'Identity' as a mereological term

Jeroen Smid

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Abstract

The mereological predicate 'is part of' can be used to define the predicate 'is identical with'. I argue that this entails that mereological theories can be ideologically simpler than nihilistic theories that do not use the notion of parthood—contrary to what has been argued by Ted Sider. Moreover, if one accepts an extensional mereology, there are good philosophical reasons apart from ideological simplicity to give a mereological definition of identity.

Keywords: parthood; identity; mereology; nihilism; ideological simplicity; theory choice.

1. Introduction

When confronted with a choice between two or more incompatible theories that are equal with respect to explanatory power, it is commonly taken to be rational to choose between the theories on the basis of theoretical virtues such as ontological parsimony and ideological simplicity. Quine (1951b) can be credited for distinguishing the ontological commitments of a theory from its ideology. Goodman (1951) analyses ideology via the primitive terms of the language, such that ideological simplicity is a measure of the number (and complexity) of the primitive terms in the language of a theory. Recently, Ted Sider (2013) argued that mereological nihilism should be preferred to any theory of mereology since the former is ideologically simpler. Nihilism is deemed to be ideologically simpler because it need not employ the predicate 'is part of'. The theory can do without this predicate because nihilism holds that there are only simple ('atomic') objects.

I argue that any theory of mereology is instead ideologically simpler than Sider's version of nihilism since mereology needs only one primitive which can then be used to define '=' (i.e., 'is identical with'), while nihilism has to take '=' and at least one other predicate as primitive. Moreover, if one accepts an extensional mereology, thus holding that composite objects are identical if and only if they have the same proper parts, then

there are good reasons to treat '=' as a mereological predicate and thus define it in terms of 'is part of'.

Ideological simplicity can be seen as a measure of unification: a theory that has many primitive (i.e., undefined) terms is less unified than a theory that has fewer primitive terms since in the latter the relevant terms are connected via definitions. An ideologically simpler theory thus expresses 'more with less' than its ideologically more complex counterpart. Sider thinks ideologically simpler theories are also more likely to be true:

[S]implicity is a guide to truth [and] (...) eliminating primitive notions makes a theory "structurally" simpler. A theory's one-place predicates correspond to the kinds of things it recognizes, and its multi-place predicates to the kinds of connections between the things that it recognizes; cutting down on kinds or connections is one way of making a theory structurally simpler. The epistemic principle [that ideologically simpler theories are more likely to be true] is most naturally paired with a metaphysical realism about ideology. Ideologically simpler theories aren't just more convenient for us. The worlds that they purport to are objectively simpler, contain less structure. Ideology is a worldly matter, not about ideas at all. (Sider 2013, 240)

Even if ideological simplicity is not—necessarily—sensitive to truth one should still strive for ideologically simpler theories. As said, ideological simplification brings unification and often enhances our understanding of the notions involved thus resulting in better theory. As Goodman puts it:

Every definition at once both increases the coherence of the system and diminishes the number of predicates that need be taken as primitive. Thus the motive for seeking economy is not mere concern for superficial neatness. To economize and to systematize are the same. (Goodman 1951, 48)

Sider's argument for nihilism is novel but does not apply to every version of nihilism. For a theory T_1 to be ideologically simpler than a theory T_2 the former should have a smaller number of primitive terms than T_2 ; or—if the number is the same— T_1 should have simpler primitive terms than T_2 .¹ Many versions of nihilism, however, are formulated using 'is part of' as primitive. Such versions of nihilism are thus not ideologically simpler than theories of mereology since both use the same primitive term(s). To get a better grasp of this, consider the following axioms for 'P', i.e., 'is part of' (universal quantifiers are omitted):²

(P1)
$$Pxx$$
(Reflexivity)(P2) $(Pxy \land Pyz) \rightarrow Pxz$ (Transitivity)(P3) $(Pxy \land Pyx) \rightarrow x=y$ (Antisymmetry)

Following Achille Varzi's (2015) exposition of mereology we can call a system incorporating these three principles 'Core Mereology'. Various other mereological predicates can be defined:

(D1) $PPxy =_{df} Pxy \land \neg Pyx$	(Proper Part) ³
(D2) $O_{XY} =_{df} \mathbf{\Xi}_{\mathcal{X}} (P_{\mathcal{X}} \land P_{\mathcal{Y}})$	(Overlap)
(D3) $Dxy =_{df} \neg Oxy$	(Disjoint)
(D4) $A_{\mathcal{X}} =_{df} \neg \exists y PP_{\mathcal{Y}\mathcal{X}}$	(Atom)

The classical nihilist accepts (P1)-(P3) and definition (D4) but adds the following principle:

(P4) A_X (Everything is an atom)

Note that this formulation of nihilism *does* use 'is part of'; hence ideologically it is no different from non-nihilistic mereologies.⁴ Sider, however, wants his version of nihilism

¹ Comparing the complexity of different primitive terms is hard, but not impossible as is shown by Goodman (1951, 46ff)

² Note that 'is part of' sometimes goes by the name 'is an improper part of' and should not be confused with 'is a proper part of' (defined in (D1)). One could instead take 'is a proper part of' as the primitive term and define 'is part of' as follows: $Pxy =_{df} PPxy \lor x = y$. However, if one wishes to define '=' and only accept Core Mereology, then one should not take 'is a proper part of' as primitive. See for an explanation footnote 9 below. Thanks here to an anonymous referee for this journal for asking me to clarify this.

³ Note that (D1) is 'strict proper part', an alternative definition of proper part is: $PPxy =_{df} Pxy \land x \neq y$. This difference does not matter for our discussion.

