INTERPRETATION OF SINGULARITIES IN GENERAL RELATIVITY AND THE INFORMATION LOSS PARADOX

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Abstract. When matter is falling into a black hole, the associated information becomes unavailable to the black hole's exterior. If the black hole disappears by Hawking evaporation, the information seems to be lost in the singularity, leading to Hawking's information paradox: the unitary evolution seems to be broken, because a pure separate quantum state can evolve into a mixed one.

This article proposes a new interpretation of the black hole singularities, which restores the information conservation. For the Schwarzschild black hole, it presents new coordinates, which move the singularity at the future infinity (although it can still be reached in finite proper time). For the evaporating black holes, this article shows that we can still cure the apparently destructive effects of the singularity on the information conservation. For this, we propose to allow the metric to be degenerate at some points, and use the singular semiriemannian geometry. This view, which results naturally from Ashtekar's new variables formulation of Einstein's equation, repairs the incomplete geodesics.

The reinterpretation of singularities suggested here allows (in the context of standard General Relativity) the information conservation and unitary evolution to be restored, both for eternal and for evaporating black holes.

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"I was borne violently into the channel of the Ström, and in a few minutes, was hurried down the coast into the 'grounds' of the fishermen."

Edgar Allan Poe, A Descent into the Maelström, 1841

1. The black hole information paradox

After an object falls into a black hole, all the information about it remains inaccessible to the external observers. Shortly after that, in its proper time, it vanishes into the singularity from the black hole's center.

On the other hand, the equations governing the physical laws are in general reversible, guaranteeing that no information can be lost. But according to Hawking [1, 2] the black hole may cause radiation, and the *no hair theorem*[1] seems to imply that Hawking radiation is independent of the information in the black hole. If the black hole evaporates completely, it seems to left behind no trace of the information it swallowed. Moreover, it seems to be possible for an originally pure state to end up being mixed, because the density matrix of the particles in the black hole's exterior is obtained by tracing over the particles lost in the black hole with which they were entangled. This means that the unitarity seems to be violated, and the contradiction becomes more acute.

In this essay it is proposed a natural interpretation of the singularities which makes them harmless for the information conservation. The conservation of information and the unitary evolution are restored both for eternal and for evaporating black holes.

2. The meaning of initial and final singularities

Despite the successes of General Relativity, at least two of its consequences seemed to question its full correctness: the initial singularity in the past – the Big Bang, and the future singularities in the black holes. It is often said that General Relativity predicts, because of these singularities, it's own breakdown [3]. Such singularities, following from Penrose and Hawking *singularity theorems* [4, 5, 6, 7], refer to the spacetime *geodesic incompleteness*.

What is the real meaning of the singularities? Do they really imply that the physical laws break down? When Schwarzschild proposed [8, 9] his solution (7) to Einstein's equation, representing a black hole, it was believed that the event horizon is singular. Years later it was understood that that singularity was only apparent, being due to the choice of the coordinate system. But the singularity at the center of the black hole remained independent of the coordinates, and the singularity theorems showed that any black hole would have such a singularity. Though, this kind of singularity has meaning only in the presence of a metric – from the topology viewpoint only there is no such problem.

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[1]According to the no hair theorem, the black holes converge asymptotically toward a Kerr or a Schwarzschild solution, being thus characterized exclusively by mass, electric charge and angular momentum.

In order to evaluate the significance of the singularities, let us start by considering a Friedmann-Lemaître-Robertson-Walker (FLRW) spacetime

$$\mathcal{F} = \left((t_i, t_f) \times \Sigma, \mathrm{d}s^2 = a(t)^2 \mathrm{d}t^2 - b(t)^2 \mathrm{d}\sigma^2\right), \tag{1}$$

where $-\infty \leq t_i < t_f \leq \infty$, Σ is S^3, H^3 or $\mathbb{R}^3$ – with the appropriate metric $\mathrm{d}\sigma^2$, and $a, b : (t_i, t_f) \to (0, \infty)$ two functions. If $t_i > -\infty$ and the timelike or null geodesics cannot be extended in the past beyond t_i, then we have a singularity at t_i. A FLRW spacetime with initial singularity is topologically equivalent with $((-\infty, t_f) \times \Sigma, \mathrm{d}s')$, and we can always change the time coordinate to $t' = t'(t)$ so that $t'_i = -\infty$. Viewed as such, the singularity at t_i corresponds to the limit at $t'_i = -\infty$, and we are not so worried with the limits of physical fields like the metric or curvature when $t' \to -\infty$. Of course, what happened at the Big Bang still requires a unification of Quantum Theory and General Relativity, the age of the universe is still finite, and the stress-energy tensor is still divergent for $t' \to -\infty$. But, viewing the initial singularity as the $-\infty$ limit in the past demystifies its image as a point where the laws of physics break down.

According to Penrose's *Cosmic Censorship Hypothesis* [10, 11], the singularities cannot be timelike, and can only be initial or final, not both simultaneously. If this is true we can, at least locally, choose a coordinate system which moves them to the past or future infinity. The metric is a field which depends on the distribution of matter in the spacetime, via Einstein's equation. The singularities are dependent of the metric, because they have meaning only as points beyond which we cannot extend geodesics. This means that an affine parameter on a geodesic has a finite bound at that point. But we can always choose a non-affine parameter going from $-\infty$ to $+\infty$. This doesn't mean that the singularities are only apparent (like the singularity on the event horizon in Schwarzschild's coordinates turned out to be), just that they can be viewed as corresponding to the infinity by an appropriate choice of coordinates. The spacetime can be covered by an atlas of coordinate maps on which all the fields behave well.

Even in the case of a Schwarzschild black hole we can perform such a coordinate change to move the singularity at $t = +\infty$. For the evaporating black holes, the singularities cannot be avoided by this method, but the incomplete geodesics can be repaired, as we shall see.

3. Minkowski spacetime and Penrose diagrams

The *Minkowski spacetime* is the spacetime $\mathbb{R} \times \mathbb{R}^3$, with a metric which in Cartesian coordinates takes the form $\mathrm{d}s^2 = -\mathrm{d}t^2 + \mathrm{d}x^2 + \mathrm{d}y^2 + \mathrm{d}z^2$. If on $\mathbb{R} \times (\mathbb{R}^3 - \{0\})$ we use the coordinates $(t, r, \vartheta, \varphi)$, where t is the time and (r, ϑ, φ) are polar coordinates, and where $\mathrm{d}\Omega^2 := \mathrm{d}\vartheta^2 + \sin^2 \mathrm{d}\varphi^2$, the metric becomes

$$\mathrm{d}s^2 = \mathrm{d}t^2 - \mathrm{d}r^2 - r^2 \mathrm{d}\Omega^2. \tag{2}$$

In *null coordinates* $u := t - r$ (retarded) and $v := t + r$ (advanced) the metric is

$$\mathrm{d}s^2 = \mathrm{d}u\mathrm{d}v - \frac{1}{4}(u - v)^2 \mathrm{d}\Omega^2. \tag{3}$$

By the conformal transformation (which does not change the light cones):

$$\begin{cases} u' := \arctan u \\ v' := \arctan v, \end{cases} \tag{4}$$

$u', v' \in [-\frac{\pi}{2}, \frac{\pi}{2}]$, the whole spacetime is stretched to a finite region. We obtain Penrose's null coordinates, in which the Lorentz metric takes the form

$$\mathrm{d}s^2 = \frac{1}{\cos^2 u'} \frac{1}{\cos^2 v'} \left(\mathrm{d}u' \mathrm{d}v' - \frac{1}{4} \sin^2(u' - v') \mathrm{d}\Omega^2 \right). \tag{5}$$

