

# Reading Newton in Early Modern Europe

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## Newton's Concepts of Force among the Leibnizians

*Marius Stan*

Sometimes we mean “Newtonian mechanics” glibly, as just another name for classical mechanics.<sup>1</sup> Then, knowing how successful the latter was (at least until 1905), we expect *Newton's* own doctrine to have spread with the speed and ease of inexorable truth. But history, ever the wily muse, teaches us a sobering lesson. To many in the early Enlightenment, Newton's dynamics seemed far from inevitable. In post-Leibnizian Germany, his foundations of mechanics struggled particularly hard to gain a foothold. Two elements in particular – Newton's concepts of force and its laws – saw a cold welcome from local natural philosophers.<sup>2</sup> While it is tempting to explain the Germans' reluctance as rooted in mistrust toward the unfamiliar (Newton's force of inertia, *vis impressa*, and distant gravity were new by any measure), I propose here a different explanation. Some prominent Leibnizians – Jacob Hermann (1678–1733), Christian Wolff (1679–1754) and his followers in Germany – turned down Newton's concepts of force because of their allegiance to a foundational agenda for dynamics ultimately inherited from Leibniz (1646–1716). This Leibnizian heritage gave them the confidence to assert that, in order to ground mechanics, Newton's foundations were dispensable if not downright mistaken.

Section 1 takes a close look at the Leibnizians' notion of *vis inertiae* and how it differs from Newton's eponymous idea. It is because they took their “force of inertia” from Leibniz, I argue. In Section 2, I document a widespread lacuna in post-Leibnizian natural philosophy: Newton's Second Law and the concept of impressed force, which it codifies. I explain this absence as, again, due to

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1 There are several distinct (but empirically equivalent) versions of classical mechanics. One is Newton-Euler dynamics, based in the Second Law generalized to forces and torques. The other is “analytic” mechanics, whose explanatory core is the Euler-Lagrange equation. In turn, analytic mechanics comes in two species, or “formalisms.” The first is rooted in the Principle of Virtual Work; the second, in the Principle of Least Action. Less significant historically are Hertz' mechanics of constraints and the twentieth-century “geometric” mechanics on symplectic manifolds.

2 So did his absolute space and time. But that is a topic for another occasion.

Leibniz. His followers hoped to ground a dynamics of interaction in his notion of “active force” not Newton’s *vis impressa*. Finally, in Section 3, I move to reconstruct and assess various arguments by Wolff against action at a distance, whether generally or as Newtonian gravitation.

My key thesis here is not wholly new. For some years now, Eric Watkins has argued cogently that “Leibniz’s philosophy of nature exerted a strong influence on the reception of Newton’s laws, particularly on a Germanic tradition” that includes Wolff essentially and culminates with Kant.<sup>3</sup> My contribution is novel, I hope, in that it spells out rigorously the subtle but deep differences between the Leibnizians and Newton on the nature of force and its laws. Moreover, I show clearly that their dissent from Newton is due to Leibniz’s legacy. Entirely new, I expect, is my account of the Leibnizians’ arguments against distant gravity, and of the costs they incur thereby. Regrettably, for reasons of space I must leave out any discussion of how they reacted to Newton’s theory of matter.<sup>4</sup>

## 1 Forces of Inertia, Newtonian and Leibnizian

Two figures central to my case are Hermann and Wolff, each of whom had been in direct epistolary contact with Leibniz soon after 1700.<sup>5</sup> Leibniz’s strategy

3 Cf. Eric Watkins (forthcoming), “The reception of Newton’s principles and concept of force”, in Scott Mandelbrote and Helmut Pulte (eds.), *The Reception of Isaac Newton in Europe* (London: Bloomsbury academic, forthcoming), Section I.A. See also his “The laws of motion from Newton to Kant”, *Perspectives on Science*, 5 (1997), 311–348.

4 R.S. Calinger, “The Newtonian-Wolffian Controversy”, *Journal of the History of Ideas*, 30 (1969), 319–330, discusses some issues associated with this topic, for the period 1740–60. For the earlier decades, see Arnold Thackray, *Atoms and Powers: an Essay on Newtonian Matter-Theory and the Development of Chemistry* (Harvard: Harvard University Press, 1970).

5 Leibniz’s correspondence with Jacob Hermann is in C.I. Gerhardt (ed.), *Leibnizens mathematische Schriften*, 7 vols. (Berlin and Halle: A. Asher and H.W. Schmidt, 1849–63), volume iv; his exchanges with Wolff are in C.I. Gerhardt (ed.), *Briefwechsel zwischen Leibniz und Chr. Wolff*, Supplement-Band zu *Leibnizens Gesammelte Werke* (Halle: H.W. Schmidt, 1860). Hermann was active at Padua, in the mid-1700s, and then at the Imperial Academy in St Petersburg. On his accession to Padua, cf. André Robinet, *L’Empire leibnizien: la conquête de la chaire de mathématiques de l’Université de Padoue* (Trieste: Lint, 1991); on his activity in Russia, cf. Fritz Nagel, “Jacob Hermann – Skizze einer Biographie”, in Fritz Nagel and Andreas Verdun (eds.), “Geschickte Leute, die was praestiren können...” *Gelehrte aus Basel an der St. Petersburger Akademie der Wissenschaften des 18. Jahrhunderts* (Aachen: Shaker, 2005), 55–76. For a biography of Wolff, by one of his ardent followers, see J.C. Gottsched, *Historische Lob-schrift des weiland hoch- und wohlgebohrnen Herrn Christians, des H.R.R. Freyherrn von Wolf* (Halle: in Verlegung der Rengerischen Buchhandlung, 1755).

had always been to cultivate the friendship of promising young talent, in the hope of shaping their views away from the two then-influential theories of mechanics, Cartesian and Newtonian. Both men went on to become major figures in distinct but adjacent fields. On the Continent, Hermann's *Phoronomia* (1716) was for two decades the leading treatise on dynamics, until Euler's *Mechanica* supplanted it.<sup>6</sup> Wolff was active in all areas of philosophy, in German then in Latin, in which he articulated his most considered views. His prodigious thorough output led admirers to hail him as *praeceptor Germaniae secundus* (with Melanchton as the first), and Voltaire to dismiss him as *un bavard germanique*.

Most relevant to my topic is Wolff's bestseller *Cosmologia Generalis*, often reissued during the 1730s.<sup>7</sup> There, he pursues a strong program of anchoring an empirical theory of motion into an *a priori* ontology of body. The program is mechanistic, in that all interactions are by contact; it is also "dynamistic", in that "force" is in the nature of body.<sup>8</sup> In Wolff's doctrine, this *endogenous* attribute comes in two kinds, active and passive force. For the latter, he has a special term: *vis inertiae*, or the (passive) force of inertia. "Every body resists motion.... In bodies, the principle of resistance to motion is called *Force of inertia*, or *passive Force*." His many followers adopt it too: "That whereby a body resists motion is called by the term, common in Mathematics, of *Force of inertia*, or also *passive force*." "In bodies, the principle of resistance to motion is called *Force of inertia*, or *passive Force*."<sup>9</sup>

This vocabulary is very much redolent of Newton's Definition III in the *Principia* introducing *vis inertiae* as that "power to resist" whereby single bodies oppose attempts to change their rest or uniform translation.<sup>10</sup> In fact, the

6 Jacob Hermann, *Phoronomia, sive de Viribus et Motibus Corporum solidorum et fluidorum libri duo* (Amsterdam: Wetstein, 1716).

7 Cf. Christian Wolff, *Cosmologia Generalis* (editio nova. Frankfurt: Renger, 1737).

8 Ibid. 125. However, I must note that Wolff distinguishes between "essence" and "nature." The essence of body "consists in the manner in which its given parts are joined to each other," whereas its nature is the "principle of actions and passions of bodies," where by "principle" Wolff means "that which contains in itself the reason of another being or of some change." For Wolff, the nature of body consists in matter and active force; matter, in turn, he understands as "extension endowed with force of inertia." Cf. 120, 125, §§ 140, 145.

9 Cf., in order, Ibid. §§ 129–130; L.P. Thümmig, *Institutiones Philosophiae Wolfianae* (Frankfurt: Renger, 1725), 84§ 31; J.F. Stiebritz, *Philosophiae Wolfianae Contractae Tomus I* (Halle: in officina libraria Rengeriana, 1744–1745), 671, § 130.

10 See Definition III in the *Principia*: "Inherent force of matter is the power of resisting by which every body, so far as it is able, *perseveres in its state* either of resting or of moving uniformly straight forward." As I. Bernard Cohen shows, Newton had added, in his personal copy of Edition II, a note he never printed: "I do not mean *Kepler's force of inertia*,

Wolffians do mention Newton or his masterpiece as they expatiate on the force of inertia:

[The law of inertia] is the first law of motion, which Mathematicians assume in their doctrine of motion as they rely on experience. An example of this is Newton, that supreme Geometer, in his *Principia*, on p. 13 in the latest London edition, where he calls our proposition above the first law of motion, and places it among his axioms, appealing to experience.<sup>11</sup>

And yet, something seems amiss. The Wolffians, almost to a man, describe their *vis inertiae* as a force resisting *motion*. In contrast, the Englishman's analogous concept denotes a force that resists *changes of state*, not "motion."<sup>12</sup> Newton's inertia appears to have a wider range – it can oppose *rest* as well; for instance, as a moving body resists a stationary one, in impact. Thus, Newtonian and Wolffian *vires inertiae* overlap partially, but they are distinct forces after all, for the latter covers less. Clearly, the Leibnizians did not adopt Newton's force of inertia.

One might be tempted to protest this conclusion as excessive, over-interpreting scant evidence. Perhaps Wolff and his followers were just a bit careless with their words? Could it not be that, by "resisting motion," they really meant resisting the addition of exogenous *velocity increments*, or changes in momentum – which would make their force of inertia dynamically equivalent to Newton's, despite their different wording of it?

I think not. On the nature of inertia, the Leibnizians differ from Newton in concept, not just *expressis verbis*. Underneath their linguistic divergence lurks a conceptual gap. This becomes apparent when we examine two types of processes that they and Newton treated, viz. 1-body motion and 2-body collision. The Leibnizians agree with Newton on the *kinematics* of a body left alone, granting that no single body self-accelerates. However, they diverge on the *dynamical* explanation for this behavior. In Newton's mechanics, *vis inertiae* is causally responsible for *both* the continued rest of resting bodies and the smooth translation of moving ones. In contrast, the Leibnizians' *vis passiva seu inertiae* merely keeps a resting body from self-moving. To explain uniform

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by which bodies *tend toward rest*, but a force of remaining in the same *state* either of resting or of moving." Cf. Sir Isaac Newton, *The Principia: Mathematical Principles of Natural Philosophy*, trs. I. Bernard Cohen and Anne Whitman (Berkeley: University of California Press, 1999), 404, fn. C; my emphasis.

11 Wolff, *Cosmologia Generalis*, 232, § 309.

12 I thank Noel Swerdlow for pressing me on this issue.

rectilinear motion, they usually introduce *another* agency, viz. an *active* “force of motion,” distinct *in kind* from the passive power to resist that is *their* force of inertia. This active force was akin to the Scholastics’ “impetus,” in that it *sustains* a body in its motion. “Motive force [*vis motrix*] consists in a continual endeavor to change place,” declares Wolff. Thümmig follows suit: “A body already in motion is endowed with force of acting [*vis agendi*]. This active force adheres to local motion. ... Hence, *active* or *motive* force [*vis motrix*] adheres to motion.” Stiebritz too: “Motive force consists in a continual endeavor to change place.” And so does Formey, verbatim.<sup>13</sup> Needless to say, no such *vis motrix* is at work in the *Principia*, for it is unnecessary. To sum up: in order to ground one and the same inertial state, the Leibnizians often employ *two* distinct forces, whereas for Newton one suffices.

The divide widens once we examine how Newton and the Leibnizians interpret the head-on collision of two bodies. Consider the case of direct impact with a body at rest. Within Newton’s framework, upon contact *both* bodies *resist* each other’s attempts to change their state. *Each* resists by its (Newtonian) *vis inertiae*. And, each such individual exercise of inertia results in an impressed force, *mutually* and equally applied by the bodies. In sharp contrast, the Leibnizians see every collision as an asymmetric clash between two bodies playing fundamentally distinct roles: an agent that *acts*, and a patient that “suffers” by *resisting*:

If a body B moves another A at rest; or changes the motion of a moving body A: then B acts, whereas A suffers. ... The action of a patient against an agent is called *Reaction*. While A resists motion, it reacts against body B, which urges it to move.<sup>14</sup>

Crucially, it is *solely* the patient that exerts Leibniz-Wolff *vis inertiae*: “A body indeed reacts as resists motion. But a body resists motion by its force of inertia. Hence, a body reacts by the force of inertia.” The agent exerts a radically different kind of force, namely, *vis motrix seu activa*:

A body in motion is endowed with a force of acting. This active force of bodies is the principle of all changes. ... Motive force [*vis motrix*], *on*

13 See, in order, Wolff, *Cosmologia Generalis*, 128, § 149; Thümmig, *Institutiones Philosophiae Wolfianae*, 85–86, §§ 35–36, 38; Stiebritz, *Philosophiae Wolfianae Contractae*, 674, § 149; J.H.S. Formey, *Elementa Philosophiae, seu Medulla Wolfiana* (Berlin: Haude and Spener, 1746), 65, § 106.

14 Wolff, *Cosmologia Generalis*, 116, 234; §§ 133, 313–314.



*which a body's action depends*, consists in a continual endeavor to change place, or in a continual striving.<sup>15</sup>

The difference is obvious. For Newton, the causal mechanism of impact rests on a *single* force *mutually* applied.<sup>16</sup> But, to handle impact, the Leibnizians invoke *two* forces, *heterogeneous* and *asymmetrically* deployed. (More on this below, in Section 2.2). *Vis inertiae* is just one of these forces. Once again, their force of inertia turns out to be narrower than Newton's. It is primarily the "force of rest," a power to resist paradigmatically found in *stationary* bodies.

At this point, the interpreter could either contend that Wolff and his followers attempted to proffer a rationalist foundation for Newton's mechanics – but a collective failure of insight led them astray – or that the Wolffians' force of inertia *is* different from Newton's, because it does not derive from him. The latter interpretation, I contend, is hermeneutically more charitable, and there exists more evidence to support it.

