Kant's Metaphysical Deduction of the Categories:
Towards a Systematic Reconstruction

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Ed. Anil Gomes & Andrew Stephenson

Nicholas Stang
University of Toronto
nick.stang@gmail.com

Introduction

In the first chapter of the Transcendental Analytic, titled 'On the Guiding Thread for the Discovery of all Pure Concepts of the Understanding,' Kant attempts to derive the complete table of fundamental pure concepts of the understanding, or, as he will later call them, the categories (A79/B105). His 'guiding thread' (*Leitfaden*, or 'clue' in the standard English translation) for this derivation is the table of logical functions in judgments, which he provides in the first section of the chapter (§9 in the B Edition).[1] The pure concepts of the understanding, Kant thinks, correspond one-to-one with logical functions of judgment (A79/B105) and can be derived from them.[2]

He elsewhere refers to this derivation of categories from logical functions as their 'metaphysical deduction' (B159), or MD for short.[3] A *deduction,* according to the legal sense of the term to which Kant alludes, is an answer to a question of right, '*quid juris*.'[4] Kant's derivation of the categories is a

[1] See, however, Hoeppner 2021, p. 53–8, who argues that the 'guiding thread' is the regulative Idea of the understanding as a capacity to judge.

[2] Kant does not talk explicitly, in the *CPR*, about 'deriving' the categories from the logical functions. The lectures on metaphysics do, however, record this remark by Kant: 'therefore I can derive [*ableiten*] the categories, and I call this their deduction [. . .] now when I derive [*ableite*] them from the logical functions [. . .]' (MvS, 28:474). See also the *Prolegomena*, where Kant talks about 'deriving' [*ableiten*] the categories from a 'principle' (Prol., 4:322) and, then a page later, identifies this principle as judging (4:323) and, two pages later, equates the categories' 'deduction' with their 'derivation' (4:323). See Schulting 2020, p.1. Due to constraints of space, I will largely limit myself to directing the reader to relevant secondary literature, rather than engaging with it in detail.

[3] Although it is controversial which part of the *CPR*, exactly, Kant is referring to here, and how its task relates to that of the Transcendental Deduction; see Horstmann 1981 for an alternative reading.

[4] A84/B116. Henrich 1989 is the classic discussion of the legal background to Kant's notion of 'deduction.' Henrich tasks the MD with answering the *quid facti* (by explaining fact of our possession of the categories) but I do not think this is right. Kant elsewhere identifies the *quid facti* as the empirical question of the conditions under which a cognition is acquired (RP, 20:275; R5636, 18:267; MMr, 29:763–4). But, as every reader of Kant knows, although every cognition, including the categories, commences with experience, pure cognitions, such as the categories, do not derive from experience (B1). The aim of the MD is to show that the categories are

deduction because it is a proof that we rightfully possess these concepts, unlike what Kant calls 'usurped' concepts like fate and fortune.[5] It is a specifically *metaphysical* deduction because it proves that we rightfully possess them as *a priori* (thus, our right to them does not derive from experience) and given concepts.[6] They are given, rather than made or invented, because they do not depend on our choice to define them appropriately (as do, for instance, certain mathematical concepts and invented empirical concepts, e.g., <*centaur*>).[7] It proves their *a priori* and given status by deriving them from the *a priori* (non-empirical) and given (not made) logical functions of judgment. The subsequent *transcendental* deduction of the categories will explain, further, why we are entitled to use these concepts in cognizing objects given to us in sensibility. Despite their non-empirical origin, they have a valid empirical use.[8]

My aim in this chapter is to reconstruct Kant's metaphysical deduction of the categories from the logical functions of judging. Every work on Kant involves some mixture of textual exegesis and systematic reconstruction; this chapter leans heavily in the direction of reconstruction. This is appropriate, given how little explicit guidance Kant gives as to the correspondence between the logical table of functions and the transcendental table of categories ('proof left to reader,' so to speak.)[9]

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pure concepts that derive from the understanding, so, although our possession of them 'commences' with experience, experience is not their origin.

[5] Cf. Proops 2003 who reads the MD as a deduction in a 'broader sense' (p. 216), rather than the strictly legal one, hence an answer to a *quid facti* (what is the origin of the categories?) but not a *quid juris*.

[6] See Kant's remarks on metaphysical and transcendental 'exposition' (*Erörterung*) at B38 and B40. A metaphysical exposition is an exposition (the 'clear representation of what belongs to a concept') that presents its concept as '*a priori* and given' (B38) while a transcendental one explains how that concept is the source of *a priori* cognitions (B40). It thus stands to reason that, if a transcendental deduction is an 'explanation of the way in which concepts relate to objects *a priori*' (thus, an explanation of our entitlement to use these concepts to cognize objects) a metaphysical deduction is an explanation of our entitlement to possess categories as *a priori* given concepts of the understanding in the first place. Cf. Horstmann 1997, p. 61–63; Longuenesse 1998, p. 4 n. 3; Caimi 2000, p. 271 n. 60; Hoeppner 2021, p. 24–5.

[7] For the distinction between given and made concepts see JL 9:93, BusL 24:268–9, DWL 24:756, VL 24:914, 918.

[8] Kohl 2018 provides a helpful guide to different interpretations of the role of the MD: those according to which (i) the MD proves both the *a priori* origin and the objective validity of the categories, or (ii) the MD proves only their *a priori* origin, or (iii) MD proves neither the *a priori* origin nor the objective validity of the categories. Most interpreters adopt (ii); Horstmann and Engstrom adopt (i); and Kohl himself defends (iii).

[9] The classic works on these issues are Reich 1932, Brandt 1991, Wolff 1995, and Longuenesse 1998. Reich controversially argues that the categories are derived from the objective unity of apperception, a claim that is sharply criticized by Brandt. Longuenesse's interpretation is sufficiently complex and subtle that I can hardly

§1. 'Exactly as many' and 'same function'

At the very beginning of the whole Transcendental Analytic (A64/B89), Kant explains this derivation project in a series of numbered claims, which I will quote and briefly explain (labeling them *M*1–4 to distinguish them from four claims labeled *L*1–4 below):[10]

'[*M*]1.That the concepts be pure and not empirical' (*Ibid*). Unlike empirical concepts, the categories do not depend on sensation (on how our sensible capacity is affected); they are grounded entirely in the nature of our cognitive capacities themselves.[11]

'[*M*]2. That they belong not to intuition and to sensibility, but rather to thinking and understanding' (*Ibid*). Kant has already uncovered, in the Transcendental Aesthetic, a set of pure concepts, but those concepts derive from the pure forms of our sensible capacity (the intuitions of space and time). The project of the '*Leitfaden*' is to uncover the pure concepts that derive, specifically, from the capacity of understanding.[12]

'[*M*]3. That they be elementary concepts, and clearly distinguished from those which are derived or composed from them' (*Ibid*). The fundamental pure concepts of the understanding are not definable through more basic concepts. They are what Kant elsewhere calls 'ancestral concepts' (*Stammbegriffe*) of the understanding.[13]

'[*M*]4. That the table of them be complete, and that they entirely exhaust the field of pure understanding' (*Ibid*).[14] Kant does not intend merely to produce a list of some pure fundamental

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do it justice in a short piece like this. Wolff's book is properly concerned only with the completeness of the logical table, but for the purpose of this chapter I am assuming that table is complete. More recently, Schulting 2020 defends Reich's controversial thesis that the categories can be deduced from the objective unity of apperception. Hoeppner 2021 is the most sophisticated and complete reconstruction of the MD of which I am aware.

