Carving Up Reality

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1. The Problem of the Many

Think of Mont Blanc, with its rabbits and foothills and its slurries of moistened rock. We can carve up the reality around Mont Blanc in different ways. If we are hunters, we might include rabbits as parts of the mountain; if we are geologists we might include only rock, perhaps together with a certain amount of air in the crevices and tunnels which have been formed beneath the mountain surface; if, on the other hand, we are soil chemists we might include also a surrounding thin layer of organic matter; if we are skiers, we will want some snow; and if we are French or Italian government surveyors, then our respective maps of the mountain might include slightly different determinations as to where, precisely, its borders lie. If we are armed with a microscope we will discover that, the closer we approach the surface of the mountain, the more questionable does the belongingness or non-belongingness of microscopic particles to the mountain itself begin to appear. What could make it true, given some atom or molecule very near the surface of the mountain, that it is, or it not, a part of the mountain? Reflection on such puzzles suggests the hypothesis – expounded in the literature on vagueness under the heading 'supervaluationism' - that there is no single answer to the question as to what it is to which 'Mont Blanc' refers. Rather there are, at any given time, many answers; many parcels of reality that deserve the name 'Mont Blanc.'

Something similar applies also to you yourself, and indeed to every other organism. When you refer to John, you do not think of all the parts of John or of his immediate surroundings. You do not think of the cells in John's arm, or the fly next to his ear, or the neutrinos that pass through his body. These things do not fall under the beam of your referential searchlight. Rather, they are traced over. John is apprehended by you as a single, unitary object. His dermatologist, though, has a different perspective, for he is all too well aware that, like

Mont Blanc, John has questionable parts and that there are at the molecular level many overlapping aggregates of matter all of which have a claim to being John. Notice that this is not an epistemological matter. Even an omniscient being would be in the same predicament as you or me as concerns where the boundaries of John, or Mont Blanc, precisely lie.

That John is losing or gaining molecules from one moment to the next is however of no consequence for our everyday purposes: it falls below our normal threshold of concern. Our cognitive habits have thus developed in such a way that they relate to reality in a *coarse-grained fashion*, and this allows us to ignore questions as to the lower-level constituents of the objects foregrounded by our referential searchlight. This in its turn is what allows such objects to be specified, not precisely, but rather in such a way that a range of alternative but nearly identical objects are simultaneously comprehended within the scope of what we see or refer to. We do not recognize this 'many' because we are focused, precisely, on those parts and moments of the matters in hand which lie above the pertinent granularity threshold. On the level of granularity we embrace in our everyday cognitive activities it is as if there is just one object which serves as the focus of our attentions.

The acts in which we make reference to objects in reality thus bring about a partitioning of reality into two domains: the foreground domain, within which the relevant object is located, and the background domain, which comprehends all entities left in the dark by the operating perceptual or referential searchlight. But how is this partitioning to be understood? Certainly it cannot be understood in terms of any simple pigeonholing of reality into jointly exhaustive and mutually exclusive parts, either of the sort that is involved in a system of categories like that of Aristotle or of the sort that underlies the periodic table of the chemical elements. Nor can the foregroundbackground partition be understood along geographical lines (by analogy with the sort of partition which might be depicted on a map). Thus it is not as if one connected, compact (hole-free) portion of reality is set into relief in relation to its surroundings, as on old maps of the known world surrounded by terrae incognitae, or on contemporary maps in which Beverly Hills is represented as something which is set into relief within the wider surrounding territory of Los Angeles. For if an object is included in the foreground domain, this does not imply that all the parts of the object are also included therein. This is because each

partition comes with its own granularity, and this means that it does not recognize parts beneath a certain size. It is for this reason that each partition is compatible with a range of possible views, and indeed with no view at all, as to the ultimate constituents of the objects included in its foreground domain.

2. The Granularity Problem Posed

If, however, partitions are effected not in any simple geographical way, but rather in such a way as to be marked by a certain granularity, then this brings a serious problem for our standard views of how reference and perception work – a new variant on older problems connected with intensionality, opacity and substitution. The problem turns on the fact that the relation of a part to its whole is transitive. Consider the question of what Mary sees when she sees John raising his hand. The following must all simultaneously be true:

- A. The molecules inside John are parts of John.
- B. John is part of what Mary sees.
- C. The molecules inside John are not a part of what Mary sees.

