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Truth and Paradox in Late XIVth Century Logic: Peter of Mantua's Treatise on Insoluble Propositions*

1. Introduction

The purpose of this paper is to present the views of the late XIVth century Italian logician Peter of Mantua (d. 1399/1400)[1] on semantic paradoxes. In the Middle Ages, the topic usually falls within the broader category of the so-called *insolubilia*-literature, a genre that covers a variety of logical puzzles whose focus is not necessarily — albeit prominently — on semantic issues. I shall offer a preliminary assessment of the contents of a treatise that Peter wrote in the early 1390s as part of his *Logica*, and analyse it on the background of some late medieval discussions concerning the relationship between truth and paradox. The text has never been edited or studied so far, and its contents are presented here in detail for the first time[2].

Medieval analyses and solutions to semantic paradoxes, i.e. such self-referential propositions involving truth and falsehood that, combined with appropriate contextual conditions, give rise to contradictions, are often connected to, if not entirely depending upon, some specific characterisation of those two fundamental semantic notions. This

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[1] A reconstruction of Peter of Mantua's profile is found in T. E. James, *Peter Alboini of Mantua: Philosopher-Humanist*, « Journal of the History of Philosophy », 12, 2, 1974, pp.161-170.

[2] After quite a lot of invaluable editorial and interpretive work, owed primarily to the efforts of P. V. Spade in the 1970s and 1980s (P. V. Spade, *The Medieval Liar: A Catalogue of the* Insolubilia-*Literature*, Pontifical Institute of Mediaeval Studies, Toronto 1975 remains, to date, the only comprehensive source book), there has been for some time a relatively quiet phase in the studies on *insolubilia*. In the last few years, however, the topic was the object of a revival of interest, after a number of articles by S. Read that culminated with his own edition of Thomas Bradwardine's treatise, one of the most relevant medieval texts in the field, see T. Bradwardine, *Insolubilia*, ed. S. Read, Peeters, Leuven 2008 (Dallas Medieval Texts and Translations, 10). I am currently preparing the critical edition of Peter of Mantua's text. This is meant to provide new materials for investigation and inject them into a debate that has a long and well-established history, but in which considerable work is still to be done in terms of the editing of texts and the understanding of their contents; the same being true of the complex geography of their transmission and mutual influence.

claim may not universally hold of all medieval accounts, but it certainly does of a significant number of them, especially from the second quarter of the XIVth century onwards. In this respect, some of the problems raised within the debate on insolubles turn out to be relevant also in connection with the development of medieval theories of truth[3]. In Peter's case, truth plays an important role (although not the definition of truth to be found elsewhere in his logical writings, namely in the treatise *De taliter et qualiter seu De veritate et falsitate propositionis*) for his own solution to the Liar, and a reconstruction of his approach — particularly of the criticisms he raises against his restricted list of opponents — offers a chance to discuss what is going on in his background.

To set the stage, I will first sketch the development of the *insolubilia* tradition, focusing in particular on the XIVth century (§2). In doing so, I will briefly introduce two lists of solutions (Bradwardine's and Paul of Venice's) that are separated by a span of about seventy years from each other and represent two important chronological boundaries for the identification of the main theoretical strands. This will give a general idea of the evolution of the most mature phase of the medieval interest in the topic. Then I will turn to Peter of Mantua (§3). I shall present an outline of his treatise by describing the three accounts that he discusses and rejects as well as a sketch of his own solution (§3.1). The latter will be then analysed and discussed in detail (§3.2) and put into connection with earlier and later sources (§3.3). Despite its apparent appeal, it will be shown (§3.4) that the solution cannot count as a general way to get rid of the paradox because of the so-called revenge problem. Finally, (§4) I shall draw some conclusions concerning the solution from the standpoint of the contemporary debate on semantic paradoxes.

2. Developments of the *insolubilia*-literature

The history of the treatment of semantic paradoxes in the Middle Ages can be divided into three main periods[4]. The first phase, starting from the late XIIth century

[3] A case in point is the discussion of truth and the Liar paradox found in Buridan's *Sophismata* (especially in part of ch. 2 *On the Causes of Truth and Falsity of Propositions*, pp. 845-862, and in ch. 8 *On Self-referential Propositions*, pp. 952-997), see J. Buridan, *Summulae de Dialectica. An Annotated Translation with a Philosophical Introduction by Gyula Klima*, Yale University Press, New Haven 2011. It should be noted that the difficulties raised by the Liar are connected with Buridan's account of the validity of consequences, which is spelled out without recourse to the notion of truth, see G. Klima, *Logic without Truth. Buridan on the Liar*, in S. Rahman, T. Tulenheimo, E. Genot eds., *Unity, Truth and the Liar. The Modern Relevance of Medieval Solutions to the Liar Paradox*, Springer, Berlin 2008 (Logic, Epistemology, and the Unity of Science, 8), pp. 86-112.

[4] An early reconstruction is found in P. V. Spade, *Insolubilia*, in N. Kretzmann, A. Kenny, J. Pinborg eds., *The Cambridge History of Later Medieval Philosophy*, Cambridge University Press, Cambridge 1982, pp. 246-252. More recently, see P. V. Spade, S. Read, *Insolubles*, in E. N. Zalta ed., *The Stanford Encyclopedia of Philosophy (Winter 2009 Edition)*, URL = <http://plato.stanford.edu/archives/win2009/entries/insolubles/>, for up-to-date bibliography and a bird's eye view of the tradition. A selection of texts, with an introduction, covering the entire chronological *spectrum* is found in L. Pozzi, *Il Mentitore e il Medioevo. Il dibattito sui paradossi dell'autoriferimento*, Zara, Parma 1987.

with independent texts (*Insolubilia Monacensia*) or with reflections connected to the early developments of the theory of obligations (*Obligationes Parisienses*)[5], sees a progressively more mature analysis that reaches a steady equilibrium by the first quarter of the XIVth century. It is characterised by a relatively contained number of solutions, which are formulated in different types of sources : commentaries on Aristotle's *Sophistici Elenchi* or independent treatises, sometimes featuring as chapters of more comprehensive logical works (Burley, Ockham).

The second phase roughly coincides with the second quarter of the XIVth century. During this period, some crucial innovations, breaking up with the earlier tradition, are introduced in the debate on paradoxes. This fact is witnessed by the complexity of the logical approaches that can be traced back to such major figures as Thomas Bradwardine, John Buridan, Roger Swyneshead, and William Heytesbury, among others[6].

The third phase — from the second half of the XIVth century on — is characterised by discussions and elaborations on this set of results, with some occasional flashes of originality. Peter of Mantua's text, not merely in terms of chronology, but also, so to speak, in terms of methodology, fits perfectly in this very last period[7].

[5] See C. Martin, *Obligations and Liars*, in S. Read ed., *Sophisms in Medieval Logic and Grammar*, Kluwer, Dordrecht 1993, pp. 357-381 (Nijhoff International Philosophy Series, 48), repr. in M. Yrjönsuuri ed., *Medieval Formal Logic. Obligations, Insolubles and Consequences*, Kluwer, Dordrecht 2001, pp. 63-94 (The New Synthese Historical Library, 49) and C. Martin, *The Logic of the* Nominales, *or*, *The Rise and Fall of Impossible* Positio, « Vivarium », 30, 1, 1992, pp. 110-126. Cf. also P. V. Spade, *The Origins of the Medieval* Insolubilia-*Literature*, « Franciscan Studies », 33, 1973, pp. 292-309, repr. in Id., *Lies, Language and Logic in the Late Middle Ages*, Variorum, London 1988. Awareness of the problem and the existence of a debate are documented also in the Arabic tradition (see A. Alwishah, D. Sanson, *The Early Arabic Liar : The Liar Paradox in the Islamic World from the Mid-Ninth to the Mid-Thirteenth Centuries CE*, « Vivarium », 47, 2009, pp. 97-127) and in the Byzantine tradition (see S. Gerogiorgakis, *The Byzantine Liar*, « History and Philosophy of Logic », 30, 4, 2009, pp. 313-330). Both possibly predate the earliest Latin accounts, which can be traced as far back as the late XIIth century (see, for instance, the *Insolubilia Monacensia*, in L. M. De Rijk, *Some Notes on the Mediaeval Tract* De Insolubilibus, *with the Edition of a Tract Dating from the End of the Twelfth Century*, « Vivarium », 4, 1966, pp. 83-115).

[6] See (1) Bradwardine, *Insolubilia* cit., (cf. also M.-L. Roure, *La problématique des propositions insolubles au XIII*e *siècle et au début du XIV*e, *suivie de l'édition des traités de W. Shyreswood, W. Burleigh et Th. Bradwardine*, « Archives d'histoire doctrinale et littéraire du Moyen Âge », 37, 1970, pp. 205-326) ; (2) J. Buridan, *Tractatus de Consequentiis*, H. Hubien ed., Publications universitaires, Louvain 1976 (Philosophes Médiévaux, 16), and, for his later view, Buridan, *Summulae* cit., pp. 952-997 ; (3) R. Swyneshead, *Insolubilia*, in P. V. Spade, *Roger Swyneshed's* Insolubilia *: Edition and Comments*, « Archives d'histoire doctrinale et littéraire du Moyen Âge », 46, 1979, pp. 177-220, repr. in Id., *Lies* cit. ; (4) W. Heytesbury, *De insolubilibus*, in Id., *Regule solvendi sophismata*, Bonetus Locatellus, Venetiis 1494, ff. 4va-7rb ; cf. also Id., *On "Insoluble" Propositions : Chapter One of His Rules for Solving Sophisms*. *Translated with an Introduction and Study by* P. V. Spade, Pontifical Institute of Mediaeval Studies, Toronto 1979 (Mediaeval Sources in Translation, 21).

[7] As regards the last phase, I shall confine myself to mentioning three sources that are relevant to the purpose of this paper : (1) J. Wyclif, *Logicae continuatio* in, Id. *Tractatus de logica*, M. H. Dziewicki ed., 3 vols., Trübner & Co. for the Wyclif Society, London 1893-1899, vol. II, ch. 8, and J. Wyclif, *Summa insolu-*

In order to understand in a better way how medievals deal with semantic paradoxes, let us briefly consider a presentation of the Liar paradox which will count for the rest of the paper as a paradigmatic example. Assume that A is the proposition « 'A' is not true ». The proposition refers to itself and says of itself that it is not true. The paradox has two legs[8].

The first part of the argument runs as follows : if we assume that (1) A is true, then since (2) in order for A to be true, A must signify something that is the case and (3) A says of itself not to be true, we conclude that (4) A is not true.

(1) $T(\mathrm{A})$
(2) $T(\mathrm{A}) \rightarrow \exists p\ (Sig\ (\mathrm{A}, p) \wedge p)$
(3) $Sig\ (\mathrm{A}, \neg T(\mathrm{A}))$
(4) $\neg T(\mathrm{A})$

By assuming A to be true, we conclude it not to be true, therefore by *reductio*, it turns out that A is not true. The second part of the argument starts from this very last step. Thus, if (5) A is not true, then — since (6) whenever A signifies something that is the case A is true, and (7) A signifies A not to be true — it turns out that (8) A is true.

(5) $\neg T(\mathrm{A})$
(6) $\exists p\ (Sig\ (\mathrm{A}, p) \wedge p) \rightarrow T(\mathrm{A})$
(7) $Sig\ (\mathrm{A}, \neg T(\mathrm{A}))$
(8) $T(\mathrm{A})$

The effect of (1)-(4) and (5)-(8), taken together, is that A is true if and only if A is not true. The force of the argument depends on a number of assumptions that are implicitly or explicitly operating in it. Alternative solutions to the paradox put into question the plausibility of such assumptions. Four elements, in particular, need to be pointed out, as they play an essential role : (*i*) the set of admitted truth values ; (*ii*) a *transmission principle* (in the above reconstruction the conjunction of (2) and (6)

bilium, P. V. Spade, G. A. Wilson eds., Center for Medieval and Renaissance Studies, Binghamton, N.Y. 1986 (Medieval & Renaissance Texts & Studies, vol. 41,) ; (2) Peter of Mantua, *Tractatus de insolubilibus* [henceforth : *Insolubilia*], in Id., *Logica*, [Johannes Herbort, Padua 1477], sig. O2rb-O4+3rb (London, British Library IB. 29939) ; (3) Paul of Venice, *Tractatus de insolubilibus*, in Id., *Logica Magna, Secunda pars*, Albertus Vercellensis, Venetiis 1499, ff. 192rb-200rb.

8 In what follows, T(A) stands for « 'A' is true », while the expression *Sig* (A, *p*) stands for « A signifies that *p* ». Step (2) below must then be read as follows : if 'A' is true, then there is a *p* such that 'A' signifies that *p* and *p* is the case. The same applies, conversely, to (6). This notation is found, with minor variations, in S. Read, *The Truth-Schema and the Liar*, in Rahman, Tulenheimo, Genot eds., *Unity, Truth and the Liar* cit., pp. 3-18.

which can be formulated in various ways, as a counterpart of Tarski's T-scheme in contemporary formulations of the paradox) ; (*iii*) a notion of signification, namely an account of what it is for a proposition to say what it says ; and finally (*iv*) the syntactic admissibility of self-reference.

The paradox can be addressed, for example, (*ad i*) by extending the set of truth values allowing for gaps or gluts, (*ad ii*) by revising the transmission principle, (*ad iii*) by providing a suitable account of the signification of propositions, (*ad iv*) by ruling out self-referential propositions ; or yet again by combining some of the above. These are indeed some of the approaches that were adopted in the medieval tradition. Bradwardine's list of solutions to the paradox gives us an idea of the options that were readily available around 1320. A number of opinions (nine in total) are presented in five main groups which supposedly represent the state of the art at his time. The table offers a summary of the opinions discussed by Bradwardine :

B1.	F. *secundum quid et simpliciter*	*restringentes*	(I)	[e.g. Burley, Ockham]
B2.	F. figure of speech			
B3.	F. non cause as a cause			
B4.	Time			
B5.	Potency	*cassantes*	(II)	[Described in *Insolubilia Monacensia*]
B6.	Act			
B7.	Denial of bivalence	*mediantes*	(III)	[Swyneshead]
B8.	F. of equivocation (*actus exercitus* vs *actus conceptus*)	*distinguentes*	(IV)	[Scotus]
B9.	F. *secundum quid et simpliciter*	'Aristotelian' approach	(V)	[Bradwardine]

The first set of opinions is that of the *restringentes*. This approach, still adopted for instance by Burley and Ockham, addresses the issue by putting constraints on the admissibility of self-referential propositions. Restrictionism solves the paradox by preventing such propositions that self-refer to be considered well-formed. Depending on how strongly the constraints are to be understood, two versions of restrictionism are possible : a strong version (all sort of self-reference is prohibited, including harmless cases in which no paradox is generated) and a weak version (only self-reference inolving semantic predicates is prohibited).

A second well-known approach is the opinion of the *cassantes*, i.e. of those who utterly reject the paradox as nonsense (already described in the *Insolubilia Monacensia*). Little is known about this solution, but it is certainly among the earliest in the Latin Middle Ages, and, as noted above, it is also found within the early developments of the theory of obligations (the procedure by means of which a *casus*, in the context of an obligational disputation, is rejected because it would lead to a contradiction is parallel to the notion of *cassatio*).

A third line is taken by those who deny bivalence (the so-called *mediantes*), while yet others endorse a fourth option by drawing a distinction between the performance of a speech act and its intended meaning (*distinguentes*).

Bradwardine gives his own alternative account as the last one in the list. It introduces some crucial innovations into the picture. Such new elements will have a profound influence on the late XIVth century debate. From roughly the 1320s, a radically different way to look at the paradox emerges: it distinctively involves a new analysis of the truth conditions of a proposition and of its signification.

