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Axiomatics and problematics as two modes of formalisation: Deleuze's epistemology of mathematics*

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1. Introduction: problematics, dialectics, and ideas

Throughout his work, Gilles Deleuze has developed a distinction between two modes of formalisation, in mathematics and elsewhere, which he terms, respectively, 'axiomatics' and 'problematics.'[1] The axiomatic (or 'theorematic') method of formalisation is a familiar one, already having a long history in mathematics, philosophy, and logic, from Euclid's geometry to Spinoza's philosophy to the formalised systems of modern symbolic logic. Although problematics has had an equally determinate trajectory in the history of mathematics, it is a more subterranean and less visible trajectory, but one that has increasingly become the object of study in contemporary philosophy of mathematics. Deleuze argues that the recognition of the irreducibility of problems and their genetic role in mathematics is 'one of the most original characteristics of modern epistemology,' as exemplified in the otherwise diverse work of thinkers such as Canguilhem, Bouligand, Vuillemin, and Lautman.[2]

Deleuze's contribution to these debates has been to take up the mathematical concept of problematics and to give it an unparalleled status in philosophy. The fundamental difference between these two modes of formalisation can be seen in their differing methods of deduction: in axiomatics, a deduction moves from axioms to the theorems that are derived from it, whereas in problematics a deduction moves from the problem to the ideal accidents and events that condition the problem and form the cases that resolve it. More generally, Deleuze characterises axiomatics as belonging to a 'major' or royal form of science, which constantly attempts to effect a reduction or repression (or more accurately, an arithmetic conversion) of the problematic pole of mathematics, itself wedded to a 'minor' or nomadic conception of science. 'What we have are two formally different conceptions of science, and ontologically *a single field of interaction* in which royal science [e.g., axiomatics] continually appropriates the contents of vague or nomad science [problematics], while nomad science continually cuts the contents of royal science loose' (Deleuze and Guattari 1987, 362, 367. Emphasis added). Many of the most important concepts in Deleuze's own philosophy – such as multiplicity, the differential, singularity, series, zones of indiscernibility, and so on – were adopted from this problematic pole of mathematics, and particularly from the history of the calculus. My primary purpose in this essay will be to elucidate the epistemological differences between problematics and axiomatics.

We should note, however, that Deleuze's interest in the mathematics of problematics is not purely epistemological, but stems from his more general interest in the status of problems in philosophy. The activity of thinking has often been conceived of as the search for solutions to problems, but this is a prejudice whose roots, Deleuze suggests, are both social and pedagogical: in the classroom, it is the mathematics teacher who poses the problems, the pupil's task being to discover the correct solution. What the notions of 'true' and 'false' serve to qualify are precisely the responses or solutions that are given to these already-formulated questions or problems. Yet at the same time, everyone recognises that problems are never given ready-made, but must be constructed or constituted – hence the scandal when a 'false' or badly-formulated problem is set in an examination. 'While it is relatively easy to define the true and the false in relation to solutions whose problems are already stated,' Deleuze writes in *Bergsonism*, '*it is much more difficult to say what the true and false consist of when they are applied directly to problems themselves*' (1988, 16–17). In fact, philosophy is concerned less with the solution to eternal problems than the constitution of problems themselves, and the means for distinguishing between legitimate and illegitimate problems, that is, between true and false problems.

In the history of philosophy, the science of problems has always had a precise name: *dialectics*. In Plato's dialectic, for instance, the appeal to a foundational realm of essence (Ideas) first appeared as the response to a particular way of posing problems, a particular form of the question – namely the question 'What is. .?' [*ti estin*?]. 'The idea,' writes Deleuze, 'the discovery of the Idea, is not separable from a certain type of question. The Idea is first of all an 'objectity' [*objectité*] that corresponds, as such, to a way of posing questions' (Deleuze 2004, 95. Translation modified). The question 'What is. .?' thus presupposes a particular way of thinking that points one in the direction of essence: for Plato, it was *the* question of essence, the *only* question capable of discovering the Idea. Aristotle in turn defined dialectics as the art of posing problems as the subject of a syllogism, while analytics gives us the means of resolving the problem by leading the syllogism to its necessary conclusion. Deleuze's own dialectic, however, is indexed primarily on Kant's 'Transcendental Dialectic' in the *Critique of Pure Reason*. Against Plato, Kant attempted to provide a purely *immanent* conception of Ideas that exposed the illusion of assigning to Ideas a transcendent object (such as the Soul, the World, or God). If the Ideas of reason give rise to illusion and lead us into false problems, Kant argued, this is first of all because reason is the faculty of posing problems in general: the object of the Idea, since it lies outside of experience, can neither be given nor known, but must be represented in a *problematic* form, without being determined. But this does not mean that Ideas have no real object; more profoundly, it means that *problems as problems are the real objects of Ideas*.[3]

This is the source of the link one finds in Deleuze's work between dialectics, problematics, and Ideas: dialectics is the science of problems, but problems themselves are Ideas. Deleuze has often been characterised (wrongly) as an anti-dialectical thinker, but it would be more accurate to say that *Difference and Repetition* (especially its fifth chapter, 'Ideas and the Synthesis of Difference') is a book that proposes *a new concept of dialectics*, one that both is indebted to but breaks with the work of the great dialectical thinkers such as Plato, Kant, and Hegel. This is where Deleuze's interest in mathematical problematics intervenes: it provides him with a model for this new conception of dialectics. If Plato found his model in Euclidian geometry, and contemporary philosophers tend to turn toward set theory and axiomatics, Deleuze has found his model for dialectical Ideas in problematics and the history of the calculus. The discussion that follows focuses primarily on the mathematical origins of Deleuze's conception of dialectics, and the problematic/axiomatic distinction that lies at its core. It will examine, in turn, the historical background of Deleuze's notion of problematics, the precise nature of the relation between axiomatics and problematics, and finally, the means by which Deleuze has attempted to provide a *formalisation* of problematics in his theory of multiplicities.

2. Problematics versus axiomatics: historical background

Although Deleuze formulates the problematic-axiomatic distinction in his own manner, it in fact reflects a fairly familiar tension within the history of mathematics, which can be illustrated by means of three historical examples.

