A critical assessment of scientific retroduction

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Abstract

We analyse Peirce's original idea concerning abduction from the perspective of a critical philosophy, the same philosophy in Peirce's background. Peirce's realism is directly related to reason and experience and has ties with the idea of abstraction. We show how the philosophical environment of science abruptly changed, specially for physics, in the last period of the XIX century and the initial period of the XX century, when science was divided in disciplines and set free from the control of philosophy. The phenotype of the physicist changed from abstract into imaginative thinker. Further, abstraction was linked to metaphysics and attacked as such. Elements of phantasy and dogmatism entered the scene in place of abstraction alongside ideas taken from the observable world. We provide evidence that the scientists of the newer kind had problems understanding those of the older school. As a consequence of the problems arisen in conciliating the idea of science grounded in experience and reason with the science actually practised, the former conceptualisation was abandoned. Abduction was then expelled from science. In the late part of the XX century and the early part of our century abduction re-emerged but without its scientific attributes.

Influenced by a constructivist, Piagetian, perspective of science, we propose and discuss a small number of conditions that we identify as characteristics of rational abduction: rules for the rational construction of theories. We show how a classical example of belief that satisfies today's most common definition of abduction does not match the standards of scientific retroduction. We further show how the same rules indicate the detachment of Special Relativity from the observable world, a fact actually known to Einstein. Finally, the same rules indicate the initial point in the path to re-conciliate Electromagnetism with the classical view of spatial relations, a matter not possible for the imaginative scientist but not extremely difficult for the abstract scientist. We close arguing that there is an urgent need to develop a critical epistemology, and to give room in science for the abstract scientist.

Keywords: theory building; abstract thinking; imaginative thinking; dialectics

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1 Introduction

Abduction or retroduction is indissolubly linked to the name of Charles Peirce (1839-1914) who studied scientific thinking from his pragmaticist perspective. Among his influential reading, Peirce indicates Kant (Peirce, 1994, CP 1.4) and Hegel, about whom he said "My philosophy resuscitates Hegel, though in a strange costume." (Peirce, 1994, CP 1.42). Peirce's works must then be contextualised within critical philosophy.

Abduction is often defined with the following logical structure (Peirce, 1994, CP 5.189):

> The surprising fact, C, is observed; But if A were true, C would be a matter of course, Hence, there is reason to suspect that A is true.

The reasoning grants the proposition of A the name of *abductive inference*. However, to raise this expression into a definition leaves abduction abandoned to free interpretation. Adoption of explanatory hypothesis is indissolubly linked to the adoption of beliefs and the cessation of doubt, the latter being the motor of thoughts. Peirce is clear in distinguishing the beliefs he pursues from religious and other common beliefs; he offers three characteristics of them:

> First, it is something that we are aware of; second, it appeases the irritation of doubt; and, third, it involves the establishment in our nature of a rule of action, or, say for short, a habit. As it appeases the irritation of doubt, which is the motive for thinking, thought relaxes, and comes to rest for a moment when belief is reached. But, since belief is a rule for action, the application of which involves further doubt and further thought, at the same time that it is a stopping-place, it is also a new starting-place for thought. [CP 5.397]

All these elements are essential to Peirce's notion of belief and as such, none of them can be dropped (not even in didactic examples) when regarding abduction, which is an act of thought. Further, Peirce sustains,

> Thought [...'s] sole motive, idea, and function is to produce belief, and whatever does not concern that purpose belongs to some other system of relations. [CP 5.396]

Thought, doubt and belief refer to our inner senses. They are to some degree defined in relation to each other. Abduction or retroduction in Peirce is also a part of scientific thinking. He provides more insight into it:

> These three kinds of reasoning are Abduction, Induction, and Deduction. Deduction is the only necessary reasoning. It is the reasoning of mathematics. ... Induction is the experimental testing of a theory. The justification of it is that, although the conclusion at any stage of the investigation may be more or less erroneous, yet the further application of the same method must correct the error. The only thing that induction accomplishes is to determine the value of a quantity. It sets out with a theory and it measures the degree of concordance of that theory with fact. It never can originate any idea whatever. No more can deduction. **All the ideas of science come to it by the way of Abduction. Abduction consists in studying facts and devising a theory to explain them. Its only justification is that if we are ever to understand things at all, it must be in that way.** (Peirce, 1994, CP 5.145) [emphasis added]

Scientific retroduction is then what the totality of these views in Peirce aims to. We highlight that it is a mandatory part for understanding, and that understanding is a name for having a theory in which we can believe.

In contrast, Magnani (1999) associates abduction to "creative inference" and defines it as:

> [...] theoretical abduction is the process of *inferring* certain facts and/or laws and hypotheses that render some sentences plausible, that *explain* or *discover* some (eventually new) phenomenon or observation; it is the process of reasoning in which explanatory hypotheses are formed and evaluated.

It has also been proposed that scientific abduction is related to "model-based reasoning" (Meheus, 1999), defined as:

> the construction and manipulation of analogue representations (representations that form structural analogues of real-world or imaginary situations). Typical forms of model-based reasoning are constructing and manipulating visual representations, thought experiments and analogical reasoning.

If this were the case, abduction would be linked to analogy. It has been already pointed out (McAuliffe, 2015) that the relatively recent re-emergence of abduction in the context of artificial intelligence (Kuipers, 1999) and of inference to the best explanation (Lipton, 2004; Oh, 2012) does not correspond with Peirce's views, hence we will not further address these matters.

Meheus argues that we urgently need to design logical systems that allow us to distinguish between sound and unsound steps in specific types of reasoning processes. We will try to address this concern in this work –although not constructing a logical system–, taking a critical step in search of the foundations of rationality.

Peirce's view was neither refuted nor adopted by the philosophy of science, nor it can be found in common practices of current scientists, as we shall explain. Epistemology followed a different mandate departing from critical philosophy. After Hegel's death and philosophical eclipse, the dominant movement in Germany became the "neo-Kantians". Beiser (2014, p. 38) explains:

> The neo-Kantians, one and all, utterly repudiated the foundationalism of the speculative idealist tradition. They very much accepted the autonomy of the empirical sciences, and they firmly believed the sciences stood in no need of a foundation from philosophy. The purpose of philosophy was not to ground the empirical sciences but to explain their logic.

In short, the movement excepted sciences from appearing in the Kantian Tribunal of Reason. A similar idea developed in the United Kingdom; Bertrand Russell wrote in 1923:

> Science may not tell us much, but it tells at least as much as we can know. Probably it will turn out in future to tell us even less than it now seems to do, since this has been the direction of past developments. But the limits of science are the limits of valid inference, and extra-scientific philosophy is only the expiring sigh of theology. (Russell, 1988, n. 35, p. 238)

Here the situation has been reverted, it is now science what puts limits to philosophy. This process developed from around 1870, with the growth of analogical thinking in physics. It intensified with Mach's criticism of Newton, the arrival of Relativity theory and the Vienna circle strong rejection of metaphysics. The situation took a dramatic turn when philosophers realised that they could not connect the observed/intuited world with the space-time of Einstein's relativity. At the time, Einstein claimed that scientific theories were produced by "free invention" (Einstein, 1936, p. 381) and the connections with empirical data were at most weak. The Gordian knot was cut by Karl Popper who simply put the creation moment (Popper, 1959, Ch. 1) outside science. Popper's epistemology bears several points of contact with Peirce's critical view and two striking differences: (a) the central role that abduction takes in Peirce is impossible in Popper for abduction is outside Popperian science; and, (b) Peirce is a critical thinker while Popper adheres to the subordination of philosophy to science-as-it-is-practised:

> A scientist, whether theorist or experimenter, puts forward statements, or systems of statements, and tests them step by step. In the field of the empirical sciences, more particularly, he constructs hypotheses, or systems of theories, and tests them against experience by observation and experiment.
>
> I suggest that it is the task of the logic of scientific discovery, or the logic of knowledge, to give a logical analysis of this procedure; that

> is, to analyse the method of the empirical sciences. (Popper, 1959, Ch. 1)

Thus, firstly the criticism of science became impossible and secondly the phenomenological moment was moved out of science as it was impossible to conciliate it with the (social?) mandate of epistemology.

The aim of this work is to explain the dismiss of retroduction following the abandonment of the critical role of philosophy of science at the same time that a new form of performing science emerged. The new approach is based upon what we will call *imaginative thinking* in contrast to abstract science. The combination of both actions drove the latter to extinction. Hence, our thesis is that Peirce's abduction can only be reintegrated to science if, at the same time, a critical epistemology is introduced, an epistemology entitled to detect and highlight the logical defects and weaknesses of scientific theories.

We will further advance in the project of a critical epistemology producing rules for the control of the rationality of retroduction, since, as far as we know, what is necessary for the cessation of doubt has not been addressed. In this regard, we will adopt a constructivist approach influenced by Piaget's work. A reason without consciousness of its own constructive efforts will consider its scientific activity to be the discovering of the laws of the universe, perhaps without noticing that their efforts are pre-formatted by their own rules of reasoning and their own criteria of considering an argument to be correct/convincing/acceptable. Thus, there is a level of meta-scientific criteria that needs to be explored and explained. We owe the idea regarding the existence of rules or norms to Piaget, as Gruber & Vonèche (1995, p. 739) write:

> Rules or norms are generally considered as dependent on structures in the subject. They do not depend on the structure of physical reality for their validation but are instead entirely determined by a principle of deduction that is not empirical in nature.

In what follows we will discus several aspects of abduction in the context of physics in an intendedly disruptive form. We start by considering the difference between physics and *phantasy*. Next we present evidence on the existence of two thinking styles, naming them *imaginative thinking* and *abstract thinking*, and present subsequently evidence of the problems that imaginative thinkers usually have in understanding abstract thinking, with an appendix concerning dialectics. Furthermore, we present a few rules of reason as recognised by our eidetic seeing (the production of ideas by intuition Husserl (1983, Ch. One)), discussing examples in several fields. Finally, we summarise the discussion in Section 5.

2 Physics and phantasy

The English spelling phantasy (Greek: $\phi\alpha\nu\tau\alpha\sigma\acute{\iota}\alpha$, Latin: *imaginatio*) is predominantly associated with "imagination, visionary notion" (Oxford). The

Latin etymology of *imaginatio* relates it to *imitare* (to copy). Aristotle describes phantasy:

> And when we come to thinking, which includes right thinking and wrong thinking, right thinking being intelligence, knowledge and true opinion, and wrong thinking the opposites of these, neither is this identical with perception. For perception of the objects of the special senses is always true and is found in all animals, while thinking may be false as well as true and is found in none which have not reason also. Imagination, in fact, is something different both from perception and from thought, and is never found by itself apart from perception, any more than is belief apart from imagination. Clearly thinking is not the same thing as believing. [...] Moreover, when we are of opinion that something is terrible or alarming, we at once feel the corresponding emotion, and so, too, with what is reassuring. But when we are under the influence of imagination we are no more affected than if we saw in a picture the objects which inspire terror or confidence. (Aristotle, 1907, p. 123, see 3.3–3.15)

Imagination in Aristotle has some familiarity with ideas in Hume:

> All the perceptions of the human mind resolve themselves into two distinct kinds, which I shall call IMPRESSIONS and IDEAS. The difference between these consists in the degrees of force and liveliness, with which they strike upon the mind, and make their way into our thought or consciousness. Those perceptions, which enter with most force and violence, we may name impressions; and under this name I comprehend all our sensations, passions and emotions, as they make their first appearance in the soul. By ideas I mean the faint images of these in thinking and reasoning; such as, for instance, are all the perceptions excited by the present discourse, excepting only, those which arise from the sight and touch, and excepting the immediate pleasure or uneasiness it may occasion (Hume, 2011, p. 7).

According to Hume we think in terms of elements pertaining to imagination which, according to Aristotle, lies close to both perception (Impressions in Hume) and belief, but not to clear thinking. We further learn from Husserl (1983, Ch. One) that the production of ideas by intuition (eidetic seeing) can proceed from data provided by both the real (observable) world as well as phantasy. Very much like Aristotle, Husserl reminds us that whatever predicate having to do with "matters of fact" must be grounded on experience ("And thus not even the most insignificant matter-of-fact truth can be deduced from pure eidetic truths *alone.*" Husserl (1983, p. 11)).

