Logique & Analyse **183–184** (2003), 447–478

A CRITICAL EXPOSITION OF ISAAC LEVI'S EPISTEMOLOGY

ALLARD TAMMINGA

Abstract

The branch of philosophical logic which has become known as "belief change" has, in the course of its development, become alienated from its epistemological origins. However, as formal criteria do not suffice to defend a principled choice between competing systems for belief change, we do need to take their epistemological embedding into account. Here, on the basis of a detailed examination of Isaac Levi's epistemology, we argue for a new direction of belief change research and propose to construct systems for belief change that can do without, but do not rule out, selection functions, in order to enable an *empirical* assessment of the relative merits of competing belief change systems.

1. *Isaac Levi's Epistemology*

Isaac Levi, professor at the Columbia University of New York since 1970, puts forward his epistemological tenets in a voluminous series of publications, using a rather uncommon terminology based on logic and probability theory. Levi defends a radical new perspective on a number of traditional epistemological issues, while firmly re-establishing the bond between logic and epistemology, which has been increasingly loosening during the past forty years. Furthermore, Levi is the main pioneer and initiator of contemporary logical-philosophical research into *belief change*. Levi places himself explicitly within the tradition of American pragmatism. Accordingly, some of the keynotes of Levi's epistemology can only be understood properly when placed against the background of American pragmatism.

1.1. *Non-Deductive Logic and the* Belief-Doubt-Belief *Model*

In January 1872, a learned society of lawyers and scientists founded the 'Metaphysical Club', the cradle of American pragmatism. The philosophically concerned members of the club set out to combine the work of the

British psychologist Alexander Bain, who defined "belief" as "that upon which a man is prepared to act", with considerations concerning the philosophy of law and the theory of evolution in order to arrive at an overall theory of human thinking. From the very beginning, the members of the club, among which William James (1842–1910) and, albeit much later, Charles Sanders Peirce (1839–1914) would gather most laurels, rejected the ubiquitous tenet that true knowledge is to be modelled after mathematics.[1]

One of the results of this heresy, which no American pragmatist would ever repudiate, was that the field of logic, as opposed to the logical inquiries of Frege and the early Russell, came to include not only mathematical reasoning, but, in Peirce's words, "the method of scientific investigation" as well. John Dewey (1859–1952) later explicitly advocated an even wider domain for logic: the very logic of inquiry which leads to scientific results was also the methodological backbone of 'common sense'. Levi, whose *continuity thesis* owes much to the American pragmatist tradition, is of the opinion that from a methodological point of view, the same mechanisms underlie both "scientific inquiry" and "practical deliberation":[2]

> The difference between theoretical inquiry and practical deliberation is a difference in goals and not a difference in the criteria for rational choice that regulate efforts to realize these goals. [Levi 1980, p. 73]

The first consequence of this wider interpretation of the field of methodological inquiry was that the pragmatists, beginning with Peirce, placed non-deductive reasoning, such as induction and abduction, at a central place of logic. However, attempts of early American pragmatists to characterize non-deductive inferences remained informal until the publication of Dewey's methodological study *Logic: The Theory of Inquiry* in 1938. By that time, the necessary groundwork had been done in Europe. Bertrand Russell and Rudolf Carnap grappled in vain with formal solutions to justify, among others, generalizations: Russell's *Our Knowledge of the External World* (1914) en Carnap's *Der logische Aufbau der Welt* (1928) showed the failure of the endeavour to uncover the mechanisms by which we acquire knowledge of the

[1] The lawyer Oliver Wendell Holmes Jr., one of the six key members of the Metaphysical Club, writes in his *The Common Law* from 1881: "The law embodies the story of a nation's development through many centuries and it cannot be dealt with as if it contained only the axioms and corollaries of a book of mathematics. In order to know what it is we must know what it has been, and what it tends to become." Quoted in [Kuklick 1977, p. 50–51].

[2] Levi sees the continuity thesis as the pivot of pragmatism: "What is 'pragmatic' about pragmatism is the recognition of a common structure to practical deliberation and cognitive inquiry in spite of the diversity of aims and values that may be promoted in diverse deliberations and inquiries." [Levi 1991, p. 78.]

world on the basis of observations and making use of the then recent instruments of mathematical logic and set theory. It emerged that the conditions under which tentative extensions of our beliefs, such as for example inductive inferences, are justified, could not be tackled with mathematical logic and set theory. In 1950, Carnap, who had been teaching at the University of Chicago since 1936, presented in his *Logical Foundations of Probability* an epoch-making treatment of non-deductive reasoning based on probability theory: inductive logic was born. Isaac Levi was one among many working within this philosophical research program which was especially strong during the seventies. The monograph *Gambling with Truth* (1967) contains the results of Levi's rather idiosyncratic research into inductive logic.

The rejection of the axiomatic ideal as a standard for all knowledge[3] in combination with the acceptance of an evolutionist perspective, also had a second consequence. While many epistemologists, including logical empiricists, focused their attention mainly on the justification of the *results* of the acquisition of knowledge, concentrating on the rational reduction of these results to their origins (basic principles, *Protokollsätze*, or sensory stimuli), the American pragmatists chose a different perspective. Mainly through their agency, the process of *belief change* became a respectable subject for epistemological study. In Levi's opinion, *pedigree epistemology* — Levi's condescending expression for the epistemological enterprise of justifying our beliefs by tracing them back to their origins by means of a rational reconstruction — has proven to be a dead end. As an alternative program, Levi suggests to investigate under which circumstances a change of our current state of knowledge is justified:

> Whatever its origins, human knowledge is subject to change. In scientific inquiry, men seek to change it for the better. Epistemologists ought to care for the improvement of knowledge rather than its pedigree. [Levi 1980, p. 1]

Levi, an advocate and pioneer of a normative approach in which adjustments of epistemic states are investigated using a *logical* apparatus, aims at

[3] Levi writes: "Following the tradition of Peirce and Dewey, I reject the requirement of self-certified first premises and principles for justifications of belief." [Levi 1991, p. 4.]

formulating criteria under which a *change* of knowledge is also an *improvement*. It is not Levi's aim to describe how our knowledge actually changes,[4] but how it reasonably *should* change:

> The central problem of epistemology ought to be [...] to provide a systematic account of criteria for the improvement of knowledge. Alternatively stated, the problem is to offer a systematic characterization of conditions under which alterations in a corpus of knowledge are legitimate or are justified. [Levi 1976, p. 1]

Although Levi was not the first to put knowledge change on the logical agenda, he is certainly the main initiator of its systematic investigation. Levi's philosophical forebears Peirce and Dewey already propagated a dynamic approach to knowledge with their *belief-doubt-belief* model, which should be understood as the first attempt at a logical description of the process of knowledge change. This model, according to Levi "the greatest insight in the pragmatist tradition" [Levi 1991, p. 163], can be broadly outlined as follows: our actual state of knowledge forms a pattern of expectations, on which we base our actions. As long as we have no reasonable cause to doubt (parts of) this current epistemic state, it makes no sense to feign some Cartesian doubt, since after all there is "much that you do not doubt, in the least. Now that which you do not at all doubt, you must and do regard as infallible, absolute truth" (Peirce, *CP* 5.416). Nevertheless, change is sometimes required. An experience "which really interferes with the smooth working of the belief-habit" (Peirce, *CP* 5.510), and which therefore, unlike an academic doubt, causes *true* doubt, shakes our opinions so that an inquiry becomes necessary to re-establish the lost equilibrium and to overcome the doubt caused by this unforeseen experience. This inquiry will lead to a new state of belief which, since it is the result of a procedure executed in accordance with the rules of methodology, will constitute a firm and sufficient basis for our thoughts and deeds, until an unforeseen experience forces us again to an inquiry. While Peirce and Dewey focussed on a meticulous

[4] Since Thomas Kuhn's *The Structure of Scientific Revolutions* (1962), in which he defends a "new historiography of science", it is accepted practice to explain belief changes in terms of 'revolutionary' paradigm shifts. Context independent rational factors are supposed to only play a marginal part in the explanation of paradigm shifts, since paradigms before and after a scientific revolution appear to be incommensurable. Levi underplays the significance of these "changes in conceptual framework" — "there are no revolutionary changes or, at any rate, there should not be" [Levi 1980, p. 68] — and actually wants to investigate, *given* a conceptual framework, into the criteria on the basis of which our beliefs ought to be changed and improved. Within such a conceptual framework all epistemic states are commensurable [Levi 1991, p. 65].

investigation of the criteria for knowledge change,[5] some of Levi's other predecessors achieved an informal description at most.

Five years after the publication of Levi's *The Enterprise of Knowledge* (1980), in which formal criteria for rational revisions of epistemic states are defended, Alchourrón, Gärdenfors and Makinson published a very elegant formalization of some of the logical prerequisites for Levi's ideas on knowledge change: the harmonious overture to what has by now built up to a discipline of symphonic proportions within philosophical logic, a discipline that is designated by the terms "belief revision", "belief change" and "theory change".[6]

In the past twenty years, logical research into *belief change* has boomed, producing a considerable amount and diversity of formal systems. Although there have been elegant and 'deep' results and successful attempts at partial systematization,[7] the reasons and motives for the construction of many systems frequently leave much to be desired: often one tiny little problem gives rise to yet another new system.[8] As Levi is not only an initiator of research into *belief change*, but also its most philosophical advocate — Levi is almost the only one to propound a philosophical embedding for formal theories of belief change —, we will subject Levi's proposals for the modelling of belief change to a critical examination, hoping that the conclusions that will be reached can be extrapolated to competing systems for *belief change*, so that we can get a clearer picture of this branch of logico-philosophical research.