### 1.1 Leibnitii Sequaces

Leibniz, I readily admit, also made room for inertia in his natural philosophy. At times, he would deploy it polemically, against Cartesians, to counter their view that the essence of a body is just extension and its modes, viz. size, shape, and motion. Addressing the French, Leibniz proclaims to have discovered "*the natural inertia of bodies*." His key evidence is from *impact with a body at rest*:

15 Ibid. 236–237, 118, 238; §§ 316, 135–136, 319; my italics.

16 Objection: according to Newton's own theory, two bodies in impact exert on each other *vires impressae*, not *vires inertiae*. It is these impressed forces that Newton's Third Law declares to be mutual, equal, and contrary. Answer: true, but incomplete. In Newton's doctrine, the *vis impressa* is caused, in its turn, by *vis inertiae*. That is to say, every impressed force applied by a body A on a body B is caused by the (Newtonian) force of inertia inherent in A. Or, A resists B's attempt to dislodge it by impressing a force on B. So, for Newton, a force (*vis inertiae*) is the origin or cause of another force (*vis impressa*) – at least for the case of collisions, where no Newtonian "active principles" efficient at a distance would be involved. For a clear account of this Newtonian idea, see Howard Stein, "Newton's metaphysics", in I. Bernard Cohen and G.E. Smith (eds.), *The Cambridge Companion to Newton*, (Cambridge: Cambridge University Press, 2002), 256–307; 283f., 289. Therefore, for Newton ultimately it is the *vis inertiae* that explains, for *both* bodies, their behavior in impact. It was precisely this Newtonian mechanism that Leonhard Euler rejected in his "Recherche sur l'origine des forces", *Mémoires de l'académie des sciences de Berlin*, 6 (1752), 419–447. There, Euler seeks to argue that a *vis impressa* is caused by a body's impenetrability not its *vis inertiae*. For an account of Euler's reasoning, see Stephen Gaukroger, "The Metaphysics of impenetrability: Euler's conception of force", *The British Journal for the History of Science*, 15 (1982), 132–154.

unless there was “natural inertia” in the resting body, “the moving would drag the resting along with it, without suffering any diminution of speed, contrary to what we see.”<sup>17</sup> Elsewhere, in a programmatic essay announcing the birth of a new discipline – “dynamics” – Leibniz reprised his claim to have discovered that matter has “a certain sluggishness or resistance to motion,” in addition to Cartesian extension.<sup>18</sup> Again, he points to collision with a resting body in order to prove that his force exists: unless we impute to matter a “resistance to motion,” it follows that “the largest body at rest would be carried away by the smallest body striking it, with no diminution in its speed,” contrary to all experience.<sup>19</sup> And, in a letter to de Volder (1643–1709), Leibniz makes clear that he understands inertia to be that force whereby a resting body resists a moving one, or a slower resists a swifter:

And so a body at rest resists every motion, and a moving body resists one that has greater motion even in the same direction, so that it weakens the force of the thing impelling it.<sup>20</sup>

It has been noted, grudgingly, that this view of inertia is not quite Newton’s concept: “Alas, Leibniz invokes the expression ‘natural inertia’ to refer specifically only to the resistance a body makes to motion and greater motion and not also to less motion and rest.”<sup>21</sup>

In his letters to Hermann, Leibniz occasionally alluded to his metaphysical dynamics, which aroused the young Swiss’ curiosity. “Your Metaphysical principles must be most excellent,” he told Leibniz while writing *Phoronomia*, “and I ask for your favor to let them adorn my little book like some precious

17 Leibniz to the editors of the *Journal des Savans*, 18 June 1691, in J.E. Erdmann, (ed.) *Leibnitii opera philosophica quae exstant latina gallica germanica omnia* (Berlin: Eichler, 1840), 113. Contrast this with Newton’s evidence for his force of inertia, all of which draws on the *motion of single* bodies, not interactions: projectiles, spinning hoops, and planets – Newton, *The Principia*, 416.

18 Leibniz, “Specimen Dynamicum 1” (1695), § 19, in Leibniz, *G.W. Leibniz: Philosophical Texts*, ed. and trs. Richard Francks and R. S Woolhouse (Oxford: Oxford University Press, 1998) 161.

19 Ibid.

20 See Leibniz to de Volder, 24 March/3 April 1699, in G. W Leibniz, *The Leibniz-De Volder Correspondence: With Selections from the Correspondence Between Leibniz and Johann Bernoulli*, ed. and trs. Paul Lodge (Yale: Yale University Press, 2013), no. 18.

21 H.R. Bernstein, “Passivity and Inertia in Leibniz’s Dynamics”, *Studia Leibnitiana*, 13 (1) (1981), 97–113; 105.

gemstones."<sup>22</sup> Hermann then read Leibniz's foundational papers in dynamics, presumably also his claim to have discovered a force of inertia. This would explain Hermann's description:

In addition to active force, there is in bodies also a certain passive Force, from which neither motion nor tendency to move results, but consists in that *Resistance* whereby it opposes any external force striving to change the bodies' state of rest or motion. ... This force of inertia is plain enough in resting bodies. Indeed, when a [moving] body A strikes another one B at rest, A loses something of its force, whereas B, drawing out some of A's force and motion, acquires it. Whence it is clear that the resting body really has some passive force, which the incoming's force must break and overcome.<sup>23</sup>

Leibniz was an early, alert reader of *Phoronomia*, which he reviewed in the *Acta Eruditorum* anonymously.<sup>24</sup> He was especially pleased with Hermann's account of *vis inertiae* as a passive force evidenced by *resting bodies in collisions*:

The inertia of matter, of which you speak in § 11, is a wonderful thing, and a topic for the deepest research; *few have grasped it so far*. Amazing consequences follow from it. For if we considered nothing in matter beside extension and impenetrability, there is no reason why *matter [at rest] in a place would resist [another] one in motion*, i.e. would *tend to remain at rest*. Hence there is no reason why there would be a struggle between agent and patient, since *matter at rest* is indifferent [to motion], and *the least motion would prevail over rest*.<sup>25</sup>

Blessed by the master in the Leibnizians' favorite venue, *Phoronomia* became authoritative for Wolff and his followers.<sup>26</sup> They seem to have adopted the

22 Hermann to Leibniz, 4 August 1712, in Gerhardt (ed.), *Leibnizens mathematische Schriften*, IV, 377.

23 Hermann, *Phoronomia*, 3.

24 See [G.W. Leibniz], "Review of Jacob Hermann, *Phoronomia*", *Acta Eruditorum*, January (1716), 1–10. More on this below, in Section 2, where I also present evidence that Leibniz reviewed Hermann's book.

25 Leibniz to Hermann, 17 Sept. 1715, in Gerhardt (ed.), *Leibnizens mathematische Schriften*, vol. IV, 398 (my emphasis).

26 And not just them. For their edition *cum perpetuis commentariis* of Newton's treatise, Fathers LeSeur and Jacquier sought help from Hermann's book as they tried to explain Newton's "force of inertia." They take over Hermann's distinction – of Leibnizian origin,

cluster of marks that distinguish *Leibnizian* inertia from its Newtonian counterpart: that it is a purely passive force; that it is evidenced by resting bodies in impact, rather than single bodies in translation; and that it is the force whereby patients resist the action of agents. Subsequently, Wolff went on in the 1730s to expand these Leibnizian marks of inertia into principles in his metaphysics of body. Earlier, in 1717, in his review of the Leibniz-Clarke correspondence, he had touted Leibniz as the winner of the debate with Newton's disciple.<sup>27</sup> Then in 1720 Wolff's disciple Thümmig went on to write a "sixth letter" against Clarke, on Leibniz's posthumous behalf.<sup>28</sup> Wolff's philosophical stature and reputation as an expert in and popularizer of the new mathematics ensured that his foundations of mechanics, with all their Leibnizian ingredients and assumptions, gained a wide following.

## 1.2 Keplerus Ex Machina

From their mentor, the Leibnizians inherited not only ideas, but also some rhetorical flourishes. More than once, Leibniz had cast himself in the paradoxical role of first discoverer and humble follower. He would proclaim in one breath that, from his new metaphysics, he could prove that bodies are more than just Cartesian *res extensae* – their essence includes inertia – and that it is the *same*

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first expounded in *Specimen Dynamicum* – between active and passive forces, and declare *vis inertiae* to be the sole passive force inherent in bodies. As evidence for it, they adopt the Leibniz-Hermann justification that bodies must have a force of inertia whereby they resist "agent bodies" in interactions "or else the moving body would be able to move [along with it] any other body [in its path], with no loss of its motion." LeSeur and Jacquier import almost verbatim Hermann's theses on *vis inertiae*, unaware that strictly speaking Hermann's distinction active/passive force and his notion of inertia is not fully compatible with Newton's mechanics, and comes from Leibniz – Thomas LeSeur, and François Jacquier (eds.), *Philosophiae Naturalis Principia Mathematica, Auctore Isaaco Newtono, Equite Aurato. Perpetuis Commentariis illustrate* (Geneva: Barillot, 1739–42), I, 4f., fn. 7–8.

27 Wolff's review is in [Wolff], "Collectio Sedarum per quas Vir Illustris Leibnitius et D. Clarkius Anno 1715 & 1716 de principiis quibusdam Philosophiae et Religionis naturalis disputarunt", *Acta Eruditorum*, October (1717), 440–447.

28 Thümmig's letter is in Gregory Sharpe, *A Defence of the late Dr. Samuel Clarke against the Reply of Sieur Lewis-Philip Thummig in Favour of Mr Leibnitz, with that Reply, in French and English* (London: Knapton, 1744); it was first written in 1720, and published in Germany with a preface by Wolff, as Samuel Clarke, *Merckwürdige Schriften, welche auf gnaedigsten Befehl Ihrer Koeniglichen Hoheit der Cron-Prinzessin von Wallis Zwischen dem Herrn Baron von Leibnitz und dem Herrn D. Clarcke ueber besondere Materien der natuerlichen Religion, in Frantzoesischer und Englischer Sprache gewechselt* (Frankfurt and Leipzig: Meyer, 1720).

force as the inertia that Kepler had posited to explain planetary motion. “[M]atter resists being moved by a certain *natural inertia* (as Kepler nicely calls it).”<sup>29</sup> Then, in *Theodicy*: “Kepler, one of the most excellent modern mathematicians, recognized a species of imperfection in matter, even when there is no irregular motion: he calls it ‘natural inertia.’”<sup>30</sup> And in 1715, writing to Clarke, he flaunts:

that inertia, ... mentioned by Kepler, repeated by Cartesius (in his letters) and made use of by me in my *Theodicy*, in order to give a notion (and at the same time an example) of the natural imperfection of creatures.<sup>31</sup>

One thing baffles the reader about Leibniz’s invocation of Kepler. Strictly speaking, Leibniz is wrong to equate his “passive force” of inertia with Kepler’s. The astronomer’s *vis inertiae* was an endogenous force of *self-deceleration*: it brought a moving body to rest from the inside, as it were.<sup>32</sup> In contrast, Leibniz grants that a body in uniform straight-line motion does *not* stop by itself. In fact, as the century draws to a close, Leibniz came to agree that his “natural inertia” *also* keeps translating bodies in motion, not just stationary ones at rest. For instance, in *De Ipsa Natura*:

Just as it is certain that matter *cannot of itself begin a motion*, so ... it is equally certain that a body considered in itself *retains any impetus imparted* to it, and that it remains constant in its mobility – that is, it has a *tendency to persevere in whatever sequence of changes* it has begun.<sup>33</sup>

29 *De ipsa natura* § 11: see “Nature Itself; or, the inherent force and activity of created things” – confirming and illustrating the author’s dynamics (1698), in G.W. Leibniz, *G.W. Leibniz; Philosophical Texts*, ed and tr. R.S. Woolhouse and Richard Francks (Oxford: Oxford University Press, 1998), 209–222; 216.

30 Cf. G.W. Leibniz, *Theodicy*, ed. Austin Farrer and tr. E.M. Huggard, (LaSalle, Ill.: Open Court, 1985), 353.

31 See Leibniz’s Fifth Letter to Clarke, § 102, in H.G. Alexander, *The Leibniz-Clarke correspondence* (Manchester: Manchester University Press, 1970), 88.

32 Kepler, *Epitome of Copernican Astronomy*, IV.ii.2: “by reason of its matter, [a celestial globe] has a natural *adynamia* or powerlessness of crossing from place to place, and it has a natural inertia or rest whereby it stays in every place where it is placed alone.” Cf. Johannes Kepler, *Epitome of Copernican Astronomy & Harmonies of the World*, tr. C.G. Wallis (Amherst, NY: Prometheus Books, 1995), 54.

33 *G.W. Leibniz: Philosophical Texts*, 217; my italics. Thereby, Leibniz comes closer to Newton’s understanding of inertia. But not too close: Leibniz still could not bring himself to accept that, by the force of inertia, a *moving* body *resists* another one *at rest*. This fact, in sharp

So, Leibnizian and Keplerian inertias are fundamentally distinct forces. What obscures this radical difference is Leibniz's phrase that, by its natural inertia, "matter resists motion."<sup>34</sup> This is true in both Kepler's doctrine and Leibniz's. However, by Keplerian inertia matter resists *its own* motion, whereas by Leibnizian inertia *a* matter, or body, resists the motion of *other* bodies.<sup>35</sup>

To deal with this discrepancy, some scholars took a drastic step and claimed that Leibniz's force of inertia *is* the same as Kepler's after all.<sup>36</sup> As evidence, they adduce a passage from a 1698 letter to Denis Papin (1647–1714), where Leibniz explains that, when a *single* body is in *uniform* translation – the very paradigm of (impressed) force-free motion, in Newton's mechanics – *two* forces act on it jointly: a fully Keplerian inertia that tends to bring the body to rest; and an active force that overcomes the former, thus keeping the body in motion:

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contrast, is perfectly acceptable to Newton – but not to Leibniz's followers in the eighteenth century, as we shall see. Likewise, Leibniz speaks of the "force of the moving body," which he takes to be *different in kind* from "natural inertia" – the latter is passive, while the former is active. This too is the seed of the Leibnizians' later notion of *vis motrix*.

34 See Leibniz's aforementioned letter to de Volder, 24 March/3 April 1699, in Leibniz, *The Leibniz-De Volder Correspondence*, no. 18.

35 There is a baffling passage in which Leibniz attributes to Kepler a different conception of inertia. In the excerpt from *Theodicy* I quoted above, Leibniz ends the sentence as follows: "[Kepler] calls it 'natural inertia,' which gives it a resistance to motion, *whereby a greater mass receives less speed from one and the same force*" (my emphasis). It is baffling because, in it, Leibniz credits Kepler with the thoroughly *Newtonian* concept of inertia as a body's *resistance – in proportion to its mass – to externally-induced acceleration*. I have no explanation for Leibniz's move here. Readers could have accused him of crassly misunderstanding Kepler, whose inertia is a force of self-resistance *qua corpora ad quietem tendunt*, as Newton put it. Or they could have charged him with giving Kepler undue credit (while denying any to Newton). Kepler's notion of inertia is pre-classical, i.e. not yet compatible with the classical mechanics of inertial forces. Newton was fully aware that his force of inertia was different from Kepler's; see I. Bernard Cohen, "Newton and Keplerian Inertia: An Echo of his Controversy with Leibniz", in A.G. Debus (ed.), *Science, Medicine and Society in the Renaissance. Essays to honor Walter Pagel*, 2 vols. (New York: Science History Publications: 1972), ii, 199–212. Had he bothered, he could have publicly excoriated Leibniz for his misattribution.