In short, concerning the study of nature (physics) there are two possible approaches at least since Aristotle: "clear thinking" and "phantasy". The former struggles for being correct while the latter may be correct or not, and its correctness cannot be established without the participation of clear thinking. Already in the second half of the XIX century, two forms of thinking had evolved without a neat distinction, differing in the relation among clear thinking and phantasy. Both of them using the word *idea* but meaning different things with it. The requisites for achieving belief, or the cessation of doubt are expected to be unlike as well. The subtleties of the different meanings may emerge when we push our reasoning to its limits. We will call *imaginative thinking* to the form originating in the Bild concept (D'Agostino, 2004), where images (often called ideas) were the central tool in developing knowledge, and *abstract thinking* to the form supported by abstraction (cognitive surpass (Piaget & García, 1989, Introduction)).

Notice that between two similar problems, analogy (the central tool in imaginative thinking) makes a direct connection between them. In contrast, abstraction makes indirect connections conditioned to the possibility of producing an universal form which can be particularised in each of the different problems originally perceived as potentially related. In so doing, abstraction opens the possibility of a manifold of connections other than those initially considered, largely enlarging the possibility of performing empirical contrastive comparisons.

There exist plenty of early examples of the first form. With respect to "understand", Lord Kelvin says:

> I never satisfy myself until I can make a mechanical model of a thing. If I can make a mechanical model I can understand it. As long as I cannot make a mechanical model all the way through I cannot understand; and that is why I cannot get the electromagnetic theory. (Thompson, 2011, p.835)

Thus Thompson puts a clear requisite for his understanding of a physical theory. Boltzmann (Boltzmann, 1974, p. 11) states

> In the end, philosophy generalised Maxwell's ideas to the point of maintaining that knowledge itself is nothing else than the finding of analogies. This once again meant that the old scientific method had been defined away and science spoke merely in similes.

Later he writes referring to ideas by Hertz:

> The view whose most extreme form has here been stated, was very variously received. Whereas some were almost inclined to regard it as a bad joke, others felt that physics must henceforth pursue the sole aim of writing down for each series of phenomena, without any hypothesis, model or mechanical explanation, equations from which the course of the phenomena can be quantitatively determined; so that the sole task of physics consisted in using trial and error to find the simplest equations that satisfied certain required formal conditions of isotropy and so on, and then to compare them with experience.(Boltzmann, 1974, p. 95)

These forms of understanding physical phenomena decline to explain why "formal conditions of isotropy" must be required, what it is meant by "simple" and so on. They request a large amount of complicity from the reader to grasp what they intend to state. Actually, Boltzmann (circa 1892) testifies of the transition from the old style into the new style of thinking during his lifetime[1] :

Another example of the different meanings associated to understanding, coupled with the introduction of auxiliary concepts, comes from mechanics. Newton's mechanics can be introduced to students in several forms. Some will rest upon "absolute space". Einstein (1924) and Boltzmann (1974, p. 102), for example, believed that absolute space was essential to Newton's mechanics. In his treatise of mechanics, Mach (1919) rejected absolute space as nonsense and introduced as reference the fixed stars (whatever they are [2]). In both cases, a first reference for motion allowed the introduction of inertial systems in which Newton's laws were claimed to hold. Perhaps more importantly, the notion of space connects directly with the intuition of the child who organises relative positions of objects in space using as reference her/himself. Yet, Newton's mechanics is based upon the notion of "absolute motion" which is *not* motion in absolute space and it is rather close to relative motion (Newton, 1687; Thomson, 1884; Solari & Natiello, 2021). Thus, while for a relevant group of physicists the auxiliary notion of space and a first reference for motion are necessary elements for the understanding of classical mechanics, yet, others can understand without those elements or with a suppressible version of space.

Absolute space was defended by Bertrand Russell in his early period as well. Russell (1901, p. 274) writes:

> En ce qui concerne les nécessités de la pensée, la théorie kantienne semble amener ce résultat curieux, que tout ce qu'on ne peut s'empêcher de croire est faux. Dans le cas actuel, ce qu'on ne peut s'empêcher de croire, c'est quelque chose qui se rapporte à la nature de l'espace, non pas à celle de notre esprit.

Despite Russell's obfuscation, it is clear that all that we can observe are spatial relations, and from them, we can construct a form of description of these spatial relations in terms of the relations between the intervening bodies and a main body and its surroundings. We owe this representation (in abstract form) to Descartes: the *space*. In the words of Kant:

> By means of the external sense (a property of the mind), we represent to ourselves objects as without us, and these all in space.(Kant, 1787, p. 51)

Thus, space is a phantasy, a representation of the real observable in the mind of the observer. By erasing the own body from the perceived scene, the observer

[1]

> I therefore present myself to you as a reactionary, one who has stayed behind and remains enthusiastic for the old classical doctrines as against the men of today; Boltzmann (1974, p. 82).

[2] Actually, Newton had considered the proposition and rejected it.

idealises (creates) the space out of the observed spatial relations with the intervening bodies (a mathematical presentation can be read in (Solari & Natiello, 2018)). Space exists only in our minds but relates to the perceived as Aristotle dictates for all phantasies. The idea of space is facilitated because it is in coincidence with the immediate perception of the child of the spatial relations of objects with respect to her/himself. Space is not real, and it is not entirely imagined either, it contains both nature and some invention as well, as such, it must be used with due caution.

Imaginative thinkers use imagination as an intermediate step in the production of abstract/symbolic forms; in contrast, abstract thinkers relate the observed to the abstract form without mediations. Both forms of thinking have been discussed in (Duhem, 1991, Ch. IV). Abstract thinking is described by Duhem as:

> THE CONSTITUTION of any physical theory results from the twofold work of abstraction and generalization. In the first place, the mind analyzes an enormous number of concrete, diverse, complicated, particular facts, and summarizes what is common and essential to them in a law, that is, a general proposition tying together abstract notions. In the second place, the mind contemplates a whole group of laws; for this group it substitutes a very small number of extremely general judgments, referring to some very abstract ideas; it chooses these primary properties and formulates these fundamental hypotheses in such a way that all the laws belonging to the group studied can be derived by deduction that is very lengthy perhaps, but very sure. This system of hypotheses and deducible consequences, a work of abstraction, generalization, and deduction, constitutes a physical theory in our definition; it surely merits the epithet Rankine used to designate it: abstractive theory. The twofold work of abstraction and generalization that goes to make up a theory brings about, we have said, a double economy of thought; it is economical when it substitutes a law for a multitude of facts; it is economical again when it substitutes a small group of hypotheses for a vast set of laws... Among these men, the faculty of conceiving abstract ideas and reasoning from them is more developed than the faculty of imagining concrete objects.

And continues characterising the imaginative scientist:

> But not all vigorously developed minds are abstract minds. There are some minds that have a wonderful aptitude for holding in their imaginations a complicated collection of disparate objects; they envisage it in a single view without needing to attend myopically first to one object, then to another; and yet this view is not vague and confused, but exact and minute, with each detail clearly perceived in its place and relative importance. But this intellectual power is subject to one condition; namely, the objects to which it is directed

> must be those falling within the purview of the senses, they must be tangible or visible. The minds possessing this power need the help of sensuous memory in order to have conceptions; the abstract idea stripped of everything to which this memory can give shape seems to vanish like an impalpable mist. ... Endowed with a powerful faculty of imagination, these minds are ill prepared to abstract and deduce.

We will address examples of misunderstandings rooted in the introduction of elements of phantasy in Section 3.

3 Caveats with imaginative thinking and retroduction

> Jo of the North Sea said, "You can't discuss the ocean with a well frog —he's limited by the space he lives in. You can't discuss ice with a summer insect —he's bound to a single season. You can't discuss the Way with a cramped scholar —he's shackled by his doctrines. Now you have come out beyond your banks and borders and have seen the great sea —so you realize your own pettiness. From now on it will be possible to talk to you about the Great Principle. (Chuang Tzu, 1968, Autumn floods)

The previous discussion confronts us with several problems. The first one is that the criteria for cessation of doubt in retroduction, i.e., the achieving of *explanation*, are now evidently subjective. Abstract and imaginative thinkers have different criteria. The second problem is that imaginative thinking does not stand by itself since the elements of phantasy do not always connect to realities. The third problem is that there is evidence of important misunderstandings when an imaginative thinker tries to relate to abstract thoughts. Actually, both families of thinkers may not even agree on what abstract means.

With respect to the first problem at least we know that models and analogies are not enough. To be acceptable, a theory must be independent of the phantasies employed, or at least there should be a form of suppressing them. In this section we provide examples that sustain our second and third claim. There is a general pattern of misunderstanding in act when thinkers reaching different levels of abstraction clash. In a first step, the abstract thought is "translated" into other forms of understanding, perhaps with some addenda to compensate for difficulties in translation. This step produces a straw-man version of the initial ideas. Next, the translated thought is criticised, (e.g., interactions between source and target transform into travelling electromagnetic actions, see Section 3.2). Concerning this generalised form we provide examples involving dialectics in Appendix A.

3.1 Physics in the late XIX century

The period of physics where the belief in the existence of the ether was prevalent is an important source of examples of different forms of understanding.

> The idea that the Sun pulls Jupiter, and Jupiter pulls back against the sun with equal force, and that the sun, earth, moon, and planets all act on one another with mutual attractions seemed to violate the supposed philosophic principle that matter cannot act where it is not. (Kelvin's preface to Hertz, 1893)

Faraday, had contested the "philosophic principle" saying:

> ... the atoms of Boscovich appear to me to have a great advantage over the more usual notion. His atoms, if I understand aright, are mere centres of forces or powers, not particles of matter, in which the powers themselves reside. If, in the ordinary view of atoms, we call the particle of matter away from the powers a, and the system of powers or forces in and around it m, then in Boscovich's theory a disappears, or is a mere mathematical point, whilst in the usual notion it is a little unchangeable, impenetrable piece of matter, and m is an atmosphere of force grouped around it. In many of the hypothetical uses made of atoms, as in crystallography, chemistry, magnetism, &c., this difference in the assumption makes little or no alteration in the results, but in other cases, as of electric conduction, the nature of light, the manner in which bodies combine to produce compounds, the effects of forces, as heat or electricity, upon matter, the difference will be very great. A mind just entering on the subject may consider it difficult to think of the powers of matter independent of a separate something to be called the matter, but it is certainly far more difficult, and indeed impossible, to think of or imagine that matter independent of the powers. Now the powers we know and recognize in every phenomenon of the creation, the abstract matter in none; why then assume the existence of that of which we are ignorant, which we cannot conceive, and for which there is no philosophical necessity? (Faraday, 1844, p. 290)

Thus, according to Faraday, we only know about matter because of its actions (forces) and we locate matter where the centre of this action lies. If the action extends through the universe, the atoms are "where they act", this is in all the universe. Matter and force cannot be separated, they are two aspects of an unity. However, Maxwell reading of Faraday contradicts the author:

> [speaking of Faraday] ...He even speaks of the lines of force belonging to a body as in some sense part of itself, so that in its action to distant bodies it cannot be said to act where it is not. This, however, is not a dominant idea with Faraday. I think he would rather have said that the field of space is full of lines of force, whose arrangement

> depends on that of the bodies in the field, and that the mechanical and electrical action on each body is determined by the lines which abut on it ([529] Maxwell, 1873) [It is worth mentioning that the reference in Maxwell is (Faraday, 1844, p. 293)].

Notice how Maxwell needs to correct what Faraday wrote ("I think he would rather have said...") giving a reading of Faraday that has been prevented against by the author ("A mind just entering..."). To understand Faraday, Maxwell needs to break the duality in terms of matter and action. He also needs to introduce space as the place for action-lines. In short, Maxwell cannot grasp Faraday without previously accommodating Faraday's thoughts into the matrix of his own (Maxwell's) thinking, something that at the end, is a common problem of the processes called assimilation and accommodation (Gruber & Vonèche, 1995).

The end of this story was the adoption of the ether as a dogma and a full attack on the positions of Gauss and his followers. The attack was based upon first asserting a notion of understanding that required actions to be carried by matter (Clausius, 1869). We now know that the ether does not lead towards empirically verifiable insights, and as such it has been disregarded. Yet, the ideas of Gauss had not been reconsidered, but rather suppressed from the history of science.

