Solving Normative Conflicts using Preference Relations

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Abstract

This article proposes a general strategy to overcome normative conflicts, namely, paradoxes represented in Standard Deontic Logic. This solution is based on preference relations between norms that circumvent situations of conflict. Pragmatic justifications of the proposed method are also given.

1 The Standard Deontic Logic

From now on, the following notation will be used:

- $\mathbb{L}$ is the propositional language over the connectives $\neg$, $\rightarrow$, $\vee$, $\wedge$;
- $\mathbb{L}_D$ is the propositional language extending $\mathbb{L}$ by adding the unary connectives O and P;
- PCL is the propositional classical logic defined over the language $\mathbb{L}$.

Consider the following list of axioms (schemes) and rules of inference:

(A0) All the instances (in the DL language) of PCL tautologies

(A1) $P\alpha \leftrightarrow \neg O \neg \alpha$

(A2) $O(\alpha \rightarrow \beta) \rightarrow (O\alpha \rightarrow O\beta)$

(A3) $O\alpha \rightarrow P\alpha$

(A4) $O\alpha \rightarrow OO\alpha$

(A5) $PO\alpha \rightarrow O\alpha$

(A6) $O(O\alpha \rightarrow \alpha)$

(A7) $O(PO\alpha \rightarrow \alpha)$

(R1) *modus ponens*: if α and $\alpha \rightarrow \beta$ then β

(R2) O-necessitation: if α is a theorem then $O\alpha$ is a theorem

As usual in the literature in the area, we will call SDL the logic consisting of A0-A3, R1 and R2.[1] One can note that the above axioms form a basis to define the well known logics OK, OM, OS4, OB, OS5, OK$^+$, OM$^+$, OS4$^+$, OB$^+$ and OS5$^+$, defined as follows (Assuming R1 and R2 for every system):

OK	=	A0-A2
OM	=	A0-A2, A6
OS4	=	A0-A2, A4, A6
OB	=	A0-A2, A6, A7
OS5	=	A0-A2, A4, A5 (hence A6 and A7 are theorems of OS5)

Let L be any logic above; so L$^+$ is the logic L plus the axiom A3.

With such settings in view, we can see that SDL is nothing else that the system OK$^+$, or the system D (also called KD) of modal logic (with the obvious differences in the interpretation of modalities). The observant reader may notice that the paradoxes presented below may be formalized using any of the ten systems above. Moreover, our proposed strategy is also applicable to them. The choice of SDL obeys to two reasons: firstly, SDL is the most notorious and studied system of deontic logic. Secondly, we believe

[1]Frequently P is not taken as a primitive connective and so A1 is presented as a linguistic definition.

that the intuitive notion of obligation is better, but not fully, represented by this logic.[2]

2 Some Normative Conflicts

2.1 The Chisholm Paradox

One could say that paradoxes and dilemmas related to deontic logic, as well as the efforts to overcome them, emerged right after the publication of von Wright's seminal paper of 1951. The Paradox of Chisholm [2] is considered by several authors as the most notable example of deontic paradoxes – so that some authors consider that any system of deontic logic unable of dealing with it is an inadequate tool to express deontic reasoning. Others (cf. [3]) suggest that this paradox was responsible for solidifying the status of deontic logic as a distinct logic from the normal modal logics. The fact is that Chisholm was right – and this is one of the few deontic logic areas in which there is a certain consensus: the type of conditional obligation expressed in [2] and other related paradoxes can not be satisfactorily expressed in SDL.

Several versions of the puzzle are known in the literature. We will consider the following version:

(1) It ought to be that John does not impregnate Suzy Mae.

(2) Not-impregnating Suzy Mae commits John to not marrying her.

(3) Impregnating Suzy Mae commits John to marry her.

(4) John impregnates Suzy Mae.

The set of sentences above can be formalized in SDL as follows:

(1') $O\neg i$

(2') $O(\neg i \to \neg m)$

(3') $i \to Om$

[2] This discussion is beyond the scope of this article, but it is of crucial importance to understand the motivations behind the present proposal – the reader only concerned with the formal part of this article may, without any problem, substitute the references to SDL to any of the ten systems presented here.