⁴ Thanks to an anonymous reviewer of *Erkenntnis* for pointing this out to me.

to be ideologically simpler than any system of mereology by eliminating the predicate 'is part of':

[N]ihilism (...) allows us to eliminate the extra-logical (or perhaps quasi-logical) notion of 'part' from our ideology, and this kind of ideological simplification is an epistemic improvement. Nihilism is an ideologically simpler theory, and so is more likely to be true. (Sider 2013, 239)

If Sider were to formulate nihilism as commitment to (P1)-(P4) (as 'standard' nihilism is formulated) he would use the predicate 'is part of' to define 'is a proper part of' and define 'is an atom' as not having proper parts. Such a nihilist thus does not eliminate 'is part of' since that definition of 'is an atom' ultimately depends on 'is part of'. Sider thus has to formulate nihilism in a different way. One such way is to have 'is an atom' as a primitive term with (P4) as (one of) its axiom(s). Formulated thus, nihilism not using the predicate 'is part of' and thus arguably ideologically simpler than any system of mereology. I will call this latter version of nihilism 'pure nihilism' since it does not even allow the term 'part' in its language.⁵

To be sure, a pure mereolological nihilist like Sider is not forced to take 'is an *atom*' as primitive.⁶ Another term can be taken as primitive instead, for example: 'is simple', 'is a particle', or 'is a point-like entity'; or a technical term like 'is an atemnei' or 'is an individium' can be coined. Taking a term from science, like 'has mass *m*', or 'has charge +1' is also possible. However, the pure nihilist has to accept *some* predicate as primitive in order to say that there are things of a certain sort that fundamentally exist. The nihilist commits herself to more than just first-order logic with identity; it is an ontological—extra-logical—view and as such in need of a predicate (in the fundamental language) that applies to the entities in the (fundamental) domain.

The nihilist needs a primitive other than '=' because she needs to say—in her fundamental language—that what fundamentally exists is an F (where 'F' is a place-

⁵ The term 'pure nihilism' is in a way unfortunate, since Sider's version of nihilism is restricted to *fundamental* reality (Sider 2013, 252–253). Since he distinguishes between what exists fundamentally and what exists derivatively Sider's nihilism may be, in a sense, impure: fundamentally, there only exist atoms; but non-fundamentally there (possibly) exist objects composed of atoms. The term 'pure fundamental nihilism' might have been more apt, but it's rather cumbersome.

⁶ Thanks to an anonymous referee of this journal for asking me to clarify this.

holder for whatever term she deems adequate).⁷ If she cannot say that everything that exists is an *F*—but instead has to remain silent on the question 'what is it that exists?— then her view is indistinguishable from that of an *ontological* nihilist (who holds that nothing exists) and her dispute with the mereologist may (seem to) be merely verbal.⁸

An example may help to further illuminate the point. Imagine that the debate between dualism and (reductive) materialism is such that the dualist has, in her fundamental language, only the term 'is mental' as a primitive and defines 'is material' as 'is not mental'. (Just as 'is an atom' is defined as 'not having proper parts'.) The materialist denies that mental objects (fundamentally) exist and may thus delete the term 'is mental' from her fundamental language. However, the materialist does think that there exists *something* and that everything that exists is material; her view is intended to be a substantial view about fundamental reality claiming that everything in fundamental reality is a material object. Simply deleting 'is mental' from the language thus does not suffice, because that leaves her with only quantification theory and no predicates to express what it is that fundamentally exists. So, the materialist needs to introduce some term or other, such that she can say what it is that (fundamentally) exists. Our situation is analogous to this example: deleting 'is part of' from the fundamental language is *not* sufficient to state the view that, fundamentally, everything is an F (where 'F' is whatever term the nihilist thinks adequately expresses what it is that fundamentally exists).

So a pure nihilist, like Sider, needs to have at least one predicate other than '=' in the fundamental language. Here I take 'is an atom' as primitive. This matters little for the discussion, except for the fact that 'is an atom' is simpler than 'is part of' since the former is a one-place predicate. To be sure: by using 'is an atom' I am not implying that Sider is implicitly committed to mereological ideology. But he is committed to at least one primitive term other than '='.

One could argue that pure nihilism (with 'is an atom' and '=' as primitive) has as many primitive terms as any system of mereology (with 'is part of' and '=' as primitive); hence pure nihilism is not ideologically simpler. However, the number of primitive terms is only one aspect of ideological simplicity; the simplicity of the primitive terms themselves should also be taken into account. Sider could argue that nihilism is still

⁷ Of course, the objects that the nihilist things that exist need not all be F; maybe some are F and others are G. In that case she would need both the terms 'is an F' and 'is a G' (and so on for any number of terms). I am assuming, for simplicity, that all the objects that according to the nihilist exist have something in common and thus all satisfy one predicate.

⁸ This is in line with what Sider writes in *Writing the Book of the World*, where he makes clear that among the primitive terms in his language there are predicates from physics (2011, 292).

simpler, because 'is an atom' is a simpler primitive than 'is part of'—I will grant him this claim.

The current situation is depicted in Table 1. Both Pure Nihilism and Core Mereology have '=' as a primitive term. The other primitive terms are 'A' (for 'is an atom') and 'P' (for 'is part of') accepted by the pure nihilist and the core mereologist, respectively. Nihilism claims that everything is an atom, which is expressed by (A1). Core Mereology states that parthood is a partial ordering, i.e., (P1)-(P3). Both Pure Nihilism and Core Mereology state that '=' is an equivalence relation subject to Leibniz's law. The most economical, or simplest, way of stating this is via Wang's law, i.e. (I1)—that '=' is reflexive, transitive, symmetric, and subject to Leibniz's law are theorems of (I1) (Quine 1963, 13). Pure nihilism thus does enjoy greater ideological simplicity *vis-à-vis* any system that incorporates (P1)-(P3)—if the story would end here.

	Pure Nihilism	Core Mereology
Primitives	'A' and '='	'P' and '='
Axioms	(A1) Ax	(P1) Pxx
		$(P2) (Pxy \land Pyz) \rightarrow Pxz$
	(I1) $\phi(y) \leftrightarrow \exists x(x=y \land \phi(x))$	$(P3) (Pxy \land Pyx) \to x=y$
		(I1) $\phi(y) \Leftrightarrow \exists x(x=y \land \phi(x))$
Definitions		<i>PP</i> , <i>O</i> , <i>D</i> , etc.

