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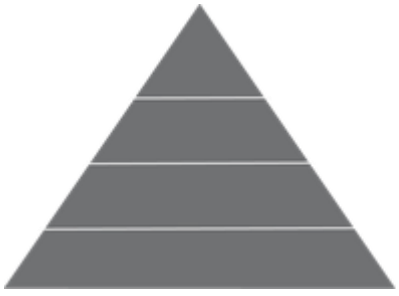

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On the Measurement of Need-based Justice

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Abstract: Need considerations play an important role in empirically informed theories of distributive justice. We propose a concept of need-based justice that is related to social participation and provide an ethical measurement of need-based justice. The β-ε-index satisfies the need-principle, monotonicity, sensitivity, transfer and several 'technical' axioms. A numerical example is given.

Keywords: Need-based Justice, Distributive Justice, Income Distribution, Social Participation, Measurement

JEL classification: I39, D63, D31

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1 Introduction

Empirically informed normative theories of distributive justice[1] usually share two properties, namely, pluralism and context dependence (Konow and Schwettmann, 2016). Justice is pluralistic if it consists of multiple fairness criteria. It is context dependent if the weight that is given to each criterion depends on 'impersonal fundamentals' (institutional factors, culture etc.) and personal traits (such as age and gender). Apart from the two top dogs equality and equity, need has been identified as one of the main ingredients of pluralistic justice theories (Konow, 2001, 2003). However, if distributive justice depends on several fairness criteria, the present dominance of egalitarian inequality measurement in applied economics fails at giving a full picture of perceived injustice of societies.

Surprisingly, as compared to the vast literature on inequality measurement, almost no attention bas been paid to the measurement of need-based justice so far. The only exemption that bears similarity to our idea is Jasso's (1978) theory of justice evaluation (surveyed in Jasso et al. 2016). According to Jasso, the justice evaluation of an income is the logarithm of the ratio of acutal reward to the just reward. The purpose of the present paper is to define a concept and to introduce an ethically compelling measurement of need-based justice. Such a measurement could make a contribution to a more complete empirically informed picture of distributive justice.

The first authors to experimentally investigate whether in addition to equity and equality need exists as a third principle of distributive justice were psychologists Lamm and Schwinger (1980, 1983). In a questionnaire experiment subjects (128 high school students) had to allocate money between two hypothetical teammates, who had contributed equally to a project but differed in their needs. The needier person was indeed allocated a significantly higher share than the less needy person even if the relationship between both was described as loose or if the needier person was responsible for her higher need. The inventors of empirical social choice (for a survey see Gaernter and Schokkaert, 2012), Yaari and Bar-Hillel (1984) asked their subjects (163 Hebrew University applicants) to divide fruit between Jones and Smith in a just way. If Jones and Smith differed in their vitamin F needs, 82% of the

[1] We do not embark here on the debate whether empirical facts can or should contribute to normative theories of distributive justice et all. This issue and many more interesting aspects of empirically informed ethics are carefully addressed in an edited volume by Christen et al. (2014), though unfortunately the perspective of economics is missing.

subjects chose the fruit allocation that exactly the same amount of vitamin F. These studies, however, did not consider need in competition with other fairness principles.

The first authors to experimentally study several competing fairness ideals at the same time were Frohlich et al. (1987). Interestingly, they found a strong prevalence of the 'Boulding principle' (Traub et al., 2005), the maximization of the average payoff subject to a floor constraint in their subjects' (220 US and Canadian students) choices. Among 44 five-person groups, 35 chose the Boulding principle, while only 7 maximized the average payoff (utilitarianism) and two groups decided for Rawlsian maximin or maximization of the average payoff subject to a range constraint. In a study by Traub et al. (2005) subjects (61 students of the University of Kiel) had to rank income distributions from under a veil of ignorance with (ignorance scenario) and without (risk scenario) known probabilities. In the ignorance scenario, the Boulding principle turned out as the top performer among all considered fairness standards; in the risk scenario it was among the top three next to average payoff maximization and randomization preferences. These studies show that need considerations are an important aspect of distributive justice.

The remainder of the paper is organized as follows. In the next Section, we introduce a concept of need-based justice. Notation and definitions are given in Section 3. Axioms for an ethical measurement of need-based justice are presented in Section 4. We derive two indices of need-based justice in Section 5. Section 6 gives numerical examples. Section 7 concludes.

2 The Concept of Need-based Justice

Like ethical poverty measurement (pioneered by Sen, 1976), the concept of need-based justice measurement is twofold. First, needs have to be acknowledged at a societal level. Second, a method is required that aggregates the need-characteristics of a group of people into an overall image of justice. Though this article focusses on aggregation, we want to briefly address the first step, the societal acknowledgement of needs, in the following.

