnice noi, precum:

- Integrarea prin divizarea diferenţialei;
- Metoda hibridă numerico-analitica → Determinarea lui K(k) cu 15 zecimale exacte;
- Metoda separării momentelor → Metoda de cinetostatică, extrem de simplă şi exactă care reduce metoda **d'Alambert,** care necesită rezolvarea unor sisteme de ecuaţii de echilibru, la o problemă simplă de geometrie elementară;
- Mişcarea circulară excentrică de excentru punct fix şi de excentru punct mobil;
- Transformarea riguroasă în cerc a diagramei polare a complianţei;
- Solutionare unor sisteme vibrante de caracteristici elastice statice neliniare;
- Întroducerea sistemelor vibrante quadrilobe / cvadrilobe.

DESCRIEREA LUCRĂRII

Cap. 1. INTRODUCERE

Este prezentat un scurt istoric al descoperirii SUPERMATEMATICII, în legatură cu cercetarile intreprinse de autor la Universitatea din Stuttgart, în perioada 1969 - 1970, la Institutul şi Catedra de Maşini-Unelte a Prof. **Karel Tuffentsammer**, în grupa de "Vibraţii la Maşini – Unelte".

Totodată, se arată că marele matematician **Leonhard Euler**, la definirea funcţiilor trigonometrice ca funcţii circulare, alegând trei puncte confundate **[Originea O(0, 0)**, **Centrul cercului**, pe atunci denumit cerc trigonometric **M**(0, 0), acum redenumit cerc unitate şi **Polul** unei semidrepte **P(0,0)]** a sărăcit din start matematica. Ea, matematica, a rămas extrem de săracă, cu un singur set de funcţii periodice (sinα, cosα, tanα, cotα, secα, cscα ş.m.a.) şi, în consecinţă, în general cu entităţi matematice unice (dreaptă, cerc, pătrat, sferă, cub, integrală eliptică, ş.m.a).

Prin simpla expulzare a polului **P** şi denumit, din această cauză, **excentrul E(e,ε)** pentru cercul oarecare C(O,R) de rază R, sau notat cu S(s,ε) pentru cercul unitate CU(O,1), pentru fiecare punct din planul cercului unitate, în care se poate plasa un pol/excentru S(s,ε), se obţine câte un set de funcţii circulare/trigonometrice denumite şi **excentrice**.

Au fost denumite **ex-centre** pentru că au fost expulzate din centrul O.

Iar pe baza acestora, se obţin o infinitate de entităţi matematice noi, denumite **excentrice**, anterior inexistente în matematică (strâmba ca extensie/generalizare a dreptei; excentrica circulară sau quqdrilobele, care completează spaţiul dintre cerc şi pătrat sau, altfel spus, realizeaza o transformare continuă a cercul într-un pătrat perfect; excentrica sferică, care transformă continuu sfera într-un cub perfect; cono-piramida; sfera-cub, ş.m.a;)

Capitolul se incheie cu o trecere în revistă a principalelor contribuţii pe care noile complemente de matematică, reunite sub denumirea de SUPERMATEMATICĂ, le aduc în domeniile matematicii, informaticii, mecanici, tehnologiei şi a altor domenii.

Cap.2. **DIVERSIFICAREA FUNCŢIILOR PERIODICE**

Simţindu-se existenţa unor "pete albe" în matematică, o serie de mari matematicieni au încercat, în trecut ca şi în prezent, şi au reuşit să remedieze parţial aceste neajunsuri. Eforturile lor, meritau să fie trecute în revistă, alături de descoperirea supermatematicii, chiar dacă nu sunt de aceeaşi anvergură, iar unele dintre ele incomplet prezentate, mai mult schiţate, au fost aduse de autor la o formă finală, compatibilă cu programele de matematică.

Este vorba de funcţiile pătratice şi funcţiile rombice ale lui **Valeriu Alaci,** funcţiile poligonale ale lui **M. Ovidiu Enulescu,** funcţiile trans-trigonometrice al **Malvinei Florica Baica** şi **Mircea Cârdu,** funcţiile pseudohiperbolice ale lui **Eugen Vişa**, toţi profesori de matematică şi concitadini cu autorul.

În acelaşi oraş Timişoara, în care, *la 3 noiembrie 1823, un tânăr ofiţer-inginer din garnizoana Timişoarei, **Ianos Bolyai**, (el avea atunci 21 de ani), trimetea tatălui său, **Farkas Bolyai**, profesor de matematică la colegiul din Târgu-Mureş o emoţionantă scrisoare. El scria, printre altele:* "***din nimic am creat o lume nouă"*** *Era lumea* ***geometriilor neeuclidiene***.

Tot astfel, prin reuniunea matematicii centrice (**MC**) ordinare, cu noua matematică excentrică (**ME**) s-a creat **supermatematica** (**SM = MC** $\cap$ **ME**). Ea multiplică la infinit toate entitaţiile **unice** ale **MC** şi, în plus, introduce în matematică noi entităţi, anterior inexistente (conopiramida, sferocubul, ş.m.a.).

Se poate afirma că şi în acest caz "**din nimic**" au fost create noile entităţi matematice, cum sunt, de exemplu, funcţiile supermatematice circulare excentrice (**FSM-CE**) amplitudine excentrică aexθ şi Aexα, beta excentrice bexθ şi Bexα, radiale excentrice rexθ şi Rexα, derivate excentrice dexθ şi Dexα, conopiramidele, cilindrii pătraţi, triunghiulari şi de alte forme, ş.m.a.

Dar se poate afirma şi că dintr-o singură entitate matematică, existentă în **MC,** au fost create o infinitate de entităţi de acelaşi gen în ME şi, implicit, şi în **SM,** sau că **SM multiplică la intfinit toate entităţile MC**.

În mod deosebit, sunt evidenţiate funcţiile evolventice ale lui **George (Gogu) Constantinescu**, creatorul sonicităţii, cosiusul românesc Corα şi sinusul românesc Sirα, care sunt, din păcate, prea puţin cunoscute ca şi funcţiile trigonometrice înclinate, ale lui **Dr. Bihringer**, pe nedrept date uitării.

Cap. 3. COMPLETĂRI ŞI REDEFINIRI CORECTE ÎN MATEMATICA CENTRICĂ

Lucrarea lui **Octavian Voinoiu,** publicată de Editura Nemira, « **ÎNTRODUCERE ÎN MATEMATICA SIGNADFORASICĂ** « a scos în evidenţă o serie de entităţi matematice, de primă importanţă, greşit introduse în matematică, în matematica centrică (**MC**).

Adept al principiului lui **Sofocle** : »Errare humanum est, perseverare diabolicum », autorul a considerat că, inainte de a fi prezentate noile complemente de matematică, e strict necesar să fie parţial evidenţiate şi eventual corectate entităţile greşite introduse şi existente în **MC**.

Un exemplu, simplu, în acest sens, este definirea greşită a semnului unei fracţii şi, ca urmare, şi a tangentei ca fiind raportul tanα = $\frac{sin\propto}{cos\propto}$, în timp ce, definirea corectă este tanvα = $\frac{sin\propto}{Abs[cos\propto]}$, tangentă care a fost numită ca tangentă centrică **Voinoiu**. În acest fel, noua **FSM-CE** tangantă excentrică **Voinoiu** texvθ a putut fi « ab initio » corect definită, ca raport între sinusul sexθ şi cosinusul cexθ excentrice, adică texvθ = $\frac{\text{sex}\theta}{Abs[\text{cex}\theta]}$.

În plus, o serie de entitaţi, noi apărute în **ME**, şi în consecinţă şi în **SM**, nu aveau echivalente în **MC**. Este cazul celor mai importante **FSM-CE**, funcţiile periodice radial excentrică **rexθ**, o adevărată funcţie « rege » şi derivată excentrică **dexθ**, care, singură, exprimă funcţia de transfer de ordinul doi, sau raportul de transmitere al vitezelor şi /sau al turaţiilor tuturor mecanismelor plane existente.

S-a constatat că echivalentele acestor **FSM-CE** în **MC** sunt funcţiile radial centric **radα** = $e^{i\alpha}$ şi derivată excentrică **derα** = $e^{i(\alpha+\pi/2)}$, care nu sunt altele decât funcţiile **Euler-Cotes** sau fazorii direcţiilor radială centrică, faţă de centrul O(0,0) şi, respectiv, fazorul, defazat în avans cu $\frac{\pi}{2}$, sau fazorul tangentei la cercul unitate în punctul W(α, 1), de coordonate polare, cu polul în originea O(0, 0).

In finalul acestui capitol a fost prezentată o aplicaţie deosebit de importantă şi originală cu privire la „Transformarea riguroasă în cerc a diagramei polare a complianţei", care vine să corecteze studiile incomplete ale celui mai studiat sistem oscilant din literatura de specialitate.

Partea I-a

FUNCŢII SUPERMATEMATICE CIRCULARE EXCENTRICE (FSM-CE)

Se ştie că în matematică, în principiu, funcţiile pot fi definite pe oricare curbă plană închisă sau deschisă, atât ca funcţii directe cât şi ca funcţii inverse. Astfel :

- Pe TRIUNGHIUL DREPTUNGHIC → Funcţii trigonometrice
- Pe TRIUNGHIUL OPTUZUNGHIC → Funcţiile trigonometrice înclinate **Bihringer**
- Pe TRILOBE → Funcţii trilobe **Şelariu**
- Pe CERC → Funcţii circulare **Euler**
- Pe ELIPSĂ → Funcţii eliptice **Jacobi**
- Pe PĂTRAT (rotit cu $\frac{\pi}{4}$) → Funcţii pătratice **Alaci**
- Pe ROMB → Funcţii rombice **Alaci**
- Pe CVADRILOBE → Funcţii cvadrilobe **Şelariu**
- $\begin{cases} \text{Pe CVADRILOBE } \left(rotite\ cu\ \frac{\pi}{4}\right) \rightarrow Funcţii\ transtrigonometrice \\ Pe\ ASTROIDE \rightarrow Funcţii\ infratrigonometrice \\ Pe\ SPIRALE \rightarrow Funcţii\ paratrigonometrice\ spirale \end{cases}$ → **Malvina Baica - Mircea Cârdu**
- Pe POLIGON → Funcţii poligonale **Enulescu**
- Pe LEMNISCATA → Funcţiile lemniscate **Marcuşevici**
- Pe EVOLVENTĂ → Funcţii evolventice **Gogu Constantinescu**
- Pe ASIMPTOTELE HIPERBOLEI → Funcţii pseudohiperbolice **Eugen Vişa**
- **Pe HIPERBOLA ECHILATERĂ**→ Funcţii hiperbolice

şi mai pot exista şi alte funcţii de acest gen.

În această lucrare au fost prezentate, în principal, funţiile supermatematice (**FSM**) definite pe cerc.

Partea I.1 FUNCŢII SUPERMATEMATICE CIRCULARE

EXCENTRICE DE VARIABILĂ EXCENTRICĂ

Cele trei puncte confundate de **Euler** (**Polul S(s,ε)** şi **centrul** cercului unitate **C(c,φ)** în **originea O**(0, 0) a unui reper) pot fi separate în următoarele trei moduri; pentru fiecare mod de separare fiind proprii alte tipuri de funcţii supermatematice (**FS**), după cum urmează:

C(0,0) ≡ **O**(0, 0) ≡ **S(0,0)** → **FCC --** Funcţii Circulare **Centrice**
C(0,0) ≡ **O**(0, 0) ≠ **S(s,ε)** → **FSM-CE** →**Funcţii Supermatematice – Circulare Excentrice**
C(c,φ) ≠ **O**(s, ε) ≡ **S(s,ε)** → **FSM-CE$_L$** →**Funcţii Supermatematice – Circulare Elevate**
C(c,φ) ≠ **O**(0, 0) ≠ **S(s,ε)** → **FSM-CEx** →**Funcţii Supermatematice – Circulare Exotice**

Toate funcţiile supermatematice pot fi, la rândul lor, de variabilă excentrică **θ** şi de variabilă centrică **α**. Primele, sunt funcţii continue doar pentr un excentru **S** interior cercului / discului unitate, adică pentru o excentricitate liniară numerică $s \leq 1$.

Funcţiile de variabilă centrică sunt continue pentru un S plasat oriunde în planul cercului unitate, adică pentru $\mathbf{s} \in [0, \infty]$.

Prin intersectarea cercului unitate cu o dreaptă ($\mathbf{d} = \mathbf{d}^+ \cap \mathbf{d}^-$) şi nu numai cu semidreapta pozitivă ($\mathbf{d}^+$), la îndemnul unor talentaţi şi autentici matematiceieni cum este Prof. dr. math. **Horst Clep**, **trigonometria excentrică** sau **FSM-CE** a fost puse de acord cu geometria diferenţială, care operează cu drepte. De aceea, toate **FSM-CE** au două determinări: una **principală**, notată cu indicele **1**, sau fără indice, când alte determinări nu se folosesc şi confuziile nu pot să apară, rezultată din intersecţia cu cecului unitate cu semidreapta pozitivă $\mathbf{d}^+$ şi una **secundară**, notată cu indice **2**, rezultată din intersecţia cercului unitate cu semidreapta negativa $\mathbf{d}^-$.

Pentru excentrul **S** exterior cercului unitate ($\mathbf{s} > 1$), apar patru determinări, dintre care intersecţia cercului cu $\mathbf{d}^+$ le generează pe primele două, de indici **1** şi **2**, iar intersecţia cu cu $\mathbf{d}^-$, pentru indicii 3 şi 4, se obţin din relaţiile pentru determinările 1 şi, respectiv, 2 pentru o variabilă **θ** defazată în avans cu **π** , adică $\boldsymbol{\theta} \rightarrow \boldsymbol{\theta} + \boldsymbol{\pi}$.

În partea **I.1** a acestei lucrări sunt prezentate / tratate cu preponderenţă **FSM-CE** de variabilă excentrică **θ**, cu preponderenţă pentru excentricitatea liniară numerică $\mathbf{s} \leq \mathbf{1}$ şi pentru excentricitatea unghiulară $\boldsymbol{\varepsilon} = \mathbf{0}$.

Sunt trecute în revistă şi definite grafic, pe cercul unitate, principalele **FSM-CE** care vor face obiectul tratării lor viitoare.

Unele **FSM-CE** sunt dependente de originea **O(0,0)** a sistemului de referinţă / reperului, iar altele sunt independente de aceasta. Prezentarea **FSM-CE** începe în Cap.4 cu o funcţie independentă de originea reperului polar sau rectangular drept şi care stă la baza definirii ulterioare şi a altor **FSM-CE.**

Cap. 4 FUNCŢIA RADIAL EXCENTRICĂ rex θ ŞI UNELE APLICAŢII MATEMATICE IMPORTANTE ALE EI

FSM-CE cu care debutează lucrarea este funcţia radial excentric de variabilă excentrică $\mathbf{rex}_{1,2}\boldsymbol{\theta}$, cea mai importantă funcţie periodică, o adevărată "**funcţie rege**", cum a numit-o Prof. dr. math. **Octav Em. Gheorghiu**, pentru că ea exprimă distanţa în plan dintre două puncte în coordonate polare: $\mathbf{W}_{1,2}$ de pe cercul unitate CU(O, 1), la intersecţia cu drepta **d** şi până la

excentrul S(s,ε). În consecinţă, această funcţie poate exprima singură ecuaţiile tuturor curbelor plane cunoscute, denumite şi **centrice**, cât şi a multor curbe noi, apărute odata cu apariţia **SM,** denumite **excentrice**.

Remarcă: Expresiile lui $rex_{1,2}\theta$ sunt soluţiile ecuaţiilor algebrice de gradul II cea ce faciliteaza rezolvarea inecuaţiilor de gradul II..

In continuare sunt definite şi prezentate succint, cu aplicatiile lor, urmatoarele funcţii supermatematice.

Cap. 5 ALTE APLICAŢII MATEMATICE ŞI TEHNICE ALE FUNCŢIEI RADIAL EXCENTRICĂ Rex θ

Determinarea oricât de exactă a unei relaţii de calcul a integralei eliptice complete de speţa I-a **K(k)** cu cel puţin 15 zecimale exacte, care a condus la elaborarea unei noi metode hibride numerice-analitice de calcul (O varianta a metodei **Landen** a mediei aritmetico-geometrice care este o metodă pur numerică, care dă **valoarea numerică** pe când **noua metodă** (sa-i zicem Şelariu) dă o relaţie analitică de calcul simplă)

Cap. 6 FUNCŢIA DERIVAT EXCENTRICĂ dex θ ŞI UNELE APLICAŢII MATEMATICE ŞI TEHNICE

Expresia acestei funcţii este si expresia generala a raportului de transmitere a mişcărilor (viteze, turaţii) a **TUTUROR** mecanismelor plane cunoscute.

Exprimă viteza unui punct pe cerc în **mişcarea circulară excentrică** (**MCE**) o generalizare a mişcării circulare centrice.

Cap. 7 ANALIZA CALITĂŢII MIŞCĂRII PROGRAMATE CU FUNCŢII SUPERMATEMATICE.

Cap. 8 METODA SEPARARII FORŢELOR ŞI A MOMENTELOR

Oferă o rezolvare simplă şi exactă a tuturor sitemelor mecanice solicitate de forţe plane sau reductibile la acestea (elastostatică) ocolind necesitatea rezolvării unor sisteme de ecuaţii de echilibru din metoda **d'Alambert**.

Volumul II al lucrării "**SUPERMATEMATICA. FUNDAMNETE"** are capitolele sale numerotate în continuarea **vol. I**, adică începând cu **Cap. 12** întitulat "**INTEGRALE ŞI FUNCŢII ELIPTICE EXCENTRICE**". El este precedat de un tabel cu privire la "**SITUAŢA ACTUALĂ A SUPER-MATEMATICII**" şi cu "**LISTA NOILOR FUNCŢII MATEMATICE INTRODUSE PRIN ACEASTĂ LUCRARE**", adică, introduse în Matematica pe care autorul a denumit-o Matematică Centrică (**MC**) şi în Matematică, în general, prin cele două volume de supermatematică (**SM**). Sunt prezentate 60 de noi simboluri de funcţii introduse de autor în matematică, prin a sa lucrare de supermatematică. Şi au fost prezentate doar funcţile principale, ca de exemplu, cosinus şi sinus eliptic excentric **ceex**, **seex**, cosinus şi sinus quadrilob/(cvadrilob) **coq** şi **siq** nu şi funcţiile compuse, cum sunt tangenta, cotangenta, secanta, cosecanta ş.m.a., dar este prezentată tangenta **Voinoiu** $\tan v\theta = \frac{sin\theta}{cos\theta}$, tangenta quadrilobă (cvadrilobă) $taq\theta = \frac{siq\theta}{coq\theta}$, ş.m.a. funcţii derivate, ca şi derivatele funcţiilor amintite.

Şi numai această observaţie cantitativă poate să divulge multe din calitaţiile acestei lucrări enciclopedice, surprinzătoare şi unică în literatura de specialitate mondială, ca şi denumirea ei de **SM**, din momentul publicării lucrării cu acest conţinut, în anul 1978 şi cu acest titlu, în anul 1993, aşa cum rezultă din bibliografia ataşată acestei lucrări.

Din primul moment, impresioneaza multitudinea de schiţe explicative, realizate cu programe de matematică, utilizând tocmai **funcţiile supermatematice FSM** descoperite de autor, precum şi numeroasele grafice ale familiilor de funcţii noi prezentate în lucrare. Pentru frumuseţea lor intrinsecă, dar şi pentru întregirea formelor funcţiilor dintr-o familie de funcţii, sunt prezentate şi numeroase familii de **funcţii SM** în **3D**.

Aici şi acum este cazul să-l cităm pe ing. **Ioan Ghiocel**, cel care a prefaţat cel de al II-lea volum: ”Să nu ne mirăm când dl. Prof. M. E. Şelariu, sub presiunea inflexiunilor şi faldurilor gândului, reuneşte cuvinte care n-au mai stat alături de la întemeierea lumii, precum ***cerc al amortizărilor vâscoase liniare***, ***funcţii elevate***, ***funcţii exotice***, ***dreapta definită ca degenerată a strâmbei*** ş.a.m.d...!”

Dacă, în vol. I, au fost introduse cu precădere funcţiile supermatematice circulare excentrice, abreviate de autor prin **FSM-CE,** dintre care amintim funcţiile **aex, bex, dex, cex, sex, rex, tex, ctex** ş.a, în vol.II, **Cap. 12** au fost introduse noi integrale eliptice excentrice de speta I-a şi de speţa a II-a care generalizează integralele eliptice centrice, pe care le poate reprezenta, pentru o excentricitate liniară numerică **s = 0**, adică pentru cazul în care excentrul **S(s, ε)** se suprapune peste originea **O(0,0)** a sitemului de coordonate sau reperului **xOy**.

Totodată sunt prezentate funcţii eliptice, hiperbolice şi parabolice excentrice, în funcţie de variabile clasice, cunoscute, dar şi în funcţie de arcul unui cerc unitate, tangent comun la hipoerbola echilateră, elipsa unitate şi la parabolă, în vârful acestora. În cel din urmă caz, sunt prezentate, totodată, şi funţiile eliptice, hiperbolice şi parabolice centrice, ca funcţie de arcul cercului unitate anterior amintit, caz unic şi în literatura matematicii centrice.

Ele sunt denumite de autor şi “***funcţii pe conice cu vârful comun***”.

Capitolul 13 este dedicat atât funcţiilor centrice cât şi a celor excentrice **autoinduse**, de forma sin[sin[sin[sin[sin[sin[…[sinx]]]]]]]]] sau cex[cex[cex[cex[…[cex[θ]]]]]]]] şi a celor **induse** de forma [cos[sin[sin[tan[tan[cos[sin[cos[tan[….sin[x]]]]]] sau cex[sex[sex[tex[tex[cex[sin[cos[tex[…sex[θ]]]]]].

Sunt prezentate şi derivatele funcţiilor induse şi autoinduse, centrice şi excentrice, precum şi derivatele funcţiilor circulare centrice şi excentrice **Voinoiu**, funcţii prezentate iniţial în primul volum, ca o corecţie necesară adusă funcţiilor tangentă şi cotangentă, introduse greşit în matematică, aşa cum a demonstrat marele matematician român **Octavian Voinoiu** în cartea sa “**INTRODUCERE ÎN MATEMATICA SIGNADFORASICĂ**”.

Pentru derivarea funcţiilor trigonometrice **Voinoiu** a fost necesară determinarea derivatei funcţiei **Abs[f(x)]**, derivată inexistentă în literatura de specialitate. Autorul demonstrează (pag. 73) că derivata acestei funcţii este $\boldsymbol{\frac{d}{dx}[f(x)] = Sign[f(x)]\,\frac{d}{dx}[f(x)]}$.

Capitolul 14 este dedicat funcţiilor hiperbolice excentrice. În prealabil sunt prezentate hiperbolele excentrice şi, în special, hiperbola echilateră excentrică, ca şi alte funcţii exponenţiale centrice şi excentrice de variabilă excentrică θ, precum şi definirea geometrică a funcţiilor hipernbolice centrice şi excentrice. Pe lângă funcţiile hiperbolice clasice, cunoscute şi în matematica centrica (**MC**) cum sunt cosinusul – **cexh -** , sinusul – **sexh -**, tangenta – **texh -** ş.a. hiperbolice excentrice, sunt prezentate şi funcţiile care au apărut odata cu **FSM-CE**, cum sunt amplitudine excentrică hiperbolică – **aexh -**, radială excentrică hiperbolică – **rexh -**, derivată excentrică hiperbolică – **dexh** – ş.a.

Pentrun funcţiile hiperbolice au fost prezentate şi cosinusul (celh) şi sinusul (selh) hiperbolice elevate. În concluzia acestui capitol sunt prezentate obiecte geometrice noi exprimate cu ajutorul acestor funcţii noi introduse în matematică.

Capitolul 15 este dedicat **FSM-CE** de variabilă centrică α, notate de autor cu majuscule (Aex, Bex, Cex, Dex, Rex, Sex, Tex, etc), pentru a fi deosebite de cele de variabilă excentrica θ (aex, bex, cex dex, rex, sex, tex ş.a). Capitolul debutează cu prezentarea schiţelor explicative de definire a **FSM-CE** pentru cazul unui excentru **S**(s, ε) plasat în discul unitate, adică în interiorul cercului unitate şi, separat, este prezentat cazul excentrului **S** plasat în exteriorul acestuia.

FSM-CE bexθ şi Bexα de excentricitate liniară numerică s = 1, a căror grafice sunt riguros în dinţi de fierestrău simetrici şi, respectiv, asimetrici au fost denumite de autor, sau ***funcţii triunghiulare Octav Gheorgiu*** în memoria şi onoarea Prof. Dr. **Octav Em. Gheorghiu**, urmaş al Prof. Dr. **Alaci Valeriu** la şefia Catedrei de Matematică a Institutului Politehnic "Traian Vuia" din Timişoara. Tot astfel cum, în onoarea matematicianului Prof. Dr. Florentin Smarandache, funcţiile în trepte, obţinute cu ajutorul **FSM-CE** au fost denumite ***funcţii în trepte Smarandache.***

În acest capitol sunt subliniate, fără tagadă, avantajele exprimării unor funcţii periodice speciale, triunghiulare, pătrate, dreptunghiulare, în trepte ş.m.a. cu ajutorul **FSM-CE** care le exprimă exact şi cu **FSM-CE** din numai doi termeni simplii, în comparaţie cu exprimarea lor aproximativă prin voltări în diverse serii. Tot aici sunt prezentate soluţiile unui sistem neamortizat de amplitudini variabile, exprimate de funcţia bexθ, a ecuaţiei diferenţiale $\Delta\ddot{\varphi} + v_0^2 sin\varphi = \varphi_0 v_0^2 sinv_0 t$.

În figura **15.28** sunt prezentate schiţele mecanismelor culisă motoare-manivelă şi manivelă motoare-culisă şi anumite **FSM-CE** exprimabile cu eceste mecanisme.

O nouă metodă de integrare, apărută graţie apariţiei **FSM-CE**, este prezentata în **Cap.16.**

Este denumită "***Metodă de integrare prin divizarea diferenţialei***" şi se bazează pe divizarea variabilei θ în variabilele α şi în β, conform relaţie cunoscute în domeniul **FSM-CE: θ = α + β,** ceea ce dă posibilitate diferenţialei **dθ** să se dividă, la rândul ei, în **dα** şi în **dβ**, adică **dθ = dα + dβ**.

În acest fel, o serie de integrale, rezolvabile în planul complex prin teorema reziduurilor, se pot rezolva direct şi cu mult mai simplu, aşa cum se ilustrează prin aplicaţiile prezentate în acest capitol. Una dintre aplicaţii este realizată împreuna cu Prof. Dr. Math. **Florentin Smarandache** şi prezentată anterior, separat, în cadrul uni articol.

Deoarece la θ = α = 0 şi pentru o excentricitate unghiulară ε = 0, indiferent de valoarea excentricităţii liniare numerice **s** ∈ [-1, 1] se obţine β = **bexθ** = arcsin[s.sin(θ - ε)] = 0 ca şi pentru θ = α = π rezultă extrem de avantajoasă integrarea între limitele 0 şi π ca şi între limitele 0 şi 2π. Cele 8 aplicaţii prezentate în lucrare sunt elocvente în acest sens.

FSM-CE bexθ, prezentată anterior şi notata în acest capitol cu **βsexθ** poate exprima şi soluţiile unor sisteme vibrante neliniare, care fac obiectul **Cap.17**.

Sunt prezentate funcţiile **bexθ = βsexθ** si **βcexθ =** arcsin[s.cos(θ - ε)] pentru un excentru **S**(**s** ∈ [-1, +1], ε = 0) sau **S**(**s** ∈ [0, +1], ε = 0 V π), ceea ce-i acelaşi lucru, precum şi derivatele lor ca şi semnificaţia geometrica a acestora (**Fig.17.2**).

Deoarece matricea wronskiana data de soluţiile
$\begin{cases} x = \boldsymbol{\beta}\mathbf{cex}\boldsymbol{\theta} \\ y = \boldsymbol{\beta}\mathbf{sex}\boldsymbol{\theta} \end{cases}$, este diferită de zero, rezultă că cele două soluţii sunt liniar independente. Sunt prezentate caracteristicile elastice statice ale acestor sisteme vibrante şi curbele integrale în spaţiul fazelor.

Capitolul 18 este dedicat **funcţiilor supermatematice** (centrice, excentrice, elevate şi exotice) **pe conice**. Atât pe conice centrice, în funcţie de arcul cercului tangent la vârful conicelor, cât şi pe conice excentrice, ca un fel de preludiu la **capitolul 19**, al **funcţiilor eliptice supermatematice de arc de cerc**. Cu aceată ocazie sunt definite **elipsele unitate** pe x, respectiv, pe y, notate $\mathbf{U_x}$ şi, respectiv $\mathbf{U_y}$, astfel încât, proiecţiiler punctelor pe axa x, respectiv, y să se înscrie în ecartul [-1, +1].

Foarte voluminos, **capitolul 19** se intinde pe 42 de pagini (254…296), în care sunt definite **funcţiile eliptice supermatematice**, proprietăţile lor, derivatele si vitezele de rotaţie ale unui punct pe eliposele unitate. Pe lângă funcţiile eliptice cunoscute în matematica centrică - cosinus cn(u,k) şi sinus sn(u,k) – aici sunt prezentate şi noile funcţii precum amplitudine eliptică excentrică, care este comparată cu funcţia eliptică **Jacobi** amplitudine sau amplitudinus - am (u,k) – şi funcţiile derivate eliptice excentrice în funcţiwe de cosinus → **dece**(α, k = s) şi în funcţie de sinus → **dese**(α, k = s).

În **figura 19.12** sunt reprezentate funcţiile eliptice **Jacobi** cn,sn dn, nu pe o elipsă, ci pe cercul unitate, gratie noilor **FSM-CE.** Funcţiile eliptice în trepte au fost denumite de autor funcţii eliptice în trepte

Smarandache, notate **smce**(α, k) şi **smse**(α,k) a căror grafice sunt prezentate în **figura 19.13** împreună cu ale derivatelor lor.

În **paragraful 19.9** sunt prezentate funcţiile intratrigonometrice, definite pe quadrilobe (cvadrilobe), care completează spaţiul dintre pătratul **Alaci Valeriu** şi cercul unitete **Euler**, ca şi domeniul dintre funcţiile circulare centrice **Euler** şi funcţiile trigonometrice pătratice **Alaci Valeriu**.

Se arată că noile curbe închise denumite de autor quadrilobe (cvadrilobe) sunt echivalentele unei "elipsei" unitate simultan pe x şi pe y (**Fig.19.19**).

Cu ajutorul acestor funcţii quadrilobe (cvadrilobe) au fost definite transformările continue ale cercului în pătrat perfect, ale sfercei în cub perfect, ca şi ale conului în piramidă perfectă cu baza un pătrat, a căror imagini în 3D sunt prezentate în **figura 19.16,** constituind, totodată, noi obiecte geometrice (super)matematice.

În **paragraful 19.11** sunt prezentate **funcţiile eliptice supermatematice** ca soluţii ale unor sisteme vibrante neliniare, iar **paragraful 19.12** este dedicat **funcţiilor eliptice de arc de cerc.**

Paragrafele 19.13 şi **19.14** se referă la funcţii **hiperbolice SM centrice** şi, respectiv, funcţii **hiperbolice SM excentrice** fiind prezentate atât funcţiile cosinus, sinus şi tangentă, cât şi noile funcţii introduse de autor şi denumite tangentă hiperbolă **Voinoiu**.

Denumit "**Găuri de vierme în matematică**", **Cap. 20** pretinde că ele pot fi realizate cu ajutorul unor **FSM-CE hibride**. În concepţia autorului, gaura de vierme ar fi o modalitate de legatură mai rapidă, posibilă, între matematica circulară centrică şi matematica eliptică. Care constituie şi visul de-o viaţă al autorului, din păcate încă ne realizat pe deplin. Sunt prezentate două "străpungeri" meritorii: funcţiile eliptice **Neville Theta C** reprezentate exact cu ajutorul **FSM-CE** cosinus excentric **cexθ** (**Fig.20.2,a** şi **Fig.20.2.,b**), precum si exprimarea functiei eliptice **Jacobi Zeta** prin **FSM-CE** modificată sin[bexθ] (**Fig.20.3**).

Paragraful 20.3 prezintă alte **funcţii matematice speciale hibride**.

Capitolul 21 se referă la funcţii **trigonometrice analitice excentrice** de variabilă reală (R-analitice **§ 21.2**) şi centrice (**§ 21.3**). **Paragraful 21.4** este dedicat funcţiilor **circulare analitice excentrice de variabilă excentrică** dependente de originea reperului (cos, sin, tan,s.a.), iar **§ 21.5** a celor independente de originea sistemului de axe de coordonate (bex, dex, rex, aex ş.a.). **Paragraful 21.10** tratează **FSM-CE dublu analitice.**

Capitolul 22 se referă la **FSM-CE de variabilă complexă (C- analitice)** şi este foarte bogat ilustrat, în special în 3D, la fel ca şi **§ 22.3** cu privire la diversele obiecte matematice reprezentate cu **FSM-CE** şi cu **FSM-CEA** care se incheie cu reprezentarea matematică a unor piese şi sisteme de piese tehnice.

În loc de postfaţă, **Cap.23** se referă la "**Materia neagră a universului matematic**" în care sunt prezentate numerele iraţionele excentrice, **excentricitatea** ca **o nouă dimensiune**, ascunsă, **a spaţiului**, **hibridarea matematică**, numerele reale excentrice şi **sistemul trigonometric excentric**, în comparaţie cu cel centric, pentru evidenţierea avantajelor nete ale primului sistem, care este unul continuu, în timp ce, cel centric este discret. De aici rezultând marile avantaje ale aproximării curbelor şi a suprafeţelor tehnice, pe lângă faptul că, odată cu apariţia supermatematicii, o serie întreagă de suprafeţe, considerate anterior nematematice, devin suprefeţe (super)matematice şi, ca urmare, pot fi reprezentate exact cu ajutorul noilor funcţii supermatematice ale lui **Mircea Eugen Şelariu**.

CONCLUZIE.

Forţa novatoare a <u>supermatematicii</u> profesorului Mircea Eugen Şelariu o recomandă ca valoroasă teorie la nivel international, care deschide noi ramuri de cercetări cu numeroase aplicaţii.

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EXTENICS

AN EXTENSION COLLABORATIVE INNOVATION MODEL IN THE CONTEXT OF BIG DATA

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The processes of generating innovative solutions mostly rely on skilled experts who are usually unavailable and their outcomes have uncertainty. Computer science and information technology are changing the innovation environment and accumulating Big Data from which a lot of knowledge is to be discovered. However, it is a rather nebulous area and there still remain several challenging problems to integrate the multi-information and rough knowledge effectively to support the process of innovation. Based on the new cross discipline *Extenics*, the authors have presented a collaborative innovation model in the context of Big Data. The model has two mutual paths, one to transform collected data into an information tree in a uniform basic-element format and another to discover knowledge by data mining, save the rules in a knowledge base, and then explore the innovation paths and solutions by a formularized model based on *Extenics*. Finally, all possible solutions are scored and selected by 3D-dependent function. The model which integrates different departments to put forward the innovation solutions is proved valuable for a user of the Big Data by a practical innovation case in management.

Keywords: Extension innovation model; Big Data; data mining; Extenics; knowledge management.

Acknowledgments

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1. Introduction

In the past few decades, a large number of scholarly efforts and theories on collaborative innovation have been developed, and many approaches have contributed to reveal the nature of innovation process in various degrees.[1,2] Nevertheless, the innovation process still largely remains a black box.[3] Although each theory can explain some of the mechanisms behind innovations, the general mysteries behind innovation process are still far from resolved. Most of the innovation models make use of a group of experts and depend on personal intelligence which would be subject to limitations of individual opinions, and therefore they keep out of step from the rapidly changing information and knowledge era.

We live in an era where a remarkable amount of new information and data is accumulated everywhere. Because of the massive amount of data that is generated almost everywhere, new tools need to be developed in order to manage and analyze the data, especially in the field of management.[4] The information environment is rapidly and continuously changing and uncertain due to global competition, information explosion, and advances in new technologies. The web and other information technologies (ITs) combined with business and lives have accumulated huge data and information. Information and other technologies are successfully bypassing the main obstacle to technological advance, and when technology support net is fully established and fixed, innovation is not a free and autonomous process of applied creativity but is technically, economically and politically subservient to the "holders and owners" of the support net.[5]

Big Data[6,7] is a collection of data sets. It is so large and complex that it becomes difficult to process using on-hand database management tools or traditional data processing applications.

Data is accumulating from almost all aspects of our everyday lives that it becomes huge and multi-structured and has hidden useful information. The era of Big Data is real and is going to stay with us.

The challenges with Big Data include capture, curation, storage, search, sharing, transfer, analysis, and visualization (http://en.wikipedia.org/wiki/Big_data). Big Data provides materials for mining hidden patterns to support innovation[8] mostly by data mining.[9,10] The interaction research with Big Data support methods for innovation is rare at present.[11] Knowledge discovered by data mining is novel and quantitative.[12,13] However, it still lacks a uniform knowledge management model to support the innovation process effectively.[14]

Extenics (formerly referred as Extension Theory) focuses to solve contradictory problems by formalization methods based on the concepts of matter-element and extension set.[15,16] Extenics uses a uniform three-dimensional (3D) matrix to express information and knowledge and utilizes extension transformation to represent properties of things and indicates things with certain attribute that can be changed into things without such attribute. It provides a new view for understanding the process of innovation. But it needs more support of IT, especially Big Data.

To improve the quality of collaborative innovation by objective hidden knowledge from Big Data, we propose a new innovation model combining IT and Extenics. The rest of this paper is organized as follows. Section 2 discusses the existing innovation methods and problems. Section 3 provides a systemic model for preparing the input for innovation from Big Data. Section 4 presents a framework and process about utilization of Extenics to generate innovative ideas or solutions with a new method to score the solutions. Section 5 gives an application of the proposed Extenics to a real-world problem and Sec. 6 suggests future research direction for our work.

2. Literature Review

There are a number of factors that affect the quality of collaborative innovation including internal factors such as the will to change, the attitude to new things, the thinking model to overcome habitual domains,[17] and external factors, such as the information, data, team work, and the policy. Among them creative thinking model[18] and IT are the most important.[19]

Creative thinking mostly relies on individuals. It cannot be understood using a single simple model and it involves multiple complex processing operations. The operation of multiple processes, multiple strategies, and multiple knowledge structures makes it difficult to understand creative thinking process.[20] However, effective creativity execution depends on the knowledge available and the strategies people employ in executing these processes.[21]

The operation of multiple processes, multiple strategies, and multiple knowledge structures makes it difficult to formulate an understanding of innovation.[20] Declarative knowledge, factual, information, and cognitive schema are commonly held to be involved in most forms of creative thinking. Information and communication technology tools are likely to provide new innovation approaches and effective means to support such new innovation processes, such as classification algorithm selection in multiple criteria decision making.[22] The new approaches for innovation will find their wide application in industry.[23]

A lot of innovation methods make use of various approaches to stimulate innovation, such as individuals or group of experts/team members, along with numerous brain storming sessions involving both financial and human resources, and has severe dependence on personal intelligence; it would be subject to limitations of individuals themselves,[24] thereby keeping out of step from the rapidly changing information and knowledge environments.

To support innovation process, there are some tools that one can be deployed. There are eight core processes for the effective execution of innovation.[21] They are: (a) problem definition, (b) information gathering, (c) information organization, (d) conceptual combination, (e) idea generation, (f) idea evaluation, (g) implementation planning, and (h) solution monitoring. Effective execution of these processes, in turn, depends on people applying requisite strategies during process execution and having available requisite knowledge and it needs to be refined in detail for practical use.

TRIZ was developed to resolve contradictions in technological inventions, with a set of 40 inventive principles and later a matrix of contradictions which indicates 39 system factors.[25] It is useful in several specific fields, such as mechanics and electronics, but limited in other fields.

One main problem presented in many existing approaches is that they have applied a deductive approach by attempting to abstract common features from historical instances to obtain general rules for invention. Although this "expert system"-like approach is not without its merit, in reality, it is not realistic, because there are so many conditions and variables to match, and for each condition or variable, there are so many possible values to compare, so it may be computationally infeasible. On the other hand, although generative approach has been proposed by some authors, there has been a lack of effective ways of generating all possible innovative solutions.

Extenics focus on solving incompatible problems by formularized methods both in management and engineering. Zhou and Li[26] put forward an Extenics-based enterprise-independent innovation model and its implementation platform. Declarative knowledge, factual information, and cognitive schema are commonly involved in most forms of complex performance including innovation.[27] By integrating methodology knowledge and information, we need a theory to guide the generation of innovative solutions from Big Data.

Big Data is the next frontier for innovation, competition, and productivity[8]; it can help to better capture, understand, and meet customer needs.[28] Kou and Lou[29] proposed a hierarchical clustering method that combines multiple factors to identify clusters of web pages that can satisfy users' information needs. It is a very important source to knowledge[30,31] and other new discoveries.[32] But the question is: how to use Big Data to support collaborative innovation effectively? The data preparation process for innovation still remains a black box. Data is big enough but the methods to handle information and knowledge are very limited. Moreover, models listed above pay little attention to data analysis during innovation processes; the methods to score innovation solutions are mostly qualitative and we need more quantitative methods.

The Big Data and information is so huge that they are beyond human mind's processing capability. So it is time to implement collaborative innovation effectively in Big Data era. It is necessary to explore the high-efficiency models that would fill up the gaps with innovation process and new methodologies. We attempt to use Extenics to bridge the innovation process with data technology and management in this work.

3. Big Data Preparation for Collaborative Innovation

3.1. *Data collection based on Extenics*

Innovation process needs data and knowledge, both explicit and tacit. There are two main sources for collecting data: internal source, such as management information system, local database, tables or other forms of flat files, and external source, such as the internet, public databases, and data from other companies with similar goals.

There is huge quantity of data and information growing dramatically. How to choose the proper data set and process is a challenging problem.

Basic-element theory describe the matter (physical existence), event, and relations as the basic elements for all the information — "matter-element," "event-element," and "relation-element." Basic element is an ordered triad composed of the element name, the characteristics, and its measures, denoted by $R = (N, c, v)$ as matter-element, $I = (d, b, u)$ as event-element, and relation-element as $Q = (s, a, w)$.[16] As the matter-element $R = (N, c, v)$ is an ordered triad composed of matter, from its characteristics and measures, we can develop new concepts as the extensibilities of one of the three sub-elements in the triad.

Multiple characteristics are accompanied with multidimensional parametric matter-element and can be expressed as:

$$M(t) = \begin{bmatrix} O_m(t), & c_{m1}, & v_{m1}(t) \\ & c_{m2}, & v_{m2}(t) \\ & \vdots & \vdots \\ & c_{mn}, & v_{mn}(t) \end{bmatrix} = (O_m(t), C_m, V_m(t)).$$

A given matter has corresponding measure about any characteristic, which is unique at nonsimultaneous moments.

Further, characteristics of matters can be divided into materiality, systematicness, dynamism, and antagonism, which are generally called matters' conjugation. According to matters' conjugation, a matter consists of the imaginary and real, the soft and hard, the latent and apparent, and negative and positive parts,[16] which are explained below:

(1) *Non-physical part and physical part*
In terms of physical attribute of matter, all matters are composed of a physical part and a non-physical part. The former is referred to the real part of matter and the latter is referred to the *non-physical* or virtual part of matter. For example, a product's entity is its real part, while its brand and reputation are its *non-physical* parts. The empty space is a cup's *non-physical* part, while the ceramic cup itself is its *physical* part.

(2) *Soft part and hard part*
Considering a matter's structure in terms of the matter's systematic attribute, we define the matter's components as the hard part of matter, the relations between the matters and its components as the soft part of the matter. In the old saying, "Three heads are better than one," the three persons are the hard parts and the cooperation relationship is the soft part. A good soft part leads to a good result.

The matter's soft part has three types of relation: (1) relations between the matter's components; (2) relations between the matter and its subordinate matters, and (3) relations between the matter and other matters.

(3) *Latent part and apparent part*
Considering matter's dynamic property, we trust that any matter is changing. A disease has its latent period, a seed has its germination period, and an egg can hatch into chicken at a certain temperature after a certain time, and so on. The matter's latent parts and apparent parts exist synchronously.

The latent part of some matters may become apparent under certain conditions; for example, a student now in class may become a teacher after 10 years. There must be a criticality in the process of reciprocal transformation between latent parts and apparent parts.

(4) *Negative part and positive part*
In terms of antithetic properties of matters, all matters have two-part antithetic properties. The part producing the positive value to certain characteristic is defined as the positive part, and the part producing the negative value is defined as the negative part.

For example, in terms of profit, an employees' welfare department, a kindergarten, publicity departments, etc., have negative measure of profits, being the negative parts of the company, but these parts will improve employees' job enthusiasm and promote company's reputation, so they are the "advantageous" parts of the company.

Conjugate analysis and basic-element theory is a guide for us to collect data and information in a systematic way. Denote physical part as ph, non-physical part as nph, soft part as s, hard part as h, apparent part as a, latent part as l, positive part as p, negative part as n, matter-element as M, event-element as E, and relation-element as R. Using the notations, for example, *the physical part of relation* can be denoted as R_{ph}. Accordingly, we form a detailed data collecting list as showing in Table 1.[19]

Innovation activities have their goals and conditions. The purpose of data collection and processing is to solve problems about how to get from the conditions to the goals. Therefore, the data we collected should also be relative to the goals.

It can be seen from the above definition that there are three paths for the process of transformation between the element (such as "positive" and "negative"), field and criteria. Denote field as F, criteria as C, and element as EL, similarly, we denote goal as g, conditions as c, and pathways as pa. Accordingly, the data and information we collect will include such three paths.

Table 1. Data collection list based on basic-element theory and its conjugate analysis.

Basic-element type	Materiality		Systematicness		Dynamic		Antithetical	
	Physical part	Non-physical part	Soft part	Hard part	Apparent part	Latent part	Positive part	Negative part
Matter-Element	M_{ph}	M_{nph}	M_s	M_h	M_a	M_l	M_p	M_n
Event-Element	E_{ph}	E_{nph}	E_s	E_h	E_a	E_l	E_p	E_n
Relation-Element	R_{ph}	R_{nph}	R_s	R_h	R_a	R_l	R_p	R_n

Table 2. Data collection list from the view of the goal on certain business.

	Goals	Conditions	Pathways
Field	F_g	F_c	F_{pa}
Criteria	C_g	C_c	C_{pa}
Element	EL_g	EL_c	EL_{pa}

Goals and conditions can be matter, event, or relations between matters and events which can be represented with basic elements. So each cell in the Table 1 can be a basic element in next level of the information tree.

$$\text{Big Data} = F + C + \text{EL},$$
$$\text{EL} = \text{EL}_g + \text{EL}_c + \text{El}_{\text{pa}} = M + E + R,$$
$$M = M_{\text{ph}} + M_{\text{nph}} + M_s + M_h + M_a + M_l + M_p + M_n.$$

All the contents in Table 2 consist of Big Data of certain business. From Tables 1 and 2, we can get a systematic cube for collecting data and information for innovation.

3.2. *Data processing paths*

The Big Data preprocess chart is shown in Fig. 1.

There are two main paths located in four levels. One path is to extract data from the database, or use web crawler to collect information from the web, then transform and filter it into data mart, and finally use data mining to discover primary knowledge. Another way is to collect documents and build an information cube by human–computer interaction, then save it as basic-elements. Using extension

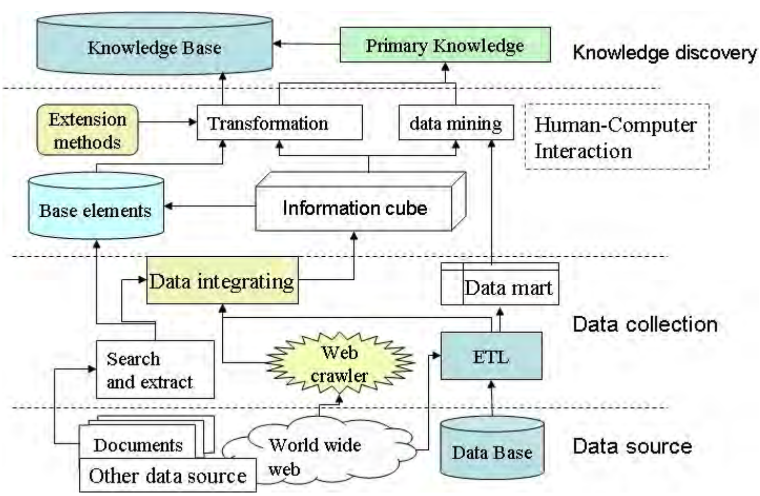


Fig. 1. Big Data collection and processing.

transformation methods, we transform the basic-elements into a knowledge base. Basic-elements and knowledge base will be the input of collaborative innovation model.

3.3. *Data transformation methods*

There are five basic transformation methods in Extenics, which can be used for information transformation by the change of a matter's object, attribute, or value.

(a) Substitution transformation
As to basic-element $B_0(t) = (O(t), c, v(t))$, if there is certain transformation T that transforms $B_0(t)$ to $B(t) = (O(t), c, v(t))$, i.e., $TB_0(t) = B(t)$, then the transformation T is referred to as substitution transformation of the basic-element $B_0(t)$.

(b) Increasing/decreasing transformations
An increasing transformation refers to the increase of certain attributes of the element. For example, as with matter-elements $M_0 = (\text{table A}_1, \text{height}, 0.8\,\text{m})$, $M = (\text{chair A}_2, \text{height}, 0.5\,\text{m})$, M is an increasable matter-element of M_0, we make $TM_0 = M_0 \oplus M = (\text{table A}_1 \oplus \text{chair A}_2, \text{height}, 1.3\,\text{m})$, then T is an increasing transformation of M_0.

A decreasing transformation refers to the decrease of certain attributes of the element. In the production process, the reduction of redundant action or work procedures belongs to the decreasing transformation of event-element, which can significantly improve production efficiency.

(c) Expansion/contraction transformations
Expansion transformation: Quantitative expansion transformation is multiple quantitative expansion of a basic-element. As for a matter-element, its quantitative expansion transformation will inevitably lead to expansion transformation of the matter. For example, the volume expansion of a balloon will inevitably lead to expansion of the balloon itself.

Contraction transformation: As for matter-element, its quantitative contraction transformation will inevitably lead to contraction transformation of the matter.

(d) Decomposition/combination transformations
Decomposition transformation refers to division of one object or attributes into several pieces. On the contrary, combination transformation refers to combination of several objects or attributes into a whole one. For example, one action can be executed in several steps.

(e) Duplication transformation
Duplication transformation refers to duplication of the basic-element to multiple basic-elements, such as photo-processing, copying, scanning, printing, optical disc burning, sound recording, video recording, the method of reuse, and reproduction of products, etc. This kind of transformation is extensively applied in the field of information, such as file copying and pasting.

Based on theory of extension set, knowledge from data mining can be mined in a second level by transformation methods, such as substitution transformation, decomposition or combination transformation, and so on. For example, decision trees can mine explainable rules, but it is only a static know-what knowledge and we may not know how to transfer class *bad* to class *good.* To improve such kind of situations, we focus on a new methodology for discovering actionable know-how knowledge based on decision tree rules and extension set theory. It is useful to re-mine rules from data mining so as to obtain actionable knowledge for wise decision making. The transformation knowledge acquiring solution on decision tree rules are practically used to reduce customer churn.[19]

4. Framework and Process of Extension Collaborative Innovation

4.1. *Framework of Extenics-based innovation*

The innovation method based on Big Data and Extenics would take advantage of specific extension methods to generate new innovative ideas or solutions. A framework is given in Fig. 2 and its relevant steps are listed as follows:

Step 1. *Multi-structure data collection*
Collect data related to the innovation goal^G and practical condition^L from data base, expertise, tacit knowledge such as experience and the web, blogs, etc., according to the method presented in Sec. 3.1.

Step 2. *Build basic-element base*
Describe and transform the information into matter-elements, event-elements, or relation-element. Meanwhile, primary knowledge is discovered from data mart by data mining. Then, we save them in the database as a basic-element tree supported by ontology. After this step, we could get a systematic cube of integrated information,[33] according to the method presented in Sec. 3.2.

Step 3. *Obtain all possible solutions by extension transformation*
Taking basic-elements and primary knowledge as input, extension transformation methods as methodology (as shown in Sec. 3.3), we transform the field, the elements, or the criteria of the goals and conditions on basic-elements that are already explored by Step 2. The detailed description to all possible solutions by human–computer interactions based on extension set theory will be presented in Sec. 4.2.

Step 4. *Scoring and evaluation by dependent function*
All the possible solutions are scored by superiority evaluation method based on dependent function quantitatively and expert's experience qualitatively. Then, results are acquired in feasible innovation proposals.

Suppose measuring indicator sets are $MI = \{MI_1, MI_2, \ldots, MI_n\}$, $MI_i = (c_i, V_i)$, $(i = 1, 2, \ldots, n)$, and weight coefficient distribution is

$$\alpha = (\alpha_1, \alpha_2, \ldots, \alpha_n).$$

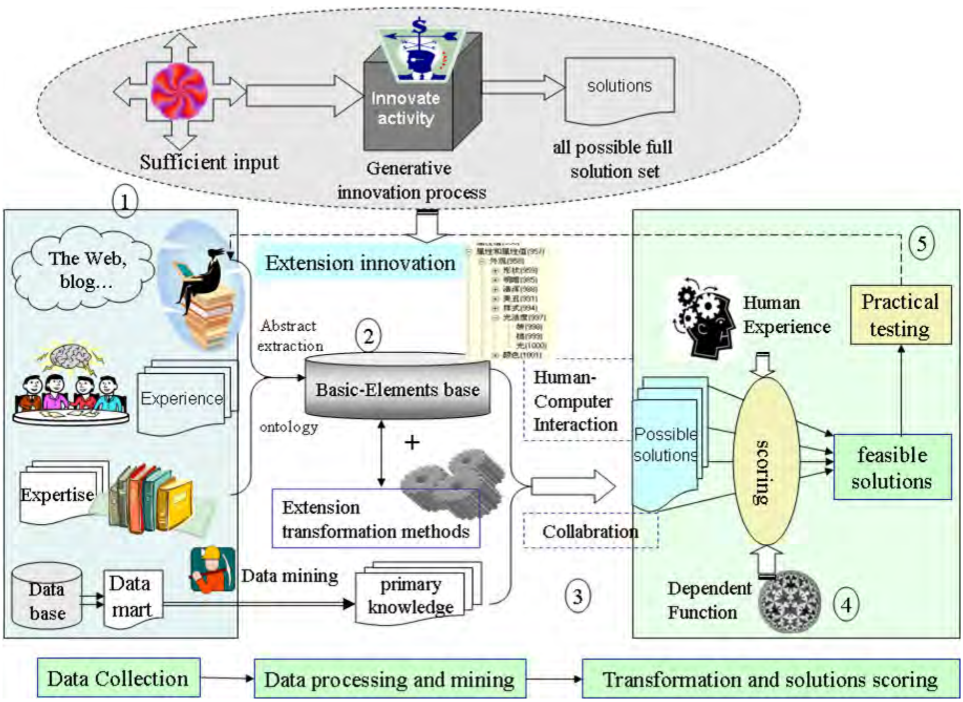


Fig. 2. Framework of extension collabrative innovation model in Big Data.

According to the requirements of every measuring indicator, dependent functions $K_1(x_1), K_2(x_2), \ldots, K_n(x_n)$ are established.

The dependent function value of object Z_j about each measuring indicator MI_i is denoted by $K_i(Z_j)$ for easy writing, and the dependent degree of every object $Z_1, Z_2, \ldots, Z_m$, about MI_i is

$$K_i = (K_i(Z_1), K_i(Z_2), \ldots, K_i(Z_m)), \quad (i = 1, 2, \ldots, n).$$

The above dependent degree is standardized as:

$$k_{ij} = \frac{K_i(Z_j)}{\max_{q\in\{1,2,\ldots,m\}}|K_i(Z_q)|}, \quad (i = 1, 2, \ldots, n; j = 1, 2, \ldots, m).$$

Then the standard dependent degree of every object $Z_1, Z_2, \ldots, Z_m$ about MI_i is

$$k_i = (k_{i1}, k_{i2}, \ldots, k_{im}), \quad (i = 1, 2, \ldots, n).$$

Step 5. *Practical testing and feedback*
We apply the feasible solutions to practices, collect new data and contrast the results, feedback to the model, and update basic-elements, knowledge base, or methods.

The above given framework of extension innovation model has been used by a reputable company in China and its case study is presented in Sec. 5.

4.2. *Directions of collaborative innovation*

All needed information and knowledge can be described as basic-element, we then take matter (one kind of basic-element) as an example. It has four characteristics and eight aspects as mentioned in Sec. 3.1. There are four main directions for innovation based on matter analysis as shown in Fig. 3.

(1) From the view of descriptions, we can extend and share our ideas from object, character, and its measure; each object has many characters, denoted as $1M:nC$. Similarly, each character can have many values, such as color can be red, green, yellow, or blue. We denote it as $1C:nV$.
(2) From the view of transformation path, we can extend our thinking from goals and conditions to both goals and conditions. Each path has many characters, measures, and objects.
(3) Based on basic extension methods, there are five methods that can be used on the matters as stated in Sec. 3.3.
(4) From the view of transformation objects, we can transform the elements, such as matter, event, or relationships, or transform the criteria and field. For example, a salesman F is regarded as good in company A, but scored as bad after he changed to company B. Here, the salesman F is an element, the rule *good* or *bad* is a criterion, and the companies A and B together are the field.

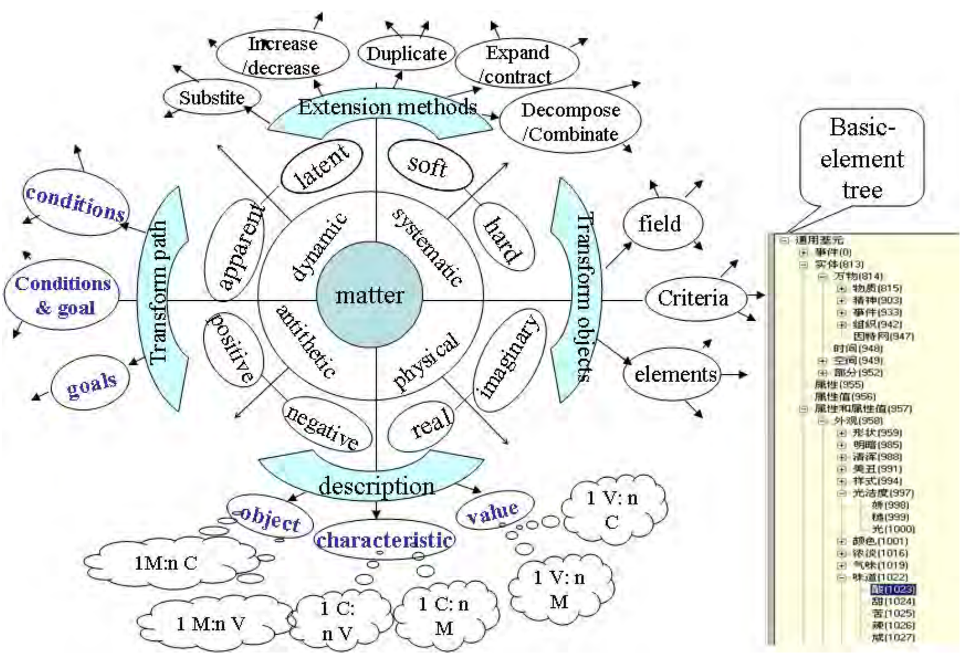


Fig. 3. Four main directions for collabrative innovation.

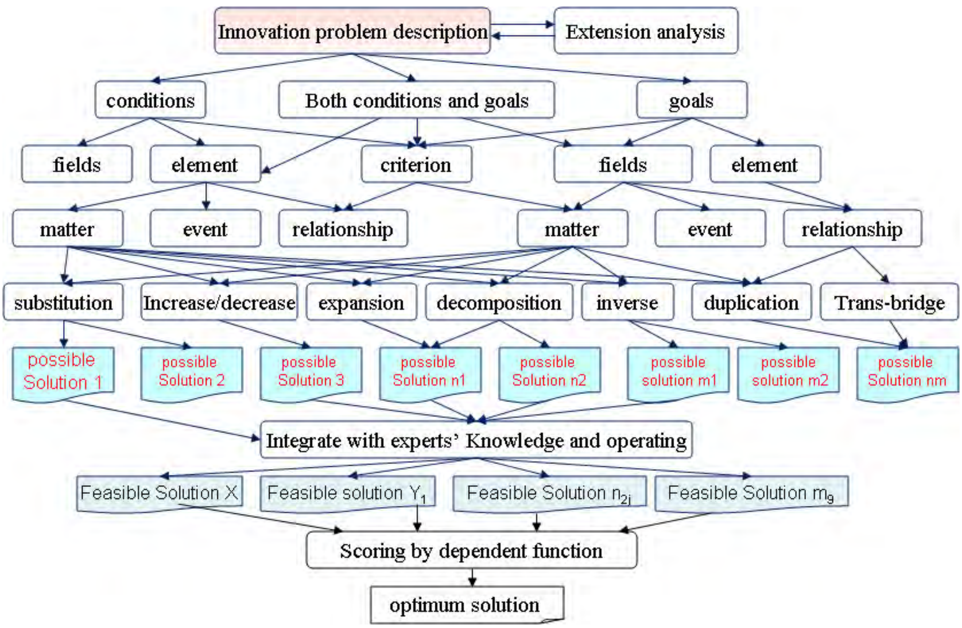


Fig. 4. Map of innovation solutions by extension transformation.

After four main transformations, we can get an information tree both for goals and conditions. Similarly, we can obtain an innovation solution tree. The solution tree is shown in Fig. 4.

4.3. *Scoring method based on Big Data*

Big Data give us a full view to score our innovation solutions. Take 2D as examples according to Extenics.[34]

Extension set theory is a set theory that describes the changing recognition and classification accordingly. Extension set describes the variability of things, using numbers in $(-\infty, +\infty)$ to describe the degrees of how the thing owns certain property, and using an extensible field to describe the reciprocal transformation between the "positive" and "negative" of things. It can describe not only the reciprocal transformation between the positive and negative of things but also the degree of how the thing owns a property.

4.3.1. *Definition of extension set*

Suppose U is universe of discourse, u is any one element in U, k is a mapping of U to the real field I, $T = (T_U, T_k, T_u)$ is a given transformation, we define

$$\tilde{E}(T) = \{(u, y, y') \,|\, u \in U, y = k(u) \in I; T_u u \in T_U U, y' = T_k k(T_u u) \in I\},$$

as an extension set on the universe of discourse $U, y = k(u)$, the dependent function of $\tilde{E}(T)$, and $y' = T_k k(T_u u)$ the extension function of $\tilde{E}(T)$, wherein, T_U, T_k, and T_u are transformations of respective universe of discourse U, dependent function k, and element u (Yang and Cai, 2013).

If $T \neq e$, we define

$\overset{E}{\cdot}_+(T) = \{(u, y, y') \,|\, u \in U, y = k(u) \leq 0; T_u u \in T_U U, y' = T_k k(T_u u) > 0\}$, as *positive extensible field of*$\tilde{E}(T)$; we define

$\overset{E}{\cdot}_-(T) = \{(u, y, y') \,|\, u \in U, y = k(u) \geq 0; T_u u \in T_U U, y' = T_k k(T_u u) < 0\}$, as *negative extensible field* of $\tilde{E}(T)$; we define

$E_+(T) = \{(u, y, y') \,|\, u \in U, y = k(u) > 0; T_u u \in T_U U, y' = T_k k(T_u u) > 0\}$, as *positive stable field* of $\tilde{E}(T)$; we define

$E_-(T) = \{(u, y, y') \,|\, u \in U, y = k(u) < 0; T_u u \in T_U U, y' = T_k k(T_u u) < 0\}$, as *negative stable field* of $\tilde{E}(T)$; and we define

$J_0(T) = \{(u, y, y') \,|\, u \in U, T_u u \in T_U U, y' = T_k k(T_u u) = 0\}$, as *extension boundary* of $\tilde{E}(T)$.

4.3.2. *Dependent function in 1D*

In 1983, Cai Wen defined the 1D-dependent function $K(y)$. Accordingly, if one considers two intervals, X_0 and X, that have no common end point, and $X_0 \subset X$, then:

$$K(y) = \frac{\rho(y, X)}{\rho(y, X) - \rho(y, X_0)}.$$

Since $K(y)$ was constructed in 1D in terms of the extension distance $\rho(.,.)$, we simply generalize it to higher dimensions by replacing $\rho(.,.)$ with the generalized $\rho(.,.)$ in a higher dimension.

4.3.3. *Extension distance in 2D*

Instead of considering a line segment AB as representing an interval $[a, b]$ in R, we consider a rectangle in R^2 enclosing the line such as $AMBN$, where AB is the diagonals of the rectangle. The mid-point of AB is now the center of symmetry O of the rectangle. Let $P(x_0, y_0)$ be a point outside of the rectangle, the coordinates of A be (a_1, a_2) and B be (b_1, b_2), and the point of intersection of the line joining P and O with the rectangle be P' be $(x_{p'}, y_{p'})$. The extension distance in 2D is denoted by $\rho(x_0, y_0)$ and is denoted by

$$\rho((x_0, y_0), AMBM) = |PO| - |P'O| = \pm|PP'|,$$

where $|PO|$ is the distance between P and O, $|P'O|$ is the distance between P' and O, and $|PP'|$ is the distance between P and P' as in coordinate geometry. The mid-point O has coordinates $(\frac{a_1+b_1}{2}, \frac{a_2+b_2}{2})$. Take $x_{P'} = a_1$, now we calculate $y_{P'}$ as

$$y_{P'} = y_0 + \frac{a_2 + b_2 - 2y_0}{a_1 + b_1 - 2x_0}(a_1 - x_0).$$

Therefore P' has the coordinates $P'\Big(x_{P'} = a_1, y_{P'} = y_0 + \frac{a_2+b_2-2y_0}{a_1+b_1-2x_0}(a_1 - x_0)\Big)$.

The distance $d(P, O) = |PO| = \sqrt{(x_0 - \frac{a_1+b_1}{2})^2 + (y_0 - \frac{a_2+b_2}{2})^2}$,
while the distance

$$d(P', O) = |P'O| = \sqrt{\left(a_1 - \frac{a_1 + b_1}{2}\right)^2 + \left(y_{P'} - \frac{a_2 + b_2}{2}\right)^2}$$
$$= \sqrt{\left(\frac{a_1 - b_1}{2}\right)^2 + \left(y_{P'} - \frac{a_2 + b_2}{2}\right)^2}.$$

Also, the distance $d(P, P') = |PP'| = \sqrt{(a_1 - x_0)^2 + (y_{P'} - y_0)^2}$.

Hence the extension 2D-distance formula:

$$\rho((x_0, y_0), AMBM) = P(x_0, y_0), A(a_1, a_2)MB(b_1, b_2)N) = |PO| - |P'O|$$
$$= \sqrt{\left(x_0 - \frac{a_1 + b_1}{2}\right)^2 + \left(y_0 - \frac{a_2 + b_2}{2}\right)^2}$$
$$- \sqrt{\left(\frac{a_1 - b_1}{2}\right)^2 + \left(y_{P'} - \frac{a_2 + b_2}{2}\right)^2}$$
$$= \pm|PP'|$$
$$= \pm\sqrt{(a_1 - x_0)^2 (y_{P'} - y_0)^2},$$

where $y_{P'} = y_0 + \frac{a_2+b_2-2y_0}{a_1+b_1-2x_0}(a_1 - x_0)$.

4.3.4. *Extension distance in n-D*

We can generalize Cai Wen's idea of the extension 1D set to an extension n-D set, and define the *extension n-D distance* between a point $P(x_1, x_2, \ldots, x_n)$ and the n-D set S as $\rho((x_1, x_2, \ldots, x_n), S)$ on the linear direction determined by the point P and the optimal point O (the line PO) in the following way:

(1) $\rho((x_1, x_2, \ldots, x_n), S) =$ the *negative distance* between P and the set frontier, if P is inside the set S;
(2) $\rho((x_1, x_2, \ldots, x_n), S) = 0$, if P lies on the frontier of the set S;
(3) $\rho((x_1, x_2, \ldots, x_n), S) =$ the *positive distance* between P and the set frontier, if P is outside the set.

We get the following properties:

(1) It is obvious from the above definition of the extension n-D distance between a point P in the universe of discourse and the extension n-D set S that:

  (i) Point $P(x_1, x_2, \ldots, x_n) \in$ Int(S) iff $\rho((x_1, x_2, \ldots, x_n), S) < 0$;
  (ii) Point $P(x_1, x_2, \ldots, x_n) \in$ Fr(S) iff $\rho((x_1, x_2, \ldots, x_n), S) = 0$;
  (iii) Point $P(x_1, x_2, \ldots, x_n) \notin$ S iff $\rho((x_1, x_2, \ldots, x_n), S) > 0$.

