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Making Sense of Paraconsistent Logic

The Nature of Logic, Classical Logic and Paraconsistent Logic

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Abstract

Max Cresswell and Hilary Putnam seem to hold the view, often shared by classical logicians, that paraconsistent logic has not been made sense of, despite its well-developed mathematics. In this paper, I examine the nature of logic in order to understand what it means to make sense of logic. I then show that, just as one can make sense of non-normal modal logics (as Cresswell demonstrates), we can make 'sense' of paraconsistent logic. Finally, I turn the tables on classical logicians and ask what sense can be made of explosive reasoning. While I acknowledge a bias on this issue, it is not clear that even classical logicians can answer this question.

1 Paraconsistent Logic Doesn't Make Sense

Over the last few decades, there have been great advances in the development of paraconsistent logic.[1] It now has well-developed proof-theories and semantics. While it has still not found a good basis in *classical* mathematics (although even this situation is changing as a result of the development of paraconsistent

[1]In this paper, I often use the term 'paraconsistent logic' as a mass noun.

set theory satisfying theorems of classical mathematics[2]), paraconsistent logic has arguably been successfully applied to many areas both within philosophy as well as such rapidly developing areas as computer science.[3] Nonetheless, paraconsistent logicians are often faced with the remark that, despite all of this development, paraconsistent logic still does not really 'make sense'. Even someone as friendly towards non-classical logics as Putnam is able to write:

> I am aware that some people think such a logic—paraconsistent logic—has already *been* put in the field. But the lack of any convincing application of that logic makes it, at least at present, a *mere* formal system, in my view. (Putnam (1994) p. 262, footnote 12)

Putnam here expresses his concern about the status of paraconsistent logic in terms of 'application'. I take it, however, that finding an application involves finding how a formal system can be used in a particular context, which, for Putnam, is knowing the 'sense' of the system.[4] That is, to give a sense to a formal system is to make it intelligible by, for example, providing an 'interpretation' outside of the formal machinery that shows how the system can be applied. It is one thing to construct an account of logical consequences by providing truth conditions for logical connectives; it is another to give it a sense (that is, to specify its use). The difference here is not just the difference between 'pure' and 'applied' logics, at least in the way that pure/applied distinction is often understood. 'Application', according to this distinction, is understood to refer to the giving of meaning often in non-formal terms.[5] This might suggest that providing truth conditions for logical connectives is enough for specifying an application, since truth conditions specify the meaning of logical connectives in non-formal terms (at least this is how they are usually understood). As we will see, however, Putnam's concern seems to be about the *conception* of logic. On this understanding of Putnam, his charge against paraconsistent logic seems to be that there is no conception of logic that can accommodate paraconsistent logic; that is, no sense has been given to paraconsistent logic.[6]

Now, this charge is reasonable if we are in the business of presenting paraconsistent logic as more than just a mere mathematical tool (even though the development of such a tool itself is an achievement worthy of praise). In this paper, I

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[2]See Weber (2010a) and Weber (2010b).

[3]See for example Priest and Tanaka (2009).

[4]See Putnam (1994) pp. 256-7. Putnam is concerned with the sense of a statement or a question rather than of a formal system. My statement is an application of his claim rather than his claim itself.

[5]See, for example, Haack (1978) pp. 30ff.

[6]A similar charge against paraconsistent logic was put to me by my former colleague, Max Cresswell (though his charge was not to do with the conception of logic but with a coherent interpretation of logical connectives). This paper is, in part, a response to him.

directly address the charge that paraconsistent logic does not 'make sense'. First, I shall argue that paraconsistent logic does 'make sense' as much as we can 'make sense' of non-normal modal logics as Cresswell (1967) demonstrates. I then turn the tables on classical logicians and ask what sense we can make of explosive reasoning. It is not clear that even classical logicians can answer that question.

2 The Tale of Non-Normal Worlds

Once upon a time, non-normal worlds were considered as mere technical devices to model non-normal systems of modal logic developed by C.I. Lewis. When Kripke (1963) provided appropriate semantics in the form of possible world semantics for Lewis systems (some of them at least), he introduced non-normal worlds to model *S2* and *S3*, the systems often called non-normal systems (as well as *E2* and *E3* of Lemmon (1957)). One of the main characteristics of a Lewis non-normal system is the failure of the rule of necessitation, i.e., it is not the case that if $\models A$ then $\models \Box A$. In order to develop a semantics for the logic which violates the rule, Kripke introduced non-normal worlds at which any formula of the form $\Box A$ fails to be true. For, then, $\not\models \Box\Box(A \vee \neg A)$, even if $A \vee \neg A$ is true at every world and so $\models \Box(A \vee \neg A)$. (By the interchangeability of $\Box\neg$ and $\neg\Diamond$, every formula of the form $\Diamond A$ takes truth as its truth value.) Non-normal worlds were introduced as technical devices to achieve this effect.