10 See Hoeppner 2021, p. 27–62, who also structures his discussion around these four claims.

11 For Kant's definitions of sensation and purity see A19/B33–4 and A20/B34, respectively.

12 A85/B118. The 'concepts' of spaces and times are (mediate, general) intellectual representations but they are themselves made possible by the pure intuitions of space and time. Thus, these concepts are not purely intellectual in the way the categories are.

13 A81/B107. The derivative concepts are what Kant calls the 'predicables' of pure understanding. The predicables of pure understanding (A82/B108) would need to be accounted for in the eventual system of metaphysics, but not in the *CPR*, the preparatory critique that specifies the architectonic plan of that system (A13/B27).

14 They exhaust the activity of the understanding because the understanding is the capacity for concepts (A124) and they specify the form of any such conceptual activity.

concepts of the understanding. His project is to produce a table that includes *all* such concepts, so he needs not only a means of deriving the table, but one that, at the same time, proves that this table, so derived, is complete (lacks no pure elementary concepts of the understanding).[15]

Immediately before providing the table of categories, Kant writes: 'there arise exactly as many pure concepts of the understanding, which apply to objects of intuition in general *a priori*, as there were logical functions of all possible judgments in the previous table [the table of logical functions of judging]' (A80/B106). I will call this the *exactly as many* claim. This is equivalent to the claim that there is a mapping of logical functions to categories that is a *one-to-one correspondence*.[16] In terms of the numbered claims *M*1–4 quoted in the previous paragraph, Kant's derivation of the table of categories from the table of logical functions will succeed in its aim *if* the following conditions are met by the table of logical functions and the mapping of that table to the table of categories:

*L*1. The logical functions of judging are pure.

*L*2. The logical functions of judging belong to understanding, not sensibility.

*L*3. The logical functions are not derivative of any more basic logical functions.

*L*4. The table of logical functions is complete; it contains every logical function that meets the first three conditions.

L-M. There is a one-to-one mapping between logical functions and categories that preserves purity (*L*1 entails *M*1), origin in the understanding (*L*2 entails *M*2), fundamentality (*L*3 entails *M*3), and completeness (*L*4 entails *M*4).[17]

It is relatively clear that conditions *L*1 and *L*2 are met: the logical functions of judgment depend upon the form of the understanding, rather than on sensation (they are pure) or on the form of sensibility (which by itself does not judge–B129). It is significantly less clear whether conditions *L*3,

[15] In fact, the table of categories must be derivable from a single Idea (A65/B90, A67/B92) that specifies all of the parts of the table and their relation to one another and 'allows the absence of any part to be noticed in our knowledge of the rest' (A832/B860).

[16] In terms that might be more familiar to some readers, it is a bijective mapping (one-to-one correspondence), i.e. one that is surjective (onto) and injective (one-one).

[17] Cf. Hoeppner 2021, p. 13–21, 358–65, 474–7.

and especially *L4*, are met.[18] In this chapter I am going to assume that they are and focus on condition *L-M*.

In the paragraph prior to the 'exactly as many' claim, Kant writes: 'The same function that gives unity to the different representations in a judgment also gives unity to the mere synthesis of different representations in an intuition, which, expressed generally, is called the pure concept of the understanding' (A79/B104–5).[19] Call this the *same function* claim. Kant had earlier defined 'function' as follows: 'by a function, however, I understand the unity of the action of ordering different representations under a common [*gemeinschaftlichen*] one' (A68/B93). Both of these sentences are quite dense, and will require significant further explication, but for now I will take 'function' to refer to the principle of unity of an act of ordering representations.[20] So I will interpret the *same function* claim to mean that the one-to-one mapping, guaranteed to exist by the *exactly as many* claim, preserves principles of unity of acts of ordering. In short: there is a one-to-one mapping of logical functions of judgment to categories that preserves such principles of unity. I will sometimes refer to this as a *structure preserving mapping*.

This mapping preserves the structure of individual logical functions. It also preserves the structure of the table as a whole. Both tables are divided into four titles—Quantity, Quality, Relation, and Modality—each of which has three moments (e.g., three logical functions of quantity, quantity, etc.). So the mapping of logical functions to categories will preserve not only number and individual structure (logical function to category), but also the structure of the table as a whole (title

[18] The most complete reconstructions of Kant's arguments for *L*1–4 are Wolff 1995 and Hoeppner 2021, Ch. 2; of the two, mt interpretation more closely resembles that of Hoeppner, with some differences noted below.
[19] Cf. Kant's remark that '[categories] are concepts of an object in general, by which its intuition is regarded as determinate [*bestimmt*] with respect to one the logical functions of judging' (B128). Although I think ultimately amount to the same thing as the *same function* claim in the main text, I will take the *same function* claim as my 'guiding thread' in what follows. Claims similar to the B128 passsage just quoted are found throughout Kant's corpus, but typically do not shed any additional light on what, exactly, it means for an intuition to be determinate in respect of a logical function; see Prol. 4:304, 324; MFNS 4:475 n.; Disc, 8:223; R5854, 18:370; R5932, 18:392; and R XLII E 24, 23:225.
[20] I am thus reading '*Einheit der Handlung*' in the same function passage as '*principle of* unity of the action.' Cf. Hoeppner 2021, p. 200–1; Reich 2001, p. 35.

to title, moment to moment).[21] In other words, it will be a structure-preserving mapping at the level of both individual logical functions/categories and at the level of the tables themselves.

§2. Logical functions of judging

Before we understand the mapping from logical functions to categories we need at least a rudimentary understanding of what is being mapped *from*: the table of the logical functions of judging. Unfortunately, Kant does very little explicitly to explain that table, much less justify its completeness. This has given rise to the (in my view, mistaken) assumption that either Kant borrows the table from the logic textbooks of his day, or that his justification for it occurs elsewhere.[22]

On the contrary, I think the entirety of the logical table of functions of judgment can be derived from Kant's discussion at the end of the section titled, 'On the logical use of the understanding in general.'[23] Kant there writes: 'Since no representation pertains to the object immediately except intuition alone, a concept is thus never immediately related to an object, but is always related to some other representation of it (whether that be an intuition or itself already a concept)' (A68/B93). An intuition represents its object immediately (not by means of some other representation of it), but concepts are mediate representations: a concept represents an object either by means of intuition (which in turn immediately represents the object) or by means of a further concept of that object. Kant renders this model slightly more determinate when he claims, in the next sentence, that 'judgment is therefore the mediate cognition of an object, hence the representation of a representation of it' (A68/B93). If we adopt temporarily the simplifying assumption (which we will suspend later) that every judgment combines only two concepts, a subject and a predicate, then the predicate-concept relates to the objects it represents by means of the subject-concept. By the

[21] Cf. Hoeppner 2021, p. 269–70. Additionally, the titles have an order: 1. Quantity, 2. Quality, 3. Relation, 4. Modality. Thus a complete understanding of the table of logical functions requires understanding why they occur in this order and a complete account of the MD requires understanding why that order is preserved in the table of categories. Cf.

[22] Hegel 1990, 225; Reich 1932, p. 93.