(2) Let S_1 and S_2 be two extension sets, in the universe of discourse U, such that they have no common end points, and $S_1 \subset S_2$. We assume they have the same optimal points $O_1 \equiv O_2 \equiv O$ located in their center of symmetry. Then, for any point $P(x_1, x_2, \ldots, x_n) \in U$ one has:

$$\rho((x_1, x_2, \ldots, x_n), S_1) \geq \rho((x_1, x_2, \ldots, x_n), S_2).$$

Then we proceed to the generalization of the dependent function from 1D space to n-D space dependent function, using the previous notations.

The dependent n-D function formula is:

$$K_{nD}(x_1, x_2, \ldots, x_n) = \frac{\rho((x_1, x_2, \ldots, x_n), S_2)}{\rho((x_1, x_2, \ldots, x_n), S_2) - \rho((x_1, x_2, \ldots, x_n), S_1)}.$$

5. Case Study

Y group is one of the world's largest menswear manufacturer group, with a production capacity of 80 million clothing items per year, includes shirts, suits, trousers, jackets, leisure coats, knitted items, and ties. Y has implemented world-class modern production lines and high-end equipment from nations including Germany, United States, and Japan. Y group's production and supply lines are using state-of-the-art comprehensive computer-operated technologies to enhance the clothing production process.

After 30 years of development, the group has many subsidiaries, including factories, sales companies, foreign trade corporation, and logistics companies. Y has forged a strong vertically integrated clothing chain, integrating the upstream components of textile and fabric production, midstream component of garment creation, and the downstream components of marketing and sales.

In recent years, the average annual sales increased by 10.1%, costs remained nearly unchanged at the previous year's level. However, the average profit of the group increased only by 0.21%, not significantly improved as shown in Table 3. To find this reason, our data analysis team worked together with the group's financial sector.

The profit of Y group is the sum of its subsidiaries and is denoted as

$$P_g = \sum_{j=1}^{n} P_j = P_f + P_s + P_l + P_t + \cdots, \quad P_g = I_{\text{income}} - C_{\text{cost}}.$$

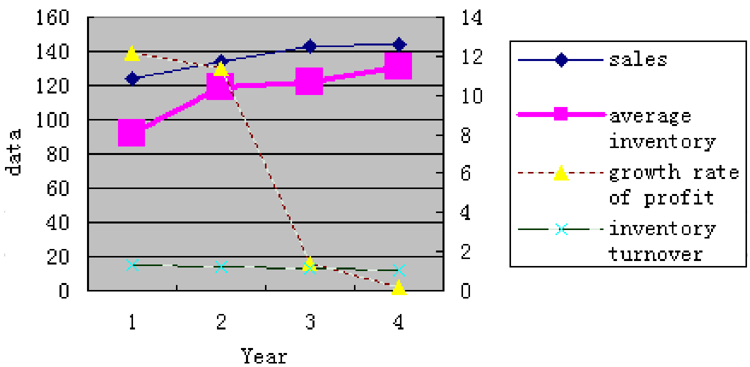
Accordingly, we made a basic-element analysis on profit-related attributes, as shown in Fig. 5.

According to the basic-element analysis, we determine the scope of the collection of Big Data as following:

(1) Total sales, inventory, and profit and cost data of the company for the past five years.

Table 3. Problem seeking on data integratation and analysis.

Items	Year 1	Year 2	Year 3	Year 4
Sales	123.9563	133.6302	143.0152	143.6497
Average inventory	91.6006	118.8934	122.4309	131.1287
Growth rate of profit	12.12	11.38	1.44	0.21
Inventory turnover	1.35	1.2	1.17	1.02



(2) Historical production records of the production plant, raw material records, employee payroll (removal of sensitive personal information), production schedules, storage records, and so on.
(3) Historical distribution data in logistics department, sales returns data, and loss of productions in self-run stores.
(4) Retail sales records in the stores and shops, records of group purchase, discounts records, and inventory balances.

We then integrate data in data warehouse and find information such as which are best-selling products, the highest inventory of suits, inventory turnover days, inventory days of supply and other indicators available as shown in Table 4.

Through analysis of various types of data in the group's departments, we found that:

(1) The main source of profit of the garment sector is self-store sales and group purchase business confirmed by group financial center.
(2) The number of inventory days of supply (similar with inventory turnover) is worse. In some areas, it is up to 536 days, which means that according to the current average sales, inventory of goods available for sale will be 536 days as shown in Fig. 6.
(3) In production scheduling, small orders that are temporarily postponed are as high as 31%, and those into priority processing sequences are mostly original equipment manufacturer (OEM) orders.

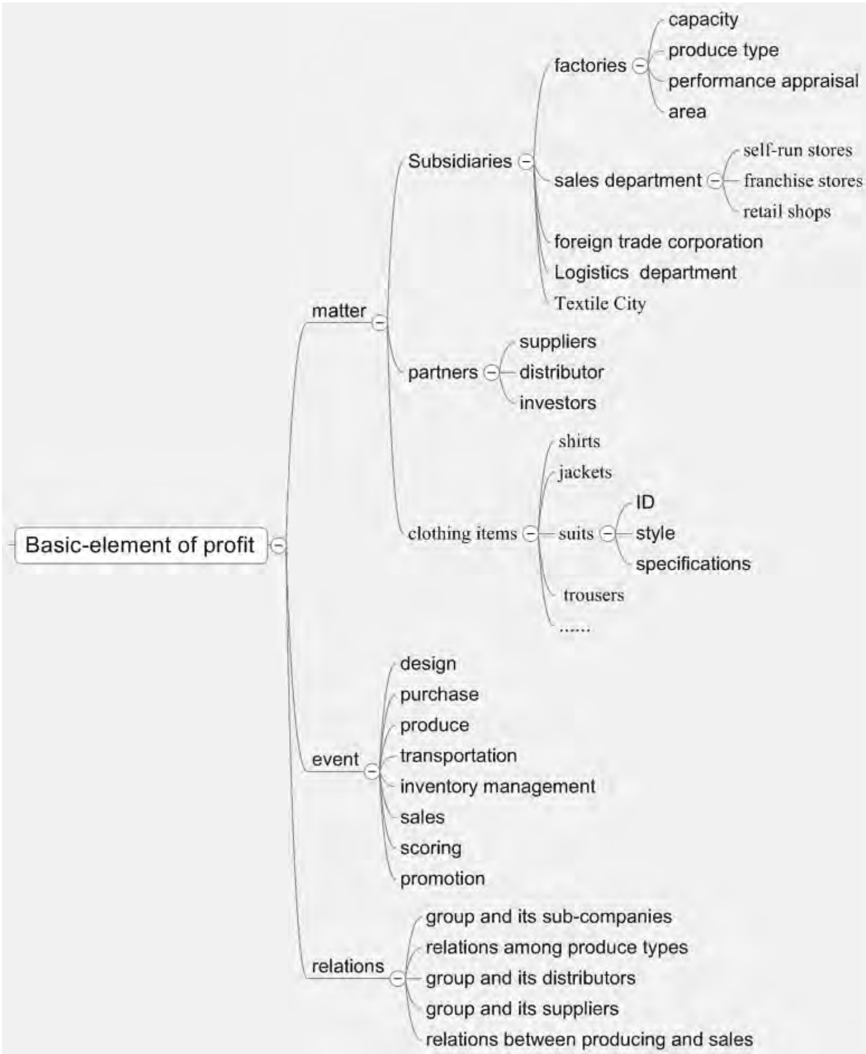


Fig. 5. The basic-element of profit analysis.

(4) In production plant, the timely delivery rates of group purchase orders have continued to decline, far below than that of the OEM production. OEM orders are usually in large quantities and the proportion in the profits of garment plants is also growing.

Based on extension set theory, we further analyzed the domain and the associated rules. After the analysis of the data, we found that:

(1) Profit is a main key performance indicator (KPI) for the appraisal of subordinate units of the group; in some garment plants annual profit targets are completed

Table 4. Sales analysis and indicators calculation.

Product ID	Sum of sales	Monthly average inventory	Days of inventory turnover	Rate of sales and delivery (%)
Y01V88417	566,566	749,788	40	43.41
Y01T8401A	272,015	272,390	30	48.38
Y01W72052	121,260	212,850	53	34.11
Y01W72059G	118,422	130,806	33	36.43
Y01V88488	110,160	61,560	17	94.44
Y01V88436	108,416	162,118	45	38.23
YC01SCT84396	94,770	42,200	13	99.93
Y01FV88417	84,798	96,788	34	50.76
Y01CA100744	83,448	104,310	38	48.09
Y01CA886112	81,404	53,298	20	87.83
Y01W72050	70,176	146,415	63	23.82
Y01W72053	67,338	192,210	86	23.43
YC01CW72048	67,250	103,125	46	31.72
YC01CW72045	65,750	168,875	77	27.80
Y01V5000	61,600	27,425	13	82.74
Y01T48506G	57,986	136,196	70	32.36
Y01CA88690	56,588	95,504	51	36.80
YC01CW72041	55,250	117,625	64	29.00
Y01T50	54,300	51,525	28	56.59
Y01CA88688	53,016	94,376	53	34.52
Y01T48505G	52,140	83,108	48	42.42
YC01CW72044	52,000	166,000	96	21.16

年 5月 区域主管 省份 大类 品牌 库存周转天数

companyID	衬衫 shirt	西服 suits	裤子 trousers	茄克 jackets	针织T恤 T shirts	领带 ties	皮制品 leathers	毛衫 sweaters	大类 average
name of managers of all sales company	168	287	110	153	157	240	289	266	187
	210	182	338	2,520	562	784	1,426	443	238
	143	537	188	510	446	826	667	1,278	262
	324	236	338	473	354	511	272	90	269
	158	353	345	1,191	2,969	1,087	222	1,462	270
	194	240	496	1,409	377	1,725	1,309	207	274
	212	237	449	302	798	844	1,341	440	285
	267	295	233	1,120	360	1,638	586	113	285
	251	295	290	331	490	246	936	196	288
	345	235	416	419	364	930	813	447	302
	233	334	509	188	921	511	2,602	1,403	321
	357	338	433	1,288	316	1,059	915	1,437	381
	279	426	593	520	830	683	2,355	339	384
	594	293	541	941	892	1,662	507	2,631	410
	279	450	481	4,864	453	1,712	936	773	436
	366	528	379	1,077	444	625	517	73	440
	311	518	413	524	493	1,408	2,100	863	443
	336	586	1,032	3,123	1,550	1,843	2,058	778	534
managers	565	421	902	1,731	1,008	358	5,071	611	536
域主管	366	444	473	765	558	1,006	1,728	505	448

Fig. 6. Inventory analysis by sales.

more than 95% and the average annual profit growth is of 3.87% — one of the best performance units.

(2) In some sales companies, sales increase at an average rate of 9.01%, while the inventory grows at a rate of 14.2%, and whereas the profit growth rate is only 0.53%.

Based on basic-element theory of Extenics, we have collected attributes information related to garment plants:

(1) The produce types are divided into three categories:
   - Make-to-stock production, for the production of inventory for all kinds of self-run stores sales;
   - Make-to-order production, especially for group purchase;
   - OEM. The plants manufacture clothing that are purchased by another international company and retailed under that purchasing company's brand name.

(2) The OEM processing fee is on average 2–4 Yuan higher than the make-to-order processing fee.

(3) By deep inquiry with managers, we found that in the production plant, the processing priority rules are: high profit, large quantities of orders will have priority processing, and OEM orders meet these two conditions.

Integrating the knowledge mined from the Big Data and information basic elements, we draw a knowledge logic chain as follows:

Profit is the primary KPIs of production plants, therefore managers give the top priority to process the high-profit orders, and since OEM orders are latent with high profits, they process OEM orders by priority, and since OEM orders are large and production capacity is limited, make-to-order productions are postponed.

It is reasonable to process high-profits OEM orders from the perspective of the garment plants, but it is not understandable from the group's perspective; OEM processing fee is so small that it is almost negligible compared to the profit of a group buy ($\leq 5\%$); to earn \$4, they lost \$80.00. One of the reasons for the cause of this "penny wise, pound foolish" phenomenon is the improper setup of profit as KPI for garment plants.

By Big Data analysis and extension model, we selected and added two new KPIs defined as *ratio of timely delivery of order* and *operation costs* to replace with single KPI profit. Therefore, we score the garment plants by 3D-dependent functions. If we only use profit as KPI, SH plants ranked A; however, if we use three KPIs and score with 3D-dependent functions, SH will be ranked C. This greatly helped the managers: the production capacity has been fully utilized in garment plants, timely delivery rate of group-buy-order is greatly improved, and OEM orders delivery rate is almost keep the same, while the group's overall profit improved from 0.21% to 12.6%.

Last year, Y group's total assets are valued at 30 billion RMB. Its annual textile and garment sales alone amount to 10 billion RMB. Y was ranked number 113 in China's list of its top 500 manufacturers and also is the only clothing company that China has recognized as one of its "Advanced Manufacturers."

6. Summary

The paper presents a framework of extension collaborative innovation model in the context of Big Data with concrete processes and a case study. The model integrates Extenics, data mining, and knowledge management, and develops a framework for collaborative innovation with team work. By collecting knowledge or information from multiple resources among all departments, we can build information tree in basic elements from various forms of data. Knowledge or information related to the problem can find relations by human–computer interaction method.

This particular method combines qualitative analysis which would take advantage of personal intelligence after formalized expression of innovation problems and quantitative analysis which follows a systematic flow based on accumulated knowledge or innovation patterns. It helps to solve management problems according to the extensibility of basic element and was applied in the innovation of management beyond data technology such as data mining and intelligent knowledge management.

By case study, we found that Extenics can serve as the starting point of a generative approach for collaborative innovation, because it focuses on solving non-compatible problems by formularized methods (Yang and Cai, 2013). The main features of using Extenics for collaborative innovation in big data can be outlined as follows:

- Extenics provides a system structure for data collection and processing. Then, it offers an opportunity for generating all possible solutions for innovative problem solving.
- In addition to generating possible solutions, Extenics also offers an effective way of evaluating solutions, so that any solutions failing the evaluation will be eliminated from further consideration. This can prevent computational explosion.
- The Extenics-based approach for innovation is a human–computer interactive process: Computers can conduct scalable data storage and mining by making use of various algorithms, far superior than what human labors can handle. Nevertheless, the real world is extremely complex and only humans can capture the dynamics of the real world beyond what any algorithms can handle.

From these general features, we provided an overview on what Extenics can offer for collaborative innovation. First, we had put forward a data collection theory in Sec. 3 to acquire sufficient input from Big Data to generate ideas, from which innovation process can take place in a systematic way, instead of by chance. Second, we presented a combined model to process Big Data — one way is to build basic-element tree by human, another way is to build knowledge base by computer such as data mining, extension transformation connect and help them to generate possible solutions in all directions by a formalized method. Last, we extend the dependent function to score multi-attribute innovation solutions in the context of Big Data. By this model, we can obtain novel ideas from several ordinary rules mined from multi-data source.

In the future, we will further test the n-D-dependent function and compare it with other methods. Moreover, the basic-element base and knowledge collaborative methods need to be integrated with agent-based system and enhance the extension innovation model. Due to the significant importance of Big Data, deep research about combination of above methods with web IT, extension data mining, and intelligent knowledge management need to be further explored. How to update basic-element base automatically and simulate the knowledge innovation process by intelligent agent is a challenging problem.

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EXTENSION HYBRID FORCE-POSITION ROBOT CONTROL IN HIGHER DIMENSIONS

VICTOR VLADAREANU, FLORENTIN SMARANDACHE, LUIGE VLADAREANU

Abstract. The paper presents an advanced method for solving contradictory problems of hybrid position-force control of the movement of walking robots by applying a 2D Extension Set. Using the linear and non-linear attraction point principle and the network of attraction curves, there is determined the 2D space Dependent Function generated by position and force in order to solve the robot real time control. The generalization of the extension distance and dependent function uses Extenics in Higher Dimensions theory eliminates the crisp logic matrix of Cantor logic which describes the position-force sequences. Thus was developed an optimization method for hybrid position-force control which ensures positioning precision and robot movement stability on rough terrain. The final conclusions lead to development of a methodology that allows obtaining high level results for hybrid position-force control using extended transformations onto the real numbers set and an optimization function generated by the extended dependence function in 2 D space.

Keywords: extenics in higher dimensions, hybrid force-position control, linear and non-linear attraction point principle, the network of attraction curves

Introduction

A safe and robust behaviour of robots and mechatronic systems in contact with objects in their environment is the basic requirement for accomplishing tasks according to the given application. Stable control of the object – robot interaction implies a difficult technical problem. Thus, for contact control a simple method called „position adaptation" is proposed by Whitney (1977) in which the contact force is used to modify the trajectory of the reference position of the robot's end-effector. Control of the arching movement, which in essence is force control implicitly based on position, was suggested by Lawrence and Stoughton (1987) and Kazerooni, Waibel and Kim (1990). Salisbury (1980) presented an active control method of apparent rigidity of the robot end-effector in Cartesian space. In this method the reference position used as input to control the contact force and no reference point for force are used. Raibert, Craig (1981) and Manson (1980) ensure force and position control when the robot interacts with the environment by decomposing it into „position sub-space" and „force sub-space". These two sub-spaces correspond to the robot's movement directions, respectively free movement or constrained environment.

There is a growing interest to this problem, based on the research done by Pelletier and Daneshmend (1990), Lacky and Hsia (1991) and Chan (1991). Pelletier and Daneshmend present a blueprint for the adaptive control device which would be used to compensate for the variations of the environment rigidity during movement. However, they then discovered this is subject to instabilities. Lasky and Hsia describe a control device system consisting of a conventional impedance control device in the inner loop and a trajectory alteration control device in the outer loop for tracking the force, but their proposal is based on the science and calculation of the manipulative dynamic model. Chan develops a control device with variable structure for impedance control under parameter uncertainty and outside distrubance, but this strategy requires exact knowledge of the location and rigidity of the environment in order to obtain good force control.

Extenics was developed by Cai Wen in 1983 and developed succesively, with a major impact in the scientific world of the last few years through results in e-learnig, data mining, image recognition, robotics, statistics and management research, among others [1, 2, 7, 10]. Extension set theory is a mathematical form for representing uncertainty which is an extension of classic set theory, with applications in many research fields [12-14]. Extenics is a field of study which aims to solve contradictory problems, as is the case of position – force control in the field of robotics, mechatronics and real time control.

Architecure of the Explicit Position-Force Control System.

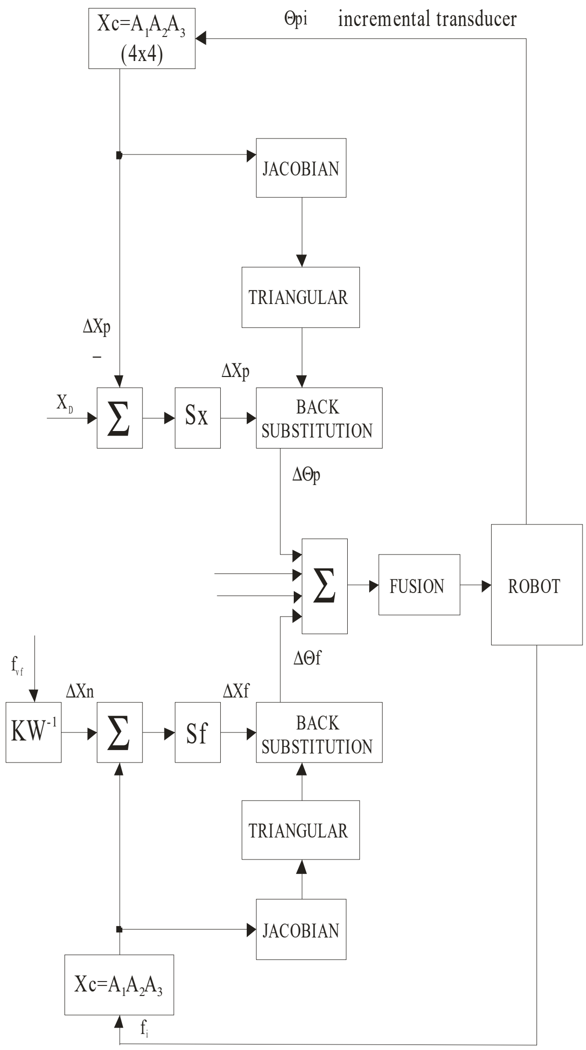

Fig.1. Architecure of the Position-Force Control System.

A hybrid position-force control system normally achieves simultaneous control of position and force. In order to determine the control relations in this situation, one divides the ΔX_P deviation measured by the command system into Cartesian coordinates into two sets: ΔX^F - corresponding to the force controlled component and ΔX^P - corresponding to position control with actuation on the axis, in accordance with the selection matrixes S_f and S_x [5, 6]. Through this approach certain Cartesian coordinates of the robot end-effector are controlled in position while others are controlled explicitly in force.

The separate processing using separate laws for position and force control requires significant preparation of the treatment of tasks and changes in the implementation of the control loops; additionaly, this method may generate instability problems, especially during the transition of free and constrained movement [8, 9].

There results the motion variation on the robot axis in relation to the end-effector motion variation from the relation: $\Delta q = J^{-1}(q)\ \Delta X_F + J^{-}1(q)\ \Delta X_P$, where $\mathbf{\Delta X_F}$ can be calculated from the relation: $\Delta X_F = K_F\ (\Delta X^F - \Delta X_D)$, and K_F is the dimensional relation of the stiffness matrix. Noting $\mathbf{F_D}$ as the desired residual force and $\mathbf{K_W}$ the physical stiffness the following relation is obtained: $\Delta X_D = K_W^{-1} * F_D$.

The architecture of the hybrid position – force control system of robots with six degrees of freedom based on the Denevit – Hartenberg transformations is presented in Figure 1. The device sensors are used in two ways. In position control, the information obtained from the sensors is used to compensate the deviation of the robots' joints, due to the load created by external forces, so that the apparent stiffness of the robot's joint system is emphasised [11]. In force control, the joint is used as a force sensor, so that the manipulator is led in the same direction as the force received from the sensors, allowing the desired contact force to be maintained.

Extension Hybrid Force-Position Robot Control

Extenics is the science that deals with solving contradictory problems. In this paper the aim is to solve the contradictory problem of hybrid position – force control of the movement of robotic and mechatronics systems by replacing the logic 0 and 1 values in the selection matrices Sx and Sf, pertaining to force – position sequences using Cantor logic, with values of the dependent function using extended distance. Moreover, through a domain extension transformation for position S_{Kx}, respectively a domain extension transformation for force S_{Kf}, a new selection matrix with correlation coefficients for position, respectively one force force, are obtained, which allows for the determination of the position error signals ε_{Kx} and force error signals ε_{Kf} with the aim of closing the control system loops in position and force.

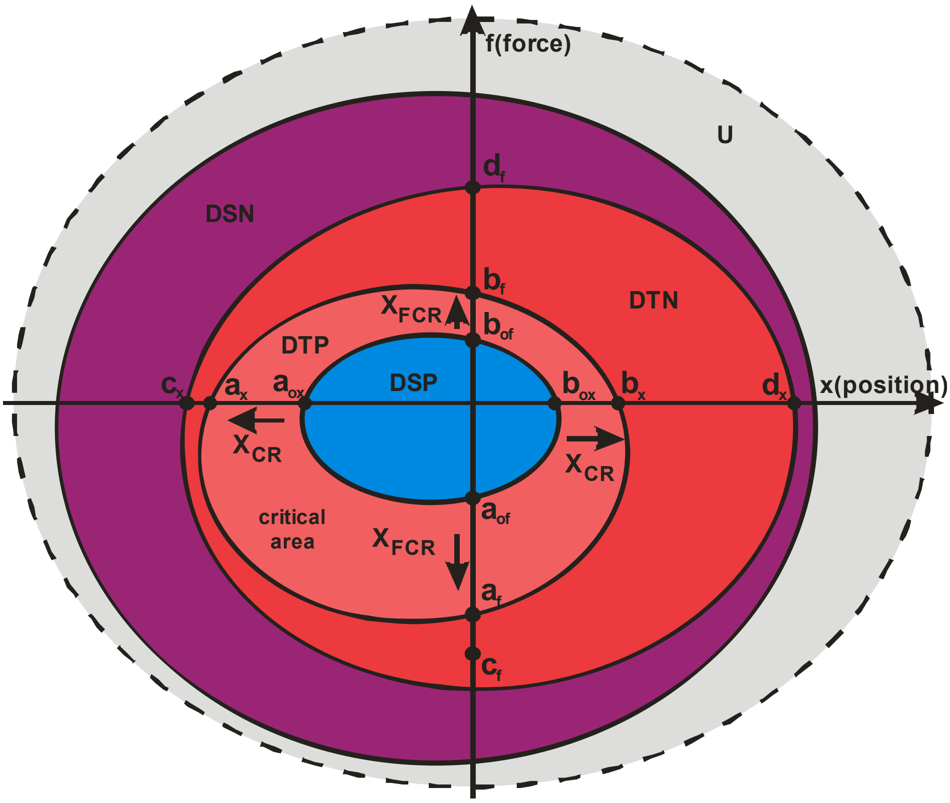

Fig. 2. Subdomains of the universe of discourse in reference to the position and force errors

In a first stage that is offline, the universe of discourse U is defined for the position and force errors presented in Figure 2. Thus, the standard positive domain DSP is defined as the projection on the **x**-axis of the acceptable position error for the reference $\mathbf{X_o}$ of the movement of the robot system, nestled on the **x**-axis in the interval (a_{ox}, b_{ox}), where a_{ox} and b_{ox} are the negative and positive maximum acceptable position errors, respectively the projection on the **f**-axis for the force reference of the movement $\mathbf{X_{Fo}}$, included in the interval (a_{of}, b_{of}), where a_{of} and b_{of} are the maximum accepted positive and negative force errors [15].

There results a positive transition domain DTP which projects onto the **x**-axis as the positive transitive position interval $\mathbf{X_{CR}}$ and onto the **f**-axis as the positive transitive force interval $\mathbf{X_{FCR}}$. The interval $\mathbf{X_{CR}}$ corresponds to the critical error in position in which it is still possible to control the position movement of the robotic system in order to bring the position error into the standard positive domain. The **x**-axis projection is limited by the interval (a_x, b_x), where a_x and b_x are the positive, respectively negative maximum accepted critical errors. The interval $\mathbf{X_{FCR}}$ corresponds to the critical error in force in which force control of the movement is still possible with the aim of bringing it into the standard positive domain and is limited by the interval (a_f, b_f), where a_f and b_f are the positive, respectively negative maximum acceptable critical force errors.

A negative transition domain DTN which is continued with the standard negative domain DSN to compose the universe of discourse U corresponds to unacceptable errors in position from which the position control of the robotic and mechatronic system SRM cannot recover to the standard positive domain, respectively unacceptable errors in force from which the force control cannot recover to the standard positive domain. Assigning values from this field would lead to saturation of the position or force reaction loop, with all negative consequences thereafter. The projection on the x-axis is nestled on the x-axis in the interval (c_x, d_x) where c_x and d_x are the negative and positive maximum unacceptable position errors, respectively the projection on the f-axis is included in the interval (c_f, d_f) where c_f and d_f are the maximum unacceptable positive and negative force errors.

A standard negative domain DSN which completes the univers of discours U and corresponds to unacceptable errors in position and force. The universe of discourse U is composed of the sum of the domains presented before.

Having the defined domains of the universe of discourse, Linear and Non-Linear Attraction Point Principle is applied to the definition of metrics with the aim of determining the Linear (or Non-Linear) Dependent *n-D*-Function of point along the curve *c* and the Extension *n-D*-Distance between a point *P(x1, x2, ..., xn)* and the *n-D*-set S as ρ*((x1, x2, ..., xn),*S*)*.

Linear and Non-Linear Attraction Point Principle

Optimal position-force control of a robot implies the tendency, in a linear or non-linear universe of discourse, towards an attraction point which ensures maximum performance.

The Linear and Non-Linear Attraction Point Principle is the following [3]:

Let S be an arbitrary set in the universe of discourse U of any dimension, and the optimal point O€ S . Then each point *P(x1, x2, ..., xn), n ≥ 1,* from the universe of discourse (linearly or non-linearly) tends towards, or is attracted by, the optimal point *O*, because the optimal point *O* is an ideal of each point. There could be one or more linearly or non-linearly trajectories (curves) that the same point *P* may converge on towards *O*. Let's call all such points' trajectories as the Network of Attraction Curves **(NAC)**.

It is a kind of convergence/attraction of each point towards the optimal point. There are classes of examples and applications where such attraction point principle may apply.

If this principle is good in all cases, then there is no need to take into consideration the center of symmetry of the set S, since for example if one has a *2D* factory piece which has heterogeneous material density, then its center of weight (barycenter) is different from the center of symmetry.

Starting from the one-dimensional extension theory of Cai Wen, generalizations were necessary for n-dimensional spaces in order to define a measurable space and n-D extension distance with the aim of applying domain extension transformations in position S_{Kx} and force S_{Kf} respectively, which would lead to solving the contradiction in robot hybrid force – position control.

1. We generalized in the track of Cai Wen's idea the extension *1D*-set to an extension *n-D*-set, and defined the Linear (or Non-Linear) **Extension *n-D*-Distance** between a point *P(x1, x2, ..., xn)* and the *n-D*-set S as ρ*((x1, x2, ..., xn),* S *)* on the linear (or non-linear) direction determined by the point *P* and the optimal point *O* (the line *PO,* or respectively the curvilinear *PO)* in the following way:

a) ρ*((x1, x2, ..., xn),* S *)* = the *negative distance* between *P* and the set frontier, if *P* is inside the set S;

b) ρ*((x1, x2, ..., xn),* S *) = 0*, if *P* lies on the frontier of the set S;

c) ρ*((x1, x2, ..., xn),* S *)* = the *positive distance* between *P* and the set frontier, if *P* is outside the set.

2. The **Linear (**or **Non-Linear) Dependent *n-D*-Function** of point *P(x1, x2, ..., xn)* along the curve *c*, is:

$$K_{nD}(x_1, x_2, \ldots, x_n) = \frac{\rho_c((x_1, x_2, \ldots, x_n), S_2)}{\rho_c((x_1, x_2, \ldots, x_n), S_2) - \rho_c((x_1, x_2, \ldots, x_n), S_1)} \quad (1)$$

which has the following **property**:

a) If point *P(x1, x2, ..., xn)* €Int(S1), then *KnD*(*x*1, *x*2,..., *xn*) > 1;

b) If point *P(x1, x2, ..., xn)* €Fr(S1), then *KnD*(*x*1, *x*2,..., *xn*) = 1;

c) If point *P(x1, x2, ..., xn)* €Int(S2-S1), then *KnD*(*x*1, *x*2,..., *xn*) € (0, 1);
d) If point *P(x1, x2, ..., xn)* €Int(S2), then *KnD*(*x*1, *x*2,..., *xn*) = 0;
e) If point *P(x1, x2, ..., xn)* €Int(S2), then *KnD*(*x*1, *x*2,..., *xn*) < 0.
Let's see in figure 3 such example in the *2D*-space.

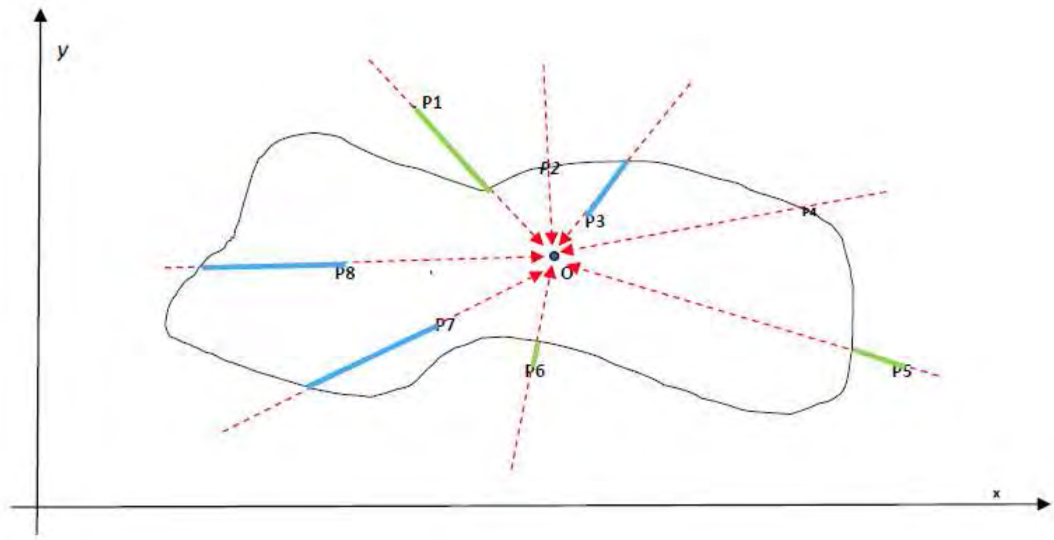

Fig. 3. The optimal point O as an attraction point for all other points P1, P2, …, P8 in the universe of discourse R2

In general, in a universe of discourse *U*, let's have an *n-D*-set S and a point *P*.
Then the **Extension Linear *n-D*-Distance** between point *P* and set S, is:

$$\rho(P,S)=\begin{cases}-\underset{P'\in Fr(S)}{d(P,P')}, & P\neq O, P\in|OP'|;\\ \underset{P'\in Fr(S)}{d(P,P')}, & P\neq O, P'\in|OP|;\\ -\underset{M\in Fr(S)}{\max}\, d(P,M), & P=O.\end{cases} \tag{2}$$

where *O* is the optimal point (or linearly attraction point); *d(P,P')* means the classical linearly *n-D*-distance between two points *P* and *P'*; *Fr(*S*)* means the frontier of set S; and |*OP'*| means the line segment between the points *O* and *P'* (the extremity points *O* and *P'* included), therefore *P*€|*OP'*| means that *P* lies on the line *OP'*, in between the points *O* and *P'*.

For *P* coinciding with *O*, one defined the distance between the optimal point *O* and the set S as the negatively maximum distance (to be in concordance with the *1D*-definition).

The **Extension Non-Linear *n-D*-Distance** between point *P* and set S, is:

$$\rho_c(P,S)=\begin{cases}-\underset{P'\in Fr(S)}{d_c(P,P')}, & P\neq O, P\in c(OP');\\ \underset{P'\in Fr(S)}{d_c(P,P')}, & P\neq O, P'\in c(OP);\\ -\underset{M\in Fr(S), M\in c(O)}{\max}\, d_c(P,M), & P=O.\end{cases} \tag{3}$$

where ρ *c*(*P*, *S*)means the extension distance as measured along the curve *c*; *O* is the optimal point (or non-linearly attraction point); the points are attracting by the optimal point on trajectories described by an injective curve *c*; *dc(P,P')* means the non-linearly *n-D*-distance between two points *P* and *P',* or the arclength of the curve *c* between the points *P* and *P'*;
*Fr(*S*)* means the frontier of set S; and c(*OP')* means the curve segment between the points *O* and *P'* (the extremity points *O* and *P'* included), therefore *P€c(OP')* means that *P* lies on the curve *c* in between the points *O* and *P'*.

For *P* coinciding with *O*, one defined the distance between the optimal point *O* and the set S as the negatively maximum curvilinear distance (to be in concordance with the *1D*-definition).

The System Architecture for the Extension Hybrid Force-Position Robot Control.

We intend to replace the S_x and S_f matrices which contain crisp logic values with K_x and K_f which contain values of the dependent function of the position error K_x and the force error K_f with respect to the standard positive field. Thus, considering the universe of discourse in figure 2 we will replace the crisp logic values 1 in matrices S_x and S_f with the K_x and K_f coefficients which result from the error positioning corresponding to the standard positive field [15].

The architecture of the Extension Position-Force Control System, presented in Figure 4, consists of a series of modules whose aim is to solve the contradictory problem of hybrid position – force control for the movement of robotic and mechatronic systems. This is obtained conceptually by replacing the 0 and 1 logic values from the selection matrices Sx and Sf, depending on the position-force sequences in Cantor logic, with values of the dependent function using extension distance. This is followed by a domain extension transformation for position S_{Kx}, respectively for force S_{Kf}, which generates a new selection matrix with correlation coefficients for position and force respectively.

Thus, a module which calculates the position extension distance CDEP, receives the current position signal X processed by a Carthesian coordinate calculation module CCC through direct cinematic, of the robotic and mechatronic system SRM and in reference to the standard positive interval of the position reference X_o, defined experimentally, calculates the position extension distance $\rho(X,X_o)$, which it sends to the module calculating the position dependence function CFDP. The extension position distance $\rho(X,X_o)$, according to extension theory, is calculated as the distance from a point, in this case the current position signal X, to an interval, in this case the standard positive interval for reference position X_o. Similarly, the data for calculating the force extension distance is calculated by the CDEF module, which works quasi-simultaneously with the CDEP module which calculates the extension position distance.

The position dependent function $K(X, X_o, X_{CR})$ of the current position signal X in relation to the standard positive interval of the reference position X_o and the transitive positive interval for position $\mathbf{X_{CR}}$ is determined according to extension theory in the CFDP module. This has the maximum value of $K(X_o)=M_P$ equal to the proportional amplification component of the position controller on the standard positive interval for the reference position X_o. Moreover, in order to not saturate the position loop, the dependent function for position $K(X, X_o, X_{CR})$ has a lower limit of 0 if the current position signal X is within the intervals X_o and $\mathbf{X_{CR}}$.

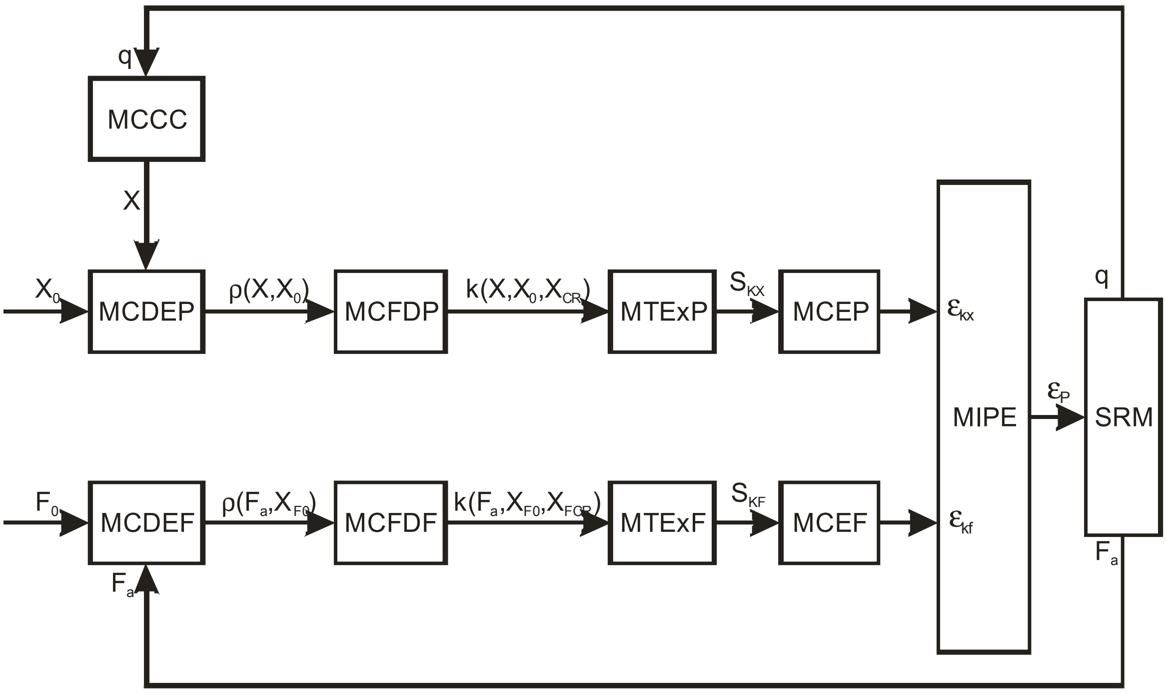

Fig. 4. Architecture of the extension hybrid force-position control system

Dependent function k(x), k(f) to enable the calculation of correlation do not have to rely on subjective judgments or statistics, it can quantitatively and objectively describe the elements having the nature or character at a certain extent and the process of quantitative change and qualitative change. This allows correlation function out of bias caused by subjective judgments.

Appling equations (1)-(3) for 2D, in a universe of discourse *U*, let's have a nest of two *n-D*-sets, S*1* ⊂ S*2*, with no common end points, and a point *P*.

Then the **Extension Linear Dependent *n-D*-Function** referring to the point *P(x1, x2, ..., xn)* is:

$$K_{nD}(P)=\frac{\rho(P,S_2)}{\rho(P,S_2)-\rho(P,S_1)} \tag{4}$$

where ρ (*P*, *S* 2) is the previous extension linear n-D-distance between the point *P* and the *n-D*-set S*2*.

The **Extension Non-Linear Dependent n-D-Function** referring to point *P(x1, x2, ..., xn)* along the curve *c* is:

$$K_{nD}(P)=\frac{\rho_c(P,S_2)}{\rho_c(P,S_2)-\rho_c(P,S_1)} \tag{5}$$

where ρ *c*(*P*,*S*2) is the previous extension non-linear n-D-distance between the point *P* and the *n-D*-set S*2* along the curve *c*.

Applications of the Extenics force-position control to 2D-Space in figure 5 is presented. We have a errors domain whose desired *2D*-dimensions should be *20 mV* x *30 mV*, and acceptable *2D*dimensions *22 mV* x *34 mV*. We define the extension 2D-distance, and then we compute the extension 2D-dependent function.

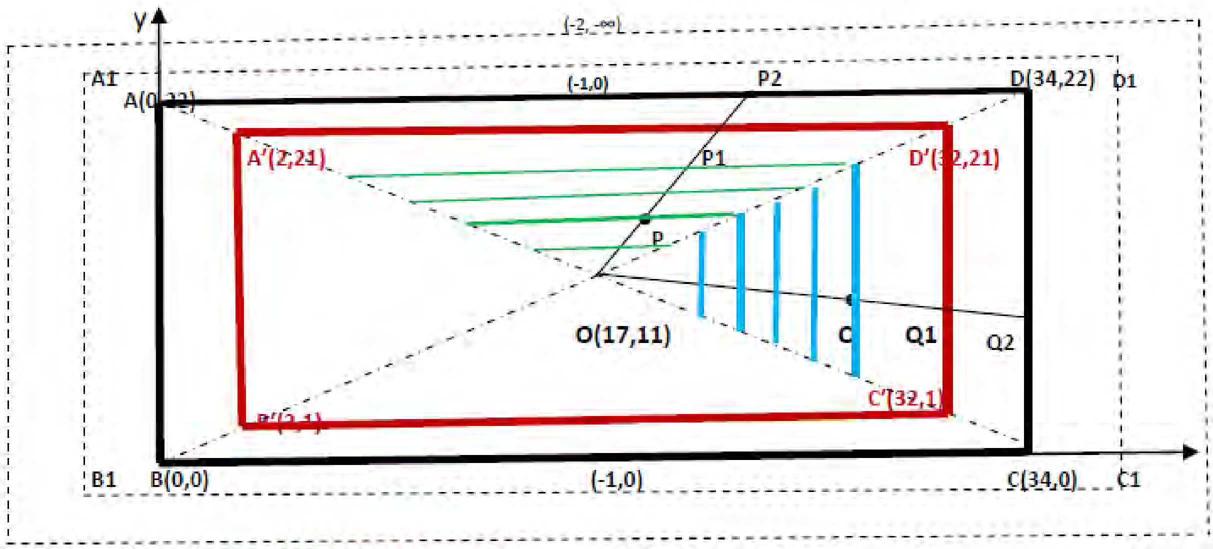

Fig. 5. Diagram of the extension 2D-dependent function

We have a desirable domain A'B'C'D' and an acceptable domanin ABCD [4]. The optimal opoint for both of them is O(17,11).

a) The region determined by the rays OA and OD.

The extension 2D-distance ρ between a point P and a set is the ± distance from P to the closestfrontier of the set, distance measured on the line OP. Whence

$$\rho\ (P, A'B'C'D') = -|PP1| \qquad (6)$$

$$\rho\ (P, ABCD) = -|PP2|. \qquad (7)$$

The extension 2D-dependent function k of a point P which represents the dependent of the point of the nest of the two sets is:

$$k(P) = \pm\frac{\rho(P,bigger\_set)}{\rho(P,bigger\_set) - \rho(P,smaller\_set)} = \pm\frac{\rho(P,ABCD)}{\rho(P,ABCD) - \rho(P,A'B'C')} = \pm\frac{|PP_2|}{|PP_2| - |PP_1|} = \pm\frac{|PP_2|}{|P_1P_2|}. \qquad (8)$$

In other words, the extension 2D-dependent function k of a point P is the 2D-extension distance between the point and the closest frontier of the larger set, divided by the 2D-extension distance between the frontiers of the two nested sets; all these 2D-extension distances are taken along the line OP.

b) The region determined by the rays *OC* and *OD*. Similar result would obtain if one gets the opposite region determined by the rays *OA* and *OB*. If one takes another region determined by the rays OC and OD and a point Q(x1,y1) in between one gets:

$$k(Q) = k(x_1, y_1) = \pm\frac{|QQ_2|}{|Q_1Q_2|} \qquad (9)$$

A complete calculation is presented in [4].

A new selection matrix S_{Kx} with the correlation coefficients for position is generated by the extension transformation module for position TExP which receives the dependent function signal for position $K(X, X_o, X_{CR})$ from the CFDP module and replaces the elements with value 1 in the position selection matrix with correlation coefficients for position , determined through an extension transformation in domain. For force similar processing takes place by using the CDEF, CFDF, TExF AND CEF modules, while quasi-simultaneously applying the position processing.

By applying the explicit sequential force-position control method on the two components, position in the CEP module and force in the CEF module, the position error and force error signals ε_{Kx} and ε_{Kf} respectively are determined with the aim of closing the system control loop in position and in force.

The implementation methodology of this advance method for hybrid position-force control of the walking robot consists in determining experimentally the standard positive field and the transient positive field for each control component, applying the transformation on the force and position error taking into account their real position in relation to the standard positive field, defined by points a_{0x} and b_{0x} for position, respectively a_{0f} and b_{0f} for force, resulting in a transformed position and force error which represents the optimized function for hybrid position-force control.

The universe of discourse is configured to admit a transient negative field, defined by points c_x and d_x for position, respectively c_f and d_f for force, so that passing these points the position and force errors will be limited so as not to lead to controller saturation and all the negative effects that derive from it.

Results and conclusions

The obtained results lead to an advanced method of solving the contradictory problem of hybrid position-force control for robot movement by applying an "Extension Set", which allows the two contradictory elements, force and position, to be controlled simultaneously in real time, allowing for improvements in the movement precision and stability of the robot. Starting from the extended distance given by Prof. Cai Wen the dependent function in 2D space generated in position and force is determined.

By replacing crisp logic values in the S_x and S_f matrices depending on the force-position sequence with values of the Extension Distance and Dependent Function for 2D space given by Smarandache, a method is developed for optimizing hybrid position-force control which ensures positioning precision and stability for the robot.

The final conclusion lead to the development of a methodology which will allow high level results for hybrid force-position control, by using an extended transformation using as an optimization function the dependence function based on extension distance, in comparison to the classical method using sequential matrices corresponding to Cantor logic.

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EXTENSION COMMUNICATION PENTRU REZOLVAREA CONTRADICŢIEI ONTOLOGICE DINTRE COMUNICARE ŞI INFORMAŢIE

FLORENTIN SMARANDACHE, ŞTEFAN VLĂDUŢESCU

Studiul se înscrie în zona interdisciplinară dintre teoria informaţiei şi extensică, în calitatea ei de ştiinţă a rezolvării contradictoriilor. În acest spaţiu se abordează problema centrală a ontologiei informaţiei relaţia contradictorie dintre comunicare şi informare. Nucleul cercetării îl reprezintă realitatea că investigaţia ştiinţifică a relaţiei comunicare-informare a ajuns într-o fundătură. Relaţia bivalentă comunicare-informare, informare-comunicare a ajuns să fie contradictorie, iar cele două concepte să se blocheze reciproc. În condiţiile în care Extensics este o ştiinţă a soluţionării problemelor contradictorii, se vor utiliza „extensical procedures" pentru a rezolva contradicţia. În acest sens, ţinând cont că matter-elements sunt definite, se vor explora proprietăţile acestora („The key to salve contradictory problems, arată Wen Cai, întemeietorul Extensics (1999, p. 1540), is the study of properties about matter-elements"). Conform „The basic method of Extensics is called extension methodology" (…), iar „the application of the extension methodology in every field is the extention engineering methods" (Weihua Li, Chunyan Yang, 2008, p. 34).

Cu metode lingvistice, sistemice, şi hermeneutice, grefate pe „extension methodology" a) sunt „open up the things", b) este marcată „divergent nature of matter-element", c) are loc „extensibility of matter-element", iar c) „extension communication" face ca să se deschidă perspectiva nouă de incluziune, să se evidenţieze o ordonare a lucrurilor la un nivel superior şi să se rezolve elementele de contradictorialitate. „Extension" este, după cum postulează Wen Cai (1999, p. 1538) „opening up carried out".

După examinarea critică a poziţiilor contradictorii exprimate de mai mulţi dintre specialiştii în domeniu, se emite ipoteza extensică şi integratoare că informarea constituie o formă de comunicare. Obiectul comunicării îl reprezintă transmiterea unui mesaj. Mesajul poate fi constituit din gânduri, idei, opinii, sentimente, credinţe, date, informaţii, intelligence sau alte elemente semnificaţionale. Atunci când conţinutul mesajului este preponderent informaţional, comunicarea devine informare sau intelligence.

Argumentele de susţinere a ipotezei sunt de natură lingvistică (cel mai important fiind acela că există „comunicare de informaţii", dar nu şi „informare de comunicări"), de natură sistemic-procesuală (în sistemul de comunicare se dezvoltă un sistem de informare;

actantul informator este un tip de comunicator; procesul de informare este un proces de comunicare), argumente practice (delimitarea elimină eforturile de înţelegere disparată şi neconcordantă a celor două concepte), argumente epistemologice (se creează posibilitatea gândirii intersubiective a realităţii), argumente lingvistice (se clarifică şi se dă forţă referentului supraordonat, acela al comunicării ca proces), argumente logico-realiste (se reţine starea de lucruri care permite a gândi coerent într-un sistem de concepte – seriile derivative sau grupările integrative) şi argumente ale experienţei istorice (conceptul de comunicare are prioritate temporală, el apare de 13 ori la Iulius Caesar). Cele mai importante argumente sunt sintetizate în patru axiome: trei sunt bazate pe observaţii pertinente ale lui Tom D. Wilson-Solomon Marcus, Magoroh Maruyama şi Richard Varey, iar cea de-a patra reprezintă o aplicaţie relevantă a teorie neutrosofice a lui Florentin Smarandache pe domeniul comunicării.

Keywords: extention communication, information, extensics, ontology, neutrosophic communication, message

I. Teza informării ca formă a comunicării

Problema relaţiei dintre comunicare şi informare ca domenii de existenţă reprezintă axa de amprentă a ontologiei comunicării şi informării. Formatul ontologic permite două formule: existenţa în act şi existenţa virtuală. Componenta ontologică a conceptelor integrează o prezenţă sau o potenţă şi un fapt existenţial sau la un potenţial de existenţă.

Pe lângă elementul categorial-ontologic, în raportul nuclear al conceptelor de comunicare-informare prezintă specificităţi comparative şi în ce priveşte atribute şi caracteristici, pe trei componente epistemologică, metodologică şi hermeneutică.

În cadrul unei ştiinţe care şi-ar fi asumat ferm un obiect de studiu, o metodologie şi un set specific de concepte, această decizie ontologică întemeietoare s-ar fi luat în cadrul unei axiome. Se ştie că, în principiu, axiomele soluţionează, în limitele acelui tip de argument numit evidenţă (situaţie clară şi distinctă), raporturile dintre conceptele sistemice, structurale, bazale. În mod specific, în cadrul Extensics, ştiinţă cu viziune avansată, fundamentată de profesorul Wen Cai, axiomele reglementează raportul între două matter-elements cu profiluri divergente. Pentru problematicile comunicării şi informaţiei care s-au constituit relativ recent (de circa trei sferturi de secol) în obiecte de studiu sau domenii de preocupare ştiinţifică nu s-a găsit o autoritate care să tranşeze problema. Slăbiciunile acestor ştiinţe de tip soft sunt vizibile şi astăzi când după propuneri neacreditate de ştiinţe („comunicologie" - communicology de Joseph de Vito, „comunicatică" – „comunicatique" de G. Metayer; informologie Klaus Otten şi Anthony Debon) s-a recurs la rămânerea în ambiguitatea de validare a disciplinei „Ştiinţele comunicării şi informaţiei" sau „Ştiinţele informaţiei şi comunicării", bucurându-se de suportul unor cursuri, cărţi, studii şi dicţionare.

Această viziune generică de unitate şi coeziune nedreptăţeşte şi comunicarea şi informarea. În practică, aparenta nedreaptă tratare globală, integrativă şi la grămadă nu are o totală şi acoperitoare confirmare. În mai toate universităţile cu profil umanist ale lumii predomină facultăţile şi cursurile de comunicare, inclusiv în cele din România şi Republica

Populară Chineză. Profesorul Nicolae Drăgulănescu constata, pentru cazul României că în 20 de facultăţi se predă comunicarea (sub diferite titulaturi) şi în numai două se predă informarea-informaţia.

Principalele perspective din care a fost abordată relaţia contradictorie comunicare-informare sunt cea ontologică, cea epistemologică şi cea sistemică. În majoritatea cazurilor, opiniile au fost incidentale. Atunci când a fost vorba de studii dedicate, cel mai frecvent demersul comparativ nu s-a făcut în mod programat pe unul sau pe mai multe criterii şi nici în mod direct şi aplicat. Fundamentele rămân contribuţiile lui Jorge Reina Schement, Brent R. Ruben, Harmut B. Mokros şi Magoroh Maruyama.

În studiul său „Communication and Information" (1993, pp. 3-31), J.R. Schement porneşte de la constatarea că „în retorica Erei Informaţiei, comunicarea şi informarea converg în înţelesuri sinonime". Pe de altă parte, reţine că dimpotrivă există specialişti ce se pronunţă pentru a se statua o distincţie fermă a semnificaţiilor acestora. Pentru a lămuri exact relaţia dintre cele două fenomene, respectiv concepte, acesta examinează definiţiile informării şi comunicării care au marcat evoluţia „studiilor informării" şi „studiilor comunicării". Pentru informare (informaţie) rezultă trei teme fundamentale: informarea-ca-lucru (information-as-thing) (M. K. Buckland), informarea-ca-proces (information-as-process) (N. J. Belkin, R. M. Hays, Machlup & Mansfield etc.), informarea-ca-produs-al-manipulării (information-as-product-of-manipulation (C. J. Fox, R. M. Hayes). Se observă, totodată, că toate aceste trei teme implică, în aprecierea emitenţilor lor, o „conexiune cu fenomenul de comunicare" („connection to the phenomenon of communication"). În paralel, din examinarea definiţiilor comunicării se desprinde că specialiştii în mod „implicit sau explicit introduc noţiunea de informare în definirea comunicării". Tot trei se desprind a fi temele centrale ale definirii comunicării: comunicarea-ca-transmitere (communication-as-transmission) (W. Weaver, E. Emery, C. Cherry, B. Berelson, G. Steiner), comunicarea-ca-proces-de-împărtăşire (communication-as-sharing-process) (R. S. Gover, W. Schramm), comunicarea-ca-interacţiune (communication-as-interaction) (G. Gerbner, L. Thayer). Comparând cele 6 noduri tematice, Schement evidenţiază că legătura dintre informare şi comunicare este „deosebit de complexă" şi dinamică: „informarea şi comunicarea sunt întotdeauna prezente şi conexate" („information and communication are ever present and connected" (Schement J. R., 1993, p. 17). În plus, pentru ca „informarea să existe trebuie să fie prezent un potenţial de comunicare" („for information to exist the potential for communication must be present").

Rezultanta în plan ontologic a acestor constatări ar fi că <u>existenţa informării este (strict) condiţionată de prezenţa comunicării</u>. Adică pentru a exista informare trebuie neapărat să fie prezentă comunicarea. Comunicarea va preceda şi întotdeauna va condiţiona existenţa informării. Şi mai detaliat: comunicarea face parte din ontologia informării. Ontologic, informarea se iveşte în cadrul comunicării şi ca potenţă a comunicării. J. R. Schement este orientat pe găsirea unei căi de catagrafiere a unei imagini coerente care să conducă spre o Teorie a comunicării şi informării („Toward a Theory of Communication and Information" – Schement J. R., 1993, p. 6). De aceea, evită să reţină conclusiv prioritatea temporală şi lingvistică, precedenţa ontologică şi amploarea comunicării în raport cu informarea. Concluzia studiului este că 1. „Informarea şi comunicarea sunt construcţii sociale" (cele „două cuvinte sunt utilizate ca interschimbabile, chiar ca sinonime" – se arată) (Schement J. R., 1993, p. 17); 2. „Studiul informării şi comunicării împărtăşesc concepte în comun" (în cadrul ambelor se vor regăsi comunicare, informare, „simbol, cogniţie, conţinut, structură, proces, interacţiune tehnologie şi sistem"- Schement J. R., 1993, p. 18); 3. „Informarea şi comunicarea formează două aspecte ale aceluiaşi fenomen" (Schement J. R., 1993, p. 18). Cu alte cuvinte, înţelegem că: a) în plan lingvistic („cuvintele", „termenii", „noţiunile", „conceptele", „ideea de") comunicarea şi informarea au o sinonimie; b) ca domeniu de studiu cele două recurg la acelaşi arsenal conceptual. Situaţia creată de aceste două elemente ale concluziei permite, în opinia noastră, o ierarhizare între comunicare şi informare. Dacă este cert că ontologic şi temporal comunicarea precedă informarea, dacă acest fenomen din urmă are o extensie mai mică decât primul, dacă eventuale ştiinţe care au obiect comunicarea, respectiv informarea, beneficiază de unul şi acelaşi vocabular conceptual, atunci informarea poate fi o formă de comunicare. În ciuda acestei direcţii pe care se înscriu în mod coerent argumentele lingvistice, categorial-ontologice, conceptual şi definiţional epistemologice aduse în argumentare, cel de-al treilea element al concluziei postulează existenţa unui fenomen unic care ar include comunicarea şi informarea (3. „Informarea şi comunicarea formează două aspecte ale aceluiaşi fenomen" - Schement J. R., 1993, p. 18). Acestui fenomen nu i se dă un nume. Panta conclusivă pe care se angajează argumentele şi elementele conclusive anterioare ne-a permis să articulăm informarea ca una dintre formele comunicării. În mod confirmativ, faptul că J. R. Schement nu numeşte un fenomen supraordonat comunicării şi informării, ne lasă posibilitatea atragem argumentul în a întări teza noastră că informarea este o formă de comunicare. Aceasta şi pentru că nu se poate găsi o categorie de fenomene care să înglobeze comunicarea şi informarea. J. R. Schement tinde către o perspectivă nivelatoare şi de convergenţă în ontologia comunicării şi informării. În schimb, M. Norton înclină către o diferenţiere pronunţată a comunicării de informare. El intră în grupul celor care văd comunicarea drept unul dintre procesele şi una dintre metodele „for making information available". Cele două fenomene „are intricately conected and have some aspects that seem similar, but they are not the same" (Norton M., 2000, p. 48 şi 39).

Harmut B. Mokros şi Brent R. Ruben (1991) fundamentează o viziune sistemică şi nivelară a înţelegerii relaţiei comunicare-informare. Luând în calcul contextul de raportare ca element de bază al structurii interne a sistemelor de comunicare şi de informare, aceştia evidenţiază informaţia drept criteriu de radiografiere a relaţionării. Metoda sistemic-teoretică non-lineară de cercetare fundamentată în 1983 de B. R. Ruben este aplicată obiectului de studiu reprezentat de fenomenele de comunicare şi informare. Cercetarea se situează în „Information Age" şi creează un tablou de raportare informaţional. Meritul principal al investigaţiei vine din relevanţa dată insubodonării dintre comunicare şi informare sub aspectul unei comunicări unipolare ce se raportează la o informaţie nivelară. Interesantă este abordarea informaţiei sub trei aspecte constitutive: „informatione" (informaţia potenţială – aceea care există într-un anume context, dar care n-a primit o semnificanţă în sistem), „information" (informaţiile active în sistem) şi „informations" (informaţiile create social şi cultural în sistem). Informarea nivelară se află în relaţie cu o comunicare unificată. Pe fiecare palier al informării există comunicare. Informarea şi comunicarea sunt coprezente: comunicarea este inerentă informării. Informarea are inerente proprietăţi de comunicare. Cercetarea aduce o clarificare sistemic-contextuală a relaţiei dintre comunicare şi informare şi doar în subsidiar o situare ontologică fermă. În orice caz: niciodată în informare nu lipseşte comunicarea.

În cele mai importante dintre studiile profesorului Stan Petrescu, „Informaţiile, a patra armă” (1999) şi „Despre intelligence. Spionaj-Contraspionaj” (2007), informaţia este înţeleasă ca „un fel de comunicare” (Petrescu S., 1999, p. 143) şi situată în contextul mai larg al „cunoaşterii despre mediul informaţional intern şi internaţional” (Petrescu S., 2007, p. 32).

II. Obiectul comunicării: mesajul. Obiectul informării: informaţia. Teza informaţiei ca specie de mesaj

Pentru definitivarea tezei noastre de bază aceea a informării ca formă de comunicare pot fi aduse noi argumente care se coroborează cu cele anterior menţionate. Ca fenomene, ca procese, comunicarea şi informarea au loc în cadrul unui sistem unic de comunicare. În cadrul comunicării, informarea şi-a dobândit un profil specializat. În câmpul informării, intelligence-ul, la rândul lui, şi-a consolidat un profil specific detectabil, discriminabil şi identificabil. Este în consecinţă de acceptat sub presiunea argumentului practic că se poate vorbi de un sistem general de comunicare care în raport de mesajul transmis şi configurat în procesul de comunicare ar putea fi imaginat ca sistem de informare (information system) sau ca sistem de intelligence (intelligence system). Sub imperiul presupoziţiei sistemice că un comunicator (unitar) transmite sau configurează tranzacţional împreună cu alt comunicator (destinatar) un mesaj, înţelegem sistemul comunicaţional ca unitatea interacţională a factorilor ce exercită şi îndeplinesc funcţia de comunicare a unui mesaj.

În cărţile sale „Messages: building interpersonal communication skills” (ajunsă în 1993 la a patra ediţie, iar în 2010 la a douăsprezecea) şi „Human communication” (2000), Joseph De Vito (reputatul specialist ce a propus pentru ştiinţele comunicării titulatura de „Communicology” - 1978), elaborează un concept de mesaj simplu şi productiv. Mesajul este, ca şi conţinut, ceea ce se comunică. Ca factor sistemic, el se profilează ca ceea ce este comunicat. De amintit în această ordine de idei că germanul Otto Kade a insistat ca ceea ce se comunică să primească titulatura de „comunicat”. În concepţia lui Joseph De Vito, prin comunicare se transmit înţelesuri. „Mesajul comunicat” constituie doar o parte a înţelesurilor (De Vito J., 1993, p. 116). Printre înţelesurile împărtăşite se regăsesc sentimente, percepţii (De Vito J., 1993, p. 298). De asemenea, se pot comunica informaţii (De Vito J., 1990, p. 42), (De Vito J., 2000, p. 347).

În cadrul unei „teorii a mesajului” numita „Angelitică” (Angelitics), Rafael Capurro exprimă opinia că mesajul şi informaţia sunt concepte ce desemnează fenomene similare, dar nu identice. În limba greacă „angelia” însemna mesaj; de aici, „Angelitica” sau teoria mesajului (Angelitica este altceva decât Angeologia care se ocupă, în câmpul religiei şi teologiei, cu studiul îngerilor) (http://www.capurro.de/angelitics.html). R. Capurro fixează 4 criterii de evaluare a raportului dintre mesaj şi informaţie. Similitudinea celor două se extinde pe trei dintre ele. Mesajul, ca şi informaţia, se caracterizează astfel: „is supposed to bring something new and/or relevant to the receiver; can be coded and transmitted through different media or messengers; is an utterance that gives rise to the receiver's selection through a release mechanism of interpretation". Diferenţa dintre cele două este următoarea: „a message is sender-dependent, i.e. it is based on a heteronomic or assymetric structure. This is not the case of information: we receive a message but we ask for information" (http://www.capurro.de/angeletics_zkm.html). A solicita informaţii înseamnă a transmite un mesaj de solicitare de informaţii. Prin urmare, mesajul este similar cu informaţia şi pe acest criteriu. În opinia noastră, diferenţa dintre ele este de la gen la specie: informaţia este o specie de mesaj. Mesajul depinde de transmiţător şi informaţia, la fel. Informaţia este, însă, o specificaţie a mesajului, este un mesaj informativ. C. Shannon apreciază că mesajul reprezintă obiectul definitoriu al comunicării. El este miza comunicării, căci „the fundamental problem of communication is that of reproducing at one point either exactly or aproximately a message selected at another point" (1949, p. 31).

Procesul de comunicare constă în fapt în „comunicarea" unui mesaj complex şi multistratificat. În mesaj se pot regăsi „gânduri, interese, talente, experienţe" (Duck S., Mc Mahan D.T., 2011, p. 222), „informaţii, idei, credinţe, sentimente" (Wood J. T., 2009, p. 19 şi p. 260). G. A. Miller, T. M. Newcomb şi Brent R. Ruben consideră că obiectul comunicării îl formează informaţiile: „Communication - arată Miller – means that information is passed from one place to another" (Miller G. A., 1951, p. 6). La rândul său, T. M. Newcomb precizează: „very communication act is viewed as a transmission of information" (Newcomb T. M., 1966, p. 66), iar Brent R. Ruben susţine: „Human communication is the process through which individuals in relationships, groups, organizations and societies create, transmit and use information to relate to the environment and one another" (Ruben B. R., 1992, p. 18).

Profesorul Nicolae Drăgulănescu, membru al American Society of Information Science and Technology, este cel mai important dintre specialiştii români în Ştiinţa informaţiei. În opinia sa, „comunicarea informaţiei" este al treilea dintre cele patru procese ce constituie „ciclul informaţional", alături de generarea informaţiei, prelucrarea/stocarea informaţiei şi utilizarea informaţiei. Procesul de comunicare, arată N. Drăgulănescu, este unul dintre procesele al căror obiect îl constituie informaţia (http://ndragulanescu.ro/publicatii/CP54.pdf, p. 8). Pe aceeaşi linie se situează şi Gabriel Zamfir; acesta gândeşte informaţia ca reprezentând „ceea ce se comunică într-unul sau altul dintre limbajele disponibile" (Zamfir G., 1998, p. 7), la fel şi profesoara Sultana Craia: comunicarea este un „proces de transmitere a unei informaţii, a unui mesaj" (Craia S., 2008, p. 53).. În general, se acceptă că informarea înseamnă transmiterea/primirea de informaţii. Cu toate acestea, atunci când se vorbeşte de transmiterea informaţiilor, procesul este considerat a fi nu informare, ci comunicare. De aceea, se creează aparenţa că informaţiile sunt produsul, iar comunicarea ar fi doar procesul de transmitere. Teodoru Ştefan, Ion Ivan şi Cristian Popa precizează: „Comunicarea este procesul de transmitere a informaţiilor, deci raportul dintre cele două categorii este de la produsul de bază la transmiterea lui" (Ştefan T., Ivan I., Popa C., 2008, p. 22). Profesorii Vasile Tran şi Irina Stănciugelu văd comunicarea ca un „schimb de informaţii cu conţinut simbolic" (Tran V., Stănciugelu I., 2003, p. 109). În fapt, comunicarea este un concept supraordonat şi o categorie ontologică mai extinsă decât informarea sau informaţia. Pe de altă parte, informaţiile se generează chiar în procesul global de comunicare. Din această perspectivă, informarea (al cărei obiect-mesaj îl alcătuiesc informaţiile) constituie o comunicare regională, sectorială. Informarea este acea comunicare al cărei mesaj este constituit din semnificaţii noi, relevante, oportune şi utile, adică din informaţii. Această poziţie o împărtăşeşte şi Doru Enache (2010, p. 26).

Poziţia fixată de Norbert Wiener, consolidată de L. Brillouin şi însuşită de mulţi alţii face din informaţie singurul conţinut al mesajului. N. Wiener arată că mesajul „conţine informaţie" (Wiener N., 1965, p. 16), L. Brillouin vorbeşte despre „informaţia conţinută de mesaj" (Brillouin L., 2004, p. 94 şi p. 28).

