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United States

RESEARCH ON THE TOPICS OF NEUTROSOPHIC OPERATIONS RESEARCH VOLUME (1)

Preparation

Florentin Smarandache
An Emeritus Professor of Mathematics
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Faculty member at the Faculty of Science, Damascus University
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2022-2023

Research on the topics of neutrosophic operations research
Volume (1)

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2022-2023

Florentin Smarandache

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Division,
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smarand@unm.edu

Born: Balcesti, Valcea district, Romania**.**
Date of Birth: 12.10.1954
Academic degree:
- PhD, PostDocs, is an Emeritus Professor of Mathematics at the University of New Mexico, United States.
- He got his MSc in Mathematics and Computer Science from the University of Craiova, Romania, Ph. D in Mathematics from the State University of Kishinev.
- Postdoctoral in Applied Mathematics from Okayama University of Sciences, Japan .And The Guangdong University of Technology, Guangzhou, China.

His scientific achievements:
- He is the founder of neutrosophy (generalization of dialectics), neutrosophic set, logic, probability and statistics since 1995.
- He published hundreds of papers and books on neutrosophic physics, superluminal and instantaneous physics, unmatter, quantum paradoxes, absolute theory of relativity, redshift and blueshift due to the medium gradient and refraction index besides the Doppler effect, paradoxism, outerart, neutrosophy as a new branch of philosophy, Law of Included Multiple-Middle, Law of Included Infinitely-Many-Middles, multispace and multistructure, HyperSoft set, TreeSoft Set, IndetermSoft Set and IndetermHyperSoft Set, SuperHyperGraph, SuperHyperTopology, SuperHyperAlgebra, SuperHyperFunction, Neutrosophic SuperHyperAlgebra, degree of dependence and independence between neutrosophic components, refined neutrosophic set, neutrosophic over-under-off-set (with degrees of membership/ indeterminacy/nonmembership less than 0 and bigger than1),

plithogenic set / logic / probability / statistics, symbolic plithogenic algebraic structures, neutrosophic triplet and duplet structures, quadruple neutrosophic structures, extension of algebraic structures to NeutroAlgebra and AntiAlgebra, NeutroGeometry and AntiGeometry, NeutroTopology and AntiTopology, Refined Neutrosophic Topology, Refined Neutrosophic Crisp Topology, Dezert-Smarandache Theory.

- He reviewed many peers and many books in international journals.
- He presented papers and plenary lectures to tenths of international conferences and seminars around the world.
- He also published many books of poetry, dramas, children's stories, literary translations, essays, a novel, folklore collections, traveling memories, and art albums.
- He wrote articles and books in four languages: English, Romanian, French, and Spanish.
- His books were translated to Arabic, Chinese, Russian, Spanish, Greek, Portuguese, Italian, German, Serbo-Croatian, and Turkish, see
https://fs.unm.edu/LiteratureLibrary.htm and
https://fs.unm.edu/ScienceLibrary.htm .
[http://fs.unm.edu/FlorentinSmarandache.htm]

Maissam Ahmad Jdid

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Born: Damascus – Syria.

Date of Birth: 27.04.1966

Origin: Tartous - Husayn al-Baher.

Academic degree:

- (Ph. D) in mathematical modeling from Tver State University - Russia

  Current place of work:

- Faculty member at Damascus University - Faculty of Science - Department of mathematics - since 2006.

  Previous place of work:

- lecturer at Al-Sham Private University - Faculty of Informatics Engineering from 2015-2023.

- lecturer at Antioch Private Syrian University in 2022-2023 College of Civil Engineering - College of Architecture.

  Honorary Membership

- Neutrosophic Science International Association –University of New Mexico, USA

- International Association of Paradoxism - University of New Mexico, USA

  Editor-in-Chief

- Journal Prospects for Applied Mathematics and Data Analysis, (ASPG), USA

https://americaspg.com/articlesvoulume/34
https://orcid.org/my-orcid?orcid=0000-0003-4413-4783
https://www.scopus.com/authid/detail.uri?authorId=57353028500
https://www.researchgate.net/profile/Maissam-Jdid
https://scholar.google.com/citations?user=-5pTuFcAAAAJ&hl=ar
https://www.facebook.com/profile.php?id=61551427142439&mibextid=9R9pXO
https://digitalrepository.unm.edu/do/search/?q=Jdid&start=0&context=8211305&facet=

In The Name of Allah, The Most Beneficent, The Most Merciful

So Exalted is Allah, the True Sovereign. Do not be hasty with the Quran before its inspiration is concluded to you, and say, "My Lord, increase me in knowledge"

(Allah Almighty is Truthful)

Dear colleagues

In this volume, we present a set of research that was published in cooperation with a number of researchers and those interested in keeping pace with the great scientific development that our contemporary world is witnessing, and one of its products was neutrosophic science, which was founded by the American scientist and mathematical philosopher Florentin Smarandache in 1995. Through it, we present a new vision for some research methods. Operations research to the concepts of this science.

It should be noted that the first research was published on November 23, 2021, and by the end of 2023, with God's help, we were able to publish 36 research papers.

We hope to God Almighty that this volume will achieve the purpose of its preparation.

Prof Florentin Smarandache & Dr- Maissam Ahmad Jdid

Search number	Subject
1	On Overview of Neutrosophic and Plithogenic Theories and Applications
2	Mysterious Neutrosophic Linear Models
3	The Graphical Method for Finding the Optimal Solution for Neutrosophic linear Models and Taking Advantage of Non-Negativity Constraints to Find the Optimal Solution for Some Neutrosophic linear Models in Which the Number of Unknowns is More than Three
4	Neutrosophic Handling of the Simplex Direct Algorithm to Define the Optimal Solution in Linear Programming
5	Neutrosophic Treatment of the Modified Simplex Algorithm to find the Optimal Solution for Linear Models
6	Neutrosophic Treatment of Duality Linear Models and the Binary Simplex Algorithm
7	Some Important Theories about Duality and the Economic Interpretation of Neutrosophic Linear Models and Their Dual Models
8	Optimal Agricultural Land Use: An Efficient Neutrosophic Linear Programming Method
9	Neutrosophic Mathematical Model of Product Mixture Problem Using Binary Integer Mutant
10	The Use of Neutrosophic Methods of Operation Research in the Management of Corporate Work
11	Inspection Assignment Form for Product Quality Control Using Neutrosophic Logic
12	Neutrosophic Mathematical Formulas of Transportation Problems
13	An Investigation in the Initial Solution for Neutrosophic Transportation Problems (NTP)
14	An Efficient Optimal Solution Method for Neutrosophic Transport Models: Analysis, Improvements, and Examples
15	Studying Transport Models with the Shortest Time According to the Neutrosophic Logic
16	Optimal Neutrosophic Assignment and the Hungarian Method
17	Neutrosophic Nonlinear Models
18	Graphical Method for Solving Neutrosophical Nonlinear Programming Models
19	Lagrange Multipliers and Neutrosophic Nonlinear Programming Problems Constrained by Equality Constraints
20	The static model of inventory management without a deficit with Neutrosophic logic
21	Important Neutrosophic Economic Indicators of the Static Model of Inventory Management without Deficit
22	The Neutrosophic Treatment of the Static Model for the Inventory Management with Safety Reserve
23	Neutrosophic Static Model without Deficit and Variable Prices
24	Neutrosophic Treatment of the Static Model of Inventory Management with Deficit
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Search number	Subject
26	Neutrosophical dynamic programming
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28	The State of Risk and Optimum Decision According to Neutrosophic Rules
29	Optimal Value of the Service Rate in the Unlimited Model $M\backslash M\backslash 1$
30	Fundamentals of Neutrosophical Simulation for Generating Random Numbers Associated with Uniform Probability Distribution
31	Using the Inverse Transformation Method to Generate Random Variables that follow the Neutrosophic Uniform Probability Distribution
32	The Basics of Neutrosophic Simulation for Converting Random Numbers Associated with a Uniform Probability Distribution into Random Variables Follow an Exponential Distribution
33	Neutrosophical Rejection and Acceptance Method for the Generation of Random Variables
34	Generating Random Variables that follow the Beta Distribution Using the Neutrosophic Acceptance-Rejection Method
35	Artificial Intelligence and Neutrosophic Machine learning in the Diagnosis and Detection of COVID 19
36	A Study of a Support Vector Machine Algorithm with an Orthogonal Legendre Kernel According to Neutrosophic logic and Inverse Lagrangian Interpolation

On Overview of Neutrosophic and Plithogenic Theories and Applications

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Abstract

We present this research to all researchers and scholars who have realized the existence of indeterminacy in all data, through the results they obtain and the values that are not accurate enough and that may cause loss to the systems and facilities under study, and we will present through it the emergence, foundations and development of Neutrosophic theories and their applications for more than two decades (1995- 2023) since it was defined and studied, along with its applications, in order to be able to present new studies and research that keep pace with the great scientific development that our contemporary world is witnessing, through the use of research that has been published by the professionals and found on the attached open links.

Keywords: Neutrosophic; origin of the word Neutrosophic; stages of development of neutrosophic science; applications of neutrosophic science; research in neutrosophic science; neutrosophic journals

1. Introduction

The beginning of the great revolution in all fields of science was in the year 1995 when the American scientist Florentin Smarandache introduced the degree of indeterminacy/neutrality (I) as independent component in 1995 (published in 1998) and he defined the neutrosophic set on three components:(T, I, F) = (Truth, Indeterminacy, Falsehood), where in general T, I, F are subsets of the interval [0, 1]; in particular T, I, F may be intervals, hesitant sets, single-values, etc. Indeterminacy (or Neutrality), as independent component from the truth and from the falsehood, is the main distinction between Neutrosophic Theories and other classical and fuzzy theory or fuzzy extension theories [1]
Neutrosophic Set is a Generalization of Intuitionist Fuzzy Set, Inconsistent Intuitionist Fuzzy Set (Picture Fuzzy Set, Ternary Fuzzy Set), Pythagorean Fuzzy Set (Atanassov's Intuitionist Fuzzy Set of second type), q-Rung Orthopair Fuzzy Set, Spherical Fuzzy Set, and n-HyperSpherical Fuzzy Set, while Neutrosophication is a Generalization of Regret Theory, Grey System Theory, and Three-Ways Decision.[2,3]

Zadeh introduced the degree of membership/truth (T) in 1965 and defined the fuzzy set. **Atanassov** introduced the degree of nonmembership/falsehood (F) in 1986 and defined the intuitionistic fuzzy set. **Smarandache** introduced the degree of indeterminacy/neutrality (I) as independent component in 1995 (published in 1998) and he defined the neutrosophic set on three components:(T, I, F) = (Truth, Indeterminacy, Falsehood), where in general T, I, F are subsets of the interval [0, 1]; in particular T, I, F may be intervals, hesitant sets, single-values, etc. See F. Smarandache, Neutrosophy / Neutrosophic probability, set, and logic", Proquest, Michigan, USA, 1998,[4,5] reviewed in Zentralblatt für Mathematik (Berlin, Germany)[6], and cited by Denis Howe in The Free Online Dictionary of Computing, England, 1999.Neutrosophic Set and Logic are generalizations of classical, fuzzy, and intuitionist fuzzy set and logic[7,8]

2. Etymology

a.   **Neutrosophy" and "Neutrosophic:**

The words "neutrosophy" and "neutrosophic" were coined/invented by F. Smarandache in his 1998 book.
Neutrosophy: A branch of philosophy, introduced by F. Smarandache in 1980, which studies the origin, nature, and scope of neutralities, as well as their interactions with different ideational spectra. Neutrosophy considers a proposition, theory, event, concept, or entity <A> in relation to its opposite <antiA>, and with their neutral <neutA>.
Neutrosophy (as dynamic of opposites and their neutrals) is an extension of the Dialectics and Yin Yang (which are the dynamic of opposites only).
Neutrosophy is the basis of neutrosophic set, neutrosophic logic, neutrosophic measure, neutrosophic probability, neutrosophic statisticsetc[9]

b.   **Neutrosophic Logic:**

Neutrosophic Logic is a general framework for unification of many existing logics, such as fuzzy logic (especially intuitionistic fuzzy logic), paraconsistent logic, intuitionist logic, etc. The main idea of NL is to characterize each logical statement in a 3D-Neutrosophic Space, where each dimension of the space represents respectively the truth (T), the falsehood (F), and the indeterminacy (I) of the statement under consideration, where T, I, F are standard or non-standard real subsets of]-0, 1+[with not necessarily any connection between them.
For software engineering proposals the classical unit interval [0, 1] may be used.
While Neutrosophic Probability and Statistics are generalizations of classical and imprecise probability and classical statistics respectively.

c.   **The Most Important Books and Papers on the Advancement of Neutrosophics:**
1980s - Foundation of Paradoxism that is an international movement in science and culture **based on excessive use of contradictions, antitheses, oxymoron, and paradoxes** [Smarandache]. During three decades (1980-2020) hundreds of authors from tens of countries around the globe contributed papers to 15 international paradoxist anthologies [10]
1995-1998: Smarandache extended the paradoxism (based on opposites) to a new branch of philosophy called Neutrosophy (based on opposites and their neutral), that gave birth to many scientific branches, such as: neutrosophic logic, neutrosophic set, neutrosophic probability and statistics, neutrosophic algebraic structures, and so on with multiple applications in all fields. Neutrosophy is also an extension of the Dialectics (which is a particular case of Yin-Yang ancient Chinese philosophy),[11]
 introduced the neutrosophic set/logic/probability/statistics;
introduces the single-valued neutrosophic set (pp. 7-8) [12,13]
1998-2001: Introduction of Nonstandard Neutrosophic Logic, Set, Probability,
https://arxiv.org/ftp/arxiv/papers/1903/1903.04558.pdf
 2002: Introduction of corner cases of sets / probabilities / statistics / logics, such as:- Neutrosophic intuitionistic set (different from intuitionist fuzzy set), neutrosophic paraconsistent set, neutrosophic fallibilists set, neutrosophic paradoxist set, neutrosophic pseudo-paradoxist set, neutrosophic tautological set, neutrosophic nihilist set, neutrosophic dialetheist set, neutrosophic trivialist set;
- Neutrosophic intuitionistic probability and statistics, neutrosophic paraconsistent probability and statistics, neutrosophic fallibilists probability and statistics, neutrosophic paradoxist probability and statistics, neutrosophic pseudo-paradoxist probability and statistics, neutrosophic tautological probability and statistics, neutrosophic nihilist probability and statistics, neutrosophic dialetheist probability and statistics, neutrosophic trivialist probability and statistics;
- Neutrosophic paradoxist logic (or paradoxism), neutrosophic pseudo-paradoxist logic (or neutrosophic pseudo-paradoxism), neutrosophic tautological logic (or neutrosophic tautologism)
https://arxiv.org/ftp/math/papers/0301/0301340.pdf
http://fs.unm.edu/DefinitionsDerivedFromNeutrosophics.pdf
2003 – Introduction by Kandasamy and Smarandache of Neutrosophic Numbers (a+bI,) where I = literal indeterminacy, $I^2 = I$, which is different from the numerical indeterminacy that is a real set), I-Neutrosophic Algebraic Structures and Neutrosophic Cognitive Maps
2005 - Introduction of Interval Neutrosophic Set/Logic , https://arxiv.org/pdf/cs/0505014.pdf
http://fs.unm.edu/INSL.pdf

2006 – Introduction of Degree of Dependence and Degree of Independence between the Neutrosophic Components T, I, F. for single valued neutrosophic logic, the sum of the components is:

$0 \leq t+i+f \leq 3$ when all three components are independent;

$0 \leq t+i+f \leq 2$ when two components are dependent, while the third one is independent from them;

$0 \leq t+i+f \leq 1$ when all three components are dependent.

When three or two of the components T, I, F are independent, one leaves room for incomplete information (sum < 1), paraconsistent and contradictory information (sum > 1), or complete information (sum = 1).

If all three components T, I, F are dependent, then similarly one leaves room for incomplete information (sum < 1), or complete information (sum = 1).

In general, the sum of two components x and y that vary in the unitary interval [0, 1] is:

$0 \leq x + y \leq 2$ - d°(x, y), where d°(x, y) is the degree of dependence between x and y, while

d°(x, y) is the degree of independence between x and y.

Degrees of Dependence and Independence between Neutrosophic Components T, I, F are independent components, leaving room for incomplete information (when their superior sum < 1), paraconsistent and contradictory information (when the superior sum > 1), or complete information (sum of components = 1).

For software engineering proposals the classical unit interval [0, 1] is used ,

https://doi.org/10.5281/zenodo.571359

http://fs.unm.edu/eBook-Neutrosophics6.pdf (p. 92)

http://fs.unm.edu/NSS/DegreeOfDependenceAndIndependence.pdf

2007:

1-   The neutrosophic set was extended [Smarandache, 2007] to neutrosophic overset (when some neutrosophic component is > 1), since he observed that, for example, an employee working overtime deserves a degree of membership > 1, with respect to an employee that only works regular full-time and whose degree of membership = 1;

and to neutrosophic underset (when some neutrosophic component is < 0), since, for example, an employee making more damage than benefit to his company deserves a degree of membership < 0, with respect to an employee that produces benefit to the company and has the degree of membership > 0;

and to and to neutrosophic offset (when some neutrosophic components are off the interval [0, 1], i.e. some neutrosophic component > 1 and some neutrosophic component < 0).then, similarly, the neutrosophic logic/measure/probability/statistics etc. were extended to respectively neutrosophic over-/under-/off- logic / measure / probability / statistics etc, https://arxiv.org/ftp/arxiv/papers/1607/1607.00234.pdf

http://fs.unm.edu/NeutrosophicOversetUndersetOffset.pdf

http://fs.unm.edu/SVNeutrosophicOverset-JMI.pdf

http://fs.unm.edu/IV-Neutrosophic-Overset-Underset-Offset.pdf

http://fs.unm.edu/NSS/DegreesOf-Over-Under-Off-Membership.pdf

2-   Smarandache introduced the neutrosophic tripolar set and neutrosophic multipolar set and consequently the Neutrosophic Tripolar Graph and Neutrosophic Multipolar Graph , http://fs.unm.edu/eBook-Neutrosophics6.pdf (p. 93)

http://fs.unm.edu/IFS-generalized.pdf

2009 – Introduction of N-norm and N-conorm https://arxiv.org/ftp/arxiv/papers/0901/0901.1289.pdf

http://fs.unm.edu/N-normN-conorm.pdf

2013:

1-   Development of neutrosophic measure and neutrosophic probability

(chance that an event occurs, indeterminate chance of occurrence, chance that the event does not occur)

https://arxiv.org/ftp/arxiv/papers/0901/0901.1289.pdf

http://fs.unm.edu/N-normN-conorm.pdf

2-   Smarandache refined / split the neutrosophic components (t, i, f) into neutrosophic subcomponents (T1, T2, ...; I1, I2, ...; F1, F2, ...) https://arxiv.org/ftp/arxiv/papers/1407/1407.1041.pdf

http://fs.unm.edu/n-ValuedNeutrosophicLogic-PiP.pdf

2014:

1-   Introduction of the Law of Included Multiple-Middle (as extension of the Law of Included Middle)(<A>; <neutA1>, <neutA2>, …; <antiA>) http://fs.unm.edu/LawIncludedMultiple-Middle.pdf

2-   Development of neutrosophic statistics (indeterminacy is introduced into classical statistics with respect to any data regarding the sample / population, probability distributions / laws / graphs / charts etc., with respect to the individuals that only partially belong to a sample / population, and so on),

https://arxiv.org/ftp/arxiv/papers/1406/1406.2000.pdf

http://fs.unm.edu/NeutrosophicStatistics.pdf

2015:

1-   Introduction of Neutrosophic Precalculus and Neutrosophic Calculus ,
https://arxiv.org/ftp/arxiv/papers/1509/1509.07723.pdf
http://fs.unm.edu/NeutrosophicPrecalculusCalculus.pdf
2-   Refined Neutrosophic Numbers (a+ b1I1 + b2I2 + … + bnIn), where I1, I2, …, In are
SubIndeterminacies of Indeterminacy I,
3-   (t,i,f)-neutrosophic graphs
4-   Thesis-Antithesis-Neutrothesis, and Neutrosynthesis, Neutrosophic Axiomatic System, neutrosophic
dynamic systems, symbolic neutrosophic logic, (t, i, f)-Neutrosophic Structures, I-Neutrosophic Structures,
Refined Literal Indeterminacy, Quadruple Neutrosophic Algebraic Structures, Multiplication Law of
SubIndeterminacies, and Neutrosophic Quadruple Numbers of the form a + bT + cI + dF ,
https://arxiv.org/ftp/arxiv/papers/1512/1512.00047.pdf
http://fs.unm.edu/SymbolicNeutrosophicTheory.pdf
5-   Introduction of the SubIndeterminacies of the form $I_0^K = \frac{K}{0}$, for k $\in$ {0, 1, 2, …, n-1}, into the ring
of modulo integers Zn - called natural Neutrosophic indeterminacies (Vasantha-Smarandache),
http://fs.unm.edu/MODNeutrosophicNumbers.pdf

6-   Introduction of Neutrosophic Topology (Salama & Smarandache) ,
http://fs.unm.edu/NeutrosophicCrispSetTheory.pdf
 2016:
1-   Introduction of Neutrosophic Multisets (as generalization of classical multisets)
http://fs.unm.edu/NeutrosophicMultisets.htm
2-   Introduction of Neutrosophic Triplet Structures and m-valued refined Neutrosophic triplet structures
[Smarandache – Ali] http://fs.unm.edu/NeutrosophicTriplets.htm
3-   Introduction of Neutrosophic Duplet Structures, http://fs.unm.edu/NeutrosophicDuplets.htm
 2017:
1-   Neutrosophic Score, Accuracy, and Certainty Functions form a total order relationship on the set of
(single-valued, interval-valued, and in general subset-valued) Neutrosophic triplets
 (T, I, F); and these functions are used in MCDM (Multi-Criteria Decision Making),
http://fs.unm.edu/NSS/TheScoreAccuracyAndCertainty1.pdf
2-   In biology Smarandache introduced the Theory of Neutrosophic Evolution: Degrees of Evolution,
Indeterminacy or Neutrality, and Involution, http://fs.unm.edu/neutrosophic-evolution-PP-49-13.pdf

3-   Introduction by F. Smarandache of Plithogeny (as generalization of Yin-Yang, Dialectics, and
Neutrosophy (as generalization of fuzzy, intuitionistic fuzzy, Neutrosophic set/logic/probability/statistics)
4-   Enunciation of the Law that: It Is Easier to Break from Inside than from Outside a Neutrosophic
Dynamic System (Smarandache - Vatuiu)
 2018:
1-   Generalization of the Soft Set to Hyper Soft Set,
http://fs.unm.edu/NSS/ExtensionOfSoftSetToHypersoftSet.pdf
2-   Introduction to Neutrosophic Psychology (Neutropsyche, Refined Neutrosophic Memory: conscious, a
conscious, unconscious, Neutropsychic Personality, Eros / Aoristos / Thanatos, Neutropsychic Crisp Personality)
, http://fs.unm.edu/NeutropsychicPersonality-ed3.pdf
2019:
1-   Theory of Spiral Neutrosophic Human Evolution (Smarandache - Vatuiu),
http://fs.unm.edu/SpiralNeutrosophicEvolution.pdf

2-   Introduction to Neutrosophic Sociology (Neutrosociology) [neutrosophic concept, or (T, I, F)-concept,
is a concept that is T% true, I% indeterminate, and F% false]
http://fs.unm.edu/Neutrosociology.pdf

3-   Refined Neutrosophic Crisp Set, http://fs.unm.edu/RefinedNeutrosophicCrispSet.pdf

4-   Generalized of the classical Algebraic Structures to NeutroAlgebraic Structures
(or NeutroAlgebras) {whose operations and axioms are partially true, partially indeterminate, and partially
false} as extensions of Partial Algebra, and to AntiAlgebraic Structures (or AntiAlgebras) {whose operations
and axioms are totally false}. And, in general, he extended any classical Structure, in no matter what field of
knowledge, to a NeutroStructure and an AntiStructure.
5-   Extension of HyperGraph to n-SuperHyperGraph

2020 –

Introduction to Neutrosophic Genetics ,http://fs.unm.edu/NeutrosophicGenetics.pdf

2021

1-   As alternatives and generalizations of the Non-Euclidean Geometries, Smarandache introduced in 2021 the NeutroGeometry & AntiGeometry. While the Non-Euclidean Geometries resulted from the total negation of only one specific axiom (Euclid's Fifth Postulate), the AntiGeometry results from the total negation of any axiom and even of more axioms from any geometric axiomatic system (Euclid's, Hilbert's, etc.), and the NeutroAxiom results from the partial negation of one or more axioms [and no total negation of no axiom] from any geometric axiomatic system.

2-   Real Examples of NeutroGeometry and AntiGeometry .

3-   Introduction of Plithogenic Logic as a generalization of MultiVariate Logic.

4-   Introduction of Plithogenic Probability and Statistics as generalizations of MultiVariate Probability and Statistics respectively

2022:

1-   SuperHyperAlgebra & Neutrosophic SuperHyperAlgebra

2-   SuperHyperGraph, Neutrosophic SuperHyperGraph

3-   SuperHyperFunction, SuperHyperTopology

4-   IndetermSoft Set, IndetermHyperSoft Set

5-   TreeSoft Set

2023 - Symbolic Plithogenic Algebraic Structures built on the set of Symbolic Plithogenic Numbers of the form $a0 + a1P1 + a2P2 + ... + anPn$ where the multiplication $Pi·Pj$ is based on the prevalence order and absorbance law.

d.   **Applications in:**

Artificial Intelligence, Information Systems, Computer Science, Cybernetics, Theory Methods, Mathematical Algebraic Structures, Applied Mathematics, Automation, Control Systems, Big Data, Engineering, Electrical, Electronic, Philosophy, Social Science, Psychology, Biology, Biomedical, Genetics, Engineering, Medical Informatics, Operational Research, Management Science, Imaging Science, Photographic Technology, Instruments, Instrumentation, Physics, Optics, Economics, Mechanics, Neurosciences, Radiology Nuclear, Medicine, Medical Imaging, Interdisciplinary Applications, Multidisciplinary Sciences etc. [Xindong Peng and Jingguo Dai, A bibliometric analysis of neutrosophic set: two decades review from 1998 to 2017, Artificial Intelligence Review, Springer, 18 August 2018 .

e.   **Important Neutrosophic Researchers:**

There are about 7,000 neutrosophic researchers, within 7489 countries around the globe, that have produced about 2,000 publications within more than two decades. Many neutrosophic researchers got specialized into various fields of neutrosophics:

Xiaohong Zhang & Yingcang Ma (neutrosophic triplet and quadruple algebraic structures), Yanhui Guo (neutrosophic image processing), Jun Ye & Peide Liu & Jianqiang Wang (neutrosophic optimization), Xindong Peng & Jingguo Dai (neutrosophic bibliometrics), Jianqiang Wang, Guiwu Wei, Donghai Liu, Xiaohong Chen, Dan Peng, Jiongmei Mo, Han-Liang Huang, Victor Chang, Hongjun Guan, Shuang Guan, Aiwu Zhao, Wen-Hua Cui, Xiaofei Yang, Xin Zhou, G.L. Tang, W.L. Liu, Wen Jiang, Zihan Zhang, Xinyang Deng, Changxing Fan, Sheng Feng, En Fan, Keli Hu, Xingsen Li, Xin Zhou, Ping Li;

Rajab Ali Borzooei & Young Bae Jun (neutrosophic BCK/BCI-algebras), Arsham Borumand Saeid (neutrosophic structures), Saied Jafari (neutrosophic topology), Prem Kumar Singh (Neutrosophic MCDM), Maikel Leyva-Vazquez (neutrosophic cognitive maps);

Amira S. Ashour, Muhammad Aslam (neutrosophic statistics), Nguyen Xuan Thao (neutrosophic similarity measures), Le Hoang Son, Vakkas Ulucay & Memet Sahin (neutrosophic quadruple structures), Irfan

Deli, Madad Khan (neutrosophic algebraic structures), Said Broumi & Muhammad Akram (neutrosophic graphs), Mohamed Abdel-Baset (neutrosophic linear and non-linear programming), Ahmed Mostafa Khalil, Ahmed Salama (neutrosophic crisp topology), etc.

f.   **Neutrosophic Journals:**

1-   Neutrosophic Sets and Systems (NSS) international journal started in 2013 and it is indexed by Scopus, Web of Science (ESCI), DOAJ, Index Copernicus, Redalyc - Universidad Autonoma del Estado de Mexico

(IberoAmerica), Publons, CNKI (Beijing, China), Chinese Baidu Scholar, etc, Submit papers on neutrosophic set/logic/probability/statistics, Simulation, etc. and their applications through our OJS, [14 – 19]

1-   International Journal of Neutrosophic Science (IJNS, in SCOPUS) , Submit papers on neutrosophic, inventory management, Linear Programming , dynamic programming,

Decision – Making , Machine learning, Queue Theory , etc , [20-29]

2-   Neutrosophic Computing and Machine Learning (NCML), in Spanish
 http://fs.unm.edu/NCML/

3-   Neutrosophic Knowledge (NK), in English and Arabic: http://fs.unm.edu/NK/

4-   Journal of Neutrosophic and Fuzzy Systems (JNFS), Submit papers on neutrosophic Machine learning, Queue Theory , etc , [30 – 33]

5-   Prospects for Applied Mathematics and Data Analysis (PAMDA) , Submit papers on neutrosophic Machine learning , Neutrosophic Simulation , etc, [34-36]

6-   Encyclopedia of Neutrosophic Researchers, To certify the names of authors who have published or presented research on Neutrosophic,[37-40]

Note : The authors who have published or presented papers on neutrosophics and are not included in the Encyclopedia of Neutrosophic Researchers (ENR), vols. 1, 2, 3, and 4 are pleased to send their CV, photo, and List of Neutrosophic Publications to smarand@unm.edu in order to be included into the next volume of ENR.

3.   Conclusion and results:

There is no absolute truth, there is nothing certain that any data presented to conduct a study in any field of science contains some kind of indeterminacy that affects the desired results of this study, which made neutrosophic science and the basic concepts of neutrosophic science the focus of attention of scholars and researchers who wish to obtain results Perfect and accurate, far from taking risks, they achieve the desired goal

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Mysterious Neutrosophic Linear Models

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Abstract:

Operations research often shortened to the initialism O.R., is a discipline that deals with the development and application of advanced analytical methods to improve decision-making. It is sometimes considered to be a subfield of mathematical sciences. The term management science is occasionally used as a synonym. It has the ability to express the concepts of efficiency and scarcity in a well-defined mathematical model for a specific issue. It has the ability to use scientific methods to solve complex problems in managing large scale systems for factories, institutions, and companies, and enables them to make optimal scientific decisions for the functioning of Its work.

Employing techniques from other mathematical sciences, such as modelling, statistics, and optimization, operations research arrives at optimal or near-optimal solutions to complex decision-making problems. Because of its emphasis on practical applications, operations research has overlapped with many other disciplines, notably industrial engineering. Operations research is often concerned with determining the extreme values of some real-world objective: the maximum (of profit, performance, or yield) or minimum (of loss, risk, or cost). Originating in military efforts before World War II, its techniques have grown to concern problems in a variety of industries. The mathematical model is the simplified image of expressing a practical system from a real life problem or an idea put forward for an executable system, as the mathematical models consist of a goal function through which we search for the maximum or minimum value subject to restrictions. Linear mathematical programming is one of the most important topics in the field of operations research due to their frequent use in most areas of life. When studying linear programming, the first step is to identify the various types of linear models and how to transition from one to the next. We realize that the ideal solution of the linear model is influenced by the coefficients

of the variables of objective function that describes a profit if the model is a maximizing model or represents a cost if the model is a minimization model, which is affected by environmental conditions. The fixed values that represent the right side of the inequalities (constraints), which express the available capital, time, raw resources, and so on, have an impact on the optimum solution. They are also affected by environmental conditions. We used to take these values as fixed values in classical logic, which does not correspond to reality and leads to erroneous solutions to the problems described by the linear model. As a result, it was essential to reformulate the classical linear models' problems, taking into consideration all probable scenarios and changes in the work environment. In this study, we will look into linear models and their kinds in view of neutrosophic logic, which takes into account all of the data and all of the changes that may occur in the issue under investigation, as well as the uncertainty that is encountered in the problem's data. We'll also look at it if the coefficients of the variables in the objective function are neutrosophic values, and the accessible options are neutrosophic values because we'll reformulate the existing linear mathematical models using neutrosophic logic, and show how to convert from one to another using some examples.

Keywords:

1. Introduction:

The primary pivot of operations research is the existence of a problem that necessitates decision-making, which becomes more necessary as the problem's complexity grows. The success of operations research's methodologies, the model's data collection procedure, and the accuracy of its depiction of reality, all of these are dependent on decision-makers. When employing quantitative analysis to solve an issue, the analyst's primary focus will be on comprehending quantitative facts and data connected to the problem, then building a mathematical model based on that understanding and familiarity. The model must accurately represent the problem's goal, limitations, and interrelationships, and one of the most important factors that aided the operations research algorithms to solve complex problems was the development of the computer, which led to the establishment of companies specializing in the development of software and systems related to operations research methods. As a result, in order to get

the most out of this development, the information we provide must be flexible with any changes that may occur in the workplace and take into account all available possibilities. Thus, in this article, we used neutrosophic logic, which is a new vision of modeling that is designed to effectively address the uncertainties inherent in the real world, by introducing a third state that can be interpreted as undetermined or uncertain, to replace the binary logic that admits merely right and wrong. Florentin Smarandache, an American mathematical philosopher [7,9-15]

In 1995, Florentin proposed neutrosophic logic as a generalization of fuzzy logic and an extension of Lotfi Zadeh's theory of fuzzy categories [6]. Ahmed A. Salama went on to present the theory of neutrosophic classical categories as a generalization of the theory of classical categories [12,16], and he used Neutrosophy to develop, introduce, and formulate new concepts in mathematics, statistics, computer science, and other fields [17].

Neutrosophic theory has gained popularity in recent years due to its use in measurement, groups, graphs, and a variety of other scientific and practical domains [8,18,21-24].

Recently, there were revolutionary movements to reformulate most mathematical branches, especially in operating research and optimization to reconstruct either its fuzzy or classical problems in the light of neutrosophic theory [19,20,25-27].

2. Discussion:

Many of the problems in our daily life have been handled by operations research topics, and one of the most essential elements that distinguish operations research science from other quantitative methods-based sciences is the description of the problem in a mathematical model. We've looked at a lot of operations research subjects using traditional logic, which relies on specific data from field studies of the case in question and leads to specific results that don't account for the changes and fluctuations we meet on the work that can lead to unexpected losses [1- 5]. This prompted the authors to conduct this study, in which they presented the most important forms of linear mathematical models known in classical logic utilizing neutrosophic mathematical relationships, which include a set of neutrosophic variables that can change as a function of the work environment. The presence of neutrosophic values ensures a safe workflow for the facility that follows these models and based on what has been clarified in the summary about the factors that influence the ideal solution, we will include all instances of the variables in the objective function as neutrosophic values., i.e. $c_j \pm \varepsilon_j; j = 1,2,\dots,n$ where ε_j is indeterminate and could be $\varepsilon_j = |\lambda_{1j}, \lambda_{2j}|$ or $\varepsilon_j \in \{\lambda_{1j}, \lambda_{2j}\}$.

Also, the values that express the available possibilities are neutrosophic values. This means that $b_i \pm \delta_i; i = 1,2,\dots,m$ where δ_i is the indeterminate part and it could be $\delta_i = [\mu_{1i}, \mu_{2i}]$ or $\delta_i \in \{\mu_{1i}, \mu_{2i}\}$. Using the above assumptions, the upcoming section dedicated to formulate the most popular forms of linear models according to the neutrosophic theory.

3. General Neutrosophic Linear Model:

The neutrosophic general form is given the linear mathematical model in the abbreviated form as follows:

$$\text{Max } or \text{ Min } f(x) = \sum_{j=1}^{n}(c_j \pm \varepsilon_j)x_j$$

Subject to

$$\sum_{j=1}^{n} a_{ij}x_j \leq b_i \pm \delta_i \quad ; \quad i = 1,2......m$$

$$\sum_{j=1}^{n} a_{ij}x_j \geq b_i \pm \delta_i \quad ; \quad i = 1,2......m$$

$$x_j \geq 0$$

Where $c_j + \varepsilon_j$, $b_i \pm \delta_i$, a_{ij}, $j = 1,2,...,n$, $i = 1,2,...,m$ are constants having set or interval values according to the nature of the given problem , x_j are decision variables.

Standard Form of the Neutrosophic Linear Model:

When the objective function is of the type maximization subject to less than or equal constraint, the neutrosophic linear model is in its standard form. Call the mathematical form of the neutrosophic linear programming:

$$\text{Max } f(x) = \sum_{j=1}^{n}(c_j \pm \varepsilon_j)x_j$$

Subject to

$$\sum_{j=1}^{n} a_{ij}x_j \leq b_i \pm \delta_i \quad ; \quad i = 1,2......m$$

$$x_j \geq 0$$

Where $c_j + \varepsilon_j$, $b_i \pm \delta_i$, a_{ij}, $j = 1,2,...,n$, $i = 1,2,...,m$ are constants having set or interval values according to the nature of the given problem , x_j are decision variables.

4. Canonical Neutrosophic Form of Linear Model:

The canonical form plays an important role in finding a solution to linear programming problems, as the issue of searching for a solution to a linear programming problem has been transferred to a process of searching for a solution to a set of linear equations consisting of the equation with anonymous. Solving this issue is useful if it is possible, that is, if it fulfils the conditions of non-negativity Hence, the optimal

solution for the linear program is the ideal values of the variables that satisfy the constraints and give the objective function the greatest or smallest possible value according to the text of the problem under solution. The conventional neutrosophic form is given as follows:

$$\text{Max } or \text{ Min } f(x) = \sum_{j=1}^{n}(c_j \pm \varepsilon_j)x_j$$

Subject to

$$\sum_{j=1}^{n}a_{ij}x_j = b_i \; ; \; i = 1,2,...,m \; , \; j = 1,2,...,n$$

$$x_j \geq 0$$

Except for the non-negative constraints, which remain inequalities, all of the constraints are of the equality type. In addition, each equality constraint's right-hand side must be non-negative, and all decision variables must be non-negative. In the neutrosophic standard form, the objective follower might be either a maximizing or a minimizing function.

5. The Neutrosophical Symmetry of the Linear Model:

The linear program is set symmetrically if all the variables are constrained to be non-negative and if all the constraints are given in the form of inequalities (the inequalities of the constraints of the maximization problem must be placed in the form ($\leq$ is less than or equal to) while the inequalities of the constraints of the minimization problem must be in the form ($\geq$ is greater than or equal to). The neutrosophic symmetric form is written in one of two ways in the following:

First pattern:

$$f(x) = \sum_{j=1}^{n}(c_j \pm \varepsilon_j)x_j \rightarrow Max$$

Subject to

$$\sum_{j=1}^{n}a_{ij}x_j \leq b_i \pm \delta_i \quad ; \quad i = 1,2......m$$

$$x_j \geq 0$$

 Second Pattern:

$$f(x) = \sum_{j=1}^{n} (c_j \pm \varepsilon_j) x_j \rightarrow Min$$

Subject to

$$\sum_{j=1}^{n} a_{ij} x_j \geq b_i \pm \delta_i \quad ; \quad i = 1, 2 \ldots\ldots m$$

$$x_j \geq 0$$

Where $c_j + \varepsilon_j$, $b_i \pm \delta_i$, a_{ij}, $j = 1,2,\ldots,n$, $i = 1,2,\ldots,m$ are constants having set or interval values according to the nature of the given problem , x_j are decision variables.

Following the presentation of the various patterns in which the neutrosophic linear model can be presented, it is worth noting that we can shift from one pattern to another by using the below primary transformations:

- Converting the minimum value of the objective function f(x) to a maximum value by taking a negative value of $f(x)$.

- If the inequalities were of the form (greater than or equal to) they will be converted to the form (less than or equal to) by multiplying both sides by (-1), and vice versa.

- The equality constraint can be converted into two inequalities of different direction.

- If the left side of an (inequality) constraint is given in absolute value, it can be converted into two regular inequalities.

- Constraint inequalities of the type (greater than or equal to) are converted to an equality constraint by subtracting an appropriate positive variable (i.e. artificial variable) from the left side of the inequality and this variable is entered into the objective function with zero coefficient.

- Constraint inequalities of the type (less than or equal to) are converted into an equality constraint by adding an appropriate positive variable (i.e. slack variable) to the left hand side of the inequality and then this variable is entered into the objective function with zero coefficient.

- If one of the decision variables x is not constrained by the non-negative condition (that is, it can be negative, positive or zero), then it can be expressed as the difference between two non-negative variables x', x'' as follows x = x' - x″ where $x' \geq 0$ and $x'' \geq 0$.

5.1 Practical Example 1:

Call the following Neutrosophic Linear Programming in its general form:

$$MinZ = (3 \pm \varepsilon_1)x_1 - (3 \pm \varepsilon_2)x_2 + (7 \pm \varepsilon_3)x_3$$

Subject to

$$x_1 + x_2 + 3x_3 \leq 40 \pm \delta_1$$
$$x_1 + 9x_2 - 7x_3 \geq 50 \pm \delta_2$$
$$5x_1 + 3x_2 = 20 \pm \delta_3$$
$$|5x_2 + 8x_3| \leq 100 \pm \delta_4$$
$$x_1 \geq 0 \quad , \quad x_2 \geq 0$$

where $c_j \mp \varepsilon_j$, $j = 1,2, \dots, n$; ε_j It is indeterminate and could be $\varepsilon_j = [\lambda_{1j}, \lambda_{2j}]$ or $\varepsilon_j \in \{\lambda_{1j}, \lambda_{2j}\}$. Also,

the values that express the available possibilities are neutrosophic values. This means that $b_i \mp \delta_i$; $i = 1,2, \dots, m$; δ_i It is indeterminate and could be $\delta_i = [\lambda_{1j}, \lambda_{2j}]$ or $\delta_i \in \{\lambda_{1j}, \lambda_{2j}\}$

To convert the above problem into the neutrosophic standard form, we perform the following

transformations:

• The objective function is a function of minimization that we turn into a function of maximization

$$MinZ = (3 \pm \varepsilon_1)x_1 - (3 \pm \varepsilon_2)x_2 + (7 \pm \varepsilon_3)x_3 \text{ transformed into}$$

$$MaxZ = -(3 \pm \varepsilon_1)x_1 + (3 \pm \varepsilon_2)x_2 - (7 \pm \varepsilon_3)x_3$$

• The second constraint is given (greater than or equal to) is converted into (less than or equal) by

multiplying both sides by (-1) we get $-x_1 - 9x_2 + 7x_3 \leq -(50 \pm \delta_2)$

$$5x_1 + 3x_2 \leq 20 \pm \delta_3$$
$$5x_1 + 3x_2 \geq 20 \pm \delta$$

• Third constraint $5x_1 + 3x_2 = 20 \pm \delta_3$ transformed into two entries

Then we turn the constraint $5x_1 + 3x_2 \geq 20 \pm \delta_3$ into $-5x_1 - 3x_2 \leq -(20 \pm \delta_3)$

• The constraint $|5x_1 + 8x_2| \leq 100 \pm \delta_4$ is equivalent to the two inequalities

$$-5x_1 - 8x_2 \leq 100 \pm \delta_4$$
$$5x_1 + 8x_2 \leq 100 \pm \delta_4$$

• The variable x3 is not restricted by the non-negative constraint, so it is replaced by the following

assumption $x_3 = x_3' - x_3''$ where $x_3'' \geq 0, x_3' \geq 0$.

The standard neutrosophic form becomes:

$$MaxZ = -(3 \pm \varepsilon_1)x_1 + (3 \pm \varepsilon_2)x_2 - (7 \pm \varepsilon_3)x_3' + (7 \pm \varepsilon_3)x_3''$$

Subject to

$$x_1 + x_2 + 3x_3' - 3x_3'' \leq 40 \pm \delta_1$$
$$-x_1 - 9x_2 + 7x_3' - 7x_3'' \leq -(50 \pm \delta_2)$$
$$5x_1 + 3x_2 \leq 20 \pm \delta_3$$
$$-5x_1 - 3x_2 \leq -(20 \pm \delta_3)$$
$$5x_2 + 8x_3' - 8x_3'' \leq 100 \pm \delta_4$$
$$-5x_2 - 8x_3' + 8x_3'' \leq 100 \pm \delta_4$$
$$x_1 \geq 0 \quad, \quad x_2 \geq 0 \quad, \quad x_3' \geq 0 \quad, \quad x_3'' \geq 0$$

5.2 Practical Example 2:

A factory produces four types of products S_4, S_3, S_2, S_1. For this purpose, the following raw materials are used M_3, M_2, M_1. The factory management wants to study the optimal organization of production during a period of time (for example, a month) and determine the monthly production for each product in order to achieve a maximum profit, bearing in mind that the profit is directly proportional to the number of units sold of the products. The available quantities of raw materials needed for each product and the profit have been showed in the following table:

Raw Materials	Product Type				Available Quantities
	S_1	S_2	S_3	S_4	
M_1	1.5	1	2.4	1	$3000 \pm \delta_1$
M_2	1	5	1	3.5	$9000 \pm \delta_2$
M_3	1.5	3	3.5	1	$7000 \pm \delta_3$
win one product	$4 \pm \varepsilon_1$	$8 \pm \varepsilon_2$	$5 \pm \varepsilon_3$	$6 \pm \varepsilon_4$	

Let us suppose x_1 is the number of units produced from the first type, x_2 is the number of units produced from the second type, x_3 is the number of units produced from the third type, and x_4 is the number of units produced from the fourth type during the production period (a month for example), and accordingly, the consumed quantity of the raw material M_1 in the production of the four varieties will be:

$1.5x_1 + x_2 + 2.4x_3 + x_4$

And it must not exceed $3000 \pm \delta_1$ from the available quantity, that is :

$$1.5x_1 + x_2 + 2.4x_3 + x_4 \leq 3000 \pm \delta_1 \tag{1}$$

Likewise, the amount of raw material M_2 consumed in the production of the four types is:

$$x_1 + 5x_2 + x_3 + 3.5x_4 \leq 9000 \pm \delta_2 \tag{2}$$

And the amount consumed of the raw material M_3 in the production of the four types is:

$$1.5x_1 + 3x_2 + 3.5x_3 + x_4 \leq 7000 \pm \delta_3 \tag{3}$$

In addition, the produced quantities must be non-negative, i.e.:

$$x_i \geq 0, i = 1,2,3,4 \tag{4}$$

And they are what is called non-negative conditions.

Thus, we have identified all the constraints imposed on the variables of the problem.

We now define the objective function. If quantified units x_1, x_2, x_3, x_4 of species are produced in order, then the profit during the productive period will be:

$$f(x) = (4 \pm \varepsilon_1)x_1 + (8 \pm \varepsilon_2)x_2 + (5 \pm \varepsilon_3)x_3 + (6 \pm \varepsilon_4)x_4$$

It represents the target function. Therefore, the mathematical model of the problem is:

$Max \ f(x) = (4 \pm \varepsilon_1)x_1 + (8 \pm \varepsilon_2)x_2 + (5 \pm \varepsilon_3)x_3 + (6 \pm \varepsilon_4)x_4$

Subject to

$1.5x_1 + x_2 + 2.4x_3 + x_4 \leq 3000 \pm \delta_1$

$x_1 + 5x_2 + x_3 + 3.5x_4 \leq 9000 \pm \delta_2$

$1.5x_1 + 3x_2 + 3.5x_3 + x_4 \leq 7000 \pm \delta_3$

Non-negative terms:

$$x_1 \geq 0 \, , \, x_2 \geq 0 \, , \, x_3 \geq 0 \, , \, x_4 \geq 0$$

We have obtained a neutrosophical standard linear model using the appropriate transformations, which can be written in the following neutrosophical standard form:

$$Max f(x) = (4 \pm \varepsilon_1)x_1 + (8 \pm \varepsilon_2)x_2 + (5 \pm \varepsilon_3)x_3 + (6 \pm \varepsilon_4)x_4 + 0y_1 + 0y_2 + 0y_3$$

Subject to

$1.5x_1 + x_2 + 2.4x_3 + x_4 + y_1 = 3000 \pm \delta_1$

$x_1 + 5x_2 + x_3 + 3.5x_4 + y_2 = 9000 \pm \delta_2$

$1.5x_1 + 3x_2 + 3.5x_3 + x_4 + y_3 = 7000 \pm \delta_3$

Non-negative terms:

$$x_1 \geq 0 \, , \, x_2 \geq 0 \, , \, x_3 \geq 0 \, , \, x_4 \geq 0$$

6. Conclusion

Due to the great interdependence that exists between the topics of linear programming, it was necessary to reformulate the forms of linear models according to the Neutrosophic logic, because the uncertainty that we added to the data described by the linear model provides us with linear models that simulate the

reality and take into account most of the changes that can occur in the working environment of the system represented by the model linear mathematics. It also enabled us to continue the study of linear programming topics, such as finding the accompanying programs that need to develop the mathematical model in a symmetrical form, and solving linear models in a simplex way that needs to model models in the standard form and other topics of linear programming, Therefore, reformulating the important forms of linear models according to neutrosophic logic is inevitable because it provides us with a more general and comprehensive study and enables us to provide quick and accurate treatment to avoid losses that may be incurred by institutions, companies, etc. that operate according to these models.

this article presents linear models and their kinds in view of neutrosophic logic, which takes into account all of the data and all of the changes that may occur in the issue under investigation, as well as the uncertainty that is encountered in the problem's data. also, we discuss the case of the variables in the objective function are neutrosophic values, and the accessible options are neutrosophic values because we reformulated the existing linear mathematical models using neutrosophic logic, and showed how to convert from one to another using some examples.

We look forward in the near future to studying other topics of linear programming and its applications in practical life using neutrosophic theory, such as accompanying programs, sensitivity analysis, transfer problem, ...etc.

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The Graphical Method for Finding the Optimal Solution for Neutrosophic linear Models and Taking Advantage of Non-Negativity Constraints to Find the Optimal Solution for Some Neutrosophic linear Models in Which the Number of Unknowns is More than Three

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Abstract:

The linear programming method is one of the important methods of operations research that has been used to address many practical issues and provided optimal solutions for many institutions and companies, which helped decision makers make ideal decisions through which companies and institutions achieved maximum profit, but these solutions remain ideal and appropriate in If the conditions surrounding the work environment are stable, because any change in the data provided will affect the optimal solution and to avoid losses and achieve maximum profit, we have, in previous research, reformulated the linear models using the concepts of neutrosophic science, the science that takes into account the instability of conditions and fluctuations in the work environment and leaves nothing to chance. While taking data, neutrosophic values carry some indeterminacy, giving a margin of freedom to decision makers. In another research, we reformulated one of the most important methods used to solve linear models, which is the simplex method, using the concepts of this science, and as a continuation of what we did in the previous two researches, we will reformulate in this research. The graphical method for solving linear models using the concepts of neutrosophics. We will also shed light on a case that is rarely mentioned in most operations research references, which is that when the difference between the number of unknowns and the number of constraints is equal to one, two, or three, we can also find the optimal solution graphically for some linear models. This is done by taking advantage of the conditions of non-negativity that linear models have, and we will explain this through an example in which the difference is equal to two. Also, through examples, we will explain the difference between using classical values and neutrosophic values and the extent of this's impact on the optimal solution.

Keywords: linear programming; Neutrosophic science; Neutrosophic linear models; Graphical method for solving linear models; Graphical method for solving neutrosophic linear models.

Introduction:

A continuation of what we have done in previous research, the purpose of which was to reformulate some operations research methods using the concepts of neutrosophic science. See [1-14] The science that made a great revolution in all fields of science, which grew and developed very quickly, as many topics were reformulated using the concepts of this Science, and we find neutrosophic groups, neutrosophic differentiation, neutrosophic integration, and neutrosophic statistics ... [14-16], and given the importance of the graphic method used to find the optimal solution for linear models, which is a graph of the model and is one of the easiest ways to solve linear programming problems, but it is not sufficient to address All linear programming problems because they often contain a large number of variables, and the use of the graphical method is limited to the following cases:

• The number of unknowns is n = 1 or n = 2 or n = 3.

• In linear models whose constraints are equal constraints, if the number of unknowns and the number of equations meet one of the following conditions: n – m = 1 or n – m = 2 or n – m = 3. Here we can transform the model into a function of one variable or two variables or three variables, respectively, by taking advantage of the non-negativity constraints that the variables of the linear model have. In this research, we will present a reformulation of the graphical method for solving linear models using the concepts of neutrosophic, as well as the graphical method for solving linear models that Its restrictions are equal restrictions, and the difference between the number of unknowns and the number of restrictions is equal to one, two, or three.

Discussion:

The graphical method is one of the important ways to find the optimal solution for the linear and nonlinear models, so in the research [3] we reformulated it for the neutrosophic nonlinear models, and in this research we will present the graphic method to find the optimal solution for the neutrosophic linear models that were presented in the research [1], we know that the model the script is written in the following abbreviated form:

$$Z = \sum_{j=1}^{n} c_j x_j \rightarrow (Max \ or \ Min)$$

Restrictions:

$$\sum_{j=1}^{n} a_{ij} x_j \begin{pmatrix} \geq \\ \leq \\ = \end{pmatrix} b_i \quad ; i = 1,2, \dots , m$$
$$x_j \geq 0 \quad ; j = 1,2, \dots , n$$

If at least one of the values c_j , a_{ij} , b_i , is a neutrosophic value then the linear model is a neutrosophic linear model.

First: The graphical method for solving linear models: [17-20]

Through the studies presented according to classical logic in many references, we know that to find the optimal solution for linear models in which the number of variables is one, two, or three graphically, we represent the area of common solutions for the constraints in one of the spaces R or R^2 or R^3This depends on the number of variables in this sentence, for example if the number of variables is two, i.e. the solution is in space R^2 (where work is done on models that contain one or three variables with the same steps)

We find the optimal solution according to the following steps:

1. We determine the half-planes defined by the inequalities of the constraints by drawing the straight lines resulting from converting the inequalities of the constraints into equals. The drawing is done by specifying two points that satisfy the constraint, and then we connect the two points to obtain the straight line corresponding to the constraint. This straight line divides the plane into two halves in order to determine the half-plane that satisfies the constraint. We choose A point at the top of the mapping from one of the two half-planes. We substitute the coordinates of this point into the inequality. If it is satisfied, then the region in which this point is located is the solution region. If it is not achieved, then the opposite region is the solution region.

2. We define the common solutions region, which is the region resulting from the intersection of the halves of the levels defined by constraint inequalities. This region must be non-empty so that we can proceed with the solution.

3. In order to represent the objective function, we note that its relationship contains three unknowns, Z , x_1 ,x_2 . Therefore, we must know a value for Z, which is unknown to us. Here we assume a value, let it be $Z_1 = 0$, draw the equation of the objective function Z_1 give another value, let it be Z_2, and represent the equation. We get a line parallel to the first line, and by continuing, we obtain a set of parallel lines representing the target function.

4. We draw ray $\vec{C} = \begin{bmatrix} c_1 \\ c_2 \end{bmatrix}$where $c_1$is coefficient of $x_1$and c_2 is coefficient of $x_2$in the objective function statement, and the direction of its increasing function is the direction of ray$\vec{C} = \begin{bmatrix} c_1 \\ c_2 \end{bmatrix}$, and the direction of its decreasing function is the opposite direction. This ray, that is, the drawing is done according to the type of objective function (maximization or minimization). In clearer terms, we find the optimal solution point by pulling the line representing Z_1parallel to itself towards the ray $\vec{C} = \begin{bmatrix} c_1 \\ c_2 \end{bmatrix}$to find the maximum value of the objective function, (And reverse this direction to find the smallest value), until it passes through the last point of the common solutions region and this point is the optimal solution point, which is located on the borders of the common solutions region and any other displacement, no matter how small, takes it out of it.

The graphical method for finding the optimal solution for neutrosophic linear models:
From the definition of neutrosophic linear models, we find that we can apply the same previous steps to obtain the optimal solution, which is a neutrosophic value suitable for all conditions. We explain the above through the following example:

Example 1:
A company produces two types of products A_1 , A_2 and uses three types of raw materials B_1 , B_2 , B_3 in the production process, if the available quantities of each of the raw materials are B_i ; $i = 1,2,3$, and the quantity needed to produce one unit of each products A_j ; $j = 1,2$, and the profit accruing from one unit of each of the products A_1 , A_2 is shown in the following table:

products ⟍ raw materials	A_1	A_2	available quantities
B_1	6	4	36
B_2	2	3	12
B_3	5	0	10
profit	$[6, 8]$	$[2, 4]$	

Table Issue data

Required:
Determine the quantities that must be produced of each products A_j ; $j = 1,2$ so that the company achieves maximum profit:
the solution :
Let x_j be the quantity produced from product j , where $j = 1,2$, then we can formulate the following neutrosophic linear mathematical model:

$$Z = [6,8]x_1 + [2,4]x_2 \rightarrow Max$$

Restrictions:

$$6x_1 + 4x_2 \leq 36 \quad (1)$$
$$2x_1 + 3x_2 \leq 12 \quad (2)$$
$$5x_1 \leq 15$$
$$x_1, x_2 \geq 0$$

The previous model is a linear neutrosophic model because there is indeterminacy in variables coefficients in objective function. To find the optimal solution for the previous model, we will use the graphical method according to the following steps:

The first constraint: We draw the straight line representing the first constraint:

$$6x_1 + 4x_2 = 36$$

We impose:

$$x_1 = 0 \Rightarrow 4x_2 = 36 \Rightarrow x_2 = 9$$

We get the first point: $A(0,9)$.

We impose:

$$x_2 = 0 \Rightarrow 6x_1 = 36 \Rightarrow x_1 = 6$$

We get the second point: $B(6,0)$

If we take a point at the top of the designation from one of the two halves of the resulting plane after drawing the straight through the two points $A(0,9)$ and $B(6,0)$, let it be the point $O(0,0)$ and substitute it in the inequality of the first entry, we find that the inequality is fulfilled, that is, the half of the plane that the point $O(0,0)$ belongs to it, which is half the solution plane of the first-constraint inequality.

We proceed in the same way for the second and third restrictions and obtain the following graphical representation: Figure No. (1)
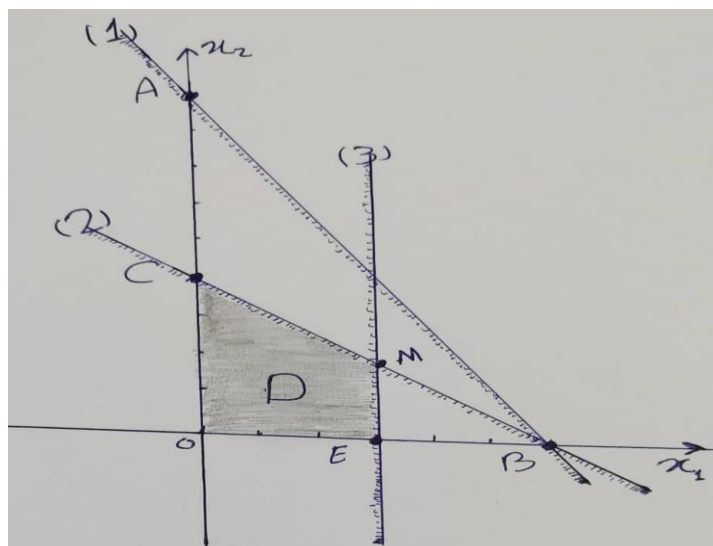

Figure No. (1) Graphic representation of the limitations of the linear model in Example 1

After representing the constraints, we notice that the common solution area is bounded by the polygon whose vertices are the points, $O(0,0)$, $E(3,0)$, M and $C(0,4)$.

The point M is the point of intersection, the second and third constraints we get their coordinates by solving the following two equations:

$$2x_1 + 3x_2 = 12$$

$$5x_1 = 15$$

We fiend**:** $M(3,2)$

Substituting the coordinates of the vertex points into the objective function expression, we get:

$$Z_O = 0$$
$$Z_E \in [12,16]$$
$$Z_M \in [22,32]$$
$$Z_C \in [8,16]$$

That is, the greatest value of the function Z is achieved at point $M(3,2)$, that is, the company must produce three units of the first product and two units of the second product, then it will achieve maximum profit.

$$Max\, Z = Z_M \in [22,32]$$

Note:
 The process of substituting the objective function with the coordinates of the points of the vertices of the common solution area is possible when the number of points is small, as we can easily replace them in the objective function, and the point that gives the best value for the objective function represents the optimal solution, but when there are a large number of constraints, we get a large number From the vertical points located on the perimeter of the common solution region. In this case, the method of finding the coordinates of all these points and substituting them into the objective function becomes impractical, so we resort to representing the objective function and determining the optimal solution point as we mentioned previously.

Second: How to take advantage of the conditions of non-negativity to find the optimal solution for some neutrosophic linear models using the graphical method:

 Example 2:
Find the optimal solution for the following linear neutrosophic model:

$$Z = x_1 - x_2 - 3x_3 + x_4 + [2,5]x_5 - x_6 + 2x_7 - [10,15] \to Max$$

Restrictions:

$$x_1 - x_2 + x_3 = 5 \qquad (1)$$

$$2x_1 - x_2 - x_3 - x_4 = -11 \qquad (2)$$

$$x_1 + x_2 - x_5 = -4 \qquad (3)$$

$$x_2 + x_6 = 6 \qquad (4)$$

$$2x_1 - 3x_2 - x_6 + 2x_7 = 8 \qquad (5)$$

$$x_1, x_2, x_3, x_4, x_5, x_6, x_7 \geq 0$$

The solution:

We note that the number of constraints $m = 5$ and the number of variables $n = 7$, meaning that $n - m = 2$. Therefore, we can, relying on the non-negativity constraints, find the optimal solution for the previous model using the graphical method according to the following steps:

1- We calculate five variables in terms of only two variables.
2- Since the variables of the linear model satisfy the non-negativity constraints, then we obtain from the variables that we calculated five inequalities of the type greater than or equal to.
3- Substituting the five variables into the objective function, we get an objective function with only two variables.
4- We write the new model, which is a linear model with two variables, so the optimal solution can be found graphically.

We apply the previous steps to Example 2:

We find:

$$x_3 = 5 - x_1 + x_2 \qquad (1)'$$

$$x_4 = 3x_1 - 2x_2 + 6 \qquad (2)'$$

$$x_5 = x_1 + x_2 + 4 \qquad (3)'$$

$$x_6 = 6 - x_2 \qquad (4)'$$

$$x_7 = 7 - x_1 + x_2 \qquad (5)'$$

Substituting in the objective function, we get:

$$Z = [1,4]x_1 + [3,6]x_2 + [8,25]$$

Since, $x_3, x_4, x_5, x_6, x_7 \geq 0$ from $(1)'$ ‹ $(2)'$ ‹$(3)'$ ‹ $(4)'$ ‹ $(5)'$, we get the following set of constraints:

$$5 - x_1 + x_2 \geq 0$$

$$-3x_1 + 2x_2 - 3 \geq 0$$

$$x_1 + x_2 + 4 \geq 0$$
$$6 - x_2 \geq 0$$
$$7 - x_1 + x_2 \geq 0$$

Then the neutrosophic linear mathematical model becomes:

Find:

$$Z = [1,4]x_1 + [3,6]x_2 + [8,25] \rightarrow Max$$

Restrictions:

$$5 - x_1 + x_2 \geq 0$$
$$3x_1 - 2x_2 + 6 \geq 0$$
$$x_1 + x_2 + 4 \geq 0$$
$$6 - x_2 \geq 0$$
$$7 - x_1 + x_2 \geq 0$$
$$x_1, x_2 \geq 0$$

The model has two variables, so the optimal solution can be found graphically by following the same steps mentioned in Example (1).

We obtain the representation. Figure No. (2) Is the required graphic representation:

'

Figure No. (2): Graphical representation of the constraints of the linear model in Example 2
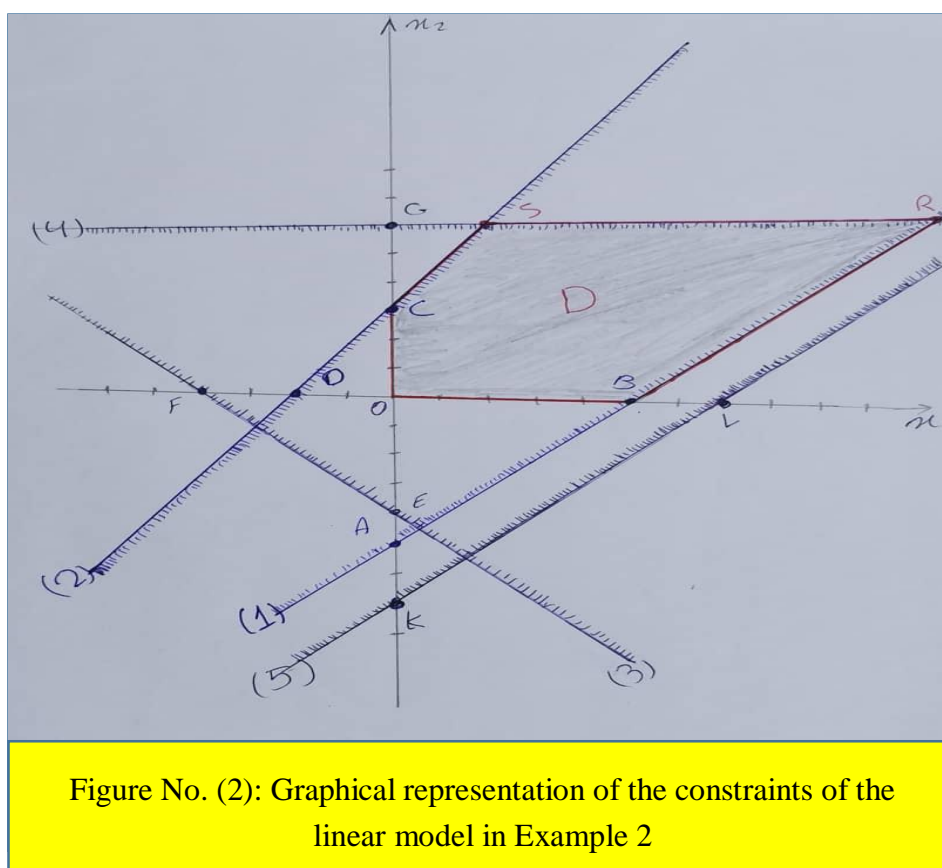
Region D is the region of joint solutions and is defined by the polygon $OBRSC$, where, $O(0,0)$, $B(5,0)$, $C(0,3)$, and for the two points R, S we find:

Point R is the point of intersection of the first and fourth entries.

We obtain its coordinates by solving the set of equations:

$$5 - x_1 + x_2 = 0$$
$$6 - x_2 = 0$$

We get: $R(11,6)$

Point S is the point of intersection of the second and fourth entries.

We obtain its coordinates by solving the set of equations:

$$3x_1 - 2x_2 + 6 = 0$$
$$6 - x_2 = 0$$

We get: $S(2,6)$

Since the optimal solution is located at one of the vertices of the common solution region, we substitute the coordinates of these points with the objective function:

At point, $O(0,0)$
$$Z_O = 0$$

At point, $B(5,0)$
$$Z_B = [13,45]$$

At point , $R(11,6)$
$$Z_R \in [37,105]$$

At point , $S(2,6)$
$$Z_S \in [28,69]$$

At point , $C(0,3)$
$$Z_C \in [17,43]$$

The greatest value of the objective function is at the point, $R(11,6)$ that is, $x_1 = 11$ and $x_2 = 6$.

We calculate the values of the remaining variables by $(1)' \cdot (2)' \cdot (3)' \cdot (4)' \cdot (5)'$.

We find, $x_3 = 0$, $x_4 = 27$, $x_5 = 21$, $x_6 = 0$, $x_7 = 2$.

Substituting in the objective function of the original model we obtain the maximum value of the Z function, which is.

$$MaxZ \in [68,126]$$

Important Notes:
1- The graphical solution applies to a vertical point in space R^n.
2- The number of components of the ideal solution is non-existent because the ideal solution applies to a vertical point, and the vertical point is the result of the intersection of a number of lines or planes, and the number of non-existent components is at least $n - m$ components.
3- The model may include some conditions that do not play a role in the solution process.
4- The ideal solution may be a single point, or it may be an infinite number of points, when one of the sides of the common solution area that passes through the point of the ideal solution is parallel to the straight line $Z = 0$. Therefore, when the straight line representing the objective function is drawn, this straight line will apply to the

parallel side, and all the points of that side, the number of which are infinite, will be they are perfect solutions.

5- If the region of acceptable solutions is open in terms of increasing the function Z then we cannot stop at a specific ideal solution, and then we say that the objective function has an infinite number of acceptable solutions that give us greater and greater values of Z.

6- The state of not having an ideal solution (acceptable solution) when the conditions contradict each other and then the region of possibilities is an empty set (the problem is impossible to solve).

Conclusion and results:

In the previous study, we presented the graphical method for finding the optimal solution for neutrosophic linear models, and also a method that is rarely discussed in classical operations research references, which is how to take advantage of non-negativity constraints to find the graphically optimal solution for some neutrosophic linear models, but we must be aware that we may encounter neutrosophic linear models with two variables, but There may be difficulty in arriving at the common solution area, or there may be difficulty in determining the optimal solution after obtaining the common solution area. Therefore, it is preferable to use the Cymex neutrosophic method. As a result, the main goal is to obtain the optimal solution, so the researcher must determine the appropriate method for the model being solved.

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Neutrosophic Handling of the Simplex Direct Algorithm to Define the Optimal Solution in Linear Programming

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Abstract

Mathematical programming can express competency concepts in a well-defined mathematical model for a particular situation or system, and the ability to derive computational methods to solve this mathematical model, it is also a mathematical tool that allows us to model, analyze and solve a wide range of problems concerned with allocating rare resources of labor, materials, machinery, and capitals. Consequently, using them in the best attainable way to minimize costs or maximize profits. In such issues, the linear programming is one of the most widely used types of mathematical programming because it is a method that helps to make good decisions and decide the best program for independent activities, considering the available sources. It does not take in consideration the continuous and rapid changes and the state of instability in data. So, this manuscript studies one of the methods to solve linear models, which is the simplex method using the neutrosophic theory that covers all the data in analysing, whether specific or not, determined or not, having consistency or not, as well as it deeming all occurring changes. However, the optimal solution is related to the variables in the objective function, which in turn are affected by the fixed quantities that express the available possibilities. This article presents a study to solve the linear model using the simplex method in which the variables and their coefficients are indeterminate values, and we will explain the effected of the indeterminate values on the optimal solution of the mathematical model. The product mixture problem has been presented as case study to demonstrate the efficiency of the proposed method.

Keywords; Simplex Algorithm; Operations Research; Mathematical Programming; Linear Programming; Neutrosophic Logic; Products Mixture Model

1. Introduction

The search for the best maximum (of profit, performance, or yield) or minimum (of loss, risk, or cost) that is optimal solutions to various problems has been the focus of human attention at all era. The great scientific development witnessed by our contemporary world was the reason for the emergence of the so-called operations research, its originating in military efforts before World War II. Operational research (OR) encompasses the development and the use of a wide range of problem-solving techniques and methods applied in the pursuit of improved decision-making and efficiency, such as simulation, mathematical optimization, queuing theory and other stochastic-process models. It is one of the modern sciences whose applications have achieved wide success in various fields of life and it was a helper for companies of all kinds of problems that deal with conditions that are characterized by continuous and very rapid change, which imposes on it a set of challenges and a continuous search for guarantees of survival and continuity through the continuous quest for the best and the increasing need to improve its performance levels and the adoption of various scientific

methods that allow achieving different performance standards. The characteristic of this science is the development of mathematical models, tools and techniques that allow modelling, analysing, and solving the issues posed by our contemporary and complex lives and looking for their optimal solutions. The major sub-disciplines in modern operational research are Computing and information technologies, financial engineering, manufacturing, service sciences and supply chain management, policy modelling and public sector work, revenue management, simulation, stochastic models, and transportation problems.

One of the most important methods that help management in making decisions is the linear programming manner, which is a mathematical tool that allows us to model, analyze, and solve a broad spectrum of problems that resulted from the great scientific development witnessed in our contemporary world, it concerned with allocating rare resources of labour, materials, machinery and capital and using them in the best possible way so as to minimize costs or maximize profits [1,2,3,4,5].

All previous traditional studies, the optimal solution for the problems were specific values resulting from specific or determined data provided by the conducted field study subject to the existed conditions, where the available resources are subject to instantaneous change. However, the reality of the situation indicates that the position of companies is not stable, the future is unpredictable, and these specific values of profits and available resources are subject to instantaneous change. Consequently, to this companies' urgent needs, it was necessary to originate a new theory that considers these fluctuations that productive companies go through and provides results that enjoy indeterminate and give companies a margin of freedom during applying linear programming models. This article gives linear model, which is the direct simplex method using neutrosophic logic, the logic that is the new vision of modelling and is designed to effectively address the uncertainties inherent in the real world founded by the Romanian mathematician Florentine Smarandache [7, 9, 10, 11, 13].

F. Smarandache presented neutrosophic logic in 1995 as a generalization of fuzzy logic that setup by Lotfi A. Zadeh [6] in 1965. And it is also a generalization for intuitionistic fuzzy logic that founded by K. Atanasove [8] in 1882. In addition to that, Ahmed A. Salama presented the theory of neutrosophic classical categories as a generalization of the theory of classical categories [12,20], also, he developed, introduced, and formulated new concepts in the various disciplinary of mathematics, statistics, computer science by neutrosophic theory [17,18,19,22,28].

In the recent past decades, the neutrosophic theory has broad and significant expanded in both its theoretical and applied aspects, where it has been applied in dozens of application fields such as but not limited to measurement, graph theory, statistics, probability...etc. [8,14,15, 16, 21, 23, 25, 26, 27].

2. Discussion

Linear programming is the method that helps in selecting decisions and choosing the best program for independent activities, considering the available resources. Linear programming is used to solve problems in which the goal is specific, such as securing a maximum profit, securing a minimum cost, saving time or effort ... etc. The linear programming problems consisting of a linear objective function subject to some constraints (inequalities or equations), is characterized by the existence of many acceptable solutions to it, the goal is to find an optimal solution among the set of those accepted solutions [1,2,3].

It is well known that to get an optimal solution for any linear programming problem using the direct simplex algorithm should be processed to be in standard form, the simplex method for solving an LP problem requires the problem to be expressed in the standard form. But not all LP problems appear in the standard form. In many cases, some of the constraints are expressed as inequalities rather than equations; at least it is most often true in the case of water resources problems. In some problems, all the decision variables may not be even nonnegative. Hence, the first step in solving an LP problem is to convert it to the standard form. The procedure to convert a general program to the standard form is outlined below:

1- Convert all inequalities to equalities.
2- Convert all decision variables unrestricted in sign to strictly non-negative.
3- Make all the right-hand side constants of the constraints nonnegative.

3. Mathematical Model of Linear Programming Having Neutrosophic Values in Its Coefficients

Using simplex method, find the optimal solution for the following linear programming problem (1):

$$\max Z = c_{1N}x_1 + c_{2N}x_2 + \cdots + c_{nN}x_n$$

$$subject\ to \begin{cases} a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n \le b_{1N} \\ a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n \le b_{2N} \\ a_{31}x_1 + a_{32}x_2 + \cdots + a_{3n}x_n \le b_{3N} \\ \quad\quad\quad\quad . \\ \quad\quad\quad\quad\quad . \\ \quad\quad\quad\quad\quad\quad . \\ a_{m1}x_1 + a_{m2}x_2 + \cdots + a_{mn}x_n \le b_{mN} \end{cases}$$

With the non-negativity conditions $x_1, x_2, \ldots, x_n \ge 0$.

It is worthy to mention that the coefficients subscribed by the index N are of neutrosophic values.

The objective function coefficients $c_{1N}, c_{2N}, \ldots, c_{nN}$ have neutrosophic meaning are intervals of possible values:

That is, $c_{jN} = [\lambda_{j1}, \lambda_{j2}]$, where $\lambda_{j1}, \lambda_{j2}$ are the upper and the lower bounds of the objective variables x_j respectively, $j = 1, 2, \ldots, n$. Also, we have the values of the right-hand side of the inequality constraints $b_{1N}, b_{2N}, \ldots, b_{mN}$ are regarded as neutrosophic interval values:

$b_{iN} = [\mu_{i1}, \mu_{i2}]$, here, μ_{i1}, μ_{i2} are the upper and the lower bounds of the constraint $i = 1, 2, \ldots, m$.

As previously mentioned, to solve any linear problem by simplex method, we should put it in its standard form, so, it should be handled using the slack, surplus, and artificial variables which have their usual meaning here,

- slack variables are additional variables that are introduced into the linear constraints of a linear program to transform them from inequality constraints to equality constraints. If the model is in standard form, the slack variables will always have a zero coefficient in the objective function and +1 coefficient in corresponding constraint. Slack variables are needed in the constraints to transform them into solvable equalities with one definite answer.
- A surplus variable represents the amount by which solution values exceed a resource. These variables are also called 'Negative Slack Variables'. Surplus variables like slack variables carry a zero coefficient in the objective function. it is added to greater than or equal to (>) type constraints in order to get an equality constraint.
- Artificial variables are added to those constraints with equality (=) and greater than or equal to (>) sign. An Artificial variable is added to the constraints to get an initial solution to an LP problem. Artificial variables have no meaning in a physical sense and are not only used as a tool for generating an initial solution to an LP problem.

After getting a well-defined for problem (1), solve it using simplex algorithm:

$$\max Z = c_{1N}x_1 + c_{2N}x_2 + \cdots + c_{nN}x_n + 0y_1 + 0y_2 + \cdots + 0y_m$$

$$subject\ to \begin{cases} a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n + y_1 = b_{1N} \\ a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n + y_2 = b_{2N} \\ a_{31}x_1 + a_{32}x_2 + \cdots + a_{3n}x_n + y_3 = b_{3N} \\ \qquad\qquad\qquad . \\ \qquad\qquad\qquad . \\ \qquad\qquad\qquad . \\ a_{m1}x_1 + a_{m2}x_2 + \cdots + a_{mn}x_n + y_m = b_{mN} \end{cases}$$

With the non-negativity conditions $x_1, x_2, \dots, x_n, y_1, y_2, y_3, \dots, y_m \geq 0$. The below tableau consists of the coefficient corresponding to the linear constraint variables and the coefficients of the objective function.

Table 1: The initial Simplex Tableau

Basic Variables	Coefficient of								Right Side b_{iN}
	x_1	x_2	$\cdots$	x_n	y_1	y_2	$\cdots$	y_m	
y_1	a_{11}	a_{12}	$\cdots$	a_{1n}	1	0	$\cdots$	0	b_{1N}
y_2	a_{21}	a_{22}	$\cdots$	a_{2n}	0	1	.	0	b_{2N}
.	.	.	$\cdots$	.	.	.	.	.	.
.	.	.	$\cdots$	.	.	.	$\cdots$	.	.
.	.	.		.	.	.	.		.
y_m	a_{m1}	a_{m2}		a_{mn}	0	0	.	1	b_{mN}
							$\cdots$ $\cdots$		
Z	c_{1N}	c_{2N}	$\cdots$ $\cdots$	c_{nN}	0	0	$\cdots$ $\cdots$.	0	$Z - q_o$

The optimal solution of a maximization linear programming model are the values assigned to the variables in the objective function to give the largest zeta value. The optimal solution would exist on the corner points of the graph of the entire model. To check optimality using the tableau, all values in the last row must contain values greater than or equal to zero. If a value is less than zero, it means that variable has not reached its optimal value. If a tableau is not optimal, the next step is to identify the pivot variable to base a new tableau on, as described below:

The added variables $\{y_1, y_2, \dots, y_m\}$ represent the first basic variables, and the variables $\{x_1, x_2, \dots, x_n\}$ are non-basic variables. Since the problem is for the maximum objective function, the maximum positive coefficient in the objective function has been selected, i.e. $Max\{c_{1N}, c_{2N}, \dots, c_{nN}\} = c_{sN}$. Suppose the coefficient c_{sN} of the term x_s is the maximum positive number. The column verses the number c_{sN} is the required column that set the variable x_s as the

enter variable, it is still required to determine the row that crossed to the selected column to determine the pivot element:

$$\theta = min\left[\frac{b_{iN}}{a_{is}}\right] = \frac{b_{tN}}{a_{ts}} > 0; \quad a_{is} > 0, b_{iN} > 0$$

The pivot element is used in row operations to identify which variable will become the unit value and is a key factor in the conversion of the unit value. The pivot element can be identified by looking at the bottom row of the tableau and the indicator. Assuming that the solution is not optimal, pick the smallest negative value in the bottom row. One of the values lying in the column of this value will be the pivot variable. To find the indicator, divide the beta values (b_{iN}) of the linear constraints by their corresponding values from the column containing the possible pivot variable. The intersection of the row with the smallest non-negative indicator and the smallest negative value in the bottom row will become the pivot element.

To optimize the pivot element, it will need to be transformed into a unit value (value of 1). To transform the value, multiply the row containing the pivot variable by the reciprocal of the pivot value.

To keep the tableau equivalent, the other variables not contained in the pivot column or pivot row must be calculated by using the new pivot values. For each new value, multiply the negative of the value in the old pivot column by the value in the new pivot row that corresponds to the value being calculated. Then add this to the old value from the old tableau to produce the new value for the new tableau. This step can be condensed into the equation on the following:

New tableau value = (Negative value in old tableau pivot column) x (value in new tableau pivot row) + (Old tableau value).

The following mathematical symbols and operations indicate the above explain:

$$\frac{a_{t1}}{a_{ts}}, \frac{a_{t2}}{a_{ts}}, \frac{a_{t3}}{a_{ts}}, \dots, \frac{a_{ts-1}}{a_{ts}}, 1, \frac{a_{ts+1}}{a_{ts}}, \dots, 0, 0, \frac{b_{tN}}{a_{ts}}.$$

$$a'_{ij} = a_{ij} - a_{tj}\frac{a_{is}}{a_{ts}} = \frac{a_{ij}a_{ts}-a_{tj}a_{is}}{a_{ts}}.$$

$$b'_{iN} = b_{iN} - b_{tN}\frac{a_{is}}{a_{ts}} = \frac{b_{iN}a_{ts}-b_{tN}a_{is}}{a_{ts}}.$$

$$c'_{jN} = c_{jN} - c_{sN}\frac{a_{tj}}{a_{ts}} = \frac{c_{jN}a_{ts}-c_{sN}a_{tj}}{a_{ts}}.$$

The $(b'_{1N}, b'_{2N}, \dots, b'_{mN}, 0, 0, \dots, 0)$ represents the non-negative basic solution which gives a value for $Z = q_{oN}$, the following tableau has been constructed:

Table 2: The Iteration Tableau of Simplex Method

Basic Variables	Coefficient of											Right Side b'_{iN}
	x	x	..	x_i	x_m	..	x_n	y_1	y_2	..	y_n	

x_1	1	0	..	0	a'_{1n}	..	a'_1	α_1	α_1	.. α_1	b'_{1N}
x_2	0	1	..	0	a'_{2n}	..	a'_2	α_2	α_2	.. α_2	b'_{2N}
.	.	.		.	.		.	.	.	.. .	.
.	.	.		.	.		.	.	.	.. .	.
.	.	.		.	.		.	.	.	.. .	.
x_m	0	0		1	a'_{mn}	..	a'_n	α_n	α_n	.. α_n	b'_{mN}
Z	0	0	..	0	c'_m	..	c'_n	q_1	q_2	.. q_n	$Z-...$

Where α_{ij} and q_{iN} are the coefficients of the added variables in the objective function and in the constraints after execute the previous iterative operations. The optimal solution is:

$x_1^* = b'_{1N}, x_2^* = b'_{2N}, \dots, x_m^* = b'_{mN}$,

The optimal maximum solution of the objective function is:

$Z^* = c_{1N}b'_1 + c_{2N}b'_2 + \cdots + c_{mN}b'_m$.

3.1 Important Notes: [2,3]

1- A basic solution is admissible if all variables of the basic solution are nonnegative.

2- Each basic solution of linear programming problem for which all variables are nonnegative, is called an admissible basic solution. This admissible basic solution corresponds to an extreme point (corner solution).

3- There are two types of unboundedness:

- The objective function is unbounded (as is the feasible region).

- The feasible region is unbounded, but the objective function is not.

" The LP is unbounded if there is a negative coefficient in the row versus of the objective function,

and all the remaining elements in the column are negative or zero ".

4- Stop if all coefficients of the row versus to the objective function are negative or zeros. The

current solution is optimal and unique.

4. Practical Example

A. The Classical Context of the Problem [3]

A company produces two types of products A, B using four raw materials F_1, F_2, F_3, F_4. The quantities needed from each of these materials to produce one unit of each of the two producers A, B, the available quantities of the raw materials, and the profit returned from one unit of both products are shown in table 3.

Table 3: the available quantities of the raw materials, and the profit returned from one unit of both products in the Classical Context

Products / Raw Materials	Required quantity per unit		Available quantities of the raw materials
	A	B	
F_1	2	3	19

F_2	2	1	13
F_3	0	3	15
F_4	3	0	18
Profit Returned per unit	7	5	

Required:

Finding the optimal production plan that makes the company's profit from the producers A, B as large as possible.

We symbolize the quantities produced from the product A with the symbol x_1, and from the product B with the symbol x_2 , after building the appropriate mathematical model and solving it, we conclude that $x_1 = 5, x_2 = 3$, and hence the maximum profit $Z^* = 50$ of monetary unit.

B. The Neutrosophic Context of the Problem

A company produces two types of products A, B using four raw materials F_1, F_2, F_3, F_4. The quantities needed from each of these materials to produce one unit of each of the two producers A, B, the available quantities of the raw materials, and the profit returned from one unit of both products are shown in table 4.

Table 4: the available quantities of the raw materials, and the profit returned from one unit of both products in the Neutrosophic Context

Products / Raw Materials	Required quantity per unit		Available quantities of the raw materials
	A	B	
F_1	2	3	[14,20]
F_2	2	1	[10,16]
F_3	0	3	[12,18]
F_4	3	0	[15,21]
Profit Returned per unit	[5,8]	[3,6]	

Required:

Finding the optimal production plan that makes the company's profit from the producers A, B as large as possible.

Symbolize the quantities produced from the product A with the symbol x_1, and from the product B with the symbol x_2 , the problem will be reformulated from the neutrosophic perspective as follow:

$$\max Z = [5,8] x_1 + [3,6] x_2$$

$$subject\ to \begin{cases} 2x_1 + 3x_2 \le [14,20] \\ 2x_1 + x_2 \le [10,16] \\ 3x_2 \le [12,18] \\ 3x_1 \le [15,21] \end{cases}$$

$$x_1 \ge 0, x_2 \ge 0$$

The above program need to reformulated to an equivalent form by adding slack variables:

$$\max Z = [5,8]\, x_1 + [3,6]\, x_2 + 0y_1 + 0y_2 + 0y_3 + 0y_4$$

$$subject\ to \begin{cases} 2x_1 + 3x_2 + y_1 = [14,20] \\ 2x_1 + x_2 + y_2 = [10,16] \\ 3x_2 + y_3 = [12,18] \\ 3x_1 + y_4 = [15,21] \end{cases}$$

$$x_1 \ge 0, x_2 \ge 0, y_1 \ge 0, y_2 \ge 0,\ y_3 \ge 0,\ y_4 \ge 0$$

Table 5: Symbolize the quantities

Non-basic var. Basic var.	x_1	x_1	y_1	y_2	y_3	y_4	R.H.S. b_i
y_1	2	3	1	0	0	0	[14,20]
y_2	2	1	0	1	0	0	[10,16]
y_3	0	3	0	0	1	0	[12,18]
y_4	3	0	0	0	0	1	[15,21]
Objective fun.	[5,8]	[3,6]	0	0	0	0	$Z - 0$

It is clear that $\max([5,8], [3,6]) = [5,8]$ versus to the column of x_1 , meaning that the variable x_1 should be placed instead of one of the basic variables. Now to demonstrate which basic variables should be ejected, the following calculation has been done:

$$\theta = min\left[\frac{[14,20]}{2}, \frac{[10,16]}{2}, \frac{[15,21]}{3}\right] = \frac{[15,21]}{3} = [5,7]$$

The value of θ indicates that the row versus to the variable y_4, and the element positioned in the cross row/column is 3 which is the pivot element, divide the elements of the row versus to y_4 yields:

$$\frac{3}{3}, \frac{0}{3}, \frac{0}{3}, \frac{0}{3}, \frac{0}{3}, \frac{1}{3}, \frac{[15,21]}{3} = [5,7]$$

Table 6: basic variables to be ejected (A)

Non-basic var.	x_1	x_2	y_1	y_2	y_3	y_4	R.H.

Basic var.							S. b_i
y_1	0	3	1	0	0	$\frac{-2}{3}$	[4,6]
y_2	0	1	0	1	0	$\frac{-2}{3}$	[0,4]
y_3	0	3	0	0	1	0	[12,18]
x_1	1	0	0	0	0	$\frac{1}{3}$	[5,7]
Object. Fun.	0	[3,6	0	0	0	$[\frac{-8}{3}, \frac{-}{3}$	$Z - [25,56]$

It still there is a non-negative value in the row of the objective function (i.e. [3,6]) which is versus to the x_2 column, this leads to the fact that the variable x_2 should be entered into the basic variables. Now, the question is: which basic variable should be ejected? Track the following calculation to answer this question, $\theta = \min\left[\frac{[4,6]}{3}, \frac{[0,4]}{1}, \frac{[12,18]}{3}\right] = \frac{[4,6]}{3} = [\frac{4}{3}, 2]$ which is versus to the slack variable y_1, the pivot element equal 3, hence the row versus to y_1 should be divided by 3, the required calculations yield table 7.

Table 7: basic variables to be ejected (B)

Non-basic var. / basic var.	x_1	x	y_1	y_2	y_3	y_4	R.H.S. b_i
x_2	0	1	$\frac{1}{3}$	0	0	$\frac{-2}{9}$	$[\frac{4}{3}, 2]$
y_2	0	0	$\frac{1}{3}$	1	0	$\frac{-4}{9}$	$[\frac{-4}{3}, 2]$
y_3	0	0	-1	0	1	$\frac{2}{3}$	[8,12]
x_1	1	0	0	0	0	$\frac{1}{3}$	[5,7]
Object. Fun.	0	0	[-2,-1]	0	0	$[\frac{-6}{9}, -1]$	$Z - [29,68]$

It is clear from the row of the objective function that all the elements are either zero or neutrosophic negative numbers, this means that we have reached to the optimal solution is:

$x_1^* = [5,7], x_2^* = \left[\frac{4}{3}, 2\right], y_2^* = \left[\frac{-4}{3}, 2\right], y_3^* = [8,12], y_1^* = y_4^* = 0$

Substitute the above optimal solution into the objective maximum function, yields:

$\max Z = [5,8].[5,7] + [3,6].\left[\frac{4}{3}, 2\right] = [25,56] + [4,12] = [29,68]$ which is identical to the result in the previous tableau.