The idea that non-normal worlds were mere technical devices was overturned by an ingenious interpretation of non-normal worlds by Cresswell (1967). Cresswell's quest was partly directed by the fact that Lewis thought of *S2* as the true system of modal logic (though Cresswell's aim was not to make Lewis' thought plausible). Cresswell notes that the laws of logic are sometimes regarded as 'laws of thought'. But that is the case only in a world in which there are 'thinking beings'. So a suggestion is that a non-normal world is a world where there are no thinking beings (and that it is not a logical truth that there are thinking beings).[7]

This seems to mean that non-normal worlds are worlds at which there are no laws of thought and hence no laws of logic. But this characterisation fails to capture the non-normal worlds introduced by Kripke. At a non-normal world, A is true if $A \wedge B$ is true. Hence, if it is a law of logic that A follows from $A \wedge B$, then it is not the case that there are no laws of logic at a non-normal world. Thus,

[7] See particularly Cresswell (1967) pp. 202-203. Note that Cresswell doesn't hold the view that the laws of logic are laws of thought. The purpose of his article is to show that different modal logics reflect different interpretations of the necessity operator, just as I am trying to show, in the first half of this paper, that paraconsistent logic (some relevant logics at least) reflects a different interpretation of the conditional operator. Thanks go to Max Cresswell for clarifying his position in personal communication.

having no laws of logic (because there are no laws of thought) is too strong a characterisation of non-normal worlds.

However, depending on how one understands 'laws of logic', the idea that there are no laws of logic seems to intelligibly cash out the notion of non-normal worlds. Consider a non-normal model (for a propositional modal language) $\mathcal{M} = \langle W, N, R, v \rangle$ where W is the set of possible worlds, N is the set of normal worlds, R is the accessibility relation on W and v is an evaluation function such that, for a propositional variable p and a world $w \in W$, $v(p, w) = 1$ or $v(p, w) = 0$, and that $v(\Box p, w) = 0$ if $w \in W - N$. For the sake of simplicity, we assume that R is the accessibility relation of *S5*, i.e., R is universal. Now, if we define validity in terms of all *normal* worlds, i.e., every $w \in N$, then we have a Lewis non-normal system. However, we can define validity in terms of *all* worlds, i.e., every $w \in W$. This gives rise to a Lemmon system of modal logic.[8] So there are two ways of defining validity in a modal logic. Let's call validity defined in terms of all normal worlds *weak validity*, represented as $\models_w$, and that defined in terms of all worlds *strong validity*, represented as $\models_s$.

A logical truth in a weak sense is defined in terms of normal worlds. So, a sentence, A, is a weak logical truth, i.e., $\models_w A$, if in every model, A is true at every normal world. A logical truth in a strong sense, on the other hand, is defined in terms of all worlds. So $\models_s A$ if A is true at every world in every model.

Now, *prima facie* at least, $\models_s A$ expresses that the necessity of A is *general*. Late in this paper, we will examine how Kant understood and Frege and Wittgenstein appropriated this feature of logic. For now, however, let's take it to mean that logic is general in the sense that a logical truth expresses a truth no matter what the situation turns out to be. So to say that the necessity of A is *general* is to say that $\Box A$ is a logical truth. In order to formalise the idea suggested here, let's represent the generality of logical truth, B, by $\models B$. Then the idea is that $\models \Box A \Leftrightarrow\ \models_s A$.

If a law of logic is understood to be expressed by a logical truth,[9] then asserting that $\models \Box A$ is asserting that $\Box A$ is a law of logic. Thus, there is a sense in which laws of logic are expressed by logical truths of the form $\Box A$. If laws of logic are delivered by formulas of the form $\Box A$, then the failure of $\Box A$ implies the failure of the laws of logic. So, the fact that any formula of the form $\Box A$ fails to be true invokes the idea that there are no laws of logic. Thus, even though the truth of $A \wedge B$ entails the truth of A, a non-normal world can be characterised as a world where laws of logic fail. A non-normal world can now be thought of as a world

[8] See Hughes and Cresswell (1996) pp. 205-6.