3.2 On the problems posed by electromagnetism

While objects and motion in mechanics refer directly to observable/intuitive phenomena, electromagnetism refers to inferred phenomena not directly accessible to observation. We observe the deflection of the needle in a galvanometer but only infer the current, for example. This intrinsic difficulty put physicists minds to test. For Gauss, Lorenz, Riemann and others, the observable was that electromagnetic action propagated from the locus of the experiment (say, the cables where currents circulated) to distant locations where no electromagnetic material connection was observable. Thus, electromagnetic actions may happen at distance, without the mediation of matter.

Gauss further speculated that the action-at-a-distance followed after a certain delay rather than instantaneously (delayed action at distance). Using these insights and his own knowledge of the mathematical form of waves, Lorenz (1867) proposed propagation equations that are identical to Maxwell's integral expressions (as acknowledged by Maxwell ([805], final note 1873)). For Lorenz, to explain a physical phenomena is to produce a fair description of the phenomena in an unified way, hence he *explained* electromagnetism without the need of the ether. Those requiring mechanical analogies as the means of explanation, could not handle Lorenz' ideas. Maxwell ([866] p. 493 1873) writes:

> But in all of these theories the question naturally occurs – if something transmitted from one particle to another at a distance, what is its condition after it has left the one particle and before it has reached the other? If this something is the potential energy of the

> two the particles, as in Neumann's theory, how we are to conceive this energy as existing in a point of space, coinciding neither with the one particle nor with the other? In fact, whenever energy is transmitted from one body to another in time, there must be a medium or substance in which the energy exists after it leaves one body and before it reaches the other, for energy... Hence all these theories lead to the conception of a medium in which the propagation takes place, and if we admit this medium as an hypothesis, I think it ought to occupy a prominent place in our investigations, and that we ought to endeavour to construct a mental representation of all the details of its action, and this has been my constant aim in this treatise.

This closing paragraph of Maxwell treatise is remarkable in may aspects. In Newton, space is the place of objects, not of interactions (or reciprocal action). Thus, it requires a discontinuity in thought to request a place in space for action. Another remarkable aspect is that Maxwell does not affirm the existence of the ether as it is usually stated in textbooks; he only considers it a rightful matter of investigation.

The idea that "matter cannot act where it is not" can be found in Prussia some years later. We read it in Helmholtz homage to Gustav Magnus:

> It must here be mentioned that Faraday, another great physicist, worked in England exactly in the same direction, and with the same object; to whom, on that account, Magnus was bound by the heartiest sympathy. With Faraday, the antagonism to the physical theories hitherto held, which treated of atoms and forces acting at a distance, was even more pronounced than with Magnus. (Helmholtz, 1908)

and later:

> Faraday and Maxwell inclined towards the simpler view that there was no action-at-a-distance; this hypothesis, which involved a complete upsetting conceptions hitherto current, was thrown into mathematical form and developed by Maxwell. According to it, the seat of changes produce electrical phenomena must be sought only in the insulators (Hertz & Walley, 1899, Preface by H von Helmholtz)

How to conciliate Helmholtz' account of Faraday's view with the original? Faraday wrote:

> The view which I am so bold as to put forth considers, therefore, radiation as a high species of vibration in the lines of force which are known to connect particles and also masses together. **It endeavours to dismiss the æther but not the vibrations**. Faraday (1855, p. 451) (emphasis added).

This is another example of how a scientist's reading of fellow scientists proceeds by accommodation to the reader's ideas, for Faraday always entertained doubts regarding action at distance as well as the ether.

Hence, the phantasy element named ether pervaded the development of electromagnetic theory, with only a few scientists allowing themselves to entertain alternatives. The more abstract nature of the alternatives was an obstacle for imaginative thinkers that could not free themselves from material or mechanical analogies in order to explain the phenomena.

The ether proved itself to be contradictory, to the point that it was "eliminated" after 1905. Albert Einstein, one of the most extraordinary imaginative thinkers, explained that the services offered by the material ether are now provided by the immaterial space (Einstein, 1924). Yet, he fails to perceive the roots of the problem, the universal form, which is the introduction of phantasies. Actually, in the same work Einstein considers absolute space necessary to understand Newton's mechanics. We will come back to the problem of absolute space in Subsection 4.2.2. It suffices to say here, that, if we consider absolute space as a privileged (original, first) reference frame for motion where Newton's laws hold, then in this abstract form it persists up to our days despite the efforts made by abstract thinkers showing that there is no need for it, the first of such thinkers being Newton himself. Even before Newton's time we were offered the fixed stars for reference to motion. Later, preferences went to some point and directions allegedly related to the universe (which must be necessarily finite for the implied imaginative calculation to be possible).

3.3 Consequences

Poincaré (1913, p. 185-186) summarised the situation at the beginning of the XX century

> Most theorists have a constant predilection for explanations borrowed from physics, mechanics, or dynamics. Some would be satisfied if they could account for all phenomena by the motion of molecules attracting one another according to certain laws. Others are more exact: they would suppress attractions acting at a distance; their molecules would follow rectilinear paths, from which they would only be deviated by impacts. Others again, such as Hertz, suppress the forces as well, but suppose their molecules subjected to geometrical connections analogous, for instance, to those of articulated systems; thus, they wish to reduce dynamics to a kind of kinematics. In a word, **they all wish to bend nature into a certain form, and unless they can do this they cannot be satisfied**. Is Nature flexible enough for this? [*Emphasis added*]

The end of the XIX century witnessed a shift in the construction of theories in physics, in which images and analogy gradually got the upper hand and predictive success became more important than solid foundations. Multiple reasons contributed to this state of affairs, among them (a) the already discussed abandonment of the critical role of philosophy of science, (b) repeated attacks to abstract thinking (as opposed to empirically based thinking) from the positivist side of philosophy, (c) the focus on predictive success, an obvious demand from technology that science felt compelled to provide, (d) intervention of the states (later private business as well) directing research towards practical purposes. We discuss these points briefly in what follows.

The abandonment of the critical role of philosophy was clear by 1870 when Jürgen Bona Meyer declared the death of philosophy Beiser (2014, p. 17,18) as part of the development of "her daughters": the sciences. Similarly, the abduction/retroduction moment was moving away from the scrutiny of reason in Popper, while Einstein supported "free invention" (Einstein, 1936, p. 381), as discussed in the Introduction. In this way, the control of scientific achievements was weakened on the side of the foundations and therefore attention moved to the results side. From this point of view, predictivism (accessed 2022-06-15, see 2.4) –the confidence on theories that predict correct results– provides a comfortable way of eliminating doubts. Popper's falsificationism (namely the rejection of theories that fail in prediction) was indeed an improvement to predictivism, although science has not moved fully away from the latter. Theories (paradigms in Kuhn's wording) are preserved as much as possible, even in front of refutations, by providing ad-hoc hypotheses that "patch" the faulty result without otherwise increasing knowledge. Indeed, the re-elaboration of falsified scientific theories opens the path for a return to Peirce and the development of scientific abduction/retroduction.

4 Science, reality and the rules of rational retroduction

Scientists conceive the world as a cosmos, a harmonious totality. For them, there is nothing as fascinating as discovering this harmony. For this task they are equipped basically with two tools: reason and experience. What they call understanding is the result of the interplay of the two, for experience does not constitute knowledge if not for the intervention of reason. These ideas (and some words) are taken from Kant (Kant, 1787) and Peirce (Peirce, 1955) and do not change in their strength if, rather than considering the world as a cosmos, we change the proposition to: the goal of the scientist is to articulate a harmonic vision of the world, to make a cosmos out of the sensorial input they receive[3].

Our views about the construction of scientific theories have been advanced in (Solari & Natiello, 2022a). In short, there are three moments in the construction of theories in order to address what we perceive through our senses: (i) A

[3]For Peirce explanation can be equated to rationalisation as it is evident in the next paragraph:

> I think I have now said enough to show that my theory – that that which makes the need, in science, of an explanation, or in general of any rationalization of any fact, is that without such rationalization the contrary of the fact would be anticipated, so that reason and experience would be at variance, contrary to the purpose of science – [that this theory] is correct, or as nearly so as we can make any theory of the matter at present." (Peirce, 1994, CP 7.201)

projection Π from the sensible world to the realm of ideas describing a version of the sensible-real –filtered by our reasoning abilities– that is relevant for understanding, (ii) analytical elaborations ϕ through mathematical logic assessing the possible consequences of the theory/model and (iii) the contrastive comparison process Γ where consequences are matched against the sensible world. These three ingredients (Π, ϕ, Γ) are integrated in the theory construction and are not independent of each other, while each has its particular reasoning requirements.

4.1 Science and reality

The task of understanding involved in scientific theories requires some precision on what we mean by reason and the requisites for inference.

4.1.1 The principle of reality

In the first place, we must indicate that the attempt of constructing a cosmos out of sensorial input implies the assumption that there is something real that reaches us through the senses, this is to say, that there are subject and object. While the truth of this statement is debatable, we can consider the dangers involved in accepting or rejecting it. Little damage is done if accepting reality were an error and it turns to be that everything is part of a unique encompassing (solipsistic) being. On the contrary, if we were in error when rejecting reality, we would become completely dysfunctional and miss one of the greatest opportunities in life. The principle is addressed in:

> "Such is the method of science. Its fundamental hypothesis, restated in more familiar language, is this: There are Real things, whose characters are entirely independent of our opinions about them; those Reals affect our senses according to regular laws, and, though our sensations are as different as are our relations to the objects, yet, by taking advantage of the laws of perception, we can ascertain by reasoning how things really and truly are; and any man, if he have sufficient experience and he reason enough about it, will be led to the one True conclusion. The new conception here involved is that of Reality."(Peirce, 1955, p. 18)

Reality in Peirce can be seen as a duality. In front of our consciousness we have what is perceived (observed) and what is elaborated by ourselves from this input: the ideated or ideal (*facts* in (Piaget & García, 1989)). Thus, to grasp reality we need the sensible world (SW) that we perceive "out there", its ideated or intuited forms or ideal world (IW) which rest within us and a form of correspondence from one world with the other: the phenomenological map, Π, Γ. We also need to include in reality as a primary element the consciousness of our mental operations as they are not ideas produced by elaboration of input from the senses –they are neither in SW nor in IW– and are not part of the phenomenological map, but rather its producer. In natural science we restrict the study to the development of the relation between IW and SW, something that is not possible if we want to study cognition or psychology ignoring self-consciousness. Notice that whatever is in IW, it is not what is sensed, it is clearly not in SW. Hence, we can think of IW as a negation of SW, and the dialogue through the phenomenological map between the two forms of the real constitutes what science is.

4.1.2 Scientific reasoning

The casting of science into the forms of dialectics, performed in the previous subsection, is leaving out an important fact. The ideal world shares a fundamental characteristic with living things: it is self-reproducing, it entails creative production, *poiesis*. We call this reproductive act **reasoning**, an activity that produces new ideas out of previous ideas or observations, it is then the poiesis of our conscience what creates IW out of SW. Then, if reasoning is the activity that institutes new ideas in IW, the sensorimotor cognitive activity of children in their earliest times in full contact with SW must be considered as reasoning, since it institutes the idea of self (ego), and alter (not ego), the idea of the permanent (the identity, what remains unchanged through perceived changes) and the transition between states of permanency (change), at the same time they conceive space and time (Piaget, 1999)[4]. These ideas frame all further knowledge. The development of early cognition just presented certainly belongs to IW and was developed by Piaget from hints obtained by the observation of SW. We can then say that it is reasoning what institutes the duality we call reality, but it does so through the sensorimotor activity of the child.

We read (Duhem, 1991, p. 55)

> The two-fold work of abstraction and generalization that goes to make up a theory brings about, we have said, a double economy of thought; it is economical when it substitutes a law for a multitude of facts; it is economical again when it substitutes a small group of hypotheses for a vast set of laws.

In Duhem, the construction of the theory proceeds by unifying steps of abstraction, from facts to laws and from laws to theories. The path can be travelled in opposite direction, particularisation or interpretation, what allows to conceive experimental tests.