Prin comunicare se „vehiculează informaţii, noţiuni, emoţii, convingeri" şi comunicarea „presupune (şi subsumează) informarea" (Rotaru N., 2007, p.10). Reputaţii profesori Marius Petrescu şi Neculae Năbârjoiu consideră că departajarea între comunicare şi informare trebuie să se realizeze în funcţie de mesaj. O comunicare ce are un mesaj informativ devine informare. Ca formă a comunicării, informarea se caracterizează printr-un mesaj informativ, iar un „mesaj rămâne informativ atât cât conţine ceva necunoscut încă" (Petrescu M., Năbârjoiu N., 2006, p. 25). Unul dintre posibilele elemente semnificaţionale ce ar putea alcătui conţinutul mesajului este deci şi informaţia. Alte componente ar putea fi gândurile, ideile, credinţele, cunoştinţele, sentimentele, trăirile, experienţele, noutăţile faptele etc. Comunicarea este „comunicare" a unui mesaj indiferent de fondul semnificaţional al acestuia.

III. Patru axiome de ontologie a comunicării-informării

3.1. Axioma mesajului. Numim axiomă ontologică de segregare privind obiectul sau axioma Tom D. Wilson-Solomon Marcus teza că nu orice comunicare este informare, dar orice informare este comunicare. Ori de câte ori mesajul conţine informaţii, procesul comunicaţional capătă profil informaţional. Totodată, sistemul comunicaţional devine sistem informaţional. În mod derivat comunicatorul devine „informator", iar relaţia comunicaţională se transformă în relaţie informaţională. Baza interacţională a societăţii, chiar în Era informaţională, o constituie interacţiunea comunicaţională. Majoritatea interacţiunilor sociale sunt non-informaţionale. În acest sens, T. D. Wilson observa: „We frequently receive communications of facts, data, news, or whatever which leave us more confused than ever. Under formal definition these communications contain no information" (Wilson T. D., 1987, p. 410). Academicianul Solomon Marcus ia în calcul existenţa incontestabilă a unei comunicări „în absenţa unui transfer de informaţie" (Marcus S., 2011, vol. 1. p. 220). Pentru comunicările ce nu conţin informaţii nu deţinem un termen separat şi specific. Comunicările ce conţin informaţii sau doar informaţii se numesc informări.

Comunicarea implică un fel de informaţie, dar, aşa cum precizează Jean Baudrillard (Apud Dâncu V.S., 1999, p. 39), „ea nu se întemeiază obligatoriu pe informaţie". Mai exact, orice comunicare conţine o cunoaştere care poate fi cunoştinţe, date sau informaţii. Prin urmare, în comunicare, informaţia poate lipsi, poate avea caracter adiacent, incident ori colateral. Comunicarea poate fi informaţională prin natura sau prin destinaţia ei. Acea comunicare ce prin natura şi organizarea ei este comunicare de informaţie poartă numele de informare.

Principalul proces derulat în Information System îl reprezintă informarea. Funcţia unui astfel de sistem este de a informa. Actanţii pot fi informatori, producători-consumatori de informaţii, transmiţători de informaţii etc. Acţiunea de informare capătă identitate prin acoperirea ce i-o aduce onto-categorial verbul „a informa”. La rândul lui, Petros A. Gelepithis consideră cele două concepte, comunicare şi informare, că sunt capitale pentru „the study of information system” (Gelepithis P. A., 1999, p. 69).

Confirmând axioma informaţiei ca mesaj reducţionist, ca obiect redus de comunicare, Soren Brier arată: „communication system actually does not exchange information” (Brier S., 1999, p. 96). Uneori, în cadrul sistemului de comunicare nu se mai schimbă informaţii. Cu toate acestea, comunicarea subzistă; sistemul de comunicare îşi păstrează validitatea, ceea ce indică şi, subsecvent, probează că poate exista comunicare care să nu incumbe informaţie.

Atunci

a) când în cadrul Information System se introduc principii funcţionale precum „need to know”/”need to share”,

b) când se derulează procese de culegere, analiză şi diseminare de informaţii,

c) când beneficiarii sunt decidenţi, „decisionmaker”, „minister”, „governement”, „polcymakers” şi

d) când intervine elementul de secretivitate,

acest Information System devine Intelligence System (vezi Gill P., MarrinS., Phytian M., 2009, p. 16, p. 17, p. 112, p. 217), (Sims J.E., Gerber B., 2005, p. 46, p. 234; Gill P., Phytian S., 2006, p. 9, p. 236, p. 88; Johnson L.K. (ed.) 2010, p. 5, p. 6, p. 61, p. 392, p. 279, Maior G.-C. (ed.), 2010). „Secrecy, arată Peter Gill, is a key to understanding the essence of intelligence” (Gill P., Marrin S., Phytian M., 2009, p. 18), iar profesorul George Cristian Maior accentuează: „în intelligence, esenţiale rămân colectarea şi procesarea informaţiilor din surse secrete” Maior G.-C., 2010, p. 11).

Sherman Kent, W. Laqueur, M. M. Lowenthal, G.-C. Maior ş.a. pornesc de la un concept de intelligence complex şi multistratificat, înţeles ca semnificând cunoaştere, activitate, organizaţie, produs, proces şi informaţie. Subsecvent, se pune problema ontologiei, epistemologiei, hermeneuticii şi metodologiei intelligence-ului. Alături de Peter Gill, G.-C. Maior face operă de pionierat în a separa abordarea ontologică a intelligence-ului de cea epistemologică şi a analiza „fundamentul epistemologic al activităţii de informaţii” (Maior G.-C., 2010, p. 33 şi p. 43). Intelligence-ul trebuie gândit şi din perspectiva axiomei ontologice a obiectului. Sub acest aspect, este de observat că una dintre semnificaţiile sale, poate cea critică, îl situează într-un fel sau altul în perimetrul informaţiilor. În opinia noastră, acele informaţii care au importanţă majoră pentru operatori acreditaţi ai puterii statale, economice, financiare, politice etc. şi deţin ori dobândesc un caracter confidenţial-secret sunt sau devin intelligence. Informaţiile din sistemele de intelligence pot constitui în sine intelligence sau ajung să fie intelligence în urma unor procesări specializate. „Intelligence-ul nu este doar informaţie care există pur şi simplu” (Marinică M., Ivan I., 2010, p. 108), reţin Mariana Marinică şi Ion Ivan, el se obţine în urma unui „act conştient de creaţie, colectare, analiză, interpretare şi modelare a informaţiilor" (Marinică M., Ivan I., 2010, p. 105).

3.2. Axioma teleologică. Pe lângă axioma de segregare a comunicării de informare în raport cu obiectul (mesajul), se poate reţine ca axiomă o contribuţie a lui Magoroh Maruyama la demitizarea informării. În articolul „Information and Communication in Poly Epistemological System" din „The Myths of Information", acesta arată: „The transmission of information is not the purpose of communication. In Danish culture, for exemple, the purpose of communication is frequently to perpetuate the familiar, rather than to introduce new information" (1980, p. 29).

Axioma ontologică de segregare în raport cu scopul determină informarea drept acel tip de comunicare cu emergenţă redusă în care scopul interacţiunii îl reprezintă transmiterea de informaţii.

3.3. Axioma lingvistică. O a treia axiomă de segregare ontologică comunicare-informare se poate desprinde în raport cu argumentul lingvistic al contextului gramatical acceptabil. Richard Varey gândeşte că a înţelege „the difference between communication and information is the central factor" şi găseşte în contextul lingivstic criteriul de a valida diferenţa: „we speak of giving information **to** while communicate **with** other" (1997, p. 220). Transmiterea de informaţii are loc „către" sau cuiva, iar comunicarea are loc „cu". Alături de această variantă de context gramatical se mai poate înregistra şi situaţia de acceptabilitate a unor enunţuri în raport cu obiectul procesului de comunicare, respectiv obiectul procesului de informare.

Enunţul „a comunica un mesaj, informaţii" este acceptabil. În schimb, enunţul „a informa comunicări" nu este. Sintagma „comunicarea de mesaje-informaţii" este validă, dar sintagma „informarea de comunicări", nu. Prin urmare, limba poartă cunoaştere şi ne „ne îndrumă" (cum spune Martin Heidegger) să observăm că, lingvistic, comunicarea are o ontologie mai extinsă şi că ontologia informării i se subsumează.

Caracterul ontic şi ontologic al limbii îi permite acesteia să exprima existenţa şi să realizeze o specificare funcţional-gramaticală. Limba nu permite decât existenţe gramaticale. Ca mesaj, informaţia poate fi „comunicată" sau „comunicabilă". Subzistă şi cazul că o informaţie să nu fie „comunicată" ori „comunicabilă". În mod conex, comunicarea nu poate fi „informată". Câmpul semantic al comunicării este deci mai extins, mai bogat şi mai versatil. Comunicarea permite „incomunicabilul".

3.4. Axioma comunicării neutrosofice. Înţelegând cadrul fixat de cele trei axiome, constatăm că unele elemente comunicaţionale sunt heterogene şi neutre în raport cu criteriul informativităţii. Dintr-un discurs pot fi suprimate anumite elemente, fără ca mesajul să sufere modificări informaţionale. Aceasta înseamnă că unele semnificaţii mesajual-discursive sunt redundante, altele sunt neesenţiale în raport de orexisul-direcţia practică sau de nuanţă practică în ordinea gândirii. Redundanţele şi elementele semnificaţionale non-nucleare pot fi elidate, iar mesajul rămâne informaţional neschimbat. Aceasta probează existenţa unor nuclee de semnificaţii neutre, neutrosofice. (În ce priveşte fundamentele epistemologice ale conceptului de neutrosofie trimitem la lucrarea lui Florentin Smarandache, *A Unifying Field in Logics, Neutrosophic Logic, Neutrosophy, Neutrosophic Set, Neutrosophic Probability and Statistics*, 1998).

Pe funcţionarea acestui fenomen se bazează procedurile de contragere textuală, de grupare, de înseriere, de asociere, de rezumare, de sintetizare, de integrare.
Propunem să se înţeleagă prin comunicare neutrosofică acel tip de comunicare în care mesajul este constituit din şi se fundamentează pe elemente semnificaţionale neutrosofice: non-informaţionale, redundante, elidabile, contradictorii, incomplete, vagi, imprecise, contemplative, non-practice, de cultivare relaţională. Comunicarea informaţională este acel tip de comunicare al cărei obiect îl alcătuieşte împărtăşirea unui mesaj informaţional. Demersul fundamental al emitentului este, în comunicarea informaţională, acela de a informa. A informa este a transmite informaţii sau, exact, cu cuvintele profesorului Ilie Rad: „să informeze, adică să transmită doar informaţii" (Moldovan L., 2011, p. 70). În linii generale, orice comunicare conţine unele ori anumite elemente neutrosofice, elemente suprimabile, redundante, elidabile, non-nucleare. Când însă elementele neutrosofice au preponderenţă comunicarea nu mai este informaţională, ci neutorsofică. Ca atare, axioma neutrosofică ne permite să delimităm două tipuri de comunicare: comunicarea neutrosofică şi comunicarea informaţională. În majoritatea timpului comunicarea noastră este neutrosofică. Comunicarea neutrosofică este regula. Comunicarea informaţională constituie excepţia. În oceanul comunicării neutrosofice se
Dobreanu C., *Preventing surprise at the strategic lever,* Buletinul Universităţii Naţionale de Apărare „Carol I", anul XX, nr. 1/2010, pp. 225-233, 2010.
Enache Doru, *Informaţia, de la primul cal troian la cel de-al doilea cal troian*, Paraşutiştii, anul XIV, nr. 27(36), pp. 25-28, 210.
Păvăloiu C., *Elemente de deontologie a evaluării în contextul creşterii calităţii actului educaţional*, Forţele terestre, nr. 1/2010.
diferenţiază insule diamantine de comunicare informaţională.

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EXTENSICA, STUDIUL SIMULTANEITĂŢILOR

FLORENTIN SMARANDACHE, TUDOR PĂROIU

Semnificaţii

Extenica nu este altceva decît mult discutata interdisciplinaritate aplicată în practică care în realitate este studiu al simultaneităţii **entităţilor/univers.** Ea nu analizează doar două sau mai multe contrarii sau "rezolvarea problemelor contradictorii" ea studiază şi încearcă rezolvarea simultaneităţii **entităţilor/univers**. Mai exact ea nu studiază doar contrariile ca elemente bipolare ci relaţia dintre două **entităţi/ univers** nu neapărat contrare. Doi oameni, două fapte, două situaţii sau două fenomene (două **entităţi/univers** diferite), dar şi continuitatea sau discontinuitatea acestora şi nu neapărat contrarii. Trebuie revenit asupra categoriei filozofice de contrarie, trebuie să extindem această categorie la întreaga **transformare/spaţiu/timp**. Dacă ţinem cont de formulele domnului Smarandache vom constata că între limitele sale orice **tranasformare/spaţiu/timp** este un raport de <A>/<antiA> unde de data aceasta prin <A> şi <antiA> nu mai definim contrariile ci limitele unei **transformări/spaţiu/timp**. Şi contrariile nu sînt altceva decît limitele transformării contrariei respective, respectiv ca exemplu trecerea de la pozitiv la negativ sau de la bine la rău. Şi trecerea de la o transformarea la alta este o contrarie sau dacă doriţi trecerea de la o contrarie la alta este o transformare a raportului <A>/<antiA> la fel cum se poate considera orice tranasformare în nelimitat. Tot ca exemplu putem defini contrarii şi transformarea noastră de la existenţă la inexistenţă sau de la viaţă la moarte sau trecerea de la o autostradă cu circulaţia pe dreapta la una cu circulaţia pe stînga, etc. În „supa" descoperită în final în Elveţia (care ar fi contrariile şi care <neutA>) ? toate sînt unul şi acelaşi lucru ca şi într-o Gaură Neagră, sînt neconvenţionale. Deocamdată neconvenţionale pentru noi şi cunoaşterea noastră, pentru că nu putem încă să le definim **transformarea/spaţiu/timp** sau contrariile (cum doriţi), dacă reuşim convenţionalizarea lor ele devin convenţii pur şi simplu. Deoarece filozofii nu au înţeles corect legătura dintre **transformare/spaţiu/timp** şi contrarii ei au condiţionat **transformarea/spaţiu/timp** de contrarii. În realitate contrariile sînt însăşi **transformarea/spaţiu/timp** sau mai exact **transformarea/spaţiu/timp** are un caz particular contrariile. Aşadar contrariile sînt cazuri particulare ale **transformării/spaţiu/timp**. Neconvenţionalul merge dincolo de aceste convenţii ale **transformării/spaţiu/timp** şi de cunoaşterea noastră dincolo de „supa" amintită.

Folosirea lui 0* şi ∞* ca şi al lui 0 şi © ne aduce mai aproape de realitate dar nu la Realitatea în Sine, doar ne măreşte limitele faţă de noi în nici un caz faţă de nelimitat.

Şi neutrosofia poate face acest lucru prin extensie. Din acest motiv trebuie să înţelegem fenomenul filozofic adică în ansamblul lui şi nu doar ştiinţific punctual, deoarece implicaţiile nu sînt doar de natura unei ştiinţe ci general valabilă adică filozofic ca simultaneitate a tuturor ştiinţelor. De la fizică cuantică la medicină, biologie, fizică, chimie, tehnologie de orice natură, chiar literatură sau artă, etc. indiferent dacă cineva consideră că rezolvarea sau nu a unei probleme contradictorii nu implică toate ştiinţele, mai mult sau mai puţim. Am să dau exemplul cu autostrăzile cu benzi diferite ce trebuiesc unite. Poate spune cineva că acest lucru nu are implicaţie, socială, artistică, fizică, biologică, tehnologică, chimică sau chiar medicală de ce nu spirituală, etc.? să nu vă grăbiţi, doar pentru faptul că nu înţelegem sau deocamdată nu vedem realitatea ci doar relativul ei nu putem nega lucrurile. Dacă ar schimba doar benzile de circulaţie este o soluţie, o soluţie convenţională sînt însă şi soluţii neconvenţionale şi nu mă refer la posibilitatea modulării maşinilor astfel ca volanul să acţioneze pe dreapta sau pe stînga în raport de necesităţi, eu privesc lucrurile mult mai neconvenţional. Dacă oamenii ar putea să moduleze totul la nivel molecular sau chiar atomic sau ar putea ajunge la teleportare nu ar mai avea nevoie să modueze şoselele şi nici oamenii. Poate că în viitor se poate modela omul şi nu autostrada printr-o simplă schimbare de ochelari sau cine ştie ce. Pentru că noi nu putem folosi realitatea în sine şi aici trebuie să respectăm regula şi să trucăm realitatea noastră pentru a păcăli Realitatea în Sine. Extenica este ca o trecere de la o filozofie teistă la una ateistă sau de la literatură la metematică sau în general de la o ştiinţă la alta. În natură şi în realitate această trecere este perfectă pentru că este neconvenţională şi se face la nivelul **entităţilor/univers** neconvenţionale simultan şi imperceptibil, nu există element neutru în simultaneitatea neconvenţională este ca naşterea sau moartea fiecăruia, noi nu ştim nici cînd ne naştem dar nici cînd murim aceasta este trecerea neconvenţională, o transformare ca trecerea de la copilărie la maturitate nu ştii niciodată cînd se face. În convenţional aşa cum spuneţi şi dumneavoastră este ca în neutrosofie, trebuie să inventăm un <neutA> care ţine locul neconvenţionalului din noi sau din Universul în Sine, (chiar dacă acest <neutA> este doar unul relativ şi probabil) care face trecerea de la o contrarie la alta, în neconvenţional aceste contrarii nu mai există sînt perfect simultane încît noţiunea însăşi de contrarie devine absurdă. Autostrăzile cu siguranţă vor avea un <neutA> este un truc al realităţii noastre. Dacă am fi neconvenţionali maşina şi individul s-ar adapta din mers şi nu şi-ar da seama decît cînd sînt pe cealaltă autostradă sau mai exact nu şi-ar da seama niciodată pentru că neconvenţionalul nu poate reflecta convenţional. În Extenică este obligatorie neutrosofia şi <neutA>, dacă nu există <neutA> trebuie să-l inventăm (aşa cum am inventat cifra 0) ca pe un truc necesar al convenţionalului la fel cum trebuie să facem cu orice **entitate/univers**, aşa cum facem cu autostrăzile sau cum fac unii cu interdisciplinaritatea unde legăturile neconvenţionale ale simultaneităţii dintre fizică şi chimie (<neutA>) le spunem **chimie/fizică** chiar dacă niciodată nu vom putea defini limita exactă dintre ele. Analog **bio/chimie**, **bio/fizică**, etc. pentru oricare două ştiinţe veţi găsi <neutA> respectiv o ştiinţă de graniţă. Să nu credeţi că între literatură şi matematică nu este o ştiinţă de graniţă, ea există dar nu am denumit-o noi încă. Toate cele prezentate sînt <neutA> convenţional ales pentru neconvenţionalul simultaneităţii **entităţilor/univers** sau mai exact Extenica lor. Din păcate sau poate din fericire dacă nu şi una şi alta simultan (deoarece în lipsa echilibrului respectiv <neutA> am înebuni cu siguranţă datorită instabilităţii şi neputinţei, ca şi datorită lipsei celorlalte elemente oblgatorii ale unei **entităţi/univers**) acest <neutA> există pentru noi special, în realitate este **pozitiv/negativul** simultan al celor două extreme doar că dimensiunile simultaneităţii sale (ale lui <neutA>) sînt din ce în ce mai mici tinzînd către 0. Elementele sale de **formă/existenţă/spirit** sînt foarte puţin perceptibile (reflectabile, convenţionalizabile) pentru noi sau **entităţile/univers** care ne ajută. Acest <neutA> aparţine domeniului numerelor foarte mici iar ca să fie o trecere (transformare) imperceptibilă trebuie ca elementele sale să fie dacă este posibil 0. Adică 0*» 0. La fel trebuie să fie şi în ecuaţiile matematice dacă se poate să fie nu doar în limtele (0*,1) ci dincolo de 0* cît mai apropiat de 0, în lumea numerelor foarte mici dintre 0 şi 0*, în acelaşi timp în care <A> şi <antiA> să aparţină mulţimii (∞*,©) adaptate cu un λ(1) sau cu ∞* în raport de posibilităţile (trucurile) convenţiilor noastre.

Mai întîi să introducem cititorul în lumea noilor convenţii mai puţin convenţionale decît toate cele anterioare, (niciodată însă neconvenţionale în totalitate, neconvenţionale doar faţă de cunoaşterea noastră convenţională) astfel vom introduce o serie de noi semnificaţii (0, 0*, ∞*, ©) chiar dacă poate simbolurile rămîn aceleaşi. Oamenii fac greşeala să încurce lucrurile, ei tind mereu să încurce realitatea lor (**iluzia/realitate**) cu Realitatea în Sine care nu le aparţine fiind reflectată de spiritul lor doar prin intermediari (simţuri, logică, instinct, etc.) niciodată direct. Din acest motive eu am introdus elemente ajutătoare (trucuri, 0, 0*, ∞*, ©) convenţionale ca să mă apropii de realitate.

Dacă discutăm filozofic, în Universul în Sine nu există cifra 1, există doar 0 şi cuantificările sau decuantificările acestuia. Cifra 0 în Universul în Sine ar trebui să fie inexistenţa dar ca pardox inexistenţa şi existenţa sînt simultane pentru Universul în Sine în toate formele lui convenţinale sau neconvenţionale. Doar noi **entităţile/univers** ni se pare că intuim existenţa şi inexistenţa separat şi le convenţionalizăm, separarea lor nu există ca realitate cum nu există nici cifra 0 sau1. Cifra 1 (este relativă) nu există nici în convenţional, doar multiplii sau submultiplii ei şi diviziunile (aceste cifre sînt limite neconvenţionale adică la nelimită) acesteia sau diverse cuantificări ale acesteia. 0 şi 1 sînt limitele Universului în Sine adică nelimitatul lui perfectul existenţei şi perfectul inexistenţei, paradoxal însă ele sînt simultane şi la limita lor dispar ca noţiuni convenţionale. Din acest motiv singurele limite pentru noi sînt cele convenţionale respectiv 0* şi ∞* (pe care le introduc eu) care în realitate nu sînt decît constante (infinit de mari sau de mici) limitate ale oricărei simultaneităţi **transformare/spaţiu/timp**. În acest caz orice **transformare/spaţiu/timp,** pentru noi, este convenţională, deci relativă, finită şi constantă raportată la Universul în Sine. Mai mult dincolo de 0* şi ∞* există limitele 0 şi © unde (0*,∞*) ∈(0,©). Asta înseamnă simultaneitatea celor două domenii de definiţie în nici un caz identitatea lor. Cum orice **transformarae/spaţiu/timp** are un domeniu de definiţie (0*, ∞*) acest lucru implică simultaneitatea oricărei **transformări/spaţiu/timp** convenţionale cu cea neconvenţională, dar şi cu cele intermediare (0n*, ∞n*). Acestă explicaţie ne arată că orice univers, orice **entitate/univers** şi ca entitate şi ca univers sînt simultane cu alte **entităţi/univers** (legea simultaneităţii). Atîta timp cît există un ∞* care respectă relaţia 0*∞*=c există şi un 0 care împreună cu nelimitatul (un 0 nelimitat de mic, deoarece şi 0* este ∞* de mic ca să respecte relaţia 0*∞*=c) respectă relaţia 0©=c diferenţa este că în timp ce în convenţional c poate lua valori în intervalul (0,1) dacă 0* şi ∞* sînt simetrice (respective 0*=1/∞*), în cazul 0©=c nu există valori în afara intervalului (0,1) pentru c, singura lui valoare este 1, este unică la fel ca şi 0 sau ©.

Trebuie ţinut cont permanent că între 0* şi 1, ca şi între 1 şi ∞* sînt ∞* subdiviziuni convenţionale iar în cazul nelimitatului, nelimitate subdiviziuni ca în realitate. De asemenea între 0* şi 0 sînt nelimitate subdiviziuni ca şi între ∞* şi ©. Acest lucru se datorează însă nu infinitului nostru convenţional (∞*) sau lui 0* ci nelimitatului ©. Se pot lua nelimitate perechi de 0*şi ∞* respectiv (0_1*, ∞_1*), (0_2*, ∞_2*), (0_3*, ∞_3*)…. (0,©), etc. şi fiecare are ∞* variante la stînga şi la dreapta lui 1 în raport de domeniul de definiţie al lui ∞* (N, R, Q, C, etc.) şi domeniile nou definite iau aceste valori. Adică între 0* şi 1 sînt numere raţionale, complexe, etc. şi între 1 şi ∞*sînt tot valori pe aceleaşi domenii de definiţie dar şi între 0_1* şi 1, sau 1 şi ∞_1*, sau între 0*şi 0_1*sau ∞_1* şi ∞* ş.a.m.d. pînă la 0 şi ©. Limita acestui şir este nelimitatul lor iar ca produs este 1, toate sînt simetrice faţă de 1. . Singura lor diferenţă este gradul de multiplicare sau demultiplicare care se reduce la adunare şi înmulţire cu şi faţă de 1 şi 0. Astfel orice număr dincolo de ∞* este un număr cuantificat prin adunare sau scădere de 1, respectiv ∞_1*=∞*+ λ(1) (indiferent de modelul funcţiei acestuia) unde λ reprezintă cuantificarea lui 1 prin adunare sau scădere de orice natură. Să nu uităm că înmulţirea sau orice operaţie este cuantificare prin adunare sau scădere de 1 şi subdiviziunile acestuia. Calculatorul şi sistemul binar al acestuia este exemplu edificator care rezolvă orice ecuaţie (fenomen, materie sau energie, etc.) prin multiplicare sau demultiplicare a lui 1 şi 0. Dacă sîntem în lumea numerelor naturale atunci ∞_1*=∞*+1, ş.a.m.d. automat se poate calcula simetricul lui ∞_1* sau valorile intermediare exterioare acestuia faţă de 0*. În acest fel constatăm că orice mulţime de valori ale produsului lor din domeniul (0_1*, ∞_1*) este valabilă şi pentru domeniul (0_1*, ∞_1*) dar şi pentru domeniile (0_1*, 0*) sau (∞*,∞_1*), diferenţa dintre ele este ordinul de cuantificare, între ∞_1* şi ∞* dat de λ(1). Unde λ poate lua toate valorile lui ∞*. Putem spune astfel că orice valoare a lui ∞_1* este o valoare a lui λ cuantificată cu ∞*. Caz particular ∞_1*=∞*+R (mulţimea numerelor reale), pentru orice număr r există un 0_1*(R). Pentru orice număr al lui R, 0_1* are un corespondent ∞_1* prin cuantuificarea cu ∞*şi evident simetric al lui 0_1*(R).

Ţinînd cont de ceea ce am adus în prim plan pînă acum nu putem nega realitatea realţiei 0*∞*=c dar nici pe cea a lui 0 unde 0©=1 cu atît mai mult că nu putem nega existenţa nelimitatului cum nu putem nega existenţa unui 0 ca nelimitat de mic. 0 şi © fiind limitele nelimitate ale lui 0* şi ∞*. Să nu uităm un aspect important, să nu facem greşeala să credem că realţiile 0=c/©, sau 0=1/© sînt relaţii neconvenţionale ele rămîn convenţionale sau mai exact neconvenţionale pentru cunoaşterea actuală dar nu neconvenţionale adică nelimitate. În nelimitat aceste convenţii devin absurde deoarece relaţia 0©=1 dispare ca noţiuni sau sensuri iar la nelimitat 0 şi 1 devin absurde. Să nu uităm de asemenea că orice relaţie, funcţie, formulă, etc. matematică sau de altă natură este o cuantificare sau decuantificare a lui 1 şi 0 ca dovadă că orice operaţie este prelucrată de un calculator oricît de sofisticată ar fi iar calculatorul nu ştie decît 0 şi 1. Ba mai mult o să constatăm că şi sentimente sau energii sînt cuantificări de 0 şi 1 şi că acestă cunoaştere este energie care produce legături sau desface lgături ceea ce este echivalent lui 0 şi 1. Fenomenul este la fel şi în creierul oricărei fiinţe raţionale sau mai puţin raţionale, doar că are alte energii şi alte sisteme de numeraţie, de legături. În convenţional 0* sau ∞* sînt de fapt o cuantificare sau decuantificare de 1, în timp ce în neconvenţional cuantificarea este pentru 0 ceea ce ne spune că universul neconvenţional este doar o multiplicare de 0 adică cuantificare de secveţe neconvenţionale 0 în nelimitat. Diferenţa între om sau orice alte **entităţi/univers** şi Universul în Sine este datorată energiei care produce procesarea datelor adică a vitezei în **spaţiu/timp** în care se produce procesarea şi modul procesării respectiv **transformarea/spţiu/timp** care produce acestă procesare. În spatele lor este doar energie în forme şi legături diferite. După toată acestă teorie cred că putem spune că în lume numerelor foarte mici sau foarte mari putem lua un ∞* (oricît de mare, dar niciodată nu va fi nelimitat) astfel încît dincolo de mulţime numerelor (0*,∞*) să putem calcula un ∞_1* =∞*+ λ(1), astfel încît să putem calcula un 0_1*=1/(∞*+ λ(1)) respectîndu-se relaţia 0_1*∞_1*=1. Este o evidenţă că Universul în Sine ca şi 0 sau nelimitatul sînt unice chiar dacă nu vom cunoaşte niciodată limitele lui în ambele sensuri.Vrem nu vrem **entităţile/univers** sîntem şi noi şi toate sînt valori intermediare ale domeniului (0, ©) unde produsul lor este 1. 0 şi © sînt tot constante dar paradoxal constante nelimitate (în timp ce ∞* este un infinit limitat şi constantă, © este o constantă nelimitată) ceea ce în convenţional nu se poate convenţionaliza, în plus acestea (0 şi ©) nu mai pot fi cuantificate dincolo de ele deşi avem tendinţa să credem acest lucru. Acestă relaţie lim 0*∞* =1 cînd 0* »o şi ∞*»© este o axiomă care nu trebuie să necesite demonstraţie şi nici nu are demonstraţie. Trebuie să ţiem cont doar că acestă limită devine 0©=1 sau 0=1/© relaţie valabilă în convenţional. O să spună unii că nu este obligatoriu 1 ci poate fi orice valoare c. Fals pentru că dacă în loc de 1 punem o altă valore 0,1 spre exemplu acest lucru se traduce prin mărirea nelimitatului (reducere la absurd) ©, adică relaţia ar fi 0=1/10© ceea ce presupune mărirea nelimitatului, (0 ar trebui să devină şi mai mic) în acest punct relaţia este absurdă pentru că nici 0 şi nici © nu mai sînt cuantificabile. Această relaţie este un adevăr recunoscut dar nedemonstrabil şi este relaţia generalizată între limitele oricărei **entităţi/univers** adică **transformare/spaţiu/timp** şi **formă/existenţă/spirit**. Un caz particular sîntem şi noi oamenii pentru om 0* este naşterea lui în timp ce ∞* al lui este moartea lui şi asemănător pentru fiecare parametru al său. Produsul lor este c ∈(0,1) pentru perioda existenţei sale (perioada convenţională) şi 1 pentru limita existenţei sale cînd el devine **entitate/univers** constantă, finită şi invariabilă în nelimitat. În acel moment toate variabilele lui devin constante mai mari sau mai mici dar invariabile definitiv. Omul devine atunci o unitate (**entitate/univers**) trecută. Pentru orice **entitate/univers** produsul 0*∞*=c în timpul existenţei dar la limita existenţei sale devine 1. Aşa cum am arătat în timpul existenţei valorile pot depăşi domeniul (0,1) pentru valori nesimetrice, în afara limitelor 0* şi ∞* şi nu în interiorul lor.

La fel ar fi şi cu Universul în Sine dacă ar apare şi dispare dar el nu are această posibilitate convenţională el este neconvenţional şi 0 şi 1 sînt simultane, noi doar convenţional avem produsul limitelor sale 1, la limita lui toate elementele sale devin constante şi invariabile şi nu ar mai putea reveni la o nouă **entitate/univers** fiind nelimitat. (ar însemna să devină limitat) Relaţia 0©=1 nu ar mai fi valabilă şi s-ar transforma în 0*∞*=c ceea ce ar contrazice realitatea deoarece dincolo de 0 şi © nu mai există, în realitate 0 şi © nu există pentru noi sau orice **entitate/univers** sînt doar o extrapolare, ele sînt ceva ce noi nu vom putea defini niciodată. 0 şi © reprezintă unicitatea, perfecţiunea universului în sine, nelimitatul lui, iar produsul lor **existenţa/inexistenţa** sa. 0 este o constantă nelimitat de mică, invarabilă, © este constantă nelimitat de mare invariabilă. Şi 0* şi ∞*sînt constante nelimitat de mici sau de mari pentru noi convenţiile cît existăm dar după finalul existenţei noastre adică în neconvenţional ele devin clar finite. Cît existăm datorită variabilităţii noastre ni se pare că ele sînt variabile, în realitate noi nu le cunoaştem doar cei care ne urmeză constată invariablitatea lor după moartea noastră.

Legea acumulării şi divizării sau legea A*+D*

Plecînd de la definiţie Extenica {=rezolvarea problemelor contradictorii in orice domenii (rezolvarea problemelor inconsistente (contradictorii)} să ne oprim la soluţiile contradictorii din matematică. Toate cazurile de nedeterminare din matematică au corespondenţe în orice ştiinţă sau neştiinţă ca şi teoria lui 0* şi ∞*. Grăbirea convenţională (accelerarea convenţională) se produce nu doar în matematică, fizică, chimie sau alte ştiinţe ci şi în neştiinţe ca şi în natura cosmică. Exemplu formarea Big-Bang nu este altceva decît acumulări succesive de planete sau alte sisteme solare sau de altă natură. Apoi acest Big-bang de la acumulare a trecvut la divizare (expansiune) a

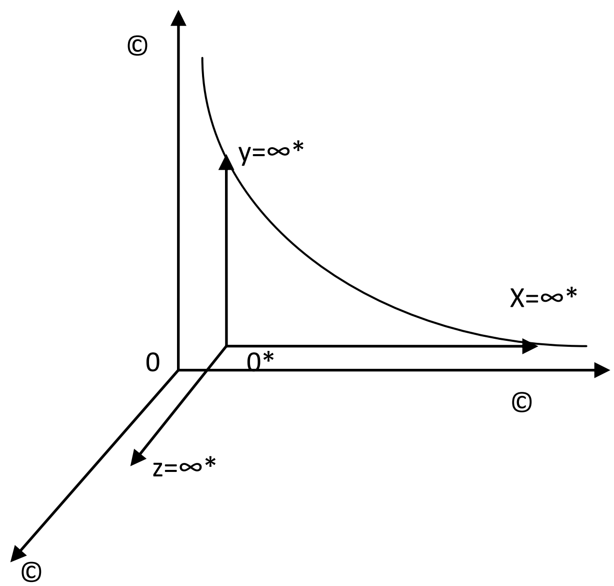
materiei/energie neconvenţionale şi nu doar în forma neconvenţională ci şi în formă convenţională. De fapt Big-bangul era deja o **materie/energie** convenţională dar nu pentru capacitatea nostră de cunoaştere actuală. Această **materie/energie** deja convenţională a accelerat procesul convenţional formînd energii convenţionale (multiplicări ale acumulărilor succesive cum sînt înmulţirea şi împărţirea faţă de adunare sau altele) depăşind limitele gravitaţiei neconvenţionale şi creează planete, vegetaţie, apă, viaţă, etc. într-un ritm mult mai mare decît acumularea gravitaţională. La fel şi omul cu energiile sale convenţionale accelerează fenomenele în mod convenţional specific **entităţii/univers** om şi elementelor sale **formă/existenţă/spirit** ca şi elementelor acestora în raport de capacităţile lui convenţionale sau de necesităţile lui convenţionale. Reamintesc că orice număr este repezentat de cifra unu şi multiplii şi submultiplii acesteia în convenţional şi de cifra 0 în neconvenţional şi că orice valoare a unei funcţii indiferent de domeniul de definiţie este un multiplu sau submultiplu al lui 1. Calculatorul este unul din cele mai sigure argumente deocamdată, el lucrînd doar cu 0 şi 1 în timp ce Universul în Sine doar cu 0 plecînd de la relaţia 0©=1, adică 1 este un multiplu nelimitat al lui 0 în neconvenţional. Relaţie valabilă şi în convenţional dacă folosim relaţia 0*∞*=1. Şi acest 1 este multiplu infinit de 0*, mai mult trebuie să ţinem cont că orice număr în orice sistem de numeraţie foloseşte aceleaşi simboluri (respectiv cifre) şi ca atare mutiplii şi submultiplii ai lui 1. Pînă şi cele 10 cifre de la 1 la 10 sînt multiplii sau submultiplii ai lui 1 iar în matematica convenţională nu există alte cifre. Am definit în acest fel o nouă lege T*, legea acumulării şi divizării Universului în Sine, adică legea A*+D* care se defineşte astfel:

- orice **entitate/univers** este acumlare sau divizare a lui 1 în convenţional sau de 0 dacă vorbim de neconvenţional.

Această lege are şi formularea matematică prin relaţia 0© =1, relaţie care se traduce prin faptul că un număr nelimitat de 0 (este vorba de un 0 neconvenţional, nelimitat de mic, secvenţa neconvenţională 0) de **entităţi/univers, entităţi/univers** 0 nelimitat de mici dau o unitate (o **entitate/univers** unitate 1). }n particular rela'ia devine convenţională şi se scrie 0*∞*= 1, care ne spune acelaşi lucru dar foloseşte valori convenţionale.

Cazuri particulare din matematică şi neconvenţionalul lor

Să analizăm cîteva cazuri de nedeterminare din matematică, ∞-∞= nedeterminat, ∞/∞= nedeterminat, sau 0/0 = nedeterminat, 0∞= nedeterminat, 0^0= nedeterminat, ∞^0= nedeterminat şi 1^n (n=∞). Trebuie să aducem în discuţie la aceste cazuri toate cazurile asimptotice ale funcţiilor care sînt de aceiaşi natură cu aceste cazuri nedeterminate. Dacă vom considera infinitul asimptotic (şi nu ∞, nelimitatul) un ∞* atunci vom şti toate valorile funcţiei inclusiv pentru ∞*. Desigur că există şi valori dincolo de ∞* dar acestea ori nu ne interesează ori devin imposibil de determinat, ori dacă este nevoie extrapolăm valoarea lui ∞* cu un λ(1. Cazul parabolelor este evident în acest sens dar nu trebuie să ne oprim doar la parabolele matematice sau fizice sau chimice relaţia cu 0* şi ∞* este valabilă oricărei parabole literare, sensibile, logice sau ilogice, ştiinţifice sau neştiinţifice. Ca principiu general ar trebui modificat sistemul de cordonate în raport de 0, 0*, ∞* şi ©, astfel graficele ar putea fi în felul următor,



Unde 0*∞*=1, iar 0© =1.

- ∞-∞= nedeterminat, este o variantă neclară pentru că noi sîntem limitaţi şi din acest motiv ∞ nu este nelimitat ci limitat, chiar dacă noi în intuiţia noastră intuim nelimitatul lui. De aceea revenim la semnificaţiile introduse de mine respectiv 0, 0*,∞*, ©, care definesc mult mai exact realitatea (chiar dacă nu Realitatea în Sine) şi totodată extindem infinitul nostru limitat la nelimitat. În acest caz dacă înlocuim ∞ cu ∞* rezultă relaţia ∞_1*-∞*= 0* unde eroarea este dată de mărimea lui ∞* şi numai dacă cele două valori ∞_1* şi ∞* sînt diferite, evident ∞_1*-∞*=0*, unde toate cele trei valori sînt numere concrete convenţionale iar relaţia este o realitate a noastră o realitate convenţională. Fiind în convenţional putem alege orice valoare pentru ∞_1*şi∞* iar diferenţa lor verifică relaţia prezentată. Dacă ∞_1*şi∞* tind către © este evident că diferenţa lor tinde către 0 neconvenţional, nelimitat de mic şi nici în acest caz nu putem verifica nedeterminarea lui.

- ∞/∞, aşa cum am arătat mai sus trebuie să facem diferenţierea între posibilităţile noastre convenţionale şi neconvenţional prin introducerea noilor convenţii. În acest caz relaţia poate fi scrisă ∞_1*/∞*=1+a,unde a ∊(0,1) dacă ∞_1*>∞*sau ∞_1*/∞*=1-a unde a ∊(0,1) dacă ∞_1*<∞*. În cazul în care ∞_1*şi ∞*»© relaţia devine ©/©=1 dar totodată devine şi imposibilă deoarece convenţional dispare totul ca sens, chiar şi sensurile.
- 0/0, este analog lui ∞/∞ doar că reaţia devine 0_1*/0*=1+a unde a ∊(0,1) dacă 0_1*>0* sau 0_1*/0*=1-a dacă 0_1*< 0*. În realitate oamenii nu caută neapărat aceste valori convenţionale ei caută nelimitatul pe care oricum nu îl vor găsi şi chiar dacă prin absurd l-ar găsi acesta este dispărut în acelaşi moment.
- 0^0, se transformă în 0^{*0*}=a iar în acest caz devine o valoare determinată.
- ∞^0, devine ∞^{*0*}=a, de asemenea valori determinate şi nu nedeterminate pe toată perioada existenţei.
- 1^n, unde n=©. Caz nedeterminat oare de ce? Dacă n= ∞* atunci valoarea 1^n= 1 pentru orice n. Dacă mergem la limita lui 1^n către nelimitat doar la limită aceasta nu este 1 dar acolo nu mai este nimic sau este totul simultan pînă la desfiinţarea convenţiilor de orice natură inclusiv 1 şi ©.
- 0∞, în realitate relaţia convenţională 0∞=nedeterminat nu este valabilă, sau este doar convenţional valabilă dacă dorim să impunem acest lucru, deoarece raprtul lor nu este o variabilă ci o constantă. În realitate 0∞=c unde c are valori în orice sistem de numeraţie şi respectă relaţia c/∞ =0. Este greu să cred că nedeterminat/∞=0 mai ales dacă nedeterminatul este ∞ sau 0 sau nelimitat, sau orice valoare între ∞ şi ©, adică dincolo de infinit în nelimitat. Îl vom analiza un pic mai special plecînd de la relaţia 0*∞*= 1, care este o relaţie perfect valabilă atîta timp cît 0* ≠ 0 şi ∞*≠ ∞ ≠ © iar 0* şi ∞* sînt simetrice faţă de 1, în aceste condiţii există o funcţie f(x,y) = 1, unde x=0*, iar y=∞* care să verifice relaţia. Evident în aceste condiţii ∞*/© = 0, dar şi c/©=0, unde © = nelimitat. În acest fel definim un 0* şi ∞* care pot fi cuantificate cu orice $\lambda \neq 0$ şi $\lambda \neq$ ©, care poate aparţine, sau nu, intervalului (0*, ∞*). Relaţia 0*∞*= 1 este un caz particular al relaţiei 0*∞*= c pentru că în matematică 0*∞*=c sau 0*∞*= nedeterminat dar acest nedeterminat este nedeterminat ca valori ale lui c şi nu că c ar fi mai puţin constantă.Ţinem cont în acest sens şi de relaţia 0*=c/∞*, adică 0*∞* = c, întrucît ∞*≠ 0, relaţie recunoscută de matematica noastră convenţională. Este de asemenea evident că 0*∈(0,1). În acest fel printr-un coeficient λ putem merge în lumea numerelor foarte mici, sau foarte mari. În orice structură, formulă, **entitate/univers**, convenţie, trebuie introdus un coeficient care face parte din universul numerelor foarte mici sau foarte mari care să corecteze relaţia convenţională şi care să reprezinte relativul oricărei relaţii, legi, etc. Trebuie să plecăm de la faptul că orice număr convenţional este fomat din cifra unu prin multiplicare sau demultiplicare adică prin adunare şi scădere şi nimic altceva. Singura diferenţă este că această multiplicare (adunare şi scădere) se poate grăbi

(convenţional) prin artificii de înmulţire şi împărţire sau alte operaţii şi funcţii, dar toate absolut toate pleacă de la ceea ce v-am prezentat. În neconvenţional toate cifrele şi relaţiile îndiferent de ştiinţă pleacă de la cifra 0. Adică cifra 1 este o sumă nelimitată de cifre 0 (0©=1 este adevărată) sau în convenţional este o multiplicare a lui 0* cu ∞*, respectiv 0*∞*= 1. Orice sistem de numeraţie pleacă de la acest număr şi putem scrie fără dubii că mulţimea numerelor naturale N este de fapt N= 1+N* sau N=1+ ∞* unde ∞*∈N* (N* =N-1). În acest fel se poate scrie că orice sistem de numeraţie dincolo de ∞* pleacă de la ∞* la care se adaugă un număr nelimitat de alte diviziuni evident dintr-un sistem de numeraţie sau altul. 0* şi ∞* teoretic nu mai sînt nedeterminate dacă le-am considerat diferite de 0 sau de ∞, ele au o valoare bine determinată dar nu le ştim valoarea şi pot lua o mulţime de valori ceea ce în convenţional este mai greu. Să presupunem că 0*= 1/a unde a ≠ (0,1), în acest caz există un număr ∞*=a astfel ca 0*∞*= 1. Dacă luăm un şir de valori ale lui 0* şi ∞*, respectiv 0n* şi ∞n*, obţinem un şir de limite (0n*,∞n*) cu relaţia dintre ele 0n*= 1/(∞*+n) şi ∞n*=∞*+n (n poate fi natural, real, raţional, etc.) asta implică faptul că pentru orice 0n*există un ∞*+n ca relaţia să rămînă valabilă. Asta presupune că pentru intervalul 0n* şi ∞n*există limitele 0,1 al produsul lor, limite între care putem lua orice valoare pentru 0n* şi ∞n*.Trebuie să remarcăm faptul că atît 0n* cît şi ∞n* nu sînt valori variabile ci constante chiar dacă ele sînt infinite către mărime nelimitată sau către un 0 nelimitat de mic. Aceste valori sînt limitele finite ale unei **entităţi/univers** (om, calculator, telescop, planetă, etc.) Filozofic vorbind produsul existenţial de la naşterea unei **entităţi/univers** (0*) pe toate direcţiile cu limita infinitului său (∞*) este 1 în realitatea convenţională dar şi la limita lor neconvenţională în condiţiile enunţate mai sus. Doar pentru Universul în Sine valoarea produsului 0©=1 în orice condiţii, în timp ce pentru orice valoare mai mică de © produsul este între (0,1) indiferent cît de mari sau de mici sînt valorile 0 şi ©. În Universul în Sine produsul 0*∞*= 1 este cuprins între (0 ,1). De aici pînă la matematica neconvenţională mai avem un pas, vorbind de neconvenţionalul cunoaşterii noastre şi nu de neconvenţionalul în sine. Putem scrie realaţia 0*∞*= c unde c ≠ (0,1) doar dacă în realitate putem presupune un ∞* nesimetric faţă de 1, în acest caz valoarea produsului este mai mare sau mai mică decît 0 sau 1. Dacă ∞n*≠∞* acesta poate fi ∞n*=∞*+a sau ∞*=∞n* -a, pentru orice a ∈C. În acest caz 0*∞*=1 devine 0*(∞n*-a) = 1, adică 0*∞n*=1+a0*.(toate operaţiile sînt valabile pentru că toate numerele sînt diferite de 0). După această explicaţie putem spune că între simetricele 0* şi ∞*produsul lor este între 0 şi 1. Adică c ∈(0,1) pentru 0* ∈(0,1) în timp ce dacă 0n*=0*+a, atunci din relaţia 0n*∞*= c rezultă a=(c-1)/ ∞* mai mic decît 1. Între 0 şi 1 sînt nelimitate subdiviziuni dar şi în afara lor întrucît nu putem lucra cu nelimitatul ne vom opri întotdeauna la un limitat 0*şi ∞* care sînt valori cuantificabile care se pot extinde dincolo de numerele mari sau mici actuale. Relaţiile rămîn valabile ca extrapolare şi în neconvenţional dar este doar o ipoteză niciodată verficabilă pentru că nelimiattul nu ne aparţine. Pardoxal însă în nelimitat existenţa şi inexistenţa nu pot fi depăşite iar ele reprezintă 0 şi 1 neconvenţional, nelimitate, adică dincolo de orice 0*şi

∞* există un singur Univers în Sine. Relaţia 0© nu este niciodată 0 şi nici valori între 0 şi 1 pînă la limita nelimitatului © şi al lui 0, care de fapt ca un paradox nu există (devin absurde ca noţiuni convenţionale). La valori nesimetrice intermediare limitelor lor (deoarece produsul lor nu permite existenţa nesimetrică a unuia dintre ele în afara lor) produsul lor nu poate fi decît 1 pentru orice valoare conform demonstraţiei anterioare. Dovada este însăşi existenţa **entităţilor/univers** şi nelimitatul lor ca număr, formă etc. cu probabilitate de apariţie 1/ © şi posibilă doar datorită relaţiei 0©=1. Este însă inutil să vorbim convenţional de neconvenţional motiv pentru care ne oprim la relaţia 0*∞*= 1 şi la convenţiile noastre. Preluînd aceste lucruri filozofic vom constata că orice realitate convenţională respectă acestă regulă şi să urmărim sentimentele care deşi au valori de la 0* la ∞* ele sînt un singur sentiment sau orice univers este o singură entitate simultan. Sentimentele, existenţa, forma, etc. sînt acest nedeterminat c cu valori între 0*şi ∞* dar în acelaşi timp nu depăşesc valoarea 1 în condiţii de simetrie ci doar anomalia lor face valori dincolo de 0 şi 1. Noi sîntem valorile nedetermnate ale Universului în Sine la fel cum pentru noi sentimentele noastre sînt aceste valori nedeterminate. Toate aceste valorii convenţionale sînt constante şi limitate (bine determinate ca **transformare/spaţiu/timp** şi **formă/existenţă/spirit** faţă de Universul în Sine) chiar dacă noi nu sesizăm acest lucru. 0©=nedeterminat mi se pare improprie pentru că în Universul în Sine nimic nu este nedeterminat faţă de Universul în Sine **entităţile/univers** sînt nedeterminate pentru noi sau alte **entităţiunivers** dar nu pentru Universul în Sine. Este greu de acceptat şi pentru că este mai greu de acceptat relaţia nedeterminat/ ©=o mai ales cînd nu ştii valoarea nedeterminatului care poate fi însăşi © şi în acest caz cu siguranţă nu mai respectă relaţia convenţională. Dacă însă nedeterminatul este o valoare constantă inclusiv ∞* atunci relaţia devine logică în convenţional. Chiar şi în cazul c/∞*=0* pentru că este o convenţie iar relaţia este logică în convenţional. Unii poate vor pune la îndoială logica ei dar atît timp cît 0* şi ∞*sînt valori simetrice relaţia este valabilă indiferent cît de mari sau de mici sînt aceste valori. A nu se confunda valoarea c cu viteza luminii. Aceste valori aparent sînt variabile dar variabilul lor este de fapt datorat nouă care sîntem variabili şi nu acestor valori constante ca şi în cazul mişcării cînd ne mişcăm noi avem senzaţia că se mişcă obiectele care stau pe loc, (în relativitatea absolută chiar nu se ştie cine stă şi cine se mişcă) noi şi convenţiile noastre sîntem relativi şi nu Universul în Sine neconvenţional şi nelimitat şi invariabil, adică perfect. Noi sîntem imperfecţiunea perfecţiunii fără de care nici perfecţiunea nu ar exista dar nici invers. În concluzie raportul c/©=0 nu este real pentru noi ci corect este 1/©=0, dar nici ∞*/©=0 sau c/∞*=0 nu sînt corecte, ele ne arată totodată un singur lucru că produsul 0*∞*sau 0© nu este nedeterminat ci o valoare constantă nedeterminată adică 0*∞*=c sau 0©=1. Această constantă reprzintă **entităţile/univers** din Universul în Sine şi valori între 0 şi 1 sau raportate într-un fel sau altul la 0 şi 1 cu probabilitatea logică de 1/∞*=0*≠ 0 sau 1/©=0.

Lumea realităţii noastre convenţionale (iluzierealitate) este aceasta, adică cea cuprinsă între 0* şi ∞*, aceasta este de fapt realitatea cunoaşterii noastre şi a existenţei noastre spirituale indiferent ce credem sau ce spunem noi sau alte **entităţi/univers**. Din întîmplare acestă realitate este simultană cu o Realiatate în Sine dar şi cu o realitate în care există 0 ca şi nelimitatul, adică şi ceea ce există dincolo de 0* şi ∞* şi ambele suprapuse (simultane) cu o lume nelimitată în care toate convenţiile noastre sau ale oricărei **entităţi/univers** chiar dacă există nu mai pot fi reflectate convenţional de nici o **entitate/univers** deorece 0 şi © devin unul şi acelaşi lucru simultan asemănător cifrei 0 care este şi pozitivă şi negativă în acelaşi timp în care nu este nici pozitivă nici negativă, nemaiputînd face o astfel de interprtare. Aceste relaţii interpretate filozofic ne spun ceea ce ne spune şi realitatea, că dincolo de limitele noastre adică între 0* şi 0 sau între ∞* şi nelimitat sînt alte limite 0n* şi ∞n*cu nelimitate subdviziuni şi variante şi **entităţi/univers** dar diferite în acelaşi **timp/spaţiu** (cuantificate în plus sau minus, **pozitiv/negativ**) şi tot aşa merg în nelimitat indiferent cît de mare sau de mic este infinitul nostru convenţional.(∞*)

O funcţie **entitate/univers**

Să ne imaginăm o funcţie pentru orice **entitate/univers**, este clar că nu putem să producem o funcţie care să înlocuie perfect o **entitatea/univers** şi că trebuie să ne folosim de trucurile convenţionale ca în cinematografie (cele 24 de imagini) sau în matematică multiplicarea rapidă sau demultiplicarea rapidă (respectiv înmulţirea şi împărţirea sau alte funcţii), în pictură perspectiva, în literatură imaginile fowlkneriene dar care sînt o mulţime şi în fizică, chimie, etc. În cinematografie ştim că mişcarea să redă prin succesiunea rapidă a 24 sau mai multe imagini, în pictură perspectiva este dată prin linii carea pleacă dintr-un punct iar paralelismul prin linii care se intersectează dincolo de peisaj, în matematică orice operaţie în afară de adunare este un truc, o cuantificare rapidă cum spun eu în încercarea de a scurta timpul sau spaţiul sau transformarea, în fizcă se fac modele mecanice, electrice sau de altă natură pentru studiul fenomenelor în timp şi spaţiu chiar dacă ştim că nu sînt realitatea în sine. Şi în cazul nostru trebuie să găsim un truc filozofic (un model de funcţie) dar să şi ţinem cont că singura legătură dintre transformările a două **entităţi/univers** este cuantificarea sau decuantificarea adică în convenţional adunarea sau scăderea în variantele lor convenţionale diverse. În acest fel orice relaţie matematică sau fizică sau de altă natură nu trece una la alta decît prin cuantificae sau decuantificare. Dar să trecem la funcţia noastră unde cea mai complexă legătură şi care doar aparent redă simutaneitatea (ca şi adunarea şi scăderea care aparent dau simultaneitate) este funcţia funcţiei adică F[fn(x)] unde n»© iar x »©.Plecînd de la această variantă să ne imagină o **entitate/univers** ca o combinaţie de două funcţii E[fn(x,y,z)]U[fn(α,β,γ)] unde U este universul iar E este entitatea iar x=forma, y=existenţa, z=spiritul, α=transformare, β=spaţiu, γ=timp . La rîndul lor fiecare din aceste variabile sînt funcţii compuse de alte variabile respectiv x=f (a_1, b_1, c_1, .. etc.) unde a,b,c, .. etc.= parametrii formei, y= f (a_2, b_2, c_2, .. etc.) unde a_2, b_2, c_2, .. etc.= parametrii existenţei (gol, plin) iar y= f (a_3, b_3, c_3, .. etc.) unde a_3, b_3, c_3, .. etc.= parametrii spiritului (memorie, gîndire, intuiţie, instinct, etc.). Toate acste funcţii şi parametrii merg în nelimitat în funcţie de alţi parametrii şi alte funcţii dar noi fiind în convenţional ne putem opri la o convenţie acceptată la care vom adăuga o funcţie de corecţie f (λ) iar λ= λ(1) care să reprezinte corecţia şi evident aprţinînd lumii numerelor foarte mici, adică relativul **entităţii/univers** datorat parametrilor necunoscuţi interiori sau exteriori şi acestă funcţie nu trebuie să lipsească de la nici o **entitate/univers**. Acestă funcţie rămîne o convenţie, limitată şi relativă pe care în raport de convenţiiile noastre o putem neglija sau nu. Plecînd de aici şi încadrînd orice **entitate/univers** în limitele ei de existenţă adică 0* şi ∞* pentru orice parametru, ţinînd cont că valorile simetrice în intervalul 0* şi ∞* pot fi stabilite avem o imagine truc a unei **entităţi/univers**. Această funcţie adaptată pentru fiecare **entitate/univers** în parte o putem utiliza pentru rezovarea contradicţiilor ei sau cel puţin pentru depistarea punctelor sensibile în raport de fiecare parametru **pozitiv/negativ**. Lumea acestor parmetri este cea prezentată în schema neconvenţională a parametrilor unei **entităţi/univers**. Cu o astfel de funcţie putem determina elementele ei neutre în raport de **spaţiu/timp** sau de elementele lor de comparaţie limitate în raport de relativul acestei funcţii f (λ). Realitatea ne spune de la început că această funcţie trebuie să fie o simultaneitate **finit/infinită** de funcţii limtate şi relative în timp ce funcţia f (λ) deşi limitată la lumea numerelor foarte mici ea este nelimiată ca diviziuni.

Lumea reală în raport de <A> şi <antiA>.

Realitatea noastră dar şi realitatea în sine sînt o simultaneitate de <A> şi <antiA> iar <neutA> nu există decît convenţional, teoretic <neutA> este tot o simultaneitate de <A> şi <antiA>, un S[(<A>/<antiA>)] unde <A> şi <antiA> au valori **pozitiv/negative** în permanenţă, convenţional spus. În neconvenţional <neutA> nu există dar ca orice paradox totul este un <neutA> ca o simultaneitate de <A> şi <antiA>. Adică să nu ne facem nici o iluzie că dacă raportul <A>/<antiA> =0,99 cele două sînt separate sau că una din ele nu există, atîta timp cît există un raport există simultaneitatea lor. <neutA> nu există dar aparţine oricărei valori ale raportului <A>/<antiA>, adică filozofic convenţional şi neconvenţional <neutA> nu există dar face parte din orice raport <A>/<antiA> al oricărei **entităţi/univers** inclusiv formule matematice, fizice, chimice, etc. ca şi în neutrosofie simultaneitatea lui <A> şi <antiA>. Orice fenomen convenţie are o reprezentare matematică, fizică, chimică, etc. adică o filozofie matematică, chimică, etc. ca şi o filozofie generală **entitate/univers** ca dovadă că în principiu pe calculator se poate studia orice fenomen sau transformare, mai bine sau mai puţin bine în raport de capacitatea convenţiilor noastre. Aceste reprezentări sînt funcţii de <A> şi <antiA> , necunoscutele lor sînt şi ele simultaneităţi de <A> şi <antiA> (ca orice **entitate/univers**). <A> şi <antiA> au acelaşi domeniu de definiţie, deoarece A ∊(0*,∞*) iar 0*∞*=1 dar şi <antiA>∊(0*,∞*), ţinînd cont că în afara lui <A> nu există <antiA> în acest caz rezultă acelaşi domeniu de definiţie iar 0*∞*=1. A crede că există un <antiA> în afara domeniului de definiţie al lui <A>, este ca şi cînd am spune că poate exista lumină fără întuneric sau **entităţi/univers** fără materie sau fără energie sau pozitivul fără negativ, sau o singură latură a oricărei contrarii, etc. în acest caz <neutA> este un element de simetrie în raportul dintre <A> şi <antiA> cum este 1 pentru produsul limtelor lor ceea ce putem spune că 1 este simetricul lui <A> şi <antiA> respectiv <neutA>=1. Doar 1 este neutru şi faţă de <A> şi faţă de <antiA> în raportul dintre ele adică <A> / <antiA> =1=<neutA>. Depinde de noi unde situăm această valoare a lui 1 pe axa dintre ele. Nu putem spune că valarea raportului este 0 sau poate fi zero niciodată deoarece valoarea fiecăreia este diferită de 0 ca să existe, chiar dacă şi 0 poate fi un neutru pentru **pozitiv/negativ** de exemplu dar nu ca produs ci ca adunere ceea ce noi nu comentăm momentan. Este evident că pentru orice valoare c ∊ (0*, ∞*) produsul 0*c =a<1. Încă un argument că limita intervalului adică ∞* verifică relaţia 0* ∞*=1. Pentru a demonstra că a <1 este suficient să luăm un 0*<∞n*<∞* pentru care relaţia nostră devine 0* ∞n*=a, dar ∞n*=∞*-k, unde 0*<k<∞* ceea ce duce la 0*(∞*-k) =a de unde rezultă 1- k0*=a ceea ce evident ne confirmă ipoteza deoareace şi 0* şi k sînt numere diferite de 0, dacă ar fi 0 a=0 adică ∞n*=∞*. În raport de acest element de echilibru fiecare valoare are un simetric în intervalele respective, nu numai atît toate elementele unei **entităţi/univers** respectă această regulă a simetriei limitelor sale. În mod convenţional putem alege alte valori pentru simetrie dar toate sînt doar cuantificări ale lui 1 şi al simetricelor acestuia. În matematică acest <neutA> există ca şi în alte ştiinţe sau neştiinţe dar nu există ca realitate neconvenţională.

Noi şi limitele noastre convenţionale

Poate unii o să-mi spună că viaţa unui om plecă de la 0 şi se termină ca exemplu la 50 de ani şi că produsul limitelor sale este ori 0 ori nedeterminat ori 50, în nici un caz 1. Cu părere de rău le spun că pe de o parte niciodată omului nu-i putem determina cu precizie de 100% anul naşterii sau al morţii (nu există sistem de măsurare perfect) iar pe de altă parte condiţia de bază este ca cele două valori să fie simetrice şi să acceptăm o anumită eroare convenţională.(eroare care ne redă relativul convenţiei) Simetricul lui 50 este 1/50, adică 0,02.Eroarea find de (0-0,02)/50 =0,0004 faţă de 0. În plus ca realitate naşterea ca şi mortea nu există este o transformare continuă. Valorile sînt realative ca orice valoare convenţională. Din cauza acestor motive putem convenţional alege oricînd un 0* în raport de ∞* (50 de ani) astfel ca relaţia să fie valabilă în raport de eroarea pe care o dorim sau o acceptăm. În condiţiile noastre relative 0* (-1,0) sau 0*∊(0,1) iar 50=∞*∊(49,50) sau (50,51). Trebuie însă luat în calcul că vorbim de valori convenţionale mici sau mari dar nu de valori convenţionale foarte mici sau foate mari care se pot obţine prin multiplicarea domeniului(0*,∞*) cu orice $\lambda(1)$. În cazul numerelor foarte mici sau foarte mari echivalenţa se menţine dar eroarea se micşorează. În cazul numerelor mici sau mari vorbim de numere dar la numere foarte mici sau foarte mari vorbim doar de simboluri ale numerelor. Orice **entitate/univers** nu poate să-şi cunoască simetria deorece nu-şi atinge limitele şi ca atare nu poate face produsul lor, valabil şi pentru Universul în Sine. Ţinînd cont că orice convenţie, **entitate/univers** este definită de domeniu de definiţie, limite, elemente de echilibru şi de comparaţie fiecare din aceste elemente are propria-i determinare şi ca atare propriile limite la care produsul lor simetric este 1. Limitele oricărui parametru sînt definite de 0*∈ (0, 1) şi ∞*∈(1, ©) sau mai exact intervalului ∞*∈(∞*-1, ∞*) sau (∞*,∞*+1), adică respectă regula neutrosofică a domnului Smarandache respectiv ∞*∈ (∞*-ε,∞*+ε) iar 0*∈ (0, 1), 0 şi 1 echivalentele lui 0*+ ε şi 0*- ε. Trebuie supus unei analize această relaţie deorece folosim un ε dar în realitate relaţia este ∞*∈ ($\infty^*-\varepsilon_1,\infty^*+\varepsilon_2$) şi doar în cazuri particulare $\varepsilon_1= \varepsilon_2$. În realitate niciodată nu este valabilă relaţia $\varepsilon_1= \varepsilon_2$ pentru că atunci ar putea fi determinat orice număr în mod perfect şi nu relativ ştiind că este media domeniului său.

Energie neconvenţională

Singura energie nelimitată este gravitaţia de fapt nu gravitaţia ci o forţă de atracţie care se transformă convenţional în gravitaţie. Dovada celor spuse de mine ste însăşi acea supă descoperită în Elveţia unde sînt convins că deşi nu mai putem separa convenţional energia de materie este şi energie şi materie iar materia este sub atracţia unor energii necunoscute încă. Această atracţie este echivalentul acumulării universale în timp şi spaţiu şi vinovatul existenţei oricărei transformări în Universul în Sine. Trebuie să ţinem cont şi de contrariul ei respectiv respingerea sau echivalentul descompunerii al împingerii materiei în afara ei echivalent al convenţionalei pierderi sau scăderi din matematică. În termeni astronomici contracţia universului şi expansiunea lui. Orice **entitate/univers** este efect al acestei acumulări şi energiei ei neconvenţionale sau în termeni convenţionali simultaneitate **materie/gravitaţie**. Nimic nu s-a format în univers fără gravitaţie chiar şi energiile convenţionale respectiv electrică, magnetică, atomică, etc. dacă ne gîndim că mai întîi trebuiau să se acumuleze particulele neconvenţionale la care nu se mai poate vorbi de energiile noastre convenţionale şi nu doar atît nu putem vorbi de energie atomică dacă atomii nu există ca şi de un cîmp magnetic dacă aceşti atomi nu mai există ca în „supa” domnilor din Elveţia. **Entităţile/univers** neconvenţionale gravitaează în Universul în Sine în formă convenţională şi neconvenţională în mod liber, acumularea lor este în timp şi spaţiu nelimitat iar după o acumulare suficientă această simulatneitate produce materii şi energii convenţionale. În final aceste **entităţi/univers** de **materie/energie** (convenţionale) prin acumulări succesive (convenţionale sau neconvenţionale) sau diviziuni ajung din nou **entităţi/univers** neconvenţionale în stadiu liber nelimitat de mici sau de mari. Trebuie să ne punem întrebări neconvenţionale şi să ne depăşim propriile limite să nu credem că energiile sînt finite, să nu credem că ceeea ce cunoaştem este Realitatea în Sine să nu credem că Big-Bangul este ultima frontieră, limita, cînd de fapt pînă acum nu am găsit limită nici măcar în interiorul atomului. Orice formă de organizare nu s-a format din inexistenţă, nici măcar din vid, ci pe o acumulare neconvenţională **materie/energie** care este în acelaşi timp **materie/energie** convenţională şi neconvenţională. O **materie/gravitaţie** dincolo de capacitatea noastră de convenţionalizare. Crede cineva că planetele sau Big-bangul sau Găurile Negre sînt posibile fără garvitaţie? se înşeală. Crede cineva că ar fi apărut viaţă sau forme de organizare fără gravitaţie (indiferent cît de mare sau de mică) fără gravitaţie? se înşeală. Crede cineva că ar fi existat existenţă fără acumulare? se înşeală. Nimic nu se putea forma în lipsa unor acumulări succesive datorită unei atracţii (gravitaţii) la fel cum totul dispare, se transformă datorită acestei energii inepuizabile, nelimitate (singura energie real neconvenţională). Nu ne referim la gravitaţia unei planete sau alta care este o gravitaţie convenţională ne referim la o gravitaţie neconvenţioanlă care îşi permite să atragă elemente (secvenţe) neconvenţionale "0" în **materie/energie** neconvenţională şi convenţională, acolo unde materia şi energia (gravitaţia) se confundă pînă la dispariţia posibilităţii de convenţionalizare. Ideea de a face structuri modulare nu este o noutate dar idea de a face structure modulare din elemente neconvenţionale este categoric nouă dar şi imposibilă pînă la proba contrarie. (depinde pînă unde convenţionalizăm noi neconvenţionalul) Teoretic putem spune că este posibil în cazul nostru să modulăm atomii şi moleculele şi nu oamenii sau şoselele, dar nu eu sînt cel care poate face sau nu acest lucru fiecare ştiinţă are această sarcină în raport de direcţia în care merge, poate nu merge dar idea de modulare neconvenţională (poate acum doar SF) va aduce mai devreme sau mai tîrziu soluţii şi modele noi neconvenţionale. Dacă nu în construcţii poate în transportarea în spaţiu şi timp a noastră sau pe alte planete. Poate şi în matematică redefinim modulul în raport de elementele neconvenţionale sau nedeterminate. Acumularea şi divizarea sînt singurele operaţii neconvenţionale (nelimitate, unice, etc.) respectiv adunarea şi scăderea în convenţional. Demonstarţia este banală dacă ţinem cont că un calculator face şi desface orice fenomen, funcţie, sistem, etc. doar prin adunare şi scădere şi doar cu 0 şi 1. Savanţii ca şi artiştii sau orice geniu au căutat cifra perfectă, această cifră este 1 pentru convenţional şi 0 pentru neconvenţional. Dacă vom ajunge la limita neconvenţională cînd vom putea aduna şi scădea doar cifre de 0 şi să obţinem aceleaşi rezultate convenţionale, atunci vom fi noi Dumnezeu şi nu vom avea limite. Ar rămîne totuşi o singură diferenţă între conenţional şi neconvenţional din acest punct de vedere, spaţiul şi timpul acestor acumulări sau divizări. Convenţionalul le face în **spţiu/timp** limitat în timp ce neconenţionalul în **spaţiu/timp** nelimitat.