36 See, for instance, J.K. McDonough, "Leibniz's Philosophy of Physics" in Edward N. Zalta (ed.), *The Stanford Encyclopedia of Philosophy* (Fall 2008 Edition): Online: <<http://plato.stanford.edu/archives/fall2008/entries/leibniz-physics/>>, Section 2.3; and Anna-Lise Rey, "The controversy between Leibniz and Papin: from the public debate to the correspondence", in Marcelo Dascal (ed.), *The Practice of Reason: Leibniz and His Controversies* (Amsterdam: John Benjamins, 2010), 75–100, 90f.

for a body, being in motion, surmounts continually *its own inertia* by means of its force, and *acts on itself* in compound proportion of the promptitude and the continuation (that is to say, the intension and extension) of the given local change.<sup>37</sup>

It seems to me that their interpretation is not tenable. It does nothing to explain a wealth of passages, ranging from 1688 to 1710, in which Leibniz spells out *his* inertia as a force incompatible with Kepler's, as I explained above. Moreover, their key evidence is not hard to discount as an ad hoc move that Leibniz was forced to make, pressed by the need to give Papin an *a priori* proof that *vis viva* is the true measure of the "force of motion" that a single body has as it moves. Bernstein, a very careful student of Leibnizian inertia, also disagrees strongly, though *avant la lettre*, with their verdict: "In any case, Leibniz rejected [Kepler's] notorious inclination to rest."<sup>38</sup>

This, however, resurrects the puzzle: why did Leibniz claim a Keplerian heritage if his inertia is not Kepler's? One might charitably assume that, knowing how novel his metaphysic of body was in replacing Cartesian extension with "force," Leibniz sought legitimacy by placing himself rhetorically in Kepler's wake, so as to seem less radical – all the while glossing over the deep differences between Kepler's inertia and his own.<sup>39</sup> Or, better yet, Leibniz engaged in an act of "creative borrowing," as Bernstein put it.<sup>40</sup>

Still, when more backdrop enters the frame, the picture changes, and it is less flattering for Leibniz. At the same time as he extolled Kepler – and even Descartes – for insight into "natural inertia," Leibniz remained studiously silent about Newton's *vis inertiae*, a key ingredient in taming planetary motion, the most spectacular feat of the age. To be sure, Leibniz was well aware of Newton's concept, having paraphrased Definition III twice in his private notes on the

37 Leibniz to Papin, 3 October 1698, in G. W. Leibniz, *Sämtliche Schriften und Briefe* (Hrsg. Berlin-Brandenburgischen Akademie der Wissenschaften und der Akademie der Wissenschaften in Göttingen. Series III, vol. VII. Berlin: Akademie Verlag, 2011), 924 (my emphasis). Unless otherwise noted, all translations are mine.

38 Cf. Bernstein, "Passivity and Inertia in Leibniz's Dynamics", 105.

39 In an unpublished note of 1702, conventionally known as *On Body and Force, against the Cartesians*, Leibniz makes clear that, for him, body does not consist in Cartesian extension *plus* forces, active and passive; rather, the essence of body *just is* force, while extension itself is a derivative attribute, resulting from "a diffusion or repetition of a certain nature," viz. "resistance diffused through body." Cf. Leibniz, *G.W. Leibniz. Philosophical Essays*, ed and tr. Roger Ariew and Daniel Garber (Indianapolis: Hackett Pub. Co., 1989), 251.

40 Bernstein, "Passivity and Inertia in Leibniz's Dynamics", 102.

*Principia* from around 1689.<sup>41</sup> And yet, not once in more than two decades of exoteric talk about “natural inertia” did Leibniz see fit to mention Newton as a co-discoverer if not predecessor. A favorite strategy of Leibniz was to narrate his intellectual evolution, letting autobiography supplant argument: *Tempus erat quo credebam*, he would begin, recounting the error of his old ways before sharing with the reader how he found the light. Reminiscing about how he discovered “natural inertia,” Leibniz strongly suggests that he recognized it at the time of his insight that *tó dynamikón*, or “force,” is the essence of body. And *that* insight Leibniz had around 1678. However, Leibniz’s first invocations of Kepler as his source are from 1689, just after Newton’s *Principia* had reached him. At least once before then, Leibniz had a chance to tell the world about his supposedly Keplerian force of inertia, but declined to do so.<sup>42</sup> Only after Newton’s masterpiece came out did Leibniz abruptly begin to claim that he had discovered, a full decade before the *Principia*, that “natural inertia” is essential to bodies and is the *same* as Kepler’s inertia – all the while keeping silent about Newton’s *vis inertiae* and its role in mechanics.

Further, we must keep in mind that Leibniz’s discovery claim arises as he was trying frantically to catch up with Newton. Starting in 1689, he published in quick succession papers on celestial dynamics and motion in resisting media, all offered ostensibly as results found before the *Principia*, from Leibniz’s idiosyncratic dynamics – though meant in fact to make up for lost time in the race against Newton.<sup>43</sup> In retrospect, Leibniz seems moved by a dubious magnanimity: eager to praise the safely dead, like Kepler, but loath to credit those, like Newton, who could have lessened his glory as an alleged discoverer.

41 Cf. Domenico Bertoloni-Meli, “Leibniz’s Excerpts from the *Principia Mathematica*,” *Annals of Science*, 45 (1988), 477–505; 479, 488.

42 In fact, at that earlier time, Leibniz had used impact with a stationary body – subsequently, his key evidence for the “passive force” of inertia – as proof that bodies have an *active* force (his later *vis viva*) not the *passive* “natural inertia.” It is only from 1689 onwards that he takes collision with resting bodies to be proof of “natural inertia,” just as he starts invoking Kepler as a predecessor.

43 In public, Leibniz claimed to have been moved to write down his results – allegedly, predating Newton – after reading a review of the *Principia* by Christoph Pfautz in the *Acta Eruditorum*. Domenico Bertoloni-Meli has shown that Leibniz was not candid about that: he had in fact read attentively the *Principia* itself, taking extensive notes. Leibniz’s notes as a whole, Bertoloni Meli asserts, “contain a private collection of thoughts and calculations which he would not have wished to see in print.... Leibniz made several attempts both to translate Newton’s terminology and arguments into the scheme of his own terms and ideas, and to attain similar mathematical results using his own tools.” Cf. Domenico Bertoloni Meli, *Equivalence and Priority: Newton versus Leibniz: Including Leibniz’s Unpublished Manuscripts on the Principia* (Oxford: Clarendon Press, 1996), 96.



It is hard to tell how much of this anti-Newtonian animus, thickly disguised behind an appeal to Kepler, Leibniz's followers were aware of. They certainly adopted his claim that Kepler first discovered their force of inertia. "The great Astronomer Johannes Kepler called this force of resistance by the most significant word *Vis inertiae*" (Hermann). "It was *Kepler* who first recognized the force of inertia, then all others, relying on experience" (Bilfinger). Wolff does not echo Leibniz's claim himself, but one of his self-professed disciples does. In his *Natural Philosophy: or, Dogmatic Physics, being a Continuation of Baron de Wolff's Philosophical System*, Michael Christoph Hanov (1695–1773) declares, "Moreover, Kepler's force of inertia is regarded by many as a passive force, and also as a principle of resisting motion, hence contrary to motion."<sup>44</sup> Georg Bernhard Bilfinger (1693–1750), may have repeated Leibniz's invocation of Kepler out of sheer partisanship; at the Imperial Academy in St Petersburg, he had stood up for Leibniz against Daniel Bernoulli (1700–1782), who was sympathetic to Newtonian attraction at a distance.<sup>45</sup> Yet he was no mere hack; though much of his natural philosophy is heavily indebted to Leibniz, his laws of motion are an equanimous mix of Leibnizian and Newtonian elements (see below, 2.2). As to Hermann, in echoing the Kepler claim he was probably moved more by deference to Leibniz than real conviction. It is unclear whether Hermann had read Kepler's *Epitome* directly. Had he done so, he would not have missed the deep differences between Kepler's inertia and that of Leibniz – and Newton's, for that matter, whom Hermann inexplicably fails to mention as the true magisterial theorist of *vis inertiae*.

However, some Wolffians seem to have been honestly puzzled by (and thus unable to accept) the deep, subtle aspects whereby Newtonian inertia suffices *alone* to underwrite interactions, hence it does not need another kind of force, *vis motrix*, as a counterpart. In thrall to age-old, deeply seated intuitions about strength and activity, weakness and passivity, motion and rest, they frankly could not see how a supposedly *active* body could exert a force of *resistance*, viz. Newton's inertia:

But not even [Newton] himself dared to claim that a body in motion resists another one at rest, for that goes against [*abhorret*] both common

44 See, in order, Hermann, *Phoronomia*, 3, § 11; G.B. Bilfinger, *Dilucidationes philosophicae de Deo, anima humana, mundo et generalibus rerum affectionibus*. (Editio tertia. Tübingen: John G. Cotta, 1746), 177, § CLXXI; Michael Christoph Hanov, *Philosophiae naturalis sive physicae dogmaticae* (Halle: Renger, 1762), 1, 4.

45 Cf. Valentin Boss, *Newton and Russia: the early influence, 1698–1796* (Cambridge: Harvard University Press, 1972), 105–106.

usage and the facts themselves. Therefore, matter endowed with a force of resistance *properly so-called resists only motion, and only while it is at rest, and only the motion of another matter, and it resists so that it not itself be set in motion.*<sup>46</sup>

For Hanov, *vis inertiae* correctly conceived is Leibniz's not Newton's concept of inertia: i.e., it explains at most the behavior of just one body, in an interaction. To account for the doings of the second body, Hanov implies, we need to invoke a different type of force, namely *vis motrix*.

And yet, among the Leibnizians, for every honest confusion about Newton, one can point to a willful misunderstanding to match. In *De Reactione*, a 1741 dissertation much admired in Germany, Christian August Hausen (1693–1743) goes as far as to credit Kepler with a *fully Newtonian* concept of inertia, and claim that Newton himself had emulated Kepler: “*Vis inertiae (as Newton named his vis insita, following Kepler) is a passive principle whereby bodies persist in their motion or rest.*”<sup>47</sup> Formey (1711–1797) likewise intimates as much:

all Bodies also have a force whereby they resist all changes of state and persevere in their own unless an external cause removes them from it by acting with a force that could defeat their resistance. *Kepler* had called this resistance *force of inertia*, and *Newton* regarded it as a force inherent in matter and proportional to its quantity.<sup>48</sup>

This is plainly wrong and unsupported by any evidence. But it is more understandable in an intellectual climate in which Leibniz and his followers had been advertising Kepler relentlessly as the true discoverer of *vis inertiae*. Such claims to primacy were more likely to arise in the tense climate of Wolff's attacks on John Keill (1671–1721) and John Freind (1675–1728), two Newtonians, and of counteroffensives by Wolffians such as Formey against Euler's anonymous attack on Leibnizian monads. In light of these facts, it is hard to deny that, in Germany, Leibniz's competing notion of inertia considerably slowed down the proper understanding and unhesitant acceptance of Newtonian inertia, and thus of Newton's powerful theory of mechanics.

46 Hanov, *Philosophiae natvralis sive physicae dogmaticae*, 2; my emphasis.

47 C.A. Hausen, *De Reactione* (Leipzig: Langenheim, 1741), xi; my emphasis. Except for his gloss in parentheses, Hausen is here quoting Newton's Latin *Optice* (London: Samuel Smith and Benjamin Walford, 1706), p. 341.

48 [J.H.S. Formey], *Recherches sur les elemens de la matiere*. (s.l., 1747), 39; original emphasis.

## 2 Interactions without Newton

If we must dig deep to find Newton's *vis inertiae* missing in early Enlightenment Germany, another key Newtonian concept is visibly absent from the Leibnizians' doctrines. The notion of *vis impressa*, which the *Principia* explicated in Definition IV and measured by the Second Law, is nowhere in their foundations of mechanics. This persistent lack is all the more astonishing as, slowly through the century's first half, the *Lex Secunda*, suitably extended, became the first general principle of classical mechanics, a fact that Euler (1707–1783) proclaimed on the Leibnizians' home turf.<sup>49</sup>

One might attempt to explain the absence of *vis impressa* as follows. The Leibnizians did not need it, for they hoped to ground mechanics in a different, non-Newtonian concept of force, namely Leibnizian *vis viva*, whose measure is  $mv^2$ , not  $mdv/dt$  as Newton's Second Law has it. There is evidence for this conjecture. At the outset of *Phoronomia*, Hermann laid out a duality of passive vs. active force, and explained that the latter comes in two kinds, "dead" and "live" force. These two are identical, in character and measure, with the *vis mortua* and *vis viva* of Leibniz's *Specimen Dynamicum*. Further, Wolff had been a combatant in the cause of *vis viva* through the 1720s. He had defended it in a paper for the first volume of transactions of the St Petersburg Academy, in which the Leibnizians fired a collective salvo against their opponents. Bilfinger

49 This was Euler's paper "Decouverte d'un nouveau principe de Mecanique", read on 3 September 1750 at the Royal Academy in Berlin, published as Leonhard Euler, "Decouverte d'un nouveau principe de Mecanique", *Histoire de l'Académie Royale des Sciences et des Belles-Lettres de Berlin*, 6 (1752), 185–217. Euler's claim to have discovered a new principle baffles, initially: what he offers is the "Newtonian" law  $F = ma$ , written in component form along three orthogonal axes fixed in space. The mystery clears once we realize that, by a "principle of mechanics," Euler and his contemporaries meant a dynamical law that yields equations of motion for *any* mechanical system. Prior to 1750, Newton's *Lex Secunda* was *not* seen as a principle in this sense. This is because in the *Principia* the Second Law applies only to forces acting on *single, unconstrained particles*. After 1687, it was not obvious to anyone that, and how, the Second Law might apply to complex situations such as forces acting on *systems* of particles *moving* relative to each other; or forces on *constrained* bodies. It took until the early 1740s to learn that the Second Law applies to elastic bodies and fluids in motion. In "Decouverte", Euler shows how to apply  $F = ma$  to forces acting on a rigid body. Thereby, he discovers that "Newton's Second Law" is a truly general principle of mechanics. The main episodes of this story are recounted in Giulio Maltese, *Introduzione alla storia della dinamica nei secoli XVII e XVIII* (Genova: Accademia Ligure di Scienze e Lettere, 1996), 179–200 and Marius Stan, (forthcoming). "Euler, Newton, and Foundations for Mechanics", in Chris Smeenk and Eric Schliesser (eds.), *The Oxford Handbook of Newton* (Oxford: Oxford University Press).

too had published in it, as had Hermann.<sup>50</sup> Johann Bernoulli (1667–1748) had taken the battle for *vis viva* to the enemy, arguing in favour of it in an influential paper twice submitted to the French Academy. In addition, he encouraged Pierre Louis Moreau de Maupertuis (1698–1759), his former student, to take on the Cartesians at the Academy in the name of *vis viva*. In the late 1730s, Wolff began an effort to sway Madame du Châtelet (1706–1749), whom Samuel König (1712–1757), his former student at Marburg, had introduced to Leibnizian force.<sup>51</sup> So, it appears, the Leibnizians had in *vis viva* a ready substitute for Newton's *vis impressa*. No wonder we cannot find the latter in their writings.

I submit that this explanation is premature and insufficient. Were it true that the Leibnizians really took *vis viva* to be *the* force, i.e. a fully general concept able to explain *all* mechanical phenomena, we ought to see them likewise adopt Conservation of Vis Viva as the fundamental principle of mechanics, viz. the law underwriting *all* processes and interactions. But that is not what we find if we inspect their doctrines. They tend to relegate Conservation of Vis Viva to the periphery of their systems, where it governs just elastic collisions, which Leibniz had peddled as the paradigm of conserved *vis viva*.<sup>52</sup> In fact, the

50 Cf. Christian Wolff, "Principia dynamica", *Commentarii Academiae Scientiarum Imperialis Petropolitanae*, 1 (1728), 217–238 and Jacob Hermann, "De mensura virium", *Commentarii Academiae Scientiarum Imperialis Petropolitanae*, 1 (1728), 1–40.