Table 1. Pure Nihilism v. Core Mereology

However, the story does not end here. Note that both the pure nihilist and the mereologist take '=' as a primitive term. But, as is well known, '=' can be defined in terms of 'is part of'. One way to do this is by adding the following axiom-schema (where ' ϕ ' is any formula in the language) to (P1) and (P2):

$$(P3') (Pxy \land Pyx) \rightarrow (\phi(x) \leftrightarrow \phi(y))$$
 (Equivalence)

and add the following definition to (P1), (P2), and (P3'):

(D5) $x = y =_{df} Pxy \land Pyx$ (Definition of Identity)⁹

⁹ One can see now that if one accepts Core Mereology (but no theory stronger than that) *and* one wishes to define '=' as in (D5), then one should not take 'is a proper part of' as the primitive

This definition is already mentioned by Leonard and Goodman (1940, 47, fn.6) while Goodman (1951, 35) gives an alternative definition of '=' in terms of 'overlaps with' in *the Structure of Appearances*.¹⁰ To be sure: this definition does not commit one to the view that the concept of identity reduces to that of parthood. The definition only introduces the predicate '=' into the language via the predicate of parthood. It thus simplifies the ideology of the language and brings more unity to the system (see also Section 2).

Given this definition, the situation is as depicted in Table 2.

	Pure Nihilism	Tightened Core Mereology (TCM)
Primitives	'A' and '='	ʻP'
Axioms	(A1) Ax	(P1) Pxx
		$(P2) (Pxy \land Pyz) \rightarrow Pxz$
	(I1) $\phi(y) \Leftrightarrow \exists x(x=y \land \phi(x))$	$(P3') (Pxy \land Pyx) \to (\phi(x) \leftrightarrow \phi(y))$
		(I1) becomes a theorem in TCM
Definitions		<i>PP</i> , <i>O</i> , <i>D</i> , etc.
		Importantly: (D5) $x = y =_{df} Pxy \land Pyx$
	Table 2 Dure Mibiliam	" Tightened Core Mereology

Table 2. Pure Nihilism *v*. Tightened Core Mereology

If '=' is defined in terms of 'is part of', then the mereologist actually has an ideologically simpler theory than the pure nihilist. Sider is aware of this definition, but objects to it. In the next section we will consider his and another possible objection to providing such a definition of identity. But before that I should clarify two things.

First, one might wonder whether the pure nihilist could define 'is an atom' in term of '=' as follows:¹¹

term of Core Mereology. If 'is a proper part of' would be the primitive term, then 'is part of' should be defined thus: $P_{XY} =_{df} PP_{XY} \vee x = y$. But since this definition already contains '=', definition (D5) would be circular. Hence, in Core Mereology one can only take 'is part of' as primitive if one wishes to define '='. In stronger systems of mereology such as Extensional Mereology (Section 3 below) other notions can be taken as primitive, for example 'overlaps with' or 'is disjoint from'. If 'is disjoint from' is taken as primitive, then one can define 'is part of' as follows: $P_{XY} =_{df} \forall_{z} D_{zY} \rightarrow D_{zX}$. This would not result in a circular definition of '=' in terms of 'is part of'. Hence, it matters what term is taken to be primitive and how strong a system of mereology one wants to formalize if one wishes to define '='. For further discussion on primitive terms of mereology, see Parsons (2014). Many thanks to an anonymous referee of this journal for asking me to clarify this.

¹⁰ Note, though, that a definition of '=' in terms of 'overlap with' is only possible when the mereology is *extensional*, i.e., in the presence of an axiom like (P5) (see below) 'overlaps with' can be taken as primitive and function as part of the definiens of '='.

¹¹ To be sure, Sider (2013) does not suggest this definition.

$(D4') \quad Ax =_{df} x = x$

Adding (D4') to Pure Nihilism would result in a theory containing only one primitive ('='). Such a theory states that everything that is self-identical is an A (i.e., is an atom). Though this might *seem* like a proper formulation of nihilism, (D4') is compatible with *any* system of merelogy. It can thus not be the proper formulation of nihilism since nihilism is—and should be—incompatible with any mereology that allows for objects to have proper parts. To see the compatibility, note that all (D4') says is that self-identical entities may be called 'A' (or: 'is an atom'), but it does not entail that 'being an atom' has the same extension as the (D4)-notion of 'being an atom'. Hence, anyone accepting Core Mereology—or any other system of mereology—can accept (D4') as a (stipulative) definition about the term 'atom' and still hold that everything that is an atom has proper parts. Hence, (D4') cannot be the definition for 'is an atom' if 'x is an atom' means that x has no proper parts.

Second, one might think that it is also possible to define 'is composed of' (a mereological predicate) in terms of '='. The mereologist could then instead take '=' as primitive, define 'is composed of' in terms of '=' and define all the other mereological notions in terms of 'is composed of'. This is what Einar Duenger Bohn (2014) suggests when he proposes to define 'x is composed of the ys' as 'x is identical with the ys' (i.e., $xCyy =_{df} x=yy$).¹² Bohn thus defends the view that a composite object is literally identical to its parts (taken collectively)—this is a substantial claim known as 'Strong Composition as Identity'. It means that it is possible for many things to be identical to one thing, and that this identity relation is the same as the relation of composition.

Bohn's definition is only possible given two conditions: first, one has to accept first-order *plural* logic since the expression 'the xs' (or 'xx') is not well formed in standard first-order logic.¹³ Second, one has to allow—within a plural logic—that the identity sign can be flanked by singular-singular, singular-plural, and plural-plural variables and constants. This 'generalized' notion of identity that allows one to express singular-plural identity (such that 'x=yy' is a well-formed expression) is not part of standard plural logics which commonly only allow '=' to be flanked by terms that denote objects with the same cardinality. Indeed, Lewis (1991, 87) thought that the problem for singular-plural identity

¹² Thanks to an anonymous referee for this journal for asking me to discuss this.