Maslow's (1943) theory of human motivation is probably the most influential approach to needs (for a survey of the psychological literature on needs see Diederich, 2017). The theory holds that there is a hierarchy of needs with physiological needs at the bottom, outranked by safety, love/belonging, and esteem needs, and with self-actualization at the top. The lower the need in

the hierarchy the stronger it is. Absolute concepts of poverty like the basic needs approach rely on expert knowledge regarding the minimum cost diet that secures physical survival of an individual (Seidl, 1988). Studying the living conditions of the working class in York, Rowntree (1901) for instance defined "families whose total earnings are insufficient to obtain the minimum necessaries for the maintenance of merely physical efficiency" (p. 86) as poor.

Living standards in a society can grow or shrink. Restricting needs to physical survival would ignore the fact that the satisfaction of both physiological and (higher level) psychological needs contributes to mental health and thus the well-being of people (Deci and Ryan, 2000; Ryan and Deci, 2000). Sociological relative deprivation theories (Runciman, 1966; Townsend, 1974) carry the subjectivity of needs to the extreme. According to Runciman (1966) a person is deprived if she does not have something, somebody else has it, she wants to have it, and she thinks that obtaining it is realistic.

Both the purely absolute and the relative view of need and poverty were harshly criticized by Sen (1983). He proposed the concept of absolute deprivation instead. It is worthwhile to repeat a quote from Adam Smith's 'The Wealth of Nations' from Sen's article in order to illustrate the meaning of absolute deprivation:

> By necessaries I understand not only the commodities which are indispensably necessary for the support of life, but what ever the custom of the country renders it indecent for creditable people, even the lowest order, to be without. [...] Custom [...] has rendered leather shoes a necessary of life in England. The poorest creditable person of either sex would be ashamed to appear in public without them. (Smith, 1776, 351–352)

Accordingly, a person is absolutely deprived if she does not have the capability, say, in terms of income in order to partake in the commonly accepted activities of the community. In other words, neediness means that a person is ashamed because she is not able to purchase leather shoes and not because she has less than other people. Hence, in the following we have the capabilities approach (Sen, 2009; Nussbaum, 2000, 2011) in mind when talking about needs.

Indeed, modern welfare states try to guarantee their citizens social participation through their tax and social systems, in particular. For example, Articles 1, 3 and 20 of Germany's constitution protect human dignity, grant equality before the law, and contain the welfare state as a national objective

('Sozialstaatsprinzip'). In its jurisdiction Germany's constitutional court has interpreted these articles in terms of a basic right to live above the breadline and to partake in common social activities. For example, the court imposed the legislator to exempt the 'subsistence level' from income taxation and to align childrens' social welfare benefits with 'actual needs' using a transparent and appropriate (statistical) method.[2] Of course, there is still some ambiguity left about what actual needs are to be taken into account.

3 Notation and Definitions

We consider a set $\mathcal{I}$ of individuals $i = \{1, \ldots, n\}$, who are endowed with $\omega_i \geq 0$ units of a good or service. Every individual needs at least a small amount ν of the good or service in order to survive or to partake in a commonly accepted social activity. As in poverty measurement, where the poverty line is the same for each individual, we assume identical needs. Differences in needs that result, for example, from household composition and size, can be taken into account by means of an equivalence scale which adjusts endowments. Let $\vec{\omega} = (\omega_1, \omega_2, \ldots, \omega_n)$ denote an allocation of endowments. $\Omega = \sum_{i=1}^{n} \omega_i$ denotes the total endowment. $N = n\nu$ denotes the total need.

We split I into two subsets, namely, those individuals whose needs have been satisfied, $\omega_i \geq \nu$, and those who are dissatisfied in their needs, $\omega_i < \nu$. The set of the satisfied is denoted by $\mathcal{S} = \{i \in \mathcal{I} : \omega_i \geq \nu\}$ and their number is given by $s = \#(\mathcal{S})$. The set of the dissatisfied is denoted by $\mathcal{D} = \{i \in \mathcal{I} : \omega_i < \nu\}$ and their number is given by $d = \#(\mathcal{D})$. The individual degree of need satisfaction is defined as follows:

Definition 1 (Need Satisfaction) *The individual degree of need satisfaction is given by* $\gamma_i = \min\{\omega_i/\nu, 1\} \, \forall \, i \in \mathcal{I}$.

$\vec{\gamma} = (\gamma_1, \gamma_2, \ldots, \gamma_n)$ is a profile of need satisfaction.

