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Importance and Explanatory Relevance: The Case of Mathematical Explanations

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Importance and Explanatory Relevance: The Case of Mathematical Explanations

Abstract: A way to argue that something (e.g. mathematics, idealizations, moral properties, etc.) plays an explanatory role in science is by linking explanatory relevance with importance in the context of an explanation. The idea is deceptively simple: a part of an explanation is an explanatorily relevant part of that explanation if removing it affects the explanation either by destroying it or by diminishing its explanatory power, i.e. an important part (one that if removed affects the explanation) is an explanatorily relevant part. This can be very useful in many ontological debates. My aim in this paper is twofold. First of all, I will try to assess how this view on explanatory relevance can affect the recent ontological debate in the philosophy of mathematics – as I will argue, contrary to how it may appear at first glance, it doesn't help very much the mathematical realists. Second of all, I will show that there are big problems with it.

1. Introduction

There is a growing interest nowadays in the philosophy of mathematics regarding the problem of the role of mathematics in scientific explanation. This interest springs mostly from the recent battleground (the role of mathematics in science) on which the clash between the mathematical realists and their opponents takes place. The battle is carried out in the following terms: there are many scientific realists who think that "to prove the existence of a kind of entity is often to show how its existence helps explain a phenomenon" (Steiner 1978, p. 17); so, if the mathematical realist can show that there are cases of explanations in which mathematics plays a genuinely explanatory role, then s/he has a good case in favour of her position because, this way, the nominalists that are scientific realists are forced to commit to the existence of mathematical objects. Now, the mathematical realist's job may seem easier than it actually is. One can be under the impression that showing that there are mathematical explanations of physical phenomena doesn't have to involve in this context more than finding examples of scientific explanations in which mathematics at least appears to play an explanatory role, because, once the mathematical realist has this, the burden of proof falls on the nominalist.[1] There

[1] This kind of strategy is explicitly suggested in Baker (2009, p. 625).

is a problem, though: not all mathematics involving explanations are automatically mathematical explanations.[2] Science is full of merely mathematized explanations, i.e. explanations in which the mathematical part doesn't play an explanatory role. So, in order for this type of minimalist strategy to work, the mathematical realist has to come up with a way of sorting out mathematical from mathematized explanations. One way to do this is by using importance to identify the explanatorily relevant parts of an explanation. The idea is simple:

> IVER: a part of an explanation is an explanatorily relevant part of that explanation if removing it affects the explanation either by destroying it or by diminishing its explanatory power, i.e. *an important part of an explanation (one that if removed affects the explanation) is an explanatorily relevant part*.

I'm calling this the Importance View on Explanatory Relevance (IVER). In order to avoid confusion, it is important to emphasize at this point that importance (a relation between a statement and an explanation) is not equated in this view with explanatory relevance (a relation between a statement and an explanandum, or between that which explains and that which is explained) but is only taken as a good sign that we are dealing with a part of an explanation that has the property of being explanatorily relevant. The key idea here is that we can determine if a certain part of an explanation has explanatory worth by looking to see what happens with the explanation if we remove it. If removing a certain part affects the explanation either by destroying it or by diminishing its explanatory power, then it is safe to say (from IVER's perspective) that it was important for the explanation and (if we consider that only the explanatorily relevant parts can be so important) that it is *ipso facto* explanatorily relevant. So, IVER is not an account of explanatory relevance, but a view about how it can be detected. What makes it very attractive is that it seems to work very well in the absence of such an account. This is, of course, very different from the way explanatory relevance is thought about in the classical theories of explanation where the detection comes after it has been determined what makes something an explanatory relevance relation (e.g. the fact that it is potentially unifying or that it specifies the cause that brought about the phenomenon to be explained).

[2] I will use "mathematical explanations" throughout this paper to refer to mathematical explanations of physical phenomena.

Something like IVER can be found, even though not explicitly, in Baker (2005, 2009, 2012). In this paper, I will focus my attention to the philosophy of mathematics and the way IVER is/can be used to argue for the explanatory value of mathematics in science, but this view on explanatory relevance can also be found in other parts of philosophy. For example, in the debate over the role of idealizations in scientific explanations, there are philosophers such as e.g. Batterman (2009), Wayne (2011), and Rice (2012, 2013) who use IVER in their arguments for the explanatory worth of idealizations. Batterman, for example, claims that "idealizations are in many instances explanatorily ineliminable. That is to say, they play an essential role in the proper explanation of the phenomenon of interest. They are not, as the traditional view of the use of idealization in modeling suggests, put in only to be subsequently removed by more detailed work" (Batterman 2009, 445). Something similar is used by Rice in the context of his argument for a new approach to understanding how optimality models explain biological phenomena: "many optimality models utilize idealizations that appear to play essential roles within their explanations—i.e. the idealizations cannot be removed from the model without consequently eliminating the explanation" (Rice 2012, 700).

IVER seems to be also among the important assumptions involved in the debate over the reality of moral properties. As one argument against moral realism has it (Harman 1977), a property is real only if it figures ineliminably in the best explanation of experience, i.e. if it is explanatorily relevant to our experience, but moral properties are not this way, so they are not real properties (Miller 2003, ch. 8; Nelson 2006). In response, moral realists such as Sturgeon (1988) argue that some moral properties are actually ineliminable from the best explanation of our experience. What is interesting is that both parties are in tacit agreement about the following point: 'ineliminable from explanation' and 'explanatorily relevant' are tied together somehow.

My main aim in this paper is to develop and analyse this view on explanatory relevance. I will argue that the explanatory merits that different parts of an explanation have cannot be assessed in terms of their importance in the context of that explanation because it can be shown that there are non-explanatory yet important parts of an explanation, i.e. I will argue that, on a closer look, IVER turns out to be a problematic view about explanatory relevance (section ***4.***). But, before doing this, since IVER occupies such an important place in Baker's strategy for arguing that mathematics plays an explanatory part in science – as I will try to show below – I believe it is important to assess how helpful relying on IVER can really be for the mathematical realist. As I will argue (section ***3.***), even if we could use importance for determining explanatory worth, that won't help the mathematical realist very much because IVER implies that there are degrees of explanatory relevance and not all such degrees are good for the needs of EIA: if mathematics is only weakly explanatorily relevant in the examples discussed (as I will try to show), EIA is in trouble since this is incompatible with the indispensability requirement that has to be satisfied by any indispensability argument. Before doing all this, I will start (section ***2.***) by saying a few words about the ontological dispute in the philosophy of mathematics over the existence of mathematical objects and about Alan Baker's contribution to it.

