Classical Electrodynamics in agreement with Newton's third law of motion

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Abstract

The force law of Maxwell's classical electrodynamics does not agree with Newton's third law of motion (N3LM), in case of *open* circuit magnetostatics. Initially, a generalized magnetostatics theory is presented that includes two additional physical fields B_Φ and B_l, defined by scalar functions. The scalar magnetic field B_l mediates a longitudinal Ampère force that balances the transverse Ampère force (aka the magnetic field force), such that the sum of the two forces agrees with N3LM for all stationary current distributions. Secondary field induction laws are derived; a secondary curl free electric field $\mathbf{E}_l$ is induced by a time varying scalar magnetic field B_l, which isn't described by Maxwell's electrodynamics. The Helmholtz' decomposition is applied to exclude $\mathbf{E}_l$ from the total electric field $\mathbf{E}$, resulting into a more simple Maxwell theory. Decoupled inhomogeneous potential equations and its solutions follow directly from this theory, without having to apply a gauge condition. Field expressions are derived from the potential functions that are simpler and far field consistent with respect to the Jefimenko fields. However, our simple version of Maxwell's theory does not satisfy N3LM. Therefore we combine the generalized magnetostatics with the simple version of Maxwell's electrodynamics, via the generalization of Maxwell's speculative displacement current. The resulting electrodynamics describes three types of vacuum waves: the Φ wave, the longitudinal electromagnetic (LEM) wave and the transverse electromagnetic (TEM) wave, with phase velocities respectively a, b and c. Power- and force theorems are derived, and the force law agrees with Newton's third law only if the phase velocities satisfy the following condition: $a >> b$ and $b = c$. The retarded potential functions can be found without gauge conditions, and four retarded field expressions are derived that have three near field terms and six far field terms. All six far field terms are explained as the mutual induction of two free fields. Our theory supports Rutherford's solution of the $\frac{4}{3}$ problem of electromagnetic mass, which requires an extra longitudinal electromagnetic momentum. Our generalized classical electrodynamics might spawn new physics experiments and electrical engineering, such as new photoelectric effects based on Φ- or LEM radiation, and the conversion of natural Φ- or LEM radiation into useful electricity, in the footsteps of Nikola Tesla and T.Henry Moray.

1 Introduction

A classical electrodynamic field theory is presented that is in good agreement with Newton's Third Law of Motion (N3LM), and that is a generalization of Maxwell's theory of electrodynamics [14]. For the development of this theory we make use of the fundamental theorem of vector algebra, also known as the Helmholtz decomposition: a vector function $\mathbf{F}(\mathbf{x})$ can be decomposed into two unique vector functions $\mathbf{F}_l(\mathbf{x})$ and $\mathbf{F}_t(\mathbf{x})$, such that:

$$\begin{aligned}
\mathbf{F}(\mathbf{x}) &= \mathbf{F}_l(\mathbf{x}) + \mathbf{F}_t(\mathbf{x}) && (1.1)\\
\mathbf{F}_l(\mathbf{x}) &= -\frac{1}{4\pi}\nabla \int\limits_V \frac{\nabla' \cdot \mathbf{F}}{|\mathbf{x}-\mathbf{x}'|}\ \mathrm{d}^3x' && (1.2)\\
\mathbf{F}_t(\mathbf{x}) &= \frac{1}{4\pi}\nabla\times \int\limits_V \frac{\nabla' \times \mathbf{F}}{|\mathbf{x}-\mathbf{x}'|}\ \mathrm{d}^3x' && (1.3)
\end{aligned}$$

The *longitudinal* vector function $\mathbf{F}_l$ is curl free ($\nabla\times\mathbf{F}_l = \mathbf{0}$), and the *transverse* vector function $\mathbf{F}_t$ is divergence free ($\nabla\cdot\mathbf{F}_t = 0$). We assume that $\mathbf{F}$ is well behaved ($\mathbf{F}$ is zero if $|\mathbf{x}|$ is infinite). The proof of the Helmholtz decomposition is based on Dirac's 3-dimensional delta function $\delta(\mathbf{x})$ and the sifting property of this function, see the following identities:

$$\begin{aligned}
\delta(\mathbf{x}) &= \frac{-1}{4\pi}\Delta\left(\frac{1}{|\mathbf{x}|}\right) && (1.4)\\
\mathbf{F}(\mathbf{x}) &= \int\limits_V \mathbf{F}(\mathbf{x}')\,\delta(\mathbf{x}-\mathbf{x}')\ \mathrm{d}^3x' && (1.5)
\end{aligned}$$

Let us further introduce the following notations and definitions.

$\mathbf{x}, t = x, y, z, t$	Place and time coordinates
$\nabla = \left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}\right)$	Del operator (in Cartesian coordinates)
$\dot{f} = \frac{\partial f}{\partial t}$	Partial derivative of time
$\Delta = \nabla \cdot \nabla$	Laplace operator
	$\Delta\Phi = \nabla\cdot\nabla\Phi, \qquad \Delta\mathbf{A} = \nabla\nabla\cdot\mathbf{A} - \nabla\times\nabla\times\mathbf{A}$
ρ	Net electric charge density distribution
Φ	Net electric charge (scalar) potential
$\mathbf{J} = \mathbf{J}_l + \mathbf{J}_t$	Net electric current density distribution
$\mathbf{A} = \mathbf{A}_l + \mathbf{A}_t$	Net electric current (vector) potential
$\mathbf{E}_\Phi = -\nabla\Phi$	Electric field
$\mathbf{E}_l = -\frac{\partial \mathbf{A}_l}{\partial t}$	Induced secondary electric field, longitudinal
$\mathbf{E}_t = -\frac{\partial \mathbf{A}_t}{\partial t}$	Induced secondary electric field, transverse
$B_\Phi = -\frac{\partial \Phi}{\partial t}$	Induced secondary scalar 'Φ' field
$B_l = -\nabla\cdot\mathbf{A}_l$	Scalar magnetic field
$\mathbf{B}_t = \nabla\times\mathbf{A}_t$	Vector magnetic field
$\phi_0 =?\ As^3/Vm^3$	Permeability of vacuum (longitudinal)
$\lambda_0 =?\ F/m$	Permittivity of vacuum (longitudinal)
$\mu_0 = 4\pi\ 10^{-7} N/A^2$	Permeability of vacuum (transverse)
$\epsilon_0 = 8.854^{-12} F/m$	Permittivity of vacuum (transverse)

The permittivity and permeability of vacuum are constants. The charge- and current density distributions, the potentials and the fields are functions of place, and not always functions of time. Time independent functions are called *stationary* or *static* functions. Basically, there are four types of charge-current density distributions to consider:

A. Current free charges	$\mathbf{J} = \mathbf{0}$
B. Stationary currents	$\dot{\mathbf{J}} = \mathbf{0}$
1. closed circuit (divergence free)	$\dot{\mathbf{J}} = \mathbf{0} \quad \wedge \quad \nabla\cdot\mathbf{J} = \mathbf{0}$
2. open circuit	$\dot{\mathbf{J}} = \mathbf{0} \quad \wedge \quad \nabla\cdot\mathbf{J} \neq \mathbf{0}$
C. Time dependent currents	$\dot{\mathbf{J}} \neq \mathbf{0}$

The following condition is supposed to be true, at each place and time, and for each type of charge-current density distribution $\rho, \mathbf{J}$:

$$\frac{\partial \rho}{\partial t} + \nabla \cdot \mathbf{J} = 0 \tag{1.6}$$

This condition is known as charge-current continuity, or the *local* conservation of charge. A current free charge density distribution is called *electrostatic*: $\dot{\rho} = -\nabla \cdot \mathbf{0} = 0$. A stationary current density distribution is called *magnetostatic*. In case a stationary current density distribution consists of closed circuits (is divergence free), then the charge density distribution has to be stationary as well: $\dot{\rho} = -\nabla \cdot \mathbf{J} = 0$. The (net) charge density distribution can even be zero everywhere, such that we have a (net) *charge free* stationary current distribution.

It is well known the Maxwell-Lorentz force law satisfies N3LM in case of electrostatic charge distributions, however, this force law violates N3LM in case of *open circuit* magnetostatic current distributions. A violation of N3LM means that momentum is not conserved by magnetostatic systems, for which there is no experimental evidence. This remarkable inconsistency in classical electrodynamics is mentioned (but not solved) by very few modern text books on classical electrodynamics, such as Griffiths' 'Introduction to Electrodynamics' [11]. Modern text books on Electrodynamics show several incorrect suggestion how to agree N3LM with the Maxwell-Lorentz force law, and this has negative consequences for developing a consistent electrodynamics that truly agrees with N3LM.