Since the constructive step in the production of theories has been called abduction or retroduction (Burks, 1946; Peirce, 1955) we can think of abduction in Peirce's sense as the production of explanatory hypothesis, where explanation must be understood in the sense of creating a duality universal-particular in which what demands to be explained is one of the particulars of the universal. Understanding something, in this conception, is to surpass the particular/singular character of the subject matter, regarding it as a realisation of the universal.

Abduction in this sense is scientific inference and not any kind of inference. This overpassing/overriding is the same operation that is observed in "cognitive

[4] We return to the issue of pairs of opposite concepts in Section 4.2.3.

surpass" (Piaget & García, 1989, Introduction). It corresponds to say that, according to the abstract mind, the cognitive activity we call science aims at the production of cognitive surpasses (see a detailed discussion in Section 4.2.4). Consequently we can call *scientific knowledge* to the outcome of this activity.

The notion of scientific understanding we have coined contrasts with the notions of assimilation and accommodation. Both these forms of cognition are present in the child's development. The idea of assimilation and accommodation belongs to the family of imitation (Gruber & Vonèche, 1995, Introduction) which is within the same realm than analogical thinking. It is present since early times in life and is in use by the time of the development of abstract thinking, likely a pre-requisite and precursor. Abstract thinking is characterised by the overpassing increasingly present in the adolescent formal thinking [5], since the universal is reached by form of abstraction which is to put the actual as a case of the possible, being then the possible the universal form of the actual.

4.2 On the relation between subjective, intersubjective and objective

4.2.1 Intersubjective science

The Oxford dictionary says of arbitrariness: the quality of being based on random choice or personal whim, rather than any reason or system. Rejection of arbitrariness appears then at the level of language as a request for reason. Yet, since ideas are supported in subjects, and even images are, the most we can assure is a coordination of ideas mediated by the sensorial world. There is then some room for arbitrariness in our enunciation of natural laws. The laws might be objective but we are only certain that they are intersubjective. Let us inspect the situation in formal terms. We consider a finite set of scientists, labelled $i = 1 \dots N$, each one observing a subset of the sensible world, SW_i, from which they intuit through the maps f_i, a set of *facts* FW_i. The facts are accommodated in terms of i's epistemic frame, and further abstracted, through maps h_i, into a common theory, IW. In order to communicate IW it is required to produce a representation of it in SW, since communication operates by exchanging elements of the sensible world, be it sounds, signs written on some material, etc. Each of the maps h_i has an inverse map $\bar{h}_i$ that produces facts out of formalised ideas. Also the $f_i's$ have inverses $\bar{f}_i$ that produce (actual,

[5]

> Formal thinking is both thinking about thought (propositional logic is a second-order operational system which operates on propositions whose truth, in turn, depend on class, relational, and numerical operations) and a reversal of relations between what is real and what is possible (the empirically given comes to be inserted as a particular sector of the total set of possible combinations). These are the two characteristics—which up to this point we have tried to describe in the abstract language appropriate to the analysis of reasoning—which are the source of the living responses, always so full of emotion, which the adolescent uses to build his ideals in adapting to society.(Gruber & Vonèche, 1995, p. 438)

performed) experiments and measurements out of facts. There are two more elements to consider, formal reasoning (as controlled by logic) that enlarges IW (an endomorphism) and an endomorphism at the level of FW_i that enlarges the facts and is sometimes called thought-experiment since it presents its results as intuitions, just like actual experiments do. We will come back to them later (see "Imperfect knowledge" below).

A well organised cognition requires that whatever is in FW_i is only the result of f_i acting on SW_i (we say that f_i is onto, meaning that the rank of f_i is FW_i) assuring that there are no elements regarded as facts that have been created by other means (such as imagination), and also that $f_i \circ \bar{f}_i = Id_i$ meaning that facts in FW_i can be used to construct the original experiments or observations. In other words, FW_i is a faithful image of SW_i. The relation between IW and FW_i is requested to be faithful as well, hence $h_i \circ \bar{h}_i = Id_i$. In this perfect situation, the maps $\Pi_i = h_i \circ f_i$ map the observable real into the ideal real, and the maps $\Gamma_i = \bar{f}_i \circ \bar{h}_i$ map the ideal real into the observable real. Finally, the maps $G_{ij} = \Gamma_i \circ \Pi_j$ are endomorphisms of SW, and SW/G (being G the group with elements G_{ij}) is an abstraction of the observable reality. The elements of the group G connect the subsets SW_i coordinating them as images of the abstract idea. For example, each one of our scientists has an experiment that can be thought as related by Π_i to the idea of "double-slit interference". Each apparatus is different, yet we collect together all particular experiments and call the set of experiments with the label of the abstract form (double-slit interference). The group G is responsible as well of expressing the equivalence of all the projections Π_i which map into a unique abstract form, $\Pi : SW \mapsto (SW/G) \equiv IW$. Thus, it is the abstract (ideal) world, IW the one that organises the observed in an objective form, and that which is communicated among the subjects i.

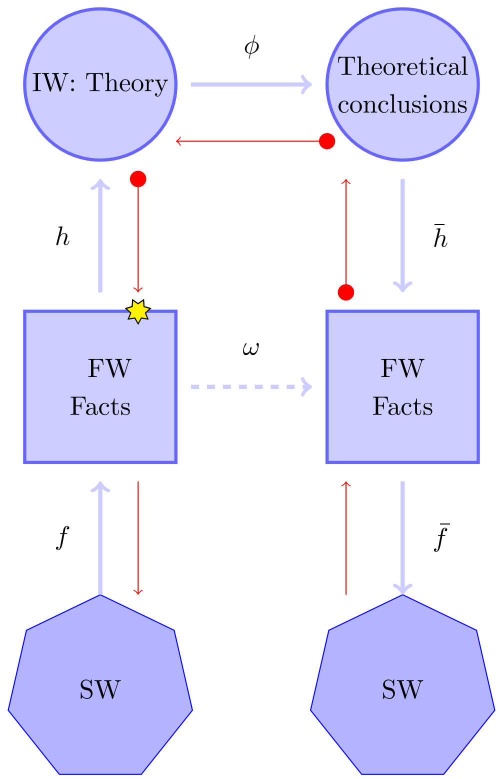

Figure 1: Proposed schema of intersubjective Science. SW, FW, IW are the observable, factual and ideal worlds. f corresponds to the intuition of the observed in terms of the epistemic frame creating facts, $\bar{f}$ corresponds to the production of observables from facts. h produces ideas through abstractions from facts, while $\bar{h}$ produces new facts from ideas. The star indicates the creation of awareness and the need to renew the epistemic frame as a consequence of the backward flow of error. It represents a change of the facts without changing the observations. It can be followed by new observations made possible by the new epistemic frame or by a new abstract form. ϕ stands for theoretical (formal) elaboration and ω informal elaboration at the level of facts (what includes thought-experiments).

The present schema of intersubjective science is depicted in Figure 1, disregarding the differences in perception, SW_i, and in construction of facts FW_i. In (Solari & Natiello, 2022a) the stage of facts is suppressed, concatenating $f \circ h = \Pi$ and $\bar{f} \circ \bar{h} = \Gamma$. For the sake of the present discussion we have emphasised the role of the subject, which will become more evident in situations of disagreement (see below). As long as different individual scientists and different ways of thinking do not influence the attainment of IW, they can be absorbed in the group G of particularities or arbitrariness.

The action of $\Pi : SW \mapsto (SW/G) \equiv IW$ can be understood in classical terms. Consider a single observation of an experiment, $x \in SW$. It is produced in a particular laboratory, at given time, with a particular apparatus, and so on. The elements of G map this experiment into any other experiment that has the same associated idea, this is, all occurrences of the experiment (it is called the orbit of x). We identify the orbit of x with a single element, collapsing all the different experiments into a "point" in a new space. Each abstract element in IW is then the set of the totality of the elements in SW that map into the abstract idea, and at the same time, it is the orbit by G of any particular element. The latter observation allows us to represent the abstract by an example in which the particularities are "blurred" (meaning we do not pay attention to them, since they are considered non essential). This form of representation of ideas can be recognised in Hume[6] and in general in empiricist philosophers like Boltzmann,[7] Hertz, [8]and Mach: [9]. At the end, abstraction and the grouping

[6]See the paragraph of Hume quoted in Section 2 .

[7]

> Nobody surely ever doubted what Hertz emphasizes in his book, namely that our thoughts are mere pictures of objects (or better, signs for them), which at most have some sort of affinity with them but never coincide with them but are related to them as letters to spoken sounds or written notes to musical sounds. Because of our limited intellects these pictures can never reflect more than a small part of objects. We can now proceed in one of two ways. The first is to leave the pictures more general, so that we run less risk of their later turning out incorrect since they will be more adaptable to new factual findings; however their generality makes these pictures more indefinite and vague and their further development will be connected with some measure of uncertainty and ambiguity. The second is to specialise the pictures and elaborate them to a measure of detail, in which case we shall have to import much more that is arbitrary (hypothetical) and might not fit new experience; but we shall have the advantage that the pictures are as clear and definite as possible so that we can draw from them all consequences fully defined and unambiguous. (Boltzmann, 1974, p. 226)

[8]

> Various images of the same objects are possible, and these images may differ in various respects.[...] we postulate in the first place that all our images shall be logically permissible [...] We shall denote as incorrect any permissible images, if their essential relations contradict the relations of external things [...] But two permissible and correct images of the same external objects may yet differ in respect of appropriateness. Of two images of the same object that is the more appropriate which pictures more of the essential relations of the object, the one which we may call the more distinct. Of two images of equal distinctness the more appropriate is the one which contains, in addition to the essential characteristics, the smaller number of superfluous or empty relations, the simpler of the two.(Hertz & Walley, 1899, p. 2)

[9]

> In mentally separating a body from the changeable environment in which it moves, what we really do is to extricate a group of sensations on which our thoughts are fastened and which is of relatively greater stability than the others, from the stream of all our sensations. Absolutely unalterable this group is not. Now this, now that member of it appears and disappears, or is altered. In its

of alike elements are two complementary views of a single process, emphasising either IW or SW.

Summarising, although science rests in part upon agreements among scientists (call them social-agreements) there is a need to involve the observable real in the agreement. To this end, the category "intersubjective" describes concepts that require both a social agreement and a supporting input from the observable real. Think, e.g., of the classical concept of time, as an intersubjective quantification of the concept of change, as opposed to permanence. Intersubjectivity requires the existence of a group (or groups) relating the different subjective perspectives.

4.2.2 The no arbitrariness principle

More generally, in (Solari & Natiello, 2018), it is shown that if we introduce some arbitrary decisions in the scientific discourse (be it for the sake of the argument or with the aim of facilitating an explanation), the set of possible arbitrary elements must have the internal structure of a group[10], being then the set of all possible presentations of the argument a representation of the group and as such equivalent. Further, we have shown that the facilitation of the relational concept of space due to Leibniz produced by the introduction of a privileged observer introduces a (useful) subjective element, the *subjective space* (the space in all elementary physics texts) along with a series of properties of this space as well as conditions that the statements regarding physical laws must satisfy if they are going to remain rational.

> full identity it never recurs. Yet the sum of its constant elements as compared with the sum of its changeable ones, especially if we consider the continuous character of the transition, is always so great that for the purpose in hand the former usually appear sufficient to determine the body's identity. But because we can separate from the group every single member without the body's ceasing to be for us the same, we are easily led to believe that after abstracting all the members something additional would remain. It thus comes to pass that we form the notion of a substance distinct from its attributes, of a thing-in-itself, whilst our sensations are regarded merely as symbols or indications of the properties of this thing-in-itself. But it would be much better to say that bodies or things are compendious mental symbols for groups of sensations—symbols that do not exist outside of thought. Thus, the merchant regards the labels of his boxes merely as indexes of their contents, and not the contrary. He invests their contents, not their labels, with real value. The same economy which induces us to analyse a group and to establish special signs for its component parts, parts which also go to make up other groups, may likewise induce us to mark out by some single symbol a whole group. (Mach, 2012, On the Economical Nature of Physical Inquiring)

[10]For example: We can say that the relations in the invariant relational space are lifted into relations in the subjective spaces by arbitrary decisions, but since the subjective statements must remain equivalent, there must be a group of transformations, T, that allows us to move from one presentation to the other. If we conceive now a theory as a space of statements, E, relating different concepts belonging to our subjective presentation, what is real in them is only the core that remains when we remove (mod out) the arbitrariness, $D = E/T$, which is the result of identifying statements that only differ by the introduced arbitrariness. Thus, D is invariant while E is equivariant with respect to T.

