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Basic Concepts of Formal Ontology

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Abstract. The term 'formal ontology' was first used by the philosopher Edmund Husserl in his *Logical Investigations* to signify the study of those formal structures and relations – above all relations of part and whole – which are exemplified in the subject-matters of the different material sciences. We follow Husserl in presenting the basic concepts of formal ontology as falling into three groups: the theory of part and whole, the theory of dependence, and the theory of boundary, continuity and contact. These basic concepts are presented in relation to the problem of providing an account of the formal ontology of the mesoscopic realm of everyday experience, and specifically of providing an account of the concept of individual substance.

1. Basic History of Formal Ontology

We owe the idea of a formal ontology to the philosopher Edmund Husserl, whose *Logical Investigations* [1] draws a distinction between *formal logic*, on the one hand, and *formal ontology,* on the other. Formal logic deals with the interconnections of truths (or of propositional meanings in general) – with inference relations, consistency, proof and validity. Formal ontology deals with the interconnections of things, with objects and properties, parts and wholes, relations and collectives. As formal logic deals with properties of inferences which are formal in the sense that they apply to inferences in virtue of their form alone, so formal ontology deals with properties of objects which are formal in the sense that they can be exemplified, in principle, by objects in all material spheres or domains of reality [2].

Husserl's formal ontology is based on mereology, on the theory of dependence, and on topology. The title of his third Logical Investigation is "On the Theory of Wholes and Parts" and it divides into two chapters: "The Difference between Independent and Dependent Objects" and "Thoughts Towards a Theory of the Pure Forms of Wholes and Parts". Unlike more familiar 'extensional' theories of wholes and parts, such as those propounded by Lesniewski (whom Husserl influenced), and by Leonard and Goodman (see [3]), Husserl's theory does not concern itself merely with what we might think of as the vertical relations between parts and the wholes which comprehend them on successive levels of comprehensiveness. Rather, his theory is concerned also with the horizontal relations between co-existing parts, relations which serve to give unity or integrity to the wholes in question.

To put the matter simply: some parts of a whole exist merely side by side, they can be destroyed or removed from the whole without detriment to the residue. A whole, all of whose parts manifest exclusively such side-by-sideness relations with each other, is called a heap or aggregate or, more technically, a purely summative whole. In many wholes, however, and one might say in *all* wholes manifesting any kind of unity, certain parts stand to each other in formal relations of what Husserl called *necessary dependence* (which is sometimes, but not always, necessary *interdependence*). Such parts, for example the individual instances of hue, saturation and brightness involved in a given instance of colour, cannot, as a matter of necessity, exist, except in association with their complementary parts in a whole of the given type. There is a huge variety of such lateral dependence relations, giving rise to a correspondingly huge variety of different types of whole which the more standard approaches of extensional mereology are unable to distinguish [4].

The topological background of Husserl's work makes itself felt already in his theory of dependence[5] . It comes most clearly to the fore, however, in his treatment of the notion of *fusion*: the relation which holds between two adjacent parts of an extended totality when there is no qualitative discontinuity between the two [6]. Adjacent squares on a chess-board array are not fused together in this sense; but if we imagine a band of colour that is subject to a gradual transition from red through orange to yellow, then each region of this band is fused with its immediately adjacent regions. In the field of what is experienced perceptually, then, we can draw a distinction,

> between *intuitively* separated contents, contents *set in relief from* or *separated off from* adjoining contents, on the one hand, and contents which are *fused* with adjoining contents, or which *flow over* into them without separation, on the other ([1], Investigation III, § 8, p. 449).

That Husserl was at least implicitly aware of the topological aspect of his ideas, even if not under this name, is unsurprising given that he was a student of the mathematician Weierstrass in Berlin, and that it was Cantor, Husserl's friend and colleague in Halle during the period when the *Logical Investigations* were being written, who first defined the fundamental topological notions of open, closed, dense, perfect set, boundary of a set, accumulation point, and so on. Husserl consciously employed Cantor's topological ideas, not least in his writings on the general theory of (extensive and intensive) magnitudes which make up one preliminary stage on the road to the formal ontology of the *Logical Investigations*. (See [8], pp. 83f, 95, 413; [1], "Prolegomena", §§ 22 and 70.)