Other problematic aspects of Maxwell's theories are: the far field inconsistency (Jefimenko's electric field solution, derived from Maxwell's theory in the Lorenz gauge, shows two longitudinal electric far field terms that do not interact by induction with other fields), and the famous 4/3 problem of electromagnetic mass. In the next sections we describe these inconsistencies of Maxwell's theory in more detail, and how to solve them.

2 Magnetostatics and Newton's third law

Let $\mathbf{J}(\mathbf{x})$ be a stationary current distribution. The vector potential $\mathbf{A}(\mathbf{x})$ at place vector $\mathbf{x}$ is given by:

$$\begin{aligned} \mathbf{A}(\mathbf{x}) &= \frac{\mu_0}{4\pi} \int_V \frac{\mathbf{J}(\mathbf{x}')}{r} \, \mathrm{d}^3 x' \qquad (2.1) \\ \mathbf{r} &= \mathbf{x} - \mathbf{x}' \\ r &= |\mathbf{x} - \mathbf{x}'| \end{aligned}$$

Since $\dot{\mathbf{A}} = \mathbf{0}$ for stationary currents, the electric field equals the primary electric field ($\mathbf{E} = \mathbf{E}_\Phi = -\nabla\Phi$), such that Gauss' law is given by

$$\nabla \cdot \mathbf{E}_\Phi(\mathbf{x}, t) = \frac{1}{\epsilon_0} \rho(\mathbf{x}, t) \tag{2.2}$$

The magnetostatic field $\mathbf{B}_t(\mathbf{x})$ is defined by Biot-Savart's law as follows:

$$\begin{aligned}\mathbf{B}_t(\mathbf{x}) &= \nabla\times\mathbf{A}_t(\mathbf{x}) \\ &= -\frac{\mu_0}{4\pi}\int\limits_V \mathbf{J}(\mathbf{x}')\times\nabla\left(\frac{1}{r}\right)\ \mathrm{d}^3x' \\ &= \frac{\mu_0}{4\pi}\int\limits_V \frac{1}{r^3}\big[\mathbf{J}(\mathbf{x}')\times\mathbf{r}\big]\ \mathrm{d}^3x' \end{aligned} \tag{2.3}$$

The magnetic field is indeed static, since the current density is stationary in time, and this shows the consistency of our definition of magnetostatics ($\dot{\mathbf{J}}=\mathbf{0}$). Practically all textbooks on electrodynamics and electromagnetism describe that the moving charges of 'magnetostatic' currents cannot accumulate (or diminish) in space, such that the current density has to be divergence free. However, static magnetic fields may be sourced by open circuit currents, such that the density of charge varies in time for some volumes in space, as long as the density of current is independent of time: $\dot{\mathbf{J}}=\mathbf{0}$. The condition $\nabla\cdot\mathbf{J}=0$ is *superfluous* for magnetostatics.

The magnetic force density $\mathbf{f}^t(\mathbf{x})$, that acts transversely on current density $\mathbf{J}(\mathbf{x})$ at place $\mathbf{x}$, is given by:

$$\begin{aligned}\mathbf{f}^t(\mathbf{x}) &= \mathbf{J}(\mathbf{x})\times\mathbf{B}_t(\mathbf{x}) \\ &= \frac{\mu_0}{4\pi}\int\limits_V \frac{1}{r^3}\mathbf{J}(\mathbf{x})\times\big[\mathbf{J}(\mathbf{x}')\times\mathbf{r}\big]\ \mathrm{d}^3x' \\ &= \frac{\mu_0}{4\pi}\int\limits_V \frac{1}{r^3}\Big[\big[\mathbf{J}(\mathbf{x})\cdot\mathbf{r}\big]\mathbf{J}(\mathbf{x}')\ -\ \big[\mathbf{J}(\mathbf{x}')\cdot\mathbf{J}(\mathbf{x})\big]\mathbf{r}\Big]\ \mathrm{d}^3x' \\ &= \frac{\mu_0}{4\pi}\int\limits_V \mathbf{F}^t(\mathbf{x},\mathbf{x}')\ \mathrm{d}^3x' \end{aligned} \tag{2.4}$$

This is the Maxwell-Lorentz force density law for magnetostatics, also known as Grassmann's force (density) law. In case this force density expression is integrated over *open* current circuit lines, then the Grassmann forces acting on these circuits do not satisfy N3LM (see Appendix A), since $\mathbf{F}^t(\mathbf{x},\mathbf{x}')\neq-\mathbf{F}^t(\mathbf{x}',\mathbf{x})$ [7]. Notice that $\mathbf{r}$ changes into $-\mathbf{r}$ by swapping $\mathbf{x}$ and $\mathbf{x}'$. By means of the following scalar function

$$\begin{aligned}B_l(\mathbf{x}) &= -\nabla\cdot\mathbf{A}_l(\mathbf{x}) \\ &= -\frac{\mu_0}{4\pi}\int\limits_V \mathbf{J}(\mathbf{x}')\cdot\nabla\left(\frac{1}{r}\right)\ \mathrm{d}^3x' \\ &= \frac{\mu_0}{4\pi}\int\limits_V \frac{1}{r^3}\big[\mathbf{J}(\mathbf{x}')\cdot\mathbf{r}\big]\ \mathrm{d}^3x' \end{aligned} \tag{2.5}$$

we define a reciprocal force density law (see also eq. 13 in [25], and [21]):

$$\begin{aligned}
\mathbf{f}(\mathbf{x}) &= \mathbf{f}^l(\mathbf{x}) + \mathbf{f}^t(\mathbf{x}) = \mathbf{J}(\mathbf{x})B_l(\mathbf{x}) + \mathbf{J}(\mathbf{x})\times\mathbf{B}_t(\mathbf{x}) \\
&= \frac{\mu_0}{4\pi}\int\limits_V \frac{1}{r^3}\Big[\big[\mathbf{J}(\mathbf{x}')\cdot\mathbf{r}\big]\mathbf{J}(\mathbf{x}) + \big[\mathbf{J}(\mathbf{x})\cdot\mathbf{r}\big]\mathbf{J}(\mathbf{x}') - \big[\mathbf{J}(\mathbf{x}')\cdot\mathbf{J}(\mathbf{x})\big]\mathbf{r}\Big]\,\mathrm{d}^3x' \\
&= \frac{\mu_0}{4\pi}\int\limits_V \mathbf{F}(\mathbf{x},\mathbf{x}')\,\mathrm{d}^3x' \qquad (2.6)
\end{aligned}$$

The volume integration of this force density satisfies N3LM for *any* stationary current density distribution, since $\mathbf{F}(\mathbf{x},\mathbf{x}') = -\mathbf{F}(\mathbf{x}',\mathbf{x})$. The additional force density $\mathbf{f}^l$ is also known as the *longitudinal* Ampère force density, and it balances the transverse Ampère force density $\mathbf{f}^t$ such that the total Ampère force density $\mathbf{f} = \mathbf{f}^l + \mathbf{f}^t$ is reciprocal ($\mathbf{f} = -\mathbf{f}'$), see Figure 1.

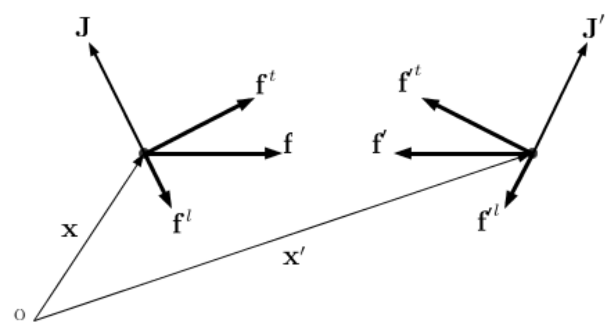

Figure 1: total Ampère force density

We conclude that the scalar function B_l has the meaning of a physical field that mediates an Ampère force, just like the vector magnetic field $\mathbf{B}_t$, and therefore it will be called the *scalar magnetic field* [27]. By means of the identities 1.4 and 1.5, and eq. 2.1, we derive the following equations:

$$\begin{aligned}
\nabla B_l(\mathbf{x}) + \nabla\times\mathbf{B}_t(\mathbf{x}) &= -\Delta\mathbf{A}(\mathbf{x}) &&= \mu_0\mathbf{J}(\mathbf{x}) \qquad (2.7)\\
\nabla B_l(\mathbf{x}) &= -\nabla\nabla\cdot\mathbf{A}_l(\mathbf{x}) &&= \mu_0\mathbf{J}_l(\mathbf{x}) \qquad (2.8)\\
\nabla\times\mathbf{B}_t(\mathbf{x}) &= \nabla\times\nabla\times\mathbf{A}_t(\mathbf{x}) &&= \mu_0\mathbf{J}_t(\mathbf{x}) \qquad (2.9)
\end{aligned}$$