The set of arbitrary decisions deserves some further consideration. It has been indicated (Margenau & Mould, 1957), in consideration of their own versions of NAP, that choosing different arbitrary sets where the statement (observation) should identically hold might lead to different theories. Therefore, a clarification of the concept of arbitrariness in this context is needed. It is important to indicate that any difference that is dictated by experimental and observational methods should *not* be considered to be arbitrary, but there is more to it. For example, the class of (idealised) mechanical isolated systems, when we disregard their internal structure, admits a group of arbitrariness. We call this class of systems *inertial* (Solari & Natiello, 2021). They are to be distinguished from those systems (bodies) that necessarily indicate the presence of a companion body. Since isolated systems are an idealisation, the same can be said for inertial systems. Thus, approximately-inertial systems must exist. In practice, if a system can be considered inertial or not, depends on the extent that the presence of other matter not being accounted for can sensibly modify the experimental outcome. This view has been held by experimentalists such as Michelson (Michelson, 1904). In Solari & Natiello (2022a) it is shown that a correspondence in all possible observations related to the two theories establishes an equivalence relation between them under the condition that the theories are faithful to the experiments. However, Margeneau's view is still possible when at least one of the theories is not faithful. Such theories will be considered in the paragraph "Imperfect knowledge".

The present view of inertia must be contrasted with the subjective view. In the subjective view the subject considers the space as "that what is surrounding me". Velocity means the velocity relative to the subject. In this form, the concept matches the intuited meaning of velocity. When these subjects learn about Newton's theory, that makes reference to certain privileged (inertial) systems where Newton's laws hold, the question arises: Who is the subject associated to those privileged systems? Thus, a first reference for "inertial system" appears as a requisite. In this form they can easily accommodate Newton's idea to their pre-existing notions of physics. If they find absolute space unacceptable, they may propose the "alpha body" (Newmann), the fixed stars, the centre of mass of the Universe or the ether, all alternatives proposed by Mach (Boltzmann, 1974, p. 103)(Mach, 1919, p. 567, 570). In order to assimilate-accommodate the notion of inertia it becomes necessary to suppress or downsize incompatible evidence. In particular, Corollary VI in Newton (1687, Ch. I) must be overlooked as it presents the case of an accelerated system in which Newton's laws are expected to be applicable. Also, the work by Lange (1886) must be disregarded. Boltzmann (1974, p. 103) argues that Lange's argument is complicated in excess. Mach disregards Lange's proposal by saying:

> In the first place, we shall not dispute the fact that the law of inertia can be referred to such a system of time and space coordinates and expressed in this form [...] It especially appeals to my mind, as a number of years ago I was engaged with similar attempts [...] I abandoned these attempts, because I was convinced that we only

> *apparently* evade by such expressions references to the fixed stars and the angular rotation of the earth. This, in my opinion, is also true of the forms in which Streintz and Lange express the law. (Mach, 1919, p. 546)

In Mach's opinion, reference to the fixed stars is needed to grasp the meaning of Lange's ideas. This further illustrates how some forms of thinking (in this case Lange's) cannot be grasped from other forms (Mach's).

When constructing a theory we have to make an early decision: are we going to introduce arbitrariness or not? The decision has not much relevance if we keep track of the introduced arbitrariness, and acknowledge the necessity of (and the methods for) removing it. However, if we lose consciousness of our constructive effort, we might inadvertently enter into the realm of arbitrariness. No amount of mathematics will take us ever out of the subjectivist cage, since the necessary step is not an analytic/deductive judgement but rather a synthetic/critical one. This is, we need to understand not what the consequences of our beliefs are, but rather which is the foundation of our beliefs.

If arbitrariness is the absence of reason, the double negation in no-arbitrariness is equivalent to reason. This is, the rejection of arbitrariness is a condition put on every rational construction (Solari & Natiello, 2018).

Principle 1 [No Arbitrariness Principle (NAP)] *No knowledge of Nature depends on arbitrary decisions.*

Imperfect knowledge Imperfect knowledge can occur in different forms, for example f_i may send some observable phenomena to the empty set, meaning that they have not been observed by the scientist, they have not been constituted as facts. More interestingly, scientists, in their effort to accommodate the observed into facts and to assimilate them to the theory, may decide to conjecture the existence of an unobserved object, thus enlarging FW_i with imaginary (imagined) entities. If the entity is actually unobservable, they will be forced to perform frustrated experiments, thoughts that cannot reach SW_i but, rather loop back into FW_i: *thought-experiments.* The most persistent of them that we know about are those associated with the early intuition called space. While we all can produce a spatial relation in SW, no one has been able to show a position in space. In identical form, nobody has succeeded in observing an action travelling through space. Every attempt of so doing rests upon observing the action on a detector and only imagining its travel through space. Neither space, nor travelling of actions are observable. We interpret that energy is absorbed/emitted in quanta and we infer that it travels through space as quanta[11].

[11] Assume a laser system emits a quantum of energy $\hbar w$ which is captured by an atom moving in relative motion with respect to it, as it is actually performed in "cooling atoms traps" (Phillips, 1998). The absorbed atom incorporates energy by an amount of $\hbar w'$ where w' depends on the relative velocity with respect to the laser and corresponds to the Doppler effect. When did the quality, w of the emitted photon changed to w'? Assume there are several atoms moving in different directions: Do identical emitted photons have several frequencies? In which sense the laser emits light quanta of frequency w? We conclude that the frequency

Yet, all evidence is in the form of absorption/emittance. What we observe then is a systematic construction incorporating imagined entities to fit the needs of our epistemic frame, akin to the construction of mechanical analogies.

When we enlarge FW_i to allocate imagined entities, a disruption, a failure in producing the necessary synapsis between f and h takes place, since the preimage of h is larger than the image of f. As a consequence, there is no longer an attempt to organise the observable world, but rather the organisation of the factual world FW_i, now enlarged with imagined entities. This organisation excludes those that do not share the same epistemic frame, the same style of thinking. This is, the real ideal will become an unanimous agreement only after expelling those that disagree, a matter of social practices. A necessity of this imperfect knowledge is that of weakening the links between theory and observation to accommodate imagined "facts". As discussed in (Solari *et al.* , 2016; Solari & Natiello, 2022a), when this has happened in history, a new epistemology was needed.

A criticism of empiricism A property is a quality proper of something. Whenever there is a property, there is something to which it belongs. If s stands for something and p for property, the basic enunciation is: "s is p". The set of properties, $\mathcal{P}$ is the set of all possible values of p irrespective of the s. It is true that the enunciation "s is" (produced after elimination of all the properties) is meaningless, an argument that is found in Carnap (1959, The significance of a sentence), but it only indicates that the search for the essence by depriving the object of its attributes is the wrong path. In the same form in which we admit a set of properties, we are forced to admit the set of objects constituted by all those things, $\mathcal{S}$, pointed by s in statements of the type "s has p", regardless of the property p. Doing otherwise is an instance of arbitrariness, since –as already discussed– the universal of something is nothing but the set of all particular forms of the matter/object under consideration. [12] It is important to notice that here as well as in Sartre, the metaphysical (and Kantian) "thing

cannot be associated to the photon. Are the quanta adjusted to whatever it is needed to match the Doppler effect?

[12] A similar discussion is found in (Sartre, 1966, p. 3):

> Force, for example, is not a metaphysical conatus of an unknown kind which hides behind its effects (accelerations, deviations, etc.); it is the totality of these effects. Similarly an electric current does not have a secret reverse side; it is nothing but the totality of the physical-chemical actions which manifest it (electrolysis, the incandescence of a carbon filament, the displacement of the needle of a galvanometer, etc.). No one of these actions alone is sufficient to reveal it. But no action indicates anything which is behind itself; it indicates only itself and the total series. The obvious conclusion is that the dualism of being and appearance is no longer entitled to any legal status within philosophy. The appearance refers to the total series of appearances and not to a hidden reality which would drain to itself all the being of the existent. And the appearance for its part is not an inconsistent manifestation of this being. To the extent that men had believed in noumenal realities, they have presented appearance as a pure negative. It was "that which is not being"; it had no other being than that of illusion and error.

in itself", the noumenon, is eliminated as in Mach and Carnap, but the abstract survives.

The elements in S bear all of them in common an undeniable property which cannot be suppressed, a essential property in the words of Mach: all members of S can be used in statements of the form "s is p" for some well selected $p \in \mathcal{P}$. The set of well selected properties regarding an object s, call them $\mathcal{P}_s \subseteq \mathcal{P}$ is the bundle of properties associated to the object in Mach. Reciprocally, the intersection of all $s \in \mathcal{S}$ having the property p is the only possible indication of the property. It then entails the same risks to admit the existence of $\mathcal{P}$ than the admission of the existence of $\mathcal{S}$, we must admit both or none of them, for otherwise we incur in arbitrariness. Mach restricts these considerations to regard observable objects, which is mere willingness and not a logical demand, for if it were true, we could never speak of the properties of mathematical objects such as vector spaces or numbers. In conclusion, the fact that the search for the essence in Mach's method fails does not mean that there is no essence, it only means what it shows: the search cannot reach the target, it is an inadequate searching method. Having failed to grasp the universal, Mach is then forced to pick one of the particulars (the "simplest one" in the words of Hertz) and to think in terms of it extending the results obtained to the whole class by invocation of a weak equivalence: analogy, an equivalence based upon opinion [13]. There is then no abolishment of metaphysics but rather what is abolished are higher levels of abstraction at the cost of introducing subjectiveness.

[13] It is interesting to compare the present view with Mach's position in (Mach, 2012, The economical nature of physical inquiry). We read:

> Nature exists once only. Our schematic mental imitation alone produces like events. Only in the mind, therefore, does the mutual dependence of certain features exist.
>
> Let us endeavor now to summarise the results of our survey. In the economical schematism of science lie both its strength and its weakness. Facts are always represented at a sacrifice of completeness and never with greater precision than fits the needs of the moment. The incongruence between thought and experience, therefore, will continue to subsist as long as the two pursue their course side by side; but it will be continually diminished. In reality, the point involved is always the completion of some partial experience; the derivation of one portion of a phenomenon from some other. In this act our ideas must be based directly upon sensations. We call this measuring.

In Mach "schematic mental imitation" corresponds to the role of abstraction, now exercised by blurred images. Rather than decorative detail or particularities, what is sacrificed –according to Mach– is completeness. This action in Mach parallels the action of Π. The restitution of the particulars that corresponds to Γ becomes in Mach "the derivation of one portion of a phenomenon from some other", which requires analogy and imagination. Thus, the central difference is Mach's relying on "imitation" as opposed to our relying on abstraction. Mach (2012, The principle of comparison in physics) explains:

> The adoption of a theory, however, always involves a danger. For a theory puts in the place of a fact A in thought, always a different, but simpler and more familiar fact B, which in some relations can mentally represent A, but for the very reason that it is different, in other relations cannot represent it. If now, as may readily happen, sufficient care is not exercised, the most fruitful theory may, in special circumstances, become a downright obstacle to inquiry.

Mach will soon name the substitution described as "analogy".

4.2.3 The mediation principle and the dialectical openings to understanding

Dialectical openings Our discussion of the construction of science is based upon repeated action of synthetic cognition:

> Synthetic cognition aims at the comprehension of what is, that is, at grasping the multiplicity of determinations in their unity. (Hegel, 2001a, §1720)

But this consideration takes as given the multiplicity of determinations, the observable. It then underplays the fact that in order to perceive as multiple what can be clearly argued that is different, we need to disregard some part of what is determined in each element of the multiple. The grasping of something as a particular case of an universal ideal and the synthesis of the particulars in universals is in fact one single operation that creates the ideal (the abstract form if we want) and the particular (the concrete forms). Such operation opens the possibility of cognition. We call this act a dialectical opening. The basic dialectical opening of natural science corresponds to the creation of the duality (SW, IW). However, the most striking form of dialectical opening is the one performed by the child conceiving space and time (see Section 4.1.2). This operation requires that there is a something (name it relative position with respect to ego, or position in space) that is not permanent, that is a non-permanent property of the object. Further, if there are non-permanent matters, the idea of non-permanent becomes the idea of change, later *time*. Any attempt to explain one element in the dualities ego-alter, permanence-change calls for the other term, any attempt to explain position requires an idea of time, and an idea of object. Thus, the elements of the descriptions are interdependent, they have been constructed by idealising (taking to extremes) perceived differences and all of them together open the possibility of organising what reaches our senses. A dialectical opening institutes the terms that make possible to organise the idealised world and with it the sensorial world, it makes understanding possible.