Concluzii

Trebuie să ţinem cont că 0 şi © sînt valori constante, nelimitate indiferent cît de mari sau de mici iar produsul lor nu poate fi decît o cifră constantă intermediară lor, regulă de altfel respectată şi în convenţional. Dacă însă în convenţional produsul limitelor poate lua orice valoare între limitele respective în neconvenţional adică nelimiat nu poate lua orice valoare ci doar una singură general valabilă componenta tuturor celorlalte valori. Nu putem concepe că 0 şi © sînt unice dar produsul lor dau valori multiple, absurd. Aceasă cifră a produsului nu poate să întrunească toate aceste condiţii decît dacă cifra este 1.Un 1 care poate reprezenta şi Universul în Sine dar şi orice **entitate/univers** prin multiplii şi submultipli lui. Pentru a studia diverse cazuri trebuie să stabilim elementele lui neutre, domeniile sale de definiţie, limitele ca şi unităţile sale de comparaţie. Orice **entitate/univers** are aceste elemente şi orice parametru al ei de asemenea are aceste elemente. Matematicienii trebuie să găsească funcţii pentru diverse **entităţi/univers** să le adapteze la realitate să le asocieze un relativ apoi pe tot parcursul cunoaşterii să completeze şi să corecteze transformarea funcţiei pînă la perfecţiunea la care nu vom ajunge niciodată dar ghidează convenţiile realităţii noastre relative. (funcţia realativităţii f (λ) este permanentă chiar şi la valorile concrete şi constante ale **entităţii/univers**). Orice funcţie în cazul general oprice **entitate/univers** convenţională placă de la elementele caracteristice, de aceea şi funcţiei noastre trebuie să îi atribuim aceste elemente ca ea să devină o convenţie (chiar dacă relativă) cu care să putem opera. La fel la orice **entiateu/univers** (om, maşină, şosea, pom, energie, materie, etc.).

MECHATRONICS

THE NAVIGATION OF MOBILE ROBOTS IN NON-STATIONARY AND NON-STRUCTURED ENVIRONMENTS

VICTOR VLADAREANU, GABRIELA TONT, LUIGE VLADAREANU, FLORENTIN SMARANDACHE

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1 Introduction

Walking robots, unlike other types of robots such as those with wheels or tracks, use similar devices for moving on the field like human or animal feet. A desirable characteristic a mobile robot is the skills needed to recognise the landmarks and objects that surround it, and to be able to localise itself relative to its workspace. This knowledge is crucial for the successful completion of intelligent navigation tasks. But, for such interaction to take place, a model or description of the environment needs to be specified beforehand. If a global description or measurement of the elements present in the environment is available, the problem consists on the interpretation and matching of sensor readings to such previously stored object models.

Moreover, if we know that the recognised objects are fixed and persist in the scene, defined as stationary environments, they can be regarded as landmarks, and can be used as reference points for self localisation. If on the other hand, a global description or measurement of the elements in the environment is not available, at least the descriptors and methods that will be used for the autonomous building of one are required (Looney, 1994). The approach of the localisation and navigation problems of a mobile robot which uses a WSN which comprises of a large number of distributed nodes with low-cost cameras as main sensor, have the main advantage of require no collaboration from the object being tracked.

The main advantages of using WSN multi-camera localisation and tracking are:

1 The exploit of the distributed sensing capabilities of the WSN.

2 The benefit from the parallel computing capabilities of the distributed nodes. Even though each node have finite battery lifetime by cooperating with each other, they can perform tasks that are difficult to handle by traditional centralised sensing system.

3 The employ of the communication infrastructure of the WSN to overcome multi-camera network issues. Also, camera-based WSN have easier deployment and higher re-configurability than traditional camera networks making them particularly interesting in applications such as security and search and rescue, where pre-existing infrastructure might be damaged (Jalilvand et al., 2009).

Robots have to know where in the map they are in order to perform any task involving navigation. Probabilistic algorithms have proved very successful in many robotic environments. They calculate the probability of each possible position given some sensor readings and movement data provided by the robot (Vladareanu et al., 2011; Kim et al., 2007). The localisation of a mobile robot is made using a particle filter that updates the belief of localisation which, and estimates the maximal posterior probability density for localisation. The causal and contextual relations of the sensing results and global localisation in a Bayesian network, and a sensor planning approach based on Bayesian network inference to solve the dynamic environment is presented. In the study is proposed a mobile robot sensor planning approach based on a top-down decision tree algorithm. Since the system has to compute the utility values of all possible sensor selections in every planning step, the planning process is very complex.

The paper first presents the position force control and dynamic control using ZMP and inertial information with the aim of improving robot stability for movement in non-structured environments. This means moving the robot in sloping terrain, on steps or uneven environments which leads to modifications in the projection of the robot support surface and variable loads on the robot legs during movement. The next chapter presents the mobile walking robot control system architecture for movement in non-stationary environments by applying wireless sensor networks (WSN) methods. Finally, there are presented the results obtained in implementing the interface for sensor networks used to avoid obstacles and in improving the performance of dynamic stability control for motion on rough terrain, through a Bayesian approach of simultaneous localisation and mapping (SLAM).

2 Dynamical stability control

The research evidences that stable gaits can be achieved by employing simple control approaches which take advantage of the dynamics of compliant systems. This allows a decentralisation of the control system, through which a central command establishes the general movement trajectory and local control laws presented in the paper solve the motion stability problems, such as: damping control, ZMP compensation control, landing orientation control, gait timing control, walking pattern control, predictable motion control (Vladareanu et al., 2011, 2012).

In order to carry out new capabilities for walking robots, such as walking down the slope, going by overcoming or avoiding obstacles, it is necessary to develop high-level intelligent algorithms, because the mechanism of walking robots stepping on a road with bumps is a complicated process to understand, being a repetitive process of tilting or unstable movements that can lead to the overthrow of the robot. The chosen method that adapts well to walking robots is the zero moment point (ZMP) method (Vladareanu et al., 2010b; Vladareanu and Capitanu, 2012). A new strategy is developed for the dynamic control for walking robot stepping using ZMP and inertial information. This, includes pattern generation of compliant walking, real-time ZMP compensation in one phase – support phase, the leg joint damping control, stable stepping control and stepping position control based on angular velocity of the platform. In this way, the walking robot is able to adapt on uneven ground, through real-time control, without losing its stability during walking (Vladareanu et al., 2009b; Capitanu et al., 2008).

Based on studies and analysis, the compliant control system architecture was completed with tracking functions for HFPC walking robots, which through the implementation of many control loops in different phase of the walking robot, led to the development of new technological capabilities, to adapt the robot walking on sloping land, with obstacles and bumps. In this sense, a new control algorithm has been studied and analysed for dynamic walking of robots based on sensory tools such as force/torque and inertial sensors (Vladareanu et al., 2010a; Raibert and Craig, 1981; Zhang and Paul, 1985). Distributed control system architecture was integrated into the HFPC architecture so that it can be controlled with high efficiency and high performance.

3 Simultaneous localisation and mapping

A precise position error compensation and low-cost relative localisation method is studied in Kim et al. (2007) for structured environments using magnetic landmarks and hall sensors. The proposed methodology can solve the problem of fine localisation as well as global localisation by tacking landmarks or by utilising various patterns of magnetic landmark arrangement. The research in localisation and tracking methods using WSN have been developed based on radio signal strength intensity (RSSI) and ultrasound time of flight (TOF). Localisation based on radio frequency identification (RFID) systems have been used in fields such as logistics and transportation but the constraints in terms of range between transmitter and reader limits its potential applications (Yoshikawa and Zheng, 1993).

Many efforts have been devoted to the development of cooperative perception strategies exploiting the complementarities among distributed static cameras at ground locations, among cameras mounted on mobile robotic platforms, and among static cameras and cameras onboard mobile robots.

Computation-based closed-loop controllers put most of the decision burden on the planning task. In hazardous and populated environments mobile robots utilise motion planning which relies on accurate, static models of the environments, and therefore they often fail their mission if humans or other unpredictable obstacles block their path. Autonomous mobile robots systems that can perceive their environments, react to unforeseen circumstances, and plan dynamically in order to achieve their mission have the objective of the motion planning and control problem (Stankovski et al., 2002; Vladareanu et al., 2010b; Deng et al., 2011; Fei et al., 2012).

To find collision-free trajectories, in static or dynamic environments containing some obstacles, between a start and a goal configuration, the navigation of a mobile robot comprises localisation, motion control, motion planning and collision avoidance. Its task is also the online real-time re-planning of trajectories in the case of obstacles blocking the pre-planned path or another unexpected event occurring. Inherent in any navigation scheme is the desire to reach a destination without getting lost or crashing into anything. The responsibility for making this decision is shared by the process that creates the knowledge representation and the process that constructs a plan of action based on this knowledge representation. The choice of which representation is used and what knowledge is stored helps to decide the division of this responsibility. Very complex reasoning may be required to condense all of the available information into this single measure (Shihab, 2005; Iliescu et al., 2010). The techniques include computation-based closed-loop control, cost-based search strategies, finite state machines, and rule-based systems (Boscoianu et al., 2008; Rummel and Seyfarth, 2008).

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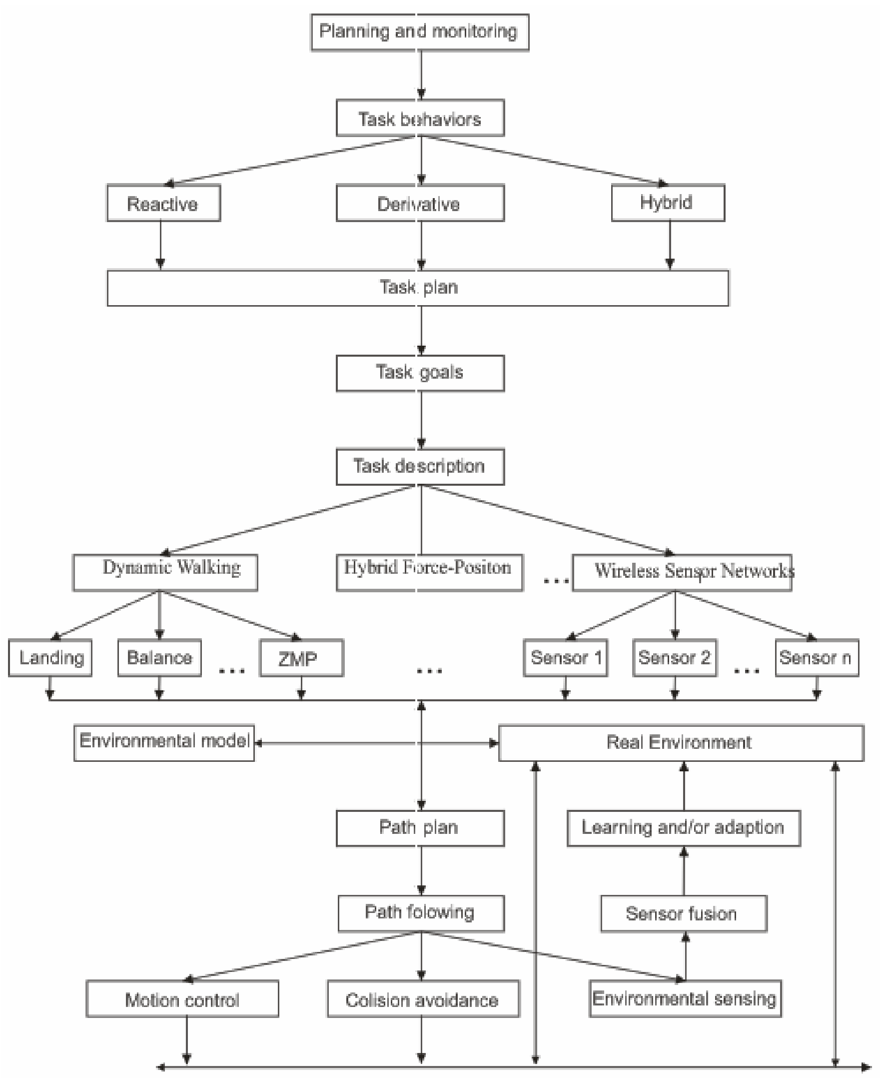
Figure 1 Mobile robot control system architect



Motion planning of mobile walking robots in uncertain dynamic environments based on the behaviour dynamics of collision-avoidance is transformed into an optimisation problem. Applying constraints based on control of the behaviour dynamics, the decision-making space of this optimisation.

4 Strategy for dynamical stability

The walking robot is considered as a set of articulated rigid bodies, which are standing as a platform and leg elements. The static stability problem is solved by calculating the extremity of each leg position according to the system of axes attached to the platform, with origin at the centre of gravity of it.

The proposed walking robot control strategy is based on three approaches, for conforming to movement characteristics: real-time balance control, walking pattern control and predictable motion control. The first main task, balance control, leads to a control model that periodically modifies the walking scheme, depending on the sensory information received from the robot transducers.

In this paper we take into account the real-time balance control.

Real-time balance control. The balance control, leads to a control model that periodically modifies the walking scheme, depending on the sensory information received from the robot transducers. Real-time balance control presented in Figure 2 contains four types of reactive loops: damping control, ZMP compensation control, walk timing control and walk orientation control. The second main task, walking scheme control, represents a real-time control of the robot equilibrium using the reactions of inertial sensors. The walking control scheme can be changed periodically in accordance with the information received from the inertial sensors during the walking cycle, by processing them into two real-time loops: platform swing amplitude control and platform rotation/advance control.

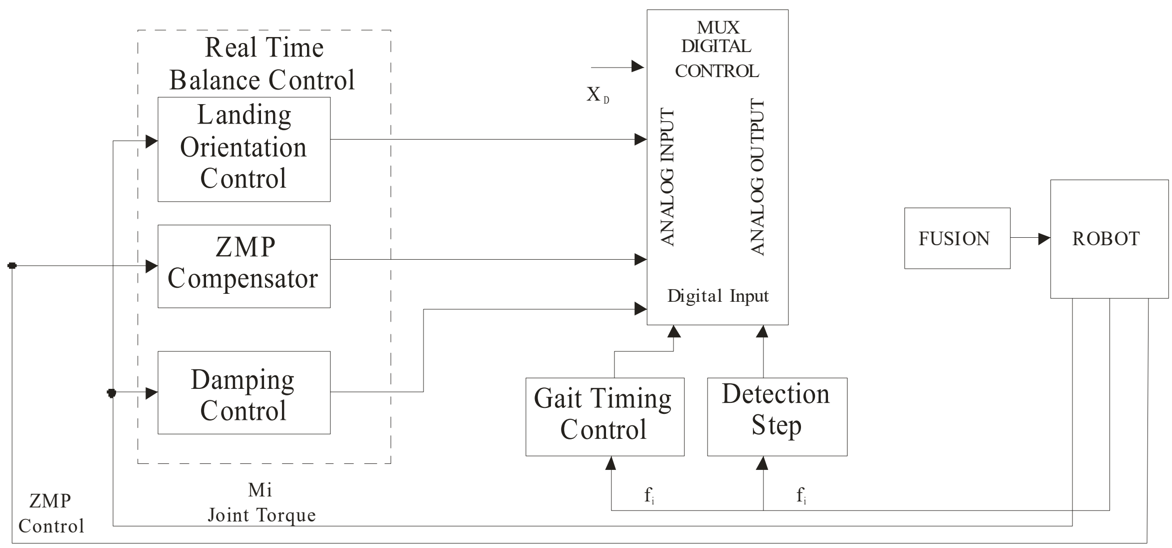
The third main task, predictable movement control, represents the control of predictable movement based on a fast decision from previous experimental data. Our research considers the following five dynamic control loops. *Landing orientation control* is achieved by integrating the torque measured for the entire gait and achieving a stable contact with the two ground surface by controlling the leg joint. A stable contact is obtained by adapting the leg articulations at ground surface, when an obstacle is preventing moving the leg on the required trajectory. The motion control will lead to a smooth walking. The control law for the landing orientation is:

$$u_c = u + \frac{T(s)}{C_L s + K_L} \quad (1)$$

where T is the measured torque, CL is the damping coefficient, KL is the rigidity, u is the leg reference angle and u_c is the leg's joint compensated reference angle.

Damping control aims to eliminate the oscillations that occur in the single support phase. The oscillations amplitude is measured in real-time by a torque transducer mounted on the robot joints, having compliant control functions of robot movement. A simple inverted pendulum equation with a joint in the single support phase, which opposes the damping forces of the leg joints was adopted for robot motion modelling. *ZMP compensation control* strategy consists in mathematical modelling of ZMP compensator through the spring-loaded inverted pendulum. A ZMP compensator is developed in single support phase (FSU), where the platform will move back and forth according to ZMP dynamics, because the damping loop is not sufficient to maintain a stable walking motion due to the ZMP movement influences.

Figure 2 Real-time balance control of the walking robots motion



Regarding the construction of mathematical model, based on quasi-dynamic analysis, each leg is considered as a generator function, with limited accuracy for displacement systems. If the number of degrees of mobility is equal to n and if the interior limitations have the following form:

$$F_j\left(x_1, x_2, \ ..., \ x_n\right) = 0, \quad j = \overline{1, \ m}, \quad n \geq m \tag{2}$$

then, in the differential equations structure:

$$\frac{\mathrm{d}x_j}{\mathrm{d}t} = f_i\left(x_1, \ x_2, ..., \ x_n, \ u_1, \ u_2, \ ..., \ u_s, \ t\right), \ i = \overline{1, \ n} \tag{3}$$

there are arbitrary u_i coefficients, which are used to obtain the 'stepping' algorithm. For differential equations, the limitations imposed by the general (platform) base, where the legs are fixed, are applied in first case, and the limitations imposed on the supporting surface, secondly.

5 Virtual projection method

A virtual projection architecture system was designed which allows improvement and verification of the performance of dynamic force-position control of walking robots by integrating the multi-stage fuzzy method with acceleration solved in position-force control and dynamic control loops through the ZMP method for movement in non-structured environments and a Bayesian approach of SLAM for avoiding obstacles in non-stationary environments. By processing inertial information of force, torque, tilting and WSN an intelligent high level algorithm is implementing using the virtual projection method.

The virtual projection method, presented in Figure 3, patented by the research team (Vladareanu et al., 2009a), tests the performance of dynamic position-force control by integrating dynamic control loops and a Bayesian interface for the sensor network. The CMC classical mechatronic control directly actions the MS1, MSm servomotors, where m is the number of the robot's degrees of freedom. These signals are sent to a virtual control interface (VCI), which processes them and generates the necessary signals for graphical representation in 3D on a graphical terminal CGD. A number of n control interface functions ICF1-ICFn ensure the development of an open architecture control system by intergrating *n* control functions in addition to those supplied by the CMC mechatronic control system. With the help of these, new control methods can be implemented, such as: contour tracking functions, control schemes for tripod walking, centre of gravity control, orientation control through image processing and Bayesian interface for sensor networks. Priority control real-time control and information exchange management between the n interfaces is ensured by the multifunctional control interface MCI, interconnected through a high speed data bus.

5.1 Bayesian Interface for sensor networks

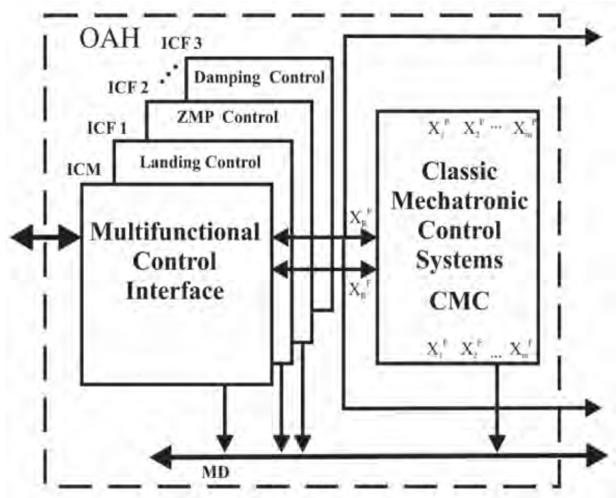
To determine the priors for the model parameters and to calculate likelihood function (joint probability) we define a given random variable x whose probability distribution depends on a set of parameters $P = (P_1, P_2, \ldots P_p)$. Exact values of the parameters are not known with certainty, Bayesian reasoning assigns a probability distribution of the various possible values of these parameters that are considered as random variables. Bayes' theory is generally expressed through probabilistic statements as following:

$$P(A \mid B) = P(A) \times \frac{P(B \mid A)}{P(B)} \tag{4}$$

$P(A \mid B)$ is the probability of A given the event B occurs or the posteriori probability. Using Bayes' theory may be recurring, that if exist an a priori distribution ($P(A)$ and a series of tests with experimental results $B_1, B_2, \ldots, B_n \ldots$, expressed according to successive equations:

$$\begin{aligned} &P\left(A \mid B_1\right) = P(A) \frac{P\left(B_1 \mid A\right)}{P\left(B_1\right)} \\ &P\left(A \mid B_1, B_2\right) = P(A) \frac{P\left(B_1 \mid A\right)}{P\left(B_1\right)} \frac{P\left(B_2 \mid A\right)}{P\left(B_2\right)} \\ &P\left(A \mid B_1, B_2, \ldots B_n\right) \\ &= P\left(A \mid B_1, B_2, \ldots B_{n-1}\right) - \frac{P\left(B_n \mid A\right)}{P\left(B_n\right)} \end{aligned} \tag{5}$$

Figure 3 The virtual projection method



A posteriori distribution called also belief, is used when the test results are known, being obtained as a new function a priori. The start of operations sequences in the Bayesian method regards the transformation γ. Recursive Bayesian updating is made under the Markov assumption: z_n is independent of $z_1, \ldots, z_{n-1}$ if we know x.

$$\begin{aligned} P\left(x \mid z_1, \ldots, z_n\right) &= \frac{P(z_n x)\, P\left(x \mid z_1, \ldots, z_{n-1}\right)}{P\left(z_n \mid z_1, \ldots, z_{n-1}\right)} \\ &= \eta \, P\left(z_n x\right) P\left(x \mid z_1, \ldots, z_{n-1}\right) \\ &= \eta_{1 \ldots n} \prod_{i=1 \ldots n} P\left(z_i \mid x\right) P(x) \end{aligned} \tag{6}$$

When there are no missing data or hidden variables the method for calculating $P(B_{si}, D)$ for some belief-network structure B_{Si} and database D is presented in Looney (1994). Let Q be the set of all those belief-network structures that have a non-zero prior probability. We can derive the posterior probability of B_{Si} given D as:

$$P\left(B_{Si} \mid D\right) = P\left(B_{Si}, D\right) \Big/ \sum B_{Si} \in QP\left(B_{Si}, D\right) \tag{7}$$

The ratio of the posterior probabilities of two belief-network structures can be calculated as a ratio for belief-network structures B_{si} and B_{sj}, using the equivalence:

$$P\left(B_{Si} \mid D\right) \Big/ P\left(B_{Sj} \mid D\right) = P\left(B_{Si}, D\right) \Big/ P\left(B_{Sj}, D\right) \tag{8}$$

which we can derive that:

$$P\left(B_{Si}, D\right) = P\left(D \mid B_{Si}\right) P\left(B_{Si}\right) \tag{9}$$

Term $P(B_{si})$ represents prior probability that a process with belief-network structure B_{Si}.

To designate the possible values of h, ca be used the Markov blanket method, $MB(h)$ (Looney, 1994; Vladareanu et al., 2011). Suppose that among the m cases in D there are u unique instantiations of the variables in $MB(h)$. Given these conditions it follows that:

$$\begin{aligned} P(D \mid B_s) = \sum\nolimits_{G_t} \cdots \sum\nolimits_{G_u} f(G_1 \ldots, G_u) \\ \int_{B_p} \left[\prod\nolimits_{t=1}^{m} P(C_t h_t \mid B_s, B_p) \right] f(B_s B_p) dB_p \end{aligned} \tag{10}$$

where G_i is a given group contains c_i case-specific hidden variables.

Recall that u denotes only the number of unique instantiations *actually realised* in database D of the variables in the Markov blanket of hidden variable h. The number of such unique instantiations significantly influences the efficiency with which we can compute equation (10). For any finite belief network, the number of such unique instantiations reaches a maximum regardless of how many cases there are in the database.

That r denotes the maximum number of possible values for any variable in the database. If u and r are bounded from above, then the time to solve equation (10) is bounded from above by a function that is polynomial in the number of variables n and the number of cases m. If u or r is large, however, the polynomial will be of high degree (Vladareanu et al., 2011).

To model a robotic system requires considering in-between the two states of operating and faulting one or more intermediate states of partial success. In Figure 4 is considered a robotic system characterised by three states: operating at full capacity (F), defect (D) and intermediate (I). A generalised diagram of states is shown in Figure 5, which included three intermediate states.

Figure 4 The model with three states for the robotic system

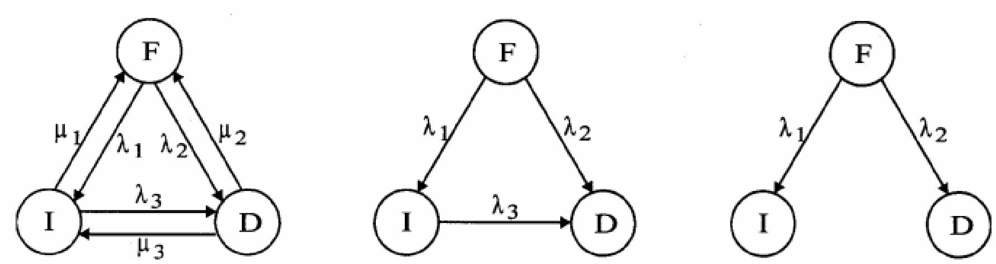
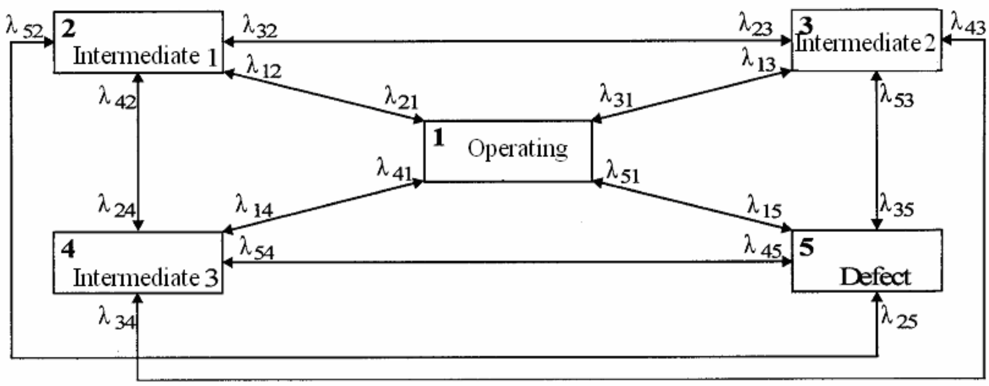


Figure 5 Generalised diagram of states with three intermediate states



The Markov modelling technique requires to identify each intermediate state (in practice, more neighbouring levels can be grouped together), to know the occupancy status of each component (T_i) and the number of transitions between states (N_{ij}), which can calculate as follows:

- occupancy probability of 'i' state: $P_i = \frac{T_i}{T_A}$
- transition intensity from state 'i' in 'j': $\lambda_{ij} = \frac{N_{ij}}{T_i}$,

  where $T_A = \sum_i T_i$ is analysed time interval.

The number of intermediate states to be modelled in order to obtain a more accurate assessment of the reliability group is necessary to consider more than one intermediate state. Figure 6 presents a model with six states to assess the predictable transitions in a robotic system. The six states of the system are:

1 operational state of robot

2 landing control

3 balance control

4 advance control

5 WSN control

6 unpredicted event.

Based on the surveillance data in operation regime of robot were determined transition probabilities using of the relationship: $\hat{p}_{ij} = \frac{n_{ij}}{n_i}$, where n_{ij} is the transition from state 'i' in 'j' in the analysis time interval; n_i is the number of all transitions from state 'i' in any other states.

Values of these transition probabilities are: $\hat{p}_{12} = 0,217$; $\hat{p}_{13} = 0,29$; $\hat{p}_{14} = 0,135$; $\hat{p}_{15} = 0,235$; $\hat{p}_{16} = 0,123$. By applying the method Markov chains are obtain the occupancy probability of the sates for the robot: $P_1 = 0.27$; $P_2 = 0.19$; $P_3 = 0.115$; $P_4 = 0.235$; $P_5 = 0.122$; $P_6 = 0.068$.

The working diagram of the petri network is presented in Figure 7 (http://www-dssz.informatik.tu-cottbus.de).

A token is assigned to P_3, and is assumed that the localiser initially knows its position. The warning event t_5 fires when the localiser fails in estimating robot's accurate position for several steps. Two navigation primitives can be modelled as P_1, P_2, respectively. Initially, the robot selects its motion by a random switch comprising the transitions t_1 and t_2 with corresponds to probabilities P_1' and P'$_2$, respectively. The transition between them takes place according to the change of localiser states.

The immediate transition t_3 means that the robot takes Contour tracking as soon as the localiser Warning event fires. The other transition between two primitives, t_2 and t_4, are modelled as timed transitions in order to express that the robot can change its current navigation primitive during the localiser Success state, if necessary.

6 Results and conclusions

The control for walking robots is achieved by a control system with three levels. The first level is to produce control signals for motor drive mounted on leg joints, ensuring the robot moving in the direction required with a given speed. The language for this level is that of differential equations.

The second level controls the walking, respectively it coordinates the movements, provides the data necessary to achieve progress. At this level, work is described in the language of algorithms types of walking.

The third level of command defines the type of walking, speed and orientation. At this level, the command may be provided by an operator who can use the control panel, in pursuit of its link with the robot, to specify the type of running and passing special orders (for the definition of the vector speed of movement).

Figure 6 Modelling the states with possible transitions for robot

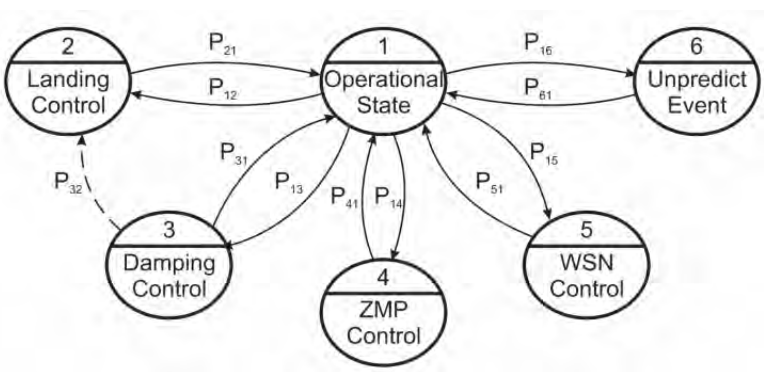
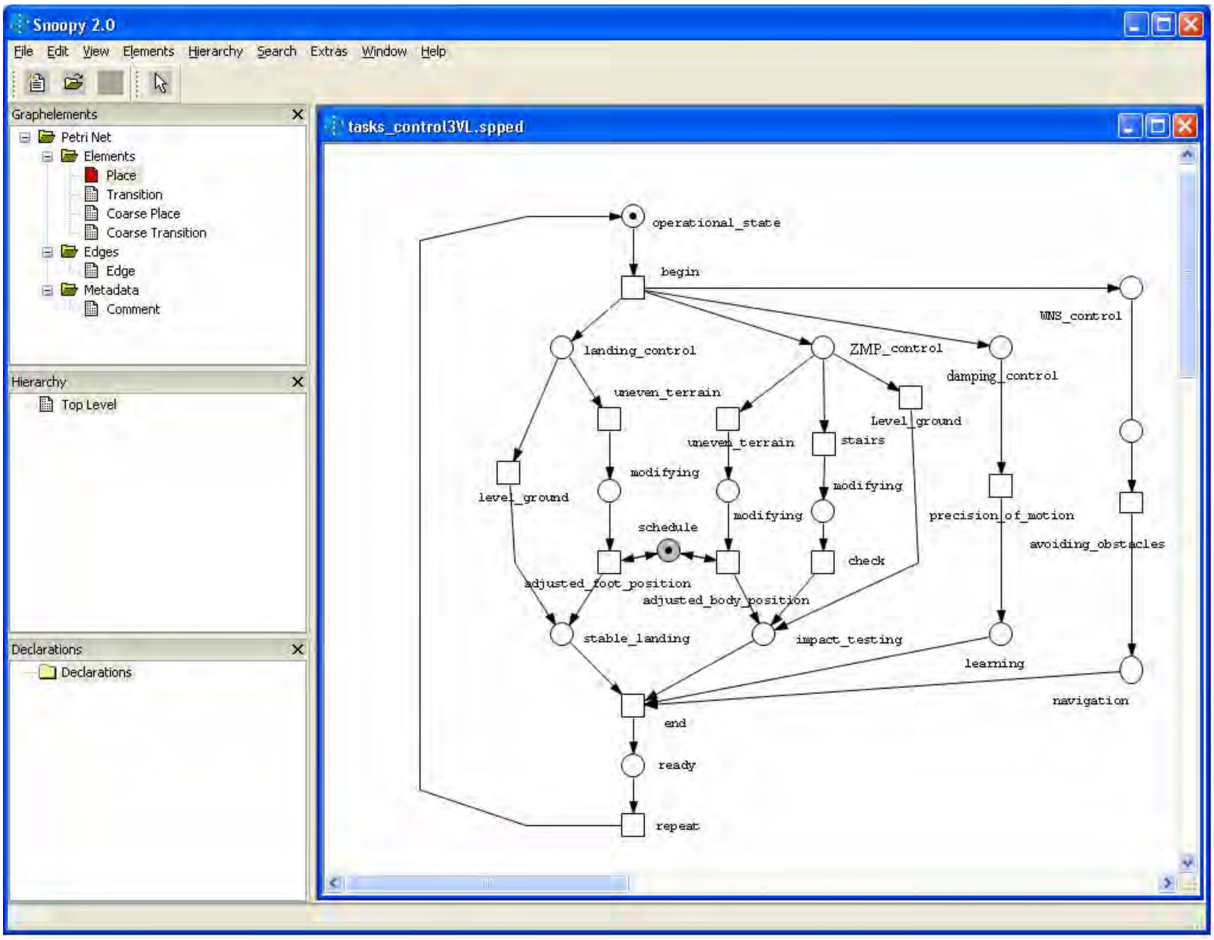


Figure 7 The petri network diagram (see online version for colours)



To maintain the platform in a horizontal position, the information provided by the horizontality transducers (or verticality) is used, that sense walking robots deviation platform to the horizontal position. Restoring the horizontal position of the platform is achieved at the expense of vertical movement of different legs of support, as decided by the block to maintain balance. Returning to the fixed height of the platform is achieved by using information provided by the height transducer of the platform and by simultaneous control of vertical movement of all legs in support phase.

From the analysis performed results the effectiveness of the proposed control strategy for a walking robot. The position of each actuator is controlled by a PD feedback loop, using encoder like transducers.

In HFPC control system, the PC system sends the reference positions to all actuators controllers simultaneously at an interval of 10 ms (100 Hz). Reference positions for the control of 18 actuators and actual positions on each axis robot obtained through interpolation are processed at an interval of 1 ms (1 kHz).

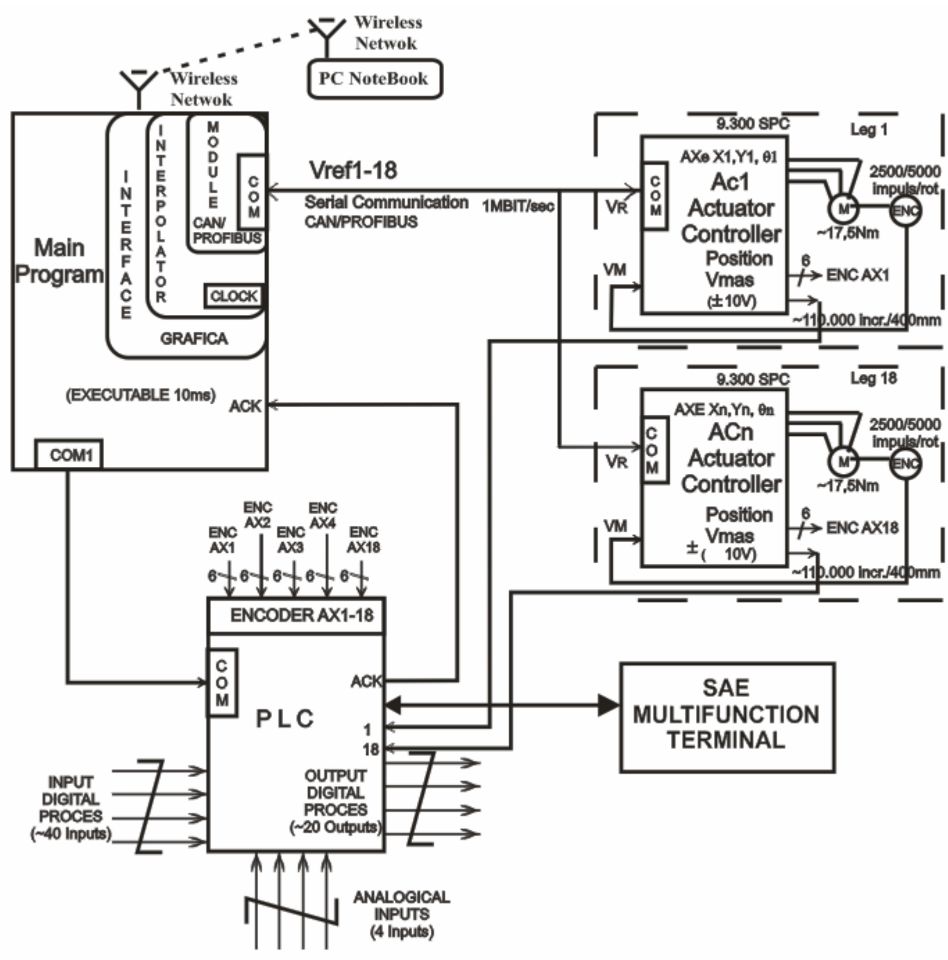
The ready to walk position is a robot base position, before the actual walk. For this position, the robot lowers the platform by bending the leg joints. The reason is to prevent the singular problem of the inverse cinematic and to achieve a stable walking with a constant platform height from the ground. To be observed that the platform height is linked to the dynamic properties of the robot. When the robot walks, it is periodically in the unique support phase. In this phase, the robot can be similar to the simple inverted pendulum model on the coronal plane and its natural frequency is:

$$f = \frac{1}{2\pi}\sqrt{\frac{g}{l}}(Hz) \tag{11}$$

where g and l are the desired acceleration due to gravity and respective the height from the ground of the robot's centre of mass.

Certainly, the natural frequency of the simple inverted pendulum exists in theory, because the robot's tilt is limited by a specific angle. Thus, one can determine the walking period for a smooth motion in two phases (for the tripod walking) and efficient power consumption. For example, for a robot with the height l of approximately 900 mm and the balance of 40 mm results the natural frequency of 0.526 Hz. Figure 8 shows the general configuration of the HFPC system for ZMP control method.

Figure 8 Open architecture system of the walking robot



The control system is distributive with multi-processor devices for joint control, data reception from transducers mounted on the robot, peripheral devices connected through a wireless LAN for offline communications and CAN fast communication network for real-time control. The HFPC system was designed in a distributed and decentralised structure to enable development of new applications easily and to add new modules for new hardware or software control functions.

The proposed petri nets and Markov chains approach provides a promising solution towards the development quantitative approach of dynamic discreet/stochastic event systems of task planning of mobile robots. For a deeper insight into control and communication of governing task assignment of the robot, the entire discrete-event dynamic evolution of task sequential process have to be linguistically described in terms of representations.

This approach has the potential to model more complex relationships between target parameters. Moreover, the short time execution will ensure a faster feedback, allowing other programs to be performed in real-time as well, like the apprehension force control, objects recognition, making it possible that the control system have a human flexible and friendly interface.

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THE NAVIGATION MOBILE ROBOT SYSTEMS USING BAYESIAN APPROACH THROUGH THE VIRTUAL PROJECTION METHOD

LUIGE VLADAREANU, GABRIELA TONT, VICTOR VLADAREANU, FLORENTIN SMARANDACHE, LUCIAN CAPITANU

***Abstract*. The paper presents the navigation mobile walking robot systems for movement in non-stationary and non-structured environments, using a Bayesian approach of Simultaneous Localization and Mapping (SLAM) for avoiding obstacles and dynamical stability control for motion on rough terrain. By processing inertial information of force, torque, tilting and wireless sensor networks (WSN) an intelligent high level algorithm is implementing using the virtual projection method. The control system architecture for the dynamic robot walking is presented in correlation with a stochastic model of assessing system probability of unidirectional or bidirectional transition states, applying the non-homogeneous/non-stationary Markov chains. The rationality and validity of the proposed model are demonstrated via an example of quantitative assessment of states probabilities of an autonomous robot. The results show that the proposed new navigation strategy of the mobile robot using Bayesian approach walking robot control systems for going around obstacles has increased the robot's mobility and stability in workspace.**

I. INTRODUCTION

Walking robots, unlike other types of robots such as those with wheels or tracks, use similar devices for moving on the field like human or animal feet. A desirable characteristic a mobile robot must have the skills needed to recognize the landmarks and objects that surround it, and to be able to localize itself relative to its workspace. This knowledge is crucial for the successful completion of intelligent navigation tasks. But, for such interaction to take place, a model or description of the environment needs to be specified beforehand. If a global description or measurement of the elements present in the environment is available, the problem consists on the interpretation and matching of sensor readings to such previously stored object models. Moreover, if we know that the recognized objects are fixed and persist in the scene, they can be regarded as landmarks, and can be used as reference points for self localization. If on the other hand, a global description or measurement of the elements in the environment is not available, at least the descriptors and methods that will be used for the autonomous building of one are required [1].

The approach of the localization and navigation problems of a mobile robot which uses a WSN which comprises of a large number of distributed nodes with low-cost cameras as main sensor, have the main advantage of require no collaboration from the object being tracked.

The main advantages of using WSN multi-camera localization and tracking are:

1) the exploit of the distributed sensing capabilities of the WSN;

2) the benefit from the parallel computing capabilities of the distributed nodes. Even though each node have finite battery lifetime by cooperating with each other, they can perform tasks that are difficult to handle by traditional centralized sensing system.;

3) the employ of the communication infrastructure of the WSN to overcome multi-camera network issues. Also, camera-based WSN have easier deployment and higher re-configurability than traditional camera networks making them particularly interesting in applications such as security and search and rescue, where pre-existing infrastructure might be damaged [2].

Robots have to know where in the map they are in order to perform any task involving navigation. Probabilistic algorithms have proved very successful in many robotic environments. They calculate the probability of each possible position given some sensor readings and movement data provided by the robot [5]. The localization of a mobile robot is made using a particle filter that updates the belief of localization which, and estimates the maximal posterior probability density for localization. The causal and contextual relations of the sensing results and global localization in a Bayesian network, and a sensor planning approach based on Bayesian network inference to solve the dynamic environment is presented. In the study is proposed a mobile robot sensor planning approach based on a top-down decision tree algorithm. Since the system has to compute the utility values of all possible sensor selections in every planning step, the planning process is very complex.

The paper first presents the position force control and dynamic control using ZMP and inertial information with the aim of improving robot stability for movement in non-structured environments. The next chapter presents the mobile walking robot control system architecture for movement in non-stationary environments by applying

Wireless Sensor Networks (WSN) methods. Finally, there are presented the results obtained in implementing the interface for sensor networks used to avoid obstacles and in improving the performance of dynamic stability control for motion on rough terrain, through a Bayesian approach of Simultaneous Localization and Mapping (SLAM).

II. DYNAMICAL STABILITY CONTROL

The research evidences that stable gaits can be achieved by employing simple control approaches which take advantage of the dynamics of compliant systems. This allows a decentralization of the control system, through which a central command establishes the general movement trajectory and local control laws presented in the paper solve the motion stability problems, such as: damping control, ZMP compensation control, landing orientation control, gait timing control, walking pattern control, predictable motion control (see ICAMechS 2011, Zhengzhou [3]).

In order to carry out new capabilities for walking robots, such as walking down the slope, going by overcoming or avoiding obstacles, it is necessary to develop high-level intelligent algorithms, because the mechanism of walking robots stepping on a road with bumps is a complicated process to understand, being a repetitive process of tilting or unstable movements that can lead to the overthrow of the robot. The chosen method that adapts well to walking robots is the ZMP (Zero Moment Point) method. A new strategy is developed for the dynamic control for walking robot stepping using ZMP and inertial information. This, includes pattern generation of compliant walking, real-time ZMP compensation in one phase - support phase, the leg joint damping control, stable stepping control and stepping position control based on angular velocity of the platform. In this way, the walking robot is able to adapt on uneven ground, through real time control, without losing its stability during walking [13].

Based on studies and analysis, the compliant control system architecture was completed with tracking functions for HFPC walking robots, which through the implementation of many control loops in different phase of the walking robot, led to the development of new technological capabilities, to adapt the robot walking on sloping land, with obstacles and bumps. In this sense, a new control algorithm has been studied and analyzed for dynamic walking of robots based on sensory tools such as force / torque and inertial sensors [3,13]. Distributed control system architecture was integrated into the HFPC architecture so that it can be controlled with high efficiency and high performance.

III. SIMULTANEOUS LOCALIZATION AND MAPING

A precise position error compensation and low-cost relative localization method is studied in [5] for structured environments using magnetic landmarks and hall sensors [6]. The proposed methodology can solve the problem of fine localization as well as global localization by tacking landmarks or by utilizing various patterns of magnetic landmark arrangement. The research in localization and tracking methods using Wireless Sensor Networks (WSN have been developed based on Radio Signal Strength Intensity (RSSI) [7] and ultrasound time of flight (TOF) [8]. Localization based on Radio Frequency Identification (RFID) systems have been used in fields such as logistics and transportation [9] but the constraints in terms of range between transmitter and reader limits its potential applications. Many efforts have been devoted to the development of cooperative perception strategies exploiting the complementarities among distributed static cameras at ground locations [10], among cameras mounted on mobile robotic platforms [11], and among static cameras and cameras onboard mobile robots [12]. Computation-based closed-loop controllers put most of the decision burden on the planning task. In hazardous and populated environments mobile robots utilize motion planning which relies on accurate, static models of the environments, and therefore they often fail their mission if humans or other unpredictable obstacles block their path. Autonomous mobile robots systems that can perceive their environments, react to unforeseen circumstances, and plan dynamically in order to achieve their mission have the objective of the motion planning and control problem [4, 9].

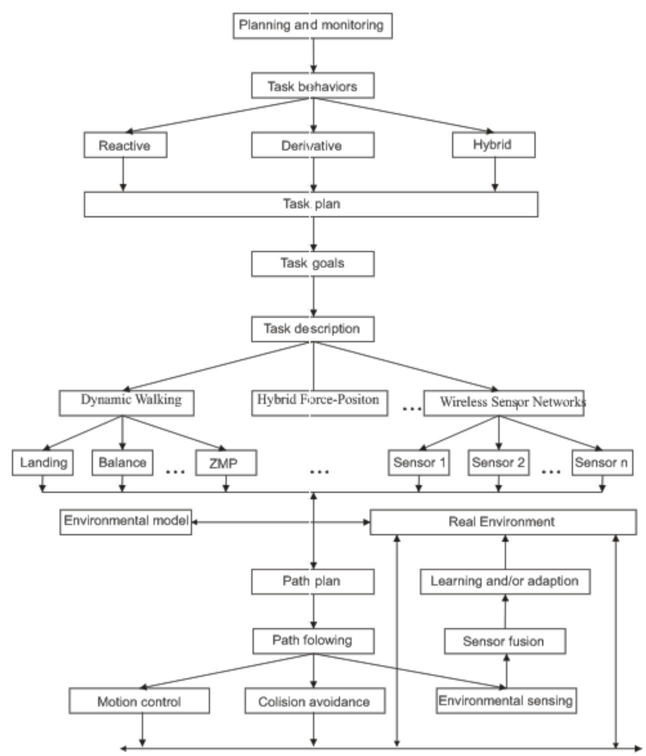

Figure 1 Mobile robot control system architecture

To find collision-free trajectories, in static or dynamic environments containing some obstacles, between a start and a goal configuration, the navigation of a mobile robot comprises localization, motion control, motion planning and collision avoidance. Its task is also the online real-time re-planning of trajectories in the case of obstacles blocking the pre-planned path or another unexpected event occurring. Inherent in any navigation scheme is the desire to reach a destination without getting lost or crashing into anything. The responsibility for making this decision is shared by the process that creates the knowledge representation and the process that constructs a plan of action based on this knowledge representation. The choice of which representation is used and what knowledge is stored helps to decide the division of this responsibility. Very complex reasoning may be required to condense all of the available information into this single measure [4, 14]. The techniques include computation-based closed-loop control, cost-based search strategies, finite state machines, and rule-based systems [17].

Computation-based closed-loop controllers put most of the decision burden on the planning task. In hazardous and populated environments mobile robots utilize motion planning which relies on accurate, static models of the environments, and therefore they often fail their mission if humans or other unpredictable obstacles block their path. Autonomous mobile robots systems that can perceive their environments, react to unforeseen circumstances, and plan dynamically in order to achieve their mission have the objective of the motion planning and control problem. To find collision-free trajectories, in static or dynamic environments containing some obstacles, between a start and a goal configuration, the navigation of a mobile robot comprises localization, motion control, motion planning and collision avoidance [15, 16]. A higher-level process, a task planner, specifies the destination and any constraints on the course, such as time. Most mobile robot algorithms abort, when they encounter situations that make the navigation difficult. Set simply, the navigation problem is to find a path from start (S) to goal (G) and traverse it without collision. The relationship between the subtasks mapping and modeling of the environment; path planning and selection; path traversal and collision avoidance into which the navigation problem is decomposed, is shown in Figure 1.

Motion planning of mobile walking robots in uncertain dynamic environments based on the behavior dynamics of collision-avoidance is transformed into an optimization problem. Applying constraints based on control of the behavior dynamics, the decision-making space of this optimization.

IV. VIRTUAL PROJECTION METHOD

A virtual projection architecture system was designed which allows improvement and verification of the performance of dynamic force-position control of walking robots by integrating the multi-stage fuzzy method with acceleration solved in position-force control and dynamic control loops through the ZMP method for movement in non-structured environments and a bayesian approach of simultaneous localization and mapping (SLAM) for avoiding obstacles in non-stationary environments. By processing inertial information of force, torque, tilting and wireless sensor networks (WSN) an intelligent high level algorithm is implementing using the virtual projection method.

The virtual projection method, presented in Figure 2, patented by the research team, tests the performance of dynamic position-force control by integrating dynamic control loops and a bayesian interface for the sensor network. The CMC classical mechatronic control directly actions the MS1, MSm servomotors, where m is the numbber of the robot's degrees of freedom. These signals are sent to a virtual control interface (VCI), which processes them and genrates the necessary signals for graphical representation in 3D on a graphical terminal CGD. A number of n control interface functions ICF1-ICFn ensure the development of an open architecture control system by intergrating n control functions in addition to those supplied by the CMC mechatronic control system. With the help of these, new control methods can be implemented, such as: contour tracking functions, control schemes for tripod walking, centre of gravity control, orientation control through image processing and Bayesian interface for sensor networks. Priority control real time control and information exchange management between the n interfaces is ensured by the multifunctional control interface MCI, interconnected through a high speed data bus.

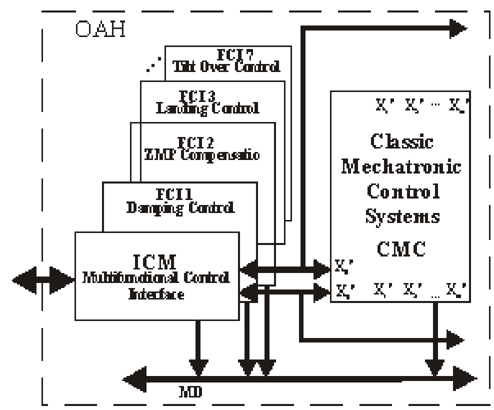

Fig. 2. The virtual projection method

Bayesian Interface for sensor networks.

To determine the priors for the model parameters and to calculate likelihood function (joint probability) we define a given random variable x whose probability distribution depends on a set of parameters $P = (P_1, P_2, \ldots P_p)$. Exact values of the parameters are not known with certainty, Bayesian reasoning assigns a probability distribution of the various possible values of these parameters that are considered as random variables. Bayes' theory is generally expressed through probabilistic statements as following:

$$P(A \mid B) = P(A) \quad \frac{P(B \mid A)}{(B)} \tag{1}$$

P (A | B) is the probability of A given the event B occurs or the posteriori probability. Using Bayes' theory may be recurring, that if exist an a priori distribution (P (A) and a series of tests with experimental results $B_1, B_2, \ldots, B_n \ldots$, expressed according to successive equations:

$$\begin{gathered} P(A \mid B_1) = P(A)\frac{P(B_1 \mid A)}{(B_1)} \\ P(A \mid B_1, B\ ) = P(A)\frac{P(B_1 \mid A)}{P(B_1)}\frac{P(B_2 \mid A)}{P(B\ )} \end{gathered} \tag{2}$$

$$P(A \mid B_1, B_2, \ldots B_n) = P(A \mid B_1, B_2, \ldots B_{-1}) - \frac{P(B_n \mid A)}{(B_n)}$$

A posteriori distribution called also belief, is used when the test results are known, being obtained as a new function a priori. The start of operations sequences in the Bayesian method regards the transformation γ. Recursive Bayesian updating is made under the Markov assumption: z_n is independent of $z_1,\ldots,z_{n-1}$ if we know x.

$$\begin{aligned} P(x \mid z_1,\ldots,z_n) &= \frac{P(z \mid x)\, P(x \mid z_1,\ldots,z_{n-})}{P(z \mid z_1,\ldots,z_{n-})} \\ &= \eta\, P(z_n \mid x)\, P(x \mid z_1,\ldots,z_{-}) \quad (3) \\ &= \eta_{1\ldots n} \prod_{i=1\ldots} P(z_i \mid x)\, P(x) \end{aligned}$$

When there are no missing data or hidden variables the method for calculating P(B_{si}, D) for some belief-network structure B_{Si} and database D is presented in [12].Let Q be the set of all those belief-network structures that have a non-zero prior probability. We can derive the posterior probability of B_{Si} given D as:

$$P(B_{Si} \mid D) = P(B_{Si}, D) / \sum B_{Si} \in QP(B_{Si}, D) \quad (4)$$

The ratio of the posterior probabilities of two belief-network structures can be calculated as a ratio for belief-network structures B_{si} and B_{sj}, using the equivalence:

$$P(B_{Si} \mid D) / P(B_{Sj} \mid D) = P(B_{Si}, D) / P(B_{Sj}, D) \quad (5)$$

which we can derive that:

$$P(B_{Si}, D) = P(D \mid B_{Si}) P(B_{Si}) \quad (6)$$

Term P(B_{si}) represents prior probability that a process with belief-network structure B_{Si}. To designate the possible values of *h*, ca be used the Markov blanket method, *MB(h)* [12, 13]. Suppose that among the *m* cases in *D* there are *u* unique instantiations of the variables in *MB(h)*. Given these conditions it follows that:

$$P(D \mid B_S) = \sum_{G_i} \ldots \sum_{G} f(G_1, \ldots, G_u) \int_{B_p} [\prod_{t=1}^{m} P(C_t h_t \mid B_S, B_P)] f(B_S \mid B_P) dB_P \quad (7)$$

where G_i is a given group contains c_i case-specific hidden variables. Recall that u denotes only the number of unique instantiations *actually realized* in database D of the variables in the Markov blanket of hidden variable h. The number of such unique instantiations significantly influences the efficiency with which we can compute Equation 7.

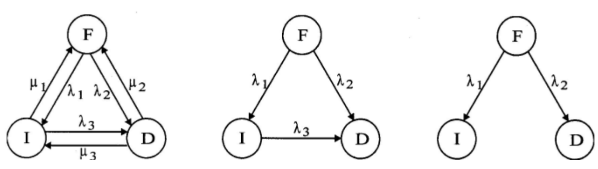

Fig.3 The model with three states for the robotic system

For any finite belief network, the number of such unique instantiations reaches a maximum regardless of how many cases there are in the database. That r denotes the maximum number of possible values for any variable in the database. If u and r are bounded from above, then the time to solve Equation 7 is bounded from above by a function that is polynomial in the number of variables n and the number of cases m. If u or r is large, however, the polynomial will be of high degree [12].

To model a robotic system requires considering in-between the two states of operating and faulting one or more intermediate states of partial success. In figure 3 is considered a robotic system characterized by three states: operating at full capacity (F), defect (D) and intermediate (I).

A generalized diagram of states is shown in figure 4, which included three intermediate states.

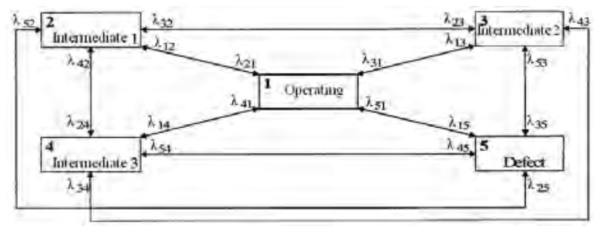

Fig. 4. Generalized diagram of states with three intermediate states

The Markov modeling technique requires to identify each intermediate state (in practice, more neighboring levels can be grouped together), to know the occupancy status of each component (Ti) and the number of transitions between states (Nij), which can calculate as follows:

- occupancy probability of "i"state: $P_i = \dfrac{T_i}{T_A}$
- transition intensity from state *"i"* in *"j"*: $\lambda_{ij} = \dfrac{N_{ij}}{T_i}$,

where: $T_A = \sum_i T_i$ is analyzed time interval.

The number of intermediate states to be modeled in order to obtain a more accurate assessment of the reliability group is necessary to consider more than one intermediate state. Figure 5 presents a model with six states to assess the predictable transitions in a robotic system. The six states of the system are:

1 - operational state of robot;
2 - landing control
3 - balance control
4 - advance control
5 - wireless sensor networks (WSN) control
6 - unpredict event

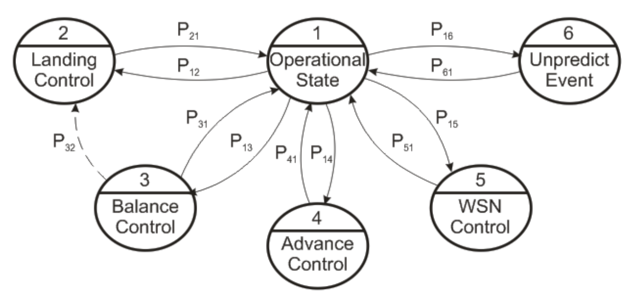

Fig.5. Modeling the states with possible transitions for robot

Based on the surveillance data in operation regime of robot were determined transition probabilities using of the relationship: $\hat{p}_{ij} = \dfrac{n_{ij}}{n_i}$, where n_{ij} is the transition from state

"i" in "j" in the analysis time interval; n_i is the number of all transitions from state "i" in any other states.

Values of these transition probabilities are: $\hat{p}_{12} = 0,247$; $\hat{p}_{13} = 0,32$; $\hat{p}_{14} = 0,125$; $\hat{p}_{15} = 0,205$; $\hat{p}_{16} = 0,103$; By applying the method Markov chains are obtain the occupancy probability of the sates for the robot: P_1=0,31; P_2=0,208; P_3=0,115; P_4=0,205; P_5=0,102; P_6=0,06.

The working diagram of the Petri network is presented in figure 6 (http://www-dssz.informatik.tu-cottbus.de). A token is assigned to P_3, and is assumed that the localizer initially knows its position. The Warning event t_5 fires when the localizer fails in estimating robot's accurate position for several steps. Two navigation primitives can be modeled as P_1, P_2, respectively. Initially, the robot selects its motion by a random switch comprising the transitions t_1 and t_2 with corresponds to probabilities P_1' and P'_2, respectively. The transition between them takes place according to the change of localizer states. The immediate transition t_3 means that the robot takes Contour tracking as soon as the localizer Warning event fires.

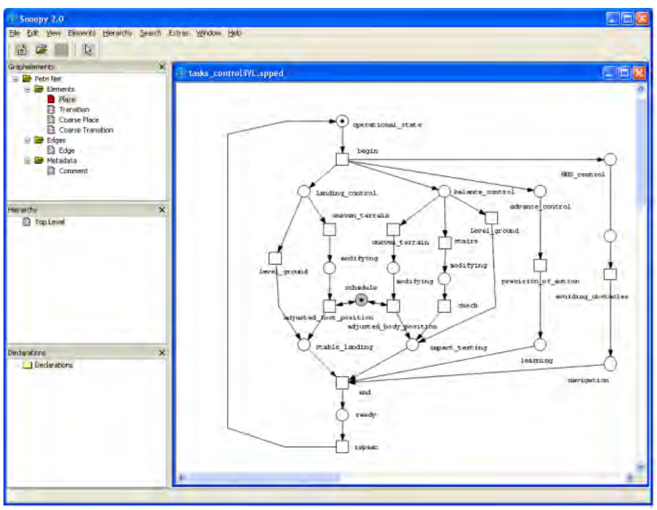

Fig.6. The Petri network diagram

The other transition between two primitives, t_2 and t_4, are modeled as timed transitions in order to express that the robot can change its current navigation primitive during the localizer Success state, if necessary.

V. Results and Conclusion

The control for walking robots is achieved by a control system with three levels. The first level is to produce control signals for motor drive mounted on leg joints, ensuring the robot moving in the direction required with a given speed. The language for this level is that of differential equations. The second level controls the walking, respectively it coordinates the movements, provides the data necessary to achieve progress. At this level, work is described in the language of algorithms types of walking. The third level of command defines the type of walking, speed and orientation.

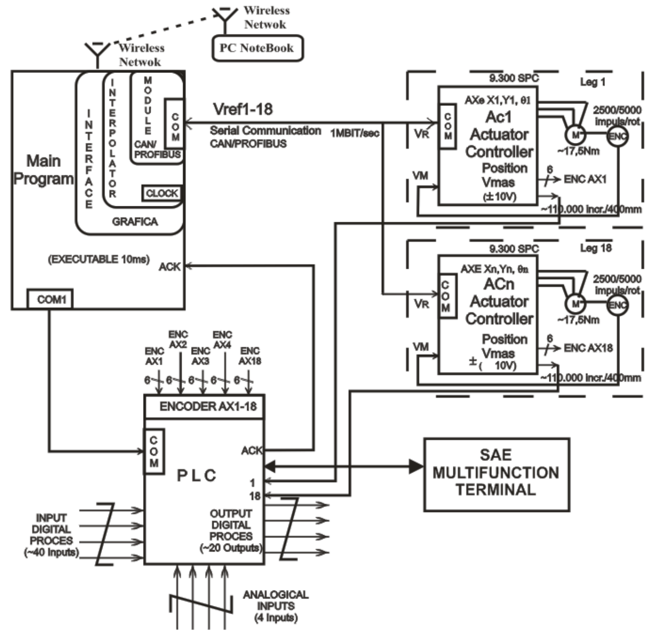

Fig.7. Open architecture system of the walking robot

At this level, the command may be provided by an operator who can use the control panel, in pursuit of its link with the robot, to specify the type of running and passing special orders (for the definition of the vector speed of movement).

To maintain the platform in a horizontal position, the information provided by the horizontality transducers (or verticality) is used, that sense walking robots deviation platform to the horizontal position. Restoring the horizontal position of the platform is achieved at the expense of vertical movement of different legs of support, as decided by the block to maintain balance. Returning to the fixed height of the platform is achieved by using information provided by the height transducer of the platform and by simultaneous control of vertical movement of all legs in support phase. From the analysis performed results the effectiveness of the proposed control strategy for a walking robot. The position of each actuator is controlled by a PD feedback loop, using encoder like transducers.

In HFPC control system, the PC system sends the reference positions to all actuators controllers simultaneously at an interval of 10 ms (100 Hz). Reference positions for the control of 18 actuators and actual positions on each axis robot obtained through interpolation are processed at an interval of 1 ms (1 kHz). Figure 1 shows the general configuration of the HFPC system for ZMP control method. The control system is distributive with multi-processor devices for joint control, data reception from transducers mounted on the robot, peripheral devices connected through a wireless LAN for off-line communications and CAN fast communication network for real time control. The HFPC system was designed in a distributed and decentralized structure to enable development of new applications easily and to add new modules for new hardware or software control functions. Moreover, the short time execution will ensure a faster feedback, allowing other programs to be performed in real time as well, like the apprehension force control, objects recognition, making it possible that the control system have a human flexible and friendly interface.