With Bradwardine, the analysis of the paradox turns out to be based on a new definition of truth, according to which a proposition is true whenever *all* it signifies is the case, and a more sophisticated account of the signification of the Liar proposition. The gist of the strategy can be summarised in two steps. First, according to Bradwardine, the Liar signifies not only itself not to be true, *but also* (and provably so) itself to be true. Second, under the assumption that in order for a proposition to be true whatever it signifies must be the case, it follows that the Liar is *false*, for what it signifies cannot be the case, because nothing is both true and not true at one and the same time, and the Liar precisely signifies those two things, i.e. itself not to be true *and* to be true[9]. Thus the strategy is that of provding an argument to establish the real truth value of Liar-like propositions. The solution has been the object of some recent controversy and it is beyond the purpose of this paper to engage in a debate concerning its plausibility[10]. The difficulties, however, have to do with some of Bradwardine's assumptions, not with the validity of the proof itself, once those assumptions are conceded. The key factor — which is also the real novelty in Bradwardine's approach to the problem — is the idea that a proposition might signify more than it is ordinarily assumed to signify. This is a crucial aspect, because variations on such a theme, namely whether there is more to the ordinary signification of propositions than meets the eye, and if so, what such an additional signification should be like and to what propositions such a standard applies, represent a much disputed subject in late XIVth century discussions on the Liar.

Along the same lines (although it is hard to establish whether because of a direct acquaintance with Bradwardine's work, or because the solution was in the air by that time), in the second quarter of the XIVth century other authors started thinking that the paradox might be solved by expanding the ordinary signification of propositions

[9] Note that its being false *does not* in turn entail its being true, because the former is no sufficient condition for the latter. The proposition is false, indeed. Moreover, it says of itself that it is false. But according to Bradwardine it also says of itself that it is true, and these two conditions can never be met together by one and the same proposition. In other words, Bradwardine is not committed to the revised antecedent of (6) with a universal quantifier, and therefore, even if he accepts (5) and (7), he is not committed to (8).

[10] See, in particular, T. Parsons, *Comments on Stephen Read's "The Truth-Schema and the Liar"*, in Rahman, Tulenheimo, Genot eds., *Unity, Truth and the Liar* cit., pp. 129-134, and S. Read, *Further Thoughts on Tarski's T-Scheme and the Liar*, *ibid*. pp. 204-225.

so as to include — unrestrictedly or only in specific cases — something more to the surface of their meaning.

In particular two approaches are relevant for the background of Peter of Mantua's treatise : the first is by Albert of Saxony, which is likely an elaboration on John Buridan's early view[11], and the second by William Heytesbury. If Bradwardine claims that insoluble propositions signify something more than what they *prima facie* signify, namely their own truth, according to Albert, such a view should be generalised : any proposition signifies its own truth, beside what it signifies according to the ordinary understanding of its propositional content. The idea is based on a suppositional characterisation of truth, and incorporates suppositional truth conditions (identity of the *supposita* of the subject-term and the predicate-term) within the signification of a proposition. In contrast with Bradwardine's narrower claim that only Liar propositions signify their own truth beside signifying their own falsity, Albert's claim is much more general and, for that matter, yields disastrous consequences on a pandemic scale, because it fails to provide proper truth conditions for *any* proposition in the language[12].

Heytesbury has yet another view on the nature a proposition's signification. If a proposition signifies *precisely* as its terms pretend (i.e. if it signifies exactly all and only what it explicitly says), then no extra content is given. But if we do not specify that the proposition signifies precisely as its terms pretend, then it could signify more. This *extra content* may well remain unspecified, but if it is in fact specified, this can obtain in two ways : either disjunctively, as if we were to say that 'every proposition is false' actually signifies that every proposition is false *or* there is a god ; or conjunctively, as if we were to say that 'Socrates is saying what is false' signifies that Socrates is saying what is false and you are running[13]. In the last two cases what happens is that we make the signification of the initial proposition precise by conjoining it or disjoining it with the extra signification in a new proposition. Heytesbury casts the solution to the Liar in an obligational framework where the respondent should refuse to admit the proposition if it is used as a *positum* only under the assumption that it precisely signifies as its terms pretend or that the extra signification, when made explicit, fulfills

[11] On Buridan's transition from the early view, according to which every proposition p signifies itself to be true, to his later claim that every proposition virtually implies another proposition that says of the former that it is true, see KLIMA, *Logic without Truth* cit., pp. 89-98. The first approach seems to be problematic, because it includes among the truth conditions of a proposition that very proposition's being true, which leads to an unwanted circularity. Albert of Saxony does not seem to be aware of the difficulty and endorses Buridan's earlier view. Peter of Mantua is unhappy with that account but does not seem to be pointing to the fact that the definition is circular.

[12] In other words, if (1) for all p, 'p' signifies (1.1) that p and (1.2) that 'p' is true, and (2) 'p' is true iff howsoever p signifies things to be, so things are, then the *howsoever*-clause in (2), which is supposed to *define* truth for an arbitrary proposition, picks out (1.2) and is therefore circular.

[13] The examples are standard and occur in Peter's treatise, see PETER OF MANTUA, *Insolubilia* cit., sig. O4+1$^{ra\text{-}rb}$.

certain criteria of compatibility or incompatibility with the *casus*. In all other cases, the Liar should be admitted and the respondent is required to follow a particular strategy according to whether and how the additional signification of the proposition is specified by the opponent at later steps of the disputation.

The struggle with the paradox in the light of new trends involving a reflection on the role of the signification of propositions accounts for some of the additional complexities that are witnessed, at the end of the XIVth century, by Paul of Venice's list of positions.

PV1.	F. figure of speech	*restringentes*	B2 [+B1]	PM3
PV2	F. non cause as a cause		B3	
PV3.	Time		B4	
PV4.	Potency	*cassantes*	B5	
PV5.	Act		B6	
PV6.	Denial of bivalence	*mediantes*	B7	
PV7.	F. equivocation (*actus exercitus* vs *actus conceptus*)	*distinguentes*	B8	
PV8.	*Non est propositio*		H2	
PV9.	*Verum vel falsum sed non verum et non falsum*		H3	
PV10.	F. *secundum quid et simpliciter*	Bradwardine	B9	
PV11.	*Significat se esse verum et se esse falsum*	Albert of Saxony		PM1
PV12.	*Casus*/obligations	Heytesbury	H4	PM2
PV13.	Mental language	Peter of Ailly, Gregory of Rimini		Digression on PM
PV14.	*Restrictio*	Walter Sexgrave		
PV15.	Self-falsifying	Paul of Venice	Swyneshead (H1)	

Paul's record presents fifteen opinions about insolubles, most of which are already to be found in Bradwardine[14]. In addition to these, however, several further views developed from the second quarter of the XIVth century onwards introduce some complications into the picture, as I have just briefly described (along with entirely different approaches — like for instance the one associated with Gregory of Rimini and Peter of Ailly, seeking an explanation from the standpoint of mental language). Albert of Saxony's and William Heytesbury's positions, as noted above, are the most important targets for Peter of Mantua. Peter's own position, without being mentioned explicitly, is in fact presented as a small cameo within the discussion of Peter of Ailly's view[15].

[14] A discussion of this list is found in C. DUTILH NOVAES, *A Comparative Taxonomy of Medieval and Modern Approaches to Liar Propositions*, « History and Philosophy of Logic », 29, 2008, pp. 227-261. For all its merits, the article fails to identify Peter's own account (generically referred to as that of a *alius magister*) that Paul of Venice discusses by way of digression in the context of his presentation of Peter of Ailly's view. The list of opinions, therefore, contains at least fifteen *plus one* items.

[15] Labels B1-9 refer to the presence of the very same *opiniones* in Bradwardine's list given above (cf. also Read's discussion in his own introduction to BRADWARDINE, *Insolubilia* cit., pp. 10-23). H1-4 refer to the opinions recorded in Heytesbury's treatise on insolubles (discussed by Spade in the study appended to HEYTESBURY, *On "Insoluble" Propositions* cit., pp. 71-95). PM1-3 refer to the views reported and criticised by Peter of Mantua in his own treatise.

3. Peter of Mantua's treatise

Let us now turn to Peter of Mantua. With the background outlined above in mind, in the following I shall present and contextualise what Peter says about insolubles, discuss why (I think) *he thinks* he is justified in saying what he says, and show why he is wrong in saying what he says (the theory is inconsistent).

The available information about Peter's biography covers the last ten years of his life, between 1389 and 1399[16]. After receiving his education in Padua, he is known to have taught in Bologna, during the 1390s, both natural and moral philosophy. His surviving works are a treatise *De primo et ultimo instanti*, which was quite popular in the XVth century (various commentaries upon it are preserved), and a huge *Logica*, which includes the treatise on *insolubilia* along with all standard chapters of an advanced logic treatise (which appears to be much more of a set of advanced notes than a textbook for teaching). This work has come down to us in its entirety in six manuscripts and four early printed editions (the latter dating from the end of the XVth century)[17]. No section has been published yet in the form of a critical edition.

A general word about Peter's sources is in order : names seldom occur in the *Logica*, but among the authors he is certainly familiar with and makes use of we should count a few Parisian figures like Albert of Saxony, Marsilius of Inghen and/or William Buser, and prominent representatives of the English tradition, above all Ralph Strode and William Heytesbury, a fact that should not be surprising, given the influence that these two figures had on Italian logic in the late XIVth century. In addition to that, Peter was also strongly acquainted, as I have recently argued, with Mesino de Codronchi, another master active in Bologna in the 1390s[18]. I shall provide new evidence (see *infra* § 3.3) in support of the claim that Peter must have been familiar with John Wyclif's logical writings as well. No mention of Peter's treatise on insolubles is found in recent literature on the subject[19]. This circumstance, probably due to the lack of a modern edition, is rather unfortunate, since Peter seems to be quite an interesting figure in the landscape of late XIVth century logic, especially in connection with the transmission

[16] See James, *Peter Alboini of Mantua* cit., and R. Strobino, Concedere, Negare, Dubitare, *Peter of Mantua's treatise on obligations*, Ph.D. Dissertation Scuola Normale Superiore, Pisa 2009, pp. 45-53.

[17] The manuscripts are : (1) [O]xford, Bodleian Library, *Canon. misc.* 219, ff. 102va-105va ; (2) [B]erlin, Staatsbibliothek Preussischer Kulturbesitz, *Hamilton* 525, ff. 93va-95va ; (3) [M]antova, Biblioteca Comunale, ms. 76 (A III 12), ff. 79^{vb}-82^{rb} ; (4) Venezia, Archivio dei [P]adri Redentoristi di Santa Maria della Fava, ms. 457, ff. 63va-66ra ; (5) [V]enezia, Biblioteca Nazionale Marciana, L.VI.128 (2559), ff. 69ra-71va ; (6) Città del Vaticano, Biblioteca Apostolica Vaticana, *Vat.* [L]at. 2135, ff. 75^{vb}-77^{rb}, 79^{rb}-80ra. I shall also refer to the aforementioned early modern edition as E (see *supra*, f. 7).

[18] See R. Strobino, *Contexts of Utterance and Evaluation in Peter of Mantua's* Obligationes, « Vivarium », 49, 2011, pp. 275-299.

[19] To do it justice, one has to go back to Spade's 1975 catalogue of the *insolubilia* literature, where Peter's position is listed and briefly described (although, even there, the connection with Wyclif is not identified).

of the English logical tradition in Italy. I am inclined to think that a reconstruction of his profile might prove to be, in several respects, even more relevant than that of Paul of Venice — who makes use of Peter as one of his (many) sources — since the latter represents a chronologically prior and independent step in the reception of that tradition (and, for want of a better term, a fairly less pedantic one).

3.1 Structure of the text

Peter of Mantua's treatise on insolubles is divided into four main sections. The first three sections deal with alternative views that are first illustrated, and then rejected by presenting a number of objections, in a customary fashion. The fourth section offers Peter's own account of the Liar — and of the Truth teller — accompanied by several objections and responses, and a discussion of a few variations on the theme of the so-called postcard paradox (in cases that do not involve semantic predicates) which are dismissed as impossible conditionals. A preliminary division of the contents is presented in the table below

1. *First section*	**2**. *Second section*	**3**. *Third section*	**4**. *Fourth section*
[Albert of Saxony] (PM, *Insolubilia* cit., sig. $O2^{rb}$- $O3^{rb}$)	[William Heytesbury] (PM, *Insolubilia* cit., sig. $O3^{rb}$ -$O4{+}1^{rb}$)	[Restrictionism] (PM, *Insolubilia* cit., sig. $O4{+}1^{rb\text{-}vb}$)	[Peter of Mantua] (PM, *Insolubilia* cit., sig. $O4{+}1^{vb}$- $O4{+}3^{ra\text{-}rb}$)
1.1 Thesis: every proposition signifies its own truth (*ibid.*, sig. $O2^{rb}$)	**2.1** Definitions : *casus* and insoluble proposition (*ibid.*, sig. $O3^{rb}$)	**3.1** Thesis : self-reference is not admitted (*ibid.*, sig. $O4{+}1^{rb}$)	**4.1** Three senses of 'true' (*ibid.*, sig. $O4{+}1^{vb}$-$O4{+}2^{ra}$)
1.2 Definition of truth (*ibid.*, sig. $O2^{va}$)	**2.2** Rules 1-5 (obligations) (*ibid.*, sig. $O3^{va\text{-}vb}$)	**3.2** Obj. 1-4 (*ibid.*, sig. $O4{+}1^{va\text{-}vb}$)	**4.2** Obj. 1-6 (*ibid.*, sig. $O4{+}2^{ra\text{-}va}$)
1.3 Obj. 1-5 (*ibid.*, sig. $O2^{va}$)	**2.3** Against definitions : Obj. 1-2 (*ibid.*, sig. $O3^{vb}$-$O4^{ra}$)		**4.3** Reply and solution to Obj. 1-6 (*ibid.*, sig. $O4{+}2^{va}$)
1.4 Arguments in support of Obj. 1-5 (*ibid.*, sig. $O2^{va}$-$O3^{ra}$)	**2.4** Against Rule 2 : Obj. 1-3 (*ibid.*, sig. $O4^{ra}$)		**4.4** Obj. 7-10 and reply (*ibid.*, sig. $O4{+}2^{va\text{-}vb}$)
1.5 Obj. 6-9 (*ibid.*, sig. $O3^{ra}$-$O3^{rb}$)	**2.5** Arguments in support of Obj. 1-3 and Obj. 4 (*ibid.*, sig. $O4^{ra}$-$O4^{va}$)		**4.5** Obj. 11-13 (impossible conditionals) (*ibid.*, sig. $O4{+}2^{vb}$-$O4{+}3^{ra}$)
	2.6 Against Rule 3 : Obj. 1-3 (*ibid.*, sig. $O4^{va}$-$O4^{vb}$)		**4.6** Reply to Obj. 11-13 (*ibid.*, sig. $O4{+}3^{ra\text{-}rb}$)
	2.7 Against Rules 2-3 (*ibid.*, sig. $O4^{vb}$-$O4{+}1^{ra}$)		
	2.8 Against Rule 4 : Obj. 1-2 (*ibid.*, sig. $O4{+}1^{ra\text{-}rb}$)		
	2.9 Against Rule 5 : Obj. 1-2 (*ibid.*, sig. $O4{+}1^{rb}$)		

3.1.1 Against Albert of Saxony

The treatise starts abruptly without any introductory statement or preamble and sets out to discuss the alternative accounts that Peter wants to target. In the first section, Albert of Saxony's position is illustrated and rejected. The core of his view is presented in the form of two general claims: a short proof that every proposition signifies its own truth and a definition of a true proposition as one which is such that howsoever it signifies so it is.

> « [**1.1**] In [discussing] the difficulties [raised by] the so-called insoluble propositions, some people have said that every insoluble proposition signifies itself to be true and to be false. For every categorical proposition signifies itself to be true, because (*i*) every categorical proposition signifies what the subject-term supposits for and [what] the predicate-term [supposits for] to be or not to be one and the same; but (*ii*) what the subject-term supposits for and [what] the predicate-term [supposits for] being or not being one and the same is for an affirmative or negative categorical proposition to be true, therefore (*iii*) every affirmative or negative categorical proposition signifies itself to be true, as they say. And since (*iv*) every insoluble proposition falsifies itself, therefore (*v*) every insoluble proposition signifies itself to be true and to be false.
> [**1.2**] In addition to that, they say that (*vi*) a true proposition is [any one which is such] that howsoever it signifies, so it is. And a false [proposition] is [any one which is such] that [it is] not [the case that] howsoever it signifies, so it is »[20].