¹³ For an exposition of plural logic, see Yi (2005; 2006).

is exactly that it is not easy to square with Leibniz's law, since the single object is *one* thing whereas the plurality are *many* things but *being one* and *being many* do not seem to be the same property. Moreover, singular-plural identity seems to distort the behaviour of 'is one of' (the key term in plural logic)¹⁴ (Sider 2007, 57–58).¹⁵ But if plural logic turns out to be unobjectionable and a notion of identity that allows for singular-plural identity can be made consistent with Leibniz's law, then the mereologist may indeed take only (generalized) identity as primitive and define the mereological notions on the basis of this primitive.

The definition of '=' in terms of 'is part of' that is suggested in this paper is far more conservative and less original. It does not suppose plural logic and it allows one to remain neutral on the question whether a plurality can be identical to a single object, since only first-order (singular) logic is needed. Such a definition *does* show that one allows for a strong connection between mereological relations and identity, but this connection need not be so intimate that composition is—literally—the many-one identity relation. (See also Section 3 below, where I argue that philosophers attracted to the idea that composition is analogous to identity should consider defining '=' in terms of a mereological predicate.)

Thus, '=' can be defined via 'is part of' and this allows the mereologist to have only one primitive term in her language. But are there reasons for not doing so?

2. Why '=' should not be defined in terms of 'is part of'

Sider objects to defining '=' in terms of 'is part of':

[C]onsider the objection that adopting parthood in fundamental theories allows the elimination of identity from ideology via the definition " $x = y =_{df} x$ is part of y and y is part of x". The savings in ideological parsimony would be outweighed by increased complexity in the laws, which I take to include laws of logic and metaphysics. The logical laws governing '=' must now be rewritten in terms of the proposed definition, making them more complex; and further, the laws of mereology will be needed. (Sider 2013, fn. 10)

¹⁴ Which in turn may make the thesis that composition is the many-one identity relation equivalent to the claim that there are no composite objects. For discussion, see Calosi (2015).

¹⁵ There is much more to say on this subject, and the relations between counting, composition, and identity are intricate. See, for a start, (Wallace 2011a; Wallace 2011b; Cotnoir 2013)

Let's go over these two objections one at the time. The first concerns the need to rewrite the laws of '=' in terms of 'is part of' and that this would increase the complexity of the former. It is, however, not clear to me why the laws governing '=' (call them 'identitylaws') should be rewritten in terms of the proposed definition when we define '=' via 'is part of'. Once '=' is defined, we can simply let the identity-laws govern it. That is to say: when we have laws in a theory, T_1 , governing A and we discover that we can define 'A' in terms of a notion, 'B', in T_2 then this does not mean that we have to rewrite the laws in T_1 in terms of 'B'. Similarly, we need not rewrite the laws of identity once we define it. The only real change is that those identity laws are not as fundamental as we might have thought: they apply to a notion that is not primitive in the theory but defined. Furthermore, given (P1), (P2), (P3'), and (D5) the laws of identity (that it is reflexive, transitive, symmetric, and obeys Leibniz's law) are *theorems*; they simply follow from (P1), (P2), (P3') and (D5). It is, however, unclear to me why we should count the number of theorems of a theory as adding to its ideological complexity.

But let us assume-for sake of the argument-that we must rewrite the identity laws in terms of 'is part of'. This would result in an increase in syntactic complexity: the number or the complexity of the laws governing '=' would increase. However, increase in syntactic complexity cannot-by itself-be a reason not to eliminate a primitive term via a definition, since any definition of a term previously taken as primitive will increase the syntactic complexity of a theory. To see this, imagine a theory T_3 that contains two primitive terms and for each primitive term two laws governing it. At a certain point, someone proves that in T_3 one can eliminate one primitive, say 'A', by defining it in terms of the other. The two laws governing 'A' now have to be rewritten in terms of the only primitive in T_3 (making these laws more complex) and the definition for 'A' has to be added. There is clearly an increase in syntactic complexity in T_3 but this is the price one has to pay when eliminating primitives. There is no way to avoid this increase in syntactic complexity if adding a definition would count as an increase in syntactic complexity and if we have to rewrite the laws of the now-defined (but previously primitive) term. The crucial question is whether the increase in ideological simplicity is worth the increase in syntactic complexity.

This is a difficult matter since I do not know of a standard test to assess how much simplification of ideology is worth a certain increase in syntactic complexity. There are, however, clear cases in which it is not worth the increase: Assume that the two laws governing 'A' as a primitive term in T_3 consist both of four atomic expressions (linked by logical constants). Assume that by eliminating A via a definition we have to rewrite the laws that govern 'A' in such a way that each of these laws would consist of 50 atomic expressions each. This is probably not worth the increase in ideological simplicity. Alternatively, if both laws are rewritten such that each law gains one atomic expression only, then this is probably worth the increase in ideological simplicity.

Going back to our problem of reformulating the laws of identity in terms of 'is part of', I would argue that this case is more like the latter given that the number of atomic expressions in each law governing identity would only increase by a factor two at most. This is evident from (D5) where '=' is defined in terms of 'is part of': each instance of 'x = y' (one atomic expression) has to be rewritten in terms of 'Pxy \land Pyx' (two atomic expressions). Thus, yes, defining '=' in terms of 'is part of' results in an increase in syntactic complexity (if we also have to count the theorems!) but eliminating a primitive term *always* increases syntactic complexity if the laws governing the now-defined term have to be rewritten in terms of the primitive(s) left. The crucial question is whether the increase in syntactic complexity is worth it, and—*pace* Sider—the case of defining '=' in terms of 'is part of' seems to be a perfect example of an instance where the simplification of ideology is worth the increase in syntactic complexity.

Before discussing the second objection of Sider, note that the objection from syntactic complexity may be a *tu qouque* for nihilism. Nihilism denies the existence of composite objects, which can make some principles of, for example, quantum mechanics quite difficult to express, since these principles *prima facie* make reference to composite systems.¹⁶ All these principles have to be rewritten in nihilistic vocabulary, which could make the syntactic complexity of these principles very complex. Sider (2013, 284ff) mentions this and his solution involves accepting set theory and its ideology (the primitive ' \subseteq ' of membership). He does not, however, think this is an additional ideological cost because everyone will have to accept set theory—except when Hartry Field's (1980; 1989) program for a nominalistic mathematics turns out to be successful (Sider 2013, 286ff). I will not dwell on this point here, but simply flag it to point out that anyone who accepts mereology and thinks she can do without set theory could point out

¹⁶ That the systems quantum mechanics speaks of can be viewed as mereological wholes is argued for by Calosi, Fano, and Tarozzi (2011); and Calosi and Tarozzi (2014). Cf. Maudlin (1998).

that Sider's nihilism is ideologically more complex than mereology since the former also needs '€' as a primitive.