(4') i

Undesirable consequence: We obtain $O\neg m$ from 1' and 2' (by A2 in 2' and MP), and Oi from 3' and 4' (by MP). Thus, we have $Oi \wedge O\neg i$ and, by A3 (and A1), it follows $\perp$, so **we lost the consistency** of (1)-(4).[3]

2.2 The minor aged murderer paradox

This paradox was originally presented by Alchourrón in [1]. Consider the following normative system:

(i) The judges must punish the murderers.

(ii) The judges should not punish minors.

(i') $h \rightarrow Op$

(ii') $m \rightarrow O\neg p$

At first glance such a system does not seem to lead to a conflict of obligations. Any lawyer and even any reasonable person non expert in law would say that the normative system above is quite clear in saying that all murderers, unless the minor aged ones, should be punished. Moreover, the norm (ii) explicitly prohibits the punishment of minors. On the other hand, by SDL, we have that $(h \wedge m) \rightarrow Op$ and $(h \wedge m) \rightarrow O\neg p$, thus, in the case of a minor aged murderer, it is simultaneously obligatory (for the judges) to punish and not to punish.

3 The preference relations approach

3.1 Pragmatic Justifications

It is usual to believe that the legislator, that is, the agent that creates the rules of a particular normative system,[4] issues rules in order to make their

[3] One may note that there are two other ways of formalizing (1)-(4):
(1") $O\neg i$, (2") $O(\neg i \rightarrow \neg m)$, (3") $O(i \rightarrow m)$, (4") i
Undesirable consequence: 3" follows from 1" in SDL.
(1"') $O\neg i$, (2"') $\neg i \rightarrow O\neg m$, (3"') $i \rightarrow Om$, (4"') i
Undesirable consequence: 2"' follows from 4"' in SDL.
That is, in both cases **we lost the logical independence** of (1)-(4).

[4] Notice that the legislator has, usually, no "real existence", that is, such entity only designates the legislative authority that set the rules.

commands achieve certain objectives. In other words, the norms encourage (require or allow) the actions taken as means to achieve the objectives of the legislator or, otherwise, restrict those actions that might bring results qualified as "undesirable".

What underlies this assumption is the expectation, or even the requirement, that the legislator does not dictate arbitrary norms, without any purpose, nor dictate norms impossible – logically or physically – to be performed. For this reason, we call this legislator a **rational** one.

The identification of the reason (the purpose) of the legislator may involve a creative process of the interpreter (usually, the judge). This identification is made, for instance, by taking into account the possible results of the actions prescribed, as well as the (moral, ethics, etc) values accepted at the time of the interpretation or at the time of promulgation of the norm. In other words, there is no method free of subjectivity for the identification of the legislator purposes. However, what interest us here is not whether or not it can be identified such purposes, but how, in view of them, the interpreter can **adapt the normative system to tailor it to specific requirements of consistency**, namely, by introducing a preference relation between norms. Let us see now how such a relation can be defined.

3.2 Main concepts

Definition 3.1. A *norm* is a sentence in the language $\mathbb{L}_D$ of the form $O\beta$, $\alpha \to O\beta$ or $O(\alpha \to \beta)$, such that α and β are sentences in the language $\mathbb{L}$. The sentence α is called the *case of the norm*, and the sentence $O\beta$ is called the *normative solution* of the norm. The case of the norms of the form $O\beta$ is the sentence $\top$. ∎

We write $\mathsf{case}(N)$ and $\mathsf{ns}(N)$ to denote the case and the normative solution of a norm N, respectively. Notice that $\mathsf{ns}(O\beta) = O\beta$.