[9] I don't take this statement to be uncontroversial. However, I am concerned with attaching a 'sense' to non-normal worlds and to non-normal modal logics and I take it that there is a sense in which a law of logic is expressed by a logical truth as was held by, for example, Frege, Russell and Hilbert.

where laws of logic fail.

Here then is the tale of non-normal worlds. What was once thought of only as technical devices (i.e., defined only in virtue of formal conditions) have now been given a conception of logic that can tell us how to think of them intelligibly. Non-normal worlds are thus given a sense. At the same time, non-normal systems of modal logic have gained proper recognition.

3 ... and Relevant Logic

A similar tale can be told for paraconsistent logic or at least some relevant logics which form a sub-class of paraconsistent logic. When Anderson and Belnap developed relevant logics, they introduced them in an axiomatic form.[10] We had to wait for Routley and Routley (1972) and Urquhart (1972) to provide the appropriate semantics for some of the Anderson and Belnap systems.[11] When the semantics for relevant logics, in particular for R, were introduced in the form of the Kripke possible world semantics based on Routley and Routley's (1972) semantics for *First Degree Entailment (FDE)*, Routley and Meyer (1973) held that the *real world*, denoted by 0, played a distinguished role. The reason is that in order to invalidate the irrelevant formula $A \to (B \to B)$, logical truths, e.g., $B \to B$, have to fail at some world. However, 0, the real world, verifies all logical truths. After all, 0 is the real world: 0 is given a "privileged status".[12] The worlds which do not have this privileged status have come to be known as non-normal worlds among (some) relevant logicians.

It is not just a historical coincidence that some worlds have come to be called non-normal worlds both in the modal logic community and in the relevant logic community. For instance, Priest (1992) claims that Routleys and Meyer generalised Kripke's notion of non-normal worlds in formulating their semantics for some relevant logics. Whether or not Priest is right, Routleys and Meyer themselves didn't explicitly refer to Kripke's non-normal worlds. Nor does Priest provide the general characteristics of Kripke's non-normal worlds that can also be attributed to relevant non-normal worlds. If the insight of Routleys and Meyer was indeed derived from Kripke's semantics for Lewis' weaker modal systems, that insight has not been made widely available.

[10] These logics are recorded in Anderson and Belnap (1975) and Anderson, Belnap and Dunn (1992).

[11] Exactly who came up with the first semantics for the Anderson and Belnap systems is a matter of dispute, just like who came up with the first semantics for Lewis modal logics is, I believe, a matter of dispute. I let historians settle the issue.

[12] Routley and Meyer (1973) p. 205. Note that there can be more than one world which has this privileged status. However, completeness doesn't force one to assume that there is more than one.

Whether or not it was an insight of Routleys and Meyer, the characterisation of non-normal worlds offered in the previous section can be generalised to include the relevant non-normal worlds.[13] Consider a formula of the form $A \to B$. Assuming that A and B do not contain $\to$, if the formula is a logical truth, then it expresses truth-preservation between A and B in every situation. This can be shown in a semantics for *FDE* which was developed to study the relationship between antecedents and consequents of implicational sentences in terms of truth-preservation between them in every situation.[14] Even if A and B contain $\to$, a formula of the form $A \to B$, if it is a logical truth, can be seen to express truth-preservation in every situation, since a semantics for a (full) relevant logic may be an extension of that for *FDE*.[15]

So, to say that $A \to B$ is a logical truth is to say that the (truth-preserving) connection between A and B is *general*. Let $\models A$ be that A is a logical truth. Then, given that the study of relevant logics is the study of the (logical) relationship between A and B, asserting that $\models A \to B$ is asserting that $A \to B$ is a law of logic. Thus, by following the line of reasoning in the modal case, there is a sense in which formulas of the form $A \to B$ express laws of logic in relevant logics.

One can show in a semantics for *FDE* that the truth-preserving connection between B and B (for any formula B) is general. This shows that $B \to B$ is a logical truth. Hence, a non-normal world where $B \to B$ fails is a world where a logical truth fails. A non-normal world in relevant logics is, thus, a world where laws of logic fail.

