

# Definitely maybe.

## Quantifiers, modality and vagueness

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The philosophical literature abounds with works on the semantics and logic of modality, and the same can be said of the semantics and logic of vagueness. It comes as a surprise, therefore, that virtually no study is available concerning the interaction of modality and vagueness—especially since the interaction of multiple kinds of modality have been studied quite extensively.<sup>1</sup>

The goal of the present paper is to start filling that gap. Section 1 is a discussion of vague modal statements, with a specific focus on the different sources of indeterminacy. By far the most interesting and least dealt with case, as it turns out, is whether a modal statement could be vague as a result of modality’s being itself vague. It will be argued that it can, and that an implicit and unexpected defense of such a thesis is to be found in David Lewis’ modal realism. Section 2 puts forward a model theory for a first-order language featuring both operators expressing metaphysical modality and operators for semantic vagueness. The interpretation of metaphysical modalities is based on counterpart theory, whereas semantic vagueness is understood in terms of precisifications. The definition of the model theory is followed by a discussion of the resulting logic. In section 3, the framework will permit us to settle an open question. Barnes and Williams [1] have claimed that a language combining expressions for both vagueness (modeled via precisifications) and modality (modeled via possible worlds) would obey an overly revisionary logic, namely by making inconsistencies satisfiable. I will argue that the claim is unwarranted.

### 1 Modal vagueness

This section is a critical examination of the ways in which modal notions could be vague. By ‘modality’ I here mean *metaphysical* modality, unless

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<sup>1</sup>For instance, Segerberg [15], Thomason [18], Gabbay [4].

otherwise stated. In particular, I assume that metaphysical modalities are *absolute*, in the sense that, if it is possible that  $p$ , in any sense of ‘possible’, then it is metaphysically possible that  $p$ . For instance, since quantum teleportation is physically possible, then it is also metaphysically possible. Likewise, since it is not a historical necessity that the Archduke Franz Ferdinand of Austria had to be killed in Sarajevo, then it wasn’t metaphysically necessary, either. On the other hand, the physical impossibility of superluminal causation need not be understood as a metaphysical impossibility. When possibility is construed in terms of existential quantification over worlds, absolute possibility is *unrestricted* existential quantification over worlds. Relative possibility is restricted existential quantification over worlds.<sup>2</sup>

First of all, a modal statement can be vague by containing a vague *predicate*, simple or complex. Given a modal language, I take the semantic value of a predicate to be a set of *possibilia*, and the semantic value of a predicate at world  $w$  to be the restriction to  $w$  of its semantic value. Now, consider a community of sloppy chemists whose use of the term ‘hydrogen’ is indeterminate between two precise meanings: the element with atomic number 1 vs. an isotope of the element with atomic number 1 which has actually been observed. Since no isotope of hydrogen has ever been observed (in nature or in a lab) with more than six neutrons (viz., hydrogen-7), the following statement is semantically vague in sloppy-chemistese:

1. No hydrogen atom could possibly have seven neutrons

For, there is one sense of ‘hydrogen’ in sloppy-chemistese—the one agreeing with our own use of the term—which allows hydrogen atoms to have more neutrons than have ever been observed, and another sense which excludes such a possibility.

It is noteworthy that the occurrence of a vague predicate, simple or complex, in a modal statement will not automatically make that statement vague—just like, in general, the occurrence of a vague expression in a statement need not make the latter vague. To wit, it can be vague whether

2. Zach is bald

and yet definitely true that

3. it is contingent whether Zach is bald

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<sup>2</sup>On absolute modalities, see Hale [6].

In order to see that, suppose there is a range of precisifications ‘bald<sub>1</sub>’, ... ‘bald<sub>m</sub>’, ‘bald<sub>m+1</sub>’, ... ‘bald<sub>n</sub>’, such that Zach is ‘bald<sub>m</sub>’ but not ‘bald<sub>m+1</sub>’. In this scenario, (2) will indeed be vague. But, as long as logical space is sufficiently plentiful, for every  $i \leq n$ ,

4. it is contingent whether Zach is bald<sub>i</sub>

Hence, (3) is definitely true.

A further scenario is one in which modal statements are vague due to the nature of *intensional identity*. I will draw on Lewis [9] in construing intensional identity in terms of a counterpart relation, in such a way that ‘ $x$  is possibly  $P$ ’ is paraphrased as ‘for some world  $w$ , the counterpart of  $x$  at  $w$  is  $P$ ’. (I make no mention of accessibility here, since modality is taken to be absolute.) The counterpart of  $x$  at  $w$  is the individual which best represents  $x$  at  $w$  in terms of content and context.<sup>3</sup>

Here is an example of vague intensional identity. Consider a world of one-way eternal recurrence  $w$  such that each epoch is a duplicate of the history of the actual world. Insofar as  $w$  contains duplicates of actual Socrates (in fact, one for each epoch), the possibility of such a world makes it intuitively true that

5. Socrates could have lived in a world of one-way eternal recurrence so-and-so

where ‘so-and-so’ is short for the above description of  $w$ . But if the actual world had been  $w$ , in which epoch would have Socrates lived? It seems sensible to say that in some sense he could have lived in the first epoch, in some sense he could have lived in the second, etc. One way to accommodate this intuition within counterpart theory is to admit the existence of infinitely many duplicate worlds  $w_1, w_2, \dots$  of  $w$ , such that in  $w_1$  the counterpart of actual Socrates is the Socrates-duplicate in the first epoch, in  $w_2$  the counterpart of actual Socrates is the Socrates-duplicate in the second epoch, etc. Each of the following will then have to be true:

- 6.1. Socrates could have lived in the first epoch of a world of one-way eternal recurrence so-and-so
- 6.2. Socrates could have lived in the second epoch of a world of one-way eternal recurrence so-and-so

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<sup>3</sup>We can safely assume that the counterpart relation is reflexive. Unlike Lewis, I assume throughout that nothing has multiple counterparts at a world. I expect my choice to make sense in light of the following remarks.

etc.

Needless to say, each  $w_i$  will make (5) true as well. Nevertheless, this solution to the above *desideratum* entails haecceitism, since there will be worlds (infinitely many, in fact) that differ in a merely non-qualitative way, viz., with respect to which of the Socrates duplicates happens to be Socrates. Since not everybody is a friend of haecceitism, it would be desirable to accommodate the above intuition in a way that does not entail such a metaphysical position. Here is how. When we say that in some sense Socrates could have lived in the  $n$ th epoch of a world like  $w$ , for every  $n$ , in counterparts theory we do not have to express such scenarios by means of possibilities. We could instead mean something different, namely that for every Socrates-duplicate  $s_n$  in  $w$ , there is a way of making the counterpart relation precise that picks  $s_n$  out. Hence, with respect to the one and only  $w$ , one precisification of the counterpart relation associates actual Socrates to the Socrates-duplicate  $s_1$  in the first epoch, another precisification of the counterpart relation associates actual Socrates to the Socrates-duplicate  $s_2$  in the second epoch, etc. Each of the following will then be *vague*:

- 6.1. Socrates could have lived in the first epoch of a world of one-way eternal recurrence so-and-so
- 6.2. Socrates could have lived in the second epoch of a world of one-way eternal recurrence so-and-so
- ...

On the other hand, (5) will be true as per the original intuition, since it remains true under every precisification of the counterpart relation.<sup>4</sup>

A third potential source of modal vagueness are *quantifier-like expressions*. Garden-variety modal languages feature two kinds of quantifier-like expressions: modal operators, ranging over worlds, and first-order variable-binding quantifiers, ranging over world-bound individuals. I will now argue that there are indeed cases in which the vagueness of modal statements stems from indeterminacy about what worlds or individuals there are.

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<sup>4</sup>The problem of vague intensional identity is reminiscent of the well-known problem of relative intensional identity discussed in Lewis [10] [11, p. 248], Gibbard [5], Stalnaker [17]. The crucial difference between the two cases is that in the latter, but not in the former, fixing the context of utterance suffices to specify a counterpart relation. Many thanks to Maite Ezcurdia for helping me see this distinction.