To more checking, the optimal solution must be satisfied the constraints:

5. Summarization

Table 8 summarizing the previous study by indicating the benefit of using neutrosophic theory.

Table 8: benefits of using neutrosophic theory

Classical logic								
issue data						results		
c_1	c_2	b_1	b_2	b_3	b_4	x_1	x_2	$max\ Z$
7	5	1 9	1 3	1 5	1 8	5	3	5 0
Neutrosophic logic								
issue data						results		
c_{1N}	c_{2N}	b_{1N}	b_{2N}	b_{3N}	b_{4N}	x_{1N}	x_{2N}	$max\ Z$
[5 , 8]	[3 , 6]	[1 4 , 2 0]	[1 0 , 1 6]	[1 2 , 1 8]	[1 5 , 2 1]	[5 , 7]	$[\frac{4}{3}, 2]$	[2 9 , 6 8]

the reader could easily noticed that, when the neutrosophic strategy has been applied in solving linear programming problem, indeterminate vales have been obtained, these indeterminate values have well simulated the reality, and takes into account most of the changes that may occur in the working environment of the system represented by the linear mathematical model, while the values we obtain when solving according to traditional classical logic are specific values and do not take into account the change that It can occur in the working environment of the system represented by the mathematical model.

6. Conclusion

The neutrosophic logic provides us with a more general and comprehensive study than the well-known classical study and does not neglect any data just because they are not explicitly specified, and therefore the presence of indeterminacy actually affects the results and these unspecified values cannot Ignoring it and keeping it away from the framework of the study in order to obtain as accurate results as possible, that is, working within the classical logic is no longer sufficient at the present time, because the development of science and the instability in the work environment of the facilities put before us a large number of cases that need quick treatment and accurate in order to avoid losses that may be incurred by the facilities, which cannot be handled using the classical logic and here comes the role of the neutrosophic logic, which provides us with a more comprehensive, it helps us to interpret the study data and helps us obtain the more logical results.

We look forward in the future to study other topics of linear programming and its applications in practical life using neutrosophic logic, accompanying programs, sensitivity analysis...etc.

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Neutrosophic Treatment of the Modified Simplex Algorithm to find the Optimal Solution for Linear Models

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Abstract

Science is the basis for managing the affairs of life and human activities, and living without knowledge is a form of wandering and a kind of loss. Using scientific methods helps us understand the foundations of choice, decision-making, and adopting the right solutions when solutions abound and options are numerous. Operational research is considered the best that scientific development has provided because its methods depend on the application of scientific methods in solving complex issues and the optimal use of available resources in various fields, private and governmental work in peace and war, in politics and economics, in planning and implementation, and in various aspects of life. Its basic essence is to use the data provided for the issue under study to build a mathematical model that is the optimal solution. It is the basis on which decision makers rely in managing institutions and companies, and when operations research methods meet with the neutrosophic teacher, we get ideal solutions that take into account all the circumstances and fluctuations that may occur in the work environment over time. One of the most important operations research methods is the linear programming method. Which prompted us to reformulate the linear models, the graphical method, and the simplex method, which are used to obtain the optimal solution for linear models using the concepts of neutrosophic science. In this research, and as a continuation of what we presented previously, we will reformulate the modified simplex algorithm that was presented to address the difficulty that we were facing when applying the direct simplex algorithm. It is the large number of calculations required to be performed in each step of the solution, which requires a lot of time and effort.

keywords: Linear programming; neutrosophic logic; neutrosophic simplex algorithm; modified neutrosophic simplex algorithm.

1. Introduction:

Operational research is the application of scientific methods to complex issues. It is the art of finding the best solutions and arriving at them through scientific methods so that nothing is left to chance or luck. With the computer, without which it would not have been possible to achieve solutions to many of these issues raised, the use of operations research methods helps us understand the foundations of selection and decision-making and reach agreement on the best ones in order to adopt the correct solutions when solutions abound and options are numerous [1,2,3,4,5], and even the studies presented complete the scientific and practical issues. It is preferable to use the concepts of neutrosophic science, the science that has proven its ability to provide appropriate solutions for all conditions that may occur in the case under study and helps decision makers to adopt ideal decisions that achieve the greatest profit and the lowest cost. Therefore, we have reformulated many research methods. Processes using neutrosophic concepts, see [6-19]. We continue to build neutrosophic processes research by reformulating the modified simplex algorithm and the synthetic algorithm used to find the optimal solution for linear models using neutrosophic concepts.

2. Discussion:

The neutrosophic linear model is given by the following general formula: [20]
The neutrosophic general form is given the linear mathematical model in the abbreviated form as follows:

$$\text{Max } or \text{ Min } f(x) = \sum_{j=1}^{n} (c_j \pm \varepsilon_j) x_j$$

Subject to

$$\sum_{j=1}^{n} a_{ij} x_j \leq b_i + \delta_i \quad ; i = 1,2,\dots,m$$

$$\sum_{j=1}^{n} a_{ij} x_j \geq b_i + \delta_i \quad ; i = 1,2,\dots,m$$

$$x_j \geq 0 \quad ; j = 1,2,\dots,n$$

Where $Nc_j = c_j \pm \varepsilon_j$, $Nb_i = b_i \pm \delta_i$, a_{ij}, $j = 1,2,\dots,n$, $i = 1,2,\dots,m$ are constants having set or interval values according to the nature of the given problem, x_j are decision variables.

It is worthy to mention that the coefficients subscribed by the index N are of neutrosophic values.

The objective function coefficients $Nc_1, Nc_2, \dots, Nc_n$ have neutrosophic meaning are intervals of possible values:

That is, $Nc_j \in [\lambda_{j1}, \lambda_{j2}]$, where $\lambda_{j1}, \lambda_{j2}$ are the upper and the lower bounds of the objective variables x_j respectively, $j = 1,2,\dots,n$. Also, we have the values of the right-hand side of the inequality constraints $Nb_1, Nb_2, \dots, Nb_m$ are regarded as neutrosophic interval values:

$Nb_i \in [\mu_{i1}, \mu_{i2}]$, here, μ_{i1}, μ_{i2} are the upper and the lower bounds of the constraint

$i = 1,2,\dots,m$.

The optimal solution is obtained using the direct simplex method, which is summarized in three basic stages, as we presented: [21]

The first stage: converting the imposed model into the formal form

The second stage: converting the regular form into a basic form to obtain the non-negative basic solutions

The third stage: searching for the ideal solution required among the non-negative basic solutions.
In this research, we will reformulate the modified Simplex algorithm using the concepts of neutrosophic science. The algorithm was presented to address the difficulty that we faced when applying the direct Simplex algorithm, which is the large number of calculations required to be performed in each step of the solution, which requires a lot of time and effort. We explain the steps of the modified simplex algorithm through the following neutrosophic linear mathematical model:

$$Max \; Z = Nc_1 x_1 + Nc_2 x_2 + \dots + Nc_n x_n$$

$$subject \; to \begin{cases} a_{11}x_1 + a_{12}x_2 + \dots + a_{1n}x_n \leq Nb_1 \\ a_{21}x_1 + a_{22}x_2 + \dots + a_{2n}x_n \leq Nb_2 \\ a_{31}x_1 + a_{32}x_2 + \dots + a_{3n}x_n \leq Nb_3 \\ \qquad\qquad\qquad\quad . \\ \qquad\qquad\qquad\quad . \\ \qquad\qquad\qquad\quad . \\ a_{m1}x_1 + a_{m2}x_2 + \dots + a_{mn}x_n \leq Nb_m \end{cases}$$

With the non-negativity conditions $x_1, x_2, \dots, x_n \geq 0$.

To find the optimal solution for this linear neutrosophic model using the modified simplex algorithm.

1- We write the neutrosophic linear model in standard form, we get the following model:

$$\max Z = c_{1N} x_1 + c_{2N} x_2 + \dots + c_{nN} x_n + 0y_1 + 0y_2 + \dots + 0y_m$$

$$subject\ to \begin{cases} a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n + y_1 = b_{1N} \\ a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n + y_2 = b_{2N} \\ a_{31}x_1 + a_{32}x_2 + \cdots + a_{3n}x_n + y_3 = b_{3N} \\ \quad\quad\quad . \\ \quad\quad\quad . \\ \quad\quad\quad . \\ a_{m1}x_1 + a_{m2}x_2 + \cdots + a_{mn}x_n + y_m = b_{mN} \end{cases}$$

With the non-negativity conditions $x_1, x_2, \dots, x_n, y_1, y_2, y_3, \dots, y_m \geq 0$. The below tableau consists of the coefficient corresponding to the linear constraint variables and the coefficients of the objective function.

We convert the regular linear model to the basal form and place the coefficients in a short table whose first column includes the basic variables and whose top row includes the non- basic variables only.

- We define the pivot column, which is the column corresponding to the largest positive value in the objective function row because the objective function is a maximization function (but if the objective function is a minimization function, it is the column corresponding to the most negative values). Let this column be the column of the variable x_s.

- We define the pivot row; The pivot row is determined by the following indicator:

$$\theta = min\left[\frac{b_{iN}}{a_{is}}\right] = \frac{b_{tN}}{a_{ts}} > 0; \quad a_{is} > 0, b_{iN} > 0$$

Let this row be the line of the base variable y_t

- Then the fulcrum element is the element resulting from the intersection of the fulcrum column and the fulcrum row, i.e., We explain the second step in the following table:

Table No. 1: Anchor element table   -

Non-Basic Variables / Basic Variables	x_1	x_2	…..	x_s	….	x_n	Nb_i
y_1	a_{11}	a_{12}	……	a_{1s}	…..	a_{1n}	Nb_1
y_2	a_{21}	a_{22}	…….	a_{1s}	….	a_{2n}	Nb_2
……..	…….	……..	…….	……	…..	……	…….
y_t	a_{t1}	a_{t2}	……	a_{ts}	……	a_{tn}	Nb_t
……	……	……	….	…..	…..	……	…….
y_m	a_{m1}	a_{m2}	…….	a_{1s}	….	a_{mn}	Nb_m
NZ	Nc_1	Nc_2	……	Nc_s	…..	Nc_n	$NZ - Nc_o$

We calculate the new elements corresponding to the pivot row and the pivot column according to the following steps:

1- We put opposite the pivot element a_{ts} the reciprocal of $\frac{1}{a_{ts}}$

2- We calculate the elements of the row corresponding to the pivot row (except the pivot row element) by dividing the elements of the pivot row by the anchor element a_{ts}

3- We calculate all the elements of the column opposite the fulcrum (except the fulcrum element) by dividing the elements of the fulcrum column by the fulcrum element a_{ts} and then multiplying them by(-1)

4- We calculate the other elements from the following relationships:

$$a'_{ij} = a_{ij} - a_{tj}\frac{a_{is}}{a_{ts}} = \frac{a_{ij}a_{ts} - a_{tj}a_{is}}{a_{ts}} \quad (1)$$

$$Nb'_i = Nb_i - Nb_t\frac{a_{is}}{a_{ts}} = \frac{Nb_ia_{ts} - Nb_ta_{is}}{a_{ts}} \quad (2)$$

$$Nc'_j = Nc_j - Nc_s\frac{a_{tj}}{a_{ts}} = \frac{Nc_ja_{ts} - Nc_sa_{tj}}{a_{ts}} \quad (3)$$

We get the following table:

Table No. 2: The first stage in searching for the optimal solution

Non-Basic Variables / Basic Variables	x_1	x_2	…..	y_t	….	x_n	Nb'_i

y_1	a'_{11}	a'_{12}	……	$\dfrac{-a_{1s}}{a_{ts}}$	…..	a'_{1n}	Nb'_1
y_2	a'_{21}	a'_{22}	…….	$\dfrac{-a_{2s}}{a_{ts}}$	….	a'_{21}	Nb'_2
……..	…….	……...	…….	……	…..	……	…….
x_s	$\dfrac{a_{t1}}{a_{ts}}$	$\dfrac{a_{t2}}{a_{ts}}$	……	$\dfrac{1}{a_{ts}}$	……	$\dfrac{a_{tn}}{a_{ts}}$	$\dfrac{Nb_t}{a_{ts}}$
……	……	……	….	…..	…..	……	…….
y_m	a'_{m1}	a'_{m1}	…….	$\dfrac{-a_{ms}}{a_{ts}}$	….	a'_{mn}	Nb'_m
NZ	Nc'_1	Nc'_2	……	$\dfrac{-Nc_s}{a_{ts}}$	…..	Nc'_n	$NZ - Nc'_0$

We apply the stopping criterion of the Simplex algorithm to the objective function row in Table (2) below:

Since the objective function is of the maximize type, the objective function row in the table must not contain any positive value, (but if the objective function is of the minimize type, the objective function row in the new table must not contain any negative value), in the event that the Criterion: We return to step No. (3) and repeat the same steps until the stopping criterion is met and we obtain the desired ideal solution.

We explain the above through the following example: (This example was presented in research [21])

Example:

A company produces two types of products A, B using four raw materials F_1, F_2, F_3, F_4. The quantities needed from each of these materials to produce one unit of each of the two producers A, B, the available quantities of the raw materials, and the profit returned from one unit of both products are shown in the following table:

Table No. 3: Issue dat

Products ／ Raw Materials	Required quantity per unit		Available quantities of the raw materials
	A	B	
F_1	2	3	[14,20]
F_2	2	1	[10,16]
F_3	0	3	[12,18]
F_4	3	0	[15,21]
Profit Returned per unit	[5,8]	[3,6]	

Required:

Finding the optimal production plan that makes the company's profit from the producers A, B as large as possible.

Symbolize the quantities produced from the product A with the symbol x_1, and from the product B with the symbol x_2, the problem will be reformulated from the neutrosophic perspective as follow:

$$\max Z \in [5,8]\, x_1 + [3,6]\, x_2$$

$$subject\ to\ \begin{cases} 2x_1 + 3x_2 \le [14,20] \\ 2x_1 + x_2 \le [10,16] \\ 3x_2 \le [12,18] \\ 3x_1 \le [15,21] \end{cases}$$

$$x_1 \ge 0,\ x_2 \ge 0$$

We apply the modified simplex algorithm:

1- The standard form of the previous linear model is:

$$\max Z \in [5,8]\, x_1 + [3,6]\, x_2 + 0y_1 + 0y_2 + 0y_3 + 0y_4$$

$$subject\ to\ \begin{cases} 2x_1 + 3x_2 + y_1 = [14,20] \\ 2x_1 + x_2 + y_2 = [10,16] \\ 3x_2 + y_3 = [12,18] \\ 3x_1 + y_4 = [15,21] \end{cases}$$

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$$x_1 \geq 0, x_2 \geq 0, y_1 \geq 0, y_2 \geq 0, y_3 \geq 0, y_4 \geq 0$$

2- We organize the previous information in the following modified simplex table:
   Table No. 2: Simplex table according to the modified Neutrosophic simplex algorithm.   -3

Non-basic var. Basic var.	x_1	x_1	b_i
y_1	2	3	[14,20]
y_2	2	1	[10,16]
y_3	0	3	[12,18]
y_4	3	0	[15,21]
Objective fun.	[5,8]	[3,6]	$Z - 0$

We know $[a, b] \leq [c, d]$ if $a \leq c$ and $b \leq d$,Therefore.

It is clear that max($[5,8], [3,6]$) = $[5,8]$ versus to the column of x_1 , meaning that the variable x_1 should be placed instead of one of the basic variables.

Now to demonstrate which basic variables should be ejected, the following calculation has been done:

$$\theta \in min \left[\frac{[14,20]}{2}, \frac{[10,16]}{2}, \frac{[15,21]}{3}\right] = \frac{[15,21]}{3} = [5,7]$$

The value of θ indicates that the row versus to the variable y_4, and the element positioned in the cross row/column is 3 which is the pivot element, divide the elements of the row versus to y_4 yields.

4- We calculate the elements of the new table using relationships (1), (2), (3), we obtain the following table

Table No. 3: Table of the first step in searching for the optimal solution

Non-basic var. Basic var.	x_1	x_2	b_i'
y_1	$\frac{-2}{3}$	3	[4,6]
y_2	$\frac{-2}{3}$	1	[0,4]
y_3	0	3	[12,18]
x_1	$\frac{1}{3}$	0	[5,7]
Object. Fun.	$\left[\frac{-8}{3}, \frac{-5}{3}\right]$	[3,6]	$Z - [25,56]$

We apply a stopping criterion in the algorithm. We find:

It still there is a non-negative value in the row of the objective function (i.e., [3,6])

This means that we have not yet reached the optimal solution, so we repeat the previous steps as follows:

which is versus to the x_2 column, this leads to the fact that the variable x_2 should be entered into the basic variables. Now, the question is: which basic variable should be ejected?

Track the following calculation to answer this question,

And here too because $[a, b] \leq [c, d]$ if $a \leq c$ and $b \leq d$,Therefore.

$$\theta \in \min\left[\frac{[4,6]}{3}, \frac{[0,4]}{1}, \frac{[12,18]}{3}\right] = \frac{[4,6]}{3} = \left[\frac{4}{3}, 2\right]$$

which is versus to the slack variable y_1, the pivot element equal 3, hence the row versus to y_1 should be divided by 3 , the required calculations yield the following tableau:

Table No. 4: Final solution table

Non-basic var. / basic var.	x_1	x_2	b'_i
x_2	$\frac{-2}{9}$	$\frac{1}{3}$	$[\frac{4}{3}, 2]$
y_2	$\frac{-4}{9}$	$\frac{-1}{3}$	$[\frac{-4}{3}, 2]$
y_3	$\frac{2}{3}$	-1	$[8,12]$
x_1	1	0	$[5,7]$
Object. Fun.	$[-6, -1]$	$[-2, -1]$	$Z - [29,68]$

We apply the algorithm stopping criterion. We find that the criterion has been met and thus we have reached the optimal solution.

The optimal solution for the linear model is:

$$x_1^* \in [5,7], x_2^* \in \left[\frac{4}{3}, 2\right], y_2^* \in \left[\frac{-4}{3}, 2\right], y_3^* \in [8,12], y_1^* = y_4^* = 0$$

The value of the objective function corresponds to:

$$\max Z \in [5,8].[5,7] + [3,6].\left[\frac{4}{3}, 2\right] = [25,56] + [4,12] = [29,68]$$

It is clear from the row of the objective function that all the elements are neutrosophic negative numbers, this means that we have reached to the optimal solution is:

$$x_1^* \in [5,7], x_2^* \in \left[\frac{4}{3}, 2\right], y_2^* \in \left[\frac{-4}{3}, 2\right], y_3^* \in [8,12], y_1^* = y_4^* = 0$$

Substitute the above optimal solution into the objective maximum function, yields:

This means that the company must produce quantity $x_1^* \in [5,7]$,of product A and quantity $x_2^* \in \left[\frac{4}{3}, 2\right]$of product B, thereby achieving a maximum profit of:

$$\max Z \in [5,8].[5,7] + [3,6].\left[\frac{4}{3}, 2\right] = [25,56] + [4,12] = [29,68]$$

It should be noted that the previous problem was solved in the research [21] using the direct simplex algorithm, and the solution tables are as follows:

Non-basic var. / Basic var.	x_1	x_1	y_1	y_2	y_3	y_4	b_i
y_1	2	3	1	0	0	0	$[14,20]$
y_2	2	1	0	1	0	0	$[10,16]$
y_3	0	3	0	0	1	0	$[12,18]$

y_4	3	0	0	0	0	1	[15,21]
Objective fun.	[5,8]	[3,6]	0	0	0	0	$Z - 0$

Table No. 5: Simplex table according to the direct neutrosophic simplex algorithm

Table No. 6: Table of the first step in searching for the optimal solution

Non-basic var. \\ Basic var.	x_1	x_2	y_1	y_2	y_3	y_4	b_i
y_1	0	3	1	0	0	$\dfrac{-2}{3}$	[4,6]
y_2	0	1	0	1	0	$\dfrac{-2}{3}$	[0,4]
y_3	0	3	0	0	1	0	[12,18]
x_1	1	0	0	0	0	$\dfrac{1}{3}$	[5,7]
Object. Fun.	0	[3,6]	0	0	0	$[\dfrac{-8}{3}, \dfrac{-5}{3}]$	$Z - [25,56]$

Table No. 7: Final solution table

Non-basic var. \\ basic var.	x_1	x_2	y_1	y_2	y_3	y_4	b_i
x_2	0	1	$\dfrac{1}{3}$	0	0	$\dfrac{-2}{9}$	$[\dfrac{4}{3}, 2]$
y_2	0	0	$\dfrac{1}{3}$	1	0	$\dfrac{-4}{9}$	$[\dfrac{-4}{3}, 2]$
y_3	0	0	-1	0	1	$\dfrac{2}{3}$	[8,12]
x_1	1	0	0	0	0	$\dfrac{1}{3}$	[5,7]
Object. Fun.	0	0	[-2, -1]	0	0	$[\dfrac{-6}{9}, -1]$	$Z - [29,68]$

It is clear from the row of the objective function that all the elements are either zero or neutrosophic negative numbers, this means that we have reached to the optimal solution is:

$$x_1^* \in [5,7], x_2^* \in \left[\frac{4}{3}, 2\right], y_2^* \in \left[\frac{-4}{3}, 2\right], y_3^* \in [8,12], y_1^* = y_4^* = 0$$

Substitute the above optimal solution into the objective maximum function:

$$\max Z \in [5,8].[5,7] + [3,6].\left[\frac{4}{3}, 2\right] = [25,56] + [4,12] = [29,68]$$

3. Conclusion and results:

From the previous study, we note that we obtained the same optimal solution that was obtained when we used the direct simplex method, but with a much smaller number of calculations than the number we did in the direct simplex algorithm, as is clear by comparing the solution tables using the modified simplex method, Tables No. (2)., No. (3), No. (4), with solution tables using the direct simplex method, Tables No. (5), No. (6), No. (7). Therefore, in order to shorten time and effort, we focus on the necessity of using the modified Simplex method to find the optimal solution for linear models. Especially when the model contains a large number of variables and constraints.

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Neutrosophic Treatment of Duality Linear Models and the Binary Simplex Algorithm

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 Abstract
One of the most important theories in linear programming is the dualistic theory and its basic idea is that for every linear model has dual linear model, so that solving the original linear model gives a solution to the dual model. Therefore, when we solving the linear programming model, we actually obtain solutions for two linear models. In this research, we present a study of the models. The neutrosophic dual and the binary simplex algorithm, which works to find the optimal solution for the original and dual models at the same time. The importance of this algorithm is evident in that it is relied upon in several operations research topics, such as integer programming algorithms, some nonlinear programming algorithms, and sensitivity analysis in linear programming...
Keywords: Neutrosophic Science; Neutrosophic Linear Models; Dual Models; Neutrosophic dual Linear Models; binary Simplex Algorithm.

1. Introduction:

Most companies and institutions rely on studies provided by experts and researchers using operations research methods in order to ensure a safe work environment away from danger and danger and to give decision makers a margin of freedom, after the studies and research presented using the concepts of neutrosophic science in most fields of science have proven their ability to provide more accurate results. From the results that we were obtaining using classical data-driven studies, we reformulated many operations research topics using neutrosophic concepts [1-20], and to complement what we have done, we present in this research a study of dual linear models and the binary simplex algorithm that gives us a solution for the original and dual models at the same time so that we can provide a clear study that helps decision makers in companies and institutions develop plans and programs through which the greatest profit and lowest cost are achieved.

Discussion:

In our practical life, we encounter many problems that are formulated in the form of linear mathematical models consisting of an objective function and a set of constraints in the form of equations or inequalities. The linear model is written in many formulas that are distinguished by the type of the objective function and the form of the constraints. Many references have been provided that studies of linear and dual linear models are based on classical data, and the appropriate algorithms for solving each of them and how to find the dual model for any model [21-25]. Since classical data results in optimal solutions, which are classical values, restricted values appropriate to the conditions in which the data were collected, and for any change in these conditions, these solutions are inappropriate and may cause the facility a large unexpected loss, so we found it appropriate to reformulate these studies using concepts neutrosophic science is done by taking data with neutrosophic values suitable for all conditions. In previous research, we presented neutrosophic linear models, the direct neutrosophic simplex algorithm, and the graphical method for finding the optimal solution for linear models,[18,19,20] . As a

continuation of what we have done previously, we present in this research a study whose purpose is to reformulate the binary simplex algorithm to find the optimal solution for the original model and the dual model at the same time.

We present this study in two stages:

The first stage:

Finding the neutrosophic dual models using the binary table.

When we want to find the model associated with any linear model, we start by placing this model in the symmetrical form, so we mention the symmetrical form.

The symmetrical form of the neutrosophic linear model: [19]

We say of a linear model that it is in the symmetrical form if all variables are constrained to be non-negative and if all constraints are given in the form of inequalities (and the inequalities of the maximization model constraints must be in the form ($\leq$ less than or equal to) while the inequalities of the minimization model constraints must be in the form ($\geq$ is greater than or equal to), then the linear model is written in the neutrosophic symmetric form in one of two cases:

First case:

$$NZ = \sum_{j=1}^{n}(c_j \pm \varepsilon_j)x_j \longrightarrow Max$$

Constraints:

$$\sum_{j=1}^{n} Na_{ij}x_j \leq b_i \pm \delta_i \quad ; \quad i = 1,2,\dots,m$$
$$x_j \geq 0$$

Second case:

$$NL = \sum_{j=1}^{n}(c_j \pm \varepsilon_j)x_j \longrightarrow Min$$

Constraints:

$$\sum_{j=1}^{n} Na_{ij}x_j \geq b_i \pm \delta_i \quad ; \quad i = 1,2,\dots,m$$
$$x_j \geq 0$$

In both cases, we have $x_j \geq 0$, which are the decision variables, unknown values that we obtain after solving the linear model.

$Nc_j = c_j \pm \varepsilon_j$ and $Nb_i = b_i \pm \delta_i$ and $Na_{ij} = a_{ij} \pm \mu_{ij}$ where (j=1,2,…,n , i=1,2,…,m) are the data of the problem under study, and they are neutrosophic values, indefinite values that enjoy a margin of freedom and are taken according to The nature of the situation represented by the linear model

First method:

Constructing neutrosophic dual linear models using tables:

If the original model is given in detailed form, we put it in the symmetrical form as stated in the previous paragraph, then we draw a binary table for the original and dual models according to the following steps:

1- The coefficients of the objective function in the original model are the constants column in the dual model, and the constants column in the original model are the coefficients of the objective function in the dual model.
2- We invert the signs of the inequalities of the constraints (if they were in the original model of type $\leq$ they become in the dual model of type $\geq$)
3- We change the objective from maximizing in the original model to minimizing in the dual model
4- We place each constraint (row) in the original model corresponding to a column in the dual model, meaning there is one variable for each constraint in the original model
5- The variables in the original model and the dual model satisfy the non-negativity constraints.

We explain the above through the following two cases:

The first case: The original model is symmetrical and of the maximization type
Find

$$NZ = Nc_1x_1 + Nc_2x_2 + \cdots + Nc_nx_n \rightarrow Max$$

Constants:

$$Na_{11}x_1 + Na_{12}x_2 + \cdots + Na_{1n}x_n \leq Nb_1$$

$$Na_{21}x_1 + Na_{22}x_2 + \cdots + Na_{2n}x_n \leq Nb_2$$

$$\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$$

$$Na_{m1}x_1 + Na_{m2}x_2 + \cdots + Na_{mn}x_n \leq Nb_m$$

$$x_1, x_2, \ldots, x_n \geq 0$$

The binary table for the original model and the accompanying model is as follows:

Table No. (1) Objective follower of the maximization type

		Original model			
		objective function constants	$Nc_1x_1 + Nc_2x_2 + \cdots + Nc_nx_n$		*Max* Constants column
	y_1	1	$Na_{11}x_1 + Na_{12}x_2 + \cdots + Na_{1n}x_n$	$\leq$	Nb_1
Dual vibrable	y_2	2	$Na_{21}x_1 + Na_{22}x_2 + \cdots + Na_{2n}x_n$	$\leq$	Nb_2
	...	...	$\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$	$\leq$	...
	y_m	m	$Na_{m1}x_1 + Na_{m2}x_2 + \cdots + Na_{mn}x_n$	$\leq$	Nb_m
		Non-negativity constraints	$x_1, x_2, \ldots, x_n$	$\geq$	0
		Dual model			
		Objective function constraints	$Nb_1y_1 + Nb_2y_2 + \cdots + Nb_iy_m$		*Min* Constants column
		1	$Na_{11}y_1 + Na_{21}y_2 + \cdots + Na_{m1}y_m$	$\geq$	Nc_1
		2	$Na_{12}y_1 + Na_{22}y_2 + \cdots + Na_{m2}y_m$	$\geq$	Nc_2
		...	$\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$	$\geq$	...
		n	$Na_{1n}y_1 + Na_{2n}y_2 + \cdots + Na_{mn}y_m$	$\geq$	Nc_n

Non-negativity constraints	$y_1, y_2, \dots, y_m$	$\geq$	0

The second case: The original model is symmetrical and of the reduction type:
Find

$$NL = Nc_1 x_1 + Nc_2 x_2 + \cdots + Nc_n x_n \longrightarrow Min$$

Constants:

$$Na_{11}x_1 + Na_{12}x_2 + \cdots + Na_{1n}x_n \geq Nb_1$$

$$Na_{21}x_1 + Na_{22}x_2 + \cdots + Na_{2n}x_n \geq Nb_2$$

$$\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots$$

$$Na_{m1}x_1 + Na_{m2}x_2 + \cdots + Na_{mn}x_n \geq Nb_m$$

$$x_1, x_2, \dots, x_n \geq 0$$

The binary table for the original model and the accompanying model is as follows:

Table No. (2) objective follower in the original model of the reduce type

			Original model			
		objective function constants	$Nc_1 x_1 + Nc_2 x_2 + \cdots + Nc_n x_n$		Min Constants column	
Dual vibrable	y_1	1	$Na_{11}x_1 + Na_{12}x_2 + \cdots + Na_{1n}x_n$	$\geq$	Nb_1	
	y_2	2	$Na_{21}x_1 + Na_{22}x_2 + \cdots + Na_{2n}x_n$	$\geq$	Nb_2	
	...	...	... ...	$\geq$	...	
	y_m	m	$Na_{m1}x_1 + Na_{m2}x_2 + \cdots + Na_{mn}x_n$	$\geq$	Nb_m	
		Non-negativity constraints	$x_1, x_2, \dots, x_n$	$\geq$	0	
			Dual model			
		objective function constants	$Nb_1 y_1 + Nb_2 y_2 + \cdots + Nb_i y_m$		Max Constants column	
		1	$Na_{11}y_1 + Na_{21}y_2 + \cdots + Na_{m1}y_m$	$\leq$	Nc_1	
		2	$Na_{12}y_1 + Na_{22}y_2 + \cdots + Na_{m2}y_m$	$\leq$	Nc_2	
		...	... ...	$\leq$	...	
		n	$Na_{1n}y_1 + Na_{2n}y_2 + \cdots + Na_{mn}y_m$	$\leq$	Nc_n	
		Non-negativity constraints	$y_1, y_2, \dots, y_m$	$\geq$	0	

The second stage: formulation of the binary neutrosophic algorithm.

The binary simplex algorithm (for the original and dual models) neutrosophic. Through this algorithm, we can find the two ideal solutions for both the original and dual models at the same time. Before starting the binary simplex algorithm, we must mention the modified simplex algorithm that we will use within the steps of the binary algorithm.

Modified simplex algorithm:

In the modified Simplex algorithm, after converting the regular linear model to the basic form, we place the coefficients in a short table whose first column includes the basic variables and whose top row includes the non-basic variables only. We define the pivot column, which is the column corresponding to the largest positive value in the objective function row if the objective function is a maximization function (but if the objective function is a minimization function, it is the column corresponding to the most negative values). Let this column be the column of the variable x_s. We define the pivot row. The pivot row is determined. Through the following indicator:

$$N\theta = min\left[\frac{Nb_i}{Na_{is}}\right] = \frac{Nb_t}{Na_{ts}} > 0; \quad Na_{is} > 0, Nb_i > 0$$

Let this line be the line of the base variable y_t, then the anchor element is the element resulting from the intersection of the anchor column and the anchor line, i.e., Na_{ts}. Then we calculate the new elements corresponding to the anchor line and the anchor column as follows:

1- We put opposite the pivot element Na_{ts} the reciprocal of $\frac{1}{Na_{ts}}$

2- We calculate the elements of the row corresponding to the pivot row (except the pivot element) by dividing the elements of the pivot row by the pivot element Na_{ts}

3- We calculate all the elements of the column opposite the pivot (except the pivot element) by dividing the elements of the pivot column by the pivot element Na_{ts} and then multiplying them by (-1)

4- We calculate the other elements from the following relationships:

$$Nb'_i = Nb_i - Nb_t\frac{Na_{is}}{Na_{ts}} = \frac{Nb_iNa_{ts} - Nb_tNa_{is}}{Na_{ts}}$$

$$Na'_{ij} = Na_{ij} - Na_{tj}\frac{Na_{is}}{Na_{ts}} = \frac{Na_{ij}Na_{ts} - Na_{tj}Na_{is}}{Na_{ts}}$$

$$Nc'_j = Nc_j - Nc_s\frac{Na_{tj}}{Na_{ts}} = \frac{Nc_jNa_{ts} - Nc_sNa_{tj}}{Na_{ts}}$$

5- We apply the stopping criterion of the direct Simplex algorithm on the objective function row. If the objective function is of the maximize type, the objective function row in the table must not contain any positive value. (But if the objective function is of the minimization type, the objective function row in the new table must not be contains any negative value), if the criterion is not met, we repeat the same steps until the stopping criterion is met and we obtain the desired ideal solution.

Steps of the binary simplex algorithm:

a. We write the two models in basic form by adding or subtracting additional variables or using synthetic variables and isolating the non-restricting variables.

Basal form of the original model:

  Find

$$NZ = Nc_1x_1 + Nc_2x_2 + \cdots + Nc_nx_n + 0u_1 + 0u_2 + \cdots + 0u_m \longrightarrow Max$$

Constans:

$$Na_{11}x_1 + Na_{12}x_2 + \cdots + Na_{1n}x_n + u_1 = Nb_1$$
$$Na_{21}x_1 + Na_{22}x_2 + \cdots + Na_{2n}x_n + u_2 = Nb_2$$
$$\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$$
$$Na_{m1}x_1 + Na_{m2}x_2 + \cdots + Na_{mn}x_n + u_m = Nb_m$$
$$x_j \geq 0 \ ; j = 1,2, \ldots, n$$
$$u_i \geq 0 \ ; i = 1,2, \ldots, m$$

Here we do not require that $Nb_i \geq 0$.

Basic form of the dual model:

Find

$$NL = Nb_1y_1 + Nb_2y_2 + \cdots + Nb_iy_m + 0v_1 + 0v_2 + \cdots + 0v_n \longrightarrow Min$$

Constans:

$$Na_{11}y_1 + Na_{21}y_2 + \cdots + Na_{m1}y_m - v_1 = Nc_1$$
$$Na_{12}y_1 + Na_{22}y_2 + \cdots + Na_{m2}y_m - v_2 = Nc_2$$
$$\ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots$$
$$Na_{1n}y_1 + Na_{2n}y_2 + \cdots + Na_{mn}y_m - v_n = Nc_n$$
$$y_i \geq 0 \ ; i = 1,2, \ldots, m$$
$$v_j \geq 0 \ ; j = 1,2, \ldots, n$$

Here we do not require that $Nc_j \geq 0$

The two models have the same coefficients, and the matrix of instances of the dual model is the transpose of the matrix of instances of the original model. We write the two models in the following binary table:

Table No. (3) Standard format for the original and companion models

			Original model		
		objective function constants	$Nc_1x_1 + Nc_2x_2 + \cdots + Nc_nx_n + 0u_1 + 0u_2 + \cdots + 0u_m$		*Max* Constants column
Dual vibrable	y_1	1	$Na_{11}x_1 + Na_{12}x_2 + \cdots + Na_{1n}x_n + u_1$	=	Nb_1
	y_2	2	$Na_{21}x_1 + Na_{22}x_2 + \cdots + Na_{2n}x_n + u_2$	=	Nb_2
	...	...	$\ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots$	=	...
	y_m	m	$Na_{m1}x_1 + Na_{m2}x_2 + \cdots + Na_{mn}x_n + u_m$	=	Nb_m
	Non-negativity constraints		$x_1, x_2, \ldots, x_n , u_1, u_2, \ldots, u_m$	$\geq$	0
			Dual model		
		objective function constants	$Nb_1y_1 + Nb_2y_2 + \cdots + Nb_iy_m + 0v_1 + 0v_2 + \cdots + 0v_n$		*Min* Constants column
		1	$Na_{11}y_1 + Na_{21}y_2 + \cdots + Na_{m1}y_m - v_1$	=	Nc_1
		2	$Na_{12}y_1 + Na_{22}y_2 + \cdots + Na_{m2}y_m - v_2$	=	Nc_2
		...	$\ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots$	=	...
		n	$Na_{1n}y_1 + Na_{2n}y_2 + \cdots + Na_{mn}y_m - v_n$	=	Nc_n
		Non-negativity constraints	$y_1, y_2, \ldots, y_m, v_1, v_2, \ldots, v_n$	$\geq$	0

b. We place the variables and coefficients of the original model in the modified simplex table, and we place the variables of the dual model outside the table as follows:

Table No. (4): The binary table for the original and dual models according to the modified Simplex algorithm

			Basic variables with a (-) sign in the dual model				
			$-v_1$	$-v_2$	...	$-v_n$	
		Non-basic vibrable basic vibrable	x_1	x_2	...	x_n	**Follow the objective of the dual model NB_i**
No n-	y_1	u_1	Na_{11}	Na_{12}	...	Na_{1n}	Nb_1

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y_2	u_2	Na_{21}	Na_{22}	...	Na_{2n}	Nb_2
...	...	...	...	...	...	...
y_m	u_m	Na_{m1}	Na_{m2}	...	Na_{mn}	Nb_m
	objective of the original model	Nc_1	Nc_2	...	Nc_n	$L - 0 \to Min$ $Z - 0 \to Max$

Binary simplex algorithm for the original and dual models:

From the modified simplex algorithm of the original model, we obtain the optimal solution of the original model when all the elements are in the last row (the objective function row of the original model) $Nc_j \leq 0$; j = 1,2, ... , n and at the same time all the elements are in the column The last (associated objective function column) $Nb_i \geq 0$; $i = 1,2, ... , m$ and we get the optimal solution for the dual model when all elements in the last column (associated objective function column) are $Nb_i \geq 0$; $i = 1,2, ... , m$ and at the same time the last row (the objective function row of the original model) $Nc_j \leq 0$; j = 1,2, ... , n (because it will correspond to $Nc_j = -v_j$) which are the two conditions Same for both models. Therefore, when searching for the optimal solution for both models together, we must work to make all elements $Nb_i \geq 0$; $i = 1,2, ... , m$ and to make all elements $Nc_j \leq 0$; j = 1,2, ... , n to achieve this, we rely on one of the two models, put its variables and coefficients in a table, and place the dual model in an external frame of that table. In general, we find that the necessity of placing the two models in a short table does not allow us to get rid of the negative constants on the right side, and therefore the general case of the previous binary table can it must include negative constants $Nb_i < 0$, and the elements of the last row can include positive elements $Nc_j > 0$, so when searching for the optimal solution for the two models, we must work to address these elements based on one of the two models.

Depending on the original model, we do this in two stages:

The first stage:

We make the constant Nb_i non-negative, which corresponds to obtaining a non-negative basic solution for the original model.

The second stage:

We make all elements of the objective function row non-positive (in the case of the objective function, maximization), and this corresponds to obtaining the optimal solution required for the original model.

Based on the dual model, we do this in two stages:

The first stage:

We must make the elements of the dual model objective function column $Nb_i \geq 0$; i = 1,2, ... , m ,The last row is non-negative

The second stage:

We must make the free constants for the dual model $-Nc_j$ non-positive, and this corresponds to obtaining the optimal solution for the dual model. We explain the above through the following example:

Find the optimal solution for both the following neutrosophic linear model and its dual using the binary algorithm

Example:

Find:

$$[5,8] \, x_1 + [3,6] \, x_2 \longrightarrow Max$$

Constans:

$$2x_1 + 3x_2 \leq [14,20]$$

$$2x_1 + x_2 \leq [10,16]$$

$$3x_2 \leq [12,18]$$
$$3x_1 \leq [15,21]$$

$$x_1 \geq 0, x_2 \geq 0$$

We form the binary table of the model and the dual model:

Table No. (5) The original model and its dual mode

Original model				
objective function constants	[5,8] x_1 + [3,6] x_2			*Max* Constants column
y_1 1	$2x_1 + 3x_2$	$\leq$		[14,20]
y_2 2	$2x_1 + x_2$	$\leq$		[10,16]
y_3 3	$3x_2$	$\leq$		[12,18]
y_4 4	$3x_1$	$\leq$		[15,21]
Non-negativity constraints	x_1, x_2	$\geq$		0
Dual model				
objective function constants	$[14,20]y_1 + [10,16]y_2 + [12,18]y_3 + [15,21]y_4$			*Min* Constants column
1	$2y_1 + 2y_2 + 3y_4$	$\geq$		[5,8]
2	$3y_1 + y_2 + 3y_4$	$\geq$		[3,6]
Non-negativity constraints	y_1, y_2, y_3, y_4	$\geq$		0

In the following table, we wrote the two models in standard form:

Original model				
objective function constants	[5,8] x_1 + [3,6] x_2 + $0u_1$ + $0u_2$ + $0u_3$ + $0u_4$			*Max* Constants column
y_1 1	$2x_1 + 3x_2 + u_1$	=		[14,20]
y_2 2	$2x_1 + x_2 + u_2$	=		[10,16]
y_3 3	$3x_2 + u_3$	=		[12,18]
y_4 4	$3x_1 + u_4$	=		[15,21]
Non-negativity constraints	$x_1, x_2, u_1, u_2, u_3, u_4$	$\geq$		0
Dual model				
objective function constants	$[14,20]y_1 + [10,16]y_2 + [12,18]y_3 + [15,21]y_4 + 0v_1 + 0v_2$			*Min* Constants column
1	$2y_1 + 2y_2 + 3y_4 - v_1$	=		[5,8]
2	$3y_1 + y_2 + 3y_4 - v_2$	=		[3,6]

Non-negativity constraints	$y_1, y_2, y_3, y_4, v_1, v_2$	$\geq$	0

Table No. (6) Standard format for the original model and the dual model

We notice from the table that the standard form of the original model includes a ready-made base of additional variables u_1, u_2, u_3, u_4, but for the dual model there is no ready-made base. Therefore, we multiply the two restrictions by (-1) and we obtain the basic form of the dual model.

The following table shows the basic form of the original and dual models:

Table No. (7): The basic shape of the original model and the dual model

		Original model			
	objective function constants	$[5,8]\, x_1 + [3,6]\, x_2 + 0u_1 + 0u_2 + 0u_3 + 0u_4$		*Max* Constants column	
y_1	1	$2x_1 + 3x_2 + u_1$	=	[14,20]	
y_2	2	$2x_1 + x_2 + u_2$	=	[10,16]	
y_3	3	$3x_2 + u_3$	=	[12,18]	
y_4	4	$3x_1 + u_4$	=	[15,21]	
	Non-negativity constraints	$x_1, x_2, u_1, u_2, u_3, u_4$	$\geq$	0	
		Dual model			
	objective function constants	$[14,20]y_1 + [10,16]y_2 + [12,18]y_3 + [15,21]y_4 + 0v_1 + 0v_2$		*Min* Constants column	
	1	$-2y_1 - 2y_2 - 3y_4 + v_1$	=	$-[5,8]$	
	2	$-3y_1 - y_2 - 3y_4 + v_2$	=	$-[3,6]$	
	Non-negativity constraints	$y_1, y_2, y_3, y_4, v_1, v_2$	$\geq$	0	

(Dual vibrable labels y_1, y_2, y_3, y_4 on the left of the Original model rows)

We put the two models in the modified Simplex algorithm table and we get the following table:

Table No. (8): The binary table for the original and dual models according to the modified Simplex algorithm

			According to the original model		
			$-v_1$	$-v_2$	
		Non-basic vibrable / basic vibrable	x_1	x_2	objective function NB_i
	y_1	u_1	2	3	[14,20]
	y_2	u_2	2	1	[10,16]
	y_3	u_3	0	3	[12,18]
	y_4	u_4	3	0	[15,21]
		objective function Nc_i	[5,8]	[3,6]	$L-0$ / $Z-0$

(Left margin label: **Non-basic vibrable**)

		According to the dual model				
		u_1	u_2	u_3	u_4	
	Non-basic vibrable basic vibrable	y_1	y_2	y_3	y_4	objective function Original model Nc_i
x_1	v_1	-2	-2	0	-3	$-[5,8]$
x_2	v_2	-3	-1	-3	0	$-[3,6]$
	objective function Dual model NB_i	$[14,20]$	$[10,16]$	$[12,18]$	$[15,21]$	$Z - 0$ $L - 0$

The first stage:

1- For the original model:

Since the values in the constant's column are all positive, we study the values in the objective function row and determine the largest positive value. We find:

$$\max([5,8], [3,6]) = [5,8]$$

It is an expression of the variable x_1. This means that it will enter the base. To determine the element that will exit from the base, we calculate the index $N\theta$, where:

$$N\theta \in min\left[\frac{[14,20]}{2}, \frac{[10,16]}{2}, \frac{[15,21]}{3}\right] = \frac{[15,21]}{3} = [5,7]$$

We find that the pivot column is the column of the non-base variable x_1, meaning that the variable x_1 will enter the base instead of the variable u_4, and the pivot element is the element resulting from the intersection of the pivot row and the pivot column, which is (3)

We perform the switching between variables using a modified simplex algorithm.

2- For the dual model:

We study the elements of the objective function row. We notice that all the values are positive. Therefore, we study the elements of the constant's column. We find that they are all negative values. We choose the most negative of them, which is $(-[5,8])$ which is the row of the base variable v_1, so its row is the pivot row. To determine the pivot column and the pivot element, we calculate the index $N\theta'$ where:

$$N\theta' \in Max\left[\frac{[14,20]}{-2}, \frac{[10,16]}{-2}, \frac{[15,21]}{-3}\right] = \frac{[15,21]}{-3}$$

So, the column of the non-base variable u_4 is the pivot column, meaning that the variable u_4 will enter the base instead of the variable v_1, and the pivot element is the element resulting from the intersection of the pivot row and the pivot column, which is (-3). We perform the switching between the variables using the modified simplex algorithm, from (1) and (2) We get the following double table:

Table No. (9): The binary table for the first stage, the solution according to the original and dual models

	According to the original model		
	$-y_4$	$-v_2$	
Non-basic vibrable basic vibrable	u_4	x_2	objective function Original model NB_i

Non-basic vibrable					
y_1	u_1	$-\dfrac{2}{3}$	3	[4,6]
y_2	u_2	$-\dfrac{2}{3}$	1	[0,4]
y_3	u_3	0	3	[12,18]
v_1	x_1	$\dfrac{1}{3}$	0	[5,7]
objective function Original model Nc_i		$\left[\dfrac{-8}{3},\dfrac{-5}{3}\right]$	[3,6]	$L-[25,56]$ $Z-[25,56]$

According to the dual model					
	u_1	u_2	u_3	x_1	
Non-basic vibrable / basic vibrable	y_1	y_2	y_3	v_1	objective function Original model Nc_i
u_4 — y_4	$\dfrac{2}{3}$	$\dfrac{2}{3}$	0	$\dfrac{-1}{3}$	$\left[\dfrac{8}{3},\dfrac{5}{3}\right]$
x_2 — v_2	-3	-1	-3	0	$-[3,6]$
objective function Original model Nc_i	[4,6]	[0,4]	[12,18]	[5,7]	$Z-[25,56]$ $L-[25,56]$

(Left labels: Non-basic vibrable; u_4, x_2)

The second phase:

We apply the stopping criterion of the algorithm

For the original model:

Since the values in the constant's column are all positive, we study the values in the objective function row. We notice that there is a positive value, which is [3,6], meaning that we have not yet reached the optimal solution. Therefore, we specify the pivot column, which is the column of the variable x_2 corresponding to the only positive value in the objective function row. [3,6] In order to determine the pivot row and the pivot element, we calculate the index $N\theta$, where:

$$N\theta \in \min\left[\frac{[4,6]}{3},\frac{[0,4]}{1},\frac{[12,18]}{3}\right]=\frac{[4,6]}{3}$$

It corresponds to the base element u_1, so its row is the pivot row and the pivot element is ($\mathbf{3}$). We swap between the variables using the modified simplex algorithm.

For the dual model:

We study the elements of the objective function row. We notice that all the values are positive. Therefore, we study the elements of the constant's column. We find that there is a single negative value, which is $(-[3,6])$, which is the line of the base variable v_2, so its row is the pivot row. To determine the pivot column and the pivot element, we calculate the index $N\theta'$ where:

$$N\theta' \in Max\left[\frac{[4,6]}{-3},\frac{[0,4]}{-1},\frac{[12,18]}{-3}\right]=\frac{[4,6]}{-3}$$

So, the column of the non-base variable y_1 is the pivot column, meaning that the variable y_1 will enter the base instead of the variable v_2, and the pivot element is the element resulting from the intersection of the pivot row and

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the pivot column, which is (-3). We perform the switching between the variables using the modified simplex algorithm, from (1) and (2) We get the following double table:

Table No. (10): The binary algorithm table for the second stage

			According to the original model		objective function Dual model NB_i
			$-y_4$	$-y_1$	
		Non-basic vibrable / basic vibrable	u_4	u_1	
Non-basic vibrable	v_2	x_2	$\dfrac{-2}{9}$	$\dfrac{1}{3}$	$\left[\dfrac{4}{3}, 2\right]$
	y_2	u_2	$\dfrac{-4}{9}$	$\dfrac{-1}{3}$	$\left[\dfrac{4}{3}, 2\right]$
	y_3	u_3	$\dfrac{2}{3}$	-1	$[8,12]$
	v_1	x_1	1	0	$[5,7]$
		objective function Original model Nc_i	$[-6,-1]$	$[-2,-1]$	$L - [29,68]$ / $Z - [29,68]$

			According to the dual model				objective function Original model Nc_i
			x_2	u_2	u_3	x_1	
		Non-basic vibrable / basic vibrable	v_2	y_2	y_3	v_1	
Non-basic vibrable	u_4	y_4	$\dfrac{2}{9}$	$\dfrac{4}{9}$	$\dfrac{-2}{3}$	-1	$[-6,-1]$
	u_1	y_1	$\dfrac{-1}{3}$	$\dfrac{1}{3}$	1	0	$[-2,-1]$
		objective function Dual model NB_i	$\left[\dfrac{4}{3}, 2\right]$	$\left[\dfrac{4}{3}, 2\right]$	$[8,12]$	$[5,7]$	$Z - [29,68]$ / $L - [29,68]$

We apply the stopping criterion of the algorithm:

1- For the original model, we study the elements of the objective function row until the criterion for stopping the algorithm is met, which is the absence of any positive element
2- For the dual model, we also study the elements of the constants column until the criterion for stopping the algorithm is met, which is the absence of any negative element
3- We find that the criterion has been met and thus we have reached the optimal solution

The optimal solution of the original model is:

$$x_2^* \in \left[\dfrac{4}{3}, 2\right] \ , u_2^* \in \left[\dfrac{4}{3}, 2\right] \ , u_3^* \in [8,12] \ , x_1^* \in [5,7] \ , u_1^* = u_4^* = 0$$

The value of the objective function corresponds to:

$$Z^* = max\, Z \in [29,68]$$

The optimal solution of the dual model is:

$$y_1^* \in [1,2] \;, y_4^* \in [1,6] \;, v_2^* = y_2^* = y_3^* = v_1^* = 0$$

The value of the objective function corresponds to:

$$L^* = MinL \in [29,166]$$

We note that:

$$Z^* = Max\, Z \in [29,68] \leq L^* = MinL \in [29,166]$$

This solution is acceptable according to the following theory:

If $(x_1, x_2, \dots, x_n)$ is an acceptable solution to the original model of type Max and $(y_1, y, \dots, y_m)$ was an acceptable solution for the dual model of type Min, so the value of the objective function of the original model does not exceed the value of the objective function of the dual model for these two solutions, that is, it is

$$\sum_{j=1}^{n} Nc_j x_j \leq \sum_{i=1}^{m} Nb_i y_i$$

This is for all acceptable solutions for both models (including the optimal solution)

5. Conclusion and Results:

From the previous study, we arrived at a solution for the original and dual models at the same time, which are neutrosophic values from which we know the minimum and maximum profit that we can obtain, because the interpretation of the optimal solution for the original model is that it gives us the best production plan that makes the value of that production as large as possible, within Available capabilities. As for the optimal solution for the dual model, it gives us the best values for the prices of raw materials, which, if used without waste, will also give us the best production plan, and the result is the maximum profit.

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Some Important Theories about Duality and the Economic Interpretation of Neutrosophic Linear Models and Their Dual Models

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Abstract

The essence of operations research is focused on creating and using models. The first step is to create the model, which requires a set of data that is determined from the aspects of the real system that the model must represent. The model is an acceptable model if it achieves the purpose for which it was formulated, and since programming issues linearity is concerned with allocating scarce resources, including labor, machinery, and capital, and using them in the best possible way, such that costs are reduced to a minimum or profits are maximized, by choosing the optimal decision from several available options. Since linear models are used in many fields, it was necessary to prepare studies that meet the needs of decision makers who have made solutions to linear programming problems a safe haven for them. The duality theory is considered one of the most important linear programming theories because it is used in many fields and is relied upon in the economic interpretation of the content of linear models. It provides a comprehensive study of the system represented by the linear model and its dual model. In this research, we present a study of the theory of neutrosophic dual and its economic interpretation by presenting a set of theorems that can be relied upon in explaining the results of solving both the original neutrosophic models and their dual.

Keywords: Neutrosophic science; Linear models; Neutrosophic Duality theory; Economic interpretation of the dual model; Applications of duality theory.

1. Introduction:

When looking at the concepts of neutrosophic science, the research and studies that have been presented by researchers and scholars in this field, and the stages of its development that can be viewed through the links available in this research [1], we find that the science of operations research, with its various methods, was in dire need of such concepts because it is the science that it is considered the applied side of mathematics and touches on the details of our lives. It is a science whose methods depend on data that is provided by experts. These data are classical values specific to the time period in which they were collected, and thus will provide specific solutions that require decision makers to prepare and prepare for any sudden change that may occur in the environment. Work, and this is what prompted us to say that the science of operations research is in dire need of the concepts of neutrosophic science because when using neutrosophic values while collecting data on the case under study, we find that we take into account the best and worst conditions and thus we obtain solutions that are neutrosophic values, values that are not completely defined and have a margin. Freedom puts us in a safe work environment. This is what we have achieved through research and studies that were presented to reformulate some operations research methods using the concepts of neutrosophic science, see [2-15].

Most companies and institutions rely on studies provided by experts and researchers using operations research methods in order to ensure a safe work environment away from danger and danger and to give decision makers a margin of freedom, after the studies and research presented using the concepts of neutrosophic science in most fields of science have proven their ability to provide more accurate results. From the results that we were obtaining using classical data-driven studies, we reformulated many operations research topics using neutrosophic concepts [1-20], and to complement what we have done, we present in this research a study of dual linear models and the binary simplex algorithm that gives us a solution for the original and dual models at the same time so that we can provide a clear study that helps decision makers in companies and institutions develop plans and programs through which the highest profit and lowest cost are achieved.

2. Discussion:

In this research, we present a study of some important theories related to linear neutrosophic models and their dual models, through whose optimal solutions we can provide an economic explanation that helps in making ideal decisions for managing systems that operate according to these models and achieve the maximum profit, based on what is stated in the following references: [16 -19]

We begin this study by presenting the matrix form of the dual models:

Find the dual model using matrices:

Here we distinguish two cases:

The first case: The original model is symmetrical and maximization type:

original model

Find

$$NZ = NC\,X \longrightarrow Max$$

Constraints

$$NA\,X \leq NB$$
$$X \geq 0$$

Where:

$$NA = \begin{bmatrix} Na_{11} & Na_{12} \dots Na_{1n} \\ Na_{21} & Na_{22} \dots Na_{2n} \\ \dots \dots \dots \dots \dots \dots \\ Na_{m1} & Na_{m2} \dots Na_{mn} \end{bmatrix} \quad NB = \begin{bmatrix} Nb_1 \\ Nb_2 \\ .. \\ Nb_m \end{bmatrix} \quad NC = \begin{bmatrix} Nc_1 \\ Nc_2 \\ .. \\ Nc_n \end{bmatrix} \quad X = [x_1 \ x_2 \dots x_n]$$

Dual model

Find

$$NL = NB\,Y \longrightarrow Min$$

Constraints

$$NA^T Y \geq NC$$
$$Y \geq 0$$

Where:

$$NA^T = \begin{bmatrix} Na_{11} & Na_{21} \dots Na_{m1} \\ Na_{12} & Na_{22} \dots Na_{m2} \\ \dots \dots \dots \dots \dots \dots \\ Na_{1n} & Na_{2n} \dots Na_{mn} \end{bmatrix} \quad NB = \begin{bmatrix} Nb_1 \\ Nb_2 \\ .. \\ Nb_m \end{bmatrix} \quad NC = \begin{bmatrix} Nc_1 \\ Nc_2 \\ .. \\ Nc_n \end{bmatrix} \quad Y = [y_1 \ y_2 \dots y_m]$$

The second case: The model is symmetrical and miniaturized
original model:

Find

$$NZ = NC\,X \longrightarrow Min$$

constraints

$$NAX \geq NB$$
$$X \geq 0$$

Where:

$$NA = \begin{bmatrix} Na_{11} & Na_{12} \dots Na_{1n} \\ Na_{21} & Na_{22} \dots Na_{2n} \\ \dots \dots \dots \dots \dots \dots \\ Na_{m1} & Na_{m2} \dots Na_{mn} \end{bmatrix} \quad NB = \begin{bmatrix} Nb_1 \\ Nb_2 \\ .. \\ Nb_m \end{bmatrix} \quad NC = \begin{bmatrix} Nc_1 \\ Nc_2 \\ .. \\ Nc_n \end{bmatrix} \quad X = [x_1 \ x_2 \dots x_n]$$

Dual model:

Find

$$NL = NB\,Y \longrightarrow Max$$

Constraints

$$NA^T Y \leq NC$$
$$Y \geq 0$$

Where:

$$A^T = \begin{bmatrix} a_{11} & a_{21} \dots a_{m1} \\ a_{12} & a_{22} \dots a_{m2} \\ \dots \dots \dots \dots \dots \dots \\ a_{1n} & a_{2n} \dots a_{mn} \end{bmatrix} \quad NB = \begin{bmatrix} Nb_1 \\ Nb_2 \\ .. \\ Nb_m \end{bmatrix} \quad NC = \begin{bmatrix} Nc_1 \\ Nc_2 \\ .. \\ Nc_n \end{bmatrix} \quad Y = [y_1 \ y_2 \dots y_m]$$

We summarize the process of finding neutrosophic conjugate models using matrices in the following steps:
1. We define a new non-negative variable for each constraint of the original model
2. We make the wind (cost) vector in the original model a column vector of constants in the companion model
3. We make the constants column vector in the original model the cost (profit) vector in the dual model
4. We transform a matrix of the parsimony of the variables of the constraints in the original model into the parsimony of the variables in the dual model
5. We reverse the direction of the constraint inequalities
6. We reverse the direction of the examples, that is, we change the increase to the maximum limit to a decrease to the minimum limit, and vice versa.

Duality theorems:
1- Weak duality theorem:
Let the following linear model:
Find

$$Z = NC \, X \longrightarrow Max$$

Constraints

$$NA \, X \leq NB$$
$$X \geq 0$$

Dual model:

$$L = NB \, Y \longrightarrow Min$$

Constraints

$$NA^T Y \geq NC$$
$$Y \geq 0$$

The value of the objective function of the dual model for any acceptable solution is always highest than or equal to the value of the objective function of the original model at the same solution, i.e.

$$NB \, Y' \geq NCX'$$

Where X′ is an acceptable solution for the original model and Y′ is an acceptable solution for the dual model.
Results from the theory:

- The value of the objective function of the original model of the maximization type for any acceptable solution represents the minimum value of the objective function in the dual model.
- The minimum value of the objective function in the dual model for any acceptable solution represents the upper limit of the maximum value of the objective function in the original model.
- If the original model and its solution are infinite (that is, the maximum value of NZ seeks infinity, $MaxNZ \longrightarrow \infty$), then the dual model does not have an acceptable solution.
- If the dual problem has an indefinite solution (that is, the minimum value of NL seeks infinity $MinNL \longrightarrow \infty$), then the original problem is infinite.

To illustrate the Weak duality theorem, we present the following example:
Original model
Find

$$Z = [1,2]x_1 + [0,1]x_2 + [3,4]x_3 + [2,3]x_4 \longrightarrow Max$$

Constraints

$$x_1 + 2x_2 + 2x_3 + 3x_4 \leq 20$$
$$2x_1 + x_2 + 3x_3 + 2x_4 \leq 20$$
$$x_1, x_2, \dots, x_4 \geq 0$$

Dual model

Find

$$L = 20y_1 + 20y_2 \longrightarrow Min$$

Constraints

$$y_1 + 2y_2 \geq [1,2]$$
$$2y_1 + y_2 \geq [0,1]$$
$$2y_1 + 3y_2 \geq [3,4]$$
$$3y_1 + 2y_2 \geq [2,3]$$
$$y_1, y_2 \geq 0$$

$x_1' = x_2' = x_3' = x_4' = [0,1]$ is an acceptable solution for the original model and
$y_1' = y_2' = [0.5,1]$ is an acceptable solution for the dual model, (the solution is Acceptable if it satisfies all constraints). We determine the value of the objective functions corresponding to the acceptable solution. We find:

The value of the objective function in the original form

$$NZ = NCX' \in [1,2]x_1' + [0,1]x_2' + [3,4]x_3' + [2,3]x_4' = [6,10].[0,1] = [0,10]$$

And the value of the objective function in the dual model

$$NL = NBY' \in 20y_1' + 20y_2' = 20[0.5,1] + 20[0.5,1] = [20,40]$$

We note that $CX' < BY'$ where $(X' = [x_1' \ x_2' \ x_3' \ x_4'], Y' = [y_1' \ y_2'])$ and this fulfills the weak duality theorem
Likewise, according to result (2), the maximum value of the objective function in the original model cannot exceed $[20,40]$, and the inverse of results (3) and (4) is also true.

2- **Optimality criterion theorem:**

If acceptable solutions x', y' are found for the symmetrical dual models such that the values of the objective functions corresponding to these solutions are equal, then these acceptable solutions are optimal solutions for these models.

 Proof:

We assume that X'' is another acceptable solution to the original model according to the previous theory, then:

$$NCX'' < NBY'$$

But according to the assumption

$$NCX' = NBY'$$

Therefore:

$$NCX' \leq NBX'$$

For all acceptable solutions of the original models, therefore, by definition, X' is an optimal solution of the original model. In the same way, we prove that Y' is an optimal solution of the companion model.

3- **Main duality theorem:**

If each of the original and companion models has an acceptable solution, then they both have optimal solutions that make the optimal values of the objective functions equal.

 Proof:

When both the original and dual models have an acceptable solution, according to results 1 and 2 of Theorem 1, we have a minimum for NL and an upper limit for ZN. In other words, the original or dual model cannot have an infinite solution. Therefore, both problems must have optimal solutions.

Economic interpretation of linear models and their dual models:

Linear programming problems are concerned with allocating the available resources of labor, materials, machinery, and capital and using them in the best possible way so that costs are reduced to their minimum and profits are maximized. What is meant by the term best is choosing the best solution from among the set of acceptable solutions.

What is meant by the economic interpretation of the linear models and their dual models is the interpretation of the results that we obtain when we find the optimal solution for both the original and dual models. We illustrate the economic interpretation of the original and dual models through the following example:

Example:

A factory wants to transfer its products from two warehouses to three retail centers at the lowest possible cost. The following table shows the data provided by the factory official:

Sales centers / Stores	B_1	B_2	B_3	Available quantities
A_1	[1,3]	[2,4]	[0,3]	300
A_2	[4,6]	[1,4]	[1,5]	600
Quantities required	200	300	400	900 / 900

The plant manager's request was for a transportation plan with a minimum cost so that the distribution centres' orders could be met from the available quantities

The previous issue is a balanced transfer issue because

$$\sum_{i=1}^{2} a_i = \sum_{j=1}^{3} b_j = 900$$

Formulating the neutrosophic mathematical model,[15].

We assume x_{ij} the quantity transported from store i where $i = 1,2$, to distribution center j, where $j = 1,2,3$. Thus, we obtain the following linear model:

Find

$$L \in [1,3]x_{11} + [2,4]x_{12} + [0,3]x_{13} + [4,6]x_{21} + [1,4]x_{22} + [1,5]x_{23} \longrightarrow Min$$

Constans:

$$x_{11} + x_{12} + x_{13} \leq 300$$
$$x_{11} + x_{21} + x_{23} \leq 600$$
$$x_{11} + x_{21} \geq 200$$
$$x_{12} + x_{22} \geq 300$$
$$x_{13} + x_{23} \geq 400$$
$$x_{ij} \geq 0 \; ; \; i = 1,2 \; , j = 1,2,3$$

We write the model in the following symmetrical form:

Since the objective function is a minimization function, all constraints must be of type greater than or equal to, so the model is written in the following symmetric form:

Find

$$L \in [1,3]x_{11} + [2,4]x_{12} + [0,3]x_{13} + [4,6]x_{21} + [1,4]x_{22} + [1,5]x_{23} \longrightarrow Min$$

Constans:

$$-x_{11} - x_{12} - x_{13} \geq -300$$
$$-x_{11} - x_{21} - x_{23} \geq -600$$
$$x_{11} + x_{21} \geq 200$$
$$x_{12} + x_{22} \geq 300$$
$$x_{13} + x_{23} \geq 400$$
$$x_{ij} \geq 0 \; ; \; i = 1,2 \; , j = 1,2,3$$

Forming the dual model, we obtain the following linear model:

Find

$$Z = -300y_1 - 600y_2 + 200y_3 + 300y_4 + 400y_5 \longrightarrow Max$$

Constans:

$$-y_1 + y_3 \leq [1,3]$$
$$-y_1 + y_4 \leq [2,4]$$
$$-y_1 + y_5 \leq [0,3]$$
$$-y_2 + y_3 \leq [4,6]$$
$$-y_2 + y_4 \leq [1,4]$$
$$-y_2 + y_5 \leq [1,5]$$
$$y_1, y_2, y_3, y_4, y_5 \geq 0$$

We formulate an appropriate text for the accompanying model based on the text of the original problem:

It is clear from the original model that the factory's goal is to transport all of its products at the lowest possible cost:

Text of the issue dual to the attached form:

A transport company submitted to a factory an offer that it would transport the entire quantity in the first warehouse, i.e., 300 units, at a price of y_1 monetary unit per unit, and transfer the entire quantity available in the second warehouse, 600, at a price of y_2 monetary units per unit. The company pledged that it would deliver $(200 , 300 , 400)$ units. To the three retail centers, respectively. These units are sold in these centers at a price of (y_3, y_4, y_5) monetary units, respectively. So that you can convince the official in the factory that if he accepts her offer, the cost of transportation in his factory will be less than the cost if he carries out the transportation process, so that he carries out the transportation process.

She used the constraints in the dual model and conducted the following discussion:

You pay the cost of transporting one unit from the first factory to the first sales center, an amount whose value belongs to the range $[1,3]$, but if you use the transport company, the cost is

$(y_3 - y_1)$, and we have from the first entry in the accompanying model

$$y_3 - y_1 \leq [1,3]$$

Here the official in the laboratory will notice that the transportation company's offer is an appropriate offer.

In the same way we discuss all the limitations of the dual model, the conclusion that the factory official will reach is that the cost of transportation on any route if the transportation company's offer is accepted is less than or equal to the cost that he would pay if he himself carried out the transportation process.

The transport company will adopt the values $(y_1, y_2, y_3, y_4, y_5)$,because it will achieve maximum profit through them, as the transport company's profit is calculated from the relationship:

$$-300y_1 - 600y_2 + 200y_3 + 300y_4 + 400y_5$$

It is the same as the objective function of the dual model, meaning that the dual model represents the transportation company that is trying to maximize its profits

The basic theorem of association states that the optimal values of the model and the dual model are always equal. The manufacturer does not save any money because he will pay the transportation company the minimum cost of transportation, but it saves the trouble of solving the original model to determine the minimum cost of transportation, and for the transportation company, it has guaranteed the deal to transport the goods with the maximum profit.

From the above, we can say that the optimal solution for the accompanying model from an economic perspective is the price that is paid for the available capabilities (resources). We know that these capabilities are limited. According to the main association theory, the value of the objective function in the original model and the dual model are equal, so if X^0 and Y^0 for two optimal solutions,

$$NZ_0 = NCX^0 = NBY^0 = NL_0.$$

In other words, the optimal value of the original or dual model is given by the following relation:

$$NL_0 = NB\, Y^0$$

Where NB is the matrix, whose elements are the limited quantities of resources, and Y^0 are the optimal values of the variables in the dual model. If any change occurs in the level of these resources, it will affect the optimal value of the objective function, that is, NL_0. Accordingly, the optimal value of the variable in the dual model (for each constraint in the original model, it gives the net change in the optimal value of the objective function. These values are called shadow prices for constrained resources. Shadow prices can be used to determine whether obtaining additional quantities of resources at prices that encourage them is economically feasible.

3. Conclusion and Results:

From the previous study, we find that it is sufficient to find the optimal solution for the original and companion models, to find the optimal solution for one of them, and using the theories found in the research to deduce the optimal solution for the other. We can also use the results to provide an economic explanation that helps in making ideal decisions, especially when we take the data as neutrosophic values, because the optimal solution for the original model gives us the best production plan makes the value of that production as large as possible within the available capabilities. As for the optimal solution for the accompanying model, it gives us the best values for the prices of raw materials, which if used without waste will also give us the best production plan, and the result is the greatest profit.

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Optimal Agricultural Land Use: An Efficient Neutrosophic Linear Programming Method

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Abstract: The increase in the size of the problems facing humans, their overlap, the division of labor, the multiplicity of departments, as well as the diversity of products and commodities, led to the complexity of business and the emergence of many administrative and production problems. It was necessary to search for appropriate methods to confront these problems. The science of operations research, with its diverse methods, provided the optimal solutions. It addresses many problems and helps in making scientific and thoughtful decisions to carry out the work in the best way within the available capabilities. Operations research is one of the modern applied sciences that uses the scientific method as a basis and method in research and study, and its basic essence is to build a model that helps management in making decisions related to difficult administrative problems. For example, the military field, financial aspects, industry, in construction for building bridges and huge projects to evaluate the time taken for each project and reduce this time, financial markets and stocks and forecasting economic conditions, in hospital management and controlling the process of nutrition and medicines within the available capabilities, in agriculture Agricultural marketing and many other problems that have been addressed using classical operations research methods. We know that the agricultural sector is one of the important sectors in every country, and the agricultural production process is regulated by those responsible for securing the needs of citizens. Also, those responsible for the agricultural sector are responsible for rationalizing the agricultural process so that the surplus is saved. Due to the difficult circumstances that the country may be going through, in this research, we will reformulate the general model for the optimal distribution of agricultural lands using the concepts of neutrosophic science.

Keywords: Operations Research; Neutrosophic Science; Neutrosophic Linear Programming; Optimal Agricultural Land Use Model.

1. Introduction

Securing the needs of citizens is necessary and one of the major responsibilities that falls on officials in the state. This matter requires a scientific study of the reality of the state's situation and optimal exploitation of the available capabilities, that is, organizing work in all sectors of the state in a way that guarantees citizens a stable life in all circumstances. This matter prompted scientists and researchers are prepare scientific studies that help decision-makers make ideal decisions to manage the work of these sectors. The classical linear programming method was one of the most widely used methods [1-3], and it was relied upon even though the solutions it provided were appropriate solutions for conditions similar to those in which, data is collected about the case under study. Any change in this data will affect the optimal solution and thus the decisions of decision-makers, which

requires us to search for a new scientific method that provides us with optimal solutions suitable for all circumstances and takes into account all changes that may occur. In the work environment, we find that using the concepts of neutrosophic science, the science that takes into account the changes that can occur in the work environment through the indeterminacy of neutrosophic values. So, we have to reformulate many practical issues using the concepts of this science, which can be viewed from what was presented by the American scientist Florentin Smarandache, the founder of this science, and many researchers in various scientific fields [4-19]. Given the importance of the linear programming method, we presented in previous research the neutrosophic linear models [20]. In another research, we presented one of the most important methods used to find the optimal solution for the models. Linear, which is the simplex neutrosophic method [21]. Among the uses of neutrosophic linear programming, we presented a study on its use in the field of education [22]. As a continuation of our previous work, we present in this research a study whose purpose is to reformulate the model of optimal use of agricultural land using the concepts of neutrosophic. This will help decision-makers obtain an optimal solution that secures the needs of citizens for agricultural crops in all circumstances that the country may go through.

The structure of this paper is organized as follows. In Section 2, we briefly examine the concept used for solving the problem. In Section 3, we apply the introduced approach to a case study and discuss the obtained results. In Section 4, we discuss the conclusions of the paper in Section 3.

2. Materials and methods

The most important stage in linear programming is the stage of creating a linear programming model, and we mean expressing realistic relationships with assumed mathematical relationships based on the study and analysis of reality. In order to formulate a linear programming model, the following basic elements must be present:

- Determine the goal in a quantitative manner
  It is expressed by the objective function, which is the function for which the maximum or minimum value is required. It must be possible to express the goal quantitatively, such as if the goal is to achieve the greatest possible profit or secure the smallest possible cost.

- Determine the constraints
  The constraints on the available resources must be specific, that is, the resources must be measurable, and expressed in a mathematical formula in the form of inequalities or equals.

- Determine the goal in a quantitative manner
  This element indicates that the problem should have more than one solution so that linear programming can be applied because if the problem had one solution, there would be no need to use linear programming because its benefit is focused on helping to choose the best solution from among the multiple solutions [1].

3. Results and discussion

3.1 Problem definition

We will apply the above in Section 2, to the model of optimal use of agricultural land, using the concepts of neutrosophic science. We will take data that is affected by the surrounding conditions, and neutrosophic values.

- Text of the issue:
  Let us assume that we have n agricultural areas (plain or cultivated), the area of each of which is equal to $A_1, A_2, \cdots\cdots, A_n$, we want to plant it with m types of agricultural crops to secure the community's requirements for it. Knowing that we need of crop i the amount b_i , if the average

productivity of one area in plain j of crop i is equal to Na_{ij} tons/ha. Where $j = 1,2,\ldots\ldots,n$ and $i = 1,2,\ldots\ldots,m$, and the profit returned from one unit of crop i equal to Np_i, Where Np_i is a neutrosophic value, an undefined non-specific value that designates a perfect and can be any neighbor of the value a_{ij}, also Np_i which can be any neighbor of p_i.

- Required:

Determine the amount of area needed to be cultivated with each crop and in all regions to achieve the greatest possible profit and meet the needs of society.

- Formulation of the mathematical model:

We symbolize by x_{ij} the amount of area in area j that must be cultivated with crop, and we place the data for the problem in Table 1.

Table 1. Issue data.

Regions ⟍ Crops	1	2	……	n	Order amount b_i	profit amount Np_i
1	Na_{11} x_{11}	Na_{12} x_{12}	……	Na_{1n} x_{1n}	b_1	Np_1
2	Na_{21} x_{21}	Na_{22} x_{22}	……	Na_{2n} x_{2n}	b_2	Np_2
……	……	……	……	……	……	……
m	Na_{m1} x_{m1}	Na_{m2} x_{m2}	……	Na_{mn} x_{mn}	b_m	Np_m
Available space a_i	a_1	a_2	……	a_n		

Then we find that the conditions imposed on the variables x_{ij} are:

1. Space restrictions

   The total area allocated to various crops in area j must be equal to a_j, that is, it must be:

$$x_{11} + x_{12} + \cdots\cdots + x_{m1} = a_1$$
$$x_{12} + x_{22} + \cdots\cdots + x_{m2} = a_2$$
$$\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$$
$$x_{1n} + x_{2n} + \cdots\cdots + x_{mn} = a_n$$

2. Conditions for meeting community requirements

   The total production of crop i in all regions must not be less than the amount b_i, that is, it must be:

$$Na_{11}x_{11} + Na_{12}x_{12} + \cdots\cdots + Na_{1n}x_{1n} \geq b_1$$
$$Na_{21}x_{21} + Na_{22}x_{22} + \cdots\cdots + Na_{2n}x_{2n} \geq b_2$$
$$\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$$
$$Na_{m1}x_{m1} + Na_{m2}x_{m2} + \cdots\cdots + Na_{mn}x_{mn} \geq b_m$$

3. Find the objective function
   - We note that the profit resulting from the production of crop i only and from all regions is equal to the product of the profit times the quantity and i.e.:

$$Np_i(Na_{i1}x_{i1} + Na_{i2}x_{i2} + \cdots\cdots + Na_{in}x_{in})$$

Thus, we find that the objective function, which expresses the total profit resulting from all crops, is equal to:

$$Z = Np_1\left(\sum\nolimits_{j=1}^{n} Na_{1j}\, x_{1j}\right) + Np_2\left(\sum\nolimits_{j=1}^{n} Na_{2j}\, x_{2j}\right) + \cdots\cdots + Np_m\left(\sum\nolimits_{j=1}^{n} Na_{mj}\, x_{mj}\right) \to Max$$

- From the above, we get the following mathematical model:

Find the maximum value of

$$Z = Np_1\left(\sum\nolimits_{j=1}^{n} Na_{1j}\, x_{1j}\right) + Np_2\left(\sum\nolimits_{j=1}^{n} Na_{2j}\, x_{2j}\right) + \cdots\cdots + Np_m\left(\sum\nolimits_{j=1}^{n} Na_{mj}\, x_{mj}\right) \to Max$$

Within restrictions:

$$x_{11} + x_{12} + \cdots\cdots + x_{m1} = a_1$$
$$x_{12} + x_{22} + \cdots\cdots + x_{m2} = a_2$$
$$\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$$
$$x_{1n} + x_{2n} + \cdots\cdots + x_{mn} = a_n$$

$$a_{11}x_{11} + a_{12}x_{12} + \cdots\cdots + a_{1n}x_{1n} \geq b_1$$
$$a_{21}x_{21} + a_{22}x_{22} + \cdots\cdots + a_{2n}x_{2n} \geq b_2$$
$$\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$$
$$a_{m1}x_{m1} + a_{m2}x_{m2} + \cdots\cdots + a_{mn}x_{mn} \geq b_m$$

$$x_{ij} \geq 0 \quad; i = 1,2,\cdots\cdots, m\,, j = 1,2,\cdots\cdots, n$$

3.2 Example

Let us assume that we want to exploit four agricultural areas A_1, A_2, A_3, A_4, and the area of each of them, respectively, is 60,150,20,10, by planting them with the following crops: wheat, barley, cotton, tobacco, and beet, from which we need the following: 800, 200,600,1000,2500 Let us assume that the regions' productivity of these crops and their prices are given in Table 2.

Table 2. Example data.

Regions / Crops	A_1	A_2	A_3	A_4	The order	Price per ton
wheat	{4,6}	4	3	6	2500	{1400,1600}
barley	7	5	4	{3,5}	1000	{900,1100}
cotton	4	{9,11 }	8	5	600	{4500,6000}
tobacco	6	{2,4}	0	0	200	{4000,5000}
beet	3	{10,14}	10	6	800	{400,700}
Space	60	150	20	10		

- Required:

Formulate the mathematical model for this issue so that the production value is as large as possible. To formulate the mathematical model, we extract the following linear conditions:

- Space restrictions

$$x_{11} + x_{21} + x_{31} + x_{41} + x_{51} = 60$$

$$x_{12} + x_{22} + x_{32} + x_{42} + x_{52} = 150$$

$$x_{13} + x_{23} + x_{33} + x_{43} + x_{53} = 20$$

$$x_{14} + x_{24} + x_{34} + x_{44} + x_{54} = 10$$

- Order restrictions

$$\{4,6\}x_{11} + 4x_{12} + 3x_{13} + 6x_{14} \geq 2500$$

$$7x_{21} + 5x_{22} + 4x_{23} + \{3,5\}x_{24} \geq 1000$$

$$4x_{31} + \{9,11\}x_{32} + 8x_{33} + 5x_{34} \geq 600$$

$$6x_{41} + \{2,4\}x_{42} + 0x_{43} + 0x_{44} \geq 200$$

$$3x_{51} + \{10,14\}x_{52} + 10x_{53} + 6x_{54} \geq 800$$

- Non-Negative restrictions

$$x_{ij} \geq 0 \ ; i = 1,2,3,4,5 \quad \text{and} \quad j = 1,2,3,4$$

- Objective function that express the value of production is:

$$Z = \{1400,1600\}(\{4,6\}x_{11} + 4x_{12} + 3x_{13} + 6x_{14})$$
$$+ \{900,1100\}(7x_{21} + 5x_{22} + 4x_{23} + \{3,5\}x_{24})$$
$$+ \{4500,6000\}(4x_{31} + \{9,11\}x_{32} + 8x_{33} + 5x_{34})$$
$$+ \{4000,5000\}(6x_{41} + \{2,4\}x_{42} + 0x_{43} + 0x_{44}) + \{400,700\}(3x_{51}$$
$$+ \{10,14\}x_{52} + 10x_{53} + 6x_{54}) \rightarrow \text{Max}$$

- Mathematical model:
  - Find the maximum value of

$$Z = \{1400,1600\}(\{4,6\}x_{11} + 4x_{12} + 3x_{13} + 6x_{14})$$
$$+ \{900,1100\}(7x_{21} + 5x_{22} + 4x_{23} + \{3,5\}x_{24})$$
$$+ \{4500,6000\}(4x_{31} + \{9,11\}x_{32} + 8x_{33} + 5x_{34})$$
$$+ \{4000,5000\}(6x_{41} + \{2,4\}x_{42} + 0x_{43} + 0x_{44}) + \{400,700\}(3x_{51}$$
$$+ \{10,14\}x_{52} + 10x_{53} + 6x_{54}) \rightarrow \text{Max}$$

  - Within restrictions:

$$x_{11} + x_{21} + x_{31} + x_{41} + x_{51} = 60$$

$$x_{12} + x_{22} + x_{32} + x_{42} + x_{52} = 150$$

$$x_{13} + x_{23} + x_{33} + x_{43} + x_{53} = 20$$

$$x_{14} + x_{24} + x_{34} + x_{44} + x_{54} = 10$$

$$\{4,6\}x_{11} + 4x_{12} + 3x_{13} + 6x_{14} \geq 2500$$

$$7x_{21} + 5x_{22} + 4x_{23} + \{3,5\}x_{24} \geq 1000$$

$$4x_{31} + \{9,11\}x_{32} + 8x_{33} + 5x_{34} \geq 600$$

$$6x_{41} + \{2,4\}x_{42} + 0x_{43} + 0x_{44} \geq 200$$

$$3x_{51} + \{10,14\}x_{52} + 10x_{53} + 6x_{54} \geq 800$$
$$x_{ij} \geq 0 \ ; i = 1,2,3,4,5 \quad \text{and} \quad j = 1,2,3,4$$

  - It is a linear model; we use the simplex method to obtain an optimal solution.

4. Conclusions

In this study, the authors presented a new formulation of the model for optimal use of agricultural land using the concepts of neutrosophic science, where we took data that are affected by

the surrounding conditions. Neutrosophic values take into account fluctuations in ambient conditions, from natural factors that can affect crop yields to price fluctuations that can affect profit. We obtained a linear neutrosophic mathematical model that can be solved using the simplex neutrosophic method that was presented in previous research. Then, the optimal solution is the values of the variables that express the areas that can be allocated in each of the regions for each crop.

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Neutrosophic Mathematical Model of Product Mixture Problem Using Binary Integer Mutant

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Abstract
Linear programming problems with integers, are issues in which some or all decision variables are restricted to be their values are correct values, and here these models can be solved by neglecting the constraints of integers and then rotating the fractional values of the optimal solution to obtain correct values, paying attention to the need for the resulting solution to belong to the accepted solutions area, but this procedure can lead to the desired purpose if the number of variables is small, but if there is a number We do not guarantee to obtain an optimal correct solution for the model, even if all the solution combinations are tested, knowing that in the model that contains a variable n, a set of solutions2^n must be tried, and if we can rotate, the correct solution will be an approximate solution, in order to obtain more accurate integer values, we present in this research a study in which we use binary integer variables to build the neutrosophic linear mathematical model. The importance of this study lies in providing solutions to many practical problems that require solutions with integers and we will clarify all of the above by building a mathematical model for the problem of the mixture of products using binary integer variables and neutrosophic values.

Keywords: Process research; Linear models; Integer models; Neutrosophic science; Neutrosophic linear mathematical model; Product mixture problem.

1. Introduction

In practical life, we encounter a lot of mathematical models that do not accept a solution unless the compounds of this solution are correct values, and one of the most important examples of these models is the product mixture model, and a distinction must be made between this model and the model of the installation of mixtures because what is meant by the mixture of products is to choose the appropriate product production from among the range of products that can be produced by a production company, it is clear that the optimal solution compounds must be integers, but in fact when solving such models The values of some or all variables may be incorrect values, which requires us to use methods to approximate them to integer values belonging to the accepted solution area, and the optimal solution is an approximate solution that may cause a loss to the company, and to obtain an accurate ideal solution for such models, use the binary integer variable that takes one of the two values, and here the one agrees with the decision to produce the product and zero the decision not to produce δ_j 0 or 1 [1,2] As a result of this use, we get linear mathematical models Optimal solution compounds have valid values, it should be noted here that this study is identical to the concepts of neutrosophic science because there is no determination in it, we do not know exactly which product we will produce, that is, these models can be considered neutrosophic linear models, also we can here from the concepts contained in the research [3] In addition to the binary variable, we can obtain neutrosophic linear models for the problem of the mixture of products using neutrosophic values with all problem data And some of them, which will enable us to obtain an optimal solution with neutrosophic values, values that the American scientist Florentin Smarandache knew when he introduced his new science neutrosophic science in 1995, a science that grew and developed rapidly thanks to the efforts of those interested in this science and scientific development [4-12].

2. Discussion:

The profit of any company that can produce a number of products is affected by the optimal choice of products that will produce it from the group of products that it can produce. To help these companies choose the products that bring them the greatest profit, operations research provided linear mathematical models called product mixtures. Since the optimal solution for any linear model may have one or all of its components' incorrect values, the researchers formulated the appropriate model δ_j for such problems. Use the binary integer variable that takes one of the two values 0 or 1 and here one corresponds to the product production decision and zero non-production decision:

Question text: [1]

A company is planning to produce Nproduct where the product j needs a fixed preparation cost or a fixed production cost K_j independent of the quantity produced, and needs a variable cost $C_j$per production unit commensurate with the quantity produced, suppose that each unit of the product j needs $\boldsymbol{a_{ij}}$ unit of the supplier $i$where there is M supplier and assuming that the product j has a sales opportunity of d_j and is sold at the price P_j of a monetary unit per unit and that only b_j unit of the supplier $\boldsymbol{i}$ is available where $i = 1,2, - - -, M$

 The goal of the issue becomes to determine the optimal product mix which makes the net profit the greatest possible

Formulation of the mathematical model:

Determination of the cost:

From the text of the problem, we note that the total cost of production is the fixed cost in addition to the variable cost, which is a nonlinear function of the quantity produced,

 But with the help of binary integer variables δ_j , the problem can be formulated in the form of a linear model with integers

We assume that the binary integer variable δ_j symbolizes the decision to produce the product j or not to produce it, in other words:

$$\delta_j = \begin{cases} 1 & j \;\; \text{if production decision was taken} \\ 0 & \text{otherwise} \end{cases}$$

Then the cost of producing one unit of the product j becomes as follows $K_j\delta_j + C_j x_j \delta_j$ where $\boldsymbol{\delta_j = 1}$ if$x_j > 0$ and $\delta_j = 0$ if $x_j = 0$ and therefore the goal function will be written as follows

$$Z = \sum_{j=1}^{N} P_j\, x_j - \sum_{j=1}^{N} \left(K_j \delta_j + C_j x_j \right)$$

Limitations of the problem:

A restriction on the supplier i is given in the following relationship:

$$\sum_{j=1}^{N} a_{ij}\, x_j \leq b_j \;\; ; i = 1,2, - - -, M$$

The restriction of the demand for the product j is given by the following relationship:

$$x_j \leq d_j \delta_j \;\; ; j = 1,2, - - -, N$$

Mathematical model: find the maximum value of the function

$$Z = \sum_{j=1}^{N} P_j\, x_j - \sum_{j=1}^{N} \left(K_j \delta_j + C_j x_j \right)$$

Within Restrictions

$$\sum_{j=1}^{N} a_{ij}\, x_j \leq b_j \;\; ; i = 1,2, - - -, M$$

$$x_j \leq d_j \delta_j \;\; ; j = 1,2, - - -, N$$
$$x_j \geq 0 \text{ and } \delta_j = 1 \text{ or } \delta_j = 0$$

And or for all values

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$$\delta_j = 1 \quad \delta_j = 0 \quad ; j = 1,2, ---, N$$

From the previous model, we notice that x_j takes a positive value only when $\delta_j = 1$, and in this case, the production of the product j is limited by the quantity d_j and the fixed production cost K_j is included in the goal function

The idea of indeterminacy is the basis of neutrosophic science represented here through the use of the binary integer variable because the optimal solution depends on the decision to produce a product or not to produce it, although we cannot guarantee the company a safe working environment in light of the great changes in the labor market through price strikes, resource availability or non-availability, and so on.

So it was necessary to reformulate this problem using neutrosophic values for the sales opportunity d_j and the cost of producing one unit of each product C_j and selling price P_j so that the sales opportunity becomes $d_j + \varepsilon_j$, production cost $C_j + \mu_j$ and selling price $P_j + \varphi_j$ where ε_j and μ_j and φ_j are the indeterminacy and is the change in the sales opportunity, cost and selling price respectively depending on the conditions of the work environment and takes one of the following forms:

$\varepsilon_j = [\lambda_{j1}, \lambda_{j2}]$ Or $\varepsilon_j = \{\lambda_{j1}, \lambda_{j2}\}$ ------- and $\mu_j = [\nu_{j1}, \nu_{j2}]$ او $\mu_j = \{\nu_{j1}, \nu_{j2}\}$ ----------and $\varphi_j = [\theta_{j1}, \theta_{j2}]$ او $\varphi_j = \{\theta_{j1}, \theta_{j2}\}$----------

Which are values close to the values d_j and C_j and can be any neighborhood to them.