Sider's second objection is that in order to define '=' in terms of 'is part of' one needs to accept the laws of mereology. Two comments on this: first, the objection might seem to beg the question against the mereologist. When we compare two theories with respect to ideological parsimony it is justified to limit in each theory the number of primitives via definitions in order to see what the minimal number of primitives is to state the theory. To decrease the number of primitives via definitions we may use the principles that are part of that theory since we are, after all, comparing the ideological simplicity of the whole theory. So, yes, in order to define '=' in terms of 'is part of' we need to make use of mereological principles. But we are comparing the ideological cost of nihilism *vis-à-vis* the ideological cost of a mereological system that uses 'is part of', thus there is nothing wrong with using mereological principles to define identity in terms of 'is part of'.

However, I think Sider's point can also be read in a non-question-begging way as analogous to the one above concerning syntactic complexity. According to this reading the concern is not about defining '=' in terms of 'is part of' but it is rather an independent though related argument in favor of pure nihilism based on syntactical rather than ideological simplicity. This argument would be that mereology needs more axioms than nihilism.

The argument is hard to evaluate because it is an open question what the tradeoff is between, on the one hand, the number and complexity of primitives, and, on the other, the number and complexity of the axioms of a theory. The difference between (tightened) Core Mereology and Pure Nihilism is minimal, as can be seen in Table 2 above. Note that the pure nihilist has to take into account one 'law of identity'; i.e., Wang's law—from which she can derive that '=' stands for an equivalence relation obeying Leibniz's law. The mereologist who defines '=' in terms of 'is part of' can also derive these properties of the relation expressed by '=', but she accepts a 'parthood-version' of Leibniz's law (P3') as an axiom.¹⁷ The nihilist thus has to accept two axioms (Wang's law and (P4)); this brings her to a total of two primitives ('is an atom' and '=') and two axioms. The mereologist, on the other hand, has only one primitive ('P') and

¹⁷ Strictly speaking, it is an axiom-schema. Since the nihilist and the mereologist both need one axiom-*schema* they are on equal footing in that respect. Hence, for convenience, I use the term 'axiom' as meaning 'proper axiom or axiom-schema'.

three axioms (P1), (P2), and (P3'). So the (core) mereologist does better than the pure nihilist with respect to the number of primitive terms but worse with respect to the number of axioms.¹⁸

We would thus need a principle that tells us under what conditions having fewer and simpler axioms trumps having fewer and simpler primitive terms. Absent such a principle, we may say that pure nihilism is syntactically simpler—strictly speaking—than any version of mereology, since nihilism has fewer axioms than mereology. But it is unclear whether mereology cannot make up for this by being ideologically simpler than pure nihilism.¹⁹

Taking stock: Sider argues that pure nihilism is ideologically simpler because it need not use the primitive 'is part of' and can thus have fewer primitives than any system of mereology. In this section I showed that the number of primitives of nihilism is actually two: next to '=' the nihilist needs to accept another primitive. If this other primitive is one-place, like 'is an atom', than it is arguably simpler than 'is part of'. Hence Sider's argument might still go through in a revised form: *both pure nihilism and mereology have two primitives, but the primitive term of nihilism is simpler than 'is part of', hence nihilism should be preferred on the basis of ideological simplicity.* This revised argument, however, is also unsound because a mereologist can reduce the number of primitives to one by defining '=' in terms of 'is part of'—hence mereology is actually ideologically simpler than pure nihilism. Yet a different version of Sider's argument, based on syntactical rather than ideological simplicity, might still to go through, since (tightened) Core Mereology has more axioms than nihilism even when '=' is defined by 'is part of'.

Thus far I have only discussed Sider's argument against defining '=' in terms of 'is part of'. The remainder of this section is devoted to a more general objection one might have to such a definition.

Objection: Identity is more fundamental (or general) than parthood and the former should thus not be defined in terms of the latter.

Reply. The objection here is that even though defining '=' in terms of 'is part of' simplifies the ideology of the theory there is something wrong with such a definition.

¹⁸ Thanks to Staffan Angere for helping me to see this more clearly.

¹⁹ This issue is further complicated by the fact that stronger theories of mereology, such as General Extensional Mereology, also known as 'Classical Mereology', can be formulated with only two axioms. See for discussion of the axiomatization of classical mereology: Hovda (2009)

What is wrong with such a definition, the objection states, is that it does not follow the order of fundamentality or that of generality.²⁰ (I will use the terms 'fundamental' and 'general' rather interchangeably in this section since the gist of the objection lies in there being some property had by identity—being more general or more fundamental—that is not had by parthood, which prohibits one from defining identity in terms of parthood.)

In discussing the objection I operate on the methodological principle that theories should strive for (recognized) theoretical virtues such as explanatory power, ontological parsimony, ideological simplicity, etc.²¹ Sider seems to hold that ideologically simpler and ontologically more parsimonious theories are better at 'carving nature at its joints' *because* they are simpler:

Quine's advice for *ontological* beliefs is familiar: believe the ontology of your best theories. Theories are good insofar as they are simple, explanatorily powerful, integrate with other good theories, and so on. (...) A good theory isn't merely likely to be *true*. Its ideology is also likely to carve at the joints. For the conceptual decisions made in adopting that theory—and not just the theory's ontology—were vindicated; (...) So we can add to the Quinean advice: regard the ideology of your best theory as carving at the joints. (Sider 2011, 12)

This puts some pressure on the objection under consideration: if '=' really is more fundamental and general *and* fundamental and general notions should be taken as primitive, then how can it be that a theory that defines '=' is ideologically simpler than a theory that does not define it? Indeed, Richard Sharvy (1983, 234) holds that—contrary to what the objection states—parthood is more basic than identity precisely because '=' can be defined in terms of 'is part of'.