In the case of relevant logics, however, there is no uniform evaluation of $\to$-formulas across all non-normal worlds. The evaluation is based on the constraints imposed on the ternary relation between worlds at a relevant non-normal world. Hence a formula of the form $B \to B$ may turn out to be true at a relevant non-normal world. Nonetheless, $B \to B$ may fail to be true even if $\models B \to B$. Hence laws of logic may fail at a relevant non-normal world. And it is because of this that the irrelevant formula of the form $A \to (B \to B)$ fails to be a logical truth. Thus, it is the (possible) failure of laws of logic that characterises relevant non-normal worlds too.

By making non-normal worlds in the relevant semantics intelligible, we have attached a sense to (some) relevant logics. Therefore, if we can claim to have made sense of non-normal systems of modal logic, we can also claim to have made sense of relevant logics. Relevant logics 'make sense' as much as non-normal modal logics do.

[13] Priest (1992) provides a different analysis of the relevant non-normal worlds. His analysis does not seem to be a generalisation of Kriple's non-normal worlds, despite his claim that Routleys and Meyer generalised Kripke's non-normal worlds.

[14] See for example Routley and Routley (1972) and Dunn (1976).

[15] See for example Tanaka (2000).

4 Making Sense of Paraconsistent Logic

In the previous sections, I provided an interpretation of non-normal worlds in the possible world semantics for non-normal modal logics of C.I. Lewis which was used to make sense of non-normal worlds in the semantics for some relevant logics. Since not all paraconsistent logics are relevant logics, I now need to provide an intelligible interpretation of paraconsistent logic in general in order to meet the charge that no sense has been given to paraconsistent logic. My strategy will be to show that the conception of logic that Putnam (1994) seems to endorse can reasonably be extended to incorporate paraconsistent logic (at least some paraconsistent logics).

Putnam (1994) tries to grasp the early Wittgenstein's thought that 'logical truths do not really say anything, that they are empty of sense (which is not the same thing as being nonsense), *sinnlos* if not *unsinning*' (p. 246). He does this by tracing the conception of logic that he thinks runs through the thoughts of Kant and Frege and finally Wittgenstein.

First, Putnam grapples with Kant's view of logic as a maximally general science. For Kant, logic is not *descriptive* of the world, whether actual or possible. That is, Kant does not present a *metaphysical* conception of logic. For Kant, according to Putnam, logic is 'a doctrine of *the form of coherent thought*' (p. 247). It is normative in the sense that 'my thought [in the normative sense of *judgement which is capable of truth*] would not be a thought at all unless it conforms to logic' (p. 247). Putnam overstates Kant's view when he writes that 'illogical thought is not, properly speaking, thought at all [for Kant]' (p. 246). For Kant's point is rather about the normative status of logic. As MacFarlane (2000) puts it nicely: for Kant, 'no activity that is not held accountable to [logical] rules can *count* as thought, and not that there cannot be thought that does not conform to these rules' (p. 87).

Kant's view that logic is a maximally general science may imply, just as Kant himself inferred, the *formal* nature of logic. Since this is exactly the inference Frege rejected,[16] it is not clear that Frege would accept Kant's view (as characterised by Putnam) that 'to say that thought, in the normative sense of *judgement which is capable of truth*, necessarily conforms to logic is not to say something which a metaphysics has to *explain*' (p. 247).[17] Nonetheless, it seems true that '[l]aws of logic are without content, in the Kant-and-possibly-Frege view, insofar as they do not *describe* the way things are or even the way they (metaphysically) *could* be' (p. 248). Even though Frege rejects Kant's *formal* view of logic that logic is abstracted from objects, Frege agrees with Kant in rejecting the view that

[16] See MacFarlane (2002).

[17] Putnam in fact declines to attribute this view to Frege (p. 247). See also Goldfarb (2001) who presents Frege as holding a different conception of logic.

logic is only a description of the way things are or could be. In the preface to the *Grundgesetze*, for example, Frege mobilises the distinction between descriptive and prescriptive laws and draws an important implication from this distinction concerning the nature of logical laws by emphasising the importance of prescriptive laws.

Putnam thinks that Wittgenstein expanded on this view of Kant and Frege. Putnam tries to put himself in Wittgenstein's shoes (as he puts it) by elaborating on the difference in status between logical and empirical laws. He invites us to consider the following three sentences:

(1) It is not the case that the Eiffel Tower vanished mysteriously last night and in its place there has appeared a log cabin.