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ランドマークとGPSによる移動ロボットのナビゲーション

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MOBILE ROBOT NAVIGATION USING ARTFICIAL LANDMARKS AND GPS

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1　はじめに

移動ロボットのナビゲーションを行うにはロボットが十分に現在位置と周囲の環境を認識する必要がある。そのために、ロボットにレーザーレンジスキャナや超音波センサ、カメラ、オドメトリ、GPS (Global Positioning System) 等のセンサを搭載することで、ロボットは現在位置・姿勢、周囲の様子、移動距離、周囲の物との距離等を知ることができるようになる。しかし、センサからの情報には誤差が含まれており、移動している環境や搭載しているセンサにより生じる誤差が累積されることで、現在の位置がわからなくなり、走行経路から外れて、目的地へたどりつけなくなることがある。正しい位置を認識するには、定期的に誤差を解消し、位置の校正を行う必要がある。位置校正を向上させるために、ロボットに SLAM (Simultaneous Localization and Mapping)[1] アルゴリズムや Kalman Filter[2] などの制御技術が導入される。

SLAM とは、自己位置認識と自己位置の校正方法の一つであり、物体と移動ロボットとの相対距離を距離測定センサによって計測しながら、得られたデータをもとにし、環境地図を作成する。また、同時にオドメトリ (Dead reckoning) 法によって移動距離を計算し、自己位置の認識を行う。

移動ロボットが屋内環境でナビゲーションする場合、建造物の構造によって行動範囲が限定され、ロボットに与えられている経路情報や障害物などの環境情報量が少なくて済む。その反面、走行経路の特徴が単調な場合が多く、現在位置を認識するのが難しくなる可能性がある。さらに、床がタイルなどであれば、タイヤがスリップを起こし、オドメトリセンサに誤差が累積されるため、より位置を認識するのが難しくなる。これらの問題に対する解決策として、床や壁に描かれている模様や天井の蛍光灯、通路上の仕切りなどから現在位置を推定する手法、画像やタグなどの人工のランドマークを設置し、これをもとに位置推定を行う手法などが提案されている [3][4]。

一方、屋外環境をナビゲーションする場合、行動範囲が非常に広く、走行経路が複雑になる。特に、屋内とは違い、走行環境は起伏が多く、障害となるものが多い。そのため、ロボットに与える環境情報量が多くなる。屋外での位置推定には、GPS や V-SLAM (Vision-based SLAM) を使った手法などがある [5]。

本研究の目標は、移動ロボットの屋外走行を行うことである。移動ロボットを屋外走行させている他の研究では [6][7]、ロボットに事前に走行環境を学習させており、高性能なセンサを搭載している。しかし、屋外の環境が変わるつどに、事前準備としての再学習が必要になり、手間がかかる。また、高性能なセンサを使用すると、ロボットシステムが高価になってしまう欠点がある。そこで、本研究では、移動ロボットに事前に学習をさせない、あるいは、少ない学習量で、屋外走行を行うことを目指す。

本稿の構成は次のようになっている。校正用ランドマークと V-SLAM を使用して、移動ロボットを屋内環境で走行させていた研究 [8][9] では、自己位置認識が困難な環境でもランドマークを設置することで、自己位置の認識ができ、環境内を正確に走行することができた。この手法を屋外走行に用いるので、移動ロボットの自己位置認識の向上を行う。そのため、2 節では、校正用ランドマークと V-SLAM を使用した移動ロボットのナビゲーションについて簡単に説明する。

次に、3 節において、移動ロボットの屋外ナビゲーション方法を提案する。提案するナビゲーション法では、ロボットにあらかじめ移動経路を渡すが、V-SLAM、

図 1: 校正用ランドマーク

ランドマークと GPS からの情報を用いて、自己位置認識及び位置の校正を行う。本手法では、GPS 信号の受信が不可能となった場合、V-SLAM とランドマークによって位置・姿勢を推定する。また、ランドマーク認識に失敗した場合、GPS と V-SLAM を使用して、高信頼性なナビゲーションを目指す。4 節に、使用するロボットの構成を説明して、行ったナビゲーション実験とその結果を示し、最後に、本研究のまとめと今後解決するべき課題について述べる。

2　位置校正用ランドマークと V-SLAM

本研究では、移動ロボットが屋内環境でナビゲーションする際に、校正用ランドマークと V-SLAM を用いて、ロボットの自己位置認識と位置の校正を行う手法を提案した [8][9]。これにより、移動ロボットは経路を正確に走行することができた。

本手法は、ロボットが経路を走行中に校正用ランドマークを認識するとその場で位置の校正を行う。ランドマークは、基準となる校正位置を示しており、ロボットがこの位置とオドメトリによって推定した現在位置との誤差をなくすことで位置の校正を行う。

校正用ランドマーク

校正用ランドマークを図 1 に示す。QR コードには位置情報が書き込まれており、ロボットは、この情報をもとに位置の校正を行う。左右に配置されている三角形は、ロボットがランドマークを認識しやすくなるためのものである。

校正手順

初めに、校正用ランドマークをロボットのカメラで撮影する。このときにランドマークまでの正確な距離や角度、特徴点の数などをモデル情報として、ロボットのモデル用データーベースに記憶させる。ロボットは、V-SLAM アルゴリズム下で、撮影した画像をもとにして、走行中に校正用ランドマークを認識すれば、ランドマークまでの距離と角度を算出して、位置を推定する。算出結果からの位置とランドマークが示している位置との誤差をなくすことによって位置の校正が行われる。

ランドマークの導入をしたことにより、特徴の数が少ない環境でも正確な屋内走行の実現ができた。そのこのとを複数の実験で検証した。

3　屋外でのナビゲーション

本研究の主な目的は移動ロボットを屋外走行させることである。移動ロボットの屋外ナビゲーションを行うために GPS を使用する。そこで、使用する GPS 受信機を３つ用意し、それぞれの性能を評価する。

次に、評価し終えた GPS 受信機を移動ロボットに搭載して、屋外走行を行う。また、2 節で述べたように校正用ランドマークと V-SLAM を使用した手法と併せてナビゲーション精度の向上を目指す。

行う屋外ナビゲーションは次のステップからなる。

1. 初期化：移動ロボットに移動経路、ランドマークモデルを転送する。
2. GPS 信号受信による走行
3. 校正用ランドマーク認識
4. ロボット・ランドマークの相対位置算出
5. ２次元コード（QR コード）の内容により自己位置校正
6. GOTO ステップ 2

なお、走行時に常に V-SLAM アルゴリズムを用いて、環境地図生成を行う。そこで、GPS 信号受信状況が悪化した場合、V-SLAM と校正用ランドマークにより、走行を続行する。また、ランドマーク認識ができないときに、GPS と V-SLAM によりナビゲーションを行うこととする。このナビゲーション方法のより信頼性が向上し、移動ロボットはキドナッピング状態を起こすことが避けられる。

GPS について

ここで、まず、GPS について説明する。GPS には大きく分けて次の２種類の測位方法がある。１つ目は、単独で測位する方法で、もう１つは相対測位である。前者は、１個の GPS 受信機が 4 つ以上の GPS 衛星を捉える

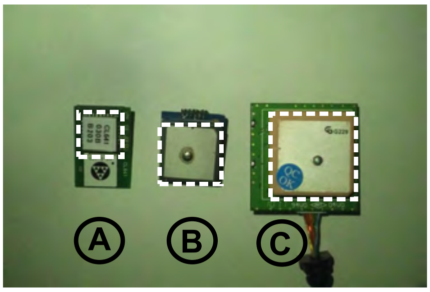

図 2: 使用する GPS

ことで位置を検出する測位方法である。相対測位は、さらに、DGPS (Differential GPS) と RTK-GPS (Real Time Kinematic-GPS) に分かれる。DGPS は相対測位のことであり、複数の受信機で 4 つ以上の GPS 衛星を観測して、受信機間の相対的な位置関係を計測する方法である。RTK-GPS とは、干渉測位のことであり、2つの受信機から、ある衛星までの距離の差を搬送波の位相を使ってもとめ、基線ベクトルを決定する計測方法である。単独測位は、測位精度が低く、相対測位は、精度が高い。特に、RTK-GPS は高い精度で測定を行うことができるが、測定に時間がかかるという欠点がある。

GPS のデータにはさまざまな要因により誤差が生じる。例えば、衛星から受信機までのあいだにある大気の影響、遮蔽物による影響、建物などによるマルチパス、また、受信データ自体にも人工的なノイズが含まれている。

GPS が出力するデータの通信規格は NMEA と呼ばれるものであり、音波探査機、ソナー、風速計 (風向風速計)、ジャイロコンパス、自動操舵装置 (オートパイロット)、GPS 受信機、その他数々の海上電子装置における通信規格のことである。NMEA 規格は、米国に本拠を置く米国海洋電子機器協会により規定されている。本研究で使用する GPS の通信規格は NMEA0183 となっている。今回は、受信データに含まれている緯度経度の情報を使用する。なお、GPS が出力する緯度経度の単位が世界測地系の度分秒（60 進数）で表示されており、地図上で表示する場合は、度 (10 進数) に変換する必要がある。

図 2 は、本研究で使用する 3 種類の GPS 受信機 (以下、GPS) である。GPS の測位方式は単独測位するタイプで、測位精度は 3m となっている。点線で囲まれている部分は GPS のアンテナである。

4 実験結果

ここで、提案したナビゲーション法を検証するために行った実験について説明する。まず、使用する車輪型移動ロボットの仕様を次節で示す。次に、屋内ナビゲーション実験、行った GPS レシーバの評価と屋外走行実験の内容について述べる。

図 3: 移動ロボットの外観と座標系

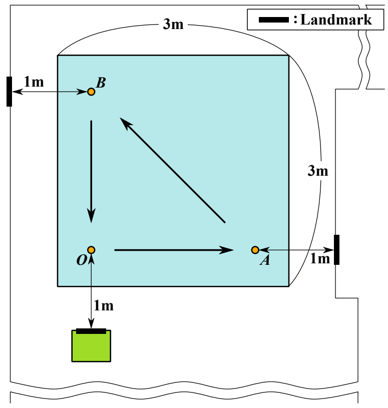

図 4: 走行環境

4.1 移動ロボットの構成

移動ロボットにはカメラ、オドメトリ、超音波センサ、GPS センサを搭載し、これらのセンサを使用して、移動ロボットのナビゲーション実験を行う。屋外環境での正確な位置推定には、GPS の精度により左右されるため、初めに複数 GPS の精度の評価を行う。次に、選定した GPS を移動ロボットに搭載して、屋外走行を行う。また、校正用ランドマークと V-SLAM を使用して、移動ロボットを屋内環境で走行させていた研究 [8][9] では、自己位置認識が難しい環境でもランドマークを設置することによって、自己位置の認識ができ、環境内を正確に走行することができた。この手法を屋外走行に用いることで移動ロボットの自己位置認識の向上を図る。

本研究では、MobileRobots 社製の P3-DX 移動ロボットを用いる [10]。移動ロボットの外観と座標系を図 3 に示す。搭載されているセンサ類は、500ppr の分解能を持つロータリエンコーダ、障害物回避のための超音

表 1: Calibration results

Method	Location error [mm]	Orientation error [deg]
Odometry	307.4	52.9
V-SLAM	342.2	51.3
Proposed	69.2	10.2

波センサ、640×480 の有効画素数を持つ 130 万画素の CCD カメラ (Logitech 社製 QuickCam Pro 4000)、自己位置認識のための GPS である。

4.2 屋内環境での走行実験

ここで、校正用ランドマークと V-SLAM を用いた屋内走行実験について説明する [8][9]。図 4 に屋内走行の 1 つの環境を示す。校正用ランドマークの有効性を確認するため、オドメトリと V-SLAM には不利な環境で実験を行った。実験環境は周囲に目立った特長が少なく、床はタイヤがスリップを起こしやすいタイルになっている。ロボットは点 O、点 A、点 B の順に循環する。移動中にランドマークを認識すると位置の校正を行う。移動と校正を繰り返し、生じる誤差の平均値を表 1 に示す。

オドメトリより V-SLAM の位置誤差が大きくなっているのが、走行環境内に目立った特徴が少ないからである。一般的に、環境の数多くの特徴点を持つ場合、V-SLAM の精度はおよそ 10cm 以下である。今回の実験でこのことを確認した。

また、同表より、本手法は正しい自己位置に復帰していることが分かる。この結果から、V-SLAM にランドマークを適用することで、校正精度の向上を図れたことが明らかである。

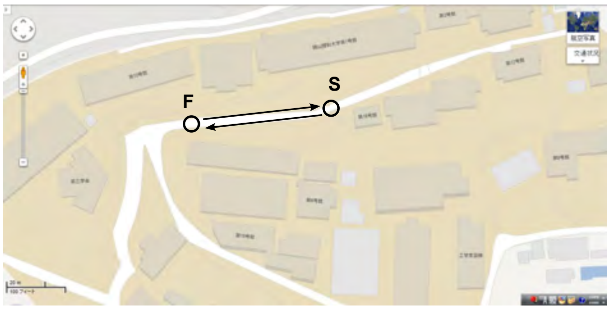

図 5: 測定環境の地図

4.3 GPS 測位性能の検証

測定方法

図 5 は測定環境である。道の幅が 3〜4m であり、移動距離が約 60m の道である。測定は、図 5 の S 点と F 点の間の道のりを往復して行った。

図 6 は、地点 S から移動しながら測定した結果である、A の GPS は蛇行して、道からそれている。B と C の GPS は道に沿うように寄ってきていることがわかる。特に、C の GPS が正確に道に沿っている。A と B の GPS 受信機の精度がよくないのは、走行環境での受信状況が頻繁に変更するからである。両 GPS のアンテナの寸法が小さいため、感度が減少する。

移動ロボットには、C の GPS を搭載して、屋外ナビゲーションを行う。

4.4 移動ロボットの屋外走行

図 5 に示した環境内に移動ロボットの屋外ナビゲーションを行う。ここで、校正用ランドマークと V-SLAM を併用した走行も行う。

まず、GPS のみで以下のように屋外走行を行った。事前に移動ロボットには走行経路の情報を与えている。移動ロボットは開始地点から目的地までの経路を移動中に定期的に一時停止し、その場で GPS による位置の

(a) A の GPS (b) B の GPS (c) C の GPS

図 6: S から F に移動しながら位置の測定をした結果

(a) A の GPS　(b) B の GPS　(c) C の GPS

図 7: F から S に移動しながら位置の測定をした結果

測定を行う。オドメトリによって移動ロボットが認識している位置と GPS が示す位置との誤差をなくすことで、位置の校正を行う。なお、今回はソフトウェアの開発が途中であるため、GPS を使用するときに一時停止するが、今後は、リアルタイムで GPS を使用できるようにして、移動ロボットのナビゲーションを行う。

次に、校正用ランドマークと V-SLAM を GPS と共に用いる。事前にロボットに与える情報は、走行経路と校正用ランドマークのモデル情報である。校正用ランドマークは経路上に一定間隔で配置した。位置の校正をするときは、最初に校正用ランドマークと V-SLAM を使用する。校正手順は 3 節と同じである。ロボットが、ランドマークを認識すると、ロボットが持っているランドマークのモデル情報とカメラで読み取ったモデル情報を比較し、ランドマークとの距離や角度などの位置を算出する。算出された位置とランドマークが示している校正位置との誤差をなくすことで、位置の校正を行う。もし、カメラがランドマークを発見できなかった場合、代わりに GPS を用いて位置の校正を行う。

GPS のみで走行した場合

図 8 は走行経路と校正位置を表している。丸印が開始地点と目的地を表し、四角の印が校正位置である。三角の印は GPS が示した位置である。道の北側にコンクリートの壁があり、南側は斜面となっている。

start 地点の周囲には木や背の高いコンクリートの壁があるため、マルチパスが発生し、GPS は壁際や斜面に近い位置を示すことが多い。そこで、受信環境の良い所に移動して、再度、走行を行った。走行場所を図 9 に示す。そこで、GPS の精度が高くなり、移動ロボットの走行が期待通りにできた。

しかし、移動ロボットが安定した走行をするには、GPS の精度をさらに向上する必要がある。Kalman Filter 等を用いて GPS の精度を向上させた上で、再度、走行させることを試みる。

校正用ランドマークと V-SLAM を併用した場合

次に、校正用ランドマークと V-SLAM を用いて屋外走行を行った。校正用ランドマークにより高精度な位置の校正はできるが、日差しの影響でランドマークの認識が困難である。。これは、カメラに自動で明るさを調節する機能が付いていないために生じた問題である。ただし、良好な受信状況下で、2 節の方法を適用するとランドマークの認識が不可能であった場合でも GPS による高精度な走行を実現できた。

5 まとめ

移動ロボットを屋外走行させるために、GPS と校正用ランドマークおよび V-SLAM を使用した。GPS を使用して移動ロボットの屋外ナビゲーションを行う前に、

図 8: 走行経路と校正位置

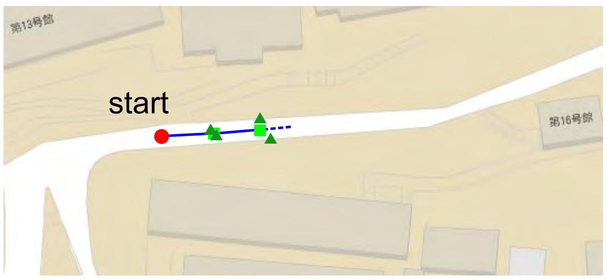

図 9: 受信状況の良い走行場所

GPS レシーバの測位精度を評価した。検証に使用したGPS は 3 種類で、いずれも単独測位するタイプである。GPS を利用したときにロボットが道に沿った高精度な走行が得られた。

しかし、安定した走行を実現するのには、GPS の精度をさらに向上させる必要がある。Kalman Filter 等の制御技術を用いて、GPS の測位精度が向上し、安定した走行をできると考えている。

次に、校正用ランドマークと V-SLAM を使用して屋外ナビゲーションを行った。この手法は、高い精度の位置校正ができたため、今回の屋外走行にも用いた。その結果、充分な精度での走行はできたが、日差しの影響でランドマークも認識が困難になることがあった。自動で明るさを調節する機能を持ったカメラを使うことで、この問題は解決でき、長距離の走行が可能になると考えている。

今後、上記の問題を解決することで、GPS と校正用ランドマークと V-SLAM を併用した、移動ロボットの長距離屋外走行させることが可能になる。

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STATISTICS

SOME RATIO TYPE ESTIMATORS UNDER MEASUREMENT ERRORS

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Abstract: This article addresses the problem of estimating the population mean using auxiliary information in the presence of measurement errors. A comparative study is made among the proposed estimators, the mean per unit estimator and the ratio estimator in the presence of measurement errors.

Key words: Auxiliary variate · Measurement error · Observational error · Mean square error · Efficiency

INTRODUCTION

In survey sampling, the properties of the estimators based on data usually presuppose that the observations are the correct measurements on characteristics being studied. Unfortunately, this ideal is not met in practice for a variety of reasons, such as non response errors, reporting errors and computing errors. When the measurement errors are negligible small, the statistical inferences based on observed data continue to remain valid. On the contrary when they are not appreciably small and negligible, the inferences may not be simply invalid and inaccurate but may often lead to unexpected, undesirable and unfortunate consequences [1]. Some authors including [2-7] have paid their attention towards the estimation of population mean μ_y of the study variable y using auxiliary information in the presence of measurement errors.

For a simple random sampling scheme, let (x_i, y_i) be observed values instead of the true values (X_i, Y_i) on two characteristics (x,y) respectively for the i^{th} (i=1.2….n) unit in the sample of size n. Let the measurement errors be

$$u_i = y_i - Y_i \quad (1.1)$$
$$v_i = x_i - X_i \quad (1.2)$$

which are stochastic in nature with mean zero and variances σ_u^2 and σ_v^2 respectively and are independent. Further, let the population means of (x, y) be (μ_X, μ_Y) population variances of (x,y) be $\left(\sigma_x^2, \sigma_y^2\right)$ and σ_{xy} and ρ be the population covariance and the population correlation coefficient between x and y respectively [4].

In this paper we study the behaviour of some estimators in presence of measurement error.

Exponential Ratio-type Estimator under Measurement Error: [8], suggested an exponential ratio type estimator for estimating $\bar{Y}$ as

$$t_1 = \bar{y}\exp\left(\frac{\mu_x - \bar{x}}{\mu_x + \bar{x}}\right) \quad (2.1)$$

Let $w_u = \frac{1}{\sqrt{n}}\sum_{i=1}^{n} u_i,\ w_y = \frac{1}{\sqrt{n}}\sum_{i=1}^{n}\left(y_i - \mu_y\right)$

$$w_v = \frac{1}{\sqrt{n}}\sum_{i=1}^{n} v_i,\ w_x = \frac{1}{\sqrt{n}}\sum_{i=1}^{n}\left(x_i - \mu_x\right)$$

$$C_x = \frac{\sigma_x}{\mu_x} \text{ and } C_y = \frac{\sigma_y}{\mu_y}$$

Expression (2.1) can be written as

$$t_1 = \left[\mu_y + \left(\bar{y} - \mu_y\right)\right]\exp\left[\frac{\mu_x - \left(\mu_x + \bar{x} - \mu_x\right)}{\mu_x + \left(\mu_x + \bar{x} - \mu_x\right)}\right]$$

$$\left(\mu_y + k_1\right)\exp\left[\frac{-k_2}{2\mu_x + k_2}\right] \quad (2.2)$$

where,

$$k_1 = \bar{y} - \mu_y = \frac{1}{\sqrt{n}}\left(w_y - w_u\right) \text{ and } k_2 = \bar{x} - \mu_x = \frac{1}{\sqrt{n}}\left(w_x - w_v\right) \quad (2.3)$$

On simplifying expression (2.2), we have

$$\left(t_1-\mu_y\right)=\mu_y\left[-\frac{1}{2}\left(\frac{k_2}{\mu_x}\right)+\frac{3}{8}\left(\frac{k_2}{\mu_x}\right)^2+..\right]$$
$$+k_1\left[1-\frac{1}{2}\left(\frac{k_2}{\mu_x}\right)+\frac{3}{8}\left(\frac{k_2}{\mu_x}\right)^2+..\right] \tag{2.4}$$

On taking the expectations and using the results

$$E\left(k_1\right)=E\left(k_2\right)=0$$

$$E\left(k_1^2\right)=\frac{\sigma_y^2}{n}\left(1+\frac{\sigma_u^2}{\sigma_y^2}\right)=V_{ym} \tag{2.5}$$

$$E\left(k_2^2\right)=\frac{\sigma_x^2}{n}\left(1+\frac{\sigma_v^2}{\sigma_x^2}\right)=V_{xm} \tag{2.6}$$

$$E\left(k_1k_2\right)=\frac{\rho\sigma_y\sigma_x}{n}=V_{yxm} \tag{2.7}$$

Taking expectation on both side of (2.4) the bias of t_1, to first order of approximation is

$$Bias\left(t_1\right)=\frac{1}{\mu_x}\left(\frac{3}{8}R_mV_{xm}-\frac{1}{2}V_{yxm}\right) \tag{2.8}$$

where $R_m=\frac{\mu_y}{\mu_x}$.

Squaring both side of (2.4) and taking expectation, the mean square error of $t_1$up to the first order of approximation, is

$$MSE=E\left(t_1-\mu_y\right)^2$$
$$=\frac{\mu_y^2}{4\mu_x^2}E\left(k_2^2\right)+E\left(k_1^2\right)-\frac{\mu_y}{\mu_x}E\left(k_1k_2\right)$$
$$=\frac{1}{4}R_m^2\left(\frac{\sigma_x^2}{n}\left(1+\frac{\sigma_v^2}{\sigma_x^2}\right)\right)+\frac{\sigma_y^2}{n}\left(1+\frac{\sigma_u^2}{\sigma_y^2}\right)-\frac{\rho\sigma_y\sigma_x}{n}R_m$$
$$=\frac{\sigma_y^2}{\mathrm{n}}\left[1-\frac{C_x}{C_y}\left(\rho-\frac{C_x}{4C_y}\right)\right]+\frac{1}{n}\left[\frac{\mu_y^2}{4\mu_x^2}\sigma_v^2+\sigma_u^2\right]=\mathrm{M}_{t_1}^*+M_{t_1} \tag{2.9}$$

where $\mathrm{M}_{t_1}^*=\frac{\sigma_y^2}{\mathrm{n}}\left[1-\frac{C_x}{C_y}\left(\rho-\frac{C_x}{4C_y}\right)\right]$ is the mean squared error of t_1 without measurement error and $M_{t_1}=\frac{1}{n}\left[\frac{\mu_y^2}{4\mu_x^2}\sigma_v^2+\sigma_u^2\right]$ is the contribution of measurement errors in t_1

Another Estimator under Measurement Error: [9], suggested a regression type estimator t_2 as-

$$t_2=\omega_1\bar{y}+\omega_2\left(\mu_x-\bar{x}\right) \tag{3.1}$$

where ω_1 are ω_2 constants that have no restriction.

Expression (3.1) can be written as

$$t_2-\mu_y=\omega_1\,k_1(\omega_1-1)-\omega_2\,\omega_2 \tag{3.2}$$

Taking expectation of both side of (3.2), we get the bias of the estimator t_2 to order $O(n^{-1})$ as

$$Bias(t_2)=\mu_y\,(\omega_1-1)$$

Squaring both side of (3.2) and taking expectation, the MSE of t_2 to the to order $O(n^{-1})$ is

$$MSE\left(t_2\right)=\mu_y^2\left(\omega_1-1\right)^2+\frac{1}{n}\omega_1^2\sigma_y^2+\frac{1}{n}\omega_2^2\sigma_x^2-\frac{2}{n}\omega_1\omega_2\rho\sigma_y\sigma_x+\frac{1}{n}\left(\omega_1^2\sigma_u^2+\omega_2^2\sigma_v^2\right) \tag{3.4}$$

$$=M_{t_2}^*+M_{t_2} \tag{3.5}$$

Where,
$M_{t_2}^*=\mu_y^2\left(\omega_1-1\right)^2+\frac{1}{n}\omega_1^2\sigma_y^2+\frac{1}{n}\omega_2^2\sigma_x^2-\frac{2}{n}\omega_1\omega_2\rho\sigma_y\sigma_x$, is the MSE of t_2 without measurement error and $M_{t_2}=\frac{1}{n}\left(\omega_1^2\sigma_u^2+\omega_2^2\sigma_v^2\right)$ is the contribution of measurement error in t_2.

Expressing MSE of t_2 as

$$MSE\left(t_2\right)=\left(\omega_1-1\right)^2\mu_y^2+\omega_1^2\left(a_1\right)+\omega_2^2a_2+2\omega_1\omega_2\left(-a_3\right) \tag{3.6}$$

where,

$a_1=(V_{ym})$, $a_2=(V_{xm})$, and $a_3=(V_{yxm})$,

Now, optimising MSE of the estimator t_2 with respect to ω_1 and ω_2 we get
where,

$$b_1 = \mu_y^2 + a_1, \quad b_2 = -a_3, \quad b_3 = a_2, \quad and \quad b_4 = \mu_y^2.$$

Using the values of ω_1^* and ω_2^* from equation (3.7) into equation (3.6), we get the minimum MSE of the estimator t_2 as

$$MSE(t_2)_{\min} = \left[\mu_y^2 - \frac{b_3 b_4^2}{b_1 b_3 - b_2^2}\right] \tag{3.8}$$

A General Class of Estimators: We propose a general class of estimator t_3 as

$$t_3 = \left[m_1 \bar{y} + m_2(\mu_x - \bar{x})\right]\left(\frac{\mu_x}{\bar{x}}\right)^{\alpha} \exp\left[\frac{\mu_x - \bar{x}}{\mu_x + \bar{x}}\right]^{\beta} \tag{4.1}$$

Expanding equation (4.1) and subtracting μ_y from both side, we have

$$(t_3 - \mu_y) = \left\{(m_1 - 1)\mu_y + m_1\mu_y\left\{-B\frac{k_2}{\mu_x} + \frac{k_2^2 A}{\mu_x^2 8}\right\} + m_1 k_1\left\{1 - \frac{Bk_2}{\mu_x} + \frac{k_2^2 A}{\mu_x^2 8}\right\}\right\} \tag{4.2}$$

Taking expectation of both side of (4.2), we get the bias of the estimator t_3 to the order $O(n^{-1})$

$$Bias(t_3) = (m_1 - 1)\mu_y + m_1\mu_y\left\{\frac{V_{xm}A}{8\mu_x^2}\right\} - m_1\left\{\frac{B}{\mu_x}V_{yxm}\right\} + m_2\left\{\frac{B}{\mu_x}V_{xm}\right\} \tag{4.3}$$

where,

$$A = \frac{1}{8}\left[4\alpha(\alpha+1) + \beta(\beta+2) + 4\alpha\beta\right], \text{ and } B = \left(\alpha + \frac{\beta}{2}\right)$$

Squaring both side of (4.2) and taking expectation, the MSE of t_3 to the to order $O(n^{-1})$ is

$$\begin{aligned} MSE(t_3) &= E(t_3 - \mu_y)^2 \\ &= (m_1 - 1)^2\mu_y^2 + m_1^2(V_{ym}) + (m_1 BR_m + m_2)^2(V_{xm}) - 2m_1(m_1 BR_m + m_2)(V_{yxm}) \\ &\quad - 2(m_1 - 1)\mu_y\left\{\frac{m_1 B}{\mu_x}V_{yxm} + \frac{1}{\mu_x}\left(\frac{m_1 R_m A}{8} + m_2 B\right)V_{xm}\right\} \end{aligned}$$

$$\begin{aligned} \mathrm{MSE}(t_3) &= m_1^2\left(\mu_y^2 + V_{xm} + B^2R_m^2 - 2BR_m V_{yxm}\right) + m_2^2(V_{xm}) + 2m_1m_2\left(BR_m V_{xm} - V_{yxm}\right) \\ &\quad - 2m_1\mu_y^2 + \mu_y^2 - 2m_1^2 BR_m Vyxm + 2m_1 R_m BV_{yxm} - \frac{m_1^2 R_m^2 A}{4}V_{yxm} \\ &\quad - 2m_2 R_m BV_{yxm} + \frac{R_m^2 A m_1}{4}V_{xm} + 2m_2 R_m BV_{xm} \end{aligned} \tag{4.4}$$

Writing MSE of the estimator t_3 as

$$MSE(t_3) = (m_1 - 1)^2\mu_y^2 + m_1^2(A_1 + 2A_3) + m_2^2 A_2 + 2m_1m_2(-A_4 - A_5) - 2m_1A_3 + 2m_2A_5 \tag{4.5}$$

where,

$$A_1 = \left(V_{ym} + B^2R_m^2V_{xm} - 2R_m BV_{yxm}\right), \quad A_2 = (V_{xm}),$$
$$A_3 = \left(AR_m^2V_{xm} - BR_mV_{yxm}\right), \ A_4 = \left(V_{yxm} - BV_{xm}R_m\right),$$
$$A_5 = (-BR_mV_{xm}).$$

Now, optimising MSE t_3 with respect to m_1 we get the optimum values of m_2 as-

$$m_1^* = \frac{B_3B_4 - B_2B_5}{B_1B_3 - B_2^2} \quad and \quad m_2^* = \frac{B_1B_5 - B_2B_4}{B_1B_3 - B_2^2} \tag{4.6}$$

where,

$$B_1 = \mu_y^2 + A_1 + 2A_3, \quad B_2 = -A_4 - A_5, \quad B_3 = A_2, \quad B_4 = \mu_y^2 + A_3 \quad \text{and} \quad B_6 = -A_5.$$

Minimum MSE of the estimator t_3 is given by

$$MSE(t_3)_{\min} = \left[\mu_y^2 - \frac{B_1B_5^2 + B_3B_4^2 - 2B_2B_4B_5}{B_1B_3 - B_2^2}\right] \tag{4.7}$$

Theoretical Efficiency Comparison: The MSE of the proposed estimator t_1 will be smaller than usual estimator under measurement error case, if the following condition is satisfied by the data set

$$\frac{\sigma_y^2}{n}\left[1 - \frac{C_x}{C_y}\left(\rho - \frac{C_x}{4C_y}\right)\right] + \frac{1}{n}\left[\frac{\mu_y^2}{4\mu_x^2}\sigma_v^2 + \sigma_u^2\right] \le \frac{\sigma_y^2}{n}\left(1 + \frac{\sigma_u^2}{\sigma_y^2}\right)$$

or

$$R_m^2 \frac{V_{xm}}{V_{yxm}} \le 4 \tag{5.1}$$

As the estimator t_3 defined in (3.1) is the particular member of the generalised estimator t_3 defined in (4.1), if the condition (5.2) is satisfied for different values of α,β the m_1, $m_2$estimator t_3(or t_2) will be better than usual estimator under measurement errors.

$MSE(t_3)_{\min} \le V(\bar{y}_m)$

if,

$$\left[\mu_y^2 - \frac{B_1B_5^2 + B_3B_4^2 - 2B_2B_4B_5}{B_1B_3 - B_2^2}\right] \le V(\bar{y}_m) \tag{5.2}$$

Empirical Study

Data Statistics: The data used for empirical study has been taken from [10].

Where, Y_i = True consumption expenditure,

X_i = True income,

y_i = Measured consumption expenditure,

x_i = Measured income.

n	μ_y	μ_y	σ_y^2	σ_x^2	ρ	σ_u^2	σ_v^2
10	127	170	1278	3300	0.964	36.00	36.00

Table 6.1: Showing the MSE of the estimators with and without measurement errors.

Estimators	MSE without measurement error	Contribution of measurement error in MSE	MSE with measurement error
Usual estimator $(\bar{y})$	319.500	9.0005	328.5005
t_1	64.8134	10.2558	75.0700
t_{reg}	22.5000	12.2401	34.7050
t_{2min}(α = 0, β = 0)	22.4687	12.1619	34.6306
t_{3min}(α = 1, β = 0)	22.4678	12.1619	34.6306
(α = 0, β = 1)	22.1033	12.0680	34.1713
(α = 1, β = 1)	20.9734	12.4075	33.3809

CONCLUSION

From Table 6.1, we observe that the MSE of the estimator t_3 (for $\alpha = 1$, $\beta = 1$) is minimum. From third column of the Table we observe that the usual estimator () is least affected by measurement errors. We also observe from that Table that, if due care of observational error is not given, the estimate of the variances that we get gives an under estimate of the true variances.

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A FAMILY OF ESTIMATORS FOR ESTIMATING POPULATION MEAN IN STRATIFIED SAMPLING UNDER NON-RESPONSE

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Abstract

Khoshnevisan et al. (2007) proposed a general family of estimators for population mean using known value of some population parameters in simple random sampling. The objective of this paper is to propose a family of combined-type estimators in stratified random sampling adapting the family of estimators proposed by Khoshnevisan et al. (2007) under non-response. The properties of proposed family have been discussed. We have also obtained the expressions for optimum sample sizes of the strata in respect to cost of the survey. Results are also supported by numerical analysis.

1. Introduction

There are several authors who have suggested estimators using some known population parameters of an auxiliary variable. Upadhyaya and Singh (1999) and Singh et al. (2007) have suggested the class of estimators in simple random sampling. Kadilar and Cingi (2003) adapted Upadhyaya and Singh (1999) estimator in stratified random sampling. Singh et al. (2008) suggested class of estimators using power transformation based on the estimators developed by Kadilar and Cingi (2003). Kadilar and Cingi (2005), Shabbir and Gupta (2005, 06) and Singh and Vishwakarma (2008) have suggested new ratio estimators in stratified sampling to improve the efficiency of the estimators.

Khoshnevisan et al. (2007) have proposed a family of estimators for population mean using known values of some population parameters in simple random sampling (SRS), given by

$$t = \bar{y}\left[\frac{a\bar{X}+b}{\alpha(a\bar{x}+b)+(1-\alpha)(a\bar{X}+b)}\right]^{g}$$

where $a \neq 0$ and b are either real numbers or functions of known parameters of auxiliary variable X. Koyuncu and Kadilar (2008, 09) have proposed family of combined-type estimators for estimating population mean in stratified random sampling by adapting the estimator of Khoshnevisan et al. (2007). These authors assumed that there is complete response from all the sample units. It is fact in most of the surveys that information is usually not obtained from all the sample units even after callbacks. The method of sub-sampling the non-respondents proposed by Hansen and Hurwitz (1946) can be applied in order to adjust the non-response in a mail survey.

In the next sections, we have tried to propose a family of combined-type estimators considering the above family of estimators in stratified random sampling under non-response. We have discussed the properties of proposed family of estimators. We have also derived the expressions for optimum sample sizes of the strata in respect to cost of the survey.

2. Sampling Strategies and Estimation Procedure

Let us consider a population consisting of N units divided into k strata. Let the size of i^{th} stratum is N_i, ($i = 1,2,..................,k$). We decide to select a sample of size n from the entire population in such a way that n_i units are selected from the N_i units in the i^{th} stratum. Thus, we have $\sum_{i=1}^{k} n_i = n$. Let Y and X be the study and auxiliary characteristics respectively with respective population mean $\overline{Y}$ and $\overline{X}$. It is considered that the non-response is detected on study variable Y only and auxiliary variable X is free from non-response.

Let $\overline{y}_i^*$ be the unbiased estimator of population mean $\overline{Y}_i$ for the i^{th} stratum, given by

$$\overline{y}_i^* = \frac{n_{i1}\overline{y}_{ni1} + n_{i2}\overline{y}_{ui2}}{n_i} \tag{2.1}$$

where $\overline{y}_{ni1}$ and $\overline{y}_{ui2}$ are the means based on n_{i1} units of response group and u_{i2} units of sub-sample of non-response group respectively in the sample for the i^{th} stratum. $\overline{x}_i$ be the unbiased estimator of population mean $\overline{X}_i$, based on n_i sample units in the i^{th} stratum.

Using Hansen-Hurwitz technique, an unbiased estimator of population mean $\overline{Y}$ is given by

$$\overline{y}_{st}^* = \sum_{i=1}^{k} p_i \overline{y}_i^* \tag{2.2}$$

and the variance of the estimator is given by the following expression

$$V(\overline{y}_{st}^*) = \sum_{i=1}^{k}\left(\frac{1}{n} - \frac{1}{N}\right)p_i^2 S_{yi}^2 + \sum_{i=1}^{k}\frac{(k_i - 1)}{n_i} W_{i2} p_i^2 S_{yi2}^2 \tag{2.3}$$

where S_{yi}^2 and S_{yi2}^2 are respectively the mean-square errors of entire group and non-response group of study variable in the population for the i^{th} stratum. $k_i = \frac{n_{i2}}{u_{i2}}$, $p_i = \frac{N_i}{N}$ and W_{i2} = Non-response rate of the i^{th} stratum in the population $= \frac{N_{i2}}{N_i}$.

2.1 Proposed Estimators

Motivated by Khoshnevisan et al. (2007), we propose a family of combined-type estimators of population mean $\overline{Y}$, given by

$$T_C = \overline{y}_{st}^{*}\left[\frac{a\overline{X}+b}{\alpha(a\overline{x}_{st}+b)+(1-\alpha)(a\overline{X}+b)}\right]^g \tag{2.1.1}$$

where $\overline{x}_{st} = \sum_{i=1}^{k} p_i \overline{x}_i$ (unbiased for $\overline{X}$)

and $\overline{X} = \sum_{i=1}^{k} p_i \overline{X}_i$.

Obviously, T_C is biased. The bias and MSE can be obtained on using large sample approximations:

$\overline{y}_{st}^{*} = \overline{Y}(1+e_0)$; $\overline{x}_{st} = \overline{X}(1+e_1)$

such that $E(e_0) = E(e_1) = 0$ and

$$E\left(e_0^2\right) = \frac{V\left(\overline{y}_{st}^{*}\right)}{\overline{Y}^2} = \frac{1}{\overline{Y}^2}\sum_{i=1}^{k} p_i^2\left[f_i S_{Yi}^2 + \frac{(k_i-1)}{n_i}W_{i2}S_{Yi2}^2\right]$$

$$E\left(e_1^2\right) = \frac{V\left(\overline{x}_{st}\right)}{\overline{X}^2} = \frac{1}{\overline{X}^2}\sum_{i=i}^{k} p_i^2 f_i S_{Xi}^2$$

$$E(e_0e_1) = \frac{Cov\left(\overline{y}_{st}^{*}, \overline{x}_{st}\right)}{\overline{Y}\,\overline{X}} = \frac{1}{\overline{Y}\,\overline{X}}\sum_{i=1}^{k} p_i^2 f_i \rho_i S_{Yi} S_{Xi}$$

where $f_i = \frac{N_i - n_i}{N_i n_i}$, S_{Xi}^2 be the mean-square error of entire group of auxiliary variable in the population for the i^{th} stratum and ρ_i is the correlation coefficient between Y and X in the i^{th} stratum.

Expressing T_C in terms of e_i $(i=0,1)$, we can write (2.1.1) as

$$T_C = \overline{Y}(1+e_0)[1+\alpha\lambda e_1]^{-g} \quad (2.1.2)$$

where $\lambda = \dfrac{a\overline{X}}{a\overline{X}+b}$.

Suppose $|\alpha\lambda e_1|$< 1 so that $[1+\alpha\lambda e_1]^{-g}$ is expandable. Expanding the right hand side of (2.1.2) up to the first order of approximation, we obtain

$$\left(T_C - \overline{Y}\right) = \overline{Y}\left[e_0 - g\alpha\lambda e_1 + \frac{g(g+1)}{2}\alpha^2\lambda^2 e_1^2 - g\alpha\lambda e_0 e_1\right] \quad (2.1.3)$$

Taking expectation of both sides in (2.1.3), we get the bias of the estimator T_C as

$$B(T_C) = \frac{1}{\overline{Y}}\sum_{i=1}^{k} f_i p_i^2\left[\frac{g(g+1)}{2}\alpha^2\lambda^2 R^2 S_{Xi}^2 - \alpha\lambda g R\rho_i S_{Yi}S_{Xi}\right] \quad (2.1.4)$$

Squaring both sides of (2.1.3) and then taking expectation, we get the MSE of the estimator T_C , up to the first order approximation, as

$$MSE(T_C) = \sum_{i=1}^{k} f_i p_i^2\left[S_{Yi}^2 + \alpha^2\lambda^2 g^2 R^2 S_{Xi}^2 - 2\alpha\lambda g R\rho_i S_{Yi}S_{Xi}\right] + \sum_{i=1}^{k} p_i^2 \frac{(k_i-1)}{n_i} W_{i2}S_{Yi2}^2 \quad (2.1.5)$$

Optimum choice of α

On minimizing $MSE(T_C)$ w.r.t. α **,** we get the optimum value of α as

$$\frac{\partial MSE(T_C)}{\partial\alpha} = 2\alpha\lambda^2 g^2 R^2\sum_{i=1}^{k} f_i p_i^2 S_{Xi}^2 - 2\lambda g R\sum_{i=1}^{k} f_i p_i^2\rho S_{Yi}S_{Xi} = 0$$

$$\Rightarrow \alpha_{(opt)} = \frac{\sum_{i=1}^{k} f_i p_i^2\rho_i S_{Yi}S_{Xi}}{\lambda g R\sum_{i=1}^{k} f_i p_i^2 S_{Xi}^2} \quad (2.1.6)$$

Thus $\alpha_{(opt)}$ is the value of α at which $MSE(T_C)$ would attain its minimum.

3. Optimum n_i with respect to Cost of the Survey

Let C_{i0} be the cost per unit of selecting n_i units, C_{i1} be the cost per unit in enumerating n_{i1} units and C_{i2} be the cost per unit of enumerating u_{i2} units. Then the total cost for the i^{th} stratum is given by

$$C_i = C_{i0}n_i + C_{i1}n_{i1} + C_{i2}u_{i2} \qquad \forall\ i = 1,2,\ldots,k$$

Now, we consider the average cost per stratum

$$E(C_i) = n_i\left[C_{i0} + C_{i1}W_{i1} + C_{i2}\frac{W_{i2}}{k_i}\right]$$

Thus the total cost over all the strata is given by

$$C_0 = \sum_{i=1}^{k} E(C_i)$$

$$= \sum_{i=1}^{k} n_i\left[C_{i0} + C_{i1}W_{i1} + C_{i2}\frac{W_{i2}}{k_i}\right] \tag{3.1}$$

Let us consider the function

$$\phi = MSE(T_C) + \mu C_0 \tag{3.2}$$

where μ is Lagrangian multiplier. Differentiating the equation (3.2) with respect to n_i and k_i separately and equating to zero, we get the following normal equations.

$$\frac{\partial\phi}{\partial n_i} = -\frac{p_i^2}{n_i^2}\left[S_{Yi}^2 + \alpha^2\lambda^2 g^2 R^2 S_{Xi}^2 - 2\alpha\lambda g R\rho_i S_{Yi}S_{Xi}\right] - \frac{p_i^2}{n_i^2}(k_i - 1)W_{i2}S_{Yi2}^2$$
$$+ \mu\left[C_{i0} + C_{i1}W_{i1} + C_{i2}\frac{W_{i2}}{k_i}\right] = 0 \tag{3.3}$$

$$\frac{\partial\phi}{\partial k_i} = \frac{p_i^2 W_{i2} S_{Yi2}^2}{n_i} - \mu n_i C_{i2}\frac{W_{i2}}{k_i^2} = 0 \tag{3.4}$$

From the equations (3.3) and (3.4) respectively, we have

$$n_i = \frac{p_i\sqrt{S_{Yi}^2 + \alpha^2\lambda^2 g^2 R^2 S_{Xi}^2 - 2\alpha\lambda g R\rho_i S_{Yi}S_{Xi} + (k_i - 1)W_{i2}S_{Yi2}^2}}{\sqrt{\mu}\sqrt{C_{i0} + C_{i1}W_{i1} + C_{i2}\frac{W_{i2}}{k_i}}} \tag{3.5}$$

and

$$\sqrt{\mu} = \frac{k_i p_i S_{Yi2}}{n_i\sqrt{C_{i2}}} \tag{3.6}$$

Putting the value of the $\sqrt{\mu}$ from equation (3.6) into the equation (3.5), we get

$$k_{i(opt)} = \frac{\sqrt{C_{i2}}\,B_i}{S_{Yi2}A_i} \tag{3.7}$$

Where $A_i = \sqrt{C_{i0} + C_{i1}W_{i1}}$

and $B_i = \sqrt{S_{Yi}^2 + \alpha^2\lambda^2 g^2 R^2 S_{Xi}^2 - 2\alpha\lambda g R\rho_i S_{Yi}S_{Xi} - W_{i2}S_{Yi2}^2}$

Substituting $k_{i(opt)}$ from equation (3.7) into equation (3.5), n_i can be expressed as

$$n_i = \frac{p_i\sqrt{B_i^2 + \frac{\left(\sqrt{C_{i2}}\,B_i W_{i2} S_{Yi2}\right)}{A_i}}}{\sqrt{\mu}\sqrt{A_i^2 + \frac{\sqrt{C_{i2}}\,A_i W_{i2} S_{Yi2}}{B_i}}} \tag{3.8}$$

The $\sqrt{\mu}$ in terms of total cost C_0 can be obtained by putting the values of $k_{i(opt)}$ and n_i from equations (3.7) and (3.8) respectively into equation (3.1)

$$\sqrt{\mu} = \frac{1}{C_0}\sum_{i=1}^{k} p_i\left(A_i B_i + \sqrt{C_{i2}}\,W_{i2} S_{Yi2}\right) \tag{3.9}$$

Now we can express n_i in terms of total cost C_0

$$n_{i(opt)} = \frac{C_0}{\sum_{i=1}^{k} p_i\left(A_i B_i + \sqrt{C_{i2}}\,W_{i2} S_{Yi2}\right)} \frac{p_i\sqrt{B_i^2 + \frac{\left(\sqrt{C_{i2}}\,B_i W_{i2} S_{Yi2}\right)}{A_i}}}{\sqrt{A_i^2 + \frac{\sqrt{C_{i2}}\,A_i W_{i2} S_{Yi2}}{B_i}}} \tag{3.10}$$

Thus $n_{i(opt)}$ can be obtained by equation (3.10) by putting different values of W_{i2} and k_i.

4. Numerical Analysis

For numerical analysis we have used data considered by Koyuncu and Kadilar (2008). The data concerning the number of teachers as study variable and the number of students as auxiliary variable in both primary and secondary school for 923 districts at 6 regions (as 1: Marmara, 2: Agean, 3: Mediterranean, 4: Central Anatolia, 5: Black Sea, 6: East and Southeast Anatolia) in Turkey in 2007 (Source: Ministry of Education Republic of Turkey). Details are given below:

Table No.4.1: Stratum means, Mean Square Errors and Correlation Coefficients S_{Yi2}

Stratum No.	N_i	n_i	$\overline{Y}_i$	$\overline{X}_i$	S_{Yi}	S_{Xi}	S_{XYi}	ρ_i	S_{Yi2}
1	127	31	703.74	20804.59	883.835	30486.751	25237153.52	.936	440
2	117	21	413.00	9211.79	644.922	15180.769	9747942.85	.996	200
3	103	29	573.17	14309.30	1033.467	27549.697	28294397.04	.994	400
4	170	38	424.66	9478.85	810.585	18218.931	14523885.53	.983	405
5	205	22	527.03	5569.95	403.654	8497.776	3393591.75	.989	180
6	201	39	393.84	12997.59	711.723	23094.141	15864573.97	.965	300

Table No.4.2: % Relative efficiency (R.E.) of T_C w.r.t. to $\overline{y}_{st}^{*}$ at $\alpha_{(opt)}$, $a = 1, b = 1$

W_{i2}	k_i	$R.E.(T_C)$
0.1	2.0	914.25
	2.5	834.05
	3.0	768.23
	3.5	713.25
0.2	2.0	768.23
	2.5	666.62
	3.0	591.84
	3.5	534.49
0.3	2.0	666.62
	2.5	561.39
	3.0	489.12
	3.5	436.42
0.4	2.0	591.84
	2.5	489.12
	3.0	421.89
	3.5	374.47

5. Conclusion

We have proposed a family of estimators in stratified sampling using an auxiliary variable in the presence of non-response on study variable. We have also derived the expressions for optimum sample sizes in respect to cost of the survey. Table 4.2 reveals that the proposed estimator T_C has greater precision than the usual estimator $\overline{y}_{st}^{*}$ under non-response.

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MULTIVARIATE RATIO ESTIMATION WITH KNOWN POPULATION PROPORTION OF TWO AUXILIARY CHARACTERS FOR FINITE POPULATION

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Abstract

In the present study, we propose estimators based on geometric and harmonic mean for estimating population mean using information on two auxiliary attributes in simple random sampling. We have shown that, when we have multi-auxiliary attributes, estimators based on geometric mean and harmonic mean are less biased than Olkin (1958), Naik and Gupta (1996) and Singh (1967) type- estimator under certain conditions. However, the MSE of Olkin(1958) estimator and geometric and harmonic estimators are same up to the first order of approximation.

Key words: Simple random sampling, auxiliary attribute, point bi-serial correlation, harmonic mean, geometric mean.

1. Introduction

Prior knowledge about population mean along with coefficient of variation of the population of an auxiliary variable is known to be very useful particularly when the ratio, product and regression estimators are used for estimation of population mean of a variable of interest. There exist situations when information is available in the form of the attribute ϕ which is highly correlated with y. For example y may be the use of drugs and ϕ may be gender. Using the information of point biserial correlation between the study variable and the auxiliary attribute Naik and Gupta (1996), Shabbir and Gupta (2006), Ab-Alfatah (2009) and Singh et al. (2007, 2008) have suggested improved estimators for estimating unknown population mean $\overline{Y}$.

Using information on multi-auxiliary variables positively correlated with the study variable, Olkin (1958) suggested a multivariate ratio estimator of the population mean $\bar{Y}$. In this paper, we have suggested some estimators using information on multi-auxiliary attributes. Following Olkin (1958), we define an estimator as

$$\bar{y}_{ap} = \sum_{i=1}^{k} w_i r_i P_i \tag{1.1}$$

where (i) w_i's are weights such that $\sum_{i=1}^{k} w_i = 1$ (ii) P_i's are the proportion of the auxiliary attribute and assumed to be known and (iii) $r_i = \frac{\bar{y}}{p_i}$, $\bar{y}$ is the sample mean of the study variable Y and p_i is the proportion of auxiliary attributes P_i based on a simple random sample of size n drawn without replacement from a population of size N.

Following Naik and Gupta (1996) and Singh et al. (2007), we propose another estimator t_s as

$$t_s = \prod_{i=1}^{k} r_i P_i \tag{1.2}$$

Two alternative estimators based on geometric mean and harmonic mean are suggested as

$$\bar{y}_{gp} = \prod_{i=1}^{k} \left(r_i P_i \right)^{w_i} \tag{1.3}$$

and

$$\bar{y}_{hp} = \left(\sum_{i=1}^{k} \frac{w_i}{r_i P_i} \right)^{-1} \tag{1.4}$$

such that $\sum_{i=1}^{k} w_i = 1$

These estimators are based on the assumptions that the auxiliary attributes are positively correlated with Y. Let $\rho_{\phi ij}$ (i=1,2,…k; j=1,2,…k) be the phi correlation coefficient between P_i and P_j and ρ_{0i} be the correlation coefficient between Y and P_i.

$$S^2_{\phi_{ij}} = \frac{1}{N-1}\sum_{i=1}^{N}\left(\phi_{ji} - P_i\right)^2,\ S^2_o = \frac{1}{N-1}\sum_{i=1}^{N}\left(Y_i - \overline{Y}\right)^2 \text{ and } C^2_0 = \frac{S^2_0}{\overline{Y}^2},\ C^2_i = \frac{S^2_{\phi_{ij}}}{P_i^2}$$

In the same way C_{0i} and C_{ij} are defined.

Further, let $\underset{\sim}{w}' = \left(w_1, w_2, ..., w_k\right)$ and $C = \left[C_{ij}\right]_{p\times p}$ $(i = 1,2,...k; j = 1,2,...,k)$

2. BIAS AND MSE OF THE ESTIMATORS

To obtain the bias and MSE's of the estimators, up to first order of approximation, let

$$e_0 = \frac{\overline{y} - \overline{Y}}{\overline{Y}} \quad \text{and} \quad e_i = \frac{p_i - P_i}{P_i}$$

such that $E(e_i) = 0\ (i = 0,1,2,...,k)$..

Expressing equation (1.1) in terms of e's, we have

$$\overline{y}_{ap} = \sum_{i=1}^{k} w_i \overline{Y}\left(1 + e_0\right)\left(1 + e_i\right)^{-1}$$

$$= \overline{Y}\sum_{i=1}^{k} w_i\left[1 + e_0 - e_i + e_i^2 - e_0 e_1 + e_0 e_i^2 - e_i^3\right] \tag{2.1}$$

Subtracting $\overline{Y}$ from both the sides of equation (2.1) and then taking expectation of both sides, we get the bias of the estimator $\overline{y}_{ap}$ up to the first order of approximation as

$$B\left(\overline{y}_{ap}\right) = f\overline{Y}\left[\sum_{i=1}^{k} w_i C_i^2 - \sum_{i=1}^{k} w_i C_{0i}\right] \tag{2.2}$$

Subtracting $\overline{Y}$ from both the sides of equation (2.1) squaring and then taking expectation of both sides, we get the bias of the estimator $\overline{y}_{ap}$ up to the first order of approximation as

$$MSE\left(\overline{y}_{ap}\right) = f\overline{Y}^2\left[C_0^2 + \sum_{i=1}^{k} w_I^2 C_i^2 - 2\sum_{i=1}^{k} w_i C_0 C_i + 2\sum\sum w_i w_j C_{ij}\right] \tag{2.3}$$

To obtain the bias and MSE of $\bar{y}_{gp}$ to the first order of approximation, we express equation (1.3) in term of e's, as

$$\bar{y}_{gp} = \prod_{i-1}^{k}\left[\bar{Y}(1+e_0)(1+e_i)^{-1}\right]^{w_i}$$

$$= \prod_{i-1}^{k}\bar{Y}\left[1+e_0 - w_i(e_i + e_0 e_i) + \frac{w_i(w_i+1)}{2}(e_i^2 + e_0 e_i^2)\right] \tag{2.4}$$

Subtracting $\bar{Y}$ from both sides of equation (2.4) and then taking expectation of both sides, we get the bias of the estimator t_{gp} up to the first order of approximation, as

$$B(\bar{y}_{gp}) = f\bar{Y}\left[\sum\frac{w_i(w_i+1)C_i^2}{2} + \sum\sum w_i w_j C_{ij} - \sum w_i C_{0i}\right] \tag{2.5}$$

Subtracting $\bar{Y}$ from both the sides of equation (2.4) squaring and then taking expectation of both sides, we get the bias of the estimator $\bar{y}_{gp}$ up to the first order of approximation as

$$MSE(\bar{y}_{gp}) = f\bar{Y}^2\left[C_0^2 + \sum_{i=1}^{k} w_I^2 C_i^2 - 2\sum_{i=1}^{k} w_i C_0 C_i + 2\sum\sum w_i w_j C_{ij}\right] \tag{2.6}$$

Now expressing equation (1.4) in terms of e's, we have

$$\bar{y}_{hp} = \sum_{i=1}^{k} w_i \bar{Y}(1+e_0)(1+e_i)^{-1}$$

$$= \sum_{i}\bar{Y}(1+e_0)\left[1 - w_i e_i + w_i e_i^2 - w_i e_i^3\right]$$

$$= \bar{Y}\left[1+e_0 - \sum_i w_i(e_i + e_0 e_i) + \left(\sum_i w_i e_i\right)^2\right] \tag{2.7}$$

Subtracting $\bar{Y}$ from both sides of equation (2.7) and then taking expectation of both sides, we get the bias of the estimator $\bar{y}_{hp}$ up to the first order of approximation will be

$$B(\bar{y}_{hp}) = f\bar{Y}\left[\sum_i w_i C_i^2 - \sum_i w_i C_{0i} + 2\sum\sum w_i w_j C_{ij}\right] \tag{2.8}$$

Subtracting $\overline{Y}$ from both the sides of equation (2.7) squaring and then taking expectation of both sides, we get the bias of the estimator $\overline{y}_{hp}$ up to the first order of approximation as

$$MSE(\overline{y}_{hp}) = f\overline{Y}^2\left[C_0^2 + \sum_{i=1}^{k} w_i^2 C_i^2 - 2\sum_{i=1}^{k} w_i C_{0i} + 2\sum\sum w_i w_j C_{ij}\right] \tag{2.9}$$

We see that MSE's of these estimators are same and the biases are different. In general,

$$MSE(\overline{y}_{gp}) = MSE(\overline{y}_{hp}) = MSE(\overline{y}_{ap}). \tag{2.10}$$

3. Comparison of biases

The biases may be either positive or negative. So, for comparison, we have compared the absolute biases of the estimates when these are more efficient than the sample mean. The bias of the estimator of geometric mean is smaller than that of arithmetic mean

$$\left|B(\overline{y}_{ap})\right| > \left|B(\overline{y}_{gp})\right| \tag{3.1}$$

Squaring and simplifying (3.1), we observe that

$$\left[\frac{1}{2}\sum_{i=1}^{k} w_i^2 C_i^2 - 2\sum_{i=1}^{k} w_i C_{0i} + 2\sum\sum w_i w_j C_{ij} + \frac{3}{2}\sum_{i=1}^{k} w_i C_i^2\right] \times$$

$$\left[\frac{1}{2}\sum_{l=1}^{k} w_i C_i^2 - \frac{1}{2}\sum_{i=1}^{k} w_i^2 C_i^2 - \sum\sum w_i w_j C_{ij}\right] > 0 \tag{3.2}$$

Thus above inequality is true when both the factors are either positive or negative. The first factor of (3.2)

$$\left[\frac{1}{2}\sum_{i=1}^{k} w_i^2 C_i^2 - 2\sum_{i=1}^{k} w_i C_{0i} + 2\sum\sum w_i w_j C_{ij} + \frac{3}{2}\sum_{i=1}^{k} w_i C_i^2\right]$$

is positive, when

$$\frac{\sum_{i=1}^{k} w_i^2 C_i^2}{\underset{\sim}{w}' C \underset{\sim}{w}} > \frac{1}{3} \tag{3.3}$$

In the same way, it can be shown that the second factor of (3.2) is also positive when

$$\frac{\sum_{i=1}^{k} w_i^2 C_i^2}{\underset{\sim}{w}' C \underset{\sim}{w}} > 1 \tag{3.4}$$

When both the factors of (3.2) is negative, the sign of inequalities of (3.3) and (3.4) reversed.

Also comparing the square of the biases of geometric and harmonic estimator, we find that geometric estimator is more biased than harmonic estimator.

Hence we may conclude that under the situations where arithmetic, geometric and harmonic estimator are more efficient than sample mean and the relation (3.4) or

$$\frac{\sum_{i=1}^{k} w_i^2 C_i^2}{\underset{\sim}{w}' C \underset{\sim}{w}} < \frac{1}{3}$$

is satisfied, the biases of the estimates satisfy the relation

$$\left|B\left(\bar{y}_{ap}\right)\right| > \left|B\left(\bar{y}_{gp}\right)\right| > \left|B\left(\bar{y}_{hp}\right)\right|$$

Usually the weights of w_i's are so chosen so as to minimize the MSE of an estimator subject to the condition

$$\sum_{i=1}^{k} w_i = 1.$$

4. Empirical Study

Data : (Source: Singh and Chaudhary (1986), P. 177).

The population consists of 34 wheat farms in 34 villages in certain region of India. The variables are defined as:

y = area under wheat crop (in acres) during 1974.

p_1 = proportion of farms under wheat crop which have more than 500 acres land during 1971.

and

p_2 = proportion of farms under wheat crop which have more than 100 acres land during 1973.

For this data, we have

N=34, $\bar{Y}$=199.4, P_1=0.6765, P_2=0.7353, S_y^2=22564.6, $S_{\phi_1}^2$=0.225490, $S_{\phi_2}^2$=0.200535,

ρ_{pb_1}=0.599, ρ_{pb_2}=0.559, ρ_ϕ=0.725.

Biases and MSE' s of different estimators under comparison, based on the above data are given in Table 4.1.

TABLE 4.1 : Bias and MSE of different estimators

Estimators	Auxiliary attributes	Bias	MSE
$\bar{y}$	none	0	1569.795
Ratio $\bar{y}\left(\frac{P_1}{p_1}\right)$	P_1	2.4767	1197.675
Ratio $\bar{y}\left(\frac{P_2}{p_2}\right)$	P_2	1.6107	1194.172
Olkin ($\bar{y}_{ap}$)	P_1 and P_2	2.0415	1024.889
Suggested $\bar{y}_{gp}$	P_1 and P_2	1.6126	1024.889
Suggested $\bar{y}_{hp}$	P_1 and P_2	1.1838	1024.889
$t_s = \bar{y}\left(\frac{P_1}{p_1}\right)\left(\frac{P_2}{p_2}\right)$	P_1 and P_2	8.4498	2538.763

5. Conclusion

From Table 4.1 we observe that the MSE's of Olkin (1958) type estimator, estimator based on harmonic and geometric mean are same. Singh (1967) type estimator t_s performs worse. However, the bias of the ratio-type estimator based on harmonic mean is least. Hence, we may conclude that when more than one auxiliary attributes are used for estimating the population parameter, it is better to use harmonic mean.

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DETERMINANTS OF POPULATION GROWTH IN RAJASTHAN: AN ANALYSIS

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Abstract

Rajasthan is the biggest State of India and is currently in the second phase of demographic transition and is moving towards the third phase of demographic transition with very slow pace. However, state's population will continue to grow for a time period. Rajasthan's performance in the social and economic sector has been poor in past. The poor performance is the outcome of poverty, illiteracy and poor development, which co-exist and reinforce each other. There are many demographic and socio-economic factors responsible for population growth. This paper attempts to identify the demographic and socio-economic variables, which are responsible for population growth in Rajasthan with the help of multivariate analysis.

1. Introduction:

Prof. Stephan Hawking (Cambridge University) was on Larry King Live. Larry King called him the "most intelligent person in the world". King asked some very key questions, one of them was: "what worries you the most?" Hawking said, "My biggest worry is population growth, and if it continues at the current rate, we will be standing shoulder to shoulder in 2600. Something has to happen, and I don't want it to be a disaster".

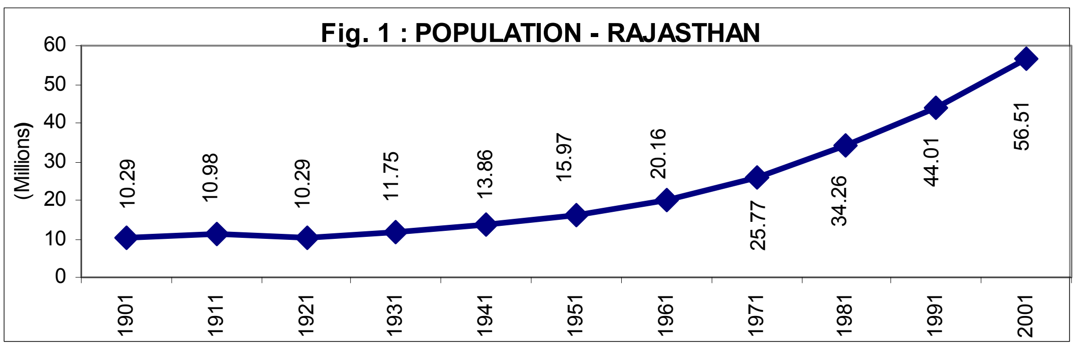
The importance of population studies in India has been recognized since very ancient times. The 'Arthashastra' of Kautilya gives a detailed description of how to conduct a population, economic and agricultural census. During the reign of Akbar, Abul Fazal compiled the Ain-E-Akbari containing comprehensive data on population, industry, wealth and characteristics of population. During the British period, system of decennial census started with the first census in 1872.

The population growth of a region and its economic development are closely linked. India has been a victim of population growth. Although the country has achieved progress in the economic field, the population growth has wrinkled the growth potential. The need to check the population growth was realized by a section of the intellectual elite even before independence. Birth control was accepted by this group but implementation was restricted to the westernized minority in the cities. When the country attained independence and planning was launched, population control became one of the important items on the agenda of development. The draft outline of the First Five Year Plan said, "the increasing pressure of population on natural resources retards economic progress and limits seriously the rate of extension of social services, so essential to civilized existence."

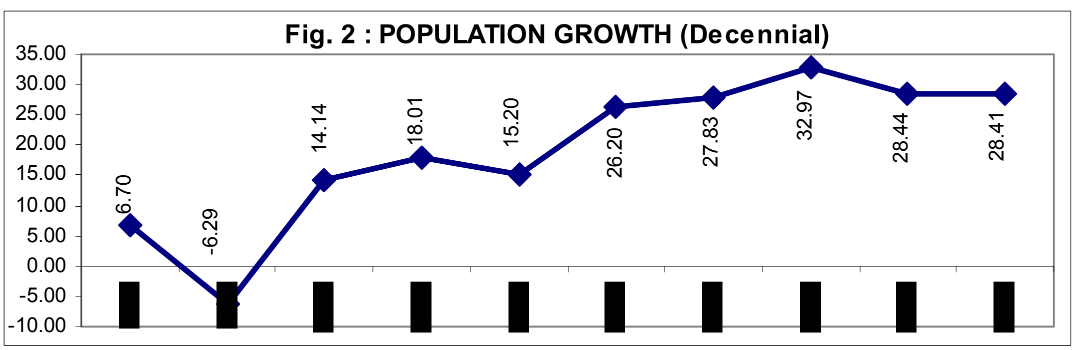
India was one of the pioneers in health service planning with a focus on primary health care. Improvement in the health status of the population has been one of the major thrust areas for the social development programs of the country in the five year plans. India is a signatory to the Alma

Ata Declaration (1978) whereby a commitment was made to achieve 'Health for All' by 2000 AD. We are in the end of the first decade of the 21st century but still have to go a long way to achieve this target. Rajasthan is lagging behind the all India average in the key parameters i.e. CBR, CDR, IMR, TFR & CPR. The state has made consistent efforts to improve quality of its people through improvement in coverage & quality of health care and implementation of disease control programs but the goals remain elusive due to high levels of fertility and mortality. According to the Report of the Technical Group on Population Projections, India will achieve the target of TFR = 2.1 (Net Reproduction Rate = 1) in 2026. Kerala & Tamilnadu had already achieved it in 1988 & 1993 respectively but Rajasthan will achieve it in 2048 & Uttar Pradesh in 2100.

Rajasthan is the largest state of the country with its area of 342239 sq. kms., which constitutes about 10.41% of the total area of the country. According to 2001 census, its population is 56.51 million. It consist 5.5 % population and ranks eighth in the country. In 1901, population of Rajasthan was 10.29 millions. In 1951, it reached to 15.97 millions with its slow growth during 1901-1951. Figure 1 shows that it increased rapidly after 1951. It reached to 34.26 million in 1981 and to 56.51 million in 2001. It has multiplied 5.5 times since 1901 and 3.5 times since 1951. Figure 2 shows decennial growth in population of the state. Before 1951, it increased by less than 20% growth per decade. In 1971-81, it shows the maximum growth rate of 32.97%. In 1981-91, it decreased by 4.53 percentage points and grew by 28.44%. The decade of 1991-2001 shows growth of 28.41%.

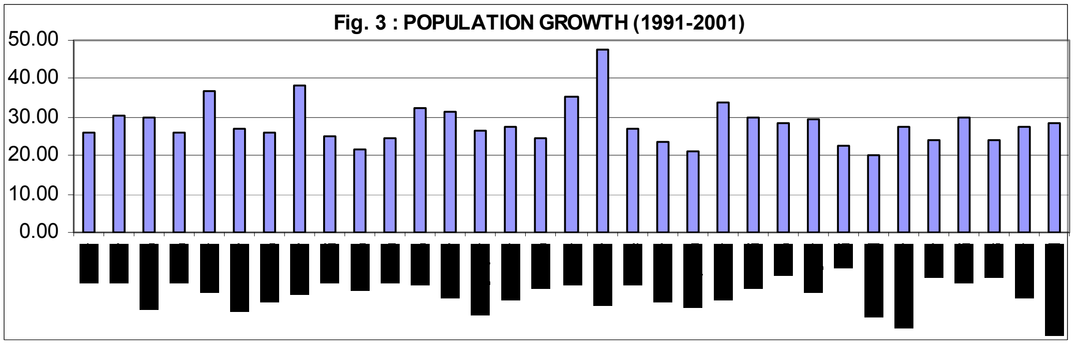


Source: Government of India, Registrar General, India, see the website www.censusindia.net



Source: Government of India, Registrar General, India, see the website www.censusindia.net

The rapid population growth in a already populated state like Rajasthan could lead to many problems i.e. pressure on land, environmental deterioration, fragmentation of land holding, shrinking forests, rising temperatures, pressure on health & educational infrastructure, on availability of food grains & on employment. Figure 3 shows the decennial growth of district-wise population during 1991-2001. Jaisalmer shows the maximum growth of 47.45% followed by Bikaner (38.18%), Barmer (36.83%), Jaipur (35.10%) and Jodhpur (33.77%). Rajasamand shows minimum growth of 19.88% followed by Jhunjhunu (20.90%), Chittorgarh (21.46%), Pali (22.39%) and Jhalawar (23.34%).



Source: Government of India, Registrar General, India, see the website www.censusindia.net

Rajasthan is currently in the second phase and is moving towards the third phase of demographic transition with very slow pace. The changes in the population growth rates in Rajasthan have been relatively slow, but the change has been steady and sustained. We are aware of the need for birth control, but too many remain ignorant of contraception methods or are unwilling to discuss them. There is considerable pressure to produce a son. However, the state's population will continue to grow for a time period.