 Then the text of the problem becomes as follows:

The text of the problem according to neutrosophic science:

A company is planning to produce N product where the product $j^{\backslash}$ needs a fixed preparation cost or a fixed production cost K_j independent of the quantity produced, and needs a variable cost $C_j + \mu_j$ per production unit commensurate with the quantity produced, we suppose that each unit of the product j needs a_{ij} a unit of the supplier i where there is M supplier. Assuming that the product j that has a sales opportunity $d_j + \varepsilon_j$ is sold at the price of $P_j + \varphi_j$ monetary unit per unit and that only b_j unit of the supplier i is available where $i = 1,2, ---, M$ the goal of the problem becomes to determine the optimal product mix that makes the net profit as great as possible.

Formulation of the mathematical model:

Determination of the cost:

From the text of the problem, we note that the total cost of production is the fixed cost in addition to the variable cost, which is a nonlinear function of the quantity produced,

But with the help of binary integer variables $\boldsymbol{\delta_j}$, the problem can be formulated in the form of a linear model with integers

We assume that the binary integer variable δ_j symbolizes the decision to produce the product j or not to produce it in other words

$$\delta_j = \begin{cases} 1 & j \text{ if production desicion was taken} \\ 0 & \text{otherwise} \end{cases}$$

Then the cost of producing one unit of the product becomes as follows $K_j \delta_j + (C_j + \mu_j)x_j$, where $\delta_j = 1$ if $x_j > 0$ and $\delta_j = 0$ if $x_j = 0$ and therefore the goal function becomes as follows:

$$Z = \sum_{j=1}^{N} (P_j + \varphi_j) x_j - \sum_{j=1}^{N} \left(K_j \delta_j + (C_j + \mu_j)x_j \right)$$

Restrictions of the problem:

A restriction on the supplier i is given in the following relationship:

$$\sum_{j=1}^{N} a_{ij} x_j \le b_j \quad ; i = 1,2, ---, M$$

The restriction of the demand for the product j is given by the following relationship:

$$x_j \le (d_j + \varepsilon_j)\delta_j \quad ; j = 1,2, ---, N$$

Mathematical model: Find the maximum value of the function:

$$Z = \sum_{j=1}^{N} (P_j + \varphi_j) x_j - \sum_{j=1}^{N} \left(K_j \delta_j + (C_j + \mu_j)x_j \right)$$

Within Restrictions

$$\sum_{j=1}^{N} a_{ij} x_j \le b_j \quad ; i = 1,2, ---, M$$

$$x_j \leq (d_j + \varepsilon_j)\delta_j \quad ; j = 1,2, - - -, N$$

$$x_j \geq 0 \text{ and } \delta_j = 1 \text{ or } \delta_j = 0$$

And or for all values

$$j = 1,2, - - -, N$$

From the previous model, we note that x_j takes a positive value only when $\delta_j = 1$ and in this case the production of the product j is limited by the quantity $d_j + \varepsilon_j$ and the fixed production cost K_j included in the goal function, by solving this model we get an optimal neutrosophic value for the goal function NZ^* through which we know the profit that the company can achieve in the best and worst conditions and enable the company to develop appropriate plans for the workflow in it.

3. Conclusion and Results

In this study, we presented important ideas for building mathematical models for practical problems that require that their solutions be correct values, and through which he showed the importance of using the binary integer variable in obtaining correct values, as it helps in converting some nonlinear mathematical programming problems into linear programming problems, and also the importance of using neutrosophic values to obtain an ideal solution that guarantees the company a safe workflow with the fluctuations of working conditions, we focus on the need to use neutrosophic values in problems that are affected by surrounding conditions, and also the use of binary integer to convert some nonlinear programming problems into linear programming problems, and this is what we will present in later researches .

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The Use of Neutrosophic Methods of Operation Research in the Management of Corporate Work

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Abstract: The science of operations research is one of the modern sciences that have made a great revolution in all areas of life through the methods provided by it, suitable and appropriate to solve most of the problems that were facing researchers, scholars and those interested in the development of societies, and the most beneficiaries of this science were companies and institutions that are looking for scientific methods that help them manage their work so that they achieve the greatest profit and the lowest cost, and one of the important methods that have been used in the management of companies we offer in this research two methods, Dynamic programming method. This method has been used in many practical matters and helped decision-makers in companies to achieve a maximum profit and less cost by formulating the reality of the state of the company and the data provided by decision-makers with a dynamic mathematical model that is solved using methods of solving dynamic models and we will provide in this research an example of this through the issue of choosing the optimal investment for the budget of a company so that it achieves a maximum profit, and the method of programming with integers: the method that provided these companies with solutions with integer values suitable for the nature of its work, through the use of the binary integer in the formulation of the appropriate mathematical model on the one hand, and on the other hand, the use of the binary integer variable helped to convert some nonlinear models that lead to some practical problems into linear models, and it should be noted here that in the previous two methods there is something indeterminable because we must make a decision in choosing or not choosing something, but the optimal solution that we will get remains A specific value because we are building the mathematical model for any realistic issue through the data provided by those responsible for the work and these data are calculated quantities and therefore they are uncertain values because their validity depends on the circumstances surrounding the work environment, they may be exposed to increase or decrease, and therefore the optimal solution on which the company will base its decision is suitable for specific values and any change in them can cause the company an uncalculated loss, so in this research we will use the concepts of neutrosophic science, the branch of science founded by the American scientist Florentin Smarandache in 1995 based on his belief that there is no absolute truth, a science that is interested in the study of ideas and concepts that are neither true nor false, but just in-between, and we will take the data (calculated quantities) neutrosophic values that are specified or unspecified values are any set close to the calculated quantities, then the resulting mathematical model is a neutrosophic model and the optimal solution has neutrosophic values and thanks to the indefinite uncertainty that these values have, companies from the development of appropriate plans for all circumstances and thus achieve the greatest profit and the lowest cost, and we will clarify the above through two issues, the issue of optimal designation of a warehouse site, which we will formulate the mathematical model of using the neutrosophic integer programming method - and the issue of capital budget, which we will present in two different forms, we use in the first form the neutrosophic integer programming method and in the second the neutrosophic dynamic programming method.

1. Introduction

The splendor of any science is completed through its scientific methods that help solve practical issues facing us in our daily lives and work on the development of societies, and one of the important methods presented by the science of operations research is the method of programming with integers, which depends on building mathematical models using the binary correct variable, which enabled researchers to convert some nonlinear mathematical models into linear mathematical models, in addition to that when solving these models, beam compounds were the optimal solution correct values that meet the nature of the issue under study and do not need to rotate fractional values for the optimal solution to obtain correct values as we used to do previously, the greatness of the science of operations research is when it meets with the concepts of neutrosophic science, the latest and most important thing that science has offered in our time, the science that brought values out of determination to indeterminacy, this margin of freedom can be greatly benefited from when the methods of operations research are reformulated using its concepts, a complement to what we have presented from research in which we formulated some methods of process research according to the concepts of neutrosophic [1−21]. Due to the importance of the dynamic programming method, we also presented a paper entitled Neutrosophic Dynamic Programming, through which we explained how to formulate dynamic models using the concepts of neutrosophic science [22]. In another paper, we used the neutrosophic integer programming method entitled Neutrosophic Mathematical Model of the product mixture problem using the binary integer variable [23]. In this research, we will use the method of dynamic neutrosophic programming and the method of neutrosophic integer programming to help companies manage their work in a scientific manner that enables them to exploit the resources available to them in an ideal way that achieves the greatest profit and the least loss by formulating two issues: the question of optimal designation of a warehouse site and the question of capital budget using the concepts of neutrosophic science It should be noted that these two issues were studied using classical values, see reference [24, 25].

2. Discussion

Since the aim of this research is to help decision-makers in companies and institutions to make the optimal decision to ensure the greatest profit and the lowest cost, we must study the issue at hand a good study through which we can determine the data that are affected by the surrounding conditions, and then we take these neutrosophic data values any unspecified values take the form

$Na_i = a_i + \varepsilon_i$ where ε_i is the indeterminacy on the data can take one of the forms $[\lambda_{i1}, \lambda_{i2}]$ أو

$\{\lambda_{i1}, \lambda_{i2}\}$.. Or----- otherwise, which is any neighborhood containing the value a_i.

We clarify the above through the following problems:

2.1 The first issue

An executive in one of the companies asked an expert in the science of operations research to help him obtain an optimal solution through which to achieve the lowest cost of transportation and operation of warehouses he wants to establish in order to expand the company's work and provided him with information through which the expert formulated the following issue: The text of the problem according to the concepts of neutrosophic science: A retail company plans to expand its activities in a specific area by establishing two new warehouses, the following Table 1 shows the potential locations, the number of customers and the possibility of meeting the demand for the sites where (∗) has been placed in the event that the site can meet the customer's request and put (×) the opposite and code Nc_{ij} For the cost of transferring one unit from site i to customer j he got the following Table:

Table 1. Transportation cost in case of location selection.

Customer site	B_1	B_2	B_3	B_4
A_1	* Nc_{11}	* Nc_{12}	×	* Nc_{14}
A_2	* Nc_{21}	* Nc_{22}	* Nc_{23}	* Nc_{24}
A_3	×	* Nc_{32}	* Nc_{33}	* Nc_{34}
Customer orders	D_1	D_2	D_3	D_4

Table 2 shows the following information available for each of the candidate locations for warehouses.

Table 2. Operation information.

Information site	Operating cost per unit (monetary unit)	Initial Invested Capital (Monetary Unit)	Site Capacity
first	Np_1	k_1	A_1
second	Np_2	k_2	A_2
third	Np_3	k_3	A_3

It is required to choose suitable locations for warehouses that make the total costs of investment, operation and transportation as small as possible.

Building the mathematical model: Each site has a fixed capital cost independent of the quantity stored in the warehouse referred to that site and also has a variable cost proportional to the quantity transported, and therefore the total cost of establishing and operating the warehouse is a non-linear function of the stored quantity and using binary integer variables can be formulated the issue of determining the location of the warehouse in a program with integers where we assume that the binary integer variable δ_i Symbolizes the decision to choose the site or not to choose it in other words

$$\delta_i = \begin{cases} 1 & \text{if we chose the site } i \\ 0 & \text{otherwise} \end{cases}$$

Suppose that x_{ij} is the quantity transferred from site i to customer j, so the constraint expressing the ability of the first site to meet the requests is as follows:

$$x_{11} + x_{12} + x_{14} \leq A_1 \delta_1$$

When $\delta_1 = 1$, the first location with capacity A_1 is chosen. The quantity transported from the first site cannot exceed the capacity of that site A_1 When $\delta_1 = 0$ the non-negative variables $x_{11}, x_{12}, x_{14} = 0$ directly, indicating that it is not possible to ship from the first location

In a similar way, we obtain the following two constraints for the second and third signatories.

$$x_{21} + x_{22} + x_{23} + x_{24} \leq A_2 \delta_2$$
$$x_{31} + x_{33} + x_{34} \leq A_3 \delta_3$$

To choose exactly two locations, we need the following restriction:

$$\delta_1 + \delta_2 + \delta_3 = 2$$

As δ_1 can take one of the values of 0 or 1 only, the new constraint will force two variables from among the three variables, δ_i to be equal to one

The restrictions for customer requests can be written as follows:

first customer $x_{11} + x_{21} = D_1$

Second customer $x_{12} + x_{22} + x_{32} = D_2$

Third customer $x_{23} + x_{33} = D_3$

Forth customer $x_{14} + x_{24} + x_{34} = D_4$

To write the objective function, we note that the total cost of investment, operation and transportation for the first site is as follows:

$$k_1\delta_1 + Np_1(x_{11} + x_{12} + x_{14}) + Nc_{11}x_{11} + Nc_{12}x_{12} + Nc_{14}x_{14}$$

When we do not choose the first site, variable $\delta_1 = 0$ And that forces the variables

$$x_{11}, x_{12}, x_{14} = 0$$

In a similar way, the cost functions of the second and third sites can be written, and thus the full formulation of the issue of assigning the location of the warehouse is reduced to the following correct mixed program: Z is meant to be made minimal

$$Z = k_1\delta_1 + Np_1(x_{11} + x_{12} + x_{14}) + Nc_{11}x_{11} + Nc_{12}x_{12} + Nc_{14}x_{14} + k_2\delta_2 + Np_2(x_{21} + x_{22} + x_{23} + x_{24}) + Nc_{21}x_{21} + Nc_{22}x_{22} + Nc_{23}x_{23} + Nc_{24}x_{24} + k_3\delta_3 + Np_3(x_{32} + x_{33} + x_{34}) + Nc_{32}x_{32} + Nc_{33}x_{33} + Nc_{34}x_{34}$$

considering the following restrictions:

$$x_{11} + x_{12} + x_{14} \le A_1\delta_1$$
$$x_{21} + x_{22} + x_{23} + x_{24} \le A_2\delta_2$$
$$x_{31} + x_{33} + x_{34} \le A_3\delta_3$$
$$\delta_1 + \delta_2 + \delta_3 = 2$$
$$x_{11} + x_{21} = D_1$$
$$x_{12} + x_{22} + x_{32} = D_2$$
$$x_{23} + x_{33} = D_3$$
$$\delta_i \text{ true variable for } i = 1, 2, 3$$
$$x_{ij} \ge 0 \; ; \; i = 1, 2, 3 \text{ and } j = 1, 2, 3, 4$$

2.2 The second issue

The second request addressed by the executive was how I can choose the appropriate projects to operate a limited capital available in the company through a number of projects presented, through the information provided by the official in charge of managing the company, the expert formulated the following issue:

The issue of the capital budget: A company plans to disburse its capital during the T_j periods. where $j = 1, 2, - - -, n$, and there is A_i A proposed project where $i = 1, 2, - - -, m$ versus a limited capital B_j Available for investment in period j and when choosing any project i becomes in need of a certain capital in each period j we denote it Na_{ij}. It is a neutrosophic value, the value of each project is measured in terms of the liquidity flow corresponding to the project in each period minus the value

of inflation, and this is called net present value (NPV), we denote it Nv_i Accordingly, the following Table 3 can be organized:

Table 3. Return on Investment during periods.

period \ project	T_1	T_2	-----------	T_n
A_1	Na_{11}	Na_{12}	----------	Na_{11}
A_2	Na_{21}	Na_{22}		Na_{2n}
--------	----------	----------	--------	----------
A_m	Na_{m1}	Na_{m2}	----------	Na_{mn}
Limited capital	B_1	B_2	-----------	B_n

What is required in this problem is to select the right projects that maximize the total value (NPV) of all selected projects. Formulation of the mathematical model:

Here we assume a binary integer variable x_j It takes the value one if the project j is selected and takes the value zero if the project j is not selected

$$x_i = \begin{cases} 1 & \text{if we chose project } i \\ 0 & \text{otherwise} \end{cases}$$

Then the objective function is given by the following relationship:

$$Z = \sum_{i=1}^{m} Nv_i \, x_i$$

Then the objective function is given by the following relationship:

$$\sum_{i=1}^{m} Na_{ij} \, x_i \leq B_j \quad ; j = 1, ---, n$$

Accordingly, we get the following mathematical model:

Find the maximum value of the function:

$$Z = \sum_{i=1}^{m} Nv_i \, x_i$$

Considering the following restrictions:

$$\sum_{i=1}^{m} Na_{ij} \, x_i \leq B_j \quad ; j = 1, ---, n$$

x_i A binary variable takes one of the values 0 or 1 for all values of $i = 1, ---, m$ in the previous two issues, we got models with integers that have special methods of solution. This research cannot be presented and we will present them in later research using the concepts of neutrosophic science.

2.3 The third issue

in the reference [25] an example of a dynamic programming problem, which is the capital budget problem using classical values, is presented. In this paper, we will reformulate the general model of this problem using neutrosophic values, and we will apply this to the example given in the reference so that we can compare the use of classical values and neutrosophic values using the optimal solution that we will get.

Another presentation of the problem of the neutrosophic capital budget in reference [25] the following question was raised: the budget of a company (5 million monetary units) wants to be distributed to four different investment projects, if you know that each investment has a certain return corresponding to each amount of one million and is shown in the attached table, it is required to find the optimal distribution of the budget if (5 million and 4 million) between the four projects in a way that obtains the greatest profit

Table 5. Periodic amounts classic values.

Invesment projects / Invested amounts	$F_1(x_1)$	$F_2(x_2)$	$F_3(x_3)$	$F_4(x_4)$
0	0	0	0	0
1	0.56	0.5	0.3	0.4
2	0.9	0.82	0.5	0.66
3	1.3	1.1	0.8	0.84
4	1.56	1.3	1.00	0.96
5	1.8	1.5	1.24	1.06

This issue is a dynamic programming issue that was addressed using the tables method and the summary table for the final solution was as follows:

Table 6. Final table of solution and classic values.

Z_4	x_4	$F_4(x_4)$	Z_3	$G_3^*(Z_3)$	$G_4(Z_4)$
5	0	0	5	2.12	2.12
	1	0.40	4	1.80	2.20
	2	0.66	3	1.40	2.06
	3	0.84	2	1.06	1.90
	4	0.96	1	0.56	1.52
	5	1.06	0	0	1.06
4	0	0	4	1.80	1.80
	1	0.40	3	1.40	1.80
	2	0.66	2	1.06	1.72
	3	0.84	1	0.56	1.40
	4	0.96	0	0	0.96

From the table, we find that the maximum return if (5) million is invested equals to $G_4^*(Z_4) = \mathbf{2.20}$

This value corresponds to $x_4 = 1$, $x_3 = 0$, $Z_3 = 4$ and therefore from the relations $Z_i = Z_{i-1} + x_i$

We find the following distribution:

$$G_4^*(Z_4) = 2.20 \Rightarrow Z_4 = 5 \Rightarrow \begin{cases} x_4 = 1 \\ x_3 = 0 \\ x_2 = 1 \\ x_1 = 3 \end{cases}$$

$x_4 = 1$ أو $x_4 = 0$:

From the table, we find that the maximum return if (4) million is invested equals $G_4^*(Z_4) = 1.80$

This value corresponds to , $x_4 = 1$ or $x_4 = 0$ and the distribution is as follows:

$$G_4^*(Z_4) = 1.80 \Rightarrow Z_4 = 4 \Rightarrow \begin{cases} x_4 = 0 \\ or \\ x_4 = 1 \end{cases} \Rightarrow \begin{cases} \begin{cases} x_4 = 0 \\ x_3 = 0 \\ x_2 = 1 \\ x_1 = 3 \end{cases} \\ 0r \\ \begin{cases} x_4 = 1 \\ x_3 = 0 \\ x_2 = 1 \\ x_1 = 2 \end{cases} \end{cases}$$

To clarify the extent to which neutrosophic values affect the final result of the solution, that is, on the maximum profit of the company In the beginning, we formulate the general model for this problem, neutrosophic science concepts, and accordingly we determine the data that are affected by the surrounding conditions of the work environment and here we take the values that express the profit from investing an amount of amounts in one of the projects neutrosophic values because they are the values that are affected by the surrounding conditions

We get the following problem:

The problem of the distribution of the neutrosophic capital budget: the budget of the company is M monetary unit that you want to distribute to n of different investment projects, $B_1, B_2, - - -, B_n$ if you know that for every amount invested in a project, there is a profit (i.e. the profit is related to the amount invested into the project), $NF_1(x_1), NF_2(x_2), - - -, NF_n(x_n)$، shown in the attached table, it

is required to find the optimal distribution of the budget between n projects so that the company gets the greatest profit

Table 4. Return on Investment by amount used.

Ivestment projects / Invested amounts	B_1	B_2	---------	B_n
1	Nc_{11}	Nc_{12}	---------	Nc_{1n}
2	Nc_{21}	Nc_{22}	---------	Nc_{2n}
----------	------	--------	--------	--------
m	Nc_{m1}	Nc_{m2}	--------	Nc_{mn}

After studying the data of this issue, we note that it can be formulated in the form of a dynamic programming problem corresponding to the case of one dimension and we mean the criterion that affects the decision-making process at a certain stage of obtaining the optimal solution and one

influencer and we mean by it the goal follower and here we note that the goal is the only one which is to achieve the greatest profit so we can represent it with the following relationships:

$$Z_i = NF_i(x_i) \quad ; i = 1, 2, ---, n$$

Z_i is the function that expresses the state reached by the problem at a certain partial point x_i is secision variable in stage i

The relationship between the state in a given phase i−1 and the next phase i is written as follows:

$$Z_i = g_i(x_i, Z_{i-1})$$

These relationships represent constraints

The goal function is written according to the partial goal dependencies in the previous stages with the following relationship:

$$NG_i(Z_i) = NF_i(x_i, Z_{i-1}, Z_i) \quad ; i = 1, 2, ---, n$$

This function takes an optimal value at each stage and this value is given by the following relationship: $NG_n^*(Z_n) = \underset{x_1 \to x_n}{opt} \left[NF_1(x_i, Z_{i-1}, Z_i) + ---- + NF_n(x_n, Z_{n-1}, Z_n) \right]$

Which can be expressed as follows:

$$NG_n^*(Z_n) = \underset{x_n}{opt} \left[NF_n(x_n, Z_{n-1}, Z_n) + NG_{n-1}^*(Z_{n-1}) \right]$$

whereas

$NG_n^*(Z_n)$ – **optimal goal function   at stage** n

$NG_{n-1}^*(Z_{n-1})$ – **optimal goal function   at stage** $n - 1$

they be solved as follows:

$$NG_i^*(Z_i) = \underset{x_i}{0pt} \, NF_i\left[(x_i, Z_{i-1}, Z_i)\right] \quad ; i = 1, 2, ---, n \quad (1)$$

Restrictions:

$$Z_i = g_i(x_i, Z_{i-1}) \quad ; i = 1, 2, ---, n \quad (2)$$

The relationships,1. and restrictions,2. It includes all one-dimensional, one-effect neutrosophic dynamic programming problems and is solved in the form of tables that we will illustrate by solving the following example: Example: (We will take some neutrosophic values) The budget of the company M=\$ 5 million wants to spread it among four projects B_1, B_2, B_3, B_4 of the various investment projects, if you know that for every million investment in a project, there is a profit (that is, the profit is related to the amount that is invested into the project), shown in the attached table, it is required to find the optimal distribution of the budget between the four projects so that the company achieves the greatest profit

Table 7. Return on Investment with neutrosophic values.

Investment projects / Invested amounts	B_1	B_2	B_3	B_4
0	0	0	0	0
1	$[0.52, 0.57]$	0.5	0.3	0.4
2	0.9	$[0.8, 0.85]$	0.5	0.66
3	1.3	1.1	$[0.79, 0.82]$	0.84
4	1.56	1.3	1.00	$[0.9, 1.3]$
5	1.8	1.5	1.24	$[1.05, 1.2]$

This problem is formulated with a dynamic programming program and the optimal solution is found using the following relationships: We denote $NF_i(x_i)$. -function at each stage (the value of the profit returned from the use of the amount x_i in project , x_1, x_2, x_3, x_4 is the amount of money distributed over the four projects

$$Z_i = Z_{i-1} + x_i$$

Solution equations:

$$NG_1^*(Z_1) = NF_1(x_1)$$

$$NG_2^*(Z_2) = \underbrace{Max}_{x_2}[NF_2(x_2) + NG_1^*(Z_1)]$$

$$NG_3^*(Z_3) = \underbrace{Max}_{x_3}[NF_3(x_3) + NG_2^*(Z_2)]$$

$$NG_4^*(Z_4) = \underbrace{Max}_{x_4}[NF_4(x_4) + NG_3^*(Z_3)]$$

1- Solution using tables

2- Table Z_1

Table 8. Z_1 table.

Z_1	x_1	$G_1^*(Z_1)$	
0	0	0	
1	1	$[0.52, 0.57]$	
2	2	0.9	$Z_0 = 0$
3	3	1.3	$Z_1 = Z_0 + x_1$
4	4	1.56	
5	5	1.8	

1- Table Z_2 where $Z_2 = Z_1 + x_2$

Table 9. Z_2 table.

Z_2	x_2	$NF_2(x_2)$	Z_1	$NG_1^*(Z_1)$	$NG_2(Z_2)$
0	0	0	0	0	0
1	0	0	1	[0.52 , 0.57]	[0.52 , 0.57]
	1	0.5	0	0	0.5
2	0	0	2	0.9	0.9
	1	0.5	1	[0.52 , 0.57]	[1.02 , 1.07]
	2	[0.8 , 0.85]	0	0	[0.8 , 0.85]
3	0	0	3	1.3	1.3
	1	0.5	2	0.9	1.4
	2	[0.8 , 0.85]	1	[0.52 , 0.57]	[1.32 , 1.42]
	3	1.1	0	0	1.1
4	0	0	4	1.56	1.56
	1	0.5	3	1.3	1.8
	2	[0.8 , 0.85]	2	0.9	[1.7 , 1.75]
	3	1.1	1	[0.52 , 0.57]	[1.62 , 1.67]
	4	1.3	0	0	1.3
5	0	0	5	1.8	1.8
	1	0.5	4	1.56	2.06
	2	[0.8 , 0.85]	3	1.3	[2.1 , 2.15]
	3	1.1	2	0.9	2.0
	4	1.3	1	[0.52 , 0.57]	[1.82 , 1.87]
	5	1.5	0	0	1.5

The values colored yellow represent the maximum values of profit returned for each amount in which the second project is supplied, so we write a table, **Z_2** the abbreviation that shows the optimal values.

Table 10. Optimal values of Z_2.

Z_2	x_2	$NG_2^*(Z_2)$	
0	0	0	
1	0	[0.52 , 0.57]	
2	1	[1.02 , 1.07]	
3	2	[1.4 , 1.42]	$Z_2 = Z_1 + x_2$
4	2	[1.7 , 1.75]	
5	2	[2.1 , 2.15]	

2- Z_3 table where $Z_3 = Z_2 + x_3$

Table 11. Z_3 table.

Z_3	x_3	$NF_3(x_3)$	Z_2	$NG_2^*(Z_2)$	$NG_3(Z_3)$
0	0	0	0	0	0
1	0	0	1	[0.52 , 0.57]	[0.52 , 0.57]
	1	0.3	0	0	0.3
2	0	0	2	[1.02 , 1.07]	[1.02 , 1.07]
	1	0.3	1	[0.52 , 0.57]	[0.82 , 0.87]
	2	0.5	0	0	0.5
3	0	0	3	[1.4 , 1.42]	[1.4 , 1.42]
	1	0.3	2	[1.02 , 1.07]	[1.32 , 1.37]
	2	0.5	1	[0.52 , 0.57]	[1.02 , 1.07]
	3	[0.79 , 0.82]	0	0	[0.79 , 0.82]
4	0	0	4	[1.7 , 1.75]	[1.7 , 1.75]
	1	0.3	3	[1.4 , 1.42]	[1.7 , 1.72]
	2	0.5	2	[1.02 , 1.07]	[1.52 , 1.57]
	3	[0.79 , 0.82]	1	[0.52 , 0.57]	[1.31 , 1.39]
	4	1.0	0	0	1.0
5	0	0	5	[2.1 , 2.15]	[2.1 , 2.15]
	1	0.3	4	[1.7 , 1.75]	[2 , 2.05]
	2	0.5	3	[1.4 , 1.42]	[1.9 , 1.92]
	3	[0.79 , 0.82]	2	[1.02 , 1.07]	[1.81 , 1.89]
	4	1.0	1	[0.52 , 0.57]	[1.52 , 1.57]
	5	1.24	0	0	1.24

The values colored yellow represent the maximum values of profit payable for each amount in which the second project is supplied, so we write a table $\boldsymbol{Z_3}$ The abbreviation that shows the optimal values.

Table 12. Optimal values.

Z_3	x_3	$NG_3^*(Z_3)$	
0	0	0	
1	0	[0.52 , 0.57]	
2	0	[1.02 , 1.07]	
3	0	[1.4 , 1.42]	$Z_3 = Z_2 + x_3$
4	0	[1.7 , 1.75]	
5	0	[2.1 , 2.15]	

3- Z_3 table where $Z_4 = Z_3 + x_4$

Table 13. Z_4 table.

Z_4	x_4	$NF_4(x_4)$	Z_3	$NG_3^*(Z_3)$	$NG_4(Z_4)$
0	0	0	0	0	0
1	0	0	1	[0.52 , 0.57]	[0.52 , 0.57]
	1	0.4	0	0	0.4
2	0	0	2	[1.02 , 1.07]	[1.02 , 1.07]
	1	0.4	1	[0.52 , 0.57]	[0.92 , 0.97]
	2	0.66	0	0	0.66
3	0	0	3	[1.4 , 1.42]	[1.4 , 1.42]
	1	0.4	2	[1.02 , 1.07]	[1.42 , 1.47]
	2	0.66	1	[0.52 , 0.57]	[1.18 , 1.23]
	3	0.84	0	0	0.84
4	0	0	4	[1.7 , 1.75]	[1.7 , 1.75]
	1	0.4	3	[1.4 , 1.42]	[1.8 , 1.82]
	2	0.66	2	[1.02 , 1.07]	[1.68 , 1.73]
	3	0.84	1	[0.52 , 0.57]	[1.36 , 1.41]
	4	[0.9 , 1.3]	0	0	[0.9 , 1.3]
5	0	0	5	[2.1 , 2.15]	[2.1 , 2.15]
	1	0.4	4	[1.7 , 1.75]	[2.1 , 2.15]
	2	0.66	3	[1.4 , 1.42]	[2.06 , 2.08]
	3	0.84	2	[1.02 , 1.07]	[1.86 , 1.91]
	4	[0.9 , 1.3]	1	[0.52 , 0.57]	[1.42 , 1.87]
	5	[1.05 , 1.2]	0	0	[1.05 , 1.2]

The values colored yellow represent the maximum values of profit returned for each amount in which the second project is supplied, so we write a table, Z-4. The abbreviation that shows the optimal values

Table 14. the optimal values of Z_4.

Z_4	x_4	$NG_4^*(Z_4)$	
0	0	0	
1	0	[0.52 , 0.57]	
2	0	[1.02 , 1.07]	
3	1	[1.42 , 1.47]	$Z_4 = Z_3 + x_4$
4	1	[1.8 , 1.82]	
5	0 *or* 1	[2.1 , 2.15]	

Conclusion of the optimal solution: Note that the maximum profit value when investing the amount (5) available in the company is achieved at two values of x_4 Therefore, the optimal distribution is as follows:

$$NG_4^*(Z_4) = [2.1, 2.15] \Rightarrow Z_4 = 5 \Rightarrow \begin{cases} x_4 = 0 \\ or \\ x_4 = 1 \end{cases} \Rightarrow \begin{cases} \begin{cases} x_4 = 0 \\ x_3 = 0 \\ x_2 = 2 \\ x_1 = 3 \end{cases} \\ or \\ \begin{cases} x_4 = 1 \\ x_3 = 0 \\ x_2 = 2 \\ x_1 = 2 \end{cases} \end{cases}$$

To obtain a maximum profit of a value belonging to the range, $[\mathbf{2.1}, \mathbf{2.15}]$ an amount of (5) million can be distributed in one of the two forms:

Figure 1: We provide the first project with (3) million dollars and the second project with (2) million dollars and cancel investment in the third and fourth projects Figure 2: We provide the first project (2) million dollars, the second project (2) million dollars, the fourth project (1) million dollars, and cancel the investment in the third project to obtain a maximum profit of a value belonging to the range$[\mathbf{1.8}, \mathbf{1.82}]$ The amount of (4) million can be distributed as follows: We provide the first project with (1) million dollars, the second project with (2) million dollars, and the fourth project with (1) million dollars, and cancel the investment in the third project

$$NG_4^*(Z_4) = [\mathbf{1.8}, \mathbf{1.82}] \Longrightarrow Z_4 = 4 \Longrightarrow \begin{cases} x_4 = 1 \\ x_3 = 0 \\ x_2 = 2 \\ x_1 = 1 \end{cases}$$

Note: When calculating $\mathbf{Z_4}$ We can only calculate for (4) and (5) million, i.e. sufficiency in the following table:

Table 15. Table of optimal values for Z_4 with the short solution.

Z_4	x_4	$NF_4(x_4)$	Z_3	$NG_3^*(Z_3)$	$NG_4(Z_4)$
4	0	0	4	[1.7, 1.75]	[1.7, 1.75]
	1	0.4	3	[1.4, 1.42]	[1.8, 1.82]
	2	0.66	2	[1.02, 1.07]	[1.68, 1.73]
	3	0.84	1	[0.52, 0.57]	[1.36, 1.41]
	4	[0.9, 1.3]	0	0	[0.9, 1.3]
5	0	0	5	[2.1, 2.15]	[2.1, 2.15]
	1	0.4	4	[1.7, 1.75]	[2.1, 2.15]
	2	0.66	3	[1.4, 1.42]	[2.06 2.08]
	3	0.84	2	[1.02, 1.07]	[1.86, 1.91]
	4	[0.9, 1.3]	1	[0.52, 0.57]	[1.42, 1.87]
	5	[1.05, 1.2]	0	0	[1.05, 1.2]

Comparison for the investment of (5) million:

Table 16. comparison in case the amount is (5) millions.

Type of profit values / Projects	Optimal distribution of amount in case that some profit values are neutrosophic values		Optimal distribution of amount in case that some profit values are classical values
Forth project	$x_4 = 1$ $0r$ $x_4 = 0$		$x_4 = 1$
Third project	$x_3 = 0$	$x_3 = 0$	$x_3 = 0$
Second project	$x_2 = 2$	$x_2 = 2$	$x_2 = 1$
First project	$x_1 = 3$	$x_1 = 1$	$x_1 = 3$
Maximal general profit	$NG_4^*(Z_4) = [2.1 , 2.15]$		$G_4^*(Z_4) = 2.20$

Comparison for the investment of (4) million:

Table 17. comparison in case the amount is (4) millions.

Type of profit values / Projects	Optimal distribution of amount in case that some profit values are neutrosophic values	Optimal distribution of amount in case that some profit values are classical values	
Forth project	$x_4 = 1$	$x_4 = 0$ or $x_4 = 1$	
Third project	$x_3 = 0$	$x_3 = 0$	$x_3 = 0$
Second project	$x_2 = 2$	$x_2 = 1$	$x_2 = 1$
First project	$x_1 = 1$	$x_1 = 3$	$x_1 = 2$
Maximal general profit	$NG_4^*(Z_4) = [1.8 , 1.82]$	$G_4^*(Z_4) = 1.80$	

3. Conclusion and Results

In this research, we presented a study whose importance lies in the following points:

1. Obtaining a general formula for the issue of establishing new warehouses that help companies and institutions expand their work at the lowest cost by choosing the ideal locations for expansion points

2. Obtaining a general formula for the issue of distributing the companies' capital budget through the ideal investment for them in selected projects within a group of proposed projects so that the company achieves the maximum profit.

3. Clarifying the role of the binary integer in converting some nonlinear models into linear models and also obtain solutions with integers appropriate to the nature of the problems under study.

4. Focusing on the need to use the concepts of neutrosophic science to obtain optimal solutions suitable for all circumstances surrounding the work environment.

Data availability

The datasets generated during and/or analyzed during the current study are not publicly available due to the privacy-preserving nature of the data but are available from the corresponding author upon reasonable request.

Conflict of interest

The authors declare that there is no conflict of interest in the research.

Ethical approval

This article does not contain any studies with human participants or animals performed by any of the authors.

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Inspection Assignment Form for Product Quality Control Using Neutrosophic Logic

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Abstract: During the production process, production companies need to monitor the finished products and ensure their quality, which imposes on them the appointment of inspectors for auditing, and this appointment costs the company amounts that affect the general profit, so it strives to make this cost as low as possible and that the audit process is carried out with high accuracy because in case that the finished products do not conform to the basic specifications of the product, the company is required to pay a fine. In this research, we will formulate the text of the problem appropriately for such a case and then we will build the appropriate mathematical model through which we can obtain the lowest possible cost of inspection, and we will present the study using classical values and neutrosophic values, and we will demonstrate them through an example that shows us the difference between them.

Keywords: Mathematical models; Neutrosophic science; Inspection issue; Product quality control.

1. Introduction

Since the genesis of the science of operations research, it has been providing solutions to the problems faced by companies through studies presented by scientists and researchers in all fields using the methods of this science, in this research we will use the method of linear programming to build a mathematical model that enables us to ensure the quality of products using inspectors and at the lowest possible cost where we will formulate the appropriate issue for that and build the mathematical model that by finding the optimal solution to it and using the methods of solution provided by the science of operations research we get on the lowest cost of inspection and high-quality products, where the matter will be displayed

First : Using classical values based on references [1,2,3]

Second: Neutrosophic values based on what researchers and those interested in this science have presented and developed through studies and research that have been published in references [4, 16]

Discussion:

1- Studying using classical values:

Formulating the problem and building the mathematical model:

Text of the issue: Through the available information about the functioning of companies, we can develop the following text:

The company has a rank for inspectors and wants to assign the task of quality control to them, and a piece of product should be audited daily during an hour of work per day, in the following table we explain the full information about the inspectors and for all mattresses nKS.

Table 1. Information on inspectors using classical values.

About the Inspector Inspector rank	Number of pieces checked (hour)	Accuracy (percent)	Inspector's remuneration (Monetary Unit per Hour)	Number of inspectors	The fine paid by the company for each fault to the inspector
1	M_1	D_1	G_1	A_1	R
2	M_2	D_2	G_2	A_2	R
----			-		-----
n	M_n	D_n	G_n	A_n	R

Required: Formulate the appropriate mathematical model through which we can assign the optimal assignment to the inspectors so that the cost of inspection is as low as possible.

Building the Mathematical Model:

To build the mathematical model, we impose the number of inspectors of each rank on the order assigned to the inspection task, then the following inequality must be met:

$x_1, x_2, - - - - x_n$

$$x_j \leq A_j \quad ; \quad j = 1,2, --, n$$

Since the company needs to audit K piece daily within S working hour per day, the following set of restrictions must be met:

$$\sum_{j=1}^{n} SM_j x_j \geq K$$

To obtain the target follower we note that the company bears two types of costs during the inspection process, the inspector's wage and the fine corresponding to the error committed by the inspector to write the target follower note the following:

The cost of the inspector from rank j hourly salary is calculated through the following relationship

$$C_j = G_j + M_j R_j \left(\frac{100 - D_j}{100}\right) \quad ; \quad j = 1,2, - - -, n$$

The total costs for all inspectors assigned to the task of quality control per hour shall be given by the following relationship:

$$TC_j = \sum_{j=1}^{n} \left[G_j + M_j R_j \left(\frac{100 - D_j}{100}\right)\right] x_j$$

The target function is then written as follows:

$$Z = S \sum_{j=1}^{n} \left[G_j + M_j R_j \left(\frac{100 - D_j}{100}\right)\right] x_j$$

From the above, we can develop the following mathematical model:

We want to find the smallest possible value for the function:

$$Z = S \sum_{j=1}^{n} \left[G_j + M_j R_j \left(\frac{100 - D_j}{100} \right) \right] x_j \rightarrow Min$$

Within Restrictions

$$x_j \leq A_j \quad ; \quad j = 1,2, --, n$$

$$\sum_{j=1}^{n} SM_j x_j \geq K$$

$$x_j \geq 0 \; ; \; j = 1,2, ---- n$$

Example:

The company has three ranks for inspectors and wants to assign the task of quality control to them, and 1500 piece should be audited daily during 8 working hours per day, in the following table we explain the full information about the inspectors and for all mattresses.

Table 2. Information on inspectors using classical values.

About the Inspector Inspector rank	Number of pieces checked (hour)	Accuracy (percent)	Inspector's remuneration (Monetary Unit per Hour)	Number of inspectors	The fine paid by the company for each fault to the inspector
1	15	95	4	10	2
2	10	90	3	6	2
3	25	98	5	8	2

Required : Formulate the appropriate mathematical model through which we can assign the optimal assignment to the inspectors so that the cost of inspection is as low as possible

To build the mathematical model, we impose the number of inspectors from the three ranks in the order assigned to the inspection task, then the following in equations must be fulfilled. x_1, x_2, x_3

$$x_1 \leq 10$$

$$x_2 \leq 6$$

$$x_3 \leq 8$$

Since the company needs to audit 1500 pieces daily during 8 working hours a day, the following set of restrictions must be met:

$$\sum_{j=1}^{n} 8 M_j x_j \geq 1500$$

That is

$$8(M_1 x_1 + M_2 x_2 + M_3 x_3) \geq 1500$$

From it we get the following restriction:

$$120 x_1 + 80 x_2 + 200 x_3 \geq 1500$$

To obtain the target function, we note that the company bears two types of costs during the inspection process, the inspector's fee and the fine corresponding to the error committed by the inspector for each piece then the target function will be written as follows:

Then the cost of the inspector's hourly salary from rank j is calculated through the following relationship :

$$C_j = G_j + M_j R_j \left(\frac{100 - D_j}{100} \right) \quad ; \quad j = 1, 2, - - -, n$$

From them we get

$$C_1 = 4 + 15 \times 2 \times \left(\frac{100 - 95}{100} \right) = 5.5$$

$$C_2 = 3 + 10 \times 2 \times \left(\frac{100 - 90}{100} \right) = 5$$

$$C_3 = 5 + 25 \times 2 \times \left(\frac{100 - 98}{100} \right) = 6$$

The total costs for all inspectors assigned to the task of quality control per hour shall be given by the following relationship:

$$TC_j = \sum_{j=1}^{n} \left[G_j + M_j R_j \left(\frac{100 - D_j}{100} \right) \right] x_j$$

We substitute the values available to us and we get:

$$TC_j = 5.5x_1 + 5x_2 + 6x_3$$

substituting the following target phrase:

$$Z = S \sum_{j=1}^{n} \left[G_j + M_j R_j \left(\frac{100 - D_j}{100} \right) \right] x_j$$

We get:

$$Z = 44x_1 + 40x_2 + 48x_3$$

From the above, we can develop the following mathematical model:

We want to find :

$$MinZ = 44x_1 + 40x_2 + 48x_3$$

Within Restrictions

$$x_1 \leq 10$$

$$x_2 \leq 6$$

$$x_3 \leq 8$$

$$120x_1 + 80x_2 + 200x_3 \geq 1500$$

$$x_j \geq 0 \; ; \; j = 1,2,3$$

To obtain the optimal solution, we use the simplex method, which is sufficiently explained in the references [1,2,3]

2- Formulation of the problem and the construction of mathematical model according to neutrosophic values:

The study concluded in the research [12] shows us how to construct neutrosophic linear models, (the linear model is a neutrosophic model if at least one of the likes of variables in the target function or neutrosophic value constraints)

The text of the issue:

The company has n rank for inspectors and wants to assign the task of quality control to them, and K pieces should be audited daily during an S hour of work per day, in the following table we explain the full information about the inspectors and for all ranks:

Table 3. Information on inspectors using neutrosophic values

About the Inspector Inspector rank	Number of pieces checked (hour)	Accuracy (percent)	Inspector's remuneration (Monetary Unit per Hour)	Number of inspectors	The fine paid by the company for each fault to the inspector
1	NM_1	ND_1	G_1	A_1	R
2	NM_2	ND_2	G_2	A_2	R
-----	--	-			-----
n	NM_n	ND_n	G_n	A_n	R

The number of pieces is a neutrosophic value $NM_j = M_j + \varepsilon_j$ where ε_j is the indeterminacy on the number of pieces, it can take one of the shapes $[\lambda_{j1}, \lambda_{j2}]$ or $\{\lambda_{j1}, \lambda_{j2}\}$ or any value close to M_j as well as the precision, neutrosophic values $ND_j = D_j + \delta_j$ where δ_j is the indeterminacy on the precision that can take one of the shapes $[\mu_{j1}, \mu_{j2}]$ or $\{\mu_{j1}, \mu_{j2}\}$ or any value close to D_j.

Required: Formulate the appropriate mathematical model through which we can assign the optimal support to the inspectors so that the cost of inspection is as low as possible

Building the neutrosophic mathematical model:

To build the mathematical model, we impose $x_1, x_2, ----, x_n$ the number of inspectors of each rank on the order assigned to the inspection task, then the following inequality must be met:

$$x_j \leq A_j \quad ; \quad j = 1, 2, --, n$$

Since the company needs to audit K piece daily within S working hour per day, the following set of restrictions must be met:

$$\sum_{j=1}^{n} S(NM_j)x_j \geq K$$

To obtain the target function, we note that the company bears two types of costs during the inspection process, the inspector's fee and the fine corresponding to the error committed by the inspector for each piece then the target follower writes as follows:

$$Z = S \sum_{j=1}^{n} G_j + NM_j R_j \left[\frac{100 - ND_j}{100}\right] x_j$$

Then the mathematical model is written as follows:

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$$Z = S \sum_{j=1}^{n} G_j + NM_j R_j \left[\frac{100 - ND_j}{100} \right] x_j \rightarrow Min$$

Within Restrictions

$$x_j \leq A_j \quad ; \quad j = 1,2, --, n$$

$$\sum_{j=1}^{n} S(NM_j) x_j \geq K$$

$$x_j \geq 0 \; ; \; j = 1,2, ---- n$$

Example 1:

A company has three ranks for inspectors and wants to assign the task of quality control to them, and 1500 pieces should be audited daily during 8 working hours per day, in the following table we explain the full information about the inspectors and for all ranks, in this example we will take the number of pieces checked by the inspectors from each rank as neutrosophic values

Table 4. Information on inspectors using neutrosophic values.

About the Inspector Inspector rank	Number of pieces checked (hour)	Accuracy (percent)	Inspector's remuneration (Monetary Unit per Hour)	Number of inspectors	The fine paid by the company for each fault to the inspector
1	{15,16}	95	4	10	2
2	{10,11}	90	3	6	2
3	{25,26}	98	5	8	2

Required : Formulate the appropriate mathematical model through which we can assign the optimal assignment to the inspectors so that the cost of inspection is as low as possible

To build the mathematical model, we impose x_1, x_2, x_3 as the number of inspectors from the three ranks in the order assigned to the inspection task, then the following inequality must be met:

$$x_1 \leq 10$$
$$x_2 \leq 6$$
$$x_3 \leq 8$$

Since the company needs to audit K pieces daily within S working hour per day, the following set of restrictions must be met:

$$\sum_{j=1}^{n} 8M_j x_j \geq 1500$$

That is

$$8(M_1 x_1 + M_2 x_2 + M_3 x_3) \geq 1500$$

From it we get the following restriction:

$$8\{15,16\} x_1 + 8\{10,11\} x_2 + 8\{25,26\} x_3 \geq 1500$$

To obtain the target function, we note that the company bears two types of costs during the inspection process, the inspector's fee and the fine corresponding to the error committed by the inspector for each piece then the target follower writes as follows:

Then the cost of the inspector is calculated from j the hourly rank through the following relationship:

$$C_j = G_j + M_j R_j \left(\frac{100 - D_j}{100}\right) \quad ; \quad j = 1,2,---,n$$

From that we get

$$C_1 = 4 + \{15,16\} \times 2 \times \left(\frac{100 - 95}{100}\right) = \{5.5,5.6\}$$

$$C_2 = 3 + \{10,11\} \times 2 \times \left(\frac{100 - 90}{100}\right) = \{5,5.2\}$$

$$C_3 = 5 + \{25,26\} \times 2 \times \left(\frac{100 - 98}{100}\right) = \{6,6.04\}$$

The total costs for all inspectors assigned to the task of quality control per hour shall be given by the following relationship:

$$TC_j = \sum_{j=1}^{n} \left[G_j + M_j R_j \left(\frac{100 - D_j}{100}\right)\right] x_j$$

$$TC_j = \{5.5,5.6\}x_1 + \{5,5.2\}x_2 + \{6,6.04\}x_3$$

substituting the following target phrase:

$$Z = S \sum_{j=1}^{n} \left[G_j + M_j R_j \left(\frac{100 - D_j}{100}\right)\right] x_j$$

We get:

$$Z = \{44,44.8\}x_1 + \{40,41,6\}x_2 + \{48,48.32\}x_3$$

From the above, we can develop the following mathematical model:
We want to find:

$$MinZ = \{44,44.8\}x_1 + \{40,41,6\}x_2 + \{48,48.32\}x_3$$

Within Restrictions

$$x_1 \leq 10$$
$$x_2 \leq 6$$
$$x_3 \leq 8$$
$$8\{15,16\}x_1 + 8\{10,11\}x_2 + 8\{25,26\}x_3 \geq 1500$$
$$x_j \geq 0 \quad ; \quad j = 1,2,3$$

Example 2:

A company has three ranks for inspectors and wants to assign the task of quality control to them, and 1500 pieces should be checked daily during 8working hours per day, in the following table we explain the full information about inspectors and for all ranks, in this example we will take the accuracy of inspection for each inspector as neutrosophic values in the form of areas whose minimum range is less accurate and the highest range is the highest accuracy that the inspector reaches by rank.

Table 5. Information on inspectors using neutrosophic values.

About the Inspector Inspector rank	Number of pieces checked (hour)	Accuracy (percent)	Inspector's remuneration (monetary unit per hour)	Number of inspectors	The fine paid by the company for each fault to the inspector
1	15	[95,97]	4	10	2
2	10	[90,92]	3	6	2
3	25	[98,99.5]	5	8	2

Required : Formulate the appropriate mathematical model through which we can assign the optimal assignment to the inspectors so that the cost of inspection is as low as possible.

To build the mathematical model, we impose x_1, x_2, x_3 the number of inspectors from the three ranks in the order assigned to the inspection task, then the following inequality must be met:

$$x_1 \leq 10$$
$$x_2 \leq 6$$
$$x_3 \leq 8$$

Since the company needs to audit K pieces daily within S working hour per day, the following set of restrictions must be met:

$$\sum_{j=1}^{n} 8M_j x_j \geq 1500$$

That is

$$8(M_1 x_1 + M_2 x_2 + M_3 x_3) \geq 1500$$

From it, we get the following entry:

$$120x_1 + 80x_2 + 200x_3 \geq 1500$$

To obtain the target function, we note that the company bears two types of costs during the inspection process, the inspector's fee and the fine corresponding to the error committed by the inspector for each piece then the target follower writes as follows:

Then the cost of the inspector is calculated from j the hourly rank through the following relationship:

$$C_j = G_j + M_j R_j \left(\frac{100 - ND_j}{100} \right) \quad ; \quad j = 1,2, ---, n$$

From that we get

$$C_1 = 4 + 15 \times 2 \times \left(\frac{100 - [95,97]}{100} \right) = [4.9,5.5]$$

$$C_2 = 3 + 10 \times 2 \times \left(\frac{100 - [90,92]}{100} \right) = [4.6,5]$$

$$C_3 = 5 + 25 \times 2 \times \left(\frac{100 - [98,99.5]}{100} \right) = [5.25,6]$$

The total costs for all inspectors assigned to the task of quality control per hour shall be given by the following relationship:

$$TC_j = \sum_{j=1}^{n} \left[G_j + M_j R_j \left(\frac{100 - ND_j}{100} \right) \right] x_j$$

$$TC_j = [4.9, 5.5] x_1 + [4.6, 5] x_2 + [5.25, 6] x_3$$

Substituting the following target phrase:

$$Z = S \sum_{j=1}^{n} \left[G_j + M_j R_j \left(\frac{100 - ND_j}{100} \right) \right] x_j$$

We get:

$$Z = [39.2, 44] x_1 + [36.8, 40] x_2 + [42, 48] x_3$$

From the above, we can develop the following mathematical model:

We want to find:

$$MinZ = [39.2, 44] x_1 + [36.8, 40] x_2 + [42, 48] x_3$$

Within Restrictions

$$x_1 \leq 10$$
$$x_2 \leq 6$$
$$x_3 \leq 8$$
$$120x_1 + 80x_2 + 200x_3 \geq 1500$$
$$x_j \geq 0 \; ; \; j = 1,2,3$$

In the two examples, and two for the optimal solution we use the neutrosophic simplex method sufficiently explained in the reference [13].

2. Conclusion and Results

Through the previous study, we note that by using the linear programming method, we can provide the optimal solution to most of the problems that can face the production companies by formulating the situation under treatment with an issue that can be converted into a linear model by solving it using the ideal solution for it, the company achieves the highest profit, and in order to obtain solutions that enjoy a margin of freedom, the concepts of neutrosophic science can be used because the indeterminacy enjoyed by the neutrosophic values can be responsible for managing the company and developing alternative plans that suit all working conditions.

Data availability

The datasets generated during and/or analyzed during the current study are not publicly available due to the privacy-preserving nature of the data but are available from the corresponding author upon reasonable request.

Conflict of interest

The authors declare that there is no conflict of interest in the research.

Ethical approval

This article does not contain any studies with human participants or animals performed by any of the authors.

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Neutrosophic Mathematical Formulas of Transportation Problems

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Abstract.

This paper comes side by side with the complement paper (Original Methods for Obtaining the Initial Solution in Neutrosophic Transportation Problems), these two papers regarded as twins, they are both dedicated to sounding the transportation problems from the perspective of neutrosophic theory, having kinds of indeterminacy in three aspects are:

1- The entries of the payment matrix are neutrosophic values (i.e. $Nc_{ij} = [c_{ij} \pm \varepsilon]$); the indexes i&j have their usual meaning representing the transportation cost of one unit from the production center i to the consumption center j. Assume the indeterminate $\varepsilon = [\lambda_1, \lambda_2]$.

2- The available and the required quantities are both having neutrosophic values represented by $Na_i = a_i \pm \varepsilon_i$, $Nb_j = b_j \pm \delta_j$ respectively, where $\varepsilon_i = [\lambda_{i1}, \lambda_{i2}]$, $\delta_j = [\mu_{j1}, \mu_{j2}]$.

3- This kind of neutrosophic transportation problem is represented gathered from the above two cases.

Keywords: Linear Programming; Neutrosophic Transportation Problem (NTP); Neutrosophic Production Quantities; Neutrosophic Consumption Quantities.

Introduction

The operation research specifically mathematical programming is used in the daily recurrent problems that appear each time when we need to transfer materials from the production centers to the consumption centers.

After the transportation problems have been formulated, the yielded linear models will be solved by simplex method and its modifications. [1-5]. This manuscript contains a modeling study transportation problems using neutrosophic logic that first adopted by F Smarandache (1995) [6-9] is took recently a huge solicitude in effectively addressing the potential uncertainties in the real world,

neutrosophic logic comes as a replacement to the fuzzy logic presented by L. Zadeh (1965) [10], intuitionistic fuzzy logic presented by K. Atanassov (1983) [11].

1. Discussion and the General Formulation of the NTP

There is no doubt of the TP importance in any Inc., because of the   high costs paid by institutions and companies to secure their needs of raw materials or through the marketing of their products or even the process of transferring their administrative and functional members, so it was necessary to present a study that keeps with the frontiers of modern science in which this article studies transportation issues using the neutrosophic logic that takes into account all the changes that can occur during work, and provides companies with a safe working environment.

Assume that one material may be transferred from the production center $A_i, i = 1,2, \dots, m$ to the consumption center $B_j, j = 1,2, \dots, n$,where $a_1, a_2, \dots, a_m$ are available quantities, $b_1, b_2, \dots, b_n$ are required quantities, C_{ij} is the transfer cost of one unit from the production center i to the consumption center j, and are represent the entries of the payment matrix $C = [c_{ij}]$. To construct the mathematical model, x_{ij} denotes the transferred amount of material from the production center i to the consumption center j . The following tableau contains the basic symbols of the any transportation problems, so any later table will be read out of this table:

CC / PC	B_1	B_2	B_3	...	B_n	**AQ**
A_1	c_{11} x_{11}	c_{12} x_{12}	c_{13} x_{13}	...	c_{1n} x_{1n}	a_1
A_2	c_{21} x_{21}	c_{22} x_{22}	c_{23} x_{23}	...	c_{2n} x_{2n}	a_2
A_3	c_{31} x_{31}	c_{32} x_{32}	c_{33} x_{33}	...	c_{3n} x_{3n}	a_3
. .	...	...	...	...	...	. .
A_m	c_{m1} x_{m1}	c_{m2} x_{m2}	c_{m3} x_{m3}	...	c_{mn} x_{mn}	a_m
RQ	b_1	b_2	b_3	...	b_n	

In any TP, there are two cases:

1- Balanced model satisfying $\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j$.

2- Unbalanced model at $\sum_{i=1}^{m} a_i \neq \sum_{j=1}^{n} b_j$.

For the purposes of generalizing any mathematical symbol in the above-mentioned tableau from its classical model to its neutrosophic meaning, it is enough to add a prefix N to indicate that this symbol will hold some indeterminacy as follow:

a. The matrix of transferring cost of one unit from the production center i to the consumption center j in its neutrosophic value is $Nc_{ij} = [c_{ij} \pm \varepsilon]$, where $\varepsilon = [\lambda_1, \lambda_2]$ represents the indeterminate value.

b. The symbol Nx_{ij} refers to the materials' amount that transported from the production center i to the consumption center j, so the matrix of the unknowns is written as $NX = [Nx_{ij}]$.

c. The neutrosophic meaning of the available quantities and the required quantities are $Na_i = a_i \pm \varepsilon_i$, $Nb_j = b_j \pm \delta_j$ respectively, where $\varepsilon_i = [\lambda_{i1}, \lambda_{i2}]$, $\delta_j = [\mu_{j1}, \mu_{j2}]$.

d. The neutrosophic encoding of the objective function in the linear programming is $NZ = \sum_{i=1}^{m} \sum_{j=1}^{n} Nc_{ij} \, x_{ij}$, or $NZ = \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij} \, Nx_{ij}$, or $NZ = \sum_{i=1}^{m} \sum_{j=1}^{n} Nc_{ij} \, Nx_{ij}$.

So, in this article, the authors assumed the representation of neutrosophic numbers as intervals such as $\boldsymbol{\varepsilon_i = [\lambda_{i1}, \lambda_{i2}]}$, $\boldsymbol{\delta_j = [\mu_{j1}, \mu_{j2}]}$. It is important to notice that the authors did not adopt (trapezoidal numbers, pentagonal numbers, or any other neutrosophic numbers which need to specify using the membership functions, this kind of neutrosophic numbers or parameters represented by intervals have been firstly introduced by Smarandache F. in his main published books [12-14]),

2. Types of Unbalanced Neutrosophic Transportation Problems

Without loss of generality, any solver can faces unbalanced NTP (i.e. $\sum_{i=1}^{m} Na_i \neq \sum_{j=1}^{n} Nb_j$) in which two types of problems can be extracted:

1. Overproduction problems occur when $\sum_{i=1}^{m} Na_i > \sum_{j=1}^{n} Nb_j$. To treat this case, the solver should balance this problem by adding an artificial consumption center B_{n+1} has need of

$Nb_{n+1} = \sum_{i=1}^{m} Na_i - \sum_{j=1}^{n} Nb_j$, where the transformation cost of one unit from the all

production centers to this artificial consumption center equal to zero (i.e. $c_{i\,n+1} = 0; i = 1,2, \dots, m$), also, in the data table, the solver should add a new column concerning the new

consumption center. The conditions of this linear programming problems that should be

satisfied are:

$$\sum_{j=1}^{n+1} Nx_{ij} = Na_i \quad ; \quad \sum_{i=1}^{m} Nx_{ij} = Nb_j \; ; \; Nx_{ij} \geq 0 \; ; \; j = 1,2, \dots n+1 \, ; i = 1,2, \dots, m$$

2. production deficient case (type of problems having production shortfall)   occur when

    $\sum_{i=1}^{m} Na_i < \sum_{j=1}^{n} Nb_j$. The same above strategy of balancing the problem has been applied by

    adding an artificial production center A_{m+1} has production power of $Na_{m+1} = \sum_{j=1}^{n} Nb_j - \sum_{i=1}^{m} Na_i$, where the transformation cost of one unit from this artificial production center to

    all consumption centers equal to zero (i.e. $c_{m+1\,j} = 0; j = 1,2, \dots, n$), also, in the data table,

    the solver should add a new row concerning the new production center. The conditions of

    this linear programming problems that should be satisfied are:

    $$\sum_{j=1}^{n} Nx_{ij} = Na_i \quad ; \quad \sum_{i=1}^{m+1} Nx_{ij} = Nb_j \; ; \; Nx_{ij} \geq 0 \; ; \; j = 1,2, \dots, n \, ; i = 1,2, \dots, m+1$$

3. Miscellaneous NT Problems

In the upcoming subsections (3.1,3.2,3.3), the problem text will be: A quantity of fuel is intended to

be shipped from three stations A_1, A_2, A_3 to four cities B_1, B_2, B_3, B_4. The available quantities at each

station, and the demand quantities in each city, with the transportation costs in each direction are

demonstrated in accompanied tables;

3.1 Balanced Neutrosophic Transportation Problem (NTP)

PC \ CC	B_1	B_2	B_3	B_4	**AQ**
A_1	$7 + \varepsilon$ Nx_{11}	$4 + \varepsilon$ Nx_{12}	$15 + \varepsilon$ Nx_{13}	$9 + \varepsilon$ Nx_{14}	$120 + \varepsilon_1$
A_2	$11 + \varepsilon$ Nx_{21}	$2 + \varepsilon$ Nx_{22}	$7 + \varepsilon$ Nx_{23}	$3 + \varepsilon$ Nx_{24}	$80 + \varepsilon_2$

A_3	$4 + \varepsilon$ Nx_{31}	$5 + \varepsilon$ Nx_{32}	$2 + \varepsilon$ Nx_{33}	$8 + \varepsilon$ Nx_{34}	$100 + \varepsilon_3$
RQ	$85 + \delta_1$	$65 + \delta_2$	$90 + \delta_3$	$60 + \delta_4$	

By assuming $\varepsilon = [0,2], \varepsilon_1 = [0,35], \varepsilon_2 = [0,10], \varepsilon_3 = [0,15], \delta_1 = [0,7], \delta_2 = [0,18], \delta_3 = [0,25], \delta_4 = [0,10]$, the above table can be rewritten as:

CC PC	B_1	B_2	B_3	B_4	**AQ**
A_1	[7,9] Nx_{11}	[4,6] Nx_{12}	[15,17] Nx_{13}	[9,11] Nx_{14}	[120,155]
A_2	[11,13] Nx_{21}	[2,4] Nx_{22}	[7,9] Nx_{23}	[3,5] Nx_{24}	[80,90]
A_3	[4,6] Nx_{31}	[5,7] Nx_{32}	[2,4] Nx_{33}	[8,10] Nx_{34}	[100,115]
RQ	[85,92]	[65,83]	[90,115]	[60,70]	

Obviously, the problem is balanced cause $\sum_{i=1}^{3} Na_i = \sum_{j=1}^{4} Nb_j = [300,360]$

The model of the linear programming is

$\text{Min } NZ = [7,9]Nx_{11} + [4,6]Nx_{12} + [15,17]Nx_{13} + [9,11]Nx_{14} + [11,13]Nx_{21} + [2,4]Nx_{22} + [7,9]Nx_{23}$

$+ [3,5]Nx_{24} + [4,6]Nx_{31} + [5,7]Nx_{32} + [2,4]Nx_{33} + [8,10]Nx_{34}$

Subject to

$\sum_{j=1}^{4} Nx_{ij} = a_i \quad ; \quad \sum_{i=1}^{3} Nx_{ij} = b_j \; ; \; Nx_{ij} \geq 0 \; ; \; j = 1,2,3,4 \; ; \; i = 1,2,3$

3.2 Unbalanced Overproduction NTP Case Study

CC PC	B_1	B_2	B_3	B_4	**AQ**
A_1	$7 + \varepsilon$ Nx_{11}	$4 + \varepsilon$ Nx_{12}	$15 + \varepsilon$ Nx_{13}	$9 + \varepsilon$ Nx_{14}	$120 + \varepsilon_1$

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A_2	$11 + \varepsilon$	$2 + \varepsilon$	$7 + \varepsilon$	$3 + \varepsilon$	$95 + \varepsilon_2$
	Nx_{21}	Nx_{22}	Nx_{23}	Nx_{24}	
A_3	$4 + \varepsilon$	$5 + \varepsilon$	$2 + \varepsilon$	$8 + \varepsilon$	$100 + \varepsilon_3$
	Nx_{31}	Nx_{32}	Nx_{33}	Nx_{34}	
RQ	$85 + \delta_1$	$65 + \delta_2$	$90 + \delta_3$	$60 + \delta_4$	

By assuming $\varepsilon = [0,2], \varepsilon_1 = [0,35], \varepsilon_2 = [0,10], \varepsilon_3 = [0,15], \delta_1 = [0,7], \delta_2 = [0,18], \delta_3 = [0,25], \delta_4 = [0,10]$, the above table can be rewritten as:

CC / PC	B_1	B_2	B_3	B_4	**AQ**
A_1	[7,9]	[4,6]	[15,17]	[9,11]	[120,155]
	Nx_{11}	Nx_{12}	Nx_{13}	Nx_{14}	
A_2	[11,13]	[2,4]	[7,9]	[3,5]	[95,105]
	Nx_{21}	Nx_{22}	Nx_{23}	Nx_{24}	
A_3	[4,6]	[5,7]	[2,4]	[8,10]	[100,115]
	Nx_{31}	Nx_{32}	Nx_{33}	Nx_{34}	
RQ	[85,92]	[65,83]	[90,115]	[60,70]	

It is obvious that $\sum_{i=1}^{3} Na_i = [315,375] > \sum_{j=1}^{4} Nb_j = [300,360]$; this lead to add an artificial consumption center b_5 has need of the value $b_5 = \sum_{i=1}^{m} a_i - \sum_{j=1}^{n} b_j = [315,375] - [300,360] = [15,15] = 15$

Hence, the value of the objective function is

$\text{Min } NZ = [7,9]Nx_{11} + [4,6]Nx_{12} + [15,17]Nx_{13} + [9,11]Nx_{14} + 0.Nx_{15} + [11,13]Nx_{21} + [2,4]Nx_{22}$

$\qquad + [7,9]Nx_{23} + [3,5]Nx_{24} + 0.Nx_{25} + [4,6]Nx_{31} + [5,7]Nx_{32} + [2,4]Nx_{33} + [8,10]Nx_{34}$

$\qquad + 0.Nx_{35}$

Subject to

$\qquad \sum_{j=1}^{5} Nx_{ij} = a_i \quad ; \quad \sum_{i=1}^{3} Nx_{ij} = b_j \ ; \ Nx_{ij} \geq 0 \ ; \ j = 1,2,3,4,5 \ ; i = 1,2,3$

3.3 Unbalanced Production Deficient NTP Case Study

CC PC	B_1	B_2	B_3	B_4	AQ
A_1	$7 + \varepsilon$ Nx_{11}	$4 + \varepsilon$ Nx_{12}	$15 + \varepsilon$ Nx_{13}	$9 + \varepsilon$ Nx_{14}	$120 + \varepsilon_1$
A_2	$11 + \varepsilon$ Nx_{21}	$2 + \varepsilon$ Nx_{22}	$7 + \varepsilon$ Nx_{23}	$3 + \varepsilon$ Nx_{24}	$80 + \varepsilon_2$
A_3	$4 + \varepsilon$ Nx_{31}	$5 + \varepsilon$ Nx_{32}	$2 + \varepsilon$ Nx_{33}	$8 + \varepsilon$ Nx_{34}	$100 + \varepsilon_3$
RQ	$85 + \delta_1$	$100 + \delta_2$	$90 + \delta_3$	$60 + \delta_4$	

Where, $\varepsilon = [0,2], \varepsilon_1 = [0,35], \varepsilon_2 = [0,10], \varepsilon_3 = [0,15], \delta_1 = [0,7], \delta_2 = [0,18], \delta_3 = [0,25], \delta_4 = [0,10]$,

the above table can be rewritten as:

CC PC	B_1	B_2	B_3	B_4	AQ
A_1	$[7,9]$ Nx_{11}	$[4,6]$ Nx_{12}	$[15,17]$ Nx_{13}	$[9,11]$ Nx_{14}	$[120,155]$
A_2	$[11,13]$ Nx_{21}	$[2,4]$ Nx_{22}	$[7,9]$ Nx_{23}	$[3,5]$ Nx_{24}	$[80,90]$
A_3	$[4,6]$ Nx_{31}	$[5,7]$ Nx_{32}	$[2,4]$ Nx_{33}	$[8,10]$ Nx_{34}	$[100,115]$
RQ	$[85,92]$	$[100,118]$	$[90,115]$	$[60,70]$	

It is worthy to mention that $\sum_{i=1}^3 Na_i = [300,360] < \sum_{j=1}^4 Nb_j = [335,395]$; this lead to add an

artificial production center a_4 has production power equal to $a_4 = \sum_{j=1}^4 b_j - \sum_{i=1}^3 a_i = [315,395] -$

$[300,360] = [35,35] = 35$

Hence, the model of the linear programming is

Min $NZ = [7,9]Nx_{11} + [4,6]Nx_{12} + [15,17]Nx_{13} + [9,11]Nx_{14} + [11,13]Nx_{21} + [2,4]Nx_{22} + [7,9]Nx_{23}$

$+ [3,5]Nx_{24} + [4,6]Nx_{31} + [5,7]Nx_{32} + [2,4]Nx_{33} + [8,10]Nx_{34} + 0.Nx_{41} + 0.Nx_{42}$

$+ 0.Nx_{43} + 0.Nx_{44}$

Subject to

$\sum_{j=1}^{4} Nx_{ij} = a_i \quad ; \quad \sum_{i=1}^{4} Nx_{ij} = b_j \; ; \; Nx_{ij} \geq 0 \; ; \; j = 1,2,3,4 \; ; i = 1,2,3,4$

4. Conclusion and Results

This manuscript presented new insights into remodeling transportation problems from classical models to the corresponding neutrosophic ones through three basic aspects containing some indeterminacies either in transformation costs, or in the available quantities existed in production centers side by side with required quantities existing in consumption centers, or in the general case study that represents the existence of indeterminacies in all previous states. The solutions to these neutrosophic transportation problems NTP give the solver a margin of freedom and reduce the loss resulting from transporting process through the availability of the indetermination notion in neutrosophic theory. This study was presented simultaneously with the paper entitled (Original Methods for Obtaining the Initial Solution in Neutrosophic Transportation Problems) regarded as complementary to this recent paper in which the authors discuss the initial solution for the NTP models and study the modifications of this initial solution aiming to get the optimal solutions.

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An Investigation in the Initial Solution for Neutrosophic Transportation Problems (NTP)

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Abstract.

Transportation issues arise often in everyday life. To ensure that the regions' needs for transported material are met at the lowest feasible cost, materials must be carried from production centers to consuming centers as quickly as possible. Operations research approaches, notably mathematical programming, are utilized to solve these recurring and daily challenges. The problem's data is transformed into a mathematical model, and then the best solution is discovered using the proper procedures. When dealing with transportation issues, we arrive at a linear mathematical model, which can be solved using the direct simplex method and its modifications. However, because of the clarity and specificity of the transportation model, scholars and researchers were able to find other methods that were easier than the simplex method.

Whatever method is used, the goal is to determine the number of units transferred for any material from the production centers to the consumption centers in order to minimize transportation costs, keeping in mind that each export center has its own capacity and cannot supply quantities of the material greater than that capacity. Furthermore, each import center has a certain requirement for which it makes a request and for which it is unable to consume further quantities. In this manuscript, the researchers will use the North-West Corner approach, the Least-Cost method, and Vogel's approximation method to discover an initial solution to the balanced neutrosophic transport problems.

The term "neutrosophic transportation problems (i.e. NTP)" refers to the transportation problems in which the required and available quantities have neutrosophic values of the form $Na_i =$

$a_i + \varepsilon_i$, where ε_i is the indeterminacy in the produced quantities, and it is either $\varepsilon_i = [\lambda_{i1}, \lambda_{i2}]$ or

$\varepsilon_i = \{\lambda_{i1}, \lambda_{i2}\}$. While the required quantities are also neutrosophic values of the form $Nb_j = b_j + \delta_j$, here

δ_j is the indeterminacy on the required quantity, and it is either $\delta_j = [\mu_{j1}, \mu_{j2}]$ or $\delta_j = \{\mu_{j1}, \mu_{j2}\}$. It is

worthy to mention that when the problem is unbalanced (i.e. the summation of the required

quantities is not equal to the produced quantities), then firstly will convert the problem to a balanced

one.

In this article, the authors assumed the representation of the neutrosophic numbers as intervals

such as $\varepsilon_i = [\lambda_{i1}, \lambda_{i2}]$, $\delta_j = [\mu_{j1}, \mu_{j2}]$. It is important to notice that the authors did not adopt

(trapezoidal numbers, pentagonal numbers, or any other neutrosophic numbers which need to

specify using the membership functions, this kind of neutrosophic numbers or parameters

represented by intervals have been firstly introduced by Smarandache F. in his main published books

[32,33].

The sections of this manuscript has been organized as follow: the introduction is the inception

of this article, section one has been dedicated to the north-west corner method containing three

subsections for three case studies depending upon the existence of the indeterminacy in the problem,

the least- cost method represents section two, while section three has been devoted to Vogel's

approximation method, section four is conclusion and results.

Keywords: Transportation Problem; Neutrosophic Transportation Problem (NTP); Initial Solution;
Consumption Centers (CC); Production Centers (PC); Available Quantities (AQ); Required
Quantities (RQ); North-West Corner Method; Least-Cost Method; Vogel's Method.

Introduction

Transportation problems are among the most prevalent linear programming problems encountered

in everyday life. These problems study the transfer of materials from production centers to

consumption centers in the shortest period of time or at the lowest cost, or the distribution of

transportation modes (such as buses, planes and ships etc.) on the imposed transportation lines in

which the requests can meet by the least cost, and since the mathematical models we obtain are linear

models, so the simplex method and its modifications can be used to obtain the optimal solution, but

the special nature of the transportation problems enabled scientists to find special ways to solve these

models that dependent on finding initial solution and then using other ways to improvement this initial solution using heuristics algorithms to find optimal solution [1-5], there are previous studies for transport models at the lowest cost using neutrosophic environments, which is the new vision of modelling and is designed to effectively address the uncertainties inherent in the real world, as it came to replace the binary logic that define merely the truthiness status and falseness status, by introducing a third, neutral state which can be interpreted as undetermined or uncertain .

This neutrosophic logic has been established in 1995 by the philosopher and mathematician Florentin Smarandache [7,9,10,11,13] introduced as a generalization to both: fuzzy logic presented by L. Zadeh in 1965 [6], and intuitionistic fuzzy logic introduced by K. Atanassov in 1983 [8]. In addition, A. A. Salama presented the theory of classical neutrosophic sets as a generalization of the classical sets. Theory [12,20], he developed, introduced and formulated new concepts in the fields of topology, statistics, computer science... etc. through neutrosophic theory [15,17-19, 22,28,29].

The neutrosophic theory has grown significantly in recent years through its application in measurement theory, group theory, graph theory and many scientific and practical fields [8,14-16, 21, 23-27,30-35], this research sheds the light on the modified that same methods used to find the initial solution for the classic transport problems in finding a preliminary solution to the neutrosophic transport problems in its three forms, the first form is that form when the cost is neutrosophic values, while the second form occurs when the demanded quantities and the supplied quantities are neutrosophic values, finally, the third form occurs when the cost of transport and the demanded quantities and the supplied quantities are all neutrosophic values.

Discussion:

It is popular that there are several ways to find a basic (initial) solution to the transfer problem, given the condition that the number of the basic variables in this initial solution must equal the number of the linear conditions (i.e. m+n-1). An initial solution can be found in several ways. In this research, we will use three methods, the North-West corner method, the least-cost method, and Vogel's approximate method, to find an initial solution to the neutrosophic transport model, and we will study the three forms in each of the methods. [1-5].

1. The North-West Corner Method

The North-West Corner Rule is a technique for calculating an initial basic feasible solution to a transportation problem. The method is called North-West Corner because the basic variables are chosen from the extreme left corner. the following three steps gives the initial basic feasible solution:

1. Find the north west corner cell of the transportation tableau. Allocate as much as possible to the selected cell, and adjust the associated amounts of supply and demand by subtracting the allocated amount.

2. Cross out the row or column with 0 supply or demand. If both a row and a column have a 0, cross out randomly row or column.

3. If one cell is left uncrossed, cross out the cell and stop. Otherwise, go to step 1.

We should not forget that the final allocated cells (nonzero cells) must equal to the value (m+n-1), where m determine how many production center that the problem have, while n refers to how many consumption center exist.

1.1 First Case Study for Neutrosophic Transportation Problem in which the Indeterminacy is in Nc_{ij}

In this case the cost of transportation will be neutrosophic values, that's mean the monetary value of transfer one unit from the production center i to the consumption center j is $Nc_{ij} = c_{ij} \pm \varepsilon$, where ε is the indeterminate value and equal to $\varepsilon = [\lambda_1, \lambda_2]$, so the payment matrix will be $Nc_{ij} = \lfloor c_{ij} \pm \varepsilon \rfloor$. Although all transportation costs have been given the same indeterminate value, it is feasible to assign a different indeterminate to each cost and use the same case study.
The Text of the Problem
A certain amount of oil is to be transported from three stations A_1, A_2, A_3 to four cities B_1, B_2, B_3, B_4. The following table shows the quantities available at each station, the demand quantities in each city, and the transportation costs in each direction:

CC \ PC	B_1	B_2	B_3	B_4	AQ
A_1	$7 + \varepsilon$ x_{11}	$4 + \varepsilon$ x_{12}	$15 + \varepsilon$ x_{13}	$9 + \varepsilon$ x_{14}	120
A_2	$11 + \varepsilon$ x_{21}	$0 + \varepsilon$ x_{22}	$7 + \varepsilon$ x_{23}	$3 + \varepsilon$ x_{24}	80
A_3	$4 + \varepsilon$ x_{31}	$5 + \varepsilon$ x_{32}	$2 + \varepsilon$ x_{33}	$8 + \varepsilon$ x_{34}	100
RQ	85	65	90	60	

In this example, the indeterminate value of $\varepsilon = [0,2]$, based on the problem's data we have, $\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j = 300$, this signifies that the issue is balanced. Substituting $\varepsilon = [0,2]$ into the preceding tableau yielded the following one:

CC PC	B_1	B_2	B_3	B_4	AQ
A_1	[7,9] x_{11}	[4,6] x_{12}	[15,17] x_{13}	[9,11] x_{14}	120
A_2	[11,13] x_{21}	[0,2] x_{22}	[7,9] x_{23}	[3,5] x_{24}	80
A_3	[4,6] x_{31}	[5,7] x_{32}	[2,4] x_{33}	[8,10] x_{34}	100
RQ	85	65	90	60	300 300

The entire calculations to find the initial solution using the North-West method have been summarized as follow:

Start with the cell located in the north-west corner of the table, i.e. that cell corresponds to the first production center crossing with the first consumption center, this cell will carry the value

$Min \{85,93\} = 85$, hence the first consumption center B_1 has the need been fulfilling of its requirement from the first production center A_1, the remaining amount in A_1 is $120 - 85 = 35$. Move to the right cell positioned in the crossing of first row with second column and put in it the value $Min \{65,35\} = 35$, so the available quantity in A_1, and the required quantity in B_1 both are being zero, but the second consumption center B_2 requires to $65 - 35 = 30$. Go down to the cell of position in the cross of second row with second column (x_{22}) and put the value $Min \{30,80\} = 30$, consequently, the second consumption center B_2 has its need been fulfilling, and the remaining value in the second production center A_2 is $80 - 30 = 50$, keep going in the same above technique till all production centers are emptied, and all consumption centers have been fulfilling, finally the following table was gotten:

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PC \ CC	B_1	B_2	B_3	B_4	AQ
A_1	[7,9] 85	[4,6] 35	[15,17]	[9,11]	120
A_2	[11,13]	[0,2] 30	[7,9] 50	[3,5]	80
A_3	[4,6]	[5,7]	[2,4] 40	[8,10] 60	100
RQ	85	65	90	60	300 \ 300

The last table illustrates: $x_{11} = 85, x_{12} = 35, x_{22} = 30, x_{23} = 50, x_{33} = 40, x_{34} = 60, x_{13} = x_{14} = x_{21} = x_{24} = x_{31} = x_{32} = 0$, we have $n = 4, m = 3, m + n - 1 = 6$, meaning that the initial conduction satisfied the necessary condition. Calculate the total cost for this initial solution by substitution the $x's$ values in the cost function:

$$NC = c_{11}x_{11} + c_{12}x_{12} + c_{13}x_{13} + c_{14}x_{14} + c_{21}x_{21} + c_{22}x_{22} + c_{23}x_{23} + c_{24}x_{24} + c_{31}x_{31} + c_{32}x_{32} + c_{33}x_{33}$$
$$+ c_{34}x_{34}$$

$$NC = [7,9] * 85 + [4,6] * 35 + [15,17] * 0 + [9,11] * 0 + [11,13] * 0 + [0,2] * 30 + [7,9] * 50 + [3,5] * 0$$
$$+ [4,6] * 0 + [5,7] * 0 + [2,4] * 40 + [8,10] * 60 = [1645,2245]$$

Which is the cost versus initial solution.

1.2 Second Case Study in which the indeterminacy is in both production center and consumption center

Problem Text

A quantity of fuel is intended to be shipped from three stations to four cities. The available quantities at each station, the demand quantities in each city, and the transportation costs in each direction are demonstrated in the following table:

CC / PC	$B1$	$B2$	$B3$	$B4$	AQ
$A1$	7	4	15	9	$120 + \varepsilon1$
$A2$	11	0	7	3	$80 + \varepsilon2$
$A3$	4	5	2	8	$100 + \varepsilon3$
RQ	$85 + \delta1$	$65 + \delta2$	$90 + \delta3$	$60 + \delta4$	

It is worthy to mention that $\varepsilon_1, \varepsilon_2, \varepsilon_3$ represent the indeterminacies that exist in available quantitiesin the fuel stations, and it can be took as intervals $[\lambda_{i1}, \lambda_{i2}]$ or as sets $\{\lambda_{i1}, \lambda_{i2}\}$ …etc.

For this case study, the following values have been picked: $\varepsilon_1 = [0,11], \varepsilon_2 = [0,9], \varepsilon_3 = [0,15]$

While the values $\delta_1, \delta_2, \delta_3, \delta_4$ are the indeterminacies in the required quantities in the four cities,also these neutrosophic values can be regarded as intervals $[\mu_{i1}, \mu_{i2}]$ or as sets $\{\mu_{i1}, \mu_{i2}\}$ …etc.

For this example, the following neutrosophic values have been took: $\delta_1 = [0,8], \delta_2 = [0,12], \delta_3 =[0,9], \delta_4 = [0,6]$. So the above table becomes:

CC / PC	$B1$	$B2$	$B3$	$B4$	A
$A1$	7	4	15	9	[120,131]
$A2$	11	0	7	3	[80,89]
$A3$	4	5	2	8	[100,115]
RQ	[85,93]	[65,77]	[90,99]	[60,66]	[300,335] / [300,335]

It is clear that the problem is balanced as $\sum_{i=1}^{3} Na_i = \sum_{j=1}^{4} Nb_j = [300,335]$.

Start with the cell located in the north-west corner of the table, i.e. that cell corresponds to the first production center crossing with the first consumption center, this cell will carry the value

Min {[85,93], [120,131]} = [85,93], hence the first consumption center $B1$ has the need been fulfilling of its requirement from the first production center $A1$, the remaining amount in $A1$ is [120,131] − [85,93] = [35,38]. Move to the right cell positioned in the crossing of first row with second column and put in it the value Min {[65,77], [35,38]} = [35,38], so the available quantity in $A1$, and the required quantity in $B1$ both are being zero, but the second consumption center $B2$ requires to [65,77] − [35,38] = [30,39]. Go down to the cell of position in the cross of second row with second column ($x22$) and put the value Min {[30,39], [80,89]} = [30,39], consequently, the second consumption center $B2$ has its need been fulfilling, and the remaining value in the second production center $A2$ is [80,89] − [30,39] = [50,50], keep going in the same above technique till all production centers are emptied, and all consumption centers have been fulfilling, finally the following table was gotten:

This table contains the following neutrosophic values:

$Nx_{11} = [85,93], Nx_{12} = [35,38], Nx_{22} = [30,39], Nx_{23} = [50,50], Nx_{33} = [40,49], Nx_{34} = [60,66], Nx_{13} = Nx_{14}$
$=$

PC \ CC	B_1	B_2	B_3	B_4	**AQ**
A_1	7 [85,93]	4 [35,38]	15 0	9 0	[120,131]
A_2	11 0	0 [30,39]	7 [50,50]	3 0	[80,89]
A_3	4 0	5 0	2 [40,49]	8 [60,66]	[100,115]
RQ	[85,93]	[65,77]	[90,99]	[60,66]	[300,335] [300,335]

$Nx_{21} = Nx_{24} = Nx_{31} = Nx_{32} = 0$. In this problem similar to the previous problem in section (1.1), $n = 4, m = 3 \Rightarrow m + n - 1 = 6$, meaning that the initial condition satisfied the necessary condition. Calculate the total cost for this initial solution by substitution the x's values in the cost function:

$NC = c_{11}x_{11} + c_{12}x_{12} + c_{13}x_{13} + c_{14}x_{14} + c_{21}x_{21} + c_{22}x_{22} + c_{23}x_{23} + c_{24}x_{24} + c_{31}x_{31} + c_{32}x_{32} + c_{33}x_{33} + c_{34}x_{34}$

$NC = 7 * [85,93] + 4 * [35,38] + 15 * 0 + 9 * 0 + 11 * 0 + 0 * [30,39] + 7 * [50,50] + 3 * 0 + 4 * 0 + 5* 0 + 2 * [40,49] + 8 * [60,66] = [1645,1779]$

Which is the cost versus to the initial solution.

1.3 Third Case Study in Which the Transportation Cost, the Available Quantities in the Production Centers, and the Demand Quantities in the Consumption Centers are all Neutrosophic Values

By taking the same context of the studied cases in the previous subsections (1.1, 1.2) subject to the following table has been considered as new example:

PC \ CC	B_1	B_2	B_3	B_4	**A**
A_1	$7 + \varepsilon$ Nx_{11}	$4 + \varepsilon$ Nx_{12}	$15 + \varepsilon$ Nx_{13}	$9 + \varepsilon$ Nx_{14}	$120 + \varepsilon_1$
A_2	$11 + \varepsilon$ Nx_{21}	$0 + \varepsilon$ Nx_{22}	$7 + \varepsilon$ Nx_{23}	$3 + \varepsilon$ Nx_{24}	$80 + \varepsilon_2$
A_3	$4 + \varepsilon$ Nx_{31}	$5 + \varepsilon$ Nx_{32}	$2 + \varepsilon$ Nx_{33}	$8 + \varepsilon$ Nx_{34}	$100 + \varepsilon_3$
RQ	$85 + \delta_1$	$65 + \delta_2$	$90 + \delta_3$	$60 + \delta_4$	

By assuming $\varepsilon = [0,2], \varepsilon_1 = [0,11], \varepsilon_2 = [0,9], \varepsilon_3 = [0,15], \delta_1 = [0,8], \delta_2 = [0,12], \delta_3 = [0,9], \delta_4 =$

[0,6], the above table can be rewritten as:

PC \ CC	B_1	B_2	B_3	B_4	AQ
A_1	[7,9]	[4,6]	[15,17]	[9,11]	[120,131]
A_2	[11,13]	[0,2]	[7,9]	[3,5]	[80,89]
A_3	[4,6]	[5,7]	[2,4]	[8,10]	[100,115]
RQ	[85,93]	[65,77]	[90,99]	[60,66]	[300,335] [300,335]

Obviously, the problem is balanced because $\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j = [300,335]$. The same north-west strategy has been implemented to get the following table:

PC \ CC	B_1	B_2	B_3	B_4	AQ
A_1	[7,9] [85,93]	[4,6] [35,38]	[15,17] 0	[9,11] 0	[120,131]
A_2	[11,13] 0	[0,2] [30,39]	[7,9] [50,50]	[3,5] 0	[80,89]
A_3	[4,6] 0	[5,7] 0	[2,4] [40,49]	[8,10] [60,66]	[100,115]
RQ	[85,93]	[65,77]	[90,99]	[60,66]	[300,335] [300,335]

Same as the previous examples we have, $Nx_{11} = [85,93], Nx_{12} = [35,38], Nx_{22} = [30,39], Nx_{23} = [50,50], Nx_{33} = [40,49], Nx_{34} = [60,66], Nx_{13} = Nx_{14} = Nx_{21} = Nx_{24} = Nx_{31} = Nx_{32} = 0$. In this problem similar to the previous problems (1.1 &1.2), $n = 4$, $m = 3 \Rightarrow m + n - 1 = 6$, meaning that the initial condition satisfied the necessary condition. Calculate the total cost for this initial solution by substitution the x's values in the cost function:

$$NC = c_{11}x_{11} + c_{12}x_{12} + c_{13}x_{13} + c_{14}x_{14} + c_{21}x_{21} + c_{22}x_{22} + c_{23}x_{23} + c_{24}x_{24} + c_{31}x_{31} + c_{32}x_{32} + c_{33}x_{33} + c_{34}x_{34}$$

NC = [7,9] $*$ [85,93] + [4,6] $*$ [35,38] + [9,11] $*$ 0 + [7,9] $*$ 0 + [11,13] $*$ 0 + [0,2] $*$ [30,39] + [7,9]$*$ [50,50] + [4,6] $*$ 0 + [5,7] $*$ 0 + [2,4] $*$ [40,49] + [8,10] $*$ [60,66] = [1645,2449]

Which is the cost versus to the initial solution.

2. The Least- Cost Method

The least cost is another method used to obtain the initial feasible solution for the transportation problem. Here, the allocation begins with the cell which has the minimum cost. The lower cost cells are chosen over the higher-cost cell with the objective to have the least cost of transportation. The Least Cost Method is considered to produce more optimal results than the north-west corner because it considers the shipping cost while making the allocation, whereas the North-West corner method only considers the availability and supply requirement and allocation begin with the extreme left corner, irrespective of the shipping cost [1-5]. We will discuss the existence types of the indeterminacies either in transportation costs, or the indeterminacy exists in both the available quantity in the production centers and in the demand quantities in the consumption centers, or in all of them by the following case studies:

2.1 Case Study Has the Indeterminacy in its Transportation Cost

The same context of example (1.1) has been resolved using least- cost method, where the least cost cell is [0,2] which is located in the position resulting from the intersection of the second row with the second column, and put the value $min\{65,80\} = 65$ in it. Thus, we have met the need of the second consumption center B_2 from the second production center A_2, and the remaining quantity in A_2 is $80 - 65 = 15$. Move to the next least cost value among the remaining costs is [2,4], which is located in the cell resulting from the intersection of the third row with the third column, and we put the value $min\{90,100\} = 90$ in it. Thus, we have met the need of the third consumption center B_3 from the third productive center A_3, and the remaining value in the third productive center is $100 - 90 = 10$. Continuing by the same strategy until all consumption centers have been saturated and all production centers have been emptied. Consequently, the following table yield:

CC / PC	B_1	B_2	B_3	B_4	AQ
A_1	[7,9] 75	[4,6]	[15,17]	[9,11] 45	120
A_2	[11,13]	[0,2] 65	[7,9]	[3,5] 15	80
A_3	[4,6] 10	[5,7]	[2,4] 90	[8,10]	100
RQ	85	65	90	60	300 / 300

Here, $x_{11} = 75, x_{14} = 45, x_{22} = 65, x_{24} = 15, x_{31} = 10, x_{33} = 90, x_{12} = x_{13} = x_{21} = x_{23} = x_{32} = x_{34} = 0.$

$NL = [7,9] * 75 + [4,6] * 0 + [15,17] * 0 + [9,11] * 45 + [11,13] * 0 + [0,2] * 65 + [7,9] * 0 + [3,5] * 15 + [4,6] * 10 + [5,7] * 0 + [2,4] * 90 + [8,10] * 0 = [1195,1795].$ As usual, it represents the cost versus to the initial solution.