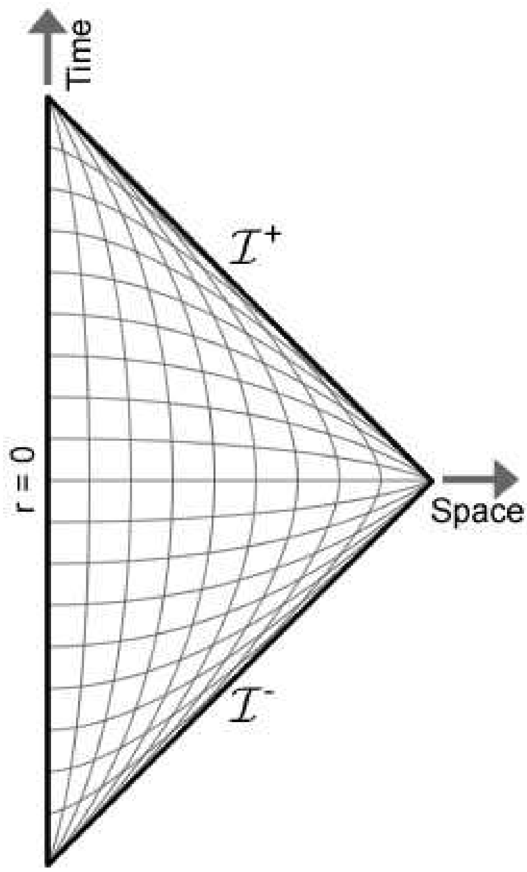

FIGURE 1. The Penrose diagram of a Minkowski spacetime. Its rotational symmetry allows us to represent it in a half-plane, and the conformal symmetry allows us to compress it to a triangle.

In timelike and spacelike Penrose coordinates $t_P := u' + v'$, $r_P := v' - u'$ we can see that the metric is conformally flat:

$$\mathrm{d}s^2 = \frac{\mathrm{d}t_P^2 - \mathrm{d}r_P^2 - \sin^2 r_P \mathrm{d}\Omega^2}{4 \cos^2 \frac{t_P + r_P}{2} \cos^2 \frac{t_P - r_P}{2}}. \tag{6}$$

4. The Schwarzschild black hole

Let us consider on $\mathbb{R} \times (\mathbb{R}^3 - \{0\})$ the polar coordinates $(t, r, \vartheta, \varphi)$, where t is the time, and let $\mathrm{d}\Omega^2 := \mathrm{d}\vartheta^2 + \sin^2 \mathrm{d}\varphi^2$. The *Schwarzschild metric* is:

$$\mathrm{d}s^2 = \left(1 - \frac{2m}{r}\right) \mathrm{d}t^2 - \frac{1}{1 - \frac{2m}{r}} \mathrm{d}r^2 - r^2 \mathrm{d}\Omega^2. \tag{7}$$

Here, $m = \frac{GM}{c^2}$, where M is the mass, G is Newton's constant, and c the speed of light.

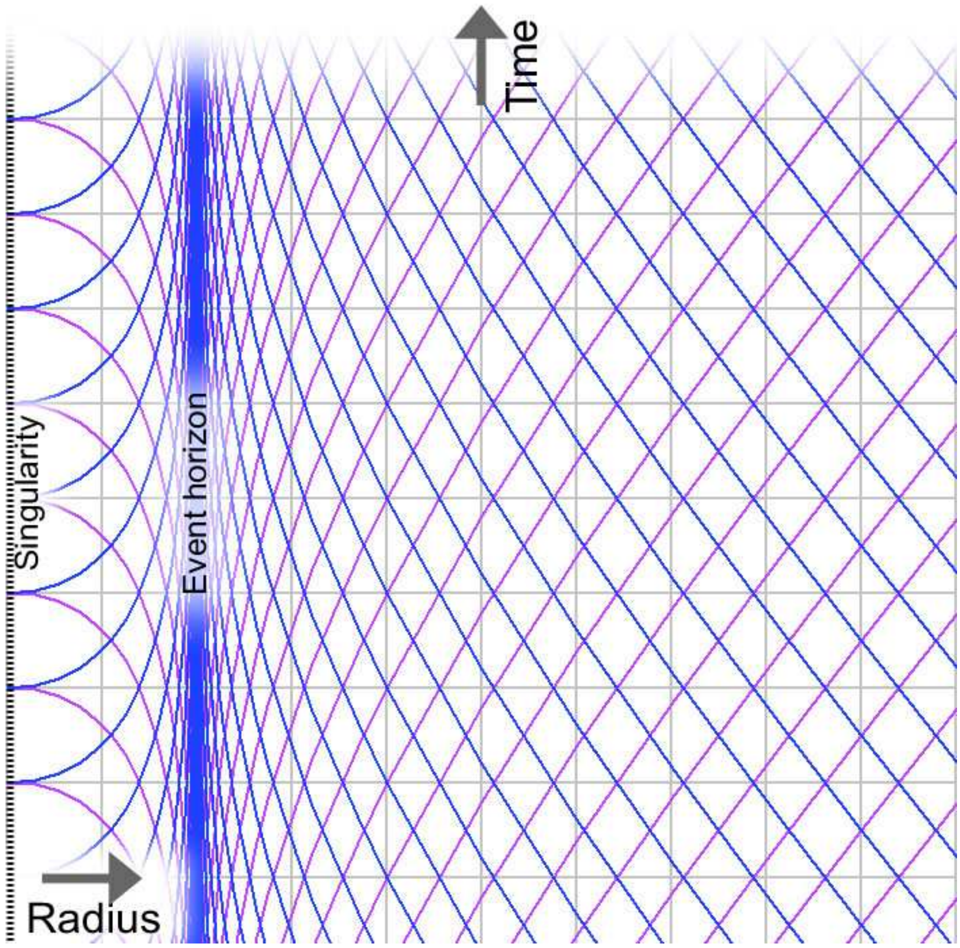

FIGURE 2. The Schwarzschild coordinates of the Schwarzschild black hole present an apparent singularity at $r = 2m$ beyond which they interchange the time with the radial direction of space. The colored curves represent ingoing and outgoing null rays.

The *Schwarzschild radius* is $r_0 = 2m$, and it is the radius of the *event horizon*, the spherical surface trapping the space inside the black hole. This coordinate system has the property that the points of constant r and t form a 2-sphere of area $4\pi r^2$.

They present an apparent singularity at $r = r_0$, and also interchanges the time with the radial direction of space (fig. 2). Both problems can be removed by using another coordinate system, but the singularity at $r = 0$ is an invariant of the semiriemannian geometry, and is independent of the coordinates.