These sentences cannot be simultaneously true in the presence of the (independently attractive-seeming) principle of the transitivity of parthood:

D. If x is part of y, and y is part of z, then x is part of z.

Counterpart triads can be constructed in regard to a host of other types of entities: truthmakers, facts, states of affairs, situations, surfaces, aspects, pluralities, shadows, visual fields, persons, Husserl's 'noemata', Kant's 'phenomenal world', Fine's 'qua objects', and so on. Thus, for example, we might have:

- A'. The molecules inside John are parts of John.
- B'. John is part of what makes it true that John is raising his hand.
- C'. The molecules inside John are not a part of what makes it true that John is raising his hand.

Clearly if we are to do justice ontologically to the facts of granularity, then some way must be found to explain how the sentences in each of these triads can be simultaneously true, and this means that some way must be found to block the transitivity of parthood (thus for example to block the move from 'x is visible' and 'y is part of x' to: 'y is visible'). We can formulate an analogous triad also in relation to entire domains, for example in relation to the domain of what might be called 'common-sense reality':

- A". The molecules inside John are parts of John.
- B". John is part of common-sense reality.
- C". The molecules inside John are not a part of common-sense reality.

One way to resolve the problem is to refuse to take expressions like 'fact,' 'sense datum,' 'what Mary sees,' or 'common-sense reality' seriously as referring to special sorts of entities. One is then simply not allowed to ask, for example, whether molecules of paint are or are not a part of the sense data which John sees when he focuses on a painted wall. To suppose that a part-whole relation might obtain (or not obtain) here is to be guilty of what some like to call a 'category mistake'.

Those interested in ontology will however persist in raising such questions nonetheless, which means that in regard to some, at least, of the types of entities mentioned, they will take such entities ontologically seriously. Sentences involving category mistakes (for example 'cardinal numbers are green') they will classify simply as unproblematic falsehoods. One standard ontological approach utilizes the phrase 'under a description' or some comparable locution. The idea is that it can be the case that some given molecule is part of John under one description (for example: physical body), but that it is not a part of John under some other description (for example: object visible with the naked eye). Again, however, ontological persistence reveals a problem with such approaches. For if John under these different descriptions is indeed one and the same entity, then he thereby also has, under each description, all the same parts. If, on the other hand, John under this description is a different entity from John under that description, then we are still in need of an account of how this difference is to be understood, and this brings us back to the puzzling triad with which we again.

Another popular starting point for the resolution of our puzzle rests on using set theory as a means of blocking the principle D. of the transitivity of parthood. The set-membership relationship is after all not transitive. But to use set theory as a means of blocking transitivity brings too great a cost. For if set theory is taken realistically, then this forces us to identify Urelemente - elements of sets which are not themselves sets – from out of which the larger worldly structures which concern us would be constructed by set-theoretical means. But what would such elements be in the case of a complex event such as John raising his hand? And even if appropriate elements – ultimate subatomic particles, for example - did present themselves for purposes of set-theoretic reconstruction, problems would still arise because we would then find that our ontology is cluttered with multiple copies of reconstructed objects existing at different times and at different levels of granularity (for example, John as set of atoms, John as set of molecules. John as set of cells; this set of atoms at a time when it constitutes John, this same set of atoms at a time when its elements are scattered to the four winds; and so on). This problem of supernumerary copies does not arise for standard theories of part and whole, since the mereological sums of the atoms, molecules, cells, and so forth constituting John at some given time are all one and the same object. It is precisely this, however, which makes the mereological approach susceptible to the puzzle captured in our triad. Mereology as an instrument of ontology is, furthermore, in no better a position than set theory when it comes to the problem of doing justice to the fact that John preserves his identity from one moment to the next even in spite of the fact that he gains and loses parts.

3. The Granularity Problem Solved

Consider what happens when you observe a chessboard. You are working with a partition of the world into that, in the region of the chessboard, which you are focusing on, and that which is traced over. Your focus brings with it, again, a certain granularity: you are interested, not in the atoms or molecules within the board and its pieces, but rather only in the board and the pieces themselves. Moreover, you are interested in the latter not as constituting a mere list, or set, but rather as they exist within a certain arrangement. The board is divided into squares. In some of these cells pieces of specific kinds are located.