2.2 Case Study in Which the Available Quantities of the Production Centers and the Demanded Quantities of the Consumption Centers are Neutrosophic Values

For the comparison purposes, the same data and problem text that used in the case study (1.2) has been considered here, so the first three tables are the same. By applying the least cost method, it seemsthe cell of the zero value is the required cell which exactly located in the intersection of the second row with the second column, so we put in this cell the value $min \{[65,77], [80,89]\} = [65,77]$, by moving to the next least cost cell that located in the intersection of the third row with the third columnof 2 value, burden this cell with the value $min \{[90,99], [100,115]\} = [90,99]$, go on with the same strategy till all consumption centers saturation at the same time all productions centers have been emptied, hence, the following table yielding:

CC / PC	B_1	B_2	B_3	B_4	AQ
A_1	7 [75,77]	4	15	9 [45,54]	[120,131]
A_2	11	0 [65,77]	7	3 [15,12]	[80,89]
A_3	4 [10,16]	5	2 [90,99]	8	[100,115]
RQ	[85,93]	[65,77]	[90,99]	[60,66]	[300,335] [300,335]

Hence, $Nx_{11} = [75,77], Nx_{14} = [45,54], Nx_{22} = [65,77], Nx_{24} = [15,12], Nx_{31} = [10,16], Nx_{33} = [90,99], Nx_{12} = N_{13} = Nx_{21} = Nx_{23} = Nx_{32} = Nx_{34} = 0$, we should not forget the problem satisfiesthe balancing condition since $m + n - 1 = 6$.

$NL = 7 * [75,77] + 4 * 0 + 15 * 0 + 9 * [45,54] + 11 * 0 + 0 * [65,77] + 7 * 0 + 3 * [15,12] + 4* [10,16] + 5 * 0 + 2 * [90,99] + 8 * 0 = [1259,1323]$

Obviously, it represents the cost versus to the initial solution.

2.3 Case Study in Which the Transportation Cost, the Available Quantities of the Production Centers, and the Demanded Quantities of the Consumption Centers are all Neutrosophic Values

In this section, the text problem and the types of the indeterminacies are same as in the case study ofthe section (1.3), but the values of $\delta_i's$, $\varepsilon_i's$ are assumed to be:

$\varepsilon = [0,2], \varepsilon_1 = [0,35], \varepsilon_2 = [0,10], \varepsilon_3 = [0,15], \delta_1 = [0,7], \delta_2 = [0,18], \delta_3 = [0,25], \delta_4 = [0,10].$

CC \\ PC	B_1	B_2	B_3	B_4	AQ
A_1	[7,9] Nx_{11}	[4,6] Nx_{12}	[15,17] Nx_{13}	[9,11] Nx_{14}	[120,155]
A_2	[11,13] Nx_{21}	[0,2] Nx_{22}	[7,9] Nx_{23}	[3,5] Nx_{24}	[80,90]
A_3	[4,6] Nx_{31}	[5,7] Nx_{32}	[2,4] Nx_{33}	[8,10] Nx_{34}	[100,115]
RQ	[85,92]	[65,83]	[90,115]	[60,70]	[300,360] [300,360]

Again, by using the least cost strategy, the least cost is the cell [0,2] located in the cell intersect the second row with second column, so we will put the value min{65,80} = 65 in it, hence the needy of the second consumption center $B2$ has been met from the second production center A_2, the remaining quantity in A_2 is 80 − 65 = 15 , by moving to the next least cost cell which is [2,4] represents the cell allocated in the intersection of the third row with the third column, burden this cell with the value min{90,100} = 90 . Thus, we have met the need of the third consumption center B_3 from the third productive center A_3, and the remaining value in the third productive center is 100 − 90 = 10. Go on with the same strategy till all consumption centers saturation at the same time all productions centers have been emptied, hence, the following table yielding:

CC \\ PC	B_1	B_2	B_3	B_4	AQ
A_1	[7,9] [75,77]	[4,6]	[15,17]	[9,11] [45,54]	[120,131]
A_2	[11,13]	[0,2] [65,77]	[7,9]	[3,5] [15,12]	[80,89]
A_3	[4,6] [10,16]	[5,7]	[2,4] [90,99]	[8,10]	[100,115]
RQ	[85,93]	[65,77]	[90,99]	[60,66]	[300,335] [300,335]

Hence, $Nx_{11} = [75,77], Nx_{14} = [45,54], Nx_{22} = [65,77], Nx_{24} = [15,12], Nx_{31} = [10,16], Nx_{33} = [90,99], Nx_{12} = Nx_{13} = Nx_{21} = Nx_{23} = Nx_{32} = Nx_{34} = 0$, we should not forget the problem satisfies the balancing condition since $m + n − 1 = 6$. The following cost represents the initial feasible solution.

NL = [7,9] ∗ [75,77] + [4,6] ∗ 0 + [15,17] ∗ 0 + [9,11] ∗ [45,54] + [11,13] ∗ 0 + [0,2] ∗ [65,77] + [7,9] ∗0 + [3,5] ∗ [15,12] + [4,6] ∗ [10,16] + [5,7] ∗ 0 + [2,4] ∗ [90,99] + [8,10] ∗ 0 = [1195,1993].

3- Vogel's Approximation Method :

Definition: The Vogel's Approximation Method or VAM is an iterative procedure calculated to findout the initial feasible solution of the transportation problem. Like Least cost Method, here also the shipping cost is taken into consideration, but in a relative sense. The following is the flow chart showing the steps involved in solving the transportation problem using the Vogel's Approximation method.

```
                        ┌──────────┐
                        │  Start   │
                        └────┬─────┘
                             ▼
     ┌───────────────────────────────────────────────────┐
     │ Find the difference between the two least cost     │
     │ cells which have not been allocated, for both the  │
     │ rows and columns                                   │
     └───────────────────────┬───────────────────────────┘
                             ▼
     ┌───────────────────────────────────────────────────┐
     │ Select the largest of the differences of both the  │
     │ rows and columns and in case there is a tie, choose │
     │ the one where the maximum quantity can be assigned  │
     │ considering the minimum cost as well                │
     └───────────────────────┬───────────────────────────┘
                             ▼
     ┌───────────────────────────────────────────────────┐
     │ Assign the largest quantity within the demand and  │
     │ supply constraint in the row or column where the   │
     │ cost is minimum                                    │
     └───────────────────────┬───────────────────────────┘
                             ▼
     ┌───────────────────────────────────────────────────┐
     │ Assign the largest quantity within the demand and  │
     │ supply constraint in the row or column where the   │
     │ cost is minimum                                    │
     └───────────────────────┬───────────────────────────┘
                             ▼
     ┌───────────────────────────────────────────────────┐
     │ Eliminate the row/ column that has been satisfied  │
     └───────────────────────┬───────────────────────────┘
                             ▼
  NO ┌───────────────────────────────────────────────────┐
     │ Are all the demand and supply requirements met?    │
     └───────────────────────┬───────────────────────────┘
                             ▼ Yes
                        ┌──────────┐
                        │  Stop    │
                        └──────────┘
```

The same cases studies that have discussed in the all previous sections (1.1,1.2,1.3,2.1,2.2,2.3) can be presented here with the same problems texts, with same data values, to be resolved in the Vogel's iterative method, this will give us a good opportunity to analyze the results which enables us to make a good comparison between (North-West Corner method, Least-Cost Method, and Vogel's Method).

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3.1 Case Study Has the Indeterminacy in its Transportation Cost

CC \ PC	B_1	B_2	B_3	B_4	AQ	Δ_1	Δ_2	Δ_3
A_1	[7,9] 75	[4,6] 45	[15,17]	[9,11]	120	3	3	3
A_2	[11,13]	[0,2] 20	[7,9]	[3,5] 60	80	3	3	11
A_3	[4,6] 10	[5,7]	[2,4] 90	[8,10]	100	2	2	1
RQ	85	65	90	60	300 / 300			
Δ'_1	3	4	5	5				
Δ'_2	3	4	5	–				
Δ'_3	3	4	–	–				

The symbols $\Delta'_1, \Delta'_2, \Delta'_3$ mean the subtractions between the columns respectively, while the symbols $\Delta_1, \Delta_2, \Delta_3$ mean the subtractions between the rows respectively.

From the above table, we have $x_{11} = 75, x_{12} = 45, x_{22} = 20, x_{24} = 60, x_{31} = 10, x_{33} = 90, x_{13} = x_{14} = x_{21} = x_{23} = x_{32} = x_{34} = 0$,

$$NL = [7,9] * 75 + [4,6] * 45 + [15,17] * 0 + [9,11] * 0 + [11,13] * 0 + [0,2] * 20 + [7,9] * 0 + [3,5] * 60$$

$$+ [4,6] * 10 + [5,7] * 0 + [2,4] * 90 + [8,10] * 0 = [1105,1705]$$

3.2 Case Study in Which the Available Quantities of the Production Centers and the Demanded Quantities of the Consumption Centers are Neutrosophic Values

CC \ PC	B_1	B_2	B_3	B_4	AQ	Δ_1	Δ_2	Δ_3
A_1	7 [75,77]	4 [45,54]	15	9	[120,131]	3	3	3
A_2	11	0 [20,23]	7	3 [60,66]	[80,89]	3	7	–
A_3	4 [10,16]	5	2 [90,99]	8	[100,115]	2	2	2
RQ	[85,93]	[65,77]	[90,99]	[60,66]	[300,335] [300,335]			
Δ'_1	3	4	5	5				
Δ'_2	3	4	5	–				
Δ'_3	3	1	13	–				

Recall the same notations $Nx_{ij}'s$ with their new values,

$Nx_{11} = [75,77], Nx_{12} = [45,54], Nx_{22} = [20,23], Nx_{24} = [60,66], Nx_{31} = [10,16], \qquad Nx_{33} = [90,99],$

$$Nx_{13} = N_{14} = Nx_{21} = Nx_{23} = Nx_{32} = Nx_{34} = 0$$

The initial feasible solution is:

$$NL = 7 * [75,77] + 4 * [45,54] + 15 * 0 + 9 * 0 + 11 * 0 + 0 * [20,23] + 7 * 0 + 3 * [60,66] + 4$$

$$* [10,16] + 5 * 0 + 2 * [90,99] + 8 * 0 = [1105,1215]$$

3.3 Case Study in Which the Transportation cost, the Available Quantities of the Production Centers, and the Demanded Quantities of the Consumption Centers are all Neutrosophic Values

Recall the same text problem in section (1.3) with respect to resolving it using Vogel's iterative procedure to conclude the following table:

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PC \ CC	B_1	B_2	B_3	B_4	AQ	Δ_1	Δ_2	Δ_3
A_1	[7,9] [75,77]	[4,6] [45,54]	[15,17]	[9,11]	[120,131]	3	3	3
A_2	[11,13]	[0,2] [20,23]	[7,9]	[3,5] [60,66]	[80,89]	3	7	–
A_3	[4,6] [10,16]	[5,7]	[2,4] [90,99]	[8,10]	[100,115]	2	2	2
RQ	[85,93]	[65,77]	[90,99]	[60,66]	[300,335] [300,335]			
Δ'_1	3	4	5	5				
Δ'_2	3	4	5	–				
Δ'_3	3	1	13	–				

So the initial feasible solution is:

$NL = [7,9] * [75,77] + [4,6] * [45,54] + [15,17] * 0 + [9,11] * 0 + [11,13] * 0 + [0,2] * [20,23] + [7,9] * 0 + [3,5] * [60,66] + [4,6] * [10,16] + [5,7] * 0 + [2,4] * [90,99] + [8,10] * 0 = [1105,1885]$

4. Conclusion and Results

This paper sheds the light on a new vision for solving Transportation problems by taking into consideration the existence of indeterminacy in many joints of the problems that have been solve nine times, each time in different method and different aspect of indeterminacy existence. With somedeep insights the reader can notice that the Vogel's iterative procedure yields minimum cost than both the costs that produced by applying North-west method, and Least-Cost method. However, themethods that used is still gives the results regarded as the initial feasible solution not the optimal solution, which mean that these methods are still need to improve to get the optimal solution.

Below table summarizes all previous solutions in comparison strategy:

Types of the problem \ The Method	North-West Method	Least- Cost Method	Vogel's Method
First type indeterminacy	[1645,2245]	[1195,1795]	[1105,1705]
Second type indeterminacy	[1645,1779]	[1259,1323]	[1105,1215]
Third type indeterminacy	[1645,2449]	[1195,1993]	[1105,1885]

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Looking forward to the further upcoming studies dedicated to implement improvements on theinitial solutions to get an optimal solution in the neutrosophic transportation problems.

As it is well known in the transportation problems, that the (North- West Corner, Least Cost Method, and Vogel's Method), all these methods are for finding the initial solutions (either these initial solutions are suffering from weak accurate by applying the North - West Corner, or having more accurate by applying Vogel's method), it still needs to investigate the optimal solutions in the transportations problems, which will be by intending to publish forthcoming papers.

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An Efficient Optimal Solution Method for Neutrosophic Transport Models: Analysis, Improvements, and Examples

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Abstract: Transport issues aim to determine the number of units that will be transferred from the production centers to consumption areas so that the cost of transportation is as low as possible, taking into account the conditions of supply and demand. Due to the great importance of these issues and to obtain more accurate results that take into account all circumstances, we conducted two research studies. In the first research, we presented a formulation of neutrosophic transport issues, and in the second research, we presented some ways to find a preliminary solution to these issues, but we do not know whether the preliminary solution is optimal or not, so we will present in this research a study whose purpose is to shed light on some important methods used to improve the optimal solution to transportation issues and then reformulating them using the concepts of neutrosophic science, a science that leaves nothing to chance or circumstances but rather provides solutions with neutrosophic values. Unspecified values take into account the best and worst conditions.

Keywords: Neutrosophic Science; Optimal Solution; Transportation; Stepping-Stone Method; Modified Distribution Method.

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1. Introduction

The linear programming method is one of the most important operations research methods that companies and institutions have benefited from in their workflows by building linear models for which science has provided ways to find the optimal solution. Given this importance, we have reformulated these models in two previous studies and found one of the most important ways to solve them using the concepts of neutrosophic science [1, 2]. The issue of transportation is one of the most important issues that have been dealt with using the linear programming method because transportation problems appear frequently in practical life. We need to transfer materials from production centers to consumption centers to secure the regions; we need the lowest possible cost, To solve these recurring and daily problems, we use operations research methods, specifically the linear programming method, where we transform the issue data into a classical linear mathematical model when the data are classical and neutrosophic model when the data are neutrosophic. And in the research [3], there is a full explanation of the neutrosophic transport issues. Since these models are linear neutrosophic, we can get an optimal solution by using the direct simplex neutrosophic method described in the research [1].

But we know that these models have special characteristics, in terms of restrictions and objective function, which enabled scientists and researchers to find special methods, the methods in which we get preliminary solutions, and we explained how to obtain a preliminary solution for the neutrosophic transport in the research [4], and we recall that the neutrosophic transport issues are

transport issues in which the required quantities and the available quantities are neutrosophic values of the form $Na_i \in a_i + \varepsilon_i$, where ε_i the indeterminacy of the produced quantities can take the forms $\varepsilon_i \in [\lambda_{i1}, \lambda_{i2}]$. In addition, the required quantities of neutrosophic values of the form $Nb_j \in b_j + \delta_j$, where δ_j is the indeterminacy of the quantities produced we take it as one of the forms $\delta_j \in [\mu_{j1}, \mu_{j2}]$. and the cost of transportation is neutrosophic values form $Nc_{ij} \in c_{ij} + \alpha_{ij}$, where α_{ij} is the indeterminacy of the cost of transportation, we take it as one of the form $c_{2j} \in \{\alpha_{1_{2j}} \alpha_{2_{2j}}\}$. In order to review the basic concepts of neutrosophic science and its stages of development and the most important topics of operations research that have been reformulated using the concepts of this science, the following research can be found [5-16].

2. Methods

The purpose of this research, as we mentioned in the abstract, is to highlight some of the methods used according to classical logic to improve the preliminary solution to transportation issues, where we will present:

   i.   The Stepping-Stone Method
   ii.   Modified Distribution Method

As stated in some references [17-20], with a focus on the scientific basis on which these methods were based, and then we will reformulate them using the concepts of neutrosophic science and use them to improve the solution of neutrosophic models after finding a preliminary solution for them using one of the methods mentioned in the reference [4].

2.1 The Stepping-Stone Method

To reach the optimal solution in this way, we follow the following steps:

   i.   We find the preliminary solution by one of the three aforementioned methods, then we calculate the total cost according to the preliminary solution.
   ii.   We identify the basic variables from the non-basic variables from the preliminary solution table.
   iii.   We determine the indirect cost by finding closed paths, as each closed path has its beginning and end as a non-basic variable and consists of horizontal and vertical lines whose pillars are basic variables, as it happens that there are two basic variables in the way of the path, so we deviate from the basic non-basic variable and in general, the closed path represents in the following Figure 1:
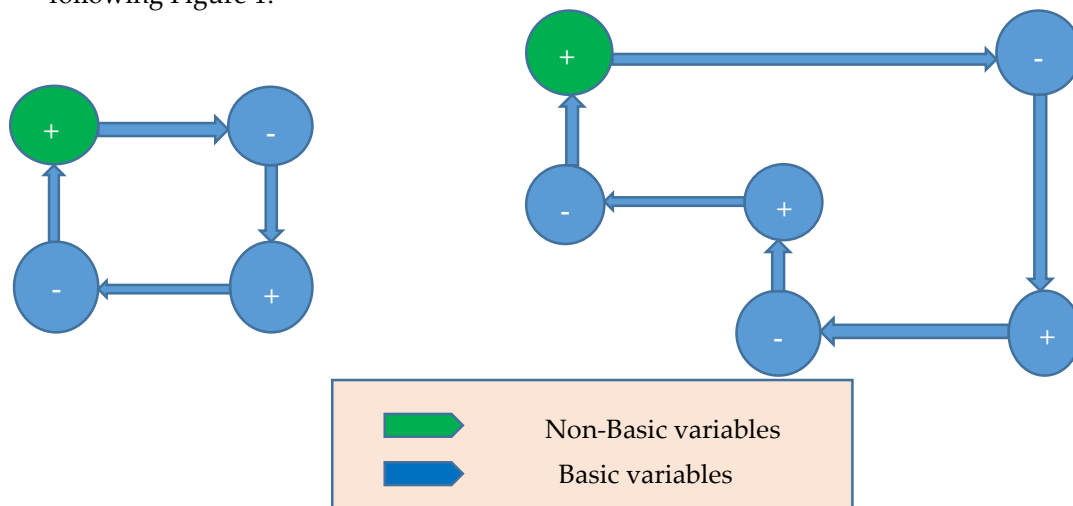
Figure 1. A representation of closed pathways.

We calculate the indirect cost of each non-basic variable by giving the cost of the non-basic variable a positive sign, and the cost of the basic variables we give it alternating negative and then positive signs, and so on. If the basic variables are positive or zero, this means that the solution that

we got is optimal and we stop. But if at least one of the indirect costs is negative, then we must develop the solution by choosing one of the non-basic variables to become basic and the exit of one of the basic variables.

Note:

To determine the basic internal variable, we take the non-basic variable that achieved the most negative in the indirect cost, and to make the solution the best possible, we try to pass in it the largest possible amount, we explain the above through the following example 1:

Example 1:

The following Table 1 represents the cost of transporting goods from sources A_i; $i = 1, 2, 3, 4$ to distribution centers B_j; $j = 1,2,3,4$ it is required to use the mobile quarantine method to improve the solution and obtain the ideal solution:

Table 1. Issue data.

Consumption center / Production centers	B_1	B_2	B_3	Available Quantities
A_1	2	4	0	150
A_2	\{3, 4.5\}	\{1, 2\}	\{5, 8\}	200
A_3	6	2	4	325
A_4	1	7	9	25
Required quantities	180	320	200	700 / 700

In this example, the cost of transportation of the product at the production center A_2 is neutrosophic values we take in forms $c_{2j} \in \left\{ \alpha_{1_{2j}} \alpha_{2_{2j}} \right\}$.

The solution:

We find the initial solution using the least cost method; we get the following preliminary solution Table 2.

Table 2. Preliminary solution.

Consumption center / Production centers	B_1	B_2	B_3	Available Quantities
A_1	2	4	0 \\ 150	**150**
A_2	\{3,4.5\}	\{1,2\} \\ 200	\{5,8\}	**200**
A_3	6 \\ 155	2 \\ 120	4 \\ 50	**325** **170** **50**
A_4	1 \\ 25	7	9	**25**
Required quantities	180 \\ 155	320 \\ 120	200 \\ 50	700 \\ 700

We note that the number of occupied squares is equal to $m + n - 1 = 6$

The total transportation cost according to the preliminary solution is:

$$Z_1 = 0 \times 150 + \{1,2\} \times 200 + 6 \times 155 + 2 \times 120 + 4 \times 50 + 1 \times 25$$

For $c_{22} = 1 \Rightarrow Z_1 = 1595$

For $c_{22} = 2 \Rightarrow Z_1 = 1795$

That is, against this preliminary solution, we have a neutrosophic value for the total transportation cost:

$$Z_1 \in \{1595, 1795\}$$

Now we see whether this solution is an optimal solution or not?

For this we define basic variables and non-basic variables, it is clear that

The basic variables are:

$$x_{13}, x_{22}, x_{31}, x_{32}, x_{33}, x_{41}$$

The non-basic variables are:

$$x_{11}, x_{12}, x_{21}, x_{23}, x_{42}, x_{43}$$

We have six basic variables and six non-basic variables, so we get six closed paths are formed it is in Figure 2:

Note: *The non-basic variables are green.*
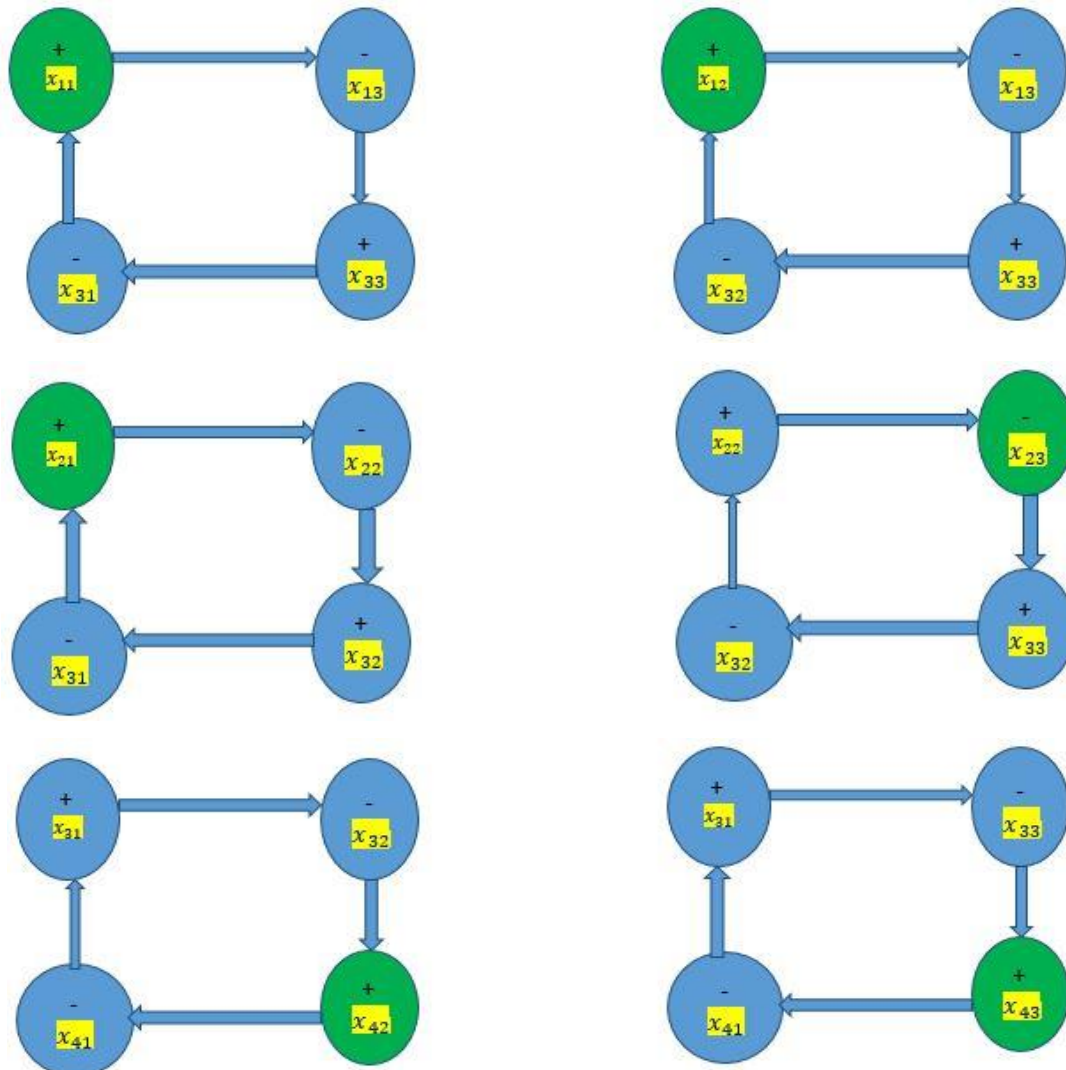
Figure 2. Possible closed paths after finding the initial solution.

The following Table 3, shows how the path is formed:

Table 3. Closed path identification.

Consumption center / Production centers	B_1	B_2	B_3	Available Quantities
A_1	2	4	0 150	150
A_2	{3,4.5}	{1,2} 200	{5,8}	200
A_3	6 155	2 120	4 50	325
A_4	1 25	7	9	25
Required quantities	180	320	200	700 700

We calculate the indirect cost, we find:

From the previous table and according to the drawn path, we explain how to calculate the indirect cost:

We start from the room (A_1B_1) we have the cost $c_{11} = 2$,_we take it with a positive sign because it is the room of the non-basic variable, then to the room (A_1B_3) we have the cost $c_{13} = 0$ We take here the minus sign and the variable in this room is a basic variable then to the room (A_3B_3)we have the cost $c_{33} = 4$, we take here the sign is positive, and the variable in this room is a basic variable then to the room (A_3B_1) we have the cost $c_{31} = 6$ We take here the minus sign, and the variable in this room is a basic variable, So the indirect cost of the non-essential variable is x_{11} is :

$$x_{11}: 2 - 0 + 4 - 6 = 0$$

In the same way, we calculate the cost for all non-essential variables we find:

$$x_{12} : 4 - 0 + 4 - 2 = 6$$
$$x_{21} : \{3,4.5\} - \{1,2\} + 2 - 6 = \{-2, -3, -0.5, -1,5\}$$
$$x_{23} : \{5,8\} - 4 + 2 - \{1,2\} = \{2,1,5,4\}$$
$$x_{42} : 7 - 1 + 6 - 2 = 10$$
$$x_{43} : 9 - 1 + 6 - 4 = 10$$

We note that the indirect cost corresponding to the basic variable x_{21} is a negative amount and it is the only one, so we enter this variable and it becomes one of the basic variables and we exit instead of x_{31}

We notice that we can pass the quantity $x_{21} = 155$, then it becomes:

$$x_{31} = 0, \quad x_{32} = 275 , \ x_{22} = 45$$

We get the following Table 4:

Table 4. The first improvement.

Consumption center / Production centers	B_1	B_2	B_3	Available Quantities
A_1	2	4	0 / 150	150
A_2	{3, 4.5} / 155	{1, 2} / 45	{5, 8}	200
A_3	6	2 / 275	4 / 50	325
A_4	1 / 25	7	9	25
Required quantities	180	320	200	700 / 700

We note that the transportation cost is according to the previous solution:

$$Z_2 \in (0 \times 150 + \{3,4.5\} \times 155 + \{1,2\} \times 45 + 2 \times 275 + 4 \times 50 + 1 \times 25)$$

For $c_{21} = 3$ and $c_{22} = 1 \Rightarrow Z_2 = 1285$

For $c_{21} = 3$ and $c_{22} = 2 \Rightarrow Z_2 = 1330$

For $c_{21} = 4.5$ and $c_{22} = 1 \Rightarrow Z_2 = 1517.5$

For $c_{21} = 4.5$ and $c_{22} = 2 \Rightarrow Z_2 = 1562.5$

Therefore:

$$Z_2 \in \{1285, 1330, 1517.5, 1562.5\}$$

$$\forall Z_2 \in \{1285, 1330, 1517.5, 1562.5\} \Longrightarrow Z_2 < Z_1 \in \{1595, 1795\}$$

That is, this solution is better than the previous one, the question now is whether this solution is the optimal solution, for this we define the basic variables and the non-basic variables we find:

The basic variables are:

$$x_{41}, x_{33}, x_{32}, x_{22}, x_{21}, x_{13}$$

The non-basic variables are:

$$x_{43}, x_{42}, x_{31}, x_{23}, x_{12}, x_{11}$$

We have six basic variables and six non-basic variables, so we get six closed paths; we form the closed paths for the six non-basic variables in Figure 3:

Note: *The non-basic variables are green.*
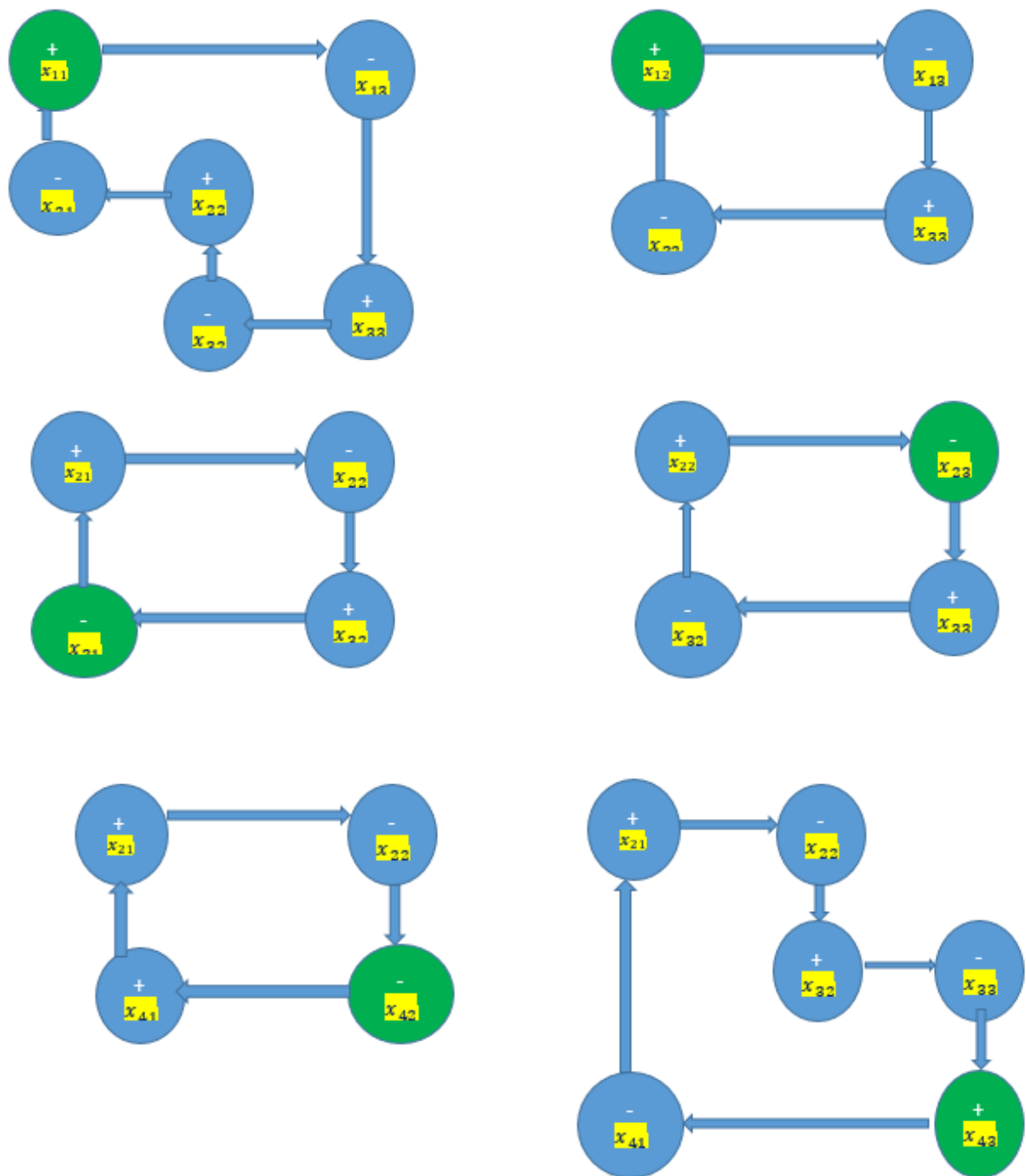
Figure 3. Possible closed paths after the first optimization.

We calculate the indirect cost:

$$x_{11} : 2 - 0 + 4 - \{1,2\} + 1 - 3 = \{3,2\}$$
$$x_{12} : 4 - 0 + 4 - 2 = 8$$
$$x_{32} : 6 - 2 + \{1,2\} - \{3,4.5\} = \{2,0.5,3,1.5\}$$
$$x_{42} : 7 - \{1,2\} + \{3,4.5\} - 1 = \{8,9.5,7,8.5\}$$
$$x_{43} : 9 - 4 + 2 - \{1,2\} + \{3,4.5\} - 1 = \{8,9.5,7,8.5\}$$

We note that the indirect cost for each non-basic variable is positive, and therefore we cannot introduce any non-basic variable to the basic rule. Therefore, the solution that we obtained in the first

improvement is an optimal vinegar, and the minimum transportation cost is the one that we obtained previously:

Therefore the optimal solution is:

$x_{13} = 150$, $x_{21} = 155$, $x_{22} = 45$, $x_{32} = 275$, $x_{33} = 50$, $x_{41} = 25$

The minimum cost of transportation is:

$$Z_2 \in \{1285 , 1330 , 1517.5 , 1562.5 \}$$

2.2 Modified Distribution Method

This method is another method of finding the optimal solution for transportation issues, and it is also similar to the previous method (the mobile stone method) conjunction.

To find the optimal solution to the transportation issue according to this method, we follow the following steps:

  i.   We find the initial solution in one of the previously mentioned ways
  ii.   We define the essential variables and non-basic variables for the solution
  iii. We associate with each line i multiplied by u_i, and with each column j multiplied, we call it v_j, so it is:

For each basic variable x_{ij} we have:

$$u_i + v_j = c_{ij} \qquad (*)$$

Where c_{ij} is the cost from A_i to B_j:

Since the number of basic variables is $m + n - 1$, we get the $m + n - 1$ equation from the previous Figure $(*)$ and by solving these equations we must find the values of u_i , v_j which have $m + n$ so we must give one of these multipliers an optional value, then we solve these equations according to this value.

After we found the values u_i , v_j , for each non-basic variable x_{ij} we calculate the quantities $c_q = c_{ij} - u_i - v_j$

In a similar way to the moving stone method, but if one of these quantities is negative, then we must introduce a non-basic variable to the set of basic variables and output instead of a basic variable, and the primary variable entered is chosen in the same previous way:

Example 2:

Let's take the previous example, where we found the preliminary solution according to the cost method:

Table 5. The preliminary solution.

Consumption center / Production centers	v_1 B_1	v_2 B_2	v_3 B_3	Available Quantities
u_1 A_1	2	4	0 150	150
u_2 A_2	{3,4.5}	{1,2} 200	{5,8}	200
u_3 A_3	6 155	2 120	4 50	325
u_4 A_4	1 25	7	9	25
Required quantities	180	320	200	700 700

Transportation cost is: $Z_1 = 1595$

Basic variables:

$$x_{13}, x_{22}, x_{31}, x_{32}, x_{33}, x_{41}$$

Non- basic variables:

$$x_{11}, x_{12}, x_{21}, x_{23}, x_{42}, x_{43}$$

Multipliers is: u_i ; i $= 1,2,3,4$ and v_j ; $j = 1,2,3.$

For basic variables, we have:

For $x_{13},$ we have $u_1 + v_3 = 0$

For $x_{22},$ we have $u_2 + v_2 = \{1,2\}$

For $x_{31},$ we have $u_3 + v_1 = 6$

For $x_{32},$ we have $u_3 + v_2 = 2$

For $x_{33},$ we have $u_3 + v_3 = 4$

For $x_{41},$ we have $u_4 + v_1 = 1$

It is six equations with seven unknowns. To solve them, we impose $u_1 = 0$, so we find the rest of the variables:

$$u_1 = 0, u_2 = 3, u_3 = 4, \qquad u_4 = -1$$
$$v_1 = 2, v_2 = -2, v_3 = 0$$

For non-basic variables, we have:

$$x_{11}, x_{12}, x_{21}, x_{23}, x_{42}, x_{43}$$

For x_{11} it is: $\bar{c}_{11} = c_{11} - u_1 - v_1 = 2 - 0 - 2 = 0$

For x_{12} it is: $\bar{c}_{12} = c_{12} - u_1 - v_2 = 4 - 0 + 2 = 6$

For x_{21} it is: $\bar{c}_{21} = c_{21} - u_2 - v_1 = \{3,4.5\} - 3 - 2 = \{-2, -3.5\}$

For x_{23} it is: $\bar{c}_{23} = c_{23} - u_2 - v_3 = \{5,8\} - 3 - 0 = \{2, 5\}$

For x_{42} it is: $\bar{c}_{42} = c_{42} - u_4 - v_2 = 7 + 1 + 2 = 10$

For x_{43} it is: $\bar{c}_{43} = c_{43} - u_4 - v_3 = 9 + 1 - 0 = 10$

We note that the quantity $\bar{c}_{21} = -2$ is a negative value, and therefore the initial solution that we got is not optimal, we must develop this solution, and for that, we form the closed path for the non-basic variable x_{21}, so we find it from the Figure 4:
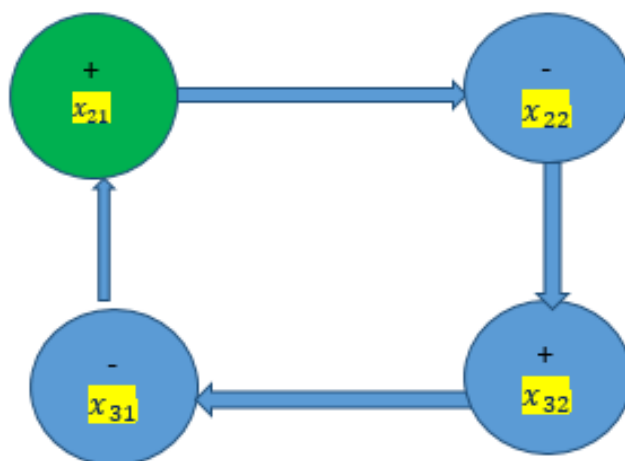
Figure 4. Possible closed pathways for the non-basic variable x_{21}.

We enter x_{21} into the set of basic variables, by giving it the value $x_{21} = 155$, and we remove the variable x_{31} so it becomes a non-basic variable, and then it becomes $x_{22} = 45$ and $x_{32} = 275$, so we get the following Table 6:

Table 6. The first improvement.

Consumption center — Production centers		v_1 — B_1	v_2 — B_2	v_3 — B_3	Available quantities
u_1	A_1	2	4	0 — 150	150
u_2	A_2	{3,4.5} — 155	{1,2} — 45	{5,8}	200
u_3	A_3	6	2 — 275	4 — 50	325
u_4	A_4	1 — 25	7	9	25
Required quantities		180	320	200	700 — 700

We enter x_{21} into the set of basic variables, by giving it the value $x_{21} = 155$, and we remove the variable x_{31} so it becomes a non-basic variable, and then it becomes $x_{22} = 45$ and $x_{32} = 275$, so we get the following Table 6:

The new transfer cost is:

$$Z_2 \in \{1285\,,1330\,,1517.5\,,1562.5\,\}$$

$$\forall\, Z_2 \in \{1285\,,1330\,,1517.5\,,1562.5\,\} \Longrightarrow Z_2 < Z_1 \in \{1595,1795\}$$

This solution is better than the previous solution, but is it the optimal solution?

For basic variables, we have:

For x_{13} we have $u_1 + v_3 = 0$
For x_{21} we have $u_2 + v_1 = 3$
For x_{22} we have $u_2 + v_2 = 1$
For x_{32} we have $u_3 + v_2 = 2$
For x_{33} we have $u_3 + v_3 = 4$
For x_{41} we have $u_4 + v_1 = 1$

We assume $u_1 = 0$ and solve the system of equations we find:

$$u_1 = 0,\ u_2 = 3,\ u_3 = 4,\ u_4 = 1$$

$$v_3 = 0,\ v_2 = -2,\ \ ,\ v_1 = 0,$$

For non-basic variables:

For x_{11} it is: $\bar{c}_{11} = c_{11} - u_1 - v_1 = 2 - 0 - 0 = 2$
For x_{12} it is: $\bar{c}_{12} = c_{12} - u_1 - v_2 = 4 - 0 + 2 = 6$
For x_{21} it is: $\bar{c}_{21} = c_{21} - u_2 - v_1 = \{3,4.5\} - 3 + 0 = \{0,1.5\}$
For x_{23} it is: $\bar{c}_{23} = c_{23} - u_2 - v_3 = \{5,8\} - 3 + 0 = \{2,5\}$
For x_{42} then: $\bar{c}_{42} = c_{42} - u_4 - v_2 = 7 - 1 + 2 = 8$
For x_{43} it is: $\bar{c}_{43} = c_{43} - u_4 - v_3 = 9 - 1 - 0 = 8$

We note that all quantities $\bar{c}_{ij}$ are positive quantities, so the solution that we got is optimal, and the minimum cost of transportation is:

$$Z_2 \in \{1285\,,1330\,,1517.5\,,1562.5\,\}$$

$$\forall\, Z_2 \in \{1285\,,1330\,,1517.5\,,1562.5\,\} \Longrightarrow Z_2 < Z_1 \in \{1595,1795\}$$

Therefor the optimal solution is:

$$x_{13} = 150\ ,\ x_{21} = 155\ ,\ x_{22} = 45\ ,\ x_{32} = 275\ ,\ x_{33} = 50\ ,\ x_{41} = 25$$

The minimum cost of transportation is:

$$Z_2 \in \{1285\,,1330\,,1517.5\,,1562.5\,\}$$

3. Conclusion

In many practical issues, we encounter cases in which we are unable to provide confirmed data on the reality of the state of the system under study; the data is affected by circumstances surrounding the working environment, and this matter affects the future and may cause large losses. In the example that was presented in this research, we noticed the study gave us a neutrosophic transport cost suitable for all conditions because it was obtained through neutrosophic data.

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Studying Transport Models with the Shortest Time According to the Neutrosophic Logic

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Abstract:

Time factor plays an important role in many issues, and the most important of these issues is the issue of transportation, when we need to transport perishable materials such as milk, medicines ,blood,….. etc. or develop war plans to secure the requirements of battle of ammunition - food - and soldiers …..etc., at maximum speed, we need a careful scientific study that enables us to avoid losses, so the researchers studied transport models in the shortest possible time using the values of classic logic and the best solution for such models is a specific value subject to increase or decrease because there is nothing certain in the real reality, all the results of the studies are related to the surrounding conditions of the system under study, due to the sensitivity of these issues had to be reformulated according to a science that takes into account all the cases that the system can go through so that we can take all possible precautions that help us reduce losses and secure the required in the shortest possible time, we have in This research formulates transport models with the shortest time and we presented a special way to solve such models using neutrosophic values, based on the concept of the neutrosophic linear mathematical model, a model that has something of non-determination (indeterminacy) and we reached transport models with the shortest time that are considered a generalization of the existing models because they give us optimal neutrosophic values for time, which are unspecified values $Nt^* = t^* \pm \varepsilon$ where ε is indeterminism and we will take it in the form then become the matrix of times $Nt^* = [t^* \pm \varepsilon]$, where Nt^* is any neighborhood of t^*, it should be noted that in this research when viewing the example we took some transfer times neutrosophic values of the form $Nt \in [t_1, t_2]$ to be able to clarify the main goal of the research.

Key words: Linear programming - Simplex - the problem of transportation at the lowest cost - the issue of transportation in the shortest time - Ways to solve the problem of transport .

Introduction:

Linear programming is one of the most methods of operations research that has been used to address many realistic issues and provided optimal solutions to them that helped reduce losses, and since the optimal solution to such issues depends on the data provided by those in charge of the work and this data lack stability , we are unable to study these issues and develop accurate future action plans It was necessary for a new science that provides us with a margin of freedom and helps us reduce losses, so it was a neutrosophic science, which was laid by the American scientist and mathematical philosopher Florentin Samarandche and came as a generalization of the fuzzy logic presented by Lutfizadeh [1], this science has received great attention by researchers as many researches and studies have been published in various fields of science using the basic concepts developed by the founder of this science [2-8], and since operations research is considered the applied aspect of mathematics, it was necessary to reformulate its topics using the basic concepts of neutrosophic science, where we published the feast of research on various topics such as static stock models, simulation, decision-making, dynamic programming, and in the topics of linear programming, and transport problems at the lowest cost. Waiting queues,

Machine Learning...etc. [9-26], a complement to our research in which we present a study of transportation models in the shortest possible time.

Discussion:

Through our study of some topics of linear programming using neutrosophic science and due to the interest that these researches have received from researchers and those interested in the development of science, we present this research as a complementary study to what we have done in two previous researches on the issue of transport [18,19] It is dedicated to transport models with the shortest time using neutrosophic values that take into account all changes that may occur during the functioning of the systems under study and provide We have optimal transit time.

Model of transporting materials in the shortest possible time using neutrosophic values:

Based on the study contained in the research [19], we can formulate the issue as follows:

The text of the issue in general form:

Suppose we want to move a material from production centers A_i where $i = 1,2,\ldots,m$,whose production capacities are respectively $Na_1, Na_2, \ldots, Na_m$, to consumption centers B_j where $j = 1,2,\ldots,n$ whose needs is $Nb_1, Nb_2, \ldots, Nb_n$ It is in order, and let the matrix of times necessary to transfer the appropriate quantity from the center i to the center j be known and equal $NT = [Nt_{ij}]$, it is required to formulate the appropriate mathematical model to transfer all the quantities ,available in the production centers and meet the needs of all consumption centers in the shortest time. In order to build the appropriate mathematical model, we denote Nx_{ij} for the quantity transferred from the production center i where $i = 1,2,3,\ldots,m$ to the consumption center j where $= 1,2,3,\ldots,n$, then we can put the problem unknowns in matrix form. $NX = [Nx_{ij}]$, and we put the information in the question in a table as follows:

consumption production	B_1	B_2	B_3	$\cdots$	B_n	quantities
A_1	Nt_{11} Nx_{11}	Nt_{12} Nx_{12}	Nt_{13} Nx_{13}	...	Nt_{1n} Nx_{1n}	Na_1
A_2	Nt_{21} Nx_{21}	Nt_{22} Nx_{22}	Nt_{23} Nx_{23}	...	Nt_{2n} Nx_{2n}	Na_2
A_3	Nt_{31} Nx_{31}	Nt_{32} Nx_{32}	Nt_{33} Nx_{33}	...	Nt_{3n} Nx_{3n}	Na_3
...	...	...	...	...	...	...
A_m	Nt_{m1} Nx_{m1}	Nt_{m2} Nx_{m2}	Nt_{m3} Nx_{m3}	$\cdots$	Nt_{mn} Nx_{mn}	Na_m
Required quant	Nb_1	Nb_2	Nb_3	...	Nb_n	

Table No. (1) Data of the issue of transport in the shortest time

To build the appropriate mathematical model, we distinguish two cases:

First case: the model is balanced:
The model is balanced if:

$$\sum_{i=1}^{m} Na_i = \sum_{j=1}^{n} Nb_j$$

 Second Case: Unbalanced Model:
The model is unbalanced if:

$$\sum_{i=1}^{m} Na_i \neq \sum_{j=1}^{n} Nb_j$$

From this case, two cases result:
Overproduction:

$$\sum_{i=1}^{m} Na_i \geq \sum_{j=1}^{n} Nb_j$$

This model is returned to a balanced model by adding an imaginary consumer center that needs it:

$$Nb_{n+1} = \sum_{i=1}^{m} Na_i - \sum_{j=1}^{n} Nb_j$$

Deficit in production:
This model is returned to a balanced model by adding a fictitious production center with a production capacity:

$$Na_{m+1} = \sum_{j=1}^{n} Nb_i - \sum_{i=1}^{m} Na_i$$

In both cases(b & a), a case of surplus production and a deficit in production, we get a balanced model.
Formulation of the mathematical model of transport models in the shortest possible time:
Our symbol for the quantity transferred from the center i to the center j with the symbol Nx_{ij} then these variables must meet the following conditions:

$$\sum_{j=1}^{n} Nx_{ij} = Na_i \qquad (i = 1,2,3,\dots,m)$$
$$\sum_{i=1}^{m} Nx_{ij} = Nb_j \qquad (j = 1,2,3,\dots,n)$$
$$Nx_{ij} \geq 0 \quad (i = 1,2,3,\dots,m)\,,(j = 1,2,3,\dots,n)$$

In these models, the objective function cannot be formulate d with a mathematical follower, so we extract its most important qualities and properties through the following discussion:
To find the optimal solution for any transport model, we must find the values of unknowns: Nx_{ij} ; $(i = 1,2,3,\dots,m)$, $(j = 1,2,3,\dots,n)$. In a previous research [18] we have presented ways to find the preliminary solution to the problem of neutrosophic transport at the lowest cost, taking advantage of the study contained in the research we find a primary basic solution using one of the methods, we know that any optimal solution that includes n+m-1 basic solution that are not equal to zero , and against this solution there is a set of times that we will symbolize as $\left[Nt_{ij}\right]_X$.

It represents the time required to transport all materials available in all production centers and meet the needs of all consumption centers.

The time required to finish the transfer process, which we will symbolize as corresponding to the largest element of the matrix $[Nt_{ij}]_X$ must achieve the following relationship:

$$Nt_x = Max_{i,j}[Nt_{ij}]_X$$

Since we have a large number of acceptable solutions, the optimal solution is given by the following relationship:

$$Nt_x^* = MinNt_x = Min(Max_{i,j}[Nt_{ij}]_X)$$

This means that we solve the model without a target function we get a base solution and then we determine the number set from the matrix $[Nt_{ij}]_X$ corresponding to this base solution.

Note 1:

If the issue is unbalanced and when adding an imaginary production center or an imaginary consumer center, we determine the time according to the following:

Since the time required to finish the transfer process achieves the following relationship:

$$Nt_x = Max_{i,j}[Nt_{ij}]_X$$

So we take the time required to transfer the quantities available in this imaginary production center to all consumption centers equal to zero.

And we take the time required to transport quantities from all production centers to the imaginary consumer center is equal to zero.

Note 2:

In order for the transport model to be a neutrosophic transport model, at least one of the data in Table 1 must be a neutrosophic value.

The general method used to obtain the smallest transfer time is to move from one neutrosophic base solution to another base solution using the simplex method to solve the neutrosophic linear models described in the research [12] ,and the goal is to make the largest elements Nt_x in the matrix $NT = [Nt_{ij}]_X$ as small as possible.

In this research, we will use a special method to solve neutrosophic transport models according to time, which we explain through the following example:

Example:

Four pharmaceutical plants distribute their production of one type to three pharmacies the available quantities, the quantities required and the times required to transport them are shown in the following table:

Consumption center Production centers	B_1	B_2	B_3	Required quantities
A_1	[1 ,1.5] Nx_{11}	[2 ,2.4] Nx_{12}	6 Nx_{13}	11
A_2	[3 ,3.2] Nx_{21}	8 Nx_{22}	[1 ,1.5] Nx_{23}	9
A_3	[7 ,7.5] Nx_{31}	10 Nx_{32}	[4 ,4.6] Nx_{33}	13
A_4	12 Nx_{41}	8 Nx_{42}	[5 ,5.1] Nx_{43}	17

Available quantity	18	10	12	40
				40

Table No. (2) Example data

We look for the smallest time in the rooms (i, j), we find that it is found in two rooms $(1, 1)$ and $(3, 2)$:

$$Min(Nt_{ij)} = Nt_{11} = Nt_{32} \in [1, 1.5]$$

We denote b.i Ω_1 for all table stone except the two rooms $(1, 1)$ and $(2, 3)$

We saturate the two chambers opposite them $(1, 1)$ and $(2, 3)$ then we put in the other stone $(*)$ we get the following table:

consumption production				Required quantities
A_1	11	$*$	$*$	11
A_2	$*$	$*$	9	9
A_3	$*$	$*$	$*$	13
A_4	$*$	$*$	$*$	7
Available quantities	18	10	12	40
				40

Table No. (3) First Step

The first solution is $x_{11}^{(1)} = 11$, $x_{23}^{(1)} = 9$ which expresses the total quantities transferred and is equal to $x_{ij} = 11 + 9 = 20$ it crosses a quantity less than the quantity required for that time solution from which we started and the author of $Nt_{11} = Nt_{23} \in [1, 1.5]$ is an imperfect solution to the problem at hand.

From the elements of the sat Ω_1, where Ω_1 equal :

$$\Omega_1 = \{ [3, 3.2], [7, 7.5], 12, [2, 2.4], 8, 10, 8, 6, [4, 4.6], [5, 5.1] \}$$

We look for the smallest time we find:

$$(Nt_{ij}) = Min\{[3, 3.2], [7, 7.5], 12, [2, 2.4], 8, 10, 8, 6, [4, 4.6], [5, 5.1]\} \in [2, 2.4]$$

Any smallest time is this $Nt_{12} \in [2, 2.4]$ means that we must transfer the quantities available in the production center A_1 to the consumption center B_2 and this is not possible because the center A_1 no longer contains any quantity and therefore this step is not useful.

We form the sat $\Omega_2 = \{ [3, 3.2], [7, 7.5], 12, 8, 10, 8, 6, [4, 4.6], [5, 5.1] \}$ is the resulting Ω_1 after deleting $Nt_{12} \in [2, 2.4]$ and we choose from Ω_2 the smallest time we find:

$$(Nt_{ij}) = Min\{[3, 3.2], [7, 7.5], 12, 8, 10, 8, 6, [4, 4.6], [5, 5.1]\} \in [3, 3.2]$$

Any smaller time is this $Nt_{21} \in [3, 3.2]$ means that we have to transfer the quantities available in the production center A_2 to the consumption center B_1 and this is not possible because the center A_2 no longer contains any quantity and therefore this step is also not useful

We form $\Omega_3 = \{[7, 7.5], 12, 8, 10, 8, 6, [4, 4.6], [5, 5.1] \}$ the sat which is the resulting sat Ω_2 after deleting $Nt_{21} \in [3, 3.2]$ and choosing from the Ω_3 smallest time we find :

$$(Nt_{ij}) = Min\{[7, 7.5], 12, 8, 10, 8, 6, [4, 4.6], [5, 5.1]\} \in [4, 4.6]$$

Any smallest time is $Nt_{33} \in [4, 4.6]$ this means that we must transfer the quantities available in the production center A_3 to the consumption center B_3 i.e. we put $x_{33} = 12$ and therefore we must shift the amount in the room $(2,3)$ with the same time $[1, 1.5]$ to a room with a time immediately followed and located in the same line i.e. to the room $(2,1)$ with the same time $[3, 3.2]$ and we put $x_{21} = 9$,then we make a balance process for the quantities that have been distributed and here we must reduce the quantity in the room $(1,1)$ becomes $x_{11} = 9$ and we put the quantity that has been reduced in the room with the lowest time immediately following the time in that room and in the same row, i.e in the room $(1,2)$ we get $x_{12} = 2$ from this step we get the following distribution:

$x_{11} = 9 \cdot x_{12} = 2 , x_{21} = 9 , x_{33} = 12$

But this solution is not ideal because the sum of the quantities that were distributed is not equal to the 40 total quantities available, that is, the time $Nt_{33} \in [4, 4.6]$ Not the shortest time, we proceed in the same way in the eighth time to reach the distribution shown in the following table:

consumption / production	B_1	B_2	B_3	Required amounts
A_1	1	10	*	11
A_2	9	*	*	9
A_3	8	*	5	13
A_4	*	*	7	7
Available quantities	18	10	12	40 40

Table No. (4) Optimal Solution

In return for $Min(Nt_{ij}) = Nt_{31} \in [7, 7.5]$ this time, the entire quantities available in the production centers have been transferred and the needs of all consumer centers have been met, the ideal solution is

$$x_{11} = 1 , \ x_{21} = 9 , \ x_{31} = 8 , \ x_{12} = 10 , \ x_{33} = 5 , x_{43} = 7$$

The rest of the variables is equal to zero, and the shortest time is $Nt^* = Nt_{31} \in [7, 7.5]$

It should be noted that the same example was presented and solved according to classical logic in reference [20], and the data of the problem were as in the following table:

The available quantities, the quantities required and the times required for their transportation are shown in the following table:

consumption / production	B_1	B_2	B_3	Required amounts
A_1	1 Nx_{11}	2 Nx_{12}	6 Nx_{13}	11
A_2	3 Nx_{21}	8 Nx_{22}	1 Nx_{23}	9
A_3	7 Nx_{31}	10 Nx_{32}	4 Nx_{33}	13
A_4	12 Nx_{41}	8 Nx_{42}	5 Nx_{43}	7

Available amounts	18	10	12	40
				40

Table No. (5) Example data according to classical logic

The optimal solution was as follows:

Time $Min(t_{ij}) = t_{31} = 7$ and in exchange for this time, the entire quantities available in the production centers have been transferred and the needs of all consumer centers have been met, the best solution is

$$x_{11} = 1 \ , x_{21} = 9 \ , \ x_{31} = 8 \ , \ x_{12} = 10 \ , x_{33} = 5 \ , x_{43} = 7$$

The rest of the variables is equal to zero, and the shortest time is $t^* = t_{31} = 7$

Conclusion and results:

The solution using neutrosophic values is an undefined neutrosophic value, which can be any value belonging to the domain $Nt^* \in [7 \ ,7.5]$, and this value is not specified shows its impact according to one time used and according to the nature of the material transported and the need of consumption centers for it in the end The difference between it and the value $t^* = 7$ obtained when solving this problem using classical values and its impact is determined by those responsible for the work, in addition to that the method used to find the shortest time is an iterative method and in this example we repeated it eight times until we reached the required despite the number of production centers and consumption centers a small number and in Real reality is more numerous, so we recommend using computers and new technologies used in programming when applying this method to real systems from reality,

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Optimal Neutrosophic Assignment and the Hungarian Method

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Abstract:

Operations research methods focus on formulating decision models, models that approximate the real work environment to mathematical models, and their optimal solution helps the decision maker to make optimal decisions that guarantee the greatest profit or the lowest cost. These models were built based on the data collected on the practical issue under study, and these data are classic values, specific values that are correct and accurate during the period in which these data are collected, which means that they remain ideal if the surrounding conditions remain the same as those in which they were collected. Data, and since the aim of this study is to make ideal decisions and to develop future plans through which we achieve the greatest profit or the least cost, It was necessary to search for a study that fits all the conditions in which the work environment passes in the present and the future. If we take data, neural values, indeterminate values - uncertain - leave nothing to chance, and take into account all conditions from worst to best. In this paper, we will present a complementary study to what we have done in previous research, the purpose of which was to reformulate some operational research methods using neutrosophic data. We will reformulate the assignment problem and one of the methods for its solution, which is the Hungarian method, using neutrosophic concepts, and we will explain the difference between using classical and neutrosophic data through examples.

Key words:

  The optimal assignment problem - The Neutrosophic optimal assignment problem - The Hungarian method for solving assignment problems - linear models - Neutrosophic logic.

Introduction:

Assignment issues are a special case of linear programming issues that are concerned with the optimal assignment of various economic, productive and human resources for the various works to be accomplished, and we encounter them frequently in practical life in educational institutions - hospitals - construction projects…..etc. In order to obtain an optimal assignment that achieves the greatest profit and the least loss in all conditions that the work environment can pass through,

it was necessary to reformulate the issue of optimal assignment and one of the ways to solve it using the concepts of the science neutrosophic , the science that has proven its ability to provide ideal and appropriate solutions in all circumstances. And in many areas, as shown in research and studies presented by researchers interested in scientific development [1-15].Since the optimal assignment model is one of the important models in the field of operations research, where we have a machines or people, and we need to assignment them to do the required work, and the number of works is equal to the number of machines or people. We need an optimal assignment for each machine so that it performs only one work and achieves the greatest profit or the lowest cost depending on the nature of the issue. This issue was studied using classical data by building the mathematical models, when we solving it, we get the desired. In this research, we will reformulate the issue using neutrosophic data that takes into account all changes that may occur in the work environment by taking costs or profit neutrosophic values, meaning that the cost (or return profit) of assignment the machine or person i to perform work j is $Nc_{ij} \in c_{ij} \pm \varepsilon_{ij}$ where ε_{ij} is the indeterminacy and $\varepsilon_{ij} \in [\lambda_{ij1}, \lambda_{ij2}]$, it is any neighborhood to the value c_{ij} that we get while collecting data on the problem then the cost (or profit) matrix becomes $NC_{ij} = [c_{ij} \pm \varepsilon_{ij}]$.

Discussion:

Assignment issues are considered a special case of linear programming issues and are concerned with the optimal assignment of various economic, human and productive resources for the different work to be accomplished, based on what we have presented in the research Mysterious Neutrosophic Linear Models [16] and the classical formulation contained in the two references [17,18]. In this research, we will reformulate the problem of optimal assignment and the Hungarian method that is used to solve these problems using the concepts of neutrosophic science, that is, we will take the costs or profit from the neutrosophic values, so that the cost of doing job j by the machine or the person i is $Nc_{ij} \in c_{ij} \pm \varepsilon_{ij}$, where ε_{ij} is indeterminacy and $\varepsilon_{ij} \in [\lambda_{ij1}, \lambda_{ij2}]$, which is any neighborhood of the value c_{ij} which we get it while collecting the data and then the cost matrix becomes equal to $NC_{ij} = [c_{ij} \pm \varepsilon_{ij}]$ and the problem text is as follows:

Standard assignment issues:

In these issues, the number of machines or people equals the number of works, which we will address in this research.

Text of the minimum cost type neutrosophic normative assignment problem:

If we have n machines, we denote them by M_1, M_2, ... , M_n and we have a set of works consisting of n different work we denote them by N_1, N_2, ... , N_n we want to designate the machines to do these jobs, cost of doing any work j on the device i, it is $Nc_{ij} \in c_{ij} \pm \varepsilon_{ij}$. Assuming that any machine can do only one job, it is required to find the optimal assignment so that the cost is as small as possible.

Formulation of the mathematical model:
To formulate the linear mathematical model, we assume:

$$x_{ij=} \begin{cases} 1 & \textit{if job j was given to machine i} \\ 0 & \textit{otherwise} \end{cases}$$

Then write the target function as follows:

$$Z = \sum_{i=1}^{n} \sum_{j=1}^{n} \left(c_{ij} \pm \varepsilon_{ij} \right) x_{ij}$$

Conditions for machines:
Since each machine accepts only one action, we find:

$$\sum_{j=1}^{n} x_{ij} = 1 \quad ; i = 1, 2, \dots ., n$$

Business terms:
Since each work is assigned to only one machine, we find:

$$\sum_{i=1}^{n} x_{ij} = 1 \quad ; j = 1, 2, \dots \dots, n$$

Accordingly, the neutrosophic mathematical model is written as follows:
Find the minimum value:

$$Z = \sum_{i=1}^{n} \sum_{j=1}^{n} \left(c_{ij} \pm \varepsilon_{ij} \right) x_{ij}$$

Machine terms:

$$\sum_{j=1}^{n} x_{ij} = 1 \quad ; i = 1, 2, \dots \dots, n$$

Business terms:

$$\sum_{i=1}^{n} x_{ij} = 1 \quad ; j = 1, 2, \dots \dots, n$$

Example 1: (The data are classic values).

Formulation of the mathematical model for the problem of standard assignment of minimum cost:

We want to find the optimal assignment for four jobs on four machines. The cost of assignment is given in the following table:

Business / The machines	N_1	N_2	N_3	N_4
M_1	10	9	8	7
M_2	3	4	5	6
M_3	2	1	1	2
M_4	4	3	5	6

Table No. (1) Table of Distribution cost table and classic values

To formulate the linear mathematical model:
We impose:

$$x_{ij} = \begin{cases} 1 & \text{if job j was given to machine i} \\ 0 & \text{otherwise} \end{cases} \quad ; \; i,j = 1,2,3,4$$

Using the problem data, we get the following objective function:

$$Z = 10x_{11} + 9x_{12} + 8x_{13} + 7x_{14} + 3x_{21} + 4x_{22} + 5x_{23} + 6x_{24} + 2x_{31} + x_{32} + x_{33} + 2x_{34} + 4x_{41} + 3x_{42} + 5x_{43} + 6x_{44}$$

Machine terms:

$$x_{11} + x_{12} + x_{13} + x_{14} = 1$$
$$x_{21} + x_{22} + x_{23} + x_{24} = 1$$
$$x_{31} + x_{32} + x_{33} + x_{34} = 1$$
$$x_{41} + x_{42} + x_{43} + x_{44} = 1$$

Business terms:

$$x_{11} + x_{21} + x_{31} + x_{41} = 1$$
$$x_{12} + x_{22} + x_{32} + x_{42} = 1$$
$$x_{13} + x_{23} + x_{33} + x_{43} = 1$$
$$x_{14} + +x_{24} + x_{34} + x_{44} = 1$$

Therefore, the mathematical model is written as follows:
Find the minimum value of the function:

$$Z = 10x_{11} + 9x_{12} + 8x_{13} + 7x_{14} + 3x_{21} + 4x_{22} + 5x_{23} + 6x_{24} + 2x_{31} + x_{32} + x_{33} + 2x_{34} + 4x_{41} + 3x_{42} + 5x_{43} + 6x_{44}$$

Within the conditions:

$$x_{11} + x_{12} + x_{13} + x_{14} = 1$$

$$x_{21} + x_{22} + x_{23} + x_{24} = 1$$
$$x_{31} + x_{32} + x_{33} + x_{34} = 1$$
$$x_{11} + x_{21} + x_{31} + x_{41} = 1$$
$$x_{12} + x_{22} + x_{32} + x_{42} = 1$$
$$x_{13} + x_{23} + x_{33} + x_{43} = 1$$
$$x_{14} + {} + x_{24} + x_{34} + x_{44} = 1$$

Where x_{ij} it is either equal to zero or one.

In the previous model, there is some indeterminacy in the assignment process, as we do not know which machine will perform a certain work. In addition to that, we will also use neutrosophic data. We will take the cost of neutrosophic values, i.e. the cost of assignment machine i to perform work j is $Nc_{ij} \in c_{ij} \pm \varepsilon_{ij}$, where ε_{ij} is the indeterminacy and $\varepsilon_{ij} \in [\lambda_{ij1}, \lambda_{ij2}]$, which is any neighborhood to the value c_{ij} then the cost matrix becomes $NC_{ij} = [c_{ij} \pm \varepsilon_{ij}]$.

Example 2: (Cost is neutrosophic values):
We want to find the optimal assignment for four jobs on four machines. The cost of assignment is given in the following table**:**

Business⟍ The machines⟍	N_1	N_2	N_3	N_4
M_1	$10 + \varepsilon_{11}$	$9 + \varepsilon_{12}$	$8 + \varepsilon_{13}$	$7 + \varepsilon_{14}$
M_2	$3 + \varepsilon_{21}$	$4 + \varepsilon_{22}$	$5 + \varepsilon_{23}$	$6 + \varepsilon_{24}$
M_3	$2 + \varepsilon_{31}$	$1 + \varepsilon_{32}$	$1 + \varepsilon_{33}$	$2 + \varepsilon_{34}$
M_4	$4 + \varepsilon_{41}$	$3 + \varepsilon_{42}$	$5 + \varepsilon_{43}$	$6 + \varepsilon_{44}$

Table No. (2) Table of allocation cost of neutrosophic values

Where ε_{ij} is the limitation on the costs of assignment and it can be any neighborhood of the values contained in Table No. (1)

To formulate the linear mathematical model we assume:

$$x_{ij} = \begin{cases} 1 & if\ job\ j\ was\ given\ to\ machine\ i \\ 0 & otherwise \end{cases} \quad ; i, j = 1, 2, 3, 4$$

Using the problem data, we get the following objective function:

$$Z \in \{(10 + \varepsilon_{11})x_{11} + (9 + \varepsilon_{12})x_{12} + (8 + \varepsilon_{13})x_{13} + (7 + \varepsilon_{14})x_{14} + (3 + \varepsilon_{21})x_{21} + (4 + \varepsilon_{22})x_{22} + (5 + \varepsilon_{23})x_{23} + (6 + \varepsilon_{24})x_{24} + (2 + \varepsilon_{31})x_{31} + (1 + \varepsilon_{32})x_{32} + (1 + \varepsilon_{33})x_{33} + (2 + \varepsilon_{34})x_{34} + (4 + \varepsilon_{41})x_{41} + (3 + \varepsilon_{42})x_{42} + (5 + \varepsilon_{43})x_{43} + (6 + \varepsilon_{44})x_{44}\}$$

Machine terms:

$$x_{11} + x_{12} + x_{13} + x_{14} = 1$$
$$x_{21} + x_{22} + x_{23} + x_{24} = 1$$
$$x_{31} + x_{32} + x_{33} + x_{34} = 1$$
$$x_{41} + x_{42} + x_{43} + x_{44} = 1$$

Business terms:

$$x_{11} + x_{21} + x_{31} + x_{41} = 1$$

$$x_{12} + x_{22} + x_{32} + x_{42} = 1$$
$$x_{13} + x_{23} + x_{33} + x_{43} = 1$$
$$x_{14} + + x_{24} + x_{34} + x_{44} = 1$$

Therefore, the mathematical model is written as follows:
Find the minimum value of the function:

$$Z \in \{(10 + \varepsilon_{11})x_{11} + (9 + \varepsilon_{12})x_{12} + (8 + \varepsilon_{13})x_{13} + (7 + \varepsilon_{14})x_{14} + (3 + \varepsilon_{21})x_{21} + (4 + \varepsilon_{22})x_{22}$$
$$+ (\ 5 + \varepsilon_{23})x_{23} + (6 + \varepsilon_{24})x_{24} + (2 + \varepsilon_{31})x_{31} + (1 + \varepsilon_{32})x_{32} + (1 + \varepsilon_{33})x_{33} + (2$$
$$+ \varepsilon_{34})x_{34} + (4 + \varepsilon_{41})x_{41} + (3 + \varepsilon_{42})x_{42} + (5 + \varepsilon_{43})x_{43} + (6 + \varepsilon_{44})x_{44}\}$$

Within the conditions:

$$x_{11} + x_{12} + x_{13} + x_{14} = 1$$
$$x_{21} + x_{22} + x_{23} + x_{24} = 1$$
$$x_{31} + x_{32} + x_{33} + x_{34} = 1$$
$$x_{41} + x_{42} + x_{43} + x_{44} = 1$$
$$x_{11} + x_{21} + x_{31} + x_{41} = 1$$
$$x_{12} + x_{22} + x_{32} + x_{42} = 1$$
$$x_{13} + x_{23} + x_{33} + x_{43} = 1$$
$$x_{14} + + x_{24} + x_{34} + x_{44} = 1$$

Where x_{ij} it is either equal to zero or one.

Since the number of works is equal to the number of machines, the issue is a standard assignment issue, and the optimal solution can be obtained using several methods, including the Hungarian method in this research.

This method was named after the scientist who created it, a mathematician D.Konig. Its principle depends on finding the total opportunity-cost matrix, references [16, 17].

Explanation of the method based on what was stated in the reference [18]:

This method is based on a mathematical property discovered by the scientist D.Konig,

If the cost is non-negative values, then subtracting or adding a fixed number of elements of any row or any column in the standard allocation cost matrix does not affect the optimal assignment, and specifically does not affect the optimal values x_{ij} .

The algorithm begins by identifying the smallest element in each row, and subtracting it from all the elements of the row , or by selecting the smallest element in each column and subtracting it from all the elements of that column, we get a new cost matrix that includes at least one element equal to zero in each row or column. We do the assignment process using cells with a cost equal to zero. If possible, we have obtained the optimal allocation. For this assignment, the cost elements (c_{ij}) are non-negative, so the minimum value of the objective function cannot be $\sum_{i=1}^{n} \sum_{j=1}^{n} c_{ij} x_{ij}$ is less than zero.

We will use the above to find the optimal assignment for the problem in Example 2 based on the following information:

Taking the indeterminacy $\varepsilon_{ij} = \varepsilon \in [0, 5]$, the problem becomes:

Example 3:

We want to find the optimal assignment for four jobs on four machines. The cost of assignment is given in the following table:

Business \ The machines	N_1	N_2	N_3	N_4

M_1	$[10,15]$	$[9,14]$	$[8,13]$	$[7,12]$
M_2	$[3,8]$	$[4,9]$	$[5,10]$	$[6,11]$
M_3	$[2,7]$	$[1,6]$	$[1,6]$	$[2,7]$
M_4	$[4,9]$	$[3,8]$	$[5,10]$	$[6,11]$

Table No. (3) Table of Example data

To formulate the linear mathematical model:
We assume:

$$x_{ij} = \begin{cases} 1 & if\ job\ j\ was\ given\ to\ machine\ i \\ 0 & otherwise \end{cases} \quad ; i,j = 1,2,3,4$$

Using the problem data, we get the following objective function:

$$Z \in \{[10,15]x_{11} + [9,14]x_{12} + [8,13]x_{13} + [7,12]x_{14} + [3,8]x_{21} + [4,9]x_{22} + [5,10]x_{23}$$
$$+ [6,11]x_{24} + [2,7]x_{31} + [1,6]x_{32} + [1,6]x_{33} + [2,7]x_{34} + [4,9]x_{41} + [3,8]x_{42}$$
$$+ [5,10]x_{43} + [6,11]x_{44}\}$$

Machine terms:

$$x_{11} + x_{12} + x_{13} + x_{14} = 1$$
$$x_{21} + x_{22} + x_{23} + x_{24} = 1$$
$$x_{31} + x_{32} + x_{33} + x_{34} = 1$$
$$x_{41} + x_{42} + x_{43} + x_{44} = 1$$

Business terms:

$$x_{11} + x_{21} + x_{31} + x_{41} = 1$$
$$x_{12} + x_{22} + x_{32} + x_{42} = 1$$
$$x_{13} + x_{23} + x_{33} + x_{43} = 1$$
$$x_{14} + +x_{24} + x_{34} + x_{44} = 1$$

Therefore, the mathematical model is written as follows:
Find the minimum value of the function:

$$Z \in \{[10,15]x_{11} + [9,14]x_{12} + [8,13]x_{13} + [7,12]x_{14} + [3,8]x_{21} + [4,9]x_{22} + [5,10]x_{23}$$
$$+ [6,11]x_{24} + [2,7]x_{31} + [1,6]x_{32} + [1,6]x_{33} + [2,7]x_{34} + [4,9]x_{41} + [3,8]x_{42}$$
$$+ [5,10]x_{43} + [6,11]x_{44}\}$$

Within the conditions:

$$x_{11} + x_{12} + x_{13} + x_{14} = 1$$
$$x_{21} + x_{22} + x_{23} + x_{24} = 1$$
$$x_{31} + x_{32} + x_{33} + x_{34} = 1$$
$$x_{41} + x_{42} + x_{43} + x_{44} = 1$$
$$x_{11} + x_{21} + x_{31} + x_{41} = 1$$
$$x_{12} + x_{22} + x_{32} + x_{42} = 1$$
$$x_{13} + x_{23} + x_{33} + x_{43} = 1$$
$$x_{14} + +x_{24} + x_{34} + x_{44} = 1$$

Where x_{ij} it is either equal to zero or one.

Solution using the Hungarian method:
From the table Number 3 we find the following:

Business / The machines	N_1	N_2	N_3	N_4
M_1	$[10,15]$	$[9,14]$	$[8,13]$	$[7,12]$
M_2	$[3,8]$	$[4,9]$	$[5,10]$	$[6,11]$
M_3	$[2,7]$	$[1,6]$	$[1,6]$	$[2,7]$
M_4	$[4,9]$	$[3,8]$	$[5,10]$	$[6,11]$

To form the opportunity cost matrix for the rows, we do the following:
We form the opportunity cost matrix for the rows as follows:

In the first row, the lowest cost is $[7,12]$ we subtract it from all the elements of the first row.

In the second row, the lowest cost is $[3,8]$, we subtract from all elements of the second row.

In the third row, the lowest cost is $[1,6]$, we subtract it from all the elements of the third row.

In the fourth row, the lowest cost is $[3,8]$, we subtract from all elements of the fourth row.

We get the opportunity cost matrix for the following rows:

Business / The machines	N_1	N_2	N_3	N_4
M_1	3	2	1	0
M_2	0	1	2	3
M_3	1	0	0	1
M_4	1	0	2	3

Table No. (4) Table of Total opportunity cost matrix table

We try to make the assignment using cells with cost equal to zero we find:
Dedicate the machine M_1 to get the job N_4 done.

Dedicate the machine M_2 to get the job N_1 done.

Dedicate the machine M_3 to get the job N_3 done.

Dedicate the machine M_4 to get the job N_2 done.

Thus, we have obtained the optimal assignment and the minimum cost:

$$Z \in \{[10,15] \times 0 + [9,14] \times 0 + [8,13] \times 0 + [7,12] \times 1 + [3,8] \times 1 + [4,9] \times 0 + [5,10] \times 0$$
$$+ [6,11] \times 0 + [2,7] \times 0 + [1,6] \times 0 + [1,6] \times 1 + [2,7] \times 0 + [4,9] \times 0$$
$$+ [3,8] \times 1 + [5,10] \times 0 + [6,11] \times 0\}$$
$$Z \in [7,12] + [3,8] + [1,6] + [3,8] = [14,34]$$

That is, the optimal allocation is:
Dedicate the machine M_1 to get the job N_4 done.

Dedicate the machine M_2 to get the job N_1 done.

Dedicate the machine M_3 to get the job N_3 done.

Dedicate the machine M_4 to get the job N_2 done.

The cost:

$$Z \in [14\,,34].$$

The Hungarian method is summarized based on what was stated in the reference [17]:

1- We determine the smallest element in each row and subtract it from the rest of the elements of that row. Thus, we get a new matrix that is the opportunity cost matrix for the rows.

2- We determine the smallest element in each column of the opportunity cost matrix for the rows and Subtract it from the elements of that column. Thus, we get the total opportunity cost matrix.

3- We draw as few horizontal and vertical straight lines as possible to pass through all zero elements of the total opportunity cost matrix.

4- If the number of the straight lines drawn passing through the zero elements is equal to the number of rows (columns). Then we say that we have reached the optimal assignment**.**

5- If the number of straight lines passing through the zero elements is less than the number of rows (Columns).Then we move on to the next step.

6- We choose the lesser element from the elements that no straight line passed through and subtract it from all the elements that no straight line. Then we add it to all the elements that lie at the intersection of two lines. The elements that the straight lines passed through remain the same without any change. We get a new matrix that we call it the modified total opportunity cost matrix.

7- We draw vertical and horizontal straight lines passing through all the zero elements in the modified total opportunity cost matrix. If the number of straight lines drawn passing through the zero elements is equal to the number of rows (columns). Then we have reached the optimal assignment solution.

8- If the number of the lines is not equal to the number of rows (columns). We go back to step (1), we Repeat the previous steps until reaching the optimal assignment that makes the total opportunity cost equal to zero.

Example 4:

We have three machines $M_1\,, M_2, M_3$ and three works $N_1,\ N_2, N_3$ and each work is done completely using any of the three machines and in return each machine can perform any of the three works as well. What is required is to allocate these mechanisms to the existing works so that we get the optimal assignment , i.e. the assignment that gives us here the minimum total cost, bearing in mind that the costs of completing these works vary according to the different mechanisms implemented for these works, and this cost is related to the performance of each work and is shown in the following table:

Business The machines	N_1	N_2	N_3
M_1	$[20,23]$	$[27,30]$	$[30,33]$

| M_2 | $[10, 13]$ | $[18, 21]$ | $[16, 19]$ |
| M_3 | $[14, 17]$ | $[16, 19]$ | $[12, 15]$ |

Table No. (5) Table of allocation cost neutrosophic values example data

Mathematical model:

Find the minimum value of the function:

$$Z \in \{[20, 23]x_{11} + [27, 30]x_{12} + [30, 33]x_{13} + [10, 13]x_{21} + [18, 21]x_{22} + [16, 19]x_{23} + [14, 17]x_{31} + [16, 19]x_{32} + [12, 15]x_{33}\}$$

Within the conditions:

$$x_{11} + x_{12} + x_{13} = 1$$
$$x_{21} + x_{22} + x_{23} = 1$$
$$x_{31} + x_{32} + x_{33} = 1$$
$$x_{11} + x_{21} + x_{31} = 1$$
$$x_{12} + x_{22} + x_{32} = 1$$
$$x_{13} + x_{23} + x_{33} = 1$$

Where x_{ij} it is either equal to zero or one.

Finding the optimal assignment using the Hungarian method:

We take Table No. (5)

Business / The machines	N_1	N_2	N_3
M_1	$[20, 23]$	$[27, 30]$	$[30, 33]$
M_2	$[10, 13]$	$[18, 21]$	$[16, 19]$
M_3	$[14, 17]$	$[16, 19]$	$[12, 15]$

1.

In the first row, the lowest cost is $[20, 23]$, which we subtract from all the elements of the first row.

In the second row, the least cost is $[10, 13]$ and we subtract it from all the elements of the second row.

In the third row, the lowest cost is $[12, 15]$, which we subtract from all the elements of the third row.

We get the opportunity cost table for the following rows:

Business / The machines	N_1	N_2	N_3
M_1	0	7	10
M_2	0	8	6
M_3	2	4	0

Table No. (6) Table of opportunity cost matrix for lines

2.

In the first column, the lowest cost is 0 we subtract it from all the items in the first column.

In the second column, the lowest cost is 4 we subtract it from all the items in the second column.

In the third column, the lowest cost is **4** we subtract it from all the items in the third column.

We get the table:

Business / The machines	N_1	N_2	N_3
M_1	0	3	10
M_2	0	4	6
M_3	2	0	0

Table No. (7) Table of total opportunity cost matrix

3. We draw as few horizontal and vertical straight lines as possible to pass through all zero elements of the total opportunity cost matrix.

4. If the number of straight lines drawn passing through the zero elements is equal to the number of rows (columns), then we say that we have reached the optimal assignment.

5. If the number of straight lines passing through the zero elements is less than the number of rows or columns, then we move on to the third step.

Business / The machines	N_1	N_2	N_3
M_1	0	3	10
M_2	0	4	6
M_3	2	0	0

Table No. (7) Total Opportunity Cost Matrix

We Note that the number of lines is less than the number of rows (columns). So we go to (6).

6.

  a. We choose the lowest element through which no straight line has passed. Smallest element is (3).

  b. We subtract it from the rest of the elements through which none of the lines drawn are passed.

  c. We add it to all the elements at the intersection of two straight lines drawn.

  d. Elements through which straight lines pass remain unchanged.

  e. We draw vertical and horizontal straight lines passing through all zero elements of the adjusted total opportunity cost matrix, and we get:

Business / The machines	N_1	N_2	N_3
M_1	0	0	7
M_2	0	1	3
M_3	5	0	0

Table No. (8) Modified Total Opportunity Cost Matrix

7. If the number of drawn lines is equal to the number of rows (columns), then we have reached the optimal assignment.

From Table No. (8), we note that the number of drawn lines is equal to the number of rows, meaning that we have obtained the optimal assignment, which is as follows:

In the third column, we have zero only in the cell that is M_3N_3, so the third machine is used to perform the third work. $M_3 \rightarrow N_3$

We delete the third row and the third column, we get the following table:

Business The machines	N_1	N_2
M_1	0	0
M_2	0	1

In the same way. The first machine is used to perform the second work, $M_1 \rightarrow N_2$.

The second machine used to perform the first work, $M_2 \rightarrow N_1$.

The minimum total cost is:

$$Z \in \{[20,23] \times 0 + [27,30] \times 1 + [30,33] \times 0 + [10,13] \times 1 + [18,21] \times 0 + [16,19] \times 0 + [14,17] \times 0 + [16,19] \times 0 + [12,15] \times 1\}$$
$$Z \in [27,30] + [10,13] + [12,15] = [49,58]$$

That is, the optimal assignment is:

Machine M_1 is assigned to do N_2 work.

Machine M_2 is assigned to perform N_1 work.

Machine M_3 is assigned to perform N_3 work.

The cost:

$$Z \in [49 , 58]$$

Important notes:
When we study the issues of optimal assignment, we come across the following:
   1. There is two types of assignment issues according to the objective function:
The first type:
It is required to obtain a minimum value for the objective function, knowing that the cost of completing any

work by a machine is a known value, and therefore the total cost is as small as possible.

The second type:

What is required is to obtain a maximum value for the objective function, and here it is known that we have the profit accruing from the completion of any work, by a machine, and therefore the total cost is the greatest possible.

In this type, we transform matter to the first type according to the following steps:

a. Multiply the elements of the cost matrix by the value (-1).

b. If some elements of the matrix are negative, we add enough positive numbers to the corresponding rows and columns so that all elements become non-negative.

c. Then the issue becomes a matter of assignment and we want to make the objective function smaller and all elements of the cost matrix are non-negative, so we can apply the Hungarian method.

2. **There are two types of customization issues according to the number of businesses and the number of machines**:

   In this research, we studied the standard assignment issues. It should be noted that there are non-standard assignment issues. In these issues, the number of works is not equal to the number of machines , and here we convert them into standard issues by adding a fictitious work or a fictitious machine and make the cost equal to zero So that the objective function does not change, then we build the mathematical model as it is in the standard models.

Conclusion and results:

Due to the importance of the issue of assignment in our practical life, it received great attention from scholars and researchers, as this issue was addressed on the basis that it is a transfer issue, and special methods were followed to solve transfer issues to find the optimal assignment ,but we find that many references deal with these issues according to the Hungarian method that It was found to be addressed. And through our study of these issues, we find that the use of the Hungarian method greatly helps to find the optimal assignment with less effort than other methods. On the other hand according to the results obtained when using the data Neutrosophic values , in issues of assignment and in all practical matters that are affected by the conditions surrounding the work environment so that decision makers can make appropriate decisions for all circumstances that achieve companies and institutions the greatest profit and the lowest cost.

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Neutrosophic Nonlinear Models

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Abstract

Nonlinear programming is an important and essential part of operations research and is more comprehensive than linear programming, its applications have spread in all branches of science, engineering, physics, chemistry, management, economic and military fields, etc. Nonlinear programming can also be used in forecasting, estimation, applied statistics and determining the costs resulting from the production, purchase and storage of goods, the mathematical model is a nonlinear model if any of the vehicles of the target or constraints are nonlinear statements and may be nonlinear statements In both, after neutrosophic science has achieved a great development in most fields of science, it was necessary to reformulate nonlinear models according to its concepts, we present in this research a study of neutrosophic nonlinear dependencies and neutrosophic nonlinear models as a prelude to future research through which we study some methods of solving neutrosophic nonlinear models.

Keywords: Nonlinear Models; Neutrosophic Logic; Neutrosophic Nonlinear Models

1. Introduction

The mathematical model consists of decision variables and the objective function and constraints and the area of possible solutions to the nonlinear mathematical model is the set of vectors whose components achieve all constraints. The optimal solution is the vector that achieves all constraints, and the function then reaches an optimal value (maximum or minimum) [1,2,3,4] In this research, we will use the concepts of neutrosophic science. To formulate some basic concepts and definitions in nonlinear programming and to complement what was presented by researchers interested in neutrosophy, and neutrosophic science is the latest findings of logical thought in its relentless quest to absorb reality and expresses the uprising of the mind when it collides with the unreasonableness of reality or false rationality scattered by petrified systems seeking stability in a world in which the pace of development and change is accelerating and to see the stages of development of this science from its inception until today see [5]. In front of this reality, we cannot say that the possibilities available during the workflow in any facility are specific amounts, and therefore we do not guarantee that the quantities produced are specific amounts that the facility can rely on in determining the profit that it can achieve during the course of its work over time, loss or costs, so it was necessary to formulate a new mathematical method used when studying realistic problems. Consider the link of the available information about the system under study with reality and the fluctuations of the conditions surrounding the work environment, as we did for linear and dynamic programming, see [6,7,8,9]. In this paper, we present a study of nonlinear programming where we will reformulate some important definitions and concepts using the concepts of neutrosophic science.

2. Discussion

The problems of mathematical examples depend on the construction of mathematical models consisting of a goal function and constraints. These models may be linear, nonlinear or ---- models, for the goal function is either a maximization function or a minimization function of a certain quantity, this quantity depends on a number of decision variables, and they may be independent of each other or linked to each other through a set of constraints and we get values for these variables By solving the mathematical model that we get, but before that, we must identify the most important concepts and definitions related to linear functions, a thorough study of this was presented using classical values [1,2] and the results of solving nonlinear models were specific values that do not take into account the changes that may occur during the work of the systems that work according to these

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models, which may lead to large losses, which prompted us to prepare this research, through which we will reformulate some concepts of nonlinear programming using the concepts of neutrosophic science to be the basis for subsequent studies through which we present some methods of solving neutrosophic nonlinear models, which provides us with a safe working environment through the indeterminism enjoyed by neutrosophic values.

Based on the references [1,2], we present the following study:

3. Neutrosophic Mathematical Model:

In the problem of examples where the objective and constraints are in the form of neutrosophic mathematical functions, then the neutrosophic mathematical model is written in the following form:

$$Nf = Nf(x_1, x_2, --, x_n) \rightarrow (Max)or(Min)$$

According to the following restrictions:

$$Ng_i(x_1, x_2, --, x_n)\begin{pmatrix}\leq\\\geq\\=\end{pmatrix}Nb_i \quad ; i = 1,2,---,m$$

$$x_1, x_2, --, x_n \geq 0$$

In this model, such variables in the objective function and in constraints are neutrosophic values as well as the second side of the relationships that represent constraints; this model is a nonlinear model if any component of the target function or constraints are nonlinear statements and the nonlinear statements may be in both.