Apart from need satisfaction, we are also interested in need-based allocation efficiency. First, we define oversupply

Definition 2 (Oversupply) *The individual oversupply is given by* $\ell_i = \max\{\omega_i - \nu, 0\} \, \forall \, i \in \mathcal{I}$. *The aggregate oversupply is given by* $L = \sum_{i \in \mathcal{I}} \ell_i$.

and then introduce our notion of need-based allocation efficiency:

[2] Judgement of the First Senate of the German Constitutional Court from February 9th, 2010.

Definition 3 (Allocation Efficiency) *The need-based efficiency of the allocation $\vec{\omega}$ is given by*

$$\Gamma_\varepsilon = \begin{cases} \left(1 - \frac{L}{\Omega}\right)^\varepsilon & \text{for } \mathcal{D} \neq \emptyset \\ 1 & \text{else}. \end{cases} \tag{1}$$

Γ_ε is an index of need-based allocation efficiency, where $\Gamma_\varepsilon \in [0, 1]$. $\varepsilon > 0$ is a parameter of inefficiency aversion. $L > 0$ means that scarce resources have been misallocated to the satisfied although there still are dissatisfied needs. The index reaches its minimum if $L = \Omega$, that is, not a single unit of the good is used for the satisfaction of needs. It reaches its maximum if $L = 0$, that is, if there is no oversupply. If $\varepsilon = 0$, inefficiency does not matter, that is, $\Gamma_0 = 1$. If $0 < \varepsilon < 1$ ($\varepsilon = 1$, $\varepsilon > 1$), we have increasing (constant, decreasing) inefficiency aversion. A graphical illustration of the gamma function is given in Figure 1 with $\Omega = 100$ (and $\mathcal{D} \neq \emptyset$).

Finally, we define an index of need-based justice.

Definition 4 (Index of Need-based justice) *An index of need-based justice is a mapping $J : (\vec{\omega}, \nu) \mapsto \mathbb{R}$.*

4 Axioms

A compelling index of need-based justice should satisfy several 'technical' and ethical axioms. Note that some of the axioms stated here (Monotonicity, Sensitivity, Transfer) have first been introduced by Sen (1976) for poverty measurement. First, we normalize J onto the $[0, 1]$ interval:

Axiom 1 (Normalization, N) *$J(\vec{\omega}, \nu) \in [0, 1]$, where $J = 1$ ($J = 0$) in the case of maximum (minimum) need-based justice.*

It appears quite natural to assume that maximum need-based justice prevails if nobody is left dissatisfied in her needs, $\mathcal{D} = \emptyset$. The case of minimum need-based justice is less straightforward because the dissatisfaction of needs may have two distinct sources, namely, a lack of goods of services and/or inefficiency of the allocation.

Second, we require J to be scale invariant:

Axiom 2 (Scale Invariance, SI) *Let $\vec{\omega}' = \lambda \cdot \vec{\omega} = (\lambda \cdot \omega_1, \lambda \cdot \omega_2, \ldots, \lambda \cdot \omega_n)$ and $\nu' = \lambda\nu$, then $J(\vec{\omega}, \nu) = J(\vec{\omega}', \nu')$.*