But ideological simplicity is but one aspect of rational theory choice. Assume that theory T_1 is ideologically simpler than theory T_2 . It may then still be that T_2 is a better theory because it does better with respect to other virtues such as ontological parsimony and explanatory power. However, by defining '=' the mereologist does not postulate

²⁰ Thanks to an anonymous referee of this journal for pressing me to discuss this objection.

²¹ To repeat: by defining identity in terms of parthood, one is not committed to the view that the concept of identity is reducible to that of parthood. The definition simply ensures that '=' is not a primitive term in the language.

extra entities; so the ontology is as parsimonious as that of the mereologist who takes '=' as primitive.²² Neither does the theory lose explanatory power by defining '='.

So none of the standard theoretical virtues concerning simplicity is lost by defining '=' in terms of 'is part of'—instead the theory becomes ideologically simpler. This means that in order for the objection to come off the ground, there should be a different theoretical virtue (call it 'virtue-X') that a mereology lacks when '=' is defined by 'is part of'. If virtue-X does not exist, then there is no reason to give up a recognized theoretical virtue. Moreover, virtue-X should be such that it *trumps* ideological simplicity, because the objection states that virtue-X is the reason for not defining '=' in terms of 'is part of'.

Unfortunately for our objector, it seems that virtue-X does not exist. Take, for example, Quine who is certainly of the opinion that identity is a very general and basic notion:

One respect in which identity theory seems a nearer neighbor to logic than to mathematics is its completeness. (...) Another respect in which identity theory seems more like logic than mathematics is universality: it treats of all objects impartially. (...) [T]he only values of the variables that matter to number theory, for instance, or set theory, are the numbers and the sets; whereas identity theory knows no preference.

This latter trait suggests that identity theory, like quantification theory, is peculiarly basic. This is suggested also by the following fact: as soon as we have merely specified the truth-functional notations, the variables, and the open sentences of a language, we have already settled enough to know what to count as an adequate definition of identity for that language. (Quine 1970, 62)

For Quine, identity theory is closer to logic (and thus more fundamental and general) than any part of mathematics; even stronger: identity 'belongs to logic' (Ibid. 64).

²² To say that a mereologist postulates more entities than the nihilist and that mereology is thus less parsimonious than nihilism is not necessarily true and, moreover, irrelevant to the objection under consideration. It is not necessarily true since, for one, the mereologist may hold that the world contains only seven objects (i.e., three atoms and four composite objects), while the nihilist may postulate the existence of eight atoms. (This is, admittedly, an unlikely scenario.) It is, more importantly, beside the point: the objection is that the mereologist who defines '=' in terms of 'is part of' does worse with respect to a certain theoretical virtue. If this were the case, then that virtue should be (re-)gained by *not* defining '='. Since the ontological commitments of mereology do not change whether '=' is defined, ontological parsimony cannot be the relevant virtue in play.

If virtue-X were to exist, or if Quine thought that there were something like virtue-X, then he would thus object to defining '='. But Quine is more than happy to provide definitions of '='; sometimes in terms of membership, at other times by exhausting the predicates of the language (Quine 1951a, 134–136; 1963, 13–15; 1970, 63–64). So Quine holds that identity is closely tied to quantification and that it is a basic and absolutely general notion. All this is, however, no reason for him to take it as a primitive term.

As another example, consider the notion of truth. As a general term that is applicable to any language (natural or formal) 'true sentence' seems indefinable. Thus, we should take 'true sentence' as primitive in those cases. Yet, when we restrict ourselves to formal languages 'true sentence' becomes definable in terms of 'satisfaction'—which is defined in terms of 'sentential functions' and 'sequences of classes' (Tarski 1956). The notions of sentential function and sequence of classes are, however, not as general and fundamental as truth: they are only applicable in formal languages, though truth applies to all kinds of languages. But this seems to be no objection to Tarski's definition of 'true sentence'.

Why do Quine and Tarski have no problems with such definitions? Arguably because there is no virtue-X that speaks against such definitions. I might be wrong on this. To prove me wrong, however, one must point out what virtue-X is. Simply saying that it is wrong *when doing metaphysics* to give definition of more basic or general concepts in terms of less basic or general concepts is an ad hoc response. There has to be a theoretical virtue that speaks in favor of keeping certain terms primitive even when they can be defined. And since ideological simplicity is a good—defeasible—guide in finding better theories, virtue-X thus has to be an even better guide.

3. Why a mereologist may want to define '='

In this section I provide positive arguments for defining '=' in terms of a mereological predicate. The obvious reason is that such a definition decreases the number of primitive terms in mereology thus making the overall theory ideologically simpler. The other reasons I provide in this section are of limited force, because one motivation for defining a term is to make it, or the entity it refers to, better understood; but I doubt that identity is better understood when it is defined. There is not much—if any—justification for defining terms other than the simplification of the ideology; theories with fewer primitive

terms might be aesthetically more appealing or better to understand, but this is not in general the case. So a general argument for defining '=' other than the gain in ideological simplicity is hard to give. Moreover, both my arguments for defining identity depend on the fact that extensional mereologies postulate an intimate connection between identity and mereological relations such as parthood. But the fact that two entities are intimately related according to a theory is not, in and of itself, a reason to define one term expressing one of these entities by the other term expressing the other entity.

Two more mereological principles need to be added to the discussion before I can present my arguments for defining '='. These principles provide the strong link between various mereological relations and identity. The first principle to add is a supplementation principle that gives expression to the idea that a whole cannot be decomposed into one proper part. A quite strong formulation of this idea is given by the following principle:

(P5)
$$\neg Pyx \rightarrow \exists \chi (Pzy \land \neg Ozx)$$
 (Strong Supplementation)

Adding (P5) to (P1), (P2), and (P3') results in a theory that is usually called 'Extensional Mereology'. Extensional Mereology can be formulated based on a different primitive (e.g., 'overlaps with', 'is a proper part of' or 'is disjoint from'). Such alternative formulations also allow one to give alternative mereological definitions of '='. This is relevant because my arguments for defining '=' in terms of a mereological predicate depend crucially on the fact that for extensional mereologists, mereological relations in *general* (not just parthood) are intimately related to identity.