Definition 3.2. Let $\mathcal{N} = \{N_1, \ldots, N_k\}$ be a set with k norms such that $\mathsf{case}(N_i) = A_i$, for $1 \leq i \leq k$. A *relevance order of cases on* $\mathcal{N}$ is a strict partial order $\gg \subseteq \{A_1, \ldots, A_k\}^2$, that is, $\gg$ is antisymmetric and transitive (and, therefore, irreflexive). ∎

Definition 3.3. Let $\mathcal{N}$ be a set of norms and let $\gg$ be a relevance order of cases on $\mathcal{N}$. The *preference order of norms on* $\mathcal{N}$ *induced by* $\gg$ is the relation $\succ \subseteq \mathcal{N}^2$ given by:

$$N \succ N' \quad \text{iff} \quad \mathsf{case}(N) \gg \mathsf{case}(N').$$

■

Definition 3.4. A *situation* is a set Σ of sentences in $\mathbb{L}_D$ that contains a non empty set $\mathcal{N} \neq \emptyset$ of norms. A situation Σ is a *conflicting situation*, or a *deontic dilemma*, if there exist situations Σ_1 and Σ_2 contained in Σ and a sentence A in $\mathbb{L}$ such that:

- $\Sigma_1 \vdash_{SDL} OA$ but $\Sigma_1 - \{B\} \nvdash_{SDL} OA$, for any $B \in \Sigma_1$;
- $\Sigma_2 \vdash_{SDL} O\neg A$ but $\Sigma_2 - \{B\} \nvdash_{SDL} O\neg A$, for any $B \in \Sigma_2$;
- $\Sigma_1 \nvdash_{SDL} O\neg A$ and $\Sigma_2 \nvdash_{SDL} OA$. ■

Definition 3.5. Let Σ_1 and Σ_2 be situations contained in a situation Σ, let N be a norm in Σ_1 and let $\succ$ be the preference order of norms induced by a preference order of cases $\gg$ defined on the set of cases of Σ. The *relevance weight of the norm N compared to* Σ_2 is the natural number

$$\mathtt{rwn}(N, \Sigma_2) = card(\{N' \in \Sigma_2 \ : \ N \succ N'\}$$

(where $card(X)$ denotes the cardinal of a set X). In the same way, let $A \in \Sigma_1$ be a formula (not a norm) which is the case of some norm in Σ (not necessarily in Σ_1); the *relevance weight of a case A compared to* Σ_2 is the natural number

$$\mathtt{rwc}(A, \Sigma_2) = card(\{A' \in \Sigma_2 \ : \ A \gg A'\}).$$

■

Definition 3.6. Let Σ_1, Σ_2 and Σ be situations as in the previous definition. The *argumentative relevance weight of Σ_1 compared to* Σ_2 is the natural number

$$\mathtt{arw}(\Sigma_1, \Sigma_2) = \sum_{N \in \Sigma_1} \mathtt{rwn}(N, \Sigma_2) + \sum_{A \in \Sigma_1} \mathtt{rwc}(A, \Sigma_2).$$

■

3.3 Analyzing some paradoxes

3.3.1 The minor aged murderer paradox revisited

This paradox is simpler than the others, so let us examine it first. Recall subsection 2.2. One can note that the intention here is to punish all murderers, unless they are minors, that is, in the case of a minor aged murderer the second norm normative solution $O\neg p$ is preferred, since the fact of being minor aged is more relevant than being murderer. Formally, let N_1 and N_2 be the norms of (i') and (ii'), respectively. That is, N_1 is $h \to Op$ and N_2 is $m \to O\neg p$. So, according to the definitions of Section 3.2:

- $\mathsf{case}(N_1) = h$;
- $\mathsf{case}(N_2) = m$;
- $\mathsf{ns}(N_1) = Op$;
- $\mathsf{ns}(N_2) = O\neg p$.

Let $\mathcal{N} = \{N_1, N_2\}$. We define a relevance order of cases on $\mathcal{N}$ as

$$m \succ h.$$

Induced by it, we built a preference order of norms

$$N_2 \succ N_1.$$

Moreover, it is easy to prove that $\Sigma = \{N_1, N_2, h \wedge m\}$ is a conflicting situation, for $\Sigma_1 = \{N_1, h \wedge m\}$ and $\Sigma_2 = \{N_2, h \wedge m\}$. Note that:

- $\mathtt{rwn}(N_1, \Sigma_2) = 0$;
- $\mathtt{rwn}(N_2, \Sigma_1) = 1$;
- $\mathtt{rwc}(h \wedge m, \Sigma_2) = 0$;
- $\mathtt{rwc}(h \wedge m, \Sigma_1) = 0$.

