

# Structural Idealism

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## 1. Introduction

We propose a new type of idealism we call *structural idealism*. Structural idealism uses computational formalisms to describe an idealist ontology. Although logical formalisms have been used to sketch the structural aspects of idealist ontologies before (Harris, 1982; Thagard, 1982), structural idealism is unique in its use of computational formalisms to describe both the structures and the processes in idealistic ontologies.

Computational formalisms have been gaining currency in philosophy, particularly in the philosophy of science (Thagard, 1988, 1992; Shrager & Langley, 1990; Slezak, 1989). Such formalisms have also been employed in metaphysics (Henry & Geertsen, 1986). Our work extends these computational formalisms to idealism. It should come as no surprise that computational formalisms are especially useful for idealism. Contemporary research in cognitive science and artificial intelligence uses computational formalisms to provide precise descriptions of the structure and processes of minds. Idealism has always concerned itself with minds and mental processes. The use of computational formalisms for idealistic ontology is thus a natural one.

Our objective here is to provide a precise, computational description of an idealist ontology. It is not our intention to present arguments for the ontology we articulate here; such arguments can be found in the classical idealist texts. Our intention here is purely descriptive. We provide both a logical description of an ontological structure (the divine hierarchy) and an algorithmic description of an ontological process (the divine process). In so doing, we are inspired by classical idealistic systems as well as by more current work.

## 2. Intentional Objects and Minds

Structural idealism posits *intentional objects* as basic. An intentional object is an ideal object. Intentional objects are analogous to Leibnizian monads (Leibniz, 1965), except that we conceive of them as typically having parts. An intentional object is always, and necessarily, the object of at least one *intentional relation*. Examples of intentional relations are affirming, denying, conceiving, perceiving, imagining, loving, and hating (Searle, 1983; Vendler, 1972). Every intentional object is an active, computational agent. We conceive of intentional object as being at least as complex as a vonNeumann machine. Every intentional object has three important parameters: (1) its selection flag; (2) its external input; (3) its activation. The selection flag is a binary variable, either on or off. The external input and activation are continuous variables whose values range between some minimum and maximum real limits. The meanings of these parameters will be explained in the sequel.

An intentional object is either complex or simple. It is complex if and only if it is a whole composed of parts; it is simple otherwise. We believe that a whole is distinct

from its parts, and is not identifiable with the set of its parts or with the set of its parts and the relations holding among those parts. We therefore reject the Plotinian principle of integral omnipresence (Plotinus, 1952, VI.4-5). A whole is a distinct object that *contains* distinct parts. Wholes sometimes contain parts that are in turn wholes containing parts. The result is a *part-whole hierarchy*.

Some intentional objects are *minds*. Every mind is itself a complex intentional object. We therefore conceive of minds as societies of intentional objects. In the *Republic*, Plato conceived of the soul as a society composed of citizens. The social model of mind was developed extensively by Nietzsche (1966, sec. 12, 17, 19, 36). In many respects, we follow the Nietzschean conception of mind as a society. Every mind is an intentional object for itself, since every mind is able to reflect on itself (Berkeley, 1988, 179-181). Some minds are intentional objects for other intentional objects. That is to say, some minds are parts of more complex intentional objects. In accordance with idealist principles, we do not distinguish between mind and world. Every mind is a world. In other words, a mind composed of intentional objects, is a world; specifically, it is an *intentional world*.

We posit seven different types of intentional objects: (1) God; (2) compossible distributions of finite minds; (3) finite minds; (4) propositions; (5) concepts; (6) sensible things; (7) sensations. God is not only a type of intentional object, but also the only instance of that type. There is only one God. All other types have an infinite number of instances. The instances of the types of intentional objects are organized into a part-whole hierarchy. God is a whole composed of compossible distributions of finite minds; compossible distributions of finite minds are wholes composed of finite minds; finite minds are wholes composed of propositions; propositions are wholes composed of concepts; concepts are wholes composed of concepts or sensible things; sensible things are wholes composed of sensations. Sensations are wholes without parts. Sensations are the only simple intentional objects.

We conceive of God as an infinite mind. God is the maximal whole that contains all intentional objects. God is a divine part-whole hierarchy. God is infinite and eternal in time. By infinite in time we mean that God exists through an infinite series of moments. By eternal in time we mean that God and all the parts of God exist eternally. We thus conceive of intentional objects and their relations as eternal objects and eternal relations, neither coming into being nor passing out of being. Nevertheless, God is not a changeless structure.

God is a structure in process. We intend our computational formalisms to capture this process in an algorithmic description. We conceive of change as change in the degrees of actuality of intentional objects. We conceive of the *degree of actuality* of an intentional object as a quantity varying from a minimum value of zero to one. The degree of actuality of an intentional object is the value of its activation parameter and is referred to simply as the activation of the object. An object whose degree of actuality is zero has no actuality and is only possible; an object whose degree of actuality is one is fully actualized. Actuality should not be confused with activity; the activity of an intentional object presupposes only the bare existence of that object and has nothing to do with its degree of actualization. From moment to moment, the distribution of degrees of actuality over the intentional objects in God changes. At every moment, therefore, God exists in a particular state. A *state* of God is a distribution of degrees of actuality over the intentional objects in God. We let the variable  $t$  range over all the moments in time. We designate the state of God at time  $t$  as  $\text{God}(t)$ .

God contains an infinite set of compossible distributions of finite minds. A compossible distribution of finite minds is a whole composed of compossible intentional worlds of finite minds. We conceive of God as containing an infinite set of finite minds. We designate the set of finite minds in God as  $Minds = \{m_1, m_2, \dots\}$ . Although we hold that Minds is infinite, we limit our discussion to finite models in which Minds has a finite cardinality. Each finite mind  $m_i$  is a set of possible intentional worlds  $\{W_1(m_i), W_2(m_i), \dots\}$ . For every finite mind in God, each compossible distribution of finite minds contains one of the possible intentional worlds for that finite mind. There are no intentional objects shared by distinct finite minds, so that  $W_h(m_i)$  and  $W_k(m_j)$  have a null intersection for all  $i \neq j$  and for all  $h$  and  $k$ . That is to say, no two distinct finite minds bear intentional relations to the same intentional object; no two finite minds ever experience the same thing. This is the principle of privacy (Ayer, 1969, ch. III, 12). The *state* of a finite mind  $m_i$  at time  $t$  is a distribution of degrees of actuality over all the intentional objects in all the possible intentional worlds of that mind. The state of  $m_i$  at time  $t$  is written  $W(m_i, t)$ .

A finite mind is a set of possible intentional worlds. Each possible intentional world is a whole composed of propositions which the finite mind entertains, affirms, or denies. A proposition is a whole composed of concepts. Specifically, it is a whole whose parts are its predicate and its arguments. Predicates and arguments are concepts. For example, "Mothers produce babies" is a proposition in which the predicate is the concept "produce" and the arguments are the concepts "mother" and "baby".<sup>1</sup> A concept is a whole composed of either concepts or sensible things. A concept composed of sensible things is a whole whose parts are its instances. Each instance of a concept is a particular sensible thing. For example, the concept "dog" is a whole composed of particular sensible things that are grouped together based on their similarity.<sup>2</sup> A sensible thing is a whole composed of sensations. We hold that every finite mind is associated with a particular sensible thing, namely, its body, that has unusual properties (Carnap, 1928, sec. 129). We follow Goodman's (1951) conception of visual sensations as color-spot-moments. Sensations in other modalities are conceived of analogously.

### 3. Compossibility and Impossibility

#### 3.1 Compossibility and Perspectival Consistency

Two possible intentional worlds  $W(X)$  and  $W(Y)$  of finite minds  $X$  and  $Y$  are either compossible or impossible. The notions of compossibility and impossibility

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<sup>1</sup>A proposition like "Mothers produce babies" is a generic proposition because its arguments are generic. Propositions can also have specific arguments, which are concepts of individuals. An individual is a concept, not a sensible thing, because an individual can have many sensible things as its instances.

<sup>2</sup>A concept is not the set of its instances. A concept is a whole composed of sensible things in accordance with a principle of composition. This principle is a kind of superposition, such as occurs in connectionist networks that learn by changing their weights on exposure to new input patterns (Rumelhart et al., 1986). In other words, sensible things are parts of concepts like ingredients are parts of cakes, not like bricks are parts of houses.

are taken from Leibniz (1956). Two possible intentional worlds  $W(X)$  and  $W(Y)$  are *compossible* if each is consistent with the other; they are *incompossible* if either is inconsistent with the other. Here we describe one type of consistency, *perspectival consistency*.

The notion of perspective was introduced into metaphysics by Leibniz (1965) and extensively refined by Russell (1952, pp. 94-100). Our treatment of perspectives and perspectival consistency is inspired by Russell's formal considerations. We deny, however, that perspectives are perspectives on the universe or real world. Finally, our treatment is also inspired by a remark of Price. According to Price (1933, p. 298): "there might *be* *sensa* [i.e. sensations] existing from the point of view P when I am at another point of view P' . . . such such *sensa* might be *sensed* though not by me; this amounts, of course, to the hypothesis that the point of view P' [*sic.*] is occupied by a mind other than self."

In order to describe perspectival consistency, we require an analysis of the perspective of a finite mind. For the sake of simplicity, we consider only visual perspective.<sup>3</sup> At any moment  $t$ , finite mind  $m_i$  experiences a visible world composed of visible things. Visible things are those sensible things experienced through the visual modality. Let  $V(m_i)$  be the set of visible things in  $W(m_i)$  along with the spatial relations among those things;  $V(m_i)$  is a set of objects embedded in a three-dimensional, Euclidean space.

Each finite mind  $m_i$  sees the visible things in  $V(m_i)$  from a certain perspective. A *perspective* is a vector or ray in  $V(m_i)$ . A vector in  $V(m_i)$  is a pair of points. A point is given by cartesian coordinates  $(x,y,z)$  in  $V(m_i)$ . The first point indicates the origin of the perspective; the second indicates the direction of the perspective. Every finite intentional world has an actual perspective and a set of possible perspectives. In the case of human minds, the *actual perspective* is a vector whose origin is a point just in front of the bridge of the nose right between the eyes. The direction of the actual perspective is that point at which the gaze terminates. In other words, the actual perspective is simply the ray traced out by the gaze. We designate the origin of the actual perspective as  $(0,0,0)$  and let all other points in the visible space  $V(m_i)$  be designated by cartesian coordinates with respect to the origin  $(0,0,0)$ .

Although we actually experience the visible things in our visible worlds only from our actual perspective, we can easily imagine seeing the things in our visible worlds from other perspectives. Every other point in  $V(m_i)$  determines a set of *possible perspectives* from which it is possible for  $m_i$  to experience  $V(m_i)$ . If  $m_i$  experienced  $V(m_i)$  from any perspective other than its actual perspective, it would experience a version of  $V(m_i)$  whose visible objects would have different spatial relations. The visible world  $V(m_i)$  as experienced from the possible perspective P is designated  $V(m_i,P)$ . Although  $V(m_i,P)$  is a different visible world than  $V(m_i)$ , it is not unrelated to  $V(m_i)$ . In fact,  $V(m_i,P)$  is just a transformation of  $V(m_i)$  by translation and rotation. Informally speaking, to derive  $V(m_i,P)$  from  $V(m_i)$ , you simply shift (i.e. translate) your head from  $(0,0,0)$  to the origin of P without turning it, and then turn (i.e. rotate) your head so that your gaze follows the direction of P. Alternatively and equivalently, we can conceive of the position of our

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<sup>3</sup>Although we defined perspectival consistency only in terms of vision, it is easy to extend the analysis to the auditory sense (in which I hear you but do not see you).

heads as fixed and the system of objects in  $V(m_i)$  as moving until the origin of P becomes (0,0,0) and then rotating until the direction of P is the direction of the actual perspective.<sup>4</sup>

The possible intentional world  $W(X)$  is *directly perspectivally consistent* with  $W(Y)$  if and only if the actual perspective of Y is some possible perspective P for X and there is a homomorphism from  $V(X,P)$  to  $V(Y)$ .<sup>5</sup> A homomorphism from  $V(X,P)$  to  $V(Y)$  is a function that preserves part of the structure of  $V(X,P)$ . Two perspectivally consistent states are therefore analogous, that is, they are similarly structured, even though they share no objects in common. Direct perspectival consistency clearly requires that X sees the body of Y. Direct perspectival consistency is reflexive. Direct perspectival consistency is also normally symmetric. If I can see your body, there is some normally some rotation or translation of your actual perspective such that you can see mine.<sup>6</sup> If  $W(X)$  is directly perspectivally consistent with  $W(Y)$ , then  $W(Y)$  is directly perspectivally consistent with  $W(X)$ . Though its formal construction requires some effort, direct perspectival consistency is a notion commonly employed in everyday life. In the visible presence of my lover's body, it is easy enough for me to determine the direction of her gaze, to imagine myself in her place, to imagine what she sees. Indeed, it is necessary for me to do this if we are to successfully communicate and interact. For I cannot reasonably expect her to deal with objects that I know she cannot see.

Consider the following illustration of direct perspectival consistency. Suppose Minds consists of just two minds, X and Y. We define a possible intentional world  $W(X)$  for X and a possible intentional world  $W(Y)$  for Y such that  $W(X)$  and  $W(Y)$  are directly perspectivally consistent. In  $W(X)$ , the mind X sees  $V(X)$ . The visible world  $V(X)$  contains the set of objects  $\{BODY_X^S, DOOR_X, BODY_X^O\}$ . The object  $BODY_X^S$  is the body of X. The object  $DOOR_X$  is a closed glass door in front of X; the right hand of X is resting on the handle of this door. The object  $BODY_X^O$  is another body like the body of X; it is the body of an other for X; this other body is on the other side of  $DOOR_X$  and is facing X. Since we supposed only two finite minds, we know that  $BODY_X^O$  is the body of Y, and that X sees the body of Y on the other side of  $DOOR_X$ . In  $W(Y)$ , the mind Y sees  $V(Y)$ . The visible world  $V(Y)$  contains the set of objects  $\{BODY_Y^S, DOOR_Y, BODY_Y^O\}$ . The object  $BODY_Y^S$  is the body of Y. The object  $DOOR_Y$  is a closed glass door in front of Y; the right hand of Y is resting on the handle of this door. The object  $BODY_Y^O$  is another body like the body of Y; it is the body of an other for Y; this other body is on the other side of  $DOOR_Y$  and is facing X. Since we supposed only two finite minds, we know that  $BODY_Y^O$  is the body of X, and that Y sees the body of X on the other side of  $DOOR_Y$ .

It is clear that  $V(X)$  and  $V(Y)$  are mirror-images of one another. That is to say,  $V(X)$  and  $V(Y)$  are perfectly isomorphic under reflection. Though neither world shares

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<sup>4</sup>Such translations and rotations are standard, for instance, in virtual reality technologies. The appearance of movement is produced by shifting the or turning the scene as we have described it.

<sup>5</sup>Isomorphism is not necessarily the case, since during a change of perspective some sensible things can block other sensible things that they did not block before.

<sup>6</sup>Excluded from this consideration are situations involving one-way glass, hidden cameras, telescopes, and other such apparatus.

any objects, both worlds share the same structure. One of the possible perspectives of X on  $V(X)$  corresponds to Y's actual perspective *as imagined by X*; likewise, one of the possible perspectives of Y on  $V(Y)$  corresponds to X's actual perspective *as imagined by Y*. If the actual perspective of X were shifted and turned to be the possible perspective of Y as imagined by X, then what X sees would be perfectly isomorphic to what Y sees; if the actual perspective of Y were shifted and turned to be the possible perspective of X as imagined by Y, then what Y sees would be perfectly isomorphic to what X sees.

Direct perspectival consistency is not transitive. It depends on the ability of each mind to see the body of other minds. Obviously, I may see your body, and you may see the body of a third person, but I may not see the body of that third person. Nevertheless, it is easy to use the notion of direct perspectival consistency to define the richer notion of indirect perspectival consistency. If my visible world is directly perspectival consistent with yours, and if yours is directly perspectival consistent with a third person's, then my visible world is *indirectly perspectival consistent* with that third person's visible world. Indirect perspectival consistency is reflexive, symmetric, and transitive. It is easy to use the transitivity of perspectival consistency to form sets of indirectly perspectival consistent finite minds in which not all minds need sensory contact with the bodies of other minds.

### 3.2 Compossible Distributions over Sets of Finite Minds

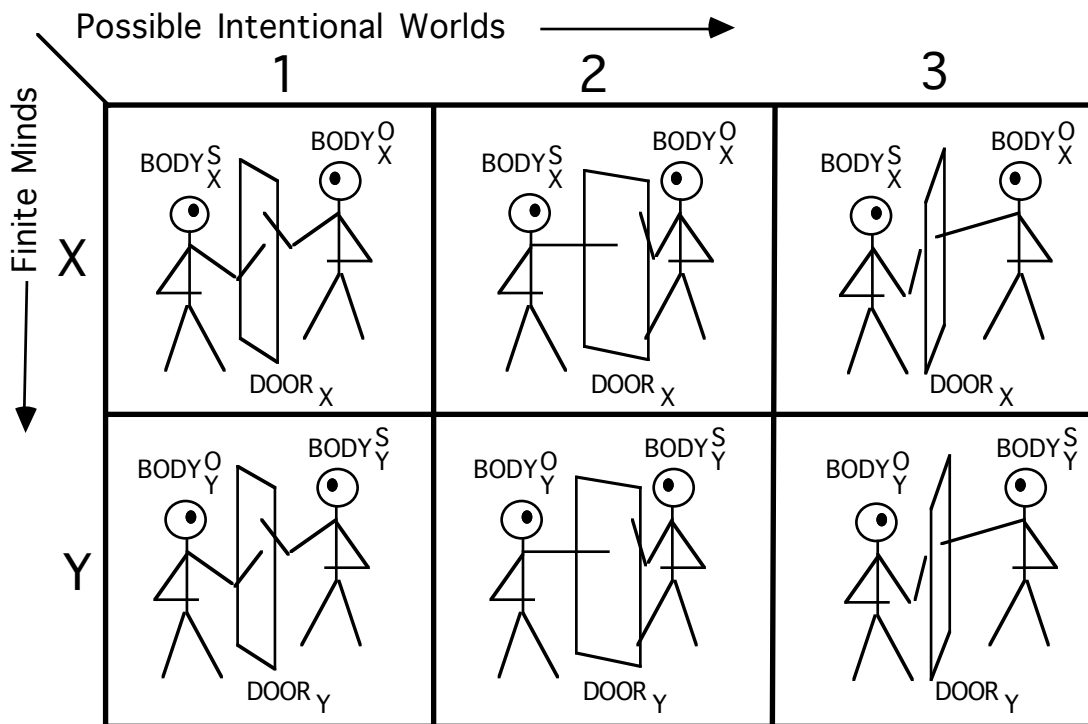
A compossible distribution over a set of finite minds is a whole whose parts are compossible intentional worlds of finite minds. An intentional object C is a compossible distribution over a set M of finite minds if and only if C contains a possible intentional world  $W(m_i)$  for each mind  $m_i$  in M and  $W(m_i)$  is compossible with  $W(m_j)$  for all  $i$  not equal  $j$ . Possible intentional worlds of distinct finite minds are compossible if and only if they are consistent; so far we have discussed only one type of consistency, perspectival consistency, and so we define compossible distributions of states over finite minds in terms of perspectival consistency.

We illustrate compossible distributions by considering all the possible intentional worlds for the minds X and Y. The first possible intentional world for X is  $W_1(X) = W(X)$  as defined earlier. The first possible intentional world for Y is  $W_1(Y) = W(Y)$  as defined earlier. Suppose that  $DOOR_X$  and  $DOOR_Y$  each swing either way. There are two other possible intentional worlds for X and Y. In the second possible intentional world  $W_2(X)$  for X, the arm of  $BODY_X^S$  is extended and  $DOOR_X$  is open in the direction of  $BODY_X^O$ . If  $W_1(X)$  is followed by  $W_2(X)$ , we say that  $BODY_X^S$  pushes  $DOOR_X$  towards  $BODY_X^O$ . In the third possible intentional world  $W_3(X)$ , the arm of  $BODY_X^S$  is retracted and  $DOOR_X$  is open in the direction of  $BODY_X^S$ . If  $W_1(X)$  is followed by  $W_3(X)$ , we say that  $BODY_X^S$  pulls  $DOOR_X$  towards  $BODY_X^S$ . In the second possible intentional world  $W_2(Y)$  for Y, the arm of  $BODY_Y^S$  is retracted and  $DOOR_Y$  is open in the direction of  $BODY_Y^S$ . If  $W_1(Y)$  is followed by  $W_2(Y)$ , we say that  $BODY_Y^S$  pulls  $DOOR_Y$  towards  $BODY_Y^S$ . In the third possible intentional world  $W_3(Y)$ , the arm of  $BODY_Y^S$  is extended and  $DOOR_Y$  is open in the direction of  $BODY_Y^O$ . If  $W_1(Y)$  is

followed by  $W_3(Y)$ , we say that  $BODY_Y^S$  pushes  $DOOR_Y$  towards  $BODY_Y^O$ . The possible intentional worlds for finite minds  $X$  and  $Y$  are shown in Figure 1.

It should be clear that the possible intentional world  $W_i(X)$  is consistent with the possible intentional world  $W_k(Y)$  for  $i = k$ , and that  $W_i(X)$  is inconsistent with  $W_k(Y)$  for  $i \neq k$ . For example,  $W_2(X)$  is consistent with  $W_2(Y)$ . That is, it is consistent for the arm of  $BODY_X^S$  to be extended and for  $DOOR_X$  to be open towards  $BODY_X^O$  and for the arm of  $BODY_Y^S$  to be retracted and for  $DOOR_Y$  to be open towards  $BODY_Y^S$ . For example,  $W_2(X)$  is inconsistent with  $W_3(Y)$ . That is, it is inconsistent for the arm of  $BODY_X^S$  to be extended and for  $DOOR_X$  to be open towards  $BODY_X^O$  and for the arm of  $BODY_Y^S$  to be extended and for  $DOOR_Y$  to be open towards  $BODY_Y^O$ .

We say that the combinations  $\{W_1(X), W_1(Y)\}$ ,  $\{W_2(X), W_2(Y)\}$  and  $\{W_3(X), W_3(Y)\}$  of possible intentional worlds are *compossible distributions of possible intentional worlds across the finite minds X and Y*. All other combinations of possible intentional worlds for  $X$  and  $Y$  are *incompossible distributions of states across the finite minds X and Y*. Since  $X$  and  $Y$  are the only finite minds in God, these sets are compossible or incompossible distributions of possible intentional worlds across all finite minds.



**Figure 1.** Some finite minds with their possible intentional worlds.

### 3.3 Compossibility and Impossibility as Constraints

Compossibility and impossibility act as *constraints* between intentional objects. God is able to simultaneously actualize compossible intentional objects. More

strongly, if two intentional objects are compossible, God is obligated to simultaneously realize both of them. This *conjunctive obligation* is a positive constraint. That is, compossibility is a positive constraint. God is not able to simultaneously actualize impossible intentional objects. If two intentional objects are impossible, God is prohibited from simultaneously realizing both of them. This *disjunctive prohibition* is a negative constraint. That is, impossibility is a negative constraint. Functioning as obligations and prohibitions, relations of compossibility and impossibility are causal relations between intentional objects. Importantly, compossibility and impossibility are also intentional relations. If  $\alpha$  is impossible with  $\beta$ , then  $\alpha$  *denies*  $\beta$ . If  $\alpha$  is compossible with  $\beta$ , then  $\alpha$  *affirms*  $\beta$ .

Two factors prevent relations of compossibility and impossibility from being fully or strictly deterministic. First, intentional objects typically participate in many relations of compossibility and impossibility. Second, intentional objects are not either strictly actualized or strictly not actualized; actuality is a matter of degree. An intentional object's relations of compossibility tend to increase its degree of actualization while its relations of impossibility tend to decrease its degree of actualization. Relations of compossibility and impossibility function not as strict logical constraints, but rather as soft logical constraints. If  $\alpha$  is impossible with  $\beta$ , then the realization of  $\alpha$  *discourages* the realization of  $\beta$ . If  $\alpha$  is compossible with  $\beta$ , then the realization of  $\alpha$  *encourages* the realization of  $\beta$ . Through their relations of compossibility, intentional objects cooperate amongst themselves for actualization. Through their relations of impossibility, intentional objects compete amongst themselves for actualization.

## 4. The Divine Hierarchy

### 4.1 Outline of the Structure of the Divine Hierarchy

The divine hierarchy is a part-whole hierarchy composed of four levels. The highest level is God. God is a whole composed of the compossible distributions over all finite minds, so the next level below God is the set of compossible distributions. Each compossible distribution is a whole composed of possible intentional worlds of finite minds. The level below compossible distributions thus consists of possible intentional worlds of finite minds. Every possible intentional world of a finite mind is a whole composed of intentional objects. We have posited the existence of many levels of intentional objects in finite minds. For example, we have posited levels of propositions, concepts, sensible things, and sensations. For the sake of simplicity, we only deal with two levels here. We treat a finite mind as a whole composed of sensible things. We further treat sensible things as wholes composed of sensations. Although the model developed here has only one level of intermediate wholes between finite minds and sensations, it is important to see that we could insert any number of levels of intermediate wholes between finite minds and sensations.

### 4.2 The Divine Hierarchy as a Hierarchy of Connectionist Networks

We use connectionist *constraint-satisfaction* techniques to model relations of compossibility and impossibility between intentional objects. Constraint satisfaction techniques have been used to model analogical cognition (Holyoak & Thagard, 1989;



Thagard et al., 1990) and explanatory coherence (Thagard, 1992). We model impossibility as an *inhibitory connection* between impossible intentional objects; we model compossibility as an *excitatory connection* between compossible intentional objects.

There are many different kinds of constraint satisfaction models. To model the divine process, we use an interactive-activation and competition (IAC) model based on McClelland and Rumelhart's interactive-activation and competition model of word-recognition (McClelland & Rumelhart, 1981, 1989; Rumelhart & McClelland, 1982, 1986). McClelland and Rumelhart's IAC model contains three levels; the top level is the word level; the level below that is the letter level; the bottom level is the feature level. Each word in the word level is composed of four letters, each in a distinct position. Each letter is selected from the Roman alphabet. Each letter is a whole composed of features. These features are taken from a single, standard set of features. The features of which a letter is composed are vertical, horizontal, and diagonal lines.

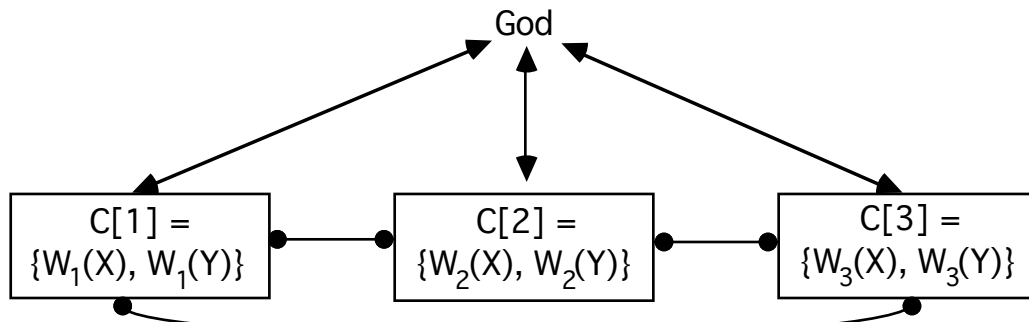
The IAC model of the divine process contains several levels. At the top level is God itself. Following McClelland & Rumelhart's linguistic analogy, we say that God is analogous to a text. Just as God is a whole composed of compossible distributions over all finite minds, so a text is composed of sentences. Each compossible distribution corresponds to a sentence. Just as a compossible distribution is a whole composed of possible intentional worlds of finite minds, so a sentence is composed of words. Each possible intentional world corresponds to a word. Just as a possible intentional world of a finite mind is a whole composed of sensible things, so a word is composed of letters. Each sensible thing corresponds to a letter. Just as a sensible thing is composed of sensations, so a letter is composed of features. Each sensation corresponds to a feature.

#### 4.3 The Array of Compossible Distributions

God is a whole composed of compossible distributions of possible intentional worlds over all finite minds. Consequently, the next level below God in the divine hierarchy is the set of compossible distributions. This level corresponds to a sentence. Compossible distributions are intentional objects for God. They are universal states of affairs in the divine mind. In our example,  $\{W_1(X), W_1(Y)\}$ ,  $\{W_2(X), W_2(Y)\}$  and  $\{W_3(X), W_3(Y)\}$  are compossible distributions of which God is composed. These sets are direct parts of God. They are also intentional objects for God, and God bears an intentional relation to each of these sets. Much as a person entertains a proposition, so God entertains compossible distributions.

The set of compossible distributions in God is designated  $C$ . Each member of  $C$  is given a number; that is, the set  $C$  is indexed to become an array. The  $i$ -th element of  $C$  is referred to by enclosing the number  $i$  in square brackets; hence the  $i$ -th element of  $C$  is  $C[i]$ . We refer to the whole array  $C$  as just  $C[]$ ; notice that the index has been dropped. We hold that  $C$  contains an infinite number of elements. However, we limit our discussion to finite models in which  $C$  contains a finite number of elements. Since any of the compossible distributions in  $C[]$  can be realized in God, there is an excitatory connection between God and each element in  $C[]$ . However, since only one element in  $C[]$  can be actualized at one time, there is an inhibitory connection between each element of  $C[]$  and every other element of  $C[]$ . A collection of objects whose members are all linked by inhibitory relations is called an. Thus  $C[]$  is an inhibitory cluster. Figure 2 shows the connections between God and the three compossible distributions in our

example. A line terminated by arrowheads is an excitatory connection; a line terminated by balls is an inhibitory connection.



**Figure 2.** God and compossible distributions.

#### 4.4 Propositions and Transition Rules

A transition rule is an intentional object. More precisely, a transition rule is a type of proposition known by God. We model propositions as semantic networks whose nodes are intentional objects (Sowa, 1990). In particular, a transition rule is a proposition that determines how the past states of God influence the next state of God. Transition rules enable the history the divine process to condition its future. Each transition rule is a proposition with the form ANTECEDENT  $\overset{P}{\square}$  CONSEQUENT.

In general, the ANTECEDENT of a transition rule is some proposition that is true or false of the current state  $God(t)$  or past states of God, that is, states  $God(t-i)$  for some positive  $i$ . In general, any intentional object can participate in the ANTECEDENT of a transition rule. Here, however, we restrict our attention to transition rules whose ANTECEDENTs are conjunctions of propositions of the form  $(x \in God(t-i))$  for some compossible distribution  $x$  and some non-negative integer  $i$ . The formula  $(x \in God(t-i))$  is true if and only if the compossible distribution  $x$  is actual in  $God(t-i)$ .

A transition rule does not automatically entail its CONSEQUENT given its ANTECEDENT. If the ANTECEDENT of a transition rule is true, then there is some probability that its CONSEQUENT is made true. If a transition rule makes its CONSEQUENT true, we say that the rule *fires*. The implication of each transition rule is labelled with a number  $P$  between 0.0 and 1.0. The number  $P$  indicates the probability that the rule fires given that its ANTECEDENT is true.

If a rule fires, its CONSEQUENT is made true. In general, the CONSEQUENT is any proposition that can be true of the next state  $God(t+1)$  of God. In general, any intentional object except a sensation is able to participate in the CONSEQUENT of a transition rule.<sup>7</sup> Here, however, we restrict our attention to transition rules whose CONSEQUENTs are conjunctions of propositions of the form  $(x \in God(t+1))$  for some compossible distribution  $x$ . If a conjunction is made true, each of its members is made true. If the proposition  $(x \in God(t+1))$  is made true, it assigns one unit of *external input* to the

<sup>7</sup>The external inputs of sensations are set by emanation only. See the description of emanation in the description of the divine process.

compossible distribution  $x$  in  $\text{God}(t+1)$ . External input is a parameter involved in the computation of new states of God; the precise use of external input is defined below.

Figure 3 shows the transition rules for our example involving the two bodies and the door. Note that the transition rules in Figure 3 involve only compossible distributions. In more complex models of the divine hierarchy, transition rules would involve intentional objects of all types except sensations. Most importantly, a more sophisticated model of the divine hierarchy would include transition rules involving concepts over sensible things; such transition rules express empirical regularities.

$$\begin{array}{l}
 C[1] \in \text{God}(t) \ \& \ C[3] \in \text{God}(t-1) \ \overset{0}{\underset{9}{\text{P}}} \ C[2] \in \text{God}(t+1) \\
 C[1] \in \text{God}(t) \ \overset{0}{\underset{9}{\text{P}}} \ C[1] \in \text{God}(t+1) \\
 C[1] \in \text{God}(t) \ \& \ C[2] \in \text{God}(t-1) \ \overset{0}{\underset{9}{\text{P}}} \ C[3] \in \text{God}(t+1) \\
 C[2] \in \text{God}(t) \ \overset{0}{\underset{9}{\text{P}}} \ C[1] \in \text{God}(t+1) \\
 C[2] \in \text{God}(t) \ \overset{0}{\underset{9}{\text{P}}} \ C[2] \in \text{God}(t+1) \\
 C[3] \in \text{God}(t) \ \overset{0}{\underset{9}{\text{P}}} \ C[1] \in \text{God}(t+1) \\
 C[3] \in \text{God}(t) \ \overset{0}{\underset{9}{\text{P}}} \ C[3] \in \text{God}(t+1)
 \end{array}$$

**Figure 3.** Transition rules for our example.

#### 4.5 The Matrix of Possible Intentional Worlds

Just as a sentence is a whole composed of words, so each compossible distribution in  $C[]$  is a whole composed of possible intentional worlds of finite minds. The level below the array of compossible distributions thus consists of possible intentional worlds of finite minds. Each compossible distribution  $C[i]$  in  $C[]$  contains one possible intentional world for each mind in  $\text{Minds}$ . Let  $\text{MAX}(\text{Minds})$  be the cardinality  $\text{Minds}$ ; then  $C[i]$  contains  $\text{MAX}(\text{Minds})$  parts, each of which is a possible intentional world for exactly one mind in  $\text{Minds}$ . If there are  $\text{MAX}(C)$  compossible distributions and  $\text{MAX}(\text{Minds})$  finite minds in our finite model, then there are  $\text{MAX}(C) * \text{MAX}(\text{Minds})$  possible intentional worlds in God.

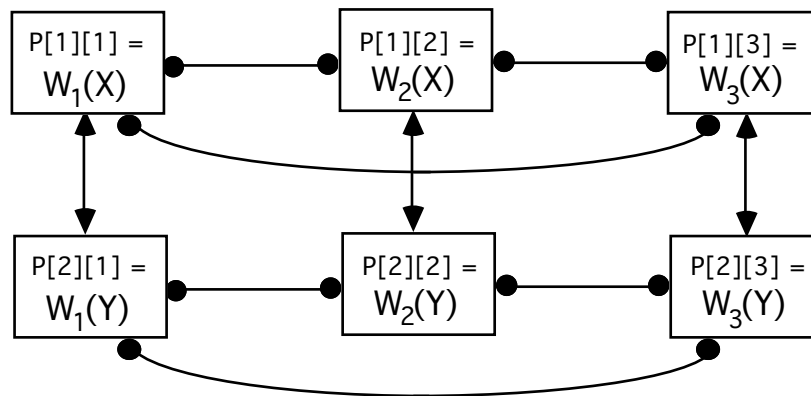
We have assigned a number to each finite mind in  $\text{Minds}$  and also to each compossible distribution in  $C[]$ . Using these numbers, we construct a matrix  $P$  whose elements are possible intentional worlds of finite minds. The numbers for finite minds are used to label the rows of the matrix  $P$ ; the numbers for compossible distributions are used to label the columns of the matrix  $P$ . We designate all the elements in the  $i$ -th row of the matrix  $P$  by the notation  $P[i][]$ ; notice that the column index is missing. The row  $P[i][]$  contains all the possible intentional worlds for the finite mind  $m_i$ . We designate all the elements in the  $j$ -th column of the matrix  $P$  by the notation  $P[][j]$ ; notice there is no row index. The column  $P[][j]$  contains all the possible intentional worlds in the  $j$ -th compossible distribution. The notation  $P[i][j]$  is used to refer to the element at the  $i$ -th row in the  $j$ -th column of  $P$ ; thus  $P[i][j]$  is the  $j$ -th possible intentional world of the finite mind  $i$ . We hold that  $P$  contains an infinite number of elements. However, in any finite model the matrix  $P$  has  $\text{MAX}(C)*\text{MAX}(\text{Minds})$  elements. It contains all possible intentional worlds of all finite minds.

Each compossible distribution  $C[j]$  is a whole that contains a possible intentional world for each finite mind  $m_i$  in  $\text{Minds}$ . If a whole is realized, then all its parts should be

realized. If compossible distribution  $C[j]$  is realized, then all its parts should be realized. The parts of  $C[j]$  are in the column  $P[][j]$ . Hence if  $C[j]$  is realized, each element in  $P[][j]$  should be realized. Thus for  $j = 1$  to  $\text{MAX}(C)$ , and for  $i = 1$  to  $\text{MAX}(\text{Minds})$ , compossible distribution  $C[j]$  is compossible with possible intentional world  $P[i][j]$ . That is to say, there is an excitatory connection between  $C[j]$  and each element of the column  $P[][j]$ .

Each element of the column  $P[][j]$  is a part of the compossible distribution  $C[j]$ . If *any* part of a whole is actualized, then *every* part of that whole should also be actualized. Hence if any element in the column  $P[][j]$  is realized, then all the other elements of the column  $P[][j]$  should also be actualized. To encourage this mutual realization of parts, there are excitatory connections between all the elements in each column  $P[][j]$  for  $j = 1$  to  $\text{MAX}(C)$ .

The row  $P[i][]$  contains all the possible intentional worlds for the finite mind  $m_i$ . Only one possible intentional world for each finite mind can be actualized at any single time  $t$ , so only one element in row  $P[i][]$  can be actualized at any single time  $t$ . To ensure this exclusivity, each element of row  $P[i][]$  has an inhibitory connection with every other element in row  $P[i][]$ . Thus row  $P[i][]$  is an inhibitory cluster (i.e. a contrast set). The excitatory and inhibitory connections among the elements in  $P[][]$  for our example are shown in Figure 4.



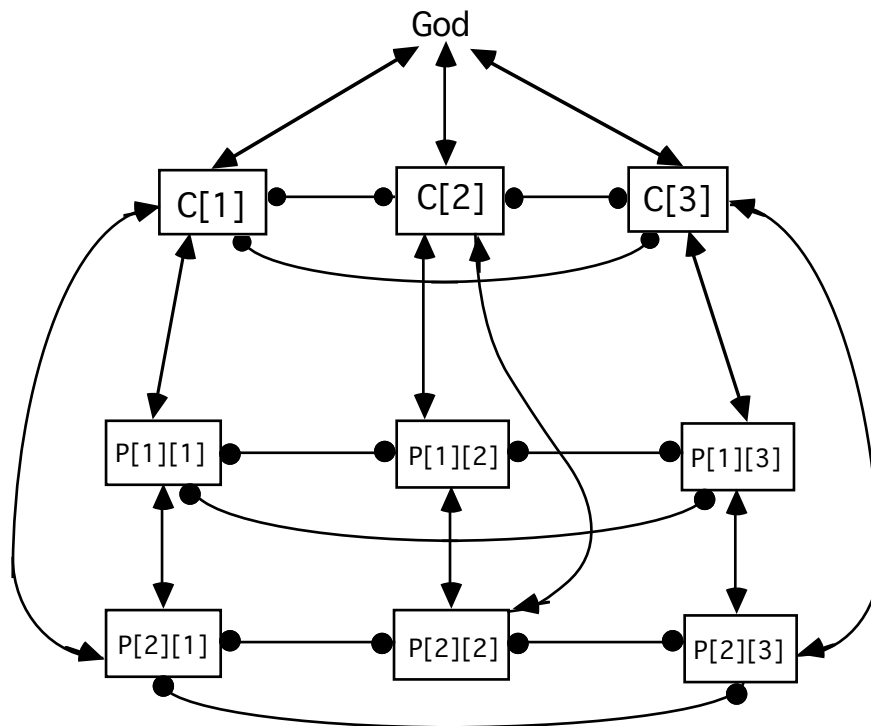
**Figure 4.** Connections among possible intentional worlds.

#### 4.6 The Structure of the Supersensible Levels of the Divine Hierarchy

Intentional objects above the level of sensible things in the divine hierarchy are supersensible. In the divine hierarchy as we have constructed it, God, compossible distributions, and possible intentional worlds are supersensible.

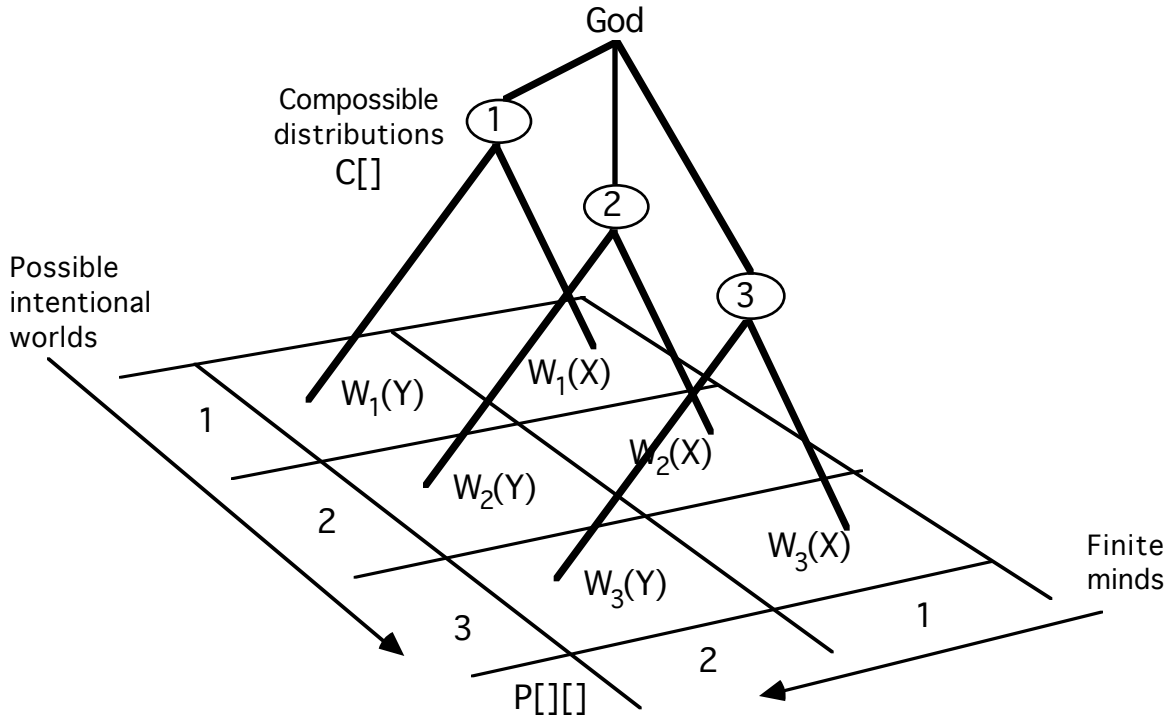
In our example, God is composed of three compossible distributions  $C[1]$ ,  $C[2]$ , and  $C[3]$ . Since a whole is compossible with all of its parts, God has an excitatory connection to each element of  $C[]$ . Since only one element of  $C[]$  can be realized at one time, each element of  $C[]$  is impossible with every other element of  $C[]$  and the elements of  $C[]$  are all linked by inhibitory connections. The matrix of possible intentional worlds contains rows for the two minds  $X$  and  $Y$  and columns for the three possible intentional worlds. Thus  $P$  has six elements,  $P[1][1]$ ,  $P[1][2]$ ,  $P[1][3]$ ,  $P[2][1]$ ,  $P[2][2]$ , and  $P[2][3]$ . The row  $P[1][]$  is the set of possible intentional worlds for mind  $X$ . Only one possible intentional world can be actualized at one time for one finite mind, so the elements of the row  $P[1][]$  are linked by inhibitory connections. The row  $P[2][]$  is the set of possible intentional worlds for mind  $Y$ . The elements of the row  $P[2][]$  are linked

by inhibitory connections. The column  $P[][1]$  is the set of possible intentional worlds in compossible distribution  $C[1]$ . These possible intentional worlds are parts of the same whole and are thus compossible. They are linked by excitatory connections. Likewise, the column  $P[][2]$  is the set of possible intentional worlds in compossible distribution  $C[2]$ . The elements of  $P[][2]$  are linked by excitatory connections. Finally, the elements of  $P[][3]$  are linked by excitatory connections. These elements and their interrelations are shown in Figure 5.

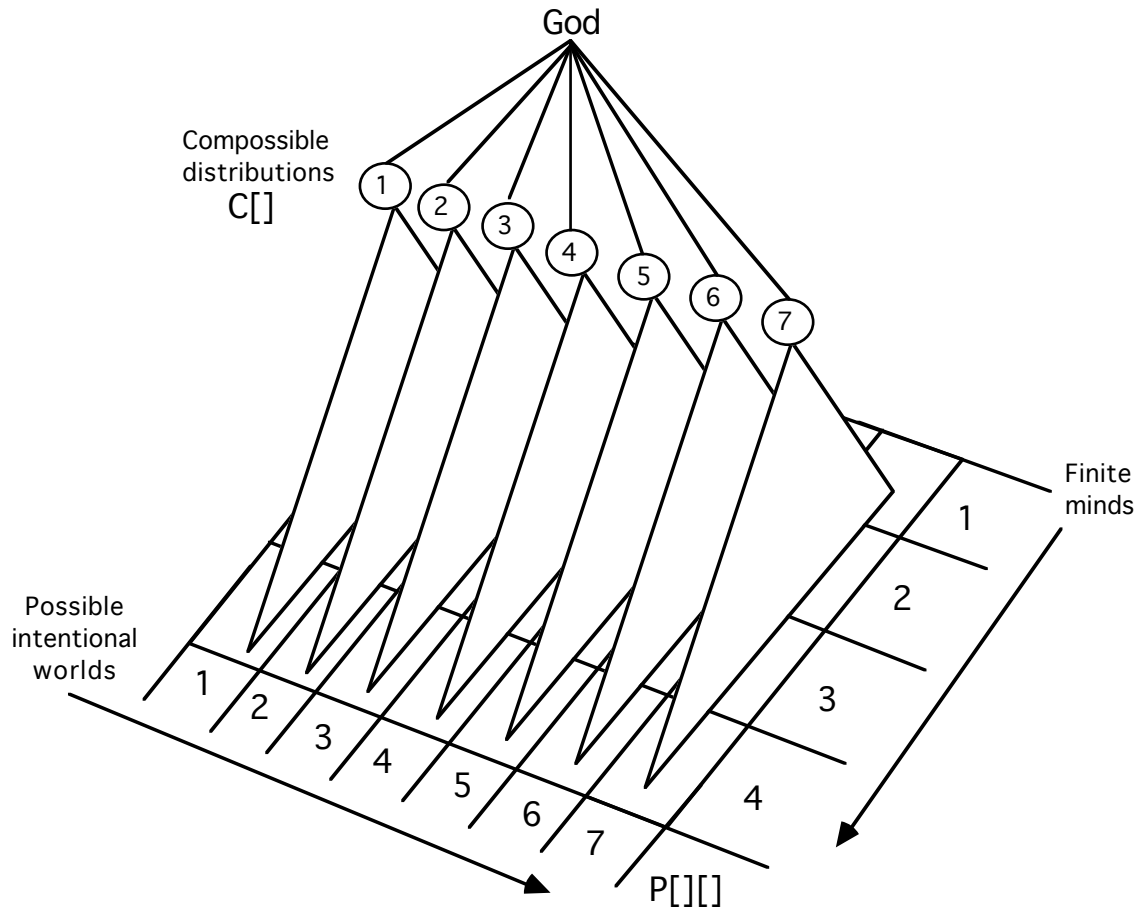


**Figure 5.** Objects and connections in the supersensible.

While displaying each intentional object with all of its connections reveals detail, it also impedes visualization. To enhance visualization, we display the matrix of possible intentional worlds without explicitly illustrating the connections. The connections are implicit in the matrix. Figure 6 shows the supersensible levels of the divine hierarchy for our example. Connections in the matrix of possible intentional worlds are implicit, as are connections between compossible distributions. Lines between levels indicate excitatory connections. The general structure of the supersensible levels of the divine hierarchy is sketched in Figure 7. In Figure 7, a triangle extending downwards from a compossible distribution indicates a set of excitatory connections to the possible intentional worlds spanned by the base of the triangle.



**Figure 6.** Connections from compossible distributions to worlds.



**Figure 7.** The general form of the supersensible.

#### 4.7 From Possible Intentional Worlds to Sensible Things

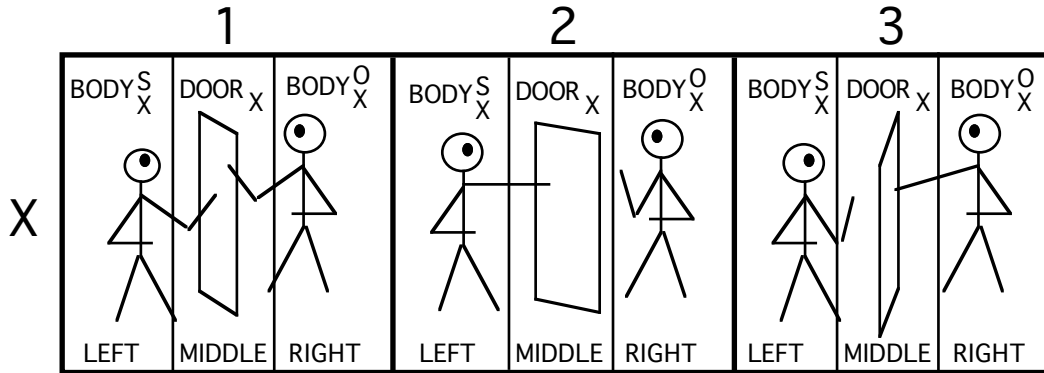
Just as a word is a whole composed of letters, so every possible intentional world of a finite mind is a whole composed of sensible things. Each possible intentional world is analogous to a word, and each sensible thing is analogous to a letter. Each letter in a word occupies a position in a word. Likewise, each sensible thing in a possible intentional world occupies a position in that world.

To model the positions in possible intentional worlds, we proceed as follows. We first model the visible world  $V(m_i)$  for finite mind  $m_i$  as a three-dimensional Euclidean space exhaustively partitioned into cubes by its cartesian coordinates, much as a piece of graph paper is exhaustively partitioned into squares. Our formal analysis at this point is inspired by Glasgow & Papadias's (1992) treatment of computational imagery. Let CUBES be set of cubes into which  $V(m_i)$  is partitioned. Let POW(CUBES) be the power set of cubes.<sup>8</sup> Each member of POW(CUBES) is a position in  $V(m_i)$ .<sup>9</sup> Each position

<sup>8</sup>The power set of a set is the set of all subsets of that set. For instance, the power set of  $\{1, 2, 3\}$  is  $\{\{\}, \{1\}, \{2\}, \{3\}, \{1,2\}, \{1,3\}, \{2,3\}, \{1,2,3\}\}$ .

<sup>9</sup>We let members of POW(CUBES) that are not strongly connected be positions occupied by sensible things because it is possible to refer to collections of sensible things (such as

occupies many cubes in  $V(m_i)$  and so has a shape. The largest position is just  $V(m_i)$ . The smallest positions are individual cubes. We assign an integer to each member of  $POW(CUBES)$ . These integers designate positions in  $V(m_i)$ . In our example, each possible intentional world for a finite mind has three positions. Basically, these positions are the left side, the middle, and the right side. In general, the right and left sides of each possible intentional world is occupied by a body, the middle is occupied by a door. Figure 8 shows the positions in the possible intentional worlds for X.



**Figure 8.** Positions in the possible intentional worlds for X.

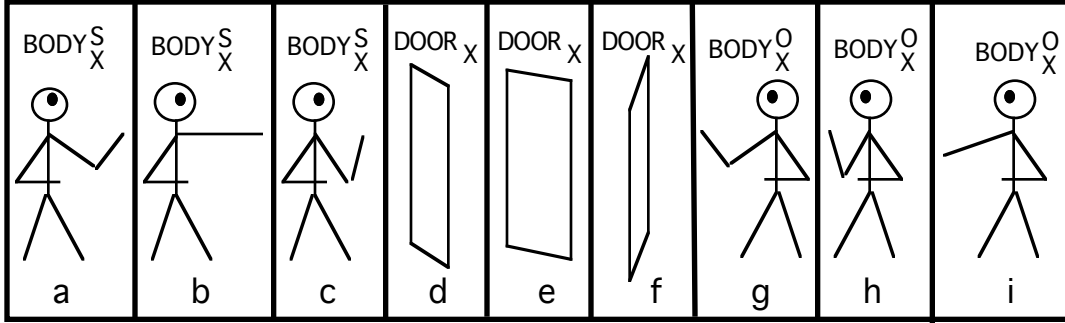
Just as there is an alphabet of letters, there is an alphabet of sensible things. A distinct alphabet  $A_i$  of sensible things is defined for each set  $W(m_i)$  of possible finite minds for each finite mind in Minds. The alphabet of sensible things for a finite mind contains all possible sensible things for that finite mind. The alphabet is an infinite set existing in God. This alphabet consists of sensible things  $\{s_1, s_2, \dots\}$ , so  $A_i = \{s_1, s_2, \dots\}$ . In our finite models, we treat the alphabet as a finite set whose cardinality is denoted by Z. The alphabet of sensible things is ordered, so it is an array rather than a set. We call an array whose elements comprise an alphabet of sensible things an *alphabetic array*.

In our example, the alphabet of sensible things for finite mind X is: "a" is  $BODY_X^S$  with its arm in a neutral position; "b" is  $BODY_X^S$  with its arm extended; "c" is  $BODY_X^S$  with its arm retracted; "d" is  $DOOR_X$  equidistant between  $BODY_X^S$  and  $BODY_X^O$ ; "e" is  $DOOR_X$  open in the direction of  $BODY_X^O$ ; "f" is  $DOOR_X$  open in the direction of  $BODY_X^S$ ; "g" is  $BODY_X^O$  with its arm in a neutral position; "h" is  $BODY_X^O$  with its arm retracted; "i" is  $BODY_X^O$  with its arm extended. Figure 9 illustrates the alphabetic array for finite mind X.

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"those bottles on the table") that occupy a position that is distributed over many disconnected cubes.



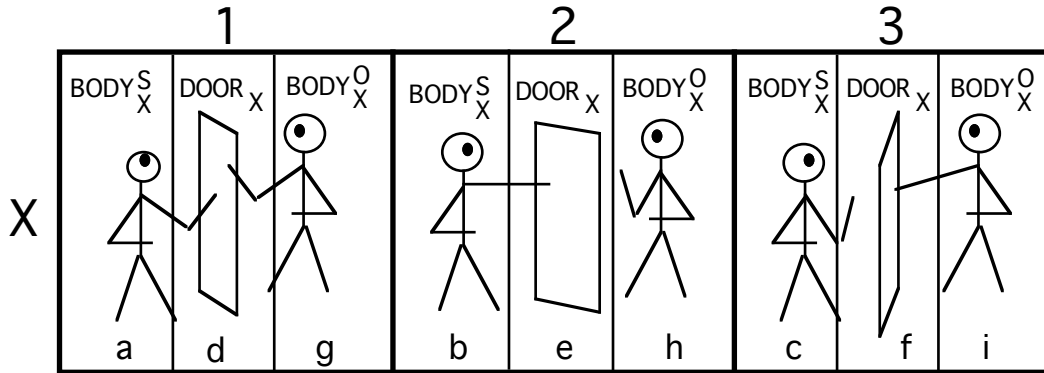


**Figure 9.** The alphabetic array for finite mind X.

The alphabet of sensible things for finite mind Y is: "A" is  $BODY_Y^S$  with its arm in a neutral position; "B" is  $BODY_Y^S$  with its arm extended; "C" is  $BODY_Y^S$  with its arm retracted; "D" is  $DOOR_Y$  equidistant between  $BODY_Y^S$  and  $BODY_Y^O$ ; "E" is  $DOOR_Y$  open in the direction of  $BODY_Y^O$ ; "F" is  $DOOR_Y$  open in the direction of  $BODY_Y^S$ ; "G" is  $BODY_Y^O$  with its arm in a neutral position; "H" is  $BODY_Y^O$  with its arm retracted; "I" is  $BODY_Y^O$  with its arm extended.

In our example, each possible intentional world is a word with three letters, one for each position. Reading from left to right, the first letter designates the sensible thing occupying the left side of the intentional world, the second letter the sensible thing occupying the middle, and the third letter the sensible thing occupying the right side. There are three possible intentional worlds for each finite mind, hence three words for each finite mind.

In the first possible intentional world  $W_1(X)$  for X,  $BODY_X^S$  has its arm in a neutral position;  $DOOR_X$  is equidistant between  $BODY_X^S$  and  $BODY_X^O$ ; and  $BODY_X^O$  has its arm in a neutral position. Consequently,  $W_1(X) = "adg"$ . In the second possible intentional world  $W_2(X)$  for X,  $BODY_X^S$  has its arm extended;  $DOOR_X$  is open in the direction of  $BODY_X^O$ ; and  $BODY_X^O$  has its arm retracted. Consequently,  $W_2(X) = "beh"$ . In the third possible intentional world  $W_3(X)$ ,  $BODY_X^S$  has its arm retracted;  $DOOR_X$  is open in the direction of  $BODY_X^S$ ; and  $BODY_X^O$  has its arm extended. Consequently,  $W_3(X) = "cfi"$ . Figure 10 shows the possible intentional worlds for finite mind X as words.



**Figure 10.** Possible intentional worlds as words.

In the first possible intentional world  $W_1(Y)$  for  $Y$ ,  $BODY_Y^S$  has its arm in a neutral position;  $DOOR_Y$  equidistant between  $BODY_Y^S$  and  $BODY_Y^O$ ;  $BODY_Y^O$  has its arm in a neutral position. Consequently,  $W_1(X) = "GDA"$ . In the second possible intentional world  $W_2(Y)$  for  $Y$ ,  $BODY_Y^S$  has its arm retracted;  $DOOR_Y$  is open in the direction of  $BODY_Y^S$ ;  $BODY_Y^O$  has its arm extended. Consequently,  $W_2(X) = "IFC"$ . In the third possible intentional world  $W_3(Y)$ ,  $BODY_Y^S$  has its arm extended;  $DOOR_Y$  is open in the direction of  $BODY_Y^O$ ;  $BODY_Y^O$  has its arm retracted. Consequently,  $W_3(X) = "HEB"$ .

#### 4.8 The Array of Matrices of Sensible Things

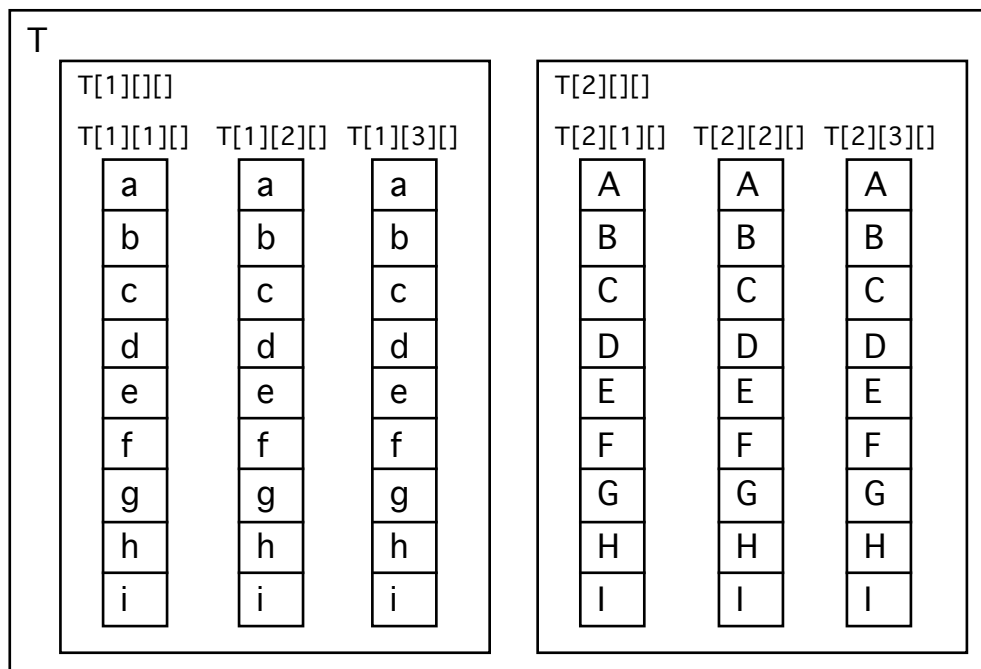
Each position in a word can be filled with a letter drawn from the alphabet. In the most complex case, each position in  $V(m_i)$  can be occupied by a sensible thing drawn from the alphabet of sensible things  $A_i$ .<sup>10</sup> Since each position in  $V(m_i)$  is filled with a sensible thing drawn from an alphabetic array, each finite mind needs one alphabetic array for each position in  $V(m_i)$  in order to hold all the possible combinations of sensible things for that finite mind. If  $K$  is the cardinality of  $POW(CUBES)$ , then each finite mind needs  $K$  alphabetic arrays to hold all its possible combinations of sensible things. For each finite mind, then, the level of sensible things consists of  $K$  alphabetic arrays. We order the alphabetic arrays in finite minds according to position in  $V(m_i)$ . The contents of each finite mind are thus stored in an array of alphabetic arrays. An array of  $K$  alphabetic arrays is called a *matrix of sensible things*.

Since there are  $MAX(Minds)$  finite minds in God, there are  $MAX(Minds)$  matrices of sensible things in the level of sensible things in God. Finite minds are ordered, so the level of sensible things in God is an array of matrices of sensible things; that is, it is an array of arrays of alphabetic arrays. We let this array of matrices of sensible things be designated "T". Thus T is a three-dimensional array. The first dimension is indexed by finite mind, so  $T[i][][ ]$  is the matrix of sensible things for the  $i$ -th finite mind. The second

<sup>10</sup>This is the most complex case because not every sensible thing can fill every position. For instance, a large spherical sensible thing cannot fill a position that has the shape of a small rectangle.

dimension is indexed by position in  $V(m_i)$ , so  $T[i][h][\ ]$  is the  $h$ -th position in finite mind  $m_i$ . The third dimension is indexed by position in the array of sensible things, so that  $T[i][h][v]$  is the  $v$ -th sensible thing in the  $h$ -th position in the  $i$ -th finite mind.

Importantly, the matrix  $T[i][\][\ ]$  of sensible things for the  $i$ -th finite mind contains all possible combinations of sensible things for finite mind  $m_i$ . The matrix  $T[i][\][\ ]$  stores all the sensible things in all the possible intentional worlds of the  $i$ -th finite mind. It is not necessary to use a distinct matrix of sensible things for each distinct possible intentional world. Note that using one array to store the contents of distinct possible intentional worlds does not violate the principle of privacy, since they are all possible intentional worlds of the same finite mind. Figure 11 shows the array of matrices of sensible things for the two finite minds X and Y; finite mind X is assigned the number 1, while Y is assigned 2.

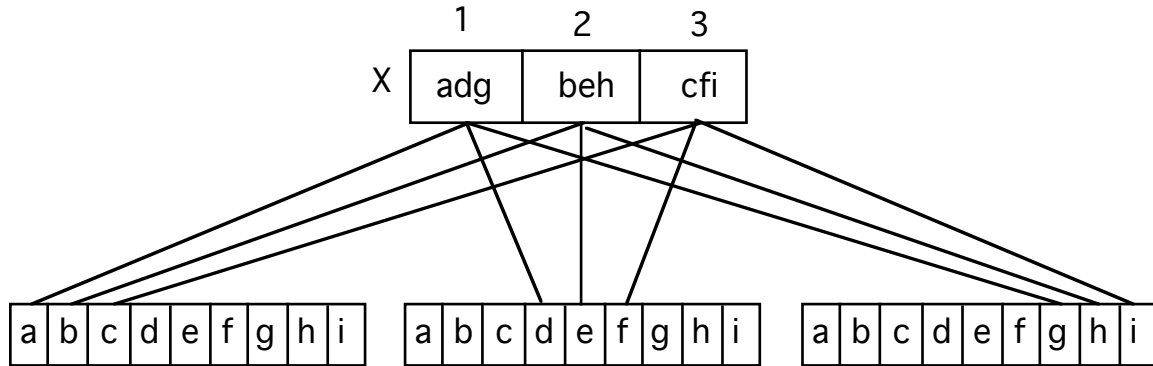


**Figure 11.** The array of matrices of sensible things.

At any single time, only one sensible thing can be realized for each position in a finite mind; hence for  $i = 1$  to  $\text{MAX}(\text{Minds})$  and for  $h = 1$  to  $K$ , each possible sensible thing in the alphabetic array  $T[i][h][\ ]$  has an inhibitory connection with every other possible sensible thing in the array  $T[i][h][\ ]$ . For  $i = 1$  to  $\text{MAX}(\text{Minds})$ , and for  $j = 1$  to  $\text{MAX}(\text{C})$ , each possible intentional world  $P[i][j]$  of the  $i$ -th finite mind contains  $K$  sensible things as its parts. For  $h = 1$  to  $K$ , the world  $P[i][j]$  has an excitatory connection with the sensible thing  $T[i][h][v]$  in  $T[i][h][\ ]$  corresponding to the  $h$ -th sensible thing in  $P[i][j]$ .

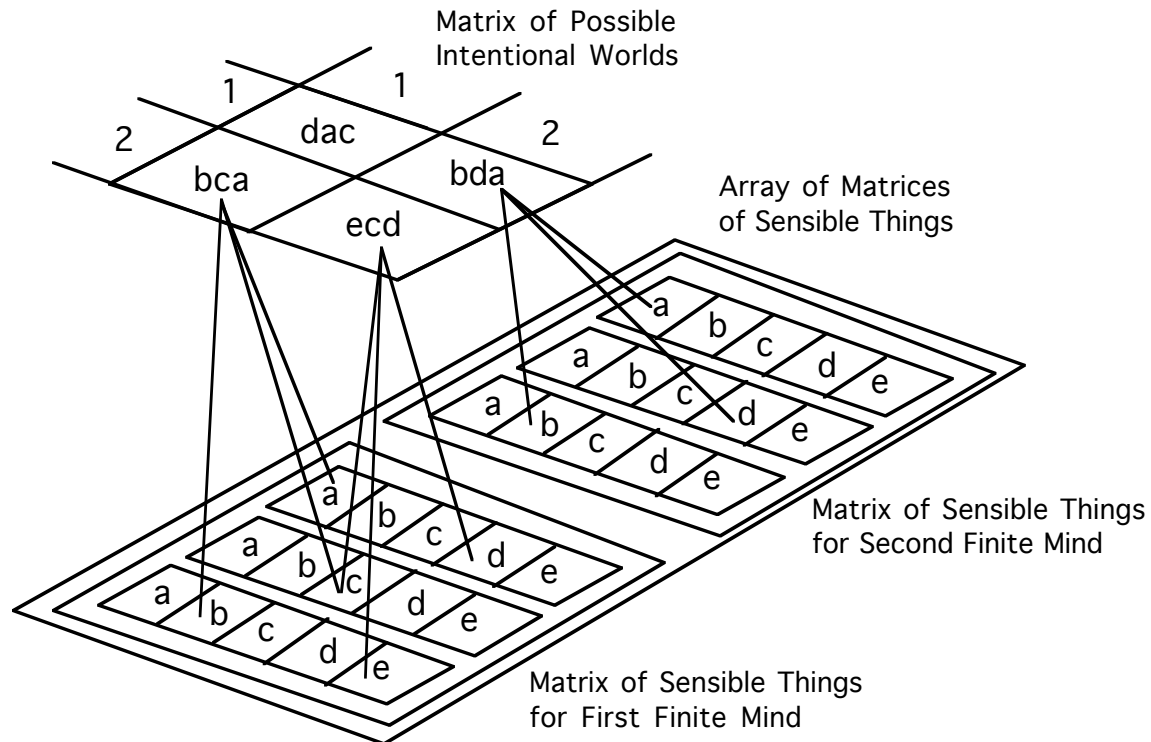
In our example,  $P$  contains six possible intentional worlds. The worlds for finite mind X are analogous to words as follows:  $P[1][1] = \text{"adg"}$ ;  $P[1][2] = \text{"beh"}$ ;  $P[1][3] = \text{"cfi"}$ . The worlds for finite mind Y are analogous to words as follows:  $P[2][1] = \text{"GDA"}$ ;  $P[2][2] = \text{"IFC"}$ ; and  $P[2][3] = \text{"HEB"}$ . There is an excitatory connection between each world and its component letters. Thus, there is an excitatory connection from  $P[1][1]$  to  $T[1][1][1]$ , to  $T[1][2][4]$ , and to  $T[1][3][7]$ . There is an excitatory connection from  $P[1][2]$  to  $T[1][1][2]$ , to  $T[1][2][5]$ , and to  $T[1][3][8]$ . There is an excitatory connection

from  $P[1][3]$  to  $T[1][1][3]$ , to  $T[1][2][6]$ , and to  $T[1][3][9]$ . Likewise, there is an excitatory connection from  $P[2][1]$  to  $T[2][1][7]$ , to  $T[2][2][4]$ , and to  $T[2][3][1]$ . There is an excitatory connection from  $P[2][2]$  to  $T[2][1][9]$ , to  $T[2][2][6]$ , and to  $T[2][3][3]$ . There is an excitatory connection from  $P[2][3]$  to  $T[2][1][8]$ , to  $T[2][2][5]$ , and to  $T[2][3][2]$ . The six excitatory connections from possible intentional worlds of  $X$  to sensible things are shown in Figure 12. Although none of the worlds for finite mind  $X$  share sensible things, this is not always the case.



**Figure 12.** Excitatory connections from worlds to sensible things.

As another example, distinct from our example involving the two minds and the two doors, let God contain two finite minds and two compossible distributions of events across finite minds. The array  $C$  has two elements  $C[1]$  and  $C[2]$ . Let  $C[1]$  contain the two possible intentional worlds  $\{P[1][1], P[2][1]\}$ ;  $C[2]$  contains the two possible intentional worlds  $\{P[1][2], P[2][2]\}$ . Each finite mind contains three sensible things, drawn from the alphabet  $\{a, b, c\}$  of sensible things. For the sake of simplicity, we let each finite mind draw from the same alphabet. We let  $P[1][1]$  be "dac";  $P[1][2]$  is "bda";  $P[2][1]$  is "bca";  $P[2][2]$  is "ecd". Figure 13 shows the excitatory connections from the possible intentional worlds  $P[2][1]$ ,  $P[2][2]$ , and  $P[1][2]$  to their component sensible things. The connections of  $P[1][1]$  are not shown so as not to cause interference.



**Figure 13.** Connections from worlds to things.

#### 4.9 The Level of Sensations

The bottom level of the divine hierarchy is the level of sensations. Just as sensible things are analogous to letters, so sensations are analogous to the features of which those letters are composed, such as horizontal or vertical lines. Sensations are the ultimate parts of finite minds and of God. For  $i = 1$  to  $\text{MAX}(\text{Minds})$ , for  $h = 1$  to  $K$ , for  $v = 1$  to  $Z$ , each sensible thing  $T[i][h][v]$  is a whole composed of sensations. Just as there is an alphabet of sensible things, there is an alphabet of sensations. This alphabet is infinite, but for simplicity we reduce it to a finite set. Sensations are ordered; an ordered set containing all the sensations in the alphabet of sensations is called an alphabetic array of sensations.

The level of sensations has a structure similar to that of the level of sensible things. The level of sensations is a three-dimensional matrix  $S$ . The first index ranges over finite minds; the second index ranges over positions in finite minds; the third index ranges over sensations themselves. For  $i = 1$  to  $M$ , and for  $h = 1$  to  $K$ , the array  $S[i][h][v]$  is an alphabetic array of sensations. That is, each alphabetic array of sensible things  $T[i][h][v]$  in  $T$  corresponds to an alphabetic array  $S[i][h][v]$  of sensations in  $S$ . Sensations are not competitive. Any particular sensation in  $S[i][h][v]$  can occur in many possible sensible things in  $T[i][h][v]$ . Since they are not competitive, sensations are not linked by inhibitory connections. Each sensible thing  $T[i][h][v]$  in  $T$  has an excitatory connection to each of its component sensations in  $S[i][h][v]$ , and an inhibitory connection to each sensation that is not a part of it.

## 5. The Divine Process

### 5.1 Overview of the Divine Process

The divine process is a series of *cognitive cycles*. One cognitive cycle occurs on every moment in time, and each cognitive cycle computes a state of God. Each cognitive cycle in the divine process consists of three phases. Following Plotinus (1952), we conceive of the first phase as the emanation phase and the second phase as the return phase. Emanation is the flow of activation from God down through the divine hierarchy to the sensations; return is the flow of activation up from the sensations back to God. To these two phases we add a third phase, the firing of transition rules. The firing of transition rules sets up the next state of God based on previous states. Each cognitive cycle therefore involves: (1) emanation; (2) return; (3) the firing of transition rules.

Although we believe that the divine hierarchy is eternal, we hold that there is an *initial state* of the divine process. We designate this state as  $God(0)$ . In terms of classical ontologies, this state corresponds to the beginning of the world, that is, it corresponds to the moment of creation. Prior to the beginning of the divine process, that is, prior to the computation of the initial state  $God(0)$ , the external inputs to intentional objects fluctuate randomly from cycle to cycle. If an intentional object has some positive external input, we say that it is primed. Prior to the computation of the initial state, activations of intentional objects are nil. Everything is possible, but nothing has ever yet been actualized.

At some moment, the divine process begins. On the first cognitive cycle, emanation selects from primed intentional objects and activates a set of sensations. After emanation, return results in the state  $God(0)$  that satisfies the constraints in the divine hierarchy. This is the initial state. The initial state is determined by the constraints in the divine hierarchy and random factors. After return has produced the initial state  $God(0)$ , transition rules fire, resulting in external inputs for the next state  $God(t+1)$ . Once transition rules have fired, the time variable  $t$  is incremented.

On moments subsequent to the beginning of the divine process, emanation randomly selects from primed intentional objects and activates a set of sensations. After emanation, return results in the state  $God(t)$  that best satisfies the constraints in the divine hierarchy given the external inputs. This is the *current state*. The current state is determined by past states, the constraints in the divine hierarchy, and random factors. After return has produced the current state, transition rules fire, resulting in external inputs for the next state  $God(t+1)$ . Once transition rules have fired, the time variable  $t$  is incremented.

### 5.2 The Emanation of Activation from God

Emanation is the flow of activation from God down through the divine hierarchy to sensations. The original source of activation is God; God is always and necessarily actual on every cycle. Since God is always actual, the selection flag of God is always turned on. Also, since God is always actual, God always receives external input from God. That is to say, God's external input parameter is always set to the maximal value. Activation flows from God at the start of every cognitive cycle. When activation passes down through an intentional object during emanation, it sets the selection flag of the object, thereby releasing it for participation in return and the firing of transition rules. It also resets the activation of the object to zero.

At the beginning of every cognitive cycle, there is some set of intentional objects with non-zero external inputs. Particularly, there is some set of compossible distributions with non-zero external inputs. On the first cognitive cycle, this set is random. On subsequent cognitive cycles, it is determined by the firings of transition rules. During emanation, God randomly *selects* a non-empty subset of the compossible distributions with non-zero external inputs. God selects these compossible distributions by passing activation down through them. When activation passes down through these compossible distributions, it sets their selection flags and resets their activations to zero. In our example, say C[1] and C[2] are selected during emanation. Their selection flags are set and their activations are reset to zero.

Activation flows from the selected compossible distributions to the possible intentional worlds in those compossible distributions. The j-th compossible distribution contains all the possible intentional worlds in the j-th column of the P matrix; that is, it contains each element of P[][j]. Activation therefore flows from each selected C[j] to each element in P[][j]. In our example, activation flows to the possible intentional worlds P[1][1], P[1][2], P[2][1], and P[2][2]. When activation passes down through these worlds, it sets their selection flags and resets their activations to zero.

Activation then flows from each selected possible intentional world P[i][j] to each of its component sensible things, selecting each sensible thing in P[i][j]. Hence activation flows from P[i][j] to one of the sensible things in T[i][h][v] for h = 1 to K, selecting each of the sensible things to which it flows. When activation passes down through these sensible things, it sets their selection flags and resets their activations to zero.

From the sensible things activation flows to the sensations. Because the sensations are the ultimate parts of the divine hierarchy, when activation flows down to sensations it does not select them but activates them. To activate a sensation, emanation sets the selection flag of the sensation and assigns a value to the external input of the sensation. Recall that all the sensible things for a single position in a single finite mind converge on the same array of sensations. For instance, say the possible intentional worlds for the first finite mind are P[1][1] = "abcd" and P[1][2] = "zbce". In this case, emanation selects "a" and "z" in T[1][1][v], "b" only T[1][2][v], "c" only T[1][3][v], and "d" and "e" in T[1][4][v]. Note that, for the second and third positions, the emanation of activation converges on the same set of sensations.

We want each intentional object to be able to exercise its freedom in the computation of the distribution of activations that emerges during a cognitive cycle; hence the sensations that are activated cannot specially favor any one of the selected compossible distributions, but must equally favor each. For instance, activating a majority of the features of "a" in the sensible array S[1][1][v] would favor P[1][1] over P[1][2]. But we want P[1][1] and P[1][2] to be equally favored, and to compete amongst themselves for activation (i.e. for actualization). Consequently, we must underdetermine the sensations activated for each sensible thing.

For each position in each finite mind, emanation therefore activates the *intersection* of the sensations of all the sensible things in that position for that finite mind. For each sensible thing T[i][h][v], let SENS(T[i][h][v]) be the component sensations of that sensible thing. For each alphabetic array of sensible things T[i][h][v], emanation computes the intersection of SENS(T[i][h][v]) for v = 1 to Z. In our example, in

S[1][1][], emanation activates the sensations in the intersection of SENS("a") and SENS("z"). This equally favors both "a" and "z", and specially favors neither.<sup>11</sup>

Once the selection flags and external inputs of sensations have been set, the emanation phase is over. The result of the emanation phase is a set of selected intentional objects whose activations are zero and whose external inputs are positive. Importantly, the process of selection through emanation is not an accidental addendum to our theory. Selection through emanation ensures that activation spreads only within hierarchies of intentional objects containing intentional objects with external inputs. Recall that intentional objects can share parts (e.g. possible intentional worlds can share sensible things, sensible things can share sensations), and that intentional objects are linked by excitatory connections expressing compossibility. If it were not for selection through emanation, activation would be able to spread from intentional objects that have received external input on a cognitive cycle to intentional objects that are not actualizable on that cognitive cycle. Selection through emanation prevents such interference.

### 5.3 The Return of Activation to God

Once sensations have been activated, activation flows upwards from the actualized sensations towards God. The flow of activation from actualized sensations back to God is the return of activation. The return phase itself takes place over a finite number of cycles. We speak of these as return cycles; they are subcycles of each cognitive cycle. The return of activation computes a distribution of activations over all the selected intentional objects in the divine hierarchy. The distribution depends on the internal activations of intentional objects, the external activations of intentional objects, and the constraints holding among intentional objects. Importantly, the distribution satisfies the constraints holding among the intentional objects in the divine hierarchy. The return of activation is thus the solution of a *constraint-satisfaction problem*. This is analogous to what Lewis (1969) calls a *coordination problem*. While activation returns, the divine hierarchy passes through many distributions of activation until it settles on one that best satisfies the constraints given the internal and external activations of selected intentional objects. That is, activation in the divine hierarchy converges on a distribution during return. We say that the divine hierarchy *runs to convergence* during return.

The return subphase runs the divine hierarchy as a constraint-satisfaction network. On each return cycle, each selected intentional object changes its activation (its degree of actualization) according to *activation updating rules* proposed by McClelland & Rumelhart (1986). In order to compute the activations of selected intentional objects, each selected intentional object is assigned a number. On each cycle, each selected intentional object  $i$  in the divine hierarchy computes its net input  $net_i$  in accordance with the rule in Formula 1.

$$[1] \text{ } net_i = \sum_j w_{ij} \text{ } output_j + \text{ } extinput_i$$

In Formula 1,  $w_{ij}$  is the weight of the excitatory or inhibitory connection from intentional object  $j$  to intentional object  $i$ ,  $output_j$  is the output of intentional object  $j$ , and  $extinput_i$  is

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<sup>11</sup>Alternatively, God could activate an equal number of the sensations of both "a" and "z" in S[1][1][].



the external input to intentional object  $i$ . The weight of an excitatory connection is +1.0. The weight of an inhibitory connection is -1.0.

Once the net input has been computed for every intentional object in the network, the change in the activation of each intentional object is given by the rules in Formulae 2 and 3.

$$[2] \Delta a_i = (max - a_i)net_i - decay(a_i - rest) \text{ if } net_i > 0$$

$$[3] \Delta a_i = (a_i - min)net_i - decay(a_i - rest) \text{ if } net_i \leq 0$$

In Formulae 2 and 3,  $a_i$  is the activation of intentional object  $i$ ,  $decay$  is a parameter indicating the rate at which activation decays,  $max$  is the maximum activation of an intentional object,  $min$  is the minimum activation of a intentional object, and  $rest$  is the resting activation of an intentional object. We define these parameters as follows:  $decay = 0.1$ ,  $max = 1.0$ ,  $min = -1.0$ , and  $rest = 0.0$ .

As activation flows upwards, it tends to activate sensible things equally. These sensible things begin to compete for activation; at this point, *external inputs* increase the activation of one sensible thing over its competitors. Thus inequities begin to creep into the upwards flow of activation. First one sensible thing tends to be favored more than its competitors, then one possible intentional world of a finite mind tends to be favored more than the other possible intentional worlds, and eventually one compossible distribution tends to be favored more than the others. The result is that a stable distribution of activations emerges over the whole selected divine hierarchy. The stable distribution of activations computed during return is the state of God( $t$ ) on the cognitive cycle  $t$ .

All actualizations that occur at any time occur with respect to and in relation to one another. All actualizations are ontologically coordinated through constraints of compossibility and impossibility. In particular, tendencies to actualization interact across distinct finite minds through the constraints connecting the different possible intentional worlds of those minds. Through interactions across their constraints, the possible intentional worlds that are actualized within finite minds are coordinated. The result is a harmony across all intentional objects, but not a pre-established harmony (Leibnitz, 1965). On the contrary, the result is an *emergent harmony* among all finite minds.

#### 5.4 The Firing of Transition Rules

Once the state God( $t$ ) of God has been computed by return, transition rules fire. The firing of transition rules proceeds as follows. First, every rule with a true ANTECEDENT is collected into RULES( $t$ ). Then RULES( $t$ ) is partitioned into sets of competing rules. A rule P competes with a rule Q if making the ANTECEDENT of P true also makes the ANTECEDENT of Q true. We ensure that competition is symmetric. If P competes with Q, then Q also competes with P. For example, the two transition rules

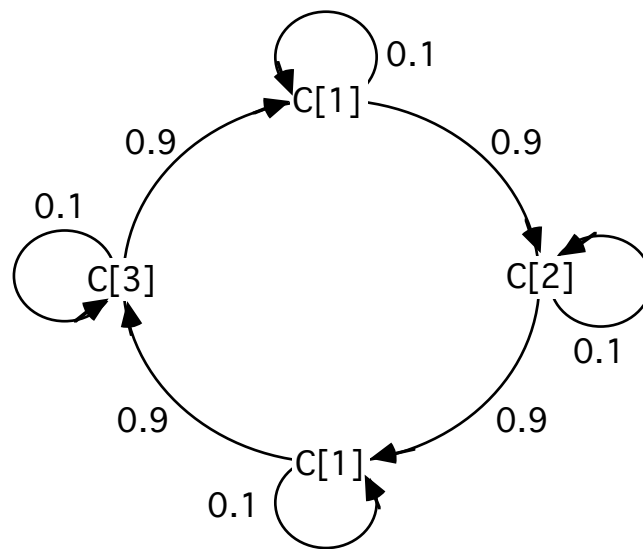
$$C[1] \in \text{God}(t) \ \& \ C[3] \in \text{God}(t-1) \Rightarrow C[2] \in \text{God}(t+1)$$

$$C[1] \in \text{God}(t) \Rightarrow C[1] \in \text{God}(t+1)$$

are competing transition rules. The probabilities of competing rules must sum to 1.0. Based on their relative probabilities, each rule in each set of competing rules is assigned a continuous range of numbers between 0.0 and 1.0. For each set of competing rules in RULES( $t$ ), the divine process generates a random number; if the random number falls in

the range for rule P, then rule P fires. The external inputs of the intentional objects in the CONSEQUENT of P are set to values indicated.

Transition rules partially determine the probability of the next state of God given the past states. Importantly, it is not possible to predict the next state of God given only the past states of God. This is because (1) emanation randomly selects from intentional objects with non-zero external inputs; because (2) transition rules fire stochastically; and because (3) running the divine hierarchy to convergence during return is a non-deterministic process. In general, therefore, it is not possible to draw anything like a state-transition diagram for the divine process. In our example, however, the non-determinism of the divine process is sufficiently restricted that we can draw a non-deterministic state-transition diagram for the divine process. The state-transition diagram for the finite non-deterministic automaton is shown in Figure 14. Each arc is labelled with the probability that it will be traversed.



**Figure 14.** State-transition diagram for the divine process.

The state-transition diagram in Figure 14 effectively defines something like a movie for both finite minds X and Y. In this movie, possible intentional worlds are frames. Importantly, because only compossible intentional worlds are actualized across distinct finite minds, the movies shown to both X and Y are coordinated; they are harmonized. If we asked X and Y what they were experiencing, their descriptions would agree. It would appear to each that he or she does not inhabit a private world, but shares a public world with another who interacts, indeed, who cooperates with him or her. The world of these two finite minds, trivial as it is, would appear to each to go through a regular process. Each person would testify that he or she pushes the door towards the other, then pulls it back through its neutral position towards himself, and that this process then repeats in a regular fashion. Each would be able to form this simple empirical generalization about the processes in this world; that is, each would be able to form a set of intersubjectively verifiable natural laws. Such would be the basis for language and empirical science in this simple world.

## 6. Conclusion

We have provided a precise, computational description of an idealist ontology. We provided a logical description of an ontological structure and an algorithmic description of an ontological process. Our descriptions are formally rigorous. We first described intentional objects and their interrelations. We provided precise criteria for consistency of visual experience across distinct finite minds. We showed how relations of compossibility and impossibility structure the intentional objects in the divine hierarchy. Finally, we gave a description of the divine process. The divine process is constrained by relations of compossibility and impossibility and by transition rules. Nonetheless, the divine process is only partially determined and is otherwise free. On each cognitive cycle, it realizes a coherent distribution of degrees of actualization across the set of its component intentional objects, including possible intentional worlds of distinct finite minds. We have thereby described a system in which distinct finite minds are harmoniously coordinated in God. We have thereby accounted for the conditions needed for the emergence of the public or intersubjectively verifiable world, and, ultimately, for language and science. While idealism has often been denigrated for its lack of precision and speculative excess, our ontology is both speculative and expressed with formal precision.

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