1. The first example comes from the Greeks. Proclus, in his *Commentary of the First Book of Euclid's Elements*, had already formulated a distinction, within Greek geometry, between problems and theorems.[4] Theorems concern the demonstration, from axioms or postulates, of the inherent properties belonging to a figure, whereas problems concern the actual construction of figures, usually using a straightedge and compass. From this viewpoint, determining a triangle the sum of whose angles is 180 degrees is theorematic, since the angles of every triangle will total 180 degrees. By contrast, constructing an equilateral triangle on a given finite straight line is problematic, since we could also construct a non-equilateral triangle or a non-triangular figure on the line (moreover, the construction of an equilateral triangle must first pass through the construction of two circles). Classical geometers struggled for centuries with the three great unresolved 'problems' of antiquity – trisecting an angle, constructing a square equal to a circle, and constructing a cube having double the volume of a given cube – although only in 1882 was it proved (theorematically) that none of these problems was solvable using only a straightedge and compass.[5]

But this is why theorematics and problematics involve two different conceptions of deduction: if in theorematics a deduction moves from axioms to theorems, in problematics a deduction moves from the problem to the ideal *events* that condition it and form the *cases* of solution that resolve it. In theorematics, for instance, a figure is defined statically, in Platonic fashion, in terms of its essence and its derived properties: Euclidean geometry defines the essence of the line in purely static terms that eliminate any reference to the curvilinear ('a line which lies evenly with the points on itself').[6] Problematics, by contrast, found its classical expression in the 'operative' geometry of Archimedes, in which the straight line is characterised dynamically as 'the shortest distance between two points.' Here, the problem (How to construct a line between two points?), with its determinate conditions, has an infinite set of possible solutions (curves, loops, etc.), and the straight line is simply the case that constitutes the 'shortest' solution. Similarly, in the theory of conic sections, the ellipse, hyperbola, parabola, straight lines, and the point are all 'cases' of the projection of a circle onto secant planes in relation to the apex of a cone. If Archimedean geometry (especially the Archimedes of 'On the Method') can be said to be an operative geometry, it is because it defines the line less as an essence than as a continuous operation or *process* of 'alignment', the circle as a continuous process of 'rounding' the square as the process of 'quadrature', and so on. In problematics, a figure is defined dynamically by its *capacity to be affected* – that is, by the ideal accidents and events that can befall the figure (sectioning, cutting, projecting, folding, bending, stretching, reflecting, rotating, and so on). As a theorematic figure, a circle may indeed be an organic and fixed essence, but the morphological variations of the circle (figures that are 'lens-shaped', 'umbelliform' 'indented', etc.) form problematic figures that are, in Husserl's words, 'vague yet rigorous' 'essentially and not accidentally inexact.'[7]

Greek thought nonetheless set a precedent that would be followed by later mathematicians and philosophers: Proclus had already pointed to (and defended) the relative triumph, in Greek geometry, of the theorematic over the problematic. The reason: to the Greeks, 'problems concern only events and affects which show evidence of a *deterioration* or a projection of essences in the imagination,' and theorematics could thus present itself as a necessary 'rectification' of thought.[8] This 'rectification' must be understood, in a literal sense, as a triumph of the rectilinear over the curvilinear. In the 'minor' geometry of problematics, figures are inseparable from their inherent variations, affections, and events (the straight line being a simple case of the curve). The explicit aim of 'major' theorematics is 'to uproot variables from their state of continuous variation in order to extract from them fixed points and constant relations,' thereby setting geometry on the 'royal' road of theorematic deduction and proof (Deleuze and Guattari 1987, 408–409).

2. For our second example, we jump ahead two millennia. By the seventeenth-century, the tension between problems and theorems, which was internal to Greek geometry, had shifted to a more general tension between geometry itself, on the one hand, and algebra and arithmetic on the other. Desargues' *projective geometry*, for instance, which was a qualitative and 'minor' geometry centred on problems-events (as developed, most famously, in Desargues' *Draft Project of an Attempt to Treat the Events of the Encounters of a Cone and a Plane*), was quickly opposed in favour of the *analytic geometry* of Fermat and Descartes – a quantitative and 'major' geometry that translated geometric relations

into arithmetic relations that could be expressed in algebraic equations (Cartesian coordinates) (Boyer 1968, 393). 'Royal' science, in other words, now entailed an *arithmetisation* of geometry itself. 'There is a correlation,' Deleuze writes, 'between geometry and arithmetic, geometry and algebra that is constitutive of major science.'[9] Descartes was dismayed when he heard that Desargues' *Draft Project* treated conic sections without the use of algebra, since to him 'it did not seem possible to say anything about conics that could not more easily be expressed with algebra than without.'[10] As a result, Desargues' methods were repudiated as dangerous and unsound, and his practices of perspective banned. Theorematics (in the form of algebra) once again triumphed, and brought about an arithmetic conversion of a problematic field.

This triumph of theorematics can be said to have reached its greatest philosophical expression in Spinoza's *Ethics*, which assumes a purely theorematic or axiomatic form of argumentation and deduction. 'In Spinoza,' Deleuze complains, '*the use of the geometric method involves no 'problems' at all*' (1994, 323, n. 21). Indeed, with regard to problematics, Deleuze suggests that in fact Descartes actually went further than Spinoza, and that Descartes the geometer went further than Descartes the philosopher. The 'Cartesian method' (the search for the clear and distinct) is a method for solving problems, but the analytic procedure that Descartes presents in his *Geometry* is focused on the constitution of problems as such ('Cartesian coordinates' appear nowhere in the *Geometry*).[11] The *Geometry* does not move from axioms to theorems, but rather starts with a problem and 'analyses' it to find a solution. 'With the [analytic] method I use,' Descartes wrote, 'everything falling under the geometers consideration can be reduced to as single class of *problem* namely, that of looking for the value of the roots of a certain equation. Nonetheless, one of the most significant innovations of Deleuze's reading of Spinoza is to have presented a *problematic* reading of the *Ethics*, which operates alongside and within Spinoza's explicit demonstrative apparatus. Rather than beginning with the axioms and following Spinoza's theorematic deductions, Deleuze starts his analysis 'in the middle', that is, with the problematic composition of finite modes and the *affections* that befall them, and undertakes a problematic deduction of the concept. Human modes of existence have affections just as geometrical figures. 'The relation between mathematics and humanity,' Deleuze writes in *Logic of Sense*, 'may thus be conceived in a new way: the question is not that of quantifying or measuring human properties, but rather, on the one hand, that of problematising human events and, on the other, that of developing as various human events the conditions of a problem' (p. 55). Spinoza's work is thus susceptible to two kinds of reading: a *conceptual* (theorematic) reading and an *affective* (problematic) reading. This is why, in his analysis of the *Ethics*, Deleuze consistently emphasised the role of the scholia (which are the only elements of the *Ethics* that fall outside the axiomatic deductions, and develop the theme of 'affections') and the fifth book (which introduces problematic hiatuses and contractions into the deductive exposition itself).[13] Pierre Macherey has complained that Deleuze, in approaching Spinoza's thought in such a manner, is attempting to introduce a new version of Spinozism that is at variance, if not completely at odds, with the model of 'demonstrative rationality' explicitly adopted by Spinoza himself.[14] But it should be clear that Deleuze's approach to Spinoza is itself a 'case' of his broader approach to philosophy from the viewpoint of problematics. 'The whole problem of reason,' Deleuze has suggested elsewhere, 'will be converted by Spinoza into a special case of the more general problem of the affects' (Deleuze 1980c).