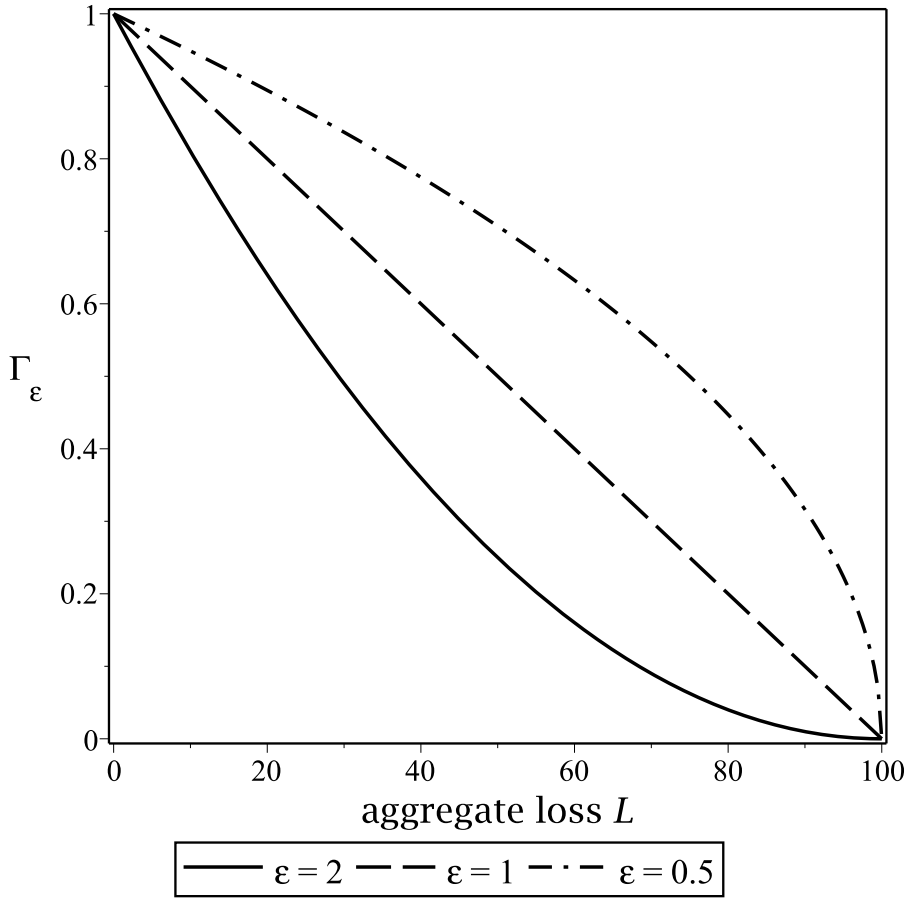

Figure 1: Graph of the Gamma Function for $\Omega = 100$

Scale invariance immunizes J to changes of the unit in which goods and services are measured. It implies homogeneity of degree zero in endowments and needs of the J function.

Monotonicity means that whenever the endowment of a dissatisfied (satisfied) individual increases, J should increase (decrease).

Axiom 3 (Monotonicity, M) $\frac{\partial J(\vec{\omega},\nu)}{\partial \omega_i} \begin{cases} > 0 \, \forall \, i \in \mathcal{D} \\ < 0 \, \forall \, i \in \mathcal{S} \, . \end{cases}$

Additionally, we think that need-based justice is raised more by increasing the degree of need-satisfaction of a very dissatisfied individual than of a less

dissatisfied individual. This precept requires sensitivity of J with respect to the degree of need-satisfaction.

Axiom 4 (Sensitivity, S) $\frac{\partial J(\vec{\omega},\nu)}{\partial \omega_i} > \frac{\partial J(\vec{\omega},\nu)}{\partial \omega_j} \, \forall \, i \neq j, i, j \in \mathcal{D}$ *if* $\gamma_i < \gamma_j$.

Next, we turn to the question how a progressive transfer, that is, a transfer of resources $\delta > 0$ from a less dissatisfied individual (donor) to a more dissatisfied individual (recipient) changes J. It is assumed that the donor remains at least as need satisfied as the recipient.

Axiom 5 (Transfer, T) *For two allocations* $\vec{\omega}$ *and* $\vec{\omega}'$ *with* $\gamma_k = \gamma'_k \, \forall \, k \in \mathcal{I}_{-\{i,j\}}$, $\gamma_i < \gamma'_i = \frac{\omega_i + \delta}{\nu} \leq \gamma'_j = \frac{\omega_j - \delta}{\nu} \leq \gamma_j$, $\delta > 0$, *we have* $J(\vec{\omega}, \nu) < J(\vec{\omega}', \nu)$.

Finally, we propose that J is the weighted sum of average individual need-satisfaction multiplied with the need-based efficiency of the allocation.

Axiom 6 (Need-Principle, NP) $J(\vec{\omega}, \nu) = \frac{\Gamma_\varepsilon}{n} \sum_{i \in \mathcal{I}} \alpha_i \gamma_i$.

The parameter $\alpha_i = \alpha(\gamma_i)$ is the weight that is given to individual i, where $\alpha_i \geq 0 \, \forall \, i \in \mathcal{I}$ and $\alpha_i > 0$ for at least one $i \in \mathcal{I}$. $A = \{\alpha_1, \alpha_2, \ldots, \alpha_n\}$ denotes a weighting scheme. Efficiency in terms of Γ_ε is an inherent property of the need-principle.

5 Indices

The following propositions contain our main results.

Proposition 1 (Incidence of Need-satisfaction (INS)) *Let* $A_{INS} = \{\alpha_i = 0 : i \in \mathcal{D}; \alpha_i = 1 : i \in \mathcal{S}\}$ *and* $\varepsilon > 0$, *then*

$$J_{INS,\varepsilon} \;\; = \;\; \frac{\Gamma_\varepsilon}{n} s \; . \tag{2}$$

satisfies NP, N, and SI. It does not satisfy M and S. T is satisfied only if the donor belongs to the set of satisfied individuals before and after the transfer.