In what follows we shall outline the basic concepts of formal ontology as Husserl conceived it, concentrating our remarks on the specific case of the world of everyday human experience. We shall thus cover some of the same ground that is covered by Patrick Hayes and others working in the territory of 'naïve physics' [9]. Most recently, formal ontology has been used as a tool of knowledge representation [10], in ways which draw on the insight that the categories of formal ontology, because they are fundamental to a wide variety of different domains, can be fruitfully employed in providing frameworks for translating between knowledge systems constructed on divergent bases.

2. Mereology vs Set Theory

When modern-day philosophers and those working in the artificial intelligence field turn their attentions to ontology, they standardly begin not with mereology or topology but with set-theoretic tools of the sort that are employed in standard model-theoretic semantics.

The rationale for insisting on a mereological rather than a set-theoretic foundation for the purposes of formal ontology can be stated as follows. The world of everyday human experience is made up of objects extended in space, objects which thus manifest a certain sort of boundary-continuum structure. The standard set-theoretic account of the continuum, initiated by Cantor and Dedekind and contained in all standard textbooks of the theory of sets, will be inadequate as a theory of this everyday 'qualitative' continuum for at least the following reasons:

1. The application of set theory to a subject-matter presupposes the isolation of some basic level of urelements in such a way as to make possible a simulation of all structures appearing on higher levels by means of sets of successively higher types. If, however, as holds in the case of investigations of the ontology of the experienced world, we are dealing with mesoscopic entities and with their mesoscopic constituents (the latter the products of more or less arbitrary real or imagined divisions along a variety of distinct axes), then there are no urelements fit to serve as our starting-point [11].

2. Experienced continua are in every case concrete, changing phenomena, phenomena existing in time; they are wholes which can gain and lose parts. Sets, in contrast, are abstract entities, existing outside time, which are defined entirely, and once and for all, via the specification of their members. Now certainly, in order to do justice to the changing relations between parts and wholes in the realm of mesoscopic objects, we might conceive of a theory analogous to the theory of sets which would be constructed on the basis of a tensed or time-indexed membership relation. To this end, however, we would need to sacrifice extensionality and those other beneficial features which had made the theory of sets attractive as a tool of ontology in the first place.

3. In the absence of points or elements, the experienced continuum seems not to sustain the sorts of cardinal number constructions imposed by the Cantor-Dedekind approach. That is to say, the experienced continuum seems not to be isomorphic to any real-number structure; indeed, standard mathematical oppositions, such as that between a dense and a continuous series, here find no application. Employing set theory as a tool of ontology thus brings new problems of its own, problems which are artefacts of the theory itself and which seem not to reflect features of the intended domain of application.

4. Most set-theoretical constructions of the continuum are predicated on the highly questionable thesis that out of unextended building blocks an extended whole can somehow be constructed (See [12], [13]). The experienced continuum, in contrast, is organized not in such a way that it would be built up out of unextended points, but rather in such a way that the wholes, including the medium of space, come before the parts which these wholes might contain and which might be distinguished on various levels within them.

Of course, set theory is a mathematical theory of tremendous power, and none of the above precludes the possibility of reconstructing topological and other theories adequate for ontological purposes also on a set-theoretic basis. Standard representation theorems indeed imply that for any precisely specified topological theory formulated in non-set-theoretic terms we can find an isomorphic set-theoretic counterpart. Even so, however, the reservations stated above imply that the resultant set-theoretic framework could yield at best a somewhat ramshackle *model* of the experienced continuum and similar structures, not a theory of these structures themselves (for the latter are after all *not sets* – in light of the categorial distinction mentioned under 2. above).

Our suggestion, then, is that mereotopology, the combination of mereology and topology, will yield a more adequate framework, allowing formal-ontological hypotheses to be formulated in a more direct and straightforward fashion, than is the case if formal ontologists are constrained to work with set-theoretic instruments. Approaches based on mereology allow one to start not with putative atoms or points but rather with the mesoscopic objects themselves by which we are surrounded in our normal day-to-day activities. The mereologist sees reality as being made up not of atoms, on the one hand, and abstract (1- and n-place) 'properties' or 'attributes' or 'worlds', on the other, but rather of you and me, of your headaches and my sneezes, of your battles and my wars, or in other words of bulky individuals of different sorts linked together by means of various different sorts of relations, including mereological and topological relations and relations of ontological dependence.