Notice that dialectical openings operate on the basis of perceived differences which are ideated into complementary options within their universe of application. Being complementary, both of them are the Universe minus its complement: the negation of each other. We emphasise that the differences are perceived, they belong to SW. For example, the duality between sensible and ideal, is related to the possibility of blocking sensorial input (as for example by closing the eyelids) being impossible to willingly stop the flow of thoughts. Notice further, that the frequently encountered duality essence-appearance is not a dialectic opening for it leaves all the perceived in appearance as opposed to a metaphysical entity: the essence, the "thing in itself". In this sense dialectical openings realise the elimination of metaphysics as the Vienna Circle seek, but preserve the abstract with a necessarily vinculum to the observable.

Mediation principle We do not usually accept as reasonable that which appears out of nothingness as self-evident assertions. We normally request a new rational belief to be derived (mediated) by acceptable argumentation from accepted beliefs. This recurrent form of reasoning cannot be pursued indefinitely. It comes to an end when we reach a point in which beliefs can no longer be derived from other accepted beliefs. At this point there seems to be only one option: Either we make explicit a layer of arbitrary assumptions (axioms) which is the opaque end that reason lets us see, or we find a set of opposing concepts and ideas that in their interplay constitute the foundation of our discussion; the dialectical openings.

Axiomatisation turns natural science into exact science, physics into mathematics, by removing the links between IW and SW. However, a purely abstract science is void. Instead of pushing physics into the exact sciences we must consider mathematics as a natural science, being the fundamental elements of mathematics the idealisation of quantitative relations in the observable world, which are always in relation to qualities (Usó-Doménech *et al.* , 2022). Thus, projecting out the quality in SW we obtain the quantity. The operation requests us to conceive the dialectical opening quality-quantity along which we make the projection.

4.2.4 Cognitive surpass

The introduction of explanatory hypothesis, the process of abduction, is subject to the control of rationality and to the condition that the newly introduced hypothesis explains a class of problems larger than the one that motivated it, this is, that the hypothesis bears some of the main ingredients of *cognitive surpass* (Piaget & García, 1989) and offers itself more openly to refutation. However, the requisites for the acceptance of explanatory hypotheses (i.e., to be able to stand in front of refutation attempts) say little about the method of production.

4.2.5 The continuity principle (reduction to the obvious/evident)

Argumentations are constructed in such a way that they rest upon small units we consider evident or obvious. Yet, what is obvious or evident for some, may not be so for others. One of the forms in which we usually identify potentially irrational arguments is by detecting hiatus or *lacunæ* in the argumentation. The request "please fill in the gap" quite often reveals a belief that cannot be supported, while being necessary for the argument. On the contrary, the rational argumentation proceeds to fill the gaps by explaining how they consist of the concatenation of smaller pieces, iterating the process until the pieces are accepted as evident or obvious. This self-similar form corresponds to what in mathematics is called *continuity.*

4.2.6 Logical action in front of contradictions

Whenever a chain of deductive reasoning arrives to a contradiction, the whole chain is rejected. When the contradiction results from comparing theoretical prediction and experimental reality we speak of experimental refutation. The logical scheme can be depicted as $A \Rightarrow$ [consequences] $\Rightarrow B$ and B evaluates to $False$. No matter how pleasant the intermediate consequences are, there is no support for them. The most evident example is the hypothesis of the ether which is fundamental for the proposition of Maxwell's displacement field. Discarding the existence of the ether (following empirical evidence) under the present principle would mean the refutation of the hypothesis as well as its consequences. Yet, in general $A = a_1 \& \ldots \& a_n$, i.e., A may be a composite statement consisting of different parts. Only the hypothesis and consequences involved in the deduction of B are necessarily affected by the falsity of B. Thus, part of the theory survives and only part needs to be constructed under new hypotheses.

There is another instance of the same logical scheme which is not usually considered, namely when the contradiction stems from the logical structure of the theory (e.g., inconsistent postulates). Assume A is True, then $-A$ is False. The construction $A \Rightarrow$ [consequences] $\Rightarrow (-A)$ discloses an internal contradiction of the theory and, as above, it forces us to reject the full chain.

Again, it is worth to realise that a refuted theory may require only some minor repair since usually A is a composite statement. It is enough for one of its terms to be wrong for the theory to be refuted. Refutation does not mean "throw away all your thoughts".

4.2.7 The rules of reason applied to the system of Law and Justice.

Reason manifests itself not just in science but in several other matters, including our social organisation (in the western civilisations this is a heritage of the French Revolution). Hence, we should be able to find the particular form that the rules of reason above discussed take in these other matters. As an exploration of the idea, we consider the Law and Justice system in modern democracies.

No arbitrariness is manifest in: "All human beings are born free and equal in dignity and rights", "Everyone is entitled to all the rights and freedoms set forth in this Declaration, without distinction of any kind, such as race, colour, sex, language, religion, political or other opinion, national or social origin, property, birth or other status", "All are equal before the law and are entitled without any discrimination to equal protection of the law". All these statements belong to the Declaration of human rights of the United Nations[14], all of them are clear rejections of arbitrariness and we hold them supported only by the conviction that reason provides.

In many periods of history, justice was a discretionally exercised attribute of a king or emperor, such as in tales about ancient China, Egypt, the Inca empire, etc. (possibly with a code of law subordinated to the ruler). A reason-guided justice presents a **dialectical opening**, replacing the discretionary power with

[14] https://www.un.org/en/about-us/universal-declaration-of-human-rights (accessed 2022-05-22)

a structure consisting of (abstract) law, judges and/or jurors and legislators. Each of these terms refers to the others.

The **mediation principle** is present as well since nothing can be considered an infraction if it is not indicated as such by a ruling law at that moment. The **continuity principle** is built in the administration of justice, since the judgment of each case (the particular) must be connected and reasonably argue in terms of the law, and laws must be rationally connected with upper statements of justice such as constitutions or international treatises as the Declaration of human rights.

Cognitive surpasses correspond to the process of construction of the law (as well as in theories). It is associated to the process of abstraction in which the fault is typified and described in general form. It is the reasoning that constitutes the law and further more, the general principles of civil law civil law (accessed 2022-05-22).

There are different instances of **action in front of contradictions**. On one hand, inconsistencies in the legal system (e.g., by errors or misunderstandings at the production instance) have a handling routine, that in ideal form starts with one individual claiming arbitrariness in the application of the specific law. If this is established as a fact, a process of reparation and law amendment starts, which may involve cognitive surpasses enhancing the range and accuracy of the new law. Further, the arguments that connect the accused with the fault must be free of contradiction to be accepted. This is e.g., the nature of an alibi.

4.2.8 Example: demarcation of a non-scientific belief

Let us show how the requisites proposed in this Section change our perception of what is acceptable as scientific and what is not. Suppose we have a belief, T such as "All swans are white", and we have a form of determining what a swan is without considering its colour, call it A, we have that $T\&A \Rightarrow A\&W$, meaning that if we believe the theory it "explains" that the swan I am observing is white. The most immediate reason why such a belief is to be called non scientific is that it is not linked (has not been explicitly linked) to observations, thus, we cannot recognise the construction of a theory, which is, at the end, what is to be subject to appraisal.

Let us now give reasons for our belief: we have been observing fowl during ten years at a lake nearby our home. In these years of observations we have recognised N swans (mostly by their neck, beak and swimming, say) and the totality of them were white. Call these observations O and we have conjectured T out of O, this is our basic theory. Would now our belief be scientific (given the fundaments)? Our observation consists of a triple: a statistics, a place and a time frame. The theory T is produced by projecting out place and time frame. In order to confront the theory with new data we have to produce statistics in other places and time frames. Let us check the requirements of rationality. The initial violation of the mediation principle has been repaired in our second attempt. Is there continuity? What allows us to go from the observed into the idealised theory? The answer is: a rudimentary version of probability theory. If we want to determine the theory, T from observations, all what we can do is to establish bounds to the probability of observing a counter-example, a refutation of our theory, in N observations. Actually, we have no support to disregard the alternative explanation that this happens only in the lake we observe or in some lakes or period of time, all doubts that tell us that our theory is not under a firm ground. We may restrict the theory to our lake and the observed period of time. During that period other bird-watchers might have collected data as well. In this case, statistics would allow no more than establishing a bound, p^*, on the probability, p, of detecting an individual not being white in the experimental situation we are exploring. Let $P(p;N)$ be the probability of observing non-white swans in N trials. Hence, for a null observation record, given P we can estimate p^* as the solution of the equation $P(p,N) = \frac{1}{2}$. Were $P(p^*,N) > \frac{1}{2}$, the null observations would be an exceptional result rather than a "confirmation" of T. But then, why we have chosen $p = 0$ in our theory T? We meet arbitrariness once again. With the sole support of statistics we cannot make such a bold theory. We would have to make then a third version of the theory which changes our belief for something in terms of probabilities, which eventually will cast doubts about the quality of our statistics: do we have bias in our sampling?

The example shows how we proceed from an initial belief, a hunch, critically searching for its fundaments using some of the rules for reasoning that are not (at least in this presentation) formal logic.

4.2.9 Example: the principle of relativity

Sapere aude! (Kant, 1783)

Most physicists, including us, accept the principle of relativity as correct. Why is it so? While Einstein does not offer any argument Einstein (1905), Poincaré does (Poincaré, 1913, Ch. VII). Poincaré states:

> **The movement of any system whatever ought to obey the same laws, whether it is referred to fixed axes or to the movable axes which are implied in uniform motion in a straight line.** This is the principle of relative motion; it is imposed upon us for two reasons: the commonest experiment confirms it; the consideration of the contrary hypothesis is singularly repugnant to the mind. [Emphasis added]

Poincaré comes to no better argument after several pages dwelling in Newton's mechanics as it was taught at his time (based upon absolute space). No search in the realm of mechanics will serve the purpose of finding the foundations of the principle after Newton's axioms. The foundations are to be found behind them, a matter Newton did not discuss at large. Moreover, the little he did discuss was misunderstood, such as the notion of "true motion" which was shadowed and finally replaced by the notion of "motion in absolute space" which is not in Newton.

The idea of the relativity principle as proposed by Poincaré and others under various names (such as "symmetry principle" (Mach, 1919)) is a belief coming from the habit instructed to physicists by the teaching of Newton's mechanics. Observational inferences such as "All swans are white" can be put to experimental test (and in this case proved wrong by e.g., displaying black swans from Australia or black-necked swans from South America). On the contrary, the relativity principle is not observational. To assess its truth value we must seek its foundation in the demands of reasoning. The principle is a particular case of a more general (universal) principle, one that when opposed produces "repugnance to the mind" (Poincaré). The principle involved is the rejection of arbitrariness, a principle that dwells in all of our reasoning, we argue. The no arbitrariness principle (NAP, Solari & Natiello (2018)) imposes conditions in the form in which theories assuming arbitrary matters must be related. Those conditions are satisfied by Galilean transformations, which can be deduced from spatial relations plus the arbitrariness of the reference point in the idea of space, but are not satisfied by Lorentz transformations (Solari & Natiello, 2022a). The validity of the relativity principle does not imply the appropriateness of Lorentz transformations. They become in this form mere analogous to the Galilean transformations [15].

[15] It is interesting to examine the argument in (Einstein, 1940):

> The so-called special or restricted relativity theory is based on the fact that Maxwell equations (and thus the law of propagation of light in empty space) are converted in equations of the same form when they undergo Lorentz transformations. This formal property of Maxwell's equations is supplemented by our fairly secure empirical knowledge that the laws of physics are the same with respect to all inertial systems. This leads to the result that the Lorentz' transformations –applied to space and time coordinates– must govern the transition from one inertial system to any other.