[5] Dewey's activities in the field of logic cover his whole philosophical career and find their culmination in his work *Logic: The Theory of Inquiry* (1938). His earlier logical-philosophical investigations are embodied in the *Studies in Logical Theory* (1903), in his more pedagogically oriented study *How We Think* (1910), and in the *Essays in Experimental Logic* (1916). [Burke 1994] gives a survey and a defence of Dewey's studies in logic.

[6] See [Alchourrón, Gärdenfors, and Makinson 1985]. For an introduction, see [Gärdenfors 1988] and [Hansson 1999a].

[7] See [Rott 1991], [Rott 1992], and [Rott and Pagnucco 1999].

[8] See for instance [Fermé and Hansson 1999], in which the system, that, unlike other systems of *belief change*, is able to accept a *part* of the new information, is illustrated with the following example: "One day when you return back from work, your son tells you, as soon as you see him: 'A dinosaur has broke grandma's vase in the living-room'. You probably accept one part of the information, namely that the vase has been broken, while rejecting the part of it that refers to a dinosaur" [Fermé and Hansson 1999, p. 331].

1.2. *Epistemic States and Their Representations*

Before we actually can write down anything sensible about the mechanisms of belief change, we must first know *what* is supposed to change. Levi distinguishes between *epistemic states* ("states of full belief"), our true states of knowledge on the one hand, and *representations of epistemic states* ("corpora") on the other. Although in his *The Fixation of Belief and Its Undoing* (1991), Levi has gone to many lengths to define and defend his preferred notion of "epistemic state", we shall adopt Levi's own policies for treating contraction and conditionals, and concentrate on his proposals concerning the *representations* of epistemic states as well as their dynamics.

It goes without saying that a choice for a certain type of representation of epistemic states has far-reaching consequences. Although there is some contention about the most suitable kinds of representation,[9] the similarities between the positions defended by logicians overrule the differences. Most protagonists, including Levi, share the presupposition that epistemic states should be represented by *structured sets of descriptive sentences*.

Why has this idea been such a resounding success? The promotion of this tendency to represent our knowledge as a structured set of descriptive sentences was mainly due to the leading role that philosophy of science, which was grafted on neo-positivism, played in the development of epistemology in the twentieth century. Originally, the members of the Vienna Circle subscribed to the traditional ideal of knowledge, although they gradually gave up the requirement to provide a *Begründung* of science. Nevertheless, the Aristotelian ideal of knowledge[10] has in its fall not dragged down the idea that our knowledge ultimately forms a coherent whole of descriptive sentences. The fall of the Aristotelian ideal has only fundamentally changed this idea. This coherence consists and consisted of 'inferential' relations between descriptive sentences. During the twentieth century, both the propositional attitude towards sentences included in a system of knowledge and their inferential relations have been reassessed. If it was formerly thought that all *true sentences* could be assigned a specific place in the fabric of our knowledge by working out which axioms and theorems (or in the case of logical empiricists such as Moritz Schlick, which *Konstatierungen*) were needed to justify them with the help of the canons of reasoning, now we give preference to the metaphor of a *web of belief* in which *sentences which are held to be true* are ordered according to their "relative likelihood, in practice, of

[9] For an already somewhat dated survey, see [Gärdenfors 1988, p. 21–46].

[10] See [Beth 1959, p. 31–32] for a detailed discussion.

our choosing one statement rather than another for revision in the event of recalcitrant experience".[11]

Secondly, there is the *holistic* approach of meaning and cognition, propagated by Quine in particular,[12] which has led many epistemologists to be inclined to think that an epistemic state is an idealization of our 'theory of the world', where, obviously, a theory consists of descriptive sentences. The logical empiricist ideal of an *Einheitswissenschaft* has only reinforced that inclination. So, Quine speaks of "[t]he totality of our so-called knowledge or beliefs", "our own particular world-theory", and "total science".[13]

So it is only natural that Levi proposes to represent the knowledge of a certain agent X that can be expressed in some (formal) language L as a deductively closed set K of sentences in L.[14] Levi calls such a deductively closed set of sentences a *corpus*. The corpus is closed under deduction, since it does not only contain the beliefs of which X is aware he has them, but the beliefs to which X is logically committed as well. Within a corpus two classes of sentences can be distinguished.

In the first place, a corpus contains sentences that X will not give up under any circumstance. For instance, those sentences that articulate the metaphysical, ontological and (classical) logical presuppositions to which X is committed. Levi labels the set of sentences which, at least for X, do not qualify for revision, as X's *urcorpus*. It forms the kernel of each corpus of X and includes at least the criteria on the basis of which proposals for changing X's corpus are to be judged. So the *urcorpus* consists at least of "those

[11] [Quine 1953, p. 43.] In recent investigations into *belief change* we come across a similar idea under the name "epistemic entrenchment". See among others [Gärdenfors and Makinson 1988], [Rott 1991], and [Levi 1991, § 4.7].

[12] In his famous 'Two Dogma's of Empiricism', Quine discredited the assumption that each meaningful sentence is equivalent to a logical-mathematical construct of observational terms, an assumption that was shared by Peirce and most logical empiricists. Instead, Quine proposes to consider *theories* instead of sentences as the primary carriers of meaning, and then to try to establish the empirical meaning of theories on the basis of the relations between theories and empirical data. Analogously, Levi stresses that "in the first instance it is not sentences or other linguistic entities that carry truth value and informational value but potential states of full belief" [Levi 1996, p. 53].

[13] [Quine 1953, p. 42]; [Quine 1960, p. 24]; and [Quine 1953, p. 42].

[14] Levi assumes that L is rich enough to express arithmetic and set theory and that it complies with (classical) first-order logic. However, L is not able to describe itself, nor does it contain modal operators to express physical (im)possibilities. For technical reasons, L does not contain conditionals either. Furthermore, L should not be understood as the language "that the agent uses or would use to communicate his convictions or other attitudes" [Levi 1991, p. 33].

assumptions which any corpus should have if an account of the revision of knowledge [...] is to stand a chance of working" [Levi 1980, p. 7]. In addition to (classical) logic, mathematics and set theory, the *urcorpus* contains a "conception of error" [Levi 1980, p. 8] and, I assume, though Levi is silent on this matter, a "conception of informational value", since Levi's criteria for belief change cannot be formulated without this latter conception.

In the second place, X's corpus contains sentences of which X does not rule out that they might one day qualify for revision, though they are, at present, infallible.[15] These sentences may be (negations of) singular statements, but also laws,[16] theories, and statistical claims. The fact that X considers a statement ϕ in his corpus at time t to be susceptible to revision, does not alter the degree to which ϕ is considered probable by X:

> From X's point of view at t, every theoretical assumption, statistical claim, universal generalization and observation report in his corpus at t is as certainly and necessarily true as any truth of logic — at least as far as the conduct of practical deliberations and scientific inquiry are concerned. [Levi 1976, p. 24]

If we now follow Levi's suggestion and represent the "credal probability" that X attributes to sentences by a function Q complying with the standard axioms of probability theory, such that Q is defined for all sentences in the language L, the above can be summed up as follows: for all sentences ϕ in X's corpus K it holds that $Q(\phi) = 1$.

Our current corpus is, according to Levi, the only standard for what we, at least for the time being, hold possible. *Logical* possibilities form much too large a class for a workable concept of possibility:

> It seems clear that in daily life and scientific inquiry, we discount utterly all sorts of logical possibilities. We do not assign them small probabilities of being true. [Levi 1976, p. 12]

Furthermore, it is for Levi a "prima facie obvious fact" that, in scientific inquiry and practical deliberation, X must consider *all* the elements from his present corpus to be certain and infallible. The tenets that our current corpus determines what we hold possible and that we consider all the elements

[15] See the quote of Peirce's (*CP* 5.416) on page 4 of this paper.

[16] In his *For the Sake of the Argument* (1996), Levi proposes to interpret conditionals along the lines of Ramsey's Test, using belief change techniques. On the basis of Gärdenfors' Triviality Theorem, Levi refuses to admit conditionals as elements of corpora. Usually, scientific knowledge is of a conditional nature, starting with "Water boils at 100 degrees Celsius". Hence, it is far from clear which laws *can* be admitted to a corpus.

from our current corpus to be certain and infallible are defended on the basis of Levi's definition of the concept "serious possibility": a sentence ϕ is a serious possibility with respect to a corpus K if and only if ϕ is consistent with K [Levi 1980, p. 5]. A corollary of this interpretation of possibility is that each element ϕ in K is necessary and therefore infallible, since $\neg\phi$ is not a serious possibility with respect to K. Levi sums up both claims with his thesis of *epistemological infallibilism* [Levi 1980, p. 13]. In short,

> X is committed to treating all items in the corpus of knowledge he adopts at t as infallibly true in the sense that the logical possibility that one of the items is false is not, as far as he is concerned, a serious one. [Levi 1976, p. 7]

Epistemological infallibility, however, does not imply that our current corpus K, our one and only standard for serious possibility, is impervious to deliberate change: "Certainty does not imply incorrigibility" [Levi 1991, p. 3]. With good reason, our current corpus can be changed and improved in order to arrive at another corpus K' which then will become our one and only standard for serious possibility. In short, knowledge is corrigible, even though we consider it to be infallible when we have no reason to change it.