Before proceeding, it is important to clarify one issue. First of all, quantification over worlds or world-bound individuals in the background language of counterpart theory can be restricted or absolute. In the case of world-bound individuals, a restricted quantifier is defined by an unrestricted quantifier and a sortal predicate. We will deal with vague unrestricted quantifiers in due course, whereas sortal predicates can be broken down to simpler constituents. Therefore, the case of vague restricted quantifiers does not need to be treated separately. As to restricted quantification over worlds, that expresses relative modality. Since we are only concerned with absolute (metaphysical) modality, this case is irrelevant for present purpose.

On the present counterpart-theoretic approach, what there is falls into two categories: worlds and world-bound individuals. Let's first consider the case in which a modal statement is vague because the domain of world-bound individuals is vague. Call *dyadism* the thesis that there are exactly two objects.<sup>5</sup> It should not be too controversial that dyadism is false. But is it at least possible? In other words, I am considering whether the following modal statement is true:

7. there could have been exactly two objects<sup>6</sup>

The answer will depend, among other things, on the underlying mereology. On the one hand, the mereological universalist believes in unrestricted composition. The range of her quantifier will therefore be closed under arbitrary fusions. In this sense of 'there is', it is impossible for there to be exactly two objects, provided that worlds are closed under fusions. At the other end of the mereological spectrum is the nihilist, denying the existence of proper parts and for whom a quantifier can only range over mereological atoms.<sup>7</sup> On the latter sense of 'there is', dyadism is possible in virtue of the existence of a world containing exactly two mereological atoms. So, as long as it can be indeterminate which mereology constrains our quantifiers, (7) will be vague. Notice that in the present case it is not vague what worlds there are, and yet it is vague what individuals there are at each world.<sup>8</sup> The moral is that modal vagueness can ensue if quantification over world-bound

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<sup>5</sup>Dyadism is modeled after monism, the thesis that there is exactly *one* object. Monism, which has famously been defended by Parmenides, should not be conflated with *priority monism*, the view that the world is prior to its parts, as advocated recently in Schaffer [14].

<sup>6</sup>This sentence can be regimented in purely-logical first-order vocabulary:  $\Diamond\exists x\exists y(\neg x = y \wedge \forall z(z = x \vee z = y))$ .

<sup>7</sup>For the sake of simplification, I am ignoring here the possibility of gunk.

<sup>8</sup>The whole discussion should be rephrased in terms of *concrete* individuals, if worlds are assumed to be closed under set-theoretic constructions or, more generally, if an in-

individuals is unsharp.

I now turn to the case in which operators expressing absolute modality are vague, which is the result of its being indeterminate what worlds there are. Consider the following example.<sup>9</sup> Let  $k$  be a universal constant occurring in some physical equation  $E(k)$  and satisfying the following two conditions:

- i) the value of  $k$  is *contingent*, so that there is a range of possible worlds which are obtainable by varying  $k$  in  $E$
- ii) the range of  $k$  is *bounded*, which is to say, there is an interval of possible values of  $k$

Now, if  $i$  is a value of  $k$  outside the interval, there exists a series from the actual value to the impossible value  $i$ . Consider a scenario in which scientists are unable to identify a sharp cutoff in the series. As a result, there ought to be some  $j$  between  $i$  and the actual value which is neither definitely possible nor definitely impossible. Consequently, it will be indeterminate whether there are worlds in which  $k$  takes on value  $j$ . Since this scenario makes it indeterminate what worlds there are, the range of modal operators will be vague. The statement

8. it is possible that both  $E(k)$  and  $k = j$

will then have to be vague, in virtue of its being true on some but not all senses of ‘possible’.

One might object that (8) can be interpreted as vague, but not in the intended way. For our goal was to show how modal vagueness can be traced to metaphysical modality itself. However, goes the objection, metaphysical modality is *absolute*, whereas the above example could equally be interpreted as providing an instance of vague *relative* modality. In order to see that, we could rephrase the story as follows. Let’s assume that absolutely every value of  $k$  is metaphysically possible, and yet there is an interval of  $k$ -values which defines the *physically* possible worlds (i.e., possible relative to the physical equation  $E(k)$ ). Statement (8) would then be definitely true, when ‘possible’ is unrestricted. But if we regard ‘possible’ as expressing physical

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finite number of abstracta exists by necessity. Otherwise, both the universalist and the mereological nihilist will regard (7) as trivially true.

<sup>9</sup>Something in the vicinity of this was suggested to me by Daniel Berntson in private conversation.

modality, then (8) will be vague.<sup>10</sup> The moral of the objection is that the story is underspecified. Unless we have independent reasons to rule out certain  $k$ -values as absolutely, rather than merely physically impossible, the above story is compatible with the weaker claim that physical modality is vague.

Whether the story could be further specified so as to avoid the above charges depends on our criteria for discriminating physical from metaphysical possibility, and in particular for identifying metaphysically possible worlds. Instead of replying directly to the objection, I will consider a new story which also aims to show that modality is vague, but which does not underdetermine whether the modality at issue is absolute or relative.

In order to guarantee that logical space be sufficiently plentiful, it is routine to assume the so-called *principle of plenitude*:

**PL:** Absolutely every way the world could be is a way a world is

However, the modal realist cannot appeal to such a principle, and for a simple reason. Since in modal realism ways a world is or could be are identified with worlds, **PL** would be tantamount to the logical truth: absolutely every world is a world.<sup>11</sup> Lewis responded by trying to capture plenitude with a *principle of unrestricted recombination*, which roughly says that every distribution of natural properties in space-time constitutes a world. The principle, on its intended application, entails

**UR:** For any objects in any worlds, there exists a world that contains any number of duplicates of all those objects<sup>12</sup>

The idea behind **UR** is that logical space should be closed under the operation of patching together copies of arbitrary collections of *possibilia* in a single world.

As it turns out, however, **UR** leads to paradox and therefore the modal realist cannot rely on it as a replacement for **PL**. The first *reductio* of **UR** was offered in Forrest and Armstrong [3], where it is argued that the principle is inconsistent with the assumption that the *possibilia* form a set. Nolan [12] has shown that, although the Forrest-Armstrong argument is invalid, a new and simpler proof is available, which goes as follows. Let  $k$  be the cardinality

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<sup>10</sup>If relative modalities are expressed model-theoretically by means of accessibility relations, as is customary, vague physical modality would then be modeled by an appropriately vague accessibility relation defined over a sharp domain of metaphysically possible worlds.

<sup>11</sup>Lewis [11, p. 86].

<sup>12</sup>Lewis [11, p. 88], Nolan [12, p. 239].

of the set of all *possibilia*. If  $a$  is an object, by **UR** there exists some world  $w$  containing  $2^k$  duplicates of  $a$ . But  $k < 2^k$  and yet the objects existing at  $w$  are a subset of all the *possibilia*. So, some *possibilia* are more than the whole. Hence, the *reductio*.

There are two main strategies available for blocking Nolan’s argument against **UR**. First, we could simply assume that the collection of all *possibilia* forms a proper class. In that case, there would be no cardinal  $k$  measuring its size, and the *reductio* would not go through. This is the road taken and defended by Nolan himself.<sup>13</sup> There is nevertheless a number of reasons for resisting the prospects of a class-sized universe. Probably the most obvious reason is that, since the modal realist identifies properties and relations with sets, and since proper classes are not members of any set, then proper class-sized properties will not have any second-order properties or relations. For instance, if the property of having mass is proper class-sized, then we won’t be able to say of that property that it is natural. In fact, there would be no (adequate) set-theoretic representative of naturalness and, therefore, no property of naturalness at all! This is of course a very unsavory outcome for the Lewisian. Nolan’s solution is to identify properties with universals, and second order properties with sets of universals. Although this approach reinstates the existence of all second-order properties, as desired, it comes at the cost of depriving modal realism of one of its main theoretical virtues, viz., its capacity to provide a nominalistic theory of properties.<sup>14</sup>

I now turn to the second strategy for resisting Nolan’s *reductio*, which is to weaken **UR**. As it turns out, this option will provide us with the instance of modal vagueness that we are seeking. A way of restricting recombination, which was put forward in Lewis [11, p. 89] and developed in Divers [2, p. 102], is to impose an upper bound to the number of objects which can coexist in any single world. The Lewis-Divers *principle of restricted recombination* states that every distribution of natural properties in space-time constitutes a world, *shape and size permitting*. The restricted principle, on its intended application, entails:<sup>15</sup>

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<sup>13</sup>Nolan [12, p. 248].