The attempt to 'arithmetise' geometry would continue well into the nineteenth-century, when Desargues' projective geometry was revived in the work of Monge, the inventor of descriptive geometry, and Poncelet, who formulated the 'principle of continuity,' which led to developments in *analysis situs* and topology. Topology (so-called 'rubber-band geometry') was initially a problematic science that concerned the property of geometric figures that remain invariant under transformations such as bending or stretching. Under such transformations, figures that are theorematically distinct in Euclidean geometry – such as a triangle, a square, or a circle – can be seen as one and the same 'homeomorphic' figure, since they can be continuously transformed into one another. This entailed an extension of geometric 'intuitions' far beyond the limits of empirical or sensible perception (à la Kant). 'With Monge, and especially Poncelet,' writes Deleuze, commenting on Léon Brunschvicg's work, 'the limits of sensible, or even spatial, representation (striated space) are indeed surpassed, but less in the direction of a symbolic power of abstraction [i.e., theorematics] than toward a trans-spatial imagination, or a trans-intuition (continuity).'[15] In the twentieth-century, computers have extended the reach of this 'trans-intuition' even further, provoking renewed interest in qualitative geometry, and allowing mathematicians to 'see' hitherto unimagined objects such as the Mandelbrot set and the Lorenz attractor, which have become the poster children of the new sciences of chaos and complexity. 'Seeing, seeing what happens,' continues Deleuze, 'has always had an essential importance, greater than demonstrations, even in pure mathematics, which can be called visual, figural, independently of its applications: many mathematicians nowadays think that a computer is more precious than an axiomatic' (Deleuze and Guattari 1994, 128. Translation modified). But already in the early nineteenth-century, there was a renewed attempt to turn projective geometry into a mere practical dependency on analysis, or so-called higher geometry (the debate between Poncelet and Cauchy).[16] The development of the theory of functions would eventually eliminate the appeal to the principle of continuity, substituting for the geometrical idea of smoothness of variation the arithmetic idea of 'mapping' or a one-to-one correspondence of points (point-set topology). Theorematics would once again triumph over problematics.

3. Finally, this double movement of major science toward theorematisation and arithmetisation would reach its full flowering in the late nineteenth-century, primarily in response to problems posed by the invention of the *calculus*. In its origins, the calculus was tied to problematics in a double sense. The first refers to the problems that the calculus confronted: the differential calculus addressed the problematic of *tangents* (how to determine the tangent lines to a given curve), while the integral calculus addressed the problematic of *quadrature* (how to determine the area within a given curve). The greatness of Leibniz and Newton was to have recognised the intimate connection between these two problematics (the problem of finding areas is the inverse of determining tangents to curves), and to have developed a symbolism to link them together and resolve them. The calculus quickly became the primary mathematical engine of what we call the 'scientific revolution' Yet for two centuries, the calculus, not unlike Archimedean geometry, itself maintained a problematic status in a second sense: it was allotted a para-scientific status, labelled a 'barbaric' or 'Gothic' hypothesis, or at best a convenient convention or well-grounded fiction. In its early formulations, the calculus was shot through with dynamic notions such as infinitesimals, fluxions and fluents, thresholds, passages to the limit, continuous variation – all of which presumed a *geometrical* conception of the continuum, in other words, the idea of a process. For most mathematicians, these were considered to be 'metaphysical' ideas that lay beyond the realm of mathematical definition. Berkeley famously ridiculed infinitesimals as 'the ghosts of departed quantities'; D'Alembert famously responded by telling his students, *Allez en avant, et la foi vous viendra* ('Go forward, and faith will come to you').[17]

The calculus would not have been invented without these notions, yet they remained problematic, lacking an adequate mathematical ground.

For a long period of time, the enormous success of the calculus in solving physical problems delayed research into its logical foundations. It was not until the end of the nineteenth-century that the calculus would receive a 'rigorous' foundation through the development of the 'limit-concept.' 'Rigour' meant that the calculus had to be separated from its problematic origins in geometrical conceptions or intuitions, and reconceptualised in purely arithmetic terms (the loaded term 'intuition' here having little to do with empirical perception, but rather the ideal geometrical notion of continuous movement and space).[18] This 'arithmetisation of analysis', as Félix Klein called it,[19] was achieved by Karl Weierstrass, one of Husserl's teachers, in the wake of work done by Cauchy (leading Guilio Giorello to dub Weierstrass and his followers the 'ghostbusters').[20] Analysis (the study of infinite processes) was concerned with *continuous* magnitudes, whereas arithmetic had as its domain the *discrete* set of numbers. The aim of Weierstrass' 'discretisation' programme was to separate the calculus from the geometry of continuity and base it on the concept of number alone. Geometrical notions were thus reconceptualised in terms of sets of discrete points, which in turn were conceptualised in terms of number: points on a line as individual numbers, points on a plane as ordered pairs of numbers, points in n-dimensional space as n-tuples of numbers. As a result, the concept of the variable was given a *static* (arithmetic) rather than a *dynamic* (geometrical) interpretation. Early interpreters had tended to appeal to the geometrical intuition of continuous motion when they said that a variable x 'approaches' a limit (e.g., the circle defined as the limit of a polygon). Weierstrass' innovation was to reinterpret this variable x arithmetically as simply designating any one of a collection of numerical values (the theory of functions), thereby eliminating any dynamism or 'continuous variation' from the notion of continuity, and any interpretation of the operation of differentiation as a process. In Weierstrass' limit-concept, in short, the geometric idea of 'approaching a limit' was arithmetised, and replaced by static constraints on discrete numbers alone (the episilon-delta method). Dedekind took this arithmetisation a step further by rigorously defining the continuity of the real numbers in terms of a 'cut': 'it is the cut which constitutes. .the ideal cause of continuity or the pure element of quantitativity' (Deleuze 1994, 172). Cantor's set theory, finally, gave a discrete interpretation of the notion of infinity itself, treating infinite sets like finite sets (the power set axiom) –

or rather, treating all sets, whether finite or infinite, as mathematical objects (the axiom of infinity).[21]