Proof. NP: The need principle is fulfilled by the construction of $J_{\mathrm{INS},\varepsilon}$. Just replace α_i by 0 for $\gamma_i < 1$ and by 1 for $\gamma_i = 1$.
N: If $\omega_i \geq \nu \;\forall\; i \in \mathcal{I}$ then $s = n$, $\Gamma_\varepsilon = 1$ and, hence, $J_{\mathrm{INS},\varepsilon} = 1$. If $\omega_i < \nu \;\forall\; i \in \mathcal{I}$ then $s = 0$ and, hence, $J_{\mathrm{INS},\varepsilon} = 0$. Moreover, if $L = \Omega$ and

$\mathcal{D} \neq \emptyset$ then $\Gamma_\varepsilon = 0$ and, hence, $J_{\text{INS},\varepsilon} = 0$.
SI:

$$\begin{aligned} J'_{\text{INS},\varepsilon} &= \frac{\left(1 - \frac{\sum_{i\in\mathcal{S}} \lambda\omega_i - \lambda\nu}{\sum_{i\in\mathcal{I}} \lambda\omega_i}\right)^\varepsilon}{n} s \\ &= \frac{\left(1 - \frac{\sum_{i\in\mathcal{S}} \omega_i - \nu}{\sum_{i\in\mathcal{I}} \omega_i}\right)^\varepsilon}{n} s \\ &= J_{\text{INS},\varepsilon} \, . \end{aligned}$$

M: $\frac{\partial J_{INS,\varepsilon}}{\partial \omega_i} = 0 \, \forall \, i \in \mathcal{D}$ contradicts M.
S: See M.
T: Without loss of generality, we can set $n = 3$, where $i = \{1 \doteq \text{recipient}, 2 \doteq \text{donor}, 3 \doteq \text{neutral}\}$. The endowment of the neutral individual is set to $\omega_3 = 0$. We have to consider four cases. (i) Let $1, 2 \in \mathcal{D}$ before and after the transfer, then $J_{INS,\varepsilon} = J'_{INS,\varepsilon} = 0$, which contradicts T. (ii) Let $1 \in \mathcal{D}$ and $2 \in \mathcal{S}$ before and $1, 2 \in \mathcal{D}$ after the transfer, then $J_{INS,\varepsilon} = \frac{\left(1 - \frac{\omega_2 - \nu}{\omega_1 + \omega_2}\right)^\varepsilon}{3} > J'_{INS,\varepsilon} = 0$, which contradicts T. (iii) Let $1 \in \mathcal{D}$ and $2 \in \mathcal{S}$ before and after the transfer, then $J_{INS,\varepsilon} = \frac{\left(1 - \frac{\omega_2 - \nu}{\omega_1 + \omega_2}\right)^\varepsilon}{3} < J'_{INS,\varepsilon} = \frac{\left(1 - \frac{\omega_2 - \delta - \nu}{\omega_1 + \omega_2}\right)^\varepsilon}{3}$, which confirms T. (iv) Let $1 \in \mathcal{D}$ before the transfer, $1 \in \mathcal{S}$ after the transfer, and $2 \in \mathcal{S}$ before and after the transfer, then $J_{INS,\varepsilon} = \frac{\left(1 - \frac{\omega_2 - \nu}{\omega_1 + \omega_2}\right)^\varepsilon}{3}$ is maximized for $\omega_1 \to \nu$ from below (the first derivative of $J_{INS,\varepsilon}$ w.r.t. ω_1 is given by $\frac{1}{3} \frac{\left(\frac{\omega_1 + \nu}{\omega_1 + \omega_2}\right)^\varepsilon \varepsilon(\omega_2 - \nu)}{(\omega_1 + \omega_2)(\omega_1 + \nu)} > 0$) and $J'_{INS,\varepsilon} = \frac{\left(1 - \frac{\omega_1 + \omega_2 - 2\nu}{\omega_1 + \omega_2}\right)^\varepsilon}{3} \cdot 2$ is minimized for $\omega_1 \to \nu$ from above (its first derivative is given by $-\frac{1}{3} \frac{2^{1+\varepsilon}\left(\frac{\nu}{\omega_1 + \omega_2}\right)^\varepsilon \varepsilon}{\omega_1 + \omega_2} < 0$). Hence, setting $\omega_1 = \nu$ yields $J_{INS,\varepsilon} = \frac{\left(1 - \frac{\omega_2 - \nu}{\nu + \omega_2}\right)^\varepsilon}{3} < J'_{INS,\varepsilon} = \frac{\left(1 - \frac{\omega_2 - \nu}{\nu + \omega_2}\right)^\varepsilon}{3} \cdot 2$, which confirms T. Obviously, changing the total number of neutral individuals does not change the result as long as $\mathcal{I}$ is a finite set. ■