2. Baker's enhanced indispensability argument

An important discussion in the philosophy of mathematics concerns the existence of mathematical entities. The participants in this discussion can be divided in two camps: those that believe that there are such abstract entities – the realists – and those that deny their existence – the nominalists. The most successful argument that the mathematical realists used in favour of their position, i.e. the indispensability argument (IA), is considered by many to have been advanced many years ago by Quine. The gist of this argument is that, if we are scientific realists, besides the concrete unobservable posits, we ought to be ontologically committed also to the existence of abstract mathematical entities because they play an indispensable role in our best scientific theories. One way to present the strategy behind this indispensability argument is this:

(1) Some scientific realists are ontologically committed only to the entities that are indispensable for our recent well confirmed scientific theories.
(2) Mathematical entities play an indispensable role in our best scientific theories.
(3) Scientific theories are confirmed or disconfirmed as a whole.

(4) Hence, if a scientific realist of the type delineated by (1) is a holist (i.e. accepts premise (3)) and accepts (2) as a fact, then s/he ought to be ontologically committed also to the existence of mathematical entities.[3]

As it turns out, there are many scientific realists that are not very happy with confirmational holism[4] and there are some, such as Hartry Field, who are not convinced that (2) is actually a fact. So, despite being so extensively discussed, the target of this indispensability argument proved to be very small.

A very peculiar thing about this argument that can be easily noticed at a closer look is the complete disregard for the actual role played by mathematics in science. All it requires is that mathematics is indispensable for science. But, as Alan Baker points out, "the phrase 'indispensability for science' is vague. What, exactly, is the scientific purpose (or purposes) for which mathematics is supposed to be indispensable?" (Baker 2005, p. 223). This is a crucial question because, depending on the answer, one can either give a final blow to the indispensability argument or can open a new path for its recasting. The final blow was attempted by Joseph Melia (2000, 2002) who argued that mathematics is not indispensable for science in the right kind of way (Baker 2005, p. 224) to rationally constrain a scientific realist to be committed to the existence of mathematical entities. For Melia, *the right way* is "the way in which the postulation of theoretical physical entities increases the utility of our scientific theories" (Melia 2002, p. 75).

Luckily for the mathematical realist, there is an answer to the above question that can be exploited to save the indispensability argument. Alan Baker (2005, 2009) provides the most extensive and influential discussion of this answer. Baker argues that *mathematics plays an indispensable explanatory role in science*. If this is the case, we can have the following modified strategy:

(1) Some scientific realists believe in the existence of any entity that plays an indispensable explanatory role in our best scientific theories.

[3] This is only a presentation of what I take to be the idea behind this indispensability argument for mathematical realism and it should not be confused with the actual argument.
[4] For example Achinstein (2001), Azzouni (2004), Psillos (2009), Maddy (1992) and Sober (1993).

(2) Mathematical objects play an indispensable explanatory role in science (i.e. there are physical phenomena which cannot be explained without appeal to some (explanatorily active)[5] mathematical objects).

(3) Hence, the scientific realists referred to in (1) ought to rationally believe in the existence of mathematical objects (Baker 2009, p. 613).[6]

Baker's enhanced indispensability argument (EIA) is more powerful than Quine's argument. First of all, its target is considerably bigger – there are many scientific realists that use inference to the best explanation to argue for their position. Secondly, it doesn't need to use something as controversial as confirmational holism[7] to prevent the scientific realist from dissociating between entities worthy and unworthy of entering one's ontology. All is needed for it to work is that inference to the best explanation is taken as central for defending scientific realism and that there is a convincing case for premise (2). The first part is trivially obtained so all that the mathematical realist is left with is the problem of showing that (2) is indeed the case. But how can s/he do that? Before discussing the options available for answering this question, it is important to understand what exactly (2) amounts to. For this, it is important to remember that the idea behind any indispensability argument is that we should be ontologically committed to those entities that are indispensable in science for a certain purpose (Field 1989, 15; Colyvan 2003, 6-7).[8] In the case of EIA, the emphasis falls on explanatory purposes. So, understanding (2) depends on understanding what exactly does "indispensable for explaining" means. The first thing that may come to mind is the effectiveness of mathematics as the language in which scientific explanations are couched. This won't do, though, partly because it doesn't create a noticeable difference between this role of mathematics and the way it is used in the rest of science, and so EIA won't represent an improvement over the type of indispensability arguments attacked by Melia. A better approach is to take "indispensable for explaining" as meaning playing a genuinely explanatory role in science (2a). This does distinguish EIA from other indispensability arguments, but it is still problematic because having the property of being explanatorily relevant doesn't make something

[5] I use "explanatorily active" for those entities (or their properties) which are mentioned in an explanatorily relevant part of an explanation.

[6] This is not, of course, Baker's argument, but what I take to be his argumentative strategy.

[7] Although see the arguments in Morrison (2012) and Marcus (2014) for a different opinion.

[8] Let's call this the IA indispensability requirement.

indispensable in science. There are dispensable but explanatorily relevant parts of scientific explanations (for an illustration, see the example discussed below, in section ***3.***) and mathematics has to be more than this in order for EIA to work properly (remember the indispensability requirement that needs to be satisfied by all indispensability arguments). So, besides being explanatorily relevant, mathematics also needs to be indispensable in the context of an explanation (2b). This is still not enough, though, because nothing prevents the possibility of there being, for those phenomena in whose explanations mathematical objects are explanatorily active, alternative mathematics-free explanations. The existence of such alternatives is, of course, incompatible with considering mathematics as indispensable for explaining. So, for it to be genuinely indispensable, one of the following two situations has to be the case: either there are no alternative explanations (2c), or there are such explanations but mathematical objects are explanatorily active in all of them (2c*). If we put all this together, we obtain the following view about what "indispensable for explaining" has to amount to in the context of EIA: ***x*** is *indispensable for explaining* some phenomenon ***P*** if

> (2a) ***x*** is an *explanatorily relevant part* of an explanation of ***P***;
>
> (2b) the explanation mentioned in (2a) cannot be given without ***x*** (i.e. ***x*** is *indispensable in the context of that explanation*);
>
> (2c) there are no other explanations for ***P*** (*the explanation is indispensable for understanding* P); or,
>
> (2c*) there are alternative explanations for ***P***, but all of them use ***x***.

Coming back to our question, in order to show that premise (2) of EIA is indeed the case, the mathematical realist has to show either that there are cases in which mathematics is an indispensable explanatorily relevant part of an explanation indispensable for understanding some physical phenomenon, or that it is an explanatorily relevant part in all the alternative explanations that scientists can/do provide for such a phenomenon.