The combined effect (*v*), i.e. an account of what the Liar signifies, and the first part of (*vi*), i.e. a definition of truth, is that the Liar turns out to be false. According to (*v*), the Liar signifies itself to be true and to be false. According to (*vi*), in order for any proposition *p* to be true, things should be in whatever way *p* signifies them to be. But things can never be as the Liar signifies them to be, for nothing is both true and false at the same time. Therefore, the Liar fails to meet the condition stated in the first part of (*vi*) — or, which is the same, it meets the condition stated

[20] Peter of Mantua, *Insolubilia* cit., sig. O2^{rb-va} « [1.1] In difficultatibus autem propositionum quas insolubiles vocant dixerunt aliqui quod omnis propositio insolubilis significat se esse veram et se esse falsam. Omnis enim propositio categorica significat se esse veram : quia (*i*) omnis propositio categorica significat idem esse vel non esse pro quo supponit subiectum et predicatum ; et (*ii*) esse idem vel non esse idem pro quo supponit subiectum et predicatum est propositionem affirmativam vel negativam esse veram ; igitur (*iii*) omnis propositio affirmativa vel negativa categorica significat se esse veram, ut dicunt. Et quia (*iv*) omnis propositio insolubilis se falsificat [O *pro* significat in E], ideo (*v*) omnis propositio insolubilis significat se esse veram et se esse falsam. [1.2] Item, addunt quod (*vi*) propositio vera est que qualitercumque significat ita est. Et falsa est que non qualitercumque significat ita est ». The numbering in the quotations follows the proposed division of the text. Wherever an emendation is required, the reading is either supported by at least one manuscript or it is a conjecture to make sense of the argument.

in the second part of (*vi*) —, and for this reason it is false. The argument can be summarised as follows:

(*i*) $\forall q\ (Sig\ (q, Supp(\mathrm{S}_q) = Supp(\mathrm{P}_q)))$[21]
(*ii*) $Supp(\mathrm{S}_q) = Supp(\mathrm{P}_q) \leftrightarrow T(q)$
(*iii*) $\forall q\ (Sig\ (q, T(q)))$
(*iv*) $Sig\ (\mathrm{A}, \neg T(\mathrm{A}))$
(*v*) $Sig\ (\mathrm{A}, \neg T(\mathrm{A}) \wedge T(\mathrm{A}))$

Two conclusions that are not explicitly drawn in Peter's report of Albert's position can be derived from the above result[22]. First, since (*v*) is equivalent to

(*v**) $Sig\ (\mathrm{A}, (Supp(\mathrm{S}_\mathrm{A}) \neq Supp(\mathrm{P}_\mathrm{A}) \wedge Supp(\mathrm{S}_\mathrm{A}) = Supp(\mathrm{P}_\mathrm{A})))$

and the revised definition of truth is

(*vi*) $T(\mathrm{A}) \leftrightarrow \forall p\ (Sig\ (\mathrm{A}, p) \rightarrow p)$

we can safely conclude that

(*vii*) $\neg T(\mathrm{A})$.

This is because the right-hand side of (*vi*) is not (and cannot be) satisfied by the Liar. The failure depends on the fact that the proposition signifies itself to be true and not to be true, and what it signifies cannot be the case.

By the same token, we can also block the inference that generates the paradox in the second leg, for from (*vii*), i.e. A's being false, its being true cannot be inferred, because again in order for A to be true the requirement is that *whatever* it signifies be the case, and A signifies something that cannot in principle be the case.

Nine objections of various length and sophistication are then raised against this view. The main point that Peter stresses is that it leads to multiple violations of the logical relations codified in the square of opposition, both in the categorical and in

[21] $Supp(\mathrm{S}_q)$ and $Supp(\mathrm{P}_q)$ stand for the supposits of S and P in q, where S and P are the subject-term and the predicate-term of that proposition.

[22] See ALBERT OF SAXONY, *Perutilis Logica*, *Tractatus sextus*, *Prima pars*, *De Insolubilibus*, Venetiis 1522, ff. 43$^{\mathrm{rb}}$-46$^{\mathrm{va}}$, repr. by Georg Olms, Hildesheim - New York 1974 (Documenta Semiotica, Serie 6 Philosophica), f. 43$^{\mathrm{rb\text{-}va}}$ (first six *conclusiones*); cf. also ALBERT VON SACHSEN, *Logik*. *Übersetzt, mit einer Einleitung und Anmerkungen herausgegeben by* H. BERGER, Felix Meiner, Hamburg 2010, pp. 1100-1177, in particular pp. 1100-1106 for an expanded version of the argument.

the modal version. In particular, Peter focuses on the role of negation as an operator and the way in which a truth value is assigned to contradictory propositions in situations in which self-reference and truth predicates are involved. An example is his fourth objection. Suppose that only two particular propositions exist, namely 'A negative particular proposition is true' = p and 'A negative particular proposition is *not* true' = q. Since q is the only existing negative proposition, q must refer to itself; therefore, in this context, q is equivalent to the Liar, because it says of itself that it is not true. According to Albert, q must be false for the reasons explained above. On the other hand, p talks about q (because q is the only negative particular proposition p can possibly refer to) and what p says is that q is true. But q is not true, therefore what p says is not the case, and so p, too, is false. But p and q are subcontraries and the argument makes them false together, which is against the standard understanding of the relations holding in the square of opposition (in particular, the above case would also entail that the corresponding contraries are true together).

Peter also seems to be willing to deny the more general claim that any proposition signifies its own truth (seventh objection), but he does not address the issue of circularity that appears to be, from a modern point of view, the most immediate problem related to this position[23].

3.1.2 Against William Heytesbury

In section two, which occupies nearly a half of the whole treatise, Peter deals with William Heytesbury's view, which came to be very popular in the second half of the XIVth century. Again, he presents his opponent's opinion by enumerating the basic theoretical principles on which it relies and by discussing separately several objections against each of them. The definition of a proposition and of a *casus* of an insoluble are laid down first. Next, Peter presents Heytesbury's famous five rules, ac-

[23] A detailed analysis of the arguments supporting each objection cannot be given here for reasons of space. I will therefore confine myself to enumerating the objections in footnote and to providing between parentheses the propositions that are made use of as counterexamples: (*Obj.* 1) an impossible proposition contradicts a contingent proposition ('a proposition is true'/'[it is] not [the case that] a proposition is true'); (*Obj.* 2) every insoluble proposition entails a contradiction ('[it is] not [the case that] a proposition is true; therefore [it is] not [the case that] a proposition is true and this is true'); (*Obj.* 3) a merely negative proposition entails a contradiction (same *casus*); (*Obj.* 4) two subcontraries are false together (see example discussed above); (*Obj.* 5) two contradictories are false together (see previous *casus*, with three additional arguments); (*Obj.* 6) the following would turn out not to be contradictories 'every proposition is false', 'not every proposition is false'; (*Obj.* 7) it is not the case that every proposition signifies itself to be true; (*Obj.* 8) against the characterisation of p as a true proposition if and only if howsoever p signifies things to be, so they are; (*Obj.* 9) it is not the case that, if every particular proposition is false, then some particular proposition is false (Albert's position would invalidate the inference 'every S is P, therefore some S is P').

cording to which one is supposed to reply to Liar-like propositions in the context of an obligational disputation. Finally, he proposes a battery of objections.

> « [**2.1**] In [discussing] these matters, other people have assumed, first, that an insoluble proposition is a proposition, mentioned in a *casus* of an insoluble, which [— i.e. the proposition — is such that], if in the same *casus* it signifies precisely as its terms commonly pretend, it follows that it is true and that it is false[24]. Secondly, a *casus* of an insoluble is a *casus* in which mention is made of a proposition which [is such that], if in the same *casus* it signifies precisely as [its] terms pretend, it follows that it is true and that it is false »[25].

An important feature of Heytesbury's position is that the whole discussion of semantic paradoxes is intentionally cast in an obligational framework. As a result, an insoluble proposition must always be seen against the background of a *casus*, i.e. an imaginary situation in which certain conditions are assumed to hold (as if we posit, for example, that Socrates is saying the following proposition and nothing else : « Socrates is saying what is false »). In order to understand better how the context of Heytesbury's discussion affects the treatment of the paradox and how Peter of Mantua reacts to it, let us introduce the following terminology borrowed from the theory of obligations[26].

[24] Heytesbury's own formulation is slightly different : « from its being true it follows that it is false, and vice versa (ad eam esse veram sequitur eam esse falsam et econtra) », see Heytesbury, *De insolubilibus* cit., f. 6rb. The same holds for the next definition. More generally it can be said that the vocabulary is used with a certain amount of flexibility. This opens a delicate question since a number of texts with a strong (Heytesburyan) family resemblance was circulating in the second half of the XIVth century (among which the so-called Pseudo-Heytesbury and Johannes Venator), so it is legitimate to ask whether Peter is actually making use of Heytesbury himself or of one of those other texts (a detailed discussion of the 'filiation' process is found in F. Pironet, *William Heytesbury and the Treatment of Insolubilia in Fourteenth-Century England Followed by a Critical Edition of Three Anonymous Treatises* De insolubilibus *Inspired by Heytesbury*, in Rahman, Tulenheimo, Genot eds., *Unity, Truth and the Liar* cit., pp. 254-334. Therefore there might be some room for maneuvre to argue that it is not Heytesbury's text but another one originating in the same family that Peter was targeting. Further investigation into the details of the lexicon might provide additional help, but it should not be forgotten that the fluidity of this kind of tradition makes it very unlikely that one might find complete consistency in the use of a certain vocabulary. Therefore, it is hard to draw firm conclusions from the presence in a later author (like Peter) of a particular nuance or lexical choice. The closer the members of the family, as in the case of this set of texts inspired by Heytesbury, the harder to produce compelling evidence of the filiation from one of them as opposed to another.

[25] Peter of Mantua, *Insolubilia* cit., sig. O3va « Acceperunt autem alii in ista materia primo quod propositio insolubilis est propositio de qua fit mentio in aliquo certo casu que, si in eodem casu precise significet sicut eius termini communiter pretendunt, sequitur se esse veram et se esse falsam. Casus autem de insolubili est casus in quo fit mentio de aliqua propositione que, si in eodem casu significet precise sicut termini pretendunt, sequitur se esse veram et se esse falsam ». Cf. also Heytesbury, *De insolubilibus* cit., f. 6rb ; Pironet, *William Heytesbury* cit., p. 284 ; and Spade's translation in Heytesbury, *On "Insoluble" Propositions* cit., p. 46.

[26] These representational conventions have become part of the common vocabulary in recent scholarship on the theory of obligations, see Strobino, *Concedere, Negare, Dubitare* cit., pp. 22-32 and 76-80. The framework that both Heytesbury and Peter of Mantua (not only here but also, more generally, in their account of obligations)

$OA(p)$	$=_{df}$	p ought to be admitted (only for *posita* that are not inconsistent)
$OC(p)$	$=_{df}$	p ought to be conceded (either it follows or it is irrelevant and true)
$ON(p)$	$=_{df}$	p ought to be denied (either it is repugnant or it is irrelevant and false)
$OD(p)$	$=_{df}$	p ought to be doubted (it is irrelevant, not known to be true and not known to be false)
$O\neg N(p)$[27]	$=_{df}$	p ought not to be denied

The solution to the Liar is justified according to standard obligational principles, and is articulated in detail by means of five rules that are expected to govern in a 'predictable' way the respondent's response in each conceivable situation. The thought is handling with the signification of an insoluble proposition in various ways according to whether it is either left undetermined, or specified fully, or only partially, and if so in what way.

> « [**2.2**] Having given these [definitions], they laid down some rules. [*Rule* 1] First, if a *casus* of an insoluble is posited, and it is not posited how it should signify, one ought to respond exactly (*omnino*) as one would have responded outside the period [of the obligation]. Accordingly, when it is posited that Socrates is saying this [proposition] 'Socrates is saying what is false' and no other, nothing else being posited, one ought to be in doubt about it, namely [about] 'Socrates is saying what is false'.
> [*Rule* 2] Secondly, if it is posited that an insoluble signifies *precisely* as the terms pretend, the *casus* ought to be rejected and not admitted.
> [*Rule* 3] But if it is posited that the insoluble signifies as the terms pretend, *without* positing [the qualification] 'precisely', the insoluble ought to be conceded as following (*tamquam sequens*) and it ought to be denied that it is true, if it is proposed that it is true. The three [rules that are] laid down [here] concern propositions that signify categorically »[28].

endorse is that of the so-called *responsio antiqua*. In the theory of obligations, rules telling the respondent whether he ought to concede, deny or doubt certain propositions are given according to a partition of propositions in two classes : relevant and irrelevant ones. On that view, relevance of a proposition p is defined in terms of logical dependence on the cumulative set consisting of the propopositions that have been previously granted and the negations of those that have been previously denied during a disputation. According to whether that set entails p or $\neg p$, respectively, p is said to be 'following' (*pertinens sequens*) or 'repugnant' (*pertinens repugnans*). If p is logically independent from what precedes it in the disputation, it is said to be irrelevant (*impertinens*) and must be evaluated according to its own status by looking at the world outside the disputation.

[27] Note that the conditions of $\neg ON(p)$ and $O\neg N(p)$ are equivalent if we accept deterministic obligational rules (i.e. if the respondent correctly $X(p)$ then he *ought* to $X(p)$). Both the former case — it is not the case that p ought to be denied — and the latter case — p ought not to be denied — are satisfied when p is neither repugnant nor irrelevant and false. For then it is either following or irrelevant and true (hence $OC(p)$), or irrelevant and doubtful (hence $OD(p)$).

[28] Peter of Mantua, *Insolubilia* cit., sig. O3$^{va\text{-}vb}$ « [2.2] Quibus datis posuerunt regulas. [*Rule* 1] Et primo quod si ponatur casus de insolubili et non ponitur qualiter istud debeat significare, respondendum est omnino sicut extra tempus fuisset responsum. Ut posito quod Sor dicat istam 'Sor dicit falsum' et nullam

The first three rules are directed to cases in which the paradoxical proposition at stake is categorical (where categorical stands for *non-molecular*). The idea is that one of the following mutually alternative situations must obtain: (1) the signification of the proposition is left entirely undetermined; (2) the proposition says exactly what it says; or (3) the proposition says what it says, but not in a strict sense, i.e. it could signify something more than what it explicitly says.

R1 if A is an insoluble and it signifies that x (where x remains unspecified), then OA(A) and OD(A)

R2 if A is an insoluble and it signifies precisely[29] what it signifies, then $O\neg A$(A)

R3 if A is an insoluble and it does not signify precisely what it signifies, then OA(A), OC(A), and $O\neg C$ (T(A))

In the first case, since it is not even clear what is being posited, the respondent should suspend his judgment and be in doubt about the proposition; in the second case, he cannot admit the *casus* without thereby committing himself to a contradiction and should therefore reject it; in the third case there is, according to Heytesbury, some room for maneuvre. The strategy behind the third rule is to 'simulate' in an obligational environment the argument that generates the paradox, and to block the inference that leads to contradiction on the basis of obligational principles. Suppose the proposition « 'A' is not true » is posited and admitted, provided that it does not signify precisely as its term pretend (otherwise we would fall within the range of the second rule). If it is proposed during the disputation, it must be conceded as following, because it is the *positum*. The respondent, therefore, ought to concede A (= « 'A' is not true »). Now, if the opponent proposes, at the second step, « "'A' is not true" is true » how is the respondent supposed to reply? Having granted A at step one, the respondent has thereby granted « 'A' is not true » (= A). Thus, a contradiction would now arise only if the respondent was committed, at step two, to conceding « 'A' is true » which is the contradictory of what has been proposed and conceded at step one. But according to Heytesbury, at step two the proposition « 'A' is true » (= « "'A' is not true" is true ») is repugnant, and consequently the respondent can (in fact ought to) deny it. Thus even if he has conceded A (= « 'A' is not true ») at step one, he will

aliam, non posito alio, ista est dubitanda, scilicet 'Sor dicit falsum'. [*Rule* 2] Si autem ponatur quod insolubile significet *precise* sicut termini pretendunt, casus est reiciendus et non admittendus. [*Rule* 3] Sed si ponitur quod insolubile significet sicut termini pretendunt, *non* ponendo 'precise', concedendum est insolubile tamquam sequens et negandum est quod sit verum, si proponitur esse verum. Que tria posita sunt de propositionibus categorice significantibus ».