The term 'Extensional Mereology' is apt, since by adding (P5) the following theorem can be proven:²³

(iv)
$$\forall z (Ozy \rightarrow Ozx) \rightarrow Pyx$$
.

²³ *Proof.* It is only the consequent of (T1) that we need to prove, which is a bi-conditional. Leftto-right is an instance of $x = y \rightarrow (\phi(x) \leftrightarrow \phi(y))$ with 'PPz_' for ' ϕ '. (Note that $x = y \rightarrow (\phi(x) \leftrightarrow \phi(y))$) follows from the definition of identity (D5) together with the parthood-version of Leibniz's Law (P3').) For the right-to-left part, we first prove that overlapping the same objects is sufficient for identity, i.e. (EO) ($O_{3x} \leftrightarrow O_{3y}$) $\rightarrow x = y$.

Assuming that $\neg Pyx$ we get via Strong Supplementation (P5) that: (i) $\exists z (Pzy \land \neg Ozx)$.

From (i) and the fact that P₂ implies O₂ we can derive: (ii) $\neg \forall_{\tilde{\chi}} (O_{2} \rightarrow O_{\tilde{\chi}})$. Thus:

⁽iii) $\neg Pyx \rightarrow \neg \forall z (Ozy \rightarrow Ozx)$; which we can contrapose to:

The antecedent of (EO) is: (v) $O_{XY} \Leftrightarrow O_{XX}$; which gives us, by (iv): (vi) $P_{XY} \land P_{YX}$. From the definition of identity (D5) and (vi) we have (EO).

(T1)
$$\exists_{\mathcal{Z}} (\operatorname{PP}_{\mathcal{Z} \mathcal{X}} \vee \operatorname{PP}_{\mathcal{Z} \mathcal{Y}}) \rightarrow (x = y \leftrightarrow \forall_{\mathcal{Z}} (\operatorname{PP}_{\mathcal{Z} \mathcal{X}} \leftrightarrow \operatorname{PP}_{\mathcal{Z} \mathcal{Y}}))$$
 (Extensionality Principle)

(The antecedent of (T1) is to ensure that a composite object is picked-out.) The Extensionality Principle (hereafter 'EP') is controversial, since it rules out objects that share all their proper parts but that fail to be identical. Thus, for example, if a statue and its piece of clay have the same proper parts, then, according to EP, the statue = the clay.

Note, also, that the left-to-right reading of the consequent is an instance of Leibniz's law using 'proper parthood'. The right-to-left reading could be called the 'Identity of Mereological Indiscernibles'. Thus, by EP (with x and y being *composite* objects):

(1) $x = y \rightarrow \forall z (PPzx \leftrightarrow PPzy)$ (Leibniz's law on Proper Parthood) (2) $(PPzx \leftrightarrow PPzy) \rightarrow x = y$ (Identity of Mereological Indiscernibles)

It might seem that (1) is too strong: we tend to think that an object can stay identical while changing parts. But doesn't (1) rule this out? However, any quarrels with (1) of this kind are similar to the more general problem of change in relation to Leibniz's law: an object can—it seems—stay identical while changing properties, but Leibniz's law seems to rule this out. However, there are various solutions to the (general) problem of change that can also be used to solve (apparent) problems with (1).

What is more substantial and controversial is (2), since it is this part of EP that entails that coincident objects are identical. Arguments for the possibility of coincident *non-identical* objects usually take the following form:

- (3) x and y share all their proper parts.
- (4) x has a property, F, that y does not have.
- (5) Hence, by Leibniz's law and (4), $x \neq y$.

Notice that from the definition of proper parthood (D1) and the definition of overlap (D2), it follows that if x and y are non-atomic objects, then if x and y have the same proper parts, anything that overlaps x also overlaps y:

 $^{(\}text{vii}) \exists \chi (\text{PP}_{\chi X} \lor \text{PP}_{\chi Y}) \rightarrow (\forall \chi (\text{PP}_{\chi X} \leftrightarrow \text{PP}_{\chi Y}) \rightarrow (\text{O}_{\chi X} \leftrightarrow \text{O}_{\chi Y})),$

given (EO) and the transitivity of implication, the consequent of T1 (right-to-left) follows. (Proof is essentially the same as the one in Cotnoir (2014, 17).)

Sharing of all proper parts, by (3), is an instance of coincidence. So, (3) and (5) are incompatible with (T1). A convinced *extensional* mereologist who accepts Leibniz's law will thus have to deny either (3) or (4). Which one of these is denied, depends on the specific form of the argument. In cases where an appeal is made to temporal notions (e.g., the clay existed before the statue) a convinced extensional mereologist often denies (3) by holding that objects also have *temporal* proper parts and that this difference in parts accounts for the difference in properties. In other cases (4) is denied by holding that, for example, we do not commonly say of y that it has property F, but this is simply due to our use of *language*; in reality y does have F, because x=y by (3) and (T1). The details of these two types of replies do not matter here.²⁴ What matters is that for the extensional mereologist mereological relations are fundamental for identity: if x and y stand in all the same mereological relations, they cannot fail to be identical.

This point becomes even clearer when we look at the more general counterpart of (2), namely the Identity of Indiscernibles. This latter principle is quite controversial at least since Max Black's (1952) two indiscernible spheres argument. I do not wish to defend (2) here—though I think it is true—but only point out its relevance for our discussion concerning the definability of '='. Someone who accepts (2) holds that standing in the same mereological relations is sufficient for identity. This means that mereological relations are given a special status in relation to identity: to determine whether x is identical with y it is sufficient to know in which mereological relations x and y stand.

If one thus defines '=' in terms of 'is part of', then the intimate relation between identity and parthood becomes more apparent than when both terms are taken as primitive. As said in the beginning of this section, the fact that two entities are intimately related in a theory does not entail that one should thus—if one can—define the term expressing one by the term expressing the other. Still, if one needs to motivate why one defines a term this seems to be quite a good one.