The most challenging thing for the mathematical realist (in part because of the success that the causal-mechanical accounts of scientific explanation enjoy among the scientific realists) is to show that mathematics can actually play an explanatory role in science, i.e. that (2a) is indeed the case. There are several strategies that s/he can use for this purpose. One way to do it is by appeal to a theory of explanation. In this case we start with a model of explanation and argue that, in a particular explanation, a certain part is doing an explanatory job because it satisfies the criteria that, from the perspective of that model, are necessary for having such a contribution.[9]

A different way is to use Steiner's account. In Mark Steiner's view, a characteristically mathematical explanation of a physical phenomenon is one in which, if "we remove the physics, we remain with a mathematical explanation – of a mathematical truth" (Steiner 1978, p. 19). The main idea here, even though not explicitly stated by Steiner, seems to be that explanatory power somehow leaks from the purely mathematical explanation into the mathematical explanation of the physical phenomenon. So, we can say that the mathematical part in a certain scientific explanation is contributing to its explanatory power if there is an internal mathematical explanation for it, i.e. an explanation within mathematics.[10]

Yet another strategy – the one that makes the object of this paper – for arguing that (2a) is the case is to try to find another more easily identifiable feature that the mathematical part can have in a mathematics using scientific explanation and that is linkable to explanatory relevance. This brings us to the importance view on explanatory relevance, because the most obvious feature that can do the trick the mathematical realist is after is importance in the context of an explanation. The idea is simple and straightforward: *if the mathematical part is important for an explanation, removing it would compromise the explanation, but how could that be if that part didn't have some sort of explanatory worth to begin with?* Basically, what the mathematical realist does if s/he adopts this strategy is to use IVER to link (2a) with (2b): if we know that (2b) is the case, given the relation between explanatory relevance and importance in the context of an explanation postulated by IVER, we also know that (2a) is the case. So, by using this point, the mathematical realist can (apparently) make a strong case for (2a) just by showing that there are examples of mathematics using scientific explanations in which the mathematical part plays an *important/essential* role. Between the most discussed such examples in the literature we can find the explanation for why honeybees build their honeycombs as hexagonal grids and the explanation for the fact that the North American

[9] See for example Aidan Lyon's (2012) program explanation account.
[10] Besides Steiner (1978), we can find this approach in Colyvan (2003) and Lyon and Colyvan (2008).

periodical cicadas (fly like insects that spend many years underground in larval form) have life-cycle period lengths that are prime. In the first case, biologists explain why honeycombs have that particular shape with the help of the following mathematical theorem: a hexagonal grid is the optimal way to divide into regions of equal area with least total perimeter a Euclidean plane. The explanation goes as follows: in order to win the natural selection fight, bees had to choose the most economic (in terms of labour and amount of wax used) way to build their honeycombs. As it is clear from the mathematical theorem presented above, from all the possible shapes, the hexagonal grid is the most economical in the relevant respects. This is why the honeycombs have that boggling shape (Lyon and Colyvan 2008, p. 228; Lyon 2012, p. 560).

In the case of North American periodical cicadas, the explanation makes essential use of the following mathematical theorem: the lowest common multiple of two numbers is maximal when the numbers are coprime. The explanation goes like this: having a life-cycle period which minimizes intersection with other periods is evolutionarily advantageous because it either helps with avoiding predators or it diminishes the chances of hybridization with similar subspecies. But, from the theorem above it follows that the prime-numbered periods minimize the intersection with other periods. So, periodic organisms such as *Magicicada* are likely to evolve prime periods (Baker 2005, p. 233).

Why should we take these and similar examples as cases of genuine mathematical explanations? If we search for an answer in Alan Baker's papers on this topic, we find something along the lines of the IVER-involving strategy discussed here.

Why should we believe that the number theoretic theorem in the cicada example plays an explanatory role and is not just a non-explanatory component of the explanation? In order to answer this question we would need to use some sort of test for the explanatoriness of a part of an explanation. But, as Baker argues, there is no "ready-made test for the explanatoriness of a piece of mathematics in a given physical situation" (Baker 2009, p. 624). A way to overcome this impasse is by figuring out on whom the burden of proof should fall. "Does the platonist need to give a positive argument for why the mathematics in the cicada case is explanatory in its own right, or does the nominalist need to give a positive argument to the contrary?" (p. 624) Baker is not shy to admit that he does "not know how to demonstrate that the mathematical component is explanatory" (p. 625), but he doesn't see this as too much of a problem for the Platonist because he considers that "it is reasonable to place the burden of proof here on the nominalist" (p. 625).[11] In order to do this he needs, of course, to offer at least some prima facie reasons for believing that mathematics has a genuine explanatory contribution in such cases. Baker comes up with two such reasons. First of all, he draws attention to the fact that biologists seem to take mathematics as playing an explanatory part:

> "The way biologists talk and write about the cicada case suggests that they do take the mathematics to be explanatory, and this provides good grounds, at least prima facie, for adopting this same point of view" (Baker 2009, p. 625).
> ... "the alleged explanatoriness of the number theoretic component of the cicada case study seems to mesh well both with our intuitions and with the intuitions expressed by biologists working in this area" (Baker 2005, pp. 235-236).

Secondly, Baker argues that the mathematical component of the cicada example is essential for the explanation and, in his opinion, this elicits the belief that it plays an explanatory role. In other words, the reason for believing that in this case we are dealing with a "mathematical explanation… [has to do with the fact that] it makes essential use of the number-theoretic theorem in premise (2)" (Baker 2012, p. 263). Now, Baker doesn't further develop this point, but, based on the claim quoted above, I think it is safe to say that he does partly rely on something at least resembling the importance view on explanatory relevance, in his case for the explanatory relevance of mathematics.

Someone can draw the attention at this point that Baker speaks about the essential, not the important role played by mathematics in the cases discussed, so his position is different from IVER. If by different what is meant is gradual difference, then I agree: "essential" is a stronger, more restrictive notion than "important." The reason I prefer to formulate IVER in terms of importance will become apparent later. For now suffice to say that, in Baker's argumentative strategy for premise (2a) in EIA, linking the explanatory relevance of a part of an explanation with its (essential) role in the context of that explanation occupies an important place.

[11] Cf. Busch and Morrison (2016).

In what follows I will try to show what is wrong with relying too much on this link.

3. *Types of explanatory relevance*

As we said above, the main idea behind one of the strategies for arguing that mathematics can play an explanatory role in science is to adopt IVER, i.e. the view according to which explanatory worth can be assessed in terms of importance in the context of an explanation which in turn can be assessed in terms of the effects that removing a certain part can have on the explanation. If things are indeed as this view suggests, we can present explanatory relevance this way:[12]

> ***ER: x*** is explanatorily relevant to ***P*** if[13] ***x*** is an *important part* of a good explanation of ***P***.

The first thing to notice if we take a closer look at ***ER*** is that linking this way explanatory relevance with importance in the context of an explanation seems to imply that there are degrees of explanatory relevance – after all, importance comes in degrees (something can be more important or less so for a certain purpose). This may be taken as evidence against IVER because we don't usually speak about degrees of explanatory relevance.[14] There are two possible ways out of this difficulty: either we argue that only the highest degree of importance is a good sign for explanatory relevance, or we can argue that explanatory relevance does actually come in degrees, so, instead of dealing with a problem, we actually stumbled upon an important feature of the explanatory relation as a result of exploring the implications of IVER.

It may seem that the first option is the best and it is more in the spirit of Baker's appeal to the essential role of mathematics (if we understand by essential the highest degree of importance). From this perspective, the main idea behind the mathematical realist's strategy fits better with the following way of thinking about explanatory relevance:

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[12] A similar formulation can be found in Cartwright (2011, p. 15).