This is the generalization of Ampère's law for magnetotatics. The following equation follows from equations 1.6, 2.2 and 3.1.

$$-\nabla\cdot\mathbf{J}_l = \frac{\partial\rho}{\partial t} = \epsilon_0\frac{\partial(\nabla\cdot\mathbf{E}_\Phi)}{\partial t} = \epsilon_0\nabla\cdot\frac{\partial\mathbf{E}_\Phi}{\partial t} = \epsilon_0\nabla\cdot\nabla B_\Phi \qquad (2.10)$$

For these magnetostatics laws, the Lorenz "gauge" condition ($\nabla\cdot\mathbf{A} + \epsilon_0\mu_0\dot{\Phi} = 0$) follows naturally from equations 2.8 and 2.10:

$$\nabla\cdot\nabla(\ B_l(\mathbf{x}) + \epsilon_0\mu_0 B_\Phi(\mathbf{x})\ ) = 0 \tag{2.11}$$

If the field B_l is physical, then the Lorenz "gauge" condition implies that the field B_Φ is physical as well. So far we have shown a generalized magnetostatics theory in agreement with N3LM. In order to derive a classical electrodynamics theory that agrees with N3LM, we will first have to review and correct Maxwell's original theory of electricity and magnetism, such that the corrected theory is far field consistent for all dynamic closed circuit current distribution.

3 Field induction and Maxwell's theory

We continue to develop the theory for the more general situation of time dependent current- and charge distributions, taking into account the physical scalar fields B_l, B_Φ and the near field equations 2.2, 2.8 and 2.9.

3.1 Induction of secondary fields

The equations of *secondary field induction* by *primary time dependent fields*, follow directly from the field definitions and the fact that the operators ∇, $\nabla\cdot$ and $\nabla\times$ commute with ∂_t, and are given by

$$\nabla B_\Phi - \frac{\partial \mathbf{E}_\Phi}{\partial t} \quad = \quad -\nabla\frac{\partial\Phi}{\partial t} + \frac{\partial(\nabla\Phi)}{\partial t} \qquad = \mathbf{0} \tag{3.1}$$

$$\nabla\cdot\mathbf{E}_l - \frac{\partial B_l}{\partial t} \quad = \quad -\nabla\cdot\frac{\partial\mathbf{A}_l}{\partial t} + \frac{\partial(\nabla\cdot\mathbf{A}_l)}{\partial t} \qquad = 0 \tag{3.2}$$

$$\nabla\times\mathbf{E}_t + \frac{\partial\mathbf{B}_t}{\partial t} \quad = \quad -\nabla\times\frac{\partial\mathbf{A}_t}{\partial t} + \frac{\partial(\nabla\times\mathbf{A}_t)}{\partial t} \qquad = \mathbf{0} \tag{3.3}$$

This is the generalization of Faraday's law of induction [8]. A divergent electrodynamic field $\mathbf{E}_l$ is induced by a time varying scalar magnetic field B_l (see eq. 3.2 and eq. 22 in [25]), similar to the induction of a divergence free electric field $\mathbf{E}_t$ by a time varying vector magnetic field $\mathbf{B}_t$ (see eq. 3.3). The total electric field $\mathbf{E}$ is sourced by static charges, or induced by time varying vector- and scalar magnetic fields, so it should be defined as $\mathbf{E} = \mathbf{E}_\Phi + \mathbf{E}_l + \mathbf{E}_t$.

3.2 Review of Maxwell's theory

Maxwell's Treatise of Electricity and Magnetism excludes the scalar functions B_l and B_Φ as physical fields, such that his electrodynamics theory disagrees with N3LM. Nevertheless Maxwell defined a total electric field $\mathbf{E}$ as the summation of primary and secondary electric fields, as follows: $\mathbf{E} = \mathbf{E}_\Phi + \mathbf{E}_l + \mathbf{E}_t = -\nabla\Phi - \dot{\mathbf{A}}$, despite the fact that Maxwell did not describe or refer to the physical induction of electric field $\mathbf{E}_l$ by a time varying scalar magnetic field B_l (see eq. 3.2). Induction of field $\mathbf{E}_l$ should be proven by experiments as well. By means of Helmholtz' decomposition the total electric field can be defined as $\mathbf{E} = \mathbf{E}_\Phi + \mathbf{E}_t$, such that the electrodynamic sources of this field are well defined and verified by those experiments that were known to Maxwell. A simple version of Maxwell's theory follows directly from this definition of total electric field:

$$\begin{aligned}
\mathbf{E}_\Phi + \mathbf{E}_t &= \mathbf{E} && (3.4)\\
\nabla\cdot\mathbf{E} = \nabla\cdot\mathbf{E}_\Phi &= \frac{1}{\epsilon_0}\rho && (3.5)\\
\nabla\times\mathbf{E} = \nabla\times\mathbf{E}_t &= -\frac{\partial\mathbf{B}_t}{\partial t} && (3.6)\\
\nabla\times\mathbf{B}_t &= \mu_0\mathbf{J}_t && (3.7)
\end{aligned}$$

From the condition of local charge conservation follows the next displacement current expression.

$$-\epsilon_0\frac{\partial\mathbf{E}_\Phi}{\partial t} = \mathbf{J}_l \qquad (3.8)$$

Contrary to popular believe, Maxwell's famous displacement current term $\epsilon_0\,\dot{\mathbf{E}}_t$ does *not* follow from local charge conservation, because its divergence is zero. However, the speculative addition of this displacement current to Ampère's law (see next equation) allowed Maxwell to derive the wave equations for the transverse electromagnetic (TEM) wave. Hertz [12] verified the TEM wave by experiment, and therefore we must add displacement current term $\epsilon_0\,\dot{\mathbf{E}}_t$ to eq. 3.7, although initially this speculative term was not inferred rationally.

$$\nabla\times\mathbf{B}_t - \epsilon_0\mu_0\frac{\partial\mathbf{E}_t}{\partial t} = \mu_0\mathbf{J}_t \qquad (3.9)$$

The Maxwell-Ampère law follows from equations 3.8 and 3.9.

$$\nabla\times\mathbf{B}_t - \epsilon_0\mu_0\frac{\partial\mathbf{E}}{\partial t} = \mu_0\mathbf{J} \qquad (3.10)$$

Rewriting eq. 3.5 and eq. 3.9, in terms of the potentials Φ and $\mathbf{A}_t$, gives

$$\begin{aligned}
-\Delta\Phi &= \frac{1}{\epsilon_0}\rho && (3.11)\\
\epsilon_0\mu_0\frac{\partial^2\mathbf{A}_t}{\partial t^2} + \nabla\times\nabla\times\mathbf{A}_t &= \mu_0\mathbf{J}_t && (3.12)
\end{aligned}$$

This simple version of Maxwell's theory does *not* require a "gauge" condition in order to find decoupled inhomogeneous differential equation for the potentials. Within the context of Maxwell's original theory, the free choice of a "gauge" condition is the direct consequence of Maxwell's unfounded (on experiments) addition of the electric field $\mathbf{E}_l$ to the total electric field. These potential equations have the following solutions.

$$\Phi(\mathbf{x},t) = \frac{1}{4\pi\epsilon_0}\int\limits_V \frac{\rho(\mathbf{x}',t)}{r}\,\mathrm{d}^3x' \tag{3.13}$$

$$\mathbf{A}_t(\mathbf{x},t) = \frac{\mu_0}{4\pi}\int\limits_V \frac{\mathbf{J}_t(\mathbf{x}',t_c)}{r}\,\mathrm{d}^3x' \tag{3.14}$$

$$r = |\mathbf{x}-\mathbf{x}'| \quad c = \frac{1}{\sqrt{\epsilon_0\mu_0}}$$

$$t_c = t - \frac{r}{c} \tag{3.15}$$

The charge potential Φ is instantaneous at a distance, while the current potential $\mathbf{A}_t$ is retarded with time interval r/c, relative to current potential sources at a distance r. The following field expressions, derived from these potentials, are simpler with respect to the Jefimenko fields.

$$\mathbf{E}(\mathbf{x},t) = \frac{1}{4\pi\epsilon_0}\int\limits_V \left[\frac{\rho(\mathbf{x}',t)\mathbf{r}}{r^3} - \frac{\dot{\mathbf{J}}_t(\mathbf{x}',t_c)}{c^2 r}\right]\mathrm{d}^3x' \tag{3.16}$$

$$\mathbf{B}_t(\mathbf{x},t) = \frac{\mu_0}{4\pi}\int\limits_V \left[\frac{\mathbf{J}_t(\mathbf{x}',t_c)\times\mathbf{r}}{r^3} + \frac{\dot{\mathbf{J}}_t(\mathbf{x}',t_c)\times\mathbf{r}}{cr^2}\right]\mathrm{d}^3x' \tag{3.17}$$

The second terms in equations 3.16 and 3.17 are far field terms of electromagnetic radiation that falls off by r, and which has the momentum $\epsilon_0(\mathbf{E}_t\times\mathbf{B}_t)$. Newton's third law of motion is not satisfied by electrodynamics that involves radiant electromagnetic fields, since N3LM describes the motion of bodies with mass, and does not take into account the momentum of massless radiation. However, the total momentum as the sum of the mass momentum and the massless radiation momentum of a radiant system, should be conserved.