[23] My interpretation is similar in some respects to that of Wolff 2017, p. 85–7, although I eschew his subsequent discussion of 'uses of concepts in judging,' predicative and non-predicative such uses, etc. My interpretation is much closer to that of Hoeppner 2021, from which I have learned a great deal.

previous sentence, the subject concept either relates immediately to intuitions (and thus mediately to their objects), or it itself is mediated by yet further concepts. But this chain of mediation has to end somewhere; some concept has to relate immediately to intuitions (and thus mediately to their objects).[24]

Kant then illustrates this model, making the further simplifying assumptions that the predicate is more general than the subject (the entire subject falls under the predicate), and that the subject relates immediately to intuition:

> In every judgment there is a concept that holds of many and, among this many, also comprehends [*begreift*] a given representation, which is then related immediately to the object. So in the judgment, e.g., 'All bodies are divisible,' the concept of the divisible is related to various other concepts; among these, however, it is here particularly related to the concept of body, and this in turn is related to certain appearances[25] that come before us. These objects are therefore mediately represented by the concept of divisibility. (A68/B93, my emphasis)

In the judgment 'All bodies are divisible' the concept <*divisible*> holds of many representations. Among them are lower-level concepts contained under it (e.g., <*body*>), but also immediate representations (intuitions) of objects (it is said to 'comprehend' those representations), the very objects in its extension. The concept <*divisible*> represents the objects contained under <*body*> in a doubly mediated fashion: by means of containing <*body*> under it and by means of that concept's relation to intuitions of such objects. Kant's reference to 'various other concepts' (underlined above) is a reminder that <*divisible*> also contains other concepts under it; since <*divisible*> is a concept, hence a general representation, it can always be further specified. Concepts other than <*body*> are contained under it, e.g., <*geometrical figure*>. There is no such thing as the *only* concept that can further specify a more general concept; no such thing as a genus that necessarily has *only* one species.

[24] On this point I agree with Hoeppner 2021, p. 96–126, and Hoeppner 2022, p. 465–8.

[25] In his own copy of the first edition, Kant changed 'appearanes' here to 'intuitions' (E XXXVI, 23:45).

Kant then extends this account one level further, to the subject of the previous judgment, the concept <*body*> itself:

> Concepts, however, as predicates of possible judgments, are related to some representation of a still undetermined object. The concept of body thus signifies something, e.g., metal, which can be cognized through that concept. It is therefore a concept only because other representations are contained under it by means of which it can be related to objects. It is therefore the predicate for a possible judgment, e.g., 'Every metal is a body.' (A69/B94)

Every concept is a predicate of a possible judgment because it is 'related to the representation of some still undetermined object,' i.e., it contains under itself some further concept, because it is a general and hence *not fully determinate* representation of an object. For instance, the concept <*body*> contains under itself <*metal*>. Iterating the model from above, we can say that <*metal*> mediately represents objects by immediately relating to intuitions, which in turn immediately represent their objects. In the judgment 'every metal is a body' the predicate <*body*> mediately represents those objects by containing a representation of them, <*metal*>, under itself. There are other ways of specifying <*body*>, other concepts contained under it other than <*metal*>, e.g., <*salt*>. This hierarchy of concepts is represented in Figure One. The solid lines represent the relation of one concept being contained under a more general concept. The dotted line represents the relation of an object falling under a concept; as these passages indicate, these dotted lines hold in virtue of the immediate relation of that concept to an intuition of the object.

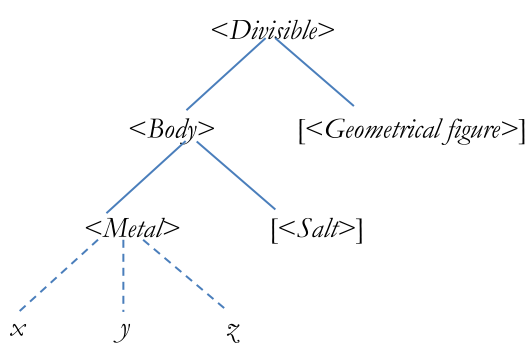

Figure One

Conceptual hierarchy (A68–9/B93–4)[26]

Two features of this hierarchy are worth noting. First, as Kant's own discussion in A68–9/B93–4 indicates, the hierarchy can be extended indefinitely in both directions. There can always be a more general concept (which contains <*divisible*> under it). That more general concept can be specified in a way distinct from the originally most general concept, generating another concept at the same 'level' of generality (e.g., <*indivisible*>). Likewise, as indicated by Kant's shift from treating <*body*> to <*metal*> as the lowest-level concept, the one immediately related to intuitions of objects, we can always further specify the object at the bottom of the hierarchy, and relate those more specific concepts immediately to intuition.[27] For instance, we can divide <*metal*> into <*precious metal*> and <*common metal*> and relate each of them immediately to intuitions.

But we need to impose one further piece of structure on this hierarchy. The hierarchy as represented so far leaves open whether two distinct determinations of the same concept are *exclusive* or not. This means that from the hierarchy so described it remains logically consistent for one and the same object to fall under both <*metal*> and <*salt*>, i.e. two distinct determinations of one and the same common concept. To prevent this, to make the two determinations logically exclusive of one another, we need to incorporate into <*salt*> the negation of whatever mark further determines <*metal*> as a specification of <*body*>.[28] If the more general concept (e.g., <*body*>) represents the genus and the more specific concept the species (e.g., <*metal*>), then the traditional name for this is the *differentia*, e.g., the differentia of the species *human* within the genus *animal* is *rational*. By incorporating the negation of differentiating marks into concepts at the same level of generality (e.g., <*brute*> = <*non-rational animal*>, we effectively make distinct conceptual determinations logically exclusive. This also has the effect of representing concepts as divided into determinations and their

[26] Again, I here agree with Hoeppner; see the references in note 24 above.
[27] JL 9:97; PöL 24:576; VL 24:927.
[28] PhL 24:461; PöL 24:576; DWL 24:762; VL 24:926–7.

negations, e.g., *conductive body* (<*metal*>) and *non-conductive body* (<*salt*>).[29] This is represented in Figure Two.[30]

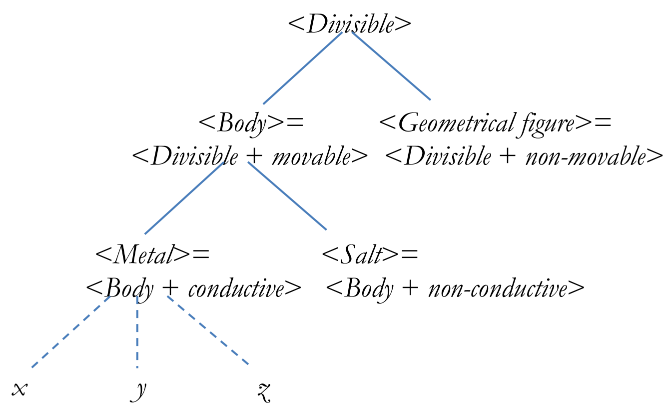

Figure Two
Conceptual hierarchy with negative differentia

Earlier in this passage Kant gives his definition of what a function is: 'by a function, however, I understand the unity of the action of ordering different representations under a common one' (A69/B93). I take this to mean that a function is a principle of unity of an act of ordering representations, i.e. of representing those representation in a certain order or structure. Specifically, the logical functions of judging are the purely logical principles of unity of ordering concepts in judgments. They are purely logical because they attend only to the form of judgment and abstract entirely from its content (i.e.., which concepts it orders, their relation to intuition, etc.).[31]

I claim we can read off the principles of unity of acts of ordering concepts in judgment (the logical functions of judging) from the concept hierarchy as schematically represented in Figure One.[32] The key thing to note is that this concept hierarchy itself already encodes judgments, i.e.,

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[29] Cf. Lu-Alder 2013, 185.
[30] Kant discusses conceptual hierarchies in his logic lectures: JL 9:95–100; PhL 24:461; PöL 24:576–7; BuL 24:660–1; DWL 24:760–2; VL 24:925–8. Anderson 2015 contains an exhaustive discussion of conceptual hierarchies. My reconstruction assumes merely that concepts stand in hierarchies of generality; it does not assume that the more general concepts are marks contained in the less general concepts, so it does not assume that the judgments encoded in these hierarchies are analytic (or that they are synthetic.)
[31] See Hoeppner 2021, p. 76–82 and 92–5, for a detailed discussion of how to read Kant's definition of function at A69/B93.
[32] Cf. Brandt 1991.