One nice coordinate system, due to Eddington and rediscovered by Finkelstein, can "repair" the ingoing null rays (figure 3). It consists in changing the time coordinate to remove the singularity of the ingoing null rays:

$$t_{EF} = t + 2m \ln |\frac{r}{2m} - 1|. \tag{8}$$

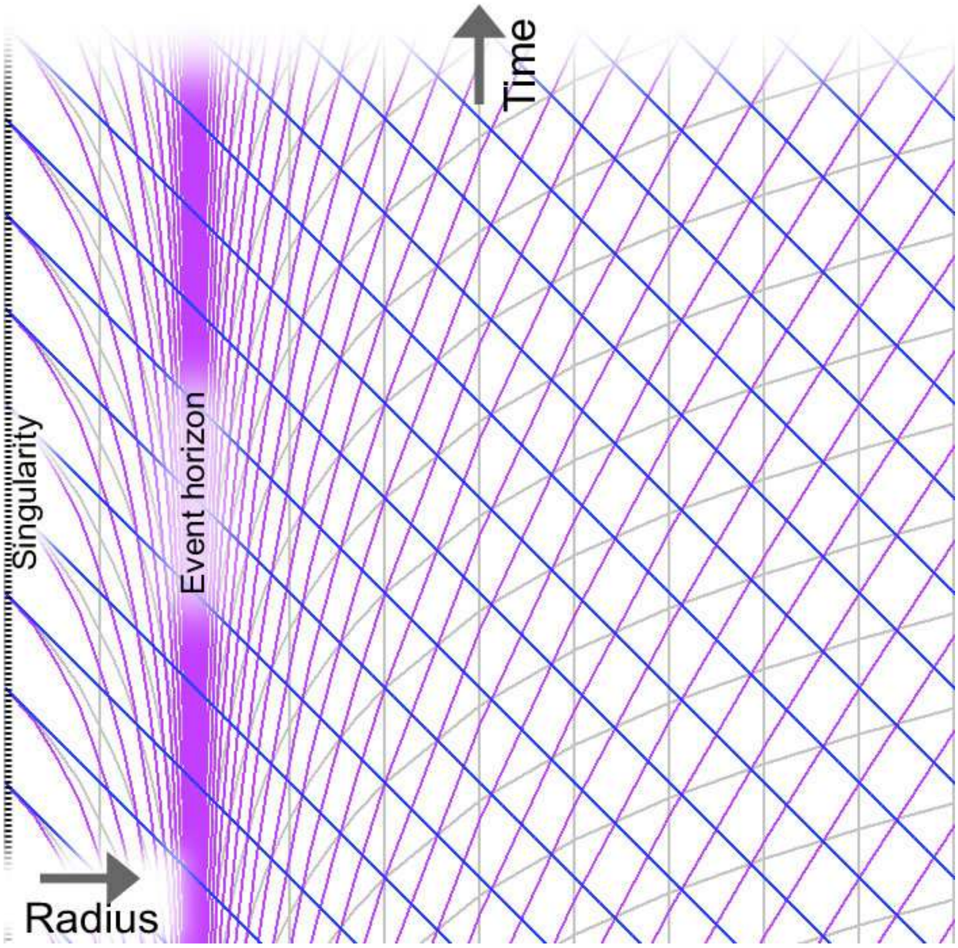

FIGURE 3. Eddington-Finkelstein coordinates allow a better description of the ingoing null rays, depicted in blue.

5. New coordinates for the Schwarzschild black hole

In both the Schwarzschild and Eddington-Finkelstein coordinates the singularity is present at $r = 0$ at any moment of time. We can choose a different coordinate system $(\tilde{t}, r, \vartheta, \varphi)$ which allows us to move the singularity at $\tilde{t} = +\infty$. In order to do this, we can subtract from the Eddington-Finkelstein time coordinate a function singular at $r = 0$, *e.g.* $\tilde{t} = t + 2m \ln |\frac{r}{2m} - 1| - 2m \ln |\frac{r}{2m}| = t + 2m \ln |1 - \frac{2m}{r}|$. We obtain

$$\tilde{t} = t + 2m \ln |1 - \frac{2m}{r}|, \tag{9}$$

as in figure 4. We can easily check that this new coordinate system delays to infinity the moment when the singularity is reached. It also maintains a good description of the null rays.

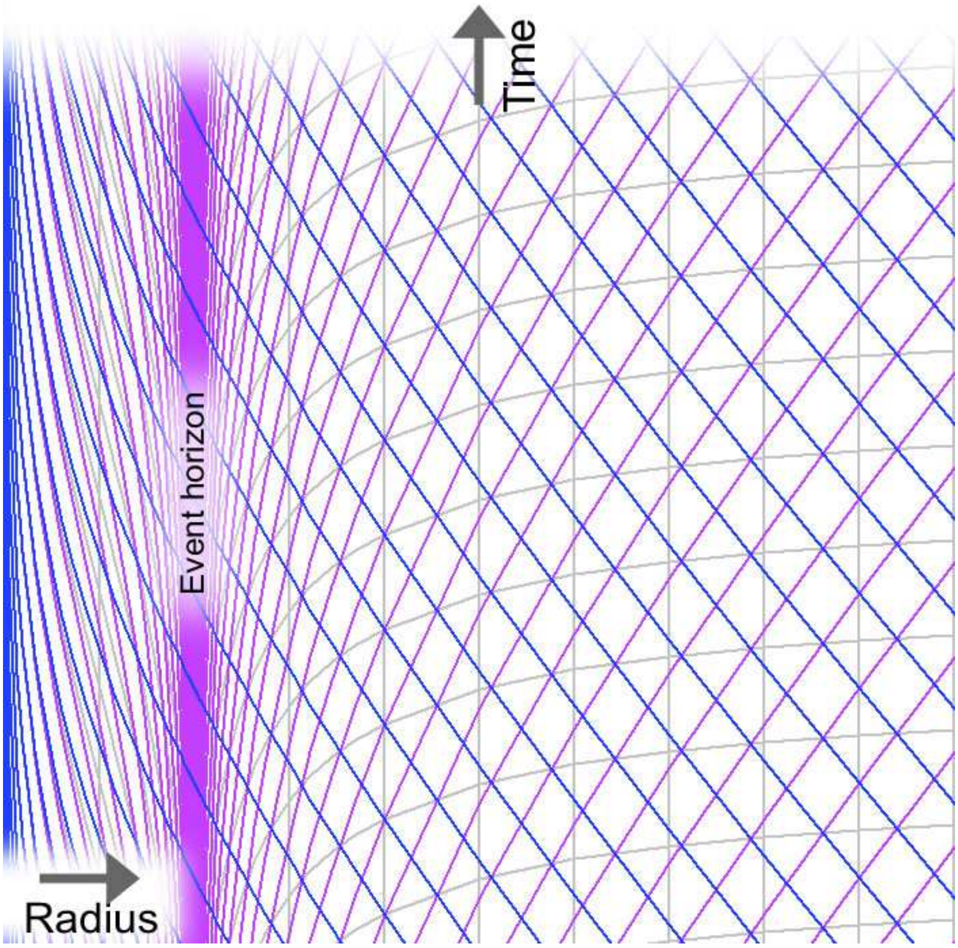

FIGURE 4. We can choose the coordinates of the Schwarzschild black hole so that the singularity is moved to future infinity.

Another coordinate system with this property can be obtained by modifying the *Kruskal-Szekeres coordinates*, which are obtained from Schwarzschild's coordinates by changing the coordinates for the exterior region $r > 2m$ to

$$\begin{cases} t_{KS} = \sqrt{\frac{r}{2m} - 1} e^{\frac{r}{4m}} \sinh \frac{t}{4m} \\ r_{KS} = \sqrt{\frac{r}{2m} - 1} e^{\frac{r}{4m}} \cosh \frac{t}{4m} \end{cases} \tag{10}$$

and for the interior region $r \in (0, 2m)$ to

$$\begin{cases} t_{KS} = \sqrt{1 - \frac{r}{2m}} e^{\frac{r}{4m}} \cosh \frac{t}{4m} \\ r_{KS} = \sqrt{1 - \frac{r}{2m}} e^{\frac{r}{4m}} \sinh \frac{t}{4m}. \end{cases} \tag{11}$$