Therefore

- $\mathtt{arw}(\Sigma_1, \Sigma_2) = 0$;
- $\mathtt{arw}(\Sigma_2, \Sigma_1) = 1$.

We conclude that the logical consequences of Σ_2 are more relevant than those of Σ_1 and so the normative solution $O\neg p$ prevails over Op.

3.4 The Chisholm paradox revisited

Recall subsection 2.1. Let N_1, N_2 and N_3 be the norms (1'), (2') and (3'), respectively. That is: N_1 is $O\neg i$, N_2 is $O(\neg i \rightarrow \neg m)$ and N_3 is $i \rightarrow Om$. Let $\mathcal{N} = \{N_1, N_2, N_3\}$. We define the relevance order of cases on $\mathcal{N}$ as

$$i \succ \neg i.$$

The induced preference order of norms is

$$N_3 \gg N_2.$$

Note that $\Sigma = \{N_1, N_2, N_3, i\}$ is a conflicting situation for $\Sigma_1 = \{N_1, N_2\}$ and $\Sigma_2 = \{N_3, i\}$. It is easy to see that

- $\texttt{rwn}(N_1, \Sigma_2) = 0$;
- $\texttt{rwn}(N_2, \Sigma_2) = 0$;
- $\texttt{rwn}(N_3, \Sigma_1) = 1$;
- $\texttt{rwc}(i, \Sigma_1) = 0$.

Therefore

- $\texttt{arw}(\Sigma_1, \Sigma_2) = 0$;
- $\texttt{arw}(\Sigma_2, \Sigma_1) = 1$.

We conclude that the logical consequences of Σ_2 are preferable to those of Σ_1 and so the normative solution Om prevails over $O\neg m$.

3.5 Preference and subjectivity

One can note that the definitions above require objective information (preference order of the cases, norms, etc). In this section we discuss the situation in which the information above is subjective, that is, provided by several agents. For this, given a group G of agents, for each agent $a \in G$ a relevance order of cases $\gg_a$ on a given set of norms $\mathcal{N}$ will be considered.

Definition 3.7. Let $\mathcal{N}$ be a set of norms and G a group of agents. For each agent $a \in G$ let $\gg_a$ be a relevance order of cases on $\mathcal{N}$. The preference order of norms $\succ_a$ on $\mathcal{N}$ of the agent a induced by $\gg_a$ is given as the definition 3.3. The functions $\mathtt{rwn}_a$, $\mathtt{rwc}_a$ and $\mathtt{arw}_a$ about the agent a are built over $\gg_a$ as the definitions 3.5 and 3.6, respectively. ■

As we've said, there is no method free of subjectivity to identify the purposes of the rational legislator. Thus, by changing the normative system to tailor it to specific requirements of consistency, that is, by introducing a relationship of preference between the norms, each agent can identify different purposes and, inevitably, introduce different orders of relevance and preference: so, different agents may get different normative solutions in a conflicting case.

4 Concluding Remarks

The use of relevance orders of cases as well as the induced preference relations between norms allows to define the notion of *argumentative relevance weight* $\mathtt{arw}(\Sigma_1, \Sigma_2)$ of a set Σ_1 compared to another set Σ_2 (cf. Definition 3.6). This parameter can be seen as a measure of the relevance of the premises of Σ_1 with respect to Σ_2 and vice-versa. Thus, if $\mathtt{arw}(\Sigma_1, \Sigma_2) > \mathtt{arw}(\Sigma_2, \Sigma_1)$ then the normative solutions inferred from Σ_1 are preferable to those derived from Σ_2. The case $\mathtt{arw}(\Sigma_2, \Sigma_1) > \mathtt{arw}(\Sigma_1, \Sigma_2)$ is symmetric. On the other hand, if $\mathtt{arw}(\Sigma_1, \Sigma_2) = \mathtt{arw}(\Sigma_2, \Sigma_1)$ then the inferences from Σ_1 and Σ_2 are equally relevant and so the normative conflict between both sets persist in this case.

We believe that argumentative relevance weight can be analyzed behind the scope of normative conflicts, as a general tool in theory of argumentation. Finally, the relationship between preference relations and Belief Revision (more specifically, revision of belief bases) deserves future research.

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