

WORDMORPH!:

A Word Game to Introduce Natural Deduction

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Abstract: Some logic students falter at the transition from the mechanical method of truth tables to the less-mechanical method of natural deduction. This short paper introduces a word game intended to ease that transition.

My first-year introductory logic course is structured like many others. We begin with logical concepts in English, then learn an artificial language for propositional logic. We use truth tables to analyze the structures of formulas and arguments in that truth-functional language, and then turn to natural deduction proofs in propositional logic.

In my first two iterations of the course, I noticed some good students struggled to make the transition from truth tables to natural deduction. End-of-semester conversations suggested a fundamental misunderstanding had slowed them down: they had been trained by their experience with truth tables to expect mechanical methods in logic, and they took their inability to see any simple mechanical method for natural deduction as evidence of their own misunderstanding. In extreme cases, their drive to discover an occulted mechanism led some students to invent for themselves bizarre constraints on proofs: the first step of every proof must be *modus ponens*, every premise must be used in order, inference rules get used up after you use them, etc. Though I had been careful to insist that natural deduction is a more flexible method than truth tables, and had selected teaching examples that falsified the confused rules students conjured for themselves, it nevertheless took some students weeks to let go of the expectation of a mechanical procedure that truth tables had established, and by that point they were behind.

The next semester I took even more care to find a variety of ways to say that there is more than one way to complete a derivation. I arranged a variety of in-class practice problems, spread over multiple meetings, designed to demonstrate that so long as the rules are followed, nothing else matters. Leave premises unused, or use them multiple times. Use

one or two rules repeatedly if you can, or use a bouquet if you want. The outcome was little changed: a handful of students who had a kind of alacrity in truth tables sank, temporarily, when we turned to natural deduction. More conversations indicated the same reasons as before. Some students, confused in the first days of a multi-week project, clung hard to their previous success with truth tables and could not hear that natural deduction involves a form of constrained creativity that is ruled out by our earlier mechanical method.

I set out to invent a game that could be introduced and mastered quickly, that would model the presentation of natural deduction proofs and highlight some fundamental contrasts with truth tables. WORDMORPH! is the result. It is word game intended to give students, in about forty-five minutes, a bird's-eye glimpse of what they will be asked to do in the coming unit on natural deduction. It is intended to help them discover for themselves the kind of freedom they'll have when writing derivations, and thus to provide a clean break with the purely mechanical method of truth tables.

This short paper sets out the rules of WORDMORPH! and explains how I use the game in the classroom.

The Rulebook for WORDMORPH!

WORDMORPH! is a word transformation game. I will give you a starting word and a target word. Your task is to transform the starting word into the target word by properly applying three transformation rules:

1. SYN: Write down a new line that is a synonym for a word on any previous line.
2. CAT: Write down a new line that concatenates (sticks together) any two previous lines.
3. ANA: Write down a new line that is a word that is an anagram made from any subset of letters from any single previous line.

To complete a word transformation, you will begin with the given word on line 1. On line 2 you will apply one of the transformation rules to line 1. Write down your new line, and note what rule you applied to line 1 to generate line 2. Line 3 can apply any of the transformation rules to any of the lines that came before. Write down your new line and, again, note what rule you applied to which line(s) to generate line 3. And so on.

Example 1: Given "Interesting" derive "cat."

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|----------------|-------|
| 1. Interesting | given |
| 2. Fascinating | SYN 1 |
| 3. Cat | ANA 2 |

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Example 2: Given “Gross” derive “time.”

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|---------------|---------|
| 1. Gross | given |
| 2. Disgusting | SYN 1 |
| 3. Dust | ANA 2 |
| 4. Grime | SYN 3 |
| 5. Dustgrime | CAT 3,4 |
| 6. Time | ANA 5 |

Note that every use of the SYN and ANA rules results in a word. The CAT rule can result in a word, but it can also result in a string of nonsense compounded from other words and strings, as with “Dustgrime” in Example 2. One way to think about the rules: the SYN rule takes in one word and outputs one word. The ANA rule can take in either one word or one string, and outputs a word. The CAT rule takes in two lines that could be words or strings, and outputs one line that could be either a word or a string.

Using WORDMORPH! in the Classroom

The game can be introduced and demonstrated in about five minutes. Set students in groups of three or four, and give them three or four WORDMORPH! problems to work on. Select one problem—I typically make it the third one—as the featured problem, and ask every group, when they finish it, to write their solution on the board. Once you have four or five solutions to the featured problem on the board, it is time to end the activity and debrief. It takes most groups about half an hour to complete three problems and write up a solution to the third.