In his 'Knowledge and Belief' from 1952, in which he provides a reinterpretation of the traditional distinction between 'knowledge' and 'belief', Norman Malcolm uses a concept of knowledge closely related to Levi's. The omission of the latter to breathe life into his rather formal conception of knowledge with a number of convincing illustrations is compensated by the five real-life examples which Malcolm puts forward to make clear that the answer to the question "Can I discover *in myself* whether I know something or merely believe it?" [Malcolm 1952, p. 69] must be in the negative.[17] Let us take a closer look at two of Malcolm's examples:

> Suppose, for example, that several of us intend to go for a walk and that you propose that we walk in Cascadilla Gorge. I protest that I should like to walk beside a flowing stream and that at this season the gorge is probably dry. Consider the following cases: [...]
>
> (4) You say "I know it won't be dry" and give a stronger reason, e.g., "I saw a lot of water flowing in the gorge when I passed it this morning". If we went and found water, there would be no hesitation at all in saying that you knew. [...]

[17] Levi writes: "In my opinion, there is no relevant difference, from X's point of view at t, between what he knows and what he fully believes" [Levi 1976, p. 5]. On the relation between 'knowledge' and 'full belief', see also [Levi 1991, p. 45].

> (5) Everything happens as in (4), except that upon going to the gorge we find it to be dry. We should not say that you knew, but that you *believed* that there would be water. And this is true even though you declared you knew, and even though your evidence was the same as it was in case (4) in which you did know. [Malcolm 1952, p. 69–70]

According to Malcolm, these examples show that "*although you knew you could have been mistaken*" [Malcolm 1952, p. 71]. Malcolm thinks that it is surely possible for a statement that we consider to be an 'absolute certainty' at present, for example "There is a heart in my body", to turn out to be false on closer examination and, hence, to be eligible for correction [Malcolm 1952, p. 76]. So, absolute certain knowledge and corrigibility are not mutually exclusive. That's all very well, but when can a change of our absolute certain knowledge be called an improvement?

1.3. *Revision of Corpora*

Now that we have represented an epistemic state by a corpus — a deductively closed set of sentences from a language L — we can start thinking of belief changes or revisions of corpora in terms of "shifts from one deductively closed set to another" [Levi 1976, p. 23]. Levi distinguishes two fundamental types of revision, namely *expansion* and *contraction*. He claims that these types are fundamental because all other kinds of revision of corpora can be understood as a series of expansions and contractions [Levi 1980, p. 65].[18] Subsequently, Levi concentrates on articulating the conditions under which these two basic types of revision are justified, starting from the following consideration:

> The kind of cognitive aim that, in my opinion, does best in rationalizing scientific practice is one that seeks, on the one hand, to avoid error and, on the other, to obtain valuable information. [Levi 1996, p. 51]

Consequently, the starting-point of Levi's ideas on belief change is formed by the twin concepts of "informational value" and "credal probability".

1.3.1. *Expansion*

In expansion, a sentence ϕ is added to a corpus K. For the sake of convenience we will denote the result of such an operation by "$K + \phi$". Logically

[18] Consequently, in belief change literature, *revision* is usually defined in terms of a *contraction* and an *expansion*: $K \times \phi = (K - \neg\phi) + \phi$. This definition of revision is called the *Levi identity*. See [Gärdenfors 1988, p. 69].

speaking, an expansion does not amount to much: just take the union of the sets K and $\{\phi\}$ and close that union under deduction. In short,

$$K + \phi \;=\; \mathit{Cn}(K \cup \{\phi\}).$$

It is plain that this definition does not answer the question under which conditions an expansion is an *improvement* of our corpus. It only indicates how we should change our current corpus once we have decided to expand it with the sentence ϕ. It tells us nothing about the reasonableness of such a decision. Unlike the great majority of researchers in the field of *belief change*, Levi formulates up a standard on the basis of which the legitimacy of the decision to implement an expansion can be judged.[19] Most of Levi's ideas on expansion stem from his *Gambling with Truth*.

Levi distinguishes two types of expansion, namely *deliberate expansion* and *routine expansion*. Both types are necessary to acquire new information. In routine expansion, an external stimulus is converted into a sentence via a previously adopted 'program'. The resulting sentence is then indiscriminately added to the corpus of the agent — what Levi has in mind here is making observations or consulting a witness or an expert. Although we only accept a program for routine expansion if we consider it to be reliable, a hundred percent reliability is an unreasonable demand. Therefore, an accepted program can inject information into our current corpus which is inconsistent with our current corpus, whereby the corpus resulting from the expansion becomes inconsistent and, hence, trivial, since Levi closes corpora under classical logic. In short, a routine expansion implemented according to the rules can unintentionally lead to the inconsistent corpus.[20] Further on, we shall see that a correctly implemented deliberate expansion does not suffer from this deficiency.[21]

[19] Friedman and Halpern rightly complain about the fact that in the bulk of belief change literature no-one takes the trouble to investigate into the conditions under which the addition of a sentence to an epistemic state is legitimate, though "deciding when a formula has come to be accepted is nontrivial. [...] Acceptance has a complex interaction with what is already believed" [Friedman and Halpern 1999, p. 404]. Levi notes the same shortcoming: "The absence of an account of the conditions under which expansion is justified is a serious lacuna in a theory of rational belief change" [Levi 1996, p. 6]. See also Levi's remarks on Gärdenfors's work on expansion [Levi 1991, p. 44 and § 3.6].

[20] See for a more detailed discussion of routine expansion [Levi 1991, § 3.4].

[21] "[I]f one is living up to one's commitments, one cannot legitimately expand into inconsistency via deliberate expansion. On the other hand, routine expansion can and sometimes does lead to inconsistency even when all commitments are fully met" [Levi 1991, p. 76].

In deliberate expansion, an agent chooses one sentence from a series of alternatives and then adds it to his corpus. Let us now take a look at what the technical ins and outs of this type of expansion are. The need for a deliberate expansion of our present corpus does not simply come out of the blue — we add new information to our corpus only for a certain purpose. What is that purpose and how do we serve it best? To clarify the issue, Levi notes down the following considerations concerning "deliberate decision making":

> In deliberate decision making, the agent identifies the options available to him, his goals, and the available relevant evidence concerning the admissibility of the options for the purpose of realizing these goals and values. The option chosen is determined relative to these beliefs and values according to principles of rational choice. [Levi 1980, p. 36]

Several aspects of our question about the conditions under which an expansion is legitimate can now be specified:

> The options are potential expansion strategies which qualify as potential answers to the question under investigation, and the aim is to gratify the demand for information occasioned by the question while at the same time avoiding error. [Levi 1980, p. 38–39]

So the aim of an expansion is answering a question with "new error-free information". However, in case ϕ as well as $\neg\phi$ are serious possibilities with respect to K, there is always the risk that if we expand our corpus K with ϕ, we allow a false sentence into our corpus. According to Levi, such an expansion is justified if and only if the information value of ϕ outweighs the risk that ϕ is false:

> On the basis of inquiries [...] we sometimes reach a point where we conclude that the trade offs between risk of error and informational benefits are such as to warrant adding some hypothesis to the corpus and so to convert its status from mere hypothesis to settled, established and infallible truth (where being settled, and established is only for the time being and not necessarily forever). [Levi 1976, p. 15]

In order to fulfil the aim of getting relevant 'new error-free information' as best as possible, we should ideally proceed as follows: if, given our current corpus K, we have to deal with a problem, we first identify, in a phase which Levi calls "abduction",[22] all seriously possible problem-solving options; then we trade off the *informational value* and the *credal probability*

[22] Levi stipulates: "Abductive logic [...] is a system of norms prescribing necessary conditions which a system of potential answers to any legitimate question should satisfy"

of all the available options; and finally, we implement an expansion of our current corpus with the negation of those options which have come up as the worst during this weighing procedure. How does it all fit together formally?

Levi calls the set U of all available options that solve a given problem an *ultimate partition*. Let $U = \{\phi_1, \ldots, \phi_n\}$ be a finite set of available options.[23] All alternative options ϕ_i in U are serious possibilities with respect to the current corpus K. No single option ϕ_i is an element of K. Moreover, from K it follows that exactly one element in U is true, though we do not know which [Levi 1967a]. Levi defines a potential answer as the rejection of a subset R of U. A potential answer can be formulated with a sentence ρ, where ρ stands for the disjunction of all alternative options in R. After choosing a potential answer R, agent X should expand his corpus K with $\neg\rho$, that is, with the statement that the correct answer in U is *not* in the subset R of rejected elements from U.[24]

For instance, let K be $Cn(\{\neg p \vee \neg q\})$ and let U be $\{p, q, \neg p \wedge \neg q\}$. Then all conditions in the previous paragraph on K and U are met. Let R be $\{p, \neg p \wedge \neg q\}$. Then $\neg\rho = \neg p \wedge (p \vee q)$. Note that $\neg\rho$ is *not* logically equivalent to the disjunction of the elements in $U \backslash R$. Expanding K with $\neg\rho$ amounts to $Cn(\{\neg p, q\})$, which contains q, the only element in U that was not rejected by R, together with its deductive consequences.

As noticed, we need to balance the risk we take of admitting a false sentence $\neg\rho$ into our corpus when choosing a potential answer R against what the choice of R brings us, namely new information: after expansion with $\neg\rho$, we indeed *know* that $\neg\rho$. In order to represent this trade off between informational value and credal probability numerically, Levi attaches to both informational value and credal probability a *separate* probability measure. (Levi maintains that these two measures should not be reducible to each other.) So, the *informational value* is fixed with an "information-determining probability measure" M, and the *credal probability* with a second probability

[Levi 1976, p. 33]. The assessment of the informational values of the potential answers is also the result of abduction. See [Levi 1980, p. 49] and [Levi 1998, p. 4].