[29] I.e. it signifies that p and nothing else.

still be able to deny « 'A' is true », and thereby avoid the contradiction. The fact that what is proposed at the second step, namely « 'A' is true », is repugnant to the *positum* and the *casus* can easily be seen. We have a *casus* of the form 'p' signifies that p and a *positum* p and we are seeking whether a given proposition q is to be conceded or not. Obviously if q is $\neg p$, as in the present case (for « 'A' is true » is the negation of A, since the latter itself is « 'A' is not true »), q is incompatible with p and the *casus* ; therefore it ought to be denied.

Casus A = « 'A' is not true » and A signifies (not precisely) that 'A' is not true

Positum	A	OA(A)
1.	A	OC(A) as following (it is the *positum*)
2.	'A' is true	$ON(T(\mathrm{A}))$ as repugnant

The whole discussion relies on a standard pattern of reasoning that is often employed within the framework of the theory of obligation : that p ought to be conceded according to obligational rules (either because it follows from previous steps or because it is irrelevant and true) *does not* entail that « 'p' is true » ought to be conceded. In fact, here the claim is even stronger, because both answers are obtained by running the standard Liar argument in the new obligational setting. In the following reconstruction it can be seen why A ($=\neg T(\mathrm{A})$) is following and must be conceded and why $\neg$A ($=T(\mathrm{A})$) is repugnant and must be denied :

R3 $Sig(\mathrm{A}, \neg T(\mathrm{A}))$ (not precisely)

1st leg of the paradox (proof of $\neg T(\mathrm{A})$, by *reductio*)

1. $T(\mathrm{A})$	hypothesis
2. $T(\mathrm{A}) \rightarrow \forall p\ (Sig\ (\mathrm{A}, p) \rightarrow p)$	def. truth (left-hand side)
3. $\forall p\ (Sig\ (\mathrm{A}, p) \rightarrow p)$	1, 2 and *modus ponens*
4. $Sig\ (\mathrm{A}, \neg T(\mathrm{A})) \rightarrow \neg T(\mathrm{A})$	3 and substitution of p with $\neg T(\mathrm{A})$
5. $Sig\ (\mathrm{A}, \neg T(\mathrm{A}))$	hypothesis
6. $\neg T(\mathrm{A})$	4, 5 and *modus ponens*
7. $\boldsymbol{\neg T(\mathrm{A})}$	1,6 by *reductio* [$\neg T(\mathrm{A})$ = A] $\boldsymbol{OC(\mathrm{A})}$ $\boldsymbol{ON(T(\mathrm{A}))}$

By assuming the Liar to be true (1), we conclude it to be not true (6), therefore, by *reductio* it is not true (7). But the claim that (7) A is not true is the Liar itself ; therefore if the Liar is proposed during a disputation, we have to concede it as following (from the *casus* and rules of logic, which is all we have at steps 1 to 6 of the proof) and we have to deny that it is true, i.e. we have to deny $T(\mathrm{A})$, if it is proposed. Furthermore, even if $\neg T(\mathrm{A})$ has been granted, this is not enough to prove, in the second leg of the paradox, $T(\mathrm{A})$, i.e. that the Liar is true :

2nd leg of the paradox (inference blocked)

8. $\neg T(\mathrm{A})$	7
9. $\forall p\ (Sig\ (\mathrm{A}, p) \rightarrow p) \rightarrow T(\mathrm{A})$	def. truth (right-hand side)
10. $Sig\ (\mathrm{A}, \neg T(\mathrm{A}))$	hypothesis

At this point, the paradox can no longer arise, for the inference from 8-10 to

11. $T(\mathrm{A})$

is not legitimate. The inference would be legitimate only if we could replace the antecedent in the antecedent of (9) with (10), and the consequent in the antecedent of (9) with (8), so as to satisfy the antecedent of (9) itself, because then we could detach the consequent, thereby proving $T(\mathrm{A})$. We would be entitled to make this move, however, only if A *precisely* signified itself not to be true. But this is not the case, because by hypothesis A signifies what it signifies *not precisely*. In other words, the inference by means of which we are trying to prove $T(\mathrm{A})$, i.e. that A is true, is not valid. This is exactly what the assumption embedded in the third rule by means of the qualification 'not precisely' is meant to avoid. Otherwise the paradox would arise again and we would find ourselves back in the situation covered by the second rule, whereby the *casus* had to be rejected.

The fourth and fifth rules are devised to deal with cases in which the additional signification that is left undetermined when the Liar is just said to signifiy not precisely what it signifies is made explicit by conjoining or disjoining it to other propositions specifying the extra signification.

> « [**2.2** (*continuation*)] [*Rule* 4] On the other hand, if an insoluble proposition is posited so as to [precisely] signify conjunctively, one should see whether the contradictory of the second conjunct is consistent with the *casus*. [1.1] If it is, the *casus* ought to be admitted. [1.2] If it is not, the *casus* ought to be rejected. Accordingly, if it is posited that Socrates is saying the [proposition] 'Socrates is saying what is false', adequately signifying that Socrates is saying what is false *and* that Socrates is speaking, since these [two propositions] are not consistent with one another — namely 'Socrates is saying what is false' and 'Socrates is not speaking' —, the *casus* ought not to be admitted. But if it is posited that this [proposition] 'Socrates is saying what is false' signifies precisely that Socrates is saying what is false and that Socrates is running, the *casus* ought to be admitted and this proposition ought to be conceded, while it ought to be denied that it is true.

> [*Rule* 5] Last, if it is posited that an insoluble signifies disjunctively, this view maintains that [2.1] if the opposite of [the second] disjunct is consistent with the *casus*, the *casus* ought to be denied. Accordingly, if it is posited that the proposition 'a falsehood exists' is any proposition adequately signifying that a falsehood exists *or* there is no god, the *casus* ought not to be admitted. But [2.2] if it is posited that the [proposition] 'a falsehood exists' is every proposition adequately signifying that a falsehood exists or there is a god, the *casus* ought to be admitted and the [proposition] 'a falsehood exists' ought to be denied, when it is proposed, and it ought to be conceded that it is true »[30].

Suppose we make the signification of an insoluble proposition A explicit by forming a new proposition in which the insoluble is conjoined or disjoined with some other proposition q. The fourth and fifth rule pick out four possible scenarios. The first division depends on whether the extra signification is made explicit conjunctively or disjunctively. The second division has to do with the logical relationships holding between $\neg q$ and the *casus*.

Provided that certain conditions are met, the strategy is analogous to the one adopted above : in certain cases we will have to reject the *casus* because it still yields a contradictory result, whereas in certain other cases we will be able to admit the *casus* and respond to the insoluble in such a way as to discharge the burden on the extra signified content. The situation for the conjunctive case is dual with respect to the disjunctive case, both in case of rejection and in case of admission of the *casus* ; and the same holds of the evaluations that are given to the propositions involved. When an (admissible) insoluble A that conjunctively signifies $A \wedge q$ is at stake, the respondent should admit A as a *positum*, concede it when it is proposed, and deny that A is true. By contrast, when an (admissible) insoluble A disjunctively signifies $A \vee q$, he should admit the proposition as a *positum*, deny it when it is proposed, and concede that it is true. So what are the conditions according to which the proposition should be admitted in one case and in the other ? If A precisely signifies conjunctively $A \wedge q$, then $\neg q$ (i.e. the contradictory of the second conjunct q) must be consistent with the *casus*. If A precisely signifies disjunctively $A \vee q$, then $\neg q$ (i.e. the contradictory of the second disjunct q) must be inconsistent with the *casus*.

[30] Peter of Mantua, *Insolubilia* cit., sig. O3^{va-vb} « [*Rule* 4] Si autem ponatur propositio insolubilis ad significandum copulative, est videndum si contradictorium secunde partis copulative stat cum casu. [1.1] Et si sic, est casus admittendus. [1.2] Et si non, est reiciendus casus. Ut si ponatur quod Sor dicat istam 'Sor dicit falsum' adequate significantem Sortem dicere falsum et Sortem loqui, quia ista non stant simul 'Sor dicit falsum' et 'Sor non loquitur', ideo non est admittendus casus. Si autem ponatur quod ista 'Sor dicit falsum' significet precise Sortem dicere falsum et Sor currere, admittendus est casus et ista propositio est concedenda et negandum est quod ipsa sit vera. [*Rule* 5] Ultimo, si ponatur insolubile significare disiunctive, dicit ista positio quod [2.1] si oppositum disiuncti potest stare cum casu, negandus est casus. Ut si ponatur quod hec propositio 'falsum est' sit quelibet propositio adequate significans quod falsum est vel nullus deus est, casus non est admittendus. Sed [2.2] si ponatur quod ista 'falsum est' sit omnis propositio adequate significans quod falsum est vel quod deus est, admittendus est casus et neganda est ista 'falsum est' cum proponitur et concedendum est quod sit vera ».

Let us call a *casus* **C**, A be the *positum* (insoluble proposition precisely signifying either conjunctively $A \wedge q$, or disjunctively $A \vee q$). Then, Heytesbury's fourth and fifth rule can be represented as follows.

R4 (*conjunctive case*) $A = \neg T(A)$ and Sig^{pr} $(A, \neg T(A) \wedge q)$[31]

1.1 If $\mathbf{C} \rightarrow \neg(\neg q)$	then **C** should be rejected[32]	[$\neg q$ inconsistent with **C**]
1.2 If $\neg(\mathbf{C} \rightarrow \neg(\neg q))$	then **C** should be admitted *and*	[$\neg q$ consistent with **C**]
	1.2.1 $OA(A)$	
	1.2.2 $OC(A)$	
	1.2.3 $ON(T(A))$	
	1.2.4 [$ON(q)$, i.e. $OC(\neg q)$]	
	[1.2.5 $OC(q)$	(PM's objection in 2.8)]

R5 (*disjunctive case*) $A = \neg T(A)$ and Sig^{pr} $(A, \neg T(A) \vee q)$

2.1 If $\mathbf{C} \rightarrow \neg(\neg q)$	then **C** should be admitted *and*	[$\neg q$ inconsistent with **C**]
	2.1.1 $OA(A)$	
	2.1.2 $ON(A)$	
	2.1.3 $OC(T(A))$	
	2.1.4 [$OC(q)$]	
	[2.1.5 $OC(A)$	(PM's objection in 2.8)]
2.2 If $\neg(\mathbf{C} \rightarrow \neg(\neg q))$	then **C** should be rejected	[$\neg q$ consistent with **C**]

The reason for the requirement that the negation of the second conjunct in $A \wedge q$ (negation of the second disjunct in $A \vee q$) — where the extra signification is made explicit — be compatible (incompatible) with the *casus* is the following. In the former case we need to be able to deny q, while in the latter case we need to be able to concede q during the disputation. The formal justification for these answers is as follows:

R4 Sig^{pr} $(A, \neg T(A) \wedge q)$

1st leg of the paradox (proof of $\neg T(A)$, by *reductio*)

1. $T(A)$ hypothesis

[31] A signifies precisely that $\neg T(A) \wedge q$, i.e. $\neg T(A) \wedge q$ is all and only what A signifies.

[32] Because it cannot be defended, just as the case of a categorical insoluble proposition precisely signifying what is signifies, covered by Rule 2.

2. $T(\mathrm{A}) \rightarrow \forall p\, (Sig\, (\mathrm{A}, p) \rightarrow p)$	def. truth (left-hand side)	
3. $\forall p\, (Sig\, (\mathrm{A}, p) \rightarrow p)$	1, 2 and *modus ponens*	
4. $Sig\, (\mathrm{A}, \neg T(\mathrm{A}) \wedge q) \rightarrow (\neg T(\mathrm{A}) \wedge q)$	3 and substitution of p with $\neg T(\mathrm{A}) \wedge q$	
5. $Sig\, (\mathrm{A}, \neg T(\mathrm{A}) \wedge q)$	hypothesis	
6. $\neg T(\mathrm{A}) \wedge q$	4, 5 and *modus ponens* [$\neg T(\mathrm{A}) \wedge q = \mathrm{A}$]	$\boldsymbol{OC(\mathrm{A})}$
7. $\neg T(\mathrm{A})$	6 and simplification	
8. $\neg T(\mathrm{A})$	1,7 by *reductio*	$\boldsymbol{ON(T(\mathrm{A}))}$

Moreover, the following also holds

9. $\neg T(\mathrm{A}) \rightarrow \neg\forall p\, (Sig\, (\mathrm{A}, p) \rightarrow p)$	def. truth (right-hand side) and contraposition	
10. $\neg\forall p\, (Sig\, (\mathrm{A}, p) \rightarrow p)$	8,9 and *modus ponens*	
11. $\exists p\, (Sig\, (\mathrm{A}, p) \wedge \neg p)$	10	
12. $Sig\, (\mathrm{A}, \neg T(\mathrm{A}) \wedge q) \wedge \neg(\neg T(\mathrm{A}) \wedge q)$	11 and substitution of p with $\neg T(\mathrm{A}) \wedge q$	
13. $T(\mathrm{A}) \vee \neg q$	12, simplification and De Morgan's laws	
14. $\neg q$	13, 8 and disjunctive syllogism	$\boldsymbol{OC(\neg q)}$ $\boldsymbol{ON(q)}$

We have, therefore, a formal justification for both scenarios covered by rule 4. The Liar proposition involved signifies conjunctively. It ought to be conceded (6), but it ought to be denied that it is true (8), just as in the case of the third rule. In addition to that, the negation of the second disjunct q should be consistent with the *casus*, because $\neg q$ ought to be conceded (14), and if it were not compatible with the *casus* we would end up again in a paradoxical situation.

Nor does the second leg of the argument generate paradox, because from $\neg T(\mathrm{A})$ we cannot derive $T(\mathrm{A})$:

2nd leg of the paradox (inference blocked)

15. $\neg T(\mathrm{A})$	8
16. $\forall p\, (Sig\, (\mathrm{A}, p) \rightarrow p) \rightarrow T(\mathrm{A})$	def. truth (right-hand side)
17. $Sig^{pr}\, (\mathrm{A}, \neg T(\mathrm{A}) \wedge q)$	hypothesis

because, again, to derive the conclusion

18. $T(\mathrm{A})$

and generate the paradox the antecedent of (16) should be satisfied. This is required to detach $T(\mathrm{A})$ in (18). The job should be done by (15) and (17). But it is immediately clear that there is something lacking. In order to satisfy the antecedent of (16), we need not only $\neg T(\mathrm{A})$ but also q, because A precisely signifies $\neg T(\mathrm{A})$ *and* q. Thus, having proved $\neg T(\mathrm{A})$ is not enough to reverse the argument and prove $T(\mathrm{A})$.

The justification for rule 5 is analogous but the responses are dual.