A well-designed WORDMORPH! problem will generate a wide variety of approaches to solving it. One problem that has worked well over multiple semesters asks students to morph “lumber” into “jack.” Three examples from last semester’s iteration of the game:

One group brainstormed words that begin with “Jack,” and then worked toward a word they could pull “Jack” from. Their resulting transformation flirted with sass:

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|------------|-------|
| 1. Lumber | given |
| 2. Bum | ANA 1 |
| 3. Ass | SYN 2 |
| 4. Jackass | SYN 3 |
| 5. Jack | ANA 4 |

Another group opted to assemble the component letters of “Jack” in different words on different lines, then stick them all together with CAT in order to finish with ANA. I initially balked at “party” as a synonym for “do,” but the class reminded me that “do” and “party” are synonymous as nouns.

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1. Lumber	given
2. Wood	SYN 1
3. Do	ANA 2
4. Party	SYN 3
5. Jamboree	SYN 4
6. Undertake	SYN 3
7. Act	SYN 3
8. JamboreeUnderake	CAT 5,6
9. JamboreeUndertakeAct	CAT 7,8
10. Jack	ANA 9

The shortest solution I've seen was accomplished by a student fortuitously named Jack:

1. Lumber	given
2. Me	ANA 1
3. Jack	SYN 2

The game does not seem terribly sensitive to the assigned words pairs—most pairs I've tried have supported the activity. Three more pairs that have generated especially clever or funny solutions:

Given: "Kind" Derive: "Goose"
Given: "Boat" Derive: "Desk"
Given: "Suffering" Derive: "Meaning"

Students generally enjoy the game and work hard at it. They are usually excited to explain their transformation to the class and interested in their peers' approaches.

Be sure, in debriefing after the activity, to identify any errors in the transformations or citations of the posted solutions. And be sure to highlight that every solution on the board that properly applies and cites the rules is equally correct, even though they diverge in a variety of ways.

How WORDMORPH! Addresses the Problem

As students work through some transformations, they learn the following things about WORDMORPH!. They need to find their own path from the given word to the target word; there is no mechanical method they are supposed to apply. Different strategies are fairly obvious from the outset, and all of them stand a good chance of working. They do not have to use all the rules; they do not have to use them in any particular order; they can use them multiple times. They can abandon lines unused; they can use lines multiple times; they can use lines in any order. Proper presentation requires a running list of citations that justify each line in the transformation, and the proper position

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for each citation is on the same line whose origin it explains. Some WORDMORPH!s are shorter than others, or cleverer than others, or funnier than others, but so long as they scrupulously follow the rules, no WORDMORPH! is *correcter* than another.

All these lessons have analogs in natural deduction. And because the entire WORDMORPH! system can be mastered in minutes, students can easily relate these specific lessons to a big-picture understanding of the word game. The full system of natural deduction will take several weeks to introduce and longer for students to master, but having the analog of WORDMORPH! allows students to grasp, right from the start, the big picture of what they are being asked to do when presented with a derivation in propositional logic.

I've used WORDMORPH! to introduce natural deduction for three years, and it seems helpful in easing the transition from truth tables. The clearest evidence of effectiveness shows up in a general shift in the kinds of questions I field during class and in office hours.¹ The questions characteristic of struggling students in my first years indicated misunderstanding of the general project: "how did you know disjunctive syllogism was the rule for line 5?" The questions characteristic of struggling students now tend to indicate misunderstanding of the patterns specified by the inference rules: "how could you use *modus tollens* on line 6 when the antecedent on line 2 is already negated?" The latter kind of question, because it is more advanced, is easier to answer.

Mastering natural deduction remains a challenge for many students. Of course this is so. But my classes now begin the unit on natural deduction with a better initial grasp of the kind of method they are expected to learn in the coming weeks, and so students may more effectively direct their study efforts from the outset.

Conclusion

WORDMORPH! is an easily acquired word game that features important similarities to natural deduction proofs. Playing a few rounds of the game allows students to discover for themselves some of the ways natural deduction proofs differ from mechanical methods such as truth tables.

Acknowledgement

Conversations with Joshua Kortbein helped me sharpen my understanding of the problem that needed to be solved and planted the seed of the word-game solution.

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Notes

1. As is usually the case in real classrooms, it isn't possible to *measure* the effectiveness of a single activity. Though student performance on end-of-semester exams has improved since I introduced the word game, in any given semester I modify, introduce, and eliminate multiple activities, and I have no way to disaggregate the contributions of individual instructional changes to student performance on that summative assessment.

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