[23] Levi also discusses, although summarily, infinite sets of options. See [Levi 1976, p. 41–42] and [Levi 1980, p. 49]. The technical problems raised by infinite sets of options are irrelevant for the purpose of my argument.

[24] There are two degenerate cases. On the one hand, X can decide not to reject any option in U. In that case, the expansion of K with the statement that the correct answer is *not* to be found in the (now empty!) set of rejected alternatives in U leaves the corpus K as it was. On the other hand, X may decide to reject all alternatives in U. Expansion of K with the statement that the correct answer in U is not to be found in the set of rejected alternatives now produces the inconsistent corpus.

measure, the "expectation-determining probability measure" Q . (We have already met the latter at our discussion of Levi's notion of a corpus.)

The probability measure M assigns to each option ϕ_i in U a probability $M(\phi_i)$, such that $0 \leq M(\phi_i) \leq 1$ and $M(\phi_1) + \ldots + M(\phi_n) = 1$. The informational value of a potential answer R equals the sum of the informational values of the options in R. Hence, if $R = \{\phi_{i1}, \ldots, \phi_{im}\}$, then $M(\rho) = M(\phi_{i1} \vee \ldots \vee \phi_{im}) = M(\phi_{i1}) + \ldots + M(\phi_{im})$. The probability measure M means to represent X's (context dependent) evaluation of the *informational value* of the available options, but says nothing about X's assessment of the *credal probability* of these options: $M(\phi_i)$ is the informational value of *rejecting* ϕ_i [Levi 1980, p. 48].

In turn, the measure Q assigns, in a given context, a probability $Q(\psi)$ to each sentence ψ in the language L, such that $0 \leq Q(\psi) \leq 1$. This probability measure is meant to represent X's assessment of the *credal probability* of the available options, but tells us nothing about X's evaluation of the *informational value* of these options. It fixes X's "credal state", a supplement to X's corpus of knowledge:

> [R]elative to his corpus of knowledge X has a "credal state" represented by a probability function assigning to all sentences in L a numerical probability consistent with the requirement that all items in his corpus bear probability 1. [Levi 1976, p. 37]

Lastly, the utilities $M(\phi_i)$ and $Q(\phi_i)$ of each option ϕ_i have to be traded off, weighed by a "degree of boldness" q. This degree of boldness, which, though it always holds that $0 < q \leq 1$, is context-dependent (as we shall see later on), represents the degree to which X is prepared to risk errors in order to acquire new information. Levi's assumptions and argumentations, based on an approach via a maximization of expected epistemic value, finally lead to the following criterion for the choice of an expansion strategy,[25] a criterion that we will designate from now on with "Rule A":

> Given a corpus $K_{X,t}$, finite ultimate partition U, information-determining probability function M defined over the Boolean algebra of elements of U, an expectation-determining probability function Q defined over the same algebra, and an index of caution q, X should reject all and only those elements of U satisfying $Q(\phi_i) < qM(\phi_i)$. [Levi 1980, p. 53][26]

[25] For technical details, see especially [Levi 1967b].

[26] Strictly speaking, the Q-function is not only defined for all boolean combinations of elements in U, but for all elements of the language L.

If the potential answer $R = \{\phi_{i1}, \ldots, \phi_{im}\}$ is the set of options which is rejected on the strength of the abovementioned criterion, then, if we subscribe to Levi's proposals, the expansion of X's current corpus K with $\neg(\phi_{i1} \vee \ldots \vee \phi_{im})$ is legitimate. The result of this expansion, X's new corpus, is then given by $K + \neg(\phi_{i1} \vee \ldots \vee \phi_{im})$, which will serve as X's new standard for serious possibility:

> To be sure, prior to expansion, there is a risk, from X's point of view, that the information to be added to his standard for serious possibility is false. Yet, sometimes X is justified in taking the risk. Once X has implemented the expansion strategy and taken the risk, he evaluates serious possibility according to a new standard relative to which the new information added is no longer possibly false. [Levi 1980, p. 57]

1.3.2. *Contraction*

In contraction, a sentence ϕ is deleted from a corpus K, such that ϕ is not a logical consequence of the remaining sentences in the corpus resulting from the contraction of K with ϕ. For the sake of convenience, we will use "$K - \phi$" to denote the resulting corpus.

Other criteria apply for contractions than for expansions. As opposed to what is the case in expansions, avoiding error cannot be a reason for deleting a sentence ϕ from a corpus K, as all the sentences in a corpus K, which after all acts as X's standard for serious possibility, cannot possibly be false: "In contraction, the concern to avoid error is vacuous" [Levi 1991, p. 79]. On the contrary, in a contraction X gives up a sentence which is definitely true: "For X to contract his corpus is for him to surrender error-free information" [Levi 1980, p. 58]. Hence, the *credal probability* of the sentences in K can play no part in the formulation of a criterion for legitimate contractions. Levi intends to formulate a theory of contraction that "seeks to show how a consistent account of justified ceasing to believe is feasible even when K is taken to be a standard for serious possibility and all members of K are true in the sense in which avoidance of error is taken to be a desideratum of efforts to improve K by revising it" [Levi 1991, p. 61].

Levi's epistemology only allows for two reasons for a contraction. First, as we have indicated briefly above, it is possible to accidentally end up in the inconsistent corpus via a legitimate routine expansion of a consistent corpus K with a sentence ϕ. Because the inconsistent corpus "fails as a standard for serious possibility to be used in inquiry and deliberation" and therefore is of no value whatsoever, an agent is obliged to once again arrive at a consistent corpus by means of a coerced contraction:

> When routine expansion injects inconsistency into the inquirer's doctrine, contraction from the inconsistent state is required. An inconsistent state of full belief or corpus fails as a standard for serious possibility for the purpose of subsequent inquiry and for practical deliberation. [Levi 1991, p. 76–77]

That we need to implement a contraction if we have landed in the inconsistent corpus is beyond dispute. In a coerced contraction, we can restrict ourselves to determining the strategies to extricate ourselves from the inconsistent corpus. The inconsistent corpus was reached by expanding a consistent corpus K already containing the sentence $\neg\phi$ with a sentence ϕ which was obtained via a program held to be reliable. Hence, according to Levi, we can do either of three things:[27] (1) we may call into question the reliability of the program which resulted in the sentence ϕ which was inconsistent with our old corpus K. In this case, we go back to the old corpus K, from which we delete with contraction the claim that the program in question is reliable; (2) we may doubt the background information present in the old corpus which is inconsistent with the sentence ϕ obtained by means of the program. In this case, we remove background sentence $\neg\phi$ with contraction from K and expand the result with ϕ; (3) we may refuse to believe both the program and the relevant background information.[28] In the last case, we take the intersection of the corpora obtained by way of the first two strategies.[29]

In the second place, an *uncoerced* contraction comes into consideration when we decide to give 'a hearing' to a hypothesis T_2, which is falsified by an element T_1 from the current, consistent corpus K. Because it initially holds that T_2 is not a serious possibility with respect to K, elements from K have to be deleted, so as "to shift to a position where judgment is

[27] Since Levi bases his system on an underlying classical logic which he considers immune to revision, he cannot account for a fourth possibility: an adjustment of the underlying logic. Von Neumann and others argued that the reconciliation of the particle theory and the wave theory of light via Bohr's *principle of complementarity* did not imply a weakening of one of the fundamental principles of the rival theories, but actually a *weakening* of the underlying logic. See [Beth 1968, p. 3–5].

[28] Although an inconsistent corpus contains all the sentences of L, it apparently does not eat away at our memory and our powers of judgment. After all, one seems not to forget from which corpus the inconsistency is reached, while the corresponding informational values which will turn out to be necessary to implement the said contraction are left undisturbed. How inconsistent is an inconsistent corpus? For a critique of Levi's views on inconsistency in belief change theories, see [Da Costa and Bueno 1998, p. 32–37].

[29] A more comprehensive account of coerced contractions can be found in [Levi 1991, § 4.8].

suspended between these rival hypotheses so that investigations can be undertaken to decide whether T_1 should be reinstated via inferential expansion or T_2 should take T_1's place" [Levi 1980, p. 60]. Not every hypothesis qualifies for such a procedure. "There must be some inducement to incur the loss of information" [Levi 1991, p. 118]:

> To be justified in ceasing to believe what is initially settled, the inquirer must regard the benefits of giving the new proposal a non-question-begging hearing to be great enough to outweigh the costs. [Levi 1991, p. 4]

For example, a hypothesis T_2, which, though it be incompatible with our current views, gives an explanation of anomalies — phenomena that cannot (yet) be explained by our current theories, whereas they should — is worth considering. The actual corpus K, which contains $\neg T_2$, prohibits an unprejudiced evaluation of T_2. If we still wish to make a fair evaluation possible between T_2 and $\neg T_2$, it is necessary to adapt our actual corpus in such a way that both T_2 and $\neg T_2$ are serious possibilities with respect to the adapted corpus. According to Levi, $K - \neg T_2$ is the best corpus for the intended evaluation, because it differs only minimally from our current corpus. Now, on the basis of this adapted corpus $K - \neg T_2$, using the criteria for expansion discussed above, we can check without prejudice whether T_2 or its negation should be added to the adapted corpus.[30]

After this brief outline of the circumstances that justify the deletion of certain sentences from our current corpus, we shall conclude our discussion of contraction with the contraction method propagated by Levi, in which the central question is: supposing that we wish to remove a sentence ϕ from a given corpus K, *how* should we implement this contraction? Levi answers:

> We need to identify the available options or strategies for contraction by removing ϕ and then examine the goals and values that ought to be promoted in order to decide among them. [Levi 1991, p. 121]

We will not be able to avoid a modest logical apparatus in order to grasp the technical details of Levi's ideas on contraction.[31] Given the aim of a contraction of a corpus K with a sentence ϕ, we can immediately impose three constraints on $K - \phi$, the result of this contraction: (1) $K - \phi$ is a corpus, a deductively closed set of sentences; (2) $K - \phi$ is a subset of K, since a sentence is removed from K; and (3), the sentence ϕ which needs to

[30] For more information, see [Levi 1991, § 4.9].