structured relations among concepts. In fact, every two concepts in the hierarchy are judgmentally related, as I will now show.[33]

Focusing for now, on what is judged, we distinguish between whether what is judged is a relation of concepts (*categorical judgment*), or a relation among judgments themselves. In the former case we represent concepts in an ordered relation. In the second case, we represent judgments themselves in such an order. Again, focusing on the former case, by inspection two distinct concepts S and P in this hierarchy can relate in one of six ways, where the sphere of a concept S is the set of all concepts *and objects* contained under it: (i) the sphere of S can be wholly contained under P (e.g., all bodies are divisible); (ii) the sphere of S can be wholly disjoint from the sphere of P (e.g., no metal is a geometrical figure) (iii) the sphere of S can be wholly contained under ~P (e.g., every geometrical figure is non-movable), (iv) the sphere of S can partially overlap the sphere of P (e.g., some divisible things are bodies); (v) the sphere of S can be partially disjoint from the sphere of P (e.g., some bodies are not conductive); and (vi) the sphere of S can partially overlap the sphere of ~P (e.g., some bodies are non-conductive) . Likewise, if *x* is an object falling under concept S, x can be related to another concept P in one of three ways: (vii) *x* can be contained under P (e.g., this salt is a body), (viii) disjoint from P (e.g., this salt is not a metal), or (ix) contained under ~P (e.g., this salt is non-conductive).

All of these possible relations of subject and predicate in a categorical judgment can be 'decomposed' into the 'quantity' of the judgment (*how much* of the subject is related to the predicate) and its 'quality' (whether the predicate is affirmed or denied of the subject, or the negation of the predicate is affirmed). In particular, we distinguish three moments of quantity:

Universal (U): *all* of the sphere of the subject S is related to the predicate P.

Particular (P)*: some* of the sphere of the subject S is related to the predicate P.

[33] My own view is thus that the table of logical functions can be derived from these conceptual hierarchies. Here I provide only a sketch of that derivation, without addressing the crucial issue of the completeness of that table. See Wolff 1995 and 2017 and Hoeppner 2021, p. 144–161, for attempts to prove the completeness of the logical table.

Singular (S)*:* a *single* instance x of the subject S is related to the predicate P.

We can then distinguish three ways the subject can be related to the predicate:

Affirmative (A): the sphere of the subject overlaps the sphere of the predicate.

Negative (N): the sphere of the subject is disjoint from the sphere of the predicate.

Infinite (I)*:* the sphere of the subject overlaps the sphere of the negation of the predicate.

By composing these three functions we get nine possibilities, which map onto the nine possibilities from above: (i) universal-affirmative, (ii) universal-negative, (iii) universal-infinite, (iv) particular-affirmative, (v) particular-negative, (vi) particular-infinite, (vii) singular-affirmative, (viii) singular-negative, (ix) singular-infinite.

From a contemporary perspective this can seem overly baroque. Whereas previously we had nine judgmental-forms, now we have six judgmental 'functions'—was the point of this trite exercise in Aristotelian logic merely to reduce nine to six? Furthermore, it might seem that we can reduce them further; given plausible assumptions, various judgment-types will be equivalent in truth-value, e.g., (ii) and (iii), (v) and (vi), (viii) and (ix).

But a logical function is not a *type* of judgment individuated by its truth-conditions. It is the principle of unity of an act of combining concepts in judgment. Furthermore, since we are interested, ultimately, in deriving the non-derivative concepts of the understanding (*M*3) from the non-derivative logical functions (*L*3, *L–M*), we are interested in non-derivative logical functions, i.e. principles of unity of acts of combining concepts that cannot be decomposed into more basic such principles of unity. This is why are concerned with U-P-S and A-N-I and how they can be composed to generate (i)–(ix). Each of U-P-S is a principle of unity of an act of combining concepts characterized in terms of how much (quantity) of the subject is related to the predicate, while each of A-N-I is a principle of unity of an act of combining concepts characterized in terms of how (quality) the subject is related to the predicate. I take it to be clear that they satisfy *L*1 and *L*2 from earlier. Insofar as (i)–(ix) are a complete specification of how two concepts can relate in a hierarchy represented in Figure 2, U-P-S & A-N-I are more fundamental principles of unity that also completely explain how two concepts can be related in a (categorical) judgment. This falls short, however, of a proof that they themselves are fundamental, i.e., are not composed of yet more basic principles of unity of acts of ordering concepts.

This hierarchy of concepts also represents structured relations among categorical judgments themselves, i.e. it represents non-categorical judgments. For instance, if an object falls under <*metal*> then it falls under <*body*>. In general, *if* any concept or object is wholly contained under another concept *then* it is wholly contained under any concept that wholly contains that other concept; it is fully disjoint from any concept fully disjoint from that concept; it partly overlaps some concepts contained under that concept, etc. Just from the hierarchy of concepts alone we can read off various conditional or *hypothetical* judgments.

The hierarchy also represents various disjunctive judgments. Each concept is exhaustively divided by its determinations, e.g., <*divisible*> is divided into <*body*> and <*geometrical figure*>. This means that we can read off from the concept hierarchy itself various disjunctive judgments, e.g., that every divisible thing is either a body or a geometrical figure. Since each of those concepts itself exhaustively divides into further determinations, these disjunctive judgments iterate. For instance, the concept hierarchy also represents the judgment that every divisible thing is either a metal, a salt, or a geometrical figure. To reiterate a point made earlier, even if each disjunctive judgment (*p* or *q*) is equivalent in truth-value to some hypothetical judgment (if ~*p* then *q*), this is irrelevant to distinguishing them as distinct principles of unity of acts of ordering concepts in judgments.

Finally, we distinguish what is judged (the content of judgment) from the mode or manner in which it relates to the capacity for judgment (the *modality* of judgment).[34] A given judgment-content can: (i) agree with the *form* of the conceptual capacity itself, i.e. with the structure of a conceptual hierarchy in general as represented in Figure Two; (ii) it can agree with the *matter* of the conceptual capacity, i.e. with the concepts actually given to, or made by, the understanding and formed in such a conceptual hierarchy; or (iii) it can be such that its negation is incompatible with the form of the

[34] Cf. Wolff 2017, p. 102–4.

actual conceptual hierarchy (the hierarchy represented in (ii)). The first kind of judgment are *problematic* judgments, i.e. judgments that do not violate the form of a conceptual hierarchy in general. Problematic judgments represent judgment-contents that can be embedded in a conceptual hierarchy of the form given in Figure Two. The second kind of judgments are *assertoric* judgments, i.e. judgments the capacity of understanding actually makes by bringing its representations into such a conceptual hierarchy. Assertoric judgments represent judgment-contents embedded in the hierarchy of concepts given to, or made by, the understanding. And the third kind of judgments are *apodictic* judgments, i.e. judgments whose negation violates the form of the actual hierarchy in which our concepts stand. For instance, given the assertoric judgment that this metal (call it *x*) is gold, the judgment that metal *x* is a body is apodictic; its negation would violate the form of any conceptual hierarchy, assuming that metal *x* is a body. This is not a proof that Kant's table of logical functions of judging is *complete* (*L4*) or that these logical functions of judging are *primitive* (*L3*), i.e. that they are irreducible to one another or some more basic functions. But I hope that it makes both claims at least plausible.