R5 Sig^{pr} (A, $\neg T(\mathrm{A}) \vee q$)

1st leg of the paradox (now proof of $T(\mathrm{A})$, again by *reductio*)

1. $\neg T(\mathrm{A})$	hypothesis	
2. $\neg T(\mathrm{A}) \rightarrow \neg\forall p\ (Sig\ (\mathrm{A}, p) \rightarrow p)$	def. truth (right-hand side) and contraposition	
3. $\neg\forall p\ (Sig\ (\mathrm{A}, p) \rightarrow p)$	1, 2 and *modus ponens*	
4. $\exists p\ (Sig\ (\mathrm{A}, p) \wedge \neg p)$	3	
5. $Sig\ (\mathrm{A}, \neg T(\mathrm{A}) \vee q)$	hypothesis	
6. $Sig\ (\mathrm{A}, \neg T(\mathrm{A}) \vee q) \wedge \neg(\neg T(\mathrm{A}) \vee q)$	4, 5	
7. $T(\mathrm{A}) \wedge \neg q$	6, simplification and De Morgan's laws	
8. $T(\mathrm{A})$	7, simplification	
9. $T(\mathrm{A})$	1, 8 by *reductio* [$\neg \mathrm{T}(\mathrm{A}) = \mathrm{A}$]	***ON*(A)** ***OC*(*T*(A))**

2nd leg of the paradox (inference blocked)

Again, the second leg does not generate paradox because this time from $T(\mathrm{A})$ we cannot prove $\neg T(\mathrm{A})$, but rather $\neg T(\mathrm{A}) \vee q$ only:

10. $T(\mathrm{A})$	9	
11. $T(\mathrm{A}) \rightarrow \forall p\ (Sig\ (\mathrm{A}, p) \rightarrow p)$	def. truth (left-hand side)	
12. $\forall p\ (Sig\ (\mathrm{A}, p) \rightarrow p)$	10, 11 and *modus ponens*	
13. $Sig\ (\mathrm{A}, \neg T(\mathrm{A}) \vee q)$	hypothesis	
14. $\neg T(\mathrm{A}) \vee q$	12, 13, substitution of p with $\neg T(\mathrm{A}) \vee q$ and *modus ponens*	
15. q	10, 14 and disjunctive syllogism	***OC*(*q*)**

As a result, in the disjunctive case, we ought eventually to concede q, i.e. the second conjunct. And this is how the requirement that $\neg q$ be inconsistent with the *casus* is to be explained.

Peter criticises Heytesbury's approach from different angles. His general strategy is to make use of analogous obligational principles to pay him back in the same coin. A few objections are directed against the two definitions but the main focus is much more extensively on the rules[33]. Peter seems to be happy with the first rule, but challenges the other four. The number and sophistication of his objections are not suitable for a detailed presentation in this context, but I shall briefly discuss at least an objection against the third rule, since it represents Heytesbury's way to solve the paradox, and two further arguments against the fourth and fifth rule.

> « [**2.6**] [*Against Rule* 3] Against the third rule [laid down within the framework] of this opinion, [and] according to which he responds to categorical insolubles, one argues as follows : (*Obj.* 1) let it be posited that Socrates is saying this [proposition] 'a falsehood is being said' and that no other proposition except for this one, or part of it, is uttered by anyone else, [and] that [the proposition] signifies that a falsehood is being said — not precisely, however, just as that view likes [to stipulate]. [...] Next, 'a falsehood is being said' is proposed and it is conceded according to this view. In addition to that, 'this [proposition] is false' is proposed, which is also conceded according to this view[34]. But to the contrary : this proposition 'a falsehood is being said' principally signifies that there is a god, therefore this proposition is necessary. The consequence is valid, known to be such and so on ; and you ought to be in doubt about the antecedent ; therefore you ought not to deny its consequent. The consequence holds according to this view ; and one ought to be in doubt about the antecedent along with the whole *casus* ; therefore you ought not to deny the consequent »[35].

Peter offers the following argument against rule 3. Let A be an insoluble signifying (not precisely) itself not to be true, without any further specification concerning its extra content, which might well be a necessary proposition, say r. Then,

Positum	A	OA(A)
1.	A	OC(A)

[33] Peter gives two arguments against the proposed definitions. The first is that it is not true to say that an insoluble proposition always needs to be understood in the setting of a *casus*. The second moves the discussion to the level of mental language, where according to him certain logical relations are problematic no matter whether a *casus* is set up or not.

[34] I.e. it must be denied that the proposition is true.

[35] Peter of Mantua, *Insolubilia* cit., sig. O4[va] « [*Against Rule* 3] Contra autem tertiam regulam istius opinionis, secundum quam ipse respondet ad insolubilia categorica, sic arguitur : (*Obj.* 1) quia ponatur quod Sor dicat istam 'falsum dicitur' et non proferatur ab aliquo alia propositio nisi ista aut eius pars, que significet falsum dici — non tamen precise, sicut illi positioni placet. [...] Deinde proponitur ista 'falsum dicitur' et conceditur secundum istam positionem. Et ultra proponitur ista 'hec est falsa', que etiam conceditur secundum istam positionem. Sed contra quia sequitur : hec propositio 'falsum dicitur' principaliter significat deum esse, igitur ista propositio est necessaria. Consequentia est bona scita esse talem etc. ; et antecedens est a te dubitandum ; igitur consequens eius non est a te negandum. Patet consequentia secundum istam positionem ; et antecendens cum toto casu est dubitandum ; igitur consequens non est a te negandum ».

2.	$\neg T(\mathrm{A})$	$OC(\neg T(\mathrm{A}))$, i.e. $ON(T(\mathrm{A}))$
3.	$Sig\,(\mathrm{A}, r) \rightarrow \Box\mathrm{A}$	valid consequence if r is necessary
4.	$\Box\mathrm{A} \rightarrow T(\mathrm{A})$	necessity entails truth
5.	$\neg T(\mathrm{A}) \rightarrow \neg\Box\mathrm{A}$	falsehood entails non-necessity
6.	$\neg\Box\mathrm{A}$	2, 5 and *modus ponens*
7.	$OD(Sig\,(\mathrm{A}, r))$	from the characteristic condition of R3
8.	$(p \rightarrow q) \rightarrow OD(p) \rightarrow O\neg N(q)$	obligational principle
9.	$O\neg N(\Box\mathrm{A})$	3, 7, 8 and *modus ponens*

The first two steps are Heytesbury's standard answers : concede the insoluble and deny that it is true[36]. Peter wants to show that the denial of $T(\mathrm{A})$ is incompatible with the assumption that the signification of the proposition is not precise, because it could signify a necessary proposition after all. In order to do so, he makes use of a rule which is to be found in his own treatise on obligations. If the respondent ought to be in doubt about the antecedent of a valid consequence, then he ought not to deny its consequent ; otherwise, by contraposition, he ought to deny the antecedent as well. Since Heytesbury's third rule applies, by definition, only in cases in which the signification of an insoluble proposition is not fully specified, Peter claims that the respondent should be in doubt about the signification of A, because he does not know in principle whether the proposition signifies that there is a god or not (or any other necessary proposition). Why is this important ? The argument, I believe, rests on the implicit assumption that if A is necessary then it is also true. But under the stipulations of the *casus*, $\neg T(\mathrm{A})$ is conceded. This entails in turn that A is not necessary. But as long as the respondent must be in doubt about what A precisely signifies (he only knows that it signifies, not precisely, itself not to be true, but nothing is said about what it could signify in addition to that), he cannot rule out that A signifies something necessary. Thus, (9) and (6) are incompatible, and their being incompatible ultimately depends on the requirements of Heytesbury's third rule and on the plausible obligational principle expressed by (8). In other words, on the one hand Heytesbury is committed to the view that A is not true, and therefore not necessary, but on the other hand, he is also committed to the view that he ought not to deny that A is necessary. And those two claims are incompatible, in one and the same obligational disputation.

[36] The terminology in Peter's example is somehow sloppy, but with some adjustments the case can be reduced to one in which all the conditions of Heytesbury's third rule apply.

As for rules 4 and 5, Peter of Mantua's strategy is to provide counterexamples to the scenarios described above (leaving out only 2.2).

Against 1.1, he presents us with a *casus* where, despite $\neg q$'s being inconsistent with **C**, the *casus* should be admitted and the insoluble is argued to be true (i.e. the rule does not avoid paradox).

> « [**2.8**] (*Obj.* 2) Again, let it be posited that this proposition 'this proposition which precisely signifies categorically is not true' precisely signifies that this proposition which precisely signifies categorically is not true and that you do not differ from yourself ; and let this proposition be B. Then B signifies conjunctively and B is true, therefore an insoluble signifying conjunctively is true and it ought to be conceded that it is true ; therefore and so on. The consequence clearly holds. And the antecedent is known, because B precisely signifies that this proposition which precisely signifies categorically is not true and that you do not differ from yourself ; and this proposition which precisely signifies categorically is not true and you do not differ from yourself ; therefore proposition B is true. And in this case, one argues (*i*) that a *casus* of an insoluble ought to be admitted when the insoluble is imposed to signify conjunctively, although the opposite of [the second] conjunct is not consistent with the *casus*, and (*ii*) that the insoluble is true »[37].

Let A be an insoluble signifying precisely that $A \wedge q$, where $\neg q$ is inconsistent with the *casus* (suppose q is a logical truth, like the example « you do not differ from yourself » : $\neg q$ is trivially incompatible with the *casus* because it is contradictory in itself), then

1.	$OA(\mathrm{A})$	
2.	$OC(\mathrm{A})$	R4
3.	$OC(\neg T(\mathrm{A}))$	R4
4.	$OC(q)$	hypothesis
5.	$Sig^{pr}(\mathrm{A}, \neg T(\mathrm{A}) \wedge q)$	hypothesis
6.	$\exists p\ (Sig\ (\mathrm{A}, p) \wedge p) \rightarrow T(\mathrm{A})$	def. truth (right-hand side)[38]
7.	$OC(T(\mathrm{A}))$	3, 4, 5, 6 and *modus ponens*

[37] Peter of Mantua, *Insolubilia* cit., sig. O4+1ra-rb « (*Obj.* 2) Item, ponatur quod hec propositio 'hec propositio precise categorice significans non est vera' significet precise quod hec propositio precise categorice significans non est vera [hec propositio[3]... vera *ms*. M] et quod tu non differs a te ; et sit B ista propositio. Tunc B significat copulative et B est vera, igitur insolubile significans copulative est verum et concedendum est esse verum ; igitur etc. Patet consequentia. Et antecedens est notum : quia B precise significat quod ista propositio precise [precise *ms*. M] categorice significans non est vera et quod tu non differs a te ; et hec propositio precise categorice significans non est vera et tu non differs a te [et hec... te *ms*. O] ; igitur B propositio est vera. Et in isto casu arguitur (*i*) quod casus de insolubili est admittendus quando insolubile imponitur ad significandum copulative, quamvis oppositum copulati non possit stare cum casu. Et arguitur (*ii*) quod insolubile est verum ».

[38] There is no need to use here the right-hand side of the revised definition : $\forall p\ (Sig\ (\mathrm{A}, p) \rightarrow p) \rightarrow T(\mathrm{A})$. Even if we did, however, the antecedent would be satisfied because the insoluble precisely signifies conjunctively, i.e. all it signifies is $\neg T(\mathrm{A}) \wedge q$, and both $\neg T(\mathrm{A})$ and q are granted, at (3) and (4), respectively. Consequently we would be still entitled to detach $T(\mathrm{A})$.

Peter's argument seems to be directed against Heytesbury's claim that whenever the contradictory of the second conjunct is inconsistent with the *casus*, the latter ought not to be admitted. In fact, according to Peter it ought to be admitted (1), in which case it can be shown that it ought to be conceded that the insoluble is true (7), which is against one of the rule's prescriptions (3). However, there seems to be a problem with this objection, for it simply asserts what it should prove and does not count as a genuine argument to prove that one ought to admit the *casus* (1) even when the contradictory of the second conjunct is inconsistent with it. Peter simply shows that, if such a *casus* is admitted, one must reply in a way other than the one suggested by Heytesbury. Heytesbury's argument in reply could be that it is exactly because an inconsistency would arise that the *casus* ought not to be admitted in the first place.

Against 1.2 Peter argues that even if $\neg q$ is compatible with **C**, Heytesbury's solution does not work because the respondent is committed to conceding q, whereas according to the rule the success of the argument ultimately relies on the fact that q is going to be denied (which is why its negation is required to be compatible with the *casus* : otherwise it could not be denied in the first place and the conjunctive case would collapse on to the case covered by rule 2).

> « [**2.8**] [*Against Rule* 4] Against the fourth rule one argues [as follows] : (*Obj.* 1) Let it be posited that Socrates is saying 'Socrates is saying what is false' which adequately signifies that Socrates is saying what is false and you are running, and let it be the case that he is saying no other [proposition] that is not part of this. Once this is posited, 'Socrates is saying what is false' is proposed. Once this is conceded, according to this view, one argues as follows : Socrates is saying what is false, therefore Socrates is saying what is false and you are running. The consequence holds, because one argues from one convertible to the other ; and the antecedent ought to be conceded ; therefore the consequent, too, [ought to be conceded]. But then, in addition to that, Socrates is saying what is false and you are running ; therefore you are running. The consequence again holds ; and the antecedent ought to be conceded ; therefore the consequent, too, [ought to be conceded]. Therefore, a conjunct ought to be conceded whose opposite is consistent with the *casus*, which is repugnant to the rule, i.e. to the view »[39].

[39] Peter of Mantua, *Insolubilia* cit., sig. O4+1[ra] « [*Against Rule* 4] Contra quartam regulam arguitur : (*Obj.* 1) quia ponatur quod Sor dicat istam 'Sor dicit falsum' adequate significantem quod Sor dicit falsum et tu curris, et non dicat aliam que non sit pars istius. Quo posito, proponitur ista 'Sor dicit falsum'. Qua concessa secundum istam positionem, arguitur sic : Sor dicit falsum, igitur Sor dicit falsum et tu curris. Consequentia patet, quia arguitur ab uno convertibili ad reliquum ; et antecedens est concedendum ; igitur et consequens. Et tunc ultra : Sor dicit falsum et tu curris, igitur tu curris. Tenet consequentia iterum ; et antecedens est concedendum ; igitur et consequens. Igitur copulatum est concedendum cuius [cuius *ms.* O] oppositum stat cum casu, quod repugnat regule sive positioni ».

Let A be an insoluble, signifying precisely that $A \wedge q$, where $\neg q$ is compatible with the *casus*, then

1.	$OA(A)$	
2.	$OC(A)$	R4
3.	$A \rightarrow (A \wedge q)$	hypothesis : $A \leftrightarrow (A \wedge q)$ by the *casus*
4.	$(p \rightarrow r) \rightarrow OC(p) \rightarrow OC(r)$	obligational principle
5.	$OC(A \wedge q)$	2, 3, 4 and *modus ponens*
6.	$(A \wedge q) \rightarrow q$	simplification
7.	$OC(q)$	4, 5, 6 and *modus ponens*

Step (7) is inconsistent with the prescription of Heytesbury's rule. The contradictory of q is compatible with the *casus* but nevertheless, according to Peter's argument, q ought to be conceded as following. Since Heytesbury's rule claims that q ought to be denied as repugnant, the rule is unable to warrant consistency.

Finally, against 2.1 Peter offers two *casus* that satisfy the condition of incompatibility (if q is necessarily true, its negation is trivially incompatible with the *casus*) but argues that one should not follow rule 5 in replying to the paradoxical propositions involved. According to Heytesbury, A (the insoluble proposition signifying disjunctively) ought to be denied. In his first objection Peter offers a proof to the contrary, namely of the fact that A ought to be conceded :

> « [**2.9**] [*Against Rule* 5] Again, one argues [as follows] against the last rule [laid down by] this view : (*Obj.* 1) let it be posited that 'every proposition is false' precisely signifies that every proposition is false or there is a god. Once this is admitted, 'every proposition is false' is proposed. If this is denied, as this view maintains, [one can argue] to the contrary because it follows 'there is a god, therefore every proposition is false or there is a god'. The consequence clearly holds because it follows 'there is a god, therefore every proposition is false or there is a god' ; and the antecedent ought to be conceded ; therefore the consequent, too, [ought to be conceded]. And in addition to that, one [can] say that every proposition is false ([arguing] from one convertible to the other) ; and the antecedent ought to be conceded ; therefore the consequent, too, [ought to be conceded]. But the consequent is an insoluble signifying disjunctively, therefore an insoluble signifying disjunctively ought to be conceded »[40].