To understand what is going on here, we need to focus in more detail on the notion of a partition and on the associated notion of cell. The first thing that we need to recognize is that partitions have their granularity built in, as it were, from the very start. An administrative map of France depicting its 91 *départements* or its 311 *arrondissements* provides a close approximation to a partition in the sense we have in mind. Such a map is the result of applying a certain coarse- or finegrained grid of cells – the minimal units of the partition – to a certain portion of reality.

A partition is the ontological analogue of the sort of labeled grid we might find in a large post office or automobile component warehouse. For a partition to do its work, it needs to have cells large enough to contain the objects that are of interest in the portion of reality which concerns us; but at the same time these cells must somehow serve to factor out the details which are of no concern. A partition is accordingly a device for focusing upon what is salient and also for ignoring or masking what is not salient. We can think of it as being laid like a net over whatever is the relevant object-domain, and, like a net or grid (or a latticed window of the type depicted in Renaissance manuals of painting), it is to a large degree transparent. Thus, importantly, it does not in any way change the objects to which it is applied. The sorts of carvings up of reality which are effected through our partitions are comparable, not to those effected by surgeons, but rather to those we find in atlases of surgical anatomy, or indeed in the various tables of categories prepared by Aristotelian philosophers and by Linnaean biologists.

A partition is like a map. It is an artifact of our perceiving, judging, classifying or theorizing activity, and it exists only as a product of the cell boundaries by which it is determined. The reality partitioned, in contrast, is what and where it is, and it has all its parts and moments, independently of any acts of human fiat and in dependently of our efforts to understand it theoretically. Granularity as it has been treated in the above is thus properly at home precisely in the realm of our partitions: granularity pertains not to the objects themselves on the side of reality, but rather to the ways we partition those objects in different sorts of contexts.

The arrangement of cells in a partition may be purely spatial, as in a map, where the relative positions of neighboring cells are determined by the corresponding positions of those portions of geographic reality to which the cells relate. Or it may be determined by a linear ordering, as for example where partitions are determined via quantitative scales reflecting age cohorts or temperature bands. The arrangement may also be determined in more complex (for example hierarchical) ways, as in

the case of a partition determined by biological kinds (for example a multi-level partition of the animals in your local zoo into *lions*, *tigers*, *spiders*, *small marsupials*, *vertebrates*, and so on). The partitions which come closest to sets (or to mere lists) are those associated with our uses of proper names, partitions which we are able to project onto reality in such a way that their cells keep track of the corresponding objects in the world as objects that are identical from one moment to the next, and this in spite of the fact that these objects change and that parts are gained and lost.

Complex multidimensional partitions arise through the combination of these different types of demarcations and cell arrangements. A map of the zoo, for example, might indicate not only the places where animals are located but also the sorts and sizes and proper names of the animals which are located in those places.

4. Towards a Theory of Partitions¹

The cells in a partition may have sub-cells. Thus, for example, the cell *rabbit* is a sub-cell of the cell *mammal* in a partition of the animal kingdom. The cell *Florida* is a sub-cell of the cell *United States* in the standard geopolitical partition of the surface of the globe. The sub-cell relation is then an analogue of the sub-set relation in standard set theory. An example of a chain of cells ordered by the sub-cell relation is your address (The Oval Office, The White House, 1600 Pennsylvania Avenue NW, Washington, DC 20500, USA).

Those cells in a given partition which have no sub-cells within that partition are called minimal cells. The closest counterparts to sets within our present framework are those ideal sorts of partitions which

¹The formal details underlying the ideas in this section are set out in Thomas Bittner and Barry Smith, "A Taxonomy of Granular Partitions," in *Spatial Information Theory, Proceedings of COSIT 2001, Morro Bay, September 2001*, ed. Daniel Montello (New York: Springer, 2001). They were inspired on the one hand by the theory of location developed by R. Casati and A. C. Varzi, *Parts and Places. The Structures of Spatial Representation* (Cambridge, MA: MIT Press, 1999), and on the other hand by the machinery of granular partitions in R. Omnès, *The Interpretation of Quantum Mechanics* (Princeton: Princeton University Press, 1994). Omnès' theory is summarized in the forthcoming B. Smith and B. Brogaard, "Quantum Mereotopology," in *Annals of Mathematics and Artificial Intelligence* (forthcoming), which also contains indications as to how the theory of partitions can be extended to deal with time and change.

can be identified as the mereological fusions of such minimal cells.² The corresponding minimal cells will, again in the ideal case, constitute a perfect tiling – a jointly exhaustive and pairwise disjoint decomposition – of the pertinent domain of objects, and each cell in such a partition is itself the mereological sum of some one or more of those minimal cells.