Weierstrass, Dedekind, and Cantor thus form the great triumvirate of the programme of discretisation and the development of the 'arithmetic' continuum (the redefinition of continuity as a function of sets over discrete numbers). In their wake, the basic concepts of the calculus – function, continuity, limit, convergence, infinity, and so on – were progressively 'clarified' and 'refined,' and ultimately given a set theoretical foundation. The assumptions of Weierstrass' discretisation problem that only arithmetic is rigorous, and that geometric notions are unsuitable for secure foundations – are now largely identified with the 'orthodox' or 'major' view of the history of mathematics as a progression toward ever more 'well-founded' positions.[22] This contemporary orthodoxy has often been characterised as an 'ontological reductionism'; as Penelope Maddy describes it, 'mathematical objects and structures are identified with or instantiated by set theorematic surrogates, and the classical theorems about them proved from the axioms of set theory.'[23] Reuben Hersh gives it a more idiomatic and constructivist characterisation: 'Starting from the empty set, perform a few operations, like forming the set of all subsets. Before long you have a magnificent structure in which you can embed the real numbers, complex numbers, quaterions, Hilbert spaces, infinite-dimensional differentiable manifolds, and anything else you like' (1997, 13). The programme would pass through two further developments. The contradictions generated by set theory brought on a sense of a 'crisis' in the foundations, which Hilbert's formalist (or formalisation) programme attempted to repair through *axiomatisation*, that is, by attempting to show that set theory could be derived from a finite set of axioms, which were later codified by Zermelo-Fraenkel (given his theological leanings, even Cantor needed a dose of axiomatic rigor). Gödel and Cohen, finally, in their famous theorems, would eventually expose the internal limits of axiomatisation (incompleteness, undecidability), demonstrating that there is a variety of mathematical forms in 'infinite excess' over our ability to formalise them consistently. Deleuze, for his part, fully recognises the position of the orthodox programme: 'Modern mathematics is regarded as based upon the theory of groups or set theory rather than on the differential calculus' (1994, 180). Nonetheless, he insists that the fundamental difference in kind between problematics and axiomatics remains, even in contemporary mathematics: 'Modern mathematics also leaves us in a state of antinomy, since the strict finite interpretation that it gives of the calculus nevertheless presupposes an axiom of infinity in the set theoretical foundation, even though this axiom finds no illustration in the calculus. What is still missing is the extra-propositional and sub-representative element expressed in the Idea by the differential, *precisely in the form of a problem*' (p. 178).

A final example can help serve to illustrate the ongoing tension between problematics and axiomatics, even in contemporary mathematics. Even after Weierstrass' work, mathematicians using the calculus continued to obtain accurate results and make new discoveries by using infinitesimals in their reasoning, their mathematical conscience assuaged by the (often unchecked) supposition that infinitesimals could be replaced by Weierstrassian methods. Despite its supposed 'elimination' as an impure and muddled metaphysical concept, the ghostly concept of infinitesimals continued to play a positive role in mathematics *as* a problematic concept, reliably producing correct solutions. 'Even now,' wrote Abraham Robinson in 1966, 'there are many classical results in differential geometry which have never been established in any other way [than through the use of infinitesimals], the assumption being that somehow the rigorous but less intuitive ε, δ method would lead to the same result.'[24] In response to this situation, Robinson developed his *non-standard analysis*, which proposed an axiomatisation of infinitesimals themselves, at last granting mathematicians the 'right' to use them in proofs. Using the theory of formal languages, he added to the ordinary theory of numbers a new symbol (which we can call i for infinitesimal), and posited axioms saying that i was smaller than any finite number $1/n$ and yet not zero; he then showed that this enriched theory of numbers is consistent, assuming the consistency of the ordinary theory of numbers. The resulting axiomatic model is described as 'non-standard' in that it contains, in addition to the 'standard' finite and transfinite numbers, non-standard numbers such as hyperreals and infinitesimals. In the non-standard model, there is a cluster of infinitesimals around every real number r, which Robinson, in a nod to Leibniz, termed a 'monad' (the monad is the 'infinitesimal neighbourhood' of r). Transfinites and infinitesimals are two types of infinite number, which characterise degrees of infinity in different fashions. In effect, this means that contemporary mathematics has 'two distinct rigorous formulations of the calculus': that of Weierstrass and Cantor, who eliminated infinitesimals, and that of Robinson, who rehabilitated and legitimised them.[25] Both these theorematic endeavours, however, had their genesis in the imposition of the notion of infinitesimals *as* a problematic concept, which in turn gave rise to differing but related axiomatisations. Deleuze's claim is that the ontology of mathematics is poorly understood if it does not take into account the specificity and irreducibility of problematics.

3. The relation between problematics and axiomatics

With these historical examples in hand, we can now make several summary points concerning the relation between the problematic and axiomatic poles of mathematics, or more broadly, the relation between minor and major science. First, according to Deleuze, mathematics is constantly producing notions that have an objectively problematic status; the role of axiomatics (or its precursors) is to codify and solidify these problematic notions, providing them with a theorematic ground or rigorous foundation. Axiomaticians, one might say, are the 'law and order' types in mathematics: 'Hilbert and de Broglie were as much politicians as scientists: they reestablished order' (Deleuze and Guattari 1987, 144). In this sense, as Jean Dieudonné suggests, axiomatics is a foundational but *secondary* enterprise in mathematics, dependent for its very existence on problematics: 'In periods of expansion, when new notions are introduced, it is often very difficult to exactly delimit the conditions of their deployment, and one must admit that one can only reasonably do so once one has acquired a rather long practice in these notions, which necessitates a more or less extended period of cultivation [*défrichement*], during which incertitude and controversy dominates. Once the heroic age of pioneers passes, the following generation can then codify their work, getting rid of the superfluous, solidifying the bases – in short, putting the house in order. At this moment, the axiomatic method reigns anew, until the next overturning [*bouleversement*] that brings a new idea.'[26] Nicholas Bourbaki puts the point even more strongly, noting that 'the axiomatic method is nothing but the "Taylor System" – the "scientific management" – of mathematics' (1971, 31). Deleuze has adopted a similar historical thesis, noting that the push toward axiomatics at the end of the nineteenth-century arose at the same time that Taylorism arose in capitalism: axiomatics does for mathematics what Taylorism does for 'work'.[27]

Second, problematic concepts often (though not always) have their source in what Deleuze terms the 'ambulatory' sciences, which includes sciences such as metallurgy, surveying, stonecutting, and perspective. (One need only think of the mathematical problems encountered by Archimedes in his work on military installations, Desargues on the techniques of perspective, Monge on the transportation of earth, and so on.)