[40] Peter of Mantua, *Insolubilia* cit., sig. O4+1^{ra-rb} « [*Against Rule* 5] Item, contra ultimam regulam istius positionis arguitur : (*Obj.* 1) quia ponatur quod hec 'omnis propositio est falsa' precise significet quod omnis propositio est falsa vel deus est. Quo admisso, proponitur ista 'omnis propositio est falsa'. Que si negatur, ut dicit positio, contra, quia sequitur 'deus est, igitur omnis propositio est falsa vel deus est'. Consequentia patet quia sequitur 'deus est, igitur omnis propositio est falsa vel deus est' ; et antecedens est concedendum ; igitur et consequens. Et ultra dicitur quod omnis propositio est falsa (ab uno convertibilium ad reliquum) ; et antecedens est concedendum ; igitur et consequens. Et consequens est insolubile disiunctive significans, igitur insolubile disiunctive significans est concedendum ».

The argument runs as follows:

1.	$OA(\mathrm{A})$	
2.	$ON(\mathrm{A})$	R5
3.	$q \rightarrow (\mathrm{A} \vee q)$	disjunction introduction
4.	$(p \rightarrow r) \rightarrow OC(p) \rightarrow OC(r)$	obligational principle
5.	$OC(q)$	hypothesis: q is a necessary proposition
6.	$OC(\mathrm{A} \vee q)$	3, 4, 5 and *modus ponens*
7.	$(\mathrm{A} \vee q) \leftrightarrow \mathrm{A}$	hypothesis
8.	$(p \leftrightarrow r) \rightarrow OC(p) \rightarrow OC(r)$	obligational principle
9.	$OC(\mathrm{A})$	6, 7, 8 and *modus ponens*

But (2) and (9) are inconsistent with one another. What is more, (9) shows that an insoluble precisely signifying disjunctively ought to be conceded, which is against the rule laid down by Heytesbury. If the second conjunct is a necessary proposition, the condition that its negation be incompatible with the *casus* will always be satisfied. But the proposition itself will also always have to be conceded during a disputation. And this leads eventually to the conclusion that the insoluble itself ought to be conceded. Therefore, we have a counterexample to one of the two situations covered by R5. Peter does not address the other, i.e. he does not argue against the claim that in the disjunctive case, when the negation of the second disjunct is compatible with the *casus*, the latter ought to be rejected.

In sum, the three objections discussed here show that Peter's approach to Heytesbury's rules focuses on their inability to maintain consistency. This is argued within the same obligational framework that Heytesbury has adopted by way of providing counterexamples such that the conditions stated by the rules are satisfied but the answer, either to the insoluble or to the other member of the conjunction/disjunction, can be different from the one prescribed by the rules.

3.1.3 Against restrictionism

After devoting a remarkable amount of space to the rejection of Heytesbury's view, Peter turns to a brief examination of his last target. In the third section, which is by far the shortest one in the treatise, the object of criticism is the opinion of the *restringentes*, in its strong version[41].

[41] Restrictionism, in its various forms, is a widely accepted position until second quarter of the XIVth century, when it is wiped off the board by the modern approaches introduced by the likes of Bradwardine, Heytesbury, or Buridan and adopted by their followers. A moderate version is endorsed, for instance, by Ockham (for a presen-

> « [**3.1**] In [discussing] these matters, some ancients said that a part of a proposition does not supposit for the whole proposition of which it is a part, nor for something convertible with it, nor for [its] contradictory, nor for something convertible with its contradictory »[42].

Peter gives a very limited number of technical objections and the general tone of this section is rather dismissive. The main claim seems to be that the view under consideration would rule out certain inferences that are taken to be valid, whereby a part must indeed supposit for the whole or for something logically related to the whole of which it is a part. First, restrictionism would fail to validate inferences like « Every particular proposition is false ; therefore some particular proposition is false » under the assumption that no other proposition exists apart from the consequent. Next, in the proposition « Every proposition is false » the subject-term 'every proposition' stands for and picks out each of its supposits, including the proposition itself, by the very definition of supposition. Last, it seems that whenever a proposition is well-formed and has a truth value its terms must supposit for something. It must be said, however, that rather than offering arguments to counter this alternative account, Peter seems to be repeating in various forms the claim that the latter is inadequate.

3.1.4 Sketch of Peter of Mantua's solution

In the fourth section, Peter finally offers his own solution. With respect to the taxonomy of solutions mentioned above, it might be said that the approach he adopts is in terms of a *secundum quid* and *simpliciter* distinction, but such a characterisation would not be very informative, since solutions of very different nature fall under this general heading. Peter's formulation is quite uncommon and seems to have been preceded by only one other example in the *insolubilia* tradition, namely that of John Wyclif.

tation, discussion and defense of the latter as a contextualist solution to the Liar see C. Panaccio, *Restrictionism : A Medieval Approach Revisited*, in Rahman, Tulenheimo, Genot eds., *Unity, Truth and the Liar* cit., pp. 229-253). Generally speaking, the main distinctive feature of strong restrictionism is that it rules out all kinds of self-reference declaring ill-formed propositions that seem to be harmless (and true) like « This is an affirmative proposition » (referring to itself). Weak restrictionism, by contrast, circumscribes the constraint only to problematic cases that generate paradox. The standard criticism against these two positions is that the former is too strong, while the latter looks *ad hoc*. It seems to me that Peter is targeting some version of strong restrictionism here, because the formulation given above seems to cover the widest variety of cases. In particular, it is worth noting that not only is it not permitted for a proposition to self-refer, but also indirect self-involvement is ruled out. In other words a part of p cannot supposit for q, if q is logically related to p (because it entails or its negation entails, either directly or indirectly, p ; or because it is entailed or its negation is entailed, either directly or indirectly, by p).

[42] Peter of Mantua, *Insolubilia* cit., sig. O4+1^{rb-vb} « Dixerunt antiqui in hac materia quod pars propositionis non supponit pro tota propositione cuius est pars nec pro convertibili nec pro contradictorio nec pro convertibili cum contradictorio ipsius [*pro* istius] ».

In this section Peter introduces three distinct senses in which a proposition can be said to be true. Only two of them, however, are relevant from a logical point of view insofar as they are used as genuine semantic predicates. Liar-like propositions are then said to be true in one sense and false in the other, so as to avoid contradiction (their being true in one sense does not entail their being false in the same sense and, conversely, their being false in one sense does not entail their being true in the same sense). As will be shown in the next section, this approach might seem to have a *prima facie* intuitive justification. However, as has been noted already in the case of Wyclif's solution[43], it does not provide a satisfactory account, because the paradox arises again as soon as it is reconstructed by replacing the semantic predicate in a suitable way.

After laying down the distinction between different senses of 'true', and considering a first block of objections, Peter also discusses a series of additional arguments that seem to be variations on the theme of the so-called postcard paradox. These are in particular identified as a class of cases that must be rejected because they are equivalent to impossible conditionals. The same view, in the very same context, is again endorsed by Wyclif among others.

3.2 Solution to semantic paradoxes

Peter of Mantua's solution may appear, to some extent, quite traditional in spirit. It seems to leave entirely out of the picture the 'modern' idea of an extra signified content of a proposition (to be found both in Albert of Saxony and William Heytesbury, the two main targets of Peter's text). It focuses instead on the notion of truth itself, grounding the solution on a conceptual distinction that determines two senses (in fact three, but the first one is not relevant for the discussion of insolubles) associated with it. His position can be described, perhaps, as that of a weak proto-hierarchist, i.e. a theorist that allows for a two-level hierarchy of semantic predicates that applies, however, only to the case of self-referential propositions. Let us examine the relevant texts.

> « [**4.1**] [$\mathbf{T_0}$] Hence, since 'true' signifies every being insofar as it is a term of first intention or imposition, and in this sense every being is true and no one is false or a *fictum*, and [again] in this sense every proposition is a true proposition and not a fictional one, we shall not care about this [sense of the term] in the present [context]. Rather we shall say that a proposition can be said [to be] true in two senses.
> [$\mathbf{T_1}$] In one sense [a proposition is said to be true] when it is verified not by the supposits of its terms, among which it itself or another proposition is [found as] a supposit — i.e. [we shall say] that a proposition is made true [in this sense] neither because a part of it supposits for that very same [proposition] nor [because a part of it supposits] for something relevant to it, like [the proposition] 'there is a god'. In this sense those

[43] See the introduction to WYCLIF, *Summa insolubilium* cit., pp. XXXI-XXXIII.

propositions that are about terms of first intention or imposition are true or false properly and absolutely (*proprie et simpliciter*). In this sense, 'proposition', according to its own etimology and [taken] literally, signifies the same as 'positing [something] for something else'. Most [propositions] are true or false in this sense.
[$\mathbf{T_2}$] In another sense a proposition is said to be true when it is verified with respect to itself or to something relevant. In this sense the self-referential proposition 'this is <not> true' is true not absolutely but in some respect (*non simpliciter sed secundum quid*). But it is false according to the first sense [i.e. *simpliciter*], because it is verified only with respect to the supposit of one of its parts, of which [part the whole proposition] itself is the supposit[44], like [in the case of] 'a human being is an ass or this disjunction is false', indicating through the subject [i.e. 'this disjunction'] the whole disjunction »[45].

According to a first general sense, namely when we take 'true' to be a transcendental notion, everything is true, so we can easily drop this from consideration in the context of a discussion concerning the Liar, because *any* proposition, insofar as it exists, is a being and is therefore true (truth being here no semantical notion).

From the logical standpoint, however, there are two other senses of 'true' that should be taken into account. We can characterise them as follows : a proposition p is true in a first and proper sense if (**Corr**) a criterion of correspondence (specified independently and depending ultimately on one's own theory of truth) is satisfied

[44] Or, perhaps in a better wording, « and the supposit of that part is [the whole proposition] itself ».

[45] Peter of Mantua, *Insolubilia* cit., sig. O4+1^{vb}-O4+2ra. « [T_0] Et ergo, cum 'verum' omne ens significet prout est terminus prime intentionis vel impositionis, et hoc modo omne ens est verum et nullum falsum seu fictum, et hoc modo omnis propositio est vera propositio et non ficta, ideo de hoc in presenti non curamus. Sed dicemus quod duobus modis propositio potest dici vera. [T_1] Uno modo quando verificatur non propter supposita suorum terminorum quorum suppositorum ipsa vel alia propositio est suppositum — scilicet quod propositio vera non reddatur vera ex eo quod pars eius supponat pro ipsamet nec pro pertinente ad ipsam, sicut hec 'deus est'. Et isto modo ille propositiones que sunt de terminis prime intentionis vel impositionis sunt proprie et simpliciter vere vel false. Quo modo propositio ex sua etimologia et sermonis virtute significat idem quod 'pro alio positio'. Et hoc modo maior pars est vera vel falsa. [T_2] Alio modo dicitur propositio esse vera quando verificatur pro se vel pro pertinente. Et isto modo hec propositio est vera 'hoc <non> est verum', seipsa demonstrata, non simpliciter, sed secundum quid. Sed est falsa primo modo, quia non verificatur nisi pro supposito sue partis cuius ipsa est suppositum, sicut ista 'homo est asinus vel ista disiunctiva est falsa' demonstrata per subiectum tota illa disiunctiva ». — The text is garbled in the manuscript tradition, and the emendation 'hoc <non> est verum' is necessary because Peter is talking here about the Liar, not the Truth teller. The claim is supported by the fact that further down in the text, when the latter is mentioned, the early printed text of E (against all manuscripts) provides a better reading. The Truth teller and the Liar receive opposite truth values, according to Peter. There is no example of a discussion of the Truth teller in the whole treatise that might suggest clearly that Peter wants it to be true in some respect (if this were to be the case, the choice would be arbitrary and that would be a problem for the theory). On the other hand, the conjecture proposed here is consistent with the reading of E against all manuscripts in the next passage, and despite the cost of intervening twice with the support of one witness only, it appears to be the only way to make good sense of the text.

and (¬**R**) the semantic value of p does not depend on the evaluation of p itself (or some other proposition q that is logically related to p), i.e. if the proposition is not self-referential and, therefore, does not act as a truth-maker for itself.

On the other hand, a proposition is true in the second sense if, again, (**Corr**) a criterion of correspondence is satisfied and (**R**) the semantic value of p does depend on the evaluation of p itself (or some other proposition q that is logically related to p), i.e. if the proposition is self-referential and does act as a truth-maker for itself.

The justification for a solution in terms of the distinction *secundum quid et simpliciter* is laid down by appealing to such a twofold characterisation of the notion of truth. All propositions — non-problematic as well as problematic ones — receive a truth value at level 1, i.e. in the object language. The former are evaluated by looking at the world and are said to be absolutely true or false according to whether what they say is the case or not. The latter receive by default the truth-value false at level 1, simply because they are self-referential (not because what they say is not the case), but are then also evaluated at level 2. At level 2, correspondence again comes into play. If what they say is the case, then they are T_2, otherwise they are F_2. Thus, on this picture, any proposition that contains a trace of self-reference is F_1, but it can still turn out to be T_2. This is precisely the case for the Liar, which, as we will shortly see, if interpreted in a suitable manner, is said to be F_1 but T_2.

Responding to some objections according to which this approach would lead to the claim that truth is an equivocal notion, Peter is prepared to concede this point, as is clear from the following passage:

> « [**4.3**] But with respect to these [issues] one should understand that the term 'true' is an equivocal term and the propositions 'this is true', 'not this is true' and the like are all propositions with multiple senses. Therefore, when 'not this is true' [or] 'this is false' [i.e. the Liar] is proposed, one should not respond according to a single response, but rather one should respond that this is false according to the first member of the division introduced [above] and true according to the other. And likewise as far as their contradictories 'not this is false', 'this is true' [i.e. the Truth teller] are concerned, [one should respond] by denying that this is true according to the second sense of the division introduced [above] and by conceding that this is false in the first sense »[46].

[46] Peter of Mantua, *Insolubilia* cit., sig. O4+2va « Pro istis intelligendum est quod iste terminus 'verum' est terminus equivocus et iste propositiones omnes sunt propositiones plures 'hoc est verum', 'non hoc est verum' et sic de aliis. Et ideo, cum proponitur 'non hoc est verum', 'hoc est falsum', non est secundum unicam responsionem respondendum, sed est respondendum quod hoc est falsum secundum primum membrum divisionis posite et verum secundum aliud. Et ita de suis contradictoriis 'non hoc est falsum', 'hoc est verum', negando quod hoc est verum [verum E, *against* falsum BLMOPV] secundo modo divisionis posite et concedendo quod hoc est falsum primo modo ».

Peter draws here a distinction between multiple senses in which propositions are said to be true. Or, to put it otherwise, he ascribes different senses of 'true' to classes of propositions identified on the basis of the syntactic property of being or not-being self-referential.

Propositions are usually (i.e. in most situations in which we are just talking about the world) said to be true only according to the first sense (T_1) and in this sense they are properly and absolutely (*simpliciter*) true or false. Truth in this sense might be labelled 'truth in the object-language'[47]. Despite being false in the first sense, some propositions can still be true in a second sense (T_2), when they act as truth-makers for themselves, and in that case they are said to be true *secundum quid,* i.e. in some respect.