[13] ***ER*** is not meant as a definition of explanatory relevance, so what comes after 'if' should not be taken as sufficient conditions for this thing but as a good sign that we are dealing with such a relation.

[14] We do speak of degrees of explanatory power, but that is a different matter.

> ***ER****: ***x*** is explanatorily relevant to ***P*** if it is an *essential part* of a good explanation of ***P***.

There is an important reason why opting for linking explanatory relevance only with the highest degree of importance, and so for ***ER**** over ***ER*** is not something an advocate of mathematical explanations should want to go for, but in order to see this we need to look for a moment at a different problem. Consider the following two cases:

(a) ***x*** can explain ***P*** by itself and without it there is no explanation;

(b) ***x*** and ***y*** together explain ***P*** and without ***x*** the explanation is impossible.

Even though it is safe to say that in both cases ***x*** is essential for the explanation (because the explanations are impossible in its absence), there is an important difference between the two cases. In (a) ***x*** is necessary and sufficient for the explanation while in (b) it is necessary but insufficient. What this suggests is that "essential part" is equivocal here (i.e. in ***ER****) between two meanings: it can mean a necessary and sufficient part of an explanation or a necessary and insufficient part of an explanation. Now, suppose we take the highest degree of importance to mean essential in the sense of necessary and sufficient and we consider that only this kind of importance can be taken as a good voucher for the explanatory relevance of a part of an explanation; we would need in this case to replace ***ER**** with something that expresses the difference between (a) and (b). We have then:

> ***ER*****: ***x*** is explanatorily relevant to ***P*** if it is a *necessary and sufficient part* of a good explanation of ***P***.

It is not hard to see that this is far from helpful to the advocate of mathematical explanations. The mathematical part in the examples presented above is not necessary and sufficient for explaining the physical phenomenon of interest. It makes no sense to say only that "The organisms with periodic life-cycles are likely to evolve periods that are prime because the lowest common multiple of two numbers is maximal when the numbers are coprime," or that "Bees build their honeycombs as hexagonal grids because a hexagonal grid is the optimal way to divide into regions of equal area with least total perimeter a Euclidean plane." So, when referring to the essential role played by mathematics in examples as those presented above, Baker must have in mind then something along the lines of case (b): the mathematical part is necessary but insufficient for the explanation of those particular physical phenomena. Can he make a case for interpreting "the highest degree of importance" only as essential in the sense of necessary and insufficient (remember that the first option for dealing with the IVER-related difficulty is meant as an alternative to accepting that there are degrees of explanatory relevance)? I don't see how – in order to do that, he would need to dismiss ***ER**** and argue that *only* a necessary and insufficient part of an explanation can be explanatorily relevant. But, if this is the case, the first way out of the difficulty discussed above (i.e. taking only the highest degree of importance as a good sign of explanatory relevance) is not something an advocate of mathematical explanations would want to opt for.

The only option left then for anyone who adheres to IVER and wants to use the (allegedly strong) link between explanatoriness and importance in the context of an explanation in an argument for the existence of mathematical explanations is to accept that explanatory relevance comes in degrees. The distinction drawn above between the (a) and (b) cases can be used to distinguish between two such degrees. We can say, for example, that a part of an explanation is *strongly explanatorily relevant* for some explanandum-phenomenon if it is necessary and sufficient[15] for an explanation of that phenomenon; and that such a part is only *moderately explanatorily relevant* if it is necessary but insufficient. What about unnecessary and insufficient parts? Can there be explanatorily relevant yet unnecessary and insufficient parts of explanations? I believe there are. To see this let's look at the following two explanations of Boyle's law.

First of all, we can explain Boyle's law by deriving it from the Ideal Gas Law this way: from the perspective of the kinetic theory, gases are composed of extremely small particles called molecules which move randomly in all directions and between which there are only negligible attractive forces, and when they collide no kinetic energy is lost (elastic collisions). Starting from here, we can produce the following explanatory derivation of the Ideal Gas Law ($\boldsymbol{P} \propto \frac{\boldsymbol{nT}}{\boldsymbol{V}}$):[16]

- the pressure of a gas is caused by the interaction between the molecules and the wall of the container. Because we are dealing with molecules, we can express this interaction in terms of the frequency of collisions and the average force that a

[15] For this to make sense, one needs to assume that an explanation can have parts that don't have explanatory value; otherwise we would not be talking about a part of an explanation in this situation, but about an explanation. I do not consider this to be a controversial assumption, but for those unconvinced, section ***4.*** of this paper can be taken as an argument supporting it.

[16] $\boldsymbol{P}$ stands for the pressure of an amount of gas, $\boldsymbol{n}$ for the moles of molecules, $\boldsymbol{V}$ for volume and $\boldsymbol{T}$ for absolute temperature.

molecule exerts when colliding to the wall (this depends on the average momentum of a molecule $\boldsymbol{mv}$, i.e. its mass $\boldsymbol{m}$ times its average speed $\boldsymbol{v}$).

- the frequency of collisions depends on (is affected by) the average speed of the molecules $\boldsymbol{v}$ (faster molecules hit the wall more often), the number of molecules $\boldsymbol{N}$ (if we have more molecules we will get more collisions), and the volume of the gas $\boldsymbol{V}$ (the larger the volume, the smaller the density and so the fewer the collisions).
- if we express formally the two points above, we get: $\boldsymbol{P} \propto (\boldsymbol{v} \times \boldsymbol{N} \times \frac{\boldsymbol{1}}{\boldsymbol{V}}) \times \boldsymbol{mv}$. From this we get $\boldsymbol{P} \propto \frac{\boldsymbol{Nmv^2}}{\boldsymbol{V}}$ (i.e. the pressure of a given amount of gas is directly proportional to the number of moles of gas and to the temperature of the gas, but inversely proportional to the volume). But, since $\boldsymbol{N}$ is proportional to the moles of molecules $\boldsymbol{n}$ and the kinetic energy ($\frac{\boldsymbol{1}}{\boldsymbol{2}}\boldsymbol{mv^2}$) is proportional to the absolute temperature[17] $\boldsymbol{T}$, we have $\boldsymbol{P} \propto \frac{\boldsymbol{nT}}{\boldsymbol{V}}$, i.e. the Ideal Gas Law. But by keeping $\boldsymbol{n}$ and $\boldsymbol{T}$ constant, we will have $\boldsymbol{P} \propto \frac{\boldsymbol{1}}{\boldsymbol{V}}$ which is nothing more than Boyle's law: at constant temperature, the pressure of a fixed amount of gas is inversely proportional to its volume.