[31] [Hansson and Olsson 1995] is an excellent study of Levi's contraction operators.

be removed is *not* an element of $K - \phi$. We shall use "$C(K, \phi)$" to denote the set of corpora that meet these three requirements:

$$C(K, \phi) = \{K' \subseteq K : K' = Cn(K') \textit{ and } \phi \notin K'\}.$$

Since each element of $C(K, \phi)$ is the intersection of one or more elements of the set $S(K, \phi)$ of "saturatable" contractions which skip a sentence ϕ from a corpus K (proving this statement is not a trivial matter),[32] we can stick without loss of generality to the set $S(K, \phi)$:

$$S(K, \phi) = \{K' \subseteq K : K' = Cn(K') \textit{ and } Cn(K' \cup \{\neg\phi\}) \textit{ is maximally consistent in } L\}.$$

The set of saturatable contractions $S(K, \phi)$ is used by Levi as a starting-point for his definition of the contraction of a corpus K with a sentence ϕ: after all, each possible contraction of K with ϕ is, on the strength of the statement mentioned above, the intersection of the elements of a subset of $S(K, \phi)$. Hence, the problem of finding the right contraction can be reduced to the question which elements of $S(K, \phi)$ we should choose for this subset. Since each subset of $S(K, \phi)$ meets the logical contraints imposed on a contraction, logical considerations alone will fall short if, like Levi, we require that "when all relevant factors in a given context are taken into account, one change at most should be legitimate or justified. Hence, given the initial corpus K and *all other relevant factors* (whatever these may be), and given that adding or deleting ϕ is legitimate or justified, the new belief state to which one shifts legitimately or with justification is uniquely determined" [Levi 1991, p. 67 — adapted notation].

Since, at the present state of logical research, no purely logical definition of a sensible contraction operator can be given within classical logic,[33] Levi resorts to an information-determining measure M which assigns an M-value $M(K')$ to all possible contractions K' in $C(K, \phi)$ and so solves the problem of making a well-founded choice from the elements of $S(K, \phi)$ in order to define the required unique subset of $S(K, \phi)$. In this way, the uniqueness of the contraction of K with ϕ is warranted, as Levi defines this contraction as the intersection of all the elements of the said subset of $S(K, \phi)$. Levi advises the agent who is planning to contract a sentence ϕ from her corpus K to use this information-determining measure M as follows:

> [T]he inquiring agent should evaluate the various contraction strategies available to her with respect to the informational

[32] See [Levi 1991, p. 122] and [Levi 1996, p. 20]. The elements of $S(K, \phi)$ are *saturatable*, because for each element K' in $S(K, \phi)$ it holds that $Cn(K' \cup \{\neg\phi\})$ is maximally consistent in L.

[33] See Observation 2.1 of [Alchourrón and Makinson 1982] and Proposition 5.3 of [Hansson and Olsson 1995].

> value incurred and should choose a contraction strategy that minimizes the loss of informational value if a minimizing strategy exists. [Levi 1991, p. 122]

In short, choose those saturatable contractions from $S(K, \phi)$ which have the highest informational value. If we now assume that an information-determining measure M assigns to all deductively closed subsets K' of K an informational value $\mathit{Cont}(K') = 1 - M(K')$, only requiring that $A \subset B$ implies $\mathit{Cont}(A) \leq \mathit{Cont}(B)$,[34] then the needed selection function γ, which chooses the elements from $S(K, \phi)$ with the highest informational value, can be defined as follows: if $S(K, \phi) = \emptyset$, then $\gamma(S(K, \phi)) = \{K\}$; if $S(K, \phi) \neq \emptyset$, then

$$\gamma(S(K, \phi)) = \{K' \in S(K, \phi) : \textit{for all } K'' \in S(K, \phi) \textit{ it holds that } \mathit{Cont}(K'') \leq \mathit{Cont}(K')\}.$$

The formal apparatus developed in this subsection allows Levi to determine the admissible contraction $K - \phi$, given a corpus K, a sentence ϕ which has to be deleted and an information-determining measure M over $C(K, \phi)$, by means of the set of saturatable contractions $S(K, \phi)$ and the selection function γ:[35]

(i) If $\phi \in K$, then $K - \phi = \cap\gamma(S(K, \phi))$,
(ii) If $\phi \notin K$, then $K - \phi = K$.

Levi's contraction operator does not have all the properties of Alchourrón, Gärdenfors and Makinson's contraction operator. The latter operator is characterized by the following six postulates:[36]

$(K-1)$	$K - \phi$ is deductively closed,	*Closure*
$(K-2)$	$K - \phi \subseteq K$,	*Inclusion*
$(K-3)$	If $\phi \notin K$, then $K - \phi = K$,	*Vacuity*
$(K-4)$	If $\phi \notin \mathit{Cn}(\emptyset)$, then $\phi \notin K - \phi$,	*Success*
$(K-5)$	$K \subseteq (K - \phi) + \phi$,	*Recovery*
$(K-6)$	If $\mathit{Cn}(\phi) = \mathit{Cn}(\psi)$, then $K - \phi = K - \psi$.	*Extensionality*

[34] On the basis of the informational values of K and of the elements of $S(K, \phi)$, Levi also defines the concept "damped informational value" to rule out the possibility that for two elements K' and K'' of $S(K, \phi)$ with $\mathit{Cont}(K') = \mathit{Cont}(K'')$, it can hold that $\mathit{Cont}(K' \cap K'') < \mathit{Cont}(K')$. We leave this extra complication aside, because it does not affect the logical characteristics of Levi's contraction operator. See [Levi 1991, § 4.4]. [Levi 1998] also presents a variant of this adapted informativity concept, which leads to a somewhat stricter contraction operator "mild contraction", characterized in [Rott and Pagnucco 1999].

[35] [Levi 1991, p. 130.] Hansson and Olsson showed that clause (ii), which is lacking in Levi's original definition, is indispensable [Hansson and Olsson 1995, p. 108]. In [Levi 1996, p. 23], this minor flaw is corrected.

[36] See [Alchourrón, Gärdenfors, and Makinson 1985, p. 513].

Levi's contraction operator meets all these postulates except $(K-5)$, which is known in the literature as *Recovery*.[37] This postulate has come under critical fire, not in the least from Levi himself.[38] Levi's contraction operator, on the other hand, is characterized by the following five postulates:[39]

$(K-1)$	$K-\phi$ is deductively closed,	*Closure*
$(K-2)$	$K-\phi \subseteq K$,	*Inclusion*
$(K-3a)$	If $\phi \notin K$ or $\phi \in Cn(\emptyset)$, then $K-\phi = K$,	*Vacuity*
$(K-4)$	If $\phi \notin Cn(\emptyset)$, then $\phi \notin K-\phi$,	*Success*
$(K-5)$	If $Cn(\phi) = Cn(\psi)$, then $K-\phi = K-\psi$.	*Extensionality*

1.4. *An Application: Conditionals*

A short outline of Levi's treatment of conditionals concludes this brief exposé of Levi's epistemological and logical positions. As we have seen, Levi imposes two requirements on a plausible theory for revising epistemic states: (1) such a theory should articulate inference rules which, unlike classical inference, allow us to draw those conclusions from an epistemic state which we would also draw in practical deliberation and scientific inquiry; and (2), it should *formally* justify transitions — expansions as well as contractions — from one epistemic state to another epistemic state. Levi's *For the Sake of the Argument* (1996) develops one general, quasi-formal framework with which formal theories that aim at meeting these two requirements can be compared on the basis of abstract characteristics.

[37] Let L be the language that consists of all truth functional combinations of p and q, and let $K = Cn(\{p, q\})$. Then $Cn(\{q \to p\}) \in S(K, p)$. Choose γ such that $\gamma(S(K, p)) = \{Cn(\{q \to p\})\}$. Then $K - p = Cn(\{q \to p\})$ and $(K - p) + p = Cn(\{p\})$. In this case, K is not a subset of $(K-p)+p$, so *Recovery* does not hold. See [Hansson and Olsson 1995, p. 112].

[38] "Consider, for example, a situation where it is believed that Jones was HIV positive, received a drug treatment and subsequently showed HIV negative. Contract the corpus by giving up 'Jones received the drug treatment.' The conviction that Jones initially showed HIV positive would be retained. But the judgment that Jones showed HIV negative later on would be abandoned. Moreover, restoring the judgment that Jones received the drug treatment would not resurrect the conviction that Jones subsequently showed HIV negative unless the inquirer had the well entrenched conviction initially that the drug treatment always eliminates the HIV virus. If this belief were not well entrenched or if all that is believed is that the drug treatment is followed by cure in some percentage of cases less than 100%, the Recovery Condition would be violated" [Levi 1998, p. 9]. [Levi 1998, p. 37] presents a second counterexample to *Recovery*. See also [Levi 1991, p. 134–135], [Levi 2003] and [Hansson 1999b].

[39] See [Hansson and Olsson 1995, p. 109] and [Rott and Pagnucco 1999, p. 512].