Rajasthan is the second state in the country to formulate and adopt its own Population Policy in January 2000. State Population Policy[5] has envisaged strategies for population stabilization and improving health conditions of people specially women and children. The policy document has clearly presented role and responsibilities of different departments actively contributing in implementation of population policy. Family Welfare Program was linked with other sectors and demands intervention and efficient policies in these sectors so that changes can be brought in the social, economic, cultural & political environment. The State Population Policy envisages time bound objectives as mentioned in table 1:

Table 1: Objectives of Population Policy of Rajasthan

Indicators	1997	2001	2004	2007	2011	2013	2016
Total Fertility Rate	4.11	3.74	3.41	3.09	2.65	2.43	2.10
Birth Rate	32.1	29.2	27.5	25.6	22.6	20.9	18.4
Contraceptive Prevalence Rate	38.5	42.2	48.2	52.7	58.8	61.8	68.0
Death Rate	8.9	8.7	8.4	7.9	7.5	7.2	7.0
Infant Mortality Rate	85.0	77.4	72.7	68.1	62.2	60.1	56.8

Rajasthan's performance in the social and economic sector has been poor in past. The poor performance is the outcome of poverty, illiteracy and poor development which co-exist and reinforce each other. State Government has taken energetic steps in last few years to assess and fully meet the unmet needs for maternal & child health care and contraception through improvement in availability and access to family welfare services but still remains a long path. The progress in these indicators would determine the year and size of the population at which the state achieves population stabilization.

2. Objectives and Methodology:

There is a major data difficulty regarding availability of annual statistics, calculations & comparisons of Crude Birth Rate (CBR), Total Fertility Rate (TFR) and Females' Mean Age at Gauna (FMAG) over time for district level study of any state and which is applied to Rajasthan also. This data problem distorts the calculations and negates the usefulness of making comparisons over time. Due to this data information problem, we use the information for different years (as per the availability of latest data, taking 2000-01 as base year) in this paper. This data problem at district level is a constraint

[5] Government of Rajsthan (1999), "Population Policy of Rajasthan", Department of Family Welfare, Jaipur.

that creates a limitation in the selection of study objectives and hypotheses. This paper attempts to identify the demographic and socio-economic variables, which are responsible for population growth in Rajasthan. The main objectives of the study are:

- To observe the characteristics of indicators of population growth in Rajasthan.
- To identify the various demographic & socio-economic variables which have causal relationship with population growth.
- To analyze the inter-relationship between the indicators of population growth and demographic & socio-economic variables.

For achieving the above objectives, the *a priori* hypotheses are as follows:

- Positive impact of infant mortality & total fertility rate and negative impact of income equality on population growth.
- Positive impact of infant mortality and negative impact of female's age at gauna and female literacy on crude birth rate.
- Negative impact of couple protection rate, income equality, female literacy and positive impact of infant mortality on total fertility rate.
- Positive impact of female literacy & income equality on female's age at gauna.
- Positive impact of female literacy, females age at gauna and income equality on couple protection rate.

To rummage the inter-relationship between indicators of population growth and demographic & socio-economic variables, a social sector model is proposed. The model is estimated by the use of Multiple Regression Analysis (Method of Ordinary Least Squares). The general form of the Multiple Regression Equation Model is as follows:

$$Y_i = \beta_1 + \beta_2 X_{2i} + \beta_3 X_{3i} + \cdots + \beta_k X_{ki} + u_i$$

where i = 1, 2, 3, … , n.

In this multiple regression equation model, Y_i is dependent variable and X_2, X_3, … , X_k are independent explanatory variables. β_1 is the intercept, shows the average value of Y, when X_2, X_3, … , X_k are set equal to zero; β_2, β_3, …, β_k are partial regression/slope coefficients; u_i is the stochastic disturbance term; i is the i^{th} observation and n is the size of population.

The model is estimated by using cross-sectional data of all 32 districts of the state (at that time, the no. of districts was 32). In this paper, we also calculated the Mean, Standard Deviation and Coefficient of Variation of the variables. The variables used in this paper, their reference year and abbreviations/identification code are given in the Appendix I (Table 9). Firstly, we regress the dependent variables with all the variables, which have theoretical relationship and then choose the appropriate variables for multiple regressions. The dependent and independent variables for the model are as follows:

Table 2: Functional Form of the Model

Dependent Variable	Independent Variables
POPGWR	CBR, TFR, FMAG, CDR, CPR, IMR, CIMM, MRANC, PWRSAP, PWETVR, MIPLP, BPGH, PCEMPH, LIT, LIT_m, LIT_f, PCEEE, PCNDDP, PPBPL, ROADSK, PHDW, PCEWS
CBR	POPGWR, FMR, $FMR_{(0-6)}$, PURPOP, FMAG, CPR, IMR, PWETVR, PCEMPH, PCEFW, LIT, LIT_m, LIT_f, PCEEE, PCNDDP, PPBPL
TFR	PURPOP, FMAG, CPR, CDR, IMR, MRANC, PWRSAP, PWETVR, PCEMPH, LIT, LIT_m, LIT_f, PCEEE, PCNDDP, PPBPL, PCESCS
FMAG	PURPOP, PWETVR, LIT, LIT_m, LIT_f, PSER, $PSER_m$ $PSER_f$, DORPS, $DORPS_m$, $DORPS_f$, PCEEE, PCNDDP, PPBPL

Dependent Variable	Independent Variables
CPR	PURPOP, FMAG, IMR, PWETVR, MIPLP, PCEMPH, PCEFW, LIT, LIT_m, LIT_f, PCEEE, PCNDDP, PPBPL, IDI, PCESCS

In this paper, we have taken 32 variables (appendix-I). All the 32 variables are relating to Population; Fertility, Reproductive Health and Mortality; Public Health and Health Infrastructure; Education and Educational Infrastructure; and Economic Growth and Infrastructure. Data used in this paper have taken from website of Census Department, State Human Development Report (Rajasthan), Various Administrative Reports of Medical, Health & Family Welfare Department, Government of Rajasthan and Plan Documents of Planning Department, Government of Rajasthan.

3. Multivariate Analysis

3.1 Mean, Standard Deviation & Coefficient of Variation

Mean, standard deviation and coefficient of variation of all the 32 variables for all 32 districts along with the figures of all Rajasthan are at appendix I (table 9). The Mean, measures the average value of the variables for all 32 districts. The Standard Deviation, measures the absolute variation in the mean and the Coefficient of Variation, measures the percentage variation in mean. The variables are divided in to five categories according to the range of Coefficient of Variation for the analysis of Standard Deviation and Coefficient of Variation.

Table 3: Range-wise Variables according to the Coefficient of Variation

Range	Variables
Less than 25%	POPGWR (19.92), FMR (5.28), $FMR_{(0-6)}$(3.26), CBR (7.02), TFR (10.20), FMAG (3.66), CPR (14.73), CDR (10.44), IMR (20.60), MIPLP (18.59), LIT (12.64), LIT_m (8.31), LIT_f (21.19), PSER (9.39), $PSER_m$ (11.17), $PSER_f$ (14.27), DORPS (12.94), $DORPS_m$ (12.89), $DORPS_f$ (17.01), PCNDDP (24.34), PHDW (21.33)
25% to 50%	MRANC (38.95), BPGH (30.61), PCEFW (38.55), PCEEE (44.54), PPBPL (46.88), PCESCS (48.75)
50% to 75%	PURPOP (53.79), PWETVR (66.94), PCEMPH (58.63), IDI (55.05)
75% to 100%	-
More than 100%	PCEWS (158.62)

Table 3 shows that variability is higher in the variables of public health & health infrastructure and economic growth & infrastructure head. There is need to reduce disparities on this front.

3.2 Regression analysis

To rummage the interrelationship between indicators of population growth and various demographic and socio-economic variables, we regress the dependent variable with the independent variables individually (independent variables are those variable which have causal relationship with dependent variable in theoretical and behavioral terms) and then pick the most influential variables and regress with the help of step-wise method and get best fitted multiple regression equation of them. Some variables with insignificant coefficients have also been kept in the model because theoretically their importance has been proved. Figures below the coefficients are 't' values. Significance of variables with the level of significance is denoted as follows:

- * Significant at 1% level of significance
- ** Significant at 2% level of significance
- *** Significant at 5% level of significance
- **** Significant at 10% level of significance

Efforts have been made to avoid the problem of multicollinearity (as it presents commonly in the analysis of cross-sectional data) but at some places, it is difficult to avoid it.

3.2.1 Population Growth (Decennial)

Population Growth (POPGWR) is regressed with different variables such as CBR, TFR, FMAG, CDR, CPR, IMR, CIMM, MRANC, PWRSAP, PWETVR, MIPLP, BPGH, PCEMPH, LIT, LIT_m, LIT_f, PCNDDP, PPBPL, ROADSK, PHDW, PCEWS.

Table 4: Regression Equations of Population Growth (Decennial)

S.No.	Intercept	Coefficient			R^2	d. f.
1.	10.0934	+	0.5643	CBR	0.0514	31
			1.2745			
2.	8.1549	+	4.1092	TFR***	0.1325	31
			2.1408			
3.	49.0047	-	1.1555	FMAG	0.0156	31
			0.6903			
4.	50.0223	-	0.5750	CPR*	0.3245	31
			3.7963			
5.	46.2904	-	2.0212	CDR****	0.1119	31
			1.9442			
6.	36.6587	-	0.0979	IMR****	0.0946	31
			1.7709			
7.	34.4649	-	0.1670	CIMM***	0.1413	31
			2.2217			
8.	278632	+	0.0061	MRANC	0.0007	31
			0.1473			
9.	13.8825	+	0.1462	PWRSAP	0.0497	31
			1.2532			
10.	30.8793	-	0.1961	PWETVR****	0.097	31
			1.8019			
11.	24.9574	+	0.1171	MIPLP	0.0118	31
			0.5995			
12.	21.9702	+	0.0768	BPGH****	0.1167	31
			1.9906			
13.	29.1035	-	0.0453	PCEMPH	0.0079	31
			0.4881			
14.	32.8303	-	0.0768	LIT	0.0106	31
			0.5664			
15.	40.5194	-	0.1629	LIT_m	0.0328	31
			1.0084			
16.	31.2097	-	0.0696	LIT_f	0.0124	31
			0.6137			
17.	26.7674	+	0.0346	PCEEE	0.0138	31
			0.6478			
18.	32.6477	-	0.0003	PCNDDP	0.0361	31
			1.0604			
19.	27.1559	+	0.0285	PPBPL	0.0057	31
			0.4138			
20.	35.2376	-	0.2308	ROADSK***	0.1539	31
			2.3363			
21.	31.0646	-	0.0465	PHDW	0.0114	31
			0.5872			
22.	28.6427	-	0.0134	PCEWS	0.0122	31
			0.6075			

Fit of the equations is with the expected signs. TFR, CPR, CDR, IMR, CIMM, PWETVR, BPGH and ROADSK have significant coefficients. PCEEE appears with opposite sign as of expected sign. In the step-wise regression, PPBPL is found more relevant in spite of PCNDDP for multiple regression.

$$POPGWR = 12.5485 + \underset{(3.0425)}{5.6405}\ TFR^* - \underset{(2.7565)}{0.1477}\ IMR^* + \underset{(0.4075)}{0.0246}\ PPBPL$$

$$R^2 = 0.3196 \qquad d.f. = 29$$

In the multiple regression analysis the coefficients of TFR and IMR are significant at 1% level of significance. This indicates that TFR influences POPGWR positively. IMR shows negative influence to POPGWR in mathematical/statistical terms but in actual terms this leads to birth to more children due to less survival. The variable PPBPL does not affect POPGWR significantly.

3.2.2 Crude Birth Rate

Crude Birth Rate (CBR) is regressed with different variables such as POPGWR, FMR, $FMR_{(0-6)}$, PURPOP, FMAG, CPR, IMR, PWETVR, PCEMPH, PCEFW, LIT, LIT_m, LIT_f, PCEEE, PCNDDP, PPBPL.

Table 5: Regression Equations of Crude Birth Rate

S.No.	Intercept	Coefficient			R^2	d. f.
1.	29.6053	+	0.0910	POPGWR	0.0514	31
			1.2745			
2.	39.4165	-	0.0079	FMR	0.0286	31
			0.9391			
3.	25.6612	-	0.0072	FMR(0-6)	0.0088	31
			0.5163			
4.	32.1109	+	0.0032	PURPOP	0.0002	31
			0.0856			
5.	50.5045	-	1.1004	FMAG****	0.0879	31
			1.7004			
6.	38.4234	-	0.1650	CPR***	0.1657	31
			2.4406			
7.	30.0598	+	0.0247	IMR	0.0372	31
			1.0766			
8.	32.2562	-	0.0059	PWETVR	0.0006	31
			0.1291			
9.	31.1170	+	0.0563	PCEMPH	0.0755	31
			1.5657			
10.	32.3631	-	0.1645	PCEFW	0.0010	31
			0.1744			
11.	32.4349	-	0.0043	LIT	0.0002	31
			0.0792			
12.	30.2455	+	0.0256	LIT_m	0.0050	31
			0.3897			
13.	32.9059	-	0.0172	LIT_f	0.0047	31
			0.3753			
14.	32.2458	-	0.0016	PCEEE	0.0002	31
			0.0748			
15.	34.6147	-	0.0002	PCNDDP	0.0689	31
			1.4901			
16.	31.1572	+	0.0321	PPBPL	0.0447	31
			1.1847			

FMAG and CPR have significant coefficients. PURPOP, PCEMPH and LIT_m are with opposite signs as of expected signs.

$$\begin{aligned} CBR = 50.2161 - & 1.0819\ FMAG^{****} + 0.0114\ IMR - 0.0234\ LIT_f \\ & (1.7123) \qquad\qquad (0.4421) \qquad (0.4701) \\ R^2 = 0.1099 & \qquad\qquad d.f. = 29 \end{aligned}$$

Fit of the multiple regression equation is with the expected signs Coefficient of FMAG is significant at 10% level of significance. This indicates that FMAG influences CBR negatively. The coefficients of IMR and LIT_f are insignificant but included due to their importance in the determination of CBR.

3.2.3 Total Fertility Rate

Total Fertility Rate (TFR) is regressed with different variables such as PURPOP, FMAG, CPR, CDR, IMR, MRANC, PWRSAP, PWETVR, PCEMPH, LIT, LIT_m, LIT_f, PCEEE, PCNDDP, PPBPL, PCESCS.

Table 6: Regression Equations of Total Fertility Rate

S.No.	Intercept	Coefficient			R^2	d. f.
1.	4.9033	-	0.0006	PURPOP	0.0002	31
			0.0744			
2.	9.2697	-	0.2629	FMAG****	1.1031	31
			1.8573			
3.	6.5736	-	0.0445	CPR*	0.2471	31
			3.1379			
4.	3.6639	+	0.1375	CDR	0.0659	31
			1.7123			
5.	4.2069	+	0.0079	IMR	0.0797	31
			1.6123			
6.	5.2989	-	0.0064	MRANC****	0.1017	31
			1.8431			
7.	5.0179	-	0.0033	PWRSAP	0.0032	31
			0.3124			
8.	51792	-	0.0073	PWETVR	0.0175	31
			0.7304			
9.	4.9694	-	0.0011	PCEMPH	0.0006	31
			0.1367			
10.	4.9951	-	0.0033	LIT	0.0025	31
			0.2719			
11.	5.4818	-	0.0139	LIT_m	0.0305	31
			0.9720			
12.	5.0591	-	0.0039	LIT_f	0.0051	31
			0.3932			
13.	4.9577	-	0.0016	PCEEE	0.0036	31
			0.3288			
14.	5.6749	-	0.00006	PCNDDP***	0.1465	31
			2.2693			
15.	4.9662	+	0.0023	PPBPL	0.0051	31
			0.3906			
16.	5.1543	-	0.0014	PCESCS	0.0664	31
			1.4618			

FMAG, CPR, MRANC and PCNDDP are with significant coefficients. All the variables show the expected signs.

$$TFR = 5.9697 - \underset{(2.9361)}{0.0412}\ CPR^{*} + \underset{(2.3704)}{0.0104}\ IMR^{***} - \underset{(0.3446)}{0.0031}\ LIT_f - \underset{(1.8641)}{0.00004}\ PCNDDP^{****}$$

$$R^2 = 0.4305 \qquad d.f. = 28$$

Coefficient of CPR is significant at 1% level of significance, IMR at 2% and PCNDDP at 10%. This indicates that CPR & PCNDDP influence TFR positively and IMR influences TFR negatively. LIT_f appears with insignificant coefficient but it has major influential role in the determination of TFR.

3.2.4 Females' Mean Age at Gauna

Females' Mean Age at Gauna (FMAG) is regressed with different variables such as PURPOP, PWETVR, LIT, LIT_m, LIT_f, PSER, $PSER_m$ $PSER_f$, DORPS, $DORPS_m$, $DORPS_f$, PCEEE, PCNDDP, PPBPL.

Table 7: Regression Equations of Females' Mean Age at Gauna

S.No.	Intercept	Coefficient			R^2	d. f.
1.	16.1794	+	0.0060	PURPOP	0.0118	31
			0.5994			
2.	15.0223	+	0.0273	PWETVR***	0.1619	31
			2.4070			
3.	15.7873	+	0.0190	LIT	0.051	31
			1.3231			
4.	14.9522	+	0.0305	LIT_m****	0.0981	31
			1.8061			
5.	151910	+	0.0126	LIT_f	0.0346	31
			1.0371			
6.	16.0412	+	0.0160	PSER	0.0456	31
			1.1974			
7.	16.9408	-	0.0028	$PSER_m$	0.0027	31
			0.2861			
8.	15.8440	+	0.0166	$PSER_f$	0.0775	31
			1.5871			
9.	17.3288	-	0.0225	DORPS	0.0796	31
			1.6104			
10.	18.1252	-	0.0268	$DORPS_m$****	0.1050	31
			1.8765			
11.	17.2214	-	0.0099	$DORPS_f$	0.0147	31
			0.6701			
12.	16.7143	+	0.0014	PCEEE	0.0018	31
			0.2328			
13.	16.4417	+	0.00002	PCNDDP	0.0074	31
			0.4713			
14.	16.6888	-	0.0010	PPBPL	0.0006	31
			0.1374			

PWETVR, LIT_m and $DORPS_m$ are with significant coefficients. Except $PSER_m$, coefficients of all are with expected Signs.

$$FMAG=13.7224 +0.0279\ PWETVR^{***} +0.0039\ LIT_f^{****} +0.00003\ PCNDDP$$
$$(2.1774) \quad (1.8126) \quad (1.0034)$$
$$R^2 = 0.1912 \qquad d.f. = 29$$

All the variables are with expected signs. Coefficient of PWETVR is significant at 5% level of significance & coefficient of LIT_f is significant at 10% level of significance. This indicates that PWETVR & LIT_f influence FMAG positively. Coefficient of PCNDDP is insignificant means the variable PCNDDP does not affect FMAG significantly.

3.2.5 Couple Protection Rate

Couple Protection Rate (CPR) is regressed on different variables such as PURPOP, FMAG, IMR, PWETVR, MIPLP, PCEMPH, PCEFW, LIT, LIT_m, LIT_f, PCEEE, PCNDDP, PPBPL, IDI, PCESCS.

Table 8: Regression Equations of Couple Protection Rate

S.No.	Intercept	Coefficient			R^2	d. f.
1.	40.2031	-	0.1133	PURPOP	0.0511	31
			1.2713			
2.	18.8647	+	1.1404	FMAG	0.0155	31
			0.6876			
3.	34.1252	+	0.0435	IMR	0.0190	31
			0.7628			
4.	35.2834	+	0.1922	PWETVR****	0.0956	31
			1.7811			
5.	35.3518	+	0.0892	MIPLP	0.0069	31

S.No.	Intercept	Coefficient			R^2	d. f.
			0.4596			
6.	36.7349	+	0.0597	PCEMPH	0.0139	31
			0.6522			
7.	33.9617	+	3.4369	PCEFW	0.0726	31
			1.5325			
8.	35.5692	+	0.1294	LIT	0.0306	31
			0.9726			
9.	29.9513	+	0.1606	LIT_m	0.0325	31
			1.0032			
10.	36.5518	+	0.0869	LIT_f	0.0197	31
			0.7762			
11.	39.5813	+	0.0402	PCEEE****	0.0189	31
			1.7607			
12.	31.0288	+	0.0005	PCNDDP****	0.0889	31
			1.7108			
13.	39.0037	-	0.1215	PPBPL****	0.1051	31
			1.8769			
14.	38.5776	+	0.0077	IDI	0.0050	31
			0.3894			
15.	36.6705	+	0.0093	PCESCS	0.0250	31
			0.8785			

PWETVR, PCEEE, PCNDDP and PPBPL are with significant coefficients and expected signs. Sign of coefficient of PURPOP is opposite of the expected.

$$CPR = 20.6541 + 0.4813\ FMAG + 0.1388\ LIT_f^{****} + 0.0006\ PCNDDP^{****}$$
$$(0.2922) \qquad (1.8065) \qquad (1.9266)$$
$$R^2 = 0.1433 \qquad d.f. = 29$$

All the variables are with expected signs of coefficients. Coefficients of LIT_f and PCNDDP are significant at 10% level of significance. This indicates that LIT_f and PCNDDP influence CPR positively. Coefficient of FMAG is insignificant means the variable FMAG does not affect CPR significantly.

4. Conclusion

The model is fit good with the expected signs. Estimated equations confirm the *a priori* hypotheses of positive impact of infant mortality & total fertility rate and negative impact of income equality on population growth; positive impact of female literacy & income equality on female's age at gauna; positive impact of infant mortality and negative impact of female's age at gauna and female literacy on crude birth rate; negative impact of couple protection rate, income equality, female literacy and positive impact of infant mortality on total fertility rate, positive impact of female literacy, females age at gauna and income equality on couple protection rate. Literacy, especially female literacy and per-capita income appeared as most influential variables to attack the poor status of socio-economic & demographic variables. There is need to emphasize on the improvement of these two variables.

Rapid population growth retards the economic, social and human development. Enhancement of women's status and autonomy has been conclusively established to have a direct bearing on fertility and mortality decline, which indirectly affects the population growth. More specifically, inter-relationships between women's characteristics and access to resources are the mechanisms through which human fertility is determined. Education is highly correlated with age at the marriage of the females and thus helps in the reduction of the reproductive life, on an average, and helps in the conscious efforts to limit the family size. The early marriage of the daughter in rural areas is an expected rational behavior, as long as there is mass illiteracy and poverty. The age at marriage for females cannot be raised by mere, legislation unless the socio-economic conditions of the rural people is improved and better educational facilities and occupational alternatives for the teenage girls are provided near their homes.

Reproductive and public health have their importance in determination of population stabilization. National Rural Health Mission (NRHM) and Rajasthan Health System Development Project (RHSDP) are ongoing programs which can improve the situation. There is need of effective monitoring of activities under these programs. Effective implementation of family welfare program will create opportunities for better education and improvement in nutritional status of family through check on population growth, which will turn in better health of mother and child and there will be less infant and maternal mortality.

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Appendix - I

Table 9: All Rajasthan Figures, Mean, Standard Deviation & Coefficient of Variation of Variables

S. No.	Variable & Year	Code	Unit	All Rajasthan	Mean	S. D.	CoV
1.	Population Growth (Decennial) 1991-2001	POPGWR	Per cent	28.33	28.25	5.63	19.92
2.	Female-Male Ratio 2001	FMR	Nos.	921	922.03	48.65	5.28
3.	Female-Male Ratio (0-6 years) 2001	$FMR_{(0-6)}$	Nos.	909	909.00	29.59	3.26
4.	Percentage of Urban Population to Total Population 2001	PURPOP	Per cent	23.38	20.69	11.13	53.79
5.	Crude Birth Rate 1997	CBR	Per '000	32.90	32.18	2.26	7.02
6.	Total Fertility Rate 1997	TFR	Nos.	4.9	4.89	0.50	10.20
7.	Females Mean Age at Gauna1996-97	FMAG	Years	17.7	16.66	0.61	3.66
8.	Couple Protection Rate 2001	CPR	Per cent	37.00	37.86	5.58	14.73
9.	Crude Death Rate 1997	CDR	Per '000	8.9	8.93	0.93	10.44
10.	Infant Mortality Rate 1997	IMR	Per '000	87	85.81	17.67	20.60
11.	Percentage of Mothers Receiving Total Ante-Natal Care 1996-97	MRANC	Per cent	72.3	63.38	24.69	38.95
12.	Percentage of Women having Exposure to TV & Radio 1996-97	PWETVR	Per cent	13.1	13.40	8.97	66.94
13.	Medical Institutions Per-Lakh of Population 1997-98	MIPLP	Nos.	27	28.13	5.23	18.59
14.	Beds Per-Lakh Population in Govt. Hospitals 1997-98	BPGH	Nos.	85	81.81	25.04	30.61
15.	Per-Capita Expenditure on Medical & Public Health 2000-01	PCEMPH	`	19.00	18.82	11.04	58.63
16.	Per-Capita Expenditure on Family Welfare 2000-01	PCEFW	`	0.97	1.13	0.44	38.55
17.	Literacy Rate 2001	LIT	Per cent	60.41	59.58	7.53	12.64
18.	Literacy Rate (Male) 2001	LIT_m	Per cent	75.70	75.31	6.26	8.31
19.	Literacy Rate (Female) 2001	LIT_f	Per cent	43.85	42.51	9.01	21.19
20.	Primary School Enrolment Ratio 1997-98	PSER	Per cent	86.50	86.75	8.15	9.39
21.	Primary School Enrolment Ratio (Male) 1997-98	$PSER_m$	Per cent	99.78	100.51	11.22	11.17
22.	Primary School Enrolment Ratio (Female) 1997-98	$PSER_f$	Per cent	71.91	71.65	10.22	14.27
23.	Drop-Out Rates at Primary Level 1996-97	DORPS	Per cent	56.60	59.13	7.65	12.94
24.	Drop-Out Rates at Primary Level (Male) 1996-97	$DORPS_m$	Per cent	54.72	57.07	7.36	12.89
25.	Drop-Out Rates at Primary Level (Female) 1996-97	$DORPS_f$	Per cent	56.96	62.68	10.66	17.01
26.	Per-Capita Expenditure on Elementary Education 2000-01	PCEPEE	`	47.00	42.86	19.09	44.54
27.	Per-Capita Net District Domestic Product 1999-2000	PCNDDP	`	12752	12831.88	3122.80	24.34
28.	Population Below Poverty Line 1999-2000	PPBPL	Per cent	30.99	31.74	14.88	46.88
29.	Infrastructure Development Index 1994-95	IDI	Nos.	100.00	93.46	51.45	55.05
30.	Percentage of Villages with Safe Drinking Water 1998-99	PHDW	Per cent	64.30	60.54	12.91	21.33
31.	Per-Capita Expenditure on Social & Community Services 2000-01	PCESCS	`	245.62	194.69	94.92	48.75
32.	Per-Capita Expenditure on Water Supply 2000-01	PCEWS	`	39.95	29.19	46.30	158.62

RATIO ESTMATORS IN SIMPLE RANDOM SAMPLING WHEN STUDY VARIABLE IS AN ATTRIBUTE

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Abstract: In this paper we have suggested a family of estimators for the population mean when study variable itself is qualitative in nature. Expressions for the bias and mean square error (MSE) of the suggested family have been obtained. An empirical study has been carried out to show the superiority of the constructed estimator over others.

Key words: Attribute · Point bi-serial · Mean square error · Simple random sampling

INTRODUCTION

The use of auxiliary information can increase the precision of an estimator when study variable y is highly correlated with auxiliary variable x. In many situations study variable is generally ignored not only by ratio scale variables that are essentially qualitative, or nominal scale, in nature, such as sex, race, colour, religion, nationality, geographical region, political upheavals (see [1]). Taking into consideration the point biserial correlation coefficient between auxiliary attribute and study variable, several authors including [2-6] defined ratio estimators of population mean when the priori information of population proportion of units, possessing some attribute is available. All the others have implicitly assumed that the study variable Y is quantitative whereas the auxiliary variable is qualitative.

In this paper we consider some estimators in which study variable itself is qualitative in nature. For example suppose we want to study the labour force participation (LFP) decision of adult males. Since an adult is either in the labour force or not, LFP is a yes or no decision. Hence, the study variable can take two values, say 1, if the person is in the labour force and 0 if he is not. Labour economics research suggests that the LFP decision is a function of the unemployment rate, average wage rate, education, family income, etc (See [1]).

Consider a sample of size n drawn by simple random sampling without replacement (SRSWOR) from a population size N. Let ϕ_i and x_i denote the observations on variable ϕ and x respectively for i^{th} unit (i=1,2,3…N). ϕ_i, if i^{th} unit of population possesses attribute ϕ and ϕ_i, otherwise. Let $A=\sum_{i=1}^{N}\phi_i$ and $a=\sum_{i=1}^{n}\phi_i$ denote the total number of units in the population and sample possessing attribute ϕ respectively, $P=\frac{A}{N}$ and $p=\frac{a}{n}$ denote the proportion of units in the population and sample, respectively, possessing attribute ϕ.

Define,

$$e_f=\frac{(p-P)}{P}, \qquad e_x=\frac{(\bar{x}-\bar{X})}{\bar{X}}$$

Such that,

$E(e_i) = 0$, $(1 = \phi, x)$

and

$$E\left(e_f^2\right)=fC_p^2, \qquad E\left(e_x^2\right)=fC_x^2, \qquad E\left(e_x e_f\right)=f\rho_{pb}C_f C_x.$$

Where,

$$f=\left(\frac{1}{n}-\frac{1}{N}\right), \qquad C_p^2=\frac{S_p^2}{P^2}, \qquad C_x^2=\frac{S_x^2}{\bar{X}^2},$$

and $\rho_{pb}=\frac{S_{fx}}{S_f S_x}$ is the point biserial correlation coefficient.

Here,

$$S_f^2=\frac{1}{N-1}\sum_{i=1}^{N}(f_i-P)^2,\ S_x^2=\frac{1}{N-1}\sum_{i=1}^{N}(x_i-\bar{X})^2 \text{ and } S_{fx}=\frac{1}{N-1}\left(\sum_{i=1}^{N}f_i x_i-NP\bar{X}\right).$$

The Proposed Estimator: We first propose the following ratio-type estimator

$$t_1=\left(\frac{P}{\bar{x}}\right)\bar{X} \tag{2.1}$$

The bias and MSE of the estimator t_1, to the first order of approximation is respectively, given by

$$B(t_1)=f\left(\frac{C_x^2}{2}-\rho_{pb}C_f C_x\right) \tag{2.2}$$

$$MSE(t_1)=f\left(C_f^2+C_x^2-2\rho_{pb}C_f C_x\right) \tag{2.3}$$

Following [7], we propose a general family of estimators for P as

$$t_2 = H(p,u) \tag{2.4}$$

Where $u=\frac{\bar{x}}{\bar{X}}$ and H(p,u) is a parametric equation of p and u such that

$$H(p,1) = P, \forall P \tag{2.5}$$

and satisfying following regulations:

- Whatever be the sample chosen, the point (p,u) assume values in a bounded closed convex subset R_2 of the two-dimensional real space containing the point (p,1).
- The function H(p,u) is a continuous and bounded in R_2.
- The first and second order partial derivatives of H(p,u) exist and are continuous as well as bounded in R_2.

Expanding H(p,u) about the point (P,1) in a second order Taylor series we have

$$t_2 = H(p,u) = p + (u-1)H_1 + (u-1)^2H_2 + (p-P)(u-1)H_3 + (p-P)^2H_4 + \ldots \tag{2.6}$$

Where,

$$H_1=\left.\frac{\partial H}{\partial u}\right|_{p=P,u=1}, \quad H_2=\left.\frac{1}{2}\frac{\partial^2 H}{\partial u^2}\right|_{p=P,u=1}, \quad H_3=\left.\frac{1}{2}\frac{\partial^2 H}{\partial p\partial u}\right|_{p=P,u=1}, \quad \text{and} \quad H_4=\left.\frac{1}{2}\frac{\partial^2 H}{\partial p\bar{y}^2}\right|_{p=P,u=1}.$$

The bias and MSE of the estimator t_2 are respectively given by -

$$B(t_2)=f\left(P\rho_{pb}C_pC_xH_3+C_x^2H_2+P^2C_y^2H_4\right) \tag{2.7}$$

$$MSE(t_2)=f\left(P^2C_p^2+H_1^2C_x^2+2H_1P\rho_{pb}C_pC_x\right) \tag{2.8}$$

On differentiating (2.8) with respect to H_1 and equating to zero we obtain

$$H_1=-\rho_{pb}P\frac{C_p}{C_x} \tag{2.9}$$

On substituting (2.9) in (2.8), we obtain the minimum MSE of the estimator t_2 as

$$\min MSE(t_2)=fP^2C_p^2\left(1-\rho_{pb}^2\right) \tag{2.10}$$

We suggest another family of estimators for estimating P as

$$t_3=\left[q_1P+q_2\left(\bar{X}-\bar{x}\right)\right]\left[\frac{a\bar{X}+b}{a\bar{x}+b}\right]^{a}\exp\left[\frac{\left(a\bar{X}+b\right)-\left(a\bar{x}+b\right)}{\left(a\bar{X}+b\right)+\left(a\bar{x}+b\right)}\right]^{\beta} \tag{2.11}$$

Where α,β,q_1 and q_2 are real constants and a and b are known as characterising positive scalars. Many ratio-product estimators can be generated from t_3 by putting suitable values of q_1, q_2, α, β, a and b (for choice of the parameters refer to [8] and [5].

$$\begin{aligned} t_3 &= \left[q_1P(1+e_0)-q_2\bar{X}\right]\left[1-\alpha\theta e_1+\frac{\alpha(\alpha+1)}{2}\theta^2e_1^2\right]\left[1-\frac{\beta\theta e_1}{2}+\frac{\beta\theta^2e_1^2}{8}(\beta+2)\right] \\ &= q_1P\left\{1+e_0-B\left(e_1+e_0e_1\right)+Ae_1^2\left(1+e_0\right)\right\}-q_2\bar{X}\left\{e_1-Be_1^2\ldots\right\} \end{aligned} \tag{2.12}$$

Where, $\theta=\frac{a\bar{X}}{a\bar{X}+b}$, $B=\left(a+\frac{\beta}{2}\right)\theta$ and

$$A=\frac{\theta^2}{8}\left[4a\left(a+1\right)+\beta(\beta+2)+4a\beta\right].$$

The bias and MSE of the estimator t_3 to the first order of approximation, are given as

$$Bias(t_3)=P(q-1)+f\left[\left(q_2\bar{X}B+q_1PA\right)C_x^2-q_1PB\rho C_pC_x\right] \tag{2.13}$$

$MSE(t_3) = E(t_3 - P)^2$

$$= (q_1 - 1)^2 P^2 + q_1^2 (M_1 + 2M_3) + q_2^2 M_2$$
$$+2q_1 q_2 (-M_4 - M_5) - 2q_1 M_3 + 2q_2 M_5$$

Where,

$$M_1 = P^2 f\left(C_p^2 + B^2 C_x^2 - 2B\rho C_p C_x\right), \quad M_2 = \bar{X}^2 f\left(C_x^2\right),$$

$$M_3 = P^2 f\left(AC_x^2 - 2B\rho C_p C_x\right), \quad M_4 = P\bar{X} f\left(-BC_x^2 + \rho C_p C_x\right),$$

$$M_5 = \bar{X} P f\left(-BC_x^2\right).$$

On minimising the MSE of t3 with respect to q1 and q2, respectively, we get

$$q_1^* = \frac{\Delta_1\Delta_4 - \Delta_2\Delta_5}{\Delta_1\Delta_3 - \Delta_2^2} \quad \text{and} \quad q_2^* = \frac{\Delta_1\Delta_5 - \Delta_2\Delta_4}{\Delta_1\Delta_3 - \Delta_2^2}$$

Where,

$\Delta_1 = (P^2 + M_1 + 2M_3)$, $\Delta_2 = (-M_4 - M_5)$,

$\Delta_3 = (M_2)$, $\Delta_4 = (P^2 + M_3)$

$\Delta_5 = (-M_5)$,

On putting these values of q_1 and q_2 in equation (2.14) we obtain the minimum MSE of t_3 as:

$$MSE(t_3)_{min} = \left[P^2 - \frac{\Delta_1\Delta_5^2 + \Delta_3\Delta_4^2 - 2\Delta_2\Delta_4\Delta_5}{\Delta_1\Delta_3 - \Delta_2^2}\right] \qquad (2.16)$$

Efficiency Comparisons: First, we compare the efficiency of proposed estimator t_3 with usual estimator.

$MSE(t_3)_{min} \leq V(\bar{y})$

If,

$$\left[P^2 - \frac{\Delta_1\Delta_5^2 + \Delta_3\Delta_4^2 - 2\Delta_2\Delta_4\Delta_5}{\Delta_1\Delta_3 - \Delta_2^2}\right] \leq P^2 f_1 C_p^2$$

On solving we observed that above conditions holds always true.

Next we compare the efficiency of proposed estimator t_3 with regression estimator.

MSE (reg)$MSE(t\alpha)_{min} \leq$ MSE(reg)

If,

$$\left[P^2 - \frac{\Delta_1\Delta_5^2 + \Delta_3\Delta_4^2 - 2\Delta_2\Delta_4\Delta_5}{\Delta_1\Delta_3 - \Delta_2^2}\right] \leq P^2 f_1 C_p^2\left(1 - \rho_{pb}^2\right)$$

Empirical Study:

Data Statistics: We have taken the data from [1].

Where

Y – Home ownership
X – Income (thousands of dollars)

n	N	P	$\bar{x}$	ρpb	Cp	Cx
11	40	0.525	14.4	0..897	0.963	0.3085

The following Table shows PRE of different estimator's with respect to usual estimator.

Table 1: Percent relative efficiency (PRE) of estimators with respect to usual estimator

				t_3		
Estimators	$\bar{y}$	t_1	t_2	$\alpha = 1, \beta = 1$	$\alpha = 1, \beta = 0$	$\alpha = 0, \beta = 1$
PRE	100	189.384	511.794	515.798	517.950	518.052

When we examine Table 1, we observe that the proposed estimators t_1, t_2 and t_3 all performs better than the usual estimator $\bar{y}$ Also, the proposed estimator t_3 is the best among the estimators considered in the paper for the choice $\alpha = 0$, $\beta = 1$.

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RURAL MIGRATION A SIGNIFICANT CAUSE OF URBANIZATION: A DISTRICT LEVEL REVIEW OF CENSUS DATA FOR RAJASTHAN

JAYANT SINGH, HANSRAJ YADAV, FLORENTIN SMARANDACHE

Discussions:

Migration witnesses a better urbanization rate and there are more districts classified in higher range of urbanization rates than the number of district classified according to total urbanization rate of the districts. At state level, the rising contribution of rural migrants in urbanization is witnessed in three successive decades.

Scale of the urbanization for some of the district that are already having higher urbanization due to rural migrants is speeding up and these district have grown tremendously due to high rate of rural migrants settling in urban areas. This in turn is resulting in big is getting bigger in recent census over previous censuses and the gap in urbanization due to rural migrants is increasing for the district that already had high urbanization from rural migrants than to districts which had small rural migrants settling in urban area. .

Introduction

Migration plays an important role in urbanization of a state. In general more the migration higher the urbanization rate though it many not necessarily true in all the situations but in general it is witnessed that migration have a fairly large share in urbanization. A district level analysis for Rajasthan state is attempted to comprehend Urbanization due to migration their interlinkages and association.

Urbanization Trend in Rajasthan State

The share of urban population inched up to 23.38% according to census 2001 from 15.06% in the census 1901 in the Rajasthan state. Number of towns in the Rajasthan state increased to 216 in the census 2001 against 133 in the 1901 census which is 62.4% growth in this period whereas at national level this growth has been 169.36% in this same period. Share of state urban population in the country urban population dropped to 4.6% from 5.98% over a century period whereas in terms of number of town state share also slipped to 4.18% from 6.94% in this same period. Therefore it can be clearly claimed that the state has to go a long way to match with national figures on account of characteristics of urbanization whether it is growth in urban population or towns, however there has been a meager improvement in the percentage share of state urban population in the national urban population as it grow to 4.1% to 4.52%, 4.52% to 4.62% and than to 4.64% in last three successive census periods.

District Level Analysis for Rajasthan

The migrants contribution in urbanization is on the rising over the decades as 16.4% of the total migrants in the Rajasthan settled in urban areas during the period 1971-80 which went up to 22.4% for the duration 1981-1990 and further advanced to 25.4% in the duration 1991-2000. This trend is evident invariably for all the districts of the state though the contribution in urbanization by the migrants vary from district to district, for some district the share of migrants moving to urban areas in total migrant is very impressive though for others it is not that much high.

In Barmer districts 7.7%, 7.1% & 4.0% of total migrants moved to urban areas in last three decades i.e. 1991-2000, 1981-90 & 1971-1980. This percentage share for Jalore was 9.6, 8.1 & 4.7%, and for Banswara 9.1,7.9 & 4.7% and these district had poor share of migrants to urban areas.

On the other side there are districts like Jaipur, Ajmer, Kota & Bhilwara where the percentage share of migrants settling in urban areas to the total migrants is comparatively very high. This percentage share of urban migrants in three last successive decades for these districts is given in table placed on next page

District / period	1991-2000	1981-90	1971-1980
Kota	56.8	54.3	50.7
Jaipur	53.2	48.5	35
Ajmer	41.4	35.6	28.7
Bhilwara	31.1	25.0	14.8
Jodhpur	26.8	18.7	12.4

Urbanization and Migration

Contribution of urban migrants in total migrants is considered as extent of urbanization by the migration in a particular category. Districts are classified in the groups where % of migrants attributing to urbanization is <20%, 20-50 and >50% for all the three durations 1971-80,1981-90 and 1991-2000 and the result is summarized below:

Range of urbanization by migrants (in%)	2001	1991	1981
	Number of Districts		
<20	10	16	28
20-50	20	14	3
>50	2	2	1

Its is evident from above classification that there is considerably shift in last three census period as number of district having high urbanization due to migration has gone up in almost all the categories of urbanization range due to migration.

Total Urbanization & Urbanization due to Migration:

An Indicator, Urbanization rate, for this comparative analysis is defined as below

Migration is an important part of the urbanization and in many cases it is attributing predominately in the urbanization. Urbanization Indicator based on two rates is defined below

1. Total Urbanization rate: is the percentage of population living in urban areas to the total population

2. Urbanization rate due migration: is the percentage share of urban migrants to the total migrants.

The comparative investigation for the last decadal period i.e. 1991-2001 between these two indicator rates is performed in coming paragraphs.

State urbanization rate is the share of urban population to the total population at state level and similarly it is calculated for districts level. Now theses two rates are compared at state and districts level to analyze the urbanization trend and its association with the migration.

At state level 23.4% of the total population is urbanized as compared to 22.9% of migrants are coming to urban areas thus at state level the urbanization rate for migrants is in line of the total urbanization rate. Barmer and Jalore are two district having migrants urbanization rate below 20% as the urbanization rate of the migrants to theses districts are mere 15 & 19%f.

Migrants urbanization rate for Jaipur (73.6%), Kota (68.2%), Ajmer (53.8%) and Udaipur (50%) districts are above 50% thus the more than half of the migrants to these districts are settling in urban areas. Bikaner and Churu are the only districts observed the migrants urbanization rate lower than total urbanization rate. This difference was more than 32% for the

Udaipur and Banswara districts and for seven districts it was more than 20%. The classification of number of districts based on the range of these two urbanization rate is classified in coming table

Range of Urbanization rate		>50%	40-50%	30-40%	20-30%	<20%
Combined (Male & female)	Total Urbanization rate	1	2	2	8	19
Male		1	1	2	9	19
Female		1	1	3	7	20
Combined (Male & female)	Urbanization rate due to migration	4	5	8	13	2
Male		12	8	4	9	12
Female		2	2	11	10	7

Clearly the migration witnesses a better urbanization rate and there are more districts classified in higher range of urbanization rates than the number of district classified according to total urbanization rate of the districts.

Technique of non-parametric test is used for district level analysis of the urbanization to see that migration to different districts is having same population. District are ranked on the basis of the total urban population and urban population due to migration and these formed two groups of Non-parametric test and Wilcoxon - Mann/Whitney Non parametric Test is employed for equality of K universes for total population and Male & Female population and results of the analysis done in Megastat is as below:

TOTAL		
n	sum of ranks	
32.00	698.00	Group 1
32.00	1382.00	Group 2
64.00	2080.00	Total
	1040.00	expected value
	74.48	standard deviation

	-4.59	Z
	0.00	p-value (two-tailed)
MALE		
n	sum of ranks	
32.00	612.00	Group 1
32.00	1468.00	Group 2
64.00	2080.00	Total
	1040.00	expected value
	74.48	standard deviation
	-5.74	Z
	0.00	p-value (two-tailed)
FEMALE		
n	sum of ranks	
32.00	775.00	Group 1
32.00	1305.00	Group 2
64.00	2080.00	Total
	1040.00	expected value
	74.48	standard deviation
	-3.55	Z
	.0004	p-value (two-tailed)
GROUP1 URBANISATION IN TOTAL POPULATION		
GROUP2 URBANISATION BY MIGRATION		

Clearly above district level analysis reveals that total urbanization and urbanization due to migration differs significantly for total, male and female population and districts have significant impact on total urbanization & urbanization due to migration. Thus the relative magnitude of total urbanization and urbanization due to migration differ significant for the districts for both genders and combined.

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URBANIZATION DUE TO MIGRATION: A DISTRICT LEVEL ANALYSIS OF MIGRANTS FROM DIFFERENT DISTANCES FOR THE RAJASTHAN STATE

JAYANT SINGH, HANSRAJ YADAV, FLORENTIN SMARANDACHE

Discussions:

Inter-state migrants share in total migrants is lagging in the state as compared to national scenario. Proportion of migrants settling in urban areas is on the rising side since last three decades however, its impact in different distances is varied. Urbanization rate due to migration is lower for intra-district migrants than to state, it is moderately high for inter-district migrants and just double for the inter-state migrants and this trend was evident for last three census periods.

Comparison of district level Urbanization rate is also skewed as on one side there are districts like Kota, Bhilwara, Jodhpur and Jaipur for which the migrant's urbanization rate has been phenomenal high than to state whereas for many districts like Jalore, Sirohi, Tonk, Karaulli, Sawai Madohur etc it is very low and this difference also varies for intra-district, inter-district and inter-state migrants. Urbanization rate for Inter-state and inter-district migrants are higher for majority of districts where as for intra-district migrants it low for most of the districts.

Introduction: People migrate to different distances and there migration is governed by different reasons. Distance of place of migration plays an important role in the migration process and an analysis based on the remoteness of the origin and destination will reveal the push and pull factors in more explicit way. However, a common phenomenon is that people do migrate to a longer distance with a more focused objective and there propensity to settle in urban areas is always higher than the small distance migration.

Census data gives details of Intra-district, Inter-districts and Inter-states number of migrants and these three categories are considered to understand the inter-relationship of distance of migration and urbanization.

State vis-à-vis National Scenario:

Migration rate (% of migrants to total population) for the Rajasthan state is in line of country migration level of around 27%. State exhibited different track on account of migration distance. At national level 60.44% of migrants were in the intra-district, 25.67% in inter-district and 14.29% in inter state categories of migrants as compared to 65.45% intra district, 23.62% inter-district and 10.92% inter-state migrant in Rajasthan. Thus the share of intra-district migrants in total migrants is higher in the Rajasthan State as compared to Country level while it is on lower side for the inter state from the country level in the inter state category.

Migration Distance and Urbanization:

The migrant's contribution in urbanization is on the rising over the decades as 16.4%, 22.4% and 25.4% of the total migrants in the Rajasthan settled in urban areas during the period 1971-80, 1981-1990 and 1991-2000. This trend is also witnessed irrespective of the distance of migration. Migrants from different distances contribute in urbanization differently. For intra-district migration (Short Distance Migration) the urbanization due to migration inched to 13.5% in the duration 1981-90 from 10.1% in 1971-80 and further scaled to 15.2% in 1991-2000. Inter-district migration (Medium Distance Migration) contribution in urbanization advanced from 28.5% to 37.2% & 37.2% to 38.2% in this same duration and, similarly, Inter-state migration (Long Distance Migration) contribution in urbanization advanced from 36.5% to 44.4% & 44.4% to 46.7% in these three consecutive decadal periods.

Looking at the urbanization due to migrants from these different places (intra-district, inter-district and inter-state) it is found that share of inter-state migrants in urbanization is way ahead to share of inter-district and intra-district migrants in urbanization, as in the duration 1991-2000, 46.75% of interstate migrants settled in urban areas as compared to 38.2% and 15.2% of inter-district and intra-districts migrants in urban areas and similar trend were also observed in 1971-80 & 1981-90 durations. Not only inter-state migration share in urbanization dominated but also its dominance is going stronger than inter-district and intra-district migration. Similarly the inter-district migration has an edge over the intra-district migration as far as urbanization is analyzed.

Share of urbanization due to migration in last three decades is considered to examine a trend in migrants in urban areas. Share of migrants in urbanization at state level and district level is compared for three consecutive decadal periods to establish the pattern in urbanization at the state and district level. For the above stated comparison six categories as given below are formed:

Category	Description
1	Higher During all the three decades
2	Higher during 1991-2000 & 1981-91 but lower in 1971-80
3	Higher during 1991-2001 but lower in last two decades
4	Lower During all the three decades
5	Lower during 1991-2000 & 1981-90 but higher in 1971-80
6	Lower during 1991-2000 but higher in 1981-90 & 1971-80

Districts falling in different categories exhibit a different trend as category 1 consist those districts, which observed higher urbanization from migration than the state level for three consecutive decadal period where as category 2 & 3 are having districts that performed better as far urbanization due to migration is concerned in last two decades and one decade respectively as compared to state level urbanization.

Category 3, 4 & 5 contain districts that have performed low in urbanization from state level in last three, two and recent decade respectively.

Cat.1	Cat.2	Cat. 3	Cat. 4	Cat.5	Cat. 6
Intra District Migration					
Hanumangarh, Jhunjhunu, Churu,Bharatpur, Dholpur, Jaipur, Ajmer,Bhilwara, Kota, Pali	Alwar, Sawai madhopur, Baran.	Bundi, Jhalawar	Bikaner, Karauli, Dausa, Nagaur,Jodhpur,Jaiselmer Barmer, Jalore, Sirohi, Tonk,Rajsamand, Udaipur Bansawara, Dungarpur, Chittorgarh.		Ganganagar, Sikar,
Inter District Migration					
Jaipur, Jodhpur, Sirohi, Ajmer, Udaipur, Kota	Bhilwara	Sawai madhopur Chittorgarh,	Bikaner, Jhunjhunu, Churu, Alwar, Karauli, Bharatpur, Dholpur, Dausa, Sikar, Nagaur, Jaiselmer, Barmer, Jalore,Tonk, Bundi, Rajsamand, Baran,Bansawara, Dungarpur, Jhalawar	HanumangarhGanganagar,	
Inter State Migration					
Bikaner, Jhunjhunu, Sawaimadhopur, Jaipur,Jodhpur, Jaiselmer,Ajmer Tonk, Bhilwara, Udaipur, Kota,			Ganganagar, Hanumangarh, Churu, Alwar, Bharatpur, Dholpur, Karauli, Sikar, Dausa, Nagaur, Barmer, Jalore, Sirohi, Bundi, Bansawara, Dungarpur, Chittorgarh, Baran, Jhalawar		Rajsamand,

Many districts are having a relationship in similar direction as far as share of urban in-migrants is concerned from different distances and they follow the trend in same direction for urbanization level due to migrants. There are districts like Jaipur & Kota where percentage contribution of urban migrants is far ahead of state urbanization due to migrants for intra & inter-districts and inter-state migration during the periods 1971-80, 1981-90 and 1991-2000. Contrary to this, there are districts where percentage contribution of urban migrants is lower than state urbanization due to migrants in intra & inter districts and inter-state migration in three consecutive decadal periods. As, Inter-state migrants are having considerably high share in urbanization in Jodhpur district and it is way ahead of state urbanization figure due to inter-state migrants though it is falling below to state urbanization due to intra-district migrants and marginal up than state urbanization share due to inter-district migrants.

Percentage Contribution of inter-state and inter-district migrants in urbanization is higher for the state than to districts namely Hanumangarh, Churu, Alwar, Bharatpur and Dholpur whereas percentage Contribution of Intra district migrants in urbanization is higher for these districts than to state. Percentage Contribution of intra-district migrants in urbanization for Udaipur district is lower than state urbanization by intra-district migrants whereas for inter-district and inter-state migrants it is differing and contribution of inter-district and inter-state migrants in urbanization is higher for Udaipur than to state figure in three consecutive decades.

Therefore districts has shown a considerable variability in terms of migration contributing in urbanization when compared to state urbanization due to migration and this volatility is visible across different type of migrants whether inter-district, intra-district or inter-state migrants.

Intra-District Migration: Ten districts observed higher urbanization share of intra-district migration than state figures of 25.3% in 1991-200, 22.4% in 1981-90 and 16.4% in 1971-80. While two districts improved in migrant's urbanization than states urban migrants share in the period 1991-2000 though it was low than state share in year 1971-80 & 1981-90 and three districts excelled the state urbanization level of migrants in the year 1981-90 and maintained it during 1991-2000.

Fifteen districts witnessed a lower share of urbanization due to migrants as compared to state level urbanization due to migrants whereas no district is classified in category 5, where share of urbanization in migrants don't witnessed downward trend in the two successive decadal periods i.e Lower during 1991-2000 & 1981-90 but higher in 1971-80. However Ganganagar & Sikar looked urbanization share of migrants lower than state share during 1991-2000 though higher in 1981-90 & 1971-80. Around half of the districts falls in categories 1 to 3 which consists the district that has performed well than the state as far as urbanization due to migrants is concerned.

Kota is having 13.6 percentage point more intra-district urban migrants than the % share of inter-district migrants of the state settled in urban areas followed by Ajmer with 8.8 more percentage points. At state level, 10.1, 13.5 & 15.2% of intra-district migrants are contributing to urbanization during 1991-2000, 1981-90 & 1971-80 period whereas Kota is having 28.8,29.1 & 22% of inter-district migrants settled in urban areas followed by Ajmer 23.4,22.4 & 17.5%; Bharatpur 26.8,22.5 & 17% in this same duration.

On the other side, Barmer with 5.1,5.3 & 3% of intra-district migrants settling in urban areas in the 1991-2000, 1981-90 & 1971-80 period followed by Jalore, Dungarpur & Bansawara were the district viewed the lowest intra-district migrants contributing in urbanization.

Inter-District Migration: There are 20 districts having inter-district migrants share in urbanization lower than the state figures of urbanization by inter-district migrants which is 38.4% in 1991-2000, 37.2% in 1981-90 and 28.6% in 1971-80. Urbanization by inter-district migrants gave that Ganganagar & Hanumangarh districts were having percentage share of inter-district urban migrants lower than state level share of inter-district urban migrants in the 1991-2000 & 1981-90 period though it was higher for these districts in 1971-80 duration. Six districts had better urbanization from inter-district migration than state.

Bhilwara improved the urbanization share in the inter-district migrants than to state in two consecutive period 1981-90 & 1991-2000 though it was low than state urbanization in migrants in the period 1971-80 whereas Hanumangarh, Sawaimadhopur & Chittorgarh improved the urbanization share in the inter district migrants than to state in period 1991-2000 though it was lower than state urbanization by inter-district migrants in the period 1981-90 & 1971-80. 71.5, 70.4 & 56.8% of inter-districts migrants having urban residence in Jaipur district during the period 1991-2000, 1981-90 & 1971-80 respectively. This urbanization share in inter district migrants, after Jaipur, is followed by Ajmer district with 52.5, 47.9 & 39%; Udaipur with 49.5, 42.8 & 32.3%; Bhilwara with 44.9, 41.8 & 28% and Jodhpur with 42.5, 39.0 & 29.9% in these durations. Therefore the urbanization by inter-district migrants has improved for these districts.

Jalore, Barmer, Nagaur, Sikar, Dausa, Karauli, Churu, Jhunjhunu & Alwar districts were having urbanization by inter district migrants 15 to 25 percent point higher than state share.

Inter-State Migration: There are only two categories of districts those have witnessed better urbanization by inter-state migrants than state share of urbanization from inter-state migrants over three consecutive decades and districts for which the urbanization share by inter state migrants remained down than to state urbanization by inter-state migrants over three consecutive decades except Rajsamand where urbanization in 1991-2000 has been lowered than state figures though it was higher in the duration 1981-90 & 1971-80.

There were 19 districts that observed the share of inter-state migrants residing in urban areas low to urbanization by inter-state migrants at state level whereas 11 districts witnessed reverse trend and there urbanization by inter-state migrants have been lowered than state figures in three successive decadal periods.

The leading districts having better urbanization by inter-state migrants in 1991-2000, 1981-90 and 1971-80 period are Kota (71.3, 79.7 & 81.3%)

Bhilwara (65.2,72.2,61.5%); Ajmer (84.4,87.6, 93.2%), Jodhpur (85.2,85.7, 86.8%) and Jaipur (87.7,89.7 & 86.2%).

Clearly the inter-state migrant's contribution in urbanization is fairly large share than any other distance migration like intra & inter-district migration share in urbanization. However the relative share of urban migrants in recent decades, in general, has gone down for these highly urban immigrants district.

Classification of Districts by Range of Percentage Share of Urban Migrants:

Districts for all the categories of migration (Intra & Inter-district and Inter-states migration) are classified in following categories where % of migrants attributing to urbanization in the census period 1971-80,1981-90 and 1991-2000 is (1) <20%, (2) 20-50% and (3) >50%. Result is summarized below:

Range of urbanization by migrants (in%)	Intra District			Inter Districts			Inter States		
	91-00	81-90	71-80	91-00	81-90	71-80	91-00	81-90	71-80
<20	22	25	31	2	4	11	4	5	5
20-50	9	6	0	27	27	20	19	15	14
>50	1	1	1	3	1	1	9	13	13

It is evident from above classification that there is stark variation in the urbanization by migrants in various categories. As number of districts are having >50% of urban migrants in total migrants are considerably high for migrants from other states and combining it with districts having 20-50% migrants it is found that eighty percent of districts fall in this class. For between district migrants most of the districts fall in the category where 20-

50% migrants are attributing to migration whereas it is quite contrary to within district migration and in this migration the urbanization share is very low.

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MISCELLANEA

ADMINISTRATION, TEACHING AND RESEARCH PHILOSOPHIES

FLORENTIN SMARANDACHE

A simple, direct, fast point of view regarding my perception of Administration Philosophy, Teaching Philosophy, Research Philosophy (including My Own Research), and What I Can Bring to This Institution.

1. Administration Philosophy

- The Department Chair is an administrator (not a ruler) in order to serve the Faculty, students, the Dean and the Provost;
- Chair is an interface between Math Department Faculty and upper level administrators;
- Collective Leadership in the department, i.e. all important actions and decisions taken by departmental discussion and vote; we thus learn to accept decisions taken by the majority;
- Delegation of responsibility and authority to Faculty (decentralization within the department);
- Analyzing the recommendations and suggestions from Faculty and staff;
- Flexibility of Chair and Faculty;
- Fairness of the Chair and Faculty;
- Active listening of Chair and Faculty;
- Students first;
- Canals of communication with departmental Faculty and staff: through emails to all of them, plus printing the email and putting it in everybody's mail box (internal mail); telephones; appointments;

- Similar communication with the upper level: Dean of Arts & Letters College, Provost; according to Confucius Theory where the order and discipline is a way of life, the Chair follows the upper level administrators.
- Short department meetings as needed;
- Meeting agenda made before the meeting and sent to everybody about one week ahead; new agenda items can be added, or other deleted as per Faculty request;
- Evaluation of performance of Chair and Faculty;
- Availability of the Chair and Faculty;
- Socializing the whole department through: pot lucks, going together to restaurants, sport if possible, hiking, swimming;
- Considering *empathy* to solve conflict, i.e. everybody should respect the other one and his/her ideas – even if not agreeing with him/her (using fuzzy logic and neutrosophic logic, where something or somebody can be partially true and partially false in the same time – so we need to work together even if we are different);
- We are influenced by each other; that's why we need to be positive to each other (because otherwise negativity would propagate); we need to rely on each other;
- Everybody has different beliefs and attitudes, therefore we need to converge all of them to the Departmental and College goals;
- It is normal in a group of people to have conflicts and contradictions; we need to bend the contradictions; we need to learn to live with contradictions and try to diminish contradictions;
- We learn to live with challenges as well;
- Collaborative team work;
- I am popular; students, faculty, staff call me Florentin.

2. Teaching Philosophy

- Infusion of Technology in the class room: graphing calculators (Texas Instruments, Casius, etc. calculators) for undergraduate and graduate students; mathematical software such as "Mathematica," "Apple, " and other computer algebra systems;
- Teaching through undergraduate or graduate research; telling students to question themselves; encouraging students to ask questions in class (to have a dialogue, not a monologue in class);

- Offer Honor Classes;

- Distance Education; teaching online more classes and programs;

- Attract students by doing math through games, math for kits, math jokes, funny math, recreational problems, showing students the math used in our everyday math;

  An example of the importance of the space in mathematics I often tell my students in various classes:

  a) On a power line there are 10 birds. A hunter kills 3 of them. How many are left?

  b) On a plain in the grass there are 10 birds. A hunter kills 3 of them. How many are left?

  c) In a cage there are 10 birds. A hunter kills 3 of them. How many are left?

  d) In the sky are flying 10 birds. A hunter kills 3 of them. How many are left?

  My students laugh when trying to guess the answer. And next times they are again asked me: can you tell us more funny problems?

- Or tell them about the Beauty of Math!

$$1 \times 8 + 1 = 9$$
$$12 \times 8 + 2 = 98$$
$$123 \times 8 + 3 = 987$$
$$1234 \times 8 + 4 = 9876$$
$$12345 \times 8 + 5 = 987\ 65$$
$$123456 \times 8 + 6 = 987654$$
$$1234567 \times 8 + 7 = 9876543$$
$$12345678 \times 8 + 8 = 98765432$$
$$123456789 \times 8 + 9 = 987654321$$

- Develop and adjust the Curriculum for the needs of the students;

- Foster students' learning;

- Being creative in teaching; continuously updating and improving the style of teaching in order to avoid monotony;

- Adjusting the teaching methods depending to the type of students: there are visual learners, and audio learners;

- Examine students learning style in order to adjusting the teaching style for their way of understanding;

- Interacting with students;

- Stimulate students by giving them extra-points towards the final grade for extra-homework and for class participation (I have students solving problems on the board during the class time and explaining them to the other students);
- Active learning, not passive learning; logical learning, not mechanical learning;
- Learning in groups;
- Learning by connecting the new knowledge with old knowledge;
- Making connections between math knowledge and other domains' knowledge;
- Exchange teaching ideas with other faculty from this institution or from others;
- Applicability of Math: make students understand that math is important in our real life;
- Bringing students off from monotony and passivity by telling them funny math stories, math curiosities, anecdotes about mathematicians, also about mathematicians' lives, etc.
- Evaluate students' critical thinking, problem-solving, technical writing, content knowledge;
- Discover students' psychology of learning;
- Challenge students' intellectuality;
- Short History of Math told to students when teaching a special topic, so the students see the evaluation of the topic, why it was needed, how it arose;

3. Research Philosophy

- Research that benefits the students and the society;
- Educate students through research;
- Be a model for the students;
- Use deductive and inductive methods of research;
- Undergraduate or graduate research projects assigned to the students;
- Attracting students to do research by involving them in our own research;
- How to generalize a problem? How to generalize a theorem? What about if the given hypotheses of a theorem are changed? Check many examples. Check corner cases. Trial and error in research

- Explore in depth the topic; do a survey of the literature
- Ask for help if not able to solve a problem, and thus co-author the research;
- Break down a bigger problem into smaller problems, and then solve each of them;
- Make connections with other subjects;
- Aboard the problem from various angles, various methods;
- A small idea sparkle can lead to a great outcome;
- Solve real problems;
- Keep a professional integrity;
- Interdisciplinary research;
- How to mathematically model a real problem?
- Research in teaching: how to better methods and strategies of teaching? How to motivate the students to learning?
- Research in pure and applied math;
- Research in order to solve existing unsolved problems, open questions, conjectures;
- Thinking differently! Sometimes a stupid apparently question can lead to a genial idea! {For example, why differentiating 2 or 3 times and not… 2.7 times? And similarly for integration. This lead to the fractional differentiation and fractional integration.}
- Question the classical theories to see if it's room for alternative or generalizations (look for example at the evolution from Euclidean Geometry to its opposite Non-Euclidean Geometry);
- What research methods to use?
- Disseminate the research results; how are they useful to the society? Theoretical research can lead to applications;
- Look for Research Grants and Fellowships for students and Faculty;
- Create a Digital Library of Math e-Books and e-Articles as support for the research;
- I partially paid for my Conferences trips; I did most of my research in my spare time (especially in weekend, or after classes);
- Research for me is a hobby.

My Own Research

- Applied Mathematics in Information Fusion (used in robotics, airspace, military, medicine);
- Granular Computing (Neutrosophic Logic and Set and their applications);
- Algebraic Structures;
- Applied Mathematics in Quantum Physics, Statistics, Economics;
- Non-Euclidean Geometry;
- Number Theory (Arithmetic Functions, Sequences, Diophantine Equations and Systems, Prime Numbers).

4. What I can bring to this institution:

- "Progress in Physics" international journal of physics and mathematics will becomes Texas A & M University-Kingsville's international journal (the correspondence address would be that of this institution); I am an associate editor of this journal since the journal was founded in 2005, and I get all work in my spare time – without asking for release time or for a penny from my university;
- Publish periodically a collective volume of research math papers of our math Faculty; then put the book in international scientific databases, such as EBSCO, CENGAGE, ProQUEST, Amazon Kindle, Amazon.com, Google Book Search, Google Scholar
- Endorse Faculty who did not yet submit papers to arXiv.org (online scientific database at Cornell University, NY);
- A Digital Library with over 300 titles of e-books and e-journal issues and over 100 scientific papers for the benefit of students, researchers and professors from around the world [for example this site of mine has presently about 7,000 hits/day from people from about 100 countries];
- Donation of books and journals periodically to the TAMUK James C. Jernigan library; (by the way I have a special collection at The University of Texas at Austin, Archives of American History);
- Attracting more students from around the world to do their graduate study in pure or applied mathematics at this university due to this Digital Library with free e-books and e-

articles; I am in touch with many people from around the world and they asked me if I can be an advisor for their future or if I know someone else to recommend to them;

- 62% of the students at TAMUK are Hispanics; I speak and understand a little Spanish (which is a romance language close to Romanian and French that I am fluent in);

- I also have a degree in Computer Science (M. Sc.), therefore I can interact with the Computer Science Department for interdisciplinary research (for example in Granular Computing);

- Search for more Grants and Fellowships for students and Faculty;

- Organizing the AMATYC [American Mathematical Association for Two Years Colleges] Competition for undergraduate math students (if it is not already in place herein; checking your website I did not find it);

- Cooperating with Dr. Reza R. Ahangar, the advisor for his the Math Club, and with other interested Faculty in order to make a similar Funny & Recreational Math Problems Club (to show the students the beauty of math!), Math jokes (to get out of the teaching monotony); this would also attract students to math;

- Setting up, if needed, of a Reconciliation Committee, within the department in order to discuss with the conflicting parties and try to reconciling them;

- Introduce Math Labs associated with many math courses [of course if approved by the Curriculum Committee] in order to assist students in doing their homework (that's, for example, what UNM does for undergraduate classes: Intermediate Algebra, College Algebra, Pre-Calculus, Trigonometry, Calculus for Business, etc.) of 1 credit hour in order to increase retention;

- Add new graduate classes to the current core of classes that I can teach, such as: Number Theory, Abstract Algebra, Neutrosophic Logic/Set (Generalization of the Fuzzy Set/Logic), Foundations of Non-Euclidean Geometry, Mathematics Applied in Information Fusion, Granular Computing; a bigger diversity of math courses and programs attracts more students;

- Try to develop a Ph D Program in Math, or in Bilingual Mathematical Education (derivative of Ph D Bilingual Education Program already existent in the College of Graduate Studies) – of course if approved by the Curriculum Committee and the upper level administrators.

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AN APPLICATION OF THE SYSTEMIC THEORY IN THE FIELD OF INDUSTRIAL COMPANIES

FLORENTIN SMARANDACHE, ŞTEFAN VLĂDUŢESCU

The enhancement of current globalisation represents the fundamental feature of world economy at the beginning of the 21st century and is characterised by emphasising the trend to reduce and remove the barriers between the national economies and enhancing the connections between these economies. The globalisation we face nowadays derives from the fact that, by starting from the technological and economical development, a significant number of human activities is situated on such a large scale and scope that they exceeded the national borders within the limits of which the sovereign states exercise their right to govern. The new actors had to cope with the challenge caused by the monopoly-type governance. Multinational corporations, global financial markets, non-governmental organisations as well as criminal organisations and international terrorist networks appeared. Their activity is not covered by international laws which are based on formal agreements between the nation-states, for they have not been able so far to find a common ground for agreements aiming the issue of globalisation.