The nature of such domains, however, is that they do not allow science to assume an autonomous power. The reason, according to Deleuze, is that the ambulatory sciences 'subordinate all their operations to the sensible conditions of intuition and construction – *following* the flow of matter, *drawing and linking up* smooth space. Everything is situated in the objective zone of fluctuation that is coextensive with reality itself. However refined or rigorous, "approximate knowledge" is still dependent upon sensitive and sensible evaluations that pose more problems than they solve: problematics is still its only mode' (Deleuze and Guattari 1987, 373). Such sciences are linked to notions – such as heterogeneity, dynamism, continuous variation, flows, and so on – that are barred or banned from the requirements of axiomatics, and consequently they tend to appear in history as that which was superseded or left behind. By contrast, what is proper to royal science, to its theorematic or axiomatic power, is 'to isolate all operations from the conditions of intuition, making them true intrinsic concepts, or 'categories' .Without this categorical, apodictic apparatus, the differential operations would be constrained to follow the evolution of a phenomenon'(p. 373–374). In the ontological field of interaction between minor and major science, in other words, 'the ambulant sciences confine themselves to *inventing problems* whose solution is tied to a whole set of collective, nonscientific activities but whose *scientific solution* depends, on the contrary, on royal science and the way it has transformed the problem by introducing it into its theorematic apparatus and its organisation of work. This is somewhat like intuition and intelligence in Bergson, where only intelligence has the scientific means to solve formally the problems posed by intuition' (p. 374).

Third, what is crucial in the interaction between the two poles are thus the processes of translation that take place between them – for instance, in Descartes and Fermat, an algebraic translation of the geometrical; in Weierstrass, a static translation of the dynamic; in Dedekind, a discrete translation of the continuous. The 'richness and necessity of translations,' writes Deleuze, 'include as many opportunities for openings as risks of closure or stoppage' (Deleuze and Guattari 1987, 486). In general, Deleuze's work in mathematical epistemology tends to focus on the reduction of the problematic to the axiomatic, the intensive to the extensive, the continuous to the discrete, the nonmetric to the metric, the nondenumerable to the denumerable, the rhizomatic to the arborescent, the smooth to the striated. Not all these reductions, to be sure, are equivalent, and Deleuze (following Lautman) analyses each on its own account. Deleuze himself highlights two of them. The first is 'the complexity of the means by which one translates intensities into extensive quantities, or more generally, multiplicities of distance into systems of magnitudes that measure and striate them (the role of logarithms in this connection)'; the second, 'the delicacy and complexity of the means by which Riemannian patches of smooth space receive a Euclidean conjunction (the role of the parallelism of vectors in striating the infinitesimal)' (p. 486). At times, Deleuze suggests, axiomatics can possess a deliberate will to halt problematics: 'State science retains of nomad science only what it can appropriate; it turns the rest into a set of strictly limited formulas without any real scientific status, or else simply represses and bans it' (p. 362; cf. p. 144). But despite its best efforts, axiomatics can never have done with problematics, which maintains its own ontological status and rigor. 'Minor science is continually enriching major science, communicating its intuitions to it, its way of proceeding, its itinerancy, its sense of and taste for matter, singularity, variation, intuitionist geometry and the numbering number. Major science has a perpetual need for the inspiration of the minor; but the minor would be nothing if it did not confront and conform to the highest scientific requirements' (p. 485–6). In Deleuzian terms, one might say that while 'progress' can be made at the level of theorematics and axiomatics, all 'becoming' occurs at the level of problematics.

Fourth, this means that axiomatics, no less than problematics, is itself an inventive and creative activity. One might be tempted to follow Poincaré in identifying problematics as a 'method of discovery' (Riemann) and axiomatics as a 'method of demonstration' (Weierstrass).[28] But just as problematics has its own modes of formalisation and deduction, so axiomatics has its own modes of intuition and discovery (axioms are not chosen arbitrarily, for instance, but in accordance with specific problems and intuitions).[29] 'In science an axiomatic is not at all a transcendent, autonomous, and decision-making power opposed to experimentation and intuition. On the one hand, it has its own gropings in the dark, experimentations, modes of intuition. Axioms being independent of each other, can they be added, and up to what point (a saturated system)? Can they be withdrawn (a 'weakened' system)? On the other hand, it is of the nature of axiomatics to come up against so-called *undecidable propositions*, to confront *necessarily higher powers* that it cannot master. Finally, axiomatics does not constitute the cutting edge of science; it is much more a stopping point, a reordering that prevents decoded flows in physics and mathematics [= problematics] from escaping in all directions. The great axiomaticians are the men of State within science, who seal off the lines of flight that are so frequent in mathematics, who would impose a new *nexum*, if only a temporary one, and who lay down the official policies of science. They are the heirs of the theorematic conception of geometry' (Deleuze and Guattari 1987, 461). For all these reasons, problematics is, by its very nature, 'a kind of science, or treatment of science, that seems very difficult to classify, whose history is even difficult to follow' (p. 361).[30]

4. *The formalisation of problematics: Deleuze's theory of multiplicities*

One of the aims of Deleuze's new concept of dialectics is to provide a *formalisation* of problematics that would constitute the basis for the theory of Ideas – a parallel to the formalisation that long ago took place in axiomatics. The difficulties of such a task, however, should be evident from the remarks above. The formalisation of theorematics has had a long history in mathematics and philosophy, and the theory of extensive multiplicities (Cantor's set theory) and its rigorous axiomatisation (Zermelo-Fraenkel, et al.) is one of the great achievements of modern mathematics. Deleuze, by contrast, is proposing to construct a hitherto non-existent (philosophical) formalisation of problematic multiplicities that are, by his own account, selected against by 'major' mathematics. In this regard, Deleuze's relation to the history of mathematics is similar to his relation to the history of philosophy: even in canonical figures there is something that 'escapes' the official histories of mathematics.[31] Nonetheless, there were a number of important precursors in mathematics who were working in this direction: Abel, Galois, Riemann, and Poincaré are among the great names in the history of problematics, just as Weierstrass, Dedekind, and Cantor are the great names in the discretisation programme, and Hilbert, Zermelo, Frankel, Gödel, and Cohen the great names in the movement toward formalisation and axiomatisation. We can therefore highlight at least three mathematical domains that have served as precursors in formalising the theory of problems in mathematics, and which Deleuze appealed to in formulating his own concept of problems as multiplicities.[32]

1. The first domain is the theory of *groups*, which initially arose from questions concerning the solvability of certain *algebraic* (rather than differential) equations. There are two kinds of solutions to algebraic equations, particular and general. Whereas a *particular* solution is given

by numerical values ($x^2 + 3x - 4 = 0$ has as its solution $x = 1$), a *general* solution provides the global pattern of all particular solutions to an algebraic equation (the above equation, generalised as $x^2 + ax - b = 0$, has the solution $x = \sqrt{a^2/2 + b} - a/2$). But such solutions, writes Deleuze, 'whether general or particular, find their sense only in the subjacent problem which inspires them' (1994, 162). By the sixteenth century, it had been proved (Tataglia-Cardan) that *general* solvability was possible with squared, cubic, and quartic equations. But equations raised to the fifth power and higher refused to yield to the previous method (via radicals), and the puzzle of the 'quintic' remained unresolved for more than two centuries, until the work of Lagrange, Abel, and Galois in the nineteenth-century. In 1824, Abel proved the startling result that the quintic was in fact *unsolvable*, but the method he used was as important as the result: Abel recognised that there was a pattern to the solutions of the first four cases, and that it was this pattern that held the key to understanding the recalcitrance of the fifth. Abel showed that the question of 'solvability' had to be determined internally by the *intrinsic* conditions of the problem itself, which then progressively specifies its own 'fields' of solvability.