A second incorrect statement (to suggest that Maxwell's force law does not conflict with N3LM) is the following: "Maxwell's electrodynamics disagrees with N3LM, *only* because of the extra momentum of massless electromagnetic radiation. This is true for closed circuit current density distributions, and it is false in case of open circuit magnetostatics, which is always free of massless electromagnetic momentum. Also our simple Maxwell theory is consistent with N3LM only in case of closed circuit current distributions.

The Jefimenko electric field, derived from Maxwell's theory in the Lorenz "gauge", is the following expression.

$$\mathbf{E}(\mathbf{x},t) = \frac{1}{4\pi\epsilon_0}\int\limits_V \left[\frac{\rho(\mathbf{x}',t_c)\mathbf{r}}{r^3} + \frac{\dot{\rho}(\mathbf{x}',t_c)\mathbf{r}}{cr^2} - \frac{\dot{\mathbf{J}}_l(\mathbf{x}',t_c)}{c^2 r} - \frac{\dot{\mathbf{J}}_t(\mathbf{x}',t_c)}{c^2 r}\right]\mathrm{d}^3x' \tag{3.18}$$

The second and third electric field terms are longitudinal far fields that fall off in magnitude by r. Since far fields are defined as two mutually inducing fields, these two longitudinal far fields are unexplained/non-existent without the definition/presence of two other far fields that interact with these two longitudinal electric far fields. We call this *the far field inconsistency* of Maxwell's theory in the Lorenz gauge. Notice that our simple Maxwell theory (see eq. 3.16) is far field consistent. The next section shows a generalized electrodynamics theory which agrees with N3LM also in case of open circuit magnetostatic currents, and which is far field consistent.

4 Generalized Electrodynamics

We will derive an electrodynamics theory that generalizes the presented magnetostatics in agreement with N3LM. This theory involves the fields B_l, B_Φ and $\mathbf{E}_l$ that are not present in our simple version of Maxwell's theory. We already showed generalized field induction laws in the previous section. Now we will generalize Maxwell's speculative addition of displacement current $\epsilon_0 \dot{\mathbf{E}}_t$ to Ampère's law (see eq. 3.9), as follows.

$$\nabla\cdot\mathbf{E}_\Phi - \frac{\phi_0}{\epsilon_0}\frac{\partial B_\Phi}{\partial t} = \frac{\phi_0}{\epsilon_0}\frac{\partial^2 \Phi}{\partial t^2} - \nabla\cdot\nabla\Phi = \frac{1}{\epsilon_0}\rho \tag{4.1}$$

$$\nabla B_l - \lambda_0\mu_0\frac{\partial \mathbf{E}_l}{\partial t} = \lambda_0\mu_0\frac{\partial^2 \mathbf{A}_l}{\partial t^2} - \nabla\nabla\cdot\mathbf{A}_l = \mu_0\mathbf{J}_l \tag{4.2}$$

$$\nabla\times\mathbf{B}_t - \epsilon_0\mu_0\frac{\partial \mathbf{E}_t}{\partial t} = \epsilon_0\mu_0\frac{\partial^2 \mathbf{A}_t}{\partial t^2} + \nabla\times\nabla\times\mathbf{A}_t = \mu_0\mathbf{J}_t \tag{4.3}$$

We introduce a displacement charge $\phi_0 \dot{B}_\Phi$ and an additional displacement current $\lambda_0 \dot{\mathbf{E}}_l$. Notice that in case of a stationary current potential ($\dot{\mathbf{A}} = \mathbf{0}$), equations 4.2 and 4.3 equal equations 2.8 and 2.9. Therefore this theory will be in agreement with N3LM for open circuit magnetostatic current distributions, if we can deduce the correct force theorem later on.

4.1 Field waves and total fields

From equations 3.1, 3.2, 3.3, 4.1, 4.2 and 4.3 the following inhomogeneous field wave equations can be derived.

$$\frac{\phi_0}{\epsilon_0}\frac{\partial^2 \mathbf{E}_\Phi}{\partial t^2} - \nabla\nabla\cdot\mathbf{E}_\Phi = -\frac{1}{\epsilon_0}\nabla\rho \tag{4.4}$$

$$\frac{\phi_0}{\epsilon_0}\frac{\partial^2 B_\Phi}{\partial t^2} - \nabla\cdot\nabla B_\Phi = -\frac{1}{\epsilon_0}\frac{\partial\rho}{\partial t} \tag{4.5}$$

$$\lambda_0\mu_0 \frac{\partial^2 \mathbf{E}_l}{\partial t^2} - \nabla\nabla\cdot\mathbf{E}_l = -\mu_0 \frac{\partial \mathbf{J}_l}{\partial t} \tag{4.6}$$

$$\lambda_0\mu_0 \frac{\partial^2 B_l}{\partial t^2} - \nabla\cdot\nabla B_l = -\mu_0 \nabla\cdot\mathbf{J}_l \tag{4.7}$$

$$\epsilon_0\mu_0 \frac{\partial^2 \mathbf{E}_t}{\partial t^2} + \nabla\times\nabla\times\mathbf{E}_t = -\mu_0 \frac{\partial \mathbf{J}_t}{\partial t} \tag{4.8}$$

$$\epsilon_0\mu_0 \frac{\partial^2 \mathbf{B}_t}{\partial t^2} + \nabla\times\nabla\times\mathbf{B}_t = \mu_0 \nabla\times\mathbf{J}_t \tag{4.9}$$

These are the wave equations of the well known transverse electromagnetic (TEM) wave and two types of longitudinal electric waves. One type of longitudinal electric wave is expressed only in terms of the electric charge potential Φ and it is not induced by electric currents, see eq. 4.4 and 4.5. Therefore this wave is called a Φ *wave*. The second type of longitudinal electric wave is associated with the curl free electric current potential, see eq. 4.6 and 4.7, which will be called a *longitudinal electromagnetic wave* (LEM wave). The following notations for the phase velocities of these wave types are used.

$$a = \sqrt{\frac{\epsilon_0}{\phi_0}} \qquad b = \sqrt{\frac{1}{\lambda_0\mu_0}} \qquad c = \sqrt{\frac{1}{\epsilon_0\mu_0}} \tag{4.10}$$

Initially we assume that the values of these phase velocities are independent constants, and that is why we introduced the new constants λ_0 and ϕ_0. Before we derive a generalized power theorem and force theorem, we first define the following 'total' fields

$$\mathbf{E} = \mathbf{E}_\Phi + \mathbf{E}_l + \mathbf{E}_t \tag{4.11}$$

$$\mathbf{E}^* = \frac{1}{a^2}\mathbf{E}_\Phi + \frac{1}{b^2}\mathbf{E}_l + \frac{1}{c^2}\mathbf{E}_t \tag{4.12}$$

$$B = \frac{1}{a^2}B_\Phi + B_l \tag{4.13}$$

such that we can rewrite equations 4.1, 4.2, 4.3 in terms of these total fields.

$$\nabla\cdot\mathbf{E} - \frac{\partial B}{\partial t} = \frac{1}{\epsilon_0}\rho \tag{4.14}$$

$$\nabla B + \nabla\times\mathbf{B}_t - \frac{\partial \mathbf{E}^*}{\partial t} = \mu_0 \mathbf{J} \tag{4.15}$$