$J_{\text{INS},\varepsilon}$ is a pure measure of the incidence of need satisfaction. It does not react to changes in endowments of the dissatisfied (unless they become fully need satisfied) and therefore does neither satisfy M nor S. Apart from the two special cases addressed in the proof to proposition 1, it does not fulfill T either. The maximization of $J_{\text{INS},\varepsilon}$ could easily be obtained by redistributing goods and services from those who are most dissatisfied in their needs to

the least dissatisfied and therefore become satisfied in their needs such that the number of satisfied individuals s increases. Accordingly, the incidence of need satisfaction is of limited use as an ethical measure of need-based justice.

Proposition 2 (β-ε-Index) *Let* $A_\beta = \{\alpha_i = \gamma_i^{-\beta} : i \in \mathcal{D}, \alpha_i = 1 : i \in \mathcal{S}\}$, $0 < \beta < 1$ *and* $\varepsilon > 0$, *then*

$$J_{\beta,\varepsilon} = \frac{\Gamma_\varepsilon}{n}\left(\sum_{i\in\mathcal{D}} \gamma_i^{1-\beta} + s\right) \tag{3}$$

satisfies NP, N, SI, M, and S. T is fulfilled if the recipient remains among the dissatisfied. T is also fulfilled if the recipient becomes satisfied and the positive satisfaction effect exceeds the negative efficiency effect.

Proof. NP: The need principle is fulfilled by the construction of $J_{\beta,\varepsilon}$. Just replace α_i by $\gamma_i^{-\beta}\,\forall\, i \in \mathcal{D}$ and by $1\,\forall\, i \in \mathcal{S}$.
N: If $\omega_i \geq \nu\ \forall\ i \in \mathcal{I}$ then $s = n$, $\Gamma_\varepsilon = 1$ and hence $J_{\beta,\varepsilon} = 1$. If $\omega_i = 0\ \forall\ i \in \mathcal{I}$ then $\gamma_i = 0$ and hence $J_{\beta,\varepsilon} = 0$. Moreover, if $L = \Omega$ and $\mathcal{D} \neq \emptyset$ then $\Gamma_\varepsilon = 0$ and hence $J_{\beta,\varepsilon} = 0$.
SI:

$$\begin{aligned} J'_{\beta,\varepsilon} &= \frac{\left(1 - \frac{\sum_{i\in\mathcal{S}} \lambda\omega_i - \lambda\nu}{\sum_{i\in\mathcal{I}} \lambda\omega_i}\right)^\varepsilon}{n}\left(\sum_{i\in\mathcal{D}}\left(\frac{\lambda\omega_i}{\lambda\nu}\right)^{1-\beta} + s\right) \\ &= \frac{\left(1 - \frac{\sum_{i\in\mathcal{S}} \omega_i - \nu}{\sum_{i\in\mathcal{I}} \omega_i}\right)^\varepsilon}{n}\left(\sum_{i\in\mathcal{D}}\left(\frac{\omega_i}{\nu}\right)^{1-\beta} + s\right) \\ &= J_{\beta,\varepsilon}\ . \end{aligned}$$

M: The first derivative of $J_{\beta,\varepsilon}$ with respect to ω_i is given by

$$\frac{\partial J_{\beta,\varepsilon}}{\partial \omega_i} = \begin{cases} \overbrace{\varepsilon\frac{\Gamma_{\varepsilon-1}}{n}\frac{L}{\Omega^2}(1-\beta)\left(\sum_{j\in\mathcal{D}}\gamma_j^{1-\beta} + s\right)}^{\text{efficiency effect } (+)} + \overbrace{\frac{\Gamma_\varepsilon}{n}(1-\beta)\frac{(\gamma_i)^{-\beta}}{\nu}}^{\text{satisfaction effect } (+)} > 0\ \forall\ i \in \mathcal{D} \\ \underbrace{\varepsilon\frac{\Gamma_{\varepsilon-1}}{n}\frac{L-\Omega}{\Omega^2}(1-\beta)\left(\sum_{j\in\mathcal{D}}\gamma_j^{1-\beta} + s\right)}_{\text{efficiency effect } (-)} < 0\ \forall\ i \in \mathcal{S}\,. \end{cases}$$