4. Some Important Neutrosophic Definitions in Nonlinear Programming

a. The quadrature form of the neutrosophic target function:

A function with variables $f(x_1, x_2, --, x_n)$ is called a quadratic form if the following is true:

$$Nf(x_1, x_2, --, x_n) = \sum_{i=1}^{n}\sum_{j=1}^{n}(Nq_{ij})x_i x_j = X^T(NQ)X$$

Where $NQ_{n\times n} = [Nq_{ij}]$ is a neutrosophic matrix, meaning that all or some of its elements are neutrosophic values of the shape $Nq_{ij} = q_{ij} + \varepsilon_{ij}$ where ε_{ij} is the indeterminism of the elements of the matrix and any neighborhood may be close to q_{ij}

It shall be written in one of the following forms: $[\lambda_{1ij}, \lambda_{2ij}]$ or $\{\lambda_{1ij}, \lambda_{2ij}\}$ or otherwise, and $X^T = (x_1, x_2, --, x_n)$

Here we can always assume without prejudice to the generality of the problem that NQ is symmetrical, we can replace the matrix NQ with the symmetrical matrix $\frac{(NQ+NQ^T)}{2}$ without changing the quadratic form.

b. Basic Minor Neutrosophic Matrices:

If the matrix NQ has dimensions $n \times n$, then the basic minimum matrix of the rank k is a mini-matrix with $k \times k$ dimensions and we obtain it by neglecting any $(n-k)$ lines and the corresponding columns of the matrix
Note: The determinant of the basic minor neutrosophic matrix is called the basic determinant and for each square matrix of the rank, $n \times n$ there is $2^n - 1$ basic determinant.

c. Main Basic Minor Neutrosophic Matrices:

The main basic minor neutrosophic matrices from rank k for matrix $n \times n$ are obtained from the matrix NQ by neglecting the last lines $n - k$ and its corresponding columns.

Example 1: Find the basic minor matrices and the main basic minor matrices from the following neutrosophic matrix:

$$NQ = \begin{bmatrix} [0,1] & [2,3] & [4,5] \\ [6,7] & [8,9] & [10,11] \\ [12,13] & [14,15] & [16,17] \end{bmatrix}$$

Basic minor matrices:
Of the first rank: the elements of the main diameter [0,1][8,9] ,[16,17]
From the second rank: the following matrices:

$$\begin{bmatrix} [0,1] & [2,3] \\ [6,7] & [8,9] \end{bmatrix}, \begin{bmatrix} [6,7] & [10,11] \\ [12,13] & [16,17] \end{bmatrix}, \begin{bmatrix} [8,9] & [10,11] \\ [14,15] & [16,17] \end{bmatrix}$$

From the third rank: NQ
Main basic minor matrices:
From the first rank, [0,1](we neglect the last two lines and the corresponding columns).

As for the main basic minor matrix of the second rank it is: $\begin{bmatrix} [0,1] & [2,3] \\ [6,7] & [8,9] \end{bmatrix}$

And the main basic minor matrix from the third rank is NQ.

Note: The number of major basic minor matrices in the matrix $n \times n$ the value n

d. **Neutrosophic function gradation:** $Nf(x_1, x_2, --, x_n)$

The neutrosophic function gradient is defined by the relation

$$\nabla f(x_1, x_2, --, x_n) = \left[\frac{\partial Nf}{\partial x_1}, \frac{\partial Nf}{\partial x_2}, ---, \frac{\partial Nf}{\partial x_n} \right]$$

e. **Hessian matrix of the neutrosophic function** $Nf(x_1, x_2, --, x_n)$:

It is a square and symmetrical matrix of the rank $n \times n$, denoted by $NH(x_1, x_2, --, x_n)$ and defined by the following relationship:

$$NH(x_1, x_2, --, x_n) = \left[\frac{\partial^2 Nf}{\partial Nx_i \partial Nx_j} \right]$$

f. **Positive defined neutrosophic matrix:**

A symmetrical neutrosophic matrix NQ is positively defined if the following is true**:**

- All diagonal elements must be positive.
- The main basic determinants should be positive.

Using the quadratic form, it is defined as follows:

We say about a matrix NQ that it is positive and defined if and only if the quadratic form satisfies the following:

$X^T(NQ)X > 0$ For all X values

g. **Positive quasi-defined neutrosophic matrix:**

A symmetrical neutrosophic matrix is NQ a quasi-positive definition if the following is true**:**

- All diagonal elements must be non-negative.
- Basic determinants must be non-negative.

Using the quadratic form, it is defined as follows:

We say about a matrix NQ that it is positive quasi-defined if and only if the quadratic form satisfies the following: $(NQ)X \geq 0X$ for all X^T values.

Note 1: To prove that a matrix NQ is negative (or negative quasi-defined) $(NQ$ -$)$ must be positive (or positive quasi-defined).

Using the quadratic form, it is defined as follows:

h. **The defined neutrosophic matrix is negative:**

We say that a matrix NQ is defined and negative if and only if $(-NQ)$ is positive and defined or if the quadratic form has satisfied $(NQ)X < 0X$ for all X^T values

i. **The quasi-defined neutrosophic matrix is negative:**

We say about a matrix, NQ that it is quasi-negative and defined if and only if $(-NQ)$ is positive and defined or if it satisfied the quadratic form: $X^T(NQ)X \leq 0$

Note 2: We say about Q matrix that it is not defined if $X^T(NQ)X$ was positive for values and negative for other values.

Note 3: The previous tests apply only to symmetrical matrices (and if the matrix NQ is asymmetrical, we replace it with the matrix $\frac{NQ - NQ^T}{2}$ and apply the tests.

j. **Convex function:** The function Nf is a convex function if the Hessian matrix of this function is positive or quasi-positive and defined for all variables.

k. **Concave Function:** The function Nf is concave if the Hessian matrix of this function has negative or quasi-negative and defined for all variables.

Example: Show whether the function Nf is convex or concave, where Nf is given by the following relationship

$$Nf = [3,3.5]x_1^2 + 4x_2^2 + 3x_3^2 + 2x_1x_2 + 3x_1x_3 + x_2x_3 - 4x_1 + 5x_2 + 3x_3$$

We find the partial derivatives of the function sNf with respect to the variable x_1, x_2, x_3

$$\frac{\partial Nf}{\partial x_1} = 2[3,3.5]x_1 + 2x_2 + 3x_3 - 4$$

$$\frac{\partial Nf}{\partial x_2} = 8x_2 + 2x_1 + x_3 + 5$$

$$\frac{\partial Nf}{\partial x_3} = 6x_3 + 3x_1 + x_2 + 3$$

Then we find the Hessian matrix of the function Nf

$$H_{Nf}(x_1, x_2, --, x_n) = \left[\frac{\partial^2 Nf}{\partial Nx_i \partial Nx_j}\right] = \begin{bmatrix} \frac{\partial^2 Nf}{\partial x_1^2} & \frac{\partial^2 Nf}{\partial x_1 \partial x_2} & \frac{\partial^2 Nf}{\partial x_1 \partial x_3} \\ \frac{\partial^2 Nf}{\partial x_2 \partial x_1} & \frac{\partial^2 Nf}{\partial x_2^2} & \frac{\partial^2 Nf}{\partial x_2 \partial x_3} \\ \frac{\partial^2 Nf}{\partial x_3 \partial x_1} & \frac{\partial^2 Nf}{\partial x_3 \partial x_2} & \frac{\partial^2 Nf}{\partial x_3^2} \end{bmatrix} = \begin{bmatrix} [6,7] & 2 & 3 \\ 2 & 8 & 1 \\ 3 & 1 & 6 \end{bmatrix}$$

And from it

$$H_{Nf} = \begin{bmatrix} [6,7] & 2 & 3 \\ 2 & 8 & 1 \\ 3 & 1 & 6 \end{bmatrix}$$

We note that H_{Nf} is symmetrical matrix and all diagonal elements are positive and the basic minimum determinants are positive because the main determinant of the third rank is equal to

$$\begin{vmatrix} [6,7] & 2 & 3 \\ 2 & 8 & 1 \\ 3 & 1 & 6 \end{vmatrix} = [6,7](8 \times 6 - 1) - 2(2 \times 6 - 3) + 3(2 - 3 \times 8) = [198,245] > 0$$

The main determinant of the second rank

$$\begin{vmatrix} [6,7] & 2 \\ 2 & 8 \end{vmatrix} = [6,7] \times 8 - 4 =$$
$$[48,56] - 4 = [44,52] > 0$$

The main determinant of the first rank $|[6,7]| > 0$

From the above, we can see that the Hessian matrix of Nf the function is positive, that is, the function Nf is a convex function.

1. The maximum value and the minimum value of a bound nonlinear function:

From the previous study, we can determine whether the value of a target function in a nonlinear model is a minimum value or a maximum value as follows:

– The value is minimal if the target is a convex function, and the set of constraints forms a convex zone.
– The value is maximal if the target is a concave function, and the set of constraints forms a convex zone.

5. Conclusion and Results

In this research, we presented a study of neutrosophic nonlinear dependencies, and what is mentioned in this research can be used when searching for optimal values for neutrosophic nonlinear models, as there are many practical problems that lead to nonlinear models and we need to obtain optimal solutions to them that take into account all the conditions and fluctuations that face us during the workflow, which is provided to us by the science of neutrosophy through the neutrosophic values that we can use to build neutrosophic nonlinear models and the specified through the realistic study of the issues under study, and we focus on the need to prepare complementary research for this research through which we present the most important methods used to find the optimal solution for nonlinear models using the concepts of neutrosophic science.

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Graphical Method for Solving Neutrosophical Nonlinear Programming Models

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Abstract: An important method for finding the optimal solution for linear and nonlinear models is the graphical method, which is used if the linear or nonlinear mathematical model contains one, two, or three variables. The models that contain only two variables are among the most models for which the optimal solution has been obtained graphically, whether these models are linear or non-linear in references and research that are concerned with the science of operations research, when the data of the issue under study is classical data. In this research, we will present a study through, which we present the graphical method for solving Neutrosophical nonlinear models in the following case: A nonlinear programming issue, the objective function is a nonlinear function, and the constraints are linear functions. Note that we can use the same method if **(i)** the objective function follower is a linear follower and the constraints are nonlinear; **(ii)** the objective function is a non-linear follower and the constraints are non-linear. In the three cases, the nonlinear models are neutrosophic, and as we know, the mathematical model is a nonlinear model if any of the components of the objective function or the constraints are nonlinear expressions, and the nonlinear expressions may be in both. At the left end of the constraints are neutrosophic values, at least one or all of them. Then, the possible solutions to the neutrosophic nonlinear programming problem are the set of rays $NX \in R^n$ that fulfills all the constraints. As for the region of possible solutions, it is the region that contains all the rays that fulfill the constraints. The optimal solution is the beam that fulfills all constraints and at which the function reaches a maximum or minimum value, depending on the nature of the issue under study (noting that it is not necessary to be alone).

Keywords: Nonlinear Models; Neutrosophic Logic; Neutrosophic Nonlinear Models; Graphical Method.

1. Introduction

Problems of mathematical examples search for maximizing or minimizing a certain quantity that we call the objective function, and this quantity depends on a number of decision variables, as these variables may be independent of each other or linked to each other through a set of constraints. Studying the methods of solving nonlinear programming problems that we encounter in many practical issues, for example when we want to determine the cost of producing or purchasing goods, as well as the cost of storing manufactured or unprocessed materials – and so on. It led to the creation of a basic structure used to find these solutions from these methods, the graphical solution method that was presented in many references using classical data, and due to the great interest in the research that was published in many international journals, which dealt with some topics of operations research using the concepts of science neutrosophic [1-11] The science that laid the foundations of the American scientist and mathematical philosopher Florentin Smarandache, which

explains the stages of its development. What was mentioned in the research [1], we will present in this research the graphic method used to find the optimal solution for nonlinear neutrosophic models, models that take data neutrosophic values of indefinite values A complete determination is not certain, and in reality it belongs to any neighborhood of the classical values and is given as follows: It is $Na = a \pm \varepsilon$ where ε is the indeterminacy and takes one of the forms $\varepsilon = [\lambda_1, \lambda_2]$ or $\varepsilon = \{\lambda_1, \lambda_2\}$ or $\varepsilon \in [\lambda_1, \lambda_2]$ -- otherwise, which is any neighborhood of the value a that we get during data collection.

2. Discussion

The importance of nonlinear models comes from the fact that many practical issues lead to nonlinear models, which prompted many researchers and scholars to search for ways to solve these models. Many methods were presented that helped the great development of computer science to find them and were presented according to classical logic, i.e. data were specific values. Appropriate for the time period in which they were collected, and since the purpose of any study of such issues is to develop plans for the course of work in the future, the decision makers faced great difficulty because of the instability of the conditions surrounding the work environment and in order to control all conditions and provide ideal decisions for the issues that turn into models In two previous researches, we presented a formulation of some concepts of nonlinear programming, and one of the ways to solve it is the method of Lagrangian multiplication for models constrained by equal constraints using the concepts of neutrosophic science [12,13].

The neutrosophic mathematical model [12]:

In the problem of examples where the objective and constraints are in the form of neutrosophic mathematical functions, then the neutrosophic mathematical model is written in the following form:

$$Nf = Nf(x_1, x_2, --, x_n) \rightarrow (Max) \, or \, (Min)$$

According to the following restrictions:

$$Ng_i(x_1, x_2, --, x_n) \begin{pmatrix} \leq \\ \geq \\ = \end{pmatrix} Nb_i \quad ; i = 1,2, ---, m$$

$$x_1, x_2, --, x_n \geq 0$$

In this model, the examples of the variables in the objective function and in the constraints are neutrosophic values, as well as the other side of the relations that represent the constraints.

Based on the information provided in the reference [14]:

The graphic method to find the optimal solution for nonlinear problems:

This method is suitable for simple problems that contain only two variables, it is impractical for problems that contain more than one variables or in which the objective function is complex in addition to the presence of restrictions that we cannot express in simple forms, so to find the optimal solution for a nonlinear model in a graphic way we represent the constraints among the coordinate axes, we define the common solution area for these constraints, so that it is the area of the accepted solutions for the mathematical model, then we represent the objective function in order to determine the optimal solution.

We have the following example using classic values [14]:

$$Maxf(x_1, x_2) = (x_1 - 2)^2 + (x_2 - 3)^2$$

$$x_1 + 2x_2 \leq 12$$

$$x_1 + x_2 \leq 9$$

$$x_1, x_2 \geq 0$$
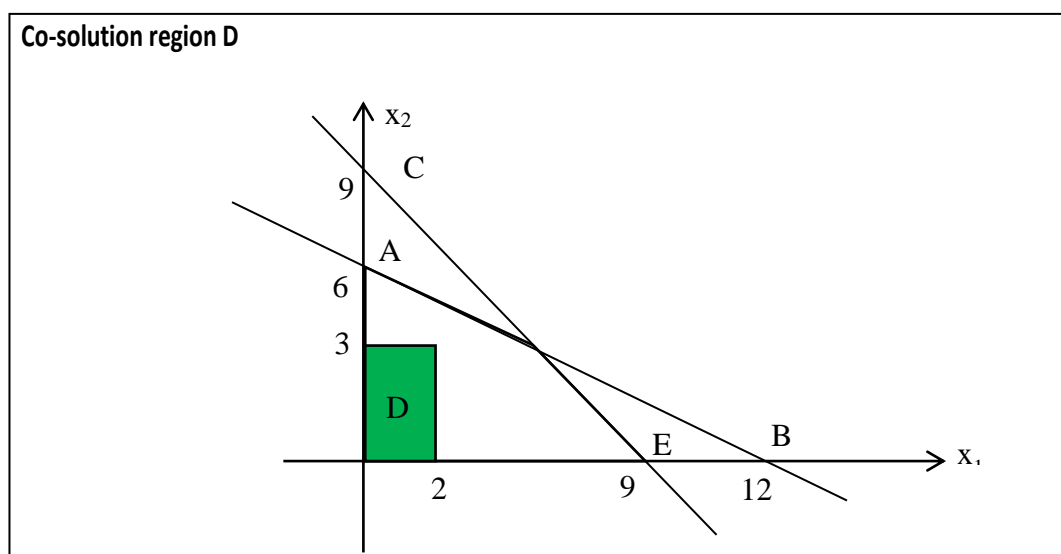
Figure 1. The first figure represents the solution to Example No (1).

From the Figure 1, it is clear $f^* = Minf = (2 - 2)^2 + (3 - 3)^2 = 0$

Whereas, the smallest value reached by the function is in the center $M(2,3)$

As for if it is required to find $Maxf$ it will be on point $E(9,0)$ and therefore
$$f^* = Maxf = (9 - 2)^2 + (0 - 3)^2 = 58$$

Neutrosophical formula for the previous example:

Finding the optimal solution for a nonlinear programming problem, where the objective function is a nonlinear function, and the constraints are linear functions:

Find the minimum value of the function:
$$MinNf(x_1, x_2) = (x_1 - 2)^2 + (x_2 - 3)^2$$

Within the restrictions:

$$x_1 + 2x_2 \leq 12 + \varepsilon_1$$

$$x_1 + x_2 \leq 9 + \varepsilon_2$$

$$x_1, x_2 \geq 0$$

Where ε_1 and ε_2 It is the indeterminacy and we take it as follows.

Then the restrictions will look like this:

In this example, we take $\varepsilon_1 \in [0, 3]$ and $\varepsilon_2 \in [0, 2]$, and then the problem is written as follows:

$$x_1 + 2x_2 \in [12, 15]$$

$$x_1 + x_2 \in [9, 11]$$

$$x_1, x_2 \geq 0$$

Clarification: The second term of the constraints expresses the available potentials taken as Neutrosophical values.

We need to find the vector $NX^* = (x_1^*, x_2^*)$ So that the inequality is fulfilled:

$$Nf(NX^*) \leq Nf(X)$$

$$\forall NX \in D$$

The solution:

1. We define the solution area D This is done by representing the constraints in the coordinate plane ox_1x_2

First constraint:

$$x_1 + 2x_2 \in [12 , 15]$$

We draw the straight line represented by the equation $x_1 + 2x_2 \in [12 , 15]$

$$x_1 = 0 \Rightarrow 2x_2 \in [12 , 15] \Rightarrow x_2 \in [6 , 7.5]$$

$$A(0 , [6 , 7.5])$$

$$x_2 = 0 \Rightarrow x_1 \in [12 , 15] \Rightarrow$$

$$B([12 , 15] , 0)$$

We define the region where the first constraint is satisfied:

We know that the line represented by equation $x_1 + 2x_2 \in [12 , 15]$ the plane defined by the first quadrant is divided into two halves of a plane. We take a point, not on the specificity, from one of the two halves of the plane, and let the point be $(0 , 0)$ we substitute in the constraint, we note that it achieves the inequality that represents the first constraint, that is, half of the plane to which this point belongs is half of the solution plane.

The second constraint:

$$x_1 + x_2 \in [9 , 11]$$

We draw the straight line represented by the equation $x_1 + x_2 \in [9 , 11]$

$$x_1 = 0 \Rightarrow x_2 \in [9 , 11]$$

$$C(0 , [9 , 11])$$

$$x_2 = 0 \Rightarrow x_1 \in [9 , 11] \Rightarrow$$

$$E([9 , 11], 0)$$

We define the region where the second constraint is satisfied:

The line represented by equation $x_1 + x_2 \in [9 , 11]$, the plane defined by the first quarter is divided into two halves of a plane. We take a point, not to be determined, from one of the two halves of the plane, let the point be $(0 , 0)$ and substitute in the constraint. We note that it achieves the inequality representing the second constraint, meaning that half of the plane to which this point belongs is half of the solution plane.

To find the optimal solution, we draw the objective function, which is a circle with a point center $M(2 , 3)$ radius $r = \sqrt{Nf}$.

From the figure, it is clear that $Min\, Nf = 0$ the minimum value reached by the objective function is at the center of the circle, i.e. at the point $M(2 , 3)$.

If required, find the maximum value of the function:

$$Max f(x_1 , x_2) = (x_1 - 2)^2 + (x_2 - 3)^2$$

Within the restrictions:

$$x_1 + 2x_2 \leq 12 + \varepsilon_1$$

$$x_1 + x_2 \leq 9 + \varepsilon_2$$

$$x_1 , x_2 \geq 0$$

In this case, we know that the optimal solution is located on the vertices of the common solution region, i.e. on the vertices of the polygon $OADE$, we have the coordinates of these points $O(0 , 0)$ ، $A(0 , [6 , 7.5])$ ، $E([9 , 11], 0)$ for the coordinates of the point D , and we determine it from the study of the intersection of the two lines represented by the following equations:

$$x_1 + 2x_2 \in [12, 15]$$

$$x_1 + x_2 \in [9, 11]$$

Solving the two equations, we get $D([6, 7], [3, 4])$ then we calculate the value of the function at these points

$$(O(0, 0)) = (0 - 2)^2 + (0 - 3)^2 = 13$$

$$f(A(0, [6, 7.5])) = (0 - 2)^2 + ([6, 7.5] - 3)^2 = 4 + ([3, 4.5])^2 \in [13, 24.25]$$

$$f(E([9, 11], 0)) = ([9, 11] - 2)^2 + (0 - 3)^2 = ([7, 9])^2 + 9 \in [58, 90]$$

$$f(D([6, 7], [3, 4])) = ([6, 7] - 2)^2 + ([3, 4] - 3)^2 \in [19, 24]$$

The Maximum value the function takes at point $E([9, 11], 0)$ and is

$$R = \sqrt{[58, 90]} \in [7.6, 9.5]$$

Therefore, the optimal solution is a circle centered at point $M(2, 3)$ and whose radius is one of the domain values $[7.6, 9.5]$.

For clarification, we draw the Figure 2, for one of the values we find:
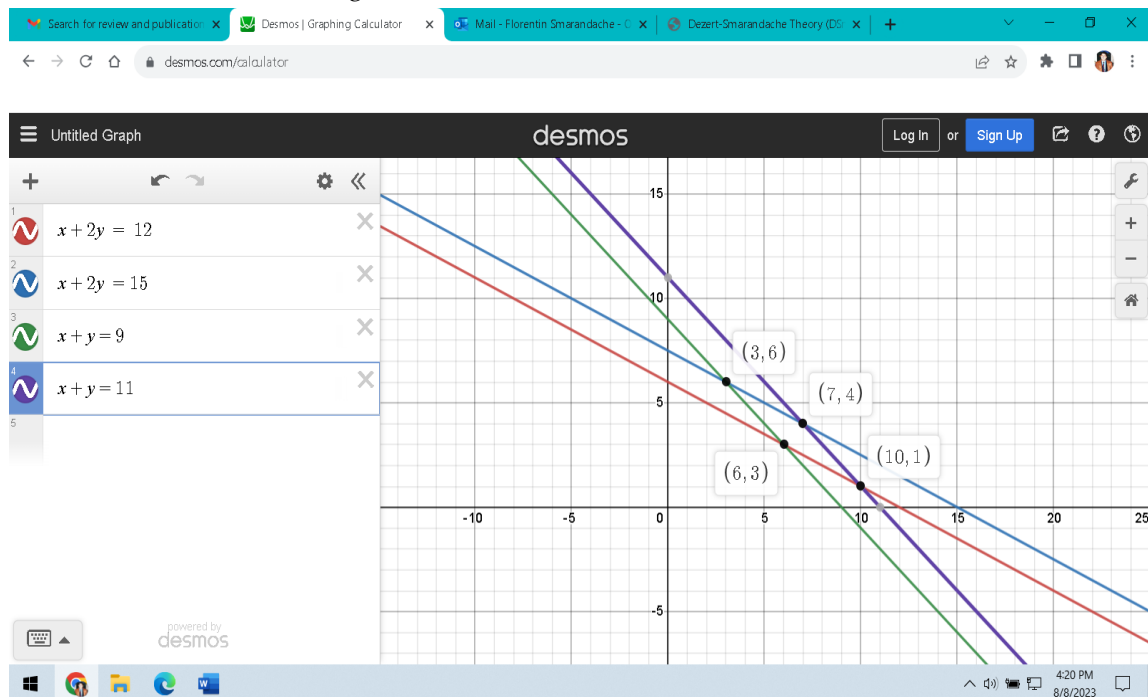
Figure 2. Determination of the joint solution region of neutrosophic constraints.

For the Neutrosophical objective function, it represents a set of circles whose center $M(2, 3)$ and radius are one of the domain values $[7.6, 9.5]$.

The following Figure 3 shows one of these circles:
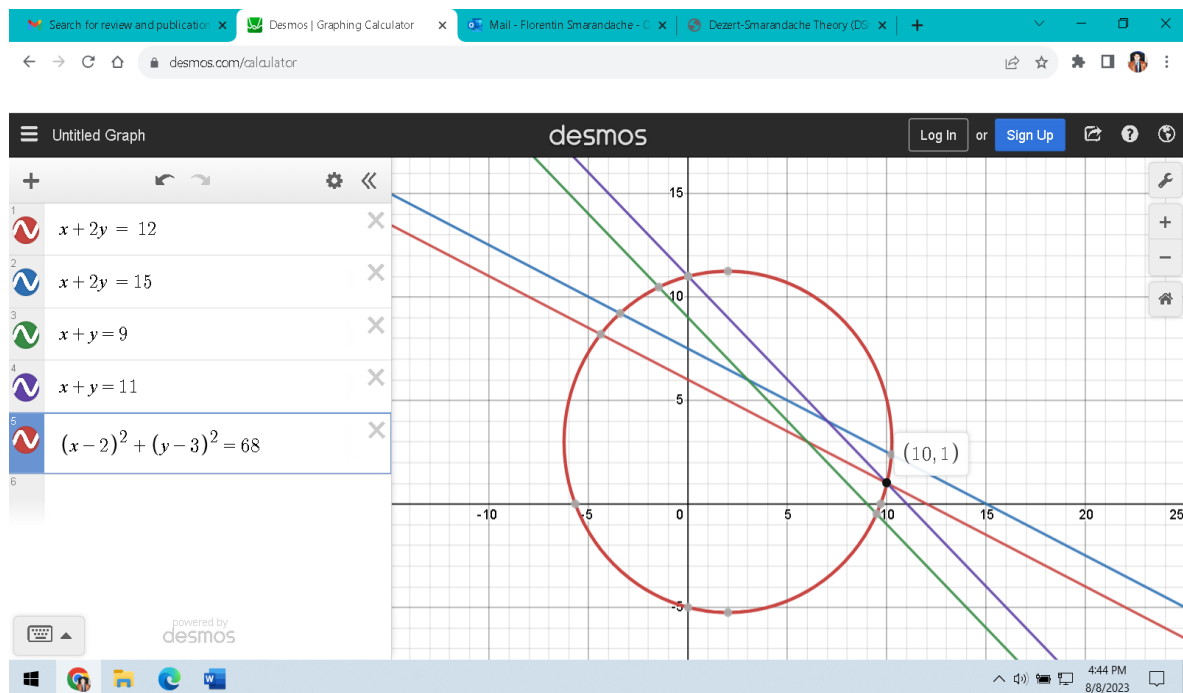
Figure 3. The graphical representation of one of the solutions of the example neutrosophical values.

We note that for both sides of the domain in both constraints is the optimal solution $f^* = Max f = 68 \in [58, 90]$, wear $[58, 90]$, the field that represents the maximum optimal solution of the neutrosophic model.

3. Conclusions

The graphical method is one of the important methods for finding the optimal solution for linear and nonlinear models. Therefore, it was necessary to present this study, which explains the difference between dealing with classical values and Neutrosophical values, and as we noticed from the results of the solution in the example, Neutrosophical values give us optimal solutions that are close to the optimal solution in the case of classical values, that is, they are in line with the conditions surrounding a work environment The system that this mathematical model represents, so it provides a safe environment that protects the systems from falling into losses and making greatest profits from them.

Data availability

The datasets generated during and/or analyzed during the current study are not publicly available due to the privacy-preserving nature of the data but are available from the corresponding author upon reasonable request.

Conflict of interest

The authors declare that there is no conflict of interest in the research.

Ethical approval

This article does not contain any studies with human participants or animals performed by any of the authors.

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Lagrange Multipliers and Neutrosophic Nonlinear Programming Problems Constrained by Equality Constraints

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Abstract: Operations research science is defined as the science that is concerned with applying scientific methods to complex problems in managing and directing large systems of people, including resources and tools in various fields, private and governmental work, peace and war, politics, administration, economics, planning and implementation in various domains. It uses scientific methods that take the language of mathematics as a basis for it and uses computer, without which it would not have been possible to achieve numerical solutions to the raised problems, those that need correct solutions, when the solutions abound and the options are multiple, so we need a decision based on correct scientific foundations and takes into account all the circumstances and changes that you can encounter the decision-maker during the course of work, and nothing is left to chance or luck, but rather everything that enters into the account and plays its role in decision-making, and we get that when we use the concepts of neutrosophic science to reformulate what the science of operations research presented in terms of methods and methods to solve many practical problems, so we will present in this research a study aimed at shedding light on the most important methods used to solve nonlinear models, which is the Lagrangian multiplier method for nonlinear models constrained by equality and then reformulated using the concepts of neutrosophic science.

Keywords: Operations Research; Nonlinear Programming; Lagrange Multiplier; Neutrosophic Science; Neutrosophic Nonlinear Programming; Lagrange Neutrosophic Multiplier.

1. Introduction

Science is the basis for managing life affairs and human activities. Operations research science is the applied aspect of mathematics. It is one of the most important modern sciences that are concerned with practical issues and meet the desire and demand of decision-makers to obtain ideal decisions through the methods it presented that are appropriate for all issues, meaning that it depends on the attached data. In every issue, these data are values that have been developed through observation, and their accuracy depends on who collects them. They are also specific for a certain period that may not be appropriate for subsequent circumstances. Therefore, they are inaccurate data that are not fully confirmed. Before the advent of neutrosophic science, we used to rely on this data. To study factual issues and accept the results as they are, but after the neutrosophic science that made a great revolution in all fields. Its ability to produce optimal results upon which ideal decisions can be built that take into account all circumstances. Many researchers interested in scientific development tended to reformulate many scientific concepts using this science, so we have neutrosophic numbers - neutrosophic groups - neutrosophic probabilities - neutrosophic statistics - neutrosophic

differentiation - neutrosophic integration - neutrosophic linear programming - neutrosophic dynamic programming - neutrosophic simulation, etc. and many more [1-28].

The method of nonlinear programming is one of the most important methods presented by the science of operations research because most of the practical issues devolve into nonlinear models. Therefore, in previous research, we formulated some basic concepts of nonlinear programming using the concepts of neutrosophy [26]. In addition to what we have done earlier, in this research we will paraphrase the Lagrangian multiplication method for nonlinear models constrained by equal constraints using the concepts of neutrosophic science, by taking the data of the issue under study, neutrosophic numbers It has the following standard form N = a+bI where real or composite coefficients, represents a and baspecified part fbI The indefinite part of the numberNindeterminacy and could be $[\lambda_1, \lambda_2]$or$\{\lambda_1, \lambda_1\}$ or something else is any set close to the true value a.

2. Discussion

We know that if the nonlinear programming issue consists of only a target function and the target function is convex or (concave), then there is a single optimal solution at a point where all the derivatives are non-existent. The only optimal, but in most realistic issues the goal is to find the maximum or minimum value of a target function that is subject to several restrictions. To calculate one of the variables in terms of the rest of the variables, then replace it with the objective function statement, thus obtaining a new objective function and a new unrestricted example problem. We will use this technique as mentioned in the references [29,30].

To convert nonlinear restricted neutrosophic problems into nonlinear non-restricted neutrosophic problems, and for that, the following information taken from the references must be recalled [27, 28]:

$$f_N'(x) = \lim_{h \to 0} \frac{[\inf f(x+h) - \inf f(x), \sup f(x+h) - \sup f(x)]}{[\inf H, \sup H]}$$

This definition is a generalization of the traditional derivative definition.

Text of the constrained neutrosophic nonlinear programming problem:

Based on what is stated in the reference [29], we offer the following:

If it is required to find the (maximum or minimum), value of a function Neutrosophic is continuous, derivable $y_0 = f_N(x_1, x_2, ---, x_n)$ and subject to the neutrosophic constraint $g_N(x_1, x_2, ---, x_n) = \alpha$ Also this function is continuous and derivable.

1. We choose the variable x_n in the constraint and express it with the remaining $n-1$ variables as follows:

$$x_n = H_N(x_1, x_2, ---, x_{n-1})$$

Then we substitute the target function we get

$$y_0 = \overline{f_N}(x_1, x_2, ---, x_{n-1}, H_N(x_1, x_2, ---, x_{n-1}))$$

Thus, the problem turned into an unrestricted one. Traditional methods can be used to obtain a maximum or minimum limit because the necessary and sufficient condition for the boundary points is that the first derivatives do not exist:

$$\frac{\partial y_0}{\partial x_j} = 0 \quad ; j = 1, 2, ---(n-1)$$

Using the chain rule, we get the following:

$$\frac{\partial y_0}{\partial x_j} = \frac{\partial \overline{f_N}}{\partial x_j} + \frac{\partial \overline{f_N}}{\partial x_n} \cdot \frac{\partial H_N}{\partial x_j} \quad ; j = 1,2, - - -(n-1)$$

Since the $g_N(x_1, x_2, - - -, x_n) = \alpha$

We get

$$\frac{\partial g_N}{\partial x_j} + \frac{\partial g_N}{\partial x_n} \cdot \frac{\partial H_N}{\partial x_j} = 0 \quad ; j = 1,2, - - -(n-1)$$

$$\Rightarrow \frac{\partial H_N}{\partial x_j} = - \frac{\dfrac{\partial g_N}{\partial x_j}}{\dfrac{\partial g_N}{\partial x_n}} \quad ; j = 1,2, - - -(n-1)$$

On condition: $\frac{\partial g_N}{\partial x_n} \neq 0$

Then we find

$$\frac{\partial y_0}{\partial x_j} = \frac{\partial \overline{f_N}}{\partial x_j} - \left[\frac{\partial \overline{f_N}}{\partial x_n} \cdot \frac{\dfrac{\partial g_N}{\partial x_j}}{\dfrac{\partial g_N}{\partial x_n}} \right] = 0 \quad ; j = 1,2, - - -(n-1)$$

If the resulting solution vector is the vector that achieves a (maximum or minimum) value, then$(x_{N1}^*, x_{N2}^*, - - -, x_{Nn}^*)$, They are the values that make the function maximum or minimum.

We symbolize λ_N the following:

$$\lambda_N = \frac{\dfrac{\partial f_N}{\partial x_n}}{\dfrac{\partial g_N}{\partial x_n}}$$

So it is:

$$\frac{\partial \overline{f_N}}{\partial x_j} - \lambda_N \frac{\partial g_N}{\partial x_j} = 0 \quad ; j = 1,2, - -, n$$

The supporting condition is:

$$g_N(x_1, x_2, - - -, x_n) = \alpha$$

And so we have obtained $n + 1$ Equation and $n + 1$ unknown, these conditions are necessary for an optimal solution, provided that all derivatives do not exist $\frac{\partial g_N}{\partial x_j} \neq 0$

when$(x_{N1}^*, x_{N2}^*, - - -, x_{Nn}^*)$

From the above, we can write the following:

$$y_0 = \overline{f_N}(x_1, x_2, - - -, x_n) - \lambda_N [g_N(x_1, x_2, - - -, x_n) - \alpha]$$

Then, we calculate the derivatives:

$$\frac{\partial y_0}{\partial x_j} = \frac{\partial f_N}{\partial x_j} - \lambda_N \frac{\partial g_N}{\partial x_j} = 0$$

$$\frac{\partial y_0}{\partial \lambda} = -[g_N(x_1, x_2, - - -, x_n) - \alpha]$$

2.1 Example

This example is shown in reference [29] (values are dependent and constraints are classic values)

2.1.1.   Find the minimum value of the function

$$f(x) = 3x_1^2 + x_2^2 + 2x_1x_2 + 6x_1 + 2x_2$$

Subject to constraint

$$2x_1 - x_2 = 4$$

The solution

We form the Lagrangian function:

$$L(x, \lambda) = 3x_1^2 + x_2^2 + 2x_1x_2 + 6x_1 + 2x_2 - \lambda(2x_1 - x_2 - 4)$$

The following optimal solution was obtained:

$$x_1^* = \frac{7}{11} \quad , x_2^* = \frac{-30}{11} \quad , \quad \lambda^* = \frac{24}{11}$$

And by testing the function using the Hessian matrix, it was found that this function is a convex function and the constraint is convex

Then the radius of the solution that we obtained is a minimum limit and the value of the function is:

$$f_N^*(x_1^*, x_2^*) = 3.55$$

As we mentioned earlier, if some of the values in the target function or constraints are undefined, ambiguous, or uncertain values, then the issue becomes a neutrosophic issue clearly)

In the previous example, we will take the coefficients x_1^2 uncertain (All values can be undefined; we will suffice with one value to convey the idea of using neutrosophic numbers clearly

2.1.2   Find the minimum value of the neutrosophic function.

$$f(x) = \{2,3,4\}x_1^2 + x_2^2 + 2x_1x_2 + 6x_1 + 2x_2$$

Subject to constraint

$$2x_1 - x_2 = 4$$

The solution

We form the Lagrangian function:

$$L(x, \lambda) = \{2,3,4\}x_1^2 + x_2^2 + 2x_1x_2 + 6x_1 + 2x_2 - \lambda(2x_1 - x_2 - 4)$$

$$\frac{\partial L}{\partial x_1} = \{4,6,8\}x_1 + 2x_2 + 6 - 2\lambda = 0$$

$$\frac{\partial L}{\partial x_2} = 2x_1 + 2x_2 + 2 + \lambda = 0$$

$$\frac{\partial L}{\partial \lambda} = 2x_1 - x_2 + 4 = 0$$

We solve the following set of equations:

$$\{4,6,8\}x_1 + 2x_2 + 6 - 2\lambda = 0$$
$$2x_1 + 2x_2 + 2 + \lambda = 0$$
$$2x_1 - x_2 + 4 = 0$$

We get

We obtained the system of three equations with three unknowns. By solving this system, we get:

$$x_1^* = \left\{\frac{7}{10}, \frac{7}{11}, \frac{7}{12}\right\} \quad, x_2^* = \left\{\frac{-13}{5}, \frac{-30}{11}, \frac{-17}{6}\right\} \quad, \quad \lambda^* = \left\{\frac{9}{5}, \frac{24}{11}, \frac{15}{6}\right\}$$

They are neutrosophic values, and the value of the target function is as follows:

$$f_N^*(x_1^*, x_2^*) = \{3.1, 3.55, 3.24\}$$

That is, we got the optimal solution vector, which is a neutrosophic value. An extreme of a subordinate, it can be a minor end or a major end. To determine its type we will use the test given in reference [26].

1.   The constraint is a linear function, as it is convex and concave at the same time.

2.   To specify the type of follower $f(x)$(convex or concave) we resort to the Hessian matrix for this function:

$$H_N(x) = \begin{bmatrix} \dfrac{\partial^2 f}{\partial x_1^2} & \dfrac{\partial^2 f}{\partial x_1 \partial x_2} \\ \dfrac{\partial^2 f}{\partial x_2 \partial x_1} & \dfrac{\partial^2 f}{\partial x_2^2} \end{bmatrix}$$

$$H_N(x) = \begin{bmatrix} \{4,6,8\} & 2 \\ 2 & 2 \end{bmatrix}$$

The matrix is symmetric and the main diagonal elements are also positive.

Major basic minor determinants are positive because:

$$|\{4,6,8\}| > 0$$

$$\begin{vmatrix} \{4,6,8\} & 2 \\ 2 & 2 \end{vmatrix} = \{8,12,16\} - 4 = \{4,8,12\} > 0$$

Hence the Hessian matrix of the function $f_N(x)$Positive knowledge, that is, the function is convex.

From the above, the Vector of the solution that we obtained is a minor limit of the value neutrosophic to follow him:

$$f_N^*(x_1^*, x_2^*) = \{3.1, 3.55, 3.24\}$$

3.   Conclusions

In the previous study, we presented an important technique for solving neutrosophic nonlinear models constrained by equal constraints. Through the study, we found that there is a difference in the values of the optimal solution when using the technique and the data are traditional values and using them and the data are neutrosophic values, since the goal of solving examples problems is to find the maximum value that expresses profit or profitability and the smallest value that expresses the amount of loss or cost for a follower of a goal within certain constraints, and since mathematical models are built using data collected on the case under study, and these data are values that represent the current reality of the work environment, and any change in the surrounding conditions leads to a change in the results of the solution, which may cause unexpected losses whose nature is

determined by the type of issue under study, it may be human, material, or otherwise, so we focus on the need to use neutrosophic values when collecting data for any realistic issue, values that take into account the worst conditions to the best, as we focus on necessity reformulation of many other techniques for solving nonlinear models using neutrosophic concepts such as projection gradients and vibration–Newton's method–Fibonacci Search.

Data availability

The datasets generated during and/or analyzed during the current study are not publicly available due to the privacy-preserving nature of the data but are available from the corresponding author upon reasonable request.

Conflict of interest

The authors declare that there is no conflict of interest in the research.

Ethical approval

This article does not contain any studies with human participants or animals performed by any of the authors.

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The static model of inventory management without a deficit with Neutrosophic logic

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Abstract

In this paper, we present an expansion of one of the well-known classical inventory management models, which is the static model of inventory management without a deficit and for a single substance, based on the neutrosophic logic, where we provide through this study a basis for dealing with all data, whether specific or undefined in the field of inventory management, as it provides safe environment to manage inventory without running into deficit , and give us an approximate ideal volume of inventory. Since the ideal size is affected by the rate of demand for inventory, we present in this paper a study of the rate of demand for inventory when it is not precisely defined, and we find that the indeterminate values of the demand rate cannot be ignored, because they actually affect determining the ideal size of inventory and calculating its costs, thus affecting on the efficiency of the facility and achieve great profits for it.

1.Introduction

 A Neutrosophic ; new view of modeling , designed to effectively address the uncertainties inherent in the real world, It came to replace the binary logic that admits right and wrong by introducing a neutral third state that can be interpreted as undetermined or uncertain. It was founded by the American mathematician Florentine Smarandache [2,4,5,6,8] . Where he presented it in 1995 as a generalization of fuzzy logic and an extension of the theory of fuzzy sets [3] presented by Lotfi Zadeh in 1965 [1]. In addition, Ahmed Salama presented the theory of classical neutrosophic sets as a generalization of the theory of classical sets [7,15,16], and he developed, introduced and for mulated new concepts in the fields of mathematics, statistics, computer science and others through neutrosophic [17,25].

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Neutrosophic has grown significantly in recent years, through its application in measurement, sets, graphs, and many scientific and practical fields [10,11,12,13,14,19,20,21,22].

In this paper, we highlight the application of neutrosophical logic to one of the inventory management models, which is the static model without deficit and for a single substance, which will open the way for us to deal with inventory management models whose formulas are expressed in a way that is not precisely defined or uncertain, as if We take ranges of values instead of a single value, as is known in the classic, and in particular we will present the static model without a deficit for a single substance according to the neutrosophic logic ,when the rate of demand for stock is defined in an undefined way, and then we will present an applied example that shows how to deal with this case.

2. Discussion:

Inventory management plays an important role in production and marketing operations, especially in production facilities and commercial establishments that have warehouses where they keep their equipment and goods[9].

 Inventory management is one of the most important functions of management, in terms of determining the ideal size of inventory and calculating its costs, as this affects the efficiency of the facility and achieves either great profits or causes it to suffer heavy losses.

It is often the responsibility of warehouse managers in production facilities or enterprises to determine the appropriate and ideal volume of Inventory of each material, to secure the demand in a certain time. We know that if the volume of the Inventory is very large, this guarantees the provision of the material on the one hand, but in return it may cause the institution losses because the value of the Inventory is frozen capital, and this large quantity needs a marketing period that depends on the rate of demand.

But if the inventory quantity is small, this may lead to a stalemate in securing materials and to various disturbances, such as price hikes and others.

Therefore, it was necessary to present a study to help us determine the ideal size of the inventory at the lowest possible cost.

When studying static inventory models in classical logic, the rate of demand for inventory was limited, and we found this rate is subject to a uniform probability distribution. Thus when the rate of demand for inventory is undetermined with the time, we use the neutrosophic uniform probability distribution, which was studied previously [18].

2.1. The main hypotheses of the study:

1. order size Q .
2. The rate of demand for inventory during a given time λ_N (unspecified) where $\lambda_N = \{\lambda_1, \lambda_2\}$ or $\lambda_N = [\lambda_1, \lambda_2]$ or … .

   so that: λ_1 minimum rate of demand for inventory.

      λ_2 Maximum rate of demand for inventory.

3. Fixed cost of order preparation $C_1 = K.$
4. Cost of purchase, delivery and pickup $C_2 = C. Q$.
5. Storage cost for the remaining quantity in the warehouse during a certain time C_3.

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6. inventory run out time is $\dfrac{Q}{\lambda_N}$ (or the duration of the storage cycle).

- We denote by q_t the quantity remaining in the warehouse at moment t during the period $\left[0, \dfrac{Q}{\lambda_N}\right]$

    (the duration of the storage cycle) , q_t is given as:

$$q_t = Q - \lambda_N\, t$$

- To calculate C_3 the "storage cost", we divide the period $\left[0, \dfrac{Q}{\lambda_N}\right]$ into n partial periods,

    of length Δt.

    If t_i is a point of partial period i , then the remaining quantity of inventory corresponding to that partial period is equal to:

$$q_i = Q - \lambda_N\, t_i$$

    Thus the storage cost in the partial period is:
$$C_i = h \cdot q_i \cdot \Delta t = h(Q - \lambda_N\, t_i)\, \Delta t$$

Therefore, the total storage cost during the period $\left[0, \dfrac{Q}{\lambda_N}\right]$ is the sum of the partial costs,

ie: $C_3 = \sum_i^n C_i$

When $\Delta t \to 0$, the number of partial periods is $n \to \infty$, then:

$$C_3 = \lim_{n \to \infty} h(Q - \lambda_N t_i)\Delta t$$

$$= \int_0^{\frac{Q}{\lambda_N}} h(Q - \lambda t)dt = \frac{hQ^2}{2\lambda_N}$$

That is, the cost of storage during $\left[0, \dfrac{Q}{\lambda_N}\right]$ is:

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$$C_3 = \frac{hQ^2}{2\lambda_N}$$

The total cost is:

$$TC(Q) = C_1 + C_2 + C_3$$
$$= K + CQ + \frac{hQ^2}{2\lambda_N}$$

To get the cost of storage at a given time, we divide the total cost $TC(Q)$ by the length of the period $\dfrac{Q}{\lambda_N}$,

denoted by the symbol C(Q):

$$C(Q) = \frac{K\lambda_N}{Q} + C\lambda_N + \frac{hQ}{2}$$

To determine the maximum values of the function C(Q), with the condition Q > 0, we find:

$$\frac{dC(Q)}{dQ} = \frac{-K\lambda_N}{Q^2} + \frac{h}{2}$$

$$\frac{dC(Q)}{dQ} = 0 \Rightarrow Q = \sqrt{\frac{2K\lambda_N}{h}}$$

To specify the type of this maximum value:

$$\frac{d2C(Q)}{dQ^2} = \frac{2\lambda_N K}{Q^2} > 0$$

So the function C(Q) achieves a minimum value when $Q = \sqrt{\dfrac{2K\lambda_N}{h}}$, The ideal size for the demand is:

$$Q_N^* = \sqrt{\frac{2K\lambda_N}{h}} \qquad ......(**)$$

3. Application example:

Assuming we have a production facility, which stores spare pieces for its production lines, and that the rate of demand for these pieces in the warehouse is not absolutely specified, but ranges between [2.4] pieces per month, According to the conditions of the facility such as working pressure, operating pressure, weather conditions, etc., and that the cost of Preparing a single demand of Q Pieces is $100, and the cost of storing one piece for a month is $20.

Let us calculate the ideal size of the demand , which makes the cost of storage as small as possible, taking into account that the rate of demand for Pieces is an indeterminate case and ranges between $\lambda_N = [2, 4]$.

And :

K = $100 (the cost of preparing the demand).

h = $20 (the cost of storing one piece for a month).

We find, based on (**):

$$Q_N^* = \sqrt{\frac{2K\lambda_N}{h}} = \sqrt{\frac{2(100)[2,4]}{20}} = \sqrt{[20,40]} = [4.5,6.3]$$

That is, the ideal size of the demand ranges from 4 to 6 spare Pieces per month, so that we remain in the safety of Inventory management and avoid reaching the stage of deficit.

- Calculating the duration of the storage cycle in hours, knowing that the number of working days per month is 23 days and the number of working hours per day is only 8 hours. We find:

$$T_N^* = \frac{Q_N^*}{\lambda_N} = \frac{[4.5,6.3]}{[2,4]} = [1.1,3.2]$$

The time period is in hours:

$$[1.1, 3.2] \times 23 \times 8 = [202.4 , 588.8]$$

That is, between 8 to 24 days.

4. Conclusion and results:

We conclude that dealing with inventory management models, under the neutrosophic logic, provides us with a more general and comprehensive study than the well-known classical study, so that it does not neglect any data just because it is not explicitly specified, especially in our dealing with the static model without a deficit , we found that the unspecified value of demand for inventory has affected the ideal size of the inventory , and it was given approximatly, so that it gives the minimum ideal size of the inventory that protects from reaching the stage of deficit and loss, and gives the maximum ideal size of the inventory so that it makes the inventory available in the best quantity without cause loss for the facility.

Thus, we find that the indeterminacy actually affects the results, and these unspecified values cannot be ignored and removed from the study framework , in order to obtain the most accurate results as possible, and thus obtain the ideal volume of inventory that meets the needs of the facility , during the period of the storage cycle at the lowest cost.

We find that working within the classical logic is no longer sufficient at the present time, as the development of science has put a large number of new problems that need more general and accurate results than the results we got it using classical logic and fuzzy logic, and here comes the role of neutrosophic logic, which provides us with more comprehensiveness to Interpret the study data and get the most accurate results possible.

In the near future, we look forward to studying the rest of the inventory management models according to the neutrosophic logic , such as inventory models with a deficit , dynamic models, and others..

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Important Neutrosophic Economic Indicators of the Static Model of Inventory Management without Deficit

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Abstract:

Inventory models are generally concerned with finding the volume of inventory to be kept to ensure the continuity of work, which achieves the facility with the lowest cost and the greatest profit, and since the ideal volume is affected by the rate of demand for inventory, in previous research [5] we studied the static model without a deficit according to the neutrosophic logic and we came to a relationship that we calculate through which the ideal size of inventory, the result was a neutrosophic value that takes into account all the fluctuations and changes that administrators may encounter during the course of work through the indeterminacy and indeterminate values of the demand rate, in this research we use the neutrosophic concepts of static model of inventory management without deficit and ideal volume which we have in last research, for most important neutrosophic economic indicators that play an important role in the administrative decision-making process about the functioning of production facilities and commercial establishments that have warehouses in which they keep their equipment, goods and manufactured and unmanufactured materials necessary for the production process and provide them with a future vision of the reality of their condition over time.

Keywords: Inventory Management; Static Model of Inventory Management; Indicators of the Static Model without Deficit; Neutrosophic Logic-;Neutrosophic Economic Indicators.

Introduction:

Operations research in its essence is a science and an art, a science of good thinking, and the art of finding the best solutions and reaching them through scientific means, so that nothing is left to chance or luck, and takes decision-making according to scientific methods that use the language of mathematics mainly it uses the computer, without which it would not have been possible to achieve this great development in the process research [2], the field that is interested in the application of scientific methods and scientific research to the complex issues facing decision-makers in all areas of life, through the topics it has presented attached to the methods of solution, such as mathematical programming - network planning - decision analysis - waiting queues --- and others as mentioned in many references that dealt with operations research [1,2,3,4]

Until we reach the subject of our research, which is inventory models, that is one of the most important topics of operations research because it is concerned with finding the ideal size of inventory so that the cost of storage is as low as possible. The storage is intended for goods and materials owned by the establishment for reselling or used in making products for sale ,Raw materials - semi-raw - spare parts ---- and others that ensure the continuity of work and achieve the desired profits, Storage costs are one of the company's capital assets, and in order to preserve the capital and achieve profits, a comprehensive economic study of the facility must be prepared based on modern scientific foundations, such as the use of the neutrosophic logic that has revolutionized all fields of science through the accuracy of the results provided by studies that use concepts The basic principles of this logic, which was clarified by research and studies that dealt with various topics that were published by those interested in this logic.$[5-25]$

Discussion:

In the classical logic, the static stock model without a deficit was studied for one substance, and based on the basic assumptions of this model and the relationship of the ideal size, a set of indicators was

calculated[1] ,we have discussed the same model with neutrosophic logic [5], Expanding on previous studies, where we will calculate some important neutrosophic economic indicators that help administrators and decision-makers to make ideal decisions through the future vision of the workflow provided by these indicators which differs from what we obtained by studying the same subject according to the classical logic because it takes into account that the rate of demand for inventory is a neutrosophic value we symbolize λ_N, which is an indefinite amount we will take in the form $\lambda_N = [\lambda_1, \lambda_2]$ where λ_1 it is the lowest value of the rate of demand for inventory during the period of the storage cycle and λ_2 is the highest value, and therefore all economic indicators that are related to the rate of demand will also be neutrosophic values that are not specific, taking into account the worst conditions that the facilities can go through to the best, which enables the responsible administrators to make decisions to develop appropriate future plans for the functioning of work in all conditions.

From the previous study of the static stock model without a deficit and for one substance according to the neutrosophic logic [5]we found the ideal volume of the order given by the following relationship:

$$Q_N^* = \sqrt{\frac{2k\lambda_N}{h}} \qquad (*)$$

where λ_N is the rate of demand for inventory in one time, K is the fixed cost of preparing the order, h is the cost of storage per unit in the warehouse during the unit of time:

We also found that the interval between one order and another is given by the following relationship:

$$T_N^* = \frac{Q_N^*}{\lambda_N}$$

Substituting Q_N^* by its equal, we get the relationship:

$$T_N^* = \frac{Q_N^*}{\lambda_N} = \sqrt{\frac{2k}{\lambda_N h}}$$

In this research to calculate the most important indicators, we make use of the basic assumptions of the static model without deficits mentioned in the reference [1]and the relationship $(*)$ we get:
1. The number of orders needed during one time:

$$n_N^* = \frac{\lambda_N}{Q_N^*}$$

2.Reorder Quantity:

In this form, the deficit is not allowed, so the quantity that is available in the warehouse must be known, and then the order must be repeated, and we will symbolize it with the code Q_N^1 so that the deficit does not occur, we can calculate it after knowing the time required to receive the order, which we will denote by the code d, then we get:

$$\frac{d}{T_N^*} = \frac{Q_N^1}{Q_N^*} \Rightarrow Q_N^1 = \frac{d}{T_N^*} Q_N^* = \frac{d}{\frac{Q_N^*}{\lambda_N}} Q_N^* \Rightarrow Q_1^* = d\lambda_N$$

3. Total Stock Accumulation in Warehouse During Storage Cycle Duration:
Calculated from the relationship:

$$Y_N^* = \int_0^{T_N^*} (Q_N^* - \lambda_N t)\, dt$$

$$Y_N^* = \int_0^{T_N^*} (Q_N^* - \lambda_N t)\, dt = \left[Q_N^* - \lambda_N \frac{t^2}{2} \right]_0^{T_N^*} = \frac{Q_N^*}{2\lambda_N}$$

$$Y_N^* = \frac{Q_N^*}{2\lambda_N}$$

4. Average Volume of Stock in the Warehouse During Unit of Time in the Period $[0, T_N^*]$:
Calculated from the relationship:

$$\bar{Y}_N = \frac{Y_N^*}{T_N^*} = \frac{Q_N^*}{2}$$

5. Smallest Storage Cost:
Calculated from the relationship:

$$NC(Q_N^*) = \frac{k\lambda_N}{Q_N^*} + C\lambda_N + h\frac{Q_N^*}{2}$$

6. Total Storage Cost During Storage Cycle Duration:

Calculated from the relationship:

$$TNC(Q_N^*) = C(Q_N^*)T_N^*$$

Substituting T_N^* by its equivalent, we get the relationship:

$$TNC(Q_N^*) = k + CQ_N^* + \frac{h}{\lambda_N}\frac{Q_N^{*2}}{2}$$

7. Total Sales During the Unit of Time:

If the selling price of one unit of the stored material equals P and we symbolize the total sales during one time by V_N, then it is calculated from the relationship:

$$V_N = P\lambda_N$$

8. Total Sales During the Storage Cycle Period:

We symbolize the total sales during the storage cycle period with W_N, then W_N is calculated from the relationship:

$$W_N = P\lambda_N T_N^* \Rightarrow W_N = P\lambda_N \frac{Q_N^*}{\lambda_N}$$

$$W_N = PQ_N^*$$

9. Average Profit During the Unit of Time:

We denote the average profit during one with the symbol B_N and calculate it from the relationship:

$$B_N = V_N - NC(Q_N^*)$$

10. Average Profit During the Storage Cycle Period:

We get the average profit over the storage cycle period B_N' from the relationship:

$$B_N' = W_N - TNC(Q_N^*)$$

11. We Get the Average Profit Per Year from the Relationship:

$$B_N^1 = 12 * B_N$$

We declare the above with the following example:

Practical Example According to Neutrosophic Logic:

A merchant wishes to invest his existing capital in storing and selling iron used in construction. Before starting the work, he conducted a study of the market movement in the area in which he wanted to invest, and obtained the following information:

1. The monthly demand rate for iron ranges between 250 and 300 tons, meaning there is no exact value for the demand rate. Therefore, the demand rate is a neutrosophic value and equals $\lambda = [250,300]$ ton.

2. The cost of preparing Q ton order is $k = 200\$$.

3. The cost of storing one ton of iron during the unit of time (month) is $h = 10\$$.

4. The cost of purchasing, delivering, receiving and arranging an order of size Q equals $C = 6000\$$.

5. The period of receiving the order is $d = 5\ day$

6. The selling price of one ton is $P = 7000\$$.

From the data we find:

The ideal size of the order we substitute in the following relationship:

$$Q_N^* = \sqrt{\frac{2k\lambda_N}{h}} = \sqrt{\frac{2 * 200 * [250,300]}{10}} = [100,109.5]\ ton$$

Let's calculate the basic indicators that give this investor a future vision of the business situation over time:

1. The ideal time interval between one order and another, we substitute in the relationship:

$$T_N^* = \sqrt{\frac{2K}{\lambda_N h}}$$

$$T_N^* = \sqrt{\frac{2k}{\lambda_N h}} = \sqrt{\frac{2 * 200}{[250,300] * 10}} = [0.4,0.4] = 0.4\ month$$

And it equals: $T_N^* = 0.4 * 30 = 12\ day$

2. The number of orders needed during a month; we substitute in the relationship:

$$n_N^* = \frac{\lambda_N}{Q_N^*}$$

$$n_N^* = \frac{[250,300]}{[100,109.5]} = [2.5,2.7] \ order$$

3.The quantity that is available in the warehouse, and then the order must be repeated, we substitute in the relationship:

$$Q_1^* = d\lambda_N = 5 * [250,300] = [1250,1500] \ ton$$

4. The total accumulation of inventory in the warehouse during the period of the storage cycle, we substitute in the relationship:

$$Y_N^* = \frac{Q_N^*}{2\lambda_N} = \frac{[100,109.5]}{2 * [250,300]} = [0.2,0.2] = 0.2 \ ton$$

5. The average volume of stock held in the warehouse during one time in the period we substitute in the relationship:

$$\bar{Y}_N = \frac{Q_N^*}{2} = \frac{[100,109.5]}{2} = [50,54.8] \ ton$$

6. The smallest storage cost we substitute in the relationship:

$$NC(Q_N^*) = \frac{k\lambda_N}{Q_N^*} + C\lambda_N + h\frac{Q_N^*}{2}$$

$$NC(Q_N^*) = \frac{10 * [250,300]}{[100,109.5]} + 6000 * [250,300] + 10 * \frac{[100,109.5]}{2} \Rightarrow$$

$$NC(Q_N^*) = [1500525,1800575] \ \$$$

7. The total cost of storage during the storage cycle period, we substitute in the relationship:

$$TNC(Q_N^*) = C(Q_N^*)T_N^*$$

$$TNC(Q_N^*) = [1500525,1800575] * 0.4 = [600210,720230] \ \$$$

8. The total value of sales during a month, we substitute in the relationship:

$$V_N = P\lambda_N$$

$$V_N = 7000 * [250,300] = [1750000,2100000] \ \$$$

9. Total sales during the storage cycle period, we compensate with the relationship:

$$W_N = PQ_N^*$$

$$W_N = 7000 * [100,109.5] = [700000,766500] \ \$$$

10. Average profit during the unit of time (month) we substitute in the relationship:

$$B_N = V_N - NC(Q_N^*)$$

$$B_N = [1750000,2100000] = [249475,299425] \ \$$$

11. The average profit during the storage cycle period, we substitute in the relationship:

$$B_N' = W_N - TNC(Q_N^*)$$

$$B_N' = [700000,766500] - [600210,720230] = [46270,99790] \ \$$$

12. Average profit per year we substitute in the relationship:

$$B_N^1 = 12 * B_N$$

$$B_N^1 = 12 * [249475,299425] = [2993700,3593100] \ \$$$

In order to be able to clarify the difference made by the concepts of neutrosophic logic and neutrosophic values when they are used in any field of science, we will re-solve this example according to classical logic based on the study mentioned in reference [1] and compare the results:

Practical Example According to Classical Logic:

A merchant wishes to invest his existing capital in storing and selling iron used in construction. Before starting the work, he conducted a study of the market movement in the area in which he wanted to invest, and obtained the following information:

1. The monthly demand rate for iron is 250 tons, $\lambda = 250$ ton.
2. The cost of preparing Q ton order is $k = 200\$$.
3. The cost of storing one ton of iron during the unit of time (month) is $h = 10\$$.
4. The cost of purchasing, delivering, receiving and arranging an order of size Q equals $C = 6000\$$
5. The period of receiving the order is $d = 5 \ day$.
6. The selling price of one ton is $P = 7000\$$

Let's calculate the basic indicators that give this investor a future vision of the business situation over time:

From the data we find:

The ideal size of the order is substituted in the following relationship:

$$Q^* = \sqrt{\frac{2k\lambda}{h}} = 100 \quad ton$$

Therefore, the main indicators are:

1. The ideal time interval between one order and another, we substitute in the relationship:

$$T^* = \sqrt{\frac{2k}{\lambda h}} = 0.4 \ month$$

$$T^* = 0.4 * 30 = 12 \ day$$

2. The number of orders needed during a month; we substitute in the relationship:

$$n_N^* = \frac{\lambda}{Q^*} = 2.5 \ order$$

3. The quantity that is available in the warehouse, and then the order must be repeated, we substitute in the relationship:

$$Q_1^* = d\lambda = 1250 \ ton$$

4. The total accumulation of inventory in the warehouse during the period of the storage cycle, we substitute in the relationship:

$$Y^* = \frac{Q^*}{2\lambda} = 0.2 \quad ton$$

5. The average volume of stock held in the warehouse during one time in the period we substitute in the relationship:

$$\bar{Y}_N = \frac{Q^*}{2} = 50 \quad ton$$

6. The smallest storage cost we substitute in the relationship:

$$C(Q^*) = \frac{k\lambda}{Q^*} + C\lambda + h\frac{Q^*}{2} = 1500525 \quad \$$$

7. The total cost of storage during the storage cycle period, we substitute in the relationship:

$$TC(Q^*) = C(Q^*)T^* = 600210 \quad \$$$

8. The total value of sales during a month, we substitute in the relationship:

$$V = P\lambda = 1750000 \quad \$$$

9. Total sales during the storage cycle period, we substitute in the relationship:

$$W = PQ^* = 700000 \quad \$$$

10. Average profit during the unit of time (month) we substitute in the relationship:

$$B = V - C(Q^*) = 249475 \quad \$$$

11. The average profit during the storage cycle period, we substitute in the relationship:

$$B' = W - TC(Q^*) = 99790 \quad \$$$

12. Average profit per year we substitute in the relationship:

$$B^1 = 12 * B = 2993700 \quad \$$$

Conclusion and Results:

We summarize our findings in this research through the following table:

Neutrosophic Logic	Classical Logic	Economic Indicators
$Q_N^* = [100, 109.5]$ **ton**	$Q^* = 100 \ ton$	Order volume
$T_N^* = 12 \ day$	$T^* = 12 \ day$	Intervals between one order and another
$n_N^* = [2.5, 2.7] \ order$	$n_N^* = 2.5 \ order$	Orders number
$Q_N^1 = [1250, 1500] \ ton$	$Q_1^* = 1250 \ ton$	Quantity to be re-ordered
$\bar{Y}_N = 0.2 \ ton$	$Y^* = 0.2 \ ton$	total inventory backlog

$NC(Q_N^*) = [1500525,1800575]$ \$	$C(Q^*) = 1500525$ \$	Smallest storage cost during the unit of time
$TNC(Q_N^*) = [600210,720230]$ \$	$TC(Q^*) = 600210$ \$	Total storage cost during the storage cycle
$V_N = [1750000,2100000]$ \$	$V = 1750000$ \$	Total sales value during the unit of time
$W_N = [700000,766500]$ \$	$W = 700000$ \$	Total sales value during the storage cycle
$B_N = [249475,299425]$ \$	$B = 249475$ \$	Average profit over the unit of time
$B_N' = [46270,99790]$ \$	$B' = 99790$ \$	Average profit over the storage cycle period
$B_N^1 = [2993700,3593100]$ \$	$B^1 = 2993700$ \$	Average profit per year

Results Comparison Table

From the previous table, we notice that dealing with neutrosophic values gives more accurate results and has a margin of freedom, which provides a safe working environment for the investor because studying according to the neutrosophic logic, gave values for economic indicators that took into account the worst and best conditions and thus the investor is aware and ready to face everything that may arise on the work environment.

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The Neutrosophic Treatment of the Static Model for the Inventory Management with Safety Reserve

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Abstract:

Institutions must store materials to ensure the continuity of their activity and to avoid incurring big losses as a result of the storage process, various models have been studied that cover all scenarios in which stock insurance is required to allow institutions to continue operating while avoiding losses. The static model with safety reserve is one of these models, and it is used in emergency and ambulance circumstances to transport medicines, food, and fuel, etc.

Those in charge of any project must estimate the quantities that will need to be stored in order to ensure that the necessary materials are available and that storage costs are minimized. As a result, a mathematical model has been developed that expresses the circumstance in which a safety reserve is necessary to meet market material demand, and the optimal answer for this model is the required solution. This model is treated in classical logic by adding the amount of the safety reserve to the ideal quantity determined through the static model without a deficit, and this quantity is a fixed amount during each storage cycle over time, which does not correspond to reality and ignores cases of fluctuations demand in the rate of demand for inventory .In this study, three scenarios were used to construct a study for the static stock model with safety reserves and for one substance utilizing neutrosophic theory through three different cases.

The First Case: Using the optimal amount of stock that determined by studying the static model with a deficit using neutrosophic logic , while assuming the safety reserve S_N as a vague value, either $S_N = \{S_1, S_2\}$ or $S_N = [S_1, S_2]$.

The Second Case: Taking the ideal value of stock that was previously determined by studying the static model with deficient using neutrosophic logic , wherein regard the safety reserve S_N as constant value.

The third Case: Taking the optimal value of stock that determined by studying the static model with deficient using classical logic, and assuming the safety reserve S_N as a vague value, either $S_N = \{S_1, S_2\}$ or $S_N = [S_1, S_2]$.

In other words, we approached the problem using neutrosophic tools, which accounts for all possible scenarios that may arise throughout the course of the job, yields more accurate results, and so ensures a safe working environment for the facilities at the lowest possible cost.

Keywords.

Inventory Management, Static Inventory Management Model, Neutrosophic Logic, Inventory Management with Safety Reserve using Neutrosophic Logic.

1- Introduction

The inventory is considered one of the assets of the facility and it can change the financial state of the facility and achieve great profits or inflict huge losses on it. The size of the stock is affected by the conditions of the facility's work, and therefore basic hypotheses are developed for the issue describing the workflow in this facility. Through these assumptions, the appropriate model is built and its solution gives the ideal size. There are several things that affect the ideal size and are related to the type of model, such as the rate of demand for inventory, the size or the available space, the amount of the deficit, the amount of the safety reserve ...etc., and the more accurately they are determined, the more we can determine the ideal size that needs to be stored during the period of the storage cycle according to the conditions of the facility's work and at the lowest cost, the classic logic consider these situations with specific values, which does not correspond to reality because the rate of demand for stock, the size or available space, the amount of the deficit, the amount of the safety reserve and others change simultaneously and unpredictably.

In this study, we will rely on the research that we had previously published [4]. Neutrosophic Logic is a logic that is a new vision of modeling designed to effectively address the uncertainties inherent in the real world, as it came to replace the binary logic that admits right and wrong only, by introducing a third, neutral state that can be interpreted as undetermined or uncertain. Neutrosophic theory is founded by the American mathematical philosopher Florentin Smarandache [8-12], who introduced it in 1995 as a generalization of fuzzy logic as an extension of the theory of fuzzy categories presented by Lotfi Zadeh [5] in 1965, as well as, Ahmed A. Salama presented the theory of neutrosophical classical categories as a generalization of the theory of classical categories [11,13,14]. Also, there were developments, introducing and formulation for new concepts in the fields of mathematics, statistics, computer science and others through neutrosophic vision [15,18,19].

The notion of neutrosophic theory has developed in popularity in recent years as a result of its use in measurement theory, set theory, graph theory, statistics, and a variety of other scientific and practical domains [6,7,19,20-23].

Recently, there were dozens of articles dedicated to reformulate most mathematical branches, especially in operating research and optimization to reconstruct either its fuzzy or classical problems in the light of neutrosophic theory [1,2,3,16,17,24-26].

In this study, we will investigate the static model with the safety reserve and for one substance using neutrosophic logic, which will allow us to determine the required amount of stock as a safety reserve during the storage cycle, more precisely, by studying the issue in three cases, in other words, we will address the issue using neutrosophic logic, which takes into account all of the possible scenarios that we may encounter during the work and provides more accurate results, ensuring secure workflow and storage at the lowest costs for facilities that rely on such models in their job, and we will provide practical examples that exemplify each of the preceding cases.

2- Discussion

It is the responsibility of the facility managers to determine the optimal policy for the work of the facility by collecting the necessary data for the work and then employing this data in managing the work of the facility. One of the most important things that must be worked on is the inventory, because inventory management is one of the most important functions of management in terms of determining the mechanism used in the process storage in terms of allowing or not allowing a deficit or securing a safety reserve ...etc., by comparing the costs in each case, large profits or huge losses, so there was an urgent requirement to procedure a study to help us determine the ideal size of the stock at the lowest possible cost. In this research, we will study the static model with a safety reserve and for one material, which is used to store materials that require a safety reserve for emergencies situations such as medicines, food, and fuel...etc.

Here it should be noted that the treatment of this model is resulting from the study prepared for the static model without deficit by adding the quantity of the safety reserve to the ideal quantity that we reached when studying the static model without deficit and also we add to the costs, the cost of storing the quantity of the safety reserve.

We will study this model according to the neutrosophic logic in the following three cases:

3- The first case: (assuming that the rate of demand for inventory is determined and the amount of the safety reserve is also not specified)

3.1- The Fundamental Assumptions:

1- The Size of demand Q.

2- The rate of demand for inventory in time unit is λ_N which is indeterminate value, i.e. $\lambda_N = [\lambda_1, \lambda_2]$, where λ_1 is the lower bound of the rate of demand for inventory, λ_2 is the upper bound of the rate of demand for inventory.

3- Fixed cost of demand preparation $C_1 = K$.

4- Cost of purchase, delivery and pickup $C_2 = C.Q$.

5- The cost of storage h is a fixed amount for each unit in the warehouse during time unit, and it includes the cost of frozen liquidity, the cost of occupied space, the cost of protection, security, insurance, taxes and various fees, then Then the storage cost during time unit is for the quantity remaining in warehouse C_3.

6- The amount of the safety reserve in each storage cycle and over time is equal to S_N and it is indeterminate , i.e. $S_N = [S_1, S_2]$, so that the lower bound of the safety reserve is S_1, while the upper bound of the safety reserve is S_2, and the cost of storing one unit of it during time unit is h, then the cost of that reserve is equal to $C_4 = hS_N$.

7-The period of running out of the stored quantity is $\frac{Q}{\lambda_N}$ (or the duration of the storage cycle).

Using the previous hypotheses and benefiting from the results we reached from the study of the static model without deficits according to the neutrosophic logic [5], we find that the neutrosophic ideal size of the first demand is given by the following relationship:

$$Q_N^{**} = Q_N^* + S_N$$

where Q_N^* represents the formula of the neutrosophic optimal size for the demand ,and it has been derived by previous article [5]. It is given by the following relationship:

$$Q_N^* = \sqrt{\frac{2\lambda_N K}{h}}$$

Consequently, the neutrosophic optimal size for the first demand is:

$$Q_N^{**} = \sqrt{\frac{2\lambda_N K}{h}} + S_N$$

After determining the amount of the first demand, the movement of inventory proceeds until it reaches to the level $S_N = [S_1, S_2]$, then it is renewed with a new demand Q_N^*. The purchase price of the safety reserve quantity is CS_N and it is one-time calculation.

The total cost of inventory during each cycle of successive cycles is:

$$TC(Q_N^*) = K + C.Q_N^* + \frac{h.Q_N^*}{2\lambda_N} + h.S_N.T^*$$

While the total cost of inventory during the time unit from successive cycles is:

$$C(Q_N^*) = \frac{K.\lambda_N}{Q_N^*} + C.\lambda_N + \frac{h}{2}Q_N^* + h.S_N$$

Finally, the amount of re demand is calculated by the relation:

$$Q_N = d.\lambda_N + S_N$$

3.2- Practical Example:

A productivity company stores flour in its warehouses, according to the following conditions and data: The safety reserve to ensure the continuity of work is absolutely indeterminate and ranges between [0,100] tons per month, and the rate of demand for flour is absolutely indeterminate, but ranges between [10,20] tons per month, depending on the company's conditions of work pressure, operating pressure, weather conditions, etc. The cost of one-ton storage equals 40 monetary units per month. The cost of

preparing one demand is 150 monetary units, the purchase price of one ton is equal to 1200 monetary units, and the period required to receive the demand is 6 days. Calculate the following:

1- calculate the optimal size of the first demand and successive demands.

2-determine the amount of re demand.

3- Calculate the total cost of inventory during a month time unit.

4-Calculating the investment cost of storage during a year.

Keep in mind that we have, $d = 6$, $C = 1200$, $K = 150$, $h = 40$, $\lambda_N = [10,20]$, $S_N = [0,100]$.

1- The optimal size of the first demand is:

$$Q_N^{**} = Q_N^* + S_N$$

$$Q_N^{**} = \sqrt{\frac{2\lambda_N K}{h}} + S_N = \sqrt{\frac{2[10,20].150}{40}} + [0,100] = [8.66,12.25] + [0,100] = [8.66,112.25]$$

The optimal size for successive demand is $Q_N^* = [8.66,12.25]$.

2- The amount of re demand is given by the following relations:

$$Q_N = d.\lambda_N + S_N = 6.[10,20] + [0,100] = [60,220].$$

3- The total cost of inventory during a month time unit is:

$$C(Q_N^*) = \frac{K.\lambda_N}{Q_N^*} + C.\lambda_N + \frac{h}{2}Q_N^* + h.S_N$$

$$= \frac{150.[10,20]}{[8.66,12.25]} + 1200.[10,20] + \frac{40}{2}.[8.66,12.25] + 40.[0,100]$$

$$= \frac{[1500,3000]}{[8.66,12.25]} + [12000,24000] + [173.2,245] + [0,4000]$$

$$= [173.210162, 244.897959] + [12173.2,6645] = [12346.4102,6889.89796]$$

4-The investment cost of storage during a year is:

$$CI_N = C(Q_N^*).12 + CS_N = [12346.4102,6889.89796].12 + 1200.[0,100] =$$

$[148156.922, 82678.7755] + [0,120000]= [148156.922,202678.776].$

4- The second case: (in which the rate of demand for inventory is indeterminate and the value of the safety reserve is fixed value S)

4.1- The Fundamental Assumptions:

1- The Size of demand Q.

2- The rate of demand for inventory in time unit is λ_N which is indeterminate value, i.e. $\lambda_N = [\lambda_1, \lambda_2]$, where λ_1 is the lower bound of the rate of demand for inventory, λ_2 is the upper bound of the rate of demand for inventory.

3- Fixed cost of demand preparation $C_1 = K$.

4- Cost of purchase, delivery and pickup $C_2 = C.Q$.

5- The cost of storage h is a fixed amount for each unit in the warehouse during time unit, and it includes the cost of frozen liquidity, the cost of occupied space, the cost of protection, security, insurance, taxes and various fees, then Then the storage cost during time unit is for the quantity remaining in warehouse C_3.

6- The amount of the safety reserve in each storage cycle and over time is equal to S, and the cost of storing one unit of it during time unit is h, then the cost of that reserve is equal to $C_4 = hS$.

7-The period of running out of the stored quantity is $\frac{Q}{\lambda_N}$ (i.e. the duration of the storage cycle).

Using the previous hypotheses and benefiting from the results we reached from the study of the static model without deficits according to the neutrosophic logic [5], we find that the neutrosophic ideal size of the first demand is given by the following relationship:

$$Q_N^{**} = Q_N^* + S$$

where Q_N^* represents the formula of the neutrosophic optimal size for the demand ,and it has been derived by previous article [5]. It is given by the following relationship:

$$Q_N^* = \sqrt{\frac{2\lambda_N K}{h}}$$

Consequently, the neutrosophic optimal size for the first demand is:

$$Q_N^{**} = \sqrt{\frac{2\lambda_N K}{h}} + S$$

After determining the amount of the first demand, the movement of inventory proceeds until it reaches to the level S , then it is renewed with a new demand Q_N^*. The purchase price of the safety reserve quantity is CS and it is one-time calculation.

The total cost of inventory during each cycle of successive cycles is:

$$TC(Q_N^*) = K + C.Q_N^* + \frac{h.Q_N^*}{2\lambda_N} + h.S.T^*$$

While the total cost of inventory during the time unit from successive cycles is:

$$C(Q_N^*) = \frac{K.\lambda_N}{Q_N^*} + C.\lambda_N + \frac{h}{2}Q_N^* + h.S$$

Finally, the amount of re demand is calculated by the relation:

$$Q_N = d.\lambda_N + S$$

4.2- Practical Example:

depending upon the previous example and adapting the data according to the hypothesis of this case:

A productivity company stores flour in its warehouses, according to the following conditions and data:

The safety reserve to ensure the continuity of work is 100 tons per month and the rate of demand for flour is absolutely indeterminate, but ranges between [10,20] tons per month, according to the company's conditions of work pressure, operating pressure, weather conditions, etc., and that the cost of storing one ton is equal to 40 monetary units in month. And the cost of preparing one demand equals 150 monetary units, and the purchase price of one ton is equal to 1200 monetary units, and the period required to receive the order is 6 days. Calculate the following:

1- calculate the optimal size of the first demand and successive demands.

2-determine the amount of re demand.

3- Calculate the total cost of inventory during a month time unit.

4-Calculating the investment cost of storage during a year.

It is obvious from the problem's information that $d = 6$, $C = 1200$, $K = 150$, $h = 40$, $\lambda_N = [10,20]$, $S = 100$.

1- $Q_N^{**} = \sqrt{\frac{2\lambda_N K}{h}} + S = \sqrt{\frac{2[10,20].150}{40}} + 100 = [8.66,12.25] + 100 = [108.66,112.25]$

While the optimal size for successive demand is $Q_N^* = [8.66,12.25]$.

2- The amount of re demand is given by the following relations:

$Q_N = d.\lambda_N + S = 6.[10,20] + 100 = [160,220]$.

3- The total cost of inventory during a month time unit is:

$$C(Q_N^*) = \frac{K.\lambda_N}{Q_N^*} + C.\lambda_N + \frac{h}{2}Q_N^* + h.S$$

$$= \frac{150.[10,20]}{[8.66,12.25]} + 1200.[10,20] + \frac{40}{2}.[8.66,12.25] + 40.(100)$$

$$= \frac{[1500,3000]}{[8.66,12.25]} + [12000,24000] + [173.2,245] + 4000$$

$$= [173.210162, 244.897959] + [16173.2, 28245] = [16346.4102, 28489.898]$$

4- The investment cost of storage during a year is:

$$CI_N = C(Q_N^*).12 + CS = [16346.4102, 28489.898].12 + 1200.(100)$$

$$= [196156.922, 341878.776] + 120000 = [316156.922, 461878.776]$$

5- The third case: (where the rate of demand for inventory is fixed value and the amount of the safety reserve is indeterminate)

5.1- The Fundamental Assumptions:

1- The Size of demand Q.

2- The rate of demand for inventory in time unit is λ.

3- Fixed cost of demand preparation $C_1 = K$.

4- Cost of purchase, delivery and pickup $C_2 = C.Q$.

5- The cost of storage h is a fixed amount for each unit in the warehouse during time unit, and it includes the cost of frozen liquidity, the cost of occupied space, the cost of protection, security, insurance, taxes and various fees, then Then the storage cost during time unit is for the quantity remaining in warehouse C_3.

6- The amount of the safety reserve in each storage cycle and over time is equal to S_N and it is indeterminate , i.e. $S_N = [S_1, S_2]$, so that the lower bound of the safety reserve is S_1, while the upper bound of the safety reserve is S_2, and the cost of storing one unit of it during time unit is h, then the cost of that reserve is equal to $C_4 = hS_N$.

7-The period of running out of the stored quantity is $\frac{Q}{\lambda}$ (i.e. the duration of the storage cycle).

Using the previous hypotheses and benefiting from the results we reached from the study of the static model without deficits according to the neutrosophic logic [5], we find that the neutrosophic ideal size of the first demand is given by the following relationship:

$Q_N^{**} = Q^* + S_N$

where Q^* represents the formula of the neutrosophic optimal size for the demand ,and it has been derived by previous article using classical logic [5]. It is given by the following relationship:

$$Q^* = \sqrt{\frac{2\lambda K}{h}}$$

Consequently, the neutrosophic optimal size for the first demand is:

$$Q_N^{**} = \sqrt{\frac{2\lambda K}{h}} + S_N$$

After determining the amount of the first demand, the movement of inventory proceeds until it reaches to the level $S_N = [S_1, S_2]$, then it is renewed with a new demand Q^*. The purchase price of the safety reserve quantity is $C.S_N$ and it is one-time calculation.

The total cost of inventory during each cycle of successive cycles is:

$$TC(Q^*) = K + C.Q^* + \frac{h.Q^*}{2\lambda} + h.S_N.T^*$$

While the total cost of inventory during the time unit from successive cycles is:

$$C(Q) = \frac{K.\lambda}{Q} + C.\lambda + \frac{h}{2}Q + h.S_N$$

Finally, the amount of re demand is calculated by the relation:

$$Q_N = d.\lambda + S_N$$

5.2- Practical Example:

depending upon the previous example and adapting the data according to the hypothesis of this case:

A productivity company stores flour in its warehouses, according to the following conditions and data:

The safety reserve to ensure the continuity of work is absolutely indeterminate, and ranges between [0,100] tons per month, and the rate of demand for flour is a specific amount equal to 20 tons per month, according to the company's conditions of working pressure, operating pressure, weather conditions, etc., and the cost of storing one ton is equal to 40 monetary units in the month. The cost of preparing one demand equals 150 monetary units, the purchase price of one ton is equal to 1200 monetary units, and the period required to receive the order is 6 days. Calculate the following:

1- Calculate the optimal size of the first demand and successive demands.

2- Determine the amount of re demand.

3- Calculate the total cost of inventory during a month time unit.

4- Calculating the investment cost of storage during a year.

It is obvious from the problem's information that $d = 6$, $C = 1200$, $K = 150$, $h = 40$, $\lambda = 20$, $S_N = [0,100]$.

1- $Q_N^{**} = Q^* + S_N$

$$Q_N^{**} = \sqrt{\frac{2\lambda K}{h}} + S_N = \sqrt{\frac{6000}{40}} + [0,100] = 12.25 + [0,100] = [12.25,112.25]$$

While the optimal size for successive demand is $Q_N^* = 12.25$.

2- The amount of re demand is given by the following relations:

$$Q_N = d.\lambda + S_N = 6.(20) + [0,100] = [120,220].$$

3- The total cost of inventory during a month time unit is:

$$C_N(Q^*) = \frac{K.\lambda}{Q^*} + C.\lambda + \frac{h}{2}Q^* + h.S_N = \frac{150.[10,20]}{[8.66,12.25]} + 1200.[10,20] + \frac{40}{2}.[8.66,12.25] + 40.(100)$$

$$= \frac{150.(20)}{12.25} + 1200.(20) + \frac{40}{2}.(12.25) + 40.[0,100]$$

$$= 244.9 + 24000 + 245 + [0,4000] = [24489.9, 28489.9]$$

4- The investment cost of storage during a year is:

$$CI_N = C_N(Q).12 + C.S_N = [24489.9, 28489.9].(12) + 1200.[0,100] = [295078.8, 461878.8]$$

6- Conclusion

From this research, we conclude that dealing with inventory management models with the perspective of the neutrosophic logic provides us with a more general and comprehensive study than the well-known classical study so that it does not neglect any data just because it is explicitly indeterminate, especially when we deal with the static model with the safety reserve. The demand for inventory, or the undetermined quantity of the safety reserve, or both of them together have affected on the optimal size of the demand, and it was expressed in an approximate form that is absolutely indeterminate ranging between two values. Therefore, we find that the presence of uncertainty actually affects the results and these unspecified values cannot be ignored or removed from the framework of the study with the aim of obtaining more accurate results, and thus obtaining the ideal volume of inventory that meets the needs of the facility during the period of the storage cycle at the lowest cost. We find that working within the classical logic in particular in the framework of inventory management is no longer sufficient at the present time, as the development of science has set in front of us a large number of new problems that need more general and accurate results than the obtained results using classical logic or fuzzy logic , and here the role of the neutrosophic logic comes in, which provides us with more comprehensiveness in interpreting the study data and obtaining the required accurate results, and enables us to deal with all forms of data that we may encounter in our practical and scientific lives.

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Neutrosophic Static Model without Deficit and Variable Prices

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Abstract:

Inventory management plays an important role in production and marketing processes, especially in production facilities and commercial institutions that have warehouses in which they store their equipment and goods. Inventory management is considered one of the most important management functions in terms of determining the ideal volume of inventory and calculating its costs, as this affects the facility's efficiency and achieves either large profits or it causes huge losses, so warehouse managers in production facilities or institutions must determine the appropriate and ideal volume of inventory, especially when they are presented with price offers from companies seeking to market their products. These offers are directly related to the volume of the order, and in this case, they must make an ideal decision through which determine the volume of the order by taking into account the following matters:

1. Securing the quantities required for production or sale so that there is no deficit.
2. The storage cost should be as low as possible.
3. Benefit as much as possible from the discounts offered by companies.

In this research, we present a complementary study to what we did in researching the neutrosophic treatment of static inventory models without deficit, through which we arrived at mathematical relationships through which we can calculate the ideal volume of the order at the lowest possible cost. We will use these relationships to determine the ideal volume of the order so that we achieve the greatest benefit from price offers and discounts. Provided by companies.

Keywords: Static inventory models without deficits - Static inventory models without neutrosophic deficits - Variable price (discounts) models without neutrosophic deficits.

Introduction:

Before the emergence of the science of operations research, decision makers relied on experience gained through the profits they obtained for correct decisions and losses for wrong decisions. With the great development witnessed by our contemporary world, we find that experience alone is not sufficient to make decisions on the scale of this development, and a comprehensive study of the reality must be conducted. The work of the system is a study that

relies on modern scientific methods, such as operations research methods. Even operations research methods are not sufficient in the face of these changes that the work environment is witnessing because they depend on restricted and specific classical data. Therefore, these methods had to be reformulated using non-specific data, completely specific Neutrosophical data that gives decision makers have a margin of freedom that enables them to face all circumstances, and this is what has been done by researchers and scholars interested in science and scientific development. In the following research, we find many studies presented using the concepts of neutrosophic science [1-15]. In this research, we will present a study using complementary neutrosophic concepts. As we have done previously, we know that most companies provide price offers and these offers are related to the volume of the order in order to market their goods, which are manufactured materials or raw materials used in the manufacturing process. This matter requires warehouse managers in production facilities or institutions that need these goods to determine the appropriate and ideal volume of the stock of each material to secure the demand at one time. We know that if the volume of the stock is very large, this guarantees the provision of the material, but in return, it may cause the organization to suffer losses because the value of the stock is frozen capital, and this large quantity requires a marketing period that depends on the rate of demand. If the quantity of inventory is small, this may lead to a bottleneck in securing materials and to various disturbances such as rising prices and others. In all cases, we find that the rate of demand for inventory is the primary control over the volume of the order. Therefore, in previous research [16], we studied the static model without a deficit using the concepts of neutrosophic science and we reached relationships through which we can calculate the ideal volume of the order and the corresponding cost was a neutrosophic value that takes into account all circumstances. In another research we also calculated a set of neutrosophic indicators for the static inventory model without a deficit [17]. In this research we will present A study based on what was presented in previous research, the purpose of which is to determine the ideal volume for students and to make the most of the price offers provided for the materials that we need to store so that no shortage occurs and the cost of storage is as low as possible.

Discussion:

Inventory models were studied in classical logic, and this study was presented on the basis that the rate of demand for inventory is a fixed value throughout the duration of the storage cycle. Therefore, the rate of demand for inventory is subject to a uniform probability distribution. This issue was addressed by relying on studies presented according to classical logic in the field of operations research, which relies on based on basic concepts in mathematics such as calculus of integration and others [21-32], we presented, in previous research [16-20], static inventory models using the concepts of neutrosophic science. We took in the study the rate of demand for inventory during the duration of the storage cycle. neutrosophic value.

Undetermined values. Completely determined. It is subject to a regular neutrosophic distribution, and we found the extent to which this value affects the ideal volume of the order. Among the research was the neutrosophic treatment of inventory models without a deficit, which is based on the following hypotheses:

Basic hypotheses of the study:

1 – Order volume Q.

2 - The rate of demand for inventory at one time λ_N (unspecified), where $\lambda_N \in \{\lambda_1, \lambda_2\}$ or $\lambda_N \in [\lambda_1, \lambda_2]$ or... so that λ_1 is the minimum rate of demand for inventory and λ_2 is the upper limit of the rate of demand for inventory.

3 – The fixed cost of preparing the order $C_1 = K$.

4 – The cost of purchase, delivery and receipt $C_2 = C.Q$.

5 - The cost of storage for the remaining quantity in the warehouse during one time C_3.