51 Bernoulli's paper was the famous "Discours sur les loix de la communication du mouvement" (1724–6). See Johann Bernoulli, "Discours sur les loix de la communication du mouvement (1724–6)" in *Opera Omnia* 4 vols (Lausanne and Geneva: Marc-Michel Bousquet, 1742), iii, 7–106 and P.M. Harman, "Dynamics and intelligibility: Bernoulli and MacLaurin" in R.S. Woolhouse (ed.), *Metaphysics and Philosophy of Science in the Seventeenth and Eighteenth Centuries* (Dordrecht: Kluwer, 1988), 213–225; and also Mary Terrall, "Vis viva revisited", *History of Science*, xlii (2004), 189–209. On Wolff's influence on du Châtelet, his plans for her as a propagator of his doctrines in France, and her advocacy of *vis viva*, see Hans Droysen, "Die Marquise du Châtelet, Voltaire und der Philosoph Christian Wolff", *Zeitschrift für französische Sprache und Literatur*, 35 (1909), 226–248 and Heinrich Oster-tag, *Naturphilosophisches aus Wolffs Briefwechsel mit Manteuffel* (Leipzig: Quelle & Meyer, 1910).

52 I offer a conjecture, to help boost this explanation. Perhaps the Leibnizians did not mean to marginalize conservation of vis viva. Maybe the reason why they limited it to elastic collisions is that academic philosophers become unable or unwilling to keep up with cutting-edge research in dynamics. During the decades in which the Wolffians tout their doctrine, Bernoulli *père et fils* and Samuel König extend *vis viva* to phenomena well beyond the range of Leibniz's original illustration. The notion of live force is now applied to constrained motions, rigid bodies, fluid dynamics, and planetary orbits. See, for instance, Johann Bernoulli's "Theoremata selecta pro conservatione virium vivarum demonstranda et experimenta confirmanda excerpts ex epistolis datis ad filium Danielem", *Comm. Acad.*

Leibnizians give place of prominence to a quite different principle. Instead of Conservation of *Vis Viva*, they offer the equality of action and reaction as the grounding law of mechanics. Consider the following statements: Hermann: "In this force of inertia of matter is grounded the law of Nature whereby *to every action there is an equal and opposite reaction.*" Wolff: "There is no action in bodies without a reaction." Thümmig: "The actions and reactions of bodies in impact are equal." Stiebritz: "There is no action in bodies without a reaction." Gottsched: "The second law of motion says, the resistance of one body is always as great as the action of the other body, which strikes it." Winckler: "There is no action of bodies on other bodies without a contrary action." Baumeister: "The actions and reactions of bodies are equal." Hausen: "Therefore, the action of a body A on B is necessarily equal to the reaction of B against A." Hanov: "The action of one body (whereby the other's resistance is to be removed) is equal to the other body's reaction."<sup>53</sup> This looks nothing like the law of Leibnizian dynamics; if anything, it seems resoundingly Newtonian.

Now a different explanation suggests itself. Given their monolithic pleading for *vis viva* and unanimity that action equals reaction, the Leibnizians might have been aiming at a synthesis, however uneasy and strained, between their master's concept of force and Newton's *Lex Tertia*, so as to reflect the growing influence of Newtonian mechanics. However, this won't do either. The truth is that Leibniz influenced his followers so deeply that their dynamical laws too, not just their concepts of force, have a distant yet demonstrable origin in his thought. To demonstrate that this was the case, I will first uncover some deep differences between the Leibnizians' law of action and reaction and Newton's eponymous principle. I will then show that these differences are due to Leibniz.

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*Sci. Imp. Petropolitanae*, 2 (1727), 200–207 and his "De vera notione virium vivarum", *Acta Eruditorum* (1735) 210–230, Daniel Bernoulli's *Hydrodynamica, sive, De viribus et motibus fluidorum commentarii* (Strasbourg : Johann Reinhold Dulsecker and Johann Henrich Decker, 1738) and *Remarques sur le principe de la conservation des forces vives pris dans un sens général* (Berlin : s.n, 1748). Yet none of these achievements makes it into the Wolffians' foundations of mechanics. On *vis viva* in Johann's mechanics, see Villaggio, "General introduction", in *Die Werke von Johann I und Nicolaus II Bernoulli*. There is as yet no good study of *vis viva* in Daniel's natural philosophy.

53 See, in order: Hermann, *Phoronomia*, 3, § 12; Wolff, *Cosmologia Generalis*, 236, § 315; Thümmig, *Institutiones Philosophiae Wolfianae*, 94, § 61; Stiebritz, *Philosophiae Wolfianae Contractae*, 714, § 315; Gottsched, *Erste Gründe der gesamten Weltweisheit*, 257, § 370; J.H. Winckler, *Institutiones Philosophiae Wolfianae, Pars I Contemplativa* (Leipzig: Fritsch, 1735), 168, § 684; Friedrich Christian Baumeister, *Institutiones Metaphysicae: Methodo Wolfii Adornatae* (Wittenberg: Zimmermann, 1774), 299, § 449; Hausen, *De Reactione*, 2; Hanov, *Philosophiae naturalis sive physicae dogmaticae*, 103, § 156.

### 2.1 Action and Reaction, Newtonian and Leibnizian

As I demonstrated above, the Leibnizians are unanimous that action equals reaction. To decide how Newtonian their principle really is, if at all, we must inspect what they mean by these terms and to what processes they apply them.

The true sense of their principle emerges only as we examine their view of interaction. They see it as an essentially *asymmetric* encounter, a struggle or wrestling match between *unequals*: the “agent,” or stronger body, and the “patient,” or weaker one. The two play *heterogeneous* dynamical roles; the agent acts, whereas the patient “suffers” by resisting. Moreover, they play these roles by means of essentially heterogeneous forces: agents exert active force, *vis motrix*, and patients resist by a passive force, *vis inertiae*. In the encounter, the patient endeavors to oppose its displacement, whereas the agent seeks to break and overcome the patient’s opposition, so as to prevail over it. Lastly, the Leibnizians restrict this account to action by *contact*. Against this backdrop, the law of action and reaction means: in *collisions*, the *agent spends as much active force as the patient has passive force to resist it, neither more nor less*.

In every respect, this view is fundamentally at odds with that of Newton. In the *Principia*, an action is a *vis impressa*, and so is a reaction.<sup>54</sup> In consequence, the full meaning of Newton’s law of action and reaction *indispensably* includes the Second Law too, not just the Third. Without a concept of impressed force and the *Lex Secunda*, no assertion that action equals reaction can possibly be Newtonian, though it might sound so. Once we take account of the conceptual link between the Second and the Third Law, the correct meaning of Newton’s principle becomes: *for every vis impressa on a body, there exists another vis impressa on another body; these forces are equal and opposite, on the line between the bodies’ centers of mass*. Seen in this light, the Newtonian view of interaction is radically unlike that of the Leibnizians. For Newton, interactions are *symmetric*: both bodies act on each other just as much, neither is stronger or weaker. Bodies play *homogeneous* dynamical roles: both act, neither is a merely resisting patient. In fact, there is no distinction between agent and patient to be drawn in Newtonian mechanics. Moreover, bodies act on each other through *homogeneous* forces: both are *vires impressae* of the same kind. Lastly, for Newton, bodies can interact *at a distance* too, not just in contact.<sup>55</sup>

54 Cf. Newton, *The Principia*, 405: “Impressed force is the *action* exerted on a body to change its state either of resting or of moving uniformly straight forward. This force *consists solely in the action* and does not remain in the body after the action has ceased.” – my emphasis.

55 I am aware that, currently, there is some debate among Newton scholars about whether Newton himself thought action at a distance was intelligible. John Henry’s recent piece has persuaded me that Newton believed it was. See John Henry, “Gravity and

Again, the interpreter could claim that Hermann, Wolff and their acolytes sought to ground *Newton's* law of action and reaction – but then s/he must accept that they were collectively wrong; an unpalatable verdict. Or s/he could surmise that their principle differs from Newton's because it is not his. The latter view is correct.

Let us recall that in *Specimen Dynamicum*, his 1695 advertisement of an alleged new science, Leibniz had boasted that, based on his metaphysical insight into force, he had discovered new, “systematic laws of motion.” Among them was, “there is no action without a reaction.”<sup>56</sup> However, his dynamics is not based on impressed forces, so his law cannot be synonymous with and equivalent to Newton's *Lex Tertia*.<sup>57</sup> Rarely, Leibniz gave faint inklings of what he meant by “action” and “reaction.” A reconstruction from these meager pronouncements yields the following account. An action is the effect of a “force of acting” exerted by an “active” body upon a “resisting” one; its measure seems to be the amount of “new force” acquired by the resisting body as a result. In turn, a reaction is the effect of a “force of resistance” exerted by the resistant on the active. It seems to be equal to the “force” lost or spent by the active body in prevailing over the resistant. It is the view implied in *Specimen Dynamicum* 1, where, upon claiming that every action has a reaction, Leibniz explains, “no new force is produced without reducing an earlier one.”<sup>58</sup> It also transpires from a letter to *Journal des Savans*, where Leibniz proclaims that, over and

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*De Gravitatione*: the development of Newton's ideas on action at a distance”, *Studies in History and Philosophy of Science*, 42 (2011), 11–27.

- 56 “... *nulla actio est sine reactione*.” Leibniz's “Specimen” was in two parts, but only Part I was published, in the *Acta Eruditorum* for 1695. An English translation is in *G.W. Leibniz: Philosophical Texts*, 153–179. In his private notes on the *Principia*, Leibniz wrote down Newton's law of action and reaction. As he copied it, Leibniz commented that the law properly applies to the action of one body on the whole universe, and he rephrases Newton's principle as the equality of action and *passion*. “We shall say, therefore, that the action of a body and the passion of the whole universe are equal.” – see Domenico Bertoloni-Meli, “Leibniz's Excerpts from the *Principia Mathematica*”, *Annals of Science*, 45 (1988), 489. Ostensibly, Leibniz recasts Newton's *single* concept – recall that, for Newton, a reaction is just an action – into a duality of fundamentally distinct notions, viz. acting and “suffering.” Earlier, in his paper “Primary truths” of 1680–4, Leibniz had claimed, “*Every created individual substance exerts physical action and passion on all others*.” See G. W. Leibniz, *Leibniz: Philosophical Papers and Letters*, ed. and trs. L.E. Loemker (Dordrecht: Reidel, 1969), 269 and Bertoloni-Meli, “Leibniz's Excerpts from the *Principia Mathematica*” 502.
- 57 Here, I rely on some results I first communicated in Marius Stan, “Kant's third law of mechanics: the long shadow of Leibniz”, *Studies in History and Philosophy of Science*, 44 (2013), 493–504, Section 2.
- 58 Leibniz, “Specimen dynamicum,” *G.W. Leibniz: Philosophical Texts*, 159.

above extension, the essence of body consists in action and force; and concludes, “*everything that acts must suffer some reaction*, hence a body at rest will not be carried away by another in motion without changing some of the speed and direction of the moving body.”<sup>59</sup> The incoming body, he explains, slows down because the quiescent opposes its advance and effort to drag it along. And the ground of this resistance is:

the *natural inertia of bodies*, or that whereby matter resists motion, indeed whereby a body already moving could not carry with it another one at rest without being slowed down.<sup>60</sup>

This inertia of a body at rest requires an incoming body to “employ some force so as to set it in motion.”<sup>61</sup> He restates the point in *Nouveaux Essais*: “natural inertia” makes matter “resistant to motion, so that force must be expended to move a body.”<sup>62</sup> In *De Ipsa Natura*, he explicates that inertia as a “passive force of resistance” that resists the “*motive force inherent in bodies*” already in motion.<sup>63</sup>

In this account, action and reaction stem from *different kinds* of powers. As *De Ipsa Natura* makes clear, Leibniz implied that his action-reaction duality overlaps with the distinction active vs. passive *force*. An active force, viz. the “*motive force inherent in bodies that move*,” exerts the action; but reaction is grounded in a “force to resist,” hence passive. Sometimes, Leibniz assigns different *dynamic roles* to the two bodies in a basic interaction. One is said to be the “agent,” or *active* body, whereas the other is the “reagent,” the body that *reacts* through its “force of reacting.” Leibniz had already embraced this duality of roles by the early 1680s. He meant to make it public in an outline of a book he planned, tentatively entitled *Elementa Physicae*: “A body is extended, mobile, and resistant. That is, it is that which can act and suffer insofar as it is extended – *acting* when it is *in motion*, *suffering* when it *resists motion*.”<sup>64</sup>

59 Leibniz, “Lettre sur la question, si l’essence du corps consiste dans l’étendue”, *Journal des Savans*, 18 June 1691, in Erdmann, (ed.) *Leibnitii opera philosophica*, 113. All emphases in the original.

60 “Extrait d’une lettre pour soutenir ce qu’il y a de lui dans le *Journal des Savans* du 18 juin 1691,” also in *Journal des Savans*, 5 January 1693, *ibid.*, 114f. All emphases in the original.

61 *G.W. Leibniz: Philosophical Texts*, 159.

62 G.W. Leibniz, *Leibniz: New Essays on Human Understanding*, eds. and trs. Peter Remnant and Jonathan Bennett (Cambridge: Cambridge University Press, 1996), 123.

63 Both quotations are from “Nature itself; or, the inherent force and activity of created things” (1698), §§11–12, *G.W. Leibniz: Philosophical Texts*, 216f.

64 See *G.W. Leibniz: Philosophical Texts*, 215–216; Leibniz, “On the elements of natural science” (1682–4), in Leibniz, *Leibniz: Philosophical Papers and Letters*, 277; my emphasis. See also Stan, “Kant’s third law of mechanics: the long shadow of Leibniz”, Section 2.



It was on these Leibnizian clues that Hermann expanded in *Phoronomia*. He introduces one passive force, which “consists in that *Resistance* whereby it opposes any external force striving to change the bodies’ state of rest or motion.”<sup>65</sup> Hermann calls it “*Force of inertia*,” and explains what it does:

In this force of inertia of matter is grounded the law of Nature whereby *to every action there is an equal and opposite reaction*. For in every action there is a struggle [*luctatio*] between an agent body and a patient one, and without such struggle no action, properly so called, of the agent upon the patient can be conceived.<sup>66</sup>

Later, he clarifies what he really means by “action” and “reaction”:

The force of a body is not the action itself. For action is just the application of some force onto a subject capable of receiving it, or to which force can be applied. Hence, we must hold that the said force is applied to that body which resists, withstands, reacts.... Hence in all corporeal action there is a clash between an agent force and the resistance of the patient body, an application of the agent’s force onto the body receiving the action; that is, action itself is equal and contrary to the resistance of the patient, which is its reaction, because this resistance – or this force of inertia – by the patient body must be removed, so that the patient might be set in motion by the agent. ... Therefore, when we say that any action is equal and contrary to the reaction of the patient body, all we mean is: *in all corporeal action, as much of the agent’s forces is lost as it is gained by the body receiving the action*.<sup>67</sup>

Note how deeply *un-Newtonian* this model of interaction is: it amounts to an asymmetric clash between heterogeneous unequals, i.e. an agent exercising active force and a patient resisting by a passive force, its *vis inertiae*. Action equals reaction because the agent spends just as much force as it needs to break the patient’s resistance. And, unlike Newton, Hermann *denies* that force and action are *the same thing*. This account of interaction, far from being Newtonian, is deeply rooted in Leibnizian doctrine. Leibniz saw that, and applauded it openly, though from behind a thin veil of anonymity. He reviewed Hermann’s *Phoronomia* for the *Acta Eruditorum*, and sanctioned its view of interaction:

65 Hermann, *Phoronomia*, 3, § 11.

66 Ibid. § 12.

67 Ibid. 378–379.

“In the introduction, Hermann notes that the rule ‘to every action there is an equal reaction’ follows from the *inertia of matter*, first discovered by Kepler.”<sup>68</sup>

Wolff was privy to Leibniz’s authorship of the review. Moreover, he too had read *Phoronomia* closely, and sent his thoughts on it to Johann Bernoulli, whose brother Jacob had been Hermann’s patron, supplanted in 1705 by Leibniz, following Jacob’s untimely death.<sup>69</sup> Wolff first discourses on action and reaction in his early best seller, the “German Metaphysics,” or *Rational Thoughts on God, the World, the Soul of Man, and all Things generally*.<sup>70</sup> He claims that action and reaction are equal and opposite, then explicates each as a pressure [*Druck*] that one body exerts on the other. So far, this is all compatible with Newton’s mechanics (except for Wolff’s restricting his principle to collisions). Then Wolff takes a sharp sudden turn away from Newton, in the direction of Leibniz and Hermann. His occasion is an alleged possible objection to his principle above; in fact, the objection is lifted from Hermann’s *Phoronomia*.<sup>71</sup> It goes thus: if action equals reaction, the result must be rest, or static equilibrium; no motion should ever ensue. Wolff lets Hermann respond on his behalf:

Herr Professor Hermann has thoroughly answered it, in an appendix to his neat work on the motion of solid and fluid bodies. We must *not* take the force of a body to be *the same* as its action – as those who see

68 Cf. *Acta Eruditorum*, no. 1 (January 1716), 2; emphasis in the original. Reviews in the *Acta* were anonymous, but the balance of evidence points to Leibniz as the author of this item. Though letters between Leibniz and Wolff in October 1715 might suggest Wolff was behind it, in a letter of 10 May 1716 to Johann Bernoulli, Wolff writes, “Leibniz wrote the review [of *Phoronomia*].” See UB Basel Ms L I a 671, Nr. 12\*. I thank Dr Fritz Nagel (*Bernoulli Forschungsstelle*, Basel) for bringing this source to my attention and graciously providing me with a copy of Wolff’s letter.