The international production, including the production of trans-national companies, branches and other companies linked to the multinational companies, by agreements and alliances, without capital participation has known a strong development. The technologic progress allows the decomposition and desegregation of production processes. Companies choose the place that meets the most favourable production factors for each of the stages of the production process.

In the globalisation era, the production environment of all countries comes to the stage of realizing the real prosperity. With the growth of markets towards globalisation, all the firms need to deal with the challenges facing it. This has resulted in the materialization of automated industries with high performance of manufacturing systems.

In this context the development of systems theory and its application in strategic management holding company becomes a necessity. The increased development of the theory of systems provides its possibility to be used in the applicative scientific research in very many fields.

Thus, the researches undertaken in the field of strategic management and strategies of companies by professors from Romania – "Valahia" University of Târgovişte, University of Petroşani, "Lucian Blaga" University of Sibiu, from Poland – Technology University of Częstochowska and from Slovakia – Technical University of Kosice, by using the systemic theory have led to some conclusions with great generalisation power in the field concerning the industrial companies that perform their activity under the conditions of current globalisation. The result of the researches was the elaboration of this book structured on the following major problems: considerations regarding the theory of systems, system management risk, organisation as system, the company on the terms of market economy, formal and informal structure of the organisation, theoretical approaches of the current strategic management, theoretical approaches of company strategies, production strategies in the mining machine and machinery manufacturing industry, strategies regarding the production quality in the company and others.

The team of authors structured scientific research that undertaken in two parts, namely:

- Part I - *The Company Dealt with Systemically*, that in the six chapters approaches these problems: information in systems theory; the risk in systems management; use of systems theory to deal with industrial companies; companies' operation environment in a global economy; companies' activity on the current market economy; companies' organisation under current globalisation;

- Part II - *Use of Systemic Theory in Strategic Management*, that in the eight chapters approaches these problems: companies' strategies – a theoretic approach; considerations on the current state of strategic management; communication in companies' development process; production strategies of companies in machine manufacturing industry; strategies used to improve industrial companies' production quality; budgeting - technique of strategic management; use of budgets to elaborate the strategy of industrial production costs; interdependence relation between industrial company's logistics and commercial strategy.

The problems treated in the two parts of the book can be grouped in three main directions synthetic:

- the presentation of the current stage of the systemic theory in the first two chapters, including the risk in the management of the systems, according to the opinions in the literature;

- the implementation of the systemic theory at the study of companies in the current globalised market economy, particularly highlighting the harmonisation of the company's structure with its development strategies. This approach is the content of the following three chapters of the book;

- dealing with the company's strategies and strategic management through the systemic theory in the following four chapters of the book. A logical dealing with the problem approached by starting from the current stage of the theory in the field of strategic management and company's strategies in the market economy was continued by dealing with the production strategies of the companies in the automotive industry, for in the end the special role would be highlighted, that the strategies have it concerning the production and quality of the production process in industrial companies that perform their activities under the conditions of current globalisation.

Such an approach widely used the econometric models in elaborating the strategies of industrial companies, particularly to optimise the managerial decisions concerning the adoption of a certain strategy. The theory of artificial intelligence was also used, meaning that of expert systems in the elaboration of production strategies.

In the same context, the quality of the activity performed by the production companies was dealt with, using the economical-mathematical models provided by the Japanese management methods, such as the Taguchi method.

Such an interdisciplinary and even international approach of the issue in the book has resulted in conclusions that address the students, MA students, entrepreneurs and specialists in the field of production companies.

Taking this into account, this book is providing further understanding the subject with more fruitful ideas to academic researchers and managers of organizations in the pipeline.

Foreward to the book Systemic Approaches to Strategic Management: Examples from the Automotive Industry, by Ioan Constantin Dima, IGI Global, September 2014.

ON GÖDEL'S INCOMPLETENESS THEOREM(S), ARTIFICIAL INTELLIGENCE/LIFE, AND HUMAN MIND

V. CHRISTIANTO and FLORENTIN SMARANDACHE

Abstract

In the present paper we have discussed concerning Gödel's incompleteness theorem(s) and plausible implications to artificial intelligence/life and human mind. Perhaps we should agree with Sullins III, that the value of this finding is not to discourage certain types of research in AL, but rather to help move us in a direction where we can more clearly define the results of that research. Gödel's incompleteness theorems have their own limitations, but so do Artificial Life (AL)/AI systems. Based on our experiences so far, human mind has incredible abilities to interact with other part of human body including heart, which makes it so difficult to simulate in AI/AL. However, it remains an open question to predict whether the future of AI including robotics science can bring this gap closer or not. In this regard, fuzzy logic and its generalization –neutrosophic logic- offer a way to improve significantly AI/AL research.[15]

Introduction

In 1931 a German mathematician named Gödel published a paper which included a theorem which was to become known as his Incompleteness Theorem. This theorem stated that:

> "To every w-consistent recursive class k of formulae there
>
> correspond recursive class-signs r, such that neither v Gen r
>
> nor Neg (v Gen r) belongs to Flg(k) (where v is the free variable of r)" [9].

In more common mathematical terms, this means that "all consistent axiomatic formulations of number theory include undecidable propositions."[9]

Another perspective on Gödel's incompleteness theorem can be found using polynomial equations [10]. It can be shown that Gödel's analysis does not reveal any essential incompleteness in formal reasoning systems, nor any barrier to proving the consistency of such systems by ordinary mathematical means.[10] In the mean time, Beklemishev discusses the limits of applicability of Gödel's incompleteness theorems.[11]

Does Gödel's incompleteness theorem limit Artificial Intelligence?

In the 1950s and 1960s, researchers predicted that when human knowledge could be expressed using logic with mathematical notation, it would be possible to create a machine that reasons, or artificial intelligence. This turned out to be more difficult than expected because of the complexity of human reasoning.[12]

Nowadays, it is widely accepted that general purpose of artificial intelligence (AI) is to develop (1) conceptual models (2) formal rewriting processes of these models and (3) programming strategies and physical machines to reproduce as efficiently and thoroughly as possible the most authentic, cognitive, scientific and technical tasks of biological systems that we have labeled Intelligent [5, p.66].

According to Gelgi, Penrose claims that results of Gödel's theorem established that human understanding and insight cannot be reduced to any set of computational rules [1]. Gelgi goes on to say that:

> "Gödel's theorem states that in any sufficiently complex formal
>
> system there exists at least one statement that cannot be proven to
>
> be true or false. Penrose believes that this would limit
>
> the ability of any AI system in its reasoning. He argues that there
>
> will always be a statement that can be constructed which is unprovable
>
> by the AI system."[1]

The above question is very interesting to ponder, considering recent achievements in modern AI research. There are ongoing debates on this subject in many online forums, see for instance [5][6][7][8][9]. Here we give a summary of those articles and papers in simple words. Hopefully this effort will shed some light on this debatable subject. Those arguments basically stand either on the optimistic side (that Gödel's theorems do not limit AI), or on the pessimistic side (that Gödel's theorems limit AI).

Mechanism and reductionism in biology and implications to AI/AL

It is known that mechanistic or closely related reductionistic theories have been part of theoretical biology in one form or another at least since Descartes.[8] The various mechanistic and reductionistic theories are historically opposed to the much older and mostly debunked theories of vitalism (see Emmeche, 1991). These theories (the former more than the latter), along with formism, contextualism, organicism, and a number of other "isms" mark the major centers of thought in the modern theoretical biology debate (see Sattler, 1986).[8]

Such mechanistic and reductionistic view of the world were discussed by F. Capra in his book: The Turning Point [13].

According to Sullins III [8], AL (Artificial Life) falls curiously on many sides of these debates in the philosophy of biology. For instance AL uses the tools of complete mechanization, namely the computer, while at the same time it acknowledges the existence of emergent phenomena (Langton, 1987, p. 81). Neither mechanism nor reductionism is usually thought to be persuaded by arguments appealing to emergence. Facts like this should make our discussion interesting. It may turn out that AL is hopelessly contradictory on this point, or it may provide an escape route for AL if we find that Gödel's incompleteness theorems do pose a theoretical road block to the mechanistic-reductionistic theories in biology.

Sullins III also writes that most theorists have outgrown the idea that life can be explained wholly in terms of classical mechanics.[8] Instead, what is usually meant is the following (paraphrased from Sattler, 1986):

1) Living systems can and/or should be viewed as physico- chemical systems.

2) Living systems can and/or should be viewed as machines. (This kind of mechanism is

also known as the machine theory of life.)

3) Living systems can be formally described. There are natural laws which fully describe

living systems.

According to Sullins III[8], reductionism is related to mechanism in biology in that mechanists wish to reduce living systems to a mechanical description. Reductionism is also the name of a more general world view or scientific strategy. In this world view we explain phenomena around us by reducing them to their most basic and simple parts. Once we have an understanding of the components, it is then thought that we have an understanding of the whole. There are many types of reductionist strategies.[8]

According to Sullins III [8], reductionism is a tool or strategy for solving complex problems. There does not seem to be any reason that one has to be a mechanist to use these tools. For instance one could imagine a causal reductionistic vitalist who would believe that life is reducible to the elan vital or some other vital essence. And, conversely, one could imagine a mechanist who might believe that living systems can be described metaphorically as machines but that life was not reducible to being only a property of mechanics.

Sullins III [8] also asserts that the strong variety of AL does not believe that living systems should only be viewed as physico-chemical systems. AL is life-as-it-could-be, not life-as-we-know-it (Langton, 1989, p. 1), and this statement suggests that AL is not overly concerned with modeling only physico-chemical systems. Postulates 2 and 3 seem to hold, though, as strong AL theories clearly state that the machine, or formal, theory of life is valid and that simple laws underlie the complex, nonlinear behavior of living systems (Langton, 1989, p. 2).

Sullins III [8] goes on with his argument, saying that at least one of the basic qualities of our reality will always be missing from any conceivable artificial reality, namely, a complete formal system of mathematics. This argument tends to make more sense when applied to strong AI claims about intelligent systems understanding concepts (see Tieszen, 1994, for a more complete argument as it concerns AI). He also concludes that it is impossible to completely formalize an artificial reality that is equal to the one we experience, so AL systems entirely resident in a computer must remain, for anyone persuaded by the mathematical realism posited by Gödel, a science which can only be capable of potentially creating extremely robust simulations of living systems but never one that can become a complete instantiation of a living system.[8]

However, Sullins III [8] also writes that the value of this finding is not to discourage certain types of research in AL, but rather to help move us in a direction where we can more clearly define the results of that research. In fact, since one of the above arguments rests on the assumption that the universe is infinite and that some form of mathematical realism is true, if we are someday able to complete the goal advanced in strong AL it would seem to cast doubt on the validity of the assumptions made above.

For a recent debate on this issue in the context of fuzzy logic, see for instance Yalciner et al. [5]. The debates on the possibility of thinking machines, or the limitations of AI research, have never stopped. According to Yalciner et al. (2010), these debates on AI have been focused on three claims:

- An AI system is in principle an axiomatic system.

- The problem solving process of an AI system is equivalent to a Turing machine.

- An AI system is formal, and only gets meaning according to model theoretic semantic (Wang 2006).[16]

More than other new sciences, AI and philosophy have things to say to one to another: any attempt to create and understand minds must be of philosophical interest.[5]

May be we will never manage to build real artificial intelligence. The problem could be too difficult for human brain over to solve (Bostrom, 2003).

Yalciner et al. [5] also write that a fundamental problem in artificial intelligence is that nobody really knows what intelligence is. The problem is especially acute when we need to consider artificial systems which are significantly different to humans.

Human mind is beyond machine capabilities

According to Gelgi [1], it follows that no machine can be a complete or adequate model of the mind, that minds are essentially different from machines. This does not mean that a machine cannot simulate any piece of mind; it only says that there is no machine that can simulate every piece of mind. Lucas says that there may be deeper objections. Gödel's theorem applies to deductive systems, and human beings are not confined to making only deductive inferences. Gödel's theorem applies only to consistent systems, and one may have doubts about how far it is permissible to assume that human beings are consistent. [1]

Therefore it appears that there are some characteristics of human mind which go beyond machine capabilities. For example there are human capabilities as follows:

a. to synchronize with heart, i.e. to love and to comprehend love;

b. to fear God and to acknowledge God: "The fear of the LORD is the beginning of knowledge" (Proverbs 1:7)

c. to admit own mistakes and sins

d. to repent and to do repentance

e. to consider things from ethical perspectives.

All of the above capabilities are beyond the scope of present day AI machines, i.e. it seems that there is far distance between human mind capabilities and machine capabilities. However, we can predict that there will be much progress by AI research. For instance, by improving AI-based chess programs, one could see how far the machine can go.

Furthermore there are other philosophical arguments concerning the distinction between human mind and machine intelligence. Dreyfus contends that it is impossible to create intelligent computer programs analogous to the human brain because the workings of human intelligence are entirely different from that of computing machines. For Dreyfus, the human mind functions intuitively and not formally. Dreyfus's critique on AI proceeds from his critique on rationalist epistemological assumptions about human intelligence. Dreyfus's major attack targets the rationalist conception that human understanding or intelligence can be "formalized".[5, p.67]

We agree with the content related to the distinctions between Human and Computer. Yet, we think that the differences (Love, God, Own mistakes, Repentance, Ethical) between Human and Computer will be in the future little by little diminished, since it would be possible to train a computer at least for partial adjustments in each of them.

In addition to the fuzzy logic in AI, neutrosophic logic provides besides truth and falsehood a third component, called indeterminacy that can be used in AI, since many approaches of reality that AI has to model or describe involve a degree of uncertainty, unknown. Neutrosophic logic is a generalization of intuitionistic fuzzy logic.[15] We have a lot of unknown and paradoxist, contradictory information that AI has to deal with in our world.

The above argument can be seen as stronger than Penrose's.

However, one should admit the differences between human intelligence and machine intelligence. There are fundamental differences between the human intelligence and today's machine intelligence. Human intelligence is very good in identifying patterns and subjective matters. However, it is usually not very good in handling large amounts of data and doing massive computations. Nor can it process and solve complex problems with large number of constraints. This is especially true when real time processing of data and information is required. For these types of issues, machine intelligence is an excellent substitute.[5]

Concluding remarks

In the present paper we have discussed concerning Gödel's incompleteness theorem(s) and plausible implications to artificial intelligence/life and human mind.

Perhaps we should agree with Sullins III, that the value of this finding is not to discourage certain types of research in AL, but rather to help move us in a direction where we can more clearly define the results of that research. Gödel's incompleteness theorems have their own limitations, but so do Artificial Life (AL)/AI systems. Based on our experiences so far, human mind has incredible abilities to interact with other part of human body including heart, which makes it so difficult to simulate in AI/AL. However, it remains an open question to predict whether the future of AI including robotics science can bring this gap closer or not. In this regard, fuzzy logic and its generalization –neutrosophic logic- offer a way to improve significantly AI/AL research. [15]

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A UNIT BASED CRASHING PERT NETWORK FOR OPTIMIZATION OF SOFTWARE PROJECT COST

PRITI SINGH, FLORENTIN SMARANDACHE, DIPTI CHAUHAN, AMIT BHAGHEL

Abstract:
Crashing is a process of expediting project schedule by compressing the total project duration. It is helpful when managers want to avoid incoming bad weather season. However, the downside is that more resources are needed to speed-up a part of a project, even if resources may be withdrawn from one facet of the project and used to speed-up the section that is lagging behind. Moreover, that may also depend on what slack is available in a non-critical activity, thus resources can be reassigned to critical project activity. Hence, utmost care should be taken to make sure that appropriate activities are being crashed and that diverted resources are not causing needless risk and project scope integrity. In this paper we want to present a technique called "Unit Crashing" to reduce the total cost of project. Unit Crashing means to crash the project duration by one unit (day) instead of crashing it completely. This technique uses an iterative approach to perform unit crashing until all activities along the critical path are crashed by desired amount. The output of this method will reduce the cost of project, and is useful at places where cost is of major consideration. Crashing PERT networks can save a significant amount of money in crashing and overrun costs of a company. Even if there are no direct costs in the form of penalties for late completion of projects, there is likely to be intangible costs because of reputation damage.

Keywords: Crashing, Uncrashing, PERT, Cost Slope

Introduction:

Complex projects require a series of activities, some of which must be performed sequentially and others that can be performed in parallel with other activities. This collection of series and parallel tasks can be modeled as a network.

In 1957 the Critical Path Method (CPM) was developed as a network model for project management. CPM is a deterministic method that uses a fixed time estimate for each activity. While CPM is easy to understand and use, it does not consider the time variations that can have a great impact on the completion time of a complex project.The *Program Evaluation and Review Technique* (PERT) is a network model that allows for randomness in activity completion times. PERT was developed in the late 1950's for the U.S. Navy's Polaris project having thousands of contractors. It has the potential to reduce both the time and cost required to complete a project.

Steps in the PERT Planning Process

PERT planning involves the following steps:

1. Identify the specific activities and milestones.
2. Determine the proper sequence of the activities.
3. Construct a network diagram.
4. Estimate the time required for each activity.
5. Determine the *critical path*.
6. Update the PERT chart as the project progresses.

1. Identify Activities and Milestones

The activities are the tasks required to complete the project. The milestones are the events marking the beginning and end of one or more activities. It is helpful to list the tasks in a table that in later steps can be expanded to include information on sequence and duration.

2. Determine Activity Sequence

This step may be combined with the activity identification step since the activity sequence is evident for some tasks. Other tasks may require more analysis to determine the exact order in which they must be performed.

3. Construct the Network Diagram

Using the activity sequence information, a network diagram can be drawn showing the sequence of the serial and parallel activities. For the original activity-on-arc model, the activities are depicted by arrowed lines and milestones are depicted by circles or "bubbles".If done manually, several drafts may be required to correctly portray the relationships among activities. Software packages simplify this step by automatically converting tabular activity information into a network diagram.

4. Estimate Activity Times

Weeks are a commonly used unit of time for activity completion, but any consistent unit of time can be used.A distinguishing feature of PERT is its ability to deal with uncertainty in activity completion times. For each activity, the model usually includes three time estimates:

- *Optimistic time* - generally the shortest time in which the activity can be completed. It is common practice to specify optimistic times to be three standard deviations from the mean so that there is approximately a 1% chance that the activity will be completed within the optimistic time.
- *Most likely time* - the completion time having the highest probability. Note that this time is different from the *expected time*.
- *Pessimistic time* - the longest time that an activity might require. Three standard deviations from the mean is commonly used for the pessimistic time.

PERT assumes a beta probability distribution for the time estimates. For a beta distribution, the expected time for each activity can be approximated using the following weighted average:

Expected time = (Optimistic + 4 x Most likely + Pessimistic) / 6

This expected time may be displayed on the network diagram.

To calculate the variance for each activity completion time, if three standard deviation times were selected for the optimistic and pessimistic times, then there are six standard deviations between them, so the variance is given by:

Variance = [(Pessimistic - Optimistic) / 6 $]^2$

5. Determine the Critical Path

The critical path is determined by adding the times for the activities in each sequence and determining the longest path in the project. The critical path determines the total calendar time required for the project. If activities outside the critical path speed up or slow down (within limits), the total project time does not change. The amount of time that a non-critical path activity can be delayed without delaying the project is referred to as *slack time*.

If the critical path is not immediately obvious, it may be helpful to determine the following four quantities for each activity:

- ES - Earliest Start time
- EF - Earliest Finish time
- LS - Latest Start time
- LF - Latest Finish time

These times are calculated using the expected time for the relevant activities. The earliest start and finish times of each activity are determined by working forward through the network and determining the earliest time at which an activity can start and finish considering its predecessor activities. The latest start and finish times are the latest times that an activity can start and finish without delaying the project. LS and LF are found by working backward through the network. The difference in the latest and earliest finish of each activity is that activity's slack. The critical path then is the path through the network in which none of the activities have slack.

The variance in the project completion time can be calculated by summing the variances in the completion times of the activities in the critical path. Given this variance, one can calculate the probability that the project will be completed by a certain date assuming a normal probability distribution for the critical path. The normal distribution assumption holds if the number of activities in the path is large enough for the central limit theorem to be applied.

Since the critical path determines the completion date of the project, the project can be accelerated by adding the resources required to decrease the time for the activities in the critical path. Such a shortening of the project sometimes is referred to as *project crashing*.

6. Update as Project Progresses

Make adjustments in the PERT chart as the project progresses. As the project unfolds, the estimated times can be replaced with actual times. In cases where there are delays, additional resources may be needed to stay on schedule and the PERT chart may be modified to reflect the new situation.

Benefits of PERT

PERT is useful because it provides the following information:

Expected project completion time.

- Probability of completion before a specified date.
- The critical path activities that directly impact the completion time.

- The activities that have slack time and that can lend resources to critical path activities.
- Activities start and end dates.

Crashing:

Crashing refers to a particular variety of project schedule compression which is performed for the purposes of decreasing total period of time (also known as the total project schedule duration). The diminishing of the project duration typically take place after a careful and thorough analysis of all possible project duration minimization alternatives in which any and all methods to attain the maximum schedule duration for the least additional cost The objective of crashing a network is to determine the optimum project schedule. Crashing may also be required to expedite the execution of a project, irrespective of the increase in cost. Each phase of the software design consumes some resources and hence has cost associated with it. In most of the cases cost will vary to some extent with the amount of time consumed by the design of each phase .The total cost of project, which is aggregate of the activities costs will also depends upon the project duration, can be cut down to some extent. The aim is always to strike a balance between the cost and time and to obtain an optimum software project schedule. An optimum minimum cost project schedule implies lowest possible cost and the associated time for the software project management

Activity time-cost relationship: A simple representation of the possible relationship between the duration of an activity and its direct costs appears in Fig. 1. Shortening the duration on an activity will normally increase its direct cost.
A duration which implies minimum direct cost is called the normal duration and the minimum possible time to complete an activity is called crash duration, but at a maximum cost. The linear relationship shown above between these two points implies that any intermediate duration could also be chosen

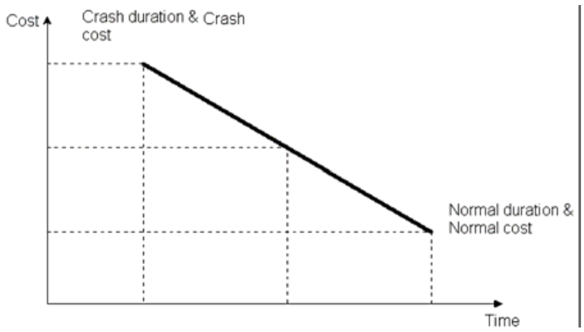

Fig. 1: Linear time and cost trade-off for an activity

It is possible that some intermediate point may represent the ideal or optimal trade-off between time and cost for this activity. The slope of the line connecting the normal point (lower point) and the crash point (upper point) is called the cost slope of the activity. The slope of this line can be calculated mathematically by knowing the coordinates of the normal and crash points:
Cost slope = (crash cost-normal cost)/ (normal duration crash duration)
As the activity duration is reduced, there is an increase in direct cost. A simple case arises in the use of overtime work and premium wages to be paid for such overtime. Also overtime work is more prone to accidents and quality problems that must be corrected,
so indirect costs may also increase. So, do not expect a linear relationship between duration and direct cost but convex function as shown in Fig. 2.

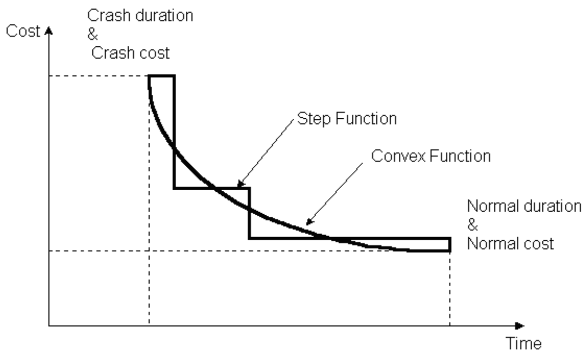

Fig. 2: Non-linear time and cost trade-off for an activity

Project time-cost relationship:

Total project costs include both direct costs and indirect costs of performing the activities of the project. If each activity of the project is scheduled for the duration that results in the minimum direct cost (normal duration) then the time to complete the entire project might be too long and substantial penalties associated with the late project completion might be incurred. At the other extreme, a manager might choose to complete the activity in the minimum possible time, called crash duration, but at a maximum cost. Thus, planners perform what is called time-cost trade-off analysis to shorten the project duration. This can be done by selecting some activities on the critical path to shorten their duration. As the direct cost for the project equals the sum of the direct costs of its activities, then the project direct cost will increase by decreasing its duration. On the other hand, the indirect cost will decrease by decreasing the project duration, as the indirect cost are almost a linear function with the project duration.

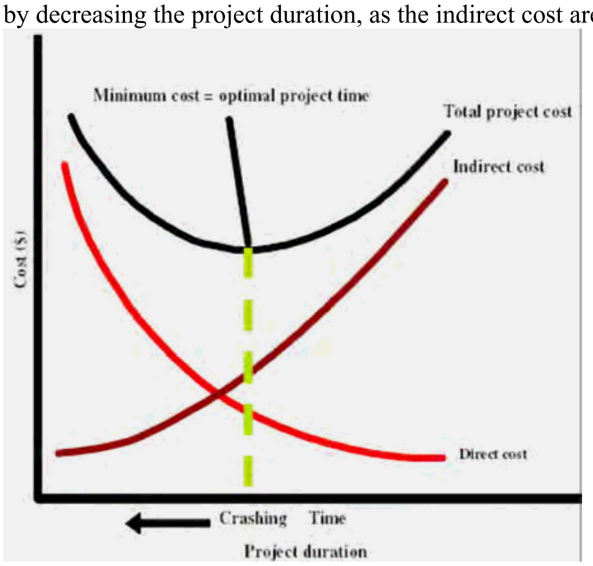

Fig. 3: Project time-cost relationship

Figure 3 shows the direct and indirect cost relationships with the project duration. The project total time-cost relationship can be determined by adding up the direct cost and indirect cost values together. The optimum project duration can be determined as the project duration that results in the least project total cost.

Literature review:

Steve and Dessouky[3] described a procedure for solving the project time/cost tradeoff problem of reducing project duration at a minimum cost. The solution to the time & cost problem is achieved by locating a minimal cut in a flow network derived from the original project network. This minimal cut is then utilized to identify the project activities which should experience a duration modification in order to achieve the total project reduction.

Rehab and Carr [4] described the typical approach that construction planners take in performing time-Cost Trade-off (TCT). Planning focuses first on the dominant characteristics and is then fine-tuned in its details. Planners typically cycle between plan generation and cost estimating at ever finer levels of detail until they settle on a plan that has an acceptable cost and duration. Computerized TCT methods do not follow this cycle. Instead, they separate the plan into activities, each of which is assumed to have a single time-cost curve in which all points are compatible and independent of all points in other activities' curves and that contains all direct cost differences among its methods.

Pulat and Horn[5] described a project network with a set of tasks to be completed according to some precedence relationship, the objective is to determine efficient project schedules for a range of project realization times and resource cost per time unit for each resource. The time-cost tradeoff technique is extended to solve the time-resource tradeoff problem. The methodology assumes that the project manager's (the decision maker) utility function over the resource consumption costs is linear with unknown weights for each resource. Enumerative and interactive algorithms utilizing Geoffrion's P (λ) approach are presented as solution techniques. It is demonstrated that both versions have desirable computational times. Walter *et al.*[6] described the application of advanced methods of process management, especially in those fields in which activity durations can be determined only vaguely, while at the same time a highly competitive market enforces strict completion schedules through the implementation of penalties. The technique presented is a new PERT-based, hybridized approach using simulated annealing and importance sampling to support typical process re-engineering, which focuses on the efficient allocation of extra resources in order to achieve a more reliable performance without changing the precedence successor-structure. The technique is most suitable for determining a time-cost trade-off based on practice relevant assumptions.

Marold[7] used a computer simulation model to determine the order in which activities should be crashed as well as the optimal crashing strategy for a PERT network to minimize the expected value of the total (crash + overrun) cost, given a specified penalty

function for late completion of the project. Three extreme network types are examined, each with two different penalty functions. Van Slyke[8] demonstrated several advantages of applying simulation techniques to PERT, including more accurate estimates of the true project length, flexibility in selecting any distribution for activity times and the ability to calculate "criticality indexes", which are the probability of various activities being on the critical path. Van Slyke was the first to apply Monte Carlo simulations to PERT. Ameen[9] developed Computer Assisted PERT Simulation (CAPERTSlM), an instructional tool to teach project management techniques. Coskun[10] formulated the problem as a Chance Constrained Linear Programming (CCLP) problem.

CCLP is a method of attempting to convert a probabilistic mathematical programming formulation into an equivalent deterministic formulation. Coskun's formulation ignored the assumed beta distribution of activity times. Instead, activity times were assumed to be normally distributed, with the mean and standard deviation of each known. This formulation allows a desired probability of completion within a target date to be entered.

Ramini[11] proposed an algorithm for crashing PERT networks with the use of criticality indices. Apparently he did not implement the algorithm, as no results were ever reported. His method does not allow for bottlenecks. Bottlenecks traditionally have multiple feeds into a very narrow path that is critical to the project's completion. Bottlenecks are the favored locations for project managers to build time buffers into their estimates, yet late projects still abound because of deviation from timetables and budgets. Johnson and Schon[12] used simulation to compare three rules for crashing stochastic networks. He also made use of criticality indices. Badiru[13] reported development of another simulation program for project management called STARC. STARC allows the user to calculate the probability of completing the project by a specified deadline. It also allows the user to enter a "duration risk coverage factor". This is a percentage over which the time ranges of activities are extended. This allows some probability of generating activity times above the pessimistic time and below the optimistic time.

Feng *et al.*[14] presented a hybrid approach that combines simulation techniques with a genetic algorithm to solve the time-cost trade-off problem under uncertainty. Grygo[15] pointed out that the habit of project managers building time buffers into non-critical paths that feed into critical ones in a project network has resulted in almost late completion of projects. The corporations are dealing firmly with time overruns that cripple their budgets, damage their reputations and tax their cash flows with paid-out penalties. It is estimated that 50 percent of the software projects that are successfully completed, are not as successful as they should be.

Jorgensen[16] emphasized that the simulation approach can be used for management of any project but he time estimates for project management of information systems are still less accurate than any other estimates in the project management cycle.

Materials and Method:

Step1: Calculate Earliest time Estimates for all the activities. It is calculated as

T_E = Maximum of all ($T_E^{\,j} + t_E^{ij}$) for all i , j leading into the event.

where $T_E^{\,j}$ is the earliest expected time of the successor event j.

$T_E^{\,i}$ is the earliest expected time of the predecessor event i. and

t_E^{ij} is the expected time of activity ij.

Step2: Calculate Latest time Estimates for all the activities. It is calculated as

T_L = Minimum of all (T_L^i - t_E^{ij}) for all i, j leading into the event
where T_L^i is the latest allowable occurrence time for event i.
T_L^j is the latest allowable occurrence time for event j and
t_E^{ij} is the expected time of activity ij.

Step3: After knowing the T_E and T_L values for the various events in the network, the critical path activities can be identified by applying the following conditions:

1) T_E and T_L values for the tail event of the critical activity are the same i.e.,
$T_E^i = T_L^i$.
2) T_E and T_L values for the head event of the critical activity are the same i.e.,
$T_E^j = T_L^j$.
3) For the critical activity, $T_E^j - T_E^i = T_L^j - T_L^i$

Step4: Find the project cost by the formula
Project cost = (Direct cost + (Indirect cost*project duration))

Step5: Find the minimum cost slope by the formula
Cost slope = (Crash cost - Normal cost)/(Normal time - Crash time)

Step6: Identify the activity with the minimum cost slope and crash that activity by one day. Identify the new critical path and find the cost of the project by formula
Project Cost= ((Project Direct Cost +
Crashing cost of crashed activity) +
(Indirect Cost*project duration))

Iteration Step:

Step7: In the new Critical path select the activity with the next minimum cost slope, and crash by one day, and repeat this step until all the activities along the critical path are crashed upto desired time.

Step8: At this point all the activities are crashed and further crashing is not possible. The crashing of non critical activities does not alter the project duration time and is of no use.

Step9 To determine optimum project duration, the total project cost is plotted against the duration time given by figure 4.

Further modification: Uncrashing

Step10 Now see if the project cost can be further reduced without affecting the project duration time. This can be done by uncrashing the activities which do not lie along the critical path. Uncrashing should start with an activity having the maximum cost slope. An activity is to be expanded only to the extent that it itself may become critical, but should not affect the original critical path.

Proposed Work:

Step1: Find Earliest time estimates for all the activities, it is denoted as T_E
Step2: Find latest time estimates for all the activities, it is denoted as T_L
Step3: Determine the Critical Path.
Step4: Compute the cost slope (i.e., cost per unit time) for each activity according to the following formula:
Cost slope = (Crash cost-Normal cost)/(Normal time-Crash time)
Step5: Among the critical path identify the activity with the minimum cost slope, and crash the activity by 1 day.
Step6: Calculate the project cost. Identify new critical path.
Project Cost= ((Project Direct Cost + Crashing cost of crashed activity) + Indirect Cost*project duration))
Step7: Now in the new critical path select the activity with the next minimum cost slope, and crash by one day.
Step8: Repeat this process until all the activities in the critical path have been crashed by 1 day.
Step9: Once all the activities along the critical path are crashed by one day, Repeat the process again i.e. goes to step5.
Step10: Find the minimum project cost and identify the activities which do not lie along the critical path
Step10: Now perform uncrashing. i.e uncrash the activities which do not lie along the critical path.

For Example:

To explain the process of crashing a network to reach the optimum project schedule, let us consider the network shown in figure 1. With each activity is associated normal direct cost and crash direct cost, the normal duration time and crash duration time. The complete data is given in table 1.The network has been drawn for normal conditions and the times shown along the arrows are normal duration times.

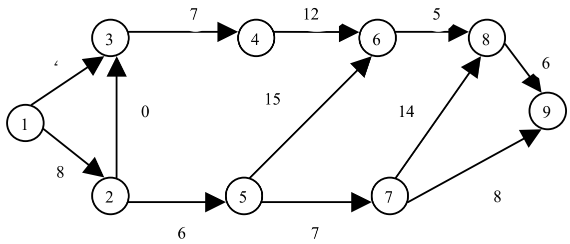

Figure 1

Activity	Normal		Crash		Δt	Δc	Δc/Δt
	Time	cost	Time	cost			
1--2	8	7000	3	10000	5	3000	600
1--3	4	6000	2	8000	2	2000	1000
2--3	0	0	0	0	0	0	0
2--5	6	9000	1	11500	5	2500	500
3--4	7	2500	5	3000	2	500	250
4--6	12	10000	8	16000	4	6000	1500
5--6	15	12000	10	16000	5	4000	800
5--7	7	12000	6	14000	1	2000	2000
6--8	5	10000	5	10000	0	0	0
7--8	14	6000	7	7400	7	1400	200
7--9	8	6000	5	12000	3	6000	2000
8--9	6	6000	4	7800	2	1800	900

Table 1

Result of Calculations based on Unit Crashing

Activity crashed	Weeks saved	Project duration	Direct cost	Indirect cost	Total cost
Nil	0	41	86500	41000	127500
7---8	1	40	87500	40000	127500
2--5	1	39	87200	39000	126200
1--2	1	38	87800	38000	125800
8--9	1	37	88700	37000	125700
5--6	0	37	89500	37000	126500
7--8	1	36	89700	36000	125700
1--2	1	35	90300	35000	125300
8--9	1	34	91200	34000	125200
1--2	1	33	91800	33000	124800
1--2	1	32	92400	32000	124400
3--4	0	32	92650	32000	124650
7--8	0	32	92850	32000	124850
2--5	1	31	93350	31000	124350
3--4	0	31	93600	31000	124600
2--5	1	30	94100	30000	124100
2--5	0	30	94600	30000	124600
1--2	0	30	94700	30000	124700
1--3	1	29	95700	29000	124700
2--5	0	29	96200	29000	125200
4--6	1	28	97700	28000	125700
2--5	0	28	98200	28000	126200
4--6	1	27	99700	27000	126700
5--6	0	27	100500	27000	127500
4--6	1	26	102000	26000	128000
5--6	0	26	102800	26000	128800
4--6	0	26	104300	26000	130300
7--8	1	25	104500	25000	129500
Uncrashing		30	93400	30000	123400

Table:2

Results and discussion:

In Table 2 the results shows how the total cost of the project is reduced as the total duration is crashed. Before uncrashing the minimum cost of project is Rs. 124100 for the project duration of 30 days and after uncrashing the minimum cost will be Rs. 123400 for the project duration of 30 days. The following graph depicts the results obtained.

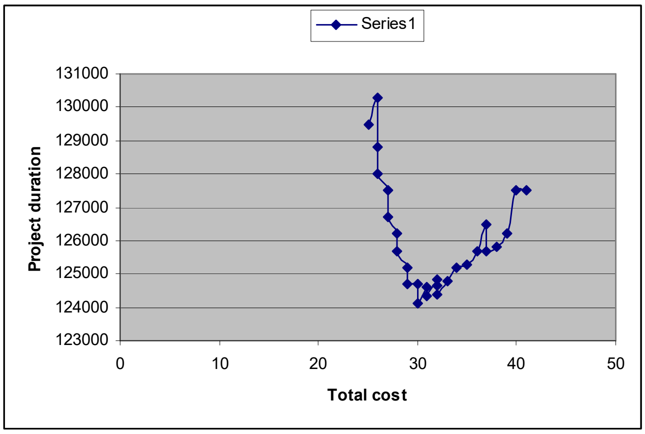

Figure: 4 Project duration Vs cost analysis

Conclusion:

In this paper the algorithm proposed for unit crashing reduces the cost of project. When activities are crashed by one day then only the crashing cost corresponding to one day is increased thereby reducing the project duration as well as cost. A C++ program is been developed to achieve the above results. This approach is well suitable for places where cost is of major consideration.

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ÖZET

Bu makalede, Saaty'nin Analitik Hiyerarşi Sürecine (AHP) alternatif olan ve onu genişleten Çok Kriterli Karar Verme için Alfa İndirgeme Yöntemi (α-İ ÇKKV) olarak adlandırdığımız yeni bir yaklaşımı sunmaktayız. Yöntem, homojen lineer eşitlikler sistemine dönüştürülebilen herhangi bir tercihler kümesi için işe yarar. Karar verme probleminin tutarlılık derecesi (ve dolaylı olarak da tutarsızlık derecesi) tanımlanmaktadır. α-İ ÇKKV, lineer ve/veya lineer olmayan homojen ve/veya homojen olmayan eşitlikler ve/veya eşitsizlikler sistemine dönüştürülebilen bir tercihler kümesine genelleştirilmiştir. Makalede birçok tutarlı, zayıf tutarsız ve güçlü tutarsız örnekler verilmektedir.

Anahtar Sözcükler: Çok Kriterli Karar Verme (ÇKKV), Analitik Hiyerarşi Süreci (AHP), α İndirgeme Yöntemi, Adillik İlkesi, Parametreleştirme, İkili Karşılaştırma, n-li Karşılaştırma, Tutarlı ÇKKV Problemi, Zayıf veya Güçlü Tutarsız ÇKKV Problemi.

1 GİRİŞ

Çok Kriterli Karar Verme için Alfa İndirgeme Yöntemi (α-İ ÇKKV), Saaty'nin Analitik Hiyerarşi Sürecine (AHP) alternatif ve onun bir genişletmesidir (Daha fazla bilgi için [1 – 11] arasındaki makalelere bakınız). Yöntem, sadece AHP'nin yaptığı gibi ikili karşılaştırmalar tarzındaki tercihler için işe yaramakta kalmayıp aynı zamanda lineer homojen eşitlikler olarak ifade edilebilen kriterlerin herhangi n-li ($n \geq 2$ için) karşılaştırmaları tarzındaki tercihler için de işe yaramaktadır.

α-İ ÇKKV'deki genel fikir; sadece sıfır çözümü olan üst taraftaki eşitliklerin lineer homojen sistemini belli bir sıfırdan farklı çözümü olan bir sisteme dönüştürmek amacıyla katsayıları azaltan veya arttıran $\alpha_1, \alpha_2, \ldots, \alpha_p$ gibi sıfırdan farklı pozitif parametreleri her bir tercihin sağ taraf katsayılarına atamaktır.

Bu sistemin genel çözümünü bulduktan sonra tüm α değerlerini belli değerler atamak için kullanılan ilkeler yöntemin ikinci önemli kısmıdır; ancak bu kısım gelecekte daha derin incelenecektir. Mevcut makalede Adillik İlkesini önermekteyiz; diğer bir deyişle, her bir katsayı ayrı yüzdeyle indirgenmelidir (Bunun adil olduğunu düşünüyoruz: Herhangi bir katsayıya adaletsizlik ya da kayırmacılık yapmama); fakat okuyucu başka ilkeler önerebilir.

İkili karşılaştırmalı tutarlı karar verme problemleri için Adillik İlkesiyle beraber kullanılan α-İ ÇKKV, AHP ile aynı sonucu vermektedir. Ancak zayıf tutarsız karar verme problemlerinde Adillik İlkesiyle beraber kullanılan α-İ ÇKKV, AHP'den farklı bir sonuç vermektedir.

α-İ/Adillik İlkesi beraber iki tercihli ve iki kriterli güçlü tutarsız karar verme problemleri için doğruluğu ispat edilebilir bir sonuç vermektedir; ancak tercih ve kriter sayısı ikiden fazla olan ÇKKV problemleri için Adillik İlkesinin yerini tüm α parametrelerine sayısal değerler atayan başka bir ilke almalıdır.

Bu makalenin konusu Saaty'nin AHP'si olmadığından sadece bu yöntemin uygulanmasındaki ana adımları hatırlatacağız, böylece α-İ ÇKKV ile AHP'nin sonuçları kıyaslanabilsin.

AHP kriterlerin sadece ikili karşılaştırmaları için işe yarayan bir yöntemdir. Bu karşılaştırmalardan n x n boyutunda bir kare Tercih Matrisi, A, oluşturulur. Bu matrise dayalı olarak A'nın maksimum öz değerini, λmax, ve ilgili öz vektörü hesaplanır.

Eğer λ_{max} kare matrisin boyutuna eşitse bu durumda karar verme problemi tutarlıdır ve ilgili normalleştirilmiş öz vektörü (Perron-Frobenius Vektörü) öncelik vektörüdür.

Eğer λ_{max} kare matrisin boyutundan kesin surette daha büyükse bu durumda karar verme problemi tutarlı değildir. Bu durumda A matrisi ikinci üssüne yükseltilir ve elde edilen matris tekrar kendi ikinci üssüne yükseltilir, vb. ki böylelikle A^2, A^4, A^8, … vb matris dizisi elde edilir. Her bir durumda, iki ardıl normalleştirilmiş öz vektörler arasındaki fark belirlenmiş eşik noktasından daha küçük oluncaya kadar maksimum öz değeri ve ilgili normalleştirilmiş öz vektörü hesaplanmaya devam eder. Belirlenmiş eşik noktasından küçük olan son öz vektör öncelik vektörü olacaktır.

Saaty, Tutarlılık Endeksini şöyle tanımlamıştır: $CI(A)=\dfrac{\lambda_{\max}(A)-n}{n-1}$, n=Kare matris A'nın boyutu.

2 Çok Kriterli Karar Verme için α-İndirgeme Yöntemi (α-İ ÇKKV)

2.1 α-İ ÇKKV Tanımı

Bu makalenin genel fikri tutarsız bir (karar verme) problemin(in) katsayılarını belli yüzdelere indirgeyerek tutarlı bir (karar verme) problem(in)e dönüştürmektir.

Kriterler kümesi, C={C1, C2, …, Cn}, n ≥ 2, ve

Tercihler kümesi, P={P1, P2, …, Pn}, m ≥ 1 olsun.

Her bir P_i tercihi yukarıda verilen $C_1, C_2, \ldots, C_n$ kriterlerinin bir lineer homojen eşitliğidir:

$$P_i = f(C_1, C_2, \ldots, C_n)$$

Aşağıdaki gibi bir temel kanı ataması (bba) oluşturmamız gerekir:

$$m: C \rightarrow [0, 1]$$

öyle ki $m(C_i) = x_i$, $0 < x_i < 1$ ve $\sum_{i=1}^{n} m(C_i) = \sum_{i=1}^{n} x_i = 1$.

P tercihler kümesiyle uyumlu tüm x_i değişkenlerini bulmamız gerekir. Bu suretle, eşlenik matrisi

$$A = (a_{ij}),\ 1 \le i \le m \text{ ve } 1 \le j \le n$$

olan m x n boyutunda eşitliklerin lineer homojen sistemini elde ederiz.

Bu sistemin sıfırdan farklı çözümlere sahip olması için A matrisinin mertebesi kesinlikle n'den küçük olmalıdır.

2.2 Lineer Karar Verme Problemlerinin Sınıflandırılması

a) Bir x_i değişkeninin bir eşitlikten diğer bir eşitliğe herhangi bir ikamesiyle tüm eşitliklerle uyumlu bir sonuç alıyorsak bu **lineer karar verme problemi tutarlıdır** deriz.

b) Bir eşitlikten diğer bir eşitliğe bir x_i değişkeninin en az bir tane ikamesiyle aşağıdaki şekillerde gösterildiği gibi en az bir eşitlikle uyumsuz bir sonuç alıyorsak bu **lineer karar verme problemi zayıf tutarsızdır** deriz:

$$WD(1)\begin{Bmatrix} x_i = k_1 \cdot x_j, k > 1; \\ x_i = k_2 \cdot x_j, k_2 > 1, k_2 \neq k_1 \end{Bmatrix}$$

veya

$$WD(2)\begin{Bmatrix} x_i = k_1 \cdot x_j, 0 < k < 1; \\ x_i = k_2 \cdot x_j, 0 < k_2 < 1, k_2 \neq k_1 \end{Bmatrix}$$

veya

$$WD(3)\{x_i = k \cdot x_i, k \neq 1\}$$

Örneğin bir x değişkeni y'den büyük olma (x > y) koşulunu farklı oranlarla sağlıyor olsun (mesela, x = 3y ve x = 5y). Bu sebepten, (WD1)-(WD3) zayıf uyuşmazlıklardır. Bu durumda, tüm uyuşmazlıklar (WD1)-(WD3) gibi olmalıdır.

c) Eğer bir x_i değişkeninin bir eşitlikten diğer bir eşitliğe en az bir tane ikamesiyle aşağıda gösterildiği gibi en az bir eşitlikle uyumsuz bir sonuç alıyorsak bu **lineer karar verme problemi güçlü tutarsızdır** deriz:

$SD(4)\begin{Bmatrix} x_i = k_1 \cdot x_j; \\ x_i = k_2 \cdot x_j, \end{Bmatrix}$, $0 < k_1 < 1 < k_2$ veya $0 < k_2 < 1 < k_1$ iken (diğer bir deyişle bir eşitlikten $x_i < x_j$ elde edilirken diğer bir eşitlikten tam tersi bir eşitsizlik olan $x_j < x_i$ elde edilir.)

Güçlü tutarsızlık için (SD4) gibi en az bir tutarsızlığın var olması gerekir; bu durum için (WD1)-(WD3) gibi tutarsızlıkların olup olmaması önem taşımaz.

A matrisinin determinantını hesapla.

a) Eğer det(A) = 0 ise karar problemi tutarlıdır zira eşitlikler sistemi bağımlıdır. Sistemi parametreleştirmek şart değildir. {Parametreleştirdiğimiz durumda Adillik İlkesini kullanabiliriz; diğer bir deyişle, tüm parametreleri birbirine eşitleriz $\alpha_1 = \alpha_2 = ... = \alpha_p > 0$}

Bu sistemi çözelim ve genel çözümünü bulalım. Parametreleri ve ikincil değişkenleri yerine koyalım, böylelikle belli bir çözüm elde edebiliriz. Bu belli çözümü (her bir bileşeni tüm bileşenlerin toplamına bölerek) normalleştirelim. Bunun sonucunda (bileşenlerinin toplamı 1 etmesi gereken) öncelik vektörünü elde ederiz.

b) Eğer det(A) ≠ 0 ise karar problemi tutarsızdır zira homojen lineer sistemin sadece sıfır çözümü vardır.

i. Eğer tutarsızlık zayıf düzeydeyse sağ taraf katsayılarını parametreleştirip sistem matrisini A(α) olarak belirt.

Parametrik eşitliği elde edebilmek için det(A(α)) = 0'ı hesapla.

Eğer Adillik İlkesi kullanılıyorsa tüm parametreleri birbirine eşitle ve α > 0 için çöz.

A(α)'daki α'yı değiştir ve elde edilen bağımlı homojen lineer sistemi çöz.

“a)”’dakine benzer şekilde her bir ikincil değişkeni 1 ile değiştir ve öncelik vektörünü elde edebilmek için ulaşılan çözümü normalleştir.

ii. Eğer tutarsızlık güçlüyse Adillik İlkesi istenildiği gibi işe yaramayabilir. Başka bir yaklaşımlı ilke tasarlanabilir veya daha fazla bilgi edilerek karar verme probleminin güçlü düzeydeki tutarsızlıkları tekrar gözden geçirilebilir.

2.3 AHP ile α-İ ÇKKV’nin Karşılaştırması

a) α-İ ÇKKV’nin genel çözümü AHP’ninki de dâhil olmak üzere tüm belirli çözümleri içerir.

b) α-İ ÇKKV sadece ikili karşılaştırmalarla sınırlı kalmayıp kriterler arasında tüm karşılaştırma türlerini kullanır.

c) Tutarlı problemler için AHP ve α-İ ÇKKV/Adillik İlkesi aynı sonucu verir.

d) Büyük girdiler için α-İ ÇKKV eşitlikleri (bazı α parametrelerine bağlı olarak) bir matris formun altına koyabiliriz ve sonra 0 olacak şekilde matrisin determinantını hesaplayabiliriz. Bundan sonra sistemi çözeriz (tüm bunlar matematik yazılımları kullanılarak bilgisayarda yapılabilir): MATHEMATICA ve MAPLE gibi yazılımlar örneğin determinant hesaplamalarını yapabilir ve bu lineer sistemin çözümlerini hesaplayabilir).

e) α-İ ÇKKV daha büyük tercihler sınıfı için işe yarayabilir; diğer bir deyişle, homojen lineer eşitliklere veya lineer olmayan eşitliklere ve/veya eşitsizliklere dönüştürülebilen türde tercihler için. Daha fazla ayrıntı için aşağıya bakın.

2.4 α-İ ÇKKV’nin Genelleştirmesi

Her bir tercih, lineer ya da lineer olmayan eşitlik veya eşitsizlik olarak ifade edilebiliyor olsun. Tüm tercihler beraber lineer/lineer olmayan eşitlikler/eşitsizlikler sistemini veya eşitlikler ve eşitsizliklerin karma bir sistemini oluştururlar.

Kesinlikle pozitif bir çözümü (yani tüm bilinmeyen $x_i > 0$) arayarak bu sistemi çözelim. Sonra çözüm vektörünü normalleştirelim. Eğer böyle birden fazla sayısal çözüm varsa bir değerlendirme yapın: Her bir durumdaki normalleştirilmiş çözüm vektörünü analiz edin. Eğer genel bir çözüm varsa en iyi belirli çözümü seçerek alın. Eğer kesinlikle pozitif çözüm yoksa sistemin katsayılarını parametreleştirin, parametrik eşitliği bulun ve α parametrelerinin sayısal değerlerini bulabilmek için uygulanacak bazı ilkeleri arayın. Bir tartışma/değerlendirme dâhil edilebilir. Belirlenemeyen sonuçlar elde edebiliriz.

3 α-İ ÇKKV/Adillik İlkesinde Tutarlılık ve Tutarsızlık Dereceleri

Tutarlı ve zayıf tutarlı karar verme problemlerindeki α-İ ÇKKV/Adillik İlkesi için aşağıdaki durumlar söz konusudur:

a) Eğer $0 < \alpha < 1$ ise o zaman α karar verme probleminin **tutarlılık derecesi**dir ve $\beta = 1 - \alpha$ da karar verme probleminin **tutarsızlık derecesi**ni belirtir.

b) Eğer $\alpha > 1$ ise o zaman $1/\alpha$ karar verme probleminin **tutarlılık derecesi**dir ve $\beta = 1 - 1/\alpha$ da karar verme probleminin **tutarsızlık derecesi**ni belirtir.

4 α-İ ÇKKV’nin İlkeleri (İkinci Kısım)

1. α-İ Yönteminin ikinci kısmında uygulamalarda diğer ilkeler Adillik İlkesi’nin yerini alabilir.

Uzman Görüşü: Örneğin, bir tercihin katsayısının uzman görüşüne dayanarak diğer bir katsayıdan iki kat daha fazla ve başka bir tercihin katsayısının da üçte biri kadar indirgeneceğine dair bir bilgimiz varsa o zaman uygun bir şekilde parametrik eşitliğimizde bu durumu belirtiriz. Örneğin; $\alpha_1 = 2\alpha_2$ ve anılan sıraya göre $\alpha_3 = (1/3)\alpha_4$.

2. α-İ/Adillik İlkesi veya Uzman Görüşü

   Buradaki başka bir görüş de bir **tutarlılık eşiği** t_c (veya dolaylı olarak bir **tutarsızlık eşiği** t_i) belirlemek olabilir. Bu durumda, tutarlılık derecesi istenen t_c değerinden azsa Adillik İlkesi veya Uzman Görüşü (hangisi kullanıldıysa) bırakılmalı ve tüm α değerlerini bulan başka bir ilke tasarlanmalıdır. Benzeri şekilde aynı durum tutarsızlık t_i değerinden çok olması durumunda da geçerlidir.

3. Tüm m tercihlerinin eşitliklere dönüştürülebildiği durum için <u>sistemin hatasızlığı</u> (veya <u>hatası</u>) ölçülebilir. Örneğin; P_i tercihi $f_i(x_1, x_2, \dots, x_n) = 0$ eşitliğine dönüştürülsün. O halde, $x_1, x_2, \dots, x_n$ bilinmeyenlerini bulmamız gerekir, öyle ki:

$$\text{(e: hata)}, \ e(x_1, x_2, \dots, x_n) = \sum_{i=1}^{m} \left| f_i(x_1, x_2, \dots, x_n) \right| \ \text{minimum olsun.}$$

   Eğer minimum değer mevcutsa Analiz (Calculus) Teorisi (kısmi türevler) kullanılarak $e : R_+^n \to R_+$ iken $e(x_1, x_2, \dots, x_n)$ gibi n değişkenli bir fonksiyonun minimum değeri bulunabilir. Tutarlı karar verme problemleri için sistemin hatasızlığı/hatası sıfırdır; böylelikle kesin sonucu elde ederiz.

   Bunu şu gerçek yoluyla kanıtlayabiliriz: Tüm i'ler için $x_i = a_i > 0$ olduğu normalleştirilmiş öncelik vektörü $[a_1\ a_2\ \dots\ a_n]$, $i = 1, 2, \dots, m$ için $f_i(x_1, x_2, \dots, x_n) = 0$ sisteminin belirli bir çözümüdür. Dolayısıyla,

$$\sum_{i=1}^{m} \left| f_i(a_1, a_2, \dots, a_n) \right| = \sum_{i=1}^{m} |0| = 0$$

   Ancak tutarsız karar verme problemleri için değişkenler için yaklaşık değerler buluruz.

5 α-İ ÇKKV için Genişletme (Lineer Olmayan α-İ ÇKKV)

Tercihlerin lineer olmayan homojen (veya hatta homojen olmayan) eşitlikler olduğu durum için α-İ ÇKKV'yi genelleştirmek zor değildir. Tercihlerin bu lineer olmayan sistemi bağımlı olmak zorundadır (bu, gene çözümün – ana değişkenlerin – en az bir tane ikincil değişkene bağlı olması anlamına gelir). Eğer sistem bağımlı değilse sistemi aynı yolla parametreleştirebiliriz. Üstelik bu lineer olmayan α-İ ÇKKV'nin ikinci kısmında (alabileceğimiz ek bilgiye bağlı olarak) ikincil değerlerin her birine bazı değerler atarız ve tüm parametreler için sayısal değerleri bulabilmemize yardım edecek bir ilkeyi tasarlamaya da ihtiyacımız vardır. (Genel çözümden böylelikle türettiğimiz) belirli bir sonuç elde ederiz. Buradan normalleştirdiğimiz sonuç bize öncelik vektörümüzü verecektir. Ancak, Lineer Olmayan α-İ ÇKKV daha karmaşıktır ve her bir lineer olmayan karar verme problemine bağlıdır.

Şimdi bazı örnekler görelim.

6 Tutarlı Örnek 1

6.1 α-İ ÇKKV ile Çözüm

α-İ ÇKKV'yi kullanarak örneğimizi çözelim. Tercihler Kümesi {C1, C2, C3} olsun ve Kriterler Kümesi ise

1. C1, C2'ye göre 4 kat önemlidir.

2. C2, C3'e göre 3 kat önemlidir.

3. C3, C1'e göre 1/12 kat önemlidir.

şeklinde belirtilmiştir. m(C1) = x, m(C2) = y, m(C3) = z olsun.

Bu karar verme problemine eşlenmiş lineer homojen sistem şöyledir:

$$\begin{cases} x = 4y \\ y = 3z \\ z = \dfrac{x}{12} \end{cases}$$

Bu sistemin eşlenik A_1 matrisi ise şöyledir:

$$\begin{pmatrix} 1 & -4 & 0 \\ 0 & 1 & -3 \\ -1/12 & 0 & 1 \end{pmatrix}$$, buradan det(A_1) = 0, bundan dolayı karar verme problemi tutarlıdır.

Bu homojen lineer sistemi çözerek [12z 3z z] vektörü olarak belirlediğimiz genel çözüme ulaşırız. z herhangi bir reel sayı olabilir (x = 12z ile y = 3x ana değişkenlerken z, ikincil bir değişken olarak addedilebilir).

z = 1 yaparak vektör değerleri olarak [12 3 1]'e ulaşırız ve akabinde normalleştirerek (her bir vektör bileşenini 12 + 3 + 1 = 16'ya bölerek) öncelik vektörünü elde ederiz: [12/16 3/16 1/16], böylelikle tercihimiz C1 olacaktır.

6.2 AHP ile Çözüm

Örneği AHP ile çözersek aynı sonucu elde ederiz. Tercih matrisimiz:

$$\begin{pmatrix} 1 & 4 & 12 \\ 1/4 & 1 & 3 \\ 1/12 & 1/3 & 1 \end{pmatrix}$$

Matrisin maksimum öz değeri λ_{max} = 3'tür ve karşılık gelen normalleştirilmiş öz vektörü (Perron-Frebenius vektörü) ise [12/16 3/16 1/16]'dır.

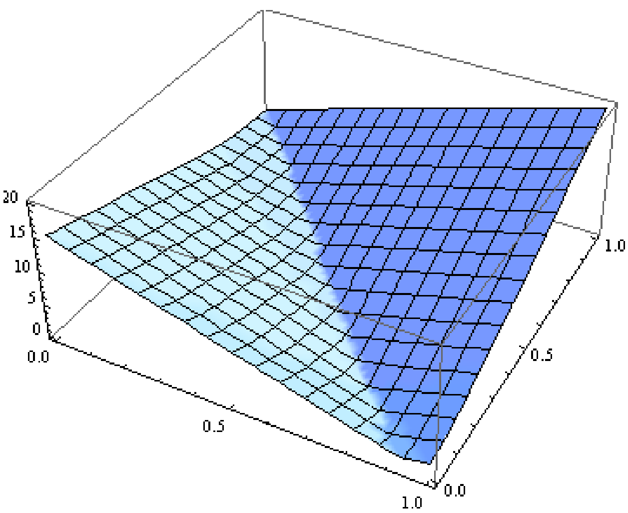
6.3 Mathematica 7.0 Yazılımıyla Çözüm

Mathematica 7.0'ı kullanarak h(x,y) = |x - 4y| + |3x + 4y - 3| + |13x + 12y - 12|; x, y ∈ [0, 1] şeklindeki fonksiyonun grafiğini çizeriz. Bu fonksiyon, tutarlı karar verme probleminin eşlenik sistemini temsil eder:

x/y = 4, y/z = 3, x/z = 12, ve x + y + z = 1, x > 0, y > 0, z > 0.

In[1]:=

Plot3D[Abs[x-4y]+Abs[3x+4y-3]+Abs[13x+12y-12],{x,0,1},{y,0,1}]



Bu fonksiyonun minimum değeri 0 ve x = 12/16, y = 3/16'dır.

Eğer h(x, y) ile eşleştirilmiş üç değişkenin orijinal fonksiyonunu ele alacak olursak o zaman

$$H(x, y, z) = |x - 4y| + |y - 3z| + |x - 12z|,\ x + y + z=1, \text{ ve } x, y, z \in [0,1].$$

Aynı şekilde H(x, y, z)'nin minimum değerini 0 olarak buluruz ve x = 12/16, y = 3/16, z = 1/16'dır.

7 AHP'nin İşe Yaramadığı Zayıf Tutarsız Örnekler

Tercihler Kümesi {C1, C2, C3} olsun.

7.1 Zayıf Tutarsız Örnek 2

7.1.1 α-İ ÇKKV Yöntemini Kullanarak Çözüm

Kriterler Kümesi,

1. C1, toplandığında C2'den 2 ve C3'ten 3 kat önemlidir.
2. C2, C1'den yarım kat önemlidir.
3. C3, C1'den üçte bir kat önemlidir.

şeklinde belirtilmiştir. m(C1) = x, m(C2) = y, m(C3) = z olsun;

$$\begin{cases} x = 2y + 3z \\ y = \dfrac{x}{2} \\ z = \dfrac{x}{3} \end{cases}$$

AHP bu örneğe uygulanamaz çünkü ilk tercihin şekli ikili bir karşılaştırma değildir. Eğer mevcut haliyle eşitliklerin bu lineer homojen sistemini çözersek x = y = z = 0 elde ederiz zira eşlenik matrisi

$$\begin{pmatrix} 1 & -2 & -3 \\ -1/2 & 1 & 0 \\ -1/3 & 0 & 1 \end{pmatrix} = -1 \neq 0$$

Ama bu sıfır sonuçlu durum kabul edilemez zira x + y + z = 1 olmalıdır. Sağ taraf katsayılarının her birini parametreleştirelim ve yukarıdaki sistemin genel çözümünü elde edelim.

$$\begin{cases} x = 2\alpha_1 y + 3\alpha_2 z & (1) \\ y = \dfrac{\alpha_3}{2} x & (2) \\ z = \dfrac{\alpha_4}{3} x & (3) \end{cases}$$

Burada $\alpha_1, \alpha_2, \alpha_3, \alpha_4 > 0$'dır.

(1)'e (2) ve (3)'teki ifadeleri yerleştirirsek şunu elde ederiz:

$$x = 2\alpha_1\left(\frac{\alpha_3}{2}x\right) + 3\alpha_2\left(\frac{\alpha_4}{3}x\right)$$

$1 \cdot x = (\alpha_1\alpha_3 + \alpha_2\alpha_4) \cdot x$ ifadesinden de $\alpha_1\alpha_3 + \alpha_2\alpha_4 = 1$ (parametrik eşitlik) (4)

Sistemin genel çözümü $\begin{cases} y = \frac{\alpha_3}{2}x \\ z = \frac{\alpha_4}{3}x \end{cases}$, buradan da öncelik vektörü: $\left[x \quad \frac{\alpha_3}{2}x \quad \frac{\alpha_4}{3}x\right] \rightarrow \left[1 \quad \frac{\alpha_3}{2} \quad \frac{\alpha_4}{3}\right]$ olur.

Adillik İlkesi: Tüm katsayıları aynı yüzdeyle indirge. O halde, (4)'te $\alpha_1 = \alpha_2 = \alpha_3 = \alpha_4 = \alpha > 0$'ı yerine koyarsak $\alpha^2 + \alpha^2 = 1$, buradan $\alpha = \frac{\sqrt{2}}{2}$ elde ederiz. Öncelik vektörümüz $\left[1 \quad \frac{\sqrt{2}}{4} \quad \frac{\sqrt{2}}{6}\right]$ olur ve bunu normalleştirirsek $\begin{matrix}[0.62923 & 0.22246 & 0.14831] \\ C1 & C2 & C3 \\ x & y & z\end{matrix}$ sonucunca ulaşırız. Tercihimiz en büyük vektör bileşeni olan C1'den yana olacaktır. Bunu doğrulayalım:

$\frac{y}{x} \cong 0.35354$ olur 0.50 yerine; yani orijinal halinin $\frac{\sqrt{2}}{2} = \%70.71$'idir.

$\frac{z}{x} \cong 0.23570$ olur 0.333 yerine; yani orijinal halinin %70.71'idir.

$x \cong 1.41421y + 2.12132z$ olur $2y + 3z$ yerine; yani, 2'nin %70.71'i ve 3'ün %70.71'i'dir. Sonuç itibariyle, her bir katsayı için adil bir indirgeme yapılmış oldu.

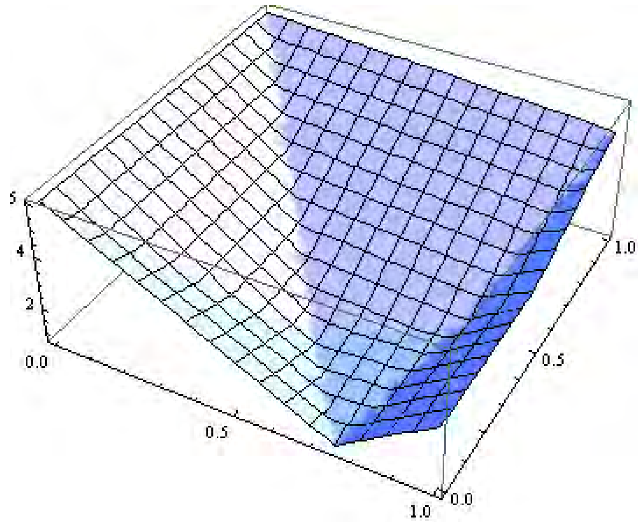
7.1.2 Mathematica 7.0 Yazılımını Kullanarak Çözüm

Mathematica 7.0 yazılımını kullanarak ilgili zayıf tutarlı karar verme problemini $x - 2y - 3z=0$, $x - 2y = 0$, $x - 3z = 0$, ve $x + y + z = 1$, $x > 0$, $y > 0$, $z > 0$ olarak temsil eden $g(x, y) = |4x - y-3| + |x - 2y| + |4x + 3y - 3|$, ve $x, y \in [0, 1]$ fonksiyonunun grafiğini çizelim.

$z = 1 - x – y$'yi çözerek ve aşağıdaki fonksiyonda yerine koyarsak $G(x, y, z) = |x - 2y - 3z| + |x - 2y| + |x - 3z|$ ve $x>0, y>0, z>0$:

In[2]:=

Plot3D[Abs[4x-y-3]+Abs[x-2y]+Abs[4x+3y-3],{x,0,1},{y,0,1}]



Eğer varsa g(x, y)'nin minimum değerini buluruz:

In[3]:=

FindMinValue[{Abs[4x-y-3]+Abs[x-2y]+Abs[4x+3y-3],x+y≤1,x>0,y>0},{x,y}]

Aşağıdaki sonuç elde edilir:

Out[3]:= 0.841235.

FindMinValue::eit: The algorithm does not converge to the tolerance of 4.806217383937354`*^-6 in 500 iterations. The best estimated solution, with feasibility residual, KKT residual, or complementary residual of {0.0799888,0.137702,0.0270028}, is returned.

7.1.3 α-İ Kullanarak Matris Yöntemiyle Çözüm

(1), (2), (3) homojen lineer sisteminin determinantı:

$$\begin{vmatrix} 1 & -2\alpha_1 & -3\alpha_2 \\ -\frac{1}{2}\alpha_3 & 1 & 0 \\ -\frac{1}{3}\alpha_4 & 0 & 1 \end{vmatrix}$$ = (1 + 0 + 0) – ($\alpha_2\alpha_4 + \alpha_1\alpha_3$) = 0 veya ($\alpha_1\alpha_3 + \alpha_2\alpha_4$) = 1 (parametrik eşitlik).

Sistemin sıfırlı olmayan çözüme sahip olması için determinant 0 olmalıdır. Matrisin mertebesi 2'dir. Böylelikle, iki değişken buluruz. Örneğin, son iki eşitlikten y ve z için x'e göre çözmek daha kolaydır:

$$\begin{cases} y = \frac{1}{2}\alpha_3 x \\ z = \frac{1}{3}\alpha_4 x \end{cases}$$ ve öncesinde olduğu gibi prosedür aynı adımları takip eder.

Çeşitli durumları incelemek için Örnek 1'i değiştirelim.