(2) It is not the case that the entire interior of the moon consists of Roquefort cheese.

(3) For all statements p, $\ulcorner\neg(p \wedge \neg p)\urcorner$ is true. (p. 250)

He argues that there is a difference of methodological significance between these three sentences. (1) and (2) are empirical hypotheses. As such, we know how to show them as false or we can adopt a 'conceptual scheme' that falsifies them (though (2) is apparently harder than (1)). However, Putnam argues that we don't know how to even begin showing that (3) is false.

It is here that Putnam appeals to the Kant-Frege-Wittgenstein view of logic. He argues that, for them, logic is not a matter of what the world is or could be like. Instead, logic is prior to all rational activities. It is logic that sets the standard for rationality. Without the logical laws, no activity can be said to be rational. Thus, any thought which violates (3) cannot be counted as rational since such a thought cannot be accounted for by the standard for rationality.

Now, one can question the legitimacy of attributing the above view to Kant, Frege and Wittgenstein. One can even question the coherence of such a view. However, it should be noted that if this is what it is to make sense of logic, then we can easily make sense of paraconsistent logic. For in many (though not all) paraconsistent logics, it is a logical truth that $\neg(p \wedge \neg p)$ for any p and so we can respect the difference in status between (3), on one hand, and (1) and (2) on the other.[18] Thus, the Kant-Frege-Wittgenstein view of logic can be used to give sense to paraconsistent logic.[19]

[18]Notable paraconsistent logics in which $\neg(p\wedge\neg p)$ is not a logical truth are the *Logics of Formal Inconsistency* (*LFIs*). See, for example, Carnielli, Coniglio and Marcos (2007). My defense of paraconsistent logic doesn't extend to *LFIs*. I let the advocates of *LFIs* provide their own defense.

[19]I note that $\neg(p \wedge \neg p)$, in fact, fails to be a logical truth in *FDE*. But this is because of truth-value gap rather than truth-gap glut that *FDE* allows: p may be assigned no truth value in which

One may argue, nonetheless, that in paraconsistent logic, for some p, $\ulcorner\neg(p \wedge \neg p)\urcorner$ may be true and hence $\ulcorner\neg(p \wedge \neg p)\urcorner$ is false. Since we know that there are false instances of logical truths, we thus know how to 'falsify' (3). The difference between (1) and (2), on one hand, and (3), on the other, is supposed to be that one can know how to falsity (1) and (2) but not (3) and it is supposed to be this difference that separates empirical statements such as (1) and (2) and logical laws such as (3). Hence, so the critique goes, we cannot give a significance to (3) different from (1) and (2) by appealing to paraconsistent logic.

Regardless of how plausible the above critique may sound to classical logicians, it misrepresents the nature of paraconsistent logic. Even if there may be some p such that $\ulcorner\neg(p \wedge \neg p)\urcorner$ is false, (3) does not cease to be a logical truth in paraconsistent logic. In the case of paraconsistent logic, there can be a difference between a statement being false and not true. Consider, for example, *FDE*. Let μ be a relation between a propositional variable and a truth value. An *FDE*-evaluation is thus a *relation* rather than a *function*. Then we can express an evaluation of p to be true as $\langle p, 1\rangle \in \mu$ and false as $\langle p, 0\rangle \in \mu$. An evaluation of p as not true is expressed as $\langle p, 1\rangle \notin \mu$ which is not equivalent to $\langle p, 0\rangle \in \mu$ as μ is a relation. Showing that p is false is no longer the same as showing that p is not true.[20]

The important point of Putnam's thought is that (3) can not be falsified in the sense that there could not be any rational thought that does not conform to logical laws. The fact that there may be some p such that $\ulcorner\neg(p \wedge \neg p)\urcorner$ is false does not change that. For it is still the case that for all p, $\ulcorner\neg(p \wedge \neg p)\urcorner$ is true in paraconsistent logic.

If we are to focus on the normative nature of logical laws, therefore, we can give a sense to paraconsistent logic which is the same as the sense that Putnam would give to classical laws. If the Kant-Frege-Wittgenstein 'sense' can be attached to classical logic, we can also attach the same sense to paraconsistent logic.