All types of sentences that have been in the spotlight of philosophical logic for decades, can be handled within the system developed by Levi. Tautologies, mathematical truths, indicative statements, laws, dispositional statements, modal statements, conditionals, counterfactuals, inductive statements, defaults — they all have their place in Levi's framework. In this subsection we will briefly discuss the way in which Levi accounts for conditionals within his system. In Levi's view, conditionals have *no* truth values, unlike the elements of a corpus, though they can be accepted or rejected on the basis of a corpus.

1.4.1. *Conditionals*

According to Levi, conditionals are statements which never are an element of a corpus. As an alternative Levi proposes to interpret them as claims *about* a corpus, as claims expressing certain *properties* of a corpus, properties which tell us how a corpus will behave under certain revisions. Levi sees it as an important advantage of his interpretation of conditionals that it can do without the Kripke semantics which abounds in philosophical logic:[40] Instead, Levi interprets conditionals on the basis of *Ramsey's Test*, which takes its name from a cursory remark by Frank Ramsey on the interpretation of conditionals:

> If two people are arguing "If p will q?" and are both in doubt as to p, they are adding p hypothetically to their stock of knowledge and arguing on that basis about q; so that in a sense "If p, q" and "If p, $\neg q$" are contradictories. [Ramsey 1929, p. 143n]

Levi now proposes to interpret conditionals on the basis of Ramsey's Test using the techniques for *belief change* outlined above, a choice which obliges him, under penalty of triviality, to refuse to admit conditionals as elements of corpora.[41] Therefore, conditionals do not have truth, but only acceptability conditions. Then Levi proposes a theory figuring three different variations

[40] Levi opposes modal extensions of the language in which our beliefs are articulated. In Levi's view, modal logic and its applications to epistemological and metaphysical problems are a "retrograde step in philosophy" [Levi 1980, p. xvi]. Possible worlds semantics is, strictly speaking, superfluous: "[M]any advocates of the usefulness of possible worlds semantics for the purpose of explicating judgments of possibility and conditionals appeal to examples that may be given a straightforward epistemic or, in the case of conditionals, belief-change treatment" [Levi 1991, p. 114]. Indeed, "[c]onditionals understood in terms of imaging become mere artifacts of the metaphysician's fevered imagination" [Levi 1996, p. 76].

[41] See [Rott 1989] for an elegant proof of Gärdenfors's Triviality Theorem from 1986.

of Ramsey's Test.[42] According to Levi, this theory provides an adequate solution to the well-known problems concerning conditional sentences: "[A]ll 'if' sentences customarily classified by contemporary philosophers as indicatives and as subjunctives are explicated by various versions of the Ramsey test" [Levi 1996, p. 13].

In order to find out whether a conditional "If ϕ, then ψ" is acceptable with respect to a consistent corpus K, we add the antecedent ϕ to our current 'stock of knowledge', that is, to the actual corpus K. If this addition produces an inconsistent set of sentences, we make minimal changes in the inconsistent set so as to make it consistent while retaining the sentence ϕ. This procedure, carried out along Levi's criteria for revisions of corpora, produces a new consistent corpus K' which contains ϕ and differs only minimally from the old corpus K. Then we check whether the consequent ψ is an element of this new corpus K'. If that indeed is the case, then the conditional "If ϕ, then ψ" is acceptable with respect to the original corpus K.[43]

All three variations of Ramsey's Test discussed by Levi can be formulated in terms of contraction and expansion. Levi argues that a conditional "If ϕ, then ψ" should ideally be interpreted on the basis of a corpus that contains neither ϕ nor $\neg\phi$. Hence, the corpus K must be processed such that both ϕ and $\neg\phi$ are serious possibilities with respect to the adapted corpus. In some cases, this may mean that ϕ has to be eliminated by contraction from K, in others (think of counterfactuals), $\neg\phi$ has to be eliminated. This variant — Levi's favourite — can be defined as follows [Levi 1996, p. 31]:

$$\text{"If } \phi\text{, then } \psi\text{" } \textit{is acceptable with respect to } K \iff \psi \in ((K - \neg\phi) - \phi) + \phi.$$

2. *A Critique*

2.1. *The Identification of Corpora*

Each application of Levi's logical theory of knowledge requires an adequate representation of the epistemic state of an agent X by a corpus K in some formal language L. Levi assumes corpora to be consistent and deductively

[42] See [Levi 1996, p. 18–50] and [Gärdenfors 1988, p. 147–148].

[43] Similar epistemic interpretations of conditionals have been formalized previously in [Rescher 1964] and [Veltman 1976]. From the beginning, the idea that counterfactuals need an epistemic interpretation has been criticized. See [Kratzer 1981], [Lewis 1973], and [Stalnaker 1968]. For a more recent discussion, see [Rott 1999] and [Stalnaker 1992].

closed. Hence, if we wish to identify the corpus K that represents X's current epistemic state, we only need to determine which sentences in L are elements of K and which are not. We already noted that for each sentence ϕ in a corpus it holds that $Q(\phi) = 1$ and that the expectation-determining probability measure Q is context dependent. Therefore, if we assume that Q_1 represents X's credal state in some context C_1, we can justifiably exclude from the corpus K all sentences ϕ for which it holds that $Q_1(\phi) < 1$. However, it cannot be concluded from $Q_1(\phi) = 1$ that ϕ is an element of K: "According to X's credal state, all items in his corpus receive probability 1 (although the converse need not hold)" [Levi 1976, p. 10]. Contextual certainties, therefore, give us little to go on in determining our current corpus K.[44]

Wouldn't it be an option to verify whether $Q_i(\phi) = 1$ holds in *all* imaginable contexts C_i in which X's corpus is equal to the current corpus K? Unfortunately, this suggestion does not give rise to a useful criterion, as a tricky question presents itself, even supposing that it is feasible to scour all imaginable contexts: how do we know that our epistemic state, to be represented by corpus K, stays the same with a context shift? It is impossible to aim, on the one hand, at finding the elements of K, and to make sure, on the other, that the corpus applied in context C_i is identical to K, because after all we have to know what K's elements are before we can make that comparison. How do we know for sure whether in such a context shift we have not inadvertently expanded or contracted K?

2.2. *The Assessment of Contextual Parameters*

The preceding discussion of Levi's theories of expansion and contraction shows that Levi's theory of knowledge can only be applied if we have at our disposal (estimations of) numerical values for the "system of contextual parameters" he uses, consisting of, among others, the information-determining measure M, the expectation-determining measure Q and the degree of boldness q. This system of contextual parameters is part of X's epistemic state at time t. In his inquiry into the mechanisms of belief change, Levi just presupposes that we have found sufficiently specific values for these parameters and argues for his criteria concerning legitimate expansions and contractions on the basis of this presupposition. Levi defends his crucial presupposition with an *ad consequentiam* argumentation:

> Of course, investigation may reveal that no system of contextual parameters can be identified such that, given specific

[44] Compare [Batens 1992, p. 202].

> values for these parameters, the legitimacy of X's modification of his corpus is determined according to adequate objective standards. In that case, such modifications of bodies of knowledge would be subjective or context dependent in a sense which put them beyond critical control. However, we would be obstructing the course of inquiry to assume that this is so at the outset. [Levi 1976, p. 2]

Following this argument, the first question we could ask ourselves is *which* course of inquiry is obstructed if we do not assume that sufficiently specific values can be found for Levi's system of contextual parameters? It would seem a bit far-fetched that Levi is alluding here to research in theoretical physics or in comparative literature. These (and other) disciplines do not need Levi's methodological considerations to get along anyway. More likely, Levi is referring to epistemological investigations which aim at grasping 'the logic of inquiry', investigations that also include Levi's own epistemology.

Second, we can check, on the basis of clues from Levi's own works, if and to what extent the presupposition that Levi's contextual parameters have been specified accurately enough is plausible. We will limit ourselves to the assessment of the probability measure M, since, in Levi's epistemological framework, this information-determining probability measure plays a crucial role in the accounts of expansion and contraction, the fundamental types of belief change.

2.2.1. *Informational Value*

Assessing an information-determining M-function is a context dependent matter. Among others, it depends on the cognitive aims pursued by an agent X in a given context. As a consequence, there are hardly any gains to be expected from the search for a universal, context independent M-function.[45] Since Levi's criteria for both expansion and contraction can only be applied once we have, among others, assessed the M-function, this function should be assessed before the intended evaluation of the proposed expansion or contraction can take place. Such assessments

> are part of the abductive task. To some extent, these assessments may be regulated by criteria which are applicable to a

[45] Levi writes: "The considerations that enter into an evaluation of informational value are diverse, often competing, and heavily context dependent. Different kinds of inquiries impose different demands for new information, so that it is not to be expected that evaluations of informational value will meet the same requirements in all contexts. And inquiries addressing the same issues may be committed to different research programs generating different demands for information" [Levi 1991, p. 83].

> large class of problems. It may, perhaps, be possible to identify certain desiderata which determine explanatory power and simplicity relevant to the assessment of informational value in inquiries where the aim is to obtain explanations of some kind. It is doubtful, however, that such desiderata can be converted into criteria for the evaluation of informational value which render it irrelevant to consider the peculiarities of the particular demands for information motivating specific inquiries. Indeed, such restrictions on the assessment of informational value are likely to be very weak. Such assessment is, in my opinion, heavily context dependent. [Levi 1980, p. 47]

In the end, in the adoption of a certain (class of) M-function(s), an "abductive logic" is the deciding factor:

> Arguments concerning the adoption of one M-function rather than another are to be evaluated (insofar as there is a right and a wrong to the matter) according to principles of abductive logic [Levi 1976, p. 40–41].