7.2 Zayıf Tutarsız Örnek 3

Örnek 3, Örnek 2'ye göre zayıf tutarsızlık derecesi arttırılmış bir örnektir.

1. Örnek 2'dekinin aynısı (Bir araya getirildiğinde C1, C2'den 2 ve C3'ten 3 kat önemlidir).
2. C2, C1'den 4 kat önemlidir.
3. Örnek 2'dekinin aynısı (C3, C1'den üçte bir kat önemlidir).

$$\begin{cases} x = 2\alpha_1 y + 3\alpha_2 z \\ y = 4\alpha_3 x \\ z = \frac{\alpha_4}{3} x \end{cases}$$

$$x = 2\alpha_1(4\alpha_3 x) + 3\alpha_2\left(\frac{\alpha_4}{3}\right)x$$

$$1 \cdot x = (8\alpha_1\alpha_3 + \alpha_2\alpha_4)x$$

$$8\alpha_1\alpha_3 + \alpha_2\alpha_4 = 1$$

$$\alpha_1 = \alpha_2 = \alpha_3 = \alpha_4 = \alpha > 0$$

$$9\alpha^2 = 1 \Rightarrow \alpha = \frac{1}{3}$$

$$\begin{bmatrix} x & 4\alpha_3 x & \frac{\alpha_4}{3} x \end{bmatrix} \to \begin{bmatrix} 1 & 4\alpha_3 & \frac{\alpha_4}{3} \end{bmatrix}$$

$$\begin{bmatrix} 1 & \frac{4}{3} & \frac{1}{9} \end{bmatrix} = \begin{bmatrix} \frac{9}{9} & \frac{12}{9} & \frac{1}{9} \end{bmatrix}$$

normalleştirme: $\begin{bmatrix} \frac{9}{22} & \frac{12}{22} & \frac{1}{22} \end{bmatrix}$

$\frac{y}{x} = 1.333$ olur 4 yerine;

$\frac{z}{x} = 0.111$ olur 0.3333 yerine;

x = 0.667y + 1z olur 2y + 3z yerine.

Her bir katsayı $\frac{1}{3} = \%33.33$ oranında indirgenmiştir. Tutarsızlık büyüdükçe (β→1) indirgeme oranı o kadar büyür (α→0).

7.3 Zayıf Tutarsız Örnek 4

Örnek 4, Örnek 3'ten bile daha tutarsız bir örnektir.

1. Örnek 2'dekinin aynısı (Bir araya getirildiğinde C1, C2'den 2 ve C3'ten 3 kat önemlidir).
2. Örnek 3'tekinin aynısı (C2, C1'den 4 kat önemlidir).
3. C3, C1'den 5 kat önemlidir.

$$\begin{cases} x = 2\alpha_1 y + 3\alpha_2 z \\ y = 4\alpha_3 x \\ z = 5\alpha_4 x \end{cases}$$

$$x = 2\alpha_1(4\alpha_3 x) + 3\alpha_2(5\alpha_4 x)$$

$$1 \cdot x = (8\alpha_1\alpha_3 + 15\alpha_2\alpha_4)x$$

$$\text{buradan} 8\alpha_1\alpha_3 + 15\alpha_2\alpha_4 = 1$$

$$\alpha_1 = \alpha_2 = \alpha_3 = \alpha_4 = \alpha > 0, 23\alpha^2 = 1 \Rightarrow \alpha = \frac{\sqrt{23}}{23}$$

$$\begin{bmatrix} 1 & 4\alpha_3 & 5\alpha_4 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & \frac{4\sqrt{23}}{23} & \frac{5\sqrt{23}}{23} \end{bmatrix}$$

Normalleştirme: $[0.34763 \quad 0.28994 \quad 0.36243]$. $\frac{\sqrt{23}}{23} = \%20.85$ oranında bir indirgemeyle;

$\frac{y}{x} \cong 0.83405$ olur 4 yerine; $\frac{z}{x} \cong 1.04257$ olur 5 yerine; x ≅ 0.41703y + 0.62554z olur 2x + 3y yerine.

Her bir katsayı $\alpha = \frac{\sqrt{23}}{23} = \%20.85$ oranında indirgenmiştir.

7.4 Tutarlı Örnek 5

α = 1 elde ettiğimizde tutarlı bir probleme sahip oluruz. Tercihlerin şöyle olduğunu varsayalım:

1. Örnek 2'dekinin aynısı (Bir araya getirildiğinde C1, C2'den 2 ve C3'ten 3 kat önemlidir).
2. C2, C1'den dörtte bir kat önemlidir.

3. C3, C1'den altıda bir kat önemlidir.

Sistemimiz şöyle olur:

$$\begin{cases} x = 2y + 3z \\ y = \dfrac{x}{4} \\ z = \dfrac{x}{6} \end{cases}$$

7.4.1 Bu Sistemi Çözmenin İlk Yolu

Bu sistemin ikinci ve üçüncü eşitliklerini birincide yerine koyarsak, şunu elde ederiz:

$x = 2\left(\dfrac{x}{4}\right) + 3\left(\dfrac{x}{6}\right) = \dfrac{x}{2} + \dfrac{x}{2} = x$ ki bu bir özdeşliktir (böylece çelişki yoktur).

Genel çözüm: $\begin{bmatrix} x & \dfrac{x}{4} & \dfrac{x}{6} \end{bmatrix}$

Öncelik vektörü: $\begin{bmatrix} 1 & \dfrac{1}{4} & \dfrac{1}{6} \end{bmatrix}$

Normalleştirme: $\begin{bmatrix} \dfrac{12}{17} & \dfrac{3}{17} & \dfrac{2}{17} \end{bmatrix}$

7.4.2 Bu Sistemi Çözmenin İkinci Yolu

Parametreleştirelim:

$$\begin{cases} x = 2\alpha_1 y + 3\alpha_2 z \\ y = \dfrac{\alpha_3}{4} x \\ z = \dfrac{\alpha_4}{6} x \end{cases}$$

Son iki eşitliğimizi birincide yerine koyarsak şunu elde ederiz:

$$x = 2\alpha_1\left(\frac{\alpha_3}{4} x\right) + 3\alpha_2\left(\frac{\alpha_4}{6} x\right) = \frac{\alpha_1\alpha_3}{2} x + \frac{\alpha_2\alpha_4}{2} x$$

$$1 \cdot x = \frac{\alpha_1\alpha_3 + \alpha_2\alpha_4}{2} x,$$

Buradan $1 = \dfrac{\alpha_1\alpha_3 + \alpha_2\alpha_4}{2}$ veya $\alpha_1\alpha_3 + \alpha_2\alpha_4 = 2$ olur.

Adillik ilkesini göz önünde bulundurun: $\alpha_1 = \alpha_2 = \alpha_3 = \alpha_4 = \alpha > 0$, öyleyse $2\alpha^2 = 1$, $\alpha = \pm 1$ ama sadece pozitif değer $\alpha = 1$'i alırız (tutarlı bir problemden beklendiği gibi). Kontrol edelim:

$\frac{y}{x} = \frac{\frac{3}{17}}{\frac{12}{17}} = \frac{1}{4}$, aynen orijinal sistemde olduğu gibi; $\frac{z}{x} = \frac{\frac{2}{17}}{\frac{12}{17}} = \frac{1}{6}$, aynen orijinal sistemde olduğu gibi;

x = 2y + 3z zira $x = 2\left(\frac{x}{4}\right) + 3\left(\frac{x}{6}\right)$; sonuç itibariyle tüm katsayılar α = 1 olarak orijinal hallerinde kaldı. Herhangi bir indirgemeye gerek görülmedi.

7.5 Genel Örnek 6

Şu genel durumu dikkate alalım:

$a_1, a_2, a_3, a_4 > 0$ iken $\begin{cases} x = a_1 y + a_2 z \\ y = a_3 x \\ z = a_4 x \end{cases}$ olsun. Parametrik hale getirelim:

$\alpha_1, \alpha_2, \alpha_3, \alpha_4 > 0$ iken $\begin{cases} x = a_1\alpha_1 y + a_2\alpha_2 z \\ y = a_3\alpha_3 x \\ z = a_4\alpha_4 x \end{cases}$ olur. İkinci ve üçüncü eşitlikleri birincide yerine koyarsak

$x = a_1\alpha_1(a_3\alpha_3 x) + a_2\alpha_2(a_4\alpha_4 x)$

$x = a_1a_3\alpha_1\alpha_3 x + a_2a_4\alpha_2\alpha_4 x$

Buradan $a_1a_3\alpha_1\alpha_3 x + a_2a_4\alpha_2\alpha_4 x = 1$ (parametrik eşitliği) elde edilir.

Bu sistemin genel çözümü, $(x, a_3\alpha_3 x, a_4\alpha_4 x)$ ve öncelik vektörü, $[1 \quad a_3\alpha_3 \quad a_4\alpha_4]$ şeklindedir.

Adillik İlkesini dikkate alırsak: $\alpha_1 = \alpha_2 = \alpha_3 = \alpha_4 = \alpha > 0$ şunu elde ederiz: $\alpha^2 = \frac{1}{a_1a_3 + a_2a_4}$, böylece $\alpha = \frac{1}{\sqrt{a_1a_3 + a_2a_4}}$ ulaşılır.

i) Eğer $\alpha \in [0,1]$ ise, o halde α problemin tutarlılık derecesiyken β = 1 – α problemin tutarsızlık derecesidir.

ii) Eğer α > 1 ise, o halde $\frac{1}{\alpha}$ problemin tutarlılık derecesiyken $\beta = 1 - \frac{1}{\alpha}$ problemin tutarsızlık derecesidir.

Tutarlılık derecesi → 0 olduğu zaman tutarsızlık derecesi → 1 olur. (Karşılıklı durum da geçerlidir).

Genel Örnek 6 için Tartışma

a_1, a_2, a_3, a_4 katsayılarının $a_1a_3 + a_2a_4 \to \infty$ olacak şekilde büyük değerler aldığını varsayalım, o zaman $\alpha \to 0$ ve $\beta \to 1$ olur.

Özel Örnek 7

a_1, a_2, a_3, a_4'ün $a_1a_3 + a_2a_4$'ü büyük yaptığı özel bir durumu görelim: $a_1 = 50$ $a_2 = 20$ $a_3 = 100$ $a_4 = 250$ olsun.

Bu durumda, tutarlılık derecesi = $\alpha = \frac{1}{\sqrt{50\cdot 100+20\cdot 250}} = \frac{1}{\sqrt{10000}} = \frac{1}{100} = 0.01$, ve tutarsızlık derecesi = β = 0.99'dur.

Özel Örnek 7'nin öncelik vektörü [1 100(0.01) 250(0.01)] = [1 1 2.5], normalleştirilmiş hali $\begin{bmatrix} \frac{2}{9} & \frac{2}{9} & \frac{5}{9} \end{bmatrix}$ olur.

Özel Örnek 8

Başka bir durum da a_1, a_2, a_3, a_4'ün $a_1a_3 + a_2a_4$'ü çok küçük yaptığı bir durumdur: $a_1 = 0.02$ $a_2 = 0.05$ $a_3 = 0.03$ $a_4 = 0.02$ olsun.

Bu durumda, $\alpha = \frac{1}{\sqrt{0.02\cdot 0.03+0.05\cdot 0.02}} = \frac{1}{0.04} = 25 > 1$ olarak elde edilir. O halde $\frac{1}{\alpha} = \frac{1}{25} = 0.04$ problemin tutarlılık derecesidir ve tutarsızlık derecesi de 0.96'dır.

Özel Örnek 8'in öncelik vektörü [1 $a_3\alpha$ $a_4\alpha$] = [1 0.03(25) 0.02(25)] = [1 0.75 0.50], normalleştirilmiş hali $\begin{bmatrix} \frac{4}{9} & \frac{3}{9} & \frac{3}{9} \end{bmatrix}$ olur. Doğrulayalım:

$\frac{y}{x} = \frac{\frac{3}{9}}{\frac{4}{9}} = 0.75$ olur 0.03 yerine; yani α = 25 kez (ya da %2500) daha büyüktür;

$\frac{z}{x} = \frac{\frac{2}{9}}{\frac{4}{9}} = 0.50$ olur 0.02 yerine; yani α = 25 kez daha büyüktür;

x = 0.50y + 1.25z olur x = 0.02y + 0.05z yerine (Hem 0.50, 0.02'den hem de 1.25, 0.05'ten 25 kez büyüktür); çünkü $\frac{4}{9} = 0.50\left(\frac{3}{9}\right) + 1.25\left(\frac{2}{9}\right)$.

8 Jean Dezert'in Zayıf Tutarsız Örnekleri

8.1 Jean Dezert'in Zayıf Tutarsız Örneği 9

$\alpha_1, \alpha_2, \alpha_3, \alpha_4 > 0$ parametrelerimiz olsun. O halde:

$$\begin{cases} \left.\begin{array}{l} (5) \quad \frac{y}{x} = 3\alpha_1 \\ (6) \quad \frac{x}{z} = 4\alpha_2 \end{array}\right\} \Rightarrow \frac{y}{x}\cdot\frac{x}{z} = (3\alpha_1)\cdot(4\alpha_2) \Rightarrow \frac{y}{z} = 12\alpha_1\alpha_2 \\ (7) \quad \frac{y}{z} = 5\alpha_3 \end{cases}$$

$\frac{y}{z} = 12\alpha_1\alpha_2$ eşitliğinin $\frac{y}{z} = 5\alpha_3$ ile tutarlı olması için $12\alpha_1\alpha_2 = 5\alpha_3$ eşitliğine veya

$2.4\alpha_1\alpha_2 = \alpha_3$ (parametrik eşitliğine) (8)

sahip olmamız gerekir. Bu sistemi çözersek:

$$\begin{cases} \frac{y}{x} = 3\alpha_1 \Rightarrow y = 3\alpha_1 \cdot x \\ \frac{x}{z} = 4\alpha_2 \Rightarrow x = 4\alpha_2 \cdot z \\ \frac{y}{z} = 5\alpha 3 \Rightarrow y = 12\alpha_1\alpha_2 z \end{cases}$$

şu genel çözümü elde ederiz:

$[4\alpha_2 z \quad 5(2.4\alpha_1\alpha_2)z \quad z]$

$[4\alpha_2 z \quad 12\alpha_1\alpha_2 z \quad z]$

Genel normalleştirip öncelik vektörü: $\left[\frac{4\alpha_2}{4\alpha_2+12\alpha_1\alpha_2+1} \quad \frac{12\alpha_1\alpha_2}{4\alpha_2+12\alpha_1\alpha_2+1} \quad \frac{1}{4\alpha_2+12\alpha_1\alpha_2+1}\right]$, burada $\alpha_1, \alpha_2 > 0$; $(\alpha_1\alpha_2 = 2.4\alpha_1\alpha_2)$.

Hangi α_1 ve α_2 en iyi sonucu verir? Bunu nasıl ölçmek gerekir? Bu en büyük zorluktur. α-İndirgeme Yöntemi tüm çözümleri (matrisi tutarlı yapan tüm olası öncelik vektörleri) içerir.

Tüm orantılarla tutarlı olmamız (yani parametrelerin sayısal değerlerini bulmak için Adillik İlkesini kullanmak) gerektiğinden tüm üç orantıya (5), (6), (7)'ye aynı indirgeme olmalıdır; buradan

$\alpha_1 = \alpha_2 = \alpha_3 > 0$ (9)

Parametrik eşitlik (8) $2.4\alpha_1^2 = \alpha_1$ veya $2.4\alpha_1^2 - \alpha_1 = 0$, $\alpha_1(2.4\alpha_1 - 1) = 0$, buradan $\alpha_1 = 0$ veya $\alpha_1 = \frac{1}{2.4} = \frac{5}{12}$. (9) ile çeliştiğinden $\alpha_1 = 0$ reddedilir. Sistemimiz şu hale gelir:

$$\begin{cases} \frac{y}{x} = 3 \cdot \frac{5}{12} = \frac{15}{12} & (10) \\ \frac{x}{z} = 4 \cdot \frac{5}{12} = \frac{20}{12} & (11) \\ \frac{y}{z} = 5 \cdot \frac{5}{12} = \frac{25}{12} & (12) \end{cases}$$

(10) ve (11)'in beraber şunu ortaya çıkardığını görüyoruz: $\frac{y}{x} \cdot \frac{x}{z} = \frac{15}{12} \cdot \frac{20}{12}$ veya $\frac{y}{z} = \frac{25}{12}$, böylece (12) ile şimdi tutarlı hale gelmiştir. (11)'den $x = \frac{20}{12}z$ ve (12)'den $y = \frac{25}{12}z$ elde ederiz. Öncelik vektörü $\left[\frac{20}{12}z \quad \frac{25}{12}z \quad 1z\right]$ ve normalleştirilmiş hali $\frac{\frac{20}{12}}{\frac{20}{12}+\frac{25}{12}+1} = \frac{\frac{20}{12}}{\frac{20}{12}+\frac{25}{12}+\frac{12}{12}} = \frac{20}{57}$, $\frac{\frac{25}{12}}{\frac{57}{12}} = \frac{25}{57}$, $\frac{1}{\frac{57}{12}} = \frac{12}{57}$; yani

$$\begin{matrix} C_1 & C_2 & C_3 \\ \end{matrix}$$
$$\begin{bmatrix} \frac{20}{57} & \frac{25}{57} & \frac{12}{57} \end{bmatrix}^T \quad (13)$$

$$\begin{matrix} & C_1 & C_2 & C_3 \\ \cong & [0.3509 & \boxed{0.4386} & 0.2105]^T \end{matrix}$$ olur.

(0.4386 ↑ en yüksek öncelik)

Sonucu inceleyelim:

$$\begin{matrix} C_1 & C_2 & C_3 \\ \frac{20}{57} & \frac{25}{57} & \frac{12}{57} \\ x & y & z \end{matrix}$$
(vector transposed: $\left[\frac{20}{57}\ \frac{25}{57}\ \frac{12}{57}\right]^T$)

<u>Oranlar:</u>

$$\frac{y}{x} = \frac{\frac{25}{57}}{\frac{20}{57}} = \frac{25}{20} = 1.25$$ olur 3 yerine;

$$\frac{x}{z} = \frac{\frac{20}{57}}{\frac{12}{57}} = \frac{20}{12} = \frac{5}{3} = 1.\overline{6}$$ olur 4 yerine;

$$\frac{y}{z} = \frac{\frac{25}{57}}{\frac{12}{57}} = \frac{25}{12} = 2.08\overline{3}$$ olur 5 yerine;

<u>İndirgeme Yüzdesi:</u>

$$\frac{\frac{25}{20}}{3} = \frac{5}{12} = \alpha_1 = \%41.\overline{6}$$

$$\frac{\frac{20}{12}}{4} = \frac{5}{12} = \alpha_1 = \%41.\overline{6}$$

$$\frac{\frac{25}{12}}{5} = \frac{5}{12} = \alpha_1 = \%41.\overline{6}$$

Sonuç itibariyle problemde sırasıyla 3, 4 ve 5'e ait eşit olan tüm orijinal orantılar aynı faktörle ($\alpha_1 = \frac{5}{12}$) çarpılarak indirgenmiştir; yani her birinin %41.6'sı alınmıştır.

Böylelikle her bir faktörü kendisinin %41.6'sına indirgemek adil olmuştur. Ama Saaty'nin yönteminde durum böyle değildir. Normalleştirilmiş öncelik vektörü: $\begin{matrix} C_1 & C_2 & C_3 \\ [0.2797 & 0.6267 & 0.0936]^T \\ x & y & z \end{matrix}$ şeklindedir. Burada,

<u>Oranlar:</u>

$$\frac{y}{x} = \frac{0.6267}{0.2797} \cong 2.2406$$ olur 3 yerine;

$$\frac{x}{z} = \frac{0.2797}{0.0936} \cong 2.29882$$ olur 4 yerine;

$$\frac{y}{z} = \frac{0.6267}{0.0936} \cong 6.6955$$ olur 5 yerine;

<u>İndirgeme Yüzdesi:</u>

$$\frac{2.2406}{3} \cong \%74.6867$$

$$\frac{2.29882}{4} \cong \%74.7050$$

$$\frac{6.6955}{5} \cong \%133.9100$$

Örneğin niye 3'e eşit olan ilk orantı %74.6867'sine indirgenirken 4'e eşit olan ikinci orantı (yakın da olsa) diğer bir yüzde olan % 74.7050'sine indirgenmiştir?

Hatta daha da şüphe çekeni 5'e eşit olan üçüncü orantımızın katsayısı %133.9100'ına yükseltgenirken önceki iki orantımız indirgenmişti. Bunlar için ne gibi bir makul açıklama vardır?

İşte bundandır ki α-İ/Adillik İlkesinin daha iyi gerekçeli olduğunu düşünüyoruz. Aynı problemi matrisleri kullanarak da çözebiliriz. (5), (6), (7) lineer parametrik homojen bir sistem oluşturmak için başka bir şekilde yazılabilir:

$$\begin{cases} 3\alpha_1 - y = 0 \\ x - 4\alpha_2 z = 0 \\ y - 5\alpha_3 z = 0 \end{cases} \tag{14}$$

Eşlenik matrisi de:

$$P_1 = \begin{bmatrix} 3\alpha_1 & -1 & 0 \\ 1 & 0 & -4\alpha_2 \\ 0 & 1 & -5\alpha 3 \end{bmatrix} \tag{15}$$

a) Eğer det(P_1) ≠ 0 o halde (10)'daki sistemin sadece x = y = z = 0 sıfırlı çözümü vardır.

b) Dolayısıyla, det(P_1) = 0 veya $(3\alpha_1)(4\alpha_2) - 5\alpha_3 = 0$ veya $2.4\alpha_1\alpha_2 - \alpha_3 = 0$'a sahip olmamız gerekir, böylelikle (8)'deki aynı parametrik eşitliği elde ederiz.

Bu durumda homojen parametrik lineer sistem (14)'ün üçlü sonsuz çözümü vardır.

Bu yöntem Saaty'nin yönteminin bir genişletmesidir, zira α_1, α_2 ve α_3 parametrelerini manipüle etme imkânımız vardır.

Örneğin, eğer ikinci bir kaynak bize $\frac{x}{z}$'nin $\frac{y}{x}$'in 2 katı kadar indirgenmesi ve $\frac{y}{z}$'nin $\frac{y}{x}$'ten 3 kat daha az indirgenmesi gerektiğini söylerse o zaman $\alpha_2 = 2\alpha_1$ ve buna bağlı olarak $\alpha_3 = \frac{\alpha_1}{3}$'e eşitleriz ve orijinal (5), (6), (7) sistemi aşağıdakine dönüşür:

$$\begin{cases} \frac{y}{x} = 3\alpha_1 \\ \frac{x}{z} = 4\alpha_2 = 4(2\alpha_1) = 8\alpha_1 \\ \frac{y}{z} = 5\alpha_3 = 5\left(\frac{\alpha_1}{3}\right) = \frac{5}{3}\alpha_1 \end{cases} \tag{16}$$

ve bunu da aynı yolla çözeriz.

8.2 Zayıf Tutarsız Örnek 10

Jean Dezert'in Zayıf Tutarsız Örnek 9'unu bir tercih daha ekleyerek karmaşıklaştıralım:

C_2'yi, C_1 ve C_3'ün toplamına 1.5 kat tercih edelim.

Yeni sistem:

$$\begin{cases} \frac{y}{x} = 3 \\ \frac{x}{z} = 4 \\ \frac{y}{z} = 5 \\ y = 1.5(x+z) \\ x, y, z \in 0,1 \\ x + y + z = 1 \end{cases} \tag{17}$$

Parametreleştirirsek:

$$\begin{cases} \frac{y}{x} = 3\alpha_1 \\ \frac{x}{z} = 4\alpha_2 \\ \frac{y}{z} = 5\alpha_3 \\ y = 1.5\alpha_4(x+z) \\ x, y, z \in 0,1 \\ x + y + z = 1 \end{cases} \tag{18}$$

$$\alpha_1, \alpha_2, \alpha_3, \alpha_4 > 0$$

Eşlenik matrisi:

$$P_2 = \begin{bmatrix} 3\alpha_1 & -1 & 0 \\ 1 & 0 & -4\alpha_2 \\ 0 & 1 & -5\alpha_3 \\ 1.5\alpha_4 & -1 & 1.5\alpha_4 \end{bmatrix} \tag{19}$$

P_2 matrisinin mertebesi (18)'deki sistemin sıfırlı olmayan bir sonuca sahip olması için kesinlikle 3'ten az olmalıdır. Eğer (19)'daki ilk üç satırı alırsak (determinantı 0 olması gereken) P_1 matrisini elde ederiz, dolayısıyla bundan önceki $2.4\alpha_1 4\alpha_2 = \alpha_3$ parametrik eşitliğini elde ederiz.

Eğer 1, 3 ve 4. satırları alırsak, bunlar C2 ve diğer C1 ve C3 kriterleri içerdiğinden determinantı 0 olması gereken şu matrisi elde ederiz:

$$P_3 = \begin{bmatrix} 3\alpha_1 & -1 & 0 \\ 0 & 1 & -5\alpha_3 \\ 1.5\alpha_4 & -1 & 1.5\alpha_4 \end{bmatrix} \tag{20}$$

$$\det(P_3) = [3\alpha_1 (1.5\alpha_4) + 5\alpha_3 (1.5\alpha_4) + 0] - [0 + 3\alpha_1 (5\alpha_3) + 0] = 4.5\alpha_1\alpha_4 + 7.5\alpha_3\alpha_4 - 15\alpha_1\alpha_3 = 0 \tag{21}$$

Eğer yandakini alırsak: $P_4 = \begin{bmatrix} 1 & 0 & -4\alpha_2 \\ 0 & 1 & -5\alpha_3 \\ 1.5\alpha_4 & -1 & 1.5\alpha_4 \end{bmatrix}$ (22)

O halde det(P_4) = [1.5α_4 + 0 + 0] – [-6$\alpha_2\alpha_4$ + 5α_3 + 0] = 1.5α_4 + 6$\alpha_2\alpha_4$ – 5α_3 = 0 (23)

Eğer şunu alırsak

$$P_5 = \begin{bmatrix} 3\alpha_1 & -1 & 0 \\ 1 & 0 & -4\alpha_2 \\ 1.5\alpha_4 & -1 & 1.5\alpha_4 \end{bmatrix} \quad (24)$$

O halde det(P_5) = [0 + 0 + 6 $\alpha_2\alpha_4$] – [0 + 12$\alpha_1\alpha_2$ – 1.5α_4] = 6$\alpha_2\alpha_4$ – 12 $\alpha_1\alpha_4$ + 1.5α_4 = 0 (25)

Böylelikle, bu dört parametrik eşitlik bir parametrik sistem oluşturur ki bunun sıfırlı olmayan bir çözümü olması gerekmektedir:

$$\begin{cases} 2.4\alpha_1\alpha_2 - \alpha_3 = 0 \\ 4.5\alpha_1\alpha_4 + 7.5\alpha_3\alpha_4 - 15\alpha_1\alpha_3 = 0 \\ 1.5\alpha_4 + 6\alpha_2\alpha_4 - 5\alpha_3 = 0 \\ 6\alpha_2\alpha_4 - 12\alpha_1\alpha_2 + 1.5\alpha_4 = 0 \end{cases} \quad (26)$$

Eğer başta elde ettiğimiz gibi $\alpha_1 = \alpha_2 = \alpha_3 = \frac{5}{12} > 0$'ı dikkate alırsak ve sonra (26)'daki sistemin son üç eşitliğindeki tüm α'ları bu değerle değiştirirsek şunu elde ederiz:

$$4.5\left(\frac{5}{12}\right)\alpha_4 + 7.5\left(\frac{5}{12}\right)\alpha_4 - 15\left(\frac{5}{12}\right)\left(\frac{5}{12}\right) = 0 \Rightarrow \alpha_4 = 0.5208\overline{3} = \frac{25}{48}$$

$$1.5\alpha_4 + 6\left(\frac{5}{12}\right)\alpha_4 - 5\left(\frac{5}{12}\right) = 0 \Rightarrow \alpha_4 = 0.5208\overline{3}$$

$$6\left(\frac{5}{12}\right)\alpha_4 - 12\left(\frac{5}{12}\right)\left(\frac{5}{12}\right) + 1.5\alpha_4 = 0 \Rightarrow \alpha_4 = 0.5208\overline{3}$$

α_4, $\alpha_1 = \alpha_2 = \alpha_3$'e eşit olamazdı çünkü α_4 ek bir tercihtir, zira satırların sayısı sütunların sayısından büyüktür.

Sonuç itibariyle, y = 1.5(x + z) dördüncü tercihini eklemek zorunda kalmadan sistem öncekiyle aynı çözüme sahiptir ve tutarlıdır.

9 Jean Dezert'in Güçlü Tutarsız Örnekleri

9.1 Jean Dezert'in Güçlü Tutarsız Örneği 11

9.1.1 Problem Tanımı

Tercih matrisimiz: $M_1 = \begin{pmatrix} 1 & 9 & \frac{1}{9} \\ \frac{1}{9} & 1 & 9 \\ 9 & \frac{1}{9} & 1 \end{pmatrix}$, böylelikle, $\begin{cases} x = 9y, x > y \\ x = \frac{1}{9}z, x < z \\ y = 9z, y > z \end{cases}$ ulaşılabilir.

Diğer üç eşitlik olan $y = \frac{1}{9}x, z = 9x, z = \frac{1}{9}y$ diğer üç eşitlikten doğrudan çıkarılabildiğinden bunları eleyebiliriz.

(Yukarıdaki birinci ve üçüncü eşitsizliklerdeki) x > y ile y > z'den x > z'ye ulaşabiliriz ancak ikinci eşitsizlik bize tam tersi olan x < z ifadesini vermektedir; bu sebepten dolayı güçlü bir çelişki/tutarsızlık ile karşı karşıyayız. Ya da, her üçünü birleştirirsek x > y > z > x'i elde ederiz ki bu da yine güçlü bir tutarsızlıktır.

Parametreleştirelim: (burada α_1, α_2, α_3 > 0'dır)

$$\begin{cases} x = 9\alpha_1 y & (27) \\ x = \frac{1}{9}\alpha_2 z & (28) \\ y = 9\alpha_3 z & (29) \end{cases}$$

(27)'den $y = \frac{1}{9\alpha_1}x$'i, (28)'den $z = \frac{1}{9\alpha_2}x$'i elde ederiz. Bu (29)'da yerine konduğunda $y = 9\alpha_3\left(\frac{9}{\alpha_2}x\right) = \frac{81\alpha_3}{\alpha_2}x$'e ulaşırız. Böylece $\frac{1}{9\alpha_1}x = \frac{81\alpha_3}{\alpha_2}x$ veya $\alpha_2 = 729\alpha_1\alpha_3$ (parametrik eşitlik) olur.

Sistemin genel çözümü: $\left(x, \ \frac{1}{9\alpha_1}x, \ \frac{9}{\alpha_2}x\right)$ ve genel öncelik vektörü de $\begin{bmatrix} 1 & \frac{1}{9\alpha_1} & \frac{9}{\alpha_2} \end{bmatrix}$'dir.

Adillik İlkesini dikkate alırsak, o zaman $\alpha_1 = \alpha_2 = \alpha_3 = \alpha > 1$ parametrik eşitlik $\alpha = 729\alpha^2$'de yerine konur. Buradan $\alpha = 0$ (iyi değil) ve $\alpha = \frac{1}{729} = \frac{1}{9^3}$'dür. Özel öncelik vektörü $[1 \quad 9^2 \quad 9^4] = [1 \quad 81 \quad 6561]$ ve normalleştirilmiş hali de $\begin{bmatrix} \frac{1}{6643} & \frac{81}{6643} & \frac{6561}{6643} \end{bmatrix}$ olarak bulunur. Tutarlılık $\alpha = \frac{1}{729} = 0.00137$ aşırı derecede düşük (ve tutarsızlık $\beta = 1 - \alpha = 0.99863$ çok büyük) çıktığından bu sonucu ihmal edebiliriz.

9.1.2 Açıklamalar:

a) Eğer M_1'de altı tane 9'u daha büyük bir sayı ile değiştirdiğimizde sistemin tutarsızlığı artar. Mesela 11'i kullanalım. $\alpha = \frac{1}{11^3} = 0.00075$ (tutarlılık) olurken tutarsızlık $\beta = 0.99925$ olur.

b) M_1'deki tüm 9'ları 1'den daha büyük ancak 9'dan küçük bir sayı ile değiştirdiğimizde sistemin tutarlılığı düşer. Mesela 2'yi kullanalım. $\alpha = \frac{1}{2^3} = 0.125$ ve $\beta = 0.875$ olur.

c) Tüm 9'ları 1 ile değiştirdiğimizde tutarlılık 1 olur.

d) Yine tüm 9'ları 1'den küçük pozitif bir sayıyla değiştirirsek tutarlılık tekrar düşer. Örneğin, 0.8 ile değiştirecek olursak $\alpha = \frac{1}{0.8^3} = 1.953125 > 1$, buradan $\alpha = \frac{1}{\alpha} = 0.512$ (tutarlılık) ve $\beta = 0.488$ (tutarsızlık) olur.

9.2 Jean Dezert'in Güçlü Tutarsız Örneği 12

M1'e benzer olan ancak tüm 9'ların yerini 5'lerin aldığı tercih matrisimiz şu şekilde olsun:

$$M_2 = \begin{pmatrix} 1 & 5 & \frac{1}{5} \\ \frac{1}{5} & 1 & 5 \\ 5 & \frac{1}{5} & 1 \end{pmatrix},\ \alpha = \frac{1}{5^3} = 0.008 \text{ (tutarlılık) ve } \beta = 0.992 \text{ (tutarsızlık) olur.}$$

Öncelik vektörü $[1 \quad 5^2 \quad 5^4]$ ve normalleştirilmiş hali $\begin{bmatrix} \frac{1}{651} & \frac{25}{651} & \frac{625}{651} \end{bmatrix}$ olarak bulunur. M2, M1'den biraz daha tutarlıdır çünkü 0.008 > 0.00137, ancak yine de bu yeterli değildir, bu yüzden bu sonuç da ihmal edilmiştir.

9.3 Jean Dezert'in Güçlü Tutarsız Örneklerinin Genelleştirmesi

Genel Örnek 13 için tercih matrisimiz şöyle olsun:

$$M_t = \begin{pmatrix} 1 & t & \frac{1}{t} \\ \frac{1}{t} & 1 & t \\ t & \frac{1}{t} & 1 \end{pmatrix},$$

$t > 0$ ve $c(M_t)$ M_t'nin tutarlılığı, $i(M_t)$ M_t'nin tutarsızlığı olsun. Adillik İlkesi için şunlara sahibiz:

$$\lim_{t\to 1} c(M_t) = 1 \text{ ve } \lim_{t\to 1} i(M_t) = 0;$$
$$\lim_{t\to +\infty} c(M_t) = 0 \text{ ve } \lim_{t\to +\infty} i(M_t) = 1;$$
$$\lim_{t\to 0} c(M_t) = 0 \text{ ve } \lim_{t\to 0} i(M_t) = 1.$$

Aynı zamanda $\alpha = \frac{1}{t^3}$, öncelik vektörü $[1 \quad t^2 \quad t^4]$ ve normalleştirilmiş hali de $\begin{bmatrix} \frac{1}{1+t^2+t^4} & \frac{t^2}{1+t^2+t^4} & \frac{t^4}{1+t^2+t^4} \end{bmatrix}$'tür.

x > y > z > x veya benzer şekilde x < y < x vb. haldeki güçlü çelişkinin bulunduğu ve tutarlılığın çok küçük olduğu durumlarda, ya x = y = z = ⅓ (böylece, Saaty'nin AHP'sinde olduğu gibi, hiçbir kriter diğerine tercih edilir durumda olmaz) ya da x + y + z = 1'i (ki C1 ∪ C2 ∪ C3 şeklinde toplam bilinmezliğe de sahip olunduğu anlamına gelir) dikkate alabiliriz.

10 Güçlü Tutarsız Örnek 14

C = {C1, C2} ve P = {C1, C2'den iki kat daha fazla önemli; C2, C1'den beş kat daha fazla önemli} şeklinde olsun. m(C1) = x, m(C2) = y şeklinde ifade edilsin. O halde, x = 2y ve y = 5x olur (burada güçlü bir tutarsızlık vardır zira birinci eşitlikten x > y elde edilirken ikincisinden y > x elde edilmektedir).

Parametreleştirelim: $x = 2\alpha_1 y$, $y = 5\alpha_2 x$, buradan $2\alpha_1 = \frac{1}{5\alpha_2}$ veya $10\alpha_1\alpha_2 = 1$ ifadesine ulaşırız.

Eğer Adillik İlkesini dikkate alırsak, o zaman $\alpha_1 = \alpha_2 = \alpha > 0$ ki bu durumda $\alpha = \frac{\sqrt{10}}{10} \approx \%31.62$ tutarlılık ile [0.39 0.61] öncelik vektörüne ulaşılır, sonuç itibariyle y > x'tir. Tutarlılık ötesi (veya nötrosofik) mantıkta olduğu gibi bir açıklama şöyle yapılabilir: Tercihlerin dürüst ancak öznel olduğunu dikkate alırız, dolayısıyla eşanlı olarak doğru olan iki çelişen ifadeye sahip olmak mümkündür, zira bir bakış açısına göre C1 kriteri C2'den daha önemli olabilirken başka bir bakış açısına göreyse C2 kriteri C1'den daha önemli olabilir. Karar verme problemimizde daha fazla bilgi sahibi olamayışımız ve hızlıca bir karar alma durumunda kalışımızdan C2'yi tercih edebiliriz, zira C2 kriteri C1'den 5 kat daha fazla önemliyken C1 kriteri C2'den ancak 2 kat daha fazla önemlidir; yani 5 > 2'dir.

Eğer bir acele yoksa, bu gibi bir ikilemde C1 ve C2 üzerinde daha fazla araştırma yaparak daha tedbirli olunmasında fayda vardır.

Eğer Örnek 14'ü x = 2y ve y = 2x şeklinde değiştirirsek (iki katsayı birbirine eşitlendi), α = ½ elde ederiz. Böylece öncelik vektörü [0.5 0.5] olur ve bu durumda karar verme problemi karar verilemez bir hale dönüşür.

11 Lineer Olmayan Eşitlik Sistemi Örneği 15

C = {C1, C2, C3} ve m(C1) = x, m(C2) = y, m(C3) = z olsun.

F de şu şekilde verilsin:

1. C1, C2 ve C3'ün çarpımından iki kat daha fazla önemli.
2. C2, C3'ten 5 kat daha fazla önemli.

Şu sistemi oluştururuz: x = 2yz (lineer olmayan eşitlik) ve y = 5z (lineer eşitlik). Bu karma sistemin genel çözüm vektörü: $[10z^2 \;\; 5z \;\; z]$, z > 0'dır. Bir irdeleme yapacak olursak:

a) y > z olduğunu kesin olarak görmekteyiz zira kesin surette pozitif z için 5z > z'dir. Ancak x'in pozisyonu ne olurdu ile ilgili bir şey görmemekteyiz.

b) Her bir vektör bileşenini z > 0 ile bölerek genel çözüm vektörünü sadeleştirelim. Sonuç olarak [10z 5 1] vektörünü elde ederiz.

   Eğer z ∈ (0, 0.1) ise o zaman y > z > x.

   Eğer z = 0.1 ise o zaman y > z = x.

   Eğer z ∈ (0.1, 0.5) ise o zaman y > x > z.

   Eğer z = 0.5 ise o zaman y = x > z.

   Eğer z > 0.5 ise o zaman x > y > z.

12 Karma Lineer Olmayan/Lineer Eşitlik/Eşitsizlik Sistemi Örneği 16

Önceki Örnek 15'in çok fazla değişik biçimleri olduğundan (önceki iki tercihe ek olarak) yeni bir tercihin sisteme dahil edildiğini varsayalım: *3. C1, C3'ten daha az önemlidir.*

Karma sistem şimdi şu duruma ulaşır: x = 2yz (lineer olmayan eşitlik), y = 5z (lineer eşitlik), ve x < z (lineer eşitsizlik).

Bu karma sistemin genel çözüm vektörü: $[10z^2 \quad 5z \quad z]$, burada $z > 0$ ve $10z^2 < z$'dir. Son iki eşitsizlikten $z \in (0, 0.1)$ elde ederiz. Buradan öncelikler $y > z > x$ olur.

13 İleriki Araştırmalar

α-İ ÇKKV ile ideal çözüme benzerlik yoluyla tercih sıralama tekniği (TOPSIS), basit toplamlı ağırlıklandırma (SAW), sıra sayılı tercihlerin bütünleştirildiği Borda-Kendall (BK) yöntemi, veri zarflama analizindeki (DEA) çapraz etkinlik değerlendirme yöntemi gibi diğer yöntemler arasındaki bağı araştırmak.

14 Sonuç

Çok Kriterli Karar Verme için "α-İndirgeme ÇKKV" adını verdiğimiz yeni bir yöntemi tanıttık. Bu yöntemin ilk kısmında her bir tercih lineer veya lineer olmayan eşitlik veya eşitsizliğe dönüştürülmekte ve hepsi beraber çözülen bir sistemi – pozitif sonuçların ortaya konduğu genel çözümü bulunur – oluşturmaktadır. Eğer sistem sadece sıfırlı bir çözüme sahipse veya tutarsızsa, sistemin katsayıları parametrik hale getirilir.

Bu yöntemin ikinci kısmında parametrelerin sayısal değerlerini bulmak için bir ilke seçilir (Biz burada Adillik İlkesi, İndirgeme için Uzman Görüşü veya Tutarlılık/Tutarsızlık Eşiği belirlemeyi önerdik).

Teşekkür

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Recreational Math: Puzzle Me!

Florentin Smarandache

Findings patterns

Input/Output Machine. Each table below represents a different rule. Look for input/output patterns to fill in the missing words, letter, or numbers. Then state each rule. [*Means the rule cannot be applied to this input.]

Input	Output
Candy	2
Juan	1
Alicia	3
George	1
Maya	--
Barbara	--
--	4

The rule: ________________

Input	Output
Short	0
Fair	*
Sad	A
Smooth	*
Shiny	I
Hard	--
--	0

The rule: ___________

Input	Output
Candy	2
Juan	1
Alicia	3
George	1
Maya	--
Barbara	--
--	4

The rule: ____________

Input	Output
Ship	Q
Boat	U
Car	S
Trolley	Z
Bus	--
Motorcycle	--
--	--

The rule: ________________

Input	Output
0	1
1	2
2	5
3	10
4	--
5	--
--	82

The rule: ___________

Input#1	Input#2	Output
4	6	4
9	3	12
2	3	6
10	8	4
7	3	--
4	4	--
--	4	10

The rule: ____________

YYUR YYUB ICUR 124C. Solution elsewhere.

Complete the following tables:

IN	OUT
math	m
zebra	7
house	h
pick	p
rose	r
school	-
mouse	-
…	
guessing	9
involves	9
taking	7
a	2
risk	5
but	4
it	3
is	3
often	6
a	-
good	-
strategy	-

IN	OUT
math	b
zebra	f
love	p
line	l
stem	f
elephant	f
sit	j
pin	-
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A Scientist and Haiku Poet

Florentin Smarandache
Phoenix, AZ, USA

FLORIN VASILIU was born in Bucharest, Romania, in March 11, 1929. He is graduated as chemist engineer, and worked in production, design, research, in higher education and in the field of the products quality. He published eight technical books, among which the most important are: *Metode de analiză a calităţii produselor*, (Methods of analyses of products quality). Ceres Publishing House, Bucharest 1983, (with N. Verciuc), *Controlul modern al calităţii produselor* (Modern control of products' quality), Ceres Publishing House, Bucharest 1985 first edition, and 1987 the second edition. He published 46 technical and scientific papers, in "*Celuloză şi Hârtie*" (Pulp and Paper), "Revista de Chimie" (Chemical Magazine), "Calitatea Producţiei şi Metrologie" (Production quality and Metrology) and "Calitatea" (Quality) magazines. Also, he presented scientific works and papers at International Conferences and Symposiums: Conference "STAQUAREL 80" from Prague, The Seventh Conference on Probability Theory, Braşov, Romania 1982, International Symposium on Cellulose Chemistry and Technology, Iaşi România 1989. He took part at Conference inter-academic exchanges: Sofia, Bulgaria 1975, Beijing, China 1983, Prague Tchechoslovakya 1984, Hanju and Pyongyang North Korea 1986, and technical and commercial contacts in Germany, Japan, Poland, Egypt, France, Italy, Bulgaria, Tchechoslovakya, China, Korea, Finland. At present he works as professor in The University PRO HUMANITAS, Bucharest, Romania.

The Academician Octav Onicescu wrote in the preface of the *Metode de analiză a calităţii produselor*, that Florin Vasiliu is situated "...among the passionate and disinterested carriers of this new form of human activity".

After his travels in Japan, China, Korea, Florin Vasiliu wrote three books about the culture, civilization and history of Japan: *Pe Meridianul Yamato*, (On The Meridian Yamato), Sport Turism Publishing House, Bucharest, 1982, *De la Pearl Harbor la Hiroshima*, (From Pearl Harbor to Hiroshima), Dacia Publishing House, Cluj-Napoca 1986, *Interferenţe lirice – Constelaţia haiku*, (Lyrics interpretations – Haiku Constellation), Dacia Publishing House, Cluj-Napoca 1988, and in this last year he published *Umbra libelulei*, (The dragonfly's shade), Anthology of Romanian haiku, and *Tolba cu licurici* (The bag with glow worms), published by Haiku Publishing House, Bucharest.

He also published 28 literary papers and poetry in many literary magazines from Romanian and Japan, and he was honored with the Prize of "Poesis" magazine, in 1990 for poetry, studies, commentaries, and for "Haiku" Magazine". Florin Vasiliu is editor-in-chief of the HAIKU magazine for Romanian-Japanese interpenetrations. He publishes also a "Collection of the Haiku Magazine", in which appear with haiku poetry of Romanian poets.

Florin Vasiliu is President of Romanian Haiku Society, Vice president of Nipponica Society, member of the Writers' Union of Romania, member of the Haiku International Association (Japan), fellow of the Paradoxist Literary Movement Association (U.S.A.)

On the occasion of the visit in Romania at a meeting of the literary circle of Romanian Haiku Society in Bucharest, Mr. Sono Uchida, the President of Haiku International Association wrote: "The style of the meeting differs somewhat from such a gathering in Japan in that winning verses are not chosen. The participants had anonymously submitted a haiku on the theme of source (…). Vasiliu read the verses out loud and the participants discussed then (…). The seriousness of purpose and enthusiasm of the gathering were most impressive (…). The economic situation in Romania is still extremely difficult (…). It seemed to me that the hardship they endured made Romanian haijins – haiku poets – even more serious about composing haiku" ("International Haiku", in *The Daily Yomiuri*, Wednesday, July 28, 1993, p. 9).

[Published on the Forth Cover of the book "Paradoxism's Main Roots" of Florin Vasiliu, 1994.]

Review of the journal "US AND THE SKY"

The Romanian Astronomical Society of Meteors, acronymed SAMD (which has a web site at http://sarm.ccs.ro, was founded by Valentin Grigore from Târgovişte in 1993, and biannually publishes de 40-page journal "Us and the Sky" (Noi şi Cerul), which is freely distributed în and outside the country. The chief-editor is Gelu-Claudiu Radu from Cluj-Napoca, whereas Andrei Dorian Gheorghe from Bucharest with Iulian Haba from Codlea are editors. Many foreign correspondents are listed, among them Dr. Ovidiu Văduvescu from the RASC Observatory in Toronto Center – my former student at Bălcescu College in Craiova.

Articles of astronomy, well-documented, in Romanian or English, interviews, biographies of astronomers, observatories, exhibitions about astronomical phenomena, international teleconferences and meetings are presented in this periodical, together with a literary part such as "astro-humor", "astro-poetry", epigrams, science fiction, "cosmo-festivals", Romanian astro-mythology. The Spring-Summer 1999 issue, that I received, is focused on the Sun Eclipse seen from Romania at August 11, 1999.

Florentin Smarandache: polymath, professor of mathematics

Scientist and writer. Wrote in three languages: Romanian, French, and English.

He did post-doctoral researches at Okayama University of Science (Japan) between 12 December 2013 - 12 January 2014; at Guangdong University of Technology (Guangzhou, China), 19 May - 14 August 2012; at ENSIETA (National Superior School of Engineers and Study of Armament), Brest, France, 15 May - 22 July 2010; and for two months, June-July 2009, at Air Force Research Laboratory in Rome, NY, USA (under State University of New York Institute of Technology).

Graduated from the Department of Mathematics and Computer Science at the University of Craiova in 1979 first of his class graduates, earned a Ph. D. in Mathematics from the State University Moldova at Kishinev in 1997, and continued postdoctoral studies at various American Universities such as University of Texas at Austin, University of Phoenix, etc. after emigration.

In U.S. he worked as a software engineer for Honeywell (1990-1995), adjunct professor for Pima Community College (1995-1997), in 1997 Assistant Professor at the University of New Mexico, Gallup Campus, promoted to Associate Professor of Mathematics in 2003, and to Full Professor in 2008. Between 2007-2009 he was the Chair of Math & Sciences Department.

In mathematics he introduced the degree of negation of an axiom or theorem in geometry (see the Smarandache geometries which can be partially Euclidean and partially non-Euclidean, 1969, http://fs.gallup.unm.edu/Geometries.htm), the multi-structure (see theSmarandache n-structures, where a weak structure contains an island of a stronger structure, http://fs.gallup.unm.edu/Algebra.htm), and multi-space (a combination of heterogeneous spaces) [http://fs.gallup.unm.edu/Multispace.htm].

He created and studied many sequences and functions in number theory.

He generalized the fuzzy, intuitive, paraconsistent, multi-valent, dialetheist logics to the 'neutrosophic logic' (also in the Denis Howe's Dictionary of Computing, England) and, similarly, he generalized the fuzzy set to the 'neutrosophic set' (and its derivatives: 'paraconsistent set', 'intuitionistic set', 'dialethist set', 'paradoxist set', 'tautological set') [http://fs.gallup.unm.edu/ebook-Neutrosophics4.pdf]. He then generalized it to Refined Neutrosophic Logic, where T can be split into subcomponents T_1, T_2, ..., T_p, and I into I_1, I_2, ..., I_r, and F into F_1, F_2, ...,F_s, where $p+r+s = n \geq 1$. Even more: T, I, and/or F (or any of their subcomponents T_j ,I_k, and/or F_l) could be countable or uncountable infinite sets.

Also, he proposed an extension of the classical probability and the imprecise probability to the 'neutrosophic probability', that he defined as a tridimensional vector whose components are real subsets of the non-standard interval]-0, 1+[, introduced the neutrosophic measure and neutrosophic integral [http://fs.gallup.unm.edu/NeutrosophicMeasureIntegralProbability.pdf], and also extended the classical statistics to neutrosophic statistics [http://fs.gallup.unm.edu/NeutrosophicStatistics.pdf].

Since 2002, together with Dr. Jean Dezert from Office National de Recherches Aeronautiques in Paris, worked in information fusion and generalized the Dempster-Shafer Theory to a new theory of plausible and paradoxist fusion (Dezert-Smarandache Theory): http://fs.gallup.unm.edu/DSmT.htm.

In 2004 he designed an algorithm for the Unification of Fusion Theories and rules (UFT) used in bioinformatics, robotics, military.

In physics he found a series of paradoxes (see the quantum smarandache paradoxes), and considered the possibility of a third form of matter, called unmatter, which is a combination of matter and antimatter - presented at Caltech (American Physical Society Annual Meeting, 2010) and Institute of Atomic Physics (Magurele, Romania – 2011).

Based on a 1972 manuscript, when he was a student in Rm. Valcea, he published in 1982 the hypothesis that 'there is no speed barrier in the universe and one can construct any speed', (http://scienceworld.wolfram.com/physics/SmarandacheHypothesis.html). This hypothesis was partially validated on September 22, 2011, when researchers at CERN experimentally proved that the muonneutrino particles travel with a speed greater than the speed of light.

Upon his hypothesis he proposed an Absolute Theory of Relativity [free of time dilation, space contraction, relativicticsimultaneities and relativistic paradoxes which look alike science fiction not fact]. Then he extended his research to a more diversified Parameterized Special Theory of Relativity (1982): http://fs.gallup.unm.edu/ParameterizedSTR.pdf and generalized the Lorentz Contraction Factor to the Oblique-Contraction Factor for lengths moving at an oblique angle with respect to the motion direction, then he found the Angle-Distortion Equations (1983): http://fs,gallup.unm.edu/NewRelativisticParadoxes.pdf.

He considered that the speed of light in vacuum is variable, depending on the moving reference frame; that space and time are separated entities; also the redshift and blueshift are not entirely due to the Doppler Effect, but also to the Medium Gradient and Refraction Index (which are determined by the medium composition: i.e. its physical elements, fields, density, heterogeneity, properties, etc.); and that the space is not curved and the light near massive cosmic bodies bends not because of the gravity only as the General Theory of Relativity asserts (Gravitational Lensing), but because of the Medium Lensing.

In order to make the distinction between "clock" and "time", he suggested a *first experiment* with different clock types for the GPS clocks, for proving that the resulted dilation and contraction factors are different from those obtained with the cesium atomic clock; and a *second experiment* with different medium compositions for proving that different degrees of redshifts/blushifts and different degrees of medium lensing would result.

In philosophy he introduced in 1995 the 'neutrosophy', as a generalization of Hegel's dialectic, which is the basement of his researches in mathematics and economics, such as 'neutrosophic logic', 'neutrosophic set', 'neutrosophic probability', 'neutrosophicstatistics'.

Neutrosophy is a new branch of philosophy that studies the origin, nature, and scope of neutralities, as well as their interactions with different ideational spectra. This theory considers every notion or idea <A> together with its opposite or negation <Anti-A> and the spectrum of "neutralities" <Neut-A> (i.e. notions or ideas located between the two extremes, supporting neither <A> nor <Anti-A>). The <Neut-A> and <Anti-A> ideas together are referred to as <Non-A>. According to this theory every idea <A> tends to be neutralized and balanced by <Anti-A> and <Non-A> ideas - as a state of equilibrium [http://fs.gallup.unm.edu/neutrosophy.htm].

He extended the Lupasco-Nicolescu's *Law of Included Middle* [<A>, <nonA>, and a third value <T> which resolves their contradiction at another level of reality] to the *Law of Included Multiple-Middle* [<A>, <antiA>, and <neutA>, where <neutA> is split into a multitude of neutralities between <A> and <antiA>, such as <neut$_1$A>, <neut$_2$A>, etc.]. The <neutA> value (i.e. neutrality or indeterminacy related to <A>) actually comprises the included middle value. Also, he extended the *Principle of Dynamic Opposition* [opposition between <A> and <antiA>] to the *Principle of Dynamic Neutrosophic Opposition* [which means oppositions among <A>, <antiA>, and <neutA>]; [http://fs.gallup.unm.edu/LawIncludedMultiple-MIddle.pdf].

Other small contributions he had in psychology [http://fs.gallup.unm.edu/psychology.htm], and in sociology [http://fs.gallup.unm.edu/sociology.htm].

Invited to lecture at University of Berkeley (2003), NASA Langley Research Center-USA (2004), NATO Advance Study Institute-Bulgaria (2005), Jadavpur University-India (2004), Institute of Theoretical and Experimental Biophysics-Russia (2005), Bloomsburg University-USA (1995), University Sekolah Tinggi Informatika & Komputer Indonesia-Malang and University Kristen Satya WacanaSalatiga-Indonesia (2006), Minufiya University (Shebin Elkom)-Egypt (2007), Air Force Institute of Technology Wright-Patterson AFB in Dayton [Ohio, USA] (2009), Universitatea din Craiova - Facultatea de Mecanica [Romania] (2009), Air Force Research Lab & Griffiss Institute [Rome, NY, USA] (2009), COGIS 2009 (Paris, France), ENSIETA (Brest, Franta) - 2010, Romanian Academy - Institute of Solid Mechanics and Commission of Acoustics (Bucharest - 2011), Guangdong University of Technology (Guangzhou, China) - 2012, Okayama University of Sciences (Japan) - 2013, Osaka University (Japan) - 2014, Universidad Nacional de Quilmes (Argentina) - 2014, Universidad Complutense de Madrid (Spain) - 2014, etc. Presented papers at many Sensor or Information Fusion International Conferences {Australia - 2003, Sweden - 2004, USA (Philadelphia - 2005, Seattle - 2009, Chicago - 2011), Spain (Barcelona - 2005, Salamanca - 2014), Italy - 2006, Belgium - 2007, Canada -2007, Germany -2008, Scotland- 2010, Singapore - 2012, Turkey - 2013}.

Presented papers at IEEE GrComp International Conferences {Georgia State University at Atlanta - 2006, Kaohsiung National University in Taiwan - 2011}, International Conference on Advanced Mechatronic Systems (Tokyo University of Agriculture and Technology, Japan) - 2012.

He received the 2011 Romanian Academy "Traian Vuia" Award for Technical Science (the highest in the country); *Doctor HonorisCausa* of Academia DacoRomana from Bucharest - 2011, and *Doctor Honoris Causa* of Beijing Jiaotong University (one of the highest technical universities of China) - 2011;

the 2012 New Mexico - Arizona Book Award & 2011 New Mexico Book Award at the category Science & Math (for Algebraic Structures, together with Dr. W. B. Vasantha Kandasamy) on 18 November 2011 in Albuquerque; also, the Gold Medal from theTelesio-Galilei Academy of Science from England in 2010 at the University of Pecs - Hungary (for the Smarandache Hypothesis in physics, and for the Neutrosophic Logic), and the Outstanding Professional Service and Scholarship from The University of New Mexico - Gallup (2009, 2005, 2001).

Very prolific, he is the author, co-author, editor, and co-editor of 180 books published by about forty publishing houses (such as university and college presses, professional scientific and literary presses, such as Springer Verlag (in print), Univ. of Kishinev Press, Pima College Press, ZayuPress, Haiku, etc.) in ten countries and in many languages, and 250 scientific articles and notes, and contributed to over 100 literary and 50 scientific journals from around the world.

He published many articles on international journals, such as: Multiple-Valued Logic - An International Journal (now called Multiple-Valued Logic & Soft Computing), International Journal of Social Economics, International Journal of Applied Mathematics, International Journal of Tomography & Statistics, Applied Physics Research (Toronto), Far East Journal of Theoretical Statistics, International Journal of Applied Mathematics and Statistics (Editor-in-Chief), Gaceta Matematica (Spain), Humanistic Mathematics Network Journal, Bulletin of Pure and Applied Sciences, Progress in Physics, Infinite Energy (USA), Information & Security: An International Journal, InterStat - Statistics on the Internet (Virginia Polytechnic Institute and State University, Blacksburg, USA), American Mathematical Monthly, Mathematics Magazine, Journal of Advances in Information Fusion (JAIF), Zentralblatt fürMathematik (Germany; reviewer), Nieuw Archief voor Wiskunde (Holland), Advances in Fuzzy Sets and Systems, Advances and Applications in Statistics, Critical Review (Society for Mathematics of Uncertainty, Creighton University - USA), Bulletin of Statistics & Economics, International Journal of Artificial Intelligence, Fuzzy Sets and Systems, Journal of Computer Science and Technology, The Icfai University Journal of Physics (India), Hadronic Journal (USA), Intelligencer (Göttingen, Germany), Notices of the American Mathematical Society, etc. and on many International Conference Proceedings. Some of them can be downloaded from the LANL / Cornell University (http://arXiv.org/find) and the CERN web sites.

During the Ceausescu's era he got in conflict with authorities. In 1986 he did the hunger strike for being refused to attend the International Congress of Mathematicians at the University of Berkeley, then published a letter in the Notices of the American Mathematical Society for the freedom of circulating of scientists, and became a dissident. As a consequence, he remained unemployed for almost two years, living from private tutoring done to students. The Swedish Royal Academy Foreign Secretary Dr.Olof G. Tandberg contacted him by telephone from Bucharest. Not being allowed to publish, he tried to get his manuscripts out of the country through the French School of Bucharest and tourists, but for many of them he lost track.

Escaped from Romania in September 1988 and waited almost two years in the political refugee camps of Turkey, where he did unskilled works in construction in order to survive: scavenger, house painter, whetstoner. Here he kept in touch with the French Cultural Institutes that facilitated him the access to books and rencontres with personalities.

Before leaving the country he buried some of his manuscripts in a metal box in his parents vineyard, near a peach tree, that he retrieved four years later, after the 1989 Revolution, when he returned for the first time to his native country. Other manuscripts, that he tried to mail to a translator in France, were confiscated by the secret police and never returned.

He wrote hundreds of pages of diary about his life in the Romanian dictatorship (unpublished), as a cooperative teacher in Morocco ("Professor in Africa", 1999), in the Turkish refugee camp ("Escaped... / Diary From the Refugee Camp", Vol. I, II, 1994, 1998), and in the American exile - diary which is still going on.

But he's internationally known as the literary school leader for the "paradoxism" movement which has many advocates in the world, that he set up in 1980, based on an excessive use of antitheses, antinomies, paradoxes in creation paradoxes - both at the small level and the entire level of the work - making an interesting connection between mathematics, philosophy, and literature [http://fs.gallup.unm.edu/a/paradoxism.htm].

He introduced the 'paradoxist distich', 'tautologic distich', and 'dualistic distich', inspired from the mathematical logic [http://fs.gallup.unm.edu/a/literature.htm].

Literary experiments he realized in his dramas: Country of the Animals, where there is no dialogue!, and An Upside-Down World, where the scenes are permuted to give birth to one billion of billions of distinct dramas! [http://fs.gallup.unm.edu/a/theatre.htm].

He stated: "Paradoxism started as an anti-totalitarian protest against a closed society, where the whole culture was manipulated by a small group. Only their ideas and publications counted. We couldn't publish almost anything.

Then, I said: Let's do literature... without doing literature! Let's write... without actually writing anything. How? Simply: literature-object! 'The flight of a bird', for example, represents a "natural poem", that is not necessary to write down, being more palpable and perceptible in any language that some signs laid on the paper, which, in fact, represent an "artificial poem": deformed, resulted from a translation by the observant of the observed, and by translation one falsifies. Therefore, a mute protest we did!

Later, I based it on contradictions. Why? Because we lived in that society a double life: an official one - propagated by the political system, and another one real. In mass-media it was promulgated that 'our life is wonderful', but in reality 'our life was miserable'. The paradox flourishing! And then we took the creation in derision, in inverse sense, in a syncretic way. Thus the paradoxism was born. The folk jokes, at great fashion in Ceausescu's 'Epoch', as an intellectual breathing, were superb springs.

The "No" and "Anti" from my paradoxist manifestos had a creative character, not at all nihilistic." Paradoxism, following the line of Dadaism, Lettrism, absurd theater, is a kind of up-side down writings!

In 1992 he was invited speaker in Brazil (Universidad do Blumenau, etc.).

He did many poetical experiments within his avant-garde and published paradoxist manifestos: "Le Sens du Non-Sens" (1983), "Anti-chambres/Antipoésies/Bizarreries" (1984, 1989), "NonPoems" (1990), changing the French and respectively English linguistics clichés. While "Paradoxist Distiches" (1998) introduces new species of poetry with fixed form.
Eventually he edited three International Anthologies on Paradoxism (2000-2004) with texts from about 350 writers from around the world in many languages.

"MetaHistory" (1993) is a theatrical trilogy against the totalitarianism again, with dramas that experiment towards a total theater: "Formation of the New Man", "An Upside - Down World", "The Country of the Animals". The last drama, that pioneers no dialogue on the stage, was awarded at the International Theatrical Festival of Casablanca (1995).

He translated them into English as "A Trilogy in pARadOXisM: avant-garde political dramas"; and they were published by ZayuPress(2004).

"Trickster's Famous Deeds" (1994, auto-translated into English 2000), theatrical trilogy for children, mixes the Romanian folk tradition with modern and SF situations.

His first novel is called "NonNovel" (1993) and satirizes the dictatorship in a gloomy way, by various styles and artifice within one same style.

"Faulty Writings" (1997) is a collection of short stories and prose within paradoxism, bringing hybrid elements from rebus and science into literature.

His experimental albums "Outer-Art" (Vol. I, 2000 & Vol. II: The Worst Possible Art in the World!, 2003) comprises over-paintings, non-paintings, anti-drawings, super-photos, foreseen with a manifesto: "Ultra-Modernism?" and "Anti-manifesto" [http://fs.gallup.unm.edu/a/oUTER-aRT.htm].

Art was for Dr. Smarandache a hobby. He did:

- graphic arts for his published volumes of verse: "Anti-chambres/ Anti-poésies/ Bizarreries" (mechanical drawings), "NonPoems" (paradoxist drawings), "Dark Snow" & "Circles of light" (covers);
- paradoxist collages for the "Anthology of the Paradoxist Literary Movement", by J. -M. Levenard, I. Rotaru, A. Skemer;
- covers and illustrations of books, published by "Dorul" Publ. Hse., Aalborg, Denmark;
- illustrations in the journal: "Dorul" (Aalborg, Denmark).

Many of his art works are held in "The Florentin Smarandache Papers" Special Collections at the Arizona State University, Tempe, and Texas State University, Austin (USA), also in the National Archives of Valcea and Romanian Literary Museum (Romania), and in the Musee de Bergerac (France).

Twelve books were published that analyze his literary creation, among them: "Paradoxism's Aesthetics" by Titu Popescu (1995), and "Paradoxism and Postmodernism" by Ion Soare (2000).

He was nominated by the Academia DacoRomana from Bucharest for the 2011 Nobel Prize in Literature for his 75 published literary books.

Hundreds of articles, books, and reviews have been written about his activity around the world.

The books can be downloaded from this Digital Library of Science:
http://fs.gallup.unm.edu/eBooks-otherformats.htm
and from the Digital Library of Arts and Letters:
http://fs.gallup.unm.edu/eBooksLiterature.htm.

As a Globe Trekker he visited more than 45 countries that he wrote about in his memories (see his Photo Gallery at:http://fs.gallup.unm.edu/photo/GlobeTrekker.html).

International Conferences:
First International Conference on Smarandache Type Notions in Number Theory, August 21-24, 1997, organized by Dr. C. Dumitrescu & Dr. V. Seleacu, University of Craiova, Romania.
International Conference on Smarandache Geometries, May 3-5 2003, organized by Dr. M. Khoshnevisan, Griffith University, Gold Coast Campus, Queensland, Australia.
International Conference on Smarandache Algebraic Structures, December 17-19, 2004, organized by Prof. M. Mary John, Mathematics Department Chair, Loyola College, Madras, Chennai - 600 034 Tamil Nadu, India.

Personal web page: http://fs.gallup.unm.edu/

[Presentation by Dmitri Rabounski, Progress in Physics, 1/2014]

This volum includes 37 papers of mathematics or applied mathematics written by the author alone or in collaboration with the following co-authors: Cătălin Barbu, Mihály Bencze, Octavian Cira, Marian Niţu, Ion Patraşcu, Mircea E. Şelariu, Rajan Alex, Xingsen Li, Tudor Paroiu, Luige Vladareanu, Victor Vladareanu, Ştefan Vladuţescu, Yingjie Tian, Mohd Anasri, Lucian Capitanu, Valeri Kroumov, Kimihiro Okuyama, Gabriela Tonţ, A. A. Adewara, Manoj K. Chaudhary, Mukesh Kumar, Sachin Malik, Alka Mittal, Neetish Sharma, Rakesh K. Shukla, Ashish K. Singh, Jayant Singh, Rajesh Singh,V.V. Singh, Hansraj Yadav, Amit Bhaghel, Dipti Chauhan, V. Christianto, Priti Singh, and Dmitri Rabounski.

They were written during the years 2010-2014, about the hyperbolic Menelaus theorem in the Poincare disc of hyperbolic geometry, and the Menelaus theorem for quadrilaterals in hyperbolic geometry, about some properties of the harmonic quadrilateral related to triangle simedians and to Apollonius circles, about Luhn prime numbers, and also about the correspondences of the eccentric mathematics of cardinal and integral functions and centric mathematics, or ordinary mathematics; there are some notes on Crittenden and Vanden Eynden's conjecture, or on new transformations, previously non-existent in traditional mathematics, that we call centric mathematics (CM), but that became possible due to the new born eccentric mathematics, and, implicitly, to the supermathematics (SM); also, about extenics, in general, and extension innovation model and knowledge management, in particular, about advanced methods for solving contradictory problems of hybrid position-force control of the movement of walking robots by applying a 2D Extension Set, or about the notion of point-set position indicator and that of point-two sets position indicator, and the navigation of mobile robots in non-stationary and nonstructured environments; about applications in statistics, such as estimators based on geometric and harmonic mean for estimating population mean using information; about Godel's incompleteness theorem(s) and plausible implications to artificial intelligence/life and human mind, and many more.

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