6 - The duration of running out of the stored quantity is $\frac{Q}{\lambda_N}$ (or the duration of the storage cycle).

Using the previous assumptions, we were able to build a non-linear mathematical model, and the optimal solution for it, i.e., the ideal volume of the order, is given by the following relationship:

$$Q_N^* = \sqrt{\frac{2K\lambda_N}{h}} \qquad (1)$$

The ideal total cost is calculated from the relationship:

$$C(Q_N^*) = \frac{K\lambda_N}{Q_N^*} + C\lambda_N + \frac{hQ_N^*}{2} \qquad (2)$$

In this research, we present a study of static inventory models without shortages after adding a new hypothesis to the basic hypotheses imposed by the reality of the market situation through the offers made by the producing companies. The content of these offers is to provide a discount whose value is determined according to the quantity that is purchased. Here the official must determine the size of the order. So that it suits the system's demand and is sufficient for the duration of the storage cycle without causing a shortage and with the lowest possible storage cost and at the same time benefiting from the companies' offer. According to the above, we present the following formulation of the issue:

Text of the issue:

An insurance company needs a certain material, so if it knows that the rate of demand for this material is λ_N during the storage cycle, and that the cost of purchasing one unit is C monetary units, the cost of storing one unit is h, the cost of preparing the order is K monetary units, and the offer provided by the company the producers of this material are:

$$
Price\ for\ one\ item
\begin{cases}
C_1 & If & 0 \leq Q < Q_1 \\
C_2 & If & Q_1 \leq Q < Q_2 \\
C_3 & If & Q_2 \leq Q < Q_3 \\
C_4 & If & Q_3 \leq Q < \infty
\end{cases}
$$

Where $C_1 > C_2 > C_3 > C_4$

What is required is to determine the ideal volume of the order so that the storage cost is as low as possible.

From the above, the basic hypotheses of this model are written as follows:

Basic assumptions of the model of variable prices without neutrosophic deficit:

1 – Order volume Q.

2 - The rate of demand for inventory at one time λ_N (unspecified), where $\lambda_N \in \{\lambda_1, \lambda_2\}$ or $\lambda_N \in [\lambda_1, \lambda_2]$ or... so that λ_1 is the minimum rate of demand for inventory and λ_2 is the upper limit of the rate of demand for inventory.

3 – The fixed cost of preparing the order $C_1 = K$.

4 – The cost of purchase, delivery and receipt $C_2 = C.Q$.

5 - The cost of storage for the remaining quantity in the warehouse during one time C_3.

6 - The duration of running out of the stored quantity is $\frac{Q}{\lambda_N}$ (or the duration of the storage cycle).

7- One of the companies producing materials to be stored provided four price levels that are inversely proportional to the volume of the order:

$$
Price\ for\ one\ item
\begin{cases}
C_1 & If & 0 \leq Q < Q_1 \\
C_2 & If & Q_1 \leq Q < Q_2 \\
C_3 & If & Q_2 \leq Q < Q_3 \\
C_4 & If & Q_3 \leq Q < \infty
\end{cases}
$$

Neutrosophic treatment of the issue:

We follow the following steps:

1 -We build a mathematical model for this issue within hypotheses 1-6, which in themselves are the basic hypotheses of the static model without neutrosophic deficit that was studied previously, and we arrived at the following:

Find

$$
C(Q) = \frac{K\lambda_N}{Q} + C\lambda_N + \frac{hQ}{2} \quad \rightarrow Min
$$

Condition:

$$
Q \geq 0
$$

It is a nonlinear neutrosophic model whose optimal solution, i.e., the ideal volume of the students, is given by the relationship (1) and the minimum storage cost is calculated from the relationship (2).

To address the issue at hand and choose the optimal volume of the order, taking into account hypothesis No. (7) of the offers presented, we calculate the following storage costs:

$$C_i(Q_N) = \frac{K\lambda_N}{Q_N} + C_i\lambda_N + \frac{hQ_N}{2} \qquad (3)$$

Where $i = 1,2,3,4, \ldots$ for all price cases

Since $C_1 > C_2 > C_3 > C_4$ the function $C_i(Q)$ satisfies the following inequality:

$$C_1(Q) > C_2(Q) > C_3(Q) > C_4(Q)$$

For $i = 1,2,3,4$.

Each of the previous functions has a minimum limit, which is the optimal solution to the nonlinear model that we arrived at in the first step, corresponding to the inventory volume, which we symbolize as Q_{0N}, calculated from the following relationship:

$$Q_{0N} = \sqrt{\frac{2K\lambda_N}{h}}$$

After calculating Q_{N0} we compare it with the given offers.

We assume that $Q_1 \leq Q_{N0} < Q_2$. Then we calculate the cost corresponding to this volume from the relationship (3). We get $C_2(Q_{0N})$. To determine the ideal volume of the order Q_N^*, we calculate the cost functions for the minimum limits of the quantity ranges specified in the discounts table. Then we choose the smallest of these costs, which is the volume. The corresponding ideal is the volume that secures inventory for the system during the storage cycle period at the lowest possible cost and taking advantage of the offers provided.

We explain the above through the following example:

    Example:

    A production institution wants to secure its need for a certain material. If it knows that the rate of demand for this material is [250,330],units per year, the cost of purchasing one unit in the market is 400 monetary units, the cost of storing one unit per year is 10% of its price, and the cost of preparing the order is equal to 150 units. In cash, the company producing this material offers the following offers:

    2% discount if quantity $50 \leq Q \leq 100$.

    3% discount if quantity $100 \leq Q \leq 200$.

    5% discount if quantity is $200 \leq Q$.

    Required: Find the optimal quantity Q_N^*,that makes the total costs of storage as small as possible.

    The solution:

    Data:

    $\lambda \in [250,350] \quad ,K = 150 \ ,C = 400$

h is the storage cost and is 10% of the price of one unit of stock in the market. Therefore:

$$h = \frac{10}{100}.\,400 = 40$$

1- We determine price levels by discounts:

    a. When $Q \leq 50$ then $C_1 = C = 400$ there is no discount.

    b. When $50 \leq Q \leq 100$ the discount is 2% and the purchase price is equal to:

$$C_2 = 400\left(1 - \frac{2}{100}\right) = 392$$

    c. When $100 \leq Q \leq 200$ the discount is 3% and the purchase price equals to:

$$C_3 = 400\left(1 - \frac{3}{100}\right) = 388$$

    d. When $Q \geq 200$ the discount is 5% and the purchase price is equal to:

$$C_4 = 400\left(1 - \frac{5}{100}\right) = 380$$

2- We calculate the initial quantity of inventory:

We study the issue based on hypotheses 1-6, and here we are faced with a storage model without a neutrosophic deficit. We calculate the ideal volume of the order through the following relationship:

$$Q_{0N} = \sqrt{\frac{2K\lambda_N}{h}} = \sqrt{\frac{2.\,[250,350].\,150}{20}} \in [61,72]$$

This means that in order for the company to provide a safe work environment without shortages and with the lowest storage cost, the volume of the order must be greater than Q_{0N}. To calculate the cost, we compare the initial quantity with the offers presented by the producing company. We find that $Q_{0N} \in [50,100]$, meaning that this quantity deserves a 2% discount. The purchase price per unit is $C_2 = 392$, and then the total storage cost is calculated from the relationship:

$$C_2(Q_{0N}) = \frac{K\lambda_N}{Q_{0N}} + \frac{hQ_{0N}}{2} + C_2\lambda_N$$

$$C_2([61,72]) = \frac{150.\,[250,350]}{[61,72]} + \frac{40.\,[61,72]}{2} + 392.\,[250,350] \in [99834,139369]$$

To benefit more from the offers presented.

3- We calculate costs for the minimum offer areas:

For the range $100 \leq Q \leq 200$ we find:

$$C_3(100) = \frac{150.\,[250,350]}{100} + \frac{40.100}{2} + 392.\,[250,350] \in [100375,139725]$$

$$C_4(200) = \frac{150.[250,350]}{200} + \frac{40.200}{2} + 380.[250,350] \in [99187.5,137262.5]$$

After obtaining the costs corresponding to the offers, we find that:

As for the cost $C_1 = C = 400$, it is offset by the order volume $Q \leq 50$, and for this volume there is no discount. In addition, this volume is less than the minimum required, meaning that it does not suit the company because the company works on the basis of not having a deficit, and this order quantity will cause a deficit to be paid. The company will face fines, which will be reflected in the total cost, so we rule out this solution.

4- We choose the lowest cost from the remaining costs:

That is, we take:

$$Min\{C_2(Q), C_3(Q), C_3(Q)\}$$

We find:

$$Min\{[99834,139369], [100375,139725], [99187.5,137262.5]\}$$
$$= [99187.5,137262.5] \quad (*)$$

It corresponds to an order size $Q = 200$. This volume is appropriate for the company's workflow.

In order to achieve the maximum benefit from the offers presented, we calculate the costs corresponding to the largest volume that the company can adopt if the storage cost is appropriate, which corresponds to an order volume equal to the rate of demand for inventory, that is:

We calculate storage costs if the order volume equals the inventory demand rate. That is, when $Q \in [250,350]$, we find that the price of one unit will be $C_4 = 380$, and the total storage costs are equal to:

$$C([250,350]) = \frac{150[250,350]}{[250,350]} + \frac{40.[250,350]}{2} + [250,350].380 \in [100150,140150]$$

We compare this cost with the cost we obtained through the comparison (*). We find that the cost is greater and there is no interest for the company in requesting this size of the order because it can ensure a safe work flow and benefit from the offers provided at a lower cost when the order volume is $Q = 200$.

From the above, we note that the minimum value of storage costs is:

$$[99187.5,137262.5]$$

And corresponds to the ideal volume of the order, which is equal to $Q^* = 200$.

Conclusion and Results:

The storage model with variable prices is considered one of the important models in inventory models because we encounter it frequently in practical life and it requires careful study from us so that we do not fall into the temptation of the offers presented. Through the offers we may be able to obtain lower prices for large quantities, but these quantities may become a burden on the company. During the costs that must be paid for the storage process, on the other hand, we find that using the neutrosophic value of the demand rate for inventory gave a careful study of the model, as we obtained the neutrosophic storage cost from which we can determine the lowest cost and the largest cost if the ideal volume of the order is adopted $Q^* = 200$.

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Neutrosophic Treatment of the Static Model of Inventory Management with Deficit

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Abstract

In this article, when the inability to manage inventory is allowed in exchange for certain fines, managers can then allow the remaining stock to run out and accumulation the order until compensation is reinstated. The idea depends on the amount of stock deficit, which could be small in comparison to the storage cost. This refers to inventory models with deficits and is used to store damageable material. To manage these models, deficit costs should pair with storage costs by applying a mathematical model that illustrates the state in which deficit occurs by satisfying a requirement for one item. All that follows the principles of static modelling with a deficit and the ideal solution would be the most appropriate. We know that the ideal amount depends on the inventory demands. In the classical logic, we define the required storage amounts by the inventory cycle which has a constant value during the inventory cycle; however, that logic overlooks the fluctuation of inventory demands. Therefore, this manuscript conducts a study about static inventory models with a deficit for one material based on neutrosophic logic, which takes into consideration the indeterminate states and hence it provides more precise results, thus, it gives institutions a stable work environment with the least costs.

1. Introduction

In our daily life, Inventory is considered foundational for an enterprise and could yield high profit or significant monetary losses. Moreover, inventory volume is affected by the demand rate and recurrence of orders through time unit, whenever the demand rate is specified meticulously throughout the storage cycle according to an enterprise's requirements and at the least cost. The classic logic takes demand rate

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as a constant value which does not represent reality due to the unpredictable and variable nature of demand rates. To obtain more precise results, we apply neutrosophic logic.

Neutrosophic logic is a new vision of modeling developed by American mathematician Florentine Smarandache and is designed to effectively address the uncertainties inherent in the real world, as it came to replace the binary logic, fuzzy logic, and intuitionistic logic that recognize the true and the false state of any idea, problem, and issue by introducing a third neutral state which can be interpreted as undefined or unconfirmed, indeterminate, vague, paraconsistent, and incompleteness [5,7,8,9,11]. Florentine Smarandache presented neutrosophic logic in 1995 as a generalization of fuzzy logic presented by Lotfi Zadeh [4] in 1965, also, it is an extension of intuitionistic fuzzy logic that set up by K. Atansove in 1983 [6]. In addition to that, Ahmed A. Salama presented the theory of neutrosophic classical categories as a generalization of the theory of classical categories [10, 18] and developed it, also he introduced and formulated new concepts in the fields of neutrosophic mathematics, neutrosophic statistics, neutrosophic computer science…etc. by the unfathomable tools of neutrosophic logic [15,16,17,20,27]. Neutrosophic logic has significantly grown in recent years through its application in measurement, collections, graphs, geometric programming, relational equations, topological spaces and dozens of scientific and practical fields [6,12-14,19,21-25,29-37].

This manuscript has been dedicated to the application of neutrosophic theory in one of the inventory management models, which is the static model without deficit and for a single material. As such, this application will allow us to deal with formulas of inventory management models that are expressed in a way that is uncertain and not precisely defined and take the range of values instead of a single value, as is done in the classical model. We will particularly present the static model without a deficit for a single substance according to the neutrosophic logic, when the rate of demand for stock is defined accordingly, and then we will present an applied example that showcases how to apply this logic.

2. Discussion

The responsibility of determining the ideal policy work of the institute based on its manager. Through, collecting necessary data for work and investing this data at management institute work, one of the important things that we have worked on is the inventory, because it is an important tour at producing and marketing, especially at firm's products which have warehouse has been dedicated to their equipment and sales. Inventory management is one of the most important functions of management, through determining the way of storing by allowed deficit or disallowed depend on comparing between deficit charge and storage cost, so the principal goal is determining the ideal inventory volume and its costing, which affected the firm efficiency and judge if it achieves huge earning or heavy losses, that we have studied and determined the ideal inventory volume with the minimal cost through studying static inventory models at classical logic, while when the rate of inventory requested was indeterminate, it is obvious that the rate is subject to a uniform probability distribution. thus when the rate of inventory request is an unlimited form at the time cycle we use neutrosophic uniform probability distribution, which has been presented previously [20]. All neutrosophic mathematical tools and strategies presented in this study are regarded as complementary to our previous work [28], hence anyone tracking our work will recognize that these articles are a series in which each article is complementing other articles.

2.1. The main hypotheses of the study:

1.　　Let Q be the demand volume.

2.　　The rate of the request on inventory at time unit λ_N (indeterminate), where $\lambda_{iN} = [\lambda_1, \lambda_2]$ is an interval or $\lambda_N = \{\lambda_1, \lambda_2\}$ is a set or any other indeterminate component., where λ_1 is the minimum rate of the request on inventory, λ_2 is the maximum rate of the request on inventory.

3.　　The constant cost for preparing request is $C_1 = K$.

4.　　The cost of purchase , conveyance, receive $C_2 = C.Q$.

5.　　The storage cost of remaining quantity at the warehouse at time unit C_3.

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6.   The deficit amount allowed at a storing cycle equal constant value S, the deficit cost is P for each unverified request at the time unit including delay penalty and losing customer, ...etc.

7.   The time period of inventory quantity run out is $\frac{Q}{\lambda_N}$ (i.e. a storing cycle time period).

The symbol of the available storage volume at the start inventory cycle is R , symbolize to the requested volume that reaching the warehouse at the end of every cycle is Q where $R < Q$ thus the deficit value is $S = Q - R$. The available inventory volume r is connected with time by the relation: $r_t = R - \lambda_t$. The validity period of the stock $\frac{Q}{\lambda_N}$, storage cycle period $\frac{Q}{\lambda_N}$,and it is greater than $\frac{R}{\lambda_N}$. For that , we divide the first inventory cycle $[0, \frac{Q}{\lambda_N}]$ into two periods:

First period $[0, \frac{R}{\lambda_N}]$ is the work period. Second period $]\frac{R}{\lambda_N}, \frac{Q}{\lambda_N}]$ is the deficit period.

Consequently, calculating the inventory costs required calculate it through two periods, the first period represent static model calculation without any deficit, and it defined by the relation:

$$C_3 = \frac{hR^2}{2\lambda_N}$$

While the deficit costs calculate as:

$$C_4 = (-1) \int_{\frac{R}{\lambda_N}}^{\frac{Q}{\lambda_N}} P(R - \lambda_N t) dt = \frac{P(Q - R)^2}{2\lambda_N}$$

Consequently, the total cost is:

$$TC(Q, R) = K + CQ + \frac{hR^2}{2\lambda_N} + \frac{P(Q - R)^2}{2\lambda_N}$$

To have the total costs by time unit, divide $TC(Q, R)$ on the inventory cycle period $\frac{Q}{\lambda_N}$,which yields :

$$C(Q, R) = \frac{K.\lambda_N}{Q} + \lambda_N.C + \frac{hR^2}{2Q} + \frac{P(Q - R)^2}{2Q}$$

From the previous discussion, the following mathematical model can be concluded:
Find

$$\min C(Q, R) = \frac{K.\lambda_N}{Q} + \lambda_N.C + \frac{hR^2}{2Q} + \frac{P(Q - R)^2}{2Q}$$

Subject to:
$Q \geq R$, $Q \geq 0, R \geq 0$

This model is nonlinear. For finding the ideal solution, the target function is twice partially derived for Q and R, these two derived formulas can be homogeneous equations when equaling them by zeros.

$$\frac{\partial C(Q, R)}{\partial Q} = \frac{-K.\lambda_N}{Q^2} - \frac{hR^2}{2Q^2} + \frac{2P(Q - R)(2Q) - 2P(Q - R)^2}{4Q^2}$$

$$= \frac{1}{Q^2}\left[-K.\lambda_N - \frac{hR^2}{2} + P(Q - R)Q - \frac{1}{2}P(Q - R)^2\right] = 0$$

$$\frac{\partial C(Q, R)}{\partial R} = \frac{2hR}{2Q} + \frac{2P(Q - R)(-1)}{Q} = 0$$

 By multiplying the above tow equations by $Q^2, 2Q$ respectively, the following two equations are gained:

$$-K.\lambda_N - \frac{hR^2}{2} + P(Q - R)Q - \frac{P(Q - R)^2}{2} = 0$$

$$2hR - 4P(Q - R) = 0$$

By solving the above system of two equations:

$$Q^* = \sqrt{\left(\frac{2K.\lambda_N}{h}\right)\left(\frac{P+h}{P}\right)} \quad , \qquad R^* = \sqrt{\left(\frac{2K.\lambda_N}{h}\right)\left(\frac{P}{P+h}\right)} \qquad (1)$$

From the equation $2hR - 4P(Q-R) = 0$, we find that R^* , Q^* having the following relationship:

$$Q^* = \frac{P+h}{P} R^*$$

To ensure that these two values verifying model conditions. We have $h > 0$, and $\frac{P+h}{P} > 0$, this implies that $Q^* \geq R^*$, then the above two solutions verify the conditions, that's mean $Q \geq R$, $Q \geq 0, R \geq 0$. So Q^*, R^* that defined in relations (1) satisfy the optimal solution of objective function. To specify the type of this objective function if it satisfy its minimum or maximum value, the Hessian matrix of the objective function is defined as below. As the variables in the nonlinear model are two, so the Hessian arrays presented as:

$$\begin{bmatrix} \dfrac{\partial^2 C}{\partial Q^2} & \dfrac{\partial^2 C}{\partial Q \partial R} \\ \dfrac{\partial^2 C}{\partial R \partial Q} & \dfrac{\partial^2 C}{\partial R^2} \end{bmatrix}$$

Calculate the derivatives and substitute its value at the point (Q^*, R^*), it is well known that hessian matrix is non negative symmetric matrix, the major mini-determinants are non-negative, hence the matrix is well defined and semi-positive, in other words, the function is concave, which implies that the optimal value is the minimum value, subsequently ,the values that we gained for (R^*, Q^*) verifies required. The total costs value is:

$$C(Q^*, R^*) = \frac{K.\lambda_N}{Q^*} + \lambda_N.C + \frac{hR^{*2}}{2Q^*} + \frac{P(Q^* - R^*)^2}{2Q^*}$$

So, the value of the total costs through the inventory cycle in which its duration $T^* = \frac{Q^*}{\lambda}$ is:

$$TC(Q^*, R^*) = K + CQ^* + \frac{hR^{*2}}{2\lambda_N} + \frac{P(Q^* - R^*)^2}{2\lambda_N}$$

and to calculate the quantity of the re-requesting, we apply the following relation:
$$Q_1 = \lambda_N d - S$$
where d is the required duration for receiving request.

2.2 Illustration [2]:

If the objective function of the constrained nonlinear programming problem is convex, and the region determined by the constraints is convex too, then, the optimal solution is the global minimum solution.

The objective function satisfies the convexity property if its Hessian matrix is well defined and positive, or, semi-defined and positive.

Any matrix will be defined and positive, if verifying the following conditions:

1.   the Matrix is symmetric.

2.   All elements of the main diameter are positive.

3.   The major mini-determinants are positive.

The matrix will be defined and semi-positive, if verifying the following conditions:

1.   the Matrix is symmetric.

2.   All elements of the main diameter are nonnegative.

3.   The major mini-determinants are nonnegative.

3. Practical Example

This example is reformation from the classical version of the same context stated in [1] to neutrosophic version by changing the inventory rate from traditional value into neutrosophic value.

3.1 Example text by Neutrosophic logic:

Assuming we have warehouse for storing and sells material, if you know that the monthly requested rate for this material is between [0,300] ton, and the period time for receiving requested after submitting the application is five days, the cost preparing a request is (200) currency unit, the price purchasing for a ton of one material is (5000) currency unit, one ton storing cost at time unit is (10) currency unit, a deficit cost for every ton at time unit is (50) currency unit, although, the warehouse system allowed definite deficit, Find:

1- the optimal quantity for both: first request R^* , and the cyclic request Q^*.

2- the deficit amount allowed.

3- the minimum storing cost at time unit (month).

4- the minimum storing cost through storing cyclic.

Solution:

From the text of the example, we have the following data:

$\lambda_N = [0,300]$, $d = 5$, $P = 50$, $h = 10$, $C = 5000$, $K = 200$, $\lambda_N = [0,300]$

Substituting the above data in the following mathematical modal:

$$\min C(Q,R) = \frac{K.\lambda_N}{Q} + \lambda_N.C + \frac{hR^2}{2Q} + \frac{P(Q-R)^2}{2Q}$$

Subject to:

$Q \geq R$, $Q \geq 0, R \geq 0$

$$\min C(Q,R) = \frac{200.[0,300]}{Q} + 5000.[0,300] + \frac{10R^2}{2Q} + \frac{50(Q-R)^2}{2Q}$$

Subject to

$Q \geq R$, $Q \geq 0, R \geq 0$

Given that:

$$Q^* = \sqrt{\left(\frac{2K.\lambda_N}{h}\right)\left(\frac{P+h}{P}\right)}$$

$$R^* = \sqrt{\left(\frac{2K.\lambda_N}{h}\right)\left(\frac{P}{P+h}\right)} \qquad , \quad (1)$$

By substituting the given data in (1), we get

$$Q^* = \sqrt{\left(\frac{2.300.[0,300]}{10}\right)\left(\frac{50+10}{50}\right)} = \sqrt{\left(\frac{[0,180000]}{10}\right)\left(\frac{60}{50}\right)} = \sqrt{[0,21600]} \cong [0,147]$$

$$R^* = \sqrt{\left(\frac{2.300.[0,300]}{10}\right)\left(\frac{50}{50+10}\right)} = \sqrt{\left(\frac{[0,9000000]}{600}\right)} = \sqrt{[0,15000]} \cong [0,123]$$

So the deficit allowed is:

$S^* = Q^* - R^* \cong [0,147] - [0,123] = [0,24]$

Minimum storing cost at time unit (month) is:

$$C(Q^* - R^*) = \frac{200.\,[0,300]}{[0,147]} + 5000\,[0,300] + \frac{10\,([0,123)^2}{2[0,147]} + \frac{50([0,147] - [0,123])^2}{2[0,147]}$$
$$= [0,408] + [0,1500000] + [0,515] + [0,116] \cong [0,1501039]$$

To calculate total costs at storing cyclic replace with the following relation:

$$TC(Q^*, R^*) = K + CQ^* + \frac{hR^{*2}}{2\lambda_N} + \frac{P(Q^* - R^*)^2}{2\lambda_N}$$

So we have:

$$TC(Q^*, R^*) = 200 + 5000[0,147] + \frac{10\,([0,123)^2}{2[0,300]} + \frac{50([0,147] - [0,123])^2}{2[0,300]}$$
$$= 200 + [0,735000] + [0,25215] + [0,48] \cong [0,760463]$$

Calculate the net profit of one cyclic storing given that sales price for one ton is (5500) price unit, then the total sales will be:

$V = 5500.\,Q^* = 5500.\,[0,147] = [0,808500]$ (price unit by storing cyclic)

The profit through storing cyclic equals:

$B = V - TC = [0,808500] - [0,760463] = [0,48037]$ (price unit by storing cyclic)

And monthly profit equals:

$B = n^*.\,B = [0,2].\,[0,48037] = [0,96074]$ (price unit by moth)

Comparing between deficit cost and storing cost for unfulfilled request:

we find that the deficit allowed equals:

$S^* = Q^* - R^* \cong [0,147] - [0,123] = [0,24]$

for this deficit cost equal (selling price this amount _ purchase price same amount)

Vended price equal: $5500\,[0,24] = [0,132000]$;

purchase price equal: $5000\,[0,24] = [0,120000]$;

deficit cost equal: $[0,132000] - [0,120000] = [0,12000]$;

fine deficit is equal: $P\,[0,24] = 50.\,[0,24] = [0,12000]$.

It is obvious that the fine of the deficit is equal to storing cost, which means the deficit didn't occur any losses for the warehouse.

Table 1 Classical data and Neutrosophic data results

Classical data and result						Neutrosophic data and result					
$\lambda = 300$	$K = 200$	$C = 5000$	$h = 10$	$P = 50$	$d = 5$	$\lambda_N = [0,300]$	$K = 200$	$C = 5000$	$h = 10$	$P = 50$	$d = 5$
$R^* = 100\,Ton$						$R^* = [0,123]\,Ton$					
$Q^* = 120\,Ton$						$Q^* = [0,147]\,Ton$					
$S^* = 20\,Ton$						$S^* = [0,24]\,Ton$					

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$C(Q^*, R^*) = 1501000 \; unit \; price$	$C(Q^*, R^*) = [0,1501039] \; unit \; price$
$TC(Q^*, R^*) = 600400 \; unit \; price$	$TC(Q^*, R^*) = [0,760463] \; unit \; price$

4. Conclusion and results:

This manuscript presented studying for inventory models with a deficit for one material, models used for storing perishable material based on neutrosophic logic, we found that using neutrosophic models presents comprehensive studying rather than known classical studying. This studying included all definite and indefinite data side by side, we found out at indefinite value for the demand rat at inventory effects on the ideal volume for request and on total costs, and it expressible on it by approximately way where the minimum value of the optimal request volume, and presents the maximum value of optimal request volume, where enables the inventory available into deficit allowed without increasing institute losses. Subsequently, we found that existence of indeterminate affects virtually on results and this indetermination values couldn't be ignoring and excluded it from studying, because the ignoring the indeterminate values will make the nonlinear program problems lacks to more accurate results, consequently, do not getting the inventory ideal volume which meet the needs of the firm during the storing cyclic with minimal cost.

Recently, the work at classical logic isn't enough anymore where the frontiers of science foreground many new problems that need more comprehensive and accurate results than the results were having by using classic logic, and fuzzy logic. Here the neutrosophic logic comes as an essential requirement giving us a more holistic explanation for studying data and procurement accuracy requested results. Looking forwards to more studying for other management inventory models based on neutrosophic logic such as inventory s modals with safety auxiliary for one material, dynamical models and probability models and others.

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The Neutrosophic Treatment for Multiple Storage Problem of Finite Materials and Volumes

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Abstract:

In this paper, we present a multi-inventory with limited size model to clarify the basic idea of multi-inventory systems in order to understand the relationships between the main variables and examine the inventory's behaviour in a very broad range. In addition to the obvious applications in physical warehouses (such as electrical equipment, supplies, raw materials used in manufacturing, etc.), there are less predictable cases in which the multi-inventory model can be used. Such a model can be applied on the number of engineers and employees in a company, also on the number of students and professors in a university, as they constitute the processes of demand, hiring, and laying off which are types of compensation. Moreover, it may be useful at times not to look at physical goods as inventory as the prior examples are both types of inventories based on the space occupied as the available space can accommodate stored materials and is considered an inventory that must be compensated when depleted. The previous examples, in addition to many others, can be classified as inventory problems indicating the abundance of inventory models application, and the possibility of benefiting from the study of inventory theory in terms of clarifying the internal structure of the systems. In this study, we used the Neutrosophic logic to solve the problem of multi-inventory and limited size, depending on the fact that the optimal volume of materials to be stored is affected by the rate of demand for inventory. Moreover, this study is considered an expansion of one of the known classical inventory models that depend on finite data and that is done by assigning a constant value to the inventory demand rate over the storage cycle time, which does not correspond with the realistic application. The limited application of classical inventory models was the motivating factor for this study as it deals with all data, whether specified or not in the inventory management process. Moreover, it considers all cases that the demand for inventory can go through, ranging from the cessation of demand for some stored materials to demand that exceeds the values provided by the real study. Through this study, we developed mathematical relationships that we used to determine the necessary quantities of each of the materials to be stored based on the rate of demand and provide us with results that are more accurate. These results that can be utilized to store many materials in appropriate quantities and available volume, ensure that there is no shortage during the storage cycle period, and enables us to calculate all the necessary costs, which will achieve great profits.

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Keyword: Inventory Management; Inventory Management Models; Neutrosophic Logic; Multiple Storage of Finite Materials and Volumes

1. **Introduction**

Inventory theory has gone through different stages since it was founded in 1920. The models were initially very simple and used a limited number of variables to determine the main parameters that affect the storage process. These models became more complex after adding more variables as they were more detailed.

One limitation was that it only dealt with one product. In contrast, the inventory that we encounter in the real word deals with different materials that are enormous and interactive to an extent that makes their management a complex problem and this led to the development of research to handle the new conditions of the inventory process.

Neutrosophic logic, which is a new vision of modeling developed by American mathematician Florentine Smarandache and is designed to effectively address the uncertainties inherent in the real world, as it came to replace the binary logic that recognizes right and wrong by introducing a third neutral state which can be interpreted as undefined or unconfirmed [5,7,8,9,11].

Florentine Smarandache presented Neutrosophic logic in 1995 as a generalization of fuzzy logic and an extension of the theory of fuzzy categories presented by Lotfi Zadeh [4] in 1965. In addition to that, Ahmed A. Salama presented the theory of Neutrosophic classical categories as a generalization of the theory of classical categories [10, 18] and developed, introduced and formulated new concepts in the fields of mathematics, statistics, computer science, and others by neutrosophic logic [15-17,20,27]. The neutrosophic logic has significantly grown in recent years through its application in measurement, collections, graphs, while in optimization especially the neutrosophic geometric programming was an unfathomable area until Huda E. Khalid et al put the basic neutrosophic mathematical concepts of it since 2014, as well as the concepts of relation equations and many other scientific and practical fields [6,12-14,19,21-25,29-37].

In this paper, we shed the light to the application of neutrosophic logic to one of the inventory management models, which is the multi-inventory model for limited materials and volumes. This application will authorize dealing with inventory management models more accurately, contrary to what is known in the classical logic given that the demand for the stock of stored materials is not precisely defined. Rather, demand may fall within a range that includes all the cases that we can face, starting from the cessation of demand for some materials to the situation in which the demand for some materials exceeds the upper limit provided by the data and market study.

2. Discussion

There are many reasons to keep inventory, including time efficiency and avoiding the cost and inconvenience caused by the continuous and infrequent compensation which requires much inventory and available space. This aids warehouse managers in production facilities or enterprises to determine the appropriate and optimal volume of inventory of each material, to secure the demand in a certain time, and to take advantage of the available space in a way that ensures securing materials throughout the duration of the storage cycle at the lowest possible cost.

Through the classic study of static inventory models, we found that the rate of demand for inventory is subject to a uniform probability distribution. Therefore, when the rate of demand for inventory is known indefinitely (not specified) during one time, we use the Neutrosophic uniform probability distribution that was previously studied [20].

This paper is a continuation of our previous study on inventory management [28].

Firstly, a summary of the studying problem according to classical logic will be presented.

The problem according to classical logic: [1]

Suppose that a production facility stores and sells an m items and the size of the storage warehouse is limited and equals to B unit. If one unit of the material i where $i = 1,2 \dots, m$ occupies S_i from the volume of the repository. The requirement is determining the optimal amount of various inventory materials so that the production facility ensures the availability of that materials during the period of the storage cycle and at the lowest cost. To solve this problem, we build the appropriate mathematical model by setting the following appropriate hypotheses:

1. The rate of demand for an item i is constant and equal to λ_i in a unit of time.

2. The available initial quantity of the item i at the beginning of the storage cycle is equal to Q_i.

3. Whenever the storage level of the item i reaches to zero, it will be compensated with the same quantity Q_i.

4. The price of one unit of item i is equal to C_i.

5. There is a fixed cost K_i of preparing each demand of item i.

6. The cost of storing item i in the time unit is equal to h_i, and that each of these items is ordered at different times and independently of each other.

Mathematical model: to find

$$Min \quad C(Q_i) = \sum_{i=1}^{m} \left(\frac{\lambda_i . K_i}{Q_i} + \frac{h_i . Q_i}{2} + \lambda_i . C_i \right)$$

Subject to:

$$S_1 Q_1 + S_2 Q_2 + \cdots + S_m Q_m \le B$$
$$Q_i \ge 0 \quad ; i = 1,2, \dots, m$$

The mathematical model is a nonlinear model used to find the optimal solution. To solve this model by one of the nonlinear programming methods, the method of Lagrange factors has been nominated by the authors. So, the following Lagrange function will be used: [1,2,3]

$$L(Q_i, \mu) = \sum_{i=1}^{m} \left(\frac{\lambda_i . K_i}{Q_i} + \frac{h_i . Q_i}{2} + \lambda_i . C_i \right) + \mu(S_1 Q_1 + S_2 Q_2 + \cdots + S_m Q_m - B)$$

The optimal solution is given by the following relationship

$$Q_i^* = \sqrt{\frac{2\lambda_i . K_i}{h_i + 2\mu^* . S_i}} \quad ; i = 1,2, \dots, m \tag{1}$$

The optimal value of μ^* let to get the optimal value of Q_i^* satisfying the equality of the constrain $S_1 Q_1 + S_2 Q_2 + \cdots + S_m Q_m \le B$, that mean, $S_1 Q_i^* + S_2 Q_2^* + \cdots + S_m Q_m^* = B$

From the relationship between the stored quantity and the rate of demand for stock, we must store quantities Q_i^* that meet demand, and at the same time, the space needed to store them does not exceed the warehouse space.

3. A Case Study: [1]

A factory stores and sells three raw materials A, B, C in its warehouse, which has an area of $700\ m^2$ only. The data for these materials are as shown in table 1.

<div align="center">Table 1 Case study example</div>

indicator		material	A	B	C
λ_i	The rate of demand for the material i		5000	2000	10000
K_i	The cost of preparing the order of the material i		500	300	200
h_i	Storage cost per unit of material i		10	15	5
S_i	The space required for one unit in m^2		0.70	0.80	0.40

The Required

Determine the optimal stock quantity of the three materials, so that the factory ensures the availability of materials during the storage cycle period and at the lowest cost.

Solution:

From studying data, we notice that the rate of inventory demand is given by specific values, therefore we must find the optimal quantities for the orders of these materials that correspond to the demand rate and are appropriate for the warehouse space. We will use the relationship (1) that gives us the optimal quantity that we have got from the above previous theoretical studying:

$$Q_i^* = \sqrt{\frac{2\lambda_i.K_i}{h_i + 2\mu^*.S_i}} \quad ; i = 1,2, \dots, m$$

By giving the qualitative values of the Lagrange factorial μ, from $\mu = 0$ then $\mu = 3$, $\mu = 5$, $\mu = 8$ to $\mu = 10$, it seems that $\mu = 10$ is the value at which the conditions of the problem are fulfilled and given the following optimal solution:

$$Q_1 = \sqrt{\frac{2\lambda_1.K_1}{h_1 + 2\mu.S_1}} = \sqrt{\frac{(2).(5000).(500)}{10 + (2).(10).(0.7)}} = 456.54 \cong 457$$

$$Q_2 = \sqrt{\frac{2\lambda_2.K_2}{h_2 + 2\mu.S_2}} = \sqrt{\frac{(2).(2000).(300)}{15 + (2).(10).(0.8)}} = 196.75 \cong 197$$

$$Q_3 = \sqrt{\frac{2\lambda_3.K_3}{h_3 + 2\mu.S_3}} = \sqrt{\frac{(2).(10000).(200)}{5 + (2).(10).(0.4)}} = 554.7 \cong 5555$$

$$B = (0.7).(456.54) + (0.8).(196.75) + (0.4).(554.7) = 699.5$$

The above area is approximately equal to the warehouse space $700m^2$, and thus we have reached the optimal solution that gives us the optimal quantities for orders from the three materials, which are: $Q_1^* = 457, Q_2^* = 197, Q_3^* = 555$

These quantities meet the demand for materials during the storage cycle and make the cost of storage as low as possible and occupy the space allocated for storage in the warehouse. However, it does not consider fluctuations in the market and applies only to the specific case with the data contained in the text of the problem. Thus, it was necessary to find a study that gives results in which a margin of freedom considers the fluctuations of the market and takes the problem out of the frame of restriction and limitation.

Therefore, in this paper, we will address the previous problem according to the Neutrosophic logic that considers all definite and indefinite cases, through which we will be able to address market fluctuations resulting from the rate of demand for materials.

4. Reformulate the Above Case Study from a Neutrosophically perspective:

We assume that a certain facility stores and sells m material and that the volume of the storage warehouse is limited and equal to B unit, if one unit of the material i, where $i = 1,2,...,m$, occupies a place S_i equal to the volume of the warehouse.

The Required:

To determine the optimal stock quantity of different materials, so that the factory ensures the availability of materials during the storage cycle period and at the lowest cost. It also takes into account that the rate of demand is indefinite. Thus, to solve this problem, we build the appropriate mathematical model by setting the following appropriate hypotheses:

1.   The rate of demand for an item i is equal to λ_{iN} (indefinite) so that $\lambda_{iN} = [\lambda_{i2}, \lambda_{i2}]$ is an interval or $\lambda_{iN} = \{\lambda_{i2}, \lambda_{i2}\}$ is a set...etc.

2.   The available initial quantity of the item i at the beginning of the storage cycle is equal to Q_i.

3.   Whenever the storage level of the item i reaches to zero, it will be compensated with the same quantity Q_i.

4.   The price of one unit of the item i is equals to C_i.

5.   There is a fixed cost K_i of preparing each order of the item i.

6.   The cost of storing item i in the time unit is equal to h_i, and that each of these items is ordered at different times and independently of each other.

5. Building the Mathematical Model:

Notice from the given problem that the main variable in the storage amount is the rate of demand in the condition that it does not exceed the volume of the warehouse. Therefore, we take λ_{iN} (indefinite) where $\lambda_{iN} = [0, \lambda_{i2}]$.

We denote the total cost of storage for material i by $C_i(Q_i)$ where we find

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$$C_i(Q_i) = \frac{\lambda_{iN}.K_i}{Q_i} + \frac{h_i.Q_i}{2} + \lambda_{iN}.C_i \qquad \text{where } i = 1,2,\dots,m$$

In which $C(Q_i)$ is the total cost of storage and gives the following formula:

$$C(Q_i) = \sum_{i=1}^{m} \left(\frac{\lambda_{iN}.K_i}{Q_i} + \frac{h_i.Q_i}{2} + \lambda_{iN}.C_i \right)$$

In order for the amount of storage does not exceed the volume B of the warehouse, it must meet the following requirement:

$$S_1 Q_1 + S_2 Q_2 + \cdots + S_m Q_m \leq B$$

From that we attain the following neutrosophic mathematical model:

$$Min \quad C(Q_i) = \sum_{i=1}^{m} \left(\frac{\lambda_{iN}.K_i}{Q_i} + \frac{h_i.Q_i}{2} + \lambda_{iN}.C_i \right)$$

Subject to

$$S_1 Q_1 + S_2 Q_2 + \cdots + S_m Q_m \leq B$$
$$Q_i \geq 0 \quad ; i = 1,2,\dots,m$$

The mathematical model is a nonlinear model used to find the optimal solution. We will use the method of Lagrange factors, so we form the following Lagrange function:

$$L(Q_i, \mu) = \sum_{i=1}^{m} \left(\frac{\lambda_{iN}.K_i}{Q_i} + \frac{h_i.Q_i}{2} + \lambda_{iN}.C_i \right) + \mu(S_1 Q_1 + S_2 Q_2 + \cdots + S_m Q_m - B)$$

To find the minimum limit, which is the optimal volume, we differentiate Lagrange function in regards to Q_i and μ where μ is the Lagrange factorial. We take its value to match the problem and we equate the differentiation to zero to get

$$\frac{\partial L(Q_i, \mu)}{\partial Q_i} = \frac{-\lambda_{iN}.K_i}{Q_i^2} + \frac{h_i}{2} + \mu S_i = 0 \qquad\qquad (*)$$

$$\frac{\partial L(Q_i, \mu)}{\partial Q_i} = S_1 Q_1 + S_2 Q_2 + \cdots + S_m Q_m \to B = 0 \qquad (**)$$

We solve the formulas $(*)$ so m equation with m variable is $Q_1, Q_2, \dots, Q_m$ and we get

$$Q_i = \sqrt{\frac{2\lambda_{iN}.K_i}{h_i + 2\mu.S_i}} \quad ; \qquad i = 1,2,\dots,m$$

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Since $\lambda_{iN}, K_i, h_i, S_i, \mu \geq 0$ so the limitation $Q_i \geq 0$ is satisfied

We compensate for the limitation

$S_1 Q_1 + S_2 Q_2 + \cdots + S_m Q_m \leq B$

We find

$$S_1 \sqrt{\frac{2\lambda_{1N}.K_1}{h_1 + 2\mu.S_1}} + S_2 \sqrt{\frac{2\lambda_{2N}.K_2}{h_2 + 2\mu.S_2}} + \cdots + S_m \sqrt{\frac{2\lambda_{mN}.K_m}{h_m + 2\mu.S_m}} \leq B \qquad (2)$$

In which the amount $Q_1, Q_2, \ldots, Q_m$ must satisfy the limitation (2), in the case that it does not

$$S_1 \sqrt{\frac{2\lambda_{1N}.K_1}{h_1 + 2\mu.S_1}} + S_2 \sqrt{\frac{2\lambda_{2N}.K_2}{h_2 + 2\mu.S_2}} + \cdots + S_m \sqrt{\frac{2\lambda_{mN}.K_m}{h_m + 2\mu.S_m}} > B$$

We notice that the left side of the formula becomes smaller when the value of μ increases, since μ is an imagined value we can find one positive value for μ. For example μ^* and that is by gradually increasing the value until we satisfy the limitation (2). This means that the optimal demand value i material is given the following relationship

$$Q_i^* = \sqrt{\frac{2\lambda_{iN}.K_i}{h_i + 2\mu^*.S_i}} \; ; \qquad i = 1,2,\ldots,m \qquad (3)$$

The optimal solution is given by the following relationship

If μ^* makes the constraint $S_1 Q_1 + S_2 Q_2 + \cdots + S_m Q_m \leq B$ equal then the quantities Q_i^* achieve the following equality:

$S_1 Q_1 + S_2 Q_2 + \cdots + S_m Q_m = B$

From the relationship between the stored quantity and the rate of demand for stock, we must store quantities Q_i^* that meet demand, and at the same time, the space needed to store them does not exceed the warehouse space.

We clarify the above aspects through the following example:

6. Practical Example

By taking the same case study that previously discussed in the classical point of view and put the rate of demand for materials indefinite, i.e. we took it in the form of intervals.

The context of the problem from neutrosophically perspective:
A factory stores and sells three raw materials A, B, C in its warehouse, which has an area of $700m^2$ only. The data for these materials were as shown in table 2.

Table 2 neutrosophic context

Indicator	material	A	B	C
λ_{iN}	The rate of demand for the material i	[0,5000]	[0, 2000]	[0, 10000]
K_i	The cost of preparing the order of the material i	500	300	200
h_i	Storage cost per unit of material i	10	15	5
S_i	The space required for one unit in m^2	0.7	0.8	0.4

The required:

Determine the optimal stock quantity of the three materials, so that the factory ensures the availability of materials during the storage cycle period and at the lowest cost.

From the study of the data, we notice that the rate of demand for inventory is given by indeterminate (i.e. indefinite) values (i.e. intervals), and therefore we must find the optimal quantities for the orders of these materials that correspond to the demand rate and are appropriate for the warehouse space. We use the formula (3) that gives us the optimal quantity that we have reached in the theoretical study:

$$Q_i^* = \sqrt{\frac{2\lambda_{iN}.K_i}{h_i + 2\mu^*.S_i}} \quad ; \quad i = 1,2,\dots,m$$

By giving qualitative values of the Lagrange factorial μ, Firstly suppose that $\mu = 0$, then

$$Q_1 = \sqrt{\frac{2\lambda_{1N}.K_1}{h_1 + 2\mu.S_1}} = \sqrt{\frac{2[0,5000].(500)}{10}} = \sqrt{[0,500000]} = [0,707.15]$$

$$Q_2 = \sqrt{\frac{2\lambda_{2N}.K_2}{h_2 + 2\mu.S_2}} = \sqrt{\frac{2[0,2000].(300)}{15}} = \sqrt{[0,80000]} = [0,282.84]$$

$$Q_3 = \sqrt{\frac{2\lambda_{3N}.K_3}{h_3 + 2\mu.S_3}} = \sqrt{\frac{2[0,10000].(200)}{5}} = \sqrt{[0,800000]} = [0,894.45]$$

substitute the above values into the equation:

$$S_1Q_1 + S_2Q_2 + \dots + S_mQ_m = B$$

$$0.7[0,707.15] + 0.8[0,282.84] + 0.4[0,894.45] = [0,495] + [0,226.27] + [0,357.78]$$

$$= [0,1079.05]$$

We need to ensure that the upper bound that we obtain from the above interval is less than or equal to the area of the warehouse $700m^2$. We note that the upper bound of the interval $[0,1079.05]$ is greater than the area of the warehouse (i.e. $1079.05 > 700\ m^2$). Thus, we must give a new value for μ that is greater than zero to reduce the stored quantity of a substance where $i = 1,2,3$. Assume that $\mu = 3$, by substituting this value into formula (3), we will obtain the following intervals:

$$Q_1 = \sqrt{\frac{2\lambda_{1N}.K_1}{h_1 + 2\mu.S_1}} = \sqrt{\frac{2[0,5000].(500)}{10 + (2).(3).(0.7)}} = \sqrt{[0,352000]} = [0,593.3]$$

$$Q_2 = \sqrt{\frac{2\lambda_{2N}.K_2}{h_2 + 2\mu.S_2}} = \sqrt{\frac{2[0,2000].(300)}{15 + (2).(3).(0.8)}} = \sqrt{[0,60600]} = [0,246.17]$$

$$Q_3 = \sqrt{\frac{2\lambda_{3N}.K_3}{h_3 + 2\mu.S_3}} = \sqrt{\frac{2[0,10000].(200)}{5 + (2).(3).(0.4)}} = \sqrt{[0,540500]} = [0,735.18]$$

$$0.7[0,593.3] + 0.8[0,246.17] + 0.4[0,735.18] = [0,415.31] + [0,196.94] + [0,294.07]$$

$$= [0,906.32]$$

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It is clear that the upper bound of the interval $[0, 906.32]$ is greater than the $700\ m^2$ area of the warehouse. Thus, we must give a new value for μ that is greater than 3 to reduce the stored quantity of a substance i where $i = 1,2,3$. For example, we substitute $\mu = 5$ into formula (3) and we obtain:

$$Q_1 = \sqrt{\frac{2\lambda_{1N}.K_1}{h_1 + 2\mu.S_1}} = \sqrt{\frac{2[0,5000].(500)}{10 + (2).(5).(0.7)}} = \sqrt{[0,294000]} = [0,542.22]$$

$$Q_2 = \sqrt{\frac{2\lambda_{2N}.K_2}{h_2 + 2\mu.S_2}} = \sqrt{\frac{2[0,2000].(300)}{15 + (2).(5).(0.8)}} = \sqrt{[0,52160]} = [0,228.39]$$

$$Q_3 = \sqrt{\frac{2\lambda_{3N}.K_3}{h_3 + 2\mu.S_3}} = \sqrt{\frac{2[0,10000].(200)}{5 + (2).(5).(0.4)}} = \sqrt{[0,444400]} = [0,666.63]$$

$$0.7[0,542.22] + 0.8[0,228.39] + 0.4[0,666.63] = [0,379.55] + [0,182.71] + [0,266.65]$$
$$= [0,828.91]$$

We note that the upper bound of the field $[0,828.91]$ is greater than the $700 m^2$ area of the warehouse. Thus, we must give a new value for μ that is greater than 5 to reduce the stored quantity of a substance i where $i = 1,2,3$. For example, we substitute $\mu = 8$ into formula (3) and we obtain:

$$Q_1 = \sqrt{\frac{2\lambda_{1N}.K_1}{h_1 + 2\mu.S_1}} = \sqrt{\frac{2[0,5000].(500)}{10 + (2).(8).(0.7)}} = \sqrt{[0,235800]} = [0,485.59]$$

$$Q_2 = \sqrt{\frac{2\lambda_{2N}.K_2}{h_2 + 2\mu.S_2}} = \sqrt{\frac{2[0,2000].(300)}{15 + (2).(8).(0.8)}} = \sqrt{[0,43160]} = [0,207.75]$$

$$Q_3 = \sqrt{\frac{2\lambda_{3N}.K_3}{h_3 + 2\mu.S_3}} = \sqrt{\frac{2[0,10000].(200)}{5 + (2).(8).(0.4)}} = \sqrt{[0,350800]} = [0,592.28]$$

$$0.7[0,485.59] + 0.8[0,207.75] + 0.4[0,592.28] = [0,339.91] + [0,166.2] + [0,236.91]$$
$$= [0,743.01]$$

Again, we note that the upper bound of the field $[0,743.01]$ is greater than the $700 m^2$ area of the warehouse. Thus, we must give a new value for μ that is greater than 8 to reduce the stored quantity of a substance i where $i = 1,2,3$. For example, we substitute $\mu = 10$ into formula (3) and we obtain:

$$Q_1 = \sqrt{\frac{2\lambda_{1N}.K_1}{h_1 + 2\mu.S_1}} = \sqrt{\frac{2[0,5000].(500)}{10 + (2).(10).(0.7)}} = \sqrt{[0,208350]} = [0,456.45]$$

$$Q_2 = \sqrt{\frac{2\lambda_{2N}.K_2}{h_2 + 2\mu.S_2}} = \sqrt{\frac{2[0,2000].(300)}{15 + (2).(10).(0.8)}} = \sqrt{[0,38700]} = [0,196.72]$$

$$Q_3 = \sqrt{\frac{2\lambda_{3N}.K_3}{h_3 + 2\mu.S_3}} = \sqrt{\frac{2[0,10000].(200)}{5 + (2).(10).(0.4)}} = \sqrt{[0,307700]} = [0,554.7]$$

$$0.7[0,456.45] + 0.8[0,196.72] + 0.4[0,554.7] = [0,319.52] + [0,157.38] + [0,221.88]$$
$$= [0,698.78]$$

We note that the upper limit of the field $[0,698.78]$ is approximately equal $700 m^2$ to the area of the warehouse, and thus we have reached the ideal solution that gives us the optimal quantities of orders from the three materials, which are:
$$Q_1^* = [0,456.45],\ Q_2^* = [0,196.72],\ Q_3^* = [0,554.7]$$

These quantities meet the demand for materials during the storage cycle and make the cost of storage as low as possible and occupy the space allocated for storage in the warehouse.

Notes:

1.   We note that the ideal quantities of orders from the three materials that we obtained from the classical study only represent one quantity from the range of optimal quantities obtained in the Neutrosophic study. This illustrates that using the Neutrosophic logic yields the best and most accurate results considering that the rate of demand for inventory changes and is affected by the market.

2.   Any kind of imprecise data such as intervals, sets or other vague (i.e. indefinite) value can be selected for λ_{iN} , and meets the limitations so that the values we obtain for the required quantities do not exceed the space needed by the available space $700m^2$.

7. A Case Study

A factory stores and sells three raw materials A, B, C in its warehouse, which has an area of $700m^2$ only. The data for these materials were as shown in table 3.

Table 3 case study on the neutrosophic context

Indicator material		A	B	C
λ_{iN}	The rate of demand for the material i	[2000,5000]	[1500, 2000]	[4000, 10000]
K_i	The cost of preparing the order of the material i	500	300	200
h_i	Storage cost per unit of material i	10	15	5
S_i	The space required for one unit in m^2	0.7	0.8	0.4

Required:

Obtaining the optimal demand values for the raw materials using the relationship that gives the optimal value, as denoted in the prior theoretical study:

$$Q_i^* = \sqrt{\frac{2\lambda_{iN}.K_i}{h_i + 2\mu^*.S_i}} \; ; \qquad i = 1,2,\dots,m$$

By giving hypothetical values for the Lagrange factorial μ from $\mu = 0$ then:

$$Q_1 = \sqrt{\frac{2\lambda_{1N}.K_1}{h_1 + 2\mu.S_1}} = \sqrt{\frac{2[2000,5000].(500)}{10}} = \sqrt{[20000,500000]} = [141.42,223.60]$$

$$Q_2 = \sqrt{\frac{2\lambda_{2N}.K_2}{h_2 + 2\mu.S_2}} = \sqrt{\frac{2[1500,2000].(300)}{15}} = \sqrt{[60000,80000]} = [244.9, 282.8]$$

$$Q_3 = \sqrt{\frac{2\lambda_{3N}.K_3}{h_3 + 2\mu.S_3}} = \sqrt{\frac{2[5000,10000].(200)}{5}} = \sqrt{[400000,800000]} = [632.46, 894.43]$$

We insert these values into the formula:

$$S_1Q_1 + S_2Q_2 + S_3Q_3 = 700$$

$$0.7[141.42,223.60] + 0.8[244.9, 282.8] + 0.4[632.46, 894.43]$$

$$= [98.99, 187.81] + [195.9, 226.24] + [252.98,357.77] = [547.87,771.82]$$

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We compare the upper bound 771,82 with the available space 700 and we note that:

$771,82 > 700$

It is clear that the gained result from $\mu = 0$ is close to the available space value , therefore, we will take $\mu = 1$, predicting to get the optimal value as follow:

$$0.7[132.45,209.43] + 0.8[232.85,268.87] + 0.4[525.23,830.45]$$
$$= [92.72,146.6] + [186.28,215] + [210,332.18] = [489,693.78]$$

We compare the upper bound of the interval $= [489,693.78]$ with the available space $700m^2$ we note that:

$693.78 \cong 700$

This means that we have used almost the entire available space, and then the ideal quantities for orders on the three materials are as follows:

$Q_1^* = [132.45,209.43], Q_2^* = [232.85,268.87], Q_3^* = [525.23,830.45]$

Note :

If we choose the Lagrange factorial and get a size smaller than the available, we continue to search for the optimal solution by taking a value of the Lagrange factorial that is smaller than the value that was used in order to increase the size. For example, if we take $\mu = 3$ we get

$[83.08,131.36] + [170.56,196.94] + [186,294.08] = [439.64,622.38]$

We compare the upper bound of the interval $[439.64,622.38]$ with the available space $700m^2$ and we note that:

$622.38 < 700$

This means that we have not used all of the available space, we must give a new value for μ that is less than 3 to enlarge the stored quantity of i substance where $i = 1,2,3$

Also, if we take $\mu = 2$ we obtain:

$[276.7,138.35] + [177.9,205.42] + [196.95,311.4] = [593,655.17]$

We compare the upper bound of the interval $[593,655.17]$ with the available space $700m^2$ and we note that: $655.17 < 700$

This means that we have not used all of the available space, we must give a new value for μ that is less than 2 to enlarge the stored quantity of i substance where $i = 1,2,3$

Practical Fact
When choosing the Lagrange factorial, if we obtain a value greater than the actual value, we increase the Lagrange factorial to reduce the result. On the other hand, if we obtain a value smaller than the actual value, we reduce the Lagrange factorial to increase the result.

8. Conclusion

Through the previous study, we note that the use of neutrosophic technique provides the production facilities, which depend on storing materials, a safe working environment that guarantees them the ability to address market fluctuations resulting from fluctuation in demand for inventory during the duration of the storage cycle. This is possible through the plans presented in this study, which included most of the cases that a production facility can go through, where the company can re-order the appropriate quantities according to the rate of demand for inventory and the volume available for it.

The results we reached through our study reflect the reality of the market, and this realistic application was lacking in the study according to the classical logic that deals with the rate of demand for inventory as a fixed amount throughout the duration of the storage cycle. The neutrosophic logic provides us with a more comprehensive study and allows us to compute the most accurate results possible.

In the near future, we look forward to studying the rest of the inventory management models according to the neutrosophic techniques, such as inventory models with a deficit of one item, dynamic models… etc.

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Neutrosophical dynamic programming

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Abstract:

The great development that science has witnessed in all fields has reduced the risks and losses resulting from undertaking any business or projects. Since the emergence of the science of operations research, many life issues have been addressed by relying on it, and by using its methods, we have been able to establish projects and businesses and use the available capabilities in an ideal manner. Which achieved great success in all areas and reduced the losses of all kinds, whether material or human, that we were exposed to because of carrying out these works or projects without prior study. We are now able to model, analyze and solve a wide range of problems that can be broken down into a set of partial problems using dynamic programming. Programming that is used to find the optimal solution in a multi-step situation that involves a set of related decisions. In this research, we study one of the operations research problems that are solved using dynamic programming. It is the problem of creating an expressway between two cities, using the neutrosophic logic. The logic that takes into account all the specific and non-specific data and takes into account all the circumstances that can face us during the implementation of the project. The goal of studying this issue is to determine the optimal total cost, which is related to the partial costs presented by the study prepared for this project. In order to avoid losses we will take the partial costs neutrosophic values of the form $[\lambda_{1i} , \lambda_{2i}]$, where λ_{1i} represents the minimum partial cost in stage i and λ_{2i} represents the upper limit of the partial cost in stage i . Through the indeterminacy offered by neutrosophic logic, we are able to find the ideal solution that will bring us the lowest possible cost for constructing this expressway. It takes into account all the circumstances that may encounter us in our study, and we will present an applied example that illustrates the study.

Keywords: Dynamic programming; neutrosophic logic; neutrosophic dynamic programming; the problem of constructing a expressway between two cities

1.Introduction

Dynamic programming is the method that helps in selecting the right decisions and approving the best program for independent activities, taking into consideration the available resources. We will use the concept of dimension, which is the criterion that affects the decision-making process at a certain stage of obtaining the optimal solution. If the criterion is the only one in making the decision, then the problem is called one-dimensional, otherwise the problem is multi-dimensional. As well as the concept of influencer

which is the target follower. That is, if the problem contains more than one goal, then we call it a multi-factor problem [19,20,21].

In our study, we present the issue of creating an expressway between two cities based on dynamic programming within the framework of the neutrosophic logic, the logic that takes into account all possible cases and enables us to deal with data, whether specific or undefined, and taking into account all the circumstances that can face us during the implementation of the study.

It should be noted that the neutrosophic logic saw the light through its founder, the American Mathematical Professor Florentin Smarandache in 1995, where he presented it as a generalization of fuzzy logic, especially the intuitive fuzzy logic [1,2,3]. This interesting logic began to spread widely among researchers of various specializations with the passage of time [4,5,6,7,8,9,10,11,12,13,14,15,16,17,18,22].

Our goal in this study is to determine the optimum total cost, which is related to the partial costs presented by the study prepared for this project in a neutrosophic manner, so that we avoid falling into any losses and complete the project at the lowest possible cost.

2. Discussion:

To formulate the dynamic program for a particular problem, we break it down into partial problems with a specific criterion according to the nature of the problem under study. In each issue we identify a set of possibilities from which a set of possible decisions branch.We define the measure of effectiveness in the form of cost, profit, time, or any other measure. We call the scale the goal function. We point out that the optimal decision in each partial problem is related to the decision that achieves the optimum value (the greatest profit - the least cost - the least time period) for the goal   function in the previous problem. We consider the stage and condition resulting from the first problem as initial conditions.

In classical logic, when the problem of dynamic programming has one dimension and one influence, then we represent the dynamic programming method with the following relationships:

$$Z_i = F_i(x_i) \qquad i = 1,2,\ldots,n$$

where:

Z_i is the state function that expresses the state the problem has reached at a particular partial stage.

x_i is the decision variable in stage i.

The relationship between the case function Z_i (in a certain stage) and the case Z_{i-1} (in the previous stage) and the decision in stage i. It takes the following form:

$$Z_i = g_i(x_i, z_{i-1})$$

- The value of the influencer that we are looking for is the goal function G. We calculate it from the following relationship:

$$G_i(Z_i) = F_i(x_i, z_{i-1}, z_i)$$

We note that it is calculated according to the partial functions of the goal in the previous stages.

- This function should take an optimal value at each stage.

We calculate the optimal value by the relationship:

$$G_n^*(Z_n) = \underset{x_1 \to x_n}{opt} \left[F_1(x_1, z_0, z_1) + \ldots + F_1(x_n, z_{n-1}, z_n) \right]$$

In addition to the general relationship of constraints:

$$Z_i = g_i(x_i, z_i)$$

Which can be expressed as follows:

$$G_n^*(z_n) = \underset{n}{\text{opt}}\left[F_n(x_n, z_n, z_{n-1}) + G_{n-1}^*(z_{n-1})\right]$$

Where:

$G_n^*(z_n)$: The ideal goal function at stage n.

$G_{n-1}^*(z_{n-1})$: The ideal goal function at stage n-1.

The previous relations take the following detailed form:

$$G_1^*(z_1) = \underset{x_1}{\text{opt}}\left[F(x_1, z_0, z_1)\right] \tag{1}$$

$$G_2^*(z_2) = \underset{x_2}{\text{opt}}\left[F(x_2, z_1, z_2)\right] \tag{2}$$

$$\dots\dots\dots\dots\dots\dots\dots\dots$$

$$G_i^*(z_i) = \underset{x_i}{\text{opt}}\left[F(x_i, z_{i-1}, z_i)\right] \tag{3}$$

$$\dots\dots\dots\dots\dots\dots\dots\dots$$

$$G_n^*(z_n) = \underset{x_n}{\text{opt}}\left[F(x_n, z_{n-1}, z_n)\right] \tag{4}$$

Restrictions for previous relationships, in order:

$$Z_1 = g_1(x_1, z_0) \tag{1'}$$
$$Z_2 = g_2(x_2, z_1) \tag{2'}$$
$$\dots\dots\dots\dots$$
$$Z_i = g_i(x_i, z_{i-1}) \tag{3'}$$
$$\dots\dots\dots\dots$$
$$Z_n = g_n(x_n, z_{n-1}) \tag{4'}$$

The previous relationships with their constraints include all issues of dynamic programming with one dimension and one influence.

If cost, profit or time are neutrosophic values then the issue becomes a matter of neutrosophic dynamic programming,

If the problem of neutrosophic dynamic programming has one dimension and one influence, then we represent the neutrosophic dynamic programming method with the following relations:

$$Z_i = NF_i(x_i) \qquad i = 1,2,\ldots,n$$

where:

Z_i is the state function that expresses the state the problem has reached at a particular partial stage.

x_i is the decision variable in stage i.

- The relationship between the state function Z_i (in a certain stage) and the state function Z_{i-1} (in the previous stage) and the decision in stage i. It takes the following form:

$$Z_i = g_i(x_i, z_{i-1})$$

The value of the influencer we are looking for is the goal function G. We calculate it from the following relationship:

$$NG_i(Z_i) = NF_i (x_i, z_{i-1}, z_i)$$

We note that it is calculated according to the partial functions of the goal in the previous stages.

This function should have an optimal value at each stage.

We calculate the optimal value by the relationship:

$$NG_n^*(Z_n) = \underset{x_1 \to x_n}{opt} \left[NF_1(x_1, z_0, z_1) + \ldots + NF_1(x_n, z_{n-1}, z_n) \right]$$

In addition to the general relationship of constraints:

$$Z_i = g_i (x_i, z_i)$$

Which can be expressed as follows:

$$NG_n^*(z_n) = \underset{n}{opt} \left[NF_n(x_n, z_n, z_{n-1}) + NG_{n-1}^*(z_{n-1}) \right]$$

Where:

$NG_n^*(z_n)$: The Neutrosophic ideal goal function at stage n.

$NG_{n-1}^*(z_{n-1})$: The Neutrosophic ideal goal function at stage n-1.

The previous relations take the following detailed form:

$$N\,nG_1^*(z_1) = \underset{x_1}{opt} \left[NF(x_1, z_0, z_1) \right] \qquad (1)$$

$$NG_2^*(z_2) = \underset{x_2}{opt} \left[NF(x_2, z_1, z_2) \right] \qquad (2)$$

$$\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$$

$$NG_i^*(z_i) = \underset{x_i}{opt} \left[NF(x_i, z_{i-1}, z_i) \right] \qquad (3)$$

$$\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$$

$$NG_n^*(z_n) = \underset{x_n}{opt}\big[NF(x_n, z_{n-1}, z_n)\big] \qquad (4)$$

Restrictions for previous relationships, in order:

$$Z_1 = g_1(x_1, z_0) \qquad\qquad (1)'$$
$$Z_2 = g_2(x_2, z_1) \qquad\qquad (2)'$$
$$\cdots\cdots\cdots\cdots$$
$$Z_i = g_i(x_i, z_{i-1}) \qquad\qquad (3)'$$
$$\cdots\cdots\cdots\cdots$$
$$Z_n = g_n(x_n, z_{n-1}) \qquad\qquad (4)'$$

The previous relationships with their constraints include all issues of dynamic programming with one dimension and one influence.

Application example:

It requires the establishment of an expressway between cities A and F. Where this expressway must pass through cities E, D, C, B. Therefore, it will consist of five sections. For each section, the cost of the various alternatives was studied and evaluated. These alternatives are represented with their costs on the attached figure. It is required to identify the road with the lowest cost to establish this expressway.

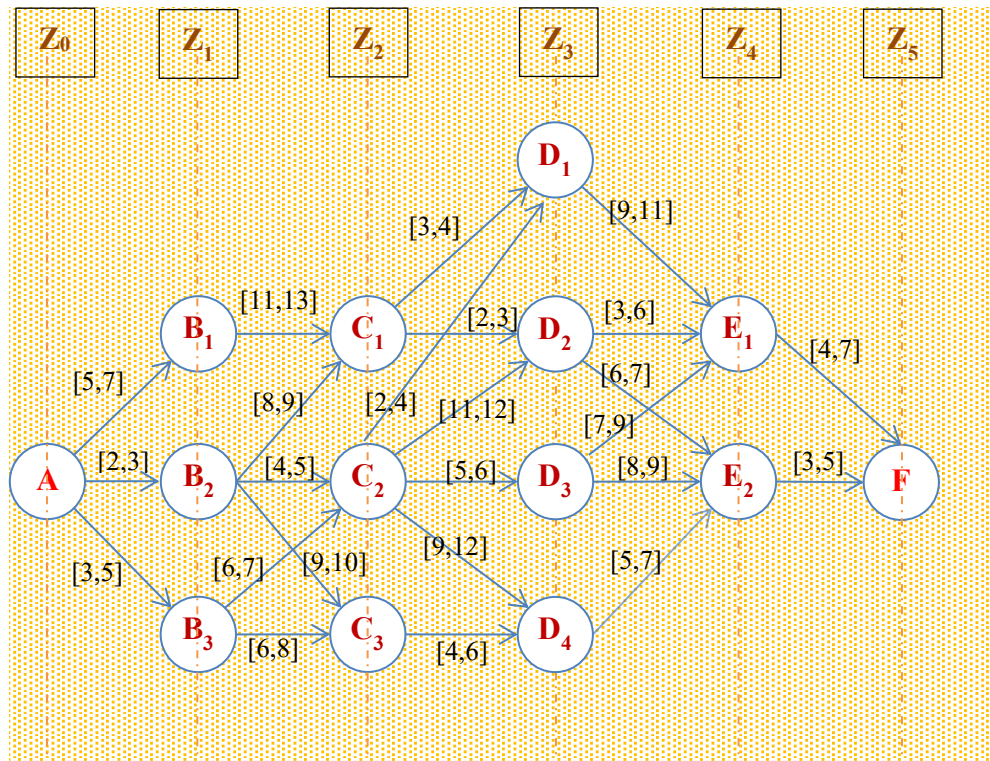The costs are neutrosophic values .

Figure 1. Application Example

Z_i represents the state function, and it is the function that expresses the state the problem has reached at a particular partial stage, so:

Z_0 : A

Z_1: B_1 B_2 B_3

Z_2: C_1 C_2 C_3

Z_3: D_1 D_2 D_3 D_4

Z_4: E_1 E_2

Z_5: F

The relationship used in the solution:

$$NG_n(Z_n) = Min\ [NG_{n-1}(Z_{n-1}) + NF_n(Z_n, Z_{n-1})]$$

Where:

$NG_n(Z_n)$: The neutrosophic minimal cost of the fractional phase from A through Z_n.

$NG_{n-1}(Z_{n-1})$: The neutrosophic minimal cost of the fractional phase from A through Z_{n-1}.

$NF_n(Z_{n-1}, Z_n)$: The neutrosophical cost of the phase or section between (Z_{n-1}, Z_n).

For n = 1. It expresses the minimum cost in the first stage:

$$NG_1(Z_1) = Min\ [NG_0(Z_0) + NF(Z_0, Z_1)]$$

For $NG_1(Z_1)$ we have three alternatives: B_1, B_2, B_3 . So we calculate the cost for each alternative:

$NG_1(B_1) = Min\ [NG_0(Z_0) + NF(A,B_1)] = Min\{\ 0\ +\ [5,7]\ \} = [5,7]$

$NG_1(B_2) = Min\ [NG_0(Z_0) + NF(A,B_2)] = Min\{\ 0\ +\ [2,3]\ \} = [2,3]$

$NG_1(B_3) = Min\ [NG_0(Z_0) + NF(A,B_3)] = Min\{\ 0\ +\ [3,5]\ \} = [3,5]$

The minimum cost in the first stage is $NG_1(B_2)$ ie the appropriate path is AB_2 at a cost $[2,3]$.

Calculating the costs for the first and second phases together:

For n = 2 the mathematical relationship is:

$$NG_2(Z_2) = Min\ [NG_1(Z_1) + NF(Z_1, Z_2)]$$

We have three alternatives for Z2 they: C1, C2, C3. We compute $G_2(Z_2)$ for each of them:

$NG_2(C_1) = Min\ [NG_1(B_1) + NF(B_1,C_1)\ , NG_1(B_2) + NF(B_2,C_1), NG_1(B_3) + NF(B_3,C_1)]$

$NG_2(C_1)$ =

$$Min\{\ [5,7]+[11,13], [2,3]+[8,9], [3,5]+\infty\} = \{[16,20], [10,12], +\infty\} = [10,12]$$

This value corresponds to $Z_1 = B_2$. The least expensive path is AB_2C_1.

$NG_2(C_2) = Min [NG_1(B_1) + NF(B_1,C_2), NG_1(B_2) + NF(B_2,C_2), NG_1(B_3) + NF(B_3,C_2)]$

$$NG_2(C_2) = Min\{[5,7]+\infty, [2,3]+[4,5], [3,5]+[6,7]\} = \{\infty, [6,8], [9,12]\} = [6,8]$$

This is for $Z_1 = B_2$. The least expensive path is AB_2C_2.

$NG_2(C_3) = Min [NG_1(B_1) + NF(B_1,C_3), NG_1(B_2) + NF(B_2,C_3), NG_1(B_3) + NF(B_3,C_3)]$

$$NG_2(C_3) = Min\{[5,7]+\infty, [2,3]+[9,10], [3,5]+[6,8]\} = \{\infty, [11,13], [9,13]\} = [9,13]$$

This value corresponds to $Z_1 = B_3$. So the least expensive path is AB_3C_3.

We choose the optimal path for the first and second stages, and the least cost is: AB_2C_2 and the cost is $[6,8]$.

Calculate the costs for the first, second and third stages i.e. n = 3:

For the Z_3 we have four alternatives, we calculate the cost for each of them:

$NG_3(D_1) = Min [NG_2(C_1) + NF(C_1,D_1), NG_2(C_2) + NF(C_2,D_1), NG_2(C_3) + NF(C_3,D_1)]$

$NG_3(D_1) =$
$$Min\{[10,12]+[3,4], [6,8]+[2,4], [9,13]+\infty\} = \{[13,16], [8,12], +\infty\} = [8,12]$$

Opposite to $Z_2 = C_2$. So the least expensive path is: $AB_2C_2D_1$.

$NG_3(D_2) = Min [NG_2(C_1) + NF(C_1,D_2), NG_2(C_2) + NF(C_2,D_2), NG_2(C_3) + NF(C_3,D_2)]$

$NG_3(D_2)=$
$$Min\{[10,12]+[2,3], [6,8]+[11,12], [11,13]+\infty\} = \{[12,15], [17,20], +\infty\} = [12,15]$$

That's when $Z_2 = C_1$. So the least expensive path is $AB_2C_1D_2$.

$NG_3(D_3) = Min [NG_2(C_1) + NF(C_1,D_3), NG_2(C_2) + NF(C_2,D_3), NG_2(C_3) + NF(C_3,D_3)]$

$$NG_3(D_3)= Min\{[10,12]+\infty, [6,8]+[5,6], [11,13]+\infty\} = \{+\infty, [11,14], +\infty\} = [11,14]$$

Opposite to $Z_2 = C_2$. So the least expensive path is $AB_2C_2D_3$.

$NG_3(D_4) = Min [NG_2(C_1) + NF(C_1,D_4), NG_2(C_2) + NF(C_2,D_4), NG_2(C_3) + NF(C_3,D_4)]$

$NG_3(D_4) =$
$$Min\{[10,12]+\infty, [6,8]+[9,12], [11,13]+[4,6]\} = \{\infty, [15,20], [15,19]\} = [15,19]$$

Opposite to $Z_2 = C_3$. So the least expensive path is $AB_2C_3D_4$.

In comparison, the optimal path for the first, second and third stages is: $AB_2C_2D_1$ and cost $[8,12]$.

Calculate the costs for the first, second, third and fourth stages i.e.: n = 4:

$$NG_4(Z_4) = Min\ [NG_3(Z_3) + NF(Z_3,Z_4)]$$

For the Z_4 we have two alternatives E_1, E_2.

For E_1:

$NG_4(E_1) = Min\ [NG_3(D_1) + NF(D_1,E_1),\ NG_3(D_2) + NF(D_2,E_1),\ NG_3(D_3) + NF(D_3,E_1),\ NG_4(D_4) + NF(D_4,E_1)]$

$NG_4(E_1) = Min\{\ [8,12]+[9,11],\ [12,15]+[3,6],\ [11,14]+[7,9],\ [15,19]+\infty\} = \{[17,23],[15,21],[18,23],+\infty\} = [15,21]$

This is for $Z_3 = D_2$. So the least expensive path is $AB_2C_1D_2E_1$.

For E_2:

$NG_4(E_2) = Min\ [NG_3(D_1) + NF(D_1,E_2),\ NG_3(D_2) + NF(D_2,E_2),\ NG_3(D_3) + NF(D_3,E_2),$
$\qquad\qquad NG_4(D_4) + NF(D_4,E_2)]$

$NG4(E2) = Min\{\ [8,12]+\infty,\ [12,15]+[6,7],\ [11,14]+[8,9],\ [15,19]+[5,7]\} =$

$\{+\infty,\ [18,22],[19,23],[20,26]\} = [18,22]$

This is for $Z_3 = D_2$. So the path has a lower cost: $AB_2C_1D_2E_2$

We proceed to the calculation of the minimum cost of the optimal total stage:

$$NG_5(Z_5) = Min\ [NG_4(Z_4) + NF(Z_4,Z_5)]$$

For the Z_5 we have one alternative which is the F.

$NG_5(F) = Min\ [NG_4(E_1) + NF(E_1,F),\ NG_4(E_2) + NF(E_2,F)]$
$NG_5(F) = Min\{\ [15,21]+[4,7],\ [18,22]+[3,5]\} = Min\{[19,28],[21,27]\} = [19,28]$

This is for $Z_4 = E_1$. So the path with the lowest cost is: $AB_2C_1D_2E_1F$ The cost is equal $[19,28]$.

4. Conclusion and results:

Through this study, we reached the minimum Cost for creating the expressways using the neutrosophic logic. Because of the uncertainty provided by the neutrosophic logic, we were able to find the ideal solution that achieves the lowest possible cost for creating the expressway. It takes into account all the circumstances that may encounter us in our study. This study is also used to calculate the costs of transportation and movement from one city to another city or one country to another. In the near future we are looking to use the Neutrosophical Dynamic Programming concept in other areas such as allocating a budget to projects or allocating engineers within a company's departments.

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Important Neutrosophic Rules for Decision-Making in the Case of Uncertain Data

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Abstract

An urgent need to make a rational decision has emerged in the world of rapid changes based on quantitative methods that reduce the proportion of risk, especially if the decisions are fateful and the decision issues are huge and complex, noting that the decision-making process and the selection of the optimal alternative depends on the quality of the data that describes the issue that the decision is intended to be taken. Because the theory of administrative decision-making depends on this data and on the type of this data if it is confirmed data or unconfirmed data and not specified with sufficient accuracy, or random data that is repeated according to a certain probability distribution law, after which the decision maker uses the methods used to obtain on the optimal decision. In this research we will study the theory of administrative decisions in the case of uncertain data, which is the situation that the decision maker faces, and he does not know anything certain about the state that nature (market or management --) will take, nor even about the possibilities of any of them, then it is assumed that the cases the possible ones are equal and they enter the analysis at the same opportunity and make a trade-off between the alternatives available to him in all circumstances. In the classical logic, a set of rules was used to help the decision maker to make the ideal decision, and since the ideal decision depends on specific classical values that do not take into account the changes that may occur in the work environment, which is represented by high prices or unavailability of materials or others, it was necessary to search for a better method that helps us to avoid dealing with specific values and gives us a margin of freedom. Therefore, in this research, we will study the theory of decision-making in the case of uncertain data using the Neutrosophic Logic, the logic that helps us to face fluctuations and changes that we may encounter during work, through uncertainty that the Neutrosophical values have, which we will take in the elements of the profit (or loss) matrix and rely on them in the decision-making process, as we will take these values in the form of fields representing the minimum field of profit (or loss) that we can get in the worst cases of nature, and represents the upper limit of the field of profit (or loss) that we can get in the best cases of nature, and we will show the most important rules used in the case of uncertain data with an applied example of each rule.

Keywords: Decision-Making Theory; Neutrosophic Logic; Decision-Making in Case of Uncertainty

1. Introduction

To avoid economic inflation and the sensitivity of political and military situations, the need to know the results of any decision appeared before taking it, which prompted mathematicians to develop mathematical models aimed at searching for optimal strategies in conditions of doubt and uncertainty. From the most prominent scientists who had a great impact in this field Von Neumann - Morgan Strain - Nash and the scientist Herbert Simon, who is considered the father of the theory of decision-making, and this is due to the great importance of the theories he developed, as it opened the way for new methods called today artificial intelligence [1,2,3]. The decision, in all fields, is increasing day by day. It was necessary for a new logic to keep pace with the great scientific development, and to limit risks and losses. The Neutrosophic logic, the logic whose emersion was a revolution in all fields of science, was founded by the American mathematical philosopher Florentin Smarandache [4,5,6,7,8], where he presented it in 1995 as a generalization of Fuzzy Logic, and as an extension of the theory of fuzzy categories, presented by Lotfi Zadeh [9] in 1965, where it came to replace the binary logic that recognizes right and wrong by entering a third neutral case, which can be interpreted as undetermined or uncertain, Neutrosophic logic has grown and developed significantly in

recent years through its application in measurement, sets, graphs, and many scientific and practical fields, [10,11,12,13,14,15,16, 17,18,19,20,21,22,23].

2. Discussion: [1, 2]

When the data is uncertain and the decision maker does not know anything about the state that nature will take, not even about the possibilities of any of them occurring, he will deal with them on the basis that they are equivalent, and enter into the analysis at the same opportunity, then he will make a trade-off between the alternatives available to him in all circumstances by using specific rules that were developed to study the state of uncertainty [1,2], we will study in this research some of the rules that were used in the classical logic to determine the optimal decision in the event of uncertainty, according to the Neutrosophic Logic, where we will take in the elements of the profit (or loss) Neutrosophic values of the matrix .

The Problem of Decision-making:

Let us have a set of alternatives, which we will symbolize with $a_1, a_2, a_3, -----a_m$, and a group of states of nature, which we will symbolize with $\theta_1, \theta_2, \theta_3, -----\theta_n$, and we have the profit (or loss) resulting from the alternative i, since $i = 1,2,3, ---m$, and the state of nature j, since $j = 1,2,3, ---n$, we will represent the above in the following table:

Table 1: Profit Matrix for Uncertainty Case

states of nature / alternatives	θ_1	θ_2	θ_3	----------	θ_n
a_1	Nx_{11}	Nx_{12}	Nx_{13}	---------	Nx_{1n}
a_2	Nx_{21}	Nx_{22}	Nx_{23}	---------	Nx_{2n}
a_3	Nx_{31}	Nx_{32}	Nx_{33}	---------	Nx_{3n}
--- ----	------------	--------	--------	----------	----------
a_m	Nx_{m1}	Nx_{m2}	Nx_{m3}	---------	Nx_{mn}

Required: Determining the appropriate alternative so that the decision maker achieves the greatest profit (i.e. the least loss).

To determine the optimal alternative, and since the situation is a state of uncertainty, we will use one of the following rules:

1- Neutrosophic Laplace's Rule (Average Gain or Loss):

In this rule we calculate the average profit (or loss) corresponding to each of the available alternatives, then we choose the largest (smallest) average, and the alternative corresponding to this value is the appropriate decision.

Table2: Neutrosophic Laplace's Rule (Average Gain or Loss):

states of nature / alternatives	θ_1	θ_2	θ_3	--------	θ_n	**average of profit or loss**
a_1	Nx_{11}	Nx_{12}	Nx_{13}	--------	Nx_{1n}	$\overline{NX_1} = \dfrac{\sum\limits_{j=1}^{n} Nx_{1j}}{n}$

a_2	Nx_{21}	Nx_{22}	Nx_{23}	--------	Nx_{2n}	$\overline{NX_2} = \dfrac{\sum\limits_{j=1}^{n} Nx_{2j}}{n}$
a_3	Nx_{31}	Nx_{32}	Nx_{33}	--------	Nx_{3n}	$\overline{NX_3} = \dfrac{\sum\limits_{j=1}^{n} Nx_{3j}}{n}$
--- ----	------- -------	-------- --------	-------- --------	-------- --------	---------- ----------	
a_m	Nx_{m1}	Nx_{m2}	Nx_{m3}	--------	Nx_{mn}	$\overline{NX_m} = \dfrac{\sum\limits_{j=1}^{n} Nx_{mj}}{n}$

In the case of profit, we have:

$$\overline{NX}^* = Max\left\{\overline{NX_1}, \overline{NX_2}, \overline{NX_3}, -\!-\!-, \overline{NX_m}\right\}$$

In case of loss, we have:

$$\overline{NX}^* = Min\left\{\overline{NX_1}, \overline{NX_2}, \overline{NX_3}, -\!-\!-, \overline{NX_m}\right\}$$

In addition, the appropriate alternative is the alternative corresponding to $\overline{NX}^*$

We illustrate the above with the following example:

Practical example:

An investor wants an amount of millions of dollars in the available investment areas, which are:

a_1 Cars trading.

a_2 Real estate trading.

a_3 Purchasing shares of a service company.

a_4 Establishing an industrial project.

It is required to determine the appropriate alternative for the investor according to the following economic conditions:

θ_1 Declining of the economic situation.

θ_2 Stabilization of the economic situation.

θ_3 Improving of the economic situation.

The annual profits of these investments, according to the economic conditions, are shown in the following table: (One thousand dollars annually).

Table 3: profit matrix for the example

states of nature alternatives	θ_1	θ_2	θ_3
a_1	$[100, 190]$	$[200, 260]$	$[300, 390]$

a_2	$[-400,-220]$	$[160,170]$	$[600,650]$
a_3	$[100,170]$	$[290,370]$	$[300,360]$
a_4	$[133,180]$	$[160,280]$	$[290,400]$

Required: find the appropriate alternative using the Neutrosophic Laplace Rule:

We calculate the average profit corresponding to each alternative, and thus we get the following table:

Table 4: Applies Neutrosophic Laplace's Rule

states of nature / alternatives	θ_1	θ_2	θ_3	average of profit
a_1	$[100,190]$	$[200,260]$	$[300,390]$	$\dfrac{[100,190]+[200,260]+[300,390]}{3} =$ $[200,280]$
a_2	$[-400,-220]$	$[160,170]$	$[600,650]$	$\dfrac{[-400,-200]+[160,170]+[600,650]}{3} =$ $[120,206.7]$
a_3	$[100,170]$	$[290,370]$	$[300,360]$	$\dfrac{[100,170]+[290,370]+[300,360]}{3} =$ $[230,300]$
a_4	$[133,180]$	$[160,280]$	$[290,400]$	$\dfrac{[133,180]+[160,280]+[290,400]}{3} =$ $[194.3,286.7]$

By comparing the elements of the last column, we notice that the largest value of the average profit is $[230,300]$ thousand dollars corresponding to the Cars trading (a_3), so we consider the alternative a_3 to be the appropriate decision for the investor according to Laplace's rule.

2- The Neutrosophic Maxi Max rule:

This rule is based on calculating the largest value of profit for each alternative (in each line), then finding the largest of these, and considering the value that corresponds to it as the appropriate decision.

Therefore, if we denote **NY** for the Maxi Max $Nxij$ for the profit from variant i versus the state of nature j, we get the following relationship:

$$NY = \underset{i}{Max}\left[\underset{j}{Max}\,Nx_{ij}\right] = \underset{i}{Max}\,NY_i$$

We explain this rule through the following table:

Table 5: The Neutrosophic Maxi Max rule

states of nature / alternatives	θ_1	θ_2	θ_3	-------	θ_n	$NY_i = \underset{j}{Max}\,Nx_{ij}$
a_1	Nx_{11}	Nx_{12}	Nx_{13}	-------	Nx_{1n}	$NY_1 = \underset{j}{Max}\,Nx_{1j}$
a_2	Nx_{21}	Nx_{22}	Nx_{23}	-------	Nx_{2n}	$NY_2 = \underset{j}{Max}\,Nx_{2j}$

a₃	Nx_{31}	Nx_{32}	Nx_{33}	-------	Nx_{3n}	$NY_3 = \underset{j}{Max}\ Nx_{3j}$
--- ----	--------- ---------	-------- --------	-------- --------	------- -------	--------- ---------	---------- ----------
aₘ	Nx_{m1}	Nx_{m2}	Nx_{m3}	------	Nx_{mn}	$NY_m = \underset{j}{Max}\ Nx_{mj}$

We illustrate the above with the following example:

Practical example: We will take the same example given in the previous rule:

Table 6: profit matrix for the example

states of nature alternatives	θ₁	θ₂	θ₃
a₁	$[100, 190]$	$[200, 260]$	$[300, 390]$
a₂	$[-400, -220]$	$[160, 170]$	$[600, 650]$
a₃	$[100, 170]$	$[290, 370]$	$[300, 360]$
a₄	$[133, 180]$	$[160, 280]$	$[290, 400]$

It is required to determine the appropriate alternative using the Maxi Max rule.

Table 7: Applies Neutrosophic Maxi Max rule

states of nature alternatives	θ₁	θ₂	θ₃	$NY_i = \underset{j}{Max}\ Nx_{ij}$
a₁	$[100, 190]$	$[200, 260]$	$[300, 390]$	$[300, 390]$
a₂	$[-400, -220]$	$[160, 170]$	$[600, 650]$	$[600, 650]$
a₃	$[100, 170]$	$[290, 370]$	$[300, 360]$	$[300, 360]$
a₄	$[133, 180]$	$[160, 280]$	$[290, 400]$	$[290, 400]$

By comparing the elements of the last column, we get: $NY = \underset{i}{Max}\left[\underset{j}{Max}\ x_{ij}\right] = [600, 650]$

That is, the largest values are the value $[600, 650]$ corresponding to alternative a₂. Thus, alternative a₂ is the appropriate alternative according to the Maxi Max rule, that is, the investor must invest the amount in real estate trading.

3- The Neutrosophic Maxi Min Rule:

This rule is based on calculating the smallest value of gain for each alternative (in each line), if the conditions of nature are bad, and then searching for the largest of these small ones, and considering the value corresponding to it as the appropriate decision.

Therefore, if we denote **NZ** for the largest of the smallest and *Nxij* for the profit from variant *i* versus the state of nature *j*, we get the following relation:

$$NZ = \underset{i}{Max}\left[\underset{j}{Min}\ Nx_{ij}\right] = \underset{i}{Max}\ NZ_i$$

We explain this rule through the following table:

Table 8: The Neutrosophic Maxi Min Rule

states of nature / alternatives	θ_1	θ_2	θ_3	-------	θ_n	$NZ_i = \underset{j}{Min}\, Nx_{ij}$
a_1	Nx_{11}	Nx_{12}	Nx_{13}	-------	Nx_{1n}	$NZ_1 = \underset{j}{Min}\, Nx_{1j}$
a_2	Nx_{21}	Nx_{22}	Nx_{23}	-------	Nx_{2n}	$NZ_2 = \underset{j}{Min}\, Nx_{2j}$
a_3	Nx_{31}	Nx_{32}	Nx_{33}	-------	Nx_{3n}	$NZ_3 = \underset{j}{Min}\, Nx_{3j}$
--- ----	--------- ---------	-------- --------	-------- --------	------- -------	--------- ---------	---------- ----------
a_m	Nx_{m1}	Nx_{m2}	Nx_{m3}	------	Nx_{mn}	$NZ_m = \underset{j}{Min}\, Nx_{mj}$

We illustrate the above with the following example:

Practical example: We will take the same example given in the previous rule:

Table 9: profit matrix for the example

states of nature / alternatives	θ_1	θ_2	θ_3
a_1	$[100,190]$	$[200,260]$	$[300,390]$
a_2	$[-400,-220]$	$[160,170]$	$[600,650]$
a_3	$[100,170]$	$[290,370]$	$[300,360]$
a_4	$[133,180]$	$[160,280]$	$[290,400]$

It is required to determine the appropriate alternative using the Maxi Min Rule.