<sup>14</sup>For the sake of completeness, I should mention that Nolan in fact proposes a second solution which does not involve commitment to universals. However, this alternative approach requires that “all and only the natural properties possess singletons”. It is questionable whether the extent of set theory should be sensitive to such metaphysical distinctions, especially since it is unclear whether there is a sharp cutoff for the (perfectly) natural properties.

<sup>15</sup>It is noteworthy that the statement of restricted recombination in Divers [2, p. 102] is unduly restricted to duplicates of two objects  $x$  and  $y$ . It is here generalized to duplicates of pluralities, as it should be.



**RR:** There is a cardinality  $k$  which is the size of all the *possibilia*, and for any objects in any worlds, there is a world that contains any number of duplicates of all those objects, as long as the total number of such duplicates does not exceed  $k$

Clearly, **RR** does not entail the existence of a world containing more things than there are absolutely.

One issue which must be considered now is whether the quantifier expression ‘there exists a cardinality  $k$ ’ in **RR** can be instantiated. Suppose it cannot. Since the background logic is precisificational, that would mean that the value of  $k$  must be vague. In other words, the statement ‘there exists a cardinality  $k$  which is the size of all the *possibilia*’ is true—and yet, for every instance  $k_n$ , the statement ‘ $k_n$  is the size of all the *possibilia*’ is untrue. This scenario is analogous to that of the proverbial heap of sand. In that case, the statement ‘there is some number  $n$  which is the least number such that  $n$  grains of sand constitute a heap’ is true, since each precisification of ‘heap’ determines a cutoff. On the other hand, the value of the cutoff varies across precisifications and, therefore, no instance of that existential statement is going to be true. Back to **RR**, if no  $k_n$  is such that it is definitely the size of the set of all *possibilia*, there will have to be multiple candidate values  $k_{n_1} \dots k_{n_j}$ . Pick one of them, say  $k_{n_1}$ . Then it is vague whether there are worlds containing  $k_{n_1}$  copies of a given object. Therefore, it will be vague what worlds there are, absolutely. As a consequence, the statement

9. there could possibly exist  $k_{n_1}$  duplicates of the Tower of Pisa

will be vague in virtue of the vagueness of ‘possibly’.

Suppose, instead, that existential instantiation can be performed on **RR**. It is reasonable to assume, without loss of generality, that the value of  $k_n$  is some uncountable cardinal. In particular, let’s suppose that  $k_n$  is the smallest uncountable cardinal  $\aleph_1$ . If the background set theory is defined by the standard Zermelo-Fraenkel axiom system, then the Continuum Hypothesis cannot be either proved or disproved, which is to say, it is indeterminate whether  $\aleph_1 < 2^{\aleph_0}$ .<sup>16</sup> Consequently, it must be vague whether there are worlds containing  $2^{\aleph_0}$  copies of a given object—and, so, what worlds there are, absolutely. The modal statement

10. there could possibly exist  $2^{\aleph_0}$  duplicates of the Tower of Pisa

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<sup>16</sup>Strictly speaking, the Platonist about sets will believe that the Continuum Hypothesis has a determinate truth value despite its independence of Zermelo-Fraenkel set theory. I will leave this further problem aside.

will be vague in virtue of the vagueness of ‘possibly’.

Suppose instead that  $k_n = \aleph_{\alpha+1}$ , for some ordinal  $\alpha > 1$ . Insofar as the Generalized Continuum Hypothesis is also indeterminate in Zermelo-Fraenkel set theory, it will be indeterminate whether  $\aleph_{\alpha+1} < 2^{\aleph_\alpha}$ . Hence, it is vague whether at some world there are  $2^{\aleph_\alpha}$  copies of a given object—for instance, whether

11. there could possibly exist  $2^{\aleph_\alpha}$  duplicates of the Tower of Pisa

We can conclude that, whether the main existential quantifier in **RR** can be instantiated or not, it will be indeterminate what worlds there are, absolutely. It is worth noting the difference between the present case, in which it is vague what worlds there are, and the previous case of vague variable-binding quantifiers, in which it was determinate what worlds there are, but vague what individuals there are at each world.

A related issue must be raised at this juncture. I just argued that quantifier expressions in the modal language, namely quantifiers proper and modals, can be vague. I have done so by exhibiting cases in which (i) quantification over worlds and world-bound individuals in the language of counterpart theory is vague and (ii) such quantifiers are absolute, as they range unrestrictedly over all worlds and *possibilia*. Moreover, I have been assuming throughout that (iii) vagueness is analyzed via precisifications.

My case, however, runs counter to an argument put forward in Sider [16], which aims to show that

- (V) if vagueness is given a precisificational account and existence is expressed by the unrestricted existential quantifier, then existence cannot be vague

If the argument for (V) is sound, the above conditions (i)-(iii) are bound to be jointly inconsistent. Nevertheless, Torza [19] has argued that Sider’s argument is compatible with a weak form of vague existence. Let us take a closer look at the dialectics.

Sider’s alleged proof has the form of a *reductio ad absurdum*. Suppose that

P.  $\ulcorner \exists x\phi \urcorner$  is vague

(where  $\phi$  is precise). As long as  $\exists$  is absolute and vagueness is construed via precisifications, it can be shown that (P) entails an inconsistency. At this point Sider applies *reductio* and infers that (P) is false. As remarked in

Williamson [20, p. 152], however, *reductio ad absurdum* is valid for bivalent languages. In this particular case, therefore, we may infer the falsity of (P) if the metalanguage of  $\ulcorner \exists x\phi \urcorner$  is precise. But notice that (P) is equivalent to

P'. In some precisification  $\ulcorner \exists x\phi \urcorner$  is true and in some precisification  $\ulcorner \exists x\phi \urcorner$  is false

which involves quantification over precisifications of the language of  $\ulcorner \exists x\phi \urcorner$ . If the *set of* precisifications is not itself precise, *reductio* may not be applied. All we could infer, then, is that (P) is untrue, i.e., either false or vague—an instance of *weak reductio* (cf. Keefe [8, p. 180]). In order to complete the original *reductio*, Sider would now have to show that (P) is not vague, i.e., that  $\ulcorner \exists x\phi \urcorner$  is not second-order vague. Torza [19] shows how to set up a *reductio* of second-order vague existence, Sider style. But if a *reductio* of vague existence presupposes that the metalanguage of the quantifier  $\exists$  be precise, likewise a *reductio* of second-order vague existence presupposes that the meta-metalanguage of  $\exists$  be precise. And so forth and so on. The upshot is that neither side has the upper hand. In particular, we have no reasons to rule out the possibility that existence be vague at all orders—i.e., vague, and second-order vague, and third order vague, etc. Following Torza [19], I call *super-vague* any instance of quantification which is vague at all orders in this sense. Accordingly, whenever I speak here of vague existence and modality, I actually mean super-vague existence and modality.

Now that the issue of the coherence of vague quantification (albeit in a weak form) has been cleared up, we can conclude that modal languages have at least four possible sources of vagueness: predication, intensional identity, quantifiers and modals. In the next Section I turn to the second goal of this paper, namely to work out a model theory for languages containing both modal operators and vagueness operators that accommodates the observation from this Section.