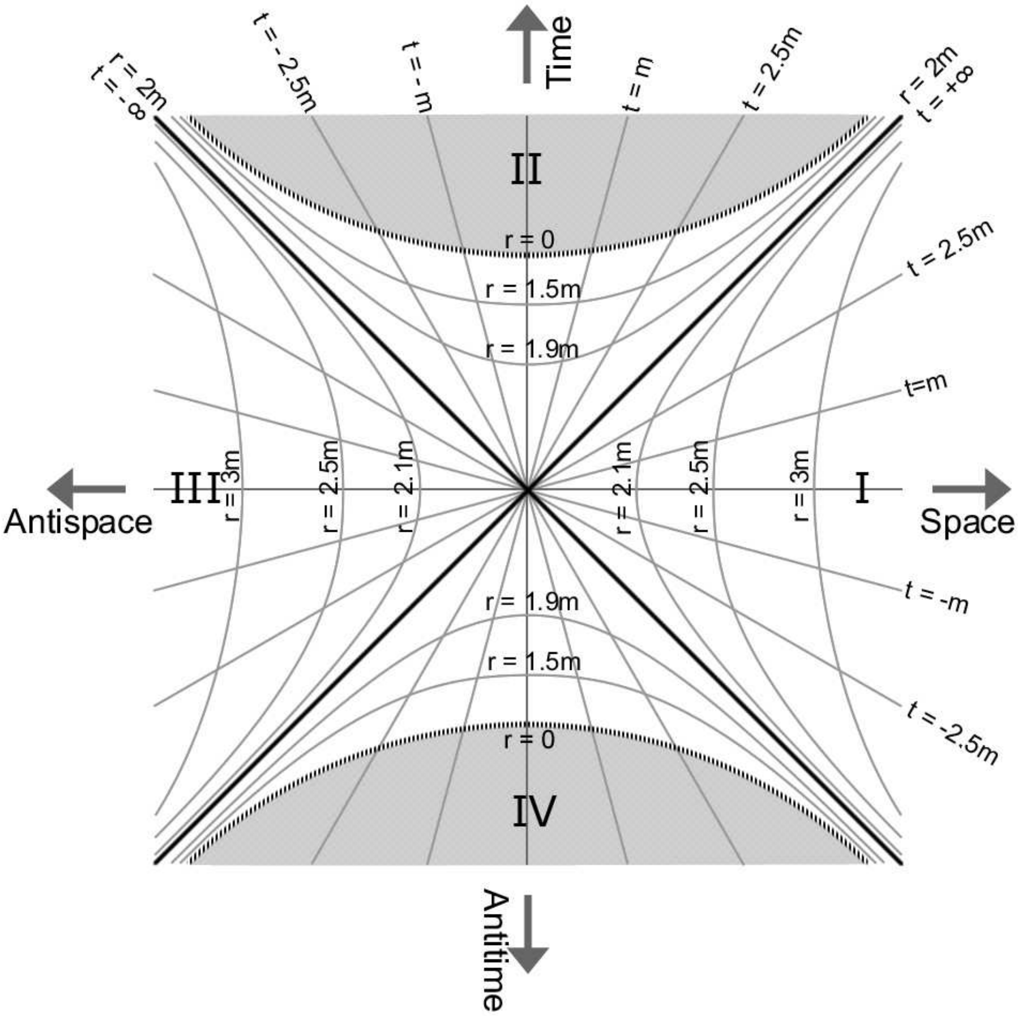

FIGURE 5. The Kruskal-Szekeres parametrization of a Schwarzschild black hole.

The *Kruskal-Szekeres coordinates* map the region $r > 2m$ from the Schwarzschild coordinates into the region I in figure 5, and the region $r \in (0, 2m)$ into the region II. The metric becomes

$$\mathrm{d}s^2 = \frac{32m}{r} e^{-\frac{r}{4m}} (\mathrm{d}t_{KS}^2 - \mathrm{d}r_{KS}^2) - r_{KS}^2 \mathrm{d}\Omega^2. \tag{12}$$

The Kruskal-Szekeres solution can be analytically extended to regions III and IV, by the same mappings but with changed signs. They have the property that the null rays are lines at 45°, and the timelike directions form angles smaller than 45° with the vertical direction. We see more clearly that the singularity is in fact a spacelike hypersurface, and any falling object will hit it in a finite proper time.

We can reparametrize the interior regions of the black hole so that the singularity is reached at the future infinity on the time coordinate, by:

$$\begin{cases} t' = \frac{2}{\pi}\tan\left(\frac{\pi}{2}\sqrt{1-\frac{r}{2m}}e^{\frac{r}{4m}}\right)\cosh\frac{t}{4m} \\ r' = \frac{2}{\pi}\tan\left(\frac{\pi}{2}\sqrt{1-\frac{r}{2m}}e^{\frac{r}{4m}}\right)\sinh\frac{t}{4m} \end{cases} \tag{13}$$

for the region II, and $-t'$, $-r'$ for the region IV. We obtain the coordinates represented in figure 6. We see that now the coordinates cover the entire spacetime, and the singularity is moved to infinity (in these coordinates).

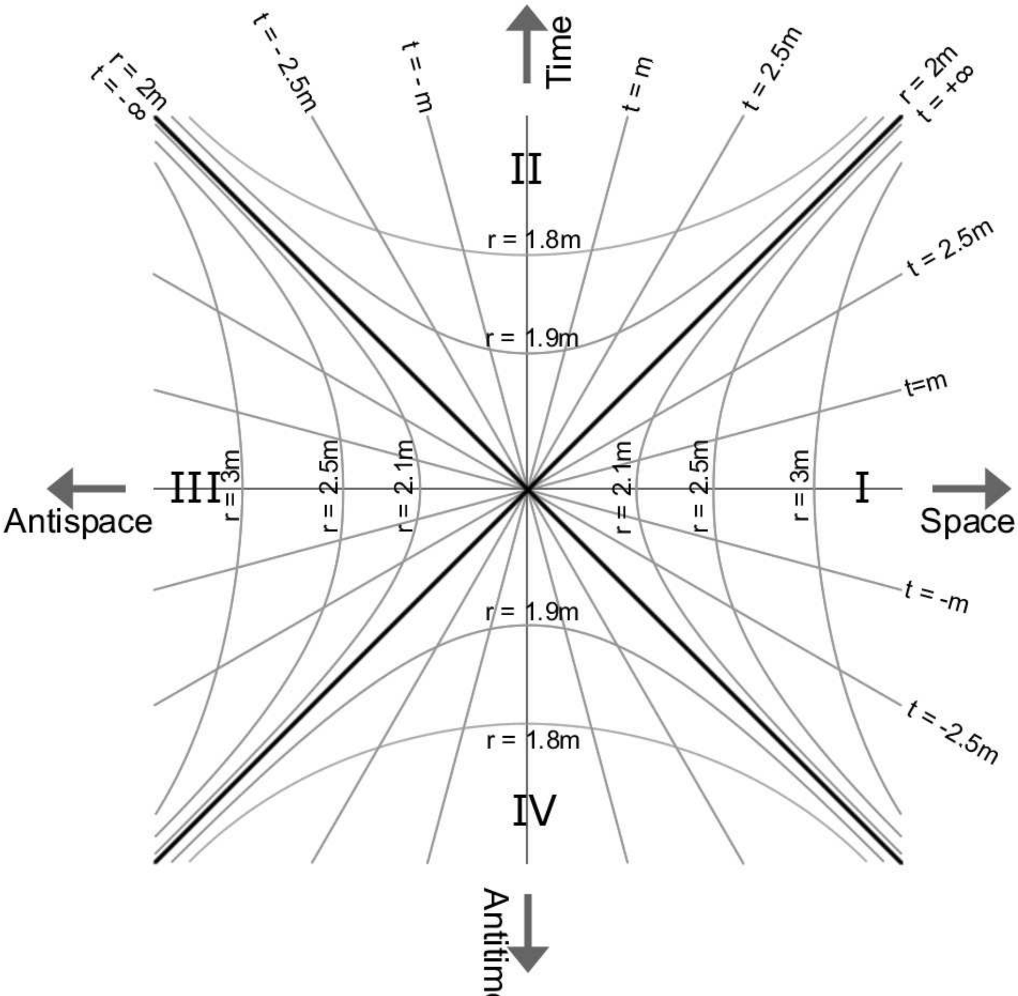

Figure 6. Moving the singularity at the future infinity.