5 Classical Logic Doesn't Make Sense!

Having shown that paraconsistent logic has an intelligible interpretation outside of its formal context, I now turn the tables and ask whether classical logic makes sense in the way that Putnam would see it. A logic is paraconsistent if it invali-

case $\neg(p \wedge \neg p)$ lacks truth value too. I take it that the concern of Putnam is with contradiction rather than indeterminacy of truth value. Even though *FDE* may not be the best paraconsistent logic to be used for the current purpose, it is the easiest to understand the nature of paraconsistency with and, hence, I have used *FDE* in my discussion. One can replace *FDE* with *LP* of Priest (1979) which imposes the *exhaustion* principle: for all p, either $\langle p, 1\rangle \in \mu$ or $\langle p, 0\rangle \in \mu$.

[20]This is also the case in *LP*.

dates *ex contradictione quodlibet* (ECQ): $\{A, \neg A\} \models B$ for any A and B.[21] Paraconsistency is thus a property that an inference may possess. Hence it must be distinguished from *dialetheism* which is a *metaphysical* view that there are true contradictions, since obtainability of a true contradiction in itself is a separate issue from the nature of an inference. With this in mind, we now examine whether or not classical logic actually makes sense.

Putnam's Kant-Frege-Wittgenstein view of logic focuses on the *formal* nature of logic. Frege (and possibly Wittgenstein) did not hold logic to be *formal* in Kant's sense (in fact Frege explicitly rejected Kant's formal view of logic). Nonetheless, Frege's view of logic, just like Kant's, does not involve a metaphysical presupposition. That is, logical laws are not derived from what there is or could be. Rather, they are conceived to be the standard for our rational thought: any activity that can count as rational can be held accountable to logical laws.

If this is the way to make sense of logical laws, what sense can we give to a thought that conforms to the classically valid inference $\{A \wedge \neg A\} \models B$ for any A and B? Can we claim it to be a *rational* thought? Consider a random thought as a result of assuming a contradiction. Could that random thought count as a rational thought?

It may be claimed that what ECQ encapsulates is not really the idea that a random thought based on assuming a contradiction is rational but the idea that a contradiction cannot be rationally obtained: ECQ encodes a 'warning' against having a contradiction in our thought or theory.[22] The issue of true contradiction, i.e., dialetheism, however, is a separate issue, as we saw above. It is primarily a metaphysical issue and not an issue about the nature of inferences as such. Moreover, according to Putnam's Kant-Frege-Wittgenstein view of logic, logic assumes no metaphysical presupposition. Thus, the issue of ECQ cannot be collapsed to that of dialetheism. But if we remove any metaphysical presuppositions from logical consideration just as Putnam claims Kant, Frege and Wittgenstein did, then it is hardly clear that the inference $\{A \wedge \neg A\} \models B$ for any A and B sets the standard for rationality. In fact, there are reasons to think that it is the thought that violates ECQ should count as rational. For example, consider the paradox of the preface. A rational person, after thorough research, writes a book in which they claim $A_1, ..., A_n$, but is aware that no book of any complexity contains only truths and so believes $\neg(A_1 \wedge ... \wedge A_n)$ too. It may be rational to revise these inconsistent beliefs. Nonetheless, no random thought should count as rational based on this inconsistent set of beliefs.

If ECQ does not set a standard for rationality and it is an integral part of

[21] Some paraconsistent logicians define this principle for *some A*. It does not make any difference for the purpose of this paper whether it is defined for *some* or *any A*.

[22] As far as I know, no one has explicitly formulated this claim on paper. However, the claim is often put to me by my colleagues, for example.

classical logic, classical logic cannot be made intelligible under Putnam's Kant-Frege-Wittgenstein conception of logic. There may be another sense that we can attach to classical logic. However, the onus is now on classical logicians to provide an intelligible conception of ECQ outside of the formal machinery. Given that ECQ is often claimed to be purely a '*formal*' principle separate from the truth of a claim (or soundness of an argument), as Lemmon, for example, claims,[23] it is not clear that classical logicians can provide such a conception. Thus, it is classical logic that has not been made sense of.

6 Conclusion

The quest in this paper has been to present paraconsistent logic as more than a mere mathematical tool. If it were to be accepted as a genuine logic, a paraconsistent logic must be made sense of. I have shown that there are senses that can be attached to paraconsistent logic. Along the way, I have shown that it is, in fact, explosive reasoning that has yet to be made sense of. It has been classical logicians who question the significance of paraconsistent logic. It is now time for paraconsistent logicians to question the legitimacy of classical logic.