## 2 Modal vagueness, regimented

### 2.1 Supervaluational counterpart semantics

Modal languages, when sharp, can be interpreted by means of *counterpart models*.<sup>17</sup> If the object language is vague, however, vanilla counterpart models are inadequate. What we need are structures with multiple precisifications, each of which will itself be a counterpart model. While in standard

<sup>17</sup>The *loci classici* of semantics based on counterparts are Lewis [9], Hazen [7].

counterpart semantics sentences are evaluated at a world  $w$ , in the super-valuationary case we want to evaluate sentences at a pair  $\langle s, w \rangle$ , where  $s$  is a precisification and  $w$  a world index. The elements of a model that are allowed to vary across precisifications will define which parts of the modal language are unsharp. Given what has been said in Section 1, we want non-logical constants to vary across precisifications, so as to allow for vague predication. We want the counterpart relation to vary, too, in order to represent the vagueness of intensional identity. We want the domain of a world to be able to vary across precisifications, if existence is to be vague. Finally, we want the whole set of worlds itself to vary across precisifications, to account for vague modality.

In order to meet the above desiderata, I start by defining a *supervaluationary counterpart frame* (*SC-frame*), which is a structure  $\mathcal{F} = \langle Q, @, R, U, Dom, c \rangle$ , where

- $Q \subseteq S \times W$ , for  $S, W$  disjoint sets
- $\langle s, @ \rangle \in Q$ , for every  $s \in S$
- $R \subseteq Q^2$  s.t.  $\langle s, w \rangle R \langle s', w' \rangle \rightarrow s = s'$
- $U$  is a set disjoint from  $S$  and  $W$
- $Dom : Q \rightarrow \mathcal{P}(U)$  s.t.
  - if  $w \neq w'$ , then  $Dom(\langle s, w \rangle) \cap Dom(\langle s', w' \rangle) = \emptyset$
  - $U = \bigcup_{\langle s, w \rangle \in Q} Dom(\langle s, w \rangle)$
- $c : U \times Q \rightarrow U$  s.t.
  - $c(a, \langle s, w \rangle) \in Dom(\langle s, w \rangle)$
  - if  $a \in Dom(\langle s, w \rangle)$  and  $b = c(a, \langle s', w' \rangle)$ , then  $s = s'$
  - if  $a \in Dom(\langle s, w \rangle)$ , then  $c(a, \langle s, w \rangle) = a$
  - if  $a, b \in Dom(\langle s, w \rangle)$  and  $a \neq b$ , then  $c(a, \langle s, w' \rangle) \neq c(b, \langle s, w' \rangle)$

A few comments are in order.  $S$  and  $W$  are sets of indices for precisifications and worlds, respectively, in such a way that each coordinate  $\langle s, w \rangle$  is identified with a world-in-a-precisification (or, simply, a world). The reason why  $\mathcal{F}$  is defined on  $Q$ , rather than the whole product-set  $S \times W$ , is that a world-coordinate  $w$  may pick out a world at some precisification  $s$  but not at some other  $s'$ . This fact captures the idea that the set of worlds, over which unrestricted modal operators range, can be vague.

Each precisification  $s$  will feature an actual world  $\langle s, @ \rangle$ .

$R$  is the accessibility relation, which relates worlds to worlds within the same precisification. Since we are interested here in absolute modalities, from now on I will simply assume that  $R$  is universal (viz.,  $\langle s, w \rangle R \langle s, w' \rangle$ , for every  $s, w, w'$ ) and omit any reference to it altogether.

$U$  are the individuals.

$Dom$  maps each world  $\langle s, @ \rangle$  to a set of world-bound individuals, and every individual exists at some world.

The function  $c$  assigns to each individual a counterpart at every world within the same precisification, so that distinct world-mates have distinct counterparts at any given world. Notice that the assumption that everything has a counterpart at every world (within the same precisification) is arguably too strong. For instance, it is reasonable to assume that some worlds are so radically different from ours that nothing over there could ever represent, say, actual Socrates. Nevertheless, for the sake of simplicity I will stick to the present choice, with the proviso that, in a fully adequate semantics, an individual may fail to have counterparts at some world.

Now, let  $\mathcal{L}$  be a first-order language endowed with identity and an infinite set of  $n$ -ary predicate constants, for each  $n > 0$ . The expansion of  $\mathcal{L}$  with the sentential necessity operator  $\Box$  (definiteness operator  $\Delta$ ) is referred to as  $\mathcal{L}_\Box$  ( $\mathcal{L}_\Delta$ ). The union of  $\mathcal{L}_\Box$  and  $\mathcal{L}_\Delta$  is  $\mathcal{L}_{\Box\Delta}$ . In  $\mathcal{L}_\Box$  the possibility operator is defined by the condition  $\Diamond\phi := \neg\Box\neg\phi$ . In  $\mathcal{L}_\Delta$ , the ‘in some sense’ operator  $\nabla$  is defined by  $\nabla\phi := \neg\Delta\neg\phi$ . The vagueness operator  $I$  is defined by  $I\phi := \nabla\phi \wedge \nabla\neg\phi$ .

A *supervaluationary counterpart model* (*SC-model*) is a structure  $\mathcal{M} = \langle \mathcal{F}, \sigma \rangle$  where

- $\mathcal{F}$  is a SC-frame
- For every  $\langle s, w \rangle \in Q$ ,
  - $\sigma(=, \langle s, w \rangle)$  is the identity relation over  $Dom(\langle s, w \rangle)$
  - $\sigma(P, \langle s, w \rangle) \subseteq Dom(\langle s, w \rangle)^n$ , for every  $n$ -ary predicate constant  $P$

Given the set  $VAR$  of variables in a language, a *value assignment* for  $VAR$  over  $\mathcal{M}$  is a set of partial functions  $\{\xi_s\}_{s \in S}$  s.t.

- $\xi_s : VAR \rightarrow Dom(\langle s, @ \rangle)$
- $\bigcup_{s \in S} \xi_s$  is a total function  $f : VAR \rightarrow \bigcup_{s \in S} Dom(\langle s, @ \rangle)$

- if  $\xi_s(x)$  and  $\xi_t(x)$  are both defined, then  $\xi_s(x) = \xi_t(x)$

The choice of breaking down an assignment for the variables into a set of partial functions aims to capture the idea that, since existence is vague, a variable may or may not successfully refer, depending on a particular precisification.<sup>18</sup>

*Local truth*, i.e. truth at a world-in-a-precisification  $\langle s, w \rangle \in Q$  in  $\mathcal{M}$  under  $\{\xi_s\}_{s \in S}$  is defined thus:

1. if  $\phi = P(x_1 \dots x_n)$ , then  $(\mathcal{M}, \langle s, w \rangle, \{\xi_s\}_{s \in S}) \models \phi$  iff  $c(\xi_s(x_i), \langle s, w \rangle)$  is defined for all  $i \in \{1, \dots, n\}$  and  $\langle c(\xi_s(x_1), \langle s, w \rangle) \dots c(\xi_s(x_n), \langle s, w \rangle) \rangle \in \sigma(P, \langle s, w \rangle)$
2. if  $\phi = \neg\psi$ , then  $(\mathcal{M}, \langle s, w \rangle, \{\xi_s\}_{s \in S}) \models \phi$  iff  $(\mathcal{M}, \langle s, w \rangle, \{\xi_s\}_{s \in S}) \not\models \psi$
3. if  $\phi = \psi \wedge \chi$ , then  $(\mathcal{M}, \langle s, w \rangle, \{\xi_s\}_{s \in S}) \models \phi$  iff  $(\mathcal{M}, \langle s, w \rangle, \{\xi_s\}_{s \in S}) \models \psi$  and  $(\mathcal{M}, \langle s, w \rangle, \{\xi_s\}_{s \in S}) \models \chi$
4. if  $\phi = \forall x \psi$ , then  $(\mathcal{M}, \langle s, w \rangle, \{\xi_s\}_{s \in S}) \models \phi$  iff, for every  $\{\xi'_s\}_{s \in S}$  such that  $\xi'_s$  is defined on  $x$  and differs from  $\xi_s$  at most on  $x$ ,  $(\mathcal{M}, \langle s, w \rangle, \{\xi'_s\}_{s \in S}) \models \psi$
5. if  $\phi = \Box \psi$ , then  $(\mathcal{M}, \langle s, w \rangle, \{\xi_s\}_{s \in S}) \models \phi$  iff, for every  $\langle s, w' \rangle \in Q$ ,  $(\mathcal{M}, \langle s, w' \rangle, \{\xi_s\}_{s \in S}) \models \psi$
6. if  $\phi = \Delta \psi$ , then  $(\mathcal{M}, \langle s, w \rangle, \{\xi_s\}_{s \in S}) \models \phi$  iff, for every  $\langle s', w \rangle \in Q$ ,  $(\mathcal{M}, \langle s', w \rangle, \{\xi_s\}_{s \in S}) \models \psi$