69 Cf. Wolff’s letter to Bernoulli of 10 May 1716, cited above.

70 See Christian Wolff, *Vernünfftige Gedancken von Gott, der Welt, und der Seele des Menschen, auch allen Dingen überhaupt* (edition XI. Halle: Renger, 1751), 414–418, §§ 669–671. I cite from the 11th edition, printed at mid-century, to give an indication of the lasting influence of Wolff’s handbook, first published in 1719.

71 “Certain distinguished men have taken exception to the well-known law of nature, viz. that action is equal and contrary to reaction. They claim that motion would never follow from action. ... For, in the example that the famous Newton adduces, of a horse yoked to a stone, ... how exactly, they ask, will the horse be able to proceed and pull the stone, if the *force of the agent* is absorbed and entirely weakened by an equal and contrary resistance? However, this objection seems to arise from an equivocation between the words ‘force’ and ‘action,’ which must be distinguished carefully.” – Hermann, *Phoronomia*, 378. Hermann then goes on to explain that an action is just the partial force spent by an agent to prevail over a patient, which then it drags along, with its remaining, unspent force.

a problem in our law of action and reaction do. For a body does not act on another with all the *force that it has*, rather only to the extent that the other body *resists* it. The action of a body A on B consists in that A *breaks the resistance* of B. Then, when B resists it no longer, A will push it along without any effort, insofar as it lies in the way of its motion. ... So, [the horse yoked to the stone] does not act on it with all the force that it uses to progress, but only with that part that is enough to break the stone's resistance.<sup>72</sup>

This is recognizably Hermann's view above, *un-Newtonian* but endorsed by Leibniz: action and force are not identical, for action is the spending of active force by an agent, reaction is the resistance of a patient. And so Wolff sides with Leibniz not Newton.

A decade later, in *Cosmologia Generalis*, Wolff names Hermann as a pioneer in the project to derive "rules of motion," or kinematic laws of impact, from dynamical laws of force. The latter, he asserts, are based in metaphysics if rightly understood. So, "mathematicians merely assume these laws, but cannot prove them." It is the proper task of metaphysicians like him, Wolff concludes, to ground the dynamical laws.<sup>73</sup> Accordingly, Wolff goes on to prove, from metaphysical premises, a law of action and reaction. His concept of the latter is fully along the *un-Newtonian* lines of the Leibniz-Hermann model of interaction I outlined above. Stepwise, Wolff proves that, in every collision, there is an agent and a patient; that the agent acts by an active *vis motrix*, while the patient resists by its passive *vis inertiae*; that an action is an exercise of active force, whereas reaction stems from passive force; that action equals reaction, and is contrary to it; and that all physical action is by contact.<sup>74</sup>

His many disciples in Germany reprise all these ideas, often repeating them nearly verbatim, without looking into their remote Leibnizian origin or wondering just how compatible they really are with Newton's mechanics. The Wolffians adopt his dual taxonomy of force, which goes back to Leibniz and Hermann: "*Motive force* is that by which a body is able to move from its place. By the *force of inertia*, a body resists motion." They also condone the Leibniz-Hermann asymmetry of interaction: "If a [resting] body is struck by another in motion, it suffers: and the latter acts on it." Further, they too declare that to be a patient is to react, by resisting: "Thus if a thing resists, its resistance is a reaction by the patient against the agent." Metaphysicians swayed by his

72 Wolff, *Vernünfftige Gedancken von Gott*, 417, § 671; my emphasis.

73 Wolff, *Cosmologia Generalis*, 228, § 303.

74 *Ibid.* 234–238, §§ 313–319.

prestige spell out his link, taken from Leibniz, between passivity, reaction and resistance: “The reaction by which the patient steadily diminishes the agent’s action... is called *resistance*.” They keep his grounding of resistance in inertia: “That whereby a body resists motion is called *Force of inertia*, or *passive force*, a term common in Mathematics.”<sup>75</sup> They too endorse his claim, adopted from Hermann and approved by Leibniz, that his law of action and reaction follows from *vis inertiae*:

No corporeal action is without a contrary action. A body’s contrary action springs from its *force of inertia*. And by a contrary action, a body *resists motion*. But it resists motion by the force of inertia.<sup>76</sup>

Like him, they make it into the second of two laws of motion: “The second law of motion is: a body’s resistance is always as great as the action of that which collides with it.” And, they repeat his point that the laws must be shown “to have their ground in general cosmology.”<sup>77</sup> Lastly, they borrow his division between rules and laws, and the restriction to impact: “Rules of motion are those whereby motive force is modified in the collision of bodies. Laws of motion we call the general principles of the rules of motion.”<sup>78</sup> Even those, like Winckler, otherwise well disposed toward Newton, cannot quite resist the lure of the Leibniz-Hermann-Wolff model of interaction.<sup>79</sup> At times, the Wolffians deploy it polemically, as they defend their philosophical ancestor. L.M. Kahle, a Göttingen don, rushes in 1741 to prove Leibniz the better metaphysician, in response to Voltaire’s *La métaphysique de Neuton, ou Parallèle des sentimens de Neuton et de Leibnitz*.<sup>80</sup> Kahle concludes his *Vergleichung der Leibnitzischen*

75 See, in order: Winckler, *Institutiones Philosophiae Wolfianae*, 152, § 624; J.Ch. Gottsched, *Erste Gruende der gesamten Weltweisheit* (Leipzig: Breitkopf, 1735), 181, § 359; Hanov, *Philosophiae naturalis sive physicae dogmaticae*, 5, § 4; N. Burkhäuser, *Institutiones Metaphysicae, Pars 1: de Ente* (Würzburg: Goebhardt, 1771), § 624; Thümmig, *Institutiones Wolfianae*, 96 § 68.

76 Winckler, *Institutiones Philosophiae Wolfianae*, 168, §§ 684–685; my italics.

77 Gottsched, *Erste Gruende*, 183, 185; §§ 366, 370.

78 Stiebritz, *Philosophiae Wolfianae Contractae*, 712, §§ 302–303.

79 In a compendium Winckler, *Institutiones Mathematico-Physicae Experimentis Confirmatae* (Leipzig: Breitkopf, 1738), §§ 448–458: cites Newton’s *vis insita* whereby “a body perseveres in its state,” and describes *vis impressa* as an action exerted “on a body to change its state of motion.” But he remains unable to see that impressed force is the key to the Third Law; when he gets to action and reaction, he re-lapses into Wolffian talk of *vis inertiae*, agent vs. patient, resistance and reaction.

80 Ludwig Martin Kahle, *Vergleichung der Leibnitzischen und Neutonischen Metaphysik* (Göttingen: University of Göttingen press, 1741); “Too bad Kahle wrote it in German, so de

*und Newtonischen Metaphysik* with an attempt to show that Leibniz was superior to Newton in dynamics too, having proved *vis viva* to be the correct measure of force. Before he extolls that alleged success, he sets the stage with an admittedly Leibnizian account of force and interaction, as Hermann and Wolff had elaborated it and I explained it above. Kahle argues that “*every body resists motion*” by a “*resisting Force, or force of inertia.*” His proof is Leibniz’s old idea, restated in Hermann’s *Phoronomia*:

Suppose there was a Body that made no *resistance* to motion. Then there would be no *Reason* why this Body could not be set in Motion by the slightest impact there is. But, we know it is not true that all Bodies can be moved in the same way by the same force. Which proves the existence of the *Force of Inertia*.<sup>81</sup>

Next, Kahle explains active force and interactions:

When body B moves body A, which is at rest; or when B changes the motion of a body already moving – we say that body B is the *Agent*, and body A is the *Patient*.... Hence it is evident that every Body has by itself a *Motive force*, and that this *active Force* is enough to explain all the *Phenomena of Motion*, with no need to resort to God that He impress that Motion to bodies every time.... To clarify what we just said about *Active Force*, it must be noted here that it consists in a *continual effort to change place*.<sup>82</sup>

This way of thinking survives into the 1760s. Hanov in his monumental compendium of Wolffian physics gave a pithy statement of the Leibniz-Hermann-Wolff law of action and reaction, with all the un-Newtonian distinctions it presupposes:

The action of one body (whereby it must *remove the resistance* of another) is equal to the other body’s reaction. And the remaining force, if there is any, is then expended to produce motion *in the direction of the stronger*.

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Voltaire can’t read it,” wrote Wolff to Baron Manteuffel in 1741; cf. Ostertag, *Naturphilosophisches aus Wolffs Briefwechsel mit Manteuffel*, 62. However, Kahle’s screed was quickly translated into French as L.M. Kahle, *Examen d’un livre intitulé, La Metaphysique de Newton, par Mr. de Voltaire*, tr. Gaultier Saint-Blancard (The Hague: P. Gosse, 1744), enabling Voltaire to read it and retort with the cutting *Courte réponse aux long discours d’un docteur allemand*.

81 Kahle, *Examen*, 112; emphasis in the original.

82 Ibid. 112–113.

Unless the second body resists, there is *no reason* for the first to act on it. Neither is there *a reason* at hand why it should continue to act on it *longer* than the second resists. Nor is there *a reason* for that resistance unless the former body *seeks to act* on it, or strives to change its state.<sup>83</sup>

This account is very much at home in Wolff's *Cosmologia*, which it echoes resoundingly. But that account, as I explained above, is not Newton's idea of interaction, nor is it really compatible with the *Principia*. Among the Wolffians, Hanov alone makes a very feeble gesture at compatibility with Newton's theory. He claims that the *Lex Secunda* is, after all, part of Wolff's system:

This second law of Newton's is a third law in the system of others, and says as follows: if a body pushing another changes the latter's state in whatever way, it too will undergo a change in motion, in the other direction. See Wolff's *Cosmologia*, § 350.<sup>84</sup>

Bent on giving his mentor all the credit, Hanov is more eager than careful. It is not true that Wolff's above proposition is equivalent to Newton's Second Law. Wolff restricts it to collisions, or action by contact, whereas the *Lex Secunda* applies to action-at-a-distance just as well. The more serious problem, however, is that Wolff's law is empirically empty. The terms "change of state," "change in motion" and "direction" are undefined. In contrast, for Newton they have all very precise meanings: "change in motion" means the time-integral of instantaneous changes in momentum reckoned in the *direction of the force*, which is *always* along the straight line *between* the interacting bodies' *centers of mass*. Without these specifications, the Wolffians' principle is useless. It is not that they forgot to add these details. Rather, their un-Newtonian conception of force makes it virtually impossible for them to do so. This is because the Wolffians' *vires motrices* are always in the directions of the bodies' *motions*. But, the direction of Newtonian forces is generally *different*, viz. on the line of centers, as I have just explained. This lacuna will plague the Wolffians at another juncture, which I detail in Section 3.2. In the meantime, I conclude that, despite Hanov's reassurances, the Wolffians' two metaphysical laws of motion are not really equivalent to Newton's dynamical principles. They are not even *compatible* with them.

However, while my results hold for the Wolffians, I do not mean to claim that all post-Leibnizians were reliably faithful to Leibniz on the nature of

83 Hanov, *Philosophiae naturalis sive physicae dogmaticae*, 103–104; my emphasis.

84 Ibid. 104.

force and its laws. Bilfinger is a peculiar case, in this respect. (And so is Israel Gottlieb Canz, who followed Bilfinger closely on this count.)<sup>85</sup> In *Dilucidationes Philosophicae*, Bilfinger puts forward no less than six laws of motion: (i) the equality of action and reaction; (ii) the Leibnizian principle that “the full effect is equipollent to the forces of the entire cause”; (iii) the same “quantity of force is conserved in the universe”; (iv) the Law of Inertia; (v) the Parallelogram of Forces, and (vi) Leibniz’s Law of Continuity.<sup>86</sup> This too *seems* an attempted synthesis between Leibniz and Newton. Bilfinger’s laws II, III and VI are recognizably Leibnizian, whereas I, IV, and V are in the *Principia*, whether as axioms or theorems.<sup>87</sup> But that is just an appearance. In fact, what Bilfinger achieves is a mere juxtaposition, not a synthesis. There are no genuine explanatory or inferential connections holding his six laws together. Rather, they form two distinct groups of three laws each, merely collocated instead of unified. This is for two reasons. First, his laws codify two distinct notions of force: Leibnizian *vis viva* is operative in laws II and III, whereas laws I and V are Newtonian impressed forces.<sup>88</sup> But these forces differ in nature, location, and measure. They cannot both be part of the same dynamics and equally fundamental. Second, Bilfinger’s six laws, as foundations, over-determine mechanical theory. Laws I, IV, and V are enough, *by themselves*, to ground an elementary mechanics of free particles; Newton had already carried out much of it. And so are the laws II, III, and IV, if we read II, as Leibniz often meant, as a work-energy principle. Thus, Bilfinger’s laws ground mechanics twice – once too many. It is unclear whether he realized it. What is clear though, is that his is no synthesis of Leibniz and Newton. If anything, Bilfinger’s laws are entirely Leibnizian, provided we take into account the full foundations of Leibnizian dynamics, so as to include Leibniz’s *vires mortuae*. The “dead forces” of *Specimen Dynamicum* do obey the parallelogram rule and the action-reaction equality as expressed in Bilfinger’s law I and IV, respectively.

In conclusion, the Leibnizians largely turned down Newton’s dynamical laws and the concept of force they express, in favor of notions and principles

85 See I.G. Canz, *Meditationes philosophicae. Pars II: Cosmologia, Psychologia* (Tübingen: Cotta, 1750), 655–663, §§ 761–772.

86 See Bilfinger, *Dilucidationes philosophicas de Deo*, 175–181.

87 Leibniz’s metaphysico-dynamical principle “there is neither more nor less *potentia* in an effect than in its cause” was known to the public from his “Specimen Dynamicum I”; see *G.W. Leibniz: Philosophical Texts*, 162. The Parallelogram of Forces is Newton’s Corollary I to the Laws of Motion, in his *Principia*; see Newton, *The Principia*, 417f.

