

# **Review of 'The Outer Limits of Reason' by Noson Yanofsky 403p (2013) (review revised 2019)**

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## **ABSTRACT**

I give a detailed review of 'The Outer Limits of Reason' by Noson Yanofsky from a unified perspective of Wittgenstein and evolutionary psychology. I indicate that the difficulty with such issues as paradox in language and math, incompleteness, undecidability, computability, the brain and the universe as computers etc., all arise from the failure to look carefully at our use of language in the appropriate context and hence the failure to separate issues of scientific fact from issues of how language works. I discuss Wittgenstein's views on incompleteness, paraconsistency and undecidability and the work of Wolpert on the limits to computation. To sum it up: The Universe According to Brooklyn---Good Science, Not So Good Philosophy.

Those wishing a comprehensive up to date framework for human behavior from the modern two systems view may consult my book 'The Logical Structure of Philosophy, Psychology, Mind and Language in Ludwig Wittgenstein and John Searle' 2nd ed (2019). Those interested in more of my writings may see 'Talking Monkeys--Philosophy, Psychology, Science, Religion and Politics on a Doomed Planet--Articles and Reviews 2006-2019 3rd ed (2019) and Suicidal Utopian Delusions in the 21<sup>st</sup> Century 4<sup>th</sup> ed (2019)

Alvy's Mom responding to his being depressed because the universe is expanding – “What has the universe got to do with it? You're here in Brooklyn! Brooklyn is not expanding!”

This famous Woody Allen joke makes a profound point about the context sensitivity of language that applies throughout philosophy and science. It's funny because it is obvious that the meaning of “expanding” in the two cases is quite different. Brooklyn might expand if the population increases or the city annexes outlying land, but the universe is said to expand due to cosmic telescopes that show a red shift indicating that stars are receding from each other or to measurements of matter density etc. Different meanings (language games) (LG's) were famously

characterized by the Austrian-British philosopher Ludwig Wittgenstein (W) as the central problem of philosophy and shown to be a universal default of our psychology. Though he did this beginning with the Blue and Brown Books (BBB) in the early 30's, left a 20,000 page nachlass, and is the most widely discussed philosopher of modern times, few understand him.

To Yanofsky's (Y's) credit, he has given much attention to philosophy and even quotes W a few times but without any real grasp of the issues. It is the norm among scientists and philosophers to mix the scientific questions of fact with the philosophical questions of how language is being used and, as W noted, — 'Problem and answer pass one another by'. Yanofsky (a Brooklyn resident like many of his friends and teachers) has read widely and does a good job of surveying the bleeding edges of physics, mathematics and computer science in a clear and authoritative manner, but when we come to the limits of scientific explanation and it's not clear what to say, we turn to philosophy.

Philosophy can be seen as the descriptive psychology of higher order thought or as the study of the contextual variations of language used to describe cognition or intentionality (my characterizations), or the study of the logical structure of rationality (LSR)(Searle). Regarding LSR, Berkeley philosopher John Searle (S) is one of the best since W and his work can be seen as an extension of W. I have reviewed many books by them and others and together these reviews constitute a skeletal outline of higher order thought or intentionality, and so of the foundations of science.

It is common for books and papers to betray their limitations in their titles and that is the case here. "Reason" and "limits" are complexes of language games. So, I should stop here and spend the whole review showing how Y's title reveals the deep misunderstanding of what the real issues are. I knew we were in for a rough time by p5 where we are told that our normal conceptions of time, space etc., are mistaken and this was known even to the Greeks. This brings to mind W: "People say again and again that philosophy doesn't really progress, that we are still occupied with the same philosophical problems as were the Greeks... at something which no explanation seems capable of clearing up...And what's more, this satisfies a longing for the transcendent, because in so far as people think they can see the 'limits of human understanding', they believe of course that they can see beyond these. - CV (1931)" and also "The limit of language is shown by its being impossible to describe a fact which corresponds to (is the translation of) a sentence without simply repeating the sentence..." So, I would say we just have to analyze the different types of language games. Looking deeper is essential but surrendering our

prior use is incoherent.

Think about what is implied by “The Outer Limits of Reason”. “Outer”, “Limits” and “Reason” all have common uses, but they are frequently used by Y in different ways, and they will seem “quite innocent”, but this can only be discussed in some specific context.

We are using the word “question” (or “assertion”, “statement” etc.) with utterly different senses if we ask “Does 777 occur in the decimal expansion of Pi?” than if we ask “Does 777 occur in the first 1000 digits of the decimal expansion of Pi?” to use one of W’s examples. In the latter case it’s clear what counts as a true or false answer but in the former it has only the form of a question. On p10 we find a group of “statements” which have quite different meanings. The first three are definitions and one could understand them without knowing any facts about their use—e.g., X cannot be Y and not Y.