In doing so, Peter is looking at propositions on the basis of the two aforementioned parameters : a criterion of correspondence (**Corr**) and the occurrence of self-reflection (**R**). The logical relationships holding between the notions of absolute truth and absolute falsehood, and truth and falsehood in some respect can be represented as follows in function of these criteria :

$\boldsymbol{T_1(p)}$	iff	$(\mathbf{Corr}) \wedge \neg(\mathbf{R})$		
$\boldsymbol{T_2(p)}$	iff	$(\mathbf{Corr}) \wedge (\mathbf{R})$		
$\boldsymbol{\neg T_1(p)}$	iff	$\neg(\mathbf{Corr}) \vee (\mathbf{R})$	iff	$\boldsymbol{F_1(p)}$
$\boldsymbol{\neg T_2(p)}$	iff	$\neg(\mathbf{Corr}) \wedge (\mathbf{R})$[48]	iff	$\boldsymbol{F_2(p)}$

These notions apply to the following categories of propositions :

(a) « There is a god », « Socrates is sitting », if Socrates is actually sitting, « The proposition "there is a god" is necessary » (T_1 only) ;

(b) « A human being is a donkey », « Socrates is running », if Socrates is actually not running, « The proposition "a human being is a donkey" is necessary » (F_1 only) ;

(c) « This is not true » (Liar), « This proposition contains exactly six words » (F_1 and T_2) ;

(d) « This is true » (Truth teller), « This proposition contains exactly twenty words » (F_1 and F_2).

[47] Note that this is only intended to distinguish the cases in which a proposition talks about itself from all other cases. If a proposition is about (the truth or falsehood of) another proposition, it is just as if it were about any other fact in the world.

[48] It should be noted that, on Peter of Mantua's account, negation can be taken to 'behave' extensionally at level 2 only because we are restricting ourselves to the class of self-referential propositions, and therefore, being F_2 in fact means simply failing to satisfy the correspondence criterion, *once* the proposition has already been established to be self-referential. If we were to define F_2 as the dual of T_2 we would be eventually forced to admit that T_1 propositions are also, by definition, F_2, which would make little sense for a semantic theory.

In the Liar case, Peter's claim is that we can consistently maintain that it is not true absolutely, while still being true in some respect. In order for this to obtain we have to assume explicitly that what the Liar says is that it itself is not true absolutely. For in that case, (*i*) it is in fact *not true absolutely* (because by definition all self-referential propositions are not true absolutely), and (*ii*) since what it says is the case, it is *true in some respect*, which means that the correspondence criterion is satisfied and that the proposition is self-referential.

In other words, even if the Liar is strictly speaking false, it still has *some amount* of truth (because what it says, namely that it is not true absolutely, satisfies the correspondence criterion : the proposition is not true absolutely because in order for it to be true absolutely it should be immune from self-reference, and it is not). Thus if A is $\neg T_1$(A), the following holds :

1. $\neg T_1$(A) [=A] is $\neg T_1$
2. $\neg T_1$(A) [=A] is T_2

A sufficient condition for (1) is (R), because by definition, whatever is $\neg T_1$ is such because ¬(Corr) ∨ (R). It is crucial here that the reason of $\neg T_1$(A)'s [=A] not being T_1 is (R) and not ¬(Corr) : otherwise the claim of absolute falsehood would rely on a failure of correspondence. But a failure of correspondence would then be a sufficient condition for the absolute truth of the proposition, which exactly says of itself that it is such that either what it says fails to obtain *or* it is self-referential, and the paradox would arise again. On the other hand, since A satisfies (R), because it is self-referential, and (Corr), because it says of itself that it is $\neg T_1$ and it is in fact $\neg T_1$, then A is T_2.

Once we accept these characterisations, the final move is consequently to deny the Liar in one sense and affirm it in the other. The solution simply amounts to classifying the proposition as true in one sense (T_2) and false in the other (F_1 or, which is the same, $\neg T_1$). At this point a contradiction no longer follows, because the sense in which the proposition is true is not the same sense in which it is not true. This can easily be seen if we look at the conditions that define these notions : in order for the Liar to be not T_1 it suffices that it be self-reflexive. But it is self-reflexive, and what it says is that it is not T_1, therefore it is T_2.

It is noteworthy that Peter himself recognises and explicitly aknowledges the fact that introducing these two senses immediately leads to the conclusion that the notion of truth is equivocal.

If we rephrase the original formulation of the paradox given above (see *supra* §2), we can see why Peter's solution might be thought to have a *prima facie* intuitive justification.

Suppose A = $\neg T_1$(A) and T = ($T_1 \vee T_2$)[49], then the following obtains:

1[st] leg of the paradox

(1*) T(A)[50]
(2*) $T(\mathrm{A}) \rightarrow \exists p\ (Sig\ (\mathrm{A}, p) \wedge p)$
(3*) $Sig\ (\mathrm{A}, \neg T_1(\mathrm{A}))$
(4*) $\neg T_1$(A)

2[nd] leg of the paradox

(5*) $\neg T_1$(A)
(6*) $\exists p\ (Sig\ (\mathrm{A}, p) \wedge p) \rightarrow T(\mathrm{A})$
(7*) $Sig\ (\mathrm{A}, \neg T_1(\mathrm{A}))$
(8*) T(A)

(9) T_2(A) by (8*), (5*), definition of T as ($T_1 \vee T_2$) and disjunctive syllogism

The conclusion of the second leg of the argument is not incompatible (as it used to be when we were using only one truth-predicate in the original formulation of the paradox) with that of the first leg. What is being said here is that one and the same proposition, A, is (4*) not true absolutely, but (8*) still true either absolutely or in some respect. The distinction drawn above makes such a move legitimate. Indeed, it immediately turns out that A *is* true in some respect, and provably so (the conclusion is derived by 5*, 8*, the hypothesis that T = ($T_1 \vee T_2$) and disjunctive syllogism).

By contrast, in the original formulation of the paradox, (4) and (8) taken together formed a contradiction.

On such an account, there is also an interesting story to be told about the Truth teller. As is well known, the Truth teller is a proposition that fails to provide proper truth conditions for its own evaluation. If it is true, it is true, and if it is false, it is

[49] What I have called earlier the *transmission principle* (i.e. the proposed definition of truth) is formulated more broadly with predicate T, which is now the disjunction of the two predicates T_1 and T_2 introduced above. The intent of this is to have the transmission depend solely on (Corr) : if things are as a proposition signifies them to be, then the proposition is either true absolutely or true in some respect, regardless of whether it is self-referential or not, and vice versa. The revenge problem will ultimately depend on the adoption of this very same predicate (or, possibly, even just of T_2). I can only think of this broader predicate as a plausible candidate for the definition of truth.

[50] The *reductio* would actually require a narrower assumption, namely T_1(A). For (4*) is not incompatible with (1*).

false, but no information conveyed by proposition itself can help us determine which is the case. On Peter's account, by contrast, the Truth teller is false absolutely — just as the Liar — simply because it is self-referential. When it comes to its evaluation at level 2, since what it says is that it itself is T_1 and this is not the case (because in order for it to be T_1 it should be non-self-referential, which it is not), then the Truth teller is not even T_2. The interesting thing about this approach is that, although both the Liar and the Truth teller get the same truth value at the first level (because they are both self-referential irrespective of what they say), the intuitive need of providing a different evaluation to those two propositions, which is grounded in the fact that, being contradictories, they seem to say opposite things, can be still preserved at level 2, where they in fact receive opposite truth values. For the Liar is T_2, while the Truth teller is not T_2. In sum Peter's approach seems to provide a viable solution, at least intuitively, out of the paradox, and also suggests a reasonable way to look at the Truth teller. This logical project, however, is doomed to fail. Before looking at why the solution must eventually be rejected, let us briefly look at the sources and the influence of Peter's text. This will help us introduce the content of the last section.

3.3 Sources and influence

Albert of Saxony and William Heytesbury are the two most clearly identifiable sources for Peter of Mantua's treatise, and this fact was already established in Spade's catalogue, where the two names are associated with that of Peter for the first (and last) time[51]. Another connection is the one that Peter has with Paul of Venice, who comes a few years later and reports his views (anonymously). Again, this fact had already been noted by Spade, and it confirms a more general connection between the two masters, since in other parts of Paul's *Logica Magna* traces of Peter's doctrines are to be found[52].

[51] See Spade, *The Medieval Liar* cit., p. 86 (*s.v.* Peter of Mantua, LII). This was, however, already obvious to contemporary readers : in one of the six manuscripts that preserve the text of Peter's *Insolubilia* (Mantova, Biblioteca Comunale, ms. 76, f. 79[va] *in margine*), the copyist, an Arts student at Ferrara in the 1420s, writes Heytesbury's name in the margin at the beginning of the passage where Peter sets out to discuss his view.

[52] See Paul of Venice, *Tractatus de insolubilibus* cit., f. 193[vb] « Alius magister ideo favens huic opinioni sed non in modo subordinationis assignate ponit quod huiusmodi termini 'verum' et 'falsum' sunt termini equivoci deducta ipsorum transcendentia, sed solum ut de signis complexe significantibus dicunt propositio ergo ait duobus modis potest esse vera. Uno modo dicitur propositio esse vera quando ipsa verificatur non propter supposita suorum terminorum quorum ipsa <propositio est suppositum>, scilicet quod propositio non redditur vera ex eo quod pars eius supponat pro ipsamet nec pro pertinente ad ipsam, sicut 'deus est'. Et isto modo propositiones que sunt de terminis prime intentionis vel impositionis sequitur simpliciter et proprie esse vere vel false. Quo modo 'propositio' ex etimologia sermonis virtute idem significat quod pro alio positio. Alio modo dicitur propositio vera quando verificatur pro seipsa aut pro alio pertinente. Et illo modo hec propositio est vera 'hoc <non> est verum', seipsa demonstrata, et hec 'hoc est falsum' seipsa demonstrata, non simpliciter sed

There are, however, a few additional facts that — to the best of my knowledge — no one seems to have noticed so far. The first shows the strong debt to another representative of the tradition of English logic. The second is a more minute consideration that counts as an interesting historical detail. Let me deal briefly with the latter, first : Spade refers in his catalogue to the position of a certain Anthony de Monte, the copyist of a number of tracts transmitted in the Oxford manuscript, *Canon. misc.* 219 (which is for the most part a collection of logic treatises among which Peter's *Logica* is preserved in its entirery[53]). This Anthony can be independently established as an associate of Peter's from the Padua circle where Peter had been educated before moving to Bologna. The Oxford manuscript contains a leaf copied by Anthony with some conclusions concerning insolubles where he discusses the positions of Albert of Saxony and William Heytesbury, explicitly quoting their names. The text dates from the mid 1390s, i.e. right after Peter is assumed to have finished his *Logica* (in the early 1390s[54]). It is likely that this very brief text derives its selection of sources from the 'portfolio' that is to be found in Peter, or that they might have had a source in common.

The main point that I want to stress here, however, is not about a 'follower', but about a source. Peter's solution in terms of the distinction of different senses of 'true' is already found in the logical writings of John Wyclif in the very same context, a strategy which is nowhere else to be found in the context of the *insolubilia*-literature.

Wyclif's discussion is much longer than Peter's and includes many different considerations, but when it comes to truth and the solution to the Liar their accounts are very close to one another :

> « If we restrict our discourse to signs, three better known degrees can be singled out according to which a proposition can be true or false.
> [$\mathbf{T_0}$] In the first broadest sense, a proposition is true because it is a being, for being and true are convertible according to the philosophers.
> [$\mathbf{T_2}$*] [...] In a second, slightly narrower sense, a proposition is said to be true, because of the truth that it primarily signifies ; no matter whether that truth consists in [the proposition] itself, or something that depends on it, or [again] an entirely distinct being ;

secundum quid. Sed est falsa primo modo, quia ipsa non verificatur nisi pro supposito sue partis cuius ipsa est suppositum. Concludit ergo quod ille propositiones sunt plures 'hoc est falsum', 'hoc est verum' et sic de aliis. Quare, cum proponitur aliqua illarum, non est secundum unam responsionem respondendum sed dicendum quod hoc est falsum secundum primum membrum <divisionis> posite, et verum secundum aliud ». Cf. also Spade, *The Medieval Liar* cit., pp. 82-84 (*s.v.* Paul of Venice, L).

[53] See Spade, *The Medieval Liar* cit., pp. 52 (*s.v.* Anthony de Monte, XXVI). For a description and analysis of the content of the manuscript, see A. Maierù, *Le ms. Oxford* Canonici misc. 219 *et la* Logica *de Strode*, in Id. ed., *English Logic in Italy in the 14th and 15th Centuries. Acts of the 5th European Symposium on Medieval Logic and Semantics, Rome 10-14 November 1980*, Bibliopolis, Napoli 1982, pp. 87-110.

[54] See James, *Peter Alboini of Mantua* cit., p. 163.

and in this sense the following propositions are true: 'this proposition exists', 'this proposition signifies', 'this proposition Socrates sees', 'there is a god' and the like.
[**T_1**] [...] Third, in a specific sense a proposition is said to be true when it primarily has a *significatum* that is independent of the proposition itself, like the following: 'there is a god', 'the sun is moved'. And Aristotle spoke according to this sense when he said: "a proposition is true or false because the thing that the proposition primarily signifies is or is not, and not by virtue of a change occurring in the proposition". And the etimology agrees with this common way of understanding a proposition, because according to the former 'proposition' derives [its name] from 'positing [something] for something else'.
[...] It is clear from the above that something false in this sense [*i.e.* $F_1 = \neg T_1$] is true both in the first [i.e. T_0, which is not relevant here] and in the second sense [i.e. T_2*]. It is also clear that if something is true in the third sense [T_1], then it is true also in the second sense [T_2*], but not the other way around. On the basis of such premises, I say that all [propositions] that are commonly called insolubles are true as well as false »[55].

It should be noted that Wyclif understands the relationship between T_1 and T_2* to be one of extensional inclusion, i.e. all T_1 propositions are T_2* propositions, because T_2* propositions include all T_1 propositions, plus all propositions that are (*i*) self-referential and (*ii*) satisfy (Corr)[56].

In other words, the conditions for T_1 are the same as in Peter's case:

$$\boldsymbol{T_1(p)} \qquad \text{iff} \qquad (\mathbf{Corr}) \wedge \neg(\mathbf{R})$$

but those for T_2 are weaker, because in the end the only requirement is that the correspondence criterion be met. This is because, by definition,

[55] Wyclif, *Logicae continuatio* cit., vol. II, ch. VIII, pp. 204-205 « Sed restringendo sermonem ad signa notantur tres gradus famosiores quibus contingit proposicionem esse veram vel falsam. [T_0] Primo modo largissime est proposicio vera, quia ens; nam ens et verum secundum philosophos convertuntur. Et cum isto famoso modo intelligendi proposicionem concordat etymologia, qua 'proposicio' dicitur a 'pro alio posicio'. [T_2*] Secundo modo, paulo contraccius dicitur proposicio vera, propter veritatem quam primarie significat; sive ipsa veritas sit ipsamet, vel ab ipsa dependens, sive ens omnino distinctum; et isto modo sunt tales vere: 'hec proposicio est', 'hec proposicio significat', 'hanc proposicionem videt Sor' [assuming at least one of them to be self-referential], 'deus est', et similia. [T_1] [...] Sed tertio specialiter dicitur proposicio vera, quando habet primarie significatum independens ab ipsa, ut sunt tales: 'deus est', 'sol movetur', etc. Et isto modo locutus est Aristoteles de proposicione, dicens: "in eo quod res est vel non est, quam proposicio primarie significat, est ipsa vera vel falsa, et non propter mutationem factam in proposicione". [...] Et ex istis patet quod falsum isto modo est verum tam primo modo quam secundo. Patet eciam quod si quicquam est verum tertio modo, tunc est verum secundo modo; sed non econtra. Istis premissis, dico quod omnia vocata communiter insolubilia sunt tam vera quam falsa ». Cf. also Id., *Summa insolubilium* cit., pp. 5-8, for a parallel discussion.