§3. 'The same function'

Recall the *same function* claim: 'the same function that gives unity to the different representations **in a judgment** also gives unity to the mere synthesis of different representations **in an intuition**, which, expressed generally, is called the pure concept of the understanding' (A79/B104–5). This sentence asserts an identity between the principle of unity (function) of two acts of combination (synthesis): combining concepts (or judgments, in the case of non-categorical judgments) in judgment and combining representations in an intuition.[35] But the principle of unity of an act of combining representations corresponds to the structure of the object of the representations so combined. If I combine representations according to a certain principle of unity, I represent the objects as having the corresponding structure. For instance, if I combine <*human*> and <*mortal*> according to the form of universal and affirmative judgment, I thereby represent humans and

[35] I take 'synthesis' and 'combination' (*Verbindung*) to be synonymous for Kant; see B130.

mortality as standing in this structure: all humans are mortal. So we can translate unproblematically between the principle of unity of an act of combination and the structure of the object represented by that act.[36]

Just as we can understand logical functions either as principles of unity of acts of combining concepts (unity of judging), or as structures of what is thereby judged (unity of judgment), so too can we understand the images[37] of these functions under the mapping from logical functions to categories (which I am about to construct) either as principles of acts of synthesis or as the correlated structures of objects so represented. Because the fundamental role of the categories is to subsume objects, I will focus on the latter. I will reconstruct the structure-preserving mapping from logical functions to structures in objects, but it would be a trivial exercise to correlate the latter, instead, with the structures of acts of synthesis involved in representing objects as so structured.[38] Finally, note how the *same function* passage ends: '[the same function] which, expressed generally, is called the pure concept of the understanding.' This means we are going to map structures in judgments (logical functions) to structures in objects and then 'represent these structures generally,' i.e. abstract from these structures concepts of objects *as standing in those structures* (categories).

The first element in my reconstruction is a mapping f from a conceptual hierarchy, including the logical objects at the bottom of the hierarchy, to a manifold of sensible objects. It will not matter which actual mapping this is (which concepts and logical objects it maps to which objects); all that will matter is its form. The mapping f will induce a mapping between relations among concepts to relations among objects: if R is a relation among concepts, and f takes R-related concepts to R′-

[36] I take this to be one of the basic principles underlying Kant's theory of cognition: the form of the act of representing corresponds to the form of the object so represented. But spelling out the nature of that correspondence is precisely what is at issue.

[37] I mean 'image' in the mathematical sense: the output of a function given an input. Kant's technical notion of 'image' (*Bild*) is not relevant here, because the metaphysical deduction of the categories must be independent of the specifically spatiotemporal form of our intuition, and thus prior to, and independent of, the Schematism and its theory of images and temporal schemata.

[38] As a result, this chapter does not include an account of synthesis, for it focuses on the intentional content of the categories, rather than the synthetic acts that enable that content. An account of synthesis along these lines must remain a project for future work.

related objects then R' is the image of R under f.[39] Likewise, f induces a mapping between structured complexes of concepts to structured complexes of objects: if $R<x,y>$ is the conceptual structure that consists in concepts x and y standing in R, then f induces a mapping from $R<x, y>$ to $R'<fx, fy>$ where R' is as above.[40] Finally, we can abstract a general concept of objects from such an induced mapping, e.g., the concept F, which applies to all objects that are the first (or second) member of an ordered pair that is itself the image of this mapping (, the concept that applies to any object that is fx, in $R'<f\text{x}, fy>$ for some x and some y). I will argue that this mapping maps logical functions to categorial structures in objects. I will then argue that the categories can be defined as concepts of objects qua standing in those structures (equivalently, qua being the images of the relevant logical function under an appropriate mapping). This is exceedingly abstract, but in the rest of this chapter I will attempt to make it more concrete.

Before we construct this mapping we need to get clear on an outstanding issue from the last section, the objects that terminate conceptual hierarchies like those represented in Figures One and Two. As we have seen, Kant thinks that (some) concepts relate mediately to objects by relating immediately to immediate representations of them, intuitions. We are now concerned with exactly that relation, i.e. with relating conceptual hierarchies to sensible objects. Even for concepts of non-sensible objects, where (ii) is lacking, we can think of them as having instances conceived purely logically. In order to do so, we are going to break down the 'mediate' relation of a concept to an object into two separate relations: (i) an immediate relation of a concept to its objects *conceived purely logically*, (ii) a relation of these purely logical objects to real (i.e., not purely logical) objects by means of intuition. We do this because we want to rigorously distinguish the realm of the purely logical (concepts and the functions of the understanding) from the sensible objects we will relate them to. As justification for this, note that we can understand concepts as having relation (i) *even if* they are

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[39] This is trivial if we represent relations as sets of ordered pairs: $f(\mathrm{R}) = \{(fx,fy) \mid (x,y)\in \mathrm{R}\} = \mathrm{R}'$.
[40] More precisely, $R<x,y>$ denotes the complex object that exists just in case x and y stand in relation R. Likewise, $R'<fx,fy>$ exists just in case fx and fy stand in relation R'.

not related to sensible objects at all. After all, Kant says that the logical form of a (universal, affirmative) judgment 'All S are P' is: for all x to which the subject S belongs, the predicate P belongs as well. [41] Since Kant accepts the traditional rule of subalternation, according to which a universal judgment immediately entails a corresponding particular judgment, this entails: for some x to which the subject S belongs, belongs also the predicate P.[42] But since every concept is the subject of some universal judgment (i.e., one that predicates a more general concept of it), every concept is also the subject of some particular judgment, i.e. it has a purely logical instance, an object conceived merely as x. Even if the concepts have no instances among objects of intuition, we can think of them as being about objects as conceived purely logically, instances of x in this schema.[43]

The mapping in question will be a mapping from conceptual hierarchies, understood as composed of concepts and purely logical objects, to objects of intuition. The relation of these purely logical objects (x, y, z in Figure Two[44]) to objects of intuition is mediated by intuition: it is by means of intuition that we assign objects to these purely logical objects and thus relate concepts to sensible objects. Which intuitions these are does not matter for our construction, just as which objects are assigned does not matter. This concerns the matter of the mapping, but we are concerned only with its form.

Three features of this mapping need to be understood at the outset. First, there is no mapping that takes *all* concepts to sensible objects and satisfies the conditions that I will put on f. This is essential to my reconstruction. Some concepts are not mapped to objects because they lack sensible extensions altogether (they have empty extensions). Other concepts are not mapped to objects,

[41] The logical form I have given is common to the logical form Kant gives for both analytic and synthetic judgments (JL, 9:111); assuming all judgments are either analytic or synthetic, it follows that it is the logical form of all (universal, affirmative) judgments in general. For more on the purely logical notion of an object see Lu-Adler 2013.
[42] JL 9:116.
[43] This does not mean the object of the concept exists (it does not constitute an ontological argument) because existence requires that the object be a thing distinct from our representation of it and causally efficacious on our minds. Existence, being a modality, does not further determine the object $=x$, but expresses the relation between our cognition of that object and our cognitive capacity: it not only agrees with the form of that capacity but is materially given in connection with perception. See A219/B266, A601/B209.
[44] Cf. R3042, 16:629.

because, while they have extensions composed of sensible objects, those extensions are too 'large' to be represented by a single object; representing such objects would require us to complete an infinite synthesis, which is impossible. Second, the correspondence does not depend upon which particular mapping *f* is used, only its formal features. The form of the mapping is given by the nature of our cognitive capacity itself; of all the formally possible mappings, the actual mapping is determined by the matter of intuition, i.e. which objects are given to us, instantiating which empirical concepts. Third, the mapping I will sketch can be extended up to the logical function of categorical judgment, and not further. To extend the analysis, we would have to consider hypothetical and disjunctive judgments, which would be represented by ordered pairs of judgments and relations between distinct manifolds (e.g., distinct perceptions at different times). That must remain a task for future work.