One issue we were faced with in definition of local semantics is how to evaluate an atomic formula  $P(x)$  at a precisification where  $x$  is non-referring. The present framework always assigns ‘false’ to such formulas. As a consequence, local truth defines a *negative free semantics*.<sup>19</sup>

We can finally define the notions of truth-in-a-model, logical consequence and validity as follows.

$\phi$  is *true in  $\mathcal{M}$  under  $\{\xi_s\}_{s \in S}$*  ( $(\mathcal{M}, \{\xi_s\}_{s \in S}) \models \phi$ ) iff, for every  $s \in S$ ,  $(\mathcal{M}, \langle s, @ \rangle, \{\xi_s\}_{s \in S}) \models \phi$ .

$\phi$  is *true in  $\mathcal{M}$*  ( $\mathcal{M} \models \phi$ ) iff, for every  $\{\xi_s\}_{s \in S}$ ,  $(\mathcal{M}, \{\xi_s\}_{s \in S}) \models \phi$ .

$\phi$  is a *consequence* of  $\Gamma$  ( $\Gamma \models \phi$ ) iff, for every  $\mathcal{M}$ , if  $\mathcal{M} \models \Gamma$  then  $\mathcal{M} \models \phi$

<sup>18</sup>This definition of an assignment for the variables is developed in Torza [19].

<sup>19</sup>For a motivation and discussion, see Torza [19]. For an elucidation of free logics, see Nolt [13].

$\phi$  is *valid* ( $\models \phi$ ) iff, for every  $\mathcal{M}$ ,  $\mathcal{M} \models \phi$

It is noteworthy that SC-frames could be enriched by adding an *admissibility relation*  $A$ , where  $A \subseteq Q^2$  and  $\langle s, w \rangle A \langle s', w' \rangle \rightarrow w = w'$ . The truth condition (6) for formulas governed by  $\Delta$  in a SC-model would then have to be revised accordingly:

6'. if  $\phi = \Delta\psi$ , then  $(\mathcal{M}, \langle s, w \rangle, \{\xi_s\}_{s \in S}) \models \phi$  iff, for every  $\langle s', w' \rangle \in Q$  s.t.  $\langle s, w \rangle A \langle s', w' \rangle$ ,  $(\mathcal{M}, \langle s', w' \rangle, \{\xi_s\}_{s \in S}) \models \psi$

In fact, such a revision in the definition of SC-models is not only possible but even necessary in the light of what has been said in Section 1 concerning vague quantification. Indeed, recall that absolute quantifiers can be vague as long as the vagueness extends to all orders, which is to say, as long as the quantifiers are super-vague. Clearly, this idea can be captured only in models that allow for higher-order vagueness. On the other hand, supervaluational models without an admissibility relation, or in which admissibility is an equivalence relation, do not admit of high-order vagueness, since they validate the schema  $I\phi \rightarrow \Delta I\phi$ . We must conclude that SC-models in which object-language quantifiers and modals are super-vague require an admissibility relation  $A$  which is not reflexive, symmetric and transitive. (Williamson [21] and Torza [19] argue that the most natural approach is to drop transitivity.) Nevertheless, I will refrain from adding the admissibility relation  $A$  as suggested, in attempt to simplify the model theory.

## 2.2 Logic

What is the logic of a language  $\mathcal{L}_{\square\Delta}$  whose behavior is defined by SC-semantics? I am going to break down the question into four subproblems. I will first consider a set of  $\mathcal{L}$ -theses, i.e., schemata and rules of inference which can be formulated in the extensional sub-language  $\mathcal{L}$ , and check which of them are validated in  $\mathcal{L}_{\square\Delta}$ . I will then repeat the test with respect to a set of  $\mathcal{L}_{\square}$ -theses, which are the purely modal theses. I will next consider a set of  $\mathcal{L}_{\Delta}$ -theses, schemas and inference rules that usually hold on a supervaluational interpretation of  $\mathcal{L}_{\Delta}$ . Finally, I consider the  $\mathcal{L}_{\square\Delta}$ -theses, which can only be formulated in a language with both modal and definiteness operators.

### 2.2.1 $\mathcal{L}$ -logic

Let us establish which schemas and inference rules, which can be formulated in an extensional first-order language  $\mathcal{L}$ , hold in the expanded language  $\mathcal{L}_{\Box\Delta}$ .

Let  $\phi$  be a  $\mathcal{L}_{\Box\Delta}$ -formula. Note that every atomic  $\mathcal{L}_{\Box\Delta}$ -formula is either locally true or locally false (under an assignment), and that sentential connectives are defined classically. Therefore, if  $\phi$  is a *classical tautology* and  $\mathcal{M}$  a SC-model,  $(\mathcal{M}, \langle s, @ \rangle, \{\xi_s\}_{s \in S}) \models \phi$ , for every  $s \in S$ . Hence,

**TAUT.**  $\models \phi$ , if  $\phi$  is a classical tautology

Moreover, *Modus Ponens* holds:

**MP.**  $\phi, \phi \rightarrow \psi \models \psi$

Other classical inference forms, however, are invalid in supervaluational counterpart semantics. As discussed in Keefe [8], *reductio ad absurdum*, *contraposition*, *conditional proof* and *argument by cases* typically fail in supervaluationism. Nevertheless, weakened versions of those forms of inference hold in general in supervaluationism and specifically in SC-semantics, namely:

**RA.** if  $\Gamma, \phi \models \perp$ , then  $\Gamma \models \neg\Delta\phi$

**CON.**  $\phi \models \psi$ , then  $\neg\psi \models \neg\Delta\phi$

**CP.** if  $\Gamma, \phi \models \psi$ , then  $\Gamma \models \Delta\phi \rightarrow \psi$

**AC.** if  $\phi \models \chi$  and  $\psi \models \chi$ , then  $\Delta\phi \vee \Delta\psi \models \chi$

A discussion and defense of these quasi-classical inference forms from a supervaluational point of view is put forward in Keefe [8, p. 179].

Let us now turn to quantified logic. As I had remarked in Section 1 already, classical *existential instantiation* fails in supervaluational frameworks. For  $\exists x\phi$  can be true at all precisifications, and yet there may be no value of  $x$  which makes  $\phi$  true at all of them. This fact remains true in supervaluational counterpart semantics.