88 For instance, the parallelogram law does *not* hold for Leibnizian *vis viva*. But, together with the Law of Inertia, it is empirically equivalent to Newton’s Second Law for dynamical forces.

ultimately traceable to Leibniz. This explains the pervasive absence of *vis impressa* from their doctrines, and should make us wary of scholarly claims that Wolff served as a conduit for Newtonian foundations in Germany.<sup>89</sup> Rather, as I have shown, the Wolffians' retention of Leibnizian foundational ideas confirms, from the particular vantage point I have adopted here, the "hesitant" and "slow" acceptance of Newtonianism that others have seen in early Enlightenment Germany.<sup>90</sup>

### 3 Unrequited Attraction

In a late piece, wisely left unpublished, Leibniz had catalogued and railed against a long list of alleged errors and heresies in the natural philosophy of his time. He denounced action-at-a-distance forces in especially stark terms: "barbarism in physics," he called them, warning that their use "may lead us back to the kingdom of darkness" populated by Scholastic occult qualities.<sup>91</sup> Thus Leibniz made clear in no uncertain terms where he stood in regard to the new doctrine of "a Universal Tendency of Matter to Matter," as disciples had called Newtonian gravity.<sup>92</sup> In opposing action-at-a-distance, Leibniz applied arguments too, not just withering scorn. Those arguments have been evaluated in our times, and generally found wanting. Scholars have concluded that, rather than making a conclusive case against Newton or else begrudgingly accept universal gravitation, Leibniz chose to dig in his heels.<sup>93</sup>

89 Cf. Martina Lorenz, "Der Beitrag Christian Wolffs zur Rezeption von Grundprinzipien der Mechanik Newtons in Deutschland zu Beginn des 18. Jahrhunderts", *Historia Scientiarum*, 31 (1986), 87–100, 100 and Dorothea Goetz, "Der naturwissenschaftliche Aspekt der deutschen Aufklärung an den Universitäten im 18. Jahrhunderts" (Diss. 1973), Potsdam, 137.

90 Cf. Jonathan Israel, *Radical Enlightenment* (Oxford: Oxford University Press, 2001), 523, 557.

91 Leibniz, "Against Barbaric Physics" (1710–16). See Leibniz, *G.W. Leibniz. Philosophical Essays*, 314.

92 John Freind, *Chymical Lectures: in which almost all the Operations of Chymistry are reduced to their True Principles, and the Laws of Nature. To which is added, an Appendix, containing the Account given of this Book in the Lipsick Acts, together with the Author's Remarks thereon*, trans. J.M. (London: William and Bowyer, 1712), 176.

93 See the limpid, helpful Gregory Brown, "'Is the Logic in London Different from the Logic in Hanover?': Some Methodological Issues in Leibniz's Dispute with the Newtonians over the Cause of Gravity", in Pauline Phemister and Stuart Brown (eds.), *Leibniz and the English-Speaking World* (Dordrecht: Springer, 2007), 145–162.



The same hostile animus imbued Wolff's feelings about gravity and forces at a distance in general. In a letter from 1710 to Leibniz Wolff had recounted with dismay that Newton in the *Queries* of his 1706 *Optice* had seriously entertained the prospect of grounding all physics in particles endowed with action-at-a-distance forces.<sup>94</sup> At the same time (anonymously, as a reviewer for the *Acta Eruditorum*), Wolff began to push back against these forces, rebuking their use in physics and chemistry by John Keill and John Freind.<sup>95</sup> When safely out of public view, on occasion Wolff's high-minded opposition to British Newtonianism dissolved into low gossip. "I learned from an Englishman who visited me recently," he tells Leibniz, about Keill's "depraved mores, who takes youths entrusted to his care to alehouses and brothels, where he spends a good deal of money in drunkenness and fornication."<sup>96</sup> The early experience of sharply polemical encounters with Newton's disciples soured his mood for decades. In the 1740s, he would still fulminate about them: "The Newtonians are arrogant creatures that despise anyone who does not sing their tune. And yet, no one who really understands philosophy could grant that their so-called Newtonian philosophy is one."<sup>97</sup>

Still, Wolff claimed to reject action-at-a-distance from principled reasons, not mere antipathy and so we must give him the benefit of the doubt. However, his case against distant forces is multifaceted, and it therefore requires patient untangling. It turns out that there are four strands of argument in his attack. I reconstruct them one by one below, and try to determine how strong they are.

### 3.1 Occult

Several times, Wolff denounces attractive force as an occult quality: "no one has claimed that gravity is a primitive force, save for those who want to bring back occult qualities in Physics," he accuses Keill publicly, in 1710.<sup>98</sup> The charge rings old, but he gives it a new twist, unbeknown to his opponents. Wolff does not mean "occult" as *insensilis*, or not manifest, as the Scholastics had it and

94 Wolff to Leibniz, 17 August 1710, in Gerhardt (ed.), *Briefwechsel zwischen Leibniz und Chr. Wolff*, 124ff.

95 I thank Professor Principe for kindly sharing his paper on Wolff's polemic with Keill and Freind with me, and for drawing my attention to Wolff's review of Freind's *Praelectiones Chymicae*.

96 Wolff to Leibniz, 1 October 1715, in Gerhardt (ed.), *Briefwechsel zwischen Leibniz und Chr. Wolff*, 174.

97 Wolff to Manteuffel, 6 January 1741, in Ostertag, *Naturphilosophisches aus Wolffs Briefwechsel mit Manteuffel*, 62.

98 See Christian Wolff, "Responsio at epistolam Viri Clarissimi Johannis Keill", *Acta Eruditorum*, February (1710), 78–80; 79.

mechanical philosophers condemned it.<sup>99</sup> Rather, he defines occult qualities, as “those for which there can be no reason except God’s will.”<sup>100</sup> Allegedly, they are occult because “no reason can be given for them, save by saying that God willed them.”<sup>101</sup> Hence, “they lack a sufficient reason why they inhere in the subject or even why they can inhere.”<sup>102</sup> I explore below what he means when he says that gravity lacks a sufficient reason. Still, we may ask Wolff, why must we ban occult qualities in *his*, non-traditional sense, as corporeal powers whose sole explanation is God’s will? It might make sense to forbid them if they were *completely* unintelligible; after all, science must be explanatory, and the incomprehensible explains nothing. But that is not Wolff’s point. He really means that “occult qualities” are inexplicable *mechanically*, i.e. as the result of contact action. (More on this, below). Yet that is less than fully unintelligible. By his own admission, they can be understood – as the direct product of divine volition. He fails to provide an argument as to why such products should not be considered part of natural philosophy.

Moreover, since Newton had proven sufficiently that distant gravity is a *real cause* of kinematic phenomena of mutual attraction (though he admittedly could not describe the cause of *that* cause), Wolff must explain why real causes, though “occult” in his peculiar sense, must not be allowed in natural philosophy. This is the gist of a common objection, voiced separately by Newton against Leibniz, and by Freind against Wolff. To Leibniz, who had dismissed Newton’s gravity as a “perpetual miracle,” Newton had planned in 1712 to answer, *Anglico sermone*:

But certainly God could create planets that should move round of themselves without any other cause than gravity that should prevent

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99 See the definitions of *qualitas occulta* in Étienne Chauvin, *Lexicon philosophicum* (Louvain: Francis Halma, 1713), 546: “An occult quality is a hidden or latent power whereby natural things act on or suffer from something, and whose a priori reason cannot be given, inasmuch as it emanates directly from the substantial form.” See also Goclenius, who classifies occult qualities into two kinds: sympathy, “which is a certain natural conspiracy, on account of some peculiar and hidden affinity,” and antipathy, “a natural enmity and dissension between physical things.” – Rudolph Goclenius, *Lexicon philosophicum* (Frankfurt: Typis viduae Matthiae Beckeri, impensis Petri Musculi & Ruperti Pistorij 1613), 929f.

100 Cf. his *Theologia naturalis: methodo scientifica pertractata. Pars prior* (editio nova. Verona: Dionysius Ramanzini, 1738), 215, § 436.

101 Ibid.

102 Ibid.

their removing through the tangent. For gravity without a miracle may keep the planets in.<sup>103</sup>

Freind made the same point more amply in an aggrieved reply to Wolff's hostile review of his *Praelectiones chymicae*:

But since the Motions of all the Bodies in the Universe do plainly evince the Existence of such a Principle [viz. a force acting at a distance], if the [Leibnizians] are of Opinion, that it is neither Essential to Matter, not to be Mechanically accounted for, I cannot think it will be either *Absurd* or *Unphilosophical* to assert, that it *depends solely on the Will of the Omnipotent Creator*: And that it is an universal Law, by which God Directs and Governs the *Universe*.<sup>104</sup>

Thus, without a further argument that divine volition must be expunged from the foundations of science, Wolff's case against action-at-a-distance seems premature.

There is another reason why his attack fails. This time, the problem comes from within, and is rather severe. In regard to physical interactions, Wolff uses the term "sufficient reason" in a strong sense. In such processes,

in every given case, there should be a a reason why the motion is *thus* rather than *otherwise*. Indeed, since with regard to motion we consider primarily its speed and *direction*, hence there should be a reason why the mobile's speed is this rather than that; and why it progresses toward *this* rather than *that direction*. Therefore, seeing as in any given case the motion must be determined according to the said reason: in the observable world, motion has its rules.<sup>105</sup>

Those "rules," to be sure, are the laws of impact. Consider what his thesis above entails. Colliding bodies do not *just* change their motions; rather, they change them to a determinate *degree* and in determinate *directions*. His strict

103 See Sir Isaac Newton, *Philosophical Writings*, ed. Andrew Janiak (Cambridge: Cambridge University Press, 2004), 117.

104 Freind, *Chymical Lectures*, 188.

105 Wolff, *Cosmologia Generalis*, 66–67, § 72; my italics; see also his "Ontologia" in Wolff, *Philosophia prima, sive Ontologia; methodo scientifica pertractata* (editio nova. Frankfurt: Renger, 1736), 653, § 883.

conception demands that these directions must have a sufficient reason too. And yet Wolff cannot account for them. In a 2-body collision, the change of motion is on the *line of centers*, i.e. the straight line between the bodies' center of mass. Then Wolff must supply a sufficient reason why bodies act along *this* line rather than any other. But he offers none, and none can be had from his doctrine. According to it, when two bodies collide, at least one is endowed with force of motion, *vis motrix*, whereby it strives to change place and so act on other bodies in its path. The direction of this Wolffian force is the line of the body's *motion*, or velocity. But, in impact, the line of motion and the line of action are *not* the same, in general.<sup>106</sup> A real sufficient reason would explain *all* features of action by contact, including the privileged line of centers above. Within Wolff's metaphysical dynamics, this aspect – that colliding bodies act *solely* on their line of centers – is just a brute fact. He has nothing to offer as an explanation for it.<sup>107</sup> And so, Wolff must face a grim reality: by his own lights, it follows that collision ultimately lacks a sufficient reason. How then can he object to Newtonian distant interactions?

It is for this reason that Wolff's dismissal of action-at-a-distance does not succeed. Still, he utilises other resources to continue his attack. A dual strategy is to counter Newton's *a posteriori* inference to universal attraction with an *a priori* argument that gravity is *not* real and *not* really intelligible. The seeds for both these moves may be found in Wolff's contention in *Ontologia* that "nothing *is* without a sufficient reason why it should *be* rather than *not be*."<sup>108</sup> This he explicates as, "if something is posited to be, one must also posit something

106 For instance, in oblique impact: the two bodies move at an angle – their velocities, and so their *vires motrices*, intersect – but the interaction is along the *straight* line between them. Wolff gives oblique impact exactly one paragraph in *Cosmologia Generalis*. All he does there is define it, utterly unconcerned by how his dynamics might be able to handle it, if at all. See Wolff, *Cosmologia Generalis*, 244, § 331.

107 Ironically, there is a way to save Wolff from his predicament, if only he would accept help from Newton. In Wolff's doctrine, bodies are physical continua, with mass continuously distributed throughout the volume of the body. As a result of this conception, it is entirely contingent whether their actions on each other will be along the line of centers. Some forces, e.g. viscosity, act *perpendicular* to this line. However, if Wolff were to conceive of bodies as composed of Newtonian atoms endowed with forces of repulsion and attraction – or, alternately, as point-sized Kantian physical monads – his problem would disappear. These forces induce conservative potentials, subject to power laws; so, their strength is always a function of the straight-line distance to the centers from which they emanate, and their action is always along the line to those centers. But, they are all distance forces, hence unacceptable to Wolff.

108 Wolff, *Philosophia prima, sive Ontologia*, 28, § 70.

else from which we can *understand* why the former should be rather than not be."<sup>109</sup> So, let us next inspect why he thinks that action-at-a-distance cannot exist and cannot be understood.

### 3.2 Impossible

With no sufficient reason for it, Wolff contends, "action-at-a-distance is impossible," a claim he makes three times, by three routes. That shows him confident, but does he have a case? He deploys his first argument as follows: all action between bodies is by contact; but action-at-a-distance is without contact; ergo, "action-at-a-distance is impossible."<sup>110</sup> The point would be conclusive, if only the first premise were true: "A body cannot act on another without mutual contact."<sup>111</sup> But is this true? Wolff infers it from another premise, viz. that "a body does not act on another, unless it presses on it."<sup>112</sup> And *this*, in turn, he justifies from his metaphysical dynamics. In his doctrine, a body acts through active force alone, which it has only if it moves. But, he contends, if the body moves freely, "it has no reason to act on another body, as long as the latter does not impede its motion."<sup>113</sup> Now, any two distant bodies can move freely; because of their separation, none impedes the other. Therefore, neither can act on the other.

Unfortunately, this begs a question since it rests on a notion by fiat: the concept of a force that *by definition* acts only in impact. Wolff's *vis motrix* is a moving body's power to dislodge other bodies in its path *upon contact* with them; and his *vis passiva seu inertiae* is a force to resist dislodgement through contact. Then no wonder that bodies cannot interact at a distance if their powers are all contact forces. Unfortunately, Wolff never proves the latter: he has no argument that *all and only* forces are contact interactions. To do so, he would first need to prove that all action is by contact – the very *quod erat demonstrandum*. Thus, this objection to Newton fails.

Wolff tries a second approach: by the Law of Inertia, no change of motion in a body A occurs except from an external cause, or some body B. But, when B changes A's state, it acts on it; in turn, A reacts to B. So, the bodies are in a state of mutual action and reaction. And, that is a state of "conflict." Ergo, no physical action occurs "except by conflict."<sup>114</sup> Wolff clearly wants "conflict" to

109 Ibid. my italics.

110 Wolff, *Cosmologia Generalis*, 240, § 323.

111 Ibid. 239, § 320.

112 Ibid. 239–240, § 321, 320.

113 Ibid. 240, § 321.

114 Ibid. 241, § 325.

denote impact, or collision, as the drawing attached to his argument makes clear.<sup>115</sup> However, his definition of “conflict” is simply, “A conflict of bodies is that state in which they act on and react to each other.”<sup>116</sup> Insert this definition into Wolff’s objection above, and it becomes toothless: “No change of motion occurs except when bodies act on and react to each other.”<sup>117</sup> But this is wholly compatible with action at a distance: Newtonian gravity is an interaction, so whenever two distant bodies gravitate mutually, they act on and react to one another – just as Wolff demands.