§4. Quantity

Consider the conceptual hierarchy from §2, but focusing only on the logical quantity of judgments, i.e. the relation of *all* of a subject concept to some predicate, the relation of *some* of a subject concept to a predicate, and the relation of a *singular* instance of a subject-concept to a predicate.

In order to represent these as inputs to the function *f* we need to represent them as structured complexes of concepts. We do so as follows, remembering that the sphere, or, to use a more familiar term, the extension (*Umfang*) of a concept refers to both the concepts *and* the objects contained under it:[45]

[45] For the equivalence of sphere and extension see PöL 24:569, VL 24:912. Longuenesse 1998 and Anderson 2015, p. 61–71 argue at length that Kant must include both 'conceptual extensions' (lower concepts contained under a concept) and 'non-conceptual extensions' (objects contained under a concept) within his notion of extension. While I agree that Kant must accept that both objects and lower concepts can *fall under* a given concept, I am not convinced Kant in fact uses the term 'extension' (*Umfang*) to include both concepts and objects, and neither Longuenesse nor Anderson provide much in the way of direct textual evidence to the contrary (the closest thing to direct evidence either of them cites is A72–3/B97–8.). The fact that Kant describes a singular judgment as one whose subject concept 'has no extension at all' (JL 9:102) sems directly contrary to their reading. To be clear, I agree with Longuenesse/Anderson on the philosophical point, and in the body of the text I go along with their usage of 'extension.' However, I have my doubts as to whether this represents Kant's usage of that term.

Universal: where S and P are concepts, the logical function of universality (symbolized $*<\mathit{All}(S), P>$) is the structured complex consisting in *all* of the sphere of S standing in relation * to P.[46]

Particular: where S and P are concepts, the logical function of particularity (symbolized $*<\mathit{Some}(S), P>$) is the structured complex consisting of *some* concept subordinated to S standing in relation * to P.[47]

I will represent singular judgment by introducing an operator This_x which takes a concept S to an object x in the extension of S. The result of applying this to a concept S, $\mathit{This}_x(S)$, is not a concept (it is a singular rather than a general representation), nor is it an intuition (it is partly conceptual), nor is it identical to the object x itself, although it designates it. It is what Kant calls a 'singular use' of the concept S. We can then represent the logical function of singular judgment as follows:

Singular: where S and P are concepts and x an object contained under S, the logical function of singular judgment (symbolized $*<\mathit{This}_x(S), P>$) is the structured complex consisting of the object x designated by $\mathit{This}_x(S)$ standing in relation * to P.[48]

To anticipate slightly, the relation * will be filled in by the logical quality of judgment (affirmative, negative, or infinite), producing a complete judgment.[49]

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[46] Kant discusses the logical quantity of judgment in his logic lectures at JL 9:102–3;

[47] It is important to note that subordination is an 'improper' relation: every concept is subordinated to itself. This is important for getting the truth-conditions for universal and particular judgment right; see below.

[48] This is why singular judgment does not need to be distinguished from universal judgment within PGL. PGL includes a purely logical concept of an object =x, but purely logical objects are only conceptually, i.e. generically, specified. A purely logical object is thus equivalent to an arbitrary instance of a concept. A singular judgment in PGL (*this (arbitrary) instance of F is thus-and-such*) is inferentially equivalent to a universal judgment (*any instance of F is thus-and-such*). Since PGL studies only the 'formal' truth of judgments (their formal relations to one another) this difference is not significant in PGL. In TL it becomes significant because we relate our concepts to singular objects that can be distinct while being conceptually indiscernible and thus study *a priori* material truth (see A271–2/B327–8). This resolves the apparent tension between Kant's claims that: (i) the table belongs to PGL (A70/B95 where the table is said to 'abstract' from all content and 'attend only to the form' of the understanding); (ii) it distinguishes logical functions (e.g., singular from universal, infinite from affirmative; A71/B96, A72/B97) that are not distinguished in PGL. For discussion of this issue see Young 1992; Wolff 1995; Allison 2004; Rosenkoetter 2017; and Hoeppner 2021, p. 60 n. 43, 140–9.

[49] The indifference of quantity to quality is made clear at JL 9:102. I thus interpret the quantity of judgment as concerning the subject (how much of the subject is related to the predicate) and quality as concerning the copula by which the predicate is predicated of the subject, nearly the reverse of Wolff 2017, p. 95, and Hoeppner 2021, p. 128–30.

We now need to relate these conceptual hierarchies and the judgments they encode to objects given in intuition. My proposal is that we map logical objects (represented by variables at the bottom of the conceptual hierarchy) to sensible objects. We then map concepts to *fusions* of the objects in their extensions. To do this, we assume that sensible objects are structured by a relation of (improper) parthood that is (*a*) reflexive (every object is a part of itself), (*b*) anti-symmetric (two distinct objects cannot be parts of one another), and (*c*) transitive (the parts of the parts of a whole are parts of the whole). We then define a *fusion* of objects as follows: a fusion of the Fs is an object x such that everything that has a part in common with x has a part in common with at least one of the Fs.[50] We assume further that fusions are unique: there is at most one fusion of the Fs. Unlike, classical extensional mereology, however, we do not assume that fusions always exist. For some concepts there is no sensible object that is the fusion of all the objects in their extension, for such a fusion would require an infinite synthesis, which we cannot perform.[51]

Having mapped logical objects to sensible objects and concepts to fusions of their extensions, it can easily be verified that, if the fusion of the As and the fusion of the Bs exist:

(i) If A is subordinated to B, then f(A) (the fusion of the As) is a part of f(B) (the fusion of the Bs).[52]

(ii) If some concept subordinated to A is subordinated to B then, f(A) overlaps f(B) (i.e., they have a common part).[53]

[50] More precisely: x is a fusion of the Fs $=_{def}$ for all y, y overlaps x if and only if x overlaps one or more of the Fs; y overlaps $x =_{def}$ for some z, z is a part of x and a part of y.

[51] The non-standard terminology notwithstanding, I think my analysis agrees here agrees with that of Longueneesse 1998, ch. 9; what Longuenesse calls the 'synthesis of (conceptually) homogenous units' is the synthesis that represents the fusion of the objects under a concept. This fusion will fail to exist in the case of infinite collections, i.e., concepts whose extensions have no number (see below).

[52] By definition of a fusion, every object that overlaps f(A) overlaps fx for some $x \in$A. But A is subordinated to B, so every such x also belongs to B. So every object that overlaps f(A) overlaps fx for some $x \in$B. But every object that overlaps fx, where $x \in$B, overlaps f(B) by definition. So every object that overlaps f(A) overlaps f(B), i.e. f(A) is a part of f(B).

[53] If some concept C is subordinated to A, which is subordinated to B, then by transitivity of subordination, C is subordinated to B. By (i), f(C) is a part of f(B), hence, trivially, overlaps it.

In other words, relations of concept-subordination are mapped to relations of parthood between objects and relations of conceptual overlap are mapped to relations of mereological overlap between objects.

Given the representation of logical functions of judging as structured complexes of concepts from above, our mapping f will induce a mapping from these structured complexes to structured complexes in objects:

Universal judgment: where *<*All*(S), P> is as above, f maps this structured complex to f^*<f(S), f(P)>), i.e. the structured complex in which the image of the entire sphere of S bears f^* to the image of P under f.

Particular: where *<*Some*(S), *P*> is as above, f maps this structured complex to f^*<f(C), f(*P*)>, i.e. the structured complex in which f(C) bears f^* to the image of P under f, where C is some concept subordinated to P.

Singular: where *<*This*$_x$(S), P> is as above, f maps this structured complex to f^*<f(*This*$_x$(S)), f(P)>, i.e. the structured complex in which the image of *This*$_x$(S) under f (namely, fx) bears f^* to the image of P under f.