4.2 Power- and force theorems

A generalized power theorem follows from eq. 4.14 and 4.15 (see also Appendix B, for power- and force theorems for each type of wave separately):

$$\mathbf{E}\cdot\mathbf{J}+c^2B\rho = \frac{1}{\mu_0}[\nabla(B\mathbf{E}+\mathbf{B}_t\times\mathbf{E}) - \mathbf{E}\cdot\frac{\partial\mathbf{E}^*}{\partial t} - \mathbf{B}_t\cdot\frac{\partial\mathbf{B}_t}{\partial t} - B\frac{\partial B}{\partial t}] \tag{4.16}$$

The derivation of the force theorem from eq. 4.14 and 4.15 is more involved, since this theorem must agree with N3LM:

$$\begin{aligned}
\mathbf{f} &= \rho\mathbf{E} + \mathbf{J}\times\mathbf{B}_t \\
&= \epsilon_0[\nabla\cdot\mathbf{E}-\frac{\partial B}{\partial t}]\mathbf{E} + \frac{1}{\mu_0}[\nabla B+\nabla\times\mathbf{B}_t-\frac{\partial\mathbf{E}^*}{\partial t}]\times\mathbf{B}_t \\
&= \epsilon_0[\mathbf{E}\nabla\cdot\mathbf{E}-\frac{\partial(\mathbf{E}B)}{\partial t}] + \frac{1}{\mu_0}[(\nabla B+\nabla\times\mathbf{B}_t)\times\mathbf{B}_t-\frac{\partial(\mathbf{E}^*\times\mathbf{B}_t)}{\partial t}] \\
&\quad + \epsilon_0\frac{\partial\mathbf{E}}{\partial t}B + \frac{1}{\mu_0}\mathbf{E}^*\times\frac{\partial\mathbf{B}_t}{\partial t}
\end{aligned} \tag{4.17}$$

The last two terms in the latter equation evaluate as follows.

$$\epsilon_0\frac{\partial\mathbf{E}}{\partial t}B = [\epsilon_0\nabla B_\Phi+\frac{\epsilon_0}{\lambda_0\mu_0}\nabla B_l+\frac{1}{\mu_0}\nabla\times\mathbf{B}_t-\frac{\epsilon_0}{\lambda_0}\mathbf{J}_l-\mathbf{J}_t]B \tag{4.18}$$

$$\frac{1}{\mu_0}\mathbf{E}^*\times\frac{\partial\mathbf{B}_t}{\partial t} = \frac{1}{\mu_0}(\nabla\times\mathbf{E}_t)\times\mathbf{E}^* = \frac{1}{\mu_0}(\nabla\times\mathbf{E})\times\mathbf{E}^* \tag{4.19}$$

In equation 4.18 the expression $(\frac{\epsilon_0}{\lambda_0}\mathbf{J}_l+\mathbf{J}_t)$ has to be equal to $\mathbf{J}$ in order to arrive at the force law of eq. 2.6 that satisfies N3LM. Therefore, the following condition has to be generally true:

$$\lambda_0=\epsilon_0 \qquad (b=c) \tag{4.20}$$

After setting $\lambda_0=\epsilon_0$, the following force theorem can be derived.

$$\begin{aligned}
\mathbf{f} &= \rho\mathbf{E} + \mathbf{J}\times\mathbf{B}_t + \mathbf{J}B \\
&= \epsilon_0(\nabla\cdot\mathbf{E})\mathbf{E} + \frac{1}{\mu_0}(\nabla\times\mathbf{E})\times\mathbf{E}^* + \frac{1}{\mu_0}(\nabla B+\nabla\times\mathbf{B}_t)\times\mathbf{B}_t \\
&\quad + \frac{1}{\mu_0}(\frac{1}{c^2}\nabla B_\Phi+\nabla B_l+\nabla\times\mathbf{B}_t)B - \epsilon_0\frac{\partial(\mathbf{E}B)}{\partial t} - \frac{1}{\mu_0}\frac{\partial(\mathbf{E}^*\times\mathbf{B}_t)}{\partial t}
\end{aligned} \tag{4.21}$$

4.3 The Lorenz condition is invalid

Further analysis of local charge conservation (see eq. 4.5, 4.7, and apply 4.20) will result into a condition for phase velocity 'a':

$$
\begin{aligned}
-\left[\nabla\cdot\mathbf{J}+\frac{\partial\rho}{\partial t}\right] &= \\
\frac{1}{\mu_0}\left[\epsilon_0\mu_0\frac{\partial^2 B_l}{\partial t^2} -\nabla\cdot\nabla B_l\right] + \epsilon_0\left[\frac{\phi_0}{\epsilon_0}\frac{\partial^2 B_\Phi}{\partial t^2} -\nabla\cdot\nabla B_\Phi\right] &= \\
\frac{\partial^2\left(\epsilon_0 B_l + \phi_0 B_\Phi\right)}{\partial t^2} - \frac{1}{\mu_0}\nabla\cdot\nabla(B_l+\epsilon_0\mu_0 B_\Phi) &= 0 \qquad (4.22)
\end{aligned}
$$

Two possible conditions for ϕ_0 come into mind to fullfill this equation, as shown in this subsection and the next subsection. The Lorenz condition is defined as the following condition for ϕ_0:

$$\phi_0 = \epsilon_0^2\mu_0 = 9.856\cdot 10^{-29}\ \frac{As^3}{Vm^3} \qquad (4.23)$$

This condition means that the phase velocities 'a', 'b' and 'c' are assumed to be equal: $a=b=c$, see definition 4.10. Applying this condition, the total fields B and $\mathbf{E}^*$ become:

$$B = \frac{1}{c^2}B_\Phi + B_l \qquad (4.24)$$

$$\mathbf{E}^* = \frac{1}{c^2}\mathbf{E} \qquad (4.25)$$

And the continuity equation 4.22 becomes:

$$\epsilon_0\frac{\partial^2(B)}{\partial t^2} - \frac{1}{\mu_0}\nabla\cdot\nabla(B) = 0 \qquad (4.26)$$

Obviously, this equation is fullfilled if $B=0$, which is known as the Lorenz "gauge". Applying the Lorenz condition, the total scalar field isn't sourced by any charge current density distribution and the resulting theory is the standard Maxwell theory in the Lorenz gauge. However, Maxwell's theory does not satisfy N3LM for the special case of open circuit magnetostatic current distributions, and therefore we conclude that the Lorenz condition 4.23 is invalid.

We also have to reject the v-"gauge" condition [1] and [26] which is the following condition for the phase velocities: $a=b$ and $b>c$. We have shown that condition $b=c$ must hold to derive the correct force law in agreement with N3LM, so the v-"gauge" is also in conflict with N3LM.

4.4 The Coulomb condition

The 'Coulomb condition' is defined as the following condition for ϕ_0

$$\phi_0 \downarrow 0 \qquad (4.27)$$

The Coulomb condition means that the phase velocity 'a' of the longitudinal Φ wave is infinite, or at least $a >> c$, see definition 4.10. The Coulomb condition further means that the resulting electrodynamics is "quasi dynamic" with respect to the instantaneous Φ potential (this resembles the Maxwell theory in the Coulomb gauge, but we avoid using the Coulomb gauge condition). Equation 4.22 becomes

$$\epsilon_0\mu_0\frac{\partial^2 B_l}{\partial t^2} \quad - \nabla\cdot\nabla B_l \quad = \quad \epsilon_0\mu_0\nabla\cdot\nabla B_\Phi \;=\; \mu_0\frac{\partial\rho}{\partial t} \tag{4.28}$$

This equation means that the LEM wave is sourced also by time varying electric charge distribution. We express the field equations as follows.

$$\begin{aligned}
\mathbf{E}_\Phi + \mathbf{E}_l + \mathbf{E}_t \quad &= \quad \mathbf{E} && (4.29)\\
\frac{1}{c^2}B_\Phi + B_l \quad &= \quad B && (4.30)\\
-\Delta\Phi \quad = \quad \nabla\cdot\mathbf{E} \quad - \frac{\partial B_l}{\partial t} \quad &= \quad \frac{1}{\epsilon_0}\rho && (4.31)\\
\epsilon_0\mu_0\frac{\partial^2\mathbf{A}}{\partial t^2} - \Delta\mathbf{A} \quad = \quad \nabla B \quad + \nabla\times\mathbf{B}_t \quad - \epsilon_0\mu_0\frac{\partial\mathbf{E}}{\partial t} \quad &= \quad \mu_0\mathbf{J} && (4.32)
\end{aligned}$$