A second, related argument stems from the fact that some philosophers have taken '=' to be some sort of mereological predicate. This is a stronger, and less precise claim than that mereology is extensional. It is stronger since it means that mereological relations and identity belong to the same kind, whereas extensionality does not commit one to such a view. It is also less precise, because it is not clear what it is for relations to be of the same kind.

²⁴ For an excellent discussion of these issues, see Varzi (2008).

Still, the view that mereology is some sort of extended logic of identity has influential adherents. David Armstrong, for example, thinks that there are cases of partial identity (expressed in terms of 'proper parthood' and 'overlap') and that 'mereology, which deals with these notions, may be thought of as an extended logic of identity, extended to deal with such cases of partial identity.' (Armstrong 1997, 18) Similarly, David Lewis holds that:

[M]ereological relations are something special. (...)[T]hey are strikingly analogous to ordinary identity, the one-one relation that each thing bears to itself and nothing else. So striking this analogy is that it is appropriate to mark it by speaking of mereological relations—the many-one relation of composition, the one-one relations of part to whole and of overlap—as kinds of identity. Ordinary identity is the special, limiting case of identity in the broadened sense. (Lewis 1991, 84–85)

These ideas form the core of the debate concerning the Composition as Identity-thesis, which mainly focuses on whether composition is (a kind of) identity relation.²⁵ A very strong formulation of this idea is that composition literally is the many-one identity relation—a claim we came across in Section 1.

But both Armstrong and Lewis make a weaker and less precise claim: mereological relations (including parthood and overlap) are *kinds* of identity relations.²⁶ If one indeed thinks that mereology is an extension of the logic of identity, one could just as well reverse the order and hold that mereology is the logic of (general) identity, whereas the logic of pure identity (the notion expressed by '=') is only a part of this general logic. In that case, it makes sense to define '=' in terms of the only primitive ('is part of') of your 'general logic of identity' (i.e., mereology). By doing so, the claim that mereological relations are very much like identity becomes more precise: they are very much alike because they are all part of one theory: mereology.

Lewis (1991, 85–86) gives various examples of the analogy between mereological relations and identity: the parts of an object (collectively) have the same location as the whole object, just as x has the same location as y if x = y. Identity is unique, such that each object is identical with exactly one object; similarly, composition is unique because,

 ²⁵ For a good overview of the debate, see Baxter and Cotnoir (2014); and Wallace (2011a; 2011b)
 ²⁶ Cowling (2013, 3906) also suggests an ideological interpretation of the Composition as Identity-thesis.

given extensionality, no two composites can have exactly the same parts. Such examples of similarities, however, merely establish an analogy between mereological relations and identity. If one is convinced that there is more than just an analogy, then—again—one can make this conviction more precise by not giving '=' a special (primitive) status, but taking it to be defined in terms of a mereological predicate.

However, Armstrong and Lewis want something with more ontological bite since they took the analogy between identity and mereological relations to be such that mereological sums are ontologically innocent (Armstrong 1997, 12; Lewis 1991, 81–85). For them, the ontological commitment to a composite object is the same as the ontological commitment to its parts. At a first glance my re-interpretation of their view in terms of the ideology of the theory seems unrelated to ontological considerations.

However, the claim that mereology is ontologically innocent is maybe best understood as saying that measuring the ontology of a theory by counting the selfidentical objects the theory postulates is—for the merologist—not the best procedure.²⁷ It is current dogma that an inventory of the world (i.e., an ontology) should be constructed on the basis of (non-)identity: if x is not identical with y (but a theory recognizes both entities), then both x and y should be listed in the inventory *even if* x is part of y. But for the (extensional) mereologist, such an inventory will contain—what she considers—redundancies: if Tibbles the cat is on the list, then adding Tibbles's tail to the list is redundant for Tibbles is simply the sum of her parts. Hence, the ontological commitment to the parts of Tibbles seems to be no additional commitment once the extensional mereologist has committed herself to the existence of Tibbles.

This motivates Varzi to propose a different way of drawing up an inventory of the world—'The Minimalist View'—which states that '[a]n inventory of the world is to include an entity x if and only if x does not [mereologically] overlap any other entity y that is itself included in that inventory.' (Varzi 2000, 285) The Minimalist View tells us that one can either list Tibbles the cat or all the parts of Tibbles (and either list would be a correct one) but one should not put both the cat and its parts on the list.²⁸

²⁷ For discussion, see Hawley (2014), Varzi (2000; 2014), and Smid (2015).

²⁸ Franz Brentano may have had something similar in mind: '[I]t would be wrong to suppose that the two parts of a thing taken together constitute an additional third thing. For where we have an addition the things that are added must have no parts in common. Thus we may say, for example, that a triangle has three angles, but not that is has three pairs of angless [sic]: angles A and Bform a pair, as do B and C, and also C and A, but each of these pairs has a part in common with each of the others.' (Brentano 1981, 16)

If the mereologist can defend the Minimalist View, then it is the mereological overlap–disjointness pair and *not* the identity–non-identity pair that determines whether or not an entity should be listed alongside another entity in one's ontology. In that case, there is no reason for her to award '=' a special status in the language since an inventory of the world depends solely upon the fact which entities (fail to) overlap with which other entities.

4. Conclusion

I have argued that, contrary to what Sider claims, pure nihilism is not ideologically simpler than any system of mereology, since the latter does not need to take '=' as primitive, whereas pure nihilism does. Pure nihilism can only be formulated with at least two primitive terms, whereas mereologies can be formulated by using only one. After neutralizing some objections to defining '=' I gave reasons in favour of such a definition.

The working assumption during the whole discussion has been that pure nihilism and systems of mereology are equal in all other respects. That is, I have assumed that they do not differ with respect to explanatory power or ontological parsimony. This is of course a gross simplification of the actual situation and a rational choice between these competing theories should also take these other matters into account. Here I have only discussed the ideological part of the debate; to decide the whole issue other parts need to be settled too.

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