3. The Ontology of Substance and Accident

3.1 Substances and Accidents

To provide an illustration of one field of application of formal ontology we consider the domain of mesoscopic reality that is given in ordinary human experience. This mesoscopic reality is divided at its natural joints into what, following Aristotle, we shall call 'substances'. Examples of substances are: animals (including human beings), logs of wood, rocks, potatoes, and forks. Substances have various properties (qualities, features, attributes) and they undergo various sorts of changes (processes, events), for all of which we shall employ, again following Aristotle, the term 'accident'. Examples of accidents are: whistles, blushes, speakings, runnings, my knowledge of French, the whiteness of this cheese, and the warmth of this stone. Other sorts of denizens of mesoscopic reality (for example holes [14], spatial regions [15], the niches into which mesoscopic substances fit, cultural and institutional objects) will not be treated here, though objects of these sorts, as well as the microscopic objects which serve in some sense as the core material of mesoscopic reality, would need to be dealt with in a more extended treatment.

Accidents, like substances, are individual denizens of reality. My headache, like my lump of cheese, exists here and now, and both will cease to exist at some time in the future. Substances and accidents are nonetheless radically different in their ontological makeup. Substances are that which can exist on their own, where accidents require a support from substances in order to exist. Substances are the *bearers* or *carriers* of accidents, and accidents are said to 'inhere' in their substances [16]. These relations between substance and accident will be defined more precisely in what follows in terms of the concept of *specific dependence.*

Substances are such that, while remaining numerically one and the same, they can admit contrary accidents at different times: I am sometimes hungry, sometimes not; sometimes suntanned, sometimes not. Substances thus endure through time. Accidents, in contrast, exist in such a way that their existence is portioned out through time, and they exist in full at every time at which they exist at all. Thus accidents have temporal parts, for example the first 5 minutes of my headache, the first three games of the match. The latter have no counterpart in the realm of substances: the first 5 minute phase of my existence is not a part of me but of that complex accident which is my life. Substances and accidents thus form two distinct though intimately interconnected orders of being.

3.2 Substances, Collectives, Relations

Substances are unities. They enjoy a certain natural completeness or rounded-offness, being neither too small nor too large – in contrast to the undetached parts of substances (my arms, my legs) and to heaps or aggregates and to complexes or collectives of substances such as armies or football teams. A substance takes up space. It is an 'extended spatial magnitude' which occupies a place and is such as to have spatial parts. It is not merely spatially extended, but also (unlike other spatially extended objects such as places and spatial regions) such as to have divisible bulk, which means that it can in principle be divided into separate spatially extended substances. The material bulkiness of substances implies also that, unlike shadows and holes, substances compete for space, so that no two substances can occupy the same spatial region simultaneously.

Substances are often joined together into more or less complex collectives, ranging from families and tribes to nations and empires. Collectives are real constituents of the furniture of the world, but they are not additional constituents, over and above the substances which are their parts. Collectives inherit some, but not all, of the ontological marks of substances. They can admit contrary accidents at different times. They have a certain unity. They take up space, and they can in principle be divided into separate spatially extended sub-collectives, as an orchestra, for example, may be divided into constituent chamber groups. Collectives may gain and lose members, and they may undergo other sorts of changes through time.

Accidents, too, may form collectives (a musical chord, for example, is a collective of individual tones). In the realm of accidents, however, we can draw another sort of distinction that offers a parallel to the distinction between collectives and individual substances. This is the distinction between relational accidents on the one hand and non-relational (or one-place) accidents on the other. Non-relational accidents are attached, as it were, to a single carrier, as a thought is attached to a thinker. Accidents are relational if they depend upon a plurality of carriers and thereby join the latter together into complex wholes of greater or lesser duration. Examples of relational accidents include a kiss, a hit, a dance, a conversation, a contract, a battle, a war. Note, again, that relational accidents, like accidents in general, are not abstract entities: all of the examples mentioned are denizens of reality which are no less individual than the substances which serve as their relata.