4.4.1 Power- and force theorems with the Coulomb condition

From these field equations the following power and force theorems follow

$$\begin{aligned}
-\mathbf{E}\cdot\mathbf{J} \; - c^2 B\rho \quad &= \quad \frac{1}{\mu_0}\nabla\cdot(\mathbf{E}\times\mathbf{B}_t - \mathbf{E}B)\\
&\quad + \frac{\epsilon_0}{2}\frac{\partial E^2}{\partial t} \; + \frac{B}{\mu_0}\frac{\partial B_l}{\partial t} \; + \frac{1}{2\mu_0}\frac{\partial B_t^2}{\partial t} && (4.33)\\
\rho\mathbf{E} \; + \mathbf{J}\times\mathbf{B}_t + \mathbf{J}B_l \quad &= \quad -\,\epsilon_0\frac{\partial(B_l\mathbf{E} + \mathbf{E}\times\mathbf{B}_t)}{\partial t}\\
&\quad + \epsilon_0\left[(\nabla\cdot\mathbf{E})\mathbf{E} + (\nabla\times\mathbf{E})\times\mathbf{E}\right]\\
&\quad + \frac{1}{\mu_0}[\nabla B + \nabla\times\mathbf{B}_t]\times\mathbf{B}_t\\
&\quad + \frac{1}{\mu_0}[\nabla B + \nabla\times\mathbf{B}_t]B_l && (4.34)
\end{aligned}$$

We finally deduced the correct force theorem that agrees with N3LM, see eq. 2.6. After applying the Coulomb condition we still have the B_Φ field power term at the left hand side of equation 4.33 that is equal to $-B_\Phi\rho = \dot{\Phi}\rho$, and a power flux of longitudinal Φ radiation that equals $-\epsilon_0\nabla\cdot(\mathbf{E}B_\Phi)$. However, after applying the Coulomb condition the longitudinal Φ radiation momentum becomes zero, see eq. 4.34.

4.4.2 The 4/3 problem of electrodynamics

David E. Rutherford published a solution for the famous $\frac{4}{3}$ problem of electrodynamics [19]. Rutherford calculated the mass of an electron by means of the total electric (self) field energy E_e of the electron, using equation 4.31 (see also [20]) and the famous equation $E = mc^2$. Next, Rutherford calculated the mass of a moving electron with a speed $\mathbf{v}$ by means of the total (self) field momentum of the electron $\mathbf{p}_e = \epsilon_0(B_l\mathbf{E} + \mathbf{E} \times \mathbf{B}_t)$, see equation 4.34. The two different methods for calculating the electron mass resulted into the *same* electron mass expression that do not differ a factor $\frac{4}{3}$, see next equations.

$$E_e = \frac{1}{4\pi\epsilon_0}\frac{q_e^2}{r_e} = m_e c^2 \tag{4.35}$$

$$\mathbf{p}_e = \frac{\mu_0}{4\pi}\frac{q_e^2}{r_e}\mathbf{v} = m_e\mathbf{v} \tag{4.36}$$

$m_e,\ q_e,\ r_e$ electron mass, electron charge, electron radius

Apparently, the momentum of the LEM fields cannot be ignored in order to calculate the correct electron 'field momentum' mass. We conclude that the presented generalized electrodynamics theory including the Coulomb condition is the correct background theory for Rutherford's solution of the $\frac{4}{3}$ problem. Rutherford's paper can also be viewed as the 'electrodynamical' derivation of the famous equation $E = mc^2$ for a moving charge with speed $\mathbf{v}$.

4.5 Retarded potentials and fields

Using the phase velocity conditions $a >> b$ and $b = c$, the scalar- and vector potential are solutions of the following decoupled inhomogeneous wave equations:

$$\frac{1}{a^2}\frac{\partial^2\Phi}{\partial t^2} - \Delta\Phi = \frac{1}{\epsilon_0}\rho \tag{4.37}$$

$$\frac{1}{c^2}\frac{\partial^2\mathbf{A}}{\partial t^2} - \Delta\mathbf{A} = \mu_0\mathbf{J} \tag{4.38}$$

which are the following retarded potentials

$$\Phi(\mathbf{x},t) = \frac{1}{4\pi\epsilon_0}\int\limits_V \frac{\rho(\mathbf{x}',t_a)}{r}\,\mathrm{d}^3x' \tag{4.39}$$

$$\mathbf{A}(\mathbf{x},t) = \frac{\mu_0}{4\pi}\int\limits_V \frac{\mathbf{J}(\mathbf{x}',t_c)}{r}\,\mathrm{d}^3x' \tag{4.40}$$

$$r = |\mathbf{x} - \mathbf{x}'| \qquad t_a = t - \frac{r}{a} \qquad t_c = t - \frac{r}{c}$$

The field expressions, derived from these potentials, are

$$\mathbf{E}(\mathbf{x},t) = \frac{1}{4\pi\epsilon_0}\int\limits_V \left[\frac{\rho(\mathbf{x}',t_a)\mathbf{r}}{r^3} + \frac{\dot\rho(\mathbf{x}',t_a)\mathbf{r}}{ar^2} - \frac{\dot{\mathbf{J}}_l(\mathbf{x}',t_c)}{c^2 r} - \frac{\dot{\mathbf{J}}_t(\mathbf{x}',t_c)}{c^2 r}\right] \mathrm{d}^3x' \tag{4.41}$$

$$B_\Phi(\mathbf{x},t) = \frac{1}{4\pi\epsilon_0}\int\limits_V \frac{-\dot\rho(\mathbf{x}',t_a)}{r}\, \mathrm{d}^3x' \tag{4.42}$$

$$B_l(\mathbf{x},t) = \frac{\mu_0}{4\pi}\int\limits_V \left[\frac{\mathbf{J}_l(\mathbf{x}',t_c)\cdot\mathbf{r}}{r^3} + \frac{\dot{\mathbf{J}}_l(\mathbf{x}',t_c)\cdot\mathbf{r}}{cr^2}\right] \mathrm{d}^3x' \tag{4.43}$$

$$\mathbf{B}_t(\mathbf{x},t) = \frac{\mu_0}{4\pi}\int\limits_V \left[\frac{\mathbf{J}_t(\mathbf{x}',t_c)\times\mathbf{r}}{r^3} + \frac{\dot{\mathbf{J}}_t(\mathbf{x}',t_c)\times\mathbf{r}}{cr^2}\right] \mathrm{d}^3x' \tag{4.44}$$

We can identify three near field terms, that fall off in magnitude by r^2, and six far field terms of the TEM, LEM and Φ waves, that fall off in magnitude by r. This theory is far field consistent. Beside electrostatics and magnetostatics, we can define two extra types of charge-current distributions with restricted behaviour: *quasi dynamics* ($a \to \infty$) and *quasi statics* ($a \to \infty \ \wedge \ c \to \infty$).

In case of quasi dynamic charge current distributions, there is no noticeable retardation of the Coulomb field and the scalar potential; the length of the circuit is much smaller than the wavelength of the Φ wave, such that detection of a far Φ field gradient is impossible. The second term in 4.41 becomes zero and $t_a = t$, which is applying the Coulomb condition (not to be confused with the Coulomb gauge condition). This is also expressed as 'instantaneous action at a distance' [3]. The induction law 3.1 is still needed, since the secondary field B_Φ does not disappear for quasi dynamics.

In case of quasi static charge current distributions, also the second term in 4.43 and 4.44 become zero. The induction laws for the secondary fields $\mathbf{E}_l$ and $\mathbf{E}_t$ are still required for quasi statics, since the third and fourth term in 4.41 do not disappear.

5 Review of Electrodynamics Experiments

5.1 Magnetostatic force experiments

First of all, the historic magnetostatic force experiments carried out by Ampère, Gauss, Weber and other famous scientist, are all examples of *open circuit* currents that are not divergence free. It is certain that batteries or capacitor banks were used as electric current sources and current sinks that typically show time varying charge densities and divergent currents at the current source/sink interface. The only example of a truly closed circuit stationary current, is the stationary current induced in a superconductor by an approaching permanent magnet, for example to demonstrate the Meissner effect. The oversimplification of treating only the integral force between *closed* circuit stationary currents is unjustified, considering the historic and modern open current circuit magnetostatics experiments, and considering Newton's third law of motion.

5.2 Longitudinal Ampère force experiments

Many magnetostatic experiments such as Ampère's hairpin experiment, demonstrated the existence of the longitudinal Ampère force [13] [22]. This force is more difficult to research than the transverse Ampère force, for instance, to demonstrate that this force is proportional to the inverse square of the distance between interacting currents. The longitudinal Ampère force is not accepted yet as a fundamental force of nature.