The singularity is at infinite, despites the fact that it can be reached in finite proper time, that is, the line element's integral for $t' \to \infty$ is still finite (because it is invariant). The strangeness of the black hole singularity consists, in fact, in having an infinite distant point at a finite distance, as measured according to the Schwarzschild metric. This should not be considered that pathological as it is sometimes suggested, because we already have such a situation in the Minkowski spacetime, where a photon reaches infinity in space and time instantaneously, according to its proper time. It is even conceivable that we can accelerate forever a particle asymptotically towards the speed of light, without exceeding it, so that the integral over the proper time for the entire particle's future history becomes finite.

6. Restoring the information conservation for the Schwarzschild black hole

The evolution equations in fundamental physics are deterministic and reversible, this meaning that the solution (and usually its partial derivatives) at t_0 determines the solution for any other t, being it in the future or in the past of t_0. This applies also for the unitary evolution in Quantum Theory taking place between two successive measurements. It seems that during quantum measurements the wavefunction collapses discontinuously[2], but the black hole information paradox refers only to the unitary evolution.

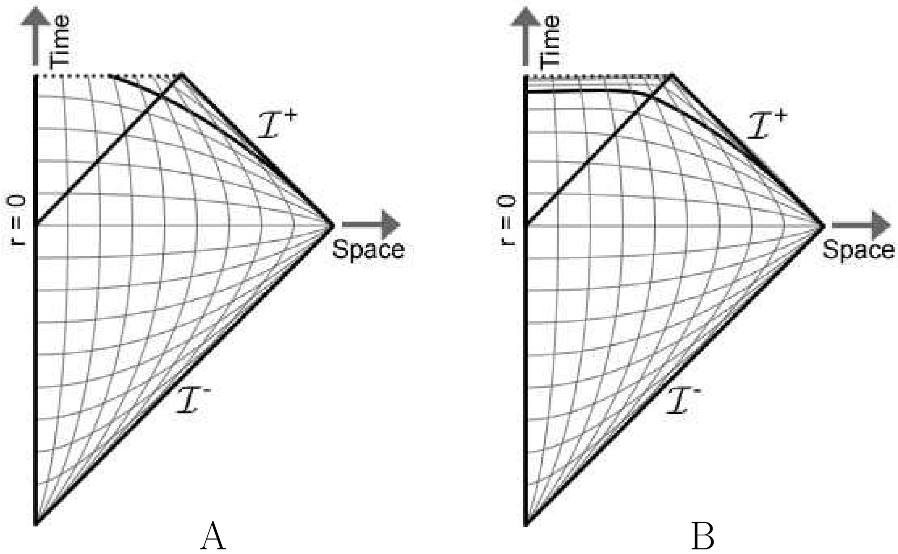

FIGURE 7. A. A black hole seems to lose information, because the time reaches its end in the singularity. B. By moving the singularity at infinite with proper coordinates, we can foliate the spacetime so that the information is preserved.

In a Schwarzschild black hole, the time reaches its end and the spacelike hypersurfaces are "eaten" by the singularity, as we can see in the Penrose diagram in figure 7.A. The information seems to be lost. But we can restore the conservation of information by choosing an appropriate foliation of spacetime in spacelike hypersurfaces. For example, we can take in the coordinates (9) the spacelike hypersurfaces $\tilde{t} =$ const. (fig. 4), or in the coordinates (13) the spacelike hypersurfaces $t' =$ const. (fig. 6). The corresponding Penrose diagram is represented in figure 7.B.

[2]It can be argued that even the wavefunction collapse can take place unitarily [12].

7. A black hole with a beginning

Schwarzschild solution represents a stationary black hole, with no beginning and no end. We can obtain a description of a black hole which begins to exist at a finite moment in time, starting with the coordinates defined at (9).

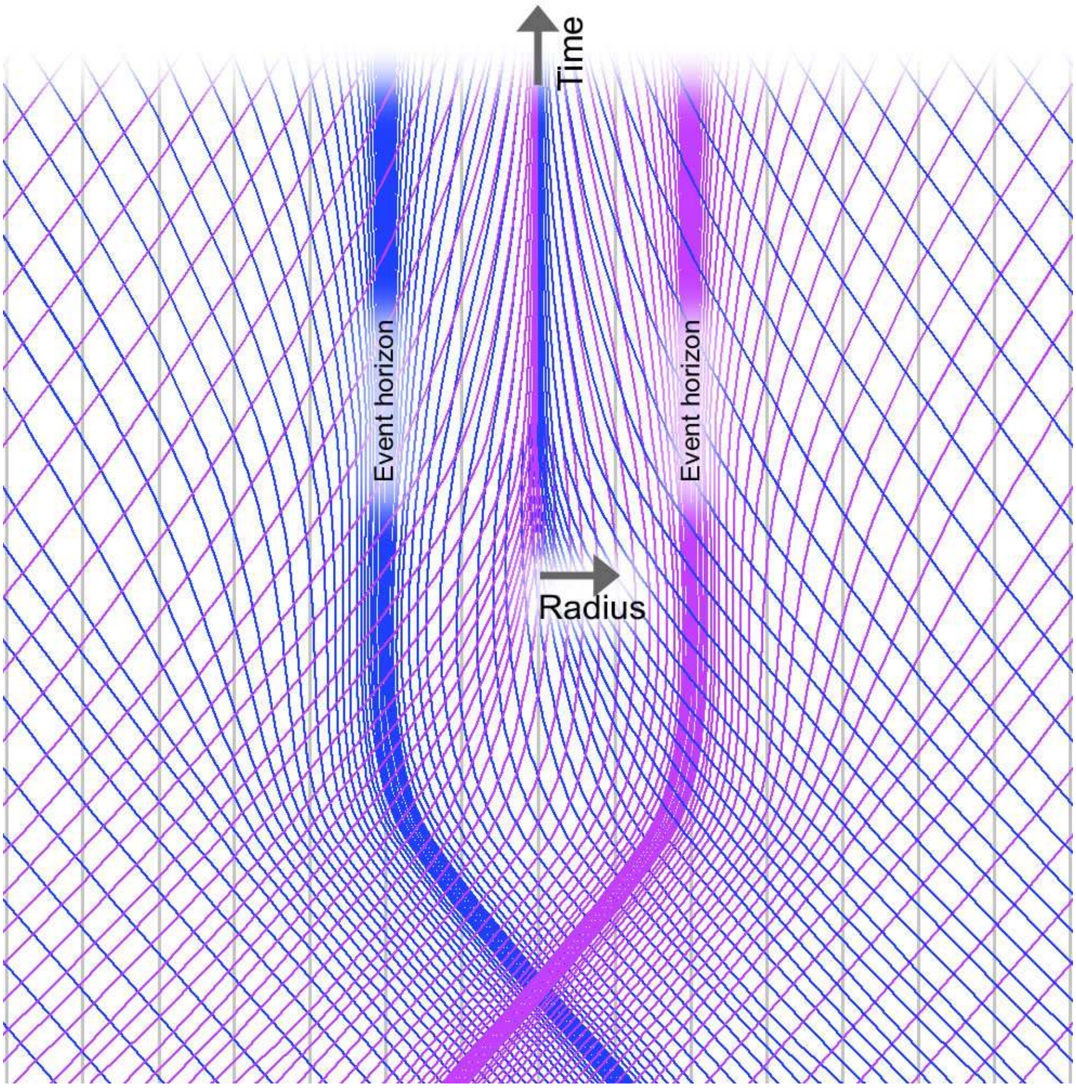

FIGURE 8. A black hole created at a finite time, with the singularity moved at $\tilde{t} = +\infty$.