4. Mereology and the Theory of Dependence

4.1 Mereology

The term 'object' in what follows will be used with absolute generality to embrace all substances, accidents, and all the scattered and non-scattered wholes and parts thereof, including boundaries. Our basic ontological categories will be defined in terms of the

primitive notion: *is part of* (for the theory of mereology) and *is necessarily such that* (for the theory of dependence). '*x is part of y*', which we shall symbolize by means of '$P(x,y)$', is to be understood as including the limit case where x and y are identical. '$PP(x,y)$' shall stand for 'x is a proper part of y' and we shall use '$x + y$' to signify the mereological sum of two objects x and y and '$x \times y$' to indicate their mereological product or intersection. If we define overlap as the sharing of common parts:

DM1 $O(x,y) := \exists z(P(z,x) \wedge P(z,y))$ *overlap*

then the axioms for standard (non-tensed) mereology can be formulated as follows ([3], [18], [20]):

AM1 $P(x,x)$ *reflexivity*
AM2 $P(x,y) \wedge P(y,x) \rightarrow x = y$ *antisymmetry*
AM3 $P(x,y) \wedge P(y,z) \rightarrow P(x,z)$ *transitivity*
AM4 $\forall z(P(z,x) \rightarrow O(z,y)) \rightarrow P(x,y)$ *extensionality*
AM5 $\exists x(\phi x) \rightarrow \exists y \forall z\, (O(y,z) \leftrightarrow \exists x\, (\phi x \wedge O(x,z)))$ *fusion*

(Here and in the sequel initial universal quantifiers are to be taken as understood.)

Parthood is a reflexive, antisymmetric, and transitive relation, a partial ordering. In addition, AM4 ensures that parthood is extensional (that no two distinct things have the same parts), where the schema AM5 guarantees that for every satisfied property or condition ϕ there exists an object, the sum or fusion, consisting precisely of all the ϕers. This object will be denoted by '$\sigma x(\phi x)$' and is defined as follows:

DM2 $\sigma x(\phi x) := \iota y \forall z\, (O(y,z) \leftrightarrow \exists x\, (\phi x \wedge O(x,z)))$

where the definite description operator 'ι' has the usual Russellian logic.

With the help of the fusion operator, other useful notions are easily defined:

DM3 $x + y := \sigma z(P(z,x) \vee P(z,y))$ *fusion*
DM4 $x \times y := \sigma z(P(z,x) \wedge P(z,y))$ *product*
DM5 $x - y := \sigma z(P(z,x) \wedge \neg O(z,y))$ *difference*
DM6 $-x := \sigma z \neg O(z,x)$ *complement*

The *complement* of an object x is that object which results when we imagine x as having been deleted from the universe as a whole.

4.2 Specific Dependence

As we have seen, substances and accidents may be compounded together mereologically to form larger wholes of different sorts. But substances and accidents are not themselves related mereologically: a substance is not a whole made up of accidents as parts. Rather, the two are linked together via the formal tie of specific dependence, which might be defined as follows (see also [5], and compare the treatment of 'notional' dependence in [3]):

DD1 $SD(x,y) := \neg O(x,y) \wedge \sim_x(E!x \rightarrow E!y)$ *specific dependence*

where '$\sim_x$' signifies the *de re* necessity operator 'x is necessarily such that' (see [17]) and 'E!' is the predicate of existence defined in the usual way in terms of the existential quantifier: $E!x := \exists y(x = y)$. Thus to say that x is specifically dependent on y is to say that x and y do not overlap and that x is necessarily such that if x exists then y exists.

We can now define mutual and one-sided specific dependence, in the obvious way, as follows:

DD2 $\mathrm{MSD}(x,y) := \mathrm{SD}(x,y) \wedge \mathrm{SD}(y,x)$ *mutual specific dependence*

DD3 $\mathrm{OSD}(x,y) := \mathrm{SD}(x,y) \wedge \neg\mathrm{MSD}(x,y)$ *one-sided specific dependence*

My headache, for example, is one-sidedly specifically dependent on me. Accidents in general stand to the substances which are their carriers in the formal tie of one-sided specific dependence only. Cases where objects are bound together via ties of *mutual* specific dependence would include the relation between the north and south poles of a magnet or the relation between individual hue, saturation and brightness mentioned above. Equally, there are cases where an object stands in a relation of specific dependence to more than one object. These are cases of what we referred to above as relational accidents – kisses and hits – which are dependent simultaneously on a plurality of substances.

4.3 Separability
A further formal tie, in some respects the converse of that of specific dependence, is the relation of separability. We define:

DD4 $\mathrm{MS}(x, y, z) := \mathrm{P}(x,z) \wedge \mathrm{P}(y,z) \wedge \neg\mathrm{O}(x,y) \rightarrow \neg{\sim_x}\exists w\mathrm{P}(w,y) \wedge \neg{\sim_y}\exists w\mathrm{P}(w,x)$ *mutual separability*

x and y are non-overlapping parts of z and x is not necessarily such that any part of y exists and y is not necessarily such that any part of x exists. z is, for example, a pair of stones, and x and y the stones themselves.