Y recommends the documentary “Into the Infinite” but actually it cannot be viewed unless you are in the UK. I found it free on the net shortly after it came out and was greatly disappointed. Among other things it suggests Godel and Cantor went mad due to working on problems of infinity—for which there is not a shred of evidence— and it spends much time with Chaitin, who, though a superb mathematician, has only a hazy notion about the various philosophical issues discussed here. If you want a lovely whirlwind “deep science” documentary I suggest “Are We Real?” on Youtube, though it makes some of the same mistakes.

W noted that when we reach the end of scientific commentary, the problem becomes a philosophical one—i.e., one of how language can be used intelligibly. Yanofsky, like virtually all scientists and most philosophers, does not get that there are two distinct kinds of “questions” or “assertions” (i.e., Language Games or LG’s) here. There are those that are matters of fact about how the world is—that is, they are publicly observable propositional (True or False) states of affairs having clear meanings (Conditions of Satisfaction --COS) in Searle’s terminology—i.e., scientific statements, and then there are those that are issues about how language can coherently be used to describe these states of affairs, and these can be answered by any sane, intelligent, literate person with little or no resort to the facts of science. Another poorly understood but critical fact is that, although the thinking, representing, inferring, understanding, intuiting etc. (i.e., the dispositional psychology) of a true or false statement is a function of the higher order cognition of our slow, conscious System 2 (S2), the decision as to whether “particles” are entangled, the star shows a red shift, a theorem has been proven (i.e., the part that

involves seeing that the symbols are used correctly in each line of the proof), is always made by the fast, automatic, unconscious System 1 (S1) via seeing, hearing, touching etc. in which there is no information processing, no representation (i.e., no COS) and no decisions in the sense in which these happen in S2 (which receives its inputs from S1). This two systems approach is now the standard way to view reasoning or rationality and is a crucial heuristic in the description of behavior, of which science, math and philosophy are special cases. There is a huge and rapidly growing literature on reasoning that is indispensable to the study of behavior or science. A recent book that digs into the details of how we actually reason (i.e., use language to carry out actions—see Wittgenstein and Searle) is ‘Human Reasoning and Cognitive Science’ by Stenning and Van Lambalgen (2008), which, in spite of its limitations (e.g., limited understanding of W/S and the broad structure of intentional psychology), is (as of mid 2016) the best single source I know.

Regarding “incompleteness” or “randomness” in math, Y’s failure to mention the work of Gregory Chaitin is truly amazing, as he must know of his work, and Chaitin’s proof of the algorithmic randomness of math (of which Godel’s results are a corollary) and the Omega number are some of the most famous mathematical results in the last 50 years.

Likewise, one sees nothing about unconventional computing such as those with membranes, DNA etc., that have no logic gates and follow the biological patterns of “information processing”. The best way to get free articles and books on the cutting edge is to visit [ArXiv.org](http://ArXiv.org), [vixra.org](http://vixra.org), [academia.edu](http://academia.edu), [citeseerx.ist.psu.edu](http://citeseerx.ist.psu.edu), [researchgate.net](http://researchgate.net), or [philpapers.org](http://philpapers.org), [libgen.io](http://libgen.io) and [b-ok.org](http://b-ok.org) where there are millions of free preprints, papers and books on every topic (be warned this may use up all your spare time for the rest of your life!).

Regarding Godel and “incompleteness”, since our psychology as expressed in symbolic systems such as math and language is “random” or “incomplete” and full of tasks or situations (“problems”) that have been proven impossible (i.e., they have no solution—see below) or whose nature is unclear, it seems unavoidable that everything derived from it—e.g. physics and math) will be “incomplete” also. Afaik the first of these in what is now called Social Choice Theory or Decision Theory (which are continuous with the study of logic and reasoning and philosophy) was the famous theorem of Kenneth Arrow 65 years ago, and there have been many since. Y notes a recent impossibility or incompleteness proof in two-person game theory. In these cases, a proof shows that what looks like a simple choice stated in plain English has no solution.