Not all partitions have these nice properties however. This is because our partitions are artifacts of our theorizing and classifying activity and thus they are often incomplete. Thus we can imagine a partition of the animal kingdom containing a cell labeled *mammal*, and other cells labeled *rabbit*, *dog*, etc., which is yet not such as to represent a complete accounting of all the species of mammal which exist. There are gaps in the partition, analogous to the no-man's-lands between the zones of civilization represented on ancient maps.

Each partition will characteristically contain cells which are empty *per accidens* – because they have no objects located in them (as a chessboard will contain squares empty of pieces, and as a hotel may, on any given night, contain rooms empty of guests). *Dodo* is an empty cell in one standard partition of the animal kingdom. There are thus many empty cells within the domain of partitions taken as a whole.

For some partitions, which we can call *distributive*, if object x is a part of object y, and if y is located in a cell z, then x is also located in that cell.³ Distributive partitions satisfy a principle to the effect that, if two objects are located in two different cells, then the sum of these objects is located in the sum of these cells.

Spatial partitions are always distributive in the sense specified. If John is in Salzburg and Mary is in Salzburg, then their sum is in Salzburg and so, too, are all their bodily parts. A set, on the other hand, is a simple example of a non-distributive partition, and a partition generated by kinds or concepts, too, is non-distributive. A partition recognizing cats does not *ipso facto* recognize parts of cats. Moreover, if Bruno is a cat and Tibbles is a cat, then the mereological sum of Bruno and Tibbles is not itself a cat.

² The latter then play the role played by singletons in the theory of classes as the mereological sums of singletons advanced in D. Lewis, *Parts of Classes* (Oxford: Blackwell, 1991).

³ Here 'part' is to be understood according to the usual axioms of classical extensional mereology, for example as set forth in P. Simons, *Parts: An Essay in Ontology* (Oxford: Clarendon Press, 1987).

We can define the notion of recognition that is at work here as follows. We shall say that x is recognized by A if and only if A is a partition and x is located in some cell in A.

Suppose John is recognized by a nominal partition A consisting of a single cell labeled 'John,' then this is consistent with its being the case that each member of a whole family of distinct though almost identical aggregates of molecules is recognized by A. This is because A does not care about the small (molecule-sized) differences between these different aggregates: it traces over John's molecular structure. The cell John captures all the aggregates which are almost identical to John, but it does so in such a way that it cannot apprehend those different aggregates as different. Only a more refined partition would have the resources necessary to apprehend the differences in question.

We now have the machinery we need in order to explain how the three sentences of our triad can be simultaneously true. All three clauses are retained; now, however, they take the following forms:

- A*. The molecules inside John are parts of John.
- B*. John is recognized by a partition associated with Mary's act of seeing.
 - C*. The molecules inside John are not recognized by any partition associated with Mary's act of seeing.

The fact that the partitions available to Mary lack appropriately finegrained cells yields a solution to our puzzle which does not require the abandonment of the transitivity of parthood.

5. Partitions, Sets and Fusions

The notion of a partition hereby turns out to be in some respects a generalization of the notion of set, and we have in effect exploited an analogue of the transitivity-blocking feature of set theory in resolving our puzzle. A set is the ontological analogue of a mere list. The elements of a set exist within the set without order or location – they can be permuted at will and the set will remain identical. A partition, in contrast, standardly comes with a specific order and location of its constituent cells. Its cells fit together in a determinate arrangement, like pieces in a jigsaw or like molecules in a strand of DNA.