To give his objection the strength it needs, Wolff needs to prove, in addition, that bodies act and react *solely* if they touch. He does try to prove that, after the fact, as it were: “There is no conflict between bodies without an impact of one into the other, or without their mutual contact.”<sup>118</sup> However, his argument for this crucial premise is only his first argument, which is entirely circular, and thus worthless. His argument that no physical action occurs without “conflict” by contact is thus seriously undermined and, as an objection to Newton does not reach its target.

His third and final argument, offered in *Cosmologia*, I construe as follows. In a body, a change is a variation of modes. All such variation of modes occurs solely “by motion.” No change in a body is induced except by another, *contiguous* body. Hence, a body A can suffer no change except from another body B that *moves* toward A and *touches* it. So, no body separated by a distance from another can suffer any change on account of the latter.<sup>119</sup> As an argument this too cannot defeat Newton. Wolff wants his second premise, viz. that all change occurs by motion, to mean: *all change of state in a body is always caused by another body moving against it*. Of course, he needs an argument for it. In response, he refers the reader to his *Ontologia*, where he claims to have proved it. The premise is there, to be sure, but it has a different meaning: *all change in a body is or reduces to a relative motion between its parts*.<sup>120</sup> Wolff is thus guilty of equivocation, which renders his key premise void, and thereby his entire third

115 In *Cosmologia*, Wolff inserts two drawings to illustrate his point; both depict a body, B suspended from a thread, colliding with A at rest in its path as B falls from a height.

116 Ibid. 241 § 324.

117 Ibid. 241, § 325.

118 Ibid. 243, § 327.

119 Ibid. 112, § 128. In this argument, premise (2) is first spelled out and defended by Wolff in his “Ontologia”, see Wolff, *Philosophia prima, sive Ontologia*, 504–505, § 667.

120 Wolff, *Philosophia prima, sive Ontologia*, 504–505, § 667: “In a composite, no change can occur except by motion. ... In a composite no intrinsic change indeed can occur except in respect to figure, size, and the position of its parts. Hence, no change can occur except by motion.”

argument fallacious. And so, his accusation that action-at-a-distance is impossible has ultimately nothing in the way of a solid foundation.

### 3.3 Unintelligible

At various junctures, Wolff castigates as unintelligible all distant forces, be they gravity or those posited by British Newtonians: “He who admits, as a cause of phenomena, an attractive force whose nature is *inexplicable*, ... admits it as an occult quality, and so falls back into the empty repetitions of the Scholastics.”<sup>121</sup> And:

From the fact that a body *A* inheres in a place and is endowed with active force we cannot understand at all [*minime intelligitur*] why a body *B*, removed from *A*, should move. ... It is not enough to posit in *A* some force such that it can act at a distance. For we ought to posit only things that can be explained intelligibly.<sup>122</sup>

Wolff's complaint, if true, would be damaging – unintelligible principles cannot yield real understanding, so they are not explanatory and hence do not belong in science. To assess his charge, however, we must look into its exact content. Allegedly, distant force is unintelligible because it is not “produced Mechanically,” as Freind saw right away. Namely, Wolff complains that attractive forces are unintelligible unless reduced to mechanism: kinematic patterns of *apparent* attraction or repulsion between distant bodies must be shown to be products of underlying contact dynamics: matter touching matter. But what exactly is the charge? Why *is* non-mechanical gravity unintelligible? On this point, Wolff is as evasive as he is forceful. The interpreter must reconstruct his views in absentia before s/he may evaluate them. His objection has two possible readings. In one sense, he might mean that distant actions are unintelligible because they lack a sufficient reason in his technical sense described above. That is, such actions cannot be understood because (1) they are not produced mechanically, rather (2) can only be brute products of God's will.<sup>123</sup> Thus analyzed, Wolff's charge is entirely parasitic upon his two objections above, which

<sup>121</sup> Wolff in his rebuttal of Freind's response to his critical review of *Praelectiones Chymicae*; see Wolff, *Meletemata mathematico-philosophica cum erudito Orbe Literarum commercio communicata* (Halle: Renger, 1755), no. XX, 56; my emphasis.

<sup>122</sup> Wolff, *Cosmologia Generalis*, 241, § 323.

<sup>123</sup> If I am not mistaken, that is also how John Heilbron understood Wolff's accusation. See J.L. Heilbron, *Electricity in the 17th and 18th Centuries: A Study of Early Modern Physics* (Berkeley: University of California Press, 1979), 45.

I have shown to fall short of their mark. His charge of unintelligibility fails in the same manner.

There is a second way to spell out Wolff's objection. He might be taking action by contact to be intelligible *simpliciter*. In the mechanical philosophy, which Wolff in some respects follows, it was a dogma that contact action is eminently intelligible. But is it? Wolff's very doctrine subverts that dogma. Consider that though action by contact seems transparent, some worried that it might be metaphysically absurd. In collision, one body appears to gain as much "motion" as the other loses; as Jacques Rohault and Locke had put it, impact seems to be "communication of motion"... But, Leibniz warned, that is metaphysically *inconceivable*: motion is a property of a moving body, and properties do not migrate from one substance into another. A century after Leibniz, Kant too cautioned against that mistaken thought.<sup>124</sup> So, if transfer of motion is inconceivable, how exactly can Wolff argue that impact is intelligible?

What is more, Wolff knew about this grave problem. Having seen Leibniz's 1686 refutation of the Cartesian doctrine of conserved "motion," Wolff went on to embrace his mentor's view that "force" is the real conserved quantity. Thereby he came to believe that impact amounts to communication of force not motion. Leibniz, however, disabused him of that idea; being a property of body, force is no more communicable than motion is: "you ought to know that forces do not cross from one body into another."<sup>125</sup> This correction, though, renews the challenge: if collision is not intelligible as a transfer (whether of motion or of force), what *is* it, then? How should we understand it?

Two accounts were available at the time. One is proto-occasionalist, and it explains collision by having God destroy some motion in a body and create some in the other as they collide.<sup>126</sup> The other is Leibnizian pre-established

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124 Leibniz, "Primae Veritates" (1680–4): "It can be said that, speaking with metaphysical rigor, no created substance exerts a metaphysical action or influence upon another. For to say nothing of the fact that *it cannot be explained how anything can pass over from one thing into the substance of another*, it has already been shown that all the future states of each thing follow from its own concept." – Leibniz, *Leibniz: Philosophical Papers and Letters*, 269; my italics. Kant makes the same point – that impact cannot be a real communication of motion, which is metaphysically absurd – in his *Metaphysical Foundations of Natural Science* (1786).

125 "...vires non transire de corpore in corpus," Leibniz explained in response to a letter from Wolff dated 31 December 1710. See Gerhardt (ed.), *Briefwechsel zwischen Leibniz und Chr. Wolff*, 131.

126 The two motions, created and destroyed, respectively, would have to be equal and opposite quantities of motion, so that conservation of momentum is ensured.



harmony of bodies: neither body acts on the other in a collision. Rather, each body acts on itself, and their self-actions are synchronized.<sup>127</sup> Both options avoid the hard task of proving that contact action is intelligible by denying the simple expedient of denying that there is any.<sup>128</sup> But Wolff does not avail himself of either. The first, or divinely-mediated impact, he rejects as an occult quality on a par with Newtonian attraction:

A likewise occult quality is the motion trans-created from one subject into another, as the Cartesians have it: viz., one whose sole reason given is that God willed motion ceased or annihilated in one subject and commenced or borne out of nothing in the other.<sup>129</sup>

As to the second, namely synchronized self-action, it seems that Wolff chose not to follow Leibniz on this count. True, Wolff did inherit pre-established harmony as an account of *mind-body* causality.<sup>130</sup> However, it is very doubtful that it was also his official view of *body-body* interaction. This is because of Wolff's conception of his "elements," the basic substances from which material bodies arise in his system. Like Leibniz's monads, elements are partless and have forces, active and passive. But Wolff diverges from Leibniz in two crucial respects. First, his elements have external states too, not just internal ones. That is radically at odds with Leibniz, for whom monads are "world-apart," i.e. have

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127 That pre-established harmony obtains between monads is conventional wisdom; whether "aggregates," or non-substantial bodies, interact has been debated, though not enough. Gregory Brown, "Is There a Pre-Established Harmony of Aggregates in the Leibnizian Dynamics, or Do Non-Substantial Bodies Interact?" *Journal of the History of Philosophy*, 30 (1992), 53–75, argues that, in metaphysical rigor, Leibnizian bodies do not interact. I follow him.

128 This, of course, raises other problems, maybe just as severe. One is: how to prove that action-at-a-distance is still unintelligible – after all, if God can create and destroy motion in collision, so can he in orbiting planets; and if bodies in contact self-act, why shouldn't bodies at a distance from each other? Another is: to explain what makes these two models intelligible. Presumably, God's action on bodies is unproblematic and intelligible *sine clausa*. And Leibnizian self-action in bodies might be comprehensible by analogy with one's mind causing itself to move to a new state.

129 Wolff, *Theologia naturalis*, 215, § 436.

130 Still, Eric Watkins has shown that as the years went by Wolff pushed pre-established harmony away toward the edges of his system, perhaps so as to avoid a repeat of the anti-Leibnizian backlash that had him banished from Prussia in the 1720s; cf. Eric Watkins, "From Pre-established Harmony to Physical Influx: Leibniz's Reception in Eighteenth-Century Germany", *Perspectives on Science*, 6 (1998), 136–203.

only internal states.<sup>131</sup> Secondly, Wolff strongly suggests that his elements act on, and suffer by, other elements.<sup>132</sup> This, again, is radically unlike Leibnizian monads, which are capable only of self-action. It is thus far from clear that Wolff took collision to reduce to harmonious synchronized self-actions, as Leibniz had explained it. How, then, should we understand it?

Wolff cannot reach for the Cartesian option either, namely claiming that action by contact is intelligible because we may grasp it by a clear and distinct perception. When we recall that, in Wolff's doctrine, the dynamics of impact rests on a clash between *vis motrix* and *vis inertiae*, we can see that he is stymied for internal reasons, for he declares of both these forces that we cannot but have *confused* notions, never clear and distinct ones. "*Vis motrix* is a phenomenon," that is to say, "we can perceive *vis motrix* in bodies only confusedly." "*Vis inertiae* is a phenomenon," because "we perceive *vis inertiae* merely confusedly."<sup>133</sup> The reason is that, to grasp these *vires* clearly and distinctly, we

131 Leibnizian monads cannot have external states in principle, as they lack real external relations, the basis for such states. "According to [Leibniz's] world-apart thesis, no state of any substance has as a real cause some state of some other created substance.... [W]hat we have is an affirmation of the ubiquity of real *intra*-substantial causality and the *denial* of inter-substantial causality." – Robert C. Sleigh Jr., "Leibniz on Malebranche on Causality" in J.A. Cover and Mark Kulstad (eds.) *Central Themes in Early Modern Philosophy* (Indianapolis: Hackett, 1990), 161–193; 162; my emphasis.

132 These claims from his "theory of simple beings," or genuine substances, in his 1730 "Ontologia," are illustrative: *Philosophia Prima, Sive Ontologia, Methodo Scientifica Pertractata, Qua Omnis Cognitionis Humanae Principia Continentur* (Frankfurt: Renger, 1730): "if the state of a thing consists in extrinsic mutables, which are relations of the thing to another, it is called *external state*" (531, § 706); "[a thing's] *external state changes if its relations to other things do not remain the same*" (533, § 709); "a *Passion* is a change of state whose reason is contained outside the subject that changes state" (537, § 714); "a *Patient* is the subject of a passion" (541, § 720); "if a being has force, its state changes continually, unless its force is being resisted" (545, § 729); "if [a substance] *A suffers on account of* [a substance] *B, then B acts on A*" (584, § 775); "the patient, insofar as it suffers, depends on another" (634, § 853); "force is the principle of actions" (646, § 869), and "force is the principle of changes" (646, § 870); "a principle is external if it exists outside the principiated" (652, § 880); "a cause is a principle on which depends the existence, or actuality, of another being distinct from it – both insofar as the latter exists and as it exists as such a thing" (652, § 881); "a cause is internal if it is an internal principle, whereas it is an external cause if it is an external principle" (§ 882); "the caused, which depends on the efficient cause, is called the effect; thus, an effect is a being, the reason for whose existence, or actuality, is the action of another being" (654, § 886); all emphases in the original.

133 Wolff, *Cosmologia Generalis*, 223–225. On the meaning of these "phenomena" in Wolff, see Mariano Campo, *Cristiano Wolff e il razionalismo precritico*, 2 vols. (Milan: Vita e Pensiero, 1939), i, 218–222.

would need a fully transparent account of how these forces of *bodies* arise out of the powers, active and passive, of “*elements*,” his physical version of Leibnizian monads. And that is just what Wolff confesses himself unable to offer. First, he is agnostic about the exact nature of the elements’ powers.<sup>134</sup> Secondly, he acknowledges that he cannot explain clearly how the force of a body results from the forces of its elements:

How exactly the force of composite substances results from the forces of the simples that compose it cannot yet be taught here ... From that which we have demonstrated so far, nothing more distinct can be taught about the forces of simple substances.<sup>135</sup>

If that is true, ipso facto our grasp of corporeal force is inexorably confused. It then follows that we cannot have a clear and distinct perception of collision, in which these forces are paradigmatically exerted. Therefore, action by contact is not really intelligible, and Wolff stands refuted – or rather, self-refuted. When we take into account all the conceptual difficulties above, Wolff’s position becomes untenable. It is plain that he cannot afford to accuse Newton of unintelligibility without resort to a double standard: expecting Newton to meet a criterion that his own dynamics fails to satisfy.

### 3.4 Hypothetical

A last complaint that Wolff lodges against action-at-a-distance forces is that they “are not sufficiently proven,” so we should not “rush to posit them.”<sup>136</sup>

<sup>134</sup> Though Wolff makes them sound like wholly physical forces, he is loath to call them so openly. And, unlike Leibniz, Wolff is not willing to say that his “elements” have a purely mentalistic *vis repraesentativa* on a par with Leibnizian monads. Eric Watkins, “On the Necessity and Nature of Simples: Leibniz, Wolff, Baumgarten, and the Pre-Critical Kant”, *Oxford Studies in Early Modern Philosophy*, III (2006), 261–314, helpfully documents and explains Wolff’s agnosticism about his basic forces.

<sup>135</sup> See Wolff, *Philosophia prima, sive Ontologia* (1730), 596, § 795. See also Wolff, *Cosmologia Generalis*, 223–224, § 295, § 297; and Campo, *Cristiano Wolff e il razionalismo precritico*, 227f.

<sup>136</sup> Wolff, “Responsio ad imputationes Johannis Freindii in *Transactionibus Anglicanis*” (1711), in Wolff, *Meletemata mathematico-philosophica cum erudito Orbe Literarum commercio communicata* (Halle: Renger, 1755), no. XX, 61. This charge reprises almost verbatim a complaint that Wolff had expressed in a letter to Leibniz: “In his *Optice*, Newton alleges at length the *hypothesis about the attractive force* of the smallest parts of mater, whose laws Keill gave in the *Philosophical Transactions*. But, it seems to me, Newton *does not prove it*

Thus, universal gravity is a mere hypothesis.<sup>137</sup> This is *prima facie* shocking, seeing as Newton had avowed that, if anything, he did not feign hypotheses. All the same, Wolff dismisses it as hypothetical, i.e. posited for the sake of saving the phenomena yet unsupported by any independent, conclusive evidence.