But there is no need to go through the Ideal Gas Law to explain Boyle's law. We can obtain a simpler explanation directly from the kinetic theory. As we said above, according to the kinetic theory, gases are composed of molecules of negligible size which move randomly in all directions. When enclosed in some container, the gas molecules interact with its walls. From this interaction results the pressure of the gas. If the volume is decreased, the concentration of molecules increases and so the interaction with the container walls increases. But pressure depends on this interaction, so it will increase also. Then a decrease in volume will determine an increase in pressure, i.e. pressure is inversely proportional to volume – and this is exactly what Boyle's law states.

It is obvious that the second explanation is simpler. All we used to explain Boyle's law was the relation between volume and molecular density: if the volume is decreased, molecular density increases and so there will be more molecules that interact with the container walls. This seems to be enough to understand the relation between volume and

[17] This is one of the postulates of the kinetic theory.

pressure expressed in Boyle's law. But what about the information about the speed of molecules, their mass or the temperature of the gas used in the first explanation? Was it just explanatory noise? Certainly not! Take, for example, temperature. According to Boyle's law, $\boldsymbol{P} \propto \frac{\mathbf{1}}{\boldsymbol{V}}$ if $\boldsymbol{n}$ and $\boldsymbol{T}$ are constant. But why does $\boldsymbol{T}$ need to be constant? What happens when it is not so? The second explanation doesn't provide enough adjacent information for understanding such a situation, but if it did, as the first explanation does, we would have a bigger and more complex picture about the relation between pressure and volume: we would understand not only why it holds, but also why it can fail to hold. Of course, in order to understand why something is the case, we don't need to know why it can fail to be so – this means that, even though minimal, the second explanation is a good explanation. But knowing why something can fail to be the case and what its relation is with some other things can greatly improve one's understanding – so, the first explanation has a greater explanatory power than the second one. Now, given that the adjacent information used in the first explanation is not indispensable for a good explanation of Boyle's law, it cannot have the same type of explanatory relevance as the information used in the second explanation, but, by contributing to the explanatory power of the first explanation, it is explanatorily relevant nonetheless. I think it is appropriate to consider this type of insufficient and unnecessary (dispensable) but nonetheless explanatorily relevant part of an explanation as being only weakly explanatorily relevant.

We can further develop these considerations by exploiting the relation between explanatory relevance and importance to distinguish between three types of explanatory relevance.

$\boldsymbol{ER^s}$ (strong explanatory relevance): $\boldsymbol{x}$ is strongly explanatorily relevant to $\boldsymbol{P}$ in case $\boldsymbol{x}$ is part of a good explanation of $\boldsymbol{P}$ and (i) $\boldsymbol{x}$ can explain $\boldsymbol{P}$ by itself; (ii) removing $\boldsymbol{x}$ will destroy the explanation. In other words, $\boldsymbol{x}$ is strongly explanatorily relevant to $\boldsymbol{P}$ just in case $\boldsymbol{x}$ *is a sufficient and necessary (indispensable) part of an explanation of* $\boldsymbol{P}$.[18]

$\boldsymbol{ER^m}$ (moderate explanatory relevance): $\boldsymbol{x}$ is moderately explanatorily relevant to $\boldsymbol{P}$ in case $\boldsymbol{x}$ is part of a good explanation of $\boldsymbol{P}$ and (i) $\boldsymbol{x}$ cannot explain $\boldsymbol{P}$ by

[18] We can dub an explanation composed only of such a part a *minimal explanation*.

itself; (ii) removing ***x*** will destroy the explanation. In other words, ***x*** is moderately explanatorily relevant to ***P*** just in case ***x*** *is an insufficient but necessary (indispensable) part of an explanation of* ***P***.

ER^w (weak explanatory relevance): ***x*** is weakly explanatorily relevant to ***P*** in case ***x*** is part of a good explanation of ***P*** and (i) ***x*** cannot explain ***P*** by itself; (ii) removing or replacing ***x*** will not destroy the explanation but will affect its explanatory power. In other words, ***x*** is weakly explanatorily relevant to ***P*** just in case ***x*** *is an insufficient and unnecessary (dispensable) part of an explanation of* ***P*** but removing or replacing it from such an explanation will be done at the expense of explanatory power.

In order to avoid confusion, it is important to emphasize at this point the fact that "indispensable," as it appears above in our formulations of ***ER***'s (i.e. as a degree of importance in a particular context), has a different sense than "indispensable," as it appears in EIA (i.e. as necessary for a certain purpose). In the first case we have *indispensable in the context of an explanation* (a part of an explanation is this way if it cannot be removed from the explanation without destroying it), in the second we have *indispensable for explaining a certain phenomenon* (a part of an explanation of some phenomenon is this way if it satisfies the conditions ((2a)-(2c)) listed in section ***2.***). These two notions are different – even if (contrary to what is argued here) one takes indispensability as a good voucher for explanatory relevance – because there can be situations in which a part of an explanation of some phenomenon is indispensable in the context of that explanation, but there is an alternative explanation for that phenomenon that doesn't use it and has a greater explanatory power. So, in such a situation, we have an indispensable part of an explanation that is not indispensable for understanding a certain phenomenon. They do overlap, though.

Getting back to the issue of mathematical explanations, we can ask now what type of explanatory relevance can the mathematical part have in the examples presented above. Let's take for example the cicada case. It is obvious from the start that the number-theoretic theorem cannot be a sufficient part of an explanation for the life-cycle periods that cicadas have, so it fails the ***ER^s*** test. What about ***ER^m***? Is the number-theoretic theorem indispensable for a good explanation of the primeness of the cicada life-cycle periods? It certainly does seem so. Remember that the explanandum in this case was the intriguing life-cycle lengths that the North American cicadas have, i.e. their primeness. As Baker (2005) notes, biologists developed two alternative evolutionary explanations for this phenomenon. One takes as the main evolutionary advantage that developing such periods is supposed to have to be the minimization of intersection with predators that also have (lower) periodic life-cycles; the other takes the avoidance of hybridization with subspecies that have different life-cycle periods to be the main advantage. What is interesting is that both these explanations have a mathematical basis in number theory, because both make use of the mathematical link between the primeness of the periods and minimizing intersection with other periods. Baker considers that since without this link there is no way to make sense of the evolutionary advantage that developing such life-cycle periods is supposed to have, the purely mathematical component is essential to the overall explanation (in both cases) and so it is genuinely explanatory[19] in its own right (Baker 2005, p. 233). This fits perfectly with ***ER^m***. There is a problem, though. As argued in Daly and Langford (2009) and in Saatsi (2011), the mathematical part in the cicada example (apparently) can be replaced with a non-mathematical alternative without destroying the explanation.