Partitions differ from sets also in this: that there are many different sorts of partitions, reflecting the many different sorts of relations between objects and cells. Set theory, in contrast, as Lewis shows, rests on just one central relation: the relation between an element and its

singleton. Unfortunately, as Lewis himself confesses, this relation is enveloped in mystery:

since all classes are fusions of singletons, and nothing over and above the singletons they're made of, our utter ignorance about the nature of the singletons amounts to utter ignorance about the nature of classes generally. . . . What do we know about singletons when we know only that they are atoms, and wholly distinct from the familiar individuals? What do we know about other classes, when we know only that they are composed of these atoms about which we know next to nothing?⁴

The machinery of partitions, in contrast, rests not just on one mysterious relation between element and singleton, but rather on a multiplicity of different (and quite unmysterious) ways we have of carving up reality. The relation between an object and its proper name is one such. Others include the relation between an object and its spatial location (for example in relation to a grid on a map), or between an object and a concept under which it falls, or between an object and a kind to which it belongs.

Objects as they exist in nature stand to each other in various relations. They have hooks of various sorts, which link them together; these include common boundaries and they include relations of dependence and of functional or causal association. The operator of mereological fusion, when properly handled, preserves these interobject relations, and it thus preserves the order and location of objects which fall within its charge: if two objects are linked together in nature, then they are linked together also within their mereological fusion.

A set is a mereological fusion of singletons, and mereological fusion preserves order and location. How can it be, then, that the elements within a set can be permuted at will and the set remain identical? The answer is that the set is built up mereologically not out of elements but out of singletons, and the latter are — according to standard philosophical conceptions of the nature of sets (which are in turn inspired by the axioms of set theory itself) — mere, homeless *somethings*, outside time and space. The singleton operator has the effect of stripping away the various sorts of linkages which obtain between the objects to which it is applied and also of setting them apart from their surroundings and from time and change.

⁴ Lewis, *Parts of Classes*, 31.

Partitions are, now, distinct from sets however (and from mereological fusions) also in this: that they are not constituted out of (not made or built out of) the objects that are located in their cells. Rather, they belong to the level of our theorizing and classifying activity. Like maps, they can in an easily understandable fashion remain the same even though the objects towards which they are directed are subject to constant change. Moreover, it is no less easily understandable how distinct partitions can arise in relation to one and the same initial stock of objects. Some partitions are like sets in the sense that they will apprehend the objects which are located in their respective cells independently of order or arrangement or linkage or time. Other partitions, however, inherit from mereology the ability to comprehend their objects in ways which map the different kinds of relations that obtain among them. The cells in such partitions project their objects not in isolation, but rather in tandem with other objects located in related cells within the same partition. Such partitions will apply to pairs of entities only in reflection of the specific relations in which they stand to each other. John and Mary, before they wed, are not, but through marriage they become, located in the two-celled partition married pair. Yet other two-celled partitions, for example the partition capturing the relation between the two terms of an ordered pair, apply to pairs of objects only in reflection of our ways of conceiving them. A three-celled partition is needed to capture the way in which, in an action of kissing or congratulating, two objects become bound together by a third object – a relational event – in which the one occurs as agent, the other as patient.

Partitions, as we have seen, have built-in granularity. The theory of partitions is thus unlike both mereology and set theory in that it has a direct and natural way of dealing with the fact that three-dimensional objects such as cats and human beings are many, but almost one.⁵ It thus also, again unlike both mereology and set theory, promises to offer a way to do justice to the ways in which three-dimensional objects such as cats or human beings can preserve their identity from one moment to the next even in spite of gaining and losing parts. A human organism is at any given time a certain family of distinct aggregates of atoms, molecules, and cells, but we are not in general aware of this fact

⁵ Compare with D.K. Lewis, "Many, but Almost One," in *Ontology, Causality and Mind: Essays in Honour of D.M. Armstrong*, ed. J. Bacon, K. Campbell and L. Reinhardt (Cambridge: Cambridge University Press, 1993), 23-38.

because, as we saw, the differences between the latter are traced over in our standard ways of dealing with them.

Partition theory does not, in and of itself, give an account of what transtemporal identity – for example the transtemporal identity of human organisms – is. Rather it tells us what is involved when cognitive subjects track identity through the sorts of changes by which human (and of course other) organisms are affected. If, however, the transtemporal identity of organisms is in part a function of the types of (small) changes by which they are affected from moment to moment, then the framework of partitions may provide a new path towards resolving the problem of what it is that makes it true that organism a at time t1 is identical to organism b at time t2, namely via an accounting of the sorts of (small) changes which affect (and which do not affect) the identity of the entity in question.