*Existential generalization*, which is instead a typically valid form of inference in supervaluational semantics, fails in the present framework, too. For example, for  $P$  a non-logical constant, it could be that  $\neg P(x)$  is true in a model (under an assignment), whereas  $\exists x\neg P(x)$  is untrue. In order to see that, just consider a model in which the variable  $x$  is undefined at  $\langle s, @ \rangle$ , for some  $s$ , and has a value in the anti-extension of  $P$  at  $\langle s', @ \rangle$ , for every other  $s'$ . The failure of classical existential generalization is clearly due to the fact



that local truth is defined in terms of negative free semantics. As it turns out, it can be proven by induction on the complexity of  $\phi$  that a weaker form of existential generalization, typical of free logics, holds in supervaluational counterpart semantics:

$$\exists\mathbf{G}. \phi(x), \exists y(x = y) \models \exists x\phi(x)$$

It is easy to show the *equivalence of self identity and existence*:

$$\mathbf{EX}. x = x \leftrightarrow \exists y(x = y)$$

Note that the first-order axiom  $x = x$ , expressing the *reflexivity of identity*, fails. However, the weaker, quantified version is valid:

$$\mathbf{SI}. \forall x(x = x)$$

*Leibniz' Law* is valid in the quantifier-free form:

$$\mathbf{LL}. x = y \rightarrow (\phi(x) \rightarrow \phi(y))$$

In fact, a stronger principle holds:

$$\mathbf{LL}^+. \diamond\nabla x = y \rightarrow (\phi(x) \rightarrow \phi(y))$$

The two laws  $\mathbf{LL}$  and  $\mathbf{LL}^+$  can be proved concurrently by induction on the complexity of  $\phi$ .

*Proof of  $\mathbf{LL}$  and  $\mathbf{LL}^+$ .* I will show only the most interesting cases of the induction. Reference to a fixed model  $\mathcal{M}$  is left implicit throughout.

1. Let  $\phi(x) = P(x)$ . Assume  $(\{\xi_s\}_{s \in S}, \langle s, w \rangle) \models P(x)$ .
  - 1.1 If  $(\{\xi_s\}_{s \in S}, \langle s, w \rangle) \models x = y$ , then  $\langle c(\xi_s(x), \langle s, w \rangle), c(\xi_s(y), \langle s, w \rangle) \rangle \in \sigma(=, \langle s, w \rangle)$ , i.e.,  $c(\xi_s(x), \langle s, w \rangle) = c(\xi_s(y), \langle s, w \rangle)$ . Since  $c(\xi_s(x), \langle s, w \rangle) \in \sigma(P, \langle s, w \rangle)$ , then  $c(\xi_s(y), \langle s, w \rangle) \in \sigma(P, \langle s, w \rangle)$ , and so,  $(\{\xi_s\}_{s \in S}, \langle s, w \rangle) \models P(y)$ .
  - 1.2 If instead  $(\{\xi_s\}_{s \in S}, \langle s, w \rangle) \models \diamond\nabla x = y$ , then  $(\{\xi_s\}_{s \in S}, \langle s', w' \rangle) \models x = y$ , for some  $\langle s', w' \rangle \in Q$ , and so  $c(\xi_{s'}(x), \langle s', w' \rangle) = c(\xi_{s'}(y), \langle s', w' \rangle)$ . Since  $c$  is 1-1, then  $\xi_{s'}(x) = \xi_{s'}(y)$ . Because  $P(x)$  is atomic,  $\xi_s(x)$  is defined. It follows that  $\xi_s(y)$  is also defined and  $\xi_s(x) = \xi_s(y)$ . Thus,  $(\{\xi_s\}_{s \in S}, \langle s, w \rangle) \models x = y$ . By (1.1),  $(\{\xi_s\}_{s \in S}, \langle s, w \rangle) \models P(y)$ .
2. Let  $\phi(x) = \Box\psi(x)$ . Assuming  $(\{\xi_s\}_{s \in S}, \langle s, w \rangle) \models \Box\psi(x)$ , take any  $\langle s, w' \rangle \in Q$ .

- 2.1 If  $(\{\xi_s\}_{s \in S}, \langle s, w \rangle) \models x = y$ , then  $\xi_s(x) = \xi_s(y)$  and, so,  $(\{\xi_s\}_{s \in S}, \langle s, w' \rangle) \models x = y$ . Since  $(\{\xi_s\}_{s \in S}, \langle s, w' \rangle) \models \psi(x)$ , by inductive hypothesis  $(\{\xi_s\}_{s \in S}, \langle s, w' \rangle) \models \psi(y)$ , thus  $(\{\xi_s\}_{s \in S}, \langle s, w \rangle) \models \Box\psi(y)$ .
- 2.2 If instead  $(\{\xi_s\}_{s \in S}, \langle s, w \rangle) \models \Diamond\nabla x = y$ , then  $(\{\xi_s\}_{s \in S}, \langle s', w'' \rangle) \models x = y$  for some  $\langle s', w'' \rangle \in Q$ . Thus,  $(\{\xi_s\}_{s \in S}, \langle s, w' \rangle) \models \Diamond\nabla x = y$ . By inductive hypothesis,  $(\{\xi_s\}_{s \in S}, \langle s, w' \rangle) \models \psi(y)$ , and therefore  $(\{\xi_s\}_{s \in S}, \langle s, w \rangle) \models \Box\psi(y)$ .
3. Let  $\phi(x) = \Delta\psi(x)$ . Assuming  $(\{\xi_s\}_{s \in S}, \langle s, w \rangle) \models \Delta\psi(x)$ , take any  $\langle s', w \rangle \in Q$ .
- 3.1 If  $(\{\xi_s\}_{s \in S}, \langle s, w \rangle) \models x = y$ , then  $(\{\xi_s\}_{s \in S}, \langle s', w \rangle) \models \nabla x = y$  and so, trivially,  $(\{\xi_s\}_{s \in S}, \langle s', w \rangle) \models \Diamond\nabla x = y$ . Since  $(\{\xi_s\}_{s \in S}, \langle s', w \rangle) \models \psi(x)$ , by inductive hypothesis  $(\{\xi_s\}_{s \in S}, \langle s', w \rangle) \models \psi(y)$ . Thus,  $(\{\xi_s\}_{s \in S}, \langle s, w \rangle) \models \Delta\psi(y)$ .
- 3.2 If instead  $(\{\xi_s\}_{s \in S}, \langle s, w \rangle) \models \Diamond\nabla x = y$ , then  $(\{\xi_s\}_{s \in S}, \langle s'', w' \rangle) \models x = y$ , for some  $\langle s'', w' \rangle \in Q$ . Hence,  $\xi_{s''}(x) = \xi_{s''}(y)$  and, so,  $(\{\xi_s\}_{s \in S}, \langle s'', @ \rangle) \models x = y$ . Consequently,  $(\{\xi_s\}_{s \in S}, \langle s', @ \rangle) \models \nabla x = y$  and, so,  $(\{\xi_s\}_{s \in S}, \langle s', w \rangle) \models \Diamond\nabla x = y$ . By inductive hypothesis,  $(\{\xi_s\}_{s \in S}, \langle s', w \rangle) \models \psi(y)$ , and therefore  $(\{\xi_s\}_{s \in S}, \langle s, w \rangle) \models \Delta\psi(y)$ .

Q.E.D.

Finally, it is worth remarking that SC-validity is not preserved under uniform substitution. For instance,  $P(x) \rightarrow x = x$  is SC-valid, whereas  $\neg P(x) \rightarrow x = x$  is not.

### 2.2.2 $\mathcal{L}_{\Box}$ -logic

The next problem is determining which typical laws and inference rules of  $\mathcal{L}_{\Box}$  carry over to  $\mathcal{L}_{\Box\Delta}$ . First of all, it is noteworthy that the *rule of necessitation* fails in  $\mathcal{L}_{\Box\Delta}$ , since  $\nabla x = x$  is valid, whereas  $\Box\nabla x = x$  is not. The same rule however is SC-valid in the sub-language  $\mathcal{L}_{\Box}$ :

$\mathbf{N}^-$ . if  $\models \phi$ , then  $\models \Box\phi$ , for  $\phi \in \mathcal{L}_{\Box}$

*Proof.* Choose a model  $\mathcal{M}$  and an assignment  $\{\xi_s\}_{s \in S}$ . Given any precisification  $s_0$ , pick out a world  $\langle s_0, w_0 \rangle \in Q_{\mathcal{M}}$ . Now, consider the one-precisification model  $\mathcal{M}'$  which is obtained by restricting  $\mathcal{M}$  to  $s_0$ , and let  $@_{\mathcal{M}'} = w_0$ . Define in  $\mathcal{M}'$  the assignment  $\xi'_{s_0}(x) = c(\xi_{s_0}(x), \langle s_0, w_0 \rangle)$ . Since  $\models \phi$ , then  $(\mathcal{M}', \xi'_{s_0}, \langle s_0, w_0 \rangle) \models \phi$ . Since  $\phi \in \mathcal{L}_{\Box}$ , the truth of  $\phi$  at a world is

independent of what is the case at any other world from a different precisification. So,  $(\mathcal{M}, \{\xi_s\}_{s \in S}, \langle s_0, w_0 \rangle) \models \phi$ . Hence,  $(\mathcal{M}, \{\xi_s\}_{s \in S}, \langle s_0, @_{\mathcal{M}} \rangle) \models \phi$ . Q.E.D.