Daly and Langford (2009) give the following mathematics-free alternative to the explanation presented above: "given that certain relevant creatures also present in the cicada habitat have periodic life-cycles of some other duration, it is advantageous for the cicada life-cycle to be of the particular duration it is, for this minimizes the encounters between the organisms" (Daly and Langford 2009, p. 657). The numbers associated with these durations depend on the unit of time used to measure them, and the reason why the North American cicadas have life-cycles periods of some particular number of units (prime number in the above example) has to do with the evolutionary advantage that having the duration indexed by that number is supposed to provide. So, a proper explanation of the life-cycle periods of the cicada should not target the primeness of the number of units that index the durations measured but the durations themselves. With this out of the way, it is easy to realize that there is no need to resort to some number-theoretic

[19] A similar kind of reasoning can be easily developed for the honeycomb example: without the mathematical theorem there is no way to make sense of the evolutionary advantage that using such shapes to build the honeycombs is supposed to have, so the honeycomb theorem is essential for the explanation and that makes it genuinely explanatory.

result in order to tie the peculiar life-cycle periods that cicadas have to some evolutionary advantage. The entire explanatory job is done by the physical phenomenon of duration.

Juha Saatsi (2011) suggests a different way of altering the cicada explanation, namely by replacing the mathematical part with the following fact about time: "For periods in the range 14–18 years the intersection minimizing period is 17" (Saatsi 2011, p. 149). The explanation we obtain as a result of this change is only a specific one about the cicadas with a 17-year life-cycle period, but there is a way to obtain a more unifying and general explanatory argument pattern by using a more abstract version of the fact about time:

> "(4) Having a life-cycle period which minimizes intersection with other (nearby/lower) periods is evolutionarily advantageous. [biological law]
> (5/6)** There is a unique intersection minimizing period T_x for periods in the range [$T_1, \ldots, T_2$] years [fact (?) about time]
> (7) Cicadas in ecosystem-type E are limited by biological constraints to periods from T_1 to T_2 years. [ecological constraint]
>
> ___________
>
> (8) Cicadas in ecosystem-type E are likely to evolve T_x-year periods" (Saatsi 2011, p. 152).

There may seem to be two argumentative options for dealing with the problem of alternative explanations open to the advocate of mathematical explanations: either dismiss the mathematics-free alternatives as not being good explanations or not being explanations at all, or argue that they are less explanatorily powerful than the mathematics involving one, i.e. either keep fighting for an $\boldsymbol{ER^m}$ type of explanatory relevance or try to save something by withdrawing to $\boldsymbol{ER^w}$. Alan Baker[20] explores both of them.

Concerning the first one, Baker's strategy is to draw attention to the fact that these alternatives are disconnected from or even in tension with the scientific practice. But many take scientists' judgements about what counts as a good explanation of a particular phenomenon to be highly reliable. So, if there are good reasons to "believe that the

[20] See Baker (2009, p. 617) and Baker and Colyvan (2011, pp. 330-331).

biologists would… reject the nominalistic alternative" (Baker and Colyvan, 2011, p. 330), we should also dismiss them as bad explanations. There are good reasons to be reluctant to accept such an argumentative strategy, though,[21] one of them being that scientists can be unhappy with an explanation for various reasons. So, just saying that the biologists would not accept the nominalistic alternatives doesn't automatically mean that they make no sense. Let's elaborate. In order to deal with the problem of alternative explanations, Baker's main strategy (Baker 2009, p. 617; Baker and Colyvan 2011, p. 330) is to point to what he takes to be a fact that scientists would not accept the nominalistic alternatives. But is this showing that replacing the mathematical part in the examples discussed actually destroys the explanation? Remember that in order to make a good case for ***ER^m***, one has to show exactly this. The straightforward meaning of "destroy the explanation" is that the explanation would not make sense anymore. So, in order for Baker's appeal to the alleged scientists' reaction to work in this context, this has to be the case: scientists reject an alternative to an explanation only when that alternative doesn't count as an explanation at all, i.e. when it makes no sense. But this is obviously false. There are several other reasons for such a reaction from the scientists, two of the most obvious being that the alternative doesn't have the same explanatory power as the initial explanation or that it doesn't sit well with the theoretic context of the present science or the actual scientific practice. So, even if it makes perfect sense, the scientists can be unhappy with an alternative explanation if the initial one has a greater explanatory power or if it uses the resources of more recent theoretical developments. Take, for example, the optimality approach in evolutionary and behavioural ecology. This approach uses optimality models to deal with the evolution and fitness of phenotypic traits without reference to the system of genetic and epigenetic transmission. Despite their utility, widespread use and the fact that they make perfect sense, not all biologists are happy with the explanatory value of these models. Marcus Feldman, for example, is unhappy, among other things, with the fact that the optimality approach disregards the intricacies of genetic transmission (Schwartz 2002). Gould and Lewontin criticize such an approach for its failure to "consider adequately such competing themes as random fixation of alleles, production of non-adaptive structures by developmental correlation with selected features, the separability of adaptation and selection, multiple adaptive peaks, and current utility as an

[21] See for example Saatsi (2011, p. 153).

epiphenomenon of non-adaptive structures" (Gould and Lewontin 1979, p. 147). As with optimality explanations, it is obvious that Daly's and Langford's and Saatsi's explanations make perfect sense, so, if biologists would reject them, they would do so for different reasons. But this is far from what Baker needs in order to make a good case for believing that the mathematical part in the examples discussed has a sort of moderate explanatory relevance.

The second option for dealing with the problem of alternative explanations is to try to save something by withdrawing to $\boldsymbol{ER^{w}}$, i.e. to grant that the alternatives are good explanations but argue that they are not as good as the mathematical explanation.[22] Baker gives two reasons for taking the mathematics-free explanations as being less powerful: they are less general and less robust. Daly's and Langford's alternative is less general because it depends on the particular ecological constraints acting upon a cicada subspecies from a certain geographic area, and so it cannot be applied to other (sub)species with different ecological histories. It is also less robust because it "misses the fact that the details of what predators are around, and their various life-cycles, are largely irrelevant to the advantage of prime periods" (Baker and Colyvan 2011, p. 331). Saatsi's explanation, on the other hand, is definitely not less general than the mathematical explanation, so, in order to be less explanatory, it has to be less robust. I do not explore this possibility here. Instead, let's suppose that we have good reasons to suspect that Saatsi's alternative is indeed less robust. Will this settle the discussion? No! Saatsi can claim that his alternative has other explanatory virtues that the mathematical explanation is lacking and therefore it is better. This will drive the discussion into an endless quarrel about what combination of explanatory virtues is better. Of course, just showing that by taking this path we run into the danger of rendering the discussion inconclusible is not the same thing as showing that Baker is actually wrong and mathematics is not even weakly explanatorily relevant in the examples discussed. So, this is far from being the worst problem for the mathematical realist.