We can, however, hardly see this reference to an 'abductive logic' as anything else but a shortcut, as long as Levi keeps us guessing about the peculiarities of such an abductive logic. Hence, the appeal to an 'abductive logic' does not contribute anything to the assessment of an M-function. Luckily, Levi gives us some rather more tangible clues for the determination of an M-function, even though the assumption of an M-function is considered by him to be "excessively unrealistic" [Levi 1976, p. 37]: different kinds of values, for example precision, simplicity and explanatory power, "constitute different dimensions that contribute to the assessment of what I call *informational value*" [Levi 1991, p. 145]. Moreover, Levi writes:

> [T]he demands for informational value that animate the inquirer's deliberations [...] may reflect commitments to research programs and ideals of explanatory adequacy, simplicity, systematicity, precision, and the like, including commitments to certain types of theoretical frameworks [Levi 1991, p. 150–151].

This does not help us make any headway either: the original problem of finding a numerical specification for one parameter is now 'reduced' to a messy multitude of problems. Is Levi's list complete? And how do we assess the different values on the list? How can the relative importance of these values be assessed? Many questions, but no answers.

A second problem with the assessment of informational values arises when we wish to test the acceptability of *counterfactuals* via Levi's criteria for conditionals. At first glance, Levi's approach seems to be preferable to David

Lewis's and Robert Stalnaker's treatments of counterfactuals with possible worlds semantics, in which a similarity relation between worlds is at the heart of the theory. After all, it is possible, within Levi's system, to sidestep counterexamples to David Lewis's and Robert Stalnaker's analyses. In 1976, Pavel Tichý described a situation in which Lewis's and Stalnaker's analyses of counterfactuals produce a result that is completely at odds with our pretheoretical intuitions:

> [C]onsider a man — call him Jones — who is possessed of the following dispositions as regards wearing his hat. Bad weather invariably induces him to wear his hat. Fine weather, on the other hand, affects him neither way: on fine days he puts his hat on or leaves it on the peg, completely at random. Suppose, moreover, that actually the weather is bad, so Jones *is* wearing his hat. [Tichý 1976, p. 271]

The statement "If the weather were fine, Jones would be wearing his hat", which, on the strength of our pretheoretical intuitions, is unacceptable, would be acceptable according to Lewis's and Stalnaker's analyses.

If we write K for the, obviously consistent, corpus that contains at least the abovementioned information, ϕ for "The weather is fine" and ψ for "Jones is wearing his hat", then it is clear that both $\neg\phi \in K$ and $\psi \in K$. To check with Levi's method whether the conditional in question is acceptable with respect to K, we first have to contract the corpus K by $\neg\phi$. (A contraction of the resulting corpus with ϕ will not be necessary, as $\phi \notin K$.) At this point it is important to arrive at a corpus that leaves open whether ψ is the case or not, because if ψ were to remain in $K - \neg\phi$, the statement "If ϕ, then ψ" would be acceptable with respect to K.

At first sight, it seems that the corpus $K - \neg\phi$ must contain ψ. As ψ is obviously relevant to the problem at hand, it seems likely that each corpus K^* such that $K^* \subseteq K - \neg\phi$ and $\psi \notin K^*$ must have a lower informational value than $K - \neg\phi$. It seems an unavoidable conclusion that $\psi \in K - \neg\phi$, because Levi demands of an admissible contraction $K - \neg\phi$ minimal loss of informational value. There is, however, an emergency exit: according to Levi's *Weak Monotonicity* postulate, it holds that $K - \neg\phi$ has at least as much informational value as all its subsets, but, on the other hand, real subsets K^* of $K - \neg\phi$ can have the same informational value as $K - \neg\phi$ itself. Hence, I propose to leave these sceptical considerations aside and to try to find, in line with Levi's proposals, an admissible corpus $K - \neg\phi$ such that both ψ and $\neg\psi$ are serious possibilities with respect to that corpus.

This we can do by finding out which element of $C(K, \neg\phi)$ is most suited to serve as a starting-point for a further expansion with ϕ. This element must be a deductively closed subset of K which contains neither $\neg\phi$ nor ψ nor $\neg\psi$ nor $\phi \rightarrow \psi$ nor $\phi \rightarrow \neg\psi$. Let K^* be this subset. Then there is a subset S^* of

$S(K, \neg\phi)$, such that $K^* = \cap S^*$. If we now choose the informational values of the elements of $S(K, \neg\phi)$ in such a way that exactly all elements in S^* have the highest informational value, then it holds that $\gamma(S(K, \neg\phi)) = S^*$ and, hence, that $K - \neg\phi = K^*$. It is now plain that, in accordance with our pretheoretical intuitions, the following sentences are both unacceptable with respect to K:

(1) "If the weather were fine, Jones would be wearing his hat."
(2) "If the weather were fine, Jones wouldn't be wearing his hat."

However, something still does not feel right. It turns out that Tichý's example is, indeed, not a counterexample to Levi's account of conditionals as long as we are free to choose the informational values of the elements of $S(K, \neg\phi)$ in such a way that we reach the result that was prescribed by our pretheoretical intuitions. Therefore, the desired result holds sway over the assessment of the informational values of the elements of $S(K, \neg\phi)$. It is even possible to generalize this observation to a theorem: Let $\neg\phi \in K$, such that $\neg\phi \notin Cn(\emptyset)$ and K is consistent. Then, for each ψ in L such that $\neg\phi \notin Cn(\neg\phi \vee \psi)$ there is a choice of informational values over $S(K, \neg\phi)$ such that "If ϕ, then ψ" is acceptable with respect to K.[46] In short, for each and every counterfactual there is a construction of the required contractions such that this counterfactual turns out to be acceptable. What would be the explanatory power of a theory of conditionals which can validate every counterfactual? 100%? (If Oswald had not killed Kennedy, Jack Ruby would have been the first man on the moon.)

3. *Conclusion*

The assessment of an information-determining M-function takes more doing than Levi wishes us to believe by simply assuming that it has already been assessed. In spite of all Levi's clues it is altogether implausible that the M-function could ever be determined with sufficient preciseness in a given context. Moreover, no credence can be attached to the often proclaimed

[46] Suppose that $\neg\phi \in K$ and that K is consistent. Since K is deductively closed, it holds that $\neg\phi \vee \psi \in K$ for all ψ in L. Suppose that $\neg\phi \notin Cn(\neg\phi \vee \psi)$. Then there is a deductively closed subset K^* of K such that $\neg\phi \notin K^*$ and $\neg\phi \vee \psi \in K^*$. There is a subset S^* of $S(K, \neg\phi)$, such that $K^* = \cap S^*$. Choose the informational values of the elements of $S(K, \neg\phi)$ in such a way that exactly all elements in S^* have the highest informational value. Then, by definition, $K - \neg\phi = K^*$. As K is consistent, it holds that $\phi \notin K$. By *Inclusion*, it holds that $\phi \notin K - \neg\phi$. By *Vacuity*, it holds that $(K - \neg\phi) - \phi = K - \neg\phi = K^*$. Hence, $\neg\phi \vee \psi \in (K - \neg\phi) - \phi$. Hence, $\psi \in ((K - \neg\phi) - \phi) + \phi$. Therefore, "If ϕ, then ψ" is acceptable with respect to K.

normative status of Levi's theory,[47] as long as Levi fails to formulate convincing criteria — criteria that are *independent* of the desired result — with which (1) the identity of a corpus K can be found out, and (2) the system of contextual parameters (the parameter q and the probability measures M and Q) can be assessed adequately.[48] It is a misleading strategy to nonchalantly *assume* that "appropriate inputs are present (such as the demand for information, ultimate partition, degree of caution, credal state, and so on)" [Levi 1991, p. 107] and then to formulate criteria for legitimate expansions and contractions on the basis of these values, since not much is gained with a 'solution' in which a given problem is reduced to the values of a number of parameters of which it is absolutely unclear how they should be assessed. Would we really be explaining much less if we limited ourselves to one parameter, l, which stipulates the legitimacy of belief changes: a "measure of legitimacy"?[49]

May Levi's dynamic epistemology have a second life, this time in the form of a descriptive theory? Descriptivity implies the obligation of testability, which Levi's theory, at least until now, fails to meet: as long as the numerical values of the parameters of our epistemic state before and after a controlled change cannot be *measured* with sufficient accuracy, a test (as opposed to the usual reference in belief change literature to the 'intuitiveness' of basic assumptions and postulates) of Levi's theory, supposing it is testable, remains a futuristic idea. Though Levi antagonizes Lewis and Stalnaker by

[47] "I am concerned with conditions under which changes in doxastic commitments are legitimate. The concern is prescriptive, not explanatory" [Levi 1991, p. 107] and "[m]y preoccupation is in the final analysis with identifying standards of rational health in reasoning. Logicians, mathematicians, and computer scientists make an important contribution to identifying what those standards might be and to the design of technologies that can contribute to enhancing our limited abilities to realize these standards. I am interested in defending a view not of what these standards might be but what they should be" [Levi 1996, p. xiii].

[48] Levi tries to justify his refusal to have anything to do with inquiries into criteria for finding adequate values for his parameters with an argument by analogy: "Thermodynamics and some branches of economic theory illustrate comparative statical theories which investigate changes in equilibrium states of systems suitably specified without scrutinizing the details of the paths such systems follow in moving from one equilibrium state to another. The normative analogue of such theories of the sort I am aiming to construct here prescribes shifts from one state of cognitive equilibrium to another without prescribing details of the psychological or social changes which are made in implementing the revision" [Levi 1980, p. 11]. This does not wash, because the assessing of the values of the required parameters in thermodynamics is done relatively unproblematically, while it is a major obstacle for Levi's logico-epistemological system. Accordingly, thermodynamic theories are usually testable, while theories in the field of *belief change* are not.