The solution requires to represent, together with the region $r > 0$ (figure 4), the region $r < 0$, which mirrors it. Because the black hole begins its existence at a finite time, the null rays go from one region to the other, and we have to combine them, as in figure 8. The outgoing null rays for $r \in (0, 2m)$ cannot come from the region $r > 2m$, because they would be ingoing rays. They come from the region $r < 0$, as ingoing rays for that region. The outgoing null rays for $r > 2m$ also come from the region $r < 0$, as ingoing rays for that region. Similarly, a part of the ingoing rays for the region $r > 0$ are going in the region $r < 0$, where they become outgoing rays for that region. This representation of a black hole also possesses the property of having the singularity delayed to $\tilde{t} = +\infty$.

8. The inevitability of singularities

In general, the singularity theorems show that, under three assumptions (varying from theorem to theorem), follows that a singularity occurs. These three assumptions are: a global condition on the spacetime, allowing well posed initial value problems, a condition saying that in a region of spacetime there is a closed trapped surface, and an energy condition on the matter, saying that the energy flows are non-negative. These conditions guarantee the occurrence of singularities in the black holes.

One may find tempting to infer from the negative energy flow due to Hawking evaporation that the singularity theorems don't apply, and the singularity may be avoided. Figure 8 may encourage us to think that the evaporation can happen before reaching the singularity. If this is true for some black hole, than it should be very short-living, so that the energy condition don't hold too long. In general the singularity occurs. Once the initial data on a spacelike section of the past cone of a point "decided" that this point will be a singularity, there is no way to avoid this without violating the causality. We represent this dependence for Eddington-Finkelstein coordinates in fig. 9.A, and for Kruskal-Szekeres coordinates in fig. 9.B.

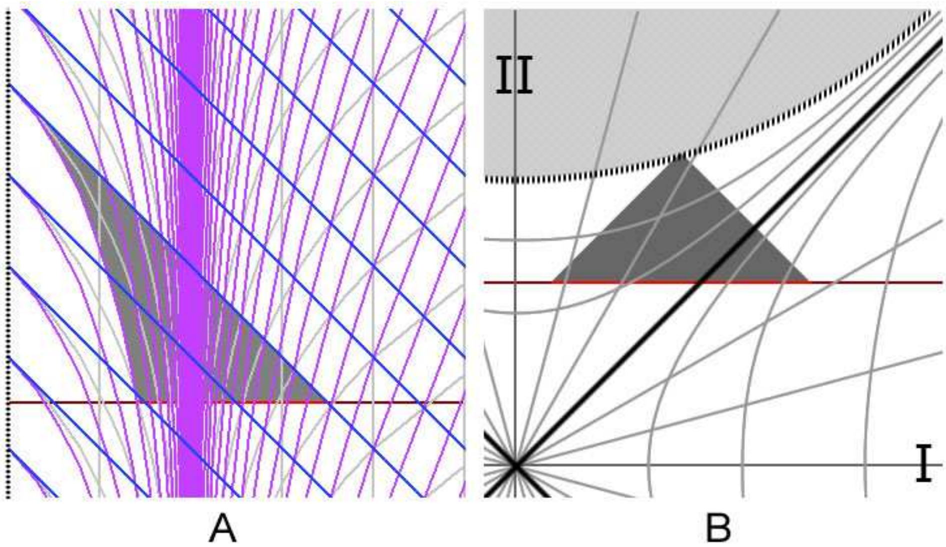

FIGURE 9. The initial data on a spacelike hypersurface decides whether a singularity will occur in the future.

9. Time reversal and black holes

Let us perform the following thought experiment. Consider a spacetime $\mathbb{R} \times \mathbb{R}^3$ with the canonical basis. The matter distribution, considered to respect Einstein's equation, is assumed to be spherically symmetric around the origin in each spacelike hyperplane $t =$ const., and symmetric to time reversal about $t = 0$. Suppose further that the overall matter distribution depends continuously on a parameter $\lambda \in [0, 1]$, so that for $\lambda = 0$ the spacetime is empty, but as λ goes to 1, a black hole appears and completely evaporates between $t = -T$ and $t = +T$. For any $\lambda \in [0, 1]$ there is no black hole at $t = \pm\infty$. The mechanism by which the matter goes into the black hole is different from the evaporation, but this doesn't matter for our purpose.

One first hint provided by this toy universe is that, because at infinity it remains basically independent of λ, the black hole should not have very violent effects on the spacetime, at global level. Maybe the information is, after all, preserved. Another hint is that there may be a symmetry at time reversal of the evaporating black hole (fig. 10).

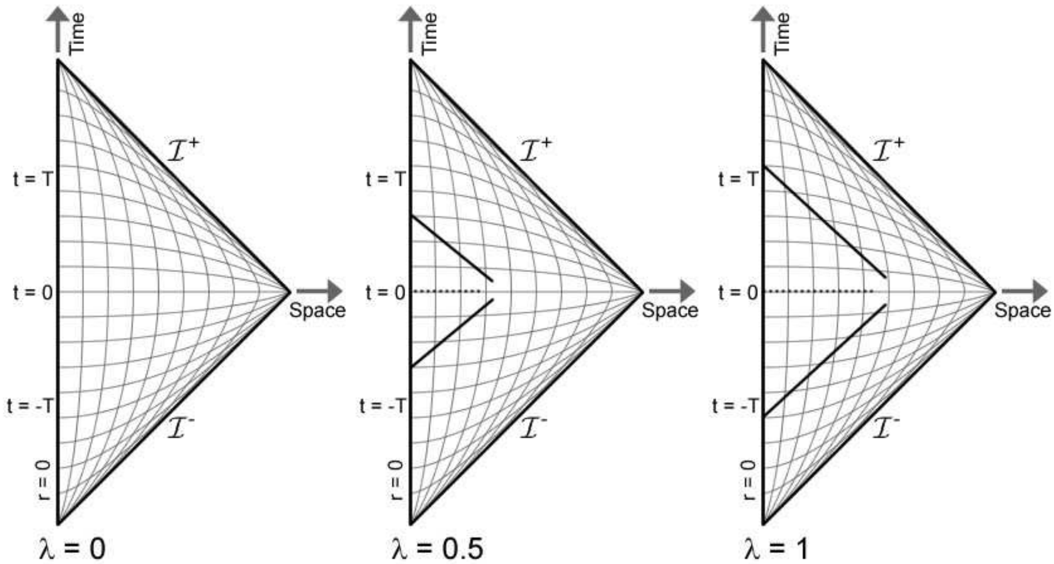

FIGURE 10. The toy spacetime containing a time symmetric evaporating black hole, parametrized by λ.

Although for a Schwarzschild static black hole the event horizon is a null hypersurface, this is no longer true when the mass changes. A null surface constitutes the event horizon so long as its area is stationary. If the mass increases around a moment t_0, the null surface which at t_0 was the event horizon (*i.e.* stationary) shrinks, and becomes trapped. Another surface, which at t_0 was divergent and enveloped the event horizon at that time, stops growing and becomes stationary. That's why the event horizon is in fact a spacelike hypersurface, during the growth period. When the black hole's mass decreases, the previously trapped null surfaces gradually become event horizons, then they become divergent. The black hole turns into a white hole, because we cannot have at the same instant a future trapped null surface and a past trapped one (this would involve a singular event horizon).

10. **Repairing the incomplete geodesics**

What about the rays reaching the singularity? During the black hole's growth, the outgoing null rays are trapped, leading to singularity. Running the time evolution backwards shows a similar history: the black hole evaporation seen in reverse becomes growing black hole, the matter density becomes large enough to trap the light, predicting a singularity in its own future (which is in fact the past). The two singularities are combined into one, in which some rays are falling and others are escaping. Combining a black hole depicted in figure 8 with its time reversed image, we obtain the figure 11.