The theory of partitions can do justice also, in a way that is precluded for set theory, to the phenomena of time and change. This is because, where sets are abstract entities existing outside time and space, partitions are human constructions which can be applied to the very same domain of objects at different points in time. In this way, we can construct entire histories of partitions tracking the temporal evolution of (for example) physical systems of different sorts at different levels of granularity.⁶ In this way also we can use the machinery of partitions to represent not only the way objects are related together at a time, but also how they evolve from one time to the next.

6. A Coda on Realism and the Objectivity of Truth

That scattered portion of the world that is made up of rabbits, that which is made up of rabbit stages, and that which is made up of undetached rabbit parts, are all three just the same scattered portion of the world. The only difference, as Quine sees the matter, "is in how you slice it." What Quine does not recognize, however, is that there are two sorts of slicings: the bona fide and the fiat. Bona fide slicings reflect boundaries existing in nature, for example the boundaries of tennis balls or planets. Fiat slicings reflect boundaries which we

⁶ See Smith and Brogaard, "Quantum Mereotopology."

⁷ W.V.O. Quine, "Ontological Relativity," in *Ontological Relativity and Other Essays* (New York: Columbia University Press), 32.

⁸ See the forthcoming B. Smith, "Fiat Objects," *Topoi* (2001).

ourselves have introduced into reality through our more or less arbitrary demarcations, for example the boundaries of census tracts or tax brackets. Both kinds of slicing are represented in our partitions. For even though the cells of the latter are entirely fiat in nature – they are artefacts of our cognitive activities – some of them are coordinated with bona fide demarcations on the side of objects in reality.

Different philosophers have different views as to which slicings are bona fide and which are fiat. Quine himself holds a view which implies that the metaphysical distinctions between continuants, stages and undetached parts belong in the realm of fiat slicings. Since reference is behaviorally inscrutable as concerns such distinctions – this is the moral of Quine's 'gavagai' fable in *Word and Object* – Quine concludes that there is no fact of the matter which they might reflect – no fact of the matter on the side of the objects themselves as these exist before we address them in our language.

Notice that this is not an epistemological thesis. Quine must hold that even an omniscient being would be in the same predicament as you or me as concerns referential inscrutability. That is, he must hold that continuants, parts and stages do not differ from each other in virtue of any corresponding (bona fide) differences on the side of the entities in reality. Rather they differ from each other in the way in which, when asked to count the number of objects in the fruit bowl, you can say either: one orange, or: two orange-halves, or: four orange-quarters, and so on – and you will give the right answer in every case. The distinctions in question are merely the products of our distinct slicings (our purely fiat partitions) of one and the same reality.

But note that Quine is being too hasty when he asserts that there is *no* fact of the matter as concerns the reality to which we are related when using singular referring terms. For it follows from his own doctrine that it is a fact of the matter that this reality is intrinsically undifferentiated as far as the mentioned metaphysical distinctions are concerned. This is just the other side of the coin from the fact that the corresponding boundaries are entirely fiat in nature.

Quine indeed comes close to a view according to which all boundaries on the side of objects in reality are of the fiat sort. Objects of reference, for him, can comprise any content of some portion of space time, however heterogeneous, disconnected and gerrymandered this may be. This is not so for Lewis, on the other hand, whose perspective on these matters we find more congenial:

Among all the countless things and classes that there are, most are miscellaneous, gerrymandered, ill-demarcated. Only an elite minority are carved at the joints, so that their boundaries are established by objective sameness and difference in nature. Only these elite things and classes are eligible to serve as referents.⁹

Elite things and classes are in our terms the things and classes captured by those partitions which track bona fide boundaries and relations in reality. It is the job of fundamental science and of fundamental metaphysics to move us in the direction of partitions of this sort. Even when scientists and philosophers have completed this job, however, there will still be room for partitions of the lesser sort, partitions which track boundaries – for example the boundary of Quebec, of Tibbles' tale, or of the No Smoking Section of your favorite restaurant – which exist only as a result of our acts of fiat.

⁹ D. Lewis, "Putnam's Paradox," Australasian Journal of Philosophy 62 (1984): 227.

¹⁰ Thanks go to Berit Brogaard for invaluable assistance in the working out of the ideas in this paper. Thanks are due also to the National Science Foundation, which supported work on the theory of partitions under Research Grant BCS-9975557: "Geographic Categories: An Ontological Investigation" and to the American Philosophical Society, for the award of a Sabbatical Fellowship.