[49] Note that this mock proposal does not differ in principle, but only *in degree* from Levi's approach.

proclaiming that their treatment of conditionals and conditional logic, based on possible worlds semantics, are "formalisms in search for an as yet undiscovered application" [Levi 1996, p. 82], they should actually be seeing eye to eye with one another.

While Levi's proposals for expansion, once it has been decided to expand a corpus K with a sentence ϕ, are unproblematic from a logical point of view, the aforementioned difficulties of determining an appropriate information-determining M-function prevent us from applying Levi's contraction operator, even if the decision has already been made to contract a corpus K with a sentence ϕ. Levi's proposal to concretize the ubiquitous reference in belief change literature to a selection function γ with an information-determining probability measure M does not yield much, since Levi's contraction operator cannot be applied without the required informational values. This problem cannot be avoided by simply assigning the same informational value to all the elements from a non-empty $S(K, \phi)$, in which case $\gamma(S(K, \phi)) = S(K, \phi)$, because then, as Hansson and Olsson demonstrated, it holds that $K - \phi = Cn(\emptyset)$. The standard approach of [Alchourrón, Gärdenfors, and Makinson 1985], in which the selection function γ is not explained in any further detail, suffers from the same deficiency.

Therefore, it is important to strive for an approach of *belief change* in which the usual reference to an extra-logical element, such as a selection function or an ordering of sentences in a corpus on the basis of their corrigibility, is avoided, so that a contraction, as well as an expansion, can be implemented directly, that is, *without* appealing to an extra-logical element. This is even necessary in cases where we have no indication of the relative corrigibility of the elements of the corpus to be contracted. Only when we have constructed such theories, can we verify on the basis of empirical tests whether the proposed contraction operator holds good. Within a classical logical framework this wish (until now) can only be fulfilled under penalty of totally unacceptable results. An underlying logic which is weaker than classical logic might open up new perspectives.[50]

Vakgroep Theoretische Filosofie
Rijksuniversiteit Groningen.
E-mail: tamminga@philos.rug.nl

REFERENCES

[Alchourrón, Gärdenfors, and Makinson 1985] Carlos E. Alchourrón, Peter Gärdenfors, and David Makinson (1985). On the Logic of Theory

[50] Translated from the Dutch by Walter van der Star.

Change: Partial Meet Contraction and Revision Functions, *Journal of Symbolic Logic*, 50: 510–530, 1985.

[Alchourrón and Makinson 1982] Carlos E. Alchourrón and David Makinson (1982). On the Logic of Theory Change: Contraction Functions and Their Associated Revision Functions, *Theoria*, 48: 14–37, 1982.

[Batens 1992] Diderik Batens (1992). *Menselijke kennis*, Garant, Leuven 1992.

[Beth 1959] Evert W. Beth (1959). *The Foundations of Mathematics*, 2d edition, North-Holland Publishing Company, Amsterdam 1965.

[Beth 1968] Evert W. Beth (1968). *Science, a Road to Wisdom: Collected Philosophical Studies*, D. Reidel Publishing Company, Dordrecht 1968.

[Burke 1994] Tom Burke (1994). *Dewey's New Logic. A Reply to Russell*, The University of Chicago Press, Chicago 1994.

[Da Costa and Bueno 1998] Newton C.A. da Costa and Otávio Bueno (1998). Belief Change and Inconsistency, *Logique & Analyse*, 161–162–163: 31–56, 1998.

[Fermé and Hansson 1999] Eduardo L. Fermé and Sven Ove Hansson (1999). Selective Revision, *Studia Logica*, 63: 331–342, 1999.

[Friedman and Halpern 1999] Nir Friedman and Joseph Y. Halpern (1999). Belief Revision: A Critique, *Journal of Logic, Language, and Information*, 8: 401–420, 1999.

[Gärdenfors 1988] Peter Gärdenfors (1988). *Knowledge in Flux*, The MIT Press, Cambridge (Mass.) 1988.

[Gärdenfors and Makinson 1988] Peter Gärdenfors and David Makinson (1988). Revisions of Knowledge Systems Using Epistemic Entrenchment, in Vardi, editor, *Theoretical Aspects of Reasoning about Knowledge*, pages 83–95, Morgan Kaufmann, Los Altos (California), 1988.

[Hansson 1999a] Sven Ove Hansson (1999a). *A Textbook of Belief Dynamics*, Kluwer, Dordrecht 1999.

[Hansson 1999b] Sven Ove Hansson (1999b). Recovery and Epistemic Residue, *Journal of Logic, Language, and Information*, 8: 421–428, 1999.

[Hansson and Olsson 1995] Sven Ove Hansson and Erik J. Olsson (1995). Levi Contractions and AGM Contractions: A Comparison, *Notre Dame Journal of Formal Logic*, 36: 103–119, 1995.

[Kratzer 1981] Angelika Kratzer (1981). Partition and Revision: The Semantics of Counterfactuals, *Journal of Philosophical Logic* 10: 201–216, 1981.

[Kuklick 1977] Bruce Kuklick (1977). *The Rise of American Philosophy*, Yale University Press, New Haven 1977.

[Levi 1967a] Isaac Levi (1967a). *Gambling with Truth*, Alfred A. Knopf, New York 1967.

[Levi 1967b] Isaac Levi (1967b). Information and Inference, *Synthese*, 17: 369–391, 1967.

[Levi 1976] Isaac Levi (1976). Acceptance Revisited, in Bogdan, editor, *Local Induction*, pages 1–71, Reidel, Dordrecht 1976.

[Levi 1980] Isaac Levi (1980). *The Enterprise of Knowledge*, The MIT Press, Cambridge (Mass.) 1980.

[Levi 1991] Isaac Levi (1991). *The Fixation of Belief and Its Undoing*, Cambridge University Press, Cambridge 1991.

[Levi 1996] Isaac Levi (1996). *For the Sake of the Argument*, Cambridge University Press, Cambridge 1996.

[Levi 1998] Isaac Levi (1998). Contraction and Informational Value, 7th version, manuscript.

[Levi 2003] Isaac Levi (2003). Counterexamples to Recovery and the Filtering Condition, *Studia Logica*, 73: 209–218, 2003.

[Lewis 1973] David Lewis (1973). *Counterfactuals*, Basil Blackwell, Oxford 1973.

[Malcolm 1952] Norman Malcolm (1952). Knowledge and Belief, in Phillips Griffiths, editor, *Knowledge and Belief*, pages 69–81, Oxford University Press, Oxford 1967.

[Peirce *CP*] Charles Sanders Peirce. *Collected Papers of Charles Sanders Peirce*, Hartshorne and Weiss, editors of volumes 1–6, Burks, editor of volumes 7–8, Belknap Press, Cambridge (Mass.) 1931–1958.

[Quine 1953] Willard Van Orman Quine (1953). *From a Logical Point of View*, Harper & Row, New York 1963.

[Quine 1960] Willard Van Orman Quine (1960). *Word and Object*, The MIT Press, Cambridge (Mass.) 1960.

[Ramsey 1929] Frank P. Ramsey (1929). General Propositions and Causality, in Ramsey, *Foundations. Essays in Philosophy, Logic, Mathematics and Economics*, pages 133–151, Humanities Press, Atlantic Highlands (N.J.) 1978.

[Rescher 1964] Nicholas Rescher (1964). *Hypothetical Reasoning*, North-Holland Publishing Company, Amsterdam 1964.

[Rott 1989] Hans Rott (1989). Conditionals and Theory Change: Revisions, Expansions, and Additions, *Synthese*, 81: 91–113, 1989.

[Rott 1991] Hans Rott (1991). Two Methods of Constructing Contractions and Revisions of Knowledge Systems, *Journal of Philosophical Logic*, 20: 149–173, 1991.

[Rott 1992] Hans Rott (1992). On the Logic of Theory Change: More Maps between Different Kinds of Contraction Function, in Gärdenfors, editor, *Belief Revision*, pages 122–141, Cambridge University Press, Cambridge 1992.

[Rott 1999] Hans Rott (1999). Moody Conditionals: Hamburgers, Switches, and the Tragic Death of an American President, in Gerbrandy, Marx, de Rijke, and Venema, editors, *JFAK. Essays Dedicated to Johan van Benthem on the Occasion of his 50th Birthday*, Vossiuspers AUP, Amsterdam 1999.

[Rott and Pagnucco 1999] Hans Rott and Maurice Pagnucco (1999). Severe Withdrawal (and Recovery), *Journal of Philosophical Logic*, 28: 501–547, 1999.

[Stalnaker 1968] Robert Stalnaker (1968). A Theory of Conditionals, in Rescher, editor, *Studies in Logical Theory*, pages 98–112, Basil Blackwell, Oxford 1968.

[Stalnaker 1992] Robert Stalnaker (1992). Notes on Conditional Semantics, in Moses, editor, *Theoretical Aspects of Reasoning about Knowledge*, pages 316–327, Morgan Kaufmann, San Mateo (California) 1992.

[Tichý 1976] Pavel Tichý (1976). A Counterexample to the Stalnaker-Lewis Analysis of Counterfactuals, *Philosophical Studies*, 29: 271–273, 1976.

[Veltman 1976] Frank Veltman (1976). Prejudices, Presuppositions and the Theory of Counterfactuals, in Groenendijk and Stokhof, editors, *Amsterdam Papers in Formal Grammar*, volume 1, pages 248–281, Centrale Interfaculteit, Universiteit van Amsterdam, Amsterdam 1976.