S: Consider two individuals $i, j \in \mathcal{D}$, then $\frac{\partial J_{\beta,\varepsilon}}{\partial \omega_i} > \frac{\partial J_{\beta,\varepsilon}}{\partial \omega_j}$ if and only if $\gamma_i < \gamma_j$, which is due to the satisfaction effect.
T: Without loss of generality, we can set $n = 3$, where $i = \{1 \doteq \text{recipient}, 2 \doteq \text{donor}, 3 \doteq \text{neutral}\}$. The endowment of the neutral individual is set to $\omega_3 = 0$. We have to consider four cases. (i) Let $1, 2 \in \mathcal{D}$ before and after the transfer, then $J_{\beta,\varepsilon} = \frac{1}{3}\left(\left(\frac{\omega_1}{\nu}\right)^{1-\beta} + \left(\frac{\omega_2}{\nu}\right)^{1-\beta}\right) < \frac{1}{3}\left(\left(\frac{\omega_1+\delta}{\nu}\right)^{1-\beta} + \left(\frac{\omega_2-\delta}{\nu}\right)^{1-\beta}\right) = J'_{\beta,\varepsilon}$ due to positive net satisfaction effect. (ii) Let $1 \in \mathcal{D}$ and $2 \in \mathcal{S}$ before and $1, 2 \in \mathcal{D}$ after the transfer, then $J_{\beta,\varepsilon} = \frac{\left(1-\frac{\omega_2-\nu}{\omega_1+\omega_2}\right)}{3}\left(\left(\frac{\omega_1}{\nu}\right)^{1-\beta} + 1\right) < \frac{1}{3}\left(\left(\frac{\omega_1+\delta}{\nu}\right)^{1-\beta} + \left(\frac{\omega_2-\delta}{\nu}\right)^{1-\beta}\right) = J'_{\beta,\varepsilon}$, due to the positive efficiency effect and the positive net satisfaction effect. (iii) Let $1 \in \mathcal{D}$ and $2 \in \mathcal{S}$ before and after the transfer, then $J_{\beta,\varepsilon} = \frac{\left(1-\frac{\omega_2-\nu}{\omega_1+\omega_2}\right)}{3}\left(\left(\frac{\omega_1}{\nu}\right)^{1-\beta} + 1\right) < \frac{\left(1-\frac{\omega_2-\delta-\nu}{\omega_1+\omega_2}\right)}{3}\left(\left(\frac{\omega_1+\delta}{\nu}\right)^{1-\beta} + 1\right) = J'_{\beta,\varepsilon}$ due to the positive efficiency effect and the positive satisfaction effect. (iv) Let $1 \in \mathcal{D}$ before the transfer, $1 \in \mathcal{S}$ after the transfer, and $2 \in \mathcal{S}$ before and after the transfer, then $J_{\beta,\varepsilon} = \frac{\left(1-\frac{\omega_2-\nu}{\omega_1+\omega_2}\right)}{3}\left(\left(\frac{\omega_1}{\nu}\right)^{1-\beta} + 1\right) < \frac{\left(1-\frac{\omega_1+\omega_2-2\nu}{\omega_1+\omega_2}\right)}{3} \cdot 2 = J'_{\beta,\varepsilon}$ only if the positive satisfaction effect exceeds the negative efficiency effect. ■

With regard to the transfer axiom, we think that case (iv) represents a sensible restriction. In terms of need-based justice it makes more sense to transfer endowments to dissatisfied individuals until they become exactly satisfied rather than diminishing need-based allocation efficiency by excessive transfers.

There are two interesting limiting cases: For $\beta = 0$ we get

$$J_{0,\varepsilon} = \frac{\Gamma_\varepsilon}{n}\left(\sum_{i\in\mathcal{D}} \gamma_i + s\right) \tag{4}$$

which is the (efficiency weighted) average degree of individual need satisfaction. $J_{0,\varepsilon}$ does not satisfy S and T, case (i). For $\beta = 1$ we get

$$J_{1,\varepsilon} = \Gamma_\varepsilon \tag{5}$$

which is a pure index of need-based allocation efficiency. Obviously, it does not satisfy M, S and T, case (i).

6 Numerical Examples

We consider a group of $n = 3$ women – Anna, Berta and Carla – who need a daily vitamin C dose of 100 mg each to stay healthy. Thus, their total need is 300 mg/day. Table 1 gives numerical examples. The table lists several vitamin C allocations, the respective degree of need satisfaction, the need-based efficiency of the allocation in terms of Γ_1, and the corresponding need-based justice indices J_{INS}, $J_{0.5}$ and J_0 (setting $\varepsilon = 1$).