Building on Abel's work, Evariste Galois developed a way to approach the study of this pattern, using the technique now known as *group theory*. Put simply, Galois 'showed that equations that can be solved by a formula must have groups of a particular type, and that the quintic had the wrong sort of group' (Stewart and Golubitsky 1992, 42). The 'group' of an equation captures the conditions of the problem; on the basis of certain substitutions within the group, solutions can be shown to be indistinguishable insofar as the validity of the equation is concerned.[33] In particular, Deleuze emphasises the fundamental procedure of *adjunction* in Galois: 'Starting from a basic 'field' R, successive adjunctions to this field (R', R", R"' .) allow a progressively more precise distinction of the roots of an equation, by the progressive limitation of possible substitutions. There is thus a succession of 'partial resolvants' or an embedding of 'groups' which make the solution follow from the very conditions of the problem' (1994, 180). In other words, the group of an equation does not tell us what we know about its roots, but rather, as George Verriest remarks, 'the objectivity of what we do *not* know about them.'[34] As Galois himself wrote, 'in these two memoirs, and especially in the second, one often finds the formula, *I don't know.* '[35] This non-knowledge is not a negative or an insufficiency, but rather a rule or something to be learned that corresponds to an *objective* dimension of the problem. What Deleuze finds in Abel and Galois, following the exemplary analyses of Jules Vuillemin in his *Philosophy of Algebra*, is 'a radical reversal of the problem-solution relation, a more considerable revolution than the Copernican.'[36] In a sense, one could say that 'unsolvability' plays a role in problematics similar to that played by 'undecidability' in axiomatics.

2. The second domain Deleuze utilises is the calculus itself, and on this score Deleuze's analyses are based to a large extent on the interpretation proposed by Albert Lautman in his *Essay on the Notions of Structure and Existence in Mathematics* (1938). Lautman's work is based on the idea of a fundamental difference in kind between a problem and its solution, a distinction that is attested to by the existence of problems *without* solution. Leibniz, Deleuze notes, 'had already shown that the calculus. .expressed problems that could not hitherto be solved, or indeed, even posed' (1994, 177). In turn Lautman establishes a link between the theory of differential equations and the theory of singularities, since it was the latter that provided the key to understanding the nature of *nonlinear* differential equations, which could not be solved because their series diverged. As determined by the equation, singular points are distinguished from the ordinary points of a curve: the singularities mark the points where the curve changes direction (inflections, cusps, etc.), and thus can be used to distinguish between different *types* of curves. In the late 1800's, Henri Poincaré, using a simple nonlinear equation, was able to identify four types of singular points that corresponded to the equation (foci, saddle points, knots, and centres) and to demonstrate the topological behaviour of the solutions in the neighbourhood of such points (the integral curves).[37] On the basis of Poincaré's work, Lautman was able to specify the nature of the difference in kind between problems and solutions. The conditions of the *problem* posed by the equation is determined by the existence and distribution of singular points in a differen*t*iated topological field (a field of vectors), where each singularity is inseparable from a zone of objective indetermination (the ordinary points that surround it). In turn, the *solution* to the equation will only appear with the integral curves that are constituted in the neighbourhood of these singularities, which mark the beginnings of the differen*c*iation (or actualisation) of the problematic field. In this way, the ontological status of the problem as such is detached from its solutions: in itself, the problem is a multiplicity of singularities, a nested field of directional vectors which define the 'virtual' trajectories of the curves in the solution, not all of which can be actualised. Non-linear equations can thus be used to model objectively problematic (or indeterminate)

physical systems, such as the weather (Lorenz): the equations can define the virtual 'attractors' of the system (the intrinsic singularities toward which the trajectories will tend in the long-term), but they cannot say in advance which trajectory will be actualised (the equation cannot be solved), making accurate prediction impossible. A problem, in other words, has an objectively determined structure (virtuality), apart from its solutions (actuality).[38]

3. But 'there is no revolution,' in the problem-solution reversal, continues Deleuze, 'as long as we remain tied to Euclidean geometry: we must move to a geometry of sufficient reason, a Riemannian-type differential geometry which tends to give rise to discontinuity on the basis of continuity, or to ground solutions in the conditions of the problems' (1994, 162). This leads to Deleuze's third mathematical resource, the *differential geometry* of Gauss and Riemann. Gauss had realised that the utilisation of the differential calculus allowed for the study of curves and surfaces in a purely intrinsic and 'local' manner; that is, without any reference to a 'global' embedding space (such as the Cartesian coordinates of analytic geometry).[39] Riemann's achievement, in turn, was to have used Gauss's differential geometry to launch a reconsideration of the entire approach to the study of space by analysing the general problem of *n-dimensional* curved surfaces. He developed a non-Euclidean geometry (showing that Euclid's axioms were not self-evident truths) of a multidimensional, non-metric, and non-intuitable 'any-space-whatever,' which he termed a pure 'multiplicity' or 'manifold' [*Mannigfaltigkeit*]. He began by defining the distance between two points whose corresponding coordinates differ only by infinitesimal amounts, and defined the curvature of the multiplicity in terms of the *accumulation* of neighbourhoods, which alone determine its connections.[40] For our purposes, the two important features of a Riemannian manifold are its variable number of dimensions (its *n*-dimensionality), and the absence of any supplementary dimension which would impose on it extrinsically defined coordinates or unity.[41] As Deleuze writes, a Riemannian multiplicity is 'an n-dimensional, continuous, defined multiplicity. . . By *dimensions*, we mean the variables or coordinates upon which a phenomenon depends; by *continuity*, we mean the set of [differential] relations between changes in these variables – for example, a quadratic form of the differentials of the co-ordinates; by *definition*, we mean the elements reciprocally determined by these relations, elements which cannot change unless the multiplicity changes its order and its metric' (1994, 182).