References

Anderson, A.R. & N.D. Belnap JR. (1975) *Entailment: The Logic of Relevance and Necessity*, Vol. I, Princeton University Press, Princeton.

Anderson, A.R., N.D. Belnap JR. & J.M. Dunn (1992) *Entailment: The Logic of Relevance and Necessity*, Vol. II, Princeton University Press, Princeton.

Carnielli, W.A., M.E. Coniglio & J. Marcos (2007). 'Logics of Formal Inconsistency', *Handbook of Philosophical Logic*, Volume 14 (Second Edition), D. Gabbay and F. Guenthner (eds.), Springer, Berlin, pp. 15-107.

Cresswell, M.J. (1967) 'The Interpretation of Some Lewis Systems of Modal Logic', *Australasian Journal of Philosophy*, Vol. 45, pp. 198-206.

Dunn, J.M. (1976) 'Intuitive Semantics for First-Degree Entailment and 'Coupled Trees'', *Philosophical Studies*, Vol. 29, pp. 149-68, reappeared in *Entailment: The Logic of Relevance and Necessity*, Vol. II, A.R. Anderson, N.D. Belnap. Jr. and J.M. Dunn, Princeton University Press, Princeton, 1992.

Goldfarb, Warren D. (2001) 'Frege's Conception of Logic', *Future Pasts – The Analytic Tradition in Twentieth-Century Philosophy*, J. Floyd and S. Shieh (eds.), Oxford University Press, Oxford, pp. 25-41.

Haack, S. (1978) *Philosophy of Logic*, Cambridge University Press, Cambridge.

[23] See Lemmon (1965), pp. 60-61.

Hughes, G.E. & M.J. Cresswell (1996) *A New Introduction to Modal Logic*, Routledge, London.

Kripke, S.A. (1963) 'Semantical Analysis of Modal Logic I. Normal Modal Propositional Calculi', *Zeitschrifft fur mathematische Logik und Grundlangen der Mathematik*, Vol. 9, pp. 67-96.

Kripke, S.A. (1965) 'Semantical Analysis of Modal Logic II. Non-Normal Modal Propositional Calculi', *The Theory of Models: Proceedings of the 1963 International Symposium at Berkeley*, J.W. Addison, L. Henkin and A. Tarski (eds.), North-Holland Publishing Company, Amsterdam, pp. 206-20.

Lemmon, E.J. (1957) 'New Foundations for Lewis Modal Systems', *The Journal of Symbolic Logic*, Vol. 22, No. 2, pp. 176-86.

Lemmon, E.J. (1965) *Beginning Logic*, Thomas Nelson and Sons, London.

Priest, G. (1979) 'Logic of Paradox', *Journal of Philosophical Logic*, Vol. 8, pp. 219-241.

Priest, G. (1992) 'What is a Non-Normal World?', *Logique et Analyse*, Vol. 35, pp. 219-302.

Priest, G. and K. Tanaka (2009) 'Paraconsistent Logic', *Stanford Encyclopeadia of Philosophy* (Summer 2009 Edition), Edward N. Zalta (ed.), Stanford.

Putnam, Hilary (1994) 'Rethinking Mathematical Necessity', *Words & Life*, J. Conant (ed.), Harvard University Press, Cambridge, MA, pp. 245-263.

Routley, R. & R.K. Meyer (1972) 'The Semantics of Entailment - II', *Journal of Philosophical Logic*, Vol. 1, pp. 53-73.

Routley, R. & R.K. Meyer (1973) 'The Semantics of Entailment - I', *Truth, Syntax and Modality*, H. Leblanc (ed.), North-Holland, Amsterdam, pp. 199-243.

Routley, R. & V. Routley (1972) 'The Semantics of First Degree Entailment', *Noûs*, Vol. 6, pp. 335-59.

Tanaka, K. (2000) *The Labyrinth of Trees and the Sound of Silence: Topics in Dialetheism*, Ph.D Thesis, University of Queensland.

Urquhart, A. (1972) 'Semantics for Relevant Logics', *The Journal of Symbolic Logic*, Vol. 37, pp. 159-169.

Weber, Z. (2010a) 'Extensionality and Restriction in Naive Set Theory', *Studia Logica*, Vol. 94, pp. 109-126.

Weber, Z. (2010b) 'Transfinite Numbers in Paraconsistent Set Theory', *Review of Symbolic Logic*, Vol. 3, pp. 1-22.