Intuitively, f maps a universal judgment to a structured complex in which the fusions of the extensions of all the concepts contained under the subject are related to the fusion of the extension of the predicate; f maps a particular judgment to a structured complex in which some part of the fusion of the extension of the subject is related to the image of the predicate; and f maps a singular judgment to a structured complex in which the image of x (the sensible object assigned to that logical object) is related to the image of the predicate.

Two comments on these definitions are in order. First, since we are still abstracting from the relation between subject and predicate and representing it with the dummy variable (since we are abstracting from the quality of judgment, e.g, whether the copula is affirmative or negative), I indicate its image with f^*. Second, given that *This*$_x$(S) has only one instance, namely, x itself, its image under f is just the image of x under f, i.e. $f(\mathit{This}_x(S)) = fx$. So f maps the structured complex $*\langle \mathit{This}_x(S), P\rangle$ to $f^*\langle fx, f(P)\rangle$.

We have specified a 'logical function' (principle of unity) of quantity in judgment and shown how it corresponds one-to-one with a 'real function' (principle of unity) of objects. In other words, we have mapped structures of concepts onto structures of objects. Now we must 'represent this function generally,' i.e. abstract a concept of objects qua standing in such structures. This is equivalent to abstracting a concept of objects as images of the appropriate mapping. These will be the categories corresponding to each logical function of quantity. Following Kant's lectures on metaphysics, I take the categories to correspond to the logical functions of quantity in the reverse order of that given in the *CPR*,[54] so I start with singular judgment and the category <*unity*>:

<*Unity*>: the concept that applies to an object *o* just in case *o* is the first element in some structured complex $f^*\langle f(\mathit{This}_x(S)), f(P)\rangle$, where this is the image of some singular judgment under *f*. Equivalently, it is the concept that applies to an object just in case the object is the image of the first term of some singular judgment under the specified mapping.

It can be easily verified that a sensible object falls under <*unity*> just in case it instantiates an empirical concept.[55, 56] Intuitively, this is because it is the concept of being a *unit* of that concept, i.e., being a *unit F* for some F.[57] There is more to the category of <*unity*> than just this, however, for it

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[54] MVo 28:396; MD, 28:626; MVi 29:985; see also Prol. 4:302–3. This reading, on which categories of quantity correspond to the logical functions of quanity in reverse order, was first proposed by Frede & Krüger 1968 and followed by Longuenesse 1998, p. 249. It is disputed by Thompson 1989, Friedman 2000, p. 20 n. 3, and Hoeppner 2021, p. 306 n. 569; Lu-Adler 2014, p. 383–6, offers a critique of some of the assumptions that have structured this debate.

[55] A sensible object *o* falls under an empirical F concept just in case it is assigned by *f* as the value of some logical object *x* that is an instance of F. But by construction of the conceptual hierarchy there will be indefinitely many singular judgments of the form This_x(F) is G for some concept G; as shown above, $f(\mathit{This}_x(F))=f(x)$ and $f(x)=o$ (by assumption) so o is the image of the first term of a singular judgment. By definition, it falls under <*unity*>.

[56] Strictly speaking this exceeds the boundaries of the Metaphysical Deduction and thus anticipates the Transcendental Deduction. I include remarks like this to show how much is already accomplished in the MD: that every object that can be judged under empirical concepts instantiates <*unity*> (likewise, the other categories). The role of (the first half of) the TD is then to prove that all objects of empirical consciousness are objects of empirical judgment (i.e., can be judged under empirical concepts.)

[57] This analysis agrees with that of Longuenesse 1998, p. 254, who also points to Prol. 4:302, where Kant identifies *Einheit* (unit(y)) with *Maß* (measure).

also involves the idea that an instance of F is a _unified_ F, i.e. it has the unity characteristic of instances of F. But further exploring this would take us too far afield, into Kant's account of acts of synthesis and their unities.

<_Plurality_>: the concept that applies to an object _o_ just in case _o_ is the first element in some structured complex _f_*<_f_(C), _f_(_P_)> where this is the image of a particular judgment under _f_. Equivalently, it is the concept that applies to an object just in case it is the image of the first term of a particular judgment.

It follows, given natural assumptions, that every sensible object falls under <_plurality_>. Assuming every sensible object can be divided into parts, all of which fall under some concept C and C is subordinated to S, then the object is the image under _f_ of the particular judgment *<_Some_(S), P>, namely one that maps _Some(S)_ to C. For example, my desk can be divided into parts all of which fall under <_desk part_>. <_Desk part_> is subordinated to subordinated to <_furniture part_>, and my desk is the image of the first term of the particular judgment *<_Some (<furniture part>)_, P> where P is any predicate. In other words, my desk is a plurality of furniture-parts. But by parity of reasoning, my desk is also a plurality of desk-parts, because <_desk part_> is (improperly) subordinated to <_desk part_>.[58]

Finally, we can abstract the concept of an object that is the image of a universal judgment:

<_Totality_>: the concept that applies to an object _o_ just in case _o_ is the first element in some structured complex _f_*<_f_(_All_(S)), _f_(P)>. Equivalently, it is the concept that applies to an object just in case it is the image of the first term of some universal judgment.

It also follows, given natural assumptions, that every sensible object falls under <_totality_>. Assuming every sensible object can be divided into parts, such that all and only the parts of that object fall under a concept S, then the object is the image under _f_ of the universal judgment *<_All_(S), P>, namely one that maps _All_(S) to the object in question.[59] For example, assuming my

[58] See previous note.
[59] Again, _sensu stricto_ some of this work is reserved for the TD.

desk is the only desk, all and only its parts fall under the concept <*desk part*> so the image of that concept under *f* will be identical to my desk itself. So my desk is the image of the first term of the universal judgment *<*All(Desk part)*, P> where P is any predicate whatsoever. In other words, my desk is the totality of the desk-parts.

These three categories may seem trivial, for necessarily every object of intuition will fall under all three of them: every object is a single instance of some concept (*unity*); every object overlaps the extension of some more general concept B (*plurality*); and every object contains all and only the As, for some concept A (*totality*). But they become significantly less trivial when we do not generalize over the concept A but specify a particular concept, i.e.

(i) <*Totality of As*> = the concept that applies to an object just in case it is the image of a universal judgment whose first term is A.

This is not trivial, because, not every concept A has an extension that can be a single object of intuition. Some concept-extensions are so 'large' that the synthesis by which we would collect all of the objects in the extension into a single object would have to be infinite, i.e. concepts with infinite extensions. Given transcendental idealism, no such object can exist in space and time, and so it cannot be the value of that concept under *f*. For example, no object can instantiate the concept <*totality of past times*> or <*totality of material parts*> because <*past times*> and <*material parts*> would require an infinite synthesis to collect all of their extension into a single object.[60]

In the case of plurality and unity, we get:

(ii) <*Plurality of As*> = the concept that applies to an object just in case it is the image of a particular judgment whose first term is A.

(iii) <*Unit of A*>= the concept that applies to an object just in case it is the image of a singular judgment whose first term is A.

For reasons given earlier, for every sensible object *o* there will be a concept A such that *o* is a plurality of As, and there will be a concept A′ such that *o* is a singular A′, a *unit* A′.

[60] Essentially the same point is made by Longuenesse 1998, p. 255.