In *Difference and Repetition*, Deleuze draws upon all these resources to develop his general theory of problematic or differential multiplicities. The fifth chapter of *Difference and Repetition* ('Ideas and the Synthesis of Difference') draws on all these resources in order to present a theory of Ideas *as* problematic (problems *are* Ideas), which in effect presents Deleuze's new concept of dialectics. The formal conditions of a problematic Idea can be briefly summarised as follows. (1) The elements of the multiplicity are merely 'determinable', their nature is not determined in advance by either a defining property or an axiom (e.g., extensionality). Rather, they are pure virtualities that have neither identity, nor sensible form, nor conceptual signification, nor assignable function (principle of determinability). (2) They are nonetheless determined reciprocally as singularities in the differential relation, a 'non-localisable ideal connection' that provides a purely intrinsic definition of the multiplicity as 'problematic'; the differential relation is not only *external* to its terms, but *constitutive* of its terms (principle of reciprocal determination). (3) The values of these relations define the complete determination of the problem, that is, 'the existence, the number, and the distribution of the determinant points that precisely provide its conditions' *as* a problem (principle of complete determination).[42] These three aspects of sufficient reason, finally, find their unity in the temporal principle of progressive determination, through which, as we have seen in the work of Abel and Galois, the problem is resolved (adjunction, etc.) (1994, 210).

The strength of Deleuze's project, with regard to problematics, is that, in a certain sense, it parallels the movement toward 'rigour' that was made in axiomatics: it presents a formalisation of the theory of problems, freed from the conditions of geometric intuition and solvability, and existing only in pure thought (even though Deleuze presents his theory in a purely philosophical manner, and explicitly refuses to assign a scientific status to his conclusions).[43] In undertaking this project, he had few philosophical precursors (Lautman, Vuillemin), and the degree to which he succeeded in the effort no doubt remains an open question. Manuel DeLanda, in a recent work, has proposed several refinements in Deleuze's formalisation, drawn from contemporary science: certain types of singularities are now recognisable as 'strange attractors'; the resolution of a problematic field (the movement from the virtual to the actual) can now be described in terms of a series of spatio-temporal 'symmetry-breaking cascades' and so on.[44] But as DeLanda insists, despite his own modifications to Deleuze's theory, Deleuze himself 'should get the credit for having adequately *posed the problem*' of problematics (2002, 102).

Notes

This essay draws on earlier work that was published in the *Southern Journal of Philosophy* 41.3 (2003). See Smith 2003.

1 See Deleuze 1994, 323 n. 22: Given the irreducibility of 'problems' in his thought, Deleuze writes that 'the use of the word "problematic" as a substantive seems to us an indispensable neologism.'

2 Deleuze 1994, 323 n. 22. Deleuze is referring to the distinction between 'problem' and 'theory' in Canguilhem 1978; the distinction between the 'problem-element' and the 'global synthesis element' in Bouligand 1949; and the distinction between 'problem' and 'solution' in Lautman 1939, discussed below. All these thinkers insist on the double irreducibility of problems: problems should not be evaluated extrinsically in terms of their 'solvability' (the philosophical illusion), nor should problems be envisioned merely as the conflict between two opposing or contradictory propositions (the natural illusion) (see Deleuze 1994, 161). On this score, Deleuze largely follows Lautman's thesis that mathematics participates in a *dialectic* that points beyond itself to a meta-mathematical power – that is, to a general theory of problems and their ideal synthesis – which accounts for the genesis of mathematics itself. See Lautman 1939, particularly the section entitled 'The Genesis of Mathematics from the Dialectic': 'The order implied by the notion of genesis is no longer of the order of logical reconstruction in mathematics, in the sense that from the initial axioms of a theory flow all the propositions of the theory, for the dialectic is not a part of mathematics, and its notions have no relation to the primitive notions of a theory' (p. 13–14).

3 When Kant says that Ideas are 'problems to which there is no solution' (Kant 1998, 319, A328/B384), he does not mean that they are necessarily false problems, and therefore insoluble; on the contrary, this means that *true problems are Ideas*, and that these Ideas do not disappear with their solutions, since they are the indispensable condition without which no solution would ever exist. See Deleuze 1994, 168.

4 Proclus 1970, 63–67, as cited in Deleuze 1994, 163; Deleuze 1987, 554 n.21; and Deleuze 1990a, 54. See also Deleuze's comments in Deleuze 1989, 174: theorems and problems are are 'two mathematical instances which constantly refer to each other, the one enveloping the second, the second sliding into the first, but both very different in spite of their union.' On the two types of deduction, see 185.

5 See E. T. Bell's comments on this issue in Bell 1937, 31–32.

6 See Deleuze 1994, 174: 'The mathematician Houël remarked that the shortest distance was not a Euclidean notion at all, but an Archimedean one, more

physical than mathematical; that it was inseparable from a method of exhaustion; and that it served less to determine the straight line than to determine the length of a curve by means of a straight line – "integral calculus performed unknowingly"' (citing Houël 1867, 3, 75). Carl B. Boyer makes a similar point in his 1968: 'Greek mathematics sometimes has been described as essentially static, with little regard for the notion of variability; but Archimedes, in his study of the spiral, seems to have found the tangent to the curve through kinematic considerations akin to the differential calculus' (p. 41).

7 Husserl 1931, 208, §74. Whereas Husserl saw problematics as 'proto-geometry,' Deleuze sees it as a fully autonomous dimension of geometry, but one he identifies as a 'minor' science; it is a 'proto'-geometry only from the viewpoint of the 'major' or 'royal' conception of geometry, which attempts to eliminate these dynamic events or variations by subjecting them to a theorematic treatment.

8 Deleuze 1994, 160. Emphasis added. Deleuze continues: 'As a result [of using *reductio ad absurdum* proofs], however, the *genetic* point of view is forcibly relegated to an inferior rank: proof is given that something cannot not be, rather than *that* it is and *why* it is (hence the frequency in Euclid of negative, indirect and *reductio* arguments, which serve to keep geometry under the domination of the principle of identity and prevent it from becoming a geometry of sufficient reason).'

9 Deleuze and Guattari 1987, 484. On the relation between Greek theorematics and seventeenth-century algebra and arithmetic as instances of 'major' mathematics, see Deleuze 1994, 160–161.

10 Boyer 1968, 394. Deleuze writes that 'Cartesian coordinates appear to me to be an attempt at reterritorialization' (Deleuze 1972).

11 See Deleuze 1994, 161 and 323 n. 21. See also Reuben Hersh's comments on Descartes in Hersh 1997, 112–113: 'Euclidean certainty boldly advertised in the *Method* and shamelessly ditched in the *Geometry*.'

12 Descartes, as cited in Hersh1997, 113.

13 For the role of the scholia, see Deleuze 1992, 342–350 (the appendix on the scholia); for the uniqueness of the fifth book, see 'Spinoza and the Three Ethics', in Deleuze 1997, 149.