Table 1: Numerical Examples

	Endowment				Need Satisfaction			Justice			
Allocation	Anne	Berta	Carla	Total	Anne	Berta	Carla	Γ_1	J_{INS}	$J_{0.5}$	J_0
1	40	70	140	250	0.400	0.700	1.000	0.840	0.280	0.691	0.588
2	10	100	140	250	0.100	1.000	1.000	0.840	0.560	0.649	0.588
3	60	70	140	270	0.600	0.700	1.000	0.852	0.284	0.741	0.653
4	40	90	140	270	0.400	0.900	1.000	0.852	0.284	0.733	0.653
5	40	70	160	270	0.400	0.700	1.000	0.778	0.259	0.640	0.544
6	50	60	140	250	0.500	0.600	1.000	0.840	0.280	0.695	0.588
7	81	70	99	250	0.810	0.700	0.990	1.000	0.000	0.911	0.833
8	60	70	120	250	0.600	0.700	1.000	0.920	0.307	0.801	0.705
9	40	100	110	250	0.400	1.000	1.000	0.960	0.640	0.842	0.768

Tables notes. Allocations in mg/day vitamin C. $\nu = 100$.

Allocation 1 endows the group with a total of 250 mg/day vitamin C. Anne and Berta take 40 and 70 mg/day, respectively, and therefore are dissatisfied. Carla takes 140. Hence $s = 1$ and $\gamma_{\text{Carla}} = 1$. Carla's oversupply leads to a loss in need-based allocation efficiency such that $\Gamma_1 = 0.840$. Computing the incidence of need satisfaction yields $J_{\text{INS}} = \frac{\Gamma_1}{n} \times s = 0.280$. The average degree of need satisfaction (J_0) is 0.588. We get $J_{0.5} = 0.691$ for the β-ε-index, evaluated at $\beta = 0.5$.

Allocation 2 demonstrates the perverted impact that taking away 30 mg/day from Anne and passing it to Berta (who then becomes need-satisfied) has on the incidence of need satisfaction. As compared to allocation 1, $J_{\text{INS}} = 0.560$ is doubled, that is, need-based justice increases.

Allocations 3 to 5 compare the impact of increasing Anne's, Berta's and Carla's vitamin C endowment by 20 mg/day. Since Ω is raised by 20 mg/day, need-based allocation efficiency increases (decreases) according to M if Anne's or Berta's (Carla's) endowment is increased. Moreover, as compared to allocation 1, J_0 increases both for Anne and Berta, but it is not sensitive towards their initial endowments. In contrast, as required by S, $J_{0.5}$ is raised more by Anna's extra 20 mg than by Berta's, because Anne has a lower initial endowment. Due to the negative efficiency effect, additionally endowing Carla with 20 mg decreases J_0 and $J_{0.5}$ as required by M.

The final four allocations demonstrate how progressive transfers affect need-based justice. In allocation 6, Berta transfers 10 mg to Anne (case (i)). $J_{0.5}$ increases as compared to allocation 1 due to the positive net satisfaction effect. Due to its linearity in need-satisfaction J_0 remains constant. In allocation 7 Carla transfers 41 mg to Anne (case (ii)). Carla therefore becomes dissatisfied, but need-based justice increase as required by T. Allocation 8 simulates an transfer from Carla to Anne (cases (iii), which increases need-based justice. Finally, allocation 9 involves an progressive transfer from Carla to Berta such that Berta becomes exactly satified (case (iv) of T).

7 Conclusion

Need considerations are an important ingredient of empirically informed normative theories of distributive justice. The present paper introduces a concept of need-based justice which is related to social participation in terms of the capabilities approach and provides a measurement of need-based justice. Borrowing from poverty measurement, we require an ethical measurement of

need-based justice to satisfy several axioms such as monotonicity, sensitivity and transfer. Need-based allocation efficiency becomes part of the measurement through the need principle. The incidence of need-satisfaction in terms of the efficiency-weighted head-count ratio of need-satisfied people does not satisfy monotonicity and sensitivity, and it satisfies transfer for special cases only. The β-ε-index satisfies all axioms. Transfer is fulfilled even if the recipient becomes satisfied as long as the positive satisfaction effect exceeds the negative efficiency effect.

Though we think that neediness is best characterized by social participation in welfare states and therefore is closely related to Sen's (1983) concept of absolute deprivation, other – more absolute or more relative – notions of need are compatible with our formal approach to the measurement of need-based justice. Of course, the catalogue of desirable properties for an index of need-based justice could be altered or amended. Other functional forms are conceivable. In so far, we see our contribution as a first step to a more complete view of distributive justice that also includes need-based justice.

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