Look more closely, and you will see an alarming glibness to Wolff's dismissal of gravity. Wolff relies *solely* on Newton's experimental arguments in *Opticks*, which he had read in the Latin translation of 1706. Though he calls the *Principia* an *opus incomparabilis*, it seems he did not try very hard to grasp Newton's argument for universal gravitation in it. Wolff's account shows no trace of understanding how Newton combined "deduction from phenomena" (his complex inference to centripetal force as the *vera causa* of Kepler motion), with inductive generalization constrained by his four *regulae philosophandi*, to conclude that any two bodies attract each other without mechanical mediation. Only by wholly ignoring the master argument in the *Principia* can Wolff afford to denounce gravity as a mere hypothesis. Wolff bypasses completely Newton's actual warrant for gravity, relying instead on two stratagems. One is to read Newton's *Optice* as giving mere hypothetical-deductive reasoning for attractive forces – which he thinks he can dismiss in favor of allegedly better hypotheses, viz. mechanical substitutes for the distant attractions that Newton had pondered in his *Queries* and that Freind claimed to have proved in chemical phenomena. Another is to ignore completely Newton's methodological concept of "deduction from phenomena" in the *Principia*, whereby the Englishman had shown that an action-at-a-distance force is both *necessary* and sufficient for planets to move in Kepler orbits.<sup>138</sup> Wolff shows little awareness of Newtonian "deduction from phenomena," and no philosophical interest in it. This lack of real engagement with Newton's methods leads him to dismiss universal gravitation, all too glibly, as just a hypothesis.

I venture a possible explanation – conjectural at this point – for Wolff's rash view of gravity as hypothetical. Newton's four "rules for philosophizing" make

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sufficiently..." – 16 July 1710, in Gerhardt (ed.), *Briefwechsel zwischen Leibniz und Chr. Wolff*, 122.

137 Wolff, *Cosmologia Generalis*, 239, 240: "He who conceives [attraction] to occur by means of the mutual actions of forces inherent in bodies, assumes something that must be proved, and yet *cannot be proved* by appeal to experience, except by a [logical] fallacy of subreption." "When two bodies seem to act on each other without contact ... we attribute to bodies mutual actions which we do not observe, *nor can we prove.*"

138 In the *Principia*, Newton proves that a centripetal force is sufficient and necessary for Keplerian motion (in an ellipse) in Propositions 1 and 2, Section 2 of Book One. For an analysis, see Steffen Ducheyne, *The Main Business of Natural Philosophy: Isaac Newton's Natural-Philosophical Methodology* (New York: Springer, 2011), 84–88.

their first appearance in the second, 1713, edition of the *Principia*.<sup>139</sup> Their ancestors in the first edition were three “hypotheses,” except for Rule III, which has no predecessor.<sup>140</sup> This fact did not escape the attention of the literate public in Germany. The anonymous reviewer of the 1713 *Principia* for the *Acta Eruditorum* signaled it, and noted the addition of Rule III.<sup>141</sup> Someone hostile to action-at-a-distance, as Wolff ostensibly was, might not have accepted Newton's epistemic upgrade of his principles for empirical reasoning, from hypotheses to *regulae*. Rather, Wolff would have taken them to be *just* that (hypotheses), and then concluded that Newton's inference from them, namely universal gravity, was likewise hypothetical.

Even if true, my conjecture would not quite absolve Wolff; he remains guilty of bad faith. Grant him, for the sake of argument, that Newton's *regulae* are nothing but hypotheses. Then all he can dismiss as hypothetical is the claim that gravity is *universal*. Yet Wolff would still have to accept that gravitation at a distance is *real*, e.g. between the Sun and the primary planets. This is because Newton's inference to gravity as a *real* force *precedes* his use of the *regulae* to generalize it to all matter. Wolff's demoting of the *regulae* to mere hypotheses is powerless against the conclusion that distant gravity is a *vera causa* in the Solar System. In Newton's doctrine, that gravity is real follows by deduction from phenomena, not by inductive ascent through his *regulae*. Moreover, Newton never referred to deduction from phenomena as hypothetical reasoning. Rather, as is well known, he thought it was the strongest source of certainty in natural philosophy and the only way to move science past the hypothetical theorizing of Descartes and his ilk. Only a massive failure of insight or sheer bad faith could drive Wolff to ignore that Newtonian idea, and call gravity hypothetical.

It turns out that, with regard to Newtonian gravity too, not just inertia and force, Wolff shows himself the obedient Leibnizian in all respects. Leibniz had

139 Sir Isaac Newton, *Philosophiæ naturalis principia mathematica* (Cambridge: Cornelius Crownfield, 1713).

140 Sir Isaac Newton, *Philosophiæ naturalis principia mathematica* (London: Royal Society, Joseph Streater and Samuel Smith, 1687).

141 “In Book III, Newton calls *regulae philosophandi* that which previously he had said to be hypotheses, from whose former ranks now a few have been restored as phenomena. Of these *regulae*, the third does not appear in the previous edition – namely, the rule that the qualities of bodies that cannot be intended and remitted and that belong to all bodies on which experiments can be made should be taken as qualities of all bodies universally.” – Anon. “Review of *Philosophiæ Naturalis Principia Mathematica*, by Isaac Newton, Second Edition, Cambridge 1713.” *Acta Eruditorum*, March (1714), 131–141; 136.

inspired him to oppose it; and, by 1709, had already suggested the several lines of attack that Wolff would later pursue against it:

It is the task of those who assert that matter by itself is heavy to *prove* it. But, if I am not mistaken, they use that thesis more like a *hypothesis*. Still, I do not admit such a hypothesis, for it violates the first principles of reason. Granted, one cannot refute their notion from merely mathematical principles. However, it conflicts with that great Metaphysical Principle (if I may call it so), viz. that *nothing is without a reason* or cause. Hence, there should be a reason why bodies are heavy, or why many bodies gravitate toward one. This reason, while we may not be able to find it – though we already have a few worthy conjectures – must be such that it can be *understood*, if some genius presumed to explain it to us. But an *intelligible* explanation is not to be had unless it rests on the better-known and more distinct marks of body, i.e. *size, figure, and motion*. If this condition were removed, the *occult qualities* are brought back, indeed qualities forever and necessarily occult.<sup>142</sup>

Here, in one terse passage, Wolff gets his marching orders. Leibniz supplies him with an anti-Newtonian agenda and all the motifs he will then go on to articulate and defend: that gravity is hypothetical, without sufficient reason, unintelligible, and occult.

In sum, Wolff's opposition to action-at-a-distance seems driven by philosophical commitments, but is not as careful and rigorous as he pretended. Leibniz's influence on him was an external cause of his hostility to distant interactions – and also a major source of his main lines of argument. In all fairness, other natural philosophers, such as Euler and Jean le Rond d'Alembert (1717–1783), rejected Newtonian gravity without any prior loyalty to Leibniz. However, unlike Wolff, they never claimed for themselves a privileged status as providers of strict conceptual foundations for the new mechanics. Hence, it is ultimately to this self-imposed higher standard that Wolff must be held. It is doubtful that he succeeds. What he did manage, though, was to slow down the acceptance of Newtonian gravity in Germany until Kant began to make a forceful case for it, just after Wolff's demise.

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142 Leibniz to Wolff, 23 December 1709, in Gerhardt (ed.), *Briefwechsel zwischen Leibniz und Chr. Wolff*, 113; my emphasis.

#### 4 Il fut leibnizien, ce demi-siècle

In conclusion, Leibniz's outsized intellectual influence and alternative foundations for dynamics did much to delay the acceptance in Germany of Newton's mechanics of impressed forces. Soon after the *Principia* reached Europe, Leibniz started claiming to have discovered a host of new dynamical concepts: inertia, force, action and reaction. As he repeated his claims, he also tried to spell out their content and defend them with heavily philosophical arguments. In parallel, Leibniz lobbied skillfully on their behalf, seeking to have them adopted and expanded by gifted theorists favorably inclined to his calculus, vision for dynamics or both. Leibniz's foundational ideas, transmitted by Hermann and then systematized and propagated by Wolff, went on to loom large in the natural philosophy of early Enlightenment Germany.

As I have shown, these Leibnizian concepts, though often homonymous with Newton's, are significantly *un-Newtonian*. In scope and meaning, Leibnizian inertia is narrower than Newtonian inertia, thus it requires another agency, namely "force of motion," to make it equivalent to Newton's *vis insita*. Leibniz's ideas of action and reaction rest on some distinctions – between agent and patient, active and passive force – that Newton's mechanics simply lacks, for it does not need them. Lastly, Leibniz had required that all apparent action-at-a-distance be shown to be mediated, chiefly through that trusty mechanical stand-in, the ether. In contrast, Newton and his followers had no qualms accepting that distant bodies could interact directly, with no material intermediary. All these Leibnizian idiosyncrasies, which Wolff then justified and promoted heavily, maintained a steady presence in German thought on the foundations of mechanics before the mid 1750s.

I hope that, based on my results here, we may now hope to solve, at least in part, what Jonathan Israel called "a meaningful historical problem on its own," namely the slow and hesitant reception of Newton, which did not take place until the 1730s in France, and, in the case of Germany, until mid-century.<sup>143</sup> Part of the answer, I suggest, lies in the legacy of Leibniz. He bequeathed to his followers an inventory of ideas and principles that, they believed, made Newton's equivalent foundations unnecessary. At the metaphysical level, Wolff and his disciples sought to ground dynamics in non-Newtonian accounts of body, force and interaction. On the level of theory building, Johann and Daniel Bernoulli, Jacob Hermann, and Samuel König endeavored to extend Leibniz's concepts of live and dead force, and his Conservation of Vis Viva – suitably

143 Israel, *Radical Enlightenment*, 523.

reinterpreted as the sum of kinetic and potential energy – to novel physical cases, including the motion of planets in orbit, formerly Newton’s exclusive province.<sup>144</sup>

The theses I defended above set me at variance with some scholars who have examined Newtonianism in early eighteenth-century Germany. Contra Goetz, who saw Wolff accepting Newton’s laws without qualification, I have argued that the similarity between their principles is merely verbal; in meaning and scope, Wolff’s and Newton’s dynamical laws are wide apart.<sup>145</sup> Unlike Lorenz, who claimed there was a gradual reception and propagation of Newtonian’s ideas in post-Leibnizian Germany, I have shown that the Wolffians blocked any serious Newtonian advances until the late 1740s.<sup>146</sup> And, I think we may doubt that Wolff was “more sympathetic” than Leibniz to Newton’s method and gravitation theory.<sup>147</sup> Rather, I have claimed that Wolff’s doctrine simmers with anti-Newtonian animus and ideas, masked carefully by a veneer of shallow appreciation. Instead, I take my lead from scholars who have emphasized the strong influence of Leibniz on Wolff’s foundations of physics and attendant reaction to Newton’s *Principia*.<sup>148</sup> As a result, I have tried here to uncover that influence and prove that Wolff’s effort to stop the spread of Newtonianism in favor of Leibnizian foundations was wide-reaching and successful.

Reservedly obsequious toward Newton in public, Wolff in private had been dismissive of the Englishman’s philosophical acumen and depth: “Much as I esteem Newton in the higher geometry, I cannot even deem him a beginner

144 Cf. Daniel Bernoulli’s *Remarques sur le principe de la conservation des forces vives pris dans un sens général* (Berlin : s.n, 1748). There, Daniel seeks to prove that the total *vis viva* of a system of bodies mutually acted on each other by gravity-like forces is constant throughout their interactions. He concludes: “We see, therefore, that nature does not ever depart from the great principle of the conservation of live forces.” – Daniel Bernoulli, “Remarques sur le principe de la conservation des forces vives pris dans un sens general”, *Histoire de l’Académie royale des sciences et des belles lettres de Berlin: avec les mémoires pour la même année*, 4 (1750), 364.

145 See Goetz, “Der naturwissenschaftliche Aspekt”, 137. Martina Lorenz too claims that Wolff “accepted Newton’s three laws without reservation,” in Lorenz, “Der Beitrag Christian Wolffs zur Rezeption von Grundprinzipien der Mechanik Newtons”, 97f. Unfortunately, both Goetz and Lorenz ignore the Leibnizian background to Wolff’s law of action and reaction, which leads them to mistake it for Newton’s *Lex Tertia*.

146 Cf. Lorenz, “Der Beitrag Christian Wolffs zur Rezeption von Grundprinzipien der Mechanik Newtons”, 100.

147 See Katherine Dunlop, “Mathematical method and Newtonian science in the philosophy of Christian Wolff”, *Studies in History and Philosophy of Science*, 44 (2013), 457–469; 458.

148 See Jean Ecole, “Cosmologie wolfienne et dynamique leibnizienne”, *Les études philosophiques*, 19 (1963), 3–9, and the groundbreaking Watkins, “The laws of motion”.



in philosophy, let alone a real philosopher," he told Manteuffel.<sup>149</sup> This condescending view of Newton as a mere mathematician, and of himself as the true provider of solid foundations for the new science, contaminated some of Wolff's disciples too. Upon his death, Georg Frederick Meier (1718–1777) reached for Pope's dictum and distorted it so as to give Wolff the greater glory:

When the famous Englishman Neuton [*sic*] died, someone wrote the following eulogy about him: "And God spoke, *Let there be light in mathematics!*, and there was Neuton." We may say, just as rightly: God spoke, *Let there be light in philosophy and all other sciences!*, and there was Wolff. Divine Providence gave us in this man a happy gift for our times.<sup>150</sup>

In 1754, such lavish praise rang hollow. It came as Euler was single-handedly extending Newton's *Lex Secunda* to fluids, thus making it the fundamental principle of classical mechanics, with the Second Law having outlived and displaced Conservation of Vis Viva. And it came a mere two years before Kant exploded the Wolffian tradition from within, by arguing that action at a distance, long reviled by Leibniz and Wolff, is in fact essential to matter – a radical form of Newtonianism.<sup>151</sup> In an important sense, Wolff's last breath was also the last gasp of Leibnizian foundations for dynamics. With him out of the way, Newtonian mechanics, triumphant elsewhere, was at long last to prevail in Germany.

149 Ostertag, *Naturphilosophisches aus Wolffs Briefwechsel mit Manteuffel*, 61; on Wolff's low opinion of Newton, cf. also Adolf von Harnack, *Geschichte der Königlich Preussischen Akademie der Wissenschaften zu Berlin*, 2 vols. (Berlin: Reichsdruckerei, 1900), 255f. Wolff on Euler, the greatest mathematical physicist of the Age of Reason: "I know that even those who regard him highly because of his calculating abilities confess ... that he understands not the least thing about philosophy, and is quite unskilled in the *methodo demonstrandi veterum*; which his tract, the *Mechanica*, amply shows." – Wolff to Manteuffel, 6 November 1746, in Ostertag, *Naturphilosophisches aus Wolffs Briefwechsel mit Manteuffel*, 75.

150 Cf. G.F. Meier, "Betrachtung bey dem Tode des Freyherrn von Wolf" in *Der Mensch, eine moralische Wochenschrift*, 8. Theil, Stück 309, (1754), 97–104.

151 Kant's argument is in his 1756 *Physical Monadology*. Euler's papers on fluid dynamics are "Principes généraux de l'état d'équilibre d'un fluide" and "Principes généraux du mouvement des fluides," both read in 1755 at the Royal Academy of Sciences in Berlin. For analysis of some of Euler's results in them, see Olivier Darrigol, and Uriel Frisch, "From Newton's mechanics to Euler's equations", *Physica D*, 237, 14–17, (2008), 1855–1869.