14 See Macherey 1996, 143. For a discussion of these issues, see Duffy 2006, 155–158.

15 Deleuze and Guattari 1987, 554 n. 23, commenting on Brunschvicg 1972. Deleuze also appeals to a text by Michel Chasles (1837), which establishes a continuity between Desargues, Monge, and Poncelet as the 'founders of a modern geometry' (Deleuze and Guattari 1987, 554 n. 28).

16 See Brunschvicg 1972, 327–331.

17 See Boyer 1959, 267. Deleuze praises Boyer's book as 'the best study of the history of the differential calculus and its modern structural interpretation' (1990a, 339).

18 For a discussion of the various uses of the term 'intuition' in mathematics, see the chapters on 'Intuition' and 'Four-Dimensional Intuition' in Davis and Hersch 1981, 391–405; as well as Hans Hahn's classic article 'The Crisis in Intuition', in Newman 1956, 1956–1976.

19 Boyer 1968, ch. 25, 'The Arithmetization of Analysis' (p. 598–619).

20 Giorello 1992, 135. I thank Andrew Murphie for this reference.

21 See Maddy 1997, 51–52, for a discussion of Cantorian 'finitism'

22 For a useful discussion of Weierstrass's 'discretisation program' (albeit written from the viewpoint of cognitive science), see Lakoff and Núñez 2000, 257–324.

23 Maddy 1997, 28. Reuben Hersh gives this a more idiomatic and constructivist characterization: 'Starting from the empty set, perform a few operations, like forming the set of all subsets. Before long you have a magnificent structure in which you can embed the real numbers, complex numbers, quaterions, Hilbert spaces, infinite-dimensional differentiable manifolds, and anything else you like' (1997, 13).

24 Robinson 1966, 83. See also p. 277: 'With the spread of Weierstrass' ideas, arguments involving infinitesimal increments, which survived particularly in differential geometry and in several branches of applied mathematics, began to be taken automatically as a kind of shorthand for corresponding developments by means of the ε, δ approach.'

25 Hersh 1997, 289. For discussions of Robinson's achievement, see Jim Holt's useful review, 'Infinitesimally Yours', in *The New York Review of Books*, 20 May 1999, as well as the chapter on 'Non-standard Analysis' in Davis and Hersh 1981, 237–254. The latter note that 'Robinson has in a sense vindicated the reckless abandon of eighteenth-century mathematics against the straight-laced rigour of the nineteenth-century, adding a new chapter in the never ending war between the finite and the infinite, the continuous and the discrete' (p. 238).

26 Jean Dieudonné, *L'Axiomatique dans les mathématiques modernes*, 47–48, as cited in Blanché 1955, 91.

27 See Deleuze 1972: 'The idea of a scientific task that no longer passes through codes but rather through an axiomatic first took place in mathematics toward the end of the nineteenth-century. .One finds this well-formed only in the capitalism of the nineteenth-century.' Deleuze's political philosophy is itself based in part on the axiomatic-problematic distinction: 'Our use of the word "axiomatic" is far from a metaphor; we find *literally* the same theoretical problems that are posed by the models in an axiomatic repeated in relation to the State' (Deleuze and Guattari 1987, 455).

28 Poincaré 1898–1899, 1–18, as cited in Boyer 1968, 601. Boyer notes that one finds in Riemann 'a strongly intuitive and geometrical background in analysis that contrasts sharply with the arithmetizing tendencies of the Weierstrassian school' (p. 601).

29 See Deleuze 1988a, 64: 'axioms concern problems, and escape demonstration.'

30 This section of the 'Treatise on Nomadology' (p. 361–374) develops in detail the distinction between 'major' and 'minor' science.

31 At one point, he even provides a list of 'problematic' figures from the history of science and mathematics: 'Democritus, Menaechmus, Archimedes, Vauban, Desargues, Bernouilli, Monge, Carnot, Poncelet, Perronet, etc.: in each case a monograph would be necessary to take into account the special situation of these scientists whom State science used only after restraining or disciplining them, after repressing their social or political conceptions.' Deleuze and Guattari 1987, 363. See Deleuze's well-known comments on his relation to the history of philosophy in 'Letter to a Harsh Critic', in Deleuze 1995, 5–6. The best general works on the history of mathematics are Boyer 1968 and Kline 1972.

32 For analyses of Deleuze's theory of multiplicities, see Durie 2002, 1–29; Ansell-Pearson 2002; and DeLanda 2002.

33 See Kline 1972, 759: 'The group of an equation is a key to its solvability because the group expresses the degree of indistinguishability of the roots. It tells us what we do *not* know about the roots.'

34 Deleuze 1994, 180, citing Verriest 1951, 41.

35 Deleuze 1997, 149, citing a text by Galois in Dalmas 1956, 132.

36 Deleuze 1994, 170. Deleuze is referring to Vuillemin 1962.

37 For discussions of Poincaré, see Kline 1972, 732–738; Lautman 1946, 41–43; and Deleuze 1980b. Such singularities are now termed 'attractors': using the language of physics, attractors govern 'basins of attraction' that define the trajectories of the curves that fall within their 'sphere of influence'

38 For this reason, Deleuze's work has been seen to anticipate certain developments in complexity theory and chaos theory. Delanda in particular has emphasized this link in his 2002 (see n. 78). For a presentation of the mathematics of chaos theory, see Stewart 1989, 95–144.

39 See Lautman 1938a, 43: 'The constitution, by Gauss and Riemann, of a differential geometry that studies the intrinsic properties of a variety, independent of any space into which this variety would be plunged, eliminates any reference to a universal container or to a center of privileged coordinates.'

40 See Lautman 1938a, 23–24: 'Riemannian spaces are devoid of any kind of homogeneity. Each is characterized by the form of the expression that defines the square of the distance between two infinitely proximate points. .It follows that "two neighboring observers in a Riemannian space can locate the points in their immediate vicinity, but cannot locate their spaces in relation to each other without a new convention." Each vicinity is like a shred of Euclidean space, *but the linkage between one vicinity and the next is not defined and can be effected in an infinite number of ways. Riemannian space at its most*

general thus presents itself as an amorphous collection of pieces that are juxtaposed but not attached to each other.'

41 See Deleuze 1994, 183, 181: A Riemannian multiplicity 'is intrinsically defined, without external reference or recourse to a uniform space in which it would be submerged. . . .It has no need whatsoever of unity to form a system.'

42 See, in particular Deleuze 1994, 183, although the entirety of the fifth chapter is an elaboration of Deleuze's theory of multiplicities.

43 See Deleuze 1994, xxi: 'We are well aware. . .that we have spoken of science in a manner which was not scientific.'

44 See Delanda 2002, 15 (on attractors), and chapters 2 and 3 (on symmetry-breaking cascades).