In the first place, the conclusion is not deducible from the premises. Secondly, the Lorentz transformations do not form a group, the composition of two of them is not a Lorentz transformation except for special cases (an observation that by itself indicates that it cannot connect different inertial systems, since such connections are automorphisms of the inertial systems and as such must form a group). The Lie algebra associated to the group of Poincaré-Lorentz symmetries has dimension six and what we need has dimension three. Hence, Einstein's inference breaks the continuity rule. A defender of relativity would argue that the three group generators not considered correspond to rotations. This is true in abstract terms, but the necessary correspondence with rotations in the sensible world is not present. The rotations correspond to $L(-(u \oplus v)) \circ (L(u) \circ L(v))$ (Gilmore, 1974, p. 503), where $\oplus$ stands for Einstein's addition of velocities and $L(u)$ is a Lorentz transformation based on the velocity u. To imagine in our minds the consecutive application of three Lorentz transformations does not enact the rotation of physical objects. There is no correspondence between actually rotating a given object and the rotation group regarded as a subgroup of the Poincaré-Lorentz group.

In turn, the failure to recognise this problem originates in the suppression of abstraction:

> An adherent to the theory of abstraction or induction might call our layers "degrees of abstraction "; but, I do not consider it justifiable to veil the logical independence of the concept from the sense experiences. The relation is not analogous to that of soup to beef but rather of wardrobe number to overcoat. (Einstein, 1936)

Einstein gives "logical independence" to the concept from its conception originated in our sense-experiences. Adopting "free invention" implies to break the connections with nature, to have purely abstract concepts, detached from their conceptualisation. Science would then

If such is the case, there is a possibility for other rules of reasoning being broken by special relativity. The exploration points to the meaning of the velocity in a Lorentz transformation. First, the meaning of velocities in the context of Electromagnetic theory changed from “relative velocities” in experiments, to velocities relative to the ether later and finally to velocities with respect to a reference frame. Such changes are a signal of trouble in itself (Assis, 1994). It is not possible to connect any observable, objective, velocity to provide meaning to the velocity involved in a Lorentz transformation (Solari & Natiello, 2022a). This matter puts us in front of the dilemma: either Special Relativity is imperfect or we have to abandon the hopes for science to be based only in observations and reasoning. Historically, the second alternative was taken. Indeed, Einstein advocates that there is no abstraction (or any other relation) between observations and the theories at the time of production (Einstein, 1936), they arise by free invention and are validated by their results.

Nevertheless, science –understood in the terms proposed in this Section– not only indicates where problems are but it also suggests where to search for solutions. Is special relativity a true necessity imposed upon us by Electromagnetism? Or is it imposed under the hidden, additional, hypothesis that we ought to understand (physics) in the terms of imaginative scientists[16]? Science suggests to reconsider the conflict issue. Thus, to achieve a “new” Electrodynamics we have to work from ether free theories; actually with theories –such as the Lorenz-Gauss theory– not based in the illusion of space. Additional doubt: what made us think that the form of ordering nearby objects developed as children can be used to order every thing in the universe, be it sensible or inferred?

Such a reconstruction of Electromagnetism is actually possible and results in a more conciliar electromagnetism where Lorentz transformations have a different meaning (Solari & Natiello, 2022b), unrelated to coordinate transformations. If we perceive (measure, detect) an Electromagnetic signal and we have no more data than the signal itself, we can only infer that it was produced by some change in a system of charges and currents (at the source) that might eventually be in relative motion with respect to our detector. For example, we cannot know about the frequencies involved as measured by a device at a fixed position relative to the source since we cannot correct for the Doppler effect. There is then some liberty for our inference about the source, any decision taken with respect to it would be arbitrary, hence all such decisions must be connected by a group, which turns to be the Poincaré-Lorentz group. Its function is to sort out arbitrariness in perception, having nothing to do with inertia. The coupling of inertia with perception was a secondary effect originated in the need of the imaginative scientist to find imagined connections with mechanics, attributing to actions a place in space.

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no longer be a matter of understanding nature, but rather a sort of game. In Einstein (1936) words:

> It is an outcome of faith that nature – as she is perceptible to our five senses– takes the character of such a well formulated puzzle.

[16]In such a case it would be just a social imposition.

But then, what made all this cooperation of changes to coincide in time? Why were German scientists and philosophers so intensely involved in them? Doubts that show how ephemeral the rest of our thoughts are.

4.3 Multiple abstract projections and the case of science

The idealisation or abstraction of an observed phenomena is performed with the aim of organising our view of it, linking the new facts to pre-existing matters in our understanding. Such operation is motivated by the need of answering questions regarding the phenomena. Thus, both the questions being posed and the pre-existing knowledge suggest which features of the phenomena carry the potential for *explanation* (providing an answer to the questions) and which do not. This process is followed by logical elaboration and interpretation that provides the opportunity for contrasting the ideal with the observed, and, in case of refutation it triggers a new attempt at producing understanding. The process is directed by the posed questions, and as such, different idealisations are possible for different sets of questions. The observable real is then crossed by several idealisations, each one can be said to correspond to a *dimension* of the phenomena. The associated projections on IW characterising these aspects are in principle considered independent one of each other, something that will later be modified by synthetic judgment, that confronts the alleged independence. Let us illustrate this process with the idea of "science as it is practised".

The notions of science so far discussed correspond to an ideal, flawless functioning, science. The practice of science develops in a society which is part of a civilisational movement, thus there is a science idealised in terms of its relation to the society at large. Science is practised by human beings that constitute a particular field of symbolic production (Bourdieu, 1999). Thus, we easily find three different dimensions in the consideration of science.

From the point of view of society in general the goals of science are often related to the production of goods and practices that enhance well being. Central to well being is techno-science, geared towards the production of new goods, enhancing comfort and capabilities. Techno-science frequently adopts the criteria proper of technology and focuses on predictive success. The quality of this science is hence rooted in prediction. If something *works*, this is taken as support for it being *correct*. The foundation of scientific theories is subordinated to their success capabilities. In the schema of Figure 1, it is $\phi, \Gamma \equiv \bar{f} \circ \bar{h}$ what matters the most.

In addition, science is requested to guide some important decisions. For example, decisions in matters of global warming, epidemics, nuclear energy safety, human environmental impact and the extinction of species. In such endeavours, the contrastive comparison of the predictions is not possible. This aspect has been called "science in the post-normal age" (Funtowicz & Ravetz, 1993; Waltner-Toews *et al.* , 2020). Such practice is forced to root its quality in the elements Π, ϕ of Figure 1, since Γ is not available.

The most traditional perspective is that of science as the search for harmony and understanding. In the scheme of Figure 1 all three elements Π, ϕ, Γ are then equally important and they cooperate (e.g., via auto-correction) to enhance understanding.

Thus, differences in epistemology are to be expected in correspondence with the demand put on science by society at large.

This situation is a constant source of misunderstanding. It may be argued that the "correct" relation between theory and observation is the one reflecting the current practices of the community of scientists, as if the practices of scientists were not conditioned by the necessity of justifying science in front of the supporting society (society at large, governments, granting institutions, etc.) or were not conditioned by the need to conform to established practices of their scientific social field. There have been attempts at explaining science as a practice directed towards the acquisition of knowledge in terms of features of the social structure of the field, such as competition for resources and social respect. A third source of misunderstanding is to believe that what has been observed for science in some age (say after World War II) can be used to explain the development of science in another age (e.g., before the second industrial revolution). To consider science as "that which is analogous to what is currently observable" is to operate against the process of abstraction, which, as Piaget taught us, is geared toward the discovery of the possible as opposite to the given. It should then be considered a political act of conservatism that deprive us of ideals and the exercising thoughts directed towards the search of the foundations: the critical thinking.

5 Final thoughts

During the late part of the XIX century two concurrent changes altered the scientific scene in physics. First, philosophy was no longer entitled to critically examine scientific results, thus giving autonomy to the sciences. Since the fundaments of any science are always out of its scope, critical reflection was in this form prevented to influence the field, or at least discouraged. At the same time, the just born theoretical physics (Jungnickel & McCormmach, 2017) adopted methods based in imagination, models and analogy as testified by Boltzmann and identified in other authors, with the approval of some philosophers such as Mach. Despite the earlier defeat regarding the ether, the transition was never reverted. After one generation, there was no more room in theoretical physics for the abstract minds. Abstraction itself was confused with metaphysics and attacked. The old science identified with experience and reasoning was left behind. The old reasoning included logical deduction as well as abduction such as the view of Peirce (based upon Kant and Hegel). The phenotype of the physicist changed in the new environment from abstract to imaginative thinker. At the same time, the imaginative scientist had difficulties understanding the old abstract work, as we have put in evidence. A fight developed against ideas in Newton's mechanics that were misunderstood in terms of absolute space. Abstract ideas such as force and inertial systems were attacked while at the same time the old adage "matter cannot act where it is not" was adopted as a fundamental principle that immediately resulted in the ether, an imagined entity bridging the gap between the observed and the dogma. Thus, in the realm of physics, abstraction was dismissed as metaphysics, and phantasy plus dogma became the foundations. We must keep in mind that if constructing with elements of phantasy is a necessary ingredient for understanding (as for example a mechanical analogy), phantasies cannot be eradicated. In the same form, it appears to us that if making abstractions is a necessity for understanding (such as for the authors) abstraction cannot be rejected. In both cases, abstraction and imagination are given in the mind; each is a central tool for organising the world according to the respective thinking style. Since abstraction comes later than imagination in the development of the child, we speculate that those that use abstraction know imagination but do not master it, being this weakness what makes them to embrace abstraction if and when they feel comfortable with it. Such motivation is absent for the bright imaginative thinker. This (wild) hypothesis of psychological order would explain why the misunderstandings detected among scientists are all of them in the form "imaginative does not understand abstract". The alternative hypothesis is that we are blind to misunderstandings by abstract thinkers, but in that case, we cannot help ourselves.

The programme of the Vienna circle proposed to eradicate metaphysics, to approve the doings of the physicists and to demonstrate that science was made of experience and deduction. Ernst Mach, one of its inspiring figures, was himself a scientist and philosopher. As scientist, he sides clearly on the extreme of the imaginative scientist. In several respects, Mach is simply unable to understand Newton and/or the relational space, he needs a first reference of position and motion, and he boldly sustains the "fixed stars" as such reference when confronted with more abstract ideas such as those of Lange. However, space is only a production of the child, made out of spatial relations and the suppression of her/himself from the total picture. In an early stage of development we fail to perceive us as one arbitrary particular of a universal, we fail to produce abstraction. Spatial relations are then abstract, the universal form of the intuited space. The idea that space does not exist as such, which is clear in Kant and later in Piaget, simply obfuscates a great mind such as a young (30 years old) Bertrand Russell. Actually, the idea that the form in which we ordered the observable in our early infancy might not be the universal form for understanding appears as inconceivable to some minds. To sustain physics on the phantasy of a space that is no longer linked to completely observable events, rather than on the observable spatial relations, "simply" requires more phantasy: things that are unobservable except for their alleged consequences, the observable facts. The phantasies in physics keep growing in the form of "fixes" that must sustain previous fantasies. A quasi-material space, populated by fields, or better by quasi-particles (bosons) carrying action so as to sustain early intuition as children and the consequent dogma of the old adage, Dirac's sea of electrons, a vacuum (contradictorily) filled with quantised springs (later strings and membranes), dark matter to compensate the failure of a loved theory, dark energy trying to complete the patching of the theory and non-mathematical operations to compensate theoretical inconsistency... the list could continue, but the rule of the list can be made explicit: preserve the imagined and the dogmas.

We have already been warned by Faraday that what began by being a conjecture too often becomes a belief just by habituation:

> But it is always safe and philosophic to distinguish, as much as is in our power, fact from theory; the experience of past ages is sufficient to show us the wisdom of such a course; and considering the constant tendency of the mind to rest on an assumption, and, when it answers every present purpose, to forget that it is an assumption, we ought to remember that it, in such cases, becomes a prejudice, and inevitably interferes, more or less, with a clear-sighted judgment. (Faraday, 1844, p. 285)

We must credit Einstein for becoming aware that physics had grown detached from the observable/experimental reality. The tie that abstraction puts between the world of ideas IW and the sensible world SW was broken, making the understanding of Nature an act of faith (in his own words). The goal of a science based upon experience and reason alone, has to be sacrificed since otherwise critical thinking would have to inspect the sciences. Critical thinking is always abstract and abstract thinking is a matter of conceiving the possible of which the given is just one particular case. Conceiving other possibilities is subversive for the given, since it challenges the rationality of the given as its only fundament. It is then a mandate for the powers of any society to oppose the abstract as much as it is a mandate for humanity to pursue it. As much as other social phenomena, the evolution of science is steered by politics. It is just a matter of political conservatism to consider science as being what is determined by the doings of those socially recognised as scientists: science is then the given science, the science as it is actually practised, the one supported by the society, the only possible one. As we have explained, in so doing the several idealisations of science are confused constructing a fallacy of ambiguity.