[56] Obviously (Corr) ultimately depends on one's favourite theory of truth.

$$\boldsymbol{T_2^{*}(p)} \quad \text{iff} \quad (\boldsymbol{T_1(p)} \quad \vee \quad (\mathbf{Corr} \wedge \mathbf{R}))$$

$$\text{iff} \quad ((\mathbf{Corr} \wedge \neg\mathbf{R}) \quad \vee \quad (\mathbf{Corr} \wedge \mathbf{R}))$$

$$\text{iff} \quad ((\mathbf{Corr}) \quad \wedge \quad (\neg\mathbf{R} \vee \mathbf{R}))$$

$$\text{iff} \quad (\mathbf{Corr})$$

i.e., no matter whether p is self-referential or not, p will be $T_2{}^{*}$ if and only if p meets the correspondence criterion, i.e. if and only if what it says is the case. As for falsehood, on Wyclif's account, it is the same notion that we find in Peter, when it is the negation of T_1

$$\neg\boldsymbol{T_1(p)} \quad \text{iff} \quad \neg(\mathbf{Corr}) \vee (\mathbf{R}) \quad \text{iff} \quad \boldsymbol{F_1(p)}$$

but it must be understood in terms of a weaker condition when it comes to the negation of $T_2{}^{*}$

$$\neg\boldsymbol{T_2^{*}(p)} \quad \text{iff} \quad \neg(\mathbf{Corr}) \quad \text{iff} \quad \boldsymbol{F_2^{*}(p)}.$$ [57]

The solution to the paradox is, just as on Wyclif's account, to declare the Liar to be true and false, but according to different senses.

Now, before looking at how both accounts fail to do what they are supposed to, as they turn out to be inconsistent, it is worth considering the main logical difference that distinguishes them. Both can argue that the Liar is $\neg T_1$ but T_2 (or $T_2{}^{*}$) ; moreover, they can also both provide a justification for the claim that the Truth teller is $\neg T_1$ and $\neg T_2$ (or $\neg T_2{}^{*}$). They only depart from one another in their respective account of negation. Wyclif has only one type of negation, Peter must have two. By definition, according to Wyclif the following relations hold :

JW1. $\neg(T_2{}^{*}(p) \rightarrow T_1(p))$

i.e. it is not the case that if p is true in some respect, it is also true absolutely, because $\neg$ (Corr $\rightarrow$ (Corr $\wedge$ R)).

[57] Which means that in Wyclif's case, negation behaves in the same way on both levels, the difference being that at level 2 the only parameter under consideration is whether the correspondence criterion is or is not satisfied.

JW2. $T_1 (p) \rightarrow T_2^* (p)$

i.e. if p is true absolutely, then it is also true in some respect, because (Corr $\wedge$ R) $\rightarrow$ Corr.

By contraposition, from JW2 we can obtain

JW3. $\neg T_2^* (p) \rightarrow \neg T_1 (p)$

i.e. if p is not true in some respect, then it is not true absolutely, because $\neg$Corr $\rightarrow$ ($\neg$Corr $\vee$ R). Theses JW2 and JW3, on Wyclif's account are equivalent.

By contrast, on Peter's account the following relations hold:

PM1. $\neg(T_2 (p) \rightarrow T_1 (p))$

as for Wyclif, it is not the case that if p is true in some respect, then it is true absolutely, because $\neg$ ((Corr $\wedge$ R) $\rightarrow$ (Corr $\wedge$ $\neg$R)).

PM2. $\neg(T_1(p) \rightarrow T_2 (p))$

But on the other hand, nor is it the case that, if p is true absolutely, then it is also true in some respect, because $\neg$ ((Corr $\wedge$ $\neg$R) $\rightarrow$ (Corr $\wedge$ R)).

PM3. $\neg T_2 (p) \rightarrow \neg T_1 (p)$

If it is not the case that p is true in some respect, must it be false absolutely? According to Peter it must, because by definition, being not true in some respect means satisfying both conditions required for being $\neg T_1$, i.e.

$$
\begin{aligned}
(\neg\text{Corr} \wedge \text{R}) &\rightarrow \neg(\text{Corr} \wedge \neg\text{R}) \\
&\rightarrow (\neg\ \text{Corr} \vee \text{R})
\end{aligned}
$$

But then, one might argue that after all from PM3 by contraposition we can derive

PM4. $T_1 (p) \rightarrow T_2 (p)$

which is not compatible with Peter's account. I believe the point at stake here is precisely that two different kinds of negation are operating on Peter's view. Standard negation would require us to apply De Morgan's laws to the conditions of $\neg T_2(p)$ and $\neg T_1(p)$ in

PM3. $\neg T_2(p) \rightarrow \neg T_1(p)$

and obtain

PM4. $T_1(p) \rightarrow T_2(p)$

Only the move from the consequent of PM3 to the antecedent of PM4, however, is legitimate, because standard negation applied to $\neg T_1(p)$ in fact yields $T_1(p)$. As for the other transformation, one cannot simply deny $\neg T_2(p)$ to obtain $T_2(p)$ because the two share a condition, namely that the proposition be self-referential. In which case, by restricting ourselves to the domain of self-referential propositions, we would restore perfect duality, but at the cost of going back to Wyclif's framework. It remains therefore unclear what progress Peter's approach is supposed to achieve.

Be this as it may, as I have already said, both projects are inevitably destined to fail. The fact has been noted already in Wyclif's case, and the argument applies also to Peter's approach, and it does not seem that their divergence on negation might be of any help in working out an alternative solution to save either of them.

3.4 Revenge

Peter of Mantua's solution is to claim that A, the Liar, says of itself that it is not true absolutely, and since it is in fact not true absolutely (because it is a self-referential proposition), what it says is the case, which makes it therefore true in some respect, according to the definitions of the two notions given above. If this solution might have a superficial appeal, because it establishes the truth value of the Liar without thereby committing itself to the paradox, it soon becomes clear that its success is not much of an advance. For it is not immune from the so-called revenge problem.

Let us assume that T stands for $T_1 \vee T_2$. $T(p)$ means that p is either true absolutely (correspondence criterion met without self-reference) *or* true in some respect (correspondence criterion met with self-reference). What happens if the paradox is proposed anew in the form $\neg T(\mathrm{A})$? It arises again, and leads us back to the original formulation. At this stage, however, there does not seem to be a way around it.

1st leg of the paradox

(1)** $T(\mathrm{A})$
(2)** $T(\mathrm{A}) \rightarrow \exists p\ (Sig\ (\mathrm{A}, p) \wedge p)$

(3)** $Sig\ (\mathrm{A}, \neg T(\mathrm{A}))$
(4)** $\neg T(\mathrm{A})$

2nd leg of the paradox

(5)** $\neg T(\mathrm{A})$
(6)** $\exists p\ (Sig\ (\mathrm{A}, p) \wedge p) \rightarrow T(\mathrm{A})$
(7)** $Sig\ (\mathrm{A}, \neg T(\mathrm{A}))$
(8)** $T(\mathrm{A})$

From a logical point of view, what is going on here (as well as in Wyclif's case, which is open to the same kind of criticism) is that we are entitled to consider the two distinct definitions modulo self-reflection. Thus, rephrasing the paradox as above is equivalent to asking of a given proposition A = « 'A' fails to meet its own correspondence criterion » whether A fails to meet its own correspondence criterion. But then of course, if it does, it does not ; and if it does not, it does. The paradox rises like a phoenix from the ashes. One might be tempted to think that revenge occurs even if we confine ourselves to rephrasing the paradox in terms of the new truth predicate T_2 (or T_2*), i.e. by asking whether A = $\neg T_2(\mathrm{A})$ is true or false[58]

1st leg of the paradox

(1*)** $T_2(\mathrm{A})$
(2*)** $T_2(\mathrm{A}) \rightarrow \exists p\ (Sig\ (\mathrm{A}, p) \wedge p)$
(3*)** $Sig\ (\mathrm{A}, \neg T_2(\mathrm{A}))$
(4*)** $\neg T_2(\mathrm{A})$

2nd leg of the paradox

(5*)** $\neg T_2(\mathrm{A})$
(6*)** $\exists p\ (Sig\ (\mathrm{A}, p) \wedge p) \rightarrow T(\mathrm{A})$

[58] This seems to be, in particular, Paul of Venice's approach. Although awareness of the revenge problem dates at least as far back as Bradwardine (part of a section of his treatise is devoted to this family of problems, see Bradwardine, *Insolubilia* cit., ch. 7), Paul of Venice addresses this particular version proposed by Peter of Mantua, see Paul of Venice, *Tractatus de insolubilibus* cit., ff. 193^{vb}-194^{ra} « Sed hec declaratio non solvit insolubilia sed potius se involvit. Nam capio 'verum' et 'falsum' secundo modo et probo quod sic sumendo hec est falsa 'hoc est falsum', se demonstrato. Nam si ipsa sit vera et significet adequate hoc esse falsum, igitur verum est hoc esse falsum. Consequentia tenet apud eum ; et ultra verum est hoc esse falsum ; igitur hoc est falsum, sic sumendo. Et sic habeo quod idem est verum et falsum secundo modo dicto [*pro* dictis], quod ipse negat ».

(7*)** $Sig\ (\mathrm{A}, \neg T_2(\mathrm{A}))$
(8*)** $T(\mathrm{A})$

In order to generate the paradox, what is needed here is in fact a much more restricted (and false) principle, i.e. that whenever *p* signifies something that is the case, *p* is true in some respect. This is obviously not the case, because in most ordinary situations — i.e. when it is not self-referential —, whenever *p* signifies something that is the case, *p* is true absolutely. Yet, since here A is indeed self-referential, it cannot be true absolutely. Therefore, we would still have a contradiction, because from (8***), (5***) and disjunctive syllogism, we could prove T_1(A). But A is self-referential and cannot in principle be true absolutely[59]. The two revenge arguments both apply in Wyclif's case because his notion of truth in some respect in fact coincides with correspondence. Peter might have a little advantage over the second formulation, but he would still have to find a way out of the first.

4. Conclusion

Paradoxes are a rather natural context for the development of discussions concerning truth, already in the framework of medieval logic. It is probably no coincidence that a rigorous characterisation — or at least the strive for a rigorous characterisation — of the notion of truth, and the systematic development of formal theories of truth in post-Fregean logic, is closely connected to the discovery or re-discovery of a variety of paradoxes between the end of the XIXth and the first half of the XXth century. In this respect, even if trying to find anything comparable to such a systematic modern attempt in medieval logic would be probably a slightly optimistic endeavour, the conceptual analysis of logical paradoxes, already in that context, prompted considerable efforts and the development of a remarkable number of different solutions. Then, more or less as nowadays, there were theorists who believed the actual truth value of Liar-like propositions could be determined and yet paradox could be avoided ; others who sought a compromise (true in one sense, but not in another, or according to contextual parameters) ; people who tried to push the problem a level further (rule out certain types of propositions, refuse to acknowledge meaningfulness to the proposition), people who claimed the paradox is semantically overdetermined (both

[59] The reason of this complication, which might be even regarded as an advantage of Peter's view over Wyclif's, lies in their different criteria for $\neg T_2$ and $\neg T_2$*. On Wyclif's account, rephrasing the paradox in the way presented in this section (A fails to meet its own correspondence criterion) or in terms of the second sense of 'true' (A is not true in some respect) has the same result, because T_2* and $\neg T_2$* are symmetrically defined in terms of correspondence vs non-correspondence. I wonder whether on this basis Peter might defend himself in a better way at least against a formulation of the revenge problem that makes use of $\neg T_2$ (as opposed to Wyclif's $\neg T_2$*) only.

true and false) or underdetermined (neither true nor false). In this paper I have tried to show how some of these approaches interact with one another, by putting them into perspective from the standpoint of a late XIVth century logician. It turned out that a particularly relevant theoretical position is taken (albeit in significantly different ways and, for that matter, with varying degrees of success) by a number of logicians such as Bradwardine, Buridan, Albert of Saxony and William Heytesbury. All of them try to solve the paradox with similar tools : in particular by adopting certain assumptions — in a restricted or unrestricted manner — on the signification of propositions, and by suitably refining their definition of truth. The last two are especially important for understanding the context of Peter of Mantua's treatise.

His work on *insolubilia* puts forward a solution that is strongly reminiscent of the traditional *secundum quid* and *simpliciter* approach, although such a characterisation embraces a broad variety of alternative accounts. The distinction between two senses of 'true', which applies to propositions according to their syntactic structure and what ultimately determines their semantic value, seems to foreshadow, as it were, a remote distinction between grounded and ungrounded propositions. It does, however, not reach much further than that. As noted above, it is understandable as a weak (proto-) hierarchical solution. Hierarchical because it attempts to solve the paradox by introducing a new sense of 'true' intended to provide a further semantic discrimination for propositions that would otherwise simply be regarded as false (some of these are still false even in the second sense, when they fail to signify things as they are ; but if they do signify things as they are, they are true at least in the second sense). Weak because it introduces a new truth predicate only for a restricted class of propositions, namely self-referential ones. Most propositions talk about the world and are unproblematic. They are either true or false in absolute terms. Some other propositions self-refer, in which case they are strictly speaking always false. Yet, according to whether what they say is the case or not, they receive an additional truth value one level up and are either said to be true in some respect or false in some respect. This allows to solve a very basic version of the paradox, but as long as the new truth predicates are rearranged in a suitable way, the contradiction resurfaces immediately.

I am doubtful about whether Peter's own version of the *secundum quid* and *simpliciter* solution can be saved. A way out could be to open the hierarchy upwards, by dropping the intuitive syntactic justification that led to the introduction of the second truth predicate. One would have to adjust the definition of truth and the introduction of each additional predicate would no longer have an intuitive justification : all propositions that have a truth value at level 1, retain that truth value throughout. Some propositions that are false at level 1, can be true at level 2, like for example, in the Liar case, A, which is $\neg T_1$ and T_2. As we have seen, if we ask of A = $\neg T_2$(A) whether it is true or not, we might run into difficulties, if the set of truth values includes only T_1 and T_2. What if we did not have such a limit ? We might tentatively want to claim that A is $\neg T_1$ and $\neg T_2$ but T_3 and redefine truth as T(A) $\leftrightarrow \exists p$ (*Sig* (A, p) $\wedge$ p), where T is ($T_1 \vee T_2 \vee T_3$). The only condition that all truth predicates would have to have in common is correspondence. This would leave open the most general formulation of the paradox once we deny T of A, but it might avoid each singular instance. Thus, for example, A = $\neg T_2$(A) is $\neg T_1$ and also $\neg T_2$, stipulating that whenever a proposition denies of itself the n-th truth predicate at level n it is $\neg T_n$ (for any n greater than 1). But this would mean, generally, that since this is what the proposition says, at the next n+1-th level, the proposition is T_{n+1}, and the definition of truth at n is T(A) $\leftrightarrow \exists p$ (*Sig* (A, p) $\wedge$ p), where T is ($T_1 \vee \ldots \vee T_{n-1} \vee T_n$).

Be this as it may, two further questions, more closely related to Peter's immediate theoretical concerns, deserve to be raised. First, in what way, if any, does the treatment of the Liar affect the notion of logical consequence (validity) that Peter endorses in his *Logica*. Secondly, how does the solution proposed here interact with his general account of truth ? These questions will have to remain open for the time being, but I believe that Peter's effort in rejecting both Albert of Saxony's and William Heytesbury's views on insolubles goes far beyond the mere fact that these two authors happen to be relevant sources for Peter's work. The ultimate target seems to be, in both cases, the characterisation of truth in terms of different *howsoever*-clauses. It will be interesting to explore how this notion is employed in the account of consequences and in the account of truth. This, however, will have to wait for next round.

ABSTRACT

This paper offers an analysis of a hitherto neglected text on insoluble propositions dating from the late XIVth century and puts it into perspective within the context of the contemporary debate concerning semantic paradoxes. The author of the text is the Italian logician Peter of Mantua (d. 1399/1400). The treatise is relevant both from a theoretical and from a historical standpoint. By appealing to a distinction between two senses in which propositions are said to be true, it offers an unusual solution to the paradox, but in a traditional spirit that contrasts a number of trends prevailing in the XIVth century. It also counts as a remarkable piece of evidence for the reconstruction of the reception of English logic in Italy, as it is inspired by the views of John Wyclif. Three approaches addressing the Liar paradox (Albert of Saxony, William Heytesbury and a version of strong restrictionism) are first criticised by Peter of Mantua, before he presents his own alternative solution. The latter seems to have a *prima facie* intuitive justification, but is in fact acceptable only on a very restricted understanding, since its generalisation is subject to the so-called revenge problem.

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