Separability, too, may be one-sided:

DD5 $\mathrm{OS}(x, y) := \mathrm{PP}(x,y) \wedge \exists w(\mathrm{P}(w,y) \wedge \neg\mathrm{O}(w,x) \wedge \mathrm{SD}(w,x)) \wedge \neg\exists w(\mathrm{P}(w,y) \wedge \neg\mathrm{O}(w,x) \wedge \mathrm{SD}(x,w))$ *one-sidedly separability*

(1) x is a proper part of y, and (2) some part of y discrete from x is specifically dependent on x, and (3) x is not specifically dependent on any part of y discrete from x. x is for example a human being and y is the sum of x together with some one of x=s thoughts. x is a conductor of electricity and y is the same conductor taken together with its momentary electric charge.

5. Topology

5.1 Substance and Topology
Intuitively, a substance is an object which is not specifically dependent and which has no separable parts. Certainly we can detach an arm or a leg from a substance, but then the results of such detachment will be new objects, distinct from their precursor objects (the relevant *attached* arm or leg) in virtue of their possession of complete boundaries. A detached arm is itself a substance. Substances in general are distinct from their attached constituent proper parts in being such as to possess complete boundaries – two-dimensional surfaces facing out, as it were, to the surrounding reality.

5.2 The Concept of Transformation
To do justice to these ideas we need to add to our formal-ontological instruments also topological tools. Standard introductions to the basic concepts of topology take as their starting point the notion of *transformation.* We can transform a spatial body such as a sheet of rubber in various ways which do not involve cutting or tearing. We can invert it, stretch or compress it, move it, bend it, twist it, or otherwise knead it out of shape. Certain properties of the body will in general be invariant under such transformations – which is to say under transformations which are neutral as to shape, size, motion and orientation. The transformations in question can be defined also as being those which do not affect the possibility of our connecting two points on the surface or in the interior of the body by means of a continuous line. Let us use the term 'topological spatial properties' to refer to those spatial properties of bodies which are invariant under such transformations (broadly: transformations which do not affect the *integrity* of the body – or other sort of spatial structure – with which we begin). Topological spatial properties will then in general fail to be invariant under more radical transformations, not only those which involve cutting or tearing, but also those which involve the gluing together of parts, or the drilling of holes through a body.

The property of being a (single, connected) body is a topological spatial property, as also are certain properties relating to the possession of holes (more specifically: properties relating to the possession of tunnels and internal cavities). The property of being a *collection* of bodies and that of being an *undetached part* of a body, too, are topological spatial properties. It is a topological spatial property of a pack of playing cards that it consists of this or that number of *separate* cards, and it is a topological spatial property of my arm that it is *connected* to my body.

This concept of topological property can of course be generalized beyond the spatial case. The class of phenomena structured by topological spatial properties is indeed wider than the class of phenomena to which, for example, Euclidean geometry, with its determinate Euclidean metric, can be applied. Thus topological spatial properties are possessed also by mental images of spatially extended bodies. Topological properties are discernible also in the temporal realm: they are those properties of temporal structures which are invariant under transformations of (for example) stretching (slowing down, speeding up) and temporal translocation. Intervals of time, melodies, simple and complex events, actions and processes can be seen to possess topological properties in this temporal sense. The motion of a bouncing ball can be said to be topologically isomorphic to another, slower or faster, motion of, for example, a trout in a lake or a child on a pogo-stick.

5.3 Boundary-Dependence
We have seen one introduction to the basic concepts of topology in terms of the notion of a topological transformation. An alternative introduction proceeds from the concept of boundary. Boundaries, too, are dependent entities, though the nature of their dependence is distinct from the specific dependence of accident on substance. To see why this is so, consider the relation between the surface of an apple and the apple itself. The surface is, clearly, dependent upon the apple which is its host. Yet it can survive the destruction of even considerable portions of the latter, providing only that the destroyed portions are confined to the interior of the apple. According to the ontology of boundaries outlined in [17], a boundary x is related to a host y as follows: (1) x is a proper part of y, and (2) x is necessarily such that either y exists or there exists some part of y properly including x, and (3) each part of x satisfies (2). In symbols:

DB1 $\mathrm{BD}(x,y) := \mathrm{P}(x,y) \wedge {\sim_x}(\mathrm{E}!y \vee \exists w(\mathrm{P}(w,y) \wedge \mathrm{P}(x,w) \wedge x \neq w)) \wedge \forall z(z \leq x \rightarrow {\sim_z}(\mathrm{E}!y \vee \exists w(\mathrm{P}(w,y) \wedge \mathrm{P}(z,w) \wedge z \neq w))$ *boundary-dependence*

Clause (2) is designed to capture the topological notion of neighborhood. Roughly, a boundary of given dimension can never exist alone but exists always only as part of some extended neighborhood of higher dimension. There are no points, lines or surfaces in the universe which are not the boundaries of higher-dimensional material things. Clause (3) is designed to exclude from the category of boundaries outliers formed by combining boundaries with non-boundary objects (thus for example an apple combined together with a thin boundary-hair growing out of its surface).

The relation of boundary-dependence holds both between a boundary and the substance which it bounds and also among boundaries themselves. Thus zero-dimensional spatial boundaries (points) are boundary-dependent both on one- and two-dimensional boundaries (lines and surfaces) and also on the three-dimensional substances which are their ultimate hosts. Note that the relation of boundary-dependence does not hold between an accident and its substantial carrier. Certainly my current thought satisfies the condition that it cannot exist unless I or some suitably large part of me exists. And certainly each part of my current thought satisfies this condition also. But my current thought is also specifically dependent upon me, and thus, by the definition of specific dependence it is not a part of me.

5.4 The Concept of Closure

The two approaches to topology sketched above may be unified into a single system by means of the notion of *closure*, which we can think of as an operation of such a sort that, when applied to an object x it results in a whole which comprehends both x and its boundaries. We employ as basis of our definition of closure the notions of mereology outlined above, which we use to provide an axiomatization of topology that is the mereological counterpart of classical topology ([18], [19], [20]).

An operation of *closure* (c) is defined in such a way as to satisfy the following axioms:

AC1 $P(x, c(x))$ *expansiveness*
(each object is a part of its closure)

AC2 $P(c(c(x)), c(x))$ *idempotence*
(the closure of the closure adds nothing to the closure of an object)

AC3 $c(x + y) = c(x) + c(y)$ *additivity*
(the closure of the sum of two objects is equal to the sum of their closures)

These axioms, the so-called Kuratowski axioms, define a well-known kind of structure, that of a *closure algebra*, which is the algebraic equivalent of the simplest kind of topological space. (See [21])

5.5 The Concept of Connectedness

On the basis of the notion of closure we can now define the standard topological notion of (symmetrical) *boundary*, $b(x)$, as follows:

DB2 $b(x) := c(x) \times c(-x)$ *boundary*

Note that it is a trivial consequence of the definition of boundary here supplied that the boundary of an object is in every case also the boundary of the complement of that object.

It is indeed possible to define in standard topological terms an asymmetrical notion of 'border', as the intersection of an object with the closure of its complement:

DB3 $b^*(x) = x \times c(-x)$ *border*

In fact, where Kuratowski's axioms were formulated in terms of the single topological primitive of closure, Zarycki showed [22] that a set of axioms equivalent to those of Kuratowski can be formulated also in terms of the single primitive notion of border, and the same applies, too, in regard to the notions of interior and boundary. A more challenging project is that of devising variant topologies which would recognize boundaries which would be genuinely asymmetric in the sense that seems to be exemplified, for example, in the figure-ground structure as this is manifested in visual perception. Here the boundary of a figure is experienced as a part of the figure, and not simultaneously as the boundary of the ground, which is experienced as running on behind the figure. Something similar applies also in the temporal sphere: the beginning and ending of a race, for example, are not in the same sense boundaries of any complement-entities (of all time prior to the race, and of all time subsequent to the race) as they are boundaries of the race itself. (On the usefulness of such variant, non-classical topologies for formal ontology see [17], [23], [24])

The notion of interior is defined as follows:

DB4 $i(x) := x - b(x)$ *interior*

We may define a *closed object* as an object which *is identical with its closure*. An *open object*, similarly, is an object which is identical with its interior. The complement of a closed object is thus open, that of an open object closed. Some objects will be partly open and partly closed. (Consider for example the semi-open interval (0,1], which consists of all real numbers x which are greater than 0 and less than or equal to 1.) These notions can be used to relate the two approaches to topology distinguished above: topological *transformations* are those transformations which map open objects onto open objects.