The Aharonov Bohm effect can be explained as a longitudinal Ampère force acting on the free electrons that pass through a double slit and pass a shielded solenoid on both sides of the solenoid. Such a force does not deflect the free electrons, and will slightly decelerate (delay) or accelerate (advance) the electrons, depending on which side the electrons pass the solenoid, which explains the observed phase shift in the interference pattern. The AB phase shift does not depend on the free electrons transit time, since shorter or longer duration of force interaction is compensated by the free electron velocity dependent Ampère force. The Ampère force exerted on the free electrons is proportional to the magnitude of solenoid current, which is observed. The scalar magnetic field B_l isn't shieldable, apparently. The *classical* interpretation of the proven *physicality* of the vector potential $\mathbf{A}$ is exactly that the vector potential exerts forces on particles through its *divergence*: $\mathbf{f}^l = e\mathbf{v}B_l = -e\mathbf{v}\nabla\cdot\mathbf{A}$. The 'AB effect is no force' conclusion by Caprez, Barwick and Batelaan [2] is premature: a theoretical time delay figure, that indicates the presence of forces, should have been calculated such that the measured time delay data can be compared with this figure. The measured data was presented as a "null" result, however, we assume that the resolution of the time interval measurements was insufficient to verify or falsify the presence of forces.

5.3 Induction of a secondary divergent electric field

To our knowledge, no experiments have been done yet to verify or falsify the induction law of equation 3.2: primary sinusoidal *divergent* (curl free) currents induce a secondary sinusoidal *divergent* (and curl free) electric field $\mathbf{E}_l$ and similar currents (depending on the resistive elements in the secondary 'circuits'), such that the secondary electric field and currents are 90 degrees (or more) out of phase with the primary currents. The first experimenter who verifies this law also verifies the existence of the scalar magnetic field B_l and the classical physicality of the vector potential $\mathbf{A}$.

5.4 Induction of B_Φ

The field B_Φ is induced simply by a time varying primary electric field. Wesley and Monstein described the induction of an electric potential Φ wave by inducing a pulsating surface charge on a centrally fed ball antenna [15]. Wesley and Monstein claimed the ball shaped send antenna does not produce divergent currents ($\nabla \cdot \mathbf{J} = 0$) and emits Φ waves only, however, this suggests a violation of charge conservation ($\nabla \cdot \mathbf{J} = 0$ and $\dot{\rho} \neq 0$). We assume that a centrally fed ball antenna also shows curl free divergent currents that induce LEM waves. We further assume that Wesley and Monstein actually observed LEM waves. Theoretically, a violation of local charge conservation is possible by means of 'macroscopic' *quantum tunneling* or *quantum teleportation* of many electrons to/from a fixed point, such that the charge distribution is varying in time and in absence of currents. A macroscopic 'teleporting' charge distribution only induces a B_Φ field via time varying primary electric fields, while the vector potential is absent for such a charge distribution. A macroscopic electron teleportation (tunneling) device might be a sender or *receiver* of longitudinal 'Φ' waves with superluminal phase velocity. A ball shaped 'monopole' send antenna usually does not show macroscopic tunneling effects, therefore it is not likely it emits Φ waves.

5.5 Longitudinal electric waves

5.5.1 Nikola Tesla, man out of time

A special tribute should be made to Nikola Tesla, the inventor of the poly phase AC motor and of the poly phase AC electricity system that is widely applied today. Dr. Tesla was also the inventor of single wire and wireless electrical signal systems, for instance, he discovered the tuned radio carrier wave technology. Tesla was ridiculed for his long distance electric energy transport system based on (according to Tesla) longitudinal electric waves. Mainstream physics models longitudinal waves only as sound waves. Sound waves are atomic particle displacement waves in a material medium that show a longitudinal electric field component. Our theory describes longitudinal electric waves that do not require a material 'atomic particle displacement' medium for establishing energy transportation. Tesla's claims should be taken seriously, considering the inconsistencies in Maxwell's theory of electrodynamics. Tesla also mentioned the observation of 'teleforce' effects by means of non-dispersive emissions from high voltage discharges, similar to Podkletnov's gravity impulse signal. Obviously, he was far ahead of his time [28] [29].

5.5.2 LEM waves

Ignatiev and Leus used a ball antenna to send wireless longitudinal electric waves with a wavelength of 2.5 km [10]. They measured a phase difference between the wireless signal and an optical fiber signal (the two signals are synchronous at the sender location) at a 0.5 km distance from the sender location. They concluded from the measured phase shift that the wireless signal is faster than the optical fiber signal, and that the wireless signal has a phase velocity of 1.12 c. Combining the results from the experiments by Wesley, Monstein, Ignatiev and Leus, we conclude that

1. The wireless signals are most likely LEM waves (and not Φ waves nor TEM waves). Wesley and Monstein showed that the polarization of the electric field wave is longitudinal, which proves the signal is not transverse electromagnetic. We ruled out that ball antennas emit Φ waves.

2. In theory the LEM wave phase velocity is c and not 1.12 c, so we assume that the 0.12 c discrepancy is most likely an incorrect interpretation of the data, for instance, the optical fiber signal has a phase velocity slower than c (in most cases it is 200,000 km/sec, depending on the refractive index of the viber).

3. The signal is definitely a far field, and not a near Coulomb field. Ignatiev and Leus could measure the electric field signal at 0.5 km distance from the sender and they also measured a phase shift between the signal at the send location and the receiver location, which cannot be expected from near fields.

The combined experimental results verify the existence of the non-Hertzian LEM wave, as predicted by our theory, that travels with luminal speed c in space.

A very efficient single wire resonant energy transport systems has been developed and tested by D.S. Strebkov et al [6]. The question is if Strebkov's single wire wave system has a 'ground return'. If yes, then the signals across the single wire are TEM waves such that the electric field is directed between the wire and ground (single wire earth return system). If no, then the electric wave component is longitudinal and the signal is most likely a LEM wave with a unidirectional energy flow of $S = \frac{1}{\mu_0} B_l \mathbf{E}_l$. Strebkov et al. described applications for their single wire resonant system that do not require grounding, so we assume their system is based on LEM waves.

5.5.3 Φ waves

Podketnov's 'impulse gravity' generator emits superluminal signals with a speed of at least 64 c [18]. The impulse gravity device is very different from a ball shaped electrical antenna: the wireless pulse is generated by means of a high voltage discharge (maximum of 2 million volt) from a *superconducting* flat surface electrode to another non-superconducting electrode. The emitted pulse travels into the direction longitudinal (parallel) to the electronic discharge direction. No TEM wave radiation was measured transverse to the direction of discharge. Podkletnov concludes the longitudinal signal isn't a TEM wave either, nor a beam of massive particles. We assume that Podkletnov's impulse 'gravity' device produces Φ waves; the measured signal speed of at least 64 c agrees with our theoretical prediction of the superluminal Φ wave phase velocity ($a >> c$). Secondly, Podkletnov expects that the signal frequency matches the tunneling frequency of the discharged electrons [17]. During the discharge pulse, a macroscopic charge tunnels through many superconducting layers before leaving the superconductor. This is probably the most efficient way to generate Φ waves, since absence of continuous currents means the absence of the vector potential $\mathbf{A}$. The electrodynamic nature of the impulse signal has not yet been fully investigated by Podkletnov and Modanese; the research is focused on the gravitational effect on objects that are hit by the superluminal signal. It is obvious that this experiment reveals a direct connection between electrodynamics and gravity.

5.6 Natural longitudinal electric waves as energy source

One of the most important applications of our theory might be the conversion of natural longitudinal electric field waves into useful electricity. The reception of Φ waves is most likely the reverse process of the generation of Φ waves: in theory the natural presence of Φ waves stimulates quantum tunneling 'teleportation' of electrons through an energy barrier on a macroscopic level. We will call Φ wave stimulated quantum tunneling the 'Φ photoelectric effect', similar to the photoelectric effect of electrons emitted by a metal surface exposed to TEM waves. This theoretical prediction has not been researched and it has not been verified nor falsified. Essential to this photoelectric effect is that it does not involve electric currents, nor does it involve the current potential $\mathbf{A}$, nor TEM waves, nor LEM waves (only as secondary effects).