Although one cannot write a book about everything, I would have liked Y to at least mention such famous “paradoxes” as Sleeping Beauty (dissolved by Read), Newcomb’s problem (dissolved by Wolpert) and Doomsday, where what seems to be a very simple problem either has no one clear answer, or it proves exceptionally hard to find one. A mountain of literature exists on Gödel’s two “incompleteness” theorems and Chaitin’s more recent work, but I think that W’s writings in the 30’s and 40’s are definitive. Although Shanker, Mancosu, Floyd, Marion, Rodych, Gefwert, Wright and others have done insightful work, it is only recently that W’s uniquely penetrating analysis of the language games being played in mathematics have been clarified by Floyd (e.g., ‘Wittgenstein’s Diagonal Argument—a Variation on Cantor and Turing’), Berto (e.g., ‘Gödel’s Paradox and Wittgenstein’s Reasons’, and ‘Wittgenstein on Incompleteness makes Paraconsistent Sense’ and the book ‘There’s Something about Gödel’, and Rodych (e.g., Wittgenstein and Gödel: the Newly Published Remarks’, ‘Misunderstanding Gödel :New Arguments about Wittgenstein’, ‘New Remarks by Wittgenstein’ and his article in the online Stanford Encyclopedia of Philosophy ‘Wittgenstein’s Philosophy of Mathematics’ ). Berto is one of the best recent philosophers, and those with time might wish to consult his many other articles and books including the volume he co-edited on paraconsistency (2013). Rodych’s work is indispensable, but only two of a dozen or so papers are free online with the usual search but it’s probably all free online if one knows where to look.

Berto notes that W also denied the coherence of metamathematics—i.e., the use by Gödel of a metatheorem to prove his theorem, likely accounting for his “notorious” interpretation of Gödel’s theorem as a paradox, and if we accept his argument, I think we are forced to deny the intelligibility of metalanguages, metatheories and meta anything else. How can it be that such concepts (words) as metamathematics and incompleteness, accepted by millions (and even claimed by no less than Penrose, Hawking, Dyson et al to reveal fundamental truths about our mind or the universe) are just simple misunderstandings about how language works? Isn’t the proof in this pudding that, like so many “revelatory” philosophical notions (e.g., mind and will as illusions –Dennett, Carruthers, the Churchlands etc.), they have no practical impact whatsoever? Berto sums it up nicely: “Within this framework, it is not possible that the very same sentence...turns out to be expressible, but undecidable, in a formal system... and demonstrably true (under the aforementioned consistency hypothesis) in a different system (the meta-system). If, as Wittgenstein maintained, the proof establishes the very meaning of the proved sentence, then it is not possible for the same sentence (that is, for a sentence with the same meaning) to be undecidable in a formal system, but decided in a different system (the meta-system) ... Wittgenstein had to reject both the idea that a formal

system can be syntactically incomplete, and the Platonic consequence that no formal system proving only arithmetical truths can prove all arithmetical truths. If proofs establish the meaning of arithmetical sentences, then there cannot be incomplete systems, just as there cannot be incomplete meanings.” And further “Inconsistent arithmetics, i.e., nonclassical arithmetics based on a paraconsistent logic, are nowadays a reality. What is more important, the theoretical features of such theories match precisely with some of the aforementioned Wittgensteinian intuitions...Their inconsistency allows them also to escape from Godel’s First Theorem, and from Church’s undecidability result: there are, that is, demonstrably complete and decidable. They therefore fulfil precisely Wittgenstein’s request, according to which there cannot be mathematical problems that can be meaningfully formulated within the system, but which the rules of the system cannot decide. Hence, the decidability of paraconsistent arithmetics harmonizes with an opinion Wittgenstein maintained throughout his philosophical career.”

W also demonstrated the fatal error in regarding mathematics or language or our behavior in general as a unitary coherent logical ‘system,’ rather than as a motley of pieces assembled by the random processes of natural selection. “Godel shows us an unclarity in the concept of ‘mathematics’, which is indicated by the fact that mathematics is taken to be a system” and we can say (contra nearly everyone) that is all that Godel and Chaitin show. W commented many times that ‘truth’ in math means axioms or the theorems derived from axioms, and ‘false’ means that one made a mistake in using the definitions, and this is utterly different from empirical matters where one applies a test. W often noted that to be acceptable as mathematics in the usual sense, it must be useable in other proofs and it must have real world applications, but neither is the case with Godel’s Incompleteness. Since it cannot be proved in a consistent system (here Peano Arithmetic but a much wider arena for Chaitin), it cannot be used in proofs and, unlike all the ‘rest’ of PA it cannot be used in the real world either. As Rodych notes “...Wittgenstein holds that a formal calculus is only a mathematical calculus (i.e., a mathematical language-game) if it has an extra-systemic application in a system of contingent propositions (e.g., in ordinary counting and measuring or in physics) ...” Another way to say this is that one needs a warrant to apply our normal use of words like ‘proof’, ‘proposition’, ‘true’, ‘incomplete’, ‘number’, and ‘mathematics’ to a result in the tangle of games created with ‘numbers’ and ‘plus’ and ‘minus’ signs etc., and with ‘Incompleteness’ this warrant is lacking. Rodych sums it up admirably. “On Wittgenstein’s account, there is no such thing as an incomplete mathematical calculus because ‘in mathematics, everything is algorithm [and syntax] and nothing is meaning [semantics]...”

W has much the same to say of Cantor's diagonalization and set theory. "Consideration of the diagonal procedure shews you that the concept of 'real number' has much less analogy with the concept 'cardinal number' than we, being misled by certain analogies, are inclined to believe" and many other comments (see Rodych and Floyd).

As Rodych, Berto and Priest (another pioneer in paraconsistency) have noted, W was the first (by several decades) to insist on the unavoidability and utility of inconsistency (and debated this issue with Turing during his classes on the Foundations of Mathematics). We now see that the disparaging comments about W's remarks on math made by Godel, Kreisel, Dummett and many others were misconceived. As usual, it is a very bad idea to bet against W. Some may feel we have strayed off the path here—after all in "The Limits of Reason" we only want to understand science and math and why these paradoxes and inconsistencies arise and how to dispose of them. But I claim that is exactly what I have done by pointing to the work of W and his intellectual heirs. Our symbolic systems (language, math, logic, computation) have a clear use in the narrow confines of everyday life, of what we can loosely call the mesoscopic realm-- the space and time of normal events we can observe unaided and with certainty (the innate axiomatic bedrock or background). But we leave coherence behind when we enter the realms of particle physics or the cosmos, relativity, math beyond simple addition and subtraction with whole numbers, and language used out of the immediate context of everyday events. The words or whole sentences may be the same, but the meaning is lost. It looks to me like the best way to understand philosophy is enter it via Berto, Rodych and Floyd's work on W, so as to understand the subtleties of language as it is used in math and thereafter "metaphysical" issues of all kinds may be dissolved. As Floyd notes "In a sense, Wittgenstein is literalizing Turing's model, bringing it back down to the everyday and drawing out the anthropomorphic command-aspect of Turing's metaphors."

W pointed out how in math, we are caught in more LG's (Language Games) where it is not clear what "true", "complete", "follows from", "provable", "number", "infinite", etc. mean (i.e., what are their COS or truthmakers in THIS context), and hence what significance to attach to 'incompleteness' and likewise for Chaitin's "algorithmic randomness". As W noted frequently, do the "inconsistencies" of math or the counterintuitive results of metaphysics cause any real problems in math, physics or life? The apparently more serious cases of contradictory statements—e.g., in set theory—have long been known but math goes on anyway. Likewise for the countless liar (self-referencing) paradoxes in language which Y discusses, but he does not really understand their basis, and fails to make clear that self-

referencing is involved in the "incompleteness" and "inconsistency" (groups of complex LG's) of mathematics as well.

Another interesting work is "Godel's Way" (2012) by Chaitin, Da Costa and Doria (see my review). In spite of its many failings—really a series of notes rather than a finished book—it is a unique source of the work of these three famous scholars who have been working at the bleeding edges of physics, math and philosophy for over half a century. Da Costa and Doria are cited by Wolpert (see below) since they wrote on universal computation and among his many accomplishments, Da Costa is a pioneer on paraconsistency. Chaitin also contributes to 'Causality, Meaningful Complexity and Embodied Cognition' (2010), replete with articles having the usual mixture of insight and incoherence and as usual, nobody is aware that W can be regarded as the originator of the position current as Embodied Cognition or Enactivism. Many will find the articles and especially the group discussion with Chaitin, Fredkin, Wolfram et al at the end of Zenil H. (ed.) 'Randomness through computation' (2011) a stimulating continuation of many of the topics here, but lacking awareness of the philosophical issues and so mixing science (fact finding) with philosophy (language games). See also Doria (Ed.), "The Limits Of Mathematical Modeling In The Social Sciences: The Significance Of Godel's Incompleteness Phenomenon" (2017) and Wuppuluri and Doria (Eds.), "The Map and the Territory: Exploring the foundations of science, thought and reality" (2018).

It is a constant struggle to keep in mind that different contexts mean different LG's (meanings, COS) for "time", "space", "particle", "object", "inside", "outside", "next", "simultaneous", "occur", "happen", "event", "question", "answer", "infinite", "past", "future", "problem", "logic", "ontology", "epistemology", "solution", "paradox", "prove", "strange", "normal", "experiment", "complete", "uncountable", "decidable", "dimension", "complete", "formula", "process", "algorithm", "axiom", "mathematics", "physics", "cause", "place", "same", "moving", "limit", "reason", "still", "real" "assumption", "belief", "know", "event", "recursive", "meta—", "self-referential" "continue", "particle", "wave", "sentence" and even (in some contexts) "and", "or", "also", "add", "divide", "if...then", "follows" etc.

To paraphrase W, most of what people (including many philosophers and most



scientists) have to say when philosophizing is not philosophy but its raw material. Yanofsky joins Hume, Quine, Dummett, Kripke, Dennett, Churchland, Carruthers, Wheeler etc. in repeating the mistakes of the Greeks with elegant philosophical jargon mixed with science. As antidotes, I suggest my reviews and some Rupert Read, such as his books 'A Wittgensteinian Way with Paradoxes' and 'Wittgenstein Among the Sciences', or go to academia.edu and get his articles, especially 'Kripke's Conjuring Trick' and 'Against Time Slices' and then as much of S as feasible, but at least his most recent such as 'Philosophy in a New Century', 'Searle's Philosophy and Chinese Philosophy', 'Making the Social World' and 'Thinking About the Real World' (or my reviews if time is short) and his recent volume on perception. There are also over 100 youtubes of Searle which confirm his reputation as the best standup philosopher since Wittgenstein.

Y does not make clear the major overlap that now exists (and is expanding rapidly) between game theorists, physicists, economists, mathematicians, philosophers, decision theorists and others, all of whom have been publishing for decades closely related proofs of undecidability, impossibility, uncomputability, and incompleteness. One of the more 'bizarre' (i.e., not so if we clarify the language games) is the recent proof by Armando Assis that in the relative state formulation of quantum mechanics one can setup a zero-sum game between the universe and an observer using the Nash Equilibrium, from which follow the Born rule and the collapse of the wave function. Godel was first to demonstrate an impossibility result and (until Wolpert) it is the most far reaching (or just trivial/incoherent) but there have been an avalanche of others. As noted, one of the earliest in decision theory was the famous General Impossibility Theorem (GIT) discovered by Kenneth Arrow in 1951 (for which he got the Nobel Prize in economics in 1972 – and five of his students are now Nobel laureates so this is not fringe science). It states roughly that no reasonably consistent and fair voting system (i.e., no method of aggregating individuals' preferences into group preferences) can give sensible results. The group is either dominated by one person and so GIT is often called the "dictator theorem", or there are intransitive preferences. Arrow's original paper was titled "A Difficulty in the Concept of Social Welfare" and can be stated like this: "It is impossible to formulate a social preference ordering that satisfies all of the following conditions: Nondictatorship; Individual Sovereignty; Unanimity; Freedom From Irrelevant Alternatives; Uniqueness of Group Rank." Those familiar with modern decision theory accept this and the many related constraining theorems as their starting points. Those who are not may find it (and all these theorems) incredible and in that case, they need to find a career path that has nothing to do with any of the above disciplines. See "The Arrow Impossibility Theorem"(2014) or "Decision Making and Imperfection"(2013) among legions of

publications.

Y mentions the famous impossibility result of Brandenburger and Keisler (2006) for two person games (but of course not limited to “games” and like all these impossibility results it applies broadly to decisions of any kind) which shows that any belief model of a certain kind leads to contradictions. One interpretation of the result is that if the decision analyst’s tools (basically just logic) are available to the players in a game, then there are statements or beliefs that the players can write down or ‘think about’ but cannot actually hold. “Ann believes that Bob assumes that Ann believes that Bob’s assumption is wrong” seems unexceptionable and ‘recursion’ (another LG) has been assumed in argumentation, linguistics, philosophy etc., for a century at least, but they showed that it is impossible for Ann and Bob to assume these beliefs. And there is a rapidly growing body of such impossibility results for 1 or multiplayer decision situations (e.g., it grades into Arrow, Wolpert, Koppel and Rosser etc). For a good technical paper from among the avalanche on the B&K paradox, get Abramsky and Zvesper’s paper from arXiv which takes us back to the liar paradox and Cantor’s infinity (as its title notes it is about “interactive forms of diagonalization and self-reference”) and thus to Floyd, Rodych, Berto, W and Godel. Many of these papers quote Y’s paper “A universal approach to self-referential paradoxes and fixed points. *Bulletin of Symbolic Logic*, 9(3):362–386, 2003. Abramsky (a polymath who is among other things a pioneer in quantum computing) is a friend of Y’s and so Y contributes a paper to the recent *Festschrift* to him ‘Computation, Logic, Games and Quantum Foundations’ (2013). For maybe the best recent (2013) commentary on the BK and related paradoxes see the 165p powerpoint lecture free on the net by Wes Holliday and Eric Pacuit ‘Ten Puzzles and Paradoxes about Knowledge and Belief’. For a good multi-author survey see ‘Collective Decision Making (2010).

One of the major omissions from all such books is the amazing work of polymath physicist and decision theorist David Wolpert, who proved some stunning impossibility or incompleteness theorems (1992 to 2008-see arxiv.org) on the limits to inference (computation) that are so general they are independent of the device doing the computation, and even independent of the laws of physics, so they apply across computers, physics, and human behavior, which he summarized thusly: “One cannot build a physical computer that can be assured of correctly processing information faster than the universe does. The results also mean that there cannot exist an infallible, general-purpose observation apparatus, and that there cannot be an infallible, general-purpose control apparatus. These results do not rely on systems that are infinite, and/or non-classical, and/or obey chaotic dynamics. They also hold even if one uses an infinitely fast, infinitely dense computer, with

computational powers greater than that of a Turing Machine.”

He also published what seems to be the first serious work on team or collective intelligence (COIN) which he says puts this subject on a sound scientific footing. Although he has published various versions of these over two decades in some of the most prestigious peer reviewed physics journals (e.g., *Physica D* 237: 257-81(2008)) as well as in NASA journals and has gotten news items in major science journals, few seem to have noticed and I have looked in dozens of recent books on physics, math, decision theory and computation without finding a reference.

It is most unfortunate that Yanofsky and others have no awareness of Wolpert, since his work is the ultimate extension of computing, thinking, inference, incompleteness, and undecidability, which he achieves (like many proofs in Turing machine theory) by extending the liar paradox and Cantors diagonalization to include all possible universes and all beings or mechanisms and thus may be seen as the last word not only on computation, but on cosmology or even deities. He achieves this extreme generality by partitioning the inferring universe using worldlines (i.e., in terms of what it does and not how it does it) so that his mathematical proofs are independent of any particular physical laws or computational structures in establishing the physical limits of inference for past, present and future and all possible calculation, observation and control. He notes that even in a classical universe Laplace was wrong about being able to perfectly predict the future (or even perfectly depict the past or present) and that his impossibility results can be viewed as a “non-quantum mechanical uncertainty principle” (i.e., there cannot be an infallible observation or control device). Any universal physical device must be infinite, it can only be so at one moment in time, and no reality can have more than one (the “monotheism theorem”).

Since space and time do not appear in the definition, the device can even be the entire universe across all time. It can be viewed as a physical analog of incompleteness with two inference devices rather than one self-referential device. As he says, “either the Hamiltonian of our universe proscribes a certain type of computation, or prediction complexity is unique (unlike algorithmic information complexity) in that there is one and only one version of it that can be applicable throughout our universe.” Another way to say this is that one cannot have two physical inference devices (computers) both capable of being asked arbitrary questions about the output of the other, or that the universe cannot contain a computer to which one can pose any arbitrary computational task, or that for any pair of physical inference engines, there are always binary valued questions about the state of the universe that cannot even be posed to at least one of them. One

cannot build a computer that can predict an arbitrary future condition of a physical system before it occurs, even if the condition is from a restricted set of tasks that can be posed to it— that is, it cannot process information (though this is a vexed phrase as S and Read and others note) faster than the universe. The computer and the arbitrary physical system it is computing do not have to be physically coupled and it holds regardless of the laws of physics, chaos, quantum mechanics, causality or light cones and even for an infinite speed of light. The inference device does not have to be spatially localized but can be nonlocal dynamical processes occurring across the entire universe. He is well aware that this puts the speculations of Wolfram, Landauer, Fredkin, Lloyd etc., concerning the universe as computer or the limits of "information processing", in a new light (though the indices of their writings make no reference to him and another remarkable omission is that none of the above are mentioned by Yanofsky either).

Wolpert says it shows that the universe cannot contain an inference device that can process information as fast as it can, and since he shows you cannot have a perfect memory nor perfect control, its past, present or future state can never be perfectly or completely depicted, characterized, known or copied. He also proved that no combination of computers with error correcting codes can overcome these limitations. Wolpert also notes the critical importance of the observer ("the liar") and this connects us to the familiar conundrums of physics, math and language that concern Y. Again cf. Floyd on W: "He is articulating in other words a generalized form of diagonalization. The argument is thus generally applicable, not only to decimal expansions, but to any purported listing or rule-governed expression of them; it does not rely on any particular notational device or preferred spatial arrangements of signs. In that sense, Wittgenstein's argument appeals to no picture and it is not essentially diagrammatical or representational, though it may be diagrammed and insofar as it is a logical argument, its logic may be represented formally). Like Turing's arguments, it is free of a direct tie to any particular formalism. [The parallels to Wolpert are obvious.] Unlike Turing's arguments, it explicitly invokes the notion of a language-game and applies to (and presupposes) an everyday conception of the notions of rules and of the humans who follow them. Every line in the diagonal presentation above is conceived as an instruction or command, analogous to an order given to a human being..."

W's prescient viewpoint of these issues, including his embrace of strict finitism and paraconsistency, is finally spreading through math, logic and computer science (though rarely with any acknowledgement). Bremer has recently suggested the necessity of a Paraconsistent Lowenheim-Skolem Theorem. "Any mathematical theory presented in first order logic has a finite paraconsistent model." Berto

continues: "Of course strict finitism and the insistence on the decidability of any meaningful mathematical question go hand in hand. As Rodych has remarked, the intermediate Wittgenstein's view is dominated by his 'finitism and his view [...] of mathematical meaningfulness as algorithmic decidability' according to which '[only] finite logical sums and products (containing only decidable arithmetic predicates) are meaningful because they are algorithmically decidable.'" In modern terms this means they have public conditions of satisfaction-i.e., can be stated as a proposition that is true or false. And this brings us to W's view that ultimately everything in math and logic rests on our innate (though of course extensible) ability to recognize a valid proof. Berto again: "Wittgenstein believed that the naïve (i.e., the working mathematicians) notion of proof had to be decidable, for lack of decidability meant to him simply lack of mathematical meaning: Wittgenstein believed that everything had to be decidable in mathematics...Of course one can speak against the decidability of the naïve notion of truth on the basis of Gödel's results themselves. But one may argue that, in the context, this would beg the question against paraconsistentists-- and against Wittgenstein too. Both Wittgenstein and the paraconsistentists on one side, and the followers of the standard view on the other, agree on the following thesis: the decidability of the notion of proof and its inconsistency are incompatible. But to infer from this that the naïve notion of proof is not decidable invokes the indispensability of consistency, which is exactly what Wittgenstein and the paraconsistent argument call into question...for as Victor Rodych has forcefully argued, the consistency of the relevant system is precisely what is called into question by Wittgenstein's reasoning." And so: "Therefore the Inconsistent arithmetic avoids Gödel's First Incompleteness Theorem. It also avoids the Second Theorem in the sense that its non-triviality can be established within the theory: and Tarski's Theorem too—including its own predicate is not a problem for an inconsistent theory "[As Priest noted over 20 years ago]. Prof. Rodych thinks my comments reasonably represent his views, but notes that the issues are quite complex and there are many differences between he, Berto and Floyd.

And again, 'decidability' comes down to the ability to recognize a valid proof, which rests on our innate axiomatic psychology, which math and logic have in common with language. And this is not just a remote historical issue but is totally current. I have read much of Chaitin and never seen a hint that he has considered these matters. The work of Douglas Hofstadter also comes to mind. His *Gödel, Escher, Bach* won a Pulitzer prize and a National Book Award for Science, sold millions of copies and continues to get good reviews (e.g. almost 400 mostly 5 star reviews on Amazon to date) but he has no clue about the real issues and repeats the classical philosophical mistakes on nearly every page. His subsequent philosophical

writings have not improved (he has chosen Dennett as his muse), but, as these views are vacuous and unconnected to real life, he continues to do excellent science.

However once again note that “infinite”, “compute”, “information” etc., only have meaning in specific human contexts—that is, as Searle has emphasized, they are all observer relative or ascribed vs intrinsically intentional. The universe apart from our psychology is neither finite nor infinite and cannot compute nor process anything. Only in our language games do our laptop or the universe compute.

However not everyone is oblivious to Wolpert. Well known econometricians Koppl and Rosser in their famous 2002 paper “All that I have to say has already crossed your mind” give three theorems on the limits to rationality, prediction and control in economics. The first uses Wolpert’s theorem on the limits to computability to show some logical limits to forecasting the future. Wolpert notes that it can be viewed as the physical analog of Godel’s incompleteness theorem and K and R say that their variant can be viewed as its social science analog, though Wolpert is well aware of the social implications. Since Godel’s are corollaries of Chaitin’s theorem showing algorithmic randomness (incompleteness) throughout math (which is just another of our symbolic systems), it seems inescapable that thinking (behavior) is full of impossible, random or incomplete statements and situations. Since we can view each of these domains as symbolic systems evolved by chance to make our psychology work, perhaps it should be regarded as unsurprising that they are not “complete”. For math, Chaitin says this ‘randomness’ (again a group of LG’s) shows there are limitless theorems that are true but unprovable—i.e., true for no reason. One should then be able to say that there are limitless statements that make perfect “grammatical” sense that do not describe actual situations attainable in that domain. I suggest these puzzles go away if one considers *W*’s views. He wrote many notes on the issue of Godel’s Theorems, and the whole of his work concerns the plasticity, “incompleteness” and extreme context sensitivity of language, math and logic, and the recent papers of Rodych, Floyd and Berto are the best introduction I know of to *W*’s remarks on the foundations of mathematics and so to philosophy.

K and R’s second theorem shows possible nonconvergence for Bayesian (probabilistic) forecasting in infinite- dimensional space. The third shows the impossibility of a computer perfectly forecasting an economy with agents knowing its forecasting program. The astute will notice that these theorems can be seen as versions of the liar paradox and the fact that we are caught in impossibilities when we try to calculate a system that includes ourselves has been noted by Wolpert, Koppl, Rosser and others in these contexts and again we have circled back to the puzzles of physics when the observer is involved. K&R conclude “Thus, economic

order is partly the product of something other than calculative rationality". Bounded rationality is now a major field in itself, the subject of thousands of papers and hundreds of books.

On p19 Yanofsky says math is free of contradictions, yet as noted, it has been well known for over half a century that logic and math (and physics) are full of them— just google inconsistency in math or search it on Amazon or see the works of Priest, Berto or the article by Weber in the Internet Encyclopedia of Philosophy. W was the first to predict inconsistency or paraconsistency, and if we follow Berto we can interpret this as W's suggestion to avoid incompleteness. In any event, paraconsistency is now a common feature and a major research program in geometry, set theory, arithmetic, analysis, logic and computer science. Y returns to this issue other places such as on p346 where he says reason must be free of contradictions, but it is clear that "free of" has different uses and they arise frequently in everyday life but we have innate mechanisms to contain them. This is true because it was the case in our everyday life long before math and science

Regarding time travel (p49), I suggest Rupert Read's "Against Time Slices" in his free online papers or "Time Travel-the very idea" in his book "A Wittgensteinian Way with Paradoxes."

Regarding the discussion of famous philosopher of science Thomas Kuhn on p248, those interested can see the work of Rupert Read and his colleagues, most recently in his book "Wittgenstein Among the Sciences" and while there, you may make a start at eliminating the hard problem of consciousness by reading "Dissolving the hard problem of consciousness back into ordinary life" (or his earlier essay on this which is free on the net).

It is in the last chapter "Beyond Reason" that philosophical failings are most acute as we return to the mistakes suggested by my comments on the title. Reasoning is another word for thinking, which is a disposition like knowing, understanding, judging etc. As Wittgenstein was the first to explain, these dispositional verbs describe propositions (sentences which can be true or false) and thus have what Searle calls Conditions of Satisfaction (COS). That is, there are public states of affairs that we recognize as showing their truth or falsity. "Beyond reason" would mean a sentence whose truth conditions are not clear, and the reason would be that it does not have a clear context. It is a matter of fact if we have clear COS (i.e., meaning) but we just cannot make the observation--this is not beyond reason but beyond our ability to achieve, but it's a philosophical (linguistic) matter if we don't know the COS. "Are the mind and the universe computers?" sounds like it needs scientific or

mathematical investigation, but it is only necessary to clarify the context in which this language will be used since these are ordinary and unproblematic terms and it is only their (lack of a clear) context which is puzzling. E.g, the “self-referential” paradoxes on p344 arise because the context and so the COS are unclear.

On p140 we might note that 1936 was not actually “long” before computers since Zeus in Germany and Berry and Atanasoff in Iowa both made primitive machines in the 30’s, though these pioneers are quite unknown to many in the field. I saw some of Zeus’s in the Deutsches Museum in Munich while the B & A machine was reconstructed from his design recently at Iowa State University, where they worked.

Wittgenstein discussed the philosophical aspects of computers some years before they existed (see Gefwert, Proudfoot etc.).

On p347, what we discovered about irrational numbers that gave them a meaning is that they can be given a use or clear COS in certain contexts and at the bottom of the page our “intuitions” about objects, places, times, length are not mistaken—rather we began using these words in new contexts where the COS of sentences in which they are used were utterly different. This may seem a small point to some, but I suggest it is the whole point. Some “particle” which can “be in two places” at once is just not an object and/or is not “being in places” in the same sense as a soccer ball, i.e., like so many terms its language games have clear COS in our mesoscopic realm but lack them (or have different and commonly unstated ones) in the macro or micro realms.

Regarding his reference on p366 to the famous experiments of Libet, which have been taken to show that acts occur before our awareness of them and hence negate will, this has been carefully debunked by many including Searle and Kihlstrom.

It is noteworthy that on the last page of the book he comments on the fact that many of the basic words he uses do not have clear definitions, but does not say that this is because it requires much of our innate psychology to provide meaning, and here again is the fundamental mistake of philosophy. “Limit” or “exist” has many uses but the important point is— what is its use in this context. “Limit of reason” or “the world exists” do not (without further context) have a clear meaning (COS) but “speed limit on US 15” and “a life insurance policy exists for him” are perfectly clear.

Regarding solipsism on p369, this and other classical philosophical ‘positions’ were



shown by W to be incoherent.

And finally, why exactly is it that quantum entanglement is more paradoxical than making a brain out of proteins and other goop and having it feel and see and remember and predict the future?

Is it not just that the former is new and not directly present to our senses (i.e., we need subtle instruments to detect it) while animal nervous systems have been evolved to do the latter hundreds of millions of years ago and we find it natural since birth? I don't see the hard problem of consciousness to be a problem at all, or if one insists then ok, but it's on all fours with endless others –why there is (or what exactly is) space, time, red, apples, pain, the universe, causes, effects, or anything at all.

Overall an excellent book provided it is read with this review in mind.