A closed object is, intuitively, an independent constituent – it is an object which exists on its own, without the need for any other object which would serve as its host. But a closed object need not be *connected* in the sense that we can proceed by a continuous path from any one point in the object to any other and remain within the confines of the object itself. The notion of *connectedness*, too, is a topological notion, which we can define as follows:

DCn1 $Cn(x) := \forall yz(x=y+z \rightarrow (\exists w(P(w,x) \wedge P(w,c(y))) \vee \exists w(P(w,c(x)) \wedge P(w,y)))$ *connectedness*

(a connected object is such that, given any way of splitting the object into two parts x and y, either x overlaps with the closure of y or y overlaps with the closure of x)

Neither of these notions is quite satisfactory however. Thus examination reveals that a whole made up of two adjacent spheres which are momentarily in contact with each other – if such is possible – will satisfy either condition of connectedness as thus defined. For certain purposes, therefore, it is useful to operate in terms of a notion of *strong connectedness* which rules out cases such as this. This latter notion may be defined as follows:

DCn2 $Scn(x) := Cn(i(x))$ *strong connectedness*
(an object is strongly connected if its interior is connected).

5.6 Substance Defined

On this basis we can now define a substance as a maximally strongly connected independent object:

DCn4 $S(x) := Scn(x) \wedge \forall y(P(x,y) \wedge Scn(y) \rightarrow x = y) \wedge \neg\exists z SD(x,z)$

This definition is still woefully incomplete. Thus in an ontology which admits spatial regions the constraint of maximal strong connectedness may be insufficient to do the job of separating substances from the spatial regions in which they are to be found. (See [14]) Moreover, there are certain problematic cases, for example a fetus inside a womb or a Siamese twin, of objects which are not maximally connected but which yet might be held to rank as substances in virtue of the intrinsic causal or dynamic integrity which they seem to possess. Substances, too, are marked essentially by the fact that they can preserve their numerical identity even in spite of changes over time. For a full treatment, therefore, the present framework needs to be supplemented by an account of spatial location, of causal integrity, and of temporal identity.

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Basic Problems of Mereotopology

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Abstract. Mereotopology is today regarded as a major tool for ontological analysis, and for many good reasons. There are, however, a number of open questions that call for an answer. Some of them are philosophical, others have direct import for applications, but all are crucial for a proper assessment of the strengths and limits of mereotopology. This paper is an attempt to put some order into this still untamed area of research. I will not attempt any answers. But I shall try to give an idea of the problems, and of their relevance for the systematic development of formal ontological theories.

1 Introduction

Mereotopology is today regarded as a major tool for formal-ontological analysis, and for many good reasons.[1] It is highly general and highly domain independent. It is ontologically neutral, treating all entities as individuals, i.e., as entities of the lowest logical type. (Set theory, by contrast, forces a distinction in ontological status between the first and second arguments of its primitive relation.) And it is intuitively attractive, dealing with formal structures (namely: structures of part and whole) that belong to the armamentarium not only of common sense and natural language but also of every empirical science.

Of course, insofar as mereotopology is only concerned with part-whole structures, it has its limits. One needs much more than mere part-whole reasoning to account for important ontological relations, for instance relations of existential dependence, causal relevance, or spatiotemporal location. One also needs to go beyond mereotopology to account for equally basic intuitions concerning, for instance, movement of parts or interaction among wholes. The world of mereotopology is, after all, a world of spheres and toruses, and we need to step into morphology or what we might also call qualitative geometry to account for basic differences in shape. We need to step into kinematics and dynamics to account for basic differences of behavior. And so on.

Even so, the fact remains that part-whole reasoning is a crucial and arguably basic ingredient of formal ontological reasoning, hence mereotopology deserves the place that it has acquired. My concern in this paper is not with the question of what comes *next*—of how mereotopology can be strengthened so as to deal with a wider range of ontological issues. Rather, I will be concerned with the loose ends of mereotopology—with issues that are still open *within* the scope of application of mereotopology itself. Some of these may just reflect the geography of the field: there is a variety of different mereotopologies by now, and in some cases the differences among these theories bear witness to genuine disagreement concerning fundamental questions. However, some of the loose ends have nothing to do with this. They are simply questions that have not yet been fully addressed in the literature of mereotopology, and my purpose here is to bring them up for discussion. I will not attempt any answers. But I shall try to give an idea of the problems, and of their relevance for the