On the other hand, it is easy to show that the *Kripke axiom*

$$\mathbf{K}. \quad \Box(\phi \rightarrow \psi) \rightarrow (\Box\phi \rightarrow \Box\psi)$$

is SC-valid in  $\mathcal{L}_{\Box\Delta}$ , unlike in some counterpart-theoretic frameworks (most notably, the one in Lewis [9]).

The following major modal theses are all SC-valid:

$$\mathbf{T}. \quad \Box\phi \rightarrow \phi$$

$$\mathbf{B}. \quad \phi \rightarrow \Box\Diamond\phi$$

$$4. \quad \Box\phi \rightarrow \Box\Box\phi$$

$$5. \quad \Diamond\phi \rightarrow \Box\Diamond\phi$$

This is another respect in which the present semantics differs from Lewis' counterpart theory. For in the latter (and restrictedly to  $\mathcal{L}_{\Box}$ ), these four theses hold only if the counterpart relation is reflexive, symmetric, transitive and euclidean, respectively. In SC semantics, on the other hand, we need not make such assumptions concerning counterparthood.

It is also easy to show that the *Barcan schema* and its *converse* hold in  $\mathcal{L}_{\Box\Delta}$ :

$$\mathbf{BF}. \quad \Diamond\exists x\phi \rightarrow \exists x\Diamond\phi$$

$$\mathbf{CBF}. \quad \exists x\Diamond\phi \rightarrow \Diamond\exists x\phi$$

Let us take a look now to the modal properties of identity. The *necessity of identity* and *non-identity* are both SC-valid:

$$\mathbf{NI}. \quad x = y \rightarrow \Box x = y$$

$$\mathbf{NN}. \quad \neg x = y \rightarrow \Box \neg x = y$$

The *necessity of self-identity*  $\Box x = x$ , on the other hand, fails (which follows immediately from **T** and the invalidity of  $x = x$ ). It follows that **NI** can't be proved in the usual way from the conjunction of **LL** and the necessity of self-identity. Nevertheless, the following weakened versions hold:

$$\mathbf{NSI}_1^-. \quad \forall x \Box x = x$$

**NSI<sub>2</sub><sup>-</sup>.**  $x = x \rightarrow \Box x = x$

The following are also SC-valid theses:

**NSD.**  $\neg x = x \rightarrow \Box \neg x = x$

**NE.**  $\exists y(x = y) \rightarrow \Box \exists y(x = y)$

**NNE.**  $\neg \exists y(x = y) \rightarrow \Box \neg \exists y(x = y)$

The following four valid SC-schemas show how blocks of modal operators can be simplified to a single modal operator:

$\Box \Diamond.$   $\Box \Diamond \phi \leftrightarrow \Diamond \phi$

$\Diamond \Diamond.$   $\Diamond \Diamond \phi \leftrightarrow \Diamond \phi$

$\Diamond \Box.$   $\Diamond \Box \phi \leftrightarrow \Box \phi$

$\Box \Box.$   $\Box \Box \phi \leftrightarrow \Box \phi$

*Proof.*  $(\Box \Diamond)$  by **(T)**, **(5)**.  $(\Diamond \Diamond)$  by **(T)**, **(4)**.  $(\Diamond \Box)$  by  $(\Box \Diamond)$ .  $(\Box \Box)$  by  $(\Diamond \Diamond)$ .

### 2.2.3 $\mathcal{L}_\Delta$ -logic

The topic of this subsection are the laws and rules of  $\mathcal{L}_\Delta$  which are SC-valid in  $\mathcal{L}_{\Box\Delta}$ .

The rule of  $\Delta$ -introduction, typical of supervaluationism, holds:

**$\Delta\mathbf{I}$ .**  $\phi \models \Delta\phi$

From  **$\Delta\mathbf{I}$**  it follows that

**$\Delta\mathbf{N}$ .** if  $\models \phi$ , then  $\models \Delta\phi$

the analog of necessitation, which guarantees that valid formulas are closed under definiteness. The analog of the Kripke axiom is SC-valid, too:

**$\Delta\mathbf{K}$ .**  $\Delta(\phi \rightarrow \psi) \rightarrow (\Delta\phi \rightarrow \Delta\psi)$

Insofar as we are presupposing that admissibility is absolute, the following are all SC-valid:

**$\Delta\mathbf{T}$ .**  $\Delta\phi \rightarrow \phi$

**$\Delta\mathbf{B}$ .**  $\phi \rightarrow \Delta\nabla\phi$

**$\Delta 4$ .**  $\Delta\phi \rightarrow \Delta\Delta\phi$

**$\Delta 5$ .**  $\nabla\phi \rightarrow \Delta\nabla\phi$

Since world domains can vary across precisifications, the analog of the Barcan Schema,  $\nabla\exists x\phi \rightarrow \exists x\nabla\phi$  fails. So does its converse  $\exists x\nabla\phi \rightarrow \nabla\exists x\phi$ , for instance when  $\phi$  is  $\neg x = x$ .

Operators for semantic (in)determinacy can be simplified as follows:

$\Delta\nabla$ .  $\Delta\nabla\phi \leftrightarrow \nabla\phi$

$\nabla\nabla$ .  $\nabla\nabla\phi \leftrightarrow \nabla\phi$

$\nabla\Delta$ .  $\nabla\Delta\phi \leftrightarrow \Delta\phi$

$\Delta\Delta$ .  $\Delta\Delta\phi \leftrightarrow \Delta\phi$

Analogously to the modal case, the proof employs a combination of  $(\Delta\mathbf{T})$ ,  $(\Delta 4)$  and  $(\Delta 5)$ . Moreover,  $(\Delta\nabla)$  and  $(\nabla\nabla)$  entail, respectively,

$\Delta\nabla^*$ .  $\Delta I\phi \leftrightarrow I\phi$

$\nabla\nabla^*$ .  $\nabla I\phi \leftrightarrow I\phi$

In particular,  $(\Delta\nabla^*)$  rules out the possibility of higher-order vagueness. As mentioned in Section 1, however, unrestricted quantification cannot be definite at any order, i.e., it can be vague only if it is super-vague. Therefore, as long as we want to capture vague quantification over worlds or world-bound individuals, the SC-semantics will need to be relaxed by introducing a suitable admissibility relation, thus obtaining a weaker logic of definiteness—arguably one in which  $(\Delta 4)$  and  $(\Delta 5)$  fail. I leave such refinements for another time.

#### 2.2.4 $\mathcal{L}_{\Box\Delta}$ -logic

This subsection is devoted to a number of conditions on the interaction of modal and determinacy operators. We will then proceed to determine which ones are SC-valid.

Although the literature does not offer any specific work on the combination of modal and supervaluational logic, there is a good deal of work on *product logics* for multi-modal languages. A product logic is defined semantically with respect to a class of models that are the cartesian products of Kripke models.<sup>20</sup> Product logics validate three key principles whose analogs in  $\mathcal{L}_{\Box\Delta}$  are:

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<sup>20</sup>See, for instance, Gabbay et al. [4], ch. 5.