Is it possible for a science based upon reason and experience to recovered the track lost more than 100 years ago? How is rational inference different from other forms of inference? We have made an effort to produce a few rules of rational thinking, and showed how they help to construct more solid beliefs. We hope it is just the beginning of a collective task long overdue, and that other scientists and philosophers will contribute their own rules. In front of us rises the most formidable task: to rethink the possibilities of humanity and the life on Earth. To believe that the same science that gave us the menace of nuclear destruction, global warming and an accelerated extinction of species will give us the means to avoid catastrophe is only the characteristic insistence of the dogmatic. Critical thinking, critical philosophy and critical science are urgently needed alongside imaginative science.

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Appendices

A Misunderstanding dialectics

As much as the difficulties of the imaginative scientists understanding the writing of more abstract predecessors, philosophers having trouble with Hegel's dialectic are not difficult to find. Let us provide three examples that illustrate how for some styles of thinking it is impossible to understand other styles.

We start with Carnap (1959, Section 6, p. 73) in his attempt to eliminate metaphysics. Carnap charges against Hegel's metaphysics in the expression "pure Being and pure Nothing, therefore, are one and the same". Thus reduced to a linguistic expression, it clearly means nothing or perhaps it is nonsense. But linguistic expressions do not stand alone, much less in Hegel who protests on the separation of the concept from the conceptualisation (Hegel, 2001b, §1):

> Philosophy has to do with ideas or realized thoughts, and hence not with what we have been accustomed to call mere conceptions. It has indeed to exhibit the one sidedness and untruth of these mere conceptions, and to show that, while that which commonly bears the name "conception", is only an abstract product of the understanding, the true conception alone has reality and gives this reality to itself. Everything, other than the reality which is established by the conception, is transient, surface existence, external accident, opinion, appearance void of essence, untruth, delusion, and so forth. Through the actual shape, which it takes upon itself in actuality, is the conception itself understood. This shape is the other essential element of the idea, and is to be distinguished from the form, which exists only as conception.

At the core of Carnap's though is that

> A word which (within a definite language) has a meaning, is usually also said to designate a concept, [...]

Thus Carnap equates words with concepts in the very same form in which a child equates names with things[17]. Further, Carnap (1959, p. 76) states:

[17]

> ... names are, to begin with, situated in objects. They form part of things in the same way as does color or form. Things have always had their names. It has always been sufficient to look at things in order to know their names.(Gruber & Vonèche, 1995, p. 131)
>
> However, at the beginning of the nineteenth century Ampère was already maintaining that sensation is simply a symbol and that those who allow it to be equated with objects are like the peasants (I would say like the children) who believe that there is a necessary correspondence between the names of things and the things which are named. (Gruber & Vonèche, 1995, p. 745, quoting Piaget)

> (Meaningful) statements are divided into the following kinds. First there are statements which are true solely by virtue of their form ("tautologies" according to Wittgenstein; they correspond approximately to Kant's "analytic judgments"). [...] Secondly there are the negations of such statements ("contradictions"). They are self-contradictory, hence false by virtue of their form. With respect to all other statements the decision about truth or falsehood lies in the protocol sentences. They are therefore (true or false) empirical statements and belong to the domain of empirical science. Any statement one desires to construct which does not fall within these categories becomes automatically meaningless. [...] Logical analysis, then, pronounces the verdict of meaningless on any alleged knowledge that pretends to reach above or behind experience.

However, the truth content of Carnap's quoted sentences is not empirically verifiable, it is indeed a knowledge that pretends to reach above experience, and therefore meaningless in Carnap's terms. It simply states his decision of rejecting as meaningless any sentence that does not belong to one of the three classes.[18]

It comes not as a surprise that in the attempt of justifying imagination as the only form of scientific thought, whatever is abstract –and hence "not scientific"–, deserves to be uprooted as metaphysical. Yet, the parallel with the early thoughts of the child suggests that there might be matters of cognitive evolution involved. Kramer and other authors (see references in (Kramer, 1983)) have explored the idea of a late stage in the development of cognition: post-formal thought. The transition to this stage would happen during adulthood and not in all human beings. It would be more frequent in the old adult than in the young and has to do with dialectic thought and inter-defined concepts as those characteristic of complex systems (García, 2011, 2006). For example, the duality matter-force in Faraday shares this kind of inter-definition, for matter is where force diverges, but force is exerted by matter. It is not possible to suppress one term without suppressing its dual. If we force the relation into the form "property of" that so nicely has worked for us when dealing with objects, we are obliged to opt between "force is the matter's action" or "matter is the thing where forces abut". Thus introducing symmetrically an arbitrariness of precedence in the terms.

In his "Conjectures and refutations" Popper makes an unsuccessful attempt to understand dialectics in his own terms. For Popper, the fundamental method of understanding is "trial and error":

[18] Logic requests that, if Carnap's classification, name it C, is meaningful and true, then C is meaningless (it is false that C is meaningful). This is a case of collapse of contradictions (see Subsection 4.2.6) and we must disregard the whole argument. However, if we assert that the classification is meaningful and false, nothing follows from it. In all options, logical analysis arrives to the same conclusion: we must disregard the classification, be it as useless, meaningless or self-contradictory. Actually, the fourth request for a phrase, S(a), to be meaningful ('The method of verification of "S(a)" is known.') (Carnap, 1959, p. 65), is not provided by Carnap for his central statement.

> We may describe the method employed in the development of human thought, and especially of philosophy, as a particular variant of the trial and error method. Men seem inclined to react to a problem either by putting forward some theory and clinging to it as long as they can (if it is erroneous they may even perish with it rather than give it up), or by fighting against such a theory, once they have seen its weaknesses. This struggle of ideologies, which is obviously explicable in terms of the method of trial and error, seems to be characteristic of anything that may be called a development in human thought. The cases in which it does not occur are, in the main, those in which a certain theory or system is dogmatically maintained throughout some long period; but there will be few if any examples of a development of thought which is slow, steady, and continuous, and proceeds by successive degrees of improvement rather than by trial and error and the struggle of ideologies.
>
> [...] And the testing of the theory proceeds by its vulnerable sides to as severe an examination as possible. This again is the trial and error method. Theories are put forward tentatively. and tried out. **If the outcome of a test shows that the theory is erroneous, then it is eliminated: the trial and error method is essentially a method of elimination**. Its success depends mainly on three conditions, namely, that sufficiently many and sufficiently different theories are offered, and that sufficiently severe tests are made. In this way we may secure, if we are lucky, the survival of the fittest theory by a process of elimination. (Emphasis added) Popper (1962)

While Popper's thoughts mirror closely Peirce's thoughts, a striking difference is that in Peirce, the backward flow of error for a theory that failed at the time of empirical contrastive comparison can be directly linked to the concept of retroduction (called abduction as well), which is responsible for the creative production of hypotheses. However, Popper has suppressed from science the moment of production of scientific theories in his "Logic of scientific discovery" (Popper, 1959), driven by his a-priori intention of accepting some physical theories whose authors claim are the product of "free invention", rejecting abstraction and induction at the time they ignore abduction (see (Solari & Natiello, 2022a) for a discussion). The consequence is that we cannot find in Popper how failure becomes new/better trials and how the process of "trial and error" produces knowledge (as distinct from "falls into knowledge by chance").

The thoughts by Popper are directed towards the political struggle and the claims of superiority and scientificality often found in followers of Marx and Engels, it is then contaminated by his political passion.

> Hegel thought that while philosophy develops, his own system has to remain the latest and highest stage of this development and cannot be superseded. The Marxists adopted the same attitude towards the Marxian system. Hence, Marx's anti-dogmatic attitude exists

> only in the theory but not in the practice of orthodox Marxism, and dialectic is used by Marxists, following the example of Engels' Anti-Dühring, mainly to defend the Marxist system against criticism and not to criticise it or to develop it. As a rule, critics are denounced as not understanding dialectic, or as unable to understand proletarian science, or as traitors.

What Popper criticises is not dialectics but rather dogmatism in Marx' followers and a lack of humility perceived in Hegel, that according to the text, makes of his philosophy the exception to the rule. In general, self-criticism is one of the most difficult tasks since it can shake the foundations of our beliefs, the same beliefs we are putting to work in performing the criticism on ourselves. It threatens annihilation, the reduction of ourselves to nothing. The problem of the followers is then, that having made such a tremendous effort to understand/accept Hegel, Marx or Einstein, just to mention three –for us– difficult authors; having been guided into their thoughts by teachers, exegetes and explanatory books, that perform (always risky) didactic transpositions, students end up being Hegelians, Marxists or Relativists that cannot criticise themselves (since the doxa is now part of their identity) with the added drama that what they believe to be Marx, Engels or Einstein's thoughts may not really be such. The point is that teaching is often structured as dogmatism. It promises: if you accept these beliefs, analytic reasoning will add this other large gift. On the contrary, if we reject the dogma, we cannot move forward in society, be it a community of dialectic philosophers, a Marxist political party or the paradigm of Relativistic Physics. For the case, to align our blindness with those of the community is a condition to belong to any paradigmatic community, a process known as enculturation (Roth, 2001).

The third example is Bunge's Critical Examination of Dialectics in (Bunge, 1975). Again, the strategies of assimilation and accommodation (Gruber & Vonèche, 1995) are present. Bunge attempts to cast dialectics into an axiomatic system (axioms D1 to D5, p. 64, although he claims that only D1 to D3 are distinctive of dialectics) and proceeds to consider the consistency and (analytic) logical content of them, not dialectics. Once again we confront opinion (surface existence, external accident, appearance void of essence, untruth, delusion as Hegel put it). Instead of perceiving dialectics as a "logical practice" an idea that goes back to Dieter Henrich (Apostol, 1985), Bunge proceeds to cast dialectics into forms he is familiar with. In so doing, he strips the concepts of the conceptualisation that furnishes them meaning and attach to them a "meaning" he forged in his attempts of assimilation. For example, we read: "D1: everything has an opposite" and its decomposition "D1a: For every thing (concrete object) there is an antithing. D1b: For every property of concrete objects there is an antiproperty". We can observe in this case a reification. All of a sudden "everything" became "every thing", and according to Bunge dialectics refers to (concrete) objects. In more general terms, if we insist in that science is empirical-deductive, with its empirical content being presented in axioms which have been stripped of their phenomenological origin (once again, concepts without conceptualisation), we cannot address –with this schema of dogma followed by analytical judgments– the reasoning (logic in Hegel) needed to address the creative thinking that is embodied in the synthetic judgments. This is, dialectic is a thinking practice that helps us in the search for the foundations, not a mechanical procedure linked to analytical judgments that, as Kant puts it, construct nothing. Dialectic is a practice conducting to critical thought. We are conscious of objective elements hinted by the senses as well as of subjective elements and lastly, we are conscious of our consciousness. Whatever we dimly consider as objective is represented as an idea hold in our conscience alongside the awareness of knowing it. This re-entrant form that knowing has is deformed by the metaphysical realists into pure knowledge of the real by suppressing from their consciousness their own presence. In such form, the knowing subjects persuade themselves that all what is in their consciousness is at least pointed by an external reality, including the constructs they need to organise the observed.

Bunge takes for granted standpoints incompatible with dialectics, as e.g., that we may know what a “thing” is in an absolute way, independently of its relation with whatever is outside the “thing” (Section 2. p. 64), or that complex and simple systems can be understood independently of each other (p. 69).

We conclude that there are different styles of understanding and moreover, that it is not always possible to explain the thoughts in one style with thoughts from another style. We have not found examples in which the abstract mind fails to understand the imaginative mind, yet, this may only indicate that we are frogs in the pond of abstract thinking.