A bigger problem with this second option for dealing with the problem of alternative explanations is that $\boldsymbol{ER^{w}}$ doesn't work well with the indispensability argument (because of the indispensability requirement):[23] the mathematical realist has to show more

[22] Mathematics is a weakly explanatorily relevant part of an explanation if that explanation has, due to its mathematical component, a greater explanatory power than any nominalistic alternatives.
[23] For more about this requirement see the discussion in section ***2.***

than the fact that the mathematical part is weakly explanatorily relevant, s/he has to show that the mathematical part is either an indispensable explanatorily relevant part of an explanation indispensable for understanding some physical phenomenon, or that it is an explanatorily relevant part in all the alternative explanations that scientists can/do provide for such a phenomenon (see the discussion in section ***2.***). If math is only weakly explanatorily relevant then it can be removed from the explanation and so it fails to satisfy conditions (2b), and (2c) or (2c*). Some mathematical realists can, of course, choose to weaken EIA by replacing premise (2) with something less demanding (see for example Mancosu's (2008, 137) understanding of EIA).[24] That doesn't concern us here, though. What is important for our discussion is that, as far as Baker's version of EIA is concerned, mathematics has to be indispensable for explaining. This means that Baker's exploration of this option for dealing with the nominalistic alternatives is pointless from the perspective of an attempt to defend a realist position.

The mathematical realist can retort at this point that there is yet another potential way of dealing with the problem of nominalistic alternatives, namely by resorting to a sort of combination of the argumentative strategies discussed above. The key idea is to use *a threshold model of explanatory goodness* and argue that the mathematics-free alternatives are not good explanations because they are less explanatorily powerful than the mathematics-involving ones. According to such a model, the putative explanations of a certain phenomenon should be taken as ranged on a continuum, based on their relative explanatory power. If we take them this way, we can establish a threshold for delimiting the good from the bad explanations of that particular phenomenon. We would then say that a good explanation is one that is above the threshold – the best one being the one with the most explanatory power. So, on this model, not being a good explanation doesn't amount to not having explanatory power, but to being below the threshold. Also, "to destroy an explanation" means here to push it below the threshold. Starting from this, the mathematical realist can try to make a case for ***ER^{m}*** by arguing along these lines: the mathematical part of some scientific explanations as those discussed above (see section ***2.***) is moderately explanatorily relevant with respect to some phenomenon of interest if

[24] Mancosu's version of EIA runs as follows:
(a) There are genuinely mathematical explanations of empirical phenomena;
(b) We ought to be committed to the theoretical posits postulated by such explanations; thus,
(c) We ought to be committed to the entities postulated by the mathematics in question. (Mancosu 2008, 137).

there is no mathematics-free alternative explanation that meets the minimum threshold of explanatory power. For this, the mathematical realist needs to show that removing/replacing the mathematical part in such cases leads to pushing the explanation below the threshold, i.e. destroying it. This is, prima facie, an easier task than that of showing that such alternatives make no sense. There are big problems, though, with this argumentative strategy. The most obvious one has to do with the fact that, in this case, there doesn't seem to be a non-arbitrary way of establishing the threshold. Assuming that they would agree that the mathematics-free alternatives are less explanatorily powerful, the nominalists can always complain about the way the threshold was established. So this way of dealing with the problem of alternative explanations is also problematic.

Mathematical realism aside, the alternatives presented by Daly and Langford (2009) and Saatsi (2011) cannot create problems for a philosopher of science concerned with e.g. the existence of non-causal scientific explanations who is content to accept even that the mathematical part in the examples discussed have a sort of weak explanatory relevance. If the idea is only to establish if mathematics can actually play an explanatory part, then showing that it can satisfy something along the lines of $\boldsymbol{ER^{w}}$ seems like a good step in that direction. In what follows I will try to block this path for the advocate of mathematical explanation by arguing that importance in the context of an explanation cannot always be taken to vouch for explanatory relevance.

4. Importance and explanatoriness

What we have until now is this: we started by presenting an interesting view about explanatory relevance (i.e. one that links the importance of a part of an explanation in the context of that explanation with its explanatory worth) and showed how this (or something like this) view was exploited (even though not explicitly) in the literature to argue that mathematics does, in some cases, genuine explanatory work. As we have seen, the advocate of mathematical explanations who is also a mathematical realist faces the following challenge: in order for this IVER-involving strategy to work, s/he has to show that mathematics is more than weakly explanatorily relevant in the examples discussed. To do this, s/he has to make a compelling case for considering that the mathematics-free alternatives are not good explanations. But, as the above discussion revealed, it is hard to drive home this point. For those advocates of mathematical explanations who are not concerned with the indispensability argument for mathematical realism things are a little bit simpler. For them, the fact that there are nominalistic alternatives to the mathematical explanations doesn't constitute a problem as long as it can be shown that they are obtained at the expense of explanatory power, i.e. as long as it can be shown that in the case of the examples presented we can have something along the lines of $\boldsymbol{ER^w}$: the mathematical part in such cases is explanatorily relevant, even though only weakly so. In the remainder of this paper I will try to show that even this position is not without problems.

Under close scrutiny, IVER turns out to be highly problematic. According to this view on explanatory relevance, an important part of an explanation is an explanatorily relevant part. This is supported by the following simple (intuitive) reasoning: if a part of an explanation is important for that explanation in the sense that removing it will affect the explanation (destroy it or diminish its explanatory power) then that part must have some sort of explanatory contribution. The hidden assumption underpinning this reasoning is that *we can destroy an explanation or diminish its explanatory power only if we remove from it explanatorily relevant parts.* But this is false. To see why, let's take a closer look at the honeycomb example. The structure of this explanation can be represented this way:

(1) Given some ecological constraints $\boldsymbol{e_i}$, it is evolutionary advantageous for cell-building organisms to be as effective as possible.

(2) Cell-building effectiveness has to do with minimizing the amount of material used.

(3) A hexagonal grid has the isoperimetric property of being the optimal way to divide a Euclidean plane into regions of equal area with least total perimeter.

(4) The most effective partition of some portion of the physical space into regions of equal area is the one done with hexagonal cells. (from (3))

(5) Honey bees are affected by the ecological constraints $\boldsymbol{e_i}$.

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(6) That is why they produce hexagonal cells.

What can we say about the mathematical theorem formulated in (3)? Is it important for this explanation? It certainly is. Without it there is no way to make the link between cell-building effectiveness and the hexagonal cell structures, i.e. we cannot get (4). But without (4) we cannot understand why under constraints $\boldsymbol{e_i}$ bees build hexagonal cells. So, removing (3) forfeits the explanation. Now, if we look closely, we notice that there is a problem with this reasoning. It is true that without (3) there is no way to go from (2) to (4), but it is false that on its own (3) can be so helpful. (3) is a mathematical theorem about the isoperimetric property of some mathematical object. In order to make it relevant in the context of this explanation, we have to assume that the Euclidean space can represent the physical space,[25] i.e. we have to add the following point to the explanation:

> (3') Euclidean geometry correctly represents the structure of the physical space and so it can be used to infer facts about the latter.

Making (3') explicit helps us realize that (3)'s role is only to point to and offer us knowledge about some relevant physical fact,[26] i.e. that a hexagonal cell structure is the most economical, in terms of the wall material used, partition of some portion of the physical space. But then, (3) doesn't do any explanatory job.[27]

A different way to argue for (3)'s explanatory irrelevancy is by pointing to the fact that (3) and (4) express structurally similar facts. But if this is the case, on pain of redundancy, it doesn't make much sense to take both as explanatorily relevant in this context. So, either (3) or (4) has explanatory value. As it is said above, (3) is a mathematical fact about the Euclidean space and is relevant for this explanation only if

[25] We are so used to applying mathematics that we sometimes forget that it is not about the physical world.

[26] A similar point is made in Saatsi (2011). Unlike Saatsi, who says that "there is no 'mathematical part' to this evolutionary explanation" (Saatsi 2011, p. 146), I take the mathematical theorem in (3) to be a genuine and very important part of the honeycomb explanation.

[27] There is a Steiner inspired possible objection that can be raised at this point: the fact that the mathematical theorem represents a physical fact doesn't make it less explanatory. Actually, if Steiner (1978) is right, this is a sort of prerequisite for mathematics to be explanatory in such a context: if the mathematical theorem represents a physical fact, then the mathematical explanation of the mathematical theorem can be transferred to the physical fact, hence we will have a mathematical explanation for it. We don't need to reject Steiner's view in order to dismiss this objection. It is sufficient to point to the fact that the explanandum in our case is the hexagonal structure of the honeycombs. But, even if the proof of the mathematical theorem formulated in (3) can be taken as an explanation for some physical fact, it will be for the fact that a hexagonal cell structure is the most economical, in terms of the wall material used, partition of some portion of the physical space. So, its role in the honeycomb structure explanation is at most that of enhancing our justification for one of the explanatory relevant facts.

(3') is the case. But, given (3'), (3) *becomes* (4). So, only (4) plays a genuine explanatory role in this explanation.

This, of course, doesn't mean that the mathematical theorem is actually not a part of the explanation. There is no way to know the relevant physical fact it stands for without its help. So, removing it from this explanation will deprive us also of the explanatory relevant physical fact. This makes (3) an indispensable part of the explanation; but then, if IVER was correct, it should also be explanatory. We have seen why that is not the case, so IVER is false. Actually, it is not completely false. It doesn't encounter the same problems if we interpret importance in the sense of $\boldsymbol{ER^s}$ (i.e. as a sufficient and necessary, so indispensable part of an explanation). But that is not very helpful for those who see in IVER a way to argue for mathematical explanations, because we cannot have minimal mathematical explanations of physical phenomena.

It is, of course, trivially true that if a part of an explanation is explanatorily relevant, then it is important for that explanation (even though in some cases only weakly so).[28] But the converse is not always true because importance in the context of an explanation is richer than it should be for such a relation to hold. So, when arguing that a part of an explanation has explanatory value, it is at least not sufficient to point to the fact that it is important in the context of that explanation. Importance is not a good voucher for explanatory relevance!

5. Two views of explanation

In the previous section I argued that there can be cases of explanations that comprise parts which are important but are not explanatorily relevant. If this is the case, then we cannot rely on the importance that a part of an explanation has in the context of that explanation for determining if it is explanatorily relevant. So, despite its intuitive appeal, the view on explanatory relevance analyzed here is problematic, to say the least. There is a very important thing that I left aside until now: in order for the argument in section ***4.*** to work and for the rest of the paper to make sense, we need to look at it from an epistemic perspective on explanation that doesn't include an account of explanatory relevance. This last part is important for the strategy discussed here because it allows us to rely

[28] See the Boyle's law example discussed above.

exclusively on importance for determining the explanatorily relevant parts of an explanation. This is something that whoever uses this strategy has to appreciate because it sits well with the possibility that the type of explanatory relevance found in mathematical explanations is different from what we encounter in other scientific explanations.

The distinction between ontic and epistemic accounts of explanation can be traced back to Salmon (1984), but, most recently, this division appears in connection with the disagreement about the nature of mechanistic explanation. Some think that (mechanistic) explanation is ontic (Machamer et al. (2000); Glennan (2002, 2005); Craver (2007)), i.e. that "the term *explanation* denotes a class of non-representational, mind-independent entities that are located within reality among its other extant spatiotemporal parts, and which scientists can discover" (Wright 2015, p. 20). Others prefer an epistemic perspective (Bechtel and Abrahamsen (2005); Bechtel (2008); Wright (2012, 2015)), i.e. they see explanations as descriptions or models aimed at making phenomena intelligible. From this perspective, "explanatory content is always and everywhere the fruit of our cognitive and epistemic labors" (Wright 2012, p. 376).

None of the main points discussed in this paper make much sense if viewed from an ontic perspective on explanation. Most importantly, it doesn't make sense to speak about unexplanatory or unimportant parts of an explanation. If explanations are "full-bloodied things" (Craver 2007, p. 27), then either something is or is not part of an explanation (i.e. it is or it is not explanatorily relevant) and if it is, it cannot fail to be important. If this is the case, then it also doesn't make sense to speak about degrees of explanatory relevance. Also, the argument in section ***4.*** (more specifically the case for (3)'s indispensability) is fallacious from this perspective, because from the fact that (3) is indispensable for knowing something doesn't follow that it is also important for the explanation (i.e. for the way things are). There is a way, though, to recast the main idea behind our discussion so as to make a bit more sense from an ontic perspective, namely by using the distinction between an explanation and its description. The view on explanatory relevance discussed here can then be modified along these lines:

IVER$^{\text{ontic}}$: *a part of a description of an explanation of some phenomenon is a genuine part of that explanation if removing it from the description affects our understanding of the phenomenon.*

I'm not going to enter into details here, but I think it is pretty clear that this is something that no advocate of the ontic conception would find very appealing.

6. Conclusion

How can one make a strong case for the idea that the mathematical part has explanatory worth in (some of) the mathematics using scientific explanations? A simple way to do this (assuming an epistemic conception of explanation) is by arguing along this lines: if ***x*** and ***y*** are parts of an explanation of ***P***, then ***x*** partially explains ***P***. In other words:

> ***ER***$^{\#}$: ***x*** is (partially?) explanatorily relevant to ***P*** if it is part of a good explanation of ***P***.

A part of an explanation partially explains the explanandum, so, if mathematics is used in some scientific explanations of physical phenomena, we can say that it partially explains these phenomena. Besides the fact that it is not easy to make clear what "to partially explain something" is meant to mean, the problem with this strategy is that it takes everything that appears in an explanation to have an explanatory contribution. Every mathematized scientific explanation would be, from this perspective, a mathematical explanation. This is obviously false.

My aim in this paper was to argue against a more sophisticated version of this strategy, i.e. one that relies on (what I'm calling) the importance view on explanatory relevance, and takes only those parts of explanations that are important (i.e. those that if removed would affect the explanatory power of the explanation) to have explanatory worth. In section ***3.*** I tried to show how this strategy can be developed and I argued that Baker's use of it fails to be of much help for the mathematical realist. In section ***4.*** I pointed to a serious problem with IVER: the central assumption underpinning it is false – we can also destroy an explanation or diminish its explanatory power if we remove from it parts that are not explanatorily relevant.

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