**Commutativity<sub>1</sub>.**  $\diamond\nabla\phi \rightarrow \nabla\diamond\phi$

**Commutativity<sub>2</sub>.**  $\nabla\diamond\phi \rightarrow \diamond\nabla\phi$

**Church-Rosser.**  $\diamond\Delta\phi \rightarrow \Delta\diamond\phi$

Are these schemas SC-valid? Consider the following conditions on a SC-frame  $\mathcal{F}$ , for  $s, s' \in S$ ,  $w \in W$ :

**C<sub>1</sub>.** if  $\langle s, @ \rangle, \langle s', @ \rangle, \langle s', w \rangle \in Q$ , then  $\langle s, w \rangle \in Q$

**C<sub>2</sub>.** if  $\langle s, @ \rangle, \langle s, w \rangle, \langle s', w \rangle \in Q$ , then  $\langle s', @ \rangle \in Q$

**CR.** if  $\langle s, @ \rangle, \langle s', @ \rangle, \langle s, w \rangle \in Q$ , then  $\langle s', w \rangle \in Q$

It is not hard to check that **Commutativity<sub>1</sub>** (**Commutativity<sub>2</sub>**, **Church-Rosser**) is true in every model based on a frame  $\mathcal{F}$  iff **C<sub>1</sub>** (**C<sub>2</sub>**, **CR**) holds in  $\mathcal{F}$ . Now, the consequent of **C<sub>2</sub>** is trivially satisfied in every SC-frame, since  $\langle t, @ \rangle \in Q$ , for every  $t \in S$ . It follows that **Commutativity<sub>2</sub>** is SC-valid. Notice, however, that the necessitation of **Commutativity<sub>2</sub>**,  $\Box(\nabla\diamond\phi \rightarrow \diamond\nabla\phi)$ , is invalid.<sup>21</sup> This is one of those cases in which the rule of necessitation fails in  $\mathcal{L}_{\Box\Delta}$ . On the other hand, neither **C<sub>1</sub>** nor **CR** are true of every SC-frame, hence both **Commutativity<sub>1</sub>** and **Church-Rosser** are invalid.

Now, call a SC-frame *complete* when  $Q = S \times W$ , i.e., when the set of all worlds contains no gaps across precisifications. It should be clear that **Commutativity<sub>1</sub>** and **Church-Rosser** (and, trivially, **Commutativity<sub>2</sub>**) are all valid with respect to the class of complete SC-frames. The moral is that those three conditions hold when the set of worlds is determinate. In fact, the following schemas are also valid with respect to the complete SC-frames:

$\Box$ -**Commutativity<sub>1</sub>.**  $\Box(\diamond\nabla\phi \rightarrow \nabla\diamond\phi)$

$\Box$ -**Commutativity<sub>2</sub>.**  $\Box(\nabla\diamond\phi \rightarrow \diamond\nabla\phi)$

$\Box$ -**Church-Rosser.**  $\Box(\diamond\Delta\phi \rightarrow \Delta\diamond\phi)$

I now turn to one of the most interesting conditions concerning the logic of  $\mathcal{L}_{\Box\Delta}$ , which is

**Locality.**  $I\diamond\phi \rightarrow \diamond I\phi$

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<sup>21</sup>Because **C<sub>2</sub>** is no longer trivially true, but in fact can be false, when @ is replaced with an arbitrary  $u \in W$ .

This schema captures the idea that any instance of indeterminacy about what is the case over the whole logical space reduces to an instance of indeterminacy at some particular world. If the schema is invalid, we say that modal vagueness can be *global*.

As it turns out, **Locality** is SC-valid with respect to the class of complete frames. On the other hand, it is easy to construct countermodels over SC-frames which are incomplete. As a matter of fact, we have already run into an example which can provide us with a false instance of **Locality**. Recall that the Lewis-Divers principle of restricted recombination **RR** entails, if the world size is bounded by  $\aleph_1$ , that

12. it is vague whether there could possibly exist  $2^{\aleph_0}$  duplicates of the Tower of Pisa

since it is indeterminate whether logical space contains worlds large enough to fit  $2^{\aleph_0}$  objects. On the other hand, it is not the case that

13. it could possibly be vague whether there exist  $2^{\aleph_0}$  duplicates of the Tower of Pisa

since there is no single world such that it is indeterminate whether *that* world does or does not contain  $2^{\aleph_0}$  objects.

### 3 Revisionism?

I have put forward a language  $\mathcal{L}_{\Box\Delta}$  with modal and determinacy operators, whose logic is defined by a combination of counterpart-theoretic and supervaluationary semantics. In Barnes and Williams [1] it has been argued, however, that a language as rich as  $\mathcal{L}_{\Box\Delta}$  will have to make some modal inconsistency satisfiable, if vagueness is interpreted via supervaluations.

Let us look at the objection in more detail. The argument in Barnes and Williams [1] is preceded by the observation that supervaluationary logic is perfectly classical with respect to an extensional language  $\mathcal{L}$ . Indeed, this fact is typically exhibited as a virtue of supervaluationism *vis á vis* alternative semantics for vagueness, especially those of the degree-theoretic variety. Some of that classicality gets ‘lost’, as it were, once the language is enriched with a determinacy operator, hence expanded to  $\mathcal{L}_{\Delta}$ . Indeed, in such languages, *reductio ad absurdum* and other classical forms of inference fail. The main charge of Barnes and Williams [1] is that, once we add modal operators as well and define a supervaluationary logic for  $\mathcal{L}_{\Box\Delta}$ , the departure from classical logic would be unacceptable insofar as some inconsistencies

become satisfiable. The argument goes as follows. Given a language  $\mathcal{L}_{\Box\Delta}$ , take some  $\phi$  such that

$$\text{a) } \nabla\neg\phi \wedge \nabla\phi$$

Since  $\phi \vee \neg\phi$  is supervaluationarily valid, we can infer

$$\text{b) } (\phi \wedge \nabla\neg\phi) \vee (\neg\phi \wedge \nabla\phi)$$

But modalities are factive, hence

$$\text{c) } \Diamond((\phi \wedge \nabla\neg\phi) \vee (\neg\phi \wedge \nabla\phi))$$

Now, assume the validity of the following inferential schema—let's call it *modal reductio ad absurdum*:

**MR.** if  $\Gamma, \phi \models \perp$ , then  $\Gamma \models \neg\Diamond\phi$

Since each disjunct in (b) is supervaluationarily inconsistent, by (**MR**) we can derive

$$\text{d) } \neg\Diamond(\phi \wedge \nabla\neg\phi)$$

$$\text{e) } \neg\Diamond(\neg\phi \wedge \nabla\phi)$$

But the following modal inference is clearly valid:

**MD.**  $\Diamond(\phi \vee \psi) \models \Diamond\phi \vee \Diamond\psi$

By (**MD**), (c) is inconsistent with the conjunction of (d) and (e). The moral is that any language with modal and determinacy operators whose logic is supervaluationary makes inconsistent statements satisfiable, if some statements are vague.

However, the supervaluationist does not have to accept that conclusion. The argument appeals to two modal inference forms, **MR** and **MD**. The Barnes-Williams objection tacitly assumes that, if such inference forms hold in the language  $\mathcal{L}_{\Box}$ , their validity should carry over to  $\mathcal{L}_{\Box\Delta}$ . Is that so? On the one hand, **MD** not only looks very natural, but is also SC-valid in  $\mathcal{L}_{\Box\Delta}$ . Therefore, we have *prima facie* reasons for accepting it. On the other hand, note that **MR** entails classical *reductio ad absurdum*, provided that modality is factive (i.e., that it satisfies **T**). But we saw that classical *reductio* fails already in  $\mathcal{L}_{\Delta}$ , therefore we should have only expected it to fail in the richer language  $\mathcal{L}_{\Box\Delta}$ . If in  $\mathcal{L}_{\Box\Delta}$  we accept the failure of classical *reductio*, *a fortiori* we should accept the failure of the stronger modal version **MR**. It can be concluded that a supervaluationary logic for  $\mathcal{L}_{\Box\Delta}$  does not have to make inconsistencies satisfiable, *pace* Barnes and Williams.



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