This explains the role of the categories of quantity in acts of counting: we want to count the As, so we start by specifying a unit of counting, i.e. we represent some object as a unit of A (i.e. a unified instance of A). We then proceed to count further As, i.e. represent the collection of objects we have already counted as a *plurality* of As. This process of counting ends when we represent our collection as containing all of the As, i.e. as being a *totality* of As. The As have a *number*, i.e. are finitely enumerable, only if this process ends, i.e. if there is a totality of them. Infinite collections, those for which there is no totality, are precisely collections that cannot be numbered, i.e. for which this process of enumeration goes on indefinitely.[61]

It also explains why a totality is a plurality that is also a unity (a unified plurality).[62] If we start with a concept A and consider various pluralities of A, when we reach the totality of As (a plurality that contains all As) we have a single object (the totality of As) which is thus a unity (*Einheit*), but not a unit (*Einheit*) of A (it does not fall under A). For instance, the totality of all crows is a plurality of crows and a singular object in its own right (the image of some singular judgment), but it is not a single crow; it is a singular instance of some other concept (e.g., the concept of massively spatially separated objects.)

In concluding this section I want to note that none of my reconstruction assumes that the sensible objects to which we map our conceptual hierarchy (the 'co-domain' of the mapping *f*) are objects of specifically *spatiotemporal* intuition.[63] This reconstruction provides a derivation of the

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[61] A432/B460; R 4756, 17:700; R 5703, 18:379; MH 28:946; MMr 29:836; MVo 28:423, 439. See Longuenesse 1998, p. 251–263.

[62] As Kant repeatedly claims in his metaphysics lectures: MVo 28:398; MvS 28:481; ML_2 28:560; MK_3 29:988, 1002.

[63] In fact, my reconstruction assumes only that the objects of intuition stand in part-whole relations, which is logically weaker than assuming they are objects of sensible intuition, and thus might potentially hold of objects of non-sensible, i.e., intellectual intuition. Why, then, have I presented this as a MD restricted to sensible objects rather than for all objects of intuition, including intellectual intuition, whatsoever?. First, considerations of space make it impossible for me to discuss the difficult question of the 'meaning' of the categories in application to noumena. Second, a mind that has intellectual intuition is also a mind that does not have concepts, or, more properly, for which the concept/intuition distinction does not hold. So the general form of this derivation, which abstracts from the concept/intuition distinction will look quite different. However, I am not sure it will not succeed. I regard it as an open question, requiring further research (see next note), whether a generalization of my reconstruction here would generate perfectly general categorical meanings and whether an analogue of my argument, where the objects are objects of intellectual intuition, would generate categorial meanings specifically for such objects (positive noumena).

categories of quantity as concepts of sensible objects in general. I take this to be a virtue of the reconstruction, for Kant originally proves that the categories are valid for all objects of sensible intuition in general (B Deduction §20) and only then explains how this is possible for specifically spatiotemporal sensible intuition.[64] This means that the derivation of the categories from logical functions (explicitly appealed to in the proof of the more general thesis in §20) must be *at least* as general as a derivation of the categories as concepts of sensible objects in general. Furthermore, in order for the derivation to be fully successful, what at various points are described as 'natural assumptions' have to be demonstrated to hold of any conceptual hierarchy related to any of sensible manifold whatsoever.

§5. The rest of the table: Quality, Relation, and Modality

Due to considerations of space I can only sketch how to extend this model to the other three moments of the table of categories. Fully working it out must remain a project for future work.[65]

Quality. In considering the logical quantity of judgment we fill in the relation * from above: overlap (affirmative), being disjoint (negative), or overlap with a negative predicate (infinite judgment).[66] In considering the quality of objects we do not consider them merely as objects of intuition, but as objects of perception. This means we consider them as objects of consciously apprehended manifolds of intuition (perception) with both a form and a matter. This, in turn, means we must consider such objects not simply as standing in part-whole relations, but as having perceptible properties. We must now introduce a new mapping g, from concepts to sensible objects and perceptible properties. In effect, we map concepts to perceptible properties shared by all of their instances (e.g., the property *red*), and we map negative predicates (the predicates of infinite judgments) to negative properties (e.g., the property *non-red*). If we construct the mapping correctly,

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[64] Likewise, immediately after the same function claim, Kant talks of 'the manifold of intuition in general' (A79/B105).

[65] The larger account is given in un published manuscript, tentatively titled *Kant's Metaphysical Deduction of the Categories: A Systematic Reconstruction*.

[66] This means that my interpretation of quantity and quality is very nearly the opposite of Hoeppner, who assigns the function of the predicate to quantity and the function of the subject to quality (Hoeppner 2021, p. 129).

affirmative judgments get mapped to ordered complexes in which the *f*-image of the subject-concept overlaps the fusion of objects that have the *g*-image of the predicate (a perceptible property); negative judgments get mapped to ordered complex in which the *f*-image of the subject-concept is disjoint from the fusion of objects that have the *g*-image of the predicate; and infinite judgments get mapped to ordered complexes in which the *f*-image of the subject-concept overlaps the fusion of the objects that have the *g*-image of the predicate, a negative property (e.g., *non-red*). If we abstract concepts of objects from these structures, we get the following: <*reality*> is the concept of objects qua having some perceptible property; <*negation*> is the concept of objects qua lacking (not having) some perceptible property; and <*limitation*> is the concept of objects qua having some negative property (i.e. being *non-red*). Again, given natural assumptions, it will follow that every object instantiates all three categories of quality: every object has some perceptible properties, lacks others, and has the negations of those it lacks (e.g., if it lacks *red* then it has *non-red*).

Relation. We now remove another abstraction from our treatment of objects: we no longer regard them only as objects of intuition and perception, but as objects of experience, i.e. as distinct from and potentially independent of any single perception of them. This means that we must think of them as being objects of multiple sensible manifolds. In effect, we map purely logical objects to objects that can be present in multiple sensible manifolds, and thus have distinct (even contrary) sensible properties according to different manifolds (e.g., at different times). The category <*substance*> is the concept of objects qua being the unified subject of different sensible properties (e.g., contrary properties at distinct times); the category <*accident*> is the concept of those sensible properties themselves. To extend this to causation, we need to map purely logical relations among concepts (e.g., antecedent-consequence, disjunction) to real grounding relations between substances and accidents. Given the centrality of the concept of <*cause-effect*> to the *CPR*, and the difficulties that have faced every extant attempt to derive it from logical functions and the form of intuition, readers may naturally feel that, until I have shown that my reconstruction can accommodate that case, I have not shown much at all. To a large extent, I agree.

Modality. Modality is actually the easiest of the four moments, because modal categories express the relation of a putative cognition to our capacity for cognition, rather than describing determinations of objects. Consequently, we do not need to introduce any further structures in objects, abstract general concepts from them, etc. To be very brief, the logical function of problematic judgment (consciousness of a judgment-content as compatible with the purely logical form of a judgment) will be mapped to the 'real' function of possibility, i.e. representing some judgment-content as compatible with the form not only of a conceptual hierarchy by itself, but with the form of sensible manifolds and the whole mapping described above in each of the other three moments. In short, the category <*possibility*> will express the agreement of a putative cognition with the form of the whole structure described above; as a consequence, it will apply to a concept just in case it is the concept of an object that can be given in intuition and brought under empirical concepts (thereby bringing it under the categories, as described above.) The logical function of assertoric judgment (consciousness of a judgment-content as compatible with the form and matter of the understanding, i.e. as true) will be mapped to the 'real function' of actuality, i.e. representing some judgment-content as compatible not only with the form described above but with the matter actually given to sensibility. In short, the category <*actuality*> will express the agreement of a putative cognition with the form and matter of the whole structure described above; as a consequence, it will apply to a concept just in case it is the concept of an object that is given to us and cognized under our empirical concepts. The metaphysical deduction of necessity, however, brings in sufficient additional complications (related to the metaphysical deduction of <*cause-effect*>) that I will not attempt to sketch it here.[67]

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