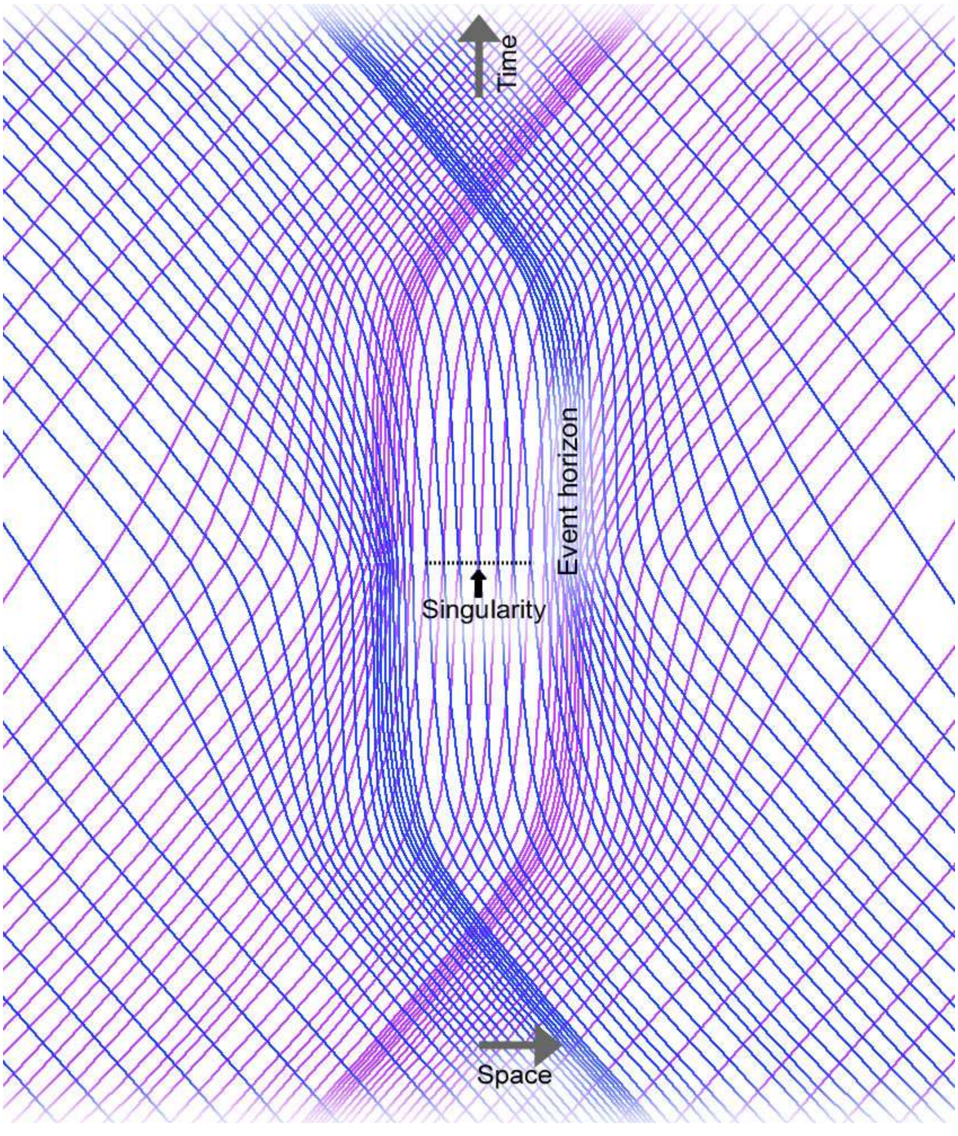

FIGURE 11. During the black hole's growth, the outgoing null rays fold around it, and while it decreases, they unfold. For both directions of time singularities are predicted, being in fact only one, with both infalling and escaping null rays.

One note about the singularity of the Schwarzschild black hole: in some coordinates (*e.g.* Kruskal-Szekeres) it is a 3-surface, and in others (*e.g.* Schwarzschild, Eddington-Finkelstein) it degenerates to a spacelike curve. The singularity in the figure 11 appears to be a spacelike 3-surface. We can gradually distort the singularity of an evaporating black hole from a curve to a 3-surface (figure 12).

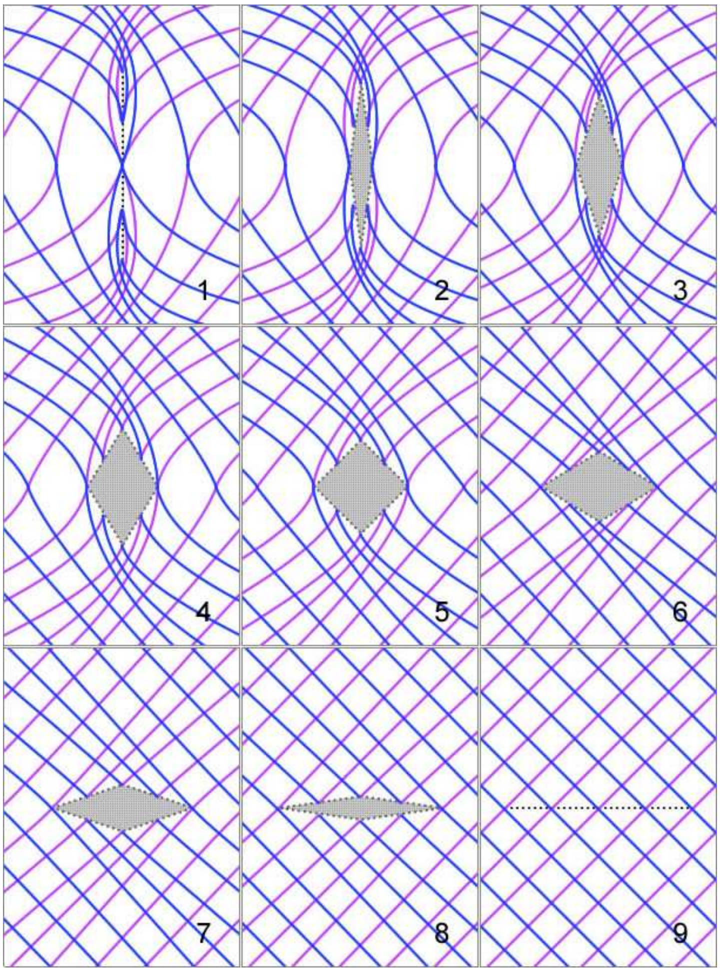

FIGURE 12. The singularity (the dotted gray region) of an evaporating black hole appears well behaved in appropriate (but singular) coordinates (9).

The singular spacelike curve (1) morphs through an intermediary 3-surface obtained by joining two cones (2-8), depicted by the diamond shape, to a 3-surface, which in the figure is a horizontal line (9). We can then identify the null rays falling into the singularity with the ones going out. For doing this, we can use the metric on the singular hypersurface. Since the metric is degenerate, we need extra information about the geodesics. For example, the metric gives the distance between the singularity's center and the points where each geodesic ends/starts, and the spherical symmetry allows us to identify the infalling geodesics with outgoing geodesics in the same radial plane, thus making the identification unambiguous.

The last geodesics entering the singularity are the first ones to leave it (LIFO – last in, first out). If the evaporation is never completed, a part of the black hole survives forever. The remaining singularity can be moved to infinity, as we did in the eternal black hole case.