5.6.1 T.H. Moray's radiant energy valve

Dr. T.H. Moray's radiant energy device converted the energy flow of natural aetheric waves into kilo Watts of useful electricity, day and night [16]. Dr. Harvey Fletcher, who was the co-discoverer of the elementary charge of the electron for which Robert Millikan won the Nobel price, filed a signed document [9] stating that Moray's radiant energy receiver functioned as claimed, so we have no reason to doubt Dr. Moray's device. The most proprietary component of Moray's receiver was a high voltage cold cathode tube containing a Germanium electrode doped with impurities, called 'the detector tube' by Moray. This tube has been described as a 'light' valve, and its huge energy reception capability might be based on the Φ photoelectric effect. Moray tuned his radiant energy device into a high frequency wave of natural 'cosmic' origin, that we assume is a Φ wave. The electric potential at the first energy receiving stage were shown to be at least 200,000 volts. Very abrupt tunneling of electrons in/out of the Germanium electrode through an energy barrier explains the observed high frequencies generated/received by Moray's valve. The importance of Dr. Moray's invention cannot be overstated; it dwarfs most Nobel prize discoveries in the field of physics. The great American inventor T. Henry Moray was the greatest electrical engineer ever.

5.6.2 Correa and Chernetsky

The same Φ photoelectric effect might explain the excess energy detected by Dr. P.N. Correa [5], and described by Correa as an anomalous and longitudinal cathode reaction force during pulsed autogenous cyclical abnormal glow discharges in a cold cathode plasma tube. Correa observed an abnormal glow discharge in a current-voltage regime (which also shows the excess energy) that is quite similar to the negative resistance coefficient regime of a tunneling diode. The same glow is also observed by Podkletnov, just before the discharge pulse occurs. We explain the 'abnormal glow' as the energetic effect of tunneled electrons on gas atoms, just before an abrupt discharge from cathode to anode. The observation of a natural self-pulsed discharge frequency might be explained by the presence of a natural background Φ wave with the same frequency. Very similar excess energy results were achieved by Dr. Chernetsky [4], by means of a self-pulsed high voltage discharge tube (filled with hydrogen), that generates longitudinal electric field waves in the electrical circuits attached to the tube.

5.7 Conclusions and discussions

We conclude that the Maxwell Lorentz force law is incorrect with respects to Newton's third law of motion. Maxwell's electrodynamics theory in the Lorenz gauge is also far field inconsistent. Two extra 'scalar' fields are required in order to express a far field consistent electrodynamics theory that satisfies N3LM as well.

We advise to include the Helmholtz theorem in the standard curriculum of physics education for treating Maxwell's theory realistically. The confusing "gauge" theory and "gauge conditions" can be avoided by defining the total electric field as the sum of only those electric field components for which the electrodynamic sources are well defined and verified by experiments.

Undoubtedly it is necessary to review standard relativity theory and standard quantum mechanics, that are characterized by the constants c and h and by the poor interpretation of experimental results, against the new background of Φ waves and LEM waves. We suspect the ratio a/c has fundamental physical meaning. Caroline H. Thompson's Φ *wave aether* comes into mind [24]. It is more important, though, that our theory inspires physicists and electrical engineers to review many experiments from the past and to perform new experiments, which may birth a new era of science and technology, and may bring a new 'balance in the force' on this planet.

6 Appendix A

In stead of integrating force densities over two closed current circuit lines (see [23], pages 4-6), we give an example of force density integration over two open current circuits, C and C', that carry the static electric currents I and I', see Figure 2. The currents I and I' are equal to the current density surface integral over a circuit line cross section of the circuits C and C'. For both circuits the current is constant for each circuit line cross section, and are 'sourced' by closed volumes that show time variable charge densities ($\partial_t \rho \neq 0$).

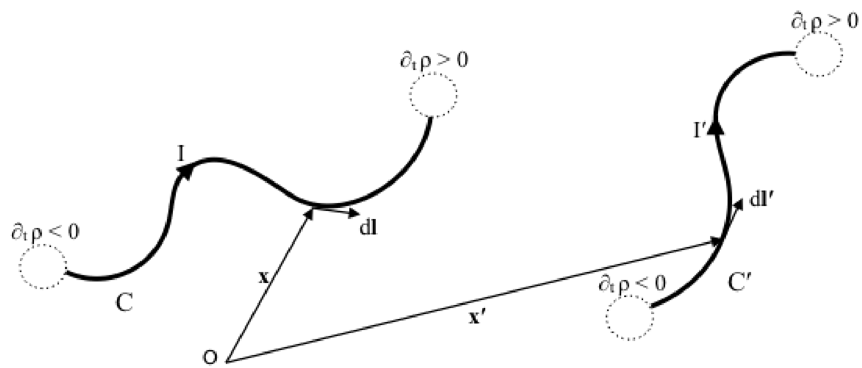

Figure 2: open circuit magnetostatics

The transverse Ampère (Maxwell-Lorentz) force acting on circuit C is given by line integration over the circuits C and C':

$$\begin{aligned}
\mathbf{F}_C^t &= \frac{\mu_0 I I'}{4\pi} \int\limits_C \int\limits_{C'} \frac{1}{r^3} d\mathbf{l} \times (d\mathbf{l}' \times \mathbf{r}) \\
&= \frac{\mu_0 I I'}{4\pi} \int\limits_C \int\limits_{C'} \frac{(d\mathbf{l} \cdot \mathbf{r}) d\mathbf{l}'}{r^3} - \frac{\mu_0 I I'}{4\pi} \int\limits_C \int\limits_{C'} \frac{(d\mathbf{l} \cdot d\mathbf{l}')\mathbf{r}}{r^3} \\
&= -\frac{\mu_0 I I'}{4\pi} \int\limits_{C'} \int\limits_C (d\mathbf{l} \cdot \nabla \left[\frac{1}{r}\right]) d\mathbf{l}' - \frac{\mu_0 I I'}{4\pi} \int\limits_C \int\limits_{C'} \frac{(d\mathbf{l} \cdot d\mathbf{l}')\mathbf{r}}{r^3} \qquad (6.1)
\end{aligned}$$

The first integral disappears in case circuit C is a closed line, since the curl of a gradient is zero. However, circuit C is not a closed line in this example, so this integral does *not* disappear, such that the two transverse Ampère forces acting on the circuits C and C' are not reciprocal ($\mathbf{F}_C^t \neq -\mathbf{F}_{C'}^t$), in violation of N3LM.

7 Appendix B

Three separate power theorems and three separate force theorems, associated with the Φ wave, LEM wave and TEM wave electrodynamics, can be derived from equations 3.1, 3.2, 3.3, 4.1, 4,2 and 4.3.

$$
\begin{aligned}
-\rho B_\Phi \ + \epsilon_0 \nabla\cdot(B_\Phi \mathbf{E}_\Phi) &= \frac{\phi_0}{2}\frac{\partial {B_\Phi}^2}{\partial t} \ + \frac{\epsilon_0}{2}\frac{\partial E_\Phi^2}{\partial t} && (7.1)\\
-\mathbf{E}_l \cdot \mathbf{J}_l \ + \frac{1}{\mu_0}\nabla\cdot(B_l \mathbf{E}_l) &= \frac{1}{2\mu_0}\frac{\partial {B_l}^2}{\partial t} \ + \frac{\lambda_0}{2}\frac{\partial E_l^2}{\partial t} && (7.2)\\
-\mathbf{E}_t \cdot \mathbf{J}_t \ + \frac{1}{\mu_0}\nabla\cdot(\mathbf{B}_t \times \mathbf{E}_t) &= \frac{1}{2\mu_0}\frac{\partial B_t^2}{\partial t} \ + \frac{\epsilon_0}{2}\frac{\partial E_t^2}{\partial t} && (7.3)
\end{aligned}
$$

$$
\begin{aligned}
\rho \mathbf{E}_\Phi \ + \phi_0 \frac{\partial (B_\Phi \mathbf{E}_\Phi)}{\partial t} &= \phi_0 (\nabla B_\Phi) B_\Phi \ + \epsilon_0 (\nabla\cdot\mathbf{E}_\Phi)\mathbf{E}_\Phi && (7.4)\\
B_l \mathbf{J}_l \ + \lambda_0 \frac{\partial (B_l \mathbf{E}_l)}{\partial t} &= \frac{1}{\mu_0}(\nabla B_l) B_l \ + \lambda_0 (\nabla\cdot\mathbf{E}_l)\mathbf{E}_l && (7.5)\\
\mathbf{B}_t \times \mathbf{J}_t \ + \epsilon_0 \frac{\partial (\mathbf{B}_t \times \mathbf{E}_t)}{\partial t} &= \frac{1}{\mu_0}\mathbf{B}_t \times \nabla\times\mathbf{B}_t \ + \epsilon_0 \mathbf{E}_t \times \nabla\times\mathbf{E}_t && (7.6)
\end{aligned}
$$

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