Table 10: Applies Neutrosophic Maxi Min Rule

states of nature / alternatives	θ_1	θ_2	θ_3	$NZ_i = \underset{j}{Max}\, Nx_{ij}$
a_1	$[100,190]$	$[200,260]$	$[300,390]$	$[100,190]$
a_2	$[-400,-220]$	$[160,170]$	$[600,650]$	$[-400,-220]$
a_3	$[100,170]$	$[290,370]$	$[300,360]$	$[100,170]$
a_4	$[133,180]$	$[160,280]$	$[290,400]$	$[133,180]$

Comparing the elements of the last column, we get:

$$NZ = \underset{i}{Max}\left[\underset{j}{Min}\, x_{ij}\right] = [133,180]$$

That is, the largest values are the value $[133,180]$ corresponding to the alternative a_4. Thus, the alternative a_4 is the appropriate alternative according to Maxi Min Rule, that is, the investor must invest the amount in establishing an industrial project.

4- The Neutrosophic Horwes Rule:

Since most of the decision-makers do not look at the future with a completely optimistic or completely pessimistic view, but rather they expect the future to take a middle state between the states of optimism and pessimism, so (Horwes) put a composite rule of the two rules given by the relationship:

$$NH = \underset{i}{Max}\left[\alpha NY_i + (1-\alpha)NZ_i\right]$$

Since this rule $0 \le \alpha \le$ gives us a certain value confined between NZ, NY, that is NZ $\le$ NH $\le$ NY, noting that NH is related to the value we give to α, and this makes it tend towards the value NY if $\alpha > 0.5$ and towards NZ if $\alpha < 0.5$, and if we put $\alpha = 0$ we get NZ and if we put $\alpha = 1$ we get NY.

We explain this rule through the following table:

Table 11: The Neutrosophic Horwes Rule

states of nature / alternatives	θ_1	θ_2	θ_3	------	θ_n	$NH_i = \alpha NY_i + (1-\alpha)NZ_i$
a₁	Nx_{11}	Nx_{12}	Nx_{13}	------	Nx_{1n}	$NH_1 = \alpha NY_1 + (1-\alpha)NZ_1$
a₂	Nx_{21}	Nx_{22}	Nx_{23}	------	Nx_{2n}	$NH_2 = \alpha NY_2 + (1-\alpha)NZ_2$
a₃	Nx_{31}	Nx_{32}	Nx_{33}	------	Nx_{3n}	$NH_3 = \alpha NY_3 + (1-\alpha)NZ_3$
--- ----	--------- ---------	-------- --------	-------- --------	------ ------	--------- ---------	---------- ----------
aₘ	Nx_{m1}	Nx_{m2}	Nx_{m3}	------	Nx_{mn}	$NH_m = \alpha NY_m + (1-\alpha)NZ_m$

Where NY_i and NZ_i are calculated as in the previous two rules, and we choose the appropriate alternative according to this rule.

$$NH = \underset{i}{Max}\left[\alpha NY_i + (1-\alpha)NZ_i\right]$$

We illustrate this rule through the example given in the previous rules and consider $\alpha = 0.2$.

Table 12: Applies Neutrosophic Horwes Rule

states of nature / alternatives	θ_1	θ_2	θ_3	NY_i	NZ_i	NH_i
a₁	$[100, 190]$	$[200, 260]$	$[300, 390]$	$[300, 390]$	$[100, 190]$	$[140, 230]$
a₂	$[-400, -22]$	$[160, 170]$	$[600, 650]$	$[600, 650]$	$[-400, -22]$	$[-200, -4]$
a₃	$[100, 170]$	$[290, 370]$	$[300, 360]$	$[300, 360]$	$[100, 170]$	$[140, 208]$
a₄	$[133, 180]$	$[160, 280]$	$[290, 400]$	$[290, 400]$	$[133, 180]$	$[164.4, 224]$

We apply the rule $NH = \underset{i}{Max}\left[\alpha NY_i + (1-\alpha)NZ_i\right]$ to the elements of the last column. We note that the largest value is $[164.4, 224]$ and it corresponds to the a₄ alternative, and therefore the a₄ alternative is the appropriate decision according to the (Horwes) rule, that is, the investor must invest the amount in establishing an industrial project.

5. The Neutrosophic Min Max Rule (Savage Rule):

This rule depends on new expressions called Savage Expressions that we symbolize with NS_{ij} and get them from the relationship:

$$NS_{ij} = \left[\underset{i}{Max}\, Nx_{ij}\right] - Nx_{ij}$$

Table 13: The Neutrosophic Min Max Rule (Savage Rule)

states of nature \ alternatives	θ_1	θ_2	θ_3	---	θ_n	θ'_1	θ'_2	θ'_3	--	θ'_n
a₁	Nx_{11}	Nx_{12}	Nx_{13}	---	Nx_{1n}	NS_{11}	NS_{12}	NS_{13}	-----	NS_{1n}
a₂	Nx_{21}	Nx_{22}	Nx_{23}	---	Nx_{2n}	NS_{21}	NS_{22}	NS_{23}	-----	NS_{2n}
a₃	Nx_{31}	Nx_{32}	Nx_{33}	---	Nx_{3n}	NS_{31}	NS_{32}	NS_{33}	-----	NS_{3n}
--- ----	------ -------	------- -------	------- -------	---	------ -------	------ -------	------ -------	------ -------	----- -----	
aₘ	Nx_{m1}	Nx_{m2}	Nx_{m3}	---	Nx_{mn}	NS_{m1}	NS_{m2}	NS_{m3}	-----	NS_{mn}

(Columns θ'_1 through θ'_n are grouped under the heading **Savage expressions**.)

That is, we take the largest value in each column and then subtract the values in that column from it, we get a new column, and from the new columns we get a new matrix that we call the Savage Expression Matrix, then we apply to the elements of this matrix the following rule $\underset{j}{Max}\, NS_{ij}$, that is, we choose from each line the largest value, and thus we get on a new column in the matrix of Savage amounts, as in the following table:

Table 14: Savage Expressions

Savage expressions					$\underset{j}{Max}\, NS_{ij}$
θ'_1	θ'_2	θ'_3	--	θ'_n	
NS_{11}	NS_{12}	NS_{13}	------	NS_{1n}	$\underset{j}{Max}\, NS_{1j}$
NS_{21}	NS_{22}	NS_{23}	------	NS_{2n}	$\underset{j}{Max}\, NS_{2j}$
NS_{31}	NS_{32}	NS_{33}	------	NS_{3n}	$\underset{j}{Max}\, NS_{3j}$
------ -------	------ -------	------ -------	------ -------		
NS_{m1}	NS_{m2}	NS_{m3}	------	NS_{mn}	$\underset{j}{Max}\, NS_{mj}$

We compare the elements of the new column and choose the smallest value for which the corresponding alternative is the appropriate decision, i.e. we apply the relationship:

$$\underset{i}{Min}\left(\underset{j}{Max}\, NS_{ij}\right)$$

We illustrate this rule through the example given in the previous rules:

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Table 15: Applies Neutrosophic Min Max Rule (Savage Rule)

states of nature \ alternatives	θ_1	θ_2	θ_3	Savage expressions		
				θ'_1	θ'_2	θ'_3
a_1	$[100,190]$	$[200,260]$	$[300,390]$	$[10,33]$	$[90,110]$	$[260,300]$
a_2	$[-400,-220]$	$[160,170]$	$[600,650]$	$[-267,-40]$	$[130,200]$	$[0,0]$
a_3	$[100,170]$	$[290,370]$	$[300,360]$	$[10,33]$	$[0,0]$	$[290,300]$
a_4	$[133,180]$	$[160,280]$	$[290,400]$	$[0,0]$	$[90,130]$	$[250,310]$

We have taken the largest value in each column and then subtracted from it the values in that column, and thus we got a new matrix that we call the Savage Amounts Matrix.

We applied the following rule $\underset{j}{Max}\ NS_{ij}$ to the elements of this matrix, and thus we got a new column in the Savage Amounts Matrix as shown in the following table:

Table 16: Savage Expressions

Savage Expressions			$\underset{j}{Max}\ NS_{ij}$
θ'_1	θ'_2	θ'_3	
$[10,33]$	$[90,110]$	$[260,300]$	$[260,300]$
$[-267,-40]$	$[130,200]$	$[0,0]$	$[130,200]$
$[10,33]$	$[0,0]$	$[290,300]$	$[290,300]$
$[0,0]$	$[90,130]$	$[250,310]$	$[250,310]$

We compare the elements of the new column and choose the smallest value for which the corresponding alternative is the appropriate decision, i.e. we apply the relationship:

$$\underset{i}{Min}\left(\underset{j}{Max}\ NS_{ij}\right)=[130,200]$$

In addition, the value $[130,200]$ is the appropriate value according to the rule (Savage), and it is corresponding to the alternative a_2, that is, the appropriate decision is the alternative a_2 according to this rule (Savage), the investor must invest the amount in the real estate trade.

6. Conclusion and results:

From the previous study, we get the following table

Table 17: Comparison results

The decision	Alternative	The profit	The rule
Cars trading	a_3	$[230,300]$	Laplace
Real estate trade	a_2	$[600,650]$	Maxi Max
Establishing an industrial project	a_4	$[133,180]$	Maxi Min

Establishing an industrial project	a_4	$\begin{bmatrix} 164.4 , 224 \end{bmatrix}$	Horwes
Real estate trade	a_2	$\begin{bmatrix} 130 , 200 \end{bmatrix}$	Savage

We note from the table that:

The alternative a_2 was chosen according to two rules, the Maxi Max rule and Min Max Rule (Savage Rule), but with a different estimate of the profit, and the reason for this is due to the optimist's view and his choice of the highest profit.

The alternative a_4 was chosen according to two rules, the Maxi Min rule and Horwes rule. The reason for this is that the rule of Horwes is related to the value we give to α, and this makes it tend towards NZ if $\alpha < 0.5$ and in this issue $\alpha = 0,2$ and $0,2 < 0,5$ were chosen, and if it was chosen is greater than $0,5$ we would have found a convergence between the base of the Horwes and the base of Maxi Max.

Alternative a_3 was chosen according to Laplace's rule only.

Alternative a_1 was not selected according to any of the rules.

The decision remains with the investor because he is the decision maker.

We also note that the Neutrosophical values of profits give us for each alternative the lowest possible profit in the event that was chosen and the greatest profit, according to the state of nature.

We are looking forward in the near future to prepare research in decision-making theory that includes other cases and decision trees.

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The State of Risk and Optimum Decision According to Neutrosophic Rules

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Abstract

In light of the current circumstances and the momentary fluctuations that we live in all domains of life, which have transformed most decision-making cases into risky cases, this study was necessary. In this research, we present a new study for decision-making in the event of risk, by formulating some rules according to the neutrosophic logic starting from the data of the problem, where we replaced the classical values in the matrix of profit and probabilities with neutrosophic values that have a margin of freedom thanks to the indeterminacy provided by the neutrosophic logic, and we got more accurate solutions than the solutions that we obtained when using the classical rules which reduces the occurrence of losses, especially if the decisions are fateful and the decision issues are huge and complex and helps us to achieve the greatest profits by adopting a rational decision based on modern quantitative ways and methods.

Keywords: Decision-Making Theory; Neutrosophic Logic; Risk Condition; Aspiration Level; Most Probable Case; Largest Expected Values.

1. Introduction:

In the world of rapid changes, there has been an urgent need to make a rational decision based on quantitative methods that reduce the risk ratio. The case of risk has been studied according to the classical logic and through the application of appropriate rules that enable us to reach the optimal decision using the amounts corresponding to the alternatives and states of nature.

Since these quantities are classical values, the decision will not be accurate enough to guarantee the greatest profits and the least losses.

 As in all the topics of operations research and other fields of life, and after the great revolution brought about by the neutrosophic logic in all fields of science, where many researchers tended to reformulate many topics in different fields of science according to the basic concepts of this logic and many researches were published on operations research and most mathematics topics [1_17], In this research, we presented a study for decision-making in the case of risk, and it is an extension of the studies presented by many researchers using neutrosophic logic [18,19], which gave more accurate results, and helped the decision maker to take a rational decision that reduces risk because it is based on accurate scientific foundations.

2. Discussion:

 the movement of nature is random and subject to a certain probability distribution, noting that the probability distribution may not be completely known, but rather a distribution assumed by experts, or by the decision maker himself, here is the risk that may be reflected in the decision itself to reduce them, the possibilities corresponding to the states of nature must be calculated, or estimated through practical facts or a statistical study taken from previous experiences and studies, so the researchers provided some rules using the concepts of classical logic

that can be relied upon during decision-making in the case of risk, by studying decision-making theory according to the classical logic as mentioned in the following references[20,21].

1- The issue of making a decision in case of risk according to the classical logic:

The decision maker has alternatives $A(a_1, a_2, ---, a_m)$ where m the number of alternatives available to the decision maker and the situations that nature can take in the future $\theta(\theta_1, \theta_2, ---, \theta_n)$ where n is the number of states that nature can take when in motion, and the amount of profit or loss that the decision maker will achieve is $X(a_i, \theta_j)$ or by short code X_{ij} Then the profit matrix is given in the following table:

Table 1: profit matrix table

States of nature / Alternatives	θ_1	θ_2	θ_3	----	θ_n
a_1	x_{11}	x_{12}	x_{13}	----	x_{1n}
a_2	x_{21}	x_{22}	x_{23}	----	x_{2n}
a_3	x_{31}	x_{32}	x_{33}	----	x_{3n}
-----	-----	-----	-----	-----	-----
a_m	x_{m1}	x_{m2}	x_{m3}	-----	x_{mn}

In addition to the law of the probability distribution that governs the possible states of nature which is a numerical series or a mathematical function corresponding to each state of nature, as follows: $P(P_1, P_2, ---, P_n)$ where $P_j \geq 0$ & $\sum_{j=1}^{n} P_j = 1$ and the goal of the decision maker is to obtain the greatest possible profit or the least possible loss.Then it is summarized in the following table:

Table 2: Profit table and odds

States of nature / Alternative	θ_1	θ_2	θ_3	----	θ_n
a_1	x_{11}	x_{12}	x_{13}	----	x_{1n}
a_2	x_{21}	x_{22}	x_{23}	-----	x_{2n}
a_3	x_{31}	x_{32}	x_{33}	----	x_{3n}
-----------	-----	-----	-----	-----	-----
a_m	x_{m1}	x_{m2}	x_{m3}	-----	x_{mn}
P_j	P_1	P_2	P_3	-----	P_n

To determine the optimal decision, the classic study presented rules, including as mentioned in

 References[20, 21]:

1-1- Ambition Level Rule:

In this rule the decision maker sets a certain level of profit and another level of loss and makes the decision to achieve both levels.

Practical example (1-1) We have the following table of alternatives and states of nature:

Table 3: profit matrix table

States of nature / Alternative	θ_1	θ_2	θ_3
a_1	300	100	400

a_2	-220	170	500
a_3	-400	200	300
a_4	160	250	200

It is required to determine the appropriate alternative using the level of ambition rule according to the following data: the level of ambition of the decision maker:

a. The profit should not be less than 300 .

b. That the loss does not exceed200 .

By studying the elements of the table, we notice that the alternative line a1 achieves the level of ambition of the decision-maker in profit.

And from the study of the alternative line a_2, we note that it achieves the level of ambition of the decision-maker in profit as well as the alternative line a_3, we note that it achieves the level of ambition of the decision maker in profit.

As for the alternative line a_4, it does not achieve the level of ambition of the decision-maker in profit, so we exclude it.

Thus, we get the following alternatives, a_1, a_2, a_3 to achieve the level of ambition of the decision maker in profit.

By studying the elements of alternative a_2, we note that it does not achieve the level of ambition of the decision-maker in the loss because of the presence of(-220) , so we exclude it.

By studying the elements of the a_3 alternative, we note that it does not achieve the level of ambition of the decision maker in the loss because of the presence of(-400), so we exclude it

Thus, the appropriate alternative that achieves the decision-maker's ambition level in profit and loss is alternative a1 and the greatest profit is 400 .

1-2 -Most Probable Case Rule (Modal): This state depends on the probability distribution of the movement of states of nature. We symbolize the corresponding probabilities of states of nature$(\theta_1, \theta_2, - - -, \theta_n)$ with symbols $(P_1, P_2, - - -, P_n)$,These possibilities must fulfill two conditions:

$$0 \leq P_j \leq 1 \ \ \& \ \sum_{j=1}^{n} P_j = 1$$

To achieve the rule we define PS where:

$$P_s = {}^{Max}_{j} [P_j]$$

Then the natural state is θ_s it is the most frequent case that we limit ourselves to when calculating, and here the issue comes to a decision in case of confirmation.

We choose the alternative that achieves the largest profit from the column$\theta_s$the corresponding decision is the appropriate decision.

Practical Example (1-2)

We have the following table of alternatives and states of nature and the corresponding probabilities of states of nature:

Table 4: Profit and Probability Matrix Table

States of nature / Alternative	θ_1	θ_2	θ_3
a_1	300	100	400
a_2	-220	170	500
a_3	-400	200	300
a_4	160	250	200
P_j	0.3	0.1	0.6

Through the table, we note that the most probable case is case θ_3 then the issue will lead to a decision in the event of confirmation according to the following table:

Table 5: Most probable case table

Alternative	Case θ_3
a_1	400
a_2	500
a_3	300
a_4	200
P_j	0.6

We select the largest value in the status column θ_3 it is 500interviews for the alternative a_3 and a_3 is the appropriate alternative

1-3 The Rule of the Largest Expected Values:
This rule depends on the following law:

$$E(a_k) = \overset{Max}{\underset{i}{}}[E(a_i)]$$

Where $E(a_k)$ is the largest expected value and $E(a_i)$ is the expected value of the profit at the alternative and a_i is calculated from the relationship:

$$E(a_i) = \sum_{j=1}^{n} P_j x_{ij}$$

Then the alternative a_k is the appropriate alternative.

Practical Example (1-3)
We have the following table of alternatives and states of nature and the probabilities corresponding to states of nature:

Table 6: profit matrix table

States of nature / Alternative	θ_1	θ_2	θ_3
a_1	300	100	400
a_2	-220	170	500
a_3	-400	200	300
a_4	160	250	200
P_j	0.3	0.1	0.6

We calculate the expected values for each of the alternatives, we get the following table:

Table 7:Table of expected values

States of nature / Alternative	θ_1	θ_2	θ_3	E(ai)
a_1	300	100	400	$E(a_1) = 0.3 * 300 + 0.1 * 100 + 0.6 * 40 = 340$
a_2	-220	170	500	$E(a_1) = 0.3 * (-220) + 0.1 * 170 + 0.6 * 5 = 251$
a_3	-400	200	300	$E(a_1) = 0.3 * (-400) + 0.1 * 200 + 0.6 * 30 = 80$
a_4	160	250	200	$E(a_1) = 0.3 * 160 + 0.1 * 250 + 0.6 * 200 = 193$
P_j	0.3	0.1	0.6	

From the table, we notice that:

$$E(a_k) = \overset{Max}{\underset{i}{}}[E(a_i)] = Max\{340,251,80,193\} = 340 \Rightarrow E(a_k) = E(a_1)$$

Therefore, the appropriate alternative is a_1 and it is the appropriate decision.

In order to obtain a more accurate decision that helps us reduce the amount of risk, we have formulated in this research the decision-making Issue and the previous rules using the concepts of neutrosophic logic:

2-   The Text of the Decision-Making Issue in the Case of Neutrosophic Risk:

At the outset, we note that the neutrosophic decision tree has been studied in light of the neutrosophic possibilities in the research [18].

In this research we will formulate the issue of decision-making in case of risk according to neutrosophic logic we get:

The decision maker has alternatives $A(a_1, a_2, ---, a_m)$ where m is the number of alternatives available to the decision maker and the cases that nature can take in the future are $\theta(\theta_1, \theta_2, ---, \theta_n)$ where n is the number of states that nature can take (They are independent of each other) when in motion and the amount of profit or loss that the decision maker will achieve, we symbolize it with the symbol $NX_{ij} = X(a_i, \theta_j) \pm \varepsilon_{ij}$ or with the short symbol NX_{ij} and they are neutrosophic values and ε_{ij} is indefinite , it could be $\varepsilon_{ij} = [\lambda_1, \lambda_2]$ or $\varepsilon_{ij} = \{\lambda_1, \lambda_2\}$ or --------- otherwise , also the law of the probability distribution to which the possible states of nature are subject, we take it as a neutrosophic numerical series or a neutrosophic mathematical function corresponding to each state of nature with its probability of occurrence:

$NP(\theta) = \big(NP(\theta_1), NP(\theta_2), NP(\theta_3), ---, NP(\theta_n)\big)$ and $^-0 \leq \sum_{j=1}^{n} NP(\theta_j) \leq 3^+$ where $NP(\theta_j) = P(\theta_j) + i_j$ and i_j it is indeterminate and we take it $i_j[\delta_{i1}, \delta_{i2}]$ or $i_j = \{\delta_{i1}, \delta_{i2}\}$ or --------- otherwise , within the previous data, the goal is to obtain the greatest possible profit or the lowest possible loss.

Organize the previous information in the following table:

Table No. 8:Table of Profit and neutrosophic probabilities

States of nature / Alternatives	θ_1	θ_2	θ_3	----	θ_n
a_1	$x_{11} + \varepsilon_1$	$x_{12} + \varepsilon_1$	$x_{13} + \varepsilon_1$	----	$x_{1n} + \varepsilon_1$
a_2	$x_{21} + \varepsilon_2$	$x_{22} + \varepsilon_2$	$x_{23} + \varepsilon_2$	-----	$x_{2n} + \varepsilon_2$
a_3	$x_{31} + \varepsilon_3$	$x_{32} + \varepsilon_3$	$x_{33} + \varepsilon_3$	----	$x_{3n} + \varepsilon_3$
----------	-----	-----	-----	-----	-----
a_m	$x_{m1} + \varepsilon_m$	$x_{m2} + \varepsilon_m$	$x_{m3} + \varepsilon_m$	-----	$x_{mn} + \varepsilon_m$
$NP(\theta_j)$	$NP(\theta_1)$	$NP(\theta_2)$	$NP(\theta_3)$	-----	$NP(\theta_n)$

To determine the optimal decision according to the neutrosophic logic, we reformulate the rules mentioned in the classical study according to the new data

2-1 Neutrosophic Aspiration Level Rule:

In this rule, the decision maker sets a certain level of profit, for example **M** and another level of loss, for example N and he studies the available alternatives and excludes every alternative that does not achieve his level of ambition then he determines the best alternative among the remaining alternatives by using one of the rules used in the case of uncertain data studied according to the neutrosophic logic in a previous research [Search I make the decision].

Practical Example (2-1)

We have the following table of alternatives and states of nature:

To be able to clarify the rule, we will take the same value of the indefinite for all alternatives lit it be $\varepsilon = [0,50]$, and we will get the following matrix of payments:

Table 9: Neutrosophic Profit Matrix Table

States of nature / Alternatives	θ_1	θ_2	θ_3
a_1	[300,350]	[100,150]	[400,450]
a_2	[−220,−170]	[170,220]	[500,550]
a_3	[−400,−350]	[200,250]	[300,350]
a_4	[160,210]	[250,300]	[200,250]

It is required to determine the appropriate alternative using the level of ambition rule according to the following data: the level of ambition of the decision maker:

a. That the profit belongs to the domain of $M = [300,350]$.

b. That the loss belongs to the domain of $N = [200,250]$.

By studying the elements of the table, we notice that the alternative line a1 achieves the level of ambition of the decision-maker in profit and from the study of the alternative line a_2, we note that it achieves the level of ambition of the decision-maker in profit.

As well as the alternative line a_3 we note that it achieves the level of ambition of the decision maker in profit.

As for the alternative line a_4, it does not achieve the level of ambition of the decision-maker in profit, so we exclude it.

Thus, we get the following alternatives, a_1, a_2, a_3 to achieve the level of ambition of the decision maker in profit.

By studying the elements of alternative a_2, we note that it does not achieve the level of ambition of the decision-maker in losing because there is $[-220, -170]$ therefore we exclude it.

By studying the elements of the a_3 alternative, we note that it does not achieve the level of ambition of the decision-maker in losing because there is $[-400, -350]$ therefore we exclude it.

Thus, the appropriate alternative that achieves the level of ambition of the decision-maker in profit and loss is alternative a_1.

2-2 Neutrosophic Most Probable Case Rule:

This state depends on the probability distribution of the movement of states of nature $\theta(\theta_1, \theta_2, - - -, \theta_n)$ we symbolize the probabilities corresponding to the states of nature with symbols $NP(\theta) = (NP(\theta_1), NP(\theta_2), NP(\theta_3), - - -, NP(\theta_n))$ which should check:

$$^-0 \leq \sum_{j=1}^{n} NP(\theta_j) \leq 3^+$$

We specify $NP(\theta_s)$ where:

$$NP(\theta_s) = \overset{Max}{_j}[NP(\theta_j)] \quad ; j = 1,2,3, - - -, n$$

Then the state of nature θ_s is the most frequent state that we restrict to when calculating, here, the issue is to make a decision in the event of certainty. We choose the alternative that achieves the greatest profit from the column θ_s and the corresponding decision is the appropriate decision.

Practical Example (2-2)

We have the following table of alternatives and states of nature:

We take the same value of the indeterminacy for all alternatives, let it be $\varepsilon = [0,50]$ and we take the indefinite $i_1, i_2, i_3 = [0,15]$, we get the following matrix of payments:

Table No. 10: Table of the matrix of profit and neutrosophic probabilities

States of nature / Alternatives	θ_1	θ_2	θ_3
a_1	[300,350]	[100,150]	[400,450]
a_2	[-220, -170]	[170,220]	[500,550]
a_3	[-400, -350]	[200,250]	[300,350]
a_4	[160,210]	[250,300]	[200,250]
$P(\theta_j)$	[0.3,0.45]	[0.1,0.25]	[0.6,0.75]

It is required to determine the appropriate alternative using the most probable rule.

Through the table, we note that the most probable case is the case θ_2 in which the issue will lead to a decision in the event of confirmation according to the following table:

Table 11: Table of the most probable Neutrosophic state

States of nature / Alternatives	θ_3
a_1	[400,450]
a_2	[500,550]

a_3	[300,350]
a_4	[250,300]
$P(\theta_j)$	[0.6,0.75]

We choose the largest value in the condition column θ_3 that [500,550] corresponds to the alternative a_2 and a_2 is the appropriate alternative.

2-3 The Rule of the Largest Expected Neutrosophic Values:

a. We calculate $E(a_i)$ the expected value of the profit at each alternative a_i where $i = 1,2, \mathbf{3} - -, \boldsymbol{m}$ from the relation:

$$NE(a_i) = \sum_{j=1}^{n} NP(\theta_j) . NX_{ij} \quad ; i = 1,2, \mathbf{3} - -, \boldsymbol{m}$$

b. We find $NE(a_k)$ which is the largest expected value, that is:

$$NE(a_k) = {}_{i}^{Max} [NE(a_i)]$$

Then, the alternative a_k is the optimal alternative according to this rule.

Example (2-3)

We take the table in the previous case and form a table of expected values. We get the following table:

Table 12: Table of Neutrosophic Expected Values

States of nature / Alternatives	θ_1	θ_2	θ_3
a_1	[300,350]	[100,150]	[400,450]
a_2	[−220,−170]	[170,220]	[500,550]
a_3	[−400,−350]	[200,250]	[300,350]
a_4	[160,210]	[250,300]	[200,250]
$NP(\theta_j)$	[0.3,0.45]	[0.1,0.25]	[0.6,0.75]

Applying the rule for all alternatives, we get the following table:

Table No. 13: Table of Expected Values

States of nature / Alternatives	θ_1	θ_2	θ_3	$NE(a_i)$
a_1	[300,350]	[100,150]	[400,450]	$NE(a_1) = [0.3,0.45] * [300,350] + [0.1,0.25] * [100,150] + [0.6,0.75] * [400,450] = [340,532.5]$
a_2	[−220,−170]	[170,220]	[500,550]	$NE(a_2) = [0.3,0.45] * [−220,−1700] + [0.1,0.25] * [170,220] + [0.6,0.75] * [500,550] = [251,391]$
a_3	[−400,−350]	[200,250]	[300,350]	$NE(a_3) = [0.3,0.45] * [−400,−350] + [0.1,0.25] * [200,300] + [0.6,0.75] * [300,350] = [80,167.5]$
a_4	[160,210]	[250,300]	[200,250]	$NE(a_4) = [0.3,0.45] * [160,210] + [0.1,0.25] * [250,300] + [0.6,0.75] * [200,250] = [193,357]$
$NP(\theta_j)$	[0.3,0.45]	[0.1,0.25]	[0.6,0.75]	

By comparing the elements of the column $NE(a_i)$ we notice that:

$$NE(a_k) = {}_{i}^{Max} [NE(a_i)] = [340,532.5] = NE(a_1)$$

It is corresponding to the alternative a_1 then the alternative a_1 is the appropriate decision according to this rule.

3. Conclusion and Results:

In any matter of making a decision, we rely on the data that we can obtain through reality, and if these data are classic values, the decision we will take will be in accordance with those data, and any change that occurs during the work cannot be controlled and may lead to unexpected losses, this cannot happen if the data are neutrosophic values, because these values have a margin of freedom that takes into account all the changes that may occur during the course of work from the worst conditions to the best, and the decision that we adopt based on these values is an appropriate decision for all circumstances, the following is a comparison between the results obtained when using the rules of decision-making in the case of risk according to the classical logic and neutrosophic logic:

Table 14: Compare the results

Largest expected values	Most probably rule	Aspiration level rule		
a_1	a_3	a_1	The appropriate alternative	Classical logic
340	500	400	The profit	
a_1	a_2	a_1	The appropriate alternative	Neutrosophic logic
[340,532.5]	[500,550]	[400,450]	The profit	

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Optimal Value of the Service Rate in the Unlimited Model M\\M\\1

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Abstract

Operations research is the applied side of mathematics, and since its inception it has helped to improve the performance of many systems that used its methods in its workflow, one of the methods of operations research is the theory of queues that have been used in many aspects of life, especially the aspects that are directly related to customer service, and the goal is to serve the customer as soon as possible and at the lowest cost, which prompted many researchers to provide   the research and studies that can be applied in systems that have a queue.These studies showed good results, from these studies this research is presented to find the optimal value of the service rate in the unlimited model $M\backslash M\backslash 1$ according to classical logic, we present in this research a study to find the relationship through which we get the optimal value of the service rate using neutrosophic logic and this study is an expansion of the study according to classical logic, through the two studies The great benefit we get when using neutrosophic logic and neutrosophic values in operations research topics is demonstrated.

Keywords: Operations Research; Neutrosophic Logic; Queues– $M\backslash M\backslash 1$ Model; Service Rate; Neutrosophic Service Rate-Optimum Value

Introduction

The search for optimization is the goal of all studies offered by operations research methods, and keeping pace with scientific development is necessary to obtain results that guarantee us a safe workflow, so many topics of operations research have been studied.

Using neutrosophic logic, such as fuzzy linear models, Static inventory models, Decision Theory, The process of converting neutrosophic random numbers into random variables tracking for exponential distribution- Artificial intelligence and neutrosophic machine learning in the diagnosis and detection of Covid-19 -Study of a support vector machine algorithm with a perpendicular logander core and reverse interpolation of Lagrange according to the neutrosophic logic [1-9] and the aim was to obtain more accurate results that ensure a secure workflow for the systems that use it, in order to provide optimal service and reduce customer waiting time in any of the service centers, researchers in the field of operations research presented the theory of neutrosophic queues and the basic indicators were reformulated using the concepts of neutrosophic logic. [10,11] Complement to previous research, we present in this research a study through which we determine the optimal value of the customer service rate according to the neutrosophic logic, because when the matter is related to the services provided to customers, we need values that give us a margin of freedom through which to get out of the restriction and embarrassment that we can fall into when the circumstances surrounding the work environment prevent the implementation of the set plan, this is what makes us agree that when replacing the classical values with neutrosophic values,   we switch the value $T(\mu)$ that express the cost of the service with a group that we denote by $T(\mu_N)$ and by which we mean (neutrosophic $T(\mu)$) or (inaccurate $T(\mu)$) or (unspecified $T(\mu)$), which may be adjacent to or may be a period containing $T(\mu)$ depending on what is mentioned in reference No. [12], in general it can be considered any group close to $T(\mu)$ that enables us to meet the customers requirements in the shortest time and lowest cost.

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Discussion:

The process of searching for the optimal solution requires building a mathematical model that expresses the situation under study and then searching for the optimal solution according to the methods of the type of this model, the type of service system in the queues greatly affects the volume of services provided and any error in the organization of these services exacerbates the problem, in the model the average number of customers or requests that the system can serve during one time, which is called the speed of the service or the ability to pass the system is one of the basic indicators in the model $M\backslash M\backslash \mathbf{1}$ [13,14] and when we can get an optimal value for it we guarantee a good functioning of the system with the lowest roll. This value is determined by building a mathematical model and solving this model is the required value in this research, we present two studies, the first is to address the problem according to classical logic and the second is to address the problem according to neutrosophic logic.

First: Formulating a problem according to classical logic based on what is mentioned in the references [13,14]:

A customer service center operates according to the unspecified model $M\backslash M\backslash 1$, the average number of customers or orders submitted during the unit of time λ, the average number of customers or orders that can be served during unit of time μ, the cost of serving one customer in the center, the cost of $\lambda\mu$ waiting for the customer in the center during the unit of time C_1 , we denote the value of the total cost of waiting and service by code $T(\mu)$, which is given by the following relationship:

$$T(\mu) = C_1\mu + C_2 L_s \qquad (1)$$

Where L_s is the average number of people inside the system (in the queue and in the service center) and is given by the following relationship:

$$L_s = \frac{\lambda}{\mu - \lambda}$$

We substitute in relation (1)

$$T(\mu) = C_1\mu + C_2 \frac{\lambda}{\mu - \lambda}$$

We get the function of total waiting cost

The mathematical model is as follows:

Find the minimum value of

$$T(\mu) = C_1\mu + C_2 \frac{\lambda}{\mu - \lambda} \rightarrow \boldsymbol{Min}$$

Within the conditions

$$\lambda < \mu$$
$$\mu , \lambda \geq 0$$

We note that the function is a nonlinear function and with one variable is μ to find the optimal value we derive this function with respect to we find:

$$T'(\mu) = C_1 + C_2 \frac{-\lambda}{(\mu - \lambda)^2}$$

$$C_1 + C_2 \frac{-\lambda}{(\mu - \lambda)^2} = 0 \Rightarrow \mu = \lambda + \sqrt{\frac{C_2.\lambda}{C_1}}$$

$$\mu^* = \lambda + \sqrt{\frac{C_2.\lambda}{C_1}} \qquad (*)$$

Through the relationship $(*)$ we can find the classical optimal value of the service rate in the model $M\backslash M\backslash 1$

Second: Formulating the issue according to the logic of neutrosophic based on what is mentioned in the article [12,13]:

The neutrosophic function that has some indefinite (indeterminacy), taking into account the definition of its field and its corresponding field, and therefore the previous mathematical model becomes a

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neutrosophic mathematical model if the goal function is a neutrosophic function by looking at the variables and constants in this function, we find that the model turns into a neutrosophic model if we take instead of the following values : μ, λ, C_1, C_2:

Neutrosophic values i.e., $\mu_N, \lambda_N, C_{1N}, C_{2N}$, which may be adjacent to or may be a period containing μ, λ, C_1, C_2 depending on what is mentioned in reference No. [12], in general it can be considered any group close to μ, λ, C_1, C_2 that enables us to meet the customers requirements in the shortest time and lowest cost

A customer service center operates according to the unspecified model $NM\backslash NM\backslash 1$, the average number of customers or orders submitted during the unit of time λ_N, the average number of customers or orders that can be served during the unit of time μ_N, the cost of serving one customer in the center C_{1N}, the cost waiting for the customer in the system during the unit of time C_{2N}, we denote the value of the total cost of waiting and service by code $T(\mu_N)$, which is given by the following relationship:

$$T(\mu_N) = C_{1N}\mu_N + C_{2N}L_{sN} \qquad (1)$$

Where L_{sN} is the average number of people inside the system (in the queue and in the service center) and is given by the following relationship:

$$L_{sN} = \frac{\lambda_N}{\mu_N - \lambda_N}$$

We substitute in relation (1)

$$T(\mu_N) = C_{1N}\mu_N + C_{2N}\frac{\lambda_N}{\mu_N - \lambda_N}$$

We get the total cost of waiting function

The mathematical model is as follows:

Find the minimum value of

$$T(\mu_N) = C_{1N}\mu_N + C_{2N}\frac{\lambda_N}{\mu_N - \lambda_N} \rightarrow Min$$

Within the conditions

$$\lambda_N < \mu_N$$
$$\mu_N, \lambda_N \geq 0$$

We note that the function is a nonlinear function and with one variable which is μ_N to find the optimal value we derive this function with respect to μ_N we find:

$$T'(\mu_N) = C_{1N} + C_{2N}\frac{-\lambda_N}{(\mu_N - \lambda_N)^2}$$

$$T'(\mu_N) = C_{1N} + C_{2N}\frac{-\lambda_N}{(\mu_N - \lambda_N)^2} = 0 \Rightarrow \mu_N = \lambda_N + \sqrt{\frac{C_{2N}.\lambda_N}{C_{1N}}}$$

$$\mu_N^* = \lambda_N + \sqrt{\frac{C_{2N}.\lambda_N}{C_{1N}}} \qquad (**)$$

Through the relationship $(**)$ we can find the optimal neutrosophic value for the service rate in the model $NM\backslash NM\backslash 1$

It should be noted that it is not necessary that all values μ, λ, C_1, C_2 are neutrosophic values in order to get an optimal neutrosophic value for the service rate, it is enough to have one neutrosophic value in the target function.

How to benefit from the previous study [10]:

It is noted that the purpose of studying neutrosophic queues is to calculate a number of important indicators such as

   1-   Average customer waiting time inside the system (in queue and in service center)

$$NW_s = \frac{1}{\mu_N - \lambda_N}$$

2-   Average customer waiting time in queue only

$$NW_q = \frac{\lambda_N}{\mu_N(\mu_N - \lambda_N)}$$

NW_s is associated with NW_q by the relationship:

$$NW_s = NW_q + \frac{1}{\mu_N}$$

3-   Average number of customers inside the system (in queue and in the service center)

$$NL_s = \lambda_N.NW_s = \frac{\lambda_N}{\mu_N - \lambda_N}$$

4-   Average number of customers in queue only

$$NL_q = \lambda_N.NW_q = \frac{\lambda_N{}^2}{\mu_N(\mu_N - \lambda_N)}$$

NL_s is associated with NL_q by the relationship:

$$NL_s = NL_q + \frac{\lambda_N}{\mu_N}$$

5-   The service rate is given by the following relationship:

$$\rho_N = \frac{\lambda_N}{\mu_N}$$

We note that these indicators are calculated after knowing λ_N and μ_N therefore in any service center, if we have the number of requests that this center accepts during the unit of time and we know the cost of serving one customer and also the cost of waiting for him in the system and we need to calculate the previous indicators, we need to calculate the optimal value of the service rate first and calculate it from the relationship (∗∗)

Then we use the resulting value to calculate the required indicators , we explain the above through the following example:

Example 1:

Customers arrive at a gas station at rate $\lambda_N = \{3,4\}$ of one customer per hour find how long the customer stays in the system if you know that the cost of customer service is $C_1 = 100$ monetary unit and the cost of waiting in the system is $C_2 = 10$ a monetary unit

The average waiting of the customer in the system is calculated from the relationship

$$NW_s = \frac{1}{\mu_N - \lambda_N}$$

This means that we need to calculate the value of μ_N which is calculated from the relationship (∗∗) because the rate of arrival of customers to the station is a neutrosophic value $\lambda_N = \{3,4\}$, and therefore we find

$$\mu_N^* = \lambda_N + \sqrt{\frac{C_{2N}.\lambda_N}{C_{1N}}} \quad = \{3,4\} + \sqrt{\frac{10.\{3,4\}}{100}}$$

For $\lambda = 3$ we find

$$\mu^* = \lambda + \sqrt{\frac{C_2.\lambda}{C_1}}$$

$$\mu^* = \lambda + \sqrt{\frac{C_2.\lambda}{C_1}} \quad = 3 + \sqrt{\frac{10 \times 3}{100}} = 3 + \sqrt{0.3} = 3.5$$

For $\lambda = 4$ we find

$$\mu^* = \lambda + \sqrt{\frac{C_2.\lambda}{C_1}} = 4 + \sqrt{\frac{10 \times 4}{100}} = 4 + \sqrt{0.4} = 4.6$$

Accordingly, it is a neutrosophic value. $\mu_N^* = \{3.5, 4.6\}$

We calculate the average length of customer waiting time in the system

$$NW_s = \frac{1}{\mu_N - \lambda_N} = \frac{1}{\{3.5, 4.6\} - \{3,4\}}$$

$$\mu = 3.5, \lambda = 3 \Rightarrow W_s = \frac{1}{3.5 - 3} = 2$$

$$\mu = 3.5, \lambda = 4 \Rightarrow W_s = \frac{1}{3.5 - 4}$$

This value is rejected because it is negative

$$\mu = 4.6, \lambda = 3 \Rightarrow W_s = \frac{1}{4.6 - 3} = 0.625$$

$$\mu = 4.6, \lambda = 4 \Rightarrow W_s = \frac{1}{4.6 - 4} = 1.7$$

Therefore, $NW_s = \{2, 0.6, 1.7\}$ and it is a neutrosophic value.

Example 2:

Car repair shop receives customers at an hourly rate $\lambda_N = [5,10]$.

Find the optimal value of the service rate if you know that the cost of customer service is $C_{1N} = [15,20]$ dollars and the cost of waiting in the system is $C_{2N} = [3,8]$ dollars.

You will be able to find the optimal neutrosophic value of the service rate through the following relationship:

$$\mu_N^* = \lambda_N + \sqrt{\frac{C_{2N}.\lambda_N}{C_{1N}}}$$

$$\mu_N^* = \lambda_N + \sqrt{\frac{C_{2N}.\lambda_N}{C_{1N}}} = [5,10] + \sqrt{\frac{[3,8].[5,10]}{[15,20]}} = [5,10] + \sqrt{\frac{[15,80]}{[15,20]}} = [5,10] + [1,2]$$

$$\mu_N^* = [5,10] + [1,2] = [7,12]$$

After obtaining the value of μ_N^* and we know in the text of the problem the value of λ_N we can calculate all the indicators

Conclusion and Results:

Through the previous study, we note that the optimal value of the rate of service provided by the classic model solution is a specific value, and the validity of this value depends on the conditions surrounding the environment in which the system operates, but the optimal value that we obtain through the neutrosophic model solution is an indefinite value that takes into account the conditions surrounding the work environment because the values that enter into the optimal solution phrase (the average number of customers or orders submitted during the unit of time λ_N and the cost of serving one customer in the center C_{1N} and the cost of waiting for the customer in the system during the unit of time C_{2N} (neutrosophic value (some or all) are suitable for the worst conditions and the best for the system working environment.

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Fundamentals of Neutrosophical Simulation for Generating Random Numbers Associated with Uniform Probability Distribution

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Abstract: The simulation process depends on generating a series of random numbers subject to the uniform probability distribution in the interval [0, 1]. The generation of these numbers is starting from the cumulative distribution function of the uniform distribution. Through previous studies in classical logic, we found any random number R_0, met with a cumulative distribution function value equal to R_0, but these specific numbers may not have sufficient accuracy, which leads to obtaining results that are not sufficiently accurate when doing the simulation. To bypass this case, in this paper, we present a study that enables us to generate as accurate as possible random numbers, using neutrosophic logic ' this Logic given by American mathematician Florentin Smarandache in 1995'. The first step in the study is, define the cumulative distribution function of the neutrosophic uniform distribution, depending on definition of the neutrosophic integral and definition of the neutrosophic uniform distribution. We used the new definition to generate random numbers subject to a neutrosophic uniform distribution on the interval [0, 1]. The result was that each random number R_0 corresponds to a interval of the distribution function related to R_0, So that it preserves enough precision for the random numbers, and thus we get a more accurate simulation of any system we want to simulate.

Keywords: Neutrosophic uniform distribution; Simulations; Cumulative distribution function of neutrosophic uniform distribution; neutrosophic random numbers.

1. Introduction

Because of the great difficulty that can face us when studying the work of any real system. As well as the high cost of studying. In addition, some systems we cannot be directly studied. Here comes the importance of the simulation process in all branches of science. The simulation depends on the application of the study on systems similar to the real systems, and then projecting these results if they are appropriate on the real system.

The simulation based on generating a series of random numbers that are subject to a uniform probability distribution in the interval [0, 1]. Then converting these numbers into random variables subject to the probability distribution in which the system to be simulated operates, based on the cumulative distribution function of the probability distribution [9].

In front of the great revolution brought about by neutrosophic logic in all fields of science, after the American philosopher and mathematician Florentin Smarandache laid its foundations in 1995 [2,4,5,6,8], who put it forward as a generalization of fuzzy logic, especially Intuitionistic fuzzy logic [3], and an extension of the theory of fuzzy sets presented by Lotfi Zadeh 1965 [1] . By extension to that, A.A. Salama presented the neutrosophic classical set theory as a generalization of the classical set theory and developed, introduced and formulated new concepts in the fields of mathematics, statistics, computer science and classical information systems using the neutrosophic [7,15,16]. The neutrosophic is the logic that studies the origin, nature and field of indeterminacy, taking into account every idea with its opposite and with the spectrum of indeterminacy.

This logic has developed in recent years, and most of the known concepts in classical logic have been reformulated according to neutrosophic logic [10,12,13,17,19,20,21,22,23,24]. Among these concepts is the study and formulation of most of the known probability distributions in classical logic [18]. In this paper, we present a study of the cumulative distribution function of the neutrosophic uniform distribution on the interval [a, b]. Depending on what researchers have found in the field of neutrosophic [11,14], such as the definition of the neutrosophic uniform distribution on the interval [a, b] and the definition of the neutrosophic integration [25]. Especially in the case where the indeterminacy is related to the upper and lower bounds of the interval [a, b]. We used the results as a basis for generating random numbers subject to a neutrosophic uniform distribution on the interval [0, 1].

The importance of this research stems from the importance of simulation in all fields of science, especially when we need accuracy in the results during the simulation process for any system. This accuracy not provided by classical logic. The limits of this study include all scientific fields that may contain indefinite cases and need simulation to represent them, and aim to reach the most accurate possible results

2. Experimental and Theoretical Part:

Based on the importance of simulation in all fields of science. Since the simulation depends primarily on generating a series of random numbers that are subject to a uniform distribution on the interval [0, 1]. It was necessary to keep pace with the neutrosophic revolution by presenting a study that generates neutrosophic random numbers that are subject to a neutrosophic uniform distribution on the interval [0, 1]. That is by finding mathematical relationships that describe the cumulative distribution function of the neutrosophic uniform distribution on the interval [a, b], especially when the indeterminacy relates to both ends of the interval. Where we studied the following three cases:

The first case: if the indeterminacy related to the lower limit of interval.

The second case: if the indeterminacy related to the upper limit of interval.

The third case: if the indeterminacy related to both the lower and upper limits of interval.

In addition, we arrived at mathematical formulas for the cumulative distribution function for each of the previous cases. We then used these formulas to generate random numbers that are subject to a

neutrosophic uniform distribution. Which provides us with the most accurate results possible while performing the simulation process for any system.

2.1. In classical logic:

To generate the random numbers subject to a uniform distribution in the interval [0, 1] (according to the Monte–Carlo method), we assume R is the continuous random variable that is subject to the continuous uniform distribution defined on the interval [a, b], which is given by:

$$f(x) = \frac{1}{b-a} \quad ; \quad a \leq x \leq b$$

Then it would be:

$$f(R) = \frac{1}{1-0} = 1 \qquad 0 \leq R \leq 1$$

The distribution function is of the form:
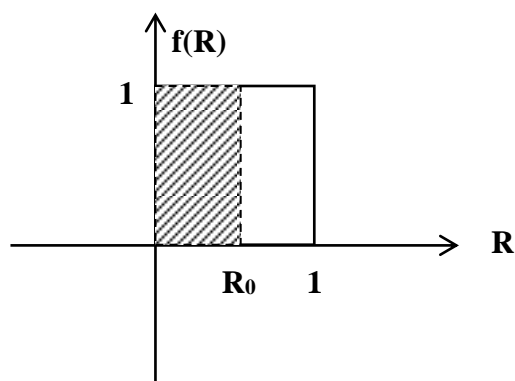
Figure (1)

We take the cumulative distribution function for this distribution. We will call it F(R), then:

$$P(R < R_0) = F(R_0) = \int_0^R f(R)dR = \int_0^{R_0} 1.dR = R_0$$

That is, every random number R_0 corresponds to a value of the distribution function equal to R_0.
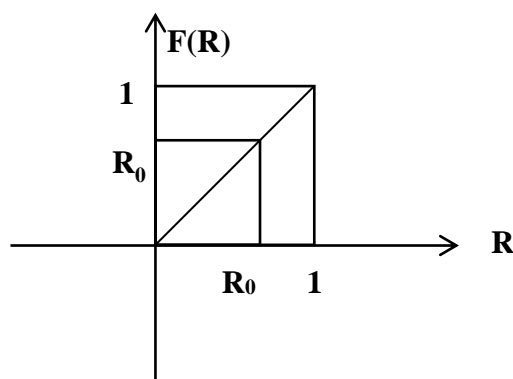
Figure (2)

2.2. In Neutrosophic logic:

The uniform distribution in neutrosophic logic takes the following form:

$$f_N(x) = \frac{1}{b-a}$$

Where b, a: one or both of them is not precisely defined, we find it in the form of set or interval, etc., we can consider all possible cases for b, a. While keeping the condition a < b.

Accordingly, we can write the interval [a, b] in one of the following forms **(*)**:

1. $[a_1 + \varepsilon , b]$; $a = a_N = a_1 + \varepsilon$

2. $[a , b_1 + \varepsilon]$; $b = b_N = b_1 + \varepsilon$

3. $[a_1 + \varepsilon , b_1 + \varepsilon]$; $a = a_N = a_1 + \varepsilon$ & $b = b_N = b_1 + \varepsilon$

Where $\varepsilon \in [0, n]$, with   a < n < b.

In order to define the cumulative distribution function of the neutrosophic uniform distribution on the interval [a, b], it is necessary to define the neutrosophic integral.

We define the neutrosophic integral in the case of indeterminacy related to the lower limit of interval as follows:

Suppose we want to integrate the function   **f: X → R**   on the interval [a, b] of X , but we are not sure of the lower limit **a** , since   **a**   has a finite part (a_1) and an indefinite part (ε), i.e.:

$$a = a_N = a_1 + \varepsilon \quad ; \quad \varepsilon \in [0, n] \; ; \; a < n < b$$

Then we write the integral of f(x) on the interval [a, b], as follows:

$$\int_a^b f(x)dv = \int_a^b f(x)dx - i_1$$

Where: $i_1 \in \left[0, \int_{a_1}^{a_1+n} f(x)dx \right]$

In another way: $\int_a^b f(x)dv = \int_{a_1+n}^b f(x)dx + i_2$

Where: $i_2 \in \left[0, \int_{a_1}^{a_1+n} f(x)dx \right]$

2.3. Neutrosophical cumulative distribution function with states (*):

First case:

The indeterminate is in the lower limit of the interval $a = a_1 + \varepsilon$; $\varepsilon \in [0, n]$; $a < n < b$

The interval becomes $[a_1 + \varepsilon , b]$, and we have:

$$a = a_N = a_1 + \varepsilon = a_1 + [0, n] = [a_1, \quad a_1 + n]$$

The cumulative distribution function for a classical uniform distribution given as:

$$P(X < x_s) = F(x_s) = \int_a^{x_s} f(x)dx$$

The Cumulative distribution function for a neutrosophic uniform distribution:

$$NP(X < x_s) = NF(x_s) = \int_a^{x_s} f(x)dv = \int_{a_1}^{x_s} f(x)dx - i$$

Where: $i \in \left[0, \int_{a_1}^{a_1+n} f(x)dx \right]$

In the case of a neutrosophic uniform distribution, we get:

$$NF(x_s) = \int_{a_1}^{x_s} \frac{1}{b-a}dx - i = \frac{x_s - a_1}{b-a} - i$$

We define the interval for **i** , let's calculate the integral:

$$\int_{a_1}^{a_1+n} \frac{1}{b-a} dx = \frac{1}{b-a}[x]_{a_1}^{a_1+n} = \frac{a_1+n-a_1}{b-a} = \frac{n}{b-a}$$

This means that the indeterminate **i** belongs to the interval:

$$i \in \left[0, \frac{n}{b-a_N}\right]$$

Second case:

The indeterminate is in the upper limit of the interval $b = b_1 + \varepsilon$; $\varepsilon \in [0, n]$; $a < n < b$

The interval becomes **[a , b₁ + ε]**, and we have:

$$b = b_N = b_1 + \varepsilon = b_1 + [0, n] = [b_1, \quad b_1 + n]$$

The cumulative distribution function for a classical uniform distribution given as:

$$P(X < x_s) = F(x_s) = \int_a^{x_s} f(x)dx$$

The Cumulative distribution function for a neutrosophic uniform distribution:

$$NP(X < x_s) = NF(x_s) = \int_a^{x_s} f(x)dv$$

$$NF(x_s) = \int_a^{x_s} \frac{1}{b-a} dx = \frac{x_s - a}{b_N - a}$$

Third case:

The Indeterminacy exists in the lower and upper limit of the interval

$$= [a_N, b_N] \quad [a_1 + \varepsilon, \quad b_1 + \varepsilon] \quad ; \quad \varepsilon \in [0, n] ; \quad a < n < b \quad . \text{ i.e.:}$$

$$a_N = a_1 + \varepsilon = a_1 + [0, n] = [a_1, a_1 + n]$$

$$b_N = b_1 + \varepsilon = b_1 + [0, n] = [b_1, b_1 + n]$$

So we write:

$$NP(X < x_s) = NF(x_s) = \int_{a}^{x_s} f(x)dv = \int_{a_1}^{x_s} f(x)dx - i$$

Where: $\quad i \in \left[0, \ \int_{a_1}^{a_1+n} f(x)dx \right]$

In addition, it will be:

$$NF(x_s) = \int_{a_1}^{x_s} \frac{1}{b-a} dx - i = \frac{x_s - a_1}{b-a} - i$$

Where: $\quad i \in \left[0, \ \frac{n}{b_N - a_N} \right]$

3. Results and Discussion

Generation of neutrosophic random numbers that are subject to a uniform distribution in the interval [0, 1]:

Based on what we have put forward, we get the following forms of the interval [0,1] with cases of the Indeterminacy:

1. $[0 + \varepsilon \ , \ 1]$

2. $[0 \ , \ 1 + \varepsilon]$

3. $[0 + \varepsilon \ , \ 1 + \varepsilon]$

Where: $\quad \varepsilon \in [0, n] \ ; \ 0 < n < 1.$

<u>First case:</u>

$$NP(R < R_0) = NF(R_0) = \frac{R_0 - \varepsilon}{1 - \varepsilon} \quad ; \varepsilon \in [0, n]$$

$$\Rightarrow NP(R < R_0) = NF(R_0) = \frac{R_0 - \varepsilon}{1 - \varepsilon} = \frac{[R_0, R_0 - n]}{[1, 1-n]} = \left[R_0, \frac{R_0 - n}{1-n} \right]$$

<u>Second case:</u>

$$NP(R < R_0) = NF(R_0) = \frac{R_0}{1 + \varepsilon} \quad ; \varepsilon \in [0, n]$$

$$\Rightarrow NP(R < R_0) = NF(R_0) = \frac{R_0}{1 + \varepsilon} = \frac{R_0}{[1, \ n+1]} = \left[R_0, \frac{R_0}{n+1} \right]$$

Third case:

$NP(R < R_0) = NF(R_0) = R_0 - \varepsilon$; $\varepsilon \in [0, n]$

$\Rightarrow NP(R < R_0) = NF(R_0) = R_0 - \varepsilon = [R_0, R_0 - n]$

From the above we conclude that each random number R_0 corresponds to an interval of the cumulative distribution function related to R_0.

4. Application example:

Using "mean of the square" method (for von Neumann), we generate the random numbers R_0, R_1, R_2, R_3, R_4.

Method explanation: We choose a fractional random number R_0. Consisting of four places (called the seed) and does not contain zero. Then we square that number (R0).   Choose the middle four digits of the fractional part then put a new fraction and consider it the random number R_1. And so on... until we get the required random numbers. i.e. we apply the rule:

$$R_{i+1} = \text{Mid} \left[R_i^2 \right] \quad i = 0, 1, 2, \ldots$$

We denote by "Mid" the middle four ranks of R_i^2.

For example, we choose, $R_0 = 0.1276$ then:

$$R_1 = \text{Mid} \left[R_0^2 \right] = \text{Mid} [0.01628176] = 0.6281$$

$R_2 = 0.4509$, $R_3 = 0.3310$, $R_4 = 0.0951$

We use these numbers to generate neutrosophic random numbers that follow a uniform distribution in the interval [0, 1], according to the three cases.

First case: In this case, each random number R_0 corresponds to $\dfrac{R_0 - \varepsilon}{1 - \varepsilon}$

We found that:

$$R_0 \to \frac{R_0 - \varepsilon}{1 - \varepsilon} \quad ; \varepsilon \in [0, n]$$

We take, for example: $[0, n] = [0, 0.03]$

$$R_0 \to \frac{0.1276 - [0, 0.03]}{1 - [0, 0.03]} = \frac{[0.1276, 0.1276 - 0.03]}{[0.97, 1]} = \frac{[0.0976, 0.1276]}{[0.971]} = [0.1006, 0.1276]$$

$$R_1 \to \frac{R_1 - \varepsilon}{1 - \varepsilon} = \frac{0.6281 - [0, 0.03]}{[1, 1 - 0.03]} = \frac{[0.5981, 0.6281]}{[0.97, 1]} = [0.6281, 0.6169]$$

$$R_2 \to \frac{R_2 - \varepsilon}{1 - \varepsilon} = \frac{0.4509 - [0, 0.03]}{[1, 1 - 0.03]} = \frac{[0.4209, 0.4509]}{[0.97, 1]} = [0.4339, 0.4509]$$

$$R_3 \to \frac{R_3 - \varepsilon}{1 - \varepsilon} = \frac{0.3310 - [0, 0.03]}{[0.97, 1]} = \frac{[0.301, 0.3310]}{[0.97, 1]} = [0.3103, 0.3310]$$

$$R_4 \to \frac{R_4 - \varepsilon}{1 - \varepsilon} = \frac{0.0956 - [0, 0.03]}{[0.97, 1]} = \frac{[0.0656, 0.956]}{[0.97, 1]} = [0.0676, 0.956]$$

Second case: In this case, each random number R_0 corresponds to $\dfrac{R_0}{1 + \varepsilon}$

Where $\varepsilon = [0, 0.03]$, $R_0 = 0.1276$

$$R_0 \to \frac{0.1276}{1 + [0, 0.03]} = \frac{0.1276}{[1, 1.03]} = [0.1238, 0.1276]$$

$$R_1 \to \frac{0.6281}{[1, 1.03]} = [0.6098, 0.6281]$$

$$R_2 \to \frac{0.4509}{[1, 1.03]} = [0.4377, 0.4509]$$

$$R_3 \to \frac{0.0956}{[1, 1.03]} = [0.0928, 0.0956]$$

$$R_4 \to \frac{0.0956}{[1, 1.03]} = [0.0928, 0.0956]$$

Third case: In this case, each random number R_0 corresponds to R_0 - ε, where:

$\varepsilon \in [0, n]$; $0 < n < 1$

For $\varepsilon = [0, 0.03]$ and $R_0 = 0.1276$, we have:

$R_0 \rightarrow 0.1276 - [0, 0.03] = [0.0976, 0.1276]$

$R_1 \rightarrow 0.6281 - [0, 0.03] = [0.5981, 0.6281]$

$R_2 \rightarrow 0.4509 - [0, 0.03] = [0.4209, 0.4509]$

$R_3 \rightarrow 0.3310 - [0, 0.03] = [0.301, 0.3310]$

$R_4 \rightarrow 0.0956 - [0, 0.03] = [0.0656, 0.0956]$

5. **Conclusions**:

Through this paper, we found that when we use neutrosophic logic to generate random numbers, we get a series of numbers that are more accurate than the numbers we get when using classical logic. This is due to the margin of freedom offered by neutrosophic logic through the indeterminacy spectrum.

We are looking forward in the near future , to preparing a study in which we can generate the random numbers that are subject to non-uniform distributions, by converting the regular random numbers in the interval [0, 1] into random numbers that are subject to the appropriate probability distribution for the case under study.

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Using the Inverse Transformation Method to Generate Random Variables that follow the Neutrosophic Uniform Probability Distribution

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Abstract

When conducting the simulation process for any of the systems according to classical logic, we start by generating random numbers belonging to the regular probability distribution on the field [0, 1] using one of the known methods, and then we convert these random numbers into random variables that follow the probability distribution that the system to be simulated works with, the simulation process that we perform it gives specific results that do not take into account the changes that may occur in the work environment of the system, to obtain more accurate results In a previous research, we prepared a study through which we reached random neutrosophic

numbers follow the uniform distribution of the neutrosophic on the field with $[0,1]$ no determination that can be enjoyed by the two parties of the field, one or both together, it may be in the form of a group or a field in another research , we converted these neutrosophic random numbers into neutrosophic random variables that follow the neutrosophic exponential distribution using the opposite conversion method that depends on the cumulative distribution function of the probability distribution by which the system to be simulated works, in this research we have useda method The opposite transformation to generate random variables that follow the neutrosophic uniform distribution and we have reached relationships through which we can convert the

neutrosophic random numbers that follow the neutrosophic uniform distribution defined on the domain $[0,1]$ with the indeterminacy enjoyed by each end of the field, one or the other, into random variables that follow the

neutrosophic uniform distribution, b , a which is a classical uniform distribution whose medians are not precisely defined values , one or both may be cognitiven in the form of a set or a domain, so that n take into

account all possible cases of mediators while maintaining the condition , and the $b , a ; a < b$ method was illustrated through an applied example and we came up with neutrosophic random variables that follow the uniform distribution that give us more accurate simulation results when used due to the indeterminacy of neutrosophic values.

Keywords: Uniform distribution; Simulation; Cumulative distribution function; Random numbers; Neutrosophic random variables

1. Introduction:

In the field of scientific research, we face many systems that cannot be studied directly, due to the great difficulty that can face us when studying, and the high cost in addition to the fact that some systems cannot be studied directly, here comes the importance of the simulation process in all branches of science, as it depends on applying the study to systems similar to real systems and then dropping these results if they are suitable on the real system, the simulation process It depends on generating a series of random numbers subject to a uniform probability distribution on the domain [0, 1], and then converting these numbers into random variables subject to the law of

probability distribution by which the system to be simulated works by using known conversion methods, in front of the revolution brought about by the science of neutrosophic in all fields of It was founded by the American philosopher and mathematician Florentin Smarandache in 1995 [1-7], and presented as a generalization of fuzzy logic, especially intuitive fuzzy logic, and as an extension of the fuzzy category theory presented by Lotfi A. Zadeh in 1965 [8]. It was necessary to keep pace with this development and formulate the basic concepts of the simulation process according to the basic concepts of neutrosophy, because the margin of freedom enjoyed by neutrosophic values gives more accurate results, which prompted many researchers to prepare many researches in various fields of science, especially in the field of mathematics and its applications [9-22], we present in this research a study aimed at obtaining neutrosophic random variables that follow the uniform neutrosophic distribution, which we will use when simulating systems that operate according to the uniform distribution in order to obtain more accurate simulation results by taking advantage of the indeterminacy enjoyed by neutrosophic values.

Discussion:

Since the aim of this paper is to obtain relationships in which we can convert neutrosophic random numbers that follow the neutrosophic uniform distribution on the field with the indeterminacy [0,1]enjoyed by both sides of the field, one or the other, into neutrosophic random variables that follow the neutrosophic uniform distribution,

which is a classical uniform distribution whose medians are not precisely defined b , a , one or both may be defined in doubt. For a group or domain, takinginto account all possible cases of intermediaries and maintaining the condition. $a < b$

2. **Using classical values to generate trace random numbers for uniform distribution on the domain** $[0,1]$**: [23,24]**

Togenerate a series of random numbers using the mean-squared method given by the following relationship: $R_1, R_2, - - -$

$$R_{i+1} = Mid[R_i^2] \; ; i = 0,1,2,3, - - \qquad (1)$$

Mid stands for the middle four ranks of R_i^2 and any R_0 random fractional number consisting of four orders (called a seed) and does not have a zero in any of its four ranks.

3. **Converting random numbers that follow for uniform distribution over the domain** $[0,1]$ **to neutrosophic random numbers: [25]**

To convert the resulting numbers from (1) to random neutrosophic numbers trace for uniform distribution on the domain [0,1] we distinguish the following cases of the domain [0,1] with a margin of indeterminacy .

The first case: the indetermincy in the minimum field $[0 + \delta, 1]$ in this case we compensate by the δ where $\delta \in [0, m]$; $0 < m < 1$, and R_i are the random numbers resulting from the relationship (1) , by the following relationship:

$$NR_i = \frac{R_{i-}\delta}{1 - \delta}$$

The second case: the indeterminacy in the maximum limit of the field in this case we replace

$[0,1 + \delta]$ in this case we compensate by the δ where $\delta \in [0, m]$; $0 < m < 1$, and R_i are the random numbers resulting from the relationship (1) ,by the following relationship:

$$NR_i = \frac{R_{i-}}{1 + \delta}$$

The third case: the indeterminacy in the upper and lower limits of the field in this case we replace $[0 + \delta, 1 + \delta]$ in this case we compensate by the δ where $\delta \in [0, m]$; $0 < m < 1$, and R_i are the random numbers resulting from the relationship (1), by the following relationship:

$$NR_i = R_i - \delta$$

1- **finding the cumulative distribution function of the neutrosophic uniform distribution: [25]**

to find the cumulative distribution function $F(x)$ of the neutrosophic uniform distribution given by the following

density probability function: $f_N(x) = \frac{1}{b-a}$

Where b, a is one or both of them not precisely defined, defined in the form of a group, domain, etc., we take into account all possible cases of intermediaries b, a while maintaining the condition $a < b$.

Accordingly, the cumulative distribution function takes one of the following forms, depending on the field of definition:

Figure I: Lack of Limitation in the Minimum Domain$[a_1 + \varepsilon, b]$

where and $a = a_N = a_1 + $; $\varepsilon \in [0, n]$; $a < n < b$ in this case the cumulative distribution function of the neutrosophic uniform distribution is given by the following relation

$$NF(x) = \frac{x - a_1}{b - a_N} - i \; ; i \in \left[0, \frac{n}{b - a_N}\right]$$

Figure II: Indeterminacy at the upper limit of the domain$[a, b_1 + \varepsilon]$

where $b = b_N = b_1 + $; $\varepsilon \in [0, n]$; $a < n < b$ in this case, the cumulative distribution function of the neutrosophic uniform distribution is given by the following relationship:

$$NF(x) = \frac{x - a}{b_N - a}$$

Figure III: Indeterminacy in the upper and lower limits of the domain $[a_{1+}\varepsilon, b_1 + \varepsilon]$ where $a = a_N = a_1 + $ and $b = b_N = b_1 + $ and $\varepsilon \in [0, n]$; $a < n < b$ this case the cumulative distribution function of the neutrosophic uniform distribution is given by the following relationship:

$$NF(x) = \frac{x - a_1}{b_N - a_N} - i \; ; i \in \left[0, \frac{n}{b_N - a_N}\right]$$

Current study:

We want to generate random neutrosophic variables that follow the structured distribution and enact the opposite transformation method that is not defined according to the following rule:

$R = F(x) \Rightarrow x = F^{-1}(R)$ where $F(x)$ is the cumulative distribution function of the uniform distribution that follows the random variables, then if $R_1, R_2, - - - - - -$ it is a sequence of random numbers then the R_i probability is defined as follows:

$$f_R(x) = \begin{cases} 1 & 0 \le x \le 1 \\ 0 & \text{otherwise} \end{cases}$$

and a cumulative distribution function:

$$F_R(x) = \begin{cases} 0 & x < 0 \\ x & 0 \le x \le 1 \\ 1 & x > 1 \end{cases}$$

From (1) and (2) the opposite conversion method to generate the random mutants that follow the neutrosophic uniform distribution distinguish three cases:

The first case: indeterminacy in the minimum field, , i.e $[a_1 + \varepsilon, b]$ where $a = a_1 + $; $\varepsilon \in [0, n]$; $a < n < b$

And we have: $a = a_N = a_1 + = a_1 + [0, n] = [a_1, a_1 + n]$

we use the relationship $R = F(x) \Rightarrow x = F^{-1}(R)$ we find

$$NF(x) = NR \Rightarrow NR = \frac{x - a_1}{b - a} - i \Rightarrow$$

$$\frac{x - a_1}{b - a} = NR + i \Rightarrow (NR + i)(b - a) = x - a_1 \Rightarrow$$

$$Nx = (NR + i)(b - a) + a_1$$

$$i \in \left[0, \frac{n}{b - a_N}\right]$$

Therefore, to generate random variables following the neutrosophic uniform distribution in the case of indeterminacy in the minimum domain we use the following relationship:

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$$Nx_j = (NR_j + i)(b-a) + a_1 \quad ; \quad j = 0,1,2,3,----$$

The second case: the indeterminacy is related to the upper limit of the domain, i.e. $[a \, , b_1 + \varepsilon]$

where $\varepsilon \in [0, n]$, $a < n < b$ and we have $b = b_N = b_1 + \varepsilon = b_1 + [0 \, , n] = [b_1 \, , b_1 + n]$

Using the relationship $R = F(x) \Rightarrow x = F^{-1}(R)$ we find

$$NF(x) = \frac{x-a}{b_N - a}$$

$$NF(x) = NR \Rightarrow NR = \frac{x-a}{b_N - a} \Rightarrow$$

$$\frac{x-a}{b_N - a} = NR \Rightarrow NR(b_N - a) = x - a \Rightarrow$$

$$Nx = NR(b_N - a) + a$$

$$Nx = NR[(b_1 + \varepsilon) - a] + a$$

Therefore, to generate random variables that follow the neutrosophic uniform distribution in the case of indeterminacy related to the upper bound we use the following rule:

$$Nx_j = NR_j[(b_1 + \varepsilon) - a] + a \quad ; \quad j = 0,1,2,3,----$$

Third case : indeterminacy in the upper and lower limits of the domain [a_1 +b , b_1 +ε] where , $a = a_N = a_1 + $

, $b = b_N = b_1 + ,$ $\varepsilon \in [0 \, , n] \quad ; \quad a < n < b$

Using the relationship $R = F(x) \Rightarrow x = F^{-1}(R)$ we find

$$NF(x) = NR \Rightarrow NR = \frac{x - a_1}{b-a} - i \Rightarrow$$

$$\frac{x - a_1}{b-a} = NR + i \Rightarrow (NR + i)(b-a) = x - a_1 \Rightarrow$$

$$Nx = (NR + i)(b-a) + a_1$$

$$i \in \left[0, \frac{n}{b_N - a_N}\right]$$

Therefore, to generate random variables that follow the neutrosophic uniform distribution in the case of indeterminacy related to the lower and upper limits we use the following relationship:

$$Nx_j = (NR_j + i)(b-a) + a_1 \quad ; \quad j = 0,1,2,3,4$$

In the following, we will present an applied example that we study according to the classical logic and neutrosophic logic to be able to compare.

Practical example of classical values:

We assume that we have a system that works according to the regular distribution on the field $[3 \, , 9]$ and we want to simulate the workflow of this system.

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We apply the previous study:

1- We generate random numbers on the domain $[0,1]$ using the average squared method, we take the seed we $R_\circ = 0,2156$ get the following numbers:

$$R_\circ = 0,2156 \quad , \quad R_1 = 0,6483 \quad , \quad R_2 = 0,0292 \quad , \quad R_3 = 0,0852 \quad , \quad R_4 = 0,7259$$

2- We know that the cumulative distribution function of the uniform distribution defined over the domain $[a,b]$ is given by the following relationship:

$$F(x) = \frac{x-a}{b-a}$$

Using the opposite conversion method, we obtain the following relationship:

$$R = F(x) = \frac{x-a}{b-a} \Rightarrow x = (b-a)R + a$$

Therefore, to convert the random numbers that follow the regular distribution on the field $[0,1]$ into random variables that follow the regular distribution on the field, $[a,b]$ we substitute in the following relationship:

$$x_i = (b-a)R_i + a \quad ; \quad i = 0,1,2,3,---$$

Therefore, the random variables that follow the regular probability distribution on the domain $[3,9]$ and corresponding to the following random numbers:

$$R_\circ = 0,2156 \Rightarrow x_\circ = (9-3) \times 0,2156 + 3 = 4.2936$$

$$R_1 = 0,6483 \Rightarrow x_\circ = (9-3) \times 0,6483 + 3 = 6,8898$$

$$R_2 = 0,0292 \Rightarrow x_\circ = (9-3) \times 0,0292 + 3 = 3.1752$$

$$R_3 = 0,0852 \Rightarrow x_\circ = (9-3) \times 0,0852 + 3 = 3.5112$$

$$R_4 = 0,7259 \Rightarrow x_\circ = (9-3) \times 0,7259 + 3 = 7.3554$$

Practical example neutrosophic values:

We suppose we have a system that works according to the uniform neutrosophic distribution on the field , where this means that it is $[3,9]$ the indeterminacy $\varepsilon = [0,3]$ that one or both ends of the field can take, $n = 3$ and we want to perform a neutrosophic simulation of the functioning of this system.

We apply the previous study:

3- We generate random numbers on the domain $[0,1]$ using the average squared method, we take the seed we get the following numbers: $R_\circ = 0,2156$

$$R_\circ = 0,2156 \quad , \quad R_1 = 0,6483 \quad , \quad R_2 = 0,0292$$
$$R_3 = 0,0852 \quad , \quad R_4 = 0,7259$$

4- We convert these random numbers to neutrosophic random numbers we will apply the example to the third case when the indeterminacy is in the upper and lower limits of the field, $[0+\delta, 1+\delta]$ i.e. where $\delta = [0, 0.02]$ with the indication that we can obtain neutrosophic random numbers if the indeterminacy is only at the upper limit or at the lower limit only, using the appropriate relation for each case

We substitute in the appropriate relationship for the third case, which is: and $NR_i = R_i - \delta \quad \delta = [0, 0.02]$

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$$NR_{\circ} = 0{,}2156 - [0\,,0{,}02] = [0.1956\,,0.2156]$$

$$NR_1 = 0{,}6483 - [0\,,0{,}02] = [0{,}6283\,,0.6483]$$

$$NR_2 = 0{,}0292 - [0\,,0{,}02] = [0.0092\,,0{,}0292]$$

$$NR_3 = 0{,}0852 - [0\,,0{,}02] = [0.0652\,,0{,}852]$$

$$NR_4 = 0{,}7259 - [0\,,0{,}02] = [0.7059\,,0.7259]$$

We get the following neutrosophic random numbers:

$$NR_{\circ} = [0.1956\,,0.2156] \quad,\quad NR_1 = [0{,}6283\,,0.6483]$$

$$NR_2 = [0.0092\,,0{,}0292] \quad,\quad NR_3 = [0.0652\,,0{,}852] \quad,\quad NR_4 = [0.7059\,,0.7259]$$

To perform neutrosophic simulation we distinguish three cases:
The first case: the uniform distribution is classic, i.e., the probability density function and the neutrosophic

$$f(x) = \frac{1}{b-a}$$

random numbers

The second case: the uniform neutrosophic distribution given by the probability density function, i.e., one or both

$$f_N(x) = \frac{1}{b-a}$$

sides of the field are not precisely defined, and random numbers are classic.

The third case: the uniform neutrosophic distribution and the neutrosophic random numbers We will apply the example to the third case, knowing that we follow the same method of solution for the first and second cases, using the appropriate laws for each case:

In the third case, the random numbers are neutrosophic, i.e., we take:

$$NR_{\circ} = [0.1956\,,0.2156] \quad,\quad NR_1 = [0{,}6283\,,0.6483]$$

$$NR_2 = [0.0092\,,0{,}0292] \quad,\quad NR_3 = [0.0652\,,0{,}852] \quad,\quad NR_4 = [0.7059\,,0.7259]$$

and uniform neutrosophic distribution with no limitation of both sides of the field, i.e. indeterminacy related to lower and upper limits

of the data we have $[a_1\,,b_1] = [3\,,9]$ and $\varepsilon = [0\,,3\,]$ and therefore

$$[a_N\,,b_N] = [a_1 + \varepsilon\,,b_1 + \varepsilon] = [3 + [0\,,3]\,,9 + [0\,,3]] = [\,[3\,,6]\,,[9\,,12]\,]$$

Here to generate random variables following this distribution we use the following relationship:

$$Nx_j = (NR_j + i)(b-a) + a_1 \quad;\quad j = 0,1,2,3,4$$

$$i \in \left[0,\,\frac{n}{b_N - a_N}\right]$$

First, we calculate and compensate i $n = 3$ $a_N = [0, 3]$ $b_N = [0, 12]$

$$i = \left[0, \frac{n}{b_N - a_N}\right] = \left[0, \frac{3}{[9, 12] - [3, 6]}\right] = \left[0, \frac{3}{[6, 6]}\right] = [0, 0.5]$$

Substitute

$$Nx_j = ((NR_j + i)(b - a)) + a_1 = ((NR_j + [0, 0.5])(9 - 3)) + 3 \Rightarrow$$

$$Nx_j = ([0, 0.5NR_j] \times 6) + 3 = ([0, 3NR_j] + 3) \Rightarrow$$

$$Nx_j = [3, 3NR_j + 3]$$

$$NR_\circ = [0.1956, 0.2156] \Rightarrow Nx_\circ = [3, 3NR_\circ + 3] = [3, 3[0.1956, 0.2156] + 3]$$
$$\Rightarrow Nx_\circ = [3, [3.5868, 3.6468]]$$

$$NR_1 = [0,6283, 0.6483] \Rightarrow Nx_1 = [3, 3NR_1 + 3] = [3, 3[0,6283, 0.6483] + 3]$$
$$\Rightarrow Nx_1 = [3, [4,8849, 4.9449]]$$

$$NR_2 = [0.0092, 0,0292] \Rightarrow Nx_2 = [3, 3NR_2 + 3] = [3, 3[0.0092, 0,0292] + 3]$$
$$\Rightarrow Nx_2 = [3, [3.0276, 3.0876]]$$

$$NR_3 = [0.0652, 0,8520] \Rightarrow Nx_3 = [3, 3NR_3 + 3] = [3, 3[0.0652, 0,8520] + 3] \Rightarrow$$
$$Nx_3 = [3, [4.1956, 5.5560]]$$

$$NR_4 = [0.7059, 0.7259] \Rightarrow Nx_4 = [3, 3NR_4 + 3] = [3, 3[0.7059, 0.7259] + 3]$$
$$\Rightarrow Nx_4 = [3, [5.1177, 5.1777]]$$

4. Conclusion and results:

We note that the values corresponding to the random variables that we obtained when using the classical values are specific values and they express states of the studied system, which does not correspond to the reality of the situation and the working conditions of the system in light of the momentary changes and fluctuations that it can go through.

But the corresponding values of the variables obtained by using neutrosophic values are field values that suit the environment and working conditions of the system that works according to the uniform distribution defined in the example, and therefore working using the concepts of neutrosophic science gives more accurate results due to the margin of freedom that we obtain through the indeterminacy provided by the science of neutrosophy, In the near future, we look forward to preparing a study through which we can generate random variables that follow other probability distributions by converting random numbers that follow the regular distribution into random variables that follow the appropriate probability distribution for the case under study.

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The Basics of Neutrosophic Simulation for Converting Random Numbers Associated with a Uniform Probability Distribution into Random Variables Follow an Exponential Distribution

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Abstract: When performing the simulation process, we encounter many systems that do not follow by their nature the uniform distribution adopted in the process of generating the random numbers necessary for the simulation process. Therefore, it was necessary to find a mechanism to convert the random numbers that follow the regular distribution over the period [0, 1] to random variables that follow the probability distribution that works on the system to be simulated. In classical logic, we use many techniques in the transformation process that results in random variables that follow irregular probability distributions. In this research, we used the inverse transformation technique, which is one of the most widely used techniques, especially for the probability distributions for which the inverse function of the cumulative distribution function can found. We applied this technique to generate neutrosophic random variables that follow an exponential distribution or a neutrosophic exponential distribution. This based on classical or neutrosophic random numbers that follow a regular distribution. We distinguished three cases according to the logic that each of the random numbers or the exponential distribution follows. We arrived at neutrosophic random variables that, when we use them in systems that operate according to an exponential distribution, such as queues and others, will provide us with a high degree of accuracy of results, and the reason for this is due to the indeterminacy provided by neutrosophic logic.

Keywords: Simulation - inverse transformation - uniform distribution - exponential distribution - neutrosophic exponential distribution - random numbers - random variables - neutrosophic logic.

1. Introduction

The generation of random variables that follow a certain distribution is the basis of the simulation. We can generate random events that simulate any real system by finding probability distributions that apply to the events and properties of that system, for example: "times between arrivals" in queues are random events that often follow an exponential distribution. There are several methods and algorithms for generating random variables from a given distribution [1,2,3].

To keep pace with the modern studies that emerged after the neutrosophic revolution, the logic laid down by the American mathematical philosopher Florentin Smarandache in 1995 [6,8,10,11,12,13,20] came as a

generalization of the fuzzy logic and an extension of the theory of fuzzy sets presented by Lotfi Zadeh in 1965 [7]. As an extension of that logic, A. A. Salama presented the theory of classical neutrosophic sets as a generalization of the theory of classical sets and developed, introduced and formulated new concepts in the fields of mathematics, statistics, computer science and classical information systems through neutrosophic. Logic that studies the origin, nature and field of indeterminacy so that it takes into account every idea with its opposite (its negation) and with the spectrum of indeterminacy [4]. In addition, there were several achievements of many researchers in the field of neutrosophic [5,9,14,15,16,17,18,19,21,22,23,24,25,26]. It was necessary to work on transforming the random numbers that follow a neutrosophic uniform distribution into random variables that follow a neutrosophic exponential distribution. In this research, we present a study on the process of converting random numbers that follow a regular distribution over the period [0, 1] to random variables that follow an exponential distribution, based on the definition of regular and exponential distributions according to neutrosophic logic.

2. Experimental and Theoretical Part:

In view of the great importance that the exponential distribution has in most fields of science, and in order to obtain more accurate results when using it in a field. the researchers defined this distribution according to the neutrosophic logic. The logic that enables us to deal with all the cases that we can come across during the study. In previous research [28] entitled " Fundamentals of Neutrosophical Simulation for Generating Random Numbers Associated with Uniform Probability Distribution" we reached mathematical formulas that help us in generating neutrosophic random numbers that follow the uniform distribution on the period [0, 1]. In this paper, we have developed a mechanism to obtain the neutrosophic random variables that follow an exponential distribution. This based on the random numbers that follow the uniform distribution on the period [0, 1]. This done by using the inverse transformation of the cumulative distribution function. The study included all the cases that we need during the simulation process for the systems that operate according to the exponential distribution.

Previous studies: [1, 2, 3, 28]

If $R_1, R_2 \ldots$ are a sequence of random numbers then R_i has a probability function defined as:

$$f_R(x) = \begin{cases} 1 & 0 \le x \le 1 \\ 0 & \text{otherwise} \end{cases}$$

Cumulative distribution function:

$$F_R(x) = \begin{cases} 0 & x < 0 \\ x & 0 \le x \le 1 \\ 1 & x > 1 \end{cases}$$

To generate $x_1, x_2 \ldots$ observations of the random variable X. follow the distribution:

$$F(x) = P(X \le x), \quad -\infty < x < \infty$$

We use the sequence of random numbers $R_1, R_2 \ldots$, and the cumulative distribution function for the random variable X. Then we apply the inverse transformation method. It is the most commonly used, especially for probability distributions in which $F^{-1}(x)$ can found. It based on matching:

$$F(X) = R \qquad (*)$$

If the random variable X follows a classical exponential distribution.

Then the probability density function is:

$$f(x) = \begin{cases} \lambda e^{-\lambda x} & x \geq 0 \\ 0 & x < 0 \end{cases}$$

The cumulative distribution function:

$$F(x) = \begin{cases} 1 - e^{-\lambda x} & x \geq 0 \\ 0 & x < 0 \end{cases}$$

We substitute in the relation (*):

$$F(X) = R$$

$$1 - e^{-\lambda x} = R \quad x \geq 0$$

By solving the previous equation, it results in:

$$x = -\frac{1}{\lambda}\ln(1 - R) \qquad (**)$$

We call the equation (**): the generator equation for the random variable that follows the exponential distribution. Are of the form:

$$X = F^{-1}(R)$$

Therefore, to obtain a sequence of observations, of the random variable X that follows the exponential distribution, we use the relationship $X = F^{-1}(R)$, and the sequence of random numbers R_1, R_2.... we write:

$$X_i = F^{-1}(R_i)$$

$$X_i = -\frac{1}{\lambda}\ln(1 - R_i) \quad ; \quad i = 1,2,..$$

It can be simplified to the form:

$$X_i = -\frac{1}{\lambda}\ln R_i$$

3. Results and Discussion

The current study: To generate random variables that follow an exponential distribution according to neutrosophic logic, we distinguish the following cases:

<u>First case</u>: the random numbers follow the neutrosophic uniform distribution on the period $[0 + \varepsilon, 1 + \varepsilon]$ and the exponential distribution in the classical form.

To generate random variables that follow the exponential distribution whose probability density function:

$$f(x) = \lambda e^{-\lambda x} \qquad x > 0$$

Cumulative Distribution Function:

$$F(x) = 1 - e^{-\lambda x} \qquad x \geq 0$$

Using the sequence of neutrosophic random numbers that follows the uniform distribution on the period $[0 + \varepsilon, 1 + \varepsilon]$, and which is given as R_1 - ε, R_2 - ε... , we apply the relationship (*):

$$F(x) = R$$

In this case, we write:

$$F(x) = R_i - \varepsilon \Rightarrow$$
$$1 - e^{-\lambda x} = R_i - \varepsilon$$
$$e^{-\lambda x} = 1 - (R_i - \varepsilon)$$
$$-\lambda x = \ln(1 - (R_i - \varepsilon)) \Rightarrow$$
$$x = -\frac{\ln(1 - (R_i - \varepsilon))}{\lambda} \qquad i = 1, 2\ldots$$

Accordingly, to obtain a sequence of observations of the random variable X using the random numbers that follow the neutrosophic uniform distribution on the period $[0 + \varepsilon, 1 + \varepsilon]$, which is given by the formula $R_i - \varepsilon$. We substitute, in the following relationship:

$$X_i = -\frac{\ln(1 - (R_i - \varepsilon))}{\lambda} \qquad ; i = 1, 2\ldots$$

It can be simplified:

$$X_{Ni} = -\frac{\ln(R_i - \varepsilon)}{\lambda} \qquad ; i = 1, 2\ldots$$

The second case: classical random numbers and a neutrosophic exponential distribution.

Let's have a sequence of random numbers R_1, R_2 … that follows a uniform distribution on the period $[0, 1]$, and we want to generate random variables that follow a neutrosophic exponential distribution.

Probability density function of the neutrosophic exponential distribution [4]:

$$f_N(x) = \lambda_N e^{-x\lambda_N} \qquad 0 < x < \infty$$

The cumulative distribution function given by:

$$NF(x) = 1 - e^{-x\lambda_N}$$

Using the relation (*):

$$NF(x) = R \Rightarrow$$
$$1 - e^{-x\lambda_N} = R \Rightarrow$$
$$e^{-x\lambda_N} = 1 - R$$

$$x = -\frac{\ln(1 - R)}{\lambda_N} \qquad \text{Or:} \qquad x = -\frac{\ln R}{\lambda_N}$$

Accordingly, to obtain a sequence of observations of the random variable X, "which follow the neutrosophic exponential distribution". Using the random numbers that follow the uniform distribution on the period $[0, 1]$, we substitute in the relationship:

$$X_{Ni} = -\frac{\ln R_i}{\lambda_N} \qquad i = 1, 2 \ldots$$

The third case: the random numbers follow the neutrosophic uniform distribution and the neutrosophic exponential distribution.

To find the relationship through which we get: random variables that follow the neutrosophic exponential distribution starting from the sequence of neutrosophic random numbers that follow the regular distribution on the period $[0 + \varepsilon , 1 + \varepsilon]$, which are given as follows:

$R_1 - \varepsilon, R_2 - \varepsilon, \ldots$

We apply the relationship (*):

$$F(x) = R$$
$$1 - e^{x\lambda_N} = R - \varepsilon \Rightarrow$$

$$X = \frac{-1}{\lambda_N} \ln[1 - (R - \varepsilon)]$$

or in the form:

$$X = \frac{-\ln(R - \varepsilon)}{\lambda_N}$$

Therefore, to obtain a sequence of observations of the random variable X that follows the neutrosophic exponential distribution using the random numbers that follow the neutrosophic uniform distribution on the period $[0 + \varepsilon , 1 + \varepsilon]$, we substitute in the relation:

$$X_{Ni} = \frac{-\ln(R_i - \varepsilon)}{\lambda_N} \qquad i = 1, 2 \ldots$$

4. Application Example:

Suppose we have a system that operates according to an exponential distribution whose probability density function is $f(x) = 2e^{-2x}$; $x \geq 0$. We want to conduct a neutrosophic simulation of this system. Where the indeterminate $\varepsilon = [0, \ 0,03]$. Here we need to generate neutrosophic random numbers. Therefore, we use one of the cases:

First case: The exponential distribution is classical, its probability density function   is $f(x) = 2e^{-2x}$; $x \geq 0$, and neutrosophic random numbers (We get it by one of the methods studied in the research [28]).

In this example, we generate random numbers that follow the neutrosophic uniform distribution on the period $[0+\varepsilon\, , 1+\varepsilon]$, That is, we generate random numbers according to one of the known methods. Here we will use the "mean-squared" method, by taking the seed $R_{\circ} = 1276$. We get the random numbers:

$$R_1 = 0,6281 \quad , \quad R_2 = 0,4509 \quad , \quad R_3 = 0,3310 \quad , \quad R_4 = 0,095$$

By using the rule that we reached in previous research [28] to convert classical random numbers into random numbers that follow the neutrosophic uniform distribution on the period $[0+\varepsilon\, , 1+\varepsilon]$. In addition, take the given indeterminacy $\varepsilon = [0\, , \ 0,03\ ]$. We get the neutrosophic random numbers:

$$R_{N\circ} = [0,0976, 0,1276] \quad , \quad R_{N1} = [0,5981, 0,6281] \quad , \quad R_{N2} = [0,4209, 0,4509]$$
$$R_{N3} = [0,3010, 0,3310] \quad , \quad R_{N4} = [0,0656, 0,0956]$$

Then we apply the following rule $\quad X_{Ni} = -\dfrac{\ln(R_i - \varepsilon)}{\lambda} = \dfrac{\ln R_{Ni}}{\lambda} \quad ; \quad i = 0,1,2,3,4$

We get:

$$X_{N\circ} = -\frac{\ln R_{N\circ}}{\lambda} = -\frac{\ln[0,0976, 0,1276]}{2} = [\,1,0294\, , 1,16343]$$

$$X_{N1} = -\frac{\ln R_{N1}}{\lambda} = -\frac{\ln[0,5981, 0,6281]}{2} = [\,0,2325\, , 0,2570]$$

$$X_{N2} = -\frac{\ln R_{N2}}{\lambda} = -\frac{\ln[0,4209, 0,4509]}{2} = [\,0,3983\, , 0,4327]$$

$$X_{N3} = -\frac{\ln R_{N3}}{\lambda} = -\frac{\ln[0,3010, 0,3310]}{2} = [\,0,5528\, , 0,6003]$$

$$X_{N4} = -\frac{\ln R_{N4}}{\lambda} = -\frac{\ln[0,0656, 0,0956]}{2} = [\,1,1738\, , 1,3621]$$

It is sequence of neutrosophic random observations, which follow an exponential distribution.

<u>The second case:</u> The neutrosophic exponential distribution, its probability density function is $f(x) = [2\, , 2,03]e^{-[2\, , 2,03]x} \quad ; \quad x \geq 0$. Random numbers are classical.

To find the required neutrosophic random observations, we will take the random numbers that follow the uniform distribution on the period $[0\, , 1]$:

$$R_{\circ} = 1276 \quad , \quad R_1 = 0,6281 \quad , \quad R_2 = 0,4509$$
$$R_3 = 0,3310 \quad , \quad R_4 = 0,0951$$

Then we apply the rule: $\quad X_{Ni} = -\dfrac{\ln R_i}{\lambda_N} \quad ; \quad i = 0,1,2,3,4$

We get:

$$X_{N\circ} = -\frac{\ln R_i}{\lambda_N} = -\frac{\ln 1276}{[2\,,\,2,03]} = [1,01421,\,1,0294]$$

$$X_{N1} = -\frac{\ln R_i}{\lambda_N} = -\frac{\ln 0,6281}{[2\,,\,2,03]} = [0,2290,\,0,2325]$$

$$X_{N2} = -\frac{\ln R_i}{\lambda_N} = -\frac{\ln 0,4509}{[2\,,\,2,03]} = [1,5045,\,0,3983]$$

$$X_{N3} = -\frac{\ln R_i}{\lambda_N} = -\frac{\ln 0,3310}{[2\,,\,2,03]} = [0,5446,\,0,5528]$$

$$X_{N4} = -\frac{\ln R_i}{\lambda_N} = -\frac{\ln 0,0951}{[2\,,\,2,03]} = [1,1590,\,1,1764]$$

It is sequence of neutrosophic random observations, which follow the neutrosophic exponential distribution.

<u>The third case:</u> The neutrosophic exponential distribution, its probability density function given by the following $f(x) = [2\,,\,2,03]e^{-[2\,,\,2,03]x}$; $x \geq 0$. The neutrosophic random numbers from the figure $R_i - \varepsilon$. To find neutrosophic random observations. We take the neutrosophic random numbers used in the first case and apply the following rule:

$$X_{Ni} = \frac{-\ln R_{Ni}}{\lambda_N} \quad ; \quad R_{Ni} = R_i - \varepsilon \quad ; \quad i = 0,1,2,3,4$$

We get:

$$X_{N\circ} = \frac{-\ln R_{N\circ}}{\lambda_N} = \frac{-\ln[0,0976,\,0,1276]}{[2\,,\,2,03]} = [1,01421,\,1,16343]$$

$$X_{N1} = \frac{-\ln R_{N1}}{\lambda_N} = \frac{-\ln[0,5981,\,0,6281]}{[2\,,\,2,03]} = [0,2291,\,0,2570]$$

$$X_{N2} = \frac{-\ln R_{N2}}{\lambda_N} = \frac{-\ln[0,4209,\,0,4509]}{[2\,,\,2,03]} = [0,3924,\,0,4327]$$

$$X_{N3} = \frac{-\ln R_{N3}}{\lambda_N} = \frac{-\ln[0,3010,\,0,3310]}{[2\,,\,2,03]} = [0,54465,\,0,6003]$$

$$X_{N4} = \frac{-\ln R_{N4}}{\lambda_N} = \frac{-\ln[0,0656,\,0,0956]}{[2\,,\,2,03]} = [1,1564,\,13621]$$

It is sequence of neutrosophic random observations, which follow the neutrosophic exponential distribution.

5. **Conclusions**:

Through the previous study, we found that, to generate a sequence of neutrosophic random observations that follow an exponential distribution, using a sequence of random numbers that follow a uniform distribution. We use one of the following cases, according to the case under study: The first

case: neutrosophic random numbers, i.e. They follow the neutrosophic uniform distribution on the period $[0 + \varepsilon \, , 1 + \varepsilon]$, and the exponential distribution in the classical form.

The second case: random numbers that follow the uniform distribution on the period $[0 \, , 1]$, and the neutrosophic exponential distribution.

The third case: the neutrosophic random numbers, i.e., they follow the neutrosophic uniform distribution on the period $[0 + \varepsilon \, , 1 + \varepsilon]$, and the neutrosophic exponential distribution.

By using techniques used in classical logic. In this paper, we used the inverse transformation technique. In addition, we found that for every random number (neutrosophic or classical) there is a random variable that follows the neutrosophic exponential distribution, which enables accurately simulate the systems that follow the exponential distribution. That is through the accuracy that neutrosophic logic provides us when studying any system according to its hypotheses.

In the near future, we are looking forward to preparing studies that will enable us to generate neutrosophic random variables that follow other probability distributions such as the Weibull distribution, the geometric distribution, and others.

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Neutrosophical Rejection and Acceptance Method for the Generation of Random Variables

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Abstract: Simulation has become a modern-day tool that helps us study many systems that its results could not have been studied or predicted through the work of these systems over time. The simulation process depends on generating a series of random numbers that are subject to a uniform probability distribution on the field [0,1], and then converting these random numbers to random variables that follow the probability distribution in which the system to be simulated, there are several methods that can be used to carry out the conversion.

In previous research, the inverse transformation method has been studied according to the Neutrosophic logic and we have reached Neutrosophic random variables, by using them we get a more accurate simulation of any system we want to simulate. It should be noted that the inverse transformation can be used if the cumulative distribution function has an inverse function, but if the system we want to simulate works according to a probability distribution, and the inverse function of the cumulative distribution function cannot be found, then we use other methods.

In this research, we are examining a study that enables us to generate Neutrosophical random variables that follow probability distributions based on Neutrosophical random numbers that follow for the uniform distribution, using the rejection and acceptance method, which depends on the largest value taken by the probability density function of the distribution in which the system to be simulated operates on its definition.

Keywords: Neutrosophic, Neutrosophical random variables, Neutrosophic logic

Introduction:

In the light of the great development in today's world, the complexity of systems and the significant material and non-material losses that can result from the operation of any system without prior study, there has to be a scientific method that enables us to know the results that we can get when operating systems and helps us to minimize these losses. The simulation process was the modern instrument through which we can predict what results we can get through the functioning of these systems over time. Since the simulation process depends on generating random numbers that follow regular distribution on the field [0, 1] and then converting these numbers into random variables that follow the probable distribution of the system to be simulated In classical logic, many methods were presented by which we were able to obtain the random variables needed for the simulation process, To keep pace with scientific development, the most important of these methods had to be reformulated using the revolutionary logic of the Neutrosophic logic established by the American philosopher and athlete Florentine Smarandache   in 1995. So in previous research we prepared a study to generate random Neutrosophic numbers on the field [a, b) Based on the Neutrosophic researchers' findings in the definition of regular distribution and definition of integration according to Neutrosophic logic, and in other research, we converted these Neutrosophic random numbers into Neutrosophic random variables tracking the exponential distribution using the opposite conversion method [1-14].

In this research we will generate Neutrosophic random variables based on previous studies and use the method of rejection and acceptance to convert Neutrosophic random numbers into random variables that follow the probable distribution of the system to be emulated.

Discussion:

During the simulation process, we encounter many systems that operate on probability distributions. The reverse function of the cumulative distribution function cannot be found. Therefore, we cannot use the opposite conversion method that we have formulated according to the Neutrosophic logic. So in this research we will present a study to convert the Neutrosophic random numbers that follow the Neutrosophic regular probability distribution into Neutrosophic random variables.

Method of rejection and acceptance according to classical logic: [15]

We take the distribution of a probability $f(x)$ defined on the field $[a,b]$ with the following relationship:

$$f(x) = \begin{cases} f(x) & ; a \leq x \leq b \\ 0 & otherwise \end{cases}$$

We assume that the greatest value $f(x)$ takes on your area of definition is M the same as Mode, then the following inequality is achieved:

$$0 \le f(x) \le M \quad ; \quad a \le x \le b$$

Thus:

$$0 \le f(x) \le M \Rightarrow 0 \le \frac{f(x)}{M} \le 1 \quad ; a \le x \le b$$

This means that the composition $\dfrac{f(x)}{M}$ is valid for comparison with random numbers that follow a uniform distribution on the field [0, 1], because it achieves the following relationship:

$$P\left(R < \frac{f(x)}{M}\right) = \frac{f(x)}{M}$$

We benefit from the above by applying the following algorithm:

1- We generate two random numbers R_1 , R_2 that follow the uniform distribution on the field

[0, 1].

We use the mean squared method to generate the two random numbers defined as follows:

To generate random numbers, we apply the following relationship:

$$R_{i+1} = Mid\left[R_i^2\right] \quad ; i = 0,1,2,3,---- \quad (1)$$

Where *Mid* stands for the middle four places of R_i^2 and any fractional $R_\circ$ random number consisting of four places (called the seed) and not containing zero in any of its four places is chosen, to generate two random numbers R_1 , R_2 from the field $[0 , 1]$.

2- We take one of the two numbers, let it be R_1 and from it a variable suitable for the uniform distribution on the field [a, b], by performing the following conversion:

$$x_1 = (b-a)R_1 + a$$

3- we check whether if R_2 achieves the inequality:

$$R_2 \le \frac{f(x)}{M} \qquad (*)$$

4- if the inequality (*) is achieved We accept that x_1 is subject to the operated distribution in the system which is being simulated.

If the inequality $(*)$ is not achieved, we reject the two numbers R_1 , R_2 and return to the first step, to generate two random numbers again.

Thus, we continue to work and compare as much as we want or as required by the simulation process.

Practical example:

We want to simulate a system that operates according to the probability distribution whose

probability density function is given by the following relationship:

$$f(x) = \begin{cases} \dfrac{1}{16}x & ; \ 2 \le x \le 6 \\ 0 & ; 0 \text{therwise} \end{cases}$$

We calculate M which is the largest value of $f(x)$ and is equal to:

$$M = \frac{1}{16} \times 6 = \frac{3}{8}$$

To perform the simulation, it is necessary to generate random variables that follow this distribution.

We will use the rejection and acceptance method to achieve the desired:

1- We use the mean squared method given by the following relationship:

$$R_{i+1} = Mid\left[R_i^2\right] \ \ ; i = 0,1,2,3,---- \quad (1)$$

Where Mid stands for the middle four places of R_i^2 and any fractional $R_\circ$ random number consisting of four

places (called the seed) and not containing zero in any of its four places is chosen, to generate two random

numbers R_1 , R_2 from the field $[0 , 1]$ we take the seed $R_\circ = 0,3176$ and from it we get

$$R_2 = 0.7551, \ R_1 = 0.0869$$

2- We take one of the two numbers, let it be $R_1 = 0,0869$ and form it a variable suitable for uniform

distribution over the field $[a , b] = [2 , 6]$ by performing the following transformation:

$$x_1 = (b-a)R_1 + a \implies$$
$$x_1 = (6-2) \times 0.0869 + 2 = 2.3476$$

We calculate $f(x_1)$:

$$f(x_1) = \frac{1}{16} \times 2.3476 = 0.1467$$

3-   We test inequality $(*)$:

$$R_2 \le \frac{f(x_1)}{M} \implies$$
$$0.7551 \le \frac{0.1467}{0.375} = 0.3912$$

4-   We note that the inequality is not achieved and therefore the random variable x_1 does not follow the

probability distribution $f(x)$ and therefore we reject the two random numbers R_1 , R_2 and return to (1).

We start again:

1-   We generate two random numbers by taking the seed $R_\circ = 0,1234$ we get:

$$R_2 = 0.3215 , R_1 = 0.5227$$

2-   We take one of the numbers, let it be $R_1 = 0,5227$ and form from it a variable suitable for the

uniform distribution on the field $[a , b] = [2 , 6]$ by doing the following conversion:

$$x_1 = (b-a)R_1 + a \implies$$
$$x_1 = (6-2) \times 0.5227 + 2 = 4.0908$$

We calculate $f(x_1)$:

$$f(x_1) = \frac{1}{16} \times 4.0908 = 0.2557$$

3-   We check the inequality (*):

$$R_2 \le \frac{f(x_1)}{M} \implies$$
$$0.3215 \le \frac{0.2557}{0.375} = 0.6819$$

4- We see that the inequality is achieved, therefor the random variable x_1 follows the probability

distribution $f(x)$, we continue to work and compare as much as we want or as required by the

simulation process.

In this research, we formulated the rejection and acceptance method according to the Neutrosophic

logic, so the previous algorithm became as follows

1- We generate two random numbers R_1 , R_2 that follow the uniform distribution on the field

[0,1].

2- We convert the two numbers R_1 , R_2 into two Neutrosophic random numbers on the field

$$\left[0 + \delta \ , 1 + \delta \right]$$

According to the following formula:

$$NP\left(R < R_i\right) = NF\left(R_i\right) = R_i - \delta \quad ; \quad \delta \in \left[0 \ , m\right]$$

Where $\delta \in \left[0 \ , m\right]$ is the indefinite, and $0 < m < 1$ let's denote them by NR_2 , NR_1

We find the cumulative distribution function for the uniform distribution, which is given by the following

relationship:

When the indeterminacy is in the upper and lower bounds of the field that means:

$$\left[a_N \ , b_N\right] = \left[a + \varepsilon \ , b + \varepsilon\right] \text{ where } \varepsilon \in \left[0 \ , n\right] \quad \text{ and } \ a < n < b \text{ then:}$$

$$NF(x) = \frac{x - a}{b_N - a_N} - i \quad ; \quad i \in \left[0 \ , \frac{n}{b_N - a_N}\right]$$

In (1) and (2) we used a study that is mentioned in reference (16).

Then we use the inverse transformation method to find the random variables that follow the uniform

distribution on the field $\left[a_N \ , b_N\right] = \left[a + \varepsilon \ , b + \varepsilon\right]$

We substitute the following relationship:

$$R = F(x) \Rightarrow x = F^{-1}(R)$$

We find:

$$NF(x) = NR \Rightarrow NR = \frac{x-a}{b_N - a_N} - i \Rightarrow$$

$$\frac{x-a}{b_N - a_N} = NR + i \Rightarrow (NR+i)(b_N - a_N) = x - a \Rightarrow$$

$$Nx = (NR+i)(b_N - a_N) + a$$

$$i \in \left[0, \; \frac{n}{b_N - a_N}\right]$$

Accordingly, to generate random variables that follow the Neutrosophic uniform distribution in the case of indeterminacy related to the lower and upper bounds

We use the following relationship:

$$Nx_j = (NR_j + i)(b_N - a_N) + a \quad ; \quad j = 0,1,2,3,4$$

Where $i \in \left[0, \; \dfrac{n}{b_N - a_N}\right]$ and ε is the indefinite where $\varepsilon \in [0, n]$ and $a < n < b$

We note that the variables have become neutrosophical values, and therefore the probability density function

$f(x)$ becomes a neutrosophical function that we denote by the symbol $f_N(x)$ and it is defined on the field

$[a_N, b_N] = [a + \varepsilon, b + \varepsilon]$.

We suppose that the largest value that this function takes in its domain is M_N, then the following inequality

is achieved:

$$0_N \le f_N(x) \le M_N ; \quad a + \varepsilon \le x \le b + \varepsilon$$

Then the following inequality is achieved:

This means that the composition $\dfrac{f_N(x)}{M_N}$ is valid for comparison with the Neutrosophical random numbers

that follow the Neutrosophic uniform distribution on the field $[0 + \delta, 1 + \delta]$ because it achieves the

following relationship:

$$NP\left(NR < \frac{f_N(x)}{M_N}\right) = \frac{f_N(x)}{M_N}$$

3- We test whether NR_2 achieves inequality:

$$NR_2 \leq \frac{f_N(Nx_1)}{M_N} \qquad (**)$$

If the inequality (**) is true, then we accept that Nx_1 is subject to the distribution $f_N(x)$ in which the system

to be simulated operates.

4-   If NR_2 does not achieve the inequality $(**)$, then we reject the two numbers R_1, R_2 and return

to the first step of generating the two random numbers again.

Thus, we continue to work and compare as much as we want or as required by the simulation process.

Note 1: In step (2) we took the indeterminacy on the two terms of the field and used the appropriate relations for that in the process of converting random numbers into Neutrosophical random numbers. Reference [16].

Note 2: When generating the Neutrosophic random variable that follows the uniform distribution on the field, we also took the indeterminacy on the two terms of the field and then applied the inverse transformation method to find the random variable. It should be noted that we follow the same method if the indeterminacy is related to one of the field terms.

Practical example:

We want to simulate a system that operates according to the probability distribution whose probability

density function is given by the following relationship:

$$f(x) = \begin{cases} \dfrac{1}{16}x & ; \ 2 \leq x \leq 6 \\ 0 & ; 0 \, therwise \end{cases}$$

1-   We use the mean squared method given by the following relationship:

$$R_{i+1} = Mid\left[R_i^2\right] \quad ; i = 0,1,2,3,---- \quad (1)$$

We take the seed $R_\circ = 0{,}3176$ and from it we form $R_2 = 0.7551$, $R_1 = 0.0869$

2- We transform the two numbers R_1, R_2 into two Neutrosophic random numbers on the field

$[0 + \delta , 1 + \delta]$ using the following relationship:

$$NR_i = R_i - \delta$$

And in this example we take $\delta = [0 , 0.03]$

$$NR_i = R_i - \delta \quad \& \quad i = 1 \Rightarrow$$
$$NR_1 = R_1 - \delta = 0.0869 - [0 , 0.03] \Rightarrow$$
$$NR_1 = [0.0569 , \ 0.0869]$$

$$NR_i = R_i - \delta \quad \& \quad i = 2 \Rightarrow$$
$$NR_2 = R_2 - \delta = 0.7551 - [0 , 0.03] \Rightarrow$$
$$NR_2 = [0.7251 , \ 0.7551]$$

We take one of the two numbers, let it be $NR_1 = [0.0569 , 0.0869]$ and from it we form a

variable suitable for the Neutrosophic uniform distribution over the field

.Here we have taken $\varepsilon = [0 , 3]$ $[a_N , b_N] = [2 + [0 , 3], 6 + [0 , 3]] = [[2 , 5], [6 , 9]]$

By performing the following conversion:

$$Nx_j = (NR_j + i)(b_N - a_N) + a \quad ; \quad j = 1,2,3,---$$

$$i \in \left[0, \ \frac{n}{b_N - a_N} \right]$$

First we calculate i :

$$i = \left[0, \ \frac{3}{b_N - a_N} \right] = \left[0, \ \frac{3}{[6 , 9] - [2 , 5]} \right] = \left[0, \ \frac{3}{4} \right] = [0 , 0.75]$$

Make up:

$$Nx_1 = (NR_1 + i)(b_N - a_N) + a \Rightarrow$$
$$Nx_1 = ([0.0569, 0.0869] + [0, 0.75])(4) + 2 \Rightarrow$$
$$Nx_1 = [2.2276, 5.3476]$$

We calculate $f_N(Nx_1)$**:**

$$f_N(x) = f_N(Nx_1)$$

$$f_N(Nx_1) = \frac{1}{16}[2.2276, 5.3476] = [0.1392, 0.3342]$$

We calculate M_N **which is the largest value of** $f_N(x)$ **and is equal to:**

$$M_N = \frac{1}{16} \times [6, 9] = [0.375, 0.5625]$$

3- We test the inequality $(**)$:

$$NR_2 \le \frac{f_N(Nx_1)}{M_N} \Rightarrow [0.7251, 0.7551] \le \frac{[0.1392, 0.3342]}{[0.375, 0.5625]} = [0.3712, 0.5941] \Rightarrow$$
$$[0.7251, 0.7551] \le [0.3712, 0.5941]$$

4- We note that the inequality is not satisfied and therefore we reject the two random numbers

R_2, R_1 and return to step (1)

We start again

1- We use the mean squared method given by the following relationship:

$$R_{i+1} = Mid[R_i^2] \quad ; i = 0,1,2,3,---- \quad (1)$$

We take the seed $R_\circ = 0,1234$ and from it we form $R_2 = 0.3215$, $R_1 = 0.5227$

2- We transform the two numbers R_1, R_2 into two Neutrosophic random numbers on the field

$[0 + \delta, 1 + \delta]$ using the following relationship:

$$NR_i = R_i - \delta$$

We take it in this example $\delta = [0, 0.03]$

$$NR_i = R_i - \delta \quad \& \quad i = 1 \Rightarrow$$
$$NR_1 = R_1 - \delta = 0.5227 - [0, 0.03] \Rightarrow$$
$$NR_1 = [0.5227, \ 0.4927]$$

$$NR_i = R_i - \delta \quad \& \quad i = 2 \Rightarrow$$
$$NR_2 = R_2 - \delta = 0.3215 - [0, 0.03] \Rightarrow$$
$$NR_2 = [0.3215, \ 0.2915]$$

We take one of the two numbers, let it be $NR_1 = [0.5227, 0.4927]$, and from it form a variable suitable for

the Neutrosophic uniform distribution over the field

$[a_N, b_N] = [2 + [0, 3], 6 + [0, 3]] = [\,[2, 5], [6, 9]\,]$. Here we have taken $\varepsilon = [0, 3]$

By performing the following conversion:

$$Nx_j = (NR_j + i)(b_N - a_N) + a \quad ; \quad j = 1,2,3,---$$

$$i \in \left[0, \ \frac{n}{b_N - a_N} \right]$$

First, we calculate i we find:

$$i = \left[0, \ \frac{3}{b_N - a_N} \right] = \left[0, \ \frac{3}{[6, 9] - [2, 5]} \right] = \left[0, \frac{3}{4} \right] = [0, 0.75]$$

Make up:

$$Nx_1 = (NR_1 + i)(b_N - a_N) + a \Rightarrow$$
$$Nx_1 = (\, [0.5227, 0.4927] + [0, 0.75] \,)(4) + 2 \Rightarrow$$
$$Nx_1 = [4.0908, 5.4927]$$

We calculate $f_N(Nx_1)$:

$$f_N(x) = f_N(Nx_1)$$
$$f_N(Nx_1) = \frac{1}{16}[4.0908, 6.9708] = [0.2557, 0.3106]$$

3- We calculate M_N which is the largest value of $f_N(x)$ and is equal to:

$$M_N = \frac{1}{16} \times [6, 9] = [0.375, 0.5625]$$

4- We test the inequality $(**)$:

$$NR_2 \leq \frac{f_N(Nx_1)}{M_N} \Rightarrow [0.3215, 0.2915] \leq \frac{[0.2557, 0.3106]}{[0.375, 0.5625]} = [0.5522, 0.6819] \Rightarrow$$

$$[0.2915, 0.3215] \leq [0.5522, 0.6819]$$

We note that the inequality is achieved, and therefore we accept Nx_1 a Neutrosophical variable that follows

the Neutrosophical probability distribution defined as follows:

$$f_N(x) = \begin{cases} \dfrac{1}{16}x & ; \quad 2 + \varepsilon \leq x \leq 6 + \varepsilon \\ 0_N & ; \ 0 \, therwise \end{cases}$$

Where $\varepsilon = [0, 3]$.

We continue in the same way until we obtain the required number of Neutrosophic random variables

needed for the simulation process.

Conclusion and Results:

From the previous study we note that we do not need the reverse function of the cumulative distribution function of the probable distribution of the system imposed when using the Neutrosophic rejection and acceptance method and therefore it is a suitable method to generate the Neutrosophic random variables needed for the simulation process when obtaining the inverse function of the cumulative distribution function is difficult or not possible and using Neutrosophic variables we get more accurate and appropriate simulation results for changes that can occur during the work of the system to be simulated.

We look forward in the near future to preparing studies in which we use the method of refusal of acceptance to generate random variables following popular and widely used potential distributions in applied fields such as beta distribution and other distributions.

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Generating Random Variables that follow the Beta Distribution Using the Neutrosophic Acceptance-Rejection Method

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Abstract:

Analysis using simulation is a natural and logical extension of the analytical and mathematical models inherent in operations research. Simulation has become a modern tool that helps in studying many systems that we could not study or predict the results that we could obtain during the operation of these systems over time before the existence of Simulation, since the main interest in statistical analysis is to obtain a series of random variables that follow the probability distribution in which the system under study operates, through a series of random numbers that follow a uniform distribution over the domain [0, 1], using scientific methods provided by the efforts of researchers. In the field of modeling and simulation, such as the reverse transformation method, the rejection and acceptance method, and other methods that we have reformulated using the concepts of neutrosophic science in previous research. In summary of what we have done previously, we present in this research a study whose purpose is to generate random variables that follow the beta distribution, which is used in many applications. In administrative processes, especially in analyzing network diagrams using the neutrosophic rejection and acceptance method.

Keywords: modeling and simulation; neutrosophic science; rejection and acceptance method for generating neutrosophic random variables; neutrosophic uniform distribution; neutrosophic random numbers; Neutrosophic beta distribution.

Introduction:

Operations research has provided many scientific methods that have contributed to the great scientific development witnessed in our contemporary world. The importance of these methods increases when they are reformulated using the concepts of neutrosophic science, the science that relies on neutrosophic data that leaves nothing to chance and takes into account all the circumstances and fluctuations facing decision makers. Therefore, researchers have presented many Among the researches through which some operations research methods were reformulated using the concepts and information presented by the founder of this science, we mention [1-18]. Neutrosophic statistical studies are an extension of traditional statistical studies, as they depend on neutrosophic data, which are groups where any a value such as a in a group denoted by a_N which means (a neutrosophic), (a imprecise), or (a non-specific). a_N may be a neighbor of a or an interval containing a, and in general it can be considered any set close to a. See [19].

In any probability distribution, if there is a quantity that contains some indeterminacy, then this distribution is a neutrosophic probability distribution. Therefore, the random numbers and random variables that we obtain based on this probability distribution are neutrosophic random numbers and random variables. See [20].

Discussion:

Probability distributions are the mainstay of the simulation process. Therefore, the interest of researchers and scholars interested in the simulation method has focused on providing scientific studies that help in obtaining random variables that follow the probability distributions most used in practical applications. In classical studies, we find many algorithms that are concerned with transforming random numbers that follow a uniform distribution into The domain [1, 0] refers to the probability distribution with which the system to be simulated operates, and based on this importance of probability distributions and after the emergence of neuroscientific science, which paid great attention to probability distributions, many probability distributions were reformulated using the concepts of this science, and we presented in previous research how Generating neutrosophic random numbers and methods for converting these numbers into neutrosophic random variables that follow the exponential distribution and others that follow the uniform distribution using the inverse transformation method, which was reformulated using the concepts of neutrosophic science [21-23]. In this research, and given the importance of the beta distribution, we present a study whose goal is to use the method Neutrosophic rejection and acceptance, which was presented in the paper [24], to generate neutrosophic random variables that follow the beta distribution, which is defined using classical values as the following probability density function:

$$f(x) = \frac{\Gamma(\alpha + \beta)}{\Gamma(\alpha)\Gamma(\beta)} x^{\alpha-1}(1 - x)^{\beta-1} \quad ; \quad 0 \le x \le 1 \qquad (1)$$

Where α and β are the medians used to define this distribution and $\alpha > 0, \beta > 0$

The symbol $\Gamma(c)$ is the value of the integral (gamma), defined by the following relationship:

$$\Gamma(c) = \int_0^\infty x^{c-1} e^{-x} dx$$

The beta distribution curve takes many shapes depending on the values of α and β and we can be classified into three types:

1- Pessimistic curve in this case is $\alpha > \beta$
2- Symmetrical curve $\alpha = \beta$
3- Optimistic curve $\alpha < \beta$

As in the following figures:
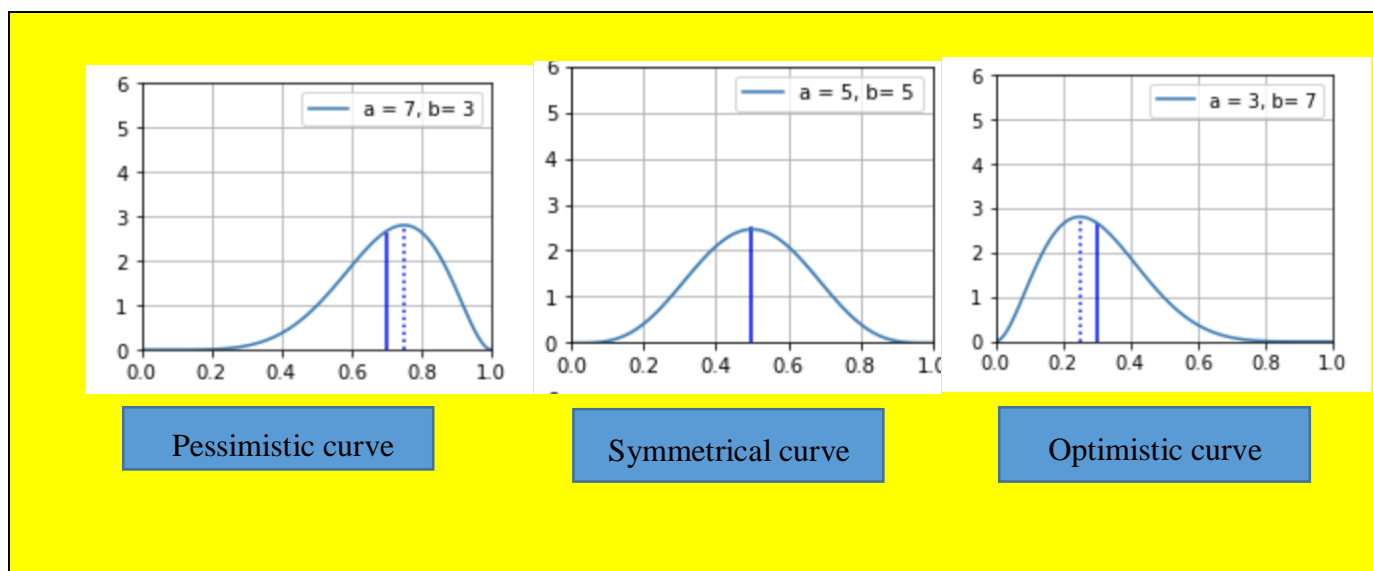
Neutrosophic function: [25]

The neutrosophic function $f: A \rightarrow B$ is a function that has some indeterminacy, taking into account the definition of its domain, its corresponding domain, and the relationship between the elements of the domain and the elements of the corresponding domain.

If α, β, or both of them carry some indeterminacy, that is, they take one or the other

Where α_N and β_N are neutrosophic values written as follows:

$$\beta_N = \beta + \delta \text{ و } \alpha_N = \alpha + \varepsilon$$

Where ε is an indeterminacy and takes one form $\varepsilon \in \{\lambda_1, \lambda_2\}$ or $\varepsilon \in [\lambda_1, \lambda_2]$ or other than that.

Where δ is an indeterminacy and takes one form $\delta \in \{\mu_1, \mu_2\}$ or $\delta \in [\mu_1, \mu_2]$ or other than that.

Then we obtain the neutrosophic beta distribution, which has a probability density function given by the following formula:

$$f(x) = \frac{\Gamma(\alpha_N + \beta_N)}{\Gamma(\alpha_N)\Gamma(\beta_N)} x^{\alpha_N - 1}(1 - x)^{\beta_N - 1} \quad ; \quad 0 \leq x \leq 1 \qquad (2)$$

Based on what has been reported about this beta distribution in classical studies, we can present these neutrosophic types of beta distribution: [24]

The graphic curve of the beta neutrosophic distribution takes many shapes depending on the values of α_N and β_N and can be classified into three types:

1- Pessimistic curve in this case is $\alpha_N > \beta_N$
2- Symmetrical curve $\alpha_N = \beta_N$
3- Optimistic curve $\alpha_N < \beta_N$

Generating random variables that follow a beta distribution:

We know that the process of neutrosophic simulation depends on generating neutrosophic random numbers that follow the regular neutrosophic distribution and then converting these random numbers into random variables that follow the probability distribution with which the system to be simulated operates. There are several methods that can be used for the conversion process, including we mentioned in previous research the inverse transformation method and the rejection method. And acceptance [21-23], where the appropriate method is used for the probability density function because in the inverse transformation method we need the inverse function of the cumulative distribution function. As we know, in many functions, the inverse function does not exist or obtaining it requires complex operations. Here we resort to the method Rejection and acceptance. As is clear from relationship (1), it is not easy to obtain the inverse function of the probability density function for the beta distribution. Therefore, we will use the inverse transformation method and apply it to the beta distribution according to the following algorithms:

Rejection and acceptance algorithm: [23,24]

We calculate $\boldsymbol{M_N}$, the maximum value taken by the probability density function over its defined domain, and we obtain it by calculating the derivative of this function and setting it equal to zero, i.e.:

$$M_N = \frac{df_N(x)}{dx}\bigg|_{x=0}$$

And in simplified form, we define the derivative of the neutrosophic function as follows [24]:

$$f_N(x) = \lim_{h \to 0} \frac{[inf\, f(x+h) - inf\, f(x) supf(x+h) - sup\, f(x)]}{h}$$

This is a generalization of the traditional derivative definition, where the function and variables are conventional. It can be written as:

$$[inf\, H\,, sup\, H] = h$$
$$inf\,(x+H) = sup(x+H) = f(x+h)$$
$$inf\, f(x) = sup\, f(x) = f(x)$$

Here, $\boldsymbol{H}$ is a closed, open, half-closed, or half-open interval.

By applying the above definition to the function defined by the relationship (2), we obtain:

$$f'(x) = c.(\alpha_N - 1)x^{\alpha_N - 2}(1-x)^{\beta_N - 1} - c(\beta_N - 1)x^{\alpha_N - 1}(1-x)^{\beta_N - 2}$$

To find the solution, we set the derivative equal to zero:

$$f'(x) = c.(\alpha_N - 1)x^{\alpha_N - 2}(1-x)^{\beta_N - 1} - c(\beta_N - 1)x^{\alpha_N - 1}(1-x)^{\beta_N - 2} = 0$$

In short, by finding the common limits, we get:

$$Nx = \frac{\alpha_N - 1}{\alpha_N + \beta_N - 2} = mod \quad (3)$$

This value of x corresponds to the maximum value of the function, and therefore:

$$M_N = \frac{\alpha_N - 1}{\alpha_N + \beta_N - 2}$$

After obtaining the values of α_N and β_N and substituting them into equations (2) and (3), we apply the following algorithm:

1. Generate two random numbers, R_1 and R_2 following a uniform distribution in the range [0, 1]. We use the method of squaring the average to generate the random numbers R_1 and R_2 as follows [24,25]:
$$R_{i+1} = Mid[R_i^2] \ ; i = 0,1,2, - - - - \qquad (4)$$

where, Mid refers to the middle four digits of R_1^2 and R_0 is an initial random number consisting of four digits (called the seed) that doesn't contain zero in any of its four digits.

2. Convert the numbers R_1 and R_2 into neutrosophic function random numbers. To convert random numbers following a uniform distribution into neutrosophic random numbers, we follow the method introduced in [22], where three forms of neutrosophic random numbers are distinguished based on the uncertainty associated with the range [0, 1]:

a. **Form 1**: Uncertainty in the lower limit, i.e., $[0 + \varepsilon, 1]$. where, each classical random number is mapped to a neutrosophic function random number using the following relationship:
$$NR_0 = \frac{R_0 - \varepsilon}{1 - \varepsilon} = \frac{[R_0, \ R_0 - n]}{[1, 1 - n]} \in [\ R_0, \frac{R_0 - n}{1 - n}]$$

b. **Form 2**: Uncertainty in the upper limit, i.e., $[0, 1 + \varepsilon]$. where, each classical random number is mapped to a neutrosophic function random number using the following relationship:
$$NR_0 = \frac{R_0}{1 + \varepsilon} = \frac{R_0}{[1, \ n + 1]} \in [R_0, \frac{R_0}{n + 1}]$$

c. **Form 3**: Uncertainty in both upper and lower limits, i.e. $[0, 1 + \varepsilon]$. where, each classical random number is mapped to a neutrosophic function random number using the following relationship:
$$NR_0 = R_0 - \varepsilon \in [R_0, R_0 - n]$$

In the three previous forms, we have $\varepsilon \in [0, n]$ *and* $0 < n < 1$.

In this study, we will use the third form: non-deterministic in the upper and lower bounds, i.e., $[0 + \varepsilon, 1 + \varepsilon]$. Here, we associate each classical random number with a non-deterministic random number using the following relationship:
$$NR_0 = R_0 - \varepsilon \in [R_0, R_0 - n]$$

Where $\varepsilon \in [0, n]$ *and* $0 < n < 1$

3. We take one of the two numbers, let it be NR_1, and transform it appropriately for the uniform distribution. We know that when the boundaries of the domain are neutrosophic values, we apply the following [22] for the upper and lower limits of the domain:

$[b + \varepsilon, a + \varepsilon]$, where $a = a_N = a + \varepsilon$ and $b_N = b + \varepsilon$, and $\varepsilon \in [0, n]$ *and* $a < n < b$.

We use the following relationship:

$$Nx_1 = (b - a)NR_1 + a$$

4. We test whether NR_2 satisfies the inequality:
$$NR_2 \leq \frac{f(NR_1)}{M} \qquad (5)$$

That is,

$$NR_2 \leq \frac{f_N(NR_1)}{M_N} = \frac{\Gamma(\alpha)\Gamma(\beta)}{\Gamma(\alpha+\beta)}(NR_1)^{\alpha_N-1}\left[1-(NR_1)^{\beta_N-1}\right]$$

5. If the inequality (5) is satisfied, then we accept that $Nx_1 = NR_1$ follows the Beta distribution defined by equation (1).

6. If NR_2 does not satisfy the inequality (5), then we reject the two numbers NR_1 , NR_2 and return to the first step to generate new random numbers.

We explain the above through the following example:

Example 1:

We have a system that operates according to the beta function defined by the following probability density function:

$$f(x) = \frac{\Gamma(10)}{\Gamma(3)\Gamma(7)}x^2(1-x)^6 \quad ; \quad 0 \leq x \leq 1$$

That is, $\alpha = 3$ and $\beta = 7$

What is required is to generate random variables that follow this distribution. In this example, we will take the beta function in the classical form and neutrosophic random numbers:

We apply the rejection and acceptance algorithm according to the previously mentioned steps:

1. We calculate the largest value of the density function over its defined field from the following relationship:

$$M = \frac{\alpha-1}{\alpha+\beta-2}$$

We find:

$$M = \frac{3-1}{3+7-2} = \frac{2}{8} = \frac{1}{4}$$

2. We use the method of squaring to generate two random numbers that follow the uniform distribution over the domain $[0 , 1]$. We take the seed $R_0 = 0.1273$ and obtain the following random numbers:
$$R_1 = 0.6205 \quad , R_2 = 0.5020$$

3. We convert the classical random numbers to random numbers that follow the non-deterministic uniform distribution over the domain $[0+\varepsilon , 1+\varepsilon]$. We take the non-deterministic bounds $\varepsilon \in [0 , 0.02]$ and obtain the following neutrosophic non-deterministic random numbers:
$$NR_1 = R_1 - \varepsilon \in [R_1 , R_1 - 0.02]$$
$$NR_1 = R_1 - \varepsilon = 0.6205 - [0 , 0.02] = [0.6205 , 0.6005] \Rightarrow NR_1 \in [0.6205 , 0.6005]$$
$$NR_2 = R_2 - \varepsilon \in [R_2 , R_2 - 0.02]$$
$$NR_2 = R_2 - \varepsilon = 0.5020 - [0 , 0.02] = [0.5020 , 0.482] \Rightarrow NR_2 \in [0.5020 , 0.482]$$

4. We take one of the numbers and form an appropriate transformation for the uniform

distribution over the domain $[0 , 1]$. Here, since the distribution is defined over the domain $[0 , 1]$, we take one of the non-deterministic random numbers, let's say $NR_1 \in [0.6205 , 0.6005]$, and calculate the value of the probability density function at that point:

$$f(NR_1) = \frac{\Gamma(10)}{\Gamma(3)\Gamma(7)} ([0.6205 , 0.6005])^2 (1 - ([0.6205 , 0.6005]))^6$$

We know that if n is a positive integer, then $(n + 1) = n!$, therefore:

$$\frac{\Gamma(10)}{\Gamma(3)\Gamma(7)} = \frac{9!}{2!\, 6!} = 252$$

$$f(NR_1) \in (252 \times [0.385, 0.3606] \times [0.003, 0.0041]) = [0.2911, 0.3726]$$

1. We test the inequality (5) and for this, we calculate:

$$\frac{f(NR_1)}{M} \in \frac{[0.2911,0.3726]}{0.25} = [1.1644,1.4904]$$

We have $NR_2 \in [0.5020 , 0.482]$. We note that:

$$[0.5020 , 0.482] \le [1.1644,1.4904]$$

Therefore, inequality (5) is satisfied, i.e.:

$$NR_2 \le \frac{f(NR_1)}{M}$$

Here we accept that $N(R_1) \in [1.1644,1.4904]$ follows the beta distribution given in the example.

Generating random numbers following the neutrosophic beta distribution from classical random numbers:

Example 2: We have a system that operates according to the beta function defined by the neutrosophic probability density function as in the following relationship:

$$f(x) = \frac{\Gamma(\alpha_N + \beta_N)}{\Gamma(\alpha_N)\Gamma(\beta_N)} x^{\alpha_N - 1} (1 - x)^{\beta_N - 1} \quad ; \ 0 \le x \le 1$$

α_N and β_N are neutrosophic values of the form $\alpha_N = \alpha + \varepsilon$, $\beta_N = \beta + \delta$, which are adjacent to the real values $\alpha = 3$ and $\beta = 7$.

Where ε and δ are the indeterminacy in these values. We take them in this example as follows: $\varepsilon \in [\lambda_1, \lambda_2] = [0 , 0.2]$ and $\delta \in [\mu_1, \mu_2] = [0 , 0.1]$ we get $\alpha_N \in [3 , 3.2]$ and $\beta_N \in [7 , 7.1]$ then the probability density function is written as follows:

$$f_N(x) = \frac{\Gamma([10 , 10.3])}{\Gamma([3 , 3.2])\Gamma([7 , 7.1])} x^{[2 , 2.2]} (1 - x)^{[6 , 6.2]} \quad ; \ 0 \le x \le 1$$

What is required is to generate random numbers that follow the previous distribution, based on classical random numbers. We apply the rejection and acceptance algorithm according to the steps mentioned previously, and as we did in Example 1, we obtain what is required.

Example 3: We have a system that operates according to the beta function defined by the neutrosophic probability density function as in the following relationship:

$$f(x) = \frac{\Gamma(\alpha_N + \beta_N)}{\Gamma(\alpha_N)\Gamma(\beta_N)} x^{\alpha_N - 1} (1 - x)^{\beta_N - 1} \quad ; \ 0 \le x \le 1$$

α_N and β_N are neutrosophic values of the form $\alpha_N = \alpha + \varepsilon$, $\beta_N = \beta + \delta$, which are adjacent to the real values $\alpha = 3$, $\beta = 7$.

And ε and δ are the indeterminacy in these values. We take them in this example as follows: $\varepsilon \in [\lambda_1, \lambda_2] = [0 , 0.2]$ and $\delta \in [\mu_1, \mu_2] = [0 , 0.1]$ we get $\alpha_N \in [3 , 3.2]$ and $\beta_N \in [7 , 7.1]$ then the probability density function is written as follows:

$$f_N(x) = \frac{\Gamma([10 , 10.3])}{\Gamma([3 , 3.2])\Gamma([7 , 7.1])} x^{[2 , 2.2]} (1 - x)^{[6 , 6.2]} \quad ; \ 0 \le x \le 1$$

What is required is to generate random numbers that follow the previous distribution, based on neutrosophic random numbers. We apply the rejection and acceptance algorithm according to the steps mentioned previously, and as we did in Example 1, we obtain what is required.

The difference between the second and third examples:

In the second example, we generate random numbers that follow a uniform distribution over the domain $[0, 1]$ using the mean square method. For example, we take the seed $R_0 = 0.1273$ from the relationship (3), which is:

$$R_1 = 0.6205 \quad , R_2 = 0.5020$$

Then we use the two numbers to implement the rejection and acceptance algorithm, as we did in the first example.

In the third example, we generate two random numbers that follow a uniform distribution over the domain $[0, 1]$ using the mean square method. For example, we take the seed $R_0 = 0.1273$ from the relationship (3), which is:

$$R_1 = 0.6205 \quad , R_2 = 0.5020$$

Then we transform them into two random numbers that follow the uniform neutrosophic distribution over the domain $[0 + \varepsilon, 1 + \varepsilon]$. We take the indeterminacy $\varepsilon \in [0, 0.02]$. We get the following two-neutrosophic random numbers:

$$NR_1 \in [0.6205, 0.6005] \quad , \quad NR_2 \in [0.5020, 0.482]$$

Then we use two-neutrosophic random numbers to implement the rejection and acceptance algorithm, as we did in the first example.

Conclusion and results:

In order to obtain more accurate results and enjoy a margin of freedom when simulating systems that operate according to the beta distribution, which is one of the important distributions that has many uses in many fields, we presented in this research a study through which we are able to obtain neutrosophic random variables that follow this distribution, using neutrosophic rejection and acceptance method. Thanks to the indeterminacy of neutrosophic values, we are able to provide simulation results suitable for all circumstances and achieve the desired goal for decision makers.

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Artificial Intelligence and Neutrosophic Machine learning in the Diagnosis and Detection of COVID 19

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Abstract

The world has always suffered and from diseases and epidemics, and the coronavirus is one of the most dangerous viruses that threatened human life that requires the use of all scientific methods and means to respond to it and reduce its spread by early detection of infections and taking necessary measures In view of the significant role that artificial intelligence plays in most fields of science, it has become one of the most important scientific methods used to resolve complex issues and has been harnessed in medical diagnosis, one of the most complex areas. Many AI and machine learning algorithms have been used to diagnose and detect diseases in general and coronavirus in particular. The support vector machine (svm) machine algorithm was one of the most important algorithms in this area and is one of the most effective compilations used in the knowledge extraction process In spite of all this, the results they present remain incomplete because classification issues do not deal with cognitive uncertainties such as ambiguity, neutrality and inconsistency associated with perception of human thinking, This adversely affects the work of a classic support vector machine and affects the accurate diagnosis of the disease To solve this problem, we have done this research using a Neutrosophic Support Vector Machine because it takes into account all possible cases during the study of the sample and it reduces the impact of extreme values. This increases the accuracy of the results when diagnosing coronavirus symptoms. The study was conducted according to the following steps:

1. We extract features from chest radiographs based on GLCM
2. We form a neutrosophic dataset.
3. We train Neutrosophic Support Machine N-SVM on new data.
4. We record the results.

Comparing the results, we got using the upgraded N-SVM algorithm with the classic SVM algorithm results we found that it gives a more accurate diagnosis of the disease.

Keywords: Corona virus; Gray-level Co-Occurrence matrix; Neutrosophic Support Vector Machine algorithm.

Studies by the World Health Organization show that the coronavirus targets the most vulnerable groups (the elderly, chronically ill, immunocompromised, cardiac and diabetic) [1.2]. coronavirus belongs to a large family of viruses (HCoV-229E, HCoV-OC43, HCoV-NL63, HKU1-CoV) [3]. More than 7.5 million cases have been diagnosed in more than 200 countries as of 11 June 2020, including some 421,000 deaths, 3.8 million recoveries, 3.2 million mild cases and 54,000 critical cases [4.5]. The high speed of the spread of the virus, the lack of testing centers and delays in showing the result of testing make many countries in the world unable to cope with the risk of these viruses and reduce their spread and unable to count positive cases of coronavirus [6]. CXR images help in early detection of infection, but cannot be relied upon because they are similar to different images of pneumonia, viral infections, infectious lung diseases and other infections, leaving radiologists unable to distinguish between COVID-19 and other viral infections.

Misdiagnosis of the disease increases the numbers of positive cases of the COVID-19 virus accelerate [7.8]. Therefore, other methods of early detection of positive cases have to be used, and due to the great success of AI across the board in the health field, where many AI algorithms have been used in diagnosis and disease detection, researchers interested in the coronavirus pandemic have used classic AI and machine learning algorithms. Classification issues are known not to deal with cognitive uncertainties such as ambiguity, neutrality and inconsistency associated with perception of human thinking. s learning algorithms because they cannot handle non-assignments. So we need a logic that takes into account all the data and addresses cases of indefinition to get the desired results. In this research, we used the concepts of neutrosophic logic that is the new vision of modeling and designed to effectively address the uncertainties inherent in the real world. and replaced the bilateral logic recognizing right and wrong by introducing a third neutral case that could be interpreted as undefined or uncertain. Founded by American mathematic philosopher Florentin Smarandache in 1995 [9,10,11,12,13], neutrosophy logic has grown considerably in recent years and its domains have diversified through research and studies by researchers and those interested in research and scientific development [13,14,15,16,17,18,19,20,21,22,23], and has been used in the medical field, especially in virus detection through the use of neutrosophic support vector machine to detect positive cases.

The main reason why we used a neutrosophic support vector machine is to know before that the most common problems affecting the decision-making process are the problem of outliers because of its negative role reflected in the accuracy of the model. we used a neutrosophic SVM in order to reduce the impact of outliers and also because it can handle data located on it.

It takes into account all possible situations and deals with cognitive uncertainties such as in contrast to the classic support vector machine, it is weaker than the neutrosophic support vector machine algorithm in reducing the impact of outliers. It is also worth noting that the classic support vector machine algorithm cannot classify the outliers data. It is unable to handle the undetermined elements. Therefore, the classic SVM does not study all possible cases, making it unable to reach the optimal decision accurately.

Discussion: There are several studies in which the classic support vector machine algorithm has been used in diagnosis and detection of COVID-19 using chest radiographs but all of them reach high accuracy.

The large number of training samples, which increases the learning of the algorithm, but in case of a problem of a very small number of data (i.e., we did not have enough data for any reason) Ref [31] made the support vector machine algorithm learn from only 30 radiographic images also based on GLCM technology in critical features where it reached resolution 57.1. In this research we applied the neutrosophic vector machine algorithm to the same few data used in [31] and compared it in accuracy. First, we configure the data we want to work on, which are chest radiographs made up of 30 images 15 including COVID19, as well as 15 images intact, and the data format is jpg and the size of the image is 25 * 25.

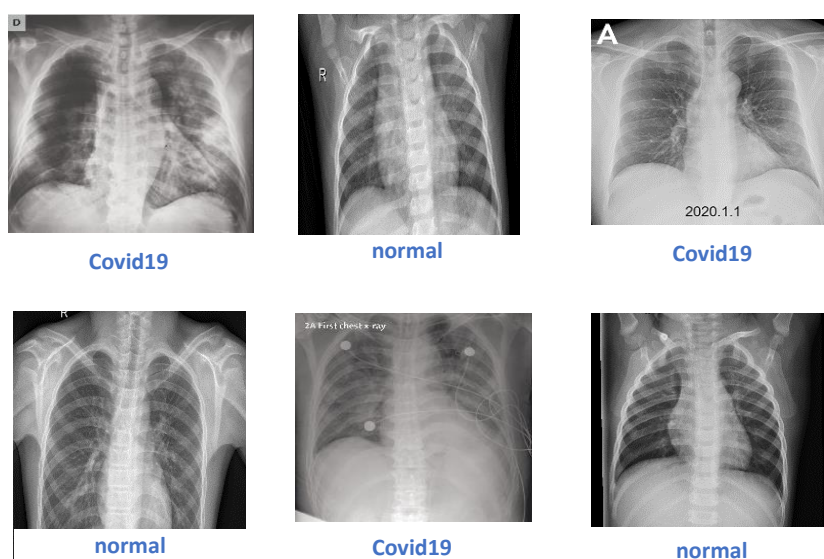| Covid19 | normal | Covid19 |
| normal | Covid19 | normal |

Figure1: Samples of two different dataset cases: patient suffering from covid-19 ,and normal healthy person

After the data is configured, the training of the model is carried out in accordance with the following stages:

1.   We enter an image of the data to be processed.

2.   We extract features based on GLCM technology.

3.   We form a neutrosophic dataset..

4.   Then we make the vector machine algorithm support neutrosophic learn from the training data.

We represent these stages with the following diagram:

Input Training images

↓

Feature extraction

By GLCM

↓

Result of GLCM

↓

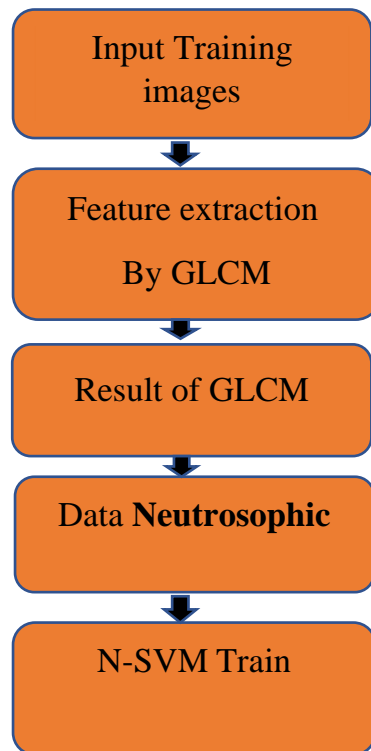Data **Neutrosophic**

↓

N-SVM Train

Figure 2: Training N-SVM Model

1.The stage of entering a picture of the data to be processed is illustrated by the following diagram:
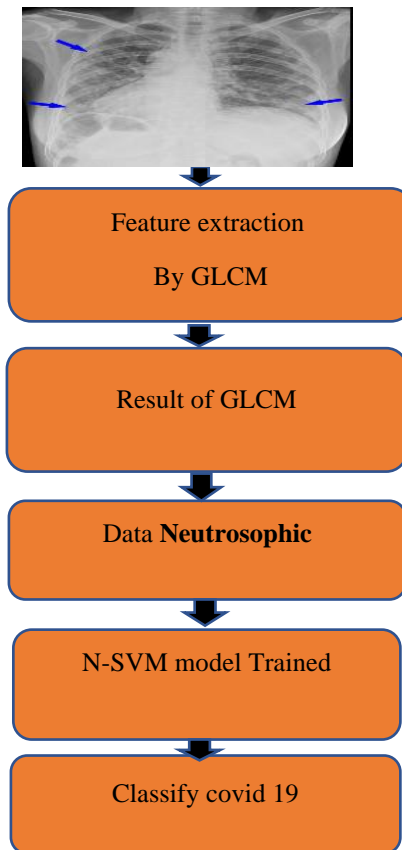
Figure 4: Classification Flow Diagram

2.Extract features from x-ray radiographs , representing energy, contrast and homogeneity we follow the next steps:[24.25.26]

- a) We enter the x-ray image that we want to process.
- b) We make the picture with a gray gradient.
  0.299R+0.587G+0.114B

c) Convert greyscale image from field 0-255 to new scale representing greyscale.

d)In this study we used the scale 0-9, i.e. we have reduced the pixel value making it easier for us to calculate GLCM.

e) we Convert the pixel value in the new scale into a matrix.

f) We compute the GLCM element that represents the appearance of gray value pairs of reference pixels, pixels and adjacent units in the distance and directions specified in the study using proximity 1 pixel and 0 direction.

g) We get the new matrix based on the account results.

h) We calculate GLCM parameters according to the relationships described in the table below:

Table1: shows the parameters of GLCM

Parameter GLCM	Arithmetic relationship
Contrast	$\sum_j \sum_i (j-i)^2 N[I,J]$
Energy	$\sum_j \sum_i N^2[I,J]$
Homogeneity	$\sum_j \sum_i \dfrac{N[I,J]}{1+[i-j]}$

N: GLCM matrix

i: the row number in N, the column number in N

So after we configured the data of 30 chest radiographs, we extracted the features from these images. Depending on parameters of Gray-level Co-Occurrence matrix represented by (Energy, Contrast, Homogeneity) As shown in Tables 2 and 3.

Table2: features extracted

N.	Energy	Cont	Hom.
1	0.0023	123.693	0.0178
2	0.0015	121.084	0.1849
3	0.0015	121.118	0.188
4	0.0016	112.858	0.1941
5	0.0015	108.897	0.1985
6	0.0027	142.454	0.1628
7	0.0025	130.193	0.1804
8	0.0019	103.057	0.1911
9	0.0019	103.710	0.2084
10	0.0017	111.213	0.1907
11	0.0015	101.739	0.2009
12	0.0018	122.323	0.1823
13	0.0021	114.802	0.1984
14	0.002	142.613	0.1669
15	0.0017	99.8898	0.1986

Table3: features extracted**.**

N.	Energy	Cont	Hom
1	0.0016	118.582	0.1881
2	0.0016	113.954	0.191
3	0.0017	118.268	0.1839
4	0.0015	125.244	0.1869
5	0.0016	113.045	0.1932
6	0.0015	113.794	0.1944
7	0.0016	124.162	0.1863
8	0.0017	112.098	0.2007
9	0.0016	100.961	0.196
10	0.0016	116.398	0.1903
11	0.0016	108.578	0.2026
12	0.0015	119.241	0.1894
13	0.0016	112.839	0.1905
14	0.0017	127.526	0.1791
15	0.0017	118.506	0.1893

3**. Formation of Neutrosophic Dataset:**

$\forall X = (x_1, x_2, x_3, \dots, x_n)^T$ set of features for binary classification data extracted using GLCM technology, but these features in this form reflect classic data and are not without cognitive uncertainties. In order to solve the problem of decisive data, we form a neutrosophic data set, which is a generalization of the classical and hazardous groups. The degree of neutrality has been introduced and added to the neutrosophic group and therefore the neutrosophic group has been defined as < T, I, F > where each element of the previous set is symbolized as [27]:

$$\forall x(t, i, f) \in < T, I, F >$$

i: represents neutrality, t: represents membership, f: represents Non-membership, t, i, f are real numbers of T, I, F respectively.

We also know that one of the disadvantages of the SVM algorithm that it is very sensitive to outliers, values, and to solve this problem we have reformulated the neutrosophic group of input samples based on the distances between the sample.

Neutrosophic group helps solve the problem of outlier values when combined with reformulated SVM.

Using the same preceding symbols that reflect data representation, we can know the input samples associated with the neutrosophic group by a set of points as follows:

$$(x_j, y_j, t_j, i_j, f_j); j = 1,2,3, \dots, n$$

$$g_j = t_j + i_j - f_j$$

For the set of points that fall on the line y = 1

$$t_j = 1 - \frac{\|x_j - \rho^+\|}{max_{x_{k \in N}}\|x_k - \rho^+\|}$$

$$i_j = 1 - \frac{\|x_j - \rho^{all}\|}{max_{x_{k \in N}}\|x_k - \rho^{all}\|}$$

$$f_j = 1 - \frac{\|x_j - \rho^-\|}{max_{x_{k \in N}}\|x_k - \rho^-\|}$$

For the set of points that fall on the line y is equal to -1.

$$t_j = 1 - \frac{\|x_j - \rho^-\|}{max_{x_{k \in N}}\|x_k - \rho^-\|}$$

$$i_j = 1 - \frac{\|x_j - \rho^{all}\|}{max_{x_{k \in N}}\|x_k - \rho^{all}\|}$$

$$f_j = 1 - \frac{\|x_j - \rho^+\|}{max_{x_{k \in N}}\|x_k - \rho^+\|}$$

where $\rho+$ represents the average of the data set on the line y = 1.

Where ρ - the average of the data set on theliney is equal to minus 1.

Where ρ^{all} represents the mean of the entire dataset.

The ρ is calculated in the form

$$\rho^{all} = \frac{1}{n}\sum_{k=1}^{n^{all}} x_k \qquad \rho^+ = \frac{1}{n^+}\sum_{k=1}^{n^+} x_k \quad \rho^+ = \frac{1}{n^+}\sum_{k=1}^{n^+} x_k$$

n^+ :the number positive s, n^- : the number negative .

We formed the neutrosophic components by calculating the g elements that represent three components of Membership degree and symbolize them with T and non-membership and symbolize them with F and neutrality and symbolize it with I, we showed the neutrosophic dataset according to the chart shown in the format fig3 where the blue line expresses Membership (T) and the orange line of Indeterminacy (I) and the gray line (Non-membership (F) as shown in figure 3.
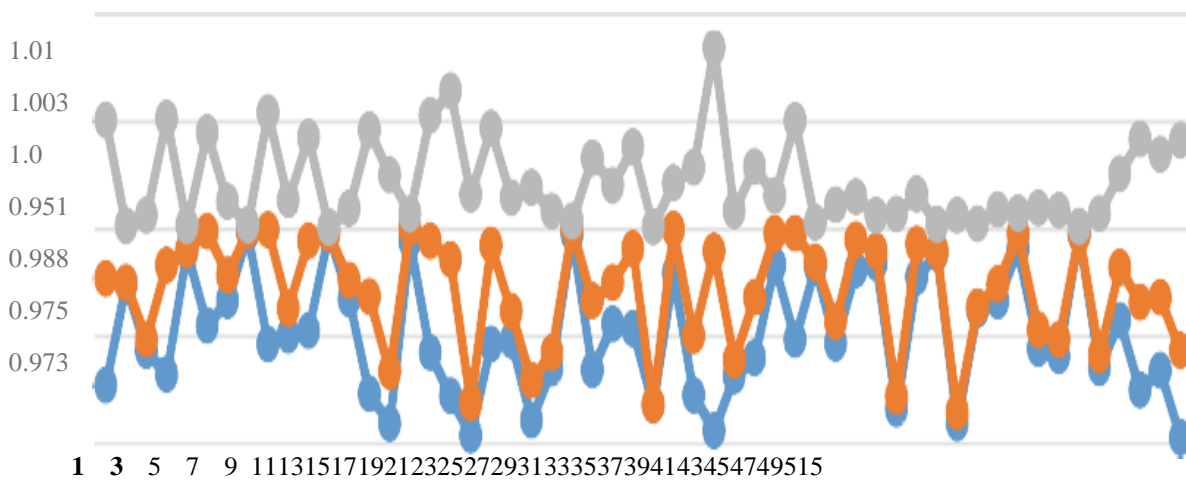

Fig3: some Neutrosophic features

4. The Machine Algorithm Vector Support Neutrosophic N-SVM:

In this research, we classified the points that are regularly distributed in the field [-1.1]. We added a neutrosophic value to determine the degree of affiliation of each entry to the intended category, and based on the fact that the training data is not linearly separable and almost not devoid of outliers, which makes the algorithm tolerate errors in classification, which requires a solution to this problem by applying the linear kernel function $\Phi : x \rightarrow \Phi(x)$ to the training data, so the set of training points became as follows [27,28,29]:

$$(\Phi(x_1), y_1, g_1), (\Phi(x_2), y_2, g_2), \dots, (\Phi(x_n), y_n, g_n)$$

We look for the best super surface that categorizes points for two categories. The problem is a nonlinear programming problem with an objective function as follows:

$$f(x) = \min \frac{1}{2}\|w\|^2 + C \sum_{i=1}^{n} g_i \,\epsilon_i \qquad \dots (1)$$

$$Subject\ to\ \ y_i(w.\Phi(x_i) + b) \geq 1 - \epsilon_i \quad ;\ \ i = 1, \dots, n$$

$$\epsilon_i \geq 0$$

Where $\Phi(x_i) = x_i x_i^t, i = 1,2,3, \dots, n$

g_i: Represents neutrosophic data and is formed in phase 3

C: a qualitative constant associated with g .

ϵ_i: mistakes committed by the decision limit through limitations

The Lagrange equation becomes:

$$L(w, b, \epsilon, \alpha, \beta) = \frac{1}{2}\|w\|^2 + C \sum_{i=1}^{n} g_i \,\epsilon_i$$

$$- \sum_{i=1}^{n} \alpha_i (y_i(w.\Phi(x_i) + b) - 1 + \epsilon_i) - \sum_{i=1}^{n} \beta_i \,\epsilon_i$$

w and b: are input arguments that the algorithm sets and any change in them affects the algorithm's output.

α, β: Lagrange factors.

To find the perfect solution we derive with respect to w,b,ϵ_i

$$\frac{\partial L(w, b, \epsilon, \alpha, \beta)}{\partial w} = w - \sum_{i=1}^{n} \alpha_i \, y_i \, \Phi(x_i) = 0 \qquad \dots (2)$$

$$\frac{\partial L(w, b, \epsilon, \alpha, \beta)}{\partial b} = - \sum_{i=1}^{n} \alpha_i y_i = 0 \qquad \dots (3)$$

$$\frac{\partial L(w, b, \epsilon, \alpha, \beta)}{\partial \epsilon_i} = g_i C - \alpha_i - \beta_i = 0 \qquad \dots (4)$$

Substituting relationships (4), (3) and (2) into relationship (1), we find the optimality problem:

$$\max W(\alpha) = \sum_{i=1}^{n} \alpha_i - \frac{1}{2} \sum_{i=1}^{n} \sum_{j=1}^{n} \alpha_i \, \alpha_j y_i \, y_j \, x_i x_i^t$$

within the terms $\qquad\qquad\qquad \sum_{i=1}^{l} y_i \alpha_i = 0$

$$0 \leq \alpha_i \leq g_i C \quad , i = 1, \dots, n \qquad \dots (5)$$

Choosing an inappropriate value for α makes the algorithm go to unsatisfactory results, , which means that we have to search for an optimal value for α, so we need the error expression ϵ, and to apply the Kuhn-Tucker conditions.

1. Error ε: the difference between the calculated output (6) and the actual output (y) in absolute terms is calculated as follows:

$$\epsilon = |(w.\Phi(x) + b) - y| \qquad \dots (6)$$

The relationship (2) shall be replaced by (6) and the following phrase shall constitute the error:

$$\epsilon_i = \left| \left(\sum_{i=1}^{n} \alpha_i \, y_i \, \Phi(x_i) . \Phi(x_i) + b \right) - y_i \right|$$

Kuhn-Tucker Terms: can be written as [30]:

$$(g \, C - \alpha) \, \epsilon = 0 \text{ :and from it } \epsilon \geq 0 \quad \text{and} \quad g \, C - \alpha \geq 0$$

Here we distinguish two situations:

Either C 0<α<g any ε=0 there is no error in classifying the support beam.

or g C-α=0 any g C=α ε>0 and there is a classification error or point located on the margin line. Where α are associated with g and C.

After we finish the algorithm training phase any new income is tested by applying the following sign function:

$$sign(w\Phi(x) + b) = \begin{cases} \text{output} \geq 0 \Rightarrow y = +1 \text{ Belonging to the first class} \\ \text{output} < 0 \Rightarrow y = -1 \text{ belonging to the second class} \end{cases}$$

Thus, any new income is classified to the appropriate output.

4. Conclusion and Results:

Initially, we extracted features using Grey-level Co-Occurrence matrix Technology because if the algorithm's income (image x-ray) is too large, it will form a surplus of data that results in the high cost of computation, processing and useless consumption of computer memory, so we converted the data into a simpler form representing the original data and this is known as feature extraction.

The advantages that we obtained are almost not devoid of outliers and vertices that fall on the decision boundary (unconfirmed values), so we used a neutrosophic support machine model in order to address the outliers and take into account the undefined values, where we trained the model on a small number of data which are 30 chest radiographs divided into 15 images for positive cases and 15 images for negative cases. We obtained a classification accuracy result of 63.8%, meaning that we obtained this accuracy after extracting the features using GLCM and training N-SVM on the extracted data. Thus, our proposed model outperforms the classic support vector machine algorithm in terms of classification accuracy as shown in Table4:

Table 4*: Comparison of accuracy between SVM and N-SVM

Accuracy	
SVM Classic [31]	57.1%
N-SVM	63.8%

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A Study of a Support Vector Machine Algorithm with an Orthogonal Legendre Kernel According to Neutrosophic logic and Inverse Lagrangian Interpolation

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Abstract

The decision-making process is greatly affected by the data collection stage. If the data collection process is not well controlled, i.e. there is some data lost due to the poor quality of the devices used or the lack of accuracy in the data entry process...etc., this will affect the work of the SVM algorithm, which is considered one of the best. Most of the workbooks suffer from the problems of missing and anomalous data. In this paper, we propose a method to treat the missing and anomalous data by reshaping the data set defined by the classical method into the neutrosophical data set by calculating the amount of true T, false F, and neutrality I in the neutrosophical set using inverse Lagrangian interpolation. We noticed the superiority of our proposed method for processing missing data over the method of [21], then we trained a support vector machine algorithm with orthogonal legender kernel on a breast cancer dataset taken from the Statistics Department of Al-Bayrouni Hospital in Damascus, where the proposed algorithm achieved a classification accuracy of 97%. The reason we chose a support vector machine classifier with an orthogonal legender kernel has two goals: the first is to eliminate the repetition of support vectors in the feature space. The second is to solve the problem of non-linear data distribution.

Keywords: Neutrosophic logic; Support Vector Machine; Orthogonal legend Kernel; Neutrosophic Group; Inverse Lagrangian Interpolation.

1. Introduction

The wide spread of information technologies led to the increase of huge data storage warehouses. The random existence of data requires finding techniques, methods and means to extract information and knowledge from it and harness it in solving issues and making decisions. Modern applications depend on making the computer simulate human thinking in solving many problems without being explicitly programmed, this is what is known as machine learning [1,2]. Many machine learning algorithms have emerged, among the most important and common of these algorithms is the support vector machine algorithm [3]. The support vector machine algorithm is one of the statistical methods that have received great attention, presented by the scientist Vladimir Vabenek in 1995, and it is a learning algorithm by a supervisor The support vector machine algorithm has successful applications in solving many complex real-world problems, including: pattern recognition, neural and medical image analysis, computer vision, robotics, computer security, text and image classification, data retrieval, bioinformatics, self-driving cars, biodiversity analysis, and the stock market [4]. One of the most important applications that made the SVM algorithm popular is handwritten digit recognition as it gave greater accuracy compared to artificial neural networks.

Based on the wide applications of the support vector machine algorithm and since the kernel is the backbone on which the support vector machine algorithm is based, we find many research that presented studies to develop

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this algorithm and solve its problems. [5] presented a new kernel for the SVM algorithm by assembling the Chebchev kernel with the Gaussian kernel to classify regular data on the range [-1,1]. [6] improved the SVM algorithm by merging the Chebyshev kernel with the Hermite kernel, as it gave better results in terms of fewer support rays and faster classification speed. [7] presented a support vector machine with an orthogonal legender kernel and demonstrated its significant role in avoiding duplication of features in space Features, however, the presence of neutral elements affecting decision-making was considered an open issue until the scientist Florentin Samarandakeh presented the nitrosophic logic as a generalization of the fuzzy logic by adding a new component to the degrees of organic and inorganic, which is the degree of indeterminacy [8,9,10,11,12,13,14, 15,16,17,18,19,20] and accordingly [21] studied the support vector machine according to the nitrosophic logic. Although the support vector machine algorithm was studied according to the nitrosophic logic, the study was limited to linearly separable data. Missing values were also processed depending on the average in the formation of neutrosophic groups, but the obtained values were far from their neighbors in the same column.

2. Discussion

In [21] the support vector machine was studied according to the nitrosophic logic. However, this study of the support vector machine algorithm according to the nitrosophic logic was limited to linearly separable data. Missing values were also processed depending on the average in the formation of nitrosophic groups, and the obtained values were far from their neighbors in the same column.

In this paper, we study the support vector machine algorithm according to the neutrosophic logic in the case of non-linear separation of data using the orthogonal legender kernel. We presented a method to treat the missing values based on the inverse Lagrangian interpolation in the formation of neutrosophic groups, as we applied the proposed method to data used in [21]. The results we obtained were as shown in the following table:

Table 1: Finding the missing values based on the inverse interpolation of Lagrangian and neutrosophic groups

Africa	Beach	Bulildeing	Bus	Dinosaur	Elephant	Flower	Horese	Mountain	Food
0.029	0.008	0.036	0.016	0.008	0.056	0.004	0.016	0.024	0.020
0.028	0.024	0.024	0.016	0.008	0.020	0.008	0.012	0.012	0.008
0.036	0.040	0.012	0.008	0.004	0.016	0.012	0.008	0.016	0.024
0.004	0.008	0.000	0.004	0.000	0.000	0.000	0.000	0.004	0.004
0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.004
0.024	0.004	0.008	0.004	0.000	0.002	0.000	0.012	0.036	0.032
0.008	0.004	0.000	0.008	0.000	0.004	0.000	0.008	0.008	0.024
0.008	0.008	0.000	0.000	0.000	0.008	0.000	0.008	0.004	0.008
0.008	0.148	0.032	0.016	0.004	0.040	0.004	0.008	0.006	0.004
0.032	0.016	0.008	0.012	0.008	0.020	0.012	0.008	0.008	0.004

We conclude that the missing values that we obtained are better than the results of the method used in [21], because each missing value that we obtained, shown in yellow above, was very close to its neighbors in the same column, unlike the missing values that were created by [21], as it was very far from the values of We formulated a support vector machine algorithm with an orthogonal Legender kernel according to neutrosophic logic and inverse Lagrange interpolation. We applied the developed algorithm to breast cancer data that we obtained from the Statistics Department of Al-Bayrouni Hospital in Damascus, which are shown as follows.:

```
  id diagnosis radius_mean texture_mean perimeter_mean area_mean \
0    842302      M      17.99       10.38        122.80    1001.0
1    842517      M       NaN        17.77        132.90    1326.0
2   84300903     M      19.69       21.25        130.00    1203.0
3   84348301     M      11.42       20.38         77.58     386.1
4   84358402     M      20.29       14.34        135.10    1297.0
..    ...       ...      ...         ...          ...        ...
564  926424      M      21.56       22.39        142.00    1479.0
565  926682      M       NaN        28.25        131.20    1261.0
566  926954      M      16.60       28.08        108.30     858.1
567  927241      M      20.60       29.33        140.10    1265.0
568  92751       B       7.76       24.54         47.92     181.0
```

```
     smoothness_mean compactness_mean concavity_mean concave points_mean \
0        0.11840         0.27760         0.30010         0.14710
1        0.08474         0.07864         0.08690         0.07017
2        0.10960         0.15990         0.19740         0.12790
3         NaN            0.28390         0.24140         0.10520
4        0.10030         0.13280         0.19800         0.10430
..        ...             ...             ...             ...
564      0.11100         0.11590         0.24390         0.13890
565      0.09780         0.10340         0.14400         0.09791
566      0.08455         0.10230         0.09251         0.05302
567      0.11780         0.27700         0.35140         0.15200
568      0.05263         0.04362         0.00000         0.00000
```

```
    ... texture_worst perimeter_worst area_worst smoothness_worst \
0   ...     17.33        184.60        2019.0       0.16220
1   ...     23.41        158.80        1956.0       0.12380
2   ...     25.53        152.50        1709.0       0.14440
3   ...     26.50         98.87         567.7       0.20980
4   ...     16.67        152.20        1575.0       0.13740
..  ...      ...           ...           ...          ...
564 ...     26.40        166.10        2027.0       0.14100
565 ...     38.25        155.00        1731.0       0.11660
566 ...     34.12        126.70        1124.0       0.11390
567 ...     39.42        184.60        1821.0       0.16500
568 ...     30.37         59.16         268.6       0.08996
```

```
    compactness_worst concavity_worst concave points_worst symmetry_worst \
0       0.66560          0.7119           0.2654             0.4601
1       0.18660          0.2416           0.1860             0.2750
2       0.42450          0.4504           0.2430             0.3613
3       0.86630          0.6869           0.2575             0.6638
4       0.20500          0.4000           0.1625             0.2364
..       ...              ...              ...                ...
564     0.21130          0.4107           0.2216             0.2060
565     0.19220          0.3215           0.1628             0.2572
566     0.30940          0.3403           0.1418             0.2218
567     0.86810          0.9387           0.2650             0.4087
568     0.06444          0.0000           0.0000             0.2871
```

```
    fractal_dimension_worst Unnamed: 32
0           0.11890             NaN
```

1	0.08902	NaN
2	0.08758	NaN
3	0.17300	NaN
4	0.07678	NaN
..	...	...
567	0.12400	NaN
568	0.07039	NaN

We note that the previous data contains missing values shown above in red color NaN, we processed this data by inverse Lagrangian interpolation and then trained a support vector machine algorithm with orthogonal Legender kernels on this data and got better results than the traditional algorithm. The results are represented in the following criteria (precision , recall, f1-score, accuracy)

3. Performance Measures

i. precision

Precision represents the ratio of the relationship of well-expected positive cases to overall expected positive cases. [22]

ii. recall

This is the magnitude relation of the properly expected positive notes to any or all operations within the actual class i.e. true positive rate. The confusion matrix is employed to assess sensitivity and is mathematically assessed as [23]

iii. f1-score

The F1 score is a comprehensive calculation of model accuracy that blends recall and accuracy[22].

iv. Accuracy

It is a parameter that tests the method's capacity by accurately calculating the proportion of cases predicted from all cases. ACC is mathematically expressed as follows [24]

Accuracy	f1-score	Recall	Precision
$\dfrac{TP + TN}{ALL}$	$2 * \dfrac{precision * Recall}{precision + Recall}$	$\dfrac{TP}{TP + FN}$	$\dfrac{TP}{TP + PF}$

Four major components are used to generate the confusion matrix

TP: It stands for values that are classified correctly and belong to positive values.

TN: It stands for values that are classified as true and belong to negative values.

FP: It stands for values that are incorrectly classified and belong to positive values.

Figure 1:Confusion matrix

FN: It stands for values that are incorrectly classified and belong to negative values.

2-Legendre polynomials

2-1 Orthogonal boundaries [25]

The set of functions $g_0, g_1, \dots, g_n$ is orthogonal to the domain [a,b] ; a,b∈R with respect to the weight function w(x) if the following is true

$$\int_a^b h(x).g_i(x).g_j(x)\,dx = \{\ 0\ ; i \neq j\ \ Number\ ; i = j\ \ \ ; i,j \in \{1,\dots,n\}$$

2-2 The weight function [25]

The function h(x) is a weight function on [a,b] if and only if it satisfies the following**:**

h(x) is continuous -1

$h(x) > 0$ -2

$\forall n \in N\ ; x \in\,]a,b[\ : \int_a^b h(x)\,|x^n|\,dx < +\infty$ -3

2-3 Basic properties of Legendre polynomials[25]

$P_n(1) = 1$

$P_n(-1) = (-1)^n$

$P_n(0) = \{0 \qquad if\ \ n\ is\ odd\ \ \frac{(-1)^m (2m)!}{2^{2m}(m!)^2}\ \ if\ \ n\ is\ even$

2-4 The Kernel [26]

It is a function that measures the distance between the training sample, that is, between the input points, and this kernel can deal with nonlinearly issues.

Table 2: The most important drugs used in data processing

Function	**Kernel**
$K(x,y) = exp^{\left(-\frac{\|x-y\|}{\delta}\right)}$	Laplacian
$K(x,y) = 1 - \dfrac{\|x-y\|^2}{\|x-y\|^2 + c}$	Rational Quadratic
$K(x,y) = \sqrt{\|x-y\|^2 - c}$	Multiduadratic
$K(x,y) = \dfrac{\theta}{\|x-y\|} \sin \sin \dfrac{\|x-y\|}{\theta}$	Wave
$K(x,y) = -\|x-y\|^d$	Power
$K(x,y) = \dfrac{J_{V+1}(\delta\|x-y\|)}{\|x-y\|^{-n(V+1)}}$	Bessel
$K(x,y) = \dfrac{1}{1 + \dfrac{\|x-y\|^d}{d}}$	Cauchy
$K(x,y) = \displaystyle\prod_{m=1}^{N} h\left(\dfrac{x_i - c}{a}\right) h\left(\dfrac{y_i - c}{a}\right)$	Wavelet

2-5 orthogonal legendar kernel [25]

The Legendar kernel is a series of Legendar bounds orthogonal to the domain [-1,+1] with weight function $h(x) = 1$, each of which can be obtained in the form:

The following iterative relation connecting the Legendar bounds is obtained:

$$P_{n+1}(x) = \frac{2n+1}{n+1} xP_n(x) - \frac{n}{n+1} P_{n-1}(x) \; ; n \geq 1 \qquad \ldots (1)$$
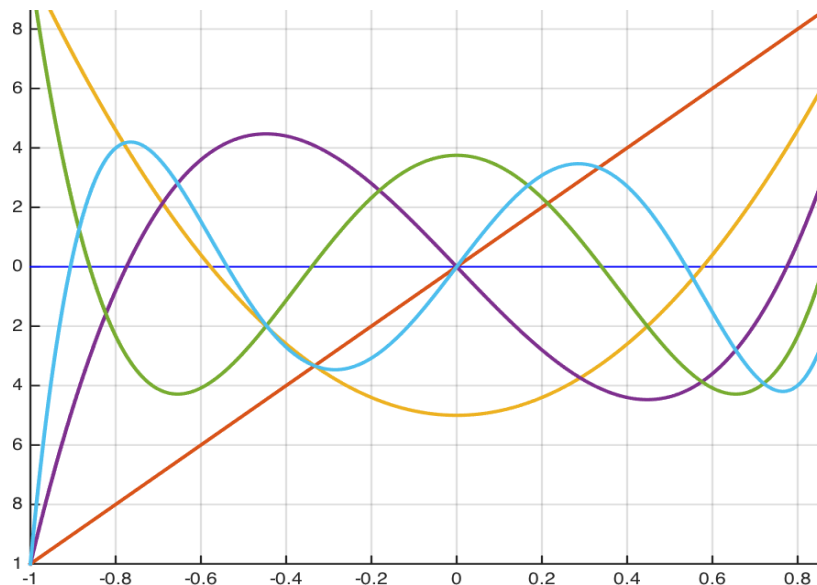
$$P_0(x) = 1$$

$$P_1(x) = x$$

Figure 2: Legendar polynomials.

4.   Inverse Lagrangian interpolation [31,32]

Inverse interpolation is to find the value of s corresponding to a given value of y of a given function in tabular form as well.

We know that the interpolation limits in Newton's direct method are given in terms of progressive differences in the form

$$y = y_0 + q\Delta y_0 + \frac{q(q-1)}{2!}\Delta^2 y_0 + \frac{q(q-1)(q-2)}{3!}\Delta^3 y_0 + \cdots + \frac{q(q-1)(q-2)\dots(q-n+1)}{n!}\Delta^n y_0$$

Where

$$q = \frac{x - x_0}{h}$$

We calculate q from this relationship, so we will adopt the method of successive approximations by writing the relationship in the form:

$$q = \varphi(q)$$

Then write the successive approximations of the relationship

$$q_m = \varphi(q_{m-1})$$

So we solve the equation with respect to q we find

$$q = \varphi(q) = \frac{1}{\Delta y_0}\left[y - y_0 - \frac{q_{m-1}(q_{m-1}-1)}{2!}\Delta^2 y_0 - \frac{q(q-1)\dots(q-n+1)}{n!}\Delta^n y_0\right]$$

We take q_0 as an initial approximation to the figure

$$q_0 = \frac{y - y_0}{\Delta y_0}$$

And after finding the q_s that achieve the required accuracy, which makes $|q_s - q_s - 1| < \varepsilon$

Where ε is the required accuracy, we adopt $q = q_s$, then we find s from the relationship

$$s = x_0 + qh \; ; h = x_{i+1} - x_i$$

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5.   Formation of the neutrosophic data set

Let $X = (x_1, x_2, x_3, \dots, x_n)^T$ be the set of features for binary classification data, since when the data falls on the decision limit it is difficult for us to classify them, and in order to solve the problem of the data located on the decision limit we form the nitrosophic data set g that It is a generalization of the classical and fuzzy groups. The degree of neutrality was introduced and added to the nitrosophic group. Accordingly, the nitrosophic group was defined as $< T, I, F >$, as each element of the previous group is denoted by the form [21]:

$$\forall x(t, i, f) \in < T, I, F >$$

i: represents neutrality, t: represents the degree of organicity, f: represents the degree of inorganicity, t,i,f are real numbers of T,I,F, respectively.

As we know that one of the disadvantages of the SVM algorithm is that it is very sensitive to outliers, and to solve this problem, we reformulated the nitrosophic group for the input samples   based on the distances between the samples.

The neutrosophic group helps solve the problem of outliers when combined with the reformatted SVM.

And using the same previous symbols that express the data representation, we can know the input samples associated with the nitrosophic group with a set of points as follows:

$$(x_j, y_j, t_j, i_j, f_j); j = 1,2,3, \dots, n$$

$$g_j = t_j + i_j - f_j$$

For the set of points on the line $y = 1$

$$t_j = 1 - \frac{\|x_j - s^+\|}{max_{x_{k \in N}} \|x_k - s^+\|}$$

$$i_j = 1 - \frac{\|x_j - s^{all}\|}{max_{x_{k \in N}} \|x_k - s^{all}\|}$$

$$f_j = 1 - \frac{\|x_j - s^-\|}{max_{x_{k \in N}} \|x_k - s^-\|}$$

For the set of points on the line y = -1

$$t_j = 1 - \frac{\|x_j - s^-\|}{max_{x_{k \in N}} \|x_k - s^-\|}$$

$$i_j = 1 - \frac{\|x_j - s^{all}\|}{max_{x_{k \in N}} \|x_k - s^{all}\|}$$

$$f_j = 1 - \frac{\|x_j - s^+\|}{max_{x_{k \in N}} \|x_k - s^+\|}$$

where s^+ is the inverse Lagrangian interpolation of the data set on the line $y = 1$

Where s^- is the inverse Lagrangian interpolation of the data set on the line $y = -1$

where s^{all} is the inverse Lagrangian interpolation of the whole data set.

s is calculated as in stage 2.

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6. Support vector machine algorithm with orthogonal legend kernel according to the nitrosophilic group

In this research, we categorized the points that are distributed uniformly over the range [-1,1] using the legendar kernel function orthogonal to the input. We were able to achieve more accurate results with fewer support beams. By adding a neutrosophilic value to determine the degree of belonging of each entry to the desired variety, the set of training points became as follows [21,27,28,29,30]

$$(\Phi(x_1), y_1, g_1), (\Phi(x_2), y_2, g_2), \dots, (\Phi(x_n), y_n, g_n)$$

We are looking for the best super surface that classifies points into two classes. Adding the neutrosophilic value g, then the problem becomes:

$$\frac{1}{2}\|w\|^2 + C \sum_{i=1}^{n} g_i \, \epsilon_i$$

where C is a qualitative constant associated with g.

And the Lagrangian equation becomes:

$$L(w, b, \epsilon, \lambda, \beta) = \frac{1}{2}\|w\|^2 + C \sum_{i=1}^{n} g_i \, \epsilon_i - \sum_{i=1}^{n} \lambda_i(y_i(w.\Phi(x_i) + b) - 1 + \epsilon_i) - \sum_{i=1}^{n} \beta_i \, \epsilon_i$$

To search for the optimal solution, we derive with respect to w, b, ϵ_i

$$\frac{\partial L(w, b, \epsilon, \lambda, \beta)}{\partial w} = w - \sum_{i=1}^{n} \lambda_i \, y_i \, \Phi(x_i) = 0$$

$$\frac{\partial L(w, b, \epsilon, \lambda, \beta)}{\partial b} = -\sum_{i=1}^{n} \lambda_i y_i = 0$$

$$\frac{\partial L(w, b, \epsilon, \lambda, \beta)}{\partial \epsilon_i} = g_i C - \lambda_i - \beta_i = 0$$

We find the question of optimization:

$$W(\lambda) = \sum_{i=1}^{n} \lambda_i - \frac{1}{2}\sum_{i=1}^{n} \sum_{j=1}^{n} \lambda_i \, \lambda_j y_i \, y_j \, K(x_i, x_j)$$

within the terms

$$\sum_{i=1}^{l} y_i \lambda_i = 0$$

$$0 \le \lambda_i \le g_i C \quad, i = 1, \dots, n$$

$$K(x, z) = \sum_{i=0}^{n} P_i(x).P_i(z)$$

After we finish the algorithm training phase, any new input is tested by applying the following sign function:

$$sign(w\Phi(x) + b) = \{The\ resulting \ge 0 \Rightarrow y = +1 \qquad The\ resulting < 0 \Rightarrow y = -1$$

Thus, any new input is classified into its appropriate output.

7. Conclusion and results:

In this research, we presented a study to classify breast cancer by obtaining study data from Al-Bayrouni Hospital. The data we used was distorted and contained abnormal and missing values.We corrected and processed the data through nitrosophic logic and inverse Lagrangian interpolation, and then we formulated a support vector machine algorithm with two kernels. Legender orthogonal according to the nitrosophic logic, and we trained it on the data, so the result was that our proposed algorithm is superior to the classical support vector machine algorithm. We were able to correlate the inputs of this algorithm, which is calculated based on the

inverse Lagrangian interpolation, in order to determine the degree of belonging to each class. We got the results as follow:

	precision	recall	f1-score	support
0	0.97	0.99	0.98	72
1	0.98	0.95	0.96	42
accuracy			0.97	114
macro avg	0.97	0.97	0.97	114
weighted avg	0.97	0.97	0.97	114

Confusion matrix:

```
[[71  1]
 [ 2 40]]
```
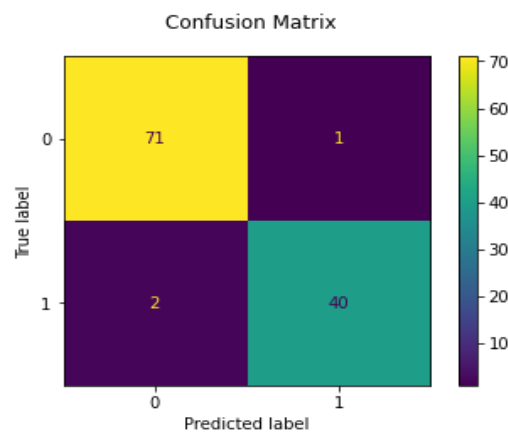

Figure 3: Confusion matrix with the predicted label

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NEUTROSOPHIC LINEAR MODELS AND ALGORITHMS TO FIND THEIR OPTIMAL SOLUTION

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