11. **Singular General Relativity**

Two distinct points p and q in a vector space V with a bilinear form can be at a zero distance in two ways. First, they can be separated by a lightlike, or null interval, as in the Minkowski spacetime. The second possibility occurs when the bilinear form is degenerate. There is a vector subspace $\tilde{V} \leq V$ made of all the elements orthogonal on V. If the bilinear form is degenerate, $\dim \tilde{V} > 0$. If the vector $q - p$ belongs to $\tilde{V}$, then the distance is again zero.

In General Relativity, the initial value problem can be formulated in a relaxed way, which can lead to a metric which is degenerate at some points. In the ADM formalism [13] this is not allowed, because the equations make use of the metric's inverse, requiring thus to be nondegenerate. In this approach, the singularities become fatal. But we can use Ashtekar's new variables [14, 15], which allow us to write the evolution equations in terms of triads which can be degenerate, allowing degenerate metric. In fact, it is even possible to avoid entirely the metric in formulating General Relativity [16]. For our purpose, it is enough to know that the metric can become degenerate, when in the usual formulation we obtain singularities.

Let us consider a singular semiriemannian manifold $(\tilde{M}, \tilde{g})$, with the property that at each point the metric signature (t', s') satisfies $t' \leq t$ and $s' \leq s$ for fixed t, s, $t + s = \dim \tilde{M}$. It follows that where the metric is nondegenerate, its signature is (t, s). For any point $p \in \tilde{M}$ there is a maximal vector subspace $\tilde{T}_p \leq T_p\tilde{M}$ of vectors orthogonal to all vectors in $T_p\tilde{M}$. When $\tilde{g}_p$ is degenerate, $\dim T_p > 0$. We define the following binary relation between two points $p, q \in \tilde{M}$:

$$p \bowtie q \tag{14}$$

if and only if $\tilde{g}_p$ is degenerate, and there is a C^1 curve $\gamma : [0, 1] \to \tilde{M}$, $\gamma(0) = p$, $\gamma(1) = q$, so that for all $t \in [0, 1]$

(1) $\tilde{g}_{\gamma(t)}$ has the same signature as $\tilde{g}_p$,
(2) $\dot{\gamma}(t) \in \tilde{T}_{\gamma(t)}$.

It is easy to check that $\bowtie$ is an equivalence relation on $\tilde{M}$.

If $p, q \in \tilde{M}$ such that $p \bowtie q$ and the signature of $\tilde{g}_p$ is (t', s) with $t' < t$, we say that p and q are *timelike separated by a zero distance*, if the signature of $\tilde{g}_p$ is (t, s') with $s' < s$, we say that they are *spacelike separated by a zero distance*. The meaning is that the zero distance is obtained by degenerating a timelike, respectively a spacelike distance.

When we solve an initial value problem for Einstein's equation, reformulated in Ashtekar's variables, we can obtain a metric which is degenerate at some points. There are two ways of looking at the singularities. One is the standard, *semiriemannian view*. This view tacitly assumes that when the distance between two spacelike separated events is zero, they coincide (figure 13.A). (Some prefer to remove the points of singularity from the spacetime.) This view is based on the implicit assumption that the metric cannot be degenerate.

The second view, the *singular semiriemannian view*[3], proposed here, comes more naturally from treating the Einstein equation in Ashtekar's formulation. It simply allows the metric to be degenerate at some points, without identifying the points at zero distance (figure 13.B). Instead of taking as a solution a *semiriemannian spacetime*, we should take the *singular semiriemannian spacetime* solution.

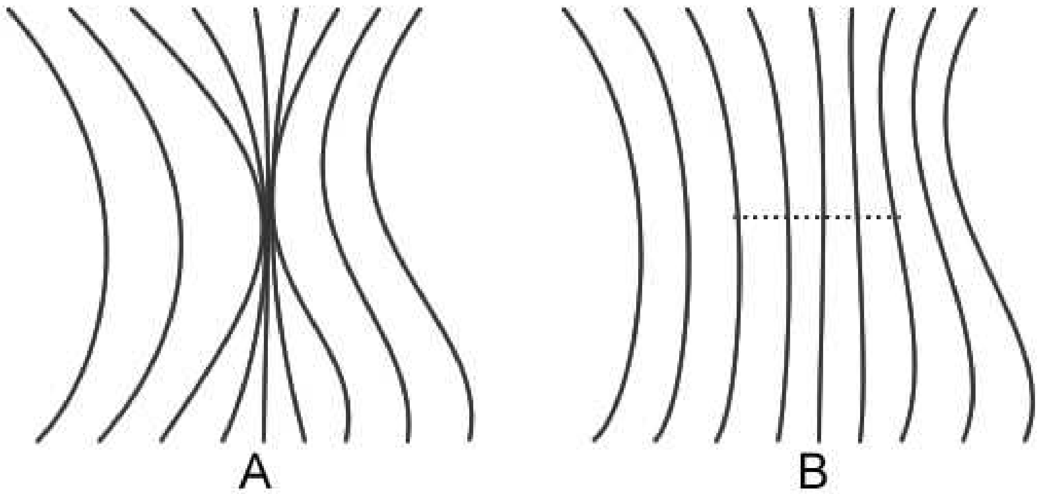

FIGURE 13. The semiriemannian view (A) identifies spacelike separated points at distance 0, while the singular semiriemannian view (B) allows them to be distinct.

Two events spacelike separated by a zero distance do not necessarily coincide. An example is given by two light rays traversing the event horizon of a Schwarzschild black hole simultaneously. They will reach the singularity at the same time, from distinct directions. Since the r coordinate goes to 0 as the two rays approach the singularity, the distance between them, measured on the sphere $r =$ const., converges to 0. They become, from semiriemannian viewpoint, identified. But the Cauchy development doesn't force us to identify them. This zero distance is simply caused by the metric being degenerate. For an evaporating black hole, the semiriemannian view overlaps the geodesics, making them to appear incomplete. Going to the singular semiriemannian view allows us to see that they do not, in fact, meet. This allows us to identify the geodesics entering the singularity with those exiting it.

The Cauchy development gives, in fact, a singular semiriemannian manifold $(\tilde{M}, \tilde{g})$, from which we can obtain the standard semiriemannian manifold (M, g) by $M = \tilde{M}/\bowtie$.

Penrose [10, 3, 11] emitted the *Cosmic Censorship Hypothesis*, stating that there are no timelike singularities, and that they can be only initial or final, but not both. On

[3]Singular semiriemannian geometry studies the differentiable manifolds with metric which is allowed to be degenerate. In general, the signature of the metric is considered to be constant, but here we need it to be variable.

the other hand, Hawking evaporation leads to a naked singularity, and maybe not all of these singularities should be ruled out. The discussion done so far allows us to revisit the Cosmic Censorship hypothesis, in order to clarify what singularities are acceptable. It seems reasonable to hypothesize that the allowed singularities are those that can be moved to infinity by a proper coordinate system, and those that can be obtained from a singular semiriemannian spacetime, by the procedure described above. Adopting the singular semiriemannian geometry interpretation seems to clarify this issue. It also allows the information to be preserved, and the unitary evolution to be restored.

12. **Conclusions**

When the conditions in the Penrose-Hawking singularity theorems are fulfilled for the entire history, the black hole exists forever, but its singularity can be moved to infinity by choosing appropriate coordinates (figures 4 and 6). If the black hole is allowed to evaporate, we can reinterpret the singularity such that each incomplete geodesic entering in it can be continued with one leaving it. This can be done by solving the Einstein equation in Ashtekar's formulation, taking care not to identify the events spacelike separated by 0-length intervals, allowing thus for the metric to be degenerate. It follows that the source of the problems related to the singularities is this identification, which is natural from semiriemannian point of view, while from the Cauchy problem viewpoint is more natural to adopt the singular semiriemannian approach, and to keep such points distinct. This way, the information is